

Modern Portfolio Optimisation Under Regime Switching

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ABSTRACT

The main objective of this assignment is to consider modern portfolio optimisation under regimes. Unobservable regimes are assumed to be modulated by a time-change Markov process. These models are well-known as Markov regime switching models. The Markov regime switching models are applied to portfolios that consist of a lagged model and a factor model. The lagged model represents a portfolio of 20 stocks which have been lagged by a day and then classified into regimes whereas the factor model uses 5 different global risk factors or measures namely, the 3 Fama-French factors, VIX and a spread between the 3-month JIBAR rate and SAFEX overnight rate to estimate the unobserved regimes for the portfolio. This assignment considers two regimes. Two regimes are classified representing the bull and bear markets, periods when the financial market is doing well and when the market is on a downturn respectively. Thereafter, optimisation is performed by taking the estimated regimes into account and obtaining the optimal portfolio allocations. Optimisation methods such as Sharpe ratio method and risk budget method are investigated. For each of these optimisation methods the portfolios were rebalanced to evaluate the financial markets at the start of the new investment period, classify it either into a new regime or remaining in the current state and then adjusting the portfolio weights. Portfolio optimisation including the regimes are then compared to classical modern portfolio optimisation without regimes consideration. Results show that portfolio optimisation with regimes obtained the highest Sharpe ratio, indicating the economic benefit of inclusion of regime switching characteristics in modern portfolio optimisation.

Key words:

Regimes, portfolio, Markov regime switching model, optimisation, lagged model, factor model, Sharpe ratio, risk budget, rebalancing.

OPSOMMING

Die hoof doel van die opdrag is om die moderne portefeulje optimalisering onder regimes te oorweeg. Dit word aangeneem dat onwaarneembare regimes gemoduleer word deur die tydveranderende Markov proses. Hierdie modelle is welbekend as Markov regime oorskakelings modelle. Die Markov regime oorskakelings modelle word toegepas op 'n portefeulje wat bestaan uit 'n sloer model en 'n faktor model. Die sloer model verteenwoordig 'n portefeulje van 20 aandele wat vir 'n dag sloer en dan eers geklassifiseer word in regimes waar die faktor model 5 verskillende globale risiko faktore gebruik of meet naamlik, die 3 Fama-French faktore, VIX en die verspreiding tussen die 3-maande JIBAR koers en SAFEX oornag koers om te bepaal wat die onwaarneembare regimes vir die portefeulje is. Hierdie opdrag oorweeg twee regimes. Twee regimes word geklassifiseer deur die bul en die beer markte, periodes wanneer die finansiële mark opwaarts neig en wanneer die mark afwaards neig. Daarna word optimalisering gedoen deur die skatting van die regimes in ag te neem en die optimale portefeulje allokasies te verkry. Optimalisering metodes soos die Sharp verhouding metode en risiko begroting metode word nagevors. Vir elkeen van die optimalisering metodes is die portefeuljes weer gebalanseer om die finansiële mark te evalueer aan die begin van die nuwe bellegings periode. Dit word dan geklassifiseer in 'n nuwe regime, of dit bly dieselfde en die portefeulje gewig word daar volgens aangepas. Portefeulje optimalisering insluitend die regimes word dan vergelyk met die klassieke moderne portefeulje optimalisering sonder om die regimes in ag te neem. Resultate wys dat portefeulje optimalisering wat regimes insluit, verkry die hoogste Sharpe verhouding, dit dui die ekonomiese voordeel om die regime oorskakelings eienskappe in 'n moderne portefeulje optimalisering.

Sleutelwoorde:

Regimes, portefeulje, Markov regime oorskakelings model, optimalisering, sloer model, faktor model, Sharpe verhouding, risiko begroting, balansering.

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LIST OF ABBREVIATIONS AND/OR ACRONYMS

MPT	Modern Portfolio Theory
CAPM	Capital Asset Pricing Model
ERC	Equally weighted risk contribution
PCA	Principal component analysis
JSE	Johannesburg Stock Exchange
CBOE	Chicago Board Options Exchange
VIX	CBOE Volatility Index
PMPT	Post-modern portfolio theory
MktRF	Market-minus-risk-free rate
SMB	Small minus big
HML	High minus low

CHAPTER 1

INTRODUCTION

1.1 AN OVERVIEW

Modern portfolio Theory optimisation has been one of the most active areas of research at the confluence of finance and mathematics, having been introduced by Markowitz's seminal paper in 1959 (see Markowitz (1959)) and intensively researched by others. This assignment will investigate the Markov regime switching models as a way of classifying the South African financial markets into different unobservable regimes. The regimes will represent the bull and bear states of the economy. Thereafter, portfolio optimisation as a whole will be investigated taking into account the regime switching features. Two different models will be chosen to fit these Markov regime switching models on. The first is a lagged model and the second is the factor model. The lagged model represents the portfolio which have been lagged by a day and then classified into regimes whereas the factor model uses 5 different risk factors such as the 3 Fama-French factors, VIX and a spread between the 3-month JIBAR rate and SAFEX overnight rate to calculate the regimes for the portfolio. First the parameters of these models have to be estimated by using a calibration method. Thereafter these calibrated parameters are used for the portfolio optimisation.

Optimising these models for the constructed portfolios is then the next task at hand. Optimisation using the Sharpe ratio method as well as the risk budget method will be put to the test and compared among one another. The higher the Sharpe ratio method the more attractive the risk-adjusted return. Risk budgeting is a type of portfolio allocation in which the portfolio's risk is spread across several asset classes with the goal of increasing overall portfolio returns while minimizing total portfolio risk (Das, 2021). For each of these optimisation methods the portfolios were rebalanced to evaluate the financial markets at the start of the new investment period, classify it either into a new regime or remaining in the current state and then adjusting the portfolio weights.

The main objective is to investigate whether applying all of these models and the estimation of the market regimes, whereafter the portfolio is then optimised taking the current state of the financial market into account, actually makes a substantial difference. This will be tested by taking a look at the annualised portfolio returns and the portfolio risk which is then used to calculate the Sharpe ratios. This is done for a portfolio taking the estimation of the regimes into account and a portfolio excluding the regime classification that were optimised with the classical optimisation methods. Thereafter the conclusion of the study can be made.

1.2 PROBLEM STATEMENT

The major question in this assignment is whether the inclusion of regimes lead to better optimisation of a portfolio than the exclusion of the regimes, which refers to the classical methods. The Markov regime switching model is the chosen model for classifying the South African market into regimes. The South African economy has gone through numerous turmoil periods and when portfolio optimisation is done taking unobservable regime switching into account it is deemed as an appropriate methodology.

1.3 CLARIFICATION OF KEY CONCEPTS

Regimes

Classification of the South African financial markets into two separate states which represent the bull and bear markets.

Portfolio

Combination of stocks that are invested in by an investor.

Markov regime switching model

Model that switches between regimes using a Markov chain.

Optimisation

To obtain the optimal weighting of stocks in the portfolio to receive the highest returns.

Lagged model

Model that are fitted on the portfolio that are lagged with one day.

Factor model

Model fitted on the portfolio based on different risk factors.

The remainder of the assignment is structured as follow: in Chapter 2 a literature review is discussed on the Markov regime switching model as well as optimisation. Continuing to Chapter 3 the models and methodologies are explained as well as some insight on the mathematical background. These methods were then applied and Chapter 4 is devoted to the empirical results continuing to the optimisation in Chapter 5. The models that have incorporated the regimes are then compared to portfolio optimisation excluding the incorporation of the unobserved regimes and the results can be obtained in Chapter 6. Whereafter, the final conclusion will follow.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter will focus on the background and survey the literature being used in the assignment. The reasoning behind the methods that are used will be discussed as well as some insights on the topic's background.

The main goal for the assignment is to assess the impact of regime switching on portfolio optimisation. To optimise a portfolio different methods can be used and should be considered to diversify the risk, and to obtain the optimal weights in each aspect of the portfolio. The founder of modern portfolio theory, Markowitz, is well regarded (Markowitz, 1959). His study was based on the concept of dividing the portfolio into a mean and variance choice. His mean-variance theorem is still the foundation of current portfolio theory. According to this theory, maintaining constant volatility maximizes anticipated return and vice versa. This principle led to the creation of the efficiency frontier, which allowed investors to select a portfolio depending on their risk tolerance. It's worth noting that the efficiency frontier is used to compare how various securities perform against one another. By taking these interactions into account, an investor may build a portfolio that has the same expected return but lower risk than if the interactions were not taken into account.

Different formulation of modern portfolios adopted in practice are for example to maximize the expected return of an asset portfolio given the exposure to risk or maximising the Sharpe ratio which is simply the excess return over risk. These fundamental assumptions are not contradicted by post-modern portfolio theory (PMPT). However, it modifies the formula for assessing risk in an investment to address problems identified by its creators in the original according to Swisher and Kasten (2005). Followers of both theories utilize software that uses either MPT or PMPT to create portfolios that fit their desired degree of risk. Risk is tied to the ever changing economic conditions therefore factors such as the Fama French factors can be considered because it adds value risk and size risk to the calculation (Womack and Zhang, 2003). Because of the ever changing state of the

economy that can be seen just by considering the global pandemic that COVID-19 has caused the well known financial market phenomenon of regime switching can be considered as one of the more appropriate ways to optimise a portfolio.

This study wants to investigate the existing link between the Markov regime switching model and the performance of an optimised portfolio. Optimisation under regime switching is not an easy task, the different regimes first needs to be estimated and identified before anything else can be done. The Markov regime switching model and optimisation methods with all their capabilities and shortcomings are explored. In the next section portfolio optimisation is investigated whereafter the Markov regime switching model is explained.

2.2 PORTFOLIO OPTIMISATION

Portfolio optimization is the process of an investor that wants to select the best asset distribution for the portfolio. Baring in mind that the best portfolio will maximise the expected return and minimise any costs associated with the risk being taken. This can be based on the principle of modern portfolio theory (MPT) that is to maximise the possible returns for the lowest risk or minimising the risk given a target return however, additional constraints are then often needed to obtain a reasonable solution. The highest Sharpe ratio will be considered because it measures the excess returns for every unit of risk taken (Sharpe, 1994). Assets should ideally have low correlation with one another because the interaction of how the assets perform relative to one another is considered. Therefore we would want to diversify the portfolio as much as possible to avoid a crash when any particular asset underperforms this forms part of risk mitigation. In other words, the portfolio needs to be diversified enough with the correct weightings in each asset that a sudden change in one wouldn't cause the investor any major losses or bankruptcy. The famous English saying says it best by not having all your eggs in one basket.

When optimisation of a portfolio is considered the most challenging aspect is to find a strategy to allocate weights to the different components in the portfolio. There are different ways to do this

that can be considered to be optimal. Skilled managers should however be careful when choosing an allocation strategy and should consider all possible information about the portfolio. According to Srivastava (2020) the most intuitive way of allocating weights to securities is based on the managers conviction for them. However from an academic point of view this can be calculated using different techniques and using programming tools to calculate the best weightings for the portfolio. In the early days the managers feel for investing was the way of doing things and they were actually profitable however after 1997 the increased volatility in the markets caused managers to turn to other methods. Srivastava (2020) also mentioned that after the 2008 Global Financial Crisis that the “risk-parity” strategy became very popular (also known as the equally weighted risk contribution (ERC)). Risk parity is a portfolio allocation approach that uses risk to decide allocations across multiple components of an investment portfolio. Through the use of leverage, the risk parity strategy modifies the modern portfolio theory (MPT) approach to investing Asness *et al.* (2012). This method assigned identical risk budgets to all participating assets and ignores investor expectations or projected return forecasts. Lohre *et al.* (2014) gives a different perspective on risk parity and states that a diverse risk parity portfolio can be built by deconstructing a portfolio using principal component analysis (PCA) depending on its underlying risk metrics. The Markowitz Mean-Variance Optimal Portfolio, based on the Capital Asset Pricing Model or CAPM, is the most prevalent strategy that incorporates viewpoints. A market-capitalization-weighted allocation is used in passive investments like the market index.

Other methods can also be considered and a few worth mentioning will be discussed. When the equally weighted strategy is considered, returns throughout the portfolio are purely random. It then simply assigns equal weights to all components. However, after the 2008 global crisis a shift to risk parity was made because in this situation the risk contribution of each asset is equal and it works best when the Sharpe ratios are the same between the asset classes according to Srivastava (2020). However, this is not realistic because equal risk contribution approaches do not use time-varying covariances or correlations. Another method exist to minimise the portfolio’s volatility and it’s called minimum variance. This would work ideally when the returns and the risk involved is not proportional to one another. This means that when the risk is reduced it doesn’t lead to lower returns. The Markovitz Portfolio is a well-known mathematical framework for piecing together a

portfolio of assets in such a way that the expected return for a given amount of risk is maximized, this is also known as mean-variance optimisation. Costa and Kwon (2019) stated that the optimal mean-variance approach is very sensitive to the level of noise that comes from the estimated input parameters, specifically referring to the covariance matrix and asset expected returns.

As in the paper of Agarwal and Lorig (2020) the Sharpe ratio of a portfolio is a simple measure of mutual fund performance, it is defined as the ratio of a portfolio's expected excess return to its standard deviation. They also mentioned that Merton used the risk-aversion parameter of a utility function to integrate the investor's risk preference. It was demonstrated that, given a constant relative risk-aversion utility function, the investor receives a higher expected utility from assets with a higher Sharpe ratio under the premise that log-asset returns are normally distributed with a constant Sharpe ratio. In other words it is considered that a market trader who seeks to maximize his expected terminal utility does so by trading risky assets.

It is also important to keep in mind that an investor should specify their risk appetite because this will have big implications on how their money is invested considering the time horizon. One should also consider the really important factor of correlation when the optimisation method is chosen because will perform better in the presence of correlation than other. Some scenarios are more optimal than others when the market switches regimes, therefore one should also consider different regimes under which investments are done. The Markov regime switching model is a good choice for this and will be discussed in the next section.

2.3 MARKOV REGIME SWITCHING MODEL

Political unrest and financial crises can cause dramatic breaks in economic time series thus the need for estimating regimes. The first application of the Markov regime switching was coined by Hamilton (1989) in his paper entitled "A new approach to the economic analysis of nonstationary time series and business cycles". If the data that are being used are the result of the alternating regimes fitting a classical linear regression model would lead to oversimplification of the market

movements because it would merge the market regimes when estimating. Therefore it is proposed to use a model that take the different states (regimes) including the current regime into account. In this paper a two-state model is used to divide the market movements into a ‘bull’ and ‘bear’ market, where the ‘bear’ market have higher volatility. The transition between these states can be represented graphically as follow:

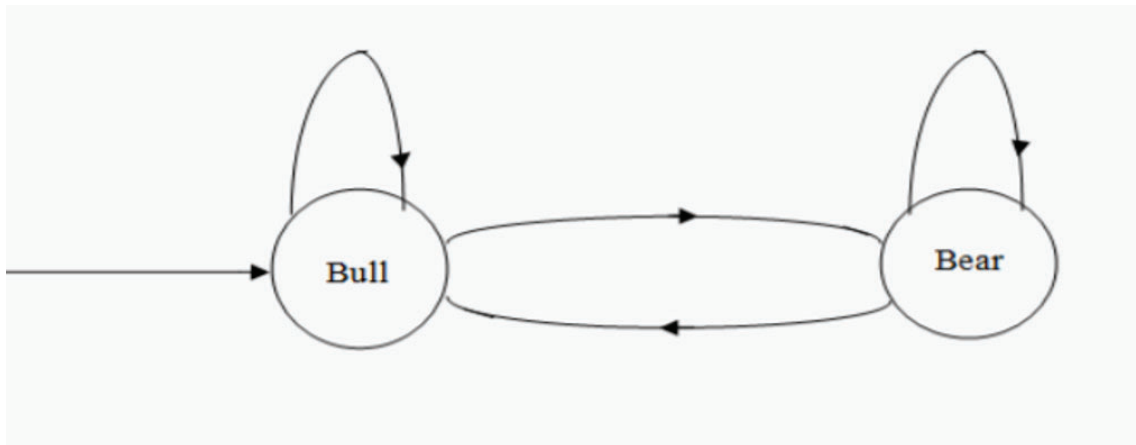


Figure 2.1: Transition from one state to another or remaining in the same state.

A bull market is one in which prices are rising and economic circumstances are typically positive. A bear market develops when the economy is weakening and the majority of stocks are losing value according to Gonzalez *et al.* (2005). It is important to note that in a market that consists of these bull and bear market cycles modern portfolio optimisation would fail to capture these significant changes in the economy.

The existence of a Markov regime-switching model can be aided by the use of a Markov chain to replicate the unseen factors that drive the dynamics of these rapid shifts according to Costa and Kwon (2019). They also continue to say that it is possible to ‘switch’ between the estimated parameters corresponding to a bull or bear market by adding a regime-switching framework to a nominal factor model of returns, therefore improving estimation accuracy. Because of the uncertainty that’s introduced by the regime estimation error the Baum-Welch algorithm is used in practice to estimate the Markov-regime switching model.

Song (2020) describes the popular Markov switching models as models that use state or regime specific values to introduce time-variation in the parameters. This time variation is driven by a discrete-valued latent stochastic process with limited memory, which is significant. More precisely, the present value of the state indicator is determined solely by the previous period's value, resulting in the Markov property and the transition matrix. Being more specific as to what this entails we define the Markov property as, the property that is an extension of the Markov process deriving the future states that are dependent on the present state but independent of the past states (Grabski, 2015). The Markov process is a memoryless process which means that it does not remember the previous states that it visited it only depends on the present state it is in.

Scherer *et al.* (2018) explains the general state-based approach according to the following three conclusive phases. The first is the “state” of the system that should be determined where the state contains both the information needed to describe the system's dynamics and to make decisions. Then the transitions between the states in the system is considered. The desired outcome for these transitions is that it should be Markovian therefore, determining the order of the model constructed is required. Another desired property is stationarity (that the transitions and time are independent). Lastly Scherer *et al.* (2018) states that the chosen decision model must be constructed and thereafter optimised, meaning that system controls must be chosen and implemented in order to maximize (or minimize) set objectives. Therefore we can see that there exist a link between the switching in market condition affecting the economy being modeled by a Markov-regime switching model and the optimisation of a portfolio. All of this information should be considered when constructing a portfolio to maximise the returns. Thus we can discuss the conclusion in the next section.

In short this study wants to investigate whether adding the regimes in optimisation is beneficial compared to optimisation without the incorporation of regimes. It will be compared by using the Sharpe ratios. The methods and results are discussed in the next sections.

CHAPTER 3

DATA AND METHODOLOGY

3.1 DATA HANDLING

The purpose of the assignment is apply a Markov regime switching model and then optimise the portfolio at hand and this is tested for South African data. Different time periods and methods of gathering data were considered until a conclusion were reached that the time period of observation that were chosen spanned from the start of 2002 until the middle of 2016. This is to incorporate the different events that is known to play a significant role in the classification of regimes for example the Global Financial Crisis of 2008. The daily returns for each asset in the portfolio were obtained. The data were retrieved from Yahoo Finance and thereafter processed. The portfolio that were chosen to be constructed were all companies whose data were publicly available (therefore on Yahoo Finance), and that were well known South African companies to invested and which were established since or before 2002.

As part of another aspect of the assignment different factors were taken into consideration. The Fama French 3-factor model were considered, it is an asset pricing model that elaborates on the CAPM by adding size risk and value risk factors to the market risk factors according to Womack and Zhang (2003). This data could be found on Kenneth R. French (2021) data library. The CBOE Volatility Index (VIX) were also considered. It is the real-time market index that represents the expected market volatility for the next 30-days. The last factor that were considered were the spread between the 3-month JIBAR rate and SAFEX overnight rate the reason for this consideration is due to the Global Financial Crisis there is still a need for a South African default-free discounting curve to capture market risk premium. The spread is used as a proxy for the South African market risk premium. The data were obtained for the same period as mentioned earlier.

3.1.1 Portfolio construction and Factors being considered.

The construction of the portfolio were done in the manner that most categories were covered to diversify the possible investment and to include most aspects of the South African market. The companies or also referred to as assets in the portfolio were chosen as stated in Table 3.1.

In Table 3.1 the code were also provided. This is the code used in Yahoo Finance to represent the corresponding company. This is a study applied on South African data therefore the .JO in the code denotes the market or index of the Johannesburg Stock Exchange (JSE). The JSE is the largest stock exchange in Africa and provides a market to deal with securities. The reason for choosing companies is traded on the JSE is because they are listed and therefore complies to certain regulations and can be seen to be more transparent and provide investors some kind of protection. This statement will be put to the test according to the different classified regimes and thereafter the optimisation of the portfolio. It is important to note that the companies might be referred to the codes as per Table 3.1.

Table 3.1: List of Assets in the Portfolio.

Consumer Goods	Code	Investments	Code
Tiger Brands	TBS.JO	Investec Group	INL.JO
RCL Foods limited	RCL.JO	Coronation Fund Management Limited	CML.JO
Consumer Services	Code	Telecommunication/Broadcasting	Code
Woolworths Holdings Limited	WHL.JO	Naspers Limited	NPN.JO
Shoprite Holdings Limited	SHP.JO	MTN Group Limited	MTN.JO
Health Care	Code	Chemicals/Mining	Code
LIFE Health Care Group Holdings Limited	LHC.JO	Sasol Limited	SOL.JO
Netcare Limited	NTC.JO	Anglo American Platinum Limited	AMS.JO
Insurance	Code	Industrials	Code
Sanlam Limited	SLM.JO	PPC Ltd.	PPC.JO
Discovery Limited	DSY.JO		
Banking	Code	Retail	Code
Absa Group Limited	ABG.JO	Mr Price Group Limited	MRP.JO
Capitec Bank Holdings Limited	CPI.JO		
FirstRand Limited	FSR.JO		
Standard Bank Group Limited	SBK.JO		

The factor selection will now be summarised. The Fama French three factors is the size of firms, book-to-market values, and excess return on the market and can be seen in Table 3.2. The other factors that were incorporate are also in Table 3.2.

Table 3.2: Summary and abbreviations of factors being considered.

	Factor	Abbreviation
Fama French Factors	Portfolio's return less the risk-free rate of return	MktRF
	Small minus big	SMB
	High minus low	HML
	Volatility Index	Vix
	3M JIBAR rate and SAFEX overnight rate	Spread

After the data of the companies in the portfolio and factors were obtained for the time period it could then be processed.

3.1.2 Data Processing

The portfolio data were collected as well as the factors data. It were then transformed to logarithmic returns that all variables can be measured in a comparable metric, thus enabling evaluation of analytic relationships amongst the different variables. The formula used for this transformation were:

$$r_{ti} = \log \left(\frac{P_{ti}}{P_{t-1,i}} \right)$$

Where r_{ti} denotes the transformed log return for the i^{th} ($i = 1, \dots, I$) asset in period t for $t = 1, \dots, T$, then P_{ti} denotes the asset price at time t for asset i and $P_{t-1,i}$ is the asset price at time $t - 1$ for asset i .

When all of the log returns were calculated the data we inspected. There were a few entries that had the value 0 included in them this were removed along with some insignificant small values. The data were then clean and ready to use. The 2 situations being explained throughout is the lagged

and factor models. Now that the data are ready to use the lag could be implemented this were done by only lagging the data by a day due to the fact that we were working with daily data. This were done and the data sets were then compiled and read into R were the rest of the coding could commence.

3.2 BASIC MODELS

The Markov regime switching model were applied to the lagged and factor models. The reason for choosing the lagged version is that it allows for different recent historical amounts to be considered in forecasting. In order for the regression model to be able to predict the future, independent variables must typically be lagged. The reason for the choice of factor models are because they are a powerful and adaptable tool for assessing current risk exposures in a portfolio. There are certain mathematical denotations that had to be made in each of the certain cases and this will be explained below thereafter the general mathematics will be explained which applies to both these models.

The financial market will be classified in one of the two regimes which will be represented by N . The time period throughout will be denoted by t . The market is considered to be in regime s_t in time period t . The remaining notation will be explained as the study continues.

3.2.1 Lagged Model

The method for a Markov regime-switching-based factor model of asset returns is now proposed for a model with lagged variables. Following the method used by Costa and Kwon (2019), the behaviour of a variable, y_t , is observed and investigated when it undergoes dramatic change at a given time, t_0 . The observed behaviour of the variable y_t for $t = 1, 2, \dots, t_0$ can be modelled by a first-order autoregression:

$$y_t = \alpha_1 + \beta_1 y_{t-1} + \sigma_1 \varepsilon_t, \quad (3.1)$$

with intercept denoted by α_1 , the autocorrelation by β_1 , the volatility is then denoted by σ_1 and $\varepsilon_t \sim \mathcal{N}(0, 1)$. If this series at time t_0 undergoes a significant change, the behaviour can then be describe by:

$$y_t = \alpha_2 + \beta_2 y_{t-1} + \sigma_2 \varepsilon_t, \quad (3.2)$$

for $t = t_0 + 1, t_0 + 2, \dots$. Rather than assuming that the two systems are independent, it is rather believed that they are governed by a larger model which consists of both states, which is then the model used for the lagged variable:

$$y_t = \alpha_{s_t} + \beta_{s_t} y_{t-1} + \sigma_{s_t} \varepsilon_t, \quad (3.3)$$

where s_t is considered to be in regime in time period t describing the state of the market. The y_t represents the returns in this case. This behaviour is that of financial cycles, where the market is observed to change abruptly between bull and bear states. The Baum–Welch procedure is used to estimate the components' regime-dependent joint distribution. As a result, this regime-switching factor model of returns proposes that the factors are non-stationary, with two unique, alternating regimes governing their behavior.

3.2.2 Factor Model

The earliest factor model is the CAPM which introduced an advantage in portfolio optimisation. However it only took one factor into account. A study done by Bartholdy and Peare (2005) discussed the formulation of the CAPM and the following equation were derived for the estimation of expected return. The expected excess return on an asset is i given by:

$$E[r_i] - r_f = \beta_i^M (E[r_m] - r_f)$$

where is r_i the return on asset i , r_f denotes the risk-free return, r_m is the return on the world market portfolio, $\beta_i^M = \frac{\text{Cov}(r_i, r_f)}{\sigma_M^2}$ is the systematic risk of asset i relative to the world market portfolio (beta), and σ_M^2 is the variance of the return on the world market portfolio. However, some issues arise due to the fact that the world market portfolio, consisting of all the assets in the world,

this is not possible therefore it is necessary to use a proxy. They continued to improve on this by introducing an alternative of the 3-factor Fama-French model. Thus, for each stock, i , to estimate excess return, beta estimates for each of the factors are first obtained from the following time series regression:

$$r_{it} - r_{ft} = a_i + \beta_{1i}(r_{It} - r_{ft}) + \beta_{2i}\text{SMB}_t + \beta_{3i}\text{HML}_t + \varepsilon_{it} \quad t = 1, \dots, t_0 \quad (3.4)$$

where r_{It} denotes the return at time t on index I , (this is used as the proxy), SMB denotes the return on a portfolio of small stocks minus the return on a portfolio of large stocks, and HML is the return on a portfolio of stocks with high book to market values minus the return on a portfolio of stocks with low book to market values (Bartholdy and Peare, 2005). Two additional factors were included to explain excess return the book to market ratio and the size. Taking this equation it was possible to derive the equation that included all the factors in the portfolio it could then be expressed as:

$$\begin{aligned} r_{it} - r_{ft} = a_i + \beta_{1i}(r_{It} - r_{ft}) + \beta_{2i}\text{SMB}_t + \beta_{3i}\text{HML}_t \\ + \beta_{4i}\text{MktRF}_t + \beta_{5i}\text{Vix}_t + \beta_{6i}\text{Spread}_t + \varepsilon_{it}, \quad t = 1, \dots, t_0 \end{aligned} \quad (3.5)$$

This equation includes all the factors mentioned in Table 3.2. However, Bartholdy and Peare (2005) concluded that this is not an appropriate approach for estimation. Therefore, a new more appropriate approach should be followed to take the economic circumstances into account in other words accounting for the unobserved regimes.

Introducing a new approach which is a similar concept than applied to lagged model, taking the unobserved regimes into account. The factor model (with a little deviation in notation from the lagged model) can estimate the returns defined by the regime-dependant linear factor model as:

$$R_t = \alpha_{s_t} + \beta_{s_t}F_t + \Gamma_{s_t}e_t \quad (3.6)$$

Where the intercept is denoted by α_{s_t} , the autocorrelation by β_{s_t} and the covariance of the residual by Γ_{s_t} . When the market is in regime s_t in period t . This were derived from research done by Ma,

MacLean, Xu and Zhao (2011). The regimes in this case are characterized by a set of J risk factors, representing the macro and micro economic indicators. Recalling that it is actually representing the factors in Table 3.2. F_{tj} denotes the value of the j^{th} risk factor for $j = 1, \dots, J$ in period t . Therefore, F_t denotes the vector of risk factors in period t . The asset returns in various market regimes are characterized with these common risk factors F_t and $e_t \sim N(0, I)$.

It is important to note that to be able to calculate the Markov regime switching models together with the matrices that will follow we first had to do linear regression of the factors or lagged returns on the model as a whole and then read that into the functions to write the regime-switching model with the results that can be seen in Chapter 4. The reason for this is that the parameters were unknown and first had to be calibrated before the Markov regime switching could be done.

The following information is applicable on both the lagged and factor model. The model parameters of the 2 models are dependent on the regime s_t . The vector $\alpha'_{s_t} = (\alpha_{1s_t}, \dots, \alpha_{Is_t})$ contains the state-dependent intercepts of the linear factor model. Then taking the second term into account we can express β_{s_t} which is the autocorrelation, as a matrix representing the relationship between the variables:

$$\beta_{s_t} = \begin{pmatrix} \beta_{11s_t} & \cdots & \beta_{1Js_t} \\ \vdots & \ddots & \vdots \\ \beta_{I1s_t} & \cdots & \beta_{IJs_t} \end{pmatrix}$$

which then defines the sensitivities of asset returns to the common risk factors in state s_t . The β value measures the systematic risk of an investment in comparison to the market as whole. The linear factor model implies that, given the factors (either lagged or risk factors), the conditional asset returns within a regime are then normally distributed with mean vectors:

$$\mu_{s_t} = \alpha_{s_t} + \beta_{s_t} F_t \quad (3.7)$$

and covariance matrix:

$$\Sigma_{s_t} = \Gamma_{s_t} \Gamma'_{s_t} \quad (3.8)$$

Estimates of the regime-dependent parameters and forecasts for the values of the factors are required for the linear factor model to be useful for forecasting returns over time. There are N distinct regimes, in this study $N = 1, 2$ and the movements of the market regimes follow a Markov chain. The regimes are denoted by n and $q_{tn} = \Pr [s_t = n]$, for $n = 1, \dots, N$. The initial regime distribution is represented as q_0 and the transition probability matrix is given as $P = \{p_{mn}\}$.

The transition probability from regime m to regime n is given by the following equation:

$$p_{mn} = \Pr (s_{t+1} = n \mid s_t = m), \quad \forall m, n. \quad (3.9)$$

For period t , given that the regime m is in the period of $t - 1$, the two-state transition matrix describing the probability of switching from one regime to another is represented by the following matrix:

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

where γ_{ij} gives the probability that state i will be followed by state j . Therefore, every row must sum up to unity as:

$$\sum_{j=1}^2 \gamma_{ij} = 1 \text{ for } i = 1, 2$$

The unconditional expected asset return is expressed as:

$$\bar{\mu}_{tm} = \sum_{n=1}^N \mu_{tn} p_{mn}, \quad (3.10)$$

and the covariance matrix as:

$$\bar{\Sigma}_{tm} = \sum_{n=1}^N \left[(\mu_{tn} - \bar{\mu}_{tm})^2 + \Gamma_n \Gamma_n' \right] p_{mn}. \quad (3.11)$$

When the regimes in each period are known, estimating model parameters from observations on returns and factors becomes quite simple. However, if the regime at each time, as well as the model

parameters for each regime, are unknown, then the model parameters must be calculated directly from the data, and the regime must be deduced.

The Baum-Welch algorithm is used for the estimation of the parameters of the Markov regime switching model, this includes the transition matrices, the β matrices, the initial state and intercept etc. according to Mitra and Date (2010). They continued defining it to also be known as the forward-backward algorithm which is a dynamic programming approach and a special case of the EM algorithm, which stands for the expectation-maximization algorithm. The forward and backward phase is part of the E-step and the update phase part of the M-step. This EM algorithm will be explained in the next section.

3.3 ESTIMATION ALGORITHM

The main research done about the topic of the EM algorithm is that of Dempster, Laird and Rubin (1977) and this combined with the work of Ma *et al.* (2011) is used as basis for the explanation of how this study implemented the EM algorithm. This process consists of 2 steps. The steps are as follow:

- The first step is regarding the E in EM algorithm. This is for the estimation of the missing data for the regimes.
- The second step refers to the M which is the maximization of the likelihood based on the estimated missing data on the regimes.

To execute the EM algorithm one have to specify the number of regimes. In this study we have defined the number of regimes $N = 2$. Then continue to denote the model parameters as $\theta = \{\alpha_{s_t}, \beta_{s_t}, \Gamma_{s_t}, q_0, P\}$, the unknown regimes at each time is defined as S , and the observed data on returns and factors is represented by X .

The iterative algorithm can then be set out to be as:

- **The E-step:** First a set an initial value θ^0 for the true parameter set θ , then the calculation

for the conditional distribution for regimes can be done as follow,

$$Q(S) = P(S | X; \theta^0).$$

Thereafter the expected log-likelihood of the data can be calculated with respect to the two regimes,

$$E_Q[\ln P(X, S; \theta)].$$

- **The M-step:** This step is were the maximization of the expected log-likelihood with respect to the conditional distribution of the regimes take place, to produce an improved estimate of θ . The improved estimate is:

$$\theta_1 = \arg \max_{\theta} \{E_Q[\ln P(X, S; \theta)]\}$$

With θ_1 denoting the new value for θ , then returning to the E-step again.

The results from applying the EM algorithm to the data are:

1. The parameter estimates as:

$$\hat{\theta} = \left\{ \left(\hat{\alpha}_n, \hat{\beta}_n, \hat{\Gamma}_n, \forall n = 1, \dots, N \right) \right\},$$

2. the estimated transition matrix, and
3. the posterior distribution of regimes. The implied regime, \hat{S} , at each time is the most likely regime.

The EM method necessitates the determination of a known number of regimes from the data. The goal is to find the model that fits the data the best, and the number of regimes is part of the fit. The number of regimes where chosen as 2 classifying the state of the economy in either an upward swing or downward swing. In the next section the results of these mathematics will be explained. Note that another amount of regimes could be chosen or investigated however taking the turmoil South African economy it was chosen to work with 2 regimes.

The data handling has been discussed in Section 3.1 as well as the chosen assets in the portfolio that were constructed. The factors that are used and their abbreviations were stated. After all of the data processing were done and the data were correct and ready to use the basic models were discussed. The lagged model and factor models along with their equations and derivations were done and then the chapter continued to the last section of the estimation algorithm. The EM algorithm is explained along side the steps and calculations that should be done. The theory is explained along with the equation and derivations and Chapter 4 will give the empirical results which were obtained by implementing the models and methods of Chapter 3.

CHAPTER 4

EMPIRICAL RESULTS

The Markov regime switching model were implemented for the two scenarios. Hereafter, the regimes were classified for the time period that the financial markets were observed. The matrix values were obtained by the methods explained in Section 3.2 completing the equations of Chapter 3. The results were obtained following the methods of Chapter 3. Thereafter, optimisation is done using different methods of optimisation for the two scenarios as well as for different categories of the portfolio.

4.1 LAGGED MODEL

As explained earlier the market were classified into regimes according to the data of 20 lagged variables. The variables were lagged with a day using daily returns. After applying Markov regime switching methods to these data the regimes could be classified into 2 regimes that is referred to as regime 1 and 2. This represents the bull and bear markets respectively, as mentioned a bull market is one where the economic circumstances are typically positive in which prices are rising and and a bear market develops when the economy is slow and most stocks are declining in value. We are interested if this would make a difference to the optimisation efficiency and mitigation of risk.

Figure 4.1 is a visual representation of the regimes after it have been classified by the lagged variables. This splits the South African market into 2 states depending on the market condition.

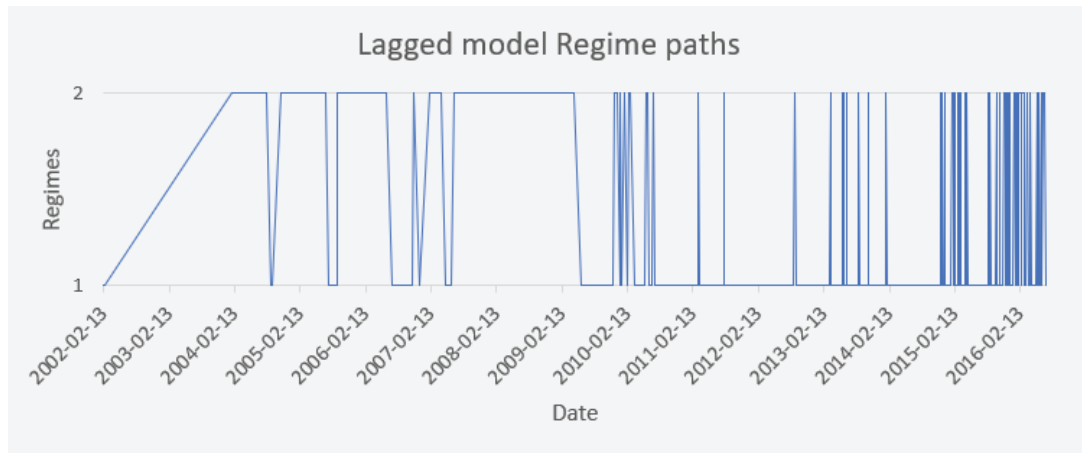


Figure 4.1: The lagged model's regime movements over the time period.

Next, the different values were calculated for each variable as explained in Chapter 3 for the equation of:

$$y_t = \alpha_{s_t} + \beta_{s_t} y_{t-1} + \sigma_{s_t} \varepsilon_t, \quad (4.1)$$

Starting with the α_{s_t} the values were calculated for the vectors representing each regime for each lagged variable. The alphas are also representing the state-dependent intercepts of the linear factor model. This can be seen in Table 4.1 below.

Table 4.1 shows the alphas (α) for regime 1 and 2, alpha measures how closely a stock's returns match or surpass those generated by the market. A negative alpha, such as Tiger brands for regime 1 seen to be -0.0011 on the other hand, shows that the asset is not generating returns at the same rate as the broader sector.

Table 4.1: Lagged model alphas for regime 1 and 2.

Lagged Alphas	Regime 1	Regime 2
Tiger brands	-0.00110	0.00176
RCL Foods	0.00125	-0.00086
Woolworths	-0.00080	0.00006
Shoprite	-0.00050	-0.00092
LIFE	-0.00012	0.00062
Netcare	-0.00090	0.00068
Sanlam	0.00068	-0.00161
Discovery	0.00136	-0.00064
Absa	-0.00091	-0.00008
Capitec	0.06374	0.00111
FirstRand	0.00006	0.00006
Standard Bank	-0.00077	-0.00016
Investec	0.00023	-0.00449
Coronation	-0.00053	0.00177
Naspers	0.00184	-0.00203
MTN	-0.06169	-0.00077
Sasol	-0.00033	0.00027
Anglo American Platinum	0.00253	-0.00220
PPC	-0.00126	-0.00085
Mr Price	-0.00466	0.00124

There is a higher chance that there will be excess returns in other words alphas in the bull regime, therefore it is expected that the knowledge of potential future regimes are up to date to allocate the the assets. The next factor in Equation 4.1 is the β variable. The most important thing to be attentive on is that the α values and β values vary across the regimes and that it doesn't stay fixed. The β represents the sensitivities of asset returns to the common risks in state s_t . It is also sometimes referred to as the autocorrelation between the variables as in the research done by Costa and Kwon (2019).

Tables 4.2 and 4.3 represents the β 's for the lagged variables. It is important to note that the numbering for 1 to 20 is simply for the 20 assets in the lagged model. These also represents the one day lags that are incorporated into the model. Thus we can assume that these values represents the autocorrelation between the variables and the respective lagged variables.

Table 4.2: Lagged Model beta coefficients for regime 1 continuing at the bottom.

	1	2	3	4	5	6	7	8	9	10
Tiger brands	-0.29332	0.10175	-0.04063	0.02499	-1.15774	-0.01855	0.01120	0.02086	-0.08555	-3.18619
RCL Foods	-0.03861	-0.02152	0.01452	0.02071	-0.20353	0.07381	-0.00535	0.02146	0.02544	-1.33226
Woolworths	-0.03705	-0.02914	0.49780	0.01599	-0.54808	0.10170	0.01160	0.00757	0.03345	-2.19770
Shoprite	0.03628	0.04444	0.09122	0.02774	-0.50966	-0.09392	-0.10012	-0.16436	-0.02799	-3.31915
LIFE	0.02239	0.01050	0.41169	0.04734	-0.18881	0.22028	-0.06334	0.04799	0.01832	9.67907
Netcare	-0.01295	-0.19990	-0.26403	0.00299	-0.87027	0.05666	0.03312	-0.05109	-0.10516	-7.36824
Sanlam	0.00560	0.14599	0.34113	0.19550	0.99708	0.14499	-0.22027	-0.06096	-0.04017	5.36067
Discovery	0.14219	0.03545	-0.19439	-0.04311	0.40594	-0.04943	0.02347	-0.48334	0.17866	1.48362
Absa	0.10038	-0.10475	-0.22304	-0.06911	0.74975	-0.12643	-0.03352	0.09681	-0.09920	-9.28193
Capitec	0.06934	0.01757	0.14270	0.09452	0.09550	-0.08135	-0.04717	-0.03554	0.15075	-0.41648
FirstRand	-0.19606	-0.02058	-0.15531	-0.27217	-0.80410	0.11358	0.00578	0.03671	-0.13517	2.29556
Standard Bank	-0.05865	0.19895	-0.14857	0.14092	0.03422	-0.13814	0.00900	0.02409	-0.07450	-6.90570
Investec	-0.10406	0.11373	0.38057	0.19211	0.08505	0.19038	-0.07632	0.00008	-0.06362	2.46532
Coronation	0.02776	-0.09405	-0.26160	0.11231	0.85198	0.05669	0.05976	0.05646	-0.08158	7.60783
Naspers	0.01830	0.09024	-0.11367	0.01880	0.54522	0.05360	-0.01929	0.04042	0.04915	-1.43607
MTN	0.03454	-0.11970	-0.01315	-0.07099	-0.66823	-0.03695	0.09360	0.00581	0.08250	-4.82672
Sasol	0.01954	-0.12499	-0.00165	-0.25943	0.03945	-0.10330	0.04885	-0.05075	0.07292	6.19480
Anglo American Platinum	0.05061	0.03233	-0.03296	0.14022	-0.07804	-0.06141	-0.03381	-0.03021	0.04349	2.26579
PPC	-0.08201	0.03916	-0.02976	0.04861	-0.50574	0.08625	0.00914	0.05740	-0.08997	-3.44336
Mr Price	-0.00578	-0.14591	-0.10307	0.07208	-0.32317	0.02736	-0.02955	0.15086	0.06261	7.67954

	11	12	13	14	15	16	17	18	19	20
Tiger brands	-0.14178	0.09420	0.07341	-0.08357	0.10963	0.13163	0.11452	0.05173	-0.04339	-0.20003
RCL Foods	0.01456	-0.00584	0.01923	-0.05255	-0.08228	-0.12755	-0.01053	0.12790	0.03781	0.35124
Woolworths	0.02195	0.02139	-0.02987	0.04211	0.01009	-3.13309	0.07883	-0.00917	-0.02200	0.38556
Shoprite	-0.02234	-0.09496	-0.01631	-0.06149	-0.05031	-0.46606	0.03750	-0.09922	0.10957	0.18883
LIFE	0.14364	0.04985	-0.00913	0.02694	0.01181	1.79727	-0.01299	0.12349	0.14220	0.03072
Netcare	-0.04055	0.14546	0.02518	-0.10600	0.02007	-1.05260	0.01761	-0.09516	-0.15710	-0.34533
Sanlam	0.12379	-0.02644	-0.02532	0.02578	-0.08168	1.94414	0.06176	0.01433	0.24751	-0.07260
Discovery	-0.18370	0.05814	0.05149	0.26261	0.06743	4.70000	0.05866	-0.29646	-0.10610	-0.64388
Absa	-0.25292	0.05646	-0.11584	0.03068	-0.04228	0.89491	-0.07825	0.01630	-0.19539	-0.38899
Capitec	0.00341	0.02988	0.01171	-0.00116	-0.02003	0.63859	0.00110	-0.03706	-0.02940	0.05417
FirstRand	0.44069	-0.08774	0.02835	-0.04583	0.05793	-0.38255	0.01591	0.15294	0.08902	-0.73560
Standard Bank	0.00883	-0.21869	0.02891	-0.06864	-0.04145	-1.46712	-0.11681	-0.16055	-0.12054	0.43369
Investec	0.08750	-0.08318	-0.14964	0.08548	0.17938	-1.34909	-0.05786	0.07370	0.28515	0.71540
Coronation	-0.06600	-0.07696	0.01559	-0.34283	0.07618	2.43118	-0.02045	0.03272	-0.03477	0.44651
Naspers	-0.00337	-0.00852	0.04524	0.03076	0.07556	-0.64444	0.04552	-0.10249	-0.17449	-0.04678
MTN	-0.08520	-0.06502	0.05317	-0.10544	-0.04749	-0.05947	-0.05823	0.04371	-0.04286	0.22806
Sasol	-0.01358	0.07227	0.07650	0.09828	0.08841	-5.60889	0.11317	-0.01297	-0.12529	-0.37609
Anglo American Platinum	0.02577	0.01015	-0.03963	-0.01821	0.04396	0.95303	-0.00338	-0.06464	0.11368	-0.08800
PPC	0.05363	0.04835	0.04416	-0.06048	0.02397	0.49355	0.01131	0.12605	0.04255	-0.06269
Mr Price	0.05363	0.04835	0.04416	-0.06048	0.02397	0.49355	0.01131	0.12605	0.04255	-0.06269

The following table represents the exact same concepts however the state referring to in this table is representing regime 2. The values of beta can give a lot of information, if the beta values are less than 1 it theoretically means that it is less volatile than the market. The inverse of this is also true. Whereas when the beta value is negative then the asset is inversely correlated to financial market benchmark.

Table 4.3: Lagged Model beta coefficients for regime 2 continuing at the bottom.

	1	2	3	4	5	6	7	8	9	10
Tiger brands	0.24364	0.07119	0.05116	0.05685	-0.00134	0.01448	0.03504	-0.02972	0.25036	0.05243
RCL Foods	0.06392	-0.38727	0.00695	0.01053	0.02776	0.01548	-0.00954	-0.04278	0.07706	0.00273
Woolworths	0.10382	-0.03672	-0.18534	0.01807	0.03762	0.01595	0.05008	0.00749	-0.02748	-0.03552
Shoprite	0.04390	-0.05931	-0.04867	-0.15376	-0.01736	0.04842	0.13591	0.00404	-0.04746	0.04595
LIFE	0.01078	-0.00843	-0.00267	0.04777	-0.05533	-0.02874	0.17471	0.00525	0.04409	0.01770
Netcare	0.02954	0.09804	0.11046	-0.05526	0.03429	-0.20977	-0.19285	0.01277	0.06523	-0.02960
Sanlam	-0.01194	-0.09526	-0.08869	-0.08832	0.00334	-0.01587	-0.02199	0.06073	0.15374	-0.00038
Discovery	-0.08332	0.00268	0.05274	-0.03293	-0.02124	0.04182	0.04376	0.32173	-0.22828	0.05106
Absa	-0.12494	0.05666	-0.02661	-0.10489	-0.06492	0.05403	-0.07062	-0.20267	-0.38196	-0.08277
Capitec	-0.17282	-0.05572	-0.05985	-0.02504	-0.03240	0.11129	0.12367	0.08570	-0.17211	-0.01684
FirstRand	0.27679	-0.00034	-0.03808	0.06242	0.04023	-0.02292	0.12880	0.16163	0.50972	0.08533
Standard Bank	-0.11636	-0.06088	0.06041	-0.00256	0.01911	-0.18849	-0.19381	-0.06029	0.22454	-0.02047
Investec	0.06920	0.09684	-0.01891	0.02230	0.01581	-0.03027	0.07912	0.03580	0.20005	-0.03135
Coronation	-0.06881	-0.04815	0.01389	-0.02710	-0.05213	-0.06276	-0.04154	0.02511	0.09778	-0.01925
Naspers	-0.10365	0.00328	-0.02328	-0.03378	-0.01993	-0.06848	-0.03145	-0.12367	-0.16784	-0.03193
MTN	-0.06074	-0.00623	-0.04664	-0.05355	-0.00928	-0.01281	-0.08013	0.04738	-0.17934	-0.01606
Sasol	0.03544	0.04196	0.02692	0.05119	0.01188	0.10669	-0.05978	0.01484	-0.16483	0.00910
Anglo American Platinum	-0.01531	-0.00747	0.00764	-0.00158	-0.01449	-0.00112	0.06585	0.00258	-0.03105	0.02584
PPC	0.04738	0.00785	0.07195	0.02607	-0.00419	-0.02523	0.00962	-0.01537	0.06572	0.00357
Mr Price	-0.01382	0.06771	-0.03049	0.00987	0.00041	-0.05827	0.00503	-0.14011	-0.05054	-0.00571

	11	12	13	14	15	16	17	18	19	20
Tiger brands	0.17687	-0.26396	0.04852	0.10423	0.05862	0.04287	-0.16674	0.02923	0.05690	0.04416
RCL Foods	0.03348	0.13373	-0.01217	0.18429	0.07450	0.05287	0.09053	-0.00163	0.05427	0.01827
Woolworths	0.06125	-0.00342	0.10668	-0.07573	0.05632	0.00934	-0.10527	-0.02892	0.02961	-0.00829
Shoprite	-0.06638	0.19073	0.08974	0.13395	-0.03836	-0.00732	-0.12452	-0.04250	-0.02834	0.06800
LIFE	0.01963	0.12595	0.18739	0.07730	0.04727	0.02415	-0.01630	-0.03424	0.02404	0.03801
Netcare	0.01593	-0.37660	-0.15329	0.10343	0.04026	0.01425	-0.11654	-0.00036	-0.08216	0.06714
Sanlam	-0.01468	0.24343	0.02953	-0.00608	0.02491	-0.00847	-0.16697	-0.06630	-0.09919	0.03066
Discovery	0.05661	-0.30026	0.01831	-0.26873	-0.07126	0.00843	-0.12341	0.07909	0.04898	0.01617
Absa	-0.02671	-0.47404	0.00781	-0.18711	-0.11617	-0.05023	-0.17023	-0.05979	0.05702	-0.06075
Capitec	0.04450	0.14048	-0.04995	0.02410	0.05670	-0.02590	0.05718	0.00980	-0.08546	-0.04787
FirstRand	-0.38129	0.36315	0.01386	0.23008	-0.10780	-0.00073	0.11600	-0.05407	-0.03532	0.03375
Standard Bank	-0.02506	0.15206	-0.23205	0.07328	0.08735	-0.03873	-0.02392	-0.02510	0.03397	-0.05574
Investec	-0.01457	0.21799	0.09274	-0.15128	-0.09286	-0.02895	0.00043	0.02646	-0.12948	-0.05283
Coronation	-0.01047	0.19103	-0.07544	0.30518	-0.11073	0.01956	0.02005	0.05895	-0.02860	-0.02185
Naspers	-0.03382	-0.18109	-0.07887	-0.11769	-0.09618	0.00398	-0.14319	0.05686	-0.00275	-0.01387
MTN	-0.04846	0.05765	-0.14147	0.08287	-0.00142	-0.05643	0.18342	-0.00339	-0.01944	-0.01274
Sasol	0.09674	-0.00784	-0.09596	-0.16445	0.01378	0.03170	-0.16797	-0.02111	0.04748	0.03612
Anglo American Platinum	-0.00983	-0.02019	0.09040	0.02279	0.03387	0.01806	0.07558	0.04639	0.01467	0.00724
PPC	0.00076	-0.13201	-0.03124	0.07636	-0.04430	0.00052	0.02432	0.00545	-0.05021	-0.00661
Mr Price	-0.04200	0.10065	-0.00268	-0.09953	0.04217	0.02239	0.19092	0.08010	0.12775	-0.07578

These coefficients can be used to determine whether or not an asset or stock moves in the same direction as the financial markets. It may also help determine how risky and volatile an asset or stock is in comparison to the rest of the market. In a nutshell, the beta is used to determine how much risk a certain asset adds to a portfolio. For example in Table 4.3 Tiger brands have a positive relationship in the same direction as the market with the value of 0.24364. Whereas Absa have a negative relationship to the market with the beta value of -0.12494.

The next result for the lagged model is the table representing the transition matrix for the transition between the bear and bull markets. Table 4.4 denotes the transition probabilities of the transition between regime 1 to 2 or of remaining in the same state or regime.

Table 4.4: Transition Matrix for the lagged model.

	1	2
1	0.6741725	0.3258275
2	0.2873179	0.7126821

When the values of the transition matrix are analysed, a few conclusions could be made. The value of 0.6741725 which is currently in regime 1 is interpreted that there is a 67.4% chance of remaining in the current regime 1 state. Still currently in regime state 1 it has a 0.3258275 chance thus 32.58275% chance of switching from regime 1 to regime 2. This matrix can be interpreted as relative stable regimes with not as high probability of switching between the two regimes.

The next section is very similar however that section is based on the factor model whereas this section were done on the lagged variables.

4.2 FACTOR MODEL

This section is very similar to that of the lagged model in Section 4.1 however the regimes are now classified according to the risk factors at hand representing the economical risks that could be present. The chosen risk factors are global proxies market conditions such as the Fama-French factors and the Vix representing the volatility that plays a significant role. The choice for the 3-month JIBAR-SAFEX overnight rate spread is due to the fact that it can be used as a measure of fear in the South African market.

Figure 4.2 is the visual representation of the switching of regimes for the observed time period at hand. It represents the different states of the South African financial markets.

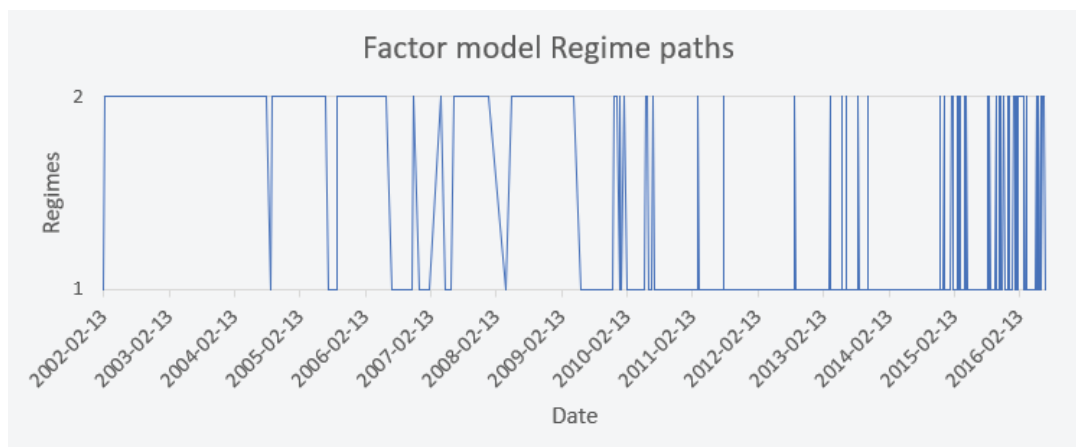


Figure 4.2: The factor model's regime movements over the time period.

For the factor model the methodology explained in Section 3.2.2 is followed to obtain the previous mentioned model of:

$$R_t = \alpha_{s_t} + \beta_{s_t} F_t + \Gamma_{s_t} e_t \quad (4.2)$$

First of all the α values are obtained for regime 1 and 2. These values represent the intercept corresponding to each factor based on the influence of the risk factors being used to represent the

market conditions.

The α 's for regime 1 and 2 are printed in the table below. It represents the intercept. When interpreting these intercepts we know that in the linear model it represents the expected mean values. Table 4.5 represents the alphas (α) for regime 1 and 2, alpha measures how closely a stock's returns match or surpass those generated by the market. A negative alpha, such as MTN for regime 1 is seen to be -0.10933 and shows that the asset is not generating returns at the same rate as the broader sector, compared to Anglo American Platinum that has a positive value for regime 1 of 0.02017.

Table 4.5: Factor model alphas for regime 1 and 2.

Factor Alphas	Regime 1	Regime 2
Tiger brands	0.01858	-0.00763
RCL Foods	0.00214	0.00897
Woolworths	-0.00380	0.00921
Shoprite	-0.00391	0.00361
LIFE	0.00050	0.03439
Netcare	-0.00869	0.00276
Sanlam	-0.00652	0.00050
Discovery	-0.00747	0.00464
Absa	0.00259	-0.01356
Capitec	0.00122	-0.95194
FirstRand	-0.00720	-0.02345
Standard Bank	-0.00978	0.00286
Investec	-0.01598	0.00498
Coronation	0.00883	0.00212
Naspers	0.00663	-0.01768
MTN	-0.10933	-0.00259
Sasol	-0.00436	0.01671
Anglo American Platinum	0.02017	-0.00510
PPC	0.00535	-0.00424
Mr Price	0.00476	-0.00483

Continuing the next values that were calculated were those of the β matrix. These values represents the sensitivities of the assets returns in the South African constructed portfolio and the common risk factors found at each state. Estimates of the regime-dependent parameters and predictions for

the values of the factors are required for the linear factor model to be useful for forecasting returns over time.

A beta coefficient can be used to relate the volatility of a single stock to the whole market's systematic risk. In statistics, beta is the slope of a line resulting from a regression of data points. According to Abdymomunov and Morley (2011) the coefficient β is an effective way of describing the movements of an asset's returns as it responds to swings in the market. Therefore, the use of regimes.

Table 4.6 as well as Table 4.7 reflects the beta coefficients for regime 1 and 2 respectively. These coefficients can help with understanding whether the asset or stock moves in the same direction as the financial markets or not. It can also assist with the process of assessing how risky and volatile the asset or stock is compared to the rest of the market. In short the beta is being used to measure how much risk is being added to a portfolio by the certain asset.

For example the -0.00130 beta value between Tiger brands and MktRF shows that there is a negative correlation or relationship between the variable and the market. It reacts different compared to the market reaction. However, it is a very small difference and not very significant. The beta value of 0.27871 between MTN and Vix have a stronger positive relationship than the previous example and could be considered more significant.

Table 4.6: Factor Model beta coefficients for regime 1.

	MktRF	SMB	HML	Vix	Spread
Tiger brands	-0.00130	0.00539	-0.00141	-0.05695	-0.02478
RCL Foods	0.00132	0.00009	-0.00018	-0.01243	-0.00162
Woolworths	0.00137	0.00269	0.00101	0.03304	-0.00143
Shoprite	0.00261	-0.00062	-0.00059	0.01921	0.00098
LIFE	0.00297	-0.00013	-0.00020	0.02259	-0.00751
Netcare	0.00447	0.00293	-0.00530	0.03687	0.00523
Sanlam	0.00221	0.00288	-0.00215	0.02546	0.00745
Discovery	0.00321	-0.00196	-0.00088	0.01036	0.00839
Absa	0.00362	-0.00600	0.00047	0.00034	-0.00835
Capitec	0.00246	0.00026	-0.00005	0.00217	-0.00086
FirstRand	0.00341	-0.00013	0.00030	0.02697	0.00828
Standard Bank	0.00134	-0.00127	0.00082	0.02613	0.01296
Investec	0.00268	0.00382	-0.00028	0.08912	0.00877
Coronation	0.00305	-0.00115	-0.00214	0.00292	-0.01941
Naspers	0.00495	-0.00102	-0.00642	-0.00768	-0.00880
MTN	0.00675	-0.01391	0.02256	0.27871	0.07967
Sasol	0.00458	-0.00102	-0.00082	0.04740	-0.00746
Anglo American Platinum	0.00373	-0.00163	0.00102	0.08184	-0.05807
PPC	0.00286	-0.00235	-0.00030	-0.01433	-0.00891
Mr Price	0.00289	0.00162	-0.00065	0.00158	-0.00815

As can be seen in Table 4.6 and 4.7 the assets doesn't deviate significantly from the market and doesn't add a significant amount of risk to the portfolio. However there is also always a trade-off between risk and return low risk is related to low returns. All of this information is also true for the beta coefficients for the lagged model.

The values of beta can give a lot of insight for example if the beta values are less than 1 it theoretically means that it is less volatile than the market. The inverse of this is also true. Whereas when the beta value is negative then the asset is inversely correlated to financial market benchmark.

Table 4.7: Factor Model beta coefficients for regime 2.

	MktRF	SMB	HML	Vix	Spread
Tiger brands	0.00272	-0.00268	0.00081	0.03644	0.00644
RCL Foods	0.00342	-0.00335	-0.00149	-0.01999	-0.00945
Woolworths	0.00709	-0.00421	-0.00364	-0.03479	-0.01324
Shoprite	-0.00204	-0.00006	0.00507	-0.01704	-0.00566
LIFE	-0.02003	0.00576	0.01699	-0.04947	-0.04762
Netcare	0.00273	-0.00091	0.00068	-0.00751	-0.00366
Sanlam	0.00542	-0.00049	0.00240	0.01461	-0.01222
Discovery	0.00192	0.00080	-0.00029	-0.00641	-0.00553
Absa	0.00328	0.00454	0.00160	0.03883	0.01339
Capitec	0.05940	0.19347	0.24428	1.57245	1.38397
FirstRand	0.00028	0.00246	-0.00144	0.04975	0.02201
Standard Bank	0.00651	0.00507	0.00167	0.02338	-0.02017
Investec	0.00533	-0.00218	0.00021	-0.05333	-0.00094
Coronation	0.00353	0.00404	0.00138	0.04239	-0.01698
Naspers	0.00228	0.00366	0.00123	0.03207	0.01783
MTN	0.00200	0.00164	0.00002	0.00560	0.00246
Sasol	0.00074	0.00333	0.00798	-0.08149	-0.00954
Anglo American Platinum	0.00150	0.00241	0.00343	0.01211	0.00260
PPC	0.00175	0.01016	0.00296	0.01224	0.00289
Mr Price	-0.00222	-0.00122	-0.00304	-0.08117	0.02519

The beta coefficient assumes that the returns are normally distributed however, this is not the case because the South African financial market is susceptible for large shocks. In reality it is also not always normally distributed therefore, one should be careful when interpreting the beta coefficients.

Table 4.8 is the result for the gamma matrix of Section 3.2.2. It denotes the transition probabilities of the transition between regime 1 to 2 or of remaining in the same state or regime. This matrix can be interpreted as relative stable regimes with not as high probability of switching between the two regimes.

With this transition matrix, one can perform matrix multiplication and determine trends if there are any present and one can use this for predictions.

Table 4.8: Transition matrix for the factor model.

	1	2
1	0.8240015	0.1759985
2	0.2772869	0.7227131

This then concludes the Markov regime switching model for both the lagged and factor portfolios. In the next chapter the optimisation will be done testing different scenarios to see if whether adding regimes to the optimisation actually improves the risk return trade-off.

4.3 OPTIMISATION

The goal is to allocate capital to risky assets such that the investment goals are met. This is usually measured in terms of risk and return. Referring to the alpha and beta matrices (earlier in this chapter) were obtained in this case the alpha measured the excess return and the regime dependent beta denoted the measurement of risk. The objective is to maximise the return incorporating risk exposure constraints. These constraints are that:

- When the weights are calculated it should add up to one. Mathematically this can be expressed as: w_{ti} for the portfolio weight for the time period t for asset i . This will then be: $\sum_{i=1}^I w_{ti} = 1$.
- The other constraint that were imposed were that the minimum sum of the weights should at least be 0.99 and the maximum sum of the weights can be 1.01.

According to Ma, MacLean, Xu and Zhao (2011) if the regime in period $t - 1$ is m and the portfolio weights for period t is $w'_t = (w_{t1}, \dots, w_{tI})$, then the one-period expected portfolio alpha is:

$$\Psi_m(w_t) = E[A'_{s_t} w_t \mid s_{t-1} = m] = \sum_{n=1}^N \sum_{i=1}^I w_{ti} \alpha_{in} p_{mn} \quad (4.3)$$

As a result, when new information is obtained at each decision point in time, the unconditional alpha with respect to the posterior probability of the regimes may be determined. The regime-dependent portfolio beta is constrained to adjust for systematic risk. Although risk aversion is

desirable, taking certain risks can result in significant gains in returns. To allow for a limited exposure to the common risk factors, a regime-dependent risk tolerance parameter δ is added. In each possible regime in the next period, the portfolio beta for factor j in regime n is defined as

$$\Phi_{jn}(w_t) = \sum_{i=1}^I w_{ti} \beta_{ijn}, \quad \forall j = 1, \dots, J, \text{ and } n = 1, \dots, N. \quad (4.4)$$

Therefore, the portfolio risk exposure is constrained as:

$$-\delta_n \leq \Phi_{jn}(w_t) \leq \delta_n, \quad \forall j = 1, \dots, J, \text{ and } n = 1, \dots, N \quad (4.5)$$

The tolerance parameter δ could depend on the particular factor and regime. With the reformulation of the constraints and objective, the portfolio optimisation for period t is determined from the following stochastic linear programming problem:

$$\begin{aligned} \max_{w_t} \quad & \Psi_m(w_t) \\ \text{s.t.} \quad & \sum_{i=1}^I w_{ti} = 1, \\ & -\xi_l \leq w_{ti} \leq \xi_u, i = 1, \dots, I. \end{aligned} \quad (4.6)$$

This will then be used to maximise the Sharpe ratio. The formula for calculating the Sharpe ratio is given by:

$$SR = \frac{R_p - R_f}{\sigma_p} \quad (4.7)$$

where: R_p is the return of the portfolio, R_f is the risk-free rate and σ_p is the standard deviation of the portfolio's excess return. This will be calculated for both cases.

Therefore, using the equations above the maximum Sharpe ratio could then be expressed as:

$$\max SR = \frac{\sum_{n=1}^N \sum_{i=1}^I w_{ti} \alpha_{in} p_{mn}}{\sum_{i=1}^I w_{ti} \beta_{ijn}} \quad (4.8)$$

which is simply Equation 4.3, divided by Equation 4.4. This is including the unobserved regimes.

The assumption of a structure for asset returns has significant ramifications for investing decisions. There is an unknown regime in each period, as well as a returns distribution for assets that are conditional on the regime. Transitions between regimes are also Markovian, with a constant probability matrix for transitions. The investment choice is made at the start of each period, based on the previous period's regime and the probability of switching to each of the current period's alternative regimes.

The constraints that were mentioned earlier were all considered when programming the outcomes and were fulfilled in the results below. As part of the Markov regime switching model it was possible to obtain the regimes for each day that were in our observation time period and these regimes were used in the optimisation.

There is an unknown regime in each period, as well as distributions for returns of the assets that are dependent on the regime. Transitions between regimes are also Markovian, with a constant probability matrix for transitions. The investment choice is made at the start of each period, based on the previous period's regime and the probability of switching to each of the current period's alternative regimes. Therefore the portfolio should be rebalanced for a constant chosen period to see whether the weights should change corresponding to the regime it is in and the possible risk appetite.

Transaction costs for rebalancing are not considered. The portfolios were rebalanced periodically, with each investment period consisting of 3 months therefore rebalancing quarterly for a total of 33 investment periods. The portfolio is then held constant or static for the duration of this investment period whereafter the process is repeated for the next period. According to Costa and Kwon (2019) these regime switching models are expected to perform better due to the fact of re-estimating the regime-dependent parameters at the start of each investment period. The implications of correctly identifying a change in the regimes at the start of the new investment period to reflect the current markets and then continuing by re-optimising the portfolio could have a tremendous positive effect on the investment goals.

In theory it would be more beneficial to rebalance the portfolio more frequently to mimic the market. This means that it takes the market conditions into consideration due to the new classification of regimes and can therefore perform better and prevent losses. However in practice it is not really the case to do it frequently due to the transaction costs and the labour needed to do this. Thus, the chosen quarter period is not unrealistic for practice.

The lagged and factor model will now be examined based off of the above explanation. For both the models these facts are true and implemented, making it possible to compare the results that were obtained. There were under each model to methods of optimisation used the first being the Sharpe ratio method and the other method being the risk budget method. For methods used the portfolio were rebalanced quarterly.

However, estimators of the means and the variance-covariance matrices of the asset classes are necessary to perform the two-moment optimisation of the classical portfolio theory. According to Seidl *et al.* (2012) the arithmetic mean is used as a mean estimator in the classical optimisation case, and the covariance is used for the variance-covariance matrix. The mean and variance-covariance matrices are computed conditional on the current state in the regime switching framework. Hence the following computation is done for the mean:

$$\mu_{s_t} = \frac{1}{n_{s_t}} \sum_{i=1}^{n_{s_t}} x_i, \quad (4.9)$$

where n_{s_t} is the number of x_i in the regime and the x_i is the asset returns in the portfolio.

$$\Sigma_{s_t} = (\text{Cov}(x_i, x_j))_{i,j=1,\dots,n_{s_t}}. \quad (4.10)$$

for the variance covariance computation.

Then the optimal unconditional portfolio weights at time $t + 1$ are then calculated as follow:

$$\begin{aligned} upw(t) = & \xi_{1t} (P(1, 1)w_{1t} + (1 - P(1, 1))w_{2t}) \\ & + (1 - \xi_{1t}) (P(2, 2)w_{2t} + (1 - P(2, 2))w_{1t}) \end{aligned} \quad (4.11)$$

with w_{it} being the optimal portfolio weights conditional on regime i . ξ_{it} being the probability of being in the regime i . $P(1, 1)$ and $P(2, 2)$ are the transition probabilities from the transition matrix.

The risk budget approach, which is very similar to the Sharpe ratio, is then used. The fundamental concept is to allow for time-varying risk budgets that can take into account changing performance projections for various assets. The paper by Chakravorty, Awasthi, Singhal, Gupta and Srivastava (2019) states that the risk budgets are proportionate to the expected Sharpe Ratio of various securities, then normalized so that the total risk budget is 1. The risk budgets are then scaled by the target risk to compare with risk contribution.

$$RB_{st} = \frac{|E[\text{Sharpe Ratio}_{st}]|}{(\sum_s |E[\text{Sharpe Ratio}_{st}]|)} * \tau \quad (4.12)$$

where, $E[\text{Sharpe Ratio}_{st}]$ is the expected Sharpe Ratio at time t for stock or asset s , τ is the target risk of the allocation, RB_{st} is the risk budget for stock or asset s at time t . For this assignment the maximum risk allocation allowed was 50%. The other measures such as the weights, mean covariance etc. is calculated the same as described in Equation 4.8 onward.

Thereafter an additional section is added where the optimal weights were calculated for each category used for investments. Thus splitting up the portfolio into categories and optimising correspondingly to each category individually. The results will be explained next.

4.4 OPTIMISATION OF THE LAGGED MODEL

The lagged model were optimised using the regimes calculated by the Markov regime switching model in the previous chapter. The results will follow for the Sharpe ratio and risk budget methods along with the rebalancing of each method.

4.4.1 Sharpe method for the lagged model.

The main objective of this method being implemented, alongside the constraints is that the returns corresponding to the risk should be maximised. In other words, the Sharpe ratio indicates how well it has performed in relation to the risk it has taken on. The Sharpe ratio may be used to compare risk-adjusted returns across all fund types since it uses the standard deviation. This method also incorporated return on investment to get the best results.

Table 4.9: Sharpe ratio as method of optimisation for regime 1 under the lagged model.

	Code	Optimal Weights
Tiger brands	TBS.JO	0.336
RCL Foods	RCL.JO	0.152
Woolworths	WHL.JO	0.024
Shoprite	SHP.JO	0.000
LIFE	LHC.JO	0.050
Netcare	NTC.JO	0.192
Sanlam	SLM.JO	0.000
Discovery	DSY.JO	0.000
Absa	ABG.JO	0.002
Capitec	CPI.JO	0.044
FirstRand	FSR.JO	0.000
Standard Bank	SBK.JO	0.042
Investec	INL.JO	0.000
Coronation	CML.JO	0.146
Naspers	NPN.JO	0.006
MTN	MTN.JO	0.000
Sasol	SOL.JO	0.000
Anglo American Platinum	AMS.JO	0.000
PPC	PPC.JO	0.000
Mr Price	MRP.JO	0.006
Total		1
Mean		0.0005771
Standard deviation		0.01109

Comparing Table 4.9 and 4.10 it can be seen that the weighting for the bull and bear regimes differ significantly. The weightings also indicate how the diversification is done and what risks could be taken. The weightings for the bull market of regime 1 differ significantly to regime 2, this is due to

the fact that regime 2 diversifies the portfolio to be safer in an unstable market whereas in regime 1 some chances can be taken because the economy is more stable and the prices are not declining dramatically in which risks should be avoided. This is also the reason why the standard deviation differs slightly for the two regimes. The standard deviation indirectly reflects the risk taken and in regime 1 the standard deviation is larger than in regime 2 as well as the means.

Table 4.10: Sharpe ratio as method of optimisation for regime 2 under the lagged model.

	Code	Optimal Weights
Tiger brands	TBS.JO	0.170
RCL Foods	RCL.JO	0.194
Woolworths	WHL.JO	0.022
Shoprite	SHP.JO	0.000
LIFE	LHC.JO	0.002
Netcare	NTC.JO	0.224
Sanlam	SLM.JO	0.000
Discovery	DSY.JO	0.116
Absa	ABG.JO	0.000
Capitec	CPI.JO	0.062
FirstRand	FSR.JO	0.002
Standard Bank	SBK.JO	0.004
Investec	INL.JO	0.076
Coronation	CML.JO	0.034
Naspers	NPN.JO	0.004
MTN	MTN.JO	0.032
Sasol	SOL.JO	0.006
Anglo American Platinum	AMS.JO	0.018
PPC	PPC.JO	0.000
Mr Price	MRP.JO	0.028
Total		0.994
Mean		0.0003592
Standard deviation		0.01089

It can be seen that Table 4.9 satisfies the first condition mentioned before Section 5.1 that the sum of the weights equal to 1. Whereas Table 4.10 satisfies the second condition that it lies between 0.99 and 1.01.

In regime 1, the weight was of Anglo American Platinum was 0, but this changed in regime 2. This gives evidence that inclusion of regime switching is economic significant. The opposite also reflects this because Tiger brands had a decline in the weight of 0.336 to 0.170 from regime 1 to regime 2 respectively. For the lagged model Sanlam had a 0 weight in both portfolios for regime 1 and 2. However when the risk budget method were used this was not the case as seen in Table 4.13 and Table 4.14. This shows the significance in the measures used to compute the optimal portfolio weights.

4.4.2 Rebalancing the lagged model for the Sharpe ratio method.

Rebalancing is were the portfolio is reclassified into a regime and according to that it gets new weightings therefore buying and selling some of the stocks to obtain the new investment strategy according to the regime the economy is classified as. The portfolio can't be held the exact same for the duration of time, it needs to change to incorporate the economic cycles.

The portfolio were rebalanced quarterly resulting in the statistics given below. Thus taking the first and last rebalancing dates into account and working with quarters, 33 rebalances were done as shown in Table 4.11.

Table 4.11: Statistics of the rebalancing of the portfolio under the Sharpe ratio method.

Number of rebalancing dates	33
First rebalance date	2007-08-31
Last rebalance date	2016-02-26
Annualized Portfolio Rebalancing Return	-0.001761423
Annualized Portfolio Standard Deviation	0.08388356

Table 4.12: Downside risk measures.

Downside Risk Measures	Portfolio Returns
Semi Deviation	0.0085
Gain Deviation	0.0070
Loss Deviation	0.0083
Downside Deviation (MAR=43.33%)	0.0136
Downside Deviation (Rf=0%)	0.0085
Downside Deviation (0%)	0.0085
Maximum Drawdown	0.3103
Historical VaR (95%)	-0.0196
Historical ES (95%)	-0.0276
Modified VaR (95%)	-0.0196
Modified ES (95%)	-0.0289

In Table 4.12 the negative Historical VaR is actually a good thing implying that the portfolio has a high probability of making a profit. When compared to standard deviation alone, downside deviation offers you a better understanding of how much an investment may lose. Standard deviation is a measure of volatility on the upside and downside, and it only gives you a partial picture. The downside deviations of two assets with the same standard deviations are likely to differ. When a “risky” investment with a high standard deviation has a low downside deviation, it is likely to be safer than it appears. In this case we can see that all the downside deviations are relatively small.

Figure 4.3 on the next page is simply a visual representation of the weightings assigned to each asset in the lagged portfolio. It gives an indication of where most of the capital were allocated to during which period. This is a reflection of the rebalancing of the weights to optimise the portfolio.

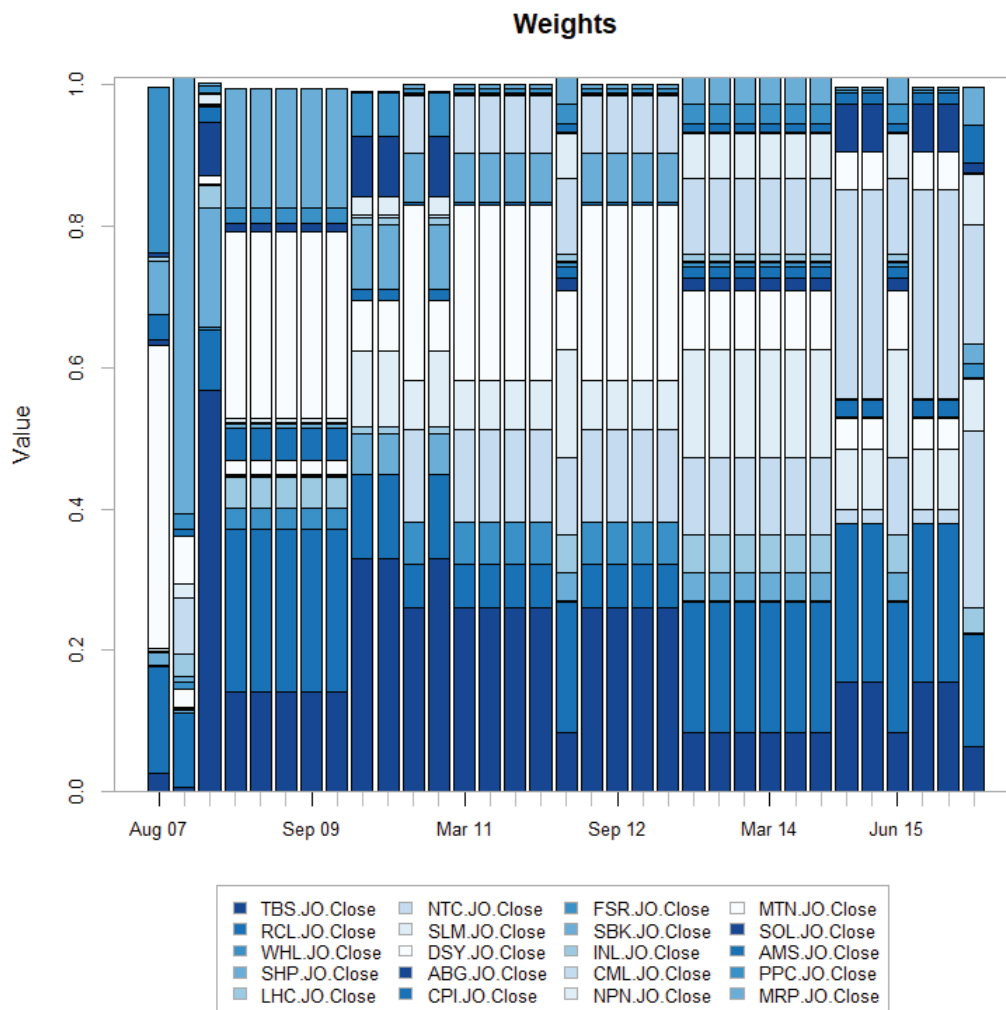


Figure 4.3: Weighting chart of the weights assigned in the portfolio for the time period.

4.4.3 Risk budget method for the lagged model.

This new method for optimising the lagged model is a popular risk-based asset allocation technique where risk budgets are assigned to each assets' risk contribution. This risk budgeting technique has the goal to diversify the risk in the portfolio by assigning target risk contributions to each component.

Table 4.13: Risk budget as method of optimisation for regime 1 under the lagged model.

	Code	Optimal Weights
Tiger brands	TBS.JO	0.066
RCL Foods	RCL.JO	0.304
Woolworths	WHL.JO	0.014
Shoprite	SHP.JO	0.002
LIFE	LHC.JO	0.010
Netcare	NTC.JO	0.106
Sanlam	SLM.JO	0.010
Discovery	DSY.JO	0.014
Absa	ABG.JO	0.006
Capitec	CPI.JO	0.066
FirstRand	FSR.JO	0.000
Standard Bank	SBK.JO	0.016
Investec	INL.JO	0.014
Coronation	CML.JO	0.144
Naspers	NPN.JO	0.014
MTN	MTN.JO	0.004
Sasol	SOL.JO	0.062
Anglo American Platinum	AMS.JO	0.024
PPC	PPC.JO	0.094
Mr Price	MRP.JO	0.030
Total		1
Standard Deviation		0.01097

The two Tables of 4.13 and 4.14 compare quite well for the two regimes. There aren't major differences as seen in the Sharpe ratio method for the lagged model. The weightings differ for the two regimes due to the risk contributions of the assets to the portfolio. The standard deviations also compare well considering that they are relatively close to one another and does not differ significantly.

Table 4.13 satisfies the condition that the sum of the weights should equal 1 whereas, Table 4.14 lies in the second condition's interval which is between 0.99 and 1.01. Both these values satisfies the necessary conditions.

Table 4.14: Risk budget as method of optimisation for regime 2 under the lagged model

	Code	Optimal Weights
Tiger brands	TBS.JO	0.084
RCL Foods	RCL.JO	0.212
Woolworths	WHL.JO	0.012
Shoprite	SHP.JO	0.040
LIFE	LHC.JO	0.054
Netcare	NTC.JO	0.074
Sanlam	SLM.JO	0.020
Discovery	DSY.JO	0.000
Absa	ABG.JO	0.020
Capitec	CPI.JO	0.034
FirstRand	FSR.JO	0.000
Standard Bank	SBK.JO	0.004
Investec	INL.JO	0.016
Coronation	CML.JO	0.188
Naspers	NPN.JO	0.008
MTN	MTN.JO	0.000
Sasol	SOL.JO	0.184
Anglo American Platinum	AMS.JO	0.054
PPC	PPC.JO	0.000
Mr Price	MRP.JO	0.004
Total		1.008
Standard Deviation		0.01121

4.4.4 Rebalancing the lagged model for the risk budget method.

The portfolio were rebalanced quarterly resulting in the statistics given below. The reason for this is that rebalancing could be a costly process, doing it very frequently would not be cost effective however it can't be ignored. Quarterly therefore is a good middle ground for doing it frequently enough to identify some cyclical trends and not too frequent that it would be considered unprofitable. Thus taking the first and last rebalancing dates into account and working with quarters, 33 rebalances were done as shown in Table 4.15.

Table 4.15: Statistics of the rebalancing of the portfolio under the Risk Budget method.

Number of rebalancing dates	33
First rebalance date	2007-08-31
Last rebalance date	2016-02-26
Annualized Portfolio Rebalancing Return	0.012159
Annualized Portfolio Standard Deviation	0.08668535

The regime at each rebalancing date could also be obtained this is given in Table A.1 in the Appendix. Together with the rebalanced regimes the optimal weightings at each rebalanced investment period were also obtained and this can be seen in Table A.2. It is important to note that these tables were obtained for all the cases however it will only be shown for this method and tests as an example of how the tables are composed, however this is not the important part to focus on. The information obtained resulting from different calculations using these weights are more essential to analyse and will be discussed throughout the study.

Table 4.16: Downside risk measures.

Downside Risk Measures	Portfolio Returns
Semi Deviation	0.0090
Gain Deviation	0.0071
Loss Deviation	0.0089
Downside Deviation (MAR=43.33%)	0.0138
Downside Deviation (Rf=0%)	0.0088
Downside Deviation (0%)	0.0088
Maximum Drawdown	0.2395
Historical VaR (95%)	-0.0201
Historical ES (95%)	-0.0285
Modified VaR (95%)	-0.0206
Modified ES (95%)	-0.0344

For all the drawdown deviations the Sharpe ratio method in Table 4.12 gave more desirable results. However, with the risk budget method the chances of making a profit due to the negative VaR is higher and the maximum drawdown is lower. Maximum drawdown seeks the most significant shift from a high to a low point before a new peak is reached. It's vital to remember, however, that it only considers the amount of the highest loss and ignores the frequency of significant losses. Maximum drawdown does not reveal how long it took an investor to recover from a loss, or even if

the investment recovered at all, because it only gauges the biggest drawdown. Maximum drawdown is a risk indicator that focuses on capital preservation, which is a major concern for most investors. It is used to compare the relative riskiness of one stock screening approach to another (Leal and de Melo Mendes, 2005).

Figure 4.4 is a graphical illustration of how the weights are assigned to each asset in the portfolio. The weights were rebalanced quarterly and therefore there are 33 columns representing the rebalanced investment periods. These values were calculated and Figure 4.4, gives an overview of how the portfolio were weighted. It can also be seen were the regimes were switched at the start of the investment period and thus the rebalanced weighting.

Figure 4.5 is to show that the risk contribution can be obtained using different measures. On the left expected shortfall were used and on the right the standard deviation. Each asset in the portfolio can then be seen as to what their contribution is to the risk of the portfolio.

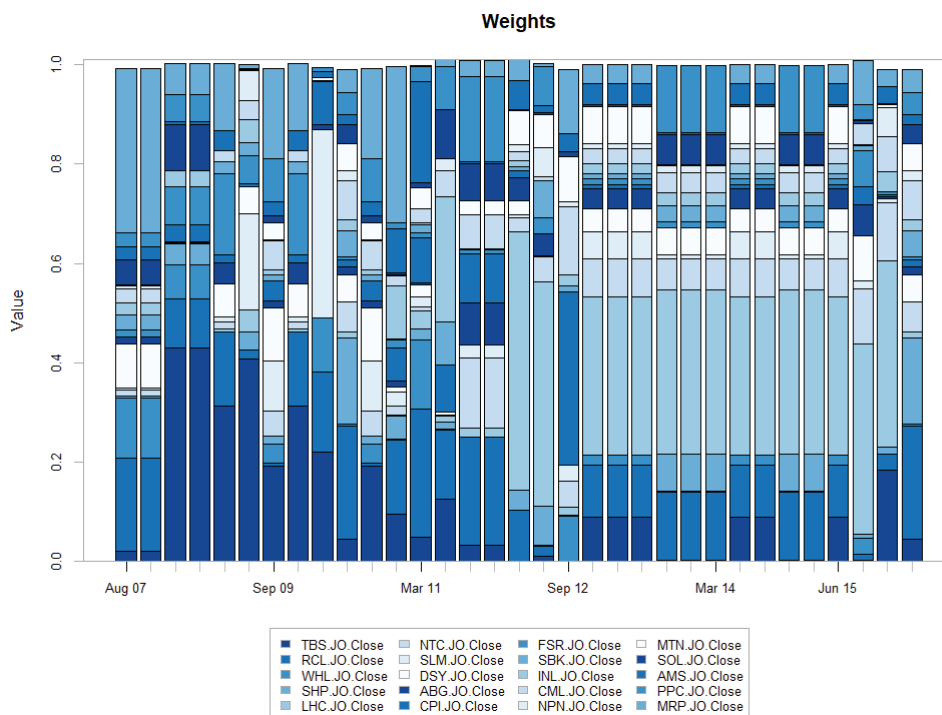


Figure 4.4: Weighting chart of the weights assigned in the portfolio for the time period using the risk budget method.

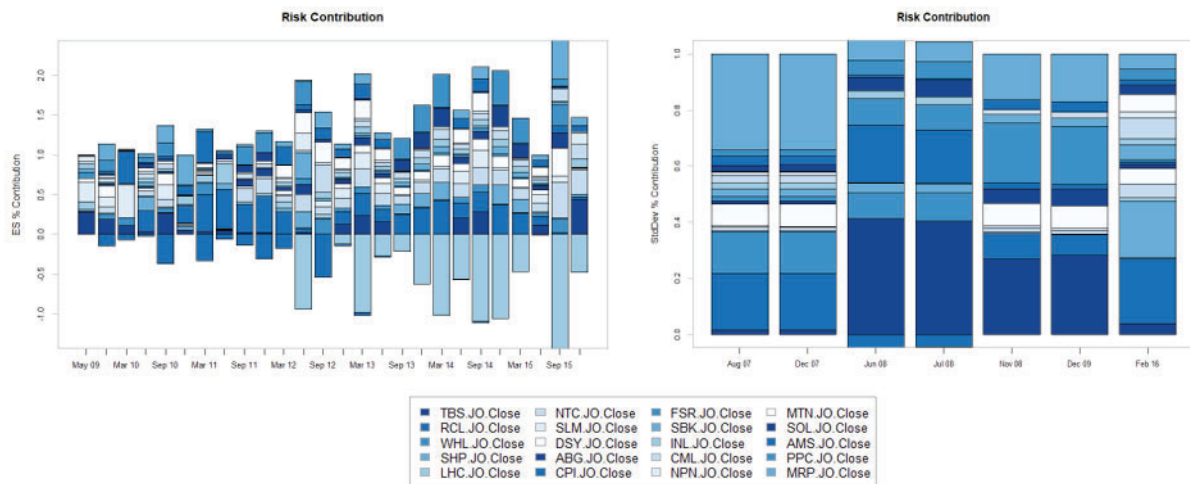


Figure 4.5: The left hand side is the risk contribution for regime 1 which were done by using Expected shortfall. The right hand side shows the risk contribution for regime 2 which were done by using the standard deviation.

4.5 OPTIMISATION OF THE FACTOR MODEL

Next we will look at the two different methods that are the Sharpe ratio method and risk budget method respectively for the factor model. This section is based on the same principles that were explained in lagged model situation. The regimes used for this optimisation were calculated from the Markov regime stitching model for the factor model. This section now incorporates the calculated regimes from the factors and uses that as a basis for the optimisation using the different methods. First the Sharpe ratio method results will be explained including the rebalancing and thereafter the risk budget method along with its rebalancing will be explained.

4.5.1 Sharpe method for the factor model.

The main objective of this method, alongside the constraints explained before Section 5.1, is that the returns corresponding to the risk should be maximised. In other words, the Sharpe ratio indicates how well it has performed in relation to the risk it has taken on. The Sharpe ratio may be used to compare risk-adjusted returns across all fund types since it uses the standard deviation.

When comparing Table 4.17 and 4.18 it can be seen that the weighting differ from regime 1 to regime 2. The weightings for the portfolio of regime 2 is more diversified than regime 1 this is due to the fact that the South African economical state is in a downturn therefore it is to risky and the investor should diversify the portfolio to mitigate some of the risk. This is reflected in the means of the two tables for the two regimes. Regime 2 has a smaller mean than regime 1 and this is to be expected. However what is interesting is that the two standard deviations are very closely related to one another.

Table 4.17: Optimal Sharpe ratio method weightings of the factor model for regime 1.

	Code	Optimal Weights
Tiger brands	TBS.JO	0.1120
RCL Foods	RCL.JO	0.1560
Woolworths	WHL.JO	0.0260
Shoprite	SHP.JO	0.0040
LIFE	LHC.JO	0.0837
Netcare	NTC.JO	0.1526
Sanlam	SLM.JO	0.0014
Discovery	DSY.JO	0.0320
Absa	ABG.JO	0.0717
Capitec	CPI.JO	0.0629
FirstRand	FSR.JO	0.0260
Standard Bank	SBK.JO	0.0042
Investec	INL.JO	0.0640
Coronation	CML.JO	0.1147
Naspers	NPN.JO	0.0068
MTN	MTN.JO	0.0000
Sasol	SOL.JO	0.0349
Anglo American Platinum	AMS.JO	0.0370
PPC	PPC.JO	0.0000
Mr Price	MRP.JO	0.0040
Total		0.9939
Mean	0.000444	
Standard deviation	0.01084	

Above is the table for the optimal weightings for regime 1 and below for regime 2. After regime 2's table the rebalancing results for the factor model will be examined for the Sharpe ratio method. This is will also be compared to the lagged model.

Table 4.17 satisfies the constraint that the optimal weights should either sum to 1 or lie inside the interval of 0.99 and 1.01. Thus Table 4.17 satisfies the interval constraint with the sum of the optimal weights adding up to 0.9939.

Table 4.18: Optimal Sharpe ratio method weightings of the factor model for regime 2.

	Code	Optimal Weighting
Tiger brands	TBS.JO	0.044
RCL Foods	RCL.JO	0.226
Woolworths	WHL.JO	0.040
Shoprite	SHP.JO	0.058
LIFE	LHC.JO	0.028
Netcare	NTC.JO	0.144
Sanlam	SLM.JO	0.004
Discovery	DSY.JO	0.036
Absa	ABG.JO	0.006
Capitec	CPI.JO	0.048
FirstRand	FSR.JO	0.028
Standard Bank	SBK.JO	0.022
Investec	INL.JO	0.004
Coronation	CML.JO	0.104
Naspers	NPN.JO	0.008
MTN	MTN.JO	0.072
Sasol	SOL.JO	0.032
Anglo American Platinum	AMS.JO	0.000
PPC	PPC.JO	0.044
Mr Price	MRP.JO	0.046
Total		0.994
Mean	0.0003223	
Standard deviation	0.01085	

The total of Table 4.18 satisfies the constraints.

4.5.2 Rebalancing using the Sharpe ratio method for the factor model.

The portfolio were rebalanced quarterly resulting in the statistics given below. Thus taking the first rebalancing date of 2007-08-31 and last rebalancing date of 2016-02-19 into account and working with quarters, 33 rebalances were done as shown in Table 4.19.

Table 4.19: Statistics of the rebalancing of the portfolio under the Sharpe ratio method.

Number of rebalancing dates	33
First rebalance date	2007-08-31
Last rebalance date	2016-02-26
Annualized Portfolio Rebalancing Return	0.00629372
Annualized Portfolio Standard Deviation	0.08241688

Table 4.20: Downside risk measures.

Downside Risk Measures	Portfolio Returns
Semi Deviation	0.0085
Gain Deviation	0.0067
Loss Deviation	0.0085
Downside Deviation (MAR=43.33%)	0.0134
Downside Deviation (Rf=0%)	0.0084
Downside Deviation (0%)	0.0084
Maximum Drawdown	0.2755
Historical VaR (95%)	-0.0186
Historical ES (95%)	-0.0272
Modified VaR (95%)	-0.0197
Modified ES (95%)	-0.0334

The negative Historical in Table 4.20 implies that the portfolio has a high probability of making a profit. When comparing this table to Table 4.12 of the Sharpe ratio method of the lagged model it can be concluded that for the Sharpe ratio method the lagged model performed better for the value at risk measures. However when examining the downside deviations the portfolio based on the factors performed better. This might be due to the fact that the factors are directly related to the markets that influence the assets in the portfolio.

Figure 4.6 on the next page is simply a visual representation of the weightings assigned to each asset in the factor portfolio. It gives an indication of where most of the capital were allocated to during which period. This is a reflection of the rebalanced weights that optimise the portfolio.

When having a closer look to Figure 4.6 it can be seen as to where the switches in the regimes too place at the start of the investment period. This is where the blue coloured size blocks differ for

each investment period compared to the investment periods where the sizes of the blocks stayed the same (in other words the weightings did not differ) and no re-optimisation and rebalancing were done.

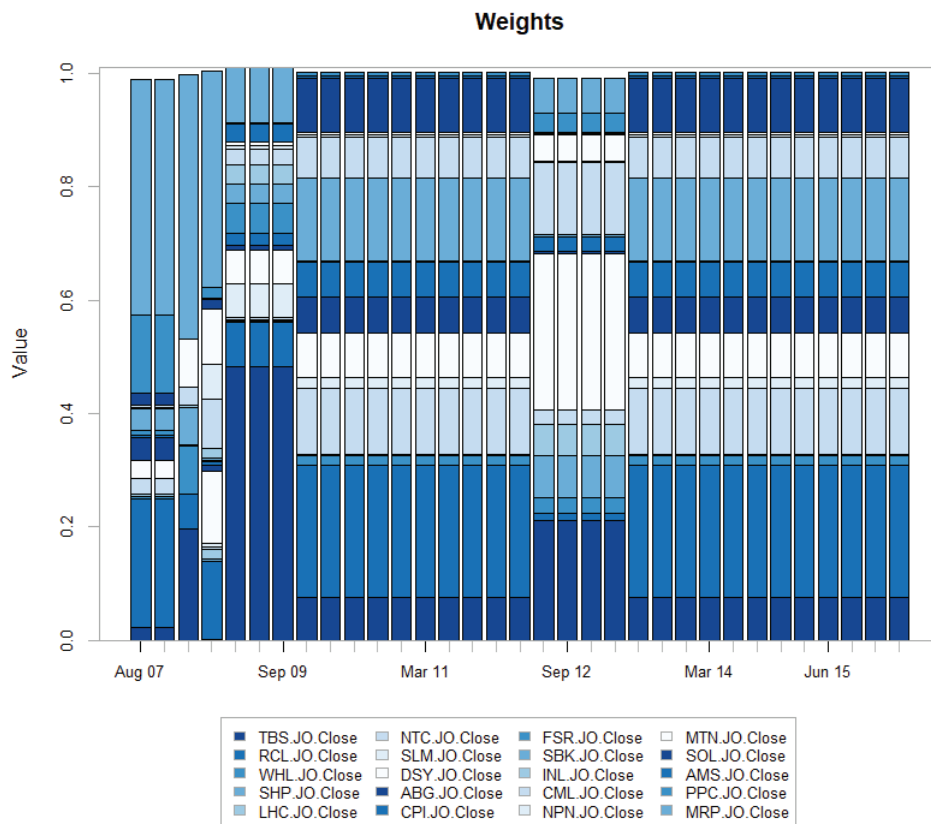


Figure 4.6: Weighting chart of the weights assigned for each asset in the portfolio for the time period using the Sharpe ratio method.

4.5.3 Risk budget method for the factor model.

The risk budget method for optimising the factor model is implemented the exact same way as for the lagged model. This is a popular risk-based asset allocation technique where risk budgets are assigned to each assets' risk contribution. This risk budgeting technique has the goal to diversify the risk in the portfolio by assigning target risk contributions to each component. It is important to note that the risk contribution has a maximum set to 0.5 for any one of the assets in the portfolio

for when the risk are calculated. The different weightings for the regimes are compared, whereafter the portfolio are rebalanced.

Table 4.21: Risk budget as method of optimisation for regime 1 under the factor model.

	Code	Optimal Weighs
Tiger brands	TBS.JO	0.190
RCL Foods	RCL.JO	0.126
Woolworths	WHL.JO	0.000
Shoprite	SHP.JO	0.018
LIFE	LHC.JO	0.092
Netcare	NTC.JO	0.134
Sanlam	SLM.JO	0.014
Discovery	DSY.JO	0.118
Absa	ABG.JO	0.016
Capitec	CPI.JO	0.016
FirstRand	FSR.JO	0.006
Standard Bank	SBK.JO	0.010
Investec	INL.JO	0.032
Coronation	CML.JO	0.012
Naspers	NPN.JO	0.086
MTN	MTN.JO	0.002
Sasol	SOL.JO	0.094
Anglo American Platinum	AMS.JO	0.006
PPC	PPC.JO	0.032
Mr Price	MRP.JO	0.000
Total		1.004
Standard Deviation	0.01125	

Comparing the optimal weights for the regime 1 and 2 there are no significant remarks worth mentioning even though the weight differ for the different states of the South African economy. However, it is important to note that the standard deviation for regime 1 is slightly higher than regime 2, this is the desired result. It is known that the South African economy is in an upwards swing when it is classified in regime 1, thus entering or currently in a bull market. Therefore the investor would be able to take on a bit more risk thus the higher standard deviation.

Compared to Table 4.22 of regime 2 with the lower standard deviation due to the fact that the market is a bull market. Therefore it is more risky to invest and it would not be advised to buy

new stocks but to rather diversify within the portfolio. The risk budget method is used due to the allocation of the risk thus this makes sense.

Table 4.22: Risk budget as method of optimisation for regime 2 under the factor model.

	Code	Optimal Weights
Tiger brands	TBS.JO	0.174
RCL Foods	RCL.JO	0.236
Woolworths	WHL.JO	0.090
Shoprite	SHP.JO	0.020
LIFE	LHC.JO	0.040
Netcare	NTC.JO	0.054
Sanlam	SLM.JO	0.000
Discovery	DSY.JO	0.000
Absa	ABG.JO	0.036
Capitec	CPI.JO	0.074
FirstRand	FSR.JO	0.056
Standard Bank	SBK.JO	0.062
Investec	INL.JO	0.000
Coronation	CML.JO	0.002
Naspers	NPN.JO	0.028
MTN	MTN.JO	0.046
Sasol	SOL.JO	0.058
Anglo American Platinum	AMS.JO	0.006
PPC	PPC.JO	0.006
Mr Price	MRP.JO	0.002
Total		0.99
Standard Deviation	0.01107	

Both Table 4.21 and Table 4.22 satisfied the previously defined constraints based on the sum of the optimal weights.

4.5.4 Rebalancing of the factor model using the risk budget method.

The portfolio were rebalanced 33 times representing the quarterly rebalanced investment periods. When comparing Table 4.23 to Table 4.19 representing the Sharpe ratio method for the factor model, the annualized portfolio standard deviation for the factor model is higher as well as the

annualized rebalancing return. The standard deviation is not a good thing it represents the risk that are taken on but therefore the returns are higher. It is the risk return trade-off that comes into play.

Table 4.23: Statistics of the rebalancing of the portfolio under the Risk Budget method.

Number of rebalancing dates	33
First rebalance date	2007-08-31
Last rebalance date	2016-02-26
Annualized Portfolio Rebalancing Return	0.009152771
Annualized Portfolio Standard Deviation	0.08744983

Table 4.24: Downside risk measures.

Downside Risk Measures	Portfolio Returns
Semi Deviation	0.0090
Gain Deviation	0.0071
Loss Deviation	0.0087
Downside Deviation (MAR=43.33%)	0.0139
Downside Deviation (Rf=0%)	0.0089
Downside Deviation (0%)	0.0089
Maximum Drawdown	0.2710
Historical VaR (95%)	-0.0190
Historical ES (95%)	-0.0279
Modified VaR (95%)	-0.0207
Modified ES (95%)	-0.0325

Comparing the table above to the Table 4.20 the maximum drawdowns are relatively close to one another however the risk budget outperforms the Sharpe ratio slightly. This can also be seen for the value at risk measures. However comparing it to the lagged model's risk budget implementation shown in Table 5.8 the lagged model outperforms the factor model when considering the maximum drawdown. The same result when comparing the value at risk measures as well.

Figure 4.7 is a graphical illustration of how the weights are assigned to each asset in the portfolio. The weights were rebalanced quarterly and therefore there are 33 columns representing the rebalanced investment periods. These values were calculated and this figure gives an visual overview of how the portfolio were weighted. It can also be seen were the regimes were switched at the start of the investment period and thus the rebalanced weighting.

Figure 4.8 is to show that the risk contribution can be obtained using different measures. On the left expected shortfall were used and on the right the standard deviation. Each asset in the portfolio's contribution to the risk of the portfolio is visualised.

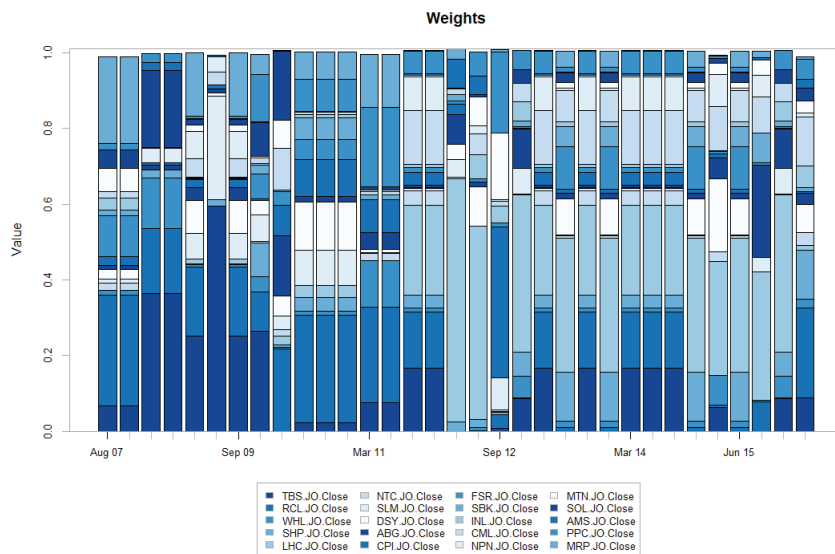


Figure 4.7: Weighting chart of the weights assigned for each asset in the portfolio for the time period using the risk budget method.

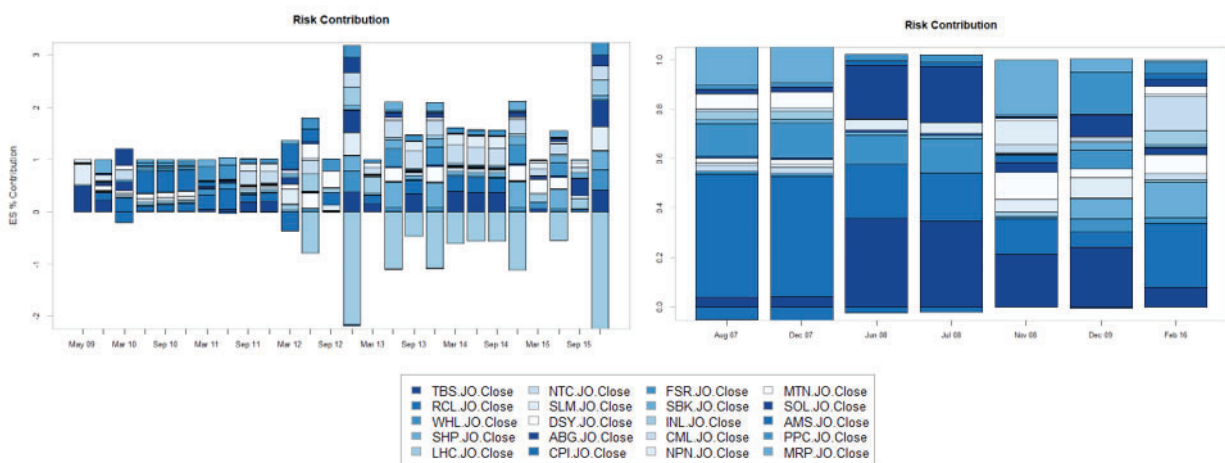


Figure 4.8: The left hand side is the risk contribution for regime 1 which were done by using Expected shortfall. The right hand side shows the risk contribution for regime 2 which were done by using the standard deviation.

4.6 OPTIMISATION FOR EACH CATEGORY

This section shows that optimisation can be applied in different ways. In this case each category of the portfolio (except those who only had one asset in the industry) were taken and optimised according to the two regimes. The optimal weightings were calculated using the risk budget method only. Therefore the standard deviation is given to show the risk obtained for each category.

Table 4.25: Optimisation for regime 1 for each category that can be invested in the portfolio. This were done by using the risk budget method. (Note that the standard deviation is for each category.)

Consumer Goods	Code	Optimal Weighting	Standard deviation
Tiger Brands	TBS.JO	0.564	0.01364
RCL Foods limited	RCL.JO	0.444	
Consumer Services			
Woolworths Holdings Limited	WHL.JO	0.488	0.01646
Shoprite Holdings Limited	SHP.JO	0.516	
Health Care			
LIFE Health Care Group Holdings Limited	LHC.JO	0.358	0.01701
Netcare Limited	NTC.JO	0.652	
Insurance			
Sanlam Limited	SLM.JO	0.492	0.01628
Discovery Limited	DSY.JO	0.518	
Banking			
Absa Group Limited	ABG.JO	0.344	0.01562
Capitec Bank Holdings Limited	CPI.JO	0.042	
FirstRand Limited	FSR.JO	0.222	
Standard Bank Group Limited	SBK.JO	0.382	
Investments			
Investec Group	INL.JO	0.526	0.01499
Coronation Fund Management Limited	CML.JO	0.476	
Telecommunication/Broadcasting			
Naspers Limited	NPN.JO	0.494	0.01854
MTN Group Limited	MTN.JO	0.508	
Chemicals/Mining			
Sasol Limited	SOL.JO	0.570	0.01924
Anglo American Platinum Limited	AMS.JO	0.436	

The weights for each industry category doesn't differ dramatically between the regimes. It is however important to note that the same constraints apply as mentioned at the start of the chapter. Therefore, when the weights are summed for each category it doesn't necessarily add up exactly to 1 due to the leniency of the minimum sum being 0.99 and maximum sum being 1.01 that is implemented by the constraints. This just shows that optimisation within an portfolio can be done in different ways.

Table 4.26: Optimisation for regime 2 for each category that can be invested in the portfolio. This were done by using the risk budget method. Note that the standard deviation represents the category.

Consumer Goods	Code	Optimal Weighting	Standard deviation
Tiger Brands	TBS.JO	0.556	0.01343
RCL Foods limited	RCL.JO	0.436	
Consumer Services			
Woolworths Holdings Limited	WHL.JO	0.482	0.01621
Shoprite Holdings Limited	SHP.JO	0.508	
Health Care			
LIFE Health Care Group Holdings Limited	LHC.JO	0.356	0.0169
Netcare Limited	NTC.JO	0.646	
Insurance			
Sanlam Limited	SLM.JO	0.490	0.01624
Discovery Limited	DSY.JO	0.518	
Banking			
Absa Group Limited	ABG.JO	0.250	0.01565
Capitec Bank Holdings Limited	CPL.JO	0.074	
FirstRand Limited	FSR.JO	0.192	
Standard Bank Group Limited	SBK.JO	0.474	
Investments			
Investec Group	INL.JO	0.524	0.01497
Coronation Fund Management Limited	CML.JO	0.476	
Telecommunication/Broadcasting			
Naspers Limited	NPN.JO	0.498	0.01869
MTN Group Limited	MTN.JO	0.512	
Chemicals/Mining			
Sasol Limited	SOL.JO	0.568	0.01921
Anglo American Platinum Limited	AMS.JO	0.436	

This chapter implemented the two scenarios of the lagged and factor models. The Markov regime switching model were applied and the regimes were classified for the time period that the financial markets were observed. The matrix values were obtained by the methods explained in Chapter 3 for the alphas, betas and transition matrices. These values were analysed and explained. Thereafter, the optimisation methods and constraints were stated. Optimisation using the Sharpe method and the risk budget method were implemented for the two scenarios and it was rebalanced quarterly. The methods were compared and the results were analysed. The optimal weights for the portfolio were obtained and some downside risk measures were provided. Lastly, optimisation were done for each category of the portfolio to show that there are different ways in which one can optimise a portfolio.

CHAPTER 5

COMPARISON OF THE RESULTS

The main objective of the assignment is to test whether adding a classification into regimes optimise a portfolio better than when optimising without classifying the economy into regimes and not incorporating that into the optimisation. In this chapter the objective is to compare the Sharpe ratios of the portfolio without the regimes with the portfolio including the regime switching model. These two options will be referred to as cases. The higher the Sharpe Ratio, the better the investment's risk-adjusted performance will be. Equation 4.7 to Equation 4.10 is used in this comparison for computation.

The Sharpe ratios using Equation 4.7 were then calculated and Table 5.3 were obtained. Note that the risk-free rate where assumed to be 5% (this value is chosen due to the fact that all the treasury bill rates varied around 5% as-well the JIBAR rates and therefore an approximated chosen value of 5% is used). The Sharpe ratio calculated for the portfolio without regimes were done by using Monte Carlo simulation and then choosing the weights out of 5000 weights that minimised the variance. The other option were to use the weights of the maximised Sharpe ratio. For the table values the minimised variance method were chosen. These weights could easily be expressed by the following equation:

$$\operatorname{argmin}_w \sigma_p^2 = \min_w \frac{1}{2} w \Sigma w^T \quad (5.1)$$

With the constraint that the sum of the weights should add up to 1 combined with an only positive constraint ($w \geq 0$) that one could only long the position.

Figure 5.1 shows the portfolio weights that minimised the variance. These weights were used in the calculation to obtain the values for the portfolio excluding the regimes in Table 5.3.

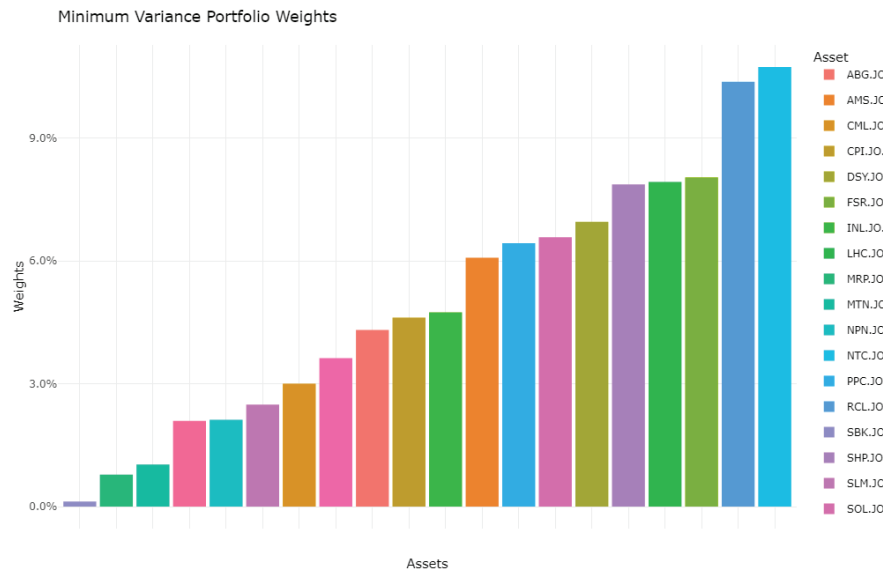


Figure 5.1: Minimum variance portfolio weights for the portfolio excluding the regimes.

Table 5.1: Weights obtained using minimum variance for the portfolio excluding the incorporation of regimes.

	Code	Portfolio Weights
Tiger brands	TBS.JO	0.089
RCL Foods	RCL.JO	0.103
Woolworths	WHL.JO	0.005
Shoprite	SHP.JO	0.081
LIFE	LHC.JO	0.043
Netcare	NTC.JO	0.041
Sanlam	SLM.JO	0.036
Discovery	DSY.JO	0.074
Absa	ABG.JO	0.080
Capitec	CPI.JO	0.057
FirstRand	FSR.JO	0.016
Standard Bank	SBK.JO	0.005
Investec	INL.JO	0.024
Coronation	CML.JO	0.077
Naspers	NPN.JO	0.037
MTN	MTN.JO	0.004
Sasol	SOL.JO	0.084
Anglo American Platinum	AMS.JO	0.016
PPC	PPC.JO	0.084
Mr Price	MRP.JO	0.044
Total		1

Table 5.1 shows the portfolio weights that were obtained by the Monte Carlo simulation using the minimum variance as criteria of Equation 5.1, that excludes the regimes.

The lagged model that were optimised using the Sharpe ratio for the case including the regimes. The values of the weights and transition matrix (Chapter 4) etc. were explained earlier in the assignment. The probability of 0.5 were used for the calculation of the unconditional portfolio weight using Equation 4.11. The following unconditional weights were obtained:

Table 5.2: Unconditional weights calculated for the lagged model including the regimes.

	Code	Unconditional Weights
Tiger brands	TBS.JO	0.250
RCL Foods	RCL.JO	0.174
Woolworths	WHL.JO	0.023
Shoprite	SHP.JO	0.000
LIFE	LHC.JO	0.025
Netcare	NTC.JO	0.209
Sanlam	SLM.JO	0.000
Discovery	DSY.JO	0.060
Absa	ABG.JO	0.001
Capitec	CPI.JO	0.053
FirstRand	FSR.JO	0.001
Standard Bank	SBK.JO	0.022
Investec	INL.JO	0.039
Coronation	CML.JO	0.088
Naspers	NPN.JO	0.005
MTN	MTN.JO	0.017
Sasol	SOL.JO	0.003
Anglo American Platinum	AMS.JO	0.009
PPC	PPC.JO	0.000
Mr Price	MRP.JO	0.017
Total		0.996

The whole assignment boils down to the following table below. Does including the regimes result in a better Sharpe ratio which measures the performance of a portfolio compared to a risk-free asset, after adjusting for risk. For the annualized portfolio returns of Table 5.3, the lag model including the regimes obtained a much higher return than the portfolio excluding the regimes. However this

comes with a trade-off therefore the portfolio risk for the lag model including the regimes has a significantly higher standard deviation than the portfolio excluding the regimes. This might be misleading because when you look at the risk return ratio the lagged model including the regimes still perform better even though the risk is much higher it has a 54.7% compared to the 41.6% of the portfolio excluding the regimes.

Comparing the Sharpe ratios show the lagged model including the regimes performed a lot better than the portfolio excluding the regimes. The Sharpe ratios are not very high but this could be due to the fact of the choice of assets in the portfolio or due to the volatile South African markets. However this still shows that including the regimes switching into the model and then optimising the portfolio outperforms the portfolio that doesn't incorporate the regimes in the model.

Table 5.3: Comparison between a model including and excluding regimes

	Portfolio excluding the regimes	Lagged Model Including Regimes
Annualized portfolio returns	0.07463949	0.288809
Portfolio Risk (Standard deviation)	0.1793876	0.5278877
Sharpe Ratio	0.1373534	0.4523859

This chapter compared two portfolios, one portfolio excluding the regimes and the other including the regimes. The results showed that the inclusion of the regimes obtained a higher Sharpe ratio and proved that incorporating the regimes is a better option for optimisation. The next and final chapter will consist of a discussion of the concluding remarks.

CHAPTER 6

CONCLUSION

This assignment investigated the Markov regime switching models as a way of classifying the South African financial markets into different regimes. Two different models were chosen to fit these Markov regime switching models on. The first was a lagged model and the second was the factor model. Optimising these models for the constructed portfolios were then the next task. Optimisation using the Sharpe ratio method as well as the risk budget method were tested and compared among one another which resulted in similar conclusions. For each of these optimisation methods the portfolios were rebalanced to evaluate the financial markets at the start of the new investment period, classify it either into a new regime or remaining in the current state and then adjusted the portfolio weights. However the lagged model were then chosen for the duration of the tested as well as the Sharpe ratio as method of comparison. The reason for choosing the lag model is because it is a key performance measure of past performance seen in the data that have an effect on the business.

The main objective was to investigate whether including the split of the financial markets into regimes, whereafter the portfolio was optimised actually makes an economic contribution. This was tested by taking a look at the annualised portfolio returns and the portfolio risk which were then used to calculate the Sharpe ratios. The inclusion of the regimes into the model and the classical way of optimising a portfolio were then compared. The finding of this assignment gives overwhelming evidence of the inclusion of regimes in portfolio optimisation to capture essential features of the financial market.

Further research could be done in different methods of optimisation beyond the Sharpe ratio and risk budget method to see how it compares. A different approach to choosing stocks in a portfolio could also be taken to see how that would possibly change the results. There are also post modern portfolio theory methods that have quite a gap for research in the market. Another approach could also be taken by adding another regime in the model which would represent the state between the bull and bear states and to investigate the optimal number of regimes.

Overall the Markov regime switching model that are included in portfolio optimisation does obtain better results than portfolio optimisation without classifying the markets into different regimes.

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APPENDIX A

ADDITIONAL TABLES

Table A.1: The regime at each rebalancing date for the Lagged model

Date	Regime
2007-08-31	2
2007-12-28	2
2008-06-23	2
2008-07-30	2
2008-11-26	2
2009-05-28	1
2009-09-28	1
2009-12-14	2
2010-03-30	1
2010-06-28	1
2010-09-24	1
2010-12-29	1
2011-03-30	1
2011-06-29	1
2011-09-29	1
2011-12-30	1
2012-03-29	1
2012-06-29	1
2012-09-27	1
2012-12-26	1
2013-03-27	1
2013-06-28	1
2013-09-27	1
2013-12-31	1
2014-03-28	1
2014-06-20	1
2014-09-30	1
2014-12-26	1
2015-03-31	1
2015-06-30	1
2015-09-30	1
2015-12-28	1
2016-02-26	2

The dates are numbered from 1 to 33 however it corresponds to the exact dates in Table A.1 above.

Table A.2: The optimal weighting at each rebalanced date for the lagged model

	TBS.JO	RCL.JO	WHL.JO	SHP.JO	LHC.JO	NTC.JO	SLM.JO	DSY.JO	ABG.JO	CPI.JO
1	0.026	0.150	0.002	0.018	0.002	0.000	0.004	0.430	0.008	0.036
2	0.006	0.106	0.000	0.004	0.002	0.000	0.002	0.026	0.000	0.000
3	0.568	0.086	0.004	0.168	0.032	0.002	0.000	0.012	0.074	0.022
4	0.142	0.230	0.030	0.000	0.042	0.002	0.002	0.020	0.000	0.046
5	0.142	0.230	0.030	0.000	0.042	0.002	0.002	0.020	0.000	0.046
6	0.142	0.230	0.030	0.000	0.042	0.002	0.002	0.020	0.000	0.046
7	0.142	0.230	0.030	0.000	0.042	0.002	0.002	0.020	0.000	0.046
8	0.142	0.230	0.030	0.000	0.042	0.002	0.002	0.020	0.000	0.046
9	0.330	0.118	0.000	0.058	0.010	0.000	0.108	0.070	0.000	0.016
10	0.330	0.118	0.000	0.058	0.010	0.000	0.108	0.070	0.000	0.016
11	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
12	0.330	0.118	0.000	0.058	0.010	0.000	0.108	0.070	0.000	0.016
13	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
14	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
15	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
16	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
17	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
18	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
19	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
20	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
21	0.260	0.062	0.060	0.000	0.000	0.130	0.070	0.248	0.000	0.004
22	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
23	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
24	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
25	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
26	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
27	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
28	0.156	0.224	0.000	0.000	0.000	0.020	0.084	0.044	0.002	0.024
29	0.156	0.224	0.000	0.000	0.000	0.020	0.084	0.044	0.002	0.024
30	0.084	0.184	0.002	0.040	0.054	0.108	0.154	0.082	0.018	0.016
31	0.156	0.224	0.000	0.000	0.000	0.020	0.084	0.044	0.002	0.024
32	0.156	0.224	0.000	0.000	0.000	0.020	0.084	0.044	0.002	0.024
33	0.064	0.158	0.000	0.002	0.036	0.250	0.074	0.002	0.000	0.000

	FSR.JO	SBK.JO	INL.JO	CML.JO	NPN.JO	MTN.JO	SOL.JO	AMS.JO	PPC.JO	MRP.JO
1	0.000	0.074	0.006	0.000	0.000	0.000	0.006	0.000	0.234	0.000
2	0.010	0.006	0.032	0.080	0.020	0.068	0.000	0.010	0.022	0.616
3	0.002	0.002	0.000	0.000	0.014	0.000	0.000	0.002	0.010	0.004
4	0.000	0.006	0.002	0.000	0.006	0.264	0.012	0.000	0.022	0.168
5	0.000	0.006	0.002	0.000	0.006	0.264	0.012	0.000	0.022	0.168
6	0.000	0.006	0.002	0.000	0.006	0.264	0.012	0.000	0.022	0.168
7	0.000	0.006	0.002	0.000	0.006	0.264	0.012	0.000	0.022	0.168
8	0.000	0.006	0.002	0.000	0.006	0.264	0.012	0.000	0.022	0.168
9	0.000	0.092	0.010	0.004	0.026	0.000	0.086	0.000	0.060	0.002
10	0.000	0.092	0.010	0.004	0.026	0.000	0.086	0.000	0.060	0.002
11	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
12	0.000	0.092	0.010	0.004	0.026	0.000	0.086	0.000	0.060	0.002
13	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
14	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
15	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
16	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
17	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
18	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
19	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
20	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
21	0.000	0.070	0.000	0.080	0.000	0.002	0.002	0.000	0.006	0.006
22	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
23	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
24	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
25	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
26	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
27	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
28	0.002	0.000	0.000	0.296	0.000	0.054	0.066	0.016	0.004	0.004
29	0.002	0.000	0.000	0.296	0.000	0.054	0.066	0.016	0.004	0.004
30	0.006	0.002	0.010	0.108	0.064	0.000	0.002	0.010	0.028	0.038
31	0.002	0.000	0.000	0.296	0.000	0.054	0.066	0.016	0.004	0.004
32	0.002	0.000	0.000	0.296	0.000	0.054	0.066	0.016	0.004	0.004
33	0.020	0.028	0.000	0.168	0.072	0.002	0.014	0.052	0.000	0.054