A PROFESSIONAL DEVELOPMENT FRAMEWORK FOR TEACHING MATHEMATICS MEANINGFULLY WITH TECHNOLOGY IN NAMIBIAN SECONDARY SCHOOLS: A DESIGN-BASED RESEARCH STUDY

by

Leena Ngonyofi Kanandjebo

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Supervisor: Dr. Christine Erna Lampen

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DECLARATION

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ABSTRACT

The aim of this Design-Based Research (DBR) was to design a professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools. The definition of the concept of meaning was modelled on Vygotsky's (1978) description of meaningful learning as that which arouses an intrinsic need and is incorporated into tasks that are relevant and necessary for life. As such, Kilpatrick, Swafford, and Findell's (2001) five strands of mathematical proficiency informed the interpretation of goals for teaching Mathematics meaningfully with technology. The study was situated within the pragmatic paradigm, with the goal to build artefacts to produce change. Three design stages were conducted.

Stage one involved a literature survey related to technology and Mathematics education, learning theories including adult professional learning, a comprehensive framework of Mathematics teaching goals, and frameworks related to professional development for teaching Mathematics with technology. The findings revealed that Technological Pedagogical and Content Knowledge (TPACK) based frameworks have no actionable guidelines for professional development and neglect the ethical dimension of professional development to teach Mathematics with technology in a developing country like Namibia. The second design stage had two phases: one, the analysis of affordances and constraints of the Namibian secondary school Mathematics curriculum in terms of teaching Mathematics meaningfully with technology; and two, had five iterative data collection cycles with technologically adept Mathematics teachers. The findings revealed that only conceptual understanding and procedural fluency are described explicitly in the Namibian curriculum. The curriculum does not provide adequate goals for the strands of strategic competence and adaptive reasoning. The objective of the development of a productive disposition is relegated to learners’ attitudes to Mathematics and does not feature explicitly in the guidelines for teaching. In terms of teaching with technology, the calculator is the only technological tool explicitly mentioned and meant for efficiency and accuracy in mathematical calculations. Further, the Namibian curriculum aligns

Mathematics teachers at the substitution level and the tasks level in terms of technology integration. In addition, the findings of Stage 2, phase 2 were that: participating teachers view technology as a tool to improve the speed and accuracy of mathematical calculations. Yet, they view teaching Mathematics meaningfully with technology as promoting strategic competence, conceptual understanding, and productive disposition. The participating teachers reported very limited access to professional development that integrates the use of technology and subject knowledge.

The findings of Design Stage 3 filled a gap in the literature on the professional development of teachers to teach Mathematics with technology, by designing a culturally responsive professional development framework with holistic and actionable design principles for goals and progression. Due to COVID-19 constraints on the study the principles and framework that are the designed products of the research, have tentative status and must be strengthened by empirical application in future research. The study recommends that the professional development of Mathematics teachers to teach Mathematics with technology should be grounded in Mathematics content didactics and should be based on the notion of meaningful learning and teaching of Mathematics which develop and change as professional developers and teachers work together to better understand the possibilities and constraints of cognitive technologies.

OPSOMMING

Die doel van hierdie ontwerpgebaseerde navorsing (OGN) was om ‘n professionele ontwikkelingsraamwerk daar te stel om Wiskunde betekenisvol te onderrig met tegnologie in Namibiese sekondêre skole. Die definisie van die begrip ‘betekenisvol’ is gemodelleer op Vygotsky (1978) se beskrywing van betekenisvolle leer as dit wat ‘n intrinsieke behoefte wek en geïnkorporeer word in take wat relevant en noodsaaklik is in die lewe. Kilpatrick, Swafford and Findell (2001) se vervlegting van vyf stringe van wiskundige vaardigheid vorm die basis vir die interpretasie van die doelwitte wat gestel word om Wiskunde betekenisvol te onderrig met tegnologie. Die studie vind plaas in ‘n pragmatiese paradigma, waar die doelwit is om artefakte te bou wat verandering teweeg sal bring. Drie ontwerpstadia is onderneem.

Stadium een het ‘n literatuurondersoek behels oor tegnologie en Wiskunde-onderrig, leerteorieë (insluitend volwasse professionele leer), ‘n omvattende raamwerk van Wiskunde onderrigdoelwitte, en raamwerke met betrekking tot professionele ontwikkeling vir die onderrig van Wiskunde met behulp van tegnologie. Die bevindinge het getoon dat ‘Technological Pedagogical and Content Knowledge’ (TPACK) gefundeerde raamwerke geen uitvoerbare riglyne bied vir professionele ontwikkeling nie, en die etiese dimensie van professionele ontwikkeling om Wiskunde te onderrig met tegnologie, afskeep in ‘n ontwikkelende land soos Namibië. Die tweede ontwerp-stadium het twee fases. Fase een behels die ontleding van voordele en beperkinge van die Namibiese sekondêre skool Wiskunde-kurrikulum met betrekking tot die betekenisvolle onderrig van Wiskunde met tegnologie. Fase twee behels vyf iteratiewe data-insamelingsiklusse met Wiskunde-onderwysers wat vaardig is met tegnologie. Die bevindinge het uitgelig dat slegs konseptuele begrip en prosedurele vaardigheid uitdruklik beskryf word in die Namibiese kurrikulum. Die kurrikulum voorsien nie voldoende doelwitte vir die komponente strategiese vaardigheid en aanpasbare beredenering nie. Die doelwit om ‘n produktiewe ingesteldheid te ontwikkel, word gerelegeer na leerders se houdings jeens Wiskunde en word nie pertinent gemeld in die onderrigriglyne nie. Met betrekking tot die gebruik van

tegnologie in onderrig, is die sakrekenaar die enigste tegnologie-instrument wat uitdruklik genoem word, en wel vir die effektiwiteit en akkuraatheid in wiskundige berekeninge. Wat onderwysers se gebruik van tegnologie in die wiskundeklas betref, beperk die Namibiese kurrikulum die gebruik tot vervanging van papiergebaseerde teks met die gee van take. Daarbenewens is die bevindinge van Stadium 2, fase 2: deelnemende onderwysers beskou tegnologie slegs as ‘n instrument om die spoed en akkuraatheid van wiskundige berekeninge te verbeter. Nogtans beskou hul die betekenisvolle onderrig van Wiskunde met tegnologie as die bevordering van strategiese vaardigheid, konseptuele begrip, en produktiewe ingesteldheid. Die deelnemende onderwysers het aangedui dat hulle baie beperkte toegang het tot professionele ontwikkeling wat die gebruik van tegnologie integreer met vakkennis.

Die bevindinge van Ontwerpstadium 3 het ‘n leemte gevul in die literatuur oor die professionele ontwikkeling van onderwysers om Wiskunde te onderrig met tegnologie, deur ‘n kultureel-sensitiewe professionele ontwikkelingsraamwerk te ontwerp met holistiese en uitvoerbare ontwerpbeginsels vir doelwitte en progressie. As gevolg van COVID-19 beperkinge op die studie, is die beginsels en raamwerk, wat die ontwerpprodukte is van die navorsing, tentatief en moet dit versterk word deur empiriese toepassing in verdere navorsing. Die aanbeveling van die studie is dat die professionele ontwikkeling van Wiskunde-onderwysers, om wiskunde met behulp van tegnologie te onderrig, gegrond moet wees in ʼn benadering tot Wiskunde inhoudsdidaktiek. Dit moet gegrond wees op die begrip van betekenisvolle leer en onderrig van Wiskunde, wat dinamies groei en ontwikkel soos wat professionele opleiers en onderwysers die moontlikhede en beperkinge van kognitiewe onderrigtegnologie saam beter leer verstaan.

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ACRONYMS

AS	Advanced Subsidiary
BETD	Basic Education Teacher Diploma
CHAT	Cultural Historical Activity Theory
EL	Experiential Learning
ETSIP	Education and Training Sector Improvement Plan
ICDL	International Computer Driver's License
ICT	Information and Communications Technology
INSTANT	In-service Training and Assistance to Namibian Teachers
JSC	Junior Secondary Certificate
MASTEP	Mathematics and Science Teachers' Extension Project
MCPD	Mathematics Continuous Professional Development
MDKT	Mathematics Digital Knowledge for Teaching
MoE	Ministry of Education
MoEAC	Ministry of Education Arts and Culture
NDP	National Development Plan
NIED	National Institute for Educational Development
NSSCO	Namibian Senior Secondary Certificate ordinary
PTK	Pedagogical Technology Knowledge
SAMR	Substitution, Augmentation, Modification and Redefinition
TPACK	Technological Pedagogical and Content Knowledge
UCLES	University of Cambridge Local Examinations Syndicate
UNESCO	United Nations Educational Scientific and Cultural Organization
ZPD	Zone of Proximal Development

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CHAPTER 1 INTRODUCTION

1.1 Introduction

This chapter introduces a design-based research study of a professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools. It begins with the context of the study, followed by an orientation on the Namibian context. Further, the chapter presents the motivation for the study, a statement of the problem, followed by the research questions, and the significance of the study. The research methodology adapted to design the framework is also presented. Toward the end, delimitations and limitations of the study, definitions of key terms, an overview of the chapters and a chapter summary are presented.

1.2 Context of the study: The concept meaningful and teaching Mathematics meaningfully

The choice of the adjective ‘meaningful’ and not the adjective ‘efficient’ to characterize the teaching and learning of Mathematics is value-laden and requires explanation. Scholars have concurred that the adjective ‘meaningful’ is unclear and not objectively defined (Skinner, 1957), dependent upon people’s views (Brownell, 1935) and used as a ‘catch-all’ phrase for justifying any pedagogical approach (Swaim, 1960). Some scholars interpreted ‘meaningful’ in relation to the rate of learning, retention, and transfer and active facilitation of knowledge (Baroody & Hume, 1991; Weaver & Suydam, 1972). Others to engagement (Kearsley & Shneiderman, 1998) and processes of acquiring knowledge and skills, as well as how one relates to ideas (Ausubel, 1960; Novak, 2011). In addition, for some ‘‘meaning is an image; for others, meaning is a response; for still others, meaning is a set of circumstances in the outside world which causes children to do something’’ (Pa, 1986, p.11). The dichotomous argument about where meaning resides is not useful because whilst humans make meaning of their activities and their environments they also impact and change them. Similarly, meaning and skills develop mutually, and the teacher plays a significant role in developing meaning (Boerst et al., 2003). The Ministry of Education, Arts and Culture (MoEAC) (2016, p.15) of Namibia noted that mathematical skills are only ‘‘meaningful’’ when they become functional life skills and are applied to the world. Pa (1986) argued that there is no consensus as to whether the term should describe something inside or outside a person.

The definition of ‘meaningful’ in this study is modelled on an interpretation of teaching writing skills to young children by Vygotsky (1978) as the researcher also described in the theoretical paper extracted from this study (Kanandjebo & Lampen, 2022, p. 118)

> ‘‘…Therefore, the issue of teaching writing in the preschool years necessarily entails a second requirement: writing must be "relevant to life"-in the same way, that we require "relevant" arithmetic. A second conclusion, then, is that writing should be meaningful for children, that an intrinsic need should be aroused in them, and that writing should be incorporated into a task that is necessary and relevant for life. Only then can we be certain that it will develop not as a matter of hand and finger habits but as a really new and complex form of speech. The third point that we are trying to advance as a practical conclusion is the requirement that writing be taught naturally’’.

Thus, teaching Mathematics meaningfully is based on reflecting on the cultural-historic status of Mathematics education, with teaching practices made relevant to learners and responsive to an intrinsic need to master Mathematics and to engage with tasks that are *necessary and relevant* in the 21st Century, with or without technology. Relevance is not a narrow requirement restricted to teaching real-life applications of Mathematics but rather includes teaching with technology to provide the means to engage with connections so that any topic or task is relevant to the wider body of Mathematics with which learners engage. Vygotsky's practical recommendation that writing be taught naturally refers to the preparation of the skill to control the hand to write as well as the skill to use writing to accomplish *communication needs*. The researcher believes that the natural need to communicate can be applied to the teaching of Mathematics meaningfully as well. Teaching that is premised on communication through Mathematics creates opportunities for natural encounters with content and articulation through cognitive technology tools.

With the advent of widespread technology, the *necessity and relevance* to life of learning content are changing. However, there is widespread agreement that the goal of teaching Mathematics with technology must be on the development of *mathematical thinking* skills (Noss & Hoyles, 1996; Pea, 1987). With the Vygotskian backing and the inclusion of 21st-century skills of communication, creativity, collaboration and critical thinking in mind, the widely adapted five strands of mathematical proficiency (Kilpatrick et al., 2001) is taken as the conceptual framework for teaching Mathematics meaningfully.

- relevance is approached through strategic competence (formulate, represent, and solve problems) when learners engage with tasks of which the relevance is premised on conceptual understanding of their relation to the larger body of Mathematics
- adaptive reasoning (logical thought, reflection, explanation, and justification) provides a natural space for communication through Mathematics
- procedural fluency (accuracy, efficiency, flexibility, appropriateness) is required to wield the mathematical thinking tool with confidence
- teaching that allows learners to see Mathematics as sensible, useful, and worthwhile
- intellectual effort (productive dispositions), evidently arouses an intrinsic need to communicate by creating, designing, and engaging in productive struggle.

The five strands of mathematical proficiency (Kilpatrick et al., 2001) are the meaning goals in the Mathematics curriculum that have rolled over, as Pa (1986) reported, even before the era that demands and uses technology in teaching. In addition, the five strands of mathematical proficiency are regarded by researchers as appropriate to capture all aspects of expertise, competence, knowledge, and facility and motivation in school Mathematics (Graven & Stott, 2012; National Research Council, 2001). Thus, they are sensible goals to posit in the endeavour of expanding teaching goals to incorporate technology. Mathematics curricula of developed countries, such as Singapore, Australia, and the United States of America (USA) have embraced mathematical goals in line with Kilpatrick et al. (2001). However, despite the widespread adoption of these mathematical proficiency strands, scholars who have linked competencies to teachers' pedagogy including Ally (2011) questioned the extent to which opportunities for developing these competencies are present in teachers' pedagogical practices. Meanwhile, Groves and Susie (2012) concluded that it requires complex changes in teachers' pedagogy. As there is an incomplete exposition of meaningful mathematical teaching (Boerst et al., 2003) so is meaningful teaching of Mathematics using technology. Consequently, it is fitting for this study to interpret and build the concept 'meaningful' around the five strands of mathematical proficiency.

The knowledge and skills teachers need to develop relate closely to those that learners need to develop throughout schooling (Tabach & Trgalová, 2019) to function effectively in the 21st century. In this study, Mathematics teachers are the main participants and their views and knowledge of meaningful teaching with technology shape all professional development interventions. Teachers' views and knowledge of teaching Mathematics meaningfully with technology are combined with document analysis protocol to develop a professional development framework for teaching Mathematics meaningfully with technology.

1.3 Orientation to the Namibia context

This section presents the Namibian context with reference to technology and teaching the Namibian secondary school Mathematics curriculum, teaching practices during the pandemic as well as professional development initiatives of Mathematics teachers.

1.3.1 Technology and teaching the Namibian secondary school Mathematics curriculum

The Namibian curriculum underwent various transformations. Before, independence, Namibia had a second-rate Cape education curriculum in which Mathematics was not regarded as universally required. Teachers were poorly trained with weak content knowledge and poor pedagogical skills (Kasanda et al., 1999). After independence, the curriculum was aligned with the University of Cambridge Local Examinations Syndicate (UCLES) but adapted for Namibia's circumstances (Iipinge & Likando, 2012), to encourage good teaching practice and setting widely recognized standards (Howarth, 1995). When the curriculum was localised in 2007, it was still benchmarked against UCLES. Mathematics was then made a compulsory subject, with emphasizing on the link between Mathematics and technology (Ministry of Education, 2009). The learner-centred teaching approach was adopted as it was believed that *'… all children can learn and develop given the right circumstances,…'* (MoE, 2009, p. 42). As part of enhancing the 'right circumstances' for developing learners, technology integration was introduced as an integral part of the learner-centred teaching approach.

Table 1.1 shows the milestone of the Namibian Mathematics curriculum transformation in terms of pedagogies and mathematical skills since independence in 1990. The summary in Table 1.1 is informed by the purpose of the study and is based on the broad curriculum document.

Table 1.1: Mathematics competencies and links to teaching with technology since independence

	Pre-conditions for successful curriculum delivery	**Numeracy core skill and competencies related to Mathematics**
After Cape Education ***(Ministry of Basic Education, 1996, p.5)***	Learner-centred approach	Promote the development of functional numeracy and mathematical thinking with a focus on: - Developing positive attitudes towards Mathematics, - Acquiring, understanding, and mastering basic number concepts, mathematical concepts, operations, and numerical notation, - Develop the ability to apply Mathematics in everyday life.
After localisation (MoE, 2009, p.11)	-Integration of Information and Communications technology (ICTs) as a tool as an integral part of the learner-centred approach - Every school is an ICT Level 2 school per the ICTs in Education policy	*Numeracy core skill:* - Create logical models for understanding and being able to think in terms of relationships of quantity, size, shape and space, and computation. - Apply numeracy skills to situations such as budgeting, simulations, and performing high-level calculations. - Understand and use mathematical language confidently and effectively. *Competencies:* estimating, approximating, measuring, calculating, tabulating, drawing graphs, charts, diagrams, shapes, and figures; using instruments; being accurate, logical; solving problems, presenting information; using mathematical language.
After curriculum revision (MoEAC, 2016, p.10)	-Integration of ICTs as a tool as an integral part of the learner-centred approach. - Every school is an ICT Level 2 school per the ICT Policy for Education (2005);	*Numeracy core skill:* - Understand and use mathematical language confidently and effectively. - Apply numeracy skills to situations such as budgeting, simulations and performing high-level calculations. - Create logical models for understanding and being able to think in terms of relationships of

	quantity, size, shape and space, and computation. *Competencies* estimating, approximating, measuring, calculating, tabulating, drawing graphs, charts, diagrams, shapes, and figures, using instruments; being accurate, logical, solving problems, presenting information; using mathematical language.

* *The name of the responsible ministry for education changes depending on its mandate*

Table 1.1 shows that after localisation, in 2007, schools were expected to be at ICT level 2 for technology integration. An ICT level 2 school is characterized by a room with ICTs, audio-visual equipment, and internet connectivity. All teachers should be at the foundation level of ICT certification, and at least two staff members should have an advanced level ICT literacy certification or a higher ICT qualification. In addition, over 20% of the communications from that school to the MoEAC should be conducted by email (MoEAC, 2016). Table 1.1 also shows that prior to the localisation of the Namibian curriculum numeracy core skills and competencies were not differentiated. Also, numeracy core skills and competencies were amended in 2007 to focus on acquiring mathematical knowledge. Although the curriculum has been revised twice, for 2007 and 2016 implementation, numeracy core skills and competencies have not changed significantly as shown in Table 1.1.

Moreover, the Namibian secondary school Mathematics syllabus (see syllabus definition in subsection 1.8 of this chapter) highlights that Mathematics: *'is more than an accumulation of facts, skills, and knowledge. The learning of Mathematics involves conceptual structures and general strategies of problem solving and attitudes towards and appreciation of Mathematics'* (National Institute for Educational Development (NIED), 2020, p.2). The teaching and learning of Mathematics strive to prepare learners to function effectively in the 21st century. Thus, aim to enable learners to:

- Further, develop their knowledge of mathematical concepts and principles, and use this knowledge for problem solving
- Devise mathematical arguments and use and present them precisely and logically
- Integrate Information Technology (IT) to enhance the mathematical experience
- Develop the confidence to apply their mathematical skills and knowledge in appropriate situations
- Develop creativity and perseverance in the approach to problem solving
- Derive enjoyment and satisfaction from engaging in mathematical pursuits, and gain an appreciation of the beauty, power, and usefulness of Mathematics
- Apply Mathematics in everyday situations and develop an understanding of the part which Mathematics plays in the world around them
- Develop the ability to apply Mathematics in other subjects, particularly science and technology
- Develop their ability to analyse problems logically, recognise when and how a situation may be represented mathematically, identify, and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem
- Experience a sufficiently wide range of mathematical topics and methods so that they can develop their appreciation of the power, elegance, and structure of the subject
- Use Mathematics as a means of communication with an emphasis on the use of clear expression (NIED, 2015, 2018, p. 5, 2020).

Although not explicitly stated in the Namibian Mathematics curriculum, the aims listed above are those encapsulated in Kilpatrick et al.'s (2001) construct of mathematical proficiency. However, there is no explicit articulation made in the curriculum guide as to how technology should be integrated to enhance the teaching of Mathematics. Hence, this study also deems it fit to analyse the Namibian secondary school Mathematics curriculum to identify affordances and constraints in terms of teaching Mathematics meaningfully with technology. The findings may influence Mathematics curriculum development and implementation.

1.3.2 Mathematics teaching in Namibia during COVID-19

In response to the demands of ensuring that teaching continues during the COVID-19 pandemic, most of the Namibian Mathematics teachers occasionally sent learning resources to learners through social media and email, without many instructions. As a result, parents took their children through educational materials and returned assessment activities as hard copies to schools for grading. Some Mathematics teachers recorded lessons and uploaded them on their newly created internet channels, whilst others created google classrooms, on which they uploaded learning materials. The variety of approaches indicates the autonomy teachers had with reference to teaching with technology, as well as with deciding whether they met the 'core objectives' set by the Namibian MoEAC. Prior to the COVID-19 pandemic, few Mathematics teachers in Namibia on a small scale had used ICT as an interface in teaching Mathematics, for example, Geometer's Sketchpad and teacher-designed videos, in their classrooms (Kanandjebo & Ngololo, 2017; Ugulu, 2019). Similarly, only a few had uploaded materials and videos on an internet-based hub (Hamilton et al., 2019; United Nations Educational Scientific and Cultural Organization (UNESCO), 2019). In all these instances, small-scale use of ICT in teaching and learning is inadequate, and the COVID-19 pandemic has proved this. The Strategic Plan of the MoEAC also highlighted that there is a need to improve the skills and competencies of educators (MoEAC, 2017). However, it does not specify further development of teachers' proficiency to teach their subjects with technology. The question thus arises regarding the existence of a professional development framework for teaching Mathematics meaningfully with technology.

1.3.3 Professional development initiatives of Mathematics teachers in Namibia

Professional development is not a new concept in the Namibian education system with two main professional development initiatives related to Mathematics teaching having taken place (Kasanda, 2015). After independence, the In-service Training and Assistance to Namibian Teachers (INSTANT) project funded by European Union (EU) ran between 1991 to 1995 (Ottevanger et al., 2005). The INSTANT project used a cascade model to contribute towards improving the effectiveness of teaching and learning Mathematics curriculum which was newly introduced (Ministry of Education and Culture, 1995). The cascade model was used with the intention that trained teachers would be agents to spread skills and knowledge to the wider Mathematics community. Later, in 2005, the EU funded the Mathematics and Science Teachers' Extension Project (MASTEP), an in-service professional development initiative. MASTEP aimed to enhance the content and pedagogical knowledge of all teachers with a Basic Education Teacher Diploma (BETD) with specialisation in Mathematics and Science.

In a study on establishing the existence and status of Mathematics Continuous Professional Development (MCPD) in Namibian schools; Kasanda (2015) reported two contradictory conclusions. One was that professional development in Mathematics had not yet taken root in Namibian schools. Another was that only informal professional development activities had taken place in the country as teachers seemed to be unaware of the benefits of MCPD. This could imply that there is a need for professional development that stimulates Mathematics teachers' imagination to teach Mathematics meaningfully. It might also imply that formalised professional development is appropriate for improving the teaching of Mathematics. However, from 2016, after the revision of the curriculum, several professional development workshops funded by the MoEAC took place. These workshops again used the cascade model to enhance Mathematics teachers' subject content knowledge in preparation for the implementation of the revised curriculum. There have been no studies conducted on the impact of the cascade professional development approach with reference to Mathematics.

In addition, to date, only one study (Kasanda, 2015) conducted on Mathematics teachers' professional development at the secondary level in Namibia, and this is before the era of the high demand for teaching with technology. Also, it focussed on establishing the existence and status of MCPD. However, Kasanda's (2015) recommendations contribute to the grounding of the need for professional development in teaching Mathematics including meaningfully with technology. That is so as it stresses the need to develop a formalised professional development framework in Mathematics.

On technology, various policy initiatives have been undertaken such as the introduction of ICT Policy in Education which aimed to help with enhancing teachers' literacy skills (Ministry of Education, 2005). Another initiative is a 15-year plan, the Education and Training Sector Improvement Plan (ETSIP) (2007) which aimed to embed ICT at all education system levels; and to integrate it as a tool for the delivery of curriculum goals and enhance the quality of teaching and learning. It also aimed to build the capacity of staff members across the education sector, including on ICT maintenance, troubleshooting, and advanced skills in the educational use of technology. The SchoolNet Namibia, a non-profit organisation, established in 2000 provided sustainable, affordable open source technology solutions, technical support, training services, empowering youths through internet and creative commons licensed educational content (Buisson, 2005; Simataa & Simasiku, 2012). Over 300 schools including educational practitioners throughout the country were reached (Simataa & Simasiku, 2012). Through SchoolNet remote parts of Namibia were provided with significantly discounted access to the internet using wireless (spread-spectrum WIFI in the ISM 2.4GHz band and now in the 2.6GHz band) (Isaacs, 2007). Reviews (Ballantyne, 2004; Buisson, 2005; Simataa & Simasiku, 2012) on the influence of SchoolNet indicated that, it had great influence in terms of technical support and training. However, it was dissolved in 2009 by its trustees. There are no data reporting the reasons for dissolution. However, earlier reports (eg. Ballantyne, 2004) pointed out that the high costs of ICT ownership in schools such as staffing, and facilities were not part of the SchoolNet model and affected the sustainable use of ICT facilities. Another reason for the dissolution of SchoolNet could be that the training courses offered focus on basic ICT skills and literacy but not on skills that schools actually need for teaching the school curriculum.

The Namibia Open Learning Network Trust (NOLNet) e-Learning Centre was established to support the development of Open and Distance Learning (ODL) in Namibia, of all the partner education bodies. The following are some of the partner institutions: Ministry of Education, Arts and Culture, Ministry of Higher and the University of Namibia (UNAM) through the Centre for Open, Distance and e-Learning (CODeL). The Trust brings together all the government funded ODL institutions in the country under one umbrella to collaborate and facilitate the sharing of resources and expertise in the ODL field. Staff members from the partner institutions have been trained on content development. The NOLNeT e-Learning committee continues to provide e-learning expertise and training to partner institutions (Namibia Open Learning Network Trust (NOLNeT), 2014). Although NOLNeT focuses on tertiary institutions, it is discussed to show that some teachers might have gained extensive ICT skills through the e-learning study mode that was used by the University of Namibia (UNAM) during the pandemic. They might have also gained significant ICT skills through ICT literacy courses and pedagogical courses during their teacher training. Similarly, in-service teachers have to some extent gained ICT skills through the online platform for teachers hosted by the University of Namibia which is aimed at building teachers' subject knowledge (Villet et al., 2020).

Moreover, a few teachers underwent an Educational Management programme which focussed on school management, administration and timetabling (referred to as the SchoolLink system) (Ministry of Education Arts and Culture, 2015). More than one hundred (103) regional master trainers-of-trainers including teacher educators and officials were trained. Regional master trainers of trainers have completed training for 559 schools up to date. The trained staff members from 559 schools were required to train all teachers at their respective schools. All regional master trainers of trainers were provided with laptops, projectors and a 4G device for internet access. Although, the SchoolLink system focuses on school management, administration and timetabling the skills acquired were necessary for ICT capacity building. The schools could also have empowered teachers' ICT skills to integrate ICT into their teaching.

In 2017 the government of Namibia in its national development plan acknowledged that a knowledge-based economy requires skills development to support and enable the full utilization of available ICTs (National Development Plan (NDP5), 2017). Also, a study carried out by the Ministry of Education Arts and Culture and UNESCO showed that Namibia has good telecommunications infrastructure and some expertise in the use of ICT for education, including e-learning, and blended learning, but have not been extensively used for teacher education and training (United Nations Educational Scientific and Cultural Organization (UNESCO), 2013). A desk research carried out by Villet et al. (2020) on technology integration found that national television, private educational institutions and radio services availed paid-up slots to broadcast lessons. Also, IT companies were enlisted to provide internet services and data bundles to teachers in order to stimulate and encourage technology integration This shows the commitment of stakeholders in the teaching and learning system.

The MoEAC has also entered into an agreement with institutions such as NAMCOL to offer International Computer Driver's License (ICDL) training. ICDL is an internationally accredited certificate aimed at raising technological competence standards. In Namibia, ICDL training focuses on certifying users to be familiar and competent in the modules on the latest or near latest Microsoft Office suites (MS Excel, Word Processing, MS PowerPoint, and MS Access), online essentials (accessing web browser and email), and computer essentials (including understanding ICT concepts, latest or near latest operating system and managing files). In 2015, 5392 teachers underwent ICDL training and only 300 received laptop incentives (Ministry of Education Arts and Culture, 2015) that are given to those who fully completed the course. In 2018 the figures of teachers trained increased from 300 to approximately 1822 teachers (Ministry of Education Arts and Culture, 2018) with available statistics that 25% of the schools are covered by broadband infrastructure (Ministry of Education Arts and Culture, 2015; NDP5, 2017). The ICDL training occurred despite the lack of facilities that most of the schools in Namibia face (Nchindo, 2019; Nendongo, 2018).

However, research in Namibia on the application of the ICDL training skills in classrooms such as by Kacelo (2018) and Kacelo et al. (2020) found more than half (53%) of teachers use computers only outside their classrooms for lesson preparation and other professional activities. Kacelo concluded by proposing that competent ICT literacy trainers should conduct administrative developments while considering varying teachers' ICT skill levels.

All initiatives discussed focused on ICT skills development not how teachers may integrate ICT skills acquired in their teaching, even the Ministry of Education Arts and Culture (2017) admitted that the use of ICT in teaching and learning is inadequate. Thus there is a need for professional development on teaching Mathematics with technology (Nchindo, 2019; Rodrigues Losada, 2012; Simataa & Simasiku, 2012). It is worth noting that professional development initiatives discussed on the use of technology are not school subject specific and thus possibilities are high that few secondary Mathematics teachers underwent training on technology. However, technologically adept Mathematics teacher participants in this study might be those who have engaged in technology related professional development activities.

1.4 Motivation for the study

Technology has become a widely used method of facilitating learning content in recent times and solidified by the COVID-19 pandemic. The pandemic forced teachers worldwide to use technology to ensure that teaching and learning continued. It brought stark attention to the affordances and constraints related to teaching with technology. It has further spurred the development of an extensive library of products for teaching and learning with technology (Mishra & Warr, 2021). There is no shortage of professionally developed mathematical software applications such as *Geogebra, Desmos, Geometers Sketchpad (GSP), Microsoft Mathematics, Gauthmath,* and *PhotoMath*. Social media platforms, such as *Facebook, Twitter, WhatsApp, Telegram,* and *YouTube*, are exploding with products by Mathematics teachers and others with a wide range of credentials who came to embrace technology affordances. As a result, classrooms are now faced with complex demands and challenges (Henriksen et al., 2021). This includes a range of top-to-bottom regulations that demands teachers to integrate technology; teachers' needs on the use of technology, and the ability to develop learners' mathematical skills through technology. Since teachers are often told to integrate technology in teaching Mathematics without the answer to 'how to implement' question (Bakker et al., 2021; Yun-Jo & Charles, 2012). Thus, teaching with technology rarely takes place in a meaningful way.

The chasm between good technology skills and meaningful, and efficient technology-mediated teaching is acknowledged widely (Tewari, 2020). However, Mishra and Koehler (2006) who conceptualised the widely used framework for Technological Pedagogical and Content Knowledge (TPACK), attest that "merely knowing how to use technology is not the same as knowing how to teach with it" (2006, p. 1033). In addition, research reports that mere knowledge of the use of computers, curriculum, theories, pedagogy, and content are no longer sufficient to plan and teach the learning content (Finger et al., 2010; Jang, 2007). Neither does the availability of technologies such as Mathematics software applications for example *Computer Algebra System, GeoGebra,* and web resources guarantee technology integration in Mathematics classrooms (Getenet, 2020; Gustafsson, 2016).

The era demands good teaching with technology, which includes promoting engagement of learners in the activities of non-routine problem solving, developing the ability to analyse, evaluate and create, and encouraging the construction of knowledge (Niess, 2006). Teaching with technology is intended to enhance learning and goes beyond substituting it in lessons at convenient times (Okojie et al., 2006). As a result, teachers need to alter their teaching approaches and processes in their classrooms. They are required to be more innovative in integrating technology in teaching and designing lessons that are in line with the technological era (Jang, 2007; National Research Council, 2001; Shulman, 1987). Also, teachers need to develop skills such as cognitive skills, meta-cognitive skills, critical thinking, and creative thinking to function effectively in the 21st century (Organisation for Economic Co-operation and Development (OECD), 2018, 2019) classrooms. They also need skills that would enable them to use technological teaching approaches to stimulate learners' inquiry and enhance mathematical experiences. Teachers' knowledge and skills must be developed through participating in professional development (Koehler & Mishra, 2009).

From the perspective of teaching Mathematics, Kilpatrick et al.'s (2001) model of five interwoven and interdependent strands of mathematical understanding was chosen as a reference. Although it is a widely acknowledged framework for the teaching and learning of Mathematics (National Research Council, 2001; Stephanus, 2014; Thames & Ball, 2010), it has however not been widely applied in teaching Mathematics with technology. The five strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. Critical engagement with this reference framework did not exclude engagement with other sources to operationalize the idea of meaningful teaching of Mathematics in this study. It is also notable that most studies centre on meaningful learning with or using technology (Howland et al., 2013; Kearsley & Shneiderman, 1998; Wiske, 2006; Wong, 2015). The assumption might be that meaningful learning emerges through meaningful teaching. Although the proposition might be valid, it is important to document practices that strive for teaching Mathematics meaningfully with technology. As teachers need to develop teaching skills and knowledge to enable them to practice effective teaching and positively facilitate learning (Rowan et al., 2002; Shulman, 1987; Villegas-Reimers, 2003) with technology.

The TPACK framework captures the knowledge teachers need to effectively integrate technology into their teaching (Garba, 2018; Getenet, 2017; Mishra & Koehler, 2006; Niess et al., 2009) cross subjects. The framework is also suitable for designing professional development interventions (Kadijevich, 2012). However, this yardstick for technology integration in the twenty-first (21st) century, TPACK framework (Garba, 2018; Gur & Karamete, 2015) cannot be used as it is for professional development of Mathematics teachers to teach meaningfully with technology. Similarly, it cannot be used as it is, for professional development concerning Mathematics education pedagogy by a technology-skilled teacher (Benson & Ward, 2013; Harris & Hofer, 2011). A professional development framework needs to develop teachers' knowledge to a scientific knowledge level (Vygotsky, 1986). Ball and Cohen (1999) attest that a comprehensive approach to professional development should be practice-based. This might imply that teachers should directly experience appropriate tasks that they will encounter when conducting the art of teaching.

The researcher believes there is a gap in how to teach Mathematics meaningfully with technology in terms of knowledge. Researchers (e.g. Getenet et al., 2016; Haßler et al., 2015) have argued the importance of context analysis to design efficient and appropriate professional development related aspects such as a framework. Thus, in the researcher's view, teachers who used technology on their own accord when face-to-face teaching became impossible, could be those who benefited from the widespread ICDL training they had likely received and were willing to apply the skills in their teaching. This provides context for studying and incorporating these teachers' views on their (a) current practices, (b) description of meaningful and teaching meaningfully with technology, (c) professional development needs, (d) knowledge necessary for teaching Mathematics meaningfully with technology, (e) aspects of teaching and learning Mathematics, that can and cannot be achieved using technology and (f) the influence of Mathematics teacher participants' participation in the study on their beliefs and views about teaching with technology.

The study added to the theoretical work of scholars such as Tabach and Trgalová (2019). It is also significant as there exist no frameworks on Mathematics teaching with technology (Chai et al., 2019; Getenet, 2015, 2020; Koehler & Mishra, 2009; Niess et al., 2009) that has conceptualized TPACK in terms of this study's description of teaching Mathematics meaningfully with technology. This study is also a quest to develop the researcher professionally in teaching Mathematics with technology especially meaningfully. Consequently, the study aims to design a professional development framework through a literature survey and research-based approaches.

1.5 Statement of the problem

Mathematics teachers fall short of proficiencies in understanding the principles and techniques required to teach Mathematics with technology (Drijvers et al., 2014; Rahayu et al., 2022; Trouche, 2016). They are also unable to adapt their teaching (Rahayu et al., 2022) and lacked the confidence to use technology during the COVID-19 pandemic (Boer & Asino, 2022). Globally, Mathematics teachers are insufficiently prepared to teach with technology (Albion et al., 2015; Getenet, 2020; Wilson, 2008). This is a situation that Agyei and Voogt (2012) as well as Benning, et al. (2018) ascribed to the lack of Mathematics-focused technological teaching knowledge and skills development. Further, based on the researcher's experience of more than eleven (11) years in teaching, professional development and teacher training observed that there is a lack of ICT knowledge integration with Mathematics subject knowledge and pedagogic knowledge after ICT training. This also created a research gap that this study attempted to fill. Thus, the study designs a professional development framework for Mathematics teachers, to teach Mathematics meaningfully with technology in Namibian secondary schools. This was done through critical engagement with the existing framework of Mathematics teaching goals (Kilpatrick et al., 2001) as a conceptualised framework for the concept meaningful teaching of Mathematics; as well as with frameworks related to professional development for teaching Mathematics with technology (Mishra & Koehler, 2006; Tabach & Trgalová, 2019; Thomas & Palmer, 2014). The literature survey formed a research focus of the first stage of design research. In the second stage, the study draws from the experiences of secondary school Mathematics teachers who showed agency to use technology in teaching Mathematics during the COVID-19 pandemic. Later, the framework is designed in stage 3.

Therefore, based on the main objective of the study to design a professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools the overarching research question and sub-research questions are presented next.

1.6 Research questions

The overarching research question is

What are the key aspects of a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology?

Sub-research questions

1. How can the aspects of meaningful teaching and learning of Mathematics as espoused by Kilpatrick et al.'s (2001) five strands of mathematical proficiency and the Namibian national curriculum be integrated with frameworks for teaching with technology (e.g., TPACK)?

   Sub-research question 1 will use Kilpatrick et al. (2001) five strands of mathematical proficiency to analyse the Namibian secondary school Mathematics curriculum as a guiding framework for teaching Mathematics; to identify affordances and constraints in terms of teaching Mathematics meaningfully with technology.

2. What technological knowledge, content knowledge and pedagogical content knowledge are considered by participating Mathematics teachers as necessary for teaching Mathematics meaningfully with technology?

   Sub-research 2 draws on the experiences of teachers who showed agency to use technology in their Mathematics teaching to inform the design process of a professional development framework. Hence, the researcher investigated their current technological pedagogical practices as well as what meaningful teaching of Mathematics and teaching of Mathematics with technology, mean to them. The researcher investigated professional development needs in relation to teaching Mathematics with technology as well as aspects of teaching and learning

Mathematics, that Mathematics teachers can and cannot achieve using technology.

3. How does Mathematics teachers' participation in the design process of a framework for teaching Mathematics meaningfully with technology influence their beliefs and views about teaching Mathematics with technology?

   Sub-research question 3 aims to investigate teachers' views about the practical value of the research engagements as participants in the design of a framework for teaching Mathematics meaningfully with technology.

1.7 Research methodology adapted to design a professional development framework

This study was qualitative, grounded within the pragmatic paradigm approach. The paradigm is suitable as it is rooted in philosophies that aim to improve and design educational practices. Specifically, the study adapted the Design-Based Research (DBR) approach. This DBR study is process-oriented, interventionist, integrative, interactive, iterative, and flexible (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003; Wang & Hannafin, 2005). The perfection of a product designed through DBR is measured by involving participants in real contexts. The product has practical value and a clear description of the design process and application (McKenney & Reeves, 2012; Nieveen, 2007; Plomp, 2013). The generic design research model by Wademan (2005) is used as a benchmark for this study's design model as it captures design features well (Nieveen, 2007). The model illustrates two complementary ‘‘successive approximations’’ of outputs of the design-based research process. According to Nieveen (2013), the two main outputs (Figure 4.2) of development design-based research are a product of practical use as an intervention and design principles.

The study adapted three main research design stages and ends with contributions to the theories (Nieveen, 2007) (Figure 4.2). The stages are:

Design stage 1: Literature survey - This stage is also known as the preliminary research stage (Plomp, 2013). This study analysed literature related to technology and Mathematics education, philosophical theories, a framework of Mathematics teaching goals, and frameworks related to professional development for teaching Mathematics with technology. This stage ended with implications for the design of the professional development framework for teaching Mathematics with technology.

Design stage 2: Context analysis – This stage is divided into two phases. In phase one the researcher analysed the content of the Namibian secondary school curriculum in terms of Mathematics teaching goals (Kilpatrick et al., 2001) by identifying affordances and constraints in terms of meaningful teaching of Mathematics with technology. In phase two the researcher co-investigated with technologically adept Mathematics teacher participants' (a) current technological pedagogical practices, (b) views and beliefs of the description of meaningful and teaching meaningfully with technology, and technology pedagogy, (c) participants' professional development needs, (d) knowledge necessary for teaching Mathematics meaningfully with technology, (e) aspects of teaching and learning Mathematics, that can and cannot be achieved using technology and (f) influence of participation on beliefs and views about teaching with technology. Phase two was carried out in five iterative phases aimed to promote research credibility, dependability, conformability, and transferability. Stage two ended with implications for the design of the professional development framework for teaching Mathematics with technology.

Design stage 3: Design – In design stage three, design principles and a professional development framework for secondary school Mathematics teachers were developed to teach Mathematics meaningfully with technology. The design principles were developed through critically engaging with literature on technology and Mathematics education

(Chapter 2), frameworks on Mathematics teaching goals, and frameworks related to professional development for teaching Mathematics (Chapter 3) (van den Akker, 1999) and incorporating them with Mathematics teacher participants' views after five iterations.

1.8 Significance of the study

The study was envisioned to be an opportunity for Mathematics teacher participants to reflect on their current technological teaching practices. Furthermore, the study process and the findings of this study:

- Provide insights into and about the teaching of Mathematics with technology meaningfully.
- Provide insights about the Namibian curriculum in relation to promoting the development of five strands of mathematical proficiency and the teaching of Mathematics with technology.
- The designed a tentative professional development framework for teaching Mathematics with technology in Namibian secondary schools has the potential to improve the conduct of professional development in other developing countries with no explicit curricula and/ or reformed curricula guidelines as well as uneven access to technology.
- The tentative design principles have the potential to guide and inform professional developers on designing holistic professional development engagement and programmes for teaching Mathematics meaningfully with technology.
- The government of Namibia through the MoEAC plans to achieve at least 65% in Mathematics performance by 2022 (National Development Plan (NDP5), 2017). The designed framework is envisioned to assist in accelerating the attainment of National development plans such as NDP5 and MoEAC strategic plan leading to the attainment of Vision 2030.

1.9 Delimitations and limitations of the study

The study intends to limit its scope to technologically adept secondary school Mathematics teachers. This implies that the findings are not generalizable to all Mathematics teachers. Research engagements were not meant to train Mathematics teachers on how to use technological tools or on Mathematics content knowledge. However, to stimulate their thinking about teaching Mathematics meaningfully with technology, and to expose them to tasks that to some extent capture teaching Mathematics meaningfully with technology. The study, however, hoped that teachers' skills and knowledge of teaching Mathematics meaningfully with technology may be heightened through their involvement in the design process. Thus, the study investigated teachers' views about the practical value of professional development research engagements as participants in the design of a framework for teaching Mathematics meaningfully with technology.

Some of the limitations of the study that must be considered when interpreting the findings, and that can provide direction for future research are:

- Internet connection might have affected Mathematics teachers' participation and involvement in virtual research engagements as sometimes the connection was poor, and this study relied hugely on access to the internet. This is so since the questionnaire was available via a link on the Stellenbosch University surveys (SUNSurveys) website, research engagements were held online, and communications were sent through emails. Hence, a WhatsApp group was created, and the researcher communicated with Mathematics teacher participants through cell phone calls and offline text messages. for further interaction to enhance data collection. Likewise, due to COVID-19 professional development research webinars took place via MS teams which required a good internet connection for better communication.

- The financial constraints might have affected the Mathematics teacher participants' participation as internet data can be costly. Mathematics teacher participants were reimbursed for the data after attending the research engagements.
- Technological tools that enhance the teaching Mathematics experience with technology were a challenge. Some teachers used cell phones to connect to the research engagement. They were also allowed to use low technology, that is pen and paper.
- Though data triangulation and an iterative approach to DBR offer trustworthy findings, concurrent data gathering and data analysis in most cases were exhaustive for a sole investigator. Thus, data were collected in stages and analysed before the next stage began to ease the pressure on the researcher.
- This study began in 2020 when the COVID-19 pandemic was at its peak this constraint the research process in terms of time the principles and framework that are the designed products of the research, have tentative status and must be strengthened by empirical application in future research.

Another limitation concerns the timeframe which might arguably still be short relative to teachers' prior experiences. It is also notable that there exist no frameworks on Mathematics teaching with technology (e.g Ball et al., 2008; Getenet, 2015, 2020; Niess et al., 2009; Tabach & Trgalová, 2019b; Thomas & Palmer, 2014) that has conceptualized the TPACK in terms of this study's description of meaningful teaching and learning of Mathematics with technology. Thus, it is entirely not possible that some of the Mathematics teachers may have had previous experience with the concept of teaching Mathematics meaningfully with technology.

1.10 Definition of key concepts

The following key terms are defined and/or explained to clarify understanding of their use in this study. The terms are arranged alphabetically:

Framework – a guiding structure for the professional development of teaching Mathematics meaningfully with technology. It outlines design principles and progression knowledge levels for the professional development of teaching Mathematics meaningfully with technology.

Mathematics Curriculum refers to the intended curriculum which encompasses policies, guidelines and principles that guide the teaching of secondary school Mathematics. The Mathematics curriculum in this study refers to all Mathematics curriculum documents including the syllabus that guide the conduct of teaching and learning Mathematics.

Meaningful curriculum is defined through Vygotsky's lens as that, that is relevant to life, arouses intrinsic need, is incorporated in tasks that are necessary and relevant for life, and beyond application but relevant to the wider body of Mathematics.

Meaningful teaching is everything relevant and necessary for life that teachers should communicate through strategic competence, conceptual understanding adaptive reasoning procedural fluency, productive dispositions, creativity, and collaboration with technology. It should challenge the learners and leads to the development of scientific knowledge.

Professional development is nurturing and enhancing knowledge and skills to promote meaningful teaching with technology.

Mathematics is the junior secondary level Mathematics, Mathematics ordinary level and Mathematics Advanced Subsidiary (AS) level of the Namibian schooling system.

Syllabus is a subject specific document that outlines the intended learning content, specific objectives, and general objectives that teachers should achieve in their conduct of teaching and learning. The syllabus consists of the intended curriculum that is to a larger extent enacted in schools. The syllabus is a frame of reference for communicating Mathematics content knowledge to the learners.

Technology includes technological tools and techniques that ease and power new ways of achieving educational goals beyond the use of a scientific calculator and beyond achieving basic mathematical procedures such as four basic operations. Technology also refers to digital technologies such as computers, laptops, cell phones, software programs, meeting conferencing software such as *MS teams* and *ZOOM*, and Mathematics software programs such as *GeoGebra*. In this study when presenting own line of thinking the researcher uses the term technology when discussing literature which refers technology to as digital tools or ICT or tools or artefacts, and then the researcher uses that.

Low- technology – use of pen and paper incorporated with technology being used to perform tasks such as communication, and four basic operation calculations.

1.11 Overview of chapters

The study is organized into 8 chapters followed by a list of references and addenda. The addenda consist of letters and communications with all involved in the study, online questionnaires, focus group discussion questions, and document analysis protocol. The addenda also consist of the professional development research tasks development guide.

Chapter 1 has introduced the study. This was done by providing the context of the concept meaningful and orientation to the Namibian context in terms of technology and teaching, Mathematics teaching as well professional development initiatives on teaching Mathematics. The motivation for the study and gaps in the existing frameworks were introduced. Research questions were presented, and the overall methodology of the study is outlined in this chapter. The significance of the study, delimitation, and limitation as well as the definition of key concepts were outlined.

Chapters 2 and **3** review literature related to design stage one. Chapter 2 presents literature relating to technology and Mathematics education. Chapter 3 reviews the literature on learning theories related to the study's research questions. It also presents a comprehensive framework of Mathematics teaching goals and frameworks related to professional development for teaching Mathematics with technology. These chapters each end with implications for the design of a professional development framework for teaching Mathematics meaningfully with technology.

Chapter 4 discusses the overarching design-based research as a methodology that was adapted to design the professional development framework for teaching Mathematics meaningfully with technology.

Chapters 5 and **6** present the data and discuss the findings of design stage two phases. Chapter 5 presented and discussed data for stage two phase one on the analysis of affordances and constraints of the Namibian secondary school Mathematics curriculum in terms of teaching Mathematics meaningfully with technology. Chapter 6 presents and discusses the data based on design stage two phase two, which draws from the experiences of secondary school Mathematics teachers who showed agency to use technology in teaching Mathematics during the COVID-19 pandemic.

Chapter 7 describes the design of the professional development framework for teaching Mathematics meaningfully with technology as well as the design principles which are the products of this design study.

Chapter 8 provides a summary of the thesis with contributions of the thesis to theories. It also provides the conclusion, contribution of the thesis to theories, recommendations for future research and framework development, and recommendations for practice.

1.12 Summary

This chapter presented the concept of teaching Mathematics meaningfully. It also presented the context of the study, followed by a discussion on technology and teaching of Mathematics curriculum. The chapter also gave an overview of professional development on technology and teaching Mathematics in Namibia. The chapter also presented the motivation of the study, statement of the problem, research questions, research methodology adapted in the study, significance of the study, delimitations and limitations, and definition of key concepts. Finally, the chapter presented an overview of the chapters in this thesis.

The following chapter (chapter 2) presents reviews of literature related to technology and Mathematics education.

DESIGN STAGE 1

Design stage one is presented in two chapters. Chapter 2 presents literature relating to technology and Mathematics education. Chapter 3 presents literature on theoretical frameworks underpinning the study. Each of these chapters ends with implications for the design of a professional development framework for teaching Mathematics meaningfully with technology.

CHAPTER 2 TECHNOLOGY AND MATHEMATICS EDUCATION

2.1 Introduction

Research into technology has gained pace in recent years, especially since the outbreak of the pandemic. Recent years demand skilled use of technology in Mathematics education to enhance schooling experience, and the acquisition of skills to ensure the continuation of activities such as schooling and research studies. This chapter thus presents literature addressing research question number one (section 2.5.1) and research question number two (sections 2.2, 2.3, 2.4, and 2.5). The first section (section 2.2) gives an overview of the transition from traditional teaching approaches to technological approaches in Mathematics education to explain current practices. Section 2.3 discusses literature related to the influence of teachers' participation in the design process on their beliefs and views about teaching Mathematics with technology. Towards the end of the chapter, literature on professional development needs of teachers concerning teaching with technology (section 2.4) is presented. Before the chapter summary (section 2.6) the chapter presents literature on the affordances and constraints of teaching Mathematics with technology (section 2.5) which aimed to determine the aspects of teaching and learning Mathematics, that Mathematics teachers can and cannot achieve using technology.

2.2 Transition from face-to-face conduct of teaching and professional development to online platforms

There are insufficient studies that thoroughly document the transition from face-to-face teaching engagements to online (Fies & Packham, 2021). Additionally, there is not so much recent research on teaching Mathematics online (Engelbrecht et al., 2020). Conversely, there is a lack of ample recent research on online professional development in Mathematics, as it is a budding phenomenon (Quinn et al., 2019). Studies (Ginsburg et al., 2004; McGraw et al., 2007; Russell et al., 2009) that are related to Mathematics, technology, and professional development, were however conducted before the high demand of transition to online modalities. Research before higher demand for the use of online teaching and professional development challenged its effectiveness and impacts (Binmohsen & Abrahams, 2020; Driscoll et al., 2012).

Technological transformation in education has been predicted long ago. In the late 1970s, Hiltz and Turoff (1978) predicted that communications through technological platforms and media would emerge, transform and disrupt individuals and human interactions. In the same period, Mathematics education placed technology applications mainly at low-level tasks of demonstration and verification of mathematical ideas in selective traditional pen and pencil teaching environments (Niess et al., 2009). Presently, technology tools have advanced and are widely available offering not only great gains but also some constraints (Drijvers, 2020b).

From 2019, without choice in most cases, the world has participated in the transformation of teaching with technologies. Educational activities (research and teaching) and professional gatherings took place fully on online modalities from face-to-face modalities (Bolton-King et al., 2022; Fies & Packham, 2021). Later in 2021, a combination of online and face-to-face modalities became prominent in many institutions. Teaching and learning online even became one of the fastest expanding fields (Philipsen, Tondeur, Pareja Roblin, Vanslambrouck & Zhu, 2019). Demand for access to technology and formations of online networks and professional grouping such as social media groups example WhatsApp groups emerged (Bolton-King, Nichols-Drew & Turner 2022; Larsen, 2019).

The shift in demand for the use of online technology challenges all educators including teachers and professional developers to broaden their technological skills and competence (Bolton-King et al., 2022). As a result, the traditional teaching community in which a teacher and the professional developers find themselves is now extended to almost infinite internet-based communities that produce innumerable artefacts, often without any qualms, as they expect to be monitored by a vast online community. Teachers are now expected to be able to create, and design technology teaching environments that would enable them to achieve teaching goals (Engeness, 2020). Teachers need to learn how to divide labour with expanded roles and how to select appropriate functional technologies (Pea, 1987) and tasks toward curriculum attainment in a technology teaching and learning environment. This demands teachers to be more than knowledgeable presenters and explainers as mathematical goals may not be achieved through learners merely imitating and practising procedures. Likewise, traditional professional developers are in the same situation as teachers; trying to work on modalities of how to deal with the impact of technology (Kanandjebo & Lampen, 2022).

2.3 Influence of Mathematics teachers' participation in the research process on their beliefs and views about teaching Mathematics with technology

Teachers' individual beliefs influence their behaviours and decisions about teaching practices (Ball et al., 2008; Campbell et al., 2014; Schoen & LaVenia, 2019). Beliefs influence teachers' cognitive processes and are likely to be predictors of their actions (Nespor, 1987; Philipp et al., 2007) including teaching practices. Further successful integration of technology in the Mathematics curricula relies on teachers' beliefs (Belbase, 2019; Schoen & LaVenia, 2019). In this section, the literature reviewed are those related to the idea of teaching with technology on teachers' views and beliefs, and online engagement. This is because the research process took place online. In addition, the researcher reviewed literature related to action research as this study involved engagement in tasks and research.

Ertmer et al., (2012) found that teachers' own beliefs and attitudes about the relevance of technology to learners' learning have a major influence on the use of technology pedagogies. Additionally, most teachers pointed out that internal factors such as support from others (personal learning networks) played key roles in shaping their technological pedagogical practices. Teachers also noted that the strongest barriers preventing other teachers from using technology were their existing attitudes and beliefs toward technology, as well as their current levels of knowledge and skills. Thus, Ertmer et al., (2012) proposed that professional development efforts should refocus on strategies for facilitating changes in teachers' attitudes and beliefs. As it influences the effectiveness of the professional development intervention. It is also worth noting that Ertmer and the team did not engage participants in any research engagement process that advocates for teaching with technology however their findings inform this study on how teachers view technology when engaged in tasks related to technology.

Moreover, in terms of conducting engagements online, Trust (2017) found that the practice empowered teachers to make changes to their teaching practices. Contrarily, McConnell et al. (2013) argued that online engagements may not be fruitful as some teachers may face challenges with software and hardware. They further added that online engagements are associated with distractions as teachers encounter distractions from their surroundings especially when they are logging in from their areas. This could mean that in a school environment where technological resources are procured mainly through a top-down approach and at a small scale at school, may hinder teaching through and/or with technological tools as well as any activity that requires technological resources. In addition, teaching through Mathematical applications that may require internet connectivity throughout may not be possible for schools. In a study, investigating learning engagement in an online environment, Zhang and Liu (2019) found that an online environment faces low teacher participation. Hence, the more valuable the online tasks are perceived by teachers the more time and energy they would invest to make contributions and be engaged.

In addition, Chrysostomou and Mousoulides (2010) investigated seventy-four Mathematics teachers' beliefs about the adoption of new technologies in a Mathematics curriculum. The researchers found that majority of the teachers held positive beliefs towards the adoption of technology however expressed concerns regarding the nature of the curriculum, organization of teaching, and effectiveness of the curriculum. The findings inform the present study, that although teachers may have positive beliefs about the innovative teaching approach; the curriculum to which the teaching approach will be applied needs to be revised to match the type of skills that need to be developed in the learners by teachers. The revision of the curricula is necessary, to inform the effectiveness of the professional development of teachers.

In a design-based research process, Mathematics teacher participants co-create interventions with the researcher to improve the teaching. This implies that teachers are validated as knowledge producers and it blurs the gap between the ''teacher participants and researchers, knowers and doers'' (Burbank & Kauchak, 2003; Cochran-Smith & Lytle, 1999, p. 22). Moreover, Manfra (2019) noted that teachers' participation in the research process situates them as learners, creates a community of practice, and promotes sustained professional learning activities. The process affords teachers an opportunity to learn how to use specific technologies situated in the context of their curricular needs (Lawless & Pellegrino, 2007). Thereby, influencing teachers' confidence in integrating the technological teaching tool in their classrooms, and highly likely to effect the change (Boer & Asino, 2022; Kubitskey et al., 2003). Notably, participation in the research process can be an extra load on the teachers and they may not be too keen to partake (Burbank &

Kauchak, 2003). Also, ongoing research on reformed and innovative teaching ideas by university-based Mathematics educators reported a gap between theory and practice due to the examination-oriented schooling system and conservative culture in school contexts (Yuan & Yang, 2020). Thus, it is challenging to bridge the gap between teaching practices and research. As a result, professional developers, university-based Mathematics educators and teachers need to work together and support each other to improve teaching practices.

In terms of teachers' professional development needs, Loucks-Horsley et al. (2010) argued that initiatives aimed to develop teachers professionally need to be sustainable. According to Loucks-Horsley et al. (2010), this can be achieved in a professional development community to ensure coherence with the subject curriculum, and provide research-based programs and resources besides focusing on building teachers' knowledge and skills. As they view the exclusion of the school component to end in partial professional development of teachers. The educational change aimed at new enhancing and enriching deep learning is almost impossible to sustain thus, sustainability has over time presented educational reformers with severe challenges (Hargreaves & Fink, 2003). Kafyulilo, et al. (2016) reported similar findings in the research study with Science and Mathematics teachers who attended a professional development program between 2010 and 2012. The report indicated that even though teachers acquired knowledge and skills through the professional development program and were positive about technology use in education, only some teachers continued to apply the technological pedagogy skills acquired. Their data further revealed that despite the challenges that all teachers in the sample encountered when using technology in their teaching (such as large classrooms, problems with electricity supply, lack of time, and lack of technology tools), the support by school management was a critical factor in teachers continued use of technology in teaching.

The present study considers sustainability in terms of skilful use of limited resources such as computers and money to secure them. The researcher argues here that little resources, that is, one computer at a school or teachers' personal smartphones can be used to promote prolonged use and enhance the use framework to teach Mathematics meaningfully with technology. Also, sustainability in this study advocates continuous professional development to accommodate organizational changes, and further skills and knowledge development. Teachers might abandon their newly developed skills if they find them "incompatible" with real teaching/learning settings (Fiszer, 2004). Thus, continuous professional development enables teachers to better comprehend their newly developed skills in light of their practices in classrooms as a key constituent of any successful plan involving the use of technology in teaching (Bradshaw, 2002; Fiszer, 2004).

Some of the literature reviewed relates to professional development interventions even though during the research design process of this study the researcher did not conduct a professional development engagement but exposed participants to tasks that informed the design of the professional development framework for teaching Mathematics meaningfully. The literature is used to inform this study of the influence of Mathematics teachers' participation in the research process on their beliefs and views about teaching Mathematics meaningfully with technology. Since the researcher anticipates that the encounters that participants had with teaching Mathematics meaningfully with technology may have influenced their views and beliefs about teaching Mathematics with technology.

2.4 Teachers' professional development needs

Teachers' knowledge relating to teaching Mathematics with technology affects the extent they engage with the technology. Appropriate professional development for Mathematics teachers on technological pedagogies is crucial if teachers are to effectively teach with technology (Jones & Moreland, 2004; Tabach & Trgalová, 2019). It is crucial to be aware of teachers' professional development needs to properly plan and design an effective professional development intervention. Various schools of thought have argued and documented different perspectives from which to approach teachers' professional development.

Some research studies concluded that teachers' professional development needs should be aligned with a specific software. Rodrigues Losada (2021) investigated Ohangwena region Mathematics teachers' learning experiences during five 2-3 hours of professional development research interventions using GeoGebra. The researcher exposed participants to different multiple representations of mathematical functions which participants were asked to explore using GeoGebra. Data were collected through semi-structured interviews focus group interviews, audiotaped discussions, observations, and field notes. The findings of the study indicated that use of technology (GeoGebra) significantly positively influenced participants' attitudes toward Mathematics. The researcher also found that teachers need more opportunities to learn to experience the pragmatic epistemologies of GeoGebra. Though the researcher used the topic of functions and sample focused on one region, this study uses GeoGebra as a benchmark to inform the design of the professional development framework for teaching Mathematics meaningfully with technology, thus Rodrigues Losada's (2021) study signals that GeoGebra has traction in Namibian schools.

Moreover, Drijvers (2020) argued that teachers need professional development on how to recognise opportunities and constraints with reference to mathematical concepts, and how to adapt teaching and learning skills to engage learners in a technology-rich learning environment. The scholar deems it appropriate since teachers step back on ‘‘teacher-driven explanations’’ when learners engage with the tools which constrain the development of mathematical skills (Drijvers, 2020, p.191). This could mean that they view technology as their ‘new best’ and could better communicate the learning content better than themselves. Trgalová and Jahn (2013) concur with Drijvers (2020) that teachers need to be able to evaluate pedagogical affordances and relevance of technological tools In addition, Wassie and Zergaw (2019) argued that teachers need prior knowledge of the syntax of programming to input some commands in GeoGebra as it may constraint them from fully exploring.

Teachers' pedagogical and content knowledge in Mathematics needs to be strengthened to be able to link Mathematics content knowledge to pedagogy and technology (Niess, 2015). Teachers need to know how to relate technology to the subject content and how to use technological software such as *Geometers' Sketchpad*, *GeoGebra,* and *Microsoft Mathematics* (Tabach & Trgalová, 2019). Moreover, teachers' professional development needs to be more strongly anchored upon the pedagogical goals of 21st-century learning, to foster 21st-century learning (Koh et al., 2017). Koh et al. (2017) views may imply that professional development should be aligned with teaching goals. Sullivan's (2012) added that teacher learning should focus on "ways of identifying tasks that can facilitate student engagement with Mathematics proficiencies" (p. 183). Leong et al. (2011)(Leong et al., 2011) further proposed that teachers' professional development programmes may include engaging teachers in re-designing the curriculum structures to stimulate a pedagogical shift. Teachers' professional development needs can also be deduced from the competencies learners need to develop to adapt to the technological era (Sarason, 1990; Tabach & Trgalová, 2019).

It is worth noting that developing teachers' knowledge and skills in integrating technology in teaching without access to technological tools lead to no integration of technology in teaching (Kafyulilo et al., 2016). Nonetheless, teachers' technological pedagogical skills need to advance to be able to teach in the computer and internet era (Garba, 2018; Koehler & Mishra, 2009). Thus, the designing of the professional development framework for teaching Mathematics meaningfully with technology should consider the ethical obligation to promote democracy, equal opportunity to learn, and promoting fairness in terms of quality teaching and learning Mathematics with technology.

2.5 Affordances and constraints of cognitive technology in teaching and learning Mathematics

This section presents literature on aspects of teaching and learning Mathematics that can and cannot be achieved using technology. The literature presented are not limited to any specific grade level and/ or specific concept in Mathematics; but focused on the technologies "that can perform mathematical tasks and/or respond to the user's actions in mathematically defined ways" (Dick & Hollebrands, 2011, p. xii). As educators' roles are being influenced by technology, they need to know the opportunities that technology affords in many folds of teaching. When technology is integrated into teaching and learning, it shares in the division of labour when it carries out some roles. The labour of recalling and practising cumbersome mathematical algorithms is now often displaced by selecting appropriate mathematical software and entering data (Pea, 1987) or setting up relationships between points on lines and planes in an interactive virtual representation system such as GeoGebra or a similar Computer-aided design (CAD) programs. Cognitive technology influence pedagogical practices in many ways. Pea (1987) argued:

> ‘‘I mean that not only do computers affect people, but people affect computers. This is true in two senses. In one sense, we all affect computers and the learning opportunities they afford students in education by how we interpret them and by what we define as appropriate practices with them. As these interpretations change over time, we change the effects the computers can have by changing what we do with them… In another sense, we affect computers when we study their use, reflect on what we see happening, and then act to change it in ways we prefer or see as necessary to get the effects we want'' (p.95).

Technologies influence and afford new ways for mathematical representations thereby influencing cognitive demand (Hollebrands, 2017). Technologies amplify and reorganise human cognitive abilities to perform Mathematics (Hollebrands, 2017; Pea, 1987). It improves precision, efficiency, and speed in comparison to performing Mathematics manually without technology; and influences mathematical representations (Hollebrands, 2017). Thus, algorithms can be carried out quickly and faster than otherwise. In addition, Ruthven and Hennessy (2002) in a design-based study to design a model to use computer-based tools to support Mathematics teaching and learning; found that technologies enable tasks to be carried out easily, rapidly, and reliably, increase engagement, and support learners' understanding. In addition, Rodrigues Losada (2021) in an analysis of Mathematics teachers' learning experiences during five 2-3 hours of professional development research intervention using GeoGebra found that it to affords ''fast and consistent feedback'' to the teachers during the intervention. Technologies restructure thinking processes by influencing how a mathematical task is perceived (Hollebrands, 2017). It affords teaching and learning opportunities to incorporate and be based on real-life problems which are beyond the confinement of the classroom environment (Trinidad, 2003). Hence, technologies provide cognitive support by engaging people to think mathematically and supporting them when they are engaged in mathematical thinking (Pea, 1987).

In a review analysis, Cullen et al. (2020) summarised that technology affords teaching and learning of Mathematics to 1) promote cycles of proof – opportunities to explore mathematical phenomena, generate hypotheses, test, review, and prove inferences. 2) Present and connect multiple representations – creating graphical representations and allowing users to link multiple representations. It 3) supports problem solving by generating a series of problem solving tasks and 4) as a tutee to communicate the learning content. Moreover, Drijvers (2020) noted that technologies offer room for designing, exploring mathematical situations, and reinventing mathematical properties. It also promotes some sense of ownership of the teaching and learning materials as users can create personalised accounts. For Hakkarainen (2009) technologies enable learners to engage in progressive enquiry that leads to expansive learning, which is characterised by the creation of new tools to engage with problem solving.

Santos-Trigo et al. (2019) analysed the affordances of dynamic software systems using GeoGebra as a reference, they noted that technology affords users to construct geometric models of problems, and to trace and communicate results. Correspondingly, GeoGebra affords teachers an opportunity to explore, construct, and move objects within the model to identify, formulate and support conjectures using geometric or algebraic arguments (Santos-Trigo et al., 2021). In this study, GeoGebra was used as a tool to aid in making inferences, aid to formulate conjectures and support them with geometric or algebraic arguments like in Santos-Trigo et al. (2019) and Santos-Trigo et al. (2021) studies. Thus, their views aid in framing teachers' views about technology and teaching Mathematics with technology.

Contrarily, in Khambari et al. (2010) teacher participants indicated that technology constrains as it drifts learners' attention from the learning content to the technological tool thus disrupting classroom teaching and learning process. Technologies have multiple affordances which make it hard to adapt (Drijvers, 2020b).

2.5.1 Frameworks for analysing technology affordances and constraints

Various framework exists to examine affordances and constraints of technology, in this thesis two research-based frameworks are reviewed as they can easily be adapted. The first one is the Substitution, Augmentation, Modification, and Redefinition (SAMR) model (Puentedura, 2010) which is used to gauge the extent to which technology transforms teaching and learning content. The second one is the pedagogical map (Pierce & Stacey, 2010) to categorise technology pedagogical affordances in terms of changes to tasks, classroom interactions, and the Mathematics subject content. The SAMR model and pedagogical map are used to help answer research question number one which is to identify affordances and constraints of the Namibian secondary school curriculum in terms of teaching with technology.

(a) The Substitution, Augmentation, Modification, and Redefinition (SAMR)

The four hierarchical SAMR models can be used to assess and categorise technological practices (Kihoza et al., 2016). It can also be used as a reflective tool of technology integration. The model has been criticised for lacking a theoretical explanation (Hamilton et al., 2016). However, several authors (Green, 2014; Hilton, 2016; Lacruz, 2018) commended it, with (Green, 2014) arguing that the model is simple to use and that it can be "…easily adapted''. For example, the SAMR has been linked to Bloom's taxonomy (Puentedura, 2014) and proficiencies (Ford, 2018) (Figure 2.1). In addition, technological integration skills required in the 21st Century can be assessed using the SAMR model (Lacruz, 2018). The SAMR levels are discussed as follows:

Substitution level: At this level, technology is directly substituted for a more traditional task and has no functional change.
Augmentation level: Here the technology is again directly substituted for a traditional role, but with significant enhancements to the student experience.
M*odification level*: this stage, is the beginning of the transformation stage in the model. Technology integration is meant to make significant changes to the teaching process.
R*edefinition level*: This is the level when technology is used to create a new educational task that would not be able to be carried out in the absence of technology (Hamilton et al., 2016). Technology use transforms learners' ability to explain, problem-solve and share their knowledge creatively and collaboratively (Ford, 2018). This level is the highest level of the SAMR model.

At the substitution and augmentation level, the curriculum advocates technology to be used to enhance pedagogies with little to no change. For example, teachers may be encouraged to use videos to explain a mathematical idea or use online or PowerPoint notes instead of the chalkboard; or use a calculator to substitute pen and paper calculations. Thus, substitution and augmentation levels are regarded as enhancement levels (Puentedura, 2010). At the modification and redefinition levels technology is suggested to be used to reform pedagogies. These levels are known as transformation levels (Puentedura, 2010).

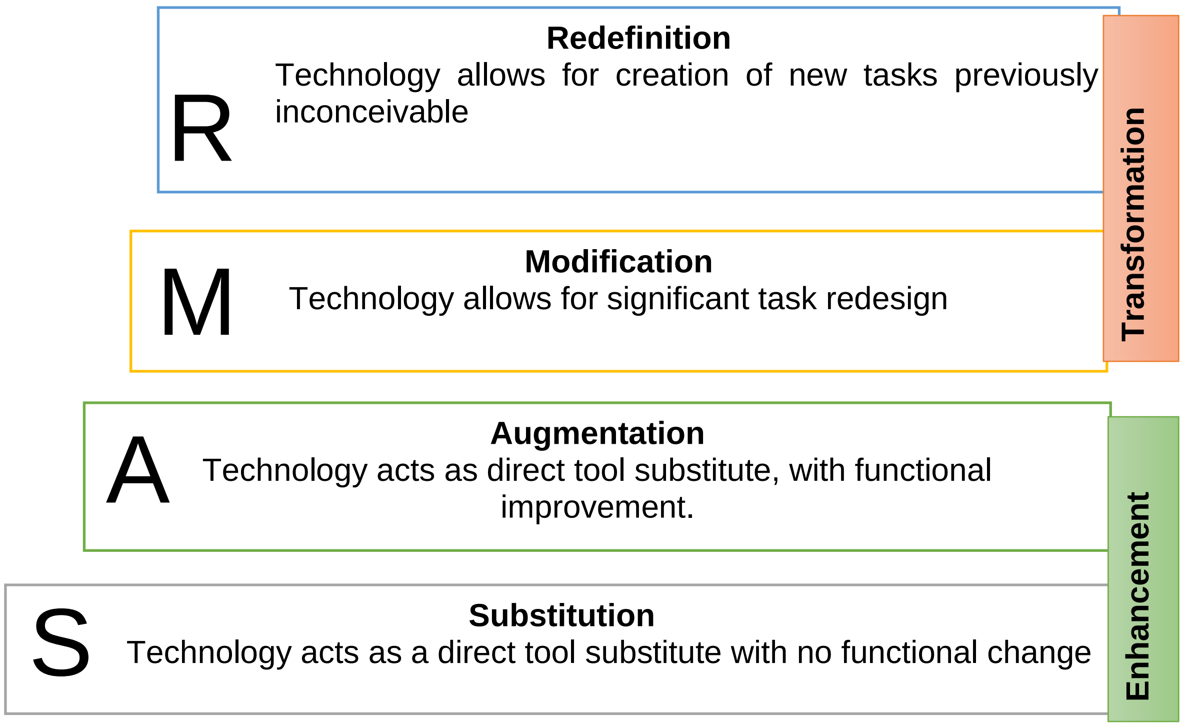

Figure 2.1: The SAMR model (Puentedura, 2010, p. 3)

(b) Pedagogical Map

The pedagogical map was developed by Pierce and Stacey (2010) to classify pedagogical opportunities afforded by a wide range of mathematical software such as computer algebra systems, calculators, dynamic geometry, or statistical packages. The map can be used to identify teachers' professional development needs and it is a catalyst for professional discussion (Goos, 2012; Pierce & Stacey, 2010). In this study, the map was used to classify pedagogical opportunities afforded by the Namibian secondary school curriculum. According to Pierce and Stacey (2010) pedagogical opportunities afforded by technological software arise at three levels:

Tasks level opportunities: at this level, mathematical tasks are enhanced by using technology; to improve speed, accuracy, and access to a variety of mathematical representations.

Classroom interaction level opportunities*:* Using technology to create opportunities that contrast with traditional classroom social dynamics and motivate a change of the didactic contract that governs students' and teachers' expectations of each other's roles.

Subject level opportunities: Using technology to provoke mathematical thinking and support new curriculum goals.

In the current study, the SAMR model and pedagogical map are used to examine pedagogical opportunities afforded by the Namibian secondary school Mathematics curriculum with reference to technology in terms of changes to tasks and mathematical thinking.

2.6 Implications for designing a professional development framework

This chapter discussed literature addressing research question number one (section 2.5.1) and research question number two (sections 2.2, 2.3, 2.4, and 2.5). It began by giving an overview of how technology influenced the changes in educational activities through the times. This was necessary to determine teachers' current practices. Literature showed that transitions in the use of technology in education have influenced a shift in teaching goals and teachers' knowledge in terms of teaching, thus a need to redefine professional development. Literature on the influence of Mathematics teachers' participation in the research process on their beliefs and views about teaching Mathematics with technology indicated that teachers may have positive beliefs and views about the innovative teaching approach; however, the curriculum onto which the teaching approach will be applied need to be revised in order to match with the type of skills that need to be developed in the learners by teachers. curricula revision is necessary to plan and design effective professional development.

In terms of teachers' professional development needs, various researchers have different views on teachers' professional development needs. Some scholars noted professional development needs with reference to knowledge of how to use certain software, the ability to do programming in relation to Mathematics and to evaluate technology affordances and constraints. Moreover, others argued that the technological skills that teachers need can be deduced from the competencies that learners need to develop and adopt in a technological era.

The last section of this chapter presented literature on aspects of teaching and learning Mathematics that can and cannot be achieved using technology. Literature noted that technology affords new ways to represent Mathematics. Technology improves precision, efficiency, visualisation, and speed in comparison to performing it manually without technology integration, Technology affords teaching and learning to incorporate real-life situations which is beyond the confinement of the classroom environment. It also offers room for designing, exploring mathematical situations, reinventing mathematical properties, and promotes some sense of ownership of the teaching and learning materials.

However, cognisance should be taken of the constraints as literature showed that teachers tend to step back to teacher-driven approaches without engaging and facilitating learning for cognitive development. Further, access to technology tools is an ethical issue because teachers have an obligation to fulfil curricula demands and objectives of teaching with technology and to ensure that all learners are taught. Moreover, literature reveals that professional development intervention should be sustainable. As a result, the design of the professional development framework for teaching Mathematics meaningfully with technology should consider ethical obligation and sustainability to promote continuity, and accommodation of all learners and to promote fairness in terms of meaningful teaching and learning Mathematics with technology.

The next chapter presents the theoretical frameworks that underpin this research study.

CHAPTER 3 THEORETICAL FRAMEWORKS UNDERPINNING THE STUDY

3.1 Introduction

This chapter discusses philosophical frameworks related to the study's research questions. The chapter is divided into two parts (part A and Part B). Part A discusses Vygotsky's sociocultural learning theory (section 3.2), third-generation Cultural Historical Activity Theory (CHAT) (section 3.2), and Experiential Learning theory (ELT) (section 3.4). These theories are compatible with one another. The theories are used as conceptual frameworks to guide the designing of design principles and professional development progression levels leading to a professional development framework for teaching Mathematics meaningfully with technology. The use of ELT was peripheral as it was only used as a theoretical guide in the design of learning tasks that were used in the research engagements. Although, learning tasks can be framed around Vygotsky's sociocultural learning theory however, ELT provides a practical and simplified theoretical foundation of designing the learning tasks for adult learning.

Part B discusses the framework of Mathematics teaching goals, known as the five strands of mathematical proficiency (Kilpatrick et al., 2001) as a conceptualised framework for the concept meaningful teaching of Mathematics. The literature presented under section 3.5 aids in answering research question number one on the affordances and constraints of Namibian secondary school curriculum in terms of Kilpatrick et al. (2001) five strands of mathematical proficiency. It also presents frameworks related to professional development for teaching Mathematics with technology. There exist different frameworks and conceptualisations of professional development for teachers to integrate technology in the teaching and learning Mathematics. Most of these frameworks are based on the construct of Technological Pedagogical Content Knowledge (TPCK) (Mishra & Koehler, 2006) which proposes integration of technological knowledge with pedagogic and subject content knowledge. Consequently, in this chapter the general and not subject related

(TPCK) (Mishra & Koehler, 2006) is discussed. Further, other frameworks related to Mathematics teaching and technology highlighted are: Pedagogical Technology Knowledge (PTK) (Thomas & Palmer, 2014), Mathematical Knowledge for Teaching with Technology (MDKT) framework (Tabach & Trgalová, 2019), and Mathematics Teacher TPACK Standards and development model (Niess, Ronau, Shafer, Driskell, Harper, Johnston, Browning, Özgün-Koca & Kersaint, 2009). Frameworks discussed in part B were used as well in chapter 6 to analyse and identify teachers' professional development needs. Critical engagement with the frameworks presented in part B helped with the planning and design of the professional development framework. The chapter concludes with implications for designing the professional development framework.

PART A

3.2 Vygotsky's sociocultural learning theory

Professional development happens in a community of teachers, never individually. In addition, professional development needs differ between cultures therefore the present study is situated in Vygotsky's socio-cultural learning theory on adult learning. According to Vygotsky's sociocultural learning theory learning is socially constructed and knowledge originates from human intelligence in culture. It further argues that learning occurs within the Zone of Proximal Development (ZPD) (Vygotsky, 1978). Learning should be integrated with practice through experiential learning in the community of practice (Lave & Wenger, 1991). It should also be contextually positioned (White, 2010), purposeful and objective. Vygotsky's notion of learning approach has been used in the learning of adults in many fields including business (Senge, 1990) and in designing professional development in education (Amolloh et al., 2018; Blair, 2016; Girvan et al., 2016; White, 1992).

In terms of knowledge, Vygotsky argues that learning should lead to the acquisition of scientific knowledge from spontaneous knowledge (everyday life knowledge) (Vygotsky, 1986). Spontaneous knowledge is grounded in everyday personal experiences without guidance. Scientific knowledge, however, results from the generalisation of human experience 'fixed' in disciplines of sciences and humanities (Karpov, 2003, p. 66). Scientific knowledge is 'fixed' in the broader sense which may imply that it is not static but well-structured. Acquisition of scientific knowledge emerges by strengthening an individual's spontaneous knowledge and it expands downwards through spontaneous knowledge, Vygotsky argues (Alves, 2014, p. 25):

> ''By forcing its slow upward trajectory, an everyday concept paves the way for a scientific concept and its descendant development. It creates a series of structures necessary for the evolution of the most primitive and elementary aspects of a concept, giving it body and vitality. Scientific concepts, in turn, provide structures for the upward development of spontaneous concepts in relation to consciousness and deliberate use by the child'' [adults].

Vygotsky's description of learning regarding a child's learning is similarly interpreted to an adult learning process in terms of professional development. This is because adult learners' learning is similarly characterised by structures and purpose which consider their preferred ways of engaging and social dimensions (Blair, 2016). Therefore implication of Vygotsky's views as quoted from Alves (2014) may mean that scientific knowledge requires other knowledge to expand. It also implies that development from the spontaneous knowledge level to the scientific knowledge level proceeds through progression levels. Scientific knowledge can also emerge from the context of related theories and frameworks (Alves, 2014).

The compositions of scientific knowledge are answers provided to the question ''what knowledge do we want the students to acquire?'' (Karpov, 2003, p. 69) and how we want them to acquire this knowledge. In this case the skills we want learners to acquire also form a reference of what the knowledgeable other should possess to be able to help the learner. Thus, the question is rephrased as 'what skills do Mathematics teachers (knowledgeable others) should have for learners to acquire the knowledge we want? In this thesis, therefore, the researcher advocates that professional development should lead to acquiring scientific knowledge. Scientific knowledge is structured, formally organised, and defined in terms of Kilpatrick et al. (2001) and technology frameworks.

Vygotsky holds that cognitive development happens through learning, and there should be development after learning, thus learning is development. The Zone of Proximal Development (ZPD) explains how intellectual capabilities can be developed through subject matter as well as through teaching (Chaiklin, 2003; Vygotsky, 1978). Vygotsky (1978, p. 86) argue that ZPD

> ''Is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under … or in collaboration with more capable peers''.

This implies the distance in terms of knowledge levels between a more knowledgeable person and a less knowledgeable person until the less knowledgeable individual becomes independently proficient. The ZPD defines functions that have not fully developed but are currently in the 'embryonic state' (Vygotsky, 1978, p. 86); which implies that it is more than just a seed and thus can already be differentiated. The embryonic state could be referenced to the ICDL training that the Ministry of Education, Arts and Culture have had provided to the teachers, and to the technology skills that teachers have acquired throughout their learning. However, these skills need to be enhanced to reach the ZPD so that teachers may integrate technology in the teaching of Mathematics.

It is worth noting that ZPD can be created in any domain skill of a task (Tharp & Gallimore, 1998) and ''expanded'' (Chaiklin, 2003, p. 41). According to Vygotsky the focus of the engagement is not on the knowledgeable other nor on the tasks used since they are means to get to the scientific knowledge level. Consequently, in the current study Mathematics teacher participants engaged in tasks (Addenda E, F) to stimulate their imaginations of teaching Mathematics meaningfully with technology. The use of tasks was necessary as Vygotsky (1978, p. 101) warns us that ‘‘separation of meaning from objects and action has different consequences, however’’. This implies that learning which is development cannot be considered as an isolated entity but embedded. Thus, learning occurs when one engages and experiences learning in the environment that learning is envisioned; that is situations that reflect teaching Mathematics meaningfully with technology.

3.3 Cultural Historical Activity Theory (CHAT)

The Cultural Historical Activity Theory (CHAT) claims that purposeful human actions are facilitated using tools. In a triangular model, referred to as first-generation activity theory, Vygotsky argued that human behaviour is mediated by and through artefacts to prompt or modulate actions, but not simply called forth by stimuli (Engeström & Miettinen, 1999; Flavin, 2017; Vygotsky, 1978). According to Vygotsky, human beings (subjects) use physical tools and abstract resources (tools) to achieve certain goals (objects) resulting in an outcome. The first-generation CHAT remained individually focused as a unit of analysis and this gave birth to the second-generation CHAT (Engeström, 2001). The second-generation activity theory argues that an action is conducted to fulfil a goal and that every activity has an object and motive (Leontiev, 1978, 1981). Later the third generation CHAT was introduced. Third-generation CHAT holds that understanding practice as meaningful and purposeful requires social mediation through tools, of how teachers think about their practices (Engeström, 2001). The third generation CHAT (Engeström, 2001) is used as a lens to compare goals, rules, and division of labour, based on the current Namibian secondary school Mathematics teaching in relation to an envisaged new activity system for teaching meaningfully with technology. Its usage is appropriate in the present study since it takes cognizance of teaching practice as socially situated (Figure 3.1) (Engeström, 2001).

The dialectic relationship between activity theory and human-technology interaction has a rich history. Already at the end of the 1900s activity theory contributed to the understanding of human-technology interaction (Engeström & Miettinen, 1999; Flavin, 2017) and to the emergence of new disciplines such as ergonomics, (Kaptelinin & Nardi, 2018). An activity system is constructed from a wider network of activities that it remains part of (Figure 3.1). Mediating artefacts can be conceptualised as both tools and signs, mediating between object, meaning-making, and the outcome of the system (Bloomfield & Nguyen, 2015). In an activity system human beings (subjects) operate within a system; use physical tools and abstract resources as mediators, in pursuit of objects (purposes), leading to the production of outcomes (Engeström, 2001; Flavin, 2017) (Figure 3.1). In an activity system, a collective journey through the ZPD is socially mediated by artefacts enabling sense-making leading to the outcome of the system. In addition, there are different roles (division of labour) and rules (which afford and constrain behaviours).

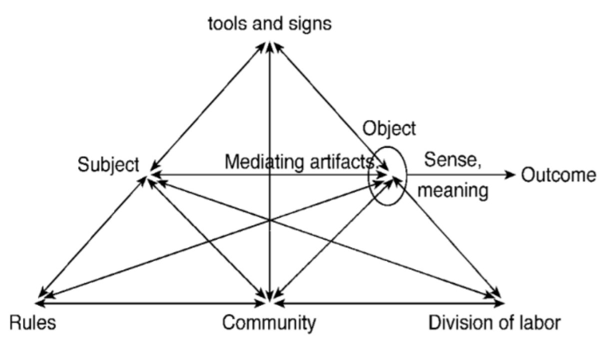

Figure 3.1: The structure of a human activity system (Engeström, 1987, p.78 cited in Engeström, 2001, p. 135)

Based on the third-generation CHAT, the unit of analysis is "a collective of the artefact-mediated and object-oriented activity system, seen in its network relations to other systems" (Engeström, 2001, p. 136). An activity system is regarded as a conglomerate of subjects with different roles, histories, and ideas that interact in complex ways. The activity system includes contradictions and innovations. Contradictions can be reconceptualised to embrace possibilities of change when they have accumulated in the system and make it unviable.

Engeström (1999) argued that an activity unfolds over time and is not equated with a brief lesson session, however, there is no theoretical objection to the use of CHAT in a session that unfolds over a short period (Bakhurst, 2009). The object is an important aspect of the activity as it signifies the rationale behind the activity system (Foot, 2014; Jonassen & Rohrer-Murphy, 1999). Thus, understanding the purpose (object) translates into understanding the activity system. The object is depicted with an oval to indicate that object-oriented actions as they arise from relationships with the other nodes do not converge to an objective certainty, but are characterised by sense-making as well as the potential for change (Engeström, 2001) and sharing. The subject (individual participant) can be transformed through the interaction of subject, tool, and object. This might imply that the goal of teaching informs the demand for the use of technology. The activity theory triangle can be interpreted depending on the particular case being investigated (Engeström, 1999). This gives room to interpret meaningful teaching as an object, technology as mediating artefacts, and teachers as subjects in the activity triangle.

Teachers-delivering-the-curriculum is interpreted as an activity triangle aimed at achieving Object 1, and researchers-and-developers-teaching-with-technology as a second activity triangle (Figure 3.2). Mathematics teachers as the subjects of one activity system and part of this study' anticipated professional development community, act on the object of their activity, which they colloquially describe as 'delivering the Mathematics curriculum'. Alongside their current activity system, teachers are confronted from outside with activity systems where researchers and curriculum developers as well as software developers strive towards teaching Mathematics with technology. Such teachers experience a clear but weakly mediated change in rules in the Namibian government's call for technology integration. True to the depiction in Figure 3.2 the intersection of Object 2 (the goal of the external activity system) hardly overlaps with the object of activity systems of most teachers in Namibia. In many instances, even reformed curricula fail to describe the changed object in ways that can expand Mathematics teaching goals.

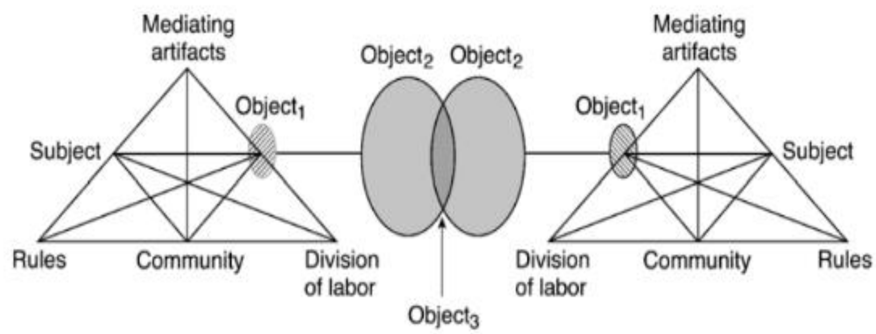

Figure 3.2: Two interacting activity system minimal model for the third generation of activity theory (Engeström, 2001, p. 136)

Owing to technology, organisations such as the National Council of Teachers of Mathematics (NCTM) which represent Mathematics educators, had been rethinking mathematical goals since the 1980s and still, these reforms are not evident in many classrooms (Pea, 1987). However, Namibian teachers may not be aware of the way the object of Mathematics with technology has changed. The change in teaching goals places teachers at the cusp of "important transformations of [their] personal lives and organizational practices" (Engeström, 2001, p. 138) since they are required to adopt new tools and rethink the object of teaching and learning Mathematics. Literally, all parts of their activity system are in flux, which brings opportunities and pitfalls that should be learned as they arise (Kanandjebo & Lampen, 2022). Engeström (2001) explains that activity in such instances cannot be mediated top-down by regulation but must necessarily emerge in new forms from expansive learning, which he describes as

> ‘‘The object of expansive learning activity is the entire activity system in which the learners are engaged. Expansive learning activity produces culturally new patterns

> of activity. Expansive learning at work produces new forms of work activity'' (Engeström, 2001, p.139).

For learning to be expansive, tools should lead to development. Vague, locally mandated change in rules will not lead to expansive changes in teaching and learning Mathematics with technology and in developing skills necessary to optimally function in a world dominated by technologies. To catalyse change, dynamics such as rules and goals need to be initiated that could cause serious transformational effort in the activity system (Engeström & Sannino, 2021). The problematic enacted curriculum is discussed (in chapter 5) and implications are drawn for professional development toward the expanded goal.

Professional development can be understood from the activity system perspective as a work activity undergoing through historical transformation; transforming and reorganizing the teaching process. Further, an activity system framework complements developmental work research for organizational change processes (Engeström, 1993) including those in education. Activity system views were therefore used as a guide for designing, implementing, analysing, and developing conclusions of a research study (Yamagata-Lynch, 2010). Using object-oriented activity, contradictions can be identified to help shift from one developmental phase to the next (Engeström, 2015). Activity systems can also be used as a conceptual tool to reach a deeper understanding of technology and its meaning to people (Kaptelinin & Nardi 2018). Marken (2006) applied an activity system approach to design and develop training programs for Human Performance Technology (HPT) practitioners to improve their practices. The activity system was used to describe participants' work-related activity (roles) and made comparisons of how systemic contradictions affected individual work roles. Though the studies (Kaptelinin & Nardi, 2018; Marken, 2006; Yamagata-Lynch, 2010) were not conducted in Mathematics education, they do show the practical application of an activity system analysis, and how helpful it can be in designing programs. The activity system is therefore used as a planning tool for holistic designing of a professional development framework.

3.4 Experiential learning theory (ELT)

This study is also informed at a peripheral by Experiential Learning Theory (ELT) by Kolb (1976a) (Kolb, 1984). Experiential learning theory is founded on reflective thought (Dewey, 1938), action learning (Kurt, 1946), and cognitive development (Piaget, 1958). Experiential learning is founded on individual experiences and encompasses methods of actively engaging individuals in the learning process (Fenwick, 2000). Thus, learning occurs through the transformation of experiences (Kolb, 1984). ELT provides a theoretical argument for work-based learning, and it can be used in the development of teachers' skills as well as in deciding how technology can aid the process of learning (Sharlanova, 2004). In the present study, ELT is used as it gives access to spontaneous knowledge networks (Vygotsky, 1978). Further, the theory has influenced and offered an approach to professional development (Zuber-Skerritt, 1992) in organisations, including schools. Kolb (1984) describes experiential learning as a four-stage cycle of experience, reflective observation, abstract conceptualisation as well as action exploration. These four stages are crucial for professional development and a general description of the process of learning (Sharlanova, 2004).

The four-cycle Figure 3.3 depicts how learning is transformed through reflection on ideas and concepts to gain and create knowledge. An individual can begin the cycle at any stage and should go through the cycle a couple of times, so it may be regarded as a spiral (McCarthy, 2010; Zuber-Skerritt, 1992). An individual therefore observes and reflects on those previous experiences, forming abstract conceptualisation and generalisation, and testing the implications of these concepts in new situations. Concrete experience and abstract conceptualisation are dimensions for grasping information; while reflective observation and active experimentation are dimensions of processing stimuli from the external environment (White, 1992). Thus, knowledge and professional growth happen through a transformation of experiences. Learning through the four-cycle happens when individuals absorb experience, knowledge, and ideas (White, 1992). Ideas are then processed through internal contemplation or active modification, and information or ideas are digested (Kolb, 1984). The process (Figure 3.3) is continuous thus at the last stage new concrete experience is created and hence the beginning of a new cycle of observation, reflection, conceptualisation, and testing implications of new actions and experiences.

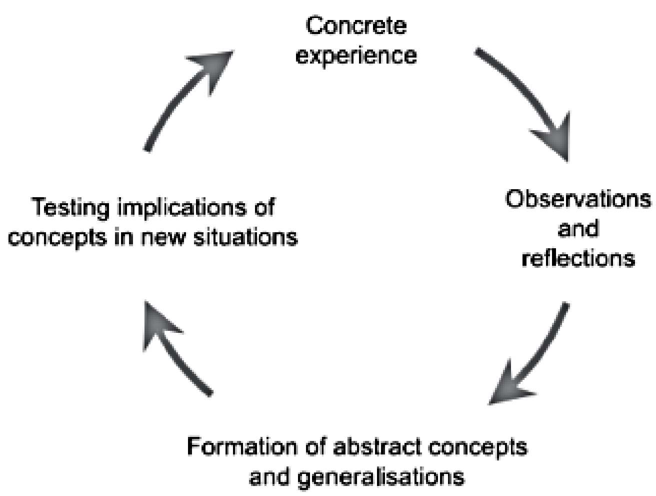

Figure 3.3: Kolb’s experiential learning model (Zuber-Skerritt, 2002; p.118)

Critics against ELT include that, it neglects the role of historical and cultural aspects of human action (Levinthal & March, 1993). Also, it does not sufficiently report on the role that non-reflective experience plays in the learning process (Cherry, 2020). Cherry (2020) further argued that ELT does not sufficiently report how individuals' interactions with the larger group influence the experiential learning process. Nevertheless, experiential learning approach has been used in the professional learning of adults including teachers (Blair, 2016; Girvan et al., 2016) and in designing professional development programmes for educators amongst others (White, 1992).

In White's (1992) study ELT was specifically used to facilitate communication between participants as the cycle makes provision for participants to pool their experienced knowledge and share their views as they approach a problem solving task. The researcher concluded that ELT provides an opportunity for professionals to reflect on their personal and professional development skills needs as well as dialogue with others about challenges they face in their work lives. Implying that experiential learning theory might assist in creating a road to building a community of practice for future engagements.

In addition, ELT was used as a theoretical framework in a qualitative analysis of Burke's (2013) study to improve teachers' practices through approaches such as demonstrations, observations, collaborations as well as reflection. The data were collected through questionnaires, written reflections by participants as well as through the researcher's observations and field notes. Burke (2013) concluded that the use of ELT in professional development provides an effective alternative to job-embedded learning.

Complementarily, ELT is used as a theoretical framework, in a survey research design by Amolloh et al. (2018) to explore the professional development of pre-service teachers. Amolloh et al. (2018) reported that for pre-service teachers to navigate through Kolb's cycle of learning, they need knowledge of teaching professional subjects. They argued that experiential learning then takes place through reflecting on possible solutions, correcting errors, and continuing the learning cycle. Through, consecutive attempts an individual arrives at possible solutions for effective teaching. Further, at the concrete stage, the teacher encounters new experiences and builds new schema. In addition, when concrete stage is achieved, an individual enters the abstract conceptualisation stage. The stage demands additional resources, support, and collaboration either with others or a more experienced person. The learning process continues in a cycle until a desired skill and knowledge are achieved.

As an extension of experiential learning theory, Senge (1990) added mental models that influence how people understand the world and take action. Mental models are discussed to understand actions involved in professional development through experiential learning. Further, Senge (1990) noted that learning of individuals in institutions happens in two forms: that is adaptive and generative. He noted that adaptive learning focuses on amending existing knowledge and it is significant to institutions seeking improvement. In terms of education, adaptive learning encompasses understanding and closing the gaps on teachers' ability to teach with technology to improve learning experience. Similarly, to Vygotsky's (1978) notion of the knowledgeable other and getting to scientific knowledge from spontaneous knowledge, adaptive learning type of experiential learning involves acquiring standard skills and knowledge and the training is trainer-designed and trainer-led (Senge, 1990; Sessa & London, 2016). Further, generative learning is socially constructed, and individuals create and apply new ideas by linking them to emerging ideas. It also implies that existing skills (spontaneous knowledge) can be developed through expansive learning to get to scientific knowledge (Engeström, 2001; Vygotsky, 1978).

In this study, ELT was only used as a theoretical frame to ground the design of learning tasks that were used in the research engagements in terms of reflection and the use of concrete experiences. Thus, although it provides a theoretical argument for work-based learning it was peripheral to the current study.
Harmonically, Vygotsky's sociocultural learning theory, CHAT, and ELT are used as planning frameworks for holistic professional development.

PART B

3.5 Lens for meaningful teaching of Mathematics: five strands of mathematical proficiency

The five intertwining strands of mathematical proficiency developed by Kilpatrick et al. (2001) capture what is necessary for anyone to learn Mathematics successfully (National Research Council, 2001). The five strands provided a direction for discussing the pedagogical content knowledge, skills, capabilities, and beliefs that teachers should constitute. The five strands of mathematical proficiency are used as a conceptual framework for the goals of teaching Mathematics (Chapter 1), thus discussed with reference to teaching as follows (Kilpatrick et al., 2001):

(a) Conceptual understanding

Conceptual understanding is a unified and functional making grasp of mathematical ideas such as concepts, operations, and techniques. It is developed through practices that promote sense-making of mathematical concepts, operations, and relations. For conceptual understanding to develop teachers need to ensure that learners know more than isolated facts and methods; that they can understand the rationale of mathematical ideas and the kinds of contexts in which those ideas are useful. Hence the teacher needs to develop learners' ability to represent different mathematical situations and to connect these representations.

(b) Procedural fluency

Procedural fluency is developed when teachers teach how to carry out mathematical processes, and when to use them appropriately, skills of how to perform them flexibly, accurately, and efficiently (National Research Council, 2001). Teachers enhance learners' efficiency and accuracy in performing computations by providing them with an opportunity to practice mathematical techniques and develop them to solve mathematical problems. Procedural fluency and conceptual understanding are often seen as competing for attention in school Mathematics (National Research Council, 2001). The implication is that teachers might mistake developing conceptual understanding for procedural fluency.

(c) Strategic competence

This strand refers to the ability to formulate mathematical problems, represent, and solve them. It is developed and enhanced when teachers engage learners in real-world mathematical problems, real-life situations which may also require learners to turn them into solvable mathematical problems. It also involves an individual ability to hypothesise and develop mathematical models.

(d) Adaptive reasoning

It is the capacity to think logically about the relationships among concepts and situations (National Research Council, 2001). Adaptive reasoning is the glue that holds everything together and the lodestar that guides learning (Kilpatrick et al., 2001). Teachers develop learners' adaptive reasoning to be able to reflect, explain and justify. It is also developed when learners are allowed to engage through mathematical facts, procedures, concepts, and solution methods. Teachers, therefore, create learning environments in which learners are expected to explain, reason, and justify mathematical claims to others. Hence, learners whose reasoning skills have developed to adaptive reasoning can think logically about the relationships among concepts and situations, consider suitable alternatives, reason correctly and justify their inferences. Teachers are sources of settling mathematical disputes and disagreements about a mathematical answer.

(e) Productive disposition

It encompasses an individual's affective aspects. Learners with productive disposition believe that their constant efforts in learning Mathematics pay off and see themselves as effective, and doers of Mathematics (Kilpatrick et al., 2001). Moreover, Siegfried (2012) regards productive disposition as eight constructs which are: affective; beliefs; goals; identity; mathematical integrity; motivation; risk-taking; self-efficacy. Productive disposition develops after other strands and influences them to develop as well. In developing productive disposition, Mathematics teachers play a critical role in providing good Mathematics teaching and in encouraging learners to maintain positive attitudes toward Mathematics (National Research Council, 2001). Thus, teachers should view Mathematics positively, as their views affect their teaching practices. Teachers are required to frequently provide opportunities for learners to make sense of Mathematics, recognize the benefits of perseverance, and experience the rewards of sense-making in Mathematics.

The properties of five strands of mathematical proficiency are discussed next section to understand their nature which would assist in aligning pedagogy.

3.5.1 Properties of five strands of mathematical proficiency

The strands of mathematical proficiency interweave and support one another. They are interwoven, mainly to denote that deep understanding requires learners to connect pieces of knowledge. Hence, each strand of mathematical proficiency should be developed in synchrony with the others as learning is not static (National Research Council, 2001). Figure 3.4 shows interwoven and intertwined strands of mathematical proficiency.

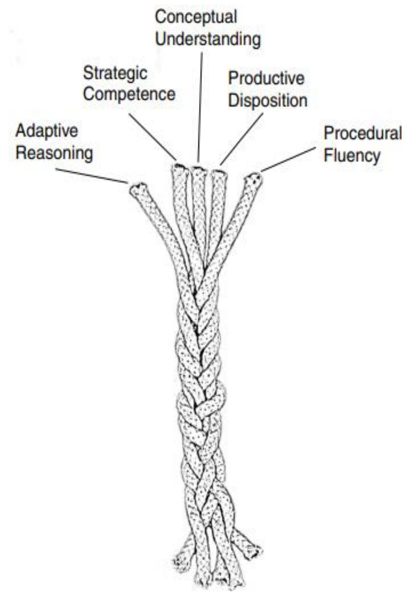

Figure 3.4: Intertwined strands of mathematical proficiency (Kilpatrick et al., 2001, p.117)

Additionally, mathematical proficiency cannot be described as simply present or absent as learners are never complete mathematical novices. Learners bring crucial mathematical concepts and skills as well as misconceptions with them to school. Teachers should also take into consideration individual learners' mathematical understanding as well as their misconceptions. Learners should not be thought of as having proficiency when one or more strands are undeveloped. Further, proficiency in Mathematics is developed over time (National Research Council, 2001) as acquiring proficiency takes time. The implication could be that Mathematics teachers should professionally develop appropriate teaching approaches, and Mathematics curricula for schools should be explicit to ease the teaching process of developing mathematical proficiencies.

3.5.2 Five strands of mathematical proficiency and Mathematics curricula of leading countries

The literature presented aids in answering research question number one on the affordances and constraints of the Namibian secondary school curriculum in terms of Kilpatrick et al. (2001) five strands of mathematical proficiency. No term captures completely all aspects of expertise, competence, knowledge, and facility in Mathematics better than the five strands of mathematical proficiency (National Research Council, 2001). Also, Graven and Stott (2012) in their study on conceptualising procedural fluency as a spectrum of proficiency, argued that Kilpatrick et al.'s (2001) model provides a rich and elaborated notion that Mathematics teachers can work towards developing in learners. In addition, strands of mathematical proficiency are key processes which inform teachers and provide a meaningful basis for the development of concepts in the learning of Mathematics (Australian Curriculum Assessment and Reporting Authority (ACARA), 2015).

Mathematics curricula of leading countries in the world have adapted five strands of mathematical proficiency. Some of those countries include Australia with explicit ideas of Kilpatrick et al. (2001) (ACARA, 2015), Singapore, the United States of America (USA) (from which the proponents of five strands of mathematical proficiency are based), and the United Kingdom (UK). Though these countries are developed and have an advantage of access to resources as compared to Namibia and other African countries; Singapore's curricula is benchmarked to the University of Cambridge Local Examinations Syndicate (Dindyal, 2006; Remillard & Reinke, 2017) similar to the Namibian secondary school Mathematics curricula under study. Table 3.1 shows strands of mathematical proficiency incorporated in four leading countries.

Table 3.1: Comparison of leading countries' Mathematics teaching goals in leading countries

Mathematics curricula	Proficiencies adapted
Singapore Mathematics Curriculum Framework (SMCF)	Pentagon model with problem solving at the core and concepts, processes, metacognition, attitudes, and skills around the sides (Ministry of Education Singapore, 2006).
Australian Mathematics curriculum (AMC)	Four proficiencies and renamed as understanding, fluency, problem solving, and reasoning (ACARA, 2015).
United States of America (USA)	Most states are based on the levels of mathematical proficiency described by Kilpatrick et al. (2001) most focus on problem-solving, reasoning, and critical thinking (Belbase, 2019; Remillard & Reinke, 2017)
Norwegian Mathematics curriculum	Five basic skills: oral, reading, writing, numeracy, and digital skills (Mullis, Martin, Foy & Hooper, 2016).

Table 3.1 shows that the Norwegian Mathematics curriculum is a step ahead with the inclusion of abilities to use digital tools in mathematical calculations, investigation, visualization, simulation, modelling, and presentation. From Table 3.1, the Singapore Mathematics Curriculum Framework (SMCF) used different terminologies from Kilpatrick et al. (2001) however Kilpatrick (2011) stated that "both their framework (SMCF) and our strand model get at the same notion, that proficiency in Mathematics is more than simply skill or understanding and that learners need to develop all five components simultaneously" (p.11). It is notable from Table 3.1 that productive disposition is not explicitly stated in the Australian and Norwegian Mathematics curricula.

Moreover, these countries have professional development goals. In Singapore for example, teachers' knowledge and skills levels are developed from a beginner teacher, through a general education office to a master teacher. A master teacher is someone with (Kaur, 2014, p. 13):

- Strong awareness of trends and issues surrounding Mathematics beyond the school setting and in industry/field,
- Knowledge of core concepts of other related subjects which integrates the learning of Mathematics to the world outside of school,
- Knowledge of significant relationships, history, structure of Mathematics and the application of this knowledge to inspire interest in Mathematics.

Notably, technological knowledge is not explicitly stated as one of the knowledge and skills teachers should acquire to be at the master teacher level. However, the structure of the curriculum informs teacher learning (Sullivan, 2012). Thus, in Australia for example, Goos (2012) analysed the AMC to determine the extent to which technology transforms teaching and learning roles as well as to classify pedagogical opportunities afforded by technology. Goos (2012) study searched the AMC document electronically for the terms: “technology”, “technologies”, “calculator”, “computer”, and “software’’; and found explicit reference to graphing and geometry software to be used in teaching topics such as number and algebra, and measurement and geometry as well as the use of the internet in statistics and probability. The researcher concluded that the AMC curriculum offers many opportunities for teachers to create and effectively use digital technologies to teach the required learning content.

However, Tabach and Trgalová (2020) documented that though official education-related policy documents acknowledge the importance of integrating Information and Communication Technology (ICT) into Mathematics education and that the teacher is central to such integration; there is dissatisfaction regarding initiatives that encourage ICT use. The researchers argued that dissatisfaction resulted from a discrepancy between teachers’ expectations and the content of these initiatives and calls to develop standards and competency frameworks geared towards Mathematics. Hence, acknowledging the benefits of technology in curriculum policies is not sufficient; there should be a benchmark in terms of a professional development framework to guide technology integration in a specific subject, namely Mathematics (Tabach & Trgalová, 2020). Their observation concurs with Drijvers et al. (2014) that the use of technology referred to in Mathematics curriculum documents is often general and meant calculators. Drijvers et al. (2014) conclusions are based on their review of the use of technology in Mathematics, in Australia, England, France, The Netherlands, New Zealand, and Singapore.

In Namibia, Mathematics teachers' pedagogies with reference to Kilpatrick et al. (2001) were already reviewed by Stephanus (2014). Stephanus (2014) found that teachers promote the development of conceptual understanding and procedural fluency even if there were opportunities for them to develop other proficiencies. This could be attributed to the lack of pedagogical skills that professional development could mediate. No study reviewed the Namibian Mathematics curriculum affordances and constraints in terms of Kilpatrick et al. (2001). In addition, although some countries have adopted mathematical capabilities in line with Kilpatrick et al.'s (2001) five strands of mathematical proficiency, Ally (2011) questions the extent to which opportunities for developing these domains are present in teachers' pedagogies.

Moreover, as part of research question one, the current study analyses the Namibian secondary school Mathematics curriculum using Kilpatrick et al. (2001) as a conceptual framework. The study also adapted the analysis approach of searching electronically for the terms "technology", "technologies", "calculator", "computer", and "software''; used by Goos (2012) in the ACM. The aim was to identify the affordances and constraints that the Namibian curriculum affords teachers to integrate technology in teaching Mathematics (discussed in Chapter 5).

The next subsection discusses frameworks for the professional development of teachers to integrate technology in the teaching and learning of Mathematics.

3.6 Professional development frameworks for teaching with technology

3.6.1 Technological Pedagogical and Content Knowledge (TPACK)

Technological Pedagogical Content Knowledge (TPCK) was changed to Technological Pedagogical and Content Knowledge (TPACK) for grammatical accuracy and also to stress the integrated use of technology, pedagogy and content knowledge for functional technology integration (Thompson & Mishra, 2007). The TPACK framework builds on the Shulman theory by integrating the knowledge of technology into pedagogical practices (Garba, 2018). It is based on the notion that models of technology integration are not sufficient on their own. Thus, the education technology component has to be added to Pedagogical Content Knowledge with an intention to improve teaching (Çam & Erdamar Koç, 2021). TPACK framework connects technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK) to represent knowledge of teaching content with appropriate pedagogical approaches and technologies. TPACK highlights the connections and interactions among content, pedagogy, and technology for the successful integration of technology in the classroom (Mishra & Koehler, 2006). It encompasses

> “knowledge of the existence, components, and capabilities of various technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as a result of using particular technologies” (Mishra & Koehler, 2006, p. 1028).

The TPACK framework proposes that effective teaching with technology lies at the intersection of Technological, Pedagogical, and Content Knowledge(Mishra & Koehler, 2006). TPACK is also useful for thinking about how teachers might develop knowledge for integrating technology in teaching (Schmidt et al., 2009) and designing professional development (Kadijevich, 2012). TPACK framework is argued as relevant for teachers in the twenty-first century (Gur & Karamete, 2015). It is also believed to be a yardstick for integrating the knowledge of technology into pedagogical practices (Garba, 2018; Mishra & Koehler, 2006). As a result, various studies have used it to describe and capture essential qualities of knowledge Mathematics teachers need to possess in order to effectively integrate technology into their teaching (Çam & Erdamar Koç, 2021; Garba, 2018; Getenet, 2017; Mishra & Koehler, 2006; Niess et al., 2009).

In Figure 3.5 the seven constructs in the TPACK model are shown as discussed as follows (Mishra & Koehler, 2006):

1. *Content Knowledge (CK)* – Koehler and Mishra, in 2009 argued that CK also entails the need to know and understand the nature of inquiry and knowledge in other subject disciplines to be able to understand the uniqueness of individual subjects. CK differs according to discipline and grade level. Thus, CK requires Mathematics teachers (scope of this study) to have a proper and in-depth understanding of the subject (they teach or are to teach). This would include concepts, values, skills, theories, and procedures within a specific subject discipline (teaching subject) (Mishra & Koehler, 2006).

2. *Pedagogical Knowledge (PK)* – Describes teachers' understanding of the practices, processes, and methods for instructional delivery. As a generic form of knowledge, PK encompasses all issues such as purposes, values, and aims of education to more specific areas relating to teaching and learning styles; classroom management; development and implementation of the lesson plan (Mishra & Koehler, 2006). The authors (Mishra & Koehler, 2006) further noted that teachers with deep pedagogical knowledge will have a good understanding of how to teach to promote understanding, construct knowledge, acquire skills and develop conduct and positive disposition toward learning.

3. *Technological Knowledge (TK)* – Describes teachers' knowledge of, and capability to use various technologies, technological tools, and related resources. TK ranges from knowledge about low-tech technologies such as pencil and paper to digital technologies such internet, hardware, and software programmes (Schmidt et al., 2009).

Koehler and Mishra (2009) as well as Mishra and Koehler (2006) noted that technology is rapidly changing thus provisions need to be made to instil pedagogical practices that would help teachers to cope with rapid changes. The ICDL training that the Ministry of Education, Arts and Culture has conducted enhanced teachers' TK.

4. *Technological Content Knowledge (TCK):* focuses on teachers' understanding of how technology relates to subject content in pedagogical practices and how knowledge of the two can be integrated and utilised to enrich pedagogical practices. It includes knowledge of how subject content can be communicated using various technology applications for classroom instructions.

5. *Technological Pedagogical Knowledge (TPK)* – is the knowledge of how technologies can be excellently applied in developing specific teaching and learning, and how teaching processes and pedagogical practices are likely to transform as a result of using such technology (Garba, 2018). It also describes teachers' understanding of how particular technologies can change both the teaching and learning experience. Another aspect of TPK concerns understanding how technology tools can be used alongside pedagogy in ways that are suitable to the discipline.

Thus, Mathematics teachers need to know and understand that lots of digital tools exist that can be used in facilitating teaching. They should, therefore, have the skills and knowledge of selecting appropriate technological tools that can fit into pedagogical designs and can facilitate the attainment of teaching goals. This requires teachers to have the following (Garba, 2018; Mishra & Koehler, 2006):

a. Knowledge of pedagogical strategies
b. Knowledge of technology and its application to educational practices
c. Knowledge and skills of integrating the two in instructional practices
d. Application of the integrated knowledge for specific instructional delivery in classroom learning.

6. *Pedagogical Content Knowledge (PCK)* – This is considered as knowledge of pedagogy relevant to a specific subject discipline. PCK entails knowing the suitable teaching methods for specific (topics) learning contents and an understanding of how learning contents of the subject disciplines can be organised. It comprises of development and representation of concepts, values, and skills, pedagogical techniques, knowledge of formation of concepts, knowledge of learners' prior knowledge in the subject area, and epistemological theories of the subject discipline (Garba, 2018). Teachers with PCK can incorporate appropriate conceptual representations of the content to address learners' challenges as far as the subject is concerned as well as their misconceptions.

7. *Technological Pedagogical Content Knowledge (TPCK):* is the complete package formed after the combination of the six knowledge domains discussed earlier (Thompson & Mishra, 2007). Schmidt et al. (2009) noted that teachers have an impetuous understanding of TPACK when they teach content using various pedagogical methods and technologies. The note might imply a research gap in the need to holistically develop Mathematics teachers' knowledge.

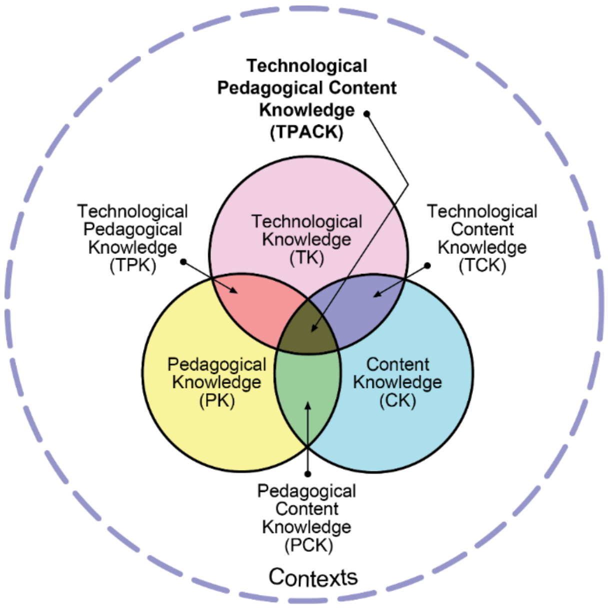

Figure 3.5: Components of the TPACK framework (http://tpack.org)

The TPACK framework is not subject-specific even though the proponents took note that the advent of digital computers has changed the nature of Mathematics (Koehler & Mishra, 2009). Correspondingly, it cannot be used as it is for professional development in relation to Mathematics education pedagogy by a technology-skilled teacher (Benson & Ward, 2013; Garba, 2018; Gur & Karamete, 2015; Harris & Hofer, 2011). Thus, various frameworks have been developed based on TPACK to enhance its effectiveness for professional development and are discussed as follows.

3.6.2 TPACK-based frameworks

The TPACK framework was extended to Pedagogical Technology Knowledge (PTK) by Thomas and Hong (2005). This was done to include mathematical content knowledge (MCK), Mathematical Knowledge for Teaching (MKT), teachers' personal orientations, and their role in influencing goal setting and decision making. PTK encompasses teachers' understanding of techniques necessary to build didactical situations and instrumental association between the teacher and technological tools. The personal orientations component of PTK emphasises teachers' beliefs and goals on the significance of technology, the essence of learning mathematical knowledge, affordances and constraints involved, as well as the affective aspect (Jafri, 2020; Tabach & Trgalová, 2019; Thomas & Palmer, 2014). PTK develops when teachers advance through the phases of instrumentation and instrumentalization of resources and gain personal understanding of the role of resources in the teaching and learning of Mathematics (Tabach & Trgalová, 2019). Hence, PTK may aid Mathematics teachers to plan and develop Mathematics teaching and learning sessions that are meaningful to 21st-century learners.

In addition, the Mathematics Digital Knowledge for Teaching (MDKT) developed by Tabach and Trgalová (2019), expanded the TPACK and PTK framework by bridging a gap between teachers' personal orientations towards technology (the affective domain) and teachers' personal instrumental genesis. The MDKT framework was necessary as teachers' personal orientations and personal instrumental genesis are not acknowledged as spontaneous sources of knowledge for teaching in the existing models (Tabach & Trgalová, 2019). It thus linked technology to the six knowledge areas that are based on Ball et al.'s (2008) Mathematics Knowledge for Teaching (MKT), as follows (Tabach & Trgalová, 2019, p. 189):

- Teachers' specialized digital content knowledge (SDCK) with respect to the Mathematics to be taught
- Knowledge of content and students, which in a technological environment includes additional aspects that may be formulated as knowledge of digital content and students (KDCS)
- Knowledge of content and teaching, which in a technological environment may be interpreted as knowledge of digital-content and teaching (KDCT)
- Knowledge of content and curriculum in a digital environment, e.g., knowledge of prescribed uses of ICT (KDCC).

In the MDKT framework, SDCK relates to personal instrumental genesis (personal use of technology) and the last three bulleted points encompass the aspects of professional instrumental genesis. Further, the MDKT framework ‘‘emphasises the decisive role played by the components of mathematical knowledge for teaching with technology that is related to teacher orientations, personal, and professional instrumental genesis’’ (Tabach & Trgalová, 2020, p. 201) (Figure 3.6).

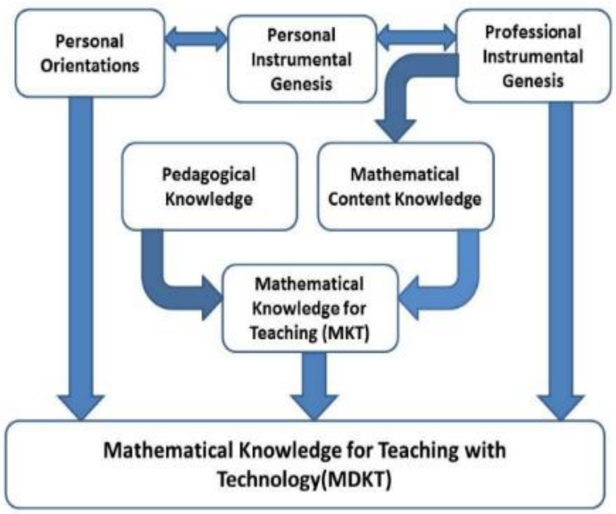

Figure 3.6: Mathematical Knowledge for Teaching with Technology (MDKT) framework (Tabach & Trgalová, 2019, p.219)

**Tabach & Trgalová (2019) rephrased MDKT as Mathematical Knowledge for Teaching with Technology as it appears in Figure 3.6.*

In the context of the standards-based curriculum for Mathematics that is widely adopted in the USA, Niess et al., (2009) developed the Mathematics teacher TPACK standards and development model. The Mathematics teacher TPACK standards and development model provide a framework for guiding professional development practice that supports the improvement of Mathematics teaching and learning with technology. The framework is developed as there is a need for content-specific professional development that addresses what teachers should know about teaching and learning Mathematics with technology (Niess et al., 2009). The absence of professional development in teaching with technology and curriculum materials for technology integration poses a risk of not using technology-rich tasks in teaching subjects.

According to Niess et al., (2009) Mathematics teacher TPACK standards are organised in themes that encompass the knowledge and beliefs that teachers need to demonstrate when incorporating technology in the teaching and learning of Mathematics. The standards shift the focus to curriculum and instructional uses of digital technologies from mainly technologies tools. Niess et al., (2009) argued that teachers progress through a five-stage developmental process when learning to integrate a particular technology in teaching and learning Mathematics. The stages are recognizing (knowledge), accepting (persuasion), adapting (decision), exploring (implementation), and advancing (confirmation). The transition from one level to another ‘‘does not display a regular, consistently increasing pattern’’ in knowledge (Niess et al., 2009, p. 10). The levels are hierarchical and iterative for the development of TPACK. The challenge with the stage levels is that technology is advancing at a rapid pace and moving through the stages might be not sustainable and may delay the full implementation of technology integration in Mathematics teaching and learning as COVID-19 proved it. As a result, teachers need to professionally develop holistically. Moreover, Niess et al. (2009, p. 13) remark on the Mathematics teacher TPACK standards and development model that,

> ‘‘moving from one level to another may require different sets of experiences for different levels and for different teachers. What are these sets of experiences? Do experiences exist that cause teachers to regress from one level to a previous one? Do teachers skip levels?’’

Niess et al. (2009) stops short of suggesting kinds of experiences that can serve as actionable guidelines for professional development.

In the same vein, Joshi et al. (2021) reviewed various other frameworks for digital competency of teachers including the Digital Competence Framework for Austria as well as UNESCO's 2011 ICT Competency Framework for teachers. Joshi et al. (2021) in the Digital Competency Framework for Teachers draw attention to the omission of the ethical aspects of teaching with technology, such as intellectual property rights, copyrights, and cyber safety, but do not consider the ethics of limited access. In addition, Joshi et al. (2021) and Tabach and Trgalová (2019) frameworks were used in Kanandjebo and Lampen's (2022) research article extracted from this thesis, to frame gaps and opportunities in existing frameworks.

3.7 Implications for designing a professional development framework

Professional development happens in a community of practice of teachers and the professional development needs of teachers differ between cultures. Vygotsky's sociocultural learning theory on adult learning argued that knowledge is socially constructed, and knowledge originates from culture. Learning occurs through experiential within the zone of proximal development. Moreover, Vygotsky argued that knowledge progresses from the spontaneous knowledge level to the scientific level. The composition of scientific knowledge are answers to the question ‘‘what knowledge do we want the students to acquire?’’ Vygotsky's sociocultural learning theory is used to explain that knowledge progresses through levels from spontaneous knowledge level to scientific knowledge level. Hence Vygotsky's views allow us to structure teachers' progression from spontaneous knowledge to scientific knowledge. Progression means relating personal instrumental genesis to spontaneous knowledge and professional instrumental genesis to link to scientific knowledge which professional development should aim to bridge

Moreover, third-generation CHAT Engeström (2001) argues that contradictions such as the absence of teaching with technology in many classrooms, the absence of reformed Mathematics teaching goals, and widely available technology resources place teachers at the cusp of “important transformations of [their] personal lives and organizational practices” (Engeström, 2001, p. 138). Since teachers are required to adopt new tools and rethink the object of teaching and learning Mathematics. As a result, all parts of their activity system are in flux, which brings opportunities and pitfalls which should be learned as they arise. In this case, for teachers’ activity system to lead to expansive learning, knowledge should be mediated bottom-up. Vaguely, locally mandated change in rules will not lead to expansive changes in teaching and learning Mathematics with technology; of developing skills necessary to optimally function in a world dominated by technologies. To catalyse change, dynamics such as rules and goals need to be initiated that could cause serious transformational effort of the activity system (Engeström & Sannino, 2021). Professional development can be understood from the activity system perspective as a work activity undergoing historical transformation; transforming and reorganizing the teaching process. Thus, professional development should expand teachers’ knowledge to the scientific knowledge level, which is organised, structured, and theorised for teaching Mathematics with technology not just personal instrumental genesis for every day.

The five strands of mathematical proficiencies are interwoven thus professional development should be holistic. Proficiencies being ‘‘not all or nothing’’ implies that an individual has prior knowledge, which is spontaneous knowledge that needs to be developed to a scientific knowledge level. The notion that proficiency develops over time, implies the gradual development of knowledge, and the need for continuous professional development. It also shows the interconnection and influence of the Mathematics curriculum and teachers’ professional development to achieve meaningful mathematical goals.

Professional development designs seem under researched; researchers define professional development according to TPACK. The reviewed frameworks for professional development on teaching with technology uniquely offer an opportunity to interpret and understand the knowledge teachers need to be able to integrate technology into teaching (Figure 3.7). The generic TPACK (Mishra & Koehler, 2006) without linking to subject pedagogy runs the risk of knowledge being used for personal genesis instead of professional genesis in the classroom. This knowledge may also take time to develop into professional instrumental genesis. In Namibia, teachers obtain general knowledge on technology integration and use it on the same old goals, when curriculum goals are revised, teachers apply old pedagogy to it. As a result, technology would be substituted (Puentedura, 2010) as an efficiency tool.

The PTK (Thomas & Palmer, 2014) affords to discuss knowledge for pedagogy, Mathematics, technology, and teachers' affective aspects. Niess et al. (2009) afford to discuss TPACK within standardised Mathematics curricula. The Mathematical Digital Knowledge for Teaching (MDKT) framework (Tabach & Trgalová, 2019) combines TPACK and PTK. It thus, allows us to discuss teachers' basic technology skills and the utilization schemes (personal instrumental genesis) as well as knowledge of how to support students in a digital environment (professional instrumental genesis) (*Figure 3.7*). Joshi et al. (2021) draw attention to the omission of the ethical aspects of teaching with technology, such as intellectual property rights, copyright, and cyber safety, but do not consider the ethics from the perspective of limited access which was considered in the design principles of a professional development framework designed in the current study. Niess et al. (2009) framework stops short of suggesting kinds of experiences that can serve as actionable guidelines for professional development. The frameworks on professional development for teaching with technology relate to each as depicted in *Figure 3.7*.

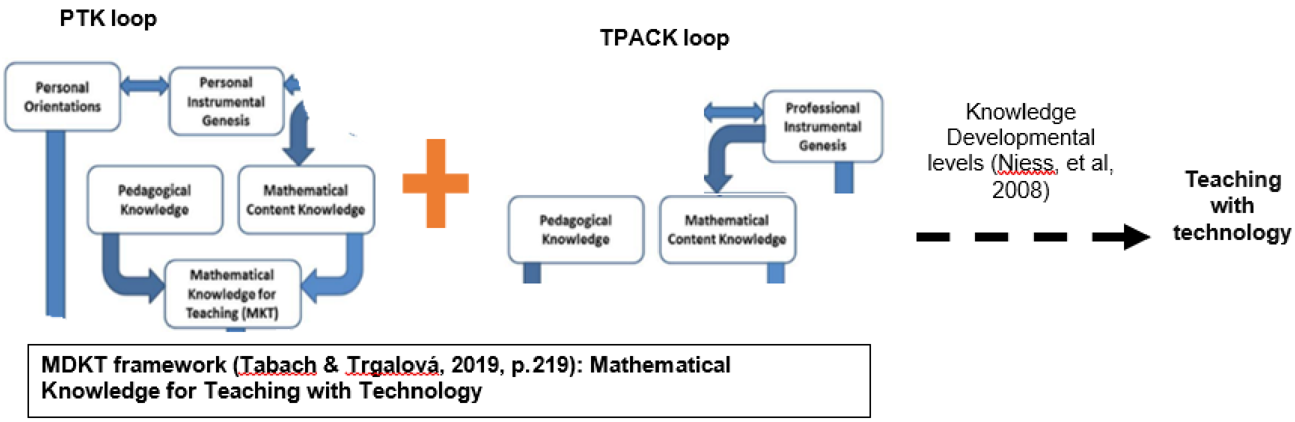

Figure 3.7: Relationship between TPACK and TPACK-based frameworks

Moreover, all existing frameworks have no explicit guidelines for professional development focusing on teachers' use of technology on promoting integrated and relational understanding of Mathematics, as well as reasoning and problem-solving. Further, teachers' personal instrumental genesis and professional instrumental genesis are more about teachers' decisions on using technology tools. These decisions are mainly left to the teachers' initiatives without guidelines. This resonates with Neubrand (2018) who argued that knowledge-driven frameworks are limited as there is a ''gap between knowing and acting''(p. 609). Likewise, Tabach (2021) argues that,

> ''One can know a subject but may not have the skills required to apply that knowledge to specific tasks since knowledge does not provide skills. A teacher may know Mathematics and pedagogy, but this only makes her knowledgeable about teaching. However, knowledge [alone] does not make one a good practitioner. To become a good teacher one must teach, practice one's techniques, and improve one's skills…''(p.100).

Consequently, towards this end, the construct of meaningful teaching of Mathematics with technology was developed. The researcher argues that professional development should have actionable guidelines, be sustainable, consider ethics in terms of access to technological resources, and Mathematics teaching goals should be explicit as well. Further, professional development should be holistic and progressive from teachers' spontaneous knowledge to scientific knowledge level.

It is worth noting that no study analysed the Namibian curriculum in terms of this thesis's conceptualisation of meaningful teaching with technology (Mateya et al., 2016; Stephanus, 2014). Further, in an international survey carried out in over forty (44) countries including Namibia on future themes of Mathematics education research before and during the pandemic by Bakker et al. (2021), professional development in relation to coordinating teaching with technology was identified as one of the themes, research should focus on. Namibia was specifically quoted to require research on professional development. Thus, these literature gaps are addressed by using the SAMR and pedagogical model to analyse the affordances and constraints of the Namibian secondary school Mathematics curriculum in chapter 5.

The next chapter presents the methodology of the study.

CHAPTER 4 RESEARCH METHODOLOGY

4.1 Introduction

This chapter presents the study's philosophical assumption (paradigm) (section 4.2) and research design of this design-based research (section 4.3). After stage one of the literature survey (chapters 2 and 3), the researcher conducted context analysis (stage two). This was done through critical engagement with the Namibian secondary school Mathematics curriculum as well as by engaging technologically adept Mathematics teachers. Thus, this chapter presents methodological aspects relating to answering research question number one on the affordances and constraints of Namibian secondary school curriculum in terms of Kilpatrick et al. (2001) five strands of mathematical proficiency. It also presents methodological aspects relating to research question number two and three based on the views and beliefs of technologically adept Mathematics teachers. In section 4.4 case selection of participants is presented detailing the selection of participants and the role of the researcher. The data collection process and organisation of research engagements during the COVID-19 pandemic are presented in section 4.5. In addition, the research methods that were used iteratively to gather the data are presented in section 4.6. Data analysis (section 4.7) and trustworthiness of the research (section 4.8) are presented followed by ethical considerations (section 4.9). The chapter concludes with a summary (section 4.10).

4.2 Research paradigm: The pragmatic paradigm

Research paradigms are common beliefs, values, and assumptions, regarding the nature and ways of conducting research (Johnson & Onwuegbuzie, 2004; Kuhn, 1977). It includes accepted theories, customs, approaches, models, frames of reference, and methodologies (Babbie, 2010; Creswell, 2007). Creswell and Creswell (2018) regard research paradigms as the worldview of the researcher. They mould the way the researcher perceives and interprets the world around them (Johnson & Onwuegbuzie, 2004). Paradigms 'guide action' (Guba, 1990, p. 17) and give rise to a particular worldview

(Nieuwenhuis, 2010). Research paradigms determine the kind of knowledge the researcher would generate. Hence, they influence approaches the researcher undertakes to construct and interpret the meaning of reality (Poni, 2014). Different scholars possess different interpretations and world views (Creswell, 2009; Poni, 2014) which influences their philosophical thinking. It is also worth noting that, researchers approach to research with certain basic assumptions about the world, research phenomena as well as how the phenomena should be studied. It is of utmost importance to select an appropriate paradigm that would inform the research design of the study in attempting to answer research questions.

Consequently, the present DBR study adapts the pragmatic paradigm which focusses on building artefacts such as frameworks as an intervention to organizational change (Goldkhul, 2012). Pragmatists acknowledge that the world is constantly changing, and it is being changed through actions; similarly, through actions, people change their being. In essence, in this study, it can be said that teachers' beliefs about teaching with technology seem to influence their teaching with technology as demonstrated in their teaching with technology during the 2020 and 2021 lockdowns. However, little is known about their professional development needs for teaching Mathematics meaningfully with technology and key aspects of a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology.

The pragmatism research paradigm resonates with the theoretical principles (Chapter 3) of the current study through three descriptions of the pragmatic paradigm argued by Morgan (2014a). The first one is that (a) ‘‘actions cannot be separated from situations and contexts in which they occur’’ (Morgan, 2014a, p. 131). Also paraphrased that human actions can never be detached from preceding experiences, and from beliefs that initiated such experiences (Kaushik & Walsh, 2019). This may imply that actions are socially constructed and situated in contexts including cultural contexts. For instance, in the current study, the Namibian secondary school Mathematics curriculum and technologically adept Mathematics teachers informed the design of the professional development framework. Second, (b) ‘‘actions are linked to consequences in ways that are open to change’’ (Morgan, 2014a, p. 132). This may imply that professional development of teaching Mathematics meaningfully with technology may result in expansive learning and a new teachers’ activity system (Engeström, 2001). In addition, (c) actions are influenced by worldviews that are socially shared. Though participants’ views are unique, they are rather connected than isolated (Kaushik & Walsh, 2019; Kivunja & Kuyini, 2017; Morgan, 2014a, 2014b) and converge to a point, to shape an action and a framework can emerge. Therefore, Mathematics teachers’ views and beliefs about teaching Mathematics meaningfully with technology and their professional development needs in unison inform the design process of the current study.

According to the pragmatic paradigm, there are multiple realities, hence multiple interpretations of the world and ways of conducting research (Barab & Squire, 2004; Kaushik & Walsh, 2019; Kivunja & Kuyini, 2017; Morgan, 2014a). The pragmatic paradigm’s research approaches are based on ‘what works in the real world situation to find answers to the research questions (Morgan, 2014b). The use of ‘what works’ could imply that the researcher can attempt to answer research questions without worrying as to whether the questions are wholly quantitative or qualitative (Kivunja & Kuyini, 2017). The pragmatic paradigm is aligned with the research methodology that is more practical and diverse (Kaushik & Walsh, 2019; Kivunja & Kuyini, 2017; van den Akker et al., 2013). Such research methodologies are those deemed appropriate for the study by the researcher and opted for based on the research questions. In addition, choices are guided by how well they help in achieving answers to research questions (Pansiri, 2005; Tashakkori & Teddlie, 2008). Thus, research approaches can be single methods, mixed methods, or a combination of methods (Dudovskiy, 2018; Kaushik & Walsh, 2019). The views could imply that methods that are best suited to answer research problems can be integrated, these can be all qualitative approaches and the researcher is flexible.

It is commonly viewed that pragmatism is somehow uniquely related to mixed methods research. However, Mertens (2012) argued against schools of thoughts that paradigms are methodological in their foundations as that disregard their constructed nature and undervalue the distinct histories and social contexts of various qualitative and quantitative research methods. Thus, Morgan (2014b) further argued that there is ‘no deterministic link that forces the use of a particular paradigm with a particular set of methods’ (p.1045). This is supported by Denzin (2012) who wrote that

> Pragmatism is not a methodology per se. It is a doctrine of meaning, a theory of truth. It rests on the argument that the meaning of an event cannot be given in advance of experience. The focus is on the consequences and meanings of an action or event in a social situation. This concern goes beyond any given methodology or any problem-solving activity (p. 81).

Therefore, basing pragmatism on mixed methods seems misplaced (Denzin, 2012; Mertens, 2012; Morgan, 2014b). The argument here is that pragmatism can serve as a philosophical paradigm for any research regardless of whether that research uses qualitative, quantitative, or mixed methods.

Pragmatists are aware that every method has its boundaries and that the different approaches can be complementary. Adopting a stance of flexibility, on one hand, helped the researcher in this study to openly engage with a wide range of perspectives (Romm, 2014) to collect and analyse substantial data. Further, flexibility ‘‘can earn researchers trust in their research endeavours insofar as they can signal to others that they are thus open’’ (Romm, 2014, p. 138), thus, contributing to the trustworthiness of the research process and data. Combined research methods on the other hand afforded the researcher confidence and wide scope to better understand the phenomenon and draw sound and informed conclusions. Kivunja and Kuyini, (2017) attested that a combination of research approaches provides a deeper understanding of the research problem. Thus, pragmatist researchers usually reason abductively, that is they move back and forth between deductive and inductive reasoning (Kaushik & Walsh, 2019). Also based on the focus of the study the pragmatic paradigm is either objective or subjective or both

(Creswell & Plano Clark, 2011; Tashakkori & Teddlie, 2008). Moreover, during the process of learning through searching and researching, researchers and the researched become ‘co-investigators’ in the research environment (Creswell & Creswell, 2018).

Moreover, the current study fits in the pragmatic paradigm because the professional development framework for teaching Mathematics meaningfully with technology is designed through a combination of iterative, and flexible research methods and aims to transform professional development. These are surveying the literature, and context analysis through iterative research engagements leading to the design of the professional development framework for teaching Mathematics meaningfully with technology. The present study aligns with the pragmatist epistemology as it views knowledge creation as a means to making a purposeful difference, to ease human existence (Goldkhul, 2012), and a ‘‘desire for a better world’’ (Maxcy, 2003, p. 53). It orientates ‘‘itself toward a prospective world, a world not yet realized’’ (Kaushik & Walsh, 2019, p. 11) and toward solving real problems in the real world. Additionally, the study aligns with the pragmatic paradigm as it argues that it is not sufficient to acknowledge the value of something but acknowledgement should lead to producing change (Barab & Squire, 2004; Goldkhul, 2012; Morgan, 2014b). Such is the quest of the current study to influence and transform the professional development of Mathematics teachers.

Though the paradigm opted in this study has remarkable benefits based on the purpose of the research, there are some shortcomings that are worth highlighting. The concurrent gathering and data analysis in most cases was exhaustive for a sole investigator. Thus, data were collected in stages and cycles, and analysed before the next stage and/ or cycle began to ease the pressure on the researcher.

4.3 Research design

This qualitative study used Design-Based Research (DBR) (van den Akker, 1999) to design a professional development framework for developing teachers' ability to teach Mathematics meaningfully with technology. The process consists of sequential flexible methodologies aimed at generating evidence-based assumptions, artefacts, and practices through iterative analysis, design, development, and improvement, in collaboration with research participants (Barab & Squire, 2004; Dede, 2005; Wang & Hannafin, 2005). Methodological activities are iterated and revised until a satisfying success in practice has been achieved (Barab & Squire, 2004; van den Akker, 1999). DBR is used for refining and developing educational practices including changing and restructuring educational frameworks. Precisely put by Plomp (2013) that DBR serves:

> ‘‘to design and develop an intervention (such as programs, teaching-learning strategies and materials, products and systems) as a solution to a complex educational problem as well as to advance our knowledge about the characteristics of these interventions and the processes to design and develop them…’’(p.15).

The intervention is designed and developed, for practical value. DBR is rooted in philosophies that combine research approaches, to transform, refine and design educational policies and practices (Ndlovu, 2019; Wang & Hannafin, 2005). Further, it advocates for the use of multiple methods to get to a single reality to transform and improve practices. It enables researchers to generate valuable instructive interventions and operative theories for addressing educational issues (Easterday, Lewis & Gerber 2014). DBR relies on approaches used in other research paradigms such as thick descriptive data sets. Moreover, DBR is characteristics (Banerjee, 2016; Cobb, Confrey, diSessa, Lehrer & Schauble, 2003; van den Akker et al., 2006; Wang & Hannafin, 2005) by:

- DBR is process oriented as it aims to develop theories about learning as well as refine interventions intended to support that learning.

- DBR is interventionist in nature; the design process takes place in ecologies as they occur in natural professional development settings.
- DBR is interactive, cyclic, and flexible; this implies the researcher is involved in the design processes and works together with research participants. Also, the development and revision form an iterative process. Several inferences about the research are occasionally challenged and alternative inferences are generated and tested. In DBR changes can take place during the design or implementation, and or evaluation stages.
- DBR is integrative as it uses multiple research methods which improves the trustworthiness of the study. The methods differ at different stages of the research process as the focus of the study evolves and new issues emerge. However, consistency is purposefully maintained and appropriate corrections and changes during the research process are applied.
- DBR is theory-oriented, the design is based on theoretical assumptions. Further, systematic evaluation of sequential patterns of the intervention contributes to theory building.
- DBR is pragmatic, both theory and practice inform the design process.

DBR can be challenging to the researcher since during the design and development process researchers multi-task, stimulate engagements, and make claims about and analyse such engagements (Barab & Squire, 2004); which could be challenging to the researcher. Moreover, data gathering in a DBR process, is carried out in complex, ‘buzzing and blooming confusion of real-life settings’ but in iterative cycles characterized by interventions to produce artefacts meant to improve educational practice (Barab & Squire, 2004; Shavelson et al., 2003).

The generic design research model by Wademan (2005) (Figure 4.1) is used as a benchmark for this study’s design model as it captures design features well (Nieveen, 2007). The model illustrates two complementary ‘‘successive approximations’’ of outputs of the design-based research process. According to Nieveen (2013), the two main outputs

(Figure 4.1) of development design-based research are a product of practical use for interventions and design principles.

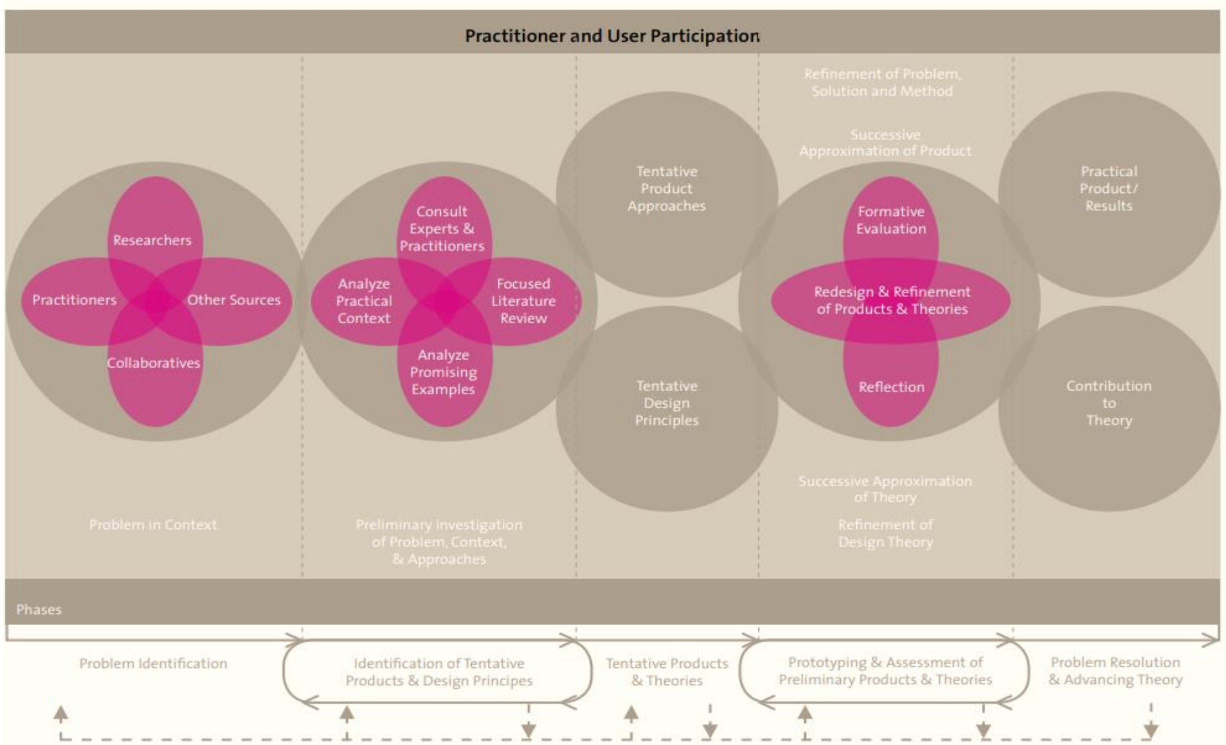

Figure 4.1: Generic Design-Based Research Model by Wademan (2005) (Nieveen, 2007, p.16)

The design principles are ‘‘heuristic statements’’ that provide substantive and methodological aspects for a specific design and development of tasks such that (van den Akker, 1999, p. 9):

> "If you want to design intervention X [for the purpose/function Y in context Z], then you are best advised to give that intervention the characteristics A, B, and C [substantive emphasis], and to do that via procedures K, L, and M [procedural emphasis], because of arguments P, Q, and R."

van den Akker (1999) cautions that design principles are intended to guide but do not guarantee success. Thus, they should continuously be improved and refined through application and critical reflection. In this study design principles (presented in chapter 7) provided the professional development framework for teaching Mathematics meaningfully with technology (intervention X), actionable guidelines for professional development (A, B, and C) and guidelines (K, L, and M) are provided through a designed professional development progression framework for teaching Mathematics meaningfully with technology. Due to the change in educational operations at schools caused by the COVID-19 pandemic and time constraints, the present study was not able to provide empirical evidence (P, Q, and R) of the design principles and product. The current research study, therefore, ended with tentative products of design principles and framework. The design stages are detailed under subsection 4.3.1.

4.3.1 Stages of Design-Based Research (DBR) of the current study

The study adapted three main research design stages and ends with contributions to the theories (Nieveen, 2007) (Figure 4.2). The stages are:

Design stage 1: Literature survey - This stage is also known as the preliminary research stage (Plomp, 2013). This study involved literature analysis related to technology and Mathematics education, philosophical theories, a framework of Mathematics teaching goals, and frameworks related to professional development for teaching Mathematics with technology. This stage ended with implications for the design of the professional development framework for teaching Mathematics with technology.

Design stage 2: Context analysis – This stage is divided into two phases. In phase one the researcher analysed the content of the Namibian secondary school curriculum in terms of Mathematics teaching goals (Kilpatrick et al., 2001) by identifying affordances and constraints in terms of meaningful teaching of Mathematics with technology. In phase two the researcher co-investigated with technologically adept Mathematics teacher participants' (a) current technological pedagogical practices, (b) views and beliefs of the description of meaningful and teaching meaningfully with technology, and technology pedagogy, (c) participants' professional development needs, (d) knowledge necessary for teaching Mathematics meaningfully with technology, (e) aspects of teaching and learning Mathematics, that can and cannot be achieved using technology and (f) influence of participation on beliefs and views about teaching with technology. Phase two was carried out in five iterative phases aimed to promote research credibility, dependability, conformability, and transferability. Stage two ended with implications for the design of the professional development framework for teaching Mathematics with technology.

Design stage 3: Design – At this development stage (Plomp, 2013) conclusions summed as implications for the design of a professional development framework from stages one and two guided the design of tentative products. The tentative products designed are (a) design principles for a professional development framework for teaching Mathematics meaningfully with technology and (b) a professional development progression framework for teaching Mathematics meaningfully with technology. The design principles for the purpose of this study were developed through critically engaging with literature on technology and Mathematics education (Chapter 2), frameworks on Mathematics teaching goals and frameworks related to professional development for teaching Mathematics (Chapter 3) (van den Akker, 1999) and incorporating them with participants' views after iterations. The products in (a) and (b) led to the design of the professional development framework for teaching Mathematics meaningfully with technology.

The process ends with contributions to theory. The study anticipates (as shown with a long dash-dot outline text box) that future research looks at the implementation and evaluation of the professional development framework for teaching Mathematics meaningfully with technology. A professional development framework for teaching Mathematics meaningfully with technology can be effectively and collaboratively developed with technologically adept Mathematics teachers through the DBR approach.

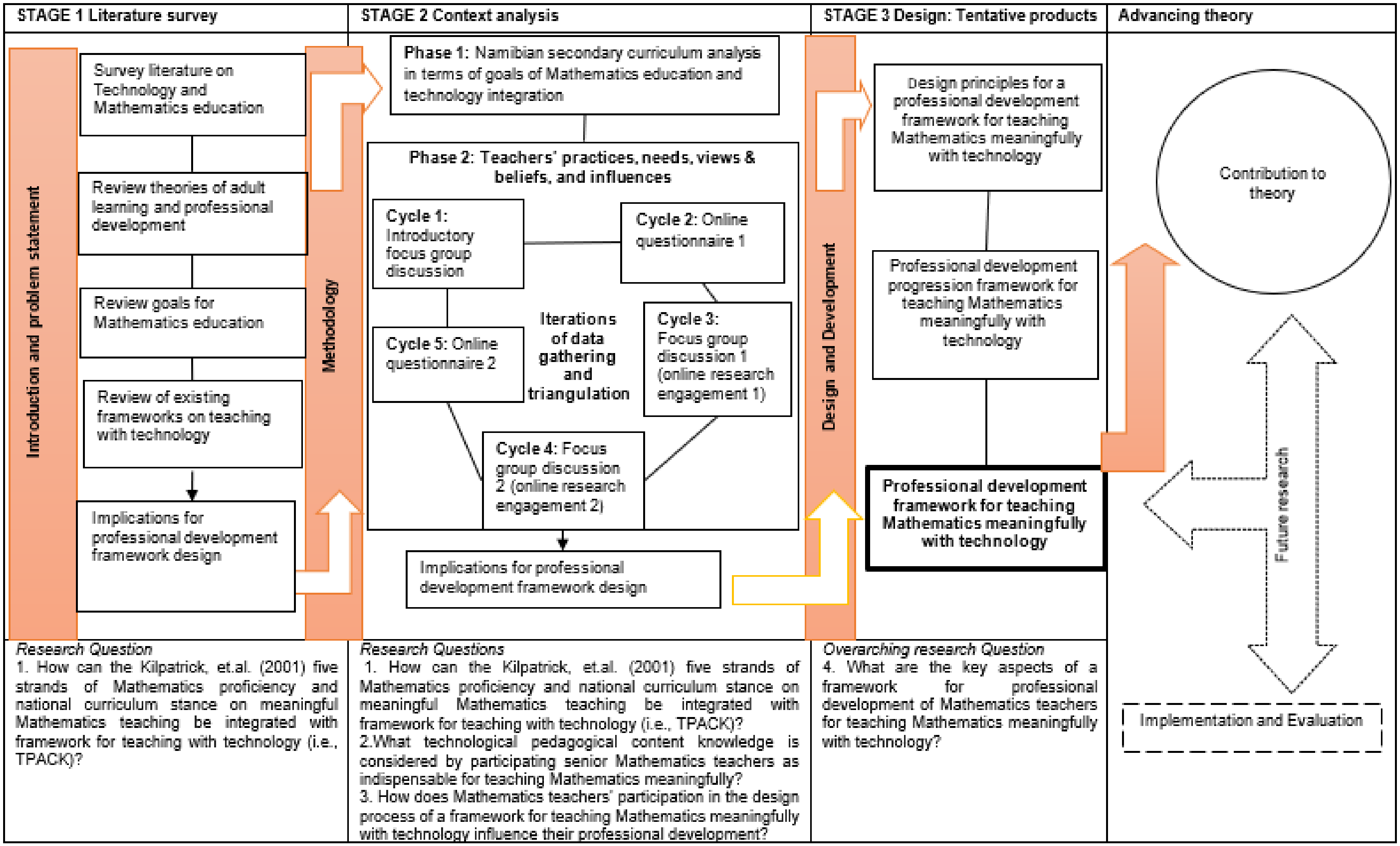

Figure 4.2: DBR process of the study

4.3.2 Suitability of the present research problem into DBR

This sub-section maps out the suitability of the DBR methodology for this research study. Through the DBR process, interventions aimed at reforming teaching are designed, developed, and improved (Bakker & van Eerde, 2013). The current study designs a professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools. It falls within the parameters of DBR because the present study is:

- Pragmatic: The design process of a professional development framework for teaching Mathematics meaningfully with technology is flexible and aimed to reform of professional development of Mathematics teachers in a world dominated by technology.
- Theory-oriented: The framework and design principles are developed from critical engagement with frameworks on Mathematics teaching goals, professional development frameworks for teaching with technology, Vygotsky sociocultural theory, third-generation CHAT, and experiential learning theory. Further, critical reflection on the design process contributed to theory building.
- Context-oriented: The design process is also informed partly by collaboration with technologically adept secondary school Mathematics teacher participants.
- Process-oriented: The study focuses on improving professional development by developing an intervention, that is a professional development framework for teaching Mathematics meaningfully with technology.
- Interventionist: The study seeks to design a professional development framework for a real-life problem – the absence of a professional development framework for teaching Mathematics meaningfully with technology.
- Integrative: In order to gather data, multiple data collection methods which were: surveying literature, online questionnaires and focus group discussions during research engagements were used. It also incorporated formative evaluation during data analysis.

- Interactive, cyclic, and flexible: The researcher in the present study obtained technologically adept Mathematics teacher participants' perspectives who are at the forefront of Mathematics curriculum implementation. The researcher also engaged with Mathematics teacher participants during focus group discussions. The stages consist of literature survey, context analysis and design. At every stage of the design process, reflective analysis is conducted. The data collection and analysis were iterative (*Figure 4.2*) Several inferences were constantly challenged during analysis and alternative inferences were generated. The repetitive sequence and critical reflection during analysis enabled the researcher to change initially planned focus group interviews to focus group discussions during research engagements to collect sufficient and informative data.

- Utility-oriented: The framework is contextualized to the Namibian secondary school Mathematics curriculum.

4.4 Case selection for stage two, phase two of the DBR

This sub-section presents how research experts (participants) were selected.

4.4.1 Selection of participants

Qualitative research typically has a small number of participants, thus it is crucial to select participants that are information rich and would provide relevant information to research questions (Miles & Huberman, 1994; Yamagata-Lynch, 2010). The researcher envisioned about forty (40) secondary school Mathematics teachers who could have taught Mathematics during the first full COVID-19 lock down, which began from 16th March 2020 to 4th August 2020. The Snowball sampling method also known as chain referral is used to locate information-rich participants. With this method, participants of interest were identified by people who know them and they can best provide answers to the questions of the study (McKenney, 1987). Snowball sampling allows the researcher to use informants to identify potential participants, who qualify for inclusion in the study; the participants further could refer others to be part of the sample (Cohen, et al., 2007). Snowball sampling was used for the case selection as participants were spread all over the country of Namibia. Hence, it was challenging for the researcher to know who took the initiative during the lock down. Thus, the sample with traits needed for the study were rare to find.

Potential Mathematics teacher participants were snowballed through senior education officers for Mathematics at regional professional development sub-divisions. These teachers were those who took the initiative to use technology platforms such as *YouTube* channels, *WhatsApp* platforms and *Google classrooms* to teach Mathematics during the lock down. Fifteen of the 34 participants who were identified responded after they were contacted through emails and follow-up cell phone calls or text messages to invite them to participate. These teachers were from the following regions: Oshana, Oshikoto, Ohangwena, Omusati, and Erongo. Figure 4.3 shows the distribution of participants in the country.

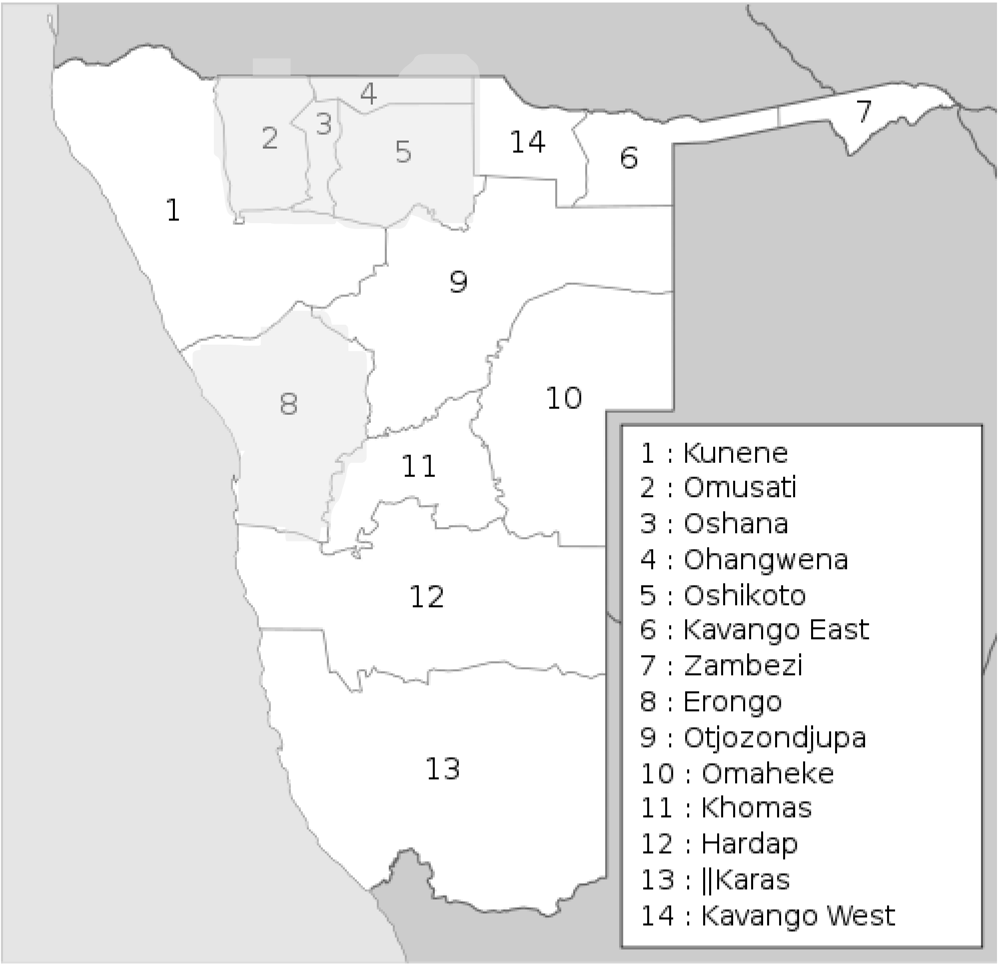

Figure 4.3: Namibian Map showing grey-shaded regions of Mathematics teacher participants

Eventually, in June 2021 a total of 15 teachers indicated their willingness to participate in the research, while six dropped out, and two passed away. The other three were promoted and withdrew their participation and one withdrew without giving any reason. The data collection process started with nine fully consenting Mathematics teacher participants. One of the nine Mathematics teacher participants got promoted to a Mathematics teacher educator and remained in the study on their own accord. Professional developers are professional developers who are envisioned to expand teachers' communities of professional practice.

From cycle three of stage two (*Figure 4.2* and/ or *Figure 4.4*) the study had a sample of four participants, though the researcher communicated to nine consent participants.

Some of the participants could not continue being part of the study due to reasons such as a heavy teaching load and not being up to date with their teaching schedule. The four participants were assigned new pseudonyms, MT1 was used to refer to Mathematics Teacher 1, MT2 to refer to Mathematics Teacher 2 and so forth. The researcher acknowledges that the participants were few in comparison to the size envisioned at the beginning of the study. Hence the study could not make conclusive and empirical statements thus the products designed have tentative status.

4.4.2 The role of the researcher

In this study, the researcher was an observer-participant. The researcher played roles of designing tasks, facilitating research engagements, and engaging with participants on WhatsApp (Addendum V). The researcher has maintained contact with participants on WhatsApp at a research-professional level only, which is to gather additional data. This was necessary since when the researcher maintains contact with the observed outside the role of a researcher it is viewed as an interference rather than an opportunity to gather data (Schwartz & Schwartz, 1955). Further, the researcher also took a reflective stance: carrying out own interpretations, reflecting on own perspectives from other's perspectives, and critically reflecting on own authority as an interpreter and author. During research engagements, the researcher acted to serve as a role model to Mathematics teacher participants as learners by modelling (a) being the 'hand that holds the mouse' while Mathematics teacher participants are the 'mind that directs the mouse' and (b) allowing participants to experience and explore mathematical concepts using a logo on GeoGebra and (c) providing guidance, stimulating questions when necessary and allowing Mathematics teacher participants to use GeoGebra without intervening.

4.5 Data collection process

After engaging in a document analysis protocol (4.6.1) of the Namibian secondary school Mathematics curriculum. Phase two data collection attempted to answer research number two and three, which are:

> **Research question 2:** What technological knowledge, content knowledge and pedagogical content knowledge are considered by participating Mathematics teachers as necessary for teaching Mathematics meaningfully with technology?
> **Research question 3:** How does Mathematics teachers' participation in the design process of a framework for teaching Mathematics meaningfully with technology influence their beliefs and views about teaching Mathematics with technology?

Data in stage two phase two were collected in five cycles (Figure 4.4. Cycle one involved an introductory focus group discussion (4.6.2) followed by an online questionnaire (cycle two) (4.6.3). Cycles three and four were focus group discussions (4.6.4) conducted during an online research engagement. This was followed by an online questionnaire (cycle five). All online questionnaires were uploaded and accessible at the Stellenbosch University Surveys platform (SUNSurveys).

For cycle one, after participants have consented, they were invited through emails and text messages to attend an introductory focus group discussion meeting via the ZOOM meeting application. This was necessary for the researcher to make acquaintances with participants; provide clarity on research information, respond to possible questions, and discuss participants' current teaching practices. The link to an online questionnaire (for cycle two data collection) was sent through an email, as a text message and posted on the WhatsApp group after the ZOOM meeting. The data collected through an online questionnaire (cycle two) lasted for four months (Figure 4.4) due to the lockdowns thus longer time allowed participants more time to complete the online questionnaire.

The focus group discussions for cycles three and four of data collection took place through MS teams. For cycle three (first group discussion (Figure 4.4), Mathematics teacher participants engaged with tasks related to geometry and thereafter had a focus group discussion to reflect on the tasks with questions during the research engagement. The tasks aimed to stimulate Mathematics teacher participants' imagination to teach Mathematics meaningfully with technology. The focus group discussions stemmed from contradictions in the responses of the participants in cycle two, the online questionnaire (Addendum C). The questions aimed to determine Mathematics teacher participants' professional development needs. That was necessary as part of data triangulation, also in a DBR design stages are informed and influenced by the previous stage's data. The researcher attempted to expose Mathematics teacher participants to tasks that would likely expand their worldview and stimulate their imagination about teaching Mathematics meaningfully and teaching Mathematics meaningfully with technology. Thereafter, focus group discussions on their professional development needs, and reflection on the rules in terms of teaching Mathematics meaningfully with technology were held.

In the cycle four group discussion, the researcher shared a word document consisting of pictures, figures, and specific objectives as per the Namibian syllabus on geometrical terms and relationships (Addendum E). During cycle four of data triangulation, Mathematics teacher participants were first shown a video about the logo (Addendum E). Secondly, they break out for 20 minutes to individually 'explore mathematical concepts embodied in the logo and present a meaningful teaching experience with technology (GeoGebra) (Addendum E). They were also to share questions that guided their explorations. Three of the Mathematics teacher participants used GeoGebra on a laptop and mobile phone. One of the four Mathematics teacher participants opted to use pen and paper due to limited access to technology. Although, the study focussed on technology and meaningful teaching, the Mathematics teacher participant without access to the laptop was allowed to use pen and paper as teaching Mathematics meaningfully is not exclusive to using technology. The research engagement was repeated on the following day as one of the four participants could not attend the first scheduled session.

In the last data collection cycle (cycle five), data were collected through an online questionnaire. An email with a SUNSurveys link was emailed to the Mathematics teacher participants a week after the research engagement. A week was necessary to allow Mathematics teacher participants to possibly adapt the 'new' teaching experiences into their present teaching and may be in a better position to make critical suggestions. The completion of the cycle five online questionnaire lasted for two weeks, as all Mathematics teacher participants who participated in the online research engagements completed the questionnaire.

All online meetings were recorded, for the researcher's reflection. It is notable that, missed opportunities to probe were remediated by contacting appropriate Mathematics teacher participants individually through WhatsApp messenger. Mathematics teacher participants who did not participate in the research engagements were not considered to complete the cycle five online questionnaire.

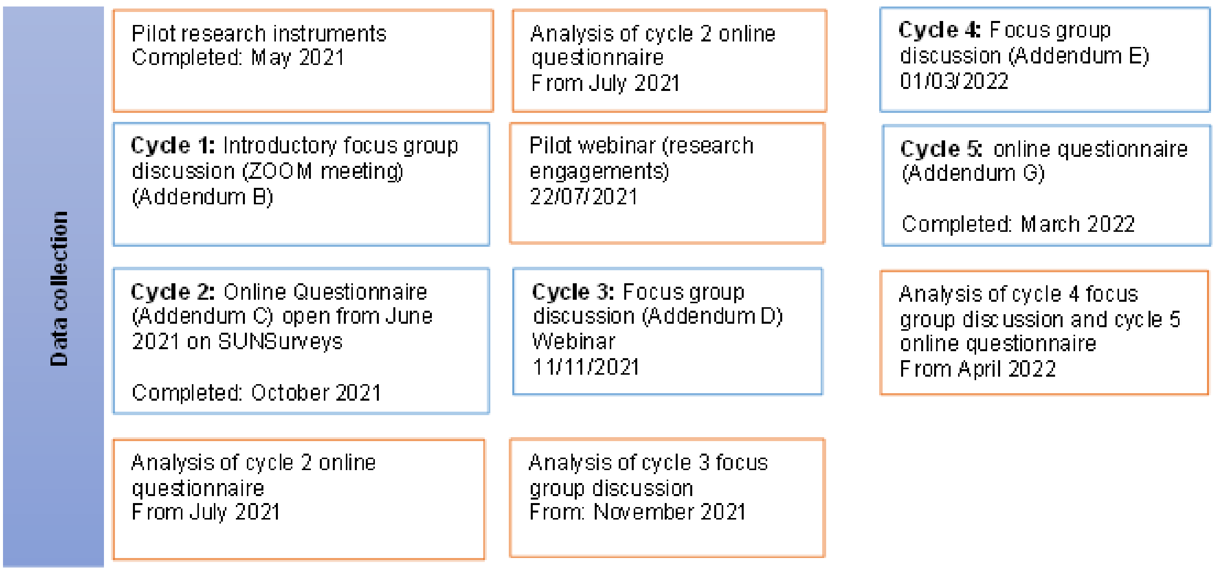

Figure 4.4: The research process of stage two phase two

4.6 Organising online research engagement during the COVID-19 pandemic

The researcher planned the study to be conducted on a face-to-face mode with Mathematics teacher participants. However, COVID-19 constraints forced the meeting to take place online instead. As a result, the 'KLeenaPhD study' WhatsApp group was created with eight consented Mathematics teacher participants, this was done after contacting them through cell phone calls and emails. Cell phone calls were necessary to remind participants of the research process, objectives, and ethics. The role of the WhatsApp group was summarised in the group description box (Addendum V). As the research process progressed, at cycle four group had six members including the researcher and research supervisor. The WhatsApp group was envisioned to be a platform for communication as well as discussions related to the research questions. However, collecting data through the group was futile as participants, although few do not partake in the topics for discussions. Thus, the group was mainly used to supplement communications that were sent via emails about professional development dates and links between the researcher and participants. It was also used as a platform to discuss viable dates and time slots (Addendum V) for research engagements.

All focus group discussions took place through the MS teams meeting application. There is insufficient literature on the benefits of meeting application platforms including MS teams (Chia et al., 2021). As seem like a common practice people opt for a meeting application depending on the host’s preference, access, convenience, and economic affordances. Correspondingly, in the current study, the MS teams meeting application was used as it was the only meeting application the researcher had access to conduct a meeting for more than forty-five minutes without being disconnected. It also allowed transcription of the live meeting.

Some participants had little knowledge of the usage of the MS teams meeting application at the beginning, the researcher observed that participants’ knowledge gradually improved throughout the research process in comparison to the first meeting. This could be attributed to guidance provided by the researcher and possibly awareness of the affordances of the meeting application intrigued self-learning. It could also be that Mathematics teacher participants might have gained the skills through engaging with the application during the research process. Tasks used during webinars were shared through emails with participants a week before the session. The intention is to give participants ample time to familiarise themselves with the resources to promote discussions. The first focus group discussion (cycle three) took place on 11 November 2021 at 17:30 for one hour and thirty minutes a month after the closing of the online questionnaire.

The second focus group discussion (cycle four) was supposed to take place in December 2021 before school closes for the academic year. The researcher also believed that the timing was appropriate as seem to be less work at the end of the academic year since most of the teaching and learning has come to halt. However, though participants confirmed and agreed on the date, only two participants connected, and others were already either on leave or busy with administrative work at school or have issues with access to technological resources. The researcher inquired participants who attended whether they had incorporated the idea of meaningful teaching with technology, to which they indicated they hadn't as there was not enough time. The question was necessary to determine the influence that the design process so far had on participants' teaching.

Eventually, the second focus group discussion webinar which aimed to immerse teachers as learners in the art of meaningful teaching and learning of Mathematics with technology took place after telephonic negotiation conversations with participants' immediate supervisors and school principals. The researcher was then allowed to conduct the research engagement from 11:00 am to ensure minimal disruption of the school day. At 11 o'clock Mathematics teachers had taught more than sixty per cent (60%) of the lessons (five lessons out of eight lessons) per day, depending on the timetabling format. Moreover, Mathematics teacher participants might not be too tired to attend the online professional development webinar. The focus group discussion was repeated the following day, for one hour and thirty minutes as one of the Mathematics teacher participants had personal commitments.

4.7 Research methods

This study used multiple qualitative research methods to gather data, meant to design a professional development framework for teaching Mathematics meaningfully with technology. Multiple methods also known as multi-method design imply merging two or more qualitative approaches to answer research questions in a single research study (Morse, 2003). Approaches can be conducting multiple focus group discussions and open-ended questionnaires in a single research study. The approaches can be implemented concurrently or sequentially (Driessnack, Sousa & Mendes, 2007) depending on the research design. The methods are selected based on research questions, theoretical framework as well as the researcher's knowledge about the research site. Different qualitative approaches that were implemented sequentially at stage two and are discussed sequentially from 4.6.1 to 4.6.4:

4.7.1 Document analysis protocol

The document analysis protocol was used to analyse the content of the texts contained in documents to draw inferences and conclusions about them (Prasad, 2008). Although, a useful way of collecting data as it is in most cases readily available in the public domain, document analysis is non-reactive and involves one-way communication. Thus, researchers in their analysis should strive to be objective, and systematic and the results obtained should be generalisable (Prasad, 2008).

The document analysis protocol (Addendum A) was used in phase one of design stage two to analyse the Namibian secondary school Mathematics curriculum documents. The curriculum documents analysed were the (a) national curriculum for basic education (Ministry of Education, Arts and Culture, 2016), (b) Mathematics subject policy for grades 4 to 12 (Ministry of Education, Arts and Culture, 2019) and (c) the secondary school Mathematics syllabuses for the junior secondary(grades 8 & 9) (NIED, 2015), ordinary level (grades 10 to 11) (NIED, 2018) and the AS (grade 12) (NIED, 2020) syllabuses. From the secondary school syllabuses, the researcher analysed the content to identify affordances and constraints in terms of Mathematics teaching goals (Kilpatrick et al., 2001). This was done by identifying Key Proficiency Terms (KPTs). Identifying KPTs is key, and educators make pedagogical decisions by engaging with proficiencies to explore the breadth and depth of mathematical concepts (Sullivan, 2012). Key proficiency terms (KPTs) extracted were those that "can be thought of as verbs" (Sullivan, 2012, p. 179) from the learning content descriptions. As a result, KPTs embedded in the learning content descriptions for grades 8 to grade 12 (National Institute for Educational Development (NIED), 2015, 2018, 2020) related to each strand of mathematical understanding were identified (Addendum A). These were marked off and assigned to a strand of mathematical proficiency. KPTs related to each strand of mathematical proficiency were extracted by marking them off using review tab in the MS word and categorized according to strands of mathematical proficiency. Categorisation was made after reading and re-reading full wording description in the grades 8 to grades 12 syllabuses. Key proficiency terms appearing more than once on the same strand of mathematical proficiency was only recorded once to avoid repetition.

In terms of affordances and constraints of teaching Mathematics with technology, the terminologies: ‘‘technology’’, ‘‘technologies’’, ‘‘calculator’’, ‘‘computer’’, ‘‘software’’ (Goos, 2012), ‘technological’, ‘Information Technology (IT)’, ‘technological applications’, and ‘mathematical software’, were electronically searched from the all the national curriculum documents (Ministry of Education Arts and Culture, 2016, 2019; National Institute for Educational Development (NIED), 2015, 2018, 2020) as well as from the three Mathematics syllabuses using ATLAS.ti.

4.7.2 Introductory focus group discussion

Focus group discussions although fall short of anonymity, they are useful for obtaining collective feedback from participants (Ndlovu et al., 2013). Focus group discussions aim to understand better how people consider an idea and/ or an experience (Ndlovu, 2019) and stimulate their imaginations about it. In the current study, the research process began with an introductory focus group discussion (Addendum B) on ZOOM. The introductory focus group discussion was planned to explain the purpose of the study (about 20 minutes) and then to get information about Mathematic teacher participants’ current teaching practices. Four out of nine Mathematics teacher participants attended the ZOOM meeting. One emailed the response on teaching practices in reaction to the invitation email sent. The remaining four were asked to talk about their current practices during the first online meeting they attended (on the MS teams meeting application, 11/11/2021). The phrase ‘teaching Mathematics meaningfully’ was visible to the participants in the title slide on the shared screen on ZOOM (Figure 4.5) (Addendum V). No theoretical information was given, and the researcher was careful to simply manage the turn taking so as not to show bias.

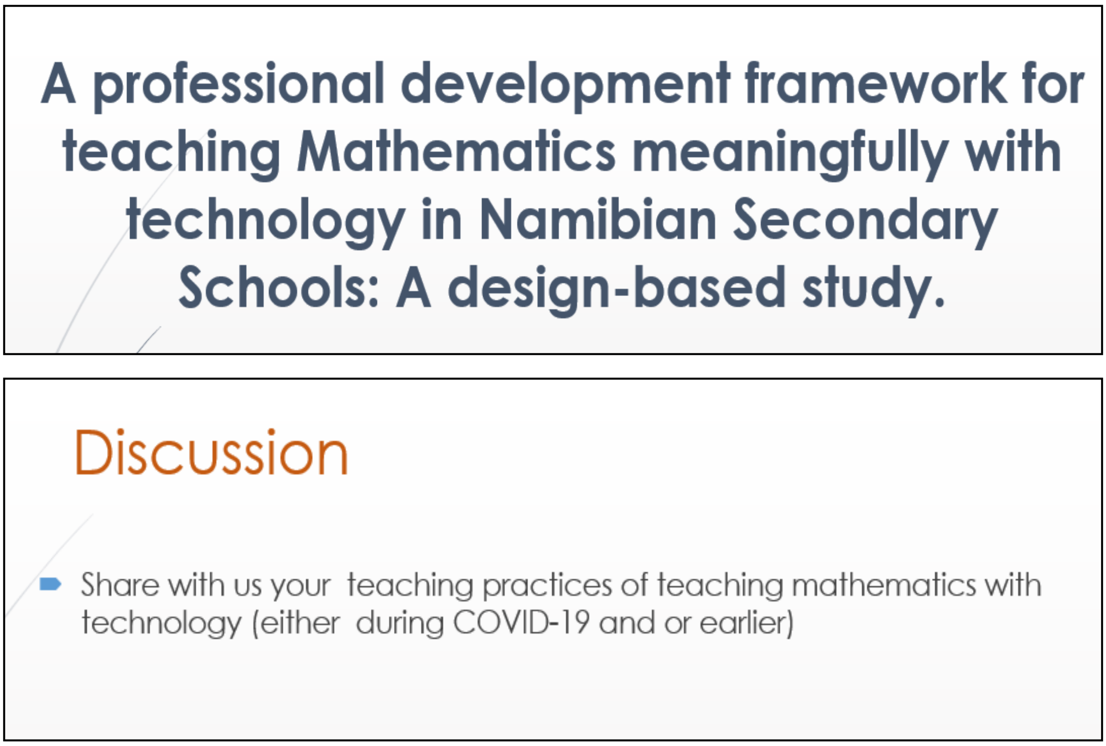

Figure 4.5: Two screenshots of the slides shown during the introductory discussion on ZOOM

4.7.3 Online questionnaires

Online questionnaires are flexible ways of collecting data. They offer an effective way to relatively inexpensively gather data from a large geographic region (Rikala, 2015). However, online questionnaires can be challenging as participants may find it difficult to navigate through the interface in case it is not user-friendly. Thus, researchers should always ensure that participants get to access the questionnaire through ‘one-click’. They should also test whether the link can easily lead to accessing the question items before sharing it with participants.

In this study two online questionnaires were used to collect data at phase two, that is cycle two and cycle five. In cycle two an online questionnaire (Addendum C) was used to investigate Mathematics teacher participants’ technological, pedagogical, and mathematical knowledge which they considered necessary for teaching Mathematics meaningfully with technology (research question number two). This was done by asking them their (a) current practices, (b) views and beliefs of the description of meaningful and teaching meaningfully with technology (c) professional development needs, (d) knowledge necessary for teaching Mathematics meaningfully with technology, and (e) aspects of teaching and learning Mathematics, that can and cannot be achieved using technology. The questionnaire was developed based on frameworks related to professional development for teaching Mathematics with technology and the framework of Mathematics teaching goals. The questionnaire had two sections A and B. Both sections consisted of open-ended items. Section A focussed on determining Mathematics teachers' beliefs on meaningful teaching of Mathematics, while Section B on Meaningful professional development needs.

Moreover, the last online questionnaire (cycle five) (Addendum F) attempted to collect data for research question number three. This is to investigate teachers' views about how their participation in the design process of a framework for teaching Mathematics meaningfully with technology influenced their views and beliefs about teaching Mathematics with technology. The questionnaire had seven open-ended items (Addendum F).

4.7.4 Focus group discussions

The focus group discussions stemmed from contradictions in the responses of the participants on cycle two an online questionnaire (Addendum C). The contradictions were used as a springboard to raise issues of 'misinterpretation' treated as such that the purpose of the study and concepts within it were not well understood and calls for addressing issues of collaboration. The researcher took a transformative orientation to schedule research engagements to conduct focus group discussions on MS teams which were recorded and analysed, to re-plan research engagements. As a result, two focus group discussions (at cycles three and four) were conducted with Mathematics teacher participants during the research engagements to ensure the trustworthiness of the data. Initially, cycle three focus group discussion was planned to be held with selected participants based on responses to cycle two online questionnaires to improve the trustworthiness of the data. However, Consequently, Mathematics teacher participants were exposed to mathematical tasks (Addendum D and Addendum E) to (a) imagine learning and teaching Mathematics in the surroundings and through objects (b) imagine a new way of teaching Mathematics concepts, (c) look at the world and its object with mathematical eyes, (d) make objects with mathematical properties for problem solving and strategic competence, (e) stimulate views to see Mathematics as useful and beautiful and (f) afford an opportunity to critically reflect on own existing teaching and learning of Mathematics.

The tasks were based on geometry as teaching geometry is not too much different from teaching Mathematics in general (Freudenthal, 1973) and geometry supports the teaching of other topics in Mathematics (Serin, 2018). Hence, the success in teaching geometry could be used in teaching other Mathematics topics too. The objective below from the Mathematics secondary syllabus (National Institute for Educational Development (NIED), 2018) was used as a guide for exploration and not limited to (NIED, 2018):

> ‘‘Define, use, and interpret geometrical terms angle, parallel, intersecting, bearing, right angle, acute, obtuse, and reflex angles, perpendicular, similarity, congruence’’ (p.15).

Additionally, in cycle four the researcher engaged Mathematics teacher participants in exploring geometrical concepts with GeoGebra to stimulate Mathematics teacher participants’ views about teaching Mathematics meaningfully with technology. The GeoGebra software was used as a benchmark to influence teachers’ knowledge to teach Mathematics meaningfully with technology. GeoGebra was chosen as it is an open source software, available free of charge for non-commercial users (Geogebra, 2021). Further, the software has some traction among Namibian teachers (e.g used in Mwiikeni, 2017; Rodrigues Losada, 2021; Waiganjo & Paxula, 2020). The use of GeoGebra is also supported by Kokol-Voljc (2007) as backing mathematical software that can be used in technological teaching approaches. The software allows teachers to design tasks that demand cognitive development. It also offers teachers an opportunity to engage in the design of teaching and learning resources, and environment, and acquire ownership of the tasks as GeoGebra is not a ‘ready-made’ material. The challenge of using GeoGebra software is, it is hard to tailor to specific teaching and learning needs however, the learning objectives and Mathematics teacher participants thinking eased and guided how the software is to be used. This concurred with Hoyles and Noss' (2009) view of the bi-directional relationship between tools and users, where thinking is shaped by both the technological tool and how the tool functions.

The geometry tasks with GeoGebra demanded Mathematics teacher participants to experience and 'explore mathematical concepts embodied in the figures and share their views about a 'meaningful teaching and learning experience with technology (GeoGebra) (Addendum E and F). The researcher also allowed flexibility to use pen and paper which deviates from the focus of the study. The deviation was necessary as two participants connected through mobile smartphones. Moreover, teaching Mathematics meaningfully with technology is not exclusive to the use of technology.

All cycles three and four focus group discussions were conducted during and after the research engagements.

4.8 Data analysis

The data gathered at stage two was analysed as follows:

4.8.1 Stage 2 Phase 1

The document analysis protocol of the Namibian secondary school curriculum was analysed using content analysis. Content analysis is a descriptive tool of messages contained in documents ranging from the basis of word counts to categorization (Cohen et al., 2007; Maier, 2017). The researcher with reference to the research problem used tools to identify the frequency of words, ideas, meanings, themes and concepts in documents of interest (Carley, 1990; Prasad, 2008). Content analysis may also include empirical analysis, describing features and drawing out implications (Carley, 1990; Maier, 2017). Content analysis was appropriate as it enabled the researcher to locate areas in the documents with concepts and themes of interest, as well as to evaluate the meaning communicated (Maier, 2017). KPTs that are separated by an 'and' were counted as two as it demands that learners are developed in the ability to carry out both actions. Further, terms such as efficiently, accurately, and appropriately are adverbs and were included as KPTs if they modified a verb in the learning content description (Mccluskey et al., 2016). KPTs were extracted and categorised according to strands of mathematical proficiency and per level. The identified KPTs from the Namibian secondary school curriculum content were summarised in graphs, and tables of frequency counts and percentages for data triangulation. KPTs per proficiency strand are added together to determine which strand the curriculum is more inclined.

In addition, two research-based frameworks were used to examine the level that the Namibian secondary school Mathematics curriculum affords and constraints Mathematics teachers to teach with technology. The first framework, SAMR, (Puentedura, 2010) is used to gauge the extent to which technology currently transforms teaching and learning roles. The second framework was the pedagogical map (Pierce & Stacey, 2010) to categorize pedagogical affordances by a technology tool in terms of changes to tasks and the teaching of Mathematics content subject.

4.8.2 Stage 2 phase 2

This stage consists of five cycles. Data collected during an introductory focus group discussion (cycle 1) was presented in a tabular form. The data collected through online questionnaires and focus group discussions (from cycles 2 till 5) was analysed as follows:

1. Firstly, within the week after data collection has ended, the researcher immersed self in the data. In the case of the online questionnaires, the researcher immersed by re-reading and taking notes of the downloaded responses in an MS excel file from the SUNSurveys platform. In terms of research engagement webinars, the researcher, immersed by listening and viewing the downloaded MS teams recording in preparation for transcription.
2. Secondly, responses were grouped per question item. Verbatim quotations were used to strengthen the reporting of data and to enhance understanding of the data.

3. Thirdly, the first round of analysis of all cycles' data was submitted to the research supervisor. After, a discussion with the research supervisor, the researcher immersed self again and performed data clean-up. Since Mathematics teacher participants are second language English speakers, some of the computer transcriptions made by the MS teams meeting application do not correlate with what participants said and these were corrected by the researcher based on the video and audio recording.

For example, this automated transcript (*Figure 4.6*) was corrected as follows:

Manipulation, 'manipulative tool'. It allows us to demonstrate and provide proof without assuming as it gives evidence on the magnitude of turn unlike using pen and paper.

4. Meaningless or clearly incorrect words and phrases from the MS teams' transcript were included in square brackets and struck through (Figure 4.6).

00:36:05.920 --> 00:36:58.820	MtA	Manipulation [~~tool~~} *to manipulate*, [~~Any blade~~] things like you can be able to move a line, To demonstrate, like assuming. You are moving this the, the, the, the (*stammering*) [~~season they~~] *scissor you* are closing, [~~their faces~~] *the scissor* which you cannot do really do on paper and, and at the same time is showing the measurement [~~I like~~] *Unlike* on paper, even if you have a [~~-season~~] *scissor* now, you the [~~then asking Nazi~~] *learners cannot see*, yeah, they can see the size of angle changing but they cannot even see the magnitude measurements. It's providing us with a lot of, through manipulation is providing us with a lot of Uh, uh, information.

Figure 4.6: Snippet of MS teams automated transcript

5. Two columns were added for analytical notes, namely aspects related to the concept meaningful as defined for the study, aspects of TPACK for professional development related to Mathematics teaching and activity theory node (aspect) (Addendum C and D).

6. Tables and figures were used to condense the data which were analysed from sentence units as a unit of analysis.

7. Lastly, the researcher harmonised the first analytical report with the refined transcript. A revised analytical report was written and shared with the research supervisor, a couple of more times.

The activity system triangular model was developed from teachers' narratives and was used as a visual and conceptual tool to help communicate and interpret the data. The activity system triangular model enhanced understanding of teachers' activity system. Further, the model helped to identify and discuss systemic contradictions and tensions that can drive changes in professional development activities (Engeström, 2001). A coding frame (Addendum C and D) was developed based on the research questions, learning theories and frameworks of the study as follows.

Development of codes and coding frame

The researcher conducted an iterative thematic analysis to develop a coding frame (Addendum C and D). A coding frame constitutes all codes through which raw data were summarised, categorised, and clustered, accompanied by a description as operationalised in the study (Cascio et al., 2019; Fonteyn et al., 2008; Geisler & Swarts, 2019). The frame helped to answer research questions two and three, that is to identify Mathematics teacher participants' (a) current technological pedagogical practices, (b) views and beliefs of the description of meaningful and teaching meaningfully with technology, and technology pedagogy, (c) participants' professional development needs, (d) knowledge necessary for teaching Mathematics meaningfully with technology, (e) aspects of teaching and learning Mathematics, that can and cannot be achieved using technology. It was also used to determine (f) how Mathematics teacher participants' participation in the design process of a framework for teaching Mathematics meaningfully with technology influenced their beliefs and views about teaching Mathematics with technology.

The coding frame was applied systematically to the data collected through online questionnaires and focus group discussions (O'Connor & Joffe, 2020). The researcher allowed conceptual frameworks to drive analysis, just as analysis will guide the design of the professional development framework for teaching Mathematics meaningfully with technology. The frameworks that illuminated meaningful teaching with technology guided the analysis. Although the frameworks informed data collection and analysis, the analysis began with inductive open coding to allow the narrative about professional development for teaching Mathematics meaningfully to develop through and from the raw data. The coding frame was developed through three phases discussed by Alhojailan (2012). Codes were assigned levels, beginning from level 1 to 3, for ease and to depict complexity:

Level 1 Initial codes were descriptive, based on making judgements about the meaning of the sections of words and phrases participants used in their responses. This was done with reference to research question number two (current practices, description of meaningful and teaching meaningfully with technology; participants' professional development needs and knowledge necessary for teaching Mathematics meaningfully with technology) and research question number three (the influence of Mathematics teacher participants' participation in the study on their beliefs and views about teaching with technology). The analysis was done by marking off, assigning codes, and adding comments using the review tab in MS word. While coding, the researcher also noted down memos. The first draft was written based on the initial codes. Emergent themes were discussed extensively and re-read to establish any evidence that could be reputed.

Level 2 codes were linked to technology, teaching and Mathematics knowledge, meaningful aspect, and professional development aspect.

Level 3 codes are broader and focused on the properties and dimensions of Level 2 codes. Level 3 codes were the nodes of the activity system theory. The activity system nodes were used as a planning tool for a holistic design of the framework.
The level 1, 2 and 3 codes all contributed to the design of the professional development framework for teaching Mathematics meaningfully with technology.

The data from stages 1 and 2 were harmonised and interpreted to design a framework (chapters 5 and 6).

4.9 Trustworthiness of the research

The trustworthiness of the research is a crucial component of the design research process. Trustworthiness also known as rigour, is the confidence that the data collection methods and analytical techniques employed ensured concurrently that the findings are accurate and reliable (Connelly, 2016; Plomp, 2013). In 1985 however, Lincoln and Guba proposed four commonly used criteria by qualitative researchers (Connelly, 2016). These criteria are credibility, dependability, conformability, and transferability (Kivunja & Kuyini, 2017; Lincoln & Guba, 1985). Similarly, Nieveen (2013) proposed four generic criteria for assessing the quality of interventions (framework and design principles) developed during a DBR. These are relevance, consistency, practicality, and effectiveness. However, these criteria are more relevant in the evaluation of DBR (Nieveen, 2007; Plomp, 2013) while the present study ended with the design. Thus, the trustworthiness of this study is discussed based on four components credibility, dependability, conformability, and transferability.

Credibility refers to confidence in the truth of the study findings. It involves the use of multiplicity to test the data, including data triangulation (using different data for one/same phenomena), methodological triangulation (using multiple data collection instruments and analysing), and theoretical triangulation (using various multiple theoretical schools of thoughts to understand the data or direct research, prolonged engagement with study participants, member checking, peer debriefing. In this study, credibility was achieved by using triangulation approaches (Kivunja & Kuyini, 2017). *Dependability* can be achieved through peer debriefing which involves colleagues or field-based researchers commenting on the study. Moreover, *transferability* refers to the replicability of the study and it is analogous to the generalisability of quantitative research (Lincoln & Guba, 1985). However, qualitative researchers are more interested in the quality of the data, thus transferability is not concerned with generalisation or sample size (Guba, 1981). *Confirmability* in research attempts to achieve objective reality (Stahl & King, 2020) to ensure that the findings are shaped by the respondents and not by the researcher’s assumptions.

The trustworthiness of the research is presented in chapter 8 under 8.4. However, the trustworthiness of the research instruments is presented under 4.8.1

4.9.1 Trustworthiness of research instruments

The research instruments were piloted to determine whether they measure what will lead to rich data and to ensure that questions are not misinterpreted. All online questionnaires were sent to a Mathematics teacher who did not participate in the completion of any of the questionnaires. That was done to help the researcher become familiar with the administration of the instruments, and how best to analyse the information obtained (Gay et al., 2009). Questions that were difficult to understand and/ or a repetition were removed and/ or rephrased. The question:

> 'What are your suggestions for successful professional development on technology to develop mathematical understanding in learners?'

Was removed as it does not relate to designing a framework but more to the effectiveness of conducting professional development. Further, in cycle five (stage 2) online questionnaire (Addendum F) questions that were worded relating to the evaluation of a professional development framework were removed as the framework was designed in stage 3.

Moreover, on the tasks that were used during cycle three and four group discussions, the researcher initially presented a task-based presentation on the Namibian secondary Mathematics AS level:

> 'Interpret the idea of the gradient of a curve, and use the notations f'(x), f''(x), $\frac{dy}{dx}$, $\frac{d^2(y)}{dx^2}$ (the technique of differentiation from first principles is not required)'' (NIED, 2020, p.9).

The task was presented to professional developers from the professional development sub-division of the MoEAC and to the research supervisor for natural validity. Presenting the task to others helped the researcher to promote credibility and dependability. Also to ensure that the researcher effect changes to any part of the plan, in case not sufficiently detailed (Barab & Squire, 2004; van den Akker et al., 2013). The professional developers observed that the content was too complex as not all Mathematics teachers teach AS Mathematics and thus more suitable for professional developers. They also observed that the pedagogical approach used was more of guiding and advising instead of developing teachers' knowledge to teach Mathematics. Thus, the specific objectives of the tasks were amended to Geometry on the ordinary level. Henceforth, the task was first presented to the research supervisor who provided support and guidance on how the tasks can be improved and questions can be phrased before they were implemented in the research engagements.

4.10 Ethical considerations

The study began with the analysis and interpretation of qualitative data from the Namibian secondary school Mathematics curriculum available in the public domain. Qualitative data was also obtained by engaging with Mathematics teachers as experts employed by the MoEAC. Ethical issues including those relating to data storage, sharing, and engaging at online platforms were addressed throughout the study as at every stage the researcher anticipates ethical issues. The researcher considered the following ethical protocols as follows (Figure 4.7).

Prior to data collection, the researcher first obtained ethical clearance from the Stellenbosch University ethics committee (Addendum H). Secondly, permission was sought from the MoEAC through the Executive Director of Education (Addendum I). After permission was obtained from the Executive Director of Education (Addendum J) regional directors of education of regions with potential technologically adept Mathematics teacher participants were communicated to through a letter sent via an email (Addendum M). After permission (Addendum H to T) was granted by regional Directors of Education, requests for permission letters and information about research (Addendum L) were emailed to principals and a supervisor where potential Mathematics teacher participants work. Requesting permission from supervisors was necessary as one of the participants was promoted to a professional developer during cycle four of data collection. Potential Mathematics teacher participants were thereafter emailed the research information (Addendum M) and consent forms (Addendum N). It is notable that during 2021 the schools experienced unpredictable

COVID-19 lock downs. The teachers were unsettled and under much pressure during this period in terms of completing teaching schedules (syllabuses). Most had inadequate access to an internet connection which affected the duration of data collection and the whole research schedule. A second formal request and letter of invitation (Addendum M) for potential Mathematics teacher participants were emailed to supervisors and school principals.

Potential Mathematics teacher participants were asked to participate in this research of their own free will. Therefore, no person was forced or threatened to participate or to continue to participate. Mathematics teacher participants who completed the first online questionnaire (phase two cycle two) of data collection were assigned a pseudonym to protect their identity such as MtA to denote Mathematics Teacher A, MtB to refer to Mathematics Teacher B and so on. During phase two cycle three, Mathematics teacher participants were assigned new codes to avoid confusion after the sample decreased. Thus, MT1 was used to denote Mathematics Teacher 1, MT2 for Mathematics Teacher 2 and so on. The number of participants remained the same until the end of data collection. The data was collected after official working hours to avoid disruptions to the everyday operations of educational institutions. Except in the instance in which a prior appropriate arrangement was made and agreed upon. The data collected is kept in encrypted files and stored on google drive and one drive. They will be maintained for five years and will be destroyed after that period using online shredders.

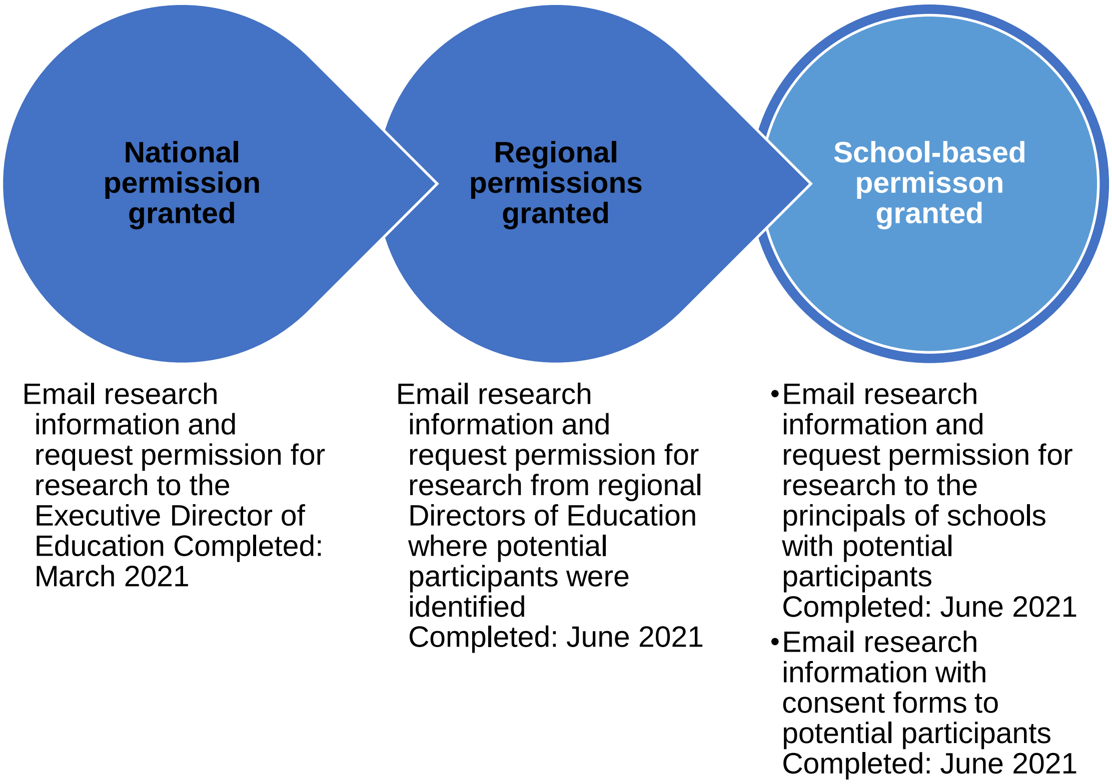

Figure 4.7: Ethical considerations sequence of events

4.11 Summary

Complementarily to the sociocultural theory, third-generation CHAT and experiential learning theory (Chapter 3), the pragmatic research paradigm view that knowledge is socially constructed, and actions cannot be separated from situations and contexts in which they take place. There are multiple realities, hence multiple interpretations of the world, and ways of conducting research, and the researcher is flexible. The design of the professional development framework for teaching Mathematics meaningfully with technology is situated within the pragmatic paradigm. The current study fits in the pragmatic paradigm because the professional development framework for teaching Mathematics meaningfully with technology is designed through a combination of iterative, and flexible research methods and aims to transform professional development. The researcher view that knowledge creation should make a purposeful difference and improve human existence for a better world. Thus, the current study attempted to contribute and improve the teaching of Mathematics through reforming the professional development of Mathematics teachers.

Qualitative iterative research methods were used to

1) Explore the Namibian curriculum in terms of Mathematics teaching goals (Kilpatrick et al., 2001) and identify affordances and constraints in terms of meaningful teaching of Mathematics with technology
2) Explore Mathematics teacher participants' (a) current technological pedagogical practices, (b) views and beliefs of the description of meaningful and teaching meaningfully with technology, technology pedagogy, (c) participants' professional development needs, (d) knowledge necessary for teaching Mathematics meaningfully with technology, (e) aspects of teaching and learning Mathematics, that can and cannot be achieved using technology and (f) the influence of Mathematics teacher participants' participation in the study on their beliefs and views about teaching with technology.

The iterative methods assisted the researcher to arrive at informative and trustworthy conclusions. The study used a document analysis protocol, two online questionnaires and three online focus group discussions. The tasks were designed to enhance engagement and understanding of the goal of 'teaching Mathematics meaningfully with technology'. Multiple iterative research instruments allowed the researcher to meet four criteria of trustworthiness.

Case selection and the researcher's role were elaborated in this chapter. Permission was sought from the MoEAC, regional directors of education, principals and supervisors where potential Mathematics teacher participants work. Consent forms were signed by the Mathematics teacher participants granting permission to participate in research activities. Mathematics teacher participants were informed of their rights to participate and withdraw at any time without any consequences, and that their information is kept confidential.

The next chapter 5 presents design stage two, an analysis of the Namibian secondary school Mathematics curriculum.

CHAPTER 5 DESIGN STAGE TWO PHASE ONE

ANALYSIS OF THE NAMIBIAN SECONDARY SCHOOL MATHEMATICS CURRICULUM

5.1 Introduction

Teachers make pedagogical decisions based on curriculum rules set out by the MoEAC. This chapter presents the findings and discussion of design stage two phase one on the Namibian secondary school Mathematics curriculum. The findings were based on the analysis carried out on the curriculum documents. These curriculum documents are (a) the national curriculum for basic education (MoEAC, 2016), (b) the Mathematics subject policy for grades 4 to 12 (MoEAC, 2019), and (c) the secondary school Mathematics syllabuses for the junior secondary (grades 8 & 9) (NIED, 2015), ordinary level (grades 10 to 11) (NIED, 2018) and the AS (grade 12) (NIED, 2020) syllabuses. The analysis aimed to answer research question one by identifying the affordances and constraints of the Namibian secondary school curriculum in terms of Mathematics teaching goals (Kilpatrick et al., 2001) with technology. The findings and discussion are presented under two headings. The first one covers pedagogical affordances and constraints of the secondary school Mathematics syllabuses in terms of meaningful teaching (section 5.2). This is followed by pedagogical opportunities afforded to teach with technology (section 5.3). The chapter ends with implications for the design of a professional development framework (section 5.4).

5.2 Pedagogical affordances and constraints of the Namibian secondary school Mathematics curriculum in terms of meaningful teaching

Key proficiency terms (KPTs) extracted were categorised under a strand of mathematical proficiency based on the full description of the learning content (NIED, 2015, 2018, 2020). For example, the KPT, *'interpret'* at the NSSCO level appear under procedural fluency as well as strategic competence. To develop procedural fluency, teachers are expected to develop learners' ability to:

"interpret personal income tax tables and determine the tax payable on an amount earned" (NIED, 2018, p.12)

Also, for strategic competence in the same syllabus teachers are expected to develop learners' ability to

"*…interpret the following set notations $A \cup B$ union of A and B $A \cap B$ intersection of A and B; n(A) number of elements in set A; $\in$ "…is an element of …;" $\notin$ "…is not an element …"; A′ complement of set A; Ø the empty set; U universal set; $A \subseteq B$ A is a subset of B; $A \subset B$ A is a proper subset of B; $A \not\subseteq B$ A is not a subset of B; $A \not\subset B$ A is not a proper subset of B"* (NIED, 2018, p.13).

As a result, some KPTs were recorded under more than one strand of mathematical proficiency (Table 5.1) depending on their complexity. KPTs appearing more than once on the same strand of mathematical proficiency were only recorded once to avoid repetition. Table 5.1 shows a summary list of key proficiency terms found in the Namibian secondary school Mathematics syllabuses.

Table 5.1: Key Proficiency Terms (KPTs)

Secondary grade level	**Strand of Mathematical proficiency**	**Key proficiency terms (KPTs)**
Grade 8-9 Junior Secondary Certificate (JSC) level (based on the summary of learning content used)	**Conceptual Understanding**	Apply, compare, order, find, use, expand, factorise, convert, identify, draw, use, construct, determine
	Procedural fluency	Determine, solve, apply, use, calculate, find, simplify
	Strategic competence	Construct, solve, determine, perform, interpret
	Adaptive reasoning	Describe, draw, interpret, discuss
Grade 10-11 Namibian Senior Secondary Certificate ordinary (NSSCO) level	**Conceptual Understanding**	Classify, identify, use, recognise, order, express, make, give, round off, define, interpret, read, measure, construct, substitute, find, describe, sketch, add & subtract, multiply, represent
	***Procedural* Fluency**	Calculate, express, perform, apply, give, solve, use, multiply, expand, factorise, simplify, manipulate, carry out, find, draw, construct, estimate, interpret,
	Strategic competence	Perform, obtain, express, solve, interpret, construct, transform, use, estimate, find, apply,

		describe, reflect, rotate, recognise, collect, classify, tabulate,
	Adaptive Reasoning	Describe, draw, interpret, discuss, construct, represent, Identify, give, inferences, distinguish, read, use, solve, prove, find, locate
Grade 12 Advanced subsidiary (AS)	**Conceptual Understanding**	Use, interpret, identify, determine, recognise
	***Procedural* Fluency**	Solve, interpret, apply, form, find, calculate, use, evaluate, divide, differentiate
	Strategic competence	Interpret, use, solve, prove, find, locate & distinguish
	Adaptive Reasoning	Complete & use, find, solve, recognise, sketch, use, perform, apply, interpret, locate

Table 5.1 shows that few KPTs aim to develop adaptive reasoning at the JSC level similarly there are few KPTs aimed to develop conceptual understanding at the AS level (Table 5.1). The KPTs relating to productive disposition are not explicitly stated in any of the Namibian secondary school syllabuses. Table 5.2 shows the distribution of strands of mathematical proficiency in terms of frequency counts and percentages.

Table 5.2: Frequencies and percentages of Key Proficiency Terms (KPTs) across the Namibian secondary school syllabus

Proficiency strand	Frequency			Total KPTs	Percentage (%) of KPTs
	JSC	NSSCO	AS		
Conceptual understanding (Understanding)	13 (45%)	24 (31%)	5 (15%)	**42**	30%
Procedural fluency (Fluency)	7 (24%)	19 (25%)	10 (29%)	**36**	26%
Strategic Competence (Problem-solving)	5 (17%)	18 (23%)	8 (24%)	**31**	22%
Adaptive Reasoning (reasoning)	4 (14%)	16 (21%)	11 (32%)	**31**	22%
Total	**29**	**77**	**34**	**140**	**100**

Table 5.2 shows that at the JSC and NSSCO level most (45% and 31% respectively) of the KPTs are those associated with conceptual understanding. Key proficiency terms relating to reasoning appear as little as four times (14%) in the JSC Mathematics curriculum. Further, from Table 5.2, KPTs relating to reasoning are represented more in the later (AS) levels of secondary schooling. Thus, at the NSSCO and AS levels, KPTs relating to reasoning represent twenty-one per cent (21%) and thirty-four per

cent (32%) respectively. Moreover, at the AS level, KPTs related to procedural fluency (29%), strategic competence (24%) and adaptive reasoning (32%) contribute with a high proportion as compared to other grade levels. The high portion could be attributed to the fact that most of the other strands of mathematical proficiencies are well catered for in the earlier grade levels, that is the JSC and NSSCO. It is also worth observing from Table 5.2 that the NSSCO level has more (77) KPTs, implying more learning content to be covered as compared to other grade levels. In totality KPTs related to conceptual understanding take precedence with thirty per cent (30%) followed by procedural fluency with twenty-six per cent (26%) (Figure 5.1). The remaining forty-four per cent (44%) are distributed equally among strategic competence (22%) and adaptive reasoning (22%). It is worth noting that, the high percentage (45%) of KPTs at the JSC level relates to conceptual understanding. Although reasoning may be critical in the development of mathematical concepts only less than a quarter (22%) of the total KPTs extracted relate to adaptive reasoning.

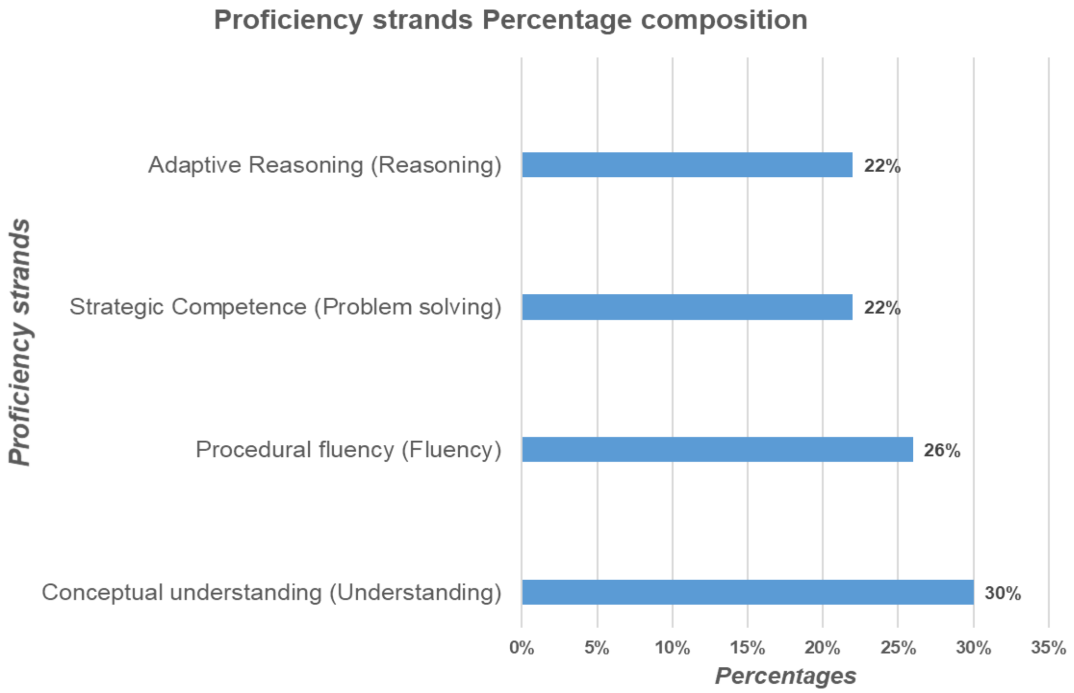

Figure 5.1: Percentage composition of the proficiency strands in the secondary school Mathematics curriculum

A high percentage (45%) KPTs at the JSC level relating to conceptual understanding could be attributed to the purpose of the JSC level as to *''extend concepts […] acquired at the primary school level''* (NIED, 2016, p.1). It is also worth noting that more KPTs relating to reasoning, problem solving and productive disposition can be inferred as strands of mathematical proficiencies are interwoven (Kilpatrick et al., 2001) and embedded. However, the embeddedness of KPTs in the Mathematics curriculum is not stated anywhere in the curriculum documents. Mccluskey et al. (2016) argued that if proficiencies are embedded this should be made explicit in the description of the learning content as Mathematics teachers might tend to work in accordance with what is provided in curriculum guidelines.

Moreover, a lack of explicit emphasis on the development of productive disposition and little emphasis on some proficiency strands such as adaptive reasoning, strategic competence, and productive disposition may lead to inconsistent curriculum implementation. Thus, teachers might assume that absent, not explicitly stated mathematical proficiencies are not crucial and cannot be developed. The implication of this is learners will leave schooling falling short of some skills and knowledge. Meanwhile, several scholars (Boerst et al., 2003; Kilpatrick et al., 2001; Perkins et al., 1995) emphasised that it is crucial for teachers to develop all mathematical understanding in the learners. Further, the little guidance on the articulation of the learning content might challenge teachers in interpreting and implementing the curriculum (Atweh et al., 2012).

5.3 Pedagogical opportunities afforded to teach with technological tools

Pedagogical opportunities afforded to teach with technology by the national curriculum for basic education, Mathematics subject policy for grades 4 to 12, and the secondary school syllabuses guide the use of technology in teaching. The content of all curriculum documents analysed consists of statements that acknowledge the importance of the use of technology and the influence of ''technology'' in Mathematics. The term *"computer", "technology",* and *"technological*" were mentioned in eight statements but under the aims, rationale, and/or assessment objectives in three secondary school Mathematics syllabuses. The term "computer" was captured in an acknowledging statement that: ''*today's learners will live in a world dominated by computers …''* (NIED, 2018, p. 5, 2020, p.2). Therefore,

> ''… the Mathematics curriculum provides insight and understanding, such as of *technological* explosion and increased connectivity; as some of the crucial global issues which affect the quality of life'' (NIED, 2016, p.3, 2018, p.1, 2020, p.1).

The secondary school syllabuses further encourage that the course in Mathematics should be able to integrate *Information Technology (IT)* to enhance learners' mathematical experience as well as develop their ability to apply Mathematics in other subjects, including *technology* (NIED, 2016,2018, 2020). In the Mathematics subject policy (Ministry of Education Arts and Culture, 2019) the phrase Information Communications and Technology (ICT) was stated as part of ''other resources'' that teachers should use to ''*enhance learning and make teaching fun''*. Moreover, the secondary school Mathematics syllabus indicates that teaching Mathematics should include, wherever appropriate, *technological applications* of Mathematics in modern society, amongst others. Across the Mathematics syllabuses, the term *mathematical software*, or the term *software* is not mentioned anywhere. It is apparent that variable messages about computers and technology are conveyed in phrases acknowledging their importance and as 'optional' tools. An unfortunate implication of this variable treatment is that technological pedagogies might be treated as conveniently using technology as a representation tool not aimed to enhance deeper understanding and exploration of mathematical concepts (Goos, 2012). The combination of ''enhancing learning and making teaching fun'' also could mean using technology as a representational tool. In that regard, technology is not used as a tool that can arouse an intrinsic need to communicate Mathematics by creating, designing, and engaging in productive struggle.

The calculator as a technological tool is the only tool explicitly explained and linked to the learning content to be developed by learners. Learners are to be introduced to a calculator ''*as a tool to handle more complex calculations as well as irrational numbers, numbers in standard form and the value of trigonometric ratios''* (NIED, 2015, p.1). Further the following frame the use of calculator and calculator skills development for JSC:

- ''Calculator skills should be taught in the contexts where the use of the calculator is appropriate to ease calculations and not necessarily as a separate topic
- Find and use prime factors, squares, cubes and their corresponding roots with… a calculator'' (NIED, 2015, p. 6).

At the entry grade (grade 8) of the JSC phase teachers are generally expected to develop learners' understanding of the features of a scientific calculator at a low level and use the calculator when appropriate.

Further, the JSC syllabus specifically requires teachers to develop learners to be able to:

- ''use the calculator for calculations involving several digits
- select the correct key sequence for calculations with more than one operation
- apply the clear, clear-entry and memory keys when appropriate'' (NIED, 2015, p. 9).

Meanwhile, at the end (grade 9) of the JSC phase in the same grade level, skills expectations are heightened to be able to use the calculator in finding ''*powers*, *square roots and cube roots of number''* (NIED, 2015, p. 18). In addition, at the senior secondary level teachers are expected to develop learners' calculator skills for *''efficiency''* and *''accuracy''* in mathematical calculations (NIED, 2018, p. 11).

The findings show a common practice in most countries (Australia, England, France, The Netherlands, New Zealand, and Singapore) in the world (Drijvers et al., 2014; Tabach & Trgalová, 2020). The use of technology is generally acknowledged but that mostly meant calculators. The findings may imply that statements in the curriculum documents suggest a narrow view of the role of the calculator, a technological tool as nothing more than an ''optional'' (Kanandjebo & Lampen, 2022, p. 3) and ''efficiency tool’’ (Olive et al., 2010, p. 138) that complements pencil and paper calculations. Further, pedagogical opportunities afforded by the Mathematics curriculum seem to align teachers at the substitution level on the SAMR model (Puentedura, 2014) and tasks level opportunities (Pierce & Stacey, 2010). Thus, the calculator, a technological tool, is directly substituted for a more traditional task. This simple, direct replacement does not lead to pedagogical changes in the teaching and learning process. This is because the calculator is meant to ease calculations such as performing more complex four basic operations calculations, irrational numbers, numbers in standard form and the value of trigonometric ratios. Consequently, technology is not incorporated in tasks that are relevant and necessary for life (Vygotsky, 1978) to enable teaching that centres on (as described in depth in Chapter 1, p.3):

- engaging learners with tasks of which the relevance is premised on conceptual understanding of their relation to the larger body of Mathematics
- providing a natural space through adaptive reasoning (logical thought, reflection, explanation, and justification) for communication through Mathematics
- wielding the mathematical thinking tool with accuracy, efficiency, flexibility, appropriateness
- allows learners to see Mathematics as sensible, useful, and worthwhile
- arousing an intrinsic need to communicate by creating, designing, and engaging in productive struggle.

The findings may imply that pedagogical practices of developing mathematical goals with technology may remain unchanged since curriculum guidelines are not explicit. The implication of this, is, technology integration in the teaching and learning of Mathematics might become “mere lip service” and may not be seen as valuable enough to inform content amplification (Atweh et al., 2012). Further, the gap in terms of technological integration between aims, rationale, and learning content may inevitably challenge teachers in planning and implementing the actual student experiences with technology in Mathematics.

5.4 Implications for designing a professional development framework

The study found that conceptual understanding takes precedence (30%) followed by fluency (26%). Strategic competence and adaptive reasoning have a composition of less than a quarter (22%) of total key proficiencies. Goals relating to productive disposition are not explicitly indicated in the curriculum. The findings concur with those of Mateya et al. (2016) that procedural fluency and conceptual understanding are best accommodated. However, there is a rare reference to the development of strategic competence and/or adaptive reasoning, similar to Stephanus' (2014) findings. There are no goals formulated for the development of productive disposition. The conclusions may imply that the teaching approaches enable the development of more basic and low-level skills. This calls for the redefinition and reformulation of pedagogical approaches as well as curriculum objectives. The redefinition may result in expansive learning and new activity (Engeström, 2001; Hardman, 2015). Thus, there is a need to restructure the Mathematics curriculum and pedagogy to suit the redefinition of knowledge and what it means to learn. Re-structuring and redesigning the curriculum stimulate a paradigm shift from instructional practices that are not congenial (Leong et al., 2011) to meaningful teaching.

In terms of teaching with technology, the Namibian Mathematics curriculum places teachers at the substitution level of technology integration according to the SAMR model (Puentedura, 2010) as well as at tasks level opportunities on a pedagogical map (Pierce & Stacey, 2010). At these levels, technology tools are narrowly used to improve speed and accuracy and to provide a greater variety of visual representations of Mathematics content. Further, the calculator specified is a single-function device that limits teachers from teaching meaningfully. The danger of such kind of inclusion of technology in Mathematics classrooms may limit the implementation of the intended curriculum even further by using technology as a crutch rather than as a tool to expand mathematical thinking.

Consequently, professional development intervention for teaching Mathematics meaningfully with technology should engage with the current mismatch of aims, rationales, visions, and goals between the Namibian curriculum and expanded goals. Such inclusion could lead to the expansion of goals for mathematical thinking and lead to new activity in a rapidly changing world. Professional developers of Mathematics teachers must promote culturally responsive, meaningful teaching of Mathematics with technology (Kanandjebo & Lampen, 2022). The authors of the extract from this thesis further argued that in the ‘‘cultural context of Namibian Mathematics education, such guidelines must be made explicit and operationalised in professional development’’ (p.6). This is like the Singaporean case where provision is made for master teachers who have a greater theoretical understanding of the subject, and they can be called upon to conduct research in certain areas of Mathematics for sustainability.

CHAPTER 6 DESIGN STAGE TWO PHASE TWO: MATHEMATICS TEACHER PARTICIPANTS' VIEWS

6.1 Introduction

The chapter builds on the design stage two phase one which reviewed the Namibian curriculum. In design-based research, the researcher and participants cooperatively and jointly construct interventions. Thus, this chapter presents and discusses the data based on design stage two phase two, which draws from the experiences of secondary school Mathematics teachers who showed agency to use technology in teaching Mathematics during the COVID-19 pandemic; in an attempt to answer the following research questions:

> ***Research Question 2:*** What technological knowledge, content knowledge, and pedagogical content knowledge are considered by participating senior secondary school Mathematics teachers as indispensable for teaching Mathematics meaningfully with technology?
>
> **Research question 3:** How does Mathematics teachers' participation in the design process of a framework for teaching Mathematics meaningfully with technology influence their beliefs and views about teaching Mathematics with technology?

The findings were obtained from three online focus group discussions and two online questionnaires. The chapter is divided into three parts. Part A presents the findings of the study, Part B discusses the findings of the study, and the chapter ends with implications for designing a professional development framework for teaching Mathematics meaningfully with technology (Part C) (Figure 6.1). The data is presented based on the research questions.

PART A

6.2 Participating Mathematics teachers' current practices

Teachers' current practices were obtained through an introductory focus group discussion (cycle 1), cycle two online questionnaire, and cycle three focus group discussions (Figure 6.1). However, some Mathematics teacher participants who could not attend the scheduled meeting sent the researcher an informal email on their current teaching practices, on their own accord.

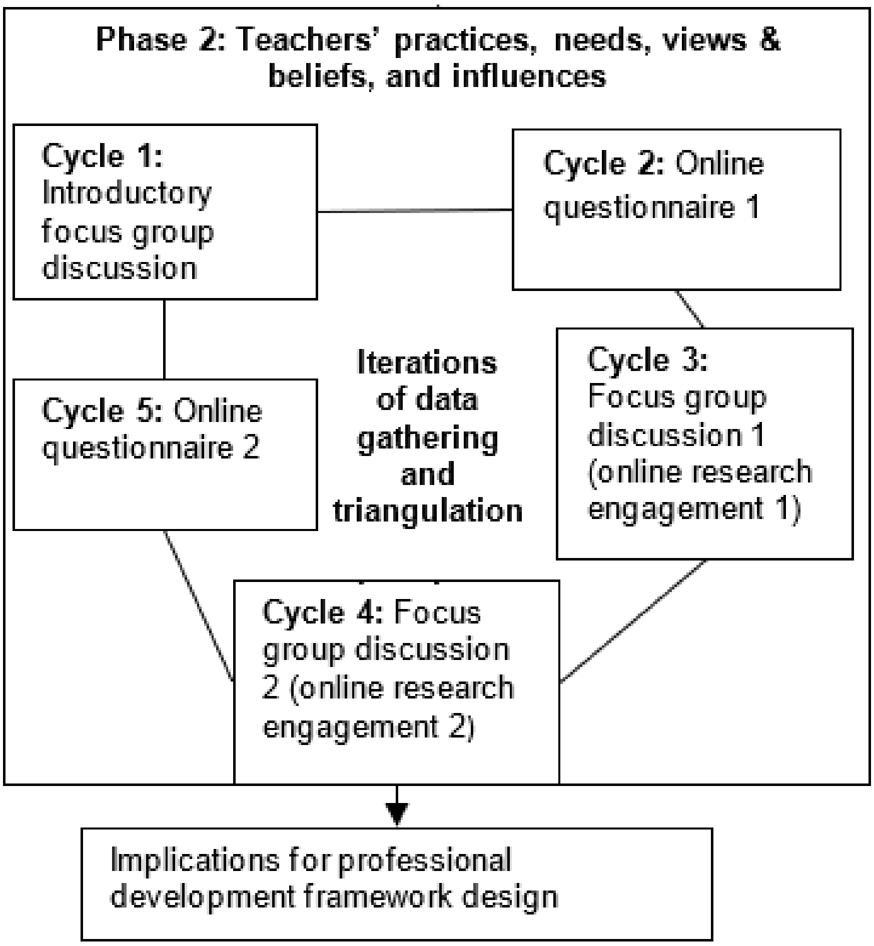

Figure 6.1: Stage two phase two research process

Mathematics teacher participants were assigned pseudonyms to protect their identity, such as "MtA" to denote Mathematics teacher A, "MtB" to refer to Mathematics teacher B, and so on during cycles one to two. From cycles three to five, Mathematics teacher participants were reassigned codes after the sample size decreased. Hence, "MT1" was used to refer to Mathematics Teacher 1, "MT2" to refer to Mathematics Teacher 2, and so on. Table 6.1 summarises the current teaching practices of Mathematics teacher participants.

Table 6.1: Mathematics teacher participants' teaching practices

Mathematics teacher participant	Technology platform used	Experiences teaching with technology
MtA	WhatsApp	Less to no participation of learners, also most learners do not know how to use smartphones.
MtB	WhatsApp	Internet issues and lack of technological devices at homes
MtC	Google Classroom and sending voice notes via WhatsApp	To increase engagement lesson sessions takes longer
MtD	Microsoft teams, ZOOM, and WhatsApp	Difficult to have all learners online at one time, due to a lack of devices and internet connection.
MtE	WhatsApp	Internet issues and lack of technological devices at homes
MtF (informal)	WhatsApp, ZOOM, and Google Classroom	Tried ZOOM and Google Classroom but only WhatsApp works Learners copy the answers to the work given without understanding. No internet data for learners
MtG	WhatsApp and ZOOM	Internet issues and lack of technological devices at homes
MtH	WhatsApp	Not shared
MtI	WhatsApp and ZOOM	Not shared

Table 6.1 shows that most Mathematics teacher participants used mostly WhatsApp during COVID-19 to ensure the continuation of teaching and learning of Mathematics. It is also evident that MS teams, ZOOM, and Google Classroom were used by only a few teachers, that is MtC, MtD, MtF, MtG, and MtI. Internet connection and shortage of technological tools were the most challenges experienced by Mathematics teacher participants. The nature of learners' work is described by MtF as learners '*copied without understanding'*. MtH and MtI experiences are not known as they did not indicate them. Notable from Table 6.1, is that challenges noted by most Mathematics

teacher participants are those that relate to learners or technological tools. However, nothing about their professional development needs with reference to their facility using all popular platforms even though they used technology for communication.

In addition, based on the cycle two online questionnaire some Mathematics teacher participants like MtC allowed learners to use the computer and '*'discover the required knowledge''* (MtC). Also, others like MtF only engage learners in assessment tasks that require them to use technology, as precisely put,

> *''I give an assignment that requires learners to search information on the internet and allow them to present their activities with the help of technology.''*

However, for some, teaching with technology is a challenge as learners do not have access to technological devices and/ or internet access. MtI attests that presently it is difficult to teach with technology as '*'learners themselves can't afford ICT tools''.* Due to the unavailability of technological tools, MtG indicated that it was only possible to highlight the importance of technology in teaching and learning Mathematics. The assumption might be that communicating the importance would instil a sense of awareness and seeing Mathematics as worthwhile by linking to other fields such as technology.

6.3 Mathematics teachers' views on meaningful teaching and meaningful teaching of Mathematics with technology

Mathematics teacher participants were asked to give their views on what 'meaningful teaching of Mathematics' and "meaningful teaching of Mathematics with technology" entail in the cycle two online questionnaire. Most of the Mathematics teacher participants (four of the nine) described the concept with relation to learners' ability to apply Mathematics knowledge that connects to their real life and learners can use such to solve societal needs. Two out of nine Mathematics teacher participants related it to teaching concepts and a deeper understanding of Mathematics to create an impact and influence learners. MtA noted that meaningful teaching of Mathematics is,

> *''Not just teaching recipes and tricks but also creating a deeper understanding of Mathematics to create problem solvers and ultimately a love for the subject''.*

In addition, three out of the other nine Mathematics teacher participants noted teaching Mathematics meaningfully in terms of teaching approaches to attend to learners' differences and different learning needs. As MtE notes:

> *''Meaningful teaching of Mathematics is a teaching that makes use of a range of teaching and learning approaches and resources to meet the different learning needs of learners. The teachers must know how their learners learn, which will enable them to effectively make their learners understand the concepts presented as well as to become fluent with the skill taught''.*

Two of the nine Mathematics teacher participants regard meaningful teaching as more of the teachers' competence to choose learning content that influences learners' views toward Mathematics as well as fulfilling curricula goals. MtG wrote that meaningful teaching,

> *''… is characterised by one(teacher) being competent as they present their lessons, making sure that learning objectives are met''.*

Furthermore, MtA described meaningful teaching of Mathematics to creating problem solvers as an output of schooling. MtB described it briefly in terms of the nature of the subject content as follows:

> *''Teaching the basic concept and its applicability to solve human problems''.*

Moreover, the concept of 'teaching Mathematics meaningfully with technology' is described by a majority (5 out 9) of the Mathematics teacher participants as a means to enhance comprehension and mastering mathematical concepts using technology. The other four Mathematics teacher participants described it as follows: (a) use of "*programming skills to promote generalisation of mathematical concepts"* (MtB); (b) the creation of a visual understanding of mathematical concepts, and (c) developing learners' knowledge of using technology. Further meaningful teaching of

Mathematics with technology also implies teaching in "*both manual and digital ways of teaching Mathematics"*, MtB noted. Additionally, Mathematics teacher participants were asked to describe teaching Mathematics meaningfully with technology. The question was necessary to further probe the pedagogical aspects of teaching Mathematics meaningfully with technology. The majority (seven out of nine) answered this question by linking meaningful teaching with technology to examinations, tests, and academic performance. MtE however indicated that teaching Mathematics meaningfully with technology is comparing it to the traditional teaching approach, putting it is as

> ''*assessing [giving tests and/ examinations to learners'] knowledge of concepts taught using technology to determine how much understanding they have acquired compared to otherwise to the traditional teaching*".

However, MtG's views were completely different from many of the other Mathematics teacher participants' views. Since the Mathematics teacher participant (MtG) related teaching Mathematics meaningfully with technology to teachers knowing the relevance of integrating technology.

The views on teaching Mathematics meaningfully and teaching Mathematics meaningfully with technology are summarised in Table 6.2 in terms of themes linked.

Table 6.2: Meaningful aspects (which could be taught, developed, and achieved) and purpose

	Meaningful Aspect	Activity theory aspects
Themes	Conceptual understanding (4) Productive disposition (2) Strategic competence (3)	Goal: attend to learners learning needs (2) Goal: Meet lesson objectives Goal: Produce problem solvers Rules: basic concepts and are applicable to solve human problems

In Table 6.2 Mathematics teacher participants viewed meaningful teaching mainly as developing learners' conceptual understanding followed by strategic competence and productive disposition. The goal of using technology in teaching Mathematics is to achieve curricula goals which are basic concepts yet to produce problem solvers.

6.4 Aspects of teaching and learning Mathematics that can and cannot be achieved using technology

Aspects of teaching and learning Mathematics that Mathematics teacher participants can and cannot achieve using technology were obtained through cycle two online questionnaire, cycle three and cycle four focus group discussions. These were those that Mathematics teacher participants view as affordances and/ or constraints of teaching and learning Mathematics with technology.

MtA noted that technology affords Mathematics teachers opportunities to recreate classroom aspects such as the demonstration of mathematical objects which would be impossible without technology. Technology also improves learners' interaction and engagement (MtE). Further, the aspects of teaching and learning Mathematics that can be achieved using technology include enhancing comprehension of concepts *"topics such as graphs (trigonometric graphs, exponential, and logarithmic graphs) through modelling the functions in Microsoft Excel" (MtC).* Two of the Mathematics teacher participants further indicated that technology enhances aspects of remedial support "*with the help of online tutorials"* (MtF). However, the medium of instruction should be comprehensible to the learners. Technology *"guarantees"* positive academic performance as it includes internet-based platforms that can be used to search for appropriate teaching guidance (MtE*)*. It also helps to cover the subject content fast and saves time, however, learners need to be ICT literate, MtI wrote. MtG acknowledges that technology saves time and elaborated as follows:

> *"… for example, when you are teaching using power point instead of having to write everything on the board(white/chalkboard). It also helps teachers to explain things that are a bit challenging (for example making use of a calculator to do calculations)".*

Similarly, findings obtained from the cycle two online questionnaire concur with the cycle three focus group discussions. Two of the four Mathematics teacher participants expressed that technology promotes productive disposition and conceptual understanding as it provides solutions to Mathematics pedagogy struggles (challenges in teaching). Teaching topics such as geometry, graphs trigonometric graphs, and exponential and logarithmic graphs are better understood with technology than in a normal classroom setting, MT3 states. As in the cycle two online questionnaire, during focus group discussion, MT4 stated that technology saves time and thus makes the teaching content be completed faster, especially when learners have access to computers or laptops at home. MT2 added:

> ‘‘… *deeper understanding of topics like functions, graphs, numerical analysis probability, differential calculus, Geometry and integration.’’*

However, MT1 expressed that technology doesn’t help develop learners’ cognition since it ‘‘*promotes laziness among teachers and learners as everything is done by the computer’’*.

In summary, aspects of teaching and learning Mathematics that can and cannot be achieved with technology in terms of strands of mathematical proficiency are summarised in Table 6.3.

Table 6.3: Themes related to meaningful aspects of teaching Mathematics with technology

	Affordances of technology	Constraints of technology
Themes	Promotes procedural fluency (problem-solving) Promotes conceptual understanding (4) Promotes productive disposition (2) Makes Mathematics understandable (2) Simplify explicitly (5) Remedial support Saves time (6) Simplify for better academic performance Speed (4)	Promotes laziness

From Table 6.3 three themes are related to meaningful aspects (Addendum U). Themes related to conceptual understanding took precedence followed by productive disposition. Other strands of mathematical proficiency such as strategic competence and adaptive reasoning were not mentioned by Mathematics teacher participants, possibly because they might not know or be aware of them. The findings concur with views expressed during focus group discussions by MT2 that adaptive reasoning is not well addressed. From Table 6.3 it is evident that affordances outweigh the constraints of teaching Mathematics with technology and that can further stimulate professional development for teaching Mathematics meaningfully with technology.

6.5 Professional development needs for teaching Mathematics with technology

The professional development needs of Mathematics teachers were examined by first presenting data on existing professional development initiatives (6.5.1). This is followed by data on professional development needs for teaching Mathematics meaningfully with technology (6.5.2). The data were obtained through cycle one online questionnaire as well as cycles three and four group discussions.

6.5.1 Existence of professional development initiatives

Mathematics teacher participants were asked to indicate the type of professional development opportunity they had received in teaching Mathematics with technology. This was necessary so that the designed professional development framework reflects the realities of situations in which it is planned to be applied and also to ensure that it considers Mathematics teachers' current capabilities. Table 6.4 shows professional development initiatives on teaching with technology attended and their relevance to teaching Mathematics. These findings were obtained through the cycle two online questionnaire.

Table 6.4: Professional development received in teaching Mathematics with technology

Mathematics teacher participant	Professional development in teaching Mathematics with technology received
MtA	Training on programming through my studies, train myself further through online learning on the use of several graphing programmes, PowerPoint, and other office programmes.
MtB	At the Mathematics conference on how to analyse data using a statistical package and Solving geometry problems using GeoGebra
MtC	ICT integration in education and Computer literacy
MtD	-
MtE	None
MtF	None
MtG	None
MtH	None
MtI	None

In Table 6.4 five of the nine Mathematics teacher participants noted that they have not received any professional development in teaching with technology. Two of the three Mathematics teacher participants received professional development training relating to topics in the Namibian Mathematics syllabus. Another one, MtC, attended an ICT literacy training and a course on general ICT integration in education. Further, Table 6.4 shows that most of the professional development received was through studies.

In addition, to further identify the current professional development needs Mathematics teacher participants were asked in cycle two online questionnaire to indicate professional development initiatives available at their schools or regions concerning teaching Mathematics with technology. Table 6.5 shows the professional development initiatives available at schools and/or regions in terms of teaching with technology for Mathematics teacher participants.

Table 6.5: Availability of professional development initiatives

Mathematics teacher participant	Availability of PD initiatives on teaching Mathematics with technology at school/region and topics enhanced	Form of technological pedagogical support provided
MtA	Informal school-based training	-On Mathematics content but not on technology - Internet
MtB	None, only at the national Mathematics congress	-Presentation on Ministry of Education: ICT for teachers -School provided internet connection and desktop computers
MtC	National Mathematics congress Regional professional development sub-division Hosted by regional professional developers on Mathematics content	Using Interactive whiteboard and GeoGebra in teaching vectors and graphs of functions by providing displays. Content knowledge
MtD	-	-
MtE	In the region by a private institution: Rossing foundations	All Mathematics content topics.
MtF	None, lack of ICT tools at school	None
MtG	None	None
MtH	None, not enough ICT tools	None
MtI	Not that I'm aware of	None

Table 6.5 shows that there are extremely few (four) initiatives available meant to develop teachers' skills in teaching Mathematics with technology. From Table 6.5 it is evident that professional development is mostly on Mathematics content knowledge which is organised by private bodies such as Rossing foundations and organisers of

the National Mathematics Congress. These private bodies provided training on the use of GeoGebra to teach topics and how to use an interactive whiteboard, Table 9 shows. Also, one of the schools conducted its school-based informal training, even though not on teaching Mathematics with technology. As MtA wrote: *"… we support each other, there are colleagues helping and training each other but no formal training"*. Further, two of the Mathematics teacher participants related professional development to the provision of technological tools. For instance, MtC's school was provided with an internet connection and desktop computers which are believed to have influenced Mathematics teaching.

6.5.2 Professional development needs for teaching Mathematics meaningfully with technology

This section presents data on the professional development needs to ensure that teachers' current needs inform the design of a professional development framework for teaching Mathematics meaningfully with technology. Based on the cycle two online questionnaire and cycle three focus group discussions, Mathematics teacher participants indicated the following as skills needed to teach Mathematics meaningfully with technology (Table 6.6).

Table 6.6: Skills needed by Mathematics teacher participants

Mathematics teacher participant	Skills needed
MtA	Training on the use of smartboards, and PowerPoint presentations Basic knowledge of how to use different technological programmes
MtB	Advanced computer training e.g., coding and syntax of programming language
MtC	Computer skills Computation skills Numeracy skills Programming and Microsoft Office programs such as Excel and PowerPoint
MtD	-
MtE	Technology integration in Mathematics
MtF	An in-depth knowledge of the digital tools (such as computers etc.) that can be used and be able to demonstrate proficiency in using them. Knowledge of how development lessons using technology
MtG	Knowledge of using technology in the teaching and learning of Mathematics
MtH	Technology integration in Mathematics
MtI	Technology integration in Mathematics

Table 6.6 shows that all Mathematics teacher participants who responded to the questionnaire need skills on how to use technology. Three of the nine Mathematics teacher participants need professional development on how to integrate technology into the teaching of Mathematics. Also, MtF indicated a need for in-depth knowledge to become proficient in using digital tools. MtC added computational and numeracy skills on top of technology integration skills. In addition, Mathematics teacher E wrote that professional development

> *‘‘… should consist of both the ability to formulate mathematical problems, represent them, and solve them and the ability to apply procedures accurately with efficiency using technology’’*.

Furthermore, professional development should *“include every Mathematics teacher”,* MtH noted. Mathematics teachers' knowledge should be developed so that it may influence their confidence to teach Mathematics with technology. This is evident from

MtF's writings that ''*not enough knowledge to teach Mathematics with technology sometimes leads to the loss of confidence in teaching Mathematics with ICT''*. Mathematics Teacher 3 stated that Mathematics teachers need professional development on how to teach the use of specific software for specific topics so that they will be able to manipulate formulas on the computer. Mathematics teachers should be developed in terms of Mathematics pedagogy to be able to identify examples in nature and connect Mathematics to real life (MT4). Two Mathematics teacher participants of four Mathematics teacher participants stated that added assessment plays an important role in developing learners' mathematical skills; thus, Mathematics teachers need to be equipped with assessment techniques related to meaningful aspects. MT1 states:

> *''… Assessments are mostly left to teachers. I mean Mathematics syllabus does not state clearly for them [meaningful aspects], it does not really come out. Only if the teacher had got skills to come up with those quality projects and investigations then their learners do not get full benefits''.*

There is a lot of free technology software, but teachers lack technological, pedagogical, and Mathematical knowledge which implies there will be no integration; teachers need to be "coached" on how to use specific software (MT4). MT3 stated that teachers need to know the syntax of programming language to help them use cognitive technologies to teach Mathematics. MT3 further stated:

> *''Equip teachers with ICT skills and how those skills are related to Mathematics subjects. Also, teachers need to be trained on how to identify mathematical proficiencies from the syllabus.*''

The data from Table 6.6 were summarised in themes relating to knowledge (as discussed in Chapter 4) in Table 6.7. Hence, Table 6.7 shows themes generated related to professional development needs based on data obtained from the cycle two online questionnaire and cycles three and four focus group discussions.

Table 6.7: Themes relating to Professional development needs

Themes	*Professional development needs*
	Mathematics knowledge (3) Mathematics technological knowledge (2) Pedagogical technological knowledge (3) Pedagogical knowledge Knowledge of teaching Mathematics with technology (11) Technological knowledge (9) Enhance personal orientations for personal and professional instrumental genesis (1)

**Instrumental genesis Instruments genesis refers to making a technological artefact into an instrument for teaching*

Table 6.7 shows that most themes on professional development needs are linked to technology knowledge and knowledge of using technology for pedagogical focus in Mathematics. This implies that teachers mainly need professional development in the knowledge of teaching Mathematics with technology and technological knowledge. Mathematics knowledge, Mathematics technology knowledge and pedagogical knowledge were least mentioned. Table 6.7 also shows the need to enhance personal orientations for personal and professional instrumental genesis even though it was mentioned once.

6.6 Aspects necessary for professional development framework for teaching Mathematics meaningfully with technology

It is necessary to holistically design a professional development framework for teaching Mathematics meaningfully with technology by considering tensions within the teachers' activity system. These tensions may hinder the successful professional development of Mathematics teachers. Thus, the data under this section (6.6) are presented as conditions necessary for teaching Mathematics meaningfully with technology.

Based on the cycle two online questionnaire, cycle three and cycle four focus group discussions with Mathematics teacher participants emphasised the need for classroom environments to be equipped with technological tools. Technological tools mentioned were whiteboards, portable overhead projectors, access to internet connection and computers, tablets, or laptops, six of the nine Mathematics teacher participants noted. In addition, *''learners need to be computer literate for them to interact with it effectively'', MtC noted*. MT4 expressed that learners should be provided with access to technology in their homes.

Seven of the Mathematics teacher participants noted that learners need to know how to use the technologies for Mathematics for teachers to be able to teach Mathematics with technology. For example, MtF writes that learners need ''*to know what they can search, where, and how". *Learners also need to perceive technology as easy to use in Mathematics, MtF added. To MtE, another condition for teaching Mathematics meaningfully with technology is for technological tools to cost reasonable cost because ''*the programmes and instruments needed for this [teaching Mathematics meaningfully] are expensive'',* MT1 argued. In the cycle two online questionnaire, seven of the nine Mathematics teacher participants acknowledged that there are plenty of online videos and links to websites in school textbooks. However, teachers may not be aware of any, due to a lack of technological skills and knowledge (MtB). Further, six of the nine Mathematics teacher participants described the syllabus that outlines the topics to be taught as '*'overcrowded'', ''too long'',* and *''too much''.* Thus, the time allocated to Mathematics lessons to enable teaching with technology is less. MtG indicated the following:

> *''The time allocated to Mathematics lessons is not enough for one to include the use of technology in their lessons. And also lack of resources such as computer and other types of technology might also not allow one to promote mathematical understanding with technology."*

MtA explicitly added:

> ''*I love teaching with technology but technology integration in Mathematics demands a lot of time, they require longer duration as compared to lessons without it.''*

As a result, MT3 suggested that the secondary school Mathematics curriculum *"must"* be amended to make the use of technology a statutory requirement in secondary schools and even part of the examination.

> *"… For teachers to use technology, it must be stated in the curriculum documents that it is a must that we must teach using Excel in Mathematics for example and it will be assessed in the examination''.*

In terms of curriculum as an enabler, MT2 expressed that the syllabus needs to be explicit about the mathematical proficiencies to be developed in the learners. This is so as currently only learners taught by teachers who are aware and able to infer strands of mathematical proficiencies from the curriculum benefit. MT2 further argues:

> *''Plus, even though we revised the curriculum, our subject policy still does not provide guidance on teaching Mathematics with technology. Strands of mathematical proficiencies are supposed to come out or be written out or spelt out in either the syllabus itself or in the subject guide''.*

MT4, however, stated that the curriculum is *''fine''* as it provides room for *'continuous learning of teaching and Mathematics learning content'*. Furthermore, professional development should *'include every Mathematics teacher',* MtH noted. Also, teachers should be self-motivated to take up leads in their professional development as there are a lot of self-learning materials online (MT4). Table 6.8 summarises the data into themes.

Table 6.8: Summary of conditions necessary for teaching Mathematics meaningfully with technology

	Activity theory nodes aspect
Themes	Rules: Explicit
	Rules: Expands for curriculum must state compulsory use of technology.
	Rules: Revision of subject policy. Subject policy needs to be explicit (2)
	Rules: Syllabus to include the competences
	Rules: More time for Mathematics teaching
	Rules: Reduce the content of the syllabus
	Community: Provide access to technologies to both learners and teachers
	Community: professional development for all, no using cascade approach

6.7 The influence of Mathematics teachers' participation in the study on their beliefs and views about teaching with technology

Participants' views and beliefs about teaching Mathematics with technology after participating in the design process were determined. Participants were asked through cycle three focus group discussion and cycle two online questionnaire to give their views on the following: (a) the influence of their participation in the design process on their views and beliefs about teaching with technology and (b) reflect on learning experiences. The question was necessary to identify a pedagogical approach to professional development. Participants indicated that the tasks were ideal, authentic, and real. Two out of four Mathematics teacher participants noted that the approach used is the "*better one*", thus *"… have no better way of teaching….*" than the approach used in the study. Some participants also expected to be given more examples of teaching Mathematics meaningfully with technology. On learning experiences, MT3 wrote that the research engagement was "*fun and constructive".* MT4 noted that the researcher and fellow participants *"seem to be experts"; ''thus, the research engagement was worth my time''.* The tasks used fit well with the content and are relevant to the context presented (MT1). MT1 added that

> *''The tasks allowed manipulation and discoveries using GeoGebra, which allowed more explorations to be made during the research engagement. The application used in this webinar helped with comprehension of the subject content as it makes concepts less abstract and easier to understand in a shorter period than normal teaching*.*''*

Three of the participants noted that they learned new skills and knowledge. For example, MT4 noted:

> ''*Some concepts that were explained I never got to understand them in high school neither at a higher institution of learning, only now during the engagement*.*''*

This is so as the demonstration approach of teaching meaningfully with technology; where the researcher is "the hand that holds the mouse" and participants are the 'mind that instructs the mouse' used; as well as opportunity afforded to them to practice provided more clarity of the content. However, MT3 indicated *no*, as the participant expected to receive *"programming skills'* during the design process. The views correspond to those shared in cycles one to four that some of the teachers should be professionally developed in understanding the syntax of programming language.

In addition, participants were also asked to provide views on (a) the chances that they would apply the skills and knowledge in their classrooms teaching, and (b) whether they would share the knowledge acquired with other Mathematics teachers. The question was necessary to determine if the design process had influenced teachers' productive dispositions. All participants noted with confidence that the probability of using the knowledge learnt and sharing it with others is very high due to the affordances of technology such as simplified content and visual representation. Concerning the question of whether participants would share their views with other Mathematics teachers, all participants indicated that they would. Three of the four participants indicated they would share the skills and knowledge gained as they view it as useful and relevant. Further, MT4 noted the modalities of how the knowledge and skills gained will be shared. Stating that:

> *''I would most definitely, during our departmental meeting of the year, I would conduct a mini continuous professional development workshop to share including how to use GeoGebra''.*

Part B presents a discussion of the findings.

PART B

6.8 Discussion

The next sub-sections discussed the findings of this study.

6.8.1 Current practices in terms of meaningful teaching

Based on the findings of current teaching practices, technology is not seen as a cognitive tool but as a tool to improve the accuracy of mathematical calculations. In addition, the findings show that challenges noted by Mathematics teacher participants are those that relate to learners or technological tools; and nothing about their professional development needs even though they used technology for communication. This could mean that professional development should be linked to teachers teaching jobs. In addition, learners' lack of technological skills posed a challenge in teaching Mathematics with technology. That is to be expected as the only technological tool learners may have access to is a scientific calculator and where possible, a computer once a week. These concur with the findings obtained from design stage one that a calculator is recommended to be used as an efficiency tool. It is also evident from teachers' views that they only inform learners of the importance and/or allow learners to engage with the software without teachers' involvement. The findings are similar to Drijvers (2020) in that teachers tend to step back on a teacher-driven approach which constrains the development of mathematical skills.

Learners could not share in the division of labour (teaching and learning process) nor are some Mathematics teachers able to teach with technology as they had no access to technological tools; thus, the teaching and learning system was perturbed. The findings concur with those of Nchindo (2019), Nendongo (2018) and Simataa and Simasiku (2012) that access to technological tools hinders teaching with technology. The study also found teachers using technological tools in teaching and learning Mathematics to merely "take Mathematics to learners" as opposed to "bringing technology into Mathematics as a cognitive tool". Hence, the use of technologies in that manner could be the reason learners copy without understanding when given mathematical tasks. As a result, no strand of mathematical proficiency will be professionally developed. Thus, there is a need to attend to issues of access where access is problematic.

6.8.2 Mathematics teachers' views on meaningful teaching and meaningful teaching of Mathematics with technology

Participants view the concept of meaningful teaching in terms of developing strategic competence, conceptual understanding, and productive disposition. The rationale of meaningful teaching is for learners to become problem solvers and to apply knowledge and understanding in their daily lives and society. The findings imply that teaching meaningfully is aligned with three proficiencies (strategic competence, conceptual understanding, and productive disposition). The findings support those obtained in design stage two phase one of this study which showed that procedural fluency and conceptual understanding are well accommodated in the Namibian curriculum. The findings also justify Stephanus's (2014) study on why teachers promoted mainly conceptual understanding and procedural fluency. The findings also reveal that teaching Mathematics meaningfully with technology is regarded as meeting the ''*lesson objectives''* for academic performance. The Mathematics curriculum content is seen as basic concepts that should be delivered to the learners; this is evident from the responses of the majority (six out of nine) of the participants. The findings reveal a partial understanding of meaningful teaching and concur with Boerst et al. (2003) that there is an incomplete exposition of meaningful mathematical teaching. The findings also concurred with the description provided in chapter 1 that, teaching Mathematics meaningfully is not exclusive to teaching with technology.

In addition, participants regard teaching Mathematics meaningfully with technology narrowly as using technology as a representation tool to improve the accuracy of mathematical calculations. However not a cognitive tool that can aid in learners' cognitive development in Mathematics. The findings show that technology is still regarded to be used for low-level tasks of demonstration and verification of mathematical ideas in selective traditional pen-and-pencil teaching environments (Niess et al., 2009). This is not unexpected, since globally, Mathematics teachers are insufficiently prepared to teach with technology (Albion, Tondeur, Forkosh-Baruch, & Peeraer, 2015; Getenet, 2020). Therefore, the findings of this study support the studies of Getenet (2020), Niess et al. (2009) and Tabach and Trgalová (2019) on the need for a professional development framework to help provide guidance and efficiency to teach Mathematics with technology.

6.8.3 Professional development needs for teaching Mathematics meaningfully with technology

Only three out of the nine participants received training related to Mathematics teaching with technology. The majority (five out of nine) of the participants attended none and some are unaware of the existence of any professional development initiatives to teach with technology. This could be attributed to the few professional development initiatives, where most are privately organised. The findings support Kasanda's (2015) view that professional development in Mathematics has yet taken root in Namibian schools and teachers seem to be unaware of the benefits of Mathematics professional development. Further, the study found professional development initiatives to mostly focus on developing teachers' Mathematics content knowledge. The findings confirm the desktop review by Villet et al. (2020) who found professional development to focus only on building teachers' subject knowledge.

Participants noted that professional development needed to teach Mathematics meaningfully should focus on developing Mathematics teachers':

- Mathematics knowledge
- Mathematics technological knowledge
- Pedagogical technological knowledge
- Pedagogical knowledge
- Knowledge of teaching Mathematics with technology and
- Technological knowledge.

In addition, professional developers should enhance Mathematics teachers' attitudes, values, and confidence in using technology (personal orientations) in sharing the role of developing their knowledge to use technology for teaching Mathematics. The findings are in line with Mishra and Koehler (2006), Niess et al. (2009), Tabach and Trgalová (2019), and Thomas and Palmer (2014) on the types of knowledge Mathematics need to teach with technology. The findings further revealed that meaningful teaching with technology is viewed as knowledge of technology, specifically knowing the syntax of programming language. The findings could imply a need to know the syntax of programming which seem to support Wassie and Zergaw's (2019) arguments. The two scholars argued that teachers need prior programming experience to input some commands as a lack of it may constrain them from fully exploring and teaching with cognitive tools.

In addition, the findings from design stage two phase one and phase two are in accord that Mathematics curriculum documents such as Mathematics subject policy and syllabus are not explicit. As a result, Mathematics teachers may tend to work towards traditional goals (Kanandjebo & Lampen, 2022). The study's findings also revealed that the teaching duration allocated for teaching Mathematics is not sufficient to allow technology integration while the subject content is compacted. The concerns about the curriculum affordances concur with the concerns about the organisation and nature of the curriculum expressed by Mathematics teachers participants in Chrysostomou and Mousoulides' (2010) study. Moreover, the findings of this study revealed a suggestion by Mathematics teacher participants that the Mathematics curriculum must be amended to make the use of technology a statutory requirement in secondary school Mathematics and even be part of the assessment. It attests that teachers are involved in a different activity system, whose object is 'delivering-the-curriculum' to the learners. It further implies that top-down rules dictate the implementation of the operative (taught) curriculum.

6.8.4 Aspects of teaching and learning Mathematics that Mathematics teachers can and cannot achieve using technology

The findings show that technology allows the development of procedural fluency, problem-solving and conceptual understanding and influences learners' productive disposition. It also further indicates that technology enhances understanding of Mathematics and simplifies concepts for better academic performance. The findings also reveal that technology affords speed and a means to provide remedial support. The affordances that teachers seem to be aware of are those of the ability of technologies to improve efficiency and speed (Hollebrands, 2017). Teachers view technology mainly as an amplifier that improves precision, efficiency, visualisation and speed in comparison to performing it manually without technology (Hollebrands, 2017). The findings concur with themes on affordances found by Ruthven and Hennessy (2002) that technology affords speed, and accuracy and can be used as a means to enhance understanding of mathematical concepts. The findings also reveal that Mathematics teachers view technology use as a promoter of laziness in Mathematics teaching and learning as most calculations are performed by the technological tool. Pea (1987) argues that technology is considered cognitive when it provides cognitive support during mathematical thinking. Even though participants listed challenges of technological tools and internet connectivity; some acknowledged that there is no paucity of technological resources, and they are agentic in the use of technology tools. Thus, the findings validate the need for professional development to enhance teachers’ understanding of how technology can be used as a cognitive tool.

6.8.5 The influence of Mathematics teacher participants’ participation in the design process on their beliefs and views about teaching with technology.

From the participants' views, it is apparent that the research engagement has influenced their views and beliefs about teaching Mathematics with technology. They described the experience as *‘‘fun and constructive’’* and *‘‘useful and beneficial’’.* They commended the demonstration approach used. The approach was that the researcher is “the hand that holds the mouse” and Mathematics teacher participants are the “mind that instructs the mouse” to support understanding even if there is a lack of facility of technology tools. Further, findings showed that the design process created to some extent a professional community of practice. This concurs with Manfra's (2019) view that teachers' participation in the research process situates them as learners and creates a community of practice. This may promote sustained professional learning activities. The findings further indicate that teachers attach the value of their teaching and learning with technology to teaching goals. Thus, further motivating that the professional development of teachers should be linked to their teaching job. The finding concurs with those of Ertmer et al. (2012) that teachers' beliefs and attitudes about the relevance of technology to learners' learning have a major influence on their technology pedagogies.

The findings also reveal that most of the participants see the concept of meaningful teaching with technology as their “new best” and suggest that there is no other better way of teaching. Teachers’ views can be explained using the pendulum notion (Figure 6.2). Before engagement, participants use technology in teaching Mathematics on a small scale (Hamilton et al., 2019; Kanandjebo & Ngololo, 2017; Ugulu, 2019). During the engagement, they might have experienced a shift in personal orientations (Tabach & Trgalová, 2019) and thus conclude that they will ‘always’ use technology. The space created between no technology to ‘always technology’ can be a space for the imagination of professional growth.

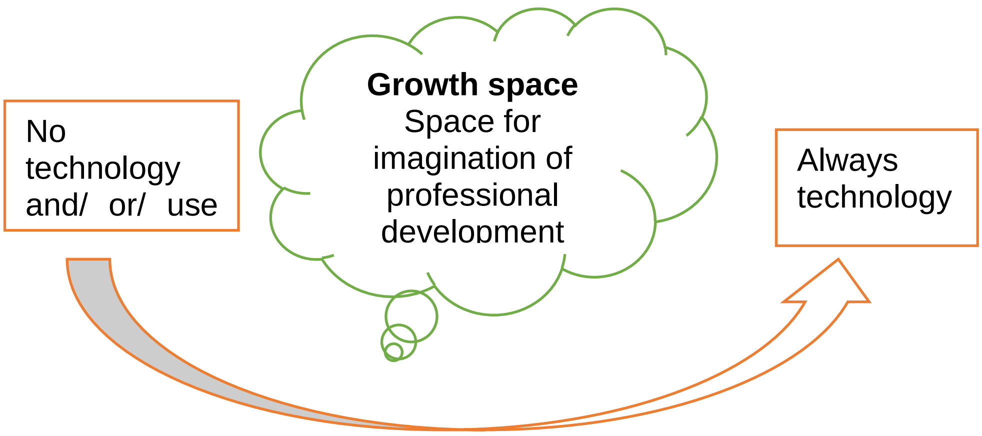

Figure 6.2: Shift of participants' personal orientations toward teaching Mathematics meaningfully with technology

PART C

6.9 Implications for Professional development for teaching Mathematics meaningfully with technology

The teachers' views are represented by an activity system showing sources of conflict (marked with dotted red lines) (Figure 6.3). In the teachers' activity system various conflicts were evident. The conflicts between,

(**A**). Rules-object relationship: Teachers are expected to use technology to enhance mathematical experiences while the curriculum rules afford procedural fluency and conceptual understanding. The rules afford Mathematics teachers to integrate technology at the substitution level and tasks level (as in Chapter 5).

(**B**). Subject-tools relationship: There are insufficient technology resources and teachers are expected to teach Mathematics with technology. Also, there is a vast array of technological resources online, yet through mere availability, technology is spurring a shift in curriculum goals. Teachers, however, may not be aware of which technology is suitable for. As a result, technology tools are merely tools to "take Mathematics to learners" as opposed to "bringing technology into Mathematics as cognitive tools".

(**C**) Division of labour: The planning of teaching Mathematics meaningfully with technology increases teachers' workload and thus teachers propose a longer duration of teaching Mathematics to meet curriculum goals.

(**D**) Subject-rules: Mathematics teachers receive unclear Mathematics goals and receive no professional development which has had actionable guidelines to teach Mathematics meaningfully with technology. But they are expected to teach Mathematics with technology.

All parts of their activity system are in flux, which brings opportunities and pitfalls that should be learnt as they arise. Engeström (2001) explains that activity in such instances cannot be mediated top-down by regulation but must necessarily emerge in new forms from expansive learning. The change in teaching goals places teachers at the cusp of "important transformations of [their] personal lives and organizational practices” (Engeström, 2001, p. 138) since they are required to adopt new tools and rethink the object of teaching and learning Mathematics.

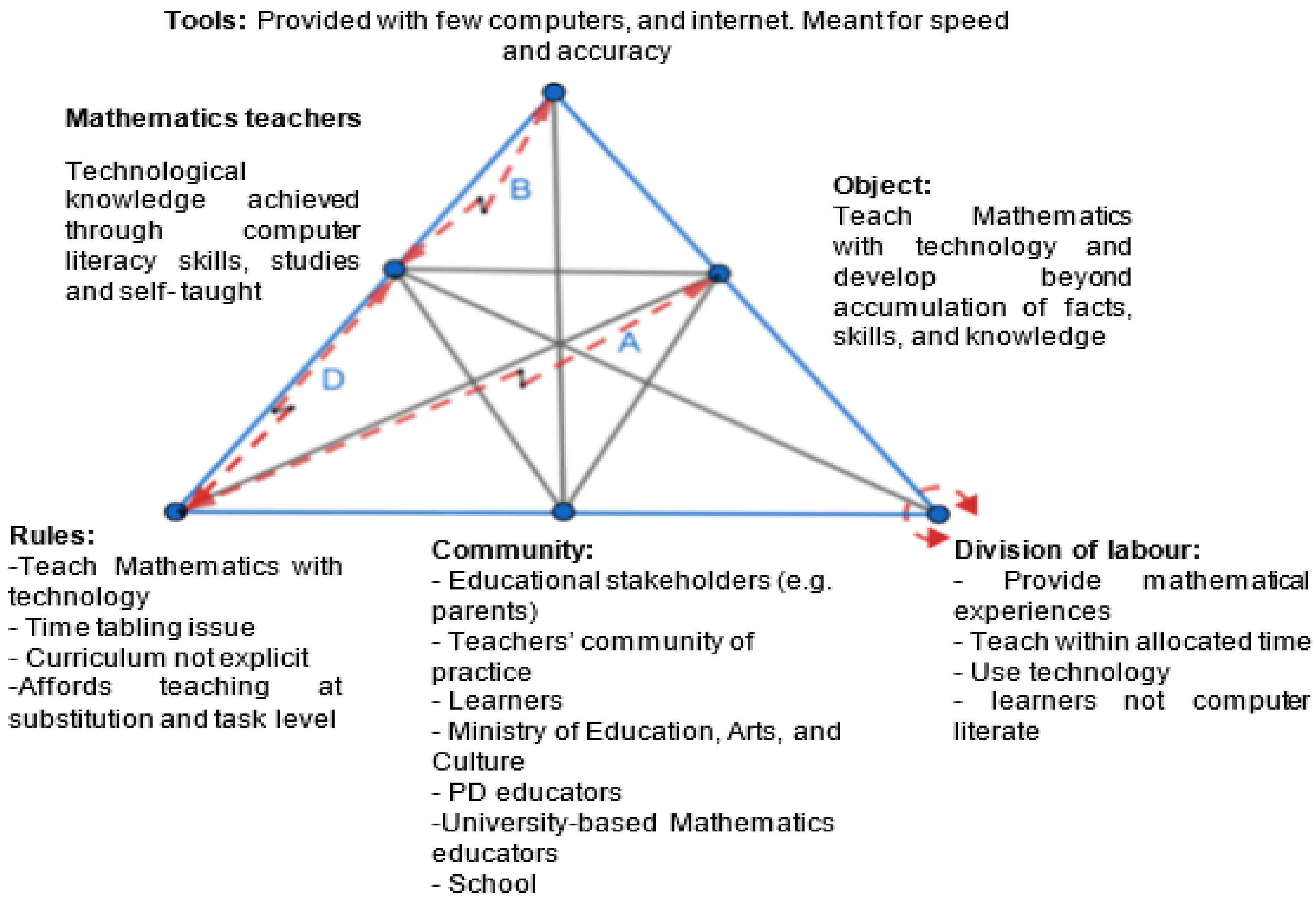

Figure 6.3: Mathematics teachers' activity system based on responses

The implication of compelling Mathematics teachers to integrate technology demands that teachers are developed beyond their current knowledge in their community of practice. The need for learners to have technological knowledge in relation to Mathematics requires that teachers have similar skills (Sarason, 1990; Tabach & Trgalová, 2019) and the Mathematics curriculum be transformed and re-imagined. Professional developers should promote reflective practices as objects of an activity system are open to change (Kanandjebo & Lampen, 2022). This may create awareness of participants' expectations and their stance on not being critical.

Consequently, teachers should be professionally developed through a structured professional development engagement to develop their knowledge of using technology for pedagogical focus. In addition, professional developers need to enhance Mathematics teachers' personal orientations. They should carefully facilitate the development of knowledge for teaching Mathematics meaningfully with technology by demonstration.

The next chapter presents stage 3, the design of the professional development for teaching Mathematics meaningfully with technology.

CHAPTER 7

CHAPTER 7 A PROFESSIONAL DEVELOPMENT FRAMEWORK FOR TEACHING MATHEMATICS MEANINGFULLY WITH TECHNOLOGY

7.1 Introduction

This chapter presents a *professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools* which is the product of this design study. A framework is a *''system of rules, ideas, or beliefs that are used to plan or decide something''* and "the ideas, information, and principles that form the structure of an organization or plan" (Cambridge University Press, 2022). The framework described in this chapter is the culmination of critical reviews of existing models for the integration of technology in Mathematics classrooms and collaboration with secondary school Mathematics teacher participants around the concept of meaningful teaching with technology. The professional development framework comprises design principles based on activity theory, and a description of progression levels based on Vygotsky's (1978) view of spontaneous and scientific knowledge. However, due to restrictions on the research process, as described in chapters 1,2 and 4, the status of the framework is tentative as in future research the framework must be implemented in professional development where it can be evaluated (Nieveen, 2007). The design process is guided by Addendum G.

7.2 Teachers' proficiency in teaching Mathematics with technology

In the cultural context of Namibian Mathematics education, guidelines for focusing on teachers' use of technology in promoting integrated and relational understanding of Mathematics, as well as reasoning and problem solving must be explicit and operationalised in professional development, based on the findings (Chapter 5) that teachers base their professional development needs on learners' knowledge and skills. Also, in following Vygotsky's argument to base ideas of how to teach on the answer to the question "What knowledge do we want the students to acquire? " (Elkonin, 1989 in Karpov, 2003, p. 69), the five strands of mathematical proficiency is adapted to include the role of technology in teaching Mathematics, and conceptualised as descriptions of the proficiency of teachers as also discussed in the extract from this thesis.

(a) Conceptual understanding: Making relevant and connecting mathematical situations to develop ways of thinking mathematically that are enabled by technology. Developing teaching skills with an understanding of which technologies are suitable to promote conceptual understanding.
(b) Adaptive reasoning: Developing teaching approaches for logical thinking, reflection, explaining, and justification with technology; adapting and comparing reasoning to different modes of enquiry with technology (algebraic, statistical, geometric); and developing ways to use technology to communicate mathematical arguments.
(c) Strategic competence: Making strategic choices about the use of technologies (or not to use technology) when formulating, representing, and solving problems (e.g., experimenting to create a database; spatial or algebraic investigation); developing the ability to discern when technology may be appropriate to use, but not necessary to learn Mathematics meaningfully, and to consider ways to facilitate investigation and reasoning also where access to technology is limited.
(d) Procedural fluency: Developing facility with a variety of programmes and applications that enable the first three competencies; developing fluency in the interpretation of technology-generated solutions.
(e) Productive disposition: Developing a positive but critical and realistic disposition through personal engagement with technology and engaging in teaching with technology.

A proficiency perspective on the goal of professional development incorporates more than knowledge, as it includes teachers' beliefs, dispositions, goals, and decision-making in the context of (new) technological tools. As argued in Chapter 3, such professional development requires expansive learning from teachers as well as professional developers. In addition, professional development should incorporate the cultural-historical position of the learning and teaching communities in Namibia in terms of the ethics of requiring teachers to teach with technology when many learners, teachers and schools have limited access to technology (as discussed in Chapter 5). In the next section, the design principles for professional development that promotes expansive learning to teach meaningfully with technology are explained.

7.3 Design principles for professional development to teach Mathematics meaningfully with technology

Design principles were developed by reflecting on existing professional development models from the CHAT perspective. As discussed in Chapter 3, CHAT enables us to holistically consider teachers' and their professional developmental needs. It also enables the identification of sources of conflicts that hold a potential for change to lead to a new activity system.

As discussed earlier (Chapter 3), the importance of personal instrumental genesis raised by Tabach and Trgalová (2019) and the knowledge necessary to teach Mathematics meaningfully with technology points to the subject-tools relationship. Teachers experiment with a vast array of technological resources and tools in their agency. However, they may not be aware of which technology is suitable for and apply it to traditional teaching goals. Professional developers of traditional Mathematics professional development may be in the same position as teachers. This motivates the need for a professional development framework to help provide guidance and efficiency to teach Mathematics meaningfully with technology (Getenet, 2020; Niess et al., 2009; Tabach & Trgalová, 2019).

Another source of conflict is the labour required from teachers to design appropriate learning tasks for teaching meaningfully with technology; it may increase the workload of teachers beyond what is considered reasonable. Moreover, this also highlights the division of labour between teacher communities and professional developers, which further motivates an expanded view of ethics in teaching with technology. Further, other sources of conflicts stem from the subject-rules relationship and rules-object relationship as Mathematics teachers receive unclear Mathematics goals and received no professional development with actionable guidelines to teach Mathematics meaningfully with technology. The subject-rules and rules-object relationship motivate the need to reform Mathematics curricula goals as in the standard-based curriculum (Niess et al., 2009) which are operationalised in professional development with sustainability goals.

In the Namibian case, professional development cannot be portrayed by two activity frameworks in a lateral relation (as in Chapter 3, Figure 3.2) as professional developers simultaneously hold and support the teachers' activity system, while purposely unsettling it for probable expansive learning. Figure 7.1 shows a bi-directional professional development activity system that encompasses and holds the activity system of teachers. The triangle E1-E5-E6 is a habitus of ethical interaction. Simply put, the professional development engagement between the professional developers and teachers (E1), in the expanded professional development community (E5) and the way the labour is divided (E6) is viewed as sociological engagement. It is an ingrained habit of working that professional developers are part of the community with shared responsibility with the teachers because both learn from each other. The sociological engagement does not ignore the bureaucracy in the school system but will work in the bureaucratic system of the school. In the habitus of ethical interaction, it's not we (professional developers/researchers) against them (teachers/schools). However, the aim is to influence, hold and support the professional development activity system. The influence is with 'meaningful rules (E4) which they could experience as meaningful.

The organisation in Figure 7.1 allows the planning of reciprocal engagements at every node between the teacher's system and the professional developers' system. As also detailed in Kanandjebo and Lampen (2022, p.8) (a publication that emanated from this study):

> *"The effects of nodal movements on distances between nodes across the triangles also enable interesting mental design experiments. For example, if the subject node of the teacher system moves closer to the subject node of the*

professional developers' system, teachers may find their knowledge of technology in teaching enhanced (the distance to the expanded tools node is shorter), but they are not yet closer to realising expanded goals (the distance across to their current goals, as well as the expanded goals, is now longer).''

The expanded activity system framework reminds professional developers of the holistic and emerging nature of professional development in situations that require both professional developers and teachers to learn expansively (Engeström, 2001).

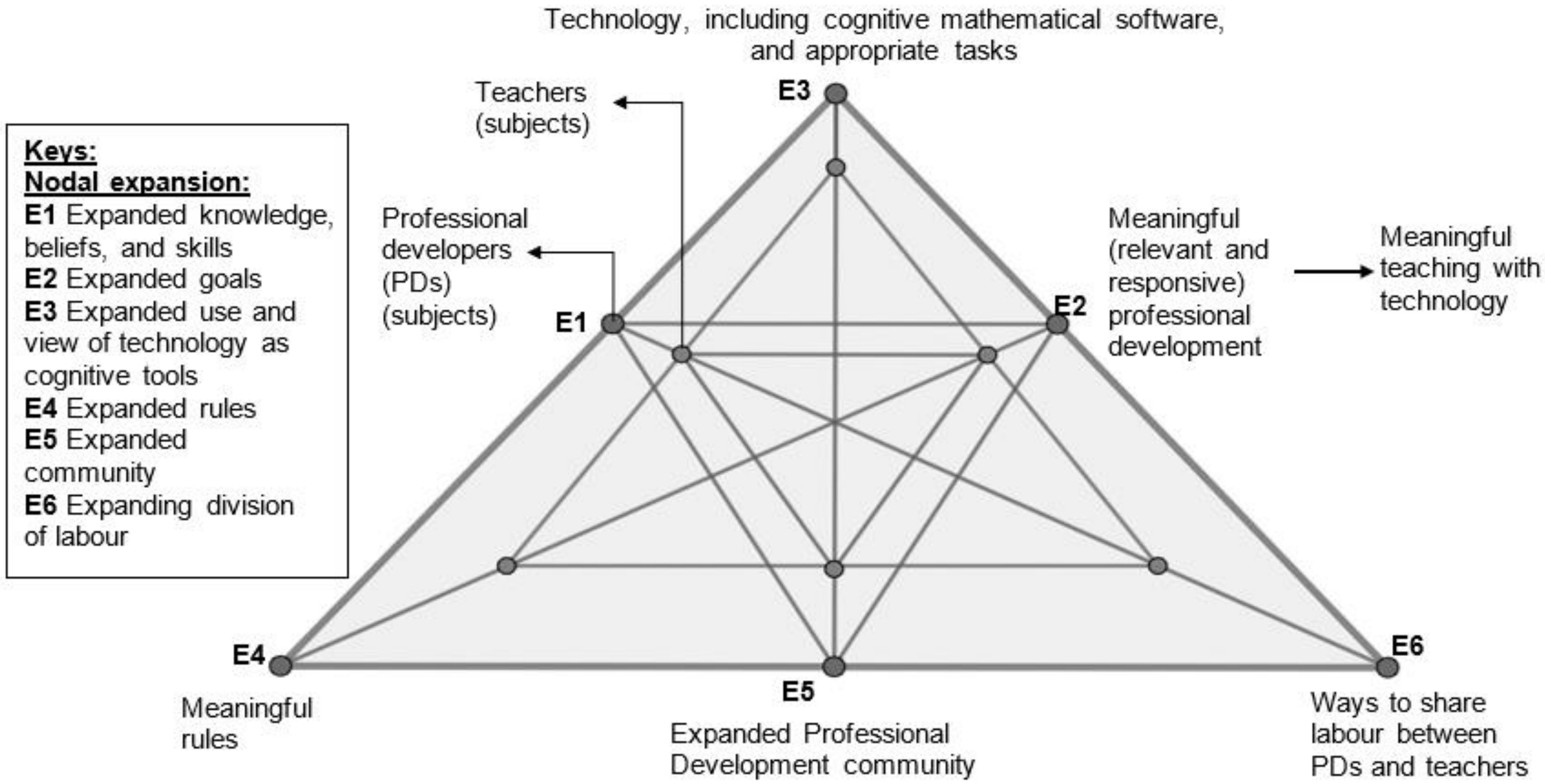

Figure 7.1: An expanded relationship between professional developers' and teachers' activity systems

The actionable principles for the professional development of Mathematics teachers to teach meaningfully with technology were described in Table 7.1. These principles were derived from the nodal interactions in Figure 7.1. The element of sustainability is spread throughout the principles to promote continuity. The design principles are linked to an 'E', which indicates the node that is expanded. These principles are also published in Kanandjebo and Lampen (2022, p.8) a publication that emanated from this thesis.

Table 7.1: Design principles for the professional development of Mathematics teachers to teach meaningfully with technology

Principle 1. Expanded beliefs, attitudes, and knowledge (E1): Expand the worldview of Mathematics teachers (personal instrumental genesis)
(a) Regularly engage with Mathematics teachers' professional development needs regarding the expanded goal. (b) Engage with Mathematics teachers' views and beliefs about technology and their mathematical histories. (c) Facilitate mathematical experiences that enhance teachers' mathematical proficiency, using technology where it is relevant and meaningful. (d) Facilitate teachers' technology use to stimulate their imagination of the relevance of technology in their lives.
Principle 2. Expanded goals (E2): Stimulate teachers' imagination to teach Mathematics meaningfully with technology (professional instrumental genesis).
(a) Promote critical reflection on teachers' existing teaching and learning of Mathematics. (b) Expose teachers to practices in the larger Mathematics education community where teaching and learning approaches are meaningful (e.g., through attending short courses and webinars).
Principle 3. Expanded use and view of technology as cognitive tools (E3): Stimulate teachers' imagination of promoting mathematical thinking with technology.
(a) Immerse teachers as learners in teaching-with-technology situations that stimulate their imagination of expanded personal goals (e.g., wanting to understand relationships beyond procedures, wanting to use technology to communicate through Mathematics). (b) Provide many opportunities for teachers to collaborate and experiment with teaching for Mathematical Proficiency using cognitive technology (e.g., by posing problems that can be solved better by the use of technology). (c) Provide many opportunities to reflect on the practicality and appropriateness of traditional tools and new technology tools for expanded goals.

Principle 4. Expanded rules for meaningful teaching with technology (E4): Stimulate teachers' imagination of how curriculum can change to promote meaningful teaching and learning.
(a) Regularly reflect on the Mathematics curriculum and contribute to curriculum reform (e.g., by co-participation of professional developers and teachers in local and international conferences). (b) Explore affordances and constraints of technology in relation to current and expanded formal assessment rules.
Principle 5. Expanded community (E5): Stimulate teachers' imagination of how technology can enable supportive communities of professional development.
(a) Create opportunities for peer observation and for professional developers to join teachers in their classrooms to learn about their natural teaching environments. (b) Expand teachers' communities of professional practice and development to include professional developers as well as education department officials to sustain long-term development. (c) Expand teachers' communities of professional practice and development to include remote online professional communities with compatible goals to expand and sustain learning on demand over the long term (e.g., through participating in the GeoGebra or Desmos communities).
Principle 6. Expanded division of labour (E6): Stimulate the development of mutually supportive communities of practice between professional developers and teachers.
(a) Engage with teachers in opportunities to plan, design, and showcase meaningful teaching experiences with technology on wider than local forums. (b) Professional developers must support teachers on an ongoing basis by developing a repository of tasks for meaningful teaching with technology, and by creating and providing non-synchronous professional development materials. (c) Professional developers with teachers must design prototype assessments with technology as a central cognitive tool and stimulate reflection on their use.
Principle 7. Ethical goal-directed action (E1-E5-E6): Promote lifelong professional development and sustainable implementation of meaningful teaching with technology.
(a) Professional developers and teachers must engage creatively with the ethics of teaching in terms of access to technology and equal opportunity for all to learn Mathematics meaningfully, with and without technology. (b) Professional developers and teachers must continuously share labour to develop culturally responsive teaching and learning materials toward the expanded goal. (c) Professional developers and teachers must engage with curriculum renewal as both become increasingly professional in their use of technology in the meaningful teaching of Mathematics.

In activity system objects are open to transformations thus the design principles are not intended to be prescriptive, complete, or final. They should remain tentative and evolving as the goals of teaching and learning Mathematics and technology evolves. They are rather emerging constantly through ongoing theoretical research, reflection, and changes in technology. They are also not intended to be a list, but to be used to develop holistic professional development for teaching Mathematics meaningfully with technology. In comparison with other models for professional development focused on teacher knowledge for teaching with technology these principles are actionable and holistic.

7.4 The progression tetrahedron for a professional development framework for teaching Mathematics meaningfully in Namibian secondary schools

Professional development designs seem under-researched; researchers define professional development according to TPACK. The reviewed frameworks for professional development on teaching with technology uniquely offer an opportunity to interpret and understand the knowledge teachers need to integrate technology into teaching. The PTK (Thomas & Palmer, 2014) afford to discuss knowledge of pedagogy, Mathematics, technology and teachers' affective aspects. Niess et al. (2009) afford to discuss TPACK within standardised Mathematics curricula. The Mathematical Digital Knowledge for Teaching (MDKT) framework (Tabach & Trgalová, 2019) allows us to discuss teachers' basic technology skills and utilisation schemes (personal instrumental genesis) as well as knowledge of how to support students in the digital environment (professional instrumental genesis). Joshi et al. (2021) draw attention to the omission of the ethical aspects of teaching with technology, such as intellectual property rights, copyright, and cyber safety, but do not consider the ethics from the perspective of limited access which was considered in this study. Niess et al.'s (2009) framework stops short of suggesting kinds of experiences that can serve as actionable guidelines for professional development.

The generic TPACK (Mishra & Koehler, 2006) (as discussed in Chapter 4) is viewed as formed by four flat-interconnected permeable triangles (Figure 7.2). The triangles represent content knowledge, pedagogy knowledge, and technology knowledge. The infusion of technology, pedagogy, and content knowledge (TPACK) is represented by the triangle at the centre. The positions of technology, pedagogy and content knowledge in the figure are of no importance, however, the triangle at the centre will always represent the infusion of content knowledge, pedagogy knowledge and technology knowledge to form TPACK. The triangles are permeable (shown with dotted lines) to imply that knowledge infuses within knowledge triangles and influences an individual's ability.

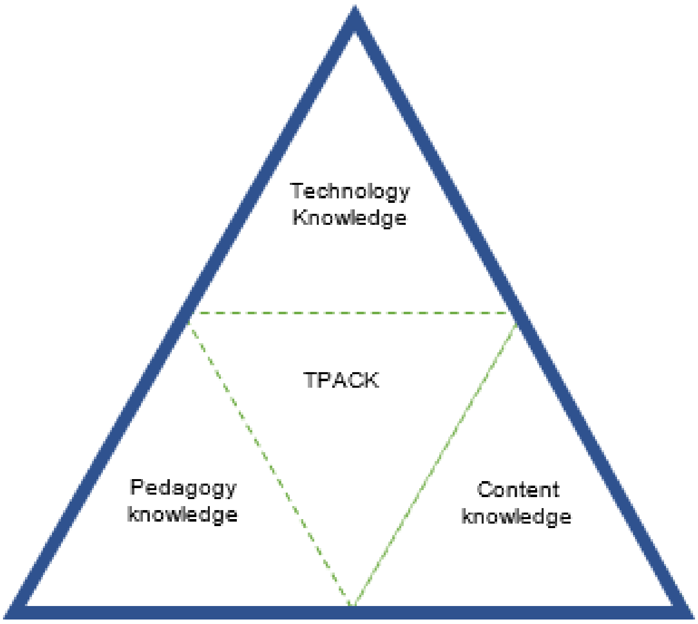

Figure 7.2: TPACK conceptualisation

The generic TPACK (Mishra & Koehler, 2006) without linking to subject pedagogy runs the risk of knowledge being used for personal genesis in the classroom which may take time to develop into professional genesis. In the case of Namibian secondary schooling, teachers obtained general knowledge of technology integration and used it on the same old goals – when curriculum goals were revised, teachers apply old pedagogy to it. The curriculum afforded teachers to integrate technology at substitution and tasks levels. As discussed, (in chapters 5 and 6) the role of curriculum, teachers' views and beliefs, ethics, and professional development needs play a greater role in the design of professional development (Figure 7.3). The teachers' personal orientations towards technology (the affective domain) as well as teachers' personal instrumental genesis are spontaneous sources of knowledge for professional development. This creates space to propose that teachers' TPACK is situated within ethics, personal orientations, and the Mathematics curriculum (context) (Figure 7.3).

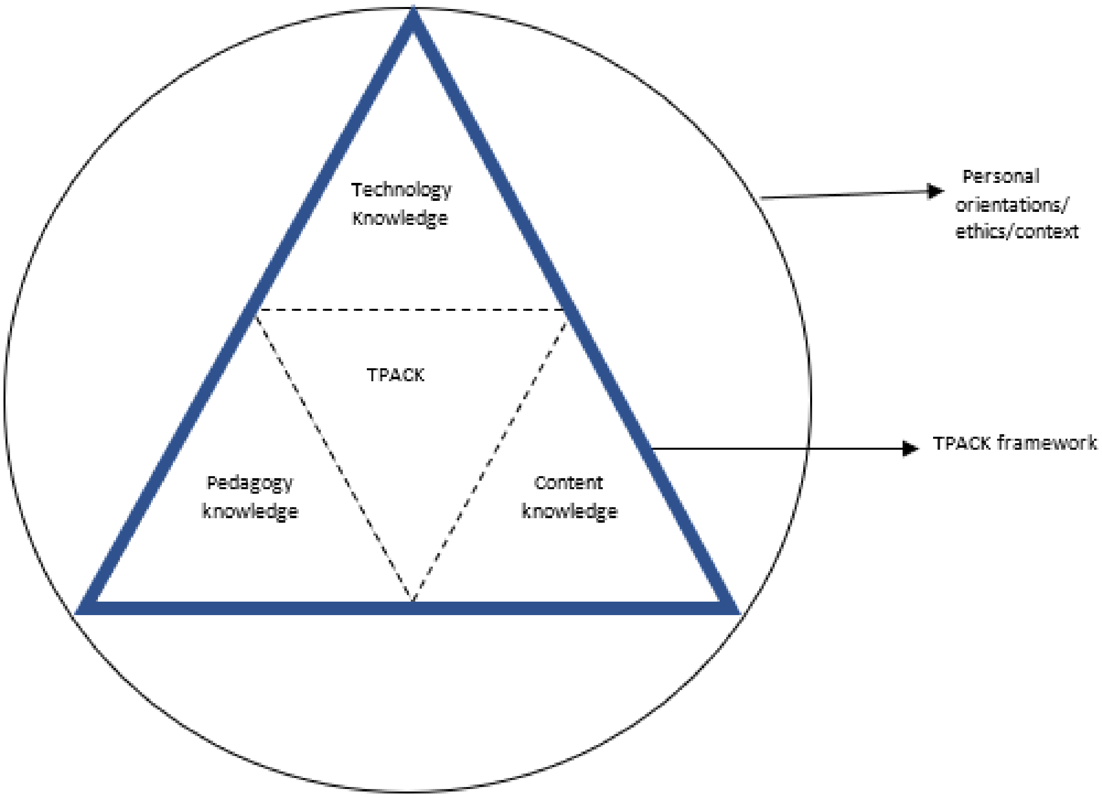

Figure 7.3: Infusion of TPACK within an individual personal orientation

A professional development session on any of the knowledge areas of the triangle will certainly lift it. For example, the Ministry of Education, Arts and Culture provided professional development on content knowledge; in the area of Mathematics, content knowledge would expand and influence the knowledge set of an individual. An individual's knowledge triangles also expanded from university training, the national

Mathematics congress, and any professional development the teacher received as per the study findings (Chapter 6). When an individual grew in all knowledge areas through professional development in teaching Mathematics meaningfully, the triangles "wing up" and form a tetrahedron (Figure 7.4).

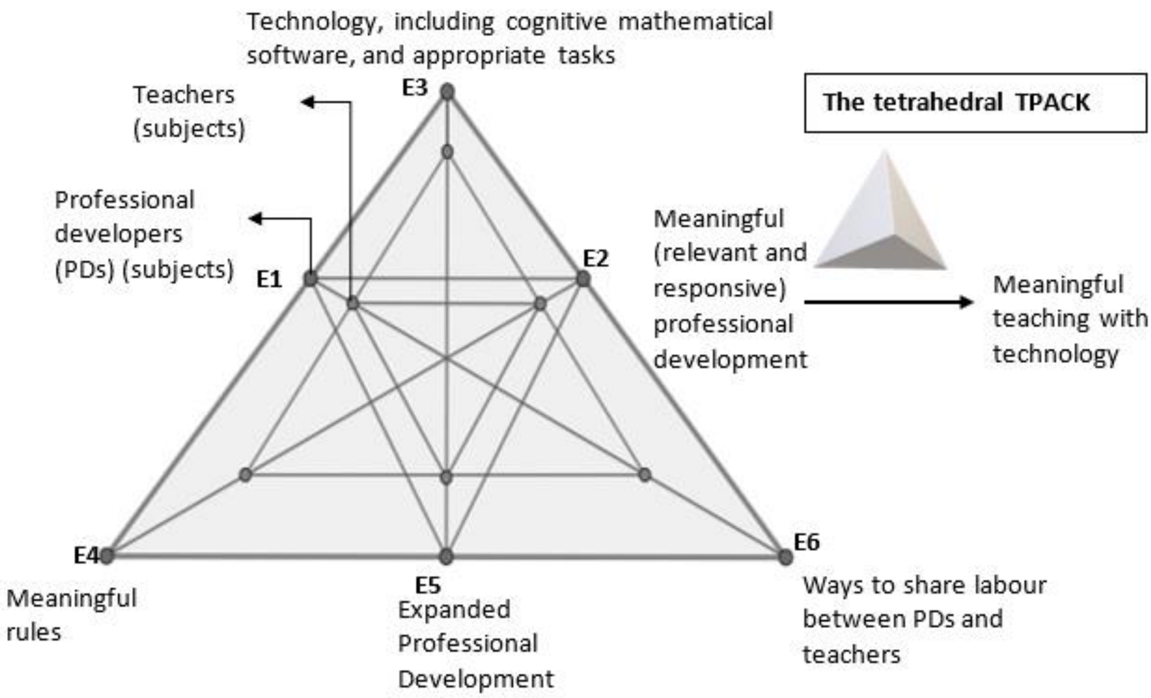

Figure 7.4: A combination of an expanded relationship between professional developers' and Teachers' activity systems, and tetrahedral TPACK

Taking cognisance of teachers' views and beliefs, their professional development needs, and the role of curriculum in the design of professional development, the researcher needed to know how knowledge develops in such a situation. Vygotsky (1978) argued that all knowledge progresses and develops from a person's spontaneous knowledge base and scientific knowledge is developed through learning. However, most TPACK-based frameworks (Koehler & Mishra, 2009; Tabach & Trgalová, 2019; Thomas & Palmer, 2014) seem to suggest that spontaneous knowledge is already scientific knowledge. Furthermore, the findings showed that teachers have some sort of spontaneous knowledge. The CHAT states that knowledge develops from the bottom up so that expansive learning can occur (Engeström, 2001).

Consequently, a holistic and sustainable professional development for developing Mathematics teachers' knowledge to teach Mathematics meaningfully with technology in Namibian schools, progresses through three levels (Figure 7.5). At each level, professional development should holistically influence Mathematics teachers' personal orientations and personal instrumental genesis for professional instrumental genesis. The knowledge is also not static but keeps changing according to the changes in technology and in the answer to "What knowledge do we want the students to acquire?". The levels are presented bottom-up with expansive learning and idealised design notion in mind. The ideal Mathematics teacher to teach Mathematics meaningfully with technology is one at the scientific knowledge level (level 3). The findings of the study allowed us to describe spontaneous knowledge level (level 1). The strategic knowledge level (level 2) and scientific knowledge level (level 3) were described theoretically (Figure 7.5).

Level 3: Scientific knowledge level – This is idealised knowledge of teaching Mathematics meaningfully with technology. Through a structured professional development, professional developers should develop teachers for teaching Mathematics meaningfully with technology to be able to:

- Influence, participate, expand, or adapt Mathematics curriculum and pedagogy when cognitive technologies become available;
- Expand or adapt applications of technology that make Mathematics meaningful for pedagogies for changes in the Mathematics curriculum;
- Carry out research as new problems and demands arise in Mathematics and technology education;
- Develop professional development interventions on different scales in different communities, e.g., develop ways to improvise where access to technologies is problematic;
- Imagine and enact ways to overcome problems of access to technology for Mathematics; and
- Reflect critically on new information about how people with technologies think and reason.

Level 2: Strategic knowledge level – At this level, a community of Mathematics teachers may emerge whose spontaneous knowledge has expanded. These Mathematics teachers can be co-demonstrators applying skills and knowledge and amending existing knowledge. These Mathematics teachers should have fully implemented learned skills and knowledge and have extracted the maximum benefits of knowledge learnt. Through a structured professional development, professional developers should develop teachers for teaching Mathematics meaningfully with technology to be able to:

- Apply mathematical pedagogies with an understanding of what technology is suitable for;
- Co-demonstrate and lead mathematical pedagogical processes with technologies to a larger extent; and
- professionally guide meaningful exploration with and without technology.

Level 1: Spontaneous engagement and knowledge level – Based on the research engagements (Chapter 6), professional development at this level was imagined and designed. Through structured professional development, professional developers create a zone of proximal development of mathematical opportunities (e.g., mathematical tasks) and various technological pedagogical environments. The opportunities should afford Mathematics teachers knowledge for teaching Mathematics meaningfully with technology to:

- Reflect on and imagine the implications of the Mathematics curriculum and teaching with technology;
- Yearn to know how technology works, by allowing their desire and their professional development needs on teaching Mathematics meaningfully with technology to drive their quest;
- Develop Mathematics teachers’ ability to “marry” technology to mathematical concepts;
- Engage in critical review and analysis of both representational and cognitive technologies;
- Engage in mathematical tasks that will result in expanded demand to reason, justify with technologies to others through social or sharing; and

- Experience expanded conceptual knowledge, and improved attitudes, interest, and enthusiasm.

Hence, professional development developers should carefully facilitate the development of knowledge for teaching Mathematics meaningfully with technology by flexible demonstration. The professional development developer hugely stimulates meaningful teaching with technology conversations. The professional development developer also plays both roles to solidify teaching conceptions and strengthen them.

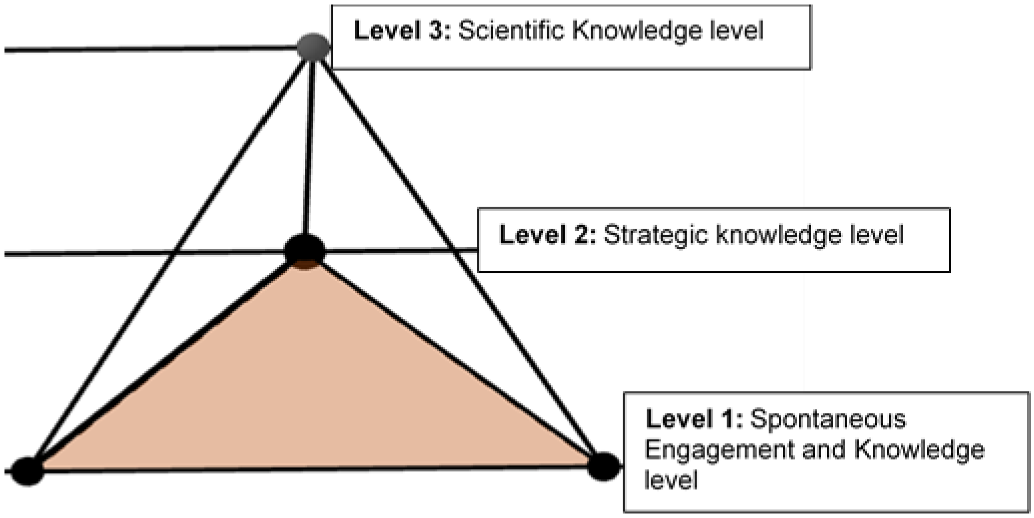

Figure 7.5: The progression tetrahedron for a professional development framework for teaching Mathematics meaningfully in Namibian secondary schools

The progression tetrahedral of a professional development framework for teaching Mathematics meaningfully holds teachers' practices and beliefs for teaching which have levels. Professional development engagement throughout the levels is structured.

7.5 A professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools

A professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools (Figure 7.6) is a culmination of (a) design principles, (b) an expanded relationship between professional developers' and Teachers' activity systems, and (c) the knowledge progression tetrahedron.

The five strands of mathematical proficiency were adapted to include the role of technology in teaching Mathematics and conceptualised as descriptions of proficiency of teachers (Section 7.4). The descriptions were based on the study findings (Chapter 6) and followed Vygotsky's urge to base ideas of how to teach on the answer to the question "What knowledge do we want the students to acquire?". The bi-directional professional development activity system that encompasses and holds the activity system of teachers was used as a planning tool for holistic professional development (subsection 7.3). The organisation of the expanded relationship between professional developers' and teachers' activity systems (Figure 7.4) allowed the planning of reciprocal engagements at every node between the teacher's system and professional developers' system. Moreover, the actionable principles for the professional development of Mathematics teachers to teach meaningfully with technology (Table 7.1) were derived from the nodal interactions shown in Figure 7.4). The design principles take cognisance of teachers' views and beliefs, their professional development needs, and the role of the curriculum.

Vygotsky (1978) argued that all knowledge develops from a person's spontaneous knowledge base and scientific knowledge is developed through learning. However, most TPACK-based frameworks (Mishra & Koehler, 2006; Tabach & Trgalová, 2019; Thomas & Palmer, 2014) seem to suggest that spontaneous knowledge is already scientific knowledge, even though some acknowledged that Mathematics teachers' TPACK progresses through developmental levels (Niess et al., 2009). Furthermore, the findings showed that teachers have some sort of spontaneous knowledge. The Activity Theory states that knowledge develops from the bottom up so that expansive learning can occur (Engeström, 2001). Consequently, a holistic and sustainable professional development for developing Mathematics teachers' knowledge to teach Mathematics meaningfully with technology in Namibian schools progresses through three levels (Figure 7.5). At each level, professional development should holistically influence Mathematics teachers' personal orientations and personal instrumental genesis for professional instrumental genesis. The knowledge progression tetrahedron is not fixed nor static but keeps changing with the changes in technology and in answer to "What knowledge do we want the students to acquire?". The levels are presented from the bottom up with expansive learning and idealised design notion in mind but develop from the spontaneous knowledge level. The culmination ends with a framework (Figure 7.6) with the desired outcome of teaching Mathematics meaningfully with technology.

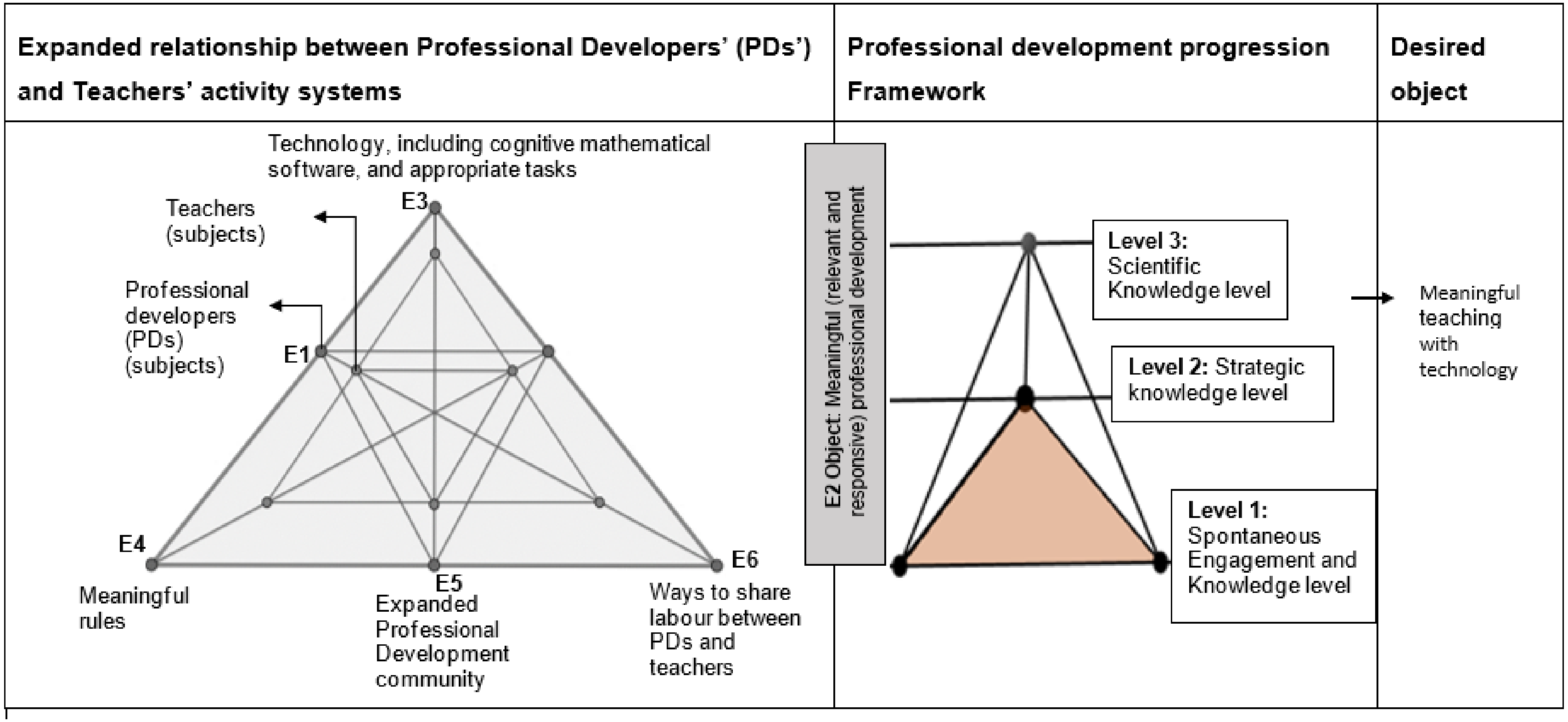

Figure 7.6: The professional development framework for teaching Mathematics meaningfully with technology

CHAPTER 8 SUMMARY AND CONCLUSIONS

8.1 Introduction

This final chapter provides a summary and conclusions of the study's main findings. First, the main findings of the study based on the research questions are discussed (Section 8.2) followed by contributions of the thesis to theories and the teaching professional development of teaching Mathematics with technology (Section 8.3). Lastly, the chapter presents the trustworthiness of the study (Section 8.4), conclusions (Section 8.5) and recommendations for future research and development (Section 8.6).

8.2 Summary of findings based on the study's research questions

This study aimed to contribute to the professional development of Mathematics teachers to teach Mathematics meaningfully with technology in Namibian schools. Research has shown that Mathematics teachers are insufficiently trained to teach with technology. This is ascribed to a lack of Mathematics-focused technological teaching knowledge and skills development. The current study focuses on designing a professional development framework for teaching Mathematics meaningfully in Namibian secondary schools. The findings were discussed and presented in chapters 2 and 3 (Stage 1) and chapters 5, 6 (Stage 2) and 7 (Stage 3). The following section 8.2.1 provides a summary of stage one and stage two phase one findings as they are both based on the same research question. Sections 8.2.2, 8.2.3 and 8.2.4 provides a summary of findings for stage two phase two.

8.2.1 Stage one: Frameworks of professional development for teaching Mathematics with technology (stage 1) and Namibian secondary school curriculum (stage two phase 1)

The sub-research question that guided stage 1 and stage two phase 1 was:

> *How can the aspects of meaningful teaching and learning of Mathematics (as espoused by Kilpatrick et al.,'s (2001) five strands of Mathematics proficiency and the Namibian national curriculum) be integrated with frameworks for teaching with technology (e.g., TPACK)?*

Stage 1: Literature survey

The study found that Technological Pedagogical and Content Knowledge (TPACK) based frameworks have no actionable guidelines for professional development and neglect the ethical dimension of professional development to teach Mathematics with technology in a developing country like Namibia. Moreover, Vygotsky (1978) argued that all knowledge develops from a person's spontaneous knowledge base and scientific knowledge is developed through learning. Some acknowledged that Mathematics teachers' TPACK progresses through developmental levels (Niess et al., 2009) fall short of suggesting kinds of experiences that can serve as actionable guidelines for professional development. Moreover, the third generation CHAT states that knowledge develops bottom-up so that expansive learning can occur (Engeström, 2001).

8.2.2 Stage 2 phase 1: Contextual analysis Namibian secondary school Mathematics curriculum

The analysis is based on reviewing the following curriculum documents related to teaching Mathematics in Namibian secondary schools which are (as discussed in Chapter 5): (a) national curriculum for basic education (Ministry of Education, Arts and Culture, 2016), (b) Mathematics subject policy for grades 4 to 12 (Ministry of Education, Arts and Culture, 2019), and (c) the secondary school syllabuses for the junior secondary (NIED, 2015), ordinary level (grades 10 to 11) (NIED, 2018) and the

AS (grade 12) (NIED, 2020) syllabuses. The findings about the Mathematics curriculum are also backed up by Mathematics teacher participants' views on the Namibian curriculum in terms of teaching Mathematics meaningfully with technology (5.4). The analysis aimed to identify the affordances and constraints of the Namibian secondary school curriculum in terms of Mathematics teaching goals (Kilpatrick et al., 2001) with technology to identify how it can be integrated with existing professional development frameworks. In chapter 6, the study found that only conceptual understanding and procedural fluency are described explicitly in the Namibian curriculum. The curriculum does not provide adequate goals for the strands of strategic competence and adaptive reasoning. The objective of the development of a productive disposition is relegated to learners' fun in Mathematics and does not feature explicitly in the guidelines for teaching. The findings concur with those of Mateya, Utete and Ilukena (2016) that procedural fluency and conceptual understanding are best accommodated. However, there is a rare reference to the development of strategic competence and/or adaptive reasoning, as was found by Stephanus (2014). In terms of teaching with technology, the Namibian curriculum aligns Mathematics teachers at the substitution level of technology integration according to the SAMR model (Puentedura, 2010) as well as at task-level opportunities on a pedagogical map (Pierce & Stacey, 2010). At these levels, technology tools and mediating artefacts are narrowly used to improve speed and accuracy and to provide a variety of visual representations of Mathematics content. The calculator is the only technological device explicitly stated to be used, and then for ‘‘efficiency’’ and ‘‘accuracy’’ in mathematical calculations.

The conclusions may imply that the teaching approaches enable the development of more basic and low-level skills which call for redefinition and reformulation of pedagogical approaches as well as curriculum objectives. The redefinition may result in expansive learning and new activity (Engeström, 2001; Hardman, 2015). Mathematics teacher participants noted a need to restructure the Mathematics school curriculum and pedagogy to suit the redefinition of knowledge and what it means to learn. Re-structuring and redesigning the curriculum stimulate a paradigm shift from instructional practices that are not congenial (Leong, et al., 2011) to meaningful teaching. Consequently, professional development intervention to enable Mathematics teaching with technology meaningfully must engage with the current mismatch of aims, rationales, visions and goals between the Namibian curriculum and expanded goals for the inclusion of technology. This is so as tools have the potential to expand goals for mathematical thinking and lead to new activity in a rapidly changing world. Professional developers of Mathematics teachers must promote culturally responsive, meaningful Mathematics teaching with technology (Kanandjebo & Lampen, 2022). The authors of the extract from this thesis further argued that in the ‘‘cultural context of Namibian Mathematics education, such guidelines must be made explicit and operationalised in professional development’’ (p.6). Similarly, to the Singaporean case, where provision is made for master teachers who have a greater theoretical understanding of the subject, and they can be called upon to conduct research in certain areas for sustainability (Kaur, 2014).

In addition, participants’ views support findings from the document analysis protocol of the Namibian secondary school Mathematics curriculum (design stage two phase one findings that Mathematics curriculum documents such as Mathematics subject policy and syllabus are not explicit in terms of proficiencies and teaching Mathematics with technology. As a result, Mathematics teachers may tend to work towards traditional goals (Kanandjebo & Lampen, 2022). The study’s findings also revealed that the teaching duration allocated for teaching Mathematics is not sufficient to allow technology integration; while the subject content is described as ‘‘overcrowded’’, ‘‘too long’’, and ‘‘too much’’. Moreover, the findings of this study revealed a suggestion by Mathematics teacher participants that the Mathematics curriculum must be amended to make the use of technology a statutory requirement in secondary school Mathematics and even be part of the assessment. This demonstrates the influence top-down rules have on the implementation of the operative (taught) curriculum.

8.2.3 Stage two phase two (cycles 2 to 4): Technological knowledge, pedagogical content knowledge and pedagogical knowledge for teaching Mathematics meaningfully

The sub-research question that guided stage two phase two

> *What technological knowledge, content knowledge and pedagogical content knowledge are considered by participating Mathematics teachers as necessary for teaching Mathematics meaningfully with technology?*

The data was collected through four iterative cycles (Chapter 6). The data is presented through subheadings.

Participating teachers' current teaching practices

Mathematics teacher participants view technology as a tool to improve the speed and accuracy of mathematical calculations. They also view meaningful teaching with technology as using technology for low-level tasks of demonstration and verification of mathematical ideas in selective traditional pen-and-pencil teaching environments. This is not unexpected, since globally, Mathematics teachers are insufficiently prepared to teach with technology (Albion et al., 2015; Getenet, 2020). Therefore, the findings of this study concur with others' (Getenet, 2020; Niess et al., 2009; Tabach & Trgalová, 2019) views on the need for a professional development framework to help provide guidance and efficiency to teach Mathematics with technology. In addition, the findings revealed that meaningful teaching is deeply aimed at answering the question prompted by Vygotsky: "What knowledge do we want the students to acquire? " (Elkonin, 1989 in Karpov, 2003, p. 69). Hence, the conclusion is that if teachers expand the goal they have for their learners towards the five strands of proficiency, then they may want their professional development to align with the expanded goals.

In addition, the lack of technological skills of learners also posed a challenge; this is to be expected as the only technological tool learners may have access to is a scientific calculator and where possible, a computer once a week. These findings correspond to the findings obtained from design stage two, phase one (chapter 5, 5.3) that a calculator is to be used as an efficiency tool. It is evident from the data that teachers only inform learners of the importance and/or allow learners to engage with the technology without teachers' involvement. The findings are similar to those of Drijvers (2020) in that teachers tend to step back on the teacher-driven approach which constrains the development of mathematical skills.

Current practices also show that learners cannot share in the division of labour (teaching and learning process) nor are some Mathematics teachers able to teach with technology as they had no access to technological tools; thus, the teaching and learning system was perturbed. The findings concur with those of Nchindo (2019), Nendongo (2018) and Simataa and Simasiku (2012) in that access to technological tools hinders teaching with technology. This could be the reason why teachers promoted mainly conceptual understanding and procedural fluency in Stephanus's (2014) study.

The findings also reveal that teachers view teaching Mathematics meaningfully with technology as meeting the ‘‘lesson objectives’’ for positive academic performance. They view the Mathematics curriculum content as basic concepts that should be delivered to the learners; this is evident from the responses of the majority (six out of nine) of the participants. Yet they also view teaching Mathematics meaningfully with technology as promoting strategic competence, conceptual understanding, and productive disposition.

Meaningful professional development needs for teaching Mathematics with technology

The participating teachers reported very limited access to professional development that integrates the use of technology and subject knowledge. That is unexpected as Kasanda (2015) found professional development in Mathematics to have not yet taken root in Namibian schools and most professional development focuses only on building teachers' subject knowledge (Villet et al., 2020). Professional development should therefore focus on developing knowledge of teaching Mathematics with technology including the syntax of programming language and computer coding skills. In addition, it should also influence Mathematics teachers' attitudes, values, and confidence in using technology (personal orientations) in sharing the role of developing their knowledge to use technology for teaching Mathematics. The findings concur with those of Mishra and Koehler (2006), Niess et al. (2009), Tabach and Trgalová (2019), and Thomas and Palmer (2014) in that teachers need pedagogy, Mathematics knowledge and technology knowledge to teach with technology. The needed syntax of programming knowledge concurs with the study by Wassie and Zergaw (2019) that teachers need prior programming experience to input some mathematical commands as a lack of it may constrain them from fully exploring and teaching with cognitive tools.

Aspects of teaching and learning Mathematics that Mathematics teachers can and cannot achieve using technology

The participants indicated that technology allows the development of procedural fluency, problem-solving, and conceptual understanding, and influences learners' productive disposition (6.8.4). The findings further indicated that technology enhances understanding of Mathematics and simplifies concepts for better academic performance. The findings also revealed that technology affords speed and a means to provide remedial support. The affordances teachers seem to be aware of are those of technologies' ability to improve efficiency and speed (Hollebrands, 2017). Teachers view technology mainly as an amplifier that improves precision, efficiency, visualisation, and speed in comparison to performing it manually without technology (Hollebrands, 2017). The findings concur with themes on affordances found by Ruthven and Hennessy (2002) that technology affords speed, and accuracy and can be used as a means to enhance understanding of mathematical concepts. The findings also revealed that Mathematics teachers view technology use as a promoter of laziness in Mathematics teaching and learning as most calculations are performed by the technological tool (6.8.4). Pea (1987) argues that technology is considered cognitive when it provides cognitive support during mathematical thinking. Participants acknowledge that there is no paucity of technological resources, and they are agentic in the use of technology tools. Thus, the finding creates a gap for professional development to enhance teachers' understanding on how technology can be used as a cognitive tool in future research.

8.2.4 Stage two phase two cycle 5: participants' views and beliefs on teaching Mathematics with technology after participation in the design process

The sub-research question was:

> *How does Mathematics teachers' participation in the design process of a framework for teaching Mathematics meaningfully with technology influence their beliefs and views about teaching Mathematics with technology?*

Participants described the research engagement as ‘‘fun and constructive’’ and ‘‘useful and beneficial’’ (6.8.5). They commended the demonstration approach used. The approach implied that the researcher is “the hand that holds the mouse”. Meanwhile, participants are the “mind that instructs the mouse” to support understanding even if there is a lack of facility of technology tools. Further, findings showed that the design process to some extent created a community of practice. This concurs with the study by Manfra (2019) that teachers' participation in the research process situates them as learners and creates a community of practice. This may promote sustained professional learning activities. The findings further indicated that teachers attach their value of teaching and learning with technology to their teaching goal, thus further motivating why the professional development of teachers should be linked to their teaching job. The finding concurs with those of Ertmer et al. (2012) that teachers' beliefs and attitudes about the relevance of technology to learners' learning have a major influence on their technology pedagogies.

The findings (6.8.5) further revealed that most of the participants see the concept of meaningful teaching with technology as their “new best”. They noted that there is no other better way of teaching Mathematics with technology, even though before the research engagement, participants used technology in teaching Mathematics on a small scale (Hamilton et al., 2019; Kanandjebo & Ngololo, 2017; Ugulu, 2019). During the engagement, they might have experienced a shift in their personal orientations (Tabach & Trgalová, 2019) and thus conclude that they will always use technology. The space created between no technology to “always technology” is the space for the imagination of strategic growth in a structured professional development. This is so as it could depict that their attitudes and values about the affordances of technology outweigh the constraints.

8.2.5 Stage three: Overview of a framework for teaching Mathematics meaningfully with technology in Namibian secondary schools

The overarching question was:

> *What are the key aspects of a framework for the professional development of Mathematics teachers for teaching Mathematics meaningfully with technology?*

The key aspects of a professional development framework for teaching Mathematics meaningfully with technology in Namibian secondary schools (Chapter 7 Figure 7.6) is a culmination of (a) conceptualised teachers’ proficiency in teaching Mathematics with technology, (b) design principles, (c) expanded relationship between professional developers' and teachers' activity systems, and (d) the knowledge progression tetrahedron.

The five strands of mathematical proficiency were adapted to include the role of technology in teaching Mathematics and conceptualised as descriptions of proficiency of teachers (Section 7.4). The descriptions were based on the study findings (chapters 5 and 6) and followed Vygotsky's urge to base ideas of how to teach on the answer to the question: "What knowledge do we want the students to acquire?'' The bi-directional professional development activity system that encompasses and holds the activity system of teachers was used as a planning tool for holistic professional development (subsection 7.3). The organisation of the expanded relationship between professional developers' and Teachers' activity systems (Figure 7.4) allowed the planning of reciprocal engagements at every node between the teachers' system and professional developers' system. Moreover, The design principles are derived from the design of embedded teacher and professional developer activity systems from a perspective of Cultural Historic Activity Theory (Engeström, 2001) for expansive learning. The design principles take cognisance of teachers' views and beliefs, their professional development needs, and role curriculum. The design principles must always remain tentative and evolving as the object of Mathematics and technology evolves. A progression model is proposed according to Vygotsky's notion of spontaneous and scientific knowledge. The object of professional development that must be expanded through professional development is the meaningful teaching of Mathematics with technology as laid out in chapter 1.

Consequently, holistic, and sustainable professional development for developing Mathematics teachers' knowledge to teach Mathematics meaningfully with technology in Namibian schools progresses through three levels (Figure 7.5). At each level, professional development should holistically influence Mathematics teachers' orientations and personal instrumental genesis for professional instrumental genesis. The knowledge progression tetrahedron is not fixed nor static but keeps changing with the changes in technology and in the answer to “What knowledge do we want the students to acquire?”. The culmination ends with a framework (Figure 7.6) with the desired outcome of teaching Mathematics meaningfully with technology.

Due to COVID-19 constraints on the research project the principles and framework that are the designed products of the research, have tentative status and must be strengthened by empirical application in future research.

8.3 Contributions of the thesis to theories and knowledge

The major contribution that the study made was a designed, culturally appropriate professional development framework with holistic and actionable guidelines for a specific developing country (Figure 7.6). The professional development framework comprises design principles based on activity theory, and a description of progression levels based on Vygotsky’s view of spontaneous and scientific knowledge. The study contributed to theories and knowledge as follows:

The study adapted the five strands of mathematical proficiency as the hallmark for meaningful teaching of Mathematics (Kilpatrick et al., 2001), including the use of technology in teaching Mathematics. These strands were conceptualised as descriptions of the proficiencies of teachers for teaching Mathematics with technology. This was done in the light of the data from the participants that show how closely these teachers align their professional development needs to the knowledge and skills they want their learners to develop. For teachers, meaningful teaching is deeply aimed at answering the question prompted by Vygotsky: “What knowledge do we want the students to acquire? ” (Elkonin, 1989 in Karpov, 2003, p. 69). Hence, the conclusion is that if teachers expand the goal they have for their learners towards the five strands of proficiency, then teachers may want their professional development to align with the expanded goals.

The theoretical relationship between professional developers and Mathematics teachers using technology was developed from third generation CHAT. The main contribution of the study is the way the activity systems were embedded to design possibilities for expanding the relationship between professional developers' and teachers' activity systems (Figure 7.1). In other studies (e.g Engeström, 2001a), CHAT seems to indicate that separate activity systems are mainly related through their objects from which a new object may develop. In the cultural context of Namibian Mathematics education, professional development cannot be portrayed by two activity frameworks in a lateral relation (as in Chapter 2, figure 3.2) as professional developers simultaneously hold and support the teachers' activity system, while purposely disturb teachers' current systems to promote expansive learning. Thus, the bi-directional professional development activity system that was developed (Figure 7.1) depicts reciprocal nodal interactions at every node between the teachers' activity system and professional developers' activity system. The embedded triangles are considered flexible and able to change shape and it reminds professional developers of the holistic and emerging nature of professional development in situations that require both professional developers and teachers to learn expansively. Another contribution to CHAT is the identification of an ethics triangle formed by the nodes Subject – Community – Division of labour. The implication of the ethics triangle is that expansive learning, especially in a developing country like Namibia, creates increased demands on the community and on teacher work, which must be shared by professional developers for sustainability.

The third theoretical contribution of the study is in the re-interpretation of TPACK as an integrated three-dimensional model that includes progress, named the knowledge progression tetrahedron. In contrast with the TPACK model of Mishra and Koehler (2006) which is depicted as three flat and interconnected circles that describe types of knowledge without depicting spaces for progression, the tetrahedron model guides the development of professional knowledge from spontaneous knowledge through the personal use of technology, to scientific knowledge that is structured and grounded in research and teachers' evolving practices. The knowledge progression tetrahedron developed in this study extends TPACK-based frameworks (Tabach & Trgalová, 2019; Thomas & Palmer, 2014) that acknowledge the role of personal instrumental genesis (spontaneous knowledge) alongside professional instrumental genesis (scientific knowledge) of using technology, but does not relate the two kinds of genesis in a developmental progression. Also based on the literature (stage 1, chapters 2 and 3) those that acknowledged that Mathematics teachers' TPACK progresses through developmental levels (Niess et al., 2009) fall short of suggesting kinds of experiences that can serve as actionable guidelines for professional development. This was through designing actionable guidelines for professional development in a cultural context like Namibia and conceptualising ethics in terms of access to technologies for developing countries, such as Namibia.

The final contribution of the study is the derivation of design principles from the nodal interactions in the professional development activity system (Table 7.1). These principles are proposed as the basis of holistic professional development for teaching secondary school Mathematics meaningfully with technology. In a lesser way, the study contributed to the design-based research methodology by adapting the generic design research model by Wademan (2005) (Figure 1.1) to DBDR stages (Figure 3.8) that end with tentative products that were used in this study.

8.4 Trustworthiness of the research

Credibility was ensured by drawing from technologically adept Mathematics teacher participants as they are the implementers of the curriculum and analysing the Namibian senior secondary school curriculum as it informs teaching and learning. Further, credibility in this study the researcher used different triangulation approaches. The researcher collected data in five cycles to identify Mathematics teacher participants' current technological teaching practices, needs and beliefs about meaningful teaching of Mathematics. Theoretical triangulation was achieved by investigating different learning theories which were Vygotsky's sociocultural learning theory, third-generation CHAT and adult ELT. In addition to the process of development described by these theories, the goal of the development of mathematical proficiency as described by Kilpatrick et al. (2001), was also triangulated with Vygotsky's notion of scientific knowledge and the TPACK goals of Mishra and Koehler (2006), Thomas and Palmer (2014), Tabach and Trgalová (2019) and Niess et al. (2009). Triangulation of data sources helped the researcher to notice and follow up on any inconsistencies in the data and signs of possible misinterpretation of questions. This was necessary to strengthen the effectiveness of the researcher's arguments. Different research instruments were used over a year, namely two online questionnaires (online due to COVID-19) and three focus group discussions on ZOOM or MS teams. All recordings and responses to questionnaires are saved securely and available for scrutiny. Directly relevant transcriptions are attached in an addendum.

For dependability, the researcher presented an overview of the study at the online African Doctoral Academy winter school in July 2022 where qualitative research experts critically commented on the study. The research supervisor’s critical views and comments throughout the study boosted the researcher’s thinking. The reviewers’ comments on the extracted paper titled “Teaching Mathematics meaningfully with technology: Design principles for professional development” from this study and accepted by the *African Research Journal in Mathematics, Science and Technology Education* (AJRMSTE) helped the researcher to re-think her writing and the data.

In addition, transferability was observed by providing a clear detailed explanation of the findings, methodology and analysis to help readers decide on the transferability of the study. Further, to establish confirmability, the researcher documented the research process flow on how data was generated, analysed, and interpreted which led to the design of the professional development framework. The documentation of the research process depicted that the framework is the result of the research data, but not the researcher’s presumptions and assumptions.

8.5 Recommendations for future research and framework development

COVID-19 restrictions have influenced the research process (chapters 2 and 4) in terms of access to more participants and the impossibility of in person professional development interventions where the framework could be applied and tested. Hence the study cannot make fully conclusive and empirical statements, thus ending the study with a framework and design principles with tentative status. In future research, the framework must be implemented in professional development where it can be evaluated. Further, future research could use ELT as a theoretical frame to ground the design of learning tasks that would be used in the research engagements to evaluate the framework. Moreover, an in-depth study should be carried out on 'what the scientific knowledge level' for meaningful teaching with technology in school as an element of the community entails. The study acknowledges that the participants were few in comparison to the size envisioned at the beginning of the study. The number of Mathematics teacher participants can be increased to obtain a wide range of views and beliefs from more Mathematics teachers. However, it is worth noting that in the small (about 2.5 million) population of Namibia, the teacher education landscape has very few variations. The Ministry of Education, Arts and Culture for example has one central professional development unit that highly influences professional development in all regions of the country. As a result, on a macro level, the way professional development is conducted in the whole country suggests that Namibian Professional Development forms a coherent activity system. As highlighted in Chapter 2, the objects of activity systems are open to transformations. They emerge constantly through ongoing theoretical research, reflection, and changes in technology. It is in such a process of future collaboration that this theoretically grounded framework for professional development must be empirically grounded.

There is a need to carry out a study on the extent to which the tetrahedral professional development framework represents a model for professional development for teaching Mathematics meaningfully with technology in Namibian schools. An in-depth study can also identify teachers' practices concerning teaching secondary school Mathematics with technology. The in-depth study can also focus on determining whether teachers in their staffrooms and classrooms speak of the five strands of mathematical proficiencies to determine their awareness and support needed thereof. The sample size should be bigger, and a longer duration can be allocated to enhance representativeness. The longer duration of the research would ensure that participants have had time to implement the knowledge and are possibly in a better position to make critical judgements.

8.6 Recommendations for practice

There is a need to structure professional development courses on the syntax of programming language and coding related to teaching Mathematics in future research. The MoEAC should invest funds to research the modalities, course content and gains. The MoEAC through the professional development division and subdivisions should enlarge the scope of professional development to professionally develop teachers' knowledge on how to teach Mathematics and other subjects with technology meaningfully. Based on the findings discussed earlier (Chapter 6) there is no paucity of technological resources, and teachers are agentic in the use of technology tools, thus the MoEAC through professional development divisions should embrace the affordances of technologies and avail necessary resources for guided self-directed professional learning and meaningful technology integration. Guided self-directed professional learning could speed up professional skills development and possibly integration into school subjects thereof. The MoEAC through the NIED should consider curriculum reform in a way that goals for teaching Mathematics are explicit. The use of cognitive technologies should explicitly be emphasised to prevent the reduction of technology affordances in education for representation purposes only.

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ADDENDA

Addendum A: Document analysis protocol for [stage 2 phase 1]

This document analysis protocol guides the analysis of Kilpatrick, et.al.'s (2001) five strands of Mathematical proficiency and the Namibian national curriculum requirements to identify affordances and constraints in terms of meaningful teaching of Mathematics with technology.

Secondary level grades	**Mathematical understanding (proficiency strand)**	**Key proficiency terms (KPTs)**
Grade 8-9 Junior Secondary Certificate (JSC) level (based on the summary of learning content used)	**Conceptual Understanding**	
	Procedural fluency	
	Strategic competence	
	Adaptive reasoning	
Grade 10-11 Namibian Senior Secondary Certificate ordinary (NSSCO) level	**Conceptual Understanding**	
	Procedural Fluency	
	Strategic competence	
	Adaptive Reasoning	
Grade 12 Advanced subsidiary (AS)	**Conceptual Understanding**	
	Procedural Fluency	
	Strategic competence	
	Adaptive Reasoning	

Documents analysed:

(a) Ministry of Education. (2009). *The National curriculum for basic Education*. NIED. http://www.nied.edu.na/assets/documents/05Policies/NationalCurriculumGuide/Po_NationalCurriculumBasicEducation_2010.pdf

Ministry of Education Arts and Culture. (2016). *The National curriculum for basic education*. National Institute for Educational Development. http://www.edu.na

(b) Ministry of Education Arts and Culture. (2019). *National subject policy guide for Mathematics Grades 4-12*. NIED. http://www.nied.edu.na

(c) Syllabuses

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National Institute for Educational Development (NIED). (2018). *Mathematics syllabus Ordinary level, grades 10-11*. NIED. http://www.nied.edu.na/documents/syllabuses/seniorsecondary/Others/

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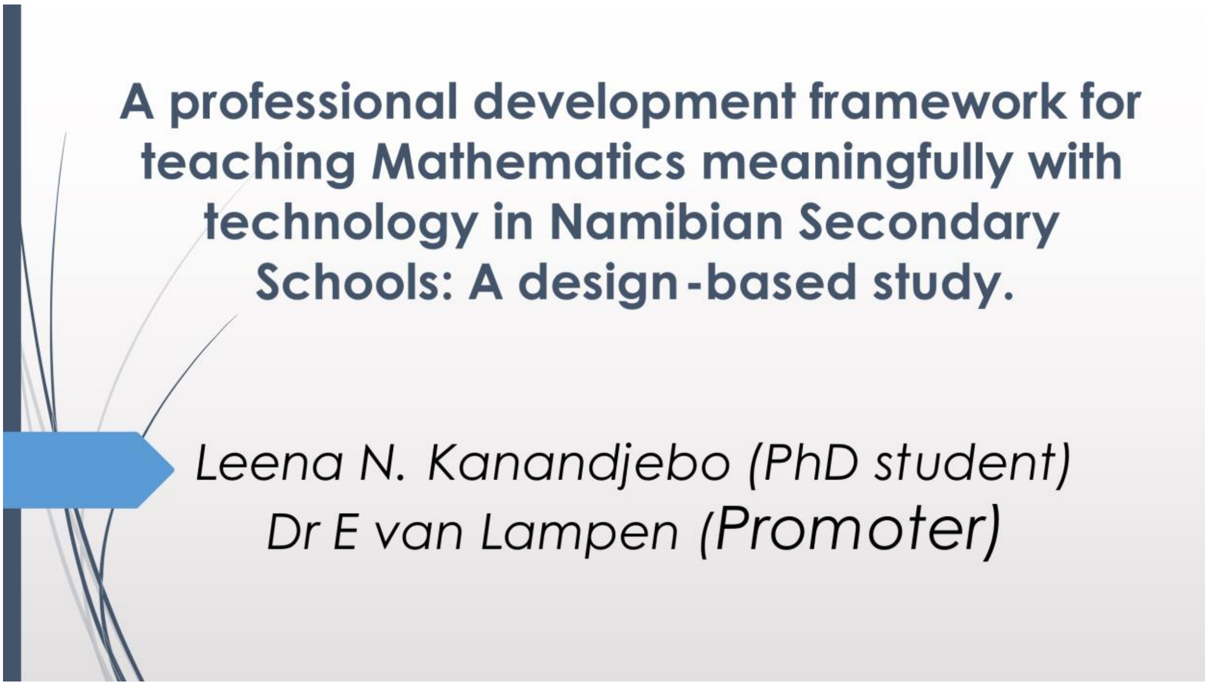
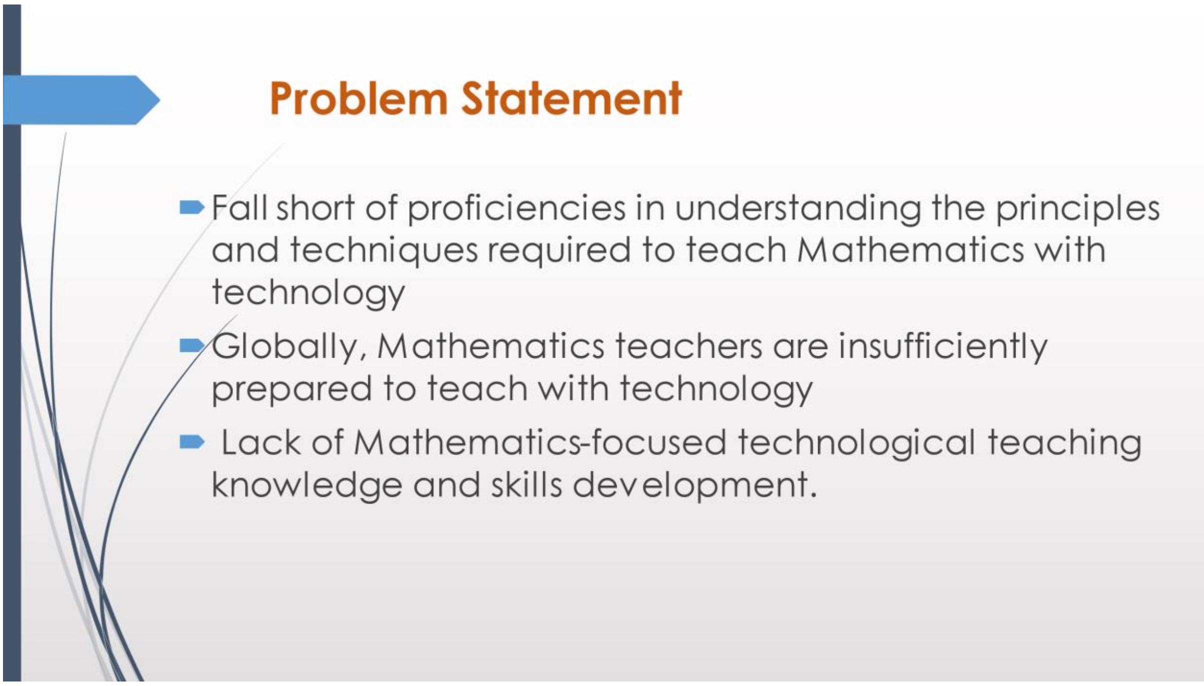
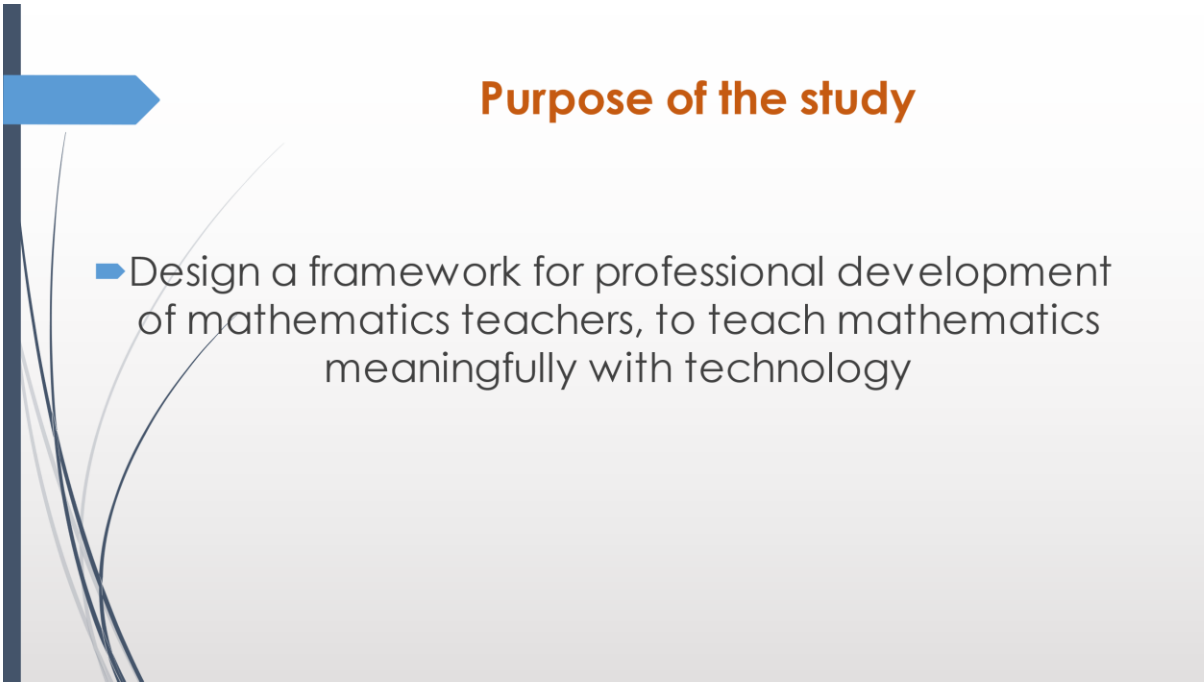
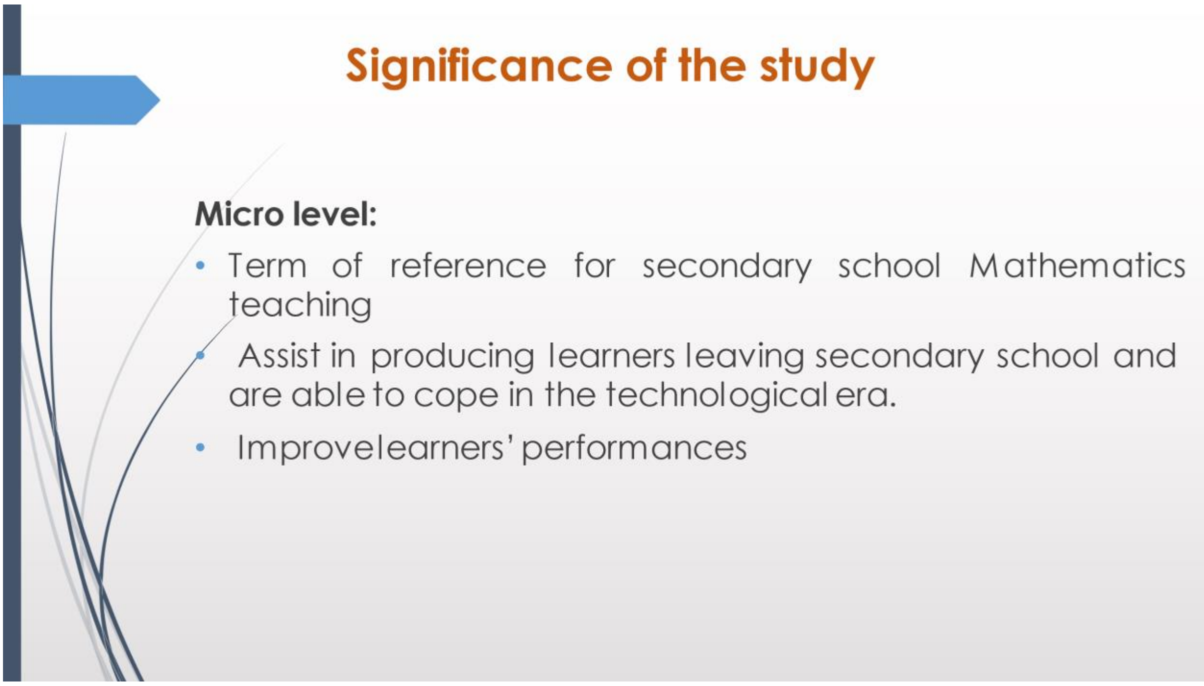
Addendum B: Introductory discussion PowerPoint presentation for [stage 2 phase 2 (cycle 1)]






Purpose of the study
Design a framework for professional development of mathematics teachers, to teach mathematics meaningfully with technology


Significance of the study
Micro level:
Term of reference for secondary school Mathematics teaching
Assist in producing learners leaving secondary school and are able to cope in the technological era.
Improvelearners' performances

Cont.

Macro level

- Contribute to literature
- Guide regional and national professional development subdivisions in the Ministry of Education on meaningful use technological in teaching secondary school Mathematics
- Influence policies and regulations.
- Assist with accelerating the attainment of National development plans e.g National Development Plan 5 (NDP 5)

STUDY PROCEDURE

Design stage and cycle	Involvement
STAGE 2: Context Analysis	Online questionnaire (20 minutes) (21/07/2021).
STAGE 4: Implementation and Evaluation	- Professional development webinar(20 minutes) - Teachers will be guided on developing technological teaching products which will be analyzed using the proposed framework. Thus participants will be asked to post their teaching products on shared Google drive folder within 5 days. [During the webinar -introduction of the prototype framework, focus group Interview (after theintroduction of the prototype) 10 minutes]

Cont...

Design stage and cycle	Involvement
STAGE 4: Implementation and Evaluation	- Online questionnaire (10 mins) - The PD framework will be revised and presented to the teachers again in a webinar that will take about 15 minutes. - Summative focus group interview and questionnaire on the usefulness and how participation in the design process influenced their professional development in teaching Mathematics meaningfully with technology (20 minutes) .

DURATION AND ENGAGEMENT

- Online interaction platform (WhatsApp). Online engagements will be after working hours **16:00-17:00 (1 hour) only** at Stage 4 (Implementation and Evaluation) after completion of Cycle 4 (PD meeting (webinars) **once a week.**

Webinars will take place via zoom cloud meetings during **weekends from 12:00 -13:00.**

Data collection is planned to take about three (3) weeks.

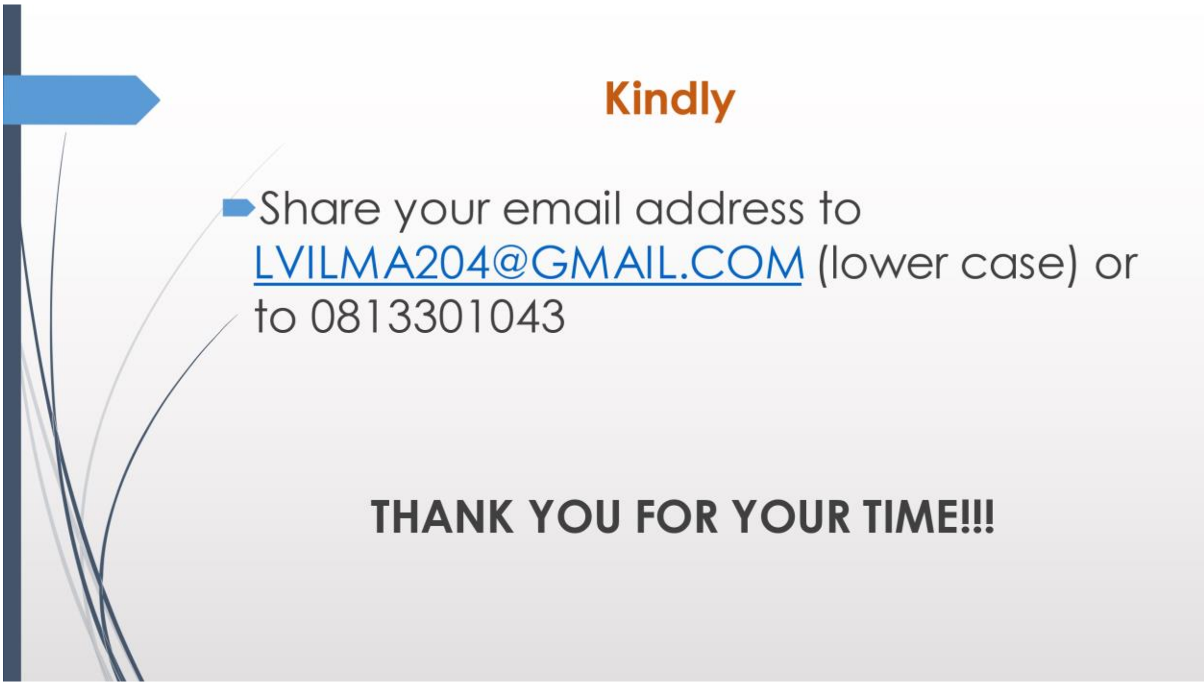
PAYMENT FOR PARTICIPATION

- Your participation will be free of payment and voluntary, that is, there will be no remuneration.
- However, the researcher might provide reimbursement for the data costs on you should funds permit.

PROTECTION OF PARTICIPANTS INFORMATION, IDENTITY AND CONFIDENTIALITY

- Assigning code names/numbers for participants which will be used on all researcher notes and documents.
- Participants will be requested to change their usernames to the codes in order to protect their names and any identification during the webinar.


Discussion
Share with us your teaching practices of teaching mathematics with technology (either during COVID-19 and or earlier)


Kindly
Share your email address to LVILMA204@GMAIL.COM (lower case) or to 0813301043
THANK YOU FOR YOUR TIME!!!

Addendum C: Cycle 2 online questionnaire and Coding [stage 2 phase 2]

QUESTIONS

Thank you for taking the time to complete this questionnaire. It will take about 10-15 minutes of your time. I am Leena Ngonyofi Kanandjebo, a Ph.D. student in the Department of Curriculum Studies at Stellenbosch University, South Africa. I am being guided and supervised by Dr. C E Lampen. As a requirement for the fulfilment of the Doctor of Philosophy, I am conducting a design-based study titled '**A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study'**. The study aims to design a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology. This questionnaire investigates Mathematics teachers' current technological teaching practices, needs, and beliefs on meaningful teaching of Mathematics to inform the design of the PD framework for teaching Mathematics meaningfully with technology. Please answer each question to the best of your knowledge. Your thoughtfulness and candid responses will be greatly appreciated.

Section A: Mathematics teachers' beliefs on meaningful teaching of Mathematics

1. What does **meaningful teaching of Mathematics** mean to you?

____

____

2. What does meaningful teaching of Mathematics with technology entails to you?

____

3. How can technology help you or hinder you to teach Mathematics meaningfully?

____

____

4. How do you want learners to understand Mathematics with technology?

____

____

5. How do you help learners develop an understanding of Mathematical concepts with technology?

____

____

6. What are the essential components that **you think must** be included and considered **when teaching** Mathematics with technology to promote meaningful teaching with technology?

____

____

7. Does the way the Mathematics curriculum is structured allow you to promote mathematical understanding with technology?

(a) If yes, how?
____
____
____

(b) If not, how can a teaching framework be designed to provide support to Mathematics teachers in order to promote mathematical understanding with technology?
____
____
____

8. What are the essential components that should be included in the **lesson planning** for teaching Mathematics with technology meaningfully?
____
____

9. How do you know that your teaching session with learners using technology is meaningful?
____
____

10. What dictates the technology to be used in teaching Mathematics?
____

11. What aspects of Mathematical understanding (e.g developing an understanding of concepts, methods/procedures, productive disposition etc) you can achieve through teaching using technology?
____
____

12. What aspects of Mathematical understanding (e.g developing an understanding of concepts, methods/procedures, productive disposition etc) cannot be achieved through teaching using technology?
____
__
____
____

Section B: Meaningful professional development needs

1. Please tell us about professional development opportunities/interventions on teaching with technology you have already received.

   ____
   ____
   ____

2. Which of the professional development opportunity you received on teaching with technology helped you most to teach Mathematics?

   ____
   ____
   ____
   ____

3. What kinds of knowledge and skills do you need in order to teach Mathematics to enhance mathematical understanding with technology?

   ____
   ____
   ____

   a. Name any professional development opportunity on teaching Mathematics with technology available at your school/region and also indicate the topics in Mathematics that they enhance or help you to teach?

   ____
   ____
   ____

   b. Is there any technological pedagogical support available in secondary school Mathematics at your school or in your region?

      i) If available, please describes the kind of support provided?

   ____
   ____

      ii) If not provide reasons why?

____
____

4. What components do you suggest a professional development framework aimed to assist Mathematics teachers to teach Mathematics meaningfully with technology should consist of?

   ____
   ____
   ____

5. How have the following supported or hindered your use of technology in teaching?
   (a) Your beliefs about teaching Mathematics using technology

   ____

   (b) your knowledge about meaningful teaching of Mathematics with technology

   ____

   (c) The way secondary level Mathematics learning content is designed/planned/structured?

   ____

   (d) Your knowledge about meaningful teaching of Mathematics with technology?

   ____

Thank you for your time

END

The questionnaire was available online at:

https://sunsurveys.sun.ac.za/surveys/a-professional-development-framework-for-teaching-Mathematics-meaningfully-with-technology-in-namibian-secondary-schools-a-design-based-study

Extract of coding Cycle two phase 1

Participant	**Q: What does meaningful teaching of Mathematics mean to you?**	**Description/Interpretation**	**Meaningful aspect**	**PD aspect (needs)**	**Activity theory aspect**
MtA	Not just teaching recipes and tricks but also creating a deeper understanding of Mathematics in order to create problem solvers and ultimately a love for the subject	Understand Love for maths	Conceptual understanding Productive disposition		Goal: Problem solvers
MtB	Teaching the basic concept and its applicability to solve human problems	Concept and apply			Rules: basic concepts and applicable to solve human problems
MtC	Meaningful teaching of Mathematics means teaching Mathematics in such a way that the teacher incorporates all learning support techniques and considers individual differences of the learners, learners' learning abilities as well as their academic backgrounds, choosing the teaching approach that best suits the learners' cognitive development and that makes teaching more fascinating and welcoming to every learner.	Learners: Connect to life learners needs -suit a child's cognitive development	Productive disposition		Goal: accommodation of every learner
MtD	It means learners understand and they can connect to their real life.	Connect to life			Goal connects to life

MtE	Meaningful teaching of Mathematics is a teaching that makes use of a range of teaching and learning approaches and resources to meet the different learning needs of learners. The teachers must know how their learners learn, which will enable them to effectively make their learners understand the concepts presented as well as to become fluent with the skill taught.	Different approaches			Goal: Learners learning needs
MtF	It means teaching Mathematics where learners gain knowledge and understanding that they can apply in their daily lives	apply	Relevance-conceptual understanding		Goals apply in their daily lives
MtG	It means teaching Mathematics with understanding, which is characterized by one(teacher) being competent as they present their lessons, making sure that learning objectives are met and the learners understand the content and assessing the learners' understanding.	Teacher need to be competent	conceptual understanding		goal meet lesson objectives, assessment
MtH	Teaching Mathematics that learners can use in their daily lives, using methods that are appropriate for all learners of different learning needs	apply			Goal use in their daily lives

MtI	It means teaching Mathematics in a manner in which all learners gain knowledge to use in their societies	apply			Goal: use in society

`

Addendum D: Cycle 3 Focus group discussion 1 and coding [stage 2 phase 2]

GUIDING TASKS

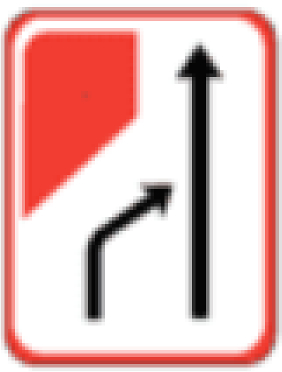
<u>Guiding Principles:</u>
Principle 1. Expanding the worldview of Mathematics teachers (personal instrumental genesis)
Principle 2. Stimulate teachers' imagination to teach Mathematics meaningfully (professional instrumental genesis)

A. **Looking at the world and its object with mathematical eyes. Imagining learning about Mathematics in surroundings and objects**

1. Activity 1
   (a) Given the three pictures, can you tell us three mathematical ideas or Mathematics concepts that can be developed or taught using them:

a.

b



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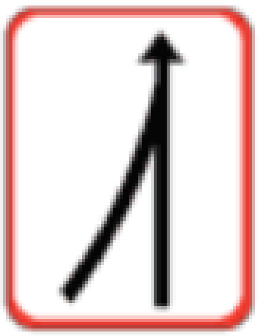
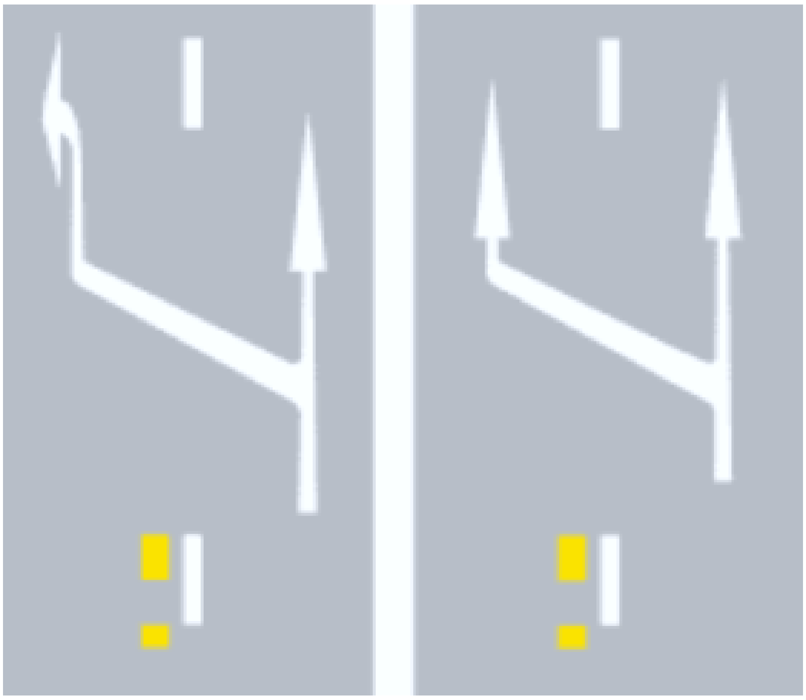


d

e

f



h

(b) Why are angles formed?

(c) Does the turn to form an angle always has to start on a horizontal or vertical line? explain

2. Use your phones to take pictures of angles in your surroundings:

(a) Identify 10 types/names of angles and share your answers on the platform.

(b) State similarities between the picture you took and others.

B. Make objects with mathematical properties (problem solving, strategic competence) Create opportunities for productive struggle, and view Mathematics as useful and beautiful)

3. Materials needed: tape/ strings, board, or black pages

On a board create intersecting lines. [Discover concepts such as opposite angles, corresponding angles, angles on a straight line, perpendicular lines and angles formed around a single point].

4. (a) Sketch angles shown in pictures by folding the papers.

(b) Estimate their values, categorise them and share conditions you used to categorise the angles

(c) Measure the angles formed by folding the paper and confirm your assumption? How can we improve it?

Reflection: What are the shortcomings of discovering angles this way? Does it provide sufficient opportunity for skills development? How can these be improved?

5. Build any real-life model using angle knowledge

DISCUSSION QUESTIONS AND CODING

	PD needs, with a purpose to teach for 5 strands of proficiency, hence meaningfully. (In the background is the session about meaningful teaching with technology) Think about teachers in Namibia including us.	Interpretation per knowledge (technology, teaching, Mathematics) And Meaningful aspects (Conceptual understanding, Productive disposition, Strategic competence, Adaptive reasoning and Procedural fluency)	Interpretation per Activity theory (Goal, Division of labour, Rules, Community and Tensions
Question: <u>What type of professional development do you think we need so that we instil these types of skills? I mean this type of teaching</u>			
MT3. (Guest) Teaching Mathematics using technology is it's very much needed in our schools. It ***makes Mathematics fun.*** *Playing around with this computer*. In the end, it Indeed *there are many topics that need computers than just normal classroom teaching.* Especially in topics on geometry, we need computers. Especially topics on graphs…trigonometric graphs, exponential and logarithmic graphs. We	Tech → maths fun (playing around) Tech replaces classroom teaching for many topics, including geometry, graphs trigonometric graphs, exponential and logarithmic graphs	Views on tech: **Promotes productive disposition** **Struggle with maths pedagogy** **Tech Promotes conceptual understanding (formula -graph)**	*Goal:* ***make maths fun*** *Division of labour: teachers and/or learners playing around* *Tech provides improved mediating artefacts* ***Goal: visual representation***

need trigonometry… I mean we need computers. **Just it, sometimes it might be not easy (even need) to make Uh, learners, understand the translation in the graphs, especially.** Let's say you know whatever graph it is. But with the computer, when it comes to programming those things you can come up with a formula. If *you want to program* this thing you can do this on the computer so that when you put this number here ***you will see the graph behaving when you see this you see the graph behaving***. I think that's the professional development that we need for teachers **to develop teaching skills on how to use specific software for a specific topic.**	Specific difficulty: Not easy to understand translation in static graphs PD need: learn to use specific software for specific topics: manipulate formula on the computer to let graph behave	**Tech provides solutions to maths pedagogy struggles)** **PD Needs: Tech, pedagogy and math knowledge**	*Division of labour: Teacher work to manipulate formulas and graphs*
MT2: That can aid in the teaching of Mathematics, I think. Also, the teachers must be made or strengthened such that they should be able to identify examples in nature and or connect the examples in nature or real life. Say for example like in nature you can take an example of say for example in a honeycomb now the man-made real-life example is like the interlocks for example and those real-life examples or man-made examples now.	PD need: Strengthened to be able to identify examples in nature and connecting Mathematics to real life	Mathematics knowledge	

MT4 ***Technology saves time.*** We are talking of software like Camtasia or GeoGebra. we tend not to use technology just because we do not know how and I believe that if we are, maybe if teachers are may be trained on how to use this latest software I believe the learners will develop a love for the subject and then they can be able to play with those software, even at home, *if they have access to a computer to a computer or maybe to a laptop* **and solve these problems *even at home,*** by doing straight home**,** they develop these skills and also **we might finish the content faster** within a short period. Yes, when the teacher is **setting out an activity the teacher must know what type of skill set.**	Tech implies time (saving) **content we might finish the content faster** Tech implies the development of the love of the subject and play with the software. Solve problems at home PD Need: **knowledge of meaningful in setting assessment tasks (that teacher setting out an activity the teacher must know what type of skill set.)**	**Promotes Productive disposition to solve problems at home** **PD Need: maths knowledge**	***Goal: technology for speed*** ***Tensions: access to a computer or laptop*** ***Division of labour: Technology shares in the division of labour by proving an opportunity for learners to solve problems in the absence of a teacher.*** ***Environment/Community: technology learning environment at home.***
Question: How can the school curriculum be modified or structured to ensure that all these strands are developed in the learners because the curriculum also has an influence, you cannot start now and go to the classroom and start saying we are developing an understanding of a concept when the curriculum does not require you to do that because we are guided *The question aimed to find teachers' views on curriculum as in chapter 6 on the analysis of the curriculum the researcher found that strands were not explicitly stated mainly direct references were referred to conceptual understanding and procedural fluency.*			

MT3. If we are to have access to all these things developers need to ensure that somehow include some of those competence in the syllabus.	Not in, provision needed		**Rules:** syllabus to include the competences
MT2 I would say ***conceptual, problem solving and procedural fluency are in, even justification sometimes yes in projects, but I don't think it's well addressed.*** Productive disposition can be addressed in those types of assessments if you are teaching in a certain way, But then Uh, those kinds of assessments are mostly **left to the teacher themselves.** If they are not well-trained. If they don't have enough examples and the curriculum, ***I mean the Mathematics syllabus does not really state clearly for them, it does not really come out.*** Only if the teacher had got skills to come up with those quality projects and investigation then the learners do not get full benefits.	Conceptual understanding, procedural fluency and partial justification objectives. Justification is not well addressed. Productive disposition can be addressed through assessment. Assessment left to the teachers. **Needs:** Assessment techniques	Curriculum accommodates: Conceptual understanding, procedural fluency and partial justification objectives. **PD Needs:** pedagogy Mathematics knowledge	**Rules:** Syllabus need to be explicit (***I mean Mathematics syllabus does not really state clearly for them, it does not really come out.)*** Division of labour: teachers set assessment tasks Community: to create an environment through assessment tasks where learners are able to set assessment tasks for teaching and learning meaningfully
Follow-up question: What do you think? How can professional development help?			
MT4 Yes, **how to set up questions or questioning techniques when we want to assess the learners using the**	The curriculum is fine as it has content combined and an	**PD Needs:** pedagogy Mathematics knowledge	Rules: Fine and provided room for continuous learning of teaching

aforementioned strands. The curriculum has been revised, extended is now ordinary level and we have Advanced Subsidiary. And you can also find that some of the things that were in the grade 10 content are also combined, but we are loving it so far. We are learning each and every day.	introduction of AS level. The curriculum has created a need for learning daily. **Needs:** Assessment techniques		
MT2 Plus even though we revised the curriculum, **but we still did not revise our subject policy** *and our subject policy maybe could have more guidance,* it is supposed to come out or be written out or spelt out in either the syllabus itself or in the subject guide.	Subject policy not revised, [current] subject policy to have explicit guidance		Rules: Revision of subject policy. Subject policy needs to be explicit
Question: What do you think about the curriculum in technology integration?			
MT4 I'm saying sometimes the reason why teachers **do not integrate Technology in there in their classroom, it's because they do not know how to use the latest** Only if now **professional development can come in, to coach**. **Educational software they are there**. Teachers can also do their part go to YouTube and learn **how to use specific software**, **not necessarily waiting all**	No knowledge implies no integration PD's role is to coach Availability of free technology software. PD Need: how to use a specific software	PD needs: technology knowledge Enhance personal genesis for professional genesis	Division of labour: professional developers should coach Teachers to self-teach

time to be fed they are available some of them are free			
Question: <u>How should professional development be and or be on?</u>			
MT3. It should ***equip teachers with ICT skills and how those skills are related to Mathematics subjects. Also, how need to be trained on how to identify mathematical proficiencies from the syllabus*** to integrate technology into our teaching of Mathematics. Again it's only if we can do programming on our own, if you know how to do **programming then you'll be able to tackle most of the Mathematics problems using ICT**. Also make **it a must, part of the curriculum** that we must teach using Excel for example and it will be assessed also	Technology skills (programming) for helping to tackle problems in Mathematics PD needs: ICT skills and implications of proficiencies to Mathematics teaching ''equip teachers with ICT skills and how those skills are related to Mathematics subject''	Tech to promote problem solving (procedural fluency) PD needs: Technology Mathematics knowledge	Division of labour: technology helps and shares a role in solving problems Rules: expands for curriculum must state Compulsory use of technology.
MT2 But now the problem is the **accessibility of those technology devices**. I think that teachers **should be exposed to different technology,**	Lack of access to technology	PD needs: Technology knowledge (create awareness)	Tensions: problem with Access to technology devices

Create awareness of equipment or devices and also programs or software that can be used in Mathematics.			

Note: *There were four participants MT1, MT2, MT3 and MT4, however, MT1 had challenges with the internet connection which was constantly tripping.*

Summary of codes

Themes	***Affordances in terms of (Meaningful Aspect)***	***Knowledge aspect***	***Activity theory nodes aspect***
	Promote problem solving (procedural fluency) Promotes conceptual understanding (formula -graph Promotes productive disposition (maths fun) Promotes productive disposition (to solve problems at home)	Enhance personal genesis for professional genesis Mathematics knowledge Maths knowledge Pedagogy Mathematics knowledge Pedagogy Mathematics knowledge Tech, pedagogy and math knowledge Technology knowledge Technology knowledge (create awareness) Technology Mathematics knowledge	*Community: technology learning environment at home.* Community: to create an environment through assessment tasks where learners can set assessment tasks for teaching and learning meaningfully *Division of labour: The teacher work to manipulate formula and graphs* *Division of labour: teachers and/or learners playing around. Tech provides improved mediating artefacts* Division of labour: professional developers should coach so that Teachers to self -teach Division of labour: teachers set assessment tasks Division of labour: technology helps and shares a role in solving problems *Division of labour: Technology shares in the division of labour by proving an opportunity for learners to solve problems in the absence of a teacher.* *Goal: make maths fun* *Goal: technology for speed* *Goal: visual representation* Rules: Fine and provided room for continuous learning of teaching Rules: expands for curriculum must state Compulsory use of technology.

			Rules: Revision of subject policy. Subject policy needs to be explicit Rules: Syllabus need to be explicit (*I mean Mathematics syllabus does not really state clearly for them, it does not really come out.)* Rules: syllabus to include the competences *Tensions: access to a computer or laptop* Tensions: problem with Access to technology devices

Addendum E: Cycle 4 Focus group discussion 2 [stage 2 phase 2]

Schedule

Time	**Activity**
11:30 – 11:40	Introduction: purpose of the Workshop
11:40 – 13:30	Experiential learning 1: Immersion and exploring to teach Mathematics meaningfully with Geogebra Figure 1-3
13:30 -14:00	Lunch Break
14:00 – 15:00	Experiential learning 2: opportunity to showcase teaching experiences that demand teaching meaningfully with technology Figure 4
15:00 - 15:15	Health break
15:15 – 16:15	Reflection on experimental tasks and meaningful teaching with technology
16:15 -17:00	Discussion and reflection

Purpose of the workshop:

- Critically explore and think together how one can learn about a mathematical concept (not limited to angle geometry as a benchmark) using technology (using Geogebra as a benchmark).

The exploration will be around angles, with reference and not limited to the following specific objectives:

- Define, use and interpret geometrical terms angle, parallel, intersecting, right angle, acute, obtuse and reflex angles, perpendicular, similarity, congruence
- Use and interpret vocabulary of triangles, circles and polygons
- Measure lines and angles

Question/objectives:

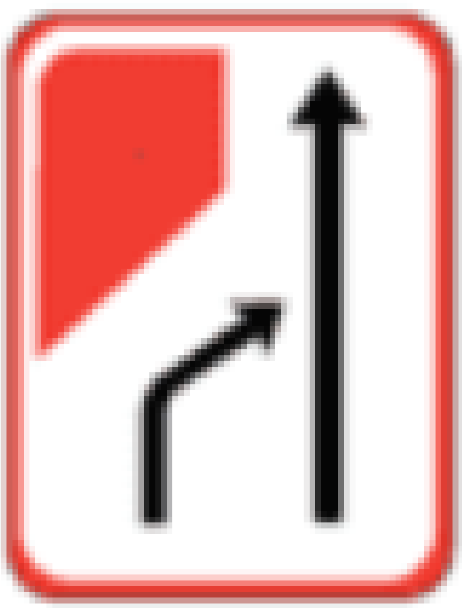
- **Using GeoGebra explore mathematical concepts embodied in the figures for meaningful teaching experience with technology**.
- Critic the questions and propose better ways of phrasing them to promote meaningful teaching with technology
- Suggestions on figures and context that can be used for teaching Mathematics with technology

- Explain and critic affordances and constraints of using technology for teaching Mathematics meaningfully

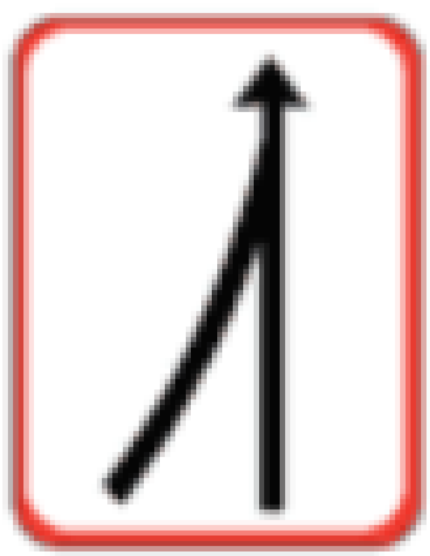
Figures:

1.

2. i)



i)



(photo credit: researcher)

3.

(photo credit: researcher)

Individually: Explore mathematical concepts embodied in the logo and present a meaningful teaching experience with technology (GeoGebra)

4.

(Photo credit: Road administration fund)

Addendum F: Cycle 5 online questionnaire on the influence of Mathematics teacher participants' participation in the study on their views and beliefs about teaching with tec [stage 2 phase 2]

1. Reflect on learning experiences during the research engagement online webinar and share your views.

2. Reflect and critique the tasks (figures and context) used in the research engagement and propose better ways (if any) of phrasing them to promote meaningful teaching with technology?
3. Reflecting on teaching Mathematics meaningfully using technology research engagement webinar, what are the chances that you will use similar views/ways of teaching with technology in your Mathematics classroom? Explain your answer.

4. Reflecting on teaching Mathematics meaningfully using technology research engagement webinar, would you share such views/ways of teaching with technology with other Mathematics teachers? Explain your answer.
5. How does your participation in the design process of a framework for teaching Mathematics meaningfully with technology influence your professional development?

6. Do you think there are limits to Mathematics concepts that can be taught meaningfully with technology? If yes, what are those concepts that cannot be taught meaningfully with technology?

7. Were your expectations and views about teaching meaningfully with technology met? Explain in detail.

END

Addendum G: Design guiding questions [stage 3]

These questions guide the researcher in developing the PD framework for teaching Mathematics meaningfully with technology.

1. What are the key aspects of a framework for the professional development of secondary school Mathematics teachers, for teaching Mathematics meaningfully with technology?
2. What design principles may be applicable to teaching Mathematics in Namibian secondary schools?

END

Addendum H: Ethical Clearance Approval Certificate

UNIVERSITEIT
STELLENBOSCH
UNIVERSITY

NOTICE OF APPROVAL

REC: Social, Behavioural and Education Research (SBER) - Initial Application Form

24 March 2021

Project number: 19500

Project Title: A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study

Dear Ms Leena Kanandjebo

Your REC: Social, Behavioural and Education Research (SBER) - Initial Application Form submitted on 27/01/2021 14:37 was reviewed and approved by the REC: Social, Behavioural and Education Research (REC: SBE).

Please note below expiration date of this approved submission:

Ethics approval period:

Protocol approval date (Humanities)	Protocol expiration date (Humanities)
24 March 2021	23 March 2024

GENERAL REC COMMENTS PERTAINING TO THIS PROJECT:

INVESTIGATOR RESPONSIBILITIES

Please take note of the General Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

If the researcher deviates in any way from the proposal approved by the REC: SBE, the researcher must notify the REC of these changes.

Please use your SU project number (19500) on any documents or correspondence with the REC concerning your project.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

CONTINUATION OF PROJECTS AFTER REC APPROVAL PERIOD

You are required to submit a progress report to the REC: SBE before the approval period has expired if a continuation of ethics approval is required. The Committee will then consider the continuation of the project for a further year (if necessary).

Once you have completed your research, you are required to submit a final report to the REC: SBE for review.

Included Documents:

Document Type	File Name	Date	Version
Research Protocol/Proposal	Kanandjebo, L_19857875_PhD proposalf	28/10/2020	Final
Data collection tool	Cycle 3 Designing	01/11/2020	first
Investigator CV (PI)	CV Kanandjebo LN, 19857578 Ph.D Curriculum Studies (Mathematics Education) - Faculty of Educati	11/11/2020	academic
Data collection tool	Focus group Interview Questions	01/12/2020	first version
Data collection tool	Stage 1 Document analysis schedule	01/12/2020	first
Data collection tool	Questionnaires	01/12/2020	first
Data collection tool	Stage 4 Cycle 4 PD webinar schedule	01/12/2020	first
Request for permission	Request for Permission from school principals	22/01/2021	final

Page 1 of 3

Informed Consent Form	Consent form for teachers	22/01/2021	final
Information sheet	INFORMATION SHEET FOR TEACHERS	22/01/2021	final
Request for permission	Request for permission from REGIONAL DIRECTOR	23/01/2021	final
Budget	Kanandjebo Yearly budget	24/01/2021	2021
Request for permission	REQUEST FOR PERMISSION FROM EXECUTIVE DIRECTOR	24/01/2021	final

If you have any questions or need further help, please contact the REC office at cgraham@sun.ac.za.

Sincerely,

Clarissa Graham

REC Coordinator: Research Ethics Committee: Social, Behavioral and Education Research

National Health Research Ethics Committee (NHREC) registration number: REC-050411-032.
The Research Ethics Committee: Social, Behavioural and Education Research complies with the SA National Health Act No.61 2003 as it pertains to health research. In addition, this committee abides by the ethical norms and principles for research established by the Declaration of Helsinki (2013) and the Department of Health Guidelines for Ethical Research: Principles Structures and Processes (2nd Ed.) 2015. Annually a number of projects may be selected randomly for an external audit.

Principal Investigator Responsibilities

Protection of Human Research Participants

As soon as Research Ethics Committee approval is confirmed by the REC, the principal investigator (PI) is responsible for the following:

Conducting the Research: The PI is responsible for making sure that the research is conducted according to the REC-approved research protocol. The PI is jointly responsible for the conduct of co-investigators and any research staff involved with this research. The PI must ensure that the research is conducted according to the recognised standards of their research field/discipline and according to the principles and standards of ethical research and responsible research conduct.

Participant Enrolment: The PI may not recruit or enrol participants unless the protocol for recruitment is approved by the REC. Recruitment and data collection activities must cease after the expiration date of REC approval. All recruitment materials must be approved by the REC prior to their use.

Informed Consent: The PI is responsible for obtaining and documenting affirmative informed consent using **only** the REC-approved consent documents/process, and for ensuring that no participants are involved in research prior to obtaining their affirmative informed consent. The PI must give all participants copies of the signed informed consent documents, where required. The PI must keep the originals in a secured, REC-approved location for at least five (5) years after the research is complete.

Continuing Review: The REC must review and approve all REC-approved research proposals at intervals appropriate to the degree of risk but not less than once per year. There is **no grace period.** Prior to the date on which the REC approval of the research expires, **it is the PI's responsibility to submit the progress report in a timely fashion to ensure a lapse in REC approval does not occur.** Once REC approval of your research lapses, all research activities must cease, and contact must be made with the REC immediately.

Amendments and Changes: Any planned changes to any aspect of the research (such as research design, procedures, participant population, informed consent document, instruments, surveys or recruiting material, etc.), must be submitted to the REC for review and approval before implementation. Amendments may not be initiated without first obtaining written REC approval. The **only exception** is when it is necessary to eliminate apparent immediate hazards to participants and the REC should be immediately informed of this necessity.

Adverse or Unanticipated Events: Any serious adverse events, participant complaints, and all unanticipated problems that involve risks to participants or others, as well as any research-related injuries, occurring at this institution or at other performance sites must be reported to the REC within **five (5) days** of discovery of the incident. The PI must also report any instances of serious or continuing problems, or non-compliance with the RECs requirements for protecting human research participants.

Research Record Keeping: The PI must keep the following research-related records, at a minimum, in a secure location for a minimum of five years: the REC approved research proposal and all amendments; all informed consent documents; recruiting materials; continuing review reports; adverse or unanticipated events; and all correspondence and approvals from the REC.

Provision of Counselling or emergency support: When a dedicated counsellor or a psychologist provides support to a participant without prior REC review and approval, to the extent permitted by law, such activities will not be recognised as research nor the data used in support of research. Such cases should be indicated in the progress report or final report.

Final reports: When the research is completed (no further participant enrolment, interactions or interventions), the PI must submit a Final Report to the REC to close the study.

On-Site Evaluations, Inspections, or Audits: If the researcher is notified that the research will be reviewed or audited by the sponsor or any other external agency or any internal group, the PI must inform the REC immediately of the impending audit/evaluation.

Addendum I: Request for permission from the Executive Director of the Ministry of Education, Arts and Culture information sheet

1. BACKGROUND AND STUDY OVERVIEW

I am Leena Ngonyofi Kanandjebo, a Ph.D student in the Department of Curriculum Studies at Stellenbosch University, South Africa. I am being guided and supervised by Dr. C E Lampen. As a requirement for the fulfilment of the Doctor of Philosophy, I am conducting a design-based study titled '**A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study'**. The study aims to design a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology. Participation in the study involves filling in online questionnaires, online webinars and focus group discussions. All technologically adept Mathematics teachers are invited to take part in this study.

You may ask the investigator if there is anything that is not clear and if you need more information.

2. PURPOSE OF THE STUDY

This study is proposed with the aim of designing a professional development framework for teaching Mathematics meaningfully using technology. Participants in the study will have an opportunity to upgrade their knowledge of meaningful technology integration and pedagogy in relation to Mathematics.

3. WHAT WILL BE ASKED OF YOU?

I would like to ask your permission to allow technologically adept Mathematics teachers in Namibian secondary schools to participate in this research after official working hours.

The following will be done during the study:

4. STUDY PROCEDURE

Secondary school Mathematics teachers will be required to partake in the following design stages:

Design stage	Involvement
STAGE 2: Context Analysis	- Complete the initial online questionnaire. This will take approximately 20 minutes. - Attend three research engagement webinars - Teachers will engage in meaningful teaching events with the researcher. Each research engagement is coupled with focus group discussions. Discussions will last about 30 minutes - For the last focus group discussion, participants will also be asked to post their teaching products on a shared Google drive folder. - After the webinar, secondary school Mathematics teachers will fill in an online questionnaire to express their views on how their participation in the design process influenced their professional development in teaching Mathematics meaningfully with technology. This will take place online after the webinar and will last for about 20 minutes.

5. DURATION

All online links to participating in the study will be shared with teachers on an online interactive platform (WhatsApp). Online engagements will be after working hours 16:00-17:00 (1 hour) only at Stage 2 (context analysis). Webinars will take place online via ZOOM cloud meetings during weekends from 12:00-13:00. Data collection is planned to take about three (3) weeks.

6. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts anticipated from participation in this research study. Teachers may decline to participate in any activity or terminate their involvement in the research at any stage and time if they wish. They may request the data they have contributed be withdrawn from the study.

7. POSSIBLE BENEFITS

The study will be an opportunity for Mathematics teachers to reflect on their current technological teaching practices. Furthermore, we hope that the study process and the findings of this study will:

- Help secondary school Mathematics teacher improve their technological, pedagogical and content knowledge. Thereby improving their teaching and enhancing the learning of Mathematics.
- Contribute to the body of knowledge in Mathematics Education by adding to the existing empirical evidence about technological instructional approaches for Mathematics specifically. Importantly, promote technological teaching that is meaningful, and in alignment with the development of skills needed in the technological era.
- Provide information for the research community and Mathematics teachers on how to successfully use technology to bring about meaningful teaching in times where technology is the only means of instructional delivery and or in blended teaching and learning situations.
- Assist in setting education reform policies, curriculum implementation and develop technological strategies that can be used to improve the teaching and learning of Mathematics.
- Support educational planners, policymakers and professional development bodies of teachers with technology integration in the Namibian educational system.
- The government of Namibia through the Ministry of Education, Arts and Culture (MoEAC) plans to achieve at least 65% in Mathematics performance by 2022

(NDP5, 2017). The designed framework is envisioned to assist in accelerating the attainment of National development plans such as National Development Plan 5 (NDP5) and MoEAC strategic plan leading to the attainment of Vision 2030.

8. PAYMENT FOR PARTICIPATION

Participation of Mathematics teachers will be free of payment and voluntary that is there will be no remuneration. However, the researcher might provide reimbursement for the data costs on participants should funds permit.

9. PROTECTION OF PARTICIPANTS' INFORMATION, IDENTITY AND CONFIDENTIALITY

Comments, reflections, and responses shared by participants during this study will be anonymous. In addition, every effort will be made by the researcher to always preserve their confidentiality including the following:

- Assigning code names/numbers for participants which will be used on all researcher notes and documents.
- Participants will be requested to change their usernames to the codes to protect their names and any identification during the webinar.
- Any paper data such as teaching products, interview transcriptions, transcribed notes and any other research materials will be converted into a soft copy before it is kept in a locked file cabinet.
- All electronic versions of the data, including responses from the questionnaire, teaching products, and webinar recordings, will be encrypted (password protected) and kept in a password protected external hard drive as well as saved on the researcher's google drive and Dropbox.
- After five years, all the research materials will be destroyed.
- All researchers may review the investigator's collected data. Information from this research will be used solely for the purpose of this study and any publications that may result from this study.
- An online group interaction platform (WhatsApp group) will be created solely for engagement, communications, and further data collection. The researcher will erase potential participants' contact numbers as soon as the

link requesting them to join the WhatsApp group is sent to them. Note: Conversations shared on WhatsApp are secured with end-to-end encryption, thus participants are in control of their data (WhatsApp LLc, 2021).

- A webinar will be recorded. Any accidental footage of the participants who have not consented or withdrawn from the study will be cut out from the recording.
- The video camera will be set off for participants during webinar sessions however if participants wish to put it on during sessions they can do so
- Although the investigator will maintain confidentiality it is not possible to guarantee that other group members will do so in focus group discussions and on WhatsApp group discussions
- Each participant has the opportunity to obtain the recording of the webinar, their contributions during focus group discussions, online questionnaire responses and any other personal informant-based data.

10. PARTICIPATION AND WITHDRAWAL

Participants are free to withdraw at any time and stage without giving a reason; even after they have consented to participate. They are free to decline to participate by either closing the web page of the questionnaire or any online research platform. They may withdraw their consent at any time and discontinue participation without penalty. In case they do agree to participate; they have the right to decline to answer any questions. In addition, they have the right to ask for their data to be withdrawn from the research if they wish to do so. However, for online questionnaires no identifying information will be collected, hence Mathematics teachers' responses will be completely anonymous. The implication of this is that if they would like to withdraw their responses after completing the study, they would need to provide the researcher with information that would help her identify their responses, i.e., specific answers to some of the questions.

11. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Leena N Kanandjebo at +264 81 3301043 or email lvilma204@gmail.com and/or the supervisor Dr. C E Lampen at ernalampen@sun.ac.za

12. RIGHTS OF RESEARCH PARTICIPANTS

This study has been approved by Stellenbosch University Research Ethics Committee. Further, you are not waiving any legal claims, rights or remedies because of your approval to participate in this research study. Any questions regarding your rights, concerns or complaints, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

If you agree to allow teachers teaching Mathematics in Namibian secondary schools to participate in this research, please also sign below. Email the signed copy to Leena N Kanandjebo at lvilma204@gmail.com . You may keep a copy for your records.

Attached is the ethical clearance from Stellenbosch University Research Ethics Committee.

Thank you very much for your help.

______________________________________ ____________________

Signature of Executive Education Director Date

Addendum J: Request for permission from the Regional Director of Education to conduct research information sheet

1. BACKGROUND AND STUDY OVERVIEW

I am Leena Ngonyofi Kanandjebo, a Ph.D student in the Department of Curriculum Studies at Stellenbosch University, South Africa. I am being guided and supervised by Dr. C E Lampen. As a requirement for the fulfilment of the Doctor of Philosophy, I am conducting a design-based study titled '**A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study'**. The study aims to design a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology. Participation in the study involves filling in online questionnaires, online webinars and focus group discussions. All technologically adept Mathematics teachers [Secondary level Mathematics teachers who used technology platforms for example WhatsApp, google classrooms, ZOOM, MS teams and YouTube channels to teach Mathematics] are invited to take part in this study.
You may ask the investigator if there is anything that is not clear and if you need more information.

2. PURPOSE OF THE STUDY

This study is proposed with the aim of designing a professional development framework for teaching Mathematics meaningfully using technology. Participants in the study will have an opportunity to upgrade their knowledge of meaningful technology integration and pedagogy in relation to Mathematics.

3. WHAT WILL BE ASKED OF YOU?

I would like to ask your permission to allow technologically adept Mathematics teachers at schools in your region to participate in this research after official working hours.

The following will be done during the study:

4. STUDY PROCEDURE

Secondary school Mathematics teachers will be required to partake in the following design stages:

Design stage	Involvement
STAGE 2: Context Analysis	- Complete the initial online questionnaire. This will take approximately 20 minutes. - Attend three research engagement webinars - Teachers will engage in meaningful teaching events with the researcher. Each research engagement is coupled with focus group discussions. Discussions will last about 30 minutes - For the last group discussion, participants will also be asked to post their teaching products on a shared Google drive folder. - After the webinar secondary school Mathematics teachers will fill in an online questionnaire to express their views on how their participation in the design process influenced their professional development in teaching Mathematics meaningfully with technology. This will take place online after the webinar and will last for about 20 minutes.

13. DURATION

All online links to participating in the study will be shared with teachers on an online interactive platform (WhatsApp). Online engagements will be after working hours 16:00-17:00 (1 hour) only at Stage 2 (context analysis). Webinars will take place online via ZOOM cloud meetings during weekends from 12:00-13:00. Data collection is planned to take about three (3) weeks.

5. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts anticipated from participation in this research study. Teachers may decline to participate in any activity or terminate their involvement in the research at any stage and time if they wish. They may request the data they have contributed be withdrawn from the study.

6. POSSIBLE BENEFITS

The study will be an opportunity for Mathematics teachers to reflect on their current technological teaching practices. Furthermore, we hope that the study process and the findings of this study will:

- Help secondary school Mathematics teacher improve their technological, pedagogical and content knowledge. Thereby improving their teaching and enhancing the learning of Mathematics.
- Contribute to the body of knowledge in Mathematics Education by adding to the existing empirical evidence about technological instructional approaches for Mathematics specifically. Importantly, promote technological teaching that is meaningful, and in alignment with the development of skills needed in the technological era.
- Provide information for the research community and Mathematics teachers on how to successfully use technology to bring about meaningful teaching in times where technology is the only means of instructional delivery and or in blended teaching and learning situations.
- Assist in setting education reform policies, curriculum implementation and develop technological strategies that can be used to improve the teaching and learning of Mathematics.

- Support educational planners, policymakers and professional development bodies of teachers with technology integration in the Namibian educational system.
- The government of Namibia through the Ministry of Education, Arts and Culture (MoEAC) plans to achieve at least 65% in Mathematics performance by 2022 (NDP5, 2017). The designed framework is envisioned to assist in accelerating the attainment of National development plans such as National Development Plan 5 (NDP5) and MoEAC strategic plan leading to the attainment of Vision 2030.

7. PAYMENT FOR PARTICIPATION

Participation of Mathematics teachers will be free of payment and voluntary that is there will be no remuneration. However, the researcher might provide reimbursement for the data costs on participants should funds permit.

8. PROTECTION OF PARTICIPANTS' INFORMATION, IDENTITY AND CONFIDENTIALITY

Comments, reflections and responses shared by participants during this study will be anonymous. In addition, every effort will be made by the researcher to preserve their confidentiality at all times including the following:

- ➢ Assigning code names/numbers for participants which will be used on all researcher notes and documents.
- ➢ Participants will be requested to change their usernames to codes in order to protect their names and any identification during the webinar.
- ➢ Any paper data such as teaching products, interview transcriptions, transcribed notes and any other research materials will be converted into a soft copy before it is kept in a locked file cabinet.
- ➢ All electronic versions of the data, including responses from the questionnaire, teaching products, and webinar recordings, will be encrypted (password protected) and kept in a password protected external hard drive as well as saved on the researcher’s google drive and Dropbox.
- ➢ After five years, all the research materials will be destroyed.

- All researchers may review the investigator's collected data. Information from this research will be used solely for the purpose of this study and any publications that may result from this study.
- An online group interaction platform (WhatsApp group) will be created solely for engagement, communications and further data collection. The researcher will erase participants' contact numbers as soon as the link requesting them to join the WhatsApp group is sent to them. Note: Conversations shared on WhatsApp are secured with end-to-end encryption, thus participants are in control of their data (WhatsApp LLc, 2021).
- A webinar will be recorded. Any accidental footage of the participants who have not consented or withdrawn from the study will be cut out from the recording.
- The video camera will be set off for participants during webinar sessions however if participants wish to put it on during sessions they can do so
- Although the investigator will maintain confidentiality it is not possible to guarantee that other group members will do so in the focus group discussions and on WhatsApp group discussions
- Each participant has the opportunity to obtain the recording of the webinar, their contribution during focus group discussions, online questionnaire responses and any other personal informant-based data.

9. PARTICIPATION AND WITHDRAWAL

Participants are free to withdraw at any time and stage without giving a reason; even after they have consented to participate. They are free to decline to participate by either closing the web page of the questionnaire or any online research platform. They may withdraw their consent at any time and discontinue participation without penalty. In case they do agree to participate; they have the right to decline to answer any questions. In addition, they have the right to ask for their data to be withdrawn from the research if they wish to do so. However, for online questionnaires no identifying information will be collected, hence Mathematics teachers' responses will be completely anonymous. The implication of this is that if they would like to withdraw their responses after completing the study, they would need to provide the researcher with information that would help her identify their responses, i.e., specific answers to some of the questions.

10. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Leena N Kanandjebo at +264 81 3301043 or email lvilma204@gmail.com and/or the supervisor Dr. C E Lampen at ernalampen@sun.ac.za

11. RIGHTS OF RESEARCH PARTICIPANTS

This study has been approved by Stellenbosch University Research Ethics Committee. Further, you are not waiving any legal claims, rights or remedies because you approved this research study. Any questions regarding your rights, concerns or complaints, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

If you agree for this research to be conducted by teachers teaching Mathematics in your region, please also sign below. Email the signed copy to Leena N Kanandjebo at lvilma204@gmail.com . You may keep a copy for your records.

Attached is the ethical clearance from Stellenbosch University Research Ethics Committee and a permission letter from the Ministry of Education, Arts and Culture.

Thank you very much for your help.

____________________________________ ____________________

Signature of Regional Director of Education Date

Addendum K: Request for permission from school principals and/ or supervisors to conduct research information sheet

1. BACKGROUND AND STUDY OVERVIEW

I am Leena Ngonyofi Kanandjebo, a Ph.D student in the Department of Curriculum Studies at Stellenbosch University, South Africa. I am being guided and supervised by Dr. C E Lampen. As a requirement for the fulfilment of the Doctor of Philosophy, I am conducting a design-based study titled '**A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study'**. The study aims to design a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology. Participation in the study involves filling in online questionnaires, online webinars and focus group discussions. All technologically adept Mathematics teachers are invited to take part in this study.

You may ask the investigator if there is anything that is not clear and if you need more information.

2. PURPOSE OF THE STUDY

This study is proposed with the aim of designing a professional development framework for teaching Mathematics meaningfully using technology. Participants in the study will have an opportunity to upgrade their knowledge of meaningful technology integration and pedagogy in relation to Mathematics.

3. WHAT WILL BE ASKED OF YOU?

I would like to ask your permission to allow technologically adept Mathematics teachers at your school to participate in this research after official working hours.

The following will be done during the study:

4. STUDY PROCEDURE

Secondary school Mathematics teachers will be required to partake in the following design stages:

Design stage	Involvement
STAGE 2: Context Analysis	- Complete the initial online questionnaire. This will take approximately 20 minutes. - Attend three research engagement webinars - Teachers will engage in meaningful teaching events with the researcher. Each research engagement is coupled with focus group discussions. Discussions will last about 30 minutes - For the last group discussion, participants will also be asked to post their teaching products on a shared Google drive folder. - After the webinar secondary school Mathematics teachers will fill in an online questionnaire to express their views on how their participation in the design process influenced their professional development in teaching Mathematics meaningfully with technology. This will take place online after the webinar and will last for about 20 minutes.

5. DURATION

All online links to participating in the study will be shared with teachers on an online interactive platform (WhatsApp). Online engagements will be after working hours 16:00-17:00 (1 hour) only at Stage 2 (context analysis). Webinars will take place online via ZOOM cloud meetings during weekends from 12:00-13:00. Data collection is planned to take about three (3) weeks

6. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts anticipated from participation in this research study. Teachers may decline to participate in any activity or terminate their involvement in the research at any stage and time if they wish. They may request the data they have contributed be withdrawn from the study.

7. POSSIBLE BENEFITS

The study will be an opportunity for Mathematics teachers to reflect on their current technological teaching practices. Furthermore, we hope that the study process and the findings of this study will:

- Help secondary school Mathematics teacher improve their technological, pedagogical and content knowledge. Thereby improving their teaching and enhancing the learning of Mathematics.
- Contribute to the body of knowledge in Mathematics Education by adding to the existing empirical evidence about technological instructional approaches for Mathematics specifically. Importantly, promote technological teaching that is meaningful, and in alignment with the development of skills needed in the technological era.
- Provide information for the research community and Mathematics teachers on how to successfully use technology to bring about meaningful teaching in times where technology is the only means of instructional delivery and or in blended teaching and learning situations.
- Assist in setting education reform policies, curriculum implementation and develop technological strategies that can be used to improve the teaching and learning of Mathematics.
- Support educational planners, policymakers and professional development bodies of teachers with technology integration in the Namibian educational system.
- The government of Namibia through the Ministry of Education, Arts and Culture (MoEAC) plans to achieve at least 65% in Mathematics performance by 2022 (NDP5, 2017). The designed framework is envisioned to assist in accelerating the

attainment of National development plans such as National Development Plan 5 (NDP5) and MoEAC strategic plan leading to the attainment of Vision 2030.

8. PAYMENT FOR PARTICIPATION

Participation of Mathematics teachers will be free of payment and voluntary that is there will be no remuneration. However, the researcher might provide reimbursement for the data costs on participants should funds permit.

9. PROTECTION OF PARTICIPANTS' INFORMATION, IDENTITY AND CONFIDENTIALITY

Comments, reflections and responses shared by participants during this study will be anonymous. In addition, every effort will be made by the researcher to preserve their confidentiality at all times including the following:

- Assigning code names/numbers for participants which will be used on all researcher notes and documents.
- Participants will be requested to change their usernames to codes in order to protect their names and any identification during the webinar.
- Any paper data such as teaching products, interview transcriptions, transcribed notes and any other research materials will be converted into a soft copy before it is kept in a locked file cabinet.
- All electronic versions of the data, including responses from the questionnaire, teaching products, and webinar recordings, will be encrypted (password protected) and kept in a password protected external hard drive as well as saved on the researcher's google drive and Dropbox.
- After five years, all the research materials will be destroyed.
- All researchers may review the investigator's collected data. Information from this research will be used solely for the purpose of this study and any publications that may result from this study.
- An online group interaction platform (WhatsApp group) will be created solely for engagement, communications and further data collection. The researcher will erase participants' contact numbers as soon as the link requesting them to join the WhatsApp group is sent to them. Note:

Conversations shared on WhatsApp are secured with end-to-end encryption, thus participants are in control of their data (WhatsApp LLc, 2021).

- A webinar will be recorded. Any accidental footage of the participants who have not consented or withdrawn from the study will be cut out from the recording.
- The video camera will be set off for participants during webinar sessions however if participants wish to put it on during sessions they can do so
- Although the investigator will maintain confidentiality it is not possible to guarantee that other group members will do so in the focus group discussions and on WhatsApp group discussions
- Each participant has the opportunity to obtain the recording of the webinar, their contribution during focus group discussions, online questionnaire responses and any other personal informant-based data.

10. PARTICIPATION AND WITHDRAWAL

Participants are free to withdraw at any time and stage without giving a reason; even after they have consented to participate. They are free to decline to participate by either closing the web page of the questionnaire or any online research platform. They may withdraw their consent at any time and discontinue participation without penalty. In case they do agree to participate; they have the right to decline to answer any questions. In addition, they have the right to ask for their data to be withdrawn from the research if they wish to do so. However, for online questionnaires no identifying information will be collected, hence Mathematics teachers' responses will be completely anonymous. The implication of this is that if they would like to withdraw their responses after completing the study, they would need to provide the researcher with information that would help her identify their responses, i.e., specific answers to some of the questions.

11. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Leena N Kanandjebo at +264 81 3301043, email lvilma204@gmail.com or

19857578@sun.ac.za and/or the supervisor Dr. C E Lampen at ernalampen@sun.ac.za

12. RIGHTS OF RESEARCH PARTICIPANTS

This study has been approved by Stellenbosch University Research Ethics Committee. Further, you are not waiving any legal claims, rights, or remedies because of your approval to this research study. Any questions regarding your rights, concerns, or complaints, contact Ms Maléne Fouché [mfouche@sun.ac.za ; 021 808 4622] at the Division for Research Development.

If you agree for this research to be conducted by teachers from your school, please sign below. Also, kindly provide the names of those Mathematics teachers and their email addresses and contact numbers as the study is online. Email to Leena N Kanandjebo at lvilma204@gmail.com or 19857578@sun.ac.za. You may keep a copy for your records.
Attached is the proof of ethical clearance from the Stellenbosch University Research Ethics Committee as well as permission letters from the Ministry of Education, Arts and Culture, and the regional education director.

Thank you very much for your help.

______________________________ ____________________

Signature of School Principal Date

Addendum L: Request for permission from School Principals and/ or supervisors to conduct research (second request)

Cell: 081…
Email: lvilma204@gmail.com or
19857578@sun.ac.za
23 February 2022

To:
The supervisors/school principal

Re: Request to conduct research engagement webinars as part of the PhD study.

I am Leena Ngonyofi Kanandjebo, a Ph.D. (Mathematics education) student at Stellenbosch University (SU), South Africa, under the supervision of Dr C. E. Lampen. As a requirement for the fulfilment of the Doctor of Philosophy, I am conducting a design-based study. The study aims to design a professional development framework for teaching Mathematics meaningfully with technology. Participation in the study involves filling in an online questionnaire, online research engagement webinars and focus group discussions. As per our cell phone conversation, in the past, engagements took place after hours as suggested. However, it became a challenge to have all participants, or at least half of the number, attend at once after teaching and learning hours and administrative work. Moreover, after working hours, the researcher observed that this is the time that participants ought to spend with their families and discussions were not fruitful.

I have thus scheduled **to conduct a one-day online professional development webinar on MS teams on March 1, 2022,** if permission is granted. Although this is research-based engagement the overall objective will benefit Mathematics educators, regions, and schools where they serve in terms of skills as the training is conceptualized within the Namibian secondary school Mathematics curriculum. Further, participants in the study will have an opportunity to enhance their knowledge of meaningful technology integration and pedagogy in relation to Mathematics.

If funds allow, the researcher may reimburse participants for the costs of data collection. The schedule of the day will be available to participants once the request has received approval.

Attached are the proofs of ethical clearance from the SU Research Ethics Committee, permission letters from the Ministry of Education, Arts and Culture, and the regional education director.

Yours sincerely
Leena Kanandjebo (student number:19857578)

Addendum M: Information sheet for teachers

1. BACKGROUND AND STUDY OVERVIEW

I am Leena Ngonyofi Kanandjebo, a Ph.D student in the Department of Curriculum Studies at Stellenbosch University, South Africa. I am being guided and supervised by Dr. C E Lampen. As a requirement for the fulfilment of the Doctor of Philosophy, I am conducting a design-based study titled '**A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study'**. The study aims to design a framework for the professional development of Mathematics teachers, to teach Mathematics meaningfully with technology. Participation in the study involves completing online questionnaires, online webinars and focus group discussions. All technologically adept Mathematics teachers are invited to take part in this study. Before you decide to participate in this study, it is important that you understand the purpose of the study and what it will involve. Kindly, take time to read the following information carefully. You may ask the investigator if there is anything that is not clear and if you need more information.

2. PURPOSE OF THE STUDY

This study is proposed with an aim of designing a professional development framework for teaching Mathematics meaningfully using technology. Participants in the study will have an opportunity to upgrade their knowledge of meaningful technology integration and pedagogy in relation to Mathematics.

3. STUDY PROCEDURE

Secondary school Mathematics teachers will be required to partake in the following design stages:

Design stage	Involvement
STAGE 2: Context Analysis	- Complete the initial online questionnaire. This will take approximately 20 minutes. - Attend three research engagement webinars

	- Teachers will engage in meaningful teaching events with the researcher. Each research engagement is coupled with focus group discussions. Discussions will last about 30 minutes - For the last focus group discussion, participants will also be asked to post their teaching products on a shared Google drive folder. - After the webinar secondary school Mathematics teachers will fill in an online questionnaire to express their views on how their participation in the design process influenced their professional development in teaching Mathematics meaningfully with technology. This will take place online after the webinar and will last for about 20 minutes.

4. DURATION

All online links to participating in the study will be shared with teachers on an online interactive platform (WhatsApp). Online engagements will be after working hours 16:00-17:00 (1 hour) only at Stage 2 (context analysis). Webinars will take place online via ZOOM cloud meetings during weekends from 12:00-13:00. Data collection is planned to take about three (3) weeks

5. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts anticipated from participation in this research study. You may decline to participate in any activity or terminate your involvement in the research at any stage and time if you wish.

6. POSSIBLE BENEFITS

The study will be an opportunity for you to reflect on your current technological teaching practices. The study will form part of your professional development in teaching Mathematics meaningfully with technology. Furthermore, we hope that the design process and findings of this study will:

- Help you as a secondary school Mathematics teacher improve your technological, pedagogical and content knowledge and thereby improve your teaching and learning process.
- Contribute to the body of knowledge in Mathematics Education by adding to the existing empirical evidence about technological instructional approaches for Mathematics specifically. Importantly, promote technological teaching that is meaningful, and in alignment with the development of skills needed in the technological era.
- Provide information for the research community and Mathematics teachers on how to successfully use technology to bring about meaningful teaching in times where technology is the only means of instructional delivery and or in blended teaching and learning situations.
- Assist in setting education reform policies, curriculum implementation, and develop technological strategies that can be used to improve the teaching and learning of Mathematics.
- Support educational planners, policymakers and professional development bodies of teachers with technology integration in the Namibian educational system.
- The government of Namibia through the Ministry of Education, Arts and Culture (MoEAC) plans to achieve at least 65% in Mathematics performance by 2022 (NDP5, 2017). The designed framework is envisioned to assist in accelerating the attainment of National development plans such as National Development Plan 5 (NDP5) and MoEAC strategic plan leading to the attainment of Vision 2030.

7. PAYMENT FOR PARTICIPATION

Your participation will be free of payment and voluntary, that is, there will be no remuneration. However, the researcher might provide reimbursement for the data costs on you should funds permit.

8. PROTECTION OF PARTICIPANTS' INFORMATION, IDENTITY AND CONFIDENTIALITY

Comments, reflections, and responses shared by you during this study will be anonymous. In addition, every effort will be made by the researcher to always preserve your confidentiality including the following:

- Assigning code names/numbers for participants which will be used on all researcher notes and documents.
- Participants will be requested to change their usernames to codes in order to protect their names and any identification during the webinar.
- Any paper data such as teaching products, interview transcriptions, transcribed notes and any other research materials will be converted into a soft copy before it is kept in a locked file cabinet.
- All electronic versions of the data, including responses from the questionnaire, teaching products, and webinar recordings, will be encrypted (password protected) and kept in a password protected external hard drive as well as saved on the researcher's google drive and Dropbox.
- After five years, all the research materials will be destroyed.
- All participants may review the investigator's collected data. Information from this research will be used solely for the purpose of this study and any publications that may result from this study.
- An online group interaction platform (WhatsApp group) will be created solely for engagement, communications, and further data collection. Note: Conversations shared on WhatsApp are secured with end-to-end encryption, thus you're in control of your data (WhatsApp LLc, 2021).
- A webinar will be recorded. Any accidental footage of the participants who have not consented or withdrawn from the study will be cut out from the recording.
- The video camera will be set off for participants during webinar sessions however if participants wish to put it on during sessions they can do so
- You should be aware that if you participate in the focus group discussions and WhatsApp group discussions, although the investigator will maintain

confidentiality it is not possible to guarantee that other group members will do so.

- Each participant has the opportunity to obtain the recording of the webinar, their contributions during focus group discussions, online questionnaire responses and any other personal informant-based data.

9. PARTICIPATION AND WITHDRAWAL

You are free to withdraw at any time and stage without giving a reason; even after you have consented to participate. You are free to decline to participate by either closing the web page of the questionnaire or any online research platform. You may withdraw your consent at any time and discontinue participation without penalty. If you do agree to participate, you have the right to decline to answer any questions. In addition, you have the right to ask for the data to be withdrawn from the research if you wish to do so. Also, note for online questionnaires no identifying information will be collected, hence your responses will be completely anonymous. The implication of this is that if you would like to withdraw your responses after completing the study, you would need to provide the researcher with information that would help her identify your responses, i.e., your specific answers to some of the questions.

10. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, or would like to withdraw your responses after participation, please feel free to contact Leena N Kanandjebo at +264 81 3301043 or email lvilma204@gmail.com and/or the supervisor Dr. C E Lampen at ernalampen@sun.ac.za

11. RIGHTS OF RESEARCH PARTICIPANTS

This study has been approved by Stellenbosch University Research Ethics Committee. Further, you are not waiving any legal claims, rights, or remedies because of your participation in this research study. If you have questions regarding your rights, concerns, or complaints as a research participant, contact

Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

Attached is the ethical clearance from the Stellenbosch University Research Ethics Committee as well as permission letters from the Ministry of Education, Arts and Culture, and the regional education director.

If you choose to be involved in online webinars with online focus group discussions and complete online questionnaires, you will be asked to sign a formal 'Statement of Informed Consent. Signing the consent form will indicate that you have read and understood all the information concerning the project. **If you wish to participate in one or more additional activities mentioned, please sign the consent form on the next page by ticking your choice(s).**

Thank you for taking the time to consider this study!

Addendum N: Consent form for teachers

Study title: A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study

Research supervisor: Dr C E Lampen
Researcher's name: Ms Leena N Kanandjebo

1. I agree to take part in the research study mentioned above.
2. I understand that the study involves the following iteration activities: completing online questionnaires, attending online webinars and participating in focus group discussions
3. I agree to participate in the following iteration activities (please tick the box on **one** or **more** or **all** the following):

   Online questionnaires ☐

   Online webinars ☐

   Online focus group discussions ☐

4. I have read and understood the Information Sheet for this study.
5. The nature and possible effects of the study have been explained to me.
6. I understand that there are no risks anticipated from participation in this study.
7. I understand that all research data will be securely stored for five years in a lockable cabinet, google drive and Dropbox from the publication of the study findings, and will then be destroyed using an electronic shredder.
8. Any questions that I have asked to have been answered to my satisfaction.
9. I understand that the researchers will maintain confidentiality and that any information I supply to the researcher will be used only for the purposes of the research.
10. I understand that in the online focus group discussions as well as WhatsApp group, although the investigator will maintain confidentiality it is not possible to guarantee that other participating members might do so.

11. I understand that the finding of the study will be published in such a way that I cannot be identified as a participant.
12. I understand that my participation is voluntary and that I may withdraw at any time without any effect. If I so wish, I may request any data I have supplied be withdrawn from the research. For an online questionnaire, I would need to provide the researcher with information that would help her identify my responses.

Participant's name: ____

Participant's signature: ______________________

Date: ________________

Email or phone number: ____

Statement by Investigator:

I have explained the research and the implications of participation in it to the participant and I believe that the consent is informed and that he/she understands the implications of participation. The participant has received the Information sheet where my details have been provided so participants have had the opportunity to contact me prior to consenting to participate in this project.

Investigator's name: ____

Investigator's signature: __________________ Date: ________________________

Email the signed copy to Leena N Kanandjebo at lvilma204@gmail.com

Addendum O: Permission letter from the Ministry of Education, Arts and Culture to conduct research in Namibia

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION, ARTS AND CULTURE

Tel: +264 61 -2933202
Fax: +264 61- 2933922
Enquiries: G. Munene
Email: gm12munene@yahoo.co.uk
File no: 13/2/9/1

Luther Street, Govt. Office Park
Private Bag 13186
Windhoek
Namibia

Ms Leena Kanandjebo
Email: lvilma2004@gmail.com

Dear Ms Kanandjebo,

SUBJECT: PERMISSION TO CONDUCT AN ACADEMIC RESEARCH IN NAMIBIA

The Ministry wishes to acknowledge receipt of your email seeking permission to conduct an academic research at schools for your PhD studies which is focussing on: *"A Professional Development Framework for Teaching Mathematics Meaningfully with Technology in Namibian Secondary Schools: A Design-Based Study."*

Permission is hereby granted to you provided you seek for further clearance from the Regional Directors of Education, Arts and Culture at the regions where you wish to conduct your research to ensure that:

- Permission is sought from the school principals and parents;
- teaching and learning should not be interrupted;
- participation is voluntary.

Furthermore, you are kindly requested to share your research findings with the Ministry after completion of the research project. You may contact Mr G. Munene at the Directorate: Programmes and Quality Assurance (PQA) for submission of your research findings at the above indicated details.

We wish you the best in conducting your research and the Ministry looks forward to hearing from you upon completion of your study.

Yours Sincerely,

Ministry of Education, Arts and Culture
Office of the Executive Director
31 MAR 2021
Tel: 061-2933523/4
Fax: 061-253671
Private Bag 13186, Windhoek, Namibia

Sanet L. Steenkamp
EXECUTIVE DIRECTOR

All official correspondences must be addressed to the Executive Director.

Addendum P: Permission letter from Erongo Regional Director of Education to conduct research

ERONGO REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS & CULTURE

Telephone : 064-4105101
Fax : 064-4105136

Private Bag 5024
SWAKOPMUND

Enquiries : Ms. E.J. Stephanus
Date : 12 April 2021

Ms. Leena Kanandjebo
Email: lvilma2004@gmail.com

Dear: Ms. Leena,

RE: PERMISSION TO CONDUCT AN ACADEMIC RESEARCH IN ERONGO REGION

The Ministry wishes to acknowledge receipt of your email seeking permission to conduct an academic research at school for your PhD studies. ***" A professional Development Framework for Teaching Mathematics Meaningfully with Technology in Namibian Secondary Schools: A Design Based Study."***

Permission is hereby granted to you provided:

- Permission is sought from the School principal and parents;
- Teaching and learning should not be interrupted;
- Participation is voluntary.

We wish you the best in conducting your research and the Ministry looks forward to hearing from you upon completion of your study.

Kind regards,

ERONGO REGIONAL COUNCIL
DIRECTORATE OF EDUCATION
12 APR 2021
OFFICE OF THE DIRECTOR
BAG 5024, SWAKOPMUND

Ms. E.J. Stephanus
REGIONAL DIRECTOR

Addendum Q: Permission letter from Ohangwena regional director of Education to conduct research

REPUBLIC OF NAMIBIA

OHANGWENA REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS AND CULTURE

Office of the Director
Tel: (+264) 65 290200
Fax: (+264) 65 290224
Enquiries: pamwanyangapo@gmail.com
Our Ref: 12/3/10/1

Hareibecke Street, Greenwell Complex Building
Private Bag, 88005
Eenhana

08 April 2021

To: Ms.Kanandjebo N L
Ivilma2004@gmail.com
0813301043

SUBJECT: REQUEST FOR PERMISSION TO CONDUCT AN EDUCATIONAL RESEARCH IN SECONDARY SCHOOLS IN OHANGWENA REGION

Receipt of your letter dated 08 April 2021 is hereby acknowledged.

1. The request has been evaluated and found to have merit.
2. Kindly be informed that permission to collect data from Secondary Schools in Ohangwena Region, research has been granted under the following conditions and requests:
   - The data to be collected only be used for the completion of your studies.
   - Kindly liaise with the concerned Principals so as to make prior arrangements before the date of the research.
   - No other data should be collected other than the data stated in the request.
   - You may share the final report of your study with the directorate.
4. It is trusted that you will find this arrangement in order while wishing you all the best with your studies.

Yours Sincerely,

Isak Hamatwi
Director

Ohangwena Regional Council
Directorate of Education Arts and Culture
Office of the Director
08 APR 2021
Tel: 065-290 200 • Fax. 065-290 224
P/Bag 88005 • Eenhana
REPUBLIC OF NAMIBIA

Addendum R: Permission letter from Omusati Regional Director of Education to conduct research

Republic of Namibia

Omusati Regional Council

Private Bag 529, OUTAPI| Tel +264 65 251700| Fax +264 65 251078

Ref: 13/2/9/1 12 April 2021

Enquiry: Apollonia Hango

To: Ms. Leena Kanandjembo
Universiteit-Stellenbosch-University

RE: PERMISSION TO CONDUCT AN ACADEMIC RESEARCH IN NAMIBIA

1. This letter serves to notify you (**Ms. Leena Kanandjembo**) that permission has been granted to carry out a research regarding an **"A professional Development Framework for Teaching Mathematics Meaningfully with Technology in Namibia Secondary School"** at any secondary school in Omusati Region.
2. Please be informed that the research to be carried out at schools should by no means whatsoever disrupt teaching and learning.
3. You are further urged to ensure that your investigation methodologies are in compliance with Covid 19 preventative protocols put in place.
4. We hope and trust this exercise will enhance quality education in the Region.

Thank you for your understanding.

Yours faithfully

12/04/202

Pauline Shapumba
Acting Director of Education, Arts and Culture

OMUSATI REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
12 -04- 2021
OFFICE OF THE DIRECTOR
PRIVATE BAG 529, OUTAPI
REPUBLIC OF NAMIBIA

Cc: Inspectors of Education

My NAMIBIA — All official correspondence must be addressed to the Chief Regional Officer

Addendum S: Permission letter from Oshikoto Regional Director of Education to conduct research

REPUBLIC OF NAMIBIA

OSHIKOTO REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE

REGIONAL COUNCIL

OSHIKOTO

Tel (065) 281900
Fax (065) 240315
Enq: Ms H Tende

Private Bag 2028
ONDANGWA
14 April 2021

Ref: 12/3/10/1

Ms Leena N. Kanandjebo
Email: Lvilma@gmail.com
Cell: 0813301043

Dear Ms Kanandjebo

RE: PERMISSION TO CONDUCT RESEARCH IN OSHIKOTO REGION

The Office of the Director acknowledges receipt of your letter seeking for permission to conduct a research study **"A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary schools"**.

Kindly be informed that permission has been granted to carry out the research in Oshikoto Region, please be guided by the following:

- You have to consult the school principals well in advance to ensure a proper co-ordination of other school activities.
- The research should not interfere with the normal teaching and learning process at the schools.
- Participation in the research should be on a voluntary basis.
- The information to be collected should be treated as confidential and only for research purposes.
- And, be advised to adhere to the Covid-19 protocols and measures.

Be further informed that some schools are private schools and the onus lies with the Board of Directors to grant you permission, in principle, we do not have an objection to your research study.

Thank you for showing interest to do the research in the Oshikoto Region. It is our sincere hope that the information you would gather will be useful towards the completion of your qualification.

Sincerely yours

15/04/2021

OSHIKOTO REGIONAL COUNCIL
Director
2021-04-15

MS ALETTA A. EISES
DIRECTOR OF EDUCATION, ARTS AND CULTURE
OSHIKOTO REGION

Addendum T: Permission letter from Oshana Regional Director of Education to conduct research

REPUBLIC OF NAMIBIA

OSHANA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION, ARTS AND CULTURE
Aspiring to excellence in Education for All

Tel: 065 229800 Fax: 065 229833
Enquiries: Hilma Nuunyango-George
E-mail: nuunyangoh@gmail.com
Ref no: 13/2/9/1

906 Sam Nujoma Road
Private Bag 5518
Oshakati, Namibia

MS LEENA KANANDJEMBO
lvilma204@gmail.com

Dear Ms Kanandjembo

RE: PERMISSION TO CONDUCT AN ACADEMIC RESEARCH IN FIVE SCHOOLS IN OSHANA REGION

1. My office hereby acknowledged receiving your request to conduct an academic research in five schools (Andimba SS, Oshakati SS, Iipumbu SS, Akuniihole PS and Cabatan Private School) in Oshana Region and it bears reference;

2. Kindly be informed that permission is hereby granted to conduct the study titled: **A professional development framework for teaching Mathematics meaningfully with technology in Namibian Secondary Schools: A design-based study.** You are hereby requested to present this letter of approval to the principals of the selected schools to validate that the research is authorised, authentic and that procedures are adhered to.

3. This permission is subject to the following strict conditions; (i) There should be no interruption of normal teaching and learning, during a class or scheduled afternoon session, (ii) Ethical issues of confidentiality and anonymity should be respected and retained throughout this activity i.e. voluntary participation, and consent from participants.

4. Both parties should understand that this permission could be revoked without explanation at any time.

5. Furthermore, we humbly request you to share with us your research findings with the Directorate of Education, Arts and Culture, Oshana Region. You may contact the office of the Deputy Director: Programs and Quality Assurance (PQA) for the provision of a summary of your research findings.

6. I wish you the best in conducting your study.

Yours Sincerely

HILENI M. AMUKANA
REGIONAL DIRECTOR

OSHANA REGIONAL COUNCIL
18 -06- 2021
Private Bag 5518
OSHAKATI
REPUBLIC OF NAMIBIA

All correspondence should be addressed to the Chief Regional Officer

Addendum U: Terms operationalisation

Kilpatrick et al. (2001) operationalisation

	Phrases about…
Conceptual understanding	understanding Mathematics concepts and terminologies, e.g understanding formulas through graphing
Productive disposition	willingness and influence to explore, experiment, problem solve, pose a problem, enjoyment, collaborate and value beauty in relation to Mathematics topics.
Strategic competence	Formulating, creating, hypothesising, representing and solving problems in relation to Mathematics topics
Adaptive reasoning	justification, logical thinking, reflection, and explaining, in relation to Mathematics topics
Procedural fluency	A relational understanding of procedures and techniques to solve mathematical problems

Operationalising Activity Theory nodes:

	Phrases that relate to…
Goal	Purpose of Mathematics and/or technology, e.g technology makes Mathematics fun, the goal here is fun (productive disposition) technology saves time, the goal is speed.
Division of labour	Specific actions by specific people, teachers, learners or others e.g teachers program on a spreadsheet, learners play around with.
Rules	Mathematics curriculum expectations and rules that regulate Mathematics
Community	The environment is community members such as educational stakeholders, government, educational institutions and parents created for the activity to be carried out.
Tensions	These are limitations. Professional development needs are envisaged for teaching Mathematics meaningfully with technology rather than the individual. E.g, if they have access to a computer to a computer or maybe to a laptop, the tension here is no access to technology tools

**The examples given are not exhaustive*

Needs Aspect: encompasses areas that are inferred as being challenges to participants, and areas that show that participants need help to effectively teach meaningfully with technology.

Meaningful Mathematics knowledge- Encompasses knowledge of **what** Mathematics learning content is meant for. It is the ability to explain the Mathematics learning content in relation to meaningful. Knowing how to relate images on the screen to communicate relevance, collaborate, critically and logically think, reflect, explain, justify, hypothesise, problem solve and persevere. Further, it is a deeper sense of understanding 'what' in relation to Mathematics. For example, they can:

- think logically about the relationship between angles formed and the situation presented, to justify one's answer.
- draw inferences for instance that the distance between lines is decreasing for conceptual understanding and procedural fluency
- make connections from previously learned information and make mathematical sense of the tasks.
- make a steady effort in exploring such as investigating mathematical concepts and re-positioning points for productive disposition.
- Represent real-life situations into solvable maths problems e.g a pair of compasses for 180 degrees angles, representing a pivot with a point to depict a centre or vertex.
- Know when to use certain procedures appropriately, and when to conclude on a mathematical assumption.
- A deeper understanding of different conceptions of mathematical concepts, e.g. stating different meanings of an angle and being aware that they imply the same.

Meaningful technology knowledge: This is knowledge of **how and when** to use technology skills and knowledge. This includes but is not limited to:

- access appropriate software applications on the internet, create an account and access online resources.
- create an environment (for example hide the label, use settings to not show grids and axes, and know different axes/grids to use when needed) for exploration.
- ZOOM out and ZOOM in

- creatively use application objects such as plot points, move, point, line segment, line, perpendicular lines, tangent lines, measure angle and length, change colour, undo and delete amongst others
- use 3 points to creatively explore rotation.
- Use objects to improve visibility
- know the difference Algebra view and the Tools view,
- to construct and or draw.

Knowledge of teaching meaningfully:

Not possible to code as the session took place in a technology environment through Geogebra. Participants and researchers were kilometres away from each other. It is not possible to draw a line between general meaningful knowledge of teaching and knowledge of teaching Mathematics meaningfully. However, on a smaller scale, the following indicates knowledge of teaching meaningfully:

(a) Ability to promote engagement constantly,
(b) collaborate and work together to explore,
(c) suggesting alternative teaching approaches to promote meaningfully,
(d) able to challenge pedagogical decisions,
(e) influence productive disposition,
(f) Ability to link topics to others i.e link geometry to estimation, and think deeply about a mathematical representation.

Knowledge of Teaching Mathematics meaningfully:

Mathematics teachers with knowledge of teaching Mathematics meaningfully know approaches and tasks that are aimed at developing *conceptual* understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. They know Mathematics curriculum requirements and can interpret what it implies for meaningful. They can:

- Able to introduce tasks, frame them and evoke mathematical reasoning and critical consideration.

- Able to associate appropriate tasks with appropriate mathematical concepts to achieve meaningful
- Able to propose alternative mathematical concepts that can be explored with available resources.
- Structure prompts using what is, when is, why that is and hypothesise to stimulate mathematical representation, creativity, fluency, reasoning and productive struggle.
- Able to restructure the learning environment necessary for justifying mathematical claims and other meaningful aspects.
- Able to link Mathematics to real life, and sees mathematical concepts as connected and thus rewarding.

Moreover, Mathematics teachers with knowledge of teaching Mathematics meaningfully are able to apply logical thinking about relationships between the line that represent an emotionless blade to teaching angle concepts. They are able to explain, provide reasons, and make connections between closing and opening linking it to an angle formed and not formed. Moreover, they think logically and consider alternatives.

Knowledge of Teaching meaningfully with Technology – Encompasses knowledge of **when and for what purpose**. This knowledge includes but is not limited to when to use technology objects to measure, drag, construct, draw, use settings, use a calculator suite and when enhance visual graphics. Further, it encompasses knowledge of what technology is used to enhance the development of learners' problem solving, concepts, mental operations, strategies, patterns, and logical thinking skills. A Mathematics teacher with knowledge of teaching meaningfully with technology knows when to use technology objects to instil desire and attitudes toward Mathematics. In addition, knowledge of using technology to measure and perform calculations to provide proof and justification, and aware when and how to use technology objects accurately. Also, able to use technology to reconstruct accurately a path by which mathematical concepts could be discovered or invented (De Villiers, 2004). Able to explain technological teaching processes to be followed to ensure meaningful.

Meaningful knowledge of Mathematics and Technology – This knowledge of **what** technology is for, to ensure meaningful. Able to use technology to reconstruct a path by which mathematical concepts could be discovered or invented (De Villiers, 2004). This knowledge includes the ability to instruct and direct which technology objects should be used to promote which mathematical meaningful aspect. For example, where to label with a point, which points are to be joined to draw a circle through three points in order to represent and justify the centre of rotation.

Knowledge of teaching Mathematics meaningfully with technology- This is knowledge of **how, when, what and for what purpose** to enhance meaningful aspects. It is the ability to use technology to execute the art of teaching Mathematics including plotting, joining and repositioning points, and persevering in doing so. Able to access and perform calculations on the calculator suite within the technology such as the GeoGebra application. Demonstrate and influence others' thinking about the pedagogical experience. They are able to combine technological and pedagogical skills to:

- create and represent real-life objects e.g model a car with a point and drag to represent moving, by which mathematical concepts can be explored, discovered or invented; in order to make connections and think about relevance.
- reconstruct and communicate to develop fluency using real-life objects e.g a roof truss and a pair of scissors by which mathematical concepts can be discovered or invented.
- appropriately and creatively use technology to construct objects such as line segments, circle through 3 points, tangent, parallel lines, measure length and angle, and triangles to explore mathematical concepts
- problem solve including providing proof and drawing mathematical inferences.
- perform Mathematics processes accurately and link concepts to communicate Mathematics, for example, length to proving parallelism, and how positions of points plotted influence the angle formed.
- Persevere and engage in the process of designing, creating and trying out teaching Mathematics with technology.

Moreover, aware of teaching opportunities and constraints afforded by technology and low technology use to enhance meaningful Mathematics teaching. Able to appropriately associate and communicate materials/figures to mathematical concepts, and technology objects. Further, able to collaborate, and promote collaboration during the process of teaching with technology.

Addendum V: Screenshot of the 'KLeena PhD study' WhatsApp group description box