Interplay of dynamical and structure effects in the observables for $^{12}$C$(p, 2p)$ near 400 MeV with polarized and unpolarized beams

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Abstract

We investigate the interplay of reaction mechanism and structure effects in the calculated cross sections and polarization observables for the direct $^{12}$C$(p, 2p)$ reaction near 400 MeV around the Quasi Free Scattering (QFS) kinematical condition. We do the first consistent comparison between the scattering observables obtained from solutions of three-body Faddeev/Alt-Grassberger-Sandhas (F/AGS) equations and from the Distorted-Wave Impulse Approximation (DWIA). We explore structure effects on the calculated observables, making use of one-nucleon spectroscopic overlaps obtained from the quantum Monte Carlo (QMC) many-body wave functions and using a Woods-Saxon parametrization with parameters adjusted to experimental $(p,2p)$ data. We show, for the first time, that the two reaction formalisms exhibit a distinct behavior depending upon the kinematic conditions. We also show that the agreement between the experimental data and the theoretical results depends on the reaction formalism, kinematical conditions and optical model parametrizations in addition to the spectroscopic factors (SFs). The agreement between the data and predictions using QMC wave functions diminishes prominently for transitions to excited states of $^{11}$B.

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1. Introduction

Particular attention has been devoted to the study of the knock-out of a nucleon from a nucleus by the collision with a proton. The reactions where a proton probe, used as target or projectile in inverse and direct kinematics, respectively, interacts with a A-body nucleus, have been effectively described as a three-body process, where the (A-1)-body residue (or core) is treated as a spectator (inert) system during the collision process. This reduction of the degrees of freedom of the many-body problem has become a standard paradigm and makes these $(p,pN)$ reactions appealing for both theoretical and experimental investigation.

A plethora of formulations for the three-body problem has been developed, like the distorted-wave Born approximation (DWBA) [1], the Faddeev/Alt-Grassberger-Sandhas (F/AGS) framework [2,3] and the distorted-wave impulse approximation (DWIA) [4,5].

The DWIA reaction formalism has been extensively used to analyze quasifree scattering (QFS) processes such as $(p,pN)$ [6–9] or $(p,p\alpha)$ [10] reactions at both medium and high energies. The F/AGS was also used in several exploratory studies of $(p,pN)$ reactions [11–13]. A common structure input for these formalisms is the one-nucleon spectroscopic overlap which is formally defined as the inner product between the wave function of the A parent nucleus and the fully antisymmetrized $(A−1)$ residue plus the knockout nucleon wave function. The importance of including many-body degrees of freedom in the scattering problem has been addressed recently. Many-body ab initio wave functions were used in the evaluation of the one-nucleon spectroscopic overlaps, lead-
ing to the understanding of the \( (p, pN) \) reactions with light systems as a function of the nucleon separation energy [14]. Generalizations of standard formulations of the three-body problem have been made to include core degrees of freedom in the scattering dynamics of the F/AGS [15] and DWBA reaction approaches [16, 17], and were found to play an important role in some cases, in particular, in the inelastic and transfer channels.

Given the widespread use of scattering formalisms for the data interpretation, it is of timely importance to provide benchmark calculations of observables using different scattering theories over a wide range of kinematical variables. Moreover, it is relevant to investigate their ability to describe the data and assess the need to introduce many-body degrees of freedom in the reaction dynamics.

\( {^{12}\text{C}} \) nucleus is a very interesting nucleus, not only because of its astrophysical importance, but also because structure approaches beyond Mean Field Approximation have been developed such as the Simple shell model [18] and \textit{ab initio} quantum Monte Carlo (QMC) [19] to describe its structure. The one-nucleon spectroscopic overlaps needed for describing the \( {^{12}\text{C}}(p, 2p) \) reaction leading to ground and low lying states of the residue, have been evaluated using QMC many-body wave functions generated from \( NN \) and \( NNN \) forces (Argonne V18 and Urbana X potentials – AV18/UX model) [20] in the work of [14].

Recently, total cross sections and angular and energy sharing distributions and polarization observables for \( {^{12}\text{C}}(p, 2p) \) were measured at GSI [21] and RCNP [8] in inverse and direct kinematics, respectively, at the same energy. Spectroscopic factors (SFs) for the low lying states were extracted from the experimental data. The SFs were obtained from comparing the experimental data with the calculated observables using the DWIA reaction framework and simplified Woods-Saxon parametrization of the one-nucleon spectroscopic overlaps. The data obtained from RCNP explore a range of different kinematical conditions which may provide a stringent constraint to theoretical reaction formalisms. In addition, polarization observables were also measured, since these might be a sensitive tool to identify the traces of nuclear correlations and test reaction formalisms. Although the calculated observables in the work of Ref. [8] follow the general trend of the data reasonably, further improvements would be desirable.

Since we want to extract reliable structure information from the experimental data, one needs to investigate whether the reaction formalism is undermining the extracted structure information. Thus, a consistent theoretical interpretation of this data together with benchmark studies of the reaction formalism is of current interest.

In this paper, we reinterpret the \( {^{12}\text{C}}(p, 2p) \) data obtained at RCNP for the transition to the ground, first and third excited states of \( {^{13}\text{B}} \) [8].

We use one-nucleon overlaps obtained by a simplified Woods-Saxon parametrization and by QMC wave functions, which are then incorporated in the standard F/AGS and DWIA reaction frameworks. Using the same underlying pair interactions we carry out a consistent analysis of the dependence of the calculated observables on the reaction formalisms. Furthermore, we also investigate the importance of many-body effects and reaction formalisms effects on the calculated observables with polarized and unpolarized beams.

### 2. Formalism

We consider the knockout reaction \( A(a, a'b)c \), where an incident particle \( a \) knocks out a nucleon or a bound cluster \( b \) from the target nucleus \( A \) resulting in three particles \( a', b \) and \( C \) in the final state. Let us call \( \ell \) the relative orbital angular momentum between \( b \) and \( C \) and \( j \) the total angular momentum obtained coupling \( \ell \) with the spin of \( b \). The differential cross section for the transition from an initial state \( i \) to a final state \( f \) of the three-body system for given \( \ell \) and \( j \) is calculated as

\[
\frac{d^\sigma_{i \rightarrow f}}{dE_d d\Omega_d d\Omega_b} = \frac{1}{2s_a + 1} \left[ \sum_{\lambda} \frac{\lambda}{|f|^2} \right]^2 ,
\]

where \( \lambda \) represents the projections of all asymptotic spins and \( \ell \), while \( \alpha_f \) is the final state phase-space factor

\[
\alpha_f = \frac{E_a}{p_a} E_d E_b P_a' P_b = \left[ 1 + \frac{E_b}{E_c} \left( 1 - \frac{P_a \cdot (P_a - P_c)\lambda}{p_d^2} \right) \right]^{-1}.
\]

In the nonrelativistic limit this phase factor reduces to the nonrelativistic result of Ref. [22].

The three-body F/AGS reaction approach [2,3] is an exact nonrelativistic formalism for a given three-body Hamiltonian. The transition amplitudes leading to the observables are obtained simultaneously for all open channels from the solution of the integral equations as described in [12,23] and references therein. In contrast to those earlier works, in the present work the amplitudes are evaluated at the core \( C \) recoil momentum and the \( a'-b \) c.m. scattering angle calculated using relativistic kinematics as in the DWIA approach. Furthermore, the relativistic phase-space factor resembling (2) is used with the normalization preserving the consistency with the data for the two-body system. We denote this approach as the pseudo-relativistic F/AGS. For the case of the knockout of a nucleon from a nucleus due to the collision with a proton target at high energies, subtle cancellations occur between the single scattering and higher order multiple scattering terms of the outgoing proton \( N \) and the heavy fragment; they are taken correctly into account [24,25].

The three-body DWIA reaction formalism for the knockout process is described in Refs. [4,5]. Formally, it can be expressed in terms of an incomplete multiple scattering series where exact cancellations between the single scattering of the proton and the heavy fragment and some higher order multiple scattering terms are merely assumed [24,25]. In addition, in practical applications of the evaluation of the scattering amplitudes (such as for example done in the case of the widely used THREED code [4,5]) a number of additional simplifications are performed. First, the final three-body wave function is assumed to be well described by the product of two distorted waves. Second, off-shell effects are not taken into account.

These three-body reaction formalisms require the three pair interactions as the input. The NC and \( pC \) pair interactions are needed in the analysis of the \( {^{12}\text{C}}(p, 2p) \) reaction. For these, we consider the Cooper EDA01 parametrization for \( {^{12}\text{C}} \) [26]. For practical reasons we take for the \( pN \) pair interaction the realistic \( NN \) CD-Bonn potential [27], since it has been shown that the observables are not sensitive to the choice of the realistic NN interaction. We shall return to this point later.

The one-nucleon spectroscopic overlaps are calculated from the QMC many-body wave functions generated using the Argonne V18 NN and the Urbana X NNN forces (AV18/UX model) as described in [14]. The corresponding theoretical SFs are collected in Table 1 together with those of Cohen and Kurath (CK) obtained from wave functions generated within a Simple Shell Model with effective two-body interactions [18]. The one-nucleon spectroscopic overlaps normalized to unit are also obtained as solutions of the one-body Schrödinger equation with the Woods-Saxon interaction. The radius, depth and diffuseness are taken from Refs. [28, 8] in which the diffuseness is set to \( a = 0.65 \text{ fm} \) and the radius to \( r_0 = [1.35, 1.65, 1.51] \text{ fm} \) for the ground and low lying excited states \( \{3/2^-, 1/2^-, 3/2^+\} \) respectively. The experimental SFs are
extracted from the (p,2p) reanalysis [8] and listed in the same Table together with those from electron scattering and transfer [28] and (p,2p) [21] data analysis. The Table shows an uncertainty of the extracted spectroscopic information from the data of nearly 25%. In addition to this, it was also found in Ref. [6] that SFs extracted from the (p,2p) data might depend on the kinematical configuration. The sum of SFs obtained from the QMC wave functions agrees fairly well with the deduced experimental values. Nevertheless, the spectroscopic strength appears to be distributed among the low lying states differently than the deduced experimental values, or those obtained from standard Mean Field Approximations [14]. A possible reason for this may be the fact that the AV18+UX Hamiltonian predicts the splitting of 2.5 MeV between the first two 3/2\(^+\) states of the \(^{11}\)B nucleus. This splitting is smaller than the experimental value of 5 MeV and might lead to a larger (smaller) SF for the ground (third excited 3/2\(^+\)) state. Nevertheless, the sum of the SFs for these two 3/2\(^+\) states is relatively closer to the experiment value than the comparison between the individual theoretical and experimental SFs. As for the first excited 1/2\(^-\) state, it has no nearby companion: The next 1/2\(^-\) excited state is more than 10 MeV higher. Therefore, it is unclear from the structure point of view, why theoretical and experimental SFs for the 1/2\(^-\) excited state differ significantly and why the former is about three times larger than the latter. We point out that the SF prediction of Cohen and Kurath [18] for the 1/2\(^-\) state is also about three times larger than the experimental value, so this discrepancy is not unique to the QMC wave functions. We also note that, as shown in Ref. [14], the observables are sensitive to a delicate interplay between the separation energies and the rms radii of both parent and residual nuclei, with a major impact on the SFs, and not to a particular combination of the geometric parameters of the Woods-Saxon parametrization.

In the work of Ref. [8] the one-nucleon overlap functions are additionally modified by a nonlocal correction with a Perey-factor of \(\rho_{NL} = 0.85\). We shall investigate the impact of the uncertainties in the description of the one-nucleon spectroscopic overlaps later.

### 3. Results and conclusions

We calculated observables for the \(^{12}\)C(p,2p) reaction at incident energy \(E_0 = 392\) MeV/u both by solving the three-body F/AGS equations and using the DWIA formalism. In a knockout reaction we have 3 particles in the final state. In our working case, these are the proton target, the knockout proton and the heavy fragment \(^{11}\)B named as p, N and C, respectively. The four momentum conservation keeps five independent kinematic variables in the final state. However, according to the experimental RCNP setup defined in Ref. [8], the emitted particles are measured in a coplanar geometry, with the azimuthal angle between the two nucleons being 180\(^\circ\). The plane geometry reduces the number of independent kinematical variables to three, chosen as energy and two polar angles of the nucleons, \((E_p, \theta_p, \theta_N)\). Following these measurements we consider two of the kinematical conditions around the QFS condition. Using relativistic kinematics, and fixing the energy of one proton in the final state at \(E_N = 251\) MeV, the theoretical QFS
Fig. 3. Scattering observables for transitions to low-lying states of $^{11}$B using the F/AGS and DWIA scattering framework with the Wakasa one-nucleon spectroscopic overlaps from Ref. [8]. The data is taken from Ref. [8].
Fig. 4. Scattering observables for transitions to low-lying states of $^{11}$B using the F/AGS and DWIA scattering framework with the ab initio QMC one-nucleon spectroscopic overlaps from Ref. [14]. The data is taken from Ref. [8].
or core non-recoil condition corresponds to the scattering angles of the protons in the final state \( \left( \theta_p^{\text{QFS}}, \theta_p^{\text{QFS}} \right) = (31.67^\circ, 50.08^\circ) \). This kinematical point is represented at the center of the contour plot in Fig. 1. The dark solid lines represent the contour lines obtained for fixed recoil momentum of the residual nucleus and different detection angles of the two outgoing protons. Also represented in the figure is the experimental point close to the core non-recoil condition \( \left( \theta_p^{\text{exp}}, \theta_p^{\text{exp}} \right) = (32.5^\circ, 50.0^\circ) \). According to Ref. [8], the kinematics labeled K4 corresponds to the set of experimental points \( \{ E_p \text{(MeV)}, 32.5^\circ, 50.0^\circ \} \). Moreover the kinematics labeled K1 \( \{ x \times 5 \text{ in Fig. 1} \} \) corresponds to the set of experimental points \( \{ 251 \text{ MeV}, 32.5^\circ, \theta_p^\text{th} \} \).

We have estimated the model uncertainties associated with the choice of the NN pair interaction. We have found that the cross sections using the NV CD-Bonn interaction [27] and the Arndt (SP07) phase shift parametrization [8,29] are very similar. It was also shown in Ref. [14] that the cross section varies less than 1% when using CD-Bonn or AV18 NN interaction. Therefore the observables do not depend on the underlying NN interaction as long as it reproduces the NN data.

We have also estimated the uncertainties associated with the use of Perey non-locality corrections to the NC interaction as done in Ref. [8]. We have found that introducing an arbitrary nonlocal effect leads to non-negligible effects on the calculated scattering observables, and might increase the cross section up to about 20%. In what follows, we do not include the DWIA Perey non-locality correction. In addition we do not use the Darwin correction factor to the scattering waves as done in Ref. [8].

We compare now the calculated differential cross sections and analyzing powers for transitions to the ground and low lying excited states of \(^{11}\text{B}\) with the experimental data for kinematics K1 and K4. We have found that the results do not change significantly when in the kinematic sets one takes the QPS no recoil point from Ref. [8], \( \left( \theta_p^{\text{QFS}}, \theta_p^{\text{QFS}} \right) = (32.5^\circ, 50.0^\circ) \), or obtained theoretically using relativistic kinematics \( \left( \theta_p^{\text{th}}, \theta_p^{\text{th}} \right) = (31.67^\circ, 50.08^\circ) \).

For the transition to the \(^{11}\text{B}\) ground state in both K1 (angular distribution) and K4 (energy-sharing distribution) kinematics we show in Fig. 2 the observables as functions of the core recoil momentum \( p_c \). We use the Woods-Saxon parametrization for the one-nucleon spectroscopic overlaps taken from the work of Wakasa et al. [8] and the NN CD-Bonn interaction. The observables were calculated using the DWIA (left) and F/AGS (right) formalisms. As follows from the figure, both the experimental data and theoretical results as functions of the core recoil momentum are quite similar for the K1 and K4 kinematics. The quantitative agreement between the data and calculations is somehow better for the K4 kinematics and slightly worse, with up to 25% deviation, for the K1 kinematics, that exhibits also larger differences between the F/AGS and DWIA results. The departure of DWIA differential cross section results from those of F/AGS may be due to the treatment of off-shell effects, as was shown prominently at lower proton beam energies [30]. Moreover, the two reaction formalisms also predict a distinct behavior of the calculated analyzing power in both kinematics.

In Figs. 3 and 4 we compare the observables calculated using DWIA and F/AGS reaction formalisms for the low lying final states of the residual taking the radial overlaps from Wakasa et al. [8] and ab initio QMC structure information, respectively. We have found that the main difference between Fig. 3 and Fig. 4 is due to the different ways the two structure models deal with the delicate interplay between the separation energies and the rms radii of the parent and residual nuclei, which has its major impact on the SFs. Therefore, the results from Fig. 4 are consistent with the results shown in Table 1 indicating that the agreement between the data and predictions derived from QMC wave functions is quite good for the transition to the ground state but diminishes prominently for transitions to the first and third excited states of \(^{11}\text{B}\). From the comparison of Figs. 3 and 4 it also follows that, in this kinematic region, the polarization observables depend significantly on the reaction formalism. In Ref. [14] it was found that the microscopic treatment of the overlaps has its biggest effect on the evaluation of the theoretical cross section through the SFs, and thus the analyzing power are essentially independent of the structure model.

Beside the expected sensitivity to the optical potential parametrization [31], we found a clear dependence of the observables, calculated with a Woods-Saxon parametrization and QMC wave functions, on the reaction formalism and the kinematic conditions. This dependence prevents conclusions on the nuclear structure inputs at this stage.

More measurements of cross sections and spin observables at different kinematical conditions and energies would be very useful to set constraints on the reaction formalism, thus allowing the extraction of reliable information on the nuclear structure from the experimental data.

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