Crafting asset allocation for a re-insurer via portfolio optimisation

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Declaration

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Abstract

One of the most challenging tasks faced by financial advisors and consultants, relates to the phenomena of portfolio selection. This process typically entails selecting asset classes based on their risk and reward attributes. Striking an optimal balance between risk and reward is no easy task, given its conflicting nature. This phenomena is referred to as *portfolio optimisation* and is commonly formulated and solved via the well-known mean-variance optimisation procedure, based on the pioneering works by Harry Markowitz. The objective function is formulated as a quadratic programming problem, that seeks to maximise expected return whilst minimising risk. While this approach presents an auspicious foundation to solve a portfolio optimisation problem, it does not incorporate the unique *liabilities* (such as future payments or claims) inherent to most institutional investors.

The aim of the study is therefore to provide a roadmap outlining how *assets* and *liabilities* are dovetailed to enhance the decision making process around portfolio optimisation. To achieve this, the notion and premise of asset-liability management (ALM) and liability-driven investing (LDI) are introduced to better manage both assets and liabilities, coherently. This would ultimately ensure an institutional investor's long term financial sustainability. To add a practical ingredient to this thesis, a real-world case study for a re-insurer is examined. Essentially, the roadmap is applied to a case study to solve a complete portfolio optimisation problem, from an LDI perspective.

The results of the unconstrained asset allocation reveal the optimiser's preference to allocate chiefly to a small range of asset classes. While this outcome may be theoretically appropriate, this presents a practical challenge given potential concentration risks, and lack of portfolio diversification opportunities. For this reason, constraints are imposed within the optimisation procedure, resulting in a more diversified and larger array of asset classes to include within a portfolio. To aid with the model validation component and to serve as credence, subject matter experts are consulted. The outcome of this validation was that the process embarked upon as well as the results produced are reasonable and resonates with industry standards. To supplement the model validation and to serve as a reasonability check, a comprehensive sensitivity analysis was undertaken on key input parameters such as expected return to assess the impact this has on the optimal portfolio of assets.

Opsomming

Een van die mees uitdagendste take wat finansiële adviseurs en konsultante in die gesig staar, hou verband met die proses van portefeuljeseleksie. Dit behels tipies die keuse van bateklasse op grond van hul risiko- en opbrengskenmerke. Gegewe die teenstrydige aard, is dit nie 'n maklike taak om 'n optimale balans tussen risiko en opbrengs te vind nie. Hierdie verskynsel word portefeulje-optimalisering genoem en word algemeen geformuleer en opgelos deur middel van die bekende gemiddelde-variansie-optimaliseringsprosedure, gebaseer op die werk van Harry Markowitz. Die doelwit funksie is geformuleer as 'n kwadratiese programmeringsprobleem wat daarop gemik is om die verwagte opbrengs te maksimeer, terwyl dit die risiko verminder. Alhoewel hierdie benadering 'n gunstige grondslag bied om 'n probleem met portefeuljeoptimalisering op te los, bevat dit nie die unieke aanspreeklikhede (soos toekomstige betalings of eise) wat inherent is aan meeste institusionele beleggers nie.

Die doel van die studie is dus om 'n padkaart te gee waarin uiteengesit word hoe bates en laste saamgevoeg kan word om die besluitnemingsproses rondom portefeuljeoptimalisering te verbeter. Om dit te bereik, word die idee en uitgangspunt van bate-aanspreeklikheidsbestuur (ALM) en aanspreeklikheidsgedrewe belegging (LDI) bekendgestel om bates en laste, samehangend, beter te bestuur. Dit sou uiteindelik 'n institusionele belegger se finansiële volhoubaarheid op lang termyn verseker. Om 'n praktiese bestanddeel by hierdie proefskrif te voeg, word 'n werklike gevallestudie vir 'n herversekeraar ondersoek. Die padkaart word in wese toegepas op 'n gevallestudie om 'n volledige probleem met die optimalisering van portefeuljes, vanuit 'n LDI -perspektief, op te los.

Die resultate van die onbeperkte batetoewysing onthul die optimiseerder se voorkeur om hoofsaaklik aan 'n klein reeks bateklasse toe te ken. Alhoewel hierdie uitkoms teoreties gepas kan wees, bied dit 'n praktiese uitdaging, gegewe moontlike konsentrasie-risiko's en 'n gebrek aan portefeuljediversifiseringsgeleenthede. Om hierdie rede word beperkings opgelê binne die optimaliseringsprosedure, wat lei tot 'n meer gediversifiseerde en breër verskeidenheid bateklasse wat in 'n portefeulje ingesluit moet word. Om hulp te verleen met die modelvalideringskomponent en om as geloofwaardigheid te dien, word deskundiges geraadpleeg. Die uitkoms van hierdie bekragtiging was dat die proses wat onderneem is, sowel as die resultate wat geproduseer is, redelik is en ooreenstem met aanvaarde industriestandaarde. Om die modelvalidasie aan te vul en as redelikheidstoetsing te dien, is 'n omvattende sensitiwiteitsanalise uitgevoer oor belangrike insetparameters, soos verwagte opbrengs, om die impak wat dit op die optimale portefeulje van bates het, te bepaal.

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CHAPTER 1

Introduction

Contents

This introductory chapter serves to provide context to this project. A brief outline of the problem description and the objectives are provided.

1.1 Background

The complex and technical nature of the financial services industry has made it a requirement for an investment practitioner or consultant to administer professional investment advice to investors[1]. Consultants should furnish investors with good, consistent, and independent guidance to enhance decision making around investment matters [31]. In addition, many institutional investor's throughout the world are governed by regulatory bodies, to ensure the investment strategy is based on sound investment principles so as not to over-expose the investors money to unmanageable risk.

A large component of the advice is centred around asset allocation decisions, as this remains one of the fundamental drivers of long-term investment performance [73]. Asset allocation may be described as a process that involves an investor selecting the most appropriate and diverse combination of asset classes (for example *cash*, *bonds*, *real estate*, and *equity*) consistent with the investor's unique goals [34]. Simply stated, *what type of assets, and what amount (percentage) thereof* should an investor invest their assets in so as to meet their unique investment objectives and goals, is an integral asset allocation question.

In practice, solving this type of problem requires a deep understanding of the investor's objectives, constraints, risk tolerance, and most importantly, the distribution of the liability cash flow profile. The liability cash flow profile provides insight into the investor's goals, such as; the estimated monetary amount the investor is expected to incur, and the time horizon thereof. The liability cash flow profile may take the form of future payment obligations or claims to policyholders. Once this is understood, the objective is to construct asset allocations *relative* to the liability cash flow profile. Stated differently, what asset allocation would best ensure the liability

[1]The term *investor* is used to describe an entity (for example, a corporate firm or individual) who allocates capital, with the chief aspiration of attaining financial returns [21].

cash flow payments are met? This type of strategy is referred to as liability-driven investing (LDI). The primary purpose of an LDI strategy is to ensure assets and liabilities are managed in conformity, and not in isolation. If assets and liabilities are managed in isolation, this may compromise the financial standing of investors, potentially leading to a state of insolvency or bankruptcy [57].

To delineate the importance of asset allocation for an investment strategy, a study was done in 1986 by Brinson *et al.* [16] that aimed to explain how much the variability of an investors returns are explained by the variability of asset allocation. The results from the study indicated that 93.6% of the portfolio's volatility can be explained by asset allocation [16]. A more recent study published by Ibbotson & Kaplan [50] revealed that 90% of a portfolio's return variability, over time, are attributed to the choice of asset allocation [50]. The results from the two studies suggest that the successful execution of an investment strategy is largely driven by the performance of asset allocation, which in turn is linked to the choice of asset allocation.

Ground-breaking quantitative frameworks to formulate an asset allocation problem were first revealed by Harry Markowitz in 1952 [65]. This quantitative framework, coined as modern portfolio theory (MPT[2]) rests on the principle that investors can construct portfolios to maximise expected return for a specified level of risk, or minimise risk for a specified level of expected return [65]. This framework rests on the diversification principle which reduces the overall riskiness of a portfolio by selecting rewarding yet unrelated assets [35]. The foundations laid back then, as well as ongoing refinements and improvements made to this framework, has led to the emergence of asset-liability management (ALM) to aid in setting the investment strategy. ALM is a risk management tool wherein investors seek to invest their assets in the financial markets that takes into account the nature of a projected set of liabilities [44]. Stated differently, ALM deals with the optimal investment of assets based on meeting a set of liabilities [83].

Solvency II[3] is a robust regulatory framework providing insurers and re-insurers with protocols on how to compute the value of its liabilities, and suitable guidelines for calculating their solvency capital requirements (SCR[4]) [47, 80]. In addition, the framework also specifies suggested calibration metrics for the common value-at-risk (VaR[5]) calculation.

A goal arising from MPT is to maximise investment returns whilst minimising investment risk. This represents a typical multiple objective optimisation problem and forms the building blocks of a sound investment strategy and will, in part, ensure the investor's financial well-being and long-term sustainability. Ultimately, the quality of the financial advice would depend on, in part, the experience of the practitioner rendering the advice, and the quantitative methods contemplated to derive the asset allocation. The advice may be viewed as integrating *art* and *science* within the decision making process.

Within the academic setting, well-known quantitative frameworks to formulate an asset allocation problem as an optimisation problem have been extensively researched [20, 65, 71, 87]. However, in the researcher's opinion, the results of practical frameworks, that integrate *assets* and *liabilities* pertaining to re-insurers within the African region have not been widely documented and researched. The shortage thereof, gives rise to the problem under study as discussed in §1.2.

[2]MPT is commonly referred to as mean-variance analysis.

[3]Solvency II is a mandate that came into effect in 2016, providing a robust solvency regulatory framework for insurers and re-insurers [80].

[4]SCR refers to the specific monetary amount of capital that an insurer or re-insurer is required to set aside, subject to a VaR calibrated at a 99.5% confidence interval, over a one-year period [7, 25].

[5]VaR measures the maximum expected loss, on an investment (in monetary terms), over a stated time horizon, with a specified confidence interval (CI) [54, 62, 64].

1.2 Problem statement

Constructing an asset allocation *relative* to a liability cash flow profile, is a central consideration to set investment strategies. This would best ensure the assets are invested optimally, with the core aim to meet its liabilities. The objective of this practical research project is to illustrate how, by utilising the principles of LDI and applying this to an ALM framework, it is possible to make key decisions pertaining to the asset allocation.

The aim of the study is therefore to provide a roadmap outlining how *assets* and *liabilities* are dovetailed to enhance the decision making process around portfolio optimisation. Specifically, the roadmap will outline a general process that practitioners may undertake to solve a portfolio optimisation problem pertaining to re-insurers. The model framework applied in this research project will leverage off the ALM developed by RisCura. To ensure the refinements and enhancements made within the proposed model framework remain practically applicable and plausible, a real world case study will be examined. For this research project, the case study is based on a Kenyan-based re-insurer with their *assets* and *liabilities* denominated in United States Dollar (USD). The scope of this research project is to provide a framework wherein re-insurers can optimise their portfolio of assets to achieve the following key objectives:

- **Financial objective:** maximise the expected return while minimising the investment risk *relative* to the liabilities, measured over a one-year period and in USD currency.

- **Strategic objective:** incorporate the balance sheet representation, by introducing various policies representing the liability. Under this framework, the portfolio of assets can be optimised, achieving specific investment objectives.

In pursuit of these objectives, a brief road map pertaining to the process of the model framework is formulated with the main components described below:

1. Discern the goals and objectives that are represented as liability cash flows as calculated by an actuarial firm *i.e.*, what is the monetary amount the re-insurer is expected to incur, and the term horizon thereof? The liability cash flows, accompanied with a set of yield curves[6], play an essential role in calculating the so-called liability-based benchmark, that sets the reference point for the portfolio optimisation.

2. Thereafter, input data parameters are extracted and quantitatively distilled into capital market assumptions (CMAs) that are used as input into the portfolio optimisation process. The CMAs comprise of two key market parameters; expected returns, and a covariance matrix for the opportunity set of asset classes used within the portfolio optimisation. These two input parameters provide insight of the *return* and *risk* profile of the different asset classes used to craft the asset allocation.

3. An opportunity set of 14 asset classes comprising of *cash*, *bonds*, *property*, and *equity* within a variety of regions are modelled relative to the liability cash flows to construct an *unconstrained* and *constrained* optimal portfolio of assets, respectively. This approach will serve as an extension of the traditional mean-variance framework as spearheaded by Harry Markowitz [65].

4. Value-at-risk (VaR), an alternative and common risk measure, is introduced with the objective of quantifying risk on a probability and monetary basis.

[6]The yield curve provides a relationship of market interest rates at various maturities [18].

5. Incorporate the balance sheet representation that characterise two policies representing the liability. These two policies are referred to as policyholder (P/H) and shareholder (S/H) establishments, respectively. The P/H portfolio comprises of the present value of the liabilities and the SCR, whereas the S/H portfolio comprises of the surplus[7].

6. For both the P/H and S/H portfolios, optimised theoretical portfolios that target an improved *return* profile (without sacrificing risk) as well as a portfolio targeting an improved *risk* profile (without sacrificing return) are proposed, as an alternative to the existing portfolio structure.

While the methodology will be elaborated upon in forthcoming chapters (§3), a brief overview is provided. In summary, the proposed roadmap is applied to a real-world case study. As a point of departure, the study will focus on gaining insight to the objectives that are expressed by the liability cash flows. Thereafter, CMAs for the key asset classes are carefully formulated that are used as an input within the portfolio optimisation procedure. The optimisation procedure draws on objective function formulations described by Panjer & Boyle [71]. Thereafter, an unconstrained and constrained portfolio of assets are produced that are measured in a typical risk and return framework, respectively. Following on, the notions of the balance sheet representation and VaR are introduced that separate the investment strategy into a P/H and S/H component respectively, that ensures applicability to re-insurers. Two suggested portfolios are crafted to improve the risk and return profile of the existing portfolio structure.

1.3 Thesis objectives and organisation

The following objectives are pursued in this thesis:

I Explore and review terminology used within the financial and insurance environment. Review literature within the optimisation and investment field to seek to understand how similar type of portfolio optimisation problems are formulated and solved.

II Draw on literature that will aid in translating an investor's objectives, goals and constraints into a model framework. Design a roadmap from an *asset* and *liability* perspective, that provides a detailed description of the factors and considerations that consultants should incorporate when contemplating a portfolio optimisation problem.

III Calibrate the model framework using liability and asset data, based on a real world case study. The purpose of the liability data will be to calculate liability analytics (such as duration[8]) of the re-insurers liability cash flows. The asset side will comprise of calculating the CMAs, and thereafter feeding this into an optimisation procedure to determine the optimal asset allocation, from an unconstrained and constrained perspective.

IV Interrogate and examine the results based on the calculated CMAs and suitable investment constraints. At this juncture, two recommendations will be made on an asset allocation that seeks to improve the existing risk and return profile. In addition, the model framework will undergo *validation* by subject matter experts to ensure the relevance and to serve with credence of the model framework and results.

V Undertake sensitivity analysis on key input parameters such as CMAs, constraints, and interest rates to understand the impact that this has on the optimal asset allocation results.

[7]Surplus is commonly referred to as the excess between the assets and liabilities.

[8]Duration is a valuable tool that is used to quantify the sensitivity of a cash flow stream against movements in interest rates [18, 49].

VI Provide some concluding remarks and ideas for future work.

Chapter 2 will comprise of a literature review pertaining to the existing asset and liability model theory. This chapter will introduce important notions used in the investment and modelling process, thus addressing Objective I.

In Chapter 3, the roadmap of the model framework, from an asset and liability perspective are provided. In addition, the data, assumptions, and methodology for the case study are furnished. The objective functions and constraints that will feed into the optimisation procedure are presented and described. Thus, Objectives II and III are the focus of this chapter.

In Chapter 4, the asset allocation results for the case study are presented and are critically examined. Model validation by various subject matter experts takes place to ensure the proposed model framework and results are consistent with industry norms and best practices. Thus, Objective IV are the focus of this chapter.

In Chapter 5, sensitivity analysis is carried out on the CMAs, investment constraints, and interest rates. Furthermore, the sensitivity analysis results are interpreted and analysed, and key findings are made thereof, in completion of Objective V.

The thesis concludes in Chapter 6 with a summary of the work presented, key contributions and ideas for future work, thus addressing Objective VI.

CHAPTER 2

Literature Review

Contents

This chapter opens with a review of general optimisation and its application from a financial perspective in §2.1. An overview of MPT that serves as a quantitative reference point in designing a portfolio optimisation framework is discussed in §2.2. The capital market assumptions are studied in depth in §2.3. An alternative and well-known measurement of risk, termed value-at-risk is discussed in §2.4. The concepts of insurance and re-insurance are contained in §2.5. The philosophy around LDI and ALM, and the importance thereof are furnished in §2.6. In addition, this section also covers approaches to formulate an asset allocation problem from a theoretical and mathematical stand point, with specific reference to so-called *liability-relative* approaches. A discussion on simulation is included in §2.7 as an alternative stochastic method to construct an optimal portfolio of assets. Lastly, a consideration around the use of meta-heuristics to solve a portfolio optimisation problem is included in §2.8. Key themes and ideas highlighted from this chapter are drawn on to develop the road map in subsequent chapters. Additional literature studies pertaining to general model building, risk, present value, duration, and covariance are covered in Appendix A.1–A.5. This will set the scene to better understand concepts and serve as a build-up in terms of how these concepts fit into the broader investment strategy.

2.1 Optimisation

The concept of *optimisation* is well established as a principle underlying the analysis of several complex decision or allocation problems [63]. The primary aim of an optimisation problem seeks to achieve the best possible outcome under a given set of circumstances [77]. The building blocks to formulate an optimisation model relies on three segments as highlighted by Winston [101].

1. an objective function(s),
2. decision variables,
3. constraints.

In most models, there is a function that a practitioner wishes to *maximise* or *minimise*. This function is referred to as a model's *objective function*. It is also common for many problems to have more than one objective. This is referred to as a *multiple objective* optimisation problem. The variables whose values need to be determined are referred to as *decision variables*. In many situations, only certain values of decision variables are possible. This may be as a result of certain practicalities given the nature of the problem. Restrictions on the values of decision variables are called *constraints* [101].

Optimisation can be applied to a number of engineering and operations research related disciplines. Examples include; allocation of resources with the aim to maximise benefit, or designing a shortest and most efficient path travelled by a salesperson touring specific destinations [77]. Rao [77] provides a comprehensive list of engineering related applications wherein optimisation is employed to formulate and solve these type of problems.

For a variety of these applications, many different mathematical programming formulations may be of use. For example; a linear program (LP), integer program (IP), and goal program (GP), to name a few. The key formulation applied within the domain of asset allocation relates to a quadratic programming (QP) problem [14, 101]. A QP problem is a non-linear programming (NLP) problem that consists of a quadratic objective function, and a linear set of constraints [77, 101]. According to Rao [77], a general QP may be formulated as follows

$$\text{Minimise } f(\boldsymbol{X}) = \boldsymbol{C}^T\boldsymbol{X} + \frac{1}{2}\boldsymbol{X}^T\boldsymbol{D}\boldsymbol{X} \tag{2.1}$$

subject to

$$\begin{aligned} \boldsymbol{A}\boldsymbol{X} &\leq \boldsymbol{B} && (2.2)\\ \boldsymbol{X} &\geq \boldsymbol{0}. && (2.3) \end{aligned}$$

where

$$\boldsymbol{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}, \quad \boldsymbol{C} = \begin{Bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{Bmatrix}, \quad \boldsymbol{B} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{Bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & & & \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix}, \text{ and } \quad \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

From Equation (2.1), $\boldsymbol{X}$ is a set of decision variables. The term $\boldsymbol{X}^T\boldsymbol{D}\boldsymbol{X}/2$ denotes the quadratic component of the objective function, with $\boldsymbol{D}$ being a symmetric positive-definite matrix. If $\boldsymbol{D} = \boldsymbol{0}$, the optimisation problem reduces to an LP problem [77].

From an asset allocation perspective, practitioners within the financial arena seek to formulate well-diversified portfolios given the advantageous benefit of reducing portfolio risk. The conceptual description of an asset allocation problem is typically formulated as *minimising* risk for a specified level of expected return, or *maximising* expected return for a specified level of risk. From an objective function perspective, risk is modelled using variance, defined as $\sigma^2_{\boldsymbol{x}}$, leading to quadratic variance and covariance terms, respectively. The expected returns component, $\mu_{\boldsymbol{x}}$, is modelled as a linear term [14]. On the other hand, constraints for a portfolio selection problem are generally linear in nature, *i.e.*, non-negativity[1] of assets are prohibited, and the total sum of all allocations must add up to 1 (100%)[2]. Therefore, assuming $\boldsymbol{D} \neq \boldsymbol{0}$ in Equation (2.1), an asset allocation problem can be stated as a QP optimisation problem. The mathematical formulation will be presented in the next section. In practice, QP problems are typically solved via numerical optimisers [71].

2.2 Modern portfolio theory

Modern portfolio theory (MPT), as spearheaded by Harry Markowitz in 1952 [65], or commonly known as mean-variance analysis, is a quantitative method for constructing a multi-asset portfolio, such that the expected return is maximised for a specified level of risk, or the risk is minimised for a specified level of expected return. The central idea is that owning a variety of different assets is less risky than being in possession of only one type, and highlights that an assets risk and return should not be assessed by itself, but by how much it contributes to a portfolios overall risk and return. The frequently used proverb, *"don't put all your eggs in one basket"* [37], lies at the forefront of MPT and diversification. The aim is to invest the assets in a well-diversified portfolio across various asset classes and geographies, leading to reduced portfolio risk [3].

Quantitatively speaking, inputs to the mean-variance framework are a set of capital market assumptions (CMAs) comprising of; *expected returns* and *covariance*. The CMAs provide insight in terms of the risk and return profile and impacts the relative attractiveness of the different asset classes used within the opportunity set. The covariance matrix is used to derive the correlation matrix for the asset classes contained within the opportunity set.

Panjer & Boyle [71] states Markowitz's [65] mean-variance framework as a multiple objective problem *i.e.*,

1. *Mean (return):* Maximise the expected portfolio return, measured as, μ_x
2. *Variance (risk)*: Minimise portfolio risk, measured as, σ^2_x.

Under this framework, only the first and second moments, μ_x and σ^2_x of the asset class returns are required[3] [71]. It should be noted that seeking a *higher return* is viewed as a desirable, whereas seeking a *lower risk* is viewed as desirable. Based on investor preferences, a practitioner assigns a risk tolerance, τ, to these conflicting objectives and *maximises* a single objective function *i.e.*,

[1] Non-negativity implies short-selling is prohibited.

[2] this constraint is referred to as a budget constraint.

[3] Studies by AracioĞlu *et al.* [2] and Lai *et al.* [58] provide alternative formulations that incorporate the third and fourth moments, namely, *skewness* and *kurtosis*, respectively.

$$\left(2\tau\mu_x - \sigma_x^2\right), \quad \text{where} \quad \tau \geq 0. \tag{2.4}$$

The parameter, τ, refers to a specified *risk tolerance*[4]. Panjer & Boyle [71] define the risk tolerance parameter as, $\tau \in [0, +\infty)$. By repeatedly specifying suitable τ values (as opposed to one single τ value), this produces an efficient opportunity set of different risk and return portfolios. Risk tolerance values closer to the upper bound prioritise return, whereas risk tolerance values closer to the lower bound increase the importance of risk prevention [20, 35].

One of the assumptions of the mean-variance framework is the normality of the asset returns [6, 20, 79]. This may be viewed as a slight drawback of this framework given that in the real world this may not always be the case. Despite this, it is commonly used to make asset allocation decisions given its practical nature, popularity and simplicity according to [58, 70, 71]. As stated by Panjer & Boyle, the traditional (also referred to as so-called "asset-only") mean-variance framework stated in general matrix form as a QP convex optimisation problem is to maximise portfolio return while minimising portfolio risk (measured as standard deviation), *i.e.*,

$$\underset{\boldsymbol{x}\in\mathcal{R}^N}{\text{maximise}} \quad \left(2\tau\boldsymbol{\mu}^T\boldsymbol{x} - \boldsymbol{x}^T\sum\boldsymbol{x}\right), \quad \text{where} \quad \tau \geq 0. \tag{2.5}$$

subject to

$$\mathbf{e}^T\boldsymbol{x} = 1, \tag{2.6}$$

$$\boldsymbol{x} \geq 0. \tag{2.7}$$

The decision variable $\boldsymbol{x}$, is defined indicating the asset class weight for the asset class in question. The parameter, $\boldsymbol{\mu}$, represents the expected returns for the asset classes, whereas, $\sum$, denotes the covariance matrix for the asset classes. The second term of the objective function (2.5), namely the variance component, results in the objective function being quadratic in nature. Hence, objective function (2.5) exhibits a similar form compared to Equation (2.1). According to Panjer & Boyle [71] the number of asset classes considered within an opportunity set should not exceed 20 asset classes. The parameter, $\boldsymbol{e}^T$ is simply a matrix of ones *i.e.*, $\boldsymbol{e}^T = (1, \ldots, 1) \in \mathcal{R}^N$. Constraint set (2.6) ensures the total portfolio allocation must be equal to 1 (100%). Constraint set (2.7) ensures that all asset classes' portfolio allocation is either zero or strictly positive. By making use of *expected returns* and a *covariance matrix* as input parameters, a practitioner is able to construct an optimal portfolio of assets, that maximises return for a given level of risk.

Figure 2.1 graphically illustrates the idea behind the mean-variance optimisation framework. The horizontal axis denotes the risk, as measured by variance, σ^2. The vertical axis denotes the expected return, measured by, μ. The so-called *efficient frontier* represented by the thick blue line, denotes the rate at which expected return increases per increase in risk. Simply stated, the efficient frontier denotes the set of optimal portfolios that maximise expected return for a specified level of risk, or minimise risk for a specified level of expected return. As an investor moves along the upper right of the efficient frontier, the investor would be compensated with a greater level of expected return, however at an increasing level of risk.

In Figure 2.1, assume that Portfolio A is an investor's current portfolio. Portfolio B is the efficient portfolio optimised at the same level of return as Portfolio A, yielding a lower risk. Portfolio C is an efficient portfolio optimised at the same level of risk as Portfolio A, yielding a higher return. Portfolios lying beneath the efficient frontier are said to be inefficient portfolios as their portfolio structure can be optimised. Portfolios lying above the efficient frontier are

[4]In §3.7 the choice of, τ, the upper bound will be discussed.

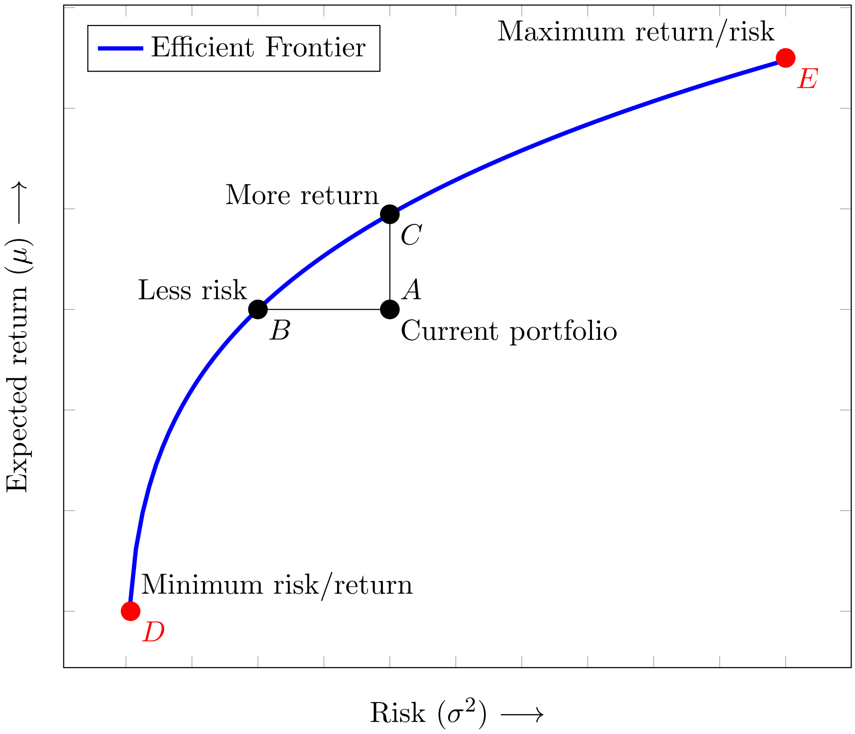


Figure 2.1: *A mean-variance framework illustration (adapted from Michaud & Michaud [68]).*

labelled infeasible, as these points does not satisfy the optimal opportunity set. Portfolio D is termed the so-called "minimum-variance" portfolio and offers the lowest optimal risk and return portfolio. Portfolio E denotes the so-called "maximum-return" portfolio, representing the portfolio that offers the greatest optimal risk and return portfolio. It is further observed that the efficient frontier curve in Figure 2.1 exhibits a parabolic-like shape.

In short, Figure 2.1, Portfolio A, presents an optimisation opportunity by improving the existing risk or return profile. If the investor's desire is to maintain the same level of *return*, but to reduce risk, then Portfolio B would be the preferred choice. However, if the investor's desire is to maintain the same level of *risk*, but to increase return, then Portfolio C would be the preferred choice. Improving an existing portfolio structure depicted by Figure 2.1, precisely represents a typical optimisation problem.

2.3 Capital market assumptions

As mentioned in §2.2, the capital market assumptions (CMAs) comprise of an expected returns matrix, and a covariance matrix for the asset classes considered. These two input parameters represent the first and second moments, respectively, and are fed into a "mean-variance" optimiser that impacts the relative attractiveness of the various asset classes that are ultimately used to formulate an optimal blend of assets. Stated differently, the CMAs aid to determine what amount (percentage) of the portfolio should comprise of *cash*, *bonds*, *real estate*, and *equity etc.* [53]. Equipped with this insight, investors can ensure that their selected combination of assets will lead to a portfolio that is consistent with the investors risk preferences and return objectives [43]. In §§2.3.1-2.3.2, references are made to literature on how the CMAs are formulated.

2.3.1 Expected returns

This section will focus on the mean (expected returns) in "mean-variance". Formulating expected returns is a key driver influencing the outcome for optimal portfolio construction. These estimated expected returns provide insight into the market pricing of assets, and acts as a broad guideline as to how the assets may behave in the future. It is well-known and documented that the predictability of expected return assumptions are difficult to estimate given the unpredictable nature of financial markets [12, 18, 43].

One method of formulating expected returns would be to calculate the mean of historical returns, for the asset classes under consideration. Black & Litterman [12] highlights that using historical mean returns often provide an investor with a misleading expectation of future returns. Since this expected returns method is calculated using historical data, it is likely that the returns would be naively skewed toward periods when the asset classes performed very well (or very poorly) [12]. Thus, providing a misleading indicator of future returns. Stated differently, if the *domestic equity* market delivered exceptional performance in the past, there is little certainty that the *domestic equity* market will continue to do so in forthcoming months. If an investor were to simply use the historical "exceptional" performance returns as an input to a mean-variance optimiser, there is a high potential for the optimiser to indicate a bias toward *domestic equity*, possibly leading to an incorrect asset allocation decision and missing out on alternative investment opportunities. The prevention of using historical means as an indicator for future expected returns is also cautioned by Panjer & Boyle [71]. Ritter [82] highlights that while no consensus is reached on how to estimate future equity expected returns, historical returns are *"irrelevant"* in predicting future expected returns. As a result of this drawback, practitioners should explore metrics beyond the mean "historical" performance returns method.

A study done by Petre [73] to determine an asset allocation, suggested using prevailing market data as an improved estimate to compute expected returns. Amongst the parameters used to determine the *equity* expected returns, include; dividend yield[5] and earnings yield[6]. The time frame associated is typically one-year. For *fixed income* asset classes such as cash and bonds, the current yield-to-maturity[7] (YTM) of a suitable bond asset is used as the expected return. The study further reveals that these parameters are reflective of a so-called equilibrium view of the financial markets, and has been shown to exhibit a positive correlation with expected returns [73]. Therefore, the usage of these parameters leads to an improved estimation of expected returns. Amongst the parameters used by Chordia & Shivakumar [24] to forecast *equity* expected returns, are dividend yield too. From a *fixed income (bond)* perspective, Bogle [15] also suggest that the current yield on a bond (or a portfolio of bonds, such as the yield curve) be used as the expected return if the bond is held for the long-term.

Fama & French [39] links three key input variables to forecasting expected returns for equity and fixed income assets. These three market indicators encompass the usage of a dividend yield, term spread, and default spread. The term spread is measured by taking the difference between the YTM of the highest quality bond and a one-month bill rate. The default spread is the difference between the YTM on a market portfolio of corporate bonds and the YTM of the highest quality bond. These variables track components of expected returns and generally provide insight on the long-term outlook of business conditions. The study further reveals, should markets be in a phase of muted economic activity, then expected returns are typically higher. Similarly,

[5]Dividend yield represents the percentage of a company's stock price that it anticipates to pay out as *dividends* over a specified period of time [42]. The time frame associated is typically one-year.

[6]The earnings yield relates to the earnings per share of a company, divided by the current market price per share. The earnings yield (which denotes the inverse of the price-to-earnings (P/E) ratio) gives insight to the percentage of a company's earnings per share [69]

[7]The yield-to-maturity denotes the total return of a bond, and is a meaningful measure of future returns for investors who wishes to hold a bond until maturity [18].

expected returns are generally lower if economic activity is well stimulated [39].

In a more recent and refined study by Fama & French [40] the authors argue that dividend yield and earnings yield should be variables used to estimate the equity expected return. These two variables are referred to as *fundamental variables* and are likely to provide a more accurate depiction compared to the mean of historical returns [40]. According to their results based on a data-set spanning from 1951 to 2000, incorporating dividend yield and earnings yield as the expected return provided a closer estimation to the actual expected value of the portfolio. The usage of earnings yield as a predictor of future expected returns is supported by Ritter [82].

James L. Farrell [53] describes the dividend discount model as a method to compute an expected return for equity. This approach starts off by contrasting the expected return of equity with that of a fixed income asset such as a government bond. The bond expected return is derived from a yield-to-maturity calculation. A practitioner then evaluates the difference (typically referred to as a spread) between the equity and fixed income asset. This spread provides insight in relative terms, how attractive the assets are compared to each other.

Black & Litterman [12] refined Markowitz's [65] mean-variance framework by incorporating factors that integrate qualitative (*art*) and quantitative (*science*) aspects in a careful optimisation process. The Black-Litterman technique allows an investor to insert so-called "expert" subjective views within the expected returns based on their investment outlook of how overvalued or undervalued an asset is (relative to one another). The views serve as a improved reference point to increase conviction and are used to tilt the asset allocation in-line with the investors sentiments. Investors market insights are a central component to the Black-Litterman approach and can be viewed as an advantageous aspect. On the contrary, if the investors market insights are not correctly captured, these potential incorrect market views would feed into the Black-Litterman model, rendering unfavourable and inconsistent asset allocation results.

It is well-known and noted by Best & Grauer, Black & Litterman, Chopra *et al.*, and Michaud & Michaud [9, 11, 23, 68] that mean-variance optimisers are very sensitive to changes in expected return assumptions. A study done by Best & Grauer [9] revealed a small increase in an expected return for one asset class resulted in a material change of 50% to the composition of the optimal asset allocation. While it is appreciated that expected returns are difficult to compute given that it represents a forecasted estimate, an investor should place emphasis on accurately estimating the expected returns as this contributes to the "quality" of the asset allocation a mean-variance optimiser produces. Poor expected returns are notorious for providing less practical and concentrated allocation results [34].

In summary, according to Philips [74] and Ritter [82] no exact consensus or universally agreed upon techniques relating to the methodology around determining expected returns is available. However, Black & Litterman [12] and Panjer & Boyle [71] has cautioned on using the "historical mean returns" as an input to a mean-variance optimisation as it provides a misleading indicator of future expected returns. As noted earlier, Ritter [82] supports this by stating historical returns are *"irrelevant"* to determine expected returns. For this reason, the "historical means" method will not be used. Instead, Fama & French [40] indicate that *fundamental* parameters such as dividend yield and earnings yield are likely to provide an improved expected return estimate as opposed to the mean stock returns. From an equity perspective, Petre [73] also mentions the usage of dividend yield and earnings yield as suitable candidates to determine a future expected return. Ritter [82] too suggest that earnings yield is a predictor of future expected returns. From a fixed income perspective, Petre [73] indicates that a yield-to-maturity be used as an expected return since it is shown to have a positive correlation with expected returns going forward. The usage of a yield as the expected return for a fixed income asset class is further supported by Bogle [15]. Therefore, a weighted average of the *dividend yield* and *earnings yield* will be used to estimate equity expected returns, whereas the *yield-to-maturity* will be used to estimate the fixed income expected returns for the case study in question.

2.3.2 Covariance and correlation

An additional key parameter for the CMAs pertains to *covariance*. This second moment is used as an input to the "variance" component of the mean-variance optimisation. A statistical method to establish the relationship between the movement of two asset prices is known as covariance. A positive covariance reveals the two asset prices tend to gravitate in the same direction. A negative covariance indicates the two asset prices tend to gravitate in opposite directions [34, 38].

It is important to note that covariance does not have a standard unit of measurement, it merely provides an indication of the direction of the relationship *i.e.*, whether the asset prices move in the same direction, or in opposite directions. For this reason, the *correlation* could aid with interpreting how strong (or weak) the relationship between two variables are. The correlation coefficient ranges between -1, and $+1$. A correlation coefficient of $+1$ indicates a strong positive relationship. A correlation coefficient of -1 indicates a strong negative relationship. A correlation of 0 implies no relationship between the movement of the two variables [34, 41]. To understand the relevance from an investment perspective, consider the scenarios below.

Assume two asset classes A and B exhibit a high correlation with one another. This means that should asset class A experience a downturn in returns, asset class B would most likely also experience a downturn in returns as well. On the other hand, assume asset classes A and B exhibits a low correlation with one another. This means that should asset class A experience a downturn in returns, asset class B has a higher likelihood to experience an upturn in returns.

In short, asset classes that are less correlated with each other are preferred alternatives to include within an optimal portfolio. This results in better portfolio diversification, that could aid with reducing portfolio risk. Simply stated, asset classes that behave differently are favoured alternatives to include within an optimal portfolio as it leads to reducing portfolio risk. For a brief review of covariance and correlation from a statistical standpoint, the reader is referred to Appendix A.5.

While prevailing market data such as dividend yield, earnings yield, and yield-to-maturity are parameters used to forecast *expected returns*, these parameters cannot be used to obtain the *covariance* of an asset class. These parameters does not provide any indication of the *risk* profile of an asset class. Instead, historical returns should be used as this will provide insight pertaining to the risk profile of an asset class. The usage of historical data to compute the risk profile (covariance) is supported by [27, 52, 61]. The most simplest method to compute the risk profile is to calculate the *standard deviation* of the historical returns over the longest time period available, per asset class. The time-period is largely dependant on the data available for an asset class. Ideally, the time-series should capture some components of a past financial crises (*eg*, the global financial crisis of 2007/2008, COVID-19 market volatility experienced in 2020) as this would capture so-called "market extremes" thus rendering a more accurate representation of the asset classes' risk profile. The result of the standard deviation for each asset class, provide insight in terms of the dispersion of a dataset relative to its mean [48]. The higher the standard deviation, the more volatile the behaviour of the asset class. Conversely, the lower the standard deviation, the less volatile the behaviour of the asset class.

Smith [88] describes a covariance matrix method that incorporates autoregressive conditional heteroscedasticity (ARCH) typically experienced in financial time series data. The ARCH effects outlines the impact of the market to fluctuate between periods of high volatility and periods of lower volatility. Smith proposed to separate the data into so-called "sub-periods". For each data series and sub-periods, the standard deviations (volatilities) are computed. These volatilities are approximated by one component related to the asset class and another component expressing the time period by fitting a linear model to log standard deviations. The time period dependent component provides insight of the relative volatilities of the periods covered. These relative volatilities are used as scaling factors to the original residuals to acquire a series of adjusted residuals with the feature that for any asset class, the volatility is largely in-line for each sub-period. According to Smith [88] this technique overcomes the ARCH effect.

2.4 Value-at-risk

Value-at-Risk (VaR) measures the maximum expected loss, on an investment (in monetary terms), over a stated time horizon, with a specified confidence interval (CI) [54, 62, 64, 81]. Simply stated, if an investor has assets worth Rx, then with probability y, of losing z, over the next n period. The n typically refers to a one-year period. It is important to recognise that VaR is merely an estimate and not a precise quantity. A method known as the *variance-covariance* technique, relies on three key parameters to estimate the VaR for a portfolio structure. This technique is known as a parametric method as it operates under the assumption that returns are normally distributed [64, 81, 91]. For the variance-covariance VaR calculation, the three parameters are listed below as stated by Steelyana [91].

1. An investors monetary amount of assets.
2. Confidence interval (α) denoting the probability (for example; 95% $\rightarrow$ z-value of 1.645, or 90% $\rightarrow$ z-value of 1.282). The confidence intervals can be sourced from the normal distribution tables.
3. Risk, typically measured as standard deviation.

The confidence interval may be specified by a firms internal policies, although the Solvency II framework recommends insurers and re-insurers calibrate their SCR using a 99.5% confidence interval, over a one-year time horizon [7, 36]. The main reason for specifying a universal confidence interval may be to ensure consistent methodology amongst all insurers and re-insurers.

The basic idea behind the variance-covariance VaR technique can be explained by means of an example. *Suppose an investor has a portfolio worth \$1,500, with a specified CI (α) of 99.5%, corresponding to a z-value of 2.576, and a portfolio variance (σ) of 4%. What is the estimated 99.5% VaR over a one-year period?*

$$\begin{aligned} 99.5\%\text{ VaR} &= (\text{Portfolio value} \times \alpha) \times \sqrt{\sigma} \\ &= (\$1{,}500 \times 2.576) \times \sqrt{0.04} \\ 99.5\%\text{ VaR} &= \text{US}\$772{,}75 \end{aligned} \tag{2.8}$$

This implies, 1/200 times, the investor is likely to lose a maximum of US\$772, 75 over a one-year period subject to a 99.5% CI. By incorporating a VaR approach as part of a risk calculation provides additional insight as it quantifies the risk on a probability basis and in monetary terms, and not merely risk in percentage terms only. A stand-alone risk number, say, 4%, may be less meaningful to an investor without an appreciation of the actual monetary impact the 4% has on the total portfolio value, hence the inclusion and relevance of a VaR calculation.

One of the drawbacks of the variance-covariance technique is the assumption of normality of returns. While a suite of alternative and more sophisticated techniques[8] to overcome this assumption are possible to incorporate, the variance-covariance technique is a relatively simple technique to implement [1, 64]. Hence, this technique will be employed for the case study under discussion.

[8] techniques such as *Monte Carlo Simulation* and *Historical Simulation* are highlighted in [28, 62].

2.5 Insurance and re-insurance

Insurance firms bear risks on behalf of their policyholders in exchange for a premium earned [45]. Insurers are typically classified in two distinct categories; *primary insurance* or *re-insurance*. A *primary insurer* is the firm selling the insurance to a policyholder. A *re-insurance* firm bears the primary insurers risk. Stated differently, the primary insurer (partially) transfers the risk to a re-insurer in exchange for a premium earned [45].

Figure 2.2 represents a simplified balance sheet framework for an insurer and re-insurer as highlighted in Pillar 1 of the Solvency II guidelines [7, 51]. The asset side of the balance sheet comprise of **investments** that are made in the financial markets, for example cash, bonds, real-estate, and equities. On the liability side of the balance sheet, the **best-estimate liabilities** represent the present value of the liability cash flows, discounted using an appropriate term-structure (yield curve) [51, 76, 90, 98]. For a review of present value, the reader is referred to Appendix A.3. The so-called **solvency capital requirement (SCR)**, refers to the specific monetary amount of capital that an insurer or re-insurer is required to set aside, subject to a VaR calibrated at a 99.5% confidence interval, over a one-year period [7, 25].

A simplified Balance Sheet

Assets:	**Liabilities:**
Investments	Surplus
	Solvency capital requirement
	Best-estimate Liabilities

Figure 2.2: *A simplified balance sheet representation for an insurer or re-insurer (adapted from Berdin & Gründl [7]).*

The **surplus** displayed in Figure 2.2, is measured by evaluating the difference between *assets* and *liabilities*, is commonly referred to as the *excess* amount. The surplus provides an indicator of the firms financial health and is key to expansion of the firm [32]. A positive surplus is desirable as this implies the firm has sufficient assets to meet its liabilities. Conversely, a negative surplus places the firm in an undesirable position as the assets are insufficient to meet its liabilities. The latter may compromise the financial standing of the firm, if prompt remedial action to rectify the negative surplus is not sought. A surplus of zero implies assets and liabilities are equal. Van Bragt & Kort [97] also describe a balance sheet approach when considering an investment strategy for insurers.

The **best-estimate liabilities** and the **SCR** are referred to as *policyholder assets* (P/H), whereas the **surplus** refers to the *shareholder assets* (S/H). Insurers and re-insurers are faced with conflicting objectives. The first goal is to ensure the value of the *shareholders assets* are maximised. The second goal is to safeguard the value of the *policyholders assets* and to ensure the future payout of policy commitments are met [75]. For this reason, two specific and separate investment strategies should be formulated to address these conflicting objectives. For the model framework under study, this will also be formulated separately, given the conflicting nature of the objectives.

2.6 Liability-driven investing and asset-liability management

Close to all institutional investors are expected to meet a unique set of liability cash flows. These liability cash flows may arise as a result of payment commitments made by a firm to its policyholders. The makeup thereof plays a crucial role in crafting the investment strategy [76]. The amount of money, and the term horizon associated thereof provide insight to the make-up of the liability profile. An investment strategy that is tailored specifically to meet an investors liabilities is known as a liability-driven investment (LDI) strategy. According to Berkelaar & Kouwenberg [8], the consideration of liabilities should be at the centre of the investment decision-making process. The most sensible approach for a liability-driven investor to consider, is to reduce the possibility of not being able to meet the specific liabilities [76]. The chief goal of an LDI strategy is to ensure an investors assets move in conformity with the value of its liabilities as interest rates fluctuate. Adverse interest rate fluctuations result in undesirable volatility between the assets and liabilities.

To establish whether assets have outperformed (or underperformed) its liabilities, an investor should first measure the performance of its liabilities. However, liabilities are not readily available and traded on a public stock exchange, so, tracking the performance thereof is not possible. Consequently, a so-called *liability-based benchmark* must be constructed that will best mimic changes in the price of its liabilities [4]. According to Babbel *et al.* [4], qualities to construct a liability-based benchmark depend on two main considerations.

1. Firstly, the liability-based benchmark must be constructed via the use of market data for which there is an active, traded, and open market. This will ensure that the firm regularly receives market appropriate data from a public stock exchange to price its liabilities, overtime.

2. Secondly, the liability-based benchmark must exhibit similar risk and return characteristics that closely resemble the market value of its liabilities (denoted by the present value) over time and under divergent economic climates. Stated differently, the benchmark should display duration, convexity[9], and sensitivity to additional market factors (for example, interest rate risk), rendering the benchmark "investable".

Babbel *et al.* [4] further indicates that constructing a liability-based benchmark is by no means a straightforward task. Despite the intricacies thereof, this approach serves as an important reference point to craft an investment strategy from an asset-liability management viewpoint. For the model framework under study, a liability-based benchmark will also be constructed.

An asset-liability management (ALM) framework refers to a risk management tool wherein an investor seeks to invest their assets in the financial markets that takes into account the nature of liabilities [44]. Stated differently, the objective of an ALM framework is to synchronise both sides of an insurers balance sheet (*i.e.,* assets and liabilities) [32]. Insurance firms and pension funds are the most notable investors incorporating an ALM as part of their investment strategy [18, 22]. A key driver behind incorporating an ALM for insurance firms is to mitigate an isolated study of the assets or liabilities which can lead to ignoring risks amidst the two sides (assets and liabilities) of the insurers balance sheet [45]. If the assets and liabilities are managed in isolation, this may compromise the financial standing of the insurer, potentially leading to a state of insolvency or bankruptcy [57].

A variety of novel techniques may be applied to solve an ALM problem. One such immunisation technique, termed *cash flow matching*, is known as a traditional form of ALM. This technique "matches" a dedicated series of cash outflows (liabilities) to a series of cash inflows (assets) [10]. The objective is to eradicate most, if not all the impact of ALM risk, specifically interest rate movements. If implemented correctly, this method will result in an equal value of the asset and liability cash flows. Drawbacks of this technique include difficulty of construction [10], and the cash flow matching exercise is expensive to execute [45].

[9] Convexity quantifies the rate of change of duration with movements in interest rates [57].

An additional immunisation strategy referred to as *duration matching* is a technique that seeks to immunise the portfolio against interest rate movements. It does this by matching asset duration with liability duration. An advantage of this technique is that it is a relatively simple strategy and is straightforward to implement [98]. There are limitations with this approach. This technique only works well if the liability cash flows are calculated with a high level of accuracy [45].

A *liability-relative* asset allocation strategy has the underlying goal of ensuring the payment of liabilities when they are due. This consists of a two-step process comprising of a *liability* and *asset* component and will be explained via two separate flow charts (Figures 2.3–2.4). Whilst the CFA Institute[10] [18] does not provide mathematical methods to formulate a liability-relative asset allocation as an optimisation problem, it does provide high-level and intuitive steps that a practitioner may draw on to aid the setting of an investment strategy.

2.6.1 Liability component

For the *liability* component, the CFA Institute details steps as adapted in Figure 2.3 that an insurance or re-insurance firm should consider when designing the investment strategy.

The first step of the liability component (Figure 2.3 Box 1) entails projecting the liability cash flows. Essentially, this step provides insight to the investors goals *i.e.,* the estimated monetary amount the investor is expected to incur, and the term horizon thereof. This is calculated via intricate actuarial mathematics and rules. Since the scope for this thesis does not delve into the schematics behind the methodology (this, in its entirety is a complex and non trivial matter) concerning the liability cash flows, this will not be elaborated upon further. Instead, the liability cash flows are assumed to be an input when designing the model framework.

The second step (Figure 2.3 Box 2) concerns deciding on the appropriate interest rate assumption that will be used as an input to calculate the present value of the liability cash flows. Guidelines by [51] suggest using a bond "risk-free[11]" term-structure to evaluate the present value. The term-structure is typically represented via a yield curve. Additional sources supporting the use of term-structures as the interest rate assumption include [76, 98].

The final step (Figure 2.3 Box 3) encompasses applying Equation (A.1) that calculates the present value. This present value, expressed in monetary terms, denotes the best-estimate liability as shown in the balance sheet representation in Figure 2.2.

2.6.2 Asset component

With the *liability* valuation components complete, the next step is to determine the optimal portfolio of assets. A suggested approach highlighted by the CFA Institute [18], is known as *liability-relative optimisation.* This is an extension of the traditional mean-variance technique that is based on asset volatility only, *i.e.*, "absolute risk". The expansion comprises of applying the mean-variance framework to an efficient frontier based on the volatility of the surplus as the measurement of risk. The volatility of the surplus, is the relative risk measure, *i.e.*, "liability-relative risk". Surplus optimisation is also referred to as *liability-relative* optimisation. The *liability-relative* risk may be characterised in monetary or percentage terms [18]. The CFA Institute [18] conceptually describes steps relating to the *liability-relative* process, as adapted in Figure 2.4.

[10]The Chartered Financial Analyst (CFA) Institute aims to set professional standards and best practices for investment specialists by offering relevant and credentialing programs within the realm of finance and investments [19].

[11]Government-bond rates may be used, as this is assumed risk-free given their lower probability of default.

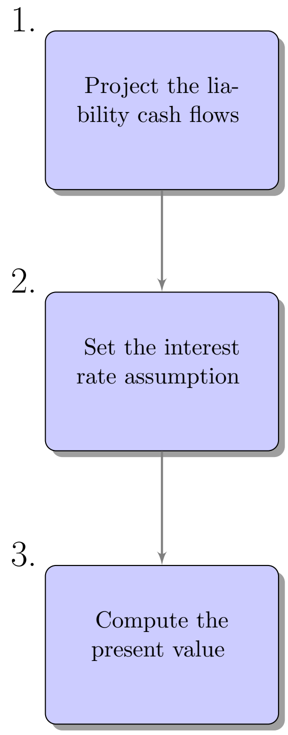


Figure 2.3: *A flow chart illustrating the basic idea of the liability-relative process, from a liability perspective (adapted from CFA Institute [18]).*

The first step concerning the *liability-relative* optimisation process (Figure 2.4 Box 1) pertains to identifying and specifying the opportunity set of asset classes. Typically, this comprises of *cash*, *bonds*, *real estate*, and *equity*, within a local and foreign context to ensure sufficient diversity of the investors assets.

The next step (Figure 2.4 Box 2) entails calculating the CMAs, comprising of expected returns and a covariance matrix for all asset classes. The varying risk and return attributes impact the relative attractiveness of the various asset classes that are ultimately used to formulate an optimal portfolio of assets.

In the third step (Figure 2.4 Box 3) the practitioner must estimate the liability returns and covariance. The liability returns (changes to the liability structure) are primarily quantified by factors such as variation in interest rates. A fixed income asset, such as a government or corporate bond, would drive changes in liability returns. Once the liability returns are computed, the risk and correlation thereof should be computed. This ultimately allows a practitioner to construct a liability-based benchmark.

The fourth step of the *liability-relative* optimisation process (Figure 2.4 Box 4) relates to a practitioner specifying suitable investment constraints to the asset classes. The constraints may arise as a result of country specific, liquidity requirements, or simply due to practicality. Common constraints imposed are; the total asset class weights must sum to 1 (100%), each asset class weights must be non-negative, as well as specifying an upper limit on foreign exposure. Chang *et al.* [20] also indicate that for practical purposes, it may be desirable to include constraints, such as imposing limits on the proportion of the portfolio devoted to any particular asset class.

The fifth step (Figure 2.4 Box 5) entails computing the *liability-relative* and the traditional asset-only efficient frontier (*non liability-relative*). This step essentially compares the optimal portfolio of assets for a *liability-relative* approach versus a *non liability-relative* approach. A further aim of this step is to understand what fixed income asset class (typically; cash, bonds,

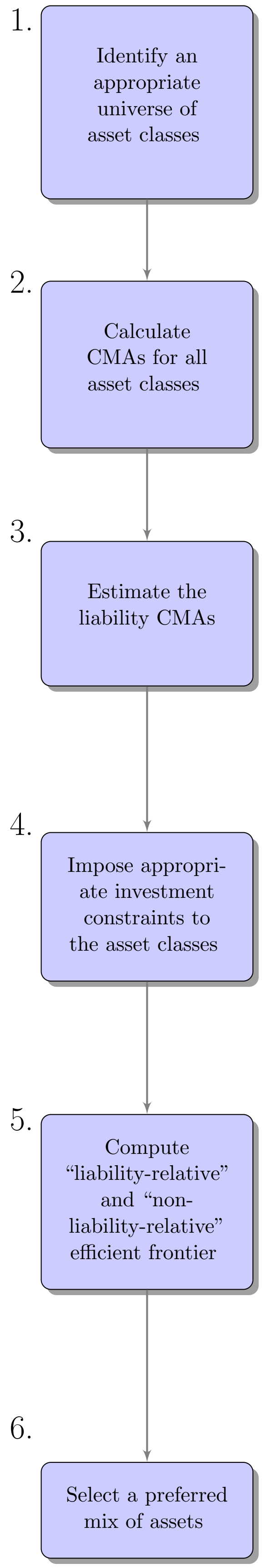


Figure 2.4: *A flow chart illustrating the basic idea of the liability-relative process, from an asset perspective (adapted from CFA Institute [18]).*

or a combination thereof) is the best liability matching instrument. Commercial optimisers are available to solve for the optimal risk and return combination of assets.

Finally, (Figure 2.4 Box 6) based on the risk preferences of the investors, the investor must select the preferred mix of assets. This determines the asset allocation.

While the CFA Institute provides intuitive steps to carry out the *liability-relative* process, little insight from a mathematical optimisation perspective to formulate the liability-relative optimisation problem is provided. From a mathematical optimisation perspective, Panjer & Boyle [71] extends objective function (2.4) that represents the traditional "asset-only" or *non liability-relative* formulation, to a *liability-relative* formulation. It is left to the reader to explore the derivation behind the *liability-relative* objective function contained in Panjer & Boyle [71]. Stated without proof, the *liability-relative* objective function stated in matrix form is given by (2.9).

$$\underset{\boldsymbol{x}\in\mathcal{R}^N}{\text{maximise}} \quad \left(2\tau\boldsymbol{\mu}^T\boldsymbol{x} - \boldsymbol{x}^T\sum\boldsymbol{x} + 2\boldsymbol{\gamma}^T\boldsymbol{x}\right), \quad \text{where} \quad \tau \geq 0. \tag{2.9}$$

A decision variable $\boldsymbol{x}$, is defined indicating the asset class weight for the asset class in question. The parameter, $\boldsymbol{\mu}$, represents the expected returns for the asset classes. Whereas, $\sum$, denotes the covariance matrix for all asset classes. The parameter, τ, refers to a specified risk tolerance. The first two terms of objective function (2.9) are noted to be identical to objective function (2.5) (*i.e.*, traditional "asset-only" optimisation). The, γ, contained in the addition of the third term, $2\boldsymbol{\gamma}^T\boldsymbol{x}$, is introduced that incorporates the covariance of the assets' returns relative to the covariance of the liability cash flows' returns, an important component to construct liability-relative asset allocations. An asset class that is highly correlated[12] with the liability cash flow, would result in the asset class featuring more prominently within the optimal asset allocation since it is seen as the preferred liability matching instrument as noted earlier. Simply stated, higher correlated asset classes relative to the liability are favoured alternatives since they share similar characteristics to the liabilities. Sharpe & Tint [87] provide a similar mathematical formulation for embedding liabilities within a portfolio optimisation problem.

Both objective functions (2.5) and (2.9) will be implemented in §4 and their results will be contrasted, to understand the impact of including (and excluding) the liability cash flow profile within the optimisation procedure.

2.7 Simulation

The concept of *simulation* may be described as a procedure that closely resembles some form of a practical real-world problem as it evolves over time [101]. Advantages of simulation include; fewer simplifying assumptions resulting in a more robust and flexible portrayal of the model. Furthermore, once the model is designed and implemented (typically on a computer), important "what-if" type of questions around different parameter estimates can be examined. Drawbacks of simulation include; writing computer code for a simulation model is not a straightforward task [66]. In addition, complex systems implemented via computer programs is often computationally expensive and is a time consuming procedure to develop [101].

Monte Carlo simulation is a stochastic method of simulation that relies on (repeated) sampling from a specified probability distribution, or via random number generation [101]. Monte Carlo simulation is a prevalent tool used within financial markets. This technique is primarily applied

[12]While the *covariance matrix* is used as an input to this objective function, the *covariance matrix* can easily be translated to a *correlation matrix* to more effectively quantify the strength of the relationship between two variables. For a brief review thereof, the reader may review Appendix A.5.

to the pricing of derivatives[13], and to estimate the VaR for financial companies such as banks and insurers [14].

Given the uncertain nature of asset prices in financial markets, practitioners may explore methods beyond traditional deterministic methods. Instead, practitioners may resort to stochastic methods that better express the uncertainty of financial markets. The term *resampled efficiency* (RE) relies on a Monte Carlo resampling approach to produce an optimal (efficient) portfolio of assets [67]. According to Michaud & Michaud [67] the RE method results in a more stable and practical mean-variance efficient frontier. Michaud & Michaud [67] describes an averaging method to derive the so-called sampled efficient frontier.

Step 1: For the opportunity set of asset classes, sample the CMAs comprising of; an expected returns (mean) matrix and a covariance (risk) matrix of returns via a probability distribution that are centred at the market implied estimates typically fed into a mean-variance optimiser.

Step 2: Compute the mean-variance efficient frontier based on these sampled risk and return estimates.

Step 3: Repeat steps 1 and 2 until sufficient observations are made for convergence in step 4.

Step 4: Determine the mean portfolio weights from step 2 − this represents the RE optimal portfolios.

Step 5: An optional step encompasses applying suitable and practical investment constraints to step 4.

This method follows an alternative (stochastic) approach compared to the QP formulation (deterministic) presented in §2.2 and §2.6.

2.8 Meta-heuristic approach

Meta-heuristics is a procedure designed to intelligently search a solution space with the primary aim of acquiring reasonable solutions to a complex optimisation problem within a reasonable amount of time [92].

A study by Zhu *et al.* [102] suggests making use of a meta-heuristic approach to solve the portfolio optimisation problem. This is done via the use of a Particle Swarm Optimisation (PSO) implementation. The objective function formulation and constraints leverage off the mean-variance framework and the Sharpe ratio[14] model. In addition, Zhu *et al.* tests their model on the inclusion and removal of the short-selling constraint (*i.e.,* (2.7)). Their PSO procedure demonstrates a high computational efficiency in composing an optimal portfolio of assets. The authors further state that preliminary results via their PSO approach exhibit outputs comparable or even preferable to that of typical solvers. The author suggest that further work encompass strengthening the efficiency of the PSO procedure to accommodate a greater amount of assets.

[13]Derivatives are contracts whose price is based on the underlying asset [34].

[14]The *Sharpe ratio* is a risk-adjusted measure that compares the difference between the *return of an asset*, R_p, and the *return of a risk-free asset*, R_f, and divides this quantity by the *standard deviation*, σ_p, of the asset [102].

2.9 Chapter summary

In this chapter, a general outline of optimisation was provided. The pioneering ideas behind the well-known mean-variance that serves as a foundation to formulate an asset allocation problem as a portfolio optimisation problem was included.

Various studies highlighting the importance and methodology for calculating CMAs, related to expected return and covariance were provided. An additional risk measure, termed, value-at-risk was studied in detail. Concepts pertaining to insurance and re-insurance and how this ties in from a balance sheet perspective was studied. The motivation behind the usage of LDI and ALM are introduced to assist in laying the foundation for a portfolio optimisation problem.

Furthermore, optimisation approaches to formulate the asset allocation problem as a QP optimisation problem was provided. Essentially, this entailed a *non liability-relative* ("asset-only") approach as well as a *liability-relative* (asset and liability) approach.

The concept and application of simulation was introduced as an alternative stochastic method to design an optimal portfolio of assets. This chapter closed with a brief discussion around the use of a meta-heuristic to solve a portfolio optimisation problem.

CHAPTER 3

Model Framework and Data

Contents

This chapter opens in §3.1 by providing a conceptual description, in the form of a roadmap of the model framework in a general setting. To add a practical ingredient to this thesis, the roadmap is applied to a real-world case study from §3.2. The actual liability cash flow data and analytics thereof are studied in §3.3. The universe of asset classes considered within this case study are furnished in §3.4. The CMAs are formulated, interpreted and studied with real-world market data obtained from RisCura, as well as Bloomberg L.P. [13] via RisCura's license in §3.5. The re-insurers current asset allocation is furnished and analysed in §3.6. The novel objective functions that form the cornerstone of the optimisation problem under study, and constraints are elaborated upon in §3.7. This chapter is rounded off with a discussion of the balance sheet representation framework, along with the VaR establishments detailed in §3.8.

3.1 Model framework

As noted in §1.1, according to the researcher's opinion, there is a shortage of novel literature sources that provide sound direction to solve a portfolio optimisation problem, that incorporates both *assets* and *liabilities*, in a coherent manner. Traditionally, studies ([20, 65]) have focused on the *asset* component with less emphasis placed on the *liabilities*. According to Broeders & Jansen [17], one of the reasons for a shortage of studies that delves into investment strategies incorporating *liabilities* is mainly due to a scarcity of detailed and comprehensive data. For this reason, the proposed model framework is positioned as a road-map that seeks to outline a careful process wherein both *assets* and *liabilities* are incorporated to aid in setting investment strategy. Investment practitioners may then draw on this approach as a reference point to aid in setting asset allocation. The model framework applied for this research project will leverage off the ALM developed at RisCura that draws on key themes studied in §2. In addition, further modifications and enhancements are incorporated within the model framework to render the results more practical and useful to re-insurers (such as incorporating *value-at-risk*, as opposed to *risk*).

The focus of this section is to provide a conceptual description in terms of how the asset and liability model components are dovetailed to ultimately solve the asset allocation phenomena. ln short, this will encompass drawing on the well-known CFA Institute guidelines, incorporating the notion of LDI to design the model framework, leveraging off MPT to solve the problem as an optimisation problem, and assimilating the balance sheet representation. Firstly, the *liability* model component will be covered, followed by a description of the *asset* model component. This will be elaborated and explained via a flow chart presented in Figure 3.1.

3.1.1 Liability component

The *liability component* will comprise of unpacking Figure 3.1 Boxes 1–2.

The first step (Figure 3.1 Box 1) consists of understanding the goals and objectives of the firm. The goals and objectives are typically measured in terms of a liability cash flow profile. The liability cash flow profile provides insight of the firms goals, such as; the estimated monetary amount the firm is expected to incur, and the term horizon thereof. These expected monetary amounts arise as a result of expected claims made by its policyholders, hence the objective is to ensure these claims are met by the firm, over a specified term horizon. In addition, a desired return objective may also form part of the firms goals. For example, the firm may wish to target an investment return of 5%, over a one-year period.

The next step (Figure 3.1 Box 2) entails constructing a liability-based benchmark as highlighted in §2.6 by Babbel *et al.* [4]. The purpose of this liability-based benchmark is to calculate the historical performance of the firms liabilities, as required for an LDI strategy. To create this benchmark, historical present values must be computed. To compute present values, Equation (A.1) must be employed. From §A.3, one of the key assumptions that is made with this calculation encompass that of the interest rate parameter. Various sources [51, 76, 90, 98] as highlighted in §2.5 suggest using a suitable term structure as the interest rate assumption.

[30] highlights two key considerations that practitioners should consider around the specific type of interest rate (risk-free) parameter to use. Firstly, whether the cash flows are based in *nominal*[1] or *real* terms. For example, if the liability cash flows are measured in *nominal* terms, then an interest rate parameter based in *nominal* terms should be used to calculate the historical present values. Conversely, if the liability cash flows are measured in *real* (*i.e.*, inflation-adjusted) terms, then an interest rate parameter that is measured in *real* terms should be selected to calculate the historical present values. Secondly, the *currency* of the liability cash flows would also inform the choice of the interest rate parameter. For example, if the currency of the liability cash flows are measured in South African Rand (ZAR), then an interest rate parameter denominated in ZAR should be chosen. Equipped with this insight, practitioners should select an interest rate parameter that adheres to these two criteria.

[1]In this context, a *nominal* figure refers to an amount unadjusted inflation rate [18].

Since the liability cash flows are assumed to be static[2], the variable parameter is the appropriate interest rate parameter (term-structure). So, by applying Equation (A.1) coupled with the liability cash flows and monthly risk-free term structures, a practitioner is able to compute a monthly historical series of present values (PVs). The change in present value, from month-to-month will reveal the current month performance (return), in percentage terms, of the firms liabilities. This iterative computation should be done monthly, going back as far as possible, data permitting. Ideally, the calculation should at least capture one or two extreme market events (*i.e.*, Global financial crises (GFC) of 2007/2008, COVID-19 *etc.*). One of the reasons why the calculation should be done going back as far as possible is so that the calculation captures these extreme market events within the risk profile. It is important to note that for markets that are less financially matured, data may not be as readily available, given the less liquid nature of those markets. In instances such as these, it would be reasonable to use the length of data that is available.

At this stage, liability analytics such as duration which should be computed to provide context of the associated interest rate risks the liability cash flows may exhibit. The duration metric provides an indication of how sensitive the liability cash flows are relative to fluctuations in interest rates [49].

3.1.2 Asset component

The *asset component* will comprise of unpacking Figure 3.1 Boxes 3–8.

The third step (Figure 3.1 Box 3) shifts the focus from the liability component to the asset component of the model framework. This step requires a practitioner to identify and specify the opportunity set of asset classes to consider incorporating within the model framework. The opportunity set of asset classes consists of a broad base of domestic and foreign classes, with the key aim to provide suitable asset class and geographical portfolio diversification. In its most basic form, the key asset classes comprise of cash, bonds, property, and equity [53].

The next step (Figure 3.1 Box 4) concerning the model framework relates to determining the CMAs, comprising of *expected returns* and a *covariance matrix* for all asset classes in question. From §2.3, these two input parameters inform the risk and return profile that impacts the relative attractiveness of the various asset classes which are fed into the mean-variance optimiser that are ultimately used to formulate an optimal portfolio of assets. From an expected return perspective, the literature study in §2.3 suggested to using *dividend yield* and *earnings yield* as key market data parameters used to estimate *equity* expected returns. For this reason, a weighted average of these two parameters (at a point in time) will be computed to derive the equity expected return for the region in question. The key market data parameter used for *fixed income* (cash and bonds) asset classes encompass a YTM, that is extracted from a suitable yield curve or fixed income index. For the covariance risk calculation, the ALM risk model will be used. Since the study is primarily focused on designing a roadmap wherein assets and liabilities are incorporated to solve an optimisation problem, the exact methodology behind the implementation around the covariance risk calculation will not be studied in depth. However, as a reasonability and sense check of the risk results produced by ALM risk model, a simple annualised[3] standard deviation risk measure will be computed to compare the possible variances thereof. In addition, by calculating the risk profile using two different approaches, would serve to provide a reasonability check between the results of the different risk profiles.

The fifth step (Figure 3.1 Box 5) entails producing an unconstrained portfolio of assets via an optimisation procedure. In its most basic form, the objective is to maximise the expected

[2]Static, in the sense of *fixed*, at a point in time.

[3]in the context of financial time-series data, the term *annualised* means the standard deviation of the historical return series will be multiplied by the positive square-root of 12.

portfolio return, whilst minimising the investment risk. This is based on the proposed quadratic objective functions stated in §2.2 and §2.6 described by Panjer & Boyle [71]. Two variants of the quadratic objective functions are formulated with different intentions. The first objective function (2.5) is employed to construct *non liability-relative* asset allocations. The second objective function (2.9) is employed to construct *liability-relative* asset allocations. Both objective functions leverage off *mean-variance* and will be used to determine the optimal portfolio of assets. However, the *liability-relative* asset allocations are an extension of *mean-variance*, given that this incorporates the correlation of the assets' returns relative to the correlation of the liability cash flows returns', as a term contained within the objective function.

Concerning the sixth step, (Figure 3.1 Box 6) the practitioner may wish to specify appropriate and relevant investment constraints to certain, or a group of asset classes. This may arise as a result of country, regulatory, practical, or liquidity specifications for the asset classes in question. For example, a firm may wish to specify a minimum or maximum percentage allocation to some or all foreign asset classes. To ensure the portfolio optimisation problem remains a QP optimisation problem (a requirement of mean-variance), the constraints imposed should be linear in nature.

The seventh step (Figure 3.1 Box 7) is complementary to the fifth step (Figure 3.1 Box 5). The practitioner merely adds the constraints decided upon (Figure 3.1 Box 6) to the optimisation procedure, to produce a constrained portfolio of assets.

The eighth step (Figure 3.1 Box 8) incorporates the balance sheet representation. Essentially this step will encompass separating the investment strategy into a P/H and S/H portfolio of assets, respectively. This will ensure each portfolio of assets are invested optimally, that achieve specific objectives. The P/H portfolio should be designed using a *liability-relative* approach, as the chief objective is to ensure the liability payments are met. The S/H portfolio should be designed using a *non liability-relative* approach. This objective is not associated with meeting any specific liability payment, however, the aim of the S/H portfolio should be to seek an appropriate return objective to grow the asset base instead. This ties back to Figure 3.1 Box 5 wherein both *liability-relative* and *non liability-relative* approaches are constructed to determine the optimal portfolio of assets, hence the requirement to produce *liability-relative* and *non liability-relative* asset allocations.

As indicated from the balance sheet representation in §2.5, the P/H portfolio consists of two terms, namely, the present value of the liabilities, and the SCR. The SCR acts as a buffer and is typically expressed as a percentage of the present value of the liabilities. For example, if the present value of the liabilities is defined as, x, and the SCR is defined as 100% of the present value of the liabilities, consequently, the SCR will also be equal to, x. Therefore, the total P/H portfolio amount would be equal to the present value of the liabilities, *plus* the SCR, yielding, $2x$.

The S/H establishment consists of one term, namely the surplus. The surplus, or excess, merely denotes the total value of the *assets* minus the total value of the *liabilities*. For example, if the value of the firms assets is defined by, y, then the surplus is equal to the value of the assets, *minus* the present value of the liabilities, *minus* the SCR. To summarise, the surplus is equal to y (value of assets) minus x (present value of liabilities) minus x (SCR). This in-turn yields, y minus $2x$.

To make the illustration simpler, fictitious numbers are provided. Suppose the present value of the liabilities is R100, and the SCR is defined once more as 100% of the present value of the liabilities. Consequently, the resulting SCR will be equal to R100. Therefore, the total P/H asset amounts to R200 (2× R100). Finally, suppose the firms total assets amount to R250. This implies the total S/H asset amount is R50 (R250 *minus* R200). In this example, the positive surplus places the hypothetical firm in a favourable position, given assets exceed liabilities.

So, to summarise the fictitious example, the investment strategy for the P/H amount of R200 should follow a *liability-relative* approach. Essentially, the P/H objective should ensure the R200 is invested into assets that will best ensure the liability payments are met, hence the reference made to *liability-relative* asset allocations. However, the S/H amount of R50 does not need to follow a *liability-relative* approach as there are no specific liability payment objectives. Instead, this R50 should be invested via a *non liability-relative* approach to allow the firm to further enhance the return objective requirement (which in turn enhances the value of the surplus).

3.2 Case study

In §3.1, the model framework was detailed as a roadmap, and explained in a conceptual, generalised sense. The focus of the study shifts by applying the model framework to a *case study* based on actual real-world data. A *case study* is a known research method that is aimed at providing in-depth understanding and context of intricate phenomena applied in a real-world setting [29]. Stated differently, a *case study* can be seen as a comprehensive study of a phenomena with an aim to generalise across a larger set of phenomena [46].

Motivation behind incorporating a case study based approach within research is useful since this yields valuable insight in terms of how the results of the model framework behave under certain conditions. Furthermore, one of the outcomes of applying the model framework to a real-world setting serves to support model validation. While certain aspects of the model framework would need to be tweaked to cater for specific circumstances, the model validation component ultimately serves as credence that the framework may be applied to a general case study.

As a point of departure for a case study, *data* pertaining to the key input parameters, required for calibration of the model framework should be collected. This is followed by an *analysis* describing the data. This may take the form of visually displaying the data in the form of charts or calculating basic descriptive statistics thereof. The aim thereof is to identify any potential outliers and to possibly recognise trends of the data. Thereafter, the model framework must be carefully calibrated with data and parameters to produce *results* of the model framework. Once results are obtained, a careful in-depth analysis and synthesis should be undertaken so as to understand what the outcomes mean from a "why" perspective. Lastly, a research report consisting of all these facets should be detailed in a coherent and logical manner such that future researchers may draw on for similar studies [5, 29].

As mentioned in §1, the case study under examination pertains to a Kenyan re-insurer. Since the case study uses real-world data, permission from the re-insurer and ethics clearance from Stellenbosch University was required to ensure the re-insurer is not placed in a compromising position. An outcome of this was approval to make use of the data within this thesis. An additional outcome, is ensuring anonymity by not stating the name of the re-insurer within this thesis. Furthermore, Crowe *et al.* [29] recommends anonymising key data descriptors so as to mitigate the risk of inadvertent disclosure thereof.

To encapsulate the model framework in its most simplistic form, Figure 3.2 illustrates the inputs and outputs of the case study, respectively. Entries 1 and 2 of the "inputs" box reflect the *liability* components' inputs, namely the liability cash flows and yield curves. Whereas entries 3 and 4 illustrate the *asset* components' inputs. The fifth and final entry of the input box reflect the constraints imposed within the *asset* component. These inputs are fed into the "optimiser" box that refer to the optimisation engine to ultimately produce an optimal portfolio of assets respectively as illustrated in the "output" box. This chapter is primarily devoted with the green "input" box illustrated in Figure 3.2, whereas Chapter 4 is concerned with the "optimiser" and "output" box.

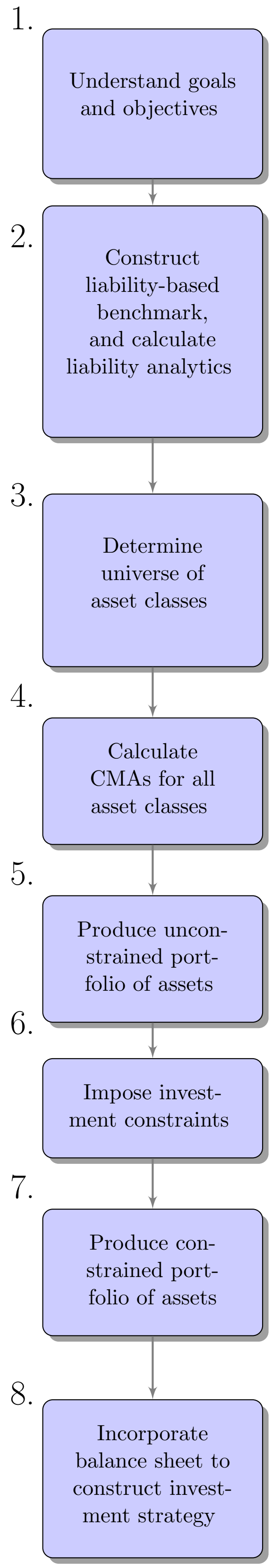


Figure 3.1: *A flow chart illustrating the basic idea of the model framework (road map).*

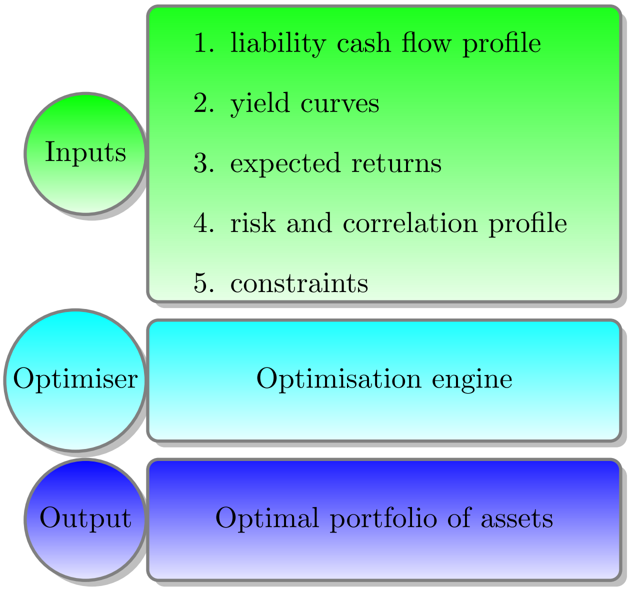


Figure 3.2: *Summarised inputs and outputs flow chart for the case study.*

3.3 Liability data analytics

As mentioned in §2.6, in accordance with an LDI strategy, an integral input is to gain insight to the nature and distribution of the re-insurers liability cash flow profile. Stated differently, the monetary amount and term horizon of the liability cash flows are a key determinant when setting the investment strategy, as this remains one of the primary drivers to construct *liability-relative* asset allocations.

Figure 3.3 shows the actual best-estimate *nominal* future liability cash flows for each of the seven service lines that the re-insurer writes business in, as calculated[4] by a Kenyan actuarial firm. To ensure anonymity of the data, the descriptions of the seven service lines are not disclosed, but instead labelled, A to G, respectively.

The *currency* is measured in USD terms, stretching 10 years into the future. While the liability cash flow profile does not span several decades into the future (such as a typical long-term investor), the liability cash flow profile spans 10 years into the future. This provides an indication that the re-insurer may be viewed as a short-to-medium term investor as opposed to a long-term investor. From Figure 3.3 it is observed that the largest cumulative cash flow amount is US\$66,024,841 (accounts for 51.55% of the total cash flow amount), and takes place in year 2. The cash flows gradually decrease from year 3 until the smallest cash flow amount of US\$1,048,987 (accounts for 0.82% of the total cash flow amount), arising in year 10.

Table 3.1 illustrates a tabular form of Figure 3.3. It is evident that the single largest service line, as a percentage of all service lines is represented by G, amounting to 37.77%. The smallest service line as represented by A, amounting to 0.20%. Table 3.1 reveals that over half (51.55%) of the total liability cash flow amounts take place in year 2, while the smallest and insignificant cash flow amount of 0.82% takes place in year 10, the final year. Upon further examination, the total sum of the liability cash flows over the 10-year period amounts to US\$128,087,930. This means, the re-insurer is expected to incur liability payments in the form of expected claims to its policyholders of US\$128,087,930 over the next 10 years.

[4]The liability cash flow data as at quarter-ends, over the 10 year period were provided. For simplicity, the quarter-end data was bucketed to year-end periods.

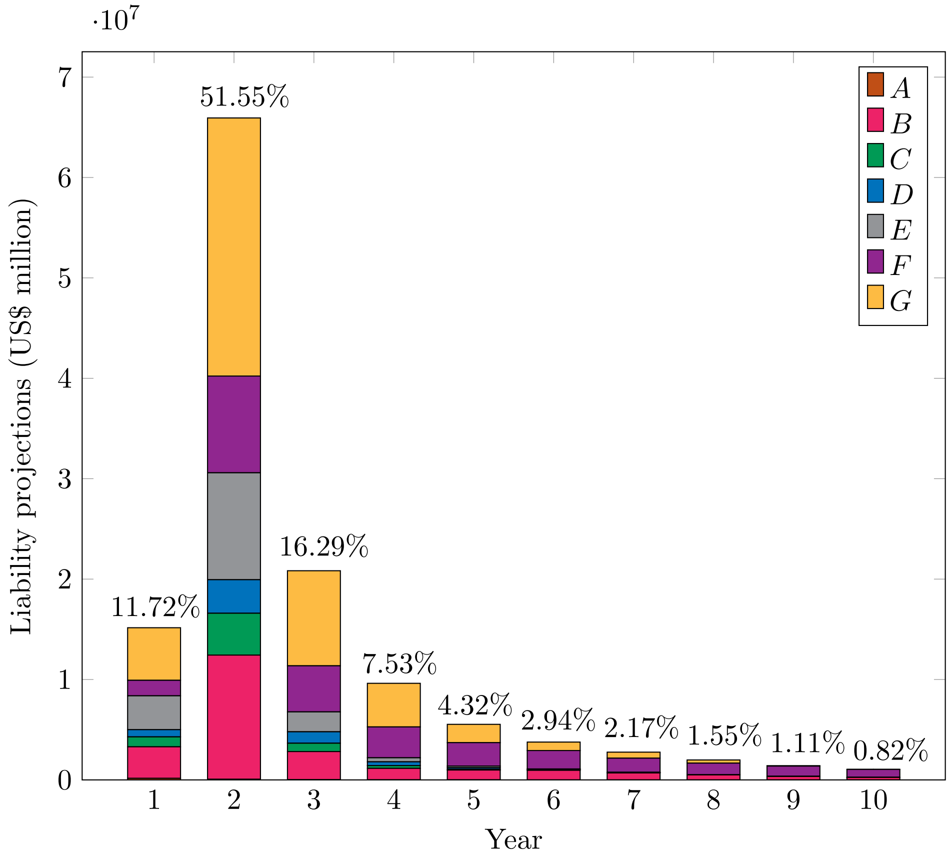


Figure 3.3: *Liability cash flow projections in monetary terms (US$ million) for each of the service lines, labelled A to G, for the re-insurer spanning 10 years into the future. The percentages denoted at the top of each bar denote the percentage of the total amount.*

Table 3.1: *Tabular reference of the re-insurers projected liability cash flows measured in Millions, USD currency, for each of the service lines A to G, from Figure 3.3.*

Liability cash flows, in Millions, measured in USD												
Service Line \ Year	1	2	3	4	5	6	7	8	9	10	Total	Total (%)
A	17	155	62	24	2	0	0	0	0	0	261	0.20%
B	3 141	12 369	2 799	1 147	982	956	693	498	341	246	23 172	18.09%
C	990	4 170	833	281	82	55	24	12	8	1	6 455	5.04%
D	719	3 340	1 137	371	156	56	31	14	1	1	5 824	4.55%
E	3 371	10 649	1 982	404	158	23	15	0	0	0	16 602	12.96%
F	1 544	9 635	4 590	3 072	2 334	1 836	1 417	1 150	1 024	793	27 395	21.39%
G	5 236	25 706	9 467	4 343	1 821	842	597	314	43	7	48 378	37.77%
Total	15 018	66 025	20 869	9 642	5 535	3 768	2 777	1 987	1 417	1 049	128 088	
Total (%)	11.72%	51.55%	16.29%	7.53%	4.32%	2.94%	2.17%	1.55%	1.11%	0.82%		100%

The liability cash flow amounts presented in Table 3.1 and Figure 3.3, represent the re-insurers *future cash flow amounts*. To calculate the *present value*, Equation (A.1) must be applied. Equation (A.1) requires an interest rate parameter assumption related to a term-structure in order to calculate the present value. For the re-insurer examined in this case study, the interest rates linked to the term-structure are the US nominal yield curve as this serves as an estimate for risk-free rates. The rationale for using the yield curve in question, is two-fold

1. The liability cash flows are measured in nominal terms, so, to ensure consistency on the liability side of the balance sheet, a nominal bond curve should be chosen.
2. The currency of the liability cash flows are denominated in USD, so, a bond curve denominated in USD terms should be chosen.

Figure 3.4 shows the full US nominal bond curve plotted[5] as a term-structure of interest rates as at 31 December 2020. Figure 3.4 suggests an upward slopping yield curve. This indicates there is a positive amount of compensation (yield) per unit of risk (tenor).

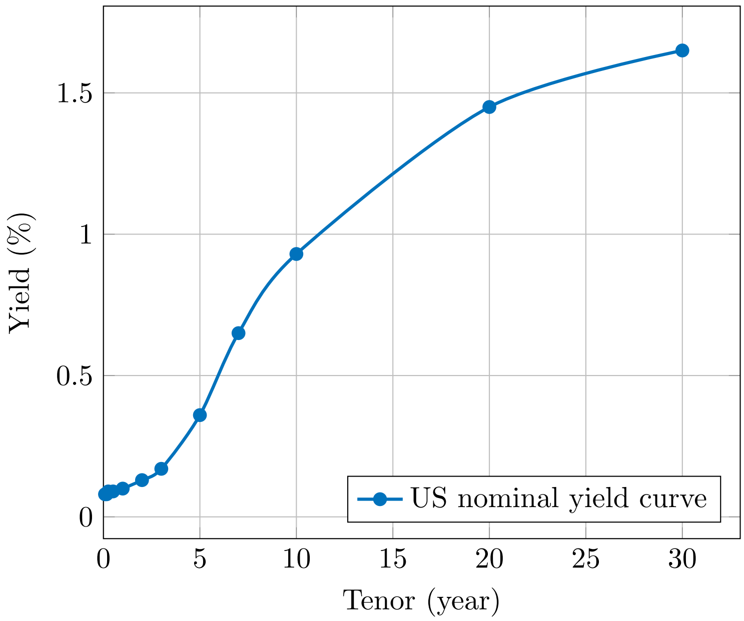


Figure 3.4: *US nominal yield curve as at 31 December 2020 (sourced from Bloomberg L.P.).*

Table 3.2 shows key liability analytics metrics. The present value amount of US$127,311,853 is identified to be very similar and marginally lower compared to the total sum of the future cash flow amount of US$128,087,930 (percentage difference of 0.6%). This relatively small similarity is not entirely unexpected. This is due, in part, to the relatively short duration of 1.77 years. The relatively short duration suggests that the present value will not be materially impacted by major shifts in interest rates. A sensitivity analysis pertaining to shifts in interest rates will be revealed in §5.4 to provide credence that the present value is not materially impacted by shocks to the interest rate parameter.

Next, as discussed in §2.6, Babbel *et al.* [4] points out that a liability-based benchmark must be constructed to estimate the historical performance (returns) of the liabilities. Once more, the US nominal yield curves will be used to compute the monthly, historical market values (historical present values) of the liabilities. Monthly US yield curves are sourced from Bloomberg L.P. beginning from 31 January 2005 until 31 December 2020. In a small number of instances, the

[5] the first 10 years (tenor) of the yield curve will be used, since the liability cash flow profile spans 10 years too.

Table 3.2: *Liability cash flow analytics.*

Parameter	Value
Future value (FV)	US$128,087,930
Present value (PV)	US$127,311,853
Macaulay duration	1.77 years

US yield curve data was not available from Bloomberg L.P.. The unavailable data was sourced online, via the U.S. department of treasury website [95]. Missing data[6] was interpolated using a simple linear interpolation. For example, if values for years 3 and 5 were known[7], but values for year 4 were not known, then a simple equally weighted average of the values of year 3 and 5, would be used as an estimate for year 4.

The time period under consideration represents 192 months in total of which the historical present values are computed. This provides insight of the historical performance of the firms liabilities. The monthly historical performance of the liability-based benchmark is shown in Figure 3.5. While Figure 3.5 may appear to have a large variation of returns (given the y-axis range), it should be noted that the minimum value of the liability-based benchmark amounts to 97.52 (as at June 2006), and the maximum value amounts to 105.85 (as at July 2020). This relatively small range subset, suggests that the variation of returns are not materially impacted by movements in interest rates. This result is largely expected, given the relatively short-to-medium term liability cash flow profile, and consequently the short duration. It is noted that the value of the index peaks during the year 2020. One of the reasons for this peak is due to the impact of COVID-19 on the financial markets. From an interpretation perspective, this means the yield curve rates were at its greatest, resulting in the highest index value, over the time period considered.

The focus of the model framework shifts from the liability component to the asset component.

3.4 Asset class framework

The next step concerning the model framework requires a practitioner to specify the opportunity set of asset classes to consider modelling within an ALM framework. In a general setting, a practitioner should consider asset classes in both a local and foreign context, so as to provide suitable geographical portfolio diversification. In its most basic form, the key asset classes generally comprise of *cash*, *bonds*, *property*, and *equity* [53]. By including asset classes that exhibit different risk, return and correlation characteristics relative to each other, this allows a practitioner to possibly enhance return and reduce the overall portfolio risk. As a result, the investors money will be invested in a broader and diversified pool of asset classes.

For the re-insurer under study, four primary geographical regions were identified, with the aim of crafting well-diversified portfolios. The specific geographical regions identified are; *Kenya markets*, *Africa markets*, *foreign-developed markets (DM)*, and *foreign-emerging markets (EM)*, respectively. In total, the opportunity set consists of 14 asset classes. For completeness, the

[6]The *missing data* refers to data that was found not to be available on Bloomberg L.P. or the U.S. department of treasury website. One of the reasons this data is currently not available is due to the U.S. department of treasury not issuing government bonds of these tenors.

[7]The total known yield curve values amounts to 60%, implying that interpolated data amounts to 40%. It should be noted that while the interpolated data may appear to be high, the interpolated data applied in the actual calculation merely accounts for approximately 13% of the total cash flow amounts. That is, year 4 (7.53%), year 6 (2.94%), year 8 (1.55%), and year 9 (1.11%). These percentages can also be found in Table 3.1 or Figure 3.3.

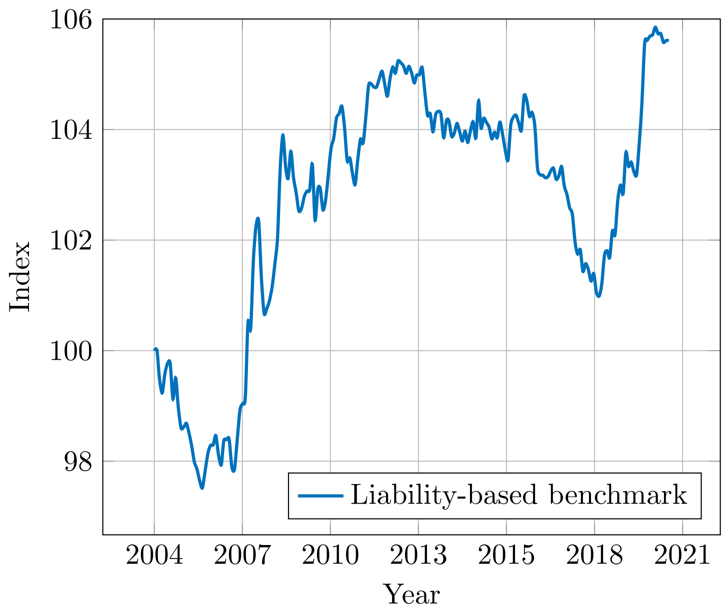


Figure 3.5: *Monthly liability-based benchmark index returns from 2005 until 2020.*

geographical regions and their asset classes are listed below. A single decision variable, x_i, (for all $i = 1, \ldots, 14$) is defined indicating the asset class weight for the asset class in question.

1. **Kenya-**
   a. cash (x_1),
   b. bonds (x_2),
   c. property (x_3),
   d. equity (x_4).
2. **Africa-**
   a. bonds (x_5),
   b. property (x_6),
   c. equity (x_7).
3. **Foreign-developed (DM) markets-**
   a. cash (x_8),
   b. bonds (x_9),
   c. property (x_{10}),
   d. equity (x_{11}).
4. **Foreign-emerging (EM) markets-**
   a. bonds (x_{12}),
   b. equity (x_{13}),
   c. China equity (x_{14}).

All *cash* and *bond* asset classes refer to its broader class description, namely *fixed income*.

3.5 Capital market assumptions

With the opportunity set of asset classes defined in §3.4, the next step concerning the model framework is to determine the CMAs, comprising of expected returns and the covariance for all asset classes in question. The CMAs are used as an input to the portfolio optimisation procedure to produce an unconstrained and constrained portfolio of assets, respectively.

To calculate the CMAs, key market data parameter inputs are required for the calibration. The data was sourced from RisCura and Bloomberg L.P. [13]. However, there are two exceptions. Kenya cash rates required for the covariance matrix calibration are sourced from the Kenyan Central Bank (KCB)[8] website, and the Kenyan inflation assumption are sourced from the *Trading Economics* [9] website. In instances, where insufficient or no data is available, assumptions will be stated and motivated. This will be discussed in §§3.5.1-3.5.4.

3.5.1 Fixed income expected returns

The literature study from §2.3 highlighted that common approaches to estimate expected returns for fixed income assets comprise of a *yield-to-maturity* (YTM) that is extracted from an appropriate yield curve or bond yield. These YTMs are expressed in annual terms *i.e.*, measured over a one-year basis.

To calculate *cash* expected returns, a 3-month YTM serves as an estimate, which are viewed as shorter-term fixed income. To calculate *bond* expected returns, a 10-year[10] YTM serves as an estimate, which are viewed as longer-term fixed income. This data as at 31 December 2020 was sourced from Bloomberg L.P.. The YTMs for *Kenya cash* and *Kenya bonds*, are extracted in Kenyan Shilling (KES), while YTMs for *Africa bonds*, *foreign-DM cash* and *foreign-DM bonds*, and *foreign-EM bonds* are extracted in USD terms, respectively.

For *Kenya cash* and *Kenya bonds*, the Kenya yield curve as illustrated in Figure 3.6a is used to derive these two expected returns. For *foreign-DM cash* and *foreign-DM bonds*, the US yield curve as displayed in Figure 3.6b is used as an estimate to derive these two expected returns. The respective 3-month and 10-year YTMs extracted from Figure 3.6 are displayed in Table 3.3. Both Figure 3.6a and Figure 3.6b suggests an upward slopping yield curve. This indicates there is a substantial and positive amount of compensation (yield) per unit of risk (tenor).

Table 3.3: *Kenya fixed income (measured in KES) and foreign-DM fixed income (measured in USD) yields, inflation expectations, and corresponding expected real returns.*

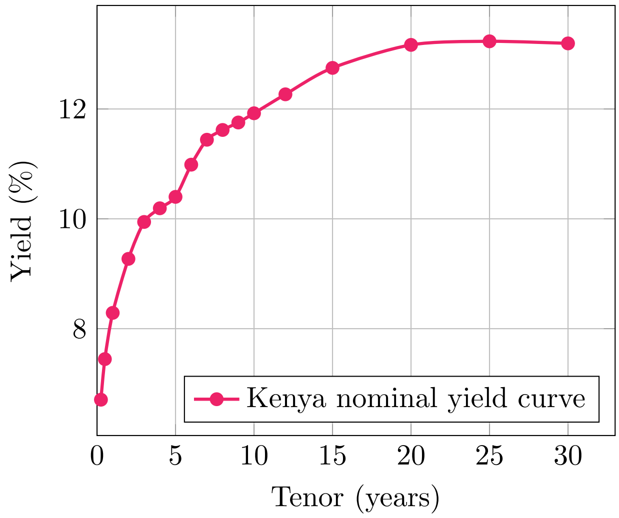
Region	Asset class	3-month yield (%)	10-year yield (%)	Inflation expectation (%)	Expected real return (%)
Kenya	cash	6.71	-	5.00	1.71
	bonds	-	11.92	5.00	6.92
Foreign-DM	cash	0.07	-	1.99	−1.91
	bonds	-	0.92	1.99	−1.07

For *foreign-EM bonds* and *African bonds*, there are no full yield curves currently available from Bloomberg L.P. to extract the YTM. In the absence of a *foreign-EM bond* yield curve, the YTM

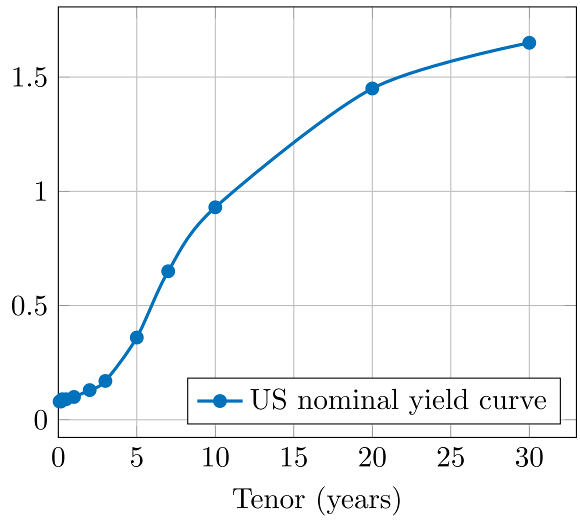
[8] https://www.centralbank.go.ke/

[9] https://tradingeconomics.com/kenya/inflation-cpi

[10] The usage of a 10-Year YTM is consistent with the most furthest term-horizon of the liability cash flows reflected in Table 3.1 and Figure 3.3.



(a) *Kenya nominal yield curve (measured in KES) as at 31 December 2020 (sourced from Bloomberg L.P.).*



(b) *US nominal yield curve (measured in USD) as at 31 December 2020 (sourced from Bloomberg L.P.).*

Figure 3.6: *Yield curves used to derive Kenya cash and Kenya bonds (measured in KES), and foreign-DM cash and foreign-DM bonds (measured in USD) expected returns (sourced from Bloomberg L.P.).*

of the *Bloomberg Barclays Emerging Market Aggregate Index* will be assumed. Whereas, in the absence of an *Africa bond* yield curve, the YTM of the *Standard Bank Africa (ex-ZA) Sovereign Bond Index* will be assumed. These two YTM values are displayed in Table 3.4.

Table 3.4: *Africa bond and foreign-EM bond YTM, inflation expectation, and corresponding expected real returns.*

Region	Asset class	YTM (%)	Inflation expectation (%)	Expected real return (%)
Africa	bonds	6.55	1.99	4.56
Foreign-EM	bonds	3.50	1.99	1.51

As mentioned earlier, the YTMs for the two Kenyan fixed income asset classes are extracted in KES terms, whereas YTMs for *Africa bonds*, *foreign-DM cash*, *foreign-DM bonds*, and *foreign-EM bonds*, are extracted in USD terms. Since the re-insurers assets and liabilities are measured in USD terms, it may seem tempting to convert the two Kenyan fixed income asset classes from KES to USD to ensure all expected returns are measured in one currency. The process of conversion would involve forecasting the exchange rate movements. [85] highlight the difficulty practitioners are faced with in attempting to predict currency movements. By incorporating a forecasted currency assumption within the expected returns, this may introduce potential unintended error as well as added complexity. In light of this, a simple assumption around *purchasing power parity* (PPP[11]) will instead be employed [89, 96]. Therefore, the Kenya expected returns does not factor a currency conversion element from KES to USD, and instead the *Kenyan* expected returns are measured in KES terms, whereas *Africa bonds*, *foreign-DM cash*, *foreign-DM bonds*, and *foreign-EM bonds*, are measured in USD terms.

[11]PPP rests on the principle that a basket of goods in two countries should be the same. Simply stated, the price level in the home country (Kenya), converted to the currency of the foreign country (US), should equate to the price level of the foreign country [85].

Since the YTMs extracted are measured in nominal terms, an appropriate inflation assumption for the various fixed income asset classes should be subtracted off the YTM to arrive at an expected real return. For the *Kenya* asset classes, a Kenya inflation assumption should be used, as the yields are represented in KES. For *Africa bonds*, *foreign-DM cash* and *foreign-DM bonds*, and *foreign-EM bonds*, a USD inflation assumption should be used, as these YTMs are represented in USD. The Kenyan inflation is sourced from *Trading Economics*[12], whereas the USD inflation assumption is sourced from Bloomberg L.P..

The expected returns in Table 3.3 reveals *Kenya bonds* as the most attractive asset class from a pure return perspective, offering the highest expected real return of 6.92%. This is mainly attributed to the high yield amount of 11.92%. The next most attractive asset class as displayed in Table 3.4 is *Africa bonds* offering an expected real return of 4.56%. The least amount of positive expected return is offered by *foreign-EM bonds* of 1.51%. It is noted that both *foreign-DM cash* and *foreign-DM bonds* provide below zero expected returns, suggesting that these two asset classes are the least attractive, relative to the other fixed income assets. Despite the negative expected real return for *foreign-DM cash* and *foreign-DM bonds*, it is appreciated that these two asset classes may exhibit attractive risk and correlation benefits, thus, potentially serving as an important diversification tool to include within a portfolio of assets.

3.5.2 Equity expected returns

One of the themes identified from the literature study detailed in §2.3 was the inclusion of dividend yield (DY) and earnings yield (EY) to estimate an equity expected return. As mentioned in §3.4, a weighted average of these two parameters will be used to calculate the expected real return for the equity asset classes. These two parameters are obtained for the regions in question and are listed in Table 3.5 as at 31 December 2020. The EY parameter denotes the inverse of the price/earnings (P/E) ratio [55].

From a pure return perspective, Table 3.5 reveals that *Kenya equity* offers the highest expected real return of 6.16%, respectively. This is followed by *Africa equity* offering an expected real return of 6.02%, respectively. The least amount of expected real return is offered by *foreign-DM equity* of 2.89%, respectively. It is observed that *foreign-EM equity* (3.56%) and *China equity* (3.43%) exhibit a similar return profile. The resemblance of return profiles amongst *foreign-EM equity* and *China equity* are largely expected, since *China* is viewed as a so-called emerging market (EM) economy, hence the return profiles exhibit some alikeness.

Table 3.5: *Equity asset classes' DY, P/E ratio, EY and corresponding expected real returns.*

Region	Asset class	DY (%)	P/E ratio	EY (%)	Expected real return (%)
Kenya	equity	4.44	12.70	7.88	6.16
Africa	equity	3.78	12.12	8.25	6.02
Foreign-DM	equity	1.84	25.39	3.94	2.89
Foreign-EM	equity	2.37	21.08	4.74	3.56
	China equity	1.42	18.37	5.44	3.43

The expected returns formulated in Table 3.3 and Table 3.5 represent a point in time (31

[12]According to the *Trading Economics* (https://tradingeconomics.com/kenya/inflation-cpi) website, the Kenya inflation rate is projected to trend around 5% in 2022. For this reason, 5% will be used as the Kenya inflation assumption.

December 2020) estimate perspective of the expected returns for fixed income and equity.

3.5.3 Property expected returns

According to the researcher's opinion, the shortage of definitive novel parameters in the literature relating to property expected returns renders the task of formulating property expected returns more challenging. Given the shortage thereof, it may be prudent to incorporate similar parameters to derive the property expected returns *i.e.*, DY and EY. There are 3 property expected returns defined for this study, namely; *Kenya property*, *Africa property*, and *foreign-DM property*. For *foreign-DM property*, the DY, P/E ratio, and the EY are displayed in Table 3.6. So, similar to all equity expected returns, an expected return for *foreign-DM property* can be computed. Once more, by calculating a weighted average of the DY and EY as this data is available on Bloomberg L.P..

However, for *Kenya property* and *Africa property* no DY nor EY data was currently available on Bloomberg L.P.. A potential reason why this type of data is currently not available for *Kenya property* and *Africa property* is mainly due to these (African) markets not exhibiting the same level of financial advancement relative to its developed market counterpart. Therefore, in the absence of DY and EY data for *Kenya property* and *Africa property*, an equal weighting (50%:50%) between the expected returns of bonds and equity for the region will be assumed [89, 96]. Essentially, this assumption denotes a linear combination of the returns of bonds and equity, for the region in question. This assumption is, in-part, as a result of property exhibiting similar characteristics of both equity and bonds. From an equity perspective, a capital appreciation component is observed. From a bond perspective, an income stream (rental income) is observed. A potential drawback of this assumption is that it may not capture the precise characteristics of property. It should be noted that the weightings for the linear combination assumption are largely a (subjective) decision made by the respective practitioners experience when faced with a shortage of data challenge such as this. The weightings thereof are largely a function of the respective practitioner's experience and serves as a so-called best-estimate in the absence of data. However, a sensitivity analysis will be performed in §5 to evaluate the impact of varying the expected return has on the results of the optimal asset allocation.

The property expected returns are summarised in Table 3.6. The "N/A" means no data was currently available. Table 3.6 reveals that *Kenya property* offers the highest expected return of 6.54%, whereas *foreign-DM property* offers the lowest expected return of 3.50%, amongst all property expected returns.

Table 3.6: *Property asset classes' DY, P/E ratio, EY and corresponding expected real return. N/A reflects lack of data at the time of study.*

Region	Asset class	DY (%)	P/E ratio	EY (%)	Expected real return (%)
Kenya	property	N/A	N/A	N/A	6.54
Africa	property	N/A	N/A	N/A	5.41
Foreign-DM	property	3.94	32.73	3.06	3.50

3.5.4 Covariance

All monthly historical data for the covariance matrix was obtained from RisCura and Bloomberg L.P., except *Kenya cash*, that is sourced from the Kenyan Central Bank (KCB).

Table 3.7 shows each benchmark index used for the 14 asset classes under consideration. From an input perspective, the covariance matrix calculation makes use of monthly historical returns of the 14 asset classes under consideration over a 15-year period, in USD terms. The start date of the data series is 31 January 2005 until 31 December 2020. The 15-year time-series history captures two pivotal market events such as the GFC of 2007/2008, and the COVID-19 market volatility experienced during 2020.

It is further noted, that no historical returns data for *Kenya property* and *Africa property* was found. Therefore, similar to the expected return assumptions for *Kenya property* and *Africa property*, a 50%:50% weighted assumption between the historical returns of bonds and equity, for its region is once more assumed [89, 96]. Essentially, this denotes a linear combination of the historical return series. Similar to the context provided for the *Kenya property* and *Africa property* expected returns in §3.5.3, it is once more noted that the weightings for the linear combination assumption are predominantly a (subjective) outlook made by the respective practitioners experience, and serves as a so-called best-estimate given the absence of data.

The benchmark indices have data commencing 31 January 2005, except; *Kenya bonds* and *Africa bonds*. The reason behind the partial lack of data or missing periods for the period in question is due to the index only starting to trade on the respective public exchange at the start date provided in Table 3.7. It is further observed that the missing periods in the data are not material time-frames within the broader 15-year period under consideration, as the existing data-set captures some elements of the GFC of 2007/2008, and the COVID-19 market volatility experienced during 2020.

An additional observation for *Kenya equity* is that over the 15-year time period, the index changed from the *MSCI Kenya* to the *Nairobi All Share Index (NASI)*. The rationale for this is that the NASI did not exist prior to March 2008, and a suitable index, *MSCI Kenya* was available and was utilised, hence the blend.

To better understand the historical performance profile and dynamics of the various indices, key descriptive statistics for all asset classes, measured in USD, including the liability as an asset class are displayed in Table 3.8. Fixed income asset classes are observed to exhibit lower annualised standard deviations relative to equity asset classes. This is consistent with the theory. *Foreign-DM cash*, a very low risk asset class is noted to have the lowest annualised standard deviation (0.46%), while *China equity*, a very risky (high risk) asset class has the highest annualised standard deviation (24.82%). Similar to the alike return profile observed between *foreign-EM equity* and *China equity*, it is once more observed that the standard deviation profiles are largely alike. This is mainly due to *China equity* being classified as a so-called emerging market economy, hence the standard deviations display some similarity.

Figure 3.7 plots the monthly fixed income asset returns under consideration and are indexed at 100 units to provide a starting base. *Foreign-DM cash* appears to grow at a fairly stable growth rate (albeit ending at the lowest cumulative end growth rate of 122.54). This trend is consistent with a low risk profile that tends to behave with a more predictable market outlook. This trend is supported by the lowest annualised standard deviation of 0.46% displayed in Table 3.8 reflecting *foreign-DM cash* with the lowest fixed income annualised standard deviation. *Africa bonds* has experienced the highest growth rate, albeit the return series only started in October 2006. It is further noted, the risk profile for *Africa bonds* appears to behave with significantly less certainty compared to *foreign-DM cash*. This outcome is supported by the higher risk profile for *Africa bonds*. The rapid decline seen with *Africa bonds* and *foreign-EM bonds* in March 2020

Table 3.7: *Asset class benchmark indices and their respective date range used to calibrate the covariance matrix.*

Region	Asset class	Date range	Benchmark index
Kenya	cash	January 2005 – December 2020	*91–day Treasury bill*
	bonds	**June 2008** – December 2020	*S&P Kenya Sovereign Bond index*
	property	No available data	
	equity	January 2005 – February 2008. March 2008 – December 2020	*MSCI Kenya* *Nairobi All Share Index (NASI)*
Africa	equity	January 2005 – December 2020	*MSCI EFM Africa ex ZA*
	property	No available data	
	bonds	**October 2006** – December 2020	*Standard Bank Africa (ex ZA) Sovereign–Bond Total Return Index (USD)*
Foreign-DM	cash	January 2005 – December 2020	*FTSE 3–Month T–bill Index*
	bonds	January 2005 – December 2020	*FTSE World Government Bond Index*
	property	January 2005 – December 2020	*FTSE EPRA Nareit Developed Index*
	equity	January 2005 – December 2020	*MSCI World Index*
Foreign-EM	bonds	January 2005– December 2020	*Barclays Emerging Market Aggregate – Total Return Index*
	equity	January 2005 – December 2020	*MSCI Emerging Markets ex–China*
	China equity	January 2005 – December 2020.	*MSCI China*

Table 3.8: *Descriptive statistics containing the minimum, maximum, mean, and standard deviation for the various asset class indices used within the study.*

Region	Asset class	Min (%)	Max (%)	Mean (%)	Stdev. (%)
Kenya	cash	−10.88	11.98	5.71	7.53
	bonds	−13.94	11.71	7.76	10.52
	property	−17.30	12.62	5.52	14.18
	equity	−29.63	21.92	8.35	23.85
Africa	bonds	−25.40	12.41	8.95	14.81
	property	−25.81	13.38	5.06	15.78
	equity	−26.23	25.35	5.23	21.73
Foreign-DM	cash	0.00	0.43	1.28	0.46
	bonds	−5.03	7.11	3.27	6.11
	property	−23.10	20.16	2.57	17.38
	equity	−18.96	12.79	7.67	15.54
Foreign-EM	bonds	−19.45	8.69	7.23	8.63
	equity	−28.15	17.93	7.48	22.13
	China equity	−22.74	19.94	12.45	24.82
Liability		−0.99	1.35	0.34	1.22

was largely driven by the COVID-19 impact. It is apparent that the COVID-19 impact had a greater negative impact on *Africa bonds* compared to *foreign-EM bonds*.

Figure 3.8 shows the monthly cumulative returns for the equity assets. It is evident that there is a higher variation amongst returns compared to Figure 3.7 over the same time period. This result is expected, as traditionally equity assets tend to behave with less certainty compared to fixed income assets, over the longer-term. *China equity*, a high risk asset class, experienced the largest growth rate over the 15-year period, ending at an index value of 647.20 in December 2020.

By making use of the monthly historical data shown in Figure 3.7–3.8, the risk profile can be established. The risk profile, also measured in annual terms to ensure consistency with the expected returns, was computed using the ALM risk model. As mentioned in §3.1.2, to ensure the risk numbers produced from this risk model are reasonable, a simple annualised standard deviation was also calculated. This sense check will aid in understanding how significant (or insignificant) the variances between the risk model and the simple annualised standard deviation may be. The results between the two approaches and differences thereof, are displayed in Table 3.9. Of the 15 asset classes (including the liability), 10 asset classes have differences amounting to less than 1%. The remaining 5 asset classes have differences between 1% and 3.13%. This differences suggest that the results from the ALM risk model are not materially different and largely in-line with the simple annualised standard deviation calculation. As such, the ALM risk model will be utilised when formulating asset allocation results in forthcoming sections.

Figure 3.9 plots a scatter plot consisting of the risk profile (horizontal axis) and expected real return (vertical axis) for all asset classes considered. The annual expected real return values as calculated in §§3.5.1–3.5.3 are plotted. The lower left shows typical lower risk and return asset classes. Generally, for assets such as *foreign-DM cash* and *foreign-DM bonds*, these are typically viewed as lower risk assets. As a result of their low risk nature, a lower return should be expected. Conversely, for a higher risk asset such as *Africa equity*, an investor can be compensated with

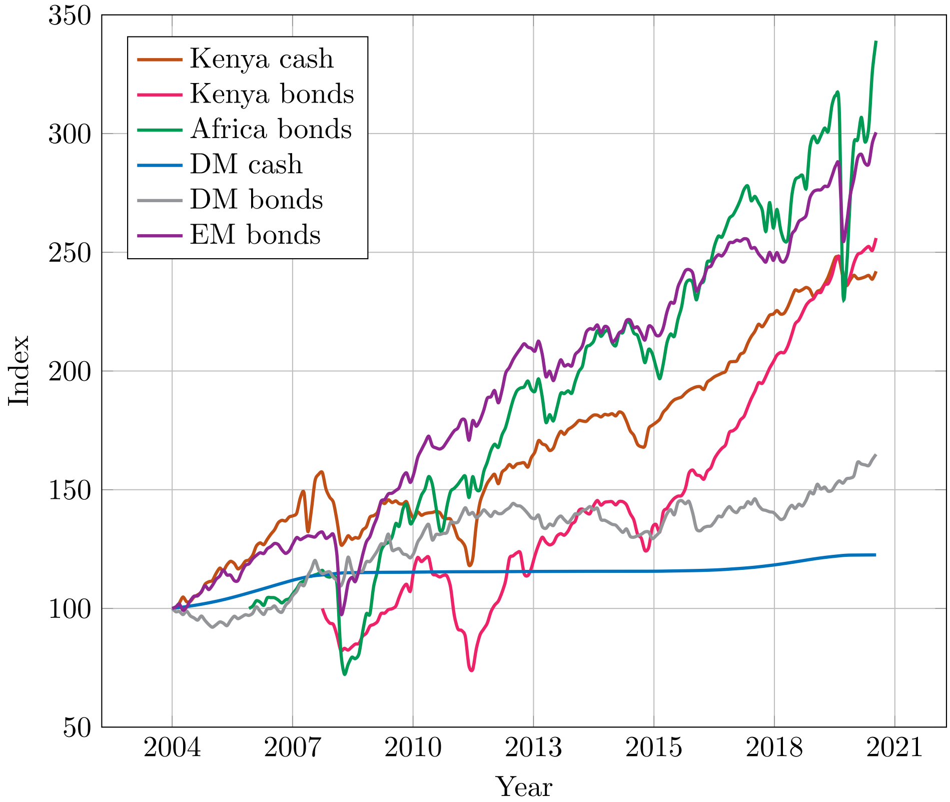


Figure 3.7: *Monthly cumulative fixed income returns over the period 2005 until 2020.*

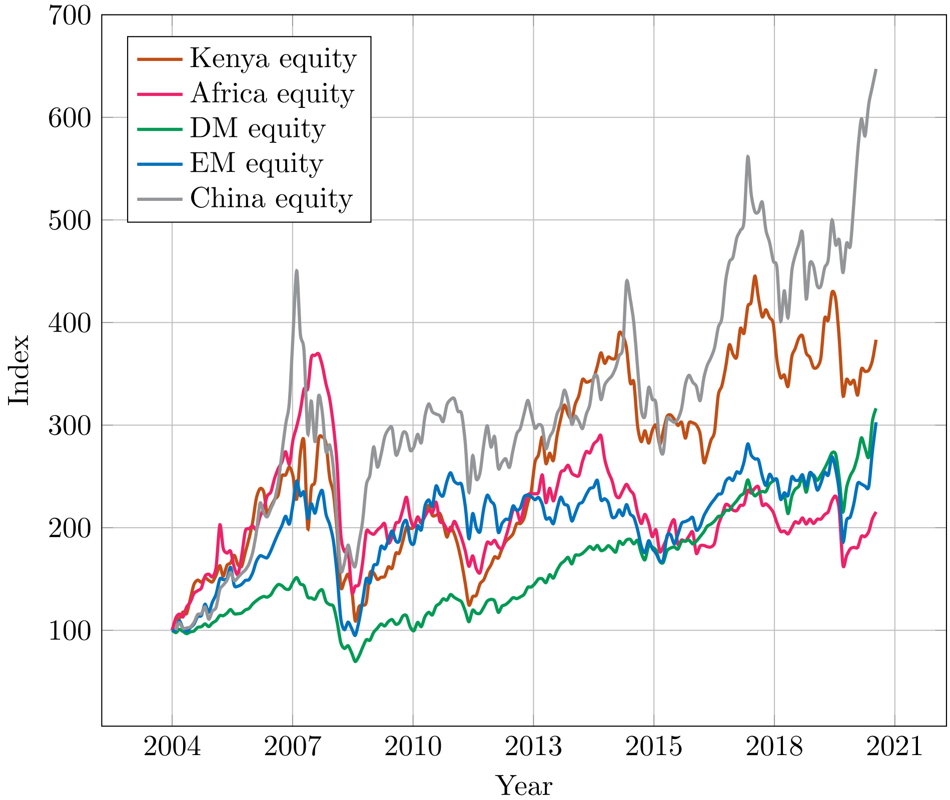


Figure 3.8: *Monthly cumulative equity returns over the period 2005 until 2020.*

Table 3.9: *Risk comparison between risk model and simple annualised standard deviation.*

Region	Asset class	Risk model (%)	Simple ann. Stdev. (%)	Difference (%)
Kenya	cash	7.35	7.53	+0.18
	bonds	13.33	10.52	−2.81
	property	17.31	14.18	−3.13
	equity	26.07	23.85	−2.21
Africa	bonds	15.06	14.81	−0.25
	property	15.53	15.78	+0.25
	equity	22.10	21.73	−0.38
Foreign-DM	cash	0.79	0.46	−0.33
	bonds	5.39	6.11	+0.72
	property	17.13	17.74	+0.61
	equity	12.51	15.54	+3.04
Foreign-EM	bonds	8.31	8.63	+0.32
	equity	22.71	22.13	−0.58
	China equity	26.40	24.82	−1.58
Liability		1.00	1.22	+0.22

a higher reward (return) for investing into this type of risky asset. It is observed that *Kenya bonds*, a fixed income asset class offers the greatest amount of expected return of all asset classes. While equity asset classes generally tend to outperform fixed income asset classes over the longer term, the expected returns formulated in this study, represent a one-year horizon, shorter-term perspective. In addition, as initially observed in §3.5.1, the high expected return of *Kenya bonds* is primarily attributed to the high *yield* amount extracted from the market data. To provide further insight to the sensitivity of this, an in-depth sensitivity analysis will be undertaken to understand the impact this may or may not have on the optimal asset allocation.

Figure 3.9 shows that *foreign-EM equity* and *China equity* exhibit a similar return profile. This is largely expected since *China equity* is a so-called emerging market economy, hence the similarity in return profiles. An additional feature of the scatter plot is the similarity of the return profile of *Africa equity* and *Kenya equity*. This too, is largely expected since Kenya forms part of the broader African economy, so these two asset classes are expected to bear similar return characteristics.

A broad trend may be observed from Figure 3.9. As an investor moves along the upper right on the return spectrum, the investor needs to be comfortable with the higher level of risk this introduces to the portfolio. A further observation in Figure 3.9 is the reference made to the "liability" asset. From an expected return perspective, the liability expected return is assumed to be equal to the expected return of *Kenya cash*. Given that the liability cash flows exhibit a relatively short duration, *Kenya cash* serves as a closer approximation for the liability expected return [96]. From a risk perspective, the liability-based benchmark represents the change in present value, from month-to-month and reveals the current month's performance (return), in percentage terms, of the firms liabilities (calculated as historical present values as noted in §3.1).

For ease of readability, the annual expected return and risk values are plotted as a scatter chart in Figure 3.9 are shown in Table 3.10.

From a correlation perspective, the correlation outputs for all asset classes are shown in Fig-

Table 3.10: *Annual expected return and risk profile assumptions for all asset classes.*

Region	Asset class	Expected real return (%)	Risk (%)
Kenya	cash	1.71	7.35
	bonds	6.92	13.33
	property	6.54	17.31
	equity	6.16	26.07
Africa	bonds	4.56	15.06
	property	5.29	15.53
	equity	6.02	22.10
Foreign-DM	cash	−1.91	0.79
	bonds	−1.07	5.39
	property	3.50	17.13
	equity	2.89	12.51
Foreign-EM	bonds	1.51	8.31
	equity	3.56	22.71
	China equity	3.43	26.40
	Liability	1.71	1.00

ure 3.11 and are once more based on the same data series from which the covariance matrix was calculated. The ones along the diagonal of the correlation matrix imply that each variable (asset class) always perfectly correlates with itself.

A negative correlation of −0.21 is observed between *foreign-DM cash* (x_8) and *foreign-DM property* (x_{10}), suggesting that these two asset class returns typically move in opposite directions. Since these two asset classes differ in terms of the type asset class (*i.e.*, cash versus property), it is largely expected that they would be negatively correlated with one another. A positive correlation of +0.42 is seen between *foreign-DM bonds* (x_9) and *foreign-EM bonds* (x_{12}). Since both of these asset class are classified as fixed income assets, merely differing in region, it is largely expected that they hold some similar correlation properties, hence the positive correlation.

Upon observation of *Africa bonds* (x_5) and *Africa property* (x_6) respectively, the correlation is revealed to be +0.86. This relatively high positive correlation is primarily driven due to the linear combination assumption used to derive the *Africa property* returns (recall, currently no historical data for *Africa property* is available on Bloomberg L.P., hence the return series is derived as a linear combination of *bonds* and *equity*, for the region in question).

Figure 3.11 further reflects an additional row of correlations, namely, x_{15}. This entry reflects the *liability* correlations with respect to the asset classes (*i.e.*, $x_1 - x_{14}$). This is one of the key inputs to construct *liability-relative* asset allocations since these values are an input within the third term of objective function (2.9). While most asset classes are revealed to be negatively correlated with the *liability* (x_{15}), *foreign-DM cash* (x_7) and *foreign-DM bonds* (x_8), both fixed income asset classes, are noted to exhibit a higher correlation (+0.65, and +0.59, respectively) relative to the *liability* (x_{15}). It is also interesting to note that the risk profile of *foreign-DM cash* (0.79%) (see Table 3.10) is largely alike to the risk profile of the *liability* (1.00%). This supports the higher correlation of *foreign-DM cash* relative to the *liability*. Essentially, this means the most suited liability matching asset classes are represented by *foreign-DM cash*, and to some degree *foreign-DM bonds* (given the correlation of +0.59, relative to the *liability*). This may mean, that the optimal asset allocation is largely expected to consist of some *foreign-DM cash* and *foreign-DM bonds* within the optimal asset allocation.

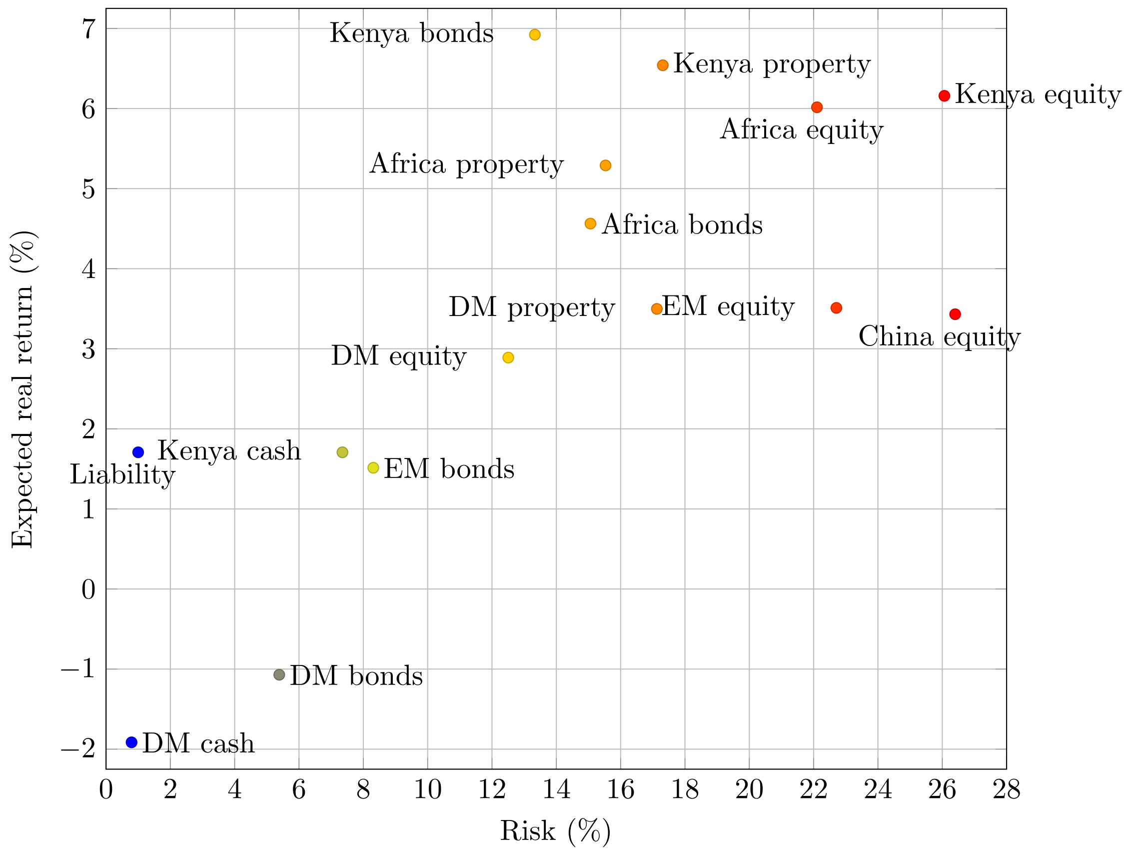


Figure 3.9: *Scatter plot of annual expected real return and risk profile for all asset classes.*

Table 3.11: *Annual correlation assumptions for all asset classes considered in the study.*

Correlations	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
x_1	1														
x_2	0.67	1													
x_3	0.56	0.86	1												
x_4	0.5	0.64	0.89	1											
x_5	0.42	0.41	0.46	0.45	1										
x_6	0.56	0.54	0.7	0.74	0.86	1									
x_7	0.57	0.47	0.65	0.78	0.53	0.82	1								
x_8	0.13	0.11	−0.08	−0.03	−0.08	0.03	0.04	1							
x_9	0.05	−0.03	−0.1	−0.03	0.29	0.21	0.02	0.37	1						
x_{10}	0.24	0.31	0.42	0.44	0.58	0.54	0.4	−0.21	−0.2	1					
x_{11}	0.37	0.32	0.56	0.69	0.62	0.81	0.73	0.03	0.13	0.44	1				
x_{12}	0.32	0.34	0.32	0.3	0.9	0.71	0.44	−0.1	0.42	0.56	0.49	1			
x_{13}	0.35	0.32	0.42	0.49	0.81	0.8	0.66	−0.06	0.25	0.46	0.8	0.77	1		
x_{14}	0.29	0.24	0.34	0.5	0.54	0.61	0.53	0.24	0.27	0.15	0.69	0.45	0.71	1	
x_{15}	−0.15	−0.11	−0.31	−0.26	−0.04	−0.16	−0.17	0.65	0.59	−0.18	−0.28	0.14	−0.15	0.05	1

In summary, this section provided insight in terms of the actual market data parameters that were used to formulate the expected returns and covariance matrix. While the expected returns and covariance matrix are a function of the quality of assumptions and input data, and are mere estimates, it is recognised that the CMAs, specifically the expected returns, are not exact forecasts, but merely a guide of future performance, over a one-year period, for the case study in question. For this reason, §5.1 will consist of a sensitivity analysis to test the impact this parameter has on the optimal portfolio of assets.

3.6 Current asset allocation

In order to suggest possible improvements and optimisation opportunities to the existing portfolio structure, a practitioner needs to gain insight into a firms existing current asset allocation. Figure 3.10 illustrates the re-insurers approximate current asset allocation, represented in percentage terms. The majority of the re-insurers portfolio exposure are invested in fixed income assets, accounting for 75.95%. The least amount of exposure of 7.73%, are invested in equity assets, with the remainder of 16.32%, invested in property assets. The high allocation to fixed income assets tends to suggest that the re-insurer has a higher preference toward less risky (fixed income) type of assets over more risky (equity) assets.

From a geographical perspective, nearly half (49.32%) of the re-insurers portfolio exposure are invested within Kenya. The least amount of exposure of 8.10%, are invested in foreign markets, whereas 42.58% is invested within Africa. Since the smallest allocation is represented by the foreign component, this may suggest that the re-insurer has a lower preference to invest their assets within foreign markets as opposed to a higher preference to Kenya or African markets. The latter two geographical regions amounting to 91.9%, collectively.

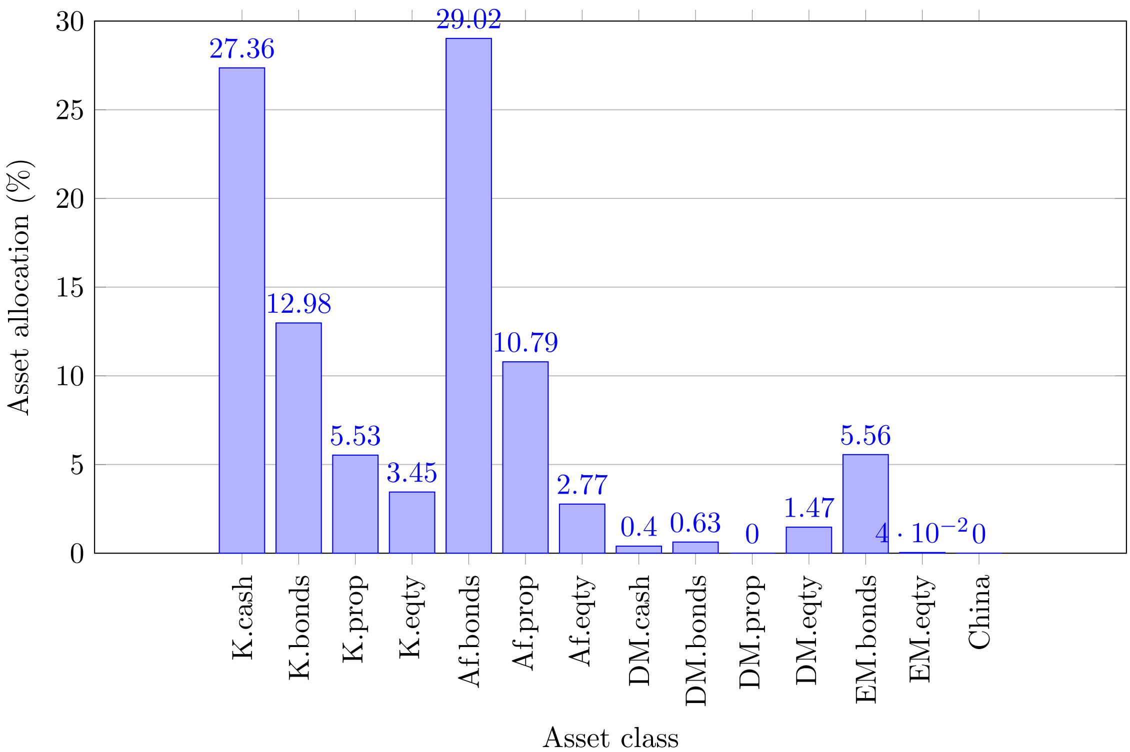


Figure 3.10: *Re-insurers estimated current asset allocation weightings mapped to the asset classes under study.*

Figure 3.10 reveals that the largest exposure to a single asset class is invested in *Africa bonds*, constituting 29.02%. The smallest non-zero amount invested to a single asset class is represented by *foreign-EM bonds*, constituting an insignificant 0.04%. It is further revealed that the re-insurer has zero exposure to *foreign-DM property* and *China equity*.

3.7 Objective functions and constraints

The mathematical framework for the *absolute framework*, in matrix form, is given by objective function (3.1). This novel objective function is provided by Panjer & Boyle [71], as discussed in §2.2. This approach does not incorporate the liability cash flows as a term in the objective function, only a matrix of expected return and covariance for the asset classes considered are used as inputs. This objective function allows a practitioner to construct "asset-only" asset allocations, or a "non liability-relative" portfolio of assets, hence the reference made to *absolute* risk and return. This is also referred to a so-called traditional approach of asset allocation. The objective is to maximise return while minimising risk, from an *absolute* perspective.

$$\underset{\boldsymbol{x}\in\mathcal{R}^N}{\text{maximise}} \quad \left(2\tau\boldsymbol{\mu}^T\boldsymbol{x} - \boldsymbol{x}^T\sum\boldsymbol{x}\right) \tag{3.1}$$

subject to

$$\mathbf{e}^T\boldsymbol{x} = 1, \tag{3.2}$$
$$\boldsymbol{x} \geq 0. \tag{3.3}$$

The variables were defined in §2.2. As a reminder, the key variables, in matrix form, are defined once more. The parameter, $\boldsymbol{\mu}$, represents the expected returns matrix for the asset classes. Whereas, $\sum$, denotes the covariance matrix for all asset classes. Furthermore, $\boldsymbol{x}$, denotes a decision variable indicating the asset class weight for the asset class in question.

Panjer & Boyle [71] extend the *absolute framework* to incorporate the liability cash flows as a term contained within the objective function. The novel objective function for the *relative framework*, in matrix form, is given by objective function (3.4) (initially introduced in §2.6). This approach allows a practitioner to construct "liability-relative" asset allocations, hence the reference made to *relative* risk and return. The objective is to maximise return while minimising risk, from a *relative* perspective.

$$\underset{\boldsymbol{x}\in\mathcal{R}^N}{\text{maximise}} \quad \left(2\tau\boldsymbol{\mu}^T\boldsymbol{x} - \boldsymbol{x}^T\sum\boldsymbol{x} + 2\boldsymbol{\gamma}^T\boldsymbol{x}\right) \tag{3.4}$$

subject to

$$\mathbf{e}^T\boldsymbol{x} = 1, \tag{3.5}$$
$$\boldsymbol{x} \geq 0. \tag{3.6}$$

Upon comparison of objective function (3.1) (the so-called "*non liability-relative framework*") and objective function (3.4) (the so-called "*liability-relative framework*"), the only change is the addition of the third term, $2\boldsymbol{\gamma}^T\boldsymbol{x}$. The, γ, contained in the addition of the term, is introduced that incorporates the correlation of the assets' returns relative to the correlation of the liability cash flows' returns. This is an important component to construct liability-relative asset allocations.

The parameter, τ (where $\tau \in [0, +\infty)$), in both objective functions (3.1) and (3.4) refers to a specified risk tolerance. Instead of assigning one single arbitrary value to the risk tolerance parameter, a matrix (vector) is defined that allows a practitioner to create an entire set of different risk and return characteristics. This vector is defined to start at 0 and end at 10, in linear increments[13] of 0.05. This yields a vector of 201 distinct risk and return sample points from which an efficient frontier curve can be constructed and subsequently plotted. Risk tolerance values closer to the upper bound prioritise return, whereas values closer to the lower bound increase the importance of risk prevention [35]. If τ equals to 0, the minimum value, this represents the minimum-variance portfolio. If τ equals to 10, the maximum value, this represents the portfolio offering the maximum return.

By making use of the CMAs as an input for all asset classes, objective function (3.1) and constraint sets (3.2)–(3.3) are fed into a mean-variance optimiser to produce a set of optimal *unconstrained* portfolio of assets (measured as risk and return), termed the unconstrained efficient frontier in an absolute framework. Whereas, objective function (3.4) and constraint sets (3.5)–(3.6) are fed into a mean-variance optimiser to produce a set of optimal *unconstrained* portfolio of assets (measured as risk and return), termed the unconstrained efficient frontier in a relative framework.

It is common practice in industry to incorporate constraints when constructing an optimal portfolio of assets [18]. This thought process should factor in practical constraints around liquidity and specific asset class limitations related to the investors investment philosophies, policies, and their risk appetite. An example thereof may relate to a higher tolerance to allocate assets *domestically*[14], as opposed to allocating assets *globally*. To incorporate such a consideration, a practitioner may wish to impose a stricter limit on *global* asset classes, as opposed to *domestic* asset classes. The stricter limit imposed may in a sense prevent the optimiser from allocating too aggressively to *global* asset classes. Consequently, this allows a practitioner to construct suitable, relevant, and meaningful asset allocations, referred to as an optimal *constrained* portfolio of assets (measured as risk and return), termed the constrained efficient frontier.

As before, a single decision variable, x_i, is defined indicating the asset class weight, for all $i = 1, \ldots, 14$. The constraints incorporated as part of the optimisation procedure are furnished by constraint sets (3.7)–(3.13). These constraints are all linear in nature, a requirement to solve an asset allocation problem as a QP optimisation problem. Given that the constraints formulated refer to specific asset classes and geographical regions, this will be listed in non-matrix form instead (initially introduced in §3.4).

$$(x_1 + x_2 + x_3 + x_4) \leq 0.5, \tag{3.7}$$
$$(x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}) \leq 0.2, \tag{3.8}$$
$$x_i \leq 0.3, \qquad \forall i = 1, \ldots, 14. \tag{3.9}$$
$$x_{11} \geq (x_{13} + x_{14}), \tag{3.10}$$
$$x_{13} \geq x_{14}, \tag{3.11}$$
$$\sum_{i=1}^{14} x_i = 1, \qquad \forall i = 1, \ldots, 14. \tag{3.12}$$
$$x_i \geq 0, \qquad \forall i = 1, \ldots, 14. \tag{3.13}$$

Constraint set (3.7) denotes a range constraint indicating an upper bound of 50% for all *Kenyan* (domestic) investments, respectively. This implies, allocations contained within the boundaries

[13] $\tau := 0, 0.05, \ldots, 9.95, 10$.

[14] by having a preference to allocate a larger amount of capital *domestically*, is commonly referred to as so-called “home-bias”.

of *Kenya* should not exceed 50%. Constraint set (3.8) ensures that the sum of all *foreign-DM*, *foreign-EM*, and *China equity* investments must not exceed 20%. Essentially, the 20% limit imposed denotes an upper bound restricting all *foreign* allocations (excluding Africa). It is further noted that no single *foreign* asset class (excluding Africa) should exceed 20% as enforced via constraint set (3.8). Upon comparison of the upper bounds imposed to constraint sets (3.7) and (3.8) suggests a higher preference toward allocating assets domestically *i.e.*, within Kenya as opposed to foreign markets (excluding Africa).

Constraint set (3.9) ensures no single asset class allocation exceeds 30%. This is imposed to mitigate a potential concentration risk whereby a small amount of asset classes may dominate the optimal portfolio of assets. Concentrated asset allocations result in a less diverse portfolio of assets that may render an impractical asset allocation. By imposing the 30% ceiling, per asset class, ensures at least 4 asset classes will form part of the opportunity set of the efficient frontier. For example, assume the optimal combination of assets comprise of *Kenya bonds* = *Africa bonds* = *Africa property* = 30%, then *foreign-DM equity* = 10%, yielding a total of 100%. From an unconstrained perspective, one of these asset classes could possibly take on a value 100%. Essentially, constraint set (3.9) moderates the exposure to dispose of this situation.

As a result of the 20% upper limit imposed on constraint set (3.8), no single foreign asset class should exceed 20%. This implicitly indicates that constraint set (3.9) should not exceed 20% for all foreign asset classes. Assume an extreme case, wherein *foreign-DM cash* = 20%. This means to ensure constraint set (3.8) is met, the allocation to all alternative foreign asset classes (excluding Africa) should be equal to zero.

While not explicitly stated, only one out of the four Kenya asset classes may attain an upper bound of 30%. The remaining maximum allocation to any alternative Kenya asset class may at most be 20%. For example, assume *Kenya cash* = 30%, and *Kenya bonds* = 20%, then *Kenya property* and *Kenya equity* should be equal to zero. This will ensure constraint sets (3.7) and (3.9) are simultaneously met. This is once more an implicit condition.

Constraint set (3.10) is a relative constraint to ensure the allocation to *foreign-DM equity* should be greater than the sum of *foreign-EM equity* plus *China equity*. Similarly, constraint set (3.11) is a relative constraint to ensure *foreign-EM equity* exposure should be greater than *China equity*. Constraint sets (3.10)–(3.11) ensures that *foreign-DM equity*, *foreign-EM equity*, and *China equity* exposure are appropriately moderated, given that the latter two asset classes exhibits a higher risk profile relative to *foreign-DM equity*. The higher risk profile of *foreign-EM equity* (22.71%) and *China equity* (26.40%), relative to *foreign-DM equity* (12.51%) are further supported by the risk numbers presented in Table 3.8 and Table 3.10. Assume the allocation to *foreign-DM equity* is zero, to ensure constraint sets (3.10)–(3.11) are satisfied, this would imply both the allocation to *foreign-EM equity* and *China equity* should also be equal to zero. Similarly, assume the allocation to *foreign-DM equity* is non-zero and positive, and assume the allocation to *foreign-EM equity* is zero, this implies the allocation to *China equity* should be equal to zero. This will ensure constraint set (3.11) is met.

Since the Kenya and foreign (excluding Africa) regions already has upper bound constraints incorporated within the optimisation procedure, no upper bound constraint concerning the full assortment of Africa has been included. Instead, the normal individual upper bound constraint (*i.e.*, constraint set (3.9)) of 30%, per asset class, is included. Essentially, the constrained asset allocation is more "relaxed" for the Africa region, given that region under study falls within Africa.

Constraint set (3.12) is a budget constraint ensuring the total exposure of all asset classes is strictly equal to 1 (100%). Constraint set (3.13) ensures each asset class allocation weight is either zero or strictly positive.

Objective functions (3.1) and (3.4) are quadratic in nature, whereas constraint sets (3.7)–(3.13) denote linearity respectively. Thus, the formulated optimisation problem represents a QP problem that must be solved, to obtain an optimal portfolio of assets.

To summarise, asset allocation optimisation results in an unconstrained setting will first be produced. Essentially, this means, objective function (3.4) and constraint sets (3.12)–(3.13) denote the unconstrained portfolio of assets. While constraint sets (3.12)–(3.13) are constraints in the traditional sense, the removal thereof would be impractical to consider, as these two general constraint sets denote binding constraints respectively, and are not unique to the re-insurer. It is noted that while the unconstrained asset allocation is merely theoretical as it does not factor in practical considerations, it is still appreciated to understand how the model behaves in the absence of constraint sets (3.7)–(3.11). Thereafter, the constrained portfolio of assets will be produced by including constraint sets (3.7)–(3.11) within the optimisation procedure.

Given the importance imposing constraints has on the optimal asset allocation, sensitivity analyses on varying the exposures in constraint sets (3.7)–(3.9) will be conducted to evaluate the impact this has on the constrained optimal asset allocation.

3.8 Balance sheet representation and VaR

The focus of §3.7 was devoted to specifying the objective functions (and constraints) as an optimisation problem, that would in turn produce asset allocations in a typical *risk* and *return* manner. Both *risk* and *return* are metrics expressed in percentage terms. While *return* expressed in percentage terms is well-known and commonly used, *risk* expressed in percentage terms is not a very tangible metric. For this reason, the *risk* will be expressed by the well-known notion of *VaR* instead. Hereafter, the investment strategy will be linked to the guidelines of the theoretical balance sheet representation initially described in Figure 2.2. The aim of this section is to provide insight in terms of how the investment strategy will be decomposed into the P/H and S/H framework of assets, respectively. In addition, the notion of VaR will be assimilated within the investment strategy.

On the asset side of the balance sheet, one parameter, namely the total value of the *investments* is required. This value refers to the actual investments that are allocated to the various list of asset classes *i.e.,* cash, bonds, property, and equity *etc.*. This total value, as provided by the re-insurer, amounts to US\$310,967,189.

Shifting the focus to the liability side of the balance sheet, three parameters are required. This comprises of the *best-estimate of the liabilities*, *SCR*, and *surplus*, respectively. The best-estimate liabilities denote the present value of the liabilities, that amounts to US\$127,311,853 as computed in §3.3. From an SCR perspective, the re-insurer defines the SCR as 100% of the present value of the liabilities. This implies that the SCR monetary amount is equal to the present value of the liabilities. Therefore, the SCR amount is equal to US\$127,311,853. From a total P/H asset perspective, recall from §3.4 this amount is equal to the present value of the liabilities *plus* the SCR. Therefore, in monetary terms, this amounts to US\$254,623,707 (2× US\$127,311,853).

Still on the liability side of the balance sheet, however, shifting the focus to the surplus, that represents the difference between the total value of assets and liabilities, yields an amount of US\$56,343,482 (US\$310,967,189 *minus* US\$254,623,707). This value represents the total S/H asset amount. As observed, the re-insurer has a positive surplus amount (and is consequently in an advantageous position). This implies the re-insurer has sufficient assets to meet their liabilities. For ease of reference, these parameters are displayed in Table 3.12.

From Table 3.12, the P/H portfolio as a percentage of the total assets amounts to 81.88% (2× 40.94%), whereas the S/H portfolio as a percentage of the total assets amounts to 18.12%. Since the P/H portfolio is primarily driven to ensure the liability payments are met, this means, that

Table 3.12: *Balance sheet parameters pertaining to the P/H and S/H and their corresponding monetary amounts of the re-insurer.*

	Parameter	P/H or S/H	Amount (US$)	Amount (%)
Assets:				
	Investments		310,967,189	100%
Liabilities:				
	Present value	P/H	127,311,853	40.94%
	SCR (100% of PV)	P/H	127,311,853	40.94%
	Surplus	S/H	56,343,482	18.12%

81.88% of the total assets should follow a *liability-relative* approach. Conversely, since there are no specific liability payments to be met from a S/H perspective, the balance of 18.12% should follow a *non liability-relative* approach, allowing the re-insurer to further enhance the return of the portfolio (and consequently enhance the surplus).

It is evident from Table 3.12 that the bulk of the strategy, in monetary and percentage terms, is tilted higher toward the P/H portfolio as opposed to the S/H portfolio. This suggests the former will have a greater impact on the overall portfolio return compared to the latter. This does not imply the S/H portfolio should be treated with less importance. Ultimately, both the P/H and S/H portfolios should be treated with equal priority to achieve optimal investment performance.

The focus shifts from describing the balance sheet representation and their respective parameters, to describing the VaR calculation procedure.

To measure risk, over a one-year period, the parametric VaR method described in §2.4 will be employed. This is presented via pseudocode in Algorithm 3.1. To initialise Algorithm 3.1, two constants, namely, the *portfolio value*, and *confidence interval* (*z-score*) are required. In addition, a third parameter, namely a *risk matrix* consisting of 201 distinctive risk values (measured by variance[15]) is also required. For each distinctive *risk value*, the *portfolio value* and *confidence interval* are multiplied by each other as listed in line 4 of Algorithm 3.1. The result thereof, yields a unique VaR matrix as opposed to a risk matrix. This allows a practitioner to measure the risk on a probability and monetary basis as opposed to risk in percentage terms.

Algorithm 3.1 Value-at-Risk matrix calculation

Input: Portfolio value (monetary amount);
Input: Confidence interval (α, denoting a $z-$score);
Input: Risk matrix obtained from ALM risk model (variance), σ_i, for $i = 1, \ldots, 201$;
Output: Value-at-Risk matrix;
1: $i \leftarrow 1$;
2: VaR(i) $\leftarrow \phi$
3: **for** each distinctive risk value contained in the risk matrix (variance), σ_i, **do**
4: VaR(i) $\leftarrow$ (Portfolio value $\times\ \alpha$) $\times \sqrt{\sigma_i}$;
5: **end for**
6: **return** VaR(i) containing 201 unique values

[15] to obtain each *standard deviation* value from each *variance* value, the positive square-root of each *variance* should be taken [38].

Table 3.13 summaries the parameters required to initialise Algorithm 3.1 that will be used to express risk in VaR terms displayed in §4. For the P/H portfolio of assets, the *portfolio value* amounts to US$254,623,707. The *confidence interval* of 99.5% will be used as suggested by Solvency II. This corresponds to a $z-$score of 2.576, as sourced from the normal distribution tables. Finally, the *risk matrix* considered will be the *liability-relative risk* measure extracted from the ALM risk model.

Table 3.13: *VaR calibration metrics for P/H and S/H portfolios.*

Parameter	P/H	S/H
Confidence interval (α)	2.576 (99.5%)	2.576 (99.5%)
Portfolio value	US$254,623,707	US$56,343,482
Risk measure	liability-relative	non liability-relative

For the S/H portfolio of assets, the *portfolio value* amounts to US$56,343,482. The same *confidence interval* of 99.5% used for the P/H portfolio of assets will once more be used for the S/H portfolio of assets. Lastly, the *risk matrix* considered will be the *non liability-relative risk* measure extracted from the ALM risk model.

3.9 Chapter summary

This chapter opened with a description of the roadmap by integrating *assets* and *liabilities*, from a theoretical and conceptualised perspective. To serve as credence and to support with model validation, the conceptual model framework was applied to a real world case study for a Kenyan re-insurer. The re-insurers actual liability cash flows were studied in depth, that provided insight of the monetary amount the re-insurer is expected to incur, as well as the term horizon thereof. In addition, liability analytics thereof was analysed and studied. The outcome of this facet of the model framework was that the duration of the liability cash flow profile represented a shorter to medium term profile.

The opportunity set of asset classes considered for the study was determined. In short, the opportunity set consisted of *cash*, *bonds*, *property*, and *equity*, within a variety of geographies to aid in portfolio diversification.

The data and derivations behind the calculations of the CMAs were covered in detail. In addition, an examination of the CMAs was uncovered, as well as a discussion around the lack of certain CMA data were provided. Essentially, for the *equity expected returns*, a weighted average of *dividend yield* and *earnings yield* was calculated to arrive at an expected return. Whereas for *fixed income expected returns*, the YTM was used. The usage of these type of parameters are supported by literature sources as detailed in §2.3. For *property expected returns*, where data was not available, a weighted average of the bonds and equity expected return, for the region in question was assumed. This was supported by input from subject matter experts. To obtain the risk profile for the asset classes in question, the ALM risk model was used wherein historical data was used as an input.

The novel objective functions to formulate an asset allocation problem as an optimisation problem, from both a *liability-relative* and *non liability-relative* perspective was furnished, once more, as initially introduced in §2. Furthermore, key constraints were incorporated within the optimisation procedure to allow for a more practical and diverse asset allocation.

This chapter closed with the P/H and S/H parameters considered for the balance sheet representation, as well as an overview of the VaR framework.

CHAPTER 4

Case Study: Results

Contents

This chapter opens with a brief overview around the choice of the programming platform in §4.1. The core focus of this chapter pertains to the results of the case study, accompanied by an examination of the analyses are furnished. A discussion around the results of the unconstrained portfolio of assets is provided in §4.2. The constraints are then incorporated into the model framework as detailed in §4.3–4.5. The results from a balance sheet and VaR perspective are presented in §4.6. The chapter closes with model validation by subject matter experts as contained in §4.7.

4.1 Model framework implementation

The proposed objective functions and practical constraints described in §3.7 represent a deterministic QP optimisation problem that must be solved. To solve the optimisation problem, a suitable programming platform should be chosen to implement the model framework. The programming platform should comfortably be able to handle large quantities of data and perform technical calculations in an efficient manner. Furthermore, the run-time of the programming platform should also be considered. The faster the run-time, the more superior the platform, as slower platforms lead to time being spent on idly waiting for code to compile, as opposed to time being spent on analysing the results of a model.

As the model framework was developed and programmed using MATLAB©, by RisCura, it is plausible to customise and modify the model framework under study in MATLAB©. In addition, MATLAB© has been used due to the robust nature in which it deals with solving optimisation problems of a higher level nature. An additional factor supporting MATLAB© is that it is specifically developed for usage by engineers and scientists [93]. Lastly, the researcher primarily has experience using MATLAB©.

The results were executed on a Dell Intel(R) Core(TM) i5-8265U at 1.60GHz with 8 GB of RAM running on Windows 10.

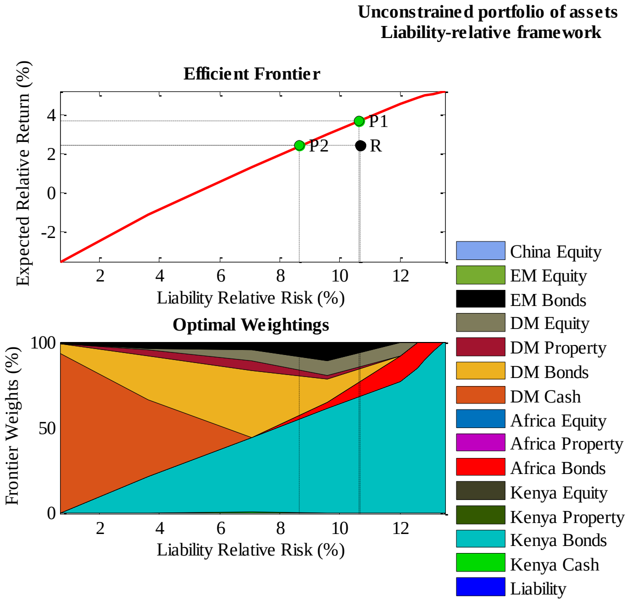
4.2 Unconstrained portfolio of assets (*liability-relative*)

In this section, the results of the *unconstrained* portfolio of assets are furnished. Essentially, objective function (3.4) along with the two binding constraints sets (3.12)–(3.13) are implemented. As mentioned in §3.7, constraint set (3.12) represents the budget constraint by ensuring the total weight of all asset classes strictly add up to 1 (100%). Whereas, constraint set (3.13) ensures each asset class weight is zero, or strictly positive. The risk and return profile as presented in Table 3.10 and correlation profile as presented in Table 3.11 are used to produce the results.

Figure 4.1 presents the *liability-relative* asset allocation results from an unconstrained perspective. The upper left of Figure 4.1 displays the efficient frontier (denoted by the thick red line). The *liability-relative risk* is shown on the horizontal axis and is expressed in percentage terms ranging from approximately 0.69% to 13.48%. The *expected relative return* is shown on the vertical axis and is once more expressed in percentage terms ranging from −3.53% to 5.21%. This means the minimum expected relative return an investor can achieve is −3.53%, subject to incurring the minimum liability-relative risk of 0.69%. However, the maximum expected relative return an investor can achieve is 5.21%, subject to incurring the maximum liability-relative risk of 13.48%. A noticeable trend can be observed from the upper left diagram of Figure 4.1. The higher the risk an investor is willing to take, the more the investor can potentially benefit from a reward (return) perspective. This trend is consistent with the theory [35].

A further observation from the upper left of Figure 4.1 are the three dots plotted along the efficient frontier. The portfolio, R, displays the re-insurers current asset allocation plotted as a black dot. The current asset allocation for the R portfolio was provided in Figure 3.10. The aim of plotting the re-insurers current asset allocation is to gain insight as to how optimal (or sub-optimal) the current portfolio may be, as well as to potentially optimise and suggest improved asset allocations that would in turn, yield a better risk and return profile. The aim is to lie on the boundary of the efficient frontier, as this would ensure the asset allocation is optimal or "efficient". It is observed that the R portfolio lies beneath the efficient frontier. This suggests that the re-insurers asset allocation is sub-optimal and can be optimised to either *increase* the *return* profile, or to *reduce* the *risk* profile. For this reason, two theoretical portfolio asset allocations are formulated, namely, portfolios $P1$ and $P2$ (plotted as two green dots). $P1$ denotes an optimised portfolio targeting a *higher return* without sacrificing any risk. On the other hand, $P2$, denotes an optimised portfolio targeting a *reduced risk* without sacrificing any return. Their respective asset allocations are shown on the upper right of Figure 4.1 and Table 4.2, and will be studied toward the end of this section.

The lower left of Figure 4.1 illustrates the optimal unconstrained portfolio of asset classes plotted as an area graph. Once more, the *liability-relative risk* is shown on the horizontal axis and is expressed in percentage terms. The optimal combination of asset classes, expressed as a percentage ranging from 0% to 100% (thus, fulfilling constraint sets (3.12)–(3.13)) is shown on the vertical axis. Essentially, the optimal area graph provides an indication of *what type of asset classes* and *what amount (percentage)* thereof should a firm invest their assets to precisely lie



Asset Allocation

	R	P1	P2
Kenya Cash	27.4%	0.0%	0.3%
Kenya Bonds	13.0%	68.2%	54.8%
Kenya Property	5.5%	0.0%	0.0%
Kenya Equity	3.5%	0.0%	0.0%
Africa Bonds	29.0%	8.7%	2.2%
Africa Property	10.8%	0.0%	0.0%
Africa Equity	2.8%	0.0%	0.0%
DM Cash	0.4%	0.0%	0.0%
DM Bonds	0.6%	7.8%	23.1%
DM Property	0.0%	0.9%	3.1%
DM Equity	1.5%	8.5%	7.8%
EM Bonds	5.6%	6.0%	8.7%
EM Equity	0.0%	0.0%	0.0%
China Equity	0.0%	0.0%	0.0%
Liability	0.0%	0.0%	0.0%

Figure 4.1: *Liability-relative, unconstrained asset allocation results.*

on the boundary of the efficient frontier.

The area graph of Figure 4.1 reveals that 6 out of the 14 asset classes are not featuring within the optimal area graph. The asset classes not featuring are *Kenya property* and *Kenya equity*, *Africa property* and *Africa equity*, *foreign-EM equity*, and *China equity*. This predominately represents equity and property asset classes, that typically exhibit a higher risk and return profile. This suggests that the optimiser finds these 6 asset classes as less appealing and sub-optimal. The alternative 8 asset classes that are included, are deemed more optimal, from a risk and return perspective. That is to say, the return profile on offer for the 6 asset classes not featuring, may not justify the level of risk the investor is expected to incur, nor is there much benefit from a correlation perspective.

The area graph of Figure 4.1 reveals that *Kenya bonds* form a significant component of the optimal portfolio of assets. At lower risk and return levels, smaller allocations of *Kenya bonds* are observed. The allocation progressively increases in an almost linear-like fashion, as the level of risk and return increases. This is primarily due to *Kenya bonds* offering the most attractive expected real return of 6.92%, hence its inclusion is so prevalent. The optimiser also prefers a substantially high allocation of *foreign-DM cash* and *foreign-DM bonds*. A possible reason why these two asset classes feature so noticeably within the optimal area graph is largely driven by the very low risk profile that these two asset classes exhibit (0.79% and 5.39%, respectively). While *foreign-DM cash* and *foreign-DM bonds* exhibit a very low risk profile, both of these asset classes yield below zero expected returns. A typical investor would consider below zero expected returns as an undesirable outcome. Despite this, it is appreciated that these two asset classes may exhibit attractive risk and correlation benefits thus serving as an important diversification tool to include within an portfolio of assets. An additional reason why *foreign-DM cash* and *foreign-DM bonds* feature so noticeably within the optimal area graph is further driven by their higher correlation (+0.65 and +0.59, respectively) that they exhibit relative to the liability. Essentially, asset classes that are higher correlated with the liability are seen as preferred liability matching asset classes.

A modest amount of *foreign-DM property*, *foreign-DM equity*, and *foreign-EM bonds* feature along most risk levels of the optimal area graph. Each of these 3 asset classes differ in terms of the "type" of asset class *i.e.*, property, equity, and bonds, respectively. This represents a less correlated range of asset classes, hence the optimisers preference to include these 3 different asset classes, to further aid in diversifying the portfolio.

From an African perspective, *Africa bonds*, the only African asset class present, features mostly at moderate to higher risk and return levels given its moderate to higher risk profile (15.06%), relative to the alternative 2 featured asset classes, namely *foreign-DM equity* (12.51%) and *foreign-EM bonds* (8.31%). Furthermore, *Africa bonds* exhibits the lowest risk profile compared to *Africa property* and *Africa equity*, hence the optimisers preference to include *Africa bonds* as opposed to the alternative, higher risk and return African asset classes.

It is further noted that *foreign-EM bonds* is the only EM asset class that features since the alternative two EM asset classes (*i.e.*, *foreign-EM equity* and *China equity*) are higher risk and return equity asset classes, relative to *foreign-EM bonds*. The latter exhibits a more conservative risk and return profile, thus the optimisers preference to include *foreign-EM bonds*, as opposed to *foreign-EM equity* and *China equity*. An insignificant amount of *Kenya cash* is seen ranging along risk levels of 5% to 8.5%.

The majority allocation of asset classes featuring in the unconstrained optimal asset allocation are fixed income type of asset classes, with *foreign-DM equity* representing the only featured equity asset class. The unconstrained optimal asset allocation results also revealed that *foreign-DM cash* and *foreign-DM bonds* featured noticeably within the optimal asset allocation. From a return perspective, these two asset classes offer below zero returns, however they exhibit attractive risk and correlation profiles. The latter was largely due to the higher correlation that these two asset classes exhibit relative to the liability, therefore prompting the optimiser to select these asset classes to feature within the optimal asset allocation. Liability correlations relative to asset class correlations are described by an explicit term within objective function (3.4), hence its inclusion. In addition, *foreign-DM property* is the only property asset class to feature. The results suggests that in an unconstrained *liability-relative* framework, fixed income asset classes are a preferred alternative as opposed to equity and property asset classes. So, higher allocations to fixed income asset classes would best ensure the re-insurers objective of the liability payment commitments are met to its respective policyholders (recall this is precisely the objective to construct *liability-relative* asset allocations). The optimisers preference to include smaller allocations of equity and property serves to further enhance the return of the portfolio and ultimately aid in better portfolio diversification.

Investments made into equity asset classes typically require a longer-term investment commitment, as opposed to fixed income asset classes typically requiring a shorter to medium-term investment commitment. The optimisers preference for an overall higher allocation to fixed income asset classes, as opposed to equity asset classes, tends to resonate with the short to medium term duration of the liability cash flows as displayed in Figure 3.3.

Table 4.1 presents the asset allocations of R, $P1$, and $P2$ and their respective differences in tabular format. A chief observation of $P1$ is the high allocation of 68.19% to *Kenya bonds*. As mentioned, this is primarily driven by the high expected return of *Kenya bonds*. However, one single asset class out of 14 asset classes comprising of a total of 68.19% may suggest that the $P1$ portfolio is too concentrated, rendering this as a non-viable and impractical portfolio. That said, in an unconstrained framework, it is appreciated that this $P1$ portfolio *increases* the *return* without sacrificing the risk, compared to the R portfolio. In contrast, moderate

Table 4.1: *Liability-relative, unconstrained optimal asset allocation results.*

Liability-relative, unconstrained efficient portfolios					
Asset class	R	$P1$	$P2$	Diff. $P1 - R$	Diff. $P2 - R$
Kenya cash	27.36	-	0.33	-27.36	-27.03
Kenya bonds	12.98	68.19	54.77	$+55.21$	$+41.78$
Kenya property	5.53	-	-	-5.53	-5.53
Kenya equity	3.45	-	-	-3.45	-3.45
Africa bonds	29.02	8.66	2.22	-20.36	-26.80
Africa property	10.79	-	-	-10.79	-10.79
Africa equity	2.77	-	-	-2.77	-2.77
DM cash	0.40	-	-	-0.40	-0.40
DM bonds	0.63	7.76	23.12	$+7.12$	$+22.49$
DM property	-	0.90	3.09	$+0.90$	$+3.09$
DM equity	1.47	8.47	7.82	$+7.00$	$+6.35$
EM bonds	5.56	6.03	8.65	$+0.47$	$+3.09$
EM equity	0.04	-	-	-0.04	-0.04
China equity	-	-	-	-	-
liability-rel. risk	10.65	10.64	8.63	-	-2.02
expected real return	4.12	5.40	4.12	$+1.28$	-

allocations are seen with respect to *Africa bonds* (8.66%), *foreign-DM equity* (8.47%), *foreign-DM bonds* (7.76%), and *foreign-EM bonds* (6.03%), whereas a small allocation is seen with respect to *foreign-DM property* (0.90%). Regionally speaking, the total allocation to Kenya amounts to 68.19%, the total allocation to Africa amounts to 8.66%, while the total allocation to the foreign component amounts to 23.15%. Given that the Africa allocation forms the smallest allocation from a regional perspective, this suggests that the optimiser finds this as the least appealing region. This is largely as a result of the higher risk and return profile associated for an African asset class, thus inclusion is less prominent. The optimiser finds the Kenya region as most appealing, given the high allocation to Kenya, albeit the full allocation is to *Kenya bonds*.

Shifting the focus to $P2$ in Table 4.1, it is observed that the *Kenya bonds* allocation remains relatively high, at 54.77%, when optimising a portfolio with the same level of return as R. Once more, this large allocation to *Kenya bonds* may suggest $P2$ is too concentrated and hence not a practical portfolio. The optimiser allocates a large amount to *foreign-DM bonds* (23.12%), while moderate to smaller allocations are seen with respect to *foreign-EM bonds* (8.65%), *foreign-DM equity* (7.82%), *foreign-DM property* (3.09%), and *Africa bonds* (2.22%). The optimiser allocates an insignificant amount to *Kenya cash* (0.33%). From a regional angle, the total allocation to Kenya amounts to 55.09%, whereas the total allocation to Africa amounts to 2.22%. The balance of 42.69% is made up by the foreign component. Similar to the African regional allocation observed with $P1$, it is once more the smallest from a regional perspective. Compared to $P1$, the optimiser prefers a much higher amount of the foreign component for $P2$. The latter is primarily driven by the optimisers preference to allocate larger amounts of *foreign-DM cash* and *foreign-DM bonds* at lower risk levels, hence $P2$ consists of a larger allocation to the foreign component.

While both $P1$ and $P2$ denote optimal and improved portfolios compared to R, respectively (from a risk and return perspective), these two optimised portfolios may not best represent well-diversified portfolios given the relatively large concentration to *Kenya bonds*. In addition, the unconstrained asset allocation does not incorporate any investment constraints. This provides further insight as to why a practitioner should incorporate constraints within an optimisation procedure. The aim is to invest a firms money in a variety of different asset classes and geographies to ensure suitable levels of diversification within the portfolio. For example, assume the *Kenya bond* market experiences a significant downturn in returns. Given the large allocation to *Kenya bonds*, this would lead to a significant drop in the overall asset value, posing a risk to the firms asset value. Assume the *Kenya bond* market experiences a significant upturn in returns. Given the large allocation to *Kenya bonds*, this would lead to a significant and favourable increase in the overall asset value. While the latter may place a firm in a desirable position, the former places a firm in an undesirable position, since there is potential exposure to a considerable amount of downside risk. This provides insight why asset allocations should be moderated via constraints to ensure moderated and diverse portfolios to limit downside risk.

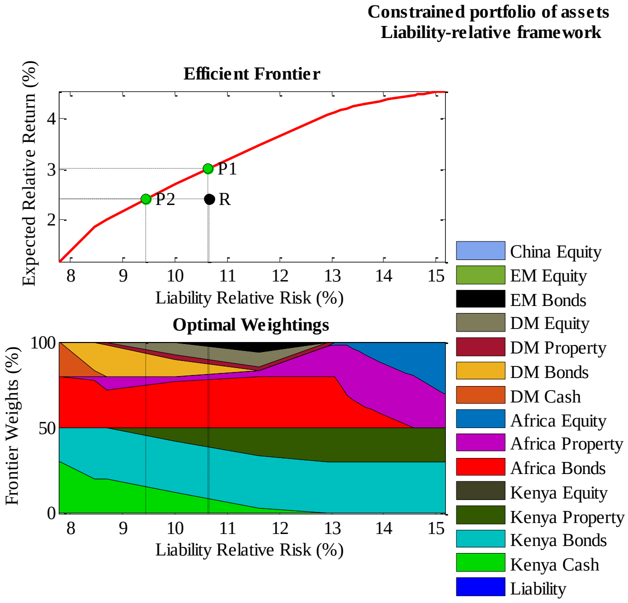
4.3 Constrained portfolio of assets (*liability-relative*)

The unconstrained asset allocation results discussed in §4.2 revealed that concentrated portfolios may arise if practical and suitable investment constraints are not incorporated with the optimisation procedure. Furthermore, by introducing investment constraints, as listed in §3.7, this in turn would allow a practitioner to construct relevant and practical portfolios. This section is devoted to producing *constrained* asset allocation outputs relative to the liability cash flow profile. In addition to constraint sets (3.12)–(3.13), constraint sets (3.7)–(3.11) will also be included within the optimisation procedure. Objective function (3.4) will be considered in this section.

Figure 4.2 shows the *liability-relative* asset allocation framework from a constrained perspective. The area graph of Figure 4.2 reveals that only 3 out of the 14 asset classes not featuring within the optimal area graph. The asset classes not featuring on the optimal area graph are *Kenya equity*, *foreign-EM equity*, and *China equity*. Similar to the unconstrained asset allocation presented earlier, these 3 asset classes did not feature either. These represent equity asset classes and are so-called "risky" asset classes, given their higher risk profile. This suggests that the optimiser finds these 3 equity asset classes as less appealing and sub-optimal, from a risk and return perspective. Stated differently, the return profile on offer for these 3 equity asset classes, may not justify the level of risk the investor is expected to incur, and there is little benefit from a correlation perspective.

It should be noted that the optimisers preference to not include any *China equity* (*i.e.,* a zero weighting along the optimal area graph) is expected. Since *foreign-EM equity* does not feature, this implies *China equity* should not feature either (as is the case). This is due to constraint set (3.11) that prevents the optimiser to allocate to *China equity*, if no allocation to *foreign-EM equity* were made. Therefore, constraint set (3.11) is satisfied.

One of the reasons why *Kenya equity* does not feature is largely due to the high risk profile associated with this asset class. Simply stated, the return profile on offer does not warrant the level of risk an investor is required to incur, thus exclusion of this asset class. To understand the *Kenya equity* exclusion empirically, the risk and return profile of *Kenya equity* and *Africa equity* should be compared in relative terms, since these are two equity asset classes exhibiting a similar return profile (and since *Kenya equity* is geographically contained within the African economy). As Table 3.10 indicates, *Africa equity*, exhibits a risk profile of 22.10% coupled with a return of 6.02% on offer. Whereas *Kenya equity* exhibits a risk profile of 26.07% coupled with a return of 6.16% on offer. Essentially, the return profiles are very similar (with *Kenya equity* offering only marginally more return), however the risk profiles are somewhat different. So, an "efficient" investor would prefer to include *Africa equity* as opposed to *Kenya equity* given that the additional risk of the latter asset class may not justify the small amount of additional return



Asset Allocation

	R	P1	P2
Kenya Cash	27.4%	8.7%	15.5%
Kenya Bonds	13.0%	30.0%	30.0%
Kenya Property	5.5%	11.3%	4.5%
Kenya Equity	3.5%	0.0%	0.0%
Africa Bonds	29.0%	28.3%	25.0%
Africa Property	10.8%	2.9%	4.9%
Africa Equity	2.8%	0.0%	0.1%
DM Cash	0.4%	0.0%	0.0%
DM Bonds	0.6%	5.7%	13.2%
DM Property	0.0%	2.7%	2.5%
DM Equity	1.5%	8.1%	4.3%
EM Bonds	5.6%	2.3%	0.0%
EM Equity	0.0%	0.0%	0.0%
China Equity	0.0%	0.0%	0.0%
Liability	0.0%	0.0%	0.0%

Figure 4.2: *Liability-relative, constrained asset allocation results.*

on offer.

A noticeable feature of Figure 4.2 is the reduced allocation to *Kenya bonds*. It is evident that the optimiser attains its upper limit of 30% for this asset class, as it features throughout the optimal area graph. Since the optimiser reaches its upper limit to *Kenya bonds*, this compels the optimiser to select alternative asset classes to include, hence allowing for a more diverse blend of asset classes to be included within the optimal area graph.

A modest amount of *foreign-DM cash* and *foreign-DM bonds* features along the lower risk levels of the optimal area graph. This is due to these 2 asset classes exhibiting a low risk and return profile, hence inclusion is mainly at lower risk and return levels. It also noted that these two asset classes exhibit a higher correlation relative to the liability, thus supporting the inclusion to include these two asset classes within the optimal asset allocation (liability correlations relative to asset classes are described by an explicit term within objective function (3.4)). For the foreign component as a whole, the optimiser attains its upper limit of 20% from the minimum risk point, until the risk level of approximately 10%. Thereafter, the foreign allocation gradually decreases until the risk level of approximately 13%. At the risk level of 13%, the optimiser no longer deems the foreign component as optimal, since there are an alternative range of asset classes offering a better risk and return profile, hence its exclusion thereof.

Instead, *Africa equity* features in the optimal area graph from higher risk levels of 13% until the maximum risk and return point of the efficient frontier. This is primarily due to *Africa equity* exhibiting a higher risk and return profile, hence inclusion is prominent at higher risk and return levels. It is observed that *Africa bonds* forms a considerable component from lower to moderate risk levels, given its moderate risk and return profile to carefully balance risk and return. The allocation to *Africa property* increases at higher risk levels at the expense of *Africa bonds* decreasing. This is primarily due to *Africa bonds* exhibiting the least amount of risk and return relative to *Africa property* and *Africa equity*, hence inclusion of *Africa bonds* at lower to moderate risk levels, whereas eminent inclusion of the former two African asset classes are seen at moderate to higher risk and return levels.

Similar to the unconstrained asset allocation, *foreign-EM bonds* is once more the only EM asset asset class that features. This is mainly due to the higher risk and return profile that the alternative two equity EM asset classes (*i.e.*, *foreign-EM equity* and *China equity*) exhibits, relative to *foreign-EM bonds*. The latter exhibits a more moderated risk and return profile, relative to the alternative two EM asset classes, thus the optimisers inclination to include some *foreign-EM bonds*.

Kenya cash, exhibiting a lower risk and return profile, starts with a high allocation and gradually decreases until higher risk levels of 13%, where it no longer features. As a result of the allocation to *Kenya cash* decreasing, the allocation to *Kenya property* starts increasing. This switch is primarily as a result of *Kenya cash* that exhibits a lower risk and return profile, as opposed to *Kenya property* that exhibits a moderate to higher risk and return profile, hence the inclusion of these two asset classes at varying levels of risk along the optimal area graph. It is further evident that the optimiser reaches its upper limit of 50% for the total Kenya component (constraint set (3.7)), once more, throughout the optimal area graph.

The unconstrained asset allocation presented in §4.2, reflected 6 out of the 14 asset classes not featuring within the optimal area graph. Given that additional asset classes feature in the constrained optimal portfolio of assets, this suggests a more diversified and moderated portfolio of assets.

Table 4.2 shifts the focus to the composition of the two optimised portfolios, namely $P1$ and $P2$. For the $P1$ portfolio, the optimiser reaches its maximum limit of 30% for *Kenya bonds*, given the attractive risk and return profile of this asset class. A large amount is allocated to the next most attractive fixed income asset class, namely *Africa bonds* (28.31%). A modest amount of *Kenya property* (11.34%), *Kenya cash* (8.66%), and *foreign-DM equity* (8.05%) is observed. Smaller allocations are seen with *foreign-DM bonds* (5.67%), *Africa property* (2.92%), *foreign-DM property* (2.71%), and *foreign-EM bonds* (2.34%). From a regional perspective, the optimiser allocates the maximum available amount of 50% to Kenya. Furthermore, the optimiser allocates 31.23% to Africa as a region, and the balance of 18.77% is attributed to the foreign region component (the maximum available amount to the foreign region is set to 20%).

Shifting the focus to $P2$ in Table 4.2, shows the optimiser once more reaches its maximum limit of 30% to *Kenya bonds*. The optimiser allocates a large amount of 24.97% to *Africa bonds*. The optimiser allocates a moderate amount to *Kenya cash* (15.48%), and *foreign-DM bonds* (13.15%). Smaller allocations are seen with *Africa property* (4.91%), *Kenya property* (4.52%), and *foreign-DM equity* (4.33%). An insignificant amount of 0.13% is allocated to *Africa equity*. From a regional perspective, the optimiser allocates the maximum available amount of 50% to Kenya. Furthermore, it allocates 20% to the foreign region component. Since the optimiser maximises its allocation to Kenya and foreign respectively, Africa as a region forms the balance of 30%.

The results of the optimisation suggest that in a constrained *liability-relative* framework, fixed income and property asset classes are a preferred alternative as opposed to equity asset classes. So, by investing the majority of assets in fixed income and property asset classes, this would best ensure the liability payments are met. Essentially, ensuring the liability payments are met is the key objective to construct *liability-relative* asset allocations, for an LDI strategy.

The results of the constrained asset allocation suggests that the optimiser tends to allocate more modestly to a broader range of asset classes (compared to the unconstrained framework). This is primarily due to the constraints imposed, which compels the optimiser to allocate to an alternative range of asset classes should the upper limit on an asset class or region on asset classes be attained. Consequently, this reduces the concentration risk by allocating to a greater range

Table 4.2: *Liability-relative, constrained optimal asset allocation results.*

Liability relative, constrained efficient portfolios					
Asset class	R	$P1$	$P2$	Diff. $P1 - R$	Diff. $P2 - R$
Kenya cash	27.36	8.66	15.48	-18.69	-11.87
Kenya bonds	12.98	30.00	30.00	$+17.02$	$+17.02$
Kenya property	5.53	11.34	4.52	$+5.81$	-1.01
Kenya equity	3.45	-	-	-3.45	-3.45
Africa bonds	29.02	28.31	24.97	-0.70	-4.05
Africa property	10.79	2.92	4.91	-7.88	-5.88
Africa equity	2.77	-	0.13	-2.77	-2.65
DM cash	0.40	-	-	-0.40	-0.40
DM bonds	0.63	5.67	13.15	$+5.04$	$+12.52$
DM property	-	2.71	2.52	$+2.71$	$+2.52$
DM equity	1.47	8.05	4.33	$+6.58$	$+2.87$
EM bonds	5.56	2.34	-	-3.21	-5.56
EM equity	0.04	-	-	-0.04	-0.04
China equity	-	-	-	-	-
liability-rel. risk	10.65	10.64	9.45	-	-1.21
expected real return	4.12	4.71	4.12	$+0.60$	-

of asset classes instead. The majority allocation of asset classes featuring in the constrained optimal asset allocation are fixed income classes, with *foreign-DM equity* and *Africa equity* representing the two featured equity asset classes. In addition, all three property asset classes feature in the optimal area graph. These three property asset classes aid in crafting diversified portfolios. Furthermore, the property asset classes generally exhibit a moderate risk and return profile, supporting the inclusion to delicately balance risk and return.

One of the key features of the constrained asset allocation are the allocations to *Kenya bonds* and *Africa bonds*. *Kenya bonds* exhibits a high return profile, whereas *Africa bonds* exhibits a moderate risk and return profile. Since the optimiser attains its upper limit of *Kenya bonds*, the optimiser allocates to the next "optimal" fixed income asset class, *i.e.*, *Africa bonds*. Since the objective function considered is to ensure the liability payments are met, the majority allocation of asset classes should be invested in fixed income and property asset classes, with a lower allocation to equity asset classes. Since the liability cash flows' duration was identified in §3.3 to be relatively shorter term in nature, fixed income asset classes are largely expected to form a greater component of the optimal portfolio of assets as opposed to equity type of asset classes. The latter typically require a longer-term commitment as opposed to the former, typically requiring a more shorter to medium-term commitment.

4.4 Combined efficient frontiers (*liability-relative*)

Figure 4.3 plots both the *unconstrained* and *constrained* efficient frontiers (*liability-relative*) on one diagram to compare the main features. Figure 4.3 reveals that the constrained efficient frontier is significantly shorter compared to the unconstrained efficient frontier. The constrained efficient frontier exhibiting a shorter curve is mainly as a result of the constraints imposed on the regional and asset classes' allocation. The rate at which the unconstrained efficient frontier increases in risk and return, resembles an almost linear relationship. This is due to the linear-like increase observed with the *Kenya bonds* allocation along the optimal area graph as depicted in Figure 4.1 (unconstrained efficient frontier). At risk levels lower than 5%, an unconstrained investor would receive below zero expected returns. This is due to the optimisers preference for low risk and return asset classes such as *foreign-DM cash* and *foreign-DM bonds*, at lower risk and return levels. These two asset classes exhibit a below zero return profile.

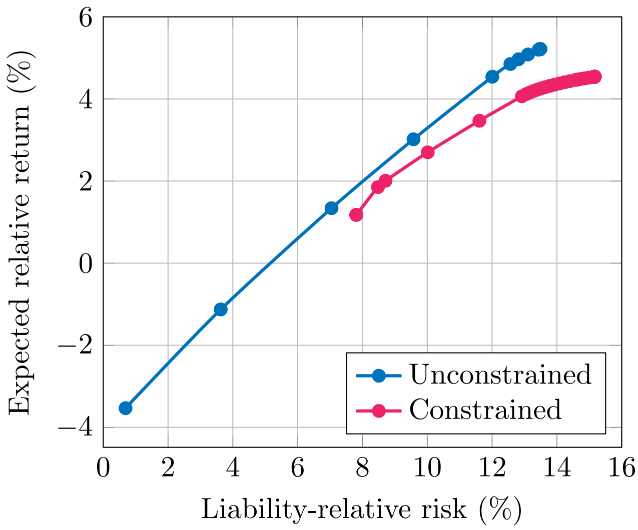


Figure 4.3: *liability-relative, unconstrained and constrained efficient frontiers, displayed in risk and return terms.*

From an unconstrained perspective, the minimum expected relative return an investor can achieve is −3.53%, subject to incurring the minimum liability-relative risk of 0.69%. However, the maximum expected relative return an investor can achieve is 5.21%, subject to incurring the maximum liability-relative risk of 13.48%. From a constrained perspective, the minimum expected relative return an investor can achieve is 1.18%, subject to incurring the minimum liability-relative risk of 7.81%. However, the maximum expected relative return an investor can achieve is 4.54%, subject to incurring the maximum liability-relative risk of 15.16%.

It is further noted that the expected relative return is strictly positive along the entire constrained efficient frontier, as there is an alternative optimal range of asset classes to ensure positive returns. However, this means an investor is expected to incur additional risk. Figure 4.3 reveals that the end-point (maximum risk and return point) of the constrained efficient frontier exhibits slightly more risk as opposed to the end-point of the unconstrained efficient frontier. Once more, this is due to the constraints imposed that compels the optimiser to allocate to an alternative range (once constraints are attained), and consequently a slightly more risky range of asset classes. In this instance, the unconstrained asset allocation end-point yields a single portfolio of 100% in *Kenya bonds* (see optimal area graph of Figure 4.1). Whereas the constrained asset allocation end-point yields a portfolio of *Kenya bonds* = 30%, *Kenya property* = 20%, *Africa equity* = 30%, and *Africa property* = 20% (see optimal area graph Figure 4.2). While the latter option exhibits a slightly more risky portfolio, it is more diversified (four asset classes) as opposed to the former, consisting of one single (very concentrated) asset class dominating the full portfolio.

From an unconstrained asset allocation angle, a practitioner may be able to select lower risk and return portfolios, however, at the expense of potentially earning negative returns. From a constrained asset allocation angle, a practitioner is expected to incur additional risk (relative to the unconstrained investor), however, this may be rewarded by earning positive returns. Ultimately, finding the appropriate balance in terms of which boundary position along the efficient frontier an investor should lie on, rests on collaboration between the consultant rendering the advice and the investor. This collaboration may include considerations such as an investor expressing a desired risk and return tolerance, given their level of "appetite" for *risk* and *return*. An investor may express a desired *risk level*, for example, to lie within a *risk level range* of between 8%–10%. On the other hand, an investor may indicate a desired *return target range*, for example, to lie within a return range of 3%–3.5%. This type of collaboration could more effectively equip the consultant to determine the most pragmatic investment strategy that is consistent and aligned with the investors *risk* and *return* objectives.

4.5 Constrained portfolio of assets (*non liability-relative*)

In this section the constrained asset allocation results will be presented, however, in the absence of the liability cash flows. Objective function (3.1) is the focus in this section. The objective is to maximise return while minimising absolute risk (not relative risk). The identical set of constraints applied in §4.3 will once more be enforced for this set of results.

Figure 4.4 shows the *non liability-relative* asset allocation framework from a constrained perspective. The area graph of Figure 4.4 indicates that 3 out of the 14 asset classes not featuring within the optimal portfolio of assets. The asset classes not featuring on the optimal area graph are *Kenya equity*, *foreign-EM equity*, and *China equity*. These 3 equity asset classes not featuring are consistent with Figure 4.2. This suggests that the optimiser deems the same amount of asset classes as optimal (and sub-optimal) when performing the optimisation in a *liability-relative* and *non liability-relative* setting. This also indicates that by including the liability cash flows (*liability-relative*) or excluding (*non liability-relative*) it, the liability cash flows used in the case study does not have a material impact on the results of the asset allocation. This is largely as a result of the relatively short duration of the liability cash flows identified in §3.3. In the event that the liability cash flows and respective duration was longer term in nature, this may have led to more noticeable shifts (such as the possibility of more equity allocations) within the results of the *liability relative* versus the *non liability-relative* asset allocation.

Upon comparison of the results displayed in Figure 4.4 (*non liability-relative*) and Figure 4.2 (*liability-relative*) suggests fairly small differences amongst the optimal area graphs. While *foreign-EM bonds* still features along the optimal area graph in Figure 4.4, it is recognised that the slither of this asset class is slightly less visible compared to the *liability-relative* asset allocation as shown in Figure 4.2. Instead, the optimiser includes slightly more *foreign-DM equity* at the expense of *foreign-EM bonds* slightly decreasing. This means in a *non liability-relative* setting, *foreign-DM equity* is a preferred alternative as opposed to *foreign-EM bonds*, given that the latter may be a better suited asset class to ensure the liability payments are met. Whereas larger allocations of the former may ensure a higher overall portfolio return (since the expected return is greater for *foreign-DM equity*, compared to *foreign-EM bonds*).

Once more, the optimisers choice to not include *China equity* is anticipated. Firstly, the risk the investor needs to incur does not justify the level of return on offer. Secondly, and more importantly, since *foreign-EM equity* does not feature along the optimal area graph (*i.e.*, the allocation is zero), *China equity* would not feature either. This is due to constraint set (3.11) that restricts an allocation to *China equity* if no allocation to *foreign-EM equity* were made.

The differences between the two optimised portfolio asset allocations, namely $P1$ and $P2$ may be seen in Table 4.3. Upon comparison of Tables 4.2–4.3 both the $P1$ and $P2$ portfolios are revealed to be largely similar with no material shifts to the composition of the asset classes. This is expected since the results of the optimal area graphs between both approaches did not reveal any material differences.

For the $P1$ portfolio, from a regional perspective, the optimiser allocates the maximum available amount of 50% to Kenya. Furthermore, it allocates an amount of 30.59% to Africa as a region,

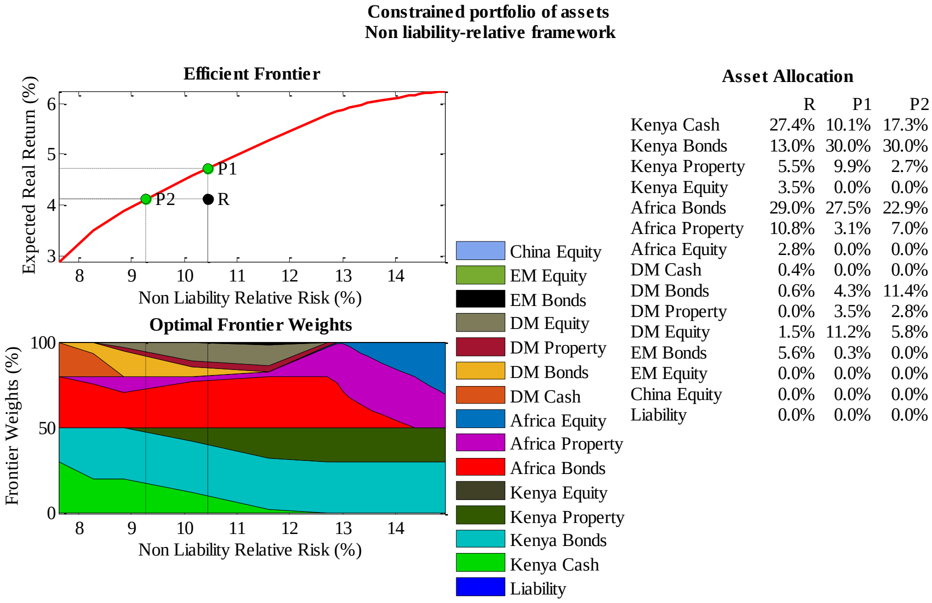


Figure 4.4: *Non liability-relative, constrained asset allocation results.*

Table 4.3: *Non liability-relative, constrained optimal asset allocation results.*

Non liability-relative, constrained efficient portfolios					
Asset class	R	$P1$	$P2$	Difference $P1-R$	Difference $P2-R$
Kenya cash	27.36	10.11	17.26	-17.25	-10.10
Kenya bonds	12.98	30.00	30.00	$+17.02$	$+17.02$
Kenya property	5.53	9.89	2.74	$+4.36$	-2.79
Kenya equity	3.45	-	-	-3.45	-3.45
Africa bonds	29.02	27.49	22.95	-1.53	-6.06
Africa property	10.79	3.10	7.05	-7.69	-3.74
Africa equity	2.77	-	-	-2.77	-2.77
DM cash	0.40	-	-	-0.40	-0.40
DM bonds	0.63	4.31	11.37	$+3.68$	$+10.74$
DM property	-	3.54	2.80	$+3.54$	$+2.80$
DM equity	1.47	11.24	5.84	$+9.77$	$+4.37$
EM bonds	5.56	0.32	-	-5.23	-5.56
EM equity	0.04	-	-	-0.04	-0.04
China equity	-	-	-	-	-
non liability-rel. risk	10.46	10.45	9.29	-	-1.17
expected real return	4.12	4.72	4.12	$+0.61$	-

and 19.41% to the foreign region component. For the $P2$ portfolio, the optimiser once more allocates the maximum available amount of 50% to Kenya. Furthermore, it allocates an amount of 30% to Africa as a region, and the maximum amount of 20% to the foreign region component.

The regional allocation of the $P1$ portfolio is recognised to be very similar for the *liability-relative* versus the *non liability-relative* asset allocation (given the relatively short duration profile), and the same set of constraints imposed. For the $P2$ portfolio, the regional allocation is revealed to be identical under both the *liability-relative* and *non liability-relative* asset allocation.

In light that the asset allocation exhibits no material shifts within its composition for the *liability-relative* and *non liability-relative* approach, it may seem unnecessary to perform separate optimisation procedures. However, the results for the *liability-relative* asset allocation are largely driven by the correlation of the liability cash flows, since this is a term contained within the objective function. Considering that the duration of the liability cash flows under study is relatively shorter-term in nature, the results under both approaches are envisaged to bear similarities. If the liability cash flows were longer term in nature, an alternative composition of asset classes may have been optimal to ensure the liability payments are met. In addition, the balance sheet representation for insurers and re-insurers consist of two separate objectives. Therefore, the investment strategy should be separated into a P/H and S/H portfolio of assets. The former should follow a *liability-relative* approach, whereas the latter should follow a *non liability-relative* approach. The results of §4.6 provides additional colour in terms of why both approaches are important when setting the investment strategy.

4.6 Balance sheet representation and VaR results

The asset allocation results of section of §4.3 (*liability-relative* framework) and §4.5 (*non liability-relative* framework) presented results in a typical *risk* and *return* manner. These earlier asset allocation results provides an auspicious foundation to craft the investment strategy. However, as mentioned in §3.8, to render the results more tangible, and to incorporate the balance sheet representation, the existing results merely need to be tweaked when formulating the investment strategy for the P/H and S/H portfolio of assets, respectively. The *liability-relative* asset allocation is represented by the P/H portfolio. Whereas the *non liability-relative* asset allocation is represented by the S/H portfolio.

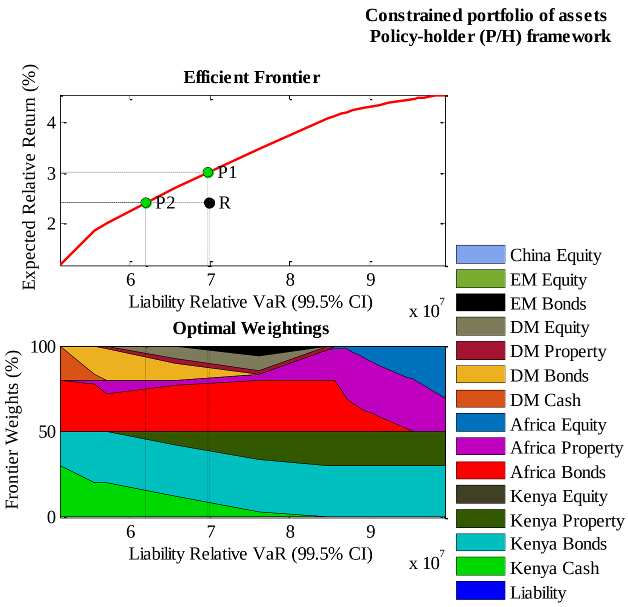
In addition, the horizontal axis (risk) seen along the efficient frontier will be expressed in VaR terms and the asset allocation results will be re-presented incorporating this modification. This modification will draw on the pseudocode described in Algorithm 3.1. By incorporating this modification, a practitioner is able to craft and view an optimal portfolio of assets in a *VaR* and *return* framework, as opposed to traditional *risk* and *return*.

4.6.1 Policy-holder (P/H) portfolio of assets

Figure 4.5 displays the P/H asset allocation in a VaR framework. This analysis is similar to the asset allocation results displayed in Figure 4.2, with the exception of the risk (horizontal) axes that is represented by VaR. The composition of asset classes displayed along the optimal area graph is revealed to be identical to the composition displayed in Figure 4.2. This is expected since the *risk* values displayed in Figure 4.2 are essentially multiplied by two constant values (namely, the *z-value* and *portfolio amount*) as highlighted in Algorithm 3.1 and listed in Table 3.13.

The upper left component of the horizontal axis of Figure 4.5 shows the minimum VaR amount of approximately US$51.21 million, whereas the maximum VaR amounts to US$99.46 million. Essentially, these values express the range of the extent of loss for the P/H framework. For example, the minimum VaR amount and corresponding asset allocation means 1/200 times, the re-insurer is likely to lose a maximum amount of US$51.21 million over a one-year period. This quantifies risk on a probability basis, as opposed to risk in percentage terms.

The upper left component of the horizontal axis of Figure 4.5 reveals that portfolio R and $P1$ yields a VaR of US\$69.86[1] million, whereas the $P2$ portfolio yields a VaR of US\$61.95 million. If the re-insurer adopts the optimised $P1$ portfolio as their P/H portfolio, it would deliver an expected real return of 4.71% at the same VaR level as the R portfolio of US\$69.86 million. From an interpretation perspective, by adopting the $P1$ portfolio, this means 1/200 times, the re-insurer is likely to lose a maximum amount of US\$69.86 million over a one-year period.



Asset Allocation

	R	P1	P2
Kenya Cash	27.4%	8.7%	15.5%
Kenya Bonds	13.0%	30.0%	30.0%
Kenya Property	5.5%	11.3%	4.5%
Kenya Equity	3.5%	0.0%	0.0%
Africa Bonds	29.0%	28.3%	25.0%
Africa Property	10.8%	2.9%	4.9%
Africa Equity	2.8%	0.0%	0.1%
DM Cash	0.4%	0.0%	0.0%
DM Bonds	0.6%	5.7%	13.2%
DM Property	0.0%	2.7%	2.5%
DM Equity	1.5%	8.1%	4.3%
EM Bonds	5.6%	2.3%	0.0%
EM Equity	0.0%	0.0%	0.0%
China Equity	0.0%	0.0%	0.0%
Liability	0.0%	0.0%	0.0%

Figure 4.5: *Liability-relative, constrained asset allocation results displayed in VaR and return.*

Alternatively, if the re-insurer adopts the optimised $P2$ portfolio as their P/H portfolio, it would deliver an expected real return of 4.12%, at a VaR of US\$61.95 million. Once more, from an interpretation perspective, this implies 1/200 times, the re-insurer is likely to lose a maximum amount of US\$61.95 million over a one-year period.

To summarise, if an investors desire is to *increase their return* (without sacrificing the current *VaR level*), then $P1$ is a preferred alternative, since in absolute terms, it delivers an additional return of +0.59% (4.12% to 4.71%). In relative terms, it delivers an increase of approximately 14%. However, if an investors desire is to *decrease their VaR* (without sacrificing the current *return level*), then $P2$ is a preferred alternative, since in absolute terms, it reduces the VaR by US\$7.91 million (US\$69.86 million to US\$61.95 million). In relative terms, it delivers a decrease of approximately 11%.

[1]To understand the inner workings of Algorithm 3.1, consider the $P1$ portfolio as an example. This portfolio exhibited a *liability-relative risk* (standard deviation) of 10.65% (as shown in Figure 4.2). So, multiplying this *liability-relative risk value* with the $z-$value of 2.576, and the *P/H portfolio amount* of US\$254,623,707, yields a VaR value for $P1$ of US\$69.86 million as reflected along the horizontal axis of Figure 4.5.

4.6.2 Share-holder (S/H) portfolio of assets

The upper left component of the horizontal axis of Figure 4.6 displays the S/H asset allocation in a VaR framework. Once more, this analysis is similar to the asset allocation results displayed in Figure 4.4, with the exception of the risk (horizontal) axes that is represented by VaR. In Figure 4.6, the minimum VaR amounts to US$11.11 million, whereas the maximum VaR amounts to US$21.66 million. Essentially, these values denote the range of the extent of loss in a S/H framework. For example, the maximum VaR amount and corresponding asset allocation means 1/200 times, the re-insurer is likely to lose a maximum of US$21.66 million over a one-year period.

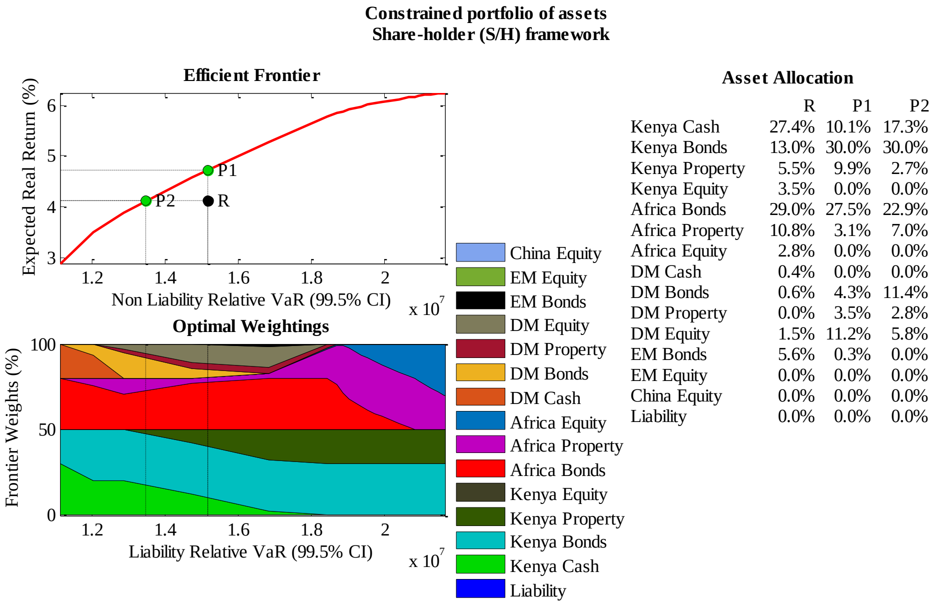


Figure 4.6: *Non liability-relative, constrained asset allocation results displayed in VaR and return.*

As reflected along the upper left component of the horizontal axis of Figure 4.6 the portfolio R and $P1$ yields a VaR of approximately US$15.49 million, whereas the $P2$ portfolio yields a VaR of US$13.48 million. If the re-insurer adopts the optimised $P1$ portfolio as their S/H portfolio, it would deliver an expected real return of 4.72% at the same VaR level as the R portfolio of US$15.18 million. From an explanation perspective, by adopting the $P1$ portfolio, this infers 1/200 times, the re-insurer is likely to lose a maximum amount of US$15.18 million over a one-year period.

As an alternative proposal, if the re-insurer wishes to adopt the optimised $P2$ portfolio as their S/H portfolio, it would deliver an expected real return of 4.12%, at a VaR of US$13.48 million. Once more, from an analysis perspective, this indicates 1/200 times, the re-insurer is likely to lose a maximum amount of US$13.48 million over a one-year period.

To put this into context, if an investors aspiration is to *increase their return* (without compromising the *VaR level*), then $P1$ is a preferred alternative, since in absolute terms, it provides an additional return of +0.60% (4.12% to 4.72%). In relative terms, it provides an additional return of approximately 14%. However, if an investors aspiration is to *decrease their VaR* (without compromising the *return level*), then $P2$ is a preferred alternative, since in absolute terms, this reduces the VaR by US$1.7 million (US$15.18 million to US$13.48 million). In relative terms, it delivers a decrease of approximately 11%.

It is noted that the S/H portfolio of assets contains a shorter range of VaR values, as opposed to the P/H portfolio of assets. This is due to the P/H portfolio of assets comprising 81.88% of the total value of assets (as detailed in Table 3.12), hence the larger range of VaR values for the P/H portfolio.

Table 4.4 summarises the return, risk, and VaR for the R, $P1$, and $P2$ portfolios. Since the asset allocations were revealed to be very similar when considering a *liability-relative* approach (P/H) versus a *non liability-relative* approach (S/H), the expected real returns displayed in Figure 4.4 are virtually the same.

Table 4.4: *P/H and S/H portfolios' expected real return, VaR, and risk.*

	Exp. real return (%)	Risk (%)	VaR (US$), mill.
P/H:			
R	4.12	10.65	69.86
$P1$	4.71	10.64	69.86
$P2$	4.12	9.45	61.95
S/H:			
R	4.12	10.46	15.18
$P1$	4.72	10.45	15.18
$P2$	4.12	9.29	13.48

4.7 Face validation

The notion of mean-variance is a popular quantitative framework (see [65, 71, 87]) used to formulate a portfolio optimisation problem. However, according to the researcher's opinion, there are limited practical applications in the literature and data availability that would allow for a full validation. As noted (§3.1) by Broeders & Jansen [17], one of the reasons for a shortage of studies that analyse investment strategies incorporating *liabilities* is mainly attributed to a scarcity of detailed and comprehensive data. While the existing literature could guide the development of the model framework in some aspects, such as the CMA parameters, and the novel objective functions, the results of a practical asset allocation optimisation problem pertaining to re-insurers within Africa are not well understood and documented. For this reason, input from subject matter experts would ensure that the roadmap of the proposed model framework and the results thereof are consistent with actual industry observed behaviours. Stated differently, face validation ensures that the assumptions, processes, and results of a model framework are reasonable and probable via knowledge from subject matter experts [56, 86]. As a result, the road map of the conceptual model framework and corresponding results was validated at face value by three[2] subject matter experts. This validation process entailed the researcher presenting a presentation to each subject matter expert, covering the following aspects

[2]Permission by the researcher was received by each subject matter expert to disclose their names within this thesis.

1. Providing background and describing the problem.
2. Describing the steps of the roadmap of the conceptual model framework.
3. Describing the liability cash flow data. In addition, the methodology and assumptions pertaining to the CMA formulation were also explained.
4. Discussing the two objective functions and constraints imposed within the optimisation procedure.
5. Carefully explaining the actual results. This included the unconstrained and constrained results.
6. Describing the balance sheet representation (P/H and S/H perspective), coupled with the VaR approach.
7. Describing selected sensitivity analysis.

The questions posed by the researcher to the various subject matter experts and their overall feedback are discussed in each sub-section below (§§4.7.1–4.7.3). A brief summary of the subject matter feedback is provided in §4.7.4.

4.7.1 Subject matter expert I

Since the model framework was developed by RisCura, and applied and modified by the researcher for this study, it is plausible to gain subject matter input and validation from one of the founding researchers and developers. Dr. van Biljon [96], a portfolio manager at RisCura, specialising in the field of portfolio management, assisted by providing input around aspects such as the CMA formulation. In addition, the road map of the conceptual model framework as well as the results displayed in this chapter were presented and discussed with Dr. van Biljon via a virtual MS Teams meeting[3]. The outcome of this validation was that the process undertaken as well as the results produced are reasonable. Examples of the discussion, between the researcher and Dr. van Biljon are listed below.

- For the calculation of the *expected return* and *covariance* for *Kenya property* and *Africa property*, is it reasonable to assume a 50%:50% weighted assumption between bonds and equity, for the respective region, given the lack of data? (see §3.5)

The response was that given the lack of data, and that *property* exhibits both capital (*equity*) and income (*bond*) like characteristics, it is a fair approximation to assume this.

- It is reasonable to assume *Kenya cash* as the choice of the *liability expected return*? (see §3.5)

The response was that given the shorter-term liability cash flow profile, it is reasonable to assume this and it serves as a closer approximation (as opposed to *Kenya bonds*).

From an output perspective, the researcher pointed out that the optimal asset allocation results from a *liability-relative* aspect consisted primarily of *fixed income* allocations (with smaller *equity* allocations) given the shorter liability duration profile. Dr. van Biljon commented that this outcome is anticipated given the shorter liability duration profile, and in the event that the liability profile were longer term in nature, one would expect more *equity* allocations to start featuring within the optimal mix of asset of classes.

[3] 3rd June 2021, one-hour meeting, 14:00–15:00. The MS Teams meeting recording is available upon request.

4.7.2 Subject matter expert II

A second subject matter expect, within the finance field also aided with model validation. Namely Dr. Snyman [89], the chief investment officer at GAIA Group. Once more, the roadmap of the conceptual model framework and the actual results in this chapter were presented and discussed with Dr. Snyman via a virtual MS Teams meeting[4]. The outcome of this validation was that the process embarked upon as well as the results produced resonates with industry standards. In addition, the questions posed by the researcher and *responses* by Dr. Snyman were as follows:

1. Calculation of *Kenya property* and *Africa property* expected return and covariance: since there is no data available for these asset classes, is a weighted estimate of 50%:50% between bonds and equity a reasonable assumption? This assumption is due to property exhibiting characteristics of both equity (capital appreciation) and bonds (rental income)? *"Yes, 50%:50% between bonds and equity is theoretically suitable in the absence of data. An alternative could be to use 25% bonds and 75% equity. The latter is partly due to industry and historical experience."*

2. The expected returns for Kenya are measured in KES, and the expected returns for the foreign (including Africa) asset classes are measured in USD. Instead of converting the Kenya expected returns to USD (as this would involve forecasting the relationship between the currency, thus introducing more complexity and potential error within the model), is it suitable to employ the assumption of purchasing power parity (PPP)? *i.e.*, the Kenya expected returns are measured in KES, whereas the foreign (including Africa) asset classes are measured in USD? *"Yes, using the assumption pertaining to PPP is reasonable given its simplicity."*

3. Does the logic of the model framework described, provide a reasonable indication of the common approach to solve an asset allocation problem? *"Yes, the approach of the model framework is common among industry related to asset allocation."*

4. Does the corresponding results presented, provide a reasonable indication of typical industry outputs? *"Yes, the outputs presented are reasonable and in-line with industry outputs."*

On the response to question 1, while the weighted estimate of 50%:50% is noted to be theoretically suitable in the absence of data and has been used for the results of the case study, the suggestion of 25% bonds and 75% equity for *Kenya property* and *Africa property* will be partly addressed in the form of a sensitivity analysis in Appendix B.

4.7.3 Subject matter expert III

A third subject matter expert within the investment field was also consulted. Namely, Mr. Lambridis [59], the principal of Axia-Investors with over 10 years of experience in multi-asset work, most recently with Prudential Investment Managers. The model framework and results were presented and discussed with Mr. Lambridis via two virtual MS Teams meetings[5]. Once more, the outcome of this validation was that the process undertaken as well as the results produced are reasonable and in-line with industry standards. Amongst the key questions the researcher posed to Mr. Lambridis, were as follows:

[4] 23rd June 2021, one hour meeting, 16:00–17:00. Since this MS Teams meeting was not recorded, all questions posed by the researcher to Dr. Snyman, and responses are in email format, and is available upon request.

[5] 1st July 2021 and 5th July 2021, respectively. Both meetings scheduled for 45 minutes each, 12:30–13:15. The MS Teams meeting recordings are available upon request.

1. Does the logic of the model framework described, provide a reasonable indication of the common approach to solve an asset allocation problem?

2. Does the corresponding results presented, provide a reasonable indication of typical industry outputs?

The responses to both these questions were "Yes".

4.7.4 Subject matter expert overview

In summary, the viewpoints and outcome from all three subject matter experts provided a common theme, *i.e.*, the outcome of this validation was that the process undertaken as well as the results produced are reasonable and are in-line with industry observed trends. This provides support and credence that the model framework approach and the corresponding results of the case study are consistent with industry observed outcomes. This serves as the face validation component for the roadmap and results of the case study in question. The sensitivity analysis of §5 serves to support and supplement the validation component. In addition, §5 also serves to provide reasonability checks with respect to key input parameters.

4.8 Chapter summary

The chief objective of this chapter was to present and examine the portfolio optimisation results for the case study in question.

The results of the unconstrained asset allocation revealed that a smaller array of asset classes, and consequently, a less diverse range of asset classes featured within the optimal portfolio of assets. One of the most notable features of the unconstrained optimal area graph was the optimisers preference to include a large amount of *Kenya bonds* within the optimal asset allocation. While this result was largely anticipated given the attractive return profile of *Kenya bonds*, the dominance of one asset class leads to concentration risk and lack of portfolio diversification opportunities. In addition, *foreign-DM cash* and *foreign-DM bonds* also featured noticeably within the optimal asset allocation. This was attributed by the low risk profile, and higher correlation profiles of *foreign-DM cash* and *foreign-DM bonds* relative to the liability, thus supporting inclusion of these two asset classes.

In light of this, suitable constraints were incorporated within the optimisation procedure so as to moderate the asset allocations. As a result of incorporating constraints within the optimisation procedure, the constrained asset allocation advocated a much more diversified and wider array of asset classes contained within the optimal area graph.

Furthermore, the results of the constrained *liability-relative* and *non liability-relative* asset allocations were revealed to be very similar. An additional feature of the results was the optimisers preference to primarily include fixed income and property type of asset classes. Despite the optimiser including smaller allocations to equity type of asset classes, the optimisers preference to primarily include fixed income asset classes was largely attributed to the relatively short duration profile of the liability cash flows. Stated differently, larger allocations of fixed income and property asset classes and smaller allocations of equity asset classes would best ensure the objective of the liability payments are met by the re-insurer. An additional reason for the similarity exhibited amongst the asset allocation was the constraints imposed within the optimisation procedure. Essentially, the nature of the constraints may have resulted in the optimiser being compelled to allocate to alternative asset class once a constraint on an asset class was attained.

Thereafter, the constrained asset allocation results were expressed in a *VaR* and *return* framework. By incorporating *VaR* not only allows a practitioner to express risk in a more tangible manner, but to aid in quantifying the *risk* on a probability and monetary basis too.

In addition, the investment strategy was presented by incorporating the balance sheet representation. This was achieved by separating the investment strategy into a P/H (*liability-relative*) and S/H (*non liability-relative*) component. Two optimised portfolios that target an improved *return* profile (without sacrificing risk (VaR)), as well as an optimised portfolio that target an improved *risk (VaR)* profile (without sacrificing return), respectively were proposed.

The chapter closed with a description of the model validation process. The outcome of this face validation was that the process undertaken as well as the results produced are reasonable and are in-line with industry observed trends.

CHAPTER 5

Case Study: Sensitivity Analysis

Contents

This chapter opens in §5.1 with a brief overview detailing the rationale of a sensitivity analysis and the parameters varied for the case study. Thereafter, results of the sensitivity analysis are presented in §5.2–5.3 to gauge the level of impact alternative parameters have on the optimal asset allocation results used for this case study. This chapter is rounded off with sensitivity analysis pertaining to interest rate shocks as furnished in §5.4.

5.1 Sensitivity analysis

As with any mathematical model framework, parameter estimates, input data, and assumptions are susceptible to change and potential error, since simplifying assumptions are made to suitably resemble a part of a real-world procedure. By its nature, model frameworks and their respective parameters are at best an approximation of a real-world phenomena, thus introducing uncertainty [100].

The procedure of studying the impact potential changes may have on outputs of a mathematical model framework is referred to as a *sensitivity analysis* [60]. The chief aim of a sensitivity analysis may aid a practitioner to make decisions and recommendations of a model more credible, understandable, and reliable [72]. Uncertainty is one of the primary drivers why sensitivity analysis is advantageous to aid in making decisions and recommendations [72, 101]. This renders sensitivity analysis as a useful tool to gauge how the model behaves under certain conditions. The sensitivity analysis may be viewed as a so-called stress or shock test on the parameters of a model framework. According to Christopher & Patil [26] a sensitivity analysis is useful to undertake model validation. For the purpose of this thesis, the sensitivity analysis serves to support with model validation, since face validation was embarked upon in §4.

Since the results of the asset allocation are largely driven by the *expected return* assumptions and *constraints* incorporated within the optimisation procedure, it seems plausible to conduct sensitivity analysis on these parameters, from an asset perspective.

From a liability perspective, the present value amount is largely driven by the assumption around the *interest rate* (yield curve) parameter. So, a sensitivity analysis pertaining to an alternative set of interest rates would be useful to discern how the present value would change. The parameter changes to *expected returns*, *constraints*, and *interest rates* will be elaborated upon further below.

1. **Expected returns**: The expected returns proposed in Table 3.10 are not exact forecasts, but instead, represent *estimates* of future returns, over a one-year period. An estimate by its nature consists of assumptions that may vary. In addition, studies by [9, 11, 23, 68] has indicated that optimisers are sensitive to the input assumptions around expected returns. For this reason, a sensitivity analysis around expected returns would be helpful to assess how the optimal asset allocation would possibly change. The one-year expected returns presented in Table 3.10 range from the lowest value of -1.91% to the highest value of $+6.92\%$.

   To study the sensitivity thereof, a single expected return was shocked, by an increase of $+1.5\%$, $+1\%$, $+0.5\%$, and a decrease of -0.5%, -1%, -1.5%, respectively. This ranges reflect possible outcomes of alternative expected returns over a one-year period. Stated differently, an increase of $+1.5\%$ reflects an optimistic outlook and an "aggressive" estimate for the asset class. Whereas, a decrease of -1.5% reflects a pessimistic outlook and a "conservative" estimate for the asset class. Essentially, by increasing a single expected return, renders the asset class in question, to appear *more* "attractive", relative to all other asset classes. Conversely, decreasing a single expected return, renders the asset class in question, to appear *less* "attractive", relative to all other asset classes. This is done for all 14 asset classes, using the constrained efficient frontier, and in a *liability-relative* framework, respectively.

2. **Constraints**: The constraints proposed in §3.7 were primarily set on the basis of assumptions, with the intent to moderate and diversify asset class exposure. Therefore, the constraint parameters applied within the optimisation procedure should be varied to test the impact alternative limits has on the area graph of the optimal asset allocation. This is done by varying the weights of constraint sets (3.7)–(3.9) to reflect *more* and *less* restrictive outcomes to test the robustness of the optimiser. The specific details pertaining to the changes will be elaborated upon in §5.3.

3. **Interest rates**: To compute the present value of the liability cash flow profile, a US nominal bond curve was used as the interest rate assumption. However, interest rates by its nature are subject to fluctuate given market conditions. For this reason, the interest rates are subjected to positive and negative shocks that express a set of optimistic and pessimistic changes, respectively. The shocks, ranges from -2% to $+2\%$, in increments of 0.4%, respectively, reflect alternative and possible ranges in the movement of the US yield curve, over a one-year period. The shocks will examine the impact on the present value.

If the model output is robust (less sensitive to changes in parameter values), assurance in the recommendation and model results are increased. Conversely, if the model output is lesser robust (more sensitive to changes in parameter values), sensitivity analysis may be used to assess the risk associated in implementing a suggested strategy [72].

5.2 Expected return sensitivity on optimised asset allocation

Figures 5.1–5.4 examines the impact of shocking the expected returns by $+1.5\%$, $+1\%$, $+0.5\%$, -0.5%, -1%, -1.5%, respectively. In addition, the unchanged expected return, referred to as "unchanged" is also displayed. The results are produced using the *constrained efficient frontier*, from a *liability-relative* perspective. Figures 5.1–5.4 are divided into four quadrants. The upper left shows the impact the shock has on the movement of the efficient frontier, plotted in terms of *risk* and *return*. The lower left shows the impact the shock has on the optimal asset allocation, plotted in terms of an *optimal area graph* and *risk* setting. The upper right displays the *legend* denoting the level of shock for the asset class under examination. The lower right reflects the *amount*, in *percentage terms* the optimiser allocates to the asset class under examination. The horizontal axis of the lower right component displays the optimal range of *risk levels*, whereas the vertical axis displays the level of the *shocked expected return*. The acronym *ER* contained in the lower right quadrant denotes *expected return*.

Figure 5.1 displays the results of the sensitivity analysis for *Kenya bonds*. The upper left reveals that from an efficient frontier perspective, as the expected return is increased, the entire efficient frontier raises slightly upward. Similarly, as the expected return is decreased, the entire efficient frontier shifts slightly downward. The lower left and right of Figure 5.1 reveals that given most expected return shocks, and along most risk levels, the optimiser tends to allocate the maximum amount (30%) to *Kenya bonds*. This is primarily due to the attractive risk and return profile of *Kenya bonds*, hence the prominence of this asset class. The prominence of *Kenya bonds* was seen in the unconstrained optimal portfolio of assets (Figure 4.1).

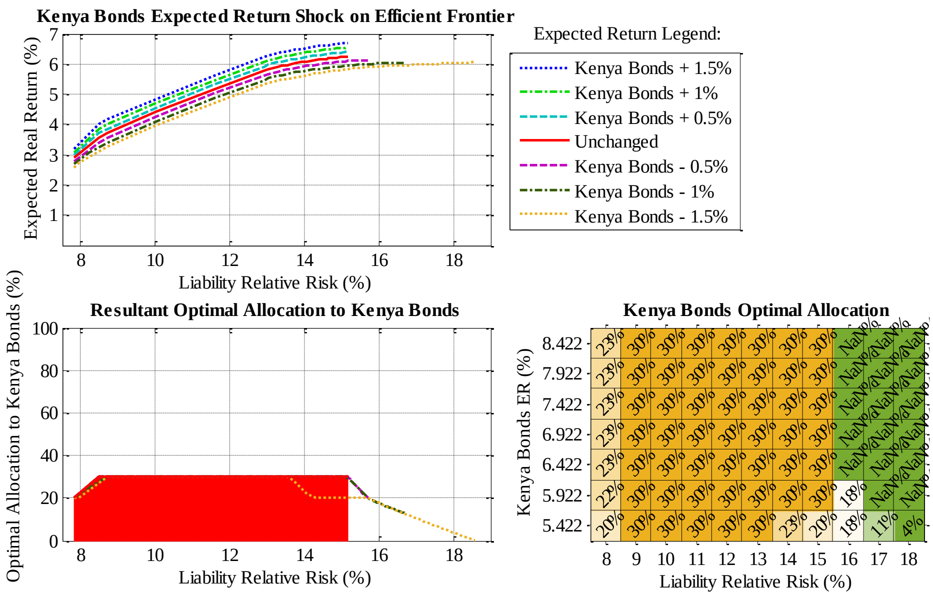


Figure 5.1: *Kenya bonds expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

If the expected return is reduced to its lowest pessimistic shock of -1.5% (*i.e.,* 5.42%), and only at higher risk levels between 14% to 18%, the optimiser starts to reduces its allocation to *Kenya bonds*. Although the *Kenya bonds* allocation still remains within a moderate range at the expected return shock of -1.5% (*i.e.,* 5.42%), the optimiser deems it as slightly less attractive to consistently attain its maximum, since this specific level of decrease in expected return is too low given the risk profile. The "NaN" seen at risk levels between 16% and 18% is not unexpected since the efficient frontier does not extend beyond these higher risk levels for the expected return shock in question, hence no values are present at these risk levels. In short, irrespective of the range of expected return shock applied, the optimiser tends to attain its maximum allocation to *Kenya bonds* along most risk levels, given the attractive risk and return profile of this asset class.

Figure 5.2 presents the results of the sensitivity analysis for *Africa equity*. The upper left reveals from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises slightly upward, at higher risk levels. This is due to the high risk and return profile associated with *Africa equity*, hence the efficient frontier raising upward slightly. As the expected return is decreased, the efficient frontier shifts immaterially downward, at higher risk levels. The lower right of Figure 5.2 reveals that as the level of expected return shock is increased (*i.e.*, +0.5%, +1%, and +1.5%, respectively), so does the allocation to *Africa equity* gradually increase. This is supported by the higher risk and return profile that *Africa equity* exhibits, hence larger allocations are more prominent toward higher risk and return levels. At expected return shocks of −0.5%, −1%, and −1.5%, and at lower risk levels of 8%–12%, respectively, the optimiser allocates a zero weight to *Africa equity*. In light of the allocations reflecting a zero weight under these reduced expected return shocks, this explains why the efficient frontier curves is largely unmoved, displayed in the upper left component of Figure 5.2. Furthermore, the risk profile for this asset class is relatively high, therefore, the lowered expected return shocks renders this asset class as unappealing for the level of risk on offer, hence the zero weight. Simply stated, the lowered expected return shocks does not justify inclusion given the higher risk profile an investor is expected to incur for *Africa equity*.

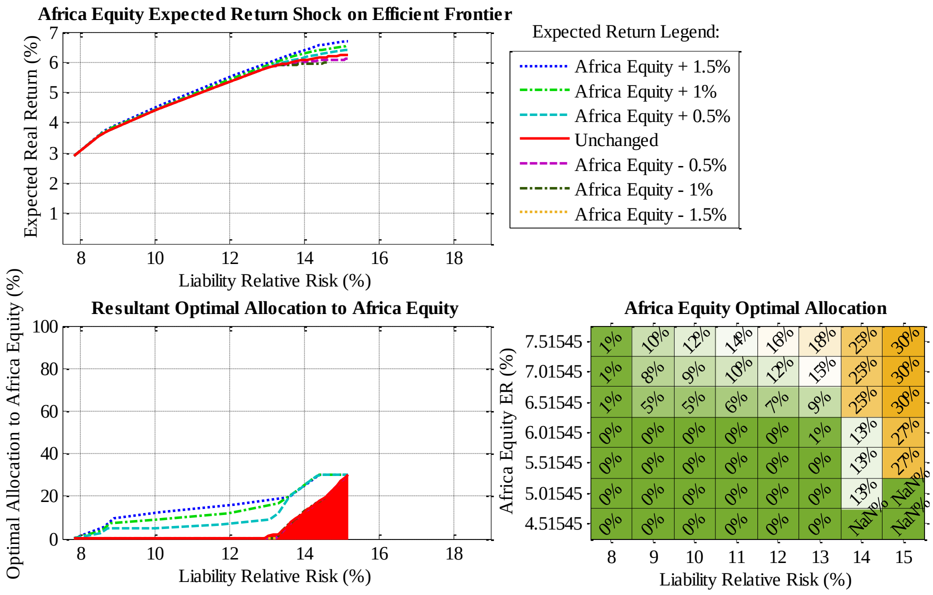


Figure 5.2: *Africa equity expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

In light of the similarity of *foreign-DM cash*, and *foreign-DM bonds*, from a risk and return perspective, this will concurrently be examined. Figures 5.3–5.4 presents the results of the sensitivity analysis for *foreign-DM cash* and *foreign-DM bonds*. The upper left reveals that from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises marginally upward. The movement is primarily seen at lower risk and return levels. This is expected since these two asset classes exhibits a very low risk and return profile. However, as the expected return is decreased, the efficient frontier shifts immaterially downward. Once more, the movement is largely seen at very low risk and return levels.

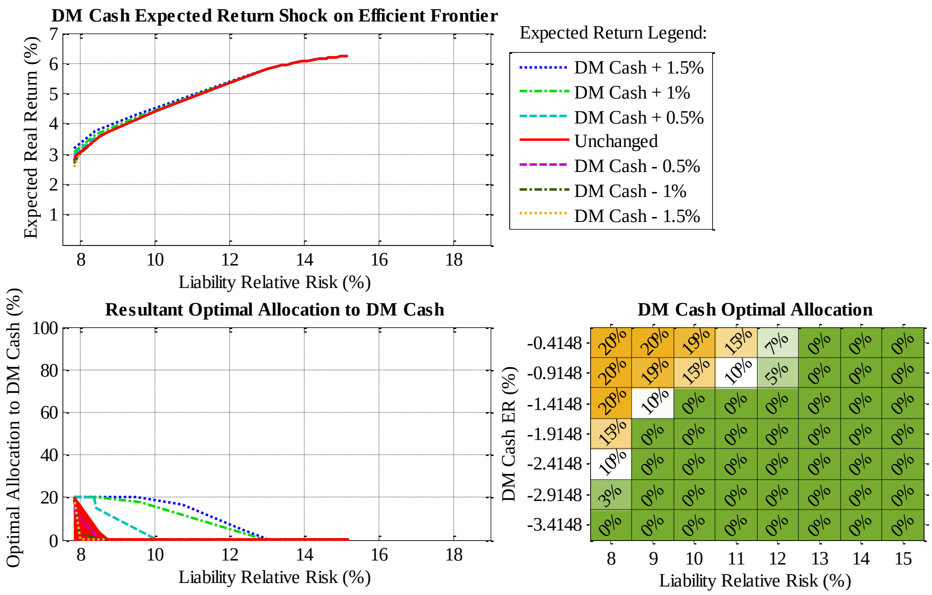


Figure 5.3: *Foreign-DM cash expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

The lower right of Figures 5.3–5.4 shifts the focus to the percentage amount of these two asset classes the optimiser deems optimal at varying risk levels. Since these two asset classes exhibits a very low risk and (negative) return profile, non-zero allocations are mostly seen at lower risk levels and gradually decrease to zero as the level of risk increases. Stated differently, allocations to *foreign-DM cash* and *foreign-DM bonds* are predominantly observed at higher expected return shocks since these shocks renders these two asset classes as more attractive, given their lower risk profile. The optimiser allocates zero weightings to *foreign-DM cash* at moderate to high risk levels of 13% to 15%, irrespective of the expected return shock applied. Similarly, the optimiser allocates zero weightings to *foreign-DM bonds* at moderate to high risk levels of 14% and 15%, irrespective of the expected return shock applied. At reduced expected return shocks (*i.e.*, more negative), the allocations are mostly zero. Despite the risk profile appearing attractive (*i.e.*, low risk), the expected return shocks are merely too low to justify allocations to these asset classes.

For ease of readability, 4 of the 14 asset class expected return sensitivity analyses are presented in this section. For the alternative 10 asset classes, the same analysis, accompanied with an examination of the results are provided in Appendix B.

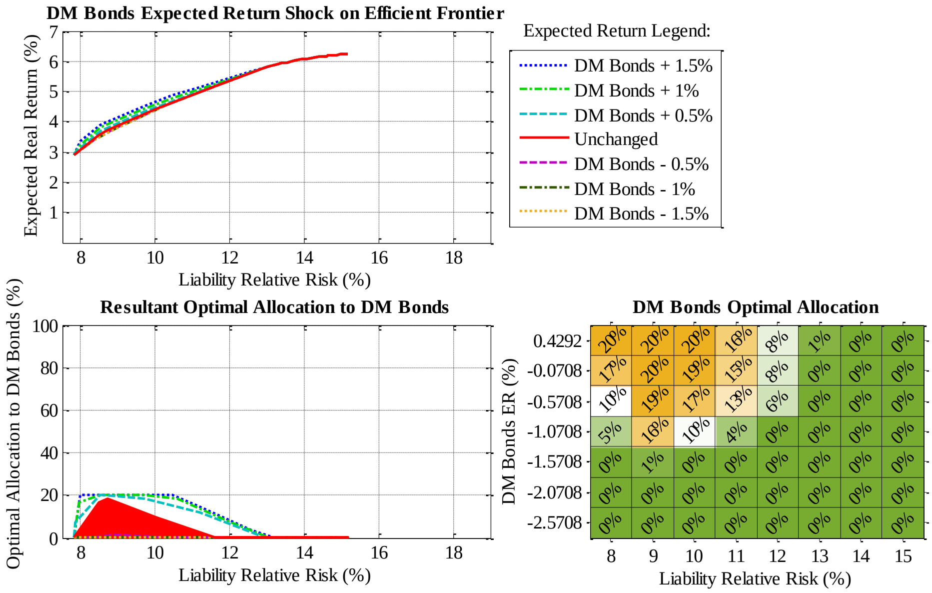


Figure 5.4: *Foreign-DM bonds expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

5.2.1 Expected return sensitivity on optimised portfolios

Tables 5.1–5.4 displays the asset allocation impact on two optimised portfolios, namely $P3$ and $P4$. The $P3$ and $P4$ denote alternative optimised portfolios targeting the same level of *risk*, and *return* for the R portfolio, respectively. $P1$ represents an optimised portfolio targeting the same level of risk as R, whereas $P2$ represents an optimised portfolio targeting the same level of return as R, both using the normal (unchanged) expected returns. To make suitable deductions, Tables 5.1–5.4 compare the differences between the two original optimised portfolios, namely $P1$ and $P2$, as initially displayed in Table 4.2. This will be analysed for the $+0.5\%$ and -0.5% expected return shock only, given that this represents a less "optimistic" and "pessimistic" sensitivity, over a one-year period.

Table 5.1 displays the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *Kenya bonds*. Given an increase in expected return, the $P3$ portfolio is identical to the $P1$ portfolio (*i.e.*, the difference between portfolios $P3$ and $P1$ are zero). This indicates that an increase of $+0.5\%$ to the expected return of *Kenya bonds* has no impact on the $P3$ portfolio. This is mainly in light of the optimiser having already reached its maximum constrained amount of 30% to *Kenya bonds*. Upon observation of $P4$, the differences between $P4$ and $P2$ are marginal. While the allocation to *Kenya bonds* in $P4$ is once more unchanged, immaterial movements are seen amongst the alternative asset classes.

The bottom of Table 5.2 shifts the focus to the decrease in expected real return. Given a decrease in expected return to *Kenya bonds*, the $P3$ portfolio is once more revealed to be identical to the $P1$ portfolio. This suggests that at a lowered expected return level for *Kenya bonds*, the optimiser still finds this asset class as an optimal and attractive asset class, from a risk and return profile perspective. It is once more observed that marginal changes are discerned with the $P4$ portfolio (compared to the $P2$ portfolio).

Table 5.2 highlights the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *Africa equity*. If the expected real return for *Africa equity* is increased, the *Africa equity* allocation is slightly increased. To compensate for the increase in allocation to *Africa equity*, the optimiser reduces its allocation to the alternative African asset classes, namely *Africa bonds* and *Africa property*, so as to carefully balance risk and return. Smaller shifts are observed with alternative asset classes. Decreasing the expected real return of *Africa equity* results in no change to the $P3$ portfolio and insignificant changes to the $P4$ portfolio. Since the $P1$ and $P2$ portfolio already had 0% and 0.13%, respectively allocated to *Africa equity*, decreasing the expected return of this asset class results in *Africa equity* appearing less attractive, hence the changes to the $P3$ and $P4$ portfolio are immaterial. To summarise, an increase in expected return renders *Africa equity* as more attractive, hence the optimiser allocates to this asset class. Whereas, a decrease to the expected return in *Africa equity* suggests that the lower return on offer does not justify the level of risk an investor would need to incur.

Table 5.3 presents the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *foreign-DM cash*. The $P3$ portfolio is unchanged. This suggests that the increase in return is still not sufficient enough to trigger a positive non-zero allocation to *foreign-DM cash* at these levels.

Under the $P4$ portfolio, increasing the expected return to *foreign-DM cash* results in the optimiser allocating a non-zero weight of 5.16% to this asset class (this was zero, prior to the increase in expected return). The increase observed to *foreign-DM cash*, is largely at the expense of a decrease to *foreign-DM bonds*. Since these two asset classes exhibit very similar low risk and return characteristics, the switch between *foreign-DM cash* and *foreign-DM bonds* is largely expected to balance risk and return. Marginal changes are seen with alternative asset classes. If the expected return is decreased for *foreign-DM cash*, the $P3$ portfolio is unchanged, with *foreign-DM cash* still containing a zero allocation. The changes for the $P4$ portfolio are trivial. Since this asset class already exhibits a negative expected return, reducing the expected return (*i.e.*, more negative), renders the asset class as less attractive, hence almost no change to the composition of the asset classes as seen with portfolio $P3$ and $P4$.

Table 5.4 shows the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *foreign-DM bonds*. If the expected real return is increased, a notable increase in *foreign-DM bonds* is seen. This increase is primarily funded by alternative foreign asset classes, such as, *foreign-DM equity* and *foreign-EM bonds*. The latter exhibiting similar correlation qualities to *foreign-DM bonds*, given that these are both bond asset classes. If the expected real return of *foreign-DM bonds* is decreased, the allocation to *foreign-DM bonds* reduces to zero for $P3$. For $P4$, the allocation to *foreign-DM bonds* is significantly reduced. The decrease observed is off-setted by an increase in the *foreign* component, such as *foreign-EM bonds*, a related bond asset class. Similar to *foreign-DM cash*, since this asset class exhibits a negative expected return, reducing the expected return (*i.e.*, more negative), renders the asset class as less attractive, hence the optimisers preference to significantly reduce its allocation to *foreign-DM bonds*.

To allow for ease of readability, 4 of the 14 asset class expected return sensitivity analyses are provided in this section. The same analysis, for the alternative 10 asset classes, coupled with a detailed examination thereof, are furnished in Appendix B.

Table 5.1: *Sensitivity analysis: Kenya bonds expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Kenya bonds + 0.5% (7.42%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	8.66	16.44	-	+0.96
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	3.56	-	−0.96
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.32	-	−0.65
Africa property	2.92	4.91	2.92	5.52	-	+0.61
Africa equity	-	0.13	-	0.17	-	+0.04
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	14.23	-	+1.08
DM property	2.71	2.52	2.71	2.36	-	−0.16
DM equity	8.05	4.33	8.05	3.41	-	−0.92
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.28	-	−0.16
expected real return	4.71	4.12	4.86	4.18	+0.15	+0.06
Kenya bonds − 0.5% (6.42%)						
Kenya cash	8.66	15.48	8.66	14.53	-	−0.96
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	5.47	-	+0.96
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	25.61	-	+0.65
Africa property	2.92	4.91	2.92	4.30	-	−0.61
Africa equity	-	0.13	-	0.09	-	−0.04
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	12.07	-	−1.08
DM property	2.71	2.52	2.71	2.68	-	+0.16
DM equity	8.05	4.33	8.05	5.25	-	+0.92
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.61	-	+0.16
expected real return	4.71	4.12	4.56	4.05	−0.15	−0.06

Table 5.2: *Sensitivity analysis: Africa equity expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Africa equity + 0.5% (6.52%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	9.58	15.69	+0.91	+0.20
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	10.42	4.31	−0.91	−0.20
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	27.01	25.07	−1.30	+0.11
Africa property	2.92	4.91	-	-	−2.92	−4.91
Africa equity	-	0.13	5.39	4.93	+5.39	+4.80
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	6.16	13.52	+0.49	+0.37
DM property	2.71	2.52	2.72	2.42	+0.01	−0.10
DM equity	8.05	4.33	4.47	2.63	−3.58	−1.71
EM bonds	2.34	-	4.24	1.44	+1.90	+1.44
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real return	4.71	4.12	4.73	4.13	+0.01	+0.01
Africa equity − 0.5% (5.52%)						
Kenya cash	8.66	15.48	8.66	15.64	-	+0.16
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.36	-	−0.16
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.73	-	−0.24
Africa property	2.92	4.91	2.92	5.27	-	+0.37
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.32	-	+0.17
DM property	2.71	2.52	2.71	2.50	-	−0.02
DM equity	8.05	4.33	8.05	4.18	-	−0.15
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.42	-	−0.03
expected real return	4.71	4.12	4.71	4.10	-	−0.01

Table 5.3: *Sensitivity analysis: DM cash expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

DM cash + 0.5% (−1.41%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	8.66	14.90	-	−0.58
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	5.10	-	+0.58
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	25.85	-	+0.89
Africa property	2.92	4.91	2.92	4.15	-	−0.76
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	5.16	-	+5.16
DM bonds	5.67	13.15	5.67	7.94	-	−5.21
DM property	2.71	2.52	2.71	2.01	-	−0.51
DM equity	8.05	4.33	8.05	4.89	-	+0.56
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real return	4.71	4.12	4.71	4.12	-	-
DM cash − 0.5% (−2.41%)						
Kenya cash	8.66	15.48	8.66	15.51	-	+0.02
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.49	-	−0.02
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.95	-	−0.02
Africa property	2.92	4.91	2.92	4.92	-	+0.01
Africa equity	-	0.13	-	0.13	-	-
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.17	-	+0.03
DM property	2.71	2.52	2.71	2.51	-	-
DM equity	8.05	4.33	8.05	4.31	-	−0.02
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real return	4.71	4.12	4.71	4.11	-	-

Table 5.4: *Sensitivity analysis: DM bonds expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

DM bonds + 0.5% (−0.57%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	3.21	12.75	−5.45	−2.74
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	16.79	7.25	+5.45	+2.74
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.60	24.85	+0.29	−0.11
Africa property	2.92	4.91	1.66	3.78	−1.26	−1.13
Africa equity	-	0.13	1.68	1.37	+1.68	+1.24
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	14.34	18.83	+8.67	+5.68
DM property	2.71	2.52	2.41	1.17	−0.30	−1.35
DM equity	8.05	4.33	1.32	-	−6.73	−4.33
EM bonds	2.34	-	-	-	−2.34	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.29	-	−0.15
expected real return	4.71	4.12	4.76	4.12	+0.05	-
DM bonds − 0.5% (−1.57%)						
Kenya cash	8.66	15.48	11.38	17.41	+2.72	+1.93
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	8.62	2.59	−2.72	−1.93
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	25.22	24.68	−3.09	−0.28
Africa property	2.92	4.91	5.60	5.32	+2.68	+0.41
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	6.27	-	+6.27
DM bonds	5.67	13.15	-	0.70	−5.67	−12.44
DM property	2.71	2.52	1.49	0.56	−1.22	−1.95
DM equity	8.05	4.33	8.14	6.11	+0.09	+1.78
EM bonds	2.34	-	9.55	6.35	+7.21	+6.35
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.52	-	+0.08
expected real return	4.71	4.12	4.71	4.11	-	-

5.3 Constraint sensitivity analysis

The focus of §§5.2–5.2.1 applied positive and negative shocks to the expected returns to identify how this impacted the optimal portfolio of assets and optimised portfolios, respectively. The focal point of §5.3 considers varying the parameters imposed on constraint sets (5.1)–(5.3) to examine the impact this has on the optimal area graph. The expected returns and risk remain unchanged as listed in Table 3.10.

For the results presented and examined in §4.3, the parameter, M_1, as seen in constraint set (5.1) were set to **20%** (total foreign constraint), M_2, as seen in constraint set (5.2) were set to **50%** (total Kenya constraint), and finally M_3, as seen in constraint set (5.3) were set to **30%** (maximum single asset class constraint). The movement of the efficient frontiers as well as the optimal area graphs will be presented and examined.

$$(x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}) \leq M_1, \tag{5.1}$$
$$(x_1 + x_2 + x_3 + x_4) \leq M_2, \tag{5.2}$$
$$x_i \leq M_3, \qquad \forall i = 1, \ldots, 14. \tag{5.3}$$

1. To measure the robustness and impact the *foreign* constraint has of the optimiser, the parameter M_1 will undergo a change comprising of $M_1 \in \{10\%, \mathbf{20\%}, 30\%, 40\%, 50\%\}$ respectively. Constraint sets (3.7) and constraint sets (3.9)–(3.13) will be included and remain unchanged.
2. Similarly, the parameter M_2 will undergo a change comprising of $M_2 \in \{30\%, 40\%, \mathbf{50\%}, 60\%, 70\%\}$ respectively. All constraint sets (3.8)–(3.13) will be included and remain unchanged.
3. Similarly, the parameter M_3 will undergo a change comprising of $M_3 \in \{20\%, 25\%, \mathbf{30\%}, 35\%, 40\%\}$ respectively. All constraint sets (3.7)–(3.8) and constraint sets (3.10)–(3.13) will be included and remain unchanged.

Figure 5.5 shows the impact of the efficient frontier when applying alternative limits to the total foreign weighting constraint set (5.1). Figure 5.5 reveals that as the limit imposed on the foreign component is increased, the length, at lower risk and return levels of the efficient frontier gradually becomes longer. Stated differently, the efficient frontier shortens as the limit imposed on the foreign component is decreased. It is noted that irrespective of the total foreign restriction being set to 10%, 20%, 30%, 40%, or 50%, the *maximum* expected real return on offer for all these restrictions is 6.25%.

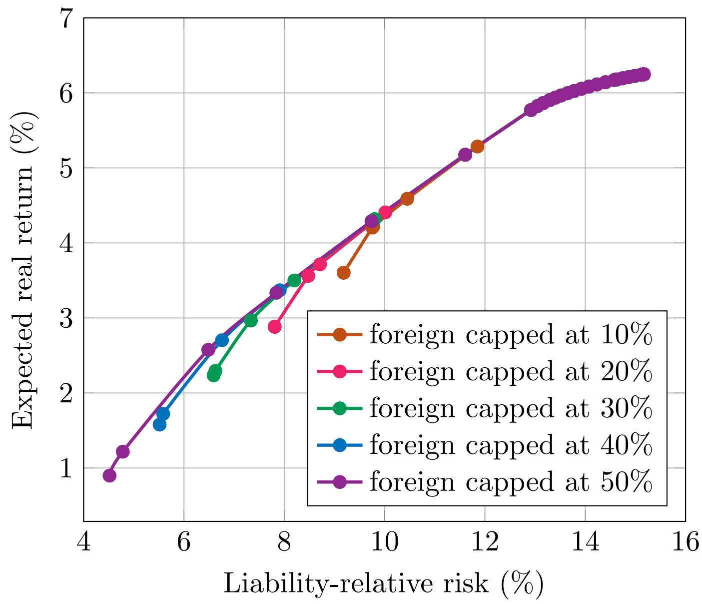


Figure 5.5: *Sensitivity analysis conducted on alternative caps pertaining to the foreign constraint, ranging from 10% to 50%, in increments of 10%, measured in risk and return terms.*

To examine this further, Figure 5.6 shows the optimal area graphs for the various limitations imposed on the total foreign constraint. While the Kenya allocation remains very similar for all variants, the foreign constraint appears to reach its maximum limit at lower risk levels. The lower risk foreign asset classes primarily consist of *foreign-DM cash* and *foreign-DM bonds*. Since these two asset classes exhibit a very low risk and return profile, the optimisers inclusion is more prominent at lower risk and return levels. In addition, the higher correlation of *foreign-DM cash* and *foreign-DM bonds* relative to the liability, supports the inclusion of these two asset classes to consistently attain its maximum limit at lower risk levels. It is further noted that as the foreign constraint is relaxed (*i.e.*, less restrictive), the allocation to *foreign-DM cash* and *foreign-DM bonds* tends to increase along the optimal area graphs. Since the allocation to *foreign-DM cash* and *foreign-DM bonds* tends to increase as the foreign constraint is relaxed, this results in the allocation to *Africa bonds* decreasing. Since *Africa bonds* exhibits a moderate to higher risk and return profile, smaller allocations of this asset class contributes to the decrease in the optimal lowest risk point observed in Figure 5.5.

At moderate and centred risk levels, the optimiser prefers *foreign-DM property*, *foreign-DM equity*, and *foreign-EM bonds*. As noted earlier, these 3 asset classes differ in terms of the "type" of asset class *i.e.*, property, equity and bonds, respectively. This represents a less correlated range of asset classes, hence the optimisers preference to include these 3 different asset classes, to aid in diversifying the portfolio, and to carefully balance risk and return. At moderate to higher risk levels, the allocation to foreign gradually decreases until it no longer features. Instead, *Africa property* and *Africa equity* starts to feature given that these two asset classes exhibit a higher risk and return profile, respectively. Hence inclusion thereof is more prominent at moderate to higher levels. The *maximum* risk and return asset allocations are identified to be consistent amongst all variants. The asset allocations, read off from the right most end-point of the optimal area graph depicted in Figure 5.6 consists of *Africa property* (20%) and *Africa equity* (30%), and *Kenya bonds* (30%) and *Kenya property* (20%), hence at the highest risk level of 15.16%, the expected real return for all variants amounts to 6.25% as shown in Figure 5.5, irrespective of the foreign constraint applied.

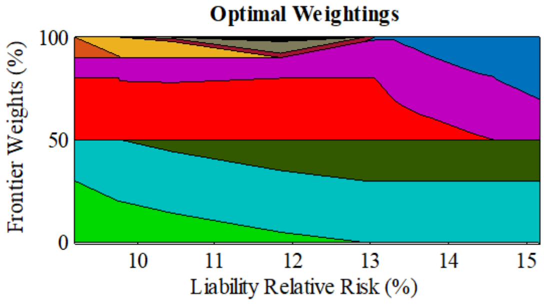
For ease of readability, 1 out of the 3 constraint sensitivity analyses are presented in this section. For the alternative 2 constraint sensitivity analyses, the same analysis, accompanied by an examination of the results are provided in Appendix C.

5.4 Interest rate sensitivity analysis

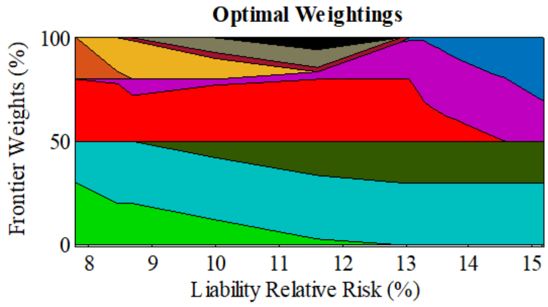
The sensitivity analysis results of §5.2–5.3 focused on the *asset* component of the model framework. The theme of this section showcases sensitivity analysis pertaining to the *liability* component of the model framework. Since the result of the present value computed in §3.3 is dependent on the term-structure of the U.S. nominal bond curve, this yield curve will be subjected to positive and negative shocks as presented in Table 5.5. The shocks range from −2% to +2%, in increments of 0.4%, reflect possible shocks in the movement of the U.S. yield curve, over a one-year period.

The *0.0% shift* in Table 5.5 represents the present value when no shock is applied to the yield curve *i.e.,* the original U.S. yield curve displayed in Figure 3.4. The +2.0% shock implies that all interest rates along the U.S. yield curve are increased by 2%. Whereas the −2.0% shock implies that all interest rates rates along the yield curve are decreased by 2%.

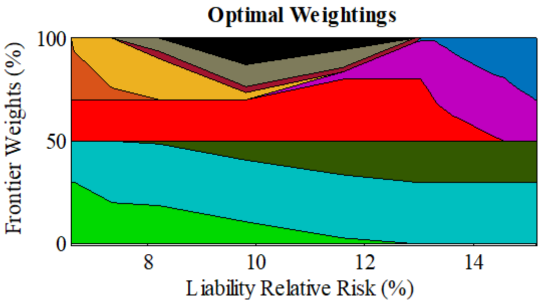
A trend from Table 5.5 may be observed. As the interest rate shocks are increased, the present value decreases. Conversely, as the interest rate shocks are decreased, the present value increases. This inverse relationship is consistent with the theory [18, 34]. The percentage differences are calculated by measuring the percentage change of the current shock versus when no shock is



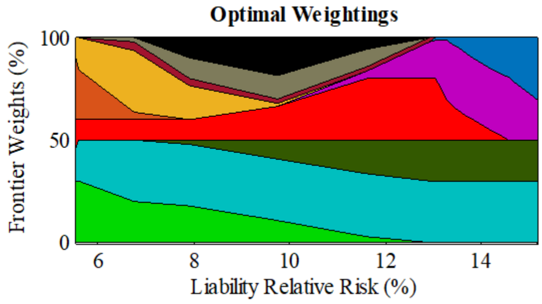
(a) *Total foreign limited to 10%.*



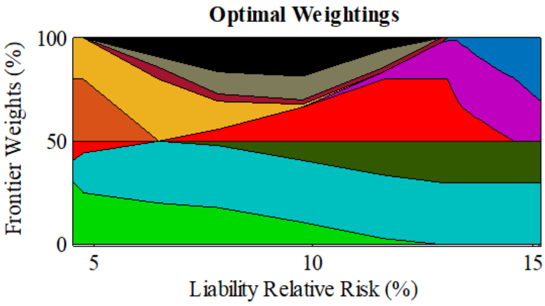
(b) *Total foreign limited to 20%.*



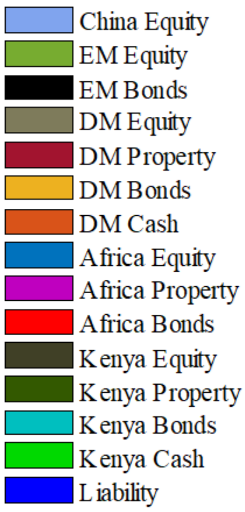
(c) *Total foreign limited to 30%.*



(d) *Total foreign limited to 40%.*



(e) *Total foreign limited to 50%.*



(f) *Colour palette legend.*

Figure 5.6: *Sensitivity analysis displayed as optimal area graphs for the foreign limits, ranging from 10% to 50%, in increments of 10%.*

Table 5.5: *Interest rate shocks and the resultant impact on present value, from a monetary and percentage difference perspective.*

Shock to yield curve (%)	PV (US$), mill.	difference (%)
−2.0% shift	132,002,066	−3.6
−1.6% shift	131,029,935	−2.8
−1.2% shift	130,075,313	−2.1
−0.8% shift	129,137,728	−1.4
−0.4% shift	128,216,721	−0.7
0.0% shift	*127,311,853*	-
+0.4% shift	126,422,698	+0.7
+0.8% shift	125,548,842	+1.4
+1.2% shift	124,689,890	+2.1
+1.6% shift	123,845,456	+2.8
+2.0% shift	123,015,169	+3.5

applied (*i.e., 0.0% shift*). It is observed that the present values do not materially change as the level of shock varies. This is largely due to the relatively short duration of the liability cash flows, causing less of an impact to the actual present value amount.

It should be noted that under these assumptions, this indicates that should the U.S. nominal bond curve change within these boundary conditions, over a one-year period, the present value would remain within an approximate range of ±3.5%. In the event that the duration of the liability cash flows were longer term in nature, the impact on the present value would be different.

5.5 Chapter summary

A sensitivity analysis on key parameters were undertaken to help understand how the results of the model framework behaves under an alternative set of inputs. The sensitivity analysis served to support with the model validation component.

Firstly, the expected returns for a single asset class were increased by +1.5%, +1%, +0.5% and decreased by −1.5%, −1%, −0.5%, respectively. From an efficient frontier perspective, the results of the expected return sensitivity analysis suggested that while the efficient frontier curve did not materially shift, the asset allocation composition changed in certain instances.

One of the most noteworthy results from the sensitivity analysis were the optimisers preference to mostly attain the maximum limit to *Kenya bonds* (30%), irrespective of the expected return shock applied. The sensitivity analysis revealed that by decreasing the expected return of *Kenya bonds* to the most "pessimistic" outlook, the optimiser would still prefer to include some *Kenya bonds* as an optimal asset class. This meant that at a reduced expected return, the risk and return profile of *Kenya bonds* was still deemed attractive, thus inclusion within the optimal asset allocation was present.

One of the other interesting sensitivity analysis results was that of *Africa equity*. Since this asset class exhibits a relatively high risk and return profile, the efficient frontier curve primarily shifted at higher risk levels, when increasing the expected return shock. The efficient frontier remained relatively unchanged when decreasing the expected return shock. As the expected return shock was increased, the allocation to *Africa equity* gradually increased. However, it was found that at reduced expected return shocks, the optimiser significantly reduced its allocation, or allocated a zero weight to this asset class. This meant, the risk profile of *Africa equity* was too high to support inclusion at the reduced expected return shocks.

Sensitivity analysis was also carried out on adjusting certain constraint sets used within the optimisation procedure. The results of the optimal area graphs generally followed a broad trend. For example, increasing the limit (less restrictive) on the total foreign constraint resulted in the optimiser preferring a greater amount of *foreign-DM cash* and *foreign-DM bonds* at lower risk levels. This was mainly due to the low risk profile as well as the higher correlation profile (relative to the liability) associated with these two asset classes. In addition, this was expected since the unconstrained asset allocation results presented in Figure 4.1 advocated a large amount of these two asset classes. So, by imposing a less restrictive limit on the foreign component, a greater amount of these two asset classes was expected.

This chapter closed with a sensitivity analysis by applying interest rate shocks to the yield curve. The results were consistent with theory, *i.e.,* a positive shock to the yield curve resulted in a decrease to the present value. Whereas a negative shock to the yield curve resulted in an increase to the present value. The results revealed that the present value did not radically change given shocks to the interest rates.

CHAPTER 6

Conclusion

Contents

This chapter opens with a summary of the research presented in this study as provided in §6.1. This is followed by a description of the main contributions of the study contained in §6.2. The chapter and thesis is concluded in §6.3 with a list of potential suggestions for further research that may stem from the study.

6.1 Thesis summary

The introductory chapter of this thesis opens with background and providing rationale to the problem under study. The notion of asset allocation is introduced, as well as the importance thereof is described. Chapter 1 also includes the formal problem description, scope and objectives of this thesis.

In Chapter 2, an overview of MPT that serves as a quantitative reference point in crafting a portfolio optimisation problem is introduced. A review of CMAs are studied to better understand how these investment assumptions are typically formulated within an academic setting. The well-known notion of VaR is introduced to quantify investment risk on a probability and monetary basis. Terminology and concepts pertaining to insurance, risk, present value, and duration are introduced that are key in setting the scene in terms of how these fit into the broader setting of asset allocation. The philosophy and premise around LDI and ALM are studied to provide perspective relating to the considerations centred around asset allocation, that incorporates *assets* and *liabilities*. Additional insights from the CFA Institute are also furnished. This is followed by a discussion on novel QP objective functions (from a *liability-relative* and *non liability-relative* perspective) that are used in practice to formulate an asset allocation problem as a portfolio optimisation problem. This chapter concludes with a brief study on alternative approaches such as simulation and meta-heuristics to formulate and solve an asset allocation problem. This is done in accordance with the Objective I as stated in §1.3.

Chapter 3 opens with the general and conceptual model framework, from an asset and liability perspective. Essentially, this is described by means of a flow chart and forms the basis of the road map of the model framework. To aid in model credence, the model framework is applied to a real-world case study. The case study opened by understanding the actual liability cash flow data of the re-insurer. This is followed by computing the so-called liability based benchmark as well as key analytic measures to better understand the potential risks around the liability cash flows. Thereafter, the universe of asset classes considered are specified to allow for asset class and geographical diversification opportunities. The data used to formulate the CMAs as guided by literature and in some instances input from subject matter experts, are calculated and presented. The re-insurers estimated current asset allocation is revealed. The novel objective functions that form the cornerstone of the optimisation procedure under study from both a *liability-relative* and *non liability-relative* are presented, once more, influenced by literature as introduced in Chapter 2. Essentially, the conceptual description of the objective functions pertains to *maximising return*, while *minimising risk* from an absolute and relative perspective. This is followed by providing rationale on practical constraints that are included within the optimisation procedure, to allow for improved portfolio diversification and to moderate exposure. The chapter closes with a discussion on incorporating VaR, and the balance sheet representation within the asset allocation process. This is done in accordance with Objectives II and III as stated in §1.3.

In Chapter 4, the results of the model framework are presented and accompanied by a detailed examination thereof. The optimisation results of the unconstrained asset allocation (in a *liability-relative* setting) indicated that fewer asset classes and consequently less asset class diversification was present. *Kenya bonds* formed a chief component of the optimal portfolio of assets given its attractive risk and return profile. Although this result is theoretically appropriate, the dominance of one asset class leads to concentration risk, and consequently, a lack of portfolio diversification opportunities. In addition, *foreign-DM cash* and *foreign-DM bonds* featured noticeably within the unconstrained asset allocation. Although the return profile was the least attractive (*i.e.,* below zero), the risk and correlation profile was favourable. The latter was largely attributed by the higher correlation that these two asset classes exhibit relative to the liability, thus supporting the inclusion thereof. Furthermore, given the relatively short duration of the liability cash flow profile, from an unconstrained perspective, fixed income asset classes were a preferred alternative to ensure the objective of the liability payments would be met. To add a further practical ingredient to the model framework and to allow for better diversification, suitable constraints were incorporated to moderate asset class allocations. The optimisation results of the constrained asset allocation (in a *liability-relative* setting) advocated a much more diversified portfolio of assets by allocating to a wider array of asset classes. This also meant, in a constrained setting, fixed income and property asset classes formed the majority component of the optimal asset allocation to ensure the objective of the liability payments would be met. This is followed by presenting constrained asset allocation results in a *non liability-relative* setting. The results of the constrained *liability-relative* and constrained *non liability-relative* approaches were revealed to be alike, from an optimal asset allocation perspective (*i.e.,* majority of the allocation comprised of fixed income and property asset classes). This was mainly due to the relatively short duration of the liability cash flow profile, resulting in a very similar array of asset class allocations. An additional factor contributing to the similarity of the *liability-relative* versus the *non liability-relative* was the constraints introduced within the optimisation procedure. Essentially, the nature of the constraints may have resulted in the optimiser being compelled to allocate to alternative asset classes, once a constraint on an asset class was reached. Thereafter, the focus shifts by integrating the notions of VaR and the balance sheet representation, to formulate the investment strategy as a P/H and S/H portfolio of assets, respectively. Two optimised portfolios that target an improved *return* profile (without sacrificing *risk*), as well as an optimised portfolio that target an improved *risk* profile (without sacrificing *return*), respectively were proposed. The chapter concludes with a discussion of the model validation process considered. Three subject matter experts were consulted. The outcome of this validation was that the roadmap presented as well as the results produced are reasonable and are in-line with industry standards. This is done in accordance with Objective IV as stated in §1.3.

In Chapter 5, several sensitivity analysis is performed to test the robustness of the optimiser given shocks made to the expected returns, one of the key input parameters, noted within literature to exhibit sensitivity (see [9, 11, 23, 68]). The results of this chapter also served to supplement the validation component of this thesis, as well as to serve as a reasonability check. One of the most noteworthy results from the sensitivity analysis was the optimisers preference to mostly attain the maximum limit to *Kenya bonds*, under most expected return shocks. The sensitivity analysis revealed that when decreasing the *Kenya bonds* expected return to the most "pessimistic" outlook, the optimiser would still prefer to include some *Kenya bonds* as an optimal asset class choice. This meant that at a reduced expected return, the risk profile of *Kenya bonds* was still deemed as a viable and attractive asset class to include within a portfolio. An additional key feature of the sensitivity analysis was that of the *Africa equity* results. As the expected return shock was increased, the allocation to *Africa equity* subsequently increased. However, it was found that at reduced expected return shocks, the optimiser significantly reduced its allocation to this asset class. This meant, the risk profile of *Africa equity* was too high to justify inclusion at the reduced expected return shocks. This is followed by sensitivity analysis on applying alternative limits to the constraints to gain insight in terms of how the optimal area composition of asset classes would be impacted. This chapter concludes with a sensitivity analysis pertaining to shocks on the interest rate assumption. This is done in accordance with the Objective V as stated in §1.3.

6.2 Contributions

The main contributions made by this study with regards to portfolio optimisation are discussed in this section. The outcome of this research is useful in the researcher's view, in that it provides investors and consultants with a road-map (coupled with a detailed real-world case study and results thereof) to solve a portfolio optimisation problem. To the best of the researcher's knowledge, there is a scarcity of documented literature sources that provides sound direction to formulate an investment strategy that incorporates both *assets* and *liabilities*, when solving a portfolio optimisation problem for a re-insurer within the African context. The approach described in this thesis paves a gateway for consultants and alike wishing to explore portfolio optimisation by incorporating both *assets* and *liabilities* within the investment decision making process.

1. *Solving a portfolio optimisation problem as a quadratic programming problem that incorporates an investors liability cash flows.* To solve the portfolio optimisation problem, the relationship between the risk and return profile for the asset classes under study had to be established. In addition, the liability cash flows of the re-insurer were also incorporated within the optimisation procedure to ensure the investors objectives are integrated within the model framework. The inclusion of the liability cash flows within the optimisation procedure represents an extension of the traditional mean-variance optimisation. The results of the constrained asset allocation of the study revealed that whether including (*liability-relative*) or excluding (*non liability-relative*) the liability cash flows within the optimisation procedure, resulted in immaterial changes to the composition of asset classes, under both approaches (*i.e.,* majority of the allocation comprised of fixed income and property asset classes). This outcome was largely expected given the relatively short duration profile of the liability cash flows, as well the same constraints (to ensure consistency) imposed within both optimisation procedures. This research demonstrated that this outcome was present within the results.

2. *Develop a road map (flow chart) to formulate and solve a portfolio optimisation problem and apply this to a real-world case study.* The roadmap of the conceptual model framework

provided direction on how to consider incorporating both *assets* and *liabilities* within an investment strategy process. This was applied to a real world case study that highlighted some of the typical intricacies faced by practitioners from a data perspective (*eg*, missing data). The results of the case study showcased a detailed examination of how typical outputs would look, from a practical and industry perspective. In addition, a detailed sensitivity analysis was undertaken on input parameters (*eg*, expected returns) to measure the impact this had on the optimal portfolio of assets.

3. *Introducing practical constraints within the optimisation procedure.* While the results of the optimal asset allocation are largely driven by the CMAs, the constraints incorporated within the optimisation procedure did have a meaningful impact on the asset allocation. The results of the unconstrained asset allocation reflected a concentrated and smaller array of asset classes, resulting in less portfolio diversification opportunities. However, as a result of imposing reasonable constraints, this subsequently led to a higher range of asset classes that featured within the constrained asset allocation. This research demonstrated that the inclusion of constraints ultimately led to more practical and moderated asset allocations, and consequently greater portfolio diversification.

4. *Decomposing the investment strategy into a P/H and S/H component.* By separating (as opposed to combining) the investment strategy into a P/H (*liability-relative*) and S/H (*non liability-relative*) component respectively, allows a practitioner to invest the firms assets such that it targets specific objectives for the two components of the investment strategy. The P/H and S/H framework conforms with the balance sheet representation, and allows a practitioner to ultimately enhance decision making around its risk and return objectives.

5. *Incorporate VaR (as opposed to risk in percentage terms) as an alternative and tangible risk measure.* *Risk* measured in percentage terms has little practical and tangible connotation. For this reason, the model framework was modified to incorporate an alternative and tangible measurement of risk, namely *VaR*. The measurement of risk along the entire efficient frontier to VaR serves as a modification to the model framework as opposed to the conventional measurement of risk (*i.e.*, in percentage terms). Essentially, the modification allows a practitioner to quantify risk on a probability and monetary basis.

6. *Improvements made to the re-insurers current asset allocation, from a risk (VaR) and return perspective (P/H portfolio).*

   (a) The suggested $P1$ portfolio was formulated off the back of optimising the existing *VaR* level of the R portfolio *i.e., increasing the return level* without sacrificing (risk) VaR. The enhancement from a return perspective, in absolute terms resulted in an additional return of 0.59% (from 4.12% to 4.71%). In relative terms, this amounts to an increase of approximately 14%.

   (b) The suggested $P2$ portfolio was devised off the back of optimising the existing *return* level of the R portfolio *i.e., decreasing the VaR level* without sacrificing return. The improvement from a VaR perspective, in absolute terms, resulted in a reduced VaR of US$7.91 million (from US$69.86 million to US$61.95 million). In relative terms, this amounts to a decrease of approximately 11%.

To summarise, if the re-insurer adopts $P1$ as the suggested portfolio, this in turn would culminate an *increase in return*. However, if the re-insurer adopts $P2$ as the suggested portfolio, this in turn would culminate a *decrease in VaR*. Both of these portfolios represent an enhanced (optimised) outcome from a *return* and *risk (VaR)* perspective, relative to the re-insurers existing current asset allocation.

6.3 Future work

As with any research study, there are multiple aspects of the model framework that can be improved and refined upon. The model framework may benefit from some refinement and the inclusion of alternative assumptions, improved data quality, or possibly alternative modelling approaches. Possible future work stemming from this study may encompass the following proposals (in partial fulfilment of Objective VI).

Incorporate Black & Litterman within the expected returns

The expected returns formulated in this study were calculated using a quantitative calculation. However, in practice, subjective refinements are often incorporated within the expected returns to better capture an investors market outlook [12]. These views are commonly referred to as so-called "expert" investment views and is the consideration described by Black & Litterman [12]. Including subjective refinements may result in a more accurate depiction of the expected returns, resulting in an optimal asset allocation that is tilted toward the views of an investor.

Assimilate the Michaud & Michaud technique

For this thesis, the efficient portfolio of assets was calculated via classical QP objective functions. As highlighted in §2.7, Michaud & Michaud [67] developed a Monte Carlo resampling approach that produces a sampled efficient portfolio of assets. It is proposed that future work incorporate this stochastic procedure to understand how different the asset allocation results may be.

Alternative VaR technique

To calculate VaR, the variance-covariance technique was employed in this study. However, a slight drawback of this parametric procedure is the assumption of normality of returns. An alternative approach could be to incorporate a non-parametric approach such as the *historical simulation* method instead. This method will align to the alternative approach discussed by [62, 64, 91].

Incorporate Environmental, Social, and Governance factors (ESG)

Investors are faced with ongoing pressure from regulators (such as the United Nations (UN)), to ensure their investments culminates a positive real-world impact on the ESG, as an entity. An example of an ESG cognisant investment includes the use of *green bonds*[1] within a portfolio.

While the optimisation under study focused on solving a portfolio optimisation problem within two realms (*i.e.*, *risk* and *return*), a third realm, namely, *real-world impact* could be embedded within the asset allocation process. Essentially, the optimisation procedure could identify portfolios of equal attractiveness (*i.e.*, from a risk and return perspective), whilst providing some level of preference to the combination of assets that offers potential for the greatest real-world impact [33]. Some of the benefits of embedding ESG within an asset allocation process include closer alignment with that of the UN's sustainability goals. In addition, including ESG factors would ultimately result in improved long-term risk and return objectives for investors.

For this reason, further work around this area could encompass embedding ESG within the portfolio optimisation process.

[1] *Green bonds* fund projects that yield positive real-world impact from an environmental or climate perspective [18].

Solve the portfolio optimisation problem via a meta-heuristic

The objective functions implemented in this thesis were based on the novel mean-variance formulations stated as a QP optimisation problem. It could be investigated to formulate and solve the portfolio optimisation problem via a meta-heuristic instead. This may align to the methods described by Zhu *et al.* [102] and Erwin & Engelbrecht [35].

Additional sensitivity analysis

While several sensitivity analysis was performed on the expected returns, constraints, and interest rates, an additional consideration can be to explore how alternative confidence intervals (z-values) would impact the range of the VaR of the P/H and S/H portfolio of assets. From §3.8 for this study, the confidence interval of 99.5% assumed was based on the guidelines of Solvency II.

Incorporate *skewness* and *kurtosis* within the optimisation procedure

As noted in §2.2, only the first two moments (*mean* and *variance*) are required to make use of the objective functions used within this study. However, advancements within the sphere of portfolio optimisation have led to researchers incorporating the third and forth (*skewness* and *kurtosis*, respectively) within the portfolio optimisation procedure. Essentially, the multiple objective problem is to *maximise return* and *skewness*, whilst *minimising risk* and *kurtosis*. This would align to the objective functions proposed by [2, 58]. Incorporating these additional moments, would add a further level of mathematical rigour and may result in a more refined asset allocation.

Attach a level of importance to the liabilities

The two objective functions considered within this thesis focused strictly on either a *liability-relative* approach or a *non liability-relative* approach. However, Sharpe & Tint [87] proposed that a level of importance be attached to the liabilities. Quantitatively speaking, a scaler k where $0 \leq k \leq 1$, may be assigned to the third term of objective function (6.1)[2]. Essentially, the k value links a level of importance to the liabilities. If k is equal to one, this merely denotes a "complete" *liability-relative* optimisation. If k is equal to zero, this merely denotes a "complete" *non liability-relative* optimisation. Values of k closer to the lower bound indicate a higher tilt toward *non liability-relative* optimisation while still embedding some level of the liabilities within the optimisation. Values of k closer to the upper bound indicate a higher tilt toward *liability-relative* optimisation while still embedding some level of the traditional, *non liability-relative* approach within the optimisation procedure.

$$\underset{\boldsymbol{x} \in \mathcal{R}^N}{\text{maximise}} \quad \left(2\tau \boldsymbol{\mu}^T \boldsymbol{x} - \boldsymbol{x}^T \sum \boldsymbol{x} + (k) \cdot 2\boldsymbol{\gamma}^T \boldsymbol{x}\right) \tag{6.1}$$

It is proposed that future work embed this "hybrid" approach as an alternative to *liability-relative* only or *non liability-relative* only.

[2]Objective function (6.1) was initially introduced in §2.6, without the k value.

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APPENDIX A

Supplementary Research

This Appendix opens with supplementary research pertaining to general model building in §A.1. This is followed by a discussion on risk contained in §A.2. Thereafter, the concept of present value is studied in §A.3. This is followed by a discussion of duration contained in §A.4. This appendix closes with a brief review of the statistical preliminaries pertaining to covariance and correlation, contained in §A.5.

A.1 Model building process

The notion of a model is widely used across a number of industries to formulate and solve a problem. A model seeks to represent an intricate real world phenomena, that is generally described by a mathematical representation of a system [78, 94]. The first step to design a model requires a researcher to provide a conceptual description of the problem. This requires the researcher to ask the following question: *what is the objective of the model?* Stated differently, *what specific problem should this model solve for?*

In the next step, a set of simplifying assumptions needs to made, as in practice, it is highly unlikely to capture each and every dimension of a problem given computing power or time constraints. The next step encompasses translating the conceptual problem description into a formalised mathematical model. While a number of choices around what type of mathematical model (*eg*, a differential equation, mathematical programming *etc.*) is best suited, the choice primarily depends on the nature of the problem. Regardless of the type of model chosen, decision variables and parameters must be defined that suitably represents the system. Once the mathematical model is formulated, the researcher calibrates the model with a set of parameters and input data. Thereafter, the model is implemented on a computer to produce outputs [94].

The next step consists of examining the outputs that yield insight in terms of how the model behaves under the specified conditions. Thereafter, model validation, a critical component to ensure the model results are consistent with theory and industry observed trends should be performed. This may take the form of *face validation*[1], or comparing the results with a similar study [94].

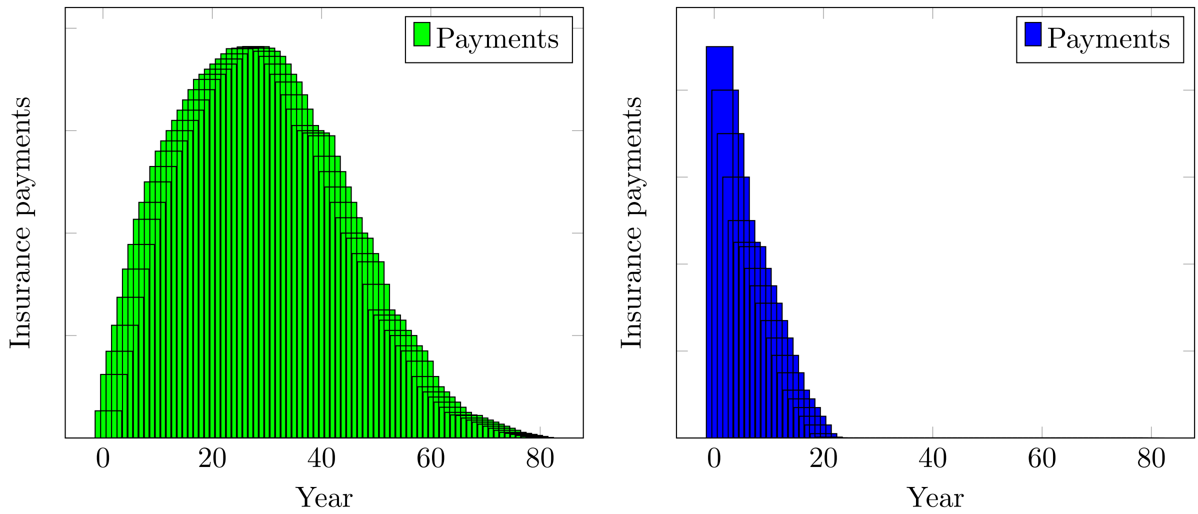
A further validation step may comprise of performing sensitivity analysis of the input parameters so as to understand the dominating factors at play influencing the results. At the final stage, the researcher draws conclusions and findings of the study [94].

[1] *Face validation* is a form of model validation wherein opinions are obtained about the reasonableness and accuracy of a model framework from people knowledgeable in a specific field relating to a system [60].

The frequently used motto *"garbage in, garbage out"* [94] lies at the heart of building and interpreting a model. This statement infers that the quality of the outputs are largely a function of the quality of the assumptions and input data [94].

A.2 Risk and investment strategy

Central to a discussion on investment strategy, is the perception of risk. Risk is the likelihood of exposure to financial losses [99]. Different investment horizons (*short-term* versus *long-term*) require a different asset allocation blend so as to meet the goals and objectives of the investor. Figure A.1 shows an example of a typical long-term (life-insurer) and short-term (non-life insurer) investor's liability cash flow profile, stretching 80 years into the future. Each bar depicts the annual monetary amount that the insurer is required to pay out to its policyholders. Figure A.1a depicts a typical long-term investors liability cash flow profile and indicates the liability payments reaches its peak between years 25 and 35 respectively. Figure A.1b shows an example of a typical shorter-term investors liability cash flow profile, only extending approximately 20 years into the future. Figure A.1 shows an eminent distinction, long-term investors have liability cash flow profiles stretching many decades into the future, whereas shorter-term investors generally have liability cash flow profiles stretching fewer decades into the future.



(a) *An example of a long-term liability cash flow stream spanning 80 years into the future.*

(b) *An example of a shorter-term liability cash flow stream spanning just over 20 years into the future.*

Figure A.1: *Example of a long-term (left) versus short-term (right) liability cash flow profile.*

It is noticeable that different investor types have contrasting investment durations, so, their tolerance to risk will be viewed differently. This suggests that a life insurer versus a non-life insurers' investment strategies are expected to be different.

A.3 Present value

The term *present value* refers to the current value of a future series of cash flows, given an appropriate interest rate or interest rates assumption [34]. Suppose an insurance or re-insurance firm has an annual[2] future cash flow stream, a practitioner should make use of Equation (A.1) to compute the present value. It is well-known and documented that the interest rate assumption, i, is typically set using a suitable bond yield or "risk-free" term-structure [18, 76, 98].

[2]the notion of *annual* is referred to as NACA, *nominal amount compounded annually*.

$$\text{Present value} = \sum_{t=1}^{n} \frac{CF_t}{(1+i)^t} \tag{A.1}$$

where:

CF_t = cash flow in the period at time t,
n = total number of periods,
t = current period in the series,
i = interest rate.

Table A.1 demonstrates a hypothetical example to compute the present value of a liability cash flow spanning five years into the future. The total present value amount of R709.56 is computed by applying Equation (A.1) for each year corresponding to its future liability cash flows and interest rates provided in Table A.1.

Table A.1: *Simple present value calculation.*

Year	Index (t)	Future liability cash flow (CF_t)	Interest rate (i)	Present value
2021	0	R50	0.5%	R50
2022	1	R100	1.0%	R99.01
2023	2	R150	1.5%	R145.60
2024	3	R200	2.0%	R188.46
2025	4	R250	2.5%	R226.49
		R750		R709.56

The key use of computing a present value is that it answers the question; what amount does a *future cash flow* equate to, in *"today's"* terms.

A.4 Duration

Duration is a valuable tool that is used to quantify the sensitivity of a cash flow stream against movements in interest rates [18, 49]. Macaulay duration, measured in years, computes the weighted average time until cash flows are received [49]. The formula to compute Macaulay duration is given by Equation (A.2)

$$\text{Macaulay duration} = \frac{\sum_{t=1}^{n} (t \times \text{PVCF}_t)}{\text{PVTCF}} \tag{A.2}$$

where:

PVCF = Present value of the cash flow at time t,
PVTCF = Present value of the total cash flow,
t = time to each cash flow (in years),
n = total number of periods.

A liability cash flow stream with a higher duration implies it is more sensitive to changes in interest rates. Whereas, a liability cash flow stream with a lower duration is less sensitive to changes in interest rates [49]. Based on the fictitious data provided in Table A.1, the Macaulay duration amounts to 2.6 years. Essentially, this means after 2.6 years, an investor would have received their initial investment amount in return.

A.5 Covariance and correlation

As noted in §2.2, a key input to a mean-variance optimiser is that of covariance. Statistically, the *covariance* between two random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$ is described by

$$\mathrm{Cov}(\boldsymbol{X}, \boldsymbol{Y}) = E[(\boldsymbol{X} - \mu_x)(\boldsymbol{Y} - \mu_y)]. \tag{A.3}$$

where μ_x and μ_y denote the mean (or expected value) of random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$. Stated without proof[3], the right hand side of Equation (A.3) can be expressed as

$$\mathrm{Cov}(\boldsymbol{X}, \boldsymbol{Y}) = E[\boldsymbol{XY}] - E[\boldsymbol{X}]E[\boldsymbol{Y}]. \tag{A.4}$$

It is noted by Ross [84] that if $\boldsymbol{X}$ and $\boldsymbol{Y}$ are independent, then $\mathrm{Cov}(\boldsymbol{X}, \boldsymbol{Y}) = 0$.

The covariance matrix, is defined in terms of covariance as

$$\sum = \begin{bmatrix} \sigma_x^2 & \sigma_{yx} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \tag{A.5}$$

where σ_x^2 and σ_y^2 denote the variances of random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$, respectively. So, to extract the risk profile for a random variable (where the random variable represents an asset class), the square root along the diagonal should be taken.

The *correlation coefficient*[4] between $\boldsymbol{X}$ and $\boldsymbol{Y}$ is defined by

$$\mathrm{Corr}(\boldsymbol{X}, \boldsymbol{Y}) = \rho(\boldsymbol{X}, \boldsymbol{Y}) = \frac{\mathrm{Cov}(\boldsymbol{X}, \boldsymbol{Y})}{\sigma_x \sigma_y}. \tag{A.6}$$

where σ_x and σ_y denote the standard deviation of random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$ [34, 41, 84]. As noted in §2.3.2, the result of Equation (A.6) provides an indication of the strength of the random variables (asset classes), and ranges between $+1$ and -1, respectively.

[3] for the proof, the reader may refer to Ross [84].

[4] referred to as the Pearson product-moment correlation coefficient [41].

APPENDIX B

Expected Return Sensitivity Analysis

In this appendix, the sensitivity analysis results and an examination of the remaining 10 asset classes §§5.2–5.2.1 are presented.

Figure B.1 presents the results of the sensitivity analysis for *Kenya cash*. The upper left reveals that from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises slightly upward. However, as the expected return is decreased, the efficient frontier shifts slightly downward. The movement is predominantly seen at lower risk and return levels, since *Kenya cash* exhibits a lower risk and return profile. The lower right of Figure B.1 shifts the focus to the percentage amount of *Kenya cash* the optimiser deems optimal at varying expected return shocks, at their corresponding risk levels. Irrespective of the expected return shock applied, *Kenya cash* forms a considerable composition of the asset allocation, at lower risk levels of 8%–9%. This is due to the optimisers preference to allocate a lower risk asset class such as *Kenya cash*, at lower risk levels. At higher risk levels between 13% and 15%, the optimiser allocates either a very small or zero weighting to *Kenya cash*. At these higher risk levels, the optimiser deems *Kenya cash* as a less optimal asset class to include given its lower risk and return profile, hence the exclusion. The optimal allocation to *Kenya cash* gradually decreases as the risk level increases. This is expected as *Kenya cash* exhibits a relatively low risk and return profile, so inclusion thereof is more prominent at lower risk levels.

Figure B.2 presents the results of the sensitivity analysis for *Kenya property*. The upper left shows that from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises upward slightly, at moderate to higher risk levels. However, as the expected return is decreased, the efficient frontier shifts slightly downward, at moderate to higher risk levels. The lower right of Figure B.2 reveals that given any expected return shock, and the lowest risk level of 8%, the optimiser allocates a zero weight to this asset class. This is due to *Kenya property* exhibiting a moderately higher risk and return profile, hence exclusion thereof at the lowest risk level. The allocation to *Kenya property* gradually increases, as the level of risk increases. This too, is largely expected since *Kenya property* exhibits a moderate risk and return profile. It is revealed that the optimiser allocates the maximum amount (30%) of *Kenya property* only once the expected return is increased by +1.5%, and at a high risk level of 15%. At the lowest expected return shock of −1.5%, the optimiser either allocates zero or a very small allocation to *Kenya property*. This means the level of decrease in the expected return does not warrant inclusion for the risk an investor is expected to incur.

Figure B.3 presents the results of the sensitivity analysis for *Kenya equity*. The upper left indicates that from the expected return shocks on the efficient frontier appears to "mirror" the "unchanged" efficient frontier. However, at increased return shocks and beyond the risk level of 15% the efficient frontier extends from a risk and return perspective. This is due to *Kenya*

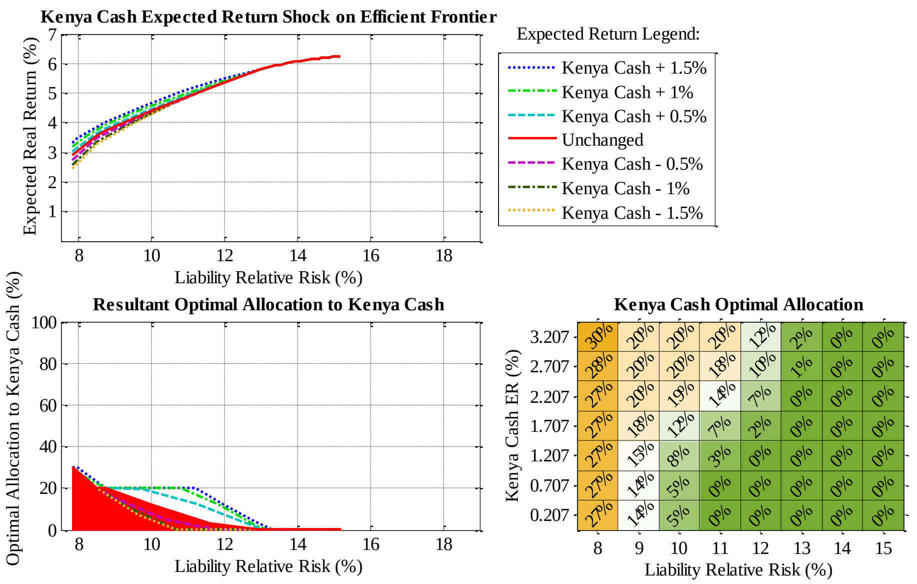


Figure B.1: *Kenya cash expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

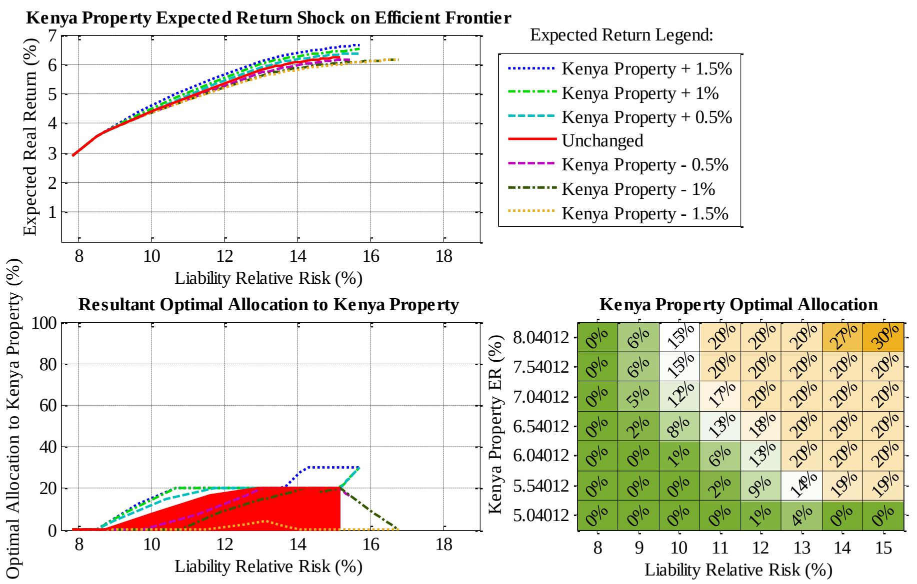


Figure B.2: *Kenya property expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

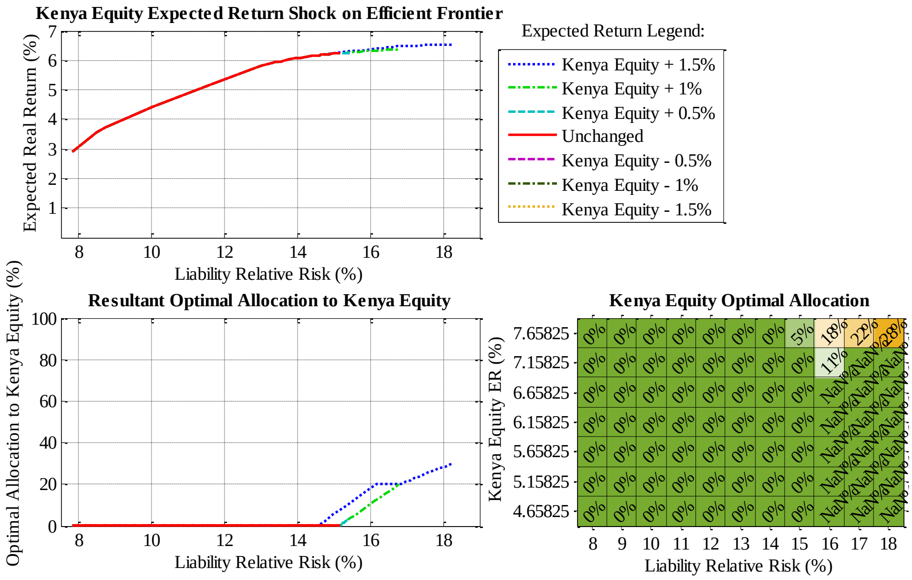


Figure B.3: *Kenya equity expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

equity exhibiting a very high risk and return profile, hence the efficient frontier adds risk to allow *Kenya equity* to feature at more attractive levels of expected return shocks. The lower right of Figure B.3 reveals that at lower to moderate risk levels between 8% to 14%, the optimiser allocates a zero weight to *Kenya equity*. For higher risk levels greater than 15%, and expected return shocks of +1% and +1.5% the optimiser rapidly starts increasing its allocation to *Kenya equity*. This means an investor is expected to incur a large amount of risk to justify inclusion of *Kenya equity*.

Figure B.4 presents the results of the sensitivity analysis for *Africa bonds*. The upper left indicates that from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises upward slightly. As the expected return is reduced, the efficient frontier shifts marginally downwards. The lower right of Figure B.4 reveals that at lowered expected return shocks, smaller allocations of this asset class are seen. This means the lowered expected return shocks does not justify large allocations given the level of risk that the investor is required to incur. At higher expected return shocks the optimiser tends to either attain its maximum, or allocate a substantially large amount to *Africa bonds*. The optimisers preference to allocate to *Africa bonds* is largely driven by its moderate risk and return profile, hence the inclusion thereof is prominent at higher expected return shocks.

Figure B.5 displays the results of the sensitivity analysis for *Africa property*. The upper left reveals that from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises upward slightly. As the expected return is reduced, the efficient frontier insignificantly shifts downwards. The lower right of Figure B.5 reveals that at lowered expected return shocks, the optimiser finds *Africa property* as less attractive since a zero weight is primarily observed. Stated differently, the reduction in return does not support the level of risk an investor is required to incur. Similar to *Africa bonds*, at higher expected return shocks the

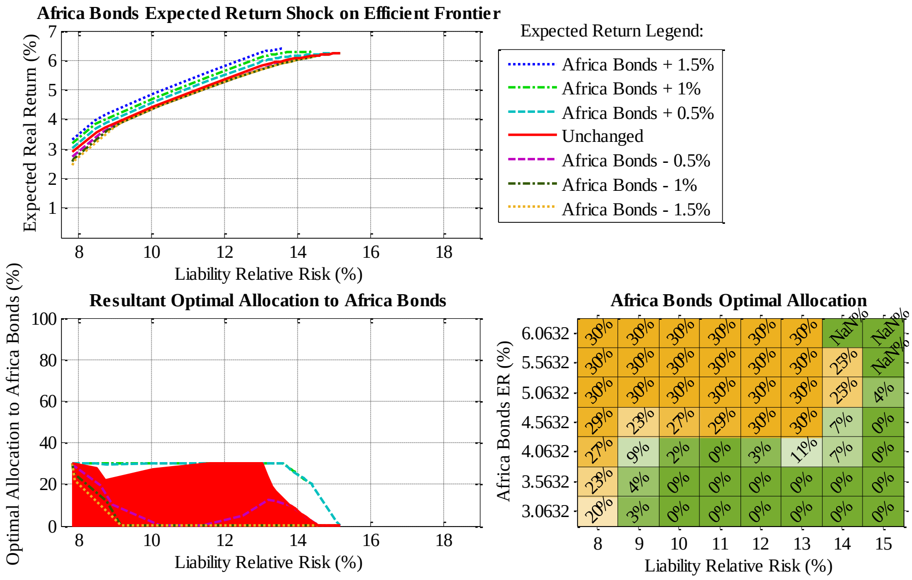


Figure B.4: *Africa bonds expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

optimiser tends to either attain its maximum, or allocate quite a large amount to this asset class. The optimisers preference to allocate to *Africa property* is spurred on by the moderate to higher risk and return profile, hence the inclusion thereof is eminent at higher expected return shocks.

Based on the similarity of the sensitivity analysis results for *foreign-DM property* and *equity*, these two asset classes will be examined with simultaneously. Figures B.6–B.7 presents the results of the sensitivity analysis for *foreign-DM property* and *foreign-DM equity*. The upper left shows that from an efficient frontier angle, as the expected return is increased, the efficient frontier raises upward marginally. The movement is primarily seen along the risk levels positioned at the centre. This is expected since these two asset classes exhibit a moderate risk and return profile.

The lower right of Figures B.6–B.7 shifts the focus to the percentage amount of *foreign-DM property* and *foreign-DM equity* the optimiser deems optimal at varying risk levels. At expected return shocks of +1.5%, +1%, and +0.5%, the optimiser rapidly increases its allocation to these asset classes, at most levels of risk. Essentially, these two asset classes are rendered more attractive, hence the increase in allocation. At expected return shocks of −1.5%, −1%, and −0.5%, the optimiser allocates a zero weight to this asset class. Since this reduction in the expected return shock renders this asset class as less attractive, the level of risk an investor is expected to incur does not warrant inclusion. Hence the optimisers preference to allocate a zero weight. Since the allocations are zero under these reduced expected return shocks, this supports why the efficient frontier curves are unmoved (at the reduced expected return shocks), displayed in the upper left of Figures B.6–B.7.

At the lowest risk level of 8%, the optimiser allocates a zero weight to both these asset classes. This is due to these asset class exhibiting a moderately higher risk and return profile, hence

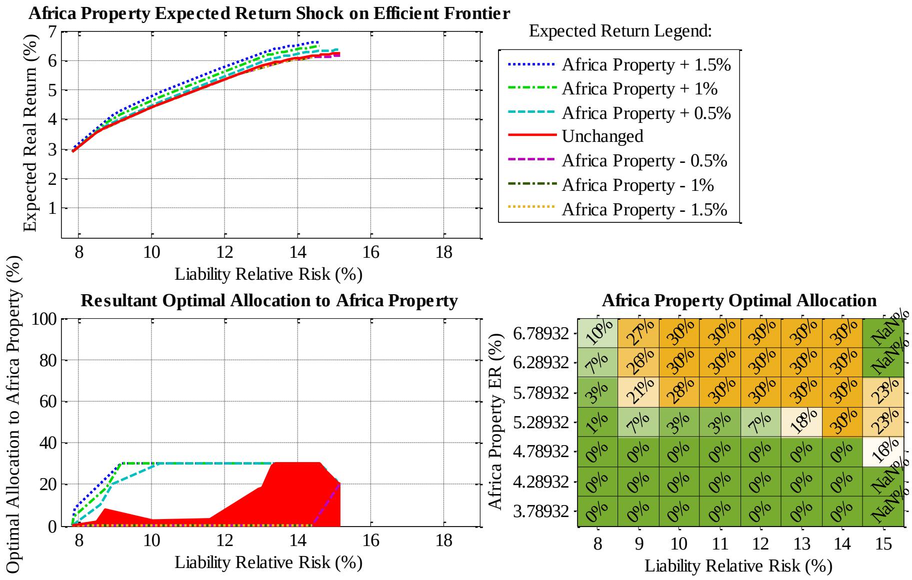


Figure B.5: *Africa property expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

exclusion at the lowest risk level. For *foreign-DM property*, the upper limit[1] is attained for the largest shock of +1.5%, mostly transpiring at moderate risk and return levels given the moderately higher risk profile of this asset class. For *foreign-DM equity*, the upper limit is attained for shocks +1% and +1.5%, transpiring at medium to higher risk levels given the moderately higher risk profile of this asset class.

Figure B.8 presents the results of the sensitivity analysis for *foreign-EM bonds*. The upper left shows that from an efficient frontier perspective, as the expected return is increased, the efficient frontier raises marginally upward. The movement is primarily seen along the risk levels positioned at the centre, since *foreign-EM bonds* exhibits a lower to moderate risk and return profile. As the expected return is decreased, no movement is seen with the efficient frontier curve. The lower right of Figure B.8 shifts the focus to the percentage amount of *foreign-EM bonds* the optimiser views optimal at varying risk levels. At expected return shocks of −1.5%, −1%, and −0.5%, the optimiser allocates a zero weight to *foreign-EM bonds*. Since this reduction in the expected return shock renders this asset class as less attractive, the level of risk an investor is expected to incur does not warrant inclusion, hence the optimisers preference to allocate a zero weight. In light of the allocations reflecting a zero weight under these reduced expected return shocks, this explains why the efficient frontier curves are unmoved (at the reduced expected return shocks), displayed in the upper left of Figure B.8. Majority of the allocations to *foreign-EM bonds* are observed when the expected return shocks are increased by +0.5%, +1%, +1.5%. This is due to the lower to moderate risk profile seen with *foreign-EM bonds*, so an increase in expected return renders this asset class as more attractive from a return profile perspective.

Based on the similarity of the sensitivity analysis results for *foreign-EM equity* and *China equity*, these two asset classes will be dealt with concurrently. Figures B.9–B.10 presents the results of

[1]In this case, the upper limit of 20% is attained to ensure constraint set (3.8) is satisfied.

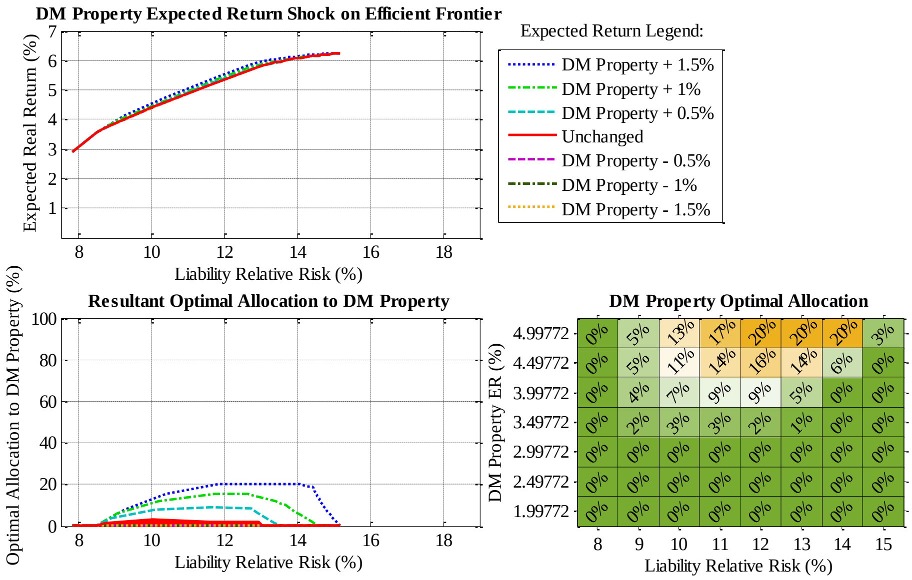


Figure B.6: *Foreign-DM property expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

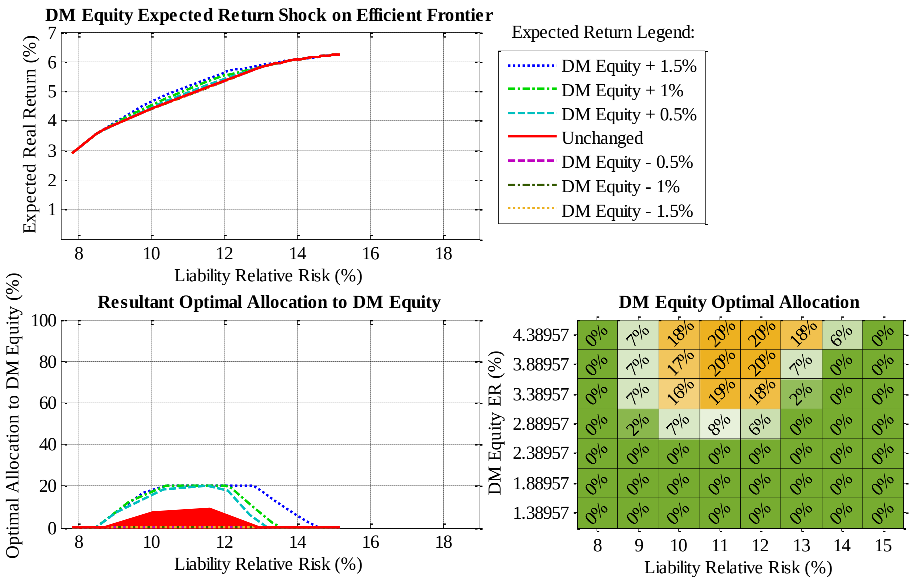


Figure B.7: *Foreign-DM equity expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

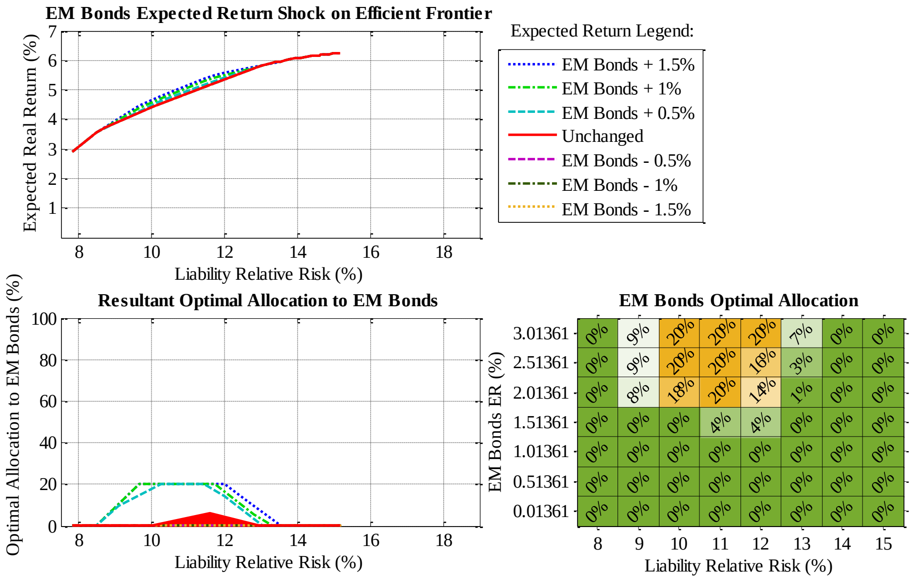


Figure B.8: *Foreign-EM bonds expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

the sensitivity analysis for *foreign-EM equity* and *China equity*. Slightly similar to the sensitivity analysis results seen with *Kenya equity*, the upper left reveals that the efficient frontier appears to "mirror" the "unchanged" efficient frontier. This indicates that no change to the asset allocation is present. While both these two asset classes exhibit a relatively high risk profile, an investor may expect these asset classes to feature at higher risk levels. However, the high level of risk an investor is expected to incur, does not justify the return on offer. In addition, since *foreign-EM equity* does not feature as an optimal asset class (*i.e.*, a zero weighting), it implies that *China equity* would not feature either. This is due to constraint set (3.11), that ensures *foreign-EM equity* should not exceed *China equity*. Since all expected return shocks indicate a zero weight to *foreign-EM equity*, this implies that the *China equity* allocation must also equal to zero. Thus constraint set (3.11) is met.

Table B.1 presents the alternative optimised asset allocations when increasing and decreasing the expected return for *Kenya cash* respectively. Given an increase in expected return, the allocation to this asset class increases, at the expense of *Kenya property* decreasing. This feature is present for both the $P3$ and $P4$ portfolio. For $P3$, positive shifts are seen with *Africa property* and *foreign-DM property*, both representing property asset class, that exhibit a moderate risk and return profile. The positive shifts are largely funded from alternative bond asset classes, namely, *Africa bonds*, *foreign-DM bonds*, and *foreign-EM bonds*. Given a decrease in expected return to *Kenya cash*, the allocation to this asset class decreases, at the expense of a Kenya asset class, namely, *Kenya property* increasing. The allocation to *Kenya cash* reducing is expected since it appears less attractive, hence the optimisers preference to allocate to an alternative Kenya asset class, namely *Kenya property*.

Table B.2 illustrates the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *Kenya property*. Recall from §3.5, a 50%:50% weighted linear

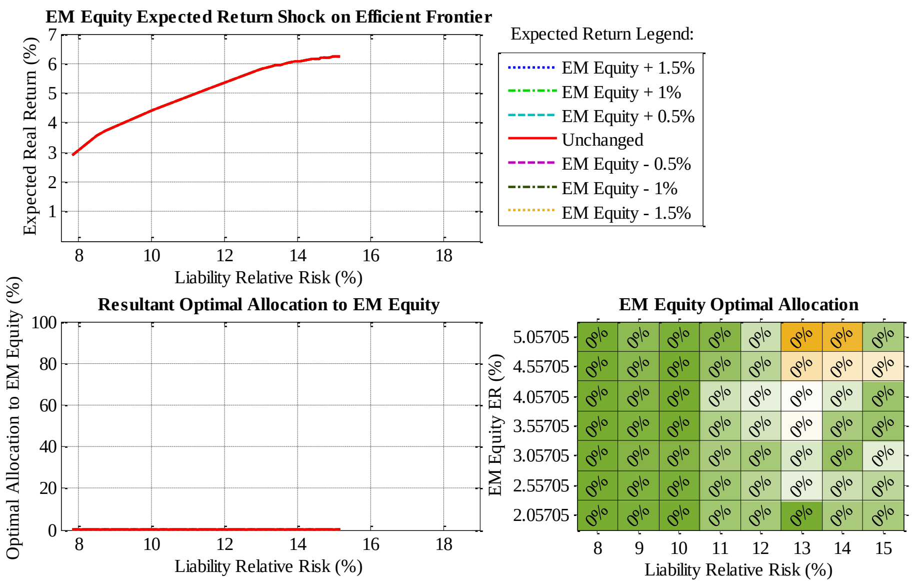


Figure B.9: *Foreign-EM equity expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

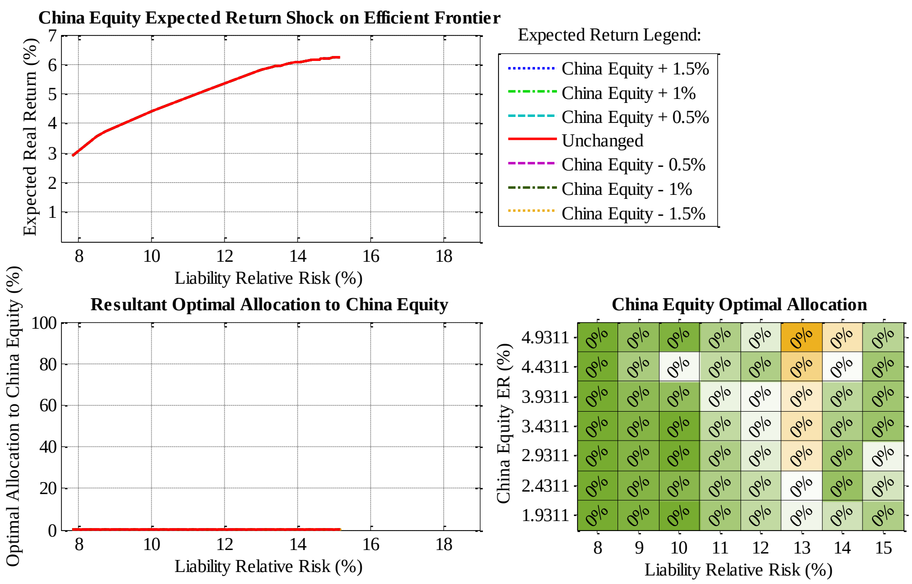


Figure B.10: *China equity expected return sensitivity analysis on efficient frontier and optimal asset allocation.*

Table B.1: *Sensitivity analysis: Kenya cash expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash + 0.5% (2.21%)						
Kenya cash	8.66	15.48	15.38	19.86	+6.72	+4.37
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	4.62	0.14	−6.72	−4.37
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	25.17	21.60	−3.14	−3.37
Africa property	2.92	4.91	10.25	8.34	+7.33	+3.44
Africa equity	-	0.13	-	0.06	-	−0.07
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	1.45	6.52	−4.21	−6.63
DM property	2.71	2.52	3.77	3.32	+1.06	+0.81
DM equity	8.05	4.33	7.88	7.07	−0.17	+2.73
EM bonds	2.34	-	1.48	3.09	−0.86	+3.09
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.54	-	+0.09
expected real return	4.71	4.12	4.78	4.25	+0.06	+0.14
Kenya cash − 0.5% (1.21%)						
Kenya cash	8.66	15.48	4.08	12.78	−4.59	−2.70
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	15.92	7.22	+4.59	+2.70
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	29.84	26.96	+1.53	+1.99
Africa property	2.92	4.91	0.12	2.19	−2.79	−2.72
Africa equity	-	0.13	0.16	0.85	+0.16	+0.72
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	10.59	17.80	+4.93	+4.65
DM property	2.71	2.52	1.86	0.86	−0.84	−1.66
DM equity	8.05	4.33	5.66	1.34	−2.39	−3.00
EM bonds	2.34	-	1.76	-	−0.58	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.65	9.32	-	−0.12
expected real return	4.71	4.12	4.69	3.98	−0.03	−0.14

combination assumption was made for this expected return, given the lack of data. This renders a sensitivity analysis useful for this asset class. Table B.2 shifts the focus to the composition of the optimal portfolios. When increasing the expected real return, the *Kenya property* allocation increases, at the expense of *Kenya cash* decreasing. The converse appears true, when decreasing the expected real return this results in a reduction to the allocation of *Kenya property*, at the expense of *Kenya cash* increasing. This may be as a result of the optimiser preferring to maximise the regional allocation to Kenya (50%), as the optimiser may deem Kenya as a more optimal region compared to the alternative regions. It is further evident from Table B.2 that as a consequence of increasing or decreasing the expected return for *Kenya property* this results in shifts to the composition of other asset classes too. For example, the decrease in expected return to *Kenya property* reveals the allocation to this asset class decreases. This in turn, results in an increase to an alternative property asset class, namely *Africa property*, differing in region.

Table B.3 presents the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *Kenya equity*. When increasing or decreasing the expected return of this asset class, the composition of asset classes shows that the optimiser nevertheless allocates a zero weight to this asset class for the $P3$ portfolio. This suggests that the optimiser finds this asset class as unattractive and sub-optimal, hence the exclusion of this asset class under these expected real return levels. Essentially, this means the return profile on offer for *Kenya equity*, may be too low to justify inclusion, given the high risk profile of *Kenya equity*. The exclusion of this asset class was evident from the results shown in the optimal area graph of Figure 4.2. Immaterial changes are seen with the composition of the $P4$ portfolio.

Table B.4 displays the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *Africa bonds*. Table B.4 shows that when increasing the expected real return, the composition remains very similar for the $P3$ portfolio, with no material changes present. This is partly due to the optimiser already reaching its maximum allocation of 30% to *Kenya bonds* and *Africa bonds*, hence no material changes. For the $P4$ portfolio, the optimiser is only +0.08% short of reaching its upper limit of 30% for *Africa bonds*. This is largely at the expense of a reduction to *Africa property*. Decreasing the expected real return for *Africa bonds* resulted in radical changes to the composition of the $P3$ and $P4$ portfolios. Most notably, a significant reduction in *Africa bonds* is observed, largely at the expense of *Africa property* increasing, and attaining its maximum allocation. The optimisers preference to maximise the allocation to *Africa property* is largely as a result of *Africa property* appearing more attractive (from a risk and return perspective) given that *Africa bonds* has a reduced return. Stated differently, the optimiser is compelled to allocate to the remaining unallocated asset class exhibiting a similar risk and return profile (albeit slightly higher), namely *Africa property*. The foreign components' composition has also shifted, with changes appearing more noticeable with the $P3$ portfolio compared to the $P4$ portfolio. Since the optimiser has radically reduced its allocation to *Africa bonds*, it is compelled to allocate to an alternative bond asset class, namely *foreign-EM bonds*. This feature is more noticeable for the $P3$ portfolio.

Table B.5 showcases the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *Africa property*. For *Africa property*, a 50%:50% weighted linear combination assumption between *Africa bonds* and *Africa equity* was assumed, given the lack of data. Once more, this renders a sensitivity analysis useful for this asset class. Table B.5 shows that when increasing the expected real return, the composition of assets shifts considerably. The most notable shift is the optimisers preference to attain its maximum limit for *Africa property* for the $P3$ portfolio. Since *Africa property* exhibits a moderate risk profile, an increase in return renders this as more attractive, hence this is viewed as a more favourable asset class to include within a portfolio. The *Africa property* allocation increasing, is largely interchanged by a significant decrease in *Africa bonds*. Additional shifts include switches amongst *Kenya cash* and *Kenya bonds*, as well as *foreign-DM bonds*, *foreign-DM equity*, and *foreign-EM bonds*. Decreasing the expected real return for *Africa property* does not result in any material movements

Table B.2: *Sensitivity analysis: Kenya property expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Kenya property + 0.5% (7.04%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	4.08	11.91	−4.59	−3.57
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	15.92	8.09	+4.59	+3.57
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	29.84	27.28	+1.53	+2.31
Africa property	2.92	4.91	0.12	1.94	−2.79	−2.96
Africa equity	-	0.13	0.16	0.78	+0.16	+0.65
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	10.59	17.29	+4.93	+4.14
DM property	2.71	2.52	1.86	0.98	−0.84	−1.53
DM equity	8.05	4.33	5.66	1.73	−2.39	−2.61
EM bonds	2.34	-	1.76	-	−0.58	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.65	9.44	-	−0.01
expected real return	4.71	4.12	4.79	4.14	+0.07	+0.03
Kenya property − 0.5% (6.04%)						
Kenya cash	8.66	15.48	15.38	19.88	+6.72	+4.39
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	4.62	0.12	−6.72	−4.39
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	25.17	21.64	−3.14	−3.32
Africa property	2.92	4.91	10.25	8.26	+7.33	+3.36
Africa equity	-	0.13	-	0.09	-	−0.03
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	1.45	8.24	−4.21	−4.91
DM property	2.71	2.52	3.77	3.09	+1.06	+0.58
DM equity	8.05	4.33	7.88	6.03	−0.17	+1.69
EM bonds	2.34	-	1.48	2.64	−0.86	+2.64
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.41	-	−0.03
expected real return	4.71	4.12	4.68	4.09	−0.04	−0.03

Table B.3: *Sensitivity analysis: Kenya equity expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Kenya equity + 0.5% (6.66%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	8.66	15.29	-	−0.19
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.71	-	+0.19
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	25.10	-	+0.13
Africa property	2.92	4.91	2.92	4.78	-	−0.12
Africa equity	-	0.13	-	0.12	-	−0.01
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	12.93	-	−0.22
DM property	2.71	2.52	2.71	2.55	-	+0.03
DM equity	8.05	4.33	8.05	4.52	-	+0.19
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.48	-	+0.03
expected real return	4.71	4.12	4.71	4.13	-	+0.02
Kenya equity − 0.5% (5.66%)						
Kenya cash	8.66	15.48	8.66	15.68	-	+0.19
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.32	-	−0.19
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.83	-	−0.13
Africa property	2.92	4.91	2.92	5.03	-	+0.12
Africa equity	-	0.13	-	0.14	-	+0.01
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.37	-	+0.22
DM property	2.71	2.52	2.71	2.48	-	−0.03
DM equity	8.05	4.33	8.05	4.15	-	−0.19
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.41	-	−0.03
expected real return	4.71	4.12	4.71	4.10	-	−0.02

Table B.4: *Sensitivity analysis: Africa bonds expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Africa bonds + 0.5% (5.06%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	8.48	15.25	−0.18	−0.24
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.52	4.75	+0.18	+0.24
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	30.00	29.92	+1.69	+4.95
Africa property	2.92	4.91	1.19	-	−1.73	−4.91
Africa equity	-	0.13	-	0.08	-	−0.05
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.54	12.52	−0.13	−0.63
DM property	2.71	2.52	2.44	1.75	−0.27	−0.77
DM equity	8.05	4.33	8.56	5.73	+0.51	+1.40
EM bonds	2.34	-	2.27	-	−0.07	-
EM equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.44	-	-
expected real return	4.71	4.12	4.86	4.26	+0.15	+0.15
Africa bonds − 0.5% (4.06%)						
Kenya cash	8.66	15.48	13.64	19.27	+4.98	+3.78
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	6.36	0.73	−4.98	−3.78
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	-	6.94	−28.31	−18.02
Africa property	2.92	4.91	30.00	23.06	+27.08	+18.15
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	-	12.54	−5.67	−0.61
DM property	2.71	2.52	2.40	1.83	−0.31	−0.68
DM equity	8.05	4.33	-	-	−8.05	−4.33
EM bonds	2.34	-	17.60	5.60	+15.25	+5.63
EM equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.31	−0.01	−0.14
expected real return	4.71	4.12	4.66	3.97	−0.05	−0.15

amongst the composition of asset classes for the $P3$ and $P4$ portfolios. Although it is noted that the optimiser allocates zero to *Africa property* if the expected real return is decreased. This is due to the reduced expected return for *Africa property* rendering this asset class as unappealing. Simply stated, the reduced expected return of *Africa property* does not advocate inclusion given the level of risk an investor is required to take.

Table B.6 show the respective movements in the composition of the asset classes when increasing and decreasing the expected real return respectively, for *foreign-DM property*. If the expected real return of *foreign-DM property* is increased, this results in an increased allocation to *foreign-DM property* under both portfolios $P3$ and $P4$. This increase, is largely funded by a reduction to the alternative *foreign* asset classes. If the expected real return is decreased, the optimiser allocates a zero weight to *foreign-DM property* under both $P3$ and $P4$ portfolios. This means *foreign-DM property* is seen as an unfavourable asset class given that its moderate risk profile does not justify inclusion at a lowered expected return level. As a result, allocations to asset classes such as *foreign-DM equity* and *foreign-EM bonds* are increased. Essentially, since the optimiser allocates a zero weight when decreasing the expected real return to *foreign-DM property*, it is compelled to allocate to alternative asset classes that appear more attractive compared to *foreign-DM property*.

Table B.7 displays the respective movements in the composition of the asset classes when increasing and decreasing the expected real return respectively, for *foreign-DM equity*. If the expected real return is increased, this results in a moderately larger increase in the allocation to *foreign-DM equity*. This is funded from the alternative *foreign* asset classes. If the expected real return is decreased, this in turn, results in the optimiser allocating a zero weight to *foreign-DM equity*. This means *foreign-DM equity* is viewed as an unfavourable asset class given that the moderate risk profile does not justify inclusion at a lowered expected return level. As a result of the optimiser allocating a zero weight to *foreign-DM equity*, this increases the allocation to alternative *foreign* asset classes.

Table B.8 shifts the focus to the composition of the asset allocation for *foreign-EM bonds*. If the expected return is increased, the $P3$ portfolio reaches its maximum attainable allocation of 20% to *foreign-EM bonds*. This is largely at the expense of alternative foreign and African asset classes. As a result of the allocation to *foreign-EM bonds* amounting to 20%, this implies that all other foreign asset classes (excluding Africa) must equal zero for the $P3$ portfolio, as is the case. This ensures the total foreign constraint set (3.8) is met (as is the case). The *foreign-EM bonds* allocation under the $P4$ portfolio increases considerably, largely at the expense of alternative foreign asset classes. By decreasing the expected return of *foreign-EM bonds*, the allocation to *foreign-EM bonds* for $P3$ reduces to zero. While no material changes amongst the composition of asset classes are seen, the reduction to the expected return of *foreign-EM bonds* suggests the return profile is too low given the level of risk an investor is required to take, for the $P3$ portfolio. Immaterial changes are seen with the $P4$ portfolio.

Tables B.9–B.10 presents the updated optimal portfolio allocations ($P3$ and $P4$) and their respective differences between $P1$ and $P2$ for *foreign-EM equity* and *China equity*. Tables B.9–B.10 indicates that neither increasing nor decreasing the expected real return has no impact on the composition of the $P3$ and $P4$ portfolios. Recall, from Figure 4.2, these two asset classes did not feature along the optimal area graph. As noted before, a possible explanation for this, is that the risk profile for these two asset classes are simply too high, to justify the level of return on offer. Hence, the optimiser deems these two asset classes as a non-viable and sub-optimal asset class to include within a portfolio. In addition, since a zero allocation to *foreign-EM equity* is seen, it implies that *China equity* would have a zero allocation too (as is the case). This is due to constraint set (3.11), that ensures the allocation to *foreign-EM equity* should not exceed *China equity*.

Table B.5: *Sensitivity analysis: Africa property expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

Africa property + 0.5% (5.79%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	15.37	19.11	+6.71	+3.62
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	4.63	0.89	−6.71	−3.62
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	3.02	6.25	−25.29	−18.72
Africa property	2.92	4.91	30.00	23.75	+27.08	+18.85
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	-	11.28	−5.67	−1.87
DM property	2.71	2.52	2.10	1.85	−0.61	−0.67
DM equity	8.05	4.33	-	-	−8.05	−4.33
EM bonds	2.34	-	14.88	6.87	+12.54	+6.87
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.40	-	−0.05
expected real return	4.71	4.12	4.81	4.17	+0.10	+0.05
Africa property − 0.5% (4.79%)						
Kenya cash	8.66	15.48	8.04	15.87	−0.62	+0.39
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.96	4.13	+0.62	−0.39
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	29.52	28.13	+1.21	+3.17
Africa property	2.92	4.91	-	-	−2.92	−4.91
Africa equity	-	0.13	0.77	1.87	+0.77	+1.74
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.52	13.72	−0.15	+0.57
DM property	2.71	2.52	2.56	2.11	−0.15	−0.40
DM equity	8.05	4.33	8.61	4.17	+0.56	−0.16
EM bonds	2.34	-	3.02	-	+0.68	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.35	-	−0.10
expected real return	4.71	4.12	4.71	4.06	-	−0.05

Table B.6: *Sensitivity analysis: DM property expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

DM property + 0.5% (4.00%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	10.56	17.10	+1.90	+1.61
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	9.44	2.90	−1.90	−1.61
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	25.22	22.42	−3.09	−2.54
Africa property	2.92	4.91	5.59	7.58	+2.68	+2.67
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.57	12.80	−0.10	−0.35
DM property	2.71	2.52	8.24	5.59	+5.53	+3.07
DM equity	8.05	4.33	5.10	1.61	−2.95	−2.72
EM bonds	2.34	-	0.28	-	−2.07	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.40	-	−0.04
expected real return	4.71	4.12	4.74	4.12	+0.03	-
DM property − 0.5% (3.00%)						
Kenya cash	8.66	15.48	8.43	15.33	−0.23	−0.16
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.57	4.67	+0.23	+0.16
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.06	25.27	−0.25	+0.31
Africa property	2.92	4.91	3.15	4.52	+0.24	−0.39
Africa equity	-	0.13	-	0.21	-	+0.08
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	4.42	11.97	−1.25	−1.18
DM property	2.71	2.52	-	-	−2.71	−2.52
DM equity	8.05	4.33	8.54	5.33	+0.49	+0.99
EM bonds	2.34	-	5.83	2.70	+3.48	+2.70
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.46	-	+0.01
expected real return	4.71	4.12	4.71	4.12	-	-

Table B.7: *Sensitivity analysis: DM equity expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

DM equity + 0.5% (3.39%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	12.19	18.56	+3.53	+3.08
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	7.81	1.44	−3.53	−3.08
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	30.00	27.60	+1.69	+2.64
Africa property	2.92	4.91	-	2.40	−2.92	−2.51
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	1.16	9.62	−4.50	−3.53
DM property	2.71	2.52	0.01	-	−2.70	−2.52
DM equity	8.05	4.33	18.83	10.38	+10.78	+6.05
EM bonds	2.34	-	-	-	−2.34	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.38	-	−0.06
expected real return	4.71	4.12	4.79	4.12	+0.08	+0.01
DM equity − 0.5% (2.39%)						
Kenya cash	8.66	15.48	8.54	15.01	−0.12	−0.47
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.46	4.99	+0.12	+0.47
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	22.53	21.35	−5.78	−3.62
Africa property	2.92	4.91	10.08	8.53	+7.16	+3.62
Africa equity	-	0.13	-	0.12	-	-
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	6.13	13.37	+0.46	+0.22
DM property	2.71	2.52	3.34	2.97	+0.63	+0.45
DM equity	8.05	4.33	-	-	−8.05	−4.33
EM bonds	2.34	-	7.92	3.66	+5.57	+3.66
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.44	-	-
expected real return	4.71	4.12	4.70	4.11	−0.01	−0.01

Table B.8: *Sensitivity analysis: EM bonds expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

EM bonds + 0.5% (2.01%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3-P1$	Diff. $P4-P2$
Kenya cash	8.66	15.48	9.18	17.99	+0.52	+2.51
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	10.82	2.01	−0.52	−2.51
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	17.90	18.67	−10.42	−6.29
Africa property	2.92	4.91	12.10	11.33	+9.19	+6.42
Africa equity	-	0.13	-	-	-	−0.13
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	-	7.32	−5.67	−5.83
DM property	2.71	2.52	-	-	−2.71	−2.52
DM equity	8.05	4.33	-	-	−8.05	−4.33
EM bonds	2.34	-	20.00	12.68	+17.66	+12.68
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.42	−0.01	−0.03
expected real return	4.71	4.12	4.80	4.14	+0.09	+0.03
EM bonds − 0.5% (1.01%)						
Kenya cash	8.66	15.48	9.59	15.80	+0.93	+0.31
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	10.41	4.20	−0.93	−0.31
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.25	24.75	−0.06	−0.21
Africa property	2.92	4.91	3.93	5.11	+1.02	+0.20
Africa equity	-	0.13	-	0.14	-	+0.01
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	6.08	13.50	+0.41	+0.35
DM property	2.71	2.52	3.30	2.46	+0.59	−0.05
DM equity	8.05	4.33	8.44	4.03	+0.38	−0.30
EM bonds	2.34	-	-	-	−2.34	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.39	-	−0.05
expected real ret.	4.71	4.12	4.71	4.09	-	−0.03

Table B.9: *Sensitivity analysis: EM equity expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

EM equity + 0.5% (4.06%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	8.66	15.48	-	-
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.52	-	-
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.97	-	-
Africa property	2.92	4.91	2.92	4.90	-	-
Africa equity	-	0.13	-	0.13	-	-
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.15	-	-
DM property	2.71	2.52	2.71	2.52	-	-
DM equity	8.05	4.33	8.05	4.34	-	-
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real ret.	4.71	4.12	4.71	4.12	-	-
EM equity − 0.5% (3.06%)						
Kenya cash	8.66	15.48	8.66	15.49	-	-
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.51	-	-
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.96	-	-
Africa property	2.92	4.91	2.92	4.91	-	-
Africa equity	-	0.13	-	0.13	-	-
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.15	-	-
DM property	2.71	2.52	2.71	2.52	-	-
DM equity	8.05	4.33	8.05	4.33	-	-
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real ret.	4.71	4.12	4.71	4.11	-	-

Table B.10: *Sensitivity analysis: China equity expected return increased and decreased by +0.5% and −0.5% respectively, and the impact on two optimised portfolios, namely P3 and P4 respectively.*

China equity + 0.5% (3.93%)						
Asset class	$P1$	$P2$	$P3$	$P4$	Diff. $P3 - P1$	Diff. $P4 - P2$
Kenya cash	8.66	15.48	8.66	15.48	-	-
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.52	-	-
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.97	-	-
Africa property	2.92	4.91	2.92	4.91	-	-
Africa equity	-	0.13	-	0.13	-	-
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.15	-	-
DM property	2.71	2.52	2.71	2.52	-	-
DM equity	8.05	4.33	8.05	4.33	-	-
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real ret.	4.71	4.12	4.71	4.12	-	-
China equity − 0.5% (2.93%)						
Kenya cash	8.66	15.48	8.66	15.48	-	-
Kenya bonds	30.00	30.00	30.00	30.00	-	-
Kenya property	11.34	4.52	11.34	4.52	-	-
Kenya equity	-	-	-	-	-	-
Africa bonds	28.31	24.97	28.31	24.97	-	-
Africa property	2.92	4.91	2.92	4.91	-	-
Africa equity	-	0.13	-	0.13	-	-
DM cash	-	-	-	-	-	-
DM bonds	5.67	13.15	5.67	13.15	-	-
DM property	2.71	2.52	2.71	2.52	-	-
DM equity	8.05	4.33	8.05	4.33	-	-
EM bonds	2.34	-	2.34	-	-	-
EM equity	-	-	-	-	-	-
China equity	-	-	-	-	-	-
liability-rel. risk	10.64	9.45	10.64	9.45	-	-
expected real ret.	4.71	4.12	4.71	4.11	-	-

APPENDIX C

Constraint Sensitivity Analysis

In this appendix, the sensitivity analysis results of the remaining two constraints discussed in §5.3 are presented. This pertains to the maximum Kenya constraint and maximum "single" asset class constraint.

Figure C.1 shows the movement of the efficient frontier when applying alternative limits to the Kenya weighting constraint set (5.1). Recall, this ranges from $M_2 \in \{30\%, 40\%, \mathbf{50\%}, 60\%, 70\%\}$. A broad trend may be observed from Figure C.1. The higher the limit imposed on Kenya, the *more* risk is incurred and added at the end of the efficient frontier. Consequently, a *higher* return can be achieved. Figure C.2 illustrates the progression of the optimal area graphs as the Kenya constraint is increased. Lower risk and return asset classes such as *foreign-DM cash* and *foreign-DM bonds* gradually decrease, as the limit imposed on Kenya increases. This is due to the optimisers preference to include additional Kenya exposure, given its favourable risk and return profile relative to these two asset classes. In Figure C.2e, a marginal amount of *Kenya equity* can be seen at higher risk levels, contributing to the increase in risk and return displayed in Figure C.1.

Figure C.3 shows the impact of the efficient frontier when changing the limit to constraint (3.9). Recall this ranges from $M_3 \in \{20\%, 25\%, \mathbf{30\%}, 35\%, 40\%\}$. Figure C.3 reveals that as the ceiling, per asset class, is increased, the *minimum-variance portfolio* is lower from both a risk and return perspective. Stated differently, a larger cap imposed per asset class delivers additional return. However, at the expense of incurring additional risk at the lowest risk levels. Figure C.4 shows the optimal area graphs for the various limitations imposed on the single asset class constraint. *Kenya bonds* attains its maximum allocation each time the limit is increased. As seen with previous results, this is due to the attractive risk and return profile of *Kenya bonds*, hence the optimisers preference to attain the limit on each occasion. From a regional perspective, the optimiser maximises the Kenya region asset allocation given its favourable risk, return profile and diversification characteristics. The foreign region also remains largely similar, with Africa dominating higher risk levels given its higher risk and return profile.

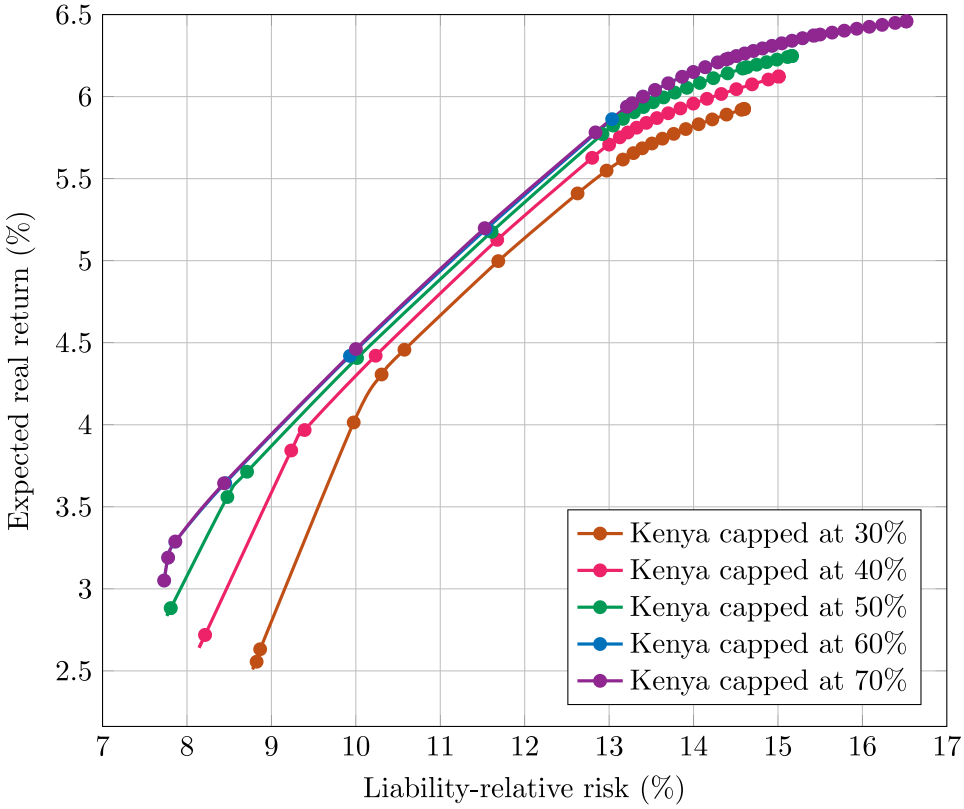
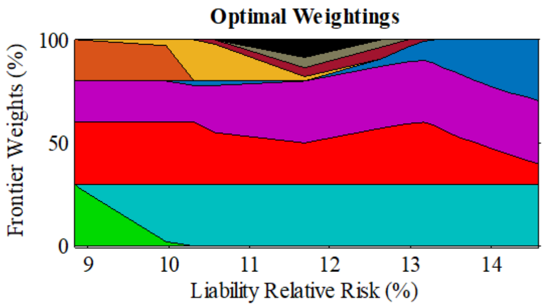
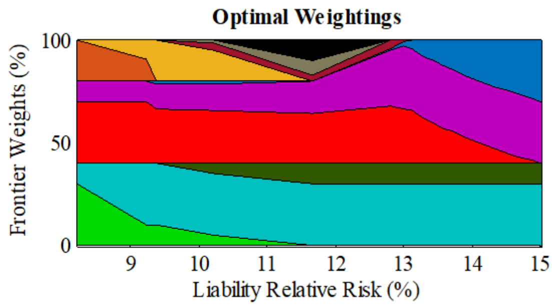


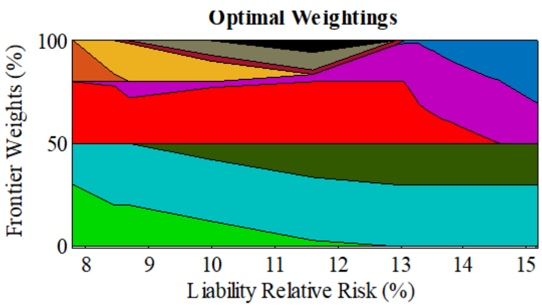
Figure C.1: *Sensitivity analysis conducted on alternative caps pertaining to the Kenya constraint, ranging from 30% to 70%, in increments of 10%, measured in risk and return terms.*



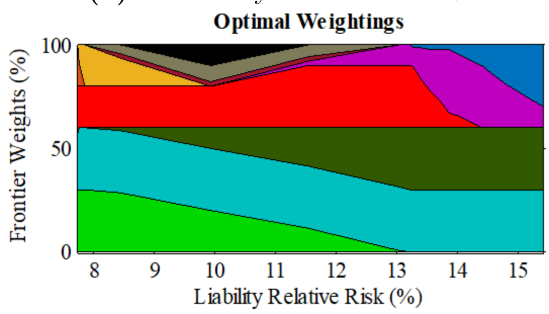
(a) *Total Kenya limited to 30%.*



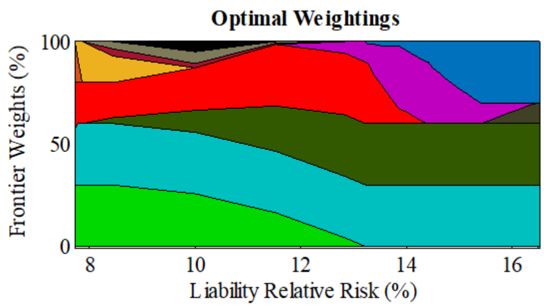
(b) *Total Kenya limited to 40%.*



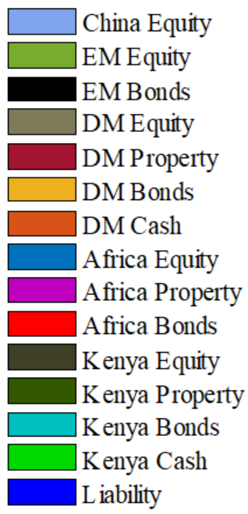
(c) *Total Kenya limited to 50%.*



(d) *Total Kenya limited to 60%.*



(e) *Total Kenya limited to 70%.*



(f) *Colour palette legend.*

Figure C.2: *Sensitivity analysis displayed as optimal area graphs for the Kenyan limits, ranging from 30% to 70%, in increments of 10%.*

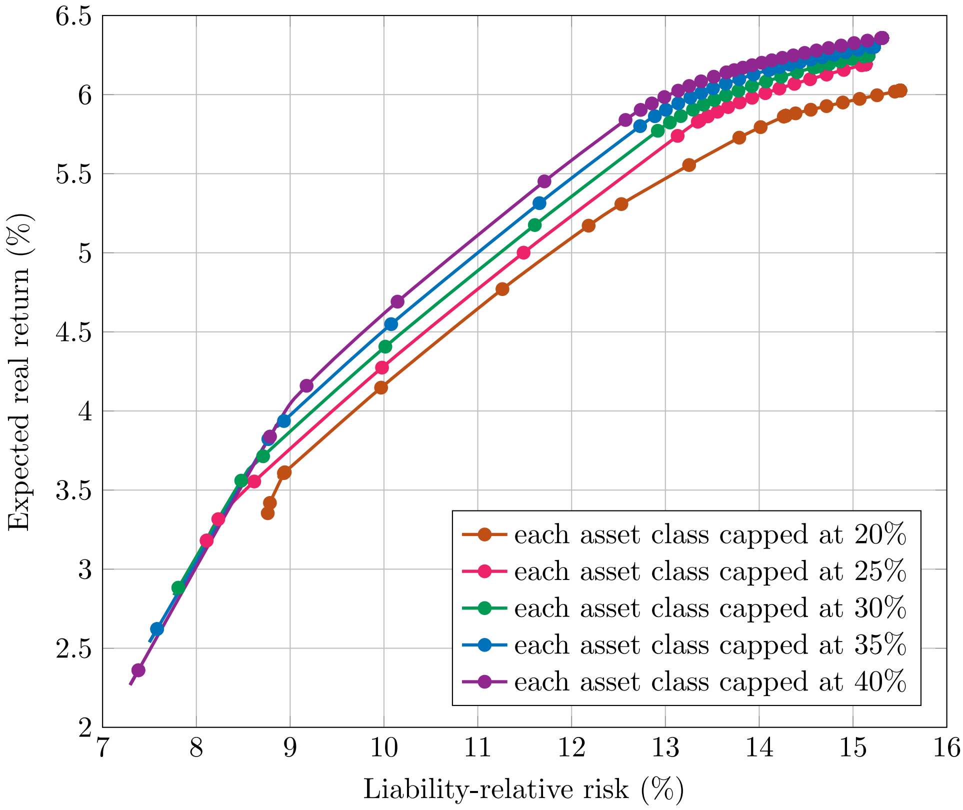
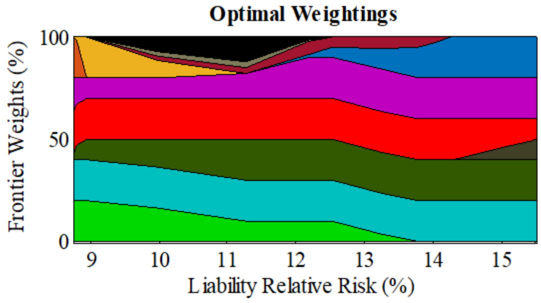
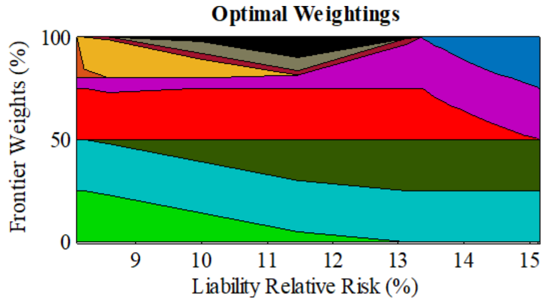


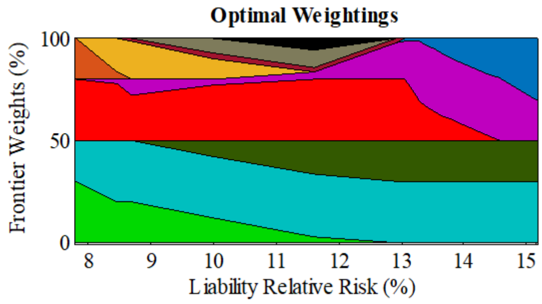
Figure C.3: *Sensitivity analysis conducted on alternative caps pertaining to the single asset class constraint, ranging from 20% to 40%, in increments of 5%, measured in risk and return terms.*



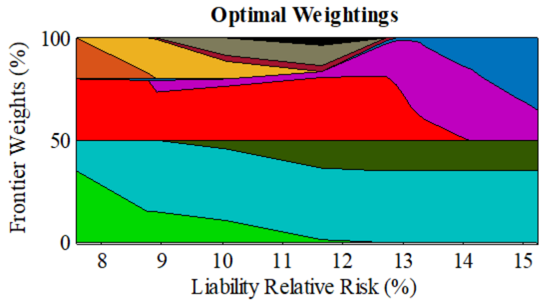
(a) *Each asset class limited to 20%.*



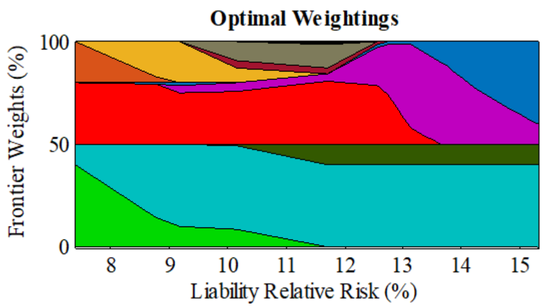
(b) *Each asset class limited to 25%.*



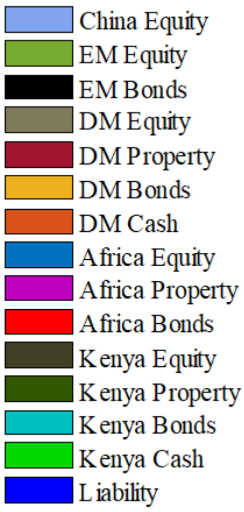
(c) *Each asset class limited to 30%.*



(d) *Each asset class limited to 35%.*



(e) *Each asset class limited to 40%.*



(f) *Colour palette legend.*

Figure C.4: *Sensitivity analysis displayed as optimal area graphs for the single asset class limits, ranging from 20% to 40%, in increments of 5%.*