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Choosing investment solutions for South African long-term investors with Prospect Theory-type risk preferences

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Supervisor: Professor Evan Gilbert

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DECLARATION

I, the undersigned Osagyefo Mazwai, hereby declare that:

(i) the work contained in this thesis is my own work; and

(ii) in instances where my thesis is based on previously submitted work, I have provided detailed information:

(a) regarding the nature, substance and origin of the overlap in the space below and throughout my thesis (using the Harvard Referencing System or footnotes),

(b) regarding content that has been added to the previous submission, and

(c) that I understand that the evaluation of my thesis will be based primarily on the new work.

Date: March 2020

Opsomming

Hierdie tesis ondersoek die optimale keuse van beleggingstyl vir 'n verteenwoordigende langtermyn Suid-Afrikaanse belegger wat 'n “Prospect Theory” tipe nutsfunksies aanvaar. Die relatiewe prestasie van Gebalanseerde Fondse, Absolute Opbrengsfondse, Algemene Aandelefondse en Buigsame Fondse is in hierdie konteks ondersoek. 'n Fundamentele insig van “Prospect Theory” is dat mense meer reageer op verliese as winste, soos gedemonstreer deur die S-vormige waardefunksie. Hierdie subtiliteite word geïgnoreer deur “Expected Utility Theory” en hul gepaardgaande risikomaatstawwe (belangriker nog, variansie) kan lei tot portefeulje-ontwerpe wat nie met beleggers se risikovoorkeure strook nie. Twee metodes om die optimale beleggingstyl vir hierdie beleggers te ondersoek is toegepas (naamlik Historiese Analise en Gefiltreerde Historiese Simulasies). Die Historiese Analise-metode toon dat Gebalanseerde Fondse die hoogste opbrengste bied, maar die Sharpe-verhouding toon dat Absolute Opbrengsfondse die beste risiko-opbrengs-afruiling het. Deur gebruik te maak van “Prospect Theory”-gebaseerde nutsfunksies met verskillende parameters wat verskillende risikovoorkeure weerspieël, onthul die Gefiltreerde Historiese Simulasie-metode dat Absolute Opbrengsfondse heel waarskynlik die optimale beleggingstyl sal wees vir 'n belegger wat óf ernstige óf ligte "verlies-aversie" toon. Vir beleggers met “geen verlies-aversie” is die optimale beleggingstyl Gebalanseerde Fondse wat die hoogste verwagte kumulatiewe nut lewer. Hierdie studie toon dat die keuse van beleggingstyl afhanklik is van die vlak van verliesaversie. Dit dui op die behoefte om dit vir Suid-Afrikaanse beleggers te skat, aangesien die parameters wat in hierdie studie gebruik word, afgelei is van internasionale studies wat relatief gedateer is.

ABSTRACT

This thesis investigates the optimal choice of investment style for a representative long term South African investor assuming a Prospect Theory-type utility functions. The relative performance of Balanced Funds, Absolute Return Funds, General Equity Funds, and Flexible Funds were investigated in this context. A fundamental insight of Prospect Theory is that human beings are more responsive to losses than gains, as demonstrated by the S-shaped value function. These subtleties are ignored by Expected Utility Theory and their associated risk measures (most importantly, variance) may lead to portfolio designs that are inconsistent with investors' risk preferences. Two methods for investigating the optimal investment style for these investors were applied (namely, Historical Analysis and Filtered Historical Simulations). The Historical Analysis method reveals that Balanced Funds offer the highest returns, but the Sharpe ratio shows that Absolute Return Funds have the best risk-return trade off. Using Prospect theory-based utility functions with differing parameters reflecting differing risk preferences, the Filtered Historical Simulation method reveals that Absolute Return Funds are most likely to be the optimal investment style for an investor exhibiting either severe or mild "loss aversion". For investors with "no loss aversion", the optimal investment style is Balanced Funds which deliver the highest expected cumulative utility. This study shows that the choice of investment style is dependent on the level of loss aversion. This suggests the need to estimate these for South African investors as the parameters used in this study are derived from international studies that are relatively dated.

Key words: Optimal invest strategies, Prospect-theory risk preference investments,

DEDICATION

I dedicate this thesis to Matumelo Khanyisa Kakana (23/02/1980 to 20/11/2005) and Belede Camagwini Mazwai (12/09/1958 to 21/04/1992). Similarly, I dedicate this work to all the ancestors who guide us, nurture us, enable us and protect us each and every day. May all their souls rest in eternal peace.

"Do not stand at my grave and weep
I am not there; I do not sleep.
I am a thousand winds that blow,
I am the diamond glints on snow,
I am the sun on ripened grain,
I am the gentle autumn rain.
When you awaken in the morning's hush
I am the swift uplifting rush
Of quiet birds in circled flight.
I am the soft stars that shine at night.
Do not stand at my grave and cry,
I am not there; I did not die."

– **Mary Elizabeth Frye**
  13 November 1905 – 15 September 2004

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- I would like to thank my Aunt, Xoliswa Kakana, for always believing in me. She has also encouraged me to believe that no bar is too high to overcome, and to push myself always to my very limits.

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ABBREVIATIONS, ACRONYMS AND GLOSSARY

AR	Autoregressive (model)
ARCH	Autoregressive Conditional Heteroscedasticity
ARIMA	Autoregressive Integrated Moving Average
ASISA	The Association for Savings and Investment South Africa
AUM	Assets under Management
CVaR	Conditional Value at Risk
EGARCH	Engle's Generalised Autoregressive Conditional Heteroscedasticity
ES	Expected Shortfall
EUT	Expected Utility Theory
EV	Expected Value
EVT	Extreme Value Theory
FHS(s)	Filtered Historical Simulation(s)
GARCH	Generalised Autoregressive Conditional Heteroscedasticity
GJR-GARCH	The Glosten-Jagannathan-Runkle Generalised Autoregressive Conditional Heteroscedasticity
i.i.d.	Independent and Identically Distributed
SA	South Africa(n)
SD	Standard Deviation
TRI (data)	Total Return Index (data)
VaR	Value at Risk

CHAPTER 1:
INTRODUCTION

1.1. RISK AND INVESTMENT MANAGEMENT

The purpose of this study is to determine the "optimal" investment portfolio for a representative long-term South African investor amongst Balanced, Absolute Turn, General Equity and Flexible Funds - which will be the portfolio that maximises utility under a prospect theory framework. This will be determined by expanding on the work done by many academics from Knight (1921) to Markowitz (1952) to Kahneman and Tversky (1992) who contributed, and continue to contribute, to the ongoing process of defining of risk related to investment decisions. "Risk" is a well-researched concept. This is because risk, or the level of risk, impacts the relative attractiveness of making a particular investment decision. According to Bernstein (2012) in his book "Against the Gods: The Remarkable Story of Risk", "risk" as a concept first came into prominence in the 1600s. In the same book, John Graunt is credited as the father of sampling and statistics. Uncertainty about the future was the basis on which he began studying the mortality rate of people living in London. It was this early discovery which began the process of people formally considering the uncertainty of the future, which laid the foundation for risk theory and modern portfolio theory developed by Markowitz in 1952.

Kahneman and Tversky (1992) made significant insights on the varying degrees of loss aversion amongst investors. The idea of loss aversion is that each individual investors has a different susceptibility/responsiveness to losses and gains and thus the relative performance of a particularly investor is slanted by an investors risk profile. The S-Shaped value function captures this by revealing the shape of the utility function when experiencing gains and losses, and it shows that investors are more receptive to losses than gains.This work was an extension of previous work on Expected Utility Theory which incorporates the behavioural and human elements of decision-making. The behavioural and human elements are captured in Prospect Theory by a loss aversion parameter (λ) in the utility function. Tversky and Kahneman (1992) classified the differing loss aversion numeric values as 1, 2.1 and 2.25 for investors with "no loss aversion", "some loss aversion" and "high loss aversion", respectively. Therefore, the varying degrees of loss aversion inadvertently impact the relative performance of a particular investment style.

In this study, we gathered historical monthly performance data for Absolute Return, Balanced, General Equity and Flexible Funds. We used a combination of Historical Analysis and Filtered Historical Simulations (see Chapter 3). The study aims to analyse the performance of these investment styles relative to Kahneman and Tversky's (1992) loss aversion parameters. We believe that these insights could potentially inform the future of portfolio construction within the South African context such that representative long-term South African investors experienced utility is maximized.

The Historical Analysis method reveals that Balanced Funds offer the highest returns, but the Sharpe ratio shows that Absolute Return Funds have the best risk-return trade-off. The Filtered Historical Simulation method reveals that Absolute Return Funds are most likely to be the optimal investment style for an investor exhibiting "loss aversion". For investors with "some loss aversion" as parameterised by Tversky and Kahneman (1992) and Baláž et al. (2013), the optimal investment style is also Absolute Return Funds. For investors with "no loss aversion" as parameterised by Tversky and Kahneman (1992), the optimal investment style is Balanced Funds which deliver the highest expected cumulative utility.

One of the limitations of this analysis is that the loss aversion parameter metrics are taken from Tversky and Kahneman (1992) which has a time bias as the metrics were determined between the 1970's and 1990's. These metrics are also USA specific where the social and economic circumstances differ from the current economic and social circumstances in South Africa.

Harrison and Swarthou (2019) are also very critical about "how" Kahneman and Tversky "came" to parametric estimates for the loss aversion parameters. They cite a number of reasons including that the testing had no salient rewards, there was no mention of a standard error and thus it is not clear if the estimates are statistically different to 1, and that the median values used as the parametric estimates might not be a true representation of the average value.

Lastly, the period under observation (2013 – 2018) was characterized by a period of high global liquidity and low global yields which might suggest that some fund classes might have benefitted from this and thus performance is slanted. This makes it difficult for the observation in this particular paper to be of use as a predictor of portfolio selection. However, the paper should contribute to the knowledge of the field of study.

In conclusion, this is the crux of the thesis – what investment portfolios will be best suited to investors' needs given that they have prospect theory-consistent utility functions? To answer this question, we need to identify and test the range of typical long-term investment solutions in available the South African investment context and establish their expected levels of utility if proxied by the prospect theory-type utility functions.

1.2. THE RESEARCH QUESTION

Which type of investment fund, between Absolute Return, General Equity, Balanced and Flexible funds, is significantly more attractive to invest in under a prospect theory framework for a representative long-term investor in South Africa?

To answer the research question, three separate sub-research questions will be explored and addressed:

a) What is meant by "risk" in the context of the investment problem?

In investment theory, "risk" is usually measured as the extent to which the actual investment outcome differs from the expected investment outcome. The variance of investment returns is one of the most prominent, measures of risk in this investment context. It was popularised by Markowitz in 1952. The mean (average) is a commonly used measure of the expected return and the risk is measured by the standard deviation (which is the extent to which the actual outcomes fluctuate around the mean). The higher the standard deviation, the greater the risk. Markowitz (1952) then presented how portfolios could be constructed to minimise the variance for each level of expected return (the efficient frontier).

Markowitz's (1952) work is based on EUT. An extension to this is "Prospect Theory". Under the prospect theory framework, mean-variance analysis as a measure of risk is invalid. The argument is that mean-variance analysis fails to capture the human and behavioural aspects of elements of preference theory. That is, a risk-reward outcome, as a preference, varies between investors due to their differing levels of loss aversion. This leads to other measures of risk which can be used to determine efficient investment portfolios. Mean-variance theory sets a platform for further and deeper analysis and is discussed alongside alternative measures of risk in Chapter 2.1. with a specific focus on long-term investors.

b) A critical review of alternative approaches to modelling investors' risk preferences: Expected Utility vs. Prospect Theory

EUT aims to portray the utility an investor derives from gains to their wealth in a risky environment by choosing alternatives that maximise the probability weighted (or expected) levels of utility (not wealth or monetary value). Prospect theory delves further into the behavioral elements of investing and illustrates how investors react to whether they achieved their intended outcome or not. This is based on their relative levels of loss aversion. Prospect theory also extends on EUT by introducing a reference point, from which you can establish how utility has changed over the sample period. Au (2014) and Tversky and Kahneman (1992) introduced loss aversion parameters which reveal a different dimension to investors' risk sensitivity. Prospect theory holds that individuals have different risk preferences, which will impact their levels of utility based on whether an outcome is above or below their reference point. In the context of this thesis, the reference point is defined as their initial state (i.e. their portfolio's investment value at the beginning of the investment period) and their utility is calculated on the difference between this and the end state of the investor. The goal would be to determine whether the investment has led to a utility gain or a utility loss for the investor over the period, and to identify which investment fund achieved the best outcome over the investment period given their levels of risk aversion. EUT is explored in detail in Chapter 2.1, and prospect theory in Chapter 2.2.

c) What is the best choice of fund, given the use of a prospect theory-based utility function?

The "optimal" investment portfolio that is chosen by an investor will be the portfolio that maximises utility under a prospect theory framework. Achieving the investment target is a function of which investment strategy maximises average, or expected, cumulative utility for a representative long-term investor given their risk preference. The portfolios under scrutiny in this research paper are Balanced, General Equity, Flexible and Absolute Return funds. These funds were chosen as proxies for the investment choices available to long-term investors in South Africa. They differ in investment style and structure of investment mandates, and the information on their returns was available from Profile Data (2018). For the purpose of this study, the investment strategy focuses primarily on a representative long-term South African investor. By this we mean investors that are using their investment returns to save for retirement which is usually of period of at least five years.

1.3. STRUCTURE OF THE DOCUMENT

Chapter 1 introduces and presents the research question and the structure of the document.
Chapter 2, contains the theoretical framework and literature review.
Chapter 3 lays out and implements the research methodology.
Chapter 4 presents and evaluates the results.
Chapter 5 contains the conclusions and sets recommendations for future research.

CHAPTER 2:
THEORETICAL FRAMEWORK

2.1 RISK RETURN FRAMEWORK

Knight (1921) describes risk as the set of outcomes which can be insured against, and uncertainty as the set of outcomes which cannot be insured against. Knight's fundamental point is that risk refers to parameters that he argued objectively exist and can be reasonably managed. Nobre and Grable (2015) articulate the definition of risk as "a set of relative stable parameters people consider when evaluating risky financial choices". What is clear about these definitions of risk is that risk refers to the problem of having to make decisions in light of an uncertain future. This thesis is primarily concerned with quantifying this uncertain future. For long-term and short-term investors, they face risk and should attempt to make investment decisions which mitigate the downside risks and maximise upside risks. For the purpose of this thesis we define "downside risk" as the risk of accumulating negative (losing) utility on average *ceteris paribus*, and "upside risk" as the risk of accumulating positive (gaining) utility on average *ceteris paribus*.

In finance, a common definition of risk is the uncertainty about future deviations from expected earnings/return or expected outcome (Economic Times, 2017). Markowitz (1952) is credited for his work on mean-variance analysis which is built on this definition of risk. While seemingly a simple and attractive definition of risk, the fact that it includes both a better-than-expected outcome and a worse-than-expected outcome (that is, what are commonly called "upside" and "downside" risks), it is a limiting attribute to both the measurement and application of risk. The backward-looking nature of mean-variance analysis has also been cited as a limitation of this measure (Coleman, 2011) as the future is never a perfect statistical replication of the past. The limitations of this approach are highlighted later in this document, as the distinction between Expected Utility Theory (EUT) and prospect theory becomes the focal point of the risk-return analysis in this paper.

The utility function is a measure of the relationship between levels of consumers' utility and gains in their wealth. It represents the various combinations of wealth and utility. The utility function is traditionally assumed to have a concave shape. This shape illustrates the diminishing rate at which an additional gain in wealth results in additional utility. The more an individual consumes of a particular good, the less additional utility the individual enjoys from the consumption of the additional unit. The concept is central to the expected utility hypothesis which is a precursor to prospect theory. Von Neumann and Morgenstern (1944) are credited as the pioneers of the expected utility hypothesis which first introduces the idea that decision-making is susceptible to behavioural biases as opposed to mean-variance analysis which is a more objective measure of risk versus return. EUT attempts to incorporate subjective elements of risk versus return. Building on from EUT, prospect theory (Kahneman and Tversky, 1979, Tversky and Kahneman, 1992) attempts to expand on the notion of EUT by more fully explaining behavioral elements of decision-making and attempting to fully capture the investor’s final state and subsequent satisfaction. They introduce an S-shaped utility function which fully explains an investor’s risk profile, thus varying degrees of risk aversion will impact changes in level of satisfaction.

Investors seek to attain a certain gain, either in wealth or in satisfaction, when making investment decisions, and the outcome determines the investors’ final state. Mean-variance captures the final state of the investor purely based on change in wealth. If there is a positive relationship between risk and return, clients are happy to accept greater levels of risk for greater reward. Nobre and Grable (2015:18) similarly state that “clients will tend to favour” an investment that maximises expected returns at the level of risk that is individually acceptable. EUT and Prospect theory expand on simple change in wealth but rather analyse a change in satisfaction. Change in satisfaction is individual specific based on their risk profiles. This highlights the need for a risk management strategy (or investment selection) that provides the most efficient exposure of risk for the desired level of returns.

2.2 EXPECTED UTILITY THEORY

Expected Utility Theory (EUT) was developed by John Von Neumann and Oskar Morgenstern in 1944 in their book “Theory of Games and Economic Behavior”. This book extended on Bernoulli’s work on utility theory. According to Bernstein (2012), Bernoulli is credited with the proposition that rational decision makers aim to maximise expected utility as opposed to levels of wealth. This idea is the foundation of EUT. The expected utility hypothesis is the basis of EUT and states that “under uncertainty, the weighted average of all possible levels of utility will best represent the utility at any given point in time” (Investopedia, 2018). EUT is a precursor to prospect theory which introduces an S-shaped utility, which is explored in Chapter 2.2.

The utility function (see Figure 2.1) has a utility property, or the essential components of the normative theory of choice, if it has finite outcomes and probabilities. The expected utility graph shows the relationship between wealth and the utility derived. The function can take various forms which illustrate the linear or non-linear relationship between the two variables and the rate of change of utility for changes in wealth. The rate of change of wealth and the shape of the utility then describe the investors risk preferences. According to Chateanuex (2008), the behaviour of an expected utility decision maker can be entirely described by the shape of the utility function. Bernoulli proposed that utility is inversely related to the amount of the good consumed (that is, from a particular point the amount of utility gained from an additional unit of a particular good diminishes). This is commonly accepted as the first formal statement of EUT. The concept of "utility" states that "utility arising from any small increase in wealth will be inversely proportionate to the quantity of goods previously possessed".

Mehra and Prescott (1985) explore the assumptions that are made with regards to an investor's risk preference. They consider the cases of a growing economy where an investor is either risk-averse or risk-loving (relative to a non-growing economy). The consideration is important in constructing the model on what asset yields greater return between a stock (representative of the S&P500 for the given time period) versus a risk-free asset (government bond). Mehra and Prescott (1985) find that a risk-averse investor discounts the future to a far greater extent than a risk-loving investor. This implies that a risk-averse investor is more doubtful about the potential for the actual return to match the expected return (mean). How risk is defined for the representative investor is important as this informs the shape of the utility function for a certain good. This is thus the crux of our analysis, how cumulative utility changes for investors exposed to the same investment portfolios but with differing levels of risk aversion.

Moy (2015) explains that the expected value (EV) of an outcome under uncertainty is the sum of the probability of a particular outcome multiplied by the actual outcome. Moy (2015) further explains that the expected value (EV) from an event is a finite measure which implies a straight-line utility function, whereas deviation from a straight-line utility function (EV) leads to the consideration of the human and behavioural elements of decision making. Thus, EV on its own fails to capture the behavioural aspects under scrutiny in prospect theory.

The purpose of this comparison leads up to what is known as the St Petersburg Paradox. The St Petersburg paradox attempts to bridge the disparity between classical economic thought on what decision a rational gambler is expected to make under uncertainty and the reality of the observed decisions made by gamblers. Thus, there is a non-linear relationship between classical economics assumptions (i.e. what a decision a rational decision maker might make) and real-life decision making. The St Petersburg paradox therefore lays the foundation of the arguments in the expected utility hypothesis in which the non-linearity of decision-making prevails.

Bernstein (2012) highlights the first appearance of the paradox in the St Petersburg Academy Proceedings in 1738. Bernoulli poses the game of flipping a fair coin and doubling the reward every time the correct side came up. Given that the expected value of the game is infinite, a representative or rational gambler looking to maximise levels of wealth would be expected to pay any amount to participate in the game. In reality, however, a different outcome is observed. An individual may be willing to pay a finite amount of dollars to participate in the game – certainly not anything. The way that this paradox has been resolved is the recognition that individuals are not acting to maximise expected levels of income, but rather levels of utility, combined with the assumption of a decreasing marginal utility of wealth.

The purpose of mapping out the axioms, assumptions, components and rules of combination is to put together a framework of the decision maker under uncertainty. Gul and Pesendorfer (2014) argue that an investor's risk perception determines how each investor's utility function is mapped differently.

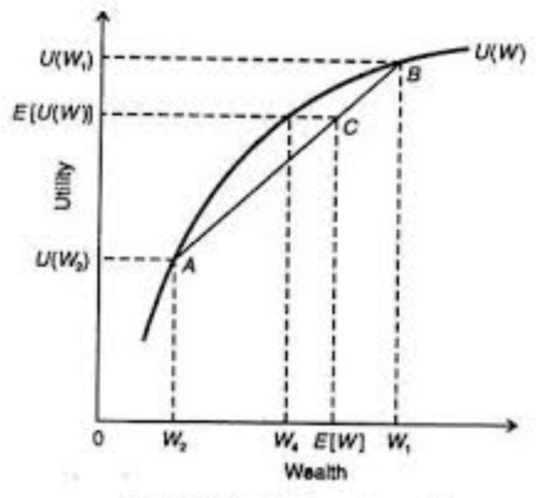

Figure 2.1: **Utility function (Microeconomicsnotes.com, 2019)**

2.2 PROSPECT THEORY

A key development in this space was the introduction of Prospect Theory by Kahneman and Tversky (1979). This was a critique/extension of the EUT described in Chapter 2.1 as the basis of making decisions under uncertainty.

Kahneman and Tversky's (1979) theory critiqued the assumptions pertaining to rational choice under uncertainty. In addition, they explored certainty and isolation effects which are explained below. Furthermore, value is assigned to losses and gains in respect to a particular reference point as opposed to the final assets (or wealth).

Levy and Levy (2002) provides a convenient summation of prospect theory. They assert that prospect theory is the key paradigm challenging EUT. They further opine that the S-shaped value function is the underpinning of Prospect Theory. The underpin of prospect theory is the S-shaped value function which is subject to the following four assumptions which are effectively extensions of EUT:

1. Investors make decisions based on change of wealth rather than on total wealth;
2. Investors maximise the expectation of a value function;
3. Investors subjectively distort probabilities; and
4. The "framing" of alternatives may strongly affect agents' choices.

The first point has significant implications for this thesis as the final outcome informs the agent's decision. Under EUT, we could define "better off" and "worse off" differently to the parameters informing prospect theory;

Kahneman and Tversky (1979) explore various effects or situations which are not captured by EUT. The first is the certainty effect which violates the substitution assumption. Various tests indicate that agents choose either an outcome with a guaranteed/certain gain or choose an outcome where the gain is larger where the probabilities of success are minimal or differ only marginally.

The reflection effect refers to the violation of the expectation principle. Based on the framing of the outcomes between gains and losses, agents place an overweight bias towards certainty in both circumstances. In the case of a certainty of loss versus a probable loss of a greater amount, the agent exhibits risk-loving behaviour and in the case of a certainty of gain versus a probable gain of a greater amount the agent exhibits risk-aversive behaviour.

The isolation effect is the final effect discussed in the paper which violates EUT. The isolation effect is an extension of the reflection effect and the certainty effect. Two examples are employed in which a two-step process applies. What is observed is that the first step is ignored by agents and only the second step considered. The second step, according to its framing, falls into the traps of certainty and reflection effects with agents choosing A in scenario one and D in scenario two although A and C are identical whilst B and D are identical choices (in terms of the expected value) but framed differently.

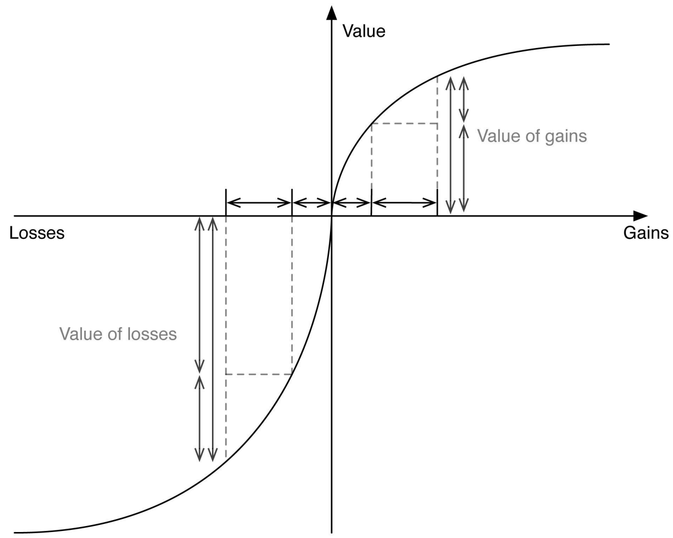

Figure 2.2: S-shaped value curve in Prospect Theory (UIPattern, 2018)

The S-shaped value function is the essential feature of Prospect Theory. The function shows that agents are more responsive to value of losses as opposed to the value of gains. The steepness and shape of the utility function shows that.

The shape of the function supports the various critiques put forward by Kahneman and Tversky (1979). The certainty and reflection effects are captured by the shape as agents choose a certain gain or the chance of a mitigated loss. Agents exhibit risk-loving and risk-averse behaviour based on the manner in which the risk is framed.

The final point of prospect theory is the addition of reference points. Utility theory does not consider a particular reference point but rather a static quantity of a good. However, prospect theory considers a particular reference point which enables the analysis of the impact of gains and losses on levels of utility. Two agents may have an equal amount of money at a particular time but prospect theory takes their initial position and therefore can distinguish differently between the impact of gains and losses.

2.3 RISK MEASURES

2.3.1 MEAN-VARIANCE HYPOTHESIS

As mentioned in the introduction, the mean-variance hypothesis developed by Markowitz in 1952 is the foundation of modern portfolio theory and has led to the use of variance becoming the standard tool for the measurement of risk in an investment context. Within the framework of mean-variance hypothesis, the mean or average of the observed values is considered the expected value (and reveals the central tendency of a distribution) while the standard deviation of the observed values is considered the extent of the risk.

The fundamental point is that these measurement tools, within the Markowitz definition, use “current and past exposures” as a means of trying to predict/control future risk. With specific reference to probability theory as laid out by Coleman (2011), the distribution function captures all the relevant information and is limited in this case to a normal distribution.

The key observation by Coleman (2011) with respect to the mean-variance hypothesis is that the observed mean and variance do not specifically imply a preference. Preferences are specific to individuals, which implies that a highly dispersed (high variance) portfolio, which is a high-risk portfolio, is not automatically a less-preferred option but the preference is specific to an individual. Coleman’s insights therefore link risk measurement to the human elements of risk-return theory and can be linked to EUT and prospect theory as per Chapter 2.2.

Coleman identifies two main problems/limitations with the mean-variance hypothesis:

Firstly, mean-variance analysis (like extreme value theory and Value at Risk), is based solely on observations of the past. The limitations of this is that we tend to fall into the trap where we assume that the future should be easy to explain and understand based on our understanding (and representation) of the past. To correct for the backward-looking nature of mean-variance analysis, simulations can be used.

Secondly, human intuition or decision making is susceptible to behavioural biases which limit the extent to which we can predict the future based on past experiences or observations. Coleman credits the inputs of Kahneman and Tversky (1979) for this insight. Bayes theorem is useful in building on the empirical evidence to factor in additional information and measure the probability of an uncertain event more carefully.

In support of Coleman’s (2011) critique, Platanakis, Sakkas and Sutcliffe (2017) demonstrate that the performance of portfolio models constructed with inputs purely based on historical values has resulted in disappointing results due to large estimation errors.

2.3.2 VALUE AT RISK

Value at Risk (VaR) measures the value of an extremely unlikely event in a pre-specified time period. It has been defined as the statistically "worst-case loss scenario" measure (Investopedia, 2018). VaR focuses on the tail-end events of a particular distribution. Coleman (2011) states that tail-end events are in their nature extremely unlikely. Defining the tail end is very important as it states what probability you are measuring. A 1% level of significance versus the 5% level of significance are very different measures. The 1% level of significance is the far end of the tail of the distribution and therefore extremely unlikely and difficult to measure. Although we have stated that the VaR measure at a 1% level of significance is the far end of the tail, it is not considered the maximum extreme loss metric but rather the minimum extreme loss metric. Once VaR has been calculated, the observer should be able to predict that at a certain level of significance, and investor will expect a certain amount of losses (for example, a 5% level of significance will imply that 5% of the time an investor will experience losses of at least -10%). As mentioned above, the VaR is considered the minimum extreme loss metric and therefore fails to capture extreme losses (that is, values beyond it which limits the credibility of the metric).

VaR is another risk measure that is conceptually consistent with the mean-variance hypothesis and thus the same critiques apply to it too. Coleman (2011) identifies a number of problems with the use of VaR:

Firstly, like mean-variance, VaR is a backward-looking measure and fails to capture human intuition or behavioural biases. VaR uses historical data in order to pre-empt/predict the future. Therefore, it is biased towards past events which do not completely capture risks that lie ahead in a rapidly changing world or human behaviour.

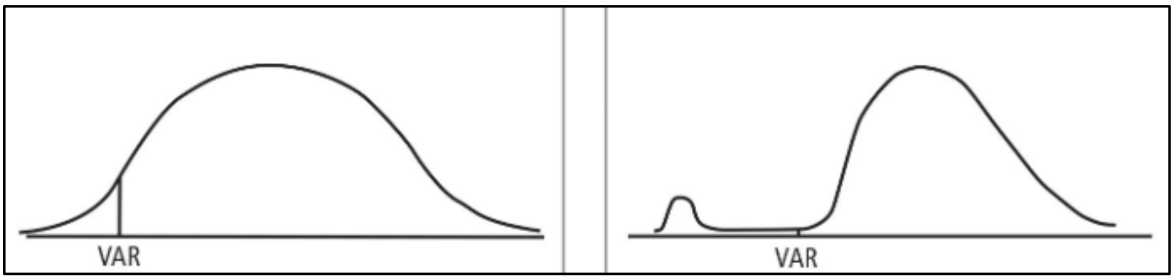

Figure 2.3: Comparison of the symmetrical and non-symmetrical VaR distributions (Risk.net, 2019)

Secondly, it is only useful as far as that the observed distribution fits certain assumptions. Non-symmetrical and skewed distributions will hinder our ability to analyse the distribution and make conclusions. In Figure 2.3, the two distributions have the exact same VaR but the risk associated with the right-hand panel are much greater than the risks with the left-hand panel. A biased VaR observation will exaggerate/underestimate the extent of losses while mean-variance is useful only for a normal distribution. To mitigate the impact of this, Coleman (2011) suggests the use of a t-distribution in addition to the normal z-distribution.

Thirdly, the illusion of certainty provides that these measures of risk tempt individuals to assumes that the finite values observed in terms of the mean, variance and VaR are enough to tell us about the future. Coleman (2011) cautions that these measures are useful and provide fuller information to the investor but do not provide full information. This links with prospect theory which forces observers to consider a variety of factors as opposed to static scenarios based on historical experience.

2.3.3 EXPECTED SHORTFALL OR CONDITIONAL VALUE AT RISK

Expected shortfall (ES) or Conditional Value at Risk (CVaR) attempts to deal with some of the limitations of the VaR model. According to CFA Institute (2016), the CVaR is the weighted average of all loss outcomes in the tail end of the statistical distribution which exceeds the VaR tail-end loss. The limitation specifically relates to the shape of the probability distribution. Hull (2006) compares the measures and concludes that CVaR is a better, coherent measure of risk. He looks at various properties of risk measures and VaR does not satisfy the sub-additivity property which applies the theories of diversification to risk measurement. The VaR of the portfolio increases with the addition of an additional portfolio which in theory should diminish due to the benefits of diversification and distributing of risk.

Breaking Down Finance (2019) stipulates that ES is calculated by ranking total monthly returns from highest to lowest (descending order). For the purpose of this literature review, we are concerned with the ES at a 95% level of confidence. To determine this, the next step is to select the 5% lowest monthly returns. The final step is to calculate the mean/average of the lowest 5% monthly returns in order to determine the percentage monthly Expected Shortfall.

The first limitation of the expected shortfall approach is that it fails to tell us about the specific values of extreme values in the sample. Extreme values or outliers (significant losses/gains) can distort the accuracy of the results, but at the same time have very different implications for investors with prospect theory-style risk preferences.

The second limitation of the expected shortfall approach (as with the other historical analysis metrics under observation in this study) is that it does not account for the timing of the observed returns. Each individual observed return is weighed in proportion to other observed returns irrespective of economic cycles and other such distortions.

2.4 INVESTMENT STRATEGIES CONSIDERED IN THE STUDY

For the purpose of this thesis, the investment strategy focuses primarily on a representative South African long-term investor. Staff (2017) defines a long-term investment as an investment with a time horizon of longer than twelve months. In this study we define it as having a five-year period.

According to data taken from ASISA (2018), the most popular investment strategies in South Africa (which we define as the investment funds with the highest Assets Under Management - AUM) are the Multi-Asset High Equity Funds (or Balanced Funds) and SA General Equity Funds. Multi-Asset High Equity Funds had a total AUM of circa ZAR512 bn while SA General Equity Funds had an AUM of circa ZAR356 bn as at 30 September 2018.

ASISA (2017) defines Multi-Asset portfolios as portfolios that invest in a wide variety of asset classes (equities, bonds, money and property) to maximise total returns over the long term. The next most preferred (or highest AUM) investment fund in South Africa is General Equity Funds. ASISA (2018) defines General Equity portfolios as portfolios invested in selected shares across all industry groups as well as across the range of large, mid and smaller market capitalisation shares. While the managers of these portfolios may subscribe to different investment styles or approaches, their intent is to produce a risk/return profile that is comparable with the risk/return profile of the overall JSE equities market. The portfolios in this category offer medium- to long-term capital growth as their primary investment objective. While this is not a multi-asset fund, it provides the greatest expected level of returns over a 15-year holding period and thus provides an example of investing which is focused primarily on return maximisation.

To expand on the usefulness of this thesis, Flexible Funds and Absolute Return Funds have been included. Kruger (2014) states that Flexible Funds are generally positioned somewhere between General Equity and Balanced Funds. The major differentiator between Flexible Funds in comparison with General Equity and Balanced Funds is the wider investment mandate, thus they are "flexible" to invest in a greater proportion of equity, debt or other financial assets while typically Equity Funds are mostly exposed to equity assets. Flexible Funds also include a wider range of funds including property, allocation, income and equity funds.

Financial Times (2019) defines an Absolute Return Fund as a type of fund which aims to deliver positive returns in all market conditions (expansions, recessions, for example) with low volatility. Absolute Return Funds are more focused on short-term portfolio returns (one-year). Through diversification, Absolute Return Funds aim to mitigate downside losses.

In the data collected from Profile Data (2018), each fund has different risk classifications. Within Balanced Funds, individual funds are classified as either high risk, medium risk or low risk. Within General Equity Funds and Flexible Funds, funds are classified as high equity, high-medium equity and medium equity. Within Absolute Return Funds, individual funds are classified as either low risk, low-medium risk or medium risk. This gives twelve fund strategy/risk level combinations:

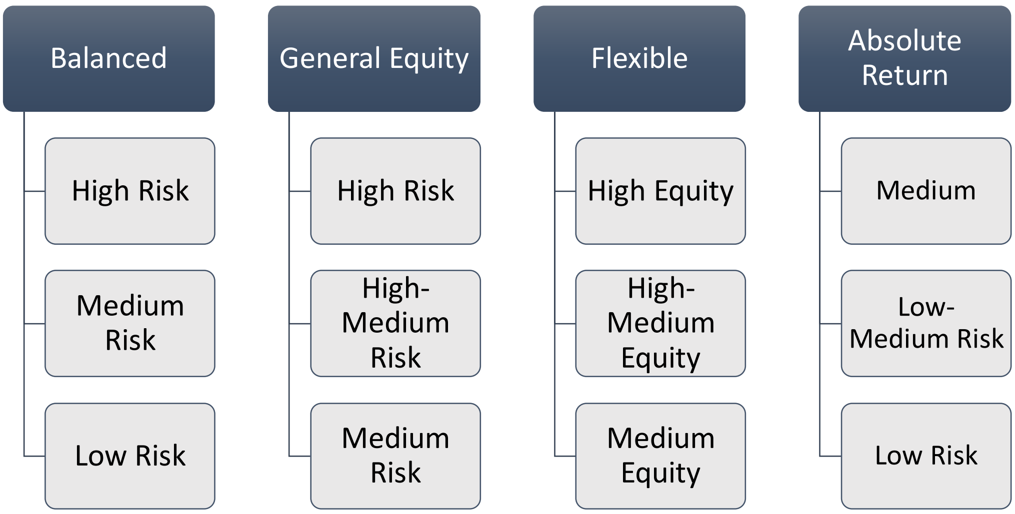

Figure 2.4: Risk category within each fund type

2.5 LITERATURE REVIEW

This section provides an overview of the various authors who have examined the relative efficacy of Prospect Theory as a measure of individual risk preferences. There are various arguments in support of Prospect Theory and various arguments against it. The arguments against Prospect Theory include the idea that stochastic dominance is present in Prospect Theory and thus gambles aren't ranked consistently, and model specification errors/bias. However, as Harrison and Swarthout (2019) point out, Prospect Theory, as developed by Kahneman and Tversky (1992) is widely accepted and considered the superior risk preference model.

Mei (2018) provides research comparing the relative usefulness of Prospect Theory versus Constant Relative Risk Aversion (CRRA) as measures of an individual's risk preferences. Under CRRA, as an individual's wealth in an asset increases or decreases, the relative risk aversion remains constant over time. The customers were presented with choosing a pension contract, considered a decision under uncertainty (risk). The three contracts under observation differ by structure and gurantees. The research found that the new contract was most attractive and consistent for a Cumulative Prospect Theory customer. Mei is thus supportive of Prospect Theory as an individual risk preference model.

Au (2014), like Hofmeyr and Kincaid (2019) observed that Prospect Theory was susceptible to the violation of first-order stochastic dominance where on gamble can be ranked superior to another gamble by decision-making agents. Prospect Theory assumes that decision-makers satisfy stochastic dominance at all times which is not necessarily satisfied at all times as show by Au (2014).

Harrison and Swarthout (2019) are amongst authors who are critical of prospect theory. They provided an analysis of the model specifications of Cumulative Prospect Theory, Rank Dependant Utility Theory (RDU), Disappointment Aversion, Dual Theory and Expected Utility Theory. Through the analysis of the various model specifications, they found that Rank Dependent Utility theory was the best individual and grouped categorization of choices. They show criticism of the parametric estimates used for the loss aversion parameter and argue that the testing of the parametric estimates had a) no significant rewards which are critical in the decision-making process of an individual under uncertainty, b) there was no mention of the standard deviation and it is not clear that the point estimates are different to

1, and c) that the median values used might not be the true mean values. Thus, Harrison and Swarthout in the study had a preference for the use of a Rank-Dependent Utility Function as opposed to Prospect Theory.

RDU was first developed by Quiggin (1982). The fundamental difference between RDU and Expected Utility Theory is that the independence assumption is relaxed. RDU also introduces decision weights as opposed to objective probability weightings under EUT. It also eliminates stochastic dominance violations observed under Expected Utility Theory and Prospect Theory and thus the ranking of gambles doesn't change.

Janeček (2004) is a paper in support of the relative efficacy of the risk aversion parameter under Constant Relative Risk Aversion. Jaeneck found that portfolios can be constructed relative to an individual risk preferences using a risk aversion parameter α. This supports the notion that differing risk aversion parameters for investors with varying degrees of loss aversion can aid in portfolio construction, or in the case of this thesis, choosing an "optimal" investment strategy for given risk aversion parameters.

Liu, Peleg and Subrahmanyan (2010) investigate the effect of the amount of information on assets an investor has on portfolio construction. Using the CRRA utility function, they derived optimal portfolios under different assumptions of the amount of information the investor had. This supports the notion that differing risk aversion parameters within some utility function has an effect on the relative performance of the investment strategies under investigation.

Lastly, Tuthill and Frechette (2004) use the RDU framework to evaluate how optimism and pessimism affects the commodity hedging price. The fundamental underpin of RDU, as mentioned above, is the introduction of decision weightings. They found that whether an agent trades in corn as a speculator or a buyer was dependent, on amongst others, their level of pessimism or optimism. This supports the idea that an investors' rank dependent utility (which is based on varying levels of optimism/pessimism) does have an effect on how they trade.

This study will use Prospect Theory to convert the monthly returns of Absolute Return, Balanced, General Equity and Flexible funds to map out cumulative utility functions for representative long-term South African investors, and thus determine the "optimal" investment strategy.

2.6 SUMMARY OF THE THEORETICAL FRAMEWORK

The various risk measures discussed in Chapter 2.3 set the framework for the risk analysis. The risk measures will determine the risk-reward framework attached to investing in Multi-Asset High Equity Funds (Balanced), Absolute Return Funds, Flexible and General Equity Funds in South Africa. Each of the risk measures will answer a different aspect of our risk-return analysis. We stipulated in the previous section that the individual risk measures have limitations that will impact our analysis.

In both historical analysis and filtered historical simulations, comparisons will be drawn between the average returns and variance of each fund type and average returns and variance within each fund type which will reveal the relative levels of risk and expected returns. In addition to risk-return analysis, we will analyse the utility frontiers attached to the investment styles.

Value at Risk (VaR) measures will reveal the chance of an extreme unlikely event, which focuses on the tail-end of the probability distributions or density plot. Comparisons will be drawn between each fund type and fund type risk classifications on their relative probabilities of extremely unlikely events. VaR is also subject to the same limitations as mean-variance.

As Coleman (2011) points out, there are several limitations to these risk measures. A density plot is used to reveal the "normality" of the observed data. If the distribution is not normal, then it is inappropriate to use the mean and variance observed on the density plot to explain risk versus return.

Throughout the analysis, the research will aim to compare the risk characteristics of each fund type, and different risk classifications within a specific fund type, as well their utility frontiers in order to determine which portfolio would be an optimal portfolio for an investor to invest in given their risk preference. The research will focus specifically on a comparison between the four different fund types. The data will enable us to map utility functions for each fund type. Certain utility characteristics will prevail for each fund type and the different risk classified funds within a fund type. A representative investor based on both EUT and

Prospect Theory will then be attached to the utility functions of the different funds. Prospect Theory has been chosen as the risk preference model in comparison with Constant Relative Risk Aversion, Disappointment Aversion, RDU and EUT. A representative risk-averse, risk-neutral and risk-loving investor as described in Chapter 2.1 can then be attached to the various fund types based on the fund's risk-return characteristics.

CHAPTER 3:
RESEARCH DATA AND METHODOLOGY

The purpose of Chapter 3 is to map out how we will firstly measure the risk-return characteristics of each investment style and secondly estimate the utilities derived under a prospect theory framework from the investment styles as set out in Chapter 2.3. relative to their risk profile: risk-averse, risk-seeking or risk-neutral. In order to distinguish between each risk profile, the numeric value of the loss aversion parameter in the value function changes as proposed by Tversky and Kahneman (1992) (see also Au, 2014). Historical data will be used to determine the risk-return characteristics of the fund types and the cumulative utilities of each investment style will be mapped. The fundamental objective of the methodology is to map the observed utilities to see which investment fund maximises utility over the sample period. Then we will implement a Filtered Historical Simulations (FHS) methodology in order to determine a multiplicity of possible return scenarios and related utility outcomes. This will allow for a conclusion as to which investment strategy would be best suitable for a long-term investor of a particular risk profile. Once we have completed the methodology, we aim to answer which investment style is the most optimal for a representative long-term investor in South Africa. Analysis was conducted in R (R Core Team, 2018) and figures were produced using the package ggplot2 (Wickham, 2009).

3.1 DATA

We use the monthly returns of various investment funds in South Africa (Appendix A.1). The South African Balanced, General Equity Fund, Flexible and Absolute Return Fund returns data were received from Profile Data (2018).

As explained previously, each fund type is grouped according to the different levels of equity or risk as defined/categorised by the Association for Savings and Investment South Africa (ASISA).

The data across funds varied in terms of dates of origination of the fund which limits the consistency of the results. 01 January 2013 was set as the starting/reference point of the investment in the various funds. The reference point was chosen in order to generate the longest commonly shared period. The data excludes funds which shutdown during the period which subjects our observations to survivorship bias, which can result in over or underestimation of past performance.

Within each investment style, the number of observed monthly returns varied by the number of funds in the category. General Equity Funds (9,052 observations), Balanced Funds (6,327 observations) and Flexible Funds (3,004 observations) had the greatest number of data points while Absolute Return Funds (681 observations) had the least. The varying number of observations amongst the investment styles could potentially impact the statistical power of our analysis.

A list of funds included in the data can be found in Appendix A2.

- The Balanced Funds data set includes 114 funds.
- The General Equity Funds data set includes 162 funds.
- The Absolute Return Funds data set includes 11 funds.
- The Flexible Funds data set includes 50 funds.

The frequency of the data is monthly returns (that is, twelve observations per year). This frequency is sufficient to generate gains and losses across different funds and investment styles. A single fund would have a maximum of 65 observed monthly returns from 01 March 2013 to 31 July 2018. This is the date that the returns data were received. The data also provided classification information in terms of risk classification. The monthly returns were calculated using each fund's total return index (TRI). The TRI for a fund measures its market returns including any share price movements, rights offers, and dividend payments – assuming they are reinvested into the stock dividend payer. Profile Data (2018) supplied us with the TRI data for each respective fund. To convert the TRI data into returns, we use the following equation to estimate the returns.

$$Monthly\ Return\ = \left(\left(\frac{TRI}{TRI_{t-1}}\right) - 1\right) \times 100 \qquad (1)$$

Each fund in our data pool was defined in Chapter 2.4.

3.2 METHODOLOGY

For the purpose of this study we will use a two-part analysis. The first part of the analysis will be an historical analysis and the second part will be the application of the Filtered

Historical Simulations (FHS) approach. This historical approach will observe and evaluate the actual history of fund returns collected from Profile Data. The salient purpose of the FHS methodology is to determine a multiplicity of possible return scenarios and related utility outcomes. This will allow for a conclusion as to which investment strategy would be best suitable for a long-term investor of a particular risk profile.

The first component, historical analysis, will break down the individual risk-return metrics defined in Chapter 2.3 to determine the mean and variance, expected shortfall, and Value at Risk (VaR) of each investment strategy. Prospect theory type-utility functions will enable us to model a cumulative utility frontier using the observed historical data for each fund in the twelve-different strategy and risk profile categories. The loss aversion parameter will vary for investors of different risk profiles. The utility frontiers will reveal the cumulative utilities of each investment style and thus reveal the optimal investment strategy for an investor of a particular risk profile. The filtered historical simulations will be conducted using a Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model along with an Autoregressive (AR) model. The FHS using Generalised Autoregressive Conditional Heteroskedasticity (GARCH) as well as an Autoregressive (AR) model have been proposed by Barone-Adesi, Giannopoulos and Vosper (1999) and Brandolini and Colucci (2012) for this type of analysis. Barone-Adesi et al. (1999) proposed the way in which the pricing of financial assets could be modelled using FHS methods and GARCH models. Brandolini and Colucci (2012) compared the efficacy of using FHS versus the filtered bootstrap method. All analysis was conducted in R (R Core Team, 2018) and figures were produced using the package ggplot2 (Wickham, 2009).

3.2.1 HISTORICAL ANALYSIS

Using historical analysis, we will estimate the mean, variance, standard deviation, and Expected Shortfall or Conditional Value at Risk (CVaR) for the four investment styles and their three risk categories. The historical analysis approach is a model-free approach which means that the analysis imposes no structure on the distribution of returns except stationarity. Accompanying these sample statistics will be density plots for each investment style (Balanced, General Equity, Flexible and Absolute Return). Within each investment style we will plot density plots for each defined level of risk. We expect the density plots to reveal dispersions around the mean and also the skewness of each investment style. The results will be scaled to one in order to increase the accuracy of the comparison between the investment styles. This analysis will show the risk return characteristics of these investment styles. As mentioned in Chapter 2.3, a limitation of the sample statistics is the assumption that the sample is a normal distribution. The next steps will attempt to deal with those limitations.

After this analysis, the cumulative utility outcomes of prospect theory-consistent utility functions will be explored. As Mehra and Prescott (1985) point out, investors of differing risk preferences discount the future to different degrees – which is the basis of this analysis. The aim is to determine what investment style is most suitable for an investor of a particular risk profile. Utility frontiers will be mapped for each investment style under each risk profile.

In order to plot these utility frontiers, we make use of Tversky and Kahneman's (1992) value function where λ is the loss aversion parameter.

Value function: $v(x) = \begin{cases} x^{\alpha} \\ -\lambda(x)^{\beta} \end{cases} \quad if\ x\ \geq 0\ and\ x < 0 \Big\}$ (2)

Where $\lambda \geq 1$, $0 \leq \beta \leq 1$, $0 \leq \alpha \leq 0$.

To the best of our knowledge, no study exists that identifies the parameter specifications for South African individuals. The loss aversion parameters used in this study are taken from the research conducted by Tversky and Kahneman (1992) and Au (2014). They are defined as follows for each risk profile:

For an investor with no loss aversion, $\lambda = 1$

For an investor with some loss aversion, $\lambda = 2.1$

For an investor with high loss aversion, $\lambda = 2.25$.

Au (2014) and Tversky and Kahneman (1992) use standard values of 0.88 and 0.88 for α and β. These parameter estimations are consistent with the experiments run by Tversky and Kahneman (1992).

The aim is to plot the utility frontier (cumulative utility frontier) for investors of a particular risk profile for the observed returns of Balanced, General Equity, Flexible and Absolute Return Funds in South Africa. The utility frontier will tell us how cumulative utilities fluctuate for each investment style under different loss aversion parameters. We would expect that an investor with no loss aversion will be primarily unconcerned with high fluctuations (that is, low standard deviation (SD)) around the mean of the observed values, while we expect investors with high loss aversion to be highly sensitive to fluctuations (that is, high SD) of the observed values around the mean.

3.2.2 FILTERED HISTORICAL SIMULATIONS

It is well understood in financial literature that the variance of asset prices varies over time. The ARCH and GARCH models attempt to model the changes in variance. Engle (1982) first introduced ARCH (Autoregressive Conditional Heteroscedasticity) which allowed for conditional variances to change over time as functions of past errors. Engle's approach was particularly popular for the modelling of high frequency financial data (Roy, 2011). Bollerslev (1986) expanded on Engle's work by introducing GARCH (Generalised Autoregressive Conditional Heteroscedasticity).

Filtered Historical Simulation (FHS) was developed by Barone-Adesi et al. (1999). The FHS method is a mixture of model-based and model-free approaches. As mentioned above, the Historical Analysis is an example of the model-free approach such that no assumptions are made on the structure of the distribution of returns, except stationarity. The FHS method is used to generate multiple scenarios of asset prices and returns, and also to evaluate the performance of the respective funds under observation. Roy (2011) states that the FHS method attempts to combine the best of model-based and model-free approaches.

Using the GARCH models, the ultimate goal is to determine/simulate, incorporating FHS, the hypothetical "nth-day" return and hypothetical "nth-day" variance. The "nth-day" mean return and variance are the proxies for risk and return. We use a methodology similar to Leigh (2018) in his paper using GARCH and ARIMA modelling to simulate returns and utility functions.

A GARCH model needs to be selected in order to use the data for calibration. We have selected the standard GARCH (Bollerslev, 1986), the EGARCH (Nelson, 1991) and the GJR-GARCH (Glosten et al., 1993) for calibration, selecting the model which has the lowest Akaike Information Criteria (AIC). Each of the fits are analysed and the model that results in independent and identically distributed (i.i.d.) standardised residuals is selected. Different ARIMA (p, 0, q) models with orders one and two of the AR and MA components are also tested, and selected for each representative fund.

Following Leigh (2018), the equations shown below demonstrate a standard ARIMA model using the three selected types of GARCH models (GARCH, eGARCH, gjrGARCH):

ARIMA (1, 0, 1)

$$r_t = \mu r_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \tag{3}$$

GARCH (1, 1)

$$h_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}^2 \tag{4}$$

EGARCH (1, 1)

$$\ln(h_t^2) = \omega + \alpha.\frac{\varepsilon_{t-1}}{h_t} + \gamma\left\{\left|\frac{\varepsilon_{t-1}}{h_t}\right| - \sqrt{\frac{2}{\pi}}\right\} + \beta\ln(h_{t-1}^2) \tag{5}$$

GJR-GARCH (1, 1)

$$h_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta h_{t-1}^2 \tag{6}$$

Where: $I = \begin{cases} 1 \text{ if } \varepsilon_{t-1} < 0 \\ 0 \text{ if } \varepsilon_{t-1} \geq 0 \end{cases}$

Where:

r_t = the conditional mean of the series,

μ = the AR(1) term,

θ = the MA(1) term,

h_t^2 = the conditional variance of the series,

ω is a constant,

ε_t = the random error residual and

I is the asymmetric indicator function for positive and negative shocks.

Once the models are fitted, the residuals are brought close to stationary i.i.d. by dividing the residuals by the volatility estimate:

$$e_t = \frac{\varepsilon_t}{\sqrt{h_t}} \tag{7}$$

The standardised residuals are then randomly drawn and scaled by the volatility values one period ahead to become suitable for simulation:

$$z_{t+1}^* = e_t^* . \sqrt{h_{t+1}} \tag{8}$$

Where: $$\sqrt{h_{t+1}^*} = \sqrt{\omega + \alpha(z_t^*)^2 + \beta h_t^*} \tag{9}$$

Finally, the various monthly return results of the representative funds are simulated as follows:

$$r_{t+1}^* = \mu r_t + \theta z_t^* + z_{t+1}^* \tag{10}$$

The number of simulations conducted is 1 000 times over a period of 60 months (five years) as per the period over which the historical analysis has been conducted. Firstly, we use the simulated returns to plot histograms in order to observe the structure of the distribution of the results. Secondly, the simulated returns are used to determine the utilities of investors that display loss aversion (λ = 2.25), some loss aversion (λ = 2.1) and no loss aversion (λ = 1). These selected parameters used to calculate the respective utilities are the same used for the historical analysis, namely, Tversky and Kahneman (1992) and Baláž et al. (2013).

CHAPTER 4:
ANALYSIS AND RESULTS

4.1 HISTORICAL ANALYSIS

4.1.1 DENSITY PLOTS

The density plots of the historical returns of each investment strategy are depicted in Figures 4.1 below.

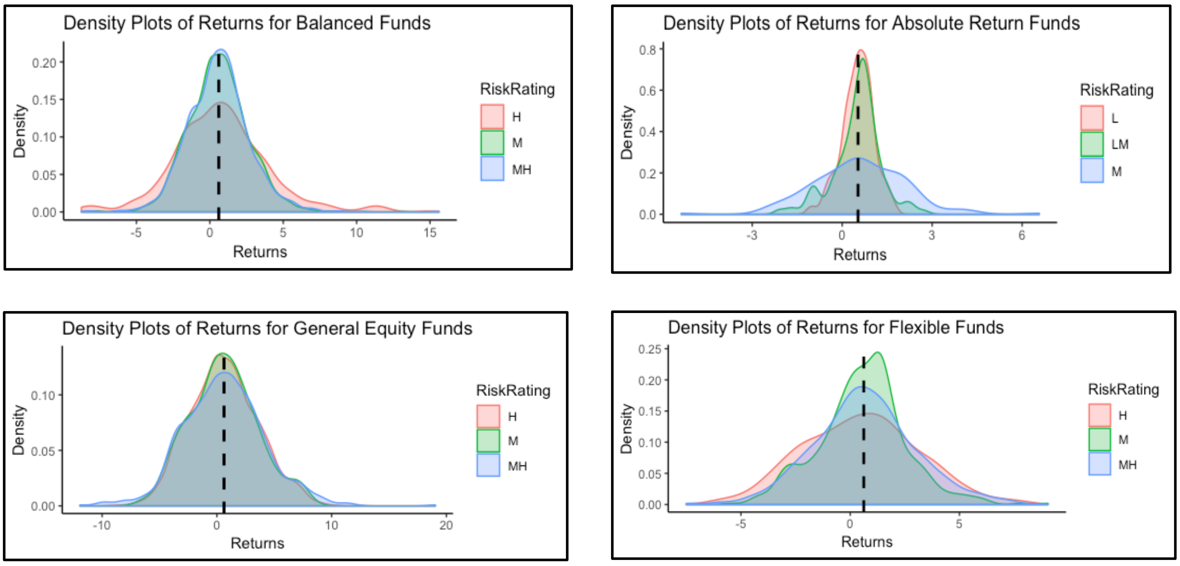

Figure 4.1: Density plots for Fund returns in South Africa

The density plots above reveal the normality of the distributions and the dispersions around the mean of each investment style. The mean variance analysis in Section 4.1.2 will more fully explain the density plots.

Absolute Return Funds appear to be more clustered around the mean than the other investment styles which suggests that they have a lower deviation around their average returns.

4.1.2 MEAN-VARIANCE ANALYSIS

Using the historical returns of Balanced Funds, the Mean-variance framework metrics for each investment strategy are summarised in Table 4.1 below.

Table 4.1: Mean-variance results

Investment Strategy	Mean return (monthly)	Standard Deviation	Variance	Sharpe Ratio	Number of Observations
Balanced High Equity	0.74 %	±3.24 %	10.60	0.225	532
Balanced High-Medium Equity	0.59 %	±2.11 %	4.45	0.280	1 713
Balanced Medium Equity	0.60 %	±2.04 %	4.16	0.295	4 082
General High Equity	0.62 %	±3.00 %	9.03	0.208	6 381
General Medium-High Equity	0.61 %	±3.52 %	12.41	0.174	2 385
General Medium Equity	0.58 %	±2.93 %	8.61	0.198	286
Flexible High Equity	0.47 %	±2.63 %	6.96	0.179	642
Flexible Medium-High Equity	0.66 %	±2.35 %	5.55	0.281	2 079
Flexible Medium Equity	0.56 %	±1.99 %	3.94	0.285	537
Absolute Return Low Risk	0.52 %	±0.52 %	0.27	0.979	260
Absolute Return Low-Medium Risk	0.50 %	0.80 %	0.64	0.626	226
Absolute Return Medium Risk	+0.58 %	1.52 %	2.32	0.378	195

The results reveal that high equity Balanced Funds had the highest overall average return over the sample period whilst high equity Flexible Funds had the lowest overall average return over the sample period. All three Absolute Return Funds had the lowest standard deviations (below 1.52 %) but showed similar levels of return to all three other investment portfolios. This result was expected as per the density plots above. We would expect risk-averse investors to invest in Absolute Return Funds and risk-loving investors to prefer Balanced Funds.

For the pool of Balanced Funds, the results reveal that the risk attached to each investment strategy increases (that is, SD increases from Low Equity Balanced Funds to High Equity Balanced Funds) as you increase the amount of equity in the respective portfolios (that is, increase the concentration of equity assets in an investment portfolio). Similarly, the variance also increases as the concentration in equity assets increases in the portfolio funds. The mean is highest for High Equity Funds (Mean = +0.74 %) which is consistent with the notion that the reward increases with the level of risk and that an all-equity portfolio is riskier than a multi-asset portfolio.

Using simple mean-variance analysis, it is difficult to infer what fund an investor should have invested in over the sample period. However, the Sharpe ratio gives insight on the combined performance of risk and return. The Sharpe ratio is highest for Absolute Return Funds, and highest for the Low-equity Absolute Return Fund. Using Markowitz's mean-variance analysis as a basis, the most attractive investment style over the period is the Low-equity Absolute Return Fund.

Using the Sharpe Ratio as a proxy for the best-performing investment strategy on a (volatility) risk-adjusted basis, we infer the following:

- Overall, the optimal investment portfolio is the Low-risk Absolute Return Fund. It is worth stating again that although Absolute Return Funds and Low-risk Absolute Return Funds returned the highest Sharpe ratios, the sample sizes are different to the sample sizes of the other investment strategies which could affect the statistical power of the analysis.
- For the pool of Absolute Return Funds, the most attractive investment style is the Low-risk Absolute Return Fund.
- For the pool of General Equity Funds, the most attractive investment style is the Medium-equity General Equity Fund.

- For the pool of Flexible Funds, the most attractive investment style is the Medium-equity Flexible Fund.
- For the pool of Balanced Funds, the most attractive investment style is the Medium-equity Balanced Fund.

4.1.3 EXPECTED SHORTFALL

The Expected Shortfall (ES) results for the funds are presented in Table 4.2.

Table 4.2: Expected shortfall for Balanced Funds in South Africa

Investment Strategy	Expected Shortfall
General High-Medium Equity	-6.55 %
Balanced High Equity	-6.04 %
Flexible High-Medium Equity	-5.60 %
General High Equity	-5.10 %
Flexible High Equity	-4.94 %
General Medium Equity	-4.85 %
Flexible Medium Equity	-4.77 %
Balanced Medium-High Equity	-3.67 %
Balanced Medium Equity	-3.48 %
Medium risk Absolute Return	-2.55 %
Low-Medium risk Absolute Return	-1.43 %
Low risk Absolute Return	-0.69 %

The results reveal that, once again, Absolute Return Funds appear the most attractive in terms of having the lowest expected shortfall. All three pools of Absolute Return Fund data have the three lowest levels of ES. It is worth stating again that although Absolute Return Funds and Low-risk Absolute Return Funds returned the lowest Expected Shortfall, the varying sample sizes between the investment strategies could be a factor affecting the statistical power of the analysis. High equity Balanced Funds and High equity General Equity Funds had the two worst ES results, -6.04 % and -6.55 %, respectively.

4.1.4 CUMULITIVE UTILITY ANALYSIS

This section investigates the cumulative utilities for the different investment styles with varying degrees of loss aversion. As discussed in Chapter 3.2.1, Au (2014) and Tversky and Kahneman (1992) use standard values of 0.88 and 0.88 for α and β in the utility function, and different values for λ (the loss aversion parameter). In this section we review three possible values: $\lambda = 1$ (no loss aversion); $\lambda = 2.1$ (Baláž et al.'s (2013) loss aversion parameter) and $\lambda = 2.25$ (Tversky and Kahneman's (1992) loss aversion parameter). As there are 36 combinations of loss aversion parameters (3), investment style (4) and investment style risk category (3), only the cumulative utility outcomes for the for the high-risk investment styles under the three loss aversion parameters are illustrated in Figures 4.5 to 4.7 below. The results for all the combinations are included in Appendix A3. It is important to highlight that, as opposed to Filtered Historical Simulations, the results represent only one set of possible return outcomes.

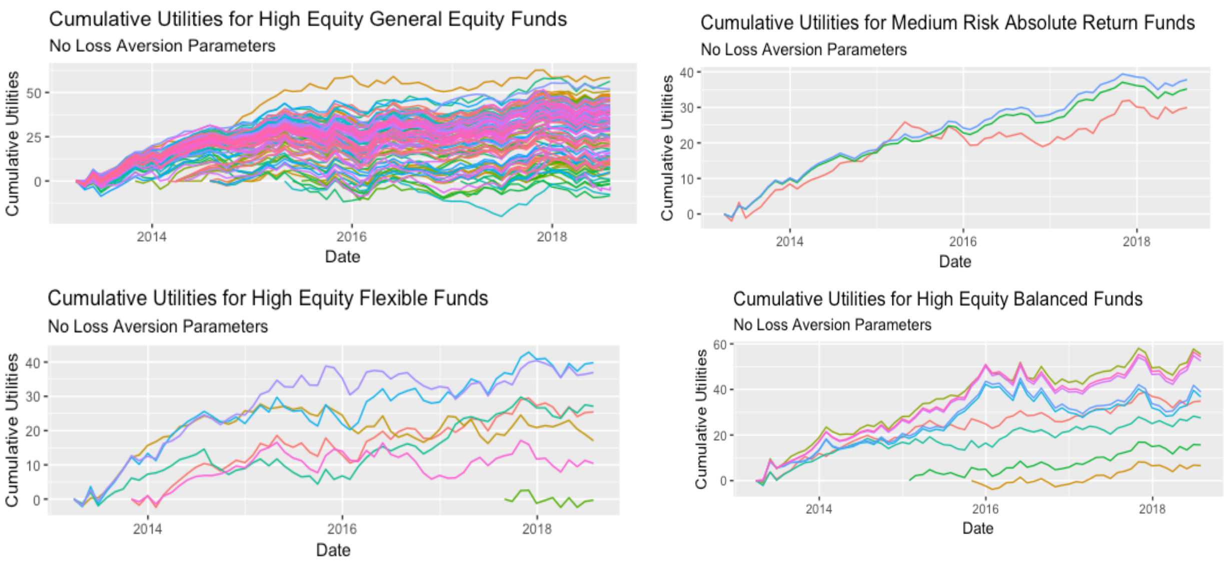

Figure 4.2: Cumulative utilities for High-Risk General Equity, Absolute Return, Flexible and Balanced Funds in South Africa with no "loss aversion" parameters ($\lambda = 1$)

Under the "no loss aversion" parameters, all High Equity Balanced, Flexible and Absolute Return investment portfolios exhibit positive utility at the end of the investment period. There appear to be cumulative utility losses in some General Equity Funds, however, most General Equity Funds result in positive utility. This shows that investors with no loss aversion should be happy to invest in any of the investment styles. An investment in one of the Absolute Return Funds in the pool for an investor with “no loss aversion” had the worst cumulative utility outcome (just below 0) while both Balanced and General Equity Funds trend close to +60 in terms of cumulative utility.

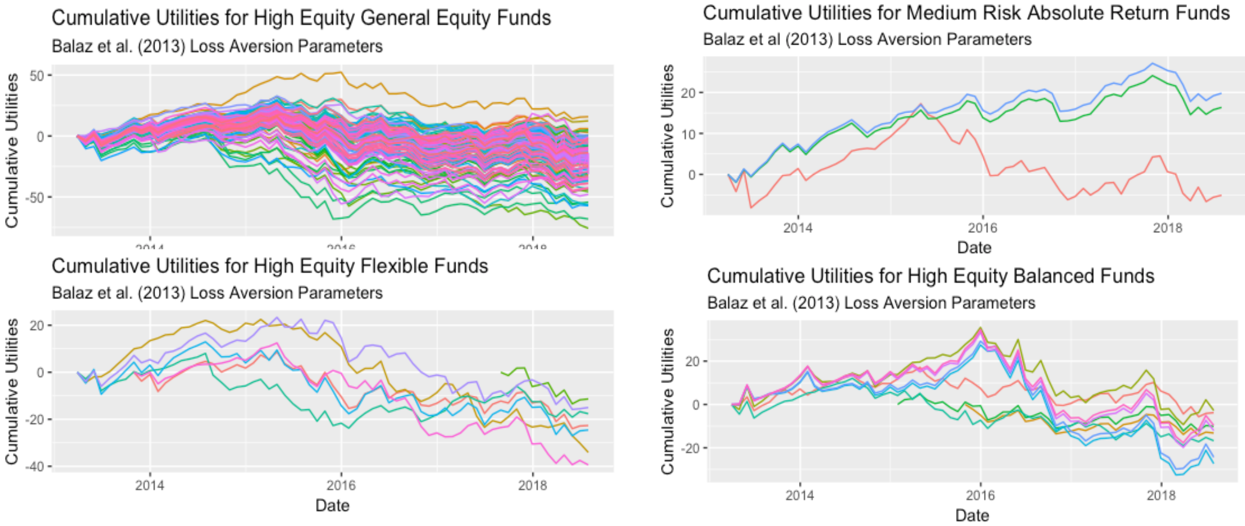

Figure 4.3: Cumulative utilities for General Equity, Absolute Return and Balanced Funds in South Africa using “some loss aversion” parameters (λ = 2.1) (Baláž et al., 2013)

The cumulative utilities for the various (high-risk categories) of the four investment styles under the “some loss aversion” condition (λ = 2.1) (Baláž et al., 2013) are presented in Figure 4.6. It appears as though, under the “some loss aversion” parameters, all High-Equity Flexible and Balanced Funds resulted in a loss of cumulative utility while a majority of General Equity Funds resulted in negative cumulative utility and a majority of Absolute Return Funds generated positive cumulative utility at the end of the investment period.

In Appendix A3 under “Some Loss Aversion”, the vast majority of General Equity Funds result in negative cumulative utility, Balanced Funds are mixed while the majority of Absolute Return Funds result in positive utility at the end of the investment period. This, therefore, shows that investors with some loss aversion would prefer to invest in Absolute Return Funds. Investing in Absolute Return Funds for an investor with “some loss aversion” has the best cumulative utility outcome (close to +40) while investing in General Equity Funds had the worst utility outcome, trending close to -120 in terms of cumulative utility.

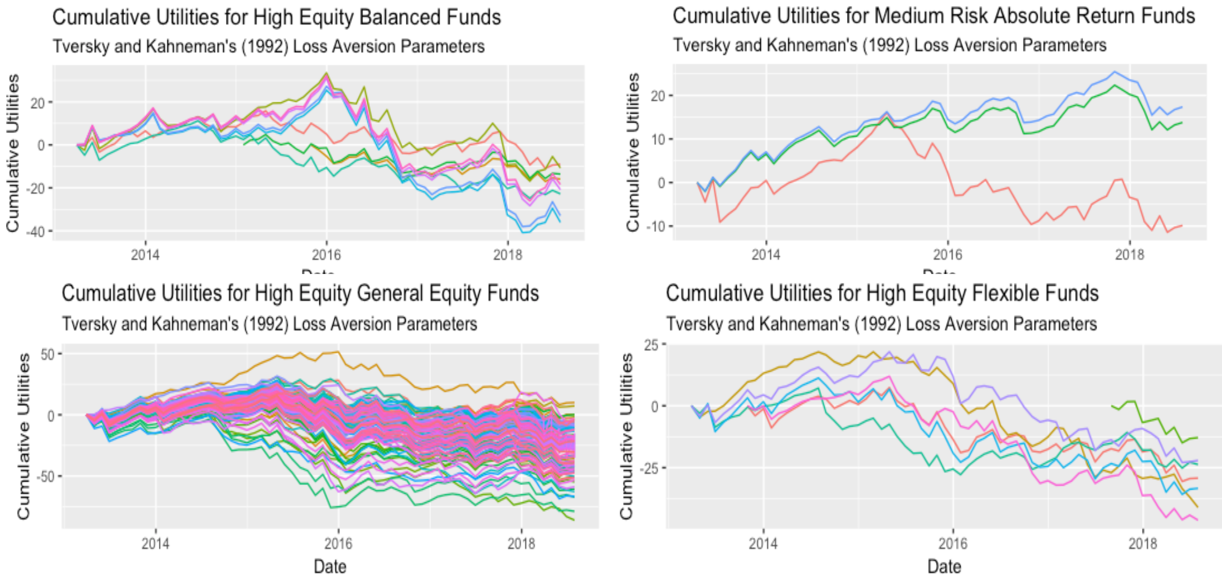

Figure 4.4: Cumulative utilities for High-Risk General Equity, Absolute Return, Flexible and Balanced Funds in South Africa using "loss aversion" parameters (λ = 2.25) (Tversky & Kahneman, 1992)

For investor with high loss aversion (λ = 2.25), all High-Equity Balanced Fund and Flexible Fund investment portfolios result in a decrease of cumulative utility at the end of the investment period whereas utility is cumulatively gained for the majority of the Absolute Return Funds. In addition, it is reported in Appendix A3 that Absolute Return Funds tend to result in higher cumulative utility gains versus Balanced Funds. Similarly, the graphs in Appendix A3 show that General Equity Fund investment portfolios tend to result in losses of cumulative utility when Tversky and Kahneman's (1992) loss aversion parameters are used in the utility function. This therefore shows that investors with high loss aversion should invest in Absolute Return Funds. Investing in Absolute Return Funds for an investor with "high loss aversion" has the best cumulative utility outcome (close to +40) while investing in General Equity Funds had the worst utility outcome, trending close to -120 in terms of cumulative utility.

4.2 FILTERED HISTORICAL SIMULATIONS

Appendix B1 displays the conditional variance dynamics outputs, which describe the parameter specifics for the respective ARIMA and GARCH models fitted to the various investment funds (Balanced, General Equity, Flexible and Absolute Return). As mentioned above, the models and orders were selected based on achieving the lowest possible AIC. Furthermore, the maximum loglikelihood was reached as well as achieving statistically significant parameter estimates. Table 4.3 summarises the selected ARIMA and GARCH models, as well as the relative orders form the conditional variance outputs:

Table 4.3: Summary of ARIMA-GARCH order specifications for investment strategies

Investment Strategy	ARIMA(p, 0, q) - GARCH(1, 1)
Balanced High Equity	ARIMA(1, 0, 2) - EGARCH(1, 1)
Balanced Medium-High Equity	ARIMA(2, 0, 1) - EGARCH(1, 1)
Balanced Medium Equity	ARIMA(2, 0, 2) - EGARCH(1, 1)
Absolute Return Low Risk	ARIMA(2, 0, 2) - EGARCH(1, 1)
Absolute Return Low-medium Risk	ARIMA(1, 0, 2) - EGARCH(1, 1)
Absolute Return Medium Risk	ARIMA(2, 0, 1) - EGARCH(1, 1)
General High Equity	ARIMA(2, 0, 2) -EGARCH(1, 1)
General Medium-High Equity	ARIMA(2, 0, 2) - EGARCH(1, 1)
General Medium Equity	ARIMA(2, 0, 1) - EGARCH(1, 1)
Flexible High Equity	ARIMA(0, 0, 1) - EGARCH(1, 1)
Flexible Medium-High Equity	ARIMA(2, 0, 2) - EGARCH(1, 1)
Flexible Medium Equity	ARIMA(2, 0, 1) - EGARCH(1, 1)

Details of the Autocorrelation Functions can be found in Appendix B5.

4.2.1 MEAN-VARIANCE ANALYSIS

The simulated returns for both strategies under the various risk profiles are provided in Appendix B2, which is a summary of the histograms of simulated returns. For all four investment strategies, the simulated returns reflect similar distributions to the historical data. It is noted that all the simulated returns have a positive means and varying distributions. Table 4.10 provides the means and standard deviations of the distributions of simulated returns shown in the histograms in Appendix B2.

Table 4.4: Simulated mean and variance for Investment Funds in South Africa

Investment Strategy	Mean (x)	Standard Deviation	Sharpe Ratio
Balanced High Equity	0.971 %	±3.50 %	0.277
Balanced High-Medium Equity	0.569 %	±2.15 %	0.265
Balanced Medium Equity	0.626 %	±2.07 %	0.302
Absolute Low Risk	0.538 %	±0.50 %	1.076
Absolute Low-Medium Risk	0.501 %	±0.62 %	0.808
Absolute Medium Risk	0.646 %	±1.50 %	0.430
General Equity High	0.54 %	±2.97 %	0.182
General Equity Medium-High	0.50 %	±3.57 %	0.143
General Equity Medium	0.61 %	±2.97 %	0.205
Flexible Equity High	0.45 %	±2.62 %	0.172
Flexible Equity Medium-High	0.63 %	±2.38 %	0.265
Flexible Equity Medium	0.59 %	±2.11 %	0.280

Using the Sharpe ratio as a proxy for the optimal investment strategy, we infer the following:

- Overall, the optimal investment portfolio is the Low-risk Absolute Return Fund.
- For the pool of Absolute Return Funds, the most attractive investment style is the Low-risk Absolute Return Fund.
- For the pool of General Equity Funds, the most attractive investment style is the Medium-equity General Equity Fund.

- For the pool of Flexible Funds, the most attractive investment style is the Medium-equity Flexible Fund.
- For the pool of Balanced Funds, the most attractive investment style is the Medium-equity Balanced Fund.

4.2.2 CUMULATIVE UTILITY ANALYSIS RESULTS

The results of the simulation analysis of the cumulative utility of the various investment strategies are presented here. Initially the average and standard deviation of the simulated cumulative utility results are presented for each investment strategy modelled under each of the three loss aversion parameters. Then the three distributions representing the various loss aversion parameters are presented for each of the investment style risk categories to illustrate the impact of increasing loss aversion in the context of each strategy. This is then followed by the focus of the study - a comparison of the cumulative utility distributions for the various investment strategies for the three categories of loss aversion. The results of the tests for the difference in means between these investment strategies is finally presented.

Table 4.5: Cumulative utility simulation results - No Loss Aversion

Fund Type	Mean	Standard Deviation
Balanced High	49.6	±19.2
Balanced High-Medium	29.5	±11.9
Balanced Medium	32.7	±13.1
Absolute Medium	35.4	±10.2
Absolute Low-Medium	30.4	± 3.9
Absolute Low	33.2	± 3.2
General High	27.6	±18.5
General Medium-High	25.8	±21.5
General Medium	30.1	±16.7
Flexible High	23.0	±15.4
Flexible Medium-High	33.2	±15.8
Flexible Medium	31.5	±11.9

The full summary of the simulation results is included in Appendix B3. Under "No Loss Aversion" parameters, High Equity Balanced Funds deliver the highest cumulative utility of +120 and also have the highest cumulative utility mean. Medium Equity General Equity Funds deliver the most negative cumulative utility of near -50. Absolute Return Funds exhibit no negative cumulative utility at any point under "No Loss Aversion" parameters. Table 4.6 reveals that Absolute Return funds have the lowest standard deviation in cumulative utilities, that is there is little fluctuation is cumulative utility. It is also observed that cumulative utilities are generally higher than for General Equity funds.

Under "No Loss Aversion" parameters, investors should prefer to invest in Balanced funds for to get the highest expected level of cumulative utility – but Absolute Return funds offer the promise of no negative cumulative utility (at the cost of a lower expected mean).

Table 4.6: Mean and standard deviations of investment strategies under "Some Loss Aversion" parameters

Fund Type	Mean	Standard Deviation
Balanced High	-1.9	±29.3
Balanced High-Medium	-10.9	±22.8
Balanced Medium	-0.6	±22.5
Absolute Medium	15.8	±16.4
Absolute Low-Medium	25.1	±4.0
Absolute Low	30.7	±4.8
General High	-26.3	±31.2
General Medium-High	-38.1	±43.4
General Medium	-22.1	±22.9
Flexible High	-26.0	±26.0
Flexible Medium-High	-5.9	±26.1

Flexible Medium	-1.9	±22.1

Based on Table 4.6 above, investors exhibiting some loss aversion should invest only in Absolute Return Funds. On average, General Equity and Balanced funds generate negative cumulative utility for investors with “some loss aversion”. Balanced Funds and General Equity Funds also exhibit high levels of risk, that is cumulative utilities fluctuate largely around the mean. High Equity Balanced funds delivered the highest level of cumulative utility at +100 whilst Medium-High Equity General Equity Funds delivered the worst negative cumulative utility in the sample at -300.

Under “Some Loss Aversion” parameters, Absolute Return funds should be most attractive to investors.

Table 4.7: Mean and standard deviations of investment strategies under “Loss Aversion” parameters

Fund Type	Mean	Standard Deviation
Balanced High	- 8.9	±30.9
Balanced High-Medium	-10.9	±24.4
Balanced Medium	- 5.2	±23.8
Absolute Medium	13.1	±17.3
Absolute Low-Medium	24.4	± 4.1
Absolute Low	30.4	± 5.1
General High	-33.7	±33.0
General Medium-High	-46.9	±46.6
General Medium	-29.2	±23.7
Flexible High	-32.7	±27.6
Flexible Medium-High	-11.3	±27.6
Flexible Medium	- 6.5	±23.7

Results observed in Table 4.7 are similar to results observed in Table 4.4, investors exhibiting loss aversion should prefer to invest in Absolute Return Funds. On average, General Equity and Balanced funds generate negative cumulative utility for investors with “loss aversion”. Balanced Funds and General Equity Funds also exhibit high levels of risk, that is an investors' cumulative utilities fluctuate largely around the mean. High Equity Balanced Funds delivered the highest level of cumulative utility at +100 whilst Medium-High Equity General Equity Funds delivered the worst negative cumulative utility in the sample at -300.

Under "Loss Aversion" parameters, it is likely that investors would prefer to invest in Absolute Return Funds.

The following section contains a review of the distribution of the simulated cumulative utility values. This illustrates the impact of differing levels of loss aversion on individual investment strategies. The different investment strategies are then compared in the context of each level of loss aversion. The section is concluded with a report of the results of the test for difference in means.

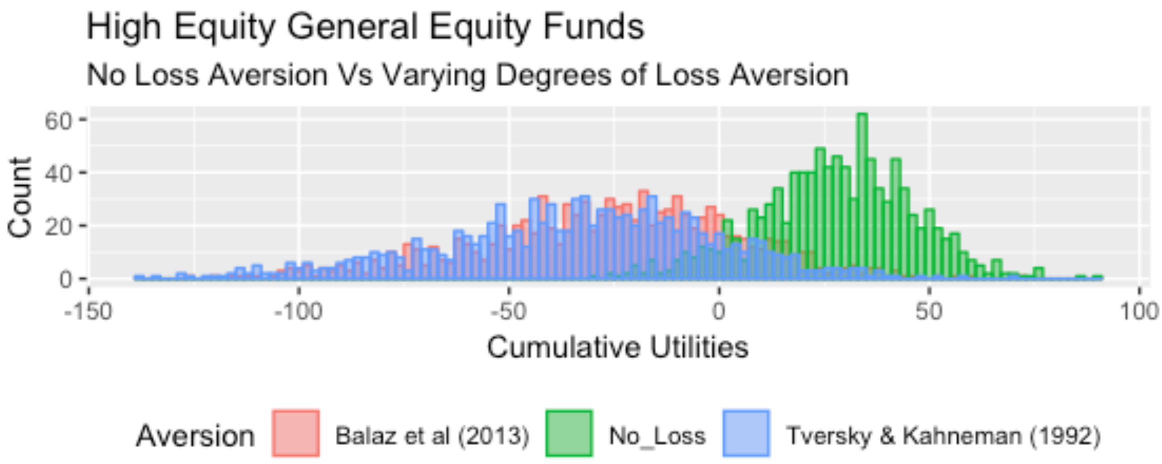

Figure 4.5: Loss aversion impact on cumulative utilities for High Equity General Equity Funds

The impact of risk aversion on cumulative utilities for General Equity Funds is presented in Figure 4.8. Higher cumulative utilities are clustered around the right of Figure 4.8 whilst the introduction of risk aversion results in shifts towards the expected value of the distribution to the left of the figure. This illustrates that, as expected, the introduction of loss aversion parameters has a negative effect on cumulative utilities

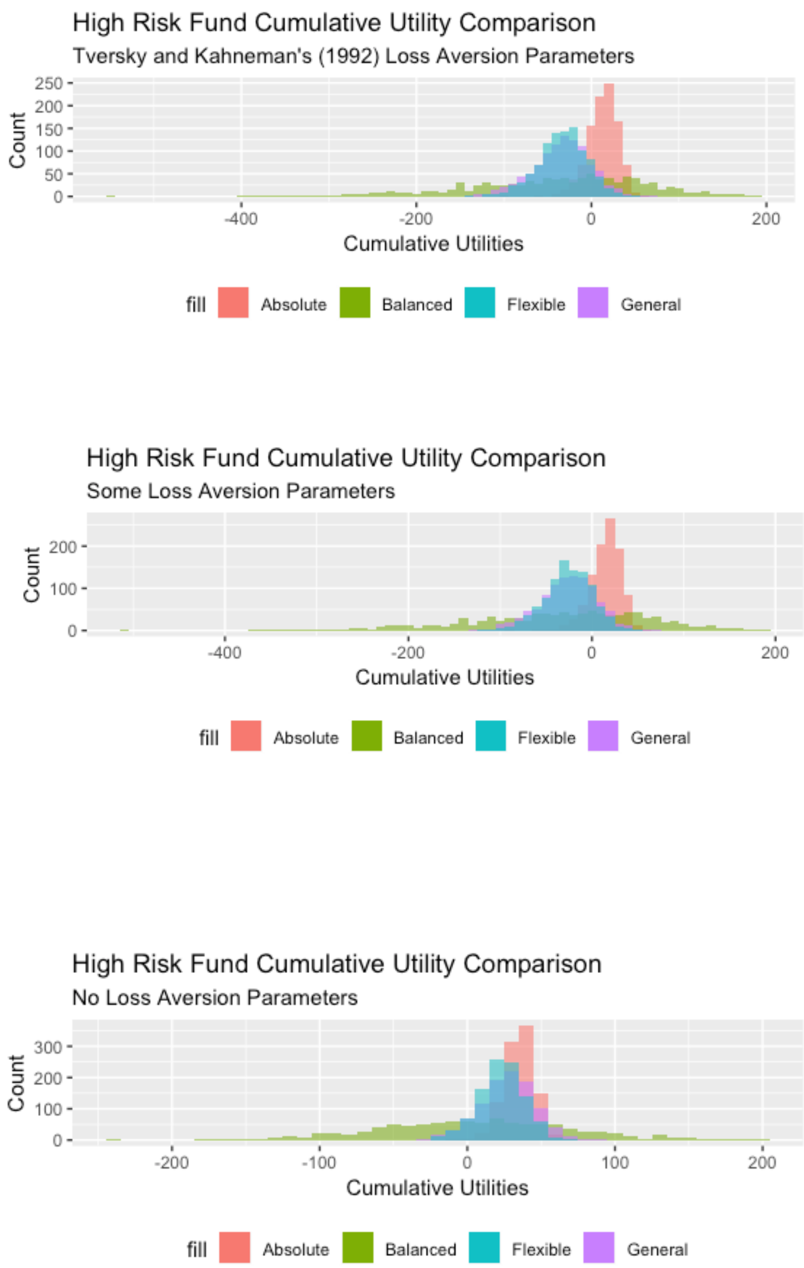

Figure 4.6: Loss aversion impact on cumulative utilities for different High-Risk investment strategies

The impact of risk aversion on cumulative utilities for High-Risk versions of the four investment strategies are presented in Figure 4.9. Higher cumulative utilities are clustered around the right of the first graph in Figure 4.9, which is "no loss aversion" parameters whilst the introduction of risk aversion results in shifts towards the expected value of the distribution to the left of the respective figures in Figure 4.9. This illustrates that, as expected, the introduction of loss aversion parameters has a negative effect on cumulative utilities.

A t-test of for the difference in means for High-Risk investment strategies under the three loss aversion parameter values show that the cumulative utilities are statistically different in the case of the high loss aversion (Tversky & Kahneman, 1992) parameters except in the case of General Equity Funds and Flexible Funds. The lack of difference becomes more apparent as the level of loss aversion drops.

Table 4.8: High Risk Funds Cumulative Utility T-test Results

Funds	p-values	Loss Aversion Parameters
Balanced v Absolute	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
Absolute v General	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
General v Flexible	0.4872	Tversky & Kahneman (1992) Loss Aversion Parameters
Flexible v Balanced	0.002777	Tversky & Kahneman (1992) Loss Aversion Parameters
Balanced v Absolute	0.00	Baláž et al. (2013) Loss Aversion Parameters
Absolute v General	0.00	Baláž et al. (2013) Loss Aversion Parameters
General v Flexible	0.8277	Baláž et al. (2013) Loss Aversion Parameters
Flexible v Balanced	0.00	Baláž et al. (2013) Loss Aversion Parameters
Balanced v Absolute	0.00	No Loss Aversion Parameters
Absolute v General	0.00	No Loss Aversion Parameters
General v Flexible	0.00	No Loss Aversion Parameters
Flexible v Balanced	0.00	No Loss Aversion Parameters

Table 4.8 confirms that the introduction of loss aversion parameters has an impact on the means of the cumulative utilities of each investment style. Under “No Loss Aversion” parameters, the funds are mostly not statistically different as they all derive some level of positive cumulative utility. However, as risk aversion parameters are introduced, the means of the investment style become increasingly statistically different. Absolute Funds are statistically different to the other fund types. Absolute return funds stay to the right of the graphs, in positive utility whilst the other funds gravitate towards the left of the graph, signalling decreasing utility.

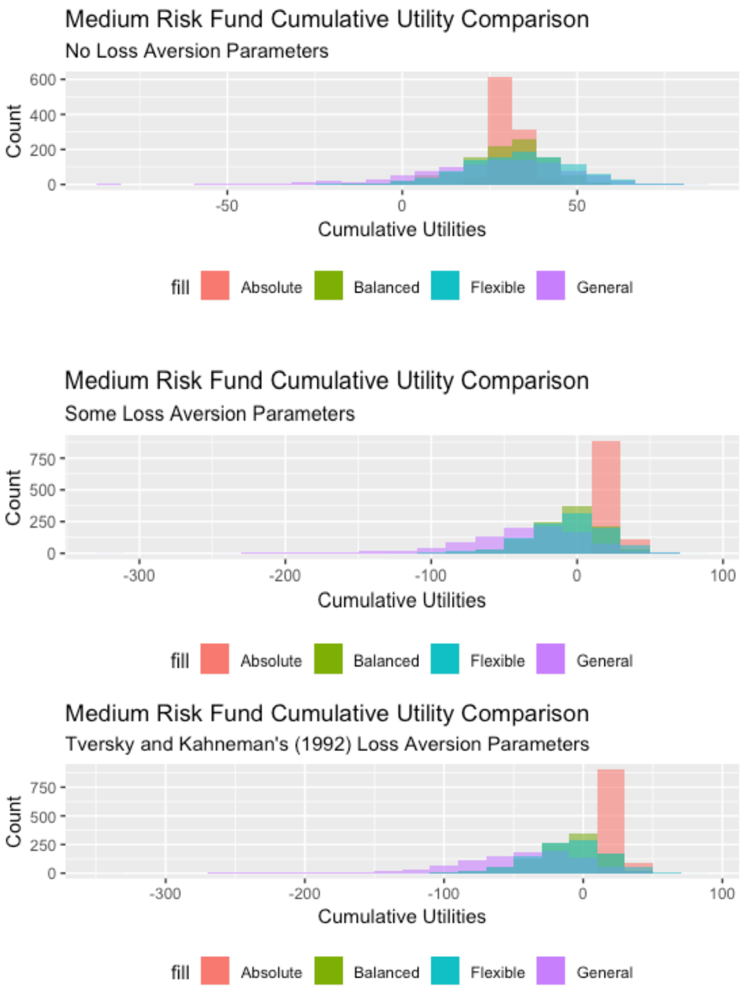

Figure 4.7: Loss aversion impact on cumulative utilities for Medium Risk Funds

The impact of risk aversion on cumulative utilities for Medium Risk Funds is presented in Figure 4.10. Higher cumulative utilities are clustered around the right of top graph in Figure 4.10, which is “no loss aversion” parameters whilst the introduction of risk aversion results in shifts towards the expected value of the distribution to the left of the respective figures in Figure 4.10. This illustrates that, as expected, the introduction of loss aversion parameters has a negative effect on cumulative utilities.

Table 4.9: Medium Risk Funds Cumulative Utility T-test Results

Funds	p-value	Loss Aversion Parameters
Balanced v Absolute	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
Absolute v General	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
General v Flexible	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
Flexible v Balanced	0.7215	Tversky & Kahneman (1992) Loss Aversion Parameters
Balanced v Absolute	0.00	Baláž et al. (2013) Loss Aversion Parameters
Absolute v General	0.00	Baláž et al. (2013) Loss Aversion Parameters
General v Flexible	0.00	Baláž et al. (2013) Loss Aversion Parameters
Flexible v Balanced	0.9401	Baláž et al. (2013) Loss Aversion Parameters
Balanced v Absolute	0.01779	No Loss Aversion Parameters
Absolute v General	0.00	No Loss Aversion Parameters
General v Flexible	0.00	No Loss Aversion Parameters
Flexible v Balanced	0.00	No Loss Aversion Parameters

Table 4.9 confirms that the introduction of loss aversion parameters has an impact on the means of the cumulative utilities of each investment style. Under “No Loss Aversion” parameters, the funds are mostly not statistically different as they derive positive utility.

However, as risk aversion parameters are introduced, the means of the investment style become statistically different. Absolute Funds are statistically different to the other fund types. Absolute Return Funds stay to the right of the graphs, in positive utility, whilst the other funds gravitate towards the left of the graph, signalling decreasing utility.

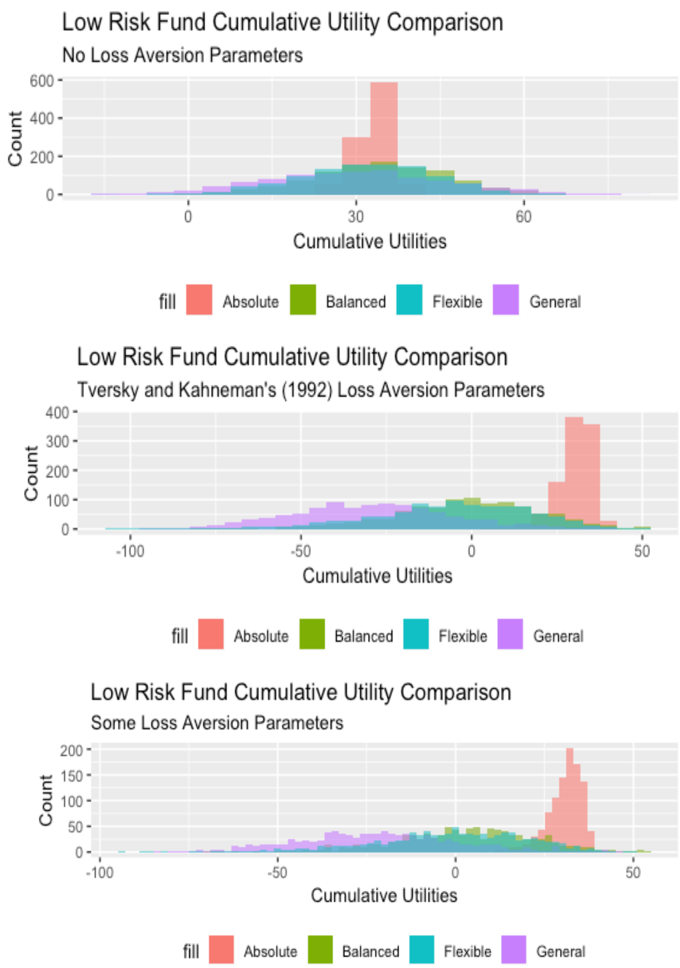

Figure 4.8: Loss aversion impact on cumulative utilities for Low Risk Funds

The impact of risk aversion is consistent across Figures 4.8, 4.9 and 4.10; that, as risk aversion is introduced, the cumulative utilities of investors shifts towards the left. Absolute Return Funds remain consistently in positive territory and thus are consistent with the observed statistical difference of other investment funds.

Table 4.10: Low Risk Funds Cumulative Utility T-test Results

Funds	p-values	Loss Aversion Parameters
Balanced v Absolute	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
Absolute v General	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
General v Flexible	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
Flexible v Balanced	0.00	Tversky & Kahneman (1992) Loss Aversion Parameters
Balanced v Absolute	0.00	Baláž et al (2013) Loss Aversion Parameters
Absolute v General	0.00	Baláž et al (2013) Loss Aversion Parameters
General v Flexible	0.00	Baláž et al (2013) Loss Aversion Parameters
Flexible v Balanced	0.9401	Baláž et al (2013) Loss Aversion Parameters
Balanced v Absolute	0.001371	No Loss Aversion Parameters
Absolute v General	0.00	No Loss Aversion Parameters
General v Flexible	0.00	No Loss Aversion Parameters
Flexible v Balanced	0.00	No Loss Aversion Parameters

Table 4.10 illustrates that the means of the cumulative utilities for the low-risk portfolios are all statistically different as loss aversion parameters are introduced.

CHAPTER 5:
CONCLUSION

The purpose of this study was to determine which investment style/strategy under observation, between Absolute Return, Balanced, General Equity and Flexible Funds would best suit investors under a Prospect Theory Framework. The Prospect Theory Framework introduces behavioural/subjective analysis to modern portfolio theory and mean-variance analysis as was developed by Markowitz (1952). The introduction of the S-shaped value function under Prospect Theory was the basis on which a distinction was made between an investor's responsiveness to gains and losses. Responsiveness to gains and losses is a function of an investor's relative risk aversion, which is determined subjectively. The subjective/behavioural analysis is anchored around the varying utility functions assigned to investors with varying levels of risk aversion. Work done by Tversky and Kahneman (1992), Au (2014) and Baláž et al. (2013) formed the basis of developing the utility functions assigned to investors with varying degrees of risk aversion. Two methods were used to find our results, namely Historical Analysis and Filtered Historical Simulations (FHS).

Under the historical analysis method, Absolute Return Funds offer a similar return to other investment styles, but for a far lower observed level of risk (SD). The observed Sharpe ratios are highest for Absolute Return Funds and thus the risk-return trade-off is in favour of investing in Absolute Return Funds. Thus, we infer a preference for Absolute Return Funds.

Looking purely at mean-variance optimisation, Expected Shortfall (ES) and observed Sharpe ratios are insufficient to answer the research question. The paper requires us to evaluate cumulative utilities in order to answer the over-arching research question.

Under the historical analysis method, incorporating Tversky and Kahneman (1992), Au (2014) and Baláž et al.'s (2013) loss aversion parameters, the majority of Absolute Return Funds were found to have generated positive cumulative utility and outperformed other investment styles over the observation period. On average, Absolute Return Funds exhibited positive cumulative utility despite rising levels of risk aversion. It is only in the case of "No Loss Aversion" parameters where Balanced Funds outperformed the other investment styles/strategies.

An interesting observation is that of Balanced Funds. Balanced Funds appear to have the highest average returns in the pool of data but also high levels of negative cumulative utility when risk aversion parameters are introduced. This shows that even though the returns of Balanced Funds might be attractive, the extent of the variation around the mean is extensive and results in a negative cumulative utility outcome for investors with some risk aversion and total risk aversion.

Under the FHS method, when loss aversion parameters are introduced, on average simulated investor cumulative utilities are negative for all investment styles (Flexible, Balanced and General Equity) with the exception of Absolute Return Funds. Absolute Return Funds under the cumulative utility simulation analysis would have been the best investment style/fund for investors with varying degrees of loss aversion. With "no loss" aversion parameters, the high-equity Balanced Funds delivered the highest return for the comparatively highest risk (SD). However, Absolute Return Funds delivered the best returns for moderate levels of risk.

It is worth noting that the funds under observation are only funds which did not close during the observation period, and thus the conclusion and recommendations thereof are open to survivorship bias.

In conclusion, this analysis suggests that Absolute Return Funds are the preferred investment style, particularly when risk-aversion parameters are introduced under the FHS method. High Equity Balanced Funds are the preferred investment style for investors with "no loss" aversion under the FHS method if investors are seeking to maximise cumulative utility and maximise simulated returns. However Absolute Return Funds had the highest Sharpe ratios which imply a better risk-return trade-off.

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APPENDICES

A. Historical Analysis

A1 Fund Types and Classifications

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APPENDIX A: HISTORICAL ANALYSIS

APPENDIX A1: FUND TYPES AND RISK CLASSIFICATIONS

Fund Type	Classification
Balanced Fund	High Equity
Balanced Fund	Medium Equity
Balanced Fund	Low Equity
General Equity	High
General Equity	High-Medium
General Equity	Medium
Absolute Return	Medium Risk
Absolute Return	Low-Medium Risk
Absolute Return	Low Risk
Flexible	High
Flexible	High-Medium
Flexible	Medium

APPENDIX A2: LIST OF FUNDS

Table A2.1: List of Balanced Funds

Fund Names – Balanced Funds		
27four Balanced Prescient Fund of Funds	Fairtree Flexible Balanced Prescient Fund	Old Mutual Core Balanced Fund
Absa Balanced Fund	FAL BCI Balanced Fund	Old Mutual Moderate Balanced Fund
Absa Balanced Fund Class A	Foord Balanced Fund Class A	Old Mutual Multi-Managers Aggressive Balanced Fund of Funds
ADB BCI Balanced Fund of Funds	Foord Balanced Fund Class B2	Old Mutual Multi-Managers Balanced Fund of Funds
Alexander Forbes Investments Balanced Fund of Funds	Foord Balanced Fund Class R	Old Mutual Multi-Managers Aggressive Balanced Fund of Funds
Allan Gray Balanced Fund Class A & B	GCI SCI Balanced Fund of Funds	Old Mutual Multi-Managers Balanced Fund of Funds
Allan Gray Tax-Free Balanced Fund Class A & B	GCI SCI Balanced Plus Fund of Funds	Optimum BCI Balanced Fund
Analytics Ci Balanced Fund of Funds	IFM Balanced Value Fund of Funds	PBi BCI Balanced Fund of Funds
Ashburton Balanced Fund Class A & B	Imalivest Sanlam Collective Investments Balanced Fund	Platinum BCI Balanced Fund of Funds
AssetMix Ci Balanced Fund of Funds	Kagiso Balanced Fund	Plexus Wealth BCI Balanced Fund
Autus BCI Balanced Fund	Kagiso Islamic Balanced Fund	Point3 BCI Balanced Fund of Funds
BCI Best Blend Balanced Fund	Kanaan BCI Balanced Fund of Funds	PPS Balanced Fund of Funds Class A1
BCI BetaPlus Balanced Fund	Kruger Ci Balanced Fund of Funds	PPS Balanced Fund of Funds Class A2
Brenthurst BCI Balanced Fund of Funds	Lynx Prime Balanced Fund of Funds Class A1	PPS Balanced Index Tracker Fund
Bridge Balanced Fund Class A	Lynx Prime Balanced Fund of Funds Class C1	PPS Global Balanced Fund of Funds
Bridge Balanced Fund Class C	Marriott Balanced Fund of Funds Class A	Prudential Balanced Fund
Cadiz Balanced Fund	Marriott Balanced Fund of Funds Class C	Prudential Namibian Balanced Fund
Caleo BCI Balanced Fund of Funds	Median BCI Balanced Fund of Funds	PSG Balanced Fund Class A
Capita BCI Balanced Fund	Momentum International Balanced Feeder Fund	PSG Balanced Fund Class E
Capstone BCI Balanced Fund	Multi Asset IP Balanced Fund	Quantum BCI Balanced Fund of Funds
Centaur BCI Balanced Fund	Multi Asset IP Balanced Plus Fund	Red Oak BCI Balanced Fund
Citadel Balanced H4 Fund	NAM Coronation Balanced Plus Fund	Rowan Capital BCI Balanced Fund of Funds
Coronation Balanced Plus Fund Class A	Naviga BCI Balanced Growth Fund of Funds	S BRO BCI Balanced Fund of Funds
Coronation Balanced Plus Fund Class P	Nedgroup Investments Balanced Fund	Sanlam Global Balanced Fund of Funds
Counterpoint SCI Balanced Plus Fund	Noble PP STANLIB Balanced Fund of Funds	Sanlam Investment Management Balanced Fund
Denker Sanlam Collective Investments Balanced Fund	Oasis Balanced Unit Trust Fund	Sanlam Investment Management Balanced Fund Class A
Discovery Balanced Fund	Oasis Crescent Balanced High Equity Fund of Funds	Sanlam Multi Managed Balanced Fund of Funds
Discovery Global Balanced Fund of Funds	Oasis Crescent Balanced Progressive Fund of Funds	Sanlam Namibia Balanced Fund
Edge BCI Balanced Fund	Old Mutual Albaraka Balanced Fund	Sanlam Private Wealth Balanced Fund
EFPC BCI Balanced Fund	Old Mutual Balanced Fund Class A	Sasfin BCI Balanced Fund

Fund Names – Balanced Funds		
Seed Balanced Fund	Old Mutual Balanced Fund Class R	
Select BCI Balanced Fund		
Sharenet BCI Balanced Fund		
Sharenet BCI Global Balanced Fund of Funds		
Signature BCI Balanced Fund of Funds		
Southern Charter BCI Balanced Fund of Funds		
STANLIB Balanced Fund Class A		
STANLIB Balanced Fund Class B1		
STANLIB Balanced Fund Class R		
STANLIB Global Balanced Cautious Feeder Fund Class B		
STANLIB Global Balanced Feeder Fund Class A		
STANLIB Global Balanced Feeder Fund Class B		
STANLIB Multi-Manager Balanced Fund Class A		
STANLIB Multi-Manager Balanced Fund Class B		
Tr_sor Sanlam Collective Investments Balanced Fund		
Warwick BCI Balanced Fund		
Warwick BCI Balanced Fund of Funds		
Wealth Associates BCI Balanced Fund of Funds		
STANLIB Global Balanced Cautious Feeder Fund Class B		
STANLIB Global Balanced Feeder Fund Class A		
STANLIB Global Balanced Feeder Fund Class B		
STANLIB Multi-Manager Balanced Fund Class A		
STANLIB Multi-Manager Balanced Fund Class B		
Tr_sor Sanlam Collective Investments Balanced Fund		
Warwick BCI Balanced Fund		
Warwick BCI Balanced Fund of Funds		
Wealth Associates BCI Balanced Fund of Funds		

Table A2.2: List of Absolute Return Funds

Fund Names – Absolute Return Funds
Absa Absolute Fund
Argon BCI Absolute Return Fund
Cadiz Absolute Yield Fund
Investec Absolute Balanced Fund Class A
Investec Absolute Balanced Fund Class B
Prescient Absolute Balanced Fund
STANLIB Absolute Plus Fund Class A
STANLIB Absolute Plus Fund Class B
STANLIB Multi-Manager Absolute Income Fund Class A
STANLIB Multi-Manager Absolute Income Fund Class B
Stewart BCI Absolute Return Blend Fund of Funds

Table A2.3: List of General Equity Funds

Fund Names – General Equity Funds		
27four Shari'ah Active Equity Prescient Fund	Coronation Equity Fund Class P	Kagiso Islamic Equity Fund
36ONE BCI Equity Fund	Coronation Top 20 Fund Class A	Kruger Ci Equity Fund
36ONE BCI SA Equity Fund	Coronation Top 20 Fund Class P	Lynx Prime Opportunities Fund of Funds
36ONE BCI SA Equity Fund Class C1	Counterpoint SCI Dividend Equity Fund	Lynx Prime Opportunities Fund of Funds Class A2
Absa Prime Equity Fund	Counterpoint SCI Value Fund	Maestro Equity Prescient Fund
Absa SA Core Equity Fund	Cratos BCI Equity Fund	Marriott Dividend Growth Fund Class B
Absa Select Equity Fund	Denker Sanlam Collective Investments SA Equity Fund	Marriott Dividend Growth Fund Class R
Alexander Forbes Investments Equity Fund of Funds	Discovery Dynamic Equity Fund	Mergence Equity Prescient Fund
Allan Gray Equity Fund Class A	Discovery Equity Fund	MI-PLAN IP Beta Equity Fund
Allan Gray Equity Fund Class C	Dotport BCI Equity Fund	Momentum Real Growth Equity Fund
Aluwani Top 25 Fund	Element Earth Equity Sanlam Collective Investments Fund	N-e-FG BCI Equity Fund
Aluwani Top 25 Fund Class A	Element Islamic Equity Sanlam Collective Investments Fund	Naviga BCI SA Equity Fund Class A1
Analytics Ci Managed Equity Fund	Excelsia Equity ACI Fund	Naviga BCI SA Equity Fund Class B
Anchor BCI Equity Fund	Fairtree Equity Prescient Fund	Nedgroup Investments Growth Fund
Anchor BCI SA Equity Fund	FG IP Mercury Equity Fund of Funds	Nedgroup Investments Growth Fund Class A
APS Ci Equity Fund of Funds	First Avenue Sanlam Collective Investments Equity Fund	Nedgroup Investments Private Wealth Equity Fund
Ashburton Equity Fund	First Avenue SCI Focused Quality Equity Fund	Nedgroup Investments Rainmaker Fund
Ashburton Equity Fund Class B1	FNB Momentum Growth Fund	Nedgroup Investments Rainmaker Fund Class A
Ashburton Multi Manager Equity Fund Class A1	Foord Equity Fund	Nedgroup Investments Value Fund
Ashburton Multi Manager Equity Fund Class B1	Foord Equity Fund Class B2	Nedgroup Investments Value Fund Class A
Autus BCI Equity Fund	Foord Equity Fund Class B4	Northstar Sanlam Collective Investments Equity Fund
Aylett Equity Prescient Fund	Gryphon All Share Tracker Fund	Oasis Crescent Equity Fund
BCI Best Blend Specialist Equity Fund	H4 Focused Wealth Fund	Oasis General Equity Fund
Benguela Equity ACI Fund	Harvard House BCI Equity Fund	Obsidian Sanlam Collective Investments Equity Fund
BlueAlpha BCI Equity Fund Class A	IFM Technical Fund	Old Mutual Albaraka Equity Fund
BlueAlpha BCI Equity Fund Class B	Imara BCI Equity Fund	Old Mutual Equity Fund
BlueAlpha BCI Equity Fund Class C	IMI IP Equity Fund	Old Mutual Growth Fund
Bridge Equity Income Growth Fund	Integral BCI Equity Fund	Old Mutual Growth Fund Class A
Bridge Equity Income Growth Fund Class A	Investec Equity Fund	Old Mutual Investors' Fund
Bridge Equity Income Growth Fund Class C	Investec Equity Fund Class A	Old Mutual Investors' Fund Class A
Cadiz Equity Fund	Investec Equity Fund Class E	Old Mutual Managed Alpha Equity Fund
Caleo BCI Equity Fund	Investec Equity Fund Class G	Old Mutual Multi-Managers Equity Fund of Funds
Cannon Equity H4 Fund	Investec Equity Fund Class H	Old Mutual RAFI 40 Index Fund
Citadel SA Equity H4 Fund	Investec Value Fund	Old Mutual Top Companies Fund
Citadel SA Multi Factor Equity H4 Fund	Investec Value Fund Class A	Old Mutual Top Companies Fund Class A
ClucasGray Equity Prescient Fund	Investec Value Fund Class E	Optimum BCI Equity Fund
Colourfield BCI Equity Fund	Investec Value Fund Class G	Personal Trust Equity Fund
Community Growth Equity Fund	Investec Value Fund Class H	PortfolioMetrix BCI Equity Fund of Funds
CoreShares Scientific Beta Multi Factor Index Fund	IP Momentum Equity Fund	PPS Equity Fund
Coronation Equity Fund Class A	Kagiso Equity Alpha Fund	PPS Equity Fund Class A2

Fund Names – General Equity Funds		
Prudential Dividend Maximiser Fund	Tower Capital Equity Prescient Fund	
Prudential Equity Fund	Truffle Sanlam Collective Investments General Equity Fund	
PSG Equity Fund	Visio BCI General Equity Fund	
PSG Equity Fund Class E		
PSG Wealth Creator Fund of Funds		
PSG Wealth Creator Fund of Funds Class C		
RECM Equity Fund		
Rezco Equity Fund		
Sanlam Investment Management General Equity Fund Class A		
Sanlam Investment Management General Equity Fund Class R		
Sanlam Multi Managed Equity Fund of Funds		
Sasfin BCI Equity Fund		
Satrix Alsi Index Fund		
Satrix Alsi Index Fund Class A1		
Satrix Alsi Index Fund Class B1		
Satrix DIVI ETF		
Satrix Mid Cap Index Fund		
Satrix Quality Index Fund		
Satrix Quality South Africa ETF		
Satrix RAFI 40 ETF		
Satrix RAFI 40 Index Fund		
Seed Equity Fund		
Select BCI Equity Fund		
Sharenet BCI Equity Fund		
SIM Top Choice Equity Fund Class A		
SIM Top Choice Equity Fund Class D		
SIM Value Fund		
STANLIB Equity Fund Class A		
STANLIB Equity Fund Class R		
STANLIB Index Fund		
STANLIB Multi-Manager Diversified Equity Fund of Funds		
STANLIB Multi-Manager Diversified Equity Fund of Funds		
STANLIB Multi-Manager Diversified Equity Fund of Funds Class A		
STANLIB Multi-Manager SA Equity Fund Class A		
STANLIB Multi-Manager SA Equity Fund Class B1		
STANLIB SA Equity Fund Class A		
STANLIB SA Equity Fund Class R		
STANLIB Sector Neutral Momentum Index Tracker Fund		
STANLIB Sector Neutral Value Index Tracker Fund		

Table A2.4: List of Flexible Funds

Fund Names – Flexible Funds		
36ONE BCI Flexible Opportunity Fund Class A	STANLIB Multi-Manager Flexible Property Fund Class A	
36ONE BCI Flexible Opportunity Fund Class B	STANLIB Multi-Manager Flexible Property Fund Class B1	
4D BCI Flexible Fund	STANLIB Quants Fund Class A	
Amity BCI Flexible Growth Fund of Funds	STANLIB Quants Fund Class B1	
Anchor Securities BCI Flexible Fund	Stringfellow BCI Flexible Fund of Funds	
Ashburton Growth Fund	Triathlon IP Fund	
Autus BCI Opportunity Fund	True North IP Flexible Equity Fund	
BlueAlpha BCI All Seasons Fund	Truffle Sanlam Collective Investments Flexible Fund Class A	
Centaur BCI Flexible Fund	Truffle Sanlam Collective Investments Flexible Fund Class B	
Citadel SA Protected Equity H4 Fund	Visio BCI Actinio Fund	
ClucasGray Future Titans Prescient Fund		
CS BCI Flexible Fund of Funds		
Denker Sanlam Collective Investments Flexible Fund		
Destiny BCI Multi Asset Fund of Funds		
Dotport BCI Flexible Fund of Funds		
Element Flexible Sanlam Collective Investments Fund		
Flagship IP Flexible Value Fund		
GCI SCI Flexible Fund of Funds		
Gryphon Flexible Fund of Funds		
IP Flexible Fund of Funds		
IP Flexible Fund of Funds Class A		
Maitland BCI Flexible Fund of Funds		
Marriott Property Equity Fund		
Melville Douglas Dynamic Strategy Fund		
N-e-FG BCI Flexible Fund		
Naviga BCI Flexible Fund of Funds		
Noble PP STANLIB All Weather Fund of Funds		
Noble PP STANLIB Flexible Fund		
Old Mutual Flexible Fund Class A		
Old Mutual Flexible Fund Class R		
Optimum BCI Flexible Fund		
Plexus Wealth BCI Flexible Property Income Fund		
PSG Flexible Fund Class A		
PSG Flexible Fund Class E		
RCI BCI Flexible Fund		
Salvo NCIS Dynamic Flexible Fund		
Select Manager BCI Flexible Equity Fund		
Sharenet BCI Flexible Fund		
STANLIB Aggressive Income Fund Class A		

APPENDIX B: FILTERED HISTORICAL SIMULATIONS

Appendix B1: Fitted GARCH Model Outputs

Table B1.1: Medium equity Balanced Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.63167776	0.02602305	24.273782	0.000000e+00
ar1	-0.77817933	0.02721715	-28.591509	0.000000e+00
ar2	-0.60920823	0.02386541	-25.526831	0.000000e+00
ma1	0.62551092	0.02756121	22.695338	0.000000e+00
ma2	0.54321111	0.02735092	19.860799	0.000000e+00
omega	0.13104523	0.02228158	5.881325	4.069956e-09
alpha1	-0.07173594	0.01416525	-5.064220	4.100763e-07
beta1	0.89996848	0.01666949	53.988973	0.000000e+00
gamma1	0.20823859	0.02526159	8.243288	2.220446e-16

Table B1.2: High equity Balanced Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.98393315	0.10233526	9.614801	0.000000e+00
ar1	-0.96559949	0.03543745	-27.247993	0.000000e+00
ma1	0.80559672	0.02681667	30.040893	0.000000e+00
ma2	-0.14940464	0.01709013	-8.742162	0.000000e+00
omega	0.44727200	0.13819528	3.236521	1.209961e-03
alpha1	0.06219609	0.05287697	1.176242	2.394983e-01
beta1	0.80465785	0.06163198	13.055848	0.000000e+00
gamma1	0.45741906	0.11671781	3.919017	8.891095e-05

Table B1.3: High-medium equity Balanced Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.56177721	0.0353565631	15.888909	0.000000e+00
ar1	-1.15998161	0.0195317603	-59.389507	0.000000e+00
ar2	-0.21517702	0.0185086582	-11.625749	0.000000e+00
ma1	0.96639139	0.0004902218	1971.334970	0.000000e+00
omega	0.06980617	0.0308838311	2.260282	2.380374e-02
alpha1	-0.12564989	0.0179464396	-7.001382	2.534417e-12
beta1	0.94905255	0.0201984809	46.986333	0.000000e+00
gamma1	0.09616128	0.0192429645	4.997218	5.816337e-07

Table B1.4: Low risk Absolute Return Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.504466019	0.0006633451	760.488058	0.000000e+00
ar1	-0.464927888	0.0836482507	-5.558130	2.726796e-08
ma1	0.376168854	0.1137705836	3.306381	9.450961e-04
ma2	-0.090273081	0.0173216585	-5.211573	1.872465e-07
omega	0.004323277	0.0004560632	9.479558	0.000000e+00
alpha1	0.071799523	0.0010209809	70.324059	0.000000e+00
beta1	0.985740470	0.0001100333	8958.564437	0.000000e+00
gamma1	-0.117079986	0.0009328892	-125.502570	0.000000e+00

Table B1.5: Low-medium risk Absolute Return Funds (gjrGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.64496510	0.09741299	6.6209350	3.569345e-11
ar1	-0.92510545	0.09588615	-9.6479570	0.000000e+00
ar2	-0.05514598	0.06843371	-0.8058307	4.203405e-01
ma1	0.92252779	0.05361063	17.2079278	0.000000e+00
omega	0.91648646	0.30529438	3.0019762	2.682332e-03
alpha1	0.14527617	0.09138864	1.5896523	1.119132e-01
beta1	0.21659025	0.16666363	1.2995652	1.937500e-01
gamma1	0.46372293	0.25375914	1.8274137	6.763759e-02

Table B1.6: Medium risk Absolute Return Funds (gjrGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.5561994	0.03572798	15.567616	0.000000e+00
ar1	-0.8425376	0.05481155	-15.371533	0.000000e+00
ar2	-0.8249226	0.04857608	-16.982074	0.000000e+00
ma1	0.7377103	0.06205645	11.887730	0.000000e+00
ma2	0.7875738	0.04582926	17.184955	0.000000e+00
omega	0.2883532	0.03940354	7.317951	2.517986e-13
alpha1	-0.1142933	0.01295522	-8.822183	0.000000e+00
beta1	0.8628472	0.01859892	46.392326	0.000000e+00
gamma1	0.1118478	0.01969897	5.677850	1.363982e-08

Table B1.7: High equity General Equity Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	5.111949e-01	0.05925249	8.627400e+00	0.000000e+00
ar1	-8.315681e-01	0.03982866	-2.087864e+01	0.000000e+00
ar2	-8.403325e-01	0.03281502	-2.560817e+01	0.000000e+00
ma1	7.149631e-01	0.04307113	1.659959e+01	0.000000e+00
ma2	8.085770e-01	0.03465382	2.333298e+01	0.000000e+00
omega	4.252022e-01	0.10976447	3.873769e+00	1.071652e-04
alpha1	2.885949e-11	0.01457890	1.979538e-09	1.000000e+00
beta1	8.856317e-01	0.02436735	3.634501e+01	0.000000e+00
gamma1	1.568183e-01	0.02702925	5.801799e+00	6.560718e-09

Table B1.8: Medium-high equity General Equity Funds (gjrGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.60324985	5.905583e-04	1021.491	0
ar1	-1.08500217	8.020267e-05	-13528.255	0
ar2	-0.12987831	1.715563e-05	-7570.593	0
ma1	0.99995022	1.006898e-04	9930.994	0
omega	0.09515897	9.913530e-06	9598.899	0
alpha1	0.03825453	6.584392e-06	5809.880	0
beta1	0.95459439	8.686456e-05	10989.458	0
gamma1	-0.17470302	2.295445e-05	-7610.855	0

Table B1.9: Medium equity General Equity Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.60324985	5.905583e-04	1021.491	0
ar1	-1.08500217	8.020267e-05	-13528.255	0
ar2	-0.12987831	1.715563e-05	-7570.593	0
ma1	0.99995022	1.006898e-04	9930.994	0
omega	0.09515897	9.913530e-06	9598.899	0
alpha1	0.03825453	6.584392e-06	5809.880	0
beta1	0.95459439	8.686456e-05	10989.458	0
gamma1	-0.17470302	2.295445e-05	-7610.855	0

Table B1.10: Medium-high equity Flexible Funds (eGARCH)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.65727759	3.975160e-02	1.653462e+01	0.000000e+00
ar1	-1.04368259	7.942929e-04	-1.313977e+03	0.000000e+00
ar2	-0.99630724	1.141304e-03	-8.729551e+02	0.000000e+00
ma1	1.02909549	1.007389e-03	1.021547e+03	0.000000e+00
ma2	0.99074251	2.277788e-07	4.349582e+06	0.000000e+00
omega	0.23106915	3.787187e-02	6.101339e+00	1.051834e-09
alpha1	-0.07001617	2.143321e-02	-3.266715e+00	1.088032e-03
beta1	0.85856895	2.332918e-02	3.680236e+01	0.000000e+00
gamma1	0.27012580	5.627802e-05	4.799845e+03	0.000000e+00

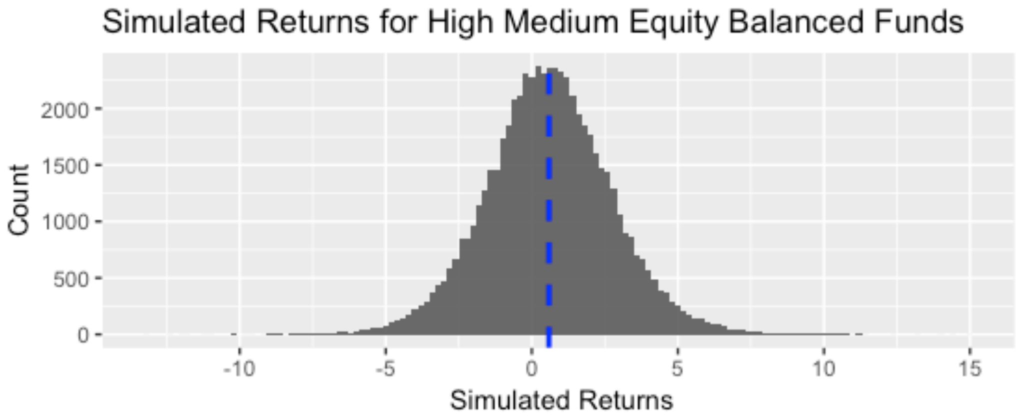
Table B1.11: Medium equity Flexible Funds (eGARCH)

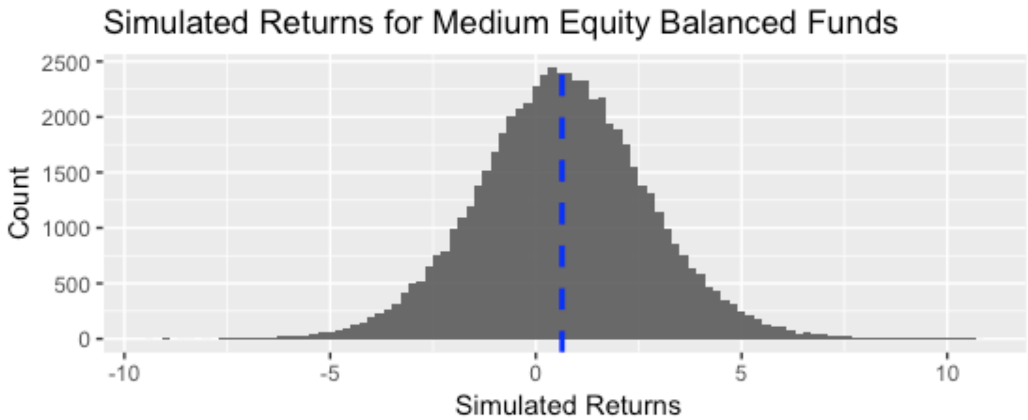
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.5955070	0.07478738	7.962667	1.776357e-15
ar1	-1.0567707	0.05417844	-19.505374	0.000000e+00
ar2	-0.1782204	0.04245561	-4.197805	2.695143e-05
ma1	0.9186720	0.03468193	26.488491	0.000000e+00
omega	0.1239834	0.03630237	3.415298	6.371236e-04
alpha1	-0.1102849	0.04328533	-2.547858	1.083864e-02
beta1	0.9010870	0.02736701	32.926025	0.000000e+00
gamma1	0.2926622	0.07499650	3.902345	9.526535e-05

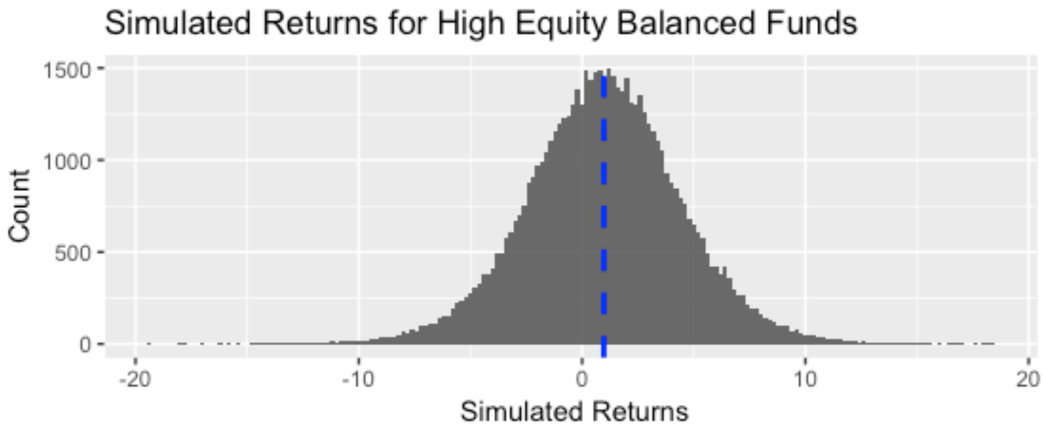
Table B1.12: High equity Flexible Funds (eGARCH)

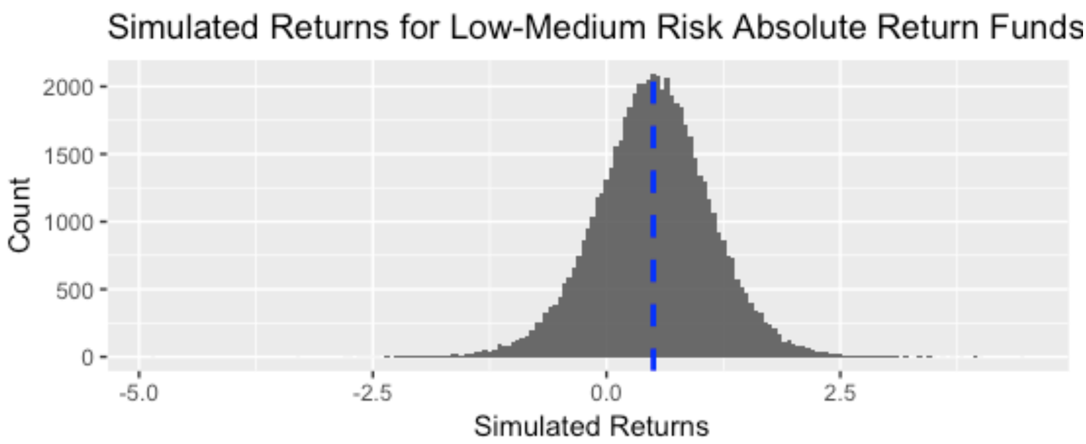
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.43921214	0.11692484	3.756363	0.0001724005
ma1	-0.09903460	0.05030238	-1.968786	0.0489776965
omega	0.26296994	0.14235538	1.847278	0.0647068701
alpha1	-0.09562485	0.05394433	-1.772658	0.0762853743
beta1	0.86045796	0.07476666	11.508578	0.0000000000
gamma1	0.10473658	0.06546471	1.599894	0.1096221514

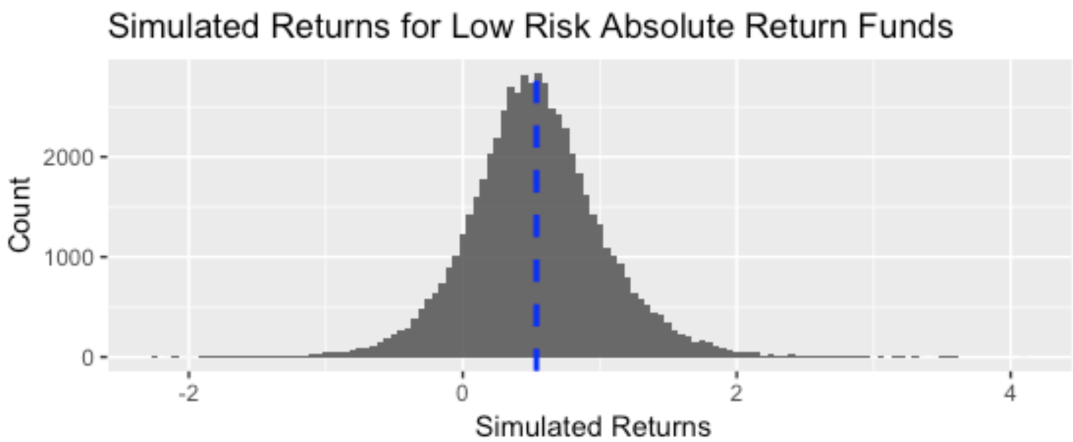
Appendix B2: Simulated Return Histograms

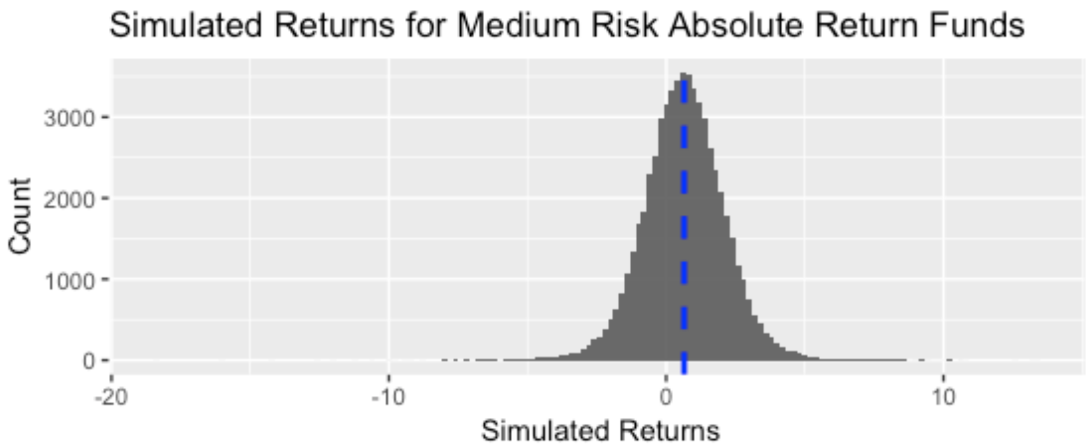


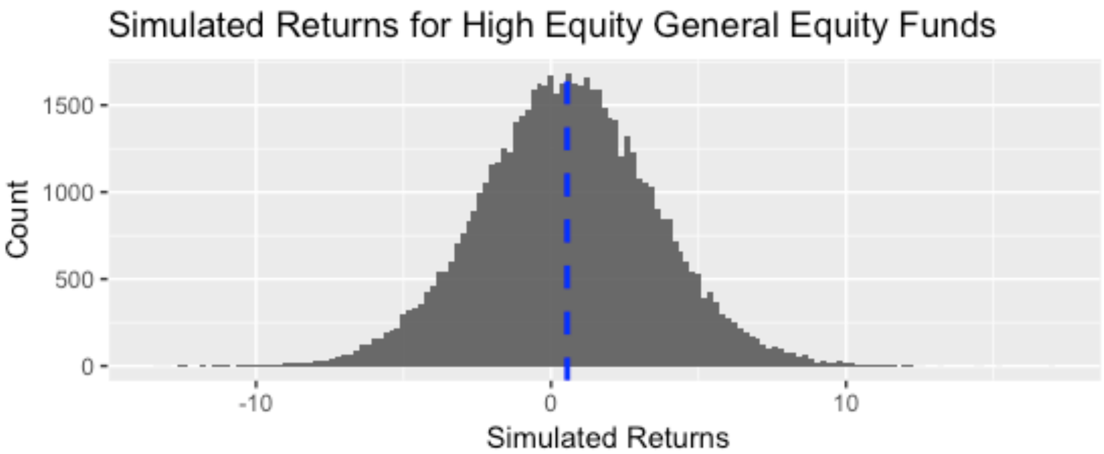


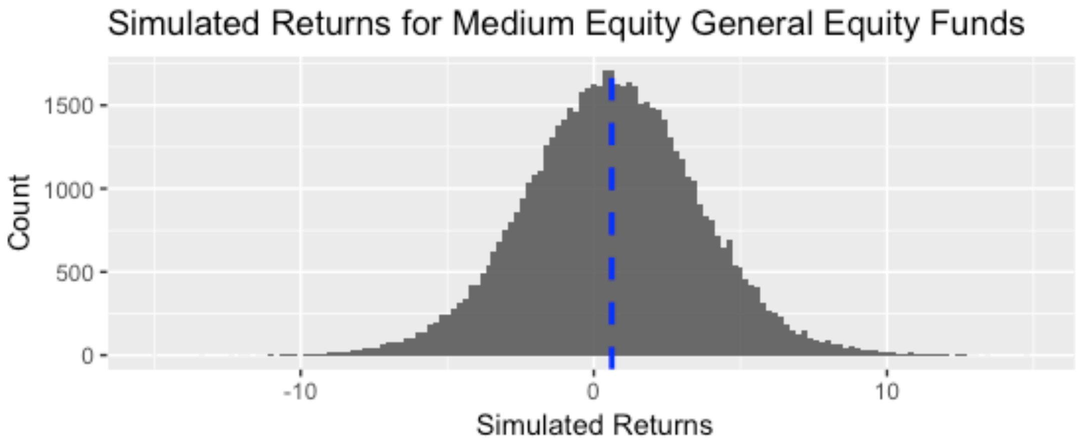


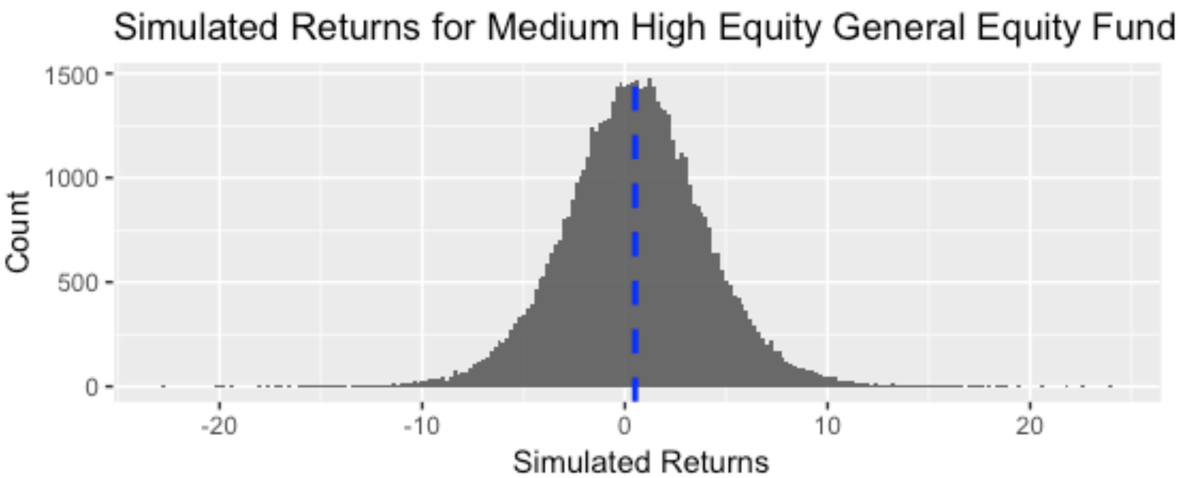

Simulated Returns for Low-Medium Risk Absolute Return Funds
Count
2000
1500
1000
500
0
-5.0
-2.5
0.0
2.5
Simulated Returns

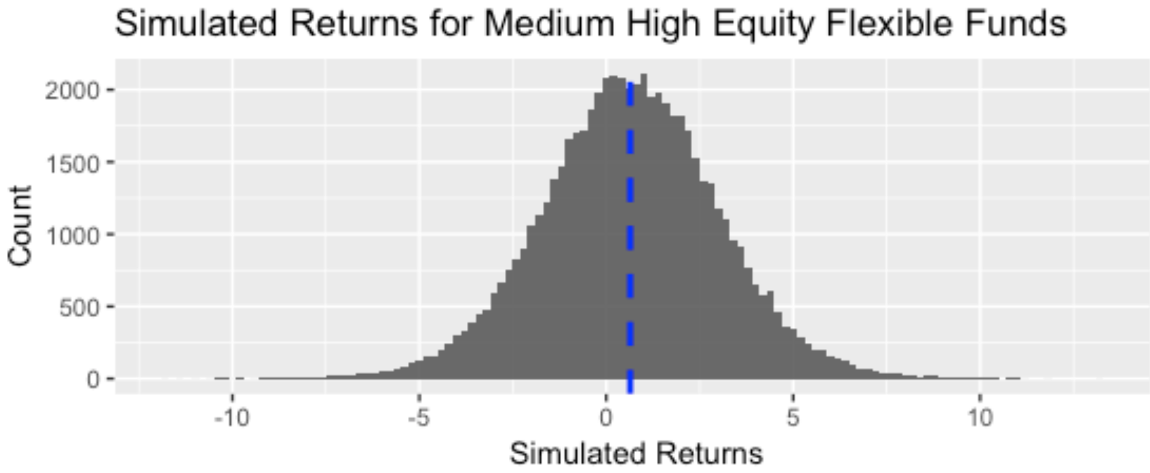

Simulated Returns for Low Risk Absolute Return Funds
Count
2000
1000
0
-2
0
2
4
Simulated Returns

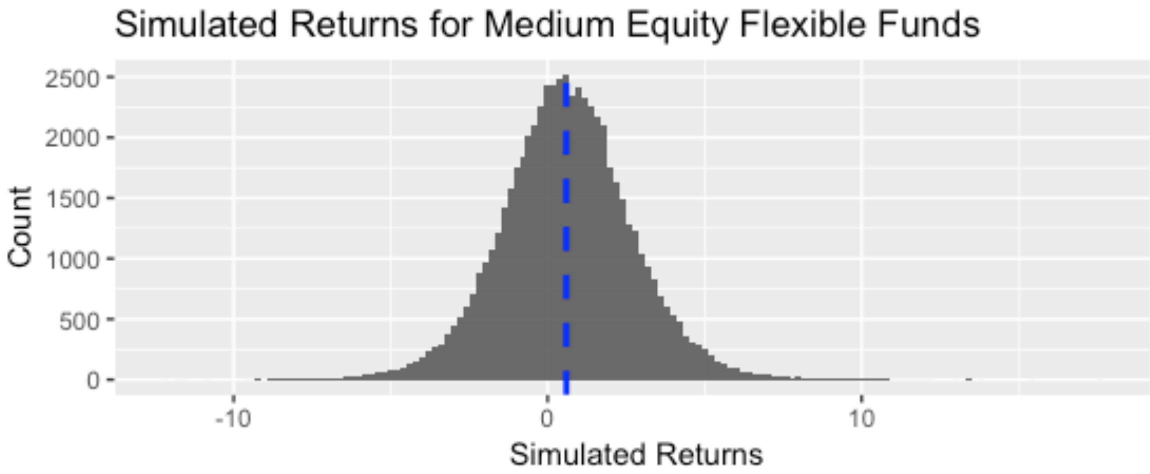

Simulated Returns for Medium Risk Absolute Return Funds
Count
3000
2000
1000
0
-20
-10
0
10
Simulated Returns

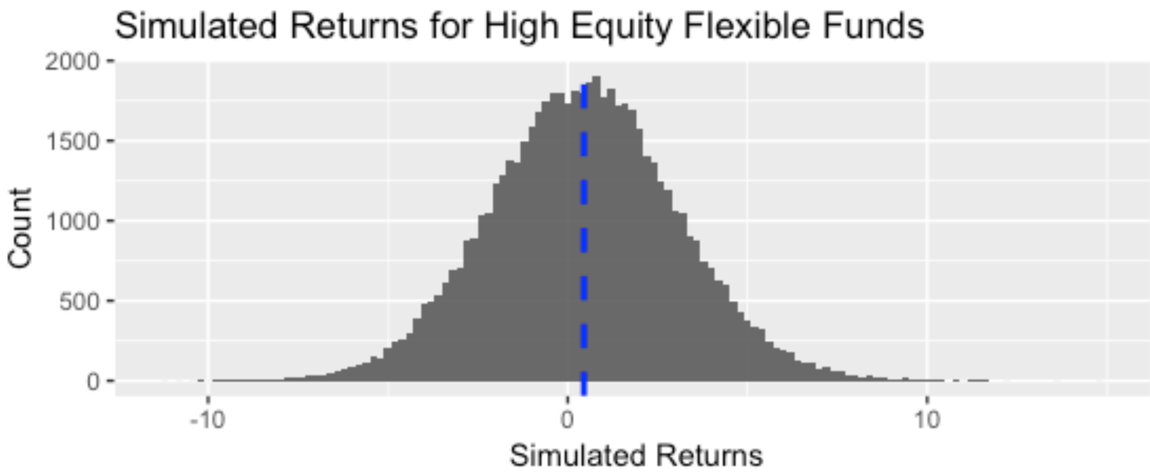

Simulated Returns for High Equity General Equity Funds
Count
1500
1000
500
0
-10
0
10
Simulated Returns


Simulated Returns for Medium Equity General Equity Funds
Count
1500
1000
500
0
-10
0
10
Simulated Returns

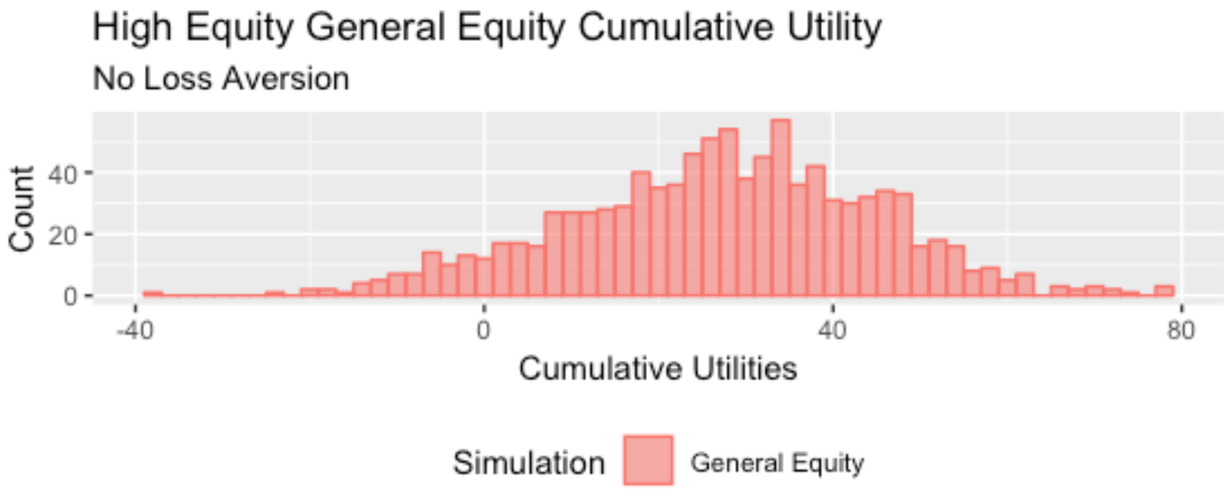

Simulated Returns for Medium High Equity General Equity Fund
Count
1500
1000
500
0
-20
-10
0
10
20
Simulated Returns

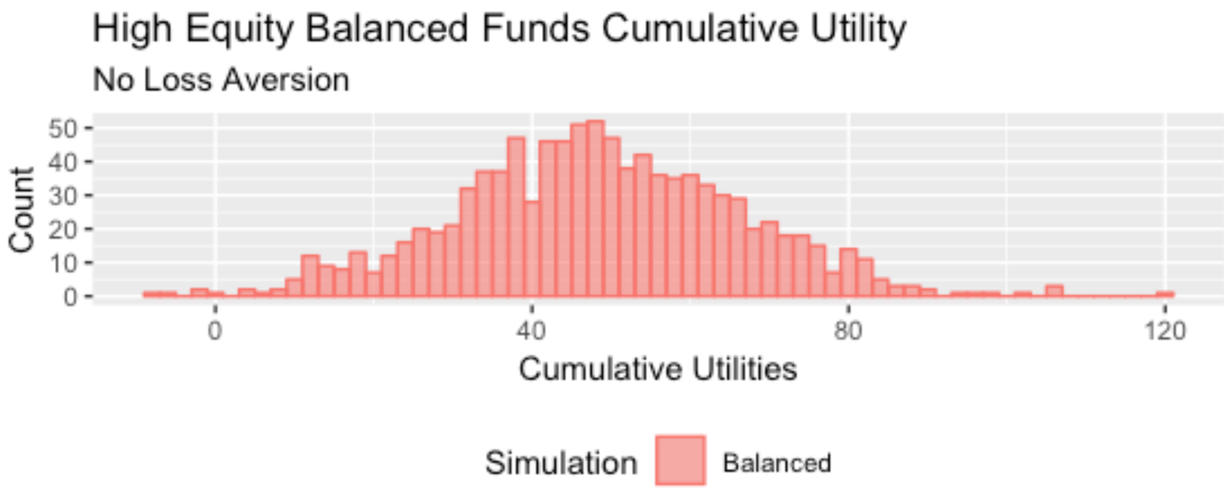

Simulated Returns for Medium High Equity Flexible Funds
Count
2000
1500
1000
500
0
-10
-5
0
5
10
Simulated Returns

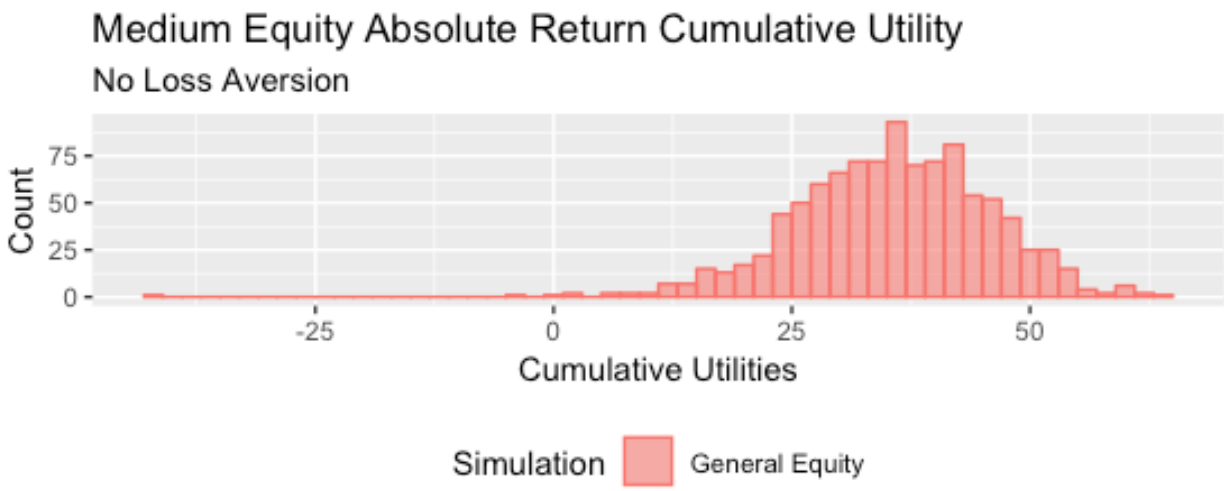

Simulated Returns for Medium Equity Flexible Funds
Count
2500
2000
1500
1000
500
0
-10
0
10
Simulated Returns

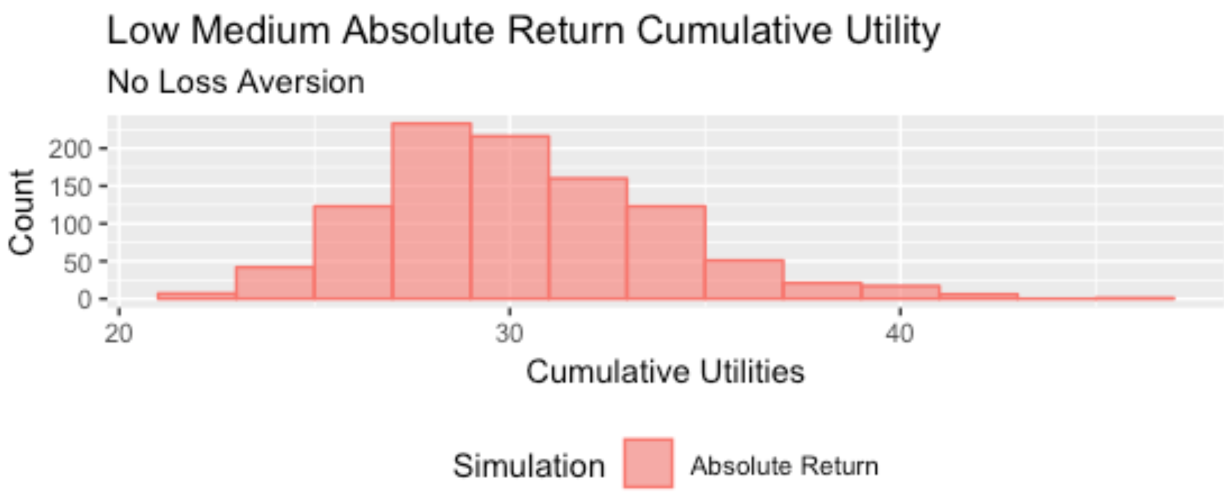

Simulated Returns for High Equity Flexible Funds
Count
2000
1500
1000
500
0
-10
0
10
Simulated Returns

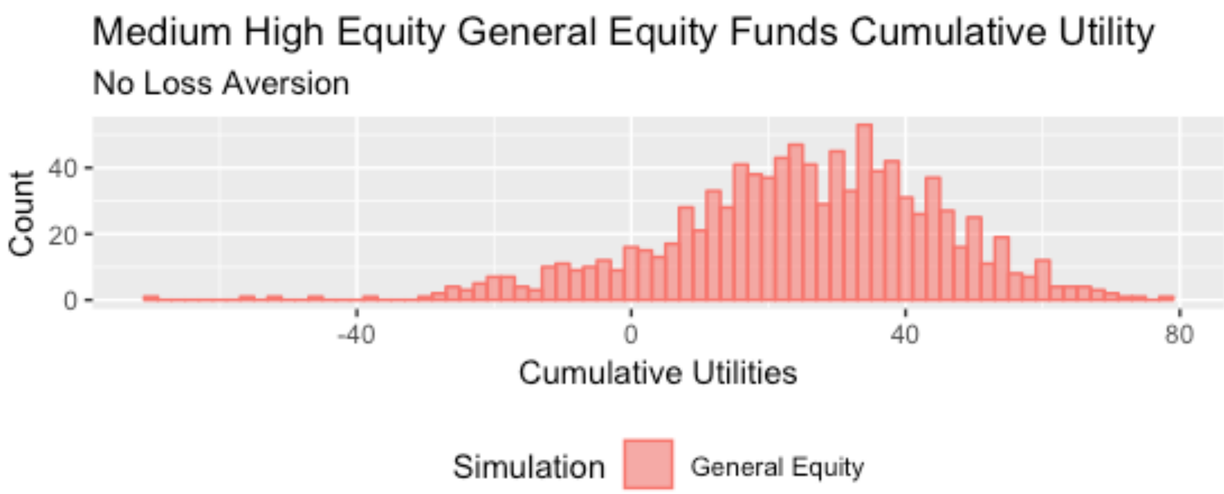
APPENDIX B3: CUMULATIVE UTILITIES

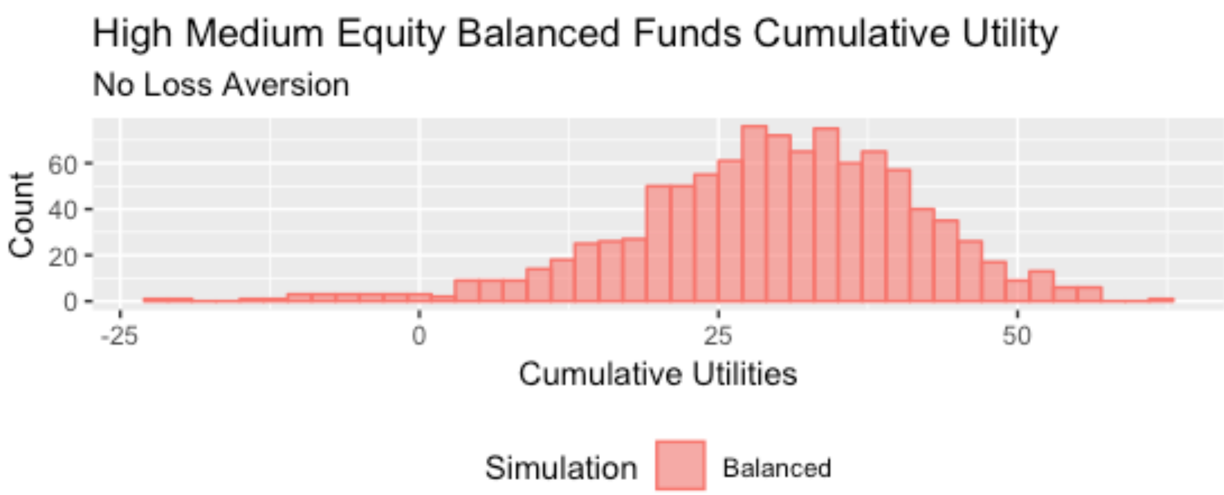


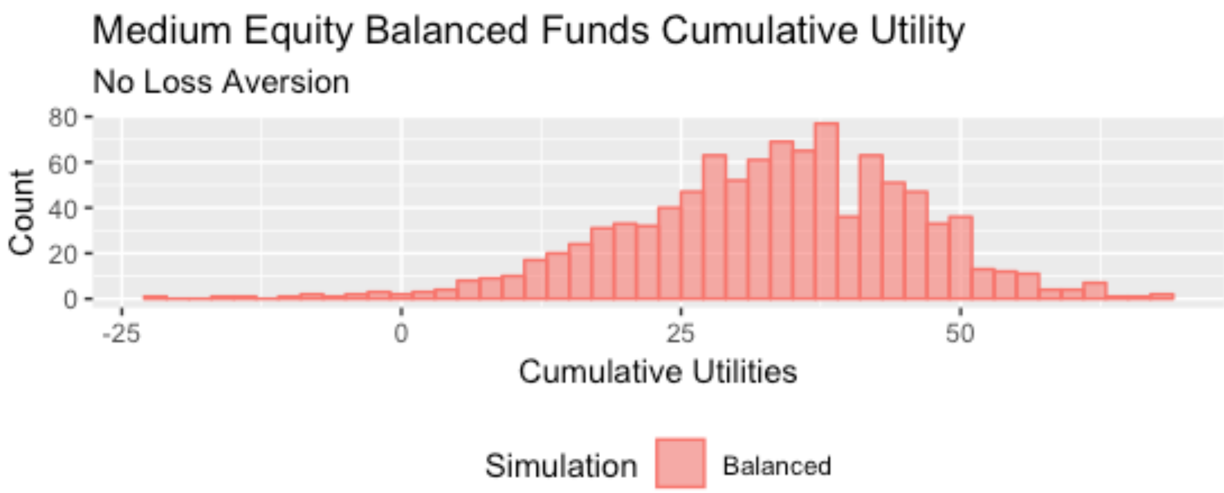


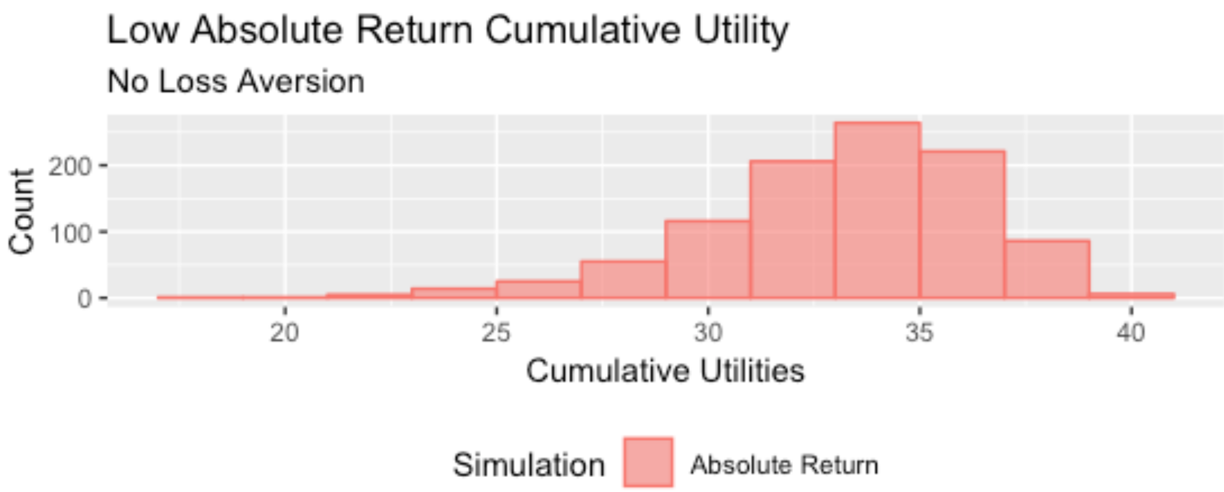


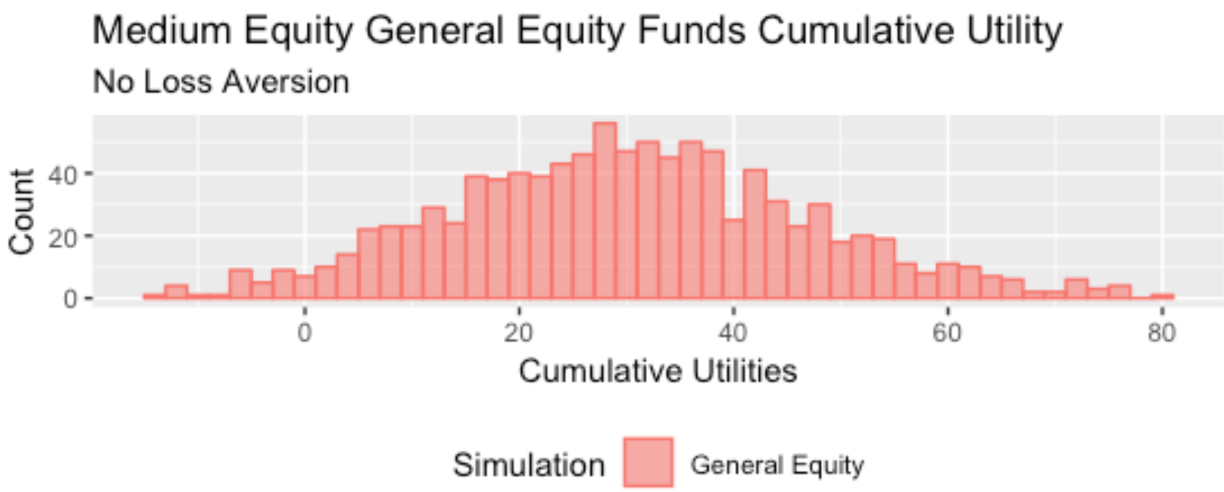

Low Medium Absolute Return Cumulative Utility
No Loss Aversion
Count
200
150
100
50
0
20
30
40
Cumulative Utilities
Simulation
Absolute Return

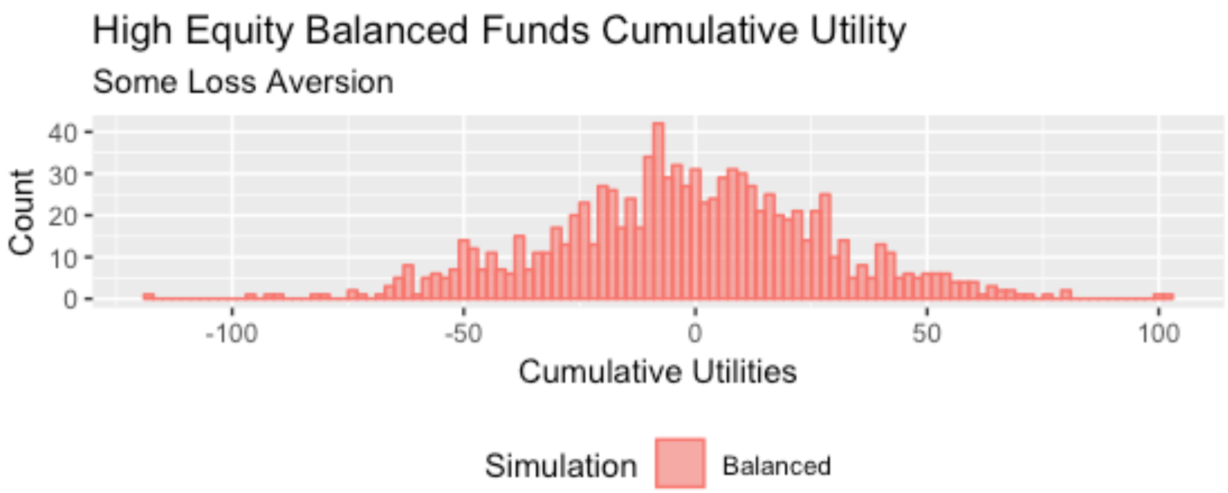

Medium High Equity General Equity Funds Cumulative Utility
No Loss Aversion
Count
40
20
0
-40
0
40
80
Cumulative Utilities
Simulation
General Equity

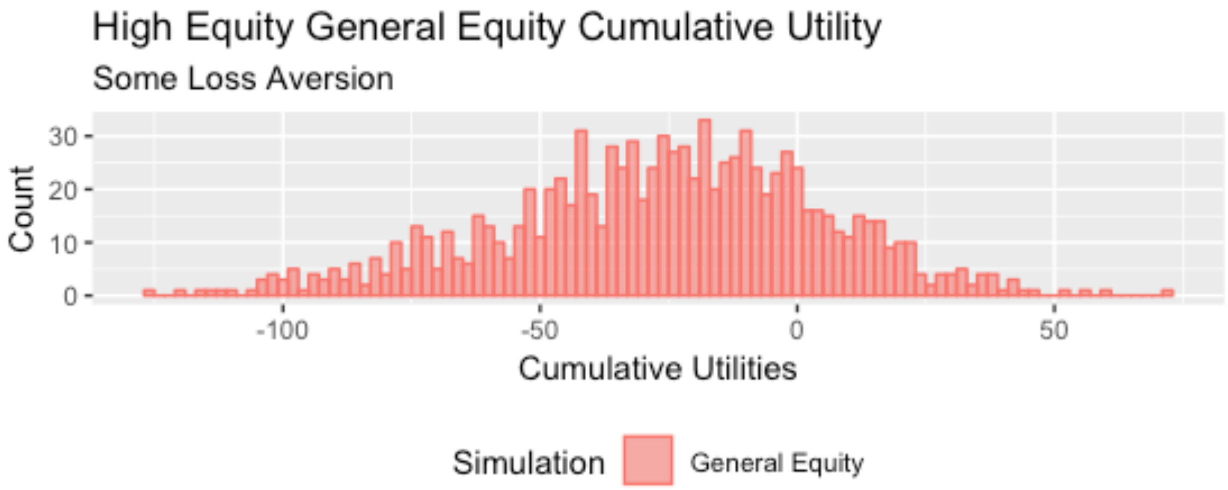

High Medium Equity Balanced Funds Cumulative Utility
No Loss Aversion
Count
60
40
20
0
-25
0
25
50
Cumulative Utilities
Simulation
Balanced

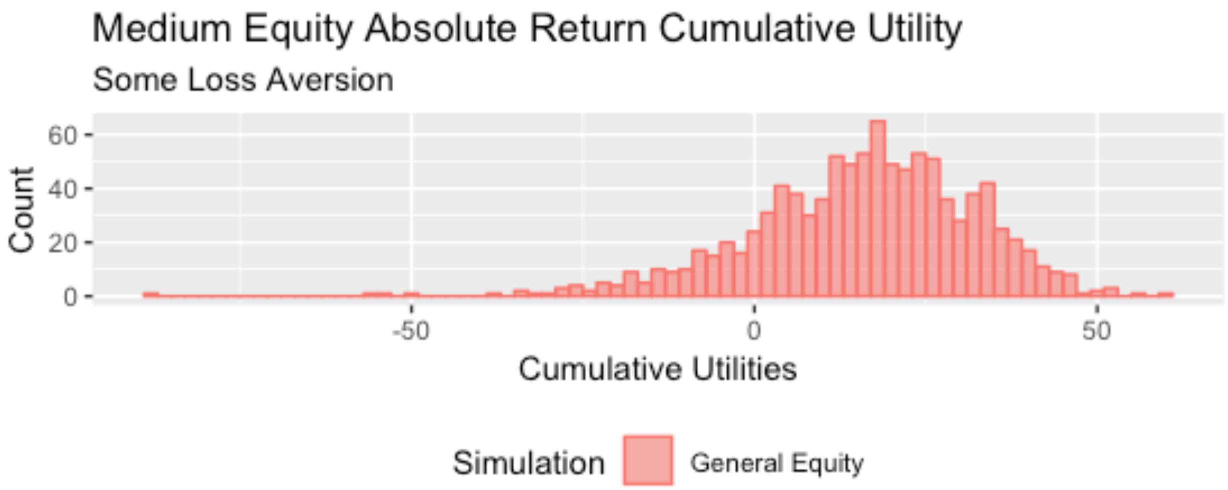

Medium Equity Balanced Funds Cumulative Utility
No Loss Aversion
Count
80
60
40
20
0
-25
0
25
50
Cumulative Utilities
Simulation
Balanced


Low Absolute Return Cumulative Utility
No Loss Aversion
Count
200
100
0
20
25
30
35
40
Cumulative Utilities
Simulation
Absolute Return


Medium Equity General Equity Funds Cumulative Utility
No Loss Aversion
Count
40
20
0
0
20
40
60
80
Cumulative Utilities
Simulation
General Equity

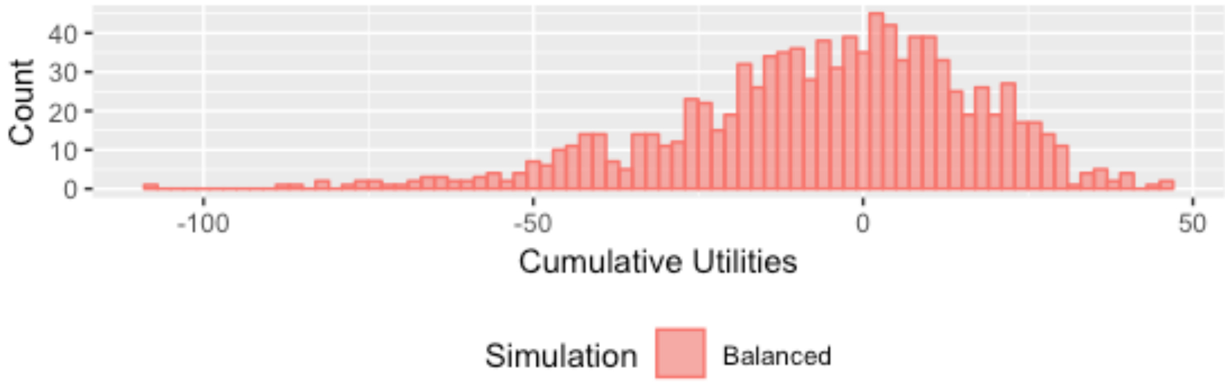

High Equity Balanced Funds Cumulative Utility
Some Loss Aversion
Count
40
30
20
10
0
-100
-50
0
50
100
Cumulative Utilities
Simulation
Balanced


High Equity General Equity Cumulative Utility
Some Loss Aversion
Count
30
20
10
0
-100
-50
0
50
Cumulative Utilities
Simulation
General Equity


Medium Equity Absolute Return Cumulative Utility
Some Loss Aversion
Count
60
40
20
0
-50
0
50
Cumulative Utilities
Simulation
General Equity

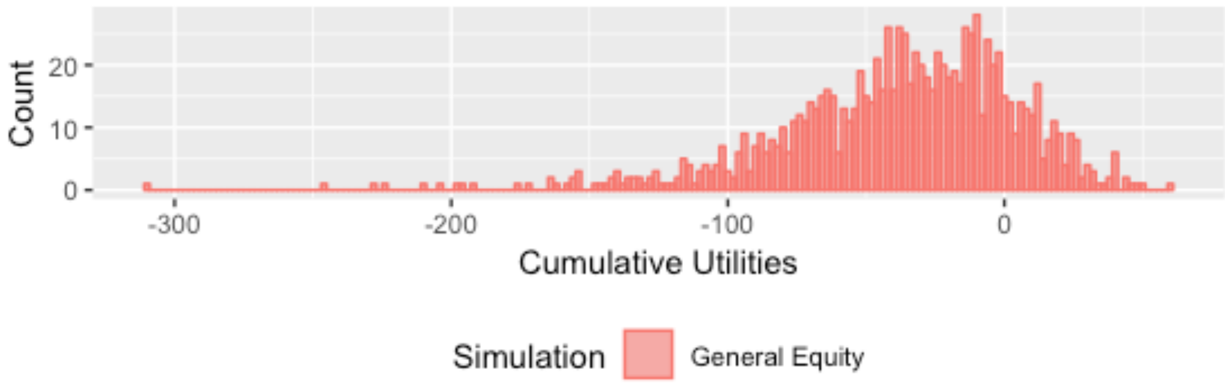
High Medium Equity Balanced Funds Cumulative Utility

Some Loss Aversion



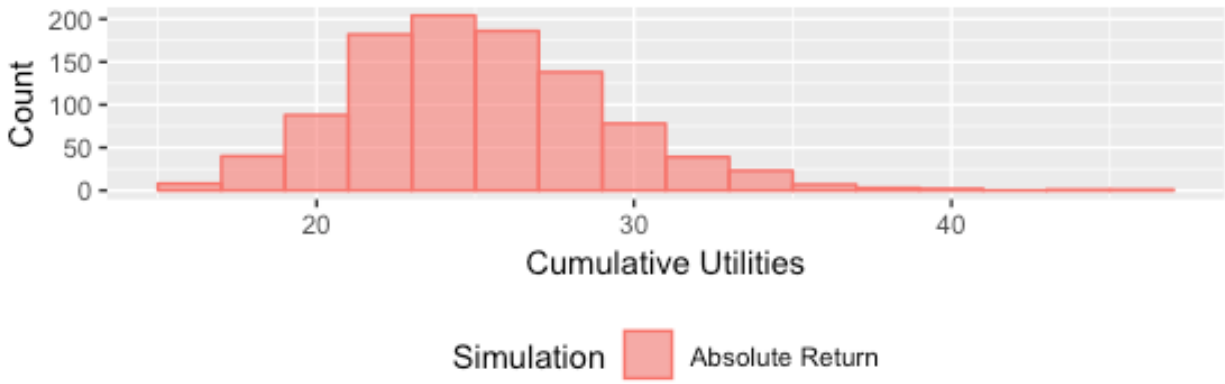
Medium High Equity General Equity Funds Cumulative Utility

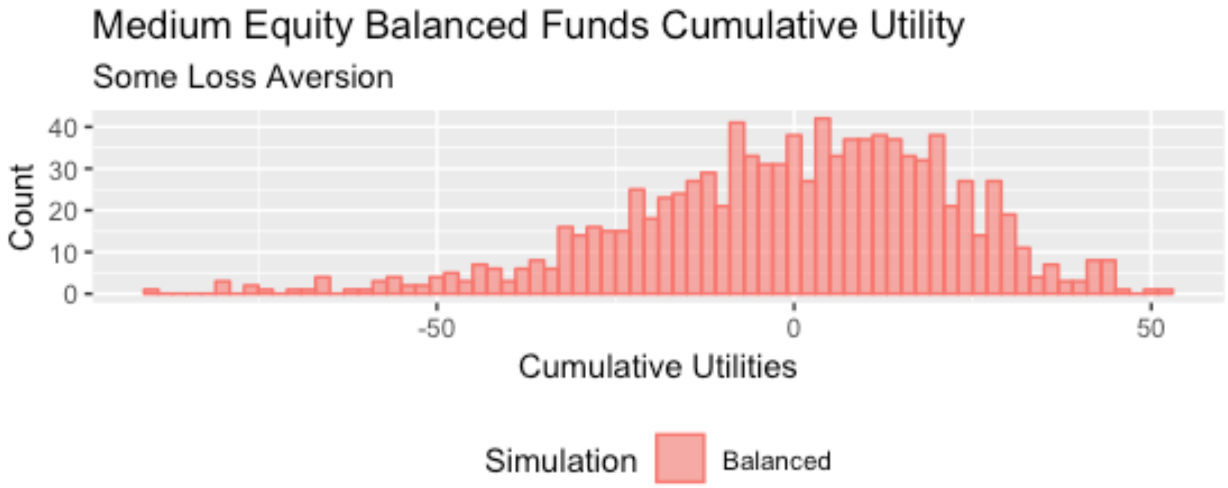
Some Loss Aversion

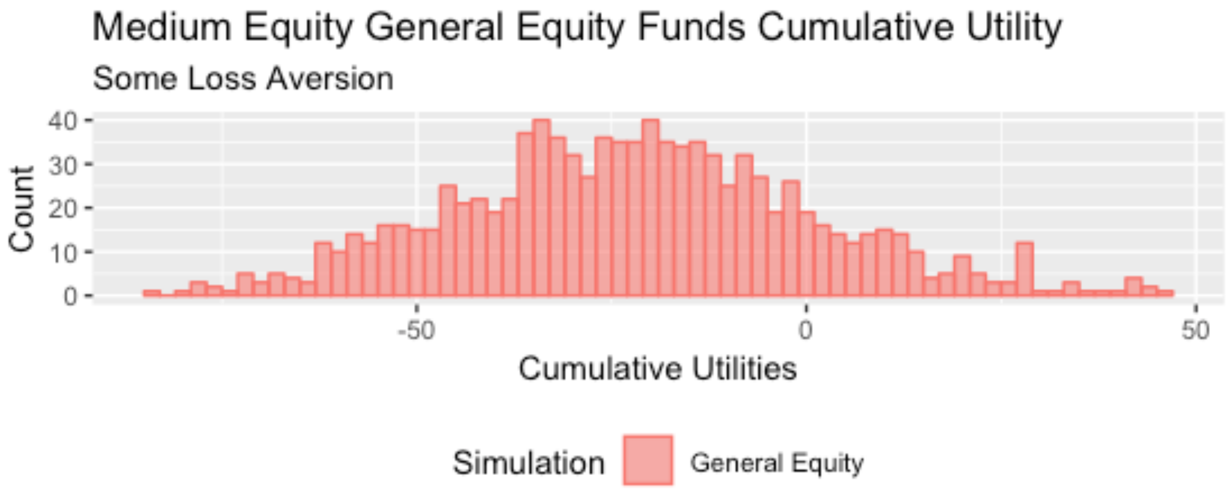


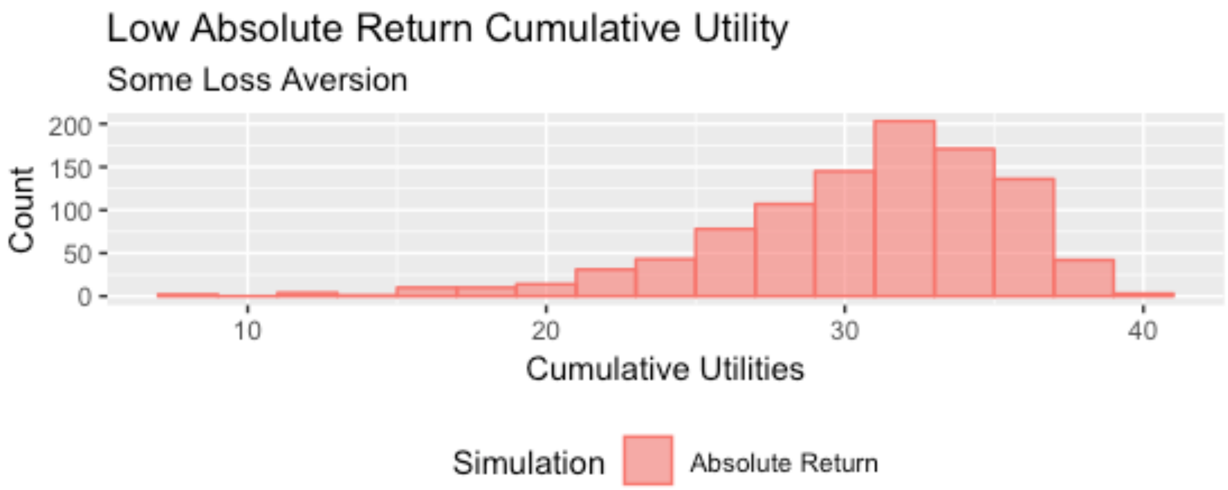
Low Medium Absolute Return Cumulative Utility

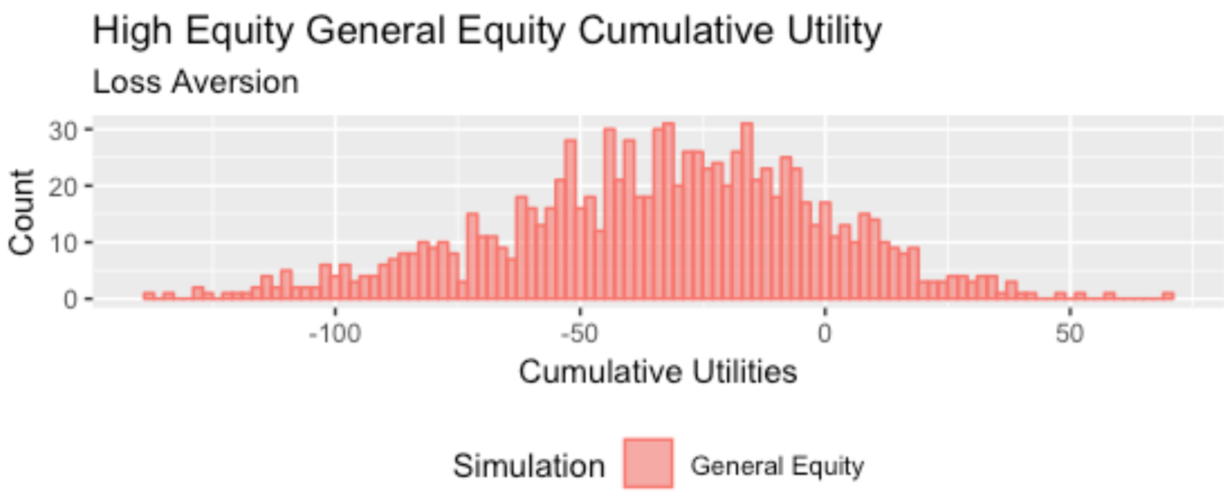
Some Loss Aversion

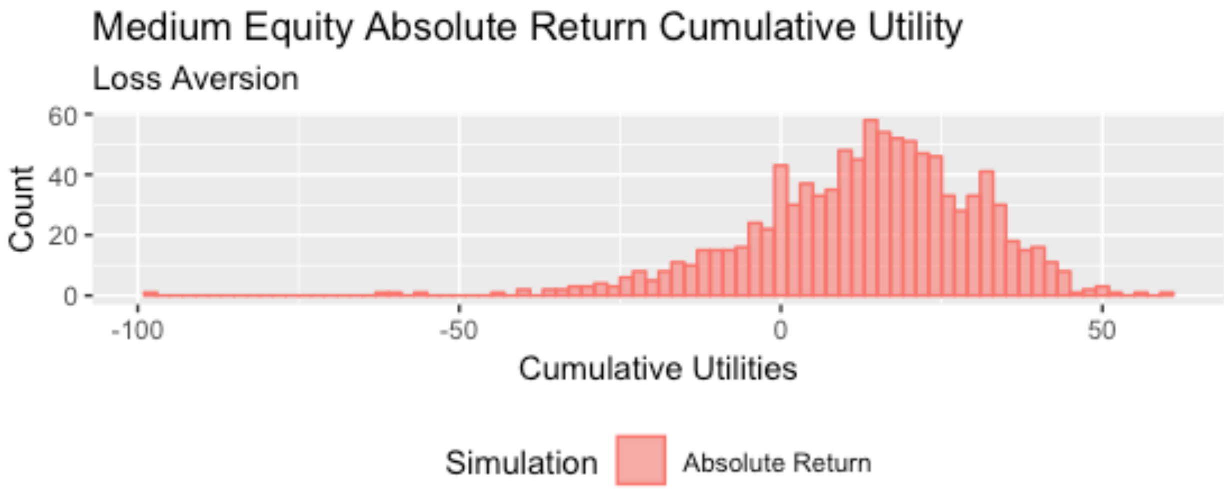


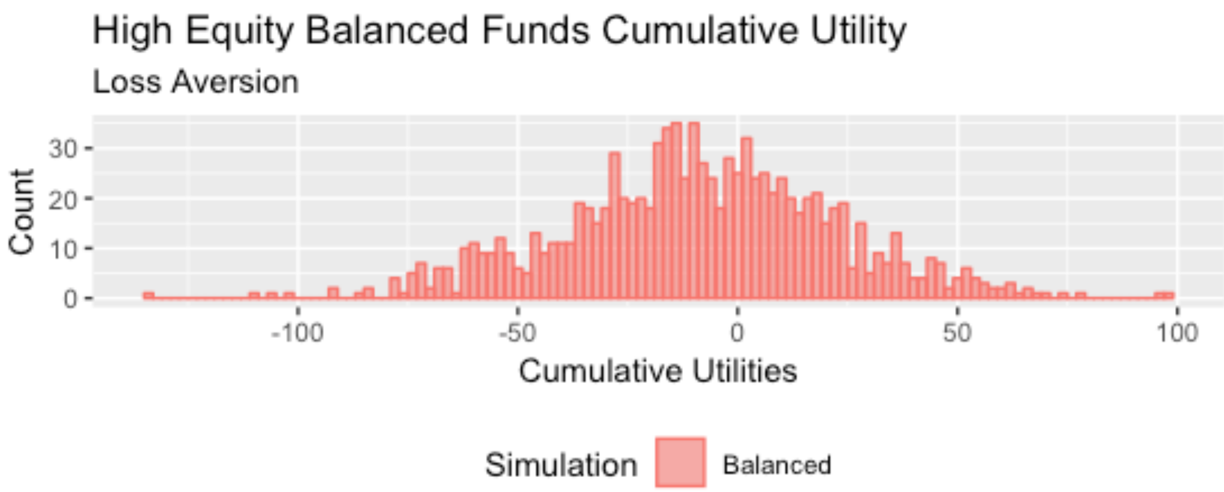

Medium Equity Balanced Funds Cumulative Utility
Some Loss Aversion
Count
40
30
20
10
0
-50
0
50
Cumulative Utilities
Simulation
Balanced

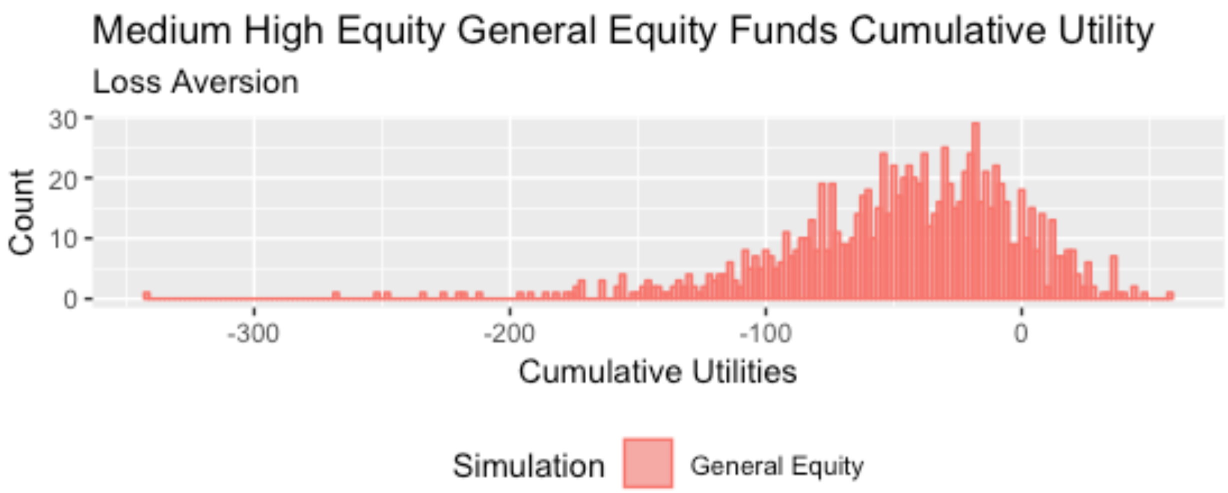

Medium Equity General Equity Funds Cumulative Utility
Some Loss Aversion
Count
40
30
20
10
0
-50
0
50
Cumulative Utilities
Simulation
General Equity

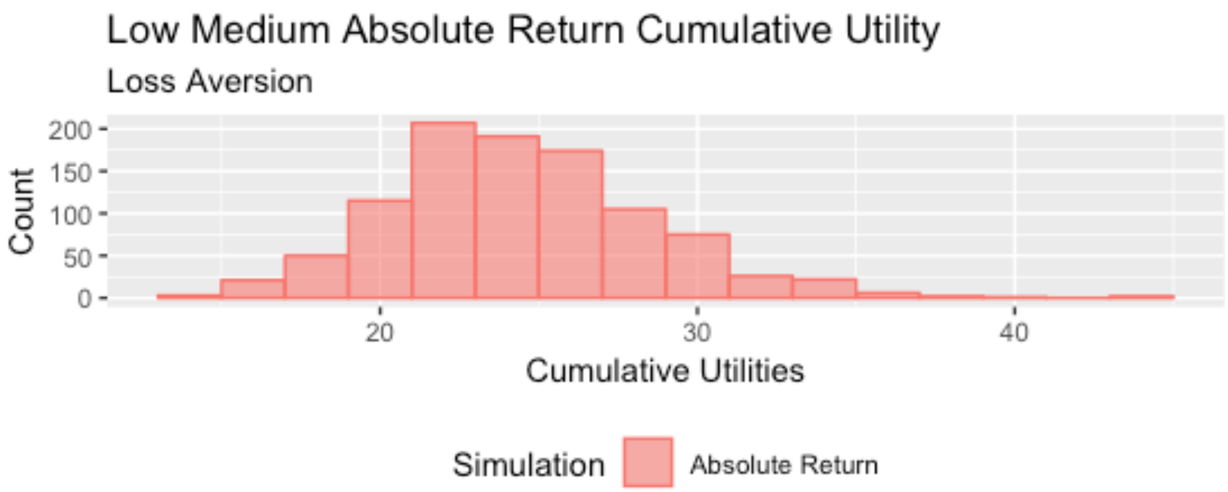

Low Absolute Return Cumulative Utility
Some Loss Aversion
Count
200
150
100
50
0
10
20
30
40
Cumulative Utilities
Simulation
Absolute Return

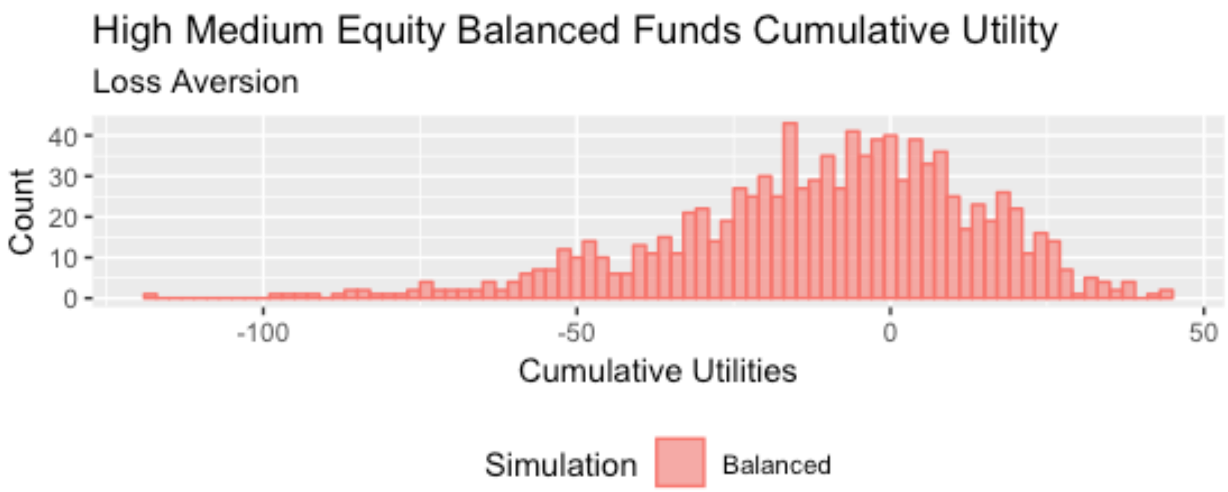

High Equity General Equity Cumulative Utility
Loss Aversion
Count
30
20
10
0
-100
-50
0
50
Cumulative Utilities
Simulation
General Equity

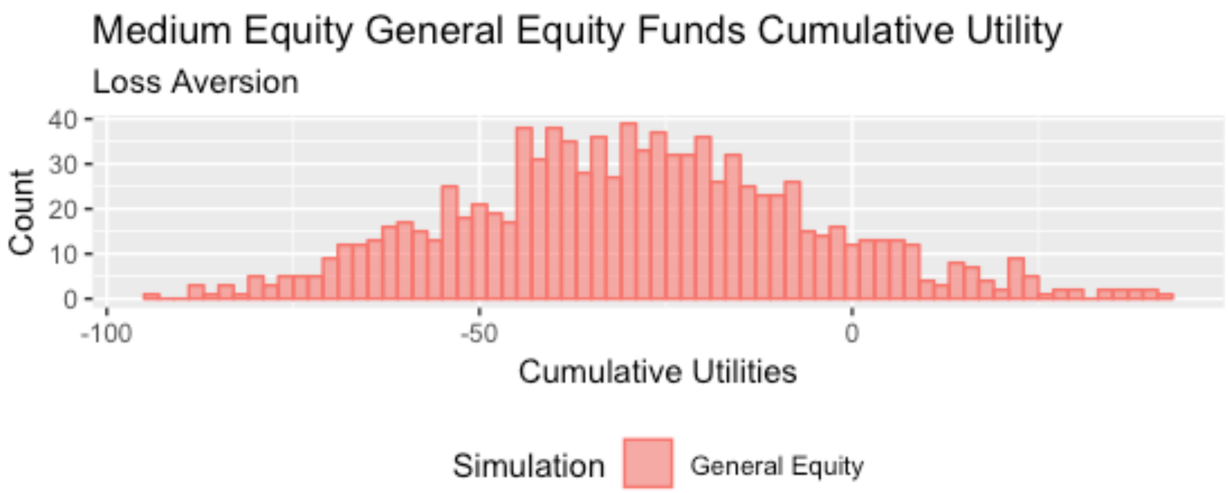

Medium Equity Absolute Return Cumulative Utility
Loss Aversion
Count
60
40
20
0
-100
-50
0
50
Cumulative Utilities
Simulation
Absolute Return


High Equity Balanced Funds Cumulative Utility
Loss Aversion
Count
30
20
10
0
-100
-50
0
50
100
Cumulative Utilities
Simulation
Balanced

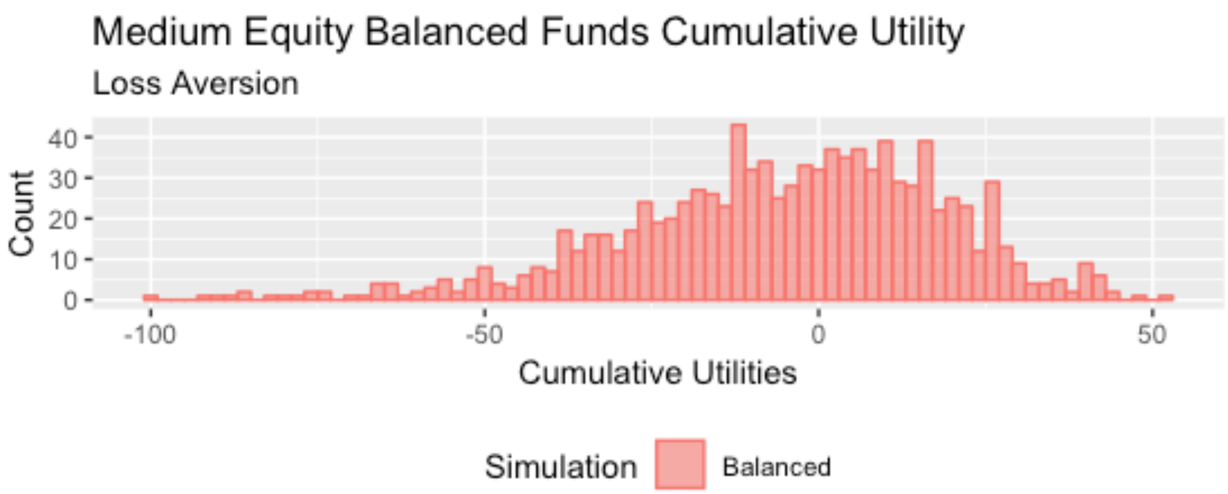

Medium High Equity General Equity Funds Cumulative Utility
Loss Aversion
Count
30
20
10
0
-300
-200
-100
0
Cumulative Utilities
Simulation
General Equity


Low Medium Absolute Return Cumulative Utility
Loss Aversion
Count
200
150
100
50
0
20
30
40
Cumulative Utilities
Simulation
Absolute Return

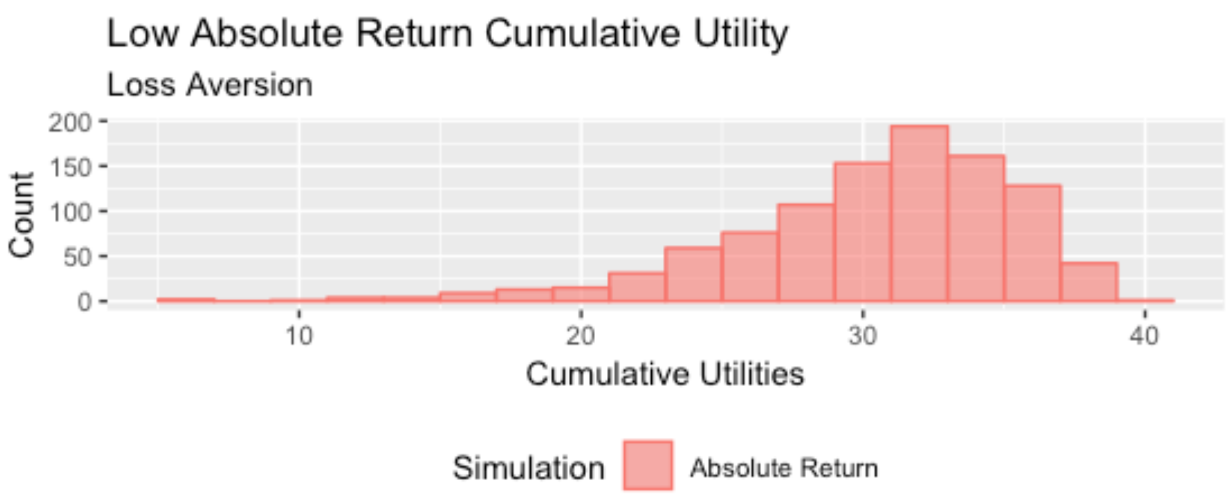

High Medium Equity Balanced Funds Cumulative Utility
Loss Aversion
Count
40
30
20
10
0
-100
-50
0
50
Cumulative Utilities
Simulation
Balanced



Medium Equity General Equity Funds Cumulative Utility
Loss Aversion
Count
40
30
20
10
0
-100
-50
0
Cumulative Utilities
Simulation
General Equity

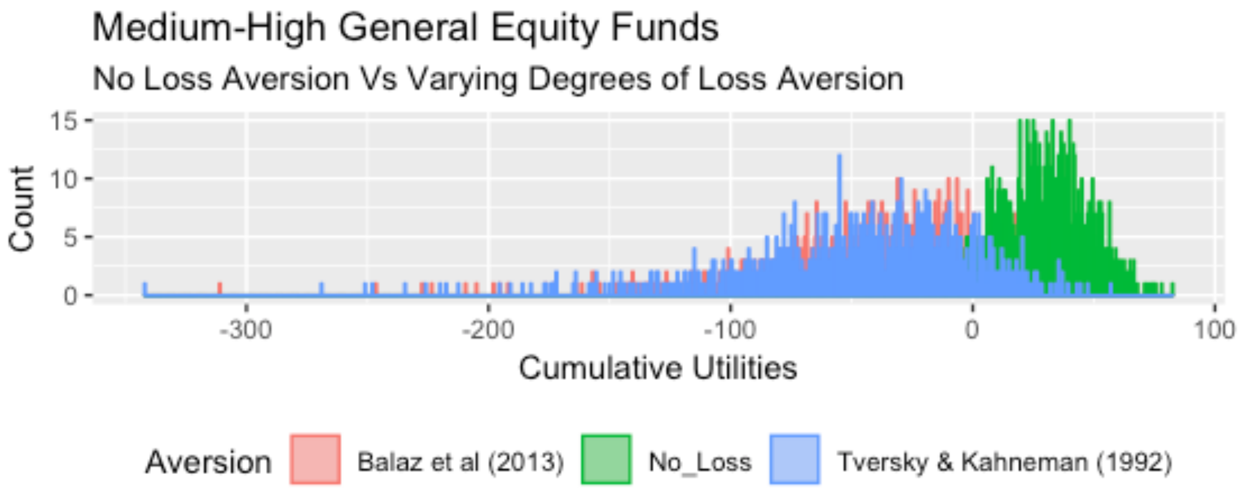


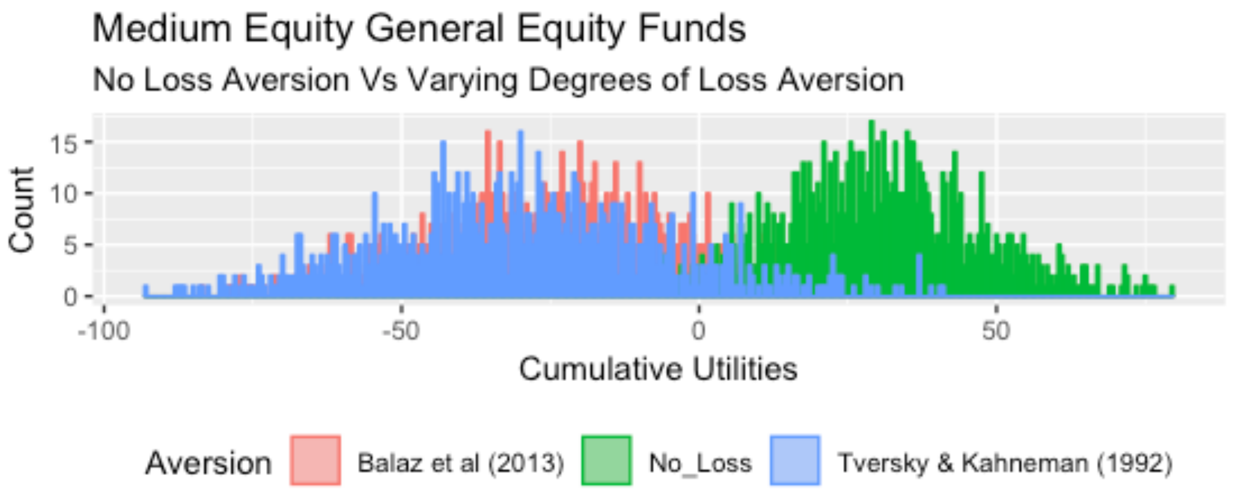
Medium Equity Balanced Funds Cumulative Utility
Loss Aversion
Count
40
30
20
10
0
-100
-50
0
50
Cumulative Utilities
Simulation
Balanced

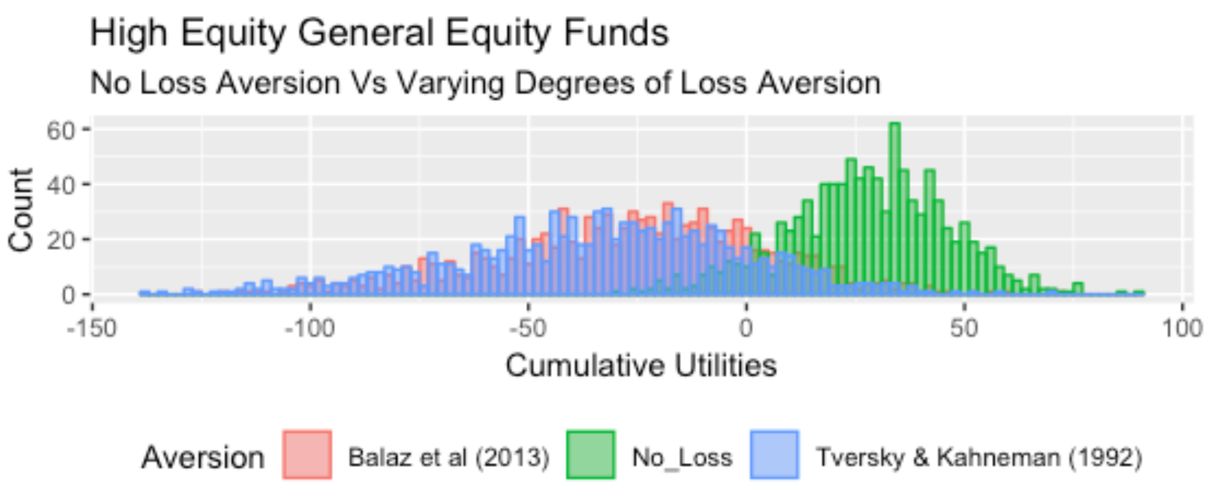


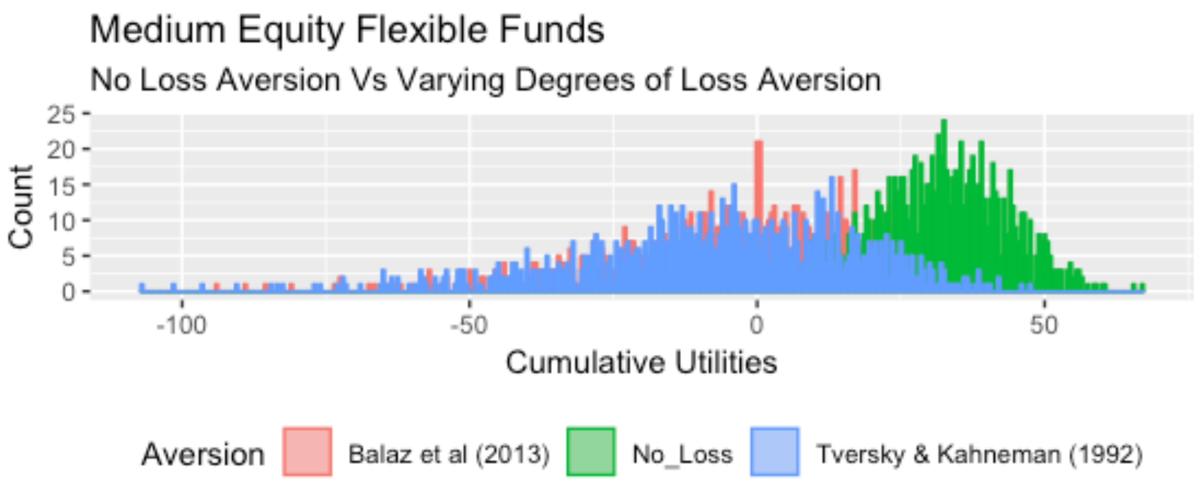
Low Absolute Return Cumulative Utility
Loss Aversion
Count
200
150
100
50
0
10
20
30
40
Cumulative Utilities
Simulation
Absolute Return

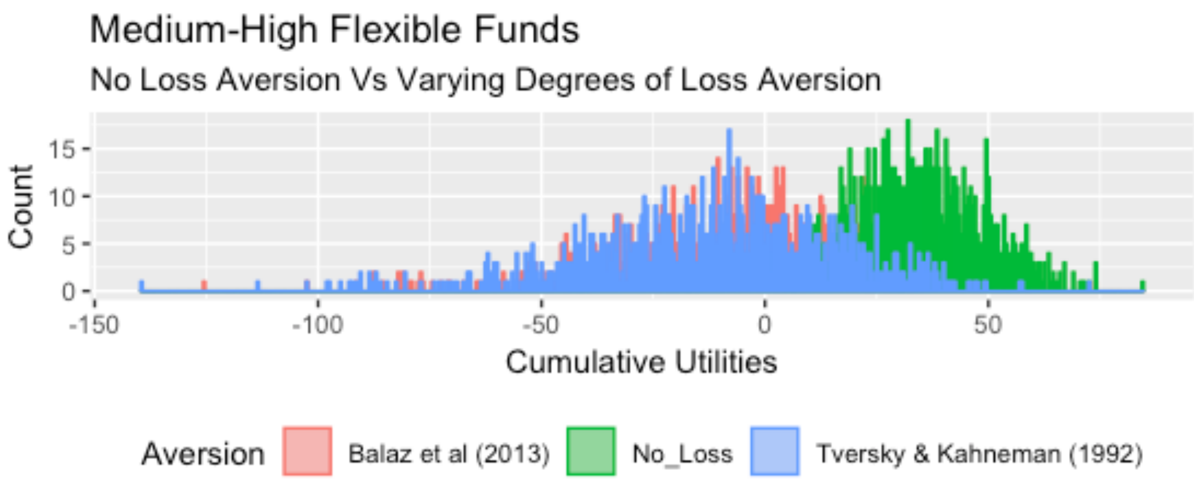
Appendix B4: Comparative Histograms of Cumulative Utilities

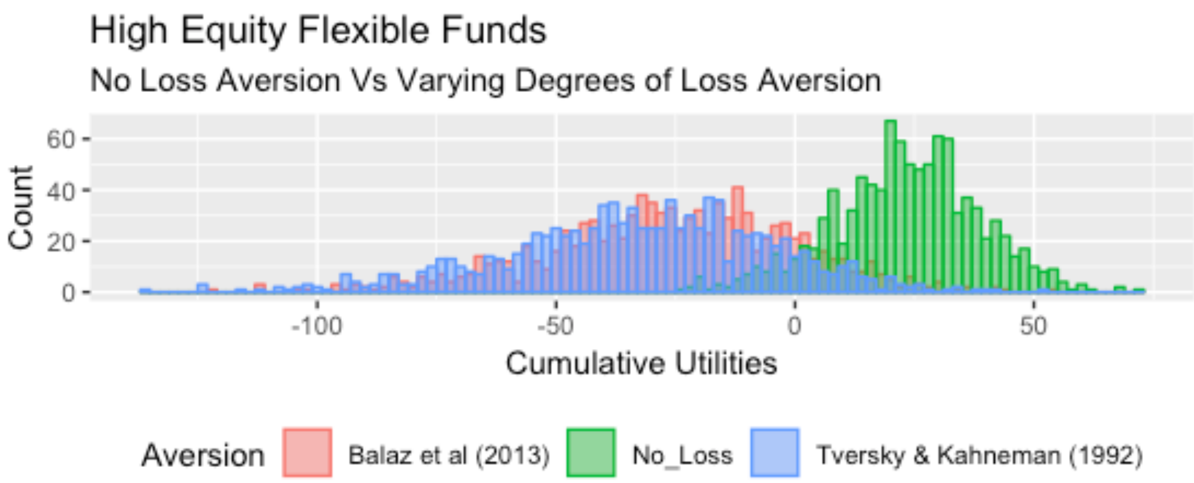


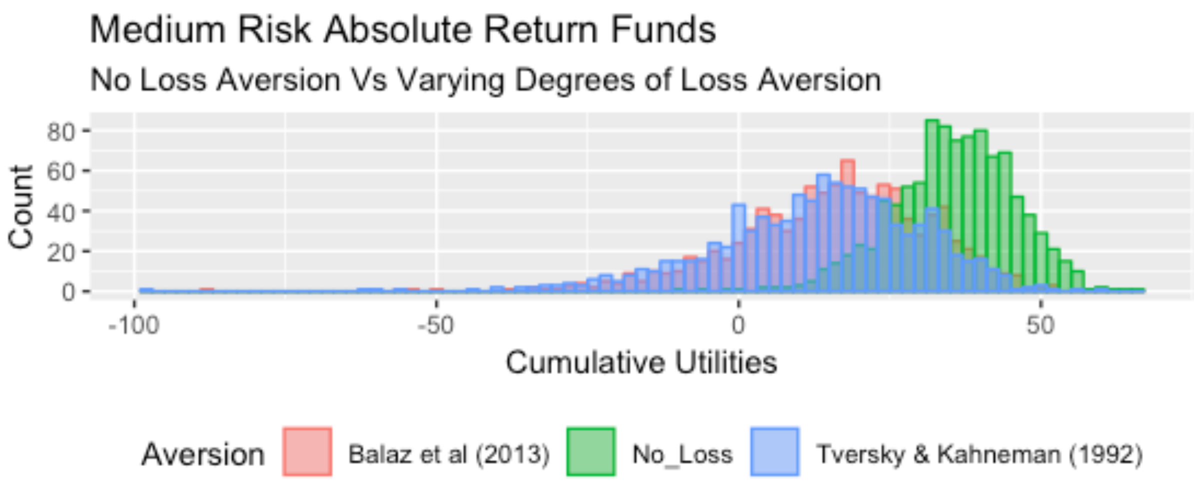


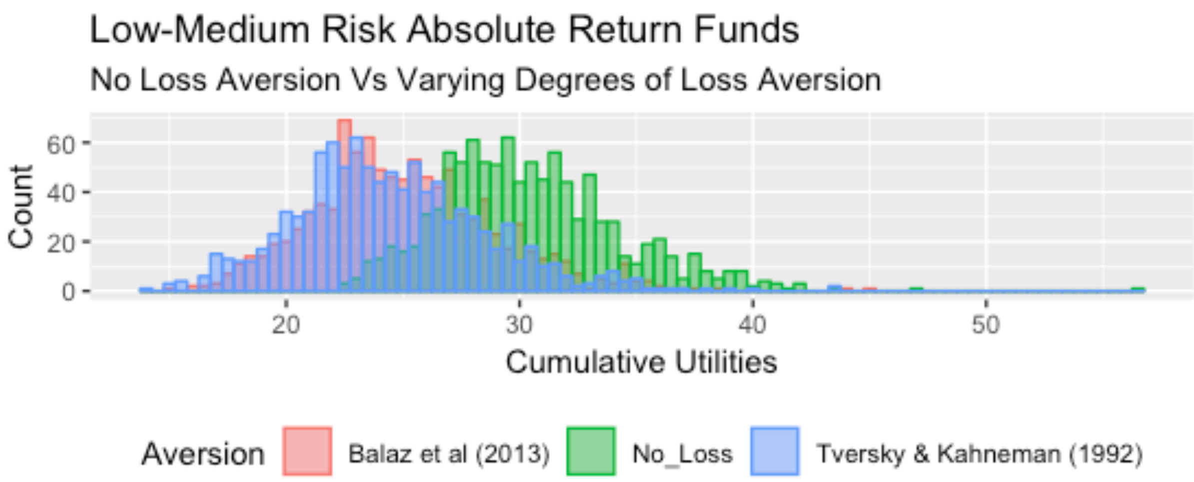


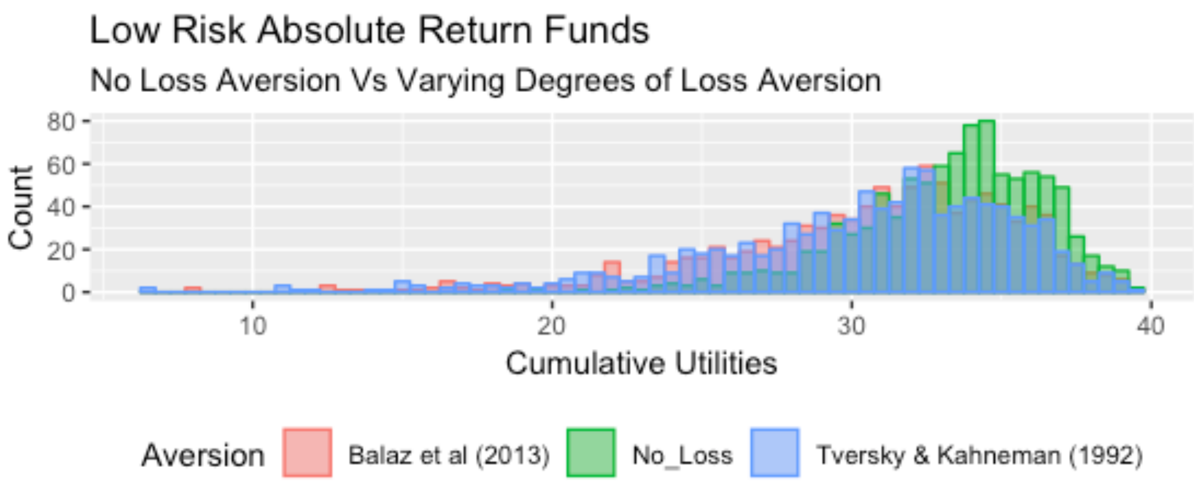

Medium Equity Flexible Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
25
20
15
10
5
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

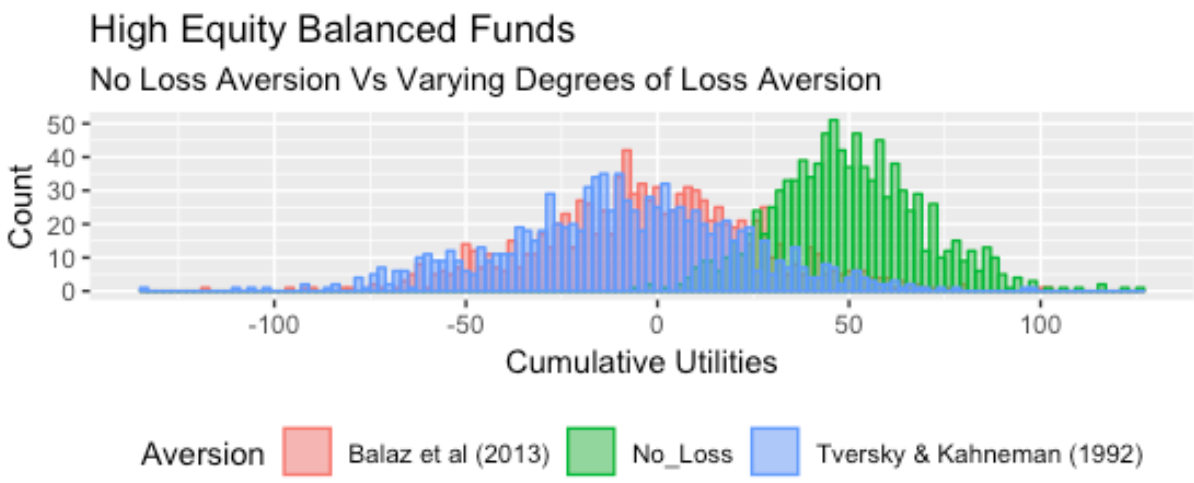

Medium-High Flexible Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
15
10
5
0
-150
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

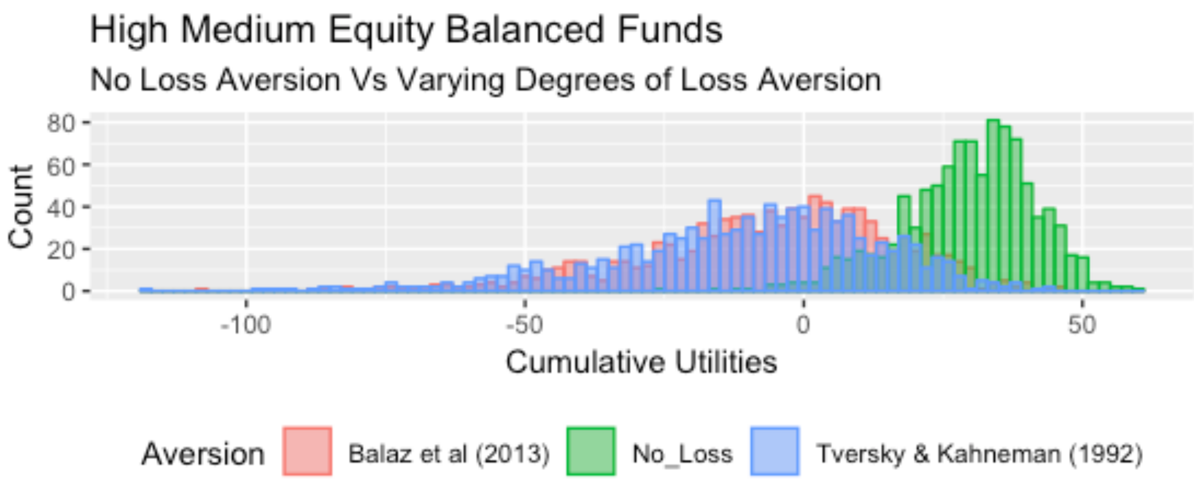

High Equity Flexible Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
60
40
20
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

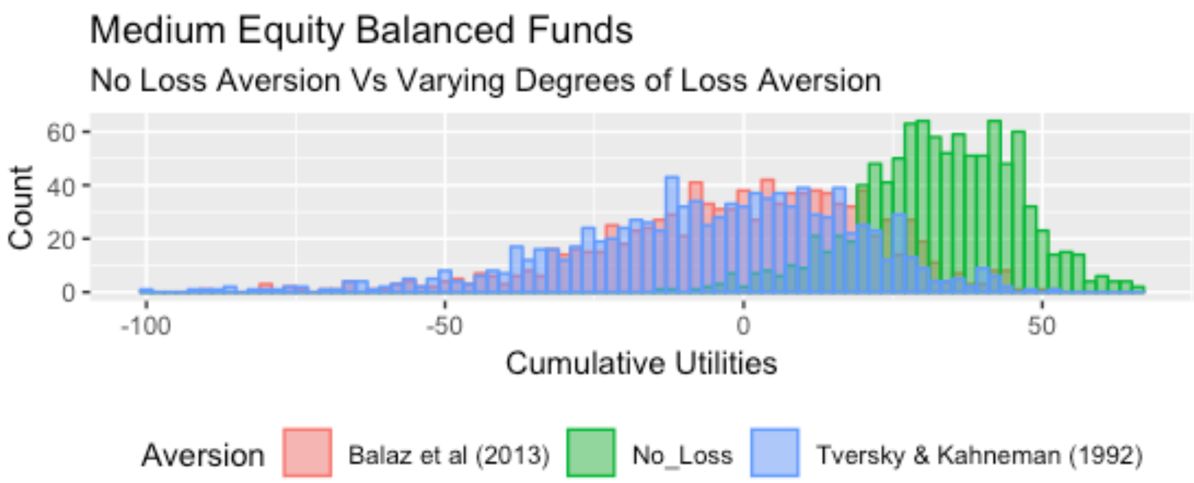

Medium Risk Absolute Return Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
80
60
40
20
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

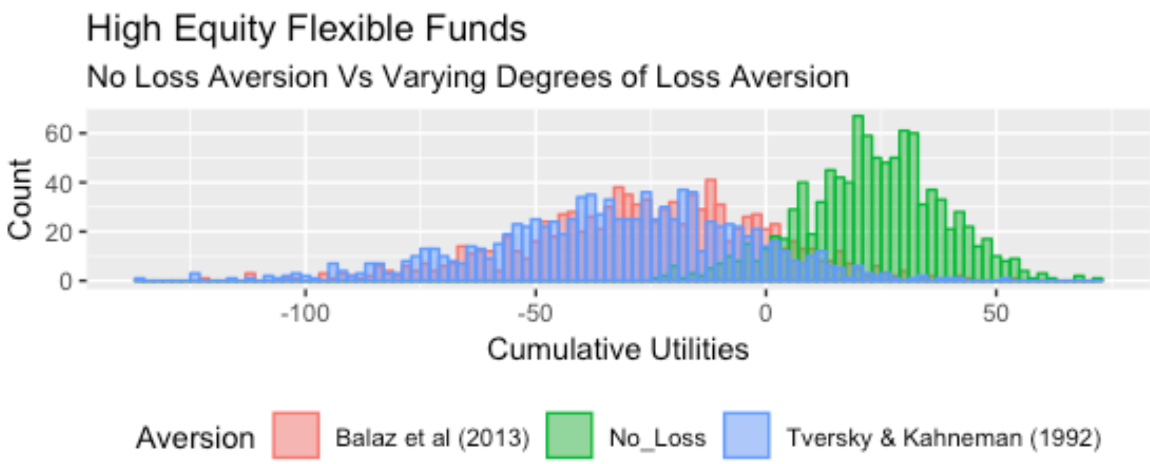

Low-Medium Risk Absolute Return Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
60
40
20
0
20
30
40
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

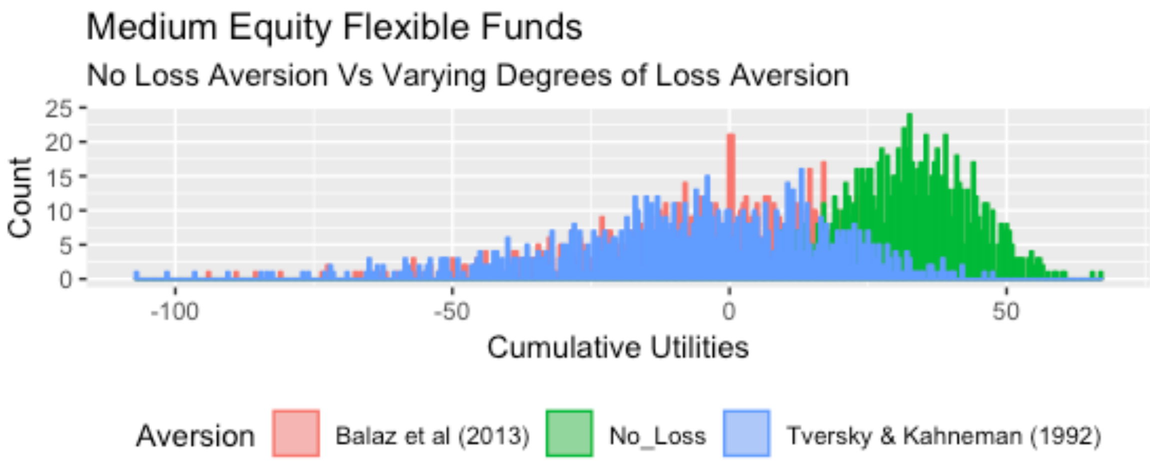

Low Risk Absolute Return Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
80
60
40
20
0
10
20
30
40
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

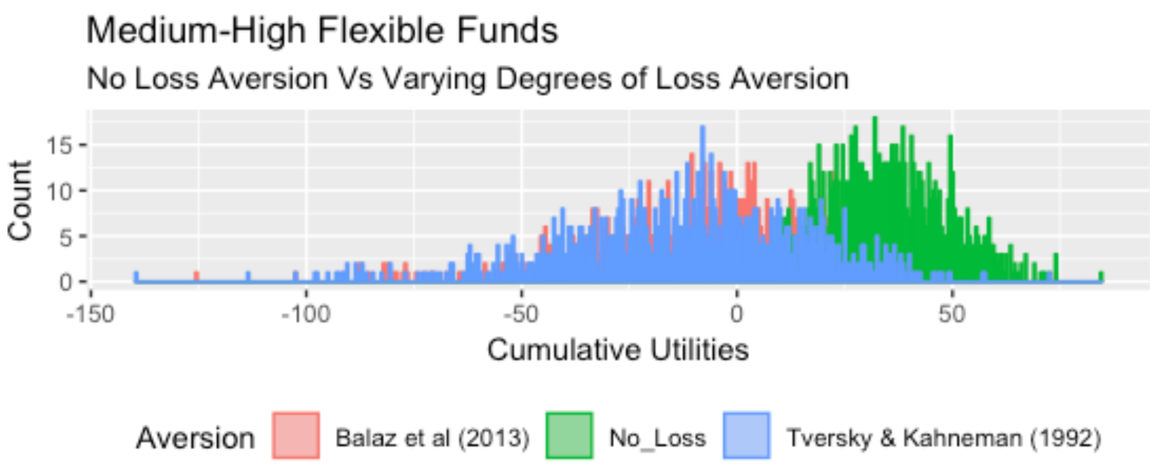

High Equity Balanced Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
50
40
30
20
10
0
-100
-50
0
50
100
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)


High Medium Equity Balanced Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
80
60
40
20
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)


Medium Equity Balanced Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
60
40
20
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)


High Equity Flexible Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
60
40
20
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)


Medium Equity Flexible Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
25
20
15
10
5
0
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)


Medium-High Flexible Funds
No Loss Aversion Vs Varying Degrees of Loss Aversion
Count
15
10
5
0
-150
-100
-50
0
50
Cumulative Utilities
Aversion
Balaz et al (2013)
No_Loss
Tversky & Kahneman (1992)

Appendix B5: T-Test Results

Results of T-tests for High Risk Samples with Tversky and Kahneman (1992) Loss Aversion

```
        Welch Two Sample t-test

data:  yy$General and zz$Flexible
t = -0.69491, df = 1936, p-value = 0.4872
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.613115  1.722534
sample estimates:
mean of x mean of y
-33.68520 -32.73991
```

```
        Welch Two Sample t-test

data:  xx$Absolute and yy$General
t = 39.656, df = 1509.3, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 44.44721 49.07305
sample estimates:
mean of x mean of y
 13.07493 -33.68520
```

```
        Welch Two Sample t-test

data:  ww$Balanced and xx$Absolute
t = -16.23, df = 1049.7, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -63.25890 -49.61293
sample estimates:
mean of x mean of y
-43.36099  13.07493
```

```
        Welch Two Sample t-test

data:  zz$Flexible and ww$Balanced
t = 2.998, df = 1127.2, p-value = 0.002777
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  3.67004 17.57212
sample estimates:
mean of x mean of y
-32.73991 -43.36099
```

Results of T-tests for Medium Risk Samples with Tversky and Kahneman (1992) Loss Aversion

```
        Welch Two Sample t-test

data:  dd$Flexible and aa$Balanced
t = -0.35645, df = 1967.2, p-value = 0.7215
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.699310  1.869002
sample estimates:
mean of x mean of y
-11.27680 -10.86165
```

```
        Welch Two Sample t-test

data:  cc$General and dd$Flexible
t = -20.775, df = 1624.3, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -38.94048 -32.22198
sample estimates:
mean of x mean of y
-46.85803 -11.27680
```

```
        Welch Two Sample t-test

data:  bb$Absolute and cc$General
t = 48.201, df = 1014.4, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 68.37592 74.17947
sample estimates:
mean of x mean of y
 24.41966 -46.85803
```

```
        Welch Two Sample t-test

data:  aa$Balanced and bb$Absolute
t = -45.172, df = 1055.2, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -36.81389 -33.74872
sample estimates:
mean of x mean of y
-10.86165  24.41966
```

Results of T-tests for Low Risk Samples with Tversky and Kahneman (1992) Loss Aversion

```
        Welch Two Sample t-test

data:  ee$Balanced and ff$Absolute
t = -47.065, df = 1116.1, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -33.23748 -30.57710
sample estimates:
mean of x mean of y
-1.502919 30.404368
```

```
data:  hh$Flexible and ee$Balanced
t = -5.0028, df = 1966.3, p-value = 6.152e-07
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.944123 -3.032966
sample estimates:
mean of x mean of y
-6.491464 -1.502919
```

```
        Welch Two Sample t-test

data:  ff$Absolute and gg$General
t = 77.685, df = 1089.4, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 58.09339 61.10402
sample estimates:
mean of x mean of y
 30.40437 -29.19434
```

```
        Welch Two Sample t-test

data:  gg$General and hh$Flexible
t = -21.421, df = 1998, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -24.78138 -20.62437
sample estimates:
 mean of x  mean of y
-29.194337  -6.491464
```

Results of T-tests for High Risk Samples with Some Aversion

```
        Welch Two Sample t-test

data:  ww$Balanced and xx$Absolute
t = -16.089, df = 1049.2, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -59.78361 -46.78654
sample estimates:
mean of x mean of y
-37.52720  15.75788
```

```
        Welch Two Sample t-test

data:  xx$Absolute and yy$General
t = 37.755, df = 1511.4, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 39.90083 44.27409
sample estimates:
mean of x mean of y
 15.75788 -26.32958
```

```
        Welch Two Sample t-test

data:  yy$General and zz$Flexible
t = -0.21769, df = 1935.8, p-value = 0.8277
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.800221  2.240683
sample estimates:
mean of x mean of y
-26.32958 -26.04982
```

```
        Welch Two Sample t-test

data:  zz$Flexible and ww$Balanced
t = 3.4027, df = 1125.1, p-value = 0.0006906
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  4.859317 18.095444
sample estimates:
mean of x mean of y
-26.04982 -37.52720
```

Results of T-tests for Medium Risk Samples with Some Aversion

```
        Welch Two Sample t-test

data:  aa$Balanced and bb$Absolute
t = -42.582, df = 1060.3, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -32.59627 -29.72447
sample estimates:
mean of x mean of y
-6.021067 25.139306
```

```
        Welch Two Sample t-test

data:  bb$Absolute and cc$General
t = 45.913, df = 1015.9, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 60.57467 65.98369
sample estimates:
mean of x mean of y
 25.13931 -38.13987
```

```
        Welch Two Sample t-test

data:  cc$General and dd$Flexible
t = -20.105, df = 1638.4, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -35.34263 -29.05963
sample estimates:
 mean of x  mean of y
-38.139872  -5.938742
```

```
        Welch Two Sample t-test

data:  dd$Flexible and aa$Balanced
t = 0.075113, df = 1962.2, p-value = 0.9401
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.067156  2.231806
sample estimates:
mean of x mean of y
-5.938742 -6.021067
```

Results of T-tests for Low Risk Samples with Some Aversion

```
        Welch Two Sample t-test

data:  hh$Flexible and ee$Balanced
t = -5.0614, df = 1970.6, p-value = 4.548e-07
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.577067 -2.903577
sample estimates:
mean of x mean of y
-1.929372  2.810950
```

```
        Welch Two Sample t-test

data:  gg$General and hh$Flexible
t = -20.021, df = 1995.9, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -22.12757 -18.17938
sample estimates:
 mean of x  mean of y
-22.082848  -1.929372
```

```
        Welch Two Sample t-test

data:  ff$Absolute and gg$General
t = 71.473, df = 1087.8, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 51.37241 54.27268
sample estimates:
mean of x mean of y
 30.73970 -22.08285
```

```
        Welch Two Sample t-test

data:  ee$Balanced and ff$Absolute
t = -43.612, df = 1118.9, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -29.18525 -26.67224
sample estimates:
mean of x mean of y
  2.81095  30.73970
```

Results of T-tests for High Risk Samples with No Aversion

```
        Welch Two Sample t-test

data:  zz$Flexible and ww$Balanced
t = 8.2219, df = 1105.4, p-value = 5.572e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 13.51931 21.99455
sample estimates:
mean of x mean of y
23.010892  5.253958
```

```
        Welch Two Sample t-test

data:  yy$General and zz$Flexible
t = 6.0518, df = 1934.1, p-value = 1.716e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 3.109785 6.091667
sample estimates:
mean of x mean of y
 27.61162  23.01089
```

```
        Welch Two Sample t-test

data:  xx$Absolute and yy$General
t = 11.721, df = 1554.8, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 6.512407 9.130041
sample estimates:
mean of x mean of y
 35.43284  27.61162
```

```
        Welch Two Sample t-test

data:  ww$Balanced and xx$Absolute
t = -14.177, df = 1045.8, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -34.35605 -26.00172
sample estimates:
mean of x mean of y
 5.253958 35.432842
```

Results of T-tests for Medium Risk Samples with No Aversion

```
        Welch Two Sample t-test

data:  dd$Flexible and aa$Balanced
t = 5.963, df = 1858, p-value = 2.957e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 2.503532 4.957473
sample estimates:
mean of x mean of y
 33.20702  29.47652
```

```
        Welch Two Sample t-test

data:  bb$Absolute and cc$General
t = 6.68, df = 1063.4, p-value = 3.844e-11
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 3.265320 5.981477
sample estimates:
mean of x mean of y
 30.41671  25.79331
```

```
        Welch Two Sample t-test

data:  cc$General and dd$Flexible
t = -8.7771, df = 1832.2, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -9.070316 -5.757095
sample estimates:
mean of x mean of y
 25.79331  33.20702
```

```
        Welch Two Sample t-test

data:  aa$Balanced and bb$Absolute
t = -2.3732, df = 1207.4, p-value = 0.01779
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.7174682 -0.1629227
sample estimates:
mean of x mean of y
 29.47652  30.41671
```

Results of T-tests for Low Risk Samples with No Aversion

```
        Welch Two Sample t-test

data:  ee$Balanced and ff$Absolute
t = 3.2086, df = 1144.9, p-value = 0.001371
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.4845539 2.0098704
sample estimates:
mean of x mean of y
 34.44599  33.19878
```

```
        Welch Two Sample t-test

data:  ff$Absolute and gg$General
t = 5.8144, df = 1072.8, p-value = 8.022e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 2.074189 4.187226
sample estimates:
mean of x mean of y
 33.19878  30.06807
```

```
        Welch Two Sample t-test

data:  gg$General and hh$Flexible
t = -2.2456, df = 1805.4, p-value = 0.02485
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.7312054 -0.1845889
sample estimates:
mean of x mean of y
 30.06807  31.52597
```

```
        Welch Two Sample t-test

data:  hh$Flexible and ee$Balanced
t = -5.4923, df = 1998, p-value = 4.475e-08
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.962694 -1.877352
sample estimates:
mean of x mean of y
 31.52597  34.44599
```