The Planning and Optimisation of a Supply Chain Network under Uncertainty

by

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Declaration

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Synopsis

This research work addresses the planning and optimisation of supply chains that are subject to conditions of operational uncertainty, i.e. fuzziness, (<, ≤, > or ≥) and stochastic's (probability), that typically exist in a supply chain operational environment. The 'planning' aspect refers to an occasional need to accommodate multiple performance objectives in the assessment and management of supply chains, and this aspect is more commonly referred to as 'multi-objectivity, which means the existence of multiple maxima, multiple minima or a combination of both maxima and minima objectives in a supply chain environment. Previous work on supply chain under uncertainty research had considered optimisation under one or two conditions of operational uncertainty, and sometimes including the planning requirement of multi-objectivity e.g. fuzzy optimisation, stochastic-fuzzy optimisation, multi-objective-stochastic-fuzzy optimisation. This current thesis is an extension of those works by considering not only relevant cases of operational uncertainty but also by considering those prevailing planning instances of multi-objectivity (i.e. maxima or minima or a combination of both maxima and minima) in a supply chain operating environment. Such capability would be tantamount to being able to deliver ***'realistic'*** *and planned supply chain solutions since all prevailing conditions of operational uncertainty would have been accommodated.*

A typical supply chain is a production and distribution network consisting of multiple production centres, distribution facilities and sales outlets. The objective of this work is to introduce and define a methodology for the optimisation of supply chains under prevailing combinational conditions of uncertainty and planning, which would be tantamount to the means of finding the best operating solution for supply chains,

Such methodology is formulated by identifying methodologies from previous research works for instances of single (e.g. fuzzy optimisation), binary (e.g. fuzzy-multi-objective optimisation) and ternary (e.g. stochastic-fuzzy-multi-objective optimisation) supply chain under uncertainty methodologies from previous research works, analysing them and then extracting the sequence of optimisation steps utilised. Such extracted optimisation methodologies, to provide a methodology for the planning and optimisation of supply chains, under conditions of uncertainty. The methodology was validated by the comparison of optimum results with those generated by a established supply chain optimisation technique, and that was subject to the same operating conditions. Both sets of optimum results were exactly the same.

This method is applied to the planning and optimisation of a NPK fertiliser production and distribution facility, which is subject to fuzzy (<, ≤, >, ≥) market demand uncertainty and which also has a multi-objective operational planning requirement to maximise the production and distribution of an entire range of NPK fertiliser in accordance with market demand, as well as to simultaneously minimise the generation and discharge of hazardous Hydrogen Fluoride (HF) gaseous effluent from the NPK fertiliser Nitrophosphate production unit. There are over 15 different blends of NPK (nitrogen, phosphorous, potassium) fertiliser available, with each blend being suited to a particular agricultural crop-type, e.g. maize, wheat, lucerne etc., and therefore the market demand uncertainty is directly translated into production uncertainty with uncertain raw material allocation in terms of the various sources of N, P and K, i.e. ammonium nitrate (NH_4NO_3), nitrophosphate ($(NH_4)_2SO_4$, $(NH_4)H_2PO_4$, NH_4NO_3, $CaSO_4.2H_2O$), superphosphate ($40\%Ca(H_2PO_4)_2$ + $60\%CaSO_4.2H_2O$) and potassium chloride (KCl). Optimum production/distribution results revealed an achievement of 99.3% of maximum possible production and distribution capability, and also in accordance with market demand.

Further, the hypothesis was satisfied by not only of the nature of the case study optimum results but also by checking the rationality of the results generated from varying the planning and operational uncertainty scenarios in the case study.

Opsomming

Hierdie navorsing handel oor die beplanning en optimalisering van verskaffingskettings wat onderworpe is aan die tipiese onseker bedryfsomstandighede van 'n verskaffingskettingomgewing, naamlik newelagtigheid (<, ≤, > of ≥) en stogastiese veranderlikheid (waarskynlikheid). Die beplanningsaspek verwys na 'n behoefte om van tyd tot tyd veelvuldige prestasiemikpunte by die assessering en bestuur van verskaffingskettings in te sluit. Hierdie multi-mikpuntaspek ("multi-objectivity") dui op die bestaan van veelvuldige maksima, veelvuldige minima of 'n kombinasie van sowel maksima as minima in 'n verskaffingskettingbestuursomgewing. Vorige navorsingswerk oor verskaffingskettings met onseker bedryfsomstandighede het ondersoek ingestel na optimalisering in slegs een of twee scenario's van bedryfsonsekerheid, en het soms ook die multi-mikpuntvereiste van beplanning ingesluit, byvoorbeeld neweloptimalisering, stogastiese neweloptimalisering of multi-mikpunt- stogastiese neweloptimalisering. Hierdie studie brei uit op daardie vorige werk deur nie net tersaaklike gevalle van bedryfsonsekerheid in 'n verskaffingskettingomgewing in ag te neem nie, maar ook heersende gevalle van multi-mikpuntbeplanning (met ander woorde maksima of minima, of 'n kombinasie daarvan). Slegs op dié manier kan 'realistiese' en beplande verskaffingskettingoplossings bedink word, aangesien dit vir alle heersende omstandighede van bedryfsonsekerheid voorsiening maak.

'n Tipiese verskaffingsketting is 'n produksie- en verspreidingsnetwerk wat uit etlike produksiesentrums, verspreidingsfasiliteite en afsetpunte bestaan. Die doel van hierdie studie was die bekendstelling en omskrywing van 'n metodologie vir die optimalisering van verskaffingskettings in 'n kombinasie van onsekerheids- en beplanningsomstandighede, om sodoende die beste bedryfsoplossing vir verskaffingskettings te vind. Hiervoor is die metodologieë uit vorige navorsingstudies oor die eenledige (bv newel-), tweeledige (bv multi-mikpunt-newel-) en drieledige (bv multi-mikpunt- stogastiese newel-) optimalisering van verskaffingskettings bepaal en ontleed, gevolg deur die onttrekking van die reeks optimaliseringstappe wat gebruik is, om 'n omvattende metodologie vir die beplanning en optimalisering van

verskaffingskettings met onseker bedryfsomstandighede te skep. Die metodologie is bevestig deur die optimale resultate te vergelyk met dié wat met 'n gevestigde tegniek vir verskaffingskettingoptimalisering behaal is en wat aan dieselfde bedryfsomstandighede onderworpe was. Die twee stelle optimale resultate was presies dieselfde.

Hierdie metode is toegepas op die beplanning en optimalisering van 'n fasiliteit vir die produksie en verspreiding van 'n NPK- (stikstof-fosfor-en-kalium-)kunsmis. Dié fasiliteit is onderworpe aan newelonsekerheid (<, ≤, >, ≥) in markvraag, en het ook 'n multi-mikpuntvereiste in bedryfsbeplanning om die produksie en verspreiding van 'n hele reeks NPK-kunsmis ooreenkomstig markvraag te maksimaliseer, en terselfdertyd die ontwikkeling en afvoer van 'n gevaarlike gasagtige waterstoffluoried-afloop uit die nitrofosfaat-produksie-eenheid te beperk. Daar is meer as 15 verskillende soorte NPK-kunsmis beskikbaar, en elkeen is bedoel vir 'n bepaalde tipe landbougewas, byvoorbeeld mielies, koring, lusern, ensovoorts. Daarom lei markvraagonsekerheid direk tot produksie-onsekerheid, met onseker grondstoftoekenning wat die verskillende bronne van stikstof (N), fosfor (P) en kalium (K) betref, met ander woorde ammoniumnitraat (NH_4NO_3), nitrofosfaat ($(NH_4)_2SO_4$, $(NH_4)H_2PO_4$, NH_4NO_3, $CaSO_4.2H_2O$), superfosfaat ($40\%Ca(H_2PO_4)_2 + 60\%CaSO_4.2H_2O$) en kaliumchloried (KCl). Met optimale produksie-/verspreidingsresultate is 99,3% van die maksimum moontlike produksie- en verspreidingsvermoë bereik, wat ook met markvraag strook.

Daarbenewens is die hipotese bevestig deur die aard van die optimale resultate in die gevallestudie, sowel as deur 'n studie van die rasionaliteit van die resultate toe die beplannings- en bedryfsonsekerheidsomstandighede in die gevallestudie afgewissel is.

Dedication

This work is dedicated my all members of my immediate family and also to all members of the greater Cole family for all their love, encouragement and support, especially in light of severe injuries that were incurred by me just prior to this thesis.

I also dedicate this work to the Lord God for all the blessings and support

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1 INTRODUCTION

1.1 Background to Study

It has become increasingly evident, especially since the beginning of the 21st century, that much international research work had been expended on supply chain planning. In this regard the emphasis has been on enhancing the operational configuration and performance of supply chains, so that they may be aligned with both the planning requirements and the prevailing conditions of operational uncertainty. Some examples of previous supply chain, under uncertainty, research works include You and Grossmann (2008), Chen and Lee (2007), Awudu and Zhang (2012), Liu and Papageorgiou (2012), Guillen and Grossmann (2009), Al-Othman et al. (2008), Tsiakis and Papageorgiou (2007), Chen et al. (2007) and Chen and Lee (2004). Other examples of related supply chain under uncertainty research work include Optimisation of Production-Distribution Systems under Uncertainty, Li et al. (2008), Optimisation of Delivery Systems under Uncertainty, Bit al. (1992), and Optimisation of Decentralised Supply Chains, Raj and Lakshminarayanan, (2008).

The term, 'Uncertainty' was not applied consistently in all cases. Sometimes it referred to 'fuzzy uncertainty' and sometimes it referred to 'stochastic uncertainty'. (Concepts and Definitions are provided in Section.2.2) In these two cases, expressions such as Fuzzy Optimisation, Stochastic Optimisation and Stochastic-Fuzzy Optimisation were coined to describe the procedures involved in determining the best operating solutions under those uncertain operating conditions. To add to the confusion, the planning term, 'Multi-Objectivity was occasionally added to these expressions whenever there was a planning requirement to consider multiple supply chain objectives, be they maxima, minima or a combination of both in nature, in

solutions for uncertain supply chain operations, e.g. Multi-Objective-Fuzzy Optimisation, Multi-Objective-Stochastic Optimisation and Multi-Objective-Stochastic-Fuzzy Optimisation.

A supply chain optimisation capability that could cater for any prevailing conditions of operational uncertainty and that could also cater for any desired number of performance objectives would be of great interest to the commercial industrialised world where various combinational instances of operational uncertainty and corresponding planning requirements frequently occur. A good example of this is ammonia, (NH_3), from coal production and also the downstream production of fertiliser and explosives. Operational uncertainty is manifest in many ways. Firstly, with the production of NH_3 from coal, the concentration of hydrogen (H) in coal is probabilistically, or stochastically uncertain, which impacts not only upon ammonia production plant design considerations, but also upon the predictability of downstream product production, i.e. nitric acid (HNO_3) and ammonium nitrate (NH_4NO_3). Secondly, there is a need to maximise the production of NH_3 and downstream fertiliser and explosives products, in accordance with market demand, whilst simultaneously minimising the generation and discharge of hazardous production effluent, i.e. carbon monoxide (CO) and nitrogen dioxide (NO_2), which, taken together, are uncertain multi-objective planning requirements. (Note: this is an example of multi-objectivity where, one objective needs to be maximised while the other needs to be minimised. More frequently, all objectives need to be either maximised or minimised). Thirdly, there is the need to maximise overall product sales in uncertain (<, ≤, > or ≥) markets, i.e. fuzzy uncertainty. There are many other examples of commercial instances where multiple instances of planning and uncertainty exist, e.g. farming with downstream product beneficiation, the precious metal mining industry etc. etc.

The availability of a planning and optimisation methodology for supply chains operating under any prevailing conditions of uncertainty, which is the intention of this research, would therefore be tantamount to being able to deliver ***'realistic'*** and planned supply chain operating solutions since all prevailing conditions of operational uncertainty would be accommodated.

1.1 Problem Statement

Planning with a Supply Chain under Uncertainty: The two different elements of operational uncertainty, i.e. stochastic and fuzzy uncertainty, can potentially influence the performance or behaviour of supply chains. Should such prevailing factors of uncertainty be taken into account when planning for the operational performance of such supply chains, which may, in itself, involve any number of performance objectives?

1.2 Hypothesis

In most business process or operational environments, there is usually a need to plan and function optimally, usually with regard to any number of operational objectives. However, since such operating environments are normally beset by problems of operational uncertainty, i.e. fuzzy uncertainty and stochastic (probabilistic) uncertainty, it is often very difficult to determine the best common operating conditions that will satisfy all process objectives, simultaneously and satisfactorily. Therefore, for this research initiative, the Hypothesis is as follows:

An operational process, i.e. supply chain network, subject to prevailing conditions of operational uncertainty, i.e. fuzzy uncertainty and/or stochastic uncertainty, will undergo a realistically improved optimum performance should the impact of these uncertainty effects be taken into account and accommodated during the process design phase, irrespective of the planned number of operational objectives involved.

1.3 Purpose Statement

The primary purpose of this research is to define and derive a planning, i.e. multi-objective, and optimisation methodology for a supply chain network that is operating under conditions of uncertainty, i.e. fuzzy and stochastic (probability) uncertainty, and that may be successfully applied to any Supply Chain Network.

The secondary purpose of this research is to successfully apply the methodology to a relatively complex production and distribution facility environment. The facility, in this particular case, has to do with a large NPK (Nitrogen, Phosphorous and Potassium) fertiliser production and distribution facility in South Africa. The operations of this facility are subject to a multi-objective planning requirement as well as to operational uncertainty. The multi-objective or dual objective in this case, planning requirement is realised by the need to maximise the production and distribution of NPK fertiliser in accordance with market demand whilst simultaneously minimising the generation and discharge of hazardous effluent, i.e. hydrogen fluoride (HF) and carbon dioxide (CO_2), from the production process. Operational uncertainty is realised by the uncertain market demand of NPK fertiliser.

1.4 Research Questions

Since the ultimate intention of this research is to plan and optimise a production, distribution and sales facility that is operating under conditions of uncertainty, the primary intention of the research itself is, therefore, to formulate a Planning and Optimisation Methodology for a Supply Chain operating under conditions of Uncertainty. In order to do this, the following research questions must be answered:

1) Which uncertainty criteria are specifically applicable to an uncertain supply chain operating environment?

2) What is the typical nature of supply chain planning requirements and in which manner would they best be constituted?

3) Can both the supply chain planning requirements and the nature of the underlying operational uncertainty be accommodated in an overall supply chain optimisation methodology that would enable the determination of a best overall operating solution under those conditions?

2 LITERATURE RESEARCH

2.1 Introduction

A supply chain subject to uncertain planning conditions, such as multi-objectivity (one or more operational performance objectives), and/or subject to uncertain operating conditions such as fuzzy uncertainty and/or stochastic uncertainty will undergo improved performance if such uncertain planning and operating conditions are taken into account during the supply chain design phase, It is the intention of this research to define and derive a planning and optimisation methodology for a supply chain network that is operating under such conditions of planning and operational uncertainty. Therefore it will first be necessary to conduct a comprehensive literature research exercise on the existence of any previously researched individual, e.g. multi-objective, fuzzy or stochastic optimisation, or on any combinational supply chain, under uncertainty, optimisation and planning methodologies, e.g. multi-objective-fuzzy or stochastic-fuzzy optimisation. This will first be necessary so that such methodologies may be identified, selected, analysed and then the core identified supply chain under uncertainty elements extracted therefrom so that they may be logically collated to ultimately create the desired methodology. However the conduct of the literature research campaign needs to be preceded by a definition of the required concepts of research, or conceptual foundation, so that literature search criteria are in alignment with the Purpose Statement (1.3).

The other key aspect to be considered in this research is that the planning and optimisation procedure that is eventually applied to an operationally uncertain Supply Chain Network environment, in determining the best operating solution(s), may have other affective considerations, apart from those uncertain conditions described above. These may need to be taken into account so that an appropriate and optimum supply chain network solution may be achieved. Such considerations could

typically include any methodology integration procedures, such as the two-stage stochastic model program (2SMP), Gupta and Maranas (2003), and/or any interdependent techniques, such as the Pareto optimality ε-constraint method utilised by Guillen and Grossmann (2009) and Liu and Papageorgiou (2013), that may need to be taken into account.

Therefore, the purpose of this 'Literature Research' section is to identify those previous research works upon which the thesis will be based. This will be achieved by defining appropriate literature search criteria for a comprehensive literature search campaign. The definition of such search criteria will require the pre-definition of the required framework of research, i.e. the 'Conceptual Foundation', which will be based on the key required concepts of the research, as defined in the Purpose Statement (1.3). Additional, and appropriate, literature search criteria will also be provided through an analysis of the various singularly uncertain and planning optimisation techniques that are available and also typically applied for uncertain supply chain network environments, i.e. fuzzy, stochastic and multi-objective optimisation. This will be supplemented by a similar analysis of some of the combinational, uncertain supply chain optimisation methodologies that are also available, e.g. multi-objective-fuzzy optimisation, stochastic-fuzzy optimisation, and multi-objective-stochastic-fuzzy optimisation. Once this has been achieved, appropriate search terms can be identified and selected for a comprehensive literature journal search exercise. This will be followed by defining the context of the research, i.e. the description and illustration of, a 'Supply Chain Network'.

2.2 Concepts and Definitions

2.2.1 Introduction

Supply Chain, under uncertainty, research, conducted since the late 20th century, has revealed the frequent use of certain terms and expressions to describe the various possible instances of planning, i.e. multi-objectivity, and operational uncertainty, i.e. fuzzy and/or stochastic uncertainty, that can exist in a supply chain operating environment. For example, the work of Bit et al. (1992) revealed the use of the term, '*fuzzy-multi-objective*' uncertainty/planning to describe an operational scenario that is subject to imprecise operational uncertainty as well as being subject to a multiple objective planning requirement. Similarly, the research of Lee and Olson (1985) and Gupta and Maranas (2003) demonstrated the use of the term *stochastic* uncertainty to describe probabilistic uncertainty in a supply chain operational environment. Therefore, in order to effectively conduct the required comprehensive literature research exercise on the existence of any previously researched, *single-case*, e.g. multi-objective, fuzzy or stochastic optimisation, or *combinational-case*, supply chain, under uncertainty and/or planning, optimisation techniques, e.g. multi-objective-fuzzy or stochastic-fuzzy optimisation, these techniques must be clearly and accurately defined up front. Similarly, the nature and scope of a supply chain network must also be correctly defined. Additionally, the concept of Pareto optimality is occasionally applied, and therefore needs to be defined as well. Two categories of supply chain, under uncertainty and planning, optimisation techniques are available, and these are:

i) Single-case Optimisation Techniques
ii) Combinational-case Optimisation Techniques
iii) Pareto optimality

2.2.2 Single-case Optimisation under Uncertainty or Planning Techniques currently available for Supply Chain Networks

It is clearly apparent, from most of the research literature listed in the Bibliography that the two instances of supply chain Uncertainty, i.e. fuzziness, stochastic uncertainty, and the one instance of supply chain planning, i.e. multi-objectivity, can exist in a supply chain operating environment. Further, it is also true that a few specific individual and combinational supply chain under uncertainty, optimisation techniques have been developed to provide optimum operating solutions for such environments, based on certain operating criteria, e.g. number and nature of suppliers, no. of distribution points etc. By *specific individual* optimisation methodologies is meant those techniques that apply to specific and definite areas, or locations of application in a supply chain operating environment, i.e. raw material procurement, production/manufacturing, distribution, sales. These various supply chain optimisation techniques include Fuzzy Optimisation, which means finding the best operating solution under uncertain, or imprecise, operating conditions. There is also Stochastic Optimisation, which means finding the best solution in a probabilistic environment, and which is usually expressed as $F(x_i)$, where $F(x)$ represents the probability distribution function.. Finally, there is Multi-objective Optimisation, which is the determination of the best operating solution in a planning environment where multiple operating objectives, either of a maxima, minima or combinational maxima and minima nature, can occur. Such specific individual uncertainty and/or planning optimisation techniques are listed and described in Table 2-1.

Table 2-1 *Individual Supply Chain Optimisation Uncertainty/Planning Techniques Available*

Optimisation Technique	Descriptive Definitions	Reference
Fuzzy Optimisation	Fuzzy Optimisation is generally regarded as '*mathematical programming*', which is normally defined as follows: $$\max f(x) = z = c^T x$$ $$\text{s.t. } Ax \leq b$$ $$x \geq 0$$ *with* $c, x \in R^n, b \in R^m, A \in R^{m \times n}$. In this model it is normally assumed that all coefficients of A, b, and c are real (crisp) numbers, that '≤' is meant in a crisp sense, and that 'maximize' is a strict imperative. This also implies that the violation of any single constraint renders the solution unfeasible and that all constraints are of equal importance (weight).	Zimmerman, H.J. (1978)
Multi-Objective Optimisation	Multi objective optimisation problem is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. A basic single-objective optimization problem can be formulated as follows: min f(x) x ∈ S, where f is a scalar function and S is the (implicit) set of constraints that can be defined as: $$S = \{x \in Rm: h(x) = 0, g(x) \geq 0\}.$$ Multi-objective optimization can then be described in mathematical terms as follows:	Caramia, M, Dell Ormo, P, (2008)

	Min [f1(x), f2(x)... fn(x)] $x \in S$, , where n > 1 and S is the set of constraints defined above. The space in which the objective vector belongs is called the objective space, and the image of the feasible set under F is called the attained set. Such a set will be denoted in the following with C = {y ∈ Rn : y = f(x),x ∈ S}	
Stochastic Optimisation	Stochastic Optimisation is the minimization (or maximization) of a function in the presence of randomness in the **optimisation** process. The randomness may be present as either noise in measurements or Monte Carlo randomness in the search procedure, or both.	Weisstein, E., (2009)

The initial phase of this aspect of the research will be concerned with the identification and application of the various specific individual optimisation techniques such as Multi-Objective Optimisation, Fuzzy Optimisation and Stochastic Optimisation, that can be found based on an appropriate literature search exercise. This will be followed by a study of the application of these various techniques with regard to the existence of any prevailing or limiting functional constraints in a typical process or production environment. The latter phase of this planning and optimisation study of supply chain under Uncertainty research will focus on the methods utilised in combining specific selected optimisation techniques into one desired methodology. Much recent research work has been conducted on combining any two specific planning and/or optimisation techniques into combinational techniques. These include Multi-Objective Fuzzy Optimisation, (Tsiakis and Papageorgiou (2007), Baky (2010)), Stochastic Fuzzy Optimisation, (Awudu and Zhang (2013), Chen and Lee (2004)), Multi-Objective Stochastic Fuzzy Optimisation (You and Grossmann (2008), Chen et al. (2007)), Bi-Objective Stochastic Fuzzy

Optimisation (Gupta and Maranas (2003)) and Multi-Objective Stochastic Fuzzy (Pareto) Optimisation (Guillen an Grossmann (2010), Liu and Papageorgiou (2013)).

No evidence of research could be found into a supply chain planning and optimisation methodology that combines all three specific planning and uncertainty optimisation techniques into one 'Planning and Optimisation of a Supply Chain Network under Uncertainty methodology that would enable the determination of planned optimum supply chain solutions under any conditions of Uncertainty.

2.2.3 Combinational-case Optimisation under Uncertainty Techniques currently available for Supply Chain Networks

As can be seen from Table 2-2, quite a few combinational uncertainty optimisation conditions are possible. For a comprehensive literature (journal) search, all possible combinations of all the various operational uncertainty terms available, i.e. multi-objective, fuzzy, stochastic, are considered in conjunction with a supply chain network infrastructure to produce the desired search results.

Table 2-2 *Combinational Optimisation Methodologies*

Combinational Optimisation Methodology	Application Conditions	Selected List
Multi-Objective Fuzzy Optimisation	Finding the 'best' planning solution for a SCN under conditions of scalar uncertainty and having multiple operational objectives	Bit et al. (1992) Tsiakis & Papageorgiou (2007) Baky (2010)
Stochastic Fuzzy Optimisation	Finding the 'best' solution for a SCN Supply Chain Network) under conditions of stochastic fuzzy uncertainty	Gupta and Maranas (2003) Santoso et al. (2005) Awudu and Zhang (2013)
Multi-Objective Stochastic Fuzzy Optimisation	Finding the 'best' planning solution for a SCN under conditions of multiple objectivity, fuzzy and probabilistic uncertainty	You and Grossmann (2008) Guillen et al. (2005)
Multi-Objective (Pareto) Fuzzy Optimisation	Finding the 'best' planning solution for a SCN under conditions of fuzzy and multiple objective uncertainty, utilising the Pareto ε-constraint method	Guillen and Grossmann (2013) Liu and Papageorgiou (2013)

Research into possible, existing optimisation methodologies was conducted by performing literature research on the various combinational optimisation methodologies listed in Table 2.2. The results that were discovered are discussed for Stochastic Fuzzy Optimisation, Multi-Objective Fuzzy Optimisation and Multi-Objective Stochastic Fuzzy Optimisation in Section 2.2.

2.2.3.1 Stochastic Fuzzy Optimisation

In the paper of Luhudjula and Gupta (1995), the applicability of optimisation models is extended to situations where both fuzzy and random data sets are involved. The major mathematical tool used to this end is the concept of fuzzy random variables, which provides a coordinated approach in dealing with these two kinds of imprecision.

2.2.3.2 Multi-Objective Fuzzy Optimisation

Researchers such as Mahapatra and Roy (2002), Tsiakis and Papageorgiou (2007), Cadona-Valdés and Ozdemir (2011) and Baky (2010) consider multi-objective reliability optimisation problem for system reliability where reliability enhancement is involved with several mutually conflicting objectives. It is conflicting to reduce the cost of the system and improve the reliability of the same system simultaneously. A new fuzzy multi-objective optimisation method is introduced and it is used for the optimisation decision making of the series and complex system reliability with two objectives. In general, cost of the components of the system's objective goals has not been stated clearly. This imprecise reliability optimisation model is solved through the fuzzy multi-objective optimisation method. Alternatively, the research work of Bit et al. (1992), which deals with a 'Fuzzy programming approach to a multi-objective solid transportation problem', includes a well-defined fuzzy multi-objective optimisation methodology that can easily be applied, and, in fact, includes a numerical example.

2.2.3.3 Multi-Objective Stochastic Fuzzy Optimisation

Papers presented by Bath et al. (2003), Gupta and Maranas (2003), Chen et al. (2007) and You and Grossmann (2008) cover a multi-objective framework, an

interactive fuzzy satisfying method, was presented to determine the optimum values of certain objectives subject to statistical uncertainties. In determining the optimum values, four objectives were simultaneously minimized. Specific techniques were put forward to convert the stochastic models into their respective deterministic equivalents. A weighting method was used to simulate the trade-off relationship between the conflicting objectives in the non-inferior domain.

The paper by Bath et al. (2004), a production generation schedule, involving NO/NO_2 pollutant emissions, system load demand and statistical uncertainties in production cost data is produced, In determining an optimum production schedule, four objectives viz. operating cost, NO/NO_2 emissions, variance of active risk and power generation mismatch are simultaneously minimised. Stochastic models are transformed into their respective deterministic equivalents by means of certain specific techniques. The trade-off between conflicting objectives is resolved by application of the weighting method. Contemporary search techniques, such as Hooke–Jeeves, are utilised to search the 'preferred' weightage pattern for a match, which would then correspond to an optimal solution. Fuzzy set theory is utilised in determining a 'preferred' optimal solution through the determination of maximum satisfaction levels of the participating membership functions. The nature of the results obtained demonstrates the validity of the proposed method.

2.2.4 Pareto Optimality

Pareto Optimal – using or dividing resources (time, money, employees etc.) in a way that results in a situation where nobody is doing worse than before, and at least one person is doing better

using or dividing resources (= time, money, employees, etc.) in a way that results in a situation where nobody is doing worse than before, and at least one person is doing better: If the operation is not yet Pareto optimal, this is because people who are powerful in the inefficient system are reluctant to change.

(Cambridge Business English Dictionary @ Cambridge University Press)(Definition of Pareto optimal from the Cambridge Business English Dictionary © Cambridge University Press)

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2.2.5 General Definitions

a) Monte Carlo simulation

A problem solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables.

b) Lamont Boiler

High pressure forced water circulation boiler that has a large helical furnace wall and helical coils above and below.

c) SAA – Sample Average Approximation method

The methodology relies on approximating the underlying stochastic program via sampling, and solving the approximate problem via a specialized optimization algorithm.

d) Benders decomposition algorithm

Benders Decomposition Algorithm: - a technique in mathematical programming that allows the solution of very large linear programming problems that have a special block structure. This structure often occurs in applications such as stochastic programming.

e) The Stackelberg leadership model

A strategic game in economics in which the leader firm moves first and then the follower firms move sequentially..

f) Lexicographic Min-Max (LMM) optimisation

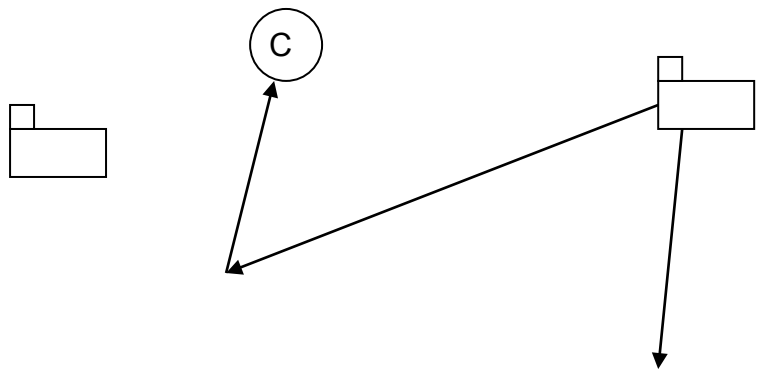
It depends on searching for solutions minimal according to the lex-max order on a multidimensional outcome space. LMM is a refinement of the standard Min-Max optimisation in that apart from determining the largest outcome, the second largest outcome is also minimised.

2.2.6 Supply Chain Network under Uncertainty

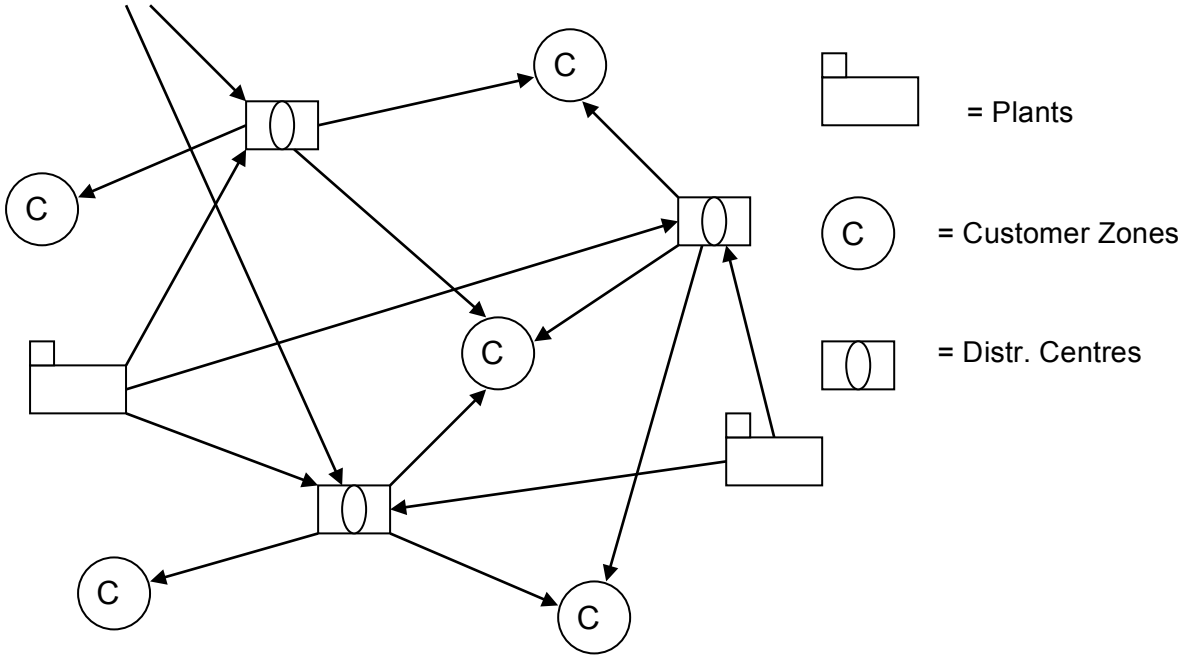
2.2.6.1 Introduction

For all the literature uncovered on supply Chain Network Optimisation under Uncertainty, research was typically based on a multi-echelon supply chain network consisting of plants, distribution centres and customer zones as depicted in Fig 2-1.

Figure 2-1 *Multi-Echelon Supply Chain Network*



The Planning and Optimisation of a Supply Chain Network under Uncertainty



Similarly, based on an overview of all the journal papers under either the 'Supply Chain Optimisation under Uncertainty' or 'Supply Chain Planning under Uncertainty' categories in the Bibliography, the fundamental keys stages involved in the Optimisation/Planning methodologies concerned are those listed in Table 2-3.

Table 2-3: *Key aspects as to the optimisation of supply chain networks under Uncertainty*

1	Illustration and description of the supply chain network (SCN) involved
2	Nature and location of SCN operational Uncertainty involved
3	Description/Explanation of the optimisation methodology utilised.
4	Application by means of a case study.

Of these various stages involved, the one entitled, 'Nature and location of SCN operational Uncertainty involved' requires further explanation, below.

Further to the work of supply chain, under uncertainty, researchers such as Bit et al. (1992), Gupta and Maranas (2003), Guillen et al. (2005), Chen et al. (2007), Tsiakis and Papageorgiou (2007), You and Grossmann (2008), Guillen and Grossmann (2009), Baky (2010), Liu and Papageorgiou (2013), Awudu and Zhang (2013) and Cadona-Valdes and Ozdemir (2013) the operational performance of supply chain networks may be affected by the existence of conditions of planning and operational uncertainty, i.e. multi-objectivity, fuzziness and/or stochastic uncertainty in the supply chain network operating environment, at certain locations, e.g. market demand, distribution, production, objectives. Such conditions of planning and/or operational uncertainty may occur either singly, e.g. fuzzy, stochastic uncertainty, or in a combinational fashion, e.g. multi-objective-fuzzy, multi-objective-stochastic-fuzzy uncertainty, in a supply chain network. The paragraphs below specifically discuss some different and previously researched cases of both single-case and combinational-case forms of supply chain operational planning and/or uncertainty that are most frequently encountered in research. However, it must be realised that in reality, any combination of supply chain planning and uncertainty is theoretically possible, provided that the context may be realised.in terms of both uncertainty type and location. Table 2-4 shows some frequently occurring instances of planning and uncertainty in terms of type and location in a supply chain environment and some of these are discussed further below.

Table 2-4: *Examples of types and locations of supply chain uncertainty and planning*

Supply Chain Planning and Uncertainty Examples		
Uncertainty/	**Uncertainty/**	**Explanation**

Planning Type	**Planning Location**	
Fuzzy	Market Demand	Market demand is imprecise
Stochastic	Market Demand	Market demand is represented by either by a continuous or discreet scenario probabilistic relationship
Multi-objective	Supply Chain Requirements	One or more supply chain performance objectives
Fuzzy	Distribution	Quantities distributed to distribution outlets are uncertain
Stochastic	Raw Material Procurement	Availability of raw materials follows a discrete or continuous probability trend
Multi-objective	Production	Production activities are usually typified by the need for multiple performance objectives, e.g. maximise production output and minimise rejects

Overviews are given, below, of some of the more frequently encountered instances of supply chain uncertainty in the research literature. These, and others, are discussed and analysed in more detail in the Literature Review.

2.2.6.2 Supply chain network with Fuzzy Demand

In a research paper by Tsiakis and Papageorgiou (2007), it was stated that enterprise optimisation could rapidly strip significant ‘‘bottom line’’ costs out of global operations, giving companies a real competitive edge. The benefits of managing supply chain networks by integrating operational, design and financial decisions have been acknowledged by the industrial and academic community.

The objective of this work was to determine the optimal configuration of a production and distribution network that was subject to operational and financial constraints. Operational constraints included quality, production and supply restrictions, and were related to the allocation of the production and the work-load balance. Financial constraints included production costs, transportation costs and duties for the material flowing within the network subject to exchange rates. As a business decision the out-sourcing of production was considered whenever the organisation could not satisfy the demand. A mixed integer linear programming (MILP) model was proposed to describe the optimisation problem. A case study for the coatings business unit of a global specialty chemicals manufacturer was used to demonstrate the applicability of the approach in a number of scenarios.

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This paper had proposed an integrated model based on a detailed mathematical programming formulation that addressed some of the complex issues relating to the design and operation of global supply chain networks. The focus was on financial and tactical operational aspects within the organisation taking into account production balancing amongst sites. Between other business benefits was the operational and distribution efficiency of the network, visibility and control of the supply chain and capability to perform towards Key Performance Indicators (KPI’s) such as operational cost, customer satisfaction and product quality. Moreover, the impact of decisions on the design and tactical operation could be quantified and evaluated. The data used was extracted directly from ERP systems, thus making such approaches easy to use and easy to update every time the application was

used. The proposed MILP model was aimed at assisting senior operations management in making production allocation, production capacity per site, purchase of raw materials and network configuration decisions, taking into account financial aspects (exchange rates, duties, etc.) and costs.

2.2.6.3 Supply chain network with a discrete probability demand distribution plus case study

In a research paper by Gupta and Maranas (2003), a two stage stochastic programming based approach was described to model the supply chain planning process as it reacts to demand realizations unfolding over time. In the proposed bi-level-framework, the manufacturing decisions are modelled as 'here-and-now' decisions, which are made before demand realization. Subsequently, the logistics decisions are postponed in a 'wait-and-see' mode to optimize in the face of uncertainty. In addition, the trade-off between customer satisfaction level and production costs is also captured in the model. The proposed model provided an effective tool for evaluating and actively managing the exposure of an enterprise's assets (such as inventory levels and profit margins) to market uncertainties. The key features of the proposed framework are highlighted through a supply chain planning case study.

Through a planning case study, the ability of the proposed framework to address key issues in managing uncertainties in supply chains was highlighted. It was shown that by utilizing the presented framework, a more realistic description of the total planning costs (in terms of a probability distribution in contrast to a point estimate) could be obtained. Consequently, this information could potentially be utilized to manage the risk exposure of the company's assets. Risk management initiatives aimed at reshaping this distribution such that the downside risk is minimized while maintaining the upside potential could be undertaken based on this information. To this end, the

use of derivative financial instruments, such as options, futures and swaps, in conjunction with the developed framework is currently being investigated. In addition to controlling risk in the supply chain, the proposed framework was also shown to provide valuable insights into the customer relationship aspects of the supply chain.

2.2.6.4 Supply chain network with a continuous probability demand distribution plus case study

According to research work of Roghanian et al. (2007), Bi-level programming, a tool for modelling decentralized decisions, consists of the objective(s) of the leader at its first level and that of the follower at the second level. Three level programming results when second level is itself a bi-level programming. By extending this idea it is possible to define multi-level programs with any number of levels. In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extreme problems under incomplete information about the random parameters is called ‘‘stochastic programming’’. Enterprise-wide supply chain planning problems naturally exhibit a multi-level decision network structure, where for example, one level may correspond to a local plant control/scheduling/planning problem and another level to a corresponding plant-wide planning/network problem. Such a multi-level decision network structure can be mathematically represented by using ‘‘multi-level programming’’ principles.

In this paper, a ‘‘probabilistic bi-level linear multi-objective programming problem’’ is considered and its application in enterprise-wide supply chain planning problem is discussed. In this research, market demand, production capacities per plant and resource availabilities for all plants are considered as random variables, whereas certain constraints are treated as joint probability distributions. This probabilistic model is at first converted into an equivalent deterministic model at each level, to

which a fuzzy programming technique is then applied to solve the multi-objective nonlinear programming problem. A compromise optimum solution is ultimately obtained.

2.2.7 Transforming Probabilistic relationships into their Deterministic equivalents

It also became evident, during preliminary research, that certain specialised programming techniques became necessary, especially in the formulation of certain stochastic-based MINLP's (Mixed Integer Non-Linear Programs), where certain constraints may be probabilistic in nature, e.g. a Normal or a Triangular Probability Distribution relationship. In cases such as these, in order to formulate a MINLP program, it becomes necessary to first transform such probabilistic relationships into their deterministic equivalents. The following two research papers discuss the transformation of probability functions into deterministic equivalents:

i) 'A gradient algorithm for chance constrained nonlinear goal programming' – Lee and Olson, (1985) and,

ii) 'Non-normal Deterministic Equivalents and a Transformation in Stochastic Mathematical Programming' – Goicoechea and Duckstein (1987)

2.2.7.1 Chance constrained nonlinear goal programming through the use of gradient algorithms and incorporating probabilistic to deterministic transformation - Lee and Olson, (1985)

The fact that certain decision environments are frequently characterised by many conflicting objectives and apart from being described by many different levels of operational uncertainty, sometimes sampling information is available as a means of describing this uncertainty. Such description can be appropriately achieved in the form of chance constraints and, in conjunction with this, nonlinear programming provides the means of resolving the multiple, conflicting objectives. It is therefore proposed that a nonlinear programming algorithm, based upon the gradient method and optimal step length, is utilised for chance constrained goal programming models. The resulting algorithm was evaluated for general applicability, reliability, precision sensitivity to parameters, data preparation and computational effort and convergence and was subsequently found to require minimal preparational effort It also displayed favourable computational time and achieved rapid convergence to the optimal solution, with exception in the case of models characterised by high degrees of' nonlinearity.

2.2.7.2 Non-normal Deterministic Equivalents and a Transformation in Stochastic Mathematical Programming – Goicoechea and Duckstein (1987)

In a paper by Goicoechea and Duckstein (1987), random variables of the continuous type are presented in two different ways, in a stochastic programming problem, to describe the treatment of objectives and constraints. Firstly, in the case of objective functions, an approach was presented for transforming them into their deterministic equivalents, and secondly, in the case of constraints, a general transformation approach was developed so that deterministic equivalents of constraints could be achieved within certain probability limits, and such an approach could be applied to continuous random variables as well. In the case of objective functions, the deterministic transformation of the stochastic program will yield a closed-form solution without resort to a Monte Carlo computer simulation. These approaches could also be extended to include stochastic problems with both discrete random variables as well as integer decision variables. An example is included.

2.3 Conceptual Foundation and Core Research Concepts

The conceptual foundation, or the key concepts, of this research can be derived from the Purpose statement (1.3), wherein it is stated: '*The primary purpose of this research is to define and derive a planning and optimisation methodology for a supply chain network that is subject to a multi-objective planning requirement and that is also subject to conditions of operational uncertainty, i.e. fuzzy and stochastic uncertainty. The intention is that it can be successfully applied to any uncertain Supply Chain Network that also has a planning requirement.* Therefore, the key defining research terms/concepts in this statement are defined in Table 2-5.

Table 2-5: *Literature research concepts*

i) Supply Chain Management, i.e. planning (multi-objectivity) and operations
ii) Supply Chain under Uncertainty, i.e. fuzzy and/or stochastic uncertainty, of which market demand, rather than supply, uncertainty is key
iii) Supply Chain Program Modelling & Optimisation, i.e. MILP/MINLP programming and solution.

Following the categorisation and analysis of the selected research literature into the three research concepts listed in Table 2-1, the supply chain optimisation technique utilised within each will be extracted and documented in a common standardised format. Methodology extraction will be achieved in chapter 3 - '*Methodology*

Development' whilst in chapter 4 – '*Methodology Integration*…', the various independent supply chain optimisation techniques will be logically integrated, according to previously researched standards and norms, to create the desired, *'Planning and Optimisation methodology for a Supply Chain under Uncertainty'*

Note (1): The letter 'U' in the term 'Uncertainty' is capitalised because the term can be comprised of up to two uncertain components, i.e. fuzzy uncertainty and/or stochastic uncertainty

2.4 Context of Research – A Supply Chain Network

A supply chain is an integrated process wherein a number of business entities (suppliers, manufacturers, distributors and retailers) work together in an effort to acquire raw materials, convert them into specified final products and deliver these final products to retailers who then sell them to customers. An example of a supply chain is given in Fig 2-2.

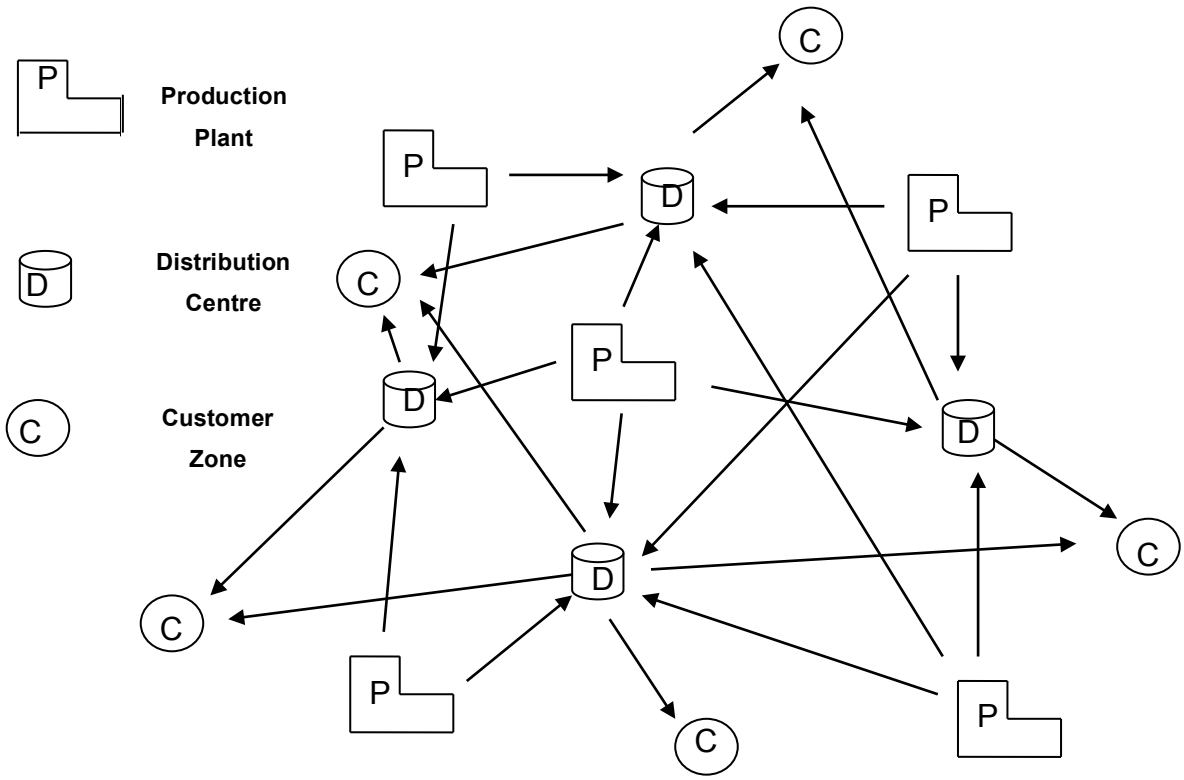


Figure 2-2 *Typical Supply Chain Network*

2.5 Establishment of Literature Research portfolio based on degree of conformance with identified Core Research Concepts

Further to the definition of Key Research Concepts, '2.3:- Conceptual Foundation', it is now possible to conduct a literature search exercise to identify potential journal research papers that conform to such Core Research Concepts. An initial and comprehensive literature exercise was conducted utilising the Core Research Concepts, either singly or combinatorially, as search criteria. This generated quite a comprehensive list of potential research journal papers (~90 in the Bibliography). These were subsequently assessed in terms of scope of operations and key features so that degree of fit, to the selected research criteria, could be established, resulting in a final list of preferred research journal papers, Table 2-6. This table also indicates the degree of fit, of the various research journal papers, to the various identified Core Research Concepts.

In order to derive the desired *'Planning and Optimisation of a Supply Chain Network under Uncertainty'* methodology, it will first be necessary to examine and analyse the selected prior supply chain under uncertainty research journal papers that, individually, may represent certain desired aspects of the required supply chain architecture. Such papers may also represent various individual or combinational aspects of planning and uncertainty in a supply chain network environment. It will also be very important that such analysis is cognisant of the integration and programming techniques utilised to develop any (full or partial) supply chain optimisation under (aspects of) uncertainty techniques. Once this has been done, identification and extraction of all necessary elements for the creation of the methodology can be achieved. This will enable the integration of all such necessary elements to create the desired 'Planning and Optimisation for a Supply Chain Network under Uncertainty' methodology.

Table 2-6: *Journal Research Literature conforming to Core Research Concepts*

Key Features of Literature Research Portfolio	Core Research Concepts		
	Supply Chain under (market demand) Uncertainty (Fuzzy, Discrete Probability, Continuous Probability, Stochastic Inventory Level based)	**Supply Chain Network Management (Planning & Operations) under Uncertainty**	**SCN Program Modelling & Optimisation under Uncertainty (programming using multi-objective, fuzzy, stochastic objectives & constraints)**
Multi-objective Fuzzy Optimisation of a Supply Chain under Uncertainty – based on the work of Bit et al. (1992)		X	X
Fuzzy Stochastic Optimisation of a SCN with probabilistic scenario's – based on the work of Gupta and Maranas (2003)	X		X
Optimisation of a Supply Chain Network, under different configuration scenarios, and subject to conditions of Multi-objective Fuzzy Uncertainty – based on the work of Tsiakis and Papageorgiou (2007)		X	X
Optimisation of a Supply Chain Network under Multi-objective Stochastic Fuzzy Uncertainty - based on the work of Chen et al. (2007)	X	X	X
A Probabilistic bi-level fuzzy linear multi-objective programming approach to supply chain planning – based on the work of Roghanian et al. (2007)		X	X
Optimisation of a Supply Chain Network under Multi-objective Stochastic Fuzzy Uncertainty – based on the work of You	X	X	X

and Grossmann (2008)			
Multiobjective supply chain design under uncertainty - based on the work of Guillen et al. (2005)	X	X	X
A Global (Pareto-) Optimisation Strategy for Chemical Supply Chains under uncertainty with environmental impact – based on the work of Guillen and Grossmann (2009)		X	X
Optimisation of a Supply chain Network under Stoch. Fuzzy uncertainty – based on the work of Santoso et al. (2004)	X		X
The Optimisation of a Bi-objective Supply Chain under Fuzzy uncertainty – based on the work of Cadona-Valdés and Ozdemir (2011)		X	X
The Optimisation of a Supply Chain Network (SCN) under Stochastic Fuzzy uncertainty – based on the work of Awudu and Zhang (2013)	X	X	X
Multi-Objective (Pareto-) Optimisation of Supply Chains in the process industry – based on the work of Liu and Papageorgiou (2013)		X	X
The Solution of Multi-Objective Linear Programs through Fuzzy Goal Programming – based on the work of Baky (2010)		X	X
A bi-objective stochastic fuzzy model for a warehouse in a supply chain network – based on the work of Razmi et al (2013)		X	X

Gradient algorithm for chance constrained fuzzy non-linear goal programming (incl. transformation of probabilistic functions into deterministic equivalents) – based on the work of Lee and Olson (1985)	**X**	**X**

Note: *The absence of a marking (**X**) in a particular area does not necessarily indicate that no research was done in that particular area by those particular researchers, but rather that the emphasis of the thesis was in other marked areas, **X.** Further, research literature selection was also based on variation in uncertainty type as well as on variation in operational type.*

Conclusion and Course of Action: Such papers, constituting the final journal research list, will be analysed and recompiled in an abbreviated format consisting of an overview plus any advantages and/or limitations associated with each paper. Such recompiled papers are analysed in the following chapter, '*Examination and Analysis of selected Review Portfolio*' Thereafter the supply chain, under uncertainty, optimisation techniques utilised therein will be extracted therefrom and then integrated to create the desired and comprehensive '*Planning and Optimisation of a Supply Chain, under uncertainty, Optimisation*' methodology. This portion of the work will be covered in a subsequent, 'Methodology Integration' chapter 3.

2.6 Examination and analysis of selected Literature Review Portfolio

Table 2-7 is a summary of the literature research journals discovered that were compatible with the three 'Core Research Concepts.

Table 2-7: *Literature Research Portfolio*

Research Paper	Core Research Concepts		
	Demand Uncertainty	Network Management	Modelling & Optimisation
The Fuzzy Programming Approach to Multi-Objective Solid Transportation Problem – Bit et al. (1992)		X	X
Managing Demand Uncertainty in Supply Chain planning - Gupta & Maranas (2003)	X		X
Optimal Production Allocation and Distribution Supply Chain Networks - Tsiakis & Papageorgiou (2007)		X	X
Multi-criteria fuzzy optimization for locating warehouses and distribution centers in a SCN - Chen et al. (2007)	X	X	X
A Probabilistic bi-level fuzzy linear multi-objective programming approach to supply chain planning – Roghanian et al. (2007)		X	X
Design of responsive supply chains under demand uncertainty – You and Grossmann (2008)	X	x	X

Multiobjective supply chain design under uncertainty Guillen et al. (2005)	X	X	X
A Global (Pareto-) Optimisation Strategy for Chemical Supply Chains under uncertainty with environmental impact – Guillen and Grossmann (2009)		X	X
A stochastic programming approach for supply chain network design under uncertainty – Santoso et al. (2005)	X		X
A bi-objective supply chain design problem with uncertainty - Cadona-Valdés and Ozdemir (2011)		X	X
The Optimisation of a Supply Chain Network (SCN) under Stochastic Fuzzy uncertainty – based on the work of Awudu and Zhang (2013)	X	X	X
Multiobjective optimisation of production, distribution and capacity planning of global supply chains in the process industry - Liu and Papageorgiou (2013)		X	X
Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach – Baky (2010)		X	X
A bi-objective stochastic fuzzy model for a warehouse in a supply chain network – based on the work of Razmi et al (2013)		X	X
A gradient algorithm for chance constrained nonlinear programming – Lee and Olson (1985)		X	X

From henceforth, each of the journal articles in Table 2-7 will be examined and analysed and, thereafter, the core '*Supply Chain, under uncertainty, Optimisation*' technique utilised within each will extracted and then, subsequently, integrated together to create the desired generic, *Supply Chain, under uncertainty,*

Optimisation' methodology that handle supply chain infrastructure environment and also handle any combinational supply chain uncertainty an/or planning requirement.

Note: in the following journal article examination section, where each article is analysed in terms of: '*Overview*', '*Advantages*' and '*Limitations*', it must be mentioned that any '*Advantages*' and '*Limitations*' identified would relate specifically to the pertinence of that particular article to this research effort, and to nothing else.

2.6.1 Multi-objective Fuzzy Optimisation of a Supply Chain under Uncertainty – based on the work of Bit et al.(1992)

Overview

A Fuzzy Multi-Objective Optimisation procedure for a supply chain under uncertainty was put together by Bit et al. (1992) and was termed, '*The Fuzzy Programming Approach to Multi-Objective Solid Transportation Problem*'. Much prior research work had been done on the design of multi-objective transportation networks but Bit et al (1992) introduced a fuzzy programming algorithm for solving the multi-objective solid transportation problem. The linear multi-objective solid transportation problem is a fuzzy linear multi-objective programming problem in which constraints are all equality type and the objectives are conflicting in nature. A generalisation of the linear multiobjective solid transportation problem, in which the supply, demand, and capacity constraints are not only of equality type but also of inequality type, is considered. This paper presents an application of fuzzy linear multi-objective programming to the linear multiobjective solid transportation problem. It gives efficient solutions as well as an optimal compromise solution.

Advantages

1) Two interactive algorithms were developed by Ringuest and Rinks (1987) to solve the fuzzy linear multi-objective programming problem to produce a preferred compromise optimal solution.

Limitations

1) The fuzzy linear multi-objective programming solution is not pareto optimal since two interactive algorithms are involved with solution determination
2) For the larger fuzzy linear multi-objective programs, it is not straight-forward in finding a preferred optimum compromise solution.

2.6.2 Fuzzy Stochastic Optimisation of a SCN under probabilistic scenarios – based on the work of Gupta and Maranas (2003)

Overview

In accordance with the paper of Gupta and Maranas (2003) entitled, '*Managing Demand Uncertainty in Supply Chain planning*', optimal supply chain network design can be achieved, under conditions of demand uncertainty, through the application of stochastic programming. A deterministic model was adopted as the benchmark formulation for highlighting the various issues involved in incorporating uncertainty in the decision making process. Importantly, the supply chain network considered in this initiative was multi-product, multi-site and multi-period in nature. Other key aspects of the model included limited capacity production equipment and conflict between inventory and customer levels. It was also shown that a framework for incorporating demand uncertainty could be constructed by appropriately partitioning the decisions variables and constraints of the deterministic model. In particular, the supply chain operation was classified into manufacturing and logistics decisions. The manufacturing decisions were made before demand uncertainty was realised, while the logistics decisions were postponed until a later time. The delay of logistics decisions was used as recourse against the evolution of uncertainty in product demand. A case study was used to highlight the ability of the proposed framework to address the key issues of uncertainty management in supply chain planning activities. It was demonstrated that, a more realistic description of the total planning costs (in terms of a probability distribution in contrast to a point estimate) could be obtained by utilising such presented framework

Advantages

1) The manufacturing decisions were made before the realization of the uncertain demand while the logistics decisions were postponed while the

option of delaying the logistics decisions was used as recourse against the evolving uncertainty in the product demand

2) The supply chain networks considered were multi-product, multi-site and multi-period in nature.

Limitations

1) The use of such bi-level solution framework is not Pareto optimal since a single MILP framework is not involved
2) Not a generic supply chain, under uncertainty optimisation methodology

2.6.3 Optimisation of Supply Chain Networks under different configuration scenarios - based on the work of Tsiakis and Papageorgiou (2008)

Overview

The paper of Tsiakis and Papageorgiou (2007) entitled, '*Optimal Production Allocation and Distribution Supply Chain Networks*', focused on the determination of optimal operating conditions for a supply network operating under different network configuration scenarios, subject to multi-objective operating conditions and also subject to certain operating and financial constraints. The production and distribution supply chain network under consideration consisted of a set of existing or potential manufacturing facilities, warehouses, distribution centres with multiple supply configurations and customer zones with varying demand. In addition to this, certain optimum operating questions need to be answered. Such questions include choice of production facility, choice of production mix per production facility, choice of which distribution centre(s) supplies which customers and then, finally, what inventory levels are necessary to maintain desired service levels.

This paper proposed an integrated model based on a detailed mathematical programming formulation that addressed some of the complex issues related to the design and operation of global supply chain networks. The focus was on certain business benefits such as the operational and distribution efficiency of the network, operational cost, customer satisfaction and product quality.

Advantages

1) The results obtained demonstrate clearly the savings of optimisation over heuristic decision-making in supply chain network structure design.
2) Different improved benefits through a number of different implementation options.
3) The single integrated and multi-faceted MILP model gives rise to a Pareto optimal solution.

Limitations

1) The model doesn't deal with either fuzzy or stochastic operational uncertainties

2.6.4 Optimisation of a SCN under Multi-objective Stochastic Fuzzy uncertainty - based on the work of Chen et al. (2007)

Overview

In a paper by Chen et al (2007) entitled, '*Multi-criteria fuzzy optimization for locating warehouses and distribution centers*', in a SCN, the authors investigated the simultaneous optimisation of multiple conflict objectives problem in a typical supply chain network with market demand uncertainties. The demand uncertainty is modelled as discrete scenarios with given probabilities for different expected outcomes. In addition to the total cost, the project considers the influence of local incentives and transport time to location decision. The problem is formulated as a mixed-integer linear programming (MILP) model to achieve minimum total cost, maximum robustness to demand uncertainties, maximum local incentives, and mini- mum total transport time. To find the degree of satisfaction of the multiple objectives, the linear increasing membership function is used; the final decision is acquired by fuzzy aggregation of the fuzzy goals, and the best compromised solution can be derived by maximizing the overall degree of satisfaction for the decision. The implementation of the proposed fuzzy decision-making method, as one can see in the case study, demonstrates that the method can provide a compensatory solution for the multiple conflict objectives problem in a supply chain network with demand uncertainties.

Advantages

1) The application of different discrete demand scenarios, of differing probabilities to describe the different expected market outcomes
2) The final best 'compromised' overall satisfaction decision is obtained by maximising the overall satisfaction level through the application of membership functions.

Limitations

1) Scenario-based modelling could result in certain operating variables being scenario dependent resulting in potential variable variation
2) Scenario based modelling does not give rise to Pareto optimal solutions

2.6.5 A probabilistic bi-level fuzzy linear multi-objective programming approach to supply chain planning – based on the work of Roghanian et al. (2007)

Overview

Research on Bi-Level Linear Programming (BLLP), which is a subset of multi-level programming, was conducted by Roghanian et al. (2007), in a paper entitled, '*A probabilistic bi-level fuzzy linear multi-objective programming approach to supply chain planning'*, in order for a formulated Mixed Integer Linear Program (MILP) problem, comprised of two interdependent parts, to be essentially broken down into those two different parts, called the leader, $F(x, y)$, and the follower, $f(x, y)$, and then to be solved independently of each other. However, the two problem scenarios are connected in a way that the leader's problem sets parameters that affect the follower's problem, and the leader's problem, in turn, is affected by the outcome of the follower's problem. A good example of this would be a decentralised company in which upper-level management decided on a budget for the company as a whole, and then lower division-level managements would compile division-level budgets based on their knowledge of the company budget. In this paper, a ''probabilistic bi-level linear multi-objective programming problem'' was considered and its application in enterprise-wide supply chain planning problem, where (1) market demand, (2) production capacity of each plant and (3) resources available to all plants for each product are random variables, was carried out. The constraints included joint and disjoint probability distributions. This probabilistic model was first converted into an equivalent deterministic model in each level, to which fuzzy programming technique was applied to solve the multi-objective nonlinear programming problem to obtain a compromise solution. This method can be applied to explicit situations by changing certain assumptions to solve the specific problem properly. Although the optimal solution is rarely possible, a compromise solution, which is acceptable for all parties with conflicting objectives, provides conflict resolution.

Advantages

1) This method can be applied to explicit situations by changing certain assumptions to solve the specific problem properly.
2) Ideal modelling procedure for a problem composed of multiple different parts where certain parts maybe dependent upon one another.

Limitations

1) Compromise optimal, and not a Pareto optimal, solution
2) Not a generic, uncertain supply chain optimisation methodology

2.6.6 Optimisation of a SCN under Multi-objective Stochastic Fuzzy Uncertainty – based on the work of You and Grossmann (2008)

Overview

The paper by You and Grossmann (2008) entitled, '*Design of responsive supply chains under demand Uncertainty*' addresses the optimisation of supply chain design and planning under responsive criterion and economic criterion with the presence of demand uncertainty. The supply chain consists of multi-site processing facilities and corresponds to a multi-echelon production network with both dedicated and multiproduct plants. The economic criterion is measured in terms of net present value, while the criterion for responsiveness accounts for transportation times, residence times, cyclic schedules in multiproduct plants and inventory management. By using a probabilistic model for stock-out, the expected lead time is proposed as the quantitative measure of supply chain responsiveness. The probabilistic model can also predict the safety stock levels by integrating stock-out probability with demand uncertainty. These are all incorporated into a multi-period mixed-integer nonlinear programming (MINLP) model, which takes into account the selection of manufacturing sites and distribution centers, process technology, production levels, scheduling and inventory levels. The problem is formulated as a bi-criterion optimization model that maximises the net present value and minimises the expected lead time. The model is solved with the ε-constraint method and produces a Pareto-optimal curve that reveals how the optimal net present value, supply chain network structure and safety stock levels change with different values of the expected lead time. A hierarchical algorithm is also proposed based on the decoupling of different decision-making levels (strategic and

operational) in the problem. The application of this model and the proposed algorithm are illustrated with two examples of polystyrene supply chains.

Advantages

1) The model integrated the long-term strategic decisions (e.g. installation of plants, selection of suppliers, manufacturing sites, distribution centres and transportation links) with the short-term operational decisions (e.g. product transitions and changeovers)
2) A Pareto optimal solution was achieved using the bi-criterion optimisation solution approach to achieve a trade-off curve between responsiveness and economics by using the ε-constraint method.
3) A hierarchical algorithm was further presented for the solution of the resulting large-scale MINLP problem based on decoupling of the decision-making levels (strategic and operational).

Limitations

1) Not a generic supply chain planning/optimisation methodology in that only probabilistic demand uncertainty was catered for

2.6.7 Optimisation of a SCN under Multi-objective Stochastic Fuzzy Uncertainty – based on the work of Guillen et al. (2005)

Overview

In a paper by Guillen et al (2005), entitled '*Multiobjective supply chain design under uncertainty*', and involving the integrated management of supply chains, the chief goals were the attainment of suitable economic returns for the Supply Chain (SC) stakeholders as well as the achievement of the desired satisfaction levels for customers. Additionally, since it was generally recognised that the operations of most supply chains are characterised by numerous sources of technical and commercial uncertainty, both deterministic and stochastic in nature, it was decided to adopt the ε-constraint method in conjunction with branch and bound techniques, to take into account not only SC profitability and customer satisfaction levels but also the uncertainty associated with financial risk, which is normally defined as the probability of not meeting targeted profit levels.

By utilising this method, the trade-off between the considered objectives (Pareto curve) can be obtained not only for the nominal case, but also when there is uncertainty about some of the parameters defining the production/distribution scenario. In this case, a Pareto stochastic curve can be obtained and the comparison with the equivalent deterministic one would demonstrate the convenience of using the stochastic formulation. The effects of these uncertainties can be accounted for as a risk associated with the NPV of the investment, which had been introduced as an additional objective into the model. Then, this risk can be managed to reduce the probability of having low earnings derived from the investment. The interaction between the design objectives has been shown. This way of generating different possible configurations will help the decision-maker determine the best design according to the selected objective

Advantages

1) A trade-off between the considered objectives (Pareto curve) can be obtained when there is uncertainty about some of the parameters defining the production/distribution scenario.
2) Risk management capability may be achieved through the comparison of Pareto stochastic behaviour with deterministic equivalents

Limitations

1) No generic supply chain under uncertainty optimisation capability in that only (two stage) probabilistic demand was considered

2.6.8 A Probabilistic bi-level fuzzy linear multi-objective programming approach to supply chain planning - based on the work of Guillen and Grossmann (2010)

Overview

In the paper by Guillen and Grossmann (2010) entitled, '*A Global (Pareto-) Optimisation Strategy for Chemical Supply Chains under uncertainty with environmental impact*', there is discussion of an emerging process engineering technique, known as Enterprise Wide Optimisation (EWO), whose main goal is the optimal integration of the supply, manufacturing and distribution activities in a supply chain environment. This is opposed to the current Supply Chain Management (SCM) ideology, which focuses more on the manufacturing stage. This is also in accordance with a recent trend of developing more sustainable processes since the type of analyses performed with regard to SCM/EWO analyses allows for any environmental impact to be determined. More specifically, the aim of this paper is to provide for a modelling and computational framework that can quantify the effect of a certain type of environmental uncertainty has on a supply chain model. This paper by Guillen and Grossmann (2010) involved the optimal design and operation of a chemical supply chain, but also taking into account certain environmental impact concerns, whose effect was assessed through the application of environment impact measuring technology, Eco-indicator 99. The overall problem was formulated as a bi-criterion chance constrained MINLP with two objective functions, one of which was to maximise NPV and the other of which was to simultaneously satisfy all the environmental targets. The resulting bi-criterion non-convex MINLP was solved by application of the ε-constraint method to guarantee a global optimum Pareto solution.

Advantages

1) Discussion of an emerging process engineering technique, known as Enterprise Wide Optimisation (EWO), whose main goal is the optimal integration of the supply, manufacturing and distribution activities in a supply chain environment
2) Global Pareto optimal solution

Limitations

1) No generic supply chain under uncertainty optimisation capability in that only environmental uncertainty, and not demand uncertainty, was considered

2.6.9 Optimisation of a SCN under Stochastic Fuzzy uncertainty – based on the work of Santoso et al. (2005)

Overview

It was evident, based on supply chain under uncertainty research conducted by Santoso et al. (2005), entitled, '*A stochastic programming approach for supply chain network design under uncertainty*' that prior stochastic programming research efforts into demand uncertainty were limited by the very small number of probabilistic scenarios that were utilised for modelling purposes. Therefore, in the paper by Santoso et al. (2005), it was decided to extend such previous research by increasing the demand sampling space. This would be achieved by providing an infinite number of demand scenarios through the integration of a sampling strategy, SAA – Sample Average Approximation method, with an accelerated Benders decomposition algorithm to solve supply chain design problems with continuous distributions for the uncertain parameter, and thereby providing for an infinite number of demand scenarios.

It was discovered that both techniques required iterative algorithms for solution and that, in the case of Benders decomposition, the number of iterations required for solution were discovered to be too large in practise.

Advantages

1) Demand sampling space in stochastic programming could be extended through the integration of a sampling strategy, SAA – Sample Average Approximation method, with an accelerated Benders decomposition algorithm, and thereby improving the accuracy of the model.

Limitations

1) Both techniques required iterative algorithm techniques for solution, but in the case of Benders Decomposition, the number of iterations required for solution was too large.
2) The SAA, Sample Average Approximation, technique is not Pareto optimal

2.6.10 The Optimisation of a Bi-objective Supply Chain under Fuzzy uncertainty – based on the work of Cadona-Valdés and Ozdemir (2011)

Overview

In the paper by Cadona-Valdés and Ozdemir (2011), entitled, `*A bi-objective supply chain design problem with uncertainty*' wherein the design of a bi-objective supply chain network is discussed, the focus of the research is on distribution facility population and location, which is considered important in defining the distribution network of the supply chain. This is because, although new distribution facilities incur both establishment and additional inventory costs, they also reduce the time to market and thus provide competitive advantage. In this work, all the cost implications of establishing new distribution centres are considered, which include the fixed costs of establishment as well as the effect on transportation nature and costs. Therefore, the intention is to minimise total cost as well as the total shipping time across the network. Consequently, the contribution of this paper is twofold: Firstly, greater operational realism is considered in the study by applying a bi-objective stochastic mixed integer linear programming approach and secondly, the solution approach is based on a fusion of the ε-constraint and L-shaped techniques.

Advantages

1) Greater levels of operational realism are realised in the study by applying a bi-objective stochastic mixed integer linear programming approach to accommodate these uncertainties
2) The solution approach is based on a fusion of the ε-constraint and L-shaped techniques in order to achieve Pareto optimality.

Limitations

2) Not a generc supply chain, under uncertainty, methodology in that only stochastic demand uncertainty is considered

2.6.11 The Optimisation of a SCN under Stochastic Fuzzy uncertainty – based on the work of Awudu and Zhang (2013)

Overview

As a result of increasingly high worldwide energy consumption, renewable energy sources, such as biofuels, have been considered as alternative energy sources, and especially in the transportation sector. However, in order to ensure a regular and consistent supply of biofuels to the market, a reliable and resilient supply chain is required. Also, since previous research works, in following a deterministic approach, did not adequately cover supply chain uncertainties, this paper will consider a combined deterministic/stochastic approach. The objective of the paper, by Awudu and Zhang (2013) and entitled, '*The Optimisation of a SCN under Stochastic Fuzzy Uncertainty*' was therefore to maximise the operating profit of a supply chain network, consisting of production plants and distribution centres, subject to uncertain prices and demand. The problem is modelled as a stochastic programming problem in two stages, with the first stage representing the production aspects and the second stage representing the distribution aspects. The Benders decomposition with Monte Carlo simulation is used to solve the proposed model. The prices of end products follow the Geometric Brownian Motion (CBM). The research effectively demonstrated that the optimal production planning of a multi-product biofuel plant could be achieved through applying an iterative two stage, master and sub-problem, stochastic linear programming procedure until tolerable optimum results were achieved. When this technique was applied to an actual biofuel supply chain in the United States, it was demonstrated that the optimum operating results achieved were better than the deterministic equivalents.

Advantages

1) The development of a stochastic linear programming model for the production planning of multi-product biofuel supply chains and whereby product demand uncertainty follows a normal distribution, the uncertainty of which is modelled through GBM's (Geometric Brown Motions)
2) The proposed stochastic model outperforms the deterministic equivalent
3) The proposed model can be applied in any biomass based biofuel supply chain

Limitations

1) The two-stage solution process is not Pareto optimal
2) Not a generc supply chain, under uncertainty, methodology in that only stochastic demand uncertainty is catered for.

2.6.12 Multi-Objective (Pareto-) Optimisation of Supply Chains in the process industry – based on the work of Liu and Papageorgiou (2013)

Overview

The paper by Liu and Papageorgiou (2013) entitled, '*Multiobjective optimisation of production, distribution and capacity planning of global supply chains in the process industry'* discussed the optimisation of a supply chain problem in which the production, distribution and capacity expansion decisions were optimised while considering total cost, total flow time and total lost sales as objectives. Most previous supply chain research models only considered a single criterion for supply chain planning and optimisation but in the last ten years, an increasing amount of research has been conducted on multi-objective problems and solution methods. Of these the ε-constraint method has been widely used to generate Pareto optimal solutions. However, since very little previous work has been done on the trade-off between supply chain cost, responsiveness and service levels, the purpose of the Liu and Papageorgiou (2013) research work is to develop a multi-objective formulation for the problem and then to adapt two solution approaches to solve the problem, i.e. ε-constraint method and the lexicographic minmax method. The ε-constraint Pareto-optimum method was applied to solve the multi-objective problem in which the 'total cost' function was assigned as the single objective to be optimised while the total flow and total lost sales functions were transformed into constraints.

Advantages

1) The ε-constraint method enabled two different capacity expansion strategies, i.e. proportional and cumulative capacity expansion, to be evaluated and compared in this research.
2) Pareto optimal

Limitations

1) Not a generic supply chain optimisation methodology

2.6.13 The Solution of Multi-Objective Linear Programs through Fuzzy Goal Programming – based on the work of Baky (2010)

Overview

The traditional approach towards the solution of mathematical linear programs only involved the inclusion of one performance objective, and also at only one decision-making level, in the overall programming mix. However, since many operational situations frequently involve the consideration of many, and often conflicting, performance objectives and this too in an hierarchical decision-making environment, the concept of Multi-Level Multi-Objective Linear Programming was introduced to account for such situations. Initial focus in this regard involved the introduction of bi-level multi-objective linear programming, to account for two decision-making levels, and this was soon supplemented by the introduction of three-level programming (TLP). However, it was soon realised that, in a hierarchical, multi-functional environment, typical of large production organisations, multiple performance objectives at multiple decision-making levels had also to be provided for. In the paper of Baky (2010) it is therefore the intention to formulate such multi-level multi-objective linear programming (ML-MOLP) capability. This paper by Baky (2010), entitled, '*Solving multi-level, multi-objective linear programming problems through goal programming approach*', presented procedures for solving multi-level multi-objective programming problems. However, since the focus of that research effort was on singular level multi-objective programming, only those aspects of this paper are focussed on in the problem example. This was achieved by minimising the group regret of degree of satisfaction of the decision makers in order to achieve the highest degree (unity) of each of the defined membership functions to the extent possible by minimising their deviational variables, and thereby obtaining the most satisfactory solution for each of the decision makers.

Advantages

1) The solution of a mathematical linear program with multiple performance objectives within a hierarchical decision level making environment
2) The presentation of procedures for solving multi-level multi-objective programming problems.

Limitations

3) This multi-objective and hierarchical decision level making MILP solution environment is not Pareto optimal

2.6.14 A bi-objective stochastic fuzzy model for a warehouse in a supply chain network – based on the work of Razmi et al (2013)

Overview

In a paper by Razmi et al. (2013) entitled, '*A bi-objective stochastic optimization model for reliable warehouse network redesign'* the fact was recognised that the traditional supply chain network design approach was changing in reflection of the increasing trend towards globalisation. As opposed to the traditional network design approach, any current redesign approach also should take into account any prevailing conditions or constraints that might impact the design. Chief amongst these is the design of warehouses in terms of their number, respective capacities and reliabilities and customer service levels. Literature research had revealed that a few shortcomings were evident with regard to prior warehousing research. These included the fact that a few important points had been ignored. Such ignored points included warehouse reliability, zone specific warehouse criteria such as environmental or capacity restrictions, and the fact that only one type of service disruption had been considered, i.e. delivery lead time (soft), but that other (hard) service level disruptions still had to be considered. The purpose of the paper by Razmi et al. (2013) was to address these shortcomings in an integrated model. The cost trade-off with two different delivery times, in conjunction with uncertain demand, is covered by means of a novel bi-objective stochastic optimisation solution approach. This was achieved by means of the formulation and solution of a two-stage bi-objective stochastic mixed integer linear programming model, with regard to simultaneously minimising operating costs and maximising customer demand levels, from the perspective of redesigning the warehouse chain in terms of consolidating capacities and minimising service disruption levels.

Advantages

1) In recognition of increasing economic globalisation, supply chain warehouse design took into account both hard and soft service level disruption issues.
2) The cost trade-off between these 'soft' and 'hard' service level disruption issues was achived by means of a novel two-stage bi-objective stochastic optimisation solution approach

Limitations

3) Such two-stage bi-objective stochastic optimisation solution approach is not Pareto optimal

2.6.15 Gradient algorithm for chance constrained fuzzy non-linear goal programming (incl. transformation of probabilistic functions into deterministic equivalents) – based on the work of Lee and Olson (1985)

The taking of management decisions often involves multiple, often conflicting, objectives against backgrounds of operational uncertainty. Although, sometimes, such uncertainty may be inconsequential in management decision-making, statistically determined measures of risk are often applied when the probability distribution of outcomes can be described. The application of more precise measures of uncertainty allows for more precise predictions of decision outcome. Goal programming has been developed as a means of analysis for a wide variety of decision problems involving multiple, potentially conflicting objectives. The gradient nonlinear goal programming algorithm of Lee and Olsen (1985), in their paper, '*A bi-objective stochastic optimization model for reliable warehouse network redesign*', provides an effective means to consider chance constrained models involving multiple goals. The ability to consider this class of decision model expands the support available to organizational decision makers. Consideration of the trade-off between opportunity and risk allows more realistic modelling of decision problems. Consideration of risk in linear models requires surrogate model constraints, such as the imposition of diversity or minimization of average variance. Use of chance constrained programming model allows more direct assessment of the impact of risk levels. Use of goal programming allows consideration of risk level as a goal, while seeking improved expected performance at the same time. Strengths of the algorithm include minimal model preparational effort. The most severe limitation of the algorithm is its specificity to a restricted class of model, as well as the limitations of all gradient based methods when highly nonlinear constraints are binding

Advantages

1) The gradient nonlinear goal programming algorithm of Lee and Olsen (1985) provides an effective means of considering chance constrained models involving multiple goals.

Limitations

1) A limitation of the algorithm is its specificity to a restricted class of model
2) Is not Pareto optimal

3 METHODOLOGY DEVELOPMENT – PLANNING AND OPTIMISATION METHODOLOGY FOR A SUPPLY CHAIN NETWORK OPERATING UNDER UNCERTAINTY

3.1 Introduction

It is the intention of this thesis to derive and then apply a 'Planning and Optimisation methodology for a Supply Chain under Uncertainty' for a ***generic*** process Supply Chain operating under any prevailing conditions of operational Uncertainty, i.e. fuzzy and/or stochastic uncertainty. By generic is meant a supply chain infrastructure that is typical of most applications in the commercial world and that can easily be expanded as represented in Fig 3-1. Such supply chain may consist of multiple production sites (P), multiple distribution centres (D) and multiple customer zones (C) with many potential network interconnections between them.

In order to derive such methodology, it is necessary to closely examine and analyse those selected prior supply chain under uncertainty research works (2.5), which, individually may represent certain aspects of a supply chain architecture and which may also, individually, represent various individual or combinational aspects of planning and uncertainty (i.e. multiple objectivity, fuzziness, stochastic uncertainty). It is very important that such analysis is cognisant of the integration and programming techniques used to compile any previously researched (full or partial) supply chain optimisation under (aspects of) uncertainty techniques. Once this has been done, identification and extraction of all necessary elements for the creation of the methodology can be achieved. This will enable the integration of all such necessary elements to create the desired 'Planning and Optimisation for a Supply Chain Network under Uncertainty' methodology.

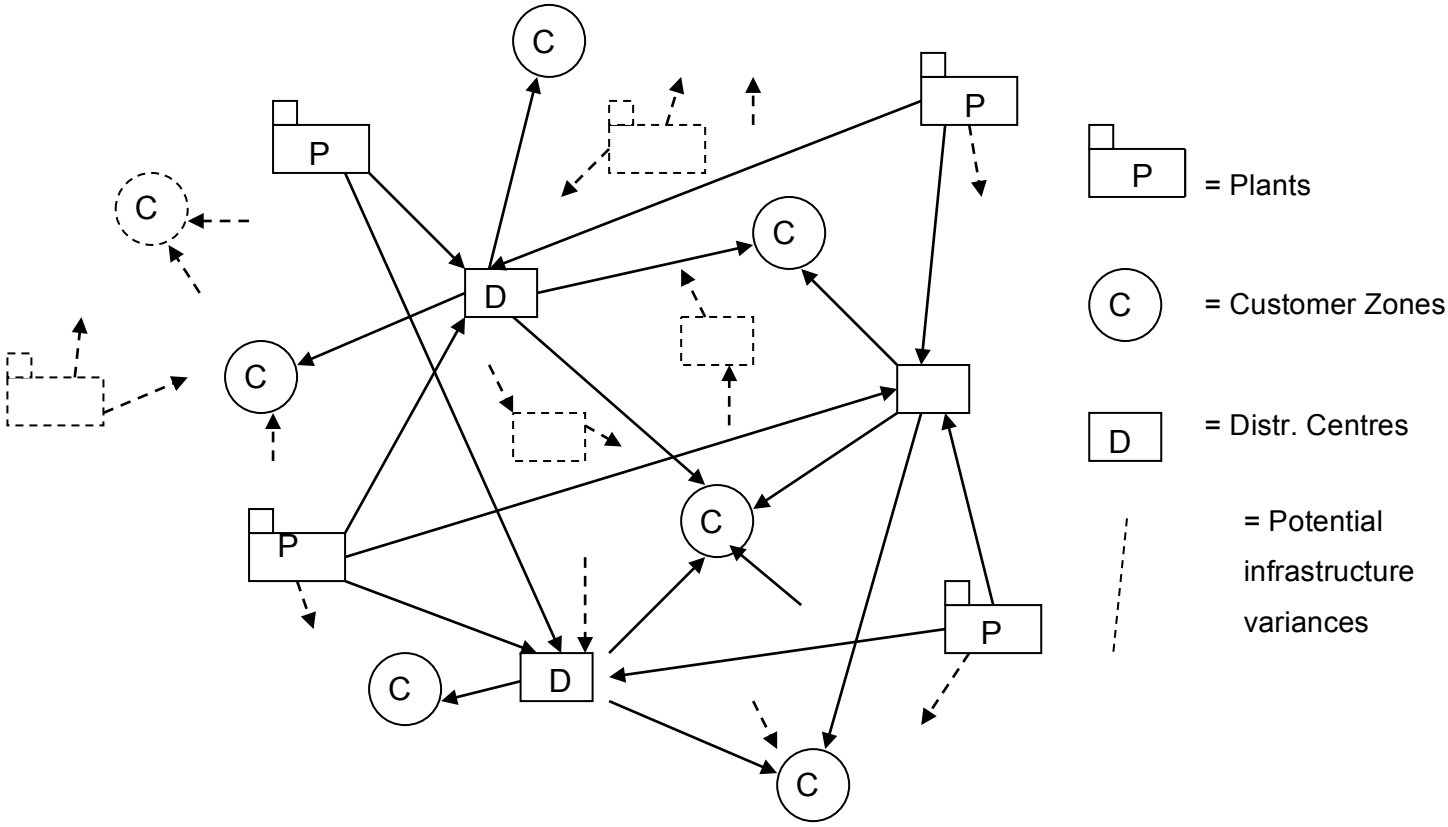


Figure 3-1 *Generic Supply Chain Network that can easily be expanded*

Once the extraction and subsequent integration of the necessary supply chain, under uncertainty elements is complete, it would then be possible to define and compile an Optimisation Methodology for a Supply Chain subject to conditions of operational uncertainty. Henceforth the research methodology to be employed is itemised in Table 3-1

Table 3-1: *Research Methodology*

a)	Diagrammatic description of generic Supply Chain Network
b)	Examination and analysis of previous and appropriate, Optimisation of Supply Chain under Uncertainty research works.
c)	Identification and extraction of all necessary elements for the creation of a Planning and Optimisation methodology for a Supply Chain Network operating under Uncertainty
d)	Definition of a Planning and Optimisation methodology for a Supply Chain Network operating under Uncertainty
e)	Integration and creation of a Planning and Optimisation methodology for a Supply Chain Network operating under Uncertainty
f)	Specification of a Planning and Optimisation methodology for a Supply Chain Network operating under Uncertainty
g)	Documentation
h)	Conclusion

3.2 Extraction of previous and appropriate Optimisation of Supply Chain Network under Uncertainty research works.

3.2.1 Introduction

Prior to examining and extracting previous and appropriate 'Optimisation of Supply Chains under Uncertainty' research works, it is at first necessary to define and then logically subdivide the scope of the research into individual and/or combinational aspects of Uncertainty applied, from an optimisation perspective, into individual and/or combinational aspects of a Supply Chain environment. By logical subdivision is meant those aspects of a Supply Chain under Uncertainty that can be dealt with independently. It is generally accepted by industry and the research community that a typical Supply Chain consists of those sequential elements listed in Table 3-2

Table 3-2: *Sequential elements of a typical Supply Chain*

1	Raw material procurement and Inventory management
2	Production
3	Warehousing of finished goods
4	Distribution
5	Sales

Further, based on a review of many previous research works into Supply Chain Optimisation under Uncertainty (see entire Bibliography), the reported forms of

Uncertainty in a Supply Chain, i.e. raw material procurement, production/manufacturing distribution, sales, environment are listed in Table 3-3 along with the planning parameter, 'Multiple Objectives'

Table 3-3: *Forms of Uncertainty and Planning in a Supply Chain environment*

Typical Sources	**Supply Chain Uncertainty (see Table 2.1 for definitions)**
Bit et al (1992) Tsiakis & Papageorgiou (2008) Chen et al. (2007) Guillen et al. (2005) Cardona-Valdés & Ozdemir (2011)	Fuzzy optimisation
Chen et al. (2007) Roghanian et al. (2007) You & Grossmann (2008) Santoso et al. (2005) Awudu & Zhang (2011)	Stochastic optimisation
Typical Sources	**Supply Chain Planning**
Bit et al. (1992) Gupta & Maranas (2003) Tsiakis & Papageorgiou (2008) Guillen et al. (2005) Liu & Papageorgiou (2013) Baky (2010)	Multi- Objective Optimisation

Now, since the intention in this thesis is to extract and examine previous research works that deal with individual and/or combinational aspects of Supply Chain Optimisation under individual and/or combinational aspects of planning and Uncertainty, it would be sensible to create a firm basis upon which this research can be conducted. This will be achieved by creating a table (Table 3-4) that illustrates many previously researched or all hypothetically possible individual and/or combinational aspects of Uncertainty, upon which optimisation is based, in various aspects of a Supply Chain environment, and then conducting the research accordingly.

3.2.2 Extraction of Supply Chain Network under Uncertainty Research journals

Table 3-4 illustrates all ***hypothetically*** possible combinations of Uncertainty applied to all key aspects of a typical Supply Chain environment (it remains 'hypothetical' until such time it is confirmed by prior research). The intention being to specify and conduct a comprehensive literature survey that will unearth all recognised previous research into Supply Chain Optimisation under ***all recognised*** forms of operational planning and/or Uncertainty, i.e. Multi-Objectivity, Fuzzy and Stochastic Uncertainty

Table 3-4: *Previously researched aspects of Uncertainty in a Supply Chain environment*

Supply Chain Operation	**Operational Uncertainty**		
	Multi-objective	**Fuzzy**	**Stochastic**
Raw material procurement and inventory management	√	√	√
Production	√	√	√
Warehousing	√	√	√
Distribution	√	√	√
Sales	√	√	√

It could well be that prior research has not been conducted on certain hypothetically defined areas of Supply Chain under Uncertainty Optimisation, listed in Table 4-4, because of operational irrelevancy or otherwise. Be that as it may, since the intention of this thesis is to formulate a Supply Chain, under conditions of Uncertainty, Optimisation methodology based on previously researched Supply Chain Optimisation under Uncertainty techniques, if any hypothetical aspect has not been previously researched, it will simply be ignored and noted accordingly. Following a comprehensive literature survey, based on the Concepts of Research defined in chapter 2, i.e.:

i) Supply Chain Management, i.e. planning (multi-objectivity) and operations

ii) Supply Chain under Uncertainty, i.e. fuzzy and/or stochastic uncertainty, of which market demand, rather than supply, uncertainty is key

iii) Supply Chain Program Modelling & Optimisation, i.e. MILP/MINLP programming and solution.

, the various research journals listed in Table 3-5 represent the final result of that survey.

Table 3-5: *Final result of literature survey – Research Journals Selected*

Research Journal paper title	Author(s)
Fuzzy programming approach to multiobjective solid transportation problem	Bit et al (1992)
Managing demand uncertainty in supply chain planning	Gupta and Maranas (2003)
Optimal production allocation and distribution supply chain networks	Tsiakis & Papageorgiou (2007)
Multi-criteria fuzzy optimization for locating warehouses and distribution centers in a supply chain network	Chen et al. (2007)
A probabilistic bi-level linear multi-objective programming problem to supply chain planning	Roghanian et al. (2007)
Design of responsive supply chains under demand uncertainty	You and Grossmann (2008)
Multiobjective supply chain design under uncertainty	Guillen-Gosalbez et al. (2004)
A global optimization strategy for the environment conscious design of chemical supply chains under uncertainty in the damage assessment model	Guillen-Gosalbez and Grossmann (2010)
A stochastic programming approach for supply chain network design under uncertainty	Santosa et al. (2005)
A bi-objective supply chain design problem with	Cadona-Valdés and

uncertainty	Ozdemir (2011)
Stochastic production planning for a biofuel supply chain under price uncertainties	Awudu and Zhang (2013)
Multiobjective optimisation of production, distribution and capacity planning of global supply chains in the process industry	Liu and Papageorgiou (2013)
Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach	Baky (2010)
A bi-objective stochastic optimization model for reliable warehouse network redesign	Razmi et al. (2013)
A gradient algorithm for chance constrained nonlinear goal programming	Lee and Olson (1985)

Now that the catalogue of necessary and appropriate research journal papers has been finalised, they can be individually analysed and the supply chain optimisation techniques described each therein can be extracted and then logically collated for the creation of a 'Planning and Optimisation of a Supply Chain under Uncertainty' methodology.

3.3 Analysis and extraction of all necessary elements for the creation of a 'Optimisation methodology for a generic Supply Chain Network operating under any prevailing conditions of Uncertainty' from selected prior research works

In this section of the work, each of the prior research journal papers that were selected, on the basis of degree of conformance to the pre-defined research concepts, will be analysed and the supply chain optimisation techniques contained each therein will be extracted and documented for subsequent processing.

3.3.1 Multi-objective Fuzzy Optimisation of a Supply Chain under Uncertainty – based on the work of Bit et al. (1992)

3.3.1.1 Introduction

In a paper by Bit et al (1992), a multi-objective fuzzy linear programming approach was developed for a supply chain distribution network consisting of multiple product origins and a supply chain network involving the transfer of homogeneous product to many different product destinations. The relevant programming aspects of this paper are fully described in 2.5.4.1. The intention of this paper was to determine an efficient and optimal compromise solution for the supply chain to accommodate the optimal multi-objective requirement. The optimisation methodology utilised to achieve this requirement is described in Table 3-6

3.3.1.2 Extraction of the Supply Chain Optimisation Methodology under Multi-objective Fuzzy Uncertainty

An analysis of the paper by Bit et al. (1992) reveals that the basic optimisation methodology for multi-objective fuzzy uncertainty employed may be summarised in Table 3-6. The relevant programming aspects of the paper are fully described in 2.5.4.1

Table 3-6: *Optimisation under Multi-objective Fuzzy Uncertainty Methodology based on the work of Bit et al. (1992)*

1	Define the supply chain problem
2	The requirement is to determine a compromise optimal solution amongst all operational objectives
3	Define Nomenclature requirements
4	Specify and define the objectives of the program
5	Specify and define the constraints and limitations for the program
6	Execute the Aspiration Level Multi-max or Multi-min subroutine i) Solve the MINLP problem, applying, each time, only one objective and ignoring the others ii) From the results of step i), determine the corresponding value for every objective for every solution derived iii) From step ii), determine, for each objective (Z_k), the best (L_k) and worst (U_k) values corresponding to the set of solutions. The 'best' (L_k) will be the highest value in a maximum scenario or lowest value (U_k) in a minimum scenario whereas the situation is reversed in a 'worst' case

	scenario iv) Formulate Aspiration Level relationship whereby, $\lambda_k \leq (Z_k - L_k)/(U_k - L_k)$ for each objective function, k Maximise λ_k in a MINLP for each objective function subject to constraints, typically using MS Excel Solver. The optimal compromise solution for the multi-objective problem is determined by the Aspiration Level (λ_k) function with the highest value v) Maximise λ_k in a MINLP for each objective function subject to constraints
7	The optimal compromise solution for the multi-objective problem is determined by the Aspiration Level (λ_k) function with the highest value
8	Record optimal compromise solution

3.3.2 Multi-objective Fuzzy Stochastic Optimisation of a Supply Chain Network utilising probabilistic scenario's – based on the work of Gupta and Maranas (2003)

3.3.2.1 Introduction

In the research work of Gupta and Maranas (2003), a bi-level, stochastic programming framework approach is adopted in modelling supply chain operations and implications as they are realised over time. With such bi-level framework approach, the manufacturing operations are modelled in a present timeframe, before demand realisations have occurred, whilst the logistical operations, which are dependent upon the manufacturing operations, are modelled in a future timeframe once demand certainty has been realised. Such an approach also enables the trade-off between customer satisfaction levels and production costs to be determined in the model. In addition to this, it also becomes possible to assess and manage an organisation's asset risks, such as inventory levels and profit margins, in the face of demand uncertainty. The model is exemplified by means of a case study.

The relevant programming aspects of the Gupta and Maranas (2003) research paper are fully discussed in 2.5.4.2 and the extracted supply chain optimisation methodology is shown in Table 3-7

3.3.2.2 Extraction of the Optimisation of the supply Chain under Multi-objective Stochastic Uncertainty Methodology

Table 3-7: *Optimisation under Multi-objective Fuzzy* Stochastic *Uncertainty Methodology based on the work of Gupta and Maranas (2003)*

1	Describe the supply chain network environment problem
2	Describe the ideal operational requirements of the system - Optimum product portfolio and utilisation level per production plant - minimisation of total annualised costs of the network
3	Define the demand uncertainty:- Based on stochastic demand uncertainty, a two-stage stochastic program (2SMP) is formulated (should discrete, probabilistic scenarios have been evident, a scenario-based approach would have been followed)
4	Nomenclature Section
5	Formulate the 2SMP model by first partitioning the overall operations into “here-and-now” (manufacturing) and “wait-and-see” (logistics) phases
6	Formulate the first “here-and-now” deterministic phase (manufacturing) MILP (mixed integer linear program) - specify the objective function (minimisation of operating costs) - define the constraints and limitations (2 – 8)

7	Formulate the second “wait-and-see” uncertain phase MILP - specify the objective function (minimisation of logistics costs) - define the constraints and limitations (9 – 12) and apply the expectation operator, E[x] to average, over all demand realisations, the costs incurred in the logistics phase
8	Solve the first “here-and-now” phase model by applying certain values, derived from the second “wait-and-see” phase through Eqn. 9
9	Record the optimum results

‘

3.3.3 ***Optimisation of a Supply Chain Network, under different configuration scenarios, and subject to conditions of Multi-objective Fuzzy Uncertainty – based on the work of Tsiakis and Papageorgiou (2007)***

3.3.3.1 Introduction

The principal objective of the research work of Tsiakis and Papageorgiou (2007) was to determine the optimal configuration of the production and distribution aspects of a supply chain network that is subject to various operational and financial constraints. Operational constraints included aspects such as product quality, production and supply restrictions while financial constraints included items such as production costs, transportation costs, logistics costs and as well as exchange rate duties for material flowing within the network.. The out-sourcing of production was also considered in the event that supply could not satisfy demand. A mixed integer linear programming (MILP) model was proposed to describe the optimisation problem and a case study, involving a business unit of a global specialty chemicals manufacturer, was utilised to demonstrate the applicability of the approach in a number of different operational scenarios.

Consequently, the work of Tsiakis and Papageorgiou (2007) proposes a strategic planning framework for multi-echelon supply chain networks that integrates all components associated with production facility location as well as that of distribution centre location.

Again, the relevant programming aspects of the Tsiakis and Papageorgiou (2007) research paper are fully discussed in 2.5.4.3 and the extracted supply chain optimisation methodology is shown in Table 3-8.

3.3.3.2 Extraction of the 'Optimisation of a Supply Chain Network under Multi-objective Fuzzy Uncertainty' Methodology

The underlying supply chain, under uncertainty, optimisation methodology utilised was extracted and is represented in Table 3-8

Table 3-8: *Optimisation under Multi-objective Fuzzy Uncertainty Methodology based on the work of Tsiakis and Papageorgiou (2007)*

1	Describe the supply chain network environment problem including an operational summary of all subcomponents, i.e. production centres, distribution centres and customer zones
2	Describe the ideal operational requirements of the system, e.g. product portfolio and utilisation level per production plant and the minimisation of total annualised costs of the network
3	Specify the notation, i.e. indices, parameters and variables, to be employed in the formulation of a MILP (mixed integer linear programming) model
4	Mathematically define all constraints of the problem - Network structure constraints - Logical constraints for transportation flows - Material balances - Production constraints

5	Mathematically define the objectives of the problem, i.e. the minimisation of:: - Fixed infrastructure costs - Production costs - Material handling costs at distribution centres - Transportation costs - Duties
6	Solution of the supply chain network MILP model via a number of optimisation scenarios using MS-Excel Solver: - Free optimisation exercise in which the model is allowed to determine all the required decisions - Fixed product allocation and customer assignment - Maximum allocation of products per plant
5	Results analysis
6	Conclusion

3.3.4 Optimisation of a Supply Chain Network under Multi-objective Stochastic Fuzzy Uncertainty based on the work of Chen et al. (2007)

3.3.4.1 Introduction

The research work of Chen et al (2007) considered the optimisation planning of a supply chain network consisting of a number of production plants, some distribution centres and a number of customer zones. Market demand uncertainty was taken into account by modelling it as a number of discrete scenarios with known probabilities. The supply chain model was formulated as a multi-objective mixed-integer linear program (MILP) with multiple conflicting objectives, from which a compensatory solution could be generated through the means of a two-phase fuzzy decision-making method. A numerical example was subsequently presented.

The programming aspects of this research paper are comprehensively described in the 2.5.4.4 and the extracted supply chain optimisation methodology is shown in Table 3-9

3.3.4.2 Extraction of the 'Optimisation of a Supply Chain Network under Multi-objective Fuzzy Uncertainty' Methodology

An analysis of the paper by Chen et al. (2007) reveals that the basic optimisation under multi-objective fuzzy uncertainty methodology employed may be summarised in Table 3-9

Table 3-9 *Optimisation under Multi-objective and Fuzzy Uncertainty based on the work of Chen et al. (2007)*

1	Problem description of the supply chain network including supply, production, warehousing, distribution and sales
2	Illustrated diagram of supply chain network
3	Discrete demand scenario's with known probabilities modelled as a MILP
4	Specification of all parameters and variables involved in the modelling of the supply chain network
5	Supply Chain modelling (MILP) with demand uncertainty 1) Specify and define all constraints i) Network structure constraints ii) Transport constraints iii) Material balance constraints iv) Cost Constraints 2) Specify and define multiple objectives for optimal planning v) Minimising total cost vi) Maximising the robustness to various demand scenarios vii) Maximising the local incentives
6	The conventional approaches for solving the multi-objective optimisation problems are usually by means of the fuzzy optimisation approach that can provide a single, yet unprejudiced,

	final decision as described as follows: i) Determine the membership function for each fuzzy objective based on the expected upper and lower bounds of the objective values ii) Considering all fuzzy objectives and using the minimum operator, maximising the degree of satisfaction for the worst situation iii) Applying the average operator, maximising the overall satisfaction level with guaranteed minimal fulfilment for all fuzzy objectives
7	Generation of optimum results

3.3.5 A Probabilistic bi-level fuzzy linear multi-objective programming approach to supply chain planning – based on the work of Roghanian et al. (2007)

3.3.5.1 Introduction

Since supply chain planning problems are concerned with the synchronizing and optimising of multiple activities involved in an enterprise from procurement through to product distribution to final customers, the research work of Roghanian et al (2007) considers such a multi-level decision network structure that may be mathematically modelled using a probabilistic bi-level linear multi-objective programming approach. With this approach, market demand, production capacity and resource availability are considered as random variables and the associated constraints may, or may not, be comprised of joint probability functions. Such probabilistic model is first converted into an equivalent deterministic model at each level, against which a fuzzy programming technique is applied to solve the multi-objective nonlinear programming problem so that an optimal compromise solution may be found.

An analysis of the paper by Roghanian et al. (2007), which is more fully discussed in 2.5.4.5 reveals that the basic optimisation under multi-objective fuzzy stochastic (probabilistic) uncertainty methodology employed may be summarised as in Table 3-10.

3.3.5.2 Extraction of the bi-level fuzzy linear multi-objective optimisation methodology put forward by Roghanian et al. (2007)

Table 3-10 *Optimisation of a Supply Chain under Multi-objective Fuzzy Stochastic Uncertainty based on the work of Roghanian et al. (2007)*

1	Problem description of the simple supply chain including production, distribution and sales
2	Simple illustrated diagram of the supply chain
3	Requirements Definition: Determine the minimal overall cost for production and distribution
4	Define type of Uncertainty involved and Solution approach: Market demand and distribution are random variables, which. follow joint probability distributions and therefore a 'probabilistic bi-level linear multi-objective programming' solution approach is followed
5	Define supply chain notation
6	Define the constraints a) Probabilistic production constraints b) Probabilistic distribution and warehousing constraints
6	Specify the Objective Functions a) Minimise production cost b) Minimise distribution and warehousing costs

7	Specify the 'probabilistic bi-level linear multi-objective (bblp) program' i) Min 'distribution and warehousing' costs' - Subject to probabilistic constraints ii) Min 'production costs' - Subject to probabilistic constraints
8	Solve the bblp problem i) Transform the bblp model into two sub-level deterministic programs, FLDM and SLDM ii) Solve both FLDM and SLDM iii) Are both solutions consistent? If not, adjust the tolerances of the objective functions & decision variables and return to pt. ii). If yes, solution is found.
9	Record the solution

3.3.6 Optimisation of a Supply Chain Network under Multi-objective Stochastic Fuzzy Uncertainty – based on the work of You and Grossmann (2008)

3.3.6.1 Introduction

In a paper by You and Grossmann (2008), the optimisation of a supply chain is considered from both a design and operational perspective, and by taking into account demand uncertainty. The intention was to determine optimal performance both in terms of financial criteria, i.e. NPV – Net Present Value, and operational criteria, i.e. supply chain responsiveness, measured in terms of expected lead time. Expected lead time is proposed as the quantitative measure of supply chain responsiveness, which may be achieved by using a probabilistic model for stock-out. Such probabilistic model can also predict the safety stock levels by integrating stock-out probability with demand uncertainty. The problem is formulated as a bi-criterion mixed-integer nonlinear programming (MINLP) model in which net present value (NPV) is maximised and the expected lead time is minimised. The relevant programming aspects of this paper are more fully discussed in Section, 2.5.4.6.

The supply chain, under uncertainty, optimisation methodology utilised in this research was extracted in and is described in Table 3-11.

3.3.6.2 Extraction of the 'Supply Chain Network under Multi-objective Stochastic Fuzzy Uncertainty' Methodology

An analysis of the paper by You and Grossmann (2008) reveals that the basic optimisation under multi-objective fuzzy uncertainty methodology employed may be summarised in Table 3-11

Table 3-11 *Optimisation of a supply chain under a Multi-objective and Fuzzy Uncertainty Methodology based on the work of You and Grossmann (2008)*

1	Problem description of the supply chain network including supply, production, warehousing, distribution and sales
2	Illustrated diagram of supply chain network
3	Specify the chief requirements of the supply chain network
4	Specification of all parameters and variables involved in the modelling of the supply chain network
5	Supply Chain modelling for a batch processing operation with demand uncertainty 1) Specify and define all deterministic constraints a) Network structure constraints b) Operational planning constraints c) Cyclic scheduling constraints d) Cost Constraint 2) Specify all probabilistic constraints related to demand uncertainty

	a) Stock-out probability b) Demand Uncertainty modelled by either Normal Distribution or Triangular Distribution c) Lead Time Definition (probabilistic) 3) Specify all process limitations 4) Specify and define multiple objectives for optimal planning a) Minimising lead time b) Maximising Net Present Value
6	Solution Procedure for batch processing operation (NB: this step wouldn't be necessary for a continuous process operation) a) Solve the MINLP (Mixed Integer Non-Linear Linear Programming) model for the entire operation by neglecting any transitions or change-overs between constituent plants b) For each plant, solve the MILP sub-problem to minimise the total transition time in a production cycle to obtain initial values for scheduling variables c) Solve the bounded model that uses convex envelopes to replace the bi-linear constraints d) Use the solutions from steps b) and c) as the starting points in solving the detailed MINLP model in the reduced space to determine the optimal scheduling and operational solution
7	Record the optimal solution

3.3.7 Optimisation of a Supply Chain Network under Multi-objective Stochastic Fuzzy Uncertainty – based on the work of Guillen et al. (2005)

3.3.7.1 Introduction

In a supply chain, under uncertainty, research paper by Guillen et al (2005), the effects of uncertainty in production were taken into account by incorporating a two-stage stochastic model in the planning process. The supply chain consisted of several production plants, a few warehouses and also some customer zones that were interconnected by means of an associated distribution network. Supply Chain performance was assessed by evaluating not only expected future profits but also the resulting demand satisfaction. Such an approach may also be utilised to obtain interesting solutions that may be obtained at different uncertainty levels. On the one hand, supply chain configuration solutions generated by means of deterministic mathematical programming considerations may be compared with those determined by different stochastic scenarios. In addition to this, such an approach would enable the determination of the financial risks associated with the different design options, which would result in a set of Pareto optimal solutions that could be used in decision-making.

The programming aspects of this paper are more comprehensively described in 2.5.4.7 and the supply chain, under uncertainty, optimisation methodology utilised is described in Table 3-12.

3.3.7.2 Extraction of the Optimisation under Multi-objective Stochastic Fuzzy Uncertainty Methodology

Table 3-12 *Optimisation of a supply chain under a Multi-objective Stochastic Fuzzy Uncertainty Methodology based on the work of Guillen et al. (2005)*

1	Problem description of the supply chain network including supply, production, warehousing, distribution and sales
2	Illustrated diagram of supply chain network
3	Requirements Definition: Determine the optimal configuration of the supply chain network in terms of the supply chain infrastructure and production rates of each product per plant
4	Define type of Uncertainty involved and Solution approach: Demand Uncertainty is represented by a set of scenarios with given probability of occurrence
5	Specify the constraints c) Mass balance constraints d) Capacity constraints
6	Specification of the Objective Function c) Maximise NPV d) Maximise the demand satisfaction e) Minimise the financial risk
7	Specification of Multi-objective Stochastic problem

	Maximise[E[NPV]; MDSat; -DRisk] per scenario Subject to constraints, eqns. (1) – (24), (27) and (31) MDSat $\geq$ 0 DRisk(x, Ω) $\leq \varepsilon_2$
8	viii) Compute the average NPV over all scenario's, E[NPV] = $\sum_s prob_s NPV_s$ ix) Compute the expected value of demand satisfaction over all scenario's, E[DSat] = $\sum_{t=2}^{T} \sum_s prob_s Dsat_{ts}/(T - 1)$ x) Determine the risk over all scenario's, DRisk = $\sum_s prob_s \delta_s$

3.3.8 A Global (Pareto-) Optimisation Strategy for Chemical Supply Chains under uncertainty with environmental impact – based on the work of Guillen and Grossmann (2009)

3.3.8.1 Introduction

With the research paper of Guillen and Grossmann (2009), the optimal design of chemical supply chains under uncertainty is discussed from the perspective of potential environmental impact. The overall problem is formulated as a bi-criterion stochastic non-convex mixed-integer nonlinear program (MINLP) and a deterministic equivalent is created by redefining the environmental impact probability constraint in the face of uncertainty. The solution to the redefined MINLP is obtained through application of the Pareto ε-constraint method, and global optimality is guaranteed through application of the new spatial branch and bound method. The capabilities and performance of the model are demonstrated by means of a case study.

The programming aspects of this paper are fully discussed in 2.5.4.8 and the extracted optimisation methodology under uncertainty is described in Table 3-13.

3.3.8.2 Extraction of the Global Pareto-Optimisation under Multi-objective Stochastic Fuzzy Uncertainty Methodology

Table 3-13: *Global Pareto-Optimisation of an environmentally conscious supply chain under a Multi-objective Stochastic Fuzzy Uncertainty Methodology - based on the work of Guillen and Grossmann (2009)*

1	Problem description of the supply chain network including supply, production, warehousing, distribution and sales
2	Requirements Definition: Determine the optimal configuration of the supply chain network that will maximise NPV and also minimise the environmental impact
4	Define type of Uncertainty involved and Solution approach: Fuzzy uncertainty is associated with the supply chain (production and distribution) aspects whilst stochastic uncertainty is associated with the environmental impact. These two aspects will be combined in a Pareto set to give a bi-criterion non-convex MINLP.
5	Formulate a standard MINLP, i.e. objective function plus constraints, for the determination of NPV in terms of the production and distribution aspects
6	Formulate an environmental impact MINLP, which is measured by the Eco-indicator 99 in terms of life cycle inventories associated with the emissions released and feedstock, manufacturing and distribution requirements
5	Integrate the environmental impact model with the production and distribution NPV MINLP to create a bi-criterion non-convex MINLP

6	Solution of the bi-criterion non-convex MINLP The global maximum of the bi-criterion MINLP will provide a Pareto solution that can be generated by application of the $\in$-constraint method by providing different instances of the MINLP and also by providing bound limits. This involves a 4-step algorithm that repeatedly calculates lower and upper bound limits until convergence occurs
7	Record the global optimum solution of the bi-criterion non-convex MINLP

3.3.9 Optimisation of a Supply chain Network under Stochastic Fuzzy uncertainty – based on the work of Santoso et al. (2004)

3.3.9.1 Introduction

The research paper of Santoso et al (2004) discusses stochastic programming model and solution modelling as a mechanism for generating realistic supply chain network design solutions. The proposed solution methodology integrates the recent sample average approximation (SAA) scheme with an accelerated Benders decomposition algorithm to rapidly generate quality solutions for comprehensive stochastic supply chain design problems with a very large number of scenarios. An appropriate supply chain network problem example, involving a large number of scenarios is provided to demonstrate the efficacy of the proposed solution strategy.

The programming aspects of this paper by Santoso et al (2004) are described in more detail in 2.5.4.9 and the extracted methodology is listed in Table 3-14.

3.3.9.2 Extraction of the programming approach for a supply chain under stochastic demand uncertainty

Table 3-14: *Extraction of the programming technique to establish demand scenarios necessary for an optimum solution in a supply chain under stochastic demand uncertainty environment based on the work of Santoso et al. (2004)*

1	Formulate a technique in order to determine the number of demand scenarios necessary for the determination of an optimum solution in a supply chain under stochastic uncertainty environment,
2	SAA – Sample Average Approximation i) Generate N realisations (demand scenarios) of the random vector, ξ, where $E[Q(y, \xi)] = \sum_{k \in K} \sum_{(i,j) \in A} q_{ijj}^{k} x_{i,,j}^{k}$ ii) Approximate the expectation vector, $E[Q(y, \xi)]$, by application of the SAA function, $N^{-1} \sum^{N}_{n=1} Q(y, \xi)$ iii) Consequently, such SAA approximation is minimised as follows: $\min \{\sum_{i \in P} c_i y_i + N^{-1} \sum^{N}_{n=1} Q(y, \xi)\}$ (Master problem) When the sample size, N, is large, it can be shown that the solution of the Master problem is convergent to an optimal solution of probability, one, and, being deterministic, it can be solved by appropriate optimisation techniques. iv) Such minimum sample size, N, is $N \geq (3\sigma^2_{max})/(\varepsilon - \delta)^2 \log(\lvert Y \rvert/\alpha)$
3	This technique guarantees that the number of probabilistic scenarios generated will provide an optimum solution

3.3.10 The Optimisation of a Bi-objective Supply Chain under Fuzzy uncertainty – based on the work of Cadona-Valdés and Ozdemir (2011)

3.3.10.1 Introduction

In the paper of Cadona-Valdés and Ozdemir (2011), a supply chain network is discussed with the intention of incorporating customer demand uncertainty in the determination of distribution centre location as well as in the determination of transport mode allocation. In addition to this, the potential impact on economic as well as service delivery considerations is discussed. The problem is formulated as a two-stage integer recourse problem so that an optimal network configuration, optimal transport mode allocation and optimal flow distribution can be determined. This is underpinned by a stochastic optimisation model, under demand uncertainty, with the inherent risk being modelled through scenarios. A solution method is proposed based on a L-shaped algorithm within an ε-optimality framework.

The programming aspects of this paper are discussed in 2.5.4.10 and the extracted supply chain optimisation methodology is listed in Table 3-15

3.3.10.2 Extraction of the optimisation methodology for a bi-objective supply chain under uncertainty

Table 3-15: *Optimisation methodology for a bi-objective supply chain under fuzzy uncertainty based on the work of Cadona-Valdés and Ozdemir (2011)*

1	Problem description of the supply chain network including supply, production, warehousing, distribution and sales and transportation
2	Requirements Definition: Determine the optimal design of the supply chain network that will maximise NPV and also minimise the delivery lead time
3	Define type of Uncertainty involved and Solution approach: Fuzzy uncertainty is associated with the supply chain (production and distribution) aspects, stochastic uncertainty is associated with uncertain demand whilst multi-objective (bi-objective) uncertainty is associated with the need to minimise both the costs and the delivery lead time in a Pareto optimal fashion
4	Formulate the bi-objective stochastic mixed integer linear program
5	Transform the bi-objective stochastic mixed integer linear program into a deterministic equivalent
6	Transform the deterministic equivalent into a single objective mixed integer linear program using the Pareto optimal $\in$-constraint method
7	Solve the resulting single objective mixed integer linear program by utilising the L-shaped technique, which handles any feasibility issues arising from stochastic programming
8	Record the Pareto optimal bi-objective result

3.3.11 The Optimisation of a Supply Chain Network (SCN) under Stochastic Fuzzy uncertainty – based on the work of Awudu and Zhang (2013)

3.3.11.1 Introduction

In the paper by Awudu and Zhang (2012), a stochastic production model is proposed for a biofuel supply chain, which is under demand and price uncertainties. Such model is orchestrated within a single-period planning framework so that expected profit may be maximised and so that all accompanying operational data can be generated. Demand uncertainty is modelled by known probability distributions and the price of end products follow Geometric Brownian Motion (GBM) and Benders decomposition (BD) with Monte Carlo simulation. The model is demonstrated through an ethanol plant case study.

The programming aspects of this paper are fully described in 2.5.4.11 and the supply chain optimisation methodology utilised is listed in Table 3-16

3.3.11.2 Extraction of the combined Benders decomposition and Monte Carlo simulation routine thus creating a 2 stage stochastic fuzzy optimisation methodology

Table 3-16: *The Two Stage Stochastic Fuzzy Optimisation Methodology based on the work of Awudu and Zhang (2013)*

Step	Description
1.	Set *i* (iteration counter) = 1, solve the Master problem, and let η = 0 in order to obtain the optimal decision values for the Master problem without cut. Also set the solution upper bound (UB) to infinity (∞) and set the solution lower bound (LB) = 0
2.	Use the Monte Carlo technique to generate N scenarios for the demand and price data by using the normal distribution and GBM respectively for all final (end) products
3.	Solve the sub-problem by using $z_{j,k}$ as a constant to determine the optimal variables, $s_{j,k}$, $F_{j,k}$, $L_{j,k}$ and $B_{j,k}$.
4.	Determine the various scenario solutions of the sub-problem and represent them as π_{ε} scenario solutions
5.	Update the upper bound setting by assigning UB_{I+1} = min[UB_I, Opt + (Sub – Opt)]
6.	Update the lower bound by applying LB_{I+1} = Opt
7.	Add equation (2) to Master problem

8.	Check whether (UB_I – LB_I) is tolerable, If so, the solution is optimal, if not, proceed with the iterations
9.	Finally solve the updated Master problem, add the updated cut, η

3.3.12 Multi-Objective (Pareto-) Optimisation of Supply Chains in the process industry – based on the work of Liu and Papageorgiou (2013)

3.3.12.1 Introduction

Since there is usually a requirement to measure the performance of supply chains in terms of multiple criteria, the paper of Liu and Papageorgiou (2013) addresses the issues of production, distribution and capacity planning in supply chains by considering cost, responsiveness and customer service level simultaneously. Consequently, a MILP (Mixed-Integer Linear Program) is developed with three objectives in mind and, additionally, two strategies are considered to expand the capacities of certain process plants in the model. The epsilon-constraint and lexicographic minimax (2.2.4) methods are therefore used to solve this multi-objective problem. The programming aspects of this paper are more comprehensively described in 2.5.4.12 and the supply chain the optimisation methodology utilised is listed in Table 3-17.

3.3.12.2 Extraction of the optimisation methodology for a bi-objective supply chain under uncertainty

Table 3-17: *Optimisation methodology for a bi-objective supply chain under fuzzy uncertainty based on the work of Liu and Papageorgiou (2013)*

1	Description and illustration of supply chain network
2	Requirements Definition: Minimise production costs, supply chain flow times and lost sales
3	Nomenclature Requirements
4	Define type of Uncertainty involved and Solution approach: Fuzzy uncertainty is associated with the supply chain (production and distribution) aspects, whilst multi-objective uncertainty is associated with the need to minimise the costs, supply chain flow times and lost sales in a Pareto optimal fashion. There is also a need to consider and incorporate capacity utilisation and expansion constraints
4	Formulate the multi-objective fuzzy mixed integer linear program – Assign the principal objective – Allocate the remaining objectives as Pareto ε-constraints – Specify constraints
5	Solve the MILP
6	Record (Pareto) optimum results

3.3.13 The Solution of Multi-Objective Linear Programs through Fuzzy Goal Programming – based on the work of Baky (2010)

3.3.13.1 Introduction

In the paper of Baky (2010), the solution to multi-level multi-objective linear programming (ML-MOLP) problems is achieved using two new fuzzy goal programming algorithms. This is achieved through a process of membership function definition of all fuzzy goals of all objective functions at all decision-making levels, as well as the membership function definition of all vectors of fuzzy goals of the decision variables, controlled by decision makers at top decision-making levels**.** This is followed by a process of minimising the deviational variables of each of the highest relative values of each of the membership functions, and thereby obtaining the most satisfactory solution for all decision-makers. Solution is achieved by means of the first algorithm grouping the membership functions at all levels and also the grouping of the decision variables at each level, except the lowest level, and then by applying the second algorithm lexicographically to solve the individual MOLP problems of the multi-level (ML-) MOLP problem.

The programming aspects of this paper are fully discussed in 2.5.4.13 and the extracted supply chain optimisation methodology is listed in Table 3-18

3.3.13.2 Extraction of the optimisation methodology for a multi-objective problem under stochastic fuzzy uncertainty

Table 3-18: *Optimisation methodology for a bi-objective supply chain network under stochastic fuzzy uncertainty based on the work of Baky (2010)*

1	Description of the problem
2	Requirements Definition: Minimise multiple objective functions
3	Nomenclature Requirements
4	Define type of Uncertainty involved and Solution approach: Fuzzy and multi-objective uncertainty is associated with the problem. The solution procedure is to transform the objective functions into membership functions and then solve the corresponding MILP model
5	Elicit the membership functions per objective function, $f_j(x)$ – Set the goals and limits, u_j and g_j per obj. fn. – Elicit the membership fns., $\mu_i(x) = [u_j - f_i(x)]/[u_j - g_j]_0^1$
6	Transform the membership function into a MILP program with corresponding objective functions and constraints
5	Solve the MILP per objective
6	Record the optimum results

3.3.14 A bi-objective stochastic fuzzy model for a warehouse in a supply chain network – based on the work of Razmi et al. (2013)

3.3.14.1 Introduction

In the paper by Razmi et al. (2013), in which a bi-objective two-stage stochastic mixed-integer linear programming model was developed for establishing a reliable warehouse network, optimal warehouse capacity that could accommodate either product phase-outs, product retention and/or product relocations, and including the delivery times, was to be determined. The proposed bi-objective model had two objectives, which were, 1) minimising supply chain costs and, 2) maximising percentage of customer demand achieved within a preferred delivery time. Pareto optimality was achieved by applying the augmented ε-constraint method. An industrial case study was used to validate the model.

The programming aspects of this paper by Razmi et al. (2013) are fully discussed in 2.5.4.14 and the optimisation methodology utilised is described in Table 3-19.

3.3.14.2 Extraction of the bi-objective stochastic optimisation methodology for a supply chain under uncertainty

Table 3-19: *Extraction of the bi-objective stochastic optimisation methodology for a Supply Chain under uncertainty based on the work of Razmi et al. (2013)*

1	Description of the supply chain network including production, warehousing, distribution and including the type and nature of two delivery times
2	Objective of the problem – Consolidation of warehouse capacity in order to minimise total cost, whilst maximising delivered demand

	satisfaction
3	Specification of all parameters and variables involved in the modelling of the supply chain network
4	List all specifications and assumptions
5	Demand uncertainty is represented by a number of different probabilistic scenarios
6	Formulation of the bi-objective stochastic program MINLP model i) Define and specify both objective functions ii) Define and specify all constraints
7	Follow the Pareto ε-constraint method for a bi-objective MINLP, as put forward by Haimes and Chankong (1979), so that both objectives, between them, achieve an optimum solution i) Select the principal objective as the single MINLP objective function ii) The other, secondary objective, is assigned as the ε-constraint iii) Solve the model
8	Record the optimum results

3.3.15 Gradient algorithm for chance constrained fuzzy non-linear goal programming (incl. transformation of probabilistic functions into deterministic equivalents) – based on the work of Lee and Olson (1985)

3.3.15.1 Introduction

The sampling of information is often used as a means of describing uncertainty in decision environments that are often characterised by such uncertainty. Such means of description are called chance constraints. A non-linear goal programming algorithm, based upon the gradient method is proposed for chance constrained goal programming models that use optimal step length. The resulting algorithm is checked for applicability, reliability, sensitivity to parameters, preparational and computational effort as well as for convergence, and was found to meet all criteria satisfactorily. The programming aspects of the Lee and Olson (1985) paper are fully discussed in 2.5.4.15 and the extracted supply chain, under uncertainty, optimisation methodology utilised is presented in Table 3-20

3.3.15.2 Extraction of the Optimisation Methodology for chance constrained non-linear goal programming

Table 3-20 *Optimisation under chance constrained goal programming – based on the work of Lee and Olson (1985)*

1	Formulate a general linear goal programming problem (for a supply chain environment) $\text{Min } Z = P_1(\sum w_i d_i^{+/-});\ldots; P_k(\sum w_i d_i^{+/-});$ $\text{s.t. } \sum a_{ij}x_j + d_i^- - d_i^+ = b_i,\ i = 1,\ldots, m$ $b_i,\ d_i^+,\ d_i^- \geq 0,\quad i = 1,\ldots,m$

	$x_j \geq 0, \quad j = 1,\ldots, n$, where x_j = j-th decision variable b_i = target for i a_{ij} = coefficient of x_j in the i-th goal constraint d_i^- = underachievement of goal i d_i^+ = overachievement of goal i P_k = k-th pre-emptive priority level w_i = weight assigned to d_i^+ and d_i^- at priority k
2	Introduce chance constraints, e.g. $\Pr(\sum a_{ij}x_j \leq b_i) > \alpha_1$, that yield deterministic equivalents that are non-linear, $E(\sum a_{ij}x_j) + F^{-1}(\beta)[V(\sum a_{ij}x_j - b_i \leq E(b_i)$, where E( ) and V( ) are the expected value and variance operators respectively
3	The gradient algorithm for non-linear goal programming a) Determine normalised constraints b) Determine an improving direction, X_p^* c) Determine optimal direction for the iteration, $U_p = X_p^* - X_{p-1}$ d) Determine optimal step length, S_p , $X_p = S_p(U_p) + X_{p-1}$ e) Evaluate for convergence (if any variable has changed by more than 0.001 go back to a))
4	Record the optimum solution point, X_p

4 INTEGRATION TO CREATE A PLANNING AND OPTIMISATION METHODOLOGY FOR A SUPPLY CHAIN UNDER UNCERTAINTY

4.1 Methodology Analysis

Following the extraction of all the 'Supply Chain, under Uncertainty, Optimisation' methodologies utilised in the various selected and analysed papers, it now remains to logically integrate them so that a generic, 'Supply Chain under prevailing conditions of planning and operational Uncertainty (multi-objectivity, fuzziness, stochastic uncertainty) Optimisation methodology' may be compiled for any uncertain supply chain environment. By 'logical' is meant being in accordance with the integration procedures utilised by some of the constituent methodologies, e.g. the 2SMP technique of Gupta and Maranas (2003), without contravening any potential conditions or limitations. The sources of the various extracted methodologies are summarised in Table 4-1. It is to be expected that, even though some of the papers may discuss identical cases of supply chain uncertainty, their optimisation techniques may differ slightly, in which case it will be important to identify and document the fundamental technique.

Table 4-1 *Extracted Optimisation under Uncertainty Methodologies*

SCN, under uncertainty, Optimisation Methodology	Explanation	Based on the work of:
Multi-Objective Fuzzy Optimisation	SCN (Supply Chain Network) Optimisation under conditions of multi-objective and/or fuzzy uncertainty	Tsiakis & Papageorgiou (2006) Baky (2010) Bit et al. (1992)
Bi-level/Objective Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of stochastic (probabilistic scenarios) fuzzy uncertainty in a bi-level solution approach	Gupta and Maranas (2003)
Single Objective Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of stochastic (large no. prob. demand scenarios) fuzzy uncertainty using Benders Decomposition & Monte Carlo simulation to solve	Awudu and Zhang (2013)
Multi-Objective (Pareto-optimal) Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of stochastic (probabilistic demand scenarios) fuzzy uncertainty using the Pareto ε-constraint method	Guillen et al. (2004)

Multi-Objective Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of multiple objectivity and stochastic (continuous probability demand + chance constraints for safety stock levels) uncertainty	You and Grossmann (2008)
Multi-Objective Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of multiple objectivity and stochastic (discrete probability demand scenarios) uncertainty	Chen et al. (2007)
Multi-Objective Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of multiple objectivity, stochastic (chance constraints) and fuzzy uncertainty	Roghanian et al. (2007)
Multi-Objective (Pareto-optimal) Fuzzy Optimisation	SCN Optimisation under conditions of fuzzy and multiple objective uncertainty, utilising the Pareto ε-constraint method	Guillen and Grossmann (2013) Liu and Papageorgiou (2013)
Single Objective Stochastic Fuzzy Optimisation	SCN Optimisation under conditions of stochastic (prob. scenarios) and fuzzy uncertainty, using SAA and Benders Decomp. to provide for infinite scenarios	Santosa et al. (2005)

A comparison between all the various extracted methodologies can best be achieved by viewing Tables 4-2, which presents all methodologies in a numbered and structurally consistent format. The development of such structurally consistent format was based on an analysis of all extracted methodologies and then the identification of all possible and important sequential steps. The identification of all possible and important sequential steps was vital to ensure that a generic 'Supply Chain, under prevailing conditions of uncertainty, Optimisation' methodology could be developed.

See the sequence of Tables 4-2 overleaf:

Table 4-2 *Comparison of Extracted* Methodologies

	Bit et al. **Multi-obj. Fuzzy Opt**	**Gupta & Maranas** **Multi-obj. Stoch. Opt.**	**Tsiakis &Papageorgiou** **Multi-obj. Fuzzy Opt**
1	Describe the supply chain environment	Describe the supply chain network	Describe the supply chain network environment
2			An illustrated diagram of the supply chain network
3		Identification of binary production/network variables	Identification of binary production/network variables
4	Requirements Definition	Requirements Definition	Requirements Definition
4	Multi-objective Fuzzy MILP programming plus Aspiration Level subroutine for multi-max, multi-min or max-min scenarios	Bi-level stochastic programming to accommodate two phases, manufacturing and logistics	Multi-objective Fuzzy MILP programming with different scenario configurations
5	Nomenclature	Nomenclature	Nomenclature
6	Formulate a MILP (Mixed Integer Linear Program0 - Define objectives - Specify constraints and limitations	Formulate a two-stage stochastic MINLP (2SMP) program - First-stage, 'variables are determined prior to resolution of the - Second-stage, variables - Define all constraints	Formulate a MILP program - defining the objectives - define all constraints
7	Execute the Aspiration level program to determine, Multi-min or Multi-max or a combination of Max-Min	Coupling between the 1st and 2nd stages is achieved by constraint variable that is required by the 2nd stage and is provided by the 1st	
8	Solve the model and record the optimal solution	This recourse optimisation problem determines optimal 2nd stage results given the optimal 1st stage results	Solution of the supply chain network MILP model based on a number of possible configuration scenario's

	Chen et al. **Multi-obj. Stoch. Fuzzy Opt**	**You and Grossmann** **Multi-obj. Fuzzy Stoch. Opt.**	**Guillen et al. (2005)** **Multi-obj. Fuzzy Stoch. Opt.**
1	Description of the SCN	Description of the SCN	Description of the SCN
2	Illustrated diagram of supply chain network	Illustrated diagram of supply chain network	Illustrated diagram of supply chain network
	Identification binary prod./network variables	Identification binary prod./network variables	
3	Requirements Definition	Requirements Definition	Requirements Definition
4	Discrete prob. Scenarios followed by fuzzy optimisation for unprejudiced solutions	MINLP modelling with probabilistic constraints	Discrete stochastic scenario's followed by MINLP programming
5	Nomenclature	Nomenclature	Nomenclature
6	Formulate a Mixed Integer Non-Linear Programming model (MINLP) - define probabilistic and deterministic objectives - Define all probabilistic and deterministic constraints - Convert all probabilistic obj. and constraints into deterministic ones	Formulate a MINLP or MILP program. - Define multiple objectives - specify all constraints for each probabilistic scenario sub-program - Convert all probabilistic objectives and constraints into deterministic equivalents	Formulate a MINLP program - Define multiple obj. - defining all deterministic and probabilistic constraints - Convert all probabilistic objectives and constraints into deterministic ones
7	Compromise soln. obtained:	Solve the MINLP program per scenario	Solve the MINLP program
8	Compute the average result for each probabilistic objective over all scenario's using the expectation operator	Generation of optimum results by summating all various weighted probabilistic scenario's	Compute the final result by the summation of prob. Weighted decision variables

	Cadona-Valdés and Ozdemir (2011) **Bi-obj. Stoch. Fuzzy Opt**	**Razmi et al. (2013)** **Bi-obj. Stoch. Fuzzy Opt.**	**Baky (2010)** **Multi-obj. Fuzzy Opt.**
1	Description SCN	Description of SCN	Description of SCN
2			Illustrated diagram of supply chain network
3	Identification binary prod./network variables	Identification binary prod./network variables	
4	Requirements Definition	Requirements Definition	Requirements Definition
5	Stochastic demand uncertainty with MILP modelling with fuzzy production and distribution uncertainty	Demand uncertainty is represented by a number of different probabilistic scenarios and fuzzy MILP solution approach	Multi-objective fuzzy uncertainty with objective functions transformed into membership functions, which then allow the formation of a MILP model
6	Nomenclature	Nomenclature	Nomenclature
7	Formulate a Mixed Integer Linear Programming model (MILP) - define probabilistic and deterministic objectives - Define all probabilistic and deterministic constraints - Convert all probabilistic obj. and constraints into deterministic ones	Formulate a MINLP program. - Define multiple objectives - specify all constraints for each probabilistic scenario sub-program - Convert all probabilistic objectives and constraints into deterministic equivalents	Elicit the membership functions per objective function - Set the goals and limits per obj. Fn. - Elicit the membership fns.
8	Transform one objective into a Pareto ε-constraint	Transform one objective into a Pareto ε-constraint	Transform membership fns into MILP with associated objective functions and constraints
9	Solve the MILP and record the Pareto optimal result	Generation of optimum solution by summating all probabilistic scenario's	Solve the MILP per objective and record the results

	Guillen and Grossmann **Multi-obj. Stoch. Fuzzy Opt**	**Liu and Papageorgiou** **Multi-obj. Stoch. Fuzzy Opt.**
1	Description of the supply chain network	Description of the supply chain
2	Identification binary prod./network variables	Identification binary prod./network variables
2	Requirements Definition	Requirements Definition
3	MINLP modelling with multi-objective /fuzzy/stochastic constraints	MILP modelling with multi-objective/fuzzy/stochastic constraints
4	Nomenclature	Nomenclature
5	Formulate a MINLP model - define probabilistic and deterministic objectives - Define all probabilistic and deterministic constraints - Convert all probabilistic obj. and constraints into deterministic ones - Integrate the environ. impact model	Formulate a MILP model - Define multiple objectives - specify all constraints for each probabilistic scenario sub-program - Convert all probabilistic objectives and constraints into deterministic equivalents
6	Transform one objective into a Pareto ε-constraint	Transform one objective into a Pareto ε-constraint and solve the MILP program per scenario
7	Solve the MINLP	Solve the MILP per scenario and obtain optimum solution by summating all probabilistic scenario's

4.2 Extracted Methodology Integration

.An analysis of the extracted Optimisation under Uncertainty methodologies from the papers listed in Tables 4-2 reveals that there is a common theme to the basic structure of those methodologies that also caters for any variations, e.g. nature of demand uncertainty and solution approach, nature of constraints etc. In addition to this, certain specific methodology steps/techniques were disregarded because they did not represent the latest accepted steps/techniques for the process involved, e.g. the Bit et al. (1992) Aspiration Level multi-objective routine that has long been surpassed by the Pareto ε-constraint routine (Guillen and Grossmann (2009) and Liu and Papageorgiou (2013) This common theme structure, with variation, for all extracted methodologies is presented in Table 4-3

Table 4-3 *Common theme structure and breakdown of extracted optimisation of supply chain under uncertainty methodologies*

Section	Common constituent sequential components
Introduction	i) Description of supply chain network' ii) Description of the problem iii) Illustrated diagram of the supply chain network *iv)* Identification of all desired supply chain linkages, i.e. production plant (Pl) – distribution centre (Dc) and Dc – Customer Zone (Cz), using binary variables, e.g. x_{ij} and y_{jk}

	v) Nomenclature Section
Nature of Uncertainty and Solution Approach	vi) fuzzy demand (sales ≤ x tonnes per period) followed by MILP/MINLP programming' solution **Or** vi) discrete probabilistic demand scenario's, followed by solution of MILP/MINLP program per scenario and then weighting of optimum results according to respective probabilities **Or** vi) Continuous probabilistic demand. Transformation of stochastic relationships into deterministic ones and then solution of MILP/MINLP program **Or** vi) A two stage stochastic modelling program (2SMP) for an inventory level-demand based management system. The two stages (production and logistics) could be any appropriate Uncertainty combination
Supply Chain Modelling, Programming and Execution (this typically represents the bulk of the work)	Dependent on the approach decided above but basically involves the creation of a Linear Program under Uncertainty Model - Formulation of operational objective relationship(s) - Formulation of constraint equation relationships for both the individual supply chain activities, i.e.

	production and distribution, as well as for the binary network constraints - If probabilistic objectives or constraints exist, then the transformation of these into deterministic equivalents e.g. using techniques of Lee and Olson (1985) **And /Or Not** - In the event of multiple objectives, the principal objective is selected as the MILP/MINLP objective whilst the others are allocated as Pareto ε (epsilon) constraints, with the value of ε being either the upper (max) or lower (min) limit of the ε-constraints - Final formulation of (deterministic) MILP/MINLP
Solution Processing	Solution of Linear Program(s) under Uncertainty (typically computer aided, e.g. Microsoft Excel Solver)
Presentation of optimum results	

It is clear from Table 4-3 that the key decision point for the Optimisation of a Supply Chain under Uncertainty is the nature and treatment of Demand Uncertainty. The reported options are:

1) Fuzzy demand (tonnes per period) followed by MILP/MINLP programming' solution.

2) Discrete probabilistic demand scenario's, followed by solution of MILP/MINLP program per scenario and then weighting of optimum results according to respective probabilities.

3) Continuous probabilistic demand - Transformation of stochastic relationships into deterministic ones and then solution of MILP/MINLP program

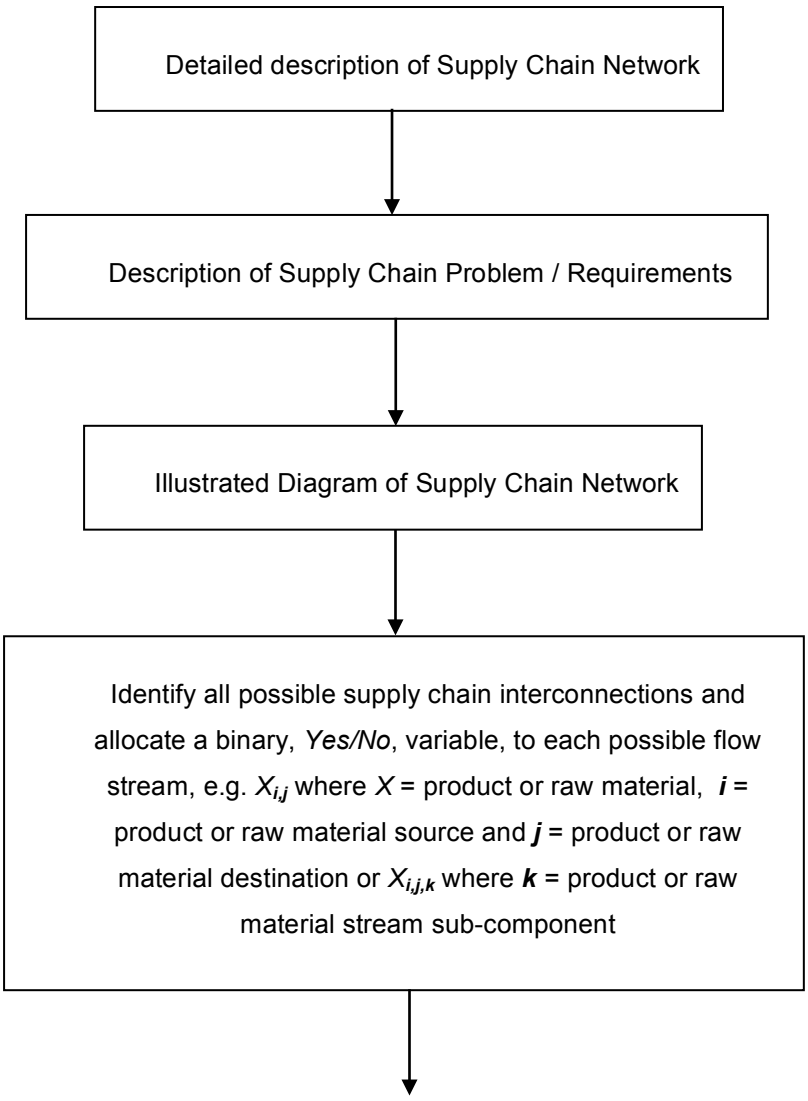
4) A two stage stochastic modelling program (2SMP) for an inventory level-demand based management system. The two stages (typically production and logistics) could be any appropriate Uncertainty combination

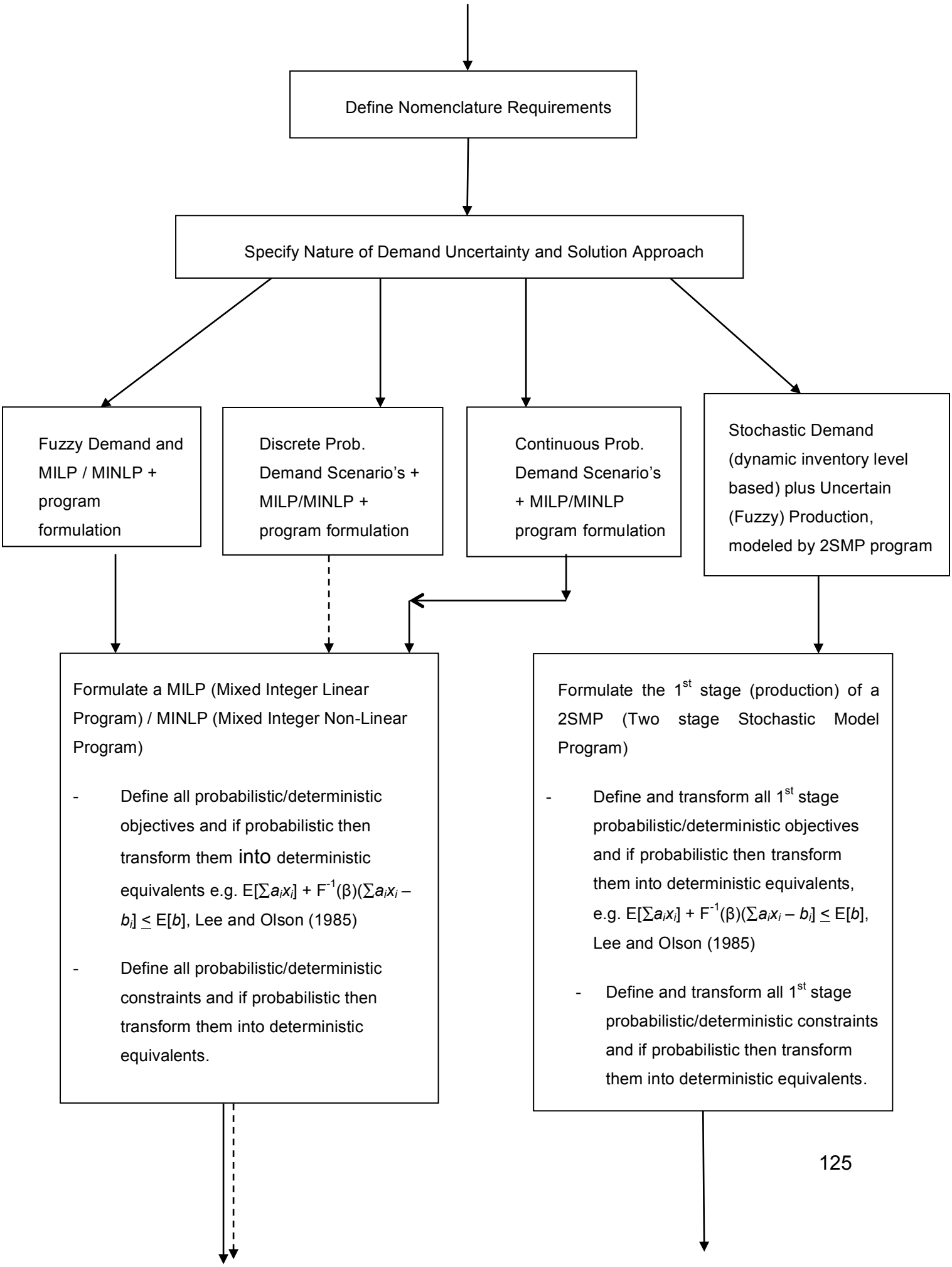
It is therefore now possible to construct a 'Planning and Optimisation Methodology' for Supply Chains that operate under conditions of Uncertainty, which, based on the contents of Table 4-3, can accommodate a variety of Demand Uncertainty options as well as accommodating some operational variations such as the nature of constraints, be they deterministic or probabilistic.

5 SPECIFICATION OF A PLANNING AND OPTIMISATION METHOD FOR A SUPPLY CHAIN NETWORK UNDER UNCERTAINTY

A process flow diagram representation of the 'Planning and Optimisation Methodology for a Supply Chain operating under Uncertainty' can now be presented (Fig 5-1), initially, to be followed by a more detailed tabular description, Table 5-1, of the various stages involved

5.1.1 Flow Diagram Description of Methodology




Define Nomenclature Requirements
Specify Nature of Demand Uncertainty and Solution Approach
Fuzzy Demand and MILP / MINLP + program formulation
Discrete Prob. Demand Scenario's + MILP/MINLP + program formulation
Continuous Prob. Demand Scenario's + MILP/MINLP program formulation
Stochastic Demand (dynamic inventory level based) plus Uncertain (Fuzzy) Production, modeled by 2SMP program
Formulate a MILP (Mixed Integer Linear Program) / MINLP (Mixed Integer Non-Linear Program)
- Define all probabilistic/deterministic objectives and if probabilistic then transform them into deterministic equivalents e.g. $E[\sum a_i x_i] + F^{-1}(\beta)(\sum a_i x_i - b_i] \leq E[b]$, Lee and Olson (1985)
- Define all probabilistic/deterministic constraints and if probabilistic then transform them into deterministic equivalents.
Formulate the 1st stage (production) of a 2SMP (Two stage Stochastic Model Program)
- Define and transform all 1st stage probabilistic/deterministic objectives and if probabilistic then transform them into deterministic equivalents, e.g. $E[\sum a_i x_i] + F^{-1}(\beta)(\sum a_i x_i - b_i] \leq E[b]$, Lee and Olson (1985)
- Define and transform all 1st stage probabilistic/deterministic constraints and if probabilistic then transform them into deterministic equivalents.

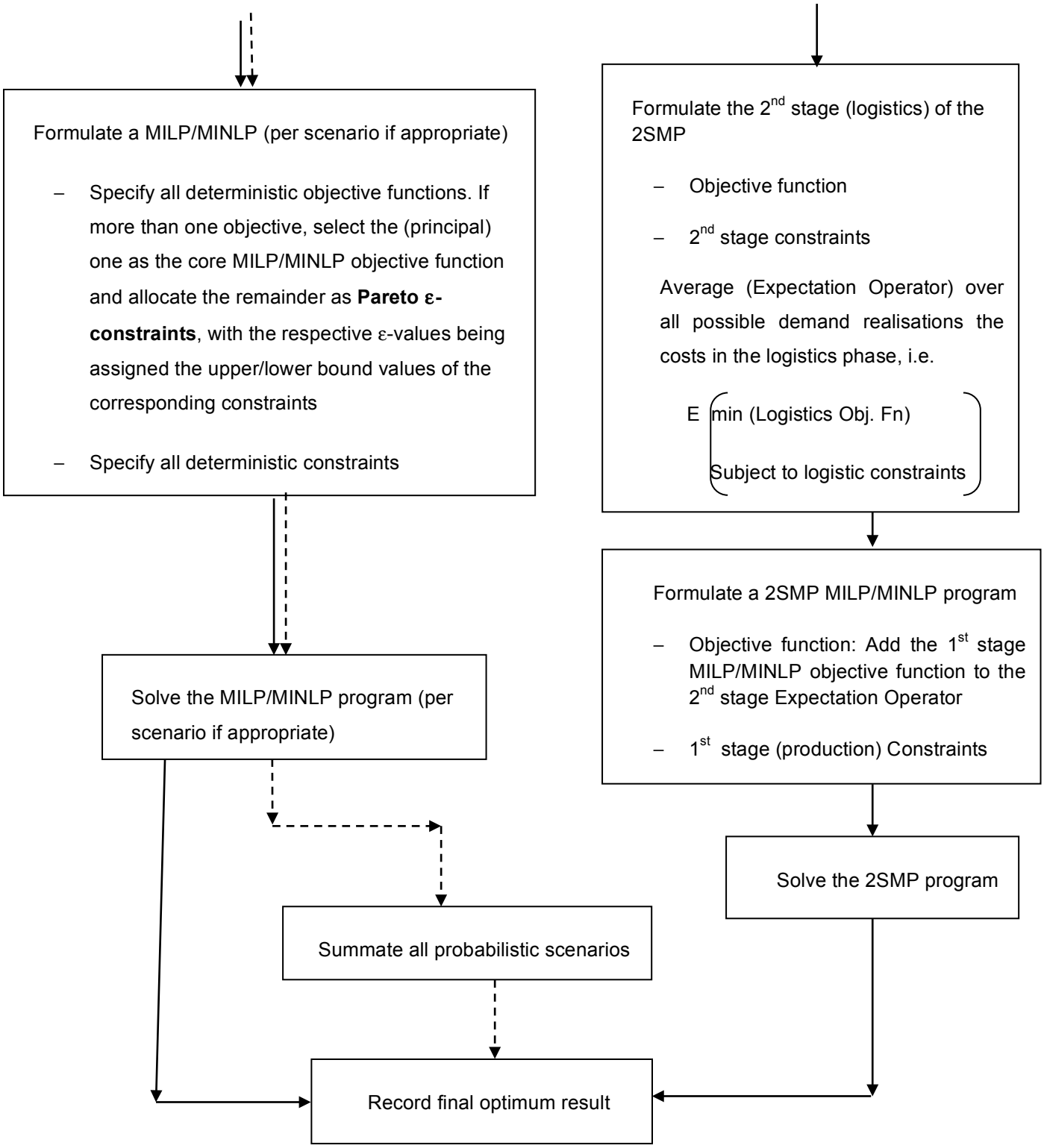


Figure 5-1: *Process flow diagram of the 'Supply Chain under prevailing conditions of Uncertainty' Optimisation methodology*

5.1.2 Tabular Description of Methodology Stages

Table 5-1 is purely a stage-wise extraction from the final 'Optimisation of a Supply Chain under prevailing conditions of Uncertainty' methodology derived, and presented in Fig 5-1, but also includes enhanced stage descriptive's to ensure greater clarity for application purposes.

Table 5-1 *Tabular specification of final 'Planning and Optimisation methodology for a Supply Chain under Uncertainty*

Stage	Stage Description
1	Detailed description of Supply Chain Network This must include a description of all supply chain components including all operational performance parameters, e.g. capacity, rate etc.
2	Description of Supply Chain Problem / Requirement These are the operational / infrastructural requirements of the supply chain network of which there could be more than one, e.g. minimum operating cost and/or maximum operating profit etc.
3	Illustrated diagram of Supply Chain Network A diagram of the entire supply chain network including an identification of all key components and all key process flow variables
4	Identify all possible supply chain interconnections and allocate a binary, *Yes/No*, variable, to each possible flow stream, e.g. $X_{i,j}$ where X = product or raw material, $\boldsymbol{i}$ = product or raw material source and $\boldsymbol{j}$ = product or raw material destination or $X_{i,j,k}$ where $\boldsymbol{k}$ = product or raw material stream sub-component
5	Nomenclature Section Definitions of all indices, parameters, variables and constants used

6	Specify nature of Demand Uncertainty and corresponding solution approach This is probably the key decision point of the entire methodology, which determines the subsequent nature of the solution approach. Options are below	
7	- Fuzzy Demand (e.g. sales <, ≤ or >, ≥ *D*) and subsequent MILP (Mixed Integer Linear Program) or MINLP (Mixed Integer Non-Linear Program) programming – most supply chain network operations would fall under this category - Discrete Probability Demand Scenarios and subsequent MILP or MINLP programming – Overall product demand could be broken down into a number of scenario's, each with a specific probability of occurrence - Continuous Probability Demand and subsequent MILP or MINLP programming – Product Demand can be represented by probability distribution function, i.e. F(*x*) - Stochastic Demand and Uncertain Production (typically fuzzy) modelled by a 2SMP (Two-Stage Model Program) program – ideal for supply chain modelling when Demand is primarily determined by inventory level dynamics, e.g. mass market commodities	
	Fuzzy, Discrete and Continuous Probability Product Demand (a)	**2SMP – Two Stage Stochastic Model Programming (b)**
8	Define a MILP or MINLP model depending on the linearity, or not, of the objective functions and constraints - Define all probabilistic and/or deterministic objectives, and if the former, transform them into deterministic equivalents, e.g. $E[\sum a_i x_i] + F^{-1}(\beta)(\sum a_i x_i - b_i] \leq E[b]$, Lee and Olson (1985) - Define all probabilistic and/or deterministic constraints, and if the former, transform them into deterministic equivalents.	Define and formulate a 1st stage 2SMP Stochastic Model - Define and formulate all 1st stage (production) probabilistic and/or deterministic objectives, and if the former, transform them into deterministic equivalents. - Define and formulate all 1st stage probabilistic and/or deterministic constraints, and if the former, transform them into deterministic equivalents.
9	Formulate a MILP or MINLP program (per prob. scenario if appropriate)	Define and formulate a 2nd stage (logistics) MILP/MINLP

	- deterministic objective function(s). If more than one objective function, then one (core) objective is selected as the single MILP/MINLP objective whilst the remaining objectives are assigned as Pareto ε-constraints, with the respective ε-values corresponding to the upper or lower bound values of the constraints concerned - deterministic constraints	- objective function - 2nd stage constraints\ Average (using Expectation Operator) over all possible demand realisations, the costs in the logistics phase, i.e. $E\begin{pmatrix} \min(\text{Logistics Objective}) \\ \text{Subj. Logistics constraints} \end{pmatrix}$
10		Formulate a 2SMP program - objective function: add the 1st stage objective function to the 2nd stage Expectation Operator (E) - 1st stage (production) constraints
11	Solve the MILP/MINLP (per scenario if appropriate)	Solve the 2SMP program
12	Summate all probabilistic scenario's (decision variables) to get final optimum result	
13	Record Final Optimal Solution	

5.2 Conclusion

The derivation of a Planning and an Optimisation Methodology for a Supply Chain under Uncertainty has now been achieved and presented in two different formats, i.e. process flowchart and descriptive tabular formats. As has been mentioned previously, the integrity of each constituent individual and/or combinational optimisation under uncertainty methodology has been maintained through the use of an integration technique that recognises and caters for all constituent procedures.

It now remains to test the derived methodology in terms of the Hypothesis (1.2), which means it needs to be checked in terms of;

a) Functionality - is it functional and are credible results obtained?
b) Validation of Results – Are the results accurate?
c) Variation in Uncertainty – can the model handle any variations in operational uncertainty

To perform these tests, it was decided to subject the methodology to a live supply chain environment, i.e. the fertiliser division of a large chemical NPK (Nitrogen, Phosphorous, Potassium) production and distribution operation in South Africa. This would be ideal because such operation(s) exhibits most elements of supply chain planning and uncertainty, and also such elements can be subject to variation.

6 APPLICATION EXAMPLE: THE PLANNING AND OPTIMISATION OF A SUPPLY CHAIN UNDER UNCERTAINTY

6.1 Introduction

As was common to all literature researched, an appropriate and comprehensive sample operation is to be provided for and solved. This is fundamental to the research as it would not only demonstrate the application of all the steps of the derived, integrated methodology, highlight the type of difficulties encountered and the suggested methods for dealing with them but also provide an ideal test bed for the derived methodology. Once this has been done, the optimum operating results achieved will be compared with those obtained from one of the existing and proven supply chain optimisation techniques (section 6.3). Further, the derived methodology will also be tested for variations in operational uncertainty (section 6.4)

One such comprehensive operating entity has been found and that is the production, distribution and sales activities of a large fertiliser and explosives chemical group in South Africa, Such group, which has a comprehensive supply chain network infrastructure both within the country and beyond. is an ideal testing environment for this research because apart from its supply chain network, it has a large integrated fertiliser production facility that produces quite a wide variety of similar product.

Most importantly, the various operations in the abovementioned example incorporate most of the elements of uncertainty, two out of three, in the planning and

optimisation of a supply chain under uncertainty, i.e. Multiple Objectivity (minima, maxima or a combination of minima and maxima) and Fuzziness. Therefore it will certainly be a useful test platform.

6.2 Example Application: A South African Fertiliser Group - Optimisation of a Supply Chain Network under prevailing conditions of operational Uncertainty

6.2.1 Introduction

A South African Fertiliser Group produces dry, liquid and speciality fertilisers, and has production plants throughout South Africa. The fertiliser division produces and sells plant nutrition products directly to farmers as well as to cooperatives and wholesalers within South Africa. This SA Fertiliser company manages exports into the rest of Africa from Johannesburg, as well as its own facilities in certain African countries. Such African operations have specialised divisions focusing on the needs of entrant farmers – an important developing market for this SA Fertiliser company.

However, the focus of this paper is on the supply chain of one particular range of speciality fertilisers produced, distributed and sold by the SA Fertiliser Group in South Africa, and that is the Nitrogen, Phosphorous, Potassium (NPK) (Appendix 11.1) range of granular fertilisers. The main reason for this is that there are about 30 to 40 different, FSSA, FAO-UN, (2005), blends of NPK fertiliser available, in the South African agricultural sector with each blend geared towards the nutrition of a specific crop type in terms of its NPK ratio, e.g. N:P:K = (2:3:2) with each number

representing the relative content of that chemical element in the fertiliser, where N (nitrogen) promotes leaf growth and forms proteins and chlorophyll, P (phosphorus) contributes to root, flower and fruit development and K (potassium) contributes to stem and root growth and the synthesis of proteins. The SA Fertiliser Group only focuses on the production and distribution of NPK fertiliser for the fifteen (15) main crop-types in SA, Table 6-4

The production and distribution of a whole variety of different NPK-based granular fertilisers varies according to market demand and an associated operational planning requirement. Market demand for the various blends of NPK fertiliser is uncertain and varies according to the distribution profile of the corresponding crops, in terms of both crop-type and concentration in the target market. Such uncertain market demand for NPK fertiliser may be regarded as fuzzy since market demand quantities for the various blends of NPK fertiliser is not precisely known. Further, there is also a multi-objective planning requirement in that there is an operational need to maximise the production of a whole range of NPK fertilisers in accordance with market demand whilst simultaneously minimising the generation and discharge of hazardous chemicals, i.e. Hydrogen Fluoride (HF) and Carbon Dioxide (CO_2), from the production process.

Therefore, it is the primary ambition of SA Fertiliser Ltd to maximise the production and distribution of NPK granular fertilisers, in accordance with market demand in an uncertain product and market environment, whilst also minimising the discharge of hazardous effluent. Therefore the optimisation methodology to be followed in this endeavour will be based on the general optimisation methodology for a supply chain under prevailing conditions of uncertainty, as given in Table 6-2. The required plant operating data and technical specifications were obtained from the corresponding operating and technical manuals on the production facilities of the SA Fertiliser Group.

6.2.2 Detailed description of Supply Chain Network

It will be evident from Fig 6-3, a comprehensive flowchart of the entire fertiliser production and distribution operations, that the final fertiliser product production processes, i.e. the various Granulators, are initiated by a number of parallel production processes (i.e. nitric acid, ammonium nitrate solution, nitrophosphate and superphosphate production plants) that constitute the production of the various raw materials required in this NPK fertiliser production. Such parallel raw material production processes, including final fertiliser production, are illustrated in Fig 6-1. All plant operational details were obtained from the various plant technical manuals.

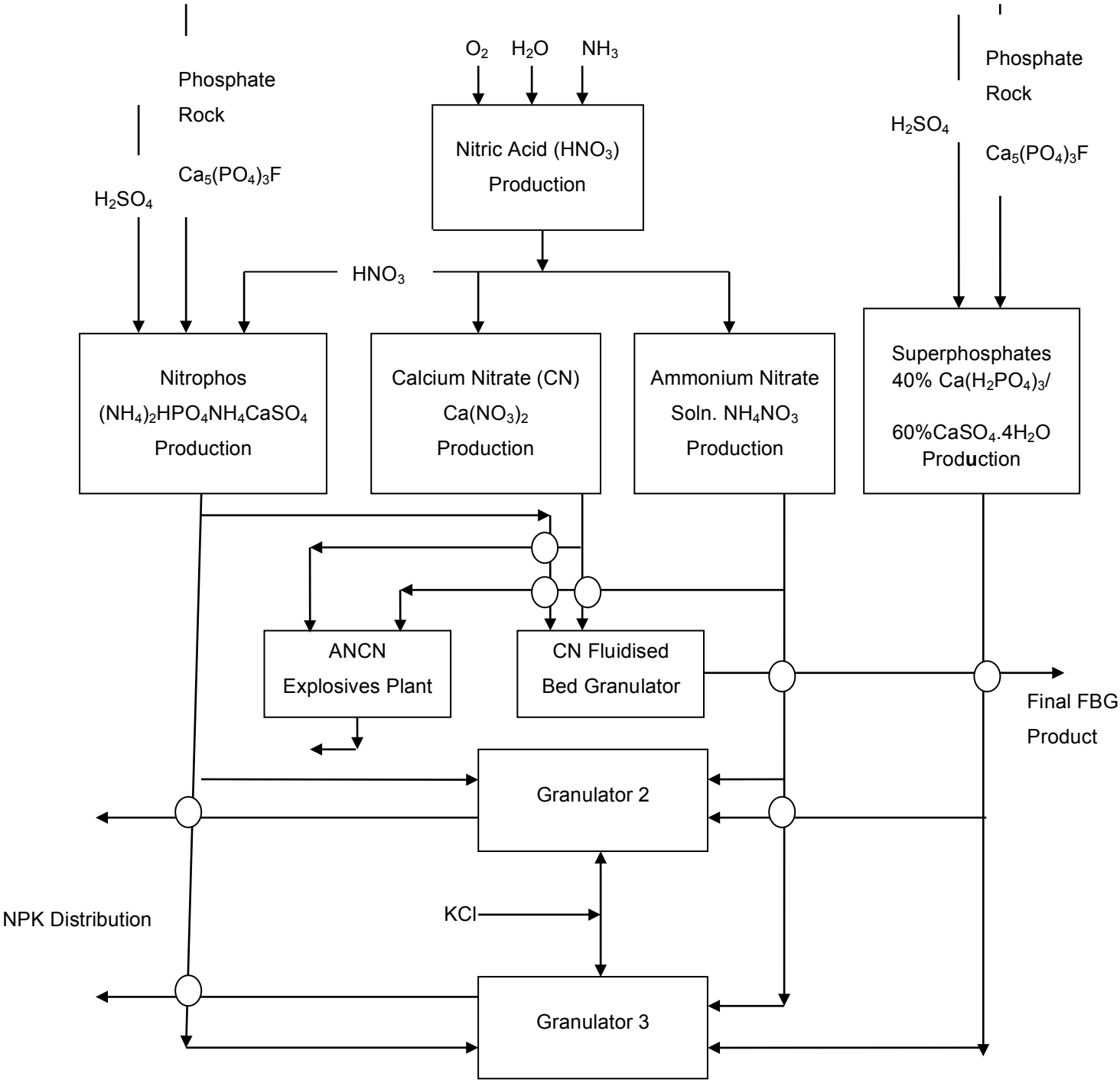


Figure 6-1 *Simplified flowchart of fertiliser production operations*

The following sections will cover a brief description of the various production processes involved

6.2.2.1 Nitric Acid (NA) Plant

Nitric Acid (HNO_3) is used, in fertiliser production, as an intermediate product for the production of ammonium nitrate solution as well as for the production of ammonium nitrate and calcium based fertilisers.

Air is extracted from the atmosphere at ambient temperature and pressure and is then compressed to a pressure of 4 bar, and at a temperature of 260^0C. The compressed air stream is split into two streams, one for the ammonia/air reactor and the other, cooled, for the bleaching tower.

Ammonia is supplied as a gas and fed, along with the air, into the ammonia/air reactor. The catalyst used in the reactor is platinum-palladium gauze. The reactor typically operates at a temperature of 890^0C and at a pressure of 4 bar and the reaction efficiency lies typically between 93% and 98%. The reactions taking place in the reactor are twofold, producing nitrous oxide (NO) and nitrogen dioxide (NO_2) as follows:

$$4NH_3 + 50_2 \rightarrow 4NO + 6H_2O;\ 866.8\ \text{kJ/mole}$$

The nitric oxide that is produced at the platinum gauze is oxidised further by means of secondary air reacting homogeneously in the gas phase in accordance with the overall equation:

$$2NO + O_2 \rightarrow N_2O_4$$

Formation of di-nitrogen tetroxide proceeds via an intermediate nitrogen dioxide, NO_2. The nitric oxide oxidation reaction is characterised by low reaction

temperatures that favour higher yields, and also, elevated operating pressures contribute to the possibility of producing higher strengths of acid.

The hot gas from the reactor is passed through a Lamont boiler where the heat is exchanged with distilled water to produce superheated steam, after which it flows through two energy recovery stages, i.e. the tail gas heater and the economiser.

The gaseous product stream, which, at that stage, is mainly nitrogen dioxide (NO_2) is then fed into the base of the absorption column whilst water and weak nitric are fed into the top of the absorption column. Di-nitrogen tetroxide is absorbed in water with the formation of nitric acid in accordance with the simplified equation:

$$3N_2O_4 + 2H_2O \rightarrow 4HNO_3 + 2NO$$

The absorption proceeds at greater rates, and with higher efficiencies, at elevated pressures and low temperatures. NO_2 is absorbed into the water and eventually 59% nitric acid is formed. Clean gas with only traces of NO is released into the atmosphere as tail gas

6.2.2.2 Ammonium Nitrate (AN) Solution Plant

According to the operating process manual of SA Fertiliser Ltd, ammonia and 59% Nitric Acid are reacted together in a forced circulation loop to produce 89% ammonium nitrate by the following reaction:

$$NH_3 + 59\%HNO_3 \rightarrow 89\%NH_4NO_3 + H_2O$$

For industrial production, this is done using anhydrous ammonia gas and concentrated nitric acid. This reaction is violent and very exothermic. After the solution is formed, typically at about 81% concentration, the excess water is evaporated to an ammonium nitrate (AN) content of between 88% to 95% concentration (AN melt), depending, primarily, on the strength of nitric acid..

The reactor operates at a temperature of 150°C, with a reactor efficiency of between 95% and 97.5%, and the resultant stream is then flashed in the flash chamber. The liquid stream is then circulated back to the reactor while the gas stream is transferred to the scrubbers before being released into the atmosphere. Ammonium Nitrate from the flash chamber is neutralised by ammonia in the neutraliser tank. Ammonium Nitrate, at a concentration of 89%, is stored in two storage tanks. Despatches to external depots are exercised from the storage tanks

6.2.2.3 Liquid Calcium Nitrate (CN) Plant

Liquid CN is produced for use in the solid CN plant as well as for use in the ANCN (Ammonium Nitrate Calcium Nitrate) plant, which delivers a mixture of ANCN ($Ca(NO_3)_2$ and NH_4NO_3 (ammonium nitrate)) for the Blasting Explosives operation.

Liquid CN is produced from a reaction of 59% nitric acid (HNO_3) with lime ($CaCO_3$ - calcium carbonate) Both reactants are both fed into an initial reactor and then overflowed into a secondary reactor for the purpose of increasing overall reactor residence time and thus enhancing the conversion of reactants as follows:

$$CaCO_3 + 2HNO_3 \rightarrow Ca(NO_3)_2 + CO_2 + H_2O$$

Product from the reactor is fed to the primary filters where the sludge is removed and the calcium nitrate solution is then stored in an intermediate tank. From this intermediate tank, calcium nitrate is pumped to a CN storage tank.

If the liquid CN is to be used in the ANCN plant, the pH is first adjusted by means of ammonia and then the liquid CN is pumped to the CN storage tank. If liquid CN is to be used in the solid CN plant, the pH is first adjusted using lime, after which it is fed through secondary filters to produce a clear CN product, which is used as a raw material for the solid CN plant.

6.2.2.4 Nitrophosphate (NP) Plant

6.2.2.5

The Nitrophosphate production process (Olanipekun (2003) involves acidifying phosphate rock with nitric acid to produce a mixture of phosphoric acid and calcium nitrate and is shown below

<u>Note</u>: It is reported by SA Fertiliser Ltd that the phosphate apatite rock that is used in the granular fertiliser production operations is sourced from a northern province in

SA and consists of the following combinational mixture of apatite rocks: 60% fluor apatite - $Ca_{10}(PO_4)_6F_2$, 40% mixture of hydroxyapatite - $Ca_5(PO_4)_3OH$ and carbonic apatite - $Ca_5(PO_4)_32CO_3$. For the sake of brevity and convenience in this report, feedstock phosphate apatite rock will, henceforth, be referred to as $Ca_{10}(PO_4)_6F_2$, on account of its 60% prevalence of occurrence and also because this is not a critical factor in this research. Additionally, since none of the final granulated NPK fertiliser products contain any -F-, -OH, -CO_3 related compounds in their composition; the precise apatite mixture in phosphate rock is irrelevant in any operational mass balance calculations since all such compounds eventually end up as some form of effluent discharge.

The Nitrophosphate reaction sequence is depicted in Fig 6-2

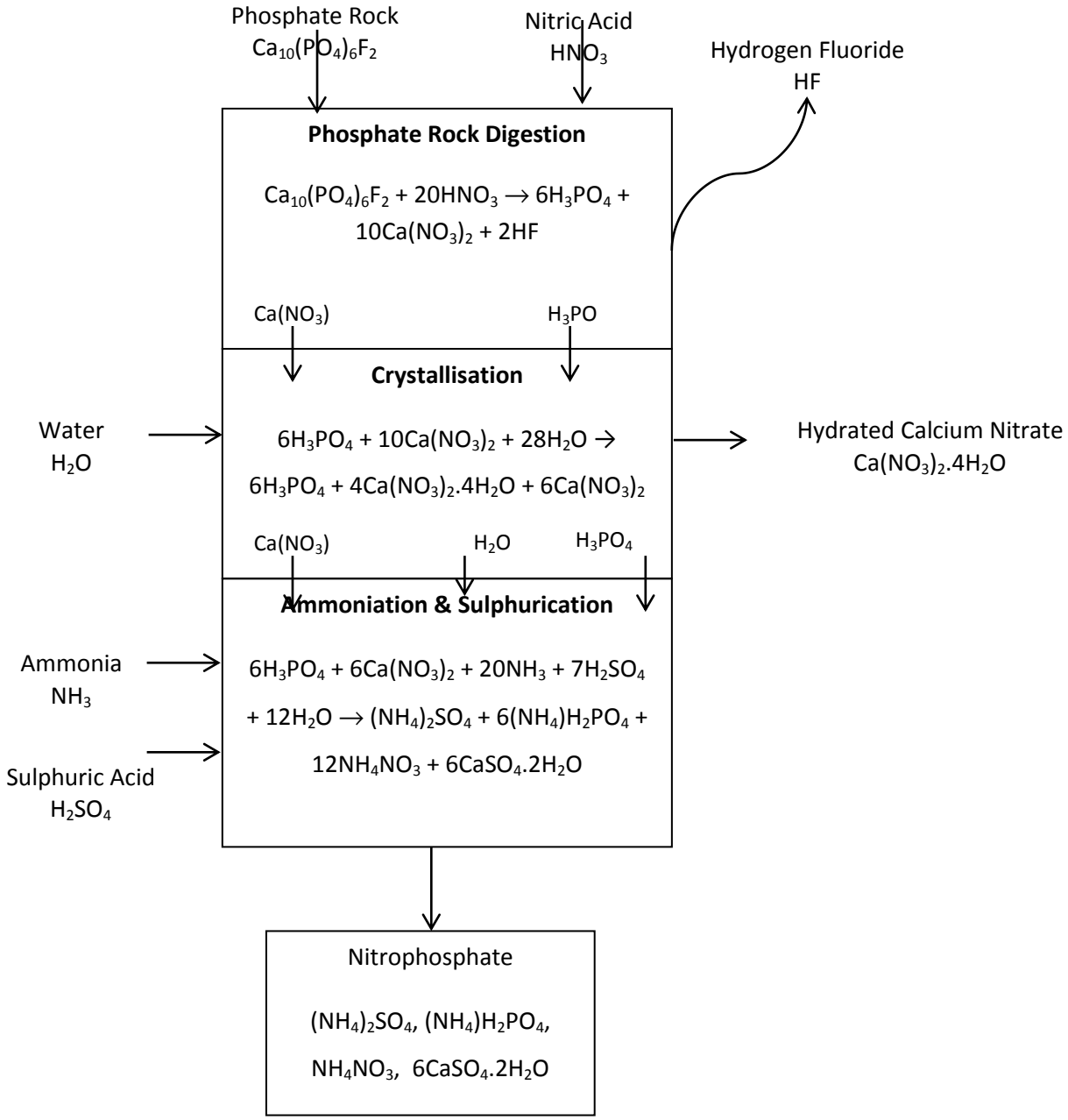


Figure 6-2: *Nitrophosphate Reaction Sequence*

As depicted in Fig 6-2, the reaction sequence in the Nitrophosphate plant is:

1) $Ca_{10}(PO_4)_6F_2 + 20HNO_3 \rightarrow 6H_3PO_4 + 10Ca(NO_3)_2 + 2HF$

The mixture is cooled to below 0 degrees Celsius, where the calcium nitrate crystallises and can be separated from the phosphoric acid. This crystallised calcium nitrate constitutes 70% of the total incoming calcium

2) $2H_3PO_4 + 4Ca(NO_3)_2 + 4H_2O \rightarrow 2H_3PO_4 + Ca(NO_3)_2.4H_2O + 3Ca(NO_3)_2$

The resulting calcium nitrate produces nitrogen fertilizer. The filtrate is composed mainly of phosphoric acid with some nitric acid and traces of calcium nitrate, and this is neutralised with ammonia and reacted with 98% sulphuric acid to produce a nitrophosphate fertiliser mixture consisting of ammonium sulphate ($(NH_4)_2SO_4$) ammonium phosphate ($(NH_4)H_2PO_4$), ammonium nitrate (NH_4NO_3) and hydrated calcium sulphate ($6CaSO_4.2H_2O$), commonly referred to as Nitrophos in the fertiliser industry

3) $6H_3PO_4 + 6Ca(NO_3)_2 + 20NH_3 + 7H_2SO_4 + 12H_2O \rightarrow (NH_4)_2SO_4 + 6(NH_4)H_2PO_4 + 12NH_4NO_3 + 6CaSO_4.2H_2O$

6.2.2.6 Ammonium Nitrate Calcium Nitrate (ANCN) Plant

ANCN (Ammonium Nitrate Calcium Nitrate) is produced by mixing 89% Ammonium Nitrate (NH_3NO_4) with 59% Nitric Acid (HNO_3) and then by evaporating off excess water. ANCN is used as an intermediate product in the manufacture of explosives and the mixture is created as follows:

$$NH_4NO_4 + Ca(NO_3)_2 \rightarrow NH_4NO_4.Ca(NO_3)_2$$

ANCN is produced in batches. Calcium nitrate from the CN plant is transferred to two tanks on the ANCN plant where it is mixed with ammonium nitrate, transferred from the AN plant. The mixture is treated with steam to evaporate water from the mixture until the product has attained the required moisture content.

The product is then transferred to an ANCN storage tank from where it is transferred to explosives manufacturing facilities

6.2.2.7 CN Fluidised Bed Granulator

Liquid calcium nitrate (CN) is transferred to one of the holding tanks in the Raw Material Storage section, where ammonium nitrate is added and mixed.

The liquid is then concentrated up by use of the steam coils in the concentrator .tanks. A fan is then used to extract the water vapour in this section. The concentrated liquid CN is injected into the fluidised bed granulator via atomising

spray nozzles using instrument air. The fluidised bed granulator is supplied with dry air, and the coolers with cold air, in the granulator, and the bed of CN granules is coated with spray. The granules that are formed move through the bed into the fluidised bed cooler before being transferred to the vibrating screens. Hot air from the granulator is scrubbed with a weak CN solution to remove CN dust from the air stream. The weak CN solution is returned to the CN storage tanks and clean air is emitted by the scrubber.

6.2.2.8 Single Super Phosphate Plant

Apatite phosphate rock [$Ca_{10}(PO_4)_6$], similarly sourced to that of the Nitrophosphate plant phosphate rock feedstock, is fed though an air swept roller mill where it is ground to $\pm$ 90% less than 53 micrometres. This ground rock is stored in a ground rock hopper from whence it is fed to a mixer on the plant. In the mixer, the ground phosphate rock is reacted with a mixture of 98% sulphuric acid and water according to the following reaction\

$$Ca_{10}(PO_4)_6 + 7H_2SO_4 + 3H_2O \rightarrow 3CaH_4(PO_4)_2H_2O + 2HF(g) + 7CaSO_4$$

The reaction also releases fluorinated gas (HF), which is removed via a scrubber system as follows:

$$4HF(g) + SiO_2 \rightarrow SiF_2(g) + 2H_2O$$

The product from the mixer is then fed through a den, which is a slow moving, fully enclosed conveyor, within which the reaction continues and the emerging product, Single Super Phosphate, 40%$Ca(H_2PO_4)_3$ plus 60%$CaSO_4.4H_2O$, forms a cake.

The Singles Super is left in store for a period of 2 – 3 weeks to allow the product to mature thus ensuring maximum conversion of the phosphorous to the water-soluble form.

6.2.2.9 Granulator Plants

The purpose of a granulator plant is to produce granules of N, NP and NPK fertiliser for an agricultural market. The product from a granulation plant is homogeneous chemical mixture of various chemical products. In some countries, fertiliser grades are usually denoted by the N (Nitrogen) to P (Phosphorus) to K (Potassium) ratios, and with the total percentage of plant food indicated in brackets thereafter, e.g. 2:3:2(22), which contains 22% NPK of which 2/(2 + 3 + 2) x 22 is N, i.e. 6.3%N. Similarly:

- ASN (mixture of NH_4NO_3 and $(NH_4)_2SO_4$ – 27%N
- MAP (mono Ammonium Phosphate) - 11%N, 22%P

There are two granulation plants at the production site of the SA Fertiliser Group for the production of complex granular N, NP and NPK fertilisers. The two plants are called Gran 1 and Gran 2 as evident in Fig 6-1 and the production processes comprise the following similar steps:

a) *Raw Material Preparation*:

For solid raw material preparation, the powder intermediates, KCl, $(NH_4)_2SO_4$, MAP and SSP are crushed, screened and put into different hoppers. Dolomitic

Lime is also put inside a hopper, but via pneumatic conveyancing. The raw materials, required for granulation, are drawn from the various hoppers, in accordance with the required granulation recipe. The combined raw materials are then fed into the granulator or pug mill.

b) *Granulation Loop*

i) In Gran 2, the granulation loop consists of the pug mill, granulator, drier, screens and crushers. For products containing high levels of Ammonium Nitrate, the pug mill is used for initial granulation. For other grades, only the granulator is used. In the granulator itself, the liquid raw materials, such as Ammonium Nitrate Solution and pipe reactor products, are combined with the solid raw material as well as recycle from the plant. Steam and water are also occasionally introduced to aid granulation. In the pipe reactor, a mixture of ammonium phosphoric acid and water is reacted together to produce a saturated solution of fertiliser salts, of which the main reaction is:

$$H_3PO_4 + NH_3 \rightarrow NH_4(H_2PO_4)$$

ii) In Gran 3, sulphuric acid and scrubbing liquid are also fed into the pipe reactor. Gran 3 also has a pipe reactor, equipped in the dryer that is used during DAP production. The product, ex-granulator, is then dried in a rotary hot drier and then passed over the screens to separate product size particles from fines. Oversize product is crushed before being recycled through the granulator.

c) *Cooling*

Product size granules are cooled down in a fluidised bed cooler after which they are given a coating of oil and kaolin dust to enhance their storage properties

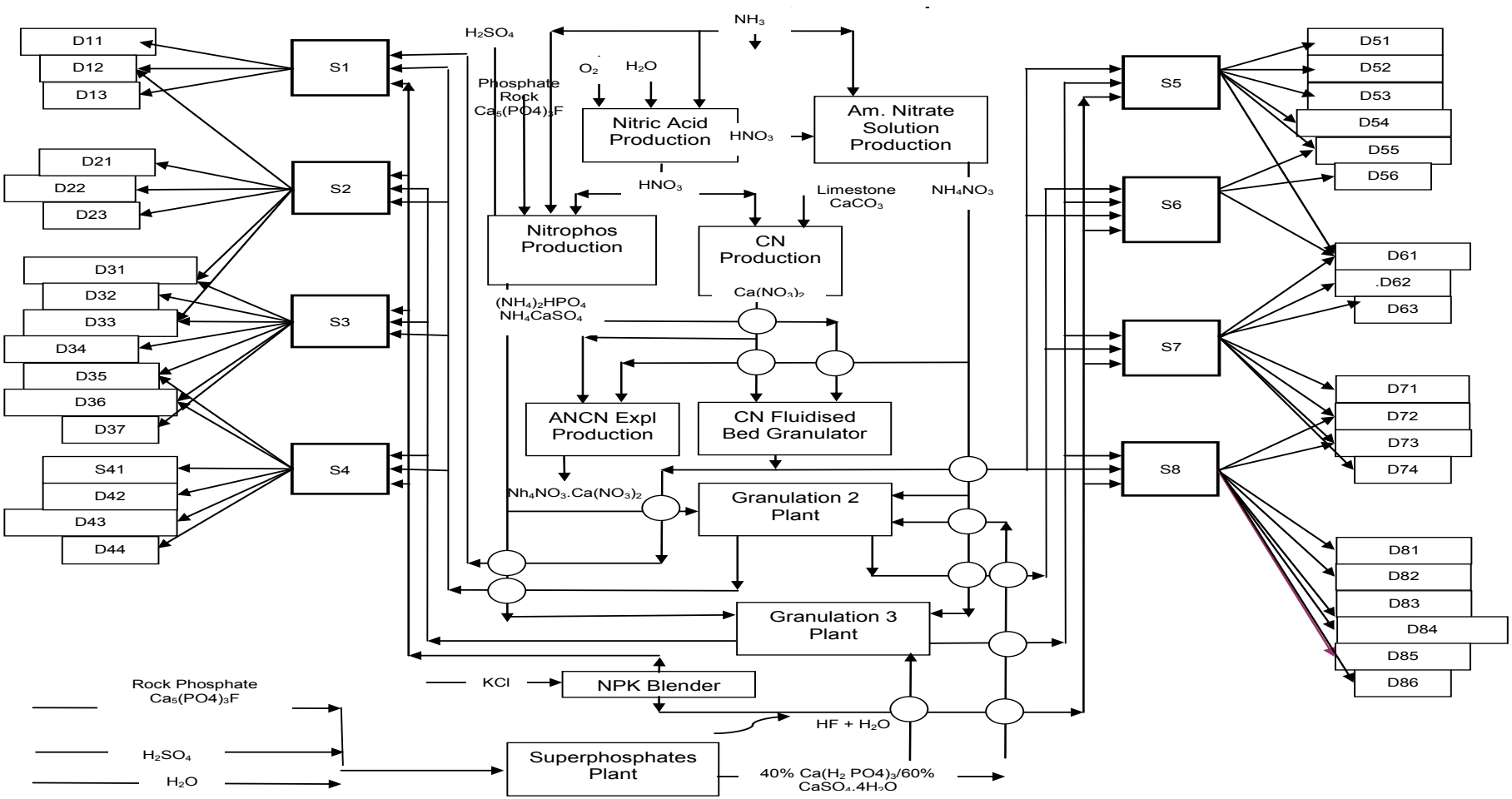
6.2.3 Description of Supply Chain Problem / Requirement

Optimisation calculations need to determine the best suite of NPK fertiliser packages, e.g. 1:2:2 (38), 1:2:3 (33), 16:1:3 (44) where *x:y:z* is the NPK ratio and (*ab*) is the mass fraction of NPK-based components, that are to be produced and distributed to maximise the satisfaction of crop/customer demand in South Africa. The supply of NPK fertiliser must be in accordance with crop/customer demand statistics published by the United Nations and the Fertiliser Society of Southern Africa, FSSA, FAO-UN, (2005), and also taking into account the fuzzy market demand of NPK fertiliser as well as abiding by an environmental requirement to minimise the discharge of hazardous chemicals.

6.2.4 Illustrated Diagram of Supply Chain Network

Fig 6-3 shows a comprehensive flow diagram of the entire fertiliser production and distribution network of the SA Fertiliser Group.

Figure 6-3: *Comprehensive flow diagram of the entire fertiliser production and distribution network of the SA Fertiliser Group*



Such comprehensive process flow diagram, Fig. 6-3 is simplified and presented in two fundamental, constituent phases in Fig. 6-4, i.e. Production and Distribution, with the corresponding process nomenclature defined in Table 6-1

Table 6-1 *Nomenclature Requirements*

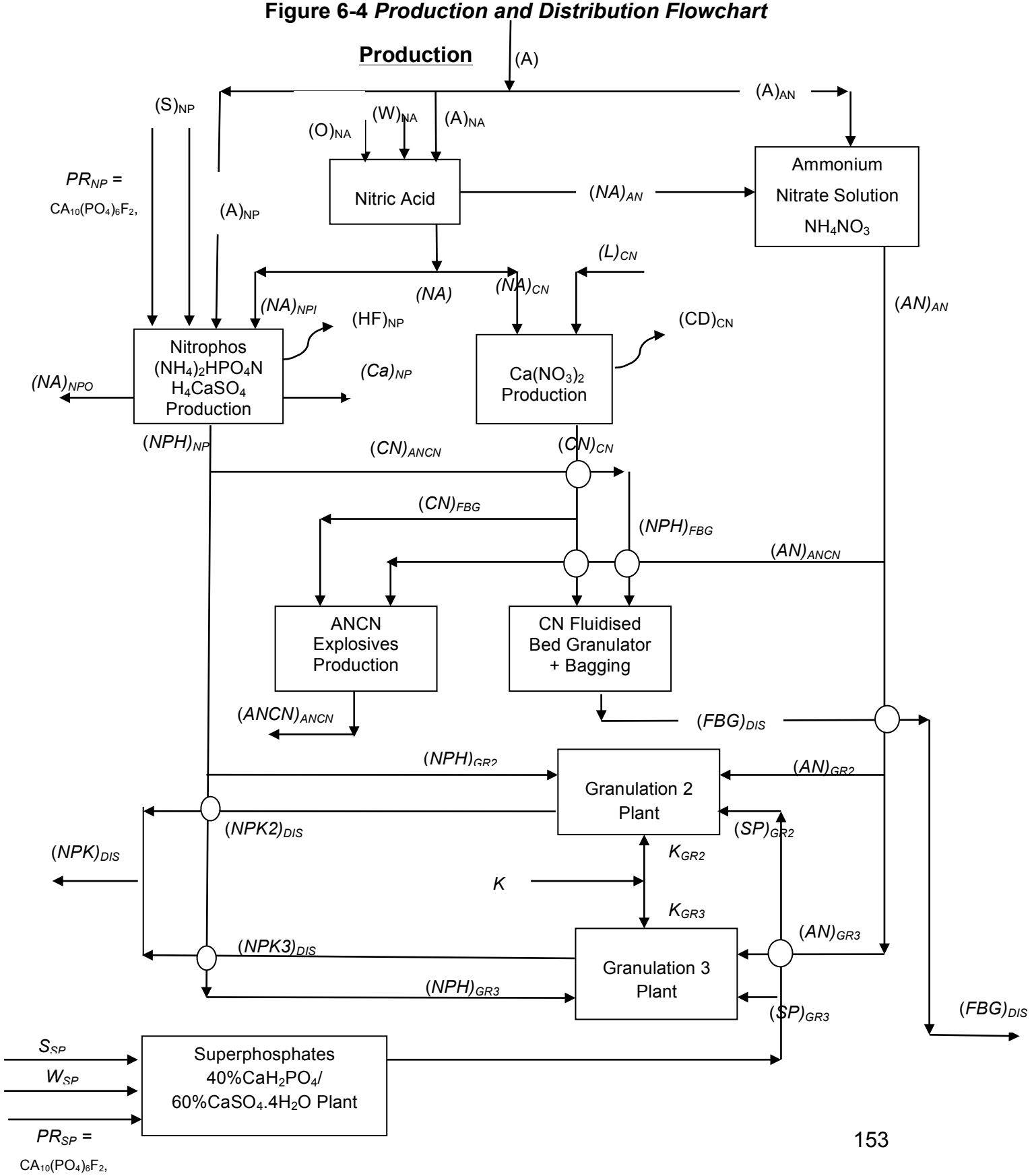
Category	Description
Indices/Sets	
Raw Materials	Ammonia (*A*), Sulphuric Acid (*S*), Rock phosphate (*RP*), Oxygen (*O*), Water (*W*), KCl (*K*), Limestone (*L*)
Intermediate Products	, e.g. Nitric Acid (*NA*), Nitrophos (*NP*), Superphosphates (*SP*), Ammonium Nitrate (*AN*), Calcium Nitrate (*CN*)
Final fertiliser products,	$(FBG)_i$, $(GR2)_j$, $(GR3)_k$ where *i, j, k* refer to the many various blends of NPK fertiliser exiting from the Granulators
P	Plants, e.g. Fluidised Bed Granulator ($_{FBG}$), Granulator 1 ($_{GR1}$) Granulator 2 ($_{GR2}$), Superphosphates Plant ($_{SP}$), Nitric Acid Plant ($_{NA}$), Ammonium Nitrate Plant ($_{AN}$), Nitrophos Plant ($_{NP}$), NPK Blender ($_{NPK}$)
S_n	Sites (n = 1,…, 8) e.g. S_1, S_2
Variable Parameters	*Description*
(A)	Flowrate of NH_3 to the Fertiliser Production complex (t/yr)
$(A)_{AN}$	Flowrate of NH_3 to Ammonium Nitrate (AN) plant (t/yr)
$(A)_{NA}$	Flowrate of NH_3 to Nitric Acid (NA) plant (t/yr)
$(A)_{NP}$	Flowrate of NH_3 to Nitrophos (NP) plant (t/yr)

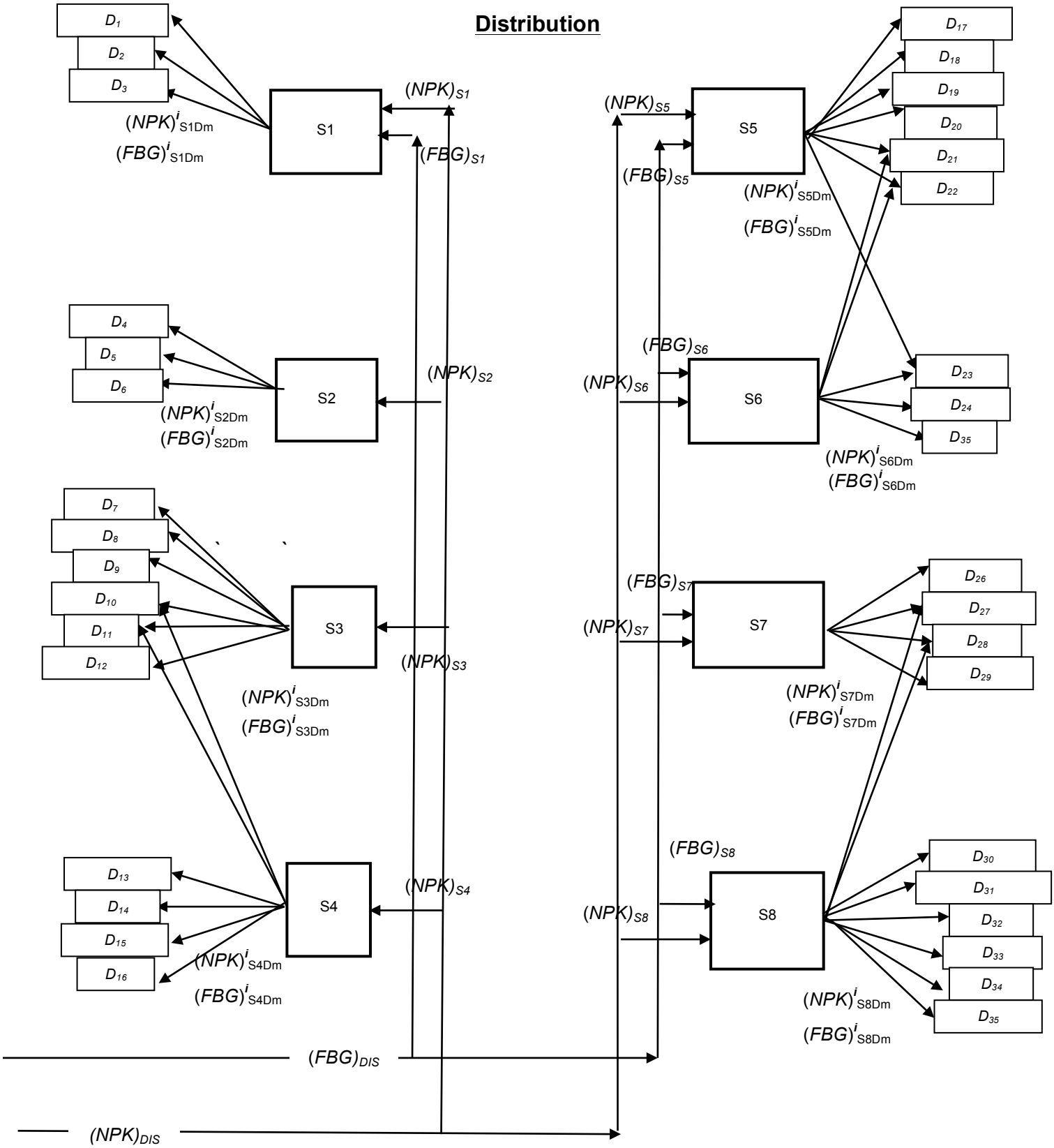
(S)	Total Flow'. of 98% Sulphuric Acid to Fertiliser complex (t/yr)
$(S)_{NP}$	Flowrate of 98%Sulphuric acid to NP plant (t/yr)
(RP)	Total Flowrate of Rock Phosphate to Fertiliser complex (t/yr)
$(RP)_{NP}$	Flowrate of Phosphate rock to NP plant (t/yr)
$(O)_{NA}$	Flowrate of Oxygen to NA plant (t/yr)
$(W)_{NA}$	Flowrate of Water to NA plant (t/yr)
$(NA)_{NA}$	Flowrate of Nitric Acid from NA plant (t/yr)
$(NA)_{AN}$	Flowrate of Nitric Acid from NA plant to AN plant (t/yr)
$(NA)_{NPI}$	Flowrate of Nitric Acid from NA plant to NP plant (t/yr)
$(NA)_{NPO}$	Flowrate of Nitric Acid from NP plant (t/yr)
$(Ca)_{NP}$	Flowrate of $Ca(NO_3)_2.4H_2O$ from NP plant (t/yr)
$(HF)_{NP}$	Discharge of Hydrogen Fluoride (HF) from NP plant (t/yr)
$(NA)_{CN}$	Flow of NA from NA plant to Calcium Nitrate (CN) plant (t/yr)
$(L)_{CN}$	Flowrate of Limestone ($CaCO_3$) to CN plant (t/yr)
$(NP)_{NP}$	Flowrate of Nitrophos from NP plant (t/yr)
$(CN)_{CN}$	Flowrate of 58%Calcium Nitrate from CN plant (t/yr)
$(CD)_{CN}$	Discharge of CO_2 from the CN plant (t/yr)
$(CN)_{ANCN}$	Flowrate of Calcium Nitrate to ANCN (explosives) plant (t/yr)
$(AN)_{ANCN}$	Flowrate of AN to ANCN (explosives) plant (t/yr)
$(ANCN)_{ANCN}$	Flowrate of ANCN from ANCN Plant (explosives) plant (t/yr)
$(NP)_{FBG}$	Flowrate of Nitrophos to Fluidised Bed Granulator (t/yr)
$(CN)_{FBG}$	Flowrate of Calcium Nitrate to Fluidised Bed Granulator (t/yr)
$(AN)_{ANCN}$	Flow of Ammonium Nitrate to ANCN (explosives) plant (t/yr)

$(FBG)_{FBG}$	Flowrate of granulated CN from FBG (t/yr)
$(NP)_{GR}$	Flowrate of Nitrophos to Granulator (t/yr)
$(AN)_{GR}$	Flowrate of Ammonium Nitrate to Granulator (t/yr)
$(SP)_{GR}$	Flowrate of Superphosphate to Granulator (t/day)
S_{SP}	Flowrate of Sulphuric Acid to Superphosphates plant (t/yr)
$(SP)_{SP}$	Flow of Superphosphate from Superphosphates plant (t/yr)
$(RP)_{SP}$	Flowrate of Phosphate Rock to Superphosphates plant (t/yr)
W_{SP}	Flowrate of Water to Superphosphates plant (t/yr)
$(NPK)_i^{Sn}$	Flowrate of *NPK* fertiliser to Site *n* (t/day) of NPK blend ***i***
$(FBG)^{Sn}$	Flowrate of fluidised bed granular product to Site *n* (t/yr)
K_{GR}	Flow of *KCl* to Granulator
$(NPK)^i_{S1}$	Flow of NPK fert. of blend *i*, from Site 1 to market
$(NPK)^i_{S2}$	Flow of NPK fert. of blend *i*, from Site 2 to market
$(NPK)^i_{S3}$	Flow of NPK fert. of blend *i*, from Site 3 to market
$(NPK)_i^{S4Dm}$	Flow of NPK fert. of blend *i*, from Site 4 to depot *m*, (m = 1, 4)
$(NPK)_i^{S5Dm}$	Flow of NPK fert. of blend *i*, from Site 5 to depot *m*, (m = 1, 6)
$(NPK)_i^{S6Dm}$	Flow of NPK fert. of blend *i*, from Site 6 to depot *m*, (m = 1, 3)
$(NPK)_i^{S7Dm}$	Flow of NPK fert. of blend *i*, from Site 7 to depot *m*, (m = 1, 4)
$(NPK)_i^{S8Dm}$	Flow of NPK fert. of blend *i*, from Site 8 to depot *m*, (m = 1, 5)
$(FBG)^{S1Dm}$	Flow of FBG fert. from Site 1 to depot *m*, (m = 1, 3)
$(FBG)^{S2Dm}$	Flow of FBG fert. from Site 2 to depot *m*, (m = 1, 3)
$(FBG)^{S3Dm}$	Flow of FBG fert. from Site 3 to depot *m*, (m = 1, 6)
$(FBG)^{S4Dm}$	Flow of FBG fert. from Site 4 to depot *m*, (m = 1, 4)

$(FBG)^{S5Dm}$	Flow of FBG fert. from Site 5 to depot *m*, (m = 1, 6)
$(FBG)^{S6Dm}$	Flow of FBG fert. from Site 6 to depot *m*, (m = 1, 3)
$(FBG)^{S7Dm}$	Flow of FBG fert. from Site 7 to depot *m*, (m = 1, 4)
$(FBG)^{S8Dm}$	Flow of FBG fert. from Site 8 to depot *m*, (m = 1, 5)
$[N]^{i}$	Application Rate of Nitrogen (kg/ha) for crop *i* in Fert. Area
$[P]^{i}$	Application Rate of Phosphor. (kg/ha) for crop *i* in Fert. Area
$[K]^{i}$	Application Rate of Potassium (kg/ha) for crop *i* in Fert. Area
Constant Parameters	*Description*
(LA)	Total Fertile Land Area involved = 4,241,000 ha
$(d)_{yr}$	Effective operational days per annum = 320 (88% uptime)
$(NA)_{NA}$	Production flowrate of 59% Nitric Acid = 1200t/d
$(AN)_{AN}$	Flowrate of 89% Ammonium Nitrate from AN plant = 700t/d
$(ANCN)_{ANCN}$	2 x 75 ton batch reactors consisting of 52% CN & 48% AN. Each batch takes 6 hrs. to complete
$(FBG)_{FBG}$	Production capacity of granulated CN from FBG = 150 t/day, BOM = 78AN/140CN
$(GR)_{GR}$	Production capacity of Granulated Fertiliser from Granulator = 1,700 t/day
$(SP)_{SP}$	Production capacity of Superphosphates plant = 450 t/day for SSP and 150 t/day for MCP

Figure 6-4 *Production and Distribution Flowchart*





6.2.5 Specify nature of Operational Uncertainty

6.2.5.1

Two forms of operational uncertainty are evident in this case study, and these are:

a) Multi-objective uncertainty evidenced by the need to maximise the production and distribution of an entire range of NPK fertiliser. In addition to this, there is also a need to minimise the discharge of hazardous chemicals whilst simultaneously maintaining the maximum production and distribution of NPK fertilisers

b) Fuzzy (<, $\leq$, >, $\geq$) market demand uncertainty of NPK fertiliser

6.2.5.2 Multi-Objective Uncertainty

The situation of multi-objectivity arises because there is a requirement to maximise the production and distribution of the entire range of NPK fertiliser, i.e. Max $[\sum(NPK)^i]$ whilst simultaneously minimising the generation and discharge of hazardous effluent, carbon dioxide (CO_2) and hydrogen fluoride (HF), i.e. Min$[(HF)_{NP} + (CD)_{CN}]$

However, the chief requirement or objective of the SA Fertiliser Group is to maximise the production and distribution of multiple blends (15) of $(NPK)^i$ fertiliser in accordance with market demand, with i being representative of crop type, in accordance with market demand in South Africa. Such a requirement can be achieved by determining Max $[\sum(NPK)^i]$.

Therefore, there is a requirement to maximise the production and distribution of NPK fertilisers whilst simultaneously minimising the generation of hazardous effluent such as Hydrogen Fluoride (HF) from the Nitrophosphate unit plus Carbon Dioxide (CO_2) from the Calcium Nitrate unit. Such dual requirement is called a multi-objective requirement. Further to the research works of both Guillen and Grossmann (2009) and Liu and Papageorgiou (2013), the method of handling potentially conflicting multiple objectives, be they either minima or maxima, or a combination of the two, objectives, is by application of the ε-constraint method. This, of course, would provide a *Pareto optimal* solution thereto, which means it would be impossible to make any one objective better off without making any other objective worse off. The principal objective of this project, i.e. Max $[\sum(NPK)^i]$, is selected as the objective of the MILP, whilst the other, $[(HF)_{NP} + (CD)_{CN}]$, is allocated as an ε-constraints with the ε-values determined by lower (min) or upper (max) bound limits

6.2.5.3 Fuzzy Demand Uncertainty

The nature of national, South African, demand for NPK-based fertiliser can best be gauged by viewing Fig 6-5, which graphically demonstrates actual NPK fertiliser sales, from the SA Fertiliser Group – the predominant NPK fertiliser supplier in the country, between the years 2006 and 2009.

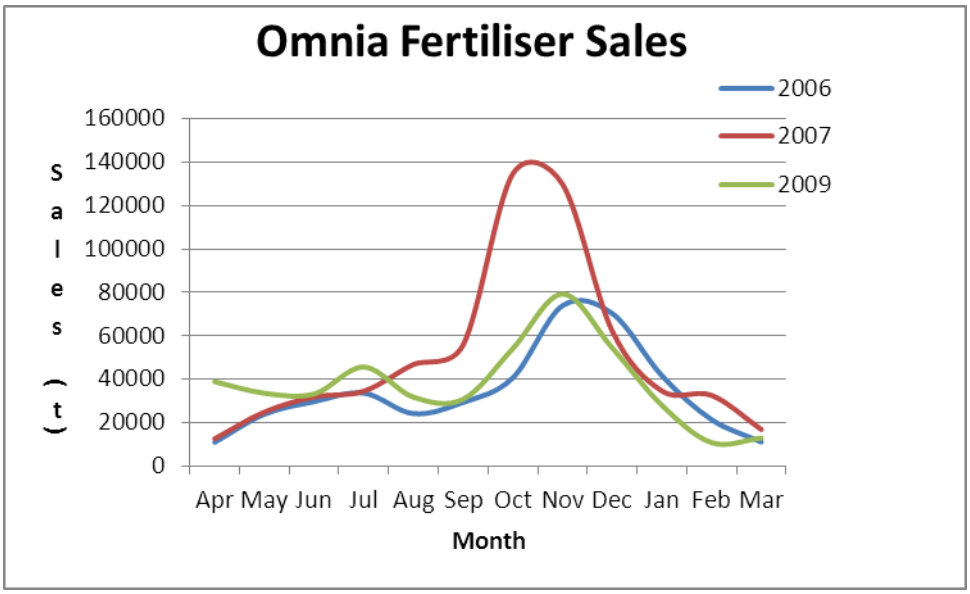


Figure 6-5: *Actual fertiliser sales from the SA Fertiliser Group*

It is clear from Fig 6-5 that, based on the actual NPK fertiliser sales figures over a four year period, NPK fertiliser demand is imprecise or *fuzzy*.

6.2.6 Specification of Solution Approach

It was decided to break the overall problem down into three separate phases, i.e. Raw Material Production, Granulation and then Distribution, as these represent the three distinct processes involved in the overall production and distribution operations. The initial raw material production process, made up of a few continuous constituent production processes, e.g. nitric acid and nitrophosphate production, is a continuous operation dependent on market demand; followed by the key NPK fertiliser granulation process, which is a batch-continuous operation, even more intimately dependent on market demand. The final distribution phase is, to a large extent, dependent on market proximity.

Therefore in accordance with the 'Supply Chain Optimisation under prevailing conditions of Uncertainty' methodology, outlined in section 6, either a MILP (Mixed Integer Linear Program) or a MINLP (Mixed Integer Non-Linear Program) programming approach is preferred depending on the linear and/or non-linear nature of the objective function(s) and constraints.

Such fuzzy demand + MILP/MINLP programming approach would, according to Table 6-2 conform to that particular aspect of the originally derived methodology which is itemised in Table 5-1.

However, since there is also a requirement to maximise the production and distribution of NPK fertilisers whilst simultaneously minimising the generation of hazardous effluent, there is a multi-objective and, preferably, a Pareto optimum solution requirement. Therefore, according to the works of Guillen and Grossmann (2009) and Liu and Papageorgiou (2013), the method of handling potentially conflicting multiple objectives, be they either minima or maxima, or a combination of the two, objectives, is by application of the ε-constraint method, which would generate a *Pareto optimal* solution, Typically, the principal objective of a project, i.e. maximum production and distribution of NPK fertiliser, $\max\sum(NPK)^{i}$ is selected as the objective of the MILP, whilst the other(s), i.e. minimum effluent discharge, $\min[(HF)_{NP} + (CD)_{CN}]$, is assigned as the Pareto ε-constraint.

Table 6-2 *Fuzzy Demand plus MILP/MINLP Programming Methodology*

No.	Programming Step	Status
1	Detailed description of Supply Chain Network	Done
2	Description of Supply Chain problem/requirements	Done
3	Illustrated diagram of Supply Chain Network	Done
4	Define Nomenclature Requirements	Done
5	Specify nature of demand uncertainty and solution approach	Done
6	Define and formulate the MILP program - Define the Objective Function - Define the ε-constraint - Define all probabilistic/deterministic objectives and if the former, transform them into deterministic ones using the Lee and Olson (1985) method - Define all probabilistic/deterministic constraints and if the former, transform them into deterministic ones	 Done Done
7	Solve the MILP program	
8	Record the optimal results, i.e. Obj. Fn., ε-constraint and process variables	

6.2.7 Formulation of a MILP (Mixed Integer Linear Program)

Further to Table 6-2, the MILP for the NPK fertiliser production and distribution operations is as follows:

Objective Function: $\max \sum(NPK)^i$

Pareto ε-constraint: $(HF)_{NP} + (CD)_{CN} \geq \varepsilon$

- Production constraints
- Granulation constraints
- Distribution constraints

But before this can be done, it is first necessary to specify all operating parameters and assumptions.

<u>Modeling Parameters and Assumptions</u>

a) The SA Fertiliser Group NPK fertiliser optimisation initiative will be based upon the NPK fertiliser demands of different national crop types as published by the FSSA, 'Fertiliser use by crop type in SA Project Area' report, Table 6-3.

b) The requirement is to determine the optimum production and distribution of a suite (*multi-objective*) of NPK fertiliser blends, which may be efficiently and effectively mathematically represented by Max $[\sum(NPK)^i]$, in accordance with SA crop fertiliser demand figures published by the FSSA. It is efficient since only one objective function is involved, and it is effective since the entire suite of NPK fertilisers will be maximised, automatically taking any interdependencies into account.

c) NPK fertiliser production and distribution figures must be in tandem with the annually reported FSSA NPK consumption figures

d) Model most be at NPK^i/crop level, i = 1,.., 16

e) Production utilises continuous processes up until the granulators and then batch thereafter

f) Raw material ratios per NPK blend can be determined from FSSA published NPK figures

g) For the purposes of this exercise, no fertiliser stock-holding is assumed, i.e. product manufactured goes directly to sales

h) Since the precise provincial distribution of crop types in SA is not essential to this study, they were estimated from FSSA graphical material, and further that provincial NPK fertiliser demand is shared equally between the sites and then equally again amongst the underlying depots

i) From a distribution perspective, there are eight (8) SA Fertiliser Group distribution sites situated strategically, from a market demand perspective, in the various SA provinces. For the purposes of this exercise, it is assumed that the sites in a particular represent, between them, the total fertiliser demand in that province. A number of strategically located depots (2 – 5) are associated with each site. Granular NPK product is distributed to these sites in accordance with customer orders, which are placed at those sites, and then transferred to the relevant depots for collection by the customer

j) It was decided, solely for the purposes of optimisation efficiency and practicality, to combine the two equal capacity (850t) granulators into one with double the capacity, 1700 t. This does not affect the optimisation process modeling at all since the model is only concerned about quantitative, and not qualitative, considerations

The formulation of an MILP program involves the definition of an Objective Function as well as the specification of all underlying constraints, both production and distribution related constraint

6.2.7.1 MILP Program Formulation

A] Objective Function

Optimum production (t) and distribution of a suite of NPK fertiliser in accordance with national FSSA, reported different crop-type demand in SA

$\text{Max}\sum(\text{NPK})^{i}_{GR}$,i = NPK blend

B] Pareto ε-constraint

Further to the research work of (Liu and Papageorgiou (2013)), (Haimes and Chankong (1979)) and (Mavrotas and Florios (2013)), the range of an ε-constraint must lie between its possible bound values, and those would be between its highest possible bound, i.e. $[(HF)_{NP} + (CD)_{CN}]_{max}$ = 41,172 t/yr and its lowest possible bound, i.e. $[(HF)_{NP} + (CD)_{CN}]_{min}$ = 38,880 t/yr

Therefore ε-constraint: $[(HF)_{NP} + (CD)_{CN}] \geq 38{,}880$ (or $[(HF)_{NP} + (CD)_{CN}] \leq 41{,}172$)

Therefore ε-constraint: $[(HF)_{NP} + (CD)_{CN}] \geq \varepsilon = 38{,}880$

C] Production Model Constraints

Note: Molar Equivalence in a reaction: $n_i = x_i/Mw_i$; n_i = no. moles ***i***

(with reference to Tables 1 & 2) $\therefore x_i/x_j = niMwi/njMwj$; Mw_i= Molec. Wt. of *i*

$\therefore x_i = (n_i/n_j)(Mw_i/Mw_j)x_j$

---

a) Overall Balances

Overall: $(A) + (RP) + (S) + (O)_{NA} + (W) + (L)_{CN} + (K)_{GR} = (NPK)_{GR} + (FBG)_{DIS} + (ANCN)_{ANCN}$

Water: $(W) = (W)_{SP} + (W)_{NA} + (W)_{CN}$

Sulphur: $(S) = (S)_{SP} + (S)_{NP}$

Ammonia: $(A) = (A)_{AN} + (A)_{NA} + (A)_{NP}$

Phosphate Rock: $(RP) = (RP)_{NP} + (RP)_{SP}$

i) Nitric Acid Production

Production rate	$(NA)_{NA} \leq 1200$ t/d $\leq 384{,}000$ t/yr
Strength of Nitric Acid produced	59%

Overall balance	$(O)_{NA} + (W)_{NA} + (A)_{NA} = (NA)_{NA}$
Chemical Processes: Oxidation	$5O_2 + 4NH_3 \rightarrow 4NO + 6H_2O$
Absorption	$2O_2 + 4NO \rightarrow 4NO_2$
Distillation	$4NO_2 + 6H_2O \rightarrow 4HNO_3 + 2H_2 + 2H_2O$
Overall	$7O_2 + 4NH_3 \rightarrow 4HNO_3 + 2H_2 + 2H_2O$
Molar equivalence:	$(A)_{NA} = (4/4)(17/63)(0.59)(NA)_{NA}$
	$\therefore (A)_{NA} = 0.159(NA)_{NA}$
Additional water $(W)_{NA}$ required during distillation to produce 59% Nitric Acid	Water in final product - water produced in reaction
Water produced in reaction (applying (W) to (NA) molar equivalence)	$(n_W/n_{NA})(Mw_W/Mw_{NA})(0.59)(NA)_{NA}$
	$= (6/4)(18/63)(0.59)(NA)_{NA}$
	$= 0.253(NA)_{NA}$
$\therefore$ Additional water required =	$= 0.41(NA)_{NA} - 0.253(NA)_{NA}$
	$\therefore (W)_{NA} = 0.157(NA)_{NA}$
Production Distribution	$(NA)_{NA} = (NA)_{NPH} + (NA)_{CN} + (NA)_{AN}$

ii) Ammonium, Nitrate Production

Reaction	$NH_3 + HNO_3 \rightarrow NH_4NO_3 + H_2O$
Production rate	$(AN)_{AN} \leq 700$ t/d $\leq 224{,}000$ t/yr
Strength of *AN*	89%
Overall balance	$(A)_{AN} + (NA)_{AN} = (AN)_{AN} + (W)_{AN}$
Nitrogen balance	$(14/17)(A)_{AN} + 0.59(14/63)(NA)_{NA} = 0.89(14/42)(AN)_{AN}$
	or $0.7(A)_{AN} + 0.13(NA)_{AN} = 0.3(AN)_{AN}$
Water Vapour removed from process:	$(W)_{AN} = (W)_{Reaction} - (1 - 0.89)(AN)_{AN}$
Applying molar equality	$(W)_{Reaction} = (n_{H2O}/n_{NH4NO3})(Mw_{H2O}/Mw_{NH4NO3})(AN)_{AN} - 0.11(AN)_{AN}$
	$= 10/42(AN)_{AN} = 0.238(AN)_{AN}$
$\therefore$ Water removed	$(W)_{AN} = 0.238(AN)_{AN} - 0.11(AN)_{AN}$
	$\therefore (W)_{AN} = 0.128(AN)_{AN}$
Production Distribution	$(AN)_{AN} = (AN)_{ANCN} + (AN)_{GR}$

iii) *CN* - Calcium Nitrate Production

Reaction	$CaCO_3 + 2HNO_3 \rightarrow Ca(NO_3)_2 + H_2O + CO_2$
Molar Equivalence	$(CN)_{CN} = (0.59)(1/2)(164/63)(NA)_{CN}$
	$\therefore (CN)_{CN} = 0.768(NA)_{CN}$
Release of water	$(W)_{CN} = (1/1)(18/164)(CN)_{CN} = 0.11(CN)_{CN}$
Release of carbon dioxide	$(CD)_{CN} = (1/1)(44/164)(CN)_{CN} = 0.268(CN)_{CN}$
Production rate	$(CN)_{CN} \leq 500$ t/d $\leq 160{,}000$ t/yr
Strength of *CN*	58%
Overall balance	$(L)_{CN} + (NA)_{CN} = (CN)_{CN} + (W)_{CN} + (CD)_{CN}$
Production Distribution	$(CN)_{CN} = (CN)_{FBG} + (CN)_{ANCN}$

iv) Nitrophosphate Production

Reactions	1) $Ca_{10}(PO_4)_6F_2 + 20HNO_3 \rightarrow 6H_3PO_4 + 10Ca(NO_3)_2 + 2HF$
	2) $6H_3PO_4 + 10Ca(NO_3)_2 + 16H_2O \rightarrow 6H_3PO_4 + 4Ca(NO_3)_2.4H_2O +$
	$+ 6Ca(NO_3)_2$
	3) $6H_3PO_4 + 6Ca(NO_3)_2 + 20NH_3 + 7H_2SO_4 + 12H_2O \rightarrow (NH_4)_2SO_4 +$
	$+ 6(NH4)H2PO4 + 12NH_4NO_3 + 6CaSO_4.2H_2O$
Effective overall Reaction	$Ca_{10}(PO_4)_6F_2 + 20HNO_3 + 28H_2O + 20NH_3 + 7H_2SO_4 \rightarrow [(NH_4)_2SO_4 +$
	$+ 6(NH_4)H_2PO_4 + 12NH_4NO_3 + 6CaSO_4.2H_2O] + 2HF + 4Ca(NO_3)_2.4H_2O$
	$= \text{Nitrophos} + 2HF + 4Ca(NO_3)_2.4H_2O$
Production rate	$(NPH)_{NP} \leq 500$ t/d $\leq 160{,}000$ t/yr
Overall balance	$(S)_{NP} + (W)_{NP} + (A)_{NP} + (RP)_{NP} + (NA)_{NP} = (NPH)_{NP} + (HF)_{NP} + (Ca)_{NP}$
Molar Equivalence (molecular weights are in Table):	
Use of 59% Nitric Acid, $(NA)_{NP}$	$(0.59)(NA)_{NP} = (n_{HNO3}/n_{RP})(Mw_{HNO3}/Mw_{RP})(RP)_{NP}$
	$\therefore (NA)_{NP} = (20/1)(63/1008)(RP)_{NP}/0.59$
	$\therefore (NA)_{NP} = 2.119(RP)_{NP}$
Similarly, for use of Ammonia $(A)_{NP}$	$(A)_{NP} = (20/1)(17/1008)(RP)_{NP}$

	$\therefore (A)_{NP} = 0.337(RP)_{NP}$
Similarly, for use of Sulphuric Acid $(S)_{NP}$	$(S)_{NP} = (7/1)(98/1008)(RP)_{NP}$
	$\therefore (S)_{NP} = 0.681(RP)_{NP}$
Similarly, for use of Water $(W)_{NP}$	$(W)_{NP} = (28/1)(18/1008)(RP)_{NP}$
	$\therefore (W)_{NP} = 0.5(RP)_{NP}$
Similarly, for use of Nitrophos	$(NPH)_{NP} = (n_{NPH}/n_{RP})(Mw_{NPH}/Mw_{RP})(RP)_{NP}$
	$\therefore (NPH)_{NP} = (1/1)(2814/1008)(RP)_{NP}$
$[(NH_4)_2SO_4\ (NH_4)H_2PO_4,\ NH_4NO_3,\ CaSO_4.2H_2O]$	$\therefore (NPH)_{NP} = 2.792(RP)_{NP}$
Hydrogen Fluoride $(HF)_{NP}$	$(HF)_{NP} = (2/1)(20/1008)(RP)_{NP}$
	$\therefore (HF)_{NP} = 0.04(RP)_{NP}$
Production Distribution	$(NPH)_{NP} = (NPH)_{GR} + (NPH)_{FBG}$

v) ANCN Production

Reaction	$NH_4NO_3 + Ca(NO_3)_2 \rightarrow NH_4NO_3.Ca(NO_3)_2$
Production rate	$(ANCN)_{ANCN} \leq 25t/hr < 192{,}000\ t/yr$
Overall balance	$(ANCN)_{ANCN} = (AN)_{ANCN} + (CN)_{ANCN}$
Nitrogen balance	$(56/244)(ANCN)_{ANCN} = (28/80)(AN)_{ANCN} + (28/164)(CN)_{ANCN}$
	$0.23(ANCN)_{ANCN} = 0.35(AN)_{ANCN} + 0.17(CN)_{ANCN}$

vi) Fluidised Bed Granulator

Production rate	$(FBG)_{DIS} \leq 150t/d \leq 48{,}000\ t/yr$
Overall balance	$(NPH)_{FBG} + (CN)_{FBG} = (FBG)_{DIS}$
Ratio of $(NPH)_{FBG}$ to $CN)_{FBG}$ flow = 1:1	$(NPH)_{FBG} = (CN)_{FBG}$

vii) Superphosphates Production

Reactions	$Ca_{10}(PO_4)_6F_2 + 7H_2SO_4 + 3H_2O \rightarrow 3Ca(H_2PO_4)_2H_2O + 2HF(g) + 7CaSO_4$
	$3Ca(H_2PO_4)_2H_2O + 7CaSO_4 + 25H_2O \rightarrow 3Ca(H_2PO_4)_2 + 7CaSO_4.4H_2O$
- overall reaction	$Ca_{10}(PO4)_6F_2 + 7H_2SO_4 + 28H_2O \rightarrow 3Ca(H_2PO_4)_2 + 7CaSO_4.4H_2O + 2HF$

Production rate	$(SP)_{SP} \leq 500$ t/d $\leq 160{,}000$ t/yr
Overall balance	$(S)_{SP} + (RP)_{NP} + (W)_{SP} = (SP)_{SP}$ $\therefore$
Phosphorous balance	$(186/1008)(RP)_{SP} = (186)/(2449)(SP)_{SP}$
	or $0.076(SP)_{SP} = 0.185(RP)_{SP}$
	or $(SP)_{SP} = 2.434(RP)_{SP}$
Sulphur balance	$(32/98)(S)_{SP} = (32/2449)(SP)_{SP}$
	$\therefore 0.327(S)_{SP} = 0.098(SP)_{SP}$
	$\therefore (S)_{SP} = 0.013(SP)_{SP}$
Water Balance	$(W)_{SP} = (28)(18)/2449$
	$\therefore (W)_{SP} = 0.206(SP)_{SP}$
Production Distribution	$(SP)_{SP} = (SP)_{GR}$

Table 6-3: *Breakdown of Fert. Cons. per Crop Type over a Year in SA Project Area*

Crop/Groups	Crop *i*	Area (*LA*)^i '000 ha	Rates, kg/ha of fertilised area				
			[N]^i	*P2O5*	[P]^i = 0.436 P2O5	*K2O*	[K]^i = 0.83 K2O
Field & Mixed Crops							
Maize	1	2,384	**55**	30	**13.08**	6	**4.98**
Wheat	2	601	**30**	40	**17.44**	4	**3.32**
Sunflower	3	455	**15**	21	**9.16**	2	**1.66**
Skybeans	4	85	**7**	25	**10.90**	8	**6.64**
Sugar cane	5	307	**92**	57	**24.85**	133	**110.39**
Lucerne	6	128	**15**	59	**25.72**	24	**19.92**
Other Pastures	7	281	**50**	44	**19.18**	7	**5.81**
Tobacco	8		**38**	144	**62.78**	98	**81.34**
Cotton	9	7	**36**	22	**9.59**	3	**2.49**

Vegetables	10	17	**170**	159	**69.32**	120	**99.60**
Potatoes	11	39	**170**	160	**69.76**	120	**99.60**
Horticulture/Fruit							
Citrus	12		**80**	35	**15.26**	60	**49.80**
Subtropical fruits/nuts	13	16	**180**	57	**24.85**	240	**199.20**
Vines	14	17	**50**	36	**15.70**	24	**19.92**
Deciduous Fruit	15		**110**	159	**69.32**	83	**68.89**
FSSA' 2005							

Source: UN/FSSA - The NPK fertiliser consumption figures in Table 7-4 for the SA Fertiliser Group were compiled from the UN/FSSA magazine

6.2.7.1.1 Granulation Constraints (with reference to Table 6-3)

i) Overall Granulator equations with crop type ***i*** blend indication

$\sum(NPK)^i_{GR} = \sum(NPK)^i_{DIS}$

Overall (1 yr) Granulator Balances: $(NPH)_{GR} + (AN)_{GR} + (SP)_{GR} = \sum(NPK)^i_{DIS} = \sum(NPK)^i_{GR}$

Overall Production Limitation (i.e. 88% uptime): $= \sum(NPK)^i_{GR} \leq (1700)320 \leq 544{,}000$ t/a

Overall (1 yr) Nitrophos Balances: $(NPH)_{GR} = \sum(NPH)^i_{GR}$

Overall (1 yr) Ammonium Nitrate Balances: $(AN)_{GR} = \sum(AN)^i_{GR}$

Overall (1 yr) Potassium Chloride Balances: $(K)_{GR} = \sum(K)^i_{GR}$

Overall (1 yr) Super Phosphate Balances: $(SP)_{GR} = \sum(SP)^i_{GR}$

NPK Ratios: {NH_4NO_3, [$(NH_4)_2SO_4$ $(NH_4)H_2PO_4$, NH_4NO_3, $CaSO_4.2H_2O$],

' KCl, (36% $Ca(H_2PO_4)_2$ + 64% $CaSO_4.4H_2O$)}

ii) Granulation Parameters

Nitrophosphate = $(NH_4)_2SO_4.6(NH_4)H_2PO_4.12NH_4NO_3.6CaSO_4.2H_2O$

Molecular Weight (Mw) = (132 + 690 + 960 + 1032) = 2814

∴ Mass percentage of N = 20(14)/2814 = 10%

∴ Mass percentage of P = 6(31)/2814 = 6.61%

Superphosphate = $3Ca(H_2PO_4)_3, 7CaSO_4.4H_2O$

Mw = 3(331) + 7(208) = 2449

∴ Mass percentage P = 3(3)(31)/2449 = 11.4%

Ammonium Nitrate = NH_4NO_3

Mw = 80 ⟹ Mass Percentage N = 2(14)/80 = 35%

Potassium Chloride = KCl

Mw = 74.5 ⟹ Mass Percentage K = 52.3%

iii) Granulator Batches - specific crop-type *NPK* fertilisers

Generic NPK fertiliser equations over the granulators per crop type, *i* :

- Overall $(NPH)^i_{GR} + (AN)^i_{GR} + (K)^i_{GR} + (SP)^i_{GR} = (NPK)^i_{GR}$
- Nitrogen (mass fraction N)$(NPH)^i_{GR}$ + (mass fraction N)$(AN)^i_{GR} \leq [N]^i(LA)^i$

  $\therefore 0.1(NPH)^i_{GR} + 0.35(AN)^i_{GR} \leq [N]^i(LA)^i$
- Phosphorous (mass fraction P)$(NPH)^i_{GR}$ + (mass fraction P)$(SP)^i_{GR} \leq [P]^i(LA)^i$

  $\therefore 0.0661(NPH)^i_{GR} + 0.114(SP)^i_{GR} \leq [P]^i(LA)^i$
- Potassium (mass fraction K)$(K)^i_{GR} \leq [K]^i(LA)^i$

  $\therefore 0.523(K)^i_{GR} \leq [K]^i(LA)^i$

Maize (i = 1): Overall Balance $(NPH)^1_{GR} + (AN)^1_{GR} + (K)^1_{GR} + (SP)^1_{GR} = (NPK)^1_{GR}$

: Nitrogen Balance $0.1(NPH)^1_{GR} + 0.35(AN)^1_{GR} \leq [N]^1(LA)^1$

$\therefore 0.1(NPH)^1_{GR} + 0.35(AN)^1_{GR} \leq 55 \times 2384 \leq 131{,}120$

:Phosphorus Balance $0.0661(NPH)^1_{GR} + 0.114(SP)^1_{GR} \leq [P]^1(LA)^1$

$\therefore 0.0661(NPH)^1_{GR} + 0.114(SP)^1_{GR} \leq 13.08 \times 2384 \leq 31{,}183$

:Potassium Balance $(K)^1_{GR} \leq 1.912(K)^1 \times (LA)^1$

$\therefore (K)^1_{GR} \leq 1.912 \times 2384 \times 4.98 = 22{,}700$

Wheat (i = 2): Overall Balance $(NPH)^2_{GR} + (AN)^2_{GR} + (K)^2_{GR} + (SP)^2_{GR} = (NPK)^2_{GR}$

: Nitrogen Balance $0.1(NPH)^2_{GR} + 0.35(AN)^2_{GR} \leq [N]^2(LA)^2$

$\therefore 0.1(NPH)^2_{GR} + 0.35(AN)^2_{GR} \leq 30 \times 601 < 18{,}030$

:Phosphorus Balance $0.0661(NPH)^2_{GR} + 0.114(SP)^2_{GR} \leq [P]^2(LA)^2$

$\therefore 0.0661(NPH)^2_{GR} + 0.114(SP)^2_{GR} \leq 17.44 \times 601 \leq 10{,}481$

:Potassium Balance $(K)^2_{GR} \leq 1.912[K]^i(LA)^i$

$\therefore (K)^2_{GR} \leq 1.912 \times 601 \times 3.32 \leq 3{,}815$

Sunflower (i = 3): Overall Balance $(NPH)^3_{GR} + (AN)^3_{GR} + (K)^3_{GR} + (SP)^3_{GR} = (NPK)^3_{GR}$

: Nitrogen Balance $0.1(NPH)^3_{GR} + 0.35(AN)^3_{GR} \leq [N]^3(LA)^3$

$0.1(NPH)^3_{GR} + 0.35(AN)^3_{GR2} \leq 15 \times 455 \leq 6{,}825$

:Phosphorus Balance $0.0661(NPH)^3_{GR} + 0.114(SP)^3_{GR} \leq [P]^3(LA)^3$

$0.0661(NPH)^2_{GR} + 0.114(SP)^2_{GR} \leq 9.16 \times 455 \leq 4{,}168$

:Potassium Balance $(K)^3_{GR} \leq 1.912[K]^3(LA)^3$

$\therefore (K)^3_{GR} \leq 1.912 \times 455 \times 1.66 \leq 1{,}444$

Skybeans (i = 4): Overall Balance $(NPH)^4_{GR} + (AN)^4_{GR} + (K)^4_{GR} + (SP)^4_{GR} = (NPK)^4_{GR}$

: Nitrogen Balance $0.1(NPH)^4_{GR} + 0.35(AN)^4_{GR} \leq [N]^4(LA)^4$

$0.1(NPH)^4_{GR} + 0.35(AN)^4_{GR} \leq 7 \times 85 \leq 595$

:Phosphorus Balance $0.0661(NPH)^4_{GR} + 0.114(SP)^4_{GR} \leq [P]^4(LA)^4$

$0.0661(NPH)^4_{GR} + 0.114(SP)^4_{GR} \leq 10.9 \times 85 \leq 927$

:Potassium Balance $(K)^4_{GR} \leq 1.912[K]^4(LA)^4$

$\therefore (K)^4_{GR} \leq 1.912 \times 85 \times 6.44 \leq 1{,}047$

Sugar cane (i = 5): Overall Balance $(NPH)^5_{GR} + (AN)^5_{GR} + (K)^5_{GR} + (SP)^5_{GR} = (NPK)^5_{GR}$

: Nitrogen Balance $0.1(NPH)^5_{GR} + 0.35(AN)^5_{GR} \leq [N]^5(LA)^5$

$0.1(NPH)^5_{GR} + 0.35(AN)^5_{GR} \leq 92 \times 307 \leq 28{,}244$

:Phosphorus Balance $0.0661(NPH)^5_{GR} + 0.114(SP)^5_{GR} \leq [P]^5(LA)^5$

$0.0661(NPH)^5_{GR} + 0.114(SP)^5_{GR} \leq 24.85 \times 307 \leq 7{,}629$

:Potassium Balance $(K)^5_{GR} \leq 1.912[K]^5(LA)^5$

$\therefore (K)^5_{GR} \leq 1.912 \times 307 \times 110.39 \leq 64{,}797$

Lucerne (i = 6): Overall Balance $(NPH)^6_{GR} + (AN)^6_{GR} + (K)^6_{GR} + (SP)^6_{GR} = (NPK)^6_{GR}$

: Nitrogen Balance $0.1(NPH)^6_{GR} + 0.35(AN)^6_{GR} \leq [N]^6(LA)^6$

$0.1(NPH)^6_{GR} + 0.35(AN)^6_{GR} \leq 15 \times 128 < 1{,}920$

:Phosphorus Balance $0.0661(NPH)^6_{GR} + 0.114(SP)^6_{GR} \leq [P]^6(LA)^6$

$0.0661(NPH)^6_{GR} + 0.114(SP)^6_{GR} \leq 25.72 \times 128 \leq 3{,}292$

:Potassium Balance $(K)^6_{GR} \leq 1.912[K]^6(LA)^6$

$\therefore (K)^6_{GR} \leq 1.912 \times 128 \times 19.92 \leq 4{,}875$

Other Pastures (i = 7): Overall Balance $(NPH)^{7}_{GR} + (AN)^{7}_{GR} + (K)^{7}_{GRj} + (SP)^{7}_{GR} = (NPK)^{7}_{GR}$

: Nitrogen Balance $0.1(NPH)^{7}_{GR} + 0.35(AN)^{7}_{GR} \leq [N]^{7}(LA)^{7}$

$0.1(NPH)^{7}_{GR} + 0.35(AN)^{7}_{GR} \leq 14{,}050$

:Phosphorus Balance $0.0661(NPH)^{7}_{GR} + 0.114(SP)^{7}_{GR} \leq [P]^{7}(LA)^{7}$

$0.0661(NPH)^{7}_{GR} + 0.114(SP)^{7}_{GR} \leq 19.18 \times 281 \leq 5{,}390$

:Potassium Balance $(K)^{7}_{GR} \leq 1.912[K]^{7}(LA)^{7}$

$\therefore (K)^{7}_{GR} \leq 1.912 \times 281 \times 5.81 < 3{,}122$

Cotton (i = 9): Overall Balance $(NPH)^{9}_{GR} + (AN)^{9}_{GR} + (K)^{9}_{GR} + (SP)^{9}_{GRj} = (NPK)^{9}_{GR}$

: Nitrogen Balance $0.1(NPH)^{9}_{GR} + 0.35(AN)^{9}_{GR} \leq [N]^{9}(LA)^{9}$

$0.1(NPH)^{9}_{GR} + 0.35(AN)^{9}_{GR} \leq 36 \times 7 \leq 252$

:Phosphorus Balance $0.0661(NPH)^{9}_{GR} + 0.114(SP)^{9}_{GR} \leq [P]^{9}(LA)^{9}$

$0.0661(NPH)^{9}_{GR} + 0.114(SP)^{9}_{GR} \leq 9.59 \times 7 \leq 67$

:Potassium Balance $(K)^{9}_{GR} \leq 1.912[K]^{9}(LA)^{9}$

$\therefore (K)^{9}_{GR} \leq 1.912 \times 7 \times 2.49 < 33$

Vegetables (i = 10): Overall Balance $(NPH)^{10}_{GR} + (AN)^{10}_{GR} + (K)^{10}_{GR} + (SP)^{10}_{GR} = (NPK)^{10}_{GR}$

: Nitrogen Balance $0.1(NPH)^{10}_{GR} + 0.35(AN)^{10}_{GR} \leq [N]^{10}(LA)^{10}$

$0.1(NPH)^{10}_{GR} + 0.35(AN)^{10}_{GR} \leq 170 \times 17 \leq 2{,}890$

:Phosphorus Balance $0.0661(NPH)^{10}_{GR} + 0.114(SP)^{10}_{GR} \leq [P]^{10}(LA)^{10}$

$0.0661(NPH)^{10}_{GR} + 0.114(SP)^{10}_{GR} \leq 69.32 \times 17 \leq 1{,}178$

:Potassium Balance $(K)^{10}_{GR} \leq 1.912[K]^{10}(LA)^{10}$

$\therefore (K)^{14}_{GR} \leq 1.912 \times 17 \times 99.6 \leq 3{,}327$

Potatoes (i = 11): Overall Balance $(NPH)^{11}_{GR} + (AN)^{11}_{GR} + (K)^{11}_{GR} + (SP)^{11}_{GR} = (NPK)^{11}_{GR}$

: Nitrogen Balance $0.1(NPH)^{11}_{GR} + 0.35(AN)^{11}_{GR} \leq [N]^{11}(LA)^{11}$

$0.1(NPH)^{11}_{GR} + 0.35(AN)^{11}_{GR} \leq 170 \times 39 \leq 6{,}630$

:Phosphorus Balance $0.0661(NPH)^{11}_{GR} + 0.114(SP)^{11}_{GR} \leq [P]^{11}(LA)^{11}$

$0.0661(NPH)^{11}_{GR} + 0.114(SP)^{11}_{GR} \leq 69.76 \times 39 \leq 2{,}721$

:Potassium Balance $(K)^{11}_{GR} \leq 1.912[K]^{15}(LA)^{15}$

$(K)^{11}_{GR} \leq 39 \times 99.6 < 3{,}884$

Subtropical fruits (i = 13): Overall Balance $(NPH)^{13}_{GR} + (AN)^{13}_{GR} + (K)^{13}_{GR} + (SP)^{13}_{GR} = (NPK)^{13}_{GR}$

: Nitrogen Balance	$0.1(NPH)^{13}_{GR} + 0.35(AN)^{13}_{GR} \leq [N]^{13}(LA)^{13}$
	$0.1(NPH)^{13}_{GR} + 0.35(AN)^{13}_{GR} \leq 2{,}880$
:Phosphorus Balance	$0.0661(NPH)^{13}_{GR} + 0.114(SP)^{13}_{GR} \leq [P]^{11}(LA)^{13}$
	$0.0661(NPH)^{13}_{GR} + 0.114(SP)^{13}_{GR} \leq 24.85 \times 16 \leq 398$
:Potassium Balance	$(K)^{13}_{GR} \leq 1.912[K]^{13}(LA)^{13}$
	$\therefore (K)^{13}_{GR} \leq 1.912 \times 16 \times 199.2 < 6{,}094$

Vines (i = 14): Overall Balance	$(NPH)^{14}_{GR} + (AN)^{14}_{GR} + (K)^{14}_{GRj} + (SP)^{14}_{GR} = (NPK)^{14}_{GR}$
: Nitrogen Balance	$0.1(NPH)^{14}_{GR} + 0.35(AN)^{14}_{GR} \leq [N]^{14}(LA)^{14}$
	$0.1(NPH)^{14}_{GR} + 0.35(AN)^{14}_{GR} \leq 50 \times 17 \leq 850$
:Phosphorus Balance	$0.0661(NPH)^{14}_{GR} + 0.114(SP)^{14}_{GR} \leq [P]^{14}(LA)^{14}$
	$0.0661(NPH)^{14}_{GR} + 0.114(SP)^{14}_{GR} \leq 15.7 \times 17 \leq 267$
:Potassium Balance	$(K)^{14}_{GR} \leq 1.912[K]^{14}(LA)^{14}$
	$\therefore (K)^{14}_{GR} \leq 1.912 \times 17 \times 19.92 \leq 647$

Now it follows from Table 7-4 that the various fertiliser crop type distribution limits can be compiled for the various distribution centres, $S_1 \ldots S_6$ of the SA Fertiliser Group and these are evident in Table 7-5

6.2.7.1.2 Distribution Constraints

$$\sum(NPK)^{i}_{GR} = \sum(NPK)^{i}_{DIS}$$

$$\sum(NPK)^{i}_{DIS} = \sum_{n=1}^{8}\sum_{i=1}^{15}(NPK)^{i}_{Sn}$$

$$(FBG)_{DIS} = \sum_{n=1}^{8}(FBG)_{Sn}$$

<u>*Distribution Limits*</u>: $(NPK)^{i}_{Sn} \leq (p)^{i}_{Sn}(LA)^{i}(NPK)^{i}$/ha
(this relationship follows the work of Chen et al. (2007), Awudu & Zhang (2013) and Razmi et al. (2013) where, in this case, $(p)^{i}_{Sn}$ is the probability of demand for that particular crop-type in that particular area

Table 6-4 *Fertiliser crop-type distribution limits*

Crop/Groups	Crop	Area	Estimated Breakdown of Fertiliser Demand in section of SA (ha)								NPK[i]/ ha
	i	'000 ha	Reg. 1	Region 2				Region 3		Reg. 4	
Sites			S3	S2	S5	S6	S7	S4	S8	S1	
Field Crops											
Maize	1	2,384	177.09	295.15	295.15	295.15	295.15	238.93	238.93	548.69	284
Wheat	2	601	44.67	74.45	74.45	74.45	74.45	60.27	60.27	138.40	231
Sunflower	3	455	33.79	56.32	56.32	56.32	56.32	45.59	45.59	104.70	119
Skybeans	4	85	6.28	10.47	10.47	10.47	10.47	8.48	8.48	19.47	125
Sugar cane	5	307	22.81	38.02	38.02	38.02	38.02	30.77	30.77	70.67	881
Lucerne	6	128	9.50	15.84	15.84	15.84	15.84	12.82	12.82	29.45	306
Other Pastures	7	281	20.86	34.76	34.76	34.76	34.76	28.14	28.14	64.62	316
		4,241	315.00	525.00	525.00	525.00	525.00	425.00	425.00	976.00	
Mixed farming											
Tobacco	8	7.06	1.75	0.67	0.67	0.67	0.67	0.68	0.68	1.25	875
Cotton	9	17.44	4.33	1.66	1.66	1.66	1.66	1.69	1.69	3.08	191
Vegetables	10	39.44	9.81	3.76	3.76	3.76	3.76	3.81	3.81	6.97	1,403
Potatoes	11	20.76	5.16	1.98	1.98	1.98	1.98	2.01	2.01	3.67	1,406
		84.69	21.06	8.07	8.07	8.07	8.07	8.19	8.19	14.97	
Horticulture/Fruit											
Citrus	12	16.00						8.00	8.00		547
Subtropical fruits/nuts	13	17.00						8.50	8.50		1,491
Vines	14	5.00						2.50	2.50		344
Deciduous Fruit	15	38.00						19.00	19.00		1,100
		76.00						38.00	38.00		
FSSA'2005											

Table 6-5: *Distribution Limits by site*

For the sake of this exercise, the probability $(p)^{i}_{Sn}$ of demand for fertiliser at all crops *I* at all sites *n* is assumed to be 1. Further to the Distribution Limit equation (6.2.7.1.2)) and Table 6-4:

S1

$(NPK)^{1}_{S1} \leq (548.69)(284) \leq 155{,}828$

$(NPK)^{2}_{S1} \leq (138.4)(231) \leq 31{,}970$

$(NPK)^{3}_{S1} \leq (104.7)(119) \leq 12{,}459$

$(NPK)^{4}_{S1} \leq (19.47)(125) \leq 2{,}438$

$(NPK)^{5}_{S1} \leq (70.67)(881) \leq 62{,}260$

$(NPK)^{6}_{S1} \leq (29.45)(306) \leq 9{,}012$

$(NPK)^{7}_{S1} \leq (64.62)(316) \leq 31{,}975$

$(NPK)^{8}_{S1} \leq (1.25)(875) \leq 1{,}094$

$(NPK)^{9}_{S1} \leq (3.08)(191) \leq 31{,}977$

$(NPK)^{10}_{S1} \leq (6.97)(1403) \leq 9{,}779$

$(NPK)^{11}_{S1} \leq (3.67)(1406) \leq 5{,}160$

S2

$(NPK)^{1}_{S2} \leq (295.15)(284) \leq 83{,}823$

$(NPK)^{2}_{S2} \leq (74.45)(231) \leq 17{,}198$

$(NPK)^{3}_{S2} \leq (56.32)(119) \leq 12{,}459$

$(NPK)^{4}_{S2} \leq (10.47)(125) \leq 1{,}309$

$(NPK)^{5}_{S2} \leq (38.02)(881) \leq 33{,}496$

$(NPK)^{6}_{S2} \leq (15.84)(306) \leq 4{,}847$

$(NPK)^{7}_{S2} \leq (34.76)(316) \leq 34{,}763$

$(NPK)^{8}_{S2} \leq (0.67)(875) \leq 586$

$(NPK)^{9}_{S2} \leq (1.66)(191) \leq 317$

$(NPK)^{10}_{S2} \leq (3.76)(1403) \leq 5{,}275$

$(NPK)^{11}_{S2} \leq (1.98)(1406) \leq 2{,}784$

S3

$(NPK)^{1}_{S3} \leq (295.15)(284) \leq 83{,}823$

$(NPK)^{2}_{S3} \leq (74.45)(231) \leq 17{,}198$

$(NPK)^{3}_{S3} \leq (56.32)(119) \leq 12{,}459$

$(NPK)^{4}_{S3} \leq (10.47)(125) \leq 1{,}309$

$(NPK)^{5}_{S3} \leq (38.02)(881) \leq 33{,}496$

$(NPK)^{6}_{S3} \leq (15.84)(306) \leq 4{,}847$

$(NPK)^{7}_{S3} \leq (34.76)(316) \leq 34{,}763$

$(NPK)^{8}_{S3} \leq (0.67)(875) \leq 586$

$(NPK)^{9}_{S3} \leq (1.66)(191) \leq 317$

$(NPK)^{10}_{S3} \leq (3.76)(1403) \leq 5{,}275$

$(NPK)^{11}_{S3} \leq (1.98)(1406) \leq 2{,}784$

S4

$(NPK)^{1}_{S4} \leq (238.93)(284) \leq 67{,}8563$

$(NPK)^{2}_{S4} \leq (60.27)(231) \leq 13{,}922$

$(NPK)^{3}_{S4} \leq (45.59)(119) \leq 5{,}425$

$(NPK)^{4}_{S4} \leq (8.48)(125) \leq 1{,}060$

$(NPK)^{5}_{S4} \leq (30.77)(881) \leq 27{,}108$

$(NPK)^{6}_{S4} \leq (12.82)(306) \leq 3{,}923$

$(NPK)^{7}_{S4} \leq (28.14)(316) \leq 8{,}892$

$(NPK)^{8}_{S4} \leq (0.68)(875) \leq 595$

$(NPK)^{9}_{S4} \leq (1.69)(191) \leq 323$

$(NPK)^{10}_{S4} \leq (3.81)(1403) \leq 5{,}345$

$(NPK)^{11}_{S4} \leq (2.01)(1406) \leq 2{,}826$

$(NPK)^{12}_{S4} \leq (8)(547) \leq 4{,}376$

$(NPK)^{13}_{S4} \leq (8.5)(1{,}491) \leq 12{,}674$

$(NPK)^{14}_{S4} \leq (2.5)(344) \leq 860$

$(NPK)^{15}_{S4} \leq (19)(1100) \leq 20{,}900$

S5

$(NPK)^{1}_{S5} \leq (295.15)(284) \leq 83{,}823$

$(NPK)^{2}_{S5} \leq (74.45)(231) \leq 17{,}198$

$(NPK)^{3}_{S5} \leq (56.32)(119) \leq 12{,}459$

$(NPK)^{4}_{S5} \leq (10.47)(125) \leq 1{,}309$

$(NPK)^{5}_{S5} \leq (38.02)(881) \leq 33{,}496$

$(NPK)^{6}_{S5} \leq (15.84)(306) \leq 4{,}847$

$(NPK)^{7}_{S5} \leq (34.76)(316) \leq 34{,}763$

$(NPK)^{8}_{S5} \leq (0.67)(875) \leq 586$

$(NPK)^{9}_{S5} \leq (1.66)(191) \leq 317$

$(NPK)^{10}_{S5} \leq (3.76)(1403) \leq 5{,}275$

$(NPK)^{11}_{S5} \leq (1.98)(1406) \leq 2{,}784$

S6

$(NPK)^{1}_{S6} \leq (295.15)(284) \leq 83{,}823$

$(NPK)^{2}_{S6} \leq (74.45)(231) \leq 17{,}198$

$(NPK)^{3}_{S6} \leq (56.32)(119) \leq 12{,}459$

$(NPK)^{4}_{S6} \leq (10.47)(125) \leq 1{,}309$

$(NPK)^{5}_{S6} \leq (38.02)(881) \leq 33{,}496$

$(NPK)^{6}_{S6} \leq (15.84)(306) \leq 4{,}847$

$(NPK)^{7}_{S6} \leq (34.76)(316) \leq 34{,}763$

$(NPK)^{8}_{S6} \leq (0.67)(875) \leq 586$

$(NPK)^{9}_{S6} \leq (1.66)(191) \leq 317$

$(NPK)^{10}_{S6} \leq (3.76)(1403) \leq 5{,}275$

$(NPK)^{11}_{S6} \leq (1.98)(1406) \leq 2{,}784$

6.2.8 Specification of ε-Constraint MILP program

A MILP (Mixed Integer Linear Program), can now be structured that describes the production and distribution operations of the SA Fertiliser Group by integrating the various program elements of the production and distribution operations, the results of which are shown in Table 6-6

Table 6-6: *ε-Constraint MILP (Mixed Integer Linear Program) Model for the production and distribution operations of the SA Fertiliser Group*

SA Fertiliser Group – MILP (Mixed Integer Linear Program) Model

Production and Distribution Operations

OBJECTIVE FUNCTION

$\text{Max}(\sum(\text{NPK})^{i}_{\text{GR}}$

PARETO ε-CONSTRAINT

$(HF)_{\text{NP}} + (CD)_{\text{CN}} \geq 38{,}880$

CONSTRAINTS

Equality

$(A) + (S) + (O)_{\text{NA}} + (W) + (L)_{\text{CN}} + (RP) + (K)_{\text{GR}} = (NPK)_{\text{GR}} + (FBG)_{\text{DIS}} + (ANCN)_{\text{ANCN}}$

$(S) = (S)_{\text{SP}} + (S)_{\text{NP}}$

$(\text{W}) = (\text{W})_{\text{SP}} + (W)_{\text{NA}} + (W)_{\text{NP}} + (W)_{\text{CN}}$

$(A) = (A)_{\text{AN}} + (A)_{\text{NA}} + (A)_{\text{NP}}$

$(RP) = (RP)_{\text{NP}} + (RP)_{\text{SP}}$

$(O)_{\text{NA}} + (W)_{\text{NA}} + (A)_{\text{NA}} = (NA)_{\text{NA}}$

$(A)_{\text{NA}} = 0.159(NA)_{\text{NA}}$

$(W)_{\text{NA}} = 0.157(NA)_{\text{NA}}$

$(NA)_{\text{NA}} = (NA)_{\text{NPI}} + (NA)_{\text{CN}} + (NA)_{\text{AN}}$

$(A)_{\text{AN}} + (NA)_{\text{AN}} = (AN)_{\text{AN}} + (W)_{\text{AN}}$

$0.7(A)_{AN} + 0.13(NA)_{AN} = 0.3(AN)_{AN}$

$(W)_{AN} = 0.128(AN)_{AN}$

$(AN)_{AN} = (AN)_{ANCN} + (AN)_{GR}$

$(L)_{CN} + (NA)_{CN} = (CN)_{CN} + (CD)_{CN} + (W)_{CN}$

$(CN)_{CN} = 0.768(NA)_{CN}$

$(L)_{CN} + (NA)_{CN} = (CN)_{CN} + (W)_{CN} + (CD)_{CN}$

$(CN)_{CN} = 0.768(NA)_{CN}$

$(W)_{CN} = 0.11(CN)_{CN}$

$(CD)_{CN} = 0.268(CN)_{CN}$

$(CN)_{CN} = (CN)_{FBG} + (CN)_{ANCN}$

$(AN)_{ANCN} + (CN)_{ANCN} = (ANCN)_{ANCN}$

$0.23(ANCN)_{ANCN} = 0.35(AN)_{ANCN} + 0.17(CN)_{ANCN}$

$(S)_{NP} + (W)_{NP} + (A)_{NP} + (RP)_{NP} + (NA)_{NP} = (NPH)_{NP} + (HF)_{NP} + (Ca)_{NP}$

$(NA)_{NP} = 2.119(RP)_{NP}$

$(A)_{NP} = 0.337(RP)_{NP}$

$(S)_{NP} = 0.681(RP)_{NP}$

$(W)_{NP} = 0.5(RP)_{NP}$

$(NPH)_{NP} = 2.792(RP)_{NP}$

$(HF)_{NP} = 0.038(RP)_{NP}$

$(NPH)_{NP} = (NPH)_{GR} + (NPH)_{FBG}$

$(NPH)_{NP} = (NPH)_{GR} + (NPH)_{FBG}$

$(NPH)_{FBG} + (CN)_{FBG} = (FBG)_{DIS}$

$(NPH)_{FBG} = (CN)_{FBG}$

$(S)_{SP} + (RP)_{SP} + (W)_{SP} = (SP)_{SP}$

$(SP)_{SP} = 2.434(RP)_{SP}$

$(S)_{SP} = 0.013(SP)_{SP}$

$(SP)_{SP} = (SP)_{GR}$

$(W)_{SP} = 0.206(SP)_{SP}$

$(AN)_{GR} = \sum(AN)^{i}_{GR}$

$(NPH)_{GR} = \sum(NPH)^{i}_{GR}$

$(K)_{GR} = \sum(K)^{i}_{GR}$

$(SP)_{GR} = \sum(SP)^{i}_{GR}$

$(NPH)^{1}_{GR} + (AN)^{1}_{GR} + (K)^{1}_{GR} + (SP)^{1}_{GR} = (NPK)^{1}_{GR}$

$(NPH)^{2}_{GR} + (AN)^{2}_{GR} + (K)^{2}_{GR} + (SP)^{2}_{GR} = (NPK)^{2}_{GR}$

$(NPH)^3_{GR} + (AN)^3_{GR} + (K)^3_{GR} + (SP)^3_{GR} = (NPK)^3_{GR}$

$(NPH)^4_{GR} + (AN)^4_{GR} + (K)^4_{GR} + (SP)^4_{GR} = (NPK)^4_{GR}$

$(NPH)^5_{GR} + (AN)^5_{GR} + (K)^5_{GR} + (SP)^5_{GR} = (NPK)^5_{GR}$

$(NPH)^6_{GR} + (AN)^6_{GR} + (K)^6_{GR} + (SP)^6_{GR} = (NPK)^6_{GR}$

$(NPH)^7_{GR} + (AN)^7_{GR} + (K)^7_{GRj} + (SP)^7_{GR} = (NPK)^7_{GR}$

$(NPH)^9_{GR} + (AN)^9_{GR} + (K)^9_{GR} + (SP)^9_{GRj} = (NPK)^9_{GR}$

$(NPH)^{11}_{GR} + (AN)^{11}_{GR} + (K)^{11}_{GR} + (SP)^{11}_{GR} = (NPK)^{11}_{GR}$

$(NPH)^{13}_{GR} + (AN)^{13}_{GR} + (K)^{13}_{GRj} + (SP)^{13}_{GR} = (NPK)^{13}_{GR}$

$(NPH)^{14}_{GR} + (AN)^{14}_{GR} + (K)^{14}_{GR} + (SP)^{14}_{GR} = (NPK)^{14}_{GR}$

$\sum(NPK)^i_{GR} = \sum_{n=1}^{8}\sum_{i=1}^{15}(NPK)^i_{Sn}$

$(FBG)_{DIS} = \sum_{n=1}^{8}(FBG)_{Sn}$

$(ANCN)_{ANCN} = 192{,}000$

$(FBG)_{DIS} = 48{,}000$

Inequality Constraints (≤)

$(NA)_{NA} \leq 384{,}000$

$(AN)_{AN} \leq 224{,}000$

$(CN)_{CN} \leq 160{,}000$

$(NPH)_{NP} \leq 160{,}000$

$(SP)_{SP} \leq 160{,}000$

$\sum(NPK)^i_{GR} \leq 544{,}000$

$0.1(NPH)^1_{GR} + 0.35(AN)^1_{GR} \leq 55 \times 2384 \leq 131{,}120$

$0.0661(NPH)^1_{GR} + 0.114(SP)^1_{GR} \leq 13.08 \times 2384 \leq 31{,}183$

$(K)^1_{GR} \leq 22{,}700$

$0.1(NPH)^2_{GR} + 0.35(AN)^2_{GR} \leq 18{,}030$

$0.0661(NPH)^2_{GR} + 0.114(SP)^2_{GR} \leq 10{,}481$

$(K)^2_{GR} \leq 3{,}815$

$0.1(NPH)^3_{GR} + 0.35(AN)^3_{GR2} \leq 6{,}825$

$0.0661(NPH)^2_{GR} + 0.114(SP)^2_{GR} \leq 4{,}168$

$(K)^3_{GR} \leq 1{,}444$

$0.1(\mathrm{NPH})^4_{GR} + 0.35(\mathrm{AN})^4_{GR} \leq 595$

$0.0661(\mathrm{NPH})^4_{GR} + 0.114(\mathrm{SP})^4_{GR} \leq 927$

$(K)^4_{GR} \leq 1{,}047$

$0.1(\mathrm{NPH})^5_{GR} + 0.35(\mathrm{AN})^5_{GR} \leq 28{,}244$

$0.0661(\mathrm{NPH})^{5}_{GR} + 0.114(\mathrm{SP})^{5}_{GR} \leq 7{,}629$

$(K)^{5}_{GR} \leq 64{,}797$

$0.1(NPH)^{6}_{GR} + 0.35(AN)^{6}_{GR} \leq 1{,}920$

$0.0661(NPH)^{6}_{GR} + 0.114(SP)^{6}_{GR} \leq 3{,}292$

$(K)^{6}_{GR} \leq 1.912 \text{ x } 128 \text{ x } 19.92 \leq 4{,}875$

$0.1(NPH)^{7}_{GR} + 0.35(AN)^{7}_{GR} \leq 14{,}050$

$0.0661(\mathrm{NPH})^{7}_{GR} + 0.114(\mathrm{SP})^{7}_{GR} \leq 5{,}390$

$(K)^{7}_{GR} < 3{,}122$

$0.1(\mathrm{NPH})^{9}_{GR} + 0.35(\mathrm{AN})^{9}_{GR} \leq 252$

$0.0661(\mathrm{NPH})^{9}_{GR} + 0.114(\mathrm{SP})^{9}_{GR} \leq 67$

$(K)^{9}_{GR} < 33$

$0.1(\mathrm{NPH})^{10}_{GR} + 0.35(\mathrm{AN})^{10}_{GR} \leq 2{,}890$

$0.0661(\mathrm{NPH})^{10}_{GR} + 0.114(\mathrm{SP})^{10}_{GR} \leq 1{,}178$

$(K)^{10}_{GR} \leq 3{,}327$

$0.1(NPH)^{11}_{GR} + 0.35(AN)^{11}_{GR} \leq 6{,}630$

$0.0661(NPH)^{11}_{GR} + 0.114(SP)^{11}_{GR} \leq 2{,}721$

$(\mathrm{K})^{11}_{GR} < 3{,}884$

$0.1(NPH)^{13}_{GR} + 0.35(AN)^{13}_{GR} \leq 2{,}880$

$0.0661(NPH)^{13}_{GR} + 0.114(SP)^{13}_{GR} \leq 398$

$(K)^{13}_{GR} \leq 6{,}094$

$0.1(\mathrm{NPH})^{14}_{GR} + 0.35(\mathrm{AN})^{14}_{GR} \leq 850$

$0.0661(\mathrm{NPH})^{14}_{GR} + 0.114(\mathrm{SP})^{14}_{GR} \leq 267$

$(K)^{14}_{GR} \leq 647$

$(NPK)^{1}_{S1} \leq 155{,}828$

$(NPK)^{2}_{S1} \leq 31{,}970$

$(NPK)^{3}_{S1} \leq 12{,}459$

$(NPK)^{4}_{S1} \leq 2{,}438$

$(NPK)^{5}_{S1} \leq 62{,}260$

$(NPK)^{6}_{S1} \leq \ \leq 9{,}012$

$(NPK)^{7}_{S1} \leq 31{,}975$

$(NPK)^{8}_{S1} \leq 1{,}094$

$(NPK)^{9}_{S1} \leq 31{,}977$

$(NPK)^{10}_{S1} \leq 9{,}779$

$(NPK)^{11}_{S1} \leq 5{,}160$

$(NPK)^{1}_{S2} \leq 83{,}823$

$(NPK)^{2}_{S2} \leq 17{,}198$

$(NPK)^{3}_{S2} \leq 12{,}459$

$(NPK)^{4}_{S2} \leq 1{,}309$

$(NPK)^{5}_{S2}) \leq 33{,}496$

$(NPK)^{6}_{S2} \leq 4{,}847$

$(NPK)^{7}_{S2} \leq 34{,}763$

$(NPK)^{8}_{S2} \leq 586$

$(NPK)^{9}_{S2} \leq 317$

$(NPK)^{10}_{S2} \leq 5{,}275$

$(NPK)^{11}_{S2} \leq 2{,}784$

$(NPK)^{1}_{S3} \leq 83{,}823$

$(NPK)^{2}_{S3} \leq 17{,}198$

$(NPK)^{3}_{S3} \leq 12{,}459$

$(NPK)^{4}_{S3} \leq 1{,}309$

$(NPK)^{5}_{S3} \leq 33{,}496$

$(NPK)^{6}_{S3} \leq 4{,}847$

$(NPK)^{7}_{S3} \leq 34{,}763$

$(NPK)^{8}_{S3} \leq 586$

$(NPK)^{9}_{S3} \leq 317$

$(NPK)^{10}_{S3} \leq 5{,}275$

$(NPK)^{11}_{S3} \leq 2{,}784$

$(NPK)^{1}_{S4} \leq 67{,}856$

$(NPK)^{2}_{S4} \leq 13{,}922$

$(NPK)^{3}_{S4} \leq 5{,}425$

$(NPK)^{4}_{S4}) \leq 1{,}060$

$(NPK)^{5}_{S4} \leq 27{,}108$

$(NPK)^{6}_{S4}) \leq 3{,}923$

$(NPK)^{7}_{S4} \leq 8{,}892$

$(NPK)^{8}_{S4}) \leq 595$

$(NPK)^{9}_{S4} \leq 323$

$(NPK)^{10}_{S4} \leq 5{,}345$

$(NPK)^{11}_{S4} \leq 2{,}826$

$(NPK)^{12}_{S4} \leq 4{,}376$

$(NPK)^{13}_{S4} \leq 12{,}674$

$(NPK)^{14}_{S4} \leq 860$

$(NPK)^{15}_{S4} \leq 20{,}900$

$(NPK)^{1}_{S5}) \leq 83{,}823$

$(NPK)^{2}_{S5} \leq 17{,}198$

$(NPK)^{3}_{S5}) \leq 12{,}459$

$(NPK)^{4}_{S5} \leq 1{,}309$

$(NPK)^{5}_{S5}) \leq 33{,}496$

$(NPK)^{6}_{S5} \leq 4{,}847$

$(NPK)^{7}_{S5} \leq 34{,}763$

$(NPK)^{8}_{S5} \leq 586$

$(NPK)^{9}_{S5} \leq 317$

$(NPK)^{10}_{S5} \leq 5{,}275$

$(NPK)^{11}_{S5} \leq 2{,}784$

$(NPK)^{1}_{S6} \leq 83{,}823$

$(NPK)^{2}_{S6} \leq 17{,}198$

$(NPK)^{3}_{S6} \leq 12{,}459$

$(NPK)^{4}_{S6} \leq 1{,}309$

$(NPK)^{5}_{S6} \leq 33{,}496$

$(NPK)^{6}_{S6} \leq 4{,}847$

$(NPK)^{7}_{S6} \leq 34{,}763$

$(NPK)^{8}_{S6} \leq 586$

$(NPK)^{9}_{S6} \leq 317$

$(NPK)^{10}_{S6} \leq 5{,}275$

$(NPK)^{11}_{S6} \leq 2{,}784$

$(NPK)^{1}_{S7} \leq 83{,}823$

$(NPK)^{2}_{S7} \leq 17{,}198$

$(NPK)^{3}_{S7} \leq 12{,}459$

$(NPK)^{4}_{S7} \leq 1{,}309$

$(NPK)^{5}_{S7} \leq 33{,}496$

$(NPK)^{6}_{S7} \leq 4{,}847$

$(NPK)^{7}_{S7} \leq 34{,}763$

$(NPK)^{8}_{S7} \leq 586$

$(NPK)^{9}_{S7} \leq 317$

$(NPK)^{10}_{S7} \leq 5{,}275$

$(NPK)^{11}_{S7} \leq 2{,}784$

$(NPK)^{1}_{S8} \leq 67{,}856$

$(NPK)^{2}_{S8} \leq 13{,}922$

$(NPK)^{3}_{S8} \leq 5{,}425$

$(NPK)^{4}_{S8} \leq 1{,}060$

$(NPK)^{5}_{S8} \leq 27{,}108$

$(NPK)^{6}_{S8} \leq 3{,}923$

$(NPK)^{7}_{S8} \leq 8{,}892$

$(NPK)^{9}_{S8} \leq 323$

$(NPK)^{10}_{S8} \leq 5{,}345$

$(NPK)^{11}_{S8} \leq 2{,}826$

$(NPK)^{13}_{S8} \leq 12{,}674$

$(NPK)^{14}_{S8} \leq 860$

Inequality Constraints ($\geq$ 0)

$(A) \geq 0$

$(A)_{AN} \geq 0$

$(A)_{NA} \geq 0$

$(A)_{NP} \geq 0$

$(S) \geq 0$

$(S)_{NP} \geq 0$

$(RP) \geq 0$

$(RP)_{NP} \geq 0$

$(O)_{NA} \geq 0$

$(W)_{NA} \geq 0$

$(NA)_{NA} \geq 0$

$(NA)_{AN} \geq 0$

$(HF)_{NP} \geq 0$

$(Ca)_{NP} \geq 0$

$(W)_{NP} \geq 0$

$(NA)_{NP} \geq 0$

$(NA)_{CN} \geq 0$

$(AN)_{AN} \geq 0$

$(L)_{CN} \geq 0$

$(NPH)_{NP} \geq 0$

$(S)_{SP} \geq 0$

$(SP)_{SP} \geq 0$

$(RP)_{SP} \geq 0$

$(W)_{SP} \geq 0$

$(W)_{CN} \geq 0$

$(CO)_{CN} \geq 0$

$(CN)_{CN} \geq 0$

$(NPH)_{GR} \geq 0$

$(AN)_{GR} \geq 0$

$(CN)_{ANCN} \geq 0$

$(AN)_{ANCN} \geq 0$

$(ANCN)_{ANCN} \geq 0$

$(NPH)_{FBG} \geq 0$

$(CN)_{FBG} \geq 0$

$(SP)_{GR} \geq 0$

$(K)_{GR} \geq 0$

$(NPH)^{i}_{GR} \geq 0$

$(AN)^{i}_{GR} \geq 0$

$(SP)^{i}_{GR} \geq 0$

$(K)^{i}_{GR} \geq 0$

$(NPK)^{i}_{GR} \geq 0$, $i = 1, 15$

$(FBG)_{DIS} \geq 0$

$(NPK)^{i}_{Sn} \geq 0$, $i = 1, 15$; $n = 1, 8$

$(FBG)_{Sn} > 0$, $n = 1.8$

6.2.9 Solution of ε-Constraint MILP program

The solution of this large MILP (Mixed Integer Linear Program), 243 program variables and 430 constraint equations required the use of an extended version of Microsoft Excel Solver ver. 2010, i.e. Premium Solver, the results from which are displayed in Table 6-7

Table 6-7: *Solution to NPK fertiliser optimisation program under prevailing conditions of uncertainty*

Pareto Optimal Solution

- Production and Distribution of NPK Fertiliser -

(Maximise total NPK Fertiliser Production)

⇒ Optimum overall production volume = 540,294 t/yr

⇒ Corresponding (ε-constraint) effluent discharge rate = 41,172 t/yr

RM Production		Granulation		Distribution			
(A)	103,882	$(NPK)^{1}_{GR}$	414,920	$(NPK)^{1}_{S1}$	-	$(NPK)^{1}_{S5}$	83,823
$(A)_{AN}$	23,514	$(NPK)^{2}_{GR}$	39,431	$(NPK)^{2}_{S1}$	-	$(NPK)^{2}_{S5}$	17,198
$(A)_{NA}$	61,056	$(NPK)^{3}_{GR}$	1,444	$(NPK)^{3}_{S1}$	6,630	$(NPK)^{3}_{S5}$	12,459
$(A)_{NP}$	19,312	$(NPK)^{4}_{GR}$	1,047	$(NPK)^{4}_{S1}$	2,438	$(NPK)^{4}_{S5}$	1,309
(S)	41,038	$(NPK)^{5}_{GR}$	64,797	$(NPK)^{5}_{S1}$	-	$(NPK)^{5}_{S5}$	33,496
$(S)_{NP}$	39,026	$(NPK)^{6}_{GR}$	4,875	$(NPK)^{6}_{S1}$	-	$(NPK)^{6}_{S5}$	4,847
(W)	136,674	$(NPK)^{7}_{GR}$	3,122	$(NPK)^{7}_{S1}$	2,880	$(NPK)^{7}_{S5}$	34,763
(RP)	120,913	$(NPK)^{9}_{GR}$	33	$(NPK)^{9}_{S1}$	398	$(NPK)^{9}_{S5}$	317
$(RP)_{NP}$	57,307	$(NPK)^{10}_{GR}$	-	$(NPK)^{10}_{S1}$	-	$(NPK)^{10}_{S5}$	5,275
$(O)_{NA}$	262,656	$(NPK)^{11}_{GR}$	3,884	$(NPK)^{11}_{S1}$	-	$(NPK)^{11}_{S5}$	2,784
$(W)_{NA}$	60,288	$(NPK)^{13}_{GR}$	6,094	$(NPK)^{1}_{S2}$	850	$(NPK)^{1}_{S6}$	83,823
$(NA)_{NA}$	384,000	$(NPK)^{14}_{GR}$	647	$(NPK)^{2}_{S2}$	259	$(NPK)^{2}_{S6}$	-

$(NA)_{AN}$	75,067	$(NPK)^{3}_{S2}$	-	$(NPK)^{3}_{S6}$	12,459
$(W)_{AN}$	11,187	$(NPK)^{4}_{S2}$	-	$(NPK)^{4}_{S6}$	1,281
$(HF)_{NP}$	2,292	$(NPK)^{5}_{S2}$	-	$(NPK)^{5}_{S6}$	-
$(Ca)_{NP}$	103,438	$(NPK)^{6}_{S2}$	-	$(NPK)^{6}_{S6}$	-
$(W)_{NP}$	28,653	$(NPK)^{7}_{S2}$	-	$(NPK)^{7}_{S6}$	-
$(NA)_{NP}$	121,433	$(NPK)^{9}_{S2}$	-	$(NPK)^{9}_{S6}$	-
$(NA)_{CN}$	187,500	$(NPK)^{10}_{S2}$	-	$(NPK)^{10}_{S6}$	-
$(AN)_{AN}$	87,395	$(NPK)^{11}_{S2}$	-	$(NPK)^{11}_{S6}$	-
$(L)_{CN}$	11,220	$(NPK)^{1}_{S3}$	-	$(NPK)^{1}_{S7}$	-
$(NPH)_{NP}$	160,000	$(NPK)^{2}_{S3}$	-	$(NPK)^{2}_{S7}$	-
$(S)_{SP}$	2,013	$(NPK)^{3}_{S3}$	-	$(NPK)^{3}_{S7}$	-
$(SP)_{SP}$	154,818	$(NPK)^{4}_{S3}$	1,309	$(NPK)^{4}_{S7}$	-
$(RP)_{SP}$	63,607	$(NPK)^{5}_{S3}$	33,496	$(NPK)^{5}_{S7}$	-
$(W)_{SP}$	31,893	$(NPK)^{6}_{S3}$	4,847	$(NPK)^{6}_{S7}$	-
$(W)_{CN}$	15,840	$(NPK)^{7}_{S3}$	34,763	$(NPK)^{7}_{S7}$	-
$(CD)_{CN}$	38,880	$(NPK)^{9}_{S3}$	317	$(NPK)^{9}_{S7}$	-
$(CN)_{CN}$	144,000	$(NPK)^{10}_{S3}$	5,275	$(NPK)^{10}_{S7}$	-
$(NPH)_{GR}$	266,171	$(NPK)^{11}_{S3}$	2,784	$(NPK)^{11}_{S7}$	-
$(AN)_{GR}$	15,395	$(NPK)^{1}_{S4}$	67,856	$(NPK)^{1}_{S8}$	-
$(CN)_{ANCN}$	120,000	$(NPK)^{2}_{S4}$	13,922	$(NPK)^{2}_{S8}$	-
$(AN)_{ANCN}$	72,000	$(NPK)^{3}_{S4}$	5,425	$(NPK)^{3}_{S8}$	-
$(ANCN)_{ANCN}$	192,000	$(NPK)^{4}_{S4}$	1,060	$(NPK)^{4}_{S8}$	-
$(NPH)_{FBG}$	24,000	$(NPK)^{5}_{S4}$	27,108	$(NPK)^{5}_{S8}$	-
$(CN)_{FBG}$	24,000	$(NPK)^{6}_{S4}$	3,923	$(NPK)^{6}_{S8}$	-

$(SP)_{GR}$	154,818			$(NPK)^{7}_{S4}$	8,892	$(NPK)^{7}_{S8}$	-
(K_{GR})	103,910			$(NPK)^{9}_{S4}$	323	$(NPK)^{9}_{S8}$	-
				$(NPK)^{10}_{S4}$	5,345	$(NPK)^{10}_{S8}$	-
				$(NPK)^{11}_{S4}$	2,826	$(NPK)^{11}_{S8}$	-
				$(NPK)^{13}_{S4}$	12,674	$(NPK)^{13}_{S3}$	-
				$(NPK)^{14}_{S4}$	860	$(NPK)^{14}_{S8}$	-
			540,294		246,460		293,834

6.2.10 Discussion of Results

The granular NPK fertiliser production and distribution operations case study represented an ideal platform for demonstrating the method and capability of the derived, 'Optimisation of a Supply Chain under Prevailing Combinational Conditions of Uncertainty' methodology. The chief reasons for this are that, firstly, all key aspects of an overall supply chain infrastructure, i.e. production, distribution and sales, were involved and that secondly, all prevailing (one) forms of operational uncertainty were accommodated, i.e. fuzziness in terms of NPK demand imprecision and expressed in terms of NPK granulation requirements and NPK distribution limits. It must also be borne in mind that the planning aspect of multi-objectivity was also successfully realised in this case study in that the production and distribution of a range of NPK fertiliser was maximised, i.e. $\max\sum(NPK)^{i}$, whilst the production and discharge of hazardous effluent, i.e. $[(HF)_{NP} + (CD)_{CN}]$, was simultaneously minimised from the production process. The overall diversity and complexity of the problem was magnified by the existence of multiple (15) and similar (*NPK*) fertiliser product types.

As described in the solution procedure section of this problem example, linear program constraint modules were separately generated for the three key phases of the overall operation, i.e. production, granulation and distribution, and then these three were integrated together to create a comprehensive single constraint-based MILP for the entire overall operation. The creation of a program module for the production operation involved the sequential analysis and integration of all participating production units. Such an analysis included interface with the two NPK granulators but these were handled in a combined fashion on account of a single market dependence on the scale and variety of NPK fertilisers required by the marketplace. The program module, generated for the NPK fertiliser distribution and sales operations, was based on a detailed analysis of the (NPK) nutritional requirements, as well as the distribution, requirements of the various crop-types involved. Crop-type nutritional requirement data was obtained from an FSSA (Fertiliser Society of South Africa) report, FSSA'2005.

Once all the program constraints for the overall MILP were generated, it then became necessary to define the operational objectives for the overall operation to finalise the specification of the complete, overall MILP (objectives plus constraints). The principal operational objective was to maximise the production and distribution of NPK fertilisers in accordance with market demand but this was accompanied by an environmental objective to minimise the discharge of hazardous effluent, which in this case constituted Carbon Dioxide (CO_2) from the Calcium Nitrate (CN) unit and Hydrogen Fluoride (HF) from the Nitrophosphate (NP) unit. The incorporation of such consequent and ***planned*** Multi-Objective requirement involved an application of the Pareto ε-constraint method (Guillen and Grossmann (2009) and Liu and Papageorgiou

(2013)) for solution whereby the one objective, i.e. minimal effluent discharge, was assigned as the ε-constraint.

The solution of this large MILP (Mixed Integer Linear Program), 243 program variables and 430 constraint equations, required the use of an extended version of Microsoft Excel Solver, and the optimum results from which are displayed in Table 6-7.

The key point that emerges from Table 6-7 is that the optimum overall production and associated distribution rate for a range of NPK fertilisers, in accordance with NPK fertiliser market demand, as described in Table 6-4, is 540,294 t/yr and the concomitant minimum total effluent discharge rate is 41,172 t/yr. This optimum production and distribution rate needs to be compared with the optimum NPK fertiliser annual production rate, optimised in isolation, to get a sense of the degree of optimality involved. Such optimum production rate, determined in isolation, is in fact exactly the same at 540,294 t/yr, i.e. a 100% degree of optimality achieved. Similarly, the calculated effluent discharge rate of 41,172 t/yr needs to be compared with the single objective, optimal minimum effluent flow MILP possible i.e. 38,880 t/yr to get a sense of the degree of optimality achieved in that area, i.e. [41,172 – (41,172 – 38,880)]/41,172 = 94.4%.

Also included in Table 6-7 are the optimum production, granulation and distribution figures for the entire range of NPK based fertilisers for the various different crop types. The Granulation figures give the total optimum annual NPK fertiliser production figures for each crop type irrespective of distribution destination and from a Distribution perspective, optimum distribution figures relate to NPK fertiliser market requirements at those particular sites (S_i), Such

NPK fertiliser site demand figures were based on NPK unitary fertilisation requirement figures (kg/ hectare) and crop-type land areas as extracted from various identified tables in the FSSA 2005 report according to procedures described in Table 5-1.

The derived Planning and Optimisation of a Supply Chain under Uncertainty methodology was proven to be very effective in that 100% optimal effectiveness was achieved on NPK production and distribution throughput and that, further, 94.4% optimal efficiency was achieved on the overall effluent discharge rate.

6.3 Validation of Supply Chain Optimality

It now becomes necessary to confirm the optimality achieved with this application of the proposed, 'Planning and Optimisation Supply Chain, under uncertainty, Optimisation', methodology thesis. This is understandable common practise with most recent and modern day 'Supply Chain, under uncertainty, Planning, Management or Optimisation' research initiatives.

6.3.1 Selection and application of supply chain optimality confirmation technique

An analysis of the research literature revealed that a number of different methods can be employed to assess or confirm global supply chain optimality. These are itemised in Table 7-8 below and described thereunder:

Table 6-8: *Assessing Supply Chain optimality*

	Author(s)	Research Work
a)	Guillen & Grossman (2010)	Global optimality is confirmed by the novel spatial branch & bound method
b)	Liu & Papageorgiou (2013)	Dual comparative solution: ε-constraint method confirmed by lexicographic minmax method
c)	Haimes & Chankong (1979)	ε-constraint method confirmed by KKK (Karush-Kuhn-Tucker) sensitivity analysis
d)	Baky (2010)	Dual comparative solution: ε-constraint method confirmed by lexicographic minmax method
e)	Mavrotas & Florios (2013)	Use Augmecon2 to generate MILP Pareto optimal solutions that can be compared with either ε-constraint and/or lexicographic minmax method and/or by KKK sensitivity analysis

f)	Razmi et al (2013)	Augmented ε-constraint method may be applied for confirmation purposes
h)	Durango-Cohen & Sarutipand (2009)	Quadratic programming MILP framework that captures the relationship between demand and facility deterioration

a) Global optimality is confirmed by the novel spatial branch & bound method, introduced by Guillen & Grossman (2010) that guarantees optimality by exploiting the mathematical structure of the MINLP model.

b) Dual comparative solution whereby the ε-constraint MINLP solution method is confirmed by the lexicographic minmax solution method - Liu & Papageorgiou (2013). Further to these research work of Liu & Papageorgiou (2013), it was shown that a key disadvantage of the ε-constraint method, i.e. the selection of the critical ε-vector, could be overcome by application of the lexicographic minmax solution method, which could generate a fair solution by not showing preference to any objectives.

c) Haimes & Chankong (1979 demonstrated that MINLP optimality could be achieved by application of KKK sensitivity analysis. This, clearly, could be confirmed by application of something like the ε-constraint method, and vice versa.

d) Further to the dual comparative research work of Baky (2010), it was also shown that the ε-constraint method could be confirmed by the lexicographic minmax method

e) The Augmecon2 MILP solution technique, developed by Mavrotas & Florios (2013), was shown to generate MILP Pareto optimal solutions and that could be compared with either ε-constraint and/or lexicographic minmax method and/or by KKK sensitivity analysis

f) Razmi et al (2013) demonstrated that their augmented ε-constraint method, that involves a bi-objective stochastic optimisation solution formulation, may also be applied for MINLP solution optimality confirmation purposes.

g) The quadratic programming MILP framework, developed by Durango-Cohen & Sarutipand (2009) and that captures the relationship between demand and facility deterioration, could also be applied to confirm optimality

It therefore appears, from the above, that the lexicographic minmax MILP optimisation technique would be ideally suited to confirm MILP optimality, as determined by the ε-constraint method. However, further research into both the research works of Liu & Papageorgiou (2013) and Baky (2010) revealed that the Lexicographic minmax multi-objective optimisation technique is, in fact, a ML-MLOP mechanism, i.e. a multi-level, multi-objective MILP optimisation technique. This means that it can manage/optimise multiple performance objectives across multiple organisational, or hierarchical, levels as well as handling multiple performance objectives within each such level. Research also revealed that it is a fairly involved mechanism. Therefore, since this current thesis is only concerned with multiple objectives at one single management or one single organisational level, it would be far preferable, simpler and effective to identify such single level MILP optimisation technique.

Further research revealed that some original (single-level) multi-objective MILP optimisation research work had originally and successfully been done by Zimmerman (1978). This work had been refined by Bit et al. (1992) to create a multi-objective MILP optimisation technique that could provide for a non-dominated optimum solution, and which could then be regarded as an '*Optimum Compromise'* solution.

6.3.2 Application of selected optimality confirmation technique

The $\in$-constraint MILP that programmatically described the original agri-fertiliser production and distribution operations in Table 6-6 has been modified a trifle, in Table 6-9, to reflect the fact that the $\in$-constraint has now been replaced by an equivalent objective function, i.e. $(HF)_{NP} + (CD)_{CN} \geq 38{,}880$.

Table 6-9: *Multi-Objective MILP (Mixed Integer Linear Program) Model for the production and distribution operations of the SA Fertiliser Group*

SA Fertiliser Group – MILP (Mixed Integer Linear Program) Model

Production and Distribution Operations

OBJECTIVE FUNCTIONS

1. $\text{Max}(\sum(\text{NPK})^{i}_{GR}$
2. $(HF)_{NP} + (CD)_{CN} \geq 38{,}880$

CONSTRAINTS

Equality

1. $(A) + (S) + (O)_{NA} + (W) + (L)_{CN} + (RP) + (K)_{GR} = (NPK)_{GR} + (FBG)_{DIS} + (ANCN)_{ANCN}$
2. $(S) = (S)_{SP} + (S)_{NP}$
3. $(W) = (W)_{SP} + (W)_{NA} + (W)_{NP} + (W)_{CN}$
4. $(A) = (A)_{AN} + (A)_{NA} + (A)_{NP}$

5. $(RP) = (RP)_{NP} + (RP)_{SP}$
6. $(O)_{NA} + (W)_{NA} + (A)_{NA} = (NA)_{NA}$
7. $(A)_{NA} = 0.159(NA)_{NA}$
8. $(W)_{NA} = 0.157(NA)_{NA}$
9. $(NA)_{NA} = (NA)_{NPI} + (NA)_{CN} + (NA)_{AN}$
10. $(A)_{AN} + (NA)_{AN} = (AN)_{AN} + (W)_{AN}$
11. $0.7(A)_{AN} + 0.13(NA)_{AN} = 0.3(AN)_{AN}$
12. $(W)_{AN} = 0.128(AN)_{AN}$
13. $(AN)_{AN} = (AN)_{ANCN} + (AN)_{GR}$
14. $(L)_{CN} + (NA)_{CN} = (CN)_{CN} + (CD)_{CN} + (W)_{CN}$
15. $(CN)_{CN} = 0.768(NA)_{CN}$
16. $(L)_{CN} + (NA)_{CN} = (CN)_{CN} + (W)_{CN} + (CD)_{CN}$
17. $(CN)_{CN} = 0.768(NA)_{CN}$
18. $(W)_{CN} = 0.11(CN)_{CN}$
19. $(CD)_{CN} = 0.268(CN)_{CN}$
20. $(CN)_{CN} = (CN)_{FBG} + (CN)_{ANCN}$
21. $(AN)_{ANCN} + (CN)_{ANCN} = (ANCN)_{ANCN}$
22. $0.23(ANCN)_{ANCN} = 0.35(AN)_{ANCN} + 0.17(CN)_{ANCN}$
23. $(S)_{NP} + (W)_{NP} + (A)_{NP} + (RP)_{NP} + (NA)_{NP} = (NPH)_{NP} + (HF)_{NP} + (Ca)_{NP}$
24. $(NA)_{NP} = 2.119(RP)_{NP}$
25. $(A)_{NP} = 0.337(RP)_{NP}$
26. $(S)_{NP} = 0.681(RP)_{NP}$
27. $(W)_{NP} = 0.5(RP)_{NP}$
28. $(NPH)_{NP} = 2.792(RP)_{NP}$
29. $(HF)_{NP} = 0.038(RP)_{NP}$
30. $(NPH)_{NP} = (NPH)_{GR} + (NPH)_{FBG}$
31. $(NPH)_{NP} = (NPH)_{GR} + (NPH)_{FBG}$
32. $(NPH)_{FBG} + (CN)_{FBG} = (FBG)_{DIS}$
33. $(NPH)_{FBG} = (CN)_{FBG}$
34. $(S)_{SP} + (RP)_{SP} + (W)_{SP} = (SP)_{SP}$
35. $(SP)_{SP} = 2.434(RP)_{SP}$
36. $(S)_{SP} = 0.013(SP)_{SP}$
37. $(SP)_{SP} = (SP)_{GR}$
38. $(W)_{SP} = 0.206(SP)_{SP}$

39. $(AN)_{GR} = \sum(AN)^{i}_{GR}$
40. $(NPH)_{GR} = \sum(NPH)^{i}_{GR}$
41. $(K)_{GR} = \sum(K)^{i}_{GR}$
42. $(SP)_{GR} = \sum(SP)^{i}_{GR}$
43. $(NPH)^{1}_{GR} + (AN)^{1}_{GR} + (K)^{1}_{GR} + (SP)^{1}_{GR} = (NPK)^{1}_{GR}$
44. $(NPH)^{2}_{GR} + (AN)^{2}_{GR} + (K)^{2}_{GR} + (SP)^{2}_{GR} = (NPK)^{2}_{GR}$
45. $(NPH)^{3}_{GR} + (AN)^{3}_{GR} + (K)^{3}_{GR} + (SP)^{3}_{GR} = (NPK)^{3}_{GR}$
46. $(NPH)^{4}_{GR} + (AN)^{4}_{GR} + (K)^{4}_{GR} + (SP)^{4}_{GR} = (NPK)^{4}_{GR}$
47. $(NPH)^{5}_{GR} + (AN)^{5}_{GR} + (K)^{5}_{GR} + (SP)^{5}_{GR} = (NPK)^{5}_{GR}$
48. $(NPH)^{6}_{GR} + (AN)^{6}_{GR} + (K)^{6}_{GR} + (SP)^{6}_{GR} = (NPK)^{6}_{GR}$
49. $(NPH)^{7}_{GR} + (AN)^{7}_{GR} + (K)^{7}_{GRj} + (SP)^{7}_{GR} = (NPK)^{7}_{GR}$
50. $(NPH)^{9}_{GR} + (AN)^{9}_{GR} + (K)^{9}_{GR} + (SP)^{9}_{GRj} = (NPK)^{9}_{GR}$
51. $(NPH)^{11}_{GR} + (AN)^{11}_{GR} + (K)^{11}_{GR} + (SP)^{11}_{GR} = (NPK)^{11}_{GR}$
52. $(NPH)^{13}_{GR} + (AN)^{13}_{GR} + (K)^{13}_{GRj} + (SP)^{13}_{GR} = (NPK)^{13}_{GR}$
53. $(NPH)^{14}_{GR} + (AN)^{14}_{GR} + (K)^{14}_{GR} + (SP)^{14}_{GR} = (NPK)^{14}_{GR}$
54. $\sum(NPK)^{i}_{GR} = \sum_{n=1}^{8}\sum_{i=1}^{15}(NPK)^{i}_{Sn}$
55. $(FBG)_{DIS} = \sum_{n=1}^{8}(FBG)_{Sn}$
56. $(ANCN)_{ANCN} = 192{,}000$
57. $(FBG)_{DIS} = 48{,}000$
58. **Inequality Constraints ($\leq$)**
59. $(NA)_{NA} \leq 384{,}000$
60. $(AN)_{AN} \leq 224{,}000$
61. $(CN)_{CN} \leq 160{,}000$
62. $(NPH)_{NP} \leq 160{,}000$
63. $(SP)_{SP} \leq 160{,}000$
64. $\sum(NPK)^{i}_{GR} \leq 544{,}000$
65. $0.1(NPH)^{1}_{GR} + 0.35(AN)^{1}_{GR} \leq 55 \times 2384 \leq 131{,}120$
66. $0.0661(NPH)^{1}_{GR} + 0.114(SP)^{1}_{GR} \leq 13.08 \times 2384 \leq 31{,}183$
67. $(K)^{1}_{GR} \leq 22{,}700$
68. $0.1(NPH)^{2}_{GR} + 0.35(AN)^{2}_{GR} \leq 18{,}030$
69. $0.0661(NPH)^{2}_{GR} + 0.114(SP)^{2}_{GR} \leq 10{,}481$
70. $(K)^{2}_{GR} \leq 3{,}815$
71. $0.1(NPH)^{3}_{GR} + 0.35(AN)^{3}_{GR2} \leq 6{,}825$

72. $0.0661(NPH)^{2}_{GR} + 0.114(SP)^{2}_{GR} \leq 4{,}168$
73. $(K)^{3}_{GR} \leq 1{,}444$
74. $0.1(\mathrm{NPH})^{4}_{GR} + 0.35(\mathrm{AN})^{4}_{GR} \leq 595$
75. $0.0661(\mathrm{NPH})^{4}_{GR} + 0.114(\mathrm{SP})^{4}_{GR} \leq 927$
76. $(K)^{4}_{GR} \leq 1{,}047$
77. $0.1(\mathrm{NPH})^{5}_{GR} + 0.35(\mathrm{AN})^{5}_{GR} \leq 28{,}244$
78. $0.0661(\mathrm{NPH})^{5}_{GR} + 0.114(\mathrm{SP})^{5}_{GR} \leq 7{,}629$
79. $(K)^{5}_{GR} \leq 64{,}797$
80. $0.1(NPH)^{6}_{GR} + 0.35(AN)^{6}_{GR} \leq 1{,}920$
81. $0.0661(NPH)^{6}_{GR} + 0.114(SP)^{6}_{GR} \leq 3{,}292$
82. $(K)^{6}_{GR} \leq 1.912 \times 128 \times 19.92 \leq 4{,}875$
83. $0.1(NPH)^{7}_{GR} + 0.35(AN)^{7}_{GR} \leq 14{,}050$
84. $0.0661(\mathrm{NPH})^{7}_{GR} + 0.114(\mathrm{SP})^{7}_{GR} \leq 5{,}390$
85. $(K)^{7}_{GR} < 3{,}122$
86. $0.1(\mathrm{NPH})^{9}_{GR} + 0.35(\mathrm{AN})^{9}_{GR} \leq 252$
87. $0.0661(\mathrm{NPH})^{9}_{GR} + 0.114(\mathrm{SP})^{9}_{GR} \leq 67$
88. $(K)^{9}_{GR} < 33$
89. $0.1(\mathrm{NPH})^{10}_{GR} + 0.35(\mathrm{AN})^{10}_{GR} \leq 2{,}890$
90. $0.0661(\mathrm{NPH})^{10}_{GR} + 0.114(\mathrm{SP})^{10}_{GR} \leq 1{,}178$
91. $(K)^{10}_{GR} \leq 3{,}327$
92. $0.1(NPH)^{11}_{GR} + 0.35(AN)^{11}_{GR} \leq 6{,}630$
93. $0.0661(NPH)^{11}_{GR} + 0.114(SP)^{11}_{GR} \leq 2{,}721$
94. $(\mathrm{K})^{11}_{GR} < 3{,}884$
95. $0.1(NPH)^{13}_{GR} + 0.35(AN)^{13}_{GR} \leq 2{,}880$
96. $0.0661(NPH)^{13}_{GR} + 0.114(SP)^{13}_{GR} \leq 398$
97. $(K)^{13}_{GR} \leq 6{,}094$
98. $0.1(\mathrm{NPH})^{14}_{GR} + 0.35(\mathrm{AN})^{14}_{GR} \leq 850$
99. $0.0661(\mathrm{NPH})^{14}_{GR} + 0.114(\mathrm{SP})^{14}_{GR} \leq 267$
100. $(K)^{14}_{GR} \leq 647$
101. $(NPK)^{1}_{S1} \leq 155{,}828$
102. $(NPK)^{2}_{S1} \leq 31{,}970$
103. $(NPK)^{3}_{S1} \leq 12{,}459$
104. $(NPK)^{4}_{S1} \leq 2{,}438$

105. $(NPK)^{5}_{S1} \leq 62{,}260$
106. $(NPK)^{6}_{S1} \leq \ \leq 9{,}012$
107. $(NPK)^{7}_{S1} \leq 31{,}975$
108. $(NPK)^{8}_{S1} \leq 1{,}094$
109. $(NPK)^{9}_{S1} \leq 31{,}977$
110. $(NPK)^{10}_{S1} \leq 9{,}779$
111. $(NPK)^{11}_{S1} \leq 5{,}160$
112. $(NPK)^{1}_{S2} \leq 83{,}823$
113. $(NPK)^{2}_{S2} \leq 17{,}198$
114. $(NPK)^{3}_{S2} \leq 12{,}459$
115. $(NPK)^{4}_{S2} \leq 1{,}309$
116. $(NPK)^{5}_{S2}) \leq 33{,}496$
117. $(NPK)^{6}_{S2} \leq 4{,}847$
118. $(NPK)^{7}_{S2} \leq 34{,}763$
119. $(NPK)^{8}_{S2} \leq 586$
120. $(NPK)^{9}_{S2} \leq 317$
121. $(NPK)^{10}_{S2} \leq 5{,}275$
122. $(NPK)^{11}_{S2} \leq 2{,}784$
123. $(NPK)^{1}_{S3} \leq 83{,}823$
124. $(NPK)^{2}_{S3} \leq 17{,}198$
125. $(NPK)^{3}_{S3} \leq 12{,}459$
126. $(NPK)^{4}_{S3} \leq 1{,}309$
127. $(NPK)^{5}_{S3} \leq 33{,}496$
128. $(NPK)^{6}_{S3} \leq 4{,}847$
129. $(NPK)^{7}_{S3} \leq 34{,}763$
130. $(NPK)^{8}_{S3} \leq 586$
131. $(NPK)^{9}_{S3} \leq 317$
132. $(NPK)^{10}_{S3} \leq 5{,}275$
133. $(NPK)^{11}_{S3} \leq 2{,}784$
134. $(NPK)^{1}_{S4} \leq 67{,}856$
135. $(NPK)^{2}_{S4} \leq 13{,}922$
136. $(NPK)^{3}_{S4} \leq 5{,}425$
137. $(NPK)^{4}_{S4}) \leq 1{,}060$
138. $(NPK)^{5}_{S4} \leq 27{,}108$

139. $(NPK)^{6}_{S4}) \leq 3{,}923$
140. $(NPK)^{7}_{S4} \leq 8{,}892$
141. $(NPK)^{8}_{S4}) \leq 595$
142. $(NPK)^{9}_{S4} \leq 323$
143. $(NPK)^{10}_{S4} \leq 5{,}345$
144. $(NPK)^{11}_{S4} \leq 2{,}826$
145. $(NPK)^{12}_{S4} \leq 4{,}376$
146. $(NPK)^{13}_{S4} \leq 12{,}674$
147. $(NPK)^{14}_{S4} \leq 860$
148. $(NPK)^{15}_{S4} \leq 20{,}900$
149. $(NPK)^{1}_{S5}) \leq 83{,}823$
150. $(NPK)^{2}_{S5} \leq 17{,}198$
151. $(NPK)^{3}_{S5}) \leq 12{,}459$
152. $(NPK)^{4}_{S5} \leq 1{,}309$
153. $(NPK)^{5}_{S5}) \leq 33{,}496$
154. $(NPK)^{6}_{S5} \leq 4{,}847$
155. $(NPK)^{7}_{S5} \leq 34{,}763$
156. $(NPK)^{8}_{S5} \leq 586$
157. $(NPK)^{9}_{S5} \leq 317$
158. $(NPK)^{10}_{S5} \leq 5{,}275$
159. $(NPK)^{11}_{S5} \leq 2{,}784$
160. $(NPK)^{1}_{S6} \leq 83{,}823$
161. $(NPK)^{2}_{S6} \leq 17{,}198$
162. $(NPK)^{3}_{S6} \leq 12{,}459$
163. $(NPK)^{4}_{S6} \leq 1{,}309$
164. $(NPK)^{5}_{S6} \leq 33{,}496$
165. $(NPK)^{6}_{S6} \leq 4{,}847$
166. $(NPK)^{7}_{S6} \leq 34{,}763$
167. $(NPK)^{8}_{S6} \leq 586$
168. $(NPK)^{9}_{S6} \leq 317$
169. $(NPK)^{10}_{S6} \leq 5{,}275$
170. $(NPK)^{11}_{S6} \leq 2{,}784$
171. $(NPK)^{1}_{S7} \leq 83{,}823$
172. $(NPK)^{2}_{S7} \leq 17{,}198$

173. $(NPK)^{3}_{S7} \leq 12{,}459$
174. $(NPK)^{4}_{S7} \leq 1{,}309$
175. $(NPK)^{5}_{S7} \leq 33{,}496$
176. $(NPK)^{6}_{S7} \leq 4{,}847$
177. $(NPK)^{7}_{S7} \leq 34{,}763$
178. $(NPK)^{8}_{S7} \leq 586$
179. $(NPK)^{9}_{S7} \leq 317$
180. $(NPK)^{10}_{S7} \leq 5{,}275$
181. $(NPK)^{11}_{S7} \leq 2{,}784$
182. $(NPK)^{1}_{S8} \leq 67{,}856$
183. $(NPK)^{2}_{S8} \leq 13{,}922$
184. $(NPK)^{3}_{S8} \leq 5{,}425$
185. $(NPK)^{4}_{S8} \leq 1{,}060$
186. $(NPK)^{5}_{S8} \leq 27{,}108$
187. $(NPK)^{6}_{S8} \leq 3{,}923$
188. $(NPK)^{7}_{S8} \leq 8{,}892$
189. $(NPK)^{9}_{S8} \leq 323$
190. $(NPK)^{10}_{S8} \leq 5{,}345$
191. $(NPK)^{11}_{S8} \leq 2{,}826$
192. $(NPK)^{13}_{S8} \leq 12{,}674$
193. $(NPK)^{14}_{S8} \leq 860$

194. Inequality Constraints (≥ 0)

195. $(A) \geq 0$
196. $(A)_{AN} \geq 0$
197. $(A)_{NA} \geq 0$
198. $(A)_{NP} \geq 0$
199. $(S) \geq 0$
200. $(S)_{NP} \geq 0$
201. $(RP) \geq 0$
202. $(RP)_{NP} \geq 0$
203. $(O)_{NA} \geq 0$
204. $(W)_{NA} \geq 0$
205. $(NA)_{NA} \geq 0$
206. $(NA)_{AN} \geq 0$
207. $(HF)_{NP} \geq 0$

208. $(Ca)_{NP} \geq 0$
209. $(W)_{NP} \geq 0$
210. $(NA)_{NP} \geq 0$
211. $(NA)_{CN} \geq 0$
212. $(AN)_{AN} \geq 0$
213. $(L)_{CN} \geq 0$
214. $(NPH)_{NP} \geq 0$
215. $(S)_{SP} \geq 0$
216. $(SP)_{SP} \geq 0$
217. $(RP)_{SP} \geq 0$
218. $(W)_{SP} \geq 0$
219. $(W)_{CN} \geq 0$
220. $(CO)_{CN} \geq 0$
221. $(CN)_{CN} \geq 0$
222. $(NPH)_{GR} \geq 0$
223. $(AN)_{GR} \geq 0$
224. $(CN)_{ANCN} \geq 0$
225. $(AN)_{ANCN} \geq 0$
226. $(ANCN)_{ANCN} \geq 0$
227. $(NPH)_{FBG} \geq 0$
228. $(CN)_{FBG} \geq 0$
229. $(SP)_{GR} \geq 0$
230. $(K)_{GR} \geq 0$
231. $(NPH)^{i}_{GR} \geq 0$
232. $(AN)^{i}_{GR} \geq 0$
233. $(SP)^{i}_{GR} \geq 0$
234. $(K)^{i}_{GR} \geq 0$
235. $(NPK)^{i}_{GR} \geq 0$, $i = 1, 15$
236. $(FBG)_{DIS} \geq 0$
237. $(NPK)^{i}_{Sn} \geq 0$, $i = 1, 15$; $n = 1, 8$
238. $(FBG)_{Sn} > 0$, $n = 1.8$

Further to the research work of Bit et al. (1992), the procedure for generating an optimal compromise solution for this multi-objective MILP is applied as in Table 6-10 below:

Table 6-10: *Optimum Compromise Solution methodology*

Step 1: The multi-objective problem is solved as a single objective problem, using each objective in turn as the single objective function whilst using all the others as criteria.

Therefore Max $\sum(NPK)_{DIS}$ = 540,294 t/yr
subject to constraints (1) – (238)

Similarly, Min[$(HF)_{NP}$ + $(CD)_{CN}$] = 38,880 t/yr
subject to constraints (1) – (238)

Step 2: From the results of step 1, the corresponding decision-variable values are determined for each optimum objective.

Effluent discharge rate corresponding to $\sum(NPK)_{DIS}^{Max}$ = 41,172 t/yr

Similarly, agri-fertiliser, (*NPK*) production and distribution rate corresponding to $[(HF)_{NP} + (CD)_{CN}]^{Min}$ = 55,776 t/yr

Step 3: From step 2, a lower and upper bound-value is determined for the set of solutions for each objective function, Z_k. The lower bound is designated L_k, whilst the upper bound is designated U_k.

	U_k	L_k
Max$(NPK)_{DIS}$	540,294	55,776
Min(Effluent)	38,880	41,172

Step 4: Formulation of required membership functions, u_k (x)

$$u_k(x) = \begin{cases} 1 & ;\text{if } Z_k \leq L_k \\ 1 - (Z_k - L_k)/(U_k - L_k) & ;\text{if } L_k < Z_k < U_k \\ 0 & ;\text{if } Z_k \geq U_k \end{cases}$$

Therefore:

Objective 1: Max[$\sum(NPK)_{DIS}$	Objective 2: Min[$(HF)_{NP} + (CD)_{CN}$]
$U_1(x) = 1 - (Z_1 - L_1)/(/(U_1 - L_1))$	$U_2(x) = 1 - (Z_2 - L_2)/(/(U_2 - L_2))$
$\therefore U_1(x) = 1 - (Z_1 - 55{,}776)/(540{,}294 - 55{,}776)$	$U_2(x) = 1 - (Z_2 - 41{,}172)/(38{,}880 - 41{,}172)$
$\therefore U_1(x) = 1 - (Z_1 - 55{,}776)/(484{,}518)$	$\therefore U_2(x) = 1 - (Z_2 - 41{,}172)/(-2{,}292)$
$\therefore U_1(x) = 0.0000021\ Z_1 + 0.115$	$\therefore U_2(x) = 1 - (-0.00044Z_2 + 17.963)$

Step 5: Maximise the membership functions, as per the same set of constraints, and, according to Bit et al. (1992), the highest membership function value corresponds to the optimum solution set, i.e.:

Max $[(u_1(x)] = 1.250$ and Max $[(u_2(x)] = 1.153$

Therefore Optimum $(u_i(x) = 1.250$ and the corresponding compromise optimal solution is set out in Table 6-11.

Table 6-11: *Compromise optimal solution set of NPK fertiliser production and distribution under uncertainty*

Compromise Optimal Solution Set
- Production and Distribution of NPK Fertiliser -
(Maximise total NPK Fertiliser Production)

⇒ Optimum overall production volume = 540,294 t/yr
⇒ Corresponding (ε-constraint) effluent discharge rate = 41,172 t/yr

RM Production		Granulation		Distribution			
(A)	103,882	$(NPK)^{1}_{GR}$	414,920	$(NPK)^{1}_{S1}$	-	$(NPK)^{1}_{S5}$	83,823
$(A)_{AN}$	23,514	$(NPK)^{2}_{GR}$	39,431	$(NPK)^{2}_{S1}$	-	$(NPK)^{2}_{S5}$	17,198
$(A)_{NA}$	61,056	$(NPK)^{3}_{GR}$	1,444	$(NPK)^{3}_{S1}$	6,630	$(NPK)^{3}_{S5}$	12,459
$(A)_{NP}$	19,312	$(NPK)^{4}_{GR}$	1,047	$(NPK)^{4}_{S1}$	2,438	$(NPK)^{4}_{S5}$	1,309
(S)	41,038	$(NPK)^{5}_{GR}$	64,797	$(NPK)^{5}_{S1}$	-	$(NPK)^{5}_{S5}$	33,496
$(S)_{NP}$	39,026	$(NPK)^{6}_{GR}$	4,875	$(NPK)^{6}_{S1}$	-	$(NPK)^{6}_{S5}$	4,847
(W)	136,674	$(NPK)^{7}_{GR}$	3,122	$(NPK)^{7}_{S1}$	2,880	$(NPK)^{7}_{S5}$	34,763
(RP)	120,913	$(NPK)^{9}_{GR}$	33	$(NPK)^{9}_{S1}$	398	$(NPK)^{9}_{S5}$	317
$(RP)_{NP}$	57,307	$(NPK)^{10}_{GR}$	-	$(NPK)^{10}_{S1}$	-	$(NPK)^{10}_{S5}$	5,275
$(O)_{NA}$	262,656	$(NPK)^{11}_{GR}$	3,884	$(NPK)^{11}_{S1}$	-	$(NPK)^{11}_{S5}$	2,784
$(W)_{NA}$	60,288	$(NPK)^{13}_{GR}$	6,094	$(NPK)^{1}_{S2}$	850	$(NPK)^{1}_{S6}$	83,823
$(NA)_{NA}$	384,000	$(NPK)^{14}_{GR}$	647	$(NPK)^{2}_{S2}$	259	$(NPK)^{2}_{S6}$	-
$(NA)_{AN}$	75,067			$(NPK)^{3}_{S2}$	-	$(NPK)^{3}_{S6}$	12,459
$(W)_{AN}$	11,187			$(NPK)^{4}_{S2}$	-	$(NPK)^{4}_{S6}$	1,281
$(HF)_{NP}$	2,292			$(NPK)^{5}_{S2}$	-	$(NPK)^{5}_{S6}$	-
$(Ca)_{NP}$	103,438			$(NPK)^{6}_{S2}$	-	$(NPK)^{6}_{S6}$	-
$(W)_{NP}$	28,653			$(NPK)^{7}_{S2}$	-	$(NPK)^{7}_{S6}$	-
$(NA)_{NP}$	121,433			$(NPK)^{9}_{S2}$	-	$(NPK)^{9}_{S6}$	-

$(NA)_{CN}$	187,500		$(NPK)^{10}_{S2}$	-	$(NPK)^{10}_{S6}$	-
$(AN)_{AN}$	87,395		$(NPK)^{11}_{S2}$	-	$(NPK)^{11}_{S6}$	-
$(L)_{CN}$	11,220		$(NPK)^{1}_{S3}$	-	$(NPK)^{1}_{S7}$	-
$(NPH)_{NP}$	160,000		$(NPK)^{2}_{S3}$	-	$(NPK)^{2}_{S7}$	-
$(S)_{SP}$	2,013		$(NPK)^{3}_{S3}$	-	$(NPK)^{3}_{S7}$	-
$(SP)_{SP}$	154,818		$(NPK)^{4}_{S3}$	1,309	$(NPK)^{4}_{S7}$	-
$(RP)_{SP}$	63,607		$(NPK)^{5}_{S3}$	33,496	$(NPK)^{5}_{S7}$	-
$(W)_{SP}$	31,893		$(NPK)^{6}_{S3}$	4,847	$(NPK)^{6}_{S7}$	-
$(W)_{CN}$	15,840		$(NPK)^{7}_{S3}$	34,763	$(NPK)^{7}_{S7}$	-
$(CD)_{CN}$	38,880		$(NPK)^{9}_{S3}$	317	$(NPK)^{9}_{S7}$	-
$(CN)_{CN}$	144,000		$(NPK)^{10}_{S3}$	5,275	$(NPK)^{10}_{S7}$	-
$(NPH)_{GR}$	266,171		$(NPK)^{11}_{S3}$	2,784	$(NPK)^{11}_{S7}$	-
$(AN)_{GR}$	15,395		$(NPK)^{1}_{S4}$	67,856	$(NPK)^{1}_{S8}$	-
$(CN)_{ANCN}$	120,000		$(NPK)^{2}_{S4}$	13,922	$(NPK)^{2}_{S8}$	-
$(AN)_{ANCN}$	72,000		$(NPK)^{3}_{S4}$	5,425	$(NPK)^{3}_{S8}$	-
$(ANCN)_{ANCN}$	192,000		$(NPK)^{4}_{S4}$	1,060	$(NPK)^{4}_{S8}$	-
$(NPH)_{FBG}$	24,000		$(NPK)^{5}_{S4}$	27,108	$(NPK)^{5}_{S8}$	-
$(CN)_{FBG}$	24,000		$(NPK)^{6}_{S4}$	3,923	$(NPK)^{6}_{S8}$	-
$(SP)_{GR}$	154,818		$(NPK)^{7}_{S4}$	8,892	$(NPK)^{7}_{S8}$	-
(K_{GR})	103,910		$(NPK)^{9}_{S4}$	323	$(NPK)^{9}_{S8}$	-
			$(NPK)^{10}_{S4}$	5,345	$(NPK)^{10}_{S8}$	-
			$(NPK)^{11}_{S4}$	2,826	$(NPK)^{11}_{S8}$	-
			$(NPK)^{13}_{S4}$	12,674	$(NPK)^{13}_{S3}$	-
			$(NPK)^{14}_{S4}$	860	$(NPK)^{14}_{S8}$	-

	540,294	246,460	293,834

6.3.3 Conclusion

The fact that the optimum solution set yielded by the 'Compromise Optimum Solution' methodology, Table 6-11, as put forward by Bit et al. (1992), is precisely the same as the optimum solution set yielded by the $\in$-constraint methodology, originally put forward by Haimes and Chankong, (1979) is valid confirmation that $\in$-constraint solution set is, indeed, optimal

6.4 Variations in Operational Planning and Uncertainty

Since the intention behind the proposed 'Planning and Optimisation of a Supply Chain Network under Uncertainty' methodology was to cater for any prevailing conditions of operational planning and uncertainty in a supply chain operating environment, the proposed methodology will now be tested against any possible variations in this regard.

The instance of planning and uncertainty considered in the case study of this research report was represented by *multi-objectivity* and *fuzziness*, where *multi-objectivity* was represented by the need to *i*) maximise production and distribution of the entire range of NPK fertiliser in accordance with market demand, and *ii*) simultaneously minimise the generation and discharge of hazardous gaseous effluent, i.e. hydrogen fluoride $(HF)_{NP}$ discharge from the Nitrophosphate unit and carbon dioxide discharge $(CD)_{CN}$ from the Calcium

Nitrate (CN) reactor. *Fuzziness* was reflected in the imprecision of certain operational variables describing the various production and distribution processes involved. Since conditions of operational *fuzziness* or imprecision are abundant in most operational supply chain projects, the decision was taken to consider only variations in *multi-objective* planning and/or *stochastic* uncertainty in this thesis, and then to assess whether the proposed methodology can withstand such variation. The manner in which this will be done will be to gradually introduce and/or remove variable planning and operational uncertainty from the base agri-fertiliser supply chain under uncertainty model, Table 6-6, in the manner put forward in Table 6-12. The above-mentioned methodology assessment will mainly take the form of checking whether a consequent set of optimum results falls within the realms of reality or not. Table 6-12 was constructed using a logical sequence of uncertainty events.

Table 6-12: *Schedule of introduction/removal of operational planning and uncertainty into/from case study model*

1. Case study model under multi-objective planning and fuzzy operational uncertainty

2. Introducing stochastic uncertainty by converting fuzzy demand uncertainty into discrete scenarios of probabilistic demand uncertainty

3. Introducing an additional element of multi-objective planning by means of an ε-constraint

4. Removing two of the, now, three instances of multi-objective planning as well as removing probabilistic uncertainty, and thereby leaving just a single objective requirement and fuzzy demand uncertainty.

The more detailed ramifications of effecting these changes in each of the supply chain planning and operational uncertainty scenarios, listed in Table 6-12, will be discussed below, if need be:

6.4.1 Introducing stochastic uncertainty by creating discrete demand scenarios of probabilistic uncertainty

The distribution of the various specific blends of *NPK* fertiliser for crop-type *i* at a particular site, S_n, i.e. $(NPK)^i_{Sn}$ is dependent on the crop-type land area, $(LA)^I$, the recommended fertilisation rate for that particular crop, $(NPK)^i$ and, of course, on the probability of demand for that particular fertiliser at that particular site, i.e. $(p)^i_{Sn}$, The relationship, according to para. 7.2.7.1.2, is, $(NPK)^i_{Sn} \leq (p)^i_{Sn}(LA)^i(NPK)^I$ t/ha.

For the purposes of the case study, it was assumed that $(p)^i_{Sn} = 1$ across all crop-types and across all sites, S_n. However, for the sake of this specific exercise, certain probabilistic uncertainty demand scenarios will be assumed for all crop-types at all sites, Table 6-13

Table 6-13: *Assumed probabilistic demand profiles - crops i and sites, S_n*

	Crop demand probability, *p* (%) at site, S_n							
Crop-Type, *i*	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**S_7**	**S_8**
Maize	80	95	60	75	115	110	90	85
Wheat	75	80	45	105	120	105	95	80
Sunflower	90	110	75	95	100\	90	110	97
Skybeans	85	90	55	60	80	75	95	95
Sugar Cane	100	110	80	70	105	95	115	110
Lucerne	110	115	95	105	120	115	100	95
Other Pastures	60	85	100	100	130	120	110	105
Tobacco	100	95	95	95	90	115	105	100
Cotton	80	75	90	95	85	90	110	95
Vegetables	110	95	95	100	95	100	105	100
Potatoes	85	85	95	110	100	105	100	95
Citrus	75	75	85	90	90	85	95	100
Subtropical	60	75	90	85	75	110	125	130
Fruits/Nuts	90	65	110	120	110	100	115	120

This new probability based demand distribution can now enable a recalculation of the new distribution constraints according to the NPK distribution relationship, $(NPK)^i_{Sn} \leq (p)^i_{Sn}(LA)^i(NPK)^I$ t/ha, which will be the same as multiplying the distribution limits in Table 6-6 by the respective probability factors in Table 6-14 and this is achieved in Table 6-15

Table 6-14: *Demand probability - amended distribution limits*

S1

$(NPK)^{1}_{S1} \leq 155{,}828(0.80) \leq 124{,}662$

$(NPK)^{2}_{S1} \leq 31{,}970(0.75) \leq 23{,}978$

$(NPK)^{3}_{S1} \leq 12{,}459(0.90) \leq 11{,}213$

$(NPK)^{4}_{S1} \leq 2{,}438(0.85) \leq 2{,}072$

$(NPK)^{5}_{S1} \leq 62{,}260(1.00) \leq 62{,}260$

$(NPK)^{6}_{S1} \leq 9{,}012(1.10) \leq 9{,}913$

$(NPK)^{7}_{S1} \leq 31{,}975(0.60) \leq 19{,}185$

$(NPK)^{8}_{S1} \leq 1{,}094(1.00) \leq 1{,}094$

$(NPK)^{9}_{S1} \leq 31{,}977(0.80) < 25{,}582$

$(NPK)^{10}_{S1} \leq 9{,}779(1.10) < 10{,}756$

$(NPK)^{11}_{S1} \leq 5{,}160(0.85) < 4{,}386$

S2

$(NPK)^{1}_{S2} \leq 83{,}823(0.95) \leq 79{,}632$

$(NPK)^{2}_{S2} \leq 17{,}198(0.80) \leq 13{,}758$

$(NPK)^{3}_{S2} \leq 12{,}459(1.10) < 13{,}705$

$(NPK)^{4}_{S2} \leq 1{,}309(0.90) \leq 1{,}178$

$(NPK)^{5}_{S2} \leq 33{,}496(0.80) \leq 26{,}797$

$(NPK)^{6}_{S2} \leq 4{,}847(1.15) \leq 5{,}574$

$(NPK)^{7}_{S2} \leq 34{,}763(0.85) \leq 29{,}549$

$(NPK)^{8}_{S2} \leq 586(0.95) \leq 557$

$(NPK)^{9}_{S2} \leq 317(0.75) \leq 238$

$(NPK)^{10}_{S2} \leq 5{,}275(0.95) \leq 5{,}011$

$(NPK)^{11}_{S2} \leq 2{,}784(0.85) \leq 2{,}366$

S3

$(NPK)^{1}_{S3} \leq 83{,}823(0.60) \leq 50{,}293$

$(NPK)^{2}_{S3} \leq 17{,}198(0.45) \leq 7{,}739$

$(NPK)^{3}_{S3} \leq 12{,}459(0.75) \leq 9{,}344$

$(NPK)^{4}_{S3} \leq 1{,}309(0.55) \leq 720$

$(NPK)^{5}_{S3} \leq 33{,}496(0.80) < 26{,}797$

$(NPK)^{6}_{S3} \leq 4{,}847(0.95) < 4{,}605$

$(NPK)^{7}_{S3} \leq 34{,}763(1.00) \leq 34{,}763$

$(NPK)^{8}_{S3} \leq 586(0.95) \leq 557$

$(NPK)^{9}_{S3} \leq 317(0.90) \leq 285$

$(NPK)^{10}_{S3} \leq 5{,}275(0.95) < 5{,}011$

$(NPK)^{11}_{S3} \leq 2{,}784(0.95) < 2{,}645$

S4

$(NPK)^{1}_{S4} \leq 67{,}8563(0.75) < 5{,}089$

$(NPK)^{2}_{S4} \leq 13{,}922(1.05) \leq 14{,}618$

$(NPK)^{3}_{S4} \leq 5{,}425(0.95) \leq 5{,}154$

$(NPK)^{4}_{S4} \leq 1{,}060(0.60) \leq 636$

$(NPK)^{5}_{S4} \leq 27{,}108(0.70) < 18{,}976$

$(NPK)^{6}_{S4} \leq 3{,}923(1.05) \leq 412$

$(NPK)^{7}_{S4} \leq 8{,}892(1.00) \leq 8{,}892$

$(NPK)^{8}_{S4} \leq 595(0.95) \leq 565$

$(NPK)^{9}_{S4} \leq 323(0.95) \leq 307$

$(NPK)^{10}_{S4} \leq 5{,}345(1.00) \leq 5{,}345$

$(NPK)^{11}_{S4} \leq 2{,}826(1.10) \leq 3{,}109$

$(NPK)^{12}_{S4} \leq 4{,}376(0.90) \leq 3{,}938$

$(NPK)^{13}_{S4} \leq 12{,}674(0.85) \leq 10{,}773$

$(NPK)^{14}_{S4} \leq 860(1.20) \leq 1{,}032$

$(NPK)^{15}_{S4} \leq 20{,}900(0.85) < 17{,}765$

S5

$(NPK)^{1}_{S5} \leq 83{,}823(1.15) \leq 96{,}396$

$(NPK)^{2}_{S5} \leq 17{,}198(1.20) \leq 20{,}638$

$(NPK)^{3}_{S5} \leq 12{,}459(1.00) \leq 12{,}549$

$(NPK)^{4}_{S5} \leq 1{,}309(0.80) \leq 1{,}047$

$(NPK)^{5}_{S5} \leq 33{,}496(1.05) \leq 35{,}170$

$(NPK)^{6}_{S5} \leq 4{,}847(1.20) \leq 5{,}816$

$(NPK)^{7}_{S5} \leq 34{,}763(1.30) \leq 45{,}192$

$(NPK)^{8}_{S5} \leq 586(0.90) \leq 527$

$(NPK)^{9}_{S5} \leq 317(0.85) \leq 269$

$(NPK)^{10}_{S5} \leq 5{,}275(0.95) < 5{,}011$

$(NPK)^{11}_{S5} \leq 2{,}784(1.00) \leq 2{,}784$

S6

$(NPK)^{1}_{S6} \leq 83{,}823(1.10) \leq 92{,}205$

$(NPK)^{2}_{S6} \leq 17{,}198(1.05) \leq 18{,}058$

$(NPK)^{3}_{S6} \leq 12{,}459(0.90) \leq 11{,}213$

$(NPK)^{4}_{S6} \leq 1{,}309(0.75) \leq 982$

$(NPK)^{5}_{S} \leq 33{,}496(0.95) \leq 31{,}821$

$(NPK)^{6}_{S6} \leq 4{,}847(1.15) \leq 5{,}574$

$(NPK)^{7}_{S6} \leq 34{,}763(1.20) < 41{,}716$

$(NPK)^{8}_{S6} \leq 586(1.15) \leq 674$

$(NPK)^{9}_{S6} \leq 317(0.90) \leq 285$

$(NPK)^{10}_{S6} \leq 5{,}275(1.00) \leq 5{,}275$

$(NPK)^{11}_{S6} \leq 2{,}784(1.05) < 2{,}923$

The effects of these changes are given and discussed in section 6.4.5

6.4.2 Maximising the production of ammonium nitrate $(AN)_{AN}$ whilst still retaining the probabilistic distribution of NPK fertiliser

An additional element of multi-objective planning will be achieved through the introduction of an additional $\in$-constraint in the agri-fertiliser production and distribution MILP. It was decided that such $\in$-constraint would take the form of maximising the production of ammonium nitrate, i.e. $(AN)_{AN}$, despite the fact that it is already a key raw material in the production of NPK fertiliser. Ammonium nitrate can either be used as an explosive or as a fertiliser, in its own right.

Such additional $\in$-constraint would take the form of $(AN)_{AN} \leq \text{Max}[(AN)_{AN}]$, where $\text{Max}[(AN)_{AN}]$, as a single objective = 224,000 t/yr. Therefore the new $\in$-constraint parameters in the Microsoft Excel Solver ver. 2010, i.e. Premium Solver program, where the primary objective function is Max $[\sum(NPK)_{\text{DIS}}]$ and the two $\in$-constraints are:

i) $[(HF)_{\text{NP}} + (CD)_{\text{CN}}] \geq \text{Min}[(\text{HF})_{\text{NP}} + (\text{CD})_{\text{CN}}] \geq 38{,}880$ t/yr
ii) $(AN)_{AN} \leq \text{Max}[(AN)_{AN}] \leq 224{,}000$ t/yr

Again, the effects of these changes are given and discussed in section 6.4.5.

6.4.3 Minimising the generation and discharge of hazardous effluent whilst retaining probabilistic demand distribution of NPK fertiliser

It will be an interesting exercise to minimise the generation and discharge of hazardous gaseous effluent, i.e. hydrogen fluoride $(HF)_{NP}$ from the Nitrophosphate unit and carbon dioxide $(CD)_{CN}$ from the CN (Calcium Nitrate) reactor, to see what impact that has on the primary production objective, i.e. the generation of multiple blends of NPK fertiliser in accordance with market demand. In this particular case, the objective function of the MILP will be $\text{Min}[(HF)_{NP} + (CD)_{CN}]$.

Again, the effects of these changes are given and discussed in section 6.4.5.

6.4.4 Maximising the production of Ammonium Nitrate whilst replacing the probabilistic demand of NPK fertiliser with the previous fuzzy demand of NPK fertiliser

It will also be interesting to see whether the replacement of probabilistic demand for (*NPK)* fertiliser, with the original case study requirement of fuzzy demand for such fertiliser, will have much impact on the optimum operating results.

Again, the effects of these changes are given and discussed in section 6.4.5.

6.4.5 Discussion of Variation Results

It was decided to display the results of the effects of the changing nature of all these supply chain operational planning and/or operational uncertainty variations in sequential tabular format, Table 6-15, so that the nature of these changing effects can be properly viewed and discussed.

Table 6-15: *The effects of changing operational planning and uncertainty on supply chain performance*

Categories / ε-constraints	Case Study - Max(*NPK*)	Max(*NPK*) + Demand Probability	Max(*AN*) + Demand Probability	Min(Eff) + Demand Probability	Max(AN)
$\sum(NPK)_{DIS}$	540,294	540,294	227,232	55,776	227,232
$(AN)_{AN}$			224,000	72,000	224,000
$(HF)_{NP} + (CD)_{CN}$	41,172	41,172	38,880	38,880	38,880
Variables					
(A)	103,882	103,882	120,673	59,018	120,673
$(A)_{AN}$	23,514	23,514	60,268	19,372	60,268
$(A)_{NA}$	61,056	61,056	60,405	39,646	60,405
$(A)_{NP}$	19,312	19,312	-	-	-
(S)	41,038	41,038	-	-	-
$(S)_{NP}$	39,026	39,026	-	-	-
(W)	136,674	136,674	75,485	54,987	75,485
(RP)	120,913	120,913	-	-	-
$(RP)_{NP}$	57,307	57,307	-	-	-

$(O)_{NA}$	262,656	262,656	259,854	170,551	259,854
$(W)_{NA}$	60,288	60,288	59,645	39,147	59,645
$(NA)_{NA}$	384,000	384,000	379,904	249,344	379,904
$(NA)_{AN}$	75,067	75,067	192,404	61,844	192,404
$(W)_{AN}$	11,187	11,187	28,672	9,216	28,672
$(HF)_{NP}$	2,292	2,292	-	-	-
$(Ca)_{NP}$	103,438	103,438	-	-	-
$(W)_{NP}$	28,653	28,653	-	-	-
$(NA)_{NP}$	121,433	121,433	-	-	-
$(NA)_{CN}$	187,500	187,500	187,500	187,500	187,500
$(AN)_{AN}$	87,395	87,395	224,000	72,000	224,000
$(L)_{CN}$	11,220	11,220	11,220	11,220	11,220
$(NPH)_{NP}$	160,000	160,000	-	-	-
$(S)_{SP}$	2,013	2,013	-	-	-
$(SP)_{SP}$	154,818	154,818	-	-	-
$(RP)_{SP}$	63,607	63,607	-	-	-
$(W)_{SP}$	31,893	31,893	-	-	-
$(W)_{CN}$	15,840	15,840	15,840	15,840	15,840
$(CD)_{CN}$	38,880	38,880	38,880	38,880	38,880
$(CN)_{CN}$	144,000	144,000	144,000	144,000	144,000
$(NPH)_{GR}$	266,171	266,171	75,232	55,776	75,232
$(AN)_{GR}$	15,395	15,395	152,000	-	152,000
$(CN)_{ANCN}$	120,000	120,000	120,000	120,000	120,000
$(AN)_{ANCN}$	72,000	72,000	72,000	72,000	72,000

$(ANCN)_{ANCN}$	192,000	192,000	192,000	192,000	192,000
$(NPH)_{FBG}$	24,000	24,000	24,000	24,000	24,000
$(CN)_{FBG}$	24,000	24,000	24,000	24,000	24,000
$(SP)_{GR}$	154,818	154,818	-	-	-
(K_{GR})	103,910	103,910	-	-	-
$(NPK)^{1}_{S1}$	-	-	-	-	-
$(NPK)^{2}_{S1}$	-	-	-	-	-
$(NPK)^{3}_{S1}$	-	-	-	-	-
$(NPK)^{4}_{S1}$	-	-	-	-	-
$(NPK)^{5}_{S1}$	-	-	-	-	-
$(NPK)^{6}_{S1}$	-	-	-	-	-
$(NPK)^{7}_{S1}$	-	-	-	-	-
$(NPK)^{9}_{S1}$	-	-	-	-	-
$(NPK)^{10}_{S1}$	-	-	-	-	-
$(NPK)^{11}_{S1}$	5,160	4,386	-	-	-
$(NPK)^{1}_{S2}$	-	-	-	-	-
$(NPK)^{2}_{S2}$	-	-	-	-	-
$(NPK)^{3}_{S2}$	-	-	-	-	-
$(NPK)^{4}_{S2}$	1,309	1,178	-	-	-
$(NPK)^{5}_{S2}$	33,496	26,297	-	-	-
$(NPK)^{6}_{S2}$	4,847	5,574	-	-	-
$(NPK)^{7}_{S2}$	34,763	29,549	-	-	-
$(NPK)^{9}_{S2}$	317	238	-	-	-
$(NPK)^{10}_{S2}$	5,275	5,011	-	-	-

$(NPK)^{11}_{S2}$	-	-	-	-	-
$(NPK)^{1}_{S3}$	83,823	50,293	-	-	-
$(NPK)^{2}_{S3}$	17,198	7,739	-	-	-
$(NPK)^{3}_{S3}$	12,459	9,344	-	-	-
$(NPK)^{4}_{S3}$	1,309	1,309	-	-	-
$(NPK)^{5}_{S3}$	33,496	26,797	-	-	-
$(NPK)^{6}_{S3}$	4,847	4,605	-	-	-
$(NPK)^{7}_{S3}$	34,763	34,763	-	-	-
$(NPK)^{9}_{S3}$	317	285	-	-	-
$(NPK)^{10}_{S3}$	5,275	5,275	-	-	-
$(NPK)^{11}_{S3}$	2,784	2,645	-	-	-
$(NPK)^{1}_{S4}$	67,856	50,892	-	-	-
$(NPK)^{2}_{S4}$	13,922	14,618	-	-	-
$(NPK)^{3}_{S4}$	5,425	5,154	-	-	-
$(NPK)^{4}_{S4}$	1,060	636	-	-	-
$(NPK)^{5}_{S4}$	27,108	18,976	-	-	-
$(NPK)^{6}_{S4}$	3,923	412	-	-	-
$(NPK)^{7}_{S4}$	8,892	8,892	-	-	-
$(NPK)^{9}_{S4}$	323	307	-	-	-
$(NPK)^{10}_{S4}$	5,345	5,345	-	-	-
$(NPK)^{11}_{S4}$	2,826	3,109	-	-	-
$(NPK)^{13}_{S4}$	12,674	10,773	-	-	-
$(NPK)^{14}_{S4}$	860	1,032	-	-	-
$(NPK)^{1}_{S5}$	83,823	96,396	-	-	-

$(NPK)^{2}_{S5}$	17,198	20,638	-	-	-
$(NPK)^{3}_{S5}$	7,621	12,459	-	-	-
$(NPK)^{4}_{S5}$	-	1,047	-	-	-
$(NPK)^{5}_{S5}$	-	35,170	-	-	-
$(NPK)^{6}_{S5}$	-	5,816	-	-	-
$(NPK)^{7}_{S5}$	-	33,334	-	-	-
$(NPK)^{9}_{S5}$	-	-	-	-	-
$(NPK)^{10}_{S5}$	-	-	-	-	-
$(NPK)^{11}_{S5}$	-	-	-	-	-
$(NPK)^{1}_{S6}$	-	-	-	-	-
$(NPK)^{2}_{S6}$	-	-	-	-	-
$(NPK)^{3}_{S6}$	-	-	-	-	-
$(NPK)^{4}_{S6}$	-	-	-	-	-
$(NPK)^{5}_{S6}$	-	-	27,581	-	27,581
$(NPK)^{6}_{S6}$	-	-	-	-	-
$(NPK)^{7}_{S6}$	-	-	-	-	-
$(NPK)^{9}_{S6}$	-	-	-	-	-
$(NPK)^{10}_{S6}$	-	-	-	-	-
$(NPK)^{11}_{S6}$	-	-	-	-	-
$(NPK)^{1}_{S7}$	-	-	-	55,776	-
$(NPK)^{2}_{S7}$	-	-	17,198	-	17,198
$(NPK)^{3}_{S7}$	-	-	12,459	-	12,459
$(NPK)^{4}_{S7}$	-	-	1,309	-	1,309
$(NPK)^{5}_{S7}$	-	-	33,496	-	33,496

$(NPK)^{6}_{S7}$	-	-	4,847	-	4,847
$(NPK)^{7}_{S7}$	-	-	34,763	-	34,763
$(NPK)^{9}_{S7}$	-	-	317	-	317
$(NPK)^{10}_{S7}$	-	-	5,275	-	5,275
$(NPK)^{11}_{S7}$	-	-	2,784	-	2,784
$(NPK)^{1}_{S8}$	-	-	67,856	-	67,856
$(NPK)^{2}_{S8}$	-	-	13,922	-	13,922
$(NPK)^{3}_{S8}$	-	-	5,425	-	5,425
$(NPK)^{4}_{S8}$	-	-	-	-	-
$(NPK)^{7}_{S8}$	-	-	-	-	-
$(NPK)^{9}_{S8}$	-	-	-	-	-
$(NPK)^{10}_{S8}$	-	-	-	-	-
$(NPK)^{11}_{S8}$	-	-	-	-	-
$(NPK)^{13}_{S3}$	-	-	-	-	-
$(NPK)^{14}_{S8}$	-	-	-	-	-
Total $(NPK)^{i}_{Sn}$	540,294	540,294	227,232	55,776	227,232
$(FBG)_{S1}$	-	-	48,000	-	48,000
$(FBG)_{S2}$	-	-	-	-	-
$(FBG)_{S3}$	-	-	-	-	-
$(FBG)_{S4}$	-	-	-	48,000	-
$(FBG)_{S5}$	-	-	-	-	-
$(FBG)_{S6}$	-	-	-	-	-
$(FBG)_{S7}$	-	-	-	-	-
$(FBG)_{S8}$	48,000	48,000	-	-	-

It is clear from a perusal of Table 6-15 that there is remarkable consistency between the various sets of optimum results presented, which correspond to the various combinations of operational planning and uncertainty discussed.

For example, considering the first two cases, i.e. the case of fuzzy demand of NPK fertiliser, $(NPK)_{Dis,}$ and the case of probabilistic demand for $(NPK)_{Dis,}$, they both yield identical production profiles and differ only in their distribution profiles since production magnitudes are consistent regardless of the subsequent distribution profile. Similarly, in the case where the production of ammonium nitrate, $(AN)_{AN}$, is maximised under fuzzy market demand conditions, identical profile results are obtained in the case where the probabilistic demand of $(NPK)_{Dis}$ fertiliser is considered. Lastly, there is also consistency between the two cases of i) maximising the production of $(AN)_{AN}$ and ii) minimising the generation and discharge of hazardous effluent, $(HF)_{NP}$ + $(CD)_{CN}$, in that, since the production of $(AN)_{AN}$ has absolutely nothing to do with the generation of effluent, the corresponding effluent figures are exactly the same at 38,880 t/yr.

7 REMARKS AND CONCLUSION

This research work has effectively demonstrated the derivation and an application of a generic 'Planning and Optimisation Methodology for a Supply Chain, under Uncertainty', in a supply chain environment.

The development of the methodology itself was very much based on the research and analysis of previously researched supply chain, under uncertainty, optimisation techniques. Such existing techniques were either single component in nature, occasionally binary and very occasionally ternary based, but never did they consider only those elements of operational uncertainty that were relevant to a particular supply chain environment. They also did not.provide for mechanisms for handling binary logic in the determination of optimum configurations for supply chain networks vis-à-vis market demand. Further, application of the ε-constraint method, in the case of multiple objectives, had only been occasionally utilised to generate Pareto optimal supply chain solutions Therefore, the intention of this thesis was to cater for the possible planning of supply chain networks that were also subject to any prevailing conditions of operational uncertainty, be they single-case or binary in nature, and by utilising the most effective and proven technology available.. Consequently, this would amount to the determination of a *'realistically planned'* optimum supply chain operational solution for such environments since all prevailing conditions of operational uncertainty would have been accommodated, and any multi-objective planning requirements would also have been accommodated.

The supply chain case study was very interesting in itself because not only did it exhibit one (fuzzy) prevailing, out of a possible two (fuzzy, stochastic), conditions of operational uncertainty but the operation was also characterised by the need of two performance objectives. Another point of interest was the specified relationships between the crop-type and the associated NPK requirement in terms of the stipulated individual N, P, K, concentrations. Such relationships made it possible to optimise both the production and raw material stream requirements. The effectiveness of this methodology was demonstrated by the nature of results achieved, i.e. 100% achievement of optimal NPK fertiliser production and distribution capability and further, a 94.4% achievement of optimum effluent discharge conditions.

The proposed methodology was validated by comparing the optimum operating results generated from the case study example with those generated from the same case study but by using an existing supply chain optimisation technique. The optimum solution suites were exactly the same for both.

The research questions were answered:

a) The uncertainty criteria that are specifically applicable to a supply chain operating environment are *i*) Fuzzy uncertainty, *ii*) Stochastic uncertainty
b) The typical nature of supply chain planning is expressed in terms of the number of required and simultaneous operating objectives, and also that any simultaneous required suite of objectives may be Pareto optimised, i.e. the best possible solution suite
c) Can both the supply chain planning requirement and the nature and extent of operational uncertainty in a supply chain environment be

accommodated in an overall methodology that would enable the determination of the best overall operating supply chain solution under those conditions?

Ans.: Yes, the proposed and proven, 'Planning and Optimisation of a Supply Chain under Uncertainty' methodology

Additionally, the Hypothesis was further validated by subjecting the proposed methodology to variations in planning and/or uncertainty, and then by obtaining reasonable results.

8 STRENGTHS, WEAKNESSES AND POSSIBLE FURTHER RESEARCH

The original rationale for the development of this new 'Planning and Optimisation of a Supply Chain under Uncertainty' methodology was because it became increasingly apparent, especially in the late 20th, and early 21st, century that more and more international research work was being done into the various aspects of 'Supply Chain, under uncertainty, Planning/Optimisation', i.e. the nature of supply chains, the nature of supply chain uncertainty and the meaning of supply chain planning/optimisation However it became clear that, since such previous international research works were neither coordinated nor conforming to a central or common theme, there was no binding rationale between these various aspects of 'Supply Chain, under uncertainty, Planning/Optimisation' research. Therefore it was decided to create such binding rationale by developing an overall supply chain planning/optimisation' under uncertainty methodology that could cater for any supply chain infrastructure and also cater for any (combinational) instances of uncertainty.

This was achieved and it now becomes necessary to discuss the strengths and weaknesses of the derived methodology against these constraints.

8.1 Strengths

1) Puts into logical and sequential perspective a lot of the previous work into supply, chain under uncertainty, planning/optimisation research.

2) The derived methodology can cater for any combined prevailing instances of operational planning and/or uncertainty.

3) For the first time ever, any and all aspects of supply chain planning and operation are catered for in one methodology.

8.2 Weaknesses

1) There is only one real key weakness in the derived methodology and that is that it caters only for the operational aspects of supply chain performance but not for any of the design aspects of supply chain operational performance. A key example of this is the design of the supply chain infrastructure, which includes the size and capacity of the production/manufacturing facility/ies, the number and capacities of warehouses, distribution centres and retail sales outlets and lastly, the orientation of the entire supply chain infrastructure, in terms of location and routes, towards market demand.

8.3 Future Possible Research

This section explores any possible areas of further supply chain, under uncertainty research and is primarily based on the observed weaknesses of the derived methodology discussed in 8.2.

It would be useful if the derived supply chain, under uncertainty, operational optimisation methodology could be expanded to include the operational implications of any supply chain infrastructure design change requirements. Such supply chain infrastructure design change requirements could typically include either quantitative and/or qualitative enhancements to the supply chain production or manufacturing facilit(y/ies), the number and capacities of warehouses, distribution centres and retail outlets and the orientation of the entire supply chain infrastructure towards market demand. This latter point could typically be qualified in terms of permissible routes and locations.

9 NOMENCLATURE

Table 9-1: *Nomenclature*

Category	Description
Indices/Sets	
Raw Materials	Ammonia (*A*), Sulphuric Acid (*S*), Rock phosphate (*RP*), Oxygen (*O*), Water (*W*), KCl (*K*), Limestone (*L*)
Intermediate Products	, e.g. Nitric Acid (*NA*), Nitrophos (*NP*), Superphosphates (*SP*), Ammonium Nitrate (*AN*), Calcium Nitrate (*CN*)
Final fertiliser products,	$(FBG)_i$, $(GR2)_j$, $(GR3)_k$ where *i, j, k* refer to the many various blends of NPK fertiliser exiting from the Granulators
S_n	Sites (n = 1,…, 8) e.g. S_1, S_2
Variable Parameters	*Description*
(A)	Flowrate of NH_3 to the Fertiliser Production complex (t/yr)
$(A)_{AN}$	Flowrate of NH_3 to Ammonium Nitrate (*AN*) plant (t/yr)
$(A)_{NA}$	Flowrate of NH_3 to Nitric Acid (*NA*) plant (t/yr)
$(A)_{NP}$	Flowrate of NH_3 to Nitrophos (*NP*) plant (t/yr)
(S)	Flowrate of 98%Sulphuric acid to fertiliser complex (t/yr)
$(S)_{NP}$	Flowrate of 98%Sulphuric acid to *NP* plant (t/yr)
$(RP)_{NP}$	Flowrate of Phosphate rock to *NP* plant (t/yr)
$(A)_{NA}$	Flowrate of Compressed air *to NA* plant (t/yr)
$(O)_{NA}$	Flowrate of Oxygen *to NA* plant (t/yr)
$(W)_{NA}$	Flowrate of Water to *NA* plant (t/yr)
$(NA)_{NA}$	Flowrate of 59% Nitric Acid from *NA* Plant (t/yr)
$(NA)_{AN}$	Flowrate of 59% Nitric Acid from *NA* plant to *AN* plant (t/yr)
$(NA)_{NP}$	Flowrate of 59% Nitric Acid from *NA* plant to *NP* plant (t/yr)
$(AN)_{AN}$	Flowrate of 89% Ammonium Nitrate from *AN* plant (t/yr)
$(NA)_{CN}$	Flow of 59% Nitric Acid from NA plant to Calcium Nitrate (*CN*) plant (t/yr)
$(L)_{CN}$	Flowrate of Limestone ($CaCO_3$) to *CN* plant (t/yr)
$(W)_{CN}$	Release of water from *CN* plant (t/yr)
$(CD)_{CN}$	Release of Carbon Dioxide from *CN* plant (t/yr)
$(NPH)_{NP}$	Flowrate of Nitrophosphate from *NP* plant (t/yr)
$(W)_{NP}$	Flowrate of Water to *NP* plant (t/yr)

$(HF)_{NP}$	Hydrogen Fluoride discharge from *NP* plant (t/yr)
$(Ca)_{NP}$	$Ca(NO_3)_2.4H_2O$ discharge from *NP* plant (t/yr)
$(NA)_{NP}$	Incoming 59%Nitric Acid to the *NP* plant (t/yr)
$(S)_{SP}$	Flowrate of Sulphuric Acid to Superphosphates plant (t/yr)
$(SP)_{SP}$	Flow of Superphosphate from Superphosphates plant (t/yr)
$(RP)_{SP}$	Flowrate of Phosphate Rock to Superphosphates plant (t/yr)
(RP)	Total Flowrate of Phosphate Rock (t/yr)
W_{SP}	Flowrate of Water to Superphosphates plant (t/yr)
$(CN)_{CN}$	Flowrate of 58%Calcium Nitrate from *CN* plant (t/yr)
$(CN)_{ANCN}$	Flowrate of Calcium Nitrate to *ANCN* (explosives) plant (t/yr)
$(AN)_{ANCN}$	Flowrate of 89% *AN* to *ANCN* (explosives) plant (t/yr)
$(ANCN)_{ANCN}$	Flowrate of *ANCN* from *ANCN* Plant (explosives) plant (t/yr)
$(NP)_{FBG}$	Flowrate of Nitrophos to Fluidised Bed Granulator (t/yr)
$(CN)_{FBG}$	Flowrate of 58% Calcium Nitrate to Fluidised Bed Granulator (t/yr)
$(NP)^i_{GR}$	Flowrate of Nitrophos for *NPK* blend *i* to Granulator (t/yr)
$(AN)^i_{GR}$	Flowrate of 89% *AN* for *NPK* blend *i* to Granulator (t/yr)
$(SP)^i_{GR}$	Flowrate of Superphosphate for NPK blend *i* to Granulator (t/yr)
K^i_{GR}	Flow of *KCl* for *NPK* blend *i* to Granulator (t/yr)
$(NPK)^i_{GR}$	Flowrate of *NPK* Fertiliser of blend *i* from Granulator (t/yr)
$(NPK)^i_{Sn}$	Flowrate of *NPK* fertiliser to Site *n* (t/yr) of NPK blend ***i***
$(LA)^i$	Land area involved for crop *i,* (ha)
$(p)^i_{Sn}$	Probability of complete demand for crop *i* at site, *n,* on account of environmental and/or economic conditions
$(FBG)_{Sn}$	Flowrate of fluidised bed granulated *CN* to Site *n* (t/yr)
$(FBG)_{DIS}$	Total flowrate of granulated *CN+NPH* from FBG to be distributed (t/yr)
Operational Constraints	
(LA)	Total Fertile Land Area involved = 4,241,000 ha
$(d)_{yr}$	Effective operational days pr annum (88%) = 320 (88% uptime)
$(NA)_{NA}$	Production flowrate of 59% Nitric Acid = 1200t/d
$(AN)_{AN}$	Flowrate of 89% Ammonium Nitrate from AN plant = 700t/d
$(ANCN)_{ANCN}$	Production capacity $\leq$ 280,000 t/yr
$(FBG)_{FBG}$	Production capacity of granulated CN from FBG = 150 t/day or $\leq$ 50,000 t/yr

$(SP)_{SP}$	Production capacity of Superphosphates plant = 450 t/day for SSP and 150 t/day for MCP ~ 500 t/day
$(CN)_{CN}$	500 t/day
$[P]^i$	Recommended application rate (kg/ha) of *P (phosphorous)* for crop type *i* (Table 7-4)
$[K]^i$	Recommended application rate (kg/ha) of *K (potassium)* for crop type *i* (Table 7-4)
$[N]^i$	Recommended application rate (kg/ha) of *N (nitrogen)* for crop type *I* (Table 7-4)
[NPK]	Equivalent daily production capacity of one Granul Granulator's 1 & 2 = 1,700 t/day

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11 APPENDICES

11.1 NPK Ratio

(1)NPK: *Unlike the N number, the numbers for P and K do not reflect the amount of elemental phosphorus and potassium in the fertilizer. Rather they represent the amount of oxide in the form of P_2O_5 and K_2O that would be present in the fertilizer if all the elemental phosphorus and potassium were oxidized into these forms. The factors for converting from P_2O_5 and K_2O values to their respective P and K elemental values are as follows:*

- *P_2O_5 consists of 56.4% oxygen and 43.6% elemental phosphorus. The percentage (mass fraction) of elemental phosphorus is 43.6% so P = 0.436 x P_2O_5*

- *K_2O consists of 17% oxygen and 83% elemental potassium. The percentage (mass fraction) of elemental potassium is 83% so $K = 0.83 \times K_2O$*

- *Nitrogen values represent actual nitrogen content so these numbers do not need to be converted.*

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