High school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra*

By

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DECLARATION

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ABSTRACT

Mathematics is a compulsory subject at all levels of pre-tertiary Namibian education and mathematical functions are key concepts of the mathematics curriculum. Personal experience has shown that teaching and understanding functions is a challenge in the Namibian high school curriculum. There are several difficulties in learning algebra due to misconceptions and errors such as misunderstanding the meaning of numerical and literal symbols, the shift from numerical data or language representation to variables or parameters with functional rules or patterns, and their cognition. Hence, there is a need to integrate the use of information and communication technologies (ICTs) in order to improve the understanding and teaching of mathematical functions. However, successful integration depends on providing teachers with learning opportunities in using ICTs. The focus of this research was to investigate a selected number of high school mathematics teachers' learning experiences during a professional development intervention aimed at improving the understanding of functions using *GeoGebra*. This study provides answers to the following questions: (1) What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using GeoGebra? (2) What does research have to say about the potential of *GeoGebra* in aiding the understanding functions? The study was conducted in the Ohangwena region in Namibia and grounded on the interpretive paradigm. The sample population consisted of ten high school mathematics teachers. Sampling of the participants was guided by convenience sampling procedures. Five workshops of 2-3 hours were organised with the selected teachers. During these workshops, guidance and time were given to the teachers to explore different activities related to multiple representations of mathematical functions. The teachers were interviewed while they interacted with a set of *GeoGebra* activities. A group discussion was held to explore and develop an understanding of the concept of functions, the nature of *GeoGebra* and its possible pedagogical affordances. Multiple methods were used to collect data, namely semi-structured interviews; focus group interviews; audiotaped discussions; observations; and field notes. Based on a qualitative analysis of the data generated, the findings indicated that teachers benefited significantly from the use of *GeoGebra* as mathematical digital software in various ways, ranging from personal mathematics exploration, attitudes toward mathematics and mathematics teaching of functions to pedagogical reflections, including the nature of mathematics and teachers interactions. These changes are well aligned with the emphases of the ongoing mathematics education reforms in

Namibia, including the integration of technology into education. The research findings also revealed that in its design *GeoGebra* affords fast and consistent feedback and that teachers need more opportunities where they learn to experience relations between the pragmatic and epistemic dimensions of *GeoGebra* use, when it comes to linear and quadratic functions, for example.

OPSOMMING

Wiskunde is ‘n verpligte vak in al die fases van voortersiêre onderwys in Namibië en wiskundige funksies is van die kernkonsepte in die wiskundekurrikulum. Persoonlike ervaring wys daarop dat die onderrig en verstaan van funksies ‘n uitdaging in die Namibiese hoërskoolkurrikulum blyk te wees. Daar is verskeie struikelblokke in die leer van algebra. Dit kan te wyte wees aan wanopvattings en foute soos die misverstaan van numeriese en lettersimbole, die skuif van numeriese data of taalverteenwoordiging na veranderlikes of grense met funksionele reëls of patrone en die herkenning daarvan. Vandaar die behoefte om die gebruik van inligting- en kommunikasietegnologieë (IKTs) te integreer ten einde die verstaan en die onderrig van wiskundige funksies te verbeter. Die sukses van hierdie insluiting hang egter af van die mate waartoe ondewysers toegang tot leergeleenthede in die gebruik van IKTs gegun word. Met hierdie navorsing is daar gefokus op die ondersoek na ‘n gekose aantal hoërskool-wiskundeonderwysers se leerervarings tydens ‘n professionele ontwikkelingsintervensie, wat daarop gemik was om die verstaan van wiskundige funksies te bevorder deur die gebruik van *GeoGebra.* Met hierdie studie is daar gepoog om antwoorde op die volgende vrae te vind: (1) Wat is hoërskool-wiskundeonderwysers se leerervarings gedurende die bywoning van ‘n professionele ontwikkelingsintervensie wat daarop gemik is om die verstaan van wiskundige funksies te bevorder deur die gebruik van *GeoGebra*? (2) Wat is die bevindings van navorsing oor die moontlikhede van *GeoGebra* in die ontwikkeling van ‘n beter begrip van funksies? Die ondersoek is onderneem in die Ohangwena-distrik in Namibië en dit is gegrond op die interpretatiewe paradigma. Die deelnemers vir die steekproef bestaan uit tien hoërskool-wiskundeonderwysers. Die keuse van deelnemers is gelei deur doelbewuste steekproefprosedures. Vyf werkswinkels van 2 tot 3 ure elk is vir die gekose onderwysers gereël. Gedurende hierdie werkswinkels is die onderwysers begelei en daar is tyd gegee om verskillende aktiwiteite met betrekking tot die veelvuldige voorstellings van wiskundige funksies te ondersoek. Onderhoude is met die onderwysers gevoer, terwyl hulle besig was met ‘n stel *GeoGebra*-aktiwiteite. ‘n Groepsbespreking het plaasgevind oor die begrip *wiskundige funksies* om die verstaan daarvan te ontwikkel. Die aard van *GeoGebra* is ook bespreek en die moontlikhede daarvan as ‘n pedagogiese hulpmiddel is ondersoek. ‘n Verskeidenheid metodes is aangewend om data in te win, soos die voer van deelsgestruktureerde onderhoude, fokusgroeponderhoude, die maak van oudio-opnames van

gesprekke, deur waarneming en met die byhou van veldnotas. Gebaseer op die kwalitatiewe ontleding van die gegenereerde data is daar bevind dat onderwysers beduidend kan baat vind by die gebruik van *GeoGebra* as 'n wiskundige, digitale grensobjek (WDGO). Dit kan op verskeie wyses aangewend word, soos byvoorbeeld tydens persoonlike ondersoeke na wiskunde, in die aanspreek van die houding jeens wiskunde, by die onderrig van wiskundige funksies of tydens pedagogiese refleksies oor die aard van wiskunde, asook tydens onderwyserinteraksies. Hierdie veranderinge klop met die volgehoue hervorming van wiskundeonderwys in Namibië wat ook die integrasie van tegnologie in opvoeding insluit. Die navorsingsbevindige bring voorts aan die lig dat *GeoGebra,* as 'n WDGO, vinnige en deurlopende terugvoer toelaat en dat onderwysers meer geleenthede behoort te kry waartydens hulle die verhouding tussen die pragmatiese en die epistemiese dimensies van *GeoGebra*-gebruik kan ervaar, veral wanneer dit kom by liniêre en drievoudige funksies.

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List of Acronyms

ABBREVIATION	DEFINITION
BECTA	British Educational Communication and Technology Agency
CAS	Computer Algebra Systems
CECS	Computer Education Community Service
DGE	Dynamic Geometry Environment
DGS	Dynamic Geometry Software
DMS	Dynamic Mathematics Software
DNEA	National Examinations and Assessment
ETSIP	Education and Training Sector and Improvement Programme
GeSCI	Global e-School Initiative
ICTs	Information and Communication Technologies
IGCSE	Cambridge International General Certificate of Secondary Education
IMTE	Integrated Methods of Technology Education
MBESC	Ministry of Basic Education, Sport and Culture
MEC	Ministry of Education and Culture
MICTSC	Ministry's ICT Steering Committee
MoE	Namibian Ministry of Education
NCTM	National Council of Teachers of Mathematic
NETA	Namibia Education Training Academy
NIED	National Curriculum for Basic Education
NPC	National Planning Commission
NSSCE	Namibian Senior Secondary Certificate Examination
NSSCO	Namibia Senior Secondary Certificate Ordinary
PD	Professional Development
TIs	Time Issues
TLEs	Teachers Learning Experiences
TPD	Teacher Professional Development

CHAPTER ONE

1.1 Introduction

This study investigated a selected number of high school mathematics teachers' learning experiences during a professional development intervention aimed at improving the understanding of linear and quadratic functions using *GeoGebra.* Additionally, it provides the background to, and explains the motivation for, the study and its methods; key terms used throughout the study are also explained. The chapter further presents the problem statement, purpose and justification for using *GeoGebra*, the significance of the study, the assumptions underlying the research questions as well as an outline of the thesis.

In the new learner-centred environment, the explosion of the information age and the prevailing high-technology environment, Namibian teachers and teachers in other developing countries need to be technologically prepared to deal with this challenge or face being marginalised in the information age (Rathedi, 2000 cited in Ipinge, 2010).

Preiner (2008:16) in his doctoral dissertation acknowledges that the professional development (PD) in the effective use of technology is needed to support secondary school mathematics teachers by teaching them not only the use of new software tools but also introducing them to new methods on how to successfully integrate this technology into their teaching and learning practices. Furthermore, teachers need to be prepared for the increasing complexity of their teaching environment, which creates more challenges for both teachers and learners than were found in 'traditional' classroom settings (Preiner, 2008). This new environment is particularly important since teachers are playing a significant role in a technology-supported mathematics classroom.

1.2 Motivation for the study

Personal experience has shown me that teaching and understanding functions is a challenge for learners in the Namibian high school curriculum. There are several difficulties in learning algebra due to misconceptions and errors such as misunderstanding the meaning of numerical and literal symbols, the shift from numerical data or language representation to variables or parameters with functional rules or patterns, and their cognition. Some of these difficulties are often caused by an approach that focuses on the calculation processes rather than on relational or structural aspects (Sajika, 2003; Sierpinska, 1992). Therefore, integrating teaching functions with information and communication technologies (ICTs) may be a solution to improve teachers' understanding of the concept of function. However, successful integration is limited by the teachers' lack of skills in ICT. Apart from this, appropriate PD is critical to assist mathematics teachers not only in the use of new software tools, but also to introduce different ways by which they could successfully utilise this technology in their teaching practices (Bower & Falkner, 2015; Hohenwarter, Hohenwarter & Lavicza, 2010).

Furthermore, teachers have to be aware of the complexity associated with ICTs compared to their 'traditional' or pencil-paper classroom practices and be equipped to deal with such complexities. Similarly, there is general agreement in the educational research community about the importance of mathematics teachers' professional development (hereafter referred to as TPD) to improve teaching and learning using ICTs. Hence, TPD can be explained as a learning process that is undertaken by mathematics teachers as well as the teacher educators, after their initial training in order to enhance their work.

Against this background, there are several implications for PD in the teaching and learning of concept of functions in this study. Also, mathematics teachers who are introduced to technology tasks as adults, even though they have not had any experience, are responsive to learning and tend engage quickly and easily. Therefore, the potential for using *GeoGebra* as an instance of ICT to teach and learn mathematics in Namibian high schools could be very beneficial, but there is still a way to go.

1.3 Problem statement

Teachers in the current study have few, if any, opportunities to investigate and to work with multiple representations of functions because of the ways they usually teach. With the affordances of *GeoGebra,* albeit in the case of PD opportunities, there is the possibility and opportunity for studying the teachers' experiences with functions as represented/inscribed in the design of *GeoGebra*. This can be justified by the fact that mathematics is a compulsory subject at all levels of pre-tertiary Namibian education and functions are a vital concept of the mathematics curriculum. Personal experience has shown that teaching and understanding a function can be a challenge for teachers and learners. The traditional way of teaching some mathematical concepts such as functions is not effective for learners to construct and understand the meaning of functions and their relevant applications.

Bautista, Cañadas, Brizuela and Schliemann (2015) investigated how 56 high school mathematics teachers used graphs in their classroom. This study is particularly important because their findings show that, although many studies have explored learners' difficulties with graphs, only a few have focused on how teachers use graphs in their classrooms. Many previous studies on graphs have focused mainly on the difficulties learners encounter when interpreting and producing function graphs (Friel, Curcio & Bright, 2001). This current study, however, was not conducted in the teachers' classrooms, because the participating teachers had not encountered or interacted with *GeoGebra* in a sustained way as they did in the five sessions that the researcher had with them (see Chapter 4 on Methodology).

Learners and teachers often struggle with functions because they may not be aware of different or multiple representations (Ainsworth, 2006). Such difficulties might be partially due to learners' and teachers' limited experience with multiple (tabular, symbolic and graphical) representations of functions, which are introduced rather late in the curriculum; see Table 1 Namibian Curriculum Syllabus Statement Concerning Function.
Furthermore, when the graphs of function finally appear in the curriculum, the emphasis is placed on learning how to plot points in the Cartesian space and on interpreting what the points represent (Bautista, *et al.,* 2015)which is also similar with the Namibian intended mathematics curriculum, i.e. guidelines provided in the official policy documents.

Table 1.1 (Namibian Curriculum Syllabus Statement on function and graphs of function)

Topic	**General objectives**	**Specific objectives**
Functions	Understand the function idea and use function notation	1. Use function notation, e.g. f(x) = 3x - 5; f : x a 3x - 5 to describe simple functions, and the notation f –1(x) to describe their inverses. 2. Form composite functions as defined by gf(x) = g(f (x))
Graphs of functions	Construct tables of values for functions,	1) Construct tables of values for functions of the form

	Draw and interpret graphs and solve equations graphically	y = ax + b, y = ±x 2 + ax + b , y = a x , (x ≠ 0) where a and b are integral constants. 2) Draw and interpret such graphs.3) Find the gradient of a straight line graph and determine the equation of a straight line in the form y = mx + c. 4) Solve linear and quadratic equations approximately by graphical methods. 5) Construct tables of values and draw graphs for functions of the form: y = ax where a is a rational constant and n = –2, –1, 0, 1, 2, 3 and simple sums of not more than three of these and for functions of the form y = a x where a is a positive integer. 6) Estimate gradients of curves by drawing tangents. 7) Solve associated equations approximately by graphical method.

Source: Adapted from NSSCO Mathematics Syllabus, NIED 2009:15-16

Mathematics teachers generally focus their teaching practices on the use of symbolic and literal algebraic representations of functions. This limits the representations of functions to the translation of the algebraic form of a function to its graphical form and vice versa (Ainsworth, 1999; Bayazit, 2011). Usually a function as taught in the secondary schools is identified with just one of its representation, either the symbolic or the graphical. One of the important aspects to the study of functions and graphs is that symbolic and graphical representations are two very different symbol systems that articulate in such a way as to jointly construct and define the mathematical concept of function (Ainsworth, 1999). Neither functions nor graphs can be treated as isolated concepts. They are systems of communication, on the one hand, and a construction and organisation of mathematical ideas, on the other.

It therefore seems essential for teachers to be strategic in the way that they work with multiple representations in class and how they establish relationships among them. Thus, an understanding of the connections between representations and between pieces of knowledge, and the ability to translate between representations, are defining aspects of conceptual knowledge and problem solving (Bayazit, 2011; Hiebert & Carpenter, 1992; Goldin, 1998).

Mathematics educators agree that meaningful learning of mathematics could be achieved when a variety of representations have been developed and the functioning relationships are established amongst them (Goldin, 2002). Hence there is a need to integrate the use of ICTs into the teaching process in order to improve the understanding and teaching of mathematical functions. However, successful integration is limited by the lack of teacher skills in ICT as well as their inability to integrate ICT into their teaching practice because the teachers themselves haven't been exposed to the use of this technology. Thus, the use of *GeoGebr*a might provide new possibilities, opportunities and challenges for teachers to deepen their understanding of functions. This may improve the professional learning and development of teachers and is crucial in transforming the leaning of functions in school mathematics. Hence, it is worth improving the PD and learning that are crucial in transforming schools mathematics as currently taught and increasing academic achievement (Darling-Hammond, Wei, Andree, Richardson & Orphanos, 2009).

1.4 Justification for using *GeoGebra*

Although to date there has been little research conducted on the effective integration of *GeoGebra* into teaching and learning mathematics, there are several reasons for selecting this software as an essential element of the suggested PD for mathematics teachers regarding technology.

GeoGebra was chosen in this study for a number of reasons.

- *GeoGebra* as an ICT tool has several features to support mathematics teaching, apart from being freeware. In the context of a developing country like Namibia, open source software is most important, since it can be downloaded without any cost.

- With appropriate structured activities, the use of *GeoGebra* has the potential to encourage discovery and visualisation in the mathematics classrooms (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008).
- *GeoGebra* is multi-platform dynamic mathematical software with its window divided into two parts: the Algebra window (left side) and the Geometry and Graphics window (right side) (Fig.1). The algebra window shows the values and the dependencies of the objects, while the geometry window (graphs, shapes, constructions etc.).
- *GeoGebra* is a dynamic geometry system, which works with points, vectors, segments, lines and conic sections (Sangwin, 2007). On the other hand, equations and coordinates can be entered directly into the grid at the bottom of the window (as shown in Fig.1) and it is easy to create sliders to change the values of variables, which can be entered as parameters in a function. This allows one to create dynamic graphs which update in real time as the parameters are adjusted.

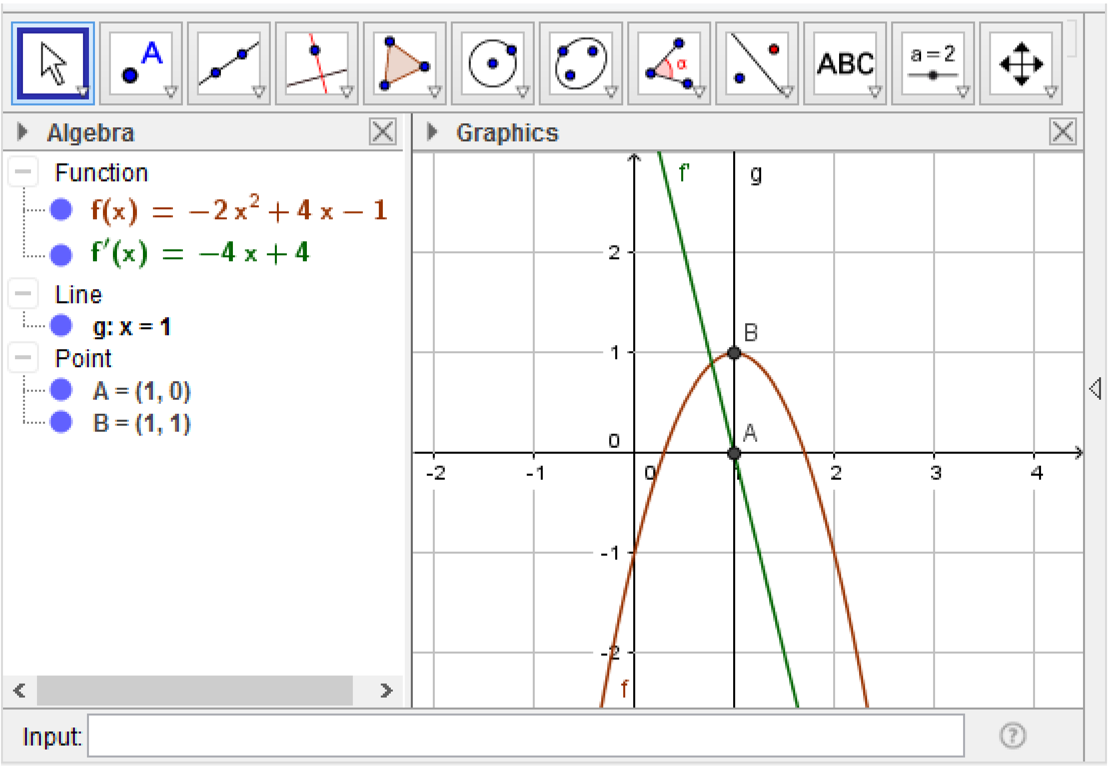


Figure 1. GeoGebra window divided into the Algebra window and the Geometry and Graphic window (Hohenwarter, 2006)

- *GeoGebra* has great potential in the teaching of algebra that lies mainly in clarifying functions and graphs. Functions can be defined algebraically and then changed dynamically afterwards (Sangwin, 2007). For example, by entering the equation $-x^2 + 4x - 1$, the corresponding graph can be seen directly. The visualisation of two windows provides a direct connection between algebraic and geometric representations. This also works the other way around, by dragging the line or curve of the graph to change the equation. The change in the equation can be seen on the algebraic window. Furthermore, with the use of this dynamics mathematics software, teachers are able to make graphical representations of the concept of functions. As the concept is introduced with pictorial representations, teachers and their learners have opportunities to make the connections between the pictures, the function concept and the symbolic representation.

In summary, central to using *GeoGebra* is the notion of affordances, and in particular the linked multiple representations of the function concept as it appears in the Namibian mathematics curriculum at the secondary level (see NSSCO Mathematics Syllabus, NIED 2009).

1.5 Aim and objectives

Aim:

- To investigate a selected number of high school mathematics teachers' learning experiences during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra.*

Objectives:

In order to achieve this aim, a group of ten high school mathematics teachers participated in 5 workshops which enabled them to interact with *GeoGebra*-based mathematical activities, focusing on different mathematical functions. Workshop activities included promoting the interactive processes of conjecture, feedback, critical thinking, investigation and collaboration (Yang & Liu, 2004). During these TPD interactions, the researcher had the dual role of teacher/research. The overall intention of the TPD intervention is to explore and to study the teachers' learning experiences and understanding of the teaching and learning of linear and quadratic functions, using *GeoGebra.*

1.6 Purpose and research questions

This study also intends to offer suggestions on ways to enhance the understanding and teaching of mathematical functions through a professional development intervention using *GeoGebra*. More specifically, the study attempts to answer one main research question and one research sub-questions. The main research question is:

1. What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using GeoGebra?

The following is the research sub-questions:

- How central is knowing mathematical functions in the Namibian secondary curriculum?
- What does research have to say about the potential of *GeoGebra (GGB)* in facilitating the understanding mathematical functions?
- What does research have to say, what are key issues in using GGB-represented mathematical functions?

1.7 Research methodology and design

The research design and methodology for this study are diagrammatically represented in Figure 1.1. The Figure indicates the research design established by identifying appropriate input from five dimensions, namely context, purpose, paradigm, methods and data collection in order to answer the research questions (Durrheim, 2006). Context is not discussed in this section because it is dealt with expansively in Chapter 4. In this chapter the purpose, paradigm, methods, data generation, data analysis, credibility and trustworthiness and ethical considerations of this study are highlighted.

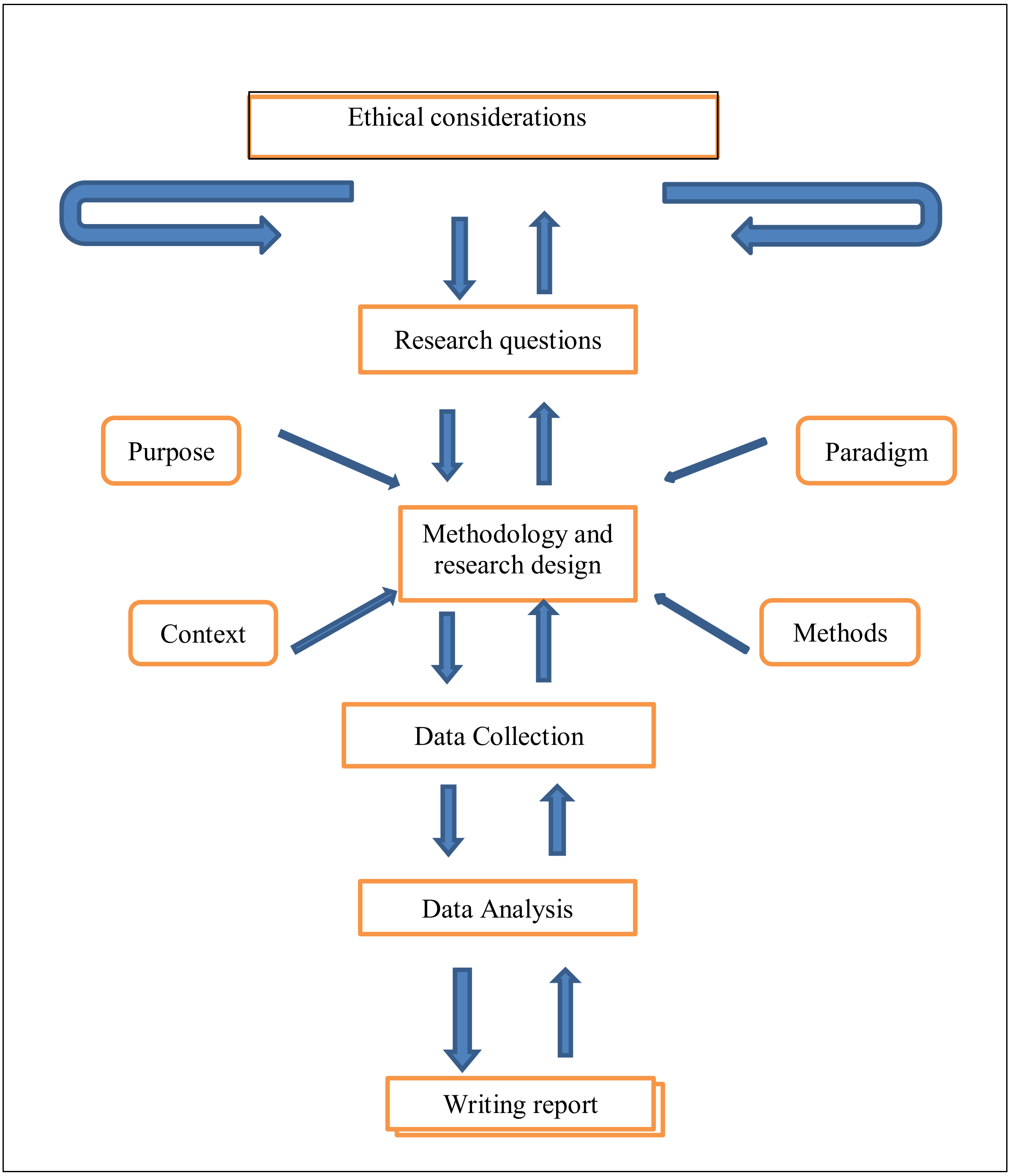


Figure 1.1: Graphic representation of the research process (Adapted from Durrheim, 2006:34)

1.7.1 Interpretive Paradigm

This study was conducted in the Ohangwena region in Namibia and grounded in an interpretive paradigm. There are three generic orientations of this paradigm, namely ontology, epistemology and methodology, which influence how researchers understand and investigate the world (Gray, 2009; Denzin & Lincoln, 2011). These three orientations guide the planning, implementation, analysis and interpretation.

A paradigm is defined as a "net that contains the researcher's epistemological, ontological and methodological premises" (Denzin & Lincoln, 2011: 13). An interpretive paradigm was considered appropriate for achieving the purpose of this research, because it assists the researcher to understand the participating teachers' learning experiences related to an intervention using the *GeoGebra* focus on functions, namely linear and quadratic functions. Within a qualitative interpretive paradigm, there are several research methodologies, each with its own underlying philosophies, practices and ways of interpretation. Creswell (2009) identifies five research methodologies, namely case study, ethnography, grounded theory, narrative research and phenomenology. For this study, case study was the appropriate way to conduct the research process, while grounded theory guided the analysis of the data.

1.7.2 Case study design

To explore and understand teachers' learning experiences interacting with the *GeoGebra* activities the study adopted a case study methodology. A case study provides a unique example of real-life situations, enabling readers to understand ideas more clearly than by simply presenting them with abstract theories or principles.

One of the advantages of using a case study is the close collaboration between the researcher and the participants, while enabling participants to tell their stories (Lather, 1992). Through these stories the participants can describe their views of reality and this enables the researcher to better understand the participants' actions' (Baxter & Jack, 2008). Additionally, a case study can assist in building up an intensive, holistic description and analysis (Merriam, 1998).

This research is a bounded, exploratory and single case study of the teachers' learning experiences when they interacted with *GeoGebra*. It is a bounded system (Stake, 1995; Merriam, 1998) because the researcher chose ten mathematics teachers from two separate high schools in Ohangwena region. This study is also an exploratory case study as suggested by Yin (1984, 1993) because the research questions are framed to explore the learning experiences of the participants. Yin (1984; 1993) indicates that in case study design an analysis can be holistic or embedded. Holistic case study involves one unit of analysis, while the embedded case study involves more than one unit of analysis (Yin, 1984; 1993).

Therefore, this study qualifies as adopting a holistic case study design with a group of high school mathematics teachers as its unit of analysis. More information on data analysis is given in Sections 1.11 and 4.5.

1.7.3 Grounded theory as an analytical tool

In this study a grounded theory approach was employed to build theory inductively through successive conceptual analyses of data with reference to the literature (Glaser & Strauss, 1967; Charmaz, 2006; Auerbach & Silverstein, 2003; Henning, et al., 2004). Grounded theory was found to be appropriate for three reasons. Firstly, grounded theory helps the researcher to proceed through the process in an inductive way without being driven by a theory, as there are no theories to be tested or verified. Secondly, the iterative nature of the theory that permits flexible movement back and forth in the data analysis process influenced the choice of grounded theory. Finally, the researchers' personal comfort in using the analytical tool made grounded theory an appropriate choice for this study.

1.8 Methods and details related to interactions with the teachers

Five successive workshops were designed to enable the participants to come together to collaborate, share ideas and find solutions related to the better understanding of the function concept using *GeoGebra* as a teaching tool . A key element to the design adopted in this study is aligning the project goals with the needs of mathematics teachers. It recognised the importance of providing teacher participants with learning opportunities that include examples of mathematical investigations related to the teaching and learning of functions, as well as opportunities to experiences this investigation as learners themselves (Putman & Borko, 2000), and to share their ideas and experiences with colleagues, including the challenges encountered and their understanding into the process.

The research was concluded by a fifth workshop (see Table 1.2).

Table 1.2 Overview of the intervention in terms of its structure and components of the workshop in this study

Date	Structure	Components
19-07-2016	Workshop 1 Introductory workshop	Familiarise participants with *GeoGebra,* demonstrations of the basis use interface, applying tool and changing properties of objects
21-07-2016	Workshop 2, focus groups Quadratic functions	Pencil and paper solutions, exploring minimum and maximum values and construction with the use of *GeoGebra*
22-07-2016	Workshop 3 Linear function	Exploring parameters and construction of linear functions with the use of *GeoGebra*
26-07-2016	Workshop 4 Quadratic polynomial	Exploring parameters and *GeoGebra* constructions
29-07-2016	Workshop 5 (final) Plotting functions (linear and quadratic) and focus groups	Exploring teachers' experiences of using *GeoGebra* in the teaching and learning of functions

1.9 Population

The sample population consisted of ten high school mathematics teachers from two high schools selected in Ohangwena Region of Namibia. Sampling of the participants was guided by the "convenience sampling procedure" (Cooksey & McDonald, 2011: 470) because of the ease of the researcher's access to these teachers and their willingness to participate in the study. Clear advantages of this sampling procedure included the availability of participants, the ease with which participation could be observed and monitored, and the quickness with which the data could be gathered for analysis. Many researchers prefer this sampling technique because of its easy accessibility, efficiency, and are free from practical constraints

(McMillan & Schumacher, 2010). As a PD opportunity for mathematics teachers in this region, teachers from other schools were invited to participate voluntarily in the study as well. Permission to conduct this study in the schools was obtained from the Permanent Secretary of the Ministry of Education in Namibia and the Ohangwena regional directorate (see Appendix A).

1.10 Data-collection strategies

To answer the main research question, multiple methods were used to collect data, namely semi-structured interviews, focus group interviews which were audiotaped or videotaped, observations and field notes. These methods assisted the researcher to collect data on teachers' learning experiences with a set of *GeoGebra* activities during the five workshops.

1.10.1 Semi-structured interviews

Semi-structured interviews were conducted over the course of the investigation (five workshops). The duration of each interview was approximately 120 minutes. The ten teachers were interviewed while they were interacting with a set of *GeoGebra* activities during the interventions. This enhanced participants' reflection on their personal professional growth and the ways in which they relate it to the project's goals and to the various activities in which they were involved. However, their written responses were collected only at the end of the last workshop with the consent of the respondents; all interview sessions were audio-recorded and transcribed later. The teachers' written responses were used as main source of evidence of teachers' learning experiences.

1.10.2 Focus group

Focus group discussions were used as a supplementary data-collection tool. According to Lederman (1990), a focus group is a technique that involves the use of in-depth group interviews with selected participants for a specific purpose. Morgan (1996:185) points out that 'the hallmark of the focus groups is their explicit use of group interaction to produce data and insights that would be less accessible without the interaction found in a group'. With this kind of interaction, focus groups enter terrains that other research methods such as the in-depth interview method or questionnaire cannot, that is, unpacking aspects of understanding which often remain unpacked by conventional methods, (Kitzinger 1995a: 109). Two focus groups were conducted, one during the second workshop and the second one during fifth workshop. The researcher's intention during the focus group interview process was to explore participants' learning experiences in the teaching and learning functions, while they interact with *GeoGebra* activities during intervention.

1.10.3 Video recordings

In addition to the above methods that generated written data, video was used in this study to capture in detail the research setting and activities. All interventions were video-recorded. These visual data were coded and analysed. However, the teacher/researcher used these data as references and observed them time and again in order to understand the process of teachers' participations.

1.10.4 Observation

Observation involves collecting qualitative information about human actions and behaviours in social activities and events in a real social environment, such as classroom teaching and learning (Cohen, Manion & Morrison, 2011; Neuman, 2007). It gives a comprehensive perspectives on the problem under investigation, and the participant-observer might discover things that no one else has paid attention to or that previously went unnoticed (Babbie & Mouton, 2000:195; Patton, 2002:263). There are two main observation strategies: participant observation and non-participant observation (Bryman, 2008; Cohen *et al*., 2011; Johnson & Christensen, 2011). Participant observation takes place when the researcher becomes part of the group under study and participates in the everyday social activities of that social system to engage with the actual feelings and experiences of the research participants, while at the same time taking notes of the actions and behaviours of the participants.

1.10.5 Field notes

Field notes I made during interviews were a valuable aid in transcribing from the recordings (Wellington, 2015). These notes should also provide information on the time, the setting and impressions of the interviewee's position, disposition, attitude, etc. Notes and audio recordings can be used together in interviewing to improve the accuracy and quality of data/evidence, and to enrich the 'texture of reality' (Stenhouse, 1975) in presenting this type of research. A format was designed to enable the researcher to take note of what was observed as well as record his personal reflections on what was being observed (see appendix I).

1.11 Data analysis

Primary data for teachers' learning experiences of teaching and learning using *GeoGebra* activities were generated from several sources, including field notes and semi-structured interviews during teachers' interactions while they were attending five workshops and two focus groups organised by the researcher. All interviews were audiotaped and then transcribed, which served as the primary data source. Figure 2 shows the coding process that was used in this study.

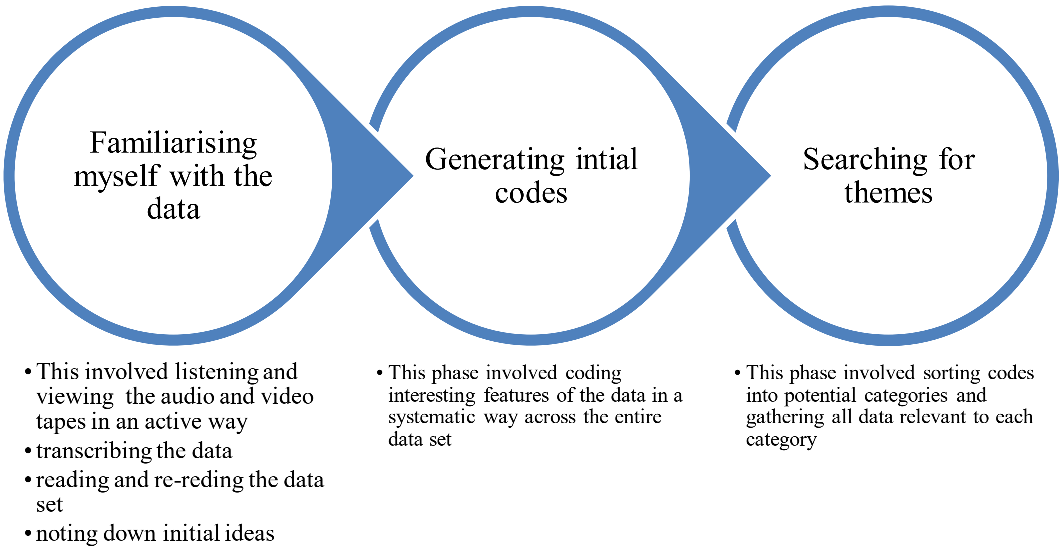


Figure 1.2 Phases of data generation and analysis (Adapted from Saldana (2015) and Creswell (2009:174))

To analyse the interview transcripts, video and audiotapes focus interviews, a constant comparison method was used to create the initial codes (Yin, 2009). Moreover, the researcher used focused coding continuously to arrange the existing codes into broader conceptual categories until the data reached saturation point (Charmaz, 2006). Throughout the coding process, I attempted to bracket my own experiences and assumptions through reflective note-taking and regular critical conversations, as well as by relying heavily on the participants' own words.

1.12 Credibility and Trustworthiness

The trustworthiness of qualitative research is often viewed with some suspicion by positivists' scholars, maybe because their concepts of validity and reliability cannot be addressed in the same way in qualitative studies. Many qualitative researchers have preferred to use different terms to distance themselves from the positivist paradigm. Guba (1981) proposes four criteria that he believes should be considered by qualitative researchers in pursuit of a trustworthy study. By addressing similar issues, Guba's constructs correspond to the criteria employed by the positivist scholars:

a) Credibility (in preference to internal validity);
b) Transferability (in preference to external validity/generalisability);
c) Dependability (in preference to reliability);
d) Confirmability (in preference to objectivity).

Validity refers to the appropriateness, meaningfulness, correctness and usefulness of any inferences a researcher draws based on data obtained through the use of an instrument (Fraenkel & Wallen, 2003).

In qualitative research the validity of interviews is dependent upon depth, honesty and extent of triangulation and objectivity of the researcher (Cohen, Manion, & Morrison, 2007 Yilmaz, 2013). Reliability stands for "the extent to which research findings can be replicated" (Merriam, 1998:205). In this study appropriate measures were taken to ensure credibility and trustworthiness during the conducting of the research, the analysis of data, as well as the reporting of the findings. To ensure sound quality in this study, the researcher utilised the trustworthiness model described by Guba (1981) and Lincoln and Guba (1985).

This model is based on the identification of four aspects of trustworthiness, namely credibility, transferability, dependability and conformability (Guba, 1981; Lincoln & Guba, 1985). Transferability was established by providing a rich description of the context within which the study occurred (Guba, 1981), so that readers could determine the extent of the similarity to their own situations (Mertens, 1998). In qualitative research, dependability refers to the consistency of the results with the data generated (Merriam, 1998). In this study dependability was ascertained by using triangulation and attaching as appendixes the instruments used to conduct the research as well as sample transcriptions of interviews to provide the reader with background information about the procedures followed in the research. Confirmability is concerned with ensuring that the researcher has acted in good faith (Bryman, 2008). To establish confirmability, the researcher provided characteristics of the respondents and the methods of data generation, as well as the methods of analysis and interpretation that were applied to show that the research findings are the result of the research data and not the researcher's assumptions and presumptions

1.13 Ethical consideration

This study involved the collection, analysis and interpretation of qualitative data from secondary mathematics teachers employed by the Ministry of Education in Ohangwena, Namibia. Ethical issues were addressed throughout the study right from the beginning with sample selection, the administration of the semi-structured interview, the data-collection phases, analysis and interpretation of data as well as the final stages of reporting, sharing and storing of the data. This implies that each phase in the proposed design research raised different ethical issues. The researcher therefore gave attention to the following ethical recommendations.

In this study the researcher followed the necessary ethical procedures set out by the Stellenbosch University Ethics Committee during every step of the planning and implementation of this research. Firstly, permission to undertake this study was obtained from the Namibian Minister of Education and regional director of education (see Appendices B and C). Secondly, the study was conducted in such a way that it caused no harm to the participants (Babbie & Mouton, 2001). The participants were informed about the aim and purpose of the study, the nature of study and the commitments involved in the study so that they could make their own decision either to participate in the study or not. All the participants signed consent forms to indicate that their participation was voluntary and a copy of the signed consent paper was given to each of them (see Appendix D). They were informed that they could withdraw from the research at any stage if they felt uncomfortable. To ensure anonymity, codes were used to protect the identity of the schools and the participants. Finally, as researchers are obliged to be honest and open wherever possible when they analyse and report their findings (Babbie & Mouton, 2001), the researcher was honest towards the research community in conducting the research and reporting the findings.

1.14 Referencing Methods

The Harvard referencing method was used in preparing this dissertation. The references are listed in alphabetical order.
In Section 1.15 the key terms used in the study are defined.

1.15 Key definitions of terminology

Applets: Applets are typically web-based having a specific conceptual focus. Well-designed interactive applets enable learners and teachers to engage in investigation of mathematical relationships without having to spend a lot of time learning how to use the tool that produces the different representations of these relationships. They thus can be used selectively by teachers to support understanding of key concepts.

Boundary objects: Boundary objects are objects which are both plastic enough to adapt to local needs and the constraints of several parties employing them, yet robust enough to maintain a common identity across sites.

Community of practice: Communities of practice are groups of people (in this case teachers) who share a concern or a passion for something they do, and learn how to do it better as they interact regularly.
Community of Interest: A community of interest involves member of distinct communities of practice coming together to solve a particular problem of common concern
Dynamic Mathematics Software: This is computer software which is combination of dynamic geometry and computer algebra systems.

Dynamic geometry environments (DGEs): DGEs allow the user to drag parts of a geometry object, while measurements of the figure change dynamically in an algebra window. A dynamic mathematics tool allows user/teachers to visualise and explore geometry and algebra.

GeoGebra is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from primary school to university level

Geometry: This is the branch of mathematics that is concerned with the properties and relationships of points, lines, angles, curves, surfaces, and solids.

Information and Communications Technology: The use of any equipment or software for processing or transmitting digital information that performs diverse general functions whose options can be specified or programmed by its user.

Ministry of Education: A government ministry responsible for leading curriculum design, policy and resourcing for state education in early childhood, elementary, middle and secondary education.

Multiple representations: Providing the same information in more than one form of external mathematical representation.

Teacher: A **teacher** is a person who helps learners to learn.

Teacher Professional development: Teacher development is a learning process that is undertaken by teachers after their initial training and preparation in order to enhance their work.

Triangulation: Involves the use of different methods and sources to check the integrity of, or extend, inferences drawn from the data.

1.16 STRUCTURE OF THE STUDY

Chapter one briefly describes the context of the research, the research methodology, the research site, the research aims, possible significance of the study, research question and objectives**.**

Chapter Two provides a brief overview of the general structure of the Namibian education system and reforms in Namibian education after independence as well as focusing on the developments of ICT policy within the context of education in Namibia, with special reference to the secondary mathematics curriculum.

Chapter Three will offer a review of the literature pertinent to the research question. This literature includes teachers' experience related to the understanding and teaching of mathematical functions using ICT *(GeoGebra*) through a professional development intervention in relation to *GeoGebra* on secondary school mathematics, mathematics teachers' professional development. The research question informs the sequence and logic of the overall literature review.

Chapter Four describes the research design and methodology employed, sampling procedures and analytical framework. Chapter Five presents the data that were generated through multiple techniques as well as the findings that emerged from the study.

Chapter six concludes with a summary of findings, the knowledge implications for practice, recommendations and limitations, suggestions for future research and a reflection on the research process .The dissertation finally includes the references and appendices.

1.17 Chapter Summary

This chapter provides an overview of this study. The underlying research problem of the study and justification for choosing *GeoGebra* as appropriate instrument to interact with participants during interventions focused in functions are presented. A qualitative collective descriptive case study was found to be the most appropriate research methodology to achieve the purpose and answer the research questions. The nature of the research questions is the reason why an interpretative paradigm was the most suitable research perspective. This paradigm can assist the researcher to understand the participating teachers' experience of using *GeoGebra* with the focus on functions.

Multiple techniques, namely, semi-structured focus group interviews, observation, video and audio recording were used to generate data.

The data generated were analysed and interpreted using the constant comparison method. Appropriate measures were taken to ensure credibility and trustworthiness during the whole research processes, including generation of data, analysis of data, as well as reporting of research findings. Finally, the appropriate ethical procedures were followed to conduct the research. The following chapter presents the literature review related to this study is provided

CHAPTER 2: THE CONTEXT OF STUDY

2.1 Introduction

The purpose of this chapter is to provide a brief overview of the general structure of the Namibian education system, reforms in Namibian education after independence as well as focusing on the developments of ICT policy within the context of education in Namibia, with special reference to the secondary mathematics curriculum.

2.2 Background of the study

This section will provide the background to the current study starting with an account of the Namibian education system with special reference to the high school mathematics curriculum and the use of ICTs.

2.2.1 An overview of the Namibian education system

Initially, the education system in Namibia prior to independence was mainly characterized by inequalities practised by apartheid. Namibian studies (Amutenya, 2002; Amkugo, 1993; Naukushu, 2011) distinguished three separate apartheid education systems prior to Namibian independence, which provided education for Whites, Blacks and Coloureds.

The black (Bantu) education system focused on the achievement of basic and minimal levels of understanding and rote learning of mathematics (Naukushu, 2016; O'Sullivan, 2004). Amutenya (2002) asserts that during apartheid the colonial rulers had a misconception that mathematics was not suitable for black minds. Consequently, according to the literature (e.g. Amutenya, 2002; MBESC, 1993; Naukushu, 2011) it was believed that blacks could be competent only up to a very basic mathematical level (arithmetic) and that beyond the arithmetic level they would not cope. As a result, there was a separate mathematics curriculum for whites only, excluding blacks, on the assumption that black learners could not be competent in mathematics. This made it difficult for many black learners of mathematics to acquire higher levels of understanding mathematical concepts at that time.

Having been subjected to such an oppressive educational system, black learners' numeric competencies and mathematical understanding suffered.

Similarly, teachers were also trained at racially segregated institutions across the country. What further aggravated the problem was that many of the students who lacked mathematical understanding became mathematics teachers in Namibia and therefore this lack of mathematical proficiency was simply recycled. As a consequence many learners do not excel in mathematics at secondary school level in Namibia. For three consecutive years the Directorate of National Examination and Assessment (DNEA, 2009, 2010, and 2011) reported the alarmingly poor performance in mathematics of Namibian students. This necessitated study of possible interventions be carried out to address the situation. But the consequences of poor mathematical teaching still prevail in Namibian education to this day.

2.2.2 Reforms in Namibian education after independence

After independence in 1990 the education system in Namibia was "reformed" to accomplish four goals: accessibility, quality, equity and democracy (Ministry of Basic Education, Sport and Culture (MBESC), 1993). In an attempt to rebuild the nation the Namibian government developed a Vision 2030 framework which anticipates that the country will be developed and industrialised by the year 2030 (National Planning Commission (NPC), 2003). The NPC (2003) further stresses the need to cultivate a knowledge-based economy underpinned by scientific and mathematical disciplines. Since mathematics and science are crucial disciplines to professions for innovations to drive economic development, it could be concluded that the development of mathematical, numerical and scientific understanding are crucial to achieve Vision 2030 and the development of the Namibian nation as a whole.

Namibia's global competitiveness is ranked 115th in the higher education and training index (Schwab, 2015), which means that it has to redouble its effort if it is to be classified as a developed and industrialised nation by the year 2030. Hence, the call for a knowledge-based economy requires new and innovative teaching and learning strategies, such as learner-centred teaching.

Consequently, the curriculum was revised in 2006 to meet the demands of a new and growing nation. The Cape Matriculation System had already been abolished and replaced by the Cambridge Matriculation System in 1995. Amkugo (1993) further contends that the Cape Matriculation Education System based on a colonial mind-set, was greatly associated with rote learning and could not educate to liberate the learners.

Although Namibia abolished the Cape matriculation system, unfortunately the standard of education seemed to have declined since the adoption of the Cambridge International General Certificate of Secondary Education (IGCSE) in 1995. This was evidenced by earlier reports of the Directorate of National Examinations and Assessment (DNEA), (1996; 1998; 2002; 2006; 2007; 2008) and other subsequent reports of DNEA (2009; 2010 and 2013) that showed declining numbers of high school graduates especially passing Mathematics.

The IGCSE was then abolished and replaced by the Namibian Senior Secondary Certificate Examination (NSSCE) in 2007. However, the NSSCE was an exact replica of the Cambridge Matriculation system that had been done away with. The National Institute for Education Development (NIED) documents such as the syllabi, assessment tools and teacher guides show that there was no difference in the high school mathematics content between the Cambridge and the new Namibian Senior Secondary Certificate Examination (NSSCE). Even, the mathematical contexts used in the teaching and learning materials were still not local, despite the call for the localisation of contents. However, there was hope with the adoption of the NSSCE, the Namibian Education System, but still there was not much improvement in the number of successful matriculates. Furthermore, in 2012 mathematics was made a compulsory subject for all learners up to Grade 12 level, regardless of whether they had passed the junior grades or not. In support of this new arrangement, the Ministry of Education took a decision that mathematics is a necessity for all learners and therefore needs to be taken by all learners (Illukena, 2011).

Furthermore the Ministry argued that the whole nation needs to be literate in the area of mathematics if national development is to be realised (Ministry of Education and Culture (MEC), 2012). However, the issue of mathematics for all seems to be a highly controversial topic in Namibia to this day. Hence, there is an on-going debate among different stakeholders as to whether making mathematics a compulsory subject for all learners at high school level was the best decision. Those in favour of mathematics for all argue that it is evident nowadays in the society that individuals with limited basic mathematical skills are at greatest disadvantage in the labour market and in terms of general social exclusion.

In furthering this debate those who believe mathematics should be taught to all argue that:

> "If the future citizens need to participate in democratic processes in an economically, and technologically advanced society, they need to have not only good literacy skills, but also good skills in mathematics. Hence, arguably, it is crucial that they receive

> better and quality education in mathematics, science and technology to meet the existing demand of a skilled workforce that will contribute to the attainment of Vision 2030'' (Illukena, 2011:12).

Illukena (2011) further argues that mathematics also plays a significant role in the lives of individuals and society as a whole. This makes it imperative that the Namibian mathematics curriculum should equip learners with the skills necessary for achieving higher education, supporting their career aspirations, and attaining personal fulfilment – hence mathematics should be compulsory at all levels of one's education. In addition research (e.g. Wolfaardt, 2003) indicates that the grades of learners in mathematics decline by an average of 2 points per grade as they progress from Grades 10 to 12. It could be argued that the inclusion of mathematics as a compulsory school subject could only aggravate the current performance of mathematics, which is already dismal. Moreover, the inclusion of mathematics as a compulsory subject in school could only be effective if the learners are equipped with skills and potential to cope with the demands posed by mathematics at high school level (Courtney-Clarke, 2012). Furthermore, in the Namibian context there seems to be a lack of research on the issue of mathematics as a compulsory school subject; there is no basis of empirical evidence for the arguments as to whether mathematics for all is a necessity. But at the very least mathematics requires learners to possess a better grasp of functions to cope with the demands of the high school mathematics curriculum.

The reform of the education system after independence did not only include structural reforms and the development of a new broad curriculum, but also the reform of teacher education at the four Colleges of Education in Windhoek, Ongwediva, Rundu and Katima Mulilo (Caprivi). Quality education requires quality teachers. Thus, a key objective of the Namibian education system should be to invest in human skills and special attention should be given to TPD, which is crucial because this will empower the teachers to be more effective and efficient in their classrooms.

The constant attention to and the continuous development of their subject, their technological knowledge and their teaching skills are of the utmost importance not only to help teachers keep abreast of new developments and changes in their subject but also equip them with the knowledge and skills needed in order to be confident and effective in their teaching.

2.3 **The Namibian structure of basic education**

The formal Namibian education system consists of a seven-year Primary phase followed by a three-year Junior Secondary phase and a two-year Senior Secondary phase. There are also special education institutions for learners with disabilities. Formal education is divided into four phases: Lower Primary Grades 1 – 4, Upper Primary Grades 5 – 7, Junior Secondary Grades 8 – 10 and Senior Secondary Grades 11 – 12.

Table 1.3 illustrates the structure of the Namibian basic education system, which is subdivided into four phases.

Table 2.1 Four phases of the Namibian basic education

First phase	Second phase	Third phase	Fourth phase
Junior Primary Pre-Primary and Grade 1-3	Senior Primary Grades 4-7	Junior Secondary Grades 8-10	Senior Secondary Grades 11- 12

Source: Adapted from The National Curriculum for Basic Education (NIED, 2016:3)

After completing the Namibia Senior Secondary Certificate Ordinary (NSSCO) level at the end of Grade 11, learners have various options: they may choose to continue with either vocational education or training, or with distance learning, or seek employment. Learners who meet the prescribed requirements may proceed to Grade 12. In Grade 12 learners will take their subjects on Advanced Subsidiary Level, which is an admission requirement for enrolment at many universities in Southern Africa and abroad.

The Junior Primary phase lays the foundation for all further learning. In Pre-Primary learners develop communication, motor and social skills and concept formation to prepare them for formal education. In Grades 1-3 the learners learn to read and write in two languages; they learn basic mathematics; they learn about the community and nature around them and how to look after their health; and they develop their creative and expressive abilities. In Pre-Primary and Grades 1-3, teaching and learning take place through the medium of the mother tongue or the predominant local language. They are also exposed to computer technology, gaining a first appreciation of information and communication technology (ICT) as a tool for learning by learning to recognise the functions and uses of ICT in their lives, and getting a basic understanding of how a computer works and how to use it in learning processes.

In the Senior Primary phase learners build on this foundation, develop literacy and numeracy, develop learning skills, gain basic knowledge and skills in natural sciences, social sciences, technology and the arts, and participate in physical education. The transition to English as medium of instruction occurs in Grade 4. Technological skills at this level require a fundamental understanding of software applications and basic navigational skills through the Windows environment. The Junior Secondary phase continues with the same learning areas as Senior Primary, consolidating previous learning and extending it to a level where the learners are prepared for young adulthood and continued formal education. In this phase all learners take English, Mathematics and another language. They will be exposed to all learning areas by taking all the science subjects, together with options of any two pre-vocational subjects. Learners then continue to formal Senior Secondary education (Grades 11-12), which provides specialisation and depth in one field of study. Those who do not meet the requirements to proceed to Grade 10 will repeat Grade 9 once. Grade 11 is the first exit point in the formal schooling system. Learners who meet the necessary requirements may continue to Grade 12. Those who do not meet these requirements have the option to continue their education through distance education.

In Grades 10-12 all learners will continue to take English, Mathematics and another language. In addition, they choose a field of study consisting of three mutually supportive subjects. ICT skills at this stage entail the confident use of applications and advanced care of a computer. In the Senior Secondary phase learners will take three to five subjects on Advanced Subsidiary Level. By the end of Grades 11 and 12, learners should be well prepared for further study or training, or entry into the job market.

Mathematics continues to be a compulsory subject from Grade 1 to Grade 11, and an elective subject in Grade 12. Mathematics is an indispensable tool for everyday life. It is also essential for the development of science, technology and commerce. Mathematical skills, knowledge, concepts and processes enable the learner to investigate, model and interpret the numerical and spatial relationships and patterns that exist in the world. The Mathematics learning area consists of Preparatory Mathematics (Pre-Primary) and Mathematics (Grades 1-12).

The revised curriculum was implemented in the Junior Primary phase in 2015, the Senior Primary phase in 2016, and in the Junior Secondary phase, Grade 8 in 2017, and Grade 9 in 2018. The Ministry is busy developing the syllabuses for the Senior Secondary phase (Grades 11-12). The implementation of the Senior Secondary revised curriculum will take place as follows: Grade 10 in 2019, Grade 11 in 2020 and Grade 12 in 2021.

2.4 Current ICT policy within the context of education in Namibia

The potential use of ICTs in the PD of mathematics teachers highlighted above. The National Council of Teachers of Mathematics states that 'Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology' (NCTM, 2008:1). In 1999 the Namibian Ministry of Education (MoE) introduced a National ICT Policy for Education. A review of this policy took place in 2005, resulting in a new National ICT Policy for Education (MoE, 2005) and the National ICT Policy Implementation Plan (MoE, 2006). Current educational policy identifies pre-service and in-service teacher education institutions as priority areas for ICT deployment.

Since 2006 the MoE in Namibia has set aside a fund in the National Budget for ICT in education. Furthermore, stakeholders such as the Global e-School Initiative (GeSCI), SchoolNet Namibia, Namibia Education Training Academy (NETA) and Computer Education Community Service (CECS) have been supporting ICT development by donating ICT resources and providing teacher training in schools. However, ICT deployment does not guarantee its use and integration into the school curriculum, even in subject areas where its use would be crucial – e.g. mathematics. Although there are several strategies outlined in the Namibian ICT policies, those touching on the curriculum and continuous preparation of teachers in ICT skills and pedagogical application of ICT seem to have been singled out as basic to the implementation of ICT. Ngololo, Howie and Plomp (2012) argue that the effects of the policy on ICT implementation in the Namibian education system are unknown, since no evaluation has taken place.

Limited studies have been conducted in Namibia focusing on ICT deployment, technical maintenance and training (see Clicherty & Tjivikua, 2005; Iipinge, 2010; Matengu, 2006). Although there have been enormous investment in education technology in many countries, ICTs are have to make a substantial impact on mathematics education (Drijvers, Kieran, Mariotti & Ainley, 2010). Nevertheless, as ICTs are becoming more integrated into education, they are providing space for new teaching, learning approaches and classroom organisation. Unfortunately, ICTs in the classroom remains a major challenge for teachers, because they have limited knowledge about how to use the technology (Donnelly, McGarr & O'Reilly, 2011). Even if the teachers have mastered the operation of the software, it is still a challenge to get teachers to successfully integrate it into their daily teaching practices.

Hence, Teacher education, development and support are critical components of a successful ICT strategy in Namibia. Unless teachers are skilled, knowledgeable about and accustomed to working with ICTs, there will be little point in placing computers in schools. Computer labs will simply become white elephants and expensive equipment will gather dust if teachers are unaware of the possibilities of ICTs in education and lack the skills and confidence in using them. The greatest challenge thus lies with teacher education and development (Chisholm, Dhunpath, & Paterson 2004:73).

2.5 Summary

This chapter provided an overview of the historical background of the Namibian national education system and information on recent reforms to the mathematics curriculum. In order to illustrate the current situation, it was necessary to understand the establishment of the revised education system to overcome the legacy of apartheid education. Teacher education, development and support are critical components of a successful ICT strategy in Namibia. But to this day very little attention has been paid to PD.

Namibia moved from an exceptionally divided and politically-sanctioned racial segregation at framework towards provided 'Instruction for All'. Statistics seem to indicate that teachers are present thought to be 'better' qualified, yet they are as yet not adequately prepared to teach the learners and to confront all the new development and difficulties in their profession (MoE, 2013; Peters, 2006). The continuous development of and improvement of their subject knowledge and training aptitudes are absolutely critical, not exclusively to assist teachers keep up-to-date with new developments and changes in their subject, but also to equip them with the knowledge and skills in order to be confident and effective in their teaching.

A key objective of the Namibian education system should be to invest in human skills and special attention should be given to TPD. It is suggested that PD is crucial because it will empower the secondary school teachers, especially mathematics teachers, to be more effective and efficient in their classrooms.

From the point of view of the current study, the ICTs are seen as a vehicle for advancing new ways of teaching and learning mathematics. Introducing the computer to mathematics teachers through a PD course might provide solutions to the current shortcomings of the system, since teachers need to have practical experience of using computer in their teaching and they need teaching models. Therefore the literature relating to teachers' experiences and understanding of teaching of mathematical functions through ICT through a professional development intervention using *GeoGebra* will be examined in the next chapter.

CHAPTER THREE

LITERATURE REVIEW

This chapter undertakes a literature review related to the research question:

1. What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using GeoGebra?

The following is the research sub-questions:

- How central is knowing mathematical functions in the Namibian secondary curriculum?
- What does research have to say about the potential of *GeoGebra (GGB)* in facilitating the understanding mathematical functions?
- What does research have to say, what are key issues in using GGB-represented mathematical functions?

The chapter is structured as follows. Firstly, there is a general overview of the literature on the use of technology in education. This review relates the literature on uses of information and communications technology (ICT) to the high schools in Namibia and around the world with a particular focus on the role of ICT (*GeoGebra)* on TPD. The review casts new light on the supporting and constraining factors that influence the integration of ICT into mathematics education. Secondly, the literature specific to the PD of teachers and high school mathematics teachers in the case of *GeoGebra* is assessed. The research question informs the sequence and logic of the overall literature review.

3.1 Introduction

The rapid technological developments bring new challenges to education. New technology has the capability to make fundamental changes in education (Bates & Bates, 2005). Many secondary schools teachers around the world already use and explore the new and existing technology for greater benefit of education. Yet there remains an increased need to promote the quality of mathematics teaching and learning around the world.

3.2 Technology use in Education

In Australia, for example, the commonwealth government has set goals for schools in relation to the adoption of ICT. The government intends that learners should complete and leave schools as confident, creative and productive users of the new technologies in the society. Schools are consequently expected to integrate ICT into their operations (Johnson, Becker, Cummins, Estrada, Freeman & Hall, 2016).

In other countries, for example, the Philippines, Indonesia, Malaysia, Uzbekistan, Vietnam – departments of education have also formulated policies for ICT use. In Asia and the Pacific, including the emerging countries, teachers in primary, secondary and tertiary institutions are being trained in the use of ICTs in education with varying degree of scope. Most of the training programmes have general objectives aimed at developing awareness, knowledge and skills in either the use of computers or the integration of computers into teaching and learning (Whelan, 2008). In a study conducted in Canada during 2003-2004 academic year school principals reported that their schools used ICTs (computers and laptops) for educational purpose such as activities directed toward lesson preparation, execution or evaluation.

This study also discovered that the percentage of secondary and elementary schools without computers was as low as 1% (Plante & Beattie, 2004). Thus ICT skills are regarded as necessary, but also as a valuable tool for the development of other skills. Actually ICTs have become an important part of a school curriculum a support tool for providing teachers and learners with enhanced teaching opportunities in the whole range of school subjects (Khatoon & Mahmood, 2011). The content of the national curriculum statements of countries such as the UK, the USA and Australia provide clear evidence of this shift from the teaching of ICT alone to the infusion of ICT as a significant tool in the school curricula (Webb & Way, 2007). In response to modern advancements in technology, the Namibian revised National curriculum (NIED, 2016:9) states that:

> The rapid spread and use of ICT in all areas of life make this skill area part of the core skills needed for a knowledge-based society. Learners must become competent in using new information and communication technologies. The specific ICT skills include the ability to appropriately choose and correctly use ICTs as tools according their purpose, to show versatility in using hardware,

> software and different media, to apply computer health and safety principles, to follow ethical norms when using ICTs, to be able to access, critically evaluate and use information, to transform information into knowledge, to distinguish between fact and opinion, and to communicate effectively using ICTs.

In addition, learners and teachers need opportunities to understand how technological systems are integral parts of social systems and political, cultural and economic frameworks, and what the limitations of these systems are. They must understand the value of information and their own roles and responsibilities as citizens in the development of information and communication technology in society (MoE, 2016:10).

Limited studies have been conducted in Namibia focusing on ICT deployment, technical maintenance and training (see Clicherty & Tjivikua, 2005; Iipinge, 2010; Matengu, 2006). While there have been enormous investments in education technology in many countries, ICTs are yet to have a significant impact on mathematics education (Drijvers, Kieran, Mariotti & Ainley, 2010). Unfortunately, adoption of ICTs in the classroom remains a major challenge for teachers, because they have limited knowledge on how to use the technology, (Ndibalema, 2014). Even if the teachers have mastered the operation of the software, it is still a challenge to get teachers to successfully integrate it into their daily teaching practices.

3.2.1 The role of technology in mathematics teachers' professional development

Research in mathematics education related to the use of digital technology has evolved over time**.** The use of technology is not new to the teaching and learning of mathematics. From the late 1980s, with the enormous improvement of the personal computer and laptop, researchers believed that ICT would have the potential to offer opportunities for progress in educating and learning in ways that were impossible before (Bransford, Brown & Cocking, 1999; Cox, 2014), and further investigations focusing on educational ICTs stressed its value in the field of educational research. Advances in educational reforms propelled further consideration of use of ICTs from government and policy makers to schools and individual teachers (Assude, Buteau & Forgasz, 2010; Sinclair, Arzarello, Gaisman, Lozano, 2010).

Many countries are in the process of reforming the teaching and learning of mathematics by including computer technology in mathematics education. Mehanovic (2009), Ndlovu, Wessels and De Villiers (2011) cited in Pfeiffer, 2017) also indicate in their studies that there is increasingly strong motivation to integrate ICTs into the teaching and learning of mathematics in various countries. They add that this worldwide enthusiasm is the result of the global advancement of digital technologies.

Many studies over the past decades suggest that the full realisation of the potential educational benefits of ICTs does not occur automatically. The effective integration of ICTs into the educational system is a complex, multifaceted process that involves not just technology but also issues of the curriculum, pedagogy, institutional readiness, teacher competencies, and long-term financing, among others (Tolani-Brown, McCormac & Zimmermann, 2011). The National Council of Teacher of Mathematics (NCTM), 2008:25) discusses the appropriate use of computers in mathematics education:

> The effective use of technology in the mathematics classroom depends on the teacher. Technology is not a panacea. As with any teaching tool, it can be used either well or poorly. Teachers should use technology to enhance their students' learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do efficiently and well by graphing, visualizing, and computing.

This statement above implied that ICTs can help to improve and develop quality of education by providing curricular support in difficult subject areas, especially mathematics, but teachers need to be trained to implement the technology effectively (Mumtaz, 2000). Teachers need to be involved in collaborative tasks and the development of intervention strategies. Consequently, teachers need to realise the potential of digital technology in their daily practices and use it well. To achieve this successfully, teachers need training and guidelines to develop their expertise when using this technology for teaching and learning (Spiteri & Rundgren, 2020).

Mathematics teachers' training and the use of new digital tools are quite challenging as technology is continuously evolving at a rapid pace. Thus, training to use the new tools must be provided continuously. Several studies stressed that teachers need the knowledge, the skills and the right attitudes to use the technology effectively (Barak 2014; Kokol-Voljc, 2007; Morsink, Hagerman, Heintz, 2010). Therefore, teachers need to develop the disposition to experiment with new technologies to capture the interest of all the learners in the class (Kinzer, 2010). This will lead to more inquiry and innovation in learning (Sun, Looi, & Xie, 2014).

As stated by Dalton (2012), the teacher must reflect on his or her own strengths and interests, activities that she or he is already comfortable with, and then develop the lessons accordingly with the use of digital technology. This requires time and collaborative training as well as feedback and a supportive school culture.

More recently Leung (2017) and Loong (2014) studied the use of technology in learning and education, specifically highlighting the use of technology for mathematics teachers. Hatlevik, Throndsen, Loi and Gudmundsdottir (2018) explain learners' beliefs and their actual achievements with regard to ICT experiences – for example, achievements with ICT in independent learning areas in addition to traditional disciplines. This is knowledge that learners can readily 'adapt and transfer to new contexts' (Fraillon, Ainley, Schulz, Friedman & Gebhardt, 2014).

Vongkulluksn, Xie, and Bowman (2018) argue that teachers' belief in the technology is one important factor in technology integration. Karadeniz and Thompson (2018) proposed the use of calculators, and Wares (2018) argues for the use of dynamic geometry software, while Martinovski (2013), Quinlan (2016), Segal, Stupel, and Oxman (2016), States and Odom (2016) advocate *GeoGebra* as a tool for technology use in mathematics teaching and learning. However, developing the roles of teachers to support the integration of digital technology into the mathematics classroom is not a simple task. Teachers have to take into account the level of mathematical knowledge, knowledge about the artefacts, didactic knowledge of mathematics, and didactic knowledge about the computer (Tapan, 2003).

A complex relationship arises when mathematical knowledge, technology and epistemology (pedagogy) come together into a fluid state with flexible boundaries (Leung, 2017). The level of this knowledge integration depends to large extent on the skills and experience of the teachers themselves. Crisan, Lerman, and Winbourne (2007) investigated the relationship between content knowledge, pedagogy and ICT exhibited by mathematics teachers in the United Kingdom. They proposed the idea of the teachers' own personal ICT pedagogical construct to conceptualise its incorporation into their classroom practices:

Learning to teach with ICT is a process. It demands doing and practice … the teachers developed their own 'expertise' with ICT, which we call here personal ICT pedagogical construct, consisting of conceptions of how the ICT tools and resources at their disposal benefited their teaching of mathematics and their pupils' understanding and learning of mathematics (Crisan, Lerman, & Winbourne, 2007:33).

Teachers need opportunities to familiarise themselves with the use of technology for a mathematical activity and the physical objects that are used as teaching and learning tools to engage learners in the learning of mathematics (Laborde, 2007; Ruthven, Deaney & Hennessy, 2009).

The Namibian national curriculum (2016) strives to prepare learners and teachers to function effectively in the 21st century by providing a basis to use mathematics in their personal and professional lives. The curriculum mandates the adoption of a scientific approach in teaching and learning, including mathematics. Through a scientific approach (observing, questioning, associating, experimenting and interacting), the learners establish the ability to think scientifically that emphasises inductive reasoning rather than deductive, and teachers guide the learners to do research themselves, rather than simply being told information. Therefore, a teacher would need to align the technology (ICT) with the intended curriculum as prescribed in the Namibian syllabus on the high school or secondary mathematics content (NIED, 2010b). This document indicates important issues related to the teaching and learning of functions; for example, learners should learn:

a) To construct tables of values for functions, as well as to draw and interpret graphs and solve equations graphically;
b) To use rectangular Cartesian coordinates in two dimensions, and understand the relationship between a graph and an associated algebraic equation;
c) To calculate the distance between two points given in coordinate form, the gradient of the line segment joining them, and the coordinates of their midpoint;
d) To find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it or one point on it and its gradient);
e) To interpret and use equations of the form $ax + by + c = 0$, including knowledge of the relationships involving gradients of parallel and perpendicular lines; and
f) To apply coordinate geometry to quadrilaterals.

Additionally, teachers' own learning experience with digital tools is a critical process in the formation of their technology-integrated knowledge. Teachers must experience for themselves as learners the potentials and pitfalls of using digital tools in the learning of mathematics, and in turn gain knowledge about how learners can learn mathematics in various digital environments (Ertmer, & Ottenbreit-Leftwich, 2010). Similarly, others studies (Pierce & Ball, 2009) show that adopting technology or ICT to promote teaching and learning is a long process and might require teachers to change their own practices, that is, working in the traditional mathematics environment controlled mainly by teaching and learning with pen and paper, to make mathematics meaningful to individual learners (Jung, 2005).

It could also be argued that the effectiveness and efficiency of technological applications depend on the teachers and the curriculum concepts. In most cases, teachers seem to be the primary mediators between technology and its integration into the educational system (Zhao, Hueyshan, & Mishra, 2001), that is, in their classrooms.

Drier (2001:173) contends that there is a need for qualified teachers who can 'utilise technology as an essential tool to developing a deep understanding' not only of mathematics but of the relevant pedagogy for their learners. Simply providing the technology itself to teachers does not always result in the successful incorporation of that technology into their teaching (Cuban, Kirkpatrick & Peck, 2001:828-829). This is the reason why this study intends to improve the understanding and teaching of mathematical functions by improving and studying teacher skills in ICT through a professional development intervention using *GeoGebra* (Hohenwarter, & Preiner, 2007), giving mathematics teachers not only access to technology, but also discussing appropriate different pedagogical approaches that may contribute to improvement of their understanding of the concept of function.

3.3 Factors limiting ICT use in mathematics classrooms

Although conditions for successful technology integration finally appear to be in place, including ready access to technology, increased training for teachers, a favourable policy environment, high-level technology use still surprisingly inadequate (Ertmer, 2005). This suggests that additional barriers, specifically related to teachers' pedagogical beliefs, may be at work.

A Possibility exists that technological tools have the potential to facilitate everyday teaching for mathematics teachers and provide numerous benefits to their learners. What are the possible reasons for teachers' failure to use such powerful tools? Wenglinsky summarises the challenges for teachers related to the integration of a new tool into teaching as follows:

> [T]teachers have historically been resistant to technological innovations when those innovations have made it more difficult for them to get through the typical school day (Wenglinsky, 1998: 8).

Using computers and learning how to work with software, in this case *GeoGebra* is definitely a challenge for teachers, especially if they have no experience with new technology. In this study *GeoGebra* was introduced for the first time to the participants. Once they have mastered the basic skills to operate this software, there is long way to go before they are actually are able to effectively integrate it into their classroom teaching practice. Numerous studies provide a long list of factors that can potentially affect the use of technology in schools. In most developing African countries, including Namibia, there are many challenges to bringing ICTs into the education process.

Researchers (for example, Jones, 2004:7-18) have found a number of barriers to the integration of ICT into lessons:

- Lack of confidence among teachers during integration;
- lack of access to resources;
- lack of time for the integration;
- lack of effective training;
- facing technical problems while the software is in use;
- lack of personal access during lesson preparation; and
- The age of the teachers.

Similarly studies by Bingimlas (2009:6), Snoeyink and Ertmer (2001:87) have shown some other obstacles, indicating these or similar variations as general barriers:

- Lack of computers;
- Lack of quality software;
- Teacher attitudes towards computers;

- Poor funding;
- Resistance to change;
- Poor administrative support;
- Lack of computer skills;
- Poor fit with curriculum;
- Scheduling difficulties;
- Poor training opportunities; and
- Lack of vision as to how to integrate ICT into instruction.

Similarly, time management and organization of schools, as well as external standardised tests, problems with the hardware, software and internet connection, limited access to school computers, and lack of communication and collaboration between teachers (Cuban *et al.,* 2001) combined with lack of support from school administration make it difficult for teachers to use the new technology in their classrooms. Conversely, the first step to support teachers in this situation should be to teach them about the basic use of appropriate software and increase their comfort level concerning its potential applications in their classrooms.

By providing prepared instructional materials and depending on connectivity or working computers per classroom, teachers can get used to the idea of integrating software into their classroom practices and teaching methods without having to spend additional time on creating materials and generating ideas on how to effectively use technology for their teaching. Thus, teachers can focus on potentially modifying their teaching methods and broaden their instructional repertoire in order to provide more effective learning opportunities for their learners in ways that wouldn't be possible without technology (Preiner, 2008). By helping teachers to treat technology as an already developed educational tool and allowing them to focus on the teaching of mathematics itself, the integration of technology into everyday teaching could be facilitated in a way that would allow teachers and learners to benefit from their new technology-enhanced teaching and learning environment.

3.4 *GeoGebra's* User Interface

Since *GeoGebra* combines dynamic geometry with computer algebra systems (CAS), its program contains further elements that do not seem to be found in pure dynamic geometry.

Apart from providing two windows containing the algebraic and graphic illustration (representations) of objects, the two views are complementary: an expression in the algebra window corresponds to an object in the geometry window and vice versa of *GeoGebra*.

On the one hand, the user is able to operate the geometric tools with the mouse to make geometric constructions on the drawing pad of the graphics window (Preiner 2008). On the other hand, the user will be able to directly enter algebraic and numerical symbols input, commands and functions into the input field by using the keyboard. Whereas the graphical illustration of all objects is displayed within the graphics window, their algebraic numerical representation is shown in the algebra window. The interface of *GeoGebra* is adaptable and might be adapted to the need of the teachers and learners. *GeoGebra* is used with the algebra window, input field, coordinate axes with grid and also the drawing pad and lots of pure geometry tools. The advantages of integrating software into mathematics teaching and learning are appreciated everywhere the world. (See Figure 3: *GeoGebra's* user interface).

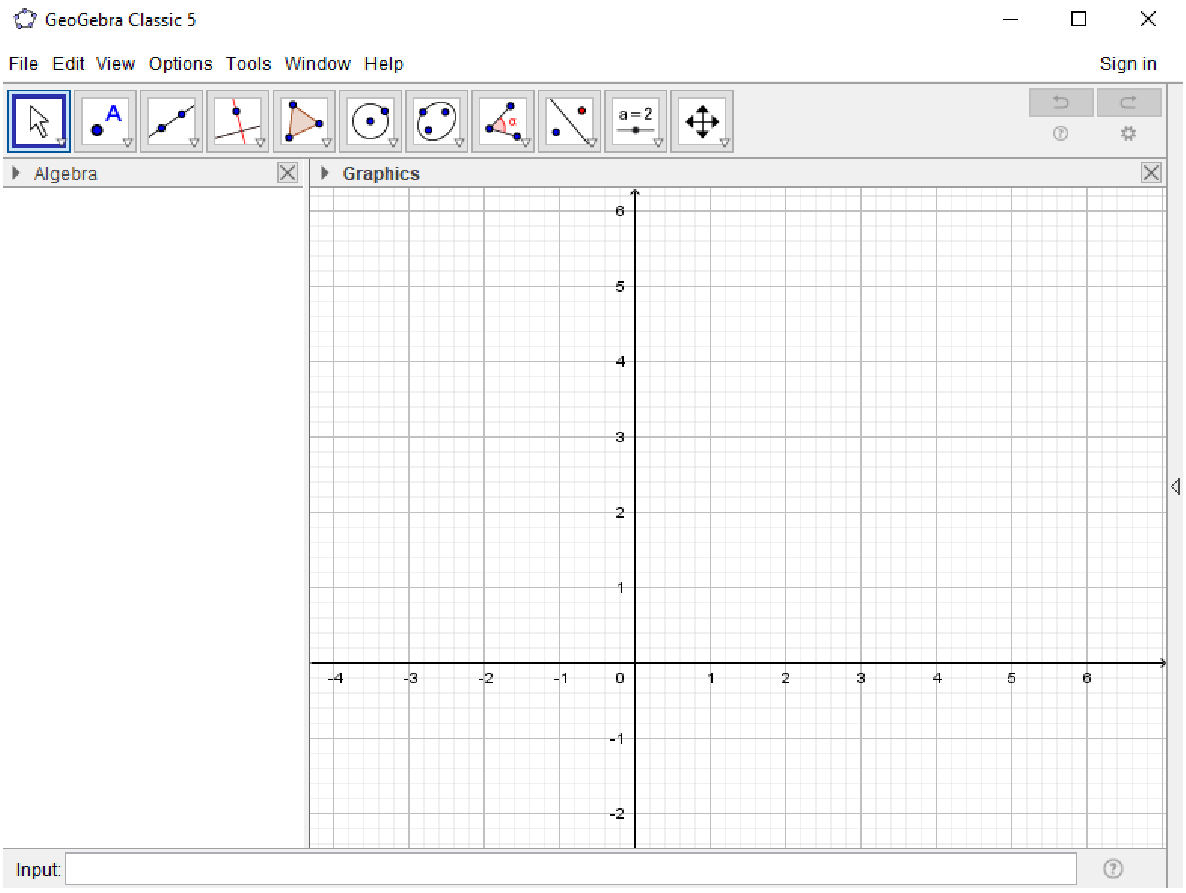


Figure 3: GeoGebra's user interface

The next section focuses on the features and interface of *GeoGebra*. *GeoGebra* has the capacity to link mathematics objects to each other in a multiple representational environment.

This design feature provides real-time dynamic changes to all corresponding representations that are simultaneously displayed on the computer screen. *GeoGebra* provides options to move between the visual mathematics options that appear on its multiple screens, and options to dynamically change the objects.

Images on the multiple screens can be enlarged for clarity, functions can be modified and colour coded (Gono, 2016; Pfeiffer, 2017) *GeoGebra* offers learners the possibility to simultaneously access different representations of the same concept. Preiner (2008) described the features of *GeoGebra* tools as outlined below.

Graphics window: The graphics window is placed on the right-hand side of the *GeoGebra* window. It contains a drawing pad on which the geometric representations of objects are displayed. The coordinate axes may be hidden and a coordinate grid may be displayed by the user. Within the graphics window, existing objects may be changed directly by dragging them with the mouse, whereas new objects may be created using the dynamic geometry tools provided within the toolbar.

Toolbar: The toolbar provides a set of toolboxes which list *GeoGebra*'s dynamic geometry devices. Devices can be activated and applied by utilising the mouse in a very intuitive way. The window on the left hand side lists all *GeoGebra* tools that are part of the chosen toolbar. If you click on one of the + symbols in front of the tool names the corresponding toolbox is opened. In the right-hand corner of the toolbar the Undo and Redo buttons can be found, which allow the user to undo and rectify mistakes step-by-step.

Algebra window: The algebra window is placed on the left-hand side of the *GeoGebra* window. It contains the numerical and algebraic representations of objects which are organized into two groups:

• Free objects can be modified directly by the user and do not depend on any other objects;

• Dependent objects are the results of construction processes and depend on 'parent objects'. Although they cannot be modified directly, changing their parent objects influences the dependent objects. Additionally, the definition of a dependent object can be changed at any time.

Additionally, both types of objects can be defined as auxiliary objects, which mean that they can be removed from the algebra window in order to keep the list of objects clearly arranged. Algebraic expressions can be changed directly in the algebra window, whereby different display formats are available (e.g. Cartesian and polar coordinates for points). If not needed, the algebra window can be hidden using the View menu.

Input field: The input field is placed at the bottom of the GeoGebra window. It permits the input of algebraic expressions directly by using the keyboard. By this means a wide range of pre-defined commands are available which can be applied to already existing objects in order to create new ones.

Construction protocol and navigation bar: Using the View menu, a dynamic construction protocol can be displayed in an extra window. It lets the user to redo a construction step-by-step with the aid of using the buttons of a navigation bar.
This function is very useful for finding out how a construction was once finished or discovering and fixing mistakes within a construction. The order of building steps can be modified as long as this does not violate the relations between established objects.
Furthermore, extra objects can be inserted at any role in order to change, extend or enhance an already existing construction. Finally the navigation bar for development steps can be displayed at the bottom of the graphics window, allowing repetition of constructions.

3.4.1 Dynamic mathematics software *GeoGebra*

GeoGebra can be viewed as software that creates a dynamic geometry environment (DGE). It is designed for teaching and learning mathematics at all levels specially it is aimed at learners and teachers in secondary school (Preiner, 2008). The software combines the ease of use of a dynamic geometry software (DGS) with certain features of a computer algebra system (CAS) and therefore allows for bridging the gap between the mathematical disciplines of geometry, algebra and even calculus (Hohenwarter & Preiner, 2007). *GeoGebra* can be used to visualise mathematical concepts as well as to create instructional materials.
The importance of a digital software visual artefact in the teaching and learning of mathematics is useful to the point that these tools have changed the idea of school mathematics practice; specifically, the utilisation of *GeoGebra* empowers a move from investing tedious time creating graphs to investing time interpreting and understanding graphs, and efficiently investigating these graphs. Consequently, the learning process in the formation of concepts can be supported by visualisation at the symbolic, graphical and numerical level (Ainsworth, 2006, Trouche & Drijvers, 2014).
In addition, *GeoGebra is* very helpful in explaining concepts and procedures through created graphics, images, and symbols. The application of *GeoGebra* software would create a conducive learning environment as it is a very dynamic educational technology with the potential to aid teachers and learner in their mathematical exploration (Zulnaidi & Zamri,

2017). For example, using *GeoGebra* features enables the teachers/learners to observe and to comment on what happens when the different parameters of a function are varied (Bayazit, Aksoy, & İlhan, 2010: 5). With the use of *GeoGebra*, leaners and teachers can have opportunities to manipulate, among many other things, tabular, graphical and algebraic representations of linear, polynomial, trigonometric and logarithmic functions (Granberg & Olsson. 2015). Figure 3.1 shows a *GeoGebra* applet that can be utilised to show different representation of straight line function $y = x + 3$ with the use of sliders.

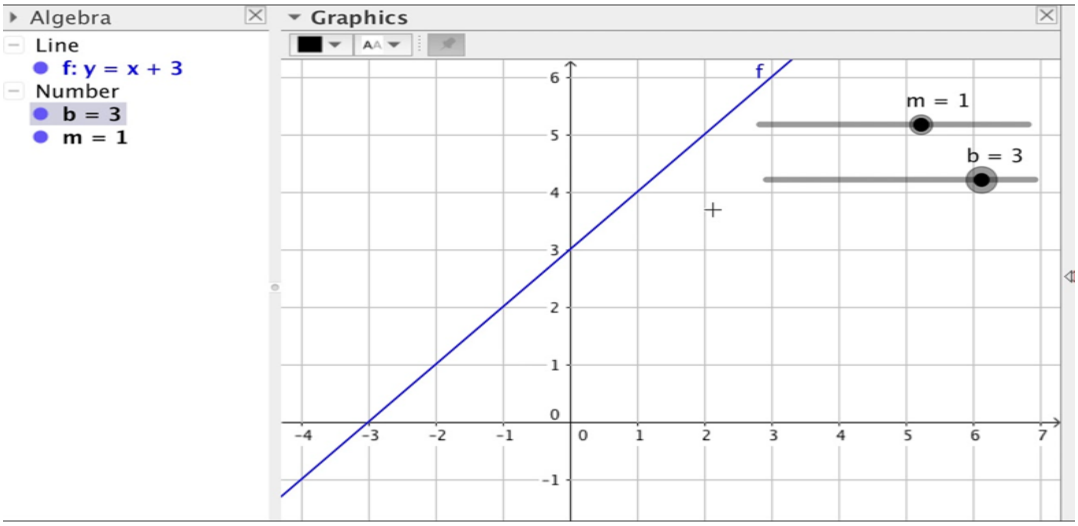


Figure 3.1: GeoGebra applet showing that the graphic view and symbolic/algebraic view are related

In both instances (Figure 3.1) 'm' and 'b' become 'variables' (Usiskin, 1988). In other words, their numerical values can be varied or changed through a 'dragging' facility (*sliders*) inscribed in the design of *GeoGebra*. Such variability of 'm' and 'b' is not easily achieved or demonstrated in a static pen-and-paper environment.

The use of *GeoGebra* makes it possible to deepen mathematical thinking ((Watson & Mason, 2007). This depends on the nature of the *GeoGebra*-designed tasks. During the workshop sections teachers can work individually or cooperatively to deepen their thinking on the mathematical symbols represented in multiple and dynamic ways. While teachers interact with the mathematical task in a pen-and-paper environment, and later working with *GeoGebra* environment they may develop and try out new structures for both pedagogy (general teaching strategies) and didactics (specific to mathematical topics) (Watson & Mason, 2007).

GeoGebra as dynamic geometry software (DGS) offers support for teaching more than only geometry (Hohenwarter, Hohenwarter, & Lavicza, 2010). In this study several mathematical tasks were given that demonstrate changeable graphs (Kaput, 2000) that can show a generalisation and support the teaching of function. Graphs produced by teachers themselves were useful for better understanding of the teachers' learning situations; manipulating the graph and working in groups improved their mathematical thinking and gave them the ability to take up the challenge of making use of *GeoGebra* to explore polynomial functions (linear and quadratic functions in the case of this study).

For example, an equation (linear equation in this case) can be manipulated with *GeoGebra*; the user can change parametric values of symbolic representation by sliding parameters *m* and *b* of that equation, and so the change of the tabular and graphical representations can be observed simultaneously (Dockendorff, & Solar, 2018). In this way, the relation between the representations can be analysed. This can potentially support teachers' and learners' understanding of the function concept and its structure in different or multiple ways.
With the use of dynamic mathematics leaning environments, sliders are increasingly used as an important pedagogical tool to create interactive mathematical activities that encourage leaners and teachers to explore mathematical ideas (Bu, & Haciomeroglu, 2010).

In a case of *GeoGebra,* a slider is usually a single mathematical object that has two representations. Firstly, in algebraic representation, a slider is just a variable that has a default or defined interval for its values. Secondly, a graphical representation, this appears as a segment which allows the user to adjust the value of the corresponding variable through dragging. In addition, a slider also has other useful properties, such as colour, position, segment length and thickness.

A variable can be called a constant variable if its value does not change in a relative sense. For example, in the general form of a linear function y = mx + b, m and b are constants, representing the slope and the y-intercept of the function, respectively. To allow learners and teachers the opportunity to explore the implications of the slope m and the y-intercept b, we could use sliders for m and b. In *GeoGebra* we could define two (constant) variables m and b with some initial values and convert them into sliders by right-clicking and making the object graphically visible (see Figure 3.1).

3.5 Professional Development (PD) of Mathematics Teachers and Their Experiences with Digital Technology (*GeoGebra*)

This section focuses on research studies that have been conducted on PD in relation to technology (*GeoGebra*). TPD has become a vital component of ongoing educational reform everywhere in the world, motivated by accountability, diversity and especially the constant advances in educational technology and innovative instruction paradigms (Darling-Hammond, Barron, Pearson, Schoenfeld, Stage, Zimmerman, and Cervetti & Tilson, 2015).

Teacher preparation in professional learning groups is widely accepted as a TPD approach that can have a substantial effect on teacher's conceptual understanding and experience (Brodie & Borko, 2016b; Horn 2010; Koellner & Jacobs, 2015).

The TPD studied and presented in the related literature in a variety of different ways. But recognising that PD is about teachers learning how to learn and translating their expertise into an experience for the good of their student's development is still at the heart of these efforts. Teacher's professional learning is a dynamic process that involves the cognitive and emotional participation of teachers individually and jointly the capacity and ability to evaluate where each person stands in terms of convictions and values and the discovery and execution of suitable alternatives for advancement or reform. Much of this happens in particular in the educational policy environment or in the school system, some of which are more appropriate and conducive to learning than others (Avalos, 2011).

The TPD process also depends on the leaners' and teachers' needs related to their classroom practice. Thus, formal structures such as workshops and other ICT interventions may serve some purposes. Not all TPD interventions are relevant to all mathematics teachers. There is a constant need to discuss, explore and reflect in TPD on the interacting links regarding their leaners' educational development, the expectations of their educational system, teachers' working conditions and the opportunities to learn that are available to them. Furthermore, while studies show that teachers are more confident in using ICTs and more convinced of the pedagogical benefits of ICTs as a result of TPD (Bennison & Goos, 2010; Jimoyiannis & Komis, 2007), it is still unclear whether teachers will eventually integrate technology into their daily teaching practices. For those who do adopt technology, some may return to a traditional teacher-directed approach (Pedersen & Liu, 2003) to teaching even after receiving educational ICT training. Possible reasons for returning to the old ways are the suitability of the ICTs in relation to the intended and examined mathematics curricula.

Hohenwarter et al. (2009) conducted a study during a three-week PD programme organised for secondary school teachers in Florida. This study described the approaches for introducing dynamic mathematics software to secondary school teachers in order to provide a basis for the development of instructional materials for TPD. The open-source program *GeoGebra* was selected from the other popular software packages for teaching and learning mathematics e.g. dynamic geometry software (DGS), computer algebra systems (CAS), spreadsheets because it is a flexible platform that integrates the ease of use of DGS with CAS features. The purpose of the researchers was firstly to assess the usability of *GeoGebra* and to recognise features and resources that could cause difficulties during the introduction of *GeoGebra*; secondly to establish the complexity criteria for assessing and categorising dynamic geometry tools and their degree of difficulty in order to better address the needs of beginner users in future workshops; and finally, to provide a basis for the improvement of introductory *GeoGebra* materials and technology-enhanced PD of secondary school teachers.

In another TPD study conducted by Mainali and Key (2012) a *GeoGebra*-based professional development course was designed and implemented. The beliefs and feelings regarding the software (*GeoGebra*) and the technological difficulties of 15 mathematics teachers in Nepal were investigated. The main purpose in that study was to advance participants' skills and confidence in interacting with *GeoGebra* tasks. This study was conducted over a period of four days. The first three days covered tasks related to technical activities that were essential to ensuring the mathematics teachers had the minimal and necessary skills needed to use *GeoGebra.* The teachers participated in morning discussions about *GeoGebra*, while in afternoon session they engaged in activities to explore more practical tasks on their own. Data-collection tools such as questionnaires, interviews and field notes were applied at the end of day four. The participants in general were excited by the use of *GeoGebra*, since it offered a dynamic graphical and symbolic/algebraic view. The participants were highly motivated and describe the software as useful tool for practical mathematics leaning, even though there were constraints and difficulties with the use of advanced technology.

Mainali and Key (2012) and Ruthven, Hennessy and Deaney (2008) also claim that DGS provides innovative tools that can support and go beyond pen-and-paper methods, enhancing the discovery process by given teachers and learners opportunities to discover many more examples on the computer screen that could be difficult to achieve in a pen-and-paper environment. The multiple related representations in dynamic mathematics software environments provide an opportunity for direct manipulation of both symbolic and graphic representations.

Bulut and Bulut (2011, cited in Kul, 2013) investigated 47 Turkish pre-service teachers with the reason to explore student teachers' views about the dynamic programming (*GeoGebra*). They were engaged with a *GeoGebra*-based course during their formal training.

The Pre-service teachers were taught how to use arcane syntax (linguistic structure) to illustrate dynamic text or images through complex worksheets. Interviews were used to collect information in the process of their investigation. According to the results of this study, the student teachers preferred the inclusion of pictures with the background of the worksheets in order to connect geometry with real-life examples. They also assumed that *GeoGebra* could be used to generate test questions, construct web pages, and calculating algebraic equations. The mathematics pre-service teachers in Bulut and Bulut's (2011) study were able to successfully use the basic computer operations, for example using the computer keyboard, the mouse and entering commands because of their previous experienced acquired in some computer courses. The participants also acquired the basic knowledge of the geometry construction command of *GeoGebra.* In addition, the teachers expressed as well that with the use of *GeoGebra* it is possible to have multiple representations of mathematical concepts. Thus, mathematics teachers had the opportunity to construct their mathematical knowledge in different way.

According to Yastrebov and Shabanova (2015), learners and teachers can also create and test dynamic models and combine them with pen-and-paper activities. In this study teachers did not always work on *GeoGebra*. In some of the designed activities the teachers used pen and paper in combination with *GeoGebra* (see Appendixes L1, L2). (This is related to the exploration of the parameters of a quadratic polynomial.) In this example the design of *GeoGebra* enables the participants/teachers to explore and then comment on the influence of 'a' as well as 'x' as variables in the case of$(a, x) = ax^2$. However, the teacher/researcher encouraged the teachers to use such combination with the expectation of developing teachers' visual thinking and reasoning abilities.

Ozyildirim, Akkuş-İspir, Güler, İpek and Aygün, (2009) researched perspectives of fourth-year pre-service teachers on teaching geometry and consolidating dynamic geometry software (including *GeoGebra* and *Geometer Sketchpad*) into geometry teaching during a multi-week course. Toward the start of the course, 75 participants answered three questionnaires. Their reactions were then utilised as the premise of semi-structured interviews conducted on a voluntary basis toward the end of the course. The course presentation was divided into two phases: (i) introductory, (ii) project development. In the introductory phase, general information about the features of DGS was presented to familiarise the participants with the program.

The students stressed that to use DGS as a sole means of teaching mathematics would be insufficient and that lessons should be reinforced with concrete manipulatives. The course participants reported that the language issue and the complexity of the commands acted as a barrier to using the system. The authors concluded that the participants had realised the significance of using DGS in geometry teaching, as it offers an experiential learning environment and enjoyable activities for students.

Another quantitative study conducted by Zulnaidi and Zakaria (2012) in 124 high schools explored the conceptual and procedural knowledge of the function concept. The study used a quasi-experimental non-equivalent pre-test/post-test control group design. The results revealed a significant difference between groups. It was concluded that *GeoGebra* improved not only the conceptual knowledge high school students but also their procedural knowledge. This study encourages teachers and leaners to use technology in teaching and learning mathematics, and specifically the use of *GeoGebra* to provide support and training to mathematics teachers.

There have been positive effects demonstrated by experimental groups using dynamic software performing better than control groups using, for example, pen and paper on post-tests (Leong, 2013; Zengin, Furkan, & Kutluca, 2012). Similar findings have been reported by Koyuncu, Akyuz, and Cakiroglu, (2015) that suggest that a combination of technology and the use of pen and paper could be useful to teachers and learners. For instance, software like *GeoGebra* that performs calculations and draws geometric figures and graphs enables teachers and learners to focus on conceptual understanding rather than executing procedures during problem solving (Hwang & Hu, 2013).

3.6 *GeoGebra*: Epistemic and pragmatic values

In the case of techniques that digital technologies such as *GeoGebra* offer for solving tasks (such as graphing or calculating), Artigue (2002:248) distinguishes two types of values that can be related to these techniques. She called these two aspects of tool techniques the pragmatic values and the epistemic values:

> A technique is a manner of solving a task and, as soon as one goes beyond the body of routine tasks for a given institution, each technique is a complex assembly of reasoning and routine work. I would like to stress that techniques are most often perceived and evaluated in terms of pragmatic value, that is to say, by focusing on their productive potential (efficiency, cost, and field of validity). But they have also an epistemic value, as they contribute to the understanding of the objects they involve, and thus techniques are a source of questions about mathematical knowledge. Artigue (2002:248)

To clarify the distinction between these values we can offer an example about the difference between variables and parameters in a formula such as $y = ax^2$ (see Appendixes L1 and L2, where x, y are variables and a is a parameter). The distinction is important, but can be difficult for learners and teachers to appreciate. However, a dynamic geometry environment can help the users make sense of the difference.

One can create a slider for the parameter a (see figure 3.2), then moving it will change the shape of the parabola. Moving the slider, the point R will change its location automatically as well.

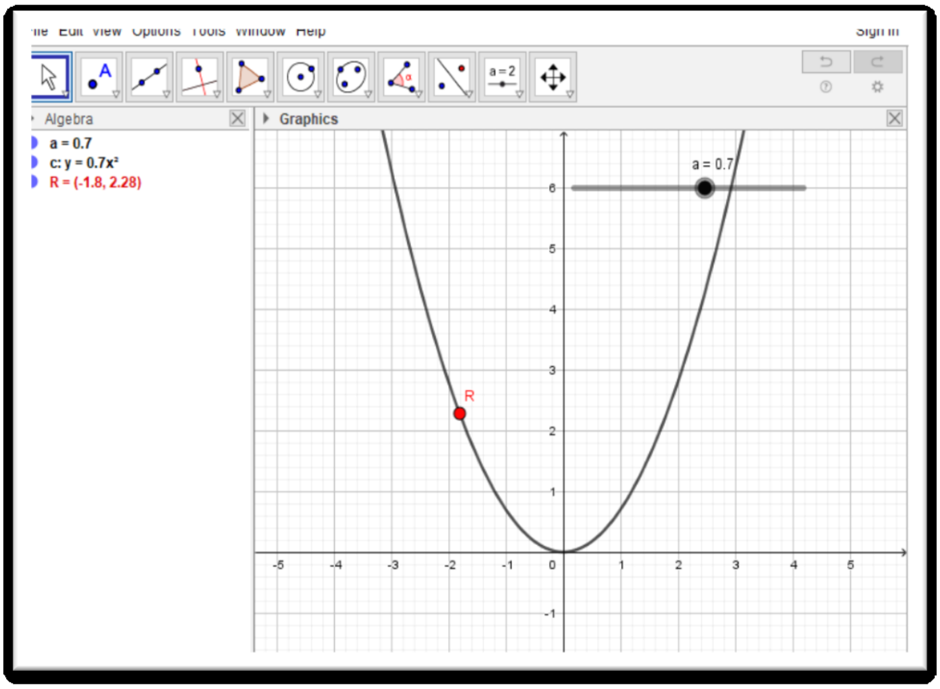


Figure 3.2 Dragging point R and parameter a in GeoGebra

There are at least two opportunities to demonstrate the movement of point R: (1) visual representation in the display window of *GeoGebra* as showed on the right, and (2) algebraic representation on the left. Each value assigned to the slider moves the point R to a different position on the function. While demonstrating this movement dynamically, teachers can verify the relation between abscissa and the ordinate (Karadag & McDougall, 2011). They can ask learners to verify those numeric values through other means such as pen-and-paper method. The link between these representations provided by *GeoGebra* could help learners and teachers transfer their visual observation to formal algebraic notation and consolidate the abstract relation between the point R and the function f(x) (Karadag & McDougall, 2011).

The utilisation of dragging parameter *a* and point R has pragmatic value, since doing this changes both *a* and the location of R and the shape of the parabola (see Figure 3.3). These activities can likewise help reveal the epistemic parts of the pragmatic action of dragging. For example, dragging *a* changes the whole shape of the curve, while dragging *R* changes the location of R on the curve. The epistemic part in this case deals with observing how the pragmatic dragging actions have epistemic effects, namely, shifting the parabola in particular directions in the graphics window. Pragmatic effects deal with 'getting answers' in a short period of time, whereas epistemic issues concern a deepening of mathematical thinking and understanding of the literal and numeric symbols encoded on the design of the sliders.

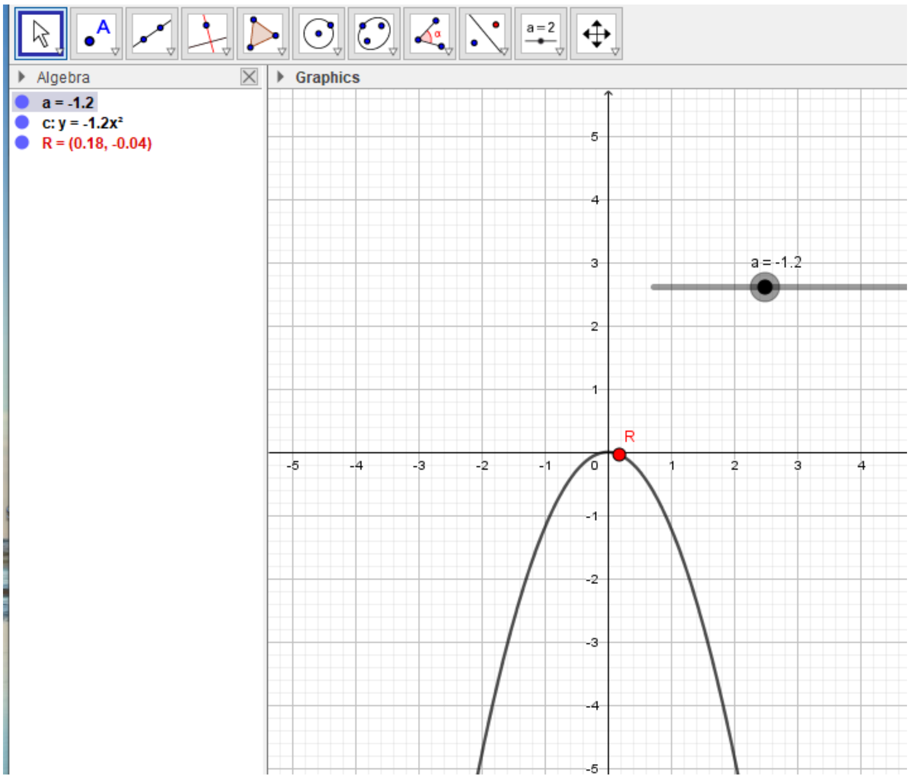


Figure 3.3: A screenshot dragging point R and parameter

Artigue (2007) pointed out that making technology genuinely scientifically helpful from an educational perspective, whatever the technology in question, requires methods of integration that harmonise the pragmatic and the epistemic values, for example, dragging a slider in *GeoGebra* requires tasks and situations that are not simple adaptations of pen-and-paper tasks. It requires situations that very frequently cannot be conceived in a pen-and-paper environment because in *GeoGebra* design, multiple linked representations of functions are built into the design. The epistemic and pragmatic values are dialectically combined (Kieran & Drijvers 2006), but there is frequently more consideration to the one than to the other. Specifically, when the focus is on learning how to utilise an instrument, the pragmatic value is chosen. In any case, since many of TPD projects that are moving towards technology-based exercises that reduce the learning on how to utilise the artefact, it is possible that the epistemic aspect gets to be foregrounded.

The demonstration of the dynamic effect in one representation, and the link between two different representations, could help learners and teachers to better process the information and develop a better understanding (Bayazit & Aksoy, 2010). For example, in *Geogebra* the pragmatic action of dragging can help to reveal epistemic aspects of this action. In the case of

GeoGebra, or whatever technologies employed, there is a need for a reasonable balance between the pragmatic and the epistemic values (Artigue, 2007:73), for example, dragging sliders. There is a challenge at the theoretical level, as some teachers (users) that do not entirely master the mathematical knowledge and methods involved in solving tasks (Chiappini & Pedemonte, 2009 cited in Losada, 2012). As result, the epistemic value of the techniques (*GeoGebra* or some other) can remain hidden. This can be a problem for the educational context where the use of technology should help not only to yield results, but also to support and promote mathematical learning and understanding (Artigues, 2007).

3.7 Instant feedback

GeoGebra provides fast and consistent feedback. This study involved participants using dynamic mathematics software (*GeoGebra*) to construct graphs of linear and quadratic functions. They manipulated the representations using sliders, getting immediate feedback on the nature of the transformed representations. Instant feedback allowed participants the choice of constructing their own understanding of the functions being studied based on the common features of the graphs that were immediately presented on the screen, or it allowed participants to monitor and manage their learning which is consistent with constructivist-based learning (Neo & Neo, 2009).

3.7.1 Speed of execution

Dynamic mathematics software enables teachers and learners to produce many examples when exploring mathematical problems. This supports observation of patterns, and fostering the making and justification of generalisations (Henningsen & Stein, 1997). The *GeoGebra*-based program allowed teachers and learners to investigate more complex problems, providing fast ways of presenting data in different forms, something that takes too long and is difficult to achieve in a static geometry environment. By reducing time, a *GeoGebra*-based program allows teachers and learners to spend more time to manipulate and develop mathematical understanding and reasoning. It provides teachers with a mathematical digital tool that can mediate their learning. In this study, the use of *GeoGebra* offered participants opportunities to investigate many cases of linear and quadratic functions at high speed, that is, feedback was immediate or instant.

It can be seen that early investigations indicated that teachers were in favour using of a skill-development package that provided quick calculations and gave immediate feedback to users to extend teachers' and learners' capacity in tackling routine problems issues or to guide users to self-learning. With expanding software showing up on the market and accessible to teachers, studies showed various ways mathematics teachers could use computers for different pedagogical purposes (Li & Ma, 2010), and the development of mathematical thinking and reasoning was highlighted. Visual representation, for instance, was highlighted as one of the foremost critical changes that computers introduced.

The difference between mental activities using traditional teaching methods (pen and paper-based) and alternative methods (computer-based) was investigated by Borba and Villarreal (2006a). They concluded in their study that the traditional teaching always valued symbolic and logical activities and ignored visualised activities which could be assisted by using computers.

In 2009 the British Educational Communication and Technology Agency published two documents for primary and secondary teachers and learners (BECTA, 2009a, 2009b) which proposed a few major ways that teachers and learners could benefit from engaging in learning with ICT in mathematics, for example, learning from feedback; observing patterns; developing visual imagery; exploring data. Several examples were indicated in the document to clearly explain each opportunity; Table 3.1 below briefly summarises the information.

Table 3.1 Opportunities provided by GeoGebra (Adapted from Becta 2009b)

Opportunities	Examples
Learning from feedback	Teachers use an improvement strategy to explore graphs of a function (e.g. $y = mx + c$) and $y = ax^2 + bx + c$) via varying coefficients and observing.
Observing patterns	Teachers drag a point on the screen and watch the movement of another point, and make a conjecture of the relationship between these two points; teachers use

	number grid to learn calculation.
Developing visual capability	Teachers manipulate diagrams dynamically by *GeoGebra*-based software to generate their own mental image; exploring 2-D shapes dynamically.
Exploring data	Working with real data and observe them in different ways; or explore relationships between different variables to gain a deep understanding of functions.

3.8 Chapter Summary

This chapter discussed research on high school mathematics teachers' learning experiences during professional development interventions aimed at improving their understanding of functions using *GeoGebra*. The literature showed that the teacher plays an important role in integrating of technology into mathematics education. This chapter reviewed relevant and current literature on the use of technology in education. It summarised the literature on uses of information and communications technology (ICT) in the high schools in Namibia and around the world, with a particular focus on the role of ICT (*GeoGebra)* on teachers' professional development. It explained the supporting and constraining factors that influence ICT integration into mathematics education, the literature specifically on *GeoGebra-based* epistemic and pragmatic values was discussed. The next chapter provides a discussion of how the research study was conducted.

CHAPTER FOUR

RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

This chapter provides an overview of the research design, methodology and methods used in this study. Section 4.2 states the purpose of the study and the research questions. Section 4.3 explains the selection of the qualitative research design, the theoretical perspective that led to the choice of an interpretive paradigm, the case study approach and the methods used in the research. Section 4.5 discusses how the data were prepared, coded and analysed. Section 4.6 indicates how the quality of the research is assured by discussing concepts related to credibility, transferability, dependability and conformability. Section 4.7 deals with the ethical issues relevant to this research: the process of obtaining permission to conduct the research, ensuring voluntary participation of the participants, procedures followed to protect the privacy of research subjects and procedures to report the research finding. Section 4.8 is a summary of the chapter.

4.2 Purpose and Research questions

The purpose of this study was to investigate the high school mathematics teachers' learning experiences during a professional development intervention aimed at improving their understanding of functions using *GeoGebra*. More specifically, the study attempts to answer one main research question and two research sub-questions. The main research question is:

1. What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using GeoGebra?

The following is the research sub-questions:

- How central is knowing mathematical functions in the Namibian secondary curriculum?
- What does research have to say about the potential of *GeoGebra (GGB)* in facilitating the understanding mathematical functions?

- What does research have to say, what are key issues in using GGB-represented mathematical functions?

4.3 Research design

Research is a systematic and complex endeavour influenced by beliefs, feelings and expectations. Because of the complexity of the undertaking, researchers utilise a range of research designs to obtain answers to their research questions (Kumar, 2019). A review of the literature reveals several distinctions and slight differences in the concepts used to describe research design. According to Bogdan and Biklen (2007: 54), research design is 'the researcher's plan of how to proceed' with the research project. Similarly, Durrheim (2006:34) describes research design as a strategic framework for action that serves as a bridge between research questions and the implementation of the research. Durrheim further indicates that a research design may be viewed as a process consisting of five stages:

Stage 1: designing the research question,

Stage 2: designing the research,

Stage 3: data collection,

Stage 4: data analysis, and

Stage 5: writing a research report.

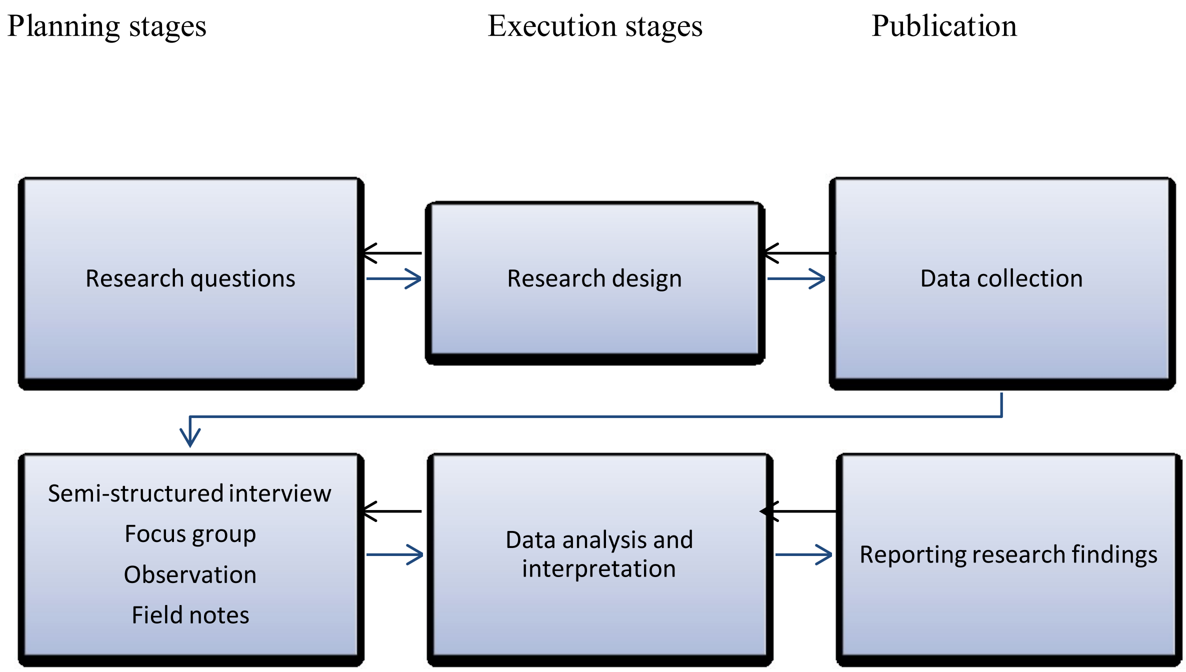


Figure 4.1 the research process (Adapted from Durrheim (2006:34))

Designing a study includes making multiple decisions about the way in which the data will be collected and analysed to ensure that the final report answers the initial research question. In Figure 4 above notes that the arrows that link the five research activities are bi-directional (Durrheim (2006:34). The research process is made up of a sequence of activities, beginning with a research question and ending with the report; one normally begins with a research project and then develops a design that will answer the research question, and finally conducts the research and writes up the findings.

Babbie and Mouton (2001) and Yin (2011) describe the research design as a blueprint for conducting research. In addition, Yin (2009:27) explains that the research design is much more than a work plan, while Flick (2014:112) is of the same opinion that research design concerns issues of how to plan a study.

There are three well-recognised research designs, namely quantitative, qualitative and mixed method research designs (Creswell, 2009). Researchers choose a quantitative research design if their philosophical assumptions are associated with a positivist paradigm, and choose qualitative research design if their philosophical assumptions are associated with an interpretive paradigm, as in the case of this study (Creswell, 2009; Mertens, 1998). For this study, a qualitative approach was considered more appropriate to explore the social reality of the teachers' experience than a quantitative approach would have been. The reason for choosing a qualitative research design is provided in Section 4.3.1.

4.3.1 Choice of qualitative research design

Most of the literature on qualitative research begins with some attempt to define qualitative research; for example, Denzin and Lincoln (2011) offer the following definition:

> "Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that makes the world visible. These practices ... turn the world into a series of representations including field notes, interviews, conversations, photographs, recordings and memos to the self ''(Denzin & Lincoln, 2011:3).

Other researchers support this definition. It is a naturalistic, holistic and inductive approach that seeks to understand phenomena in context-specific settings, such as a "real-world setting where the researcher does not attempt to manipulate the phenomenon of interest" (Durrheim, 2006; Patton, 2002: 39). Some other writers describe qualitative research as "any kind of research that produces findings not arrived at by means of statistical procedures or other means of quantification" (Strauss & Corbin, 1990: 17).

To define qualitative research Bogdan and Biklen (1982), Merriam (1998: 5), Lincoln and Guba (1985) and Patton (1990) list the following key features of qualitative research that are relevant to this study:

- It is usually conducted in natural settings (classrooms, schools, school venues such as computer laboratory, library);
- Qualitative research is concerned with accurately capturing the participants' perspectives;
- The researcher is the key instrument for generating data rather than inanimate inventories or impersonal questionnaires;

- Qualitative research is an inductive approach, that is the researcher builds theory from observations and intuitive understanding;

- Qualitative research uses the natural setting as the source of data the researcher attempts to observe, describe and interpret settings as they are, keeping what Patton (1990:55) calls an "emphatic neutrality" where the researcher attempt to be non-judgmental when compiling findings;

- Qualitative research is interpretive research, aimed at discovering the meaning of events for the individuals who experience them, and the interpretations of those meanings by the researcher;

- The meaning and understanding obtained through qualitative research are richly descriptive (Merriam, 1998)

Babbie and Mouton (2001), Bogdan and Biklen (2007) and Creswell (2009) agree with the above characteristics of qualitative research. However, Creswell and Poth (2017:45) provide the following additional features of qualitative research:

- Natural setting: qualitative researchers often collect data in the field at the site where participants experience the issue or problem. They do not typically send out instruments for individual to complete, such as in survey research. Instead, qualitative researchers collect information by actually talking directly to people and seeing them behave and act within their context;

- Data are generated through multiple sources. The researcher typically collects data through multiple forms, such as interviews, observations and documents rather than relying on a single data source, and then the researcher reviews all the data to make sense of it, organizing it into categories or themes that cut across all the data sources;

- Qualitative research is an emergent design. The original plan for the research may change after the researcher enters the field; data gathered may lead to a change in the research focus or questions (Creswell & Poth 2017:47). Although all steps of the research are planned before the researcher gathers any data, what has been planned originally may change and thus the original plan for the research cannot be strictly followed and has to be modified. Correspondingly, the research methods are often adjusted, which why it is often called an "emergent design" (Ary, Jacobs, Razavieh & Sorensen, 2006:454);

- Qualitative researchers often use a theoretical lens to investigate the subject of their studies.

Qualitative research strives to provide a holistic picture of the problem or issue under study. This involves reporting multiple views and identifying the many factors involved in a situation. In the light of the characteristics mentioned above, a qualitative research design was considered to be the most appropriate approach for this study. Firstly, this study qualifies to be qualitative because it took place in computer laboratory school venue. This research setting also provided an opportunity to more generally investigate participants' experiences (Taylor, Bogdan, & DeVault. 2015), views and expectations about the use of ICTs (*GeoGebra)* to acquire deeper understanding of function concept. Since interventions were held in a computer lab, teachers faced the challenges of working and communicating in a lab environment. Secondly, this study is specifically concerned with understanding the mathematics teachers' subjective learning experience related to an intervention addressing the use of *GeoGebra* to focus on functions as they participated in the five workshops planned for this study. The series of workshops and interventions for the teachers generated in-depth and descriptive information about the participants' perspectives through the application of multiple research techniques. Thirdly, in contrast to quantitative research design, where research instruments are inanimate (e.g. questionnaires), in qualitative research design the researcher is an instrument in selecting the designs, generating data through a series of qualitative methods (including interviews and questionnaires that are also used in qualitative research), analysing and interpreting data (Bogdan & Biklen, 2007; Merriam, 1998; Patton, 2002). Fourthly, the intention of this study is not to produce generalisable findings, as is common in quantitative research. Lastly, as this study is there is no theory to be tested. Instead, the intention is to build theory using the data generated from the field through a series of interpretations and interaction with the participants instead of using a deductive approach that tests existing theory as often in the quantitative approach.

Furthermore, qualitative research is particularly useful for obtaining insights into common or problematic experiences and the meaning attached to these experiences by selected individuals (e.g. biography, autobiography, case study, oral history, life history and autoethnography) (Leech & Onwuegbuzie, 2007).

4.3.2 Dimensions of the research design

Creswell (2009) specifies that the development of a research design involves decisions in three interconnected dimensions namely paradigm methodology and methods as shown in figure 4.2. Depending on the research question the paradigm affects the methodology, which in turn determines the methods used to produce and evaluate data. The paradigm also tells about the choice of methods. An interpretive paradigm was found to be sufficient in this analysis. Centered on this model case studies and grounded analytical methodologies were used to carry out the analysis and interpret the results respectively. In exchange, these methodologies affect the approaches used to produce data. Different methods have been used to produce data in this study. These three dimensions of research design are further expanded in section 4.3.2.1.

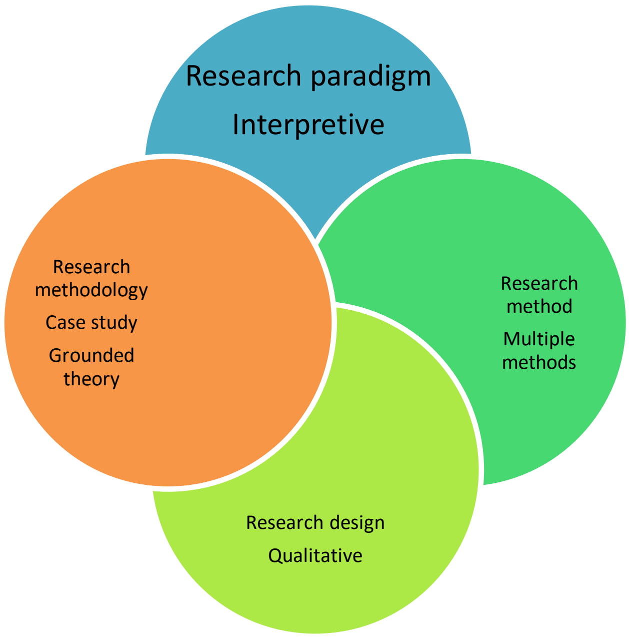


Figure 4.2: The interconnectedness of research paradigm, methodology and method (Adapted from Durrheim (2006) and Creswell (2009)

4.3.2.1 Research paradigms

4.3.2.2 Ontology, epistemology and methodology

Three orientations affect the way researchers interpret and view the universe, namely ontolog y, epistemology and methodology (Gray, 2009; Denzin & Lincoln, 2011). Ontology is a field of philosophy that discusses the essence of existence and asks whether there is a true world b eyond our understanding (Guba, 1990; Guba & Lincoln, 1994; Nieuwenhuis, 2007a; Gray, 20 09; Denzin & Lincoln, 2011; Newman, 2011).

According to Neuman (2011), within ontology there are two opposing positions, namely the p ositivist and the nominalist interpretivist positions. Positivists believe the universe is full of th eories waiting to be found out there. In comparison, the interpretivists believe that truth is con structed by interpretations informed by personal experience (Neuman, 2011).

The latter position is deemed important because this research focuses on the subjective experience of the participants while they were interacting with the *GeoGebra* activities. That is why the researcher chose to perform inductive analysis, rather than testing a hypothesis. Epistemology explains the researcher's connection to what's being examined.

Epistemology is a field of philosophy concerned with knowledge learning, and it focuses on what we can know about the universe and how we can know it (Guba, 1990; Guba & Lincoln , 1994; Nieuwenhuis, 2007a; Gray, 2009; Denzin & Lincoln, 2011; Neuman, 2011).With a po sitivist point of view, the researcher holds a distance from what is experienced and the acquisi tion of knowledge is objective, free from any kind of value judgments. In comparison, the int erpreters assume that understanding is acquired by observation, perception, and reflection (G uba, 1990; Guba & Lincoln, 1994; Denzin & Lincoln, 2011; Neuman, 2011), guided by comp lex beliefs, values , and assumptions (Carr & Kemmis, 1986). This research is situated within an interpretive paradigm from an epistemological point of view and recognises that reality is subjective and value-laden.

This study is thus based on the learning experience of the participants linked to an intervention that focuses on functions with the use of *GeoGebra*. That is why the researcher chose to do inductive research for this purpose. An interpretative researcher is therefore in constant search for meaning rather than evidence that can be shown in numbers or performed under experimental condition or testing a hypothesis.

Methodology deals with how a researcher obtains the desired information and understanding of the phenomenon under study (Denzin & Lincoln, 2011; Gray, 2009; Guba & Lincoln, 1994; Neuman, 2011). Quantitative researchers utilise primarily deductive or experimental methods that focus on verification of a hypothesis, whereas qualitative research utilises inductive approaches such as interviews, observations and document analysis to generate theory (Guba & Lincoln, 1994). From a methodological perspective, this study utilises a qualitative design that is inductive and exploratory in nature.

The above reflects my epistemological position that knowledge emerges through human interactions and negotiations with other peers, the social environment and technological tools, as in the current study. It is assumed that knowledge is dependent on and an outcome of social construction. Accordingly, the main research question "What are high school mathematics teachers' learning experiences during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra* was directed to the participants. Through this question, the current study mainly concentrates on the process of the social phenomena, as is the case with most qualitative studies (Robson, 2011). More specifically, in this study, an attempt was made to gain insights into high school mathematics teachers' learning experience and reflection processes within a PD intervention. Since the current study seeks to understand the mathematics teachers' experience of attempting to understand the concept of functions with the use of *GeoGebra*, the teachers were given the opportunity to experience and to interact through the of use *GeoGebra*, which is a technological and pedagogical tool.

From an epistemological point of view, this study is situated within an interpretive paradigm and acknowledges that reality is subjective and value-laden. This paradigm makes it possible to gauge the teachers' learning experiences in the case of interventions and conversations around *GeoGebra* software and it affordability to mathematics teachers understanding of function concept.

In summary the three generic orientations discussed above direct researchers thought and the way they see the universe are thus collectively referred to as paradigm (Gray 2009; Denzin & Lincoln 2011). The notion of a paradigm is explored further in section 4.4 below. Table 4.1 describes the various paradigms in terms of their ontological epistemological and methodological orientation.

Table 4.1: The three orientations of a paradigm

	Orientation		
	Ontology	Epistemology	Methodology
Paradigm			
Positivist	Single reality that can be measured and examined. The aim is to predict and track belief in the relationship between cause and effect	Objective The only knowledge is scientific knowledge which is truth, reality is apprehensible	Deductive Experimental Hypothesis testing Quantitative
Interpretive	Multiple realities The focus is on understanding what is happening	Subjective Observer is part of what is being observed Co-created, multiple realities	Inductive Interpretive Qualitative
Critical	Multiple realities shaped by social, political, cultural, economic, and ethnic and gender values. The focus is on uncovering institutional structures and helping people to change conditions.	Subjective Observer constructing versions Finding are based on values, local example of truth	Participatory Emancipatory Qualitative

Source: Adapted from Terre Blanche and Durrheim (2006:6)

4.3.2.3 Paradigm

The paradigm of any research acts as a lens to understand and view the world. the first thing a researcher has to outline is the paradigm that underpins the study, the researchers' point of view or the frame of reference for looking at life or interpreting reality (Delport, Fouche & Schurink 2011:297).Thomas Kuhn was the first to present the term paradigm to apply to the worldview that guides a research process (Kuhn cited in Guba, 1990). Although similar in their descriptions, many scholars have attempted to re-define paradigm. For example, to Denzin and Lincoln (2011:13) paradigm is a “net that contains the researcher’s epistemological, ontological and methodological premises”. Guba defines paradigm as “a basic set of beliefs that guide action” (Guba, 1990: 17). To Mertens (1998:6) paradigm is “a way of looking at the world". Similarly, Bogdan and Biklen (2007:24) describe paradigm as "loose collection of logically related assumptions, concepts, or propositions that orient thinking and research".

Three research paradigms particularly positivist interpretive and critical paradigms are widely recognised (Carr & Kemmis 1986; Henning, Van Rensburg & Smit 2004; Merriam 1998; Waghid, 2000). Selecting an appropriate paradigm is a crucial part of any research that influences and guides a research process. Since the paradigm has its own strengths and disadvantages, it is necessary to select the one that best serves the purpose (Neuman, 2011). The three paradigms will be described below, followed by a reason for selecting the interpretive paradigm for this study.

Positivists explore social processes by applying research methods and procedures used in natural sciences (Mertens, 1998). Positivists assume that reality emerges independently of human reasoning and perception (Guba, 1990; Neuman, 2011), which suggests that everyone experiences reality in the same way. A researcher following a positivist paradigm keeps a distance from what is being researched. Knowledge acquisition is believed to be objective with no space for any sort of value judgments (Guba & Lincoln, 1994). Positivists are monitoring the situations of the research project in order to explore the relationship between cause and effect. In addition, they rely on statistical inferences to generalise their results to a broader community (Guba 1990; Neuman 2011). However, for several reasons, the positivist paradigm is not suitable for this research which aims to investigate the subjective learning experiences of high school mathematics teacher intervention based on the use of *GeoGebra* with an emphasis on function. Firstly, the intention of this study is not to separate and control variables. Because this research is done in a school or classroom setting it is difficult to establish controlled situations and to examine a single variable at a time. Studying school or classroom environments requires contextual analysis rather than numerical descriptions. Secondly, the purpose of this research is not to predict and evaluate a hypothesis but rather to consider describe and explain the learning experiences values and assumptions of high school mathematics teachers. Thirdly it is not possible to observe and examine an incident or intervention in a school or classroom environment in an impartial manner without influencing or interacting with it. Finally, it is not the intention of this researcher to generalise the findings to a wider population.

Neuman 2011:108 describes the critical paradigm as a process of inquiry that moves beyond the surface illusion to uncover the real structures in the material world to help people improve conditions and build a better world for themselves. Unlike advocates of the positivist paradigm advocates of the critical paradigm believe in the notion of multiple realities (Mertens 1998: 20). These realities are shaped by values implicit in cultural societal political, economic, ethnic, and gender hegemonies which are taken for granted as unchallengeable and natural (Guba & Lincoln, 1994; Mertens, 1998). In a critical paradigm, a researcher assumes a "political role by cultivating critical consciousness and breaking down institutional structures that reproduce oppressive traditions and social disparities" (Henning, *et al.,* 2004: 23). Unlike the positivist paradigm, the critical paradigm a researcher and participants engage with each other and the researcher influences an inquiry, resulting in "value driven" findings (Guba & Lincoln, 1994:110). While more focus is placed on qualitative design in the critical paradigm, quantitative design within the emancipatory framework can also be used (Mertens, 1998).

Contextual factors are described to help the researchers in uncovering injustice and deprivation and advocating for social justice (Guba & Lincoln, 1994; Mertens, 1998). As can be seen from this debate, the critical research paradigm is not suitable for this study; alternatively, the goal was to improve the improve of mathematics teachers understanding of functions, ultimately improve the skills of ICT teacher's kills in ICT, through a professional development intervention using *GeoGebra*, and examine their knowledge as they engage with it. Knowledge and concrete reality are created inside and through the relationship between people and their environment and are formed and communicated in a social sense (Crotty, 1998: 42). Thus, the social world can only be understood from the point of view of people who engage in it (Cohen, Manion & Morrison, 2007:190). Interpretivism attempts to put hidden structural forces and processes into consciousness. Interpretive approach is intended to explain the phenomena from a human standpoint, to investigate the relationship between persons and the historical and cultural backgrounds in which people exist (Creswell, 2009: 8). Examples of methodologies include case studies in-depth analysis of events or procedures over an extended period of time phenomenology analysis of direct reality from the viewpoint of participants without interfering with current preconceptions hermeneutics deriving levels of context from language and ethnography the study of cultural communities over a longer period of time.

The interpretive analysis seeks to establish a connection between the researcher and the participants that are the core elements of the researcher's study. (Neuman 2011:101-102) identifies an interpretive paradigm as a systematic study of socially meaningful action through the direct and thorough examination of individuals in natural settings in order to explain how individuals create and preserve their social contexts. Like the critical paradigm the interpretive paradigm considers multiple realities and logical constructions Mertens 1998; Neuman, 2011). This means that reality is contextual and different individuals construct different realities on the basis of their intentions convictions values and reasons (Henning *et al*., 2004; Mertens, 1998).

The methodological consequence of having to address several realities is that evolving study designs that change over time are important (Bogdan & Biklen, 2007; Merriam 1998; Mertens, 1998). In comparison to the positivist interpretation where the observer is independent of what is observed the observer is part of what is observed in an interpretive paradigm and takes a more dynamic and contextual role Neuman 2011 in the context of these comparative analyses this study is guided by an interpretive paradigm. Firstly an interpretive paradigm is an acceptable option since the purpose of this analysis is to understand describe and interpret selected mathematics teacher's experiences of the use of *GeoGebra* with the aim of improving their understanding of the concept of function. Secondly, the interpretive paradigm is well adapted to the task of interpreting the diverse relational and social dimensions of education such as teacher development. Finally, qualitative approaches such as interviews observation audio and videotaped recording that originate from the interventions were used to generate data. The next section explains the research methodology and offers the rationale for using the case study as the methodology for this study.

4.3.3 Research methodology – case study

In any research, methodology embodies the principles motivating the use of a particular research design in (Creswell & Poth, 2017). It provides the basis for decisions about what to do and how to do it (Mason, 2002). Several research methodologies have been identified in the context of an interpretive research design, each with its own underlying philosophies, methods and interpretations. Creswell (2009) describes five qualitative research methodologies, including case studies, ethnography, grounded theory, narrative research and phenomenology. In this study, the case study was selected as the methodology for carrying out the research process.

While the case study is a common and recognisable concept and approach, there are differences of opinion as to its intent and application.(Merriam 1998:7) defined the case study as an investigation of a particular phenomenon, such as a program event, an organisation, an individual practice or a social group. The researcher is intensively researching the subject or action of one or more individuals (Creswell 2009). Yin 1984:23) describes the case study as an analytical examination of a contemporary phenomenon within its real-life context. This explanation shows that a case study is the method of choice where an in-depth analysis of a phenomenon is required, as in the case of this study. In addition (Yin, 1984) suggests that a case study is necessary where the boundaries between the event and the context are not easily evident; and where several sources of evidence are used.

A descriptive single case study (Yin 2003; Stake 2005:445) was used as the descriptive case study endeavours to describe, analyse and interpret a particular phenomenon, which was required for this particular study. Case study can be either descriptive also referred to as an intrinsic instrumental or collective stake 1995 an intrinsic case study refers to a review that is conducted on the basis of the aim to clarify the particular case in question (Stake, 1995: 3; Yin 2003) this study is not an intrinsic case study since the goal was not to understand a specific situation. Similarly, this is not an instrumental case study as the intention was not to look at aspects other than focus as indicated by Stake (1995).

The approach includes an empirical examination of a particular contemporary phenomenon by drawing upon various sources of evidence (Robson, 2011). As a single case study, the current study examines the learning experiences of ten mathematics teachers within a PD intervention as they engage with *GeoGebra* activities to focus on the concept of function and interact with their peers and the researcher.

The reason for undertaking this research with ten mathematics teachers from two schools is not to generalise the results of the study to the larger population, but to evoke the diverse views of teachers working in different environments. Creswell (2007) points out that the topic of generalisability is of little significance in qualitative research, even a case study, since it is not supported by the sampling procedures used in qualitative research. Diversity and

distinction are believed to be more important in qualitative research than homogeneity and generalisation (Auerbach & Silverstein, 2003).
According to Yin (1984; 1993), a case study can be either holistic or embedded based on the nature of the analysis used. A holistic case study involves one unit of analysis, while an embedded case study involves more than one unit of analysis (Yin, 1984; 1993). A unit of analysis is the object being studied and it could either be individuals, groups, organisations, processes, social artefacts, cultural objects, action or interventions (Babbie & Mouton, 2001). Thus, this study is considered as a holistic case study with a group of high school mathematics teachers as its unit of analysis. In addition, Yin (1984) indicates that the form of research question is another criterion that defines any research methodology. It indicates that the "how" and "why" questions are exploratory in nature and relevant to the case study. He further points out those two possibilities emerge when research questions are presented as "what" questions, as in the case of this study.

The "what" questions may be either exploratory questions or a separate form of "how many" or "how much" questions. The "what" questions of this study are exploratory in nature and are intended at exploring what are high school mathematics teachers' learning experiences during a professional development intervention aimed at improving their understanding of mathematical functions using *GeoGebra*. As research questions are intended to examine the learning experience of the participants this study is an exploratory case study as indicated by (Yin, 1984; 1993) rather than a confirmatory study as indicated by Merriam (1998). Based on the above discussion the case study was deemed appropriate for this study.

To summarise, while ICTs is present in Namibian schools, especially in the northern part of the country, they are not integrated into the school curriculum to enhance teaching, even in subject areas such mathematics, where they have been proved to make the teaching process more effective as well as improve the learners' understanding of the basic concepts of mathematics, including the concept of function (Ittigson & Zewe, 2003).

I have used an exploratory case study because I wanted to examine a contemporary issue related to the understanding of the concept of function with the use of ICTs *(GeoGebra)*. These teachers don't use *GeoGebra* in their teaching. This case study concentrates on high school mathematics teachers encountering and interacting with *GeoGebra* for PD in a "real-life" context. I will not try to generalise from the study, but a primary purpose is to generate an in-depth understanding of how mathematics teachers experience PD to generate knowledge and skills in order to use ICTs (i.e. *GeoGebra*) with the purpose of better understanding of the concept of functions, looking at it in depth, a case where the teachers have the opportunities to explore the meaning of mathematical symbols that make up different types of functions/polynomial, functions found in the secondary mathematics curriculum.

4.3.3.1 Research method

A research method is the technique used to generate data (Henning, *et al*., 2004). To serve the purpose of this study, data were generated from mathematics teachers through a combination of different methods, namely semi-structured interview, focus group interview,, observations and field-notes. By using multiple sources of data and multiple methods of generating data, the researcher could use triangulation, as suggested by Patton (2002). The data generation was carried out from the 19 to 29 July 2016. The processes and methods used to generate data during this period are discussed in the next sections.

4.3.3.2 Sampling strategy

Sampling in research involves the process of selecting "a portion of the population for study" (Nieuwenhuis, 2007b:79). According to Merriam (1998), there are two methods for sampling: probabilistic and non-probabilistic sampling. The former is predominantly used in quantitative research, while the latter is used in qualitative research. Samples chosen for a quantitative study are typically broad, random and representative. These populations are chosen using such statistical methods in order to be able to generalise the findings of the study to a broader population (Merriam, 1998; Patton, 2002). The intention of this research was not to generalise the results to a wider population. Thus, the sampling technique used in this study focused primarily on the purposefully selecting small information-rich cases from which much could be learned (Merriam, 1998; Patton, 2002). Since the purpose of this study was to find rich, descriptive and in-depth information rather than quantity and breadth, a purposeful sampling technique was found to be more suitable than a probabilistic sampling method.

I intended to have at least four to ten participants who teach mathematics Grades 10 to 12 from different schools in order to provide an environment where they can work both in pairs and in groups. Therefore, five mathematics teachers were selected from Okamu Secondary

School (pseudonym) and five teachers from Galileo H (pseudonym) from the Ohangwena region in Namibia.

4.3.3.3 Obtaining access and selecting schools

According to Bogdan and Biklen (2007), the first duty of a field researcher is getting permission to conduct a study. Request for permission to conduct this study were sent via email to the permanent secretary of the Ministry of Education (MoE) of Namibia on 24 November 2015 (see Appendix A). On 21 December 2015 the MoE approved the request and granted permission to conduct the research in two selected high school in Ohangwena region (see Appendix B). This letter of approval was presented to the Regional Director and to inform the Inspectors of Education of the selected schools to ensure that research ethics are adhered to and disruption of curriculum delivery is avoided. On 30 December 2015 permission was also granted from the Ohangwena Regional Council, Directorate of Education, Arts and Culture, Director's office (see Appendix C). Having been granted permission to access the schools and the teachers in Ohangwena region, I paid a visit to the schools where they were working at the time of the study. First, I had an appointment with the Inspectors of Education of Ohangwena and Ongha circuit to provide them with some details of my study. Ethical issues were discussed.

The Inspectors of Education and the principals from both schools selected accepted the request to ask the mathematics teachers in their schools to participate in the study. Further, the principals showed their gratitude for their schools being selected, adding that this can be a useful opportunity for their mathematics teachers to improve their technological-mathematical skills, knowledge and commitment to the subject. In order to identify suitable schools from the 12 existing secondary schools in the region, the researcher carried out a preliminary review. Selection of the schools was based on the fact that the schools had at least four to six mathematics teachers willing to participate in this study especially schools that were offering grade 11 and grade 12 classes. as the researcher had to work in schools for a long period of time financial constraints limited the researcher in selecting schools within travel distances. in order to preserve the identity of the participants and the schools the two schools in which the research took place were named Okamu S.S and Galileo H.S (pseudonyms).

4.3.3.4 Selecting participants and ensuring voluntary participation

After obtaining permission from the Ministry of Education (MoE), the researcher visited the schools on 14 and 15 July 2016. The schools welcomed these visits by the researcher. During these visits the researcher explained the aim of the research, the nature of the research, the duration of the research, the number of teachers required and the procedures for selecting participant teachers to the principals of the selected schools.

Together with the participants, we agreed to attend the workshops planned for this study in afternoon sessions for a period of two weeks on a voluntary basis. The participants of Okamu SS and I also came to an agreement that the study would be carried out in another school's computer laboratory, Galileo H.S computer laboratory, which is located 20 km from their school. The reason for this decision was that the computer laboratory of Galileo H was much better equipped in terms of number of computers and internet connection. Although I intended to work with 10 mathematics teachers in the first place, one teacher had to withdraw due to her participation in another workshop related to examinations that took place during this time. There was one female teacher and nine male teachers in the group. More information on these participants is provided in the form of mini-profiles in Table 4.2.

Table 4.2: Details on participants

Participants names (pseudonym)	Number of years teaching mathematics	Grade level of teaching	Gender	Numbers of year using ICT	Name of school (pseudonym)
Petrus	16	8-12	Male	5	Galileo H
Henry	4	8-12	Male	4	Galileo H
Michael	4	8-12	Male	4	Galileo H
Rob	3	11-12	Male	3	Galileo H
Robert	10	8-12	Male	4	Galileo H
Peter	8	8-12	Male	7	Okamu SS
Charles	6	8-12	Male	3	Okamu SS
Oveka	6	8-12	Male	4	Okamu SS

Ali	10	8-12	Male	7	Okamu SS
Emma	6	8-12	Female	6	Okamu SS

Choosing a small number of participants was a deliberate decision because it is difficult to find time that is convenient for all the participants (Stepanek, Appel, Leong, Mangan, & Mitchell, 2007). Moreover, it was not the intention of this research to generalise the findings, but instead to generate in-depth information and understand the subjective experience of high school mathematics teachers as they participate in the workshop sessions (Merriam, 1998). This is why only 10 teachers were initially selected to participate in this study.

4.4 Organising the workshops for professional development (PD)

4.4.1 Conducting training workshops

This study was designed to investigate the high school mathematics teachers' learning experiences during a professional development intervention aimed at improving their understanding of functions using *GeoGebra.*

To achieve this aim a professional development intervention was created to offer the participants an opportunity to experience alternative ways of learning functions through multiple representations with the use of ICTs *(GeoGebra*), which ultimately improved their skills in ICT. Educational technology training is necessary to prepare teachers to use technology effectively in their teaching (Russell, Bebell, O'Dwyer & O'Connor 2003:308). There is no doubt that technology leads to significant changes in a number of school practices, and teachers always play a central role in instituting and sustaining these changes in classroom practices.

After identifying the schools and selecting the participants, five introductory workshops (see Table 4.3) were planned by negotiating suitable dates with the school principals and the participants prior to the workshops. The participants identified for this study were invited to attend to these workshops in order to interact with the mathematical *GeoGebra* activities planned to investigate the concept of functions more deeply.

4.4.2 Objectives for the introductory workshops

- Offering teachers opportunities to experience alternative ways of learning about functions through multiple representations.

- Preparing teachers for innovative approaches to management of learning mathematics (particularly the use of *GeoGebra* to acquire a deeper understanding of the concept of functions).
- Teachers will become familiar with the basis use of *GeoGebra* (use interface, applying tools, and changing properties of objects).
- Teachers will learn about fundamental differences between paper and pencil constructions and dynamic software (e.g. a drawing is different from the construction).
- Teachers will learn to enter algebraic expressions (e.g. to crate points, functions).
- Teachers will learn how to use sliders to explore the impact of parameters on algebraic expressions and their graphical representations.

The introductory workshop was conducted on 19 July 2016.The venue for the workshops was the computer lab of Galileo H, which was assumed to be convenient for all the participants. During this workshop the researcher introduced himself and briefly explained to participants the research topic, purpose, objective, aims and the focus group details. The researcher asked the participants to introduce themselves. This was done to ensure that all participants have contributed something from the beginning of the discussion. This helped to the researcher to differentiate between the voices in the discussion (Kitzinger, 2005, Hennink, 2007). Only first names were used in order to ensure some protection of the privacy of the participants. However, names were removed during the transcription period. Pseudonyms were used to transcribe their interventions to preserve their confidentiality and anonymity. The first name was used for the group members and the researcher builds rapport. Referring to each other by the first names helps to create a greater sense of group identity and cohesiveness (Stewart & Shamdasani, 2014:103). Consent letter were signed; the researcher explained in detail the content of the letter, emphasizing the right of withdraw their participation at any time without any consequences for them. Permission was asked to audio and video record the interventions.

The researcher handed out the *GeoGebra* introduction booklet (adapted from Introduction to *GeoGebra* (www.geogebra.org) and workshops materials related to the activities planned for the five sessions (see Appendix H). Availability of participants for the rest of the interventions were discussed before the first workshop commence, because this study took place during schools' preparation for the August 2016 examination in Namibia secondary schools and some of the participants attended also examination workshop during that time. Yin (1984:23) describes case study as "an empirical inquiry that investigates a contemporary phenomenon within its real-life context". *GeoGebra* software was installed in all the teachers' laptops.

In order to identify difficulties that occur during the introduction process of dynamic mathematics software (*GeoGebra)* to mathematics teachers, semi-structured interview questions were administered at the end of every intervention to give feedback from the teachers about the design of the workshops, as well as about the usability of the software *GeoGebra.* Some questions that were addressed in every intervention are the following:

- Were the information and materials useful? Can you say more?
- It is alleged that some teachers tend to emphasise some representations more than
- Others. What is your view on this?
- Were the information and materials relevant?
- Did participant learn what they were intended to learn?
- Did participant learn something new? If yes/no, can you explain please?

This process took place at the end of every intervention session.

Table 4.3: Log of data gathering workshops activities

Date	Place	Activity	Who	What to explore
19-07-2016 Workshop 1 Introductory	Computer lab Galileo H	Activities 1.1, 1.2,1.3 Participants introduced themselves Researcher introduced briefly *GeoGebra* software and the study Ethics issues were explained, teachers signed consent letter	Ten high school mathematics teachers Six teachers from Galileo H and four teachers from Okamu SS	Familiarity with *GeoGebra.* Teachers become familiar with the basis use interface, applying tools, and changing properties of objects. Teachers learned how to enter algebraic expressions (e.g. to create points and functions
21-07-2016	Computer lab Galileo H	Activities 2.1,2.2; 2.3,2.4,2.5	Eight high school	Teachers engaged with possible

Workshop 2		(Activity 2.3) a video were shown of how *GeoGebra* can be used to solves problems in Science Focus groups interview	mathematics teachers (two teachers were excused, as they attended examination workshop)	solutions exploring pencil-and-paper methods and teachers' preferences on the solution of quadratic functions and reasons behind of their preferences. Exploring minimum or maximum values of the quadratic functions. Repeat the construction with the use of *GeoGebra.*
22-07-2016 Workshop 3	Computer lab Galileo H	Discussion of activity 2.5	Ten high school mathematics teachers Six teachers from Galileo H and four teachers from Okamu SS	Exploring parameters And construction of linear functions using *GeoGebra*
26-07-2016 Workshop 4	Computer lab Galileo H	Discussion of activities 4.1 and 4.2	Ten mathematics teachers from the two schools selected	Exploring parameters of a quadratic polynomial
29-07-2016 Workshop 5	Computer lab Galileo H	Final focus group interview	Seven teachers from the two schools selected	Review of what teachers had learned Exploring teachers' experiences with the use of *GeoGebra* in the teaching and learning of functions

4.4.3 Data-collection strategies

Yin (2009) indicated that "case study research is not limited to a single source of data in collecting case study data". The reason behind this is to support the evidence and findings as much as possible. Instead of relying on a single method, data were systematically generated using a combination of several methods, namely semi-structured interview, focus group interviews, audio and video recordings, observations and field notes. In this study the primary data sources included a selected number of high school mathematics teachers' learning experiences, during a professional development intervention aimed at improving their understanding of functions using *GeoGebra.* Some of the advantages and disadvantages of the data-generating methods used in this study are summarised in Table 4.4.

Table 4.4: Qualitative methods, advantages and disadvantages

Methods	Advantage	Disadvantages
Semi-structured interview and focus group discussion	Records feelings and thoughts that cannot easily be detected through observations. Participants can provide background information. Allows researcher to clarify questions.	Provides indirect information filtered through informant's perspective. Takes place in contrived places rather than the real-life setting. Researcher's presence may bias responses. Respondents cannot express their thoughts equally well.
Observation and field notes	Researcher gets first-hand account. Researcher better understands the situation. Takes place in its natural setting. Unusual aspects can be noticed. Useful in exploring topics that participants are not able to or willing to discuss.	Is subjective. Researcher's presence may bias observation. Researcher may be seen as intrusive. Requires good attending and observing skills. Establishing rapport with some participants may be difficult.
Videos	May be an unobtrusive method of collecting data. Make behaviour patterns more visible. A relatively holistic record can be made of the situation. It can be revisited to see things not noticed during the presentation	Analysis takes much time and is technically demanding. Data are partial and can be misleading as some elements are selectively included or excluded. A video can be edited to represent the order of events in new ways

Source: Adapted from Altrichter, Posch and Somekh (1993) and Creswell (2009)

The data-generating methods used in this study are further discussed in Sections 4.4.3.1, 4.4.3.2, 4.4.3.3, 4.4.3.4

4.4.3.1 Semi-structured interview

The qualitative interview attempts to understand the world from the subjects' point of view, to unfold the meaning of their experiences, to uncover their lived world prior to scientific explanations (Kvale & Brinkmann, 2008). It is important to underline that "case study research is not limited to a single source in generating case study data" (Yin, 2009). The reason for this is to support the evidence and finding as much information as possible.

Interviews are one of the most commonly used methods of evidence collection in qualitative research (Babbie & Mouton 2001). Nieuwenhuis (2007b) explains the interview as a two-way dialogue in which the interviewer asks the participants to obtain data and learn about the participants thoughts, beliefs, experiences, views and their behaviors (Nieuwenhuis, 2007b: 87). In the same vein, Patton (2002) indicates that the purpose of conducting interviews is to find out participants' perspectives, thoughts, interpretations, feelings and intentions that could not easily be detected through observation.

However, the quality of information obtained from interviews depends on the interviewer's ability to ask probing questions (Merriam, 1998; Babbie & Mouton, 2001) in clear and understandable language (Patton, 2002). Skilled interviewers are good listeners, never criticise the logic of their respondents, never judge the perspectives of their respondents and never push respondents to talk about topics that upset, hurt or humiliate them (Bogdan & Biklen, 2007; Nieuwenhuis, 2007b). It is usually best practice to begin the conversation with questions that participants can answer easily and then continue to more difficult or focused questions. This can help put respondents at ease, build up confidence and rapport, and often generates rich data that subsequently develops the interview further (Gill, Stewart, Treasure & Chadwick, 2008).

Merriam (1998) advises interviewers to avoid double questions, multiple-choice questions, leading questions and yes-or-no questions because such types of questions jeopardise the richness of information (Merriam, 1998). As far as possible this advice was adhered to while conducting interviews in this study. Based on their degree of structure, interviews are classified into structured, semi-structured and unstructured categories (Jennings (2005:99); Merriam, 1998; Nieuwenhuis, 2007b) and Patton, 2002. The use of unstructured and semi-structured interviews is related to the interpretive paradigm, which is based on an ontology (worldview) that distinguishes multiple perspectives in regard to the research focus, an epistemological stance that is subjective in nature, and a methodology which is predicated on principles of equality. Axiologically, the research process is value laden, and the research purpose is intrinsic in nature. Semi-structured interviews differ from unstructured interviews in that the former have a flexible list of themes to focus the interview.

The unstructured interview is more open and more conversation-like with no set questions, just a theme, so the interviewer and interviewee will become co-researchers with respect to topic treatment (Jennings, 2005:99). In semi-structured, interviews the interviewer asks key questions in the same way each time and does some probing for further information, but this probing is more limited than in unstructured interviews. This study followed a semi-structured interview schedule (see Appendix E) where the researcher prepared in advance a list of interview issues in the form of an outline as indicated in the literature (Merriam, 1998; Patton, 2002). Firstly, it allowed some flexibility in changing the wording and sequencing of questions depending on the answers given by participants. In fact, during the interview further questions were asked as follow-up to the responses obtained from participants. Secondly, by asking similar questions to all respondents, comprehensive data could be generated more consistently (see Appendix G). Thirdly, it prevented information from being overlooked as may occur when a structured interview or a completely unstructured interview schedule is used. Finally, the researcher's assumption impacted on the methodological choice. He thought he could conduct a semi-structured interview more effectively than an unstructured interview. Table 4.5 shows a comparison of structured, semi-structured and unstructured interviews.

Table 4.5: Comparison of structured, semi-structured and unstructured interviews

	Structured interview	Semi-structured interview	Unstructured interview, In-depth interview
Style	Question and answer	Conversation	Conversation
Design	Structured	Semi-emergent	Emergent
Researcher stance	Objective	Subjective	Subjective
Researcher perspective	Outsider	Insider	Insider
Consequence of researcher Stance and perspective	Limited reflexivity	Reflexivity	Reflexivity
Exchange issues	Limited reciprocity	Reciprocity	Reciprocity

during the research process			
Language used	Subject/respondent	Informant, participant Co-researcher	Informant, participant Co-researcher
Data collection	Data representation Checklist Some open-ended questions	Empirical materials Slice of life Field notes transcription and recording	Empirical materials Slice of life Field notes transcription and recording
Basis of analysis	Mathematical and statistical analysis	Textual analysis	Textual analysis
Findings expressed as	Numerical representation	Depth and thick descriptions	Depth and thick descriptions
Reporting research	Scientific report	Narrative	Narrative

Source: Adapted from Jennings (2005:101)

A date, time and venue for the interviews were arranged with the participants. All five sessions were held in the computer laboratory. Before the interviews were conducted, informed consent was obtained (Appendix D). During this time I introduced myself and explained the aim of my study. I assured the teachers that confidentiality would be maintained and that no information would be attached to them personally or used against them or their institution in any way.

Pseudonyms were used to protect their identity. Semi-structured interviews were conducted over the course of the interventions. The duration of each interview was approximately 120 minutes. The ten teachers were interviewed while they were interacting with a set of *GeoGebra* activities (see Appendix I). However, the writing responses were only collected at the end of the last workshop. With the consent of the participants all interview were audio and video recorded and transcribed later. The teachers written and focus group responses were used as main sources of evidence to explain and analyses teachers' experiences. Appendix G provide examples of some of the interview questions that were used to explore teachers' experiences while they were interacting with some *GeoGebra* activities planned for this study.

4.4.3.2 Focus group discussion

Focus groups are forms of group interview that focus on the interaction between research participants with the purpose of investigating a topic that is not well known in order to generate data (Mason, 2017). This means that instead of the researcher asking each participant to respond to a question in turn, people are encouraged to talk to one another, asking questions, exchanging anecdotes and commenting on each other's experiences and points of view (Kitzinger, 1995). Kitzinger further argued that:

> The idea behind the focus group method is that group processes can help people to explore and clarify their views in ways that would be less easily accessible in a one to one interview. Group discussion is mainly appropriate when the interviewer has a series of open-ended questions and wishes to encourage research participants to explore the issues of importance to them, in their own vocabulary, generating their own questions and pursuing their own priorities. When group dynamics work well the participants work together with the researcher, taking the research in new and often unexpected directions (Kitzinger, 1995:299).

The researcher used the focus group interview as a strategy for the following reasons. Firstly, since the participants were all mathematics teachers working in groups and sharing common experiences, a group interview is suitable for gauging the collective experience of these teachers. Secondly, respondents resound to each other's experience, because when one comes up with an idea, other members of the group express similar ideas in their own words. Finally, group interviews save time (Auerbach & Silverstein, 2003). The researcher was aware that in conducting a group interview there is a danger of losing some valuable information for the sake of the above advantages (Babbie & Mouton, 2001). This method seemed to be very convenient, because it gave the researcher the possibility to intervene in the conversation and pose questions to confirm what a participant had said, and also gave the participants the opportunity to engage actively in mathematical tasks which they might never have experienced before, in this case using *GeoGebra*. Participants also had the opportunity to construct and explore the meanings of mathematical symbols that made up different types of functions/polynomials interacting with the mathematics activities using *GeoGebra.*

The mathematics teachers were interviewed as a group. Two focus group interviews were conducted in the course of the investigation (see Appendix F). The first group interview was conducted after completing the first workshop. The second was conducted at the end of workshop 5.

These sessions lasted for approximately 90-120 minutes. The interviews took place in free spaces such as the computer lab room, where the mathematics teachers interacted with the researcher and engaged with the *GeoGebra* activities. All the sessions were video and audio recorCrarded. Recording the interviews relieved the researcher to some extent of taking extensive notes. This helped him to develop a rapport with the respondents. Seidman (1997) argues that interviews become more effective if they are used in combination with observation (Seidman, 1997). Hence, in this study observation was used to complement the other data-generating methods.

4.4.3.3 Videos recordings

In addition to methods that generated textual data, videos recording were used to capture the details of the activities conducted during the research. Altrichter, Posch and Somekh (1993) indicate that video recordings make the context and casual relationships more accessible and behaviour patterns more visible than other methods of data generation. Patton (2002) indicates that videos can highlight significant facts if the researcher has the skills that go beyond pressing the record button (Patton, 2002). In this study all workshops were video recorded. The researcher used them as supplementary material in order to either watch or listen to the events that took place during the fieldwork. The videos were saved for further reference.

4.4.3.4 Observation and field notes

Observation is the primary source of generating data in qualitative research (Merriam, 1998). Nieuwenhuis (2007b) describes observation as a "systematic process of recording the behavioral patterns of participants, objects and occurrences without necessarily questioning or communicating with them" (Nieuwenhuis, 2007b:83-84). Observation as used in research is different from observation in everyday life. Observation becomes a research method if the researcher takes notes on the observable behaviour patterns and activities of participants at the research site (Creswell, 2009). In this study the researcher recorded field notes during observations, interviews and discussions held with the participants. According to Patton (2002), field notes are descriptions of the researcher's feelings, reactions to what has been observed and what the researcher believes to be important. Depending on the time and place of recording, researchers could use different styles of taking field notes (Patton, 2002). Silverman (2008) highlights a number of different questions we should consider when conducting observations and writing field notes

- What are people doing? What are they trying to accomplish?
- How exactly do they do this?
- How do people characterise and understand what is going on?
- What assumptions do they make?
- Analytic questions: What do I see going on here? What did I learn from these notes? Why did I include them?

The researcher recorded events during interventions (*GeoGebra* workshops) on a format prepared for observation (Appendix J). The observation format for the researcher was designed in such a way that it enables the researcher to note the description of what is actually observed and personal reflection of what is being observed simultaneously on the same page. Although the data obtained through this observation format were not coded for analysis, they formed an invaluable source of information during data analysis. The researcher used them to support other data. The procedures used to analyses the data in this research are discussed in Section 4.5.

4.5 Data analysis

The preceding section discussed qualitative data generation and the instruments used.
Analysis refers to a continuing process of "giving meaning" to impressions about data (Stake, 1995:71). Qualitative data analysts tend to generate data by interpreting what they see or hear from the participants throughout the field work (Denscombe, 2007). Researchers (Bryman, 2004; Froggatt, 2001; Silverman, 2000) describe this as an "iterative" process which suggests an interrelationship between collecting and analysing the data.

According to Bogdan and Biklen (2007:159), analysis in qualitative research is the process of "working with the data, organising them, breaking them into manageable units, coding them, synthesising them and searching for patterns". In this study the process of data analysis took place in two major steps, namely preparing and organising the data (Section 4.5.1), and then coding the data into categories (Section 4.5.2).

4.5.1 Preparing and organising the data

In the case of the current research, the following steps were taken to analyse the data:

- Preparing the whole data set for analysis
- Coding
- Categorising
- Identifying broader themes.

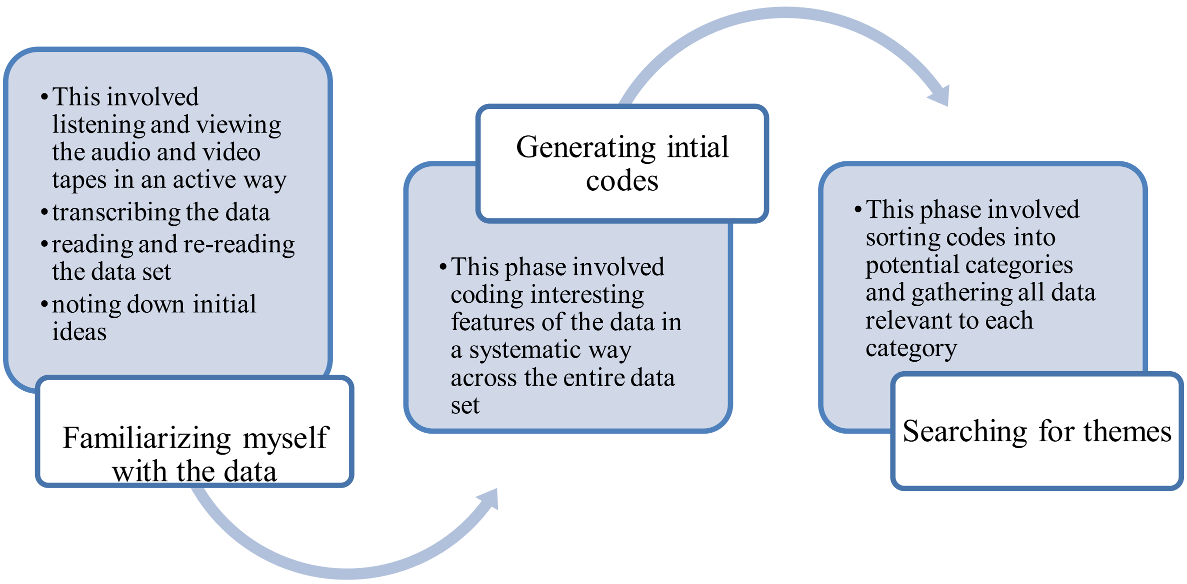


Figure 4.3: Data-generation process (Adapted from Saldana, (2015) and Creswell, (2013)

The first task in data analysis is to make sure that data are in a form that can be easily analysed. The semi-structured interview and the focus group video were viewed, transcribed and viewed multiple times after the end of the two weeks of data collection. In this way the researcher was familiar with the data set by re-reading the whole of the transcribed data. This made it possible to obtain a gradually deeper understanding of the participant's perspectives.

4.5.2 Coding data into categories

In this study the grounded theory approach, which helps to build theory through successive conceptual analyses of data, was employed as an analytical tool to analyse the data as suggested in the literature (Glaser & Strauss, 1967; Auerbach & Silverstein, 2003; Henning, *et al.,* 2004). Unlike in the deductive approaches that test existing theories, grounded theory is an inductive approach that develops theory (Auerbach & Silverstein, 2003). Influenced by an objectivist epistemology, the notion of building a theory through the grounded theory approach was originated by Glaser and Strauss (1967). However, in the course of time Strauss and Corbin (1990) made changes to the classic objectivist grounded theory by elaborating and specifying the coding procedures with the intention of making them more explicit. Subsequently, a more pragmatic grounded theory with a constructivist underpinning emerged (Charmaz, 2008). Charmaz (2006) challenges the assertions of classic objectivist grounded theory for: 1) detaching the relationship of a researcher from what is being researched, 2) assuming the emergence of theories from data independently without being influenced by the observers and the methods used to produce them, and 3) being didactic, prescriptive and structured. However, constructivist grounded theory asserts that theory emerges from data through interactions between the researcher and what is being researched and through interpretations of data guided by flexible and emergent guidelines as opposed to structured and rigid prescriptions (Charmaz, 2008).

As these two versions of grounded theory follow contrasting epistemologies, the researcher had to choose the version that best fits his research. A constructivist grounded theory was chosen as an analytical tool to analyse data and construct theory in this study.

The choice of using grounded theory as an appropriate analytical framework was influenced by three factors. Firstly, grounded theory helps to conceptualise and build theory in an inductive way without being driven by theory, as there were no theories to be tested or verified. Secondly, its iterative nature permits flexible movement back and forth in the data-analysis process. Finally, it was chosen because of the simplicity of the analytical tool in the data-analysis process (Charmaz, 2006, 2008).

Coding is defined as a process of categorising segments of data and then assigning names or labels to them in order to attach meaning to the pieces of data (Punch, 2005). In this study the data analysis was begun by reducing textual data that were obtained through the semi - structured and focus group interviews into manageable units called codes and then categorising them into data sets in order to identify emerging patterns as suggested by Bogdan and Biklen (2007).

Data generated through an observation format designed for the researcher. Three types of coding were used, namely open coding, axial coding and selective coding as suggested in the literature (Saldana, 2015; Strauss, 1987; Strauss & Corbin, 1990; Henning, *et al.,* 2004). Open coding was the first step in the coding process to break the raw data into segments called codes and then sort similar codes into subcategories. This was done by listening to the interviews and reading the transcripts repeatedly sentence by sentence, line by line and word for word in order to generate codes inductively from the raw data in the tradition of grounded theory. This process helped to familiarise the researcher with the raw data and understand the concepts. Axial coding, the second step in the coding process, was focused on the codes created during the open coding rather than on the raw data. This was an iterative process in which the initial codes were reviewed and examined again in order to enable the researcher to generate additional ideas and codes, merge closely related concepts into one or eliminate some of the ideas (Neuman, 2011). At this stage the sub-categories created during open coding were compared to one another and then similar subcategories were grouped into higher-order concepts to form generic categories. Selective coding was the final stage where the generic categories were matched to the research questions to form the main categories that were mainly used to structure the presentation and discussion of data.

A diagrammatic representation of the coding process is presented in Figure 4.4. The key elements of the procedure are: raw data, codes and main categories. By following these steps, the teacher/researcher started the coding process with the raw data and then moved from the ground up in small steps, where each step builds on the previous one, in order to answer the research questions. This process enabled the teacher/researcher to move inductively from the lowest-level concepts to a highest level of theorising and abstraction (Auerbach & Silverstein, 2003; Punch, 2005). Table 4.6 provides an example of coding.

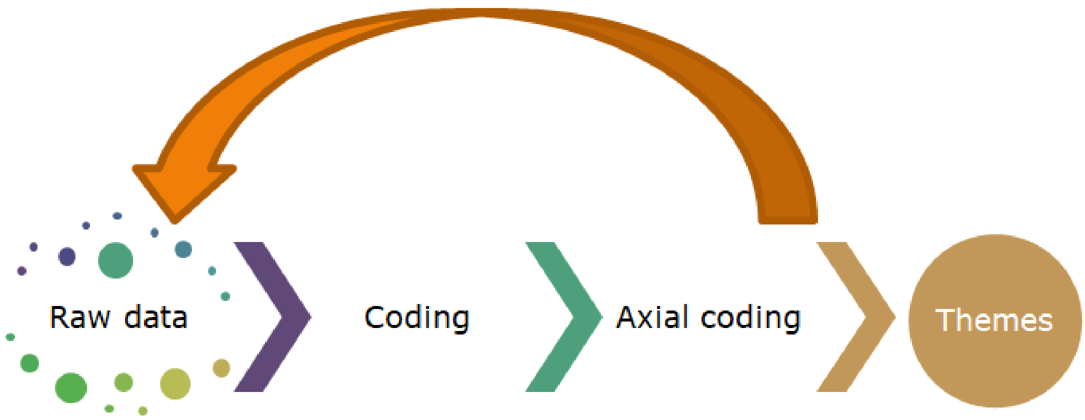


Figure 4.4: Grounded analytical framework and coding process (Adapted from Saldana (2015) and Creswell (2009)

Table: 4.6 Example of coding

11. Do you believe that there is a role for ICT/*GeoGebra* to deepening functions learning? Please elaborate.				
Participants responses	**Textual data excerpt (Raw data)**	**Coding**	**Axial coding**	**Main themes**
Petrus (Pseudonym)	*Yes, ICT/GeoGebra will/can make preparation easier and faster. ICT can be used to supplement pen-paper work for learners to visualise what actually on the screen. Also we can record*	Easy and faster. Supplement pen-paper work. Visualise what is on the screen.	Time concern and constraints. GeoGebra affordance (visual capability). Supporting pen-paper learning of functions	**Time concerns** **Supporting pen-paper learning of function**

	lessons and in the case of not being around, learners can just view lessons themselves on repeat			
Henry	*Yes, after getting confident with GeoGebra through this project I realised how easier is to come up with graphs and draw the functions by means of ICT.* *It is more interesting plotting points and drawing lines with GeoGebra. It deepens the teaching and learning of functions. GeoGebra in connection with ICT is quicker and simplify the methodology of learning.*	Easy use of the software. Deepens the teaching and learning of functions.	GeoGebra has epistemic and pragmatic values.	**Epistemic and pragmatic dimensions**
Michael	*I believe there is a big role that GeoGebra will play if it's to be used in the learning of functions, because arouse leaners' interest and makes them to love the topic as much as the program, it brings*	Role of *GeoGebra* in the teaching and learning of functions. Excited about *GeoGebra.* Fast answers. Better	Time saving. *GeoGebra* has quick feedback (Dynamic). Epistemic and pragmatic value. Deepening of mathematical	**Deepening mathematical thinking** **GeoGebra affordances**

	theory into practice, also is faster and the answers are ready available, learners understanding of functions will be much more enriched.	understanding.	thinking	
Rob	*I definitely believe that GeoGebra program will play a major role to deepening learning on the ground that: learners will understand the topic very well since GeoGebra is more visual. I believe that it will make mathematics perfect.* *It also deepening functions learning in the sense that its faster and one can always have a picture in mind on how problem were solved by GeoGebra*	Deepening learning. Visual practices. Faster.	Discovery learning. Visualise abstract concepts	**Epistemic and pragmatic dimensions**
Robert	*Yes, GeoGebra is very effective and save a lot of time in drawing graphs.it also will help the learners to viewing the effects of nature of gradient and the*	Saving time in drawing graphs. Graph behaviour. Learner interest	Time concerns and constraints. Epistemic values. Multiple representations	**Time concerns**

	general behaviour of graphs as different input values are entered into the input bar . it will generate learners interest as the concept is introduced			
Peter	*Yes, the program is fast and time saving that will gives the learners the shape of the graphs upon entering the function, deepening leaners understanding of different functions and easy ways of manipulating functions to find things such as, midpoint, turning points or solutions to a given function.*	Deeper understanding of function, epistemic values	GeoGebra affordances. Time concerns. Function manipulation. Epistemic and pragmatic dimension	
Charles	*Yes, because using GeoGebra in the teaching of function is much faster that using a chalkboard. Is very easy to find a maximum and minimum of functions unlike completing the squares first or finding the x intercept and memorising the formula of turning point; tp = (-*	*GeoGebra* much faster And *GeoGebra* affordances Comparing ICT with pen-paper environment. Time concern		

	p;q)			
Oveka	*Yes, that can deepen functions learning because learners will be willing to explore and understand better, they will find it easy to draw and interpret any given graph. Learners can observe what is happening to the graph as soon as we keep on changing the parameters of the function.*	Draw and interpret graph and graph behaviour. Visualization	Particularities around functions. Multiple representations of function.	Particularities around functions
Ali	*Yes, because this will help the learners to get deeper understanding of functions as they can observe at the same time the behaviour of the graphs when the parameters are changing and it also create curiosity for them to learn on their own, giving opportunity to explore and discover new things on their own,*	Deeper understanding of functions. Explore and discover. Own learning , discovering and exploring	Multi-representation of function. Discover teaching and learning. *GeoGebra* affordances.	Epistemic and pragmatic values

4.6 Credibility and Trustworthiness

In important work in the 1980s, Guba and Lincoln substituted reliability and validity with the parallel concept of "trustworthiness," containing four aspects: credibility, transferability, dependability and confirmability.
Merriam (1998:198) asserts that both quantitative and qualitative researches are "concerned with producing valid and reliable knowledge in an ethical manner". It is the duty of any researcher to indicate that the information generated from the research is authentic and trustworthy (Mertens, 1998). In this study appropriate measures were taken to ensure credibility and trustworthiness during conducting the research and reporting the findings. Although there is general agreement on the importance of ensuring quality, the criteria used for assessing quality in qualitative research are different from those used for quantitative research (Creswell, 2009; Cohen, *et al.*, 2011).

Guba (1981) and Lincoln and Guba (1985) suggest that if researchers are concerned about the trustworthiness of their research they must answer the following four questions:

- How credible or believable are the findings?
- How applicable and transferable are the findings to other contexts or with other subjects?
- How can you be sure that the findings could be repeated if the study were to be conducted with the same or similar participants in the same or similar contexts?
- How can we be sure that the findings are reflective of the subjects and the conditions of the enquiry rather than a creation of the researcher? (Guba, 1981; Lincoln & Guba, 1985)

According to Guba (1981), Lincoln and Guba (1985), these four questions establish truth-value, applicability, consistency and neutrality respectively. Initially, they matched these four concepts with four terms borrowed from the traditional positivist paradigm, namely internal validity, external validity, reliability and objectivity respectively. Soon after, they developed alternative constructs that ensure credibility and trustworthiness within a qualitative research epistemology and replaced them with qualitative parallels, namely credibility, transferability, dependability and conformability (Guba, 1981; Lincoln & Guba, 1985). These are summarised in Table 4.7.

Table 4.7: Criteria for increasing qualitative in research

Quantitative	Qualitative	Strategies used to enhance quality in this study
Internal validity	Credibility	Prolonged engagement in the research sites Triangulation using multiple sources and methods Collection of referential material Peer debriefing Clarifying researcher's position
External validity	Transferability	Thick description of data Detailed description of context and research process
Reliability	Dependability	Clarifying researcher's position Triangulation Establishing audit trail
Objectivity	Confirmability	Detailed description of context and research process Corroboration with literature

Source: Guba (1981:80) and Lincoln and Guba (1985) the four aspects of credibility and trustworthiness used to ensure quality in this study are discussed in the subsequent sections.

4.6.1 Credibility

According to Guba (1981) and Lincoln and Guba (1985), credibility refers to the reality of data and its interpretations. Strategies that could improve credibility are triangulation and collection of referential adequacy materials and peer debriefing (Guba, 1981; Lincoln & Guba, 1985). Merriam (1998) and Mertens (1998) share the same idea. Creswell (2009) also proposes explicitly clarifying researcher's bias or reflectivity as a viable strategy to enhance credibility.

All these strategies were employed in this study to enhance credibility. Firstly, the researcher used triangulation (Guba, 1981; Lincoln & Guba, 1985; Merriam, 1998; Patton, 2002; Cohen, *et al.,* 2011). Data were generated from the mathematics teachers by using multiple techniques. The rationale behind triangulation is that using multiple sources of data and methods are better than using a single source of data and method. Patton (2002) reminds us that "Studies that use only one method are more vulnerable to errors linked to that particular method […] than studies that use multiple methods in which different types of data provide cross-data consistency checks" (Patton, 2002:556).

Collection of referential materials is the second strategy for ensuring credibility (Guba, 1981; Lincoln & Guba, 1985). For example, documents related to participant exploring parameters of a quadratic polynomial (see Appendix L1 and L2). To ensure credibility of this study, the documentation of data was done in such a way that they would be accessible when a need arise. All interviews and their transcriptions were appropriately documented. Data that emerged from the audio-video, written semi-structured interviews and focus group were appropriately documented and archived. Peer debriefing is the third strategy for ensuring credibility (Guba, 1981; Lincoln & Guba, 1985). Peer debriefing involves asking colleagues to comment on the data and findings (Merriam, 1998).

Consulting with peers was a consistent aspect of this study. Colleagues provided the researcher with the necessary input during proposal writing, data generation and data analysis. They helped the researcher in reading the drafts before submitting his work to the promoter. The constant meeting with the supervisor, his comments and critiques were a consistent part of the research process that enhanced the researcher's interpretations and thinking. Finally, the researcher ensured credibility by indicating his position in relation to the phenomena being studied, for example, by indicating the rationale for selecting methodology, methods, participants and research sites.

4.6.2 Transferability

Transferability in qualitative research parallels generalisability in quantitative research (Guba, 1981; Lincoln & Guba, 1985). But generalisability is of little importance in qualitative research (Creswell, 2007) as the sampling techniques are not designed to enable the researcher to generalise the findings to a larger population (Merriam, 1998; Patton, 2002). So transferability is not about whether a study includes a representative sample or not, but it is about how well the study provides a rich description of the context within which it occurred (Guba, 1981) in order to enable readers to determine the extent of similarity to their own situations (Mertens, 1998). In this study the issue of transferability was addressed by providing a detailed description of the context and working conditions of the informants so that readers could make their own judgements about how well this corresponded with their own situations.

4.6.3 Dependability

Dependability in qualitative research parallels reliability in quantitative research (Guba, 1981; Lincoln & Guba, 1985). From the perspectives of quantitative research, reliability refers to the extent of possible replication of a study in order to produce the same results (Merriam, 1998). This is because quantitative researchers assume that there is a single reality and studying that reality repeatedly produces the same results. On the contrary, qualitative researchers believe in multiple realities that make the logic of replication a "misfit" (Merriam, 1998:206), because different people interpret their reality differently. Echoing the same view, Bogdan and Biklen (2007:40) describe reliability as "fit between what [is] recorded as data and what actually occurs in the setting under study, rather than the literal consistency across different observations". Therefore, what is important in qualitative research is not whether the results can be replicated, "but whether the results are consistent with the data collected" (Merriam, 1998:206).

There are strategies to be used to enhance dependability. For example, the researcher used multiple sources and multiple methods in order to satisfy the principles of triangulation (Guba, 1981; Lincoln & Guba, 1985; Merriam, 1998; Patton, 2002; Cohen, *et al.,* 2011). The use of colleagues and methodological experts (peer examination) to check the research plan and implementation is another means of ensuring dependability. One can enhance stability over time by repeated observation of the same event and questioning informants again about major issues; these are similar strategies to those that enhance credibility (Lincoln & Guba, 1985). Finally, the researcher attached as appendices the instruments used to conduct the research and sample transcriptions of interviews to provide the reader with background information about the procedures followed in this study. This documenting process also provides readers with opportunities to track the whole process, and attests to the quality and appropriateness of the study (Mertens, 1998).

4.6.4 Confirmability

Confirmability in qualitative research is comparable to objectivity or neutrality in quantitative research (Guba, 1981; Lincoln & Guba, 1985). In qualitative research confirmability is concerned with ensuring that the researcher has acted in good faith and requires the researcher to ensure that his personal values and biases are addressed and monitored (Bryman, 2008). To establish confirmability, the researcher outlines the characteristics of the respondents and methods of data generation, analysis and interpretation used to show that the research findings are the result of the research and not based on the researcher's assumptions and preconceptions. Furthermore, the promoter of this study, through questioning various aspects of the research, enabled the researcher to reflect on his personal biases and values in terms of how he was conducting the research process. Confirmability also refers to the degree to which the results could be corroborated by others. Confirmability in this study was established by substantiating the findings of this study with perspectives found in the literature.

4.7 Research design plan (Data-generation strategies)

A research design is a strategy that gives the researcher a detailed plan for achieving the research objectives. A plan like this is defined by De Vos, Strydom, Fouche, and Delport (2005) as "a logical strategy for gathering evidence about the knowledge desired." According to Yin (1994), research design addresses four issues: what questions to investigate, what data is relevant, what data to collect, and how to analyse the findings. The table 4.8 illustrate the strategies that were used to generate data from the participants.

Table 4.8: Show the strategies that were used to generate data from the participants in order to answer the research questions

Issues	**Strategies**
Empirical situation of data collection	A series of 5 workshops aimed at familiarising teachers with an in-depth study of functions using *GeoGebra*. (See Appendix I). These workshops were conducted outside their classrooms at an agreed upon venue (computer lab), affording opportunities for an engagement with mathematics, i.e. deepening mathematical thinking and reflecting on their experience of doing mathematics-related tasks

	by themselves, or cooperatively, including becoming aware of multiple approaches (Watson & Mason, 1998, 2007).
Main research question and sub-questions	1. What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using GeoGebra? The following is the research sub-questions: • How central is knowing mathematical functions in the Namibian secondary curriculum? • What does research have to say about the potential of *GeoGebra (GGB)* in facilitating the understanding mathematical functions? • What does research have to say, what are key issues in using GGB-represented mathematical functions?
Unit of analysis	Teacher learning experiences (TLEs)
There are at least two themes of TLEs based on the design and content of the different workshops	*1) Pragmatic experiences:* • Speed of calculations and representation on the screen; there is an instant, immediate visibility and display of the specific functions. • These functions are part of secondary mathematics content in the Namibian curriculum. The teachers 'know' this content from the classroom practices where they use a pen-and-paper medium.
	2) Epistemic experiences: • *GeoGebra* design enables user to alter the parameters of the various functions. Effects of particular changes are displayed through tables, graphs and symbols on the screen. • *GeoGebra* design enables the user to syntactically change different parameters of functions and observe different effects on the screen; researcher collected data on teachers' semantic comments on the different parameter changes they made.

	Researcher/teacher interacts with teachers by having them engage with *GeoGebra* (workshops) and collected data on their ways of taking and *using GeoGebra* during the workshop sessions.
Grounded theory approach	A constructivist grounded theory was chosen as an analytical tool to analyse data and construct theory in this study. Data generated through multiple techniques were coded and analysed using the grounded theory approach and the results were presented in accordance with the research questions.

In the following section 4.8 is a discussion of ethical considerations undertaken in this research.

4.8 Ethical Considerations

When human subjects are involved in any kind of research, researchers have the obligation to apply sound ethical standards in relation to their research subjects while conducting the research, as well as to the scientific community while analysing and reporting the findings (Neuman, 2011). They have a responsibility to protect the participants from harm (Babbie & Mouton, 2001). A researcher has both "moral and professional obligation to be ethical even when research subjects are unaware of or unconcerned about ethics" (Neuman, 2011:143). Therefore, this researcher employed the necessary ethical procedures in the planning and implementation of the study. In this research five ethical issues were addressed, namely obtaining permission from the MoE to conduct this study in selected secondary schools of Ohangwena region in Namibia, obtaining informed consent from the research participants, protecting the anonymity and confidentiality of the participants, avoiding harm or damage to the participants, and being honest during the analysing and reporting of the study. These issues are briefly discussed in the sections below.

4.8.1 Obtaining permission

After the research proposal had been approved by the Research Committee of the Department of Curriculum Studies of the Faculty of Education at Stellenbosch University, on 23 March 2016, the researcher worked towards obtaining permission from the MoE to conduct the research in selected secondary schools of Ohangwena region in Namibia. Subsequently, the MoE granted him permission to conduct the research in the schools (see Appendix B and C).

4.8.2 Informed consent and voluntary participation

According to Neuman (2011), research subjects must agree voluntarily to participate in research without any coercion. They must decide to participate in the research after obtaining full information about the nature of the research and any possible dangers that may arise (Babbie & Mouton, 2001; Patton, 2002; Neuman, 2011). There is abundant information on ensuring the voluntary participation of research subjects. This researcher ensured the voluntary participation of the research subjects before the commencement of the study, as suggested in the literature (Babbie & Mouton, 2001; Patton, 2002; Bogdan & Biklen, 2007; Neuman, 2011).

All the participants signed consent forms to indicate that their participation was voluntary and a copy of their signed consent forms was given to each of them (see Appendix D). The participants were fully informed about the purpose, and nature of the study and the commitments involved before signing the consent paper so that they could make an informed decision to participate in the study or not. They were informed that they are free to withdraw from the research at any stage without any consequences. Moreover, the participants were also assured of anonymity. Pseudonyms were given to protect their privacy and the confidentiality of data (Babbie & Mouton, 2001; Patton, 2002; Bogdan & Biklen, 2007; Neuman, 2011). All the participants who were selected for the study were willing to participate in this study hoping that they would develop their professional skills and gain additional knowledge.

4.8.3 Anonymity and confidentiality

According to Neuman (2011), anonymity entails protecting the privacy of participants in such a way that the identity of the participants cannot be traced. For Cohen *et al*. (2011), anonymity is achieved when "the researcher or another person cannot identify the participant or subject from the information provided" (Cohen, *et al.,* 2011: 91). Although it was possible for the researcher to identify the participants and their schools from the information given, confidentiality was maintained by presenting the data in such a way that the identities of the participants and their schools could not be traced by others. The identity of the schools and participants were protected by giving pseudonyms (Babbie & Mouton, 2001; Patton, 2002; Neuman, 2011). Protecting the identity and confidentiality gave the participants confidence in order to participate in the research without fear. Data was stored in locations not accessible unauthorised persons.

4.8.4 Avoiding harm to participants and schools

According to Babbie and Mouton (2001), it is imperative that a research project should never cause emotional, psychological or physical harm to the research subjects, regardless of whether they have volunteered for the study or not. They added that researchers should be careful not to reveal information that would cause harm to the research subjects (Babbie & Mouton, 2001). In this study the research subjects were protected from any damage and harm by protecting their identity (see Section 4.7.3). Moreover, the researcher tried to minimise harm to the school by disruptions in the schools to a minimum (Creswell, 2009).

4.8.5 Honesty during analysis and reporting

In addition to being ethical in relation to research subjects, researchers are ethically obliged to be honest and open when they analyse and report the findings of their research (Babbie & Mouton, 2001). In this research the researcher was as honest and open as possible while conducting the research and reporting the findings. He reported on his personal limitations and the limitations and strengths of the study openly and honestly.

4.9 Chapter Summary

This chapter discussed the purpose, research design, research paradigm and research methodology for this study. The purpose in this study was to investigate the high school mathematics teachers' learning experiences during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra*. The interpretative methodology provides a framework to understand, describe and interpret the experiences of the high school mathematics teachers encountering and interacting with *GeoGebra* for PD in a "real-life" context. This is a qualitative study that used the interpretive case study approach.

The case study format was found to be most appropriate because it enables the researcher to generate in-depth information and generate in-depth understanding of how mathematics teachers experience PD to generate knowledge and skills in order to use ICTs (i.e. *GeoGebra)* with the purpose of enhancing understanding of concept of functions. This study took place in two purposefully selected high schools in Ohangwena region in Namibia. Triangulation was applied by using multiple sources of data and multiple of methods of data generation. Triangulation also enhanced the credibility of the study. Finally, the data generated through multiple techniques will be categorised into themes in order to find answers to the research questions. Chapter Five presents and discusses the results of the research study.

CHAPTER FIVE

DATA ANALYSIS AND FINDINGS

5.1 Introduction

This chapter presents the analysis of data and the discussion of the research findings of the study. It has five main sections. The chapter begins highlighting salient features of the chapter in Section 5.1. In Section 5.2 the teachers' background information includes their familiarity with ICT. The initial reaction of the participants towards the workshops can be found in Section 5.3. Section 5.4 includes the analysis of the data produced from the semi-structured and focus group interviews related to teachers' learning experiences (TLEs) during the five *GeoGebra* workshops sessions. This will be followed by a discussion of the findings around the three main themes related to the research question(s) in section 5.5. Finally the chapter is provided in Section 5.6.

In this chapter primarily presents the analysis and findings of teachers' learning experiences (TLEs). This study addressed one main research question and three sub-questions:

1. What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra?*

The following is the research sub-questions:

- How central is knowing mathematical functions in the Namibian secondary curriculum?
- What does research have to say about the potential of *GeoGebra (GGB)* in facilitating the understanding mathematical functions?
- What does research have to say, what are key issues in using GGB-represented mathematical functions?

To answer these questions, various types of data-collection methods were used, as shown in Chapter Four (see section 4.6). A semi-structured interview and two focus groups interview schedule were used. The questions in the interviews gave the teachers the opportunity to share their experiences of the potential benefit of *GeoGebra*. All teachers interviewed were teaching mathematics at two different secondary school in Namibia.

Several definitions of data analysis can be found in the literature. One of them, related to qualitative analysis, states that:

> Qualitative analysis is the segmenting of data into relevant categories/cases and the naming of these categories with codes while simultaneously generating the categories from the data. In the reconstructing phase the categories are related to one another to generate theoretical understanding of the social phenomenon under study in terms of the research question (Boeije, 2010: 76).

5.2 The teachers' background information related to their familiarity with ICT

This sub-section provides an overview of and some insights into the participants' educational experience and their familiarity with ICTs (see Table 5.2).

Table 5.1 Profile of the ten participants' educational experience and their familiarity with ICTs

Particip ants names (pseudo nym)	**Years of teaching**	**Level of teaching**	**Gender**	**ICT familiarity – Examples**	***GeoGebra* familiarity**	**School (pseud onym)**
Petrus	16	8-12	Male	Yes, I usually Google to download question papers for test and exams purposes to use it later in my classroom	Yes, I first learned about it during my participatio n in the Namibian mathematic s congress 2015	Galileo H
Henry	4	8-12	Male	Yes, but I'm not using it regularly, sometimes I Google and print material that I use	No	Galileo H

				later in my mathematics lessons		
Michael	4	8-12	Male	Yes, but not often due to limited facilities in the school. But when I used it is mainly looking for extra explanation of a particular topic.	Yes, I got introduced to this software in 2005 at national mathematics congress in Swakopmund, Namibia	Galileo H
Rob	3	11-12	Male	Yes, I did ICT module when I was in the university but I'm not using it often because the lack of facilities	No	Galileo H
Robert	10	8-12	Male	Yes, I'm using Excel, PowerPoint, MS words, internet	No	Galileo H
Peter	8	8-12	Male	Yes, I have a certificate in international computer programming, I always looking for a better way to teach functions	**No**	Okamu SS
Charles	6	8-12	Male	Not familiar at all	**No**	Okamu SS
Oveka	6	8-12	Male	I'm not so familiar with ICT, only for typing tests, exercises and assignments	**No**	Okamu SS
Ali	10	8-10	Male	Yes, I'm	**No**	Okamu

				using spreadsheet, Excel, power point, MS Word, internet searching		SS
Emma	6	8-12	Female	I'm setting exams papers and also, I'm using power point in my lessons	**No**	Okamu SS

All of the participants except Charles indicated that they had already taken courses on computer literacy during their in-service and pre-service period and had developed only their basic computer skills such as word processing, using *Excel* and *PowerPoint* and preparing printing materials to reinforce their mathematics teaching. Through observation and participants' semi-structured interview the study revealed that mathematics teachers selected were not using *GeoGebra* before this study. Only Peter and Michael reported that they had initial experience with *GeoGebra*. Apart from these two, they had very little familiarity with the use of ICT in mathematics education.

In a conversation with the participants about their background in computers (ICT), none of the participants had been offered an opportunity to use computer-supported applications during their teaching years. The main reason for this, according to all participating teachers, was the lack of appropriate computer technology equipment availability in their schools and lack of sufficient experience. For example, Okamu Secondary School, one of the schools selected for this study, has 20 computers in their computer lab but none of them is functioning at all due to the lack of maintenance and connectivity.

5.3 The initial reaction of the participants towards the workshops and the description of the interventions

To analyse the teachers' written responses, some background was needed. The workshops with the teachers were divided into five main sessions. During the first session, the teacher/researcher introduced himself and briefly explained to participants the research topic, purpose, objective, aims and the focus groups details. To obtain the participants' initial reactions to the workshops the teacher/researcher asked the participants to indicate their expectations about the workshops prepared to implement the *GeoGebra* interventions. The first session was used to ensure that all participants would contribute something from the beginning of the interventions. It helped the researcher to differentiate between the voices in the discussion (Kitzinger, 2005; Hennink, 2007). Only first names were used in order to ensure some protection of the privacy of the participants. However, names were removed during the transcription phase. Pseudonyms were used to transcribe their interventions to preserve their confidentiality and anonymity. The first name was used to provide a basis for the group members and the researcher to build rapport. Referring to each other by first names helps to create a greater sense of group identity and cohesiveness (Kitzinger, 2005; Hennink, 2007).

According to Guskey (2000), initial reactions of teachers to a teacher development initiative are prerequisites to experiences higher-level learning. Higher levels of thinking or learning include concept formation, understanding the big picture, creative thinking and visualisation. The premise is that if participants develop positive feelings at the first level, then it becomes likely that they will reap benefits from the teacher development programme at higher levels. These are example of the initial reactions of teachers from the two different high schools, namely Okamu SS and Galileo HS.

> **Petrus:** I was waiting for this opportunity for long time. I'm a teacher and HOD of math & science at Galileo High. I'm expecting to learn more about GeoGebra in these workshops and how to use it in my mathematics teaching.
>
> **Peter:** I'm a mathematics teacher at Okamu SS. I'm really looking forward to getting the basics on how to draw graphs and come to know … this complicated mathematical drawing using this program, so...? I hope I will be using it in my mathematics teaching.
>
> **Henry**: I'm teaching mathematics Grade 8 to 12 at Galileo High. I have high expectations of how to use *GeoGebra* in my mathematics teaching and to make drawing in class using it.
>
> **Robert**: I'm teaching mathematics Grade 11 and 12 at Galileo High. My expectations from these workshop is to at least to raise the level of my teaching and to be able to use technology in my teaching, because, honestly, I never use it in my previous teaching, for mathematics lessons we never bring learners to the computer lab.

Charles: I am teaching mathematics Grade 11 and 12 at Okamu SS. My expectation from these workshops is to by the end of the sessions feel confident on how to draw functions, functions that we used.

Oveka: I am a mathematics teacher at Okamu SS. Yes, my expectation from these workshops is just to gain some skills on how to use *GeoGebra* in my mathematics teaching.

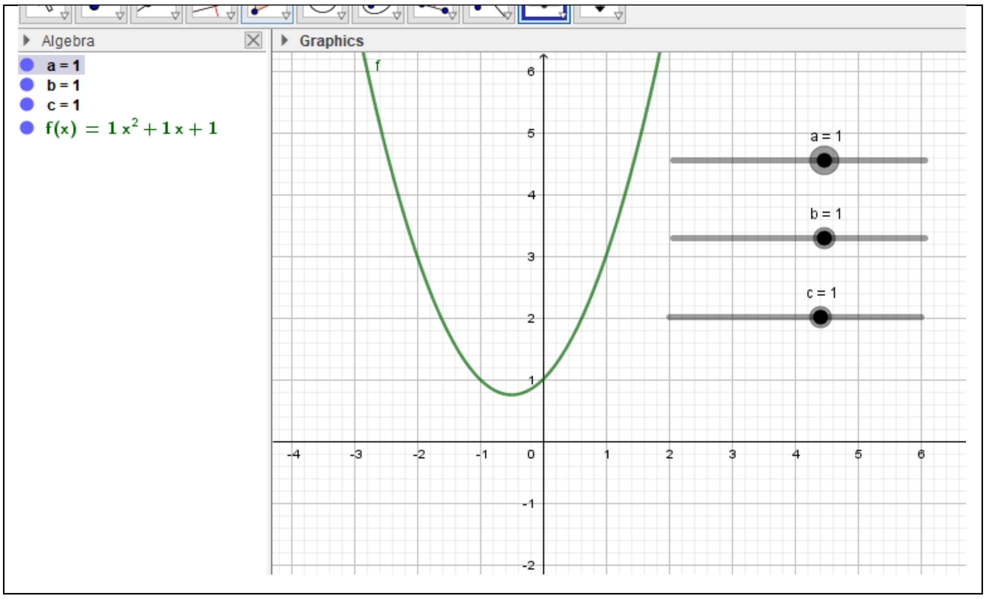
Ali: I am teaching mathematics Grade 8 to 10 at Okamu SS. My expectations are to learn something new in these workshops and to gain knowledge on how to draw graphs of functions using *GeoGebra*.

Michael: My learners used to call me Dr Michael; I am teaching mathematics Grade 11 and 12 at Galileo High. I love these things…., I want to learn the best from what this gentleman brought to us.

Emma: I am a mathematics and science teacher at Okamu SS. My expectation from these workshops is to learn more on how to use *GeoGebra* in my mathematics and science teaching.

The researcher introduced general information about the development, potential and design of *GeoGebra* and explained that *GeoGebra* is dynamic mathematics software, open source in terms of its design. It integrates multiple dynamic representations, various domains of mathematics, and a rich variety of computational utilities for modeling and simulations invented in the early 2000s by Markus Hohenwarter. Next, the teacher/researcher explained the general tools and menu items; for example, entering, extracting and modifying coordinate points (see activities 1.1, 1.2, 1.3 in Appendix I). This introduction was particularly important because these teachers (the participants) were using *GeoGebra* for the first time in a focused way and so one of the aims of the first intervention was for teachers to become accustomed to the software. To accomplish this, the researcher began the intervention by demonstrating how to operate the software: how to open the software and how to use the dragging function of *GeoGebra* in order to update positions of a figure dynamically.

Teachers were given a demonstration of the main features of *GeoGebra* software. The construction of sliders in the case of quadratic and linear functions was shown. For example, slider m and b (see Figure 3.1) and sliders a, b and c were created to view the dynamic changes in the parameters of a quadratics function (see Figure 5.1 below).



. Figure 5.1: A screenshot of manipulation of a quadratic function by sliders

This applet in Figure 5.1 shows an example of how the researcher demonstrated how the main features of *GeoGebra* software work. During the introductory workshop, the researcher explored through manipulation the use of sliders in the case of a quadratics function. The sliders are shown in the upper-right part of the screen. This affords control of the equation. The slider can also be automated so that every possibility is cycled through repeatedly, enabling the teachers/users basically watch the resulting changes in the graph. Three sliders, namely, a, b and c, were created to dynamically change or vary the values of the respective variables. With all values being set at zero, no graph or function would result. Each value needs to be explored one at a time to see the resultant effects in relation to a quadratic function in the form $y = ax^2 + bx + c$ and $y = a(x - p)^2 + q$ where:

a - represents a change in orientation (increasing the value narrows the parabola – decreasing the value widens the parabola). Negative will flip the graph (reflection)

b - helps determine the axis of symmetry (and turning point) for a parabola

q - represents the maximum value or minimum value of the turning point

c - represents a vertical change of the graph (y-intercept)

p - represents the horizontal shift

By viewing the general forms of the function and the movement of the slider from side to side simultaneously, the participants would be able to see the resultant outcome. By moving the slider "a" to the right, a minimum value function would result, and a maximum value graph would result should the slider be moved in the opposite direction.

This also indicates to the teachers the effect of positive "a" as compared to negative "a". The manipulation of "c" results in the vertical shift up or down. The manipulation of "b" results in the axis of symmetry being changed, confirming the formula $x = -b/2a$. This move simultaneously shows the effect the maximum or minimum value has on the turning point of the function. The resulting outcomes after several manipulations of the variables ought to convince the teachers of their effect in the general form.

During the second workshop session the participants engaged with the activities 2.1 and 2.2 (see Appendix I). The aim of this task was to engage teachers in finding possible solutions to the solution of a quadratic equation exploring pen-and-paper methods and teachers' preferences to the solution of quadratic equations, and to explore the reasons behind their preferences, time consuming and other particularities related to symbol sense. Issues related to quadratic functions were discussed. The first focus group interview was conducted after this task was completed.

Figure 5.2 shows an example of one of the activities that the teachers completed during the first workshop using *GeoGebra* and how the screen appears

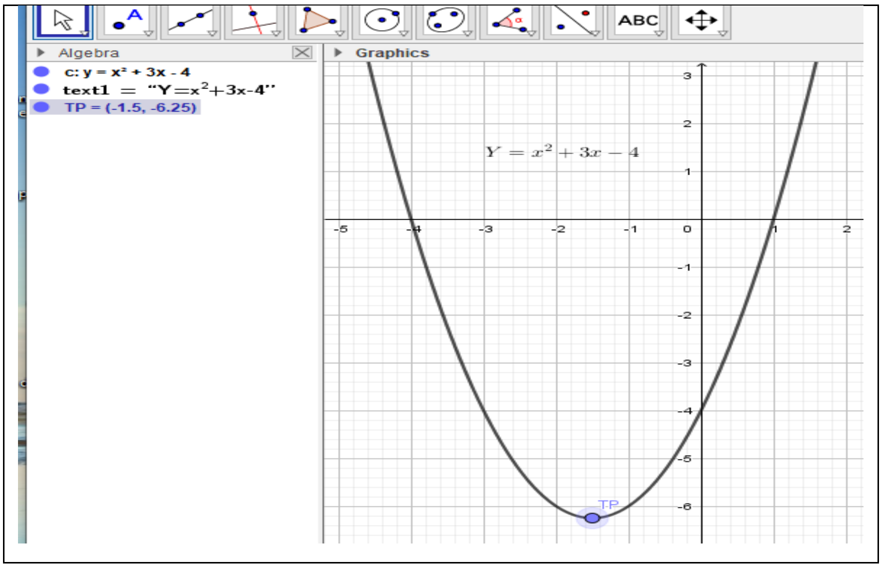


Figure: 5.2: A screenshot of a quadratic equation (activity 2.1) in GeoGebra

Figure 5.2 shows the outcome of a quadratic equation produced by the teachers during the first workshop with the use of *GeoGebra* in the form of $f(x) = x^2 + 3x - 4$; teachers successfully completed this task of drawing the quadratic equation typed into the input bar.

This activity was relevant because it served as the teachers' preparation for engaging with the activities planned for the next session.

During the third and fourth workshop sessions (see Appendix I) the teachers continued their discussion related to activity 2.5 (exploring parameters of linear functions – more details in section 5.4.2). This activity was introduced as a homework exercise for the participants at the end of the second intervention to be discussed during the third intervention. At the beginning of the third intervention teachers/researcher recapitulated what had been learned about the *GeoGebra* function content.

This activity of exploring the parameters of linear functions was supposed to be completed during workshop three, but the discussions were so intense that the time allocated for this session extended beyond the time allocated. At the beginning of workshop 4 some teachers joined the discussion because they were absent the day before. One of the teachers (Michael) who was present at the previous session offered to explain to the rest of the teachers what they did during workshop 3, In this session the activity related to the exploration of the parameters of linear functions was analysed (see Appendix I).

During the fifth workshop (see Appendix I) the last activity on cubic polynomials (see Appendix I) was supposed to be completed and discussed, but unfortunately the focus group prepared for this day was prolonged and the activity was not completed.

5.4 Analysis of the teacher learning experiences (TLEs) related to the five *GeoGebra* workshops sessions

This section responds to the research question: What are high school mathematics teachers' learning experiences during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra*?

Analysing the data thematically across this case study revealed four salient dimensions (themes) in relation to *GeoGebra* intervention: time issues, teachers' mathematical thinking around the particularities of functions, other mathematical issues and the *GeoGebra* affordances in the teaching and learning of linear and quadratic functions.

The methodology adopted in this study enable the teacher/research to examine participants' experiences during this *GeoGebra* intervention.

Merriam (1998:7) described case study as an "examination of a specific phenomenon such as a program, an event, a person, a process, an institution, or a social group". The researcher intensively investigates the issue or activity related to one or more individuals (Creswell, 2009). Therefore the case study enables the teacher/research to examine and collect data from participants during their interaction with *GeoGebra* activities.

The ten teachers were interviewed while they were interacting with a set of *GeoGebra* activities (see Appendix I). However, their semi-structured responses were collected only at the end of the last workshop. The teachers' written responses and interactions with *GeoGebra* activities were guided by the following questions (see Table 5.1 in sub-section 5.2 summarising responses to questions 1 to 5):

1. Familiarity with information communication and technology (ICT)? Give examples:
2. Are you involved in any ICT network learning/ network? If yes, give details.
3. How long have you been using ICT in your mathematics teaching? For what purposes?
4. Are you familiar with *GeoGebra*? If yes, where and how?
5. Have you ever used *GeoGebra* in relation to the secondary school mathematics curriculum? If yes, provide details.

The focus group interviews conducted during the workshop two and workshop five were guided by the following questions.

- What do you think about this content of linear /quadratic function now that you have experience through *GeoGebra*?
- Are you thinking about implementing *GeoGebra* in your teaching? And when?
- After you have been exposed to *GeoGebra*, what insights have you gained?
- Did you learn something new?

5.4.1 Case 1: Teachers' responses during the semi-structured and focus interviews with regards to deepening functions learning.

GeoGebra interventions were influenced by time concerns. The teachers were asked the following question:

Do you believe that there is a role for ICT/*GeoGebra* to deepen their learning about functions? Please elaborate. (Question 11, see Appendix E).
This section presents qualitative analysis regarding the teachers' responses to the question about their view on whether there is a role for ICT/*GeoGebra* in deepening their learning about functions.

> **Petrus**: Yes, ICT/*GeoGebra* will/can make preparation easier and faster. ICT can be used to supplement pen and paperwork for learners to visualise what on the screen. Also, we can record lessons and in the case of not being around, learners can just view lessons themselves on repeat.

> **Henry**: Yes, after getting confident with *GeoGebra* through this project I realised how easier it is to come up with graphs and draw the functions by means of ICT. It is more interesting plotting points and drawing lines with *GeoGebra.* It deepens the teaching and learning of functions. *GeoGebra* in connection with ICT is quicker and simplifies the methodology of learning.

The above responses show that Petrus and Henry raised a time concern in addition to the use of *GeoGebra* comparing with the traditional method of teaching functions. The claim that could be made is that Petrus realised that the lesson preparation can be done "easier and faster" in term of plotting points and draw a graph of an algebraic (symbolic) equation. Henry indicated that the teaching and learning of functions can be enhanced with the use of ICT (*GeoGebra* in this case). He is also aware of the *GeoGebra* potentialities to make learners' work less tedious compared with the "traditional classroom". It was therefore pragmatic.

> **Michael**: I believe there is a big role that *GeoGebra* will play if it's to be used in the learning of functions, because it arouse leaners' interest and makes them to love the topic as much as the program. It brings theory into practice, also is faster and the

> answers are ready available, learners understanding of functions will be much more enriched.

This teacher realised that time could be "saved" when using the *GeoGebra* program and learners' motivation could be increased to learn the topic of functions. It seems that his comment "readily available" is referring to the *GeoGebra* applet ability to dynamically represent the function solution in both windows simultaneously (algebraic and graphic). For example, the user can directly enter numerical values, algebraic input, commands and functions into the input field by using the keyboard. While the graphical representation of all objects is displayed in the graphics window, their algebraic numeric representation is shown in the algebra window.

The *GeoGebra* screen has multiple windows (algebraic, graphical, input and toolbar) that are epistemically represented and connected. On the screen or window, there are scrollbars connected to and representing various parameters that the user can manipulate by typing in inputs or dragging a scrollbar and thereby effect graphical, tabular and symbolic changes; for example, it is possible for the teachers and learners to investigate changes in the parameters of the equation of a curve by dragging the curve in the graphics window and observe the changes to the equation in the algebraic window. Furthermore, teachers can change the equation of the curve directly and observe the way the representation in the graphics window changes (Hohenwarter & Jones 2007). However, no claim could be made about Michael understanding the relationships between the different representations. The time issue here was more pragmatic than epistemic, because his comments were not about understanding the symbols or objects in this specific program he observed during the workshop sessions.

> **Rob**: I believe that *GeoGebra* program will play a major role to deepening learning on the ground that: learners will understand the topic very well since *GeoGebra* is more visual. I believe that it will make mathematics perfect. It also deepens functions learning in the sense that it's faster and one can always have a picture in mind of how problem were solved by *GeoGebra.*

Rob indicated that G*eoGebra* being "more visual" and faster. This is an epistemic comment because it linked multiple representations. This excerpt probably suggests that learning from dynamic visual representations has a great impact on learning, because it allows learners to interact with the content and manipulate the application, which promotes connections of symbolic and iconic representations with formal definitions (Dockendorff & Solar, 2018). Researchers have demonstrated that graphical and symbolic representations through visualisation have a great impact on learning functions. For example, Karadag and McDougall (2011) show that 'visual learning' is more natural compared to learning through any other type of representations. Visual learning can be understood in terms of learners' preference for seeing (think in pictures; visual aids such as overhead slides, diagrams, handouts, diagrams, written text and pictures viewed on a computer screen (Schmeck, 2013).

Learners are more familiar with visual learning because they learn from an early age how to use almost all technological tools such as computers, the internet and cell phones visually. Therefore, it might be very challenging for them to learn mathematics through symbolic algebra; rather, it could be easier for them to understand algebraic notations after they develop a visual understanding of mathematical concepts. Thus, great importance can be given to the use of ICT tools. Hence, the learning process of the formation of concepts can be supported by visualisation at the symbolic, graphical and numerical level (Ainsworth, 2006; Trouche & Drijvers, 2014). Similarly, the use *of GeoGebra* can play a role not only in stimulating and shaping learners and teachers' visual images, but in providing access to new forms of representations as well as to multiple and linked representations (Kaput, 1989).

> **Robert**: Yes, *GeoGebra* is very effective and saves a lot of time in drawing graphs also will help the learners to viewing the effects of nature of gradient and the general behaviour of graphs as different input values are entered into the input bar. It will generate learners' interest as the concept is introduced.

Robert similarly valued the time saved when drawing graphs with *GeoGebra*. He was apparently aware of the pragmatic (focusing in the productive potential) and epistemic value (contribute to the understanding of the objects that involves, for example, 'nature of the gradient and general behaviour of the graph') of teaching functions with *GeoGebra* and he might have been thinking about the multiple representations that 'could help learners understand better the linear and quadratic functions.' He was interested in the need to help learners to view the effects of the nature of a gradient and the general behaviour of graphs as different input values are entered into the input bar. These epistemic issues can be attributed to him noticing that *GeoGebra* can serve as a tool that teachers can use to involve more learners in the process of teaching and learning, or keep them more motivated in what they learn; for example, leaning the nature of the gradient and the general behaviour of the graph. In other words, increasing the time available for the learners during the possible interaction and use of *GeoGebra* could give them more opportunity to engage more deeply in understanding the behaviour of graphs as different input values are entered into the input bar and eventually improve the epistemic value of the mathematical object involved.

Saving time as a pragmatic issue can assist teachers to find other alternatives of thinking about different solutions. Robert's assumption was that if the learners take more time to analyse their own hypothesis, they might be more motivated to learn. Consequently, the feedback available in the *GeoGebra* task environment might enable leaners and teachers to take greater responsibility for thinking through mathematical situations for themselves. Thus, the use of *GeoGebra* can potentially serve as a mediation processes between the teachers and the body of mathematical knowledge to be learned (Leung, 2017). For example, a teacher using *GeoGebra* can dynamically visualise his/her mental images of geometrical objects such that s/he can reason the correctness (mediate by dynamic feedback tool like dragging) of the mental images with respect to the Euclidean world embedded in *GeoGebra*. Naidoo (2012) also stated that the visual image is a symbolic demonstration of the visual appearance of an object. Visuals help to break down abstract Mathematics concepts leading to better understanding and comprehension and advanced mathematical skills (Kosa, 2016).

It is evident that TLEs were influenced by the time that could be gained and saved when using *GeoGebra* to explore graph functions. This pragmatic issue is understandable because of pressure of time to teach curricular content. According to the participants' comments below, less time will be used in the classroom environment than when using pen-and-paper methods.

> **Peter:** Yes, the program is fast and time saving, that will give the learners the shape of the graphs upon entering the function, deepening leaners understanding of different functions and easy ways of manipulating functions to find things such as midpoint, turning points or solutions to a given function.
>
> **Charles:** Yes, because using *GeoGebra* in the teaching of function is much faster than using a chalkboard. Is very easy to find a maximum and minimum values of

functions unlike completing the squares first or finding the x intercept and memorising the formula of turning point; tp = (-p;q).

Peter and Charles similarly valued the time saved and more quicker, a pragmatic issue of getting answers quickly when teaching with *GeoGebra*. They were concerned with the pragmatic value of teaching with *GeoGebra* which took "a short time," and they might have been thinking about the multiple representations that "could help learners understand better the quadratic functions" more quickly.

Oveka: Yes, that can deepen functions learning because learners will be willing to explore and understand better; they will find it easy to draw and interpret any given graph. Learners can observe what is happening to the graph as soon as we keep on changing the parameters of the function.
Ali : Yes, because this will help the learners to get deeper understanding of functions as they can observe at the same time the behaviour of the graphs when the parameters are changing and it also create curiosity for them to learn on their own, giving opportunity to explore and discover new things on their own.

Oveka and Ali similarly valued the affordances of the *GeoGebra* environment. However, they were worried about the pragmatic and epistemic values of teaching and learning with *GeoGebra* as they could "observe the behaviour of the graph when the parameter is changing". Observing the behaviour of the graph can contribute to the understanding of mathematical function. They might have been thinking about the multiple representations as well that "could help learners understand better the linear and quadratic functions". The design of some dynamic geometry software (DGS), including *GeoGebra* provides teachers and learners with wide-ranging opportunities for mathematical exploration and sense-making. With these tools, teachers and learners are encouraged to make mathematical conjectures and use the dynamic capabilities to visualise an idea under a wide variety of situations (Niess, 2006).

Ali's comment may suggest, for example, when he states "behaviour of the graphs when the parameters are changing" appeared more in favour of having learners to explore mathematical concepts and aspects of the learning of function for themselves; this will in turn stimulate their curiosity and interactive explorations, as well as learning by discovery. *GeoGebra* can possibly develop active and learners-centred learning, where the teaching shifts the focus of instruction from the teacher to the learner by allowing for mathematical experiments, interactive explorations as well as learning by discovery, for example, through the ways that learners think and comment on what they know and see on the screen for themselves.

5.4.2 Case 2: Exploring parameters of linear functions

Teachers' deals with learning experiences (TLEs) related to the five *GeoGebra* interventions. Their responses had pragmatic and epistemic dimensions. The teachers' interactions revealed some of *GeoGebra*'s potential in the teaching and learning of mathematical functions.
Five workshops were conducted during this project (see Section 4.5.1). This section will present only the results of the third and the fourth workshops. Information sought from respondents was linked to the main research question. This question was: What are high school mathematics teachers' learning experiences during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra*?

Teachers were instructed to construct four linear equations in such a way that they form a square when they are drawn in *GeoGebra.* This activity was designed to create an intellectual challenge for the teachers (Schoenfeld, 1992), and included the construction and interpretation of the algebraic and graphical representations of linear functions. Thus, the researcher's intention with this activity was to deepen the teachers' leaning experiences with respect to varying the symbols of the gradients, perpendicular gradients when they occur, why they occur based on the affordances of the scrollbars in the design of *GeoGebra*.

In this task (activity 2.5, see Appendix I) the participants needed to construct for themselves four linear functions in the form [y = mx+c] in such a way that lines will form a square when they are drawn using the *GeoGebra* coordinate system.

This activity provided an opportunity for teachers to explore the properties of a square as a geometric object as in Analytical Geometry with its Cartesian coordinatisation as well. No additional instructions were given to the participants related to the constructions of the four linear functions, because an example related to the square construction appears in the adapted *GeoGebra* manual (see Appendix M) given to them during the introduction session. However, the way teachers approached the solution of this task showed evidence that teachers did not stick to the instructions from their *GeoGebra* guides manual. They improvised and followed their own approaches.

The aim of this activity was to engage teacher/researcher, participants and the resource (*GeoGebra*) to mutually enrich each other through collaboration in producing mathematical experiences (Leung & Bolite-Frant, 2015:192). This intended collaboration also gave teachers an opportunity to practise their newly gained knowledge in preparation for the next session in workshop 4 (See Appendix I).

This specific activity was designed taking into consideration the Namibian mathematics curriculum in the secondary school that addresses the development of learners' general mathematical skills, such as the content knowledge of functions, the curriculum requires outcomes regarding the learners understanding of graphical and algebraic representations of functions as well the use of digital tools to improve the teaching and learning of mathematics (NIED, 2016:10).

All the teachers began by engaging in this activity to initiate their collaborative participation. The group initiated their discussion by creating a shared goal. They negotiated and agreed on the appearance of the graphical representations of the four functions needed to create a square.

Figure 5.4 shows the representations of the four linear functions in the symbolic/algebraic and graphical window as done by Peter (one of the participants).

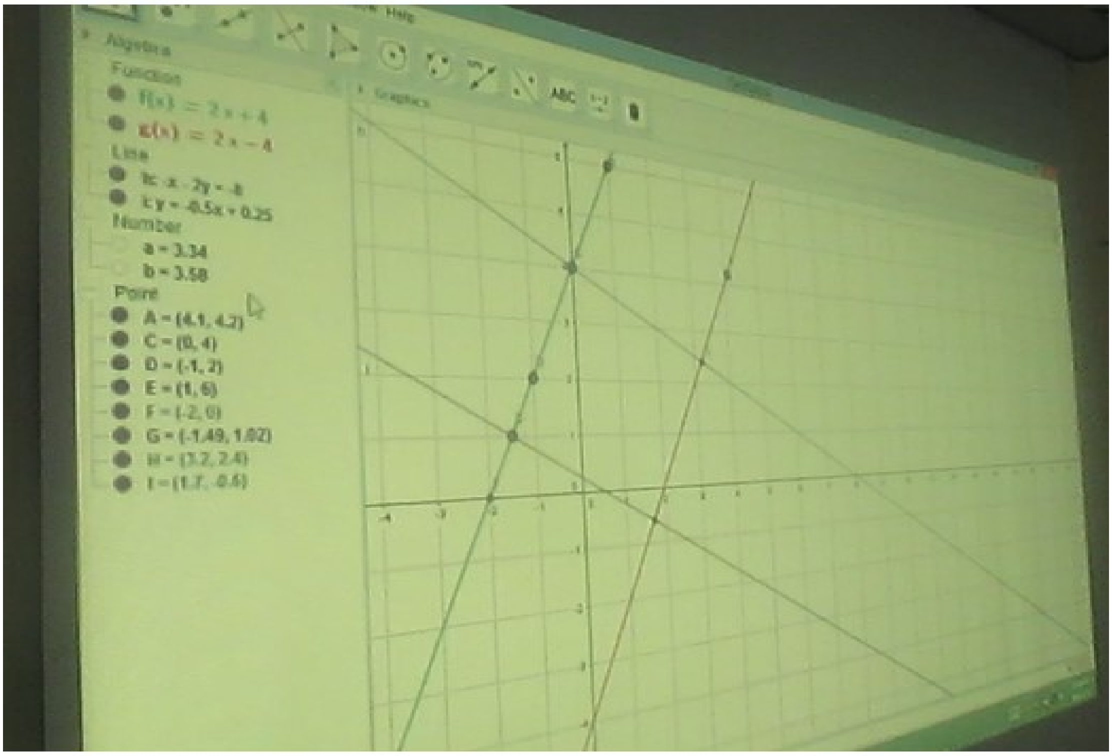


Figure 5.4: A GeoGebra screenshot of the algebraic and graphical representation of four linear functions forming a square done by Peter

Evidence of a teacher's construction (Figure 5.4) related to the four linear function drawn in such a way that they will form a square when they are drawn in *GeoGebra* coordinate system is given next.

In this specific activity there are pragmatic and epistemic issues. Pragmatic issues include speed in making the drawing with *GeoGebra* and the epistemic issues related to the construction of parallel and perpendicular lines and comparing or interpreting the values of the gradients. The gradients can then be manipulated using the mouse and this task also provided the teachers with the opportunity to drag objects (parallel and perpendiculars lines) in the geometry window and see the corresponding changes in the algebraic representation; or they changed the algebraic representations in the algebra widow and saw the object change in the geometry window (Hohenwarter & Jones, 2007). Teachers might thus be able to explore mathematical concepts and dynamically connect algebraic, graphic and numerical representations of those concepts without having to spend a significant amount of time of drawing figures or functions.

Researcher: What are you going to do with this specific activity? (This refers to activity 2.5 in Figure 5.4, see Appendix I).

> **Michael:** In this activity we must come out with four linear functions such as when you draw the four linear equations using *GeoGebra* you come out with a square [see Figure 5.4]**.** Now we have drawn the lines, the shape that comes out looks like a square. Now it is matter of verifying if it really is a square, because if this is really a square, the dimensions should be equal.

Researcher: Do the four lines you constructed form a square? Can you elaborate more?

> **Michael**: Well … we did it by inspection, it looks like a square. We came out with a parallel line first. Parallel lines have same gradient but different intercepts, and then we can find the other two equations reversing the gradients.

In this excerpt Michael is guessing or assuming mentally that his construction can be a square without verifying it "we did by inspection" (referring to pen-and-paper types of construction).

> **Robert**: One gradient is positive, and one is negative.

The following session shows how the teachers discussed the findings on perpendicular lines being valid for slopes or gradients different than 1.

Researcher: What will happen if you multiply the values of the two gradients**?**

> **Michael:** It will be negative 1 (-1) perpendiculars.
>
> **Henry**: [Showing with his hands. Indicating that the two lines are perpendiculars each other] (See Figure 5.5).
>
> **Charles:** Yes, they are perpendiculars.
>
> **Michael**: This line? This line? (Pointing to the computer screen)

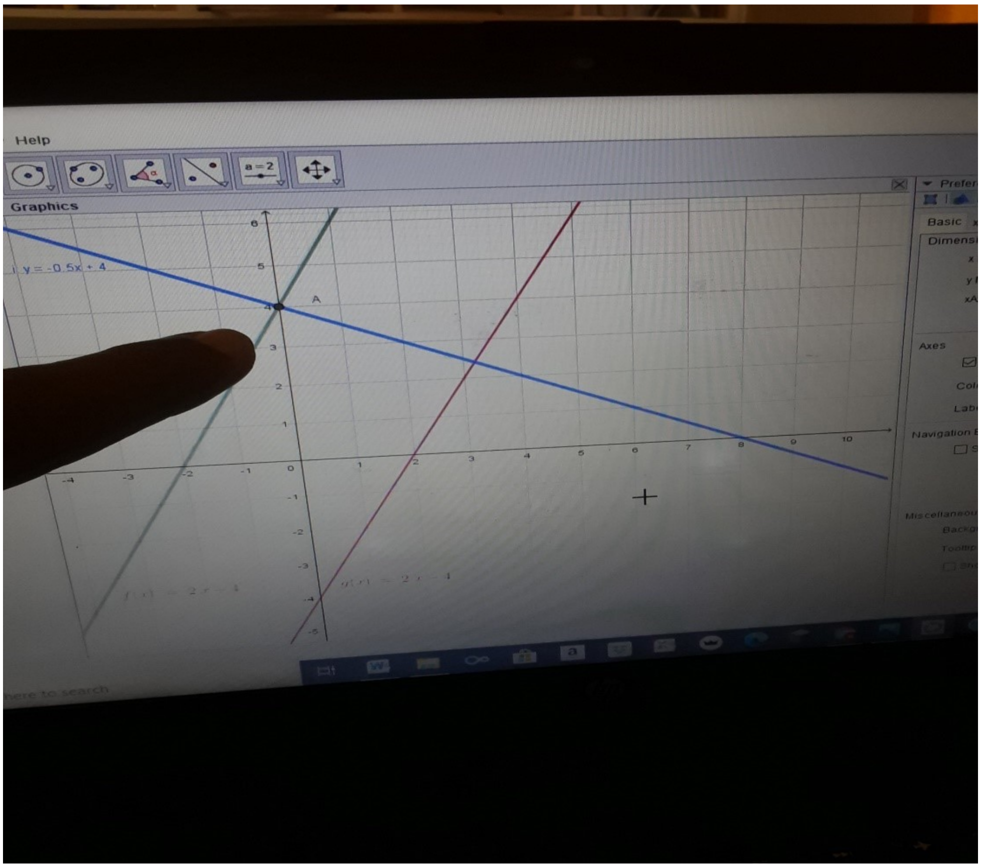


Figure 5.5: Henry showing with his hands that the lines $F(x) = 2x + 4$ and $Y = -0.5X + 4$ are perpendicular to each other

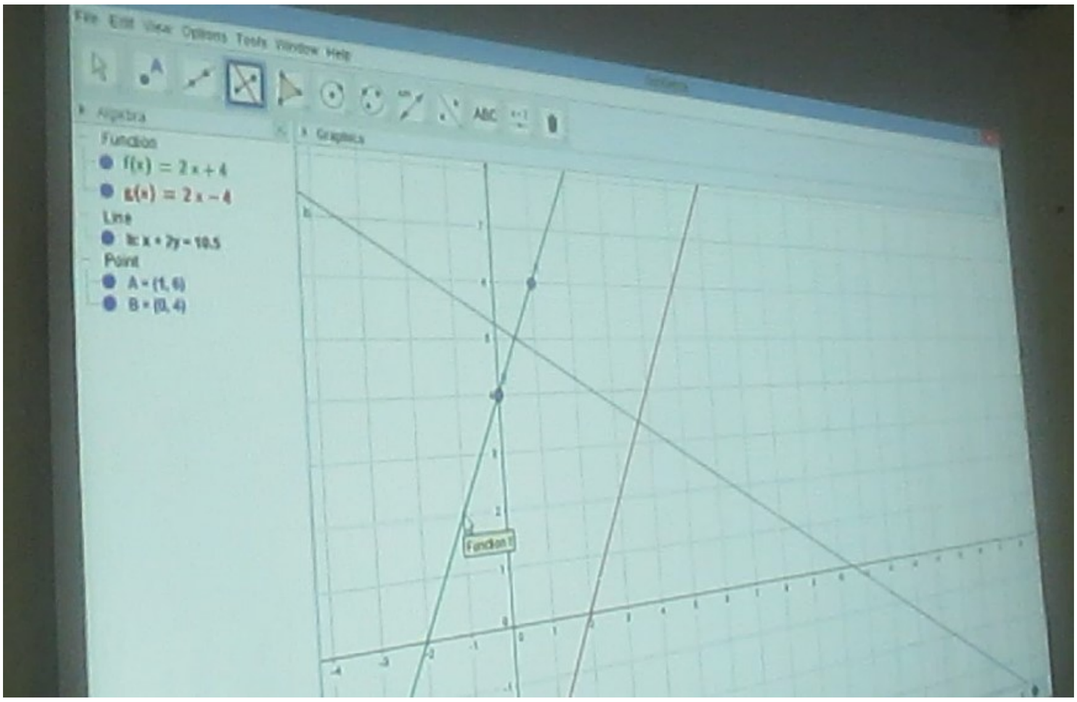


Figure 5.6: A screenshot of two parallel lines $F(x) = 2x + 4$ and $G(x) = 2x - 4$ with GeoGebra

Michael: Yes! I can multiply these two gradients together. The gradient of function (F) and the gradient of function (G), that is, multiply 2 by negative half $(2 * (-\frac{1}{2})$, we are getting -1, so … the two line are perpendicular to each other. So …. this is a square! (Referring to what Figure 5.4 shows).

Researcher: Did you plot on *GeoGebra* the four lines forming the square?

Michael: Yes

From the above statement where Michael says 'Yes,' it is evident that Michael was familiar with the concept of perpendicular lines in this specific activity, but did not necessarily understand how to verify whether his construction was a square. In other words, Michael did not draw on the properties of a square as a 2D geometric figure. The researcher was hoping they would draw on their thinking about the properties of a square as well.

There is no clear evidence about how well the teachers understood the features embedded in *GeoGebra* to produce the two perpendicular lines (see Figure 5.6). The possible causes of the graph behaviour remained hidden. However, this specific interaction, that is, the conversation between the teacher/researcher, Michael and the rest of the participants (Robert, Henry, Charles) created an opportunity for Michael and probably to the rest of teachers to explore how *GeoGebra* is used and how other representations (multiple representations) could or might enable them to find the necessary condition to construct the two perpendicular lines. In these instances *GeoGebra* means utilising its affordances and related conversations with teachers as ways to direct the teachers' attention to the properties of a square, gradients and lengths of line segments that intersect. Here the 'boundary' entails the epistemic issues/properties of a square as featured on the screen windows.

From the analysis of the conversation between Michael, Henry, Charles and the rest of the participants, it is possible to identify what could be one of the essential benefits of the use of *GeoGebra* in this case. It provides the possibility of building mathematical knowledge and mathematical thinking among the participants based on a platform that enables them to discover patterns and regularities, and identify the relations and properties of diverse objects, for example, a square in this case.

The dynamic nature of the straight lines makes *GeoGebra* effective as a learning strategy, which is difficult to achieve for teachers who only use the traditional classroom resources (pen-and-paper environment). Thus, the possibility of visualising the four linear equations in such a way that they form a square when they are drawn in *GeoGebra* enabled Michael and the rest of the teachers to identify the correct criteria based on parallel and perpendicular lines, because the use of *GeoGebra* enables the teachers to recognise a certain relative position (parallelism) while at the same time checking the parameter values that produce this graphic situation.

Researcher: What do you think about this content on the linear function now that you have experience based on a *GeoGebra* representation?

> **Michael**: If I have to do this, teaching functions using *GeoGebra*, I have the feeling, not a feeling, I'm positive that learners' understanding will be much more enhanced rather than use theories, because now you bring theories to practice. For instance, if you are telling the learners "the lines are perpendicular", learners can see that there is a line A and a line B and those lines meet exactly at 90^0. It will make your explanation easier, so you ask the learners to work on it.

The above statement where Michael says the "learners can see" whether the two lines A and B are perpendicular (pragmatic issue) is demonstrating the geometric (a square) as well as the algebraic/epistemic issues of two gradients being perpendicular.

Researcher: Michael, during our first session you mentioned something about "learners' visualisation". What did you mean**?**

> **Michael:** Actually, I said that *GeoGebra* it will give insight and understanding, learners visualise the problem. If you introduce it to them, they will use it, yes, it will be easy for them to think back when they go to the classroom, to reflect on what they were doing in the computer lab. That will still be motivating them**.** The visualisation that I was referring to maybe will explain how *GeoGebra* will make learners understand the function concept. For example, if you put a function, sin x or $\mathrm{y} = 2\mathrm{x} + 1$ in your laptop (meaning type in the "input" bar) at the bottom of the screen in the form of $\mathrm{f(x)} = 2\mathrm{x} + 1)$ (see Figure 5.7) then you will be able to share with your

learners how this function was brought up. Ok, put any point on the line or anywhere else, now using the tool that allows the point to move, that point will move up and down on the line (see Figure 5.6) to make the explanation easy for the learners as you drag the point through the line.

In the above comment Michael is making epistemic and pragmatic comments on what he is seeing on the screen pragmatic because typing a function, for example $y = 2x + 1$, will produce a result, a graph related to this specific linear function but without a deep understanding as to why this linear function was produced. On the other hand, he is also referring to 'dragging the point through the line' which has an epistemic effect; the symbols of the gradient and the y- intercept whichever on the screen, takes variable or varying values. This may be contributing to a conceptual understanding of the linear function.

Researcher: The values are changing also in the algebra window? (Figure 5.7)

Michael: Yes

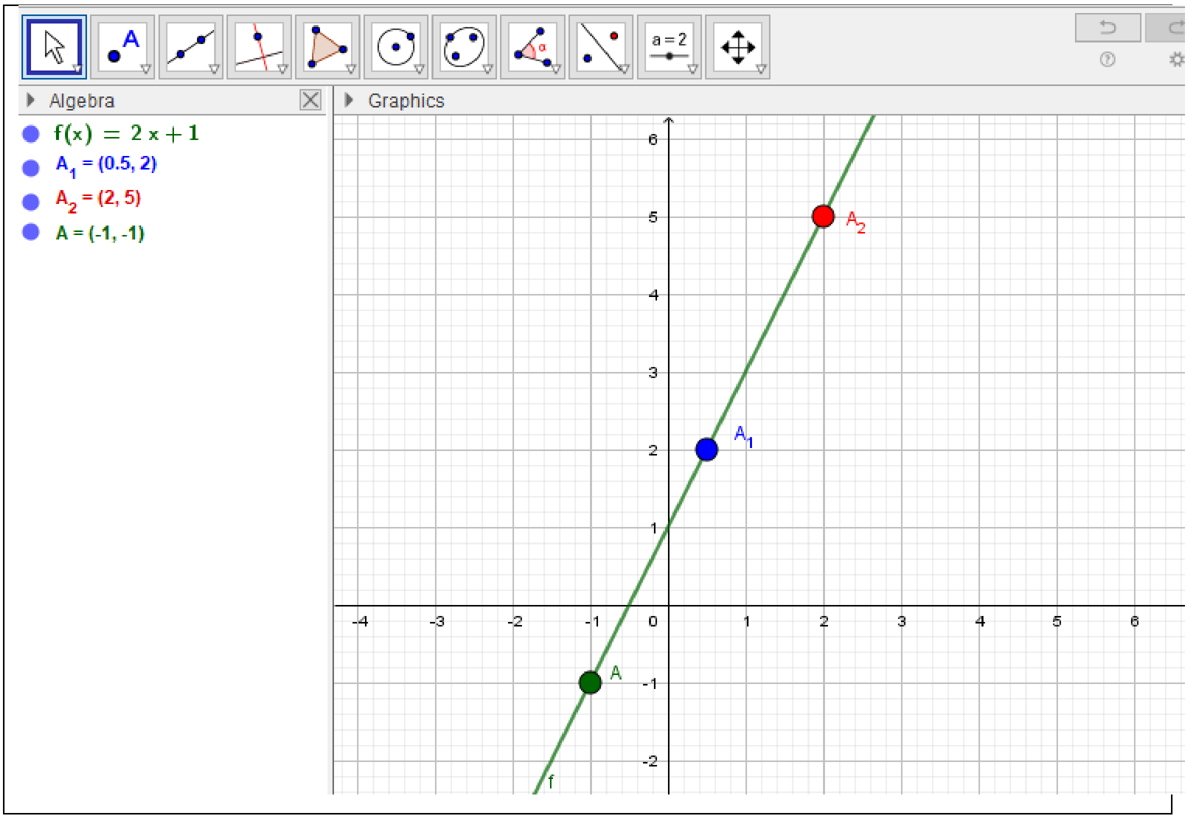


Figure 5.7: A screenshot of a change of point A through a linear equation, according to Michael's explanation above

Researcher: What is the meaning of these changes?

Michael: Yes, the meaning of this is that for every value of x entered, it will be a new value of (y) so … changing the parameters, obviously the graph will change.

In Figure 5.7 *GeoGebra* was used by Michael to explore the change of the input and output values in a linear equation. He typed the equation in the "input" bar at the bottom of the screen in the form [f(x) $= 2x + 1$]. There are at least two opportunities to show the movement of point A: (1) visual representation on the display window of *GeoGebra* as illustrated on the right part (graphic window), and (2) algebraic representation on the left (algebra window). When point A was clicked on and dragged through the line (f) (green line) the position of the point and the corresponding coordinate of the new position were changed dynamically in both windows (algebraic and graphic window). These eventually enable the users to see what happens to point A when moving through the linear function. In other words, Michael emphasised that for every value of (x) entered, it will be a new value of (y), according to a symbolically represented rule '2x +1' Teachers can verify those numeric values of point A (see Figure 5.7) through others means such as the pen-and-paper method or calculators.

Dragging point A with a mouse has pragmatic value, since changes both of the location of point A and the coordination A(x, y), these coordinates also have an epistemic component, namely, they lie on the green line which has a fixed gradient (see Figure 5.7). In other words, the gradient is a ratio that remains the same, which is an epistemic issue. These actions can also help reveal the epistemic aspects of the pragmatic action of dragging. The epistemic value is usually not apparent to the user and often requires strong teacher mediation (Schneider, 2000). *GeoGebra* has both pragmatic and epistemic value (Artigue, 2000). Pragmatically it allows a user to make computations relatively quickly, that is, there is an instant display in the algebra window. Epistemically it enhances understanding of the mathematical objects such as variables and parameters.

The use of *GeoGebra* in this specific example potentially supports the teachers' interpretation of the parameters or variables represented via *GeoGebra*. For example, the *GeoGebra* applet manipulated made the parameters dynamic and represented epistemically in tabular, graphic and symbolic forms, which in turn has the potential to contribute to the teachers' conception of the idea of mathematical function (Lloyd & Wilson, 1998). This is something that would be difficult to promote in a pen-and-paper environment.

With the use of *GeoGebra*, different representations of the same function (graphs, tables) are connected dynamically, allowing users to go back and forth between them, thereby making the relationships among those representations more easily understandable by teachers and learners (Pfeiffer, 2017).

The experience of using *GeoGebra* tended to modify teachers' ideas towards accepting a more visual approach to mathematics, generating an increasing acknowledgement of the importance of representations for learning. Similarly, Michael recognises the importance of promoting competence in visualisation in learning mathematics. Michael believes that "*GeoGebra* will give insight and understanding, when learners visualise the problem".

The discussion on the construction related to the four linear functions drawn in such a way that the lines will form a square when they are drawn in *GeoGebra* coordinate system activity 2.5 (see Appendix I) continued at the beginning of workshop four.

Robert, one of the participants, demonstrated to all teachers present his own construction of the four linear functions [$y = mx + c$] in such a way that lines will form a square when they are drawn in *GeoGebra* coordinate system. This activity was designed to provide the teachers with multiple opportunities to engage in exploration, reflection and discussion related to linear function.

All the teachers present contributed to summarise what was done on the day before related to this activity. Later on, the teacher/researcher explained the use of the tool perpendicular line and perpendicular bisector as one of the possible tools to use in order to construct the four lines in such a way that they will form a square when they are drawn in *GeoGebra* coordinate system.

The researcher introduced the content of workshop four related to the use of *GeoGebra* focus on quadratic functions. He explained the use of sliders once again, activity 2.5 was mentioned this time again because some participants were absent from the previous intervention.

The following excerpt from the teachers' written responses and from interview transcripts during workshop four related to linear and quadratic functions convey a typical picture of their experiences interacting with *GeoGebra*.

Researcher (to Michael, one of the teachers): Can you share your experience about activity 2.5 with us?

> **Michael**: Ok, gentlemen, the one for drawing four linear equations, such as if we draw those four linear equations on the *GeoGebra* sheet we are going to come up with a very nice square. Ok, obviously the way to come up with the square is to draw two parallel lines first, parallel lines that have two different (y) intercepts but the same gradients (m). Let say, for example, our first equation is $(2x + 4)$, that is my first equation, another equation that will be parallel to this one, it needs to have same gradient $(m = 2)$, but different (y) intercepts, let say $(2x - 4)$. Are those two lines parallel? Yes, they are parallel [see Figure 5.8].

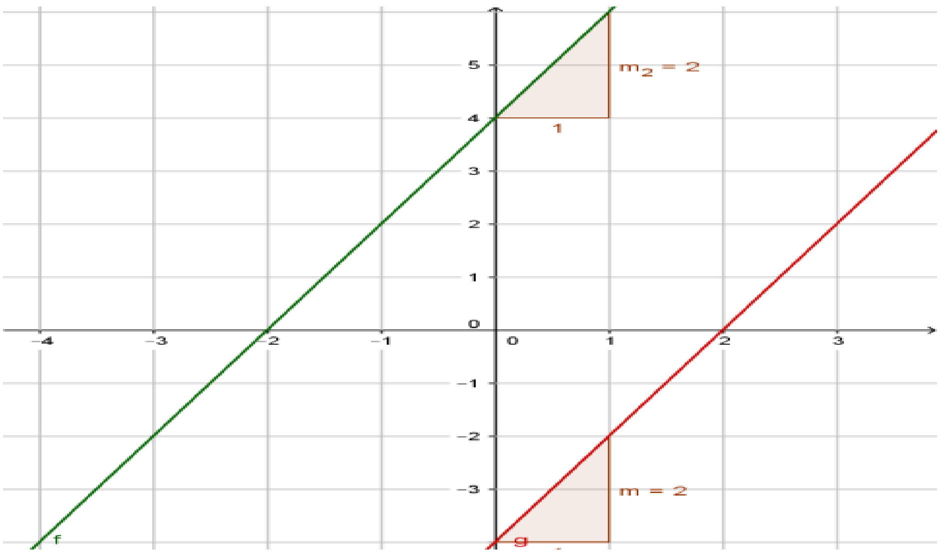


Figure 5.8: A screenshot of the construction of two parallel lines with GeoGebra by Michael

> **Michael**: The next part is to find two line again, two equations, let find an equation perpendicular to the first function, a function which is perpendicular to the first one, the one we have already , we have to look at the gradients, obviously it should have the reverse (reciprocal) of the first gradient [referring to the first line], yes, if the first gradient is (2x) I mean (2), then the reverse of (2) is $-(\frac{1}{2})$, such as if we multiply $(2)(-\frac{1}{2})$ we are going to get (-1), because the product of two perpendicular lines is minus 1 (-1) [see Figure 5.9].

Michael recognised the pragmatic aspects when he entering the value of the gradient that instantaneously produced a perpendicular line to the first line (m=2) (Figure 5.9) when using *GeoGebra*. He was referring to the lines being perpendicular because the product of their gradient is (-1). This showed an epistemic issue, that is, the analytic geometrical instance of the product of the gradient.

Michael: My third function it will be …? Minus x (–x) out of 2, that is ($-\frac{x}{2}$) that is the function! The function that passed through the Origin, the intercept is zero (m = 0), then I need the fourth equation that is going to complete the square, so I should have a function which will be perpendicular to this one, again the gradient of this line is (2), so, it means that the gradient of the line which is perpendicular to function (G) it will be a negative 1 out of 2, ok, fine. I can have negative x (–x) divided by 2 and then maybe I can add the (y) intercept. Maybe I can make it 4, let's see…? Is that a square? [see Figure 5.9].

Michael in his presentation used the term "reverse" mathematically referring to the additive inverse of the reciprocal of 2, the gradient. He knows that the product of the gradients must be -1.

The teachers' discussion on how to construct a square in *GeoGebra* showed that this task was relatively difficult for them, but provides an opportunity to think about both the mathematical properties of the figure that they are to construct (a square) and how to use the features embedded in *GeoGebra* to construct them.

Michael for example, discovered the perpendicular line tool allowing him to construct perpendiculars lines, to plot points in the *GeoGebra* graphing screen, draw segments and measure the distance between the lines with *GeoGebra* to verify that a square is square.

Researcher: How do you know that is a square?

When the teacher/researcher asks the teachers to assess this solution, everybody agrees that control must be exerted on the particular drawing; according to the well-known definition of a square, they suggested measuring the sides of the mathematical figure produced. The main elements arising from the discussion are the use of a measure and the precision related to it.

Michael: How do I know that is a square? Obviously, it looks like a square to me, but I wish that is a square.

Petrus: Check the intercepts

Michael is focusing on the graphic or geometric aspect of the four lines intercepting displayed on the screen. His comment has a mix of pragmatic and epistemic inputs,

> **Michael:** Ok, let me check the intercepts, the interceptions we have, let me just put the points of interceptions. I have a point A (-1.6; 0.8), B (0; 4), then I have another point there which is C, then I have point D, yes (A, B, C, D) [see Figure 5.8].

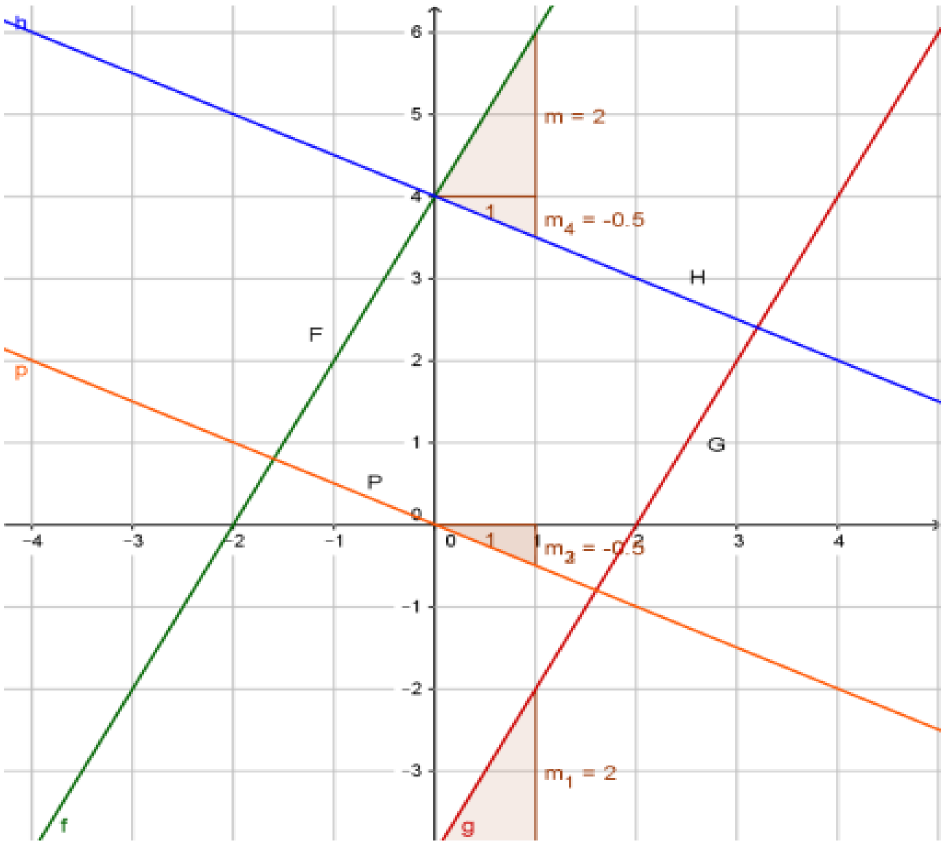


Figure 5.9: A screenshot of the constructed four linear functions by Michael with the help of other participants

> **Michael:** How can I test that this is a square?

These statements imply that Michael and Petrus are alluding to the properties of a square. They are thinking of ways to verify that the quadrilateral is a square.

In geometry research mathematicians usually operate with mental entities, which contain an image as an essential component (Fischbein, 1993). This helps them in thinking, but it often does not coincide with the formal definition of a square in this case. When doing research in geometry (especially within natural axiomatic geometry) we deal with and manipulate mental entities called figural concepts, which reflect spatial characteristics (shape, position, quantity), as well as conceptual qualities such as ideality, abstraction, generality and completeness (Fischbein, 1993).

> **Robert:** Find the distance between those two lines [referring to the distance between the lines F and G or distance between line H and P [see Figure 5.10].

Robert seems to be interested in using the "distance formula" as in Analytical Geometry. He is therefore thinking of checking whether the length measurements are going to be equal. This is one way to begin to find out or verifying whether a square is square. What Robert said is on the epistemic route, because it relates to understanding "what makes a square a square." The researcher's intention with this activity was to deepen the teachers' leaning experiences with respect to varying the symbols of the gradients, to understand perpendicular gradients when they occur, and why they occur based on the affordances of the scrollbars in the design of *GeoGebra*.

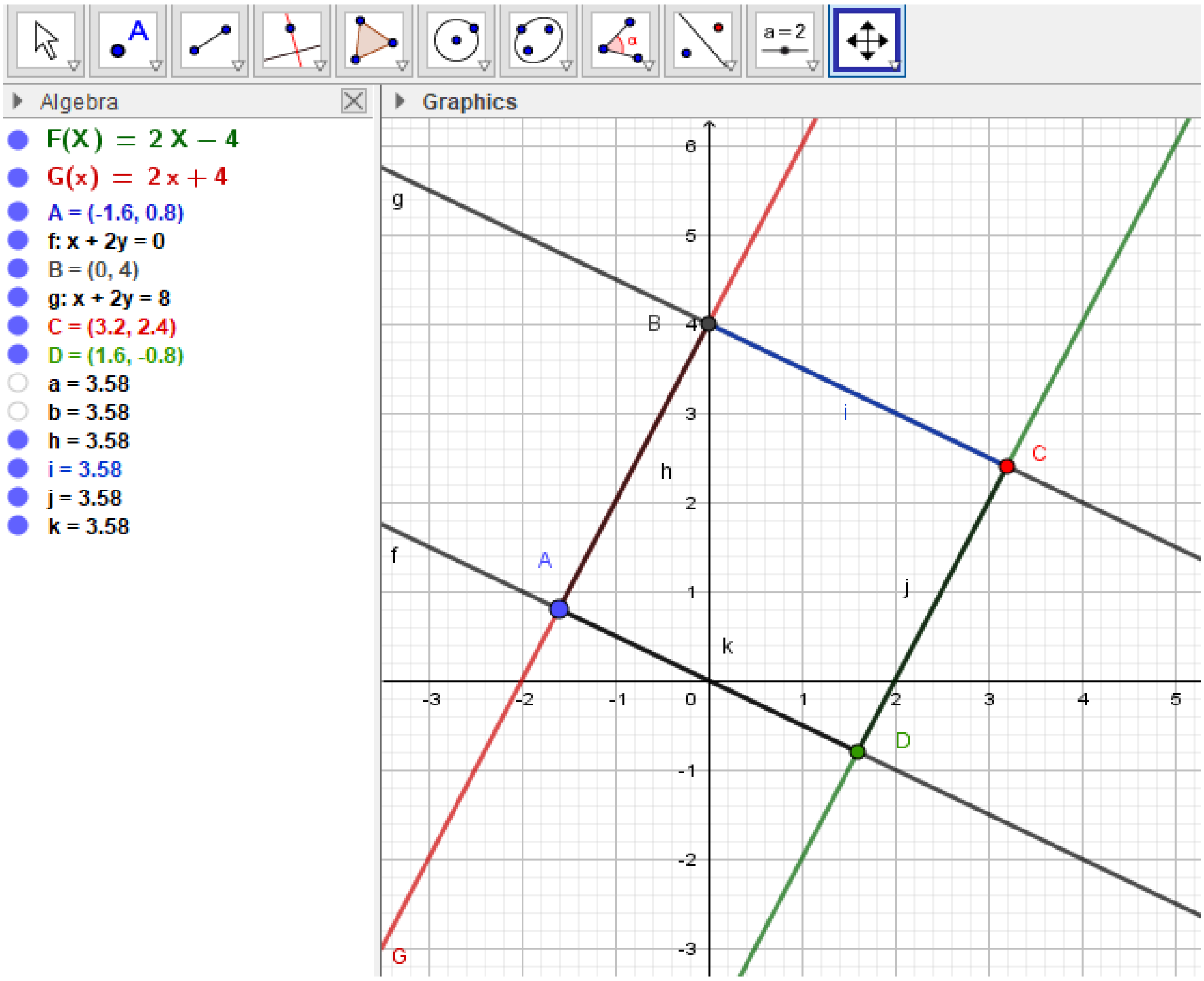


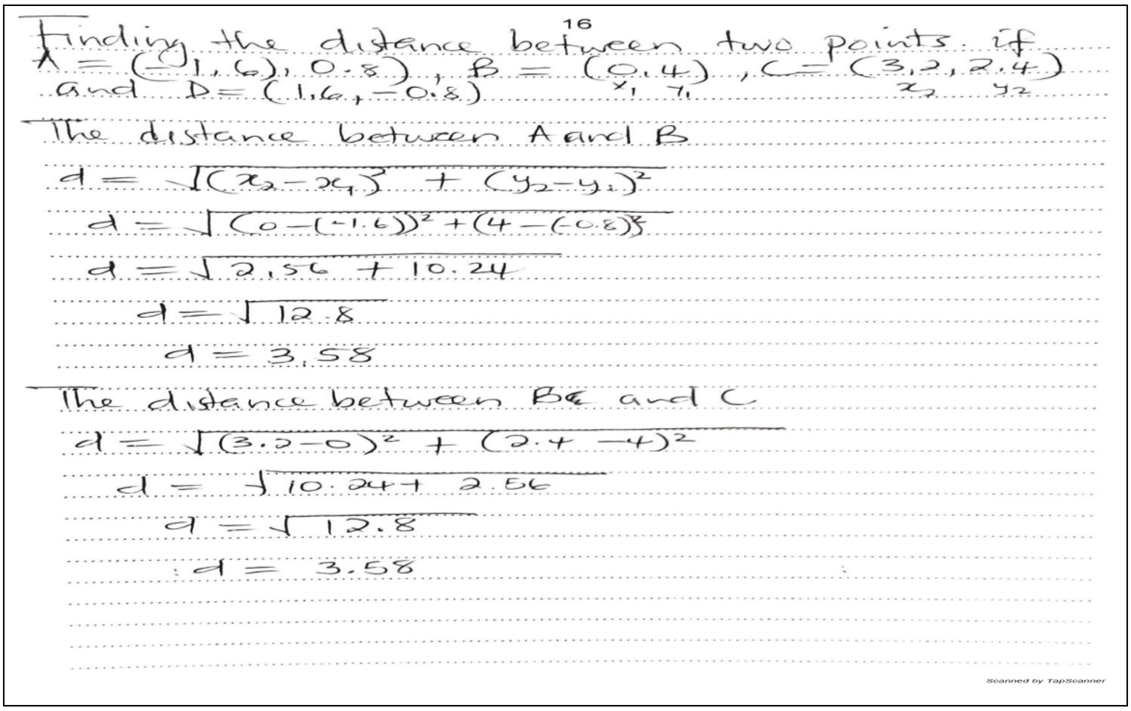
Figure 5.10: A GeoGebra screenshot of the constructed four-line functions by Michael verifying that the distances between the vertices (A, B, C and D) of the square are the same

> **Michael**: The points are labels. Point A to point B, point B to point C, it is that what you are saying? So? The distance from point A to point B it will be the same distance from point D to point C, also will the same distance from point C to D, and now definitely will be the same distance from A to D [see Figure 5.10].
>
> **Petrus**: If we test two distance?

Peter also suggested to test the distances between the sides of the possible square produced; it seems like the teachers were not sure about if the geometrical object produced by Michael was a square.

> **Michael**: If we test two distances, it is fine, we can conclude that is a square. Obviously, this one is parallel to that one and equal [referring to the lines (f) and (g) – see Figure 5.10] and same applies to the other two lines. Can we use a piece of paper and test to find the distance between the two points?

Oveka suggested a shift to a pen-and-paper option (see Figure 5.11), while working with *GeoGebra* following Michael's suggestion: "Can we use a piece of paper and test to find the distance between the two points?" Oveka used the coordinate of points A, B and D to find the distances between these points to verify if the lengths of the sides of the square constructed by Michael are equal as the minimum condition to make the square a square. Oveka's calculations of the distances between the points and Michael distances proved to be equal (see algebraic window left side in Figure 5.10). Teachers are familiar with using pen and paper to check their work. This is a combination of pragmatic and epistemic concerns. It is not surprising that teachers are more familiar with pen and paper compared to *GeoGebra;* this is not negative practice because *GeoGebra* is in a sense foreign to their classroom practice.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Finding the distance between two points if
A = (−1,6), 0.8) , B = (0.4) , C = (3,2, 2.4)
and D = (1.6, −0.8) x_1 y_1 x_2 y_2

The distance between A and B

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(0-(-1.6))^2 + (4-(0.8))^2}$

$d = \sqrt{2,56 + 10.24}$

$d = \sqrt{12.8}$

$d = 3,58$

The distance between B and C

$d = \sqrt{(3.2-0)^2 + (2.4-4)^2}$

$d = \sqrt{10.24 + 2.56}$

$d = \sqrt{12.8}$

$d = 3.58$

Scanned by TapScanner

Figure 5.11: A screenshot pen-and-paper calculation by Oveka of the distance between point A and B and point A and D

The above shows evidence of how Michael and Petrus ran into difficulty trying to verify if the geometrical object produced is a square. However, Michael suggested using the distance formula computation, which is acceptable, because the teachers' participation framework is their classroom or local practices. Pragmatically the teachers can use pen-and-paper methods to find distance by using distance formula. Using *GeoGebra* features to do the same distance calculation is new to most of them, because they are not familiar with the design features of *GeoGebra*. The pen-and-paper environment is their daily classroom environment, where use of technology is generally limited.

Researcher: Can you use *GeoGebra* to find the distance?

> **Michael***:* How can we find the distance?
>
> **Peter**: Ok, let's go to distance [referring to the *GeoGebra* input bar]
>
> **Michael:** Yes
>
> **Peter:** Then type in the input bar "dist. (A, B)"

This tool calculates the distance between two points, two lines, or a point and a line as a number, and shows a dynamic text in the graphics view . It can also be used to measure the length of a segment (or interval), the circumference of a circle, or the perimeter of a polygon.

Peter used this *GeoGebra* facility to find the distances between A, B, C and D. He typed in the input bar tool "Dist (A, B)".

Michael: Oh yes, it is 3.58, now the distance (B, C), very nice Mr Peter, thanks, you just saved my life, can we conclude, that is a square? The trick was just to come out with the first two lines, because the first two lines play a role, then we inspected the gradients. The gradients should be reciprocal.

The distances were measured by Peter using the *GeoGebra* input bar, allowing Michael to make his own assumption that the quadrilateral produced is in fact a square according to the minimum conditions that will make a square a square. This confirms that ABCD is a square at a low level (Van Hiele level 1). This level is restricted to the physical, global attributes of a figure (Gutiérrez & Jaime, 1998). Teachers at some point use geometry jargon, yet this term conveys a visual meaning more than a numerical one. For this situation, they can see some mathematical properties of figures (the square in this case) accurately, yet these are simple properties, such the number of sides. Teachers need to measure the sides and the angles to confirm that all four sides are equal in length, and all four angles are right angles.

Researcher: Can you use *GeoGebra* to find the perpendicular lines?
Teachers were encouraged to exploit the design features of *GeoGebra* to check whether or when lines are perpendicular or not. In the *GeoGebra* design there is a combination of the pen-and-paper classroom practice approaches that coincides with the design of *GeoGebra* in the case of distances between coordinates, perpendicular lines and other concepts and procedures related to geometry and analytical geometry that they know from their teaching.

> **Michael**: We can use it, we can use *GeoGebra* to find when the two lines intercept, so whether two lines are perpendicular.
> **Michael**: Oh, those two lines are not perpendicular to each other; we can help each other again here.
> **Petrus**: He is not guessing, because he already has the gradients.
> **Michael**: Can I delete those two lines again?
> **Peter**: Ok.
> **Michael:** Ok, let me check this one [he opens the tool perpendicular line], Mr Peter. Can I put a point in this line?

Michael: Then I want the line perpendicular to this one, but it is not working out.

Michael: Let me check my manual again.

Michael's statement "Let me check my manual again"(referring to *GeoGebra* Manual given to the teacher-participants at the beginning of the study, see Appendix M) is important because it shows that it is necessary to become familiar with the design embedded in *GeoGebra* in relation to the problem of what makes a square a square.

Ali: What about if we find the perpendicular bisector between two point, for example, A and B on the parallel line?

Ali is referring to further properties of what will make a square a square, i.e. ways of defining a square in a *GeoGebra* environment.

My own observations as the researcher, based on the teachers' conversation on finding a minimum condition of making a square a square, indicate that the participants became motivated while using *GeoGebra* and they continued within the context of investigating the graphs of linear function. This motivation was evident as well from the way participants persisted when faced with difficulties of finding the correct perpendiculars lines using *GeoGebra* commands.

Researcher: What Ali is trying to say is that we can put two points on the parallel lines and look for the perpendicular bisector between the two points, then it will give us the perpendicular line between those point A and B, so...? That is what you are looking for? That is what Mr Ali said.

Researcher: Peter, can you come to show us what you mean using *GeoGebra* construction?

Peter: This is point C [see Figure 5.12] $f(x) = 2x + 4$ and $g(x) = 2x - 4$ clearly these two functions are parallel.

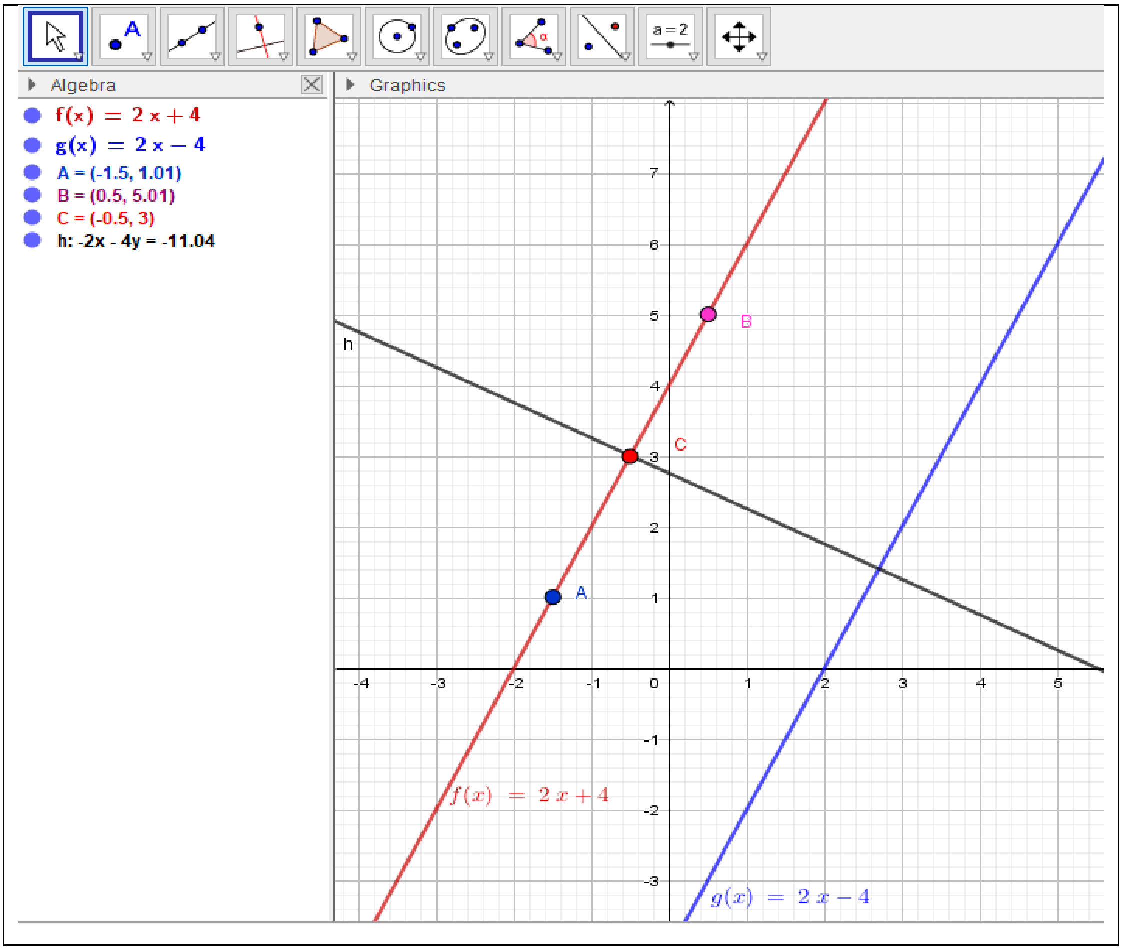

Figure 5.12: A screenshot of GeoGebra construction representing point C as midpoint between point A and B

Figure 5.12 represents Peter's way of referring to point C as the midpoint, which is the point on the segment halfway between endpoints A and B.

It may be the case that the midpoint of a segment can be found simply by counting two blocks up and down from point C.

If the segment is vertical or horizontal, you can find the midpoint by dividing the length of the segment by 2. The next comment shows evidence of how Peter in referring to that way of finding the midpoint C.

> **Peter**: Can you see there? There are two blocks up [referring to the divisions from point C to point B, see Figure 5.12]. Even from point C down [referring to the divisions from point C to point A] there are two blocks also. I can put a point there two blocks down and another point two blocks up, right? Meaning the point C will be my midpoint [see Figure 5.12] and then I can verify writing in the input bar [perpendicular point to point] my first point is A and the other one is B then we have

a line perpendicular to this line. Figure 5.13 shows how Peter is also trying to find the perpendicular line between the points A and B using the toolbar feature perpendicular line).

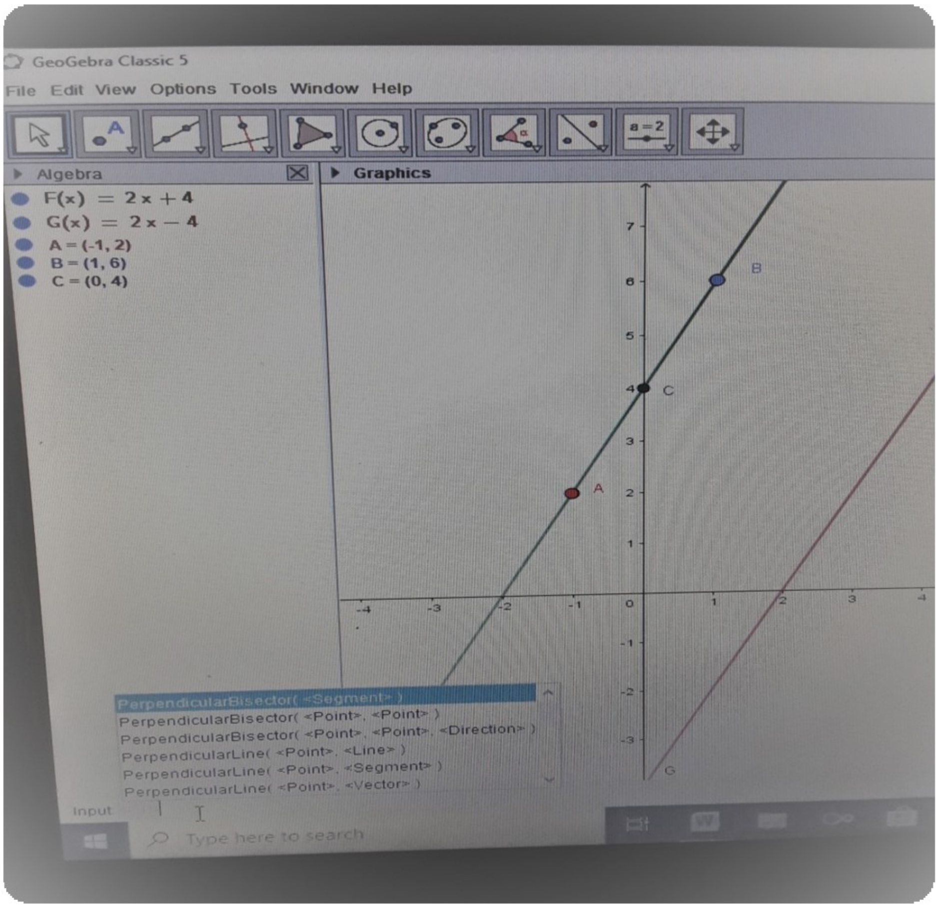

Figure 5.13: A screenshot of GeoGebra showing how Peter is trying to find a perpendicular line between point A and B using the input bar features embedded in GeoGebra

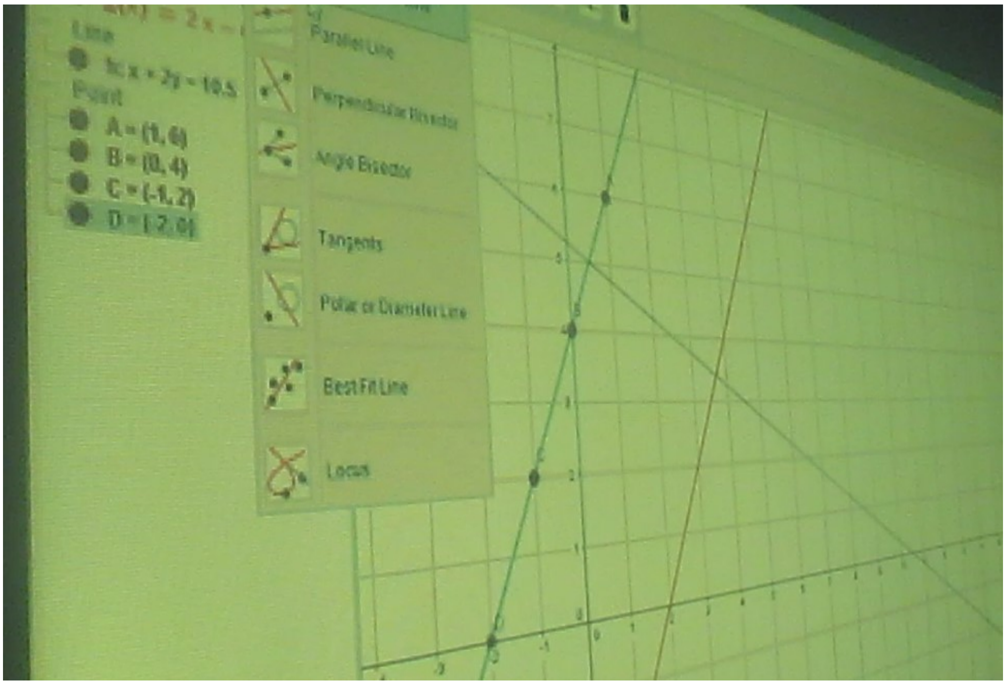

Figure 5.14: A screenshot of GeoGebra toolbar features showing Peter's construction of a perpendicular line

Peter was trying to verify his conjecture with the collaboration of other participants to validate that the two lines have the same distances and are parallel to each other, verifying in that way the general properties of a square. Peter therefore used *GeoGebra* to empirically verify his statement: "I can put a point there two blocks down and another point two blocks up, right? This means that point C will be my midpoint [see Figure 5.12] and then I can verify writing in the input bar [perpendicular point to point] my first point is A and the other one is B, then we have a line perpendicular to this line".

The fact that *GeoGebra* allows teachers to move the objects (lines) afforded them an opportunity to empirically verify or test their results and statements (De Villiers, 2004). The teachers' interactions and cooperation are beneficial, because they could learn from each other and enhance their mathematical thinking, namely, dealing with epistemic issues with respect to gradients, properties of squares, straight lines and parallel lines as represented in *GeoGebra* and drawing on their pen-and-paper background.

> **Peter:** The distances are equals; we did some calculation with *GeoGebra* to fine that the distances are the same.

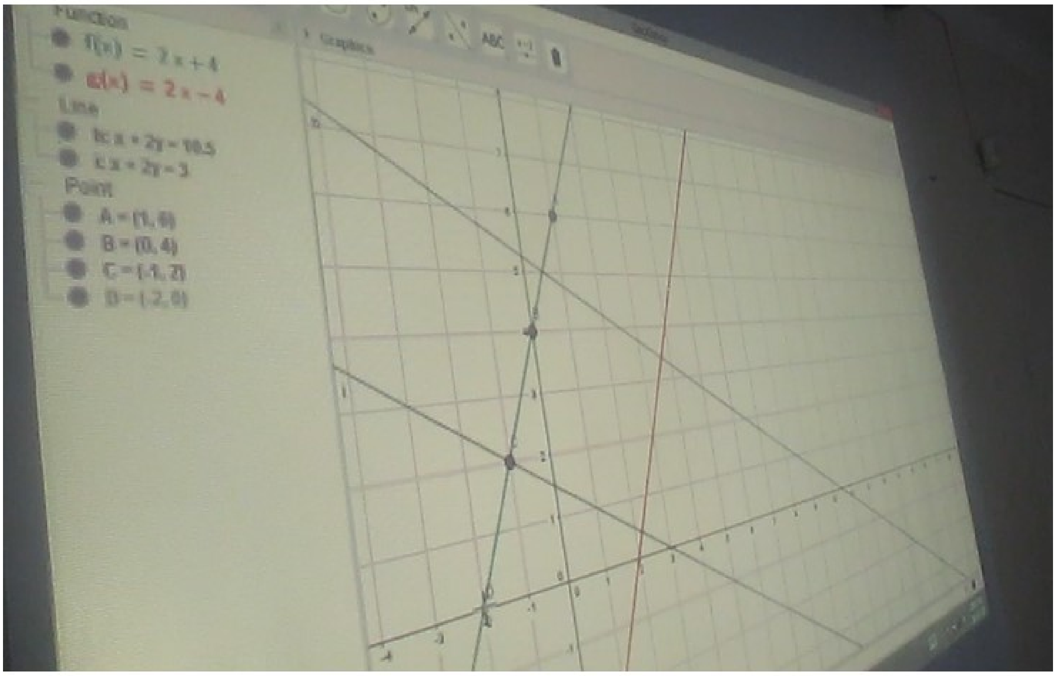

Figure 5.15: A GeoGebra screenshot shows Peter square construction

> **Peter:** I can see there is square. You can construct a line parallel to line AB by constructing a perpendicular to a perpendicular.

In the beginning of the series of five workshop sessions, teachers found it difficult to establish the minimum conditions for the four lines to become a square after it had been drawn in *GeoGebra*. Participants' independence seemed to increase as they became more familiar with *GeoGebra* and they developed the confidence to try things out in an experimental way, for example, trying to find whether the distances between the points allocated are the same. They were motivated to seek justifications for their descriptions. Teachers indeed needed more time to be more familiar with the features embedded in *GeoGebra.* Thus it becomes important to share with teachers the standard terminology in the *GeoGebra*-based software. Teachers may have their own way of naming geometric objects. But for the sake of consistent communication they have to know how the majority of people using the same tool and also the computer software they are using the terminology. These will help them to communicate with other people, understand each other and get the computer software to work more effectively. For example, it is useful to know the meaning of the terms segment, line, circle, midpoint, perpendicular, parallel, angle bisector, intersection, on the *GeoGebra*-based tool menu and the diagrams associated with them. The teachers/learners may not be able to explain these words verbally, but they should form an appropriate mental image (Leung, 2017) associated with each of them. This is obviously easier with *GeoGebra* than with the traditional pen-and-paper approaches that they used.

Understanding the dynamic connections between multiple representations of mathematical objects, for example, gradients, square, parallel and perpendicular lines seemed to open up for them a range of capabilities of *GeoGebra* software, which can be used for teaching and learning mathematics (Preiner, 2008). *GeoGebra* design features include multiple representations of various mathematical concepts, which have been strongly connected with the complex process of learning in mathematics, and more particularly with advancing the teachers'/learners' better understanding of important mathematical concepts such as graphical representation of linear and quadratic function.

> **Ali:** What about if we find the perpendicular bisector between two point, for example A and B on the parallel lines? [see Figure 5.13].

Ali is referring to further properties of what will make a square a square, i.e. ways of defining a square in the *GeoGebra* environment.

Researcher: What Ali is trying to say is that, if we can put two points on one of the parallel lines and look for the perpendicular bisector between the two points [see Figure 5.13], then it will give us the perpendicular line between A and B, so...? That is what you are looking for? That is what Mr Ali asked.

> **Petrus:** Draw another perpendicular line using the same idea, so that you can form a square.

Petrus is now giving a further instance of a property, a perpendicular line, which is necessary to make a square a square.

> **Peter**: Another perpendicular line to form a square? Maybe we can do some calculation here, so that we can find where we can place the mid-point, I think we have to make some calculations.
>
> **Petrus:** The lines are perpendicular but they are not forming a square, it is a rectangle now.

Petrus makes an analytical distinction between a rectangle and a square. Analytically, according to the Van Hiele levels, a square is a 'special rectangle', which is also a quadrilateral parallelogram, according to minimum conditions.

At level one of the Van Hiele hierarchies, the analysis of geometric concepts begins with visualisation at level one. For example, through observation and experimentation learners begin to discern the properties of 2D figures. These emerging properties are then used to conceptualise classes of shapes. Learners at level one cannot yet explain relationships between properties; interrelationships between figures are still not seen and definitions are not yet understood. At the level of informal deduction (level two) learners are able to establish the interrelationship of properties both within figures (e.g. in a quadrilateral opposite sides being parallel necessitates opposite angles being equal) and among figures (a square is a rectangle because it has all the properties of a rectangle). Thus, they can deduce properties of a figure and recognise classes of figures (Webb, & Feza, 2005).

Researcher: Remember there are more option tools. (The teacher/researcher is pointing them to tools embedded in *GeoGebra* that they can possibly use to find out what makes a square a square).

> **Peter***:* Yes, it true.
>
> **Peter:** What about if we change the value manually in the algebraic window to make the line with same distance?
>
> **Michael:** Exactly.

Peter is referring to instances where the user types in a value, relying on the analytical geometry inscribed in the *GeoGebra* design.

GeoGebra is an interpreter between the phenomenological geometrical world and the axiomatic geometrical world (Leung, 2017). In other words, the use of *GeoGebra* mediates between users and the mathematical objects; for example, a square with all its properties in terms of lengths, gradients and perpendicular lines segments and 'proving' a square in a dynamic environment.

The appropriate use of *GeoGebra* potentially enables users to construct geometric figures according to Euclidean principles and then dynamically changing those (Hall & Chamblee, 2013: 14). Users can construct geometric figures using tools embedded in the *GeoGebra* feature and 'drag' the figure, which will maintain its given properties, to make observations and predictions. For example, a user can plot two points on a straight line and afterwards find the midpoint between those points and construct a perpendicular bisector on that line. This process is quick and simple compared to traditional pen-and-paper method. This ability to dynamically manipulate figures saves time (a pragmatic issue) and provides a responsive visualisation of an object's properties, which allows for immediate visual feedback to the user (Hall & Chamblee, 2013). These aspects of DGS enhance visual representation and increase teachers'/learners' cognitive capacities during learning; they encourage greater mathematical discourse, which is the mathematical communication that takes place when the teachers articulate their own ideas and consider their peers' mathematical perspectives as a way to construct mathematical understanding. Computers foster mathematical discourse, extending communication from teacher to teacher, or computer to teachers. However, teachers encounter challenges as they try to incorporate a *GeoGebra* language with mathematics content related to constructing and discussing the properties of a square. In addition, *GeoGebra* allows discussion of geometric objects (a square in this case) in a way that was once impossible with traditional pen-and-paper representations.

Researcher: Do you think differently now about your previous way to construct the square with the use of *GeoGebra*?

> **Michael**: Yes, we did not think about bisecting the lines; it was mainly the idea of pencil and paper.

Researcher: You can still use different tools.

> **Michael:** Yes, maybe we did not approach it well. Using the same idea, we can try to bisect the line.

"Bisect the line" has deep implications. Bisecting the line enables ways of checking for perpendicular lines. The design of *GeoGebra* enables user/teacher to use pen-and-paper analytical geometry concepts and procedures in a dynamic environment.

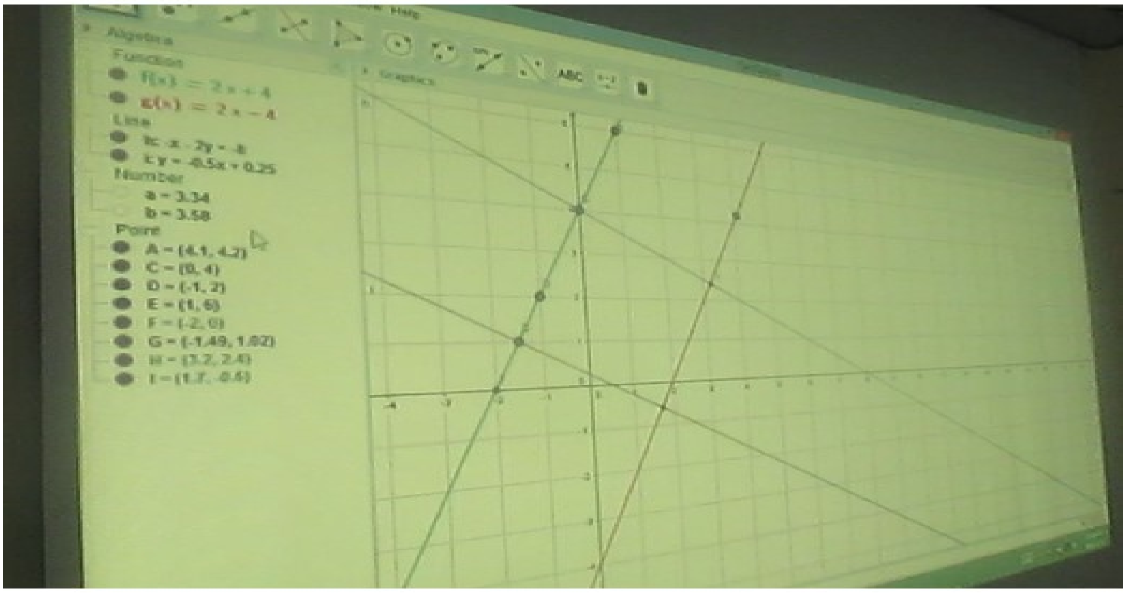

Figure 5.16: A screenshot of Peter's final square construction.

Peter: For us to come out with a square we have to plan our points very well.

Michael: Yes

Researcher: After you have been exposed to *GeoGebra* what insight have you gained?

Michael: Now, we came out with this square using *GeoGebra*, using perpendicular bisector, but previously we were using pencil-and-paper ideas, maybe we can actually combine the two methods to make learners understand much better, because if we use the traditional way (theoretical) maybe we can use *GeoGebra* as practical, maybe learners will understand much better.

Researcher: What is your idea about linear functions now?

Michael: Actually enhanced, because we have discovered new things.

Researchers: Can you elaborate more?

Michael: After using *GeoGebra* I can primarily say that my understanding of linear functions is enhanced. I have learned new behaviour of linear functions. For instance, the effect of changing the parameters in a linear function, the changing of the slope of the graph. I also learned the easy way of solving simultaneous equations using the graph of function. I can now also find any value of f(x) without necessarily using the calculator and most of all I can find the domain and its range of any linear function using *GeoGebra.*

The teachers' conversation with the teachers/researcher in the form of questions and prompts enabled them to pair and to compare their classroom practices related to analytical geometry and algebra. Consequently, the design of *GeoGebra* as digital artefact shows that there is a potential to deepen mathematical thinking with respect to "geometry" as well as "algebra" as the name *GeoGebra* indicates.

Working in a *GeoGebra* environment not only gave participants appropriate diagrams, but also allowed them to experience more while enabling a shift in focus from spending time producing graphs to spending time interpreting and understanding graphs, and systematically exploring these representations, all of which gives a more central role to the formulation of mathematical conjectures (Gono, 2016; Bozkurt & Ruthven, 2017) for example, the geometrical construction of four linear functions in such a way that the four lines will form a square when they are drawn in *GeoGebra* coordinate system (see Appendix I). Teachers repeating the same process frequently helped to confirm that their construction was linked with a physical square with *GeoGebra.* The focus shifted from sketching correct graphs using pen and paper to investigating the properties of the graphs of linear functions. Information obtained through use of multiple representations was a step towards the generation of conjectures, which in turn served as a basis for enhancing their understanding of the concept of linear functions.

The teachers were exposed to exploring transformations of functions with *GeoGebra* and could make conjectures and verify their statements about the condition of drawing the four linear functions in such a way that after using *GeoGebra* they successfully constructed a square. This was consistent with the quasi-empirical methods such as applying conjectures and verifications (De Villiers, 2004; Swanepoel & Gebrekal, 2010).

5.4.3 (Case 2.1). Exploring parameters of a quadratic function

In this activity participants were working in groups to complete the activity 4.1 (see Appendix I) following the instructions; this activity first required working out in paper and pen to submit their answer sheets by the end of the session. Since all of the participants were familiar with the construction tables of values for functions, drawing and interpreting graphs and solving equations graphically in their teaching.

The teacher/researcher decided to provide various conceptually engaging activities for them to develop a holistic view of *GeoGebra*-based quadratic polynomial relations and to deepen their mathematical thinking related to quadratic functions. The initial construction starts with the function $f(x) = x^2$ and changing the equation by typing in different values for the parameter (e.g. 0.5, -2, -0.8, and 3), followed by *GeoGebra*-based modeling, reasoning and higher-level analyses (see activity 4.1 Appendix I). This activity focused on the pragmatic and epistemic issues related to deepening teachers' mathematical thinking.

Parameters are literal or numerical objects that influence the output or behaviour of a mathematical object, for example, a polynomial function. Parameters are closely related to variables, meaning that they can assume varying numerical values in a *GeoGebra* environment. The symbols a, b and c are parameters that determine the behaviour of a function.

Participants were required to follow the instructions on the worksheet and write down their results and observations based on working with *GeoGebra*.

Questions for Discussion

- Did you experience any problems or difficulties when using *GeoGebra*?
- How can a setting like this (*GeoGebra* in combination with instructions on paper) be integrated into a 'traditional' teaching environment?
- In what way could the dynamic exploration of parameters of a polynomial function possibly affect your students' learning?

At a pragmatic level, when using *GeoGebra*, teachers can change the numerical values of parameter (*a)* and automatically see the results. Changing this value of (*a)* in the classroom context could support and promote the learning and understanding of algebra (Chiappini & Pedemonte, 2009). Artigue (2007: 73) argues for a balance between pragmatic and epistemic values when using a digital tool such as *GeoGebra* to teach and learn algebra. Thus, in this case the assumption is that teachers and perhaps learners will develop a better understanding of quadratic functions, which is important in algebra teaching and learning.

The technique used is to change the parameters that define the graphs of a quadratic function. This technique has the pragmatic value of using the sketch, since the action produces different configurations of the objects that can allow teachers to assess how many solutions there might be. However, when changing the values of the parameters, teachers must try to understand how each parameter affects the shape and location of the corresponding object, and eventually what changes can contribute to producing cases with two, one or no solutions.

According to Yastrebov and Shabanova (2015), teachers and learners can also create and test dynamic models and combine them with pen-and-paper activities. In this study teachers did not always work on *GeoGebra*; in some of the designed activities teachers used the pen-and-paper method in combination with *GeoGebra* (see Figures 5.17 and 5.18) (for the activity related to the exploration of the parameters of a quadratic function, see Appendix I). In this example the design of *GeoGebra* enables the teachers to explore and then comment on the influence 'a' as well as 'x' as variable in the case o$f(a, x) = ax^2$. The reason why teachers constructed these pen and paper graphs is because the teachers' 'participation framework' is the operative curriculum, which is based exclusively on pen-and-paper approaches, in other words, no use of any ICT. All the time the participants were comparing and contrasting the activities planned in this study with this operative curriculum. The teacher/researcher encouraged teachers to use such a combination to promote opportunities for a deeper engagement with mathematics, i.e. deepening mathematical thinking (Watson & Mason, 1998).

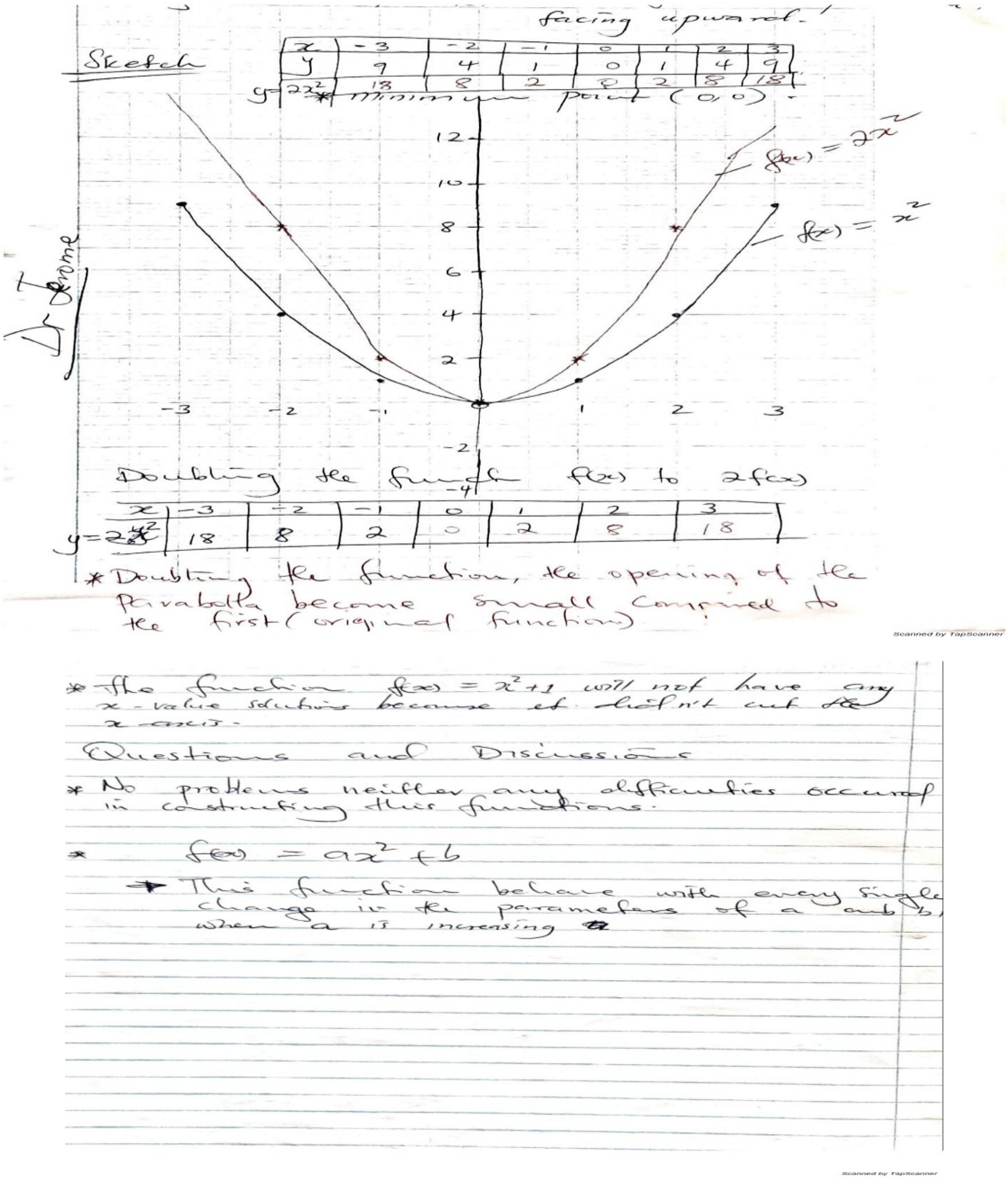

Figure 5.17: A screenshot of the pen-and-paper activity of one of the participant to explore parameters of the quadratic function $f(x) = ax^2 + b$*, a, b* $\varepsilon\ \mathbb{R}$

Figure 5.18 shows the same activity but in this case teachers explored the behaviour of the graph x^2 with the use of *GeoGebra*; they explored the changes in the general behaviour of a graph of a function in accordance with the changes in its algebraic form. The teachers could sketch the graph of $f(x) = x^2$ and then they could change the literal coefficient of x^2 (see Figure 5.18).

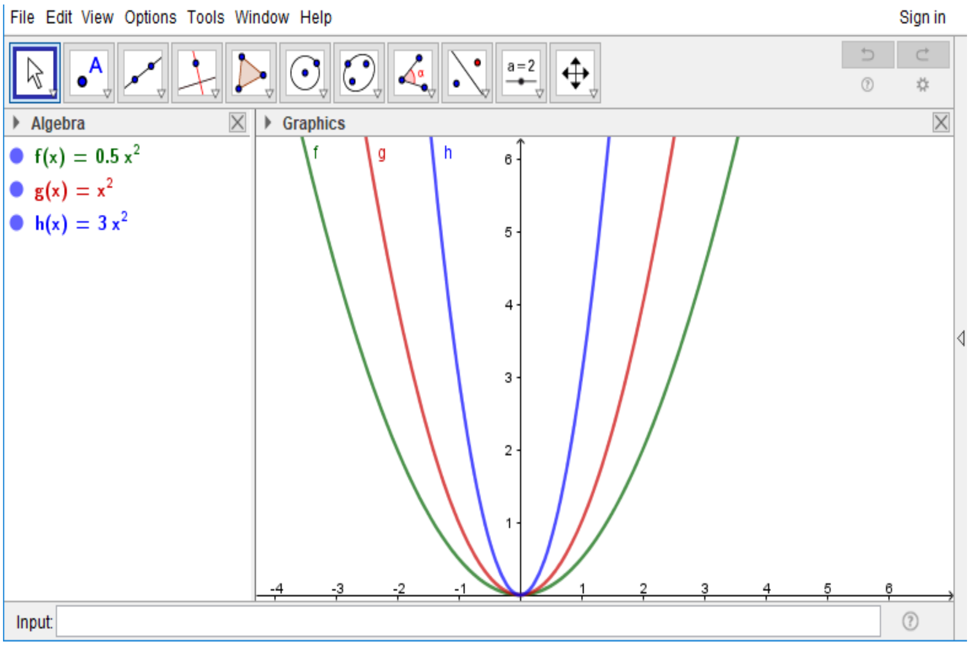

Figure 5.18: A screenshot an investigation of the changes in the general behaviour of a parabola in accord with the changes in its algebraic form.

In the above figure the action taken upon the coefficient of x^2 causes changes in the graph function. *GeoGebra* allows learners and teachers to observe this change in a more general form and find out more about the idea that as the coefficient a increases, the graph of parabola $f(x) = x^2$ gets steeper to the y-axis (in the case of $f(x) = 3x^2$). As the coefficient decreases, the parabola gets shallower. If we plot $0.5\,x^2$,x^2 *and* $3x^2$ the results look like Figure 5.18 above. This connection between the representations of a mathematical function is difficult to achieve in a static environment (see Figure 5.18) (Nathan, Kintsch, & Young, 1992). Thus, a primary affordance of representational software like *GeoGebra* is that actions taken in one representational window are dynamically automated in another, because of the embedded design, which helps learners and teachers to make important connections between them (Kaput, 1992; Sherman, 2010).

The following excerpt from the teacher's participation and from the second focus group transcripts during workshop four related to quadratic function gives an example of the pragmatic and epistemic comments and participants' experiences while interacting *with GeoGebra* as an activity.

Researcher: Do you have something to share about this activity? (Referring to activity 4.1: Exploring Parameters of a Quadratic function (see Appendix I).

Michael: Basically with this activity we found that function x^2 is a parabola, since the coefficient of (a) is positive, then the parabola is faced upward and we are trying to move it horizontal; we also found out that the Y-intercept is changing as well, then we move the function about the x axis, we found that this function does not have any solution, but we shifted down the function cut the x axis in two points, so in this case the function has two solutions.

Researcher: Ok

Michael: Now when we increased the coefficient of x for example $3x^2$, the functions become narrower (see Figure 5.18).

Figure 5.18 shows Michael's own construction with the use of *GeoGebra* related to the solution of activity 4.1 (see Appendix I)

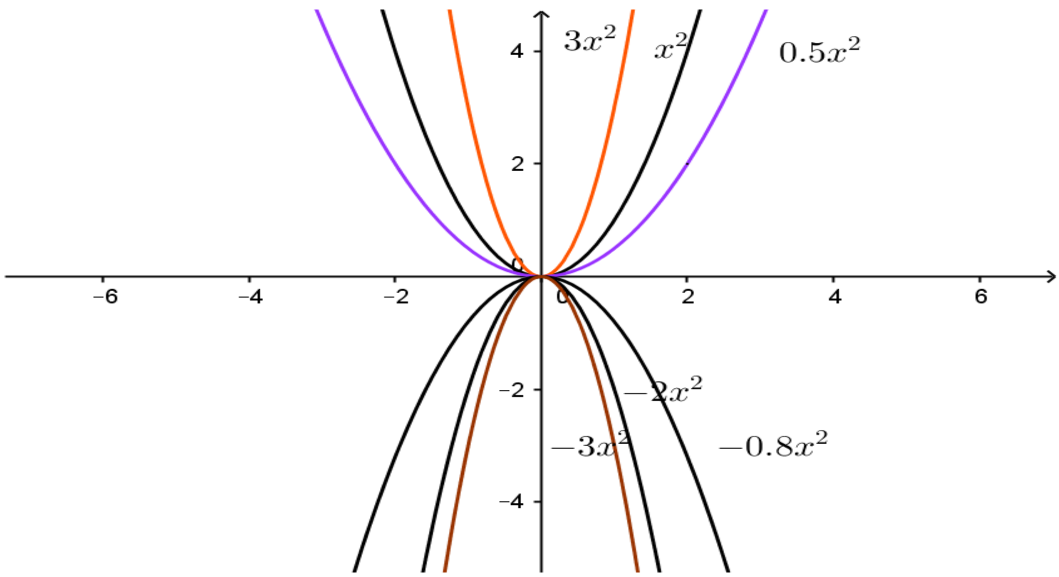

Figure 5.19: A GeoGebra screenshot of th the exploration of a quadratic monomial by Michael

Petrus: What did you say? You mean when the coefficient is 3 the parabola become narrower? Ok, when the coefficient is half, the parabola become wider again; if the coefficient is (-2) faced down and with coefficient (-0.8) wider again, then with a coefficient (-3) the parabola is the same, but faced downward [see Figure 5.19].

Researcher: What will happen when the coefficient increases or decreases?

Michael: If the coefficient of a is increasing the parabola becomes narrower; if the coefficient decreases, it become wider. Learners can observe and can see the changes, which is also much interesting. They are going to enjoy it.

Michael and Petrus recognised the pragmatic aspects of seeing the display instantly when entering different values of a using the *GeoGebra* tool bar. Epistemically they identified the actual behaviour of the graphs when the values of parameter a are positive or negative. According to my observations, Petrus and Michael were excited about how *GeoGebra* could give more than one representation at the same time. Nothing like that is possible in the traditional methods of teaching algebra. Exploring the general behaviour of the parabola involving sketching the graph $f(x) = ax^2$ and then changing the parameter of x^2 (see Figures 5.17 and 5.18) might enhance their structural conception of function. As shown in the literature review, a structural conception entails the ability to manipulate a function as if it were a single entity. Thus, the use of *GeoGebra* in this instance allowed teachers to observe this change in a more general way and discover that as the coefficient of x^2 increases, the graph of $f(x) = ax^2$ gets narrower and closer to the y-axis, and as the coefficient of $f(x) = x^2$ decreases, the graph gets wider; therefore through this activity the epistemic value of the tool is emphasised.

Researcher: In what way could the dynamic exploration of parameters of a quadratic function of the type that we have been exploring possibly affect your learners' learning?

Michael: Yes, the parameters will change the way they think, it will change their mind.

Researcher: Can you add something more in this regard?

Michael: Definitely, this will help to get better insight, to get whole picture of how these parameters will change the behaviour of the graph, because explaining to them if the coefficient of a is less than zero the graph will face down, maybe showing them it with the use of *GeoGebra*, they will have better understanding, because *GeoGebra* is dynamic, actually this will open up the learners' mind.

Researcher: Did you learn something new?

Ali: We learned to find a solution of functions using *GeoGebra*, we also learned that when we change the parameters of the function, for example the quadratic and linear functions, the behaviour of the graph also changes, we learned how to use different tools in *GeoGebra*, bisect lines and find perpendicular lines.

Oveka: I have learned drawing graphs using *GeoGebra*, which I did not know at the beginning. Meaning that it was so wonderful. I've also learned how easy it is to find a turning point of the graph, to find a solution of two equations, the roots within few minutes.

Michael: I think we learned a lot here and we can now integrate *GeoGebra* into our teaching, that also will be interesting for learners, allow them to explore and discover a lot of things at the same time; as you go you can see changes on the functions, learners can observe the behaviour of the graphs. I think *GeoGebra* has allowed us to get a deeper understanding of functions concept that we can integrate into our teaching.

In the above comments Ali and Oveka valued the pragmatic value that *GeoGebra* might offer them in their future teaching and personal development to 'find a solution of functions and drawing graphs'. They might have been thinking about the multiple representations that 'change the parameters of the function, example the quadratic and linear functions, the behaviour of the graph also change'. On the other hand, Michael was excited about the possibility of integrating *GeoGebra* into his teaching because this software could enhance his learners' visual and exploratory capabilities. Michael was also emphasising the epistemic issues embedded in *GeoGebra* that will allow him and his learners to 'see changes on the functions' and the behaviour of the graphs (curves) while parameters are changing.

The above conversations with the teachers showed evidence that *GeoGebra* compared to their traditional teaching realities might provide teachers and learners with the opportunity to develop new strategies in deepen mathematical functions thinking 'the following epistemic concern made by Michael support this, "*GeoGebra* will help to get better insight, to get whole picture of how these parameters will change the behaviour of the graph, because explaining to them if the coefficient of a is less than zero the graph will face down, maybe showing them it with the use of *GeoGebra* they will have better understanding, because *GeoGebra* is dynamic".

That approach could not be developed in traditional environments (Artigue, 2002; Watson & Mason, 1998). Thus the software might provide a technology learning environment (Galbraith & Haines, 1998) in which teachers can develop alternative strategies and find a dialectical balance between the pragmatics and epistemic values of the tool and explore more ideas in geometry. Teachers compared this activity with their operative curriculum; they used dynamic and traditional methods to justify and analyse their solutions. However, the pragmatic responses predominated compared with the epistemic comments.

Ali, Oveka and Michael believed that the experiences gained during the workshops might enable them to implement *GeoGebra* activities in their future mathematics teaching. They thought that the interactive nature of *GeoGebra* could make learners more familiar with mathematical concepts, when they observe transformations with shapes in different positions on the screen. Robert indicated that, based on his own experiences, learners and teachers get bored because they do not have much time in the classroom to work through more examples for them to gain more insight on into how a different range of numbers might fit the function.

Researcher*:* What insights have you gained from the *GeoGebra* functions content and the way you teach functions?

> **Robert:** In my case, teaching functions, especially Grade 12, it is so difficult to come out with different equations because we depend on what is in the textbook; it is difficult to come out with graphs of different functions, but now it will be much better and easy for me, because now I have more clear view of how the graphs will behave.

In Robert's comments there is an emphasis on teachers' classroom practice where the textbook serves as a resource on a daily basis. Pragmatically they rely on textbooks; if *GeoGebra*-designed tasks are ready to use, they can thus serve as an additional resource to teach functions.

Researcher: Do you think that the use of *GeoGebra* will enhance your understanding of the concept of function?

> **Peter:** Yes, it's very relevant to use *GeoGebra* to give the learners a better understanding of function. The good thing about *GeoGebra* is that, the moment you place in the number, you will see the line changing, the moment you change the parameters you will see how the graph will change, so it creates a better picture in the learners mind and also in my mind. *GeoGebra* gives me a better understanding of some of the formulas; sometime I use some of the formulas but I don't know really from where they are coming from; for example, in the activities we analysed we noticed how the graph was changing but also was changing in the algebraic window on the other side … so it wonderful, it gave me a lot of why things are happening and the way happened.

In Peter's comments there is a combination of pragmatic ("the moment you place in a number") and epistemic issues ("you see the line changing).
Peter is referring to instances of deepening mathematical thinking about the various different symbols that are common to the pen-and-paper world**.** This interpretation of the data might be in line with or similar to a study conducted by Nocar and Zdráhal (2016). In their investigation the ICT tool used was the dynamic mathematics software *GeoGebra.* In *GeoGebra*, as indicated by them, the various representations of the same function (graphs, tables) are connected dynamically, permitting users to go back and forth between them, hence making connections among those representations more reasonable for teachers. Whenever one of the representations is modified, all others adapt automatically in order to maintain the relations between the different objects. Objects can be created either by using dynamic geometry tools or algebraic keyboard input. Thus, the dynamic geometry technology (like *GeoGebra)* could be used to maximise teachers understanding the concept of function and help to visualise functions' graph being studied. This action might foster a learning process of negotiation meaning, during this investigation, teachers can find patterns and develop functional thinking while dealing with dynamic input–output dependencies. All these possibilities might serve to give teachers better understanding of the concept of function.

> **Petrus:** Now we are one step ahead compared with what we are doing in pencil and paper; we can explore functions, we can explore further a particular function rather than what we do in pencil and paper. It will raise learners' interest more and more.

Petrus made a distinction between the way teachers explore function in their actual classroom practice compared with their use of *GeoGebra*, This is a pragmatic comment based on the design features of *GeoGebra*, namely its graph and algebraic windows.

Researcher: Do you want to add something else?

> **Peter:** Yes, using *GeoGebra*, you can show the leaners how to crate parallel and intercept lines and more; leaners will be motivated to learn more, but our curriculum won't allow them to use even a scientific calculator to draw the graph, that will be a problem. This software will give the answers, but the learners won't know what is happening.

This comment is about the relationship between the operative curriculum and the design features of *GeoGebra*. This is an implementation issue. Thus, any possible use of *GeoGebra* interacts with the operative or 'intended' and 'implemented curricula.' As Peter indicated, probably the function plotters and *GeoGebra* present the whole graph of a function at once. Sometimes, this is an educational disadvantage because it hides the process of drawing such a graph.

> **Petrus**: Ok, obviously the program is a wonderful one, because, you see like in my case when I introduce a lesson I have to give the keywords: this is a turning point, this is a stationary point, this is what, and this is what, so…? If I have *GeoGebra*, for example, I can introduce this topic using this program ... saying, this is a turning point, the computer will show "this is the turning point", this is what, this is what, and the learners will see some number also [referring to the algebraic window] they will like to ask "how do you get this number?" And I will tell them that is what we are going to do with paper and pencil, that is the way we get the turning point, that is the way we are getting the interception; that will be good for them because they will see; one thing very important is when you asking them to draw a curve graph using freehand, most of them will use rulers, then they will see how the software draws the curve and compare that with their construction.

This teacher is hypothetically sketching what can happen when he/she introduces the quadratic function. 'Keywords' can be displayed and pointed out on the screen, graphically and in a linked tabular form (multiple representations), because it is key feature of the design of *GeoGebra*. *GeoGebra* might similarly help uncover the epistemic parts of the pragmatic action of finding and interpret 'the turning point'. The epistemic parts in this case deal with observing how the pragmatic action of 'getting the turning point' has epistemic effects, namely, a visual representation of the turning points on the computer screen. A pragmatic approach deals rather with 'getting answers' in short period of time.

5.5 Summary of the main findings

Findings emerging from teachers learning experiences (TLEs) related to the five *GeoGebra* workshops' session were divided in two cases:

Case 1: Teachers' responses during the semi-structured and focus group interviews with regards to deepening functions learning. *GeoGebra* interventions were influenced by time concerns.

Case 2: Teachers' responses had pragmatic and epistemic dimensions. Their interactions revealed some of the *GeoGebra* potential in the teaching and learning of mathematical functions.

In this section, the researcher will discuss and interpret the findings of the study and the main themes which emerged from the data obtained from the semi-structured and focus group interviews. Below is a summarised representation (figure 5.20) of the dominant themes identified from the data obtained from the semi-structured interviews and focus group interviews.

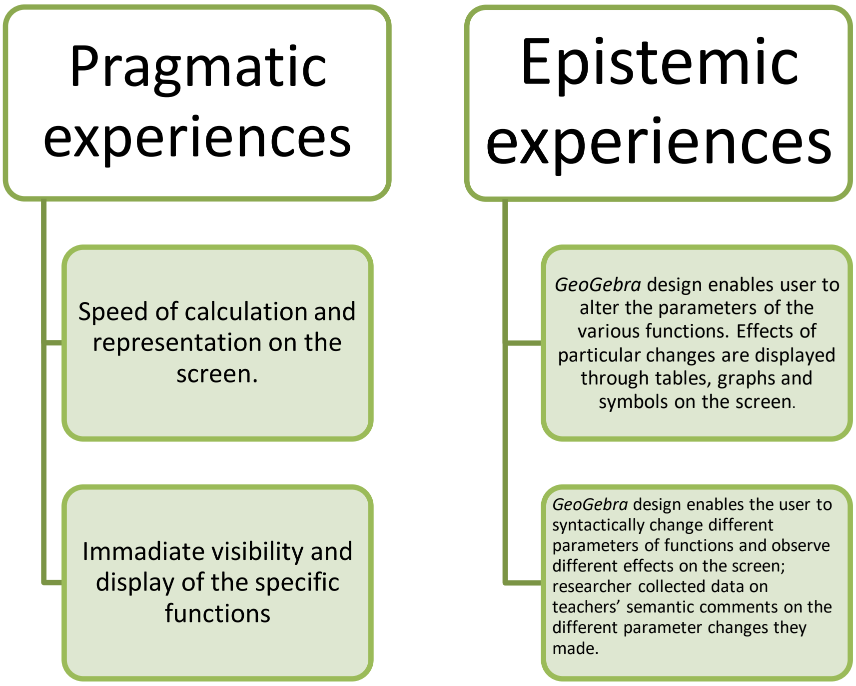

Figure: 5.20 dominant themes identified from the data obtained from the semi-structured interviews and focus group interviews

5.5.1 Raising time concern

An important finding about teachers' learning experiences (TLEs) during this professional development intervention aimed at improving their understanding of functions using *GeoGebra* was the issue concerning time (time issues abbreviated as TIs).

We should note that TIs are related to the context of teachers' place of work, such as secondary schools and institutions and organisations where teaching and learning are intended to take place (Watson, 2008). Also, time is a crucial resource (Adler, 2000) in schools and thus must be used in optimal ways so that mathematics teaching and learning can happen. In other words, teachers are quite aware how much time they have at their disposal and the degree to which the possible use of *GeoGebra* can help or hinder them in their teaching. The teachers' semi-structured interview responses were collected towards the end of the last workshop. Time issues (TI) emerged as a common issue, combined with epistemic and pragmatic issues. Figure 5.21 illustrates what is meant.

Time issues/pragmatic concerns	Mixture of pragmatic & epistemic issues	Epistemic value
Spectrum of time issues (TIs) in the data excerpts		

Figure: 5.21: Relationships between time constraints, epistemic and pragmatic issues

Those pragmatic concerns/issues (Figure 5.3) are related to getting quick answers, the day-to-day events and infrastructure of the teachers' classrooms, time availability, connectivity, seating arrangements in classroom, a working computer laboratory in the school. Epistemic issues, on the other hand, deal with interpreting the various mathematical objects computerised or inscribed in *GeoGebra* and their dynamic meanings.

Compared to pen-and-paper activities, time was a crucial issue in terms of what the course participants thought about the use of *GeoGebra.* They referred to *GeoGebra* as a time-saver, providing the teacher with more time to complete a task in their future mathematics lessons and also to the easier lesson preparations compared to their traditional way, in turn enabling them to produce more accurate graphics and figures, which meant that they could allocate more time to understand more of the behaviours of the linear and quadratic function when its parameters changed (epistemic values). We also could not discount "researcher effects", i.e. instances where the teachers picked up or appropriated the researcher's discourse regarding *GeoGebra*, such as the general behaviour of graphs as different input values are entered into the input bar. It is important to highlight the fact that the different sessions the teacher/researcher had with the teachers entailed an exposure to Math Education as a research domain with its peculiar ways of talking and working with 'the same' content that they knew from a pen-and-paper perspective.

5.5.2 Speed of making the drawing with *GeoGebra*

GeoGebra in its design affords fast and consistent feedback. This study involved participants using dynamic mathematics software (*GeoGebra*) to construct graphs of linear and quadratic functions and manipulating the representations using sliders, and getting immediate feedback on the nature of the transformed representations. Instant feedback allowed teachers the opportunity to develop their own comprehension of the functions being studied based on the common features of the graphs that were promptly presented on the screen. It likewise permitted teachers to monitor and manage their learning. In addition, *GeoGebra* use has the potential to broaden and deepen the ideas of dynamic geometry and extend them to the fields of algebra and mathematical investigation. This encourages legitimisation and generalisation by allowing quick, accurate sketches and exploration of multiple representational forms (Gono, 2016).

The category related to teachers' learning experiences (TLEs) related to the five *GeoGebra* interventions and time saving with its pragmatic and to a lesser extent epistemic dimension, identified a range of perceived practical, motivational and educational benefits which underpin the teachers' evident enthusiasm for using *GeoGebra* to enhance their understanding of the concept of functions. Thus, teaching linear and quadratic graphs using *GeoGebra* might enable teachers to gain insights that to construct richer and more coherent graphing concepts and to develop important techniques needed to reason through mathematical algebraic ideas (Asp, Dowsey & Stacey, 1992). It is also true that any mathematical software has both potential and pitfalls. A digital tool's pedagogical potential for one teacher may be a pitfall for another teacher.

5.5.3 *GeoGebra* (pragmatic and epistemic dimensions)

Teachers' interest was stimulated while they continued to interact cooperatively with the *GeoGebra* software. While teachers were learning how to use the *GeoGebra* software, they also acknowledged the possible future impact of the software in their algebra teaching, especially the teaching and learning of mathematical functions.

These are among the difficult topics in the operative mathematics curriculum in their classrooms. The participants were excited about using this new didactic tool. The finding also revealed that they need more opportunities where they can learn to experience relations between the pragmatic and epistemic dimensions of *GeoGebra* use when it comes to linear and quadratic functions. In particular, there are relationships between numerical, graphical and symbolic representations, which are among the affordances of *GeoGebra.* The teacher interaction with multiple representations of the algebraic objects in this software can enhance learning, thus enabling broader insights than might have been gained from the single representation of the same algebraic object (Calder, 2010 cited in Losada, 2012). Hence, the use of *GeoGebra* could help mathematics teachers to play an important role in influencing learners' mathematical thinking, particularly in the context of algebra learning, through multiple representations with respect to linear and quadratic functions

The findings of the study indicate that there was significant motivation and enthusiasm about using *GeoGebra* in analysing linear and quadratic functions and their hope to use this software in future teaching and learning about linear and quadratic functions. Furthermore, as Laborde (2002) pointed out, the use of DGS evolved over time from being a visual amplifier to a fundamental component that enhances conceptual understanding.

The use of DGS decreases the need for traditional methods. However the use of DGS does not replace but improve and complement them (Kokol-Voljc, 2007). Although there are many advantages of constructions made with DGS, the value of construction activities with pen and paper should not be discounted because both DGS and pen-and-paper environments make important contributions to teachers' and leaners' conceptual development in understanding functions. Thus, the *GeoGebra*-based mathematics in the case of this study can assists to supplement the teachers' traditional way of teaching (pen-and-paper reality) which is the teachers' main frame of reference.

5.6 Chapter Summary

The findings of this study are consistent with those of other studies, for example, the investigations conducted by Mainali and Key (2012), Ruthven, Hennessy and Deaney (2008 cited in Gono, 2016), Bulut and Bulut (2011), Ozyildirim *et al*. (2009); Zulnaidi and Zakaria (2012), and Koyuncu, Akyuz and Cakiroglu, (2015). These studies also found a positive impact of utilising mathematical learning software (*GeoGebra*), thus this software might enhance teachers' learning and understanding of multiple representations of mathematical concepts. Thus, teachers had the opportunity to construct their mathematical knowledge in a different way. The use of *GeoGebra* demonstrates the instructional effectiveness of *GeoGebra* compared to the traditional construction methods. However, teachers' reasoning during their interaction with the *GeoGebra*-based activities was predominantly supported by a traditional pen-and-paper way of thinking. With drawing tools different from pen-and-paper tools implemented in *GeoGebra*, it should be clear from this study that the use *GeoGebra* certainly mediates the way linear and quadratic functions is learned.

In conclusion, in the event that *GeoGebra* can be utilised to improve teachers' and learners' understanding of mathematical functions, it makes sense to believe that there is a need to adjust the *GeoGebra-based tasks* as a way to scaffold teachers' or learners' existing mathematical thinking. On the other hand, mathematics teachers should not be expected to create and develop their own *GeoGebra* activities, but as they gain familiarity and their repertoire of abilities develops, this could become possible.

The next chapter presents a summary of main findings, the knowledge implications for practice, recommendations and limitations, suggestions for future research and a reflection on the research process.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

This chapter provides a brief summary of the research and some general conclusions drawn from the findings, as described in the preceding chapter. This chapter is divided into seven sections. Section 6.1 highlights keys points of the chapter. Section 6.2 gives a summary of the major findings of the study in relation to the research questions. Section 6.3 discusses the contribution of this study to the field. Section 6.4 presents the factors that limited this study. Based on the research findings, Section 6.5 makes recommendations for improving teacher development in Namibia. Recommendations for further study are presented in section 6.6. The researchers' personal reflections can be found in Section 6.7.

6.1 Introduction

This study has its roots in the researcher's interest, as a secondary school mathematics teacher, in making a positive contribution to TPD in Namibia. The purpose of the study was to improve the understanding and teaching of mathematical functions by improving teacher skills in ICTs through a TPD intervention using *GeoGebra*.

The problem addressed by this study was that teachers have few, if any, opportunities to view and to work with multiple representations of functions, because the methods they commonly use to teach are not coupled with the availability of information and communication technologies (ICTs). With the affordances of *GeoGebra*, albeit in the case PD opportunities, there is an opportunity chance to study the teachers' experiences with functions as represented/inscribed in the design of *GeoGebra.*

Learners and teachers often struggle with functions because they may not be aware of different or multiple representations (Ainsworth, 2006). Such difficulties could possibly be partially because of learners' and teachers' limited experience with more than one (tabular, symbolic and graphical) representation of functions. Additionally, when the graphs of function eventually appear in the curriculum, the emphasis is placed on acquiring knowledge of how to plot points in the Cartesian area and on translating what the points symbolise (Bautista, *et al.*, 2015), which is the case with the Namibian mathematics curriculum.

Therefore, teacher education, development and support are critical components of a successful ICT strategy in Namibia.
To this day very little attention has been paid to TPD. Peters (2016) conducted a study on PD based on realistic mathematics education (RME) principles and the process of lesson study (LS) in a primary school in Namibia, while in his study Iipinge (2010) explored the need for the Namibian government to exploit the new opportunities created by ICTs and the development of new policies for integrating ICTs into the education curriculum. No study has been conducted so far on PD that used *GeoGebra* as mathematical digital software to improve teachers' understanding of mathematical functions. This was another factor that motivated the researcher to conduct this study in order to support mathematics teachers and in this way make positive contributions towards the promotion of ICTs and PD.

This is a qualitative case study that took place in two purposefully selected high schools in the Ohangwena region of Namibia and involved 10 mathematics teachers. A series of 5 workshops aimed at familiarising teachers with an in-depth understanding of functions using *GeoGebra* (See Appendix I). These workshops were conducted outside their classrooms at an agreed upon venue (a computer lab of one of the school selected), affording opportunities for an engagement with deeper mathematical thinking. The unit of analysis was the leaning experiences (TLEs) of a group of 5 mathematics teachers from each school, who volunteered to participate in the study for a period of two weeks. Data generated through multiple techniques were coded and analysed using the grounded theory approach and the results were presented in accordance with the research questions.

6.2 Summary of Findings

This study focused on ways of understanding and teaching mathematical functions by improving the teacher skills in ICT through a professional development intervention using *GeoGebra*. More specifically, the study attempts to answer one main research question and one research sub-question. The main research question is:

1. What are high school mathematics teachers' learning experiences, during a professional development intervention to improve their understanding of linear and quadratic functions using GeoGebra?

The following is the research sub-questions:

- How central is knowing mathematical functions in the Namibian secondary curriculum?
- What does research have to say about the potential of *GeoGebra (GGB)* in facilitating the understanding mathematical functions?
- What does research have to say, what are key issues in using GGB-represented mathematical functions?

In order to achieve this aim, a group ten high school mathematics teachers participated in 5 workshops which enabled them to interact with *GeoGebra*-based mathematical activities, focusing on different mathematical functions. Workshop activities included promoting the interactive processes of conjecture, feedback, critical thinking, investigation and collaboration (Yang & Liu, 2004). During these TPD interactions, the researcher had the dual role of teacher/research. The overall intention of the TPD intervention is to explore and to study the teachers' learning experiences and understanding of the teaching and learning of linear and quadratic functions, using *GeoGebra.*

Objectives:

- To investigate the ten high school mathematics teachers' learning experiences during a professional development intervention aimed at improving their understanding of linear and quadratic functions using *GeoGebra*.

- To investigate what the research has to say about the potential of *GeoGebra* in aiding the understanding of functions?

The mathematics teachers' leaning experiences during a professional development intervention to improve the understanding of linear and quadratic functions using *GeoGebra* were explored in the following cases:

> Case 1: Teachers' responses during the semi-structured and focus interviews with regards to deepening learning about linear and quadratic functions. *GeoGebra* interventions were influenced by time concerns.

Case 2: Teachers' learning experiences (TLEs) related to the five *GeoGebra* interventions. Their responses had pragmatic and epistemic dimensions. The teachers' interaction revealed some of the potential of *GeoGebra* in the teaching and learning of mathematical functions.

The researcher/teacher interacted with teachers by having them engage with *GeoGebra* in workshops, and collected data on their ways of talking about and using *GeoGebra* during the workshop sessions. There are at least two themes of TLEs based on the design and content of the different workshops.

1) Pragmatic experiences:

- Speed of calculations and representation on the screen; there is instant, immediate visibility and display of the specific functions.

2) Epistemic experiences:

- *GeoGebra* design enables user to alter the parameters of the various functions. Effects of particular changes are displayed through tables, graphs and symbols on the screen.

- *GeoGebra* design enables the user to syntactically change different parameters of functions and observe different effects on the screen; researcher collected data on teachers' comments of the different parameter changes they made.

Based on the findings, the participants perceived that saving time can assist teachers to find other alternatives to thinking about different solutions. The assumption on the part of the teachers was that if the learners spent more time on analysing their own assumptions, they might be more motivated to learn (see section 5.4.1). They felt that using ICTs, such as *GeoGebra*, made it easier and less tedious for learners to look at more examples in a short time. They considered that the quick feedback available in the *GeoGebra* task might enable teachers and learners to have more opportunities to thinking through mathematical situations related to function. Therefore less time will be used in the classroom environment compared to when they were using pen-and-paper methods. Thus, the utilisation of *GeoGebra* can possibly contribute in promoting for teachers a mediating role between technology and the body of mathematical knowledge to be learned. For instance, a teacher utilising *GeoGebra* can gradually visualise his/her mental pictures of geometrical object (a square) to such an extent that she/he can reason the accuracy (mediated by a dynamic feedback tool like dragging) of the mental pictures concerning the Euclidean world embedded in *GeoGebra* (Leung, 2017).

There was a perceived indication of interest and disposition among the teachers to use ICTs to support the teaching and learning of algebra, especially mathematical functions (linear and quadratic functions as instances of polynomial functions). Nonetheless, a primary barrier to teachers' readiness and confidence in using *GeoGebra*, despite their general enthusiasm and belief in the benefits for learners, was their lack of relevant preparation. The teachers in this study were glad to use *GeoGebra* ICT in their teaching practice. They felt that by using ITCs they could provide genuine learning experiences, show new applications and offer opportunities to the learners to work even more positively in the new technological environment.

The experience of using *GeoGebra* tended to modify teachers' ideas towards a more visual approach to mathematics, generating an increasing acknowledgement of the importance of representations for learning. Michael (one of the teacher) recognised the importance of promoting visualisation competence in learning mathematics. Michael believes that '*GeoGebra* will give insight understanding, learners visualise the problem' (see section 5.4.2) which in turn favours processes such as conjecturing and generalising (Pedemonte & Buchbinder, 2011). The teachers' interactions and cooperation were beneficial, because they could learn from each other and enhance their mathematical thinking.

The findings revealed that teachers need more opportunities where they might figure out how to understand relations between the pragmatic and epistemic components of *GeoGebra* use with regards to linear and quadratic functions. The teachers' interactions with multiple representations of algebraic objects with *GeoGebra* can promote teachers' learning, accordingly empowering them with more extensive experiences than might have been gained from the single representation of the equivalent algebraic object (Ainsworth, 2006). The utilisation of *GeoGebra* could help mathematics teachers to play a significant role in influencing leaners' mathematical thinking, especially with regards to algebra, through offering multiple representations.

This study revealed differences between the presentation of *GeoGebra* and performing transformations of the linear and quadratic functions by hand. The teachers were creating and testing more conjectures more quickly. In these instances the use of *GeoGebra* as acted as an amplifier (Denton, 2017) permitting teachers to accomplish combinations of transformation at a speed which would have not been possible with pen-and-paper method. The dragging facility enabled teachers to physically move the graphical representations of the four functions needed to create a square to see the changing effect on the image (see Appendix I, activity 2.5), allowing them to see many examples as opposed to just one. In this specific activity there are pragmatic and epistemic issues.

Pragmatic issues included the speed of making the drawing with *GeoGebra* and the epistemic issues related to the construction of parallel and perpendicular lines. Another epistemic issue is related to interpreting the values of the gradients which can be manipulated using the mouse, this activity (see activity 2.5, Appendix I) provides teachers with the opportunity to drag objects (parallel and perpendiculars lines) in the geometry window and see the changes in the algebraic representation.

GeoGebra can support concept development (Schaffer & Kaput, 1999) which is achieved through the dragging and measurement facilities acting as both amplifiers, speeding up processes, and moving the teachers from the particular to the general in a way that is not achievable using pen and paper alone. *GeoGebra* can serve two major purposes; first as tools to support the interaction and collaboration between different communities of practice, and second, they can serve to enhance the interaction between the users (teachers) and the computational environment.

The "researcher effects" should not be discounted, i.e. instances where the teachers picked up or appropriated the researcher's discourse regarding *GeoGebra*, such as the general behaviour of graphs as different input values are entered into the input bar. It is important to highlight the fact that the different sessions the teacher/researcher had with the teachers entailed an exposure to mathematics education as a research domain with its peculiar ways of talking and working with 'the same' content that they know from a pen-and-paper perspective.

6.3 Contribution of this study

Research shows that in spite of the various advantages of utilising ICT in mathematics education, inserting ICT into classroom practice and for teacher professional development (TPD) is a moderate and complex procedure. Most teachers need something other than being provided with technology if the advantages of ICT are to be substantially realised (Hohenwarter & Lavicza, 2007). No research has been done in the area of the mathematics teachers' leaning experiences during a professional development intervention aimed at improving their understanding of functions using *GeoGebra*. This implies that the literature on TPD related to teachers' leaning experiences is limited. In this sense, this study is significant because it contributes towards this limited body of knowledge by exploring the effects of using *GeoGebra* on teachers' learning and understanding the concept of function. Although this was a small-scale study, its findings contribute towards enriching the knowledge base regarding teacher development in general and in particular to creating a new knowledge base regarding the advantages in using *GeoGebra* for teaching and learning of mathematics, especially in the attempts to improve the teaching of mathematical concepts in the Namibian education context.

An important reason for conducting a study of this nature was to contribute towards the limited literature on this topic, specifically with reference to Namibia. Namibia, like other developing countries, is challenged by a lack of qualified personnel to promote ICT, especially in mathematics education. Moreover, it is believed that this study has the potential to contribute to teacher' involvement in curriculum decision making in order to consider ways in which *GeoGebra* might support the teaching and learning of linear and quadratic functions.

6.4 Limitations of the study

The fundamental limitation of the research related to the high school mathematics teachers' learning experiences during a professional development intervention to improve their understanding of linear and quadratic functions using *GeoGebra* can be highlighted as follows:

1. Due to the nature of the study, the researcher was limited in several respects, from the time available to the resources available. Although mathematics teachers of two schools participated in the study, the data were collected just for 10 teachers who usually teach Grade 11 and 12, and not taking into account the rest of the mathematics teachers from the schools selected or from other secondary schools in Ohangwena region; therefore the sample size of the study was still relatively small. It is believed that richer findings would emerge if more schools from more regions participated in the study the researcher was aware that the two schools selected for the study cannot be taken as representative of other areas of the country, and therefore the findings of the study cannot be automatically extended to the whole country to describe the

teachers' experiences related to an intervention aimed at improving their understanding of functions using *GeoGebra.*

2. The participant teachers are beginner users of the software; it is necessary to add this as a possible limitation, because participants were exposed to *GeoGebra* for the first time. Although the teacher/researcher provided help in order for them to get to know *GeoGebra*, they needed more time to understand the basic concepts and design features inscribed in *GeoGebra*. Part of the time in this study was dedicated simply to teaching them how to use the software. Furthermore, teachers' limited experience in the use of software led to many of them using the wrong strategies to solve tasks, or adopt strategies more naive than would have been used in a more familiar environment.

3. Additional time should have been spent interviewing and observing the teachers' cooperation during the workshops. This may have enabled the investigation to arrive at more in-depth discoveries and more similarities and differences could have been identified.

4. The was poor or practically no support for teachers' professional development (TPD) with respect to the utilisation of ICTs in mathematics education, even though this was stressed by the government through educational strategies (ICT Policy for Education, 2005; Education Training Sector Improvement Program, ETSIP, 2007).

6.5 Recommendations for improving teacher development in Namibian secondary schools

As shown in the interviews and focus groups discussion in Chapter Five, the mathematics teachers who participated in this study benefited significantly from the use of *GeoGebra* in various ways, ranging from personal mathematics exploration, attitudes toward mathematics and mathematics teaching of functions (linear and quadratic), to pedagogical reflections, including the nature of mathematics and the teachers' interactions. These changes are well aligned with the emphases of the ongoing mathematics education reforms in Namibia, including the integration of technology into education (see NIED, 2016:10-13). Technology can provide a means of offering new form of TPD and support. Many of the features of *GeoGebra* and other educational software technology are particularly promising for overcoming some of the limitations presented in traditional static methods of teaching and learning about functions. The finding of this study support the use of *GeoGebra* in TPD that seeks to improve teachers mathematical thinking and to empower them with (*GeoGebra* that deepens their understanding of mathematical functions and enhances the future teaching and learning of mathematics in general. Thus, looking forward, the teacher/researcher makes the following recommendations:

1. The findings of this study can be used to encourage secondary school teachers to use *GeoGebra* in their mathematics classes, especially for teachers who may have had initially negative reaction to *GeoGebra* or any other educational software. Furthermore, schools can utilise these results to provide support and training to teachers on the use *GeoGebra* in teaching mathematics, especially the teaching and learning of functions. A supportive subsequent stage would to include the option to offer professional development for teachers and facilitate research activities in relation to *GeoGebra*.

2. Part of teachers' development is to offer in-service professional development in the technology that is used in mathematics. This helps teachers to learn how the knowledge and skills in teaching and learning of functions could be used in the classroom more effectively in order to save time. It is recommended that teachers should not only learn about the hardware and software of the technology, but also learn the practical skills that they can use in teaching (based on the findings of this study). Using ICTs such as *GeoGebra* or other ICTs mathematical tools correctly might help them to carry out their particular algebra teaching more effectively and efficiently.

3. Acceptance of technology as a pedagogical tool is highly inadequate among mathematics teachers in Namibia. The Ministry of Education has introduced calculators for use at secondary school level, which opened the opportunity for use of technology in our mathematics classrooms. It is recommended that the Ministry of Education, through the Namibian government, further directs and supports the use of computers for learning mathematics, therefore, breaking away from the traditional methods of learning mathematics. This will make mathematics more exciting and interesting for teachers and learners.

4. There should be continuous in-service training of mathematics teachers to promote awareness of modern, ICT-related teaching methods in line with the evolving times.
5. Future professional development programmes have to be designed to stimulate and promote teachers' willingness to construct an understanding of the characteristics of ICTs applications and ways in which they can promote mathematical understanding of the operative curriculum.

6.6 Recommendations for further research

The results of this study show that there is a potential in using *GeoGebra* in teaching and learning functions in the secondary schools mathematics in Namibia. However, further research investigating teachers' experiences through the use of *GeoGebra* is necessary.
This study was restricted to a specific geographical location and teacher population. It was also limited to a short time period of investigation with only ten participants providing data on teacher learning experiences related to an intervention aimed at promoting the use of *GeoGebra* to deepen an understanding of functions. Hence, it would be advisable to conduct future studies with a larger scope and a more in-depth focus to explore what might happen in other regions that were excluded from this study. This would include more participants who are interested and who show a commitment to using *GeoGebra* in their teaching and learning of functions or mathematics in general. This may be a way to motivate mathematics teachers to discover the potential benefits (and limitations) of *GeoGebra* as tools to provide rich and deep understandings of important algebraic concepts such as variables, expressions, equations and solutions in secondary school mathematics.

This study recommends that in-service professional development must include the use of ICTs, such *GeoGebra* or other dynamic geometry software (DGS) in mathematics teaching and learning. This may help teachers to learn how their knowledge and skills could be used in the classroom more effectively in order to save time. The latter is a pragmatic concern. This can be achieved with the help of teachers who have acquired the necessary technical and pedagogical skills related to ICT (*GeoGebra*) helping others. If the purpose of in-service professional development is to produce real solutions, it needs to be delivered over a longer period of time in order to effectively change the confidence levels of teachers in the use of *GeoGebra*. Therefore, one- to three-day workshops are not sufficient to address this problem effectively.

Research shows that ICT may meaningfully enhance teachers' development of important skills, foster better attitudes towards mathematics, and stimulate an extensive vision of the nature of this subject (Putnam & Borko, 2000). Teachers are important players in making this development happen. It is recommended that further research into this field regarding teachers' experiences of *GeoGebra* should be undertaken. It would be of great help to find out more about how to minimise the actual difficulties of learning about functions in Namibian secondary/high schools.

ICT-related studies should focus more on specific areas or topics of mathematics that are poorly performed rather than looking at mathematics from a general point of view; this could help resolve the problem of poor performance in mathematics .

A final observation that emerged from this study was that time and access to ICT need to be made available to mathematics teachers. This access to ICT would mean that teachers would be better informed, skilled, ready and possess the correct tools to contribute more effectively to the teaching of mathematics in high schools.

6.7 A reflection on the potential of *GeoGebra* in the teaching and learning of functions

I continue to be concerned with the challenges teachers and learners face in school mathematics, especially functions. This research has shed light on how to approach these difficulties in teaching and learning about functions with the use of *GeoGebra*. Through my research I have discovered that *GeoGebra* is important digital software which can significantly reduce the challenges faced in secondary (high) schools offering mathematics as a subject. These challenges contribute to the increasing difficulty in motivating learners to sustain their interest in mathematics.

The findings showed some commonality across the teachers' interactions in that the majority of them, according to their comments, revealed a willingness to possibly use *GeoGebra* in their future mathematics teaching. The findings of the research also reveal that the use of *GeoGebra* can provide rich technological mathematics environments in which participating teachers are engaged in a professional development. It appears to have the potential to facilitate teachers' interaction and communication related to the activities focus on functions, as well as to focus that interaction on learning. They see themselves as learners, develop confidence to try things out in a new way and are motivated to seek justifications for their conjectures (Schoenfeld, 1989).

The use of *GeoGebra* can motivate teachers and learners; its use is motivational because it enables teachers and learners to make improvements to the quality of their work. This study showed that as teachers became more deeply involved in the activities, the subject became more attractive and enjoyable, and they thereby learn more from the activities in a relative short period of time. Peter (one of the participants) suggests this in his comment:

> Yes, it's very relevant the use of *GeoGebra* to give the learners a better understanding of function. The good thing about *GeoGebra* is that the moment you place in the number you will see the line changing, the moment you change the parameters you will see how the graph will change, so it create a better picture in the learner's mind and also in my mind; the *GeoGebra* give me a better understanding of some of the formulas. Sometimes I use some of the formula but I don't know really where they are coming from, for example, in the activities we analysed we noticed how the graph was changing but also was changing in the algebraic window on the other side … so it was wonderful, it gave me a lot about why things happened and the way they happened.

Teachers' interaction with *GeoGebra* allowed me to identify a range of practical, motivational and educational benefits that underpin the use of such programs. On the other hand, there is a need to take into consideration that the pedagogical and technical abilities of the teachers are absolutely critical. Thus to use *GeoGebra* in their future functions teaching, the teachers should know how to use this didactic tool properly.

In the case of this research the participating teachers showed that the use of *GeoGebra* might be good practice for mathematics teachers in addressing some of the difficulties that occur when teaching functions. The teachers encountered some difficulties in manipulating this software because it was their first time using this particular tool (*GeoGebra)* during a professional development intervention. Finally, the teachers' classroom world is different from the *GeoGebra* environment. They do not have *GeoGebra* in their mathematics curriculum; they do not have this digital tool in their everyday teaching. Therefore a main recommendation is search for ways to bridge the two worlds, namely, that of the teachers and the facilitators that use *GeoGebra* by working towards mutual intelligibility. By the latter I mean that facilitators should 'translate' *GeoGebra*-representations of functions in ways that relate to the classroom practices of teachers. This is not a once-off exercise. It will involve mutual interactions that go –back-and-forth, in mutual ways, between ways the teachers speak about and work in their classrooms in relation to ‘the same’ functions represented via *GeoGebra*. In this regard, a ‘long-term intelligibility or understanding between teachers and facilitators, will take time and will need coordination and alignment between the two practices, namely, school mathematics teaching and research-informed professional development that employs *GeoGebra* to teach functions.

The *GeoGebra* activities that are used in future workshops have to be carefully selected, ones that ‘open up the mathematical content so that opportunities arise for the teachers to learn and for the facilitator learn to what it takes to facilitate between GeoGebra-represent mathematical content and ‘the same’ mathematical content as configured in the teachers’ classrooms.

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APPENDIXES

Appendix A:
Request for permission to conduct research in Namibians schools

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

The Permanent Secretary
Ministry of Education, Arts and Culture
Government Park 3rd Floor
Windhoek

24/11/2015

Stellenbosch University
Faculty of education
P. Bag X1 Matieland
7602
Stellenbosch
South Africa

RE: APPLICATION FOR PERMISSION TO CONDUCT RESEARCH ON MATHEMATICS EDUCATION IN TWO SCHOOLS AT OHANGWENA REGION

Dear Mrs. Sanet Steenkamp

I Ricardo J. Rodrigues Losada, PhD (mathematics education) candidate at Stellenbosch University hereby kindly request permission to conduct my research in two selected schools in Ohangwena region namely, Ongha Secondary school and Ponhofi secondary school. I have selected these schools because I am a H.O.D of mathematics & science at Ongha SS and familiar with many mathematics teachers in the area. I hope that they will not be hesitant to share their experiences with me.

The tittle of my research is: ***"High school mathematics teachers' learning experiences related to an intervention based on the use GeoGebra with a focus on functions"***
The need for the study is coupled with efforts of Vision 2030 to build an information society and knowledge-based economy in Namibia.

The focus of the study is on the investigation of a selected number of high school mathematics teacher learning experiences during a professional development intervention aimed at improving the understanding of functions using *GeoGebra*.

Personal experience has shown that teaching and understanding a mathematical function is a challenge hence there is a need to integrate the use of ICTs in order to improve the understanding and teaching of this concept. However successful integration is limited by the lack of teacher skills in ICT. This study intends to improve the understanding and teaching of mathematical functions by improving the teacher skills in ICT through a professional development intervention using *GeoGebra*.

In order to explore and understand teachers' learning experiences interacting with the *GeoGebra* activities, the study will use case study methodology. The use of *GeoGebr*a might provide new possibilities, opportunities and challenges for teachers to conceive a strong mathematical concept related to functions. This improves professional learning and development of teachers and is crucial in transforming school mathematics leaning of functions.

The field work will take a period of three months beginning on the second term, from June to August. The researcher designed the workshops in a way that participants will not be affected in their everyday teaching. The interventions will take place during afternoon sessions. The researcher will meet the participants selected for this study twice a month with a period of work away in between, one in the beginning and one at the end of every month.

I am confident that the findings of my study will make valuable contributions to the current ongoing project of Education and Training Sector Improvement Programme (ETSIP), aimed to embed ICT at all levels of the education system and to integrate the use of ICT as a tool in the delivery of curriculum and learning, thereby leading to the market improvement in the quality of learning and teaching process across all levels.

The University of Stellenbosch requires that the participants of this study be protected in terms of keeping their identity anonymous and the information be kept confidential. Upon completion of this study, a copy of the report will be made available to the Ministry of Education offices and other government agencies.

I hope my request shall be thoroughly considered.

Yours sincerely

Ricardo J. Rodrigues Losada
PhD candidate and HOD mathematics & science.
r.losadaricardo@yahoo.com
Cell: +264818740554 or +27712395850

Appendix B:

Permission letter from Ministry of Education, Namibia

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION, ARTS AND CULTURE

Tel: (061) 2933286
Fax: (061) 2933922
Enquiries: Ms. C. Dentlinger
Email: C.Dentlinger@moe.gov.na

Luther Street, Govt. Office Park
P/Bag 13186
WINDHOEK

To: Mr. Ricardo J. Rodrigues Losada
Cell: 0813820277
r.losadaricardo@yahoo.com

Dear Mr. Ricardo J. Rodrigues Losada

SUBJECT: PERMISSION TO CONDUCT RESEARCH ON MATHEMATICS EDUATION IN TWO SCHOOLS IN OHANGWENA REGION: "HIGH SCHOOL MATHEMATICS TEACHERS'LEARNING EXPERIENCES RELATED TO AN INTERVENTION BASED ON THE USE GEOGEBRA WITH A FOCUS ON FUNCTIONS".

Your letter dated 24 November 2015 requesting permission to conduct research in education bears reference:

Kindly be informed that permission to conduct research study for your doctorate in two schools (Ongha and Ponhofi Senior Secondary Schools) in Ohangwena Region resorting under the Directorate: Education, Arts and Culture is herewith granted. You are further requested to present this letter of approval to the Regional Director and engage with the Inspectors of Education of the selected schools just to ensure that research ethics are adhered to and disruption of curriculum delivery is avoided.

Furthermore, we humbly request you to share your research findings with the Ministry. You may contact Mr. C. Muchila at the Directorate: Programmes and Quality Assurance (PQA) for provision of summary of your research findings.

Accompanying this permission are best wishes for your endeavor and that the Ministry is looking forward to the outcome of your research.

Sincerely yours,

....
Sanet L. Steenkamp
PERMANENT SECRETARY

Ministry of Education, Arts and Culture
Office of the Permanent Secretary
22/12/15
Date 2015 -12- 21
Private Bag 13186
Windhoek, Namibia

All official correspondence must be addressed to the Permanent Secretary

Appendix C

Permission granted from the Ohangwena Regional Council, Directorate of Education, Arts and Culture, Director's office

OHANGWENA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
DIRECTOR'S OFFICE

1st Floor Greenwell Complex Private Bag 88005 Eenhana Tel: 065 – 290 201 Fax: 065 -290 224

Enquiries: Magano Gaoses
Email: mcnotto@yahoo.com
Ref: 12/3/10/1

30 December 2015

Mr. Ricardo J Rodriques Losada

Subject: Approval granted to conduct a research at schools in Ohangwena Region – Ongha and Ponhofi Secondary Schools.

1. Receipt of your letter via email on the 28th December 2015 on the above subject matter, is hereby acknowledged.
2. The Ohangwena of Education, Arts and Culture indeed supports and herewith grants approval to you to carry out the envisaged research on Mathematics Education in schools in Ohangwena Region.
3. This office has noticed that your research will not disrupt the normal teaching and learning however you are advised to liaise with the Inspector of Education to put proper planning and also to adhere to research ethics as per the with the directive from the office of the Permanent Secretary.
4. This would also like to wish you all the best in your endeavors and may this be a fruitful research and has faith that it will be of great benefit to the Namibian child.

Thank you

30/12/2015

Armas P Kashiimbi
Acting Director: MEAC
Ohangwena Region

Ohangwena Regional Council
30 DEC 2015
Tel: 065-290 200 • Fax: 065-290 224
P/Bag 88005 • Eenhana
REPUBLIC OF NAMIBIA

Appendix D:

Participants' consent form

STELLENBOSCHUNIVERSITY

CONSENT TO PARTICIPATE IN RESEARCH

Project title: High school mathematics teachers' learning experiences related to an intervention based on the use *GeoGebra* with a focus on functions.

You are asked to participate in a research study conducted by Ricardo Jacinto Rodrigues Losada *(Qualifications: Med, Licentiate in Education & Bed Honours) under the supervision of Doctor GierdienFaaiz*from the Faculty of education (curriculum studies) *at* Stellenbosch University. You were selected as a possible participant in this study because your contribution will help me to get the information required for the completion of my thesis.

1. **PURPOSE OF THE STUDY**

The purpose of the proposed study isto investigate a selected number of high school mathematics teacher learning experiences during a professional development intervention aimed at improving the understanding of functions using *GeoGebra*.

2. **PROCEDURES**

Data will be collected by using semi-structured and focus group interviews, audiotaped, observations and field notes. Semi-structured interviews will last no longer than 45 minutes and focus groups interviews not longer than 90 minutes. All interviews will be recorded and note will be taken. The focus group interviews will also be recorded. Information obtains from the interviews or observations will be kept strictly confidential. Participation in interviews is voluntary and participants can choose not to answer any question. You may also stop the interview at any point.

3. POTENTIAL RISKS AND DISCOMFORTS

In the interview, you will be asked to speak freely about your experiences using GeoGebra regarding difficulties and learning of functions and reflection as experienced by them. You will not share the information with anyone except members of the focus group. The risks are that other members of the focus group might share sensitive issues with other individuals. In this regard, you will be ask not to share. If you become tired during the exercise you can rest any time. The possibility that you may be emotionally upset when answering some questions is ruled out since the questions are all of a non-emotional nature.

4. POTENTIAL BENEFITS TO SUBJECTS AND/OR TO SOCIETY

It is hoped that you as participant in this study will assist the researcher to be able to find out more about the understanding of function concept and the potentiality of ICT *(GeoGebra*) in the learning of function. The project goals is aligning with the needs of mathematics teachers. In particular, it recognised the importance of providing mathematics teacher with learning opportunities that include examples of mathematical investigations related to the teaching and learning of functions, opportunities to experience this investigation as learners themselves.

5. PAYMENT FOR PARTICIPATION

No financial remuneration is involved for participating in this research study.

6. CONFIDENTIALITY

Confidentiality will be maintained all times. Any information that is obtained about this study and that can be used to identify with you will remain confidential and will be reported in a scholarly manner by means of a thesis and possible publications. Transcriptions' of the semi-structured interviews and the focus interviews as well as notes by the researcher will be coded and sorted into categories in relation to the study goals. From these categories I aim to develop themes and subthemes according to the research questions. All the data will be safely kept in a personal computer to which only the researcher and the supervisor have access. After research all audio-taped and recorded records will be kept save for a maximum of five years thereafter before destroying it.

After the interview, the audio-tape will be transcribed and a written copy will be provided to each participant for review. The participant may edit or delete anything he/she does not want included in the interview data.

7. PARTICIPATION AND WITHDRAWAL

You can choose whether participate in this study or not. If you volunteer to be in this study, you may withdraw at any time without consequences of any kind. You may also refuse to answer any questions you don't want to and remain in the study. The investigator may withdraw you from this research if circumstances arise which warrant doing so. Any complain by a subject that shows unexpected risk or which cannot be resolved by the researcher. Anticipated circumstances under which the subject's participation may be terminated includes personal reasons for example when participant is sick.

8. IDENTIFICATION OF INVESTIGATORS

If you have any questions or concerns about the research, please feel free to contact Mr Ricardo J. Rodrigues Losada at r.losadaricardo@yahoo.com or Tel no: +27712395850. Supervisor: Doctor Gierdien Faaiz at faaiz@sun.ac.za or Tel no: +27846199639.

9. RIGHTS OF RESEARCH SUBJECTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research subject, contact Ms MaléneFouché [mfouche@sun.ac.za .27 21 808 4622] at the Division for Research Development. Stellenbosch University

SIGNATURE OF RESEARCH SUBJECT OR LEGAL REPRESENTATIVE

The information above was described to me, ___________________________ by Mr Ricardo J. Rodrigues Losada in *English and* I *am* in command of this language. *I was* given the opportunity to ask questions and these questions were answered to my satisfaction.

I have read, understand, and received a copy of the above consent and desire of my own free will and decision to partake in this study.

____

Name of Subject/Participant

____

Name of Legal Representative (if applicable)

____ _______________

Signature of Subject/Participant or Legal Representative **Date**

SIGNATURE OF INVESTIGATOR

I declare that I will explain the information given in this document to all participants in this study. *Each participant* will be encouraged and given ample time to ask me any questions. This conversation will be conducted in *English* and no *translator will be needed since all participants understand English.*

____ _______________

Signature of Investigator

Appendix E

The Semi-structured interview schedule for the participants in the study

Research project:

High school mathematics teachers' learning experiences related to an intervention based on the use of *GeoGebra* with a focus on functions.

Background information

1. Respondent's name and contact details:…………………………………..
   ………………………………………………………………………………
2. Sex: Male or female ……………………………………………………….
3. Number of years teaching ……………………………………………………
4. Grade level you are teaching ……………………………………………….
5. Name of the school/ location ……………………………………………..
6. Familiarity with information communication and technology (ICT)? Give examples:

…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………

7. Are you involve in any ICT network learning/ network? If yes, give details.

…………………………………………………………………………………………………
…………………………………………………………………………………………………

8. How long have you using ICT in your mathematics teaching? For what purposes?

…………………………………………………………………………………………………
…………………………………………………………………………………………………
………………………………………………………………………………………………..

9. Are you familiar with GeoGebra? If yes, where and how?

…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………

10. Have you ever used GeoGebra in relation to the secondary school mathematics curriculum? If yes, provide details.

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

11. Do you believe that there is a role for ICT/GeoGebra to deepening functions learning? Please elaborate.

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Appendix F

Focus group interview

1. How do you think now about this particular content that you have experience through *GeoGebra?*

2. How do you think about implementing GeoGebra in your teaching? And when?

3. After you have been exposed to GeoGebra what insight have you gained?

4. Did you learn something new?

Appendix G

Questions to address in every intervention are the following:

- It is alleged that some teachers tend to emphasize some representations more than Others- What is your view on this?
- Were the information and materials useful? Can you say more?
- Were the information and materials relevant?
- Did participant learn what they were intended to learn?
- Did participant learn something new? If yes/not, can you explain please?

Appendix H

Teachers semi-structured interview responses and example of coding

11. Do you believe that there is a role for ICT/*GeoGebra* to deepening functions learning? Please elaborate.				
Participants responses	**Textual data excerpt (Raw data)**	**Coding**	**Axial coding**	**Main themes**
Petrus (Pseudonym)	*Yes, ICT/GeoGebra will/can make preparation easier and faster. ICT can be used to supplement pen-paper work for learners to visualise what actually on the screen. Also we can record lessons and in the case of not being around, learners can just view lessons themselves on repeat*	Easy and faster. Supplement pen-paper work. Visualise what is on the screen.	Time concern and constraints. GeoGebra affordance (visual capability). Supporting pen-paper learning of functions	**Time concerns** **Supporting pen-paper learning of function**
Henry	*Yes, after getting confident with GeoGebra through this project I realised how easier is to come up with graphs and draw the functions by means of ICT.* *It is more interesting plotting points and drawing lines with GeoGebra. It deepens*	Easy use of the software. Deepens the teaching and learning of functions.	GeoGebra has epistemic and pragmatic values.	**Epistemic and pragmatic dimensions**

	the teaching and learning of functions. GeoGebra in connection with ICT is quicker and simplify the methodology of learning.			
Michael	*I believe there is a big role that GeoGebra will play if it's to be used in the learning of functions, because arouse leaners' interest and makes them to love the topic as much as the program, it brings theory into practice, also is faster and the answers are ready available, learners understanding of functions will be much more enriched.*	Role of *GeoGebra* in the teaching and learning of functions. Excited about *GeoGebra.* Fast answers. Better understanding.	Time saving. *GeoGebra* has quick feedback (Dynamic). Epistemic and pragmatic value. Deepening of mathematical thinking	**Deepening mathematical thinking** **GeoGebra affordances**
Rob	*I definitely believe that GeoGebra program will play a major role to deepening learning on the ground that: learners will understand the topic very well since GeoGebra is more visual. I believe that it*	Deepening learning. Visual practices. Faster.	Discovery learning. Visualise abstract concepts	**Epistemic and pragmatic dimensions**

	will make mathematics perfect. *It also deepening functions learning in the sense that its faster and one can always have a picture in mind on how problem were solved by GeoGebra*			
Robert	*Yes, GeoGebra is very effective and save a lot of time in drawing graphs.it also will help the learners to viewing the effects of nature of gradient and the general behaviour of graphs as different input values are entered into the input bar . it will generate learners interest as the concept is introduced*	Saving time in drawing graphs. Graph behaviour. Learner interest	Time concerns and constraints. Epistemic values. Multiple representations	**Time concerns**
Peter	*Yes, the program is fast and time saving that will gives the learners the shape of the graphs upon entering the function, deepening leaners understanding of different functions and easy ways of manipulating functions*	Deeper understanding of function, epistemic values	GeoGebra affordances. Time concerns. Function manipulation. Epistemic and pragmatic dimension	

	to find things such as, midpoint, turning points or solutions to a given function.			
Charles	*Yes, because using GeoGebra in the teaching of function is much faster that using a chalkboard. Is very easy to find a maximum and minimum of functions unlike completing the squares first or finding the x intercept and memorising the formula of turning point; tp = (-p;q)*	*GeoGebra* much faster and*GeoGebra* affordances Comparing ICT with pen-paper environment. Time concern		
Oveka	*Yes, that can deepen functions learning because learners will be willing to explore and understand better, they will find it easy to draw and interpret any given graph. Learners can observe what is happening to the graph as soon as we keep on changing the parameters of the function.*	Draw and interpret graph and graph behaviour. visualisation	Particularities around functions. Multiple representations of function.	Particularities around functions

Ali	*Yes, because this will help the learners to get deeper understanding of functions as they can observe at the same time the behaviour of the graphs when the parameters are changing and it also create curiosity for them to learn on their own, giving opportunity to explore and discover new things on their own,*	Deeper understanding of functions. Explore and discover. Own learning , discovering and exploring	Multi-representation of function. Discover teaching and learning. *GeoGebra* affordances.	Epistemic and pragmatic values

Appendix I

Workshop activities

Workshop 1

Activity 1.1: Entering, extracting and modifying coordinate points.

- Features introduced: algebra window, free and dependant objects, coordinate axes, grid, and labels of objects.
- Features introduced: construction protocol, navigation bar.
- Tools introduced: sliders, Slope

Activity 1. 2: constructing line bisector on a paper using pencil, straightedge, and compass.

Activity 1. 3: constructing a line bisector with *GeoGebra.*

- Tools introduced: segment between two points, circle with centre through point, intercept two objects, line through two points.
- Features introduced: construction protocol, navigation bar.
- Tools introduced: sliders, Slope

Workshop 2:

Activity 2. 1: The purpose of this activity is to engage teachers with possible solutions exploring paper-pencil methods and teachers' preference on the solution of quadratics equations and reasons behind of their preferences, time consuming, other particularities related with symbol sense.

Draw the graph of $y = x^2 + 3x - 4$ for values of x between -4 and 1

I. Find the coordinate of the minimum point of the curve.

Activity 2.2: State whether the functions has a minimum or a maximum value and find the value.

G(x) = $-2x^2 + 4x - 1$

Activity 2.3: A ball thrown straight up from the top of a 128-foot-tall building with an initial speed of 32 feet per second. The height of the building as a function of time can be modelled by a function H(t) =$-16t^2 + 32t + 128$ (YouTube video)

How long will it take for a ball to hit the ground?

Activity 2.4: Repeat the construction above with the use of *GeoGebra*

Activity 2. 5: Polynomials Functions (this activity will be discussed in workshop 3). This also will give teachers an opportunity to practise their newly gain knowledge in preparation of the next session.

1. Construct four linear function ($y = mx + c$) in such a way that they will form a square when they are drawn in *GeoGebra* coordinate system.

Workshop 3:

Discussion of activity 2.5

Workshop 4:

Activity 4. 1:Exploring Parameters of a Quadratic Polynomial

Follow the instructions on the paper worksheet and write down your results and observations while working with *GeoGebra*.

1. Open a new *GeoGebra* file

2. Type in f(x) = x^2 and hit the *Enter* key. Which shape does the function graph have? Write down your answer on paper.

3. In *Move* mode, highlight the polynomial in the algebra window and use the ↑ up and ↓ down arrow keys.

4. Again, in *Move* mode, highlight the function in the algebra window and use the ← left and → right arrow keys.
   a. How does this impact the graph of the polynomial? Write down your observations.

   b. How does this impact the equation of the polynomial? Write down your observations.

5. In *Move* mode, double click the equation of the polynomial. Use the keyboard to change the equation to f(x) = 3 x^2.

   <u>Hint</u>: Use an asterisk * or space in order to enter a multiplication.

   a. Describe how the function graphs changes.
   b. Repeat changing the equation by typing in different values for the parameter (e.g. 0.5, -2, -0.8, and 3). Write down your observations

Questions for Discussion

- Did any problems or difficulties concerning the use of *GeoGebra* occur?
- How can a setting like this (*GeoGebra* in combination with instructions on paper) be integrated into a 'traditional' teaching environment?
- In which way could the dynamic exploration of parameters of a polynomial possibly affect your students' learning?

Activity 4. 2: Constructing graphs of the line $x + y = 4$ and a quadratics equation y $=$ x^2 $+$ 5x $+$ 10 in paper and pencil method.

Activity 4. 3: Repeat the construction Plotting a line (x + y = 4), a quadratic equation (y = x^2+5x+10) using *GeoGebra*.

Tools introduced: Algebraic input and command item tools.

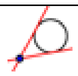
Activities 4.3 will be discussed in the beginning of workshop 5.

Workshop 5 (adapted from introduction to *GeoGebra* manual)

Activity 5. 1: Create your sketch

Open a new *GeoGebra* document and make sure the algebra window, input field, and coordinate axes are shown.

1	**f(x) = 0.5x^3 + 2x^2 + 0.2x – 1**	Enter the cubic polynomial *f*
2	**R = Root [f]**	Create the roots of polynomial *f* <u>Hint</u>: If there are more than one root *GeoGebra* will produce indices for their names if you type

		in R = (e.g. R_1, R_2, R_3).
3	**E = Extremum [f]**	Create the extrema of polynomial *f*
4		Create tangents to *f* in E_1 and E_2
5	**I = Inflection Point [f]**	Create the inflection point of polynomial *f*

Teachers will be always encouraged to observe corresponding changes that appear in the algebraic window when the object is change in the graphical window.

Appendix J

Observation form for the researcher

Time & date ………………..

School/venue……………….

Content /situation	What teachers are doing (activities)	Reflections

Adapted from Nieuwenhuis (2007b) & Silverman (2015)

Appendix K

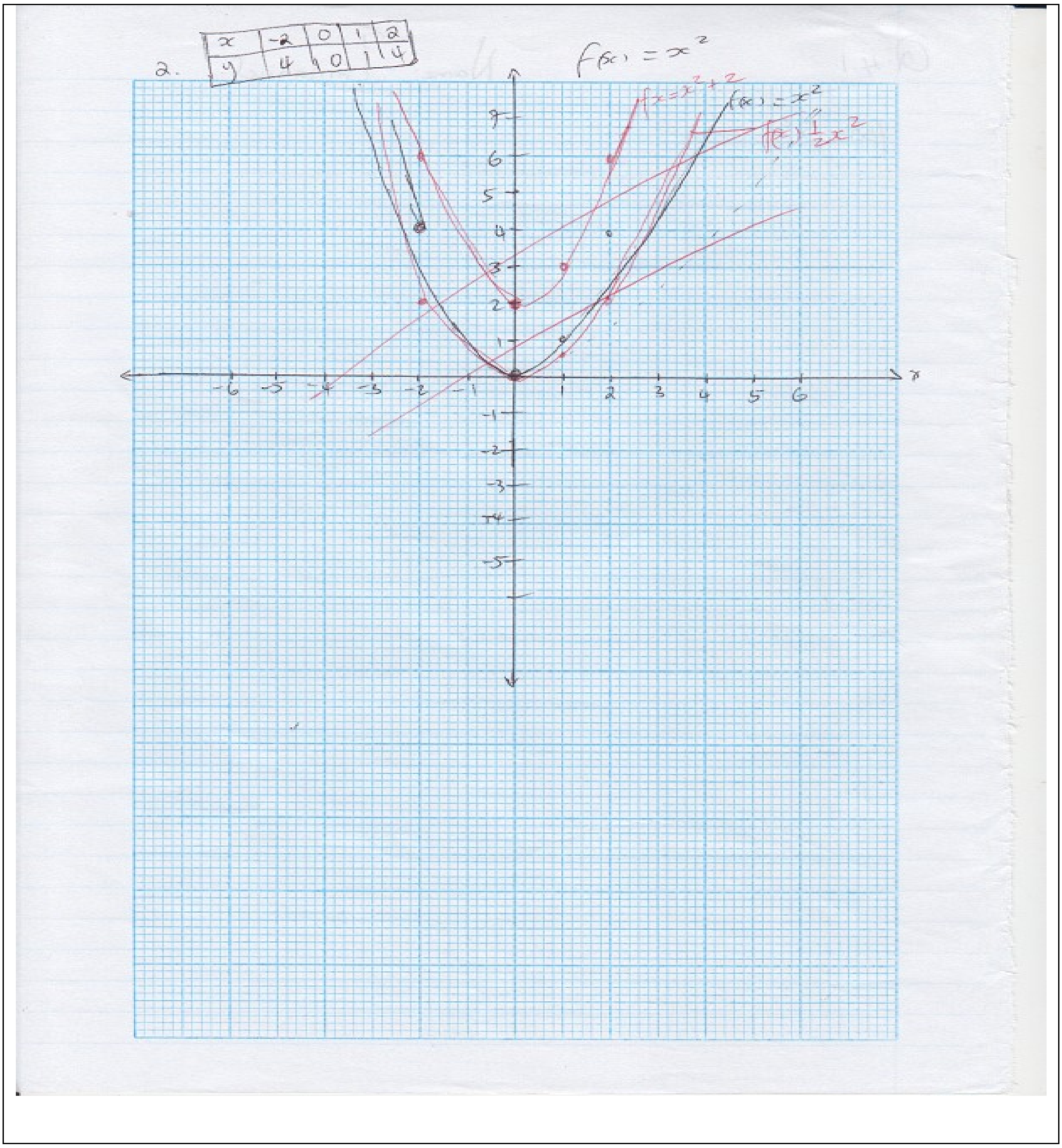
Details on 5 Workshops and log of data gathering activities per workshop

DATE	**Place**	Activity	Who	What to explore
19-07-2016 Workshop 1 introductory	Computer lab Galileo H.S	Activities 1.1, 1.2,1.3 Participants introduced themselves Researcher introduced briefly GeoGebra software and the study Ethics issues were explained, teachers signed consent letter	Ten High school mathematics teachers Six teachers from Galileo H.S, four teachers from Okamu S.S	Familiarity with *GeoGebra*. Teachers become familiar with the basis use interface, applying tools, and changing properties of objects. Teachers learned how to enter algebraic expressions (e.g to create points and functions.
21-07-2016 Workshop 2	Computer lab Galileo H.S	Activities 2.1,2.2; 2.3,2.4,2.5 (Activity 2.3) a video were shown of how *GeoGebra* can be used to solves problems in Science (quadratics function applications) Focus groups interview	Eight High school mathematics teachers (two teachers were excused, they attended examination workshop)	Teachers engaged with possible solutions exploring pencil-paper methods and teachers' preferences on the solution of quadratic functions and reasons behind of their preferences. Exploring minimum or maximum values of the quadratic functions. Repeat the construction with the

				use of *GeoGebra*.
23-07-2016 Workshop 3	Computer lab Galileo H. S and Okamu S. S	Discussion of activity 2.5	mathematics teachers from schools selected	Exploring parameters And construction of linear functions using *GeoGebra*
26-07-2016 Workshop 4	Computer lab Galileo H. S	Discussion of activities 4.1 and 4.2 And focus group interview	Ten mathematics teachers from the two schools selected	Exploring parameters of a quadratic polynomial
29-07-2016 Workshop 5	Computer lab Galileo H. S	Discussion of activity 4.3 and Focus group interview	Ten mathematics teachers from the two schools selected	Review of what teachers had learned Exploring teachers' experiences with the use of *GeoGebra* in the teaching and learning of functions

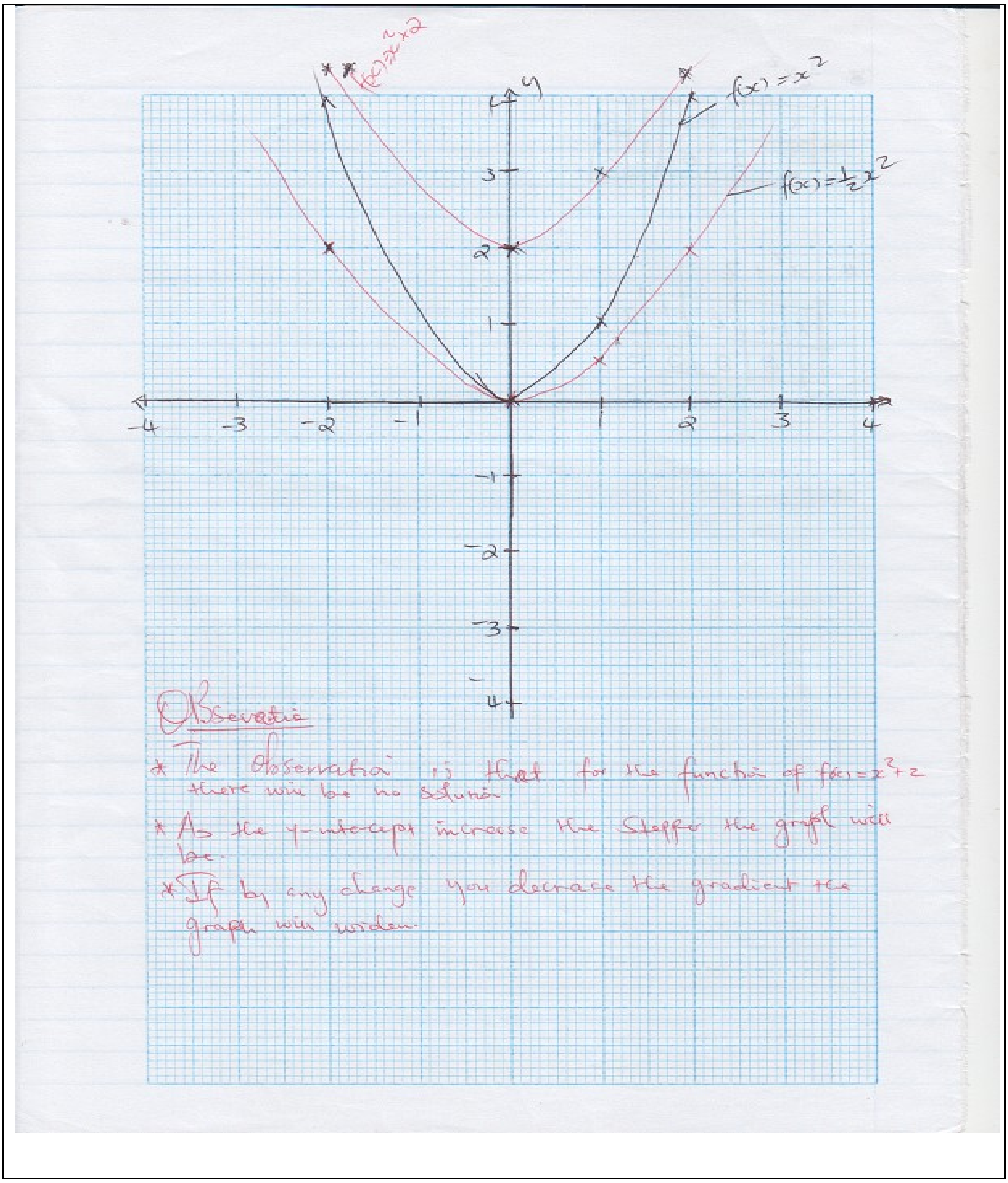
Appendix L1

Participant construction of the quadratics function $f(x) = x^2$ Workshop 4, activity 4.1



Appendix L2

Participant exploring the parameters of a quadratic polynomial. Activity5 workshop 4



Appendix M

Introduction to GeoGebra

Installing GeoGebra

Preparations

Create a new folder called *GeoGebra_Introduction* on your desktop.
Hint: During the workshop, save all files into this folder so they are easy to find later on.

GeoGebra Installers

- Download the installer file from www.geogebra.org/download into the created *GeoGebra_Introduction* folder on your computer.
  Hint: Make sure you have the correct version for your operating system.
- Double-click the GeoGebra installer file and follow the instructions of the installer assistant.

2. Basic Use of GeoGebra

How to operate GeoGebra's geometry tools

- Activate a tool by clicking on the button showing the corresponding icon.
- Open a toolbox by clicking on the lower part of a button and select another tool from this toolbox.
  Hint: You don't have to open the toolbox every time you want to select a tool. If the icon of the desired tool is already shown on the button it can be activated directly.
  Hint: Toolboxes contain similar tools or tools that generate the same type of new object.
- Click on the icon at the right of the Toolbar to get help on the currently active tool.

How to save and open GeoGebra files

Saving GeoGebra Files

- Open the *File* menu and select *Save*.
- Select the folder *GeoGebra_Introduction* in the appearing dialog window.
- Type in a *name* for your GeoGebra file.
- Click *Save* in order to finish this process.

Hint: A file with the extension '*.ggb*' is created. This extension identifies GeoGebra files and indicates that they can only be opened with GeoGebra.

Hint: Name your files properly: Avoid using spaces or special symbols in a file name since they can cause unnecessary problems when transferred to other computers. Instead you can use underscores or upper case letters within the file name (e.g. First_Drawing.ggb).

Opening GeoGebra Files

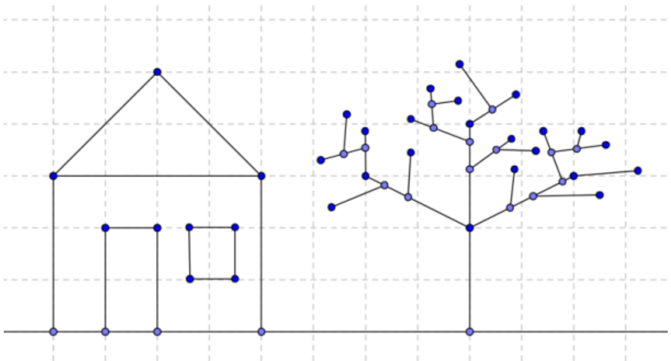
- Open a **new GeoGebra window** (menu *File – New window).*
- Open a **blank GeoGebra interface** within the **same window** (menu *File – New).*
- Open an **already existing GeoGebra file** (menu *File – Open).*
  - Navigate through the folder structure in the appearing window.
  - Select a GeoGebra file (extension '*.ggb*') and click *Open*.

Hint: If you didn't save the existing construction yet GeoGebra will ask you to do so before the blank screen / new file is opened.

3. Creating drawings with GeoGebra

Preparations

- Click on the arrow at the right side of the *Graphics View* and select *Basic Geometry* from the *Perspectives* Sidebar.
- Right-click (MacOS: *Ctrl*-click) on the *Graphics View* and choose *Grid* to show the grid lines



Drawing Pictures with GeoGebra

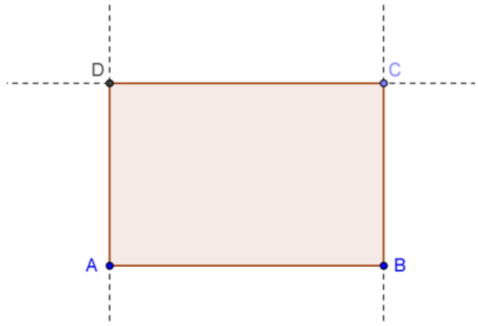
Use the mouse and the following selection of tools in order to draw figures in the *Graphics View* (e.g. square, rectangle, house, tree,…).

	Point **New!** Hint: Click on the *Graphics View* or an already existing object to create a new point.
	Move **New!** Hint: Drag a free object with the mouse.
	Line **New!** Hint: Click on the *Graphics View* twice or on two already existing points.
	Segment **New!** Hint: Click on the *Graphics View* twice or on two already existing points.
	Delete **New!** Hint: Click on an object to delete it.
	Undo / Redo **New!** Hint: Undo / redo a construction step by step (on the right side of the Toolbar).
	Move Graphics View **New!** Hint: Click and drag the *Graphics View* to change the visible part.
	Zoom In / Zoom Out **New!** Hint: Click on the *Graphics View* to zoom in / out.

Hint: Move the mouse over a tool to show a tooltip on how to use the tool.

5. Rectangle Construction

Preparations



- Summarize the properties of a rectangle before you start the construction.
  Hint: If you don't know the construction steps necessary for a rectangle you might want to open the link to the dynamic worksheet "Rectangle Construction" http://www.geogebratube.org/student/m25907. Use the buttons of the *Navigation Bar* in order to replay the construction steps.
- Open a new GeoGebra window.
- Switch to *Perspectives* - *Basic Geometry*.
- Change the labeling setting to *New Points Only* (menu *Options – Labeling*).

Introduction of new tools

	Perpendicular Line **New!** Hint: Click on an already existing line and a point in order to create a perpendicular line through this point.
	Parallel Line **New!** Hint: Click on an already existing line and a point in order to create a parallel line through this point.
	Intersect **New!** Hint: Click on the intersection point of two objects to get this one intersection point. Successively click on both objects to get all intersection points.
	Polygon **New!** Hints: Click on the *Graphics View* or already existing points in order to create the vertices of a polygon. Connect the last and first vertex to close the polygon! Always connect vertices counterclockwise!

Hints: Don't forget to read the Toolbar help if you don't know how to use a tool. Try out all new tools before you start the construction.

Construction Steps

1		Create segment *AB*.
2		Create a perpendicular line to segment *AB* through point *B*.

3		Insert a new point *C* on the perpendicular line.
4		Construct a parallel line to segment *AB* through point *C*.
5		Create a perpendicular line to segment *AB* through point *A*.
6		Construct intersection point *D*.
7		Create the polygon *ABCD*. <u>Hint</u>: To close the polygon click on the first vertex again.
8		Save the construction.
9		Apply the drag test to check if the construction is correct.

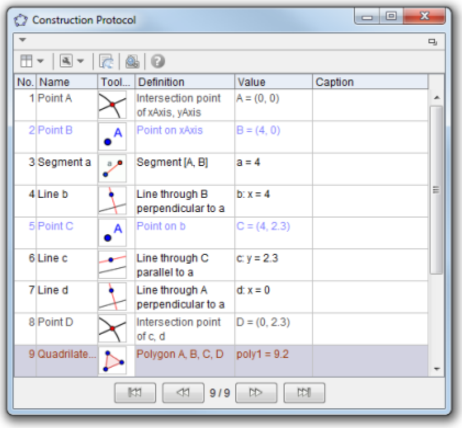
6. Navigation Bar and Construction Protocol

Right-click (MacOS: *Ctrl*-click) the Graphics View to show the *Navigation Bar* to review your construction step-by-step using its buttons.

In addition, you can open the *Construction Protocol* (*View* menu) to get detailed information about your construction steps.

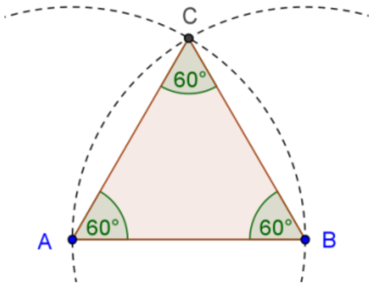
What to practice

- Try to change the order of some construction steps by dragging a line with the mouse. Why does this NOT always work?
- Group several constructions steps by setting breakpoints:
    - Show the column *Breakpoint* by checking *Breakpoint* in the *Column* drop-down menu
    - Group construction steps by checking the *Breakpoint* box of the last one of the group.
    - Change setting to *Show Only Breakpoints* in the *Options* drop-down menu
    - Use the *Navigation Bar* to review the construction step-by-step. Did you set the breakpoints correctly?



7. Equilateral Triangle Construction

Preparations



- Summarize the properties of an equilateral triangle before you start the construction.
  Hint: If you don't know the construction steps necessary for an equilateral triangle you might want to have a look at the following link to the dynamic worksheet "Equilateral Triangle Construction" http://www.geogebratube.org/student/m25909.
  Use the buttons of the *Navigation Bar* in order to replay the construction steps.
- Open a new GeoGebra window.
- Switch to *Perspectives* - *Geometry*.
- Change the labeling setting to *New Points Only* (menu *Options – Labeling*).

Introduction of new tools

	Circle with Center through Point **New!** Hint: First click creates center, second click determines radius of the circle.
	Show / Hide Object **New!** Hints: Highlight all objects that should be hidden, then switch to another tool in order to apply the visibility changes!
	Angle **New!** Hint: Click on the points in counterclockwise direction! GeoGebra always creates angles with mathematically positive orientation.

Hints: Don't forget to read the Toolbar help if you don't know how to use a tool. Try out all new tools before you start the construction.

Construction Steps

1		Create segment *AB*.
2		Construct a circle with center *A* through *B*. Hint: Drag points *A* and *B* to check if the circle is connected to them.
3		Construct a circle with center *B* through *A*.
4		Intersect both circles to get point *C*.

5		Create the polygon *ABC* in counterclockwise direction.
6		Hide the two circles.
7		Show the interior angles of the triangle by clicking somewhere inside the triangle. Hint: Clockwise creation of the polygon gives you the exterior angles!
8		Save the construction.
9		Apply the drag test to check if the construction is correct.

8. GeoGebra's Object Properties

Graphics View Stylebar

You can find a button showing a small arrow to toggle the *Stylebar* in the upper left corner of the *Graphics View*. Depending on the currently selected tool or objects, the *Stylebar* shows different options to change the color, size, and style of objects in your construction. In the screenshot below, you see options to show or hide the *axes* and the *grid*, adapt *point capturing*, set the *color*, *point style*, etc.

Hint: Each view has its own *Stylebar*. To toggle it, just click on the arrow in the upper left corner of the view.

Object Preferences Dialog

For more object properties you can use the *Preferences* dialog. You can access it in different ways:

- Click on the symbol on the right side of the Toolbar. Then choose *Objects* from the appearing menu.
- Right-click (MacOS: *Ctrl*-click) an object and select *Object Properties*...
- In the *Edit* menu at the top select *Object Properties…*
- Select the *Move* tool and double-click on an object in the *Graphics View*. In the appearing *Redefine* dialog, click on the button *Object Properties*.

1. Square Construction

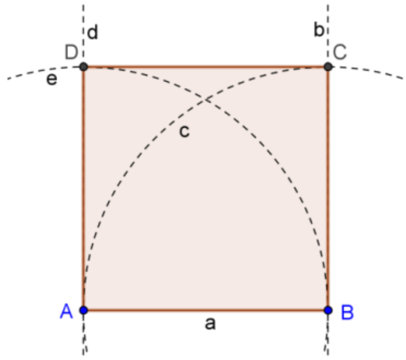
In this section you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction of the square:

Segment	Polygon
Perpendicular Line	Show / Hide Object
Circle with Center through Point	Move
Intersect	

<u>Hint:</u> You might want to have a look at the link to the dynamic worksheet "Square Construction" http://www.geogebratube.org/student/m25910 if you are not sure about the construction steps.

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* – *Geometry*.
- Change the labeling setting to *New Points Only* (menu *Options* – *Labeling*).



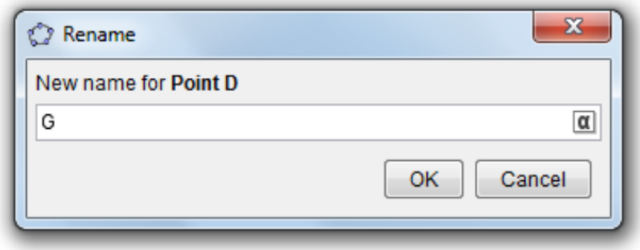
Construction Steps

1		Draw the segment *a* = *AB* between points *A* and *B*.
2		Construct a perpendicular line *b* to segment *AB* through point *B*.
3		Construct a circle *c* with center *B* through point *A*.
4		Intersect the perpendicular line *b* with the circle *c* to get the intersection points *C* and *D*.
5		Construct a perpendicular line *d* to segment *AB* through point *A*.
6		Construct a circle *e* with center *A* through point *B*.
7		Intersect the perpendicular line *d* with the circle *e* to get the intersection points *E* and *F*.
8		Create the polygon *ABCE*. <u>Hint</u>: Don't forget to close the polygon by clicking on point *A* after selecting point *E*.

9	Hide circles and perpendicular lines.
10	Perform the drag test to check if your construction is correct.
11	Enhance your construction using the *Stylebar*.

Challenge: Can you come up with a different way of constructing a square?

Hint: To rename an object quickly, click on it in *Move* mode and start typing the new name on the keyboard to open the *Rename* dialog.



2. Regular Hexagon Construction

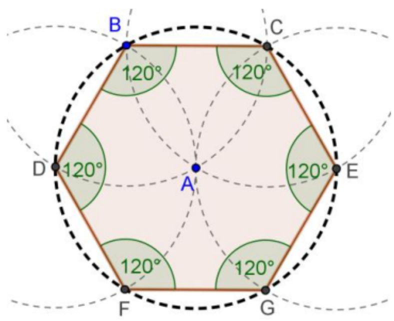
In this section you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction of the hexagon:

Circle with Center through Point	Angle
Intersect	Show / Hide Object
Polygon	Move

Hint: You might want to have a look at the link to the dynamic worksheet "Regular Hexagon Construction" http://www.geogebratube.org/student/m25912 if you are not sure about the construction steps.

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives – Geometry*.
- Change the labeling setting to *All New Objects* (menu *Options – Labeling*).



Construction Steps

1	Draw a circle *c* with center *A* through point *B*.
2	Construct a new circle *d* with center *B* through point *A*.
3	Intersect the circles *c* and *d* to get the hexagon's vertices *C* and *D*.
4	Construct a new circle *e* with center *C* through point *A*.
5	Intersect the new circle *e* with circle *c* in order to get vertex *E*. Hint: Selecting circle *e* and circle *c* creates both intersection points. If you just want a single intersection point, click on the intersection of the two circles directly.
6	Construct a new circle *f* with center *D* through point *A*.
7	Intersect the new circle *f* with circle *c* in order to get vertex *F*.
8	Construct a new circle *g* with center *E* through point *A*.
9	Intersect the new circle *g* with circle *c* in order to get vertex *G*.
10	Draw hexagon *FGECBD*.
11	Hide the circles.
12	Display the interior angles of the hexagon.
13	Perform the drag test to check if your construction is correct.

Challenge: Try to find an explanation for this construction process.
Hint: Which radius do the circles have and why?

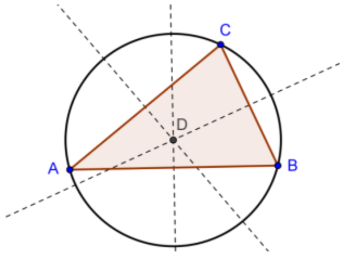
3. Circumcircle of a Triangle Construction

In this section you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction:

Polygon	Circle With Center through Point
Perpendicular Bisector **New!**	Move
Intersect	

Hint: If you are not sure about the construction steps, you might want to have a look at the link to the dynamic worksheet "Circumcircle of a Triangle Construction" http://www.geogebratube.org/student/m25916.

Preparations



- Open a new GeoGebra window.
- Switch to *Perspectives* – *Geometry*.
- Change the labeling setting to *New Points Only* (menu *Options – Labeling*).

Introduction of new tool

Perpendicular Bisector ***New!***

Hints: Don't forget to read the Toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

Construction Steps

1		Create an arbitrary triangle *ABC*.
2		Construct the perpendicular bisector for each side of the triangle. Hint: The tool *Perpendicular Bisector* can be applied to an existing segment.
3		Create intersection point *D* of two of the line bisectors. Hint: The tool *Intersect* can't be applied to the intersection of three lines. Either select two of the three line bisectors successively, or click on the intersection point and select one line at a time from the appearing list of objects in this position.
4		Construct a circle with center *D* through one of the vertices of triangle *ABC*.
5		Perform the drag test to check if your construction is correct.

Back to school…

Modify your construction to answer the following questions:

1. Can the circumcenter of a triangle lie outside the triangle? If yes, for which types of triangles is this true?
2. Try to find an explanation for using line bisectors in order to create the circumcenter of a triangle.

6. Exploring Parameters of a Quadratic Polynomial

Back to school…

In this activity you will explore the impact of parameters on a quadratic polynomial. You will experience how GeoGebra could be integrated into a 'traditional' teaching environment and used for active and student-centered learning.

Follow the construction steps of this activity and write down your results and observations while working with GeoGebra. Your notes will help you during the following discussion of this activity.

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* – *Algebra & Graphics*.

Construction Steps

1		Type `f(x) = x^2` into the *Input Bar* and hit the *Enter* key. Task: Which shape does the function graph have?
2		Click on the polynomial in the *Algebra View*.

3	↑ ↓	Use the ↑ up and ↓ down arrow keys. Task: How does this impact the graph and the equation of the polynomial?
4		Again click on the polynomial in the *Algebra View*.
5	← →	Use the ← left and → right arrow keys. Task: How does this impact the graph and the equation of the polynomial?
6		Double-click the equation of the polynomial. Use the keyboard to change the equation to `f(x) = 3 x^2`. Task: How does the function graph change? Repeat changing the equation by typing in different values for the parameter (e.g. 0.5, -2, -0.8, 3).

Discussion

- Did any problems or difficulties concerning the use of GeoGebra occur?
- How can a setting like this (GeoGebra in combination with instructions on paper) be integrated into a 'traditional' teaching environment?
- Do you think it is possible to give such an activity as a homework problem to your students?
- In which way could the dynamic exploration of parameters of a polynomial possibly affect your students' learning?
- Do you have ideas for other mathematical topics that could be taught in similar learning environment (paper worksheets in combination with computers)?

7. Using Sliders to Modify Parameters

Let's try out a more dynamic way of exploring the impact of a parameter on a polynomial `f(x) = a * x^2` by using sliders to modify the parameter values.

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* – *Algebra & Graphics*.

Construction Steps

1	Create a variable `a = 1`.

2		Display the variable *a* as a slider in the *Graphics View*. Hint: Click on the symbol ○ next to number *a* in the *Algebra View*. Change the slider value by dragging the appearing point on the line with the mouse.
3		Enter the quadratic polynomial `f(x) = a * x^2.` Hint: Don't forget to enter an asterisk * or space between `a` and `x^2`.
4	a=2	Create a slider *b* using the *Slider* tool Hint: Activate the tool and click on the *Graphics View*. Use the default settings and click *Apply*.
5		Enter the polynomial `f(x) = a * x^2 + b.` Hint: GeoGebra will overwrite the old function *f* with the new definition.

Tips and Tricks

- **Name a new object** by typing in `name =` into the *Input Bar* in front of its algebraic representation.
  Example: `P = (3, 2)` creates point *P*.
- **Multiplication** needs to be entered using an asterisk or space between the factors.
  Example: `a*x` or `a x`
- **GeoGebra is case sensitive**! Thus, upper and lower case letters must not be mixed up.
  Note:
  - Points are always named with upper case letters.
    Example: `A = (1, 2)`
  - Vectors are named with lower case letters.
    Example: `v = (1, 3)`
  - Segments, lines, circles, functions… are always named with lower case letters.
    Example: circle `c: (x – 2)^2 + (y – 1)^2 = 16`
  - The variable *x* within a function and the variables *x* and *y* in the equation of a conic section always need to be lower case.
    Example: `f(x) = 3*x + 2`
- If you want to use an **object within an algebraic expression** or command you need to create the object prior to using its name in the *Input Bar*.
  Examples:
  - `y = m x + b` creates a line whose parameters are already existing values *m* and *b* (e.g. numbers / sliders).
  - `Line[A, B]` creates a line through existing points *A* and *B*.

1. Parameters of a Linear Equation

In this activity you are going to use the following tools, algebraic input and commands. Make sure you know how to use them before you begin with the actual construction.

Slider	Intersect
`a: y = m x + b`	Slope New!
Segment	Move
`Intersect[a, yAxis]`	Delete

Hint: You might want to have a look at the link to the dynamic worksheet "Parameters of a linear equation" http://www.geogebratube.org/student/m25968 first.

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* - *Algebra & Graphics*.

Construction Step 1

Enter: `a: y = 0.8 x + 3.2`

Tasks

- Move the line in the *Algebra View* using the arrow keys. Which parameter are you able to change in this way?
- Move the line in the *Graphics View* with the mouse. Which transformation can you apply to the line in this way?

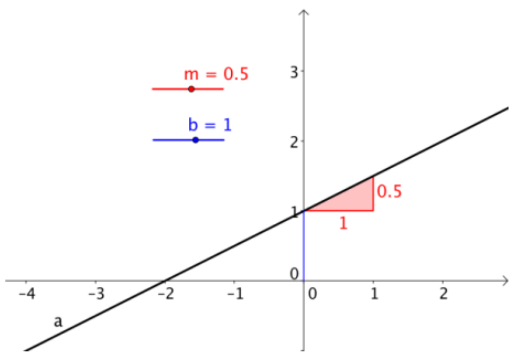
Introduction of new tool

Slope **New!**

Hints: Don't forget to read the Toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

Construction Steps 2

1	Delete the line created in construction step 1.
2	Create sliders *m* and *b* using the default settings of sliders.
3	Enter `a: y = m x + b`.
4	Create the intersection point *A* between the line a and the *y*-axis. Hint: You can use the command `Intersect[a, yAxis]`.
5	Create a point *B* at the origin.
6	Create a segment between the points *A* and *B*. Hint: You might want to increase the line thickness make the segment visible on top of the *y*-axis.
7	Create the slope (triangle) of the line.
8	Hide unnecessary objects. Hint: Instead of using this tool, you can also click on the appropriate symbols in the *Algebra View* as well.
9	Enhance the appearance of your construction using the *Stylebar*.



Task

Write down instructions for your students that guide them through examining the influence of the equation's parameters on the line by using the sliders. These instructions could be provided on paper along with the GeoGebra file.

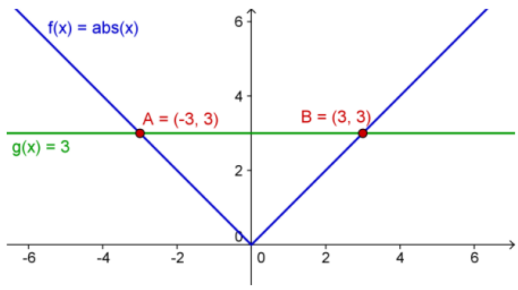
2. Library of Functions – Visualizing Absolute Values

Apart from polynomials there are different types of functions available in GeoGebra (e.g. trigonometric functions, absolute value function, exponential function). Functions are treated as objects and can be used in combination with geometric constructions.

<u>Note</u>: Some of the functions available can be selected from the menu next to the *Input Bar*. Please find a complete list of functions supported by GeoGebra in the GeoGebra Wiki (http://wiki.geogebra.org/en/).

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* – *Algebra & Graphics*.



Construction Steps

1		Enter the absolute value function `f(x) = abs(x)`.
2		Enter the constant function `g(x) = 3`.
3		Intersect both functions. <u>Hint:</u> You need to intersect the functions twice in order to get both intersection points.

<u>Hint</u>: You might want to close the *Algebra View* and show the names and values as labels of the objects.

Back to school…

(a) Move the constant function with the mouse or using the arrow keys. What is the relation between the *y*-coordinate and the *x*-coordinate of each intersection point?
(b) Move the absolute value function up and down either using the mouse or the arrow keys. In which way does the function's equation change?
(c) How could this construction be used in order to familiarize students with the concept of absolute value?
<u>Hint</u>: The symmetry of the function graph indicates that there are usually two solutions for an absolute value problem.

3. Library of Functions – Superposition of Sine Waves

Excursion into physics

Sound waves can be mathematically represented as a combination of sine waves. Every musical tone is composed of several sine waves of form $y(t) = a\,\text{sine}(\omega t + \varphi)$.

The amplitude *a* influences the volume of the tone while the angular frequency *ω* determines the pitch of the tone. The parameter *φ* is called phase and indicates if the sound wave is shifted in time.

If two sine waves interfere, superposition occurs. This means that the sine waves amplify or diminish each other. We can simulate this phenomenon with GeoGebra in order to examine special cases that also occur in nature.

Preparations

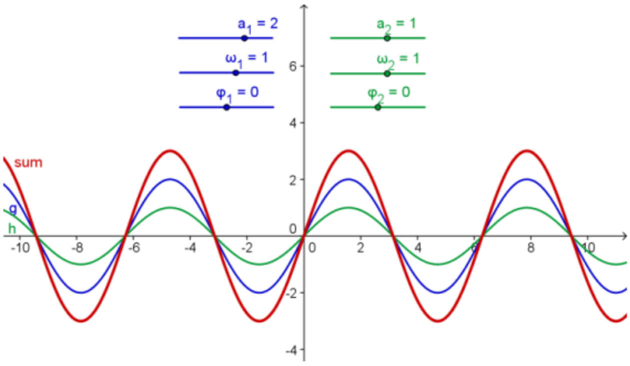
- Open a new GeoGebra window.
- Switch to *Perspectives* – *Algebra & Graphics*.

Construction Steps

1	a=2	Create three sliders *a_1*, *ω_1* and *φ_1*. Hints: *a_1* produces an index. You can select the Greek letters from the menu α next to the text field *Name* in the *Slider* dialog window.
2		Enter the sine function `g(x)=a_1 sin(ω_1 x + φ_1)`. Hint: Again, you can select the Greek letters from a menu next to the text field *Name*.
3	a=2	Create three sliders *a_2*, *ω_2* and *φ_2*. Hint: Sliders can be moved when the *Slider* tool is activated.
4		Enter another sine function `h(x)=a_2 sin(ω_2 x + φ_2)`.
5		Create the sum of both functions `sum(x) = g(x) + h(x)`.
6		Change the color of the three functions so they are easier to identify.

Back to school…

(a) Examine the impact of the parameters on the graph of the sine functions by changing the values of the sliders.



(b) Set $a_1 = 1$, $\omega_1 = 1$ and $\varphi_1 = 0$. For which values of a_2, ω_2 and φ_2 does the sum have maximal amplitude?
Note: In this case the resulting tone has the maximal volume.

(c) For which values of a_2, ω_2, and φ_2 do the two functions cancel each other?
Note: In this case no tone can be heard any more.

4. Introducing Derivatives – The Slope Function

In this activity you are going to use the following tools, algebraic input, and commands. Make sure you know how to use them before you begin with the actual construction.

`f(x) = x^2/2 + 1`	`S = (x(A), m)`
Point	Segment
Tangents	Move
`m = Slope[t]`	

Hint: You might want to have a look at the link to the dynamic worksheet "Introducing Derivatives - The Slope Function" http://www.geogebratube.org/student/m25969 first.

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* - *Algebra & Graphics*.

Introduction of new tool

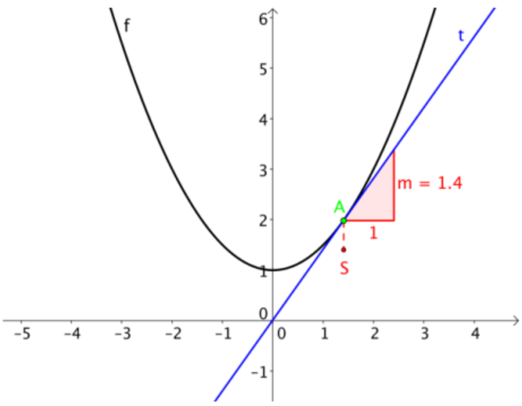
Tangents	**New!**
Hint: Click on a point on a function and then on the function itself.	

Hints: Don't forget to read the Toolbar help if you don't know how to use the tool. Try out the new tool before you start the construction.

Construction Steps

1		Enter the polynomial `f(x) = x^2/2 + 1`.
2		Create a new point *A* on function *f*. Hint: Move point *A* to check if it is really restricted to the function graph.
3		Create tangent *t* to function *f* through point *A*.
4		Create the slope of tangent *t* using: `m = Slope[t]`.
5		Define point *S*: `S = (x(A), m)`. Hint: `x(A)` gives you the *x*-coordinate of point *A*.
6		Connect points *A* and *S* using a segment.

Back to school…



(a) Move point *A* along the function graph and make a conjecture about the shape of the path of point *S*, which corresponds to the slope function.

(b) Turn on the trace of point *S*. Move point *A* to check your conjecture.
Hint: Right-click point *S* (MacOS: *Ctrl*-click) and select *Trace on*.

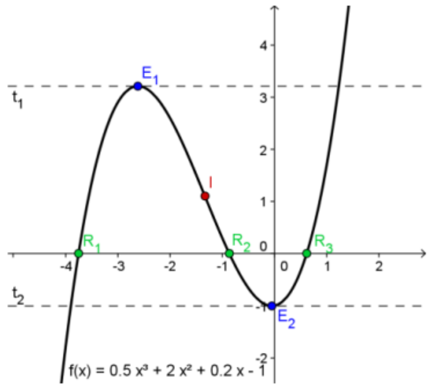
(c) Find the equation of the resulting slope function. Enter the function and move point *A*. If it is correct the trace of point *S* will match the graph.

(d) Change the equation of the initial polynomial *f* to produce a new problem.

5. Exploring Polynomials

Preparations

- Open a new GeoGebra window.
- Switch to *Perspectives* – *Algebra & Graphics*.



Construction Steps

1	Enter the cubic polynomial `f(x) = 0.5x^3 + 2x^2 + 0.2x - 1`.
2	Create the roots of polynomial *f*: `R = Root[f]` Hint: If there are more than one root GeoGebra will produce indices for their names if you type in `R =` (e.g. R_1, R_2, R_3).
3	Create the extrema of polynomial *f*: `E = Extremum[f]`. English UK: Create the turning points of polynomial *f*: `E = TurningPoint[f]`.
4	Create tangents to *f* in E_1 and E_2.
5	Create the inflection point of polynomial *f*: `I = InflectionPoint[f]`.

Hint: You might want to change properties of objects (e.g. color of points, style of the tangents, show name and value of the function).

6. Exporting a Picture to the Clipboard

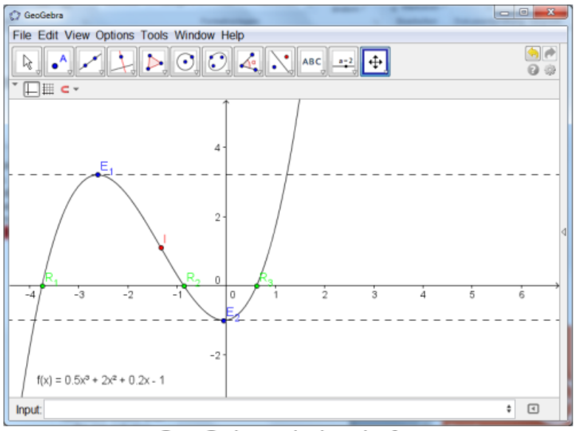
GeoGebra's *Graphics View* can be exported as a picture to your computer's clipboard. Thus, they can be easily inserted into text processing or presentation documents allowing you to create appealing sketches for tests, quizzes, notes or mathematical games.

GeoGebra will export the whole *Graphics View* into the clipboard. Thus, you need to make the GeoGebra window smaller in order to reduce unnecessary space on the drawing pad:

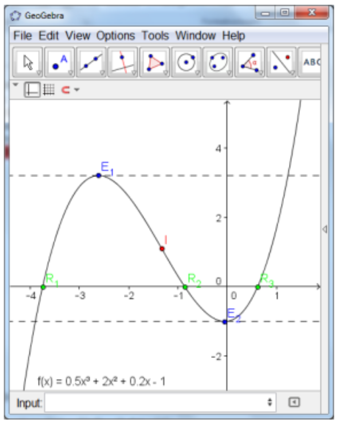
- Move your figure (or the relevant section) to the upper left corner of the *Graphics View* using the *Move Graphics View* tool (see left figure below).

Hint: You might want to use tools *Zoom in* and *Zoom out* in order to prepare your figure for the export process.

- Reduce the size of the GeoGebra window by dragging its lower right corner with the mouse (see right figure below).
  Hint: The pointer will change its shape when hovering above an edges or corner of the GeoGebra window.



GeoGebra window before the size reduction



GeoGebra window after the size reduction

Use the *File* menu to export the *Graphics View* to the clipboard:

- *Export – Graphics View to Clipboard*
  Hint: You could also use the key combination *Ctrl – Shift – C* (MacOS: *Cmd – Shift – C*).
- Your figure is now stored in your computer's clipboard and can be inserted into any word processing or presentation document.