# Development of fuzzy rule-based systems for industrial flotation plants by use of inductive techniques and genetic algorithms

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### Synopsis

Control of flotation processes is mostly managed by plant operators, who assess the performance of the plant based on their own experience and other heuristic rules. These rules tend to be subjective or ill-defined, since most of them are concerned with the structure of the flotation froth, such as colour, bubble size and shape distributions, froth mobility and froth stability. These phenomena are very difficult to quantify objectively, and inexperience on the part of the operators, human error, etc., can lead to significant inefficiencies in plant operation. In this paper the development of a fuzzy system to support the control decisions of plant operators is described, which leads to significantly smoother control action and more stable plant operation than could be obtained with crisp sets of rules or manual control strategies.

Keywords: flotation, fuzzy logic, image analysis, genetic algorithms, rule induction

## Background

Flotation processes are difficult to model at a fundamental level and at present automatic monitoring and control of industrial plants have met with limited success. In practice these processes are most often controlled by human operators who tend to assess the performance of the plant based on their own experience and other heuristic rules. As a result plants are usually not controlled optimally, owing to lack of experience on the part of the operators, human error, etc. Indeed, considerable variation is sometimes observed between different shifts, or during different times of the day. These operational instabilities are considered to play a significant role in the cost-effective operation of flotation plants.

For these reasons, some attempts at the development of decision support systems that would aid the operator controlling the plant have recently been made[1,2]. Although these systems have met with varying degrees of success, it was only with the advent of on-line sensors such as those based on digital image analysis, that effective data-driven development of expert systems could be initiated. For example, Moolman *et al.*[3] have shown that by making use of computer vision systems, it is possible to characterize the performance of flotation plants with a high degree of accuracy. Other investigations have since followed[4–7] and several commercial systems have been developed in the last few years. These systems typically make use of features extracted from digitized images of the structure of the flotation froths. Although these features are representative of the behaviour of the process, the data still have to be interpreted by an expert prior to incorporation into a decision support system. Unfortunately experts such as these are in short supply and even when available, it is doubtful whether they would be able to interpret all the subtle nuances that complex plants can exhibit. Moreover, different experts also exhibit cognitive biases such as over-confidence and over-simplification[8,9]. It could therefore take months, if not years, to develop an accurate, comprehensive expert system for a flotation plant and could require a large capital investment.

This paper describes the design of a fuzzy logic decision support system for use in controlling industrial flotation plants. Rules obtained through probabilistic induction, using both froth features and physical plant parameters as input attributes, and grade of the floated minerals as the classification output, are fuzzified and incorporated into a fuzzy expert system shell, *Fuzzy-CLIPS*, together with heuristic rules obtained from a flotation domain expert. A description of the induction algorithm, used to obtain the classification rules, is given. The methodology of rule fuzzification and optimization is discussed. Thereafter the techniques are

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applied to a set of industrial flotation data and the results are considered in terms of classification accuracy, complexity and suitability for decision-making on an industrial flotation plant.

## Methodology of fuzzy system development

The general approach towards the development of the hybrid inductive fuzzy inference system is outlined in Figure 1. As indicated in this Figure, the first step of the algorithm is to generate a classification tree that partitions the input variable or feature space. The C4.5 induction algorithm[10] was used for this purpose. The output of the classification tree and rule generation algorithms is a set of explicit, but not mutually exclusive, IF-THEN rules, which provide a near-optimal partitioning of the feature space[10]. The structure of the decision tree is subsequently used as a basis for the derivation of fuzzy rules, after which the membership functions of the fuzzified rules are optimized. Finally the set of fuzzy rules, as well as rules obtained from a domain expert, are integrated into a fuzzy expert system shell, as described in more detail below.

### *Partitioning of the feature space through induction*

The partitioning of the feature space is based on the principles of information entropy. In particular, generalized rules are derived from the data, which link the features, or independent variables, of a set of exemplars to the predefined classes or dependent variables of each of the exemplars.

The commercially available C4.5 algorithm is descended from the ID3 induction algorithm[11] and is based on an information gain criterion. C4.5 uses an improved version of this criterion, described below, to recursively partition a set of exemplars to form a decision tree. A decision tree is composed of branches and nodes, the latter being either terminal or decision nodes. Branches represent connections between nodes, connecting antecedent nodes to descendent nodes when moving from top to bottom in the tree and descendent nodes to antecedent nodes when moving from bottom to top. Terminal nodes have no descendents and represent all of the possible solutions that can be derived from the tree. Each decision node represents a question which when decided determines the appropriate branch to follow. The node at the top of the tree has no antecedent node and is called the root node.

The data used to construct decision trees (for partitioning of the feature space) typically consists of examples of the behaviour of the process, each comprised of a number of features (the independent variables) and a class (the dependent variable). This classification approach is in accordance with the way industrial plants are currently being controlled by human operators. Plant personnel tend to interpret the process in terms of several classes (normal, abnormal, etc.), rather than attempting to quantify the performance of the plant.

The decision tree can be converted into disjunctive normal form (DNF) rules[12] with little effort, where each class is specified by a disjunctive set of production rules. One such rule can be obtained for each leaf in the decision tree. The conjunction of all the branching attributes leading to a leaf in the tree gives a description of the examples in the particular branch of the tree. The number of levels, or depth, of the induced tree is determined by the maximum number of conjunctive conditions allowed in a rule, as specified by the user.

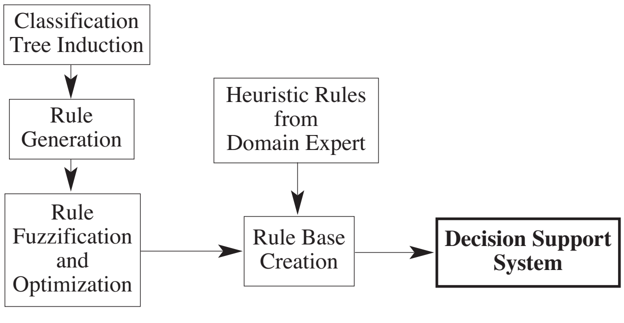


**Figure 1–Methodology for the development of the fuzzy decision support systems**

### *Fuzzification of crisp rules and integration with other heuristics*

In the next step the rules inferred by C4.5 are fuzzified. Only the structure of the decision tree is used and the actual split values (on which the tree splits the data into N subsets) are ignored. C4.5 is therefore used to determine the structure of the training data, so that more densely data populated regions are defined in finer detail than those regions where fewer data reside.

The derivation of fuzzy rules associated with the partitions in the feature space (nodes of the decision tree) can be explained by way of example as follows: Consider a crisp rule, obtained with C4.5, with the structure IF $A < X$ and $B > Y$ THEN Class = *High*. If a given feature or attribute, say A, only occurs once in the antecedent of the rule, A is split into two groups, *High* and *Low*, each with its accompanying membership function, with *High* representing the upper range of the given attribute and *Low* the lower range of the feature. The split-value X is ignored. If A occurs twice, three membership functions are created, *Low, Medium* and *High*, with *Medium* representing the intermediate partition of the feature's possible range.

In general, if a feature occurs N times in the antecedent of a rule, N+1 membership functions are formed. Similar operations are performed on the consequents of the rules. The output of exemplars with M possible output classes was partitioned into M membership functions.

### *Selection of fuzzy membership functions*

In mineral processes operated in steady state, the observed data are assumed to have a Gaussian distribution around the set point. Accordingly Gaussian membership functions were used to represent the distribution of data around the set point, while sigmoidal membership functions were used to model control action.

$$f(z) = 1/\{1 + exp(-a(z-c))\} \qquad [6]$$

Sigmoidal membership functions have only two parameters that need to be optimized, viz. $a$ and $c$ as shown in Equation [6]. This is in contrast to other types of membership functions, such as triangular ones, which are computationally more expensive, and which require the specification of more than 2 parameters. Although the choice of membership functions such as those above does not appear to be an important issue, it remains to be investigated more systematically.

If the process were to deviate from the given set point, the control action would move it back to its selected set point. The rate of the movement back to the set point would depend on the extent of the process deviation.

### *Optimization of membership functions and weights associated with rules*

The positions and shapes of the previously defined membership functions are subsequently optimized, as well as the weights (indicative of the relative importance of each rule) of the entire set of fuzzy rules. In this case, the optimization procedure used was a differential evolution (DE) algorithm developed by Storn and Price[13].

The algorithm is comprised of a heuristic approach that can accommodate non-linear and non-differentiable continuous response surfaces and performs least squares optimization of the membership functions and rule weights, based on the original training data, combined with a genetic algorithm.

The individuals in the population are based on real-valued representations of so-called parameter vectors. These are the parameters of the given objective function that must be optimized. The principal difference between this algorithm and standard genetic algorithms is that instead of random mutation of the population members, it uses the difference of two randomly selected population members (parameter vectors) as the source of the random variations of an individual. This leads to a directed component in the evolutionary search of the variable space. Table I shows the parameter values used for the differential evolution algorithm.

The rest of the fuzzy inference system uses standard concepts to determine the system output. Fuzzy conjunction is accomplished by taking the minimum of the conjunctives, disjunction by taking the maximum of the disjunctives. Correlation-minimum inference is used to determine the membership value of the consequent of a given rule and aggregation of the rules is applied by taking the maximum membership value of the rule consequents. The centroid method is used for defuzzification.

### *Acquisition of heuristic rules from human experts*

Other heuristic rules that are not necessarily reflected in the process data can consequently be incorporated. These rules are usually related to process constraints pertaining to the operational limits of equipment, HAZOP analyses, aspects of the systems that cannot be measured, etc., and are not changed as frequently as the induced rules that track the dynamics of the process itself. The fuzzy inference system is thus designed to generate coherent output over the entire feature space.

*Table I*

**DE algorithm parameters**

| | |
|---|---|
| Number of individuals in a population | 80 |
| Number of generations trained | 120 |
| DE step size: F | 0.5 |
| Crossover probability constant (CR): | 0.7 |

Once a satisfactory set of fuzzy rules has been derived from the process data and other sources, all the rules can be incorporated into a suitable expert system shell. This fuzzy decision support system can consequently be used to assist operators controlling flotation plants, while changing process conditions can be accommodated with relatively little cost by periodically updating the set of process rules.

## Fuzzy decision support system

The fuzzy decision support system runs on top of a machine vision system typically monitoring the flotation froth in the second cell in a flotation bank. The first cell is usually not monitored, since conditioning of the ore in the first cell tends to be incomplete, and the structure of the surface froth is therefore not representative of process deviations. Most of the valuable mineral in the bank is floated in the first few cells, so that stabilization of the process at an early stage tends to stabilize the system in its entirety.

When the monitoring system is in use, images are first acquired by the video camera or CCD system, followed by so-called grey level processing of the images. Various features are subsequently extracted from the digitized images, characterizing among others the structure of the froth, as well as the froth dynamics, such as mobility and stability. These features constitute the raw process data from which process rules can be induced and integrated with the knowledge-based system

## Experimental data

Data were obtained from an industrial platinum group metals (PGM) flotation plant and the optimization problem was to find the decision tree or trees (i.e. a set of heuristic rules) that would be best able to classify the appearance of the froths, given a set of froth features. In the data obtained from an industrial platinum group metals (PGM) flotation plant, only three classes of froth structures were identified, which were associated with the grade of the PGMs in the fourth cell in the flotation bank. The grades were classified as ***high***, *medium* or *low*. The fourth cell was chosen by a domain expert as the best indicator of PGM grade.

Videographic images of the froths in a single flotation cell were obtained and then reduced to a set of compact statistical image variables or froth features with the use of the neighbouring grey level dependence matrix (NGLDM) method, originally developed by Sun and Wee[14] and adapted for use on flotation plants by Moolman, *et al.*[15]. Although a variety of image variables can be derived by use of the NGLDM method, only four were used in this analysis, namely a so-called small number emphasis feature (SNE), entropy feature (ENT), the average grey level of the image (AGL) and the instability of the froths (INSTAB) derived from several images captured in rapid succession at specified intervals. The scores of the first two principal components of these four image variables are shown in Figure 2. Colective, these two principal components accounted for more than 93% of the variance in the image features.

Other plant variables were also measured in addition to

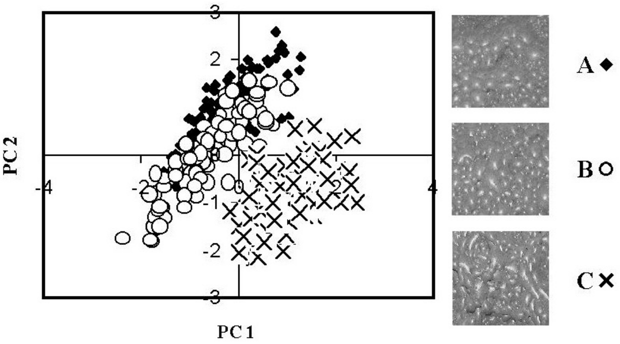


Figure 2—Principal component map of the platinum froth data set and froth structures with 60.0% and 33.8% of the variance explained by the first (PC1) and second principal component (PC2) respectively

these froth features, namely the pulp density (SG), the copper sulphate addition rate ($CuSO_4$) and wet ore feed rate to the flotation circuit (T/h_wet). Based on $\chi^2$ significance tests, these five image variables were found to have the most significant influence on the classification capability of the techniques considered. The $\chi^2$ tests were performed by grouping the values of a given variable into two categories (branches) so as to give the highest $\chi^2$ value possible when the two branches were used to classify the froth appearance. The instability was a measure of the fluctuation in the patterns of grey scales in the image in a known time interval, owing to the formation and bursting of bubbles on the froth surface.

## Discussion of results

### *Accuracy and classification*

As can be seen from the results in Figure 3, the inductive algorithm was capable of classifying the froth structures with a high degree of accuracy, both before and after pruning. Pruning typically resulted in a reduction in the number of rules of 20% to 30%. Three-fold cross validation was used in determining the classification accuracy of the fuzzy rules in order to allow the DE algorithm to be initialized with different random seed values. As indicated in Figure 3, fuzzification of these rule-bases yielded results that were not significantly different from the results obtained with the crisp data, even though the split values used in the original rules were ignored when the fuzzy rules were generated. Note that the classification output of the fuzzy rules was taken as the class with the highest membership value.

### *User input*

The quality of the fuzzy rules was evaluated qualitatively by an expert on flotation, who had to classify the different platinum froths without prior knowledge of the rules derived by the inductive algorithm. No direct comparisons could be made, since the human expert could not formulate his knowledge explicitly. However, an analysis of the classification approach followed by the human expert suggested that most of his classifications were effectively based on the average bubble sizes of the froth structures. This corresponds with the analysis of the image parameters for the PGM froth, which indicated that the SNE parameter (related to the fineness of the froth image) had the most significant influence on the classification of the froth under investigation.

### *Comparison of C4.5 crisp and fuzzy response surfaces*

In Figure 4 and Figure 5 the response surfaces obtained from the induced rules before and after fuzzification are shown for the PGM data set. Only the input features SNE and SG are compared with the output grade. The grade is indicated on the vertical axes, and assumes the ordinal values of *high*, *medium* and *low*. As can be seen from these Figures, both types of responses are roughly similar, as can be expected. However, the surfaces associated with the fuzzy rules are notably smoother than those associated with the crisp rule sets, which contain large discontinuities. This can clearly lead to unstable decisions in plant operating regimes close to these discontinuities.

For example, in Figure 4 it is shown that if the value of the SNE parameter is high (around a normalized value of 0.60) and the value of SG is low (around 0.25), giving output classification *low*, any decrease in SNE or increase in SG will change the output classification to either *high* or *medium*. This means that if these two parameters fluctuate around those values, it will result in continuous on-off action as far as control based on the crisp decision surface is concerned. As can be seen from Figure 5, the fuzzy decision rules give a

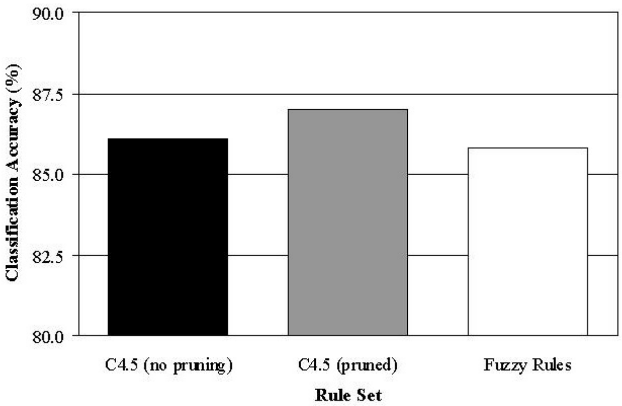


Figure 3—Classification accuracy of rules at different stages of development process

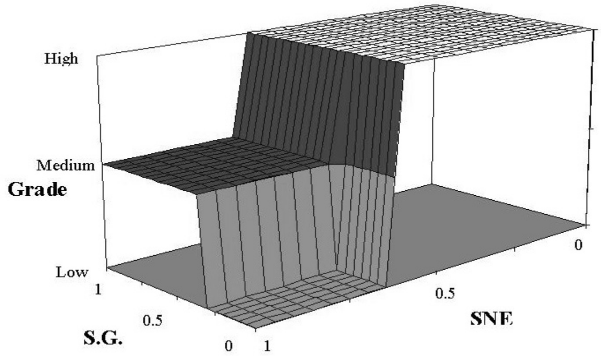


Figure 4—Response surface generated by the set of crisp rules derived by the C4.5 algorithm

response that is significantly smoother. At any point on the response surface of the fuzzy rules the smoothly varying gradient may be determined whereas with the crisp rule response surface the gradient is either zero or undefined (providing no new information).

### *Fuzzy inferencing*

The small optimized subset of rules shown in Figure 6, with their corresponding weights shown in parentheses, is an example of the output derived by the fuzzy inferencing procedure shown in Figure 7. The weight of a rule is an indication of the relative importance attached to each rule by the inference engine. In these rules GRADE refers to the grade of the PGMs in the flotation cell. Figure 7 can be described as follows. Each row of pictures is a representation of the membership functions of the given fuzzy rule shown in Figure 6. The vertical lines drawn over the first four columns of pictures indicate a typical input vector of feature values. The shaded sections of the membership functions are an indication of the membership values of the given input vector. The picture at the bottom right is the result of the centroid summation of the output of the five rules which in turn gives the final output.

At present the system is only being used for the monitoring and diagnosis of plant conditions, and further work is required to enable it to advise on appropriate control actions in order to move the plant from one state to another. Moreover, the NGLDM features extracted from the digitized images of the froth have to be related more meaningfully to physical variables of the froth, such as average bubble size, mineral loading, the texture of the froth, etc. This will enable the plant operator or human expert to better interpret decisions suggested by the rule-based system. The relationships between the statistical features extracted from images of the froths and other physical variables and plant characteristics do not affect the performance of the rule-based system, but merely serve to make decisions more intelligible to the plant operator.

## Conclusions

In this paper the development and use of a fuzzy decision support system for industrial flotation plants have been described. In particular, a strategy for the development of a fuzzy knowledge base representative of uncertain,

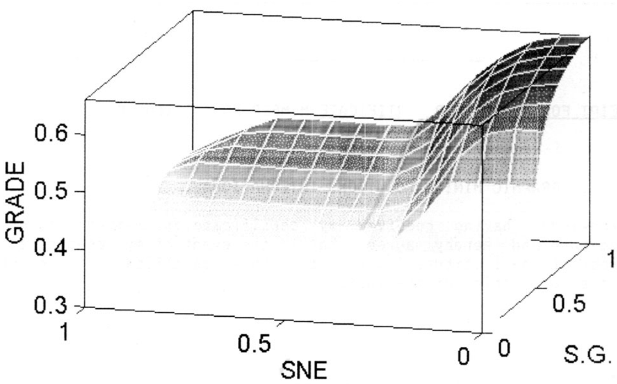


**Figure 5—Smooth response surface generated by set of fuzzy rules**

1. IF (SNE=*high*) AND (T/h_wet=*low*) THEN (Grade=*low*) (0.84)
2. IF (T/h_wet=*low*) AND (SG=*low*) AND ($CuSO_4$=*low*) THEN (Grade=*low*) (0.99)
3. IF (SNE=*low*) AND (SG=*high*) THEN (Grade=*high*) (0.72)
4. IF (T/h_wet=*high*) THEN (Grade=*high*) (0.55)
5. IF (SNE=*low*) AND ($CuSO_4$=*high*) THEN (Grade=*high*) (0.64)

**Figure 6—Typical subset of fuzzy rules. The reliability of a particular rule is indicated by the weight (probability of being correct) in brackets following the rule**

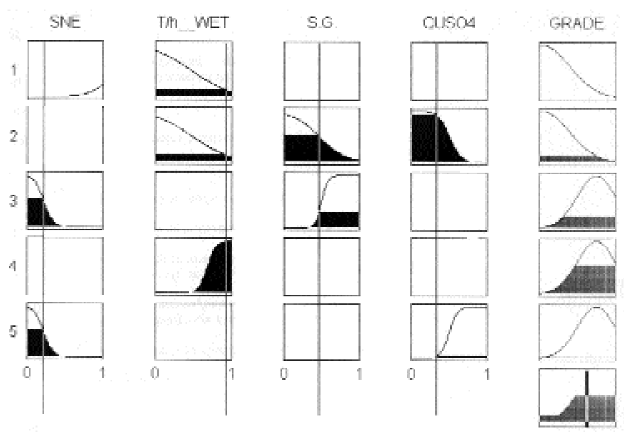


**Figure 7—Typical fuzzy inferencing for the industrial PGM data**

fluctuating plant conditions was described. It was shown that the system is capable of accurately predicting the performance of a flotation plant on-line. More specifically,

- ➤ The smooth response surfaces associated with the fuzzy rule-base of the decision support system allow for more stable decision-making than is possible with the use of crisp rules
- ➤ Vague or ill-defined heuristic rules and soft constraints are readily incorporated into the system, which is particularly important on flotation plants, where quantification of the effects of plant variables is often difficult
- ➤ The induction of rules allows rapid partitioning of the feature space, and focusing on areas of importance, so that continuous changes in plant conditions can be tracked and captured sufficiently fast to keep the rule base of the system current
- ➤ The fuzzy character of the system allows it to be adapted relatively easily to widely different plant conditions, and the system is sufficiently generic to be used on different plants with minor modification.

## References


1. JAMSA, S.L. and HERBST, J.A. A simulation study of expert control system for flotation. *Proc. of IFAC Software for Computer Control*, Pergamon Press, Johannesburg, South Africa. 1988. pp. 67–74.
2. YNCHAUSTI, R.A. and HALES, L.B. Object oriented expert system control of flotation plants. In: *Emerging Computer Techniques for the Minerals Industry*. Eds: Scheiner, B.J., Stanley, B.J. and Karr, C.L., vol. I-2, pp. 13–20, Society for Mining, Metallurgy, and Exploration, Inc., Littleton, Colorado, U.S.A. 1993.
3. MOOLMAN, D.W., ALDRICH, C., and VAN DEVENTER, J.S.J. The monitoring of

froth surfaces on industrial flotation plants using connectionist image processing techniques, *Min. Eng.*, vol. 8 no. 1/2, 1995a. pp. 23–30.

4. NIEMI, A.J., YLINEN, R., and HYÖTYNIEMI, H. On characterization of pulp and froth in cells of flotation plant, *International Journal of Mineral Processing*, vol. 51, 1997. pp. 51–65.
5. HARGRAVE, J.M. and HALL, S.T. Diagnosis of concentrate grade and mass flowrate in tin flotation from colour and surface texture analysis. *Minerals Engineering*, vol. 10, no. 6, 1997. pp. 613–621.
6. CIPRIANO, A., GUARINI, H., VIDAL, R., SOTO, A., SEPÚLVEDA, C., and MERY, D. A real time visual sensor for supervision of flotation cells. *Minerals Engineering*, vol. 11, no. 6, 1998. pp. 489–499.
7. LENCZOWSKI, S. and GALAS, J. Froth image analysis in a flotation control system. In: *Frothing and Flotation II*, Chap 9, pp. 275–308. Eds: Laskowski, J.S. and Woodburn, E.T. Gordon and Breach Science Publishers, The Netherlands. 1998.
8. JOHANNSEN, G. and ALTY, J.L. Knowledge engineering for industrial expert systems, *Automatica*, vol. 27, no. 1, 1991. pp. 97–114.
9. KATTAN, M. Inductive expert systems vs. human experts, *AI Expert*, vol. 9, no. 4, 1994. pp. 32–38.
10. QUINLAN, J.R. 'C4.5: *Programs for Machine Learning*'. Morgan Kaufmann Publishers, San Mateo, California, U.S.A. 1993.
11. QUINLAN, J.R. Induction of Decision Trees, *Machine Learning*, vol. 1, no. 1, 1986. pp. 81–106.
12. WEISS, S.M. and INDURKHYA, N. Optimised rule induction, *IEEE Expert*, vol. 8, no. 6, 1993. pp. 61–69.
13. STORN, R. and PRICE, K.P. 'Differential Evolution—A simple and efficient adaptive scheme for global optimization over continuous spaces', Report TR-95-012, International Computer Science Institute, 1947 Center Street, Berkeley, CA, USA, 94704-1198, 1995.
14. SUN, C. and WEE, W.G. Neighbouring grey level dependence matrix for texture classification, In: Comp. Vision, *Graphics and Image Proc.*, vol. 23, 1983. pp. 341–352.
15. MOOLMAN, D.W., ALDRICH, C., VAN DEVENTER, J.S.J., and BRADSHAW, D.J. The interpretation of flotation froth surfaces by using digital image analysis and neural networks, *Chem. Eng. Sci.*, vol. 50, no. 22, 1995b, pp. 3501–3513. ◆