A case study of mental mathematics lessons: Analysing early grade teachers' perceptions of their practice

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Declaration

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Abstract

Research has shown that South African primary school students are performing below international and national grade level expectations for mathematics. It can be argued that a root cause is students' reliance on inefficient counting-based versus reasoning-based strategies for calculating. Personal experience with supporting teachers in a wide range of classrooms resonates with the literature, which reveals that the reliance on counting-based strategies hinders the development of more efficient, number range appropriate strategies in the later years. Teaching approaches that favour learning through the memorisation of facts, rules and procedures in the early grades encourage the use of learnt procedures over the development of reasoning-based strategies. Alternatively, teaching for understanding and reasoning negates the need for memorisation and supports the development of reasoning-based calculating strategies. In '*Adding it up: Helping children learn mathematics'*, Kilpatrick, Swafford and Findell (2002) refer to mathematical proficiency to convey what they think it means to be successful in mathematics. Mathematical proficiency describes success in mathematics as the ability not only to calculate accurately, but also to understand, apply, reason and engage with mathematics. Mathematical proficiency consists of an interrelated set of actions that are equally important in contributing to success in mathematics. These actions, or strands of mathematical proficiency, are comprised of a combination of calculating, understanding, applying, reasoning and engaging.

Mental mathematics forms an integral part of the development of number sense in the early years and can mean so much more than the recall of facts. If taught for understanding and reasoning, mental mathematics can support the development of mathematical proficiency, and result in the development of reasoning-based calculating strategies.

This is a qualitative case study that analyses how early grade teachers perceive their mental mathematics teaching practice. Using Activity Theory as the analytical framework, the study considers the interrelated dimensions that contribute to the activity of teaching. Two Grade 3 teachers participated in the study. Data were collected through introductory interviews (semi-structured), lesson observation video recordings, self-reflection checklists and reflective interviews (unstructured). There was a workshop session where mathematical proficiency and the resultant implications for teaching mental mathematics were discussed. The teachers then reflected on their own practice via video recordings using the lens of mathematical proficiency. The reflections were for both of their lesson observations: one that took place before the workshop session (pre-workshop lesson) and one that took place after (post-workshop lesson). The analysis explored how the teachers perceived their own teaching through self-reflection. Using Activity Theory as the analytical framework allowed for the analysis of relationships within the activity of teaching that described the teachers' perceptions of practice.

The analysis revealed that both teachers have an awareness of where and how they should adapt their practice, and their perceptions of practice revealed similar themes, namely:

1. **Object (lesson objective)**

   Move beyond the constraints of 'knowing' and 'calculating' during mental mathematics lessons: to create opportunities to develop understanding, application and reasoning.
2. **Tools**

   Move beyond the 'knowing' level of mental mathematics task items: to elicit application and reasoning through the use of more cognitively demanding tasks that are purposefully utilised to allow noticing of patterns relationships.
3. **Division of labour**

   Move beyond 'answer only' questioning in mental mathematics lessons: to facilitate student discussion and reflection through more deliberate planning.

The findings of this study highlighted tensions between existing practice and desired practice as reflected on through the lens of mathematical proficiency. These tensions, if further explored and supported with ongoing reflection, may lead to professional learning opportunities that enable transformative teacher practice.

Opsomming

Volgens navorsing voldoen Suid-Afrikaanse laerskoolleerders nie aan internasionale en nasionale graadvlakverwagtinge in wiskunde nie. Een oorsaak hiervoor is moontlik leerders se afhanklikheid van ondoeltreffende telgebaseerde in plaas van denkgebaseerde berekeningstrategieë. Persoonlike ervaring van onderwysersteun in ’n wye verskeidenheid klaskamers bevestig wat die literatuur sê, naamlik dat die afhanklikheid van telgebaseerde strategieë die latere ontwikkeling van doeltreffender, getalreeksgepaste strategieë verhinder. Onderrigbenaderings in die vroeër grade wat op leer deur die memorisering van feite, reëls en prosedures afgestem is, moedig die gebruik van aangeleerde prosedures bo die ontwikkeling van denkgebaseerde strategieë aan. Daarteenoor skakel onderrig met die oog op begrip en redenering die behoefte aan memorisering uit en ondersteun die ontwikkeling van denkgebaseerde berekeningstrategieë. In ‘*Adding it up: Helping children learn mathematics’* verwys Kilpatrick, Swafford en Findell (2002) na ‘wiskundige vaardigheid’ om te verwoord wat dit volgens húlle beteken om suksesvol in wiskunde te wees. Die konsep van wiskundige vaardigheid doen aan die hand dat sukses in wiskunde nie net daaroor gaan om akkuraat te kan bereken nie, maar ook om wiskunde te verstaan, te kan toepas, te bestudeer en daaroor te kan redeneer. Wiskundige vaardigheid bestaan uit ’n onderling verwante stel aksies wat elk ewe veel tot sukses in wiskunde bydra. Hierdie aksies, of onderdele, van wiskundige vaardigheid behels ’n kombinasie van berekening, begrip, toepassing, redenering en studie.

Kopwiskunde (‘mental mathematics’) maak ’n kerndeel uit van die ontwikkeling van getalbegrip in die vroeë jare, en kan uit veel meer as die blote oproep van feite bestaan. Indien kopwiskunde met die oog op begrip en redenering onderrig word, kan dit die ontwikkeling van wiskundige vaardigheid ondersteun en tot denkgebaseerde berekeningstrategieë lei.

Hierdie navorsing is ’n kwalitatiewe gevallestudie wat vroeëgraadonderwysers se opvattings oor hulle eie kopwiskundeonderrigpraktyk ontleed. Met aktiwiteitsteorie as die analitiese raamwerk ondersoek die studie die onderling verwante aspekte wat tot die onderrigaktiwiteit bydra. Twee graad 3-onderwysers het aan die studie deelgeneem. Data is deur (semigestruktureerde) inleidende onderhoude, selfbesinning oor leswaarnemings (kontrolelys) en (ongestruktureerde) nadenkende onderhoude ingesamel. ’n Werksessie is gehou waar wiskundige vaardigheid en die gevolglike implikasies vir die onderrig van kopwiskunde bespreek is. Daarna het die onderwysers deur die lens van wiskundige vaardigheid oor video-opnames van twee van hulle eie lesse besin: een wat voor die werksessie plaasgevind het, en die ander ná die tyd. Deur hierdie selfbesinning kon die onderwysers se opvattings oor hulle eie onderrig verken word. Met aktiwiteitsteorie as die analitiese raamwerk is ’n ontleding onderneem van die verhoudings binne die onderrigaktiwiteit wat die onderwysers se praktykopvattings rig.

Die ontleding toon dat albei onderwysers weet waar en hoe hulle hulle praktyk behoort aan te pas. Hulle praktykopvattings bring ook soortgelyke temas aan die lig, naamlik:

1. **Doelstelling**

   Gaan verder as ‘ken’ en ‘bereken’ gedurende kopwiskundelesse. Skep geleenthede om werklik te verstaan, toe te pas en te redeneer.

2. **Gereedskap**

   Gaan verder as die ‘ken’-vlak van kopwiskundetake. Gebruik kognitief uitdagender take om toepassing en redenering aan te moedig.

3. **Arbeidsverdeling**

   Gaan verder as vrae wat slegs ’n antwoord vereis in kopwiskundelesse. Beplan doelbewus om leerdergesprekke en -besinning in die hand te werk.

Die bevindinge van hierdie studie beklemtoon die spanning tussen bestaande en gewenste onderrigpraktyk deur die lens van wiskundige vaardigheid. Voortdurende besinning oor, en verdere verkenning van, hierdie spanning kan tot professionele leergeleenthede lei wat transformerende onderrigpraktyk moontlik maak.

Dedications

Leo Rust: For supporting me, loving me and giving me the space … with endless patience, presence and encouragement.

Aarnout Brombacher: For showing me the way, and showing me how … ‘knowing what to do when you don’t know what to do.’

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CAPS: Curriculum and Assessment Policy Statement

ANAs: Annual National Assessments

DBE: Department of Basic Education

IRE: Initiate-Respond-Evaluate

Chapter 1: The Context and Purpose

1.1 Introduction

This chapter introduces the study and provides the context and motivation for analysing early grade teachers' reflections on, and perceptions of, their mental mathematics teaching practice.

A primary school teacher by profession, with the majority of my teaching practice in the remedial environment, I have now shifted my focus from supporting students to supporting teachers. My current employment includes working with early grade mathematics teachers as a teacher-coach. This coaching support occurs in a group of low, or no-fee state (public) schools across the Western Cape (South Africa), once a week, during the mathematics lessons. The role of the coach in this context is to facilitate the adoption of mathematics routines that promote the teaching and learning of mathematics for understanding and reasoning. These routines consist of structured counting, manipulating number (mental mathematics) and problem-solving activities. The routines are guided by the NumberSense Mathematics Programme[1], which has been created to support the development of the foundational skills (Aunio and Räsänen, 2015), focusing on developing a strong sense of number and an understanding of mathematics. The design of the NumberSense Mathematics Programme is informed by current research on how children learn mathematics (Brombacher, 2012; Kilpatrick, Swafford and Findell, 2002).

It is through this work, exposure to, and involvement in early grade classrooms in South Africa, that my curiosity for *what is* and *what could be*, in terms of the teaching (and the resultant learning) of mathematics, has grown.

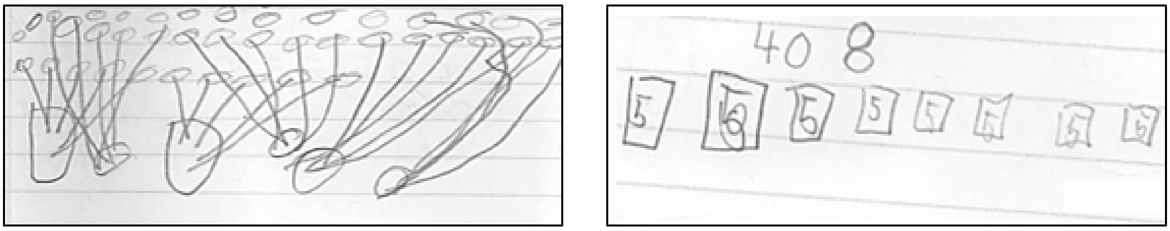
[1] www.NumberSense.co.za The NumberSense Mathematics Programme is appropriate for the South African context, and is available in all the official languages.

1.2 Getting it 'right' in the early grades: The foundation for future mathematical success

It is no secret that South African students are underperforming in mathematics, as reported by McCarthy and Oliphant (2013). For many students their mathematical future is predetermined around Grade 4 level (Spaull and Kotze, 2015; Van der Berg, 2015; Venkat and Spaull, 2015). The research suggests that students in the early grades (Grades 1-3) do not develop the foundational skills in mathematics that enable them to be successful beyond Grade 4. According to McCarthy and Oliphant (2013) and Venkat and Spaull (2015), much of the reason that students do not develop the necessary foundational skills has to do with how they are taught mathematics and has little to do with the mathematics that they are taught. With reference to an abundance of evidence from South African and international research (Ensor *et al.*, 2009; Gervasoni, 2011; Graven *et al.*, 2013; Weitz and Venkat, 2013), it has and can be argued that a cause of this low performance in the early grades is students' reliance on inefficient counting-based strategies (concrete representations of number) for calculating. Not moving beyond counting-based strategies to more sophisticated reasoning-based strategies (more abstract representations of number) hinders the development of the foundational skills needed to progress to more complex concepts and tasks addressed in the later years. The examples of student work in figure 1.1. illustrate the difference between counting and reasoning-based strategies, and are from a Grade 2 class that forms part of my teacher-coaching.

Figure 1.1: Grade 2 students' worked examples of the two different strategies, for the same question: "If 40 oranges are put into packets with 5 oranges in a packet, how many packets will there be?"

Counting-based strategy | **Reasoning-based strategy**



Counting-based strategies are inefficient for higher number ranges and result in incomplete, inefficient and confusing strategies.

Reasoning-based strategies use the knowledge of the 'basic' facts that are applied with understanding and reasoning.

Reliance on counting-based strategies tends to continue into the Intermediate Phase (Grades 4-6), and contributes to the underperformance of mathematics in the later years (Venkat and Askew, 2012). The dependence on counting-based strategies is encouraged by teaching practices that promote concrete strategies over the more abstract ways of working with number (Ensor *et al.*, 2009), and inhibit opportunities for students to develop more symbolic representations of number that characterise mathematics in the later years. The Centre for Development and Enterprise reports that deficits in learning acquired in the early years are cumulated rather than reduced as the students advance through the grades (CDE, 2014; McCarthy and Oliphant, 2013). This accumulation of deficits is evident in that, of the students who start school in South Africa, many do not complete their schooling. As shown in table 1.1, of those students who do enter into matric (the final year in the South African schooling system), there are very few who experience success with the mathematics at this level.

Table 1.1: Analysis of the 2019 matric results for mathematics (Department of Basic Education, 2020)

# Started Grade 1 in 2008	# Wrote 2019 matric	# Wrote 2019 mathematics	# > 30% for mathematics	# > 40% for mathematics	# > 80% for mathematics
1 122 114	504 303	222 034	121 179	77 751	4 415

The quote below, summarises the analysis in table 1.1.

> *"Of the 42% who passed matric, 19% attempted mathematics, only 0,4% achieved an 'A' for mathematics."* (Brombacher, 2020)

Recognising that the link between teaching and learning is complex, ongoing research supports the claim that different teaching approaches tend to strongly predict different kinds of learning (Hiebert and Grouws, 2007). It could therefore be suggested that the prevalence and reliance on counting-based calculating strategies, as opposed to the development of reasoning-based strategies, indicates a predominance of teaching approaches that result in learning that does not progress beyond counting-based strategies. In South Africa, the predominant view on teaching mathematics is to teach through rules, formulas and procedures which result in learning through memorisation often without understanding (Rossouw, Rhodes and Christiansen, 2000). The teaching of rules, formulas and procedures does not support the development of more sophisticated and efficient reasoning-based strategies (Venkat and Spaull, 2015). Indeed, *"It may be more important in the mathematics class how you teach, not what you teach."* (Polya, 1981, p. 118). With reference to Polya's quote, and the 'how' mathematics is taught, it is clear that in many cases the teaching practices in the early grades are not resulting in learning that supports students' access to mathematics in the later grades.

My own learning and teaching experience of mathematics has been that of coming to know facts through memorisation. Over the past six years, having been exposed to the teaching of mathematics for understanding and reasoning has led to a shift in my own mathematical understandings and beliefs of what it means to do and be successful in mathematics. Reflecting on my own shifts in practice furthers my curiosity for *what is* and *what could be* in terms of teaching mathematics, and the implications this has on the perceptions of the objectives, teacher roles and resources, beliefs and outcomes of mathematics teaching.

1.3 Using the right tools in the right way: The role of mental mathematics

Modern day technology means that at any given time we have access to a multitude of devices that can assist with any number of calculations, making us less dependent on our own ability and need to calculate (Wolfram, 2010). As the rote learning of mathematical procedures no longer has the clear utility it once had, Wolfram (2010) argues for prioritising the development of critical thinking, problem-solving and reasoning skills as the objective of mathematics teaching and learning.

For the purposes of this study and for consistency, I choose to use the term *mental mathematics* to draw attention to the core meaning of the word *mathematics* - *to know how to know*. We can compare the term in Germanic languages – *wiskunde* – *knowledge of knowing* and in contrast to the phrase *mental arithmetic* (as in *arithmetic* from the Greek *arithmos*, meaning number) which draws attention to the direct number-like utility of mental calculations, and vitiates the reasoning involved in mental mathematics.

My experience with supporting teachers in a wide range of classrooms resonates with the literature (Rossouw, Rhodes and Christiansen, 2000) that indicates that the dominant approach to teaching mathematics in the early grades still relies largely on memorised facts and the use of rote procedures. This is most evident in the way in which mental mathematics is taught. Mental mathematics is traditionally viewed as the drill and memorisation of facts, which overlooks the role that mental mathematics can play in developing reasoning-based, as opposed to counting-based, calculating strategies (Gürbüz and Erdem, 2016). Although the South African Curriculum and Assessment Policy Statement[2] (CAPS) document indicates that mental mathematics *"features strongly in both the counting and the number concept development sections"* (Department of Basic Education, 2011, p. 12), it is described with phrases such as *"brisk mental starters"*; *"to know or recall fairly quickly"*; *"rapidly recall"*; and *"use the following calculation strategies"* (Department of Basic Education, 2011, pp. 11, 12, 23). These phrases indicate little suggestion or support for the role that mental mathematics plays in developing reasoning-based calculating strategies.

There is much research that supports the argument for the importance of mental mathematics in developing reasoning-based calculating strategies, and problem-solving skills

[2] Curriculum and Assessment Policy Statement (CAPS) is the current South African teaching curriculum and is a revision of the previous National Curriculum Statement (NCS). CAPS provides teachers with detailed guidelines as to what to teach (Department of Basic Education, 2011).

that promote success in later written calculations (Gürbüz and Erdem, 2016; Swan and Sparrow, 2001; Threlfall, 2002). Gürbüz and Erdem (2016) draw attention to the importance of mathematical patterns and relationships in developing proficiency with mental calculations. They argue for mental mathematics as a way of calculating which is processed 'with your head' (with understanding and reasoning), whether written down or verbalised, rather than processed 'in your head' with no representation other than the answer (with limited understanding and reasoning). Mental calculation processed with understanding and reasoning, 'with your head', can be illustrated by using the example of 17x8: for those who have learnt through the memorisation and recall of number facts, this may seem impossible to calculate mentally, as the last memorised fact in the 8 times table is traditionally 12x8. But those who have developed understanding and reasoning skills (with your head) may calculate 17x8 with an awareness that to multiply by 8 is to double (multiply by 2) and double again (to multiply by 4) and double again (to multiply by 8). This strategy does not rely only on memory in order to find the solution: 17 doubled (x2) = 34, double again (x4) = 68, and double again (x8) = 136, so 17x8 = 136.

Recording the process of strategy selection and implementation used when calculating mentally 'with your head', is important in supporting the development of reasoning (Gürbüz and Erdem, 2016). The process of calculating can be reflected on, tracked and recorded through discussion, or by way of written notes (Gürbüz and Erdem, 2016; MacLellan, 2001; Torbeyns and Verschaffel, 2013). Mental mathematics, if taught for understanding, can mean so much more than the memorisation and recall of number facts and is key to developing a range of reasoning-based calculating strategies that can be applied fluently with flexibility and success. Swan and Sparrow (2001) argue that the traditional rapid-fire questioning approach of mental recall produces anxiety in many students and reduces flexibility of thinking, and does little to support the development of 'doing mathematics'. Swan and Sparrow (2001) suggest that students who make use of mental strategies are 'doing mathematics' and developing number sense rather than remembering procedures. A strong sense of number (number sense) means that students can work flexibly and fluently with numbers and number combinations, and they have a range of effective calculating strategies (Brombacher *et al.*, 2002; Swan and Sparrow, 2001). Number sense is the basis of mathematical learning in the early grades, and the development of number sense, mental strategies and fluency must be prioritised in order to improve future success in mathematics (Graven *et al.*, 2013).

The teaching of mental mathematics, with your head (with reasoning) has a profoundly different teaching objective (and outcome) to the teaching of mental mathematics in your head (with limited reasoning). Following the argument that mental mathematics plays a role in developing reasoning-based calculating strategies adds to my curiosity for what is and what could be in terms of the teaching (and the resultant learning) of mathematics. The what is refers to the predominant, more traditional outlook of teaching mental mathematics for memorisation and recall (with limited reasoning), and the what could be references an alternative objective of teaching mental mathematics for supporting the development of understanding and reasoning.

1.4 Teaching Practice: What is and what could be

Few teachers will disagree with the fact that they want their students to progress mathematically and to develop understanding, yet there is a difference in how this is perceived, practised and achieved. Working on the premise that teachers have the best interests of their students at heart, then it is the beliefs and understandings of what doing mathematics for 'the best interests' entails that would lead them to make the teaching choices that they make (Schoenfeld, 2017). In my experiences as a teacher-coach I have noticed that many teachers lack the confidence to seek and implement alternative teaching approaches that support the development of understanding and reasoning, are not aware of alternatives, or do not hold a belief that motivates the need for a different approach.

Through my observations as a teacher-coach I distinguish between drill and memorisation teaching approaches, and between understanding and reasoning approaches with specific reference to mental mathematics. These practices tend to be distinguishable by: the objective or purpose for the teaching; the teacher actions or role of the teacher in the teaching process; and the use of tasks or tools. While the observations and details of the teaching approaches summarised in table 1.2 stem from my experiences, they resonate with the literature which describes and represents shifts in teaching practice on a continuum (McGatha *et al.*, 2018).

The indicators of practice in table 1.2 can be likened to, and linked to the main tenets of Activity Theory (object, subject, tools). Within Activity Theory the subject uses mediated tools to reach the object (Pather, 2012). Activity Theory will be described in detail in chapter 2.

Table 1.2: An initial framework for the study of mental mathematics teaching practices

Practice	***Drill and memorisation teaching approach*** →	***Understanding and reasoning teaching approach***
Purpose	Focus on memorisation. Mastering already formulated knowledge or facts	Focus, or emerging focus, on understanding Exploring mathematical concepts through facilitation and participation
Teacher Role	Teacher instructs or 'tells'; students respond with an answer; teacher confirms if response is correct or incorrect	Teacher invites and directs student participation; students discuss their thinking; teacher facilitates discussion to clarify understanding
Tasks	Mental mathematics tasks are used in a random way, with limited connection that encourage an answer only Inconsistent practice and exposure to mental mathematics	Mental mathematics items are deliberately structured and used to reveal patterns, and encourage application in that they go beyond just requiring an answer Consistent, daily practice

During my observations and interactions, I noted that many teachers do not fall into one approach or the other, and some are in between approaches, somewhere on the continuum from 'drill and memorisation' to 'understanding and reasoning'. These two approaches therefore signify opposite ends of a spectrum rather than an either or divide. My observations and interactions during the coaching sessions tell only part of the story and do not include the teachers' perspectives. The questions that are raised for me are, "How do teachers perceive their practice? How do they define their objectives and action them in a mental mathematics lesson?" Furthermore, teachers make these decisions as part of a system, subject to many interrelating relationships and demands, for example, parent demands and curriculum requirements. Apart from these systems (communities) that shape practice, teachers also bring their own beliefs, experiences and rules to their practice.

1.5 Research Purpose: To analyse teachers' perceptions of their mental mathematics teaching practice

The purpose of this study is to analyse teachers' reflections on, and perceptions of their mental mathematics teaching practice and to answer, "How do teachers' perceive their practice?" The teachers who participated in this study are also engaged participants in the coaching programme. They receive weekly mathematics coaching during one of their mathematics lessons. The coaching aims to facilitate the adoption of mathematics routines that promote the teaching of mathematics for understanding and reasoning. The participating teachers have all been introduced to the routines and are in various stages of adopting and implementing these routines as part of their practice. While the teachers are receptive to and in agreement with the routines that support the development of understanding and reasoning, the uptake and adoption of these routines as a regular part of their practice has been erratic. With this research I want to probe their motivations and objectives for the decisions they make regarding their practice, and the use of materials and interactions with the students. Using the basic tenets of Activity Theory (object, subject, tools), table 1.3 serves to structure the question, "How do teachers perceive their mental mathematics teaching practice?" in order to guide my study of teachers' actual perceptions in comparison with my views on their lessons.

Table 1.3: Structuring the question, "How do teachers perceive their mental mathematics teaching practice?"

Practice	***Drill and memorisation teaching approach*** → ***Understanding and reasoning teaching approach***
How do teachers perceive their mental mathematics teaching practice?	
Purpose	How do teachers describe their teaching objectives?
Teacher Role	How do teachers assign roles (teacher role versus student role) during the teaching of mental mathematics?
Tasks	How do teachers choose and use tasks for teaching mental mathematics?

In order to foreground the rationale for adopting the routines suggested during the coaching support, the teachers participated in a workshop session as part of this study. The workshop introduced the five strands of mathematical proficiency (Kilpatrick, Swafford and Findell,

2002) as a research-based framework that underpins the foundations of the teaching routines that support the development of understanding and reasoning.

In *'Adding it up: Helping children learn mathematics*' (Kilpatrick, Swafford and Findell, 2002), the authors use the term *mathematical proficiency* to describe what it means to learn and do mathematics, and describe mathematical success as more about developing understanding and reasoning than about producing the correct answers. Mathematical proficiency consists of five interrelated strands, referred to as: calculating, understanding, applying, reasoning and engaging. These strands apply not only to the learning of mental mathematics, but to the learning of mathematics in general. The five strands of mathematical proficiency are:

- **Calculating (Procedural fluency):** carrying out mathematical procedures with flexibility, accuracy and efficiency.
- **Understanding (Conceptual understanding):** understanding of mathematical concepts, operations and relations.
- **Applying (Strategic competence):** the necessary application and representation of numbers and number facts in order to solve mathematical problems (using what you know to solve what you do not know).
- **Reasoning (Adaptive reasoning):** ability to reflect, explain and justify the thinking used, and the strategy that was applied and executed.
- **Engaging (Productive disposition):** a self-belief in one's ability to 'do mathematics', and a positive attitude towards and willingness to engage in mathematical problem solving (a growth mind set versus a fixed mind set).

The teaching objective of mathematical proficiency is to create mathematical learning opportunities and environments using all five strands in an interrelated way: students **engage** in doing mathematics to develop a wide range of **calculating** strategies that they can **apply** with **understanding**, and they are able to **reason** about what they have done.

In order to explore and analyse teaching practice, this interpretive, qualitative study focused on early grade (Grades 1, 2 and 3) teachers in order to investigate their perceptions and interpretations of their own practice in teaching mental mathematics. Their reflections were analysed through a case study of perceptions of mental mathematics teaching practice in early grade classrooms. It is hoped that this study will encourage the participating teachers to continue reflecting on their practice (and beliefs) and what this means for their teaching. Through teachers' reflections on practice by the teachers in this study, and through analysing their perceptions, there may be a commonality, or difference, in perceptions that can be shared with other teachers and researchers that may inspire further research.

1.6 Chapter Conclusion

In this chapter I argued for research into teachers' practices of teaching mental mathematics with understanding. I proposed that such a study should investigate teachers' perceptions of their mental mathematics teaching practice with specific reference to the lesson objective, the teachers' role and the tasks used in the lesson. In the next chapter, I will review the literature that supports my argument for the importance and complexity of teaching for understanding and reasoning, and for the role that mental mathematics plays in supporting this development. The review of this literature leads into the introduction and discussion of how I use Activity Theory as the analytical framework for this study. In the chapters that follow, the reflections, perceptions and interpretations of practice will be analysed and discussed in order to describe the relationships and outcomes of mental mathematics teaching practice.

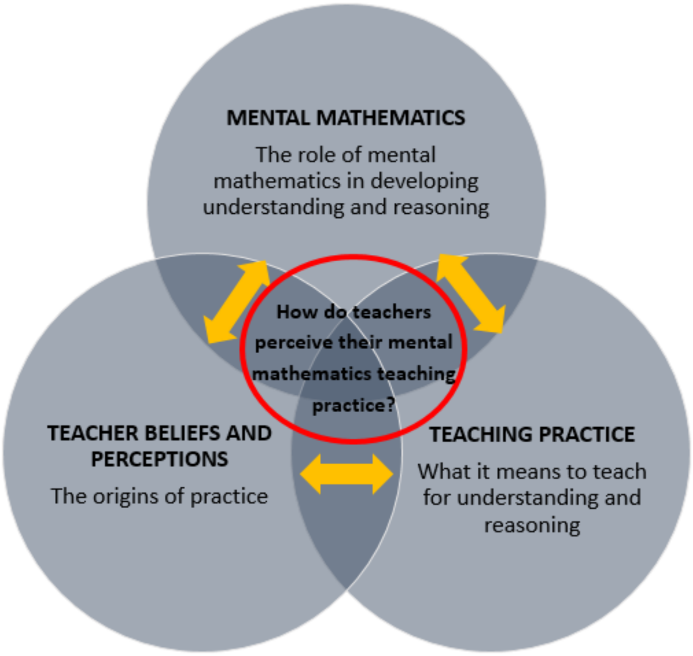
Chapter 2: Literature Review and Activity Theory

2.1 Introduction

In the introductory chapter the context and motivation behind the research purpose and question were introduced. Developing mathematical proficiency and being successful in mathematics rely on more than just the recall of answers (Kilpatrick, Swafford and Findell, 2002). Teaching that favours memorisation over understanding seldom encourages the development of calculating strategies beyond counting-based strategies (Weitz and Venkat, 2013). The predominant teaching approach in South African early grade classrooms appears to involve little or no encouragement or support for students to develop understanding and work more abstractly with numbers (Venkat and Spaull, 2015) and is coupled with the popular belief that the purpose of mental mathematics is to memorise and recall facts. As learning is a function of teaching, in that students' learning depends largely on decisions teachers make in the classroom (Ball and Forzani, 2011), it follows that teachers' beliefs and perceptions about learning and mathematics will influence their decisions and also the learning outcome. I argue for the importance of teaching for understanding and reasoning, and for the role that mental mathematics can play in supporting this development (Gürbüz and Erdem, 2016). While arguing for this teaching objective, I consider that beliefs shape practice and that the roots of beliefs stem from a variety of contexts and influences (Clements and Sarama, 2009; Rossouw, Rhodes and Christiansen, 2000). To support this argument and to frame the research question, "How do teachers perceive their mental mathematics teaching practice?", the main discussions within the literature review are listed below and are summarised in figure 2.1. The discussions reflect South African and international research on:

- Teaching practice: what it means to teach for understanding and reasoning.
- Teacher beliefs and perceptions: the origins of practice.
- Mental mathematics: the role of mental mathematics in developing understanding and reasoning.

Figure 2.1: The main discussions within the literature review



The discussions within the literature review provide the arguments for the kind of teaching practice and the role of mental mathematics in supporting the development of understanding and reasoning. The literature that foregrounds the beliefs and perceptions that shape teaching practice argues for teaching practice as a complex endeavour and highlights the challenges that characterise initiating and sustaining shifts in practice.

2.2 Teaching Practice: What it means to teach for understanding and reasoning

Research has shown that South African primary school students are performing below grade level expectations, with strategies that involve inefficient and developmentally inappropriate counting strategies (Graven *et al.*, 2013; Venkat *et al.*, 2019). Further research has tracked these counting strategies back to teaching in the early grades where concrete counting methods have been accepted as a developmentally appropriate calculating strategy across the early grades (Weitz and Venkat, 2013). The predominant teaching approach appears to favour the memorisation of facts and procedures (Venkat and Spaull, 2015). Number sense (fluent, flexible and effective ways of working and calculating with numbers), is the basis of mathematical work in primary school and it is clear that the development of number sense in the early grades should be prioritised (Graven *et al.*, 2013). Kamii (1996; 1998) argues that the challenge for teachers is to distinguish between knowledge that must be told (social knowledge) and knowledge that students should (and can) construct for themselves (conceptual knowledge) and refers to Piaget's description of three different types of mathematical knowledge:

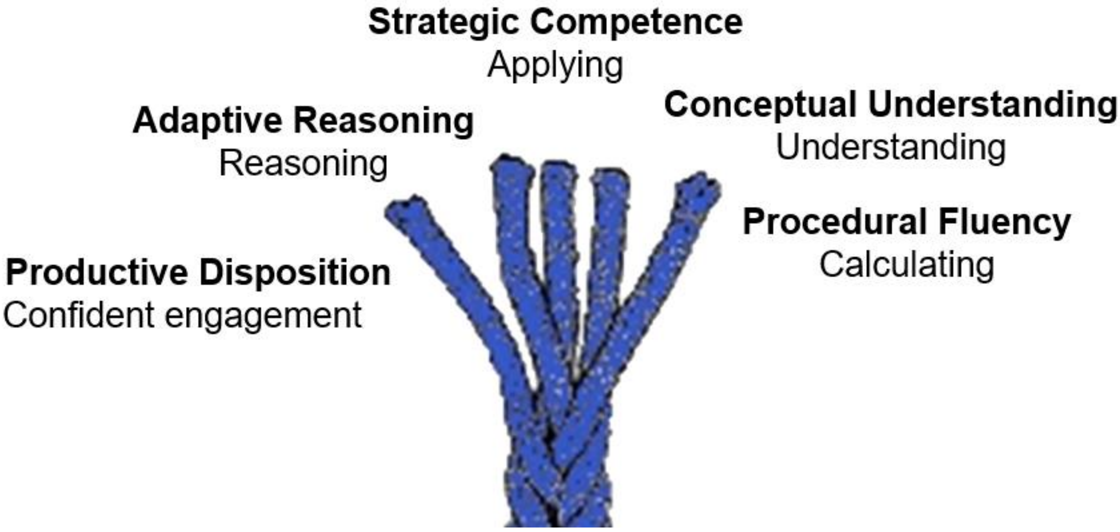
- Social Knowledge: knowledge that must be told and remembered; such as the number names and symbols.
- Physical Knowledge: knowledge that results from physical experiences; such as counting using objects.
- Conceptual Knowledge: knowledge that is constructed when students reflect on activities and begin to see patterns, relationships, sameness and difference within mathematical concepts.

If teachers view mental mathematics as facts they can tell and which students must remember, they treat mental mathematics as social knowledge. In contrast, teachers who view mental mathematics as thinking and reasoning strategies acknowledge that mental mathematics can and should be constructed by students and cannot be transmitted by telling (Kamii & Dominick, 1998).

In order to be clear on the meaning and intention of teaching for understanding, it is helpful to look at the difference in meanings between knowledge and understanding, as defined by Wiggins and McTighe (2005). They describe knowledge (to know) as the correct reproduction of facts. In contrast, they describe understanding (to know how) as the application and transfer of known facts. For understanding, the focus is on the process of producing a solution rather than on the solution itself. It can be seen from these differences that understanding and teaching for understanding require more than just the learning and recall of facts. One of the most significant features of understanding and teaching for understanding, is transfer. Transfer can be explained as 'using what I know to solve what I do not know' (Wiggins and McTighe, 2005). Knowing which facts (knowledge) to use when, requires more than just another fact. Understanding is about application and the ability, or know-how, to transfer what is known to unknown situations.

Wiggins and McTighe (2005) describe teaching for understanding as a combination of processes: knowing facts and knowing how to use, apply and transfer these known facts to unknown situations. This complexity is well-defined by Kilpatrick, Swafford and Findell (2002) as mathematical proficiency and describes their view of success in mathematics as being more than getting the correct answer. They emphasize performance (product) **with** mastery (process). In *"Principles to action: Ensuring mathematical success for all"* (National Council of Teachers of Mathematics, 2014), mathematical proficiency is used to describe a shared vision for mathematics teaching that integrates all five strands of proficiency (illustrated in figure 2.2) in a coherent teaching practice.

Figure 2.2: The interrelated strands of mathematical proficiency (adapted from Kilpatrick, Swafford and Findell, 2002)



To further describe the integrated approach to the teaching of mathematics for understanding, the terms used by Hiebert and Lefevre (1986) to define the different knowledge types are illustrated in table 2.1. Their definitions are similar to those used by Kilpatrick, Swafford and Findell (2002) to describe strands (knowledge types) of procedural fluency and conceptual understanding. Similarities between the definitions for knowledge and understanding by Wiggins and McTighe (2005) can also been seen.

Table 2.1: Summary of terms to define teaching for 'knowledge' versus teaching for 'understanding'

Mathematical Knowledge (strand)	**Procedural Knowledge**	**Conceptual Knowledge**
Wiggins and McTighe (2005)	Involves facts, and student response is either correct or incorrect	Involves the meaning of facts, and student response requires an explanation as to why it is correct
Hiebert and Lefevre (1986)	Learning (knowing) the language (symbol representation) and learning (knowing) the rules and algorithms for completing mathematical procedures Follows a linear sequence	Knowledge that is characterised by relationships (a network of knowledge) that allow for further connections to be made – *using what I do know to solve what I do not know*
Kilpatrick, Swafford and Findell (2002)	The carrying out of mathematical procedures with flexibility, accuracy and efficiency	The awareness and comprehension of mathematical concepts, operations and relationships Knowing the meaning of these concepts and relationships and recognising when to use which facts The transfer of knowledge - *using what I do know to solve what I do not know*

Hiebert and Lefevre (1986) argue that the teaching of the two knowledge types requires different approaches. Procedural knowledge can be learnt without meaning and developed through direct instruction with regular opportunities to practise the taught procedures. On the other hand, conceptual knowledge cannot be taught through direct instruction, and cannot be learnt without meaning. It is developed through the construction of relationships between pieces of information and by creating relationships between existing knowledge and new information. As previously mentioned, Kamii and Dominick (1998) reference Piaget's different knowledge types (social, physical and conceptual) and identify that the challenge for the mathematics teacher is to distinguish between knowledge that can be told (to know) and knowledge that students should construct for themselves (to know how).

Kilpatrick, Swafford and Findell (2002) make the point that it is not a case of one or the other, but more a case of one with the other. Procedural knowledge that is informed by conceptual knowledge results in symbols and rules that have meaning and procedures that can be remembered better and used (transferred and applied) more effectively (Hiebert and Lefevre, 1986). Conversely, the development of conceptual knowledge of numbers relies on the knowledge of the basic number combinations and facts, and being able to recall and use them – procedural knowledge. With the introduction of the interrelatedness of the five strands Kilpatrick, Swafford and Findell (2002) avoid the debate around the division of the knowledge types (which is better and which comes first) by highlighting the importance and value of each strand (knowledge type) and its role in supporting the development of the other. While we may consider procedural knowledge without conceptual knowledge, it is not so easy to imagine conceptual knowledge without some procedural knowledge. It is the relationships and interrelatedness between the knowledge types that hold the key to teaching for understanding.

> *"Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy."* (Kilpatrick, Swafford and Findell 2002, p. 122)

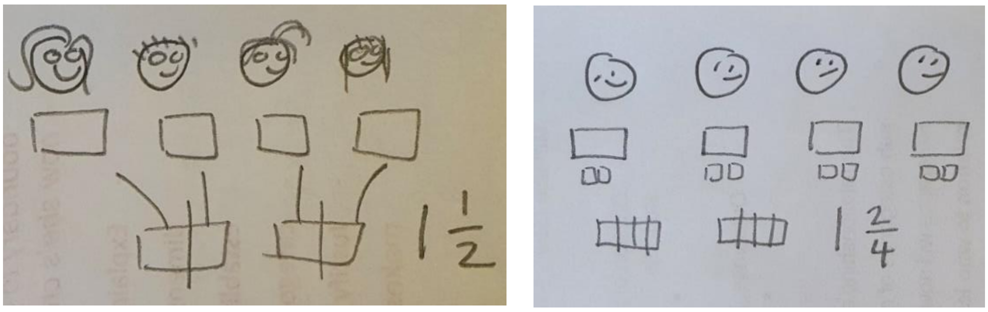
With a focus on developing understanding and reasoning, the interactions between the teacher and the students within a lesson can predict the way in which knowledge is developed. The role of social interaction is crucial for the development of conceptual knowledge (Kamii, 2014), and posing questions that require students to think in order to answer supports the development of conceptual knowledge. Swan and Sparrow (2001) argue for the role that discussion plays in developing reasoning-based mental calculating strategies. This is important in that: students need to think about their thinking (metacognition) in order to put it into words; listening to a variety of approaches can inspire students to adapt and adopt strategies; discussion of how strategies are linked encourages students to think about the structure of number; and discussions develop mathematical vocabulary. Considering the role of discussion in developing understanding and reasoning, teachers are being asked to add non-traditional discussions that stimulate and support 'higher-order thinking' in their mathematics lessons. As the emphasis on teaching shifts to developing understanding and reasoning (conceptual knowledge), so teachers are being asked to develop new patterns of questioning and interaction (Cazden, 2001).

Cazden (2001) describes the traditional pattern of questioning as a three-part pattern: initiate - respond - evaluate (IRE). The teacher initiates the interaction by asking a question, the student responds, the teacher evaluates – the response is either right or wrong. This teacher-student interaction best fits the transmission of facts and learnt procedures. The criticism of the IRE structure is that the teacher asks questions to which the answers are known, and therefore student responses are evaluated as being either correct or incorrect, and there is no further opportunity for interaction. Mathematics teaching practice, irrespective of teaching approach, can be described by looking at the interactions between the teacher and the students: what teachers are doing (teacher action) in relation to what students are doing (student action). How teachers listen (and respond) can be used to describe mathematics teaching practice (Davis, 1997). Three variations of listening that can be used to describe teacher and student action during a mathematics lesson are detailed by Davis (1997):

- Evaluative listening: listening for as opposed to listening to (this describes the pattern of IRE).
    - Teacher action: teacher asks a question (initiate). “8+7?”
    - Student action: student responds (respond). “15”
    - Teacher action: teacher evaluates (evaluates). “Correct”
- Interpretive listening: listening for information as opposed to listening for an answer.
    - Teacher action: teacher asks a question where the responses or answers from the students are not fully anticipated. “Tell me how you would calculate 8+7.”
    - Student action: student responds. “If I know that 8+2=10 ... then I just need to add 5 more to find 8+7 … 8+2 = 10 … plus 5 = 15”
    - Teacher action: teacher probes for further explanation of thinking. “So how would you use this to solve 28+37?”
    - Student: student responds with explanation. “I know that 8+7=15, so if I add this to 20+30, then I’ll have the answer. 15+50=65”

- Hermeneutic listening: listening as a joint exploration of mathematical ideas and concepts. Hermeneutic in this context means to interpret for the objective of deeper understanding. This is an enquiry-based interaction that aims to make connections and reveal the mathematical concepts, rather than questioning and anticipating already known facts.
    - Teacher action: teacher poses a question that requires an interaction and discussion in order to reveal the intended concept. Concept to be revealed: equivalent fractions, in this case introducing the notion that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent. "Share 6 chocolates between 4 children. Show me how to do it and, tell me, how much will each child get?"
    - Student action: students make a plan and show their thinking. Figure 2.3 illustrates an example of two of the possible responses that the students may produce.

Figure 2.3: An example of variation in student responses for the same question



    - Teacher action: teacher probes for explanation of thinking.
    - Student action: students respond with explanation.
    - Teacher action: teacher probes for further 'noticing' from the students in order to reveal the equivalence between the two responses.

It is evident that when teacher action is limited to evaluative listening, there is little opportunity to engage with students in ways that lead to the development of understanding and reasoning. It follows that a teacher's engagement habits and listening habits influence the opportunities they can create for developing mathematical proficiency. It is generally accepted that different kinds of teaching practice tend to result in different kinds of learning.

However, Hiebert and Grouws (2007, pp. 380) state that there is no rigorous evidence that shows a *"simple correspondence"* between teaching approaches and the associated learning. Hiebert and Grouws (2007) therefore refer to **features of teaching that facilitate the intended learning**. In this study, the focus is on the features of teaching that are most likely to facilitate the development of understanding and reasoning. Using the research of Hiebert and Grouws (2007, pp. 383-390), McGatha et al. (2018, pp. 6-8) and the statements of the National Council of Teachers of Mathematics of the United States of America (National Council of Teachers of Mathematics, 2014, pp 9-10), the features of 'good' teaching can be characterised as teaching that creates learning opportunities. The descriptions of these learning opportunities used by the authors are well-aligned to the integration of the five strands of mathematical proficiency which I phrase as:

- Creating opportunities for students to make connections where the connections and relationships within and between facts and concepts are revealed;
- Creating deliberate and structured activities where the facts can become known through frequent use in relation to patterns and a variety of situations;
- Creating opportunities for discussion that allow the students to notice the patterns and relationships, and to justify why strategies are appropriate and accurate. Discussion where the teacher's role shifts from that of listening for an answer, to that of listening to an answer (Davis, 1997);
- Creating opportunities to engage students in the 'struggle', where students are required to respond with more than a memorised fact, to make an effort to make sense of what they are presented with and to find a solution that is not immediately apparent. This does not mean 'struggle' in the sense of *"needless frustration or extreme levels of challenge"* (Hiebert and Grouws, 2007, p. 387). The question (challenge) needs to be accessible rather than easy. Vygotsky's Zone of Proximal Development (Vygotsky, 1978a) describes the space where tasks are accessible rather than easy, and present enough challenge so there is something new to resolve that results in a productive struggle.

The design of mental mathematics lessons and resources needs to be deliberate in providing teaching opportunities to enable the development of understanding (conceptual knowledge). As described by Hiebert and Grouws (2007) and McGatha et al. (2018), this can be achieved by designing tasks that set up patterns, encourage the noticing and making of these connections, and encourage reasoning through productive struggle. Creating opportunities for 'struggle', as well as for application and reasoning, relies on the level of cognitive demand of the tasks. The cognitive demand should be enough that the answer or strategy is not obvious, but not too demanding that the task is not accessible. In terms of the cognitive demand of tasks, Stein et al. (2000) describe various levels which are detailed in table 2.2.

Table 2.2: Levels of cognitive demand

Knowing (low level of cognitive demand)	Characterised by straightforward tasks that rely on the recall of previously learnt (memorised) facts where the answer is the outcome
Using Procedures (low level of cognitive demand)	Requires the use of a learnt procedure or algorithm where the selection of required procedure is obvious and the focus is on producing correct answers that require no explanations
Applying Procedures (high level of cognitive demand)	Requires the application of procedures, as the procedure selection is not obvious, and requires an application of current knowledge and explanations to develop and deepen understanding
Reasoning (high level of cognitive demand)	Requires non-procedural thinking where students discuss and explain their thinking in order to explain and justify their solution strategy

The level of demand of the tasks depends on the objective of the lesson, the developmental level of the students and the amount of experience that they have had with a certain concept.

A study by Eisenhart et al. (1993) looked at novice teachers' practice related to the teaching of mathematics for understanding. For this study, Eisenhart defined teaching for understanding as teaching for the development of both procedural and conceptual knowledge – and with a pedagogy that creates opportunities for students to be exposed to the different situations and practices that lead to the development of both procedural and conceptual knowledge types. The findings indicated that teachers thought they were better prepared to teach for procedural knowledge than conceptual knowledge. The research revealed tensions between the belief in the importance of teaching mathematics for understanding and in the need to teach for both procedural and conceptual knowledge.

Further tensions arose in the teacher's own content and pedagogical knowledge, which was more suited to teach procedures. So, while the teachers agreed on teaching for understanding, in practice they were proficient at teaching procedures. This study and the findings resonate with my current observations that even though teachers agree on understanding as an important goal for teaching, they favour more procedural ways of teaching.

Teaching for understanding involves an integrated approach that creates opportunities for each of the knowledge types or strands to be developed. This integrated approach differs from the more direct teaching approach of teaching through 'telling' and learning formulas, which only caters for the development of the procedural fluency (calculating) strand. The reviewed research has highlighted the importance of, and defined and described, teaching for understanding and reasoning (mathematical proficiency). This study analyses how teachers perceive their practice through reflections on their mental mathematics teaching practice and uses mathematical proficiency as the over-arching description of teaching for understanding and reasoning. The discussions within the literature review provide the arguments for teaching practice that supports the development of reasoning-based calculating strategies.

2.3 Teacher Beliefs and Perceptions: The origins of practice

South African teachers have predominantly been educated and trained in a system that supports the more traditional approach of teaching mathematics through the memorisation of facts, rules and formulas (Venkat and Spaull, 2015), and these teachers use the memorisation of facts, rules and formulas to teach mathematics because they come from a belief system that favours procedural learning and teaching over the development of conceptual knowledge and understanding.

Baroody (2006) describes two different approaches or beliefs with regard to teaching mathematics, with specific reference to mental mathematics. He refers to the *conventional wisdom approach* as being that which supports the belief that students learn mathematics through memorising basic facts by rote and drill. This approach relies on the memorising of isolated procedures and methods in order to calculate and arrive at an answer and is based on the 'transmission or absorption' view of teaching and learning. In this view students are passive recipients of facts, rules and procedures which are transmitted (taught) by the teacher. In contrast, Baroody (2006) refers to the *number sense approach* which follows a belief that mathematical proficiency can be achieved through instruction that creates deliberate learning situations that support the recognition of patterns and relationships which connect the basic number combinations.

Teachers' beliefs, experiences and understandings of what it means to do and be successful in mathematics influence how they will teach mathematics. If teachers' experiences and understandings of success in mathematics involve facts, rules and procedures that need to be memorised, then they will teach in this manner. With this approach there is no belief, or experience, that students are able to construct their own knowledge or that they can think abstractly (Kamii, 1996). However, if a teacher regards being successful in mathematics as developing understanding and sense-making, then their teaching approach will be different (Clements and Sarama, 2009). This belief results in practice that creates opportunities for students to reflect on and describe their thinking and construct their own knowledge, as opposed to passively receiving it (Clements and Battista, 2009).

McGatha et al. (2018, p. 9) support the idea of a continuum of teaching objectives and practice, rather than a dichotomy. They describe teaching *"as a complex and intellectually stimulating endeavour"* and argue that teaching objectives and practice are continually developing. McGatha et al. (2018) believe that teachers are not positioned at one end or the other, but are rather on the continuum somewhere trying to move in a desired direction.

There are few teachers who disagree that students need to reach mastery for the efficient and accurate recall of facts and production of answers. There is, however, disagreement as to how this is achieved, but it is rudimentary to view this disagreement as binary and frame the conventional wisdom approach in contrast to the number sense approach (Baroody, 2006). A study by Purnomo (2017) revealed the complex nature of teaching approaches by illustrating how teachers tend to be number sense approach orientated, although their practice does not match. Purnomo's (2017) research findings suggesting possible factors that may influence the inconsistency, and complexity between beliefs and practice are:

- Past experiences in learning and teaching.
- Mathematics knowledge for teaching: when beliefs are not supported by the knowledge about how the content should be taught, the practice tends to follow prior experiences and fixed rules.
- Habit strength: teachers are reluctant to break their existing habits (practice).

More fundamentally Polya warns:

> *"We cannot judge the teacher's performance if we do not know the teacher's aim. We cannot meaningfully discuss teaching, if we do not agree to some extent about the aim of teaching."* (Polya, 1981, p. 100)

Hiebert and Grouws (2007) argue that while the teaching-learning relationship is complex, different teaching approaches may be effective for different learning objectives: if the objective is to develop understanding and reasoning, then a different more progressive approach to teaching would make sense. But, if the objective is to recall facts, then a more traditional approach would be adopted. Yet, teachers may have both objectives in sight and use a combination or integration of teaching approaches (Hiebert and Grouws, 2007).

Observing and analysing practice can therefore not be done in isolation to both beliefs and objectives. In a study conducted by Wood, Cobb and Yackel (1991), in which teachers were part of the research team, their findings indicated changes in teacher beliefs about the nature of mathematics moving from rules and procedures to meaningful activity. As members of the research team, teachers had the opportunity to express their concerns, receive support and learn in the setting of their classroom. Wood, Cobb and Yackel (1991) conclude that the focus of support should be on the nature of mathematics and developing an understanding of the way students learn, and that in turn would change the way in which teachers teach. If beliefs are changed then it follows that teachers' objectives and practices are also more likely to change. But Wood et al. (1991) determine that ongoing support is necessary to introduce the teachers to an alternative approach and materials for teaching mathematics. This ongoing support also allows for reflective discussion on teachers' practice, which proves to be instrumental in changing beliefs, objectives, habits and practice. From their research that investigated teacher practice, Clements and Sarama (2009) conclude that professional development must address beliefs as well as knowledge – and should be ongoing and reflective – if a shift in practice along the continuum (McGatha *et al.*, 2018) is expected. The teachers we are asking and working with to change, are the products of the system that they are being asked to change. These shifts require carefully designed professional development and long-term support (Rossouw, Rhodes and Christiansen, 2000).

A teacher's action and role in the lesson is an obvious part of teaching practice. Machaba and Mokhele (2014) investigated the approaches that teachers use when teaching mental mathematics in South African classrooms. They found that the lesson was dominated by the teacher with no awareness from the teacher that students were not engaging. While large class sizes and whole class teaching as well as language of instruction were contributing factors, there was no attempt at discussion and no attempt to enable student responses, other than to give an answer. The teacher action in this study described the pattern of IRE (initiate-respond-evaluate). Another South African study examined teacher perceptions of effective mathematics teaching and concluded that teachers tended to focus on generic content and what is being taught as best practice, rather than the development of understanding and how this is taught (Stols, Ono and Rogan, 2015). Although South African policy documents advocate teaching for understanding, and although teachers will 'talk this talk', these views are often not seen in practice. Stols, Ono and Rogan (2015) conclude that reflection on practice should play a more significant role, as any change in practice will need the teacher to aquire new knowledge and beliefs. Through reflection, the teachers have the opportunity to acknowledge the shifts that are necessary and create new knowledge and form new beliefs. The importance of reflection as reported in this research resonates with my study design. Although the purpose of my study is not to anticipate a shift in practice, the importance of reflection in the development of teaching practice is echoed.

Findings from research by Weitz and Venkat (2013) support the idea that teaching objectives shape learning. They report that teaching for assessment is a predominant objective that leads to reliance on inefficient counting strategies. This can be attributed to the assessment-driven objective in many of the early grade South African classrooms where teachers believe that they need to teach for assessment. One such assessment example is the Annual National Assessments (ANAs) that take place in the Western Cape, South Africa. These are standardised mathematics assessments that are administered across the province's Grade 3s, Grade 6s and Grade 9s at the end of each school year. The researchers compared the results of the ANAs to the results of an assessment described in the work by Robert Wright (Wright, Martland and Stafford, 2006). What they found was that the participating students performed better in the ANAs (that awards marks for a correct answer) than in the Wright assessment (that awards marks for efficient and appropriate strategies as well as for the answer). Weitz and Venkat (2013) concluded that until the goal of teaching and assessment changes, there is little to no motivation for teachers to change their practice.

The Department of Basic Education (DBE) acknowledges that a shift in practice and approach to the teaching of mathematics needs to be adopted in order to remedy the current under-performance of South African students (Department of Basic Education, 2018). With this as the focus, a framework was developed for the teaching and learning of mathematics in South African primary schools. The purpose of the framework is to guide the teaching of mathematics in a way that improves outcomes (Department of Basic Education, 2018). The framework uses the five strands of mathematical proficiency to illustrate what it means to teach for understanding. The framework concludes that the most obvious implications for teaching are the application of the curriculum, assessment (currently, teaching for assessment is a dominant goal), the use of materials that support the development of mathematical proficiency, and teacher development.

The general (international) trend in mathematics teaching is to shift away from the more traditional approach which promotes the drill and memorisation of procedures and algorithms. Research and recent student performance is driving and supporting this change (Stipek *et al.*, 2001). Asking teachers to shift their practice is not without its complexities, as teachers filter new knowledge through existing beliefs. A further consideration for shifts in practice is that the two dominant teaching approaches are not necessarily contradictory, there being place for both conceptual understanding and procedural fluency. This highlights that teachers' subject knowledge cannot be the only consideration for good teaching: knowledge of which mathematical concepts (knowledge) can be 'taught' and which should be 'constructed' is crucial in determining a teaching objective. But if teachers do not hold the belief that mathematical knowledge can and should be constructed (understanding and sense-making), as opposed to be being taught (drill and memorisation), then there is no motivation for change. In a study by Stipek et al. (2001), they conclude that influencing beliefs may be essential in changing practice. The aim of their research was to better understand the nature of teacher beliefs about mathematics teaching and learning and the links between beliefs and practice. Their findings indicated that beliefs are coherent with practices. They found reflecting on practice to be a significant contributor to shifting beliefs and subsequently practice. Without reflection, any new input (readings, research and professional development) was not effective as the new knowledge was filtered through existing beliefs.

The use of reflection on practice as a catalyst for change is well supported in this review. As the focus of this study is on teachers' perceptions of their practice, it is important to understand the origins of these perceptions and the beliefs that ultimately shape practice. Reviewing the literature that foregrounds the beliefs and perceptions that shape teaching practice argues for teaching practice as a complex endeavour and highlights the challenges that characterise initiating and sustaining shifts in practice.

2.4 Mental Mathematics: The role of mental mathematics in developing understanding and reasoning

There is current South African research that points to a lack of the development of number sense due to an absence of teaching mental mathematics (amongst other factors), and teaching mental mathematics with understanding. Venkat and Askew (2012) make note of the importance of teacher talk and mediation (interpretive and hermeneutic listening) in developing more abstract and compressed ideas of number and calculation strategies. Venkat and Naidoo (2012) support the need for teacher-student conversations that mediate as opposed to evaluate within mental mathematics lessons.

> *"Mental computation is used for computing that is based on understanding and knowledge of mathematical properties and relationships due to calculations **with your head** rather than **in your head**."* Gürbüz and Erdem (2016, p. 2)

My working definition of mental mathematics emphasises calculating that is based on understanding and knowledge of mathematical patterns and relationships, which are processed with understanding and reasoning (with your head). In support of calculations performed mentally, Swan & Sparrow (2001) claim that the teaching of written methods (formulas and algorithms), particularly at an early age, can hinder the development of mental strategies. By making use of mental strategies, and discussing these, students are working with understanding and thinking about numbers rather than remembering procedures.

> *"The process of mental calculation cannot be traced as it is happening, it can only be remembered and described afterwards."* Threlfall (2000, p. 79)

The quote above highlights the importance that reflection and discussion play in mental mathematics, when the focus is on the development of calculating strategies and not just the production of an answer. Threlfall, (1998, p. 71) uses the examples below, from three different students, to illustrate reflections on the mental calculation of 374 + 58:

- Student 1: "374 add 6 is 380, add 2 is 382, add 50 is 432."
- Student 2: "I added 370 and 50 then 8 and 4 then added them all together."
- Student 3: "Make 374 up to 380 by adding on 6. That means you have 2 units left from 58. Then add the 50 onto 380 to make 430, then add your 2 units on to make 432."

The descriptions (verbal or written) of calculations performed mentally, enable the students to track and reflect on their thought processes (Threlfall, 2000; Verschaffel, Greer and De Corte, 2007).

Threlfall (2002) argues for the role that mental mathematics can play in supporting the learning of number facts with understanding by developing both fluency and flexibility:

- Fluency (calculating): knowledge of number facts. This means that students have a base of known facts that can be recalled and applied.
    - If I know that 5+2=7, then I also know that 25+2=27 and 500+200=700.

- Flexibility (understanding and reasoning): having a range of age-appropriate strategies available for application across a variety of situations. This relies on an awareness of how numbers can be broken up and how number combinations can be changed.
    - Consider solving 256÷8 with mental calculating. In this case breaking up 256 into its constituent 100s, 10s and 1s (200+50+6) is not helpful, as only 200 is divisible by 8. But using what is known, if I know 24÷8, then I also know that 240÷8, creates the awareness that 256 can also be broken up into 240+16. If I know that 24÷8=3, then I know that 240÷8=30 and I also know that 16÷8=2. So, 256÷8=32.

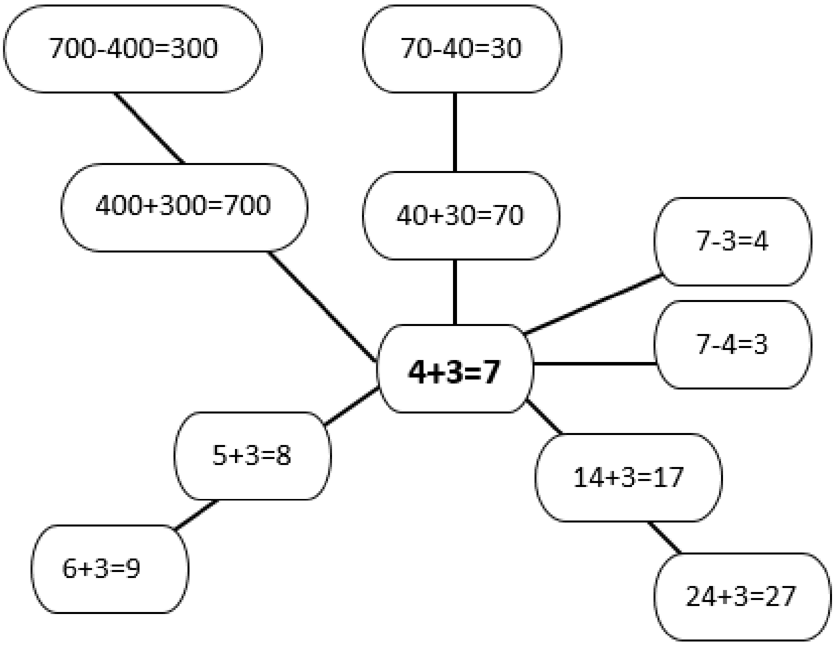
Fluency is developed through regular experience and practice with the various number facts, and flexibility develops through the use, connection and application of these number facts to other number combinations and situations. Discussion plays a vital role in developing flexibility and the associated reasoning-based calculating strategies (Swan and Sparrow, 2001). Mental mathematics can therefore be further defined as the mathematics that develops fluent and flexible calculating strategies through discussion in order to develop understanding and reasoning.

As previously discussed, the prevalent teaching of mental mathematics in South African classrooms is focused on the production of answers through the memorisation of facts, rules and formulas. This stems from the belief that mental mathematics is the ability to do calculations quickly and accurately, but it can be more than that. Mental mathematics should also involve conceptual understanding, problem solving and reasoning so that calculations can be performed flexibly, efficiently and accurately with understanding (Olsen, 2015). The development of number sense is connected to developing mental representations and mental strategies for calculating. These mental strategies follow on from and develop with concrete counting strategies (physical knowledge), and proceed and develop alongside flexible and reasoning-based calculating strategies (logico-mathematical knowledge) (Kamii, 1996). Although research and the CAPS curriculum emphasise the importance mental mathematics plays in developing number sense, it is not included in the ANAs. As assessment often drives teaching, this aspect of the mathematics lesson is being overlooked. While the CAPS curriculum emphasises the importance of this routine and details what needs to be taught, it does not give guidance on how this should be taught. It is possible that many teachers interpret mental mathematics as a random selection of mental questions rather than a systematic variation of questions aimed at getting students to see and consolidate patterns and relationships (Graven *et al.*, 2013).

How mental mathematics is taught can mean the difference between developing counting-based calculating strategies and the development of reasoning-based strategies. Mental mathematics is not only about learning the number facts, but also crucial in developing number sense (Boaler, 2015). Number sense, which is critical to mathematical development, is inhibited by over-emphasis on the memorisation of mathematics facts; and memorisation leads to students giving up on sense making (Boaler, 2015). Boaler's study revealed that students who learnt through the use of strategies showed superior performance, worked at the same speed as the 'memorisers', and showed better transfer to new problems. The best way to develop fluency and flexibility with numbers is to develop a sense of number through working with numbers in different ways and not to just memorise without meaning (Boaler, 2015).

Anghileri (2006) suggests that number sense, and having a sense of number, includes the ability to notice, use and generalise about patterns and connections between number facts, and to link new information to existing knowledge. Having a sense of number enables students to identify relationships and to work flexibly with numbers. This can be developed through mental mathematics if taught with the objective not only to recall isolated facts, but also to see the patterns and make connections (understanding). There is a choice: 4+3=7 can be just that, one isolated fact, or it can be connected to and applied as part of a wider network of facts that lead to and connect to other facts that can be derived from knowing 4+3=7, as illustrated in figure 2.4.

Figure 2.4: A web of facts derived from 4+3=7 (adapted from Anghileri, 2006, p. 61)



Evidence shows that although flexible, fluent, efficient and reasoned knowledge of number facts is associated with mathematical success in the later years, South African students are reliant on inefficient counting strategies. Mental mathematics and mathematical reasoning are two interrelated cognitive processes as mathematical reasoning is necessary in deciding which strategy to use (Gürbüz and Erdem, 2016). To develop mathematical reasoning, students should be encouraged to do mental mathematics and to discuss the strategies that they use. Over time this can lead to the development of flexible and fluent calculation strategies that are executed competently and confidently. Swan and Sparrow (2001, p. 242) argue that as well as being fluent, flexible, competent and confident, students will also develop the following:

- A good depth of factual knowledge, with the ability to answer automatically.
- A broad range of mental strategies, with the ability to select the most appropriate strategy.
- An ability to verbalise their thinking.

Heirdsfield (2005) conducted a study that investigated teacher actions that promoted the development of mental computation. The teachers in this study had worked with the researcher previously and had been exposed to research and practice that supported the development of this. The teachers already had an awareness and shift in belief regarding their practice towards teaching for understanding. The teacher actions found to support the learning were sound planning, questioning by the teacher and encouragement for discussion. The tasks provided contributed in that they had been designed in a deliberate and careful sequence used to establish connections and strategic thinking. The devised tasks also allowed for enough opportunities to practise, and produced an appropriate challenge to encourage the development of application and reasoning. Too often the tasks presented in early grade mathematics classrooms rely on the recall of known facts (low level of cognitive demand), and do not provide enough opportunities for practice or exposure (too little attention is given to mental mathematics in the bigger scheme of the curriculum), and often too much time is spent on counting. In South African classrooms it has been noted that the time allocation of counting activities did not shift over a three-year period, and the number of mental computations over time were also not increased: the Grade 3 students were exposed to the same amount of counting and mental computations as the Grade 1 students. Teachers are not presenting students with enough mathematics at the right level to develop understanding successfully (Ensor *et al.*, 2009). Graven et al. (2013) have noted through their work and research in early grade mathematics that there are challenges in performance which relate to the absence of the development of number sense and mental flexibility. Flexibility in mental calculation is better approached through teaching that focuses on understanding, and not just calculating.

Taught at the correct level of cognitive demand, mental mathematics can develop an understanding of working and calculating with numbers and an ability to discuss and to do mathematics. Mental mathematics is not only important for developing mental calculating strategies, but also for developing higher order thinking, reasoning and sense-making of numbers and number operations. For mental mathematics, a challenging level of cognitive demand would encourage thinking and application, but the demand would still be able to be met by using mental calculating strategies. Students need exposure to and practice with cognitively demanding tasks if the goal is to develop understanding and reasoning, and reasoning-based strategies (Stein *et al.*, 2000). The levels of cognitive demand were detailed in table 2.2.

In '*Adding it Up: Helping children learn mathematics*' (Kilpatrick, Swafford and Findell, 2002), mental mathematics is shown to strengthen not only the procedural fluency (calculating) strand, but also the other four strands. Mental mathematics, at the appropriate level of cognitive demand, can be used to strengthen all five strands necessary for mathematical proficiency, and not just the calculating strand. The five strands can be strengthened by mental mathematics as follows:

- Calculating: provide students with regular meaningful opportunities to experience and practice calculations.
    - Level of cognitive demand: knowing and using procedures.
- Understanding: number facts are related, and using these patterns and connections can make learning of number facts more meaningful and less reliant on memory. For example: if I know that 8+2=10, then 8+7 → 8+2+5=15. The interrelatedness of addition and subtraction and multiplication and division are also key connections here. For example: one can solve a subtraction calculation by thinking of it as addition, so 13-8 can also be thought of as what must be added to 8 to get 13?
    - Level of cognitive demand: knowing and applying procedures.
- Applying: students are encouraged to use number facts that they know in order to plan and execute calculations that they do not know. For example, students can use 5x8 to solve 6x8. 6x8 is just 5x8 (which is 40), plus another 8.
    - Level of cognitive demand: applying procedures.
- Reasoning: as students discuss how they figured out a number problem, they explain their thinking and strategy. By explaining their strategies, they demonstrate and refine their understanding of the mathematical relationships and connections.
    - Level of cognitive demand: reasoning.
- Engaging: when students notice the connections and patterns among number facts they make sense of them and they begin to see themselves as capable, and are thus equipped to draw on their own resources instead of just memorising facts.

Mathematical proficiency can be achieved through planning deliberately structured and developmentally appropriate activities at the right level of cognitive demand, aimed at supporting students to notice patterns and relationships (Kamii and Dominick, 1998; Kilpatrick, Swafford and Findell, 2002; Swan and Sparrow, 2001).

While the reviewed research has addressed the importance of mental mathematics in developing understanding, there is little research that investigates teachers' reflections on their practice of teaching mental mathematics. This study analyses teachers' perceptions of this practice and considers their teaching objectives and beliefs as to what it means to do mental mathematics. As discussed in the review, an important consideration in the role of mental mathematics in developing understanding and reasoning is the level of cognitive demand of the tasks used and the role of discussion. The tasks need to be demanding enough to encourage and generate the discussion that is crucial to developing understanding and reasoning.

The discussions within the literature review provide the arguments for the role that mental mathematics plays in the development of understanding and reasoning.

2.5 Summary of Literature Reviewed

The literature review confirmed the role of mental mathematics in developing number sense through reasoning about numbers and relationships. The literature also confirmed that such understanding requires teaching aimed at integrating the five strands of mathematical proficiency. To structure the descriptions of practice and guide the reflections and analysis of the perceptions of practice for this study, three main categories were derived from the literature: mathematical proficiency (How do teachers describe their objectives?); teacher action (How do teachers assign roles during the teaching of mental mathematics?); and cognitive demand (How do teachers choose and use tasks for teaching mental mathematics?). These categories are summarised in table 2.3. For the category of mathematical proficiency only four of the five strands are listed. If teaching integrates calculating, understanding, applying and reasoning, then opportunities for engagement should follow, hence the reference to these four strands.

Table 2.3: Summary of categories used to describe practice

MATHEMATICAL PROFICIENCY (Kilpatrick, Swafford and Findell, 2002)	
Calculating	Provide students with regular meaningful practice Provide students with the opportunity to learn with understanding
Understanding	Provide situations that allow students to notice the patterns and relationships Provide situations that allow for the representation of mathematical situations in different ways
Applying	Provide opportunities for students to use what they do know in order to solve what they do not know Provide the opportunity for 'struggle'
Reasoning	Provide opportunities for students to reflect on, explain and justify their thinking Provide opportunities for enquiry-based interaction in order to make connections and reveal the mathematical concepts

TEACHER ACTION (Cazden, 2001; Davis, 1997; Hiebert and Grouws, 2007; McGatha *et al.*, 2018; Van de Walle *et al.*, 2001)	
Random items	Using random resources and calculations - i.e. items that do not vary systematically in a way that allows students to notice and use the patterns and relationships between numbers (Graven *et al.*, 2013), with little regard for cognitive demand of task Initiate-respond-evaluate pattern of interaction: evaluative listening
Structured items	Planning for and using tasks that set up a pattern, with an awareness that tasks with a higher cognitive demand should require reasoning Teacher probes students for their responses (response is not anticipated by teacher): interpretive listening
Asking related questions	Teacher asks questions to facilitate discussion in order to elicit which strategies the students have applied, and to help students notice a pattern or relationship between successive tasks Teacher probes students for their responses (response is not anticipated by teacher): interpretive listening
Building on responses	Through questioning, discussion and listening the teacher facilitates and uses the students' responses to promote understanding and reasoning Teacher-student interaction is enquiry-based: hermeneutic listening

COGNITIVE DEMAND (Stein *et al.*, 2000)	
Knowing	*Lower-level demands (memorisation):* Memorisation (the answer is the outcome) Straightforward tasks that rely on previously learnt facts and are the exact reproduction of previous tasks
Using procedures	*Lower-level demands (procedures without connections):* Using procedures without meaning Requires the use of a learnt procedure or algorithm Selection of required procedure is obvious Focus is on producing correct answers and requires no explanations
Applying procedures	*Higher-level demands (procedures with connections):* Applying procedures with meaning Procedure selection is not obvious as standard procedure may not lead to correct solution Application of current knowledge and explanations develop and deepen understanding
Reasoning	*Higher-level demands (doing mathematics):* Requires students to reflect on their thinking (and strategy), mentally organise the steps they took, and discuss and explain their thinking in order to explain and justify their solution strategy

Although the sub-categories may be interpreted as descriptions of levels, the organisation of these sub-categories is intended to highlight the complexity of teaching practice rather than promote one sub-category over the other. Each of the sub-categories has a place in the teaching of mathematics depending on the desired objective for learning. In chapter three I will describe how I used the notions of proficiency, associated teacher action and cognitive demand to design a workshop and checklist to guide the teachers' reflections on practice.

2.6 Using the Literature to Frame the Analysis

Understanding that teaching is a complex endeavour, and does not happen in isolation, prompted me to look for an analytical framework that would accommodate the interrelating and contributing factors when analysing the data. The teacher, context, resources and objectives, and the way in which these all interact and interrelate, contribute to the act of teaching. The discussions in the literature review relate and integrate to form the 'system' of teaching, in that how and what is taught are informed by the purpose of the teaching and the beliefs that shape the teaching practice. This relationship is key when answering the research question, "How do teachers perceive their mental mathematics teaching practice?"

The first round of data analysis for this study, with the focus on using mathematical proficiency as the analytical framework, did not allow me to explore the teaching of mental mathematics in a system that considers the relationships between different aspects of teaching practice. As mathematical proficiency describes the learning outcome of mathematics, it did not give me the 'teacher-lens' for which I was looking. Hiebert and Grouws (2007) argue the need for a more robust theory of teaching by considering:

- The complexity of the relationship between one teaching method and a particular type of learning.
- The view that teaching is a system of interrelated features, and should be described as part of the system in which it occurs.
- The effect of mediating variables on teaching, such as students' thinking and responses.

Acknowledging that teaching is a dynamic and complex activity, led me to Activity Theory as a means to analyse teachers' perceptions of practice.

2.7 Activity Theory: Analysing perceptions of mental mathematics teaching practice (who is doing what, why and how)

Activity Theory is grounded in Vygotsky's insights that human actions are socially constructed through relationships within and around their environments (Vygotsky, 1978b). Activity Theory, sometimes referred to as Cultural-Historical Activity Theory, provides the framework to describe who is doing what, why and how (Hasan and Kazlauskas, 2014). It provides a framework for analysing and understanding human interaction through their use of tools and artefacts (Jones and Hashim, 2007).

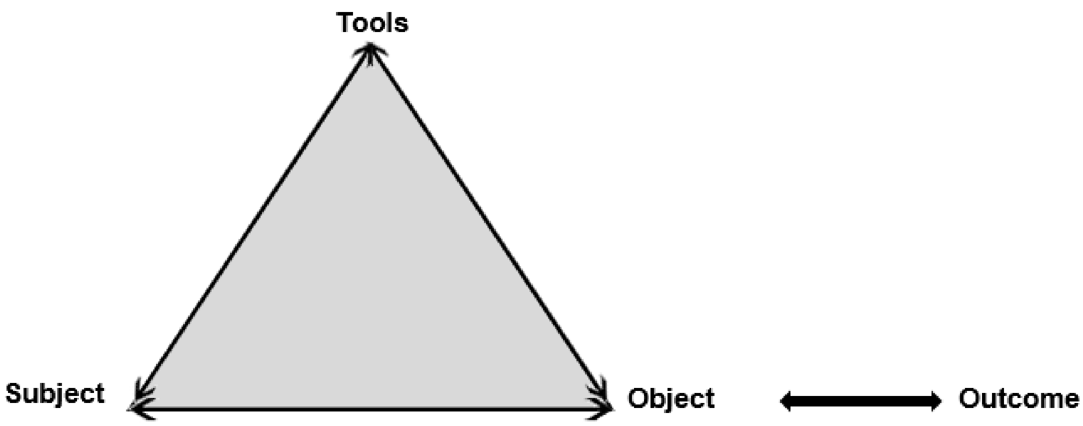
The perspectives of the 'who is doing what, why and how' can be linked to the literature review as follows:

- How? Teaching practice: what it means to teach for understanding and reasoning.
- Why? Teacher beliefs and perceptions: the origins of practice.
- What? Mental mathematics: the role of mental mathematics in developing understanding and reasoning.

Activity Theory provides a useful framework with which to analyse and describe perceptions of practice from a holistic viewpoint that considers the what, why and the how of teaching practice.

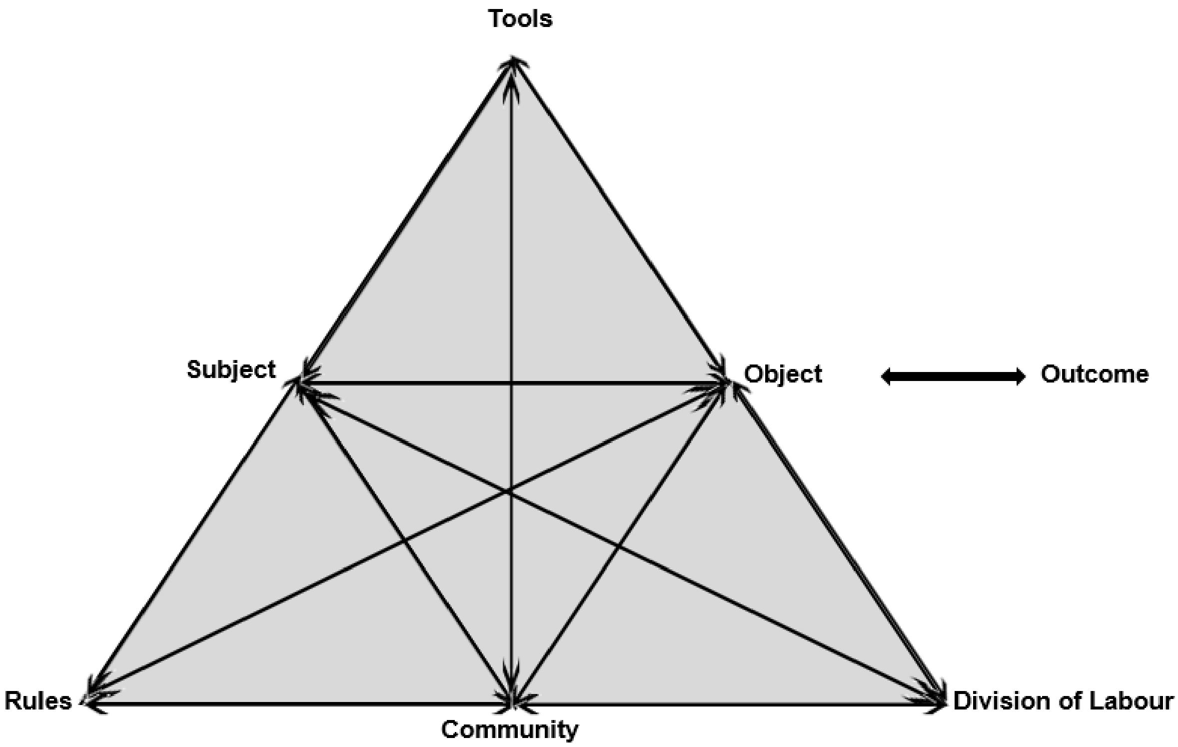
Activity Theory is grounded in almost a century of research and has been successfully applied within education research, and more recently to research in mathematics education (Jones and Hashim, 2007). Activity Theory allows for the analysis of relationships and provides the framework with which to better explore, understand and describe human activity and interaction within an activity. Vygotsky's early model uses the idea that human activity is not just a response to a stimulus, but rather the result of a mediated action and that tools (for example, physical artefacts and language) facilitate the interaction and relationships within human activity (Vygotsky, 1978b). Leont'ev further developed Vygotsky's notion of a mediated activity to form what is known today as the first generation model of Activity Theory (Engeström, 2001). Acknowledging the tool as a mediating artefact, the activity could be modelled as a system (Jones and Hashim, 2007) which includes the dimensions of the subject (doing the activity), the object (purpose of the activity), and the tools (devices by which the activity is carried out). This is illustrated in figure 2.5. An example of this could be a mathematics activity or lesson as the activity system, the teacher as the subject, the resources (both material and conceptual) as tools, and the goal or purpose of the lesson as the object. The way in which the teacher uses and implements the tools will affect the outcome, the realising of the purpose of the lesson.

Figure 2.5: Leont'ev's model of Activity Theory based on Vygotsky's early model of a mediated act (Engeström, 2001; Vygotsky, 1978, p. 40)



Using Activity Theory as the analytical framework for this study, takes *activity* as the unit of analysis, where *activity* is defined as the analytical relationship between the subject and object: the 'who is doing what for what purpose?' (Hasan and Kazlauskas, 2014). This first generation model does not take into account contextual factors that influence the selection and implementation of tools, and the forming of teaching objectives. In reality, teachers are not using tools and forming objectives in isolation. Engeström (2001) adds a layer of 'community' to the first generation model (Engeström, 2001) which considers the social and contextual aspects that influence the way in which activities are planned and executed and tools are selected and implemented. The addition of community (the context in which the activity is grounded) to the first generation model considers the influences of rules (norms that determine how and why humans may act), and the division of labour (distribution of actions and power relations within an activity). This provides a far richer description of practice and the relationships within an activity, and it is this second generation model of Activity Theory that is used in this study. The second generation model of Activity Theory is illustrated in figure 2.6.

Figure 2.6: Engeström's second generation Activity Theory (adapted from Engeström, 2015, p. 63)



Engeström's second generation model can be summarised as follows: in an activity a person (the subject) is motivated towards a specific purpose (the object). The object can further be defined as *"the purpose and motive of the activity"* (Engeström *et al*., 2015, p. 93). The interaction between the subject and the object is mediated by resources (the tools). The tools within an activity can be either material (for example, physical artefacts) or conceptual (for example, language) (Foot, 2014). The mediated activity happens within the constraints of cultural factors and social conventions (the community). These relationships inform the outcome of the activity (Bandara, 2018). As an analytical framework, Activity Theory takes into account the influences of the context and community on the subject within the system and provides the structure to guide and make sense of 'Who is doing what, why and how?'.

Bakhurst (2009) offers a cautionary note on the neat triangles of Activity Theory's structural representation as they may hide the complexity of the relationships as they play out in an activity system: "*... be very cautious about given, stable, structural representations where you aspire to understand dynamism, flux, reflexivity, and transformation.*" (Bakhurst, 2009, p.

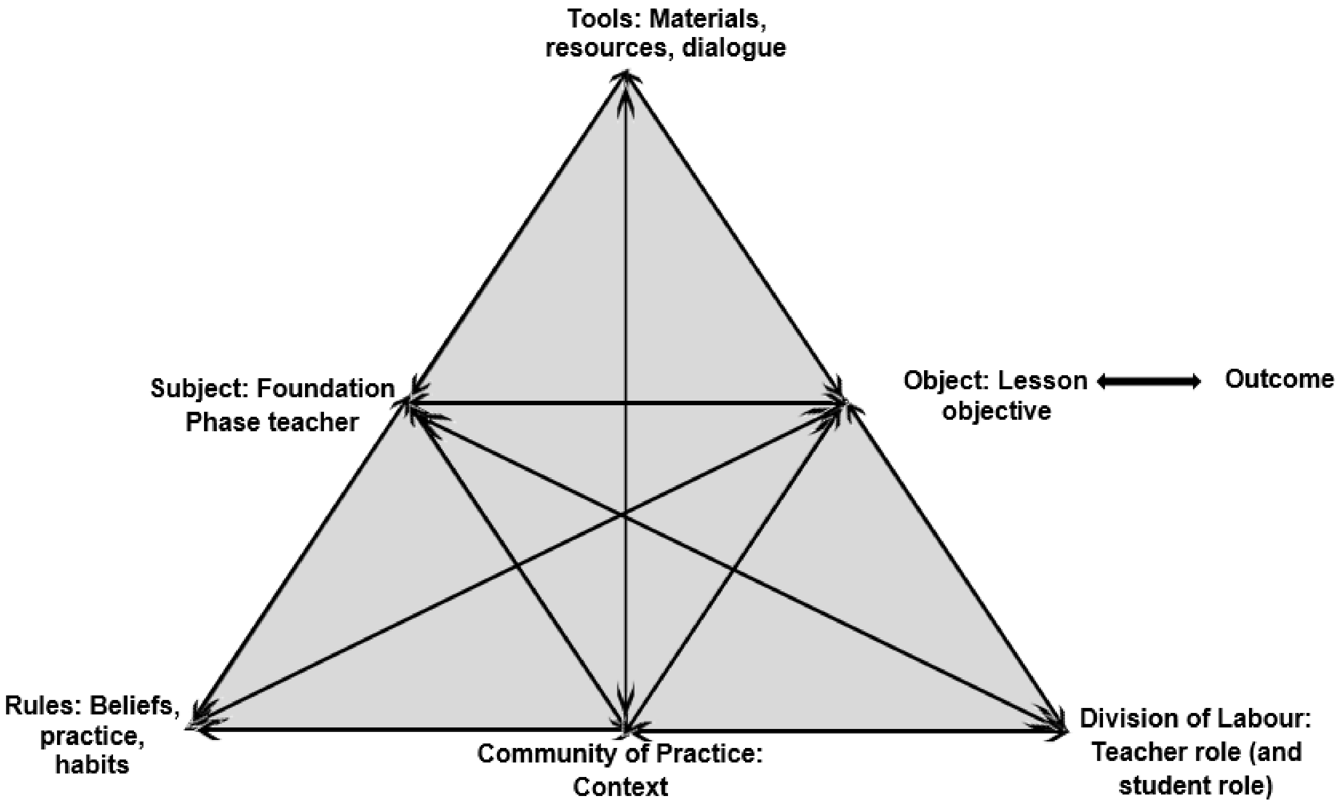
207). Venkat and Adler (2008) foreground the need to include both the context and the participants' reflections in the activity system discussions in order to reveal and elaborate on this complexity of relationships. The activity system definitions and contexts for this study are described in table 2.4.

Table 2.4: Activity Theory for this study

Dimension	Explanation	Description
Activity	What sort of activity is being carried out?	Teaching early grade mental mathematics
Subject (Who)	Who is involved in carrying out this activity? Individual whose viewpoint is adopted and the perspective from which the activity is analysed	Participating Foundation Phase teachers
Tools (What)	By what means are the subjects carrying out this activity?	Materials, resources and dialogue
Object (Why)	Why is this activity taking place? The purpose (objective) of the activity This precedes and motivates the activity	The lesson objective as communicated by the participating teachers
Rules	Are there any regulators and norms influencing the actions within this activity?	Beliefs, practice (curriculum interpretation and implementation) and habits
Division of labour	Who is responsible for what during this activity?	Teacher (and student) role in the classroom
Community of practice	What is the environment in which this activity is being carried out?	The teaching context, in relation to school practices and curriculum demands (Education Department, subject advisors, school management)
Outcome	What were the results of this activity?	Participant teachers' reflections on their actual versus intended results

The relationships between the aspects of Activity Theory that are represented in table 2.4 can be diagrammatically represented using Engeström's second generation model of Activity Theory as illustrated in figure 2.7.

Figure 2.7: Activity Theory for this study



An activity system includes the person who acts (here the teacher as the subject), the tools by which the action is accomplished (teaching resources, materials, dialogue), and the goal that the subject aims to reach by means of the tool. In Figure 2.7 the top triangle represents this basic relationship. This 'first order' triangle represents the aspects of practice that are most visible, and that encompass the activity. The basic first order activity triangle is then extended to include the context of the teachers' intrinsic reasons for acting as they do, as well as the social context (the wider community in which the action is located). This 'second order' layer encompasses the less visible aspects of practice. Although these dimensions may not be 'seen' in practice, they influence the decisions made and actions taken in the first order triangle (Roth and Radford, 2011). Activity Theory provides a framework for organising the various dimensions of teaching as a system so that data can be interpreted in reference to the dimensions and the relationships between dimensions. The 'nodes' of the activity system indicate the various dimensions of an activity. The framework that comprises these dimensions allows for the interactions between them to be acknowledged (Roth and Radford, 2011). Through analysing the relationships between the dimensions, the dominant relationships can be detailed and the outcome of the activity described.

Hence, Activity Theory is itself a tool that enabled me to systematically consider the relationships between the dimensions of the system, and to analyse 'Who is doing what, why and how?' as the teachers reflected on their lessons and detailed their perceptions of practice.

2.8 Chapter Conclusion

Although the discussions in the literature review are captured under separate headings, they relate and integrate to form the 'system' of teaching in that teaching practice (how), teacher beliefs and perceptions (why), and what is taught (mental mathematics) is informed by what teachers are wanting to achieve (purpose of teaching practice):

- Teaching practice: what it means to teach for understanding and reasoning.
- Teacher beliefs and perceptions: the origins of practice.
- Mental mathematics: the role of mental mathematics in developing understanding and reasoning.

Mathematical proficiency provides a research-based description of the interrelatedness of the various types of mathematical knowledge needed for mathematical success, and is supported by the research that describes the relationship between procedural and conceptual knowledge. Teachers who embrace the development of understanding and reasoning as a teaching objective, and have the pedagogical knowledge, are able to provide learning opportunities and experiences that develop mathematical proficiency (Gervasoni, 2011). We cannot assume that the participating teachers have or do not have the pedagogical knowledge necessary to teach for understanding. With this in mind, this study included teachers in a discussion and workshop session around teaching for the purpose (lesson objective) of developing understanding and reasoning (mathematical proficiency), and to analyse their practice and experiences of this. Using Activity Theory as the framework allowed for the consideration of many of the factors that influence and shape practice, including the selection and use of mental mathematics teaching materials, and the use of 'teacher talk' or dialogue (Brodie, 2007; Cazden, 2001). Being cognisant of the findings of the existing research, the teachers were asked to reflect on their teaching at various times during their participation in the research. It is their perceptions of their practice which is key to this study. My analysis of the teachers' perceptions of practice used the framework of Activity Theory to reveal and describe the insights of *who is doing what, why and how*.

Chapter 3: Methodology

3.1 Introduction

The purpose of this chapter is to provide a description of and rationale for the qualitative research methodology used to analyse the teachers' reflections on and perceptions of their mental mathematics teaching practice. This chapter addresses the research design, data collection and analysis procedures, trustworthiness and ethics used in this study.

3.2 Research Design

3.2.1 Purpose

The purpose of this study was to analyse teachers' reflections on their mental mathematics teaching practice. A case study of perceptions of mental mathematics teaching practice was used for the purpose of analysing teachers' reflections on their practice, and answering, "***How do early grade teachers perceive their mental mathematics teaching practice?"***

3.2.2 Paradigm

This is a qualitative, interpretative study that investigates in order to understand (Connole, 1993). The interpretive research paradigm is characterised by a need to understand the world as it is from a subjective perspective within the frame of reference of the participant, and not the observer (Ponelis, 2015, p. 538). Using Connole's (1993, p. 62) detail, the interpretive paradigm can be further described as subjective understandings which do not assume one reality, and therefore meaning-making precedes fact in order to discover the meanings and beliefs underlying the actions of others.

In keeping with the characteristics of the interpretive paradigm, a case study was used to investigate teachers' perceptions of mental mathematics teaching practice. The case study approach fits the purpose of this study as it lends itself to answering 'how' and 'why' questions (Baxter and Jack, 2008) and allowed me to explore the case within the contexts and realities that contribute to the practice of teaching (Tellis, 1997). In a case study, triangulation refers to the protocols that are used to ensure accuracy and lower the probability of alternative accounts (Tellis, 1997). Data source triangulation was used for this study, namely:

- Introductory interviews (semi-structured).
- Lesson observation recordings (videos).
- Lesson observation reflections (checklist).
- Reflection interviews (unstructured).

This triangulation enabled me to collate the data to provide a rich description of the participant teachers' perceptions of practice. Activity Theory was the analytical framework used for this study because it allowed me to capture mathematics teaching practice as an interrelated human and social activity, and to analyse the teachers' reflections within the contexts that contribute to the practice of teaching (Jones and Hashim, 2007).

3.2.3 Context

The participating teachers form part of the teaching team at a low-fee state primary school in the Western Cape, South Africa. It is one of the schools where I am currently involved in early grade mathematics coaching support. I have been involved in the coaching programme at this school since its inception five years ago. The coaching support was initiated at the school as the teachers were willing to participate in the programme and were ready to engage with the support. All the early grade (Grades 1, 2 and 3) teachers were invited to participate. Two Grade 1 teachers and two Grade 3 teachers accepted the invitation. The Grade 3 teachers participated for the duration of the study and their data were included in the analysis. The data from the two Grade 1 teachers were excluded on the following grounds:

- Grade 1 Teacher 1: due to ill health, this teacher was absent for six weeks of the term and missed the workshop session with the other participating teachers. This was later caught up on a one-to-one basis, but it was agreed that her experience of the workshop session was significantly different to the other participating teachers and, although she continued to participate in the research, it was decided to exclude her data as a conclusive comparison would not be able to be made.
- Grade 1 Teacher 2: during the reflection session of the second lesson, this teacher admitted that she had not prepared at all for this lesson and did not want to reflect

on it as she felt that it was not representative of the objective and of what she usually achieves with her lessons. This sentiment was confirmed in her feedback on the reflection of both lesson observations. It was decided to exclude her data as a conclusive comparison would not be able to be made.

The school principal and teachers were willing to participate in the study; they acknowledged that their participation was voluntary and could be terminated by them during any stage of the study. Data collection commenced once the study had been introduced and the participants had given consent and knew their rights, and were clear on the purpose and implications of the data collection methods. My role as teacher-coach and, for a while, as researcher in the teachers' classrooms, placed me in the teachers' activity systems. The analysis and interpretation of the data is sensitive to the fact that my involvement in their teaching context (community of practice) may have unwittingly imposed rules with which the teachers might have felt they needed to comply. That said, the focus of the study was the teachers' perceptions of their practice, thus I foregrounded the perspective of the teachers, and used their reflections to support my analysis and findings.

The participating teachers in this study are known to me through the coaching support I provided and a relationship of trust and collaboration had already been established with the school management and teaching team. The existing relationship worked well for this study as my presence in the school and classroom environment was familiar and accepted by all the research participants. The necessary steps to mitigate both ethical and validation risks were taken and are discussed in detail later in this chapter.

3.3 Data Collection

Throughout the study it was important to be mindful of the characteristics of an interpretive approach to research that strives to consider the perspective of the participants from their frame of reference. The data collected needed to be rich and qualitative with a focus on the teachers' perceptions of practice. The interviews, lesson observation video recordings and teacher reflections relied on conversation and collaboration with the participating teachers, and served to foreground their perspectives in the data interpretation and analysis.

As part of the study, the participating teachers engaged in a two-hour workshop session. During this session, I introduced research about teaching for understanding and reasoning. We discussed the notion of mathematical proficiency (Kilpatrick, Swafford and Findell, 2002) and interpreted it in relation to mental mathematics specifically. Although the teachers had been receiving weekly coaching sessions and had previously attended a variety of professional development workshops aimed at early grade mathematics teaching, they had not yet been part of a workshop that explicitly discussed mathematical proficiency and the implications for teaching mental mathematics for understanding and reasoning. The workshop was not a classic intervention to test or measure a ‘pre’ and ‘post’ phenomenon, but rather a way of sharing a methodology and vocabulary that teachers could use to think about and explain their goals, motivations and actions. While data were not collected from the workshop, the mathematical proficiency framework served as a lens for reflecting, and was used in the design of a self-reflection checklist which the teachers used to guide their reflections on practice. The schedule and stages of data collection are outlined in table 3.1.

Table 3.1: Schedule and stages of data collection

Phase	What	When
Pre-Workshop	Step 1: Individual lesson observation video recordings (15-20 min)	March 2019
	Step 2: Individual introductory interviews (30 min)	
	Step 3: Individual lesson observation reflection interviews (30 min) *The teachers viewed the video recording of their pre-workshop lesson during the interviews and reflected on their practice (without the shared lens of mathematical proficiency)*	April 2019
Step 4: Workshop session (2 hours)		
Post-Workshop	Step 5: Individual lesson observation video recordings (15-20 min)	June 2019
	Step 6: Individual reflections, using the self-reflection checklist, on both the pre- and post-lesson observation video recordings *(teachers' own time)*	August 2019
	Step 7: Individual lesson observation reflection interviews (1 hour) *The teachers were given the opportunity to add thoughts and comments regarding the self-reflection checklists* *The teachers viewed the video recording of their post-workshop lesson during the interviews and reflected on their practice (with the shared lens of mathematical proficiency)*	September 2019

From table 3.1 the data collection time frame can be further detailed as such: the lesson observations (step 1) were followed by the introductory interviews (step 2) in the last week of the first term (end of March). The reflection interviews (step 3) were conducted in the second week of the second term (middle of April), and the workshop (step 4) followed a week later. In order to accommodate the teachers' teaching and assessment schedules, as well as to give the teachers the opportunity to process and discuss (informally amongst themselves) the workshop content, the next step of data collection (step 5 – the second lesson observation) was scheduled for the end of the second term (end of June). At the start of the third term (beginning of August) the teachers were given both of their lesson observation recordings and the self-reflection checklists. The teachers were allowed to complete the self-refection checklists at home without a time constraint, and step 7 of the data collection (the reflection interview for the second lesson) took place at the end of the third term (end of September).

All interviews were audio recorded, and the lesson observations were video recorded, with permission from the participants. The steps and methods of data collection as outlined in table 3.1 are now described in further detail.

3.3.1 Step 1: Pre-workshop lesson observation video recording

The pre-workshop lesson observation was video recorded for the purpose of creating a record of teaching practice on which the teachers could later reflect. The focus of this observation video recording was the teacher, and I (the recorder) sat at the back of the classroom and maintained the focus on the teacher. The vantage point from the back of the classroom allowed me to capture the student-teacher interaction without compromising the students' identity. The lesson observation video recording was scheduled individually at convenient times for each teacher. No duration suggestion or limit was dictated for the lesson, but half an hour was scheduled as the teachers all agreed that this formed part of their daily routine, and would be sufficient time for them to incorporate and teach a mental mathematics lesson (activity) within their mathematics lesson[3].

This observation was done as the first step of data collection, as I wanted it to be a direct reflection of the teachers' practices and therefore recorded before any interviews, discussions or the workshop were scheduled.

3.3.2 Step 2: Pre-workshop interview

The purpose of this interview was to gain an overall representation of the teachers' context, teaching experience and experiences of teaching and of their own mathematics learning and teaching, focusing on teaching mental mathematics. This interview was conducted after the lesson observation had taken place and was scheduled individually at a convenient time. In line with the qualitative, interpretive approach to this research, a semi-structured interview was used which allowed the participant teachers to access a broad range of experiences and perspectives that may not have been explored using a structured and fixed set of questions (Adams, 2010). A bank of questions was prepared (see Addendum A) in order to initiate and guide conversation. The order and delivery of these questions was flexible. Questions were also omitted if I felt them to be inappropriate or non-productive at the time, and additional

[3] Although mental mathematics activities exist throughout the bigger mathematics lesson, the term 'lesson' has been used to describe the mental mathematics activity because the participating teachers talk about teaching a 'lesson' when referring to the mental mathematics activity within a mathematics lesson.

questions were added to probe for further detail if deemed appropriate and necessary. These questions refer specifically to mathematics and the teaching of mental mathematics. A few sample questions taken from Addendum A:

- Think back to your own school experiences:
    - Your own schooling /education?
    - How do you remember your primary school years?
    - How do you remember being taught mathematics (mental mathematics)?
- What are your views on how children learn mathematics, and in particular how they learn mental mathematics?
- How does this currently influence how you teach?
- What is your current classroom routine when teaching mathematics (mental mathematics)?

3.3.3 Step 3: Pre-workshop lesson reflection interview

An unstructured interview was used for this reflection as the teachers controlled when and what they wanted to comment on when viewing their observation video. In keeping with the characteristics of an interpretive paradigm, which seeks to understand the world as it is from a subjective perspective within the frame of reference of the participant, an unstructured interview was used rather than structured questioning. The purpose of this interview was to elicit the teachers' perceptions through dialogue (Connole, 1993) in order to gain insight into the teachers' perceptions of their practice. This reflection interview took place after the first lesson observation and introductory interview, but before the workshop and the introduction of the shared lens of mathematical proficiency. The interview was scheduled individually at a convenient time for each teacher. As the lesson video recording played, the teacher indicated when she wanted to comment, and the recording was paused while the teacher commented. My role was to audio record the dialogue and verify my understanding by asking clarifying questions in response to the teachers' comments.

3.3.4 Step 4: Workshop session

The workshop session introduced the five strands of mathematical proficiency (Kilpatrick, Swafford and Findell, 2002) as a research-based framework that underpins teaching for understanding and reasoning: in this case mental mathematics teaching. The purpose of the workshop session was to provide the teachers with the language and lens through which to reflect on their practice.

I presented the workshop in a two-hour session after school in one of the teacher's classrooms. All the Foundation Phase teachers attended the session and not just those participating in the research. The five strands of mathematical proficiency (calculating, understanding, applying, reasoning and engaging) were introduced and the resultant implications for teaching were highlighted and discussed (see Addendum B for an example of the workshop material). The implications for mental mathematics teaching that arose from the discussion focused on creating opportunities to integrate the strands of mathematical proficiency, teacher actions and the cognitive demand of the tasks. A brief overview of the implications for teaching drawn from Kilpatrick et al. (2002) was shared and discussed in the workshop and is presented in table 3.2.

Table 3.2: Overview of the implications for teaching as presented in the workshop

Implications for teaching when considering mathematical proficiency as a teaching objective
Mathematical Proficiency: - Provide opportunities for regular practice - Provide opportunities for students to notice patterns and connect facts - Provide opportunities for application of known facts to discover unknown facts or problems - Provide opportunities for students to discuss and talk about their thinking - Provide opportunities for students to experience success, and gain confidence through sense-making and understanding
Teacher Action: - Provide opportunities for reflection by asking questions (and not just listening for correct answers) - Facilitate discussion amongst students
Cognitive Demand: - Provide structured items that set up patterns and connect facts - Provide enough cognitive demand to encourage productive struggle

With mathematical proficiency as the goal for instruction, selecting and using materials that have been intended for this purpose is crucial to achieving this outcome (Kilpatrick, Swafford and Findell, 2002; Smith and Stein, 1998). The teachers were already familiar with and had been using materials designed to promote mathematical proficiency, namely, the NumberSense Mental Mathematics materials (Brombacher and Associates, 2011). The workshop session foregrounded the design principles of these materials. During the workshop session the teachers worked in pairs with the NumberSense materials to identify the elements of the five strands of mathematical proficiency within the tasks. This became a group activity where all the teachers engaged with the same page of the materials and discussed how the design of the tasks for that page supported the development of the various strands of mathematical proficiency. An example from the materials, with the description of the task design, is summarised in table 3.3.

Table 3.3: Tasks and their design (adapted from the NumberSense Mental Mathematics Guide[4]*, 'NumberSense Workbook 9' – early Grade 3)* (Brombacher and Associates, 2011)

<table>
<tr><th>#</th><th>Task Structure
(what the teacher says)</th><th>Task Design in relation to Mathematical Proficiency
(the purpose of task structure)</th></tr>
<tr><td>1</td><td>What is 4 plus 4?</td><td rowspan="28">The materials support the development of understanding and reasoning in that their design provides opportunities for students to engage with mental mathematics and not just 'do' mental mathematics

Calculating (with understanding):
Provide reasoning-based opportunities for practice (that develop in complexity over time)

Understanding:
Provide structured items that set up patterns and connect facts

Applying:
Encourage students to use what they do know to work out what they do not know. Provide items that are unknown to encourage the use of known facts (4+4; 8+4), and reveal their relationship (connectedness and patterns) to the unknown fact
Items appropriate to students' current level of development

Reasoning:
Provide opportunities for students to reflect on what they have done, forcing them to reason and develop their understanding of what they are doing
Items are demanding enough to encourage productive struggle

(Brombacher, 2012; Kilpatrick, Swafford and Findell, 2002)</td></tr>
<tr><td>2</td><td>What is 24 plus 4?</td></tr>
<tr><td>3</td><td>What is 34 plus 4?</td></tr>
<tr><td>4</td><td>What is 44 plus 4?</td></tr>
<tr><td>5</td><td>What is 54 plus 4?</td></tr>
<tr><td colspan="2">"What do you notice?"</td></tr>
<tr><td colspan="2">"How was this activity similar to or different from previous activities?"</td></tr>
<tr><td>6</td><td>What is 94 plus 4?</td></tr>
<tr><td>7</td><td>What is 104 plus 4?</td></tr>
<tr><td>8</td><td>What is 8 plus 4?</td></tr>
<tr><td>9</td><td>What is 28 plus 4?</td></tr>
<tr><td>10</td><td>What is 38 plus 4?</td></tr>
<tr><td>11</td><td>What is 48 plus 4?</td></tr>
<tr><td colspan="2">"Explain how you got your answer."</td></tr>
<tr><td>12</td><td>What is 68 plus 4?</td></tr>
<tr><td>13</td><td>What is 88 plus 4?</td></tr>
<tr><td>14</td><td>What is 6 plus 4?</td></tr>
<tr><td>15</td><td>What is 16 plus 4?</td></tr>
<tr><td>16</td><td>What is 26 plus 4?</td></tr>
<tr><td>17</td><td>What is 36 plus 4?</td></tr>
<tr><td>18</td><td>What is 46 plus 4?</td></tr>
<tr><td>19</td><td>What is 86 plus 4?</td></tr>
<tr><td>20</td><td>What number plus 8 equals 48?</td></tr>
<tr><td>21</td><td>What number plus 7 equals 47?</td></tr>
<tr><td>22</td><td>What number plus 6 equals 36? Explain.</td></tr>
<tr><td>23</td><td>What number plus 5 equals 55?</td></tr>
<tr><td>24</td><td>What number plus 50 equals 53?</td></tr>
<tr><td>25</td><td>What number plus 60 equals 67?</td></tr>
</table>

[4] The NumberSense Mental Mathematics Guides are freely available for Grades 1 to 3. These can be accessed and downloaded at no charge from www.NumberSense.co.za

While the materials have been designed to support the development of understanding and reasoning, their design alone does not guarantee effective and efficient teaching towards this goal. The success of these materials, of any tools, is in their implementation and as Activity Theory maintains, the tools are used by the subject to pursue the outcome (Jones and Hashim, 2007).

The workshop session was not planned as a data collection method, but the impact of this session on teachers' reflections contributed to the data collected in the subsequent interviews and self-reflections. A shared, coherent view of mathematical proficiency is important if teachers are to reflect productively on their lessons in order to describe their perceptions of practice. The workshop session, and the shared view of mathematical proficiency as an objective for teaching for understanding and reasoning, provided a common lens for reflection.

3.3.5 Step 5: Post-workshop lesson observation video recording

The purpose of this observation was to record the post-workshop mental mathematics teaching practices. This observation was done roughly eight weeks after the workshop session in order to accommodate the teachers' teaching and assessment schedules, as well as to give the teachers the opportunity to process and discuss the workshop content in their community of practice at school. As during the first lesson video recording, my role as observer (and recorder) was to focus on the teacher, in order to capture the lesson for the purpose of reflection at a later stage. The lesson observation video recording was scheduled individually at a convenient time for each teacher. As with the first lesson video recording, no duration suggestion or limit was dictated for the lesson, but half an hour was scheduled to teach a mental mathematics lesson (activity) within their mathematics lesson.

3.3.6 Step 6: Self-reflection checklist

The purpose of the self-reflection checklist was to provide the teachers with a common means of reflecting on both the pre- and the post-workshop lessons and, in doing so, provide detail and insight into their perceptions of practice. A self-reflection checklist was provided for the purposes of guidance and recording (see figure 3.1). The teachers received two copies of this checklist: one to record their pre-workshop reflections, and the other to record their post-workshop reflections. With the completion of the self-reflection checklists, the teachers had the opportunity to add written comments and to reflect on the pre-workshop lesson with the same lens as the post-workshop lesson.

The workshop session and the shared view of mathematical proficiency provided a common language and lens for reflection and highlighted the implications for teaching when considering mathematical proficiency as a teaching objective. I consciously used the descriptors of practice developed in the literature review and listed in table 3.4 to design the self-reflection checklist.

Table 3.4: Self-reflection checklist categories as described in the literature review

MATHEMATICAL PROFICIENCY (Kilpatrick, Swafford and Findell, 2002)	Calculating Understanding Applying Reasoning
TEACHER ACTION (Davis, 1997; Hiebert and Grouws, 2007; McGatha *et al.*, 2018; Van de Walle *et al.*, 2001)	Random items Structured items Asking related questions Building on responses
COGNITIVE DEMAND (Stein *et al.*, 2000)	Knowing Using procedures Applying procedures Reasoning

The teachers were given a copy of both their lesson observation video recordings (pre and post), and two copies of the self-reflection checklist. The teachers were asked, in their own time, to review the video recordings and complete the checklists for each lesson recording. The purpose of the self-checklists was to give the teachers the opportunity to review and reflect on their lessons in their own time and space. The self-reflection checklists also provided the teachers with the opportunity to reflect on both lessons with the lens of mathematical proficiency. To ensure consistency across and within the completion of the self-reflection checklists, the teachers reviewed their lessons at two minute intervals. Figure 3.1 illustrates the checklist used in this study.

Figure 3.1: The self-reflection checklist used by the teachers to reflect on both of their lessons

Mental Mathematics Lesson Observation Self-Reflection							
MATHEMATICAL PROFICIENCY		2	4	6	8	10	12
Record with an "x" alongside where in the video you identify your actions as contributing to the development of …	Calculating						
	Understanding						
	Applying						
	Reasoning						
	None of the above						
TEACHER ACTION		2	4	6	8	10	12
Record with an "x" alongside where in the video you identify your actions as …	Random items/sums (where the answer is the outcome)						
	Structured items/sums that set up a pattern (where noticing the pattern is the outcome)						
	Asking related questions						
	Building on responses						
	None of the above						
COGNITIVE DEMAND		2	4	6	8	10	12
Record with an "x" alongside where in the video you identify your lesson content as …	Knowing (memorisation)						
	Using procedures (without meaning)						
	Applying procedures (with meaning)						
	Reasoning						
	None of the above						
COMMENTS							
What was your focus for this lesson? What were you hoping to achieve?							
Did you achieve this? If no, what could you have done differently?							
Other comments							

The teachers were asked to pause the video recording every two minutes to reflect and record one sub-category for each category they felt dominated those two minutes of their lesson. This was done over a 12-minute time frame (the average duration of the mental mathematics lessons). Only one sub-category for each category could be observed for each interval. This interval recording can be likened to a 'snapshot'. If you took a photo (snapshot) at each two-minute recording interval, what would it reveal about the teacher's practice and lesson content at that time? Only one of the sub-category options was possible for that 'snapshot'. Allowing more than one response per interval would have resulted in teachers having varying responses in the same time frame. This process can be likened to that of a time-motion study (Baumgart and Neuhauser, 2009).

The time-motion study was originally a method developed by Frank and Lillian Gilbreth (based on the work of Frederick Taylor) in the early 1900s (Baumgart and Neuhauser, 2009) for collecting data to establish employee productivity standards in large factories where production and profit margins are time-sensitive and every second saved resulted in money made. Time-motion studies are generally appropriate for repetitive tasks, where a complex task is broken into small, simple steps and the sequence of performance of execution of these steps is tracked in order to detect and eliminate redundant actions within the production cycle.

I adapted this method of data collection in order for the teachers to track their observations and reflections of their mental mathematics teaching practice in which instances of mathematical proficiency, teacher action and cognitive demand, and their relevant sub-categories, were observed, tracked and recorded over the lesson duration. The teachers also had the chance to add additional thoughts and comments to the checklist through the few structured questions that were asked at the end of the checklist:

- What was your focus for this lesson? What were you hoping to achieve?
- Did you achieve this? If no, what could you have done differently?
- Other comments?

The checklists were returned to me once the teachers had reflected on both lessons and completed the self-reflection process. The data from the checklists were tracked on a graph which represented the occurrence of the relevant sub-category for each two-minute interval, and used in the analysis which will be described in further detail in the next chapter.

3.3.7 Step 7: Post-workshop lesson reflection interview

The first part of this interview was used to capture the teachers' holistic reflections on their practice with reference to the completion of the self-reflection checklists. A semi-structured interview was used for this discussion as it allowed the teachers to access a broad range of experiences and perspectives that may not have been explored using a structured and fixed set of questions (Adams, 2010). This was not a long discussion and was prompted and guided by these few questions:

- Any comments regarding your completion of the self-reflection checklists?
- Having observed and reflected on both lessons, what are the significant similarities and differences?

The second part of the interview focused on the post-workshop video recorded lesson in order to gain insight into the teachers' perceptions of their practice. Although the teachers had already completed the self-reflection checklist for this lesson, this interview did not refer to the checklist but followed the format of an unstructured interview. As per the pre-workshop reflection interview (step 3), an unstructured interview was used to elicit the teachers' perceptions through dialogue (Connole, 1993). There were no set questions and the teachers controlled when and on what they wanted to comment when reflecting on their observation video. This gave me the opportunity to collect the teachers' reflections in relation to the exact moment in the lesson that these reflections happened. These were used for the purposes of triangulation in that these reflections could be verified against the reflections tracked in the self-reflection checklist. The interview was scheduled individually at a convenient time for each teacher. In line with the structure and purpose of an unstructured interview, the teachers reflected spontaneously and led the interview. As with the pre-workshop reflection interview, while the video recorded lesson played, the teacher indicated when she wanted to comment, and the recording was paused while the teacher commented. My role was to audio record the dialogue and verify my understanding by asking clarifying questions in response to the teachers' comments.

3.3.8 Summary of data collected

The various kinds of data collected is summarised in table 3.5.

Table 3.5: Summary of data collected

Step	Data Source	Data Format	Data Description
1	Lesson 1	Video recording	Pre-workshop lesson observation video recording
2	Introductory interview	Audio recording	The context for and orientation to the teachers' mental mathematics teaching practice
3	Pre-workshop lesson reflection interview	Audio recording	Teachers' reflections (without the lens of mathematical proficiency) on and perceptions of the pre-workshop lesson observation video recording
***4** Workshop (no data collected from the workshop session)*			
5	Lesson 2	Video recording	Post-workshop lesson observation video recording
6	Self-Reflection Checklist	Checklist	Teachers' completed self-reflection checklist (with the lens of mathematical proficiency) for both the pre- and the post-workshop lessons
7	Post-workshop lesson reflection interview	Audio recording	Teachers' reflections (with the lens of mathematical proficiency) on and perceptions of the post-workshop lesson observation video recording

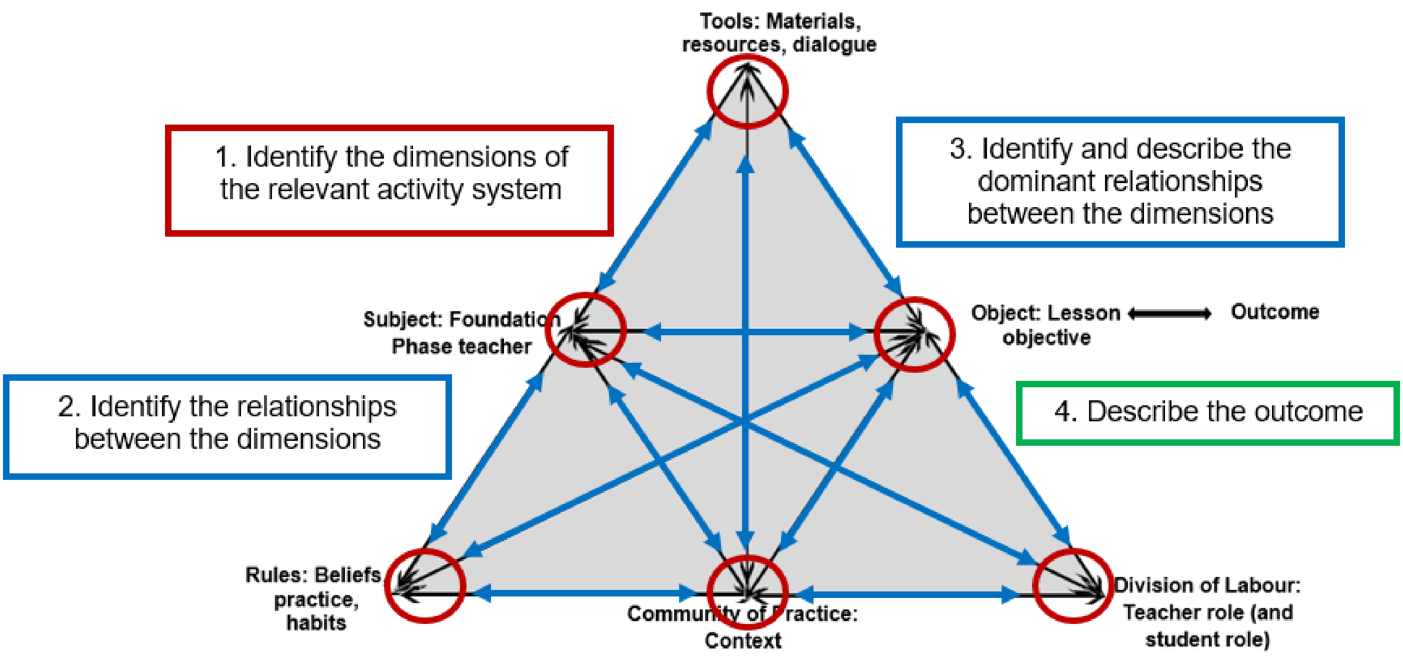
3.4 Data Analysis

The aim of data analysis in a qualitative study, following the interpretive paradigm, is to look for meaning in subjective understandings of lived experiences (Connole, 1993). The analysis involves the coding of data, identifying units of analysis (meaning), and producing themes. In this study Activity Theory allowed for the analysis of relationships between the following dimensions of the activity: subject, object, tools, rules, community, and division of labour. It provided the framework from which to explore and better understand and describe human activity and interaction during an activity (Bandara, 2018). The pre-workshop reflection data was analysed as Activity System 1 for each teacher, and the post-workshop reflection data was analysed as Activity System 2 for each teacher. The data also needed to be interpreted in relation to the context of the activity, as well as by including and acknowledging my own experiences and subjectivity in the data analysis process.

Activity Theory as an analytical framework acknowledges that teaching as an activity does not happen in isolation and allows for the analysis of the relationships between the dimensions of the system to answer, "How do teachers' perceive their mental mathematics teaching practice?". Applying Activity Theory as the framework for this study involved the systematic identification and description of data in relation to each of the dimensions and their relationships with each other (Davydov, 1999; Hasan and Kazlauskas, 2014). This systematic process is described below and illustrated in figure 3.2:

1. Identify and describe the dimensions specific to the activity system: subject, object, tools, rules, community of practice, division of labour.
2. Identify the relationships between the dimensions of the activity system.
3. Identify and describe the dominant relationships between the dimensions of the activity system.
4. Analyse and describe the outcome as a consequence of the dimensions and the dominant relationships.

Figure 3.2*: Activity Theory as an analytical framework for this study* (adapted from Engeström, 2015, p. 63)



Using the data collected from the lesson videos, interviews and from the self-reflection checklists, and following the systematic identification and description of each of the dimensions and their relationships with the activity systems, the analysis was structured as follows:

3.4.1 Preparing the data

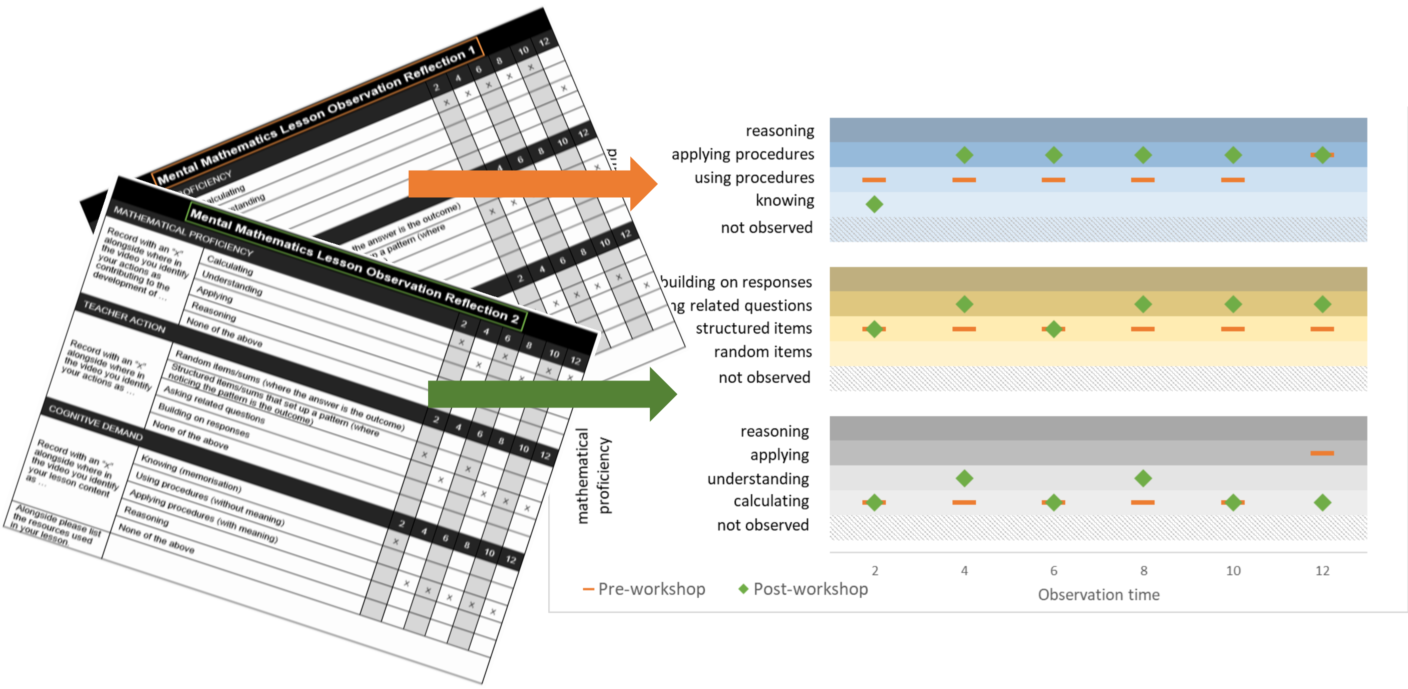
The content of each interview (audio) and lesson recording (video) was transcribed. The original recordings were saved and filed. During the transcriptions of the video recordings, and for the purpose of distinguishing student interaction versus teacher interaction during the analysis, I did an 'interaction-response' audit for each teacher, for each lesson. I noted the instances of 'asking *answer only* questions' and 'asking questions to facilitate discussion', and tracked and recorded them in a table (see table 3.6). The audit data was tallied every two minutes for the 12-minute lesson duration.

Table 3.6: The 'interaction-response' audit table

Action / Time	**02:00**	**04:00**	**06:00**	**08:00**	**10:00**	**12:00**	**TOTAL**
Asking 'answer only' questions							
Asking questions to facilitate discussion							

The data from the self-reflection checklists were tracked onto time-motion graphs. After the second (post-workshop) lesson observation the teachers were asked to reflect on both of their lesson observation video recordings (pre and post) using the self-reflection checklist (figure 3.1). This provided the teachers with the opportunity to reflect on both lessons, this time with the lens of mathematical proficiency as introduced in the workshop session. The categories used to reflect on practice were: mathematical proficiency, teacher action, and cognitive demand. Once the reflections and checklists had been completed, and in order to analyse this data, I tracked the teachers' reflections on their practice onto time-motion graphs. The graphs provided a visual representation of the teachers' perceptions of their actions, cognitive demand of the tasks, and mathematical proficiency. An example of the self-reflection checklist to time-motion graph tracking can be seen in figure 3.3. where the pre-workshop reflections were tracked in orange and the post-workshop in green.

Figure 3.3: Self-reflection checklist to time-motion graph tracking



Using the data collected from the transcribed lesson videos and interviews, and from the reflection checklists, both the pre- and post-workshop lessons were analysed in depth as described below.

3.4.2 Analysing the data

3.4.2.1 Part 1: The analysis of Activity System 1 and Activity System 2

1. **Identify and describe the dimensions specific to the activity system** (data source: introductory interview and reflection interviews)

    - Activity: an early grade mental mathematics lesson.
    - Subject: a Foundation Phase teacher.
    - Object: the lesson objective, purpose of the teaching as stated by the teacher.
    - Tools: materials, resources and dialogue used in the mental mathematics lesson.
    - Rules: beliefs, practice (curriculum interpretation and implementation) and habits.
    - Community of practice: the teaching context in relation to school practices and curriculum demands (Education Department, subject advisors, school management).
    - Division of labour: teacher (and student) role in the classroom.

2. **Identify the relationships between the dimensions of the activity system** (data source: reflection interviews and self-reflection time-motion graphs)

- Look for the main ideas within the teachers' reflections, where the reflections establish relationship connections.
- Select the direct quotes from the reflection interviews that reveal the main ideas.
- Isolate the vignettes in the lesson video recordings that indicate the time in the lesson where the reflections were made.
- Map the direct quotes to the relevant vignettes.
- Use the self-reflection time-motion graphs to analyse the dominant reflections using the categories and sub-categories from the self-reflection checklists (the shared lens of mathematical proficiency).
- Map the dominant reflections to the dimensions in the activity system to identify the relationships between the relevant dimensions.

3. **Identify and describe the dominant relationships between the dimensions of the activity system**

Describe and analyse the dominant relationships between the dimensions to provide a rich description of the teachers' perceptions of practice, and of their perceptions of the outcome of their practice in relation to their objective.

4. **Analyse and describe the outcome as a consequence of the dimensions and the dominant relationships**

Describe the outcome as revealed through the relationships within the activity system.

3.4.2.2 Part 2: The comparison analysis of Activity System 1 and 2

1. **Detail a comparison of Activity System 1 and Activity System 2 for each teacher**

Compare and analyse the similarities and differences of the dominant relationships (themes) across Activity System 1 and Activity System 2 for each teacher.

The data from the two Grade 3 teachers, pseudonyms Beyoncé and Lorraine, were analysed in depth and interpreted to present the findings of this study. The two Grade 3 teachers presented with similarities in context and experience and therefore fewer variables when interpreting and comparing the data in order to present the findings. The data analysis is described in further detail in the following chapter.

3.5 Validity and Reliability

Interpretive research generates data that is rich in detail. It is important to collect, analyse and report on this data in a manner that is credible: there is enough detail to ensure that readers are able assess the validity or credibility of the findings (Baxter and Jack, 2008). In order to ensure that this research was valid and reliable (trustworthy), I followed the measures suggested by Baxter and Jack (2008). These measures are described below:

- Coherent design: the research design was coherent, in that the purpose, paradigm, context and method all followed a qualitative and interpretive thread.
- Data management: the data were collected and managed systematically. The data described and revealed the reflections and perceptions of the teachers.
- Self-reflexivity: I demonstrated an awareness of how biases may emerge through my own subjectivity and attempted to minimise the impact of my experiences and context on data collected. Acknowledging my employment at Brombacher and Associates, who have developed the NumberSense Mathematics Programme, there was continual reflection on subjectivity during the data collection and analysis. The reflection was key to 'listening to the data' and to analysing and interpreting the teachers' perceptions of practice. The self-reflexivity enabled me to report on and interpret 'what is' in terms of the teachers' perceptions of practice and not what 'could', or 'should be' based on my own perceptions and judgements. In order to minimize the risk of analysing the data against my own judgements and opinions, I referred to the teachers' direct quotes in order to frame my interpretation and analysis.
- Triangulation: I used a variety of data sources to ensure that the analysis, explanations and findings were done through a variety of lenses. The data sources were cross-referenced as follows:

- The teachers' contexts and realities (direct quotes from the audio recordings of introductory interviews) were used to frame their perceptions of practice (direct quotes from the audio recordings of the reflection interviews).
- The teachers' perceptions of practice (direct quotes from the audio recordings of the reflection interviews) were mapped to exact moments in their lessons (vignettes from the video recordings of the lesson observations).
- The teachers' perceptions of practice were also mapped onto time-motion graphs representing their self-reflections on the lesson observations (data from the self-reflection checklists).
- Direct quotes (from the audio recordings of the reflection interviews) and the corresponding vignettes (from the video recordings of the lesson observations) were used as the basis of interpretation for the time-motion graphs (data from the self-reflection checklists).
- The final analysis summary was supported by direct quotes from the teachers (from the audio recordings of the introductory and reflection interviews).

- Double-coding: the data were coded and, after a period of time, the data were recoded, and the results were compared.
- Verification: references from the literature review were used (where relevant) to support my findings and analysis of the teachers' perceptions.

3.6 Ethics

My study was low-risk from an ethical perspective since data were not gathered from minors. I asked the participating teachers to provide their own pseudonyms so that their identities could be protected. I also refrained from mentioning the school's name and location. In order to mitigate any other ethical risk, the following steps were followed:

- Application to, and acceptance from, the relevant authorities to do the research:
  - Ethical Clearance from Stellenbosch University Ethics Committee (Human Research Department): project number 8514.

  - Ethical Clearance from the Western Cape Education Department.
- Application to, and acceptance from, the relevant authorities to use the NumberSense Mathematics Programme materials:
  - Permission granted from Aarnout Brombacher, Founder and CEO of Brombacher and Associates (the developers of the NumberSense Mathematics Programme).
- Application to, and acceptance from, the school management, teachers and parents to do the research:
  - I explained research to the principal of the school and got her permission.
  - I met with teachers and explained the research and invited participation.
  - Formal letters stating the research outline and asking for consent were given to the principal and teachers, and signed acceptance copies received.
- I ensured by means of formal letters that participants had given consent and knew their rights, and that they were clear on the following:
  - The purpose of the lesson observations and interviews and topic for discussion: teachers' perceptions of their mental mathematics teaching practice.
  - The length and format of the lessons and interviews (discussion generator, questions do not have to be answered).
  - Confidentiality, anonymity and autonomy: while anonymity cannot be guaranteed, the school and teachers have been given pseudonyms in an attempt to lessen the risk.
  - Permission to audio record the interviews and to video record the lesson observations.
  - Teacher participation in the research through participating in interviews and to the agreement of the use of their narratives in the study.
  - Permission (from teachers and parents) to video record lessons and use this data as part of the interview discussion.
  - Option of contributing to the narrative: expanding on thoughts prior to and post the interview.
  - How data will be managed once collected (anonymous, where stored, who has access, when will data be disposed of, how will data be disposed of).

(See Addendum C for letters and ethical clearance documents)

3.7 Key Considerations and Limitations

3.7.1 Key considerations

When conducting interviews and observations, in order to allow the data to be collected and analysed as close to the teachers' realities as possible, and to minimise subjective interpretation, the following were considered:

- Interviews:
    - To listen, and note responses (audio record) as they happened, and not assume or pre-empt responses.
    - To guide the conversation with a logical progression of questions, and be mindful of interviewer bias.
    - To create a conducive interview environment.
- Observation:
    - To be aware of the effects of the observer on those observed, for example, putting on a show for the observer that may not reveal the actual practice.

I chose to conduct two (pre and post) lesson observation video recordings in order to avoid once-off observations that might not be typical and could be misleading.

3.7.2 Limitations

My own subjectivity and bias need to be acknowledged, particularly as I am part of the teachers' activity systems, in that I had an existing mentoring relationship with the participants at the time of the study. In order to mitigate any bias, I have included a variety of data sources and provided direct quotations from the teachers' perspective with any claims I have made. Another possible limitation is that the mentoring relationship might have influenced the way the participating teachers responded. By asking the teachers to reflect on their lessons using the checklist in the privacy of their own time and space, I intended to give the teachers the opportunity to organise, process and express their own thoughts and observations well before the reflection interviews.

This case study provided me with an excellent opportunity to gain insight into teachers' perceptions of their practices as it enabled me to gather data from a variety of sources (Baxter and Jack, 2008). Yet, I acknowledge that the data from this case study is not generalisable as it is confined to a short activity in community of practice at a single school (Tellis, 1997).

3.8 Chapter Conclusion

In this chapter, I have described the methodological approach for this study. I have introduced the participants and context of the research and how and when the data were collected and analysed. I have also described how this was done in relation to the framework of Activity Theory. The validity, trustworthiness and ethical considerations were also described in this chapter. In the following chapter I provide more detail about how the data were analysed and I report on and describe the relationships and outcomes of the teachers' perceptions of their mental mathematics teaching practice.

Chapter 4: Data Analysis and Presentation of Findings

4.1 Introduction

This chapter analyses the data from the two Grade 3 teachers, pseudonyms Beyoncé and Lorraine, using the framework of Activity Theory (subject, object, tools, division of labour, rules, community of practice). Activity Theory, as the framework for this study, allows for the acknowledgment that teaching as an activity does not happen in isolation, but considers the influences of the community and the use of tools on the subject and object to answer the wider question of: *who is doing what, why and how*? as discussed in chapter 2. Using the relationships within and between the dimensions of the activity provided a rich description of each teachers' perception of practice. The pre-workshop lesson data were analysed as Activity System 1, and the post-workshop lesson data as Activity System 2.

The analysis was done in two parts:

Part 1: the analysis, for each teacher, of Activity System 1 and Activity System 2

1. Identify and describe the dimensions specific to the activity system: subject, object, tools, rules, community of practice, division of labour.
2. Identify the relationships between the dimensions of the activity system.
3. Identify and describe the dominant relationships between the dimensions of the activity system.
4. Analyse and describe the outcome as a consequence of the dimensions and the dominant relationships between the different dimensions of the activity systems.

Part 2: the comparison analysis between Activity System 1 and Activity System 2

1. Detail a comparison of Activity System 1 and Activity System 2 for each teacher.

4.2 The Analysis of the Activity Systems

The activity systems for each teacher are analysed using the data gathered from teachers' reflections on, and perceptions of practice. Both teachers have similar teaching experience, across similar contexts, and currently share a teaching context in that they teach at the same school, in the same grade. With reference to Activity Theory, the teachers share their community of practice, which is described below. The other dimensions specific to each teacher and each activity are described as part of the analysis for each teacher and their activity systems that follow.

4.2.1 Community of practice

Within Activity Theory the community considers the social and contextual influences on the activity. These influences extend to the dimensions of rules and division of labour. The way in which activities are planned and executed, and tools are selected and implemented, are influenced by the rules (beliefs, practices and habits) and the division of labour (teacher role versus student role) (Hasan and Kazlauskas, 2014). Teachers' beliefs may have roots in their own educational experiences as well, and their practices may be shaped by their teaching contexts and their experiences of curriculum demands. Hence, the community of practice for Beyoncé and Lorraine is detailed with reference to their school context, teaching context and curriculum demands (Education Department, subject advisors, and curriculum coverage).

School context: Beyoncé and Lorraine both form part of the teaching staff at a Western Cape Education Department primary school in Cape Town, South Africa. Teaching at a state school means that both teachers are constrained to following a prescribed curriculum and to meeting assessment and progression criteria as determined by the DBE. Both Beyoncé and Lorraine were educated in state schools in the Western Cape, received teacher training at the same institution in the Western Cape, and have been teaching in state schools in the Western Cape for the duration of their careers.

Teaching context: Beyoncé and Lorraine have both been teaching for more than thirty years, and are both currently Grade 3 classroom teachers. Their classes comprise just under thirty students and not all students have English as a first language. As well as being a class teacher, Beyoncé is the Foundation Phase Head of Department. The grade teachers have weekly planning meetings where they plan what to teach. The participant teachers have an easy-going relationship and will often consult one another to share and reflect on their mathematics teaching. While there is a strong emphasis on following the CAPS curriculum and using the prescribed resources, the school management and teaching team are open to methodologies and resources that support the teaching and learning of mathematics for understanding and reasoning.

Curriculum demands: My involvement as teacher coach poses contrasting demands on the teachers' practices compared to the CAPS curriculum. Curriculum Advisors from the DBE pay regular visits to the school where progress in relation to curriculum coverage is monitored. The CAPS curriculum suggests a sequenced programme for covering curriculum content and details this coverage on a day-to-day and week-to-week planner. Although the DBE advocates for mathematical proficiency as a teaching objective (Department of Basic Education, 2018), the suggested sequencing and structuring of the curriculum coverage does little to support the realisation of this. The tension of curriculum coverage is real for many teachers across South Africa, and research in this field confirms that teachers experience the sequencing and pacing as 'too much', with little time to revise and consolidate (Du Plessis and Marais, 2015). While the sequence for curriculum coverage is a 'suggested sequence' (Department of Basic Education, 2011, p. 36) many teachers interpret this sequence as inflexible.

The CAPS curriculum guidelines for mental mathematics gives broad indications of number ranges and operations. The curriculum guidelines also specify that mental mathematics forms part of the whole class and small group activities for ten minutes during the daily mathematics lesson (Department of Basic Education, 2011, pp. 11, 12, 23). Other than the general content and classroom organisation suggestions, there is no official guidance for teaching mental mathematics from the DBE. A formal Grade 3 mathematics lesson plan for the first term, developed by the Western Cape Education Department, suggests the following mental mathematics activities for the week: *"Engage the whole class in rapid recall of addition and subtraction facts to 20, change the nos. [sic] daily to challenge learners with the same question type; and bonds of 20."* (Western Cape Education Department, 2020, no page number). This presents a challenge and tension from a rule perspective for the teachers to spend the time on structured and mathematical proficiency-orientated mental mathematics tasks, as so little 'weight' and attention is detailed for mental mathematics in the curriculum documents and associated resources. Although mental mathematics is cited within the curriculum as being important, the lack of curriculum guidance, materials and specific task examples for mental mathematics suggests little to support the role that mental mathematics plays in developing reasoning-based calculating strategies.

The analysis and interpretation of the data considers the community of practice and the influence that this has on the various aspects (dimensions) of teaching practice and the decisions and implementation (object, tools, division of labour) that comprise teaching practice. The analysis is sensitive to the fact that my involvement in Beyoncé and Lorraine's community of practice, through the coaching support, may have unwittingly posed rules with which the teachers might have felt they needed to comply. That said, I foregrounded the perspective of the teachers, and used their reflections to support my analysis.

The analysis of Beyoncé's and Lorraine's perceptions of practice started with the identification of the dimensions of the activities for Activity System 1 (pre-workshop lesson) and Activity System 2 (post-workshop lesson). From here the relationships within these activities are identified through quotes from the reflective interviews and from the corresponding lesson vignettes from the video recordings. From the data collected from the reflections and interviews, the activity systems are described and the relationships within them analysed and the activity outcome revealed.

4.2.2 Beyoncé: Analysis of Activity System 1 and Activity System 2

4.2.2.1 Activity System 1

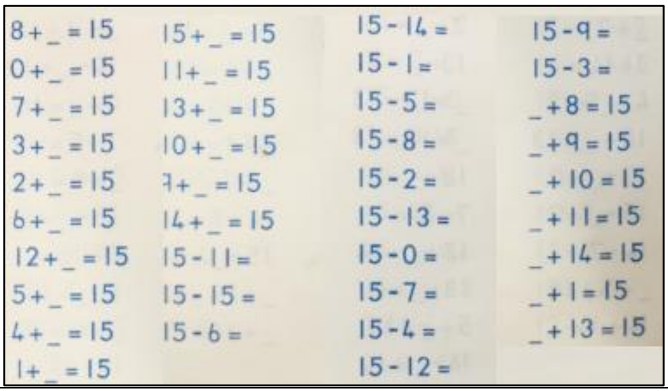
1. **Identify and describe the dimensions specific to Beyoncé's activity system**

The dimensions of the pre-workshop lesson, Activity System 1, are detailed in table 4.1.

Table 4.1: Beyoncé's Activity System 1 dimensions

<table>
<tr><td>Activity</td><td>12 minute Grade 3 mental mathematics lesson</td></tr>
<tr><td>Subject</td><td>Beyoncé: Grade 3 class teacher as well as the Head of Department for the Foundation Phase</td></tr>
<tr><td>Object</td><td>Rapid recall of the bonds of 15, with speed and accuracy

"Ok this is now the mental maths, speed and accuracy is important …"

(Data source: pre-workshop reflection interview)</td></tr>
<tr><td>Tools</td><td>Random items: bonds of 15
An example of the items used from 'Bonds and Tables 3' (Bonds and tables 3, 2001)

8+_=15   15+_=15   15-14=   15-9=
0+_=15   11+_=15   15-1=   15-3=
7+_=15   13+_=15   15-5=   _+8=15
3+_=15   10+_=15   15-8=   _+9=15
2+_=15   9+_=15   15-2=   _+10=15
6+_=15   14+_=15   15-13=   _+11=15
12+_=15   15-11=   15-0=   _+14=15
5+_=15   15-15=   15-7=   _+1=15
4+_=15   15-6=   15-4=   _+13=15
1+_=15   15-12=</td></tr>
<tr><td colspan="2">Community of practice: context as described earlier in the chapter</td></tr>
<tr><td>Rules</td><td>Rule derived from beliefs, practice, habits: facts that only need to be known

Beliefs: Beyoncé believes that her own mathematics is not very strong and never has been. Beyoncé believes that to be good at mathematics you need to know the basic facts, and the recall of facts needs to be quick.

"My maths is not very strong and it never has been … Children need to know mental maths, the bonds, and they also need to be quite quick about it."

Practice: Beyoncé admits the she is not very consistent in how she teaches mental mathematics and acknowledges that she needs to be more consistent. She knows that she needs to do mental mathematics daily and that her delivery pace of these lessons is not fast enough.</td></tr>
</table>

	"I must admit I am not very consistent in how I teach – I need to be consistent – it does impact positively when you are consistent. I do mental mathematics at least 3 times a week although it should be every day of the week. I also don't think my pace is fast enough." **Habits**: Beyoncé acknowledges that she still teaches through telling *"I have to stop myself showing the children ... although I know that they should be telling me and discussing."* (Data source: introductory interview)
Division of labour	Teacher-led, whole-class teaching with oral presentation of tasks *"I'm just reading out the sum and they give the answer."* (Data source: pre-workshop reflection interview)

2. **Identify the relationships between the dimensions of Beyoncé's activity system**

Setting the scene: the students were seated at their desks, which were two-seater desks, arranged in three rows of five. The students did not have any workbooks or stationery on their desks. Beyoncé was at the front of the classroom with the book from which she was using examples of the bonds of 15 for the mental mathematics lesson. She addressed the whole class of just under thirty students for the twelve-minute duration of the mental mathematics lesson. Beyoncé walked up and down the rows of desks, asking questions (in this lesson the questions were 'sums' (calculations) that required an answer) from the book that she was using, pointing at various students to answer.

The collection of vignettes that follow represent all the instances where Beyoncé chose to stop the video recording to reflect on her practice during the pre-workshop lesson reflection interview. I looked out for cases where her reflections established relationship connections between the dimensions within the activity system.

T	*Teacher*
Ss	*All or a group of students responding together*
S#	*# indicates the number of the student who responded: tallies the student responses*
Italics	*Indicates the actions / non-verbal responses*

Beyoncé Vignette 1.1

T: 7 plus how many is equal to 15?
Teacher points to one student.
S3: 8.
T: 3 plus how many is equal to 15?
Hands go up and the teacher points to one student.
S4: *No answer.*
T: 3 plus how many is equal to 15? 3 plus how many is equal to 15?
S4: *No answer.*
T: OK, 3 plus how many is equal to 15?
Teacher asks a different student.
S5: 12.
T: Alright … *And starts to walk up the aisle between the first two rows of desks. The teacher is now facing 10 children with her back to the rest of the class.*
T: 2 plus how many is equal to 15? *Hands go up and the teacher points to one student.*
S6: 13.
T: OK, so now 6 plus how many is equal to 15?
Hands go up and the teacher points to one student.
S7: *No answer.*
T: Come … *And clicks fingers.*
S7: 9.
T: 12 plus how many is equal to 15?
Hands go up and the teacher points to one student.
S8: 3.
T: 5 plus how many is equal to 15? *And points to a student.*
S9: 10.
T: Lovely.
T: 4 plus how many is equal to 15? *And points to a student as she starts to walk backwards down the aisle.*
S10: 11.
T: Right.
T: 1 plus how many is equal to 15? *And she turns her attention to the second row of desks, with her back now to the first row and points to a student.*
S11: 14.

Beyoncé's reflection moment 1*:* Roughly two minutes into the lesson Beyoncé stopped the recording to comment:

"There must be a better way of doing this as the lesson is ***monotonous and boring****. There must be a* ***better way to engage*** *the children. I know* ***speed and accuracy are the aim of mental maths*** *but* ***there is more*** *than just add and subtract. I could have rephrased things to get* ***more from the children****. I can see that I need* ***more from them than just the answer****."*

(Data source: pre-workshop reflection interview)

My analysis:

Referring to the literature on the types of listening, this vignette seems to illustrate the pattern of IRE (Cazden, 2001) and evaluative listening (Davis, 1997) in that Beyoncé poses only knowing-type 'sums' (low-level of cognitive demand) for which she anticipates just an answer that will be either correct or incorrect (Stein *et al.*, 2000). The only student engagement in this section of the lesson is to respond with an answer. Beyoncé's reflections indicate that she became aware that the division of labour at the time was not conducive to developing reasoning. She was aware that she elicited only calculation or knowledge of known facts. Beyoncé indicates a shift in rules from where the answer and recall are the aims, to her reflections on needing more from the students, and more than just the answer.

Synopsis: Students need to know and recall facts; expected student response is to give an answer – recall a known fact

<table>
<tr><td colspan="2">Beyoncé Vignette 1.2</td></tr>
<tr><td>T: 2 plus how many is equal to 15?
S24: Long pause.
T: 2 plus how many is equal to 15?
Hands go up.
S24: Long pause.
Teacher turns and points to another student.
S25: 13.
T: Lovely, lovely, alright …now are we ready for the next one?
Ss: Yes, ma’am.
T: 15 minus 11? The teacher is still at the front of the class and points to a student.
S25: 4.
T: 15 minus 15? … hands go up … Come, come … and the teacher points to a student.
S26: 0.
The teacher starts to walk up the last aisle, facing the middle row with her back to the last row.
T: 15 minus 6? And points to a student.
S27: 9.
T: How did you get that?
S27: I know that 9 plus 6 is 15.
T: Very good.
T: 15 minus 10? And points to a student.
S28: 5.
T: Right, 15 minus 14? The teacher turns to face the last row and points to a student.
S29: 1.
T: 15 minus 1? And points to a student.</td>
<td>S30: Long pause … 14.
T: 15 minus 5? And points to a student.
S31: 10.
T: 15 minus 8? And points to a student. The teacher is now back at the front of the class, but still facing the last row.
S32: Long pause.
T: Remember what he said? The teacher points vaguely over to middle or first row.
S32: 7.
T: Very good, how did you get your answer? Explain to us.
S32: Because 8 plus 7 is 15, so 15 minus 8 is 7.
T: So, 15 minus 8 is 7 – thank you.
T: How many plus 8 is equal to 15? And points to a student.
S33: 7.
T: How many plus 9 is equal to 15? Hands go up and the teacher points to a student.
S34: Long pause.
T: Come!
S34: Long pause.
T: How many plus 9 is equal to 15? The teacher repeats this louder and slower.
S34: Long pause.
T: How many plus 9 is equal to 15? And points to another student.
S35: 6.
T: 6, lovely!</td></tr>
<tr><td colspan="2"><u>Beyoncé’s reflection moment 2:</u> Roughly four minutes into the lesson Beyoncé stopped the recording to comment:
“Everything seemed so disconnected. This shouldn’t be a whole-class lesson – not all the children are actively engaged and I’m not catering for all the children’s needs. Not enough student engagement.”
(Data source: pre-workshop reflection interview)</td></tr>
</table>

My analysis:
Although Beyoncé probes for student thinking by asking, "How did you get that?" in the vignette above, the tasks need no further thinking as these are facts that the students just know. Both the cognitive demand of the facts and the random structure of these facts do not support the development of understanding, application or reasoning. Beyoncé's comments indicate an awareness that the activity was 'disconnected' and that she needs to reconsider not only her tasks (tools) but also her classroom arrangement (division of labour) in order to encourage more active engagement.

Synopsis: Each question was an isolated (disconnected) fact in a low number range

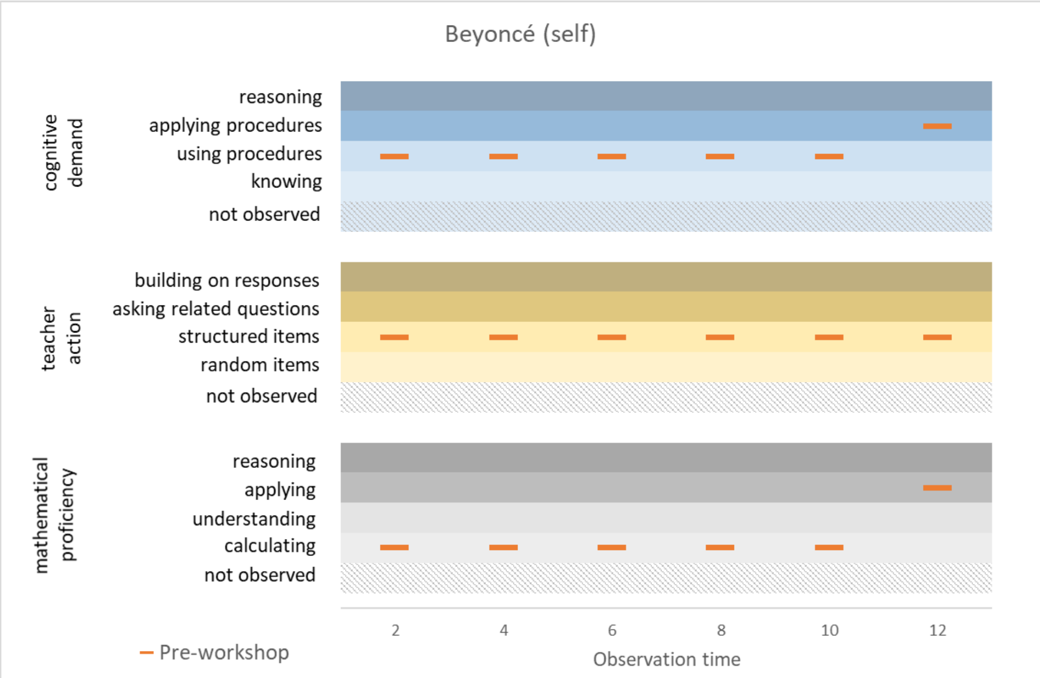
Beyoncé's reflections on her practice, as quoted above, were made before the workshop session, and before sharing the lens of mathematical proficiency. In her second reflection on the pre-workshop lesson video recording, this time with the knowledge of mathematical proficiency, Beyoncé added the following reflections:

*"Throughout the mental maths lesson I focused on **one developmental skill** – just **one-dimensional** add and subtract. I realised that **mat work with a group would have been of more benefit** to the learners. Some children are **not paying attention**. There were **lots of opportunities** for me as the teacher to **engage the children in a more meaningful discussion**."*
(Data source: self-reflection checklist for the pre-workshop lesson video recording)

My analysis:
With reference to Beyoncé's reflection of 'one developmental skill' she now voices her understanding that there is more than one developmental strand (to use the shared language of mathematical proficiency) and that she is not managing to address the other strands. Yet she is developing a vision of change of practice: in the first reflections Beyoncé only saw what she was not doing. Here, she now begins to see what she could be doing to improving practice towards creating more opportunities for the students to engage and discuss.

These reflections of practice and initial analyses can be further interpreted by referring to the time-motion graph generated from Beyoncé's reflections of practice, using the completed self-reflection checklist for the pre-workshop lesson, Activity System 1, and as illustrated in figure 4.1.

Figure 4.1: Time-motion graph representing Beyoncé's pre-workshop reflections on practice



Beyoncé's reflective comments regarding the lesson being monotonous, one-dimensional and addressing one developmental skill can be verified by the lack of variation across any of the categories. However, Beyoncé is realising that mental mathematics can be about more than just recall and calculation. This awareness became evident towards the end of her lesson as the graph shows: Beyoncé recognised an increase in the level of cognitive demand and attention to applying. Although Beyoncé's reflections on practice in the last two minutes indicate a change in levels in two of the categories, other than her reflections of mental mathematics needing to be "about more than just the answer", no further comments were made about this change during the reflection interview.

Mathematical Proficiency: Beyoncé reflects that the dominant strand of mathematical proficiency in this lesson was calculating. As the vignettes illustrated, she realises that the question-response style created no opportunities for application and reasoning. The 'calculating' in this lesson consisted of recalling known facts.

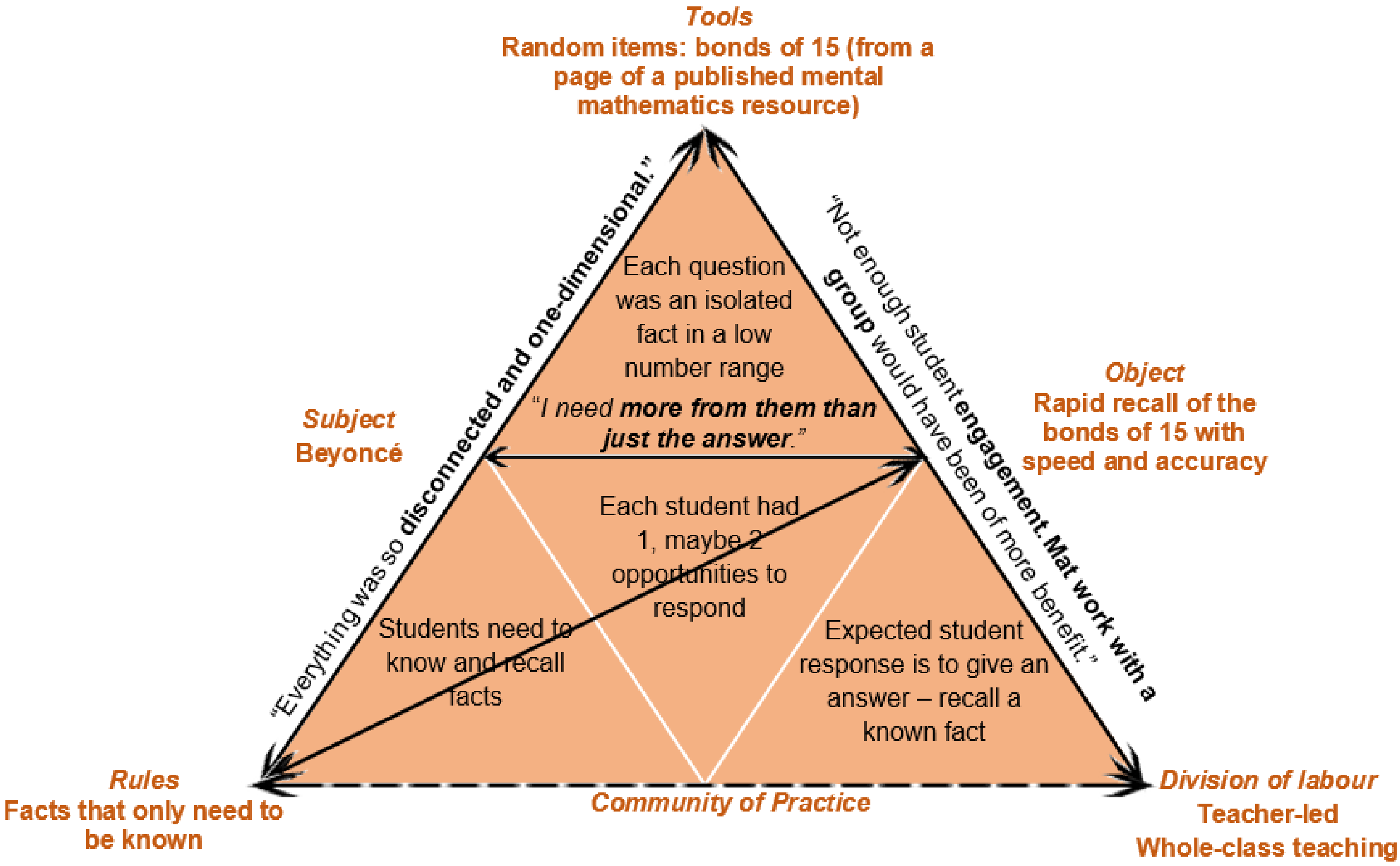
Teacher Action: Beyoncé perceives her teacher action as presenting calculations, or 'sums' (structured items) for the recall of the bonds of 15 (calculating). The calculations were not presented in any particular pattern (except that she led with addition) and did not include application of the bonds of 15 in an extended number range, but only relied on the students recalling the answers. Beyoncé views her tasks as structured items, but there is no indication of the structure that she refers to: it may simply be that all the calculations are related to 'bonds of 15'. In her reflection Beyoncé recognises, in hindsight, opportunities where she could have built on responses, and could have asked better questions.

Cognitive Demand: Beyoncé has perceived the level of cognitive demand of tasks used in the lesson to be that of 'using procedures'. This supports the reflection that the lesson was 'one-dimensional' as, not only is there no variation across the levels of each sub-category, but the levels observed by Beyoncé are also the 'lower-levels' for each category.

3. **Identify and describe the dominant relationships between the dimensions of Beyoncé's activity system**

In order to investigate the *what, why and how* of Beyoncé's activity system, I summarised her perceptions of practice in terms of the relationships between the dimensions of her activity system as illustrated in figure 4.2.

Figure 4.2: Activity System 1 relationships as interpreted through the analysis of Beyoncé's reflections



I positioned Beyoncé's quotes (from her reflections) along the arrows that connect the appropriate dimensions in the activity system. The dimensions and their relationships are explained and described with the use of Beyoncé's own words in table 4.2. The analysis synopses of Beyoncé's reflections fill the triangle areas in order to structure my analysis into the activity system. In table 4.2 I explain how I arrived at the activity system of figure 4.2.

Table 4.2: Description of Beyoncé's Activity System 1 relationships

<table>
<tr><th>Subject: Beyoncé</th><th>Perception</th></tr>
<tr><td>Tools: Random items, covering the bonds of 15</td><td>"Everything was so disconnected and one-dimensional."
(Data source: self-reflection checklist for the pre-workshop lesson video recording)</td></tr>
<tr><td colspan="2">Analysis summary: Each question was an isolated (disconnected) fact in a low number range
Beyoncé is aware that her use of the tool leads to disconnected and one-dimensional teaching, and this realisation indicates Beyoncé's awareness of an alternative.</td></tr>
<tr><td>Object (stated lesson objective): Rapid recall of bonds of 15 with speed and accuracy</td><td>"I need more from them than just the answer."
(Data source: pre-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary:
Beyoncé is becoming aware that mental mathematics is more than just the answer and is dissatisfied with her current objective. There is an awareness that she needs to engage the students in all of the strands of mathematical proficiency.
Relationship:
Subject-Object
The 'need more' indicates Beyoncé's realisation that the students could be engaging further with these facts and reveals an emerging awareness that the students can and should be doing 'more'; more in terms of engaging with more of the strands of mathematical proficiency, such as understanding and reasoning, and more in terms of cognitive demand. This indicates Beyoncé's dissatisfaction with the current objective.</td></tr>
<tr><td>Rules: Facts that only need to be known</td><td>"I know speed and accuracy are the aim of mental maths but there is more than just add and subtract."
(Data source: pre-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary: Students need to know and recall the facts
Beyoncé had these rules before she taught this lesson, but these rules, and the resultant objective, are no longer satisfactory.
Relationship:
Rules-Object
In the literature review it was argued that beliefs shape practice (Clements and Sarama, 2009) and beliefs (rules) regulate a subject's actions towards an objective. Beyoncé's current beliefs focus on recall indicating</td></tr>
</table>

there is little belief, or experience, that students are able to construct their own knowledge or are capable of 'more'. Yet Beyoncé has indicated a shift in this belief through an awareness that more student engagement is favourable for learning, and that she needs more from them, which indicates that a change in objective is necessary.

Rules-Subject-Tools

Beyoncé uses the words 'disconnected' and 'one-dimensional' to describe this lesson and indicates an awareness that there is more to a mental mathematics lesson than recall. Beyoncé's belief that being good at mathematics involves the recall of facts has a direct influence on the selection of materials (tools) and how they are used. It is plausible to believe that recall is best achieved through a great variation of random challenges. Beyoncé's reflections indicate that the tools used need to create more opportunities for connections to be made and need to be more varied in cognitive demand.

Division of labour: Teacher-led (whole class)	*"Not enough student engagement. Mat work with a group would have been of more benefit to the learners."* (Data source: self-reflection checklist for the pre-workshop lesson video recording)

Analysis summary: **Expected student is to give an answer – recall a known fact**

Beyoncé posed a question ('sum'), and the student to whom she pointed provided an answer (correct or incorrect). Other than the individual students who were responding, the rest of the class sat quietly and waited their turn. Beyoncé acknowledges that, in order to create more opportunities for student engagement, working with a small group of students would be more beneficial. An audit from the lesson regarding the teacher role can be seen below and this supports Beyoncé's reflections on the need for more student engagement.

Action / Time	02:00	04:00	06:00	08:00	10:00	12:00	TOTAL
Asking 'answer only' questions	11	19	9	5	3	2	49
Asking questions to facilitate discussion	2	1	1	0	2	1	7

Relationship:

Rules-Object-Division of labour

As per the lesson objective, Beyoncé asked questions and the individual students responded with the answer when called upon to do so. Although Beyoncé's practice demonstrated teacher-led delivery (linked to her belief about what mental mathematics is) with the answer as the expected student response, she indicated that she wants 'more' and that she wants to encourage the students to think about their responses (this goes against her prior belief that mental mathematics is about speed and recall). Although Beyoncé does not elaborate on the 'more' that she would like, this does imply that she is dissatisfied with the outcome of her lesson and, hence, with her objective of rapid recall of facts.

Tools-Object-Division of labour

An audit of practice on this lesson reveals that, across the lesson, Beyoncé posed 56 questions ('sums' posed as questions – and some "How did you get that?" questions) to a class of 30 students. This means that there were 56 direct response opportunities. Of the 56 questions, 49 required merely a one-word answer. Beyoncé's selection and implementation of the tools for this lesson answered to the objective of rapid recall, but Beyoncé realises that, in order for there to be a shift in the division of labour or 'more student engagement', a change is needed. Here she mentions mat work versus whole class teaching as a possible solution.

4. **Analyse and describe Beyoncé's lesson outcome as a consequence of the dimensions and the dominant relationships**

On reflection of this lesson, Beyoncé has identified that a shift in objective is necessary in order to create the opportunities for 'more', as described in her reflections: mental mathematics is about 'more' than just the answer, 'more' engagement is needed from the students, and 'more' opportunities for connections. Although the outcome of the lesson suggests that the objective of the recall of facts was achieved, there is an awareness that the objective could, and should, be different in order to include **more student engagement and more opportunities for students to make connections.** This awareness results in a conflict between Beyoncé's existing system of rules and achieved outcome. With the awareness that things could be different, Beyoncé is challenging her current rules and the resultant impact of this on her lesson objective, selection and use of tools, and the division of labour which is clearly evident in the description of the dominant relationships.

4.2.2.2 Activity System 2

1. **Identify and describe the dimensions specific to Beyoncé's activity system**

The dimensions of the post-workshop lesson, Activity System 2, are detailed in table 4.3.

Table 4.3: Beyoncé's Activity System 2 dimensions

Activity	12 minute Grade 3 mental mathematics lesson	
Subject	Beyoncé: Grade 3 class teacher as well as the Head of Department for the Foundation Phase	
Object	Use known facts to calculate the answer, and to notice patterns and relationships *"To use the facts of 5+3=8, and 7+6=13, to apply to different number ranges and to use tasks that allowed the children to notice patterns and relationships."* (Data source: post-workshop reflection interview)	
Tools	What is 5 plus 3? What is 25 plus 3? What is 35 plus 3? What is 45 plus 3? "What do you notice?" What is 65 plus 3? What is 15 plus 3? "Explain how you got your answer." What is 7 plus 6? What is 27 plus 6? What is 37 plus 6? What is 67 plus 6? What is 107 plus 6? "Explain how you got your answer." What is 127 plus 6? What is 7 plus 6?	Structured items: facts of 5+3 and 7+6, as well as linked facts An example of the items used from '*NumberSense Workbook 9'* mental mathematics materials (Brombacher, 2012)
Community of practice: context as described earlier in the chapter		
Rules	**Rule derived from beliefs, practice, habits:** facts that need to be known and applied **Beliefs:** emerging belief that mathematical success is about more than just the answer, and includes applying known facts.	

	"I am more aware of mathematical proficiency. Children must have a good understanding so that they are ***able to connect what they know, and can use*** *it in solving problems. This cannot be achieved by focusing on one or two strands. I have to include all aspects of the five strands when teaching mental mathematics – they cannot be developed in isolation."* **Practice**: an emerging awareness that there are still changes in practice that are needed in order to match the emerging belief of success in mathematics being more than the answer. *"Reflecting on the video, I realised that I did not allow for the representation of mathematical situations in different ways. What I did was very limited."* **Habits:** awareness that although the lesson objective and tools may have shifted, practice is still rooted in previous habits. *"I used tasks that allowed the children to notice patterns and relationships, but I did not present opportunities for the children to connect what they know to other situations."* (Data source: post-workshop reflection interview)
Division of labour	Teacher-led, small-group teaching *"There has been an improvement in how I manage the class – differentiation with the group work and meeting the children developmentally …"* (Data source: post-workshop reflection interview)

2. Identify the relationships between the dimensions of Beyoncé's activity system

Setting the scene: Beyoncé had a group of ten students on the mat with her, while the rest of the class was engaged with independent mathematics work at their desks for which they had been prepared. Beyoncé was using the NumberSense mental mathematics materials that had been introduced during the workshop, and she was on page 33 of Workbook 9 (this class was working in Workbook 10 at the time of this lesson). The lesson (with the group of students on the mat) started as follows:

T: We are going to do mental maths. OK? Fold your arms and sit up straight. T: What is 5 plus 3? S1: *Doesn't answer.* T: I can't hear you. S1: 8. T: Correct. T: 25 plus 3? S2: *Puts hand up* … 28.	T: 35 plus 3? S3: 38. T: What is 45 plus 3? S4: 48. *The teacher is going around the circle in order, starting from her left. The students are all putting up their hands to respond, but she is going in order around the circle.*

As before, the collection of vignettes that follow represent all the instances where Beyoncé chose to stop the video recording to reflect on her practice during the post-workshop lesson reflection interview. I looked out for cases where her reflections established relationship connections between the dimensions within the activity system.

T	*Teacher*
Ss	*All or a group of students responding together*
S#	*# indicates the number of the student who responded: tallies the student responses*
Italics	*Indicates the actions / non-verbal responses*
Beyoncé Vignette 2.1	
T: Right. Can you give me another sum? S5: 55 plus 3. T: Can't hear – 55 plus 3 is equal to? Is equal to? I can't hear! S5: Is equal to 58. T: Right … Can you give me another one? *Teacher points to S6.* S6: 135 plus 3 is equal to 138. T: Right … Now what did you notice, what did you notice? About the sums?	*Teacher gets out the whiteboard and a marker.* T: Are you ready? Yes? S7: When you're adding 3 to the 5 – then you get 8 … there were only these sums: 5 plus 3 equals 8. T: Yes … You are very right … Remember we started with 5 plus 3 which equals 8 and then we went on to 45 plus 3 equals 48. Now just remember that. *The teacher has written these sums down on the whiteboard.*

Beyoncé’s reflection moment 1: Roughly two minutes into the lesson Beyoncé stopped the recording to comment:

“I did not present opportunities for the children to ***connect what they know to other situations****. I realise that I could have* ***extended the children more*** *and stretched their* ***understanding by continuing into a higher number range****.”*

(Data source: post-workshop reflection interview)

My analysis:

Beyoncé has expressed an awareness that in order for students to move beyond calculating and to develop understanding, applying and reasoning, not only is the structure of the tasks important, but so is the level of cognitive demand. Both the structure and level of the tasks are critical in developing mathematical proficiency (Hiebert and Grouws, 2007; McGatha *et al.*, 2018). Beyoncé’s selection of tasks from a Workbook lower than the Workbook the students are currently busy with can explain her reflections regarding the inappropriately low number range number range. While her reasons for this lower level of task selection were not expressed, they could be rooted in her rules regarding mathematics being about the rapid recall of answers, therefore requiring the recall of previously learnt, known facts.

Synopsis: Low number range - limited opportunities for application

<table>
<tr><td colspan="2">Beyoncé Vignette 2.2</td></tr>
<tr><td>T: Now what is 7 plus 3?
At this point the teacher picks up the mental mathematics booklet and continues on a different page.
T: What is 7 plus 3 … 7 plus 3?
S8: 10.
T: Now what is 7 times 3 … Pause while no one answers and teacher does not select anyone …
T: Think about it.
S9: 21.
T: Now OK, 7 times 3 is 21 – OK now what is 7 times 6?
T: What is 7 times 6?
S10: 42.
T: How did you get 42?
S10: I know that 3 times 7 is 21, 6 times is just double that.</td><td>T: Hold on hold on … Right and how do you get it?
Another student carries on.
S11: 20 plus 20 is 40 and 1 plus 1 is 2 …
T: She says she knows that 21 plus 21 equals 42 …
The teacher writes this on the whiteboard … So when she saw 7 times 6, what did she do? She doubled the 21. Why did she double the 21?
S12: Because 6 is double 3.
T: Thank you – right. The teacher realises that she was on the wrong page.
T: So, let us go back to the page that we are going to do, let us go back to the page that we are going to do.
T: What is 65 plus 3? Remember?</td></tr>
</table>

<u>Beyoncé's reflection moment 2:</u> Roughly four minutes into the lesson Beyoncé stopped the recording to comment (this vignette picks up directly from where the previous one left off). **To note:** although this vignette illustrates a part of the lesson where Beyoncé continued on a different page that deviated from her stated objective, her reflections focus on her practice and not the actual task items:

"I could be more fluent in my delivery and pace in order to get through the range of examples in a quicker time. ***This is all in isolation and not meaningful*** *- there is no opportunity for the children to use their knowledge."*

(Data source: post-workshop reflection interview)

My analysis:

Beyoncé's reflection on the need to be fluent indicates an awareness that an increase in delivery pace is needed to work through the structured collection of items so that there is more opportunity for connections to be made. Her reflection regarding the lack of meaning and connections supports this awareness. Although there is evidence of emerging awareness that points to the development of mathematical proficiency, Beyoncé has noticed that her practice is not yet achieving this.

Synopsis: Students know more than, and need to do more than just recall the facts

Beyoncé Vignette 2.3

T: 7 plus 6? 7 plus 6?
S27: 13.
T: 27 plus 6? Take your time … 27 plus 6?
S28: 33.
T: How did you get your answer so quick?
The teacher writes 27 + 6 on the white board.
T: 27 plus 6? How did you get your answer? Come, listen to her.
S28: I know that 7 plus 6 is 13. And 20 plus 13 is 33.
T: Right, yes OK. Anybody got another way? Explain quickly.
S29: *Inaudible explanation and spoken at the whiteboard with the teacher and not to the group.*
T: OK anybody got another way?
S30: *Inaudible explanation and spoken at the whiteboard with the teacher and not to the group. The teacher's focus is on the second half of the circle, and has been for a while … the students are getting restless.*
T: OK thank you – I see what you do. You fill up the 10 and then you minus – all right? That's also fine. Right, where we now …
T: 37 plus 6?
S31: 43.
T: 67 plus 3?
S32: 70.
T: 127 plus 6?

S33: 133
T: *Settles the students as they are now restless.*
T: 7 plus 6? Remember?
S34: 13.
T: Listen nicely. If 7 plus 6 is 13 - what will 8 plus 6 be? 8 plus 6? Remember 7 plus 6 is 13, so 8 plus 6 is …
S35: 14.
T: Very good. How do you know that?
S36: *Says nothing and the students around him start to get involved.*
S37: But its 15 …
T: Remember 7 plus 6 is 13, so 8 plus 6 is?
S38: 14.
T: How did you get your answer so quickly?
S38: *Inaudible explanation.*
T: 106 plus … 106 plus … *looks at book …* 108 plus 6?
S39: 114.
T: How did you get that? *And writes it on the whiteboard.*
S39: I know that 8 plus 6 is equal to 14 so I added that to the 100. *Spoken to the teacher at the whiteboard.*
T: Very good. 8 plus 6?
S40: 14.
T: So what will 38 plus 6 be?
S41: 44.

<u>Beyoncé's reflection moment 3:</u> Roughly 10-12 minutes into the lesson Beyoncé stopped the recording to comment:

*"I could have used the children's **explanations to engage more** and elaborate further. I also don't engage with the children – I just ask them a question and move on. I am also **missing opportunities for application** – 'Now if we know this, then can we do that …?' **I need to plan** for this and have prompts to ask questions. I can see they are losing interest."*

(Data source: post-workshop reflection interview)

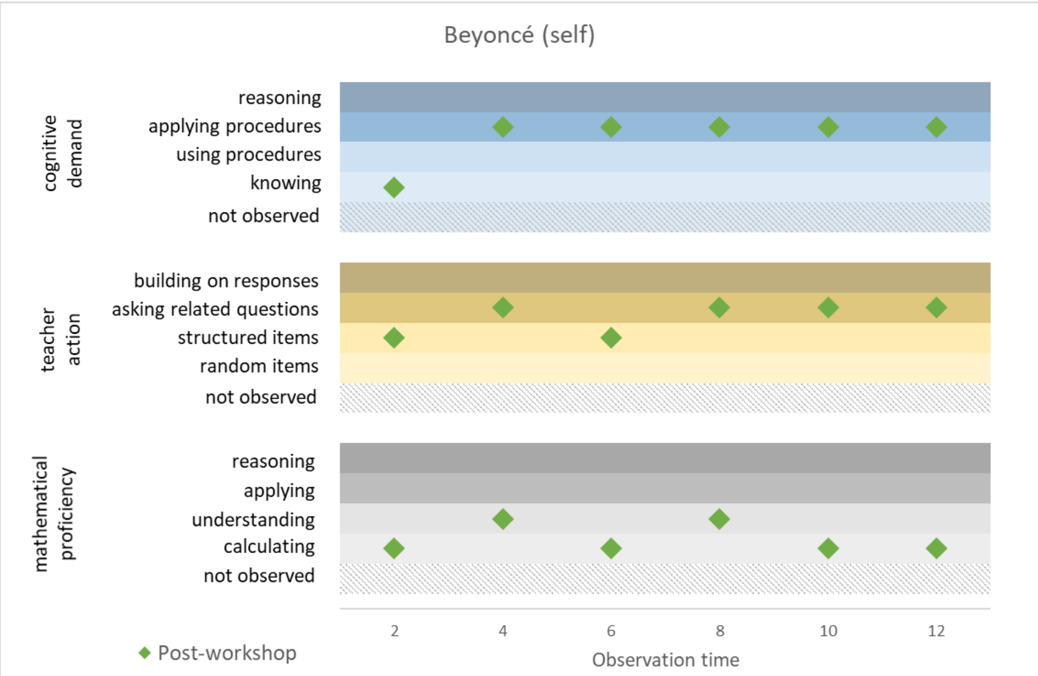
My analysis:

Although the tasks for this lesson included question prompts for the teacher, Beyoncé's reflections reveal that she is still missing opportunities for application and further engagement. The teacher-student interaction still follows the evaluative listening (Davis, 1997) and the IRE pattern (Cazden, 2001). Beyoncé's use of the word 'explanations' as opposed to 'answers' illustrates a significant shift in objective, even if her practice in this instance is not yet realising this. Beyoncé sees what she could be doing, and recognises planning as a solution for this shift in student engagement and her own practice.

Synopsis: Asking questions but not prepared for what these responses should 'look like'

Beyoncé's reflections on her practice as quoted above were made after the workshop session, and with the shared lens of mathematical proficiency. These reflections of practice and initial analyses can be further interpreted by referring to the time-motion graph generated from Beyoncé's reflections of practice using the completed self-reflection checklist for the post-workshop lesson, Activity System 2 and as illustrated in figure 4.3.

Figure 4.3: Time-motion graph representing Beyoncé's post-workshop reflections on practice



The graph depicts some variation across the sub-categories, although still predominantly in the lower levels across the sub-categories. Beyoncé's reflections, by means of the checklist, indicate no instances of reasoning, and support her awareness that she is missing opportunities to further engage the students.

Mathematical Proficiency: Beyoncé's comments of 'isolated' and 'not meaningful' indicate an awareness that there were more opportunities for connections to be made. Here she noted the instances of calculating and understanding, and the absence of applying and reasoning.

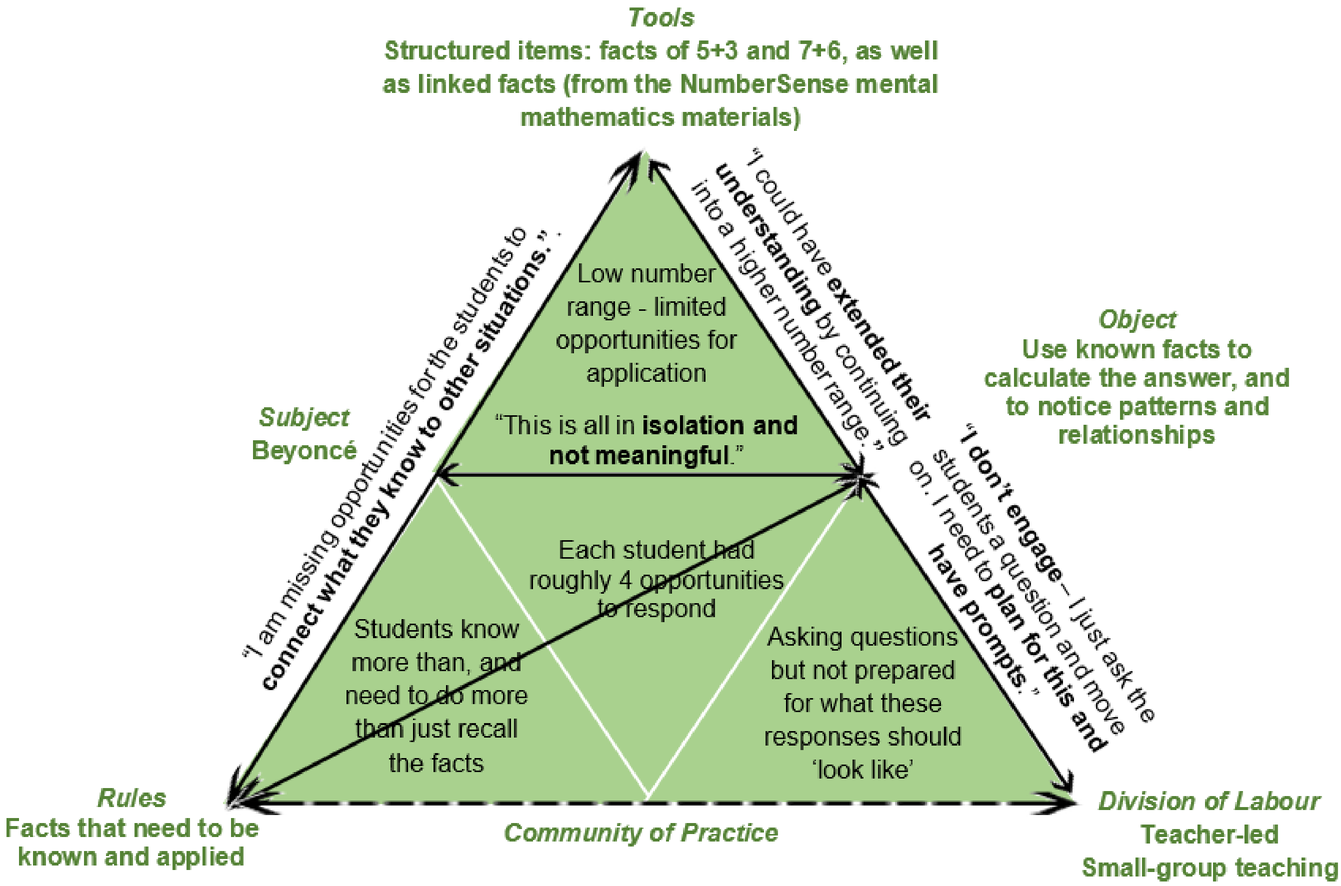
Teacher Action: Although Beyoncé observed that she asked related questions and encouraged the applying of procedures, she acknowledged that she needed to plan in order to build on student responses.

Cognitive Demand: The observed levels of cognitive demand seem to reflect Beyoncé's objective for this lesson more than her actual practice. Her reflections across the other two categories indicated a lack of application and, even though she was using tasks that were designed to support the development of application and reasoning, she acknowledged that the number range of the items was too low to achieve this.

3. **Identify and describe the dominant relationships between the dimensions of Beyoncé's activity system**

In order to investigate the *what, why and how* of Beyoncé's activity system, I summarised her perceptions of practice in terms of the relationships between the dimensions of her activity system illustrated in figure 4.4.

Figure 4.4: Activity System 2 relationships as interpreted through the analysis of Beyoncé's reflections



I positioned Beyoncé's quotes (from her reflections) along the arrows that connect the appropriate dimensions in the activity system. The dimensions and their relationships are explained and described with the use of Beyoncé's own words in table 4.4. The analysis synopses of Beyoncé's reflections fill the triangle areas in order to structure my analysis into the activity system. In table 4.4 I explain how I arrived at the activity system of figure 4.4.

Table 4.4: Description of Beyoncé's Activity System 2 relationships

<table>
<tr><th>Subject: Beyoncé</th><th>Perception</th></tr>
<tr><td>Tools: Structured items, facts of 5+3 and 7+6, as well as linked facts</td><td>"I could have extended their understanding by continuing into a higher number range."
(Data source: post-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary: Low number range - limited opportunities for application
Beyoncé noticed that the number range was too low to develop understanding effectively. There was a new awareness that, in order to develop mathematical proficiency beyond calculating, the level of cognitive demand of the tasks used needed to include more than the 'knowing' level which requires only the recall of known facts.</td></tr>
<tr><td>Object (stated lesson objective): Use known facts to calculate the answer, and to notice patterns and relationships</td><td>"This is all in isolation and not meaningful."
(Data source: post-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary:
While Beyoncé's objective for this lesson includes more than just recall, her reflections indicate that this was not achieved. Although she had aimed for application (using known facts) and understanding (notice patterns and relationships), her experiences of this being 'in isolation and meaningless' indicate that the outcome was not successful.
Relationship:
Tools-Object
Beyoncé's selection of tools for this lesson aligned with her objective in that their design supported the development of understanding as the tasks were deliberately sequenced to facilitate the noticing of patterns. However, her awareness of the importance of the number range in facilitating application (and reasoning) is evident in her comments that the number range could have been greater in order to 'extend their understanding'.

Subject-Object
The 'in isolation and not meaningful' indicated Beyoncé's realisation that, although her objective included application and understanding, there were still more opportunities for connections and meaning to be made.</td></tr>
</table>

Rules: Facts that need to be known and applied	*"I am missing opportunities for the students to connect what they know to other situations."* (Data source: post-workshop reflection interview)

Analysis summary: **Students know more than, and need to do more than just recall the facts**

Beyoncé's awareness that there is more to mental mathematics than just the recall of answers was indicated by the inclusion of application and understanding in her objective and in her task selection.

Relationship:

Rules-Subject-Tools

There was an aspect of Beyoncé's rules that was causing some tension. There was an emerging awareness that there is more to mental mathematics than just recall, but Beyoncé's practice was still not creating the opportunities to fully achieve this. Even with the use of tasks that support the development of understanding and reasoning, we saw a tension in Beyoncé's awareness that her lesson was missing opportunities for students to make the connections.

Rules-Object

Beyoncé seemed to prescribe to a rule that suggests that, even if arranged in a manner to reveal a pattern, each fact was perceived as an isolated unit, and not connected to those facts within the pattern. The limited number range and the focus on the 'facts' and not the 'pattern' continue to speak to the belief that indicates the 'answer' as the objective, but there was an emerging awareness of both the importance of increasing the number range and of delivering tasks that reveal a pattern.

Division of labour: Teacher-led (small group)	*"I don't engage with the students – I just ask them a question and move on. I need to plan for this and have prompts to ask questions."* (Data source: post-workshop reflection interview)

Analysis summary: Asking questions but not prepared for what these responses should 'look like'

Although Beyoncé is following the prompting questions within the materials that go beyond just the recall of an answer, her questioning pattern still follows that of IRE (Cazden, 2001). Beyoncé recognised this in her reflections on just moving on after asking questions and not engaging further. An audit from the lesson regarding the teacher role can be seen below and this supports Beyoncé's reflections on the need to engage further with the students.

Action / Time	02:00	04:00	06:00	08:00	10:00	12:00	TOTAL
Asking 'answer only' questions	5	3	1	6	9	5	29
Asking questions to facilitate discussion	0	1	2	3	3	2	11

Relationship:

Object-Division of labour

The audit of practice on this lesson reveals that across the second lesson Beyoncé posed 40 questions (items that were structured to reveal a pattern and related questions) to a group of ten students. There were 40 response opportunities. This means that on average each student had the possibility of responding to roughly four questions across a 12-minute lesson. Of the 40 questions, 11 of these required more than an answer – although Beyoncé reflected that she was missing opportunities for discussion with the students in that she did nothing with their responses. She noticed that she asked the students questions, but terminated the interaction almost immediately. Beyoncé's reflections here revealed that she felt that this was still a teacher-led lesson, and that further planning on her part was needed in order to facilitate the engagement of the students and her response to their offers. Although Beyoncé's selection and use of tools included teacher-prompts in the form of questions to facilitate discussion, she indicated an awareness that planning for these prompts and what may happen beyond these prompts was necessary for student engagement.

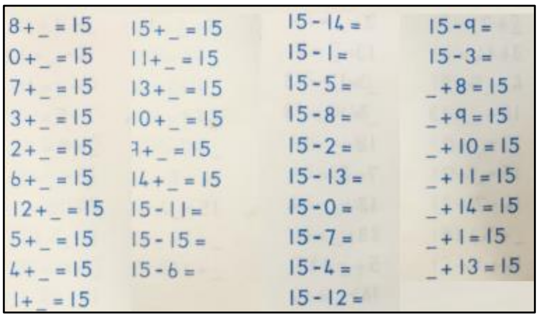
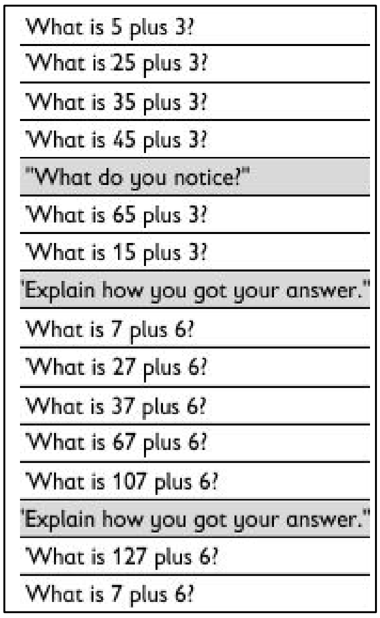
4. **Analyse and describe Beyoncé's lesson outcome as a consequence of the dimensions and the dominant relationships**

Reflecting on the video, Beyoncé realised that what she did was *"… **very limited**. There was no opportunity for the students to connect and use their knowledge in other situations."* This suggests that Beyoncé did not feel that the outcome of the lesson achieved the objective. The stated objective for this lesson included application and the noticing of patterns and relationships (understanding). Beyoncé's reflection on the outcome revealed an awareness that there were **missed opportunities for the students to make connections and engage**. Although the objective may not have been realised for this lesson, a positive outcome was the growing awareness Beyoncé had of her own practice and the shifts that were necessary to reach the objective of teaching towards developing understanding and reasoning.

4.2.2.3 Comparison of Beyoncé's Activity Systems

Beyoncé's reflections for both of the lessons, Activity System 1 and Activity System 2, have been analysed as separate systems. In order to uncover similarities, differences, patterns and variations across these activity systems, they were placed alongside each other and compared for their similarities and differences. To keep Beyoncé's voice in the overview that follows, her closing reflections are included in these interpretations. As an overview of the two activity systems, the similarities and differences across the two can be tracked as detailed in table 4.5.

Table 4.5: The similarities and differences across the dimensions of Beyoncé's activity systems

Dimensions	Activity System 1	Activity System 2
Subject	Beyoncé	Beyoncé
Object	Rapid recall	Using known facts, and noticing patterns
Tools	Random items: bonds of 15  8+_=15 0+_=15 7+_=15 3+_=15 2+_=15 6+_=15 12+_=15 5+_=15 4+_=15 1+_=15 15+_=15 11+_=15 13+_=15 10+_=15 9+_=15 14+_=15 15-11= 15-15= 15-6= 15-14= 15-1= 15-5= 15-8= 15-2= 15-13= 15-0= 15-7= 15-4= 15-12= 15-9= 15-3= _+8=15 _+9=15 _+10=15 _+11=15 _+14=15 _+1=15 _+13=15 *Focus*: facts; practice and recall *Design*: bonds of 15; variation in position of the unknown	Structured items: facts of 5+3 and 7+6, as well as linked facts  What is 5 plus 3? What is 25 plus 3? What is 35 plus 3? What is 45 plus 3? "What do you notice?" What is 65 plus 3? What is 15 plus 3? "Explain how you got your answer." What is 7 plus 6? What is 27 plus 6? What is 37 plus 6? What is 67 plus 6? What is 107 plus 6? "Explain how you got your answer." What is 127 plus 6? What is 7 plus 6? *Focus*: apply known facts to other number ranges *Design*: structured to reveal a pattern; reflection prompts
Rules	Facts that need to be known	Facts that need to be known and applied
Community of practice	As described earlier in this chapter	
Division of labour	Teacher-led, whole-class teaching (Each student had one, maybe two response opportunities)	Teacher-led, small-group teaching (with the rest of the class working independently at their desks) (Each student had roughly four response opportunities)

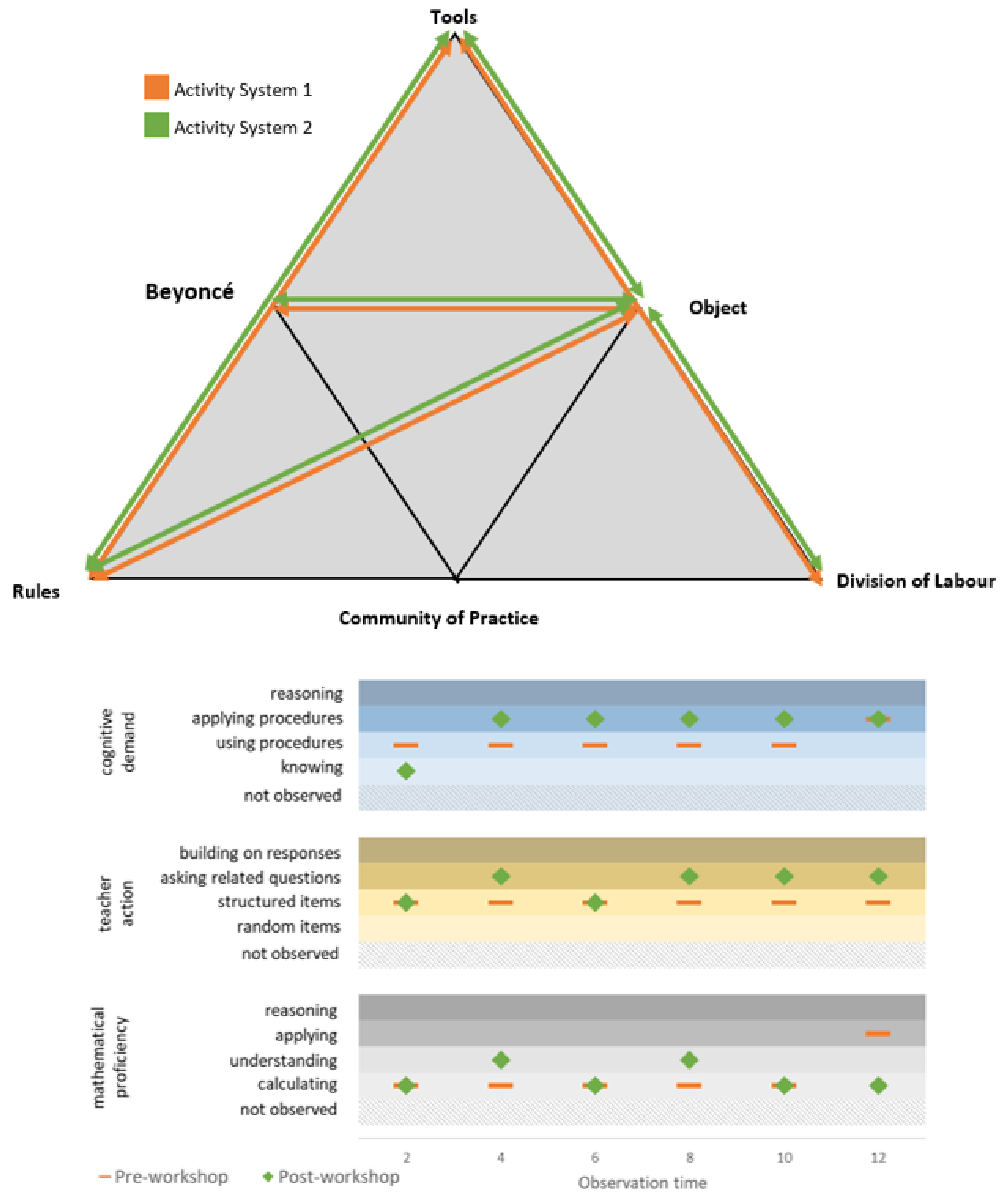
Dominant relationships	**Subject-Object** **Rules-Object** **Rules-Subject-Tools** Rules-Object-Division of labour **Tools-Object-Division of labour**	**Subject-Object** **Rules-Object** **Rules-Subject-Tools** **Tools-Object** **Object-Division of labour**
Outcome	Dissatisfied with objective of rapid recall and identifies the need for: more student engagement, and more opportunities for students to make connections	Felt that the objective to develop understanding and reasoning was not achieved

Comparing the dimensions of the two activity systems, differences can be noted across the objects, tools, rules and division of labour. While the rules still indicate a focus on isolated facts, there is a difference in that some application is now anticipated in the second system. With a shift in these dimensions in the second system, the division of labour, although still teacher-led, has allowed for more student response opportunities. The dominant relationships across the two activity systems can be summarised as:

- Subject-Object
- Rules-Object
- Rules-Subject-Tools
- Tools-Object
- Object-Division of labour

The dominant relationships are very similar across both activity systems as illustrated in figure 4.5. Activity System 1 is indicated in orange, and Activity System 2 in green. Although the dominant relationships are similar, they reveal marked differences in perceptions of practice as illustrated by the time-motion graph in figure 4.5. The time-motion graph shows these differences in the increase in both the variation across the categories, and the increased level at which these categories have been observed for the second (post-workshop) activity system.

Figure 4.5: Beyoncé's summary relationships and reflections across both activity systems



The relationships in Beyoncé's Activity System 1 are characterised by her awareness that there is more to mental mathematics than learning isolated facts, more than recall and more student engagement is needed than simply just responding with an answer. In Activity System 2, these relationships are characterised by a different set of perceptions. Beyoncé's awareness from the first system, that students can and should be engaging more, resulted not only in an emerging shift in beliefs, but also in an objective that included the application of, and not just merely the recall of, known facts.

Due to the small-group teaching in the second system, the students had more response opportunities than in the first lesson. Despite this, Beyoncé reflects that she could have created more opportunities for student engagement, and that she could have done more with their responses. While Beyoncé's beliefs (rules), object and tool changed from Activity System 1 to Activity System 2, how she used the tool and interacted with the students (division of labour) seems to need more time, and perhaps support and effort, to change. Beyoncé's comments in the closing interview acknowledged her awareness that her planning needed more attention; not only the planning of what to teach, but also of how to teach it:

"I need to do a lot more planning. I need to plan to ***use the children's explanations and let them reflect and discuss more****. Planning is not just picking up a book and doing random examples. You need to know what you want to achieve and to plan questions and examples that will lead to this."* (Data source: post-workshop reflection interview)

In the first activity system, based on Beyoncé's rules and lesson objective, the materials used provided little chance for anything other than an answer only response. This was very much a teacher-led lesson that could have been done in a written pen-and-paper format with no benefit from the interaction. In Activity System 2 Beyoncé selected different materials, yet she observed that she was still missing opportunities for connections. In her closing reflections, Beyoncé has acknowledged the need to increase the number range in order to promote application, understanding and reasoning:

"I need to ***increase the number range and level of difficulty*** *in order to engage application and reasoning and therefore develop understanding. I see the need for* ***tasks to become more difficult in order to integrate the other strands*** *and not just the knowing type sums and calculating."* (Data source: post-workshop reflection interview)

A concluding remark made by Beyoncé during the closing interview signifies new awareness of the importance of teaching for understanding and reasoning:

"I have noticed a shift in my own thinking … and my own understanding has also improved. The introduction to mathematical proficiency has helped me to see the ***importance of***

***interrelatedness**, and that has been a major instigator in changing my practice. **I need to move away from knowing**. I tended to focus on that in the past but there is no opportunity for application."* (Data source: post-workshop reflection interview)

Beyoncé's perceptions of practice across both systems can be summarised as follows:

- Needs to move beyond knowing, and create opportunities for understanding, application and reasoning (interrelatedness).
- Needs to increase the complexity of tasks in order to integrate all the strands of mathematical proficiency.
- Needs to plan for building on students' responses to further reflection and discussion.

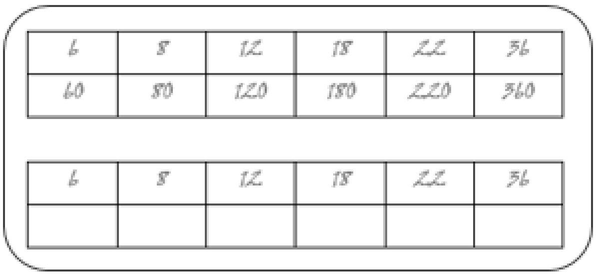
4.2.3 Lorraine: Analysis of Activity System 1 and Activity System 2

4.2.3.1 Activity System 1

1. **Identify and describe the dimensions specific to Lorraine's activity system**

The dimensions of the pre-workshop lesson, Activity System 1, are detailed in table 4.6.

Table 4.6: Lorraine's Activity System 1 dimensions

Activity	12 minute Grade 3 mental mathematics lesson
Subject	Lorraine: Grade 3 class teacher
Object	To halve x10 in order to calculate x5 *"When I said to them use what you know, I wanted them to remember that x10 is easy so I can use this to x5 ... just halve it."* (Data source: pre-workshop reflection interview)
Tools	 Whiteboard on which Lorraine tracked the examples (even numbers ranging from 6 to 36) and dialogue (responses to questions posed) *Lorraine wrote her examples in the top row and tracked the students' responses in the bottom row*
Community of practice: context as described earlier in the chapter	
Rules	**Rule derived from beliefs, practice, habits:** recognise the patterns in number facts (understand) in order to apply to other connected facts

	Beliefs: Lorraine believes that having a sense of number, and knowing how to work with numbers, is the focus of mathematics teaching and not just knowing the answer. *"…they can **understand what to do with numbers in that they would be able to get an answer quickly by using an efficient strategy** and they can verbalise it …"* **Practice:** Lorraine works on being consistent with doing a little bit of mental mathematics every day. *"I have generally three ability groups – in an ideal situation I would take each group on the mat on a daily basis – 20 minutes per group and spending 5 to 7 minutes doing mental maths daily."* **Habits**: although Lorraine holds the belief that doing mathematics is more than just about the answer, there is an awareness that her practice in this regard is still evolving. *"I need to focus on doing this differently … I need to rather guide them to see and notice things …"* (Data source: introductory interview)
Division of labour	Teacher-led, small-group teaching with limited opportunities for student engagement *"I talk too much … I need to see more chances for the children to verbalise and explain …"* (Data source: pre-workshop reflection interview)

2. **Identify the relationships between the dimensions of Lorraine's activity system**

Setting the scene: there were ten students seated on the mat at the back of the classroom with the teacher. The rest of the class were engaged with independent mathematics work at their desks, for which they had been prepared. Lorraine's mental mathematics lesson formed part of her daily mathematics routine of counting, manipulating numbers (mental mathematics) and problem solving. The lesson content was based on the page in the NumberSense Workbook that the students were going to complete once the mat work session was completed, but Lorraine did not explicitly refer to this resource during the lesson. Lorraine had led the group through counting in 10s and 5s and had then gone around the group with a few 'times 10' questions, "So, I'm counting in 10s, 6 times?", and so on. While she posed these questions to various students in the group, she tracked their responses in a table on a whiteboard. As she filled in the students' responses in the table, she confirmed as follows, "So, 6 times 10 is 60". Once the table was completed she asked the students why it had been so quick to get these answers – they very quickly responded that they "… just added a zero to the number … times-ing by 10 is just adding a zero …" Once this had been established, Lorraine drew a second table underneath the first one as illustrated in figure 4.6. The reflection of practice and the vignettes below continue from here.

Figure 4.6: Illustration of the tables that Lorraine drew on the whiteboard during her lesson

6	8	12	18	22	36
60	80	120	180	220	360

6	8	12	18	22	36

The collection of vignettes that follow represent all the instances where Lorraine chose to stop the video recording to reflect on her practice during the pre-workshop lesson reflection interview. I looked out for cases where her reflections established relationship connections between the dimensions within the activity system.

T	*Teacher*
Ss	*All or a group of students responding together*
S#	*# indicates the number of the student who responded: tallies the student responses*
Italics	*Indicates the actions / non-verbal responses*

Lorraine Vignette 1.1

T: OK, so 2 hands, have 10 fingers. Now I just want to focus on one hand. So that's 5 fingers. So we'll be counting in?

Ss: 5s … *The teacher draws a new table underneath the existing one (as per figure 4.6).*

T: So 6 hands?

Ss: 30.

T: That was quick?

Ss: We know 6 times 5 is 30.

T: Ok now 8 hands? If 8 of us hold up 1 hand – how many fingers?

Ss: 40.

T: Are you sure of that? *Filling in the table as they go.*

T: Now if 12 of you show me 1 hand?

Ss: 60.

T: Now if 18 of you show me? How many fingers?

Ss: 90.

T: How did you get that?

S11: Half of 180 is 90. And 10 fingers are 180 so 5 fingers must be half of that – 90.

T: *Pointing to the tables at the top and bottom …* The bottom table is half of the top table - you also know that half of 60 is 30, half of 80 is 40, half of 120 is 60 … Have you noticed that?

6	8	12	18	22	36
60	80	120	180	220	360

6	8	12	18	22	36
30	40	60	90		

T: So, how many fingers if I have 26 hands?

Ss: 130!

T: That is right! So what you are actually saying is 26 times 5 is 130.

T: Now do you notice that you use two things that you know – 10 times 26 – and then half of that?

<u>Lorraine's reflection moment 1:</u> Roughly 8-10 minutes into the lesson Lorraine stopped the recording to comment:

*"I wanted them to **notice** and use x10 and halve to get x5 … I **guided them to notice** this …and at first I thought they had noticed this because that child said 'Half of …'"*

(Data source: pre-workshop reflection interview)

My analysis:

Lorraine wants to use what the children know (x10) to help them with what they don't know (x5). Her objective is to help students notice the halving relationship between x10 and x5, yet the students already have efficient strategies of their own to x5, so her choice of tasks enabled their strategies rather than challenged them to make the new connection. Here her level of cognitive demand is not sufficient to facilitate noticing and application of the relationship between x10 and x5 (Stein *et al.*, 2000). In terms of division of labour, although Lorraine is posing questions that seem to require more than a quick answer, she is dominating the interaction by curbing student offerings. With reference to 'listening' and the IRE pattern (Cazden, 2001), Lorraine seems to be somewhere between evaluative and interpretive listening (Davis, 1997): although Lorraine is asking and listening for information, she is still expecting a set answer of 'I x10 and then halved to find out what x5 was …'.

Synopsis: Use a particular strategy – through 'noticing', facilitated 'noticing' through questioning

Lorraine Vignette 1.2	
T: 28 children - how many fingers – now use what you know – what do you know? T: 28 children – each with 1 hand – 28 times 5? 28 times 5? S12: Half of 28 is 14 … 140. T: 28 times … what did you say? T: OK, wait I hear what you say – what did you say? You said this - 280: 28 children will have 280 fingers. This needs to be halved because instead of 10 fingers we are only counting half of them, 140 …	… So 28 times 5 is 140 … You used what you know to do this. T: So can you tell me what is 32 times 5? Remember, I said 32 times 5? *No response from students.* T: 32 times 5 … Use what you know … S13: 32 times 5 … But I know that 32 times 10 is 320 and if I halve that I get … mmm … half of 32 is 16 and add the zero makes half of 320 to be 160. S14: I just halved 32 and then times by 10 … 16 times 10 is 160.

<u>Lorraine's reflection moment 2:</u> Roughly 10-12 minutes into the lesson Lorraine stopped the recording to comment:

*"I had **wanted them to use this strategy** (x10 and then halve), but they are **doing something else**. I notice that **they are doing something different** (halving first and then x10) … **I hadn't thought of this** … not sure how it will work with odd numbers? **I should have introduced an uneven number earlier**."*

(Data source: pre-workshop reflection interview)

My analysis:

Lorraine's reflections here indicate an awareness that her tasks (tools) were not achieving the objective of application. Her intention was for the students to make use of the halving relationship between x10 and x5. Without the introduction of an uneven number the students were not provoked to apply the intended strategy of x10 and then halve. The level of demand was also such that the students were relying on what they knew, without needing to apply this further.

Synopsis: Planning, including tool selection and use, did not cater for alternative strategies

Lorraine's reflections on her practice, as quoted above, were made before the workshop session, and before sharing the lens of mathematical proficiency. In her second reflection on the pre-workshop lesson video recording, this time with the knowledge of mathematical proficiency, Lorraine added the following reflections:

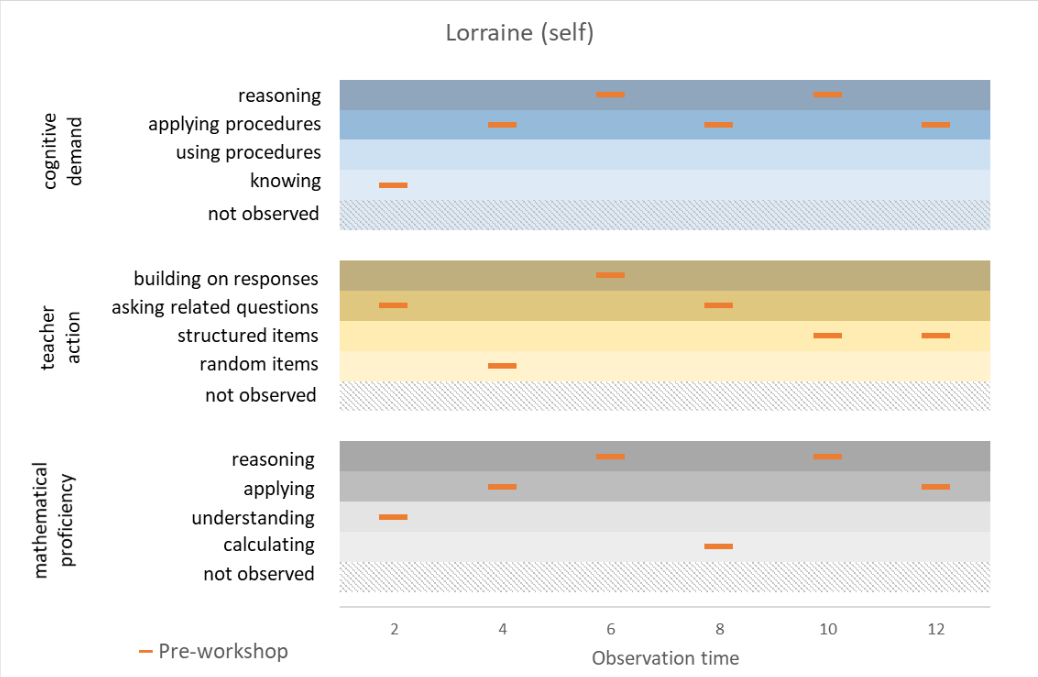
*"My granddaughter was watching this with me and she said – 'Granny, you talk too much!' – I must agree – I do! **I repeat the question over again without giving the children a chance to respond**."* (Data source: self-reflection checklist for the pre-workshop lesson video recording)

My analysis:

With reference to Lorraine's reflection of 'without giving the children a chance to respond', she voices an awareness of improving practice towards creating more opportunities for the students to engage and discuss.

These reflections of practice and initial analyses can be further interpreted by referring to the time-motion graph generated from Lorraine's reflections of practice using the completed self-reflection checklist for the pre-workshop lesson, Activity System 1 and as illustrated in figure 4.7.

Figure 4.7: Time-motion graph representing Lorraine's pre-workshop reflections on practice



Mathematical Proficiency: The graph depicts some variation across the sub-categories of mathematical proficiency and, although Lorraine's reflections by means of the checklist indicate instances of applying and reasoning, Lorraine's interview reflections on this lesson suggest that the students were using known strategies, rather than applying a new strategy (Lorraine's intended strategy) and reasoning.

Teacher Action: Lorraine reflects that she does not always give the students the opportunity to respond and, without the opportunity to respond and discuss, the students will be unlikely to develop the application and reasoning for which Lorraine is aiming. Had her focus been on the students' responses, she may have picked up earlier that they were not applying the intended strategy, and she may have been able to build on the students' responses by including an item that would have 'forced' them to use the intended strategy – introducing an uneven number would have made halving first, before multiplying by 10, uncomfortable. On reflection, this is something that Lorraine observed and highlighted.

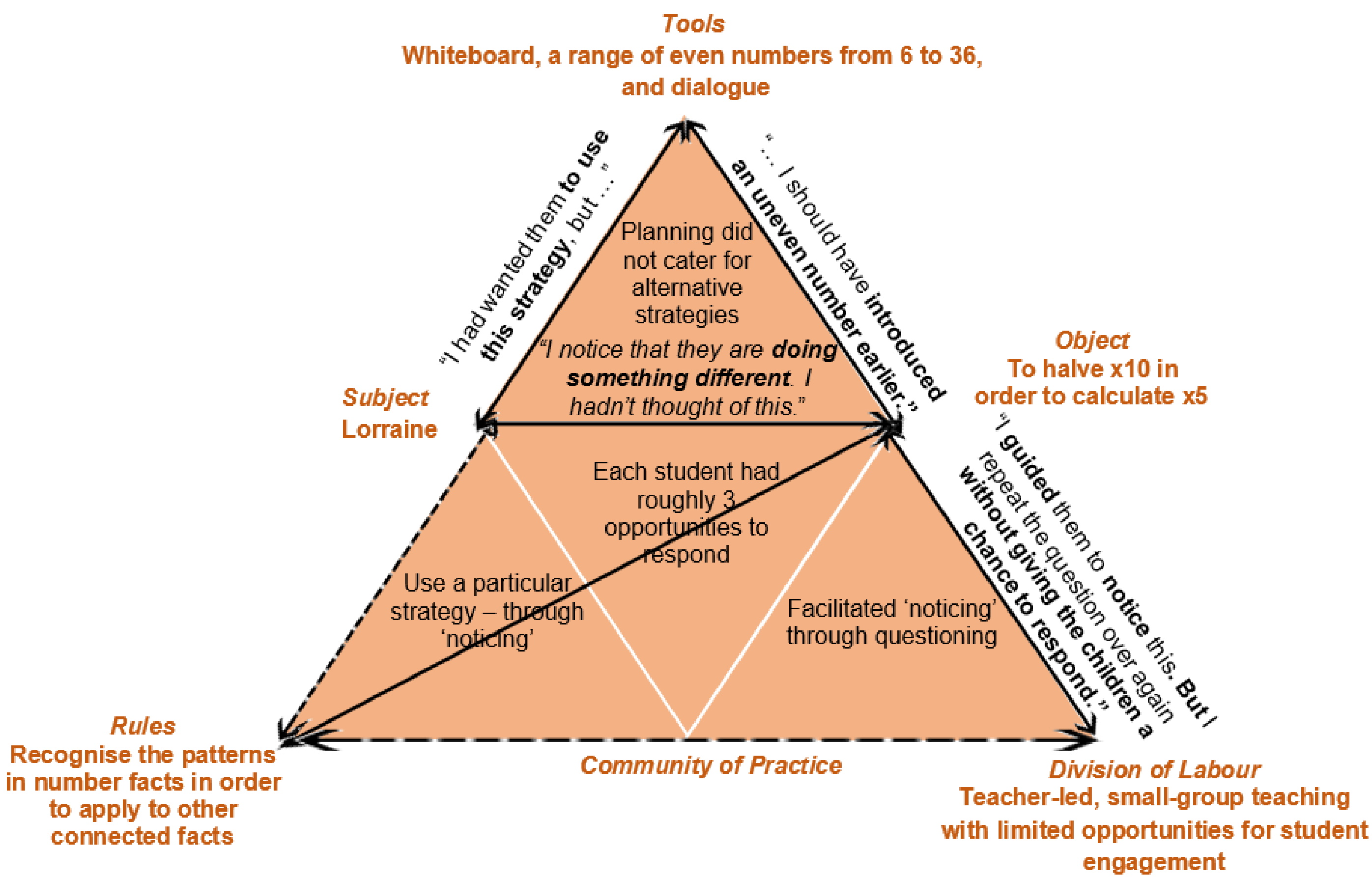
Cognitive Demand: Lorraine checked the higher level sub-categories of applying procedures and reasoning. My interpretation of this is that the cognitive demand was lower than Lorraine gauged, as many of the 'sums' in the vignettes were facts that the students already knew (6x5, 8x5, 12x5). It was only towards the end of the lesson, when application was necessary, that

Lorraine noticed that the students were using an alternative strategy to the intended one. Up until that point, the students had just known the answers, and had not needed a strategy.

3. **Identify and describe the dominant relationships between the dimensions of Lorraine's activity system**

In order to investigate the *what, why and how* of Lorraine's activity system, I summarised her perceptions of practice in terms of the relationships between the dimensions of her activity system illustrated in figure 4.8.

Figure 4.8: Activity System 1 as interpreted through the analysis of Lorraine's reflections



I positioned Lorraine's quotes (from her reflections) along the arrows that connect the appropriate dimensions in the activity system. The dimensions and their relationships are explained and described with the use of Lorraine's own words in table 4.7. The analysis synopses of Lorraine's reflections fill the triangle areas in order to structure my analysis into the activity system. In table 4.7 I explain how I arrived at the activity system of figure 4.8.

Table 4.7: Description of Lorraine's Activity System 1 relationships

<table>
<tr><th>Subject: Lorraine</th><th>Perception</th></tr>
<tr><td>Tools: Whiteboard on which to track the examples (even numbers ranging from 6 to 36) and dialogue (responses to questions posed)</td><td>"I should have introduced an uneven number earlier."
(Data source: pre-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary: Planning, including tool selection and use, did not cater for alternative strategies
Although Lorraine had prepared well enough knowing what she wanted to achieve, she reflects that her choice of tools could have been more deliberate in terms of assisting to achieve the objective. Planning for only one possible strategy suggests that Lorraine's practice could still be rooted in an old belief that suggests that mental mathematics is just about the answer even if the answer is a specific strategy.
Relationship:
Subject-Tools
Lorraine's planning and tool selection did not ensure that her tools would result in the students having to use the desired strategy of using x10 and then halving to calculate x5. Not only were the tasks in the lower levels of cognitive demand (knowing and using procedures) but their structure (even numbers) did not provide enough discomfort (struggle) for the students to apply the desired strategy (Stein et al., 2000).</td></tr>
<tr><td>Object (stated lesson objective): To halve x10 in order to calculate x5</td><td>"I had wanted them to use this strategy, but they are doing something else ...I hadn't thought of this."
(Data source: pre-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary:
In her objective of wanting the students to apply a specific strategy, Lorraine had not considered the possibility that the students would use a different approach.
Relationship:
Subject-Object
Lorraine had not planned for an alternative strategy and had not planned for examples to lessen the use of an alternative, nor to strengthen the case for the intended strategy. In this case the use of uneven numbers would have assisted the students to apply the strategy that Lorraine had planned.

Tools-Object
The students applied a different strategy to the intended one as the range of items allowed for an alternative strategy.</td></tr>
</table>

Rules: Recognise the patterns in number facts in order to apply to other connected facts	*"When I said to them use what you know, I wanted them to remember that x10 is easy so I can use this to x5 ... just halve it … 28x5 … 28x10=280 and half of this is 140 …"* (Data source: pre-workshop reflection interview)

Analysis summary: **Use a particular strategy – through 'noticing'**

Lorraine had a strategy planned that she wanted the students to 'notice' and then apply. Her use of the word 'notice' indicates that she has established rules that go beyond the expectation of an answer only response from students to noticing patterns over a range of examples.

Relationship:

Rules-Object

Lorraine's rules indicate that she holds the belief that students can and should learn mental mathematics beyond an answer, and this is evident in her lesson objective which includes application.

Division of labour: Teacher-led, small-group teaching with limited opportunities for student engagement	*"I guided them to notice this, but I repeat the question over again without giving the children a chance to respond."* (Data source: pre-workshop reflection interview)

Analysis summary: **Facilitated 'noticing' through questioning**

Although Lorraine intentionally asks questions that require a response beyond the answer, there is the anticipation of an expected response and Lorraine does little with the actual student responses. An audit from the lesson regarding the teacher role can be seen below and this supports Lorraine's reflections of not giving the students sufficient opportunity to respond.

Action / Time	02:00	04:00	06:00	08:00	10:00	12:00	TOTAL
Asking 'answer only' questions	4	2	5	2	4	7	24
Asking questions to facilitate discussion	1	3	0	2	2	0	8

Relationship:

Object-Division of labour

Lorraine is aware that she talks too much, and that she is doing the work and does not always give the students the chance to respond. In this lesson, although the 'answer only' questions may have included a more detailed explanation from the students, the response was treated like a 'correct' versus 'incorrect' answer only question, in that nothing further was done with the student responses. The audit of practice on this lesson reveals that Lorraine posed 32 questions (random items, structured items and related questions) to a group of ten students across the lesson. Only 8 of the 32 questions (that is, 25%) required more than just an answer.

4. **Analyse and describe Lorraine's lesson outcome as a consequence of the dimensions and the dominant relationships**

Lorraine was very clear about her objective of wanting the students to use the strategy of x10 and then halve in order to x5 (application with little reasoning). This objective was achieved in that the students did **engage in some level of application** by using the x10 facts in order to solve the x5 facts. Lorraine's reflections identify that **a shift in the division of labour is necessary in order to create opportunities for the students to respond**, and for her to build on those responses instead of assume them. Lorraine also reflected on the **tool (item) selection in being crucial** to achieving the objective as, had she included uneven numbers in her items, the students may have noticed and applied the intended strategy sooner.

4.2.3.2 Activity System 2

1. **Identify and describe the dimensions specific to Lorraine's activity system**

The dimensions of the post-workshop lesson, Activity System 2, are detailed in table 4.8.

Table 4.8: Lorraine's Activity System 2 dimensions

Activity	12 minute Grade 3 mental mathematics lesson	
Subject	Lorraine: Grade 3 class teacher	
Object	Working with the multiples of 10, and familiarising students with correct vocabulary *"I wanted them to complete the 10 and work with multiples of 10 … to introduce the vocab."* (Data source: post-workshop reflection interview)	
Tools	What is 60 plus 5? What is 80 plus 5? What is 100 plus 5? What is 40 plus 5? What is 140 plus 5? What is 50 plus 5? What is 150 plus 5? What is 35 plus 5? What is 135 plus 5? What is 75 plus 5? What is 175 plus 5? What is 15 plus 15? What is 25 plus 25? What is 35 plus 35? What is 25 plus 35? What is 25 plus 45? What is 25 plus 75? What must be added to 70 to get 100? What must be added to 75 to get 100? Explain. What must be added to 90 to get 100? What must be added to 95 to get 100? What must be added to 40 to get 100? Explain.	Structured items: adding a '5' or a '10' to a '10'; and adding a '5' to a '5' An example of the items used from '*NumberSense Workbook 10'* mental mathematics materials (Brombacher, 2012)
Community of practice: context as described earlier in the chapter		
Rules	**Rule derived from beliefs, practice, habits:** teaching the 'names' (vocabulary) is important **Beliefs**: in this lesson Lorraine spent time in teaching the vocabulary 'the multiples of 10'. This indicates an existing belief that learning the 'names' of mathematical operations and concepts is important.	

	*"… I realise that **the 'name' does not impact the ability for children to work with it. I was trying to introduce the vocab**, but taking too long with it, in the wrong way."* **Practice**: Lorraine's practice indicates an awareness that she is making a deliberate attempt to teach through noticing and discovery rather than through telling. *"… I was really trying not to tell them! I spend too long on what they already know."* **Habits:** Lorraine is able to identify that her practice is not yet realising her objective and that she reverts to her old habits of 'telling' and spending too long on the 'knowing' (refer to table 4.6). *"… I am stuck in my routines and habits."* (Data source: post-workshop reflection interview)
Division of labour	Teacher-led, small-group teaching with limited opportunities for student engagement *"I do take too long and talk too much to get through things."* (Data source: post-workshop reflection interview)

2. **Identify the relationships between the dimensions of Lorraine's activity system**

Setting the scene: there was a group on 10 students on the mat with Lorraine. The rest of the class were engaged in independent mathematics work at their desks. Lorraine was using the mat work time to prepare the students to work independently in the NumberSense Workbooks on return to their desks. Lorraine was using the NumberSense mental mathematics materials, and she was on page 6 of Workbook 10 (this was the page that this group were going to be working on when returning to their desks). The examples consisted of adding to a '10', and adding to a '5'. Lorraine was posing questions and working her way around the circle of students selecting the students from whom she wanted responses.

As before, the collection of vignettes that follow represent all the instances where Lorraine chose to stop the video recording to reflect on her practice during the post-workshop lesson reflection interview. I looked out for cases where her reflections established relationship connections between the dimensions within the activity system.

T	*Teacher*
Ss	*All or a group of students responding together*
S#	*# indicates the number of the student who responded: tallies the student responses*
Italics	*Indicates the actions / non-verbal responses*

Lorraine Vignette 2.1

T: Yes, good. Listen learners …
T: 5 plus 5?
S15: 10.
T: 10 plus 5?
S16: 15.
T: 10 plus 10?
S17: 20.
T: Well done - I just want you to look at the answers they get. *Picks up the whiteboard and repeats the questions (sums) and writes the questions and answers on the whiteboard and repeats the questions.*
T: 25 plus 10?
S18: 35.
T: 25 plus 10? And you say it is? 35 … *And adds this to the whiteboard.*
T: OK. 40 plus 40?
S19: 80.
T: It is 80. OK learners, I want you to look at the numbers that I have written and the answers.

The teacher holds up the whiteboard with the sums written down and shows the group.
T: Look at this … *The teacher adds 15 +15 =30 to the whiteboard.*
T: Look at this … What do you notice? What do you notice?
Ss: *No response.*
T: Just anything that you notice? Look at the last digits of the numbers. *The teacher points to the numbers on the whiteboard that end in zero.*
T: Look at the numbers and the answers. What do you notice? Anything?
S20: There's a zero.
T: I like that – when is there a zero?
T: *Points to the friendly numbers (end in a zero) on the whiteboard.*
S21: When it's a friendly number?
T: Ah – ends in a zero when you add two friendly numbers, or add two 5s.

<u>Lorraine's reflection moment 1:</u> Roughly six minutes into the lesson Lorraine stopped the recording to comment:

*"I spent a lot of time **setting up the pattern** … I spent too much time on this, on something that the children already knew. I **spend too long on what they already know …** I should have moved on and worked with more examples of the multiples of 10."*

(Data source: post-workshop reflection interview)

My analysis:

Lorraine acknowledges that she spent too long on 'knowing' type examples. This lack of a planned 'struggle' proposes that Lorraine was working below (outside of) the students' Zone of Proximal Development (Vygotsky, 1978a) which resulted in a space where tasks are easy rather than accessible, and did not present enough challenge (Hiebert and Grouws, 2007; McGatha *et al.*, 2018). This vignette illustrates a point in the lesson which was teacher-led. Although Lorraine was attempting to get the students to 'notice', she did not engage or probe for further student responses. Lorraine is engaging with the students in the typical IRE pattern of 'answer' only responses (Cazden, 2001).

Synopsis: Facilitated discussion through questioning, although level of demand was knowing

Lorraine Vignette 2.2	
T: Give me another word for a friendly number? We call them friendly numbers because they end in a zero. S22: Tens. T: No … *The teacher points to the numbers on the whiteboard* … Is this a friendly number? Is this a friendly number? Why do you call them friendly numbers? Ss: *Respond by calling out and pointing to the numbers on the whiteboard.* T: Why do we call them friendly numbers? Ss: *Lots of interaction and hand raising and calling out among students.* T: It's easy to add on to a friendly number, but look at my friendly numbers, what can I also call them man?	Ss: *Calling out.* T: Multiples of …? Ss: Of 10. T: Multiples of 10. Multiples of 10. So you notice that when I add multiples of 10, if there are zeros then the answers will always have a zero … *Pointing the numbers on the whiteboard.* T: Now look at this answer that ends in a 5 … *Points to the whiteboard …* What do you notice? Ss: *Calling out.* T: uh-uh no, look carefully – no man … *Points again* … When one number is not a friendly number, ends in 5, and this is added to a friendly number, the answer …? S23: … will end in a 5! T: Wonderful!
<u>Lorraine's reflection moment 2:</u> Roughly 8-10 minutes into the lesson Lorraine stopped the recording to comment: *"I spent too long on trying to get them to discover the 'name' …* ***and the 'name' does not impact children's ability to work with it****. I was trying to introduce the vocab, but taking too long with it, in the wrong way. I was really trying* ***not to tell*** *them!"* (Data source: post-workshop reflection interview)	

My analysis:

This reflection from Lorraine is supported by the argument made by Kamii and Dominick (1998) that the challenge for teachers is to distinguish between knowledge that must be told (social knowledge) and knowledge that students should (and can) construct for themselves (conceptual knowledge). In this instance Lorraine, in her attempt not to 'tell', has treated the teaching of social knowledge (the name) as conceptual knowledge.

Synopsis: Students need to know that 'friendly numbers' are called multiples of 10

Lorraine Vignette 2.3	
T: Look carefully – when I have two 5s – what happens? S24: The answer is a friendly number. T: Because 5 plus 5 is equal to …? Ss: … 10. T: And 10 is a friendly number and 10 ends in zero you see that? Does it make sense to you? T: Now quick, quick, quick … T: So, so what is … 70 plus 5? S25: 75. T: And 50 plus 5? S26: 55. T: And 40 plus 5? S27: 45. T: And 100 plus 5? S28: 105. T: And 90 plus 5? S29: 95. T: And 40 plus 15? S30: 55.	T: So this is still a friendly number and a number ending in 5. Can you see that? Alright, now … T: 25 plus 45? Now listen carefully 25 plus 45 – so you know already your answer must end with a ….? Ss: … zero. S32: 70. T: Is that the answer? 45 plus 25? Ss: *All shouting out answers.* T: Now listen – 45 plus 25? Ss: He said 70! T: I'm sorry my child, I didn't listen properly. So you said 70. 20 plus 40 and 5 plus 5 is 10 – and that makes 70. T: Right OK – now what must I add to 70 to get to 100? S33: 30. T: And what must I add to 40 to get to 100? S34: 60.
<u>Lorraine's reflection moment 3:</u> Roughly 10-12 minutes into the lesson Lorraine stopped the recording to comment: *"I went on too long and should have moved on. I kept going* ***because I felt they didn't know****. I do take too long and talk too much to get through things. Gosh –* ***I am stuck in my routines, and habits****."* (Data source: post-workshop reflection interview)	

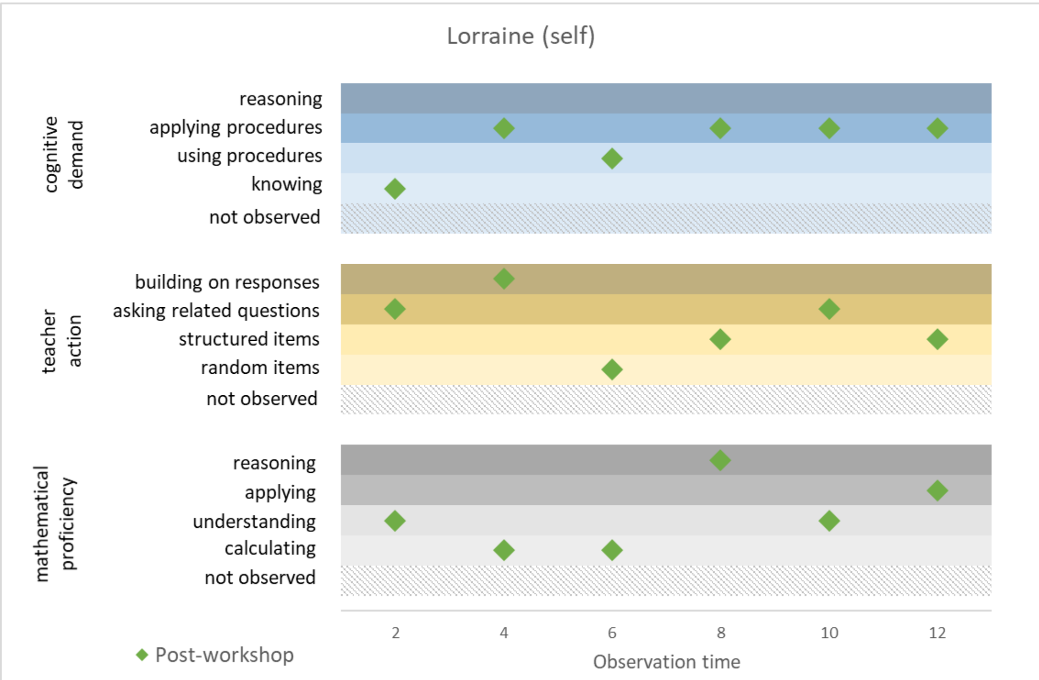
My analysis:

Lorraine's reflections reveal that she did not think the students were capable of more demanding tasks. Nonetheless, she shows an awareness that this low perception of what students can do is rooted in her old routines and habits of 'telling' and spending too long on the 'knowing', and is not necessarily part of her current beliefs.

Synopsis: Time spent on what the students know

Lorraine's reflections on her practice, as quoted above, were made after the workshop session, and with the shared lens of mathematical proficiency. These reflections of practice, and initial analyses can be further interpreted by referring to the time-motion graph generated from Lorraine's reflections of practice using the completed self-reflection checklist for the post-workshop lesson, Activity System 2 and as illustrated in figure 4.9.

Figure 4.9: Time-motion graph representing Lorraine's post-workshop reflections on practice



Mathematical Proficiency: Lorraine's reflections that she spent too long on what the students knew are supported by her observations of calculating from four to six minutes. Although she observes an instance of reasoning, it seems to come from nowhere and go nowhere as it is not supported by instances of application.

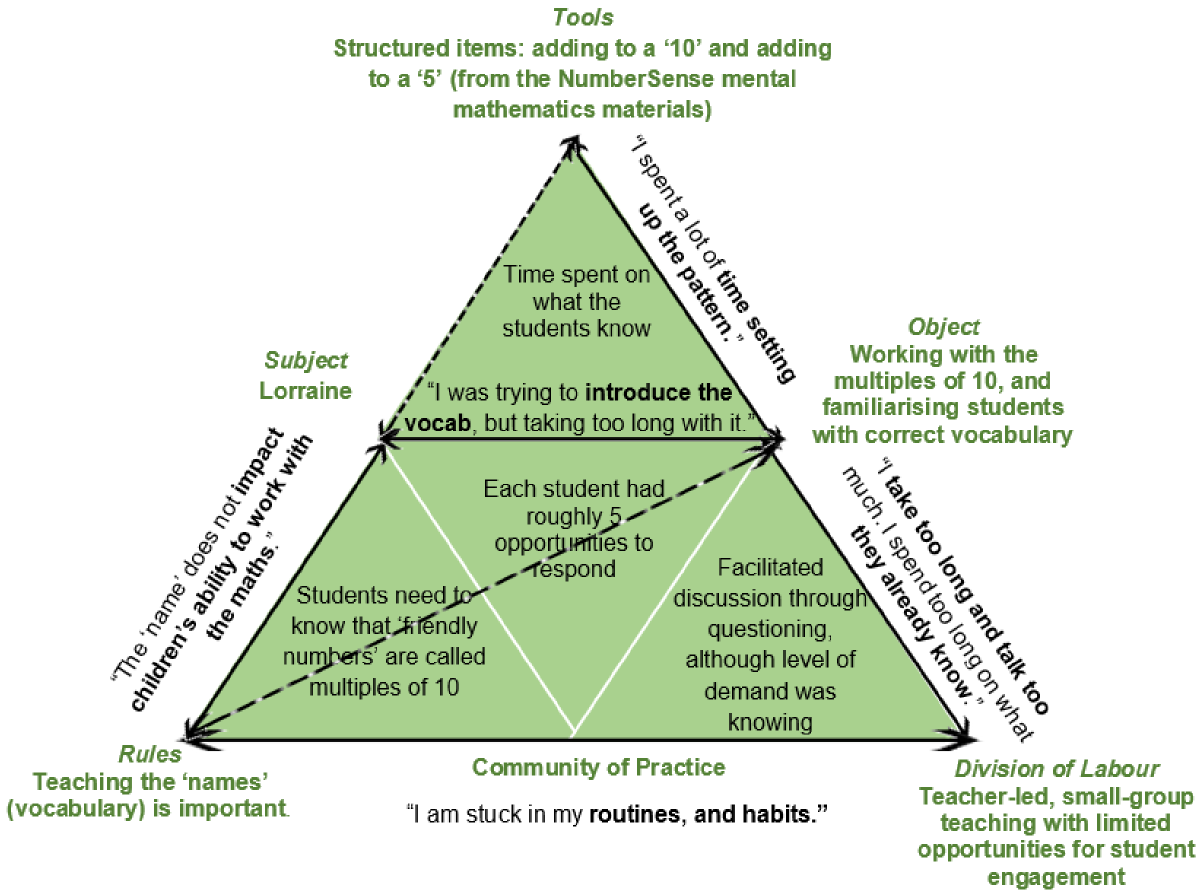
Teacher Action: There is an inconsistent attempt to ask related questions and build on responses. This interpretation is supported by Lorraine's reflections that she talks too much and takes too long.

Cognitive Demand: Although Lorraine has indicated observations of applying procedures, my interpretation, with reference to her reflection of taking too long to set up a pattern for something the students already know, is that the level of demand is not sufficient to create opportunities for reasoning, which could explain why Lorraine only indicated one observation of building on the students' responses. There is no opportunity for the students to deepen their understanding through application and reasoning as the level of demand of the questions is too low – they just know the answer.

3. **Identify and describe the dominant relationships between the dimensions of Lorraine's activity system**

In order to investigate the *what, why and how* of Lorraine's activity system, I summarised her perceptions of practice in terms of the relationships between the dimensions of her activity system illustrated in figure 4.10.

Figure 4.10: Activity System 2 relationships as interpreted through the analysis of Lorraine's reflections



I positioned Lorraine's quotes (from her reflections) along the arrows that connect the appropriate dimensions in the activity system. The dimensions and their relationships are explained and described with the use of Lorraine's own words in table 4.9. The analysis synopses of Lorraine's reflections fill the triangle areas in order to structure my analysis into the activity system. In table 4.9 I explain how I arrived at the activity system of figure 4.10.

Table 4.9 Description of Lorraine's Activity System 2 relationships

<table>
<tr><th>Subject: Lorraine</th><th>Perception</th></tr>
<tr><td>Tools: Structured items, adding to a '10' and adding to a '5'</td><td>"I spent a lot of time setting up the pattern."
(Data source: post-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary: Time spent on what the students know
Lorraine's awareness that she spent too long on what the students knew suggests that at the time of the lesson she did not think that the students were capable of more demanding tasks.</td></tr>
<tr><td>Object (stated lesson objective): Working with the multiples of 10, and familiarising students with correct vocabulary</td><td>"I was trying to introduce the vocab, but taking too long with it."
(Data source: post-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary:
The lesson was characterised by working with multiples of 10 but the lesson objective was unclear. It appeared as if introducing the vocabulary of the 'multiples of 10' was important but, during reflection, Lorraine noted that knowing the name of a concept did not influence the students' ability to work with it. The stated objective of 'working with multiples of 10' does not allude to what the lesson comprised, as the term 'working with' lacks clarity.
Relationship:
Subject-Object
Lorraine's awareness that she spent too long on introducing the vocabulary suggests that at the time of the lesson she considered this important for the students' ability to 'work with the multiples of 10'.

Tools-Object
Lorraine's awareness that she spent too long on 'setting up the pattern' indicates that she was not using the tool optimally. The tool did not get the opportunity to 'reveal the pattern' for which it was designed.</td></tr>
<tr><td>Rules: Teaching the 'names' (vocabulary) is important.</td><td>"The 'name' does not impact children's ability to work with the maths."
(Data source: post-workshop reflection interview)</td></tr>
<tr><td colspan="2">Analysis summary: Students need to know that 'friendly numbers' are called multiples of 10
Lorraine took this belief into her lesson but, on reflection, has an awareness that the name bears no relevance to the mathematics in this instance.</td></tr>
</table>

Relationships:

Rules-Subject

Lorraine observed that she spent too much time trying to introduce the 'name' through 'noticing', but on reflection indicates an awareness of a shifting rule that the vocabulary bears no relation to the students' ability to be able to do the mathematics.

Division of labour: Teacher-led, small-group teaching with limited opportunities for student engagement.	*"I take too long and talk too much. I spend too long on what they already know. I am stuck in my routines, and habits."* (Data source: post-workshop reflection interview)

Analysis summary: **Facilitated discussion through questioning, although level of demand was knowing**

Lorraine is aware that her habit of telling is keeping her from creating opportunities for the students to reflect and discuss. An audit from the lesson regarding the teacher role can be seen below and this supports Lorraine's reflections that she talks too much as there is still a predominance of 'answer only' questions.

Action / Time	**02:00**	**04:00**	**06:00**	**08:00**	**10:00**	**12:00**	**TOTAL**
Asking 'answer only' questions	6	11	8	8	6	5	44
Asking questions to facilitate discussion	3	0	0	2	2	2	9

Relationship:

Rules-Division of labour

Lorraine is aware that her 'rules' are keeping her 'stuck'. In this lesson she focuses on the vocabulary which does not create the opportunities for discussion or the progression into a higher number range. Although her beliefs have shifted, her practice is still evolving and not yet congruent with her beliefs and expectations of her practice.

Object-Division of labour

Lorraine spent too much time setting up the pattern which revealed what the students already knew and she did not create enough opportunities for the students to respond. Lorraine posed 53 questions (random items, structured items and related questions) to a group of ten students. This means that each student had roughly five opportunities to respond to questions across a 12-minute lesson, but while this may seem sufficient in terms of the 53 questions, only 9 of these (that is, 17%) required more than an answer only. This resonates with Lorraine's reflections that she spent too much time on what the students already knew.

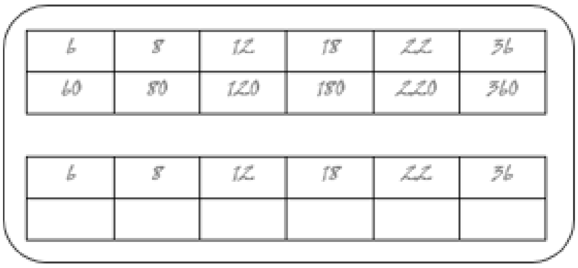
4. **Analyse and describe Lorraine's lesson outcome as a consequence of the dimensions and the dominant relationships**

Lorraine's awareness that she is still **'stuck' in her old habits in that this lesson consisted of too much 'knowing' and not enough opportunities to apply and engage in reflective discussion** contribute to the conclusion that this lesson did not achieve what she had planned. In this lesson, if 'working with' meant learning the vocabulary, then Lorraine's reflections indicate that she took too long with this, and it was not necessary to know the vocabulary in order for the students to access the mathematics. If 'working with' meant calculating, understanding, applying and reasoning, then Lorraine's reflections that too much time was spent on what the students already knew, indicate that this was not achieved either. Lorraine cited her old habits as having played a role in her observed practice and **what was envisioned and planned looked different in practice**.

4.2.3.3 Comparison of Lorraine's activity systems

Lorraine's reflections for both lessons, Activity System 1 and Activity System 2, have been analysed as separate systems. In order to uncover similarities, differences, patterns and variations across these activity systems, they were placed alongside each other and compared for their similarities and difference. To keep Lorraine's voice in the overview that follows, her closing reflections are included with these interpretations. As an overview of the two activity systems, the similarities and differences across the two can be tracked as detailed in table 4.10.

Table 4.10: The similarities and differences across the dimensions of Lorraine's activity systems

Dimensions	**Activity System 1**	**Activity System 2**
Subject	Lorraine	Lorraine
Object	To halve x10 in order to calculate x5	Working with the multiples of 10, and familiarising students with correct vocabulary
Tools	Whiteboard on which to track the examples (even numbers ranging from 6 to 36) and dialogue (responses to questions posed)  *Focus*: to reveal the relationship that x5 is half of x10 *Design*: even numbers and in a low number range	Structured items: adding to a '10' and adding to a '5' What is 60 plus 5? What is 80 plus 5? What is 100 plus 5? What is 40 plus 5? What is 140 plus 5? What is 50 plus 5? What is 150 plus 5? What is 35 plus 5? What is 135 plus 5? What is 75 plus 5? What is 175 plus 5? What is 15 plus 15? What is 25 plus 25? What is 35 plus 35? What is 25 plus 35? What is 25 plus 45? What is 25 plus 75? What must be added to 70 to get 100? What must be added to 75 to get 100? Explain. What must be added to 90 to get 100? What must be added to 95 to get 100? What must be added to 40 to get 100? Explain. *Focus*: adding multiples of 5 and 10 to a '10'; and adding multiples of 5 to a '5' *Design*: structured to reveal a pattern; variation in position of the unknown
Rules	Recognise the patterns in number facts in order to apply to other connected facts	Teaching the vocabulary is important
Community of practice	As described earlier in this chapter	
Division of labour	Teacher-led, with opportunities for student response (Each student had roughly three response opportunities)	Teacher-led with opportunities for student response (Each student had roughly five response opportunities)
Dominant relationships	Subject-Tools **Subject-Object** **Tools-Object** Rules-Object **Object-Division of labour**	**Subject-Object** **Tools-Object** Rules-Subject Rules-Division of labour **Object-Division of labour**

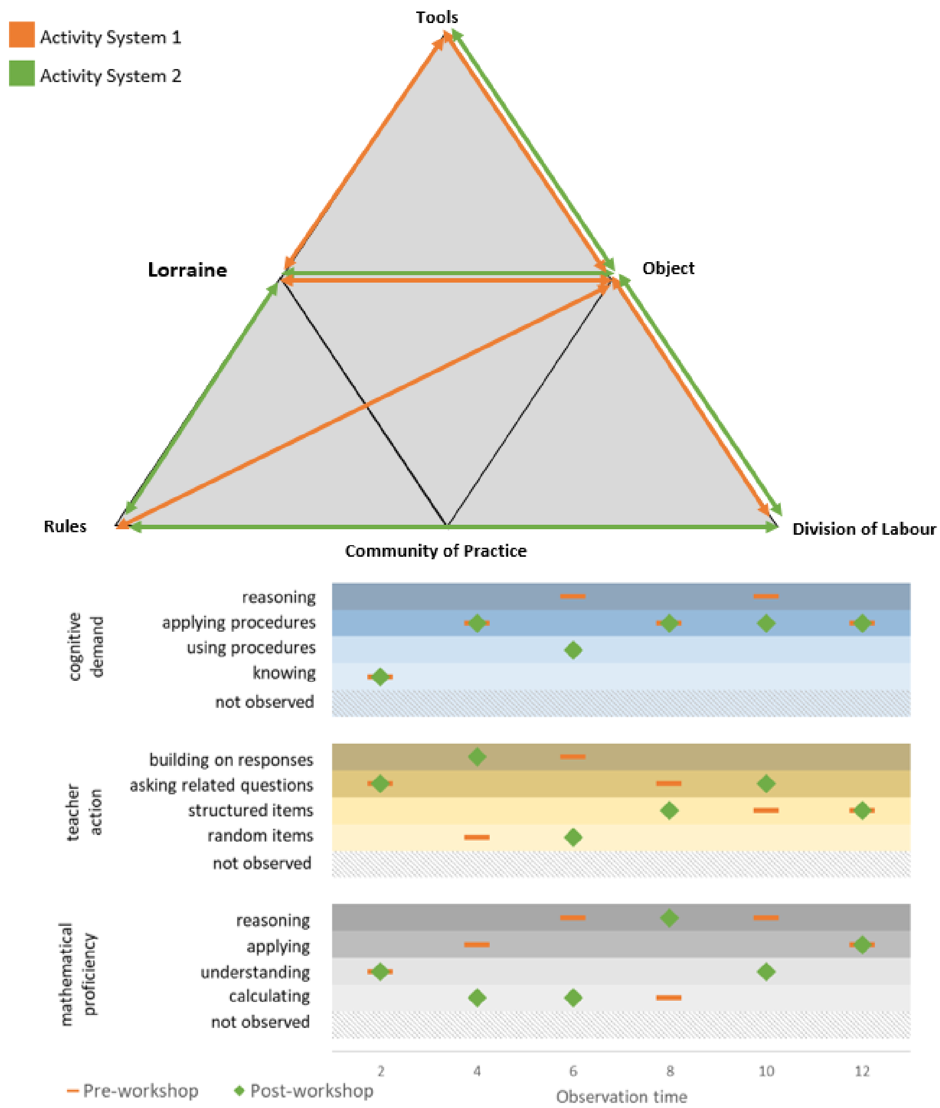
Outcome	The objective was achieved in that the students engaged with the x10 facts in order to solve the x5 facts, but a better selection of tasks and more discussion opportunities were needed to foreground the application 'times 5 is half of times 10'	The objective of 'working with' was unclear but the dominant reflection is that the objective was not met due to old habits ('telling' and spending too long on the 'knowing') in practice

Comparing the dimensions of the two activity systems, differences can be noted across the object, tools and rules. The most pertinent difference is in the object and rules: in contrast to the first lesson where the object and rules supported reasoned application of knowledge, in the second lesson the object and rules were narrowed to accepting a new term. Whilst the division of labour in the second system provided an increase in student response opportunities, the number of teacher questions that may have allowed for further student engagement decreased from Activity System 1 (8 out of 32, or 25%) to Activity System 2 (9 out of 53, or 17%). The dominant relationships across the two activity systems can be summarised as:

- Subject-Object
- Tools-Object
- Object-Division of labour

There are three similar dominant relationships across both activity systems as illustrated in figure 4.11. Activity System 1 is indicated in orange, and Activity System 2 in green. Although there are differences across the dominant relationships, they reveal similarities in perceptions of practice as illustrated by the time-motion graph in figure 4.11. The time-motion graph shows variation across the categories for both systems, although there is no pattern or consistency to this variation and so comparison is difficult.

Figure 4.11: Lorraine's summary relationships and reflections across both activity systems



Lorraine's Activity System 1 and Activity System 2 had very different objectives, and bore no connection to one another. The second system was not a progression from what Lorraine had reflected on and perceived in the first system. Looking at the time-motion graph variation across the sub-categories for both lessons can be seen and, while there are similarities and differences in perception between the two systems, there is no consistent pattern or progression in these.

Lorraine's reflections on the objective for both systems indicates that she had the expectation that the lesson would achieve what she had planned, and that she perceived aspects of her practice as counterproductive. Lorraine acknowledges that her planning needs more attention: not only the planning of what to teach but also how to teach it, and to be clear on what it is that she wants to achieve, as different objectives require different practice. One of Lorraine's closing reflections reveals her awareness of this:

"I do need to be more ***adaptive to what comes up in the lesson from the learners, and this means that my planning needs to be thorough to create opportunities for the children to reflect on their thinking, and discuss.*** *I need to be clear about what it is I want to accomplish, how I am going to do this, and what I need to use in order to achieve this."* (Data source: post-workshop reflection interview)

From Activity System 1, it is clear that Lorraine attempted to facilitate the students' noticing connections rather than telling them, and that inviting student responses that go beyond just the answer is important. Across both systems Lorraine noticed that she took too long and talked too much, and spent too much time on what the students already knew. She also acknowledged that the level of cognitive demand of the tasks was too easy and, in the first system, this resulted in the students being able just to 'know' the answer and not have to apply the intended strategy, as the nature and number range of the tasks did not 'force' this. In the second system Lorraine acknowledged the low number range by way of reflecting that she spent too long on setting up the pattern, instead of creating opportunities to use and apply it. A concluding remark made by Lorraine signifies an awareness of the importance of teaching for understanding and reasoning:

"I can see that I ***spend too long on the knowing, on a low number range****. I realise the importance of application and reasoning and need to make more opportunities for this in my teaching.* ***Creating opportunities in my teaching where all of the strands can be addressed*** *is important and part of this means giving the children* ***the opportunity to use what they know in another situation."*** (Data source: post-workshop reflection interview)

Lorraine's perceptions of practice can be summarised as follows:

- Needs to create opportunities in teaching to integrate all five strands.
- Needs to spend less time on the knowing and low number range tasks and create more opportunity for application and reasoning.
- Needs to be more flexible with student contributions and plan for more opportunities for student reflection and discussion.

4.3 Chapter Conclusion

This chapter has detailed the analysis of the data collected from Beyoncé and Lorraine. The analysis investigated how the teachers perceived their practice. Using the relationships between the dimensions of the activity provided a rich description of practice - *who is doing what, why and how.* The analysis of both activity systems for both teachers reveals that, although Beyoncé and Lorraine are each on their own continuum towards teaching for understanding and reasoning, they both demonstrate a belief, or emerging belief, that supports the teaching for understanding and reasoning, and they were able to identify aspects of their practice that did not currently support this. They were also able to communicate what it was that they needed to do in order to improve their practice, namely: create more opportunities for students to understand, apply and reason and, hence, develop mathematical proficiency (Kilpatrick, Swafford and Findell, 2002); adapt the tasks to elicit application and reasoning; and engage students more extensively in reflection and discussion.

Chapter 5: Discussion and Conclusion

5.1 Introduction

The purpose of this study was to analyse teachers' reflections on their mental mathematics teaching practice and to answer the question, "How do teachers perceive their practice?" The teachers who participated in this study are also engaged participants in a mathematics coaching programme. They receive weekly mathematics coaching during one of their mathematics lessons. The coaching aims to facilitate the adoption of mathematics routines that promote the teaching of mathematics for understanding and reasoning. In order to foreground the rationale for adopting the routines suggested during the coaching support, the teachers participated in a workshop session as part of this study. The workshop introduced the five strands of mathematical proficiency (Kilpatrick, Swafford and Findell, 2002) as a research-based framework that underpins the foundations of the teaching routines that support the development of understanding and reasoning. The shared, coherent view of mathematical proficiency provided a common lens for reflection. The findings discussed in this chapter describe the teachers' perceptions of their practice through the lens of mathematical proficiency.

5.2 Discussion of Findings

The analysis of Beyoncé's and Lorraine's reflections on practice revealed similar perceptions of practice across both of their activity systems for both teachers, as summarised at the end of chapter 4. While this study did not intend to investigate shifts in practice, the answering of the research question, "How do teachers perceive their practice?" revealed a tension between their actual practice and their desired practice. This indicates some discomfort with how the teachers are currently perceiving their practice. Not only have both teachers acknowledged that they are not achieving the desired outcome of mathematical proficiency (Kilpatrick, Swafford and Findell, 2002), but they have also identified an alternative, a vision of what they should be doing, and a shift in practice is emerging.

Their perceptions of practice, presented in chapter 4, are summarised in table 5.1 and clearly show three dominant themes: object (to integrate the five strands of mathematical proficiency); tools (task design and use); and division of labour (interaction with students).

Table 5.1: Summary of the teachers' perceptions of practice and the dominant themes

Beyoncé	Lorraine
Object: Needs to move beyond knowing, and create opportunities for understanding, application and reasoning (interrelatedness)	**Object**: Needs to create opportunities in teaching to integrate all five strands
Tools: Needs to increase the complexity of tasks in order to integrate all the strands of mathematical proficiency	**Tools**: Needs to spend less time on the knowing and low number range tasks, and to create more opportunity for application and reasoning
Division of labour: Needs to plan for building on students' responses to further reflection and discussion	**Division of labour**: Needs to be more flexible with student contributions, and to plan for more opportunities for student reflection and discussion

The analysis revealed that both teachers have an awareness of where and how they should adapt their practice and that, even though structured tasks provided in the NumberSense materials (Brombacher, 2012) were selected, their use of these did not achieve the objective of mathematical proficiency. Indeed, *"… good choice of tasks is important, but the way they are enacted makes the difference in bringing mathematics to life."* (Askew, 2016, p. 11). The dominant themes that emerged from their perceptions of practice are further detailed:

1. **Object (lesson objective)**

   Move beyond the constraints of 'knowing' and 'calculating' during mental mathematics lessons: to create opportunities to develop understanding, application and reasoning.

2. **Tools**

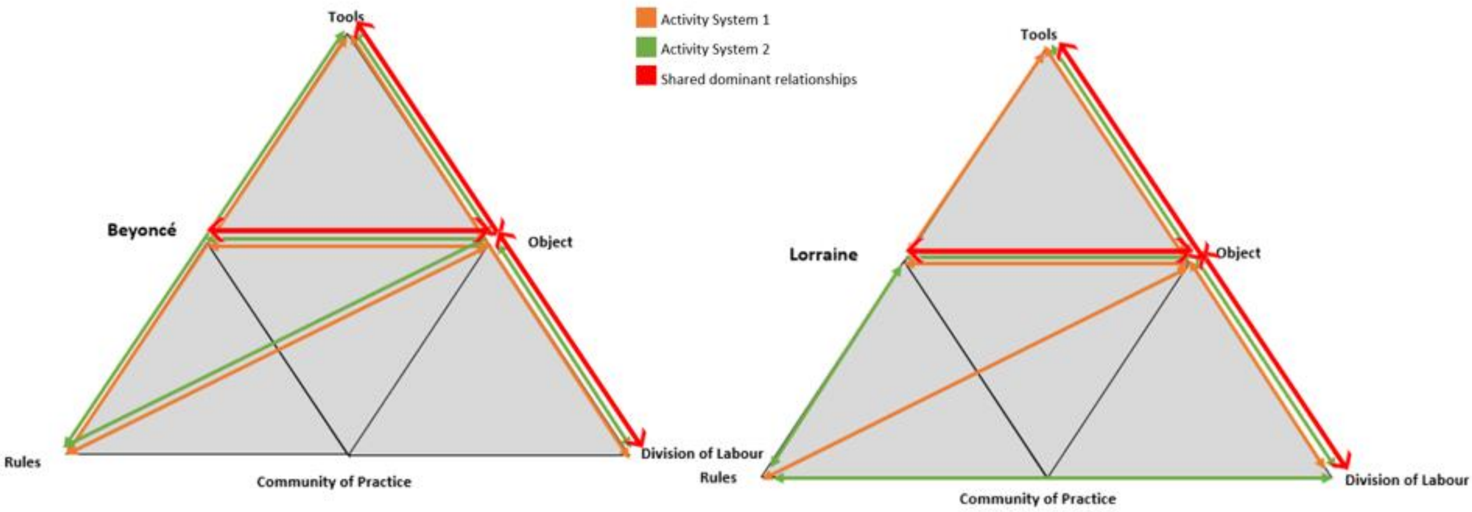
   Move beyond the 'knowing' level of mental mathematics task items: to elicit application and reasoning through the use of more cognitively demanding tasks that are purposefully utilised to allow noticing of patterns relationships.

3. **Division of labour**

   Move beyond 'answer only' questioning in mental mathematics lessons: to facilitate student discussion and reflection through more deliberate planning.

The dominant relationships across Beyoncé's and Lorraine's activity systems reveal that for both teachers the tool-object-division of labour and the subject-object relationships are shared, as illustrated in red in figure 5.1.

Figure 5.1: The shared dominant relationships for Beyoncé and Lorraine



Although the perceptions reflected by the teachers were similar, their sources and the implications for teaching have different meanings and result in different practices and shifts in practice for each teacher.

The analysis of Beyoncé's activity systems (first summarised in table 4.5) revealed the dominance of rules-based relationships between the object, subject and the tools. This suggests the introduction to research-based principles caused tension around her rules (beliefs, practices and habits), and particularly her beliefs. This tension is seen in the emerging awareness that Beyoncé has of her own teaching, and the students' learning needing 'more'. This tension was further highlighted in the second activity system where, even with the use of materials designed for the development of understanding and reasoning, the materials on their own were not enough to achieve this, and Beyoncé still perceived that her interaction with the students did little to move the discussion beyond the answer.

For Lorraine the relationships that are shared in her two activity systems (first summarised in table 4.10) are between the division of labour, object and tools, and did not incorporate the rules. However, in an activity system rules are always at play (Engeström, 2015), and it is plausible to conclude that Lorraine's rules came to include beliefs that subscribed to teaching for understanding and reasoning. Hence, Lorraine’s perceptions focussed on the division of labour and the perception that she needed to create learning opportunities that promote more meaningful student discussion and reflection. Over both activity systems Lorraine perceived that the interaction between the students and herself was imbalanced. Although Lorraine, from the first lesson, subscribed to rules that supported questioning beyond the answer, she still perceived her interactions as falling short of facilitating sufficient student interaction and discussion. Lorraine perceived again in the second lesson that she fell short of generating the desired student reflection and discussion. It is not clear if Lorraine was aware that the way in which she used the structured tasks (tools) did not promote her vision of appropriate student interaction.

While I did not think that the construct of mathematical proficiency would be a tool for change within this study, it gave the teachers another perspective and a shared language with which to communicate their perceptions. Although the teachers are each on their own continuum towards teaching for understanding and reasoning, their shared perceptions of practice reveal that not only have they used mathematical proficiency as a lens through which to reflect, but they have also identified these principles as aspects of their practice to which to aspire. Both teachers were able to acknowledge what they were not achieving in their practice, and were able to identify opportunities and a vision of what they should be doing. Although the shifts were subtle, and may have been initiated before the teachers’ participation in the study, they were revealed in the teachers’ perceptions through the lens of mathematical proficiency. Thus, a marked shift in practice could be anticipated. Both teachers were able to acknowledge and envision the practices that support the teaching for understanding and reasoning. The themes that emerged from the teachers’ perceptions, the implications of these, and the emerging beliefs and practices are detailed in table 5.2.

Table 5.2: The teachers' perceptions (as themes) → implications → beliefs → practice

Object: Move beyond the constraints of 'knowing' and 'calculating' during mental mathematics lessons: to create opportunities to develop understanding, application and reasoning	
Implication	Students can and should be engaging with mathematics beyond what they know
Emerging belief	Students are able to make connections, make sense of, and engage with mathematics
Emerging practice	Create more opportunities for students to develop understanding (make connections), application and reasoning (using what they do know to solve what they do not, and reflecting on their actions)
Tools: Move beyond the 'knowing' level of mental mathematics task items: to elicit application and reasoning through an increased cognitive demand of the tasks	
Implication	Level and nature of the 'sums' are too low to encourage application and reasoning
Emerging belief	Increasing the cognitive demand of 'sums' will provide opportunities for students to apply what they know
Emerging practice	Use materials that have been designed to support the development of understanding and reasoning (purposefully structured to allow noticing of patterns relationships), and use them in a manner to develop this through 'struggle', and the opportunity to use what they do know to solve what they do not
Division of labour: Move beyond 'answer only' questioning in mental mathematics lessons: to facilitate student discussion and reflection through more deliberate planning	
Implication	Less teacher-led discussion, and more student reflection and engagement in discussion
Emerging belief	Students learn from discussion and reflection
Emerging practice	Create more opportunities for students to discuss and reflect

The features of 'good' teaching (Hiebert and Grouws, 2007; McGatha *et al.*, 2018) that facilitate the development of understanding and reasoning were detailed in chapter 2, and are listed below in relation to the emerging practice described in table 5.2:

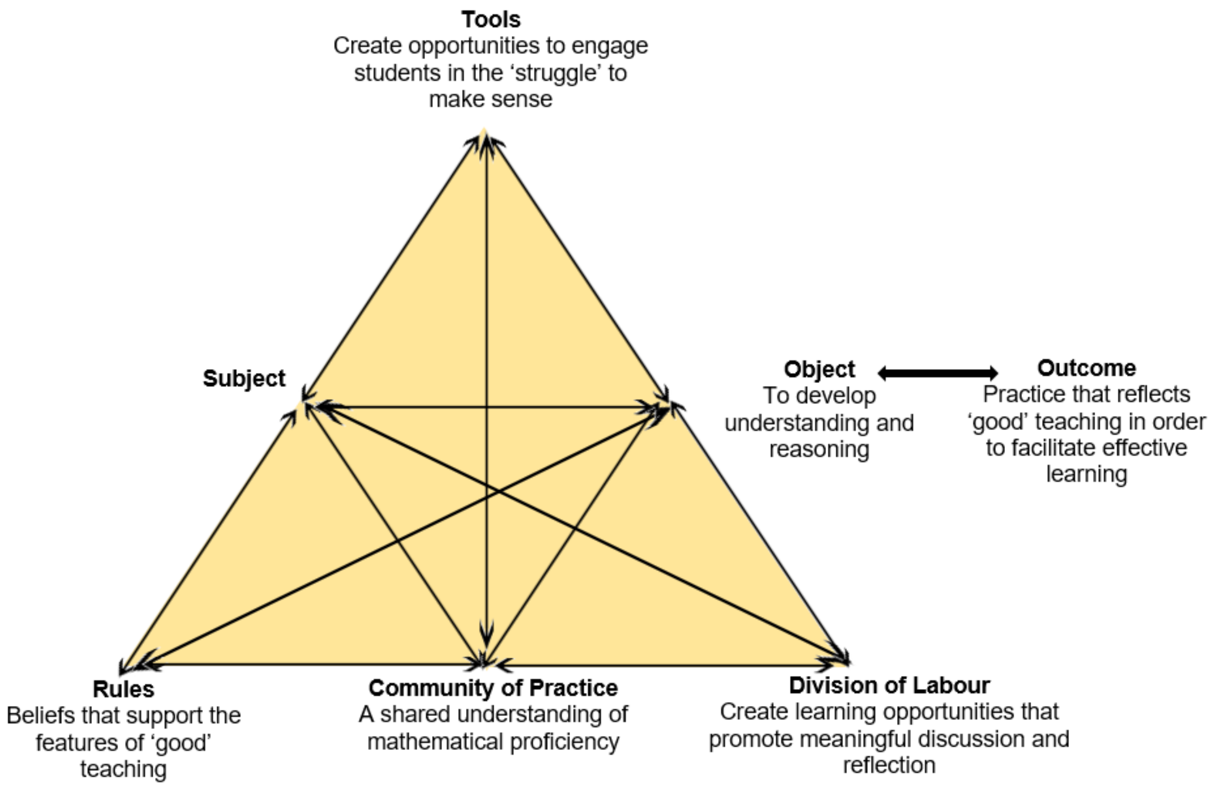
- *Create opportunities to make connections*: create more opportunities for students to develop understanding (make connections), application and reasoning (using what they do know to solve what they do not, and reflecting on their actions).
- *Create opportunities to engage students in the 'struggle' to make sense*: use materials that have been designed to support the development of understanding and reasoning (purposefully structured to allow noticing of patterns relationships),

and use them in a manner to develop this through 'struggle', and the opportunity to use what they do know to solve what they do not.

- *Create opportunities for discussion*: create more opportunities for students to discuss and reflect.

Through the shared language and lens of mathematical proficiency, both teachers are communicating an awareness and an emerging shift in practice that aligns with the research-based features of good teaching. Providing descriptions of alternative practices gave the teachers not only the lens and language to describe their current practice, *what i*s, but also the lens and language to describe the opportunities for alternatives, *what could be*. While the shifts are delicate, the awareness of mathematical proficiency supported Beyoncé and Lorraine in becoming aware of and communicating a practice that moves towards the teaching of understanding and reasoning. Relating these shifts to Activity Theory and to the features of 'good' teaching, we see the base of a new activity system forming for both of these teachers, as illustrated in figure 5.2. Yet, the neat triangles of Activity Theory's structural representation may hide the complexity of the relationships as they play out in an activity system (Bakhurst, 2009).

Figure 5.2: The formation of a new activity system based on the teachers' perceptions of practice of 'what could be'



5.3 Future Considerations: Teachers as learners

Teachers are at the heart of teaching and learning. As the quote below states, any change in education is dependent on what teachers do (practice) and think (believe).

> *"Educational change depends on what teachers do and think – it's as simple and as complex as that."* (Fullan, 2007, p. 129)

Identifying aspects of practice that need shifting is one matter, but initiating, supporting and effecting this shift is another. In order to support teachers' efforts to sustain a shift in practice, learn and grow, it may be useful to view teachers as learners themselves and refer to the literature on how adults learn. Viewing teachers as adult learners may shift the focus from 'change' to 'learn'. Transformative learning, defined by Mezirow as *"a deep shift in frame of reference"* (Mezirow, 2000, p. 104) may offer a lens through which to view teachers as adult learners. Mezirow's transformative learning theory presents a process of transformation which leads the adult learner through a progression starting with an experience of tension, or conflict, and ending in reflection that results in the transformation of the initial perspective (Calleja, 2014). This process is illustrated in figure 5.3.

Figure 5.3: The transformative learning process (adapted from Jarvis, 2008)

Mezirow's work on transformative learning acknowledges that existing frames of reference or perspectives exist. These frames of reference are the perspectives that are used to make sense of experiences. Transformative learning occurs when existing frames of reference are challenged and restructured by conflicts and tensions within experiences. The restructuring (transformation) of existing interpretations occurs through reflection and dialogue, and new meanings are formed (Jarvis, 2008). Just as discussion and reflection are important aspects of mathematical learning for students, so too is reflection crucial in adult learning. This transformation, or learning, occurs in four different ways:

- Existing frames of reference are elaborated on.
- New frames of reference are established.
- Points of view (beliefs) are transformed.
- Habits are transformed (practice).

Future research that investigates, plans and reviews teacher support and professional development through the lens of transformative learning may be useful in better understanding the complexities of transformations in teaching practice. The investigating and understanding of adult transformative learning through case studies of teacher practice could explicate the shifts experienced by teachers who allow themselves to learn from their own experiences and reflections on practice within a community of practice. Adult learning goes beyond just the acquisition of new knowledge; it transforms practice and, in turn, transforms the community in which learning takes place.

This study was designed to analyse and describe, and not to initiate or claim change in practice. Yet, what can be seen is that, with the appropriate tools, support and reflection on own practice, change (transformation) is a natural process of teachers' learning. Not only has this study answered the question of, "How do teachers perceive their mental mathematics teaching practice?", the findings have also revealed possible avenues for future research. The findings revealed that the teachers' perceptions of their **existing practice** (*what is*) versus their perceptions of their **desired practice** (*what could be*) may be a foundation for change, and may provide insight into teachers' beliefs in order to support and guide professional development programmes that could bridge the actual (*what is*) and the possible (*what could be*). Future research should ask the question, "What lenses on existing teacher practice enable transformative teacher learning?"

Now, more than ever, there is a call to support teachers to reflect on their practice and investigate transformation that aims at teaching mathematics towards the development of understanding and reasoning. The findings of this study have highlighted tensions between existing practice, *what is,* and desired practice, *what could be.* This conflict, if further explored and supported by opportunities for continued reflection and discussion, may lead to learning outcomes for teachers that result in a sustained shift in practice and true transformation.

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Addenda

Addendum A

Pre-workshop interview questions (data collection)

The points below served as conversation-starters (ice-breakers) in establishing a relaxed and conversational atmosphere in which to conduct the interview:

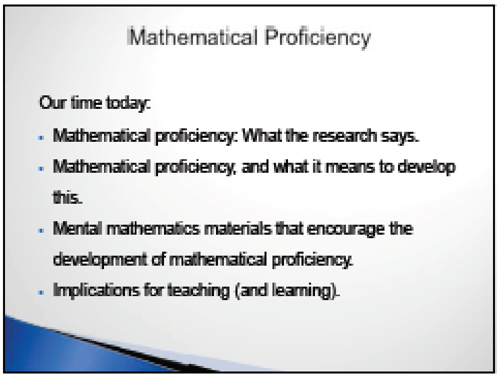
- Please select a pseudonym as you are aware that throughout your participation you will remain anonymous.
- This discussion is to get background information to better plan the workshop and understand you. The discussion and responses that the questions evoke simply need to be your reality. A few guidelines, for me the interviewer, to ensure that the discussion is the priority:
  - The order and delivery of these questions is flexible and aim to generate and guide discussion
  - Questions may also be omitted if felt to be inappropriate or non-productive at the time
  - Additional questions may also arise in order to explore resultant views and experiences further
- While there is reference to mathematics and mathematics teaching in general, constant attention will be drawn to mental mathematics.

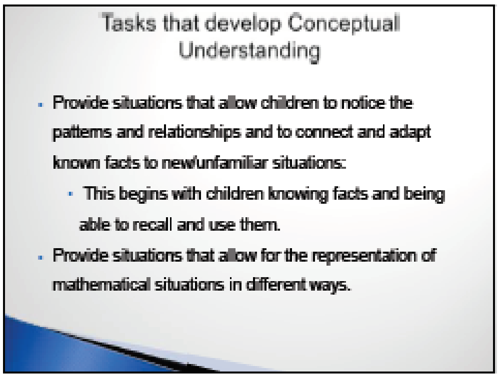
Context for Questions	**Questions and Prompts**
Teacher introduction and context	Think back to your own school experiences … - Your own schooling /education? - How do you remember your primary school years? - How do you remember being taught mathematics (mental mathematics)? - How do you remember coming to know your number facts … bonds and tables? Give me a few examples or stories to illustrate your experience …. - How much input on teaching mathematics were you exposed to during your teacher training? Did you receive any information about number concept development and how children learn mathematics? - Think back to your first few years of teaching – with specific reference to mathematics (mental mathematics) … how is your teaching practice the same? And how is it different?

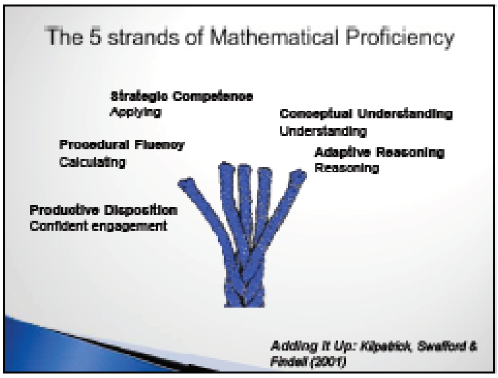
	- What successes and challenges are you presented with in the teaching of mathematics? How (if at all) has this changed over the last few years?
Teaching and learning mathematics	- What are your views on how children learn mathematics, and in particular how they learn mental mathematics? - How does this currently influence how you teach? With reference to … The curriculum Teaching and learning materials Planning and preparation - What aspects of your own schooling, teacher training and teaching and life experiences have been the most valuable in preparing you to teach mathematics? - How would you explain your own mathematical understanding? And your own mathematical thinking? - What do you view as the role of mental mathematics?
Classroom management/routines	- What is your current classroom routine when teaching mathematics (mental mathematics)? - How do you currently plan for your mathematics lessons/mental mathematics routines? - For your mental mathematics lesson, how do you record this planning? - What is the outcome of your planning/teaching? What you do want the children to learn?
Flexibility/responsiveness	- How flexible/adaptive is your planning? By this I mean how do you adapt to meet the levels in your classroom, and go “off script” in order to respond to in the moment needs and levels? - How flexible/adaptive is your teaching? In particular, how do you: Respond to in the moment errors/misconceptions Scaffold a task to make it more accessible Extend a task to make it more complex - What strategies do you use to calculate mentally?
Measure progress	- How do you assess learning in mathematics? - How do you deal with the errors that children make? - Do you incorporate discussion in your mental mathematics lessons and encourage the children to explain their thinking? How?
General	- What would you rate as an ideal environment for teaching mathematics? - What would you need in order to implement this? - What do you think children need in order to be successful in mathematics? - What do you think you need in order to teach mathematics successfully? - What is your expectation for the workshop?
Closure	- Is there anything you would like to add?

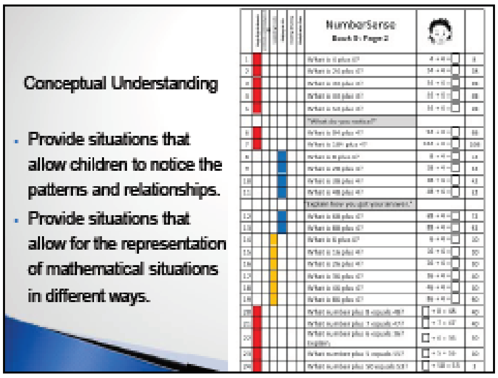
Addendum B

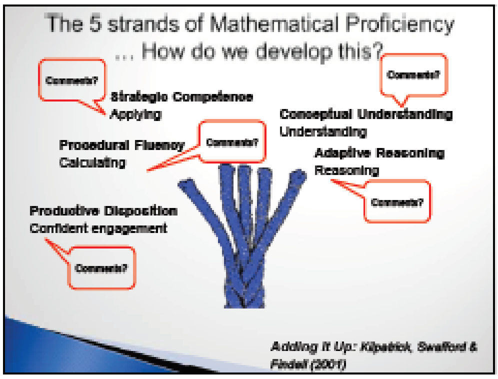
An example of the workshop material

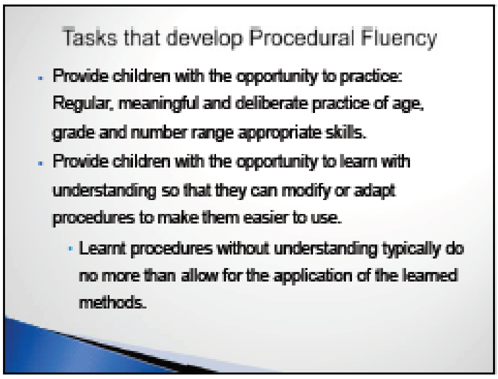


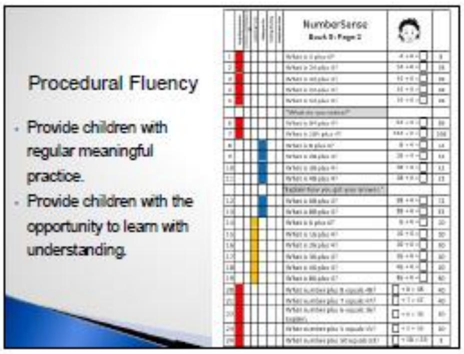


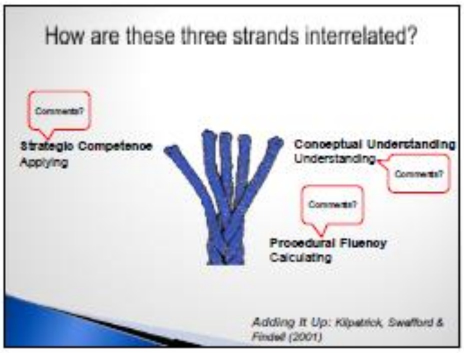


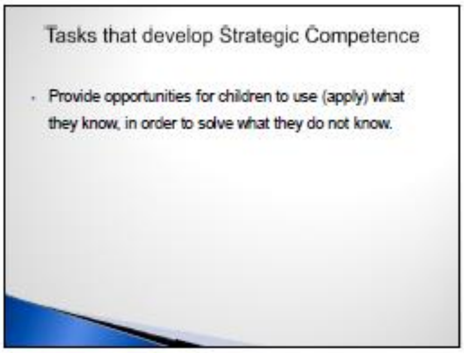


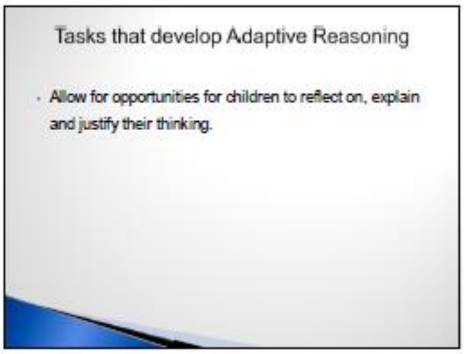




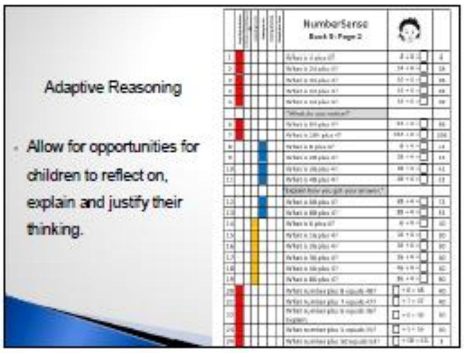

Procedural Fluency
Provide children with regular meaningful practice.
Provide children with the opportunity to learn with understanding.
NumberSense

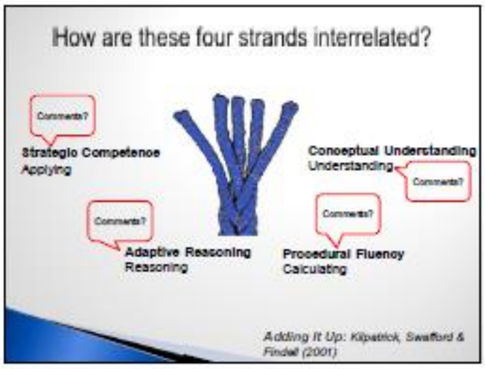

How are these three strands interrelated?
Comments?
Strategic Competence
Applying
Conceptual Understanding
Understanding
Comments?
Comments?
Procedural Fluency
Calculating
Adding It Up: Kilpatrick, Swafford & Findell (2001)

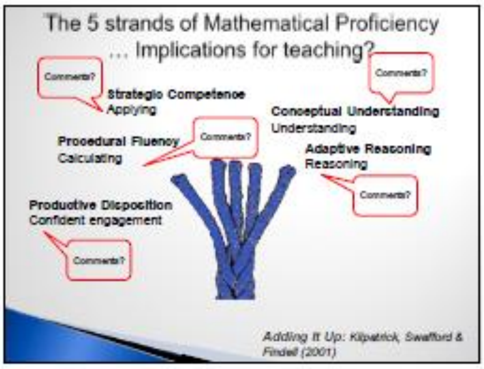

Tasks that develop Strategic Competence
Provide opportunities for children to use (apply) what they know, in order to solve what they do not know.

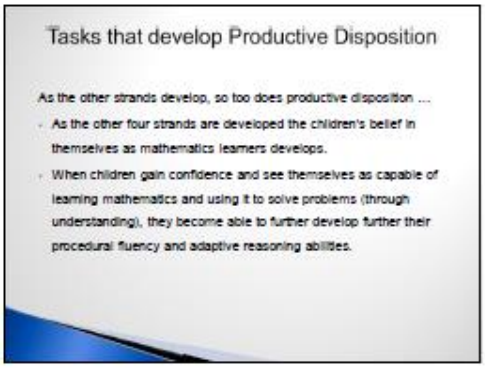

Tasks that develop Adaptive Reasoning
Allow for opportunities for children to reflect on, explain and justify their thinking.

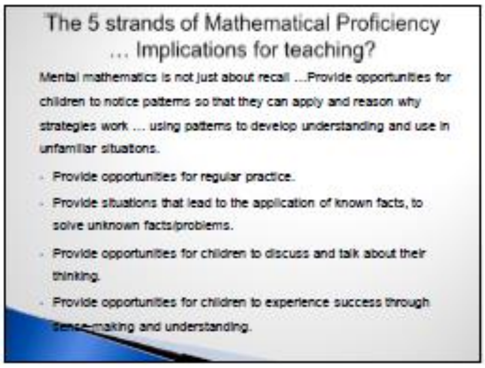

Strategic Competence
Provide opportunities for children to use what they know, in order to solve what they do not know.
NumberSense

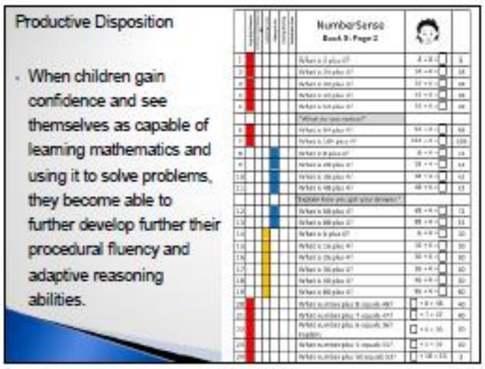

Adaptive Reasoning
Allow for opportunities for children to reflect on, explain and justify their thinking.
NumberSense


How are these four strands interrelated?
Comments?
Strategic Competence
Applying
Conceptual Understanding
Understanding
Comments?
Comments?
Comments?
Adaptive Reasoning
Reasoning
Procedural Fluency
Calculating
Adding It Up: Kilpatrick, Swafford & Findell (2001)


The 5 strands of Mathematical Proficiency
… Implications for teaching?
Comments?
Strategic Competence
Applying
Comments?
Conceptual Understanding
Understanding
Procedural Fluency
Calculating
Comments?
Adaptive Reasoning
Reasoning
Comments?
Productive Disposition
Confident engagement
Comments?
Adding It Up: Kilpatrick, Swafford & Findell (2001)


Tasks that develop Productive Disposition
As the other strands develop, so too does productive disposition …
As the other four strands are developed the children's belief in themselves as mathematics learners develops.
When children gain confidence and see themselves as capable of learning mathematics and using it to solve problems (through understanding), they become able to further develop further their procedural fluency and adaptive reasoning abilities.


The 5 strands of Mathematical Proficiency
… Implications for teaching?
Mental mathematics is not just about recall …Provide opportunities for children to notice patterns so that they can apply and reason why strategies work … using patterns to develop understanding and use in unfamiliar situations.
Provide opportunities for regular practice.
Provide situations that lead to the application of known facts, to solve unknown facts/problems.
Provide opportunities for children to discuss and talk about their thinking.
Provide opportunities for children to experience success through sense-making and understanding.


Productive Disposition
When children gain confidence and see themselves as capable of learning mathematics and using it to solve problems, they become able to further develop further their procedural fluency and adaptive reasoning abilities.
NumberSense


The next step …
Your role …
Teacher as co-researcher and implementation of mental mathematics activities as workshopped.
Teacher journal notes and peer discussions.
Interview, observation, observation reflection.
Time-line …
Implementation of mental mathematics activities: April/May
Teacher journal notes and peer discussions: April/May
Interview, observation: June (end of term 2)
Observation reflection: July (start of term 3)

Addendum C

1. Ethical clearance: Stellenbosch University

UNIVERSITEIT
STELLENBOSCH
UNIVERSITY

NOTICE OF APPROVAL

REC Humanities New Application Form

14 March 2019

Project number: 8514

Project Title: Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency

Dear Ms Melanie Gow

Your response to stipulations submitted on 11 December 2018 was reviewed and approved by the REC: Humanities.

Please note the following for your approved submission:

Ethics approval period:

Protocol approval date (Humanities)	Protocol expiration date (Humanities)
6 December 2018	5 December 2021

GENERAL COMMENTS:

Please take note of the General Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

If the researcher deviates in any way from the proposal approved by the REC: Humanities, the researcher must notify the REC of these changes.

Please use your SU project number (8514) on any documents or correspondence with the REC concerning your project.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

FOR CONTINUATION OF PROJECTS AFTER REC APPROVAL PERIOD

Please note that a progress report should be submitted to the Research Ethics Committee: Humanities before the approval period has expired if a continuation of ethics approval is required. The Committee will then consider the continuation of the project for a further year (if necessary)

Included Documents:

Document Type	File Name	Date	Version
Informed Consent Form	Section 5_Consent letters x3_MelGow	28/10/2018	Final
Information sheet	Section 5_Information Sheet_MelGow	28/10/2018	Final
Data collection tool	Section 7_Data collection guidelines_MelGow	28/10/2018	Final
Data collection tool	Section 7_Data collection guidelines_MelGow	28/10/2018	Final
Proof of permission	Section 8_ Research approval letter_WCED	28/10/2018	Final
Default	Section 9_Materials_Permission_MelGow	28/10/2018	Final
Research Protocol/Proposal	Section 2_MelGow Proposal_21510636_ETHICS_Oct2018	28/10/2018	Final
Default	Response to REC Stipulations_MelGow21510636	10/12/2018	Final
Proof of permission	Proof of permission from participating school	11/12/2018	Final

If you have any questions or need further help, please contact the REC office at cgraham@sun.ac.za.

Sincerely,

Clarissa Graham

REC Coordinator: Research Ethics Committee: Human Research (Humanities)

National Health Research Ethics Committee (NHREC) registration number: REC-050411-032.
The Research Ethics Committee: Humanities complies with the SA National Health Act No.61 2003 as it pertains to health research. In addition, this committee abides by the ethical norms and principles for research established by the Declaration of Helsinki (2013) and the Department of Health Guidelines for Ethical Research: Principles Structures and Processes (2nd Ed.) 2015. Annually a number of projects may be selected randomly for an external audit.

2. Ethical clearance: Western Cape Education Department

Western Cape Government
Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za
tel: +27 021 467 9272
Fax: 0865902282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20181024–7821
ENQUIRIES: Dr A T Wyngaard

Ms Melanie Gow
149 Pluto Road
Plumstead
7800

Dear Ms Melanie Gow

RESEARCH PROPOSAL: FOUNDATION PHASE TEACHERS' LIVED EXPERIENCES OF TEACHING MENTAL MATHEMATICS IN A MANNER THAT SUPPORTS MATHEMATICAL PROFICIENCY

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **04 March 2019 till 27 September 2019**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

    The Director: Research Services
    Western Cape Education Department
    Private Bag X9114
    CAPE TOWN
    8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 24 October 2018

Lower Parliament Street, Cape Town, 8001
tel: +27 21 467 9272 fax: 0865902282
Safe Schools: 0800 45 46 47

Private Bag X9114, Cape Town, 8000
Employment and salary enquiries: 0861 92 33 22
www.westerncape.gov.za

3. Ethical clearance: Brombacher and Associates (NumberSense)

Brombacher and Associates cc

Unit E23, Prime Park
Mocke Rd, Diep River, 7800

T. +27 (0)21 706 3777
F. +27 (0)21 706 3779
E. info@brombacher.co.za
W. www.brombacher.co.za

Reg No.: 2003/026360/23
VAT No.: 4660208978

Member: A A Brombacher

GeoGenius
www.GeoGenius.co.za

NumberSense
www.NumberSense.co.za

To whom it may concern,

I hereby give permission to **Melanie Gow (ID 721123 0013 084)** to use the *NumberSense Mathematics Programme* materials and resources in her Masters research programme entitled: ***Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency***.

Brombacher and Associates is the sole developer and owner of the copyright for the *NumberSense Mathematics Programme* materials and resources.

Yours sincerely

Aarnout Brombacher
15 October 2018

4. Ethical clearance: Principal consent letter

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jou kennisvennoot • your knowledge partner

STELLENBOSCH UNIVERSITY

CONSENT TO PARTICIPATE IN RESEARCH

Date

Dear

As per our meeting discussion, I would like to invite the Foundation Phase teachers to participate in a research project entitled *Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency.*

We share the concern that many children do not seem to know their bonds and tables, and that this has a detrimental effect on their overall mathematical progression. Mental mathematics is crucial in providing the platform from which children come to know their number facts. We also experience the challenge of teaching mental mathematics in a way that children can memorise their number facts and use them in a range of situations. The basis of my study is to investigate Foundation Phase teachers' experiences of teaching mathematics in a way that aims to support children to know their number facts and be able to use these facts fluently, flexibly and in a range of different situations.

The teachers' participation is **entirely voluntary** and they are free to decline to participate. Please note that there will be no payment for participation in this research. If they say no, this will not affect them negatively in any way whatsoever. They are also free to withdraw from the study at any point, even if they do agree to take part.

What will the teachers be expected to do as part of their participation?

- Participate in a pre- and post-interview session with me.
- Participate in a mental mathematics lesson observation, pre- and post-workshop. I will video record these observations and we will meet to view and reflect on this afterwards.
- Participate in a workshop session.
- Keep a teacher journal of their own experiences and observations during the workshop and implementation.

When will this participation take place?

- Interviews: At a time, convenient to the participating teachers (approximately 30 minutes per interview).
- Workshop: At a time, convenient to the participating teachers (approximately 2 hours).
- Observations: At a time, convenient to the participating teachers (observation approximately 15 minutes and reflection approximately 30 minutes).
- Teacher journal: At the participating teachers' discretion (on going).

What will this look like in terms of a timeline?

- Week 1: Interview, observation and reflection meeting.
- Week 2: Workshop.
- Week 3-8: Implementation and teacher journal (the duration here to be decided on during the workshop session).
- Week 9: Interview, observation and reflection meeting.

How will this benefit the participating teachers?

- The workshop session and materials may be a professional development opportunity.

If you have any questions or concerns about the research, please feel free to contact me or my supervisor, Dr Erna Lampen: melanie@brombacher.co.za /083 229 2267 and ernalampen@sun.ac.za / 021 808 2292

If you are willing to have the Foundation Phase teachers participate in this study, please sign the attached Declaration of Consent and I will collect it at a time convenient to you.

Regards

Melanie Gow

RIGHTS OF RESEARCH PARTICIPANTS: You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché (mfouche@sun.ac.za; 021 808 4622) at the Division for Research Development.

DECLARATION BY PRINCIPAL

By signing below, I …………………………………….…………… agree that the Foundation Phase teachers may take part in a research study entitled *Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency* and conducted by Melanie Gow.

I declare that:

- I am aware that study and participation will run from March 2019 to September 2019.
- I attended the information meeting and have read and understand the detail in the information sheet and it is written in a language with which I am fluent and comfortable.
- I have had a chance to ask questions and all my questions have been adequately answered.
- I understand that taking part in this study is **voluntary** and the teachers will not be pressurised to take part.

- The teachers may choose to leave the study at any time and will not be penalised or prejudiced in any way.
- I understand that my name, the participating teachers' names and school name will not be used in any reporting and my identity and that of the school will remain anonymous. All issues related to privacy and the confidentiality and use of the information provided have been explained to my satisfaction.
- I agree that the data gathered in this study may be used to publish in academic journals and conferences.

Signature of principal **Date**

SIGNATURE OF RESEARCHER

I declare that I explained the information given in this document to <<>>. She was encouraged and given ample time to ask me any questions. This conversation was conducted in English and no translator was necessary.

Signature of researcher **Date**

5. Ethical clearance: Teacher consent letter

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jou kennisvennoot • your knowledge partner

STELLENBOSCH UNIVERSITY

CONSENT TO PARTICIPATE IN RESEARCH

Date

Dear Foundation Phase teachers

As per the meeting discussion, and detail on the information sheet, I would like to invite you to participate in a research project entitled *Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency.*

We share the concern that many children do not seem to know their bonds and tables, and that this has a detrimental effect on their overall mathematical progression. Mental mathematics is crucial in providing the platform from which children come to know their number facts. We also experience the challenge of teaching mental mathematics in a way that children can memorise their number facts and use these in a range of situations. The basis of my study is to investigate your experiences as teachers, of the teaching of mathematics in a way that aims to support children to know their number facts and be able to use these facts flexibly and in a range of different situations.

Your participation is **entirely voluntary** and you are free to decline to participate. Please note that there will be no payment for participation in this research. If you say no, this will not affect you negatively in any way whatsoever. You are also free to withdraw from the study at any point, even if you do agree to take part.

What will I be expected to do as part of the participation?

- Participate in a pre- and post-interview session with Melanie.
- Participate in a mental mathematics lesson observation, pre- and post-workshop. Melanie will video record these observations and then meet with you to view and reflect on this afterwards.
- Participate in a workshop session.
- Keep a teacher journal of your own experiences and observations during the workshop and implementation.

When will this participation take place?

- Interviews: At a time, convenient to you (approximately 30 minutes per interview).
- Workshop: At a time, convenient to you and the other participating teachers (approximately 2 hours).
- Observations: At a time, convenient to you (observation approximately 15 minutes and reflection approximately 30 minutes).
- Teacher journal: At your discretion (on going).

What will this look like in terms of a timeline?

- Week 1: Interview, observation and reflection meeting.
- Week 3: Workshop.
- Week 4-9: Implementation and teacher journal (the duration here to be decide on during the workshop session).
- Week 10: Interview, observation and reflection meeting.

How will this benefit me?

- The workshop session and materials may be a professional development opportunity.

If you have any questions or concerns about the research, please feel free to contact me or my supervisor, Dr Erna Lampen: melanie@brombacher.co.za /083 229 2267 and ernalampen@sun.ac.za / 021 808 2292.

If you are willing to participate in this study, please sign the attached Declaration of Consent and I will collect it at a time convenient to you.

Regards

Melanie Gow

RIGHTS OF RESEARCH PARTICIPANTS: You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché (mfouche@sun.ac.za; 021 808 4622) at the Division for Research Development.

DECLARATION BY PARTICIPANT

By signing below, I …………………………………..……………. agree to take part in a research study entitled *Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency* and conducted by Melanie Gow.

I declare that:

- I am aware that the study and my participation will run from March 2019 to September 2019.
- I attended the information meeting and have read and understand the detail in the information sheet and it is written in a language with which I am fluent and comfortable.
- I have had a chance to ask questions and all my questions have been adequately answered.
- I understand that taking part in this study is **voluntary** and I have not been pressurised to take part.

- I may choose to leave the study at any time and will not be penalised or prejudiced in any way.
- I understand that my name and school name will not be used in any reporting and my identity and that of the school will remain anonymous. All issues related to privacy and the confidentiality and use of the information provided have been explained to my satisfaction.
- I agree to participate in the pre- and post-interview sessions.
- I agree to be videoed as part of observation on mental mathematics activities with my class and I will attend a reflection meeting with Melanie.
- I agree to keep a teacher journal and that the information within this will be used as data.
- I agree that the data gathered in this study may be used to publish in academic journals and conferences.

____ _______________

Signature of participant **Date**

SIGNATURE OF RESEARCHER

I declare that I explained the information given in this document to ____________________ [*name of the participant] [H*e/she] was encouraged and given ample time to ask me any questions. This conversation was conducted in English and no translator was necessary.

____ _______________

Signature of researcher **Date**

6. Ethical clearance: Parent consent letter

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STELLENBOSCH UNIVERSITY

PARENT CONSENT FOR CHILD TO PARTICIPATE IN RESEARCH

Date

Dear Parents

My name is Melanie and I would like to invite your child's teacher to participate in a research project entitled *Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency*.

Please take some time to read the information presented here, which will explain the details of this project and contact me if you require further explanation or clarification of any aspect of the study. Also, your child's participation is **entirely voluntary** and you are free to decline to participate. If you say no, this will not affect you negatively in any way whatsoever. You are also free to withdraw your child from the study at any point, even if you do agree to take part.

We share the concern that many children do not seem to know their bonds and tables (number facts), and that this has a detrimental effect on their overall mathematical progression. Mental mathematics is crucial in providing the platform from which children come to know their number facts. We also experience the challenge of teaching mental mathematics in a way that children can memorise their number facts and use them in a range of situations. The basis of my study is to investigate the experiences of your child's teacher of teaching mental mathematics in a way that aims to support children to know their number facts and be able to use these facts flexibly and in a range of different situations.

Part of the study will involve a mental mathematics lesson observation which will involve a video recording of your child's teacher teaching. This will happen at the start of the study and again at the end of the study. These video recordings will be used for reflection discussions with myself and teacher. The video recording is necessary to engage in discussion with the teachers to better clarify their experiences. My focus is the teacher and I will be recording the lesson from the back of the classroom so that the children's faces are not in focus. The video recording will not be used directly in the study, and the teachers' experiences thereof, and the experiences of their teaching that they observe through the video material will be used. The school and teacher will not be named in the study and their identities will remain anonymous. The video footage will be stored in a cloud space with password access.

If you are willing for your child to participate in the research (with the focus of the research being your child's teacher), please sign the attached Declaration of Consent and return it to your child's class teacher.

Regards

Melanie Gow

DECLARATION BY PARENT

By signing below, I ……………………………………………… agree to the participation of my child ……………………………………………………. (with the focus of the research being my child's teacher), in the research study entitled *Foundation Phase teachers' lived experiences of teaching mental mathematics in a manner that supports mathematical proficiency* conducted by Melanie Gow.

I declare that:

- I have read the information within this letter and it is written in a language with which I am fluent and comfortable.
- I have had a chance to ask questions and all my questions have been adequately answered.
- I understand that my child's participation in this study is voluntary and he/she has not been pressurised to take part.
- I may choose to remove my child from the study at any time and will not be penalised or prejudiced in any way.
- All issues related to privacy and confidentiality and the use of the information provided have been explained to my satisfaction.
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____ _______________

Signature of parent **Date**

SIGNATURE OF RESEARCHER

I declare that the information in this document was given to the parents of the children in the classes of those teachers who are participating in this study. They were encouraged and given ample time to ask any questions. This information was delivered in English and no translator was used.

____ _______________

Signature of researcher Date