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# Conversations reflecting boundary-objects-related details of a teacher's local practices with spreadsheet algebra programs on variables

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The ways teachers converse about their work in relation to information and communications technologies (ICTs) are worth studying. We analyse how a teacher converses about her local practices in relation to two spreadsheet algebra programs (SAPs) on variables. During the conversations we noticed that the teacher keeps different policy documents – boundary objects – firmly in view, in relation to the design of the two other boundary objects, namely the two SAPs. The policy documents provide details on the operative curricula which entail the intended, implemented and examined curricula. Of these curricula, the teacher regarded the examined curriculum and associated examinations as most important. Also, she conversed about how she intends to align the design features of the two SAPs with particular policy documents, especially in the context of the South African high-stakes National Senior Certificate examinations and the attendant examination pressure. Our results confirm current professional development (PD) literature suggestions that emphasise fostering coherence, for example between policy boundary objects details and what university-based PD providers do when they interact with school teachers.

**Contribution:** The results provide guidelines for university-based PD providers to integrate SAPs or other ICTs related to algebra and variables by keeping teachers' local practices in view. These providers should note that different policy-related boundary objects shape the ways teachers understand and converse about their local practices, namely their work at the classroom level.

**Keywords:** boundary objects; professional development; local practices; information and communications technologies (ICTs); spreadsheet algebra programs; algebra; variables; nature of the roots.

## Introduction

The topic of this study is boundary-objects-related details. We unpack this topic by outlining the meanings of boundary objects. In this article the concept of boundary objects does not take on a singular meaning. First, we define boundary objects as 'objects which inhabit several intersecting social worlds and satisfy the informational requirements of each of them' (Star & Griesemer, 1989, p. 393). In the South African education system we find different policy documents that are used to provide details about schools and the education system to different stakeholders, for example teachers and learners, teachers and parents and principals, and teachers and subject advisors and academic institutions and the provincial education departments. Examples of such boundary objects include programmes of assessment (POA), subject assessment guidelines (SAG), the curriculum and assessment policy statement (CAPS) and, more recently, annual teaching plans (ATPs) and the National Protocol for Assessment (NPA) (DBE, 2012; 2017). These boundary objects aim at coordinating activities in schools on a district, provincial and national basis (Wenger, 1998). Another boundary object, the high-stakes National Senior Certificate (NSC) Mathematics examination, and its associated question papers, provide details on content at the national level and downwards to the school and classroom levels. Second, we define boundary objects also as technologies, for example spreadsheet algebra programs (SAPs) on variables, as an instance of information and communication technologies (ICTs). In the current study the two SAPs, *Discriminant* and *Factoring*, inhabit the intersecting world of the teachers and the university-based mathematics educators (UMEs), and have enough in common as representations of algebra with respect to variables (Gelfand & Shen, 1993). The two SAPs have enough in common between the communities of UMEs and teachers to make them 'recognizable' (Star & Griesemer, 1989, p. 393). In our case the SAPs have been designed in ways that facilitate knowledge sharing and knowledge generation between UMEs as PD providers, and teachers. Boundary-objects-related details therefore relate either to the mentioned policy documents or the SAPs or both.

Another key notion in this article is teachers' local practices. We define teachers' local practices as ways teachers converse about their work within the schooling system and at the classroom level. Teachers respond to and interact with different stakeholders, for example parents, school principals and education department officials, to name a few. Also, teachers contend with operative curricula that include:

- The intended curriculum: CAPS details per grade level.
- The interpreted and implemented curricula: what they understand and do, that is, teach in their classrooms, informed by CAPS details.
- The examined curriculum*:* the (mathematics) content present in examinations and assessments (Julie, 2013).

Operative curricula details are spelled out in policy documents, that is, the boundary objects we listed in the first paragraph. The operative curricula inform teachers' 'logic of practice' and their ways of working (Bourdieu, 1990; Julie, 2013). On a related point, we read about teachers' practical rationality as well as their practical rationality of mathematics teaching (Herbst & Chazan, 2003). Together these analytical constructs can be used to understand how teachers converse about their work, namely their local practices.

## Problem statement and research questions

In terms of the professional development (PD) literature, we do not know much about ways teachers who work under conditions of high-stakes examinations in the greater Cape Town area, South Africa, converse on the work they do (Julie et al., 2019a). When UMEs converse with teachers who work under such conditions about the design of the two SAPs based on variables, other boundary-objects-related details are bound to emerge. The teachers are likely to converse in ways where they seek coherence and alignment between the two SAPs and boundary objects such as the various policy documents that inform their local practices. Also, during conversations, UMEs and teachers can differ in their terms of reference, that is, their perspectives. Moreover, a boundary crossing occurs when UMEs take and introduce the SAPs from the university to the school (Akkerman & Bakker, 2011). For this article we pursue the main research question:

> *What boundary-objects-related details about teacher local practices emerge during conversations with SAPs on variables?*

Two sub-questions are:

- What other studies set a foundation for this main research question?
- How do the article's findings relate to other studies?

## Rationale for the study

We justify the main research question with its conversation focus as follows. First, in terms of working in the school and the education system, this study aims to bring to the fore ways teachers converse about boundary-objects-related details that impact on their work at the local, that is, classroom and school levels. Generally, teachers are stakeholders in mathematics education research (Krainer, 2014). The analysis therefore offers ways for UMEs to better understand the teachers' local practices in their school. As noted, we find few such studies in the greater Cape Town low socioeconomic areas (Julie et al., 2019a). By analysing such conversational exchanges, the UMEs are likely to identify curricular details that signal to teachers what to teach, how to assess and what will be examined (Gölogلu & Kaplan Keles, 2021; Jonsson & Leden, 2019). Such details provide another way of discovering how teachers converse about the intended, implemented, interpreted and examined curricula (Julie, 2013). Second, based on the PD literature, the analysis can shed light on ways UMEs can better understand their role as knowledge brokers or interlocutors when they cross the boundary between the university and the schools in a general sense (Rycroft-Smith, 2022; Wenger, 1998).

Here, the analysis can bring to the fore curricular details about what the two parties – UMEs and teachers – know relative to each other. Such details become helpful for 'working with' as opposed to a deficit view of 'working on' teachers (Setati, 2005). Working with teachers is an attempt to counter a 'reduced analytical representation' of teachers and the boundary-objects-related details they deal with in their schools (Liberman, 2012, p. 277). In addition, teachers are likely to share 'instructional norms and professional obligations to the stakeholders of school mathematics', for example themselves, parents and principals (Herbst & Chazan, 2012, p. 610). Third, also taken in part from PD literature, UMEs need to know that their conversations with teachers reflect a 'boundary encounter' generally between the university and the school (Akkerman & Bakker, 2011). Also, within the school, we find boundary objects, which we listed in the first paragraph. Fourth, this boundary encounter also becomes one between mathematics education research and school mathematics teaching concerning variables. A boundary encounter of this kind also involves boundary objects, for instance ICTs and algebra (Robutti et al., 2019) and policy documents. Here, the analysis has implications for current calls for using digital technologies (Clark-Wilson et al., 2020) as a 'resource approach' in mathematics education (Chazan, 2022; Trouche, et al., 2019). In the current context, digital technology or ICT use is not widespread. More interestingly, the design of the two SAPs differs from the ways that algebra appears in the operative curricula, namely the curriculum structure of algebra spelled out in policy documents and the examined curriculum (Potari et al., 2019). In particular, the analysis can illuminate how teachers converse about the cell-variables inscribed in the design of the two SAPs concerning the high school operative curricula (Haspekian, 2005). The analysis thus has implications for

UMEs on ways to enhance their role as knowledge brokers, that is, interlocutors, between research knowledge on variables and ways teachers converse about variables (Rycroft-Smith & Stylianides, 2022). Of particular interest was how teachers converse about the design of the two SAPs, which breaks boundaries by representing learning trajectories that connect factors, products, trinomials and ways of interpreting the discriminant through an expansive view of variables and parameters (Confrey & Maloney, 2014; Epp, 2012; Göbel, 2021).

In the operative curricula, we find boundaries, that is, separations, between products, factors, trinomials and parabolas. The design features of *Factoring* and *Discriminant* show connections between these separate ideas and concepts found in the South African operative curricula (see the section on Data, methodology and analysis). Policy documents, that is, boundary objects such as various diagnostic reports on the high-stakes Grade 12 NSC Mathematics examinations, note learners' poor algebraic and manipulation skills, their struggles with the concept of a variable, and interpreting the discriminant in the case of quadratic functions (DBE, 2017; 2018; 2020).

# Literature review

Two sub-questions inform the literature review.

## Other studies that set a foundation for the main research question

From the PD literature we find several studies that reference boundary-objects-related details. In South Africa, Julie et al. (2019b) make the argument for examination-driven teaching as an underpinning of their PD initiative. This project takes its cue from the high-stakes NSC Mathematics examinations, a boundary object integral to the schooling system. Similarly, in the United States, Boardman and Woodruff's (2004) results suggest that some teachers may use 'high-stakes' assessments as their primary reference point when it comes to PD that focus on innovative teaching practices, for example using SAPs in our case. The teachers in Boardman and Woodruff's study viewed the statewide assessment as the reference point by which they gauged both student learning and their teaching effectiveness. In other words, the statewide assessment, as a policy detail, serves as a boundary object. Also, Wideen et al. (1997) note that high-stakes examinations as a form of summative assessment in mathematics are not an uncontested area, but proponents have argued that they have 'become a permanent and vital part of education' (p. 430). In other words, education systems cannot survive or do without the boundary object, namely high-stakes examinations.

Boundary-objects-related details do not only refer to the high-stakes Grade 12 NSC examinations. These details also include references to school-based end-of-year summative assessments. In a recent survey on assessment in mathematics, Suurtamm et al. (2016) view the last-mentioned assessments as 'increasingly play(ing) a prominent role in the lives of students and teachers as graduation or grade promotion often depend on students' test results' (p. 4). Boundary objects such as the CAPS and ATP documents provide details on school-based assessments for the different grade levels.

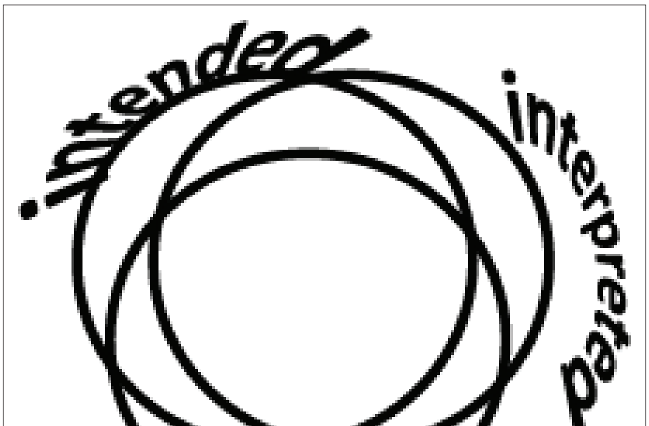
From the effective PD literature we also find references to boundary-objects-related details. Garet et al. (2001) note the following core features of professional development activities that have significant, positive effects on teachers' self-reported increases in knowledge and skills and changes in classroom practice: (1) focus on content knowledge, (2) opportunities for active learning and (3) coherence with other learning activities. The relevant core feature in the current study is: *fostering coherence.* Fostering coherence means that there must be alignment with state and district standards and assessments (Hochberg & Desimone, 2010). Desimone (2009) also notes that PD activities must be aligned with and directly related to 'state academic content standards, student academic achievement standards, and assessments' (coherence) (p. 184). Desimone (2011) elaborates the core feature, coherence, as follows: what teachers learn in any professional development activity should be consistent with other professional development, with their knowledge and beliefs, and with school, district and state reforms and policies (p. 69). In our case, these state and district standards and assessments become the various South African policy documents, that is, boundary objects, we outlined in the first paragraph. In the South African PD literature, coherence and alignment become synonymous with ecological relevance (Julie et al. 2019a). Ecological relevance implies that teachers deem the implementation of ideas offered during PD workshops and institutes as doable within the functioning milieu of their schools and classrooms with their varying demands (Julie, 2019).

The operative curricula become key to understanding ways teachers converse about the varying demands on their local practices, especially the examined curriculum. We define operative curricula as the intertwining intended, interpreted, implemented and examined curricula. In practical ways, in South African schools, and on a daily, weekly and monthly basis, the different policy documents, that is, boundary objects, seek to impose 'order' and provide details on the intended curriculum, namely the Curriculum and Assessment Policy Statements (CAPS) documents. Teachers interpret, that is, make sense of, and implement CAPS details in their classrooms. During the school year teachers also prepare their learners for the examined or assessed curriculum, in other words, for examinations. Examinations exert an ordering effect on teachers' local practices because they occur during stipulated times and dates during the school year. Such examination details inform us that teachers will likely bring up issues of examinations and assessment. Examinations also operationalise significant components of the intended curriculum spelled out in policy documents (Julie, 2013). Bishop et al. (1993, p. 11) note that examinations tend to determine the implemented curriculum, that is, what teachers do in their classrooms. For UMEs who intend to work with teachers, it therefore becomes necessary to note and to study the interactions of curricular variation between the intended, interpreted, implemented and examined curricula. In the schooling system, the intended and interpreted curricula provide only boundaries of the content to be taught, but the implemented curriculum, that is, what teachers do in their classrooms, is heavily driven by the examined curriculum (Julie, 2013). Examinations reflect the content of the examined curriculum. They become high-stakes occasions, because there are consequences for learners and other stakeholders, for example principals, parents and politicians. Examinations determine whether learners proceed to the next grade level or whether they can enter higher education. These curricular variations can be displayed as shown in Figure 1.

The overlapping circles in Figure 1 emphasise interlocking relationships between the curricular variations. The examined curriculum at the bottom of Figure 1 signals a foundational role and a permanent and vital part of the education system, which can be characterised as 'examination-driven' (Julie, 2013; Wideen et al., 1997).

### How do the article's findings relate to other studies?

First, the article's findings relate to studies that mention teachers' awareness of examinations or assessment issues. Pong and Chow's (2002) study on examinations in Hong Kong reports on examination pressure. Although historically different from Hong Kong, South African teachers also deal with the emphasis on examinations, which creates all kinds of pressures on teachers. The teachers in the current study work in high schools located in a low-income socioeconomic area in the Western Cape, South Africa. South African teachers contend with newspaper reports that publish the high-stakes Grade 12 NSC Mathematics (matric) results. These reports list the schools into halls of fame and halls of shame, based on ranking of examination results (Keitel, 2005).




*Source*: Julie, C. (2013). Can examination-driven teaching contribute towards meaningful teaching? In D. Mogari, A. Mji & U.I. Ogbonnaya (Eds.), *Proceedings of the ISTE International Conference on Mathematics, Science and Technology Education* (pp. 1–14). UNISA Press

**FIGURE 1:** Outline of the operative curricula showing interlocking interactions between curricular variations.

On a similar issue, Gregory and Clarke (2003) did a study on high-stakes assessment in England and Singapore. They speak of 'league tables' that rank schools according to examination results (p. 67). Here the boundary object is newspaper reports or league tables that become recognisable by the different stakeholders, for example teachers, students, parents and the general public. It would be out of bounds for teachers to ignore the content specified for high-stakes examinations (Julie, 2013; Wall, 2000). The other important boundary object is the CAPS policy documents that spell out details on the assessment, that is, examinations. In the South African PD literature, we find studies on examination-driven teaching as an underpinning of a PD project (Julie et al., 2019b). In Melbourne's high schools, in Australia, Hagan (2005) did a study on examination-driven mathematics teaching, in which assessment plays a key role in determining a certain style and approach to teaching. Clearly, when UMEs interact and converse with teachers over protracted periods, as in PD initiatives, the teachers are likely to reference the operative curricula, which include examinations or the assessed content, that is, the examined curriculum (Göloglu Demir & Kaplan Keles, 2021; Jonsson & Leden, 2019).

Second, the article's findings relate to policy documents and studies in algebra and variables. As noted above, diagnostic reports in South Africa note that learners struggle with the concept of a variable. Unsurprisingly, we also read about ways of 'making algebra work' in schools, which includes focusing on meanings of variables based on instructional strategies that deepen student understanding, within and between algebraic representations (Star & Rittle-Johnson, 2009). The design features of *Factoring* and *Discriminant* aim at deepening learners' understanding of variables. In particular, these design features break boundaries between factors and products, by representing learning trajectories that connect factors, products, trinomials and ways of interpreting the discriminant through an expansive view of variables and parameters (Confrey & Maloney, 2014; Epp, 2012; Göbel, 2021). These design features can also be used to address learner errors or challenges in graphing polynomial functions and the discriminant formula (Hasanah et al., 2021).

# Methodology

## Research design

This study followed a qualitative research design approach in which we adopted a case study. For the case study we examined the particularity and complexity of the case, namely the topic of boundary-objects-related details (Tomaszewski et al. 2020). Regarding the case, we wanted to understand the complex nature of its activities and particular circumstances, for example a high-stakes examinations

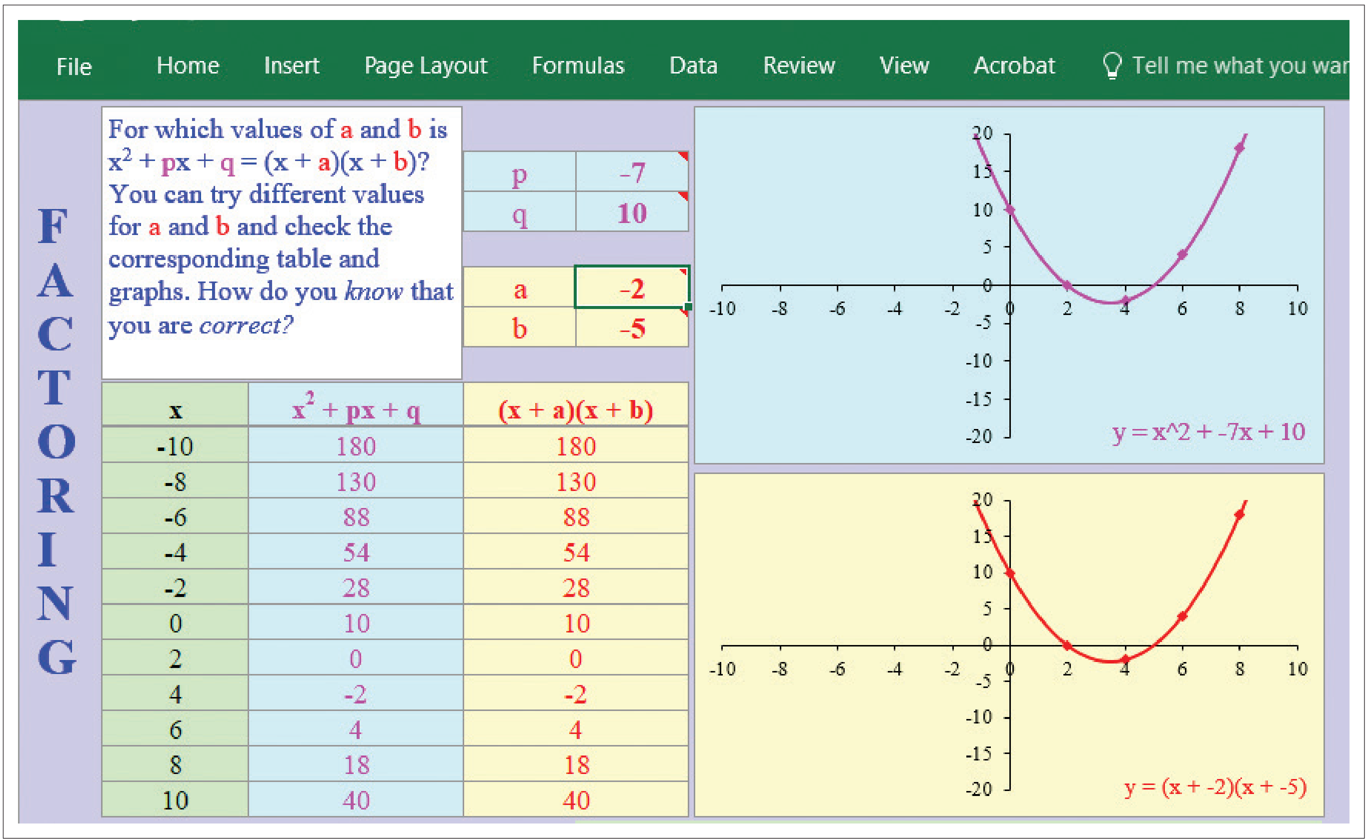



**FIGURE 2:** Screenshot of *Factoring*.

environment (Stake, 1995). Also, as PD providers and researchers, we became aware of the interlocking nature of the operative curricula in a real-life context for the participating teachers, and results from related studies. To address the full complexity of the case, we drew sources of evidence from multiple sources, namely the policy documents that outline the operative curricula, relevant PD literature as well as literature on ways variables feature in the design of the two SAPs. Our case study investigates 'a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident' (Yin, 2017, p. 18).

## Sampling

The idea for this study comes from teachers who participated in a small-scale PD initiative, on a voluntary basis. Activities with the teachers included discussions that focused on the design of different SAPs as instances of the application of ICTs (Leung, 2006). During informal and formal conversations with the teachers, they conversed about their ways of working in their schools and commented on the design of the two SAPs. The teachers work in high schools located in a low-income socioeconomic area in the Western Cape, South Africa. They did not use any ICTs in a concerted way for mathematics teaching. As the time-restricted high-stakes matric examinations approached, however, they used school computers or their laptops to display and to work through matric examinations papers (past papers) in their preparing for the examinations.

For the study, we sampled conversation excerpts from one teacher because they reflected the particularity and complexity of the case. During the conversations, this teacher referenced and compared the boundary objects, namely the POA, SAG and CAPS, with the design features of the two SAPs. In addition, she conversed about other important boundary-objects-related details such as preparing for high-stakes NSC Mathematics examinations.

## Data collection

We collected data in the form of audio-taped conversations that focused on boundary objects, namely the two SAPs, *Factoring* and *Discriminant*, which we briefly outline below.

The design of *Factoring* is based on the process-object duality of mathematical objects and the meanings of variable (Moschkovich, Schoenfeld & Arcavi, 1993; Usiskin, 1988). (See Figure 2.) The mathematical objects $(x + a)(x + b)$ and $(x^2 + px + q)$ as expressions are constructed from operational mathematical processes (Sfard, 1991). Mathematics education researchers view $a$, $b$, $p$ and $q$ as parameters (Epp, 2012). These parameters as cell-variables act as placeholders for different numerical values (see three columns on the left in Figure 2) designed in ways that aim at deepening learners' understanding of variables and parameters (Bills et al., 2006; Haspekian, 2014; Siagian et al., 2021). Different Excel affordances also make it possible to represent the factored quadratic equations:

$y = (x + a)(x + b)$ and $y = (x^2 + px + q)$ [Eqn 1]

Equation 1 is displayed as functional relationships in tabular and graphical formats (Epp, 2012) (see right-hand side of Figure 2). As we can see, through the use of cell-variables, the different literal symbols as mathematical objects become dynamic computational processes. This design breaks boundaries between factors and products or trinomials by representing them graphically.

Furthermore, the instructions in the upper left corner become key to understanding the variables or parameters in the case of *Factoring* (see Figure 2). Through inductive design heuristics, the user (learner) is asked to type in different numerical values for *a* and *b* with the goal of discovering relations between, *a, b, p* and *q* (How do you know you are correct?). The goal is to make the user discover when $x^2 + px + q = (x + a)(x + b)$ is true. This equality occurs when $p = (a + b)$ that is, the sum of the roots, and when, $q = (a \times b)$, namely the product of the roots. This 'discovery' becomes possible because of cell-variables and linked symbolic, tabular and graphical representation affordances. A variable can represent 'unknowns' that have symbolic value (Matz, 1980). The script needs to be viewed as a response to diagnostic reports on learners' struggles with the concept of a variable. The design or script reflects a UME's or designer's perspective anchored in multiple representations of 'polynomials of degree 2' or quadratic functions (Freudenthal, 1973). From a school mathematics perspective, the design breaks curricular, grade-level boundaries between factors, products, trinomials and the sum and product of roots, and associated graphs, for example. On a related point, Julie (2014) refers to 'pieces of mathematics', in the case of algebra.

Figure 2 shows a screenshot of a particular instance, namely where $p = (-2 + -5)$, that is, the sum of the roots, and, $q = (-2 \times -5)$, the product of the roots, of quadratic equations.

As before, key to understanding the design of *Discriminant* is the process-object duality of mathematical objects and meanings of variables. The policy documents note learners' challenges with interpreting the discriminant, namely $b^2 - 4ac$. As a mathematical object this discriminant also represents processes; for different or variable input values for the parameters *a, b* and *c* there will be different output values. The mathematics education research literature shows no agreement regarding the meaning of variable (Schoenfeld & Arcavi, 1988; Usiskin, 1988). The designer used Excel's cell-variable affordance, which makes it possible to vary these parameters (Epp, 2012). Typing in or 'entering' values for these parameters enables the user to interpret the effects and changes in the value of the discriminant as well as what 'the graph looks like'. In turn, these actions help with interpreting the 'nature of the zeros' or the roots of the general quadratic function, given as $y = ax^2 + bx + c$ (see Figure 2). As we can see, the symbolic and graphical connections in the design can also be used to address 'student errors or challenges in graphing polynomial functions' and the discriminant formula (Hasanah et al., 2021).

In addition, the script starting with 'consider the standard form of a quadratic function' enables interpretive flexibility with respect to the cell-variables. This script addresses policy concerns about learners' challenges with interpreting the discriminant, for example (see Figure 3). The 'how' and 'why' prompts make this script a 'technology-assisted guided discovery to support learning' and help in 'investigating the role of parameters in quadratic functions' (Göbel, 2021). The question 'What relationships do you find between the discriminant and the zeros of the graph?' shows another instance of guided discovery. Extreme instances in this boundary object (*Discriminant*) can occur when the parameters take on the values $a = 0$, $b = 0$ or $c = 0$. Here we find the null solution of $y = 0$, which amounts to the *x*-axis (Freudenthal, 1973). To orientate the reader, we show a particular instance of the script, namely the discriminant (delta) value where $a = 2$, $b = 7$, $c = 0$, the zeros or roots, and the associated graphical representation of the quadratic function (see Figure 3).

## Data analysis

Based on the case study, we used a 'constant comparative method' to analyse the data excerpts, namely the transcriptions of audio-taped recordings (Tomaszewski et al., 2020, p. 2). We noticed that during every meeting with the participating teachers, they made comparisons between the boundary-objects-related details coming from policy documents and the high-stakes NSC examinations context wherein they work, and the design features of the two SAPs. In particular, we applied the conversation analysis (CA) tool 'epistemic order' to answer the main research question (Heritage, 2009). In all conversation exchanges 'persons continually position themselves with respect to the epistemic order: what they know relative to others, what they are entitled to know, and what they are entitled to describe or communicate' (Heritage, 2009, p. 309). With reference to the transcriptions, epistemic order refers to instances where the teacher or the UME takes the conversation in the same or a different direction, informed by their respective ways of speaking and working. In the conversation excerpts the teacher conversed about different boundary-objects-related details endemic to the school. These include policy documents detailing the operative curricula, the high-stakes NSC Mathematics examinations questions on algebra with respect to variables per grade level, for example.

The case study calls for a main and embedded unit of analysis (Yin, 2009). As for the primary unit of analysis, the teacher makes no immediate references to the two SAPs. Instead, she provides details on the operative curricula with their attendant boundary objects, and what it takes to work in her school and its circumstantial or entangled conditions. As for the second or embedded unit of analysis, she converses specifically on how and where algebra and variables feature in the operative curricula and attendant boundary objects, in relation to the design of the two SAPs. We present answers to the main research question starting with conversation excerpts related to *Factoring* followed by *Discriminant*.

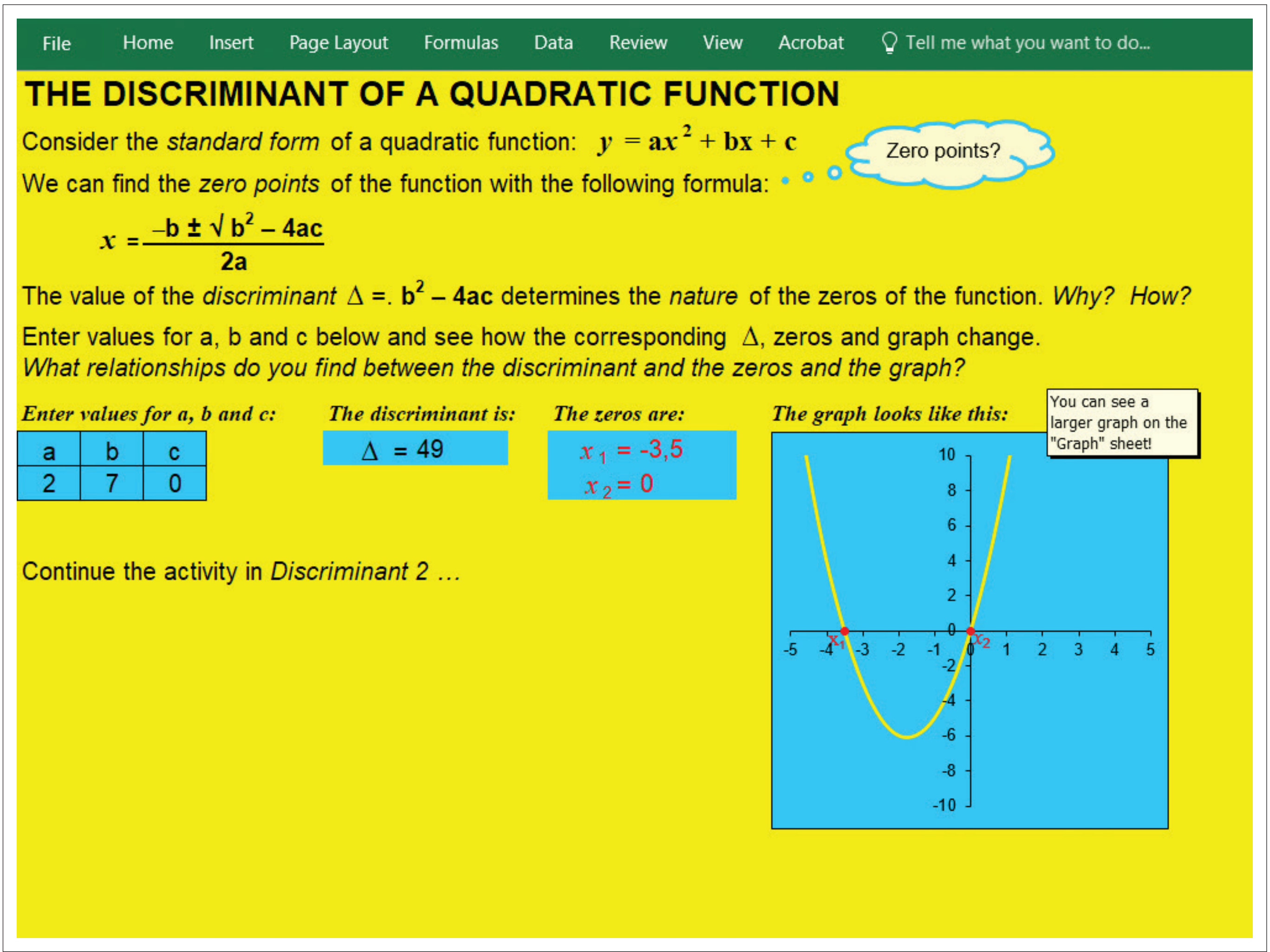



**FIGURE 3:** Screenshot of *Discriminant*.

Applying the CA analytical tool for analysing the epistemic order of the conversation turns in the transcripts enabled us to identify the evidence for the main and embedded units of analysis. Applying CA is most appropriate because, during conversational exchanges, the UME and the teacher can have different reference points, or perspectives. In general, UMEs use mathematics education research, while school mathematics teaching informs teachers' local practices. The two parties can differ in terms of awareness of boundary-objects-related details. For instance, UMEs may not be aware of the various policy documents, such as the POA, that inform the operative curricula or the teachers' examination pressure. Also, teachers may not be aware of how changing variables or parameters can transform a parabola into a straight line or linear function, for instance when $a = 0$ in the case of the parabola.

## Research framework

We used what Niss (2007) calls a research framework to answer the main research question. We used this framework because it consists of an organised network of concepts, namely the various policy boundary-objects-related to the education system and any school in general, and the two SAPs (see Table 1). The main and embedded units of analysis inform the outline of this framework. In the case of the first unit of analysis, the teacher makes no immediate references to the two SAPs. Instead, she provides details on the operative curricula with attendant boundary objects, and what it takes to work in her school and its circumstantial or entangled conditions. In the case of the second unit of analysis, she comments specifically on how and where algebra and variables feature in the operative curricula with attendant boundary objects, in relation to the design of the two SAPs. The left-hand column labelled as 'layers' denotes the subtleties of multi-layered engagements, namely the empirical situation (top row) and two interrelated 'analytical layers' (Zeiss & Groenewegen, 2009). The second row (Analytical layer 1) indicates the main unit of analysis. The third row (Analytical layer 2) indicates the embedded unit of analysis, for example the SAG on algebra and variables and the design of the SAPs. We opt for a main and an embedded unit of analysis to avoid a 'reduced analytical representation' of this teacher and her school site, as noted earlier. This means we avoided selecting conversation excerpts that focus solely on the SAPs. More interestingly, a separation between the two units of analysis becomes difficult, because during conversation the teacher can reference details outlined in layers 1 and 2.

**TABLE 1:** Research framework outlining the two data incidents.

| Empirical situation | University-based mathematics educator (UME) meets with the teacher, with the design of the two spreadsheet algebra programs serving as a focus of conversation. |
|---|---|
| Analytical layer 1: Main unit of analysis | • During these conversations the teacher provides boundary-object-related details about the operative curricula with associated boundary objects that influence and structure her local practices; these include references to the intended, implemented and examined or operative curricula as well as ways that variables feature in the operative curricula.<br>• These meetings and conversations also instantiate a boundary encounter between two discursive practices: university-based mathematics education and school mathematics teaching. |
| Analytical layer 2: Embedded unit of analysis | • Also, during conversation exchanges the teacher directly or indirectly compares and contrasts the operative curricula with the design of the two spreadsheet algebra programs with respect to variables. |

**TABLE 2:** Excerpt 1.

| Turn | Speaker | Utterance |
|---|---|---|
| 1 | UME | 'You were saying …' |
| 2 | T1 | 'The way we are basically guided by curriculum, and when the advisors come, we go to meetings etc. They will tell us, there's the paper, I want the paper in March, this is the stuff you have to teach and then we set up our POA, call the parents to the office …' |
| 3 | UME | 'What is a POA?' |
| 4 | T1 | 'Programme of assessment. We set it up according to those lines; at the start of the year we will sit down, teachers will have a meeting and we'll say okay we have two assessments for the quarter and a March examination. What do we teach? Now we open the SAG and the SAG will state, no graphs, it is multiplication, products, it is factorisation. Then we do the factorisation, that is $x^2 + 10x + 25$. We'll do it but as a separate entity and then next quarter we'll sit down for a programme of assessment meeting. Then we get … ideas. So it is done, the kids see it as: first quarter work, factorisation; second quarter, parabola. What I see now is the real connection between … we can actually use this [*points to the SAP*]. So, I can set up a tutorial now for quarter one; doing your programmes, multiplication and your graphs all in one without them having knowledge, inductively without them having knowledge of graphs. After just teaching the graphs, I can now do that. I can do it for you; I will show you that … we can do this. There is your multiplication, there is your factorisation. Let's say $y$ equal to … and put it in a table and do a plotting, just plotting and then describing the behaviour of whatever this says. Don't tell them what it is, it's just joining of points etcetera; describe the points, in your own words …' |
| 5 | UME | 'You said something about what I was doing was reinterpreting the SAG, what does that mean?' |
| 6 | T1 | 'The SAG has certain guidelines that state what we have to do in a specific way etc. Very seldom do they say that we like, you saying connecting or taking three things, your variables, your numerical values and your sketch and doing it as a one completed lesson. What they have is almost like an apart session only for multiplication. That's a concept that they need to understand. They will say, the learner must be able to, the learner must be able to … that's what the SAG states. The learner must be able to …, the learner must be able to … They have been taught that way, but what I see now is, what I can do, is, you can give your products then you must be able to give you your whatever …' |
| 7 | UME | 'Factorise.' |
| 8 | T1 | '… and connect it to the graphs all in one etc. and connect it inductively; they will see, hopefully they will see the picture emerging between numerical values and the things over there [*pointing to 'Factoring'*].' |
| 9 | UME | 'Okay.' |
| 10 | T1 | 'That graph will go well with the projector. I am not really *au fait* with using technology yet. I am going for whiteboard training now, the second, third and the fourth of October with the department.' |

T1, teacher; UME, university-based mathematics educator.

## Results and discussion

In each case, there is a table with three columns labelled: turns, speaker (T1 for 'teacher' and UME for 'university-based mathematics educator') and utterance. We use the acronym UME to emphasise the distance and boundary encounter between the university and the school.

### Conversation excerpts related to *Factoring*

Excerpt 1 (see Table 2) contains evidence related to the main unit of analysis. The intended curriculum, spelled out in policy documents, guides the teacher's local practices and that of her colleagues. We note this from the change in epistemic order between Turns 1 and 2, where she notes, 'we are basically guided by curriculum'. As boundary objects, the POA and SAG provide content details for each school subject as the academic year progresses, which includes algebra (see Turns 1, 2 and 3). The POA and SAG details indicate the intended as well as the assessed or examined curricula. This should be noticed from the A in POA and in SAG. At the beginning of the school year, she attends meetings organised by the curriculum or subject advisor. During these meetings the POA and SAG become boundary objects between these advisors, teachers and parents, and thus a means of communication between the school and the district office (see Turn 4). The subject advisor's visits to schools thus aim at helping teachers interpret and implement these details outlined in these boundary objects. From here we should note that teachers have professional obligations to the stakeholders of school mathematics, namely learners, the principal and parents, for example. Subject advisor visits aim at fostering coherence in teachers' classrooms. For example, she notes 'we set up our POA'. Interestingly, the epistemic order in Turn 3 shows that the UME was not familiar with the 'POA' per se, as a boundary object between the stakeholders, namely teachers, parents and subject advisors. Turn 3 thus signals a boundary encounter where the UME was ignorant of details about the teacher's local practice.

Excerpt 1 (see Table 2) also contains evidence of the embedded unit of analysis. The teacher provides details about how the SAG 'separates' (separate entity) understandings of algebra with respect to variables' (see Turn 4). The SAG provides details on 'subject assessment guidelines' for 'first quarter' and 'second quarter' work. She notes 'no graphs, it's multiplication, products, it is factorisation', for the 'March examinations'. We interpret 'multiplication' as referring to the procedure for finding 'products,' that is, trinomials. Her words point to 'pieces of mathematics' in the case of algebra. For example, during the first quarter she hones her learners' skills with respect to finding products and factors of trinomials. During the second quarter the 'products' represent the 'parabola' (see Turn 4). As for the first quarter, the variables ($x$'s) represent 'unknowns' that have symbolic value (Matz, 1980), that is, the variables serve as placeholders for numerical values. Here the numerical and literal symbols present in factors and products (trinomials) represent mathematical objects. She then comments on how cell-variables capabilities of *Factoring* and its tabular and graphical affordances make it possible to produce tabular and related graphical representations of the SAG 'factors' and 'products'. These representations correspond to her words: 'put it in a table and do a plotting' (see Turn 4). Also, she notes how the mathematical objects – factors and products – become placeholders for 'your variables, your numerical values and your sketch' (see Turn 6).

The epistemic order in Turn 5 merits attention in terms of embedded unit of analysis. The teacher notes that the design of *Factoring* with respect to variables amounts to 'reinterpreting the SAG'. As its name indicates, the SAG (subject assessment guidelines) contains 'certain guidelines'. She further notes that it says nothing about 'connecting or taking three things, your variables, your numerical values and your sketch and doing it as a one completed lesson' (see Turn 6). In Turn 8 especially, she elaborates how this 'reinterpretation' can become possible ('and connect it to the graphs all in one') to the point where her learners 'will see the picture emerging between numerical values and the things over there' [*pointing* to *Factoring*]. Put differently, she notices *Factoring's* boundary-breaking design features, namely its linked numerical, symbolic, tabular and graphical representations. As we noted earlier, Excel's cell-variables, together with its tabular and graphical design features, make these different representations possible (see Turn 8). The teacher recognises and notices this design as one that is structurally arranged in which the sections of algebra come together (see Turn 4: 'connect it to the graphs all in one') and become relevant to her local practices. We also note a circumstantial condition she contends with at her school, namely about going for 'whiteboard training' in October. This training can eventually help with representing 'the graph' which involves a placeholder view of variables (see Turn 10), and thus breaking boundaries in the operative curricula in the case of algebra. From the main and embedded units of analysis, we should note how the teacher compares and wants to align the design of *Factoring* with the operative curricula she contends with in terms of her local practices.

From the two units of analysis, we should notice how the teacher converses about a coherence she sees between two different boundary objects, namely the POA and SAG on the one hand and *Factoring* on the other hand. As policy documents, POA and SAG contain and provide an outline of implementation and assessment details. She is therefore concerned with a consistency between what she needs to teach and assess in algebra in her classroom, and the design of *Factoring*, in particular.

In Excerpt 2 (see Table 3) we find evidence of the main unit of analysis. The teacher comments on two boundary objects – textbooks and examination papers ('any exam paper') – which point to the intended, implemented and examined curricula. Textbooks 'carry' information about the intended and implemented curricula, whereas 'exam papers' incorporate details about the examined curriculum. As noted earlier, the contents of 'any exam paper' operationalise significant components of the implemented curriculum, that is, what teachers do in their classrooms. As texts, these

**TABLE 3:** Excerpt 2.

| Turn | Speaker | Utterance |
|---|---|---|
| 1 | T1 | 'In any textbook, if you go to any textbook, even an exam paper. I am busy moderating at the moment. In any exam paper, we have separate chapters in books, where they treat products separate from factors, factorisation. They would do products as a separate entity, factors as a separate entity. Then they'll go over to equations, different types of equations, then they'll do something on the function. There is never an integrated approach, where they do everything together. As teachers, we need to make that connection known to learners.' |
| 2 | UME | Making algebra work: Instructional strategies that deepen 'Why?' |
| 3 | T1 | 'Because ultimately, if you can understand that the graph is actually a visual of a function on a table or a visual of an algebraic, given in rubric form, they have to select their own input values. They can actually put everything together, because then they have the picture in terms of the graph.' |
| 4 | UME | 'Can I say something? You said that the learners will bring everything together. Now that can only happen, I would say, on the encouragement of teachers. Do you agree?' |
| 5 | T1 | 'Yes, or otherwise you must have a directed worksheet for them or a work programme for them like this that will lead them into that.'<br>'Then they can make their own deductions or inductions for that matter.' |
| 6 | UME | 'You said directed worksheets; are there particular worksheets that you have that you have designed in the past where you have such an integration, to use your words, or …' |
| 7 | T1 | 'You mean like the connection here?' [*pointing*] |
| 8 | UME | 'Yes.' |
| 9 | T1 | 'We normally do this. We have the given function, $f(x) = y$ equal to the table. But then the only thing, the only technology we use at the moment is the Casio, the Casio calculator. It has this operation where you can actually do a table. You can do a table on the calculator. You get your input values, starting point and your end point.' |
| 10 | UME | 'Right.' |
| 11 | T1 | 'Your parameters, it can quickly give you your coordinates and from there they quickly do the plotting. At the end, I will give them the shortcut.' |
| 12 | UME | 'Why?' |
| 13 | T1 | 'Because I know, the pressure of the exams doesn't allow for them to more or less work out a table. Some of them are slow actually. Some of them don't have the necessary capacity in terms of technology. They don't have a calculator.' |

T1, teacher; UME, university-based mathematics educator.

two objects are used to coordinate activities in schools on a school, district, provincial and even national basis (see Turn 1). It is difficult to think of schools without textbooks. It would be out of bounds for teachers not to mention textbooks or not to reference examination papers, that is, the content of the examined or assessed curriculum. This teacher signals her awareness of the time-restricted, high-stakes nature of examinations ('the pressure of the exams') and therefore mentions giving her learners 'the shortcut.' (see Turns 11 and 13). Like teachers in Hong Kong and Melbourne, Australia, she faces examination pressure and thus refers to a kind of examination-driven mathematics teaching.

In Excerpt 2 (see Table 3) we also find evidence of the embedded unit of analysis. Here the teacher compares the ways that the meanings of variables vary in the operative curricula, although she does not directly mention the variables. She notes the boundaries between products and factors ('separate chapters', 'separate entity') and the operative curricula. When her learners find products and factors, the variables do not necessarily reflect graphical or tabular representations. In the operative curricula or paperpencil environment, the variables represented in binomials or trinomials do not function as placeholders for the set of real numbers, say. At the Grade 8 level, her learners simply find factors and products. Furthermore, she comments on how the design of *Factoring* breaks these boundaries between factors, products, 'the graph,' 'visual of a function' and a 'table' (Turn 3). She notes a connection between the Casio calculator, with its design feature of a 'table' and *Factoring*'s table (see Turn 9). Evidence for this observation on boundary breaking is her reference to 'an integrated approach'. She thus proposes to design 'directed worksheets' and a 'work programme', which would align with *Factoring*'s 'integrated approach.' Also, in Excerpt 1 (see Table 2) she proposes a 'tutorial' wherein her learners can 'inductively' experience, and have her learners make their 'own deductions or inductions for that matter'. Key to this kind of worksheet design would be the meaning of variables ranging from indeterminate objects to placeholders. Evidently, she sees and wants to make *Factoring* an ecologically relevant resource for the operative curricula that she contends with in her local practices.

As in the case of Excerpt 1 (see Table 2), the teacher sees value in the design of *Factoring*. First, this design brings together tabular and graphical representations of variables in the case of the parabola. In the implemented curriculum, for example the textbooks she uses, these representations are not considered at the same time. The representations appear in separate chapters. Second, she finds these multiple representations helpful when it comes to the examined curriculum, for example time-restricted examinations. Hence, she mentions providing her learners with a shortcut, when answering the related examination questions.

## Conversation excerpts related to *Discriminant*

In Excerpt 3 (see Table 4) we find evidence of the main unit of analysis. The teacher looks at the design of *Discriminant* by keeping an eye on the intended curriculum, namely the 'new CAPS' (see Turns 1 to 4). This policy boundary object outlines mathematics content, which she cannot ignore in her teaching. She is thus noting a change in circumstantial conditions in Grades 10 and 11 and 'the new question' related to 'your discriminant' that her learners 'will need to know' (see Turn 4). In other words, she is also pointing to particular content of the examined curriculum in these grade levels. As we can see, her utterances intimate interactions of curricular variations between the intended and examined curricula (see Turn 4).

In Excerpt 3 (see Table 4) we also find evidence of the embedded unit of analysis. The teacher makes connections between the design of *Discriminant* with respect to parameters or variables and 'the matric paper' of 'last year' (see Turn 8). The script prompts the user to 'enter' or type in different integer values or signed numbers for the parameters $a$, $b$, and $c$, and to then comment on simultaneous

**TABLE 4:** Excerpt 3.

| Turn | Speaker | Utterance |
|---|---|---|
| 1 | UME | 'I want you to look at the screen here … and just tell me what you are looking at? Anyone. You have the heading there: 'The discriminant … quadratic function …' |
| 2 | T1 | 'Basically, it's information concerning the type of function being quadratic and then also the means of the method of finding the $x$ value, the roots of that function also identifying the idea that these things or points is known as your discriminant [*pointing to the screen*] and we can therefore find or discuss the nature of the roots. We just are coming back with the new CAPS now.' |
| 3 | UME | 'Is this the new CAPS?' |
| 4 | T1 | 'Yes, this is the new CAPS. Next year when they go to Grade 11, this is the new question that they will need to know.' |
| 5 | UME | 'Now below on the screen there you've got a table there for $a$, $b$, $c$, then you've got delta and then you have the red parts which shows you $x$ 1 and $x$ 2, then you've got this graph.' |
| 6 | T1 | 'Smart.' |
| 7 | UME | 'Why do you say it is smart?' |
| 8 | T1 | 'Because, again you can clearly see the bridge between the numerical values $a$, $b$ and $c$ and the contact or the bridge between them … the visual is important over here. The visual aspect over here … if your discriminant is 41 the learner can pick up the discriminant found under the square root of 41. This is also important. I saw last year in the matric paper that the question there they didn't give the numerical values of $a$, $b$ and $c$; they give them where $a$ is positive, $b$ is positive and $c$ is negative like that. They had to give the shape of the graph. This program will assist them … it will assist them.' |

T1, teacher; UME, university-based mathematics educator.

graphical effect changes ('the shape of the graph') (see Figure 3). Excel's cell-variable affordance makes it possible to vary these parameters. We say 'integer values' because the teacher noted that the high-stakes NSC Mathematics (matric) question required her learners to make observations on the signs of these parameters (see Turn 8). Diagnostic reports point out learners' struggles with the visual syntax of the discriminant and interpreting its numerical meaning and associated graphical meaning. The design of *Discriminant*, in other words, 'will assist' her learners with answering such questions on interpreting the discriminant.

As before, we need to note the teacher's comments on how the design of *Discriminant* fosters a coherence between policy-related boundary objects, for example the intended curriculum ('new CAPS'), as well as the examined curriculum (matric paper), that is, the examinations content.

In Excerpt 4 (see Table 5) we also find evidence of the main as well as the embedded unit of analysis. The teacher refers to using 'lead questions' based on the script for *Discriminant* (see Figure 3). She notes how this script might help learners with 'discussing the nature of the roots' (see Turn 1). As a topic, the latter appears in the 'new CAPS', a policy-related boundary object. She therefore envisions the *Discriminant* – a different type of boundary object – serving as a 'resource' for the intended curriculum, spelled out in the policy boundary object. More interestingly, studying the 'nature of the roots' requires varying the parabola's parameters, namely, $a$, $b$ and $c$ (see Turn 2 in Excerpt 3 [see Table 4], as well as Turns 4 and 6 in Excerpt 4 [see Table 5]). In Turns 7 and 9, for example, the UME breaks a boundary by taking the instance where $a = 0$,

**TABLE 5:** Excerpt 4.

| Turn | Speaker | Utterance |
|---|---|---|
| 1 | UME | 'I think this is the second time I hear you mention lead questions. What is it you want to lead them to?' |
| 2 | T1 | 'You see I have certain objectives … I have certain objectives.' |
| 3 | UME | 'Which are?' |
| 4 | T1 | 'If I teach the parabola, right, without giving them any numerical values, right, and only give them the signs of *a,* the signs of *b,* the signs of *c*.' |
| 5 | UME | 'By that you mean?' |
| 6 | T1 | 'The positive or negative sign of *a*, greater than zero, less than zero, *b* greater than zero, less than zero, and *c* greater than zero, less than zero.' |
| 7 | UME | 'What about the zeros? I am thinking of the case where *a* is equal to zero, *b* is equal to zero and *c* is equal to zero, or not?' |
| 8 | T1 | 'Yes, that can also be included as a lead.' |
| 9 | UME | 'Let me get to the instance if *a* is equal to zero, *b* is equal to zero and *c* is equal to zero. I notice you don't agree with me.' |
| 10 | T1 | 'Because then we are moving away from the fact that it is a parabola, a parabolic function. It won't have two roots, it won't have the characteristics of a parabola.' |

T1, teacher; UME, university-based mathematics educator.

**TABLE 6:** Excerpt 5.

| Turn | Speaker | Utterance |
|---|---|---|
| 1 | UME | 'I want to get back to your use of drilling. Drilling has been used in particular ways and sometimes it has negative meanings. Is that how you …' |
| 2 | T1 | 'Obviously drilling has a positive meaning.' |
| 3 | UME | 'Say a little bit more about that. |
| 4 | T1 | 'Ask them whether they understand, they will say yes, and when it comes to an exam. But then ask them the next day, then are not that sure.'<br>'But then as a teacher to make sure that you've consolidated that particular topic.'<br>'The other way is to make sure that you as a teacher have consolidated that topic is, you should drill them.'<br>'For me it's very important for the, ultimately, for me it's very important for them to make their own observations, even in class, I want them to confirm the answers, because many learners think differently. If you ask them, if you allow them that opportunity to …'<br>'If you allow them that opportunity to confirm an answer, you give him that confidence.' |
| 5 | UME | 'So, is that how you use drilling? Namely?' |
| 6 | T1 | 'To consolidate, to consolidate certain topics, especially in your teaching. You see our whole education system is based on that, it's based on preparing learners for examinations.' |

T1, teacher; UME, university-based mathematics educator.

$b = 0$ and $c = 0$. This leads to the null function, $y = 0$. In the school's operative curricula boundaries between a parabola, a straight line and the null function are rigid. By typing in $a = 0$, the parabola can be changed to a polynomial of degree one, namely a straight line. The teacher prefers her 'lead questions' to focus on varying the signs of the discriminant's parameters ($a$, $b$, $c$) (see Turn 1).

We interpret 'signs' to mean different integer values, which are needed to compute the discriminant and to decide on the nature of the roots. She keeps the content of the examined curriculum in mind or in view (see Excerpt 1). The UME asks her to be specific about her lead questions (see the epistemic order in Turns 3 and 5). The UME continues to ask her to consider boundary instances, that is, where the parameters as placeholder variables assume the values of zero (see the epistemic order in Turns 7 and 9). She makes clear that as for her local practices, the resulting changes of the parameters will not display the 'characteristics of a parabola' or a 'parabolic function' (see Turn 10). As we can see, in the ecology of studying and teaching the mathematical object – a parabolic function, in this case – she keeps the content of the examined curriculum in view (see Turn 4 in Excerpt 3 [see Table 4]).

Excerpt 5 (see Table 6) shows further evidence of the main unit of analysis. The teacher notes boundary-object-related details about her local practices, namely making use of 'drilling' 'when it comes to an 'exam.' Examinations ('exam') impose an ordering effect, namely they occur during set times. She and her colleagues therefore need to 'consolidate certain topics'. Examinations entail texts in the form of question papers. The latter become boundary objects, that is, a means of communication between learners, teachers, the principal and parents, for instance. 'Exam' details are specified in the intended policy documents, for example different question types. It would be out of bounds, that is, not doing her work, for her not to prepare or 'drill' for an 'exam,' namely the content of the examined curriculum. She notes the education system's examination-driven ecology. She contends with the high-stakes NSC (matric) examinations every year, and the associated pathologies that group schools into halls of fame and halls of shame, in annual newspaper reports. Policy-related boundary objects such as CAPS (DBE, 2011) provide details on the intended and examined curricula, which include examinations. She is aware of examination pressure, that is, the impact of high-stakes testing on teaching and learning on 'our whole education system' (see Turn 6). Hence, she refers to the need for 'drilling'. The epistemic order in Turns 1 and 3 shows her elaborating 'drilling'. For example, she wants her learners to 'make their own observations' and allow them an opportunity 'to confirm an answer'. These details need to be viewed in the light of her earlier comments on having her learners 'inductively' connect products, factors and 'graphs' (see Excerpt 1 on *Factoring* [see Table 2]). Clearly these comments point to an intention to deepen learners' ways of knowing with respect to the behaviour of the discriminant and its associated parameters. In addition, she outlines specific ways of 'consolidating' for the 'exam' and 'preparing learners for examinations', a key feature of the education system's ecology (see Turns 4 and 6). Here UMEs should note, for purposes of ecological relevance, how the teacher wants to align *Discriminant's* design features with the intended, implemented and examined curricula in her school. Her comments align with what Julie and colleagues call examination-driven teaching.

## Conclusion

Answers to the main research question – What boundary-objects-related details about the teacher's local practices emerge during conversations that focus on the design of SAPs based on research related to variables? – offer ways for UMEs involved in PD to understand their work better. Concerning the main or primary unit of analysis, we gain insights into ways the teacher speaks about her local practices in a challenging socioeconomic school environment. In this regard, she keeps in view different boundary objects integral to the school's operations, namely the different policy documents that outline the intended, implemented, interpreted and examined curricula. These entangled curricula form what we called the operative curricula. More importantly, she works in a high-stakes examinations environment where examination pressure, for example examination contents, counts. Hence, she makes it clear that she keeps the examinations firmly in mind, for instance the contents of the examination papers in the high-stakes NSC (matric) Mathematics examinations. The reality of the high-stakes examinations reflects the teacher's experiential world, which has parallels, in the case of Hong Kong and the United Kingdom, for example. This examination pressure reality, in turn, should help UMEs to better understand what it takes to cross boundaries to school when they do PD work. In this regard, we ask: Can UMEs afford a reduced analytical representation of teachers? In schools, teachers contend with boundary objects.

In the case of the embedded unit of analysis, UMEs need to note the following. The teacher mentions how the features of the *Discriminant*'s cell-variables can help her learners understand questions related to interpreting the *Discriminant*. This design of the SAP aligns with the content of the examined and the implemented curricula. Also, she articulates how *Factoring* can help reinterpret the policy boundary object – SAG. Currently, teachers face similar reinterpretation issues when it comes to the ATPs. Current boundary-objects-related details include outlines in the ATPs. Therefore, teaching with technology means keeping the ATPs in mind. In particular, she values the boundary-breaking design affordances that connect factors, products and their related graphs. Such design affordances reflect an instance of the two SAPs becoming recognisable boundary objects and, hence, as possible resources. For example, she refers to using a work programme or directed worksheets and lead questions, triggered by the design of the two SAPs.

In conclusion, from these topic boundary-objects-related details, we should note that the teacher aims at coordinating and aligning different boundary objects such as policy document details, which impact on her local practices. She works in a school located in a low-income socioeconomic environment, combined with the reality of the high-stakes NSC Mathematics examinations and associated examination pressure. As noted before, high-stakes examinations are not an uncontested area but have become a permanent and vital part of education. Such entangled and circumstantial conditions should signal to UMEs what is at stake for teachers as stakeholders in the education system. Boundary encounters between SAPs – or other types of ICTs, for that matter – and teachers, provide UMEs as PD providers and knowledge brokers with opportunities to improve their work with teachers' functioning milieu of their schools and classrooms with their varying demands.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## Authors' contributions

F.G. is the principal investigator, W.D. and A.A are collaborators. F.G. was responsible for data collection and analysis. The two collaborators contributed to the writing process.

## Ethical considerations

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## Data availability

Data sharing is not applicable to this article as no new data were created or analysed in this study.

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