A CRITICAL THEORY ENQUIRY IN THE DEVELOPMENT OF NUMBER SENSE IN NAMIBIAN FIRST YEAR PRE-SERVICE SECONDARY MATHEMATICS TEACHERS

by

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DECLARATION

By submitting this dissertation, I declare that the entirety of the work contained herein is my own, original work, I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in in its entirety or in partial submitted it for obtaining any qualification.

March 2016

DEDICATION

This thesis is dedicated to:

My wife:	Rachel Ndilimeke Tuwilika Naukushu
My Children:	Peter Shiningashike and Shiwana Teeleleni Naukushu (Jnr)
My mother:	Ndapunikwa Naukushu
My late father:	Petrus Shiningashike Naukushu

I dedicated this thesis to them for their patience, love, support and encouragement during the time that I stole from them as I was carrying out this study.

ABSTRACT

This study, *"A Critical Theory enquiry in the development of number sense in Namibian first year pre-service secondary mathematics teachers,"* inquired into the effectiveness of a Critical Theory informed intervention on the number sense training of Pre-service Secondary Mathematics Teachers in Namibia.

The study proposed and evaluated a number sense training CRENS model based on Critical Theory of pre-service secondary mathematics teachers at the University of Namibia. A convenient sample of sixty (60) pre-service secondary mathematics teachers was selected. The study utilised both qualitative and quantitative methods with a pre-test-post-test control design to draw data from the participants. The study utilised a five tier-number sense test, an in-depth focus group interview, document analysis as well as a questionnaire with both open ended and closed ended questions to draw data from participants.

Regarding the question about the level of number sense comprehension both the qualitative and quantitative findings revealed that the number sense of the preservice mathematics secondary teachers was below basic before the intervention. Regarding the association of the independent variable number sense the dependent variable academic performance of pre-service secondary mathematics teachers the quantitative results showed a moderate positive association. The study also found out that the changes in academic performance could be attributed to number sense up to 23% and vice-versa.

The Multiple Linear Regression analysis results revealed that the individual contribution of the number sense proficiency variable was statistically significant while that of number sense reasoning was not. The number sense proficiency variables meaning and size of numbers, meaning and effect of operations and estimation, were found to have statistically significant relationship with the academic performance of preservice secondary mathematics teachers.

The qualitative data presented in section 6.3 indicate that the majority of the students described their number sense experiences to be relevant to their academic performance

in mathematics, with a few of the students who felt their number sense was not relevant and did not really impact their academic performance.

Regarding the impact of Critical Theory intervention the study found out that there were statistically significant differences in the performance of the students before and after the intervention, particularly, in both the number sense reasoning and proficiency variables. For the number sense reasoning variables, meaning and size of numbers, counting and computational strategies and estimation the study found that the impact of a Critical Theory intervention was statistically significant. For the variables of number sense reasoning statistically significant differences were observed in the estimation and counting and computational strategies only.

Overall the study found out that the impact of Critical Theory was effective. The results of the Cohen's *d* effect size indicated a very large effect size in both number sense proficiency and reasoning. The qualitative data showed some improved responses to the number sense items for both number sense reasoning and number sense proficiency.

It is therefore recommended that the CRENS based intervention could be used to improve the number sense of pre-service secondary mathematics teachers. It is also recommended that the number sense be incorporated on the training of pre-service secondary mathematics teachers. Both the primary and secondary school curricula should consider integrating number sense components to help the learners to understand mathematics better.

This study considered an important contribution that it made in an African context where the quality of both primary and secondary mathematics education constantly falls short of international benchmark standards such as TIMSS and PISA. That is, incorporating number sense training in the curricula for preservice teachers should not just be at primary school only but also even at secondary school level.

By developing the multiple linear regression analysis model presented in the analysis of data, it can also be argued that the study makes a methodological contribution to the research on number sense. This relationship can be used to give guidance on the relationship between number sense and academic performance which from the cited literature did not appear to have been explored.

By applying Critical Theory and therefore introducing the CRENS model which as a unique characteristic of this study, the study fills a gap doing away with a monotonous conceptual frame work of constructivism that seems to be existing in the development of number sense. The nature of resources utilised in the study were suitable for the level of the participants, as a result, these could always be utilised in offering guidance on the number sense training by the other teacher training institutions or the University of Namibia in developing a practical number sense course as per recommendations of this study.

OPSOMMING

Hierdie studie, *A Critical Theory enquiry in the development of number sense in Namibian first year pre-service secondary mathematics teachers*, het die doeltreffendheid van 'n Kritiese Teorie ingeligte ingryping op die getalbegrip opleiding van voordiens Sekondêre Wiskunde-onderwysers in Namibië ondersoek.

Die studie het ŉ getalbegrip opleiding CRENS model, gegrond in die Kritiese Teorie, van voordiens sekondêre wiskunde-onderwysers by die Universiteit van Namibië voorgestel en geëvalueer. 'n Gerieflike monster van sestig (60) voor-diens sekondêre wiskunde-onderwysers is gekies. Die studie het beide kwalitatiewe en kwantitatiewe metodes, met 'n voor-toets-na-toets kontrole ontwerp gebruik om data van die deelnemers te bekom. Die studie het 'n vyf-vlak getalbegrip toets, 'n in-diepte fokusgroep-onderhoud, dokument-analise asook 'n vraelys met beide oop einde en geslote vrae benut om data van die deelnemers in te samel.

Ten opsigte van die vraag oor die vlak van getalbegrip vaardighede, het beide die kwalitatiewe en kwantitatiewe bevindinge getoon dat die getalbegrip van die voordiens sekondêre wiskunde-onderwysers onder die basis vlak was, voor die intervensie. Ten opsigte van die verhouding van die onafhanklike veranderlike, getalbegrip, en die afhanklike veranderlike, akademiese prestasie, het die kwantitatiewe resultate 'n matige positiewe assosiasie getoon. Die studie het ook bevind dat die veranderinge in akademiese prestasie toegeskryf kan word aan getalbegrip tot 23% en andersom.

Die meervoudige lineêre regressie analise resultate het getoon dat die individuele bydrae van die getalbegrip vaardigheid veranderlike statisties betekenisvol was, terwyl dié van getalbegrip redenering nie statisties betekenisvol was nie. Die getalbegrip vaardigheidsveranderlikes, betekenis en die grootte van getalle, betekenis en die effek van bewerkings en beraming, het statisties beduidende verbande met die akademiese prestasie van voordiens sekondêre wiskunde-onderwysers getoon.

Die kwalitatiewe data in afdeling 6.3 aangebied, dui daarop dat die meerderheid van die studente hul getalbegrip ervaringe relevant tot hul akademiese prestasie in wiskunde beskryf het, terwyl 'n paar van die studente gevoel het dat hul getalbegrip nie relevant was nie en nie regtig 'n impak op hul akademiese prestasie gehad het nie.

Aangaande die impak van die Kritiese Teorie intervensie het die studie bevind dat daar statisties beduidende verskille was in die prestasie van die studente voor en na die intervensie, veral in beide die getalbegrip redenering en vaardigheid veranderlikes. Vir die getalbegrip redenering veranderlikes, betekenis en grootte van getalle, tel en berekeningstrategieë en beraming, het die studie bevind dat die impak van 'n Kritiese Teorie intervensie nie statisties beduidend was nie. Vir die veranderlikes van getalbegrip redenering is statisties beduidende verskille slegs waargeneem in die beraming en tel en berekeningstrategieë.

Die studie het bevind dat die impak van die kritiese teorie oor die algemeen doeltreffend was. Die resultate van die Cohen *d* effekgrootte het aangedui dat daar 'n baie groot effekgrootte in beide getalbegrip vaardigheid en redenering is. Die kwalitatiewe data het enkele verbeterde reaksies op die getalbegrip items vir beide getalbegrip redenering en getalbegrip vaardigheid getoon.

Dit word dus aanbeveel dat die CRENS intervensie gebruik kan word om die getalbegrip van voordiens sekondêre wiskunde-onderwysers te verbeter. Dit word ook aanbeveel dat getalbegrip opgeneem word in voordiensopleiding van sekondêre wiskunde-onderwysers. Beide die primêre en sekondêre skool kurrikulums behoort te oorweeg om getalbegrip komponente te integreer om die leerders te help om wiskunde beter te verstaan.

Hierdie studie is'n belangrike bydrae in 'n Afrika konteks waar die kwaliteit van beide primêre en sekondêre wiskunde-onderwys voortdurend kort val van internasionale standaarde soos TIMSS en PISA. Dit is om getalbegrip opleiding te inkorporeer in die kurrikulum vir indiensopleiding vir onderwysers nie net op laerskool nie, maar ook selfs op sekondêre skoolvlak.

Deur die ontwikkeling van die meervoudige lineêre regressie-analise model in die ontleding van data, kan dit ook aangevoer word dat die studie'n metodologiese bydrae tot die navorsing oor getalbegrip maak. Hierdie verhouding kan gebruik word om leiding te gee oor die verhouding tussen getalbegrip en akademiese prestasie. Dit blyk asof die teorie nie genoeg ondersoek is nie.

Deur die toepassing van die Critical Theory en dus die bekendstelling van die CRENS model wat 'n unieke kenmerk van hierdie studie is, kan die leemte wat te doen het met 'n eentonige konseptuele raamwerk van konstruktivisme wat blyk in die ontwikkeling van getalbegrip bestaan,gevul word. Die aard van die hulpbronne wat gebruik word in die studie was geskik vir die vlak van die deelnemers. As gevolg van die bekombaarheid van die hulpbronne in die aanbieding oor die getalbegrip studie opleiding deur enige onderwys opleidingsinstelling of die Universiteit van Namibië in die ontwikkeling van 'n praktiese getalbegrip kurses is daar n sterk aanbeveling van hierdie studie.

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ACRONYMS

ANOVA	Analysis of Variance
ASEI-PDSI	Activity Students Experiment Improvisation-Practise Do See Improve
BED	Bachelor of Education
BETD	Basic Education Teachers Diploma
CME	Critical Mathematics Education
CRENS	Critical Realistic Number Sense
DNEA	Directorate of National Examinations and Assessment
E L O s	Exit Learning Outcomes
FTNST	Five Tier Number Sense Test
HLT	Hypothetical Learning Trajectory
IGCSE	International General Certificate for Secondary Examination
JICA	Japan International Cooperation Agency
LCE	Learner Centred Education
MBESC	Ministry of Basic Education Sport and Culture
MEC	Ministry of Education and Culture
MRA	Multiple Regression Analysis
MRC	Multiple Regression Coefficient
NCTM	National Council Teachers of Mathematics
NSSAT	Number Sense Standardised Achievement Tests
NSSCE	Namibian Senior Secondary Certificate Examinations
NVCI	Naukushu's Vicious Cycle of Innumeracy
PNSA	Practical Number Sense Activities
RME	Realistic Mathematics Education
SMASE	Strengthening of Mathematics and Science Education
TIMSS	Third International Mathematics and Science Study
UNAM	University of Namibia

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CHAPTER ONE
INTRODUCTION AND ORIENTATION OF THE STUDY

1.1 Introduction

This study, *"A Critical Theory enquiry in the development of number sense in Namibian first year pre-service secondary mathematics teachers"* investigates the effectiveness of a Critical Theory informed intervention into the number sense training of Pre-service Secondary Mathematics Teachers (PSMTs) in Namibia. This chapter begins with a discussion of the background of the Namibian education system before independence, the reforms and transformations that took place in the Namibian education after independence.

The chapter further presents the problem statement, purpose of the study, significance of the study, assumptions of the study and research questions. Additionally, delimitations, limitations, the definitions of key concepts as well as the outline of the thesis are also presented in this chapter. The chapter concludes with a brief summary of what was discussed and a preview of what to expect in Chapter 2.

1.2 Background to the study

1.2.1 An overview of the Namibian education system before independence

Before independence in Namibia the education system was mainly characterised by inequalities brought about by apartheid. Literature (e.g., Amutenya, 2002; Ministry of Basic Education Sport and Culture (MBESC), 1993; Naukushu, 2011; Amkugo, 1999) distinguishes three separate education systems that existed based on the apartheid regime scheme prior to the attainment of independence in Namibia. The three education systems that existed before independence were: education for Whites, Blacks and for Coloureds. The black (Bantu) education put emphasis on achieving basic and minimal levels of understanding and hence focused on rote learning more especially in mathematics. Amutenya (2002) asserts that prior to independence based on the colonial mind mathematics education had a misconception that black minds were not meant for mathematics.

Moreover, according to literature (e.g. Amutenya, 2002; Ministry of Education Sport and Culture (MBESC), 1993; Naukushu, 2011) it was believed that blacks could only be competent up to a very basic mathematical level (arithmetic) and that once taught further mathematics beyond the arithmetic level they would not cope. Thus there were separate mathematics curricula for whites and for blacks. The teachers were also trained at racially segregated institutions across the country.

The assumption that black learners could not be competent in mathematics made it difficult for many black learners of mathematics to acquire higher levels of understanding mathematical concepts at that time. Having been subjected to an oppressive educational system, black learners' numerical competencies and mathematical understanding suffered. Such strains of mathematical deprivation still exist in the Namibian education to date.

The foregoing mathematical denial is in line with Dewey (1999) who notes that black people have been oppressed and deprived a chance to learn Mathematics and Science at the initial stage of colonialism. Having been subjected to oppression learners' numerical competencies and mathematical understanding were compromised to an extent, also, the scars still exist to date. Many of the students that lacked mathematical understanding became mathematics teachers in Namibia and therefore this lack of Mathematical facility was viciously recycled. As a consequence many learners do not excel in Mathematics in Namibia at secondary school level. For three consecutive years the Directorate of National Examination and Assessment (DNEA, 2009; 2010; 2011) reported the alarmingly poor performance in mathematics of Namibian students. It is therefore imperative that studies of possible interventions be carried out to alleviate the situation.

1.2.2 Reforms in the Namibian education after independence

After independence in 1990, the education system in Namibia was "reformed" to accomplish four goals: accessibility, quality, equity and democracy (Ministry of Basic Education, Sport and Culture (MBESC), 1993). In an attempt to rebuild the nation the Namibian government developed a Vision 2030 framework which anticipates that the country will be developed and industrialised by the year 2030 (National Planning Commission (NPC), 2003). The NPC (2003) further stresses the need to cultivate a knowledge-based economy underpinned by scientific and mathematical disciplines.

Since mathematics and science are innovation crucial disciplines to professions driving economic development it could be concluded that development of mathematical, numerical and scientific understanding are crucial in drawing closer to vision 2030 and development of the Namibian nation as a whole. Also, in terms of global competitiveness Namibia is ranked 115th in the higher education and training index (Schwab, 2015). This therefore means Namibia has to redouble its effort if it is to be classified as a developed and industrialised nation by the year 2030. The call for a knowledge based economy hence requires new and innovative teaching and learning strategies such as learner-centred teaching. Therefore the curriculum was further revised in 2006 to suit the demands for a new and growing nation.

It was therefore against the foregoing background that the Cape Matriculation System was abolished and replaced by the Cambridge Matriculation System in 1995. Amkugo (1999) further contends that the Cape Matriculation Education System based on colonial mind, was greatly associated with rote learning and could not educate to liberate the learners.

Unfortunately, the standard of education seemed to have declined since the adoption of the Cambridge International General Certificate of Secondary Education (IGCSE), in 1995. This was evidenced by earlier reports of Directorate of National Examinations and Assessment (DNEA), (1996; 1998; 2002; 2006; 2007; 2008) and other subsequent reports of DNEA (2009; 2010 and 2013) that showed diminishing numbers of high school graduates especially passing Mathematics.

Consequent to the foregoing the IGCSE was abolished and replaced by the Namibian Senior Secondary Certificate Examination (NSSCE) in 2007. However, the NSSCE was an exact replica of the Cambridge Matriculation system that had been “done away with”. The National Institute for Education Development (NIED) documents such as the syllabi, assessment tools and teacher guides show that there is no difference between the Cambridge end of high school mathematics contents and the new Namibian one. The mathematical contexts used in the teaching and learning materials were still not local, despite the localisation of contents.

Since the adoption of NSSCE in the hope to alleviate the situation, the whole Namibian Education System has never enjoyed improvement in the number of successful matriculates. Learners continued to underperform at high school level particularly, in mathematics, allegedly due to lack of number sense DNEA (2009; 2010; 2013).

Furthermore, commencing with the academic year 2012 Mathematics was made a compulsory subject for every learner up to grade 12 level regardless of whether it was passed at junior grades or not. In support of this the ministry of Education took a decision that mathematics is a necessity for all learners and therefore needs to be taken by each and every learner (Illukena, 2011). Furthermore the ministry argued that the whole nation needs to be literate in the area of mathematics if national development is to be realised (Ministry of Education and Culture (MEC), 2012).

It therefore appears that the issue of mathematics for all is among the controversial issues in the Namibian education system. There is hence a debate among different stakeholders as to whether making mathematics a compulsory subject for all learners at high school level was the best decision. Those in favour of mathematics for all argue that it is evident, nowadays, in the society that individuals with limited basic mathematical skills are at greatest disadvantage in the labour market and in terms of general social exclusion from the social set up.

> Therefore, if the future citizens need to participate in democratic processes in an economically, and technologically advanced society, they need to have not only good literacy skills, but also good skills in mathematics. It is crucial that they receive better and quality education in mathematics, science and technology to meet the existing demand for such skills in the workforce and contribution to the attainment of Vision 2030 (Illukena, 2011, p.12).

Illukena (2011) further argues that Mathematics also plays a significant role in the lives of individuals and society as a whole. This makes it imperative that Namibian mathematics should equip learners with skills necessary for achieving higher education, career aspirations, and for attaining personal fulfilment hence mathematics should be compulsory at all levels of one's education.

In addition research (e.g. Wolfaardt, 2003) indicates that the grades of learners in mathematics diminish by an average of 2 points as they progress from grade 10 to 12. It could be argued that the inclusion of mathematics as a compulsory school subject could only aggravate the current performance of mathematics which is already dismal.

Moreover, the inclusion of mathematics as a compulsory subject in school could only be effective if the learners are equipped with skills and potential to cope with the demands posed by mathematics at high school level (Courtney-Clarke, 2012). Such mathematical skills and potential include a better numerical facility which could be enhanced by improving learners' number sense. Moreover, in the Namibian context there seem to be a lack of research on the issue of mathematics as a compulsory subject to position the basis of these arguments on empirical evidence as to whether mathematics for all is a necessity. However, it remains certain that mathematics requires learners to possess a better grasp of numerical understanding to cope with the demands of the high school mathematics curriculum.

Additionally, echoing the strains of colonial education as articulated by Amutenya (2002) as well as Amkugo (1999) that was not educating to liberate, the fact that the blacks were perceived as mathematically handicapped and that mathematics education then focused only on arithmetic. It can be argued that teachers were therefore not adequately trained to handle the mathematical content after independence. The researcher thus argues that these mathematical deficiencies could be passed on to the learners currently in school.

It can therefore be concluded from the foregoing that if learners possess a poor number sense their performance in mathematics will be compromised to a great extent. It also appears that there is a need to take into consideration that the learners have no choice in taking mathematics as a school subject and that they are from a previously disadvantaged mathematical background as referred to above.

It was hence against the foregoing background that the researcher deemed it necessary to investigate a Critical theory intervention in the number sense of training of mathematics teachers to help them gain a better understanding of numerical skills such as number sense. This was done in anticipation that once equipped with the better numerical skills teachers will help produce learners that can bring about improved performance in the senior secondary certificate results in topics involving number sense.

Moreover; the concept of number sense is a relatively new strand in the Namibian mathematics education context (Courtney-Clarke, 2012; Naukushu, 2012). There is therefore a need to embark on this and other similar studies to investigate how pre-service teachers' number sense can be enhanced as part of a long term strategy to remedy the problem of poor academic performance in mathematics. This might ultimately contribute to a successful implementation of the mathematics for all policy in the Namibian high school curriculum.

1.3 Problem Statement

This study was fuelled by a problem of persistent poor performance in Mathematics at grade 12 level since the phasing out of the Cape Matriculation system after independence in Namibia. This poor academic performance in Mathematics could be partly attributed to a lack of number sense (DNEA, 2009) owing to the apartheid legacy of inferior mathematical education for the blacks referred to by Amkugo (1993).

Ever since the indigenisation of the curriculum in 2007 the Namibian education system has reported an alarming failure rate among mathematics learners at high school level associated with a weak understanding of number sense (DNEA, 2008; 2009; 2010; 2011; 2012). Moreover, the Namibian education system seems also to be trapped in a three stage vicious cycle of numerical deficiency as identified by Naukushu (2012).

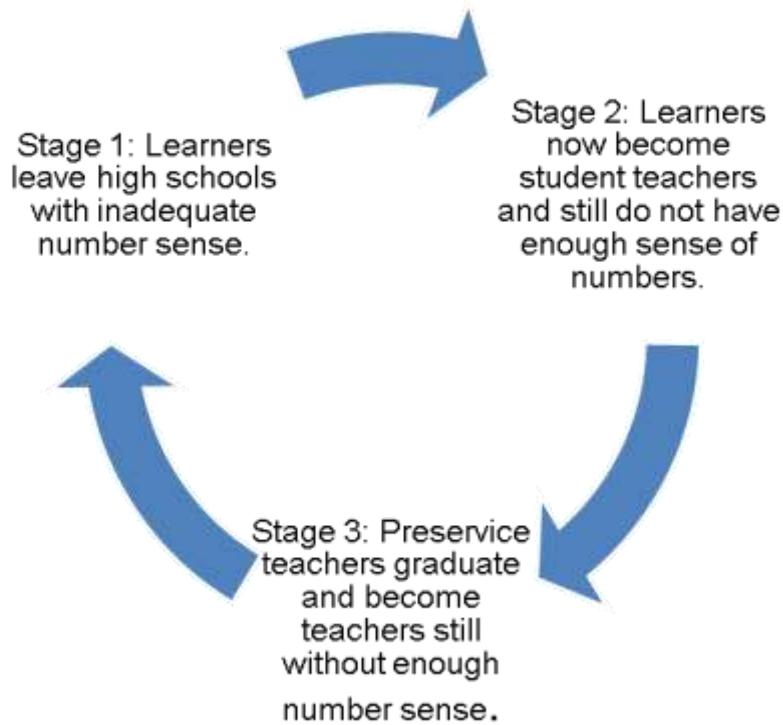

Figure 1.1: Naukushu's three stage vicious cycle of lack of number sense

The limited related studies in the Namibian context failed to suggest possible means of alleviating the problem of lack of number sense among high school learners. This study assumes that once pre-service teachers are equipped with numerical skills they have a better chance to develop the same number sense skills among their learners and the problem of lack of number sense and ultimately the poor performance in mathematics could be mitigated.

The study anticipates that the cycle of innumeracy explained by Naukushu (2012) could probably be broken by a Critical Theory intervention in the number sense training of pre-service secondary teachers of mathematics in Namibia as early as their first year of study. Therefore pre-service teachers could be emancipated by utilising Critical Theory in training them to be numerically competent. This could be aided by employing new approaches of educating to liberate teachers such as Critical Mathematics Education, Realistic Mathematics Education and Ethnomathematics.

Consequently; this study utilises Critical Theory to critically enquire into assessing the level of number sense of pre-service secondary mathematics teachers at first year level in Namibia. The study will also employ Critical Theory in developing and assessing the impact of a number sense intervention programme for secondary pre-service teachers of mathematics at first year in Namibia.

1.4 Purpose of the study

This study seeks to determine how a Critical Theory informed intervention can aid the number sense training of pre-service secondary mathematics teachers in Namibia. More specifically, the study sought to achieve the following objectives:

- To determine the number sense competency levels of first year pre-service secondary teachers of mathematics in Namibia.
- To investigate the relationship between the number sense of pre-service mathematics teachers and their academic performance in mathematics.
- To develop and evaluate a model curriculum (based on Critical Theory) of number sense training for first year pre-service secondary mathematics teachers in Namibia.

1.5 Significance of the study

The role of number sense in the comprehension of school mathematics can neither be underestimated. Research (e.g. Naukushu, 2012; Emmanuelsson & Johansson, 1996; McIntosh, Reys, Reys, Bana, & Farell, 1997; National Council of Mathematics teachers (NCTM), 2000; Sowder, 1992; Yang, 2003) holds the idea that number sense is internationally considered to be a key ingredient in mathematical skill, and therefore should be integrated in the school mathematics curriculum. Additionally the foregoing literature suggests that individuals with strong numerical facility are likely to perform well in mathematics as a school subject.

However, these never attempted to assess the strength of the extent to which number sense impacts the academic performance of students in mathematics. It is therefore envisaged that this study could contribute to the body of knowledge by identifying the strength of the extent to which the number sense of the preservice secondary mathematics teachers contributes to their academic performance in mathematics.

Moreover, the fact that mathematics was made a compulsory subject in the Namibian high school curriculum was fuelled by a desire to produce a mathematically literate nation (Illukena, 2011). It can therefore be assumed that if number sense is properly included in the training of preservice mathematics teachers, the development of number sense could trickle down to their learners.

It is therefore plausible to argue that this number sense intervention investigation is imperative and should be carried out to find means of developing the number sense of pre-service teachers. Thus this study was carried out in anticipation that an improved comprehension of number sense could help them cope with the demands of high school mathematics curriculum. It is for this reason therefore that this study was carried out to assess the role that critical theory could play to facilitate in the number sense training of pre-service secondary mathematics teachers in Namibia.

This study also seeks to contribute to the existing knowledge in the training of mathematics teachers in number sense in the Namibian context as it has been explored by limited literature (e.g. Courtney-Clarke & Wessels, 2014 & Potgieter, 2014). The study therefore envisages contributing to the ideas of the foregoing researchers by unfolding different strategies of developing number sense of pre-service secondary mathematics teachers. It is hoped that the study will propose a model that could be used to develop the number sense of pre-service secondary mathematics teachers in developing country contexts like Namibia. The model could also be adapted and adopted for in-service primary teachers of mathematics and therefore be made a national one.

This study was deemed important in contributing to the global knowledge bank by availing new insights on the Critical Theory intervention in the number sense training of pre-service secondary mathematics teachers. Recommendations will be made to policy makers to ensure reinforcement of number sense development in Namibian education system to mitigate the problem of poor academic performance in mathematics.

1.6 Assumptions of the study

The study holds the assumptions of the Critical Theory as a conceptual framework. Pioneered by Max Horkhemeir at the Institute for Social Research in the late 1800s Critical Theory seeks to understand how human values are affected by the hegemonic oppressive powers. Critical Theory claims that the truth very often serves the status quo; i.e. the truth is made and unmade by human beings (Venter, Higgs, Jeevavanthan, Letseka & Mays, 2007).

This study acknowledges that pre-service teachers being considered are from a previously disadvantaged section of Namibian society where education was, as outlined by Nkomo (1990), a tool of domination. The apartheid system considered blacks to be inferior to whites in mathematical ability. Such a superiority complex accounted for the inferior resourcing of the Bantu Education system from which the majority of pre-service teachers came leading to the underdevelopment of numerical competencies (Nkomo, 1990). It is therefore envisaged that the study in some way or the other could contribute to addressing the issue of emancipating the previously disadvantaged education system from which these learners come.

Higgs and Smith (2002) remark that human thoughts are linked to power relations hence humans have no neutral thoughts. According to Higgs and Smith (2002) Critical Theory anticipates seeing humans empowered and free of oppression and domination. This study therefore holds the assumption that developing pre-service teachers' numeracy skills at first year could empower and emancipate them and thereby break the vicious cycle of innumeracy. The assumption of developing number sense of pre-service secondary mathematics teachers could therefore be addressed by applying critical theory to the development of number sense of pre-service secondary mathematics teachers.

Critical Theory favours practicing new approaches such as Critical Mathematics Education (CME) as outlined by Skovsmose, (2002) which can lead to the development of critical thinking. Thus another assumption of this study is that the first year pre-service teachers will be receptive to efforts to develop their critical thinking abilities. Additionally, Critical Theory supports principles of Realistic Mathematics Education (RME) as outlined by van den Heuvel-Panhuizen (2003) based on Freudenthal's idea of mathematics as a human activity. That is, mathematics must have human value and must stay connected to reality, stay close to the children and should be relevant to the society (Freudenthal, 1983). Yet another assumption of this study, therefore, is that the number sense concepts to be dealt with will be as close to the experiential realities of the pre-service teachers and their learners as possible. During the intervention the study utilised the number sense questions that are relevant and within the context of the students.

D'Ambrosio's (2006) ethno mathematics also places mathematical content within the context of the learners, and links to their daily lives, which relates it closely to Critical Theory. CME theorists (e.g. Pais, Fernandes, Matos & Alves, 2007; Skovsmose, 2010; Jett, 2012; Simson & Bullock, 2012) hold the view that mathematics education should emphasise the critique concept. The latter concept calls for students to critically reflect on the reasonableness of their work and answers thereof. This study thus holds the additional assumption that by developing their number sense as early as first year pre-service teachers may develop into critical mathematics educators able to reflect on their own work critically. Therefore the activities to be utilised during the intervention were challenging to ensure that critical thinking was fostered during the number sense intervention.

Critical Theory at it manifests in CME, ethno mathematics and RME, addresses the role of the learner in the learning process, the role of the educator, the role of the curriculum and that of pedagogy. The learner should play an active role in the learning process, be able to challenge the content, the learning method and the teacher, if need be.

In addressing this assumption the study therefore takes into account that the teacher should play the role of facilitator of knowledge construction as an equal and not a superior transmitter of readymade knowledge. The pedagogy should be learner-centred while the mathematical content should be drawn from cultural and experiential contexts that are familiar to the learners.

1.7 Research questions

The study seeks to address the following key research question:

How might a Critical Theory intervention inform the enhancement of Namibian first year pre-service secondary mathematics teachers' competencies in number sense?

In order to address the main question the study utilised the following sub-questions:

1. What is the level of number sense comprehension of first year pre-service secondary mathematics teachers?
2. What is the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics?
3. What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers?

1.8 Delimitations of the study

The study draws data from first year pre-service secondary mathematics teachers at of the University of Namibia. The number sense training was covered in five components as indicated in chapter 3 section 3.2. The number sense training focused on those number sense principles that were perceived relevant to secondary school mathematics curriculum.

1.9 Limitations of the study

In the number sense test, guess work might have interfered with the outcome of the study. The questions on number sense are multiple choice and students might have just guessed without actually trying the questions. To mitigate this limitation the thinking through tier was made provision for, in this tier the respondents were required to indicate the thinking that they carried out during the time they were attempting the questions.

During teaching of the number sense intervention programme pre-service teachers might not have been comfortable and may have changed their normal way of learning knowing that someone is capturing their responses, this might have resulted in them being shy or putting extra effort, knowing that they are being observed. In response to the call for mitigating the Hawthorne effect as alluded to, the researcher endeavoured to inform the participants that the information collected from the study was to be treated with strictest confidentiality. To some extend this reduced the anxiety and or over enthusiasm of respondents. The respondents seem to have behaved in a normal way during the training, probably because the purpose was explained to them clearly.

However, in both situations confidentiality and the importance of honest responses on the findings of this study was explained to the respondents in order to try and address these situations.

Other limitations in terms of methodological implications could have limited the study to some extent. For example it was difficult for the researcher to claim that all academic performance in mathematics was attributable to number sense training. It was also not easy for the researcher to hold constant other factors that could contribute to the academic performance in mathematics when determining the impact of number sense on academic performance in mathematics. In response to this the study attempted to study the impact of each of the number sense variables on the academic performance in mathematics. The researcher also attempted to understand the combined association between all the variables of number sense and the academic performance of preservice secondary teachers in mathematics.

1.10 Definition of terms

Number sense: In general number sense means a sound practical judgment of numbers (Robert, 2002). In this study number sense should be understood as the: understanding of meaning and size of numbers, ability to recognise the equivalence of numbers, abilities to understand the effects of operations, ability to do computations using numbers; ability to make accurate estimations using reasonable benchmarks (McIntosh, Reys & Reys, 1997, 356).

Number sense proficiency: In general number sense proficiency is interpreted as the abilities of individuals to understand the number sense concepts (Bana, 2009; Farrell, 2007). For the purpose of this study number sense proficiency should be understood as the score obtained on the number sense question items.

Number sense reasoning: In general number sense reasoning means the individuals' abilities to reason and understand the reasoning with the numbers (Bana, 2009; and Farrell, 2007). For the purpose of this study number sense reasoning should be understood as the individual's score obtained in the reasoning section of the number sense test that was issued.

Number sense components: The term number sense components in this study refers to the five areas believed to constitute the meaning of the term number sense (McIntosh, Reys and Reys 1992; Jonassen 2004; Zanzali 2005; Burn 2004; & Hilbert 2001).These components are: understanding of meaning and size of numbers, ability to recognise the equivalence of numbers, abilities to understand the effects of operations, ability to do computations using numbers; ability to make accurate estimations using reasonable benchmarks.

Development of number sense: Generally development means the process of producing or creating something so that it becomes advanced and stronger (Robert, 2007). In this study development of number sense should be understood as the process of training first year preservice secondary teachers of mathematics to obtain or advance the following skills: understanding of meaning and size of numbers, ability to recognise the equivalence of numbers, abilities to understand the effects of operations, ability to do computations using numbers; ability to make accurate estimations using reasonable benchmark (McIntosh, Reys, & Reys, 1997, p. 356).

Academic performance: Generally means achievement in education. In this study academic performance means final mark/percentage obtained by each of the first year preservice secondary mathematics teacher in their first semester core module Basic Mathematics for Teachers.

Pre-service secondary mathematics teachers: In this study this concept should be understood as teacher trainees of mathematics at secondary school level (grade 8-12) that are pursuing a B.Ed. degree at the University of Namibia.

Critical Realistic Ethno Number Sense (CRENS): Is a model proposed by this study that was used as a framework for the number sense training of preservice secondary mathematics teachers. This model holds the assumption that the development of number sense should be "Critical" training the preservice secondary mathematics teachers to be critical thinkers, "Realistic" that it should be comprised of real life situations common to daily lives of preservice secondary mathematics teachers, Ethno that is should be part of the context and cultural setting up of preservice secondary mathematics teachers.

CRENS intervention: for the purpose of this study should be understood as the number sense training of preservice secondary mathematics teachers based on the Critical Realistic Ethno Number Sense (CRENS) which is developed from Critical Theory.

1.11 Thesis outline

This thesis is sub-divided into seven (7) chapters:

Chapter 1 provides an introduction and orientation of the study. The chapter begins by giving the synopsis of the Namibian education system before independence, the reforms and transformations that the Namibian education went through are discussed. Chapter one further presents the problem statement, purpose of the study, significance of the study, a brief overview of assumptions of the study and research questions. Additionally, delimitations, limitations, the definitions of key concepts as well as the outline of the thesis are also presented in this chapter. The chapter concludes with a brief summary.

Chapter 2 discusses the theoretical framework (or conceptual framework) underpinning the study. It discusses the general ideas and thinking supported by Critical Theory as the philosophical assumption underpinning this study. This chapter also gives a justification of why Critical Theory is the main theory that influences the design of the intervention in this study, how it will be used to understand the respondents' point of view. In this chapter an outline of how Critical Theory is used to guide the researcher in carrying out data collection and analysis is also outlined. Also, the chapter presents the synergy and coherence between Critical Theory and the other frameworks such as CME, Ethnomathematics and RME in greater detail. The chapter concludes by presenting the CRENS model that was used in the number sense training of preservice secondary mathematics teachers.

Chapter 3 reviews literature relevant to number sense. It analyses the ideas of different researchers in an attempt to define the concept of number sense. Furthermore, Chapter 3 tries to bring out some issues pertaining to teaching and learning of number sense from the literature. The chapter will present as analysis of the possible inclusion of Critical Theory in the number sense in the training of preservice secondary mathematics teachers in Namibia. Since the number sense training intervention will be started as a curriculum the chapter envisages highlighting some aspects of curriculum design.

Chapter 4 discusses how the study was carried out. The mixed methods approach that was used in collecting the empirical data for this study is discussed in this chapter. Hence chapter four attempts to justify the choice of this mixed approach. The population, sample as well as the sampling procedures that are utilised are also outlined in Chapter 4. Chapter 4 further outlines the different phases of the study and their corresponding data collection methods. Issues of validity and reliability were addressed in chapter 4. A discussion of the pilot study and its impact on the conduct of main study will be provided in the first part of this chapter. Chapter 4 also discussed the methods utilised to analyse the data.

Chapter 5 presents and analyses the quantitative findings of the study. Chapter 5 reports on quantitative findings from the pre-test-post-test number sense test as questionnaire, well as the number sense test that assessed factual information of preservice secondary mathematics teachers.

Chapter 6 presents findings from the qualitative part of the study. The qualitative data used in this study were collected by means of a questionnaire, interview guides and the number sense test.

Chapter 7 provides a discussion on the findings, conclusions and relevant recommendations from qualitative and quantitative parts of the study. Both the qualitative and quantitative findings are pooled and compared, and triangulation since triangulation was the thrust of this study. The chapter also attempts to explain the contribution to knowledge the study makes, the research gap it attempts to fill as well as recommendations for further research. Conclusions are also drawn from the data and recommendations were specified regarding the study.

1.12 Summary

This chapter outlined the orientation to the study "*A Critical Theory enquiry in the development of number sense in Namibian first year pre-service secondary mathematics teachers*". The chapter gave a synopsis of the Namibian education system before independence, the reforms and transformations that the Namibian education went through are discussed. Chapter one further presented the problem statement, purpose of the study, significance of the study, assumptions of the study and research questions. Additionally, delimitations, limitations, the definitions of key concepts as well as the summary are also presented in this chapter. The next chapter examines literature related to Critical Theory.

CHAPTER TWO

ARTICULATION OF CRITICAL THEORY AS A CONCEPTUAL FRAMEWORK

2.1 Introduction

This chapter outlines the historical perspective of Critical Theory, its assumptions and a justification of the choice thereof as the main theoretical framework underpinning the study. A summary of other supplementary theoretical perspectives that directly link with Critical Theory also forms part of this chapter. The chapter is hence divided into sections such as: The history of Critical Theory, assumptions underlying Critical Theory, Applications of Critical Theory to research, other paradigms related to Critical Theory such as: Critical Mathematics Education, Ethno mathematics, Realistic Mathematics Education (RME) and Hypothetical Learning Trajectory (HLT).

The chapter also gives a justification why the foregoing theories were deemed necessary within the scope of this study. From Critical Theory and other supporting theories the researcher proposes a conceptual model for the understanding of number sense among first year preservice teachers of mathematics. In this chapter, the researcher discusses the research methodologies that are utilised by critical theorists when conducting research. Finally, the chapter concludes with a conceptual framework developed from existing theories to support the development of number sense. Through this new theory emanating from Critical Theory and its supporting paradigms, by proposing a new theory that is proposed in this study the researcher envisages to contribute to the already existing body of knowledge by combining the ideas carried in several theories into one presumption.

2.2 The history of Critical Theory

Venter, Higgs, Jeevavanthan, Letseka & Mays (2007) trace the origin of Critical Theory back to the 1800s in Germany. However, it became prominent in the 1920s from the Frankfurt School. Geuss (1981) and Shirikov (1937) add that the Frankfurt School is a collective term signifying a group of German philosophers, sociologists and economists associated with the “Institut für Sozialforschung” (Institute for Social Research) that was set up in Frankfurt with the pioneering intellectuals referred to as the critical theorists. The most prominent members pioneering Critical Theory are Horkhemeir, Adorno, Marcuse and Benjamin who is considered by literature as an overarching figure in Critical Theory. Literature also considers Hambermas as the most imperative contemporary representative of the Frankfurt School.

The Frankfurt School was then established as an affiliate of the University of Frankfurt with its primary purpose of research. The prominence of Critical Theory commenced in the 1930s with the appointment of Horkhemeir as a director of the institute. Geuss (1981) and Shirikov (1937) indicate that with the Nazi party's assumption of power in 1933 the Frankfurt Scholars were forced into exile due to the crash of religion as most of them were Jewish. The Frankfurt scholars fled to Switzerland and later the United States of America where the institute was reformed at the University of Columbia. After almost a decade from 1950 to 1958 the institute relocated back to Germany with its director as Adorno.

Upon the relocation of the Institute back to Germany in the 1950s it adopted the Marxist ideology which originated from the work of Karl Max. This adoption of Marxism later on led to the defeat of the working class in Germany and developed an intuition that the proletariats were not necessarily the agents of revolutionary changes. Later on Critical Theory developed the idea that the Frankfurt School itself was also perpetuating the German tradition of philosophical idealism (Guba & Lincoln, 1994).

The younger generation however posed a challenge to the ideology of Critical Theory which led to the protests at the Institute in the late 1960s which later on resulted in the accusation that Critical Theory was a form of mandarin elitism (Kincheloe, 1994). The latter refers to the fact that the critical theorists of that time attempted to create a thinking group which was somehow characterised by superiority of the mind. As Critical Theory advanced it acquired some ideas from elsewhere. For instance, Freud's *Psychoanalytic theory* became an integral part of the Critical Theory (Guba & Lincoln, 1994).

Despite its origin that traces back to the 1800s, Critical Theory still impacts how some present scholars think in their quest for truth (Kincheloe, 1994, p.165).

> The historical moments to which Critical Theory responded are still unfolding. Critical Theory has shifted through a number of distinguishable phases since the first generation of Marxist social theorists gathered into a loosely associated research group under the auspices of the Frankfurt Institute for Social Research in the first part of the 19th Century. The principal members of what came to be known as the Frankfurt School were Max Horkheimer (1895-1973), Theodore Adorno (1903-1969), and Herbert Marcuse (1898-1979).

There are however some other theorists that are believed to have impacted Critical Theory. The remaining part of this section briefly discusses their contributions. Horkheimer (1940's) is one of the pioneering critical theorists. His prominence in Critical Theory is demonstrated in the book, *Dialectic of Enlightenment* (1940s), with Adorno. In this manuscript argues that enlightenment and modern scientific thought had become an irrational force that dominated nature and humanity. The rationalisation of human society had ultimately led to Fascism and other totalitarian regimes that represented a loss of human freedom. Adorno (1940's) on the other hand who worked with Horkheimer focused on rationalisation and its impact on human society. Adorno too like Horkheimer believed rationalization led to Fascism and other totalitarian regimes that represented a loss of human freedom (Raune, 1988).

Another prominent critical theorist was Fromm (1920's - 1950's) who came into the spheres of Critical Theory from the angle of the Psychoanalytic Theory (Raune, 1988). He was a social philosopher who explored the interaction between psychology and society. Fromm believed that an understanding of the psychological needs of human beings was necessary to build a healthy society. Moreover more of the contributions of Fromm's work to the Critical Theory include the book titled Escape from Freedom (1941), which examined the growth of human freedom and self-awareness from the Middle Ages to modern times. In this book Fromm argues that the power relations directly in some way impact human growth.

The other critical theorists that are believed to have played a role in the development of Critical Theory include Apple (1980's) who wrote the book, *Education and Power* (1982) in which he discusses how school systems are set up to keep power under the control of the dominant order which is structured to be the formal way of shaping a person’s future. In this manuscript Apple (1982) argues that people placed much emphasis on the school as the only means of surviving in the future however there are other means by which individuals could make a living.

In the foregoing argument it follows that since education is viewed as the only means of surviving it could be used as a tool of oppressing and forcing people to follow a normal school set up which could mean young people are enclosed by the school set up which compromises their freedom to carry out other activities as deemed necessary by them. More of Apple’s work is also about critical theory and critical pedagogy.

Recent critical theorists include Giroux whose work on Critical theory gained fame around the 1980's to 1990's. The work of Giroux was more noticeable in an attempt to push the philosophies of critical pedagogy away from a Marxism viewpoint which saw teachers and students as prey, and debated that educational communities are sovereign and are capable of resistance and optimism. Giroux has also authored numerous books and articles on critical pedagogy.

Another recent critical theorist is Habermas who worked on Critical Theory around 1980's. Due to his contributions to Critical Theory he was regarded as "Best-known" critical theorist more especially for developing an epistemology that shows how human interests relate to knowledge, medium and science. Habermas also developed the "theory of communicative action" where he indicated that the validity of claims about truth and authenticity are tested through argumentation. Only arguments that are approved by those involved can be accepted.

2.3 Tenets of Critical Theory

Critical Theory seeks to understand how human values are affected by the hegemonic oppressive powers. Critical Theory claims that the truth very often serves the status quo; i.e. the truth is made and unmade by human beings (Venter, Higgs, Jeevavanthan, Letseka & Mays, 2007). Ideally what was perceived as the truth sometime back is not necessarily what will be perceived as truth in the days to come.

Venter, et al. (2007), further contend that Critical Theory perceives a link between power and human thought. This implies that the critical theorists hold the argument that human societies are structured around some power relationships. This henceforth means that power dominates all forms of knowledge inclusive of moral and normative knowledge. The thrust of Critical Theory is protecting humankind from hegemonic powers of oppression. In this study it could also therefore be argued that the application of Critical Theory in this study is to free the teachers from the vicious cycle of innumeracy.

Another, but vital, characteristic of Critical Theory is that it emphasises democracy as a celebration of differences within and amongst individuals (Higgs and Smith, 2002). Therefore, the researcher takes into account the fact that the preservice teachers under investigation are from different social, economic and educational backgrounds that could even be traced back to the time before Namibia's independence. The investigator in this study thus took the ideas of preservice secondary mathematics teachers and applied them to the real world.

Critical Theory holds that it is deceptive to divorce "real-life testing" from scientific theory (Higgs and Smith, 2002). This implies that scientists do not exist in a vacuum, but are rather part and parcel of the society which is bound by social reality and norms as much as anyone else in the society is. Critical literature also argues that theorists identify the shortcoming of science in that it tends to exclude certain areas of science such as political science or philosophy which cannot be studied as pure sciences such as mathematics and physics (Adorno, 1998; Held, 1980; Callewaert, 1999). The fact that scientific objectivity excludes certain areas of science defeats the primary purpose of Critical Theory of protecting humankind against all forms of oppression as it implies that huge areas of human life are ignored.

This study further acknowledges that pre-service teachers being considered are from a previously disadvantaged section of Namibian society where education was a tool of domination. The apartheid system considered blacks to be inferior to whites in mathematical ability (Amkugo, 1993). Such a superiority complex accounted for the inferior resourcing and less preference for the Bantu education system prior to the Namibian independence from which the majority of pre-service teachers came. From the foregoing one could therefore argue that such a superiority complex was the cause of underdevelopment of numerical competences as well as other mathematical competencies among many Namibian learners. Therefore, by using Critical Theory, the researcher envisages freeing secondary preservice mathematics teachers from the previous hegemonic powers of the apartheid era that the education system was subjected to.

Higgs and Smith (2002) hold a common idea with Jessop (2012), Venter et al. (2007), Tyson (2006), Wiggershaus (1986), Freire (1970), Freire (1973), Freire (1976) and Freire (1993) that human thoughts are linked to power relations hence humans have no neutral thoughts. Higgs and Smith (2002) affirm that Critical Theory anticipates seeing humans empowered and free of oppression and domination. The idea of a vicious cycle of innumeracy that the secondary preservice mathematics teachers face as proposed by Naukushu (2012) could be another form of oppression as one is not considered functional when lacking mathematical competencies.

One of the beacons of Critical Theory is Paulo Freire (1970) who coined the term "culture of silence" which indicates that within a culture of silence, people are unable to distance themselves from their life activities, so it is impossible for them to become empowered or to control their own destiny. In his prominent work “The Pedagogy of the Oppressed” Freire viewed learning and literacy as political projects, and believed that poverty and illiteracy were related to the oppressive social structures and dominant powers in society. He instigated adult literacy campaigns that aimed at empowering individuals to engage in social analysis and political activism as opposed to formal schooling which he termed as a tool of oppression. The work of Freire will be discussed in subsequent sections of this chapter.

Paulo Freire's legendary work in critical theory contributed to the issue of education of the oppressed (e.g. Freire (1973); Freire (1976) and Freire (1993)) by perceiving education as a tool for oppression. According to Freire education is a tool of oppression due to the mere fact that it is a tool in the hands of the powerful to oppress the powerless.

Freire further argues, that a poor child's inevitable lack of success in school (partially as a result of malnourishment) could create a belief that he or she is inferior to the other counterparts who are well off and therefore must accept and assume the role of a poorly paid manual labourer. Thus young girls from a poor socio-economic background could be accustomed to accept that they will have to spend their era as domestic drudges and feed their families on the skimpy earnings of their husbands. Since the secondary preservice mathematics teachers under investigation in this study were taught by teachers exiting from a system that identified them as weak and incapable of competing in mathematics due to the fact that they were black could definitely be considered to be a tool of oppression in education.

The researcher therefore hopes that by using Critical Theory secondary preservice mathematics teachers' numeracy skills at first year could empower and emancipate them from Naukushu's vicious cycle of innumeracy. Hence Critical Theory has the potential to benefit the researcher in comprehending the socio-cultural circumstances and background of the secondary preservice mathematics teachers whose number sense is being explored.

In addition to the foregoing Forester (2010) indicates that critical theorists attempt to seek out contradictions and social inequalities in a variety of disciplines in order to empower those who are oppressed. Forester (2010) viewed empowerment as a crucial element of Critical Theory. Ojelanki (2010)'s principal argument is that certain people in society are oppressed and need to be empowered by others. All fundamental categories of all disciplines should be questioned to achieve emancipation of the oppressed. The human capacities of individuals must be developed and linked to democracy to improve society. Therefore, the researcher envisages using the method of student centred approach when conducting the number sense training of secondary preservice mathematics teachers.

According to the learner centred paradigm the student is to assume the role of an active participant and should under no circumstances be viewed as an empty vessel to be filled with information. Learner centred pedagogy requires learners to actively participate, critique and contribute collaboratively to their learning and training of number sense as suggested by Ojelanki (2010) and Forester (2010).The researcher thus anticipates that by actively involving the secondary preservice teachers in their own number sense training, has a duty to mean they could be empowered. Once these get empowered, it is anticipated that they can become independent thinkers that might also be able to empower their students once they join the job market.

Additionally, Critical Theory assumes that knowledge is not static but dynamic. Horkheimer (1937, p.134) in an attempt to refer to the dynamic nature explains the concept of mind as:

> The mind is liberal. It tolerates no external coercion, no revamping of its results to suit the will of one or other power. But on the other hand it is not cut loose from the life of the society; it does not hang or suspend over it. Truth is therefore liable to change but under no circumstances is it an illusion. The abstract reservation that one day a justified critique of one's own epistemic situation will be put into play, that it is open to correction expresses itself among materialists not in a tolerance for contradictory opinions or even in a sceptical indecision, but in watchfulness against one's own error and in the mobility of thought a later correction does not imply that an earlier truth was an earlier untruth.

In addition to the debate on the dynamic nature of truth, Higgs and Smith (2002) argue that Critical Theory rejects classical philosophy in that it uses some form of independent, objective, critical thinking to reflect on society:

> According to critical theory, reason is immanent, not transcendent. For, according to critical theorists, certain rational principles (e.g. the law of deduction, formal logic and mathematical reasoning) are not timelessly true. They are only "true" because we human beings at this point in human history believe that this principle achieves certain things. The idea that 1+ 1 = 2 is simply an arrangement of symbols that are pronounced to be a truth at this era. In deed the whole mathematics system which is seemingly unassailable has a history. The fact that the year 2000 is more important to the west means nothing to the Chinese or the Indians, neither of whom rely on the Christian dating system (p.10).

The idea that knowledge is not static aids the researcher when conducting the number sense training; secondary preservice mathematics teachers are in possession of some prior knowledge of number sense which could then be used as a foundation to extend what they already know. Therefore, the investigator envisages using instructional paradigms that value the existence of prior leaning and experience such as the learner centred method which is the core of the number sense training as it allows individualised flexibility, democracy and freedom as required by the assumptions of Critical Theory.

2.4 Applications of Critical Theory

This section provides an outline of the possible practical applications of the Critical Theory for number sense to emerge in particular. The quest to find practical applications of Critical Theory is not a new phenomenon. This is evidenced by Raune (1988) who reasons that the ultimate goal of Critical Theory has always been to find its practical applications to the real world situations since the foundation of the Frankfurt School. On the contrary, Dallmayr (1990) argues that despite their pursuit to find the practical application of Critical Theory, the early critical theorists such as Horkheimer, Adorno and Max stressed the practical relevance of their project but their most influential work was highly theoretical thereby making it difficult to find the practical applications in the theory.

Nonetheless Dallmayr, (1990) further contends that Habermas the prominent critical theorist developed Critical Theory into a systematic social theory of impressively wide scope and has related it more closely to the empirical social sciences which led to a growing influence of the theory to empirically oriented social sciences. The argument in the literature that Critical Theory seeks to orient itself to empirical evidence forms the basis on which the researcher deemed it necessary to choose Critical Theory as the main theory underpinning this study. That is to find empirical evidence on how number sense instruction can be structured.

In advocating Critical Pedagogy Freire's (1970) pedagogy of literacy education involves not only reading the word, but also reading the world. This involves the development of critical consciousness (a process known in Portuguese as "conscientização") (Freire, 1970). In advocating the formation of critical consciousness Freire (1970) allows people to question the nature of their historical and social situation to read their world with the goal of acting as subjects in the creation of a democratic society. This is why the researcher in this study tries to create that same democratic society among the preservice secondary teachers of mathematics. For education, Freire implies a dialogic exchange between teachers and students, where both learn, both question, both reflect and both participate in meaning-making.

The fact that Critical Theory begins by inquiring into what prevents the realization of this enlightenment makes it ideal for both the quantitative and qualitative research paradigms which inform the basis of data collection for this study. Additionally, Critical Theory questions and challenges the seemingly obviousness, naturalness, immediacy, and simplicity of the world around us, and, in particular, of what we are able to perceive through our senses and understand through the application of our powers of reason (Kincheloe, 1994). It is therefore envisaged that the researcher will apply these ideas about Critical Theory to ensure that the concept of number sense is practically comprehended by the secondary preservice teachers of Mathematics.

Another, but vital, application of Critical Theory by Raune (1988) is that it involves questioning and challenging the passive acceptance of "the way things are" or "the way things seem" simply "is" the "natural" way they necessarily "should" or "must" be. In other words, based on this, the researcher interprets Critical Theory as a theory that questions and challenges the conviction that what is, or what is in the process of becoming, or what appears to be, or what is most commonly understood to be, or what is dominantly conveyed to be, is also at the same time right and true, good and just, and necessary and inevitable.

> Critical Theory has additionally always particularly been concerned with inquiring into the problems and limitations, the blindness and mistakes, the contradictions and incoherence, the injustices and inequities in how we as human beings, operating within particular kinds of structures and hierarchies of relations with each other, facilitated and regulated by particular kinds of institutions, engaged in particular kinds of processes and practices, have

> formed, reformed, and transformed ourselves, each other, and the communities, cultures, societies, and worlds in which we live (Raune, 1988. p.125).

The quotation above can be interpreted to mean that Critical Theory is mainly concerned with the way humans operate on some kind of structures, hierarchies that exist within the structures they operate. The researcher henceforth, from the fore going interpretation, views Critical Theory and its applications to be ideal to inform this study for the researcher to understand the nature of the problem being studied and participants thereof.

Moreover, the uniqueness of Critical Theory appears to be that it has always occupied tenuous positions within traditional (academic) disciplines, and has always moved restlessly across disciplinary borders (Guba & Lincoln, 1994). After all, when we think of what Critical Theory has influenced, we must include such diverse disciplines such as sociology, political science, philosophy, economics, history, anthropology, psychology, and even biology and physics, as well as studies in English and other national, regional, and ethnic languages and literatures (Guba & Lincoln, 1994).

Guba &Lincoln (1994) further argue that Critical Theory, in sum, is by no means merely a province of aspects of a human being, and neither need it be, should it be, nor can it be confined to a single aspect of human being alone. From this it can be understood that Critical Theory involves a holistic conceptual framework it can therefore be deduced that the assumptions held by the Critical Theory are not focusing on only one aspect of human beings but rather on development and empowerment of humans that follow a variety of human aspect. The researcher therefore envisages understanding the nature of the participants from a broader perspective and not merely from one aspect.

The researcher therefore draws understanding that a study informed by Critical Theory as a means granting learners an opportunity not only to theoretical and critical approaches of what might often at least initially seem like an elite caste of distant and specialized others specific, and frequently famous, named "theorists" and "critics". It also, and more importantly, reflects upon how and why a specific study draws thoughts from the beliefs of Critical Theory with the kinds of theoretical and critical approaches to research.

In addition to the above, it can also be interpreted that Critical Theory allows for the reflection on the literature review, data collection and analysis involved, where these critical approaches come from and what gives rise to them; where they lead and what follows from them. Therefore such approaches to research predominate in what areas of everyday life today, in what places within what societies and cultures, with what uses and effects, toward the advancement of what ends and toward the service of what interests. These approaches also allow for the questions on what alternative approaches are possible, what alternatives are desirable as well as what alternatives are necessary, to conduct research.

In sum, the researcher took account of the probable practical applications of the Critical Theory to research as a theoretical perspective, the uniqueness it possesses and its idyllic nature in dealing with the emancipation and empowerment of individuals (pre-service secondary mathematics teachers in this case) practically with skills to empower others upon joining the teaching job market. Besides, for education (mathematics education in particular) critical theory facilitates the development and improvement of teachers and learners' ability to participate as critical citizens.

Critical Theory could also be interpreted to imply, that empowered agents are able to effectively question, challenge, and contribute toward the progressive transformation of the prevailing status quo within the communities, societies, and cultures that they function in. Within the context of this study preservice secondary mathematics teachers could be empowered and therefore they will possess skills to challenge, and contribute towards the learning and development of number sense.

Such chances of challenging, and contribution to the transformation of communities learners were initially deprived of prior to independence. Therefore, it is against that background that the tenets of Critical Theory are envisaged by the researcher to have a potential to underpin this study in developing the participants and allow them to challenge, and contribute to the harmonisation of their learning environment.

2.5 Mathematics Education theories related to the Critical Theory

This section discusses the paradigms along the lines of Critical Theory that support mathematics education. Critical Theory favours practicing new approaches such as Critical Mathematics Education (CME) as outlined by Skovsmose (2002), which can lead to the development of critical thinking and consequently empowering learners which is the greatest attribute of Critical Theory. The Realistic Mathematics Education (RME) as outlined by Freudenthal's (1991) calls for the teaching and learning of mathematics in situations that are familiar to the learners; hence mathematics should be realistic in nature and connected to their daily life experiences.

Ethno mathematics on the other hand advocates that the teaching and learning of mathematics should be related the learners' cultural experiences (D'Ambrosio, 2006)

2.5.1 Critical Mathematics Education (CME)

Critical mathematics education traces its roots from the necessity to foreground the role of mathematics education in educating for citizenship and empowerment fostered by the desire to produce critical citizens (Skovsmose, 2002). Skovsmose (2005, p. 3) puts forward the view that the acknowledgement of the critical nature of mathematics education, including the uncertainties associated with mathematics, is a feature of critical mathematics education. The effective learning of mathematics may result in empowerment, citizenship and democratic participation or it may result in disempowerment, marginalisation and exclusion. Critical mathematics education aims at implementing a more equitable mathematics education and mathematics education for democracy and citizenship.

In concurrence with the above claims about CME, another vital attribute of Critical Theory is empowerment of an individual. In this study the researcher therefore envisages to utilise the underlying assumptions carried by the Critical Theory, CME as well as other mathematics education paradigms related to the Critical Theory such as

CME to aid the development of number sense. Along these lines preservice secondary mathematics teachers may perhaps be equipped with skills that might impart on their learners once they join the profession. Moreover, the researcher reckons that it is necessary to combine contemporary and innovative paradigms of mathematics education for emancipating the learners as well as the whole education system from the aforementioned vicious cycle of innumeracy illustrated in section 1.3.

CME in its setting, challenges the role of mathematical consciousness, the rationale of mathematics education and, sequentially, CME questions mathematics curricula along with classroom practice. Scholarly work by Ernest (2001), Frankenstein (2000), Knijnik (2002), Skovsmose (2005) and Vithal (2003) in discusses education policies, curricula and teaching practices that position mathematics as an absolute, neutral, accepted body of knowledge that possesses pint-sized relationship with the socio-political milieus we live in. Such curricular do less to develop students as participating, thinking citizens. This study therefore, holds a contrasting view to such curricula, which is the belief that mathematics curriculum making is not a static dynamic exercise which is divorced from the environment of the learners.

Ernest (2001, p. 278) argues that CME shifts the focus on approaching mathematics and its teaching from a critical viewpoint. Per se, the fundamental purpose for critical mathematics education is to stimulate critical thinking allied to mathematics amongst students so that they could use mathematics in their lives to empower themselves both personally and as functional citizens in the society. CME also requires learners to appreciate the role of mathematics in history, culture and in the contemporary world, which reverts back to the idea of ethnomathematics that mathematics should not be divorced from culture (Ernest, 2001, p. 285). Therefore, in the context of this study it could be suggested that the development of number sense should also not be isolated from the environment of the students, their culture and contemporary world. The researcher in this study therefore hopes that by applying Critical Theory and the components to the development of number sense it could offer a chance for the students to develop the number sense that is real within the context of the pre-service secondary mathematics from whom the data is drawn.

Skovsmose (1994, p. 16) similarly states that it is critical to draw attention to circumstances or a progression in a crisis, to ascertain, to try to grasp, to recognise and to react to it. Skovsmose's approach to critical mathematics education centres on the role and formatting power of mathematics in highly technological societies and on how these are frequently veiled and left unchallenged.

Mathematics has primed affirmative contributions to humanity but then again has shaped what Skovsmose (1994) and Skovsmose and Nielson (1996) label as crunches with respect to prejudiced dissemination and access to means, autonomous partaking and acting justly. This therefore means that if the pre-service secondary mathematics teachers are equipped with number sense it will not only help their professional practice but also improve their access to means, i.e. they will also understand the practical applications of number sense in other life situations. Skovsmose (2005, pp. 4-5) cites problematical outcomes of mathematics education such as the concern about globalisation, ghettoising and the occurrence of what is contemporarily a Fourth World. The development of number sense for that reason means the pre-service teachers become aware of the fact that they are being exposed to the mathematical skills that will make them functional citizens in the society.

Likewise, the ideologies and ethos that reinforce critical mathematics education as presented in the preceding literature also appear in some authors work (e.g. Skovsmose, 1994; 2005; Nielson, 1996; Ernest, 2001; Frankenstein, 2000; Knijnik, 2002) support the idea that through mathematics and science, an opportunity exists to engage students with issues from the standpoints of social justice, equity, sustainability and ethical action. This study envisages offering interactive and collaborative learning opportunities to the pre-service teachers of mathematics during their number sense training.

The literature on critical mathematics education offers a complex, progressing concept with a variety of assessments around what constitutes critical mathematics education. Johnson (1994) formalised the idea of critical numeracy as a way to make meaning with mathematics that extends beyond the functional or utilitarian application of mathematics to include socio-historical perspectives and the potential to explore the power relations when applying mathematics. The issue of critical numeracy appears to possess ample relation with the number sense.

Frankenstein (2000) established the notion of critical mathematics literacy based on questioning the use of number and statistics in particular to interpret and challenge inequalities in society. This study acknowledges therefore that there is an acute need for this and other similar studies to bring about the number sense that allows learners to use number sense beyond understanding mathematics and apply number sense to real life situations.

In concurrence with critical mathematics education, his study therefore assumes that by training the pre-service secondary mathematics teachers in number sense concepts taking into account their cultural experiences, their quality could be compromised as mathematics teachers but may also use their number sense for the solution of real life problems. For instance, a number sense literate preservice mathematics teacher or student will figure out what to put in a shopping trolley when in possession of only N$ 20-00.

Through working on CME the concept of Mathemacy was made known by Skovsmose (1994) as a vital competency in the interior of critical mathematics education that echoes Freire’s proficiency of literacy for social equality. The term Mathemacy as coined by Skovsmose (1994; 1998, p. 200) is proficiency with mathematics to interpret social life, to act in a world structured by mathematics. It draws together three ways of knowing; mathematical, technical and reflective knowing with reflective knowing being the potential catalyst for critical awareness.

Skovsmose (2005) defines what a critical mathematics education should inspire through alertness or comprehension that in numerous ways replicate the goals of politics of mathematical knowledge and mathematics of political knowledge Therefore, the investigator in this study acknowledges the equal but collaborative role that number sense can play along with these foregoing contemporary concepts in CME to comprehend and convey the thoughts of mathemacy and mathematics in action with regards to science and technology (Frankestein, 1998; Skovsmose, 1994; 1998; 2005).

Posing a challenge to each of the above constructs for critical mathematics education Skovsmose (2005) has underscored features common to CME. The investigator in this study has henceforth preferred few elements as the basis for answering the question of interrelatedness of number sense to CME. The elements favoured are: authentic, interdisciplinary learning experiences; the significance of mathematics in empowerment of an individual, reflection on the reasonableness of one's mathematical work and answer obtained thereof (self-critique), general critique and action nature of mathematics. Therefore using the fore going elements the researcher envisages defining the intersection of number sense and CME.

Similar to the above, contemporary CME advocates such as (Pais, Fernandes, Matos & Alves 2007; Skovsmose 2010; Jett 2012; Simson & Bullock 2012) hold the view that mathematics education should put emphasis on the critique concept. The latter concept calls for students to critically reflect on the reasonableness of their work and answers thereof, which is among the greatest attributes of possessing number sense.

In view of the foregoing characteristics of CME this study seeks to develop preservice teachers' number sense as early as first year to increase opportunities to develop into critical mathematics educators which may well help them perform better in mathematics and produce learners that are able to cope with the demands posed by the high school mathematics curricula that they will be teaching.

2.5.2 Realistic Mathematics Education (RME)

In addition to CME, critical theory proponents perceive a connection between Critical Theory and the neo-constructivist theory of Realistic Mathematics Education (RME) as argued by van den Heuvel-Panhuizen (2001; 2003; 2010) and Gravemeijer (1994). This subsection therefore explores ideas of RME and its origin as well as its relation to Critical Theory. Ultimately, the researcher attempts to justify how RME is envisaged to play a role in the design and implementation of a number sense intervention programme which forms the basis for the data that is used in this study.

Of Dutch origin, RME was developed some decades ago (Freudenthal, 1977). The foundations of RME were laid in the Netherlands by Freudenthal and his colleagues at the former Institute for the Development of Mathematical Education (IOWO), a preceding institute to the current Freudenthal Institute. The actual process of the restructuring started with the launch of the Wiskobas project in 1968, initiated by Wijdeveld and Goffree. The present form of RME is mostly determined by Freudenthal's (1977) view about mathematics, who advocated a connection between mathematics and reality. Freudenthal (1977) argued that mathematics must stay close to learners and should be relevant to society within which it is learned. The connection between mathematics and reality as reasoned by Freudenthal (1977) aims at making mathematics to be of human value.

Despite seeing mathematics as subject matter that has to be transmitted and as a human activity, Freudenthal reinforced the idea that mathematics is a human activity. In RME Freudenthal deduced that the teachers have a duty to play a facilitative role to give students the "guided" opportunity to "re-invent" mathematics through practicing it (Freudenthal, 1991). Taking into account that the recent critical theorists also call for the new and innovative methods of teaching and learning mathematics, where the teacher plays the role of facilitator and the learners are active participants guided towards unveiling mathematical ideas, the researcher therefore deemed it necessary to infer a connection between critical theory and RME. Consequently, RME is envisaged to play a complementary role in this study alongside other paradigms of teaching mathematics that are of fame at the current period.

In mathematics education, the thrust is on the activity as opposed to being in a closed system and ultimately leading to the process of mathematisation (Freudenthal, 1968). Treffers (1987) proposed the idea of two types of mathematisation explicitly in an educational context and distinguished between "horizontal" and "vertical" mathematisation. In horizontal mathematisation, Treffers (1987) contents that the students ought to come up with mathematical implements which can help to organise and solve problems located in a real-life situation contemporary to the environment where they find themselves. Vertical mathematisation as perceived by Treffers (1987) is the process of reorganisation within the mathematical structure itself, i.e. discovery of shortcuts and noticing associations amongst concepts and approaches and then relating these discoveries. In an attempt to articulate the number sense idea, RME and its two kinds of mathematisation were therefore taken into consideration by the researcher when designing the training intervention programme that the participants went through.

The design of an intervention programme based on RME has been researched by Ndlovu (2014) on similar population as the targeted on in this study.

> The result of teacher professional learning based on RME was acknowledged to contribute to improvement in teaching and learning to address the concern about unsatisfactory learner achievement in mathematics. The response was that teachers experienced the sessions positively in relation to all but one of all of the six RME principles (Ndlovu, 2014, p.10).

From this it can be noted that RME seems to support positive learning hence the researcher envisages utilising RME as one of the conceptual framework of conducting number sense training among preservice secondary mathematics teachers in this study.

Freudenthal (1991) holds the view that "horizontal mathematisation comprises of going from the world of life into the world of symbols, whereas vertical mathematisation could be perceived as moving within the world of symbols." Even though this foregoing discrepancy between the two varieties of mathematisation seems to be free from ambiguity, it could under no circumstances imply that Freudenthal assumed a clear cut difference between these two. Freudenthal (1991) further stressed that there exists equivalence between the values of these two forms of mathematisation. Likewise mathematics educators must therefore bear in mind that mathematisation can come about on different altitudes of understanding. At this stage the researcher notes that horizontal mathematisation is similar to mathematical modelling which in turn is similar to context-based problem solving, hence a link with Critical Theory.

Literature (e.g. Freudenthal, 1991; Streefland, 1991; Treffers, 1991; Gravemeijer,1994) suggests that RME further calls for mathematics education to be structured in a manner of "*guided reinvention"*; where students are exposed to a similar process associated with the process by which mathematics was invented; i.e., the meaning of invention refers to the steps in the learning processes while the meaning of guided is the instructional environment of the learning process offered by the teachers. Further literature argues that for that reason, the history of mathematics can be used as a source of inspiration for course design (Heuvel-Panhuizen, 2003).

The researcher therefore acknowledges that Critical Theory and its tenets that it supports in its totality being a theoretical perspective that holds a history of several decades could also make it uniquely applicable theoretical perspective to underpin this study. Moreover, van den Heuvel-Panhuizen (1996; 2001; 2003) argues that the reinvention principle can also be stimulated by informal solution procedures. Informal strategies of students can often be interpreted as anticipating more formal procedures, which the researcher in this study accepts as being ideal to the development of number sense among the preservice teachers of mathematics.

RME cannot be isolated from the worldwide reform movement in mathematics education that occurred in the last few decades (Treffers, 1978; Freudenthal, 1991; van den Heuvel-Panhuizen, 1996; 2001; 2003; 2010; van den Heuvel-Panhuizen, 2001 Gravemeijer & Doorman 1999 and Drijvers et al. 2010, 2013).

It can logically be concluded that RME has much in common with reform oriented approaches to mathematics education including Critical Theory and critical pedagogy. Therefore, this study further acknowledges that the teaching and learning of mathematics is not static but dynamic in terms of theoretical perspectives as well as paradigms associated with the teaching and learning of mathematics number sense being part of mathematics is not excluded either.

Nevertheless, RME involves a number of core principles for teaching mathematics which are inalienably connected to Critical Theory. Most of these core teaching principles were articulated originally by Treffers (1978) but were reformulated over the years. In total six principles can be distinguished (e.g. Streefland, 1991; Treffers, 1991; Van den Heuvel-Panhuizen, 1996; van den Heuvel-Panhuizen, 2001).

The first principle holds the idea of activity implying that in RME students are treated as active participants in the learning process, which in the light of this study comes back to the whole notion of empowerment as one of the entreaties for the Critical Theory. It also emphasises that RME is a product of its time and cannot be isolated from the worldwide reform movement in mathematics education that occurred in the last decades Freudenthal (1991). The activity principle further states that in RME mathematics is best learned by doing mathematics, which is strongly reflected in Freudenthal's interpretation of mathematics as a human activity, as well as in Freudenthal's and Treffers' idea of mathematisation (Heuvel-Panhuizen, 2003). This study also holds a strong feeling that number sense can be best developed if the preservice secondary mathematics teachers are afforded an opportunity to do number sense as opposed to the traditional approach of teaching mathematics.

The second principle is the reality principle which is understood in two ways in RME (Treffers 1978; Freudenthal 1991; Gravemeijer 1994; Streefland 1991; Treffers 1991). Firstly, it articulates the importance that is attributed to the goal of mathematics education including students' ability to apply mathematics in solving "real-life" problems. Secondly, of the principle proposes that activity in mathematics education should start from problem situations that are meaningful to students, which offers them opportunities to attach meaning to the mathematical constructs they develop while solving problems. Rather than beginning with teaching abstractions or definitions to be applied later, in RME, teaching starts with problems in rich contexts that require mathematical organisation or, in other words, can be mathematized and put students on the track of informal context-related solution strategies as a first step in the learning process (Treffers, 1987 and Freudenthal,1991). Thus taking account that RME refers to starting with problems with socially rich contexts it comes back to the whole point of empowering individuals; which is also one of the tenets of CME and that these socially rich problems aim at empowering the students, it could be argued that RME, CME and Critical Theory hold the same ground in this regard.

The third principle is the level principle which refers to the idea that learning mathematics requires students to transit through various levels of comprehension: from informal context-related solutions, through creating various levels of shortcuts and schematisations, to acquiring awareness into how ideas and approaches are interrelated as assured by Treffers (1978) and Freudenthal (1991). Moreover, simulations are important for bridging the gap between the informal, context-related mathematics and the more formal mathematics. This means that when students are taught under simulated conditions it brings the contents to their world and therefore they are able to grasp that.

The fourth principle is the intertwinement principle which means mathematical content domains such as number, geometry, measurement, and data handling are not considered as isolated curriculum chapters but as heavily integrated. Students are offered rich problems in which they can use various mathematical tools and knowledge. This principle also applies within domains (Streefland, 1996; Gravemeijer, 1994 and Van den Heuvel-Panhuizen, 2003). For example, within the domain of number sense, mental arithmetic, estimation, and algorithms are taught in close connection to each other.

Bearing in mind the foregoing principle of intertwinement, this study therefore defers to the fact that the components of number sense are interrelated this informed the researcher for the duration of the design and implementation of number sense intervention programme that was used in this study. Therefore, the researcher considered various concepts in mathematics, similar to number sense and consequently used such notions as ingredients in the number sense training of preservice secondary mathematics teachers from which the data of this study was drawn.

The fifth principle is the interactivity which principle which signifies that learning mathematics is not only an individual activity but also a social activity (Streefland, 1996; Gravemeijer, 1994 and Van den Heuvel-Panhuizen, 2003).Therefore, RME favours whole class discussions and group work which offer students opportunities to share their strategies and inventions with others. In this way students can get ideas for improving their strategies.

Moreover, interaction evokes reflection of their work, which enables students to reach a higher level of understanding. The researcher therefore realised that the best methods to use on the number training of preservice secondary mathematics teachers are those that call for the students to interact with each other and the teacher so as to create a discourse during the teaching and learning of number sense.

The sixth and last principle is the guidance principle which refers to Freudenthal's idea of "guided re-invention" of mathematics. Hence the guidance principle implies that in RME teachers should have a proactive role in students' learning and that educational programs should contain scenarios which have the potential to work as a lever to reach shifts in students' understanding. To realize this, the teaching and the programs should be based on coherent long-term teaching-learning trajectories. It is therefore for this reason that the researcher deemed it necessary to use a hypothetical learning trajectory. This was also done to keep record and track of the progressive student progress as they move from one level of number sense to another.

To sum up, this study carries the assumption that despite the significant role in mathematics education reform, RME is not so unique that it could be used on its own to develop the number sense among the preservice secondary teachers of mathematics. The study therefore assumes that there are yet other conceptual frameworks aligned to Critical Theory that could inform the number sense training of preservice secondary mathematics teachers. Hence the next section discusses Ethnomathematics more fully.

2.5.3 Ethnomathematics

Pioneered by D'Ambrosio, the origin of the term ethnomathematics within the sphere of mathematics education can be traced back to the 1980s. This was necessitated by the desire by to illustrate the mathematical practices of identifiable cultural groups and perhaps regarded as the study of mathematical ideas found in any culture

(D'Ambrosio, 1985). Moreover, in an attempt to define ethnomathematics step by step D'Ambrosio (1990) brings forward the following ideas:

> The prefix ethno of ethnomathematics is in the present day acknowledged as a very broad term that refers to the social-cultural perspective and therefore, includes language, jargon, and codes of behaviour, myths, and symbols. The origin of mathema is complicated, but tends to mean to give explanation, to be on familiar terms with, to comprehend, and to perform activities such as ciphering, measuring, classifying, inferring, and modelling. The suffix tics is derived from techné, and has the same root as technique (D'Ambrosio, 1990 p. 81).

This therefore means that by practising mathematics the elements of social and cultural, language and symbols should be included. Therefore in this manner the teaching of mathematics will make sense to the learners. The ultimate aim of ethnomathematics is therefore to bring the contents to the context of the students so that they benefit from the teaching of mathematics. Therefore, by including the ethno component on the development of number sense the researcher envisages that the number sense content will be unpacked and be brought within the context of the students, thus this will potentially accelerate the development of number sense.

In addition to the way D'Ambrosio (1990), Rosa & Orey (2003; 2007; 2009) hold a view that in defining ethnomathematics:

> The ethno part of ethnomathematics would imply the members of a cluster within a cultural milieu, whose brand is their cultural customs, codes, symbols, mythology, taboos and unambiguous ways used in such cultural milieu to reason and deduce. Mathema on the other hand could imply to put in plain words and identify with the human race in order to surpass, deal with and muddle through with certainty so that the members of cultural groups can carry on and thrive, and tics refers to techniques such as counting, ordering, sorting, measuring, weighing, ciphering, classifying, inferring, and modelling.

Rosa and Orey (2003) further point out that the mathema notion develops the tics within the circumference of ethnos for the reason that it consists of daily problems that people come across, larger problems of humanity, and comings and goings of humans to create an eloquent world. In other words, the concept of ethno mathematics could be perceived as the mathematics that revolves in our daily life as humans, our society and the environment we live in.

Regarding the ideas put forward by D'Ambrosio (1990), the quest for solutions designed for solving specific problems with the purpose of assisting the development of mathematics are at all times embedded in a cultural perspective. In contribution to the debate Rosa and Orey (2011) argue that in order to understand how mathematics (tics) is created, it is necessary to understand the problems within which it was created from. In a sense, it is necessary to develop an intellectual capacity on those problems (mathema) by considering the cultural context (ethnos) that drives them.

Besides, D'Ambrosio (1993) asserts that the mission of the ethnomathematics programme may possibly be to understand that there is a variety of ways of exploiting mathematics by taking into consideration the appropriation of the scholarly mathematical knowledge nurtured by a variety of the sectors of society as well as by considering different approaches in which diverse cultures discuss their mathematical practices. Therefore, when designing ethnomathematics lessons one ought to consider the environment that it is going to be presented. Also, it could be interpreted therefore from this that the learning activities may thus differ as one moves from one environment to another.

Barton (1996) contributes to our understanding of concept the of ethnomathematics by asserting it to be a program that looks into the habits in which a variety of cultural groups comprehend, articulate, and apply concepts and practices that can be distinguished as mathematical practices. Likewise, ethnomathematics could possibly be expressed as a manner in which individuals from a particular way of life use mathematical ideas and concepts for handling quantitative, relational, and spatial aspects of their lives (Borba, 1997). This manner of perceiving mathematics authenticates and maintains all individuals' understanding of mathematics for the reason that it shows that mathematical thought is inherent in their lives.

Further substantiation of this contention of ethnomathematics is given by Orey (2000) who points out that various cultures use or work surrounded by distinctive interactions between their language, culture and environment. Within these circumstances, D'Ambrosio (2006) argues that in an ethno mathematical standpoint, mathematical philosophy is extended in a variety of cultures in acceptance of regular problems that are encountered within a cultural context. Therefore it can be argued that ethnomathematics is not a static but a dynamic process. That is, the ideas of ethnomathematics should not only be confined to one cultural setting but should rather vary as one move from one cultural setting to the other.

Another crucial assumption of ethnomathematics is that it authenticates all practices of mathematical clarifying and understanding conveyed and accrued by different cultural groups (Rosa & Orey, 2011). Other literature also asserts that this same accumulated cultural knowledge is viewed as a portion of an evolutionary practice of change that is part of the same cultural dynamism by way of each traditional set reaching into communication with each other (D'Ambrosio, 1993). The idea of evolutionary practice of change therefore implies that the concept of ethnomathematics is not of a static but is of a dynamic nature, which also carries the perception of mathematics curriculum from the perspective of Critical Theory, RME as well as CME, that curriculum is dynamic but not static.

Hence the assumption of the researcher in the development of number sense is that the number sense curriculum should embrace the dynamic nature of mathematics in terms of potential for multipurpose utilisation – critical democratic citizenship, sensitivity to cultural diversity, and local experiential contexts, etc. To this end the researcher in this study assumes that the development of number sense should be considerate to the cultural diversities that exist among the preservice secondary mathematics teachers. Therefore, the researcher appreciates the role that number sense could play to ensure socialisation as well as harmonisation of cultural interactions amongst the preservice secondary mathematics teachers.

Rosa and Orey (2011) hold the view that there is a necessity to conduct a study of various ways in which groups of peoples resolve problems and the practical algorithms upon which they base these mathematical perspectives becomes relevant for any real comprehension of the concepts and the mathematical they have. For instance Kasanda and Kapenda (2011) conducted a study on different algorithms used by vendors at the open market and found that there exists a variety of mathematical concepts, such as computation, measurement, estimation, etc. Ethnomathematics discusses to forms of mathematics that differ as a consequence of being surrounded by cultural events whose initial motive differs from doing mathematics. In this connection, Orey (2000) asserts that ethnomathematics could be regarded as a device to act in the domain and by itself, sheds some light into the social role of academic mathematics.

Moreover, among the arguments supported by this study is that ethnomathematics rejects the suppositions that, the learning of mathematics has always been linked with the schooling process, i.e., it was assumed that mathematical conceptions and skills are only acquired in school. By contrast, the exploration of students' mathematical facility has led educators and researchers to conclude that mathematical understanding could also be acquired outside the spheres of the structured systems of mathematics learning such as schools (Bandeira & Lucena, 2004; Duarte, 2004; Knijnik, 1993; Rosa & Orey, 2010).

It is therefore against this background that, mathematical ideas applied in exceptional sociocultural contexts refer to the use of mathematical concepts and procedures acquired outside of schools as well as the attainment of mathematical abilities other than from schools. Bandeira and Lucena (2004) and Lean (1994) paid a special attention to school mathematics and the impact of cultural factors on teaching and learning academic mathematics.

Dossey (1992) & Orey (2000) hold the argument that mathematical knowledge results from social interactions in which relevant mathematical ideas, facts, concepts, principles, and skills are acquired as a result of cultural context. Therefore, it is vital when developing mathematical ideas of preservice teachers in general and number sense in particular to consider their cultural experiences form which their mathematical knowledge might have originated.

Stigler & Baraness (1988) argue that contrary to the idea that mathematics is a universal formal domain of knowledge it is in fact a collection of culturally constructed symbolic illustrations and processes that aid the manipulation of these representations. Rosa and Orey (2008) similarly hold that students advance illustrations and techniques into their rational systems in the context of socially constructed activities.

Alternatively, and within the context of this study, the foregoing idea can be interpreted to mean that number sense skills secondary preservice teachers of mathematics learn in the intervention programme were not logically constructed based on abstract cognitive structures but rather forged out of a combination of previously acquired knowledge and skills and new cultural inputs. Therefore, this study reaffirms the ideas of D'Ambrosio (1990) that mathematics arose out of the needs of organised society, which cannot be divorced from the activities and practices developed by people in a globalised society. Thus this holds a component of empowerment which is among the tenets of Critical Theory.

2.5.4 Hypothetical Learning Trajectories

Literature suggests the genesis of Hypothetical Learning Trajectories (HLT's) to have originated from Simson (1995), who proposed the term hypothetical learning trajectory to identify and describe relevant aspects associated with a mathematics lesson plan. Simson (1995) argues that a hypothetical learning trajectory should comprise of: An explanation of the students' mathematical objectives (what is intended for students to learn); the mathematical responsibilities or problems that students will work on to accomplish the goals; and a hypothetical path that describes the students' learning processes.

Simon and Tzur (2004) also acknowledge the prominence of choosing and examining the tasks that stimulate the students' expansion of new mathematical concepts in order to construct a hypothetical learning trajectory to frame their mathematical learning. The construction of hypothetical learning trajectories can be seen as the tools to guide and foster students' learning and thinking. This is evidenced by Clements and Sarama (2004) who define the hypothetical learning trajectories as:

> A description of children's thinking and learning in a specific mathematical domain, and a related conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain (Clements & Sarama, 2004, p 83).

The primary use of the construct of learning trajectory is to structure, organise, and systematise mathematical practices or knowledge as well as to keep track of the learning progression in mathematical training. Researchers, curriculum designers and planners, though, understand and utilise this construct in a variety of ways (Clements and Sarama 2004).

This study adopts the meaning of HLT's as understood by literature (Clements & Sarama, 2004; Confrey, Maloney & Nguyen, 2011; Simson, 1995; Steffe, 2003 & Sarama, Clements, Barrett, Van Dine, & McDonel, 2011). The ideas presented in the foregoing literature were studied and bears an influence in the development of the HLT that was used in the number sense training of preservice secondary mathematics teachers. The fore going literature also hold the common opinion that there should be a track record of the students' progression so that this can be studied at independent time from the data collection of the study.

The researcher, therefore, in response to this and the results of the pilot study, developed a training manual where the students worked independently each at their own pace and this manual was used to analyse the data and offered rich grounds of data essential for the purpose of this study.

In particular, research (e.g. Clements & Sarama, 2004; Confrey, Maloney & Nguyen, 2011) suggests that a distinction is essential between an intended or hypothetical learning trajectory and an actual learning trajectory. The intended HLT lays a basis for designing, branding and detecting possible instructional routes to approach mathematical instructional objectives as well as developing students' mathematical thinking during the instructional period. The actual learning trajectory aims at indicating the actual techniques endeavoured by the student consequent to working on activities or tasks that were perhaps established in the use of an HLT.

Firstly, Simon and Tzur (2004) suggest three assumptions that justify the use of the HLT construct; these include the idea that the generation of a hypothetical learning trajectory is based on understanding of the current knowledge of the students involved. Secondly, the HLT should be used as a tool to plan and learn particular mathematical concepts. Thirdly, mathematical tasks in the HLT should serve as tools for promoting learning of particular mathematical concepts.

Therefore, for the purpose of this study, such mathematical tasks are considered a key part of the instructional process and the hypothetical learning trajectory thereof. Because of the hypothetical and inherently uncertain nature of this process, the researcher in this study acknowledges that the HLT's involve a dynamic process; henceforth, the teacher is regularly involved in modifying every aspect of the HLT.

Research (e.g. Sarama, Clements, Barrett, Van Dine, & McDonel, 2011; Confrey, Maloney & Nguyen, 2011; Steffe, 2003; Clements and Sarama, 2004; Simson, 1995) shows that mathematical tasks used in the HLT should be sequenced in the order of the developmental progressions to complete the assumed learning trajectory. The main theoretical argument is that such tasks might constitute a particularly successful educational/scaffolding program.

However, the researcher in this study holds a view that the task sequence is not merely the best path for learning and teaching but also a route to ensure that the envisaged outcomes are achieved. Besides, Clements and Sarama (2004) recommend that the influence of the learning trajectory lies in connecting the students' psychological developmental progression and the instructional sequences to stimulate mathematical thinking in this particular case development of number sense among preservice mathematics teachers.

The concept of learning trajectories could be fitted within a greater theoretical framework referred to as “Hierarchic Interactionism” (HI) (Clements & Sarama, 2004). By the concept of HI Clements and Sarama (2004) argue that the synthesis of contemporary techniques to comprehending how human beings learn and develop mathematical concepts. Based on the work of Clements and Sarama (2004) it therefore holds that cognitive development, both general and domain specific, precisely proceeds through a hierarchical sequence of levels of concepts and others develop by continuously repeating the concepts. These levels grow within domains and in interaction with each other across domains and their growth also reflect the interaction between innate competencies, dispositions and internal resources.

Literature (e.g. by Sarama, Clements, Barrett, Van Dine, & McDonel, 2011; Confrey, Maloney, & Nguyen 2011; Steffe, 2003) seems to acknowledge the role of experience, comprising of the idea that practices of culture as well as cautious training is crucial in the development of HI. HI would propose, by means mathematics, no less than the idea that, the progressive levels portrayed in trajectories are doubtlessly best understood and observed within detailed mathematical domains or topics, yet they also are influenced by more broad, cross-domain development.

The development of mathematical principles and number sense in particular, has been advocating for the cross-domain development as per cognitive domains (Sarama, Clements, Barrett, Van Dine & McDonel, 2011; Confrey, Maloney, & Nguyen, 2011). Therefore the development of number sense among preservice secondary mathematics teachers therefore considered the activities that cut across different domains. The levels of sophistication and cross domain interaction for the development of number sense activities were also designed according to “increasing sophistication, complexity, abstraction, power, and generality” as recommended by Maloney, and Nguyen (2011); i.e., that the teaching using the ideas of trajectory involves the sequencing of tasks and these increase the level of difficulty as one goes deep into the subject matter.

Greenough, Black, and Wallace (1987) note that learners have important, but often inchoate, prior mathematical and general cognitive competencies and predispositions at birth or soon thereafter, these could enormously support or constrain, their successive development of mathematical knowledge. Greenough et al.(1987) refer to these as the “experience-expectant processes”, in which universal experiences lead to an interaction of innate aptitude and environmental inputs that guide the development of mathematical and in this case the number sense of preservice secondary mathematics teachers in similar ways across cultures and individuals. They are not built-in representations or knowledge, but predispositions and pathways to guide the development of numerical facility.

Karmiloff-Smith (1992) notes other general cognitive and meta-cognitive competencies which require learners to be active participants in their learning and development, these general and meta-cognitive competencies. These idas are also acknowledged in some literature such as Tyler and McKenzie (1990) and Clements & Sarama (2007). However, HI also assumes that the speed with which individuals' knowledge and skill develop, and the particular intermediate alleyways they go after from level to level along with whether they definitely reach later levels at all, can vary considerably with variations in experiences in addition to individual differences. Therefore, HI does not claim that any particular progression is inevitable, but to a certain extent affirms that some progressions could be more fruitful than others.

Tyler & McKenzie (1990) and Clements & Sarama (2007) note that HI makes a profound hypothetical claim with respect to the organisation of instruction and the design of HLTs. That is, progressions of instructional experiences and tasks that follow and exploit the more likely developmental paths will prove to be more effective and efficient in helping most students to move towards desired instructional goals. This could be done in short term paths that leave the learners with deeper and more flexible understanding. Hence throughout the development of the number sense of preservice mathematics teachers the researcher must consider that the number sense content has to be broken into chewable chunks that can be grasped at their.

Further to the foregoing this study envisages that the number of short term learning paths used in the development of number sense of preservice mathematics teachers needed to be small enough for the preservice teachers to handle, as recommended by Murata and Fuson (2006). However, Mutara and Fuson also argue that HI would assume that the influence of more universal and internal factors relative to the discrepancy in exterior occurrence would diminish as students get older and the mathematics gets more advanced. The range of deviation due to individual differences in experience will certainly increase.

HI makes clear that a lot of interacting and potentially compensating factors are normally at work in a student's response to an instructional experience (Tyler & McKenzie 1990 and Clements & Sarama 2007). Thus instruction at any given time possibly will relate to multiple levels of a learning trajectory for each student, in this case preservice secondary mathematics teacher. However, this study holds a strong view that the use of HLTs should be for the full implementation of curriculum and harmonisation of mathematics education, in particular the number sense of preservice secondary mathematics teachers whose number sense learning is being problematised.

2.6 Critical theory and research methodology

Critical Theory as illustrated by Habermas (1971; 1973), states that critical theorists believe the perpetuation of the subjective-objective controversy is problematic. That is, the subjective-objective label is socially contrived. The literature by Habermas (1971; 1973) suggests that critical theorists have shown that 'objective' practices are those that have been shown to be the most 'subjective.' Therefore the research enterprise for critical theorists acknowledges the association of 'objectivity' with natural sciences and less association of 'subjectivity' to interpretive sciences.

The investigator in this study, therefore, considers that Critical Theory does not under any circumstances underestimate the quantitative paradigm of research. Moreover the issue of subjectivity is also taken into account so as to minimise the bias in the data collected for the purpose of this study. Therefore the researcher envisages utilising methods that will keep away from bias by minimising the issue of subjectivity so as to attempt producing results that are free of bias about the number sense of secondary preservice mathematics teachers.

Kincheloe and McLaren (1994) on the other hand argue that the subject-object distinction according to the Critical Theory could be seen as an artifact of a system defined to privilege the 'objective' label and the natural sciences. Moreover, Kincheloe and McLaren argue that the subject-object distinction affords identity protection and privileges for powerful groups in the academic world. Otherwise it will lead to misleading beliefs about the presumed relation between qualitative and quantitative research paradigms. If the subject-object dualism gets motivated, researchers will start noticing that objects in both quantitative and qualitative research methods are socially shared, historically produced and general to a social group. From this it could be interpreted that Critical Theory supports the idea of applying a mixed methods approach when conducting research.

For the foregoing reason and for the benefit that exists in the usage of a dual research paradigm as advised by the literature (e.g. Loraine 1998, p.60; MacMillan and Schumacher, 2006; Polit & Hungler, 1997) the researcher in this study therefore utilised both qualitative and quantitative research paradigm.

Critical theoretical approaches tend to rely on dialogic methods that combine observation and interviewing with approaches that foster conversation and reflection (Skovsmose, 2002; Sinson & Bullock, 2012 and Venter, Higgs, Jeevavanthan, Letseka, & Mays, 2007). This reflective dialogic allows the researcher and the participants to question the 'natural' state and challenge the mechanisms for the maintenance of order. The data collection methods utilised in this study were similar to those utilised by the Critical Theorists when collecting data for research. Critical theorists are not just trying to describe a situation from a particular vantage point or set of values but are trying to challenge the situation and its understanding thereof (Venter et al., 2007). Based on this argument, and by utilising a Critical Theory approach the researcher envisages assuming a neutral role, utilising interview as well as questionnaires of number sense lessons as recommended by the literature on Critical Theory.

Moreover, Venter et al. (2007) hold a view that Critical Theory expects researchers to discuss the meaning and implications of the concepts developed. Venter, Higgs, Jeevavanthan, Letseka, & Mays (2007) continue to argue that bearing in mind that Critical Theory aims at protecting human values against all forms of oppression, the criteria for research methodology in Critical Theory should be considerate of ethical issues related to the participants.

Henceforth, researchers have the responsibility to justify in their work that it does not cause any form of oppression to participants in any manner. In addition, the task of the researcher is to address and resolve any tension that manifests itself in the research endeavour so as to safeguard the participants. Generally, the complete philosophical grounds for the research decisions made in Critical Theory during a research project cannot be expressed in a manuscript, but some attempt should be made to articulate these briefly (Skovsmose, 2002; Sinson & Bullock, 2012). Some general description of alternative research orientations, approaches or ways of seeing should be at the discretion of the researcher to foster accountability.

2.7 Justification of Critical Theory

This section gives a justification why this study has adopted Critical Theory as the underpinning theoretical perspective. Also, there exists a link between supremacy and human thought (Higgs &Smith, 2002; Venter et al. 2007). Furthermore, the fact that the discriminatory practices of the apartheid system considered blacks to be inferior to whites in mathematical ability (Amkugo, 1993) could justify the fact that the majority of secondary preservice mathematics teachers come from a background that had in the past suffered some kind of oppression and disadvantage.

Therefore, considering the ideas outlined above, the main argument of the researcher in justifying the choice of Critical Theory as the theoretical perspective underpinning the study was that the preservice teachers come from historically disadvantaged backgrounds. Therefore, by utilising Critical Theory and other supporting theories the number sense of preservice teachers could be developed to empower them with the competencies of effectively developing the number sense of their learners. The researcher, for that reason, acknowledged the role of Critical Theory in emancipating humankind from hegemonic powers of oppressors. Also, Critical Theory will facilitate the investigator's understanding of how the oppressive system from which these preservice teachers are derived could have inhibited their numerical facility.

2.8 Conceptual Framework of the Study

From the delimitations of this study, and the literature (e.g. Freire, 1978; Freire, 1979; Skovsmose, 2002; Pais, Fernandes, Matos & Alves, 2007; Skovsmose, 2010; Jett, 2012; Sinson & Bullock, 2012; Heuvel-Panhuizen,2002; Freudenthal, 1983; D'Ambrosio, 2006; Heuvel-Panhuizen, 2003) it is evident that Critical Theory favours practicing contemporary approaches such as Critical Mathematics Education (CME), Realistic Mathematics Education (RME), Ethnomathematics, Hypothetical Learning Trajectories (HLT). Therefore, the researcher hopes that the development of number sense of preservice secondary mathematics teachers could be best achieved by utilising Critical Theory and its foregoing supportive paradigms.

This study therefore proposes an eclectic conceptual framework that might be termed "Critical Realistic Ethno Number Sense (CRENS)". This conceptual framework combines Critical Theory with RME, Ethnomathematics and therefore number sense. This implies that these could be used in developing number sense among preservice secondary mathematics teachers to ultimately foster number sense development in learners. Consequently, Critical (C) in the notion of Critical Realistic Ethno Number Sense (CRENS) advocates two ideas; i.e., it implies that the Critical Theory could be combined with Critical Mathematics Education along with the ideas portrayed from Critical Pedagogy as advocated by Freire (1978; 1979). The Realistic part (R) indicates the that the advancement of number sense can be best developed when they are brought to the real lives of the preservice secondary mathematics teachers whose number sense is being enhanced.

The Ethno (E) it thus refers to the notion that the development of number sense should pay attention to the cultural aspects of preservice secondary mathematics teachers. The latter idea implies that preservice secondary mathematics teachers came from different cultural backgrounds and therefore these should be considered when developing their number sense curriculum. Number sense refers to the idea that when these aspects of Critical theory, CME, RME as well as ethnomathematics are combined there is a greater possibility for the preservice mathematics teachers to gain a better understanding. I.e., empowerment of one could mean to empower others, which is the key component of the Critical Theory.

The researcher is of the view that the ideas portrayed by CRENS can be generalizable to the development of number sense as a whole, and therefore the CRENS approach should play its equal but collaborative role in the development of number sense of either preservice secondary mathematics teachers or their learners that they will interact with once they join the teaching profession. Moreover, the conceptual framework of CRENS sees the learner as an active participant but not an empty vessel to be filled with information. The teacher’s role as perceived by the framework is of an equal but collaborative facilitator of knowledge not as a source or centre to which knowledge is centred. The content according to the framework is dynamic and should guide the learners towards desired outcomes.

The researcher believes the foregoing conceptual framework not only has the potential to develop the number sense of the preservice secondary mathematics teachers but could also become a permanent model and paradigm that could find its equal but collaborative role within the mathematics education. The researcher is also of the strong opinion that the ideas of contemporary mathematics educationists revolve around the paradigms supporting Critical Theory.

To gain a better understanding of the CRENS framework, as well as its role, the following diagram shows how the development of number sense could be facilitated according to the CRENS conceptual framework. The CRENS conceptual framework could be summarised to obtain a better grasp of its meaning. This study also assumes that CRENS theory could not be restricted to the number sense only but could be rather generalised to other areas of mathematics education and the teaching and learning of mathematics in general. Figure 2 summarises the proposed framework of CRENS.

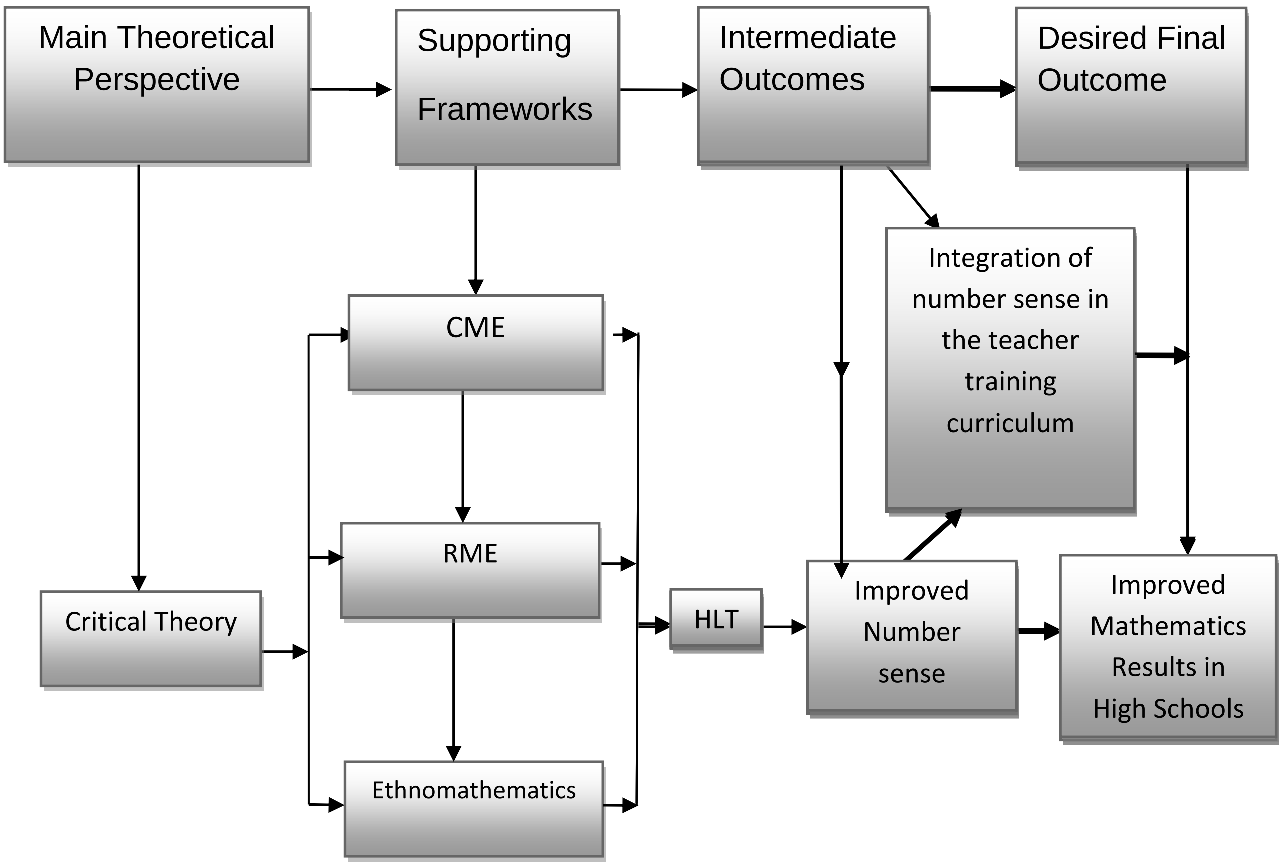

Figure 2.1: Interpretation of Critical Realistic Ethno Number sense (CRENS) framework

The CRENS framework suggests that the development of number sense should comprise of four stages; namely, the theoretical perspective in this case the Critical Theory. The supporting theoretical frameworks refer to CME, RME, Ethnomathematics as well as the ideas portrayed by the Hypothetical Learning Trajectories.

The foregoing therefore means that the Critical Theory encompasses other paradigms that hold similar ideas to the Critical Theory and therefore these were used to influence the number sense training of pre-service secondary mathematics teachers.

The intermediate outcomes refer to the idea that the development of number sense does not only consider good facility of pre-service secondary mathematics teachers with numerical concepts but also calls for ultimate impact on learners' academic achievement in mathematics.

2.9 Pre-service teacher curriculum based on Critical Theory

Several curriculum models exist; these models of curriculum are of several styles. For instance the Tyler's model which was created by Tyler (1949) is guided by four key ideas; the educational aims and objectives of the curriculum strive to achieve, educational context it sees, the issue of learning activities the educational experiences as well as some means of assessment.

Clandinin and Connelly (2002) are also of the view that curriculum models have some areas they define, each looking at education from a different perspective. The focus concept looks at students and centers instruction on them (Craig, 2006). From this it can be argued that the integration of Critical Theory supports instruction that is student-centred, that is the students should be given a chance to actively participate in their own development of number sense.

The approach to curriculum development could be a traditional or modern approach and looks at the type of instruction that will be used (Ornstein, Allan, Francis, Hunkins. 2009). Following this the researcher argues that the content could be topic based or content based, asking how units or strands will be written. This study applying the CRENS model to the development of number sense allows for the role of content, that it should be dynamic and not static, more so the content according to the CRENS model should be challenging and stimulating the thought among the pupils.

Finally, Shawer (2010) argues that the curriculum should possess some structured focus on the system of review, determining how the curriculum will come up for revision. In this study the structure was organised in the curriculum to cater for the five components of number sense development based on a critical theory intervention.

From the arguments presented above, activities that had critical, ethno, and realistic parts were used in the number sense training of pre-service secondary mathematics teachers. Therefore the number sense curriculum that was used was a typical of a Critical Theory nature.

2.10 Summary

This chapter has reviewed the main tenets of Critical Theory which are that human-kind should be freed from all kinds of oppression and that truth is not value free but may be subjectively interpreted to serve a status quo in which the dominant class perpetuates its hold on illegitimate power. Critical Theory in itself, manifests in CME, Ethnomathematics, RME, as well as Hypothetical Learning Trajectories, and ultimately addresses the role of the learner as an active participant, not merely an empty vessel to be filled with the teacher's wisdom. The role of the educator is that of facilitator of learning and not merely the sole source of knowledge. This study also endorses the emancipatory role of the curriculum to produce functional citizens of the society.

The study also examined the research methods aligned to Critical Theory. The study concluded that there is no specific research paradigm prescribed to hold advantage over the other within Critical Theory. Therefore, the researcher envisaged the use of own discretion in choosing the research paradigm.

The chapter proposes an eclectic CRENS conceptual framework that facilitates understanding of number sense and ultimately enable students to effectively develop number sense in their learners. CRENS presupposes that the learner should play an active role in the learning process, be able to challenge the content where needed, the learning method and the teacher, if need be. CRENS further presupposes that the teacher should play the role of facilitator of knowledge construction as an equal and not a superior transmitter, or source of readymade knowledge. The pedagogy according to CRENS should be learner-centred while the mathematical content should be drawn from cultural and experiential contexts that are familiar to the learners as proposed by the RME, CME and Ethnomathematics from which CRENS was derived. The next chapter presents the review of literature regarding the number sense.

CHAPTER THREE

REVIEW OF LITERATURE RELATED TO NUMBER SENSE

3.1 Introduction

Number sense is a relatively new concept within the context of the Namibian mathematics education (Coutney-Clarke, 2012; Naukushu, 2013; Potgieter, 2014). Nonetheless, researchers (Naukushu, 2013; Coutney-Clarke & Wessels, 2014; Menon, 2004; Morris, 2001; & Jackson, 1986) argue that number sense has learning benefits in Mathematics education. Based on the foregoing literature the researcher argues that the role of number sense in mathematics education cannot be disregarded. In this chapter, therefore, the researcher presents a review of the literature about number sense and its role in the teaching and learning of mathematics.

The chapter commences with the articulation of the meaning of number sense, misconceptions around the notion of number sense as well as the characteristics of number sense. Moreover, the researcher in this chapter further attempts to discuss ways of measuring and evaluating number sense, the development of number sense in Namibia and elsewhere, the relationship between number sense and academic mathematical proficiency as well as teaching to enhance the development of number sense.

In the midst of the chapter, the researcher articulates the relative position of number sense on the training of preservice mathematics teachers in Namibia and other parts of the world. The chapter further presents some research findings from the related literature on the development of the number sense of preservice teachers. Drawing closer to the end of the chapter, the researcher discusses the issues related to the development of a number sense curriculum and hence a proposal of a number sense curriculum model that was utilised to inform the design of the number sense training intervention programme that the preservice secondary mathematics teachers.

The chapter concludes with the presentation of the context of this study in an endeavour to illustrate the uniqueness of this study among the existing literature as it utilises the CRENS model derived from Critical Theory as well as its supporting frameworks, for this reason the study could be an eclectic approach to the development of number sense as opposed to the existing literature. In addition to the foregoing, the researcher further presents the key issues from literature that triggered the researcher's interest as well as the knowledge gaps that the study envisages to address in the conclusion at the end of the chapter.

3.2 Defining number sense

Number sense is perceived differently by different researchers and or mathematics education scholars. Daniel (2005) argues that it is rare for two researchers or authors to define number sense in exactly the same way despite the fact their definitions might possess some similarities. Daniel (2005) further points out that when defining number sense, the diversity is even more aggravated by the fact that cognitive scientists and mathematical educators define the concept of number sense in very different ways. It can therefore be argued that number sense might be perceived in different ways as one endavours to understand manuscripts of different researchers. It is for this reason that in this study the researcher presents the interpretation of number sense from different literature to draw closer to a cohesive meaning of number sense.

For instance Zanzali (2005) perceives number sense as a faculty permitting the numerical recognition that something has changed in a small collection when, without direct knowledge, an object has been removed or added to the collection elementary abilities or intuitions about numbers and arithmetic. From this definition one could argue that an individual with number sense should be able to note small amounts added to or subtracted from a quantity. In most cases where there is poor numerical reasoning individuals might not notice small changes to added to or subtracted from an amount and in some cases these are often neglected.

For example, if a person loses 3 cents every day it might accumulate to larger amount if viewed in the light of a longer period such as one year. Similarly, a learner that is late by less than a minute for every lesson might find himself or herself absent for a whole day if these small elements of time are looked at from the point of view of a lengthy period such as an academic year.

In addition, (Burn, 2004, p. 23.; Hilbert, 2001, p. 17.; Bana, 2009, p. 64; Farrell, 2007, p. 54) hold a view that number sense should incorporate the ability do one of the following or more: to approximate or estimate, make meaningful numerical magnitude comparisons, decompose numbers naturally, develop useful computational strategies, solve complex problems, use the relationships among arithmetic operations, understand the base-ten number system, use numbers and quantitative methods communicate, process, and interpret information. From the foregoing literature the researcher therefore holds the argument that number sense could be manifested by the individual's abilities or facility to manipulate numbers, to understand them and recognise the meaning they hold; that is, numerical reasoning and understanding.

One can therefore further argue that number sense can as a result be developed and nurtured. Thus it can be concluded that number sense is not a static but a dynamic concept. This study therefore seeks to understand what methods can be best utilised to develop a better grasp of number sense among secondary pre-service mathematics teachers being studied so as to harmonise their abilities to teach mathematics once they join the job market.

Fennell and Landis (2004) on the other hand equate number sense to the individual's awareness of various levels of accuracy and sensitivity for the reasonableness of calculations. It therefore follows that individuals with number sense should be able to make sense of numerical answers without necessarily having to perform algorithmic methods. For instance for a number sense fluent individual, there is no need to perform any algorithm when working out 10% of an amount, since it just means one out of ten equal parts. It can therefore be concluded that by looking at the calculation even without determining the answer through algorithmic methods an individual with number sense fluency should be able to discriminate the reasonable solution from the alternative solutions that do not make sense within the context of the problem being studied.

Case and Sowder (2000) view number sense as equivalent to possessing *fluency and flexibility with numbers, understanding number meanings and the quantities they may represent as well as multiple relationships among numbers;* Kalch-man and Moss (2001) acknowledge that number sense means recognising benchmark for numbers, number patterns, and gross numerical errors. In addition, Greeno (2006) number sense involves comprehension and use of equivalent forms and representations of numbers as well as equivalent expressions.

From the above it can be argued that the individuals who possess number sense should show some kind of fluency in terms of how numbers are related to each other and their representations in multiple ways as well as how they can be used in the aspects of measurements. Dehaene (2007) on the other hand concurs with the above literature to argue that number sense can be viewed as the ability to make referents to measure quantities in the real world.

On the basis of the said argument the researcher hence adopts the view that one of the greatest attributes of a meaningful definition of number sense is the idea that an individual with number sense should be able to develop reasonable benchmarks and referents for measurement of objects their relationship, space and order. In this study it is acknowledged, for that reason that by paying attention to the common ideas presented by literature on the articulation and meaning of number sense a cohesive definition could be achieved with some precision.

Based on the arguments carried in the literature such as by (Jonassen, 2004; Zanzali, 2005; Burn 2004; Hilbert 2001; McIntosh, Reys & Reys, 1992: Menon, 2004) the researcher argues that a cohesive definition of number sense should consist of five key components:

- understanding meaning and size (magnitude) of numbers,
- understanding equivalence of numbers,
- understanding meaning and effects of operations,
- understanding counting and computation strategies and,
- understanding estimation using appropriate benchmarks without calculations.

Furthermore, despite the diversity in attempting to define number sense, this study concurs with the meaning of number sense as articulated in the above literature. This is for the reason that it encompasses the key components stated above perceived to be crucial when defining number sense within the context of this study.

Consequently this study articulates number sense from a student's point of view, including preservice teachers as learners, as:

> An intuition about number sense that aids students to make predictions about rationality of their computational outcomes and proposed solutions to numerical problems. i.e. students with superior number sense possess: An improved grasp of number meanings, develop numerous relationship among numbers, recognise the relationships among numbers, recognise the relative magnitudes of numbers, know the relative effects of operating numbers, develop referents for measures of common objects and situations in their milieu (McIntosh, Reys and Reys 1992, p. 356).

This section paid attention on the meaning of number sense as described from different perspectives in the literature. The study adopted the meaning of number sense as largely stated by McIntosh, Reys and Reys (1992, p. 356). The next section attempts to present some of the misconceptions that are perceived to hold an equivalent meaning to that of number sense.

3.3 Features of number sense components

Studies on the subject of numbers sense (e.g. Dehaene, 2007; Kalch-man and Moss, 2001; Case and Sowder, 2000) suggest that the characteristic features of number sense lies within the components of number sense. This section therefore discusses the number sense characteristics that could be possessed by each individual component of number sense.

3.3.1 Understanding meaning and size (magnitude) of numbers

Despite the fact that different literature presents different characteristics of number sense, there are remarkably similar properties of number sense in the literature around this component. To single out, the idea that individuals with number sense about the understanding of the meaning and size of numbers go beyond the ability to perform calculations accurately, these have a feel with numbers, and are comfortable with numbers seems to be the attribute mostly endorsed by a variety of the literature (e.g. Dehaene, 2007; Kalch-man and Moss, 2001; Case and Sowder, 2000; Fennell and Landis, 2004; Burn, 2004; Hilbert, 2001; Bana, 2009 and Farrell, 2007) in this component. This could further be interpreted to imply that the students whose number sense is sound hold the understanding of what magnitude is represented by different numbers.

Sowder (2002) and Verschaffel & De Corte (2006) also affirm that the attributes of number sense about the meaning and size of numbers includes developing a mental number line on which analog representations of numerical quantities can be manipulated mentally. This means being able to process and conceptualise the meaning and size of number and developing a mental perception of such. Therefore, being in possession of number sense means one should be able to make a rough estimate of how big or small any number presented to them is.

3.3.2 Understanding equivalence of numbers,

Gersten and Chard (2009) argue that among the attributes of an individual with number sense particularly in the component of understanding the equivalence of numbers is that s/he can move seamlessly between the real world of quantities, the mathematical world of numbers and numerical expressions. It can be argued that number sense seems to suggest an idea that individuals should not unnecessarily experience problems when presented numbers in different forms.

In search for commonalities in the attributes of number sense it appears that number sense could be portrayed by the ability of relating and identifying equivalence amongst numbers represented in different forms. For instance Courtney-Clarke and Wessels (2014) stress the importance of recognising how a different number can be represented in different forms but it still holds the same meaning and magnitude; e.g. half can be represented as fraction, as 0.5 in terms of decimal fraction, 50% in terms of percentages and 1:2 in terms of ratio. Therefore, an individual whose numbers sense is reasonable should be able to recognise that the foregoing representations are only different notations for one and the same number. Additionally, one does not need a complicated algorithm to recognise these multiple representation for a single number.

3.3.3 Understanding meaning and effects of operations

Howden (2009) holds the view that the feature of number sense in this area is the ability to think or talk in a sensible way about the general properties of a numerical problem or expression, how they are implicated by operations. It can thus be concluded that number sense cannot be merely equated to the ability to follow algorithmic methods of solving number sense problems.

A variety of literature (e.g. Zanzali and Ghazali, 1999; McIntosh, Reys and Reys, 1992; Menon, 2004 and Markovits and Sowder, 2004) also suggests that individuals with distinct number sense follow a non-algorithmic approach to solving number sense problems but rather hold an understanding of how the numbers get affected by a variety of operations around them. It is anticipated therefore that pre-service secondary mathematics teachers who emerge from this study with adequate understanding that could aid them develop the understanding of how numbers are affected by operations and enable them to gain a better understanding of numbers and ultimately aid them to perform better in mathematics generally.

There is thus a noteworthy difference between number sense which is based on the principle of numerical understanding and doing normal mathematics which could be based on normal algorithms using numbers, without necessarily attempting to understand the meaning and effect of operations on numbers, but without necessarily paying attention following given algorithm or set of rules on a mathematical calculation. Thus in number sense it is not the algorithm that matters but rather the understanding or reasoning behind the answer as well as being able to assess the reasonableness of the solution that is obtained.

The National Council of Teachers of Mathematics (NCTM) (2000) presents the attribute of number sense as equivalent to a well organised conceptual network that enables a person to relate numbers and their operation, a conceptual structure that relies on many links among mathematical relationships, mathematical principles, and mathematical procedures. From this it can be interpreted that the individuals whose number sense is sound could relate the numbers and how they relate with mathematical operations as well as being able to understand different kinds of numbers and how they are affected by operations.

3.3.4 Understanding counting and computation strategies

Menon (2004) asserts that number sense can be recognized by an individual's ability to invent procedures (i.e. come up with new and accurate counting and computation strategies). This seems to suggest that there is not only one way of performing numerical manipulation. Henceforth, for numerical sense facility to be established an individual should be capable of employing different ways of calculating and manipulating numerical information precisely. It can also be argued that the individual with number sense does not stick to some rigid rule-bound mathematical calculation but rather develops flexibility in counting and computation strategies.

However, this notion of using a variety of ways to manipulate numerical information should go beyond and include being able to come up with best and most efficient methods of representing a mathematical calculation. For instance when asked to work out 25% of 500 instead of following the normal algorithm: $\frac{25}{100} \times 500 = 125$ a student with number sense could notice that 10% equals 50 and therefore 20% is 100. The 5% could also be obtained as half of 10% which is 25 and adding together 20% + 5% will give 100 + 25 = 125. It is therefore assumed in this study that the pre-service mathematics teachers would be able to develop and internalise their own methods and approaches to solving number sense problems, so as to demonstrate evidence of numerical facility. As a result, by so doing they would be empowered which is yet again the greatest attribute of the Critical Theory that underpins this study. This empowerment is then hoped to improve their mathematical understanding and as a result become better teachers.

Several authors (e.g. Number Sense Research Group, 1995; Okamoto & Case, 1994; Reys & Reys, 2009) believe that number sense can be recognised by the skill or kind of knowledge about numbers that an individual is capable of performing rather than being an intrinsic process; that is, a process that develops and matures with experience and knowledge. This could be interpreted to imply that number sense relies on the flexibility within the individual about performing a mathematical concept. Thus an individual with a better number sense should be able to understand when a certain method is applicable and when it could not be used to solve a particular number sense problem.

3.3.5 Understanding estimation using appropriate benchmarks without calculations.

Individuals with a better sense of numbers also possess nonverbal, evolutionarily ancient, innate capacity to process approximate numerosities (Hilbert, 2001). This could be interpreted to imply that an individual with a sound number sense can process and approximate different numbers accurately (estimation using appropriate benchmarks). When given to carry out a calculation of 2.1×49 the student with a sound sense of numbers, by looking at the sizes of numbers in the calculation should be able to reasonably recognise that the answer is nearly 100.

To conclude this section the researcher took note of the fact that there is nevertheless a consensus in the literature. That is, number sense is demonstrated in the ability to feel comfortable with numbers in respect of five components namely, to be able to recognise their magnitude and being able to manipulate the numbers as well as assessing different methods of solving numerical problems, developing referents for measurements and estimations, following non-algorithmic and being able to evaluate the reasonableness of the solution obtained from calculations.

3.4 Misconceptions around the concept of Number Sense

The previous section endeavoured to draw the definition of number sense from different literature. This section presents the alternative conceptions of number sense, or encountered among the pre-service teachers and learners when dealing with number sense problems.

First and foremost it should be clear that the understanding of an individual's number sense indicates the depth of acquisition of the concept. Several studies have placed emphasis on the children's understanding of number sense as well as the number sense of their teachers (Mack, 2010; Markovits & Sowder, 2001; Reys, Reys, Nohda & Emori, 2014; Reys & Yang, 2008; Yang, 2007).

A few of these studies focused on the pre-service teacher's understanding of rational numbers (Nohda and Emori, 2005; Reys and Yang, 1998 and Yang, 1997). Simon and Blume (1992) have targeted learning about the teaching of how and why rational number concepts should be utilised in the numeral system. The foregoing studies suggest that being fluent in number sense means feeling at home with both rational and irrational numbers since they are mostly utilised in the numeral system. However, teachers and learners need to understand how each piece of rational and irrational number knowledge fits into the larger mathematical meaning system and therefore number sense.

The misconceptions of children and adults are often difficult to undo due to the fact that in most cases they are often hidden behind computationally correct answers, but correct answers might be obtained due to one of several wrong reasons (Mack, 2010). Teachers also fail to analyse why particular mistakes were made and thus miss the information that is critical to do appropriate remediation and just consider the computation that yields the correct answer (Galbraith, 1995). The meaning of number sense calls for the students to be aware of the reasonableness, for instance for a calculation 36 x 0.49 the student should be able to note that the reasonable answer is slightly less than 18 looking at 0.49 as slightly less than half of their solutions.

It is therefore important that teachers are empowered with skills that enable them to aid the development of number sense among their learners. Therefore, number sense is not merely an art of producing the correct answer but rather being able to go beyond that; i.e. to be able to understand the reasonableness and the meaning of that answer thereof. From the previous example it could be argued that a number sense researcher is not interested in the mere calculation of 36×0.49 but rather in a reasonable answer that is slightly less than 18.

In addition both teachers and learners need to know how previously learned procedures may obstruct the learning and therefore comprehension of the next learning material, and to clearly rule out new elements from those that are similar to old elements during the student's instruction and evaluation (Behr, Lesh and Post, 2001 and Mack, 2000). From the literature above the researcher can strongly argue that teachers that lack the awareness of potential problem areas of number sense may design instruction and assessment activities that reveal the presence of the known number sense misconceptions without knowing. This could jeopardise the development of number sense of their learners. More so teachers hardly give a thought to such potential academic risks; they hardly analyse and generally are unable to use such knowledge and awareness of such misconceptions in planning, instruction as well as evaluation of student performance (Behr et al., 2001).

It could also be argued that a variety of complications may also occur when students use decimals when they are required to use fractions for instance when presented with a calculation: $2 \times \frac{1}{3}$ students might opt to give the answer of 0.667 which is not equivalent to two thirds but a mere approximation of two thirds. In this case students might not see the implication of rounding off that it alters the answer, however with number sense this should to be noticed. For instance Ball (2000) and Mack (2005) note that students often struggle with the significance of the four basic arithmetic operations. In whole number arithmetic for instance, the concept of division is perceived to mean just making smaller, and multiplication is equally understood to have a magnifying effect. The rules for addition, subtraction, multiplication, and division of fractions are easily confused and misapplied.

From this illustration by Ball (2000) it is reasonable to argue that the students have misconceptions of associating numbers and expressing them in another equivalent expanded. Students also hold a misconception of comparing the relative sizes of numbers and based on that misconception they overlook the fact that the forms compared must be the same size.

Other studies reflect that students hold a misconceptions about translating one rational number model, for instance whether verbal, pictorial, or symbolic, into another and that fractions are symbols illustrating rational numbers therefore they may not be associated with real world situations and consequently these may not have practical implications than merely computation (Johnson, 2008). It appears therefore that in these cases students do not link mathematics and number sense in particular to the quantities they represent. It is therefore envisaged that by conducting this study new approaches of bringing mathematics to a real life context will be explored.

Johnson (2008) also found that middle school pre-service teachers have poor conception in their rational number understanding and that they rely on the use of algorithms when approaching non-standard number sense problems. For instance in the first example 36×0.49 student might struggle to identify the correct answer if it is given with plausible distractors, in some cases they may refer to a choice that is informed by algorithm. That is for that particular sum (36×0.49) students tend to follow the multiplication algorithm learned in school to support their choice of answer. The misconceptions they exhibit tend to be similar across different representations of rational numbers as per findings by Rasch (2002) and Hungerford (2004) who suggest that preservice middle school teachers exhibit difficulties with rational numbers that may be indicative of a lack of intuitive conceptual understanding of the meaning and properties of the number system.

It can be argued that such pre-service teachers with limited intuitive conceptual understanding of the meaning and properties of the number system go beyond the scope of number to perform mathematical calculations and computations without reasoning to comprehend the computation. Such teachers therefore, merely base their arguments on algorithms without the conceptual understanding of the number concept.

The misconception of taking number sense to be equivalent to the algorithms and computations without necessarily assessing the reasoning behind the calculation has been acknowledged by some authors (e.g. Berch, 2005; Markovits & Sowder, 2004; McIntosh, Reys and Reys, 1992; National Council of Teachers of Mathematics

(NCTM), 2000; Verschaffel, 2007; Yang & Li, 2008). In such cases students turn to pay more attention to the algorithmic way of responding to mathematical questions, this is also fuelled by examination driven curriculum. The researcher therefore argues that the examination driven teaching of mathematics does not support the development of number sense.

Other studies on number sense explain the features observed among the individuals who hold misconceptions around the concept of number sense in different perspectives. For instance individuals with poor conceptions of number sense usually make no sense of numbers (whole numbers, fractions and decimals) as well as the impact of their related basic arithmetic operations (+, -, x, and ÷), (McIntosh, Reys and Reys, 1992 and Yang and Tsai, 2010). Students whose number sense is poor might acknowledge the fact that in the number 48 500, 8 occupies the place value of thousands but might not be able to recognise that it adds a value of 8000 to the whole number.

Having number misconceptions also to some extent implies that there is a lack of ability to use multiple representations of numbers and operations. This inability includes a lack of competencies when dealing with different numbers and representing numbers in different ways (McIntosh, Reys and Reys, 1992; NCTM, 2000). For instance, an individual with a poor conception of number sense might face difficulties to comprehend the equivalence of numbers in the relation: $\frac{1}{5} = 0.2 = 20\%$ of 1.

The inability to recognise the meaning and size (magnitude) of numbers is among the greatest manifestations of poor conception of number sense. The ability to recognise the relative magnitude of numbers as defined by McIntosh, Reys and Reys (1992) and Yang and Tsai (2010) is the ability to compare and order numbers correctly. For example, individuals with poor conception of numbers at this stage will face problems explaining that $\frac{23}{50}$ is less than $\frac{21}{40}$ since $\frac{21}{40}$ is more than half and that $\frac{23}{50}$ is less than half. Often when individuals are presented with such a comparison problem they would refrain from thinking about the sense behind the numbers and will draw conclusions based on the algorithmic methods such as converting to the same denominators, such an approach leads to less efficient solution methods.

The students also could feel challenged by composing and decomposing numbers flexibly. This implies a poor conception of numbers that the ability to break down numbers for the convenience of computational fluency (McIntosh, Reys and Reys, 1992). For example, when preservice mathematics teachers are asked to solve 44×25, they could decompose the multiplicand 44 into 11× 4 to get 11×4×25, and this is equal to 11×100 which is more convenient to carry out. This helps students to solve problems efficiently.

Lack of ability to judge the reasonableness of a computational outcome by means of different approaches is also a manifestation of poor conceptions of numbers. This concerns students' ability to utilize strategies such as estimation and mental computation to solve problems appropriately and to know if the result is reasonable (NCTM, 2000 and Yang and Tsai, 2010). For example, students should be able to estimate the height of the classroom from the ceiling to the floor being about 3 to 4 metres.

This section discussed some misconceptions that are perceived to be equivalent to the concept of number sense. The section also presented closer to its end, some manifestations of poor conception of numbers. In the light of these, it was deemed necessary within the context of this study for teachers to possess a better grasp of numbers in order to deliver the effective teaching of mathematics as a subject. This study therefore attempts to explore ways of facilitating a better understanding with a view that it will enable pre-service teachers' to deliver the mathematical content.

The researcher holds the view that a better understanding of number sense could have a positive impact on the development of pre-service mathematics teachers' numerical understanding and therefore the teaching programmes which will ultimately aid teachers to become better practitioners. The researcher anticipates therefore that by carrying out this study he will explore avenues of clearing misconceptions and therefore ensuring better conception of numbers and ultimately an improved practice among the pre-service secondary mathematics teachers upon joining the teaching careers. The next section therefore attempts to explain the characteristic features that should be developed when attempting to develop the number sense of pre-service secondary mathematics teachers.

3.5 Measuring number sense

The process of determining the number is both quantitative and qualitative. Literature (e.g. Dehaene, 2007; Kalch-man and Moss, 2001; Fennell & Landis, 2004; Burn, 2004; Hilbert, 2001; Bana, 2009; Farrell, 2007 & Menon 2004) shares a common view that this dual nature creates a controversy when attempting to measure the number sense abilities of learners. Therefore there is no specific recipe of measuring the number sense of the preservice secondary mathematics teachers prescribed by literature.

McIntosh, Reys and Reys (1992) attempted to quantify number sense of learners by categorising the scores of students in the Number Sense Standardised Achievement Tests (NSSAT) in different levels of achievements which were then transcribed with a meaning assigned to each and every level. The number sense of individuals could be quantified by considering four levels that depict different numerical abilities; i.e. (levels 1- 4) which are in the order of decreasing strength (McIntosh, Reys, & Reys, 1992, Reys & McIntosh, 2002; Markovits & Sowder, 2004) The classification according to the four levels of number sense depended on the percentage scores in a validated NSSAT:

- **Advanced** *(75 % and above)*
- **Proficient** *(60 - 74 %)*
- **Basic** *(50-59 %)*
- **Below Basic** (less than 50%)

Contrary to the foregoing, other literature suggests that number sense is an internal qualitative phenomenon it does not orient itself to quantifiable means (Number

Sense Research Group, 2005; Verschaffel & De Corte, 2006; Okamoto & Case, 2006; Sowder, 2002; Menon, 2004). On the basis of the literature above it follows that the quest to find the magnitude of number sense should not be merely based on the scores of an individual in the number sense test but in addition to that add qualitative interpretation on what the level of number sense among the students is. This could also be due to the fact that tests can be affected by various factors. Thus in an attempt to find out how much numerical sense an individual has grasped should go a step further to also gather qualitative information.

In support of the foregoing some scholars, (e.g. Verschaffel & De Corte, 1996; Burn, 2004; Hilbert, 2001 & Bana, 2009) stress that the importance of integrating qualitative aspects in determining the level of grasp of number sense cannot be underestimated. This study adopts the view that most of the evidence of the number sense ability cannot be unveiled by means of quantitative means only but largely means of both quantitative and qualitative as opposed to quantitative only. It is therefore imperative that the evaluation of number sense should comprise of both the qualitative and quantitative data to infer from.

3.6 Development of number sense in Namibia and elsewhere

A study number sense informed by ‘mathematical proficiency’, which had five strands, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition was conducted in Namibian by Courtney-Clarke and Wessels (2014). Using sample size of 47 preservice teachers at primary school level respectively the study revealed an acute need for intervention to enhance a better sense making of numbers. The overall results of a study by Courtney-Clarke and Wessels (2014), reveal that the final year pre-service primary mathematics teachers demonstrated limited number sense and possession of very few of the indicators of number sense.

> The lack of a sound foundation in the domain of numbers and operations may be the root cause of the low standards of performance of Namibian learners in mathematics at all levels and the lack of improvement over the last decade or more, teacher training institutions need to re-assess the mathematics education curriculum and include

> programmes to develop the number sense of pre-service teachers (Courtney-Clarke & Wessels, 2012, p. 8).

From the findings above the researcher can claim that there is a gap in knowledge that requires studies to inquire in the number sense development of preservice teachers, particularly at all levels of teacher training (including secondary preservice teachers). This should be done to ensure that mathematics teachers graduating from universities possess competencies that will allow them to cope with the demands of teaching mathematics upon joining the teaching profession.

The other issue emanating from Courtney-Clarke and Wessels (2014) is that teachers seem to have indicated high confidence and willingness to learn. It also appears therefore that it is essential to study the ways of enhancing the confidence of pre-service teachers about their number sense in anticipation of improving the number sense of learners once they start teaching to avoid the vicious cycle of innumeracy already alluded to in this study (see Figure 1.1, section 1.3).

Potgieter (2014) on a constructivist based number sense study of 35 middle school teachers' abilities to use manipulatives in teaching number sense in Namibia also made similar findings with respect to middle school preservice teachers:

> This study has shown that the middle school preservice teachers included in this study had a very poor understanding of the following: number sense, how it developed in young children, how to select suitable manipulatives for their lessons on number sense skills and place value and use it effectively in teaching and learning activities. Furthermore the study reveals that the teachers' poor understanding of number sense and ineffective use of manipulatives had an overall negative outcome on the Grade 2 children's performance in selected number sense skills (p.45).

The findings by Potgieter (2014) are yet but another call for research to search for solutions in the effective integration of number sense in the Namibian mathematics teacher education curriculum.

Additionally, Naukushu (2013) also conducted a study based on a constructivist framework on the number sense of 150 high school leavers in one of the Oshana Education Region of Namibia, results revealed a poor grasp of number sense.

Therefore there was an acute need to intervene at teacher training levels to strengthen the number sense of pre-service teachers in anticipation that they will impart these skills to their learners upon joining the job market.

What remains concealed is however the number sense for pre-service secondary mathematics teachers in the Namibian context. Therefore there exists a need for this and similar studies to harmonise the development of number sense of preservice secondary mathematics teachers and ultimately improving teaching and learning of mathematics at secondary school level.

In addition to the number sense in the Namibian context, number sense has been studied widely in other countries. In South Africa for instance some universities make number sense an integral component of the both primary and secondary mathematics teacher training package (Venkat, 2013). Despite this effort the curriculum content of some universities that offer number sense shows that number sense training is more theoretical than practical. That is, it is centred more on how number sense can be developed as opposed to exposing preservice teachers to the practical activities where they can practice using problem solving activities to develop their own number sense (North West University, 2014).

Singapore, which achieved first place at grade 4 in the Third International Mathematics and Science Study (TIMSS) of 2011set emphasis on the importance of number sense right from the initial stage of their education. Singaporean mathematics teachers put emphasis on the development of number sense to enhance a better academic performance in mathematics Mullis et.al (2012). The development of number sense also appeared to be the main emphasis in the development of Mathematical understanding in the Netherlands which claimed the first position way back in TIMSS 1999 (Schoenfeld, 2006).

3.7 The relationship between number sense and mathematical proficiency

From the literature number sense appears to be a vital ingredient in the development of mathematical proficiency. Kilpatrick (2001) discusses a research verification that demonstrates a strong association between basic facts of numerical fluency and strong mathematics achievement, but at the same time shows that successful mathematical skills are based on a mixture of memory and techniques. Hence mathematical proficiency cannot be equated to number sense because it is more than number sense; nonetheless, it is plausible to argue that number sense is an aspect of mathematical proficiency.

The positive connection between number sense and mathematical proficiency is also noted by other researchers (e.g. Reys & Reys, 1998; Anghileri, 2006; Heirdsfield & Cooper, 2004). The foregoing literature maintains that mathematical knowledge and number sense develop together. The literature also reflects that the relationship between number sense and mathematical proficiency is such that “one cannot exist without the other” (Griffin, 2003, p. 306). On the grounds of the evidence provided by the foregoing literature, this study maintains that number sense plays a crucial role in facilitating academic achievement in mathematics. For that reason, the researcher believes that learners need a better grasp of number sense if they are to do well in school mathematics.

There seems however to be a need to establish how much number sense could possibly contribute to academic achievement in mathematics, as most of the studies seem to be silent on this. This study consequently, also sought to establish the extent to which secondary teachers’ number sense facility is relevant to and influences mathematical facility. This was carried out in an endeavour to find ways of broadening the number sense of pre-service mathematics teachers in anticipation that they will impart such numerical sense facility to their learners upon joining the teaching career and ultimately mitigating the problem of poor academic performance in high school mathematics discussed in chapter one of this study.

3.8 Teaching strategies with the potential to develop number sense

Kamii and Dominick (1997, p. 60) argue that teaching for the development of number sense depends on key components that are constituent to the development of number sense. For that reason, acknowledging that the ability to perform mental calculations is one of the key components of defining number sense, it can be concluded that the teaching that develops mental calculation facility could bear a positive influence on the development of number sense. Griffin (2003) concurs that number sense can be built by moulding students' abilities to perform mental calculations using numerical reasoning, thereby empowering them to develop abilities to think in their own ways. Griffin (2003) further affirms that moulding student's mental calculations using numerical reasoning will enable them to elevate their numerical sense making to higher levels of comprehension that are rooted in their own knowledge as well as the ability to assess the reasonableness of their obtained solutions.

It can therefore be argued that contrary to proficiency in using standard paper and pencil algorithms, which might be harmful to children's development of numerical thinking, number sense can be acquired by strengthening mental calculation linked to numerical reasoning. The emphasis is therefore not merely about getting the correct answer by following rigid and structured algorithms but rather, it is about making use of flexible sense of numbers in order to yield the required mathematical solution to a problem as well as the reasonableness of that solution.

In addition literature (e.g. Kamii and Dominick, 1997; Griffin 2003) also contends that mental calculations for operations with a miscellany of single and multi-digit numbers could guide students on how numbers work, how to make decisions about numerical sense procedures as well as how to create a variety of strategies to solve both numerical and mathematical problems. The use of written algorithms encourages students to follow different steps without thinking (Carpenter, 2008; Veloo, 2010). Therefore this should be discouraged as it does not have a positive bearing to the development of number sense.

Carpenter (2008) also claims that relevant mental calculations could encourage children to reflect on the process and to think about what numbers mean in relation to the problem, ultimately developing their own number sense and therefore mathematical proficiency. Students certainly need pen and paper very often, to precisely work out the answers. However in some cases where the estimated but not an exact answer is desired this is more related to number sense. For example, 59×61should be around $60 \times 60 = 3600$ roughly. An answer that is way out of this (e.g. 36, 360, or 36000) would sensibly be incorrect. However performing this calculation mentally only without pen and paper could be difficult, which is not really what the development of number sense seeks to address in the component of estimation.

The fore going argument and example indicate a connection between estimation using a relevant benchmark and the development of number sense. Therefore the researcher concludes that mental calculation has some bearing on the estimated answer and therefore the development of number sense.

From a cognitive science perspective, Carpenter (2008) and other earlier researchers (e.g. Griffin, 2003; Heirdsfield & Cooper 2002; Reys & Reys 1998; Pesek & Kirshner, 2000; Heirdsfield and Cooper, 2004) analyse factors influencing the ability to carry out meaningful and accurate estimations as an attribute of number sense that once developed could yield some positive development of number sense to some extent. Moreover, ability to carry out meaningful estimation has come to be viewed as a significant manifestation of number sense (Heirdsfield & Cooper, 2004). This therefore means that the individuals whose ability to carry out accurate estimations is relatively stronger show better characteristics of number sense.

In an endeavour to scrutinise the processes involved in computational estimation, the literature (e.g. Heirdsfield & Cooper 2002; Reys & Reys 1998; Pesek & Kirshner, 2000) identifies conceptual understanding as the ultimate goal of number sense. Firstly, numerical comprehension should encompass a comprehension about the role of approximate numbers in the estimation of quantities. Secondly, estimation may encompass numerous processes and consequently numerous answers. However, the most important fact to consider is that the final outcome of the estimated answer should draw closer to the anticipated solution.

Understanding computational estimations should take into account that the appropriateness of an estimate depends on the context that is under consideration (Sowder & Wheeler, 1989). Most children and adults lack the basic computational estimation skills largely because of the limited exposure to these skills at school (Alajmi & Reys, 2007; Dowker, 2002; Hanson & Hogan, 2000; Sowder & Wheeler, 2009; LeFevre, 2003). From this, the researcher argues that this lack of estimation skills might compromise students' ability to assess the reasonableness of the solutions.

Hanson and Hogan (2000) found that even college students experience difficulties with estimating answers to fraction and decimal problems. They were found to be better at estimating answers to addition and subtraction of whole numbers than estimating answers to multiplication and division problems. Bana and Dolma (2004) confirm these findings from a constructivist informed study with 60 pre-service secondary teachers in Australia.

Besides, Alajmi and Reys (2007) report that most Kuwaiti middle school teachers view a reasonable answer this shows lack of competencies in the component of estimation using a relevant benchmark to be an exact answer and maintain that the heavy emphasis should be placed on procedural rules as opposed to the development of number sense. Based on these findings it is clear that such a teaching and development of number sense inhibits rather than facilitates the development of number sense.

It is sensible to argue that there is an element of guess work which could also hamper the development of estimation skills among the learners and ultimately the development of number sense from a broader perspective. A disposition to make sense of a situation to produce reasonable answers instead of wild guesses is a prerequisite to the effective development of number sense (Dowker, 2002). Consequently, teaching students to make meaningful estimations could actually lead to the development of number sense.

Kilpatrick (2001) argues that computational estimation is a complex activity that ought to integrate necessary competencies of numerical proficiency. Furthermore, it requires a flexibility of a calculation that emphasises logical reasoning and systematic competence. This flexibility should therefore be guided by students' conceptual understanding of both the problematic situation and the mathematics underlying the calculation characterised by fluency in computational procedures.

Alajmi and Reys (2007) and Dowker (2002) contend that when studying about the knowledge and facility with numbers the emphasis should be placed on the students' ability to acquire some alertness to several approaches to solving number problems exist. However, some have nothing to do with number sense even if they yield the same solution as the ones related to the development of number sense. Therefore points towards the idea that the students who possess a better grasp of numbers must be able to employ efficient methods to review answers and therefore the reasonableness of the solution that is obtained.

Reys and Reys (2008) favour a problem-centred approach to teaching counting and computation skills to enhance knowledge and facility with numbers. Thus the thrust when teaching to develop the number sense should be on utilising rich mathematical activities that engage and advance preservice teachers' thinking to facilitate the sense making of numerical situations and ultimately develop their number sense.

Dowker (2002) argues that number sense students should be taught and motivated to creatively come up with numerical calculations techniques that are based on their emerging understanding of numbers and operations so that they can build a conceptual foundation of number and numerical understanding. However, conceptual and procedural knowledge ought to develop simultaneously as argued by Rittle-Johnson, Siegler and Alibali (2001). The students should therefore be exposed to number sense activities that help them to develop both their understanding of the effects of operations on numbers and ultimately the whole development of number sense. The argument that can be extracted from the foregoing literature therefore does not favour following algorithmic procedures as opposed to the utilisation of methods based on numerical understanding and comprehension.

This section endeavoured to discuss teaching strategies that enhance the development of number sense. Literature holds a common understanding and argument that the teaching to develop number sense should be centred on the key components that constitute to the development of number sense. The following section presents the review of relevant literature on the number sense training of teachers.

3.9 Teacher training versus the development of number sense

This section outlines research about number sense training of preservice teachers. Researchers (e.g. Alajmi & Reys, 2007; Dowker, 2002; Hanson & Hogan, 2000; Sowder & Wheeler, 2009; LeFevre, 2003; Number Sense Research Group, 2005; Verschaffel & De Corte, 2006; Okamoto & Case, 2006; Sowder, 2002; Fishebein, 2007; Menon, 2004) seems to have been carried out mainly in the context of training of primary mathematics teachers. The foregoing literature, gives the impression that number sense can only be developed at the initial (primary) stage of education; i.e. literature on the number sense training of teachers seems to have paid little or no attention to the number sense training of secondary mathematics teachers. In this study the researcher however holds a different argument that number sense is a phenomenon that needs to be integrated at all stages of mathematics education.

Chrysostomou, Pitta-Pantazi, Tsingi, Cleanthous & Christou (2012) administered a mathematical test on number sense and algebraic reasoning, a self-report Object–Spatial Imagery and Verbal cognitive style questionnaire to 83 pre-service mathematics teachers. The results of this study indicated that spatial imagery, in contrast to the object imagery and verbal cognitive styles, is related to achievement in number sense and algebraic reasoning. In addition to this, the results revealed that the higher the pre-service mathematics teachers' tendency towards spatial imagery cognitive style, the more conceptual and flexible strategies they employ in algebraic reasoning and number sense tasks. From the findings of Chrysostomou et.al. (2012) it could be argued that there is a need to development the number sense of pre-service teachers to ensure that number sense helps pre-service teachers to understand other concepts in mathematics which are related to number sense as indicated in the findings above.

On the other hand some researchers (e.g. Menon, 2004; Hilbert, 2001; Greeno, 2006) argue for the need for pre-service mathematics teachers across the board to take up a number sense course as part of their training curriculum across the board. In Namibia the status of number sense training among mathematics teachers especially secondary mathematics is not clearly known. It is therefore necessary to carry out research in order to explore and find the best means of incorporating number sense in the teacher education curriculum.

The University of Namibia used to train senior secondary school mathematics teachers in the Bachelor of Education (BEd.) programme while colleges of education trained junior secondary and primary school mathematics teachers in the Basic Education Teachers Diploma (BETD) programme. None of the said institutions explicitly offered number sense as part of the teacher training curricula, the curricula followed by these institutions did not make provision for integrating number sense in the teacher education either (Clegg, 2008).

Currently, mathematics teachers are trained at the University of Namibia opting for the BEd degree specialising in the primary or secondary phase. The new teacher education curriculum shows that the number sense training is only offered for pre-service mathematics teachers at primary school level. The number sense training content appears to be more theoretical than practical (University of Namibia, 2014). There is therefore a need to come up with some practical number sense activities which pre-service teachers can work thorough as opposed to just learning theories about how number sense develops.

The idea of learning mathematics and particularly number sense from a practical point of view has been advocated by Ngololo and Kapenda (2014), Otieno (2012) and Kwamboka (2013) when assessing in-service teachers' abilities to include the ASEI-PDSI concept in their lessons. The Japan International Cooperation Agency (JICA) (2013) also argues through the Strengthening of Mathematics and Science Education (SMASE) programmes that the learning of mathematics and science should focus on practical activities. From the view point of JICA the learning of mathematics can therefore follow the Activity Students Experiment Improvisation-Practise Do See and Improve (ASEI-PDSI).

The number sense that is practical could therefore follow the ASEI-PDSI approach with the understanding that once teachers are equipped with the necessary skills they will improvise activities for their students which are experiment based and these will give their learners a chance to practise and ultimately improve. Literally speaking the concept of ASEI-PDSI is equivalent to Learner Centred Education (LCE). The concept has been utilised in Kenya, Japan, Nigeria, Namibia and Botswana. Despite the fact that this process takes time to implement, it was still considered viable in this study due to the fact that the study supports the tenets of LCE. Through its Critical Theory and CRENS model the integration of practical number sense on the development of number sense is also anticipated to yield improved number sense and ultimately improved results in mathematics.

It is also therefore in the view of this study that the concept of number sense is two dimensional; that is, there exist two kinds of number sense training that can be given to teachers, theoretical and practical.

The theoretical number sense therefore could refer to the learning of number sense that focuses on theories of how number sense is learned or developed. Often such theories have too much theoretical content as opposed to activities that involve students can work through and ultimately develop their number sense (Hilbert, 2001).

Practical number sense on the other hand as argued by Hilbert (2001) could involve practical activities such as the ones suggested by Otieno (2012) and Kwamboka (2013) in the assessment of preservice secondary mathematics teachers' implementation of ASEI-PDSI that students could work with and ultimately develop their number sense by working through these pre planned activities. These activities approach the number sense training by addressing the aspects covered in the key constituents to the definition and meaning of number sense alluded to earlier in section 3.2.

By including practical number sense it can be argued that students could strike a balance on the time spent interacting with practical number sense activities and the learning theories on how number sense can develop (Otieno, 2012). Based on the view presented by Otieno (2012) it can be argued that the ideas carried by practical number sense are linked to those of ASEI-PDSI and therefore LCE. It is also both logical and reasonable to argue that the ideas portrayed by theoretical number sense are more of rote learning and depict teacher centeredness.

Therefore, it could be beneficial to incorporate the practical number sense, ASEI-PDSI which is equivalent to LCE into number sense curriculum design. By contrast, literature shows theoretical number sense to be less beneficial when compared to practical number sense (e.g. Kasanda, 2010; Makari, 2009; Sinson & Bullock, 2012; Heuvel-Panhuizen, 2002; Freudenthal, 1983; D'Ambrosio's, 2006; Heuvel-Panhuizen, 2010). In other words teaching based on the LCE paradigm appears to be more beneficial as opposed to the rote learning. The notion of a practical number sense learning approach and its components is illustrated in the following diagram which should therefore be developed alongside with theoretical number sense.

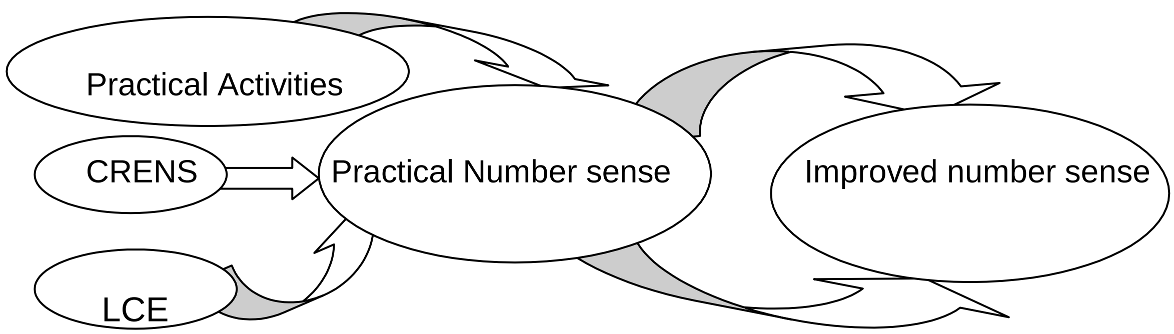

Figure 3.1: The illustration of practical number sense

Figure 3.1 demonstrates that once the necessary ingredients are put in the practical number sense it will eventually yield the benefit of improved number sense. Therefore, this study argues in favour of combining both the practical and theoretical number sense in order to yield the benefit of improved number sense.

3.10 Research findings on issues pertaining to the number sense of pre- and in-service teachers

The researcher acknowledges that this study cannot exist in a vacuum; i.e., there are other similar studies preceding this one in the area of number sense and mathematics education. Therefore it is imperative to discuss some findings, methodologies and conceptual frameworks of other studies already carried out in the area of number sense training of teachers. This was carried out with an ultimate purpose of positioning this study within the already existing literature to identify the gaps in knowledge that the study seeks to fill.

Informed by the constructivist paradigm, Leinhardt and Smith (2005) carried out a study on 65 “expert mathematics intermediate teachers” in the Belgium. The findings indicated considerable variability in teachers' knowledge of fundamental for both rational and irrational number concepts. This shows that there is diversity in the number sense comprehension of teachers despite the fact that they could have been trained on the same curriculum.

Gliner (2001) conducted a study on the estimation performance of 45 pre-service teachers at Madison University and the results indicated a lower performance than would be reasonably expected of a numerically literate eighth grader. The study also showed that school learned algorithms could block the student's ability to perform tasks presented symbolically or mathematically. An analysis of the prospective teachers' explanations of their solution efforts indicated that the difficulties lie not in the ability to estimate, but instead in a lack of number sense (Gliner, 2001). Furthermore, the research data implies emphasis on rule-bound mathematics restricts the growth of a number of computational skills and keeps understanding of both rational and irrational numbers isolated from realistic applications and models.

Based on the findings by Gliner (2001), it appears that the preservice teachers did not possess number sense as in most cases they utilised algorithms and rigidly following the rules as opposed to numerical reasoning and comprehension, which is a poor characteristic of number sense according to section 3.3

Also, recent behavioural genetic research suggests that although the heritability of number sense is not high, the links between number sense and mathematical ability are largely mediated by genetic factors. The developmental course, the direction of causal links between number sense and mathematics, and the effects on education on these links remain unclear. The results suggest uneven relationship between number sense and mathematics across development and quality of educational provision (Dyyadova, Pica, Lemer, Izard, & Dehaene, 2013).

It has been unclear, however, whether number sense plays a role in the uniquely human ability to learn higher mathematics such as algebra, calculus and trigonometry. A study by Stein (2008) involving 64 pre-service secondary teachers found that among the students who did well on a test that measured their "number sense" were much more likely to have gotten good grades in math classes however there were inconsistencies in the results therefore such expectation was not accounted for.

Based on the constructivist theory, Markovits' study (2009) focused on 50 preservice middle school mathematics teachers at the Michigan University, that were not expected to make any decisions or judgment in school mathematics, so they showed very limited number sense according to the study. This technically implies that the number sense of students according to Markovits (2009) could be a nurtured concept, i.e. had they been exposed to number sense these teachers would have performed better. This study assumes that (see section 2.9) number sense could be nurtured through Critical Theory intervention.

Post, Harel, Behr and Lesh (2001) carried out a phenomenological study to assess the extent to which 80 intermediate in-service teachers understood rational and irrational number concepts. The study probed teachers not only to solve problems but also to determine the conceptual and pedagogical adequacy of their explanations. The results of the study indicated that there was a limited numerical understanding especially in the irrational number concepts that were assessed. For instance many teachers did not exhibit enough substantive mathematics and only a few of them were able to solve problems. This latter fraction of teachers was also able to provide coherent pedagogical explanations.

Based on the findings of this study it is imperative to suggest a turnaround on the entire mathematics education process more especially by incorporating number sense in the training of pre-service mathematics teachers. This will provide time for them to develop their own understanding over a long period of time since the study assumes that the comprehension of number sense appears to be a time consuming concept.

Peck and Connell (2011) carried out an ethnographic study that investigated teachers’ knowledge about problems and situations involving the part-whole interpretation of rational numbers. Both pre- and in-service intermediate teachers in this study were unable to recognize and utilize important links between concepts, and were unable to effectively help their students to construct mathematical concepts. The findings suggested a poor comprehension of number sense among both the pre- and in-service teachers. This study confirms weak number sense abilities among preservice mathematics teachers as found in Namibia together with others (e.g. Courtney-Clarke, 2012; Potgieter, 2014). Teacher weaknesses in number sense are therefore not only a Namibian but also a global phenomenon since it has been referred to in other literature outside the Namibian context presented earlier in this section.

Joyner (2004) on the other hand carried out a study informed by a constructivist paradigm to analyse senior primary school teachers' knowledge of rational number concepts through an instrument that was designed to gather information about computational processes and reasoning. Elementary teachers were required to carry out numerical computations and provide symbolic, pictorial or word models for real life and symbolic numerical problems. Models of typical student numerical sense misconceptions were presented where teachers were asked to judge them for their reasonableness. Practicing teachers showed no better number sense comprehension. Joyner (2004) concluded that the teachers confused place value concepts with whole number addition and fraction addition, that they had a poorly developed referent system for rational numbers, and that they lacked number sense relating to fractions.

This study suggests that there is virtually no difference in the number sense of pre- and in-service mathematics teachers. This technically implies that the experience acquired by teachers during their practice did not appear to yield any effect of improving the number sense of in-service teachers. The findings of Joyner (2004) also appear to suggest that both pre and in-service teachers lacked very important components of number sense such as ability to carry out numerical computations, estimation skills as well as knowledge and facility with numbers. Therefore, it is plausible to conclude that such pre- and in-service teachers lacked number sense.

In a study of preservice middle school teachers' understanding of the operation of division, Ball (2000) found that their understanding relied on rules and was unrelated to other mathematical operations. Five of the eighteen participants when asked to design a model to illustrate a division, a third generated inappropriate representations for division by fractions, while only five were able to provide appropriate representations. Eight participants were unable to construct any representation at all. They either recognized the conceptual problem or recognized that their initial response represented division by 3 rather than by 1/3.

Thus researcher arrives at a conclusion that the research reveals that preservice teachers can apply deeply-ingrained whole number rules, in addition to weakly understood fraction and decimal concepts, to draw false conclusions about both rational and irrational number representations. For example they these participants presented 0.45 as greater than 0.5 because 45 is greater than 5. In the same manner these students concluded that $\sqrt{0.25}$ is greater than $\sqrt{0.9}$ since 25 is greater than 9 (Ball, 2000). The results also suggest that there was a weak conception of fractions among the preservice middle school mathematics teachers investigated by Ball (2000).

Generally most, if not all of the findings, revealed a weak numerical conception among the pre- and in-service mathematics teachers. It also emerges that most of the studies were based on the constructive paradigm and it was rare to encounter studies based on the Critical Theory which despite the fact that it suggests critical thinking, learners centred teaching and other contemporary methods of mathematics education. The researcher also concludes that most of the studies appeared to be of a quantitative nature. Thus it is envisaged that by carrying out a study that utilises a mixed approach both the quantitative and qualitative information about number sense will add more value.

3.11 Designing a number sense curriculum for preservice secondary mathematics teachers

Researchers (e.g. Carpenter, 2008; Griffin, 2003; Heirdsfield & Cooper, 2002; Reys & Reys, 1998) give the impression that if one is to design and implement a number sense curriculum then it should be centred on the key components that are constituent to the meaning of the development of number sense. However, none of these studies have suggested a prospective curriculum for the number sense development. Nonetheless, some researchers (e.g. Bobbit, 2000; Oyedele, 2003; Schoenfeld, 2006) argue that curriculum development should assume the existence of prerequisite knowledge which should then be laid as a foundation for cultivating new knowledge that is to be developed.

From the above this study assumes that the preservice secondary mathematics teachers possess some number sense prior to entry into the number sense intervention programme. Therefore, this prior number sense knowledge will be used as a foundation upon which the new number sense will be built. It is therefore for that reason that a pre-test was conducted to assess this baseline number sense knowledge to assess the prior number sense that existed before the Critical Theory intervention.

Oyedele (2003) also concurs that a curriculum should contain exit learning outcomes and the learned knowledge. The exit learning outcomes (E L O s) will be derived from the five key components that constitute number sense. In addition the post-test could test the learned number sense from the number sense intervention programme. Based on insights from the literature on number sense and curriculum development (see also section 2.9), the researcher proposes the following curriculum model for the development of number sense among the preservice secondary mathematics teachers.

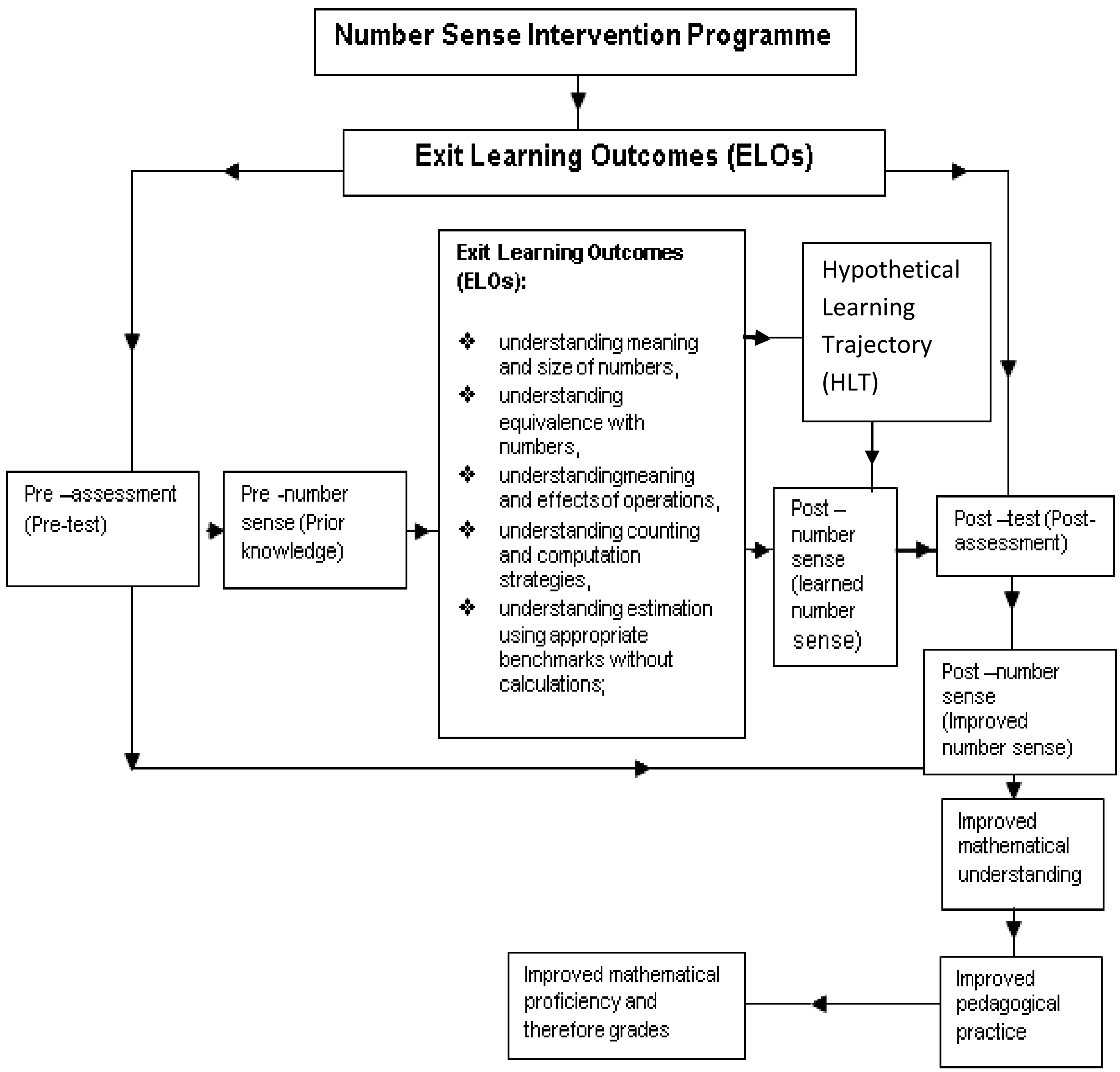

Figure 3.2: A number sense curriculum model derived from a CRENS framework

The model acknowledges prerequisite knowledge (prior number sense), which was assessed by means of a pre-test. In addition to the prior number sense there must be ELOs that the curriculum seeks to address whose attainment is assessed by the post-test. The assumption is therefore that once given the training, pre-service secondary mathematics teachers will improve their number sense level of comprehension. Thus the post-test is intended to determine the extent to which the number sense of preservice secondary mathematics teachers improved.

If the number sense competencies of preservice secondary mathematics teachers improve, that could imply the possibility of improved pedagogical practice which should lead to improved mathematical proficiency and ultimately better grades will be achieved by the learners of these preservice secondary mathematics teachers after graduation.

3.12 The context of the study in the existing literature

This section presents the context of this study, on the issues of the number sense training of preservice secondary mathematics teachers. The section further presents the key issues emerging from the reviewed literature that triggered the researcher's interest to carry out this study as well as the knowledge gaps that this study is seeking to address. For instance most of the studies carried out followed a constructivist paradigm. Based on one of the main constructivist tenets that individuals learn by interacting with their environment, it can be observed that most of the studies cited in this chapter assume that number sense can develop by interacting with the environment only.

The researcher therefore argues that by taking a Critical Theory enquiry and the CRENS model thereof, the study will employ an eclectic approach to the subject of number sense. It is also therefore envisaged that by employing this eclectic approach to the number sense training of preservice mathematics teachers they will gain a better understanding of number sense which is the anticipated outcome of the intervention in this study.

It is envisaged that by employing Critical Theory and the CRENS model as presented in Chapter 2 (specify sections) preservice secondary mathematics teachers could be empowered. Empowerment is at the centre of the Critical Theory and it is hoped that this study will contribute to empowering teachers.

Most of the studies succeeded at giving the impression that there is poor numerical grasp among preservice mathematics teachers. That is, at least all, if not most of the studies cited revealed that there is a poor numerical grasp among both the pre- and in-service teachers. In the Namibian context studies by Potgieter (2014) and Courtney-Clarke (2012) also revealed a poor grasp of number sense among the preservice primary mathematics teachers. What remains unknown however is the impact of a number sense training intervention programme such as the one based on Critical Theory and the CRENS on the number sense training of preservice mathematics teachers at secondary school level.

Most of the studies cited were also carried out on pre and in-service mathematics teachers at a primary school level. The researcher therefore argues that most of the researchers appear to give the impression that the concept of number sense only develops during the primary stage of education. This study henceforth assumes that number sense is a concept that develops across the whole curriculum at all stages of the mathematics education. By addressing the number sense of preservice secondary mathematics teachers, the study undertook a wide-ranging approach to seek means and ways by which the problem of lack of number sense alluded to in this and previous chapters could be mitigated.

The researcher also envisages that by carrying out this study, the researcher in this study could come up with a mechanism to introduce the concept of number sense in the training of preservice mathematics teachers particularly at secondary school level where the development of number sense appears to have been taken for granted. Thus the main assumption is once the teachers are equipped with the necessary number sense skills they will be able to impart such skills to their mathematics learners and ultimately the proficiency as well as the performance of learners in mathematics could improve.

The study also takes into consideration that the emphasis on number sense as mental calculation, estimation even justification of numerical results are still relevant at the age of calculators and computers. That is, estimation and assessment of reasonable of answers has become even more important in a technological world because of the garbage in garbage out concept. Human judgment remains extremely important and the role played by human judgement cannot be compromised or be replaced by computers or calculators. However operations on number technology have greatly reduced the need for mental mathematics but more towards the plausibility of answers obtained through these new tools. Therefore the emphasis of number sense should not completely shift to modelling situations mathematically and interpreting the mathematical results in real life situations but should rather also pay attention to human judgement.

3.13 Summary

This chapter paid attention to the literature on number sense, definitions, feature or components, and training of different learner populations including both pre- and in-service mathematics teachers. The chapter adopted the definition by McIntosh et al.'s (1992) definition of number sense. The chapter also acknowledged that teaching strategies to develop number sense as well as the number sense curriculum could be developed by relying on key components that are constituent to the definition of number sense.

It is also envisaged that by employing the CRENS model which is derived from the Critical Theory, it is an ideal opportunity to come up with an eclectic model to identify means of incorporating the number sense training in the training of secondary mathematics teachers. Therefore this study anticipates that by adopting the ideas of Critical Theory and the CRENS model, teachers at secondary school level will possess improved number sense competencies that could ultimately improve their pedagogical practice and improve the number sense capabilities of their learners. Therefore the improved number sense and pedagogical practice will help the learners gain better mathematical proficiency leading to improved achievement in mathematics. The next chapter presents the methodology for the study.

CHAPTER FOUR
METHODOLOGY

4.1 Introduction

This chapter discusses the methodology used to collect and analyse the data for this study. The chapter presents the research design which is a mixed methods approach, as well as the justification for opting to take up the mixed methods approach for the purpose of collecting and analysing data in this study. The chapter further presents the sample for the study; the research instruments that were utilised to gather the data from participants.

A summary of the pilot study that was conducted, the data collection and analysis procedures in the pilot stage are also discussed in this chapter. The researcher also attempts in this chapter to explain the nature of the experimental research design utilised in this study. Finally, the chapter presents the ethical issues that were observed in this study, the validity and the reliability issues that were observed to ensure the trustworthiness of the results of this study.

4.2 Research design

Some writers (e.g. Berry and Otley 2004; Creswell 2009; Saunders, Lewis & Thornhill 2009; Neuman 2011) emphasise that it is important to initially question the research paradigm to be applied in conducting research because it substantially influences how one undertakes a study from the way of framing and understanding a phenomenon. Following this suggestion, two research paradigms (qualitative and quantitative) that were utilised are discussed below to enable a justification of the theoretical assumptions and fundamental beliefs underpinning a social research.

Thus the first part (4.2.1.) of this subsection attempts to justify the choice for the mixed approach. The second part (4.2.2.) of this section explains and justifies the experimental research design that was adopted to gather data from participants. It also explains and justifies the subsequent statistical tests carried out to analyse and create meaning from the data.

4.2.1 Research paradigm

A research paradigm is a set of fundamental assumptions and beliefs as to how the world is perceived which then serves as a thinking framework that guides the behaviour of the researcher (Jonker & Pennink, 2010). In order to gain an insight about the number sense comprehension of preservice secondary mathematics teachers, the study opted to follow a combined research paradigm; i.e. both quantitative and qualitative research methods were utilised. The quantitative and qualitative data are presented in chapters 5 and 6 respectively. Patton (2000) and Polit and Hungler (2007) argue that the choice of the research paradigm is more beneficial if the mixed paradigm is utilised as opposed to confining to a single paradigm, that it collects and allows the two data to complement each other and form a strong basis of findings.

4.2.1.1 Quantitative approaches

Moreover, literature (e.g. Loraine, 1998; Patton, 2000; Polit & Hungler, 2007) highlights the fact that quantitative research data focus on information that can be represented by means of numerical values such as frequencies, percentages, proportions and averages. Quantitative studies provide the data that can be expressed in numbers hence the name quantitative which is derived from the word quantity. Because the data are in a numeric form, statistical tests can be applied in making statements about the data. These include descriptive statistics like the mean, median, and standard deviation, but can also include inferential statistics like *t*-tests, analysis of variance ANOVAs, or multiple regression correlations (MRC) (Polit & Hungler, 2007). These too were utilised in this study (see sections 5.2 to 5.4).

Patton (2000) says the strength of the quantitative method is that by performing statistical analysis the researcher can obtain essential facts from research data, including preference trends, differences between groups, and demographics. In addition to this, multivariate statistics like the MRC or stepwise correlation regression break the data down even further and determine what factors could be attributed to differences between specific groups. Also, quantitative studies often employ automated means of collecting data such as surveys, but can also use other static methods such as examining preferences through two alternative, forced choice studies or examining error rates and time on task using competitive benchmarks. Quantitative studies' also have the advantage of providing the empiricist data that is descriptive and therefore is viewed to be credible.

Loraine (1998) contends that despite its weakness that it cannot enter into the world of the respondents and draw meaning form it, the use of quantitative data has gained favour in the sense that it orients itself to quantifiable means therefore it possesses proof grounded on the logical empiricism or positivism as conceptual frameworks. The average performances of pre-service teachers in the number sense pre-test and post-test will be presented as quantitative information and therefore the scores gathered were believed to possess empirical evidence. This was done to respond to the question: "How much number sense was possessed by preservice secondary mathematics teachers" (sub-research question 1 (RSQ1)).

Moreover, to respond to the question of how effective the number sense intervention programme was, the pre- and post-test results were paired and these were tested as illustrated in Figure 4.6. The correlations between the number sense results of preservice secondary mathematics teachers and their performance in mathematics were also presented using scatter plots and their respective correlation coefficients were calculated and relevant regression analysis was utilised in this study as argued from above. This was done to respond to the question of the relationship between number sense and the academic performance of pre-service secondary mathematics teachers.

4.2.1.2 Qualitative approaches

Data from qualitative studies describe the qualities or features of something. Qualitative descriptions are not easily reduced to numbers though this can be achieved by encoding process (Loraine, 1998). Contrary to quantitative research which requires the standardization of data collection to allow statistical comparison, qualitative research requires flexibility, allowing the researcher to respond to user data as it emerges during a session.

Thus, qualitative research usually takes the form of either some form of naturalistic observation such as ethnography or structured interviews (Loraine, 1998). In this case, a researcher observed and documented opinions in the responses, patterns as presented in the extracts of their answers, and without yet fully understanding what data will be meaningful, after gathering this data the researcher then analysed it to draw meaning from it.

Patton (2000) emphasises that qualitative research studies provide details about human behaviour, emotion, and personality characteristics that quantitative studies cannot match. Patton (2000) further states that the difference between quantitative and qualitative data analysis is that rather than performing a statistical analysis as in quantitative research, a qualitative researcher looks for trends in the data. When it comes to identifying trends, researchers look for statements that are identical across different research participants. Neuman (2011) explains that the rule of thumb is that hearing a statement from just one participant is an anecdote; from two, a coincidence; and hearing it from three makes it a trend. The trends thus identified can then direct the researcher’s decisions and conclusions. In this case the ideas presented by Neuman (2011) could be interpreted to imply that since these trends cannot be subjected to statistical analysis, they cannot be validated by performing statistical computations or testing calculating e.g. a *p*-value or an effect size as one could validate quantitative data.

Neuman (2011) also points out that the conceptual frameworks that utilise qualitative research such as interpretivists inspired by the theory of hermeneutics reject objectivism and a single truth as proposed in post-positivism. To understand the social world from the experiences and subjective meanings that people attach to it, interpretivist researchers favour to interact and to have a dialogue with the studied participants. Therefore they prefer to work with qualitative data which provides rich descriptions of social constructs. As opposed to generalisation or the nomothetic approach adopted by post positivist researchers, interpretivists use a narrative form of analysis to describe specifics and highly detailed accounts of a particular social reality being studied, which is termed the idiographic approach. Taking into account the idiographic approach, the qualitative data gathered would then be used lead to inductive approach by using the said narrative form of analysis.

Nonetheless despite its weakness that it cannot produce the empirical numerical data from which inferences are drawn, the study also employed the qualitative research paradigm as referred to fill the gap that could not be filled by the quantitative data, and therefore to allow for triangulation between the two data sets. During the Critical Theory intervention by means of the Critical Realistic Ethno Number Sense (CRENS) model, the pre-service secondary mathematics teachers' thoughts, responses and written work as presented by them before, during and after intervention were captured.

In utilising the qualitative data the researcher's quest was driven by the concern of how well as opposed to the question of how much as explained by Patton (2000). By assessing the confidence levels of the preservice mathematics teachers, the researcher anticipates that this evaluates their experiences and reasoning towards number sense and therefore describe the level number sense comprehension they possessed. The researcher thus also holds a belief that by capturing the thinking processes of preservice secondary mathematics teachers as they worked through the number sense items during their training, that would answer the question of 'How well?' which is at the centre of the qualitative research paradigm. Therefore, this can be seen as the advantage of the qualitative studies which therefore complements the quantitative data.

The researcher further concurs with Loraine (1998) who affirms that a high level of comprehension is achieved by means of approaching research from a qualitative point of view. According to Loraine (1998) the reason is that it allows the researcher to get into a deep conversation with the participants to gain a better understanding of the data as well as the nature of the participants. Qualitative approaches are also advantageous in that they allow the investigator to get a better understanding of the world of participants and therefore obtain results that are of high quality (Patton, 2000). The researcher therefore envisages that by utilising qualitative methods he will understand the participants better provided their responses are clear.

4.2.1.3 Justification of a mixed methods design

Informed by the Critical Theory the study also recognises that Critical Theory supports the use of mixed methods. For instance the issue of subject-object dualism is also advocated for by Critical Theory (Moreover, Venter et al., 2007; Kincheloe & McLaren, 1994); therefore, researchers informed by Critical Theory take into account that objects in both quantitative and qualitative research methods are socially shared, historically produced and general to a social group. From this it could be interpreted that Critical Theory supports the idea of applying a mixed methods approach when conducting research.

The idea of using a mixed approach has been advocated for by some researchers (e.g. Patton, 2000; Polit & Hungler 2007; MacMillan & Schumacher, 2006; Neuman, 2011). These researchers argue for the need to avoid conflict between the two kinds of data but rather to allow them to complement each other it becomes necessary to present them in two distinct chapters.

In as much as authors (e.g. Polit & Hungler, 2007; MacMillan & Schumacher; 2006) assert that there is a need to distinguish between qualitative and quantitative data, there is a precaution that has to be taken to ensure that the splitting of two methods does not compromise the quality of the data obtained. Polit and Hungler (2007) as well as MacMillan and Schumacher (2006) assert that the two research paradigms (qualitative and quantitative) do not exist to conflict each other but rather to complement each other. In addition to the above, in this study it should be understood that the two research paradigms were used to collectively yield responses to the research question and to offer means of triangulation to provide a better outcome for the study. It is against this background that the two chapters 5 and 6 keep on referring to each other to enhance cohesion between the qualitative and quantitative data.

To understand how the mixed methods approach was utilised in this study, it was deemed necessary to illustrate it in a form of a diagram (Figure 4.1). Therefore, theoretically, the study proposes to utilise the following model in collecting and analysing both quantitative and qualitative.

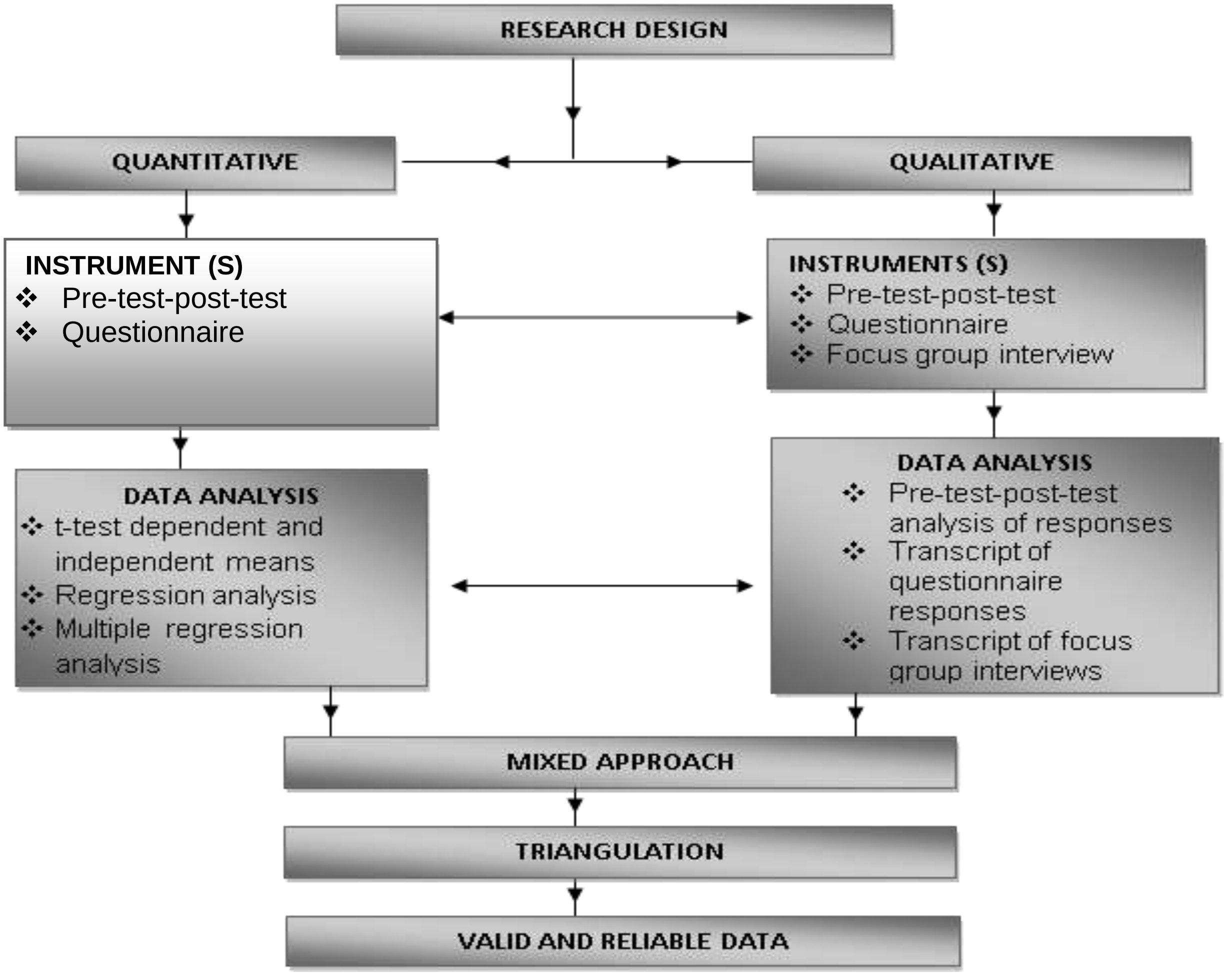

Figure 4.1: The mixed methods approach utilised in this study

4.2.2 Pre-test-post-test control group mixed methods approach design

The researcher took into account that this study adopted an experimental design where the emphasis is on testing the cause and effect relationships between and amongst the variables (MacMillan & Schumacher, 2006; Polit & Hungler, 2007; Lorraine, 1998; Patton, 2000). The researcher also concurs with the view of Lorraine (1998) and Patton (2000) that the classic experimental design specifies an experimental group and a control group. The independent variable is administered to the experimental group and not to the control group, and both groups are measured on the same dependent variable.

The researcher minimised threats to validity by ensuring that the experiment had control, randomization, and manipulation of variables to ensure reliable, valid and credible data. A pre-test-post-test control group experimental design is classified by literature as a strong research design. A research design is considered to be strong if it controls for the influence of confounding extraneous variables (MacMillan & Schumacher, 2006; Lorraine, 1998). The most important of these control techniques was random assignment of participants to both the control and the experimental group.

Based on the foregoing arguments it can be argued that the pre-test-post-test control group design controls for most if not all of the standard threats to external and internal validity. Therefore, the researcher holds the view that by employing the pre-test-post-test control group experimental design, other threats to validity such as differential attrition, maturation or history may also be overcome.

Taking into consideration the nature of pre-test-post-test control group research design, Figure 4.2 below summarise the design for this study.

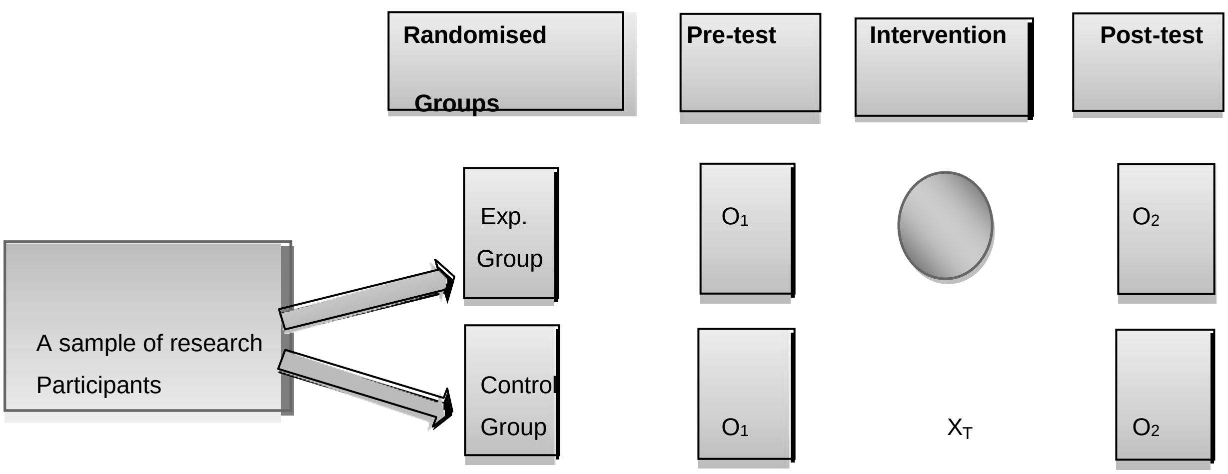

Figure 4.2: A diagrammatical representation of the pre-test-post-test experimental design

In Figure 4.2, O_1 and O_2 refer to the pre-test or pre-experimental observation and the post-test or post-experimental observation, whereas, X_T refers to the experimental conditions. The researcher organised the data analysis and therefore drawing conclusions from carrying out the statistical analysis (t-tests) as illustrated in Figure

4.2. This was carried out to ensure that the results obtained were free from bias.

4.3 Population

The population of this study included all first year pre-service secondary mathematics teachers pursuing the Bachelor of Education (B.Ed.) programme at the University of Namibia. Accordingly this study had a population of 60 students and 80 participants at the central and northern campuses respectively, yielding a total population of 140 students.

4.4 Sample

As it appeared from the previous section, pre-service first year secondary mathematics teachers being considered were located at two campuses of the University of Namibia. One campus is in the northern part of the country, the other is at the central part, in the capital city, Windhoek.

Since both groups of students study the same curriculum and are admitted and taught under similar conditions and following the same curriculum the researcher found it convenient to utilise the students from the campus situated in the northern part of the country for the purposes of the pilot study only and the students from the central campus were therefore utilised for the main study.

This was done in the light of the fact that when following a pilot study the researcher selects participants that contain typical characteristics present in the population of interest recommended by MacMillan and Schumacher (2006). Thus this study therefore adopted the view that the participants in the pilot study should possess typical attributes of the population of interest to the researcher. Essentially; all participants were asked to voluntarily sign up for participation in this study with informed consent. A convenient sample of sixty (60) pre-service secondary mathematics teachers was selected, i.e. participants were asked to volunteer in taking part in the study and the response rate was 100%. The participants of the sample were then further randomly assigned two equal groups: the experimental and the control group.

All 60 sampled participants took part in both pre-and post-tests. Moreover, ten participants from the experimental group were randomly selected for in-depth interviews; this was done because the researcher sought to compare the number sense comprehension, experiences and perceptions of the preservice secondary mathematics teachers before and after the intervention programme. The questionnaire was administered to all thirty (30) participants from the experimental group. This sought to assess the experiences and perceptions of preservice mathematics teachers about the number sense intervention programme. Therefore, the in-depth interviews and questionnaire were not administered to the control group. This is because these instruments only targeted to understand the views of participants on the number sense intervention they went.

4.5 Research instruments

The study used a variety of research instruments in order to draw the data from the sampled participants. This was done in the light of the recommendation by MacMillan and Schumacher (2006) that the quality of the data is enhanced if obtained with triangulated tools. Thus this section presents different data collection instruments that were utilised in drawing the data from the participants.

4.5.1 A Five-Tier Number sense test (FTNST)

The use of number sense tests has been used designed by Yang, 1997 and has been utilised by other researchers (e.g. Courtney-Clarke, 2012 McIntosh et.al 1992; Joyner, 2004; Peck & Connell, 2011; Yang, 1997) and others. This study designed a unique test based on the ideas of the fore going authors. The uniqueness of the test used in this study is that it consists of five tiers, hence the name “Five-Tier-Number Sense Test”.

The name “Five-Tier-Number Sense Test” therefore refers to the fact that the test consisted of five tiers. To gain a better view of how the items looked like Table 4.1 shows an example of a question in the Five Tier Number Sense Test.

Table 4. 1: An example of a question in the number sense test

Question (Tier 1)	Think through (Tier 2)	Answers (proficiency) (Tier 3)	Reason (Reasoning) (Tier 4)	Confidence level (Tier 5)
Which statement makes more numerical sense on what the relationship between pi and $\frac{22}{7}$ is?		a) $\pi = \frac{22}{7}$ b) $\pi > \frac{22}{7}$ c) $\frac{22}{7} >>> \pi$ d) $\pi \approx \frac{22}{7}$	a) π is the same as $\frac{22}{7}$ but π is a decimal and $\frac{22}{7}$ is a common fraction. b) $\pi = 3.1414$ but $\frac{22}{7} = 3.142857$ c) $\frac{22}{7} = 3.142857$ $\pi = 3.142$ as result of $\frac{22}{7} = 3.142857$ is more larger than $\frac{22}{7} = 3.142857$ d) The values of $\frac{22}{7}$ and π only differ by a thousandth therefore $\frac{22}{7} \approx \pi$.	a) Very confident b) Confident c) Not sure d) Not confident

Ensuing to presentation of the five tiers as illustrated above here is a description of the 5 tiers of the number sense test:

4.5.1.1 The question tier

This tier carries the question that the preservice teachers are required to work out. As illustrated in the diagram above the question part carries out the information and it demands the desired competency that the respondent is expected to demonstrate to come up with the correct answer.

4.5.1.2 The thinking thorough tier

This tier was deemed necessary by the researcher informed from the result of the pilot study. In this study the focus was not only on assessing whether the respondents obtained the correct answers, but also on the thinking that led to the choice of each the preferred answer. In this section the researcher captured the working through and ultimately the qualitative data in the think through column. This capturing of the working through was done to gain a better understanding of the thinking perspective of the pre-service secondary mathematics teachers being studied, this provided more qualitative information.

4.5.1.3 The answer tier

This tier consisted of the correct answer as well as the three distractors as suggested by literature (Peck & Connell, 2011; Yang, 1997) that the use of distractors should take into account of the plausibility concept. I.e., according to the foregoing literature the concept of plausibility is the fact that distractors should make a lot of sense to reasonably hide the correct answer. Therefore in constructing the test items the researcher also took into account that the alternative answers accompanying the correct answer made reasonable sense to hide the correct answer.

4.5.1.4 The reason tier

Researchers (e.g. Polit & Hungler 2007; Lorraine, 199; Patton, 2000) note that the testing instruments that are of a multiple choice nature are prone to guess work. Therefore to avoid this pitfall the researcher introduced the reasoning tier. Some researchers (e.g. Courtney-Clarke and Wessels, 2014; Ball, 2000) recommend that since the choice of a number sense test consisting of multiple choice test is prone to guess work it needs to be designed with distractors of a high quality.

However, in addition to the distractors being plausible the researcher deemed it necessary to include the think through section. The researcher therefore anticipated that the addition of think through does not only help in mitigating the effect of guess work and therefore gets a broader and fair view to assess the participants' way of thinking that motivated the student to make the choice of the answer from all four alternatives.

4.5.1.5 The confidence level tier

This section aimed at assessing how confident the respondents were when selecting that particular answer. The section also assessed whether the respondents chose their answers with self-confidence. The researcher also studied, compared and contrasted the confidence of the preservice teachers in the pre- and post-tests in the experimental group.

The expression Five-Tier-Number Sense Test bears a resemblance to the standardised number sense tiers as adopted from (Joyner, 2004; Peck and Connell, 2011; Yang, 1997). The number sense test was designed based on the framework for considering the number sense as outlined by McIntosh et.al. (1992) from which the definition of number sense used in this study was derived as (indicated in section 3.1).

During the piloting when the inclusion of suggestions from validators was done the pre-test-post-test was tested for split half-reliability as well as the reliability of the Kuder-Richardson coefficient in accordance with the guidelines specified by MacMillan and Schumacher (2006). The pre-test-post-test Kuder-Richardson coefficient reliability was found to be 0.762, therefore the researcher reasoned that the test would measure what it is supposed to measure consistently.

The number sense sections (components) were derived from the key components of number sense namely: a) *the meaning and size of numbers both rational and irrational numbers; b) equivalence of numbers both rational and irrational numbers; c) meaning and effects of operations; d) counting and computational strategies as well as e) the estimation without calculating.*

During the data collection a pre-test-post-test a Five-Tier Number Sense Test (FTNST) was administered to the preservice secondary mathematics teachers for both the experimental and control group (see appendix F). The FTNST included both rational numbers such as whole number, fraction and decimal items, the four basic operations as well as some concepts of irrational numbers such as pi, square roots of prime numbers and the concept of infinity that are related to the number sense contents related to the secondary mathematics curriculum as well as the teacher training of preservice secondary mathematics teachers.

The test was validated by three colleagues and two supervisors believed to possess the expertise in the field of number sense; the suggestions made by the experts were considered. Hence some statements were rephrased to ensure that they made sense from the pre-service teachers' point of view as suggested by the validators. Additionally, some refinements were made after the piloting phase and these are discussed in the pilot study section of this chapter. Issues of validity and reliability were also considered and these were equally summarised in the reliability and validity sections.

After the pre-test phase, the experimental group was given a ten week number sense training intervention (based on the CRENS model anchored in Critical Theory). Post-testing followed the intervention for both the control and experimental groups. The participants for the control group were however exempted from the number sense training intervention programme and all its activities, for comparison purposes.

4.5.2 Focus group interviews

A focus group interview could be defined as a group of interacting individuals having some common interest or characteristics, brought together by a moderator or interviewer, who uses the group and its interaction as a way to gain information about a specific or focused issue, (Kitzinger, 2004; Race, Hotch & Parker, 2004; Lloyd, 2006). The role of focus group in collecting data has been acknowledged in the literature (e.g. Race, Hotch & Parker, 2004; Kitzinger, 2004; Powell, 2006) as a means for researchers to discuss and comment from personal experience, on the topic that is the subject of the research.

Different authors also argue that focus groups are often perceived as forms of group interviewing but it is important to distinguish between the two. Group interviewing involves interviewing a number of people at the same time, the emphasis being on questions and responses between the researcher and participants. Focus groups however rely on interaction within the group based on topics that are supplied by the researcher, (Merton & Kendall, 2007; Race et al., 2004; White & Thomson 2005; Morgan, 2004). Merton and Kendall (2007) also add that there is a difference between group interviews and focus group interviews, in that focused group interviews set the parameters for focus group development. This is in terms of ensuring that participants have a specific experience of or opinions about the topic under investigation ask each other questions, as well as to re-evaluate and reconsider their own understandings of their specific experiences.

Thus it was necessary to utilize the focus group interviews in the light of the argument that they allow for interaction among pre-service secondary mathematics teachers, that an explicit interview guide was used, and that the subjective experiences of participants were explored in relation to predetermined research questions. Based on this the researcher therefore argues that the preservice secondary mathematics teachers have their subjective experiences with regards to the issue of number sense and these will be assessed by the researcher through focus groups.

Interestingly, Morgan and Krueger (2003) and Morgan (2008) identify the interaction among the respondents as the crucial feature of focus groups because the interaction between participants highlights their view of the world, the language they use about an issue and their values and beliefs about a situation. Interaction also enables participants to

The focus group interviews were therefore preferred because they allow the participants to interact not only with the researcher but also with each other.

Researchers (e.g. Race, Hotch & Parker, 2004; White & Thomson, 2009; Powell, Single & Lloyd, 2006) also indicate that the main purpose of focus group research is to draw upon respondents' attitudes, feelings, beliefs, experiences and reactions in a way in which it would not be feasible using other methods, for example observation, one-to-one interviewing, or questionnaire. These attitudes, feelings and beliefs may be partially independent of a group or its social setting, but are more likely to be revealed via the social gathering and the interaction which being in a focus group entails.

This study used a pre-test-post-test focus group interview schedule as another instrument of drawing data from respondents. The pre-test, focus group interview (see appendix D) was administered to the experimental group only. Five (5) interviewees were randomly selected using a random numbers. After the intervention and the post-test number sense test the experimental group students were subjected to the same focus group interview since it aimed at assessing the views, experiences and perceptions of preservice secondary mathematics teachers on the intervention.

The interview procedure was to utilise a displayed series of number sense items that were already assessed in the test as a means of drawing out responses and probing for further information to enhance a deep comprehension of the thoughts of participants built on the five components of number sense. The interview questions were not changed based on the responses of the subjects to ensure the consistency among the other subsequent interviews.

Some literature (e.g. MacIntosh, 2011; Lankshear, 2003; Krueger, 2008) asserts that focused groups have several advantages over other methods of collecting information. For instance compared to individual interviews, which aim to obtain individual attitudes, beliefs and feelings, focus groups elicit a multiplicity of views and emotional processes within a group context. The individual interview is easier for the researcher to control than a focus group in which participants may take the initiative.

Compared to observation, a focus group enables the researcher to gain a larger amount of information in a shorter period of time. Observational methods tend to depend on waiting for things to happen, whereas the researcher follows an interview guide in a focus group. In this sense focus groups are not natural but organised events (Morgan, 2008; Lankshear, 2003).

Another benefit is that focus groups elicit information in a way which allows researchers to find out why an issue is salient, as well as what is salient about it (Morgan, 2008). As a result, the gap between what people say and what they do can be better understood (Lankshear, 2003). If multiple understandings and meanings are revealed by participants, multiple explanations of their manners and attitudes will be more readily articulated. The researcher in this study aimed at understanding these multiple understandings as well as filling the knowledge gaps that might exist between what the preservice secondary mathematics teachers say and what they can do about number sense.

Although focus group interviews have several advantages, the researcher also acknowledged the fact that with all research methods there are limitations therefore focus group interviews could also have their weaknesses. For instance one shortcoming of focus group interviews is that the researcher has less control over the data produced than in either quantitative studies or one-to-one interviewing (Morgan, 2008). Thus the researcher attempted to overcome this by moderating and validating the instrument with the respondents during the pilot study phase to ensure the focused group interview instruments measured what they are supposed to measure.

Some weaknesses of research methods can be unavoidable and this is not peculiar to the focus group interview approach. For instance, the researcher had to allow participants to talk to each other, ask questions and express doubts and opinions, while having very little control over the interaction other than generally keeping participants focused on the topic. By its nature focus group questions are open ended and cannot be entirely predetermined (Morgan & Krueger, 2003; Lankshear, 2003). Therefore, at some intervals in this study the researcher had to guide the participants to ensure that they made sense of the questions and that they were presented with so that the discussion is within the desired context.

4.5.3 Questionnaire

A questionnaire may be defined as a research instrument consisting of a series of questions and other prompts for the purpose of gathering information from respondents, (Gillham, 2008; Foddy, 2004). Questionnaires are often designed for generating both quantitative and qualitative data for statistical analysis of the responses (Mellenberg, 2008). In this study a questionnaire that consisted of both open ended and closed ended questions was administered on the pre-post-intervention basis to the experimental group only, for the reasons explained in the sampling section of this study. This was done to assess and evaluate how the participants' views, perceptions and experiences changed before and after the number sense intervention.

In this study the researcher took advantage of the fact that potentially large information can be collected from a large portion of a group as this is not the case with other methods of data collection such as one-to-one interviews (Munn & Drever, 2004). Therefore, the questionnaire in this case ~~it~~ was used to generate another pool of data that could be triangulated with focused group interview results. Oppenheim (2000) reasons that the potential of questionnaires in creating a larger pool of data is not often realized, as returns from questionnaires are usually low. However the researcher addressed this weakness by ensuring that the return rates improved dramatically by delivering the questionnaires to respondents by hand.

Thus all participants that were issued with the questionnaires returned them. If the questionnaires are standardized so it is not possible to explain any points in the questions that participants might misinterpret (Leung, 2001). This was partially dealt with by piloting the questions on a small group of students during the pilot study the amendments that were made are explained on the pilot study section of this chapter.

Gillham (2008) and Foddy (1994) assert that respondents may not be willing to answer the questions. They might not wish to reveal the information or they might think that they will not benefit from responding perhaps even be penalised by giving their real opinion (Foddy, 1994). In response to this the researcher explained to the respondents that the information being collected could be beneficial to them and might not by any means affect their formal academic marks. The respondents were thus asked to reply honestly and were also told that if their response was negative it could just be as useful as a positive opinion. Moreover, anonymity was guaranteed by the use of student numbers as opposed to the real names that could reveal the identity of the respondents.

4.5.4 Document analysis

Document analysis is a form of qualitative research in which documents are interpreted by the researcher to give voice and meaning around an assessment topic Platt (2001). Payne and Payne (2004) describe the role of the document analysis method as the techniques used to categorise, investigate, interpret and identify the attributes of a certain variable. This study also employed the method of document analysis in which scripts of students to the responses of test items as well as to the response of CRENS activities teachers were studied to generate data to compare with the number sense of preservice secondary mathematics teachers.

Payne and Payne (2004) assert that one of the key advantages in conducting documentary research is that the researcher can by using documents eliminate the effect that, as an individual, might have on a person or situation where research is conducted research (“the researcher effect”). This means in effect the researcher has on a situation or subject may be partly due to the knowledge that s/he is there as a researcher. That is, respondents could also be affected by how the researchers conduct themselves and how they are perceived as participants might affect their academic performance if the researcher had prepared research tools to assess their performance in mathematics. In this case the researcher opted to utilise the scripts to ensure that there is no subjectivity in the assessment of the preservice mathematics teachers’ achievement. The researcher deemed it also fit to use the marks from their first semester of their first year module (Basic Mathematics for Teachers) since they lack “the researcher effect” and therefore could be considered subjective. Nevertheless, the shortcoming of documents is that they are usually not designed with research in mind. The information recorded may be eccentric or incomplete (Platt, 2001). The researcher in response to this therefore made sure that the information collected was that related to the academic performance of preservice secondary mathematics teachers in their official assessment tasks only but not on every assessment task.

Platt (2001) further argues on the disadvantages of documents as source of data that they get misfiled, left on people's desks for long periods or simply just do not get fully completed at all therefore there may be information that is available for one period of time and not another. All of this will create gaps in data (missing data) as well as coding difficulties. The researcher in this study therefore collected the scripts as each number sense assessment task was done to ensure that no information was lost and to avoid the gaps in the data.

4.6 Pilot Study

The pilot study was conducted among ten participants out of all the first year pre-service secondary mathematics teachers from the campus situated at the northern part of Namibia. However, due to the fact that they are at the same level and their learning curriculum was the same as that of participants from the main campus, these were deemed fit for participating in the main study.

These participants were conveniently selected and none of them took part in the main study. All instruments earmarked for data collection procedures were administered under similar conditions to the main data. Based on the pilot study several changes were done; i.e. with necessary alterations depending on the results of the pilot study; thus the rest of this section presents the changes that were made to the research instruments depending on the results of the pilot study.

The researcher realised a need to capture the thinking through tier that led to the choice of answers given by the pre-service secondary mathematics teachers, which is part of qualitative data. A tier that was meant to capture the thinking of pre-service secondary mathematics teachers for the number sense pre-test-post-test was introduced. Therefore, the test was revised and this fifth tier was introduced and consequently the name changed from a “Four Tier Number Sense Test” to a “Five Tier Number Sense Test”.

During the piloting of the intervention the researcher noticed that there were a lot of qualitative data being lost as the teaching focused more on verbalised data such as discussions and excluded the thinking processes of the preservice teachers. The researcher additionally provided participants with worksheets on the Number Sense Training Manual (NSTM) where the preservice teachers recorded their thinking processes as they worked through the activities of the intervention. These thinking processes were also presented as part of the qualitative data (also see section 6.2.7). The initially allocated time of 1 hour 30 minutes to do each of the pre- and the post-tests was increased to 2 hours. Questions deemed to be ambiguous from the perspective of the participants were also revised to make them clearer.

4.7 The nature of CRENS Intervention

At the beginning of the second semester a ten week number sense intervention was given to the participants in the experimental group only. This was not done as an extracurricular activity however it was a normal lesson with activities number sense and CRENS related during the normal syllabus coverage.The teaching therefore covered the whole syllabi the only difference it had with the control group was that the activities used for this lesson were based on the Critical Realistic Ethno Number Sense (CRENS). This was also done to avoid contaminating the sample as the other group was not aware of the methodology utilised in the control group lessons.

The activities used were realistic, i.e., representing the real life situations, and were within the cultural context of Namibia hence the ethno term. They were also critical in since they probed for the learners to think beyond providing the correct answer. The CRENS activities were also considered to be practical activities as they required students to work with each other and individually. Figure 4.3 illustrates nature of the CRENS activities used during the intervention.

The diagram shows a traditional fence of a kraal in the shape of a circle. Indicate how far the fence of a homestead had gone if it covered two thirds of the circle in an anti-clockwise direction from the starting point indicated in the diagram.

Figure 4.3: An example of a CRENS activity used in the intervention

From the foregoing illustration the CRENS framework comes in the several ways. For instance the critical part of the question is due to the fact that the students were required to think from reality to abstract thinking as they are required to produce a diagram of a circle and shade off two thirds of that circle. The other issue that reveals the critical part has to do with the fact that they have to go in an anti-clock wise direction the direction was also purposefully chosen to provoke the students' thinking.

The realistic part of this activity is that it is a real life situation, thus the issue of bringing a real object that they see will bring closer the meaning and competencies of number sense. Namibia is a developing country where agriculture and in particular keeping cattle is a practise of every culture; therefore by relating the activity to keeping cattle it brings the ethno part. The issue of number sense is also exhibited in the previous activity in the sense that the preservice teachers had to use number sense to work out that 66% is almost two thirds and therefore the distance they had to divide the whole circumference in 3 parts and walk off two out of these three parts in an anti-clockwise direction.

The issue of LCE is not ignored by the activity. The fact that preservice mathematics teachers had to work independently and share their findings with their classmates' means they played an active role in their own learning. Additionally the researcher had to play a facilitative role and the students were interacting with each other freely and with the researcher it affirms that leaner centred teaching took place. With regards to the explanations above, the researcher therefore attempted to ensure that the activities presented to the preservice secondary mathematics teachers during their number sense training portrayed elements of both the CRENS and LCE similar to those indicated in the activity above.

Also, as mentioned in chapter 3 (3.9) a Critical Theory intervention based on the Critical Realistic Ethno number Sense (CRENS) model which is derived from Critical Theory as discussed in chapter 2 (2.8) was implemented following a Hypothetical Learning Trajectory (HLT) that was built into a normal university module on Basic Mathematics for Teachers.

A three phase HLT was utilised to facilitate the Critical Theory intervention with its CRENS framework. The intervention programme endeavoured to facilitate the number sense training of preservice secondary mathematics teachers following recommendations of Clements and Sarama (2009).

More explicitly, the Critical Theory intervention followed the Hypothetical Learning Trajectory (HLT) with three components: *a learning goal or domain* (in this particular case the components of number sense), *a set of the learning activities*, based on CRENS), as referred to in chapter 3 (3.9) and *the hypothesized learning process*, and the CRENS framework.

The researcher opted to carry out a Critical Theory intervention therefore enacted 10 one-hour teaching sessions spread over 10 weeks during the main study. The sequence of training followed the five components of number sense. This marked the implementation phase of the intervention based on CRENS framework for the experimental group. Table 4.3 illustrates the HLT that was utilised to conduct the number sense intervention.

Before presenting Table 4.3 it is vital to recap the questions of the study to gain a better understanding of how they relate to the HLT that was utilised and to obtain a better view of the role of the HLT in this study.

Table 4. 2: A recap of questions addressed by the study

Main Question (MQ)	How can a Critical Theory intervention be utilised to enhance the Namibian first year pre-service secondary teachers' competencies in number sense?
Sub-Question 1 (SQ1)	What were the pre-intervention levels of number sense comprehension of first year preservice secondary mathematics teachers?
Sub-Question 2 (SQ2)	What was the relationship between the number sense of preservice secondary mathematics teachers and their academic performance in mathematics?
Sub-Question 3 (SQ3)	What was the impact of a Critical Theory intervention programme on the development of number sense of the first year pre-service secondary mathematics teachers?

Having observed the research questions, Table 4.3 links the HLT domains, the research questions, the research instruments, the learning activities, the theoretical framework developed and the statistical tests used in the study.

Table 4.3: The proposed HLT for developing the number sense of preservice secondary mathematics teachers in Namibia

Components of number sense	**Supporting Theories or paradigms**	**Intervention**	**Section in literature review**	**Pre-testing methods**	**Research question addressed**	**Post-testing Instruments**	**Statistical tests/Data analysis techniques**
1. Understanding the meaning of number sense. 2. Understanding meaning and size of numbers, 3.Uderstanding equivalence with numbers, 4. Understanding meaning and effects of operations, 5. Understanding counting & computation strategies,	1. LCE (ASEI-PDSI) 2. CRENS	**Activity 1** Introduction to the meaning of number sense. **Activity 2** Number sense strands. Worksheet on the **Activity 3** understanding meaning and size of numbers, **Activity 4** Questions on the understanding of equivalence	**3.5** **3.7**	Pre-test Questionnaire Interview	**SQ1**		Descriptive statistics
6.Understanding estimation without necessarily carrying out calculations;		**Activity 5** A quiz on understanding meaning & effects of operations, **Activity 6** Questions Understanding counting & computation strategies, **Activity 7** Understanding estimation, **Activity 8** Wrap-up: Students come up with their own problems and pose them to their classmates.	**3.7**	Document analysis Pre-test	**SQ2**	Post-test Questionnaire Interview	Correlation analysis Regression analysis Multiple regression analysis Multiple analysis of variance Z-test for dependent samples
		Activity 9 Revision: Students pose any number sense related question. **Activity 10** Miscellaneous activities	**3.8** **3.9**	Pre-test	**SQ3**	Post-test Interview Questionnaire	z-tests for dependent samples z-tests for independent samples
			3.6		**MQ**	Lesson observation	Qualitative data analysis methods

Table 4.3 is an attempt to only explain how the HLT which is a teaching experiment links with the instruments, research questions, supporting theories as well as the literature review and the data analysis in this study. Thus the hypotheses and their

related statistical tests that were utilised are presented on the data analysis section of this chapter. Furthermore, Table 4.3 indicates the components of number sense, the supporting theories to the teaching experiment (the HLT), and a series of activities that were used to conduct the practical number sense activities (PNSA) that were used in the intervention programme. The HLT assumes the number sense components can be achieved by carrying out a ten (10) week number sense training based on practical number sense activities as illustrated in the table.

The number sense training draws ideas from the CRENS model initially developed. Table 4.3 also shows the data collection instruments such as the Five Tier Number Sense Test (FTNST), the questionnaire and the focus group interview prior and after the intervention. The number sense training intervention programme was given to all 30 students that were randomly assigned to the experimental group at the beginning of the semester. The training followed the stipulations for normal half a semester module for the university, and was timetabled accordingly.

In general the experimental group in the first lesson were asked to indicate why they need to have number sense and why it was so important to apply critical nature, why it is also important for Critical theory to play a role in their training, and in the society. During the consolidation stage, the idea that is important to conscientise preservice teachers that often numbers can be abused, inflated or misinterpreted to justify wrong decisions with negative consequences to society. Critical citizenship requires a critical consumption of numbers. Even democracy, is often defined in terms of number of votes. A constituency-based electoral system can therefore be a disadvantaged compared to a proportional representation one.

4.8 Data collection procedures

The data collection procedure followed in this study occurred in three phases:

4.8.1 Pre-test data

At the beginning of the semester, the Five Tier Number Sense Test (FTNST) was administered to all participants just after the information sharing session which included participants from both the comparison and the experimental group. The rules and instructions for the test were explained prior to the testing procedure. The pre-test-post-test, in-depth interviews and questionnaires were also administered to the experimental group participants as explained in section 4.4.

4.8.2 Intervention as a method of gathering data

During the semester the intervention was given to the participants in the experimental group only and there after the post-test as explained in the next subsection. During the intervention the worksheets completed by the students were also collected at the end of every lesson for further data analysis. It should be noted that the intervention took place during the normal lessons only that the group was taught with a different method.

4.8.3 Post-test data

At the end of the semester the testing procedure was repeated to the same participants as in the initial data collection process at the beginning of the semester. The pre-test results (numerical test results, test scripts, focus group interviews responses and responses to questionnaires) were compared with the post-test results these comparisons and their hypotheses are summarised and tested in the data analysis section as well as in detail in the next two chapters (chapters 5 and 6).

To sum up the procedure of the three phase data collection procedure that took place, Figure 4.4 illustrates a brief summary:

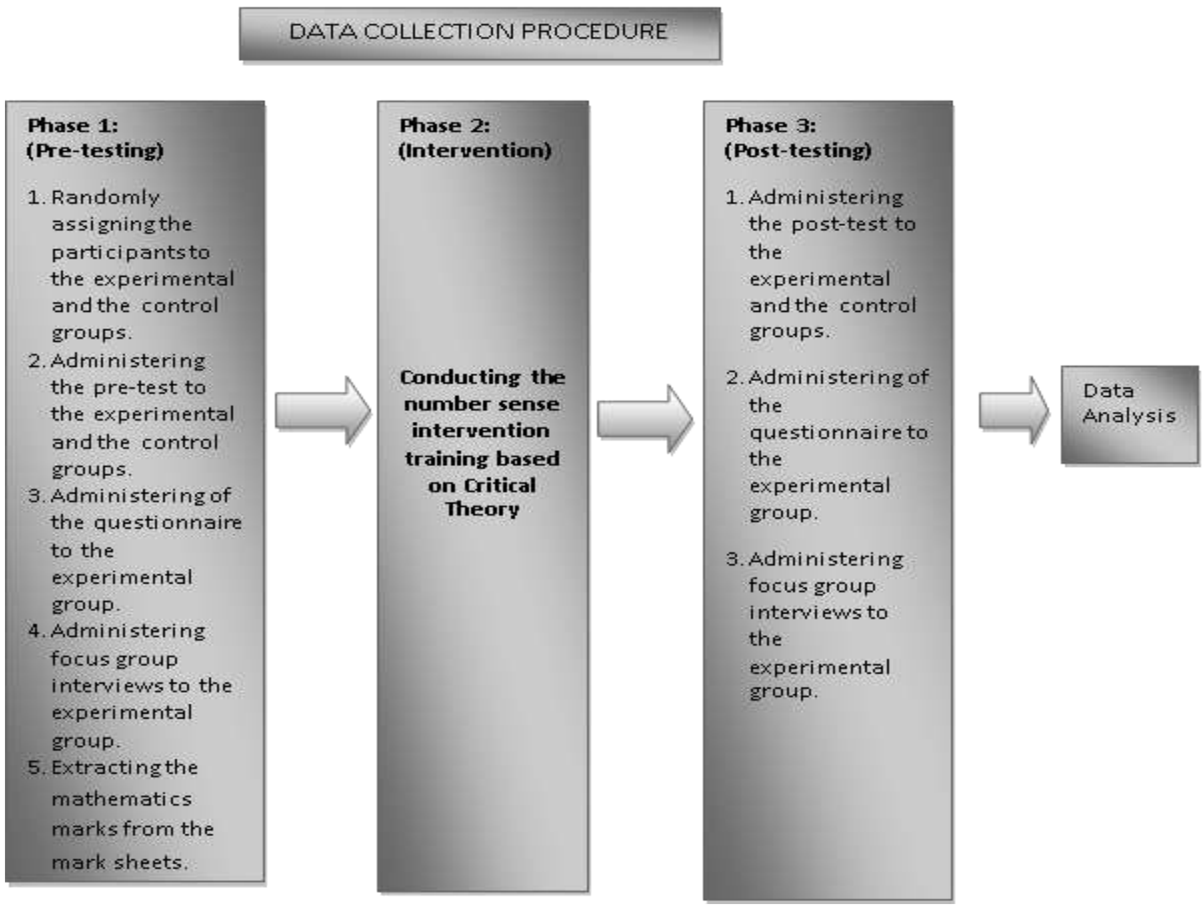

Figure 4.4: A summary of the data collection procedures

4.9. Data analysis

To prevent the qualitative and quantitative data from compromising each other, the researcher deemed it necessary to present the two in separate chapters (chapters 5 and 6).

4.9.1 Quantitative data

This section presents the methods that were utilised to analyse the quantitative data. The section thus is divided into sub-topics such as: the level of number sense comprehension, the relationship between the number academic performance in mathematics as well as the impact of a Critical Theory intervention. These sections as well as the qualitative data attempt to give answers to the three research sub-questions questions.

4.9.1.1 The level of number sense comprehension

In an attempt to assess and evaluate the level of number sense comprehension Greeno (2001); Hope (2008); Howden (2009); Reys & McIntosh (1992) & Reys & Yang (2008) argue that the measure of measurement of the number sense comprehension should rely on both the quantitative and qualitative data. It is clear therefore from the foregoing literature that to comprehensively assess the number sense grasp both quantitative and qualitative data is important.

Therefore, this sub section explains how the quantitative data related to the first sub question: "What was the level of number sense comprehension of first year preservice secondary mathematics teachers?" was analysed. In order to determine the current levels of number sense McIntosh et.al. (1992), grouped the levels of number sense into four levels depending on the percentage scores in a standardised number sense test. These same levels or categories of number sense proficiency were also proposed by Reys and McIntosh (1992) and refined by Markovits and Sowder (2004):

- ❖ **Advanced** *(75 % and above);*
- ❖ **Proficient** *(60 - 74 %);*
- ❖ **Basic** *(50-59 %);*
- ❖ **Below Basic** *(0 - 49 %).*

In addition to the fore going levels, (Reys and Yang, 2008; Sowder, 2002; Markovits and Sowder, 2004; and Yang, 2002) recommend a new method of assessing number sense using a rubric could aid the process of qualifying the number sense comprehension as it adds a qualitative value to the process of measuring number sense. According to the rubric for assessing the number sense comprehension it should consist of levels that are ordinal attempting to classify the number sense of learners, each level is then interpreted to attach a meaning to it (see appendix G for the rubric).

4.9.1.2 The relationship between the number sense and academic performance in mathematics

Using Statistical Package for Social Science (SPSS) analysis, regression analysis was performed to study the relationship that existed between number sense of preservice secondary mathematics teachers and their academic performance in mathematics being their first semester results of the core module Basic Mathematics for Teachers. This was done in the quest for the answer to the second research sub-question for this study: What is the relationship between the number sense of preservice secondary mathematics teachers and their academic performance in mathematics?

The study tested statistical significance of the relationship between the number sense of first year pre-service mathematics teachers and their academic performance in mathematics. These were all tested at 95% level of significance articulating to a value of $\alpha = 0.05$ on the hypotheses that tested the existence of the relationship between number sense and academic performance in mathematics. The measures on effect size by using the coefficient of determination were also worked out and interpreted to assess how much of the variations on the dependent variable academic performance were attributable to the independent variable number sense.

The results on the degree of relationship as hypothesised are presented in the next chapter where quantitative data is presented in section. The results on the degree of relationship are presented in the next chapter (see section 5.4). The independent variables of number sense that best predicted (best predictor variables) the dependent variable academic performance in mathematics were also identified as presented and discussed in section 5.4. Multiple linear regression and its subsequent model of predicting academic performance using these predictor variables was developed.

In addition to the tests of significance, another comparison between the number sense of preservice secondary mathematics teachers and their academic performance in mathematics was carried out using descriptive statistics. I.e. the measures of dispersion for the two variables (number sense and academic performance) were presented using a box-and-whisker plot. This was done to compare the two variables fairly well. These data presentations and their interpretation could be obtained in detail in section 5.4.

4.9.1.3 The impact of a Critical Theory intervention

This section presents the quantitative data regarding the impact of the intervention on the number sense of preservice mathematics secondary teachers based on the CRENS model. This section therefore undertakes to provide the answer to the research question: What was the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers? Consequent to the question above a series of *t*-tests for both dependent and independent samples was carried out to test for significance of differences between the control and the experimental groups on their respective hypotheses (see section 5.5).

In the light of the above, the following diagram indicates a series of *t*-tests for statistical significance on the differences of the means that were carried out in section 5.4.

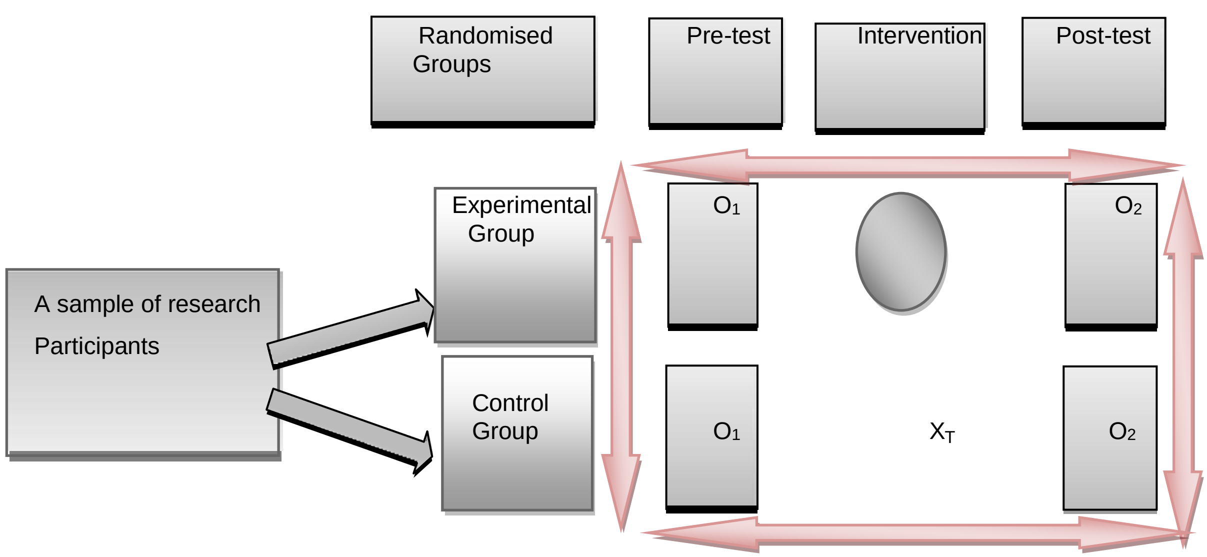

Figure 4.5: A diagrammatical representation of the tests on hypotheses.

Figure 4.5 illustrates that a series of tests for the four hypotheses that were carried out as represented by the arrows (see section 5.5). This series was employed to free the study from the threats to both the internal and external validity in this manner to improve the generalizability of the results to the population of the study. The study also used Cohen's *d* to assess the effect size of the Critical Theory intervention.

4.9.2 Qualitative Data

The qualitative data chapter was presented into four sub-sections, each of these represents a research question area; the first sub-section presents methods of analysing qualitative data related to the first research sub-question: What is the level of number sense comprehension of first year preservice secondary mathematics teachers?. The second sub-section similarly presents data analysis methods related to the second research sub-question: What is the relationship between the numbersense of preservice secondary mathematics teachers and their academic performance in mathematics?.

In the same manner the third section endeavours to respond to the third research sub-question: What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers? Finally the last subsection in this section presents the data analysis methods of the qualitative data related to the main question of the study that deals with how Critical Theory intervention could be used to development of number sense preservice secondary mathematics teachers.

4.9.2.1 The level of number sense comprehension

The responses of the students to the number sense test items were studied and analysed. At this point the researcher analysed the reasoning that caused the students to choose the answers they chose. The worksheets of the students during the first few lessons were also analysed. The responses of students to the questionnaire and the focus group interviews were also studied. This analysis was carried regarding the first research sub-question (SQ1).

4.9.2.2 The relationship between the number sense and academic performance in mathematics

The qualitative data in this section were presented in terms of responses of the preservice secondary mathematics teachers on how the number sense impacted their mathematics performance and or comprehension in mathematics. This was thus considered as qualitative data as argued by Loraine (1998) that qualitative data deals with reflections from the personal experiences of the participants and that this should then be used to aid the quantitative data to enhance the data that were reliable with a considerable level of depth.

4.9.2.3 The impact of a Critical Theory intervention

This subsection presents the data analysis methods related to the impact of the critical theory intervention that was offered to the experimental group. The responses in test scripts of the preservice secondary mathematics teachers during the pre-test were compared with those of the pre-test for some number sense question in the pre-test-post-test number sense.

The researcher paid attention to the interpretation of written work as well as the numerical sense presented in the written work of the pre-test versus the post-test. The responses of the preservice secondary mathematics teachers to the interview questions as well as the questionnaire before and after the intervention were also studied. At this stage the researcher attempted to analyse how they experienced the number sense intervention. Therefore this analysis focused on the experiences and or skills which the preservice secondary mathematics teachers had prior to the intervention, the acquired skills after the intervention as well as the shortcomings of the intervention programme if there were any.

In this section, the researcher also compared the methods of responding to the number sense test items in the pre-test with the post-test. This then was used to establish whether or not there was any improvement of number sense strategies as opposed to making decisions based on the quantitative method of merely following average scores.

4.10 Ethical considerations

The researcher sought and obtained institutional permission from UNAM and ethical clearance from Stellenbosch University respectively for this study to be carried out by observing their ethical codes. The informed consent from participants in this study was sought using the Stellenbosch University approved format and guidelines (see appendices A to D). Participants were at every point of the study assured of the strictest confidentiality and privacy as per recommendations of Brink and Wood (2001, p.301).

Participants were informed of their rights to withdraw at any stage (see appendix C) of the study should they so wish but were also informed of the impact of their withdrawal thereof; for instance that they will not be allowed to come back again if they once dropped off and the impact they will make on the validity of the study. Therefore they were advised to withdraw with valid reasons.

4.11 Validity and Reliability

Validity as defined by Loraine (1998) is the degree to which a measurement instrument assesses what it is supposed to measure. Taking into account of the two kinds of validity, in this study both the face and content validity were determined. For the face validity the tools were validated by three experts in the field of study. The instruments were also given to the two supervisors/promoters, to evaluate questions and outline in relation to the research questions stipulated in chapter 1 of the study, as recommended by (Polit & Hungler 1997, p. 374). This was carried out to ensure that questions actually assessed the characteristics that were targeted by the investigator.

For content validity the literature and existing policies on number sense were utilised to ensure that the content being asked was up to date and the questions actually assessed the number sense attributes. Moreover, the validations that were suggested were incorporated to ensure that the tools actually contained the desired contents.

The issue of contamination was also dealt with considering that both the control and experimental group participants were from the same cohort. It is against this background that each of the groups (control and experimental) were informed not to share their information with other people from a different group as theirs. Moreover, the material used for each lesson was collected for each and every lesson since it was used as part of data collection, hence it was not easy for the control group participants to access the information utilised during the training.

Loraine (1998) defines reliability of a tool as its ability to measure what it is supposed to measure consistently. During the piloting stage, the instruments were subjected to a test-retest technique, based on the assumptions that the phenomenon to be measured remains the same at two testing times and that any change is a result of random error (Dipoy & Gitlin 1998, p. 203). For the same reason the questionnaires and the interviews were also pre-piloted.

4.12 Summary

This chapter presented the methodology that was utilised. The chapter presented the justification of the dual nature of the data collected in this for both the pilot and the main study. The chapter also then presented the population, the sample, research instruments, qualitative and quantitative data analysis, validity and reliability issues. The next chapter presents the quantitative data for the study.

CHAPTER FIVE

PRESENTATION, INTERPRETATION AND DISCUSSION OF QUANTITATIVE DATA

5.1 Introduction

This chapter presents and analyses the quantitative data for the study. The quantitative data presented in this chapter are being triangulated with the qualitative data presented in chapter 6. The idea of presenting data in two chapters has been justified and accounted for earlier in this study (see section 4.2.1.).

The chapter has sections presented according to the research questions:

1. What is the level of number sense comprehension of first year pre-service secondary mathematics teachers?
2. What is the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics?
3. What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers?

In pursuit of the answers to the foregoing questions the chapter presents the quantitative data in four themes: Biographic or demographic information of participants, analysis of results regarding the level of number sense comprehension, analysis of results regarding the relationship between number sense and academic performance in mathematics as well as the analysis of results regarding the impact of a Critical Theory intervention. Each of these three last themes attempts to address the research questions posed in that section.

5.2 Demographic information of participants

This section gives demographic information of the respondents in a sample of 60 preservice secondary mathematics teachers that was selected. Figure 5.1 gives the age groups of participants for the sample.

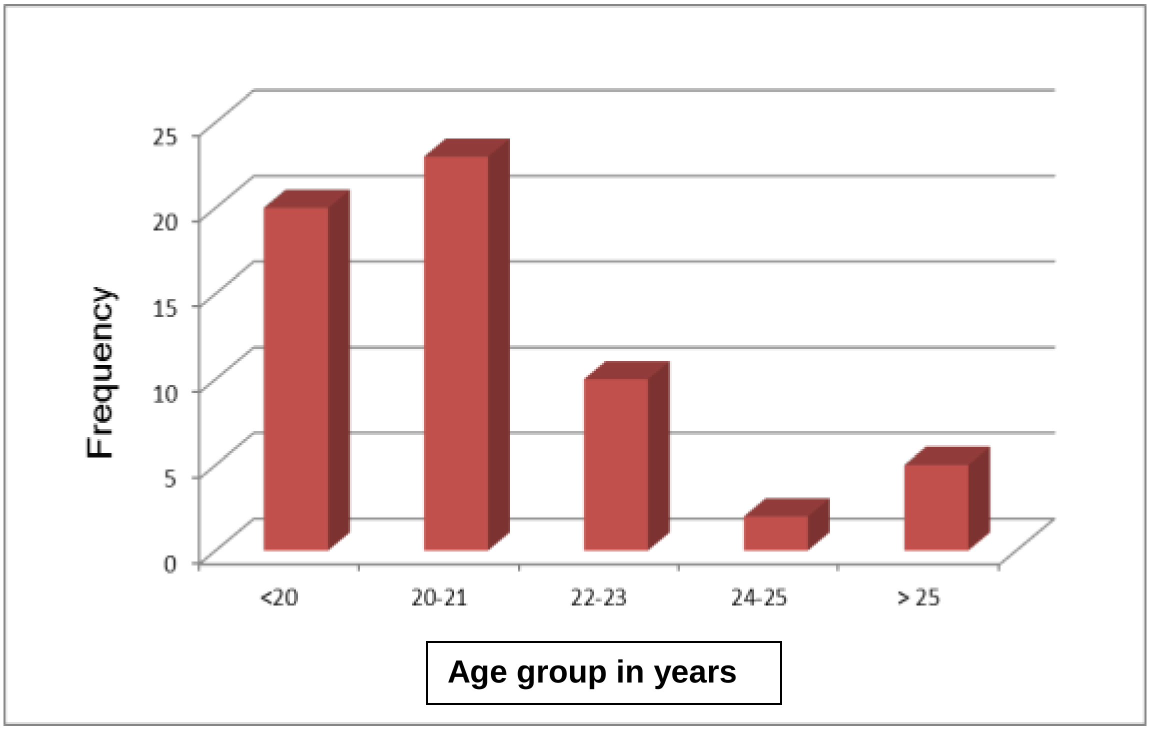

Figure 5.1: Age distribution of the total sample (N=60)

Figure 5.1 shows that 20 (33.3%) of the preservice secondary mathematics teachers were less than the age of 20, while 23 (38.3%) were aged between 20 and 21. Furthermore, 10 (16.7%) of the participants were aged between 22 and 23, two (3.3%) were aged between 24 and 25, whereas there were five (8.4%) respondents more than 25 years of age.

All 60 participants were randomly assigned to the two groups; i.e., the control and the experimental groups. Thus Figure 5.2 shows the comparison of the age groups of the experimental and the control groups.

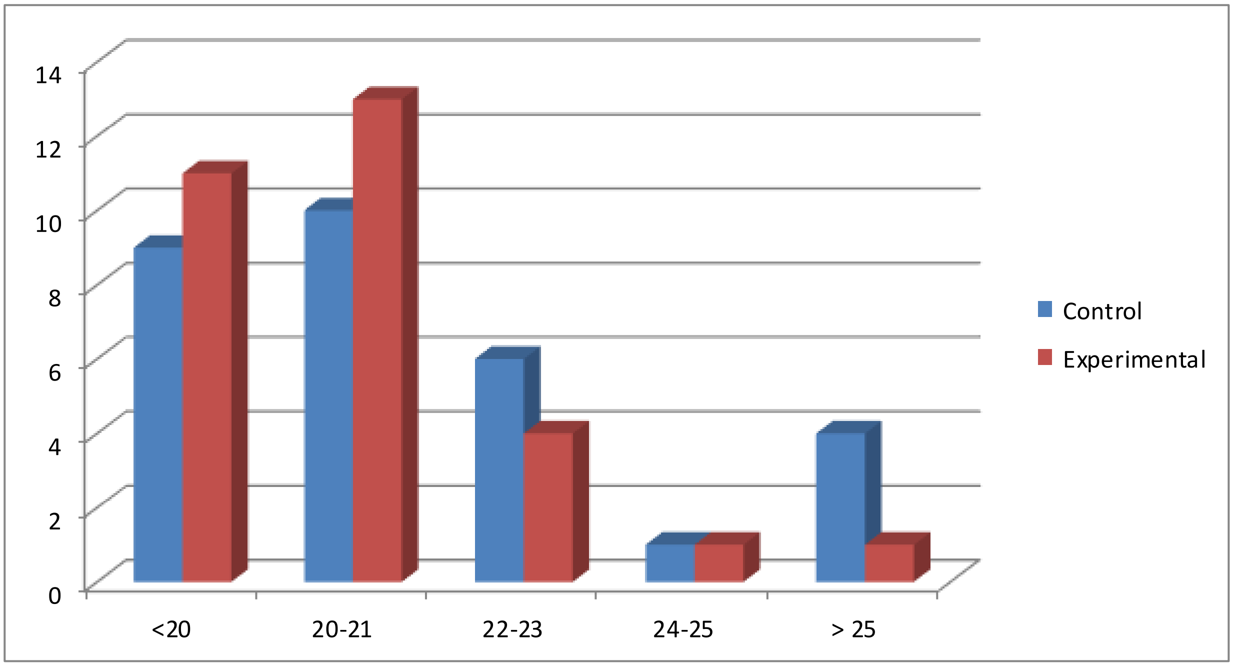

Figure 5.2: Age distribution of the experimental versus the control group (n=60)

Figure 5.2 shows that 9 respondents from the control group were aged less than 20 as opposed to 11 from the experimental group. Figure 5.2 also shows that 10 of the participants from the control group were aged between 20 and 21, whereas from the experimental group these were 13. Six of the control group participants were aged between 22 and 23 while the same age group from the experimental group was represented by four participants. In the age group of 24 to 25 there was only one participant in both the experimental and the control groups. Moreover, 4 participants were aged more than 25 in the control group as opposed to one from the experimental group.

The participants were randomly assigned to the control and the experimental groups thus the age distribution approximates to a normal distribution curve as it appears from above. There were also requirements for admission to the university; therefore it could be assumed that some of the participants had to improve their grades before gaining admission, thus they appear to be in different age groups. Different age groups could also be a result of the preservice secondary mathematics teachers not starting school at the same age.

The sample consisted of both male and female participants. The participants' gender is presented in Figure 5.3.

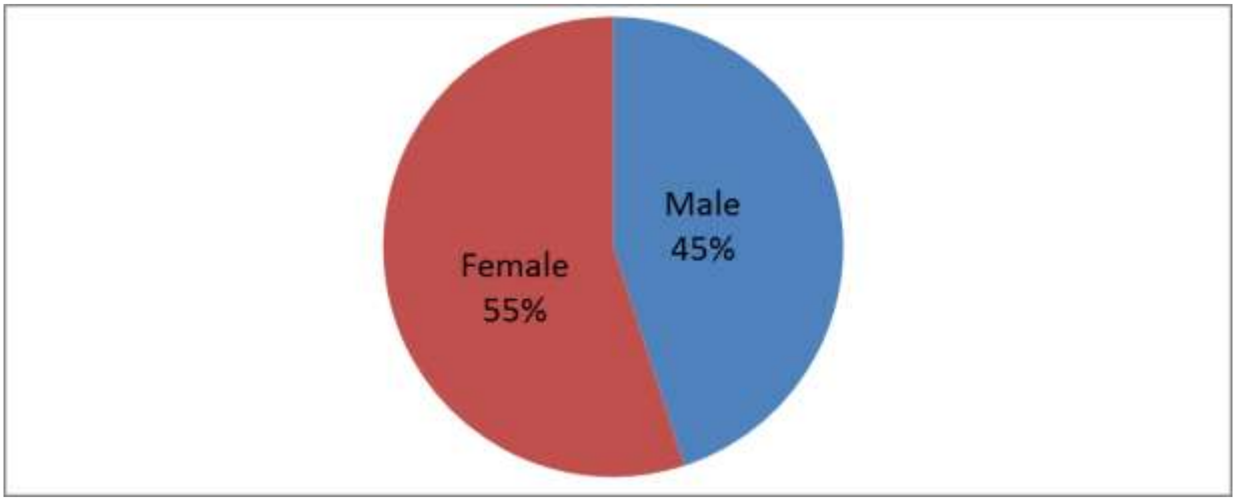

Figure 5.3: Distribution of participants according to gender (n=60)

Figure 5.3 suggests that the gender is dominated by females 33 (55%) of the participants were female while 27 (45%) were male.

Figure 5.4 gives a comparison for the gender representation between the experimental and the control groups.

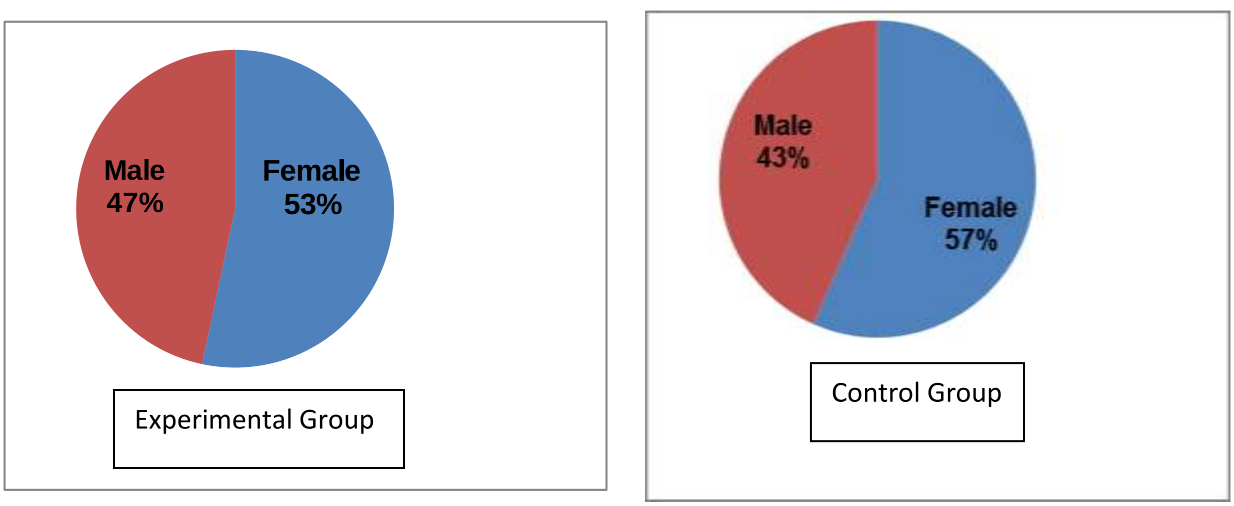

Figure 5.4: Comparison of gender experimental versus control group (n=30)

Figure 5.4 indicates that 14 (47%) males and 16 (53%) females represented the experimental group, whereas for the control group the gender distribution is 13 (43%)

males and 17 (57%) females. The participants' genders as well as the age groups they represent for both the control as well as the experimental groups are also compared in Table 5.1.

Table 5.1: Comparative analysis of demographic information of respondents for the experimental and control groups n= 60

Group	Female	Male	<20	20-21	22-23	24-25	>25
Experimental	16	14	9	10	6	1	4
Control	17	13	11	13	4	1	1
Total	**33**	**27**	**20**	**23**	**10**	**2**	**5**

Table 5.1 indicate the gender proportion of 33 (55%) females and 27 (45%) for males. Furthermore it can be observed that the proportion of females to males in both experimental and the control groups are 16 (53.3%) to 14 (46.7%) and 17 (56.7%) to 13 (43.3%) respectively, considering the fact that the total students in each of the groups (experimental and control group) is 30.

Thus Table 5.1 shows that the males were slightly outnumbered by females in both the control and the experimental groups. Thus it can be said that the B.Ed. secondary mathematics attracted more females as compared to males for this academic year.

5.3 The level of number sense comprehension prior to the Critical Theory intervention

This section presents the quantitative results regarding the number sense comprehension of pre-service secondary mathematics teachers. The section sought to respond to the research question: What is the level of number sense comprehension of first year pre-service secondary mathematics teachers prior to the intervention? The quantitative data presented in this section was only drawn from the pre-test that was administered to both the control and the experimental groups.

By assessing the level of number sense intervention prior to the critical theory intervention the researcher draws ideas from the main tenets of Critical Theory that it attempts to free humans from all hegemonic powers and that empowerment is the central point. Thus the researcher holds the argument that by assessing the level of number sense prior to the intervention the researcher will establish ways of helping the students to even elevate their number sense to greater heights, i.e. finding means of empowering them.

The number sense test as explained in section 4.5.1 (also see appendix F) consisted of five tiers: the question, think through, the answer, the reason as well as the confidence. The researcher assigned score of 1 mark to each correct answer in the answer section of the pre-test this score was regarded as number sense proficiency. Similarly, each correct reason was also assigned a mark of 1; this constituted the number sense reasoning scores. Thus in this chapter the term number sense should be interpreted as encompassing a combination of number sense proficiency scores with those of number sense reasoning (see also definition of terms).

This section hereafter, in conjunction with section 6.2 presents the quantitative data related to the first sub question (SQ1): "What is the level of number sense comprehension of first year preservice secondary mathematics teachers prior to the intervention?". Thus in order to determine the current levels of number sense the study utilised the ideas of McIntosh, Reys, and Reys, (1992), Reys and McIntosh (2002) and Markovits and Sowder (2004) which grouped the levels of number sense into four categories depending on the percentage scores in a validated number sense test:

- ❖ **Advanced** *(75% and above)*
- ❖ **Proficient** *(60–74 %)*
- ❖ **Basic** *(50-59 %)*
- ❖ **Below Basic** *(0 - 49 %)*

The mean scores of the total sample in the pre-test were found to be 41.1% and 35.5% for the number sense proficiency and reasoning respectively. Following the rubric (see sections 4.9.2.1 and 6.2.1) it can be said that the mean number sense scores of pre-service secondary mathematics teachers in both the number sense proficiency and reasoning show that the competencies of pre-service secondary mathematics teachers were below basic in both number sense proficiency and number sense reasoning.

The mean scores of pre-service secondary mathematics teachers in the number sense proficiency and the number sense reasoning were also computed for each of the five sections of the number sense test and these are presented in Table 5.2.

Table 5:2: Mean score for each section of pre-test (N=60)

Number sense component	**Number sense proficiency Mean score (%)**	**Number sense reasoning Mean score (%)**
The meaning and size of numbers both rational and irrational numbers	36.9	27.5
Equivalence of numbers both rational and irrational numbers	54.5	49.6
Meaning and effects of operations;	46.8	45.0
Counting and computational strategies	38.7	31.4
Estimation using relevant benchmarks without calculating	28.6	24.0
Overall total mean score (%)	**41.1**	**35.5**

Table 5.2 indicates that the number sense proficiency as well as reasoning scores of 54.5% and 49.6% in the section of equivalence of both rational and irrational numbers. Using the rubric referred to in 4.9.2.1 it is clear that these scores suggest that the competencies of pre-service secondary mathematics teachers were (before the intervention) at basic level for number sense proficiency and below basic for the number sense reasoning for the meaning and effects of operations component.

Furthermore, Table 5.2 indicates that in all the other four components the preservice secondary mathematics teachers scored below the basic level. Their scores were found to be between 46.8% and 28.6% for the number sense proficiency and 45.0% and 24.0% for the number sense reasoning respectively. At a glance the mean scores presented in Table 5.2 clearly suggest that the preservice secondary mathematics teachers' number sense was below basic.

The box-and-whisker plots in Figure 5.5 gives further summary of the data collected with regards to the level of number sense comprehension.

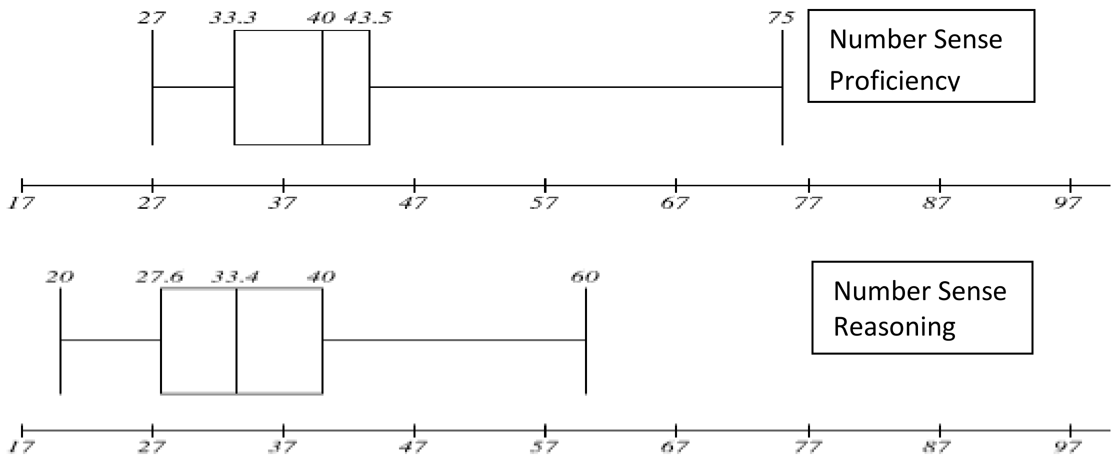

Figure 5.5: Five number summaries for number sense proficiency and reasoning

Figure 5.5 shows that the minimum score in the number sense proficiency was 27% whereas for number sense reasoning it was 20 %. The lower quartiles were 33.3% and 27.6% for the number sense proficiency and reasoning respectively. The median scores for the number sense proficiency and reasoning were 40% and 33.4% respectively; this implies that at least 50% of the participants scored less than 40% and 33.4% (which is below basic) in the number sense proficiency and reasoning respectively. At glance the results of Figure 5.5 suggest that the participants tend to perform better at number sense proficiency items as compared to number sense reasoning. The results from Figure 5.5 also concur with the rest of the results presented in this section as well as the qualitative data presented in section 6.2 in it that the data clearly indicate that both the number sense proficiency and reasoning of preservice secondary mathematics teachers were below the basic level.

Since the number sense test consisted of five sections which formed the components of number sense, there is a need to look at how many of the preservice secondary mathematics teachers fell in each of the levels of number sense. The frequencies of preservice secondary mathematics teachers in each of the four levels of number sense for the number sense proficiency and reasoning are summarised in Figure 5.6.

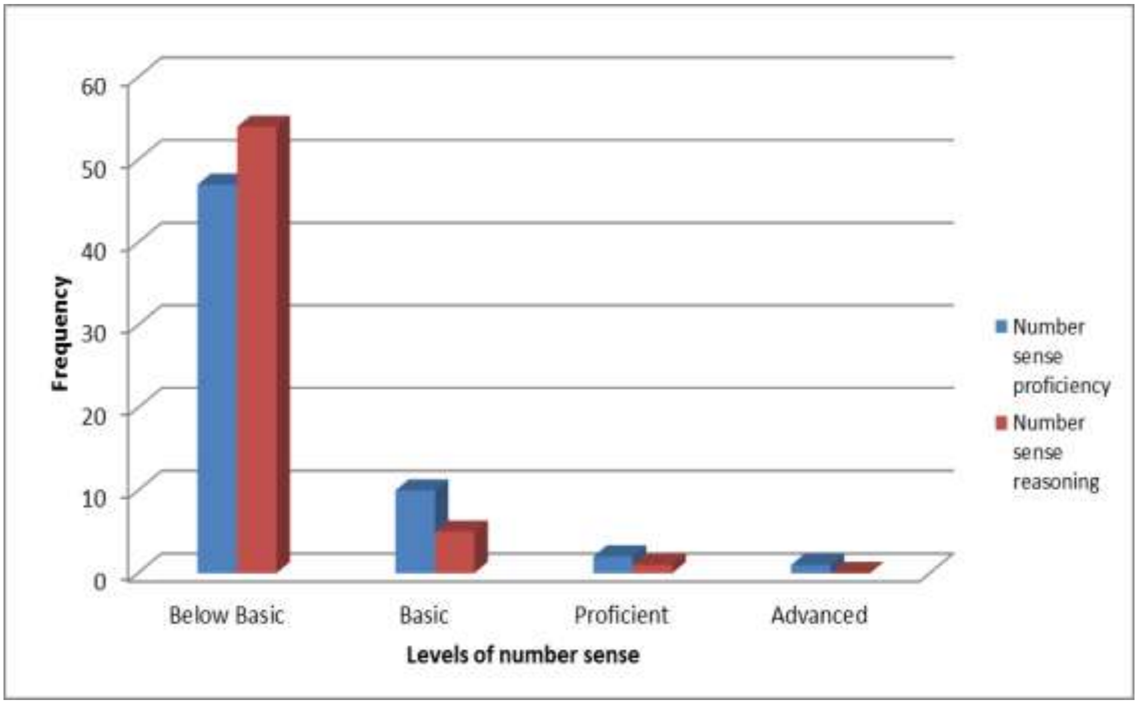

Figure 5.6: Frequencies of pre-service teachers' levels of number sense proficiency and reasoning

Figure 5.6 shows that 47 (78.3%) and 54 (90%) of the participants scored overall number sense proficiency and reasoning scores below basic level respectively. While 10 (16.7%) and 5 (8.3%) respondents achieved the scores in the basic level for the number sense proficiency and reasoning respectively. The proficient levels for the number sense proficiency and reasoning were achieved by 2(3.3%) and 1 (1.7%) of the participants respectively. Only one (1.7%) of the participants achieved a number sense proficiency in the advanced level and none of the participants achieved a score in the advanced level for the number sense reasoning.

Figure 5.6 clearly shows a weak number sense among the participants. Figure 5.6 also shows that many participants performed better on number sense proficiency than on number sense reasoning.

More results on the levels of number sense are indicated by Table 5.3 which shows the frequencies and percentages of each of the level of number sense for the number sense components.

Table 5.3: Distribution of levels of number sense proficiency by number sense

component (n=60)

Level of number sense reasoning	components of number sense proficiency				
	Meaning and size of numbers	**Equivalence of numbers**	**Effects of operations**	**Counting and computation**	**Estimation**
Advanced	2(3.3%)	8(13.3%)	4(6.7%)	3 (5%)	1(1.7%)
Proficient	4(6.7%)	18(30%)	11(18.3%)	6 (10%)	3(5%)
Basic	11(18.3%)	19(31.7)	23(38.3%)	14 (23.3%)	4(6.7%)
Below Basic	43(71.7%)	15(25%)	22(36.7%)	37 (61.7%)	52(86.7%)
Total	60 (100%)	60 (100%)	60 (100%)	60 (100%)	60 (100%)

As indicated in Table 5.3, for the components: meaning and size of numbers, counting and computation as well as estimation, the highest frequencies of students were below basic while the lowest frequency of students have scored advanced levels of number sense. For the equivalence of numbers and the effect of operations on numbers the highest frequency of students achieved the basic level whereas the lowest achieved the advanced.

Table 5.4 shows the distributions of number sense reasoning levels for each of the number sense components.

Table 5.4: Distribution of levels of number sense reasoning by number sense component (n=60)

Level of number sense proficiency	components number sense reasoning				
	Meaning and size of numbers	**Equivalence of numbers**	**Effects of operations**	**Counting and computation**	**Estimation**
Advanced	2(3.3%)	4(6.7%)	3(5%)	1 (1.7%)	0(0%)
Proficient	1(1.7%)	11(18.3%)	10(16.7%)	3 (5%)	3(0%)
Basic	2(3.3%)	27(45)	22(36.7%)	10 (16.7%)	2(3.3%)
Below Basic	55(93.3%)	18(30%)	25(41.6%)	46 (76.6%)	58(96.7%)
Total	60 (100%)	60 (100%)	60 (100%)	60 (100%)	60 (100%)

Table 5.4 shows that most of the participants' number sense reasoning was below basic in the components except for the equivalence of numbers. Table 5.4 also shows

that the least number of pre-service secondary mathematics teachers achieved the advanced level for all the number sense reasoning components.

The participants were asked to indicate how confident they were as they worked out the items in the test when the responded to the questionnaire. Their confidence for each section is recorded in Figure 5.7.

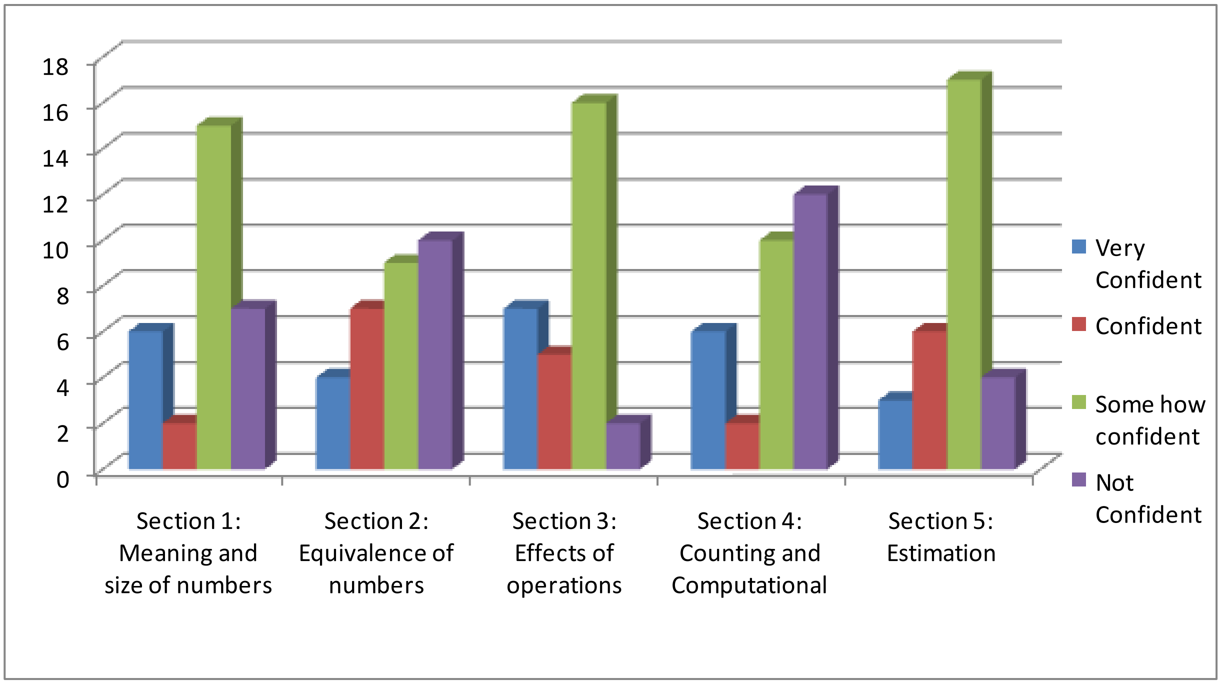

Figure 5.7: A Summary of confidence of the experimental group

Figure 5.7 gives the indication that at least half of the students indicated a lack of confidence for all the five sections of their pre-test number sense test. These either chose not confident or somehow confident. In many of the sections the proportion of the respondents who indicated regarded themselves as confident or very confidence is relatively lower than that of the students who indicated a lack of confidence. It can therefore be concluded that these have attempted the questions in the number sense pre-test with a lack of confidence.

The researcher asked the students to reflect on their own before the intervention and indicate the extent to which they agree with (20 general statements) general number sense statements believed to portray the number sense characteristics as derived from characteristics of number sense presented in section 3.3. These responses are summarised in Figure 5.8.

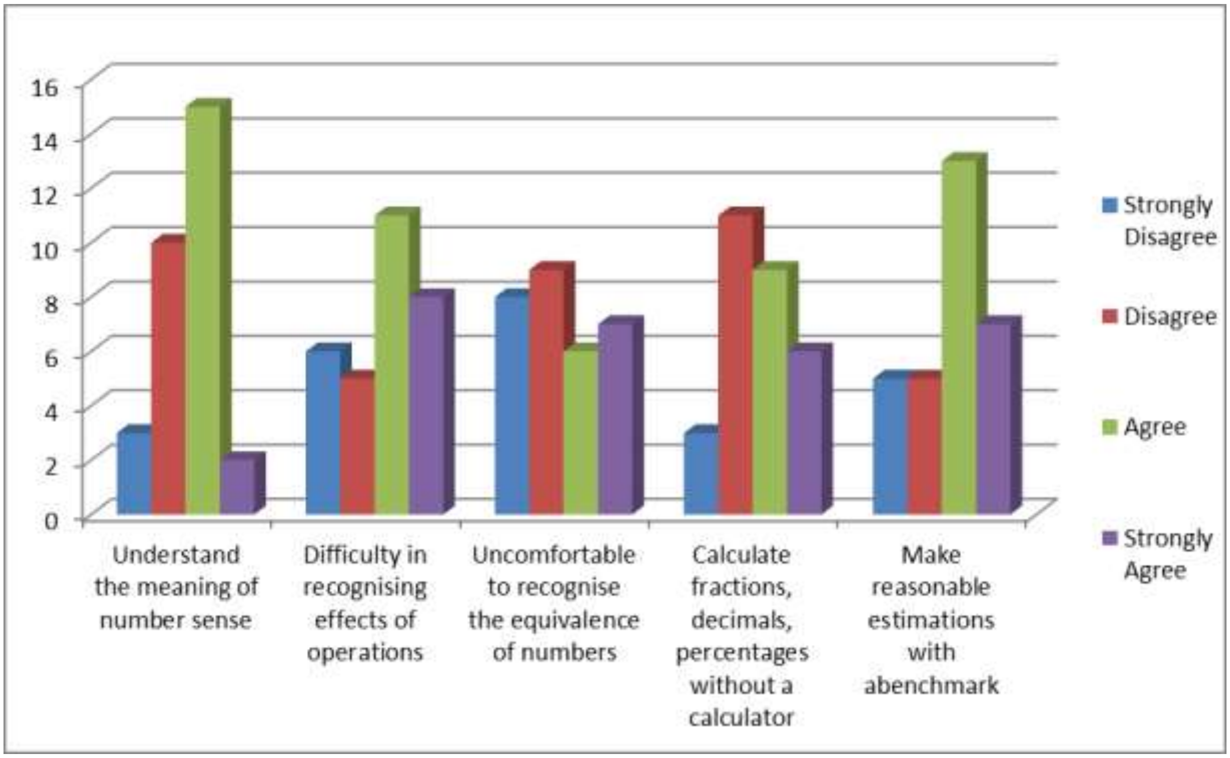

Figure 5.8: Self-evaluation of experimental group respondents: number sense attributes (1- 5)

The data presented in Figure 5.8 more than half (16 out 30) felt they possessed a lack of understanding of the meaning of number sense. In addition, more than half (17 out of 30) expressed that they experienced difficulties in recognising the effects of operations on numbers. Figure 5.8 further shows that two thirds (20 out of 30) of the participants admitted that they would still be uncomfortable if they were asked to recognise the equivalence among the numbers. In addition, half of the respondents indicated that they could not calculate fractions, decimals and percentages without a calculator. Also, half of the students indicated that they could not make reasonable and accurate estimations with a valid benchmark.

At a glance the comparison of the results presented in Figure 5.8 indicate that the self-evaluation shows that the students are not comfortable working with number sense items. More of the self-evaluations on other number sense attributes of pre-service secondary mathematics teachers are presented in Table 5.5.

Table 5.5: Self-evaluation of experimental group respondents: Attributes of number sense (6-15) n=30

Number sense attribute	SA	A	D	SD
Not confident to predict the reasonableness of the answer based on sizes of numbers	5	4	15	6
Not confident to explain calculations without using a calculator	2	4	17	7
I have trouble using alternative methods for explaining a mathematical calculation.	4	5	11	10
I cannot easily do mental calculations in my head.	5	4	14	10
I feel confident using a calculator when doing computations.	5	8	4	13
I don't have the math skills to work out percentages in my head.	5	3	11	11
When given calculations I can only do them with a pen and paper if there is no calculator.	7	6	9	8
I cannot figure out the relative size of my final answer before I finish calculating the whole sum.	3	2	18	7
I hate mental arithmetic	4	14	12	10

The results of Table 5.5 reveal that about half (14 out of 30) of the respondents indicated a lack of confidence in predicting the reasonableness of the answer based on sizes of numbers presented in the problem. In addition, many of the students (19) lacked confidence to explain calculations without using a calculator. Similarly, 18 out of 30 respondents indicated that they had trouble using alternative methods for explaining mathematical calculations. Also 17 out of 30 respondents could not easily do mental calculations in their heads.

Table 5.5 continues to indicate that about three quarters (22) of the respondents felt confident using a calculator when doing mathematical computations. Table 5.5 further indicates that about half (16) of the participants felt that they lacked the mathematical skills to work out percentages in their heads. Most of the respondents (24) indicated that when required to calculate without a calculator, they could only do with a pen and paper. The results of Table 5.5 further indicate that more than half, (18 out of 30) of the respondents revealed a lack of interest on mental arithmetic. Furthermore, half of the respondents (15 out of 30) indicated that they could not figure out the relative size of their final answer before they finish calculating the whole sum.

Table 5.5 indicates that at least half of the respondents lacked most of the attributes of number sense. It was also observed from Table 5.5 that the respondents relied on calculators and were not really familiar with the normal methods of solving number sense problems. Table 5.5 also gives the impression that the students possessed inadequate abilities to assess the reasonableness of their solutions which could be attributed to the fact that they relied so much on the use of a calculator. All these therefore suggest a lower attributes of number sense among the participants.

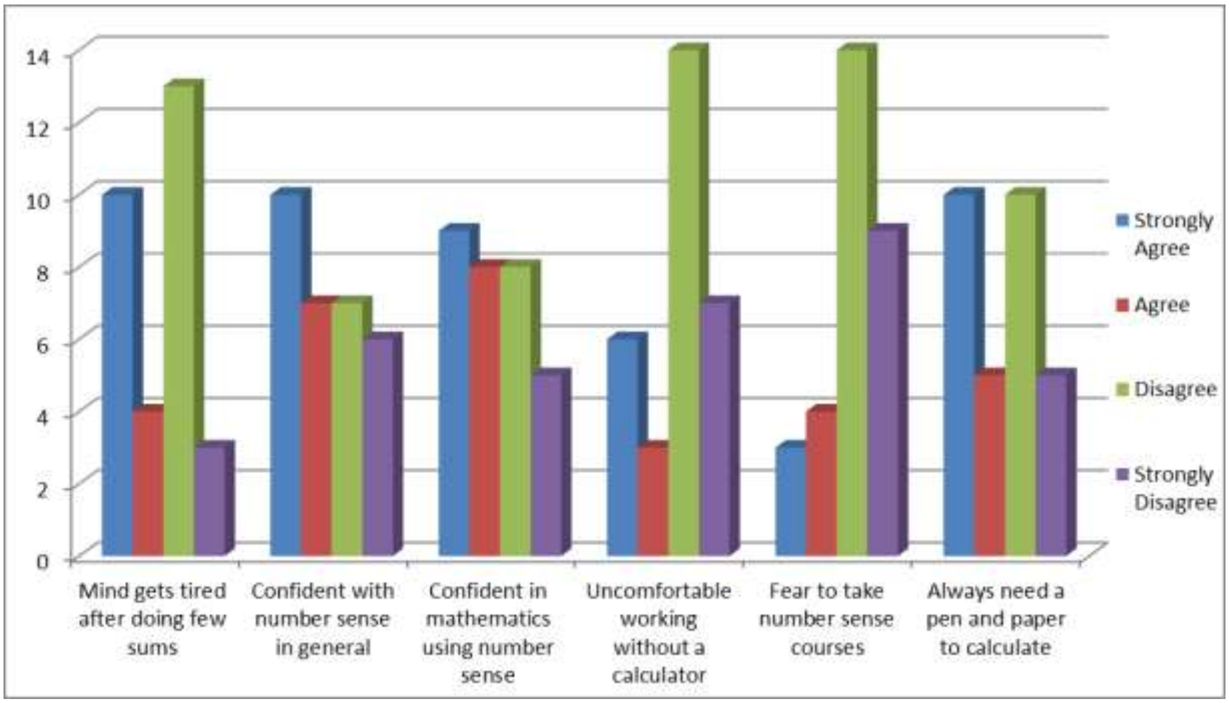

Figure 5.9 Self-evaluation of experimental group respondents before the intervention: number sense attributes (17-20)

Figure 5.9 indicates about half (14 out of 30) of the participants indicated that they felt very tired in their minds after doing few sums in their heads. More than half (17out of 30) of the participants indicated that they felt confident with number sense. Figure 5.9 continued to reveal that only 12 out of 30 respondents after the intervention felt confident in applying number sense concepts to their mathematical content. Two thirds (20 out of 30) had an anxiety of the number sense course that they had to take.

More than two thirds of the respondents believed that they always needed a pen and paper to calculate even if there is a calculator prior to the intervention. From Figure 5.9 it is clear that a reasonable proportion of respondents had poor number sense attributes or lacked number sense prior to the intervention. It is also clear that these participants also revealed that their confidence was low prior to the intervention.

This section presented the quantitative results of the pre-test regarding the number sense proficiency and reasoning of preservice secondary mathematics teachers. The data presented in this section were triangulated with the qualitative data presented in section 6.2 of chapter 6. Most of the data presented in this section showed a lack of number sense. Most of the participants scored marks below the basic level, it was therefore concluded that the pre-service secondary mathematics teachers' number sense was below basic level. The next section presents the quantitative data regarding the relationship between the number sense of preservice secondary mathematics teachers and academic performance in Mathematics.

5.4 The relationship between the number sense and academic performance in mathematics

This section presents the quantitative data regarding the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics. The section is divided into two subsections. The first subsection (5.4.1) presents the linear regression analysis between the number sense and the academic performance of pre-service secondary mathematics teachers in their core module of mathematics (Basic Mathematics for Teachers). The second subsection on the other hand provides a multiple linear regression analysis between the sections of number sense versus the academic performance. All the data presented in this section endeavour to respond to the question: What is the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics?

The variable academic performance was measured at the end of the first semester after the intervention (from the final semester 1 mark of the Basic Mathematics for Teachers). It should thus be noted that the number sense data (both proficiency and reasoning) used in establishing the correlation or the linear model was collected after the intervention (post-test number sense scores). This was done to ensure that the variations that occurred in the number sense levels comprehension of each individual student during the intervention did not contaminate the sample. That is, the researcher at this stage holds the assumption that every individual student would perform according to their acquired experiences and abilities regardless of their number sense. At this juncture therefore the endeavour was to establish the relationship between the number sense and the academic performance in mathematics.

5.4.1 Linear regression analysis of number sense versus academic performance in mathematics

This sub-section presents linear regression analysis between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics, i.e. academic performance is a final mark obtained by preservice secondary mathematics teachers in their core first semester module (Basic Mathematics for Teachers). The sub-section firstly presents the regression analysis between the number sense proficiency of pre-service secondary mathematics teachers and their academic performance in mathematics. Secondly the sub-section presents a linear regression analysis for the number sense reasoning versus academic performance in mathematics. A linear model from both analysis is also worked out to assess how much is the contribution of each of the two independent variables (number sense proficiency and reasoning) to the prediction of the dependent variable academic performance in mathematics.

5.4.1.1 Linear regression analysis between number sense proficiency and academic performance in mathematics

This sub-section presents the regression analysis results regarding the correlation between number sense proficiency and the academic performance in mathematics (Final marks for the first semester in the core module Basic Mathematics for Teachers). The section presents the data on the analysis of the correlation between number sense proficiency and academic performance, the statistical significance and the coefficient of determination (effect size) for the correlation as well as the linear regression model. The section sought to test for the following hypothesis.

Hypothesis

H_0: There is no relationship between number sense proficiency of preservice secondary mathematics teachers and their academic performance in mathematics; $(r = 0)$.

H_1: There is a relationship between the number sense proficiency of preservice secondary mathematics teachers and their academic performance in mathematics; $(r \neq 0)$.

Figure 5.10 shows a scatter plot that attempts shows the association between number sense proficiency and academic performance in mathematics.

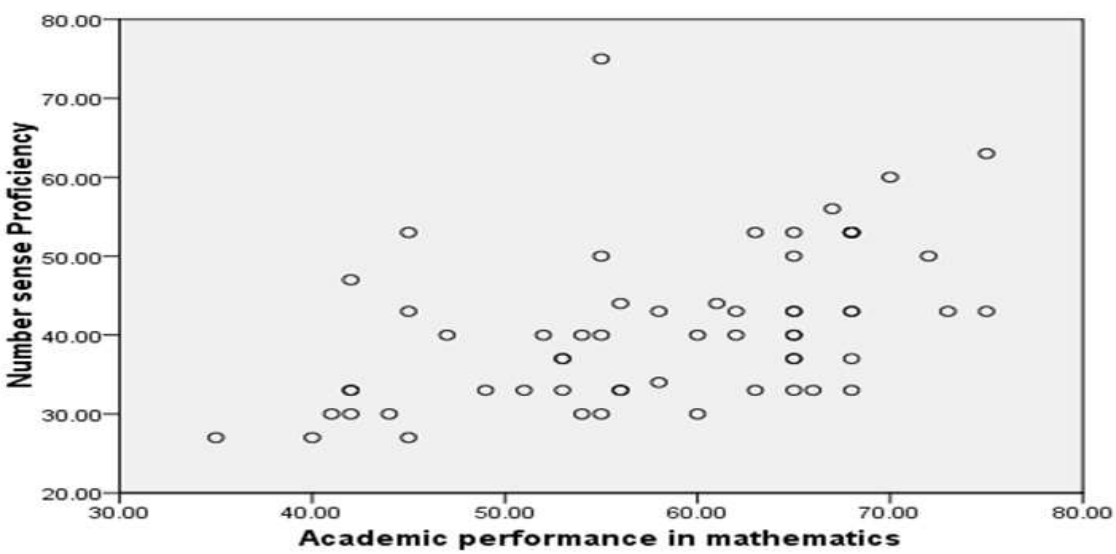

Figure 5. 10: A scatter plot between number sense proficiency and academic performance in mathematics

Figure 5.10 shows that there is a positive correlation between number sense proficiency and academic performance in mathematics. The value of the correlation coefficient for Pearson's correlation coefficient is summarised in Table 5.6. Table 5.6 also presents the results of statistical test for significance that was carried out to study the statistical significance of the correlation between the number sense proficiency of preservice secondary mathematics teachers and their academic performance on hypothesis aforementioned.

Table 5.6: Summary of correlation analysis: number sense proficiency and academic performance in mathematics.

Correlations		Academic performance in mathematics	Number sense Proficiency
Academic performance in mathematics	Pearson Correlation	1	.486**
	Sig. (2-tailed)		.000
	N	60	60
Number sense Proficiency	Pearson Correlation	.486**	1
	Sig. (2-tailed)	.000	
	N	60	60

**. Correlation is significant at the 0.01 level (2-tailed).

The results of Table 5.6 indicate a positive and moderate correlation coefficient (*r* = .486) between number sense proficiency and academic performance in mathematics. MacMillan and Schumacher (2006) explain the rule of the thumb that the relationship is very weak if $r < 0.2$, weak when it is between 0.2 and 0.4, greater than 0.4 but less than 0.6 is moderate, less than or equal to 0.8 as a strong and very strong when greater than 0.8. Also, the 95% confidence for *r* interval is $0.347 < r < 0.710$. It can thus be concluded with 95% confidence that the value of *r* is positioned in that interval. This means that there was a slight chance that the individuals who did well in the number sense test were also likely to do well in the mathematics module from which the variable academic performance was derived.

In addition, the statistical test on the significance of $p = 0.000$ which is less than 0.05 this shows that *r* is also statistically significant at 99% level of significance therefore the null hypothesis was rejected at even at 99% level of significance. It was therefore concluded that there was a statistically significant moderate positive correlation between number sense proficiency and academic performance in mathematics.

To triangulate the coefficients of correlation, the value of Spearman's ranking correlation coefficient, rho (ρ), was also determined as presented in Table 5.7 with its result for the test for statistical significance.

Table 5.7: Summary of Spearman's rank correlation coefficient (ρ)

			Academic performance in mathematics	Number sense Proficiency
Spearman's rho	Academic performance in mathematics	Correlation Coefficient	1.000	.552**
		Sig. (2-tailed)	.	.000
		N	60	60
	Number sense Proficiency	Correlation Coefficient	.552**	1.000
		Sig. (2-tailed)	.000	.
		N	60	60

**. Correlation is significant at the 0.01 level (2-tailed).

Table 5.7 the value of Spearman's ranking correlation coefficient of ρ = 0.552. This shows a moderate positive correlation between number sense proficiency and academic performance in mathematics according to the thumb rule as outlined in the work of MacMillan and Schumacher (2006). The 95% interval of confidence for ρ is 0.272 < ρ < 0.659. From this interval of confidence it can be claimed with 95% confidence that the value of ρ is situated on the interval 0.272 to 0. 659.

In addition, the statistical test on the significance of rho (ρ) shows that the correlation was statistically significant at 95% level of significance, since the value $p = 0.00$ is less than 0.05. The study therefore rejects the null hypothesis and adopts the research hypothesis. It was therefore concluded that the moderate positive Spearman's rank correlation (ρ = 0.552) is statistically significant, i.e. $\rho \neq 0$. From this moderate positive correlation it appeared that there was a reasonable chance that the individuals who did well in the number sense proficiency and might also perform well in mathematics.

To study the effect size the linear regression was performed in SPSS to determine the coefficient of determination (r^2). This was done to assess the contribution of number sense proficiency to academic performance of pre-service secondary mathematics teachers in mathematics as indicated in Table 5.8.

Table 5.8: Summary of linear regression analysis between the number sense proficiency and the academic performance in mathematics

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.486	.236	.223	8.98428

The linear regression analysis model echoes the already presented moderate positive correlation (R=0.486) between the number sense proficiency of preservice secondary mathematics teachers and their academic performance in mathematics. The adjusted coefficient of determination of (R^2 = 0.223) shows that about 22% of the variations in academic performance in mathematics can be attributed to the number sense proficiency and vice-versa. This could be interpreted that the variations that the number sense could explain on the academic performance of preservice secondary mathematics teachers to some de

5.4.1.2 Linear regression analysis regarding the relationship between number sense reasoning and academic performance in mathematics

As indicated earlier the number sense test assessed the number sense proficiency alongside with the number sense reasoning. The number sense reasoning scores were also correlated with academic performance in mathematics to assess the association between the two. This sub-section, therefore, presents the result regarding the linear regression between the number sense reasoning and the academic performance of pre-service secondary mathematics teachers in mathematics. The simple linear regression model for predicting the academic performance in mathematics using the number sense reasoning is also presented in this sub-section.

Hypothesis

H_0: There is no association between the number sense reasoning and the academic performance of preservice secondary mathematics teachers in mathematics ($r = 0$);

H_1: There is an association between the number sense reasoning and the academic performance of preservice secondary mathematics teachers in mathematics ($r \neq 0$).

The scatter diagram about the correlation between the number sense reasoning and the academic performance in mathematics is presented in Figure.

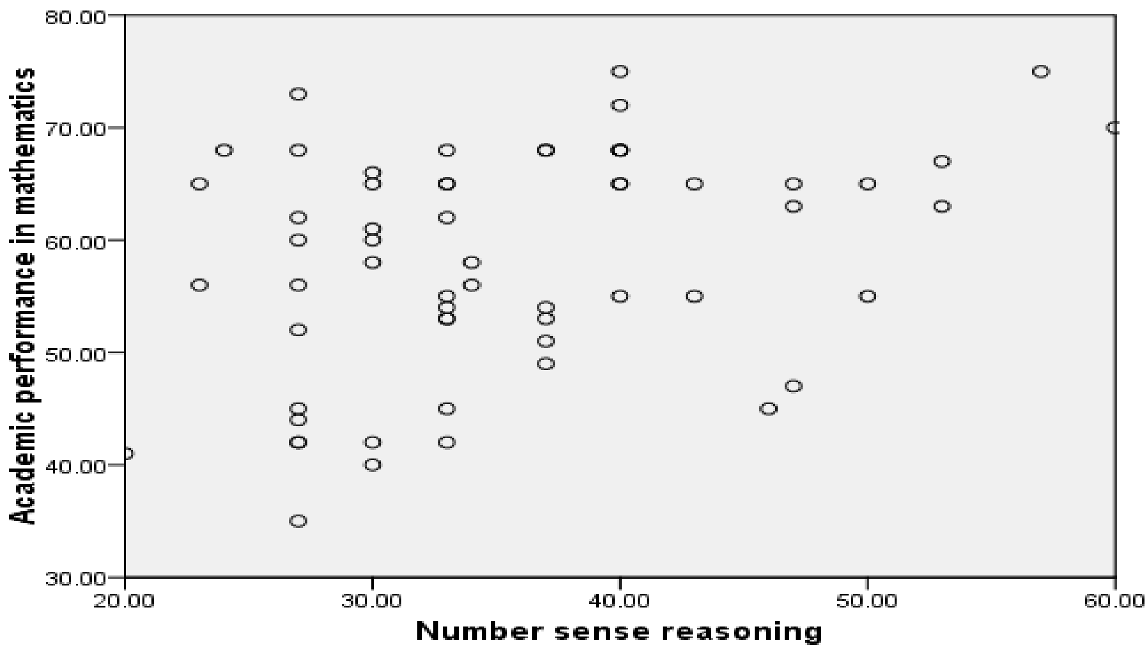

Figure 5.6 : A scatter plot between number sense reasoning and academic performance in mathematics

The results of Figure 5.10 show a positive correlation between the number sense reasoning and the academic performance in mathematics. In addition to Figure 5.10 a further analysis on the nature of this positive correlation is illustrated in Table 5.9.

Table 5.9: Summary of correlation between number sense reasoning and academic performance in mathematics

		Number sense reasoning	Academic performance in mathematics
Number sense reasoning	Pearson Correlation	1	.374**
	Sig. (2-tailed)		.005
	N	60	60
Academic performance in mathematics	Pearson Correlation	.374**	1
	Sig. (2-tailed)	.005	
	N	60	60

**. Correlation is significant at the 0.01 level (2-tailed).

Following the interpretations of the correlation coefficient illustrated by MacMillan and Schumacher (2006) on the rule of thumb, the results of Table 5.9 show that there was a weak positive correlation between the number sense reasoning and the academic performance in mathematics. However the statistical test for significance performed on the significance of the Pearson's correlation coefficient (*r*) shows that p = 0.005 which is within the margin of statistical significance. Therefore the null hypothesis is rejected and the conclusion is that the weak correlation between the number sense reasoning and the academic performance in mathematics is statistically significant.

Therefore, the research hypothesis that there is a statistically significant correlation coefficient between the number sense reasoning of preservice secondary mathematics teachers and their academic performance in mathematics is valid; i.e. To triangulate the weak positive correlation observed above the Spearman's ranking correlation coefficient is summarised in Table 5.10.

Table 5. 10: Summary of Spearman's rank correlation between number sense reasoning and academic performance in mathematics

			Number sense reasoning	Academic performance in mathematics
Spearman's rho	Number sense reasoning	Correlation Coefficient	1.000	.329**
		Sig. (2-tailed)	.	.010
		N	60	60
	Academic performance in mathematics	Correlation Coefficient	.329**	1.000
		Sig. (2-tailed)	.010	.
		N	60	60

**. Correlation is significant at the 0.01 level (2-tailed).

Table 5.10 also shows a weak positive correlation between the number sense reasoning and the academic performance of pre-service teachers as shown by the value of the Spearman's rank correlation coefficient (ρ=*0*.329). Also, the statistical test for significance performed at 95% level of significance shows that the value of p = 0 which is statistically significant. Therefore at this level of significance the null hypothesis was rejected and the research hypothesis, i.e., the conclusion that there is a statistically significant correlation is valid.

To closely analyse the nature of this positive weak correlation, a linear regression model is illustrated in table 5.11.

Table 5.11: Summary of linear regression model between the number sense reasoning and the academic performance in mathematics

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.374[a]	.140	.128	9.57558

From the linear regression model presented in Table 5.11 the value of r = 0.374 and the adjusted square of the correlation coefficient show that about 13% of the variation in the academic performance in mathematics can be explained by number sense reasoning and vice-versa.

The first two sub-sections presented the results regarding the influence of the variables number sense proficiency as well as number sense reasoning on the academic performance. A statistically significant moderate positive correlation was observed between number sense proficiency and academic performance in mathematics. While a weak positive correlation was observed between the number sense reasoning and the academic performance in mathematics. From this it appears that the number sense proficiency predicts the academic performance of pre-service secondary mathematics teachers better than number sense reasoning.

However, the fact that both correlation coefficients were still found to be statistically significant there was a need to be certain about the best predictor variable of academic performance in mathematics. A multiple regression analysis on the impact of the combination of number sense proficiency and reasoning on the academic performance in mathematics was carried out. The next section thus presents this multiple regression analysis between the independent variables number sense proficiency and number sense reasoning and the dependent variable academic performance in mathematics.

5.4.2 Multiple regression analysis on the independent variables of number sense and the academic performance

This section presents the data regarding the multiple linear regression analysis of the independent variables number sense proficiency and number sense reasoning on the dependent variable academic performance. This was done to identify the best number sense predictor of academic performance. The section is divided into two sub-sections; the first subsection, through multiple linear regressions evaluates the effect each of the independent variables number sense proficiency and number sense reasoning on the dependent variable academic performance in mathematics. This therefore means that the independent number sense variable has no statistically significant contribution to the dependent variable number sense academic performance.

The second sub-section deals with the multiple regression analysis of each of the components of the best predictor variable on the dependent variable academic performance.

5.4.2.1 Multiple regression analysis of number sense proficiency and reasoning academic performance in mathematics

This sub-section presents quantitative data regarding the impact of combination independent variables (number sense number sense proficiency and number sense reasoning) on the dependent variable academic performance in mathematics. This was done to test for the following hypothesis for significance and effect size.

Hypothesis

H_0: There is no association between the independent variables number sense proficiency and number sense reasoning on the dependent variable academic performance of preservice secondary mathematics teachers in mathematics ($R = 0$).

H_1: There is an association between the independent variables number sense proficiency and number sense reasoning on the academic performance of preservice secondary mathematics teachers in mathematics ($R \neq 0$).

For this hypothesis the study also assessed the significance of the effect size of each of the variables number sense proficiency and number sense reasoning on the academic performance in mathematics.

To assess the feasibility of employing the multiple regression analysis the study checked the necessary assumptions such as dependent variable is continuous scale, the presence of two or more independent variables, which are continuous, the presence of independence of observations. The other conditions such as the existence of a linear relationship between the dependent variable and each of the independent variables, the homoscedasticity, the absence of show multicollinearity and significant outliers as well as the assumption of normality were also assessed using SPSS as recommended by Macmillan and Schumacher (2006).

Satisfied that the fore going assumptions hold, the study ran a test for the possibility of the multiple regression analysis by analysing the satisfaction of the minimum ratios of the valid cases to the number of independent variables, the result of this test is summarised in Table 5.12.

Table 5.12: Summary of feasibility of the sample size on the multiple regression analysis

	Mean	Std. Deviation	N
Academic performance in mathematics	58.1833	10.03129	60
Number sense Proficiency	52.5000	9.84800	60
Number sense reasoning	36.6000	8.87660	60

The minimum ratio of valid cases to independent variables for multiple regression analysis is 5:1. Table 5.12 indicates a ratio of 60 valid cases to two independent variables which simplifies to 30:1 for this multiple regression analysis. In addition this ratio satisfies the preferred ratio of 15:1. Therefore it was feasible to carry out the multiple regression analysis in this case.

In addition and as part of this multiple linear regression analysis Table 5.13 shows the summary of the results of the analysis of variance (ANOVA).

Table 5. 4: The summary of multiple linear regression analysis: ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1458.100	2	729.050	9.278	.000[b]
	Residual	4478.883	57	78.577		
	Total	5936.983	59			

a. Dependent Variable: Academic performance in mathematics
b. Predictors: (Constant), Number sense reasoning, Number sense proficiency

The results of Table 5.13 on the statistical significance of the F ratio show that the probability of the F(2, 57) = 9.278 statistics for the overall relationship between the independent variables number sense proficiency and number sense reasoning on the academic performance in mathematics is statistically significant since the value of p = 0.00. Therefore the null hypothesis that there is no relationship between the independent variables ($R^2 = 0$) is rejected.

Thus by rejecting the null hypothesis the study adopts the research hypothesis that there is a statistically significant relationship between the set of independent variables number sense proficiency as well as number sense reasoning and the dependent variable academic performance in mathematics. Thus it can be concluded that the overall regression model is a good fit for the data. The model summary for this multiple linear regression analysis is presented in Table 5.14 for a further analysis.

Table 5.14: The multiple linear regression analysis: Model Summary

					Change Statistics				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig.(p) F Change
1	.484[a]	.236	.219	8.86436	.246	9.278	2	57	.000

a. Predictors: (Constant), Number sense reasoning, Number sense Proficiency
b. Dependent Variable: Academic performance in mathematics

Table 5.14 shows that the multiple correlation coefficient (R) between the set of independent variables and the dependent variable is 0.484 which is a moderate according to the rule of thumb. The relationship of each individual independent variable to the dependent variable is summarised in Table 5.15.

Table 5.15: Summary of relationship between number each individual independent variable to the dependent variable: Coefficients

Model		Unstandardised Coefficients B	Unstandardised Coefficients Std. Error	Standardized Coefficients Beta (β)	t	Sig. (p)	95.0% CI for B Lower Bound	95.0% CI for B Upper Bound
1	(Constant)	35.311	5.542		6.371	.000	24.213	46.409
	Number sense Proficiency	.403	.136	.396	2.952	.005	.130	.676
	Number sense reasoning	.179	.151	.158	1.180	.243	-.125	.482

a. Dependent Variable: Academic performance in mathematics

The results of Table 5.15 show that the probability of *t* statistics for the *t* = 2.952 on the B coefficient for the independent variable number sense proficiency is statistically significant since p = 0.005<0.05. Therefore for the independent variable number sense proficiency the null hypothesis that the slope associated with the number sense proficiency is equal to zero, i.e. $\beta = 0$ is rejected. In rejecting the null hypothesis the study concludes that there is a statistically significant combined relationship between the independent variable number sense proficiency and the dependent variable academic performance in mathematics.

The unstandardised β coefficient regarding the independent number sense proficiency is 0.403 which is positive, this could be interpreted to imply that for each one percentage increase the there is an increase of 0.403% in the academic performance in mathematics for preservice secondary mathematics teachers provided that all the other parameters that bear an influence on the academic performance in mathematics are held constant.

Regarding the association of the independent variable number sense reasoning on the dependent variable academic performance the results of Table 5.15 show that the probability of *t* statistics for the *t* = 1.180 on the β coefficient for the independent variable number sense reasoning is not statistically significant since the value of p = 0.243 is not statistically significant.

Therefore for the independent variable number sense reasoning the null hypothesis that the slope associated with the number sense reasoning is equal to zero is accepted, i.e. the idea that $\beta = 0$ is accepted. By so doing the study concludes that the independent variable number sense reasoning is not statistically significant in predicting the dependent variable academic performance in mathematics.

5.4.2.2 Multiple regression analysis of number sense proficiency components on the academic performance in mathematics

This sub-section therefore presents the data regarding the multiple linear regression analysis of the five number sense proficiency components only on the academic performance in mathematics. These number sense proficiency components which form the independent predictor variables of the dependent variable academic performance in mathematics are number sense proficiency in: Meaning and size of numbers, Equivalence of numbers, Effects of operations, Counting and computation as well as Estimation.

The independent variable number sense reasoning was not considered to be statistically significant on predicting the academic performance of pre-service secondary mathematics teachers in mathematics as concluded in the previous subsection. Therefore the number sense reasoning components were eliminated from the linear model. The tests for statistical significance carried out in this section were aimed at assessing the significance of the contributions of each of the number sense proficiency variable on the following hypothesis:

Hypothesis 5.4

H_0: There is no association between number sense proficiency components of preservice secondary mathematics teachers and their academic performance in mathematics $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

H_1: There is an association between the number sense proficiency components on of preservice secondary mathematics teachers and their academic performance in mathematics; ($\beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_4 \neq \beta_5 \neq 0$).

The test results on the sample size for the feasibility of performing multiple regression analysis are summarised in Table 5.16.

Table 5.16: Summary of feasibility of the sample size for multiple regression analysis

	Mean	Std. Deviation	N
Academic performance in mathematics	58.2	10.03129	60
Proficiency on meaning and size of numbers	57.5	15.93719	60
Proficiency on equivalence of operations	67.8	18.16163	60
Proficiency on meaning and effects of operations	57.2	18.80416	60
Proficiency on Counting and computational strategies	43.3	18.89639	60
Proficiency on Estimation using a relevant benchmark	54.5	15.23914	60

The minimum required ratio of valid cases to independent variables for multiple regression analysis is 5:1. Table 5.16 indicates a ratio of 60 valid cases to five independent variables which simplifies to 12:1. In addition this is closer to the preferred ratio of 15:1 though it is marginally lower; it is still feasible to carry out the multiple regression analysis in this case. The model summary of the multiple linear regression analysis on the five variables of number sense proficiency and academic performance is presented in Table 5.17.

Table 5.17: Model summary of the multiple linear regression analysis on the five variables of number sense proficiency and academic performance

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.526[a]	.277	.219	8.97440

a. Predictors: (Constant), Proficiency on Estimation using a relevant benchmark, Proficiency on equivalence of operations, Proficiency on meaning and effects of operations, Proficiency on Counting and computational strategies, Proficiency on meaning and size of numbers

Table 5.17 shows a moderate multiple correlation coefficient (R=0.526) between the set of independent variables and the dependent. The value of the adjusted coefficient of determination, (R^2) is 0.22. From this value it can be claimed that the effect size of the combination of number sense proficiency variables is 22%. It is therefore concluded that among the variations in academic performance 22% could be accounted for by a combination of the five independent predictor variables and vice-versa.

The results of the multiple linear regression analysis (MRA) of the analysis of variance (ANOVA) are presented in Table 5.18.

Table 5.18: The summary of multiple linear regression analysis: ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig. (*p*)
1	Regression	1587.834	5	317.567	3.943	.004[b]
	Residual	4349.149	54	80.540		
	Total	5936.983	59			

a. Dependent Variable: Academic performance in mathematics

b. Predictors: (Constant) plus i) Proficiency on Estimation using a relevant benchmark, ii) Proficiency on equivalence of operations, ii) Proficiency on meaning and effects of operations, iv) Proficiency on Counting and computational strategies, v) Proficiency on meaning and size of numbers

Table 5.18 on the statistical significance of the F ratio shows that the probability of the F (5, 54) = 3.943 statistics for the overall relationship between the five independent variables on the academic performance in mathematics is statistically significant since the value of p = 0.004 which is within the rejection region. Therefore the null hypothesis that there is no relationship between the independent variables number sense proficiency and number sense reasoning and the academic performance in mathematics ($R^2 = 0$) is rejected.

Thus by rejecting the null hypothesis the study adopts the research hypothesis that there is a statistically significant relationship between the set of five variables of number sense proficiency and the dependent variable academic performance in mathematics. Thus it can be concluded that the overall regression model is a good fit for the data. In addition to Table 5.18, the relationship of each individual independent variable to the dependent variable is summarised in Table 5.19.

Table 5.19: The summary of multiple linear regression analysis coefficients a

Model: Academic achievement=39.746+ .307numbermeaning+.055equiv+.126operations+.082countcompute+.242estimation		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta (ß)		
1	(Constant)	39.746	5.190		7.659	.000
	Proficiency on meaning and size of numbers	.193	.081	.307	2.394	.020
	Proficiency on equivalence of operations	.031	.069	.055	.444	.659
	Proficiency on meaning and effects of operations	.167	.064	.126	1.041	.003
	Proficiency on counting and computational strategies	.054	.068	.082	.680	.236
	Proficiency on estimation using a relevant benchmarks	.128	.080	.242	1.895	.004

a. Dependent Variable: Academic performance in mathematics

The results of Table 5.19 show that for the independent variables: Proficiency on meaning and size of numbers, Proficiency on meaning and effects of operations, Proficiency on estimation using a relevant benchmark, the *t* statistics (2.394, 1.041, 1.895) for the ß coefficients for these independent variables were statistically significant since the values of *p* were 0.020, 0.003 and 0.004.

Therefore for these independent variables: Proficiency on meaning and size of numbers, Proficiency on meaning and effects of operations as well as Proficiency on Estimation using a relevant benchmark, the null hypothesis that the individual coefficients associated with each of these variables is equal to zero, (i.e. $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ is rejected.

For the independent variables proficiency on counting and computational strategies as well as proficiency on equivalence of operations the *p* values were 0.659 and 0.236 these values were not statistically significant. Therefore for these variables the study failed to reject the null hypothesis and therefore concluded that the β coefficients regarding these variables were not statistically significant; i.e., $\beta_1 = \beta_2 = 0$. Thus from this it can be established that the independent variables proficiency on counting and computational strategies as well as proficiency on equivalence of operations have no statistically significant impact on predicting the dependent variable academic performance in mathematics.

The multiple linear model of predicting the dependent variable (Y) academic performance in mathematics using the number sense independent variables that were statistically significant is:

$Y = 39.746 + 0.307x_1 + 0.055x_2 + 0.507x_3 + 0.082x_4 + 0.242x_5 + \varepsilon$. Where, Y, represent the dependent variable academic performance in mathematics, x_1, x_2, x_3, x_4, x_5 the independent variables of number sense proficiency (i.e. proficiency on meaning and size of numbers, proficiency on estimation using a relevant benchmarks and proficiency on meaning and effects of operations) whereas ε is the residual or standard error respectively.

A study carried out by Griffin, Case, and Siegler (1994) argues that number sense is in many cases informally acquired prior to formal schooling and is a necessary condition for learning formal arithmetic in the early elementary grades. It has been unclear, however, whether number sense plays a role in the uniquely human ability to learn higher mathematics such as algebra, calculus and trigonometry. A study by Stein (2008) involving 64 adult students found that among the students who did well on a test that measured their "number sense" were much more likely to have gotten good grades in math classes. However it was discovered that a person's ability to quickly estimate significantly predicts their performance in school mathematics. This study too, found out that for every 1% increase the standard deviation of the individual's abilities to carry out accurate estimations could increase the standard deviation of academic performance by 0.242%.

Placing the position of this study to the existing debate on the impact of number sense and the academic performance in mathematics, this study, using a full bouquet of regression analysis investigated the relationship between number sense of preservice secondary mathematics teachers and their academic performance in mathematics. Therefore in the light of the findings presented in this section it can be argued that the number sense of students to some extent bears an impact on the academic performance in mathematics.

The study established from the value of $R^2 = 0.219$ in Table 17 that number sense could only impact the performance of preservice secondary mathematics up to at most 22%, given that the other factors bearing effect on academic performance of pre-service secondary mathematics teachers are held constant.

One of the aims of this study is to establish if there was a relationship between the number sense of preservice secondary mathematics teachers and their academic performance in mathematics. Therefore, based on the findings of this study it can be argued that the impact of number sense on the academic performance in mathematics is medium sized suggesting that the impact of number sense proficiency on academic performance could not be under estimated.

5.5 The impact of a Critical Theory intervention on the development of number sense

This section presents the quantitative data regarding the impact of the Critical Theory intervention on the number sense of preservice secondary mathematics teachers based on the CRENS model. The section embarked on the quest for the answer to the research question: What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers?

The section is divided in two sub-sections; the first sub-section presents the comparison of the pre-test and post-test scores for both the number sense proficiency and the number sense reasoning using descriptive and inferential statistics.

The second sub-section presents the subsequent statistical tests for significance (sequence of t-tests) that were carried out to assess the impact of Critical Theory intervention on the number sense training of preservice secondary mathematics teachers.

5.5.1 An overview of the pre-test versus post-test results

This sub-section presents the descriptive statistics on the comparison of the pre-test and post-test results of preservice secondary mathematics teachers for both the control and the experimental groups.

For the number sense proficiency the comparison of frequencies of pre-test with the post-test Figure 5.12 gives a summary.

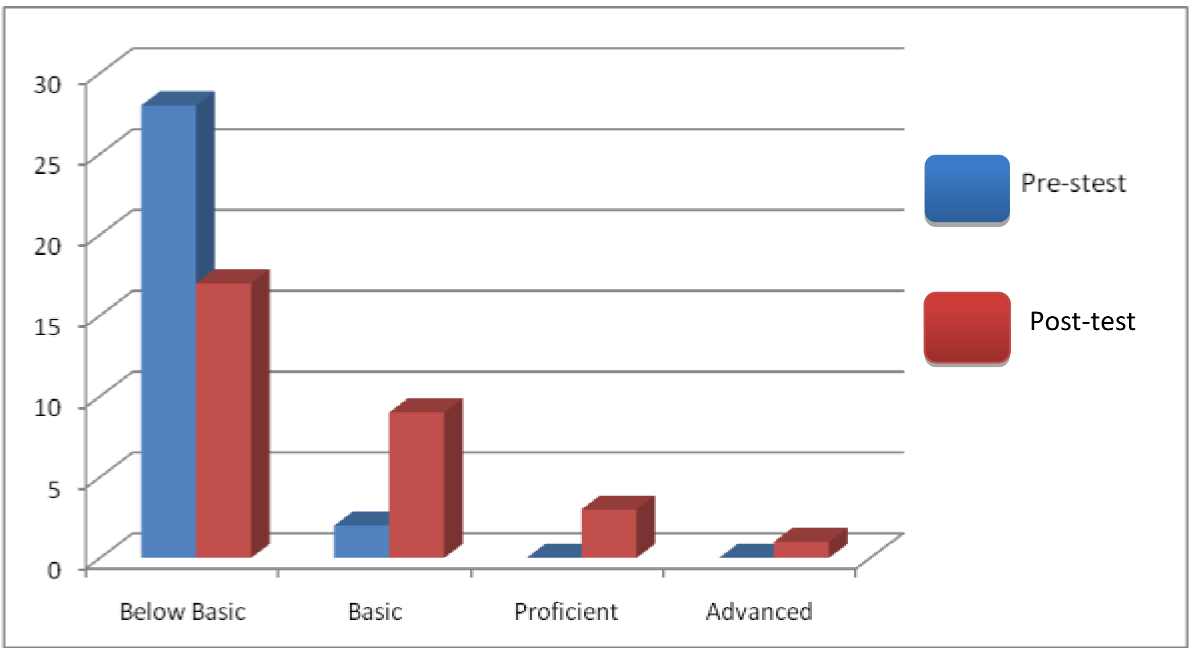

Figure 5.12: Comparison of frequencies of pre-test-post-tests of number sense proficiency levels for the experimental group

Figure 5.12 shows that the frequency of preservice secondary mathematics teachers who scored below basic is high. At least two thirds scored marks in the below basic level for the pre-test. The frequency of preservice secondary mathematics teachers who scored below basic scores however is reduced by at least third in the post-test.

Looking at Figure 5.12 from broader view there is an increase in the frequency of preservice teachers from below basic to proficient level. Therefore the results suggest an improvement in the number sense levels of comprehension after the treatment.

Figure 5.13 shows the comparison of the number sense reasoning levels of the pre-test and the post test for the control group.

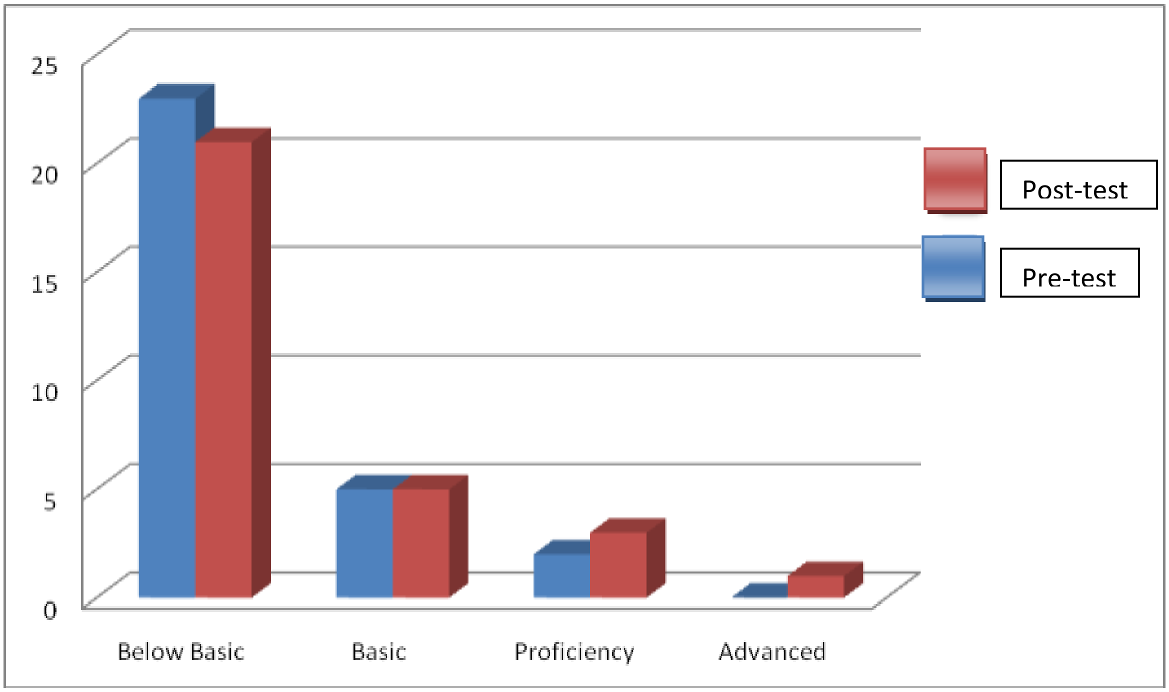

Figure 5.13: Comparison of pre- and post-test number sense reasoning levels for the control group

Figure 5.13 shows that the frequencies of number sense levels of post-test are by a very small margin higher than that of pre-test in the Proficient and the Advanced levels of number sense with less than 10%. At glance the comparison gives the impression that the participants performed fairly the same. However, as to whether these indifferences observed in terms of frequencies are statistically significant the next section will present these.

Figure 5.14 shows the comparison of the number sense reasoning frequencies of pre-test and post-test levels of number sense but for the experimental group.

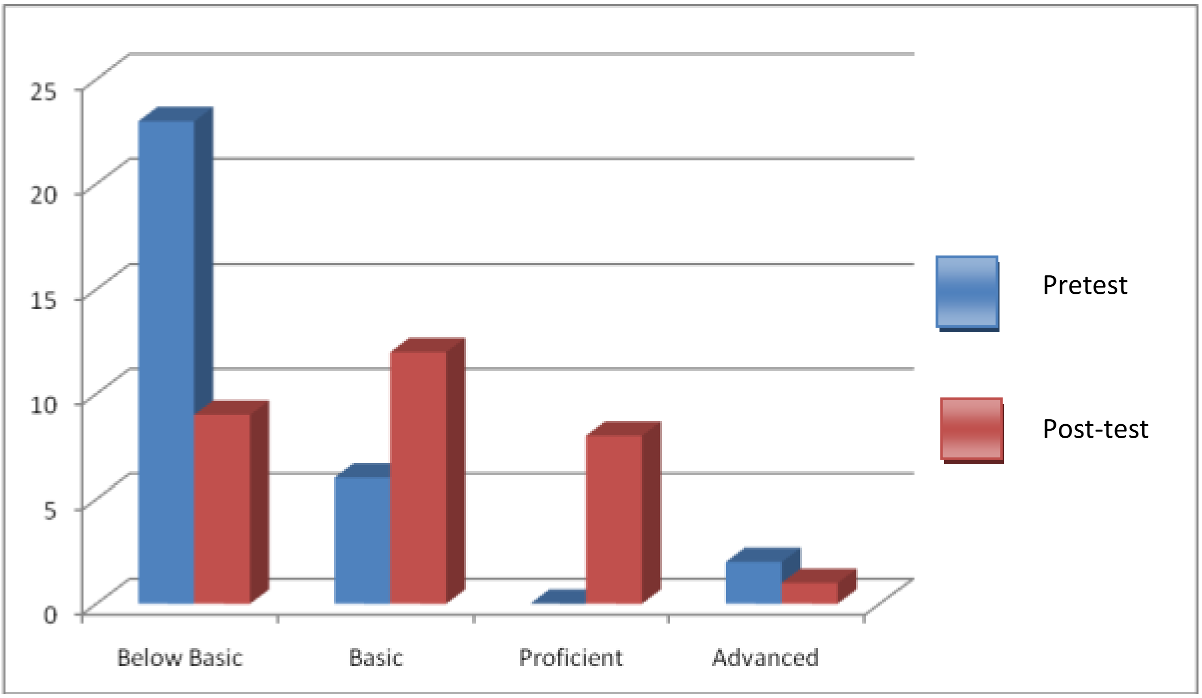

Figure 5.14: Comparison of pre- and post-test number sense reasoning experimental group

Figure 5.14 shows that the frequencies of post-test number sense resoning for the experimental group approximate to a normal distribution. However the pre-test number sense scores for the control group are skewed to the left. This sugests that most of the control group participants performed poorly in their post-test. It could be concluded that that there is a slight improvement in the number sense reasoning of the experimental group.

Figure 5.15 gives the five number summaries the preservice secondary mathematics teachers' pre- and post-test results on number sense proficiency for the control and the experimental groups.

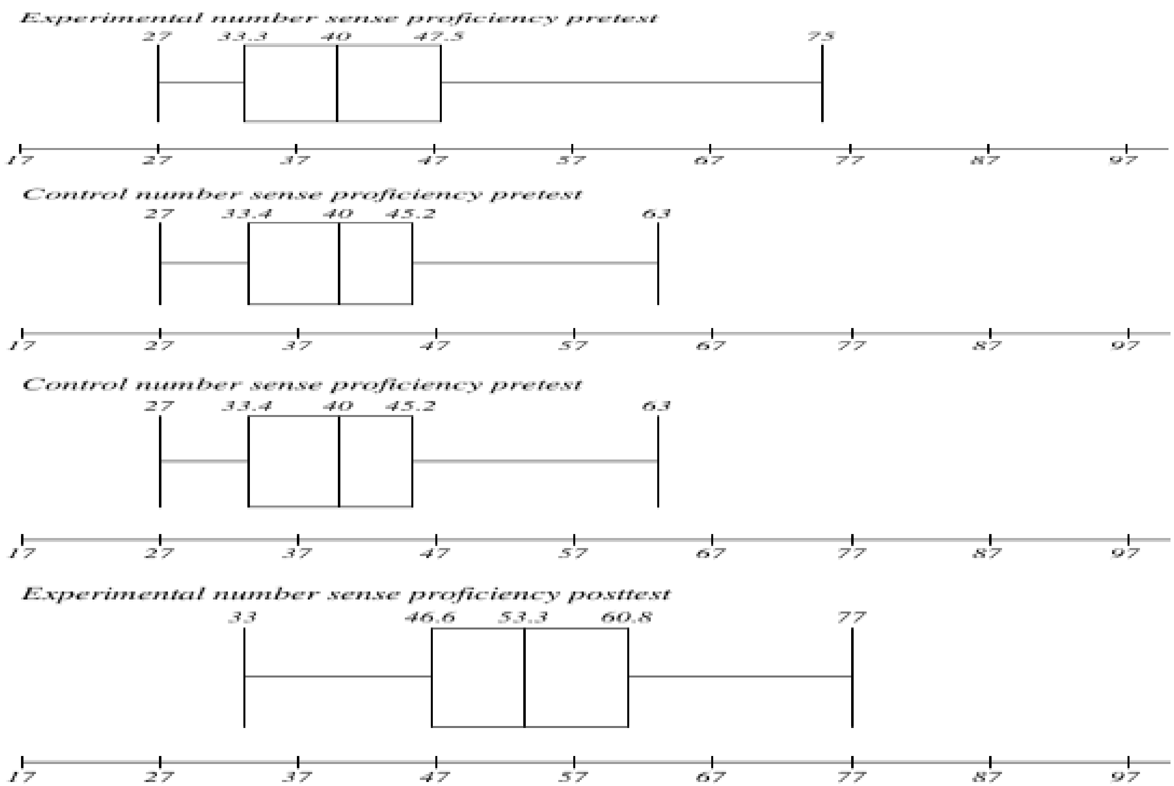

Figure 5.15: Comparison of five number summaries for number sense proficiency in the control and experimental groups

Figure 5.15 shows that the pre-test number sense proficiency scores for the control group have a minimum and maximum value of 27 and 63%, a lower and upper quartile of 33.4 and 45.5% and a median of 40%. The pre-test number sense proficiency scores for the experimental group have a minimum and maximum value of 27 and 75%, a lower and upper quartile of 33.4 and 47.5% while the median is also 40%. From this comparison it is clear that the performance of the pre-test number sense proficiency scores for both the control and the experimental groups was reasonably the same.

The post-test results for the experimental group show some observable differences when compared to the pre-test results of the control group. For instance there is a difference of 6% in the minimum values, a difference of 13% for both the median and lower quartile scores, a difference of 3.3% upper quartile values as well as a difference of 2% in the maximum scores.

All these differences were observed in favour of the post-test scores for the experimental group. These differences suggest an improvement in the performance of the pre-service secondary mathematics teachers who belonged to the experimental group. Therefore it could be argued that these differences were due to the treatment that was administered to the experimental group.

Figure 5.16 shows how the number sense reasoning scores of preservice secondary mathematics teachers were dispersed for both the control and experimental groups.

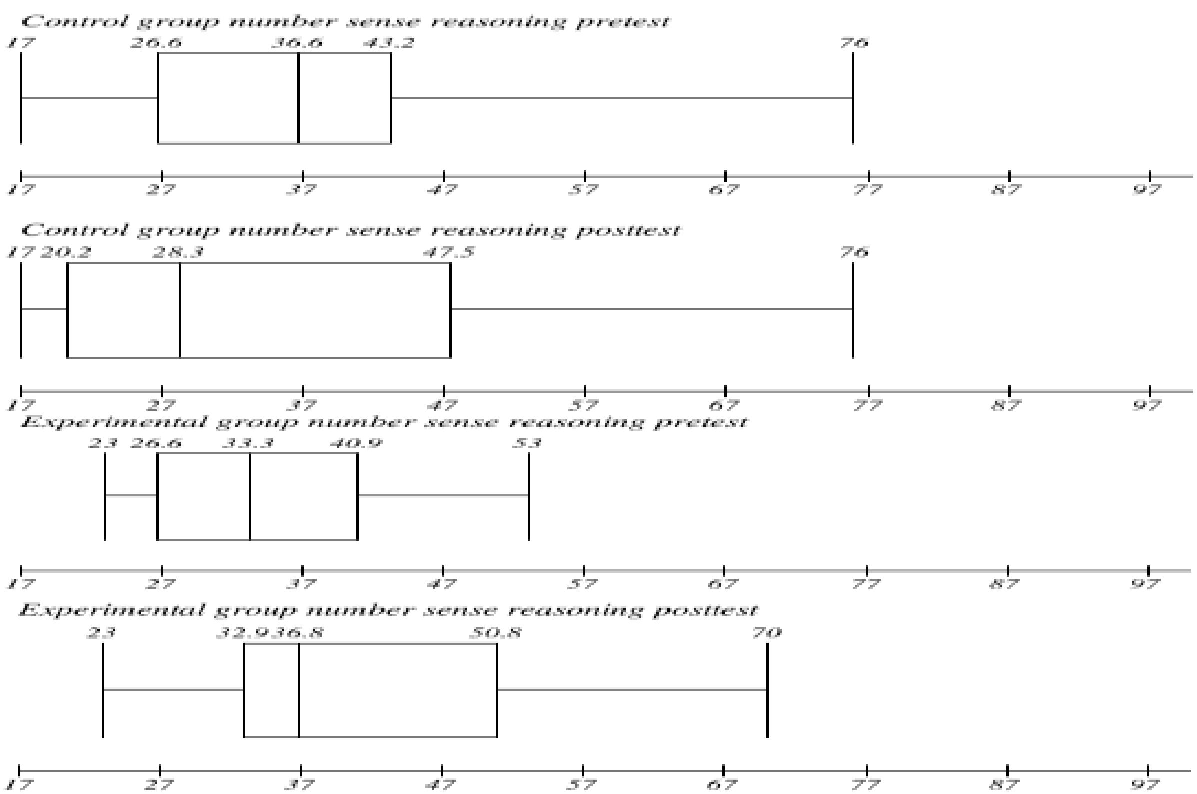

Figure 5.16: Five number summaries control versus experimental: number sense reasoning

Figure 5.16 shows that the pre-test number sense reasoning scores in the control group had a minimum and maximum value of 17 and 76%, lower and upper quartiles of 26.6 and 43.2%, while the median is 36.6%. For the post-test number sense reasoning scores the control group had a minimum and maximum value of 17 and

76%, a lower and upper quartile of 20.2 and 47.5% while the median was 28.3%.

This comparison shows trivial differences between the pre-test number sense reasoning scores for both the control and the experimental groups.

The comparison of the pre- and post-tests for the experimental groups had a minimum of 23% each; the lower quartiles of 33.3 and 36.8%, the medians of 33.3 and 36.8% the upper quartiles were 40.9 and 50.8%. These results show an improvement in favour of the post-test experimental groups.

The results presented in this sub-section show some improvement in favour of the post-test experimental group. From these results it can be concluded that impact of a Critical Theory intervention on the number sense training of pre-service secondary mathematics teachers seems to improve their number sense. The next sub-section presents the results regarding the tests for statistical significance that were carried out to assess the impact of a Critical theory intervention on the number sense training of pre-service secondary mathematics teachers.

5.5.2 Results of the statistical tests for significance on the impact of Critical Theory Intervention on the development of number sense

This section presents the results on the statistical tests for significance that were carried out to assess the significance of the impact of the Critical Theory intervention in the number sense training of preservice secondary mathematics teachers. The thrust of this section was to present the data to respond to the research question: What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers?

The section starts by presenting the dependent results of the four statistical tests of significance as suggested in the previous chapter, (see section 4.8) regarding the intervention that was given to the experimental group. The section then presents the independent t-test results for statistical significance on whether or not the preservice secondary mathematics teachers in the experimental group performed differently in their post-test.

5.5.2 Dependent t-test results on pre- and post-tests

This sub-section presents the results that respond to the question of what the impact of a Critical Theory intervention on the development of number sense training of preservice secondary mathematics teachers was. A series of *t*-tests for the dependent samples was carried out to test for the statistical significance of differences between the control and the experimental group means. Therefore this sub section presents the dependent t-tests conducted on the control group and those conducted on the experimental group.

5.5.2.1 Dependent tests administered to the control group

This section presents the t-tests conducted to the control group. These tests sought to test the following hypothesis:

Hypothesis

H_0: There is no significant difference between the pre-test number sense mean and post-test number sense mean for the control group ($\mu_1 = \mu_2$).

H_1: There is a significant difference between the pre-test number sense mean and post-test number sense mean scores for the control group ($\mu_1 \neq \mu_2$).

The summary of the test for significance in the mean number sense proficiency scores is shown in Table 5.20.

Table 5.20: Comparison of paired samples pre- and post-test results for number sense proficiency control group

		Mean	N	Std. Deviation	Std. E. Mean
Pair 1	Pre-test mean score	40.8333	30	9.43611	1.72279
	Post-test mean score	41.0333	30	12.95478	2.36521

Table 5.20 shows that the means for pre- and post-tests for the control group were 40.83 and 41.03% while the standard deviations were 9.43611 and 12.95478 respectively. This table suggests that the mean performance for the pre-test and post-test were more or less the same. Table 5.20 gives a further comparison of the pre-test and post-test scores for the number sense proficiency of the control group.

Table 5.21: Paired sample correlation pre- and post-test number sense proficiency control group

		N	Correlation	Sig.
Pair 1	Pre-test mean score & Post-test mean score	30	.765	.000

Table 5.21 illustrates a high correlation coefficient of 0.765 indicating a high association between the number sense proficiency pre-test and post-test scores. In addition the corresponding value of p = 0.000 shows that the correlation is statistically significant. Therefore from Table 5.21 the study concludes that there is a statistically significant high positive correlation between the pre- and post-test number sense test scores of the pre-service secondary mathematics teachers in the control group. This also suggests that the instrument is reliable since it measured what it was supposed to measure consistently.

From these highly correlated scores it can be claimed that there is a high chance that the performance of individuals to be the same for both the pre- and the post-test. That is; from the value of the coefficient of determination (r^2 = 0.585) there is a 58.5% chance that preservice secondary mathematics teachers who scored higher scores in the pre-test have scored high also in the post and vice-versa.

A further analysis on the test for the statistical significance on the mean is presented in Tables 5.22.

Table 5.22: Comparison of number sense proficiency pre- and post-test control group

		Paired Differences					t	df	Sig. 2-tailed
					95% CI				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	Pre-test total score - Post-test total score	.200	8.360	1.5263	-3.321	2.921	.131	29	.897

The data from Table 5.22 indicates that t (29) =.131, p = 0.897 is not statistically significant and therefore the study fails to reject the null hypothesis and concludes that the two means were not statistically significant; there is no statistically significant difference in the number sense proficiency of mean scores of the pre- and post-test for the control group.

Due to the mean difference and the of t it was concluded that there was a non-statistically significant improvement of 0.2% between the control group pre- and post-tests at 95% confidence level. Since the study concluded that there were no statistically significant differences in the two means it was not deemed necessary to compare whether there was statistical significance in different components of the number sense proficiency for both the pre-test and post-test control group.

The comparison of paired statistics for the comparison pre-test-post-test number sense reasoning results for the control group is summarised in Table 5.23.

Table 5.23: The paired sample statistics for the pre- and post-test total number sense reasoning scores of the control group

		Mean	N	Std. Deviation	Std. E. Mean
Pair 1	Pre-test total score	36.0333	30	9.20076	1.67982
	Post-test total score	36.3000	30	10.62252	1.93940

Table 5.23 shows the means of 36.0333 and 36.3000 as well as standard deviations 9.20076 and 10.62252 were observed for the pre- and post-test respectively. According to Table 5.23 it can be observed that the means for number sense reasoning of the pre- and post-test are the same with a mean difference of 0.2667%. To test whether there is a statistically significant association between the pre- and post-test number sense reasoning means of the participants in the control group Table 5.24 gives a further comparison of the pre-test and post-test scores for the number sense reasoning of the control group.

Table 5.24: The paired sample correlation for the pre-test-post-test number sense reasoning for the control group

		N	Correlation	Sig.
Pair 1	Pre-test total mean score & Post-test total mean score	30	.655	.000

Table 5.24 shows a correlation coefficient of 0.655 which according to the thumb rule as explained by MacMillan and Schumacher (2006) is regarded as a high positive

correlation. In addition the corresponding value of p = 0.000 which means the correlation is statistically the significant. From this observation the study rejects the null hypothesis which claims there is no association between the number sense test reasoning pre-test scores and post-test scores.

By rejecting the null hypothesis the study adopts the research hypothesis and concludes that there is a strong association between the number sense reasoning pre-test scores and post-test scores. From this correlation coefficient between the pre- and post-test scores and the coefficient of determination it can be deduced that there is a 42.9% chance that the preservice secondary mathematics teachers who scored lower scores in the pre-test of number sense reasoning from the control group will score lower scores also in the post and vice-versa.

A further analysis on the test for the statistical significance on the mean difference of the pre- and post-test number sense reasoning scores of the control group is presented in Table 5.25.

Table 5.25: Paired sample test for significant difference on pre-test-post-test number sense reasoning for the control group

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% CI Lower	95% CI Upper	t	df	Sig.(2-tailed)
Pair 1	Pre-test total score - Post-test total score	.2667	8.374	1.522	-3.379	2.8465	.175	29	.862

Table 5.25 indicates a mean difference of 0.2667 percentage points in favour of the post-test scores and a standard deviation of 8.374. From Table 5.25 it can also be inferred that since t (29) = 0.175, the value of p= 0.862, then the null hypothesis can be accepted. By accepting the null hypothesis it can be concluded that the two means did not statistically differ; i.e., there was no statistically significant difference between the mean pre-test and post-test number sense reasoning of the control group.

Deducing from the mean difference and t it was concluded that there was an improvement of 0.2667percentage points from the pre- to the post test that was not statistically significant at 95% confidence level. Since the study concluded that there were no statistically significant differences in the two means it was not deemed necessary to test for a statistical significance in different components of the number sense reasoning for both the pre-test and post-test control group.

5.5.2.2 Dependent t-tests administered to the experimental group

This section presents the statistical tests for significance on the differences in the means of the pre- and post-test number sense proficiency scores for the experimental group. The section sought to test for the following hypothesis:

Hypothesis

H_0: There are no significant differences between the mean pre-test and post-test number sense for the experimental group ($\mu_1 = \mu_2$).

H_1: There are significant differences between the mean pre-test and post-test number sense for the experimental group ($\mu_1 \neq \mu_2$).

The summary of the test for significance in the mean number sense proficiency scores is shown in Tables 5.26.

Table 5.26: Paired sample statistics on the number sense proficiency mean differences for the experimental group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre-test total score	41.1667	30	10.40253	1.89923
	Post-test total score	54.5667	30	10.29792	1.88013

Table 5.26 presents the paired sample statistics means for the pre- and post-test number sense proficiency for the experimental group. The means of 41.1667 and 54.5667 as well as standard deviations 10.40253 and 10.29792 were obtained for the pre and post-tests respectively. According to Table 5.26 it can be observed that the mean difference between the pre- and post-test number sense reasoning is 13.4% in favour of the post test.

Table 5.27 gives a further comparison of the pre-test and post-test scores for the number sense proficiency of the experimental group.

Table 5.27: Paired correlations on the number sense proficiency pre- and post-test scores of the experimental group

	N	Correlation	Sig.
Pair 1 Pre-test mean score & Post-test total score	30	0.252	.178

From table 5.27 is a weak correlation of 0.252 between the pre-test and post-test scores for the number sense proficiency of the experimental group was observed. The corresponding value of p = 0.178, therefore from this the study failed to reject the null hypothesis. By failing to reject the null hypothesis the study therefore rejected the research hypothesis and concluded that there was no statistically significant correlation between the number sense proficiency pre-test scores and post-test scores. This implies that the students who scored lower marks in the pre-test need not be the ones scoring lower marks in the post-test.

To test whether the differences observed in the means were statistically significant the results of the statistical test for significance are summarised in Table 5.28.

Table 5.28: Summary paired sample test pre- and post-number sense proficiency experimental group

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	SD	Std. Error Mean	95% CI of the Difference Lower	95% CI of the Difference Upper			
Pair 1 Pre-test total score - Post-test total score	13.40	12.656	2.310	8.674	18.125	5.79	29	.000

From the results on the tests of statistical significance presented in Table 5.28 it can be interpreted that t (29) = 5.799, p = 0.000 was statistically significant. Therefore the study rejects the null hypothesis and adopts the research hypothesis. By rejecting the null hypothesis the study concludes that there are statistically significant differences between the scores of pre- and post-test number sense proficiency of the experimental group.

Given the direction of the difference in the means and the direction of t it can be deduced that there is a statistically significant improvement of 13.4 % in the post-test scores of the number sense proficiency of the experimental group. The study henceforth concludes that the development of number sense based on Critical Theory, i.e. developing number sense through CRENS model had an effect of increasing the scores of preservice secondary mathematics teachers' number sense by about 13%.

Since the study concluded that the impact of Critical Theory on the development of number sense is statistically significant, it was deemed necessary to enquire into the statistical significance of each of the pre- and post-test components of number sense proficiency for the experimental group. The aim of this closer analysis was to identify the components of number sense proficiency where the differences in the pre- and post-test were statistically significant.

From the results of this analysis the researcher will then be able to establish which components of number sense proficiency were more affected by the Critical Theory intervention that was given to the participants in the experimental group. Table 5.29 shows the summary of paired sample statistics for the pre- and post- test number sense test scores.

Table 5.29: Paired sample statistics for the number sense proficiency components for the experimental group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre-test Score Meaning and Size of numbers	36.066	30	14.54347	2.65526
	Post-test Score Meaning and Size of numbers	57.666	30	19.97642	3.64718
Pair 2	Pre-test Score Equivalence of Operations	54.966	30	17.72099	3.23539
	Post-test Score Equivalence of Operations	61.566	30	15.68369	2.86344
Pair 3	Pre-test Score Meaning and Effects of Operations	49.100	30	17.32120	3.16240
	Post-test Score Meaning and Effects of Operations	57.266	30	15.08124	2.75344
Pair 4	Pre-test Score Counting and Computational Strategies	36.700	30	16.39838	2.99392
	Post-test Score Counting and Computational Strategies	49.500	30	21.59366	3.94245
Pair 5	Pre-test Score Estimation	29.400	30	12.78361	2.33396
	Post-test Score Estimation	48.900	30	19.52426	3.56463

Table 5.29 shows the mean differences in different components of pre- and post-test number sense proficiency. For the meaning and size of numbers the mean difference was 21.60%, equivalence of operations 6.6%, meaning and effects of operations 8.16%, counting and computational strategies 12.8 % and for the estimation is 19.50%.

For all the sections presented above the observed differences in the means were in favour of the post-test. Thus it can be observed that the post-test scores for the experimental group were higher than the pre-test scores. This suggests an improvement in the pre-test number sense proficiency scores.

Table 5.30 thus presents the paired sample correlations for each individual section for number sense proficiency.

Table 5.30 Summary of paired sample correlations for number sense proficiency components for the experimental group

		N	Correlation	Sig.
Pair 1	Pre-test Score Meaning and Size of numbers & Post-test Score Meaning and Size of numbers	30	.389	.021
Pair 2	Pre-test Score Equivalence of Operations & Post-test Score Equivalence of Operations	30	.092	.628
Pair 3	Pre-test Score Meaning and Effects of Operations & Post-test Score Meaning and Effects of Operations	30	.036	.472
Pair 4	Pre-test Score Counting and Computational Strategies & Post-test Score Counting and Computational Strategies	30	.221	.040
Pair 5	Pre-test Score Estimation & Post-test Score Estimation	30	.321	.002

Table 5.30 indicates that the correlation coefficients and the corresponding *p* values between the pre- and post-test of the number sense components were; for the meaning and size of numbers r = .389, p = .021, equivalence of operations r =.092, p = .628, for the meaning and effects of operations r =.036, p =.472, for the section on counting and computational strategies r =.221, p =.040 and for the estimation is r =.321, p = .002. These correlation coefficients values give an indication that the pre- and post-test of each number sense proficiency component were weak or very weak. The combined correlation was found to be (0.2318) which also suggest a weak correlation. This weak correlation suggests that these experimental group participants might not have performed the same in the pre- and post-tests.

Additionally, three of the sections (equivalence of operations, meaning and effects of operations as well as counting and computational strategies) the tests for statistical significance showed a weak association between the pre- and post-test number sense proficiency scores.

Table 5.31 indicates the paired samples test on the pre- and post-test number sense proficiency for the experimental group mean differences of each of the five individual components. In this case Table 5.31 establishes whether the differences in the means of each of the individual components of the pre- and post-test number sense proficiency presented were statistically significant or not.

Table 5.31: Summary of the paired sample dependent t-test on the pre- and post-test number sense proficiency for the experimental

		Paired Differences							
					95% CI of the Difference				
		Mean	SD	SE Mean	Lower	Upper	*t*	df	Sig. (2-tailed)
Pair 1	Pre-test Score Meaning and Size of numbers - Post-test Score Meaning and Size of numbers	21.60	21.034	3.840	29.454	13.745	5.624	29	.000
Pair 2	Pre-test Score Equivalence of Operations - Post-test Score Equivalence of Operations	2.800	24.722	4.513	22.031	3.568	2.836	29	.063
Pair 3	Pre-test Score Meaning and Effects of Operations - Post-test Score Meaning and Effects of Operations	4.166	20.296	3.705	15.745	.5879	2.204	29	.119
Pair 4	Pre-test Score Counting and Computational Strategies - Post-test Score Counting and Computational Strategies	8.600	22.516	4.110	15.007	1.807	1.605	29	.008
Pair 5	Pre-test Score Estimation - Post-test Score Estimation	19.500	21.829	3.985	-27.651	11.348	4.893	29	.000

Table 5.31 shows that for the meaning and size of numbers *t* (29) = 5.624, *p* = .000, equivalence of operations *t* (29) = 2.836, *p* = .063 meaning and effects of operations *t* (29) =1.605, *p* = .119 counting and computational strategies *t* (29) = 2.204, *p* = .008, and for the estimation is *t* (29) = 4.893, *p* =.000. For the sections meaning and effects of operations, as well as the equivalence of operations the *p* values were statistically significant, therefore for these the study rejected the null hypotheses and concluded that the differences in the pre- and post-test number sense proficiency means for these components were statistically significant. Thus for these sections the study

concluded that the impact of Critical Theory on the number sense training was statistically significant.

Hence for the components (meaning and size of numbers, counting and computational strategies and estimation) the observed differences in the values were considered to be statistically significant. For the components meaning and effects of operations as well as equivalence of operations the values of *p* did not fall within rejection region therefore the study failed to reject the null hypothesis. Thus for these two components it was concluded that there were no statistically significant differences. This means that the Critical Theory intervention did not have an impact on these sections.

Table 5.32 shows the paired *t*-test results for the tests for significance of pre- and post-test number sense reasoning for the experimental group.

Table 5.32: Summary of paired sample statistics means: pre- and post-test number sense reasoning for the experimental

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre-test mean score	35.0667	30	8.66994	1.58291
	Post-test mean score	47.4000	30	9.81413	1.79181

Table 5.32 presents the paired sample statistics means for the pre- and post-test number sense reasoning for the experimental group. Table 5.32 thus shows means of 35.0667 and 47.4000 as well as standard deviations 8.66994 and 1.79181 for the pre- and post-tests respectively. It can be observed that the mean difference between the pre- and post-test number sense reasoning is 12.3%. This difference was observed in favour of the post-test. This suggests an improvement in the post-test scores of pre-service secondary mathematics teachers in the experimental group.

Table 5.33 gives a further comparison of the pre-test and post-test scores for the number sense reasoning of the experimental group.

Table 5.33: The paired samples correlation for number sense reasoning pre- and post-test scores for the experimental group

		N	Correlation	Sig.
Pair 1	Pre-test mean score & Post-test mean score	30	.500	.005

Table 5.33 indicates that the pre- and post-test number sense reasoning scores were correlated by r = 0.500 which according to the rule of the thumb is a moderate correlation. The value of p = 0.005 indicating that there is moderate correlation between the pre- and post-test number sense reasoning scores. Thus it can be deduced from the coefficient of determination that there is a 25% chance that the individuals could repeat the same state of performance in the post-test.

A further analysis of whether the difference of 12.3% observed between the pre- and post-test mean number sense reasoning scores of the experimental group is presented in Table 5.34.

Table 5.34: Summary statistics of paired samples tests pre- and post-test number sense reasoning scores for the experimental group

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% CI of the Difference Lower	95% CI of the Difference Upper			
Pair 1 Pre-test total mean score - Post-test total mean score	12.333	9.293	1.696	8.861	15.803	7.26	29	.000

Table 5.34 indicates a mean difference of 12.3 % in favour of the post-test scores and the standard deviation of 9.293. From the results presented in Table 5.34 it can also be deduced that for the mean difference the t value, t (29) = 7.269, at p= 0.000, which implies that the study rejects the null hypothesis which claims that there are no statistically significant differences in the pre- and post-test number sense means for number sense reasoning. Thus the study adopts the research hypothesis and concludes that there are statistically significant differences in the pre- and post-test

number sense means for number sense reasoning.

The study established that there were statistically significant differences in the number sense reasoning means. It was therefore deemed necessary to test for a statistical significance in different components of the number sense reasoning for both the pre-test and post-test experimental group.

Table 5.35 shows the paired samples statistics for the different sections of the five number sense reasoning components. Table 5.35 aims at testing for the statistical significance of each between the pre- and post-test for the experimental group.

Table 5.35: Paired samples statistics for different components of number sense reasoning pre- and post-test scores for the experimental group

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre-test Score Meaning and Size of numbers	23.466	30	10.122	1.848
	Post-test Score Meaning and Size of numbers	44.400	30	20.674	3.774
Pair 2	Pre-test Score Equivalence of Operations	50.466	30	17.371	3.171
	Post-test Score Equivalence of Operations	61.733	30	15.319	2.796
Pair 3	Pre-test Score Meaning and Effects of Operations	45.533	30	18.552	3.387
	Post-test Score Meaning and Effects of Operations	51.133	30	16.384	2.991
Pair 4	Pre-test Score Counting and Computational Strategies	31.133	30	14.896	2.719
	Post-test Score Counting and Computational Strategies	38.366	30	21.342	3.896
Pair 5	Pre-test Score Estimation	24.466	30	8.11866	1.4822
	Post-test Score Estimation	41.133	30	18.4030	3.3599

Table 5.35 shows paired means in different sections of pre- and post-test number sense reasoning. For example in the section of the meaning and size of numbers the mean difference is 20.9%, equivalence of operations 11.3%, meaning and effects of operations 5.6%, counting and computational strategies 7.2 % and for estimation is 16.7%.

All the number sense reasoning sections presented above the observed differences in the means were in favour of the post-test. This suggests that there is an improvement in the post-test scores of number sense reasoning. To test whether this improvement was statistically significant, the individual results of *t*-test for statistical significance are presented as follows. Table 5.36 presents the paired sample correlations for each individual component for pre- and post-test number sense reasoning.

Table 5.36: Summary of paired sample correlations for each individual component for pre and post-test number sense reasoning

		N	Correlation	Sig.
Pair 1	Pre-test Score Meaning and Size of numbers & Post-test Score Meaning and Size of numbers	30	.132	.488
Pair 2	Pre-test Score Equivalence of Operations & Post-test Score Equivalence of Operations	30	.197	.297
Pair 3	Pre-test Score Meaning and Effects of Operations & Post-test Score Meaning and Effects of Operations	30	.096	.118
Pair 4	Pre-test Score Counting and Computational Strategies & Post-test Score Counting and Computational Strategies	30	.238	.015
Pair 5	Pre-test Score Estimation & Post-test Score Estimation	30	.291	.016

Table 5.36 indicates that the values for the correlation coefficient between the pre- and post-test of the number sense sections were as follows: for the meaning and size of numbers r =.132, *p*= .488, equivalence of operations r =.197, *p* = .297, for the meaning and effects of operations r =.096, *p* =.118, for the section on counting and computational strategies r =.238, *p* = .015 and for the estimation is r =.291, *p* = .016. The values for these correlation coefficients appear to give an indication that the pre and post-test correlation coefficients of each individual number sense reasoning sections like those of number sense proficiency seem to possess a weak relationship to each other approximating their combined correlation which is also weak.

Additionally, deducing from the corresponding values of *p* to each of the correlation coefficient values it can be seen that three of the sections (equivalence of operations, meaning and size of numbers as well as meaning and effects of operations) the tests for statistical significance did not show a statistically significant association between the pre- and post-test number sense proficiency scores. The other two sections (estimation as well as the counting and computational strategies) the correlation was considered statistically significant.

Table 5.37 indicates the results of the paired sample test on the pre- and post-test number sense reasoning for the experimental group mean differences of each of the five individual sections. Table 5.37 establishes whether the differences in the means of each of the individual sections of the pre- and post-test number sense reasoning sections presented were statistically significant or not.

Table 5.37: Summary of experimental group t-test results for paired samples in number sense reasoning scores

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	SD	Std. Error Mean	95% C I of the Difference Lower	95% C I of the Difference Upper			
Pair 1	Pre-test Score Meaning and Size of numbers - Post-test Score Meaning and Size of numbers	20.933	21.790	3.978	12.796	29.070	5.26	29	.000
Pair 2	Pre-test Score Equivalence of Operations - Post-test Score Equivalence of Operations	11.266	20.777	3.793	3.508	19.025	2.97	29	.006
Pair 3	Pre-test Score Meaning and Effects of Operations - Post-test Score Meaning and Effects of Operations	5.600	20.868	3.810	2.192	13.392	1.47	29	.152
Pair 4	Pre-test Score Counting and Computational Strategies - Post-test Score Counting and Computational Strategies	7.233	22.935	4.187	1.331	15.797	1.72	29	.095
Pair 5	Pre-test Score Estimation - Post-test Score Estimation	16.666	20.811	3.799	8.895	24.437	4.38	29	.000

Table 5.37 illustrates the respective *t* and *p* values; for meaning and size of numbers t (29) = 5.26, p = .000, equivalence of operations t (29) = 2.97, p = .006 meaning and effects of operations t (29) =1.47, p = .152 counting and computational strategies t (29) = 1.72, p = .095, and for the estimation is t (29) = 4.38, p =.000.

It can further be inferred that for the components for meaning and size of numbers, equivalence of operations, and for the estimation the *p* values were statistically significant, therefore for these, the study rejected the null hypotheses and concluded that the differences in the pre- and post-test number sense proficiency means for these particular components were statistically significant. Hence for those components where the observed mean differences were considered to be statistically significant, it can be concluded that the Critical Theory and in particular the CRENS model developed from it had a significant impact on the development of number sense.

For the components meaning and effects of operations as well as counting and computational strategies, the corresponding values of *p* were not statistically significant. Therefore for these the study accepted the null hypothesis and concluded that the impact of the CRENS and the Critical Theory did not have a statistically significant impact for these sections.

This subsection presented the tests for statistical significance to assess whether or not the Critical Theory intervention had a statistically significant impact on the development of number sense. The tests conducted and the results presented in this section give the impression that the Critical Theory and the CRENS model had to some extend impacted the development of number sense of pre-service secondary mathematics teachers. The next subsection presents the independent tests carried out to assess the significance of the differences that exist in the means of the control and experimental groups.

5.5.3 Independent t-test results on experimental versus control groups

This sub-section presents the results of tests for statistical significance of the independent *t*-test between the experimental and control groups. The first part of this sub-section presents the comparison of number sense proficiency as well as number sense reasoning results regarding the independent *t*-test for statistical significance that was performed on the pre-tests. The first part presents results for statistical significance performed on the following hypothesis:

5.5.3.1 Independent t-test results regarding the pre-tests for the control and experimental groups

This section presents the t-test result regarding the number sense pre-tests that were administered to the control and experimental groups. The section sought to test the following hypothesis:

Hypothesis

H_0: There are no significant differences between the mean pre-test number sense for the control and experimental groups ($\mu_1 = \mu_2$).

H_1: There are differences between the mean pre-test number sense for the control and experimental groups ($\mu_1 \neq \mu_2$).

Table 5.38: Group statistics for pre-test number sense proficiency of experimental and control group

Control Experiment		N	Mean	Std. Deviation	Std. Error Mean
Number sense proficiency pre-test Control versus Experimental	Control	30	41.0333	12.95478	2.36521
	Experiment	30	41.1667	10.40253	1.89923

Table 5.38 indicates the group statistics for the pre-test number sense proficiency for the control and the experimental groups. The table indicates the means of 41.0333 and 41.1667 as well as standard deviations of 12.95478 and 10.40253 for the control and experimental groups respectively. From this a mean difference of 0.13% can be deduced this suggests that the two groups performed fairly the same.

Table 5.39 presents the results of statistical tests for significance of the difference in the means of the control and experimental groups.

Table 5.39: Summary independent sample t-test pre- and post-test number sense proficiency experimental versus control group

		Levene's Test for Equality of Variances		*t*-test for Equality of Means						
		F	Sig.	*T*	df	Sig. 2-tailed	MD	Std. Error D	95% CI of the Difference Lower	Upper
Number sense proficiency pre-test Control versus Experimental	Equal variances assumed	2.14	.149	.044	58	.965		3.0333	5.938	6.205
	Equal variances not assumed			.044	55.5	.965	.133	3.0333	5.944	6.211

According to the Levene's test for equal variance the corresponding *p* value for equal variance test is *p* = 0.149, which indicates that there are no statistically significant differences in the variances. This implies that the variances are equal. Thus Table 5.39 indicates that assuming the equality of variances the *t* value is *t* (58) = 0.044, at *p*= 0.965 which is statistically not significant. Therefore for the mean differences the study fails to reject the null hypothesis and ultimately rejects the research hypothesis. It can therefore be concluded that there was a statistically non-significant difference of 0.13 percentage points in the pre-test number sense proficiency means for the control and the experimental group.

An independent *t*-test for statistical significance on the pre-test for the experimental and control groups was carried out.

Table 5.40: Group statistics for pre-test number sense reasoning of experimental and control groups

	Control Experiment	N	Mean	Std. Deviation	Std. Error Mean
Number sense reasoning pre-test Control versus Experimental	Control	30	36.033	9.200	1.679
	Experiment	30	35.066	8.669	1.582

Table 5.40 presents the group statistics for the pre-test number sense reasoning for the control and the experimental groups. From this table it can be observed that the means of 36.033 and 35.06 as well as standard deviations of 12.954 and 10.402 were obtained for the control and the experimental groups respectively. A mean difference of 0.97% can be deduced in favour of the control group from Table 5.40. Table 5.41 presents the results of statistical tests for significance of the difference in the means of control and experimental groups.

Table 5.41: Independent samples t-test results on number sense reasoning in the pre-test for the control and experimental groups

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% CI of the MD	
		F	Sig.	t	df	Sig. 2-tailed	MD	SE.	Lower	Upper
Number sense reasoning pre-test Control versus Experimental	Equal variances assumed	.179	.674	.419	58	.677	.966	2.308	3.63	5.58
	Equal variances not assumed			.419	57	.677	.966	2.308	3.67	5.58

Table 5.41 illustrates that the Levene's test for equality of variances yields p = 0.674, which implies that there were no statistically significant differences in the variances, thus the test assumes equality of variances. Table 5.41 also indicates that t (58) = 0.419, p= 0.677. Accordingly the study accepts the null hypothesis and concludes that there is a statistically non-significant difference of 0.97% in the pre-test number sense reasoning means for the control and the experimental group.

5.5.3.2 Independent t-test results regarding the post-tests for the control and experimental groups

This section presents the independent t-test results regarding the post-tests that were administered to the control and experimental groups. The section sought to test for the hypothesis:

Hypothesis

H_0: There is no difference between the post-test number sense means for the control and experimental groups ($\mu_1 = \mu_2$).

H_1: There is a significant difference between the post-test number sense means for the control and experimental groups ($\mu_1 \neq \mu_2$).

Table 5.42 presents the results of the group statistics for the post-test number sense proficiency for the control and the experimental groups.

Table 5.42: Group statistics for the post-test on number sense proficiency for the control and the experimental groups

	Control Experiment	N	Mean	Std. Deviation	Std. Error Mean
Number sense proficiency post-test Control and Experimental	Control	30	41.0333	12.95478	2.36521
	Experimental	30	54.5667	10.29792	1.88013

Table 5.42 shows the means of 41.033% and 54.567% as well as standard deviations of 12.95 and 10.29 for the control and the experimental groups respectively. Table 5.42 also shows a mean difference of 13.53% in favour of the experimental group. From this mean difference it can be deduced that on average the participants of the experimental group performed better than those of the control group by 13.5%.

Table 5.43 presents the results of statistical tests for significance of the difference in the means of control and experimental groups.

Table 5.43: Summary of tests for statistical significance in the differences between the post-test means of the control and experimental groups

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. 2-tailed	Mean Difference	Std. Error D.	Lower	Upper
Number sense proficiency post-test Control versus Experimental	Equal variances assumed	1.82	.182	4.47	58	.000	13.53	3.02	7.48	19.58
	Equal variances not assumed			4.47	55.1	.000	13.53	9.58	3.02	7.47

Table 5.43 illustrates the Levene's test for equal variance *p* = 0.182, which shows that there were no statistically significant differences in the variances of number proficiency post-test mean scores for the experimental and control groups. Henceforth, the results in this table are presented on the assumption of equal variances. The results presented in Table 5.43 also indicate that *t* (58) = 4.47, *p* = 0.000, which was statistically significant. Accordingly the study rejects the null hypothesis and adopts the research hypothesis to conclude that there was a statistically significant difference of 13.53% in the post-test number sense proficiency means of the control and experimental groups even though there was no difference in variances.

The study also performed a test for statistical significance of the differences in the post-test number sense reasoning means of the control and experimental groups.

Table 5.44: Summary of group statistics for the post-test number sense reasoning means: control versus experimental group

Control Experiment		N	Mean	Std. Deviation	Std. Error Mean
Number sense reasoning post-test Control versus Experimental	Control	30	36.30	10.622	1.939
	Experiment	30	47.40	9.814	1.791

Table 5.44 shows the means of 36.30% and 47.40% as well as standard deviations of 10.622 and 9.814 for the control and the experimental groups respectively. Table 5.44 also shows a mean difference of 11.10% in favour of the experimental group. A summary of tests on the statistical significance of the 11.10% mean difference between the post-test number sense reasoning of the experimental and the control group is presented in Table 5.45.

Table 5.45: Summary of independent samples t-test for post-test on number sense reasoning means of the control and experimental groups

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% CI of the Difference	
		F	Sig.	t	df	Sig. 2-tailed	MD	Std. Error D.	Lower	Up-per
Number sense reasoning post-test Control and Experimental	Equal variances assumed	.219	.641	4.20	58	.000	11.1	2.64	-16.38	-5.81
	Equal variances not assumed			4.20	57.6	.000	11.1	2.64	-16.38	-5.81

Table 5.45 illustrates the Levene's test assuming for equality of variances p = 0.641, which denotes that there were no statistically significant differences in the variances. Henceforth, the results are presented on the assumption of equal variances. Assuming the equality of variances Table 5.45 indicates t (58) = 4.20, p= 0.000, which is statistically significant. Accordingly, the study rejects the null hypothesis and

concludes that there was a statistically significant difference of 11.10% between the two means.

To sum up this section the tests for significance carried out on the number sense proficiency of the experimental and control groups were summarised in two tables. Table 5.46 shows the summary of the scores for number sense proficiency for the experimental and control groups.

Table 5.46: Summary of mean scores for number sense proficiency for experimental and control group

Component	Experimental					Control			
	Pre-test	Post-test	t-critical. For α=0.05	t-calc.	Accept/Reject H_0	Pre-test	Post-test	t-calc	Accept/Reject H_0
The meaning and size of numbers both rational and irrational numbers	36.1	57.7	1.96	4.78	Reject	37.6	45.0	1.54	Accept
Equivalence of numbers both rational and irrational numbers	55.0	67.8	1.96	2.96	Reject	53.9	52.1	0.35	Accept
Meaning and effects of operations;	48.1	57.2	1.96	1.97	Reject	44.5	47.1	0.47	Accept
Counting and computational strategies	36.7	43.3	1.96	1.33	Accept	40.6	37.	1.04	Accept
Estimation using relevant benchmarks without calculating	29.4	54.5	1.96	4.58	Reject	27.9	30.1	0.51	Accept

Table 5.46 shows the number sense proficiency means for the experimental and control groups. Table 5.46 suggests that for the experimental group the post-test means are higher than those of the pre-test in all five components. For the control group the means fluctuate between the pre and post-tests. The largest difference in the means of 25.1% between the pre- and post-test of the experimental group was recorded in the estimation component and the least difference of 6.6% was observed

in the counting and computational strategies.

For the control groups the mean difference between the pre- and post-test does not seem to follow one direction, as is with the experimental group. However, comparing the mean performance of the post-tests for both the experimental and the control groups it could be observed that the means for the control groups are lower than those of the experimental group.

The *t*-test pre and post test results indicate that there statistically significant differences in the number sense proficiency in four components except for the Counting and computational strategies where the study failed to reject the null hypothesis. For the control group the t-test results show that there were no statistically significant differences in all sections before and after the intervention. From these it could be deduced that the Critical Theory intervention had an impact on the number sense training of pre-service secondary mathematics teachers.

Table 5.47 indicates the t-test results that were carried out to test for the significant differences in the means for the number sense reasoning components. Table 5.47 sought to establish whether the Critical Theory intervention had an impact on the number sense reasoning of preservice secondary mathematics teachers.

Table 5.47: Comparison of number sense reasoning means for experimental and control groups

Component	**Experimental**					**Control**			
	Pre-test (%)	**Post-test (%)**	**t-critical. For α=0.05**	**t-calc.**	**Accept /Reject H_0**	**Pre-Test (%)**	**Post-test (%)**	**t-calc**	**Accept /Reject H_0**
The meaning and size of numbers both rational and irrational numbers	33.4	36.6	1.96	4.98	Reject	31.6	42.3	1.89	Accept
Equivalence of numbers both rational and irrational numbers	33.5	52.5	1.96	2.66	Reject	42.9	42.7	1.59	Accept
Meaning and effects of operations;	29.4	42.5	1.96	1.23	Accept	44.5	42.2	0.33	Accept
Counting and computational strategies	22.5	45.9	1.96	1.52	Accept	31.7	26.6	1.32	Accept
Estimation using relevant benchmarks without calculating	25.2	52.5	1.96	4.53	Reject	23.5	27.3	1.14	Accept

In Table 5.47 the number sense reasoning means for the experimental group suggest that the post-test means are higher than those of the pre-test in all five components. For the control group the mean difference fluctuate between the pre- and post-tests. The largest mean difference of 27.3% was recorded between the pre and post-test of the experimental group in the estimation section while the least mean difference of 6.6% was observed in the counting and computational strategies.

Table 5.47 for the pre and post-test number sense reasoning results indicate that there statistically significant differences in the means of number sense reasoning three components except for the counting and computational strategies as well as Meaning and effects of operations; where the study failed to reject the null hypothesis. For the post test the t-test results show that there were no statistically significant differences in the means for the number sense reasoning of preservice secondary mathematics teachers before and after the intervention. Hence it could be concluded that the Critical Theory intervention did not make a significant impact on the number sense reasoning training in these areas.

This section presented results for tests of statistical significance on the pre- and post-test number sense proficiency and reasoning. It was observed that both the number sense proficiency and reasoning were statistically significant for the experimental and the control groups. Hence the Critical Theory had an impact on both the number sense proficiency and reasoning among the participants in the experimental groups.

The effect sizes for the CRENS based intervention were also worked out by using Cohen's *d* and these are summarised in Table 4.48.

Table 5.48 results of the Cohen's *d* effect size

Number sense Component	Meaning and size of numbers	Equivalence of different representations of numbers	Meaning and effect of operations	Counting & computational strategies	Estimation with relevant benchmarks	Overall Effect
Cohen's *d* for number sense Proficiency	1.25	0.22	0.15	0.34	1.55	1.3
Effect size	Large	Small	Trivial	Small	Very large	Very large
Cohen's *d* for number sense Reasoning	0.2	0.16	0.23	1.29	0.76	1.34
Effect size	Small	Trivial	Small	Large	Medium	Very large

For the number sense proficiency components meaning effect of numbers and Estimation with relevant benchmark the effect size was very large, the small effect size was noticed in counting and computational strategies and equivalence of different representations of numbers. The overall effect size was also found to be very large.

For the number sense reasoning components the study found a large effect on counting and computational strategies as well as estimation with a relevant benchmark. A small effect size was observed on meaning and size of numbers as well as on the meaning and effects of operations component. For counting and computational strategies while for equivalence of different representation of numbers the effect size was found to be trivial. The overall effect size on the number sense proficiency was found to be very large.

5.6 Summary

This chapter presented the quantitative data for the study; these quantitative data presented in this chapter were triangulated with the qualitative data presented in chapter 6.

Based on the data presented about the level of number sense comprehension of preservice secondary mathematics teachers it can be claimed that the preservice secondary mathematics teachers' number sense was below basic and after the intervention it improved to the basic level.

On the subject of the relationship between the number sense of preservice secondary mathematics teachers the study found moderate positive correlations of 0.486 and 0.374 for number sense proficiency and reasoning respectively, which were still statistically significant.

The multiple correlation coefficient (R) between the set of independent variables of number sense proficiency was found to be R = 0.484 which was also a positive moderate correlation. From this value or R using the coefficient of determination (R^2) it could claimed that the combination of number sense proficiency variables that 23% of the variations in academic performance could be accounted for by a combination of the five independent predictor variables of number sense proficiency.

The number sense reasoning variables were not considered to bear a statistically significant impact of the academic performance of preservice secondary mathematics teachers.

The dependent t-tests for statistical significance showed that there were no statistically significant differences in the control group while for the experimental groups there were statistically significant differences observed. The independent t-test performed on the pre-test number sense proficiency and reasoning for the experimental and control group showed non-statistically significant differences. It was also observed that the both the number sense proficiency and reasoning post-test were statistically significant for the experimental and the control group. The collected data thus gave the impression that the CRENS framework improved the number sense of pre-service secondary mathematics teachers. The next chapter presents the qualitative data for the study.

CHAPTER SIX
PRESENTATION AND INTERPRETATION OF QUALITATIVE DATA

6.1 Introduction

This chapter presents the qualitative data for the study. The qualitative data presented in this chapter were triangulated with the quantitative data presented in chapter 5, thus here and there it refers to similar sections of chapter 5 where the qualitative data on the same subject were presented. As indicated in section 5.1 the reasons of presenting data in two chapters have been given in the earlier chapters (see section 4.2.1.3).

The chapter presents three sections according to the research questions:

1. What was the level of number sense comprehension of first year pre-service secondary mathematics teachers before the intervention?
2. What was the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics?
3. What was the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers?

In pursuit of the answers to the foregoing questions the chapter presents the qualitative data in three themes: analysis of qualitative results regarding the level of number sense comprehension, analysis of qualitative results regarding the relationship between number sense and academic performance in mathematics as well as the analysis of qualitative results regarding the impact of a Critical Theory intervention. Each of these themes addresses the research questions posed in that section.

It should also be understood that Critical Theory is characterised by a critical nature. It could not be sufficient therefore to just stop at which students provided correct or wrong answers. Thus by applying Critical Theory to the study it also attempted to understand why the students responded the way they did. Hence in this chapter the thrust is more on what influenced the thinking of the students as they went through the test items, why did they preferred that particular answer as opposed to the other.

6.2 The level of number sense comprehension prior to the intervention

This section presents the qualitative data regarding the level of number sense comprehension for the pre-service secondary mathematics teachers. The qualitative data presented in this section aim to provide the answer to the question: What is the level of number sense comprehension of first year pre-service secondary mathematics teachers before the intervention? The section presents the qualitative data from the number sense pretest, lesson observations as well as the interview. Therefore this section is divided into subsections that present qualitative data drawn from each of the instruments presented above (see section 4.5).

As referred to in section 4.9.2.1 a rubric was developed to explain and give a qualitative meaning to the number sense competencies portrayed by the four levels of number sense. Therefore this rubric (see section 4.9.2.1) was utilised to add a qualitative meaning to the number sense levels illustrated in section 5.2. Consequently, the following qualitative interpretations were made:

- ❖ Pre-service secondary mathematics teachers possessed a level of number sense that was below basic of the meaning and size of number sense proficiency and reasoning.
- ❖ Preservice secondary mathematics teachers also experienced difficulties or struggled to recognise equivalence of numbers, the quantitative data could estimate the level that is just basic.
- ❖ Pre-service secondary mathematics teachers had a very minimum or no understanding was demonstrated and often participants struggled to recognise and understand the meaning and effect of operations, hence their level of number sense in this area is below basic as it appeared from the qualitative data presented herein.
- ❖ Pre-service secondary mathematics teachers lacked knowledge of counting and computing strategies and relied on a calculator heavily to do most if not all the sums, for this section the level could be estimated at the basic level.
- ❖ Pre-service secondary mathematics teachers could neither make any reasonable estimation nor identify a benchmark as a reference point of their

estimation, for this item the preservice secondary mathematics teachers could be estimated to be at the basic level.

In addition the study also gathered (by means of the think through tier in the number sense test) the work of the pre-service secondary mathematics teachers. This work was examined and any information related to or depicting the level of number sense of pre-service secondary mathematics teachers was extracted and therefore reviewed to examine the number sense presented in their responses. The researcher chose the wrong and correct answers produced by students in order to understand the thinking that led to such responses.

The study utilised the following themes as recommended by MacMillan & Schumacher (2006) to analyse the responses of the students:

WA: **W**rong **A**nswer. This theme was utilised in the event the respondents provided a wrong answer. The researcher's interest at this juncture was to understand why the student provided a wrong answer but not to just the wrong answer itself. Therefore this theme is analysed into the following sub-themes:

WCU: This refers to the **W**rong answer with some **C**onceptual **U**nderstanding. In this case the student provided the wrong answer and in addition still shows some number sense understanding.

WAA: **W**rong **A**nswers with no conceptual understanding but merely **A**lgorithms. In this case the student provided the wrong answer and has merely followed algorithms to work out the number sense items.

WGA: **W**rongly **G**uessed **A**nswer. In this case the student merely guessed the answer wrongly.

CA: This refers to the **C**orrect **A**nswer provided by the student. In the same vein the researcher went a step ahead to understand why the student provided the correct answer. Hence under this theme there are sub-themes as follow.

CCU: This refers to the **C**orrect answer with **C**onceptual **U**nderstanding.

CAA: This refers to the **C**orrect **A**nswer but the student provided mere **A**lgorithms.

CGA: This refers to the situation where the student has given a **C**orrect answer but there is no work to support the correct answer. This was referred to as a **C**orrect **G**uessed the **A**nswer.

In order to analyse the responses the researcher aligned the responses of the students according to the five number sense components. Therefore the rest of this subsection presents the sample of responses of pre-service secondary mathematics teachers' responses to the number sense pre-test items in the five sections which formed the number sense domains number sense:

6.2.1 Qualitative results regarding the responses of participants in the pre-test

This section presents the qualitative data regarding the level of number sense comprehension in terms of the responses of pre-service secondary mathematics teachers to the number sense pre-test.

6.2.1.1 Section 1 of pre-test-post-test on the meaning and size of numbers component

The pre-service secondary mathematics teachers were asked to state which fraction is larger than the other between $\frac{8}{15}$ and $\frac{3}{7}$, for this question, the pre-service secondary mathematics teachers showed a variety of answers as illustrated in Figure 6.1. Despite the variety in their answers, the anticipated explanation and answer was $\frac{8}{15}$ is larger than $\frac{3}{7}$ because $\frac{8}{15}$ is larger than a half and $\frac{3}{7}$ is less than a half.

To this question the pre-test analysis for both the control and experimental group participants indicated that 15 out of 60 respondents gave a correct answer with the correct reason, while 24 out of 60 gave the correct answer without the correct reason whereas for 21 out of 60 respondents the answer and the reason were both wrong.

The responses of the participants to this question were grouped in two according to the way they attempted the question. Some pre-service secondary mathematics teachers approached this question by performing calculations and based on these calculations they made inferences were then made. The calculations performed by

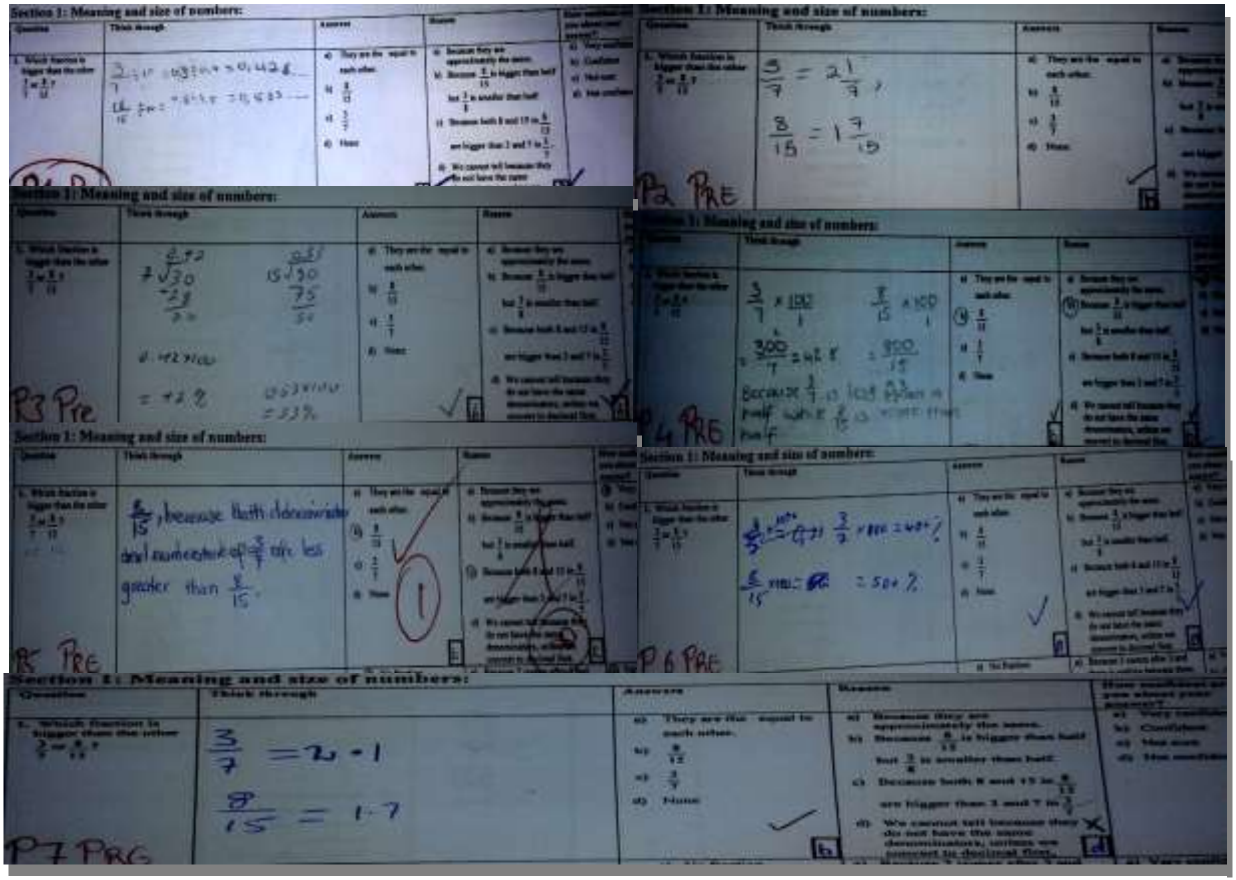

Figure 6.1: Samples of algorithmic responses to the meaning and sizes of numbers

pre-service secondary mathematics teachers are shown in Figure 6.1.

Figure 6.1 indicates that among the participants who approached the problem by performing mathematical calculations as opposed to number sense most of the participants converted to decimal fraction and or percentage as illustrated by the work of P1, P2, P5 these could therefore be regarded as (CAA). In addition Figure 6.1 also illustrates that P2 and P7 also performed wrong algorithms when attempting to convert the given fractions to decimal faction and therefore made wrong inferences from these wrongly performed algorithms. The fact that P2 and P7 chose a correct answer but had performed a wrong algorithm could also be regarded as a CGA.

Despite the fact that some students attempted to convert the two fractions to percentage as illustrated in the work of P3, P 4 and P6 they were still not able to interpret their answers meaningfully. Figure 6.1 shows that the answer of P6, portrayed some number sense reasoning (CCU) and concluded using the estimated percentage value without actually working out the exact percentage value that the fraction $\frac{3}{7}$ is about 40% + while $\frac{8}{15}$ was 50% + and therefore made an inference that the latter is larger. P4 made an effort to explain the size of two fractions that $\frac{8}{15}$ is larger than a half and $\frac{3}{7}$ is smaller than a half, therefore $\frac{8}{15}$ is larger than $\frac{3}{7}$, this could be regarded as CCU.

For this item it could be noted that most of the participants were classified as CCA as they did not show the number sense in their correct answers. Few participants were CCU than CCA. This gives the impression that the students approached this number sense question from an algorithm perspective despite that it required number sense.

Despite the fact that the Number sense in this question required students to understand the magnitude of the two fractions in question but not necessarily to perform algorithms some students still based their decision of the sizes of two fractions on the performed algorithm. Other participants represented the fractions in question by means of diagrams and inferences were made from these diagrams as shown in Figure 6.2.

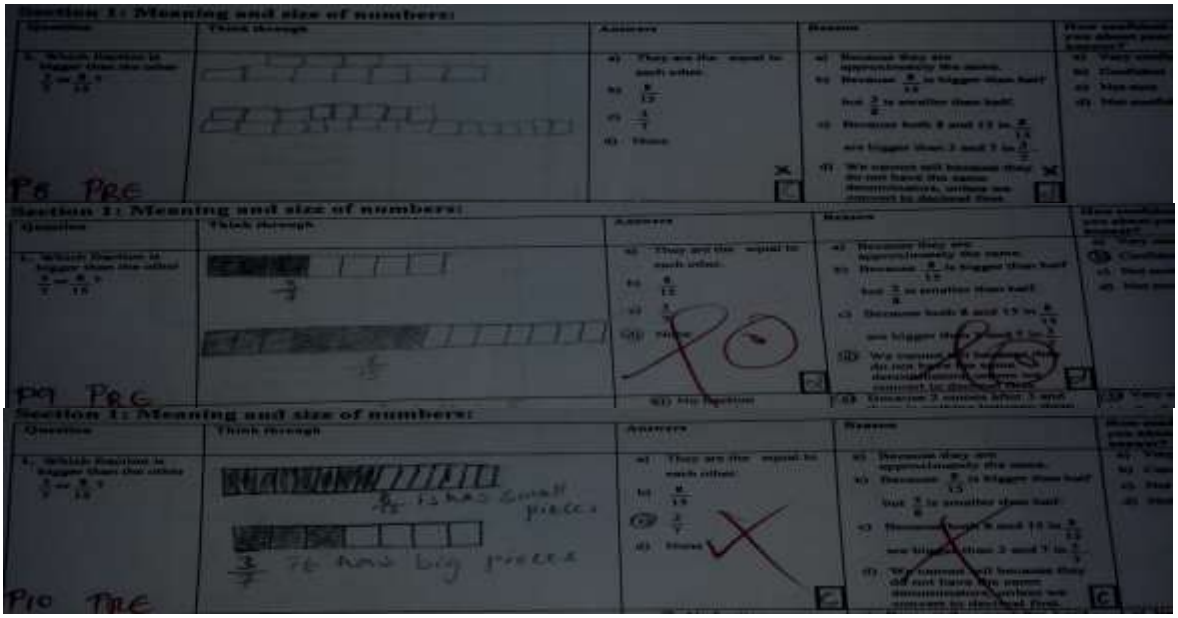

Figure 6.2: Samples of diagrammatical responses to the meaning and size of numbers

P8 opted to represent the fractions' both numerator and denominators using boxes. This shows the participant lacked the understanding of the meaning of the concept of fraction and means of illustrating a fraction using diagrams. There is however no link between the diagram and the way such a student used the diagrams to produce the correct answer. Hence for this participant it was not clear as to how the diagram guided the student to arrive at the desired outcome implying the student guessed the answer wrongly; therefore this is a typical of a WGA.

Some of the students opted to illustrate the diagrams using rectangles e.g. P 9 and P 10, for these students there is an understanding that a fraction represents the number of equal parts of a whole and therefore these managed to illustrate the fractions fairly well. However, a close analysis gives the impression that the challenge was now to use the representation to produce the desired answer. For instance P 9 attempted to illustrate $\frac{8}{15}$ and $\frac{3}{7}$ using accurate drawings, yet, these could not make correct inferences from their diagrammatical representations of fractions. P10 illustrated the fractions fairly well on the diagram by shading off the represented fractions but could not use this diagram to make correct decisions on the relative size of the two fractions. According to this respondent when representing $\frac{8}{15}$ one has to reduce the size of the equal blocks in a whole and for $\frac{3}{7}$ the blocks are relatively larger than the ones for $\frac{8}{15}$, therefore for these respondents $\frac{8}{15}$ was considered to be the larger fraction. Therefore for P 9 and P 10 these are typical of WCU. For P10 there is also an element of WAA since the student made inferences based on the relative sizes of the blocks utilised to represent each of the fractions.

For this particular situation illustrated in the work of P9 and P10 it gives an account for the reason why the number sense proficiency scores are relatively higher than those of number sense reasoning as illustrated in chapter 5 Tables 5.2 to 5.5. Some of the participants have acquired the number sense proficiency answers since their misconceptions coincidentally led to the correct answer, but these did not get correct answers for the number sense reasoning.

The respondents that represented their answers with a diagram did not present correct answers. In addition to that most of them either WAA, or WGA this suggests that there was a lack understanding of number sense. Some students attempted to illustrate the fractions correctly despite that these did not use their diagrams to produce correct answers. The overall general impression of this section is that many of the students lacked number sense, only a few students presented characteristics of number sense as presented in section 3.3. Therefore their level of number sense for this section could be estimated to be below basic. The next section presents captions of participants' work on a number sense question drawn from the second component, equivalence of numbers.

6.2.1.2 Section 2 on the equivalence of numbers component

Participants were asked to find the value of n that will make the two sides equal in the equation: $0.3 \times 10^2 = 30000 \times 10^n$, for this particular question the respondents were required to carry out a simple comparison of the left hand side with the right hand side. Participants were expected to recognise that the two sides are equal since it is an equation. By comparing the two sides, students were also expected to recognise that the 30 000 is 1000 times larger than the 0.3×10^2 on the left hand side, hence there was a need of a thousandth to balance the two sides. Therefore, the value of n = - 3 which gives a thousandth will balance the equation.

For this item 14 out of 60 respondents gave a correct answer with the correct reason, while 16 out of 60 gave the correct answer without the correct reason whereas for 30 out of 60 respondents had both the wrong answer and reason. The researcher upon screening the answers chose two samples; for correct responses and for wrong answers. To find out the reasoning that the respondents used to reach a correct answer, these are presented in Figure 6.3.

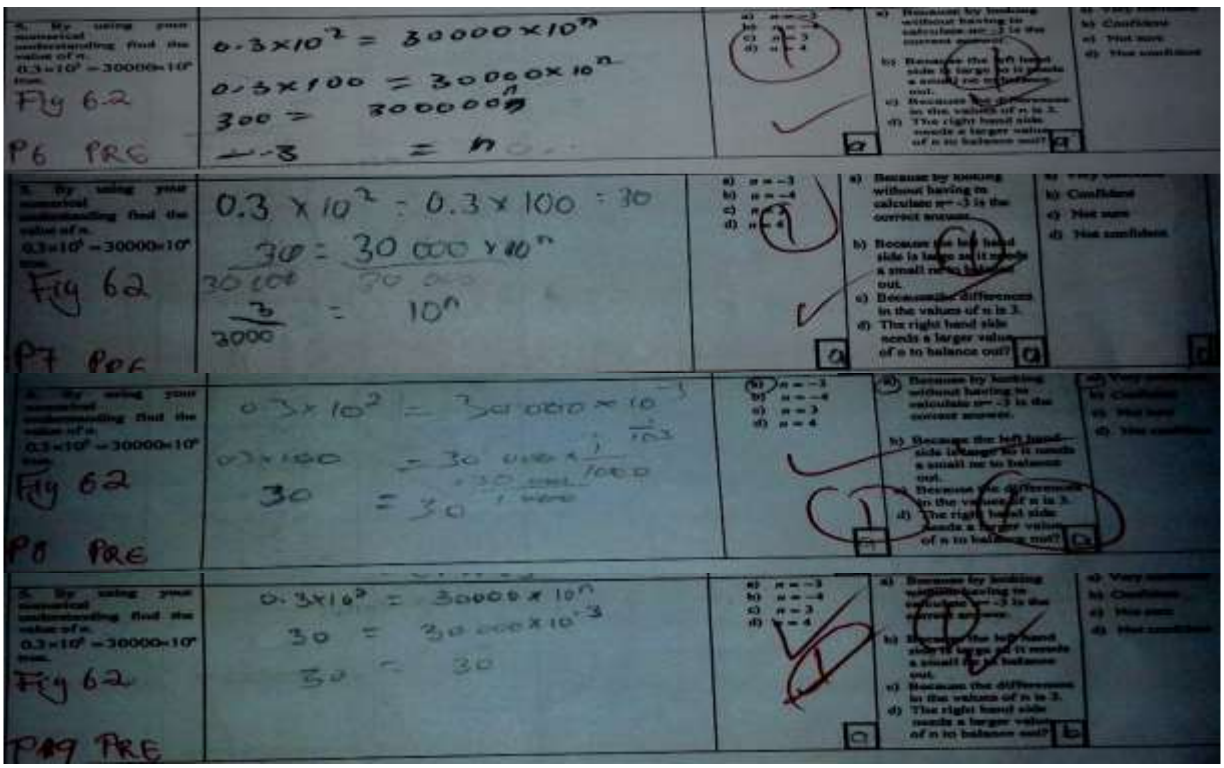

Figure 6.3: Sample of the correct responses to the equivalence of numbers

Figure 6.3 reveals that despite the fact that the students presented the correct answers, some utilised algorithm as opposed to number sense ideas (P6 and 7). Since answers provided by these were correct they could be regarded as (CAA). These respondents (P6 and 7) perceived the number sense problem as an equation attempted to solve an equation.

For P6 was not clear as to how the solved equation led to the correct answer of n = -3 it could therefore be concluded that this respondent guessed the answer. Also, for P6 it can also be observed that out of performing algorithm the student ended up getting the wrong answer for the first step on the left hand side, i.e. $0.3\times10^2=300$ which is not correct. From nowhere the student, added an extra zero on the right hand side in the second step, thus the comparison of zeros on the left with those on the right hand side had a difference of 3 therefore n = -3. There was no clear evidence of where the extra zero added to the right hand side came from as one moved from steps 1 to 2. This somehow seems to suggest that this student did not grasp the number sense reasoning behind the correct answer, which was an indication of lack of number sense as per section 3.3. Since the algorithm of the student did not also support the correct answer, it was regarded as a CGA.

For the participant P8 there was a plausible effort to apply number sense in recognising that there is a need of a factor of a thousandth to be introduced to the right hand side and once this was introduced the two sides could be equal. This was regarded as an evidence of number sense reasoning (CCU). Similar to P8, P9 also have proven to have used number sense to decide that there is a difference of 3 in the powers on the right hand side with the powers on the left hand side. Therefore for this particular respondent, the difference was a factor of thousand and hence by introducing a thousandth on the right hand side would balance the equation. Thus for this student (P9) it was reasonable to classify their work as CCU.

From this analysis it can be observed that despite some of the respondents' getting the correct answer there was still a lack of number sense among some of the responses presented by the participants that is why many of the answers in the foregoing item were classified as either CAA or CGAs. It also could be observed that some students attempted to apply reasonable number sense to the question and therefore arrived at correct answers (CCU) however these were few. Thus from this item it was plausible to claim that many of the students still regarded number sense items as though they were mathematical questions. Therefore these approached the question using an algorithm while in an actual fact they were required to use number sense understanding and reasoning.

Some students presented the wrong answers; the responses of these students are presented in Figure 6.4.

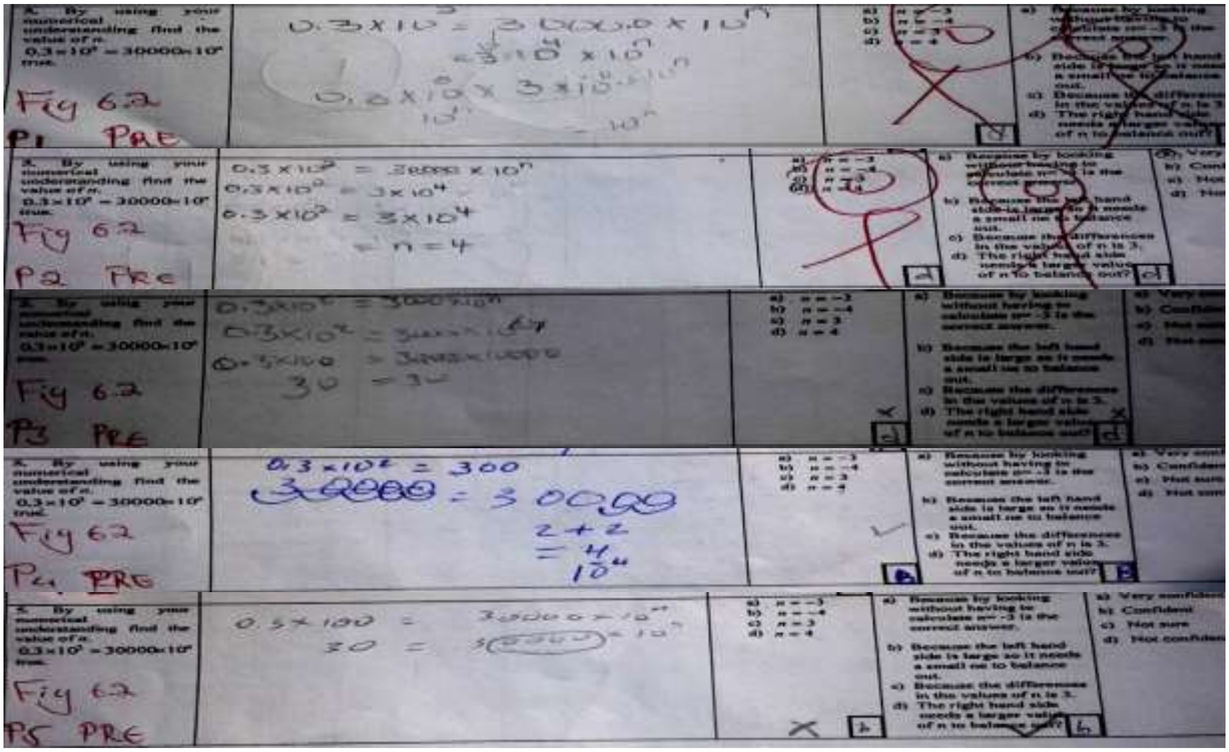

Figure 6.4: Samples of the wrong responses to the equivalence of numbers

The work of P1 also shows the same trend of making invalid conclusions that were based on neither mathematical nor numerical facts (WGA). For this respondent the problem presented as an equation, but as steps went ahead the equality was lost between the two sides and hence the equal sign disappeared. As a result the respondent could not make sense of the problem anymore and therefore could not make sound and reliable conclusions.

P3 and P2 also overlooked the fact that an equation could be viewed as a balance. For these particular respondents the power of n was disregarded, and then the student basically converted the right hand side to standard form, since a power of 4 was obtained on the right hand side then it was established that the value of n = 4 was the correct answer this shows a WAA typical characteristics. This conclusion holds neither numerical nor mathematical basis on which it is anchored; i.e. the conclusion that n = 4 was based on a mere guess and not a mathematical idea or misconception at least therefore it is plausible to also regard this as a WGA.

The work illustrated by P3 however suggests that the respondent did not pay attention to the relevance of the magnitude of a number. Moreover, for this respondent there was a poor facility of number sense and lack of comprehension of the concept of equations. This student lacked the understanding that an equation can be viewed as a balance where an operation done on one side should be counter acted against on the other side of the equation to maintain the balance (WAA).

In addition P3 multiplied one side of the equation with ten thousand and did nothing on the other side, yet maintained that the sides are equal despite the fact that one side is 10 000 times larger than the other after carrying out a multiplication, this was regarded as a WAA and the fact that the student attempted to identify an answer it also shows that the student guessed the answer therefore this could also be regarded as a WGA. For this particular student it can be interpreted that the student had a misconception that even if a number is made to be ten thousand times the original value it still remains exactly the same size.

P5 put a circled around the number of zero on the right hand side and therefore concluded that the correct answer is n = 4 there since there are 4 zeros on the right hand side. The responses of this student has similarity with the traditional high school mathematics teaching which is characterised by following rigid rules and paid no attention to mathematical flexibility, i.e., “one must check the number of zeros”. This background of following rules and algorithms is however contrary to the characteristics of number sense described in section 3.3 of chapter 3 therefore this shows some WAA.

P4 followed the pattern of P3. This student recognised the fact that there are 4 zeros on the right hand side and divided these zeros into two groups each containing two zeros. From this a conclusion that each group of two zeros can be classified as exponent 2 was drawn and therefore since exponent 2 appears two times it means the exponents must be added together thus 2 + 2 = 4. Henceforth a power of n = 2 was achieved.

The foregoing situation yet again suggests that the P4 only attempted to follow the rules (WAA) of indices that multiplication is associated with adding the powers without necessarily attempting to make sense of how large the numbers presented in the question were. In this case however the number sense item in question did not ask the rules but rather attempted to seek the students' abilities to recognise the equivalence of numbers.

The results presented in this sub-section show a lack of number sense among the students. Most of the students approached the number sense items by following rules WAA or just by guessing the work WGA. Few students in this section attempted to use number sense to obtain the correct answer (CCU). Taking into account the characteristics of number sense presented in 3.3, it could therefore be reasonable to conclude that for this particular sub-section the students have shown characteristics of poor number sense. Thus for this section it could be estimated based on the qualitative data that the students are at a level that is below basic.

6.2.1.3 Section 3 on the effects of operations on numbers component

When asked to find the number that will give an answer of 1 in the calculation: $\frac{2}{5}\times[\ \ ]=1$, respondents presented a variety of answers. However the desired solution for this question was to use number sense in order to recognise that $2\frac{1}{2}$ is the reciprocal of $\frac{2}{5}$, and therefore once multiplication is carried out the answer is 1.

In this item 21 out of 60 respondents gave a correct answer with the correct reason, while 15 out of 60 gave the correct answer without the correct reason whereas for 24 out of 60 respondents the answer and the reason were both wrong. It can also therefore be said that more than half of the participants passed this question despite the fact that some of the preservice secondary mathematics teachers did not pass the question with the correct reason.

The researcher, after scrutinising the answers attempted to categorise them in two main categories (wrong or correct answers) with a sample of four from each category, to try and probe what influenced their choice of answers. The correct responses to this item are summarised in Figure 6.5. In Figure 6.5 the researcher also sought to understand the reasoning carried out by respondents who gave a correct answer.

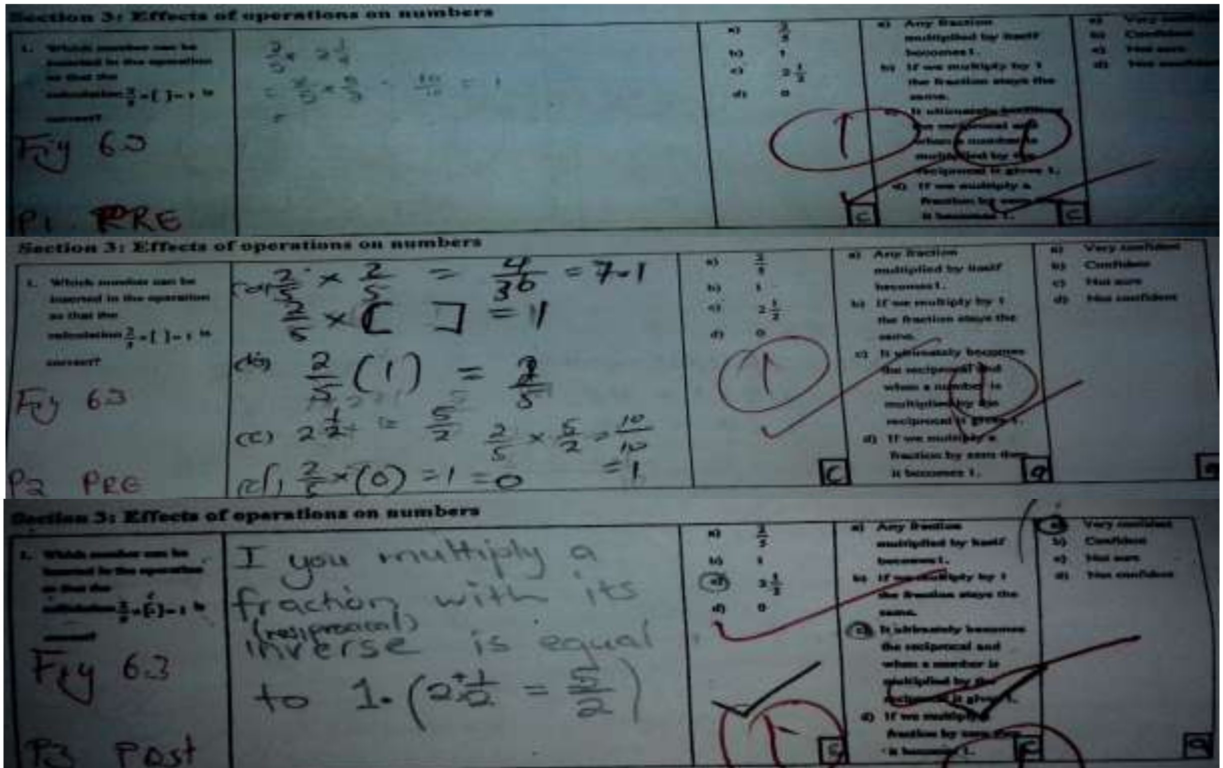

Figure 6.5: Samples of correct responses to effects of operation on numbers

Despite the fact that the respondents obtained the correct answer there is still evidence that some participants still align themselves to the algorithmic approach when solving number sense items For instance P1 attempted to identify the reciprocal of the fraction presented in the question from the available options. However for this particular student a further attempt of verifying the solution was carried out. This was however aligned to the algorithms without understanding the numerical reasoning as opposed to the use of number sense (CAA). The evidence of having utilised algorithms for this particular respondent is evident from the cancelling out the 5s and the 2s from the calculation as this is a common rule and algorithm practiced in school mathematics.

In addition to practicing the algorithm some students approached this problem by attempting to see the impact of each of the alternative answers provided for. P2 for example attempted to assess whether the first alternative answer could give the desired outcome. However for this student there was an algorithm (CAA) used and this led to the wrong working out of the missing number as seen from the part labeled “(a)” in the respondent’s work, the student holds an opinion that 5 times 5 is 36 which is wrong. However by following trial and error substitution provided the participant arrived at a desired outcome. The ideas emanating from this respondent’s work is that there is no evidence of utilising umber sense in order to achieve the desired outcome but the student merely guessed the answer using the alternative answers given (CGA).

For P3 on the other hand there seem to be an evidence of number sense since the student attempted to identify the reciprocal. The respondent also made a plausible effort based on number sense to have gone an extra mail and explain that if a number is multiplied with its reciprocal it gives 1. This respondent therefore attempted to answer the question in a desired manner. It was therefore plausible to recognise this answer as a CCU.

At glance the responses of these three responses seem to vary in terms of their numerical reasoning and the way they attempted to answer the question. However these are dominated by the CAA or CGA, therefore these responses suggest a lack of number sense comprehension. The analysis also suggests that similar to the above situations most of the students that obtained the correct answer approached the question using the algorithm. There were very few students who attempted to use number sense in order to solve the problem presented in the question correctly.

Some respondents did not manage to provide the correct answer for this item, in an endeavour to understand the reasoning that led to the choice of their wrong answer; their responses are presented in Figure 6.6.

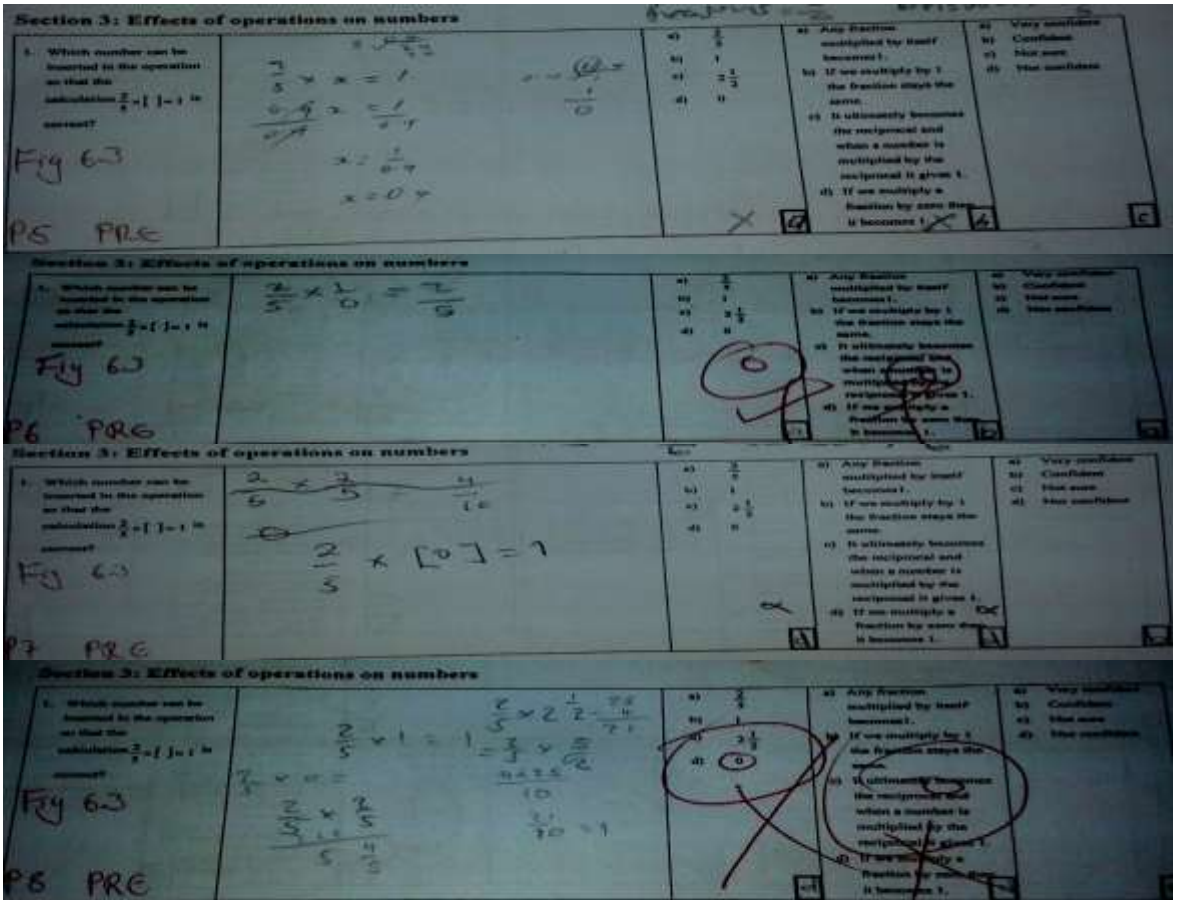

Figure 6.6: Samples of wrong responses to the effects of operations on numbers

The participants with wrong answers did not seem to be aware of how the number sense can be used to solve the question. Respondent, P1 attempted this question by formulating a mathematical equation instead of attempting to seek the relationships between the numbers. Moreover the fact that the participant attempted to convert the common fraction to decimal seems to suggest a lack of fluency in dealing with common fractions (WAA).

However, respondent made a plausible effort to carry out the steps involved in following the equation. The final step carried out seems to suggest that there is a lack of number sense, the fact that the respondent performed a calculation $1 \div 0.4 = 1$ seems to suggest that the respondent lacks the ability to assess the recognise the effect of the multiplication on a fraction to get the number 1. The student in attempting to solve the equation failed to give the correct answer which influenced a wrong choice of the answer (WAA).

P2 on the other hand approached the question from the perspective of carrying out an algorithm as opposed to applying number sense (WAA). However, the algorithm carried out was wrongly done and lacked the feel or sense of numbers. For instance for this particular respondent the fact that there is there is an evidence that there is a lack of understanding of the zero concept. P2 seem to have shown some knowledge of number facts due to the fact that there is some knowledge that the identity of multiplication is 1 and is achieved when the number is multiplied with its reciprocal (WCU). This is regardless the fact that the participant had the understanding that when 1 is divided by zero the answer is still a unit; this implies that the student had a misconception about the effect of dividing by zero. I.e., the student lacked the understanding that dividing by zero is undefined.

P3 approached the problem using an algorithm (WAA). The fact that the respondent attempted to carry out a calculation $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$ and realised it does not bring the desired result and henceforth cancelled it seems to suggest a lack of understanding of the effects that operations have on numbers. This also suggests that the respondent seems to be taking chances to see which number once substituted in the box will give the desired result (WGA). Moreover, the participant lacks the understanding of the zero concept, i.e., the fact that the student carried a multiplication by zero and still recorded an answer of 1 suggests a lack of ability to assess the reasonableness of the solution obtained and therefore the number sense since the ability of assessing the reasonableness of the solution is among the attributes of number sense described in section 3.3.

P4 had a strategy to substitute all the given options into the equation (WGA) but failed to handle the substitution by 2 and a quarter because the student could not convert the latter to improper fraction (WAA). Moreover, it was not clear how the student obtained 5 out of 2 from the working presented. It also appears that the respondent attempted to carry out the calculation using the correct answer as well as the alternative distractors, though, the correct answer was obtained the respondent still failed to recognise the correct answer among the available distractors (WGA).

The fact that the student attempted to carry out most calculations correctly using all the available distractors except the multiplication by zero could also be interpreted that the student lacks the concept of multiplying by zero. The fact that the according to the respondent calculation $\frac{2}{5}\times\frac{2}{5}=\frac{4}{5}$ also shows that there is a lack of computational skills which is among the components of number sense. Additionally this student seems to lack the understanding of the meaning and size of numbers since the student could not recognise that the answer obtained is double the fraction but the multiplier is not 2 (WAA).

The responses presented in this section also suggest a lack of number sense. In most cases the students were classified as WAA, WAA or CCA. This was clear evidence that the students did not have a better grasp of number sense. There were very few students that attempted to approach this problem according to the number sense characteristics (CCU). For this particular item it could thus be estimated that the students are at a level of number sense that is below basic.

6.2.1.4 Section 4 on the counting and computational strategies component

For this particular task the emphasis was on finding the most reasonable answer for the calculation: $35\times 11\times 2$ with alternatives to choose the correct answer from 700, 765, 800 and 385. The number sense behind this question was to recognise the fact that 35×11 gives an answer which is slightly more than 350 and once a number that is slightly more than 350 is doubled the answer will be above but within 700s. Alternatively the number sense behind this question could be recognised if one attempted to multiply 35 by 2 to get 70 and once 70 is multiplied by 11 it will give an answer that is more than 700 but less than 800.

For this question 19 out of 60 respondents gave a correct answer with the correct reason, 17 out of 60 gave the correct answer without the correct reason and 24 out of 60 presented both wrong answer and the reason.

For this item the correct responses were studied separately from wrong responses. This was carried out with an ultimate aim of understanding the thinking that led to the responses of participants in each case. After a identifying a variety of arguments carried out by different participants the researcher obtained a set of two correct responses and these are presented in Figure 6.7.

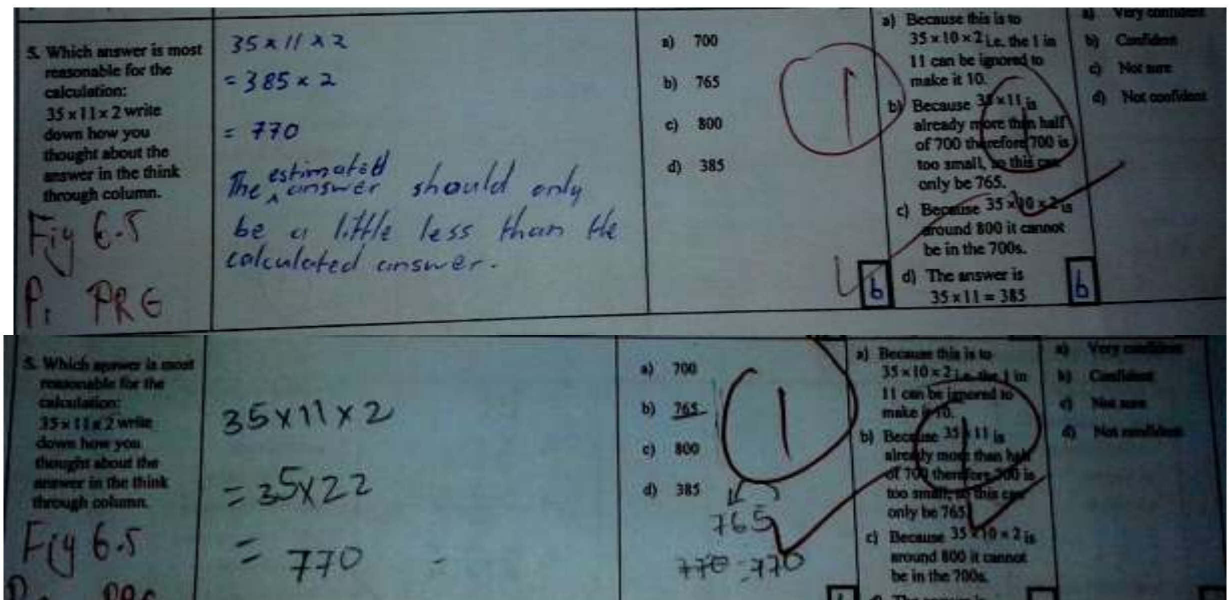

Figure 6.7: Samples of correct responses to the section on computational strategies

As illustrated in Figure 6.7 appears that P1 attempted to use number sense reasoning since after obtaining the exact answer to the calculation the respondent attempted to establish the idea that the estimated answer has to be little less than the calculated answer (CCU). The response of P1 shows that the student possesses some awareness of estimation; however the respondent did not establish the reason and the base of that estimation.

P2 attempted to work out the value of the calculation and obtained an exact answer. However this particular respondent appeared to have indicated some number sense by reasoning that 765 must have been the number that is rounded off to produce an answer of 770 as illustrated in the work of the student in the answer column (CCU).

The responses presented by the students show that among the participants that presented the correct answer the student have shown good characteristics of number sense. Most of the students whose work is presented in the foregoing, their work was classified as a CCU which shows some positive characteristics of number sense.

Figure 6.8 shows analysis of the three wrong responses of three other participants to this number sense item.

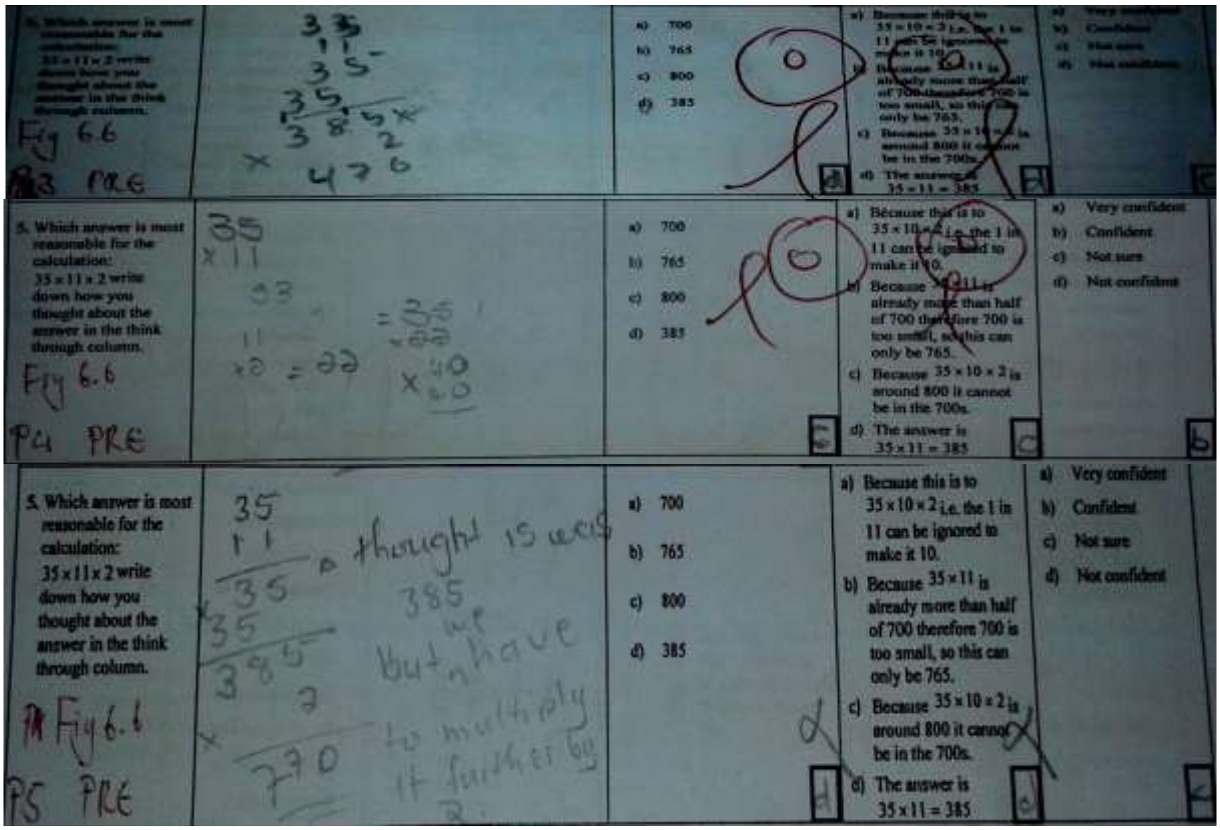

Figure 6.8: Samples of wrong responses to counting and computational strategies

The reasoning that guided the students' answers according to Figure 6.8 seemed to vary. P1 simply carried out the calculation, however this was wrongly performed (WGA). The fact that this respondent attempted the calculation $385 \times 2 = 470$ suggests that the respondent did not attempt to assess whether the answer matches the given calculation of the solution that is obtained. The inability of a student to assess the numerical reasonableness of a solution that is obtained is regarded as a negative indicator of number sense. Figure 6.8 further illustrates that the student performed a calculation $35 \times 11 = 53$ which lacks numerical reasonableness implying a lack of number sense (WAA).

Respondents P3, P4 and P5 on the other hand performed the calculation: $35 \times 11 = 385$. The respondents P3 and P5 opted to choose the obtained answer of 385 (these were typical of WAA). In addition to the answer P5 made an additional statement *"thought it was 385 but we have to multiply further by 2"*. From this statement it could be interpreted that the student lacked confidence dealing with this problem (WGA). Despite the fact that P3 still doubted the reasonableness of the answer 385 it was still the preferred as the best answer.

The responses of students to that item did not seem to show number sense. In most cases the participants were either WAA or WGA. This suggests that the participants either attempted to guess the answers, wrongly performed the algorithms. The responses of the participants that presented the correct answer showed characteristics of good number sense. It could thus be estimated that for this section the number sense is at proficient level.

6.2.1.5 Section 5 on the estimation using relevant benchmarks component

The question asked the respondents to state the relationship between the number π and $\frac{22}{7}$. The researcher by asking this question expected the students to utilise the fact that pi is not exactly the same as but it just approximates to $\frac{22}{7}$. Ideas such as pi is an irrational number and $\frac{22}{7}$ is a rational number were also anticipated as acceptable arguments in the view point of number sense for this particular question.

For this question 18 out of 60 respondents gave a correct answer with the correct reason, while 20 out of 60 gave the correct answer without the correct reason whereas 22 out of 60 respondents had both wrong answer and the reason. The responses of seven participants form a sample representation for this question as illustrated in figure 6.9.

The results of Figure 6.9 indicate that most, (57, 95%) of the respondents could not

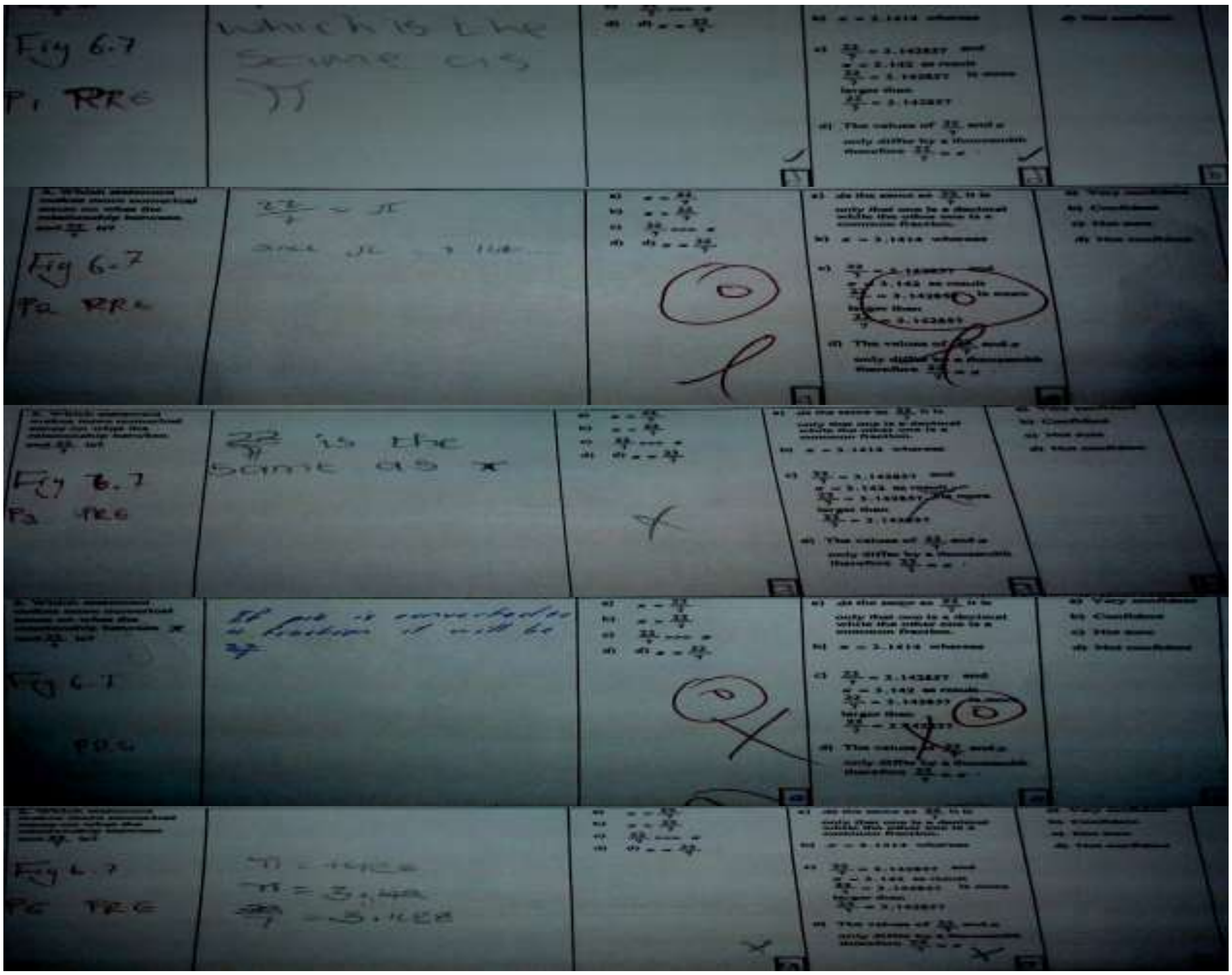

Figure 6.9: Samples of responses to estimating using benchmarks

state the relationship between pi and $\frac{22}{7}$ accurately. P1 for instance indicated that once $\frac{22}{7}$ is worked out it gives 3.142 which is the same as pi (WAA). In the same manner P2 equated pi to $\frac{22}{7}$ and then reasoned that the equality is true since $\pi = 3.142$ (WAA). Also, P3 holds the idea that π is the same as $\frac{22}{7}$ without any reason (WGA).

In addition, P4 argued that when pi is converted to fraction it will give $\frac{22}{7}$ and therefore pi equals to $\frac{22}{7}$ WAA. Taking an overall look at the results of Figure 6.9 the results clearly suggest that the respondents possess a poor grasp of the number pi. Most of the respondents have a misconception that pi equals 3.142 (WAA), which is an approximated value used in high school mathematics.

It could be argued that if this misconception is not rectified these preservice secondary mathematics teachers might carry on teaching this wrong idea once they join the teaching career. This again reverts to the vicious cycle of innumeracy represented in section 1.3. Students also had no understanding that pi is an irrational number since it is inexpressible as a quotient of two integers, while $\frac{22}{7}$ could be expressed as a quotient of two integers and therefore $\frac{22}{7}$ is a rational number.

This sub-section presented the number sense pre-test results of preservice secondary mathematics teachers' results regarding their level of number sense comprehension. The results presented give the impression that there is little or insufficient number sense comprehension among the preservice secondary mathematics teachers (as most of the responses given by students were WGA, WAA or CAA). This low level of number sense comprehension was also manifested by the quantitative data presented in section 5.2.

The results presented in this subsection also suggest that the respondents had a tendency to approach number sense questions from an algorithmic (WAA or CAA) perspective as opposed to reason using numerical understanding. In some instances the students attempted to give answers without correct reasons (CGA or WGA), this is probably why the number sense reasoning results were relatively lower than those of the number sense proficiency as indicated in sections 5.3 and 5.4. It could therefore be estimated that the students for this level have shown competencies that could be classifies as below basic.

6.2.2 Results on the level of number sense comprehension from focus group interviews

This sub-section presents the qualitative data of the pre-test focus group interviews regarding the level of number sense comprehension of preservice secondary mathematics teachers in the experimental group. The section presents the responses given by 5 interviewees regarding the level of number sense comprehension of respondents during the pre-test interviewing session.

The interview questions were asked as a follow-up on the questions taken from the number sense pre-test, one question per section. This was done to try and understand the participants further by offering them a chance to express themselves. To ensure anonymity, the respondents are coded with numbers preceded by the abbreviation "IP" which could simply be interpreted as "Interview Participant":

6.2.2.1 Responses to the meaning and size of numbers component

This section presents the responses of interviewees regarding the question derived from the first section of the pre-test number sense. In this section participants were asked:
Which fraction is larger than the other between 8 fifteenths and 3 sevenths? What was your preferred answer? Why do you think the answer you opted was the correct answer?

IP 1: *I chose three over seven, if you represent fractions using boxes for three over seven you use larger boxes compared to eight over fifteen therefore three over seven.* (WAA)

IP2: *Three over seven, because if you simplify eight over fifteen you get two over five which is smaller than three over seven.* (WAA)

IP3: *I chose three over seven, I have no reason I just know it is the correct one.*

(WGA)

IP4: *My preferred answer for this question is three over seven. I just know it sir.* (WGA)

IP5: *I chose eight over fifteen, because if we compare the denominators and we divide them by the same number for example 2, for seven divided by 2 we get 3.5 but for eight over fifteen we get 7.5.*(CGA)

For this item among the interviewed participants many (IP 1, 2, 3 and 4) of the interviewees chose the wrong answer with either no or wrong reason these were either WAA or WGA. IP1 and 2 opted for wrong answers with wrong reasons based on wrong algorithms therefore these were regarded as a typical of (WAA). For participants IP 3 and IP4 it could be inferred that the students merely guessed the preferred answer WGA. IP 5 managed to perform mathematical algorithm of converting to decimal and therefore concluded from the decimal representation of fractions, nonetheless performing algorithm is not really an attribute of number sense as indicated in section 3.3 (CAA). The responses of students also show that they lacked number sense reasoning, this seems to support the data presented in chapter 5 which indicates that the number sense reasoning of preservice secondary mathematics teachers seem to be weak.

How confident were you when you were giving this answer?

IP 1: *I was CONFIDENT sir; I think I was very confident.*

IP2: *NOT CONFIDENT I think.*

IP3: *I was NOT CONFIDENT, it is correct sir.*

IP4: *SOMEHOW, YEAH, it's correct, I am NOT SURE.*

IP5: *SOMEHOW CONFIDENT.*

The confidence of the student according to their self-evaluation seems to be relatively low. This lower confidence however seem to agree with the self-evaluation for the results presented in section 5.1. In this case students tended to choose either not confident of somehow not confidence. It could be argued that these students appeared to be less confident with regards to this number sense test component.

6.2.2.2 Responses to the equivalence of numbers component

Find the value of *n* that will make the two sides equal in the equation: $0.3\times10^{2}=30000\times10^{n}$. What was your preferred answer? Why do you think the answer you opted was the correct answer?

> **IP 1:** *n = - 4 because the left hand side equals 30 and the 30 000 on the right hand side should be multiplied by 1 over 10 000 to give the 30 on the left hand side.* (WAA)
>
> **IP2:** *n= - 4, if you start to solve the equation by taking ten to the power of n to the left it becomes negative. You also transfer 0.3 to the right hand side it becomes negative. Then you compare the powers you get a difference of 4.* (WAA)
>
> **IP3:** *n = 3, I will explain it like this; ten squared equals 100, 0.3 multiplied by 100 is 30, so we have 30000 multiplied by ten to the power of negative three.* (WAA)
>
> **IP4:** *n = 4, 0.3 times 10, three… n is four.* (WGA)
>
> **IP5:** *n = equals negative 4, it is the correct answer.* (WAA)

For this item students seem to have performed poorly as well. Most of the interviewees (except IP3) neither gave the correct answer nor the reasoning (WAA or WGA). Students tended to have utilised more calculations (WAA) despite the fact that the question attempted to understand their ability to recognise the equivalence between the numbers on the right hand side and the left hand side without necessarily having to perform a calculation.

How confident were you when you attempted to respond to this question?

> **IP 1:** *I was CONFIDENT sir; I think I was very confident.*
>
> **IP2:** *SOMEHOW NOT CONFIDENT I think.*
>
> **IP3:** *I was SOMEHOW NOT CONFIDENT, that it is correct sir.*
>
> **IP4:** *I was CONFIDENT sir.*
>
> **IP5:** *SOMEHOW NOT CONFIDENT.*

Most students indicated a lack of confidence when they attempted this question.

Accordingly, the researcher concludes that their self-confidence seemed to be low. Some students indicated a confidence as they attempted this question. There were some contradictions observed as well, for instance IP3 lacked confidence but still managed to give the correct response.

6.2.2.3 Responses to the effects of operations component

Find the number that will give an answer of 1 in the calculation: $\frac{2}{5}\times[\ \ ]=1$. What was your preferred answer? Why do you think the answer you opted for was the correct one?

> **IP 1:** *The answer is 2 over 5. We need to multiply by two over five, since five over two will give us one over one.* (WAA)
>
> **IP2:** *I preferred 0, yes d. I just know it is the only one.* (WGA)
>
> **IP3:** *It is zero; I just know it is zero.* (WGA)
>
> **IP4:** *The answer is zero, two over five multiplied by zero over five equals one over x equals two over five.* (WAA)
>
> **IP5:** *Is two over 5, two over 5 multiplied by two over five it gives ten over ten which equals 1.* (CAA)

How confident were you when you were giving this answer?

> **IP 1:** *I was VERY CONFIDENT sir; I think I was VERY CONFIDENT.*
>
> **IP2:** *NOT CONFIDENT, NOT AT ALL sir.*
>
> **IP3:** *I was CONFIDENT.*
>
> **IP4:** *I was CONFIDENT.*
>
> **IP5:** *very CONFIDENT.*

In this component students tended to have rated themselves with higher confidence, as most of them were either very confident or just confident. It could therefore be argued that confidence of the students according to their self-evaluation seems to be relatively high as they attempted to respond to this number sense item. Despite this self-rating of high confidence most of the students did not manage to give the correct answer. There is hence a mismatch between the number sense response and the self-evaluation on the confidence of the pre-service secondary mathematics teachers.

6.2.2.4 Responses to the computational skills component

Find the answer that is closest to the correct answer for the calculation: $35 \times 11 \times 2$. What was your preferred answer? Why did you opt for that as your correct answer?

IP 1: *I chose b), 765, if you multiply 11 by 2 you get 22 and if the 22 is multiplied by 35 we get 385.* (CAA)

IP2: *765,* $35 \times 11 = 385$ *and then* $385 + 385 = 770$ *which is closer to 765.* (CAA)

IP3: *It is c) 800, 35 times 2 equals 70. 70 times 10 equals 700.* (WAA)

IP4: *Is 800, if you make 5 lots of 70s you get 140 plus 140 plus 70 it gives 350, plus 35 it gives 385 times 2 it gives 700.* (WAA)

IP5: *It is 800, 55 plus 35 equals 88, therefore the answer is 880.* (WAA)

The responses of students to this item clearly indicate a poor grasp of number sense. Very few students managed to obtain the correct answer, in addition to these the correct answer was still obtained by means of performing algorithms. Moreover, among the students that obtained the wrong answer, most of them simply carried out the algorithms.

How confident were you when you were giving this answer?

IP 1: *CONFIDENT I suppose.*

IP2: *VERY CONFIDENT.*

IP3: *I think I was CONFIDENT.*

IP4: *I was SOMEHOW NOT CONFIDENT.*

IP5: *SOMEHOW NOT CONFIDENT.*

This section gives the impression that many of the respondents have rated themselves with higher confidence, with either very confident or just confident. Based on these self-evaluation results of respondents for this section, the researcher can conclude that the confidence of the students was high for to this number sense item. There was however a considerable number of students who felt somehow not confident for this particular item. The most confided student (IP2) also got the correct answer.

6.2.2.5 Responses to the estimation using relevant benchmarks component

What is the relationship between pi and 22 out of 7? What was your preferred answer? Why do you think the answer you opted was the correct answer?

> **IP 1:** *Pi is equal to 22 over 7. Pi is always 22 over seven.* (WGA)
>
> **IP2:** *Pi equals 22 over seven. They are the same.* (WGA)
>
> **IP3:** *Pi equals 22 over seven no reason needed.* (WGA)
>
> **IP4:** *Pi equals 22 over 7.* (WGA)
>
> **IP5:** *Pi equals to 22 over 7.* (WGA)

In this section, respondents did not show understanding of the relationship between pi and 22 over 7. In most cases they failed to give reasons why they thought pi is the same as 22 out of 7, hence it was plausible to conclude that for this item students simply guessed their answers wrongly. Therefore these students did not show good characteristics of number sense.

How confident were you when you were giving this answer?

IP 1: *I was VERY CONFIDENT.*

IP2: *CONFIDENT.*

IP3: *CONFIDENT.*

IP4: *VERY CONFIDENT.*

IP5: *VERY CONFIDENT.*

Interestingly, despite the fact that all of the students did not opt for the correct answer for this item, none of them showed a lack confidence according to their self-evaluation. It can be argued that these students despite the fact that they possessed this misconception they were still so confident to protect and defend this misconception. It is very difficult to undo the wrong conception, and it is likely that this and other similar misconceptions could be carried to their classrooms once they become teachers. Such teachers may also confidently teach this misconception to their learners once they start teaching, a situation that will revert back to the vicious cycle of innumeracy alluded to in section 1.3.

What other material did you need to work out these questions that we have been discussing?

IP 1: *A CALCULATOR or at least a PEN and PAPER.*

IP2: *CALCULATOR.*

IP3: *PEN and PAPER.*

IP4: *I need a CALCULATOR of course*

IP5: *Yes a CALCULATOR is definitely needed.*

In response to this question most of the respondents preferred to have a calculator, while others preferred to have a pen and paper methods. It should however be understood that within the perspective of number sense, the algorithms of pen and paper as well as the calculators are not a necessity for one to work out number sense questions. A proficient student in the area of number sense need not have a pen, paper or a calculator or pencil in order to produce a correct answer.

6.2.3 Qualitative data of document analysis regarding the level of number sense comprehension

This sub-section presents the qualitative data of the documents that were completed over the first few lessons (at the initial stage of the intervention) regarding the level of number sense comprehension of preservice secondary mathematics teachers. The data in this sub-section therefore refer to the number sense shown in the work of pre-service secondary mathematics teachers during the first few lessons of the intervention. The thrust here was to understand the level of number sense portrayed by students during the initial stage of the intervention.

Therefore activities presented here attempted to assess the initial level of number sense as the students started with the intervention as observed from the lessons that were spread over a few weeks of the intervention. For the first activity students were asked to study the diagram of a grain storage used in the traditional society (ethnomathematics) and reflect on the number sense around it. Students were required to analyse the diagram, define the term number sense and carry out a number sense survey by critically looking at the diagram to explain the number sense depicted in it.

The researcher believed by carrying out this activity the students could be exposed to the CRENS model. The fact that the question asked the students to establish the number sense from a diagram that is derived from their context the researcher argues that this brought in the realistic nature of the activity. Also, the activity drew ideas from the cultural set up since this is a part of the doings in their societies and therefore it goes back to the ethno part. Additionally the critical thinking was also included in this activity in it that it asked the students to reflect on the number sense shown in the diagram. The students were asked to think openly hence they were at liberty of thinking in any direction; this therefore comes closer to critical thinking. The tenets of Critical Theory also appear in this question as there was no limit in the thinking required from the learners. Therefore looking at these elements the researcher could establish the effect of the CRENS model in this activity.

The diagram below shows the storage container for storing Mahangu grains. Study the diagram carefully and answer the following questions:

1. Give the meaning of the term number sense from your own point of view.
2. Explain in as much detail as you can by giving examples from the diagram the number sense ideas that can be seen in the diagram in the diagram.

Figure 6.10: Activity 1, number sense portrayed by a traditional Mahangu storage.

The following is a sample of their responses of pre-service secondary mathematics teachers to the activity:

- *Number sense means making sense of numbers. The diagram shows some of the ideas in geometry, I see a circle, I see a sphere whose formula is … and In my culture there is no mathematics we only count things and find how many they are.*
- *Number sense to me means to learn to calculate without a calculator. I see the diagram looks like a graph of a quadratic function parabola facing upwards. I also see that the patterns are going in circular shapes; therefore I see a circle there, circle geometry.*
- *Number sense means understanding different kinds of numbers. I see the ideas of fractions, decimals, ratios and other types of numbers to compare the finished part to the part still under construction.*
- *Number sense to me means to be able to answer mathematics without a calculator. I expect to learn mathematics more, I also expect to learn what numbers mean. In the class room we have lots of mathematics, different kinds of numbers, ratios, percentages, algebra etc.*
- *Number sense means how different kinds of numbers can be manipulated. I expected to learn how to solve equations, surds and fractions. The number sense we have in our surrounding is much, from different shapes of our traditional rooms, the size of the house, the number of the people in the family etc.*

The responses of the students suggest that they did not understand what number sense could possibly mean. The students also appeared to find it difficult to draw a line between number sense and their formal mathematics curriculum. A closer look at the meanings they attached to the concept of number sense gives the impression that they perceived number sense to be part of their formal mathematics, which was a positive thing. Students also expected to learn contents as taught in the high school curriculum. Many students could not bring up the number sense that exists in their cultural setup as well as surroundings. Few students however made a plausible effort to indicate practical situations where number sense can be utilised e.g. when shopping or cooking. Based on the responses of students it can be argued that the students did not seem to understand fully what number sense was and what its components were. The researcher also concluded that very few students were able to recognise the number sense existing in their cultural setup or everyday life situations.

For the second activity the students were asked to decide which leather/skin mat covers more or less twelve-thirteenths of the green surface and which one covers more or less thirteen-fourteenths.

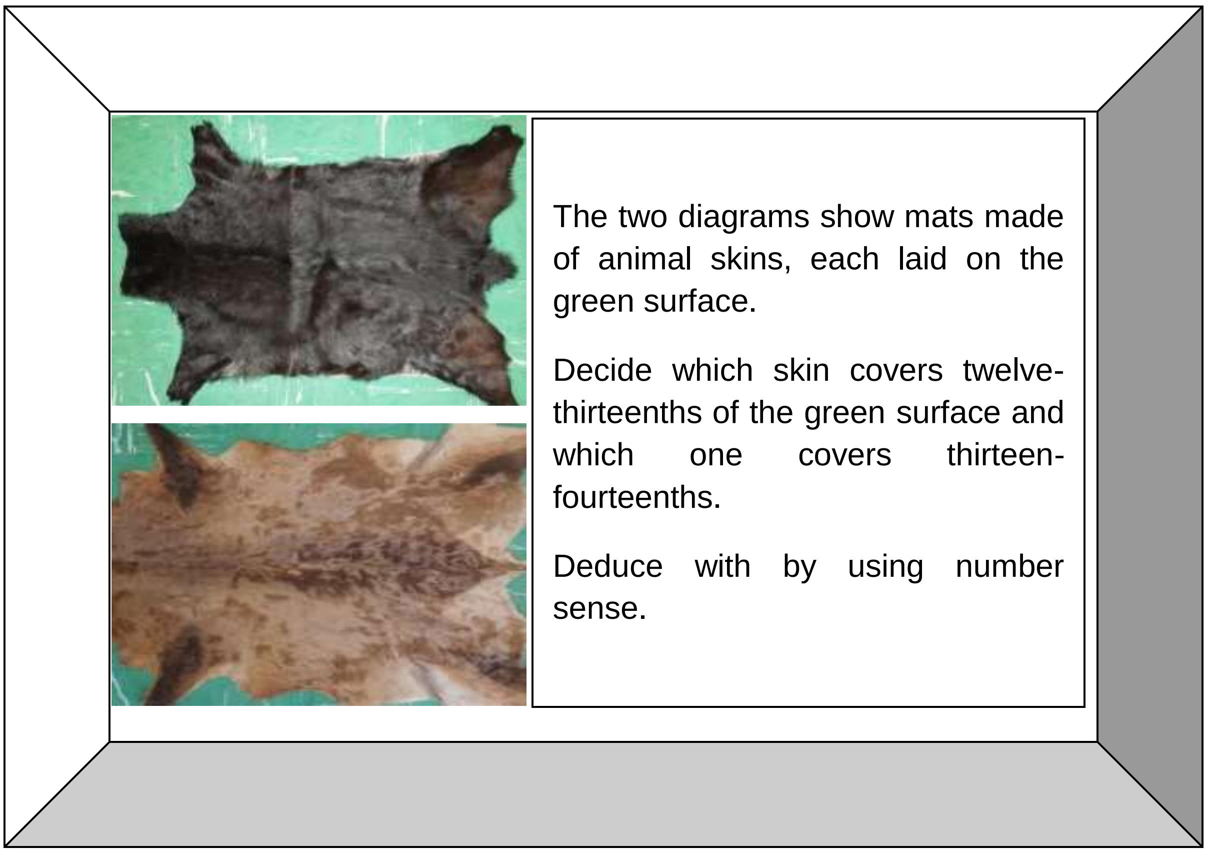

Figure 6.11: Activity 2, comparing two fractions with different denominators

This activity is believed to be realistic in the sense that leather mats are part and parcel of the student's lives; they see these mats being produced in their culture. In addition to this every culture makes their traditional carpets throughout the country therefore the question also possesses some ethno number sense element. The fact that the fractions given were not of the same denominators also suggests come critical thinking as part of CRENS comes in. The question anticipated that the students would identify the green surface where the leathers lie flat as a whole also probed for their critical thinking.

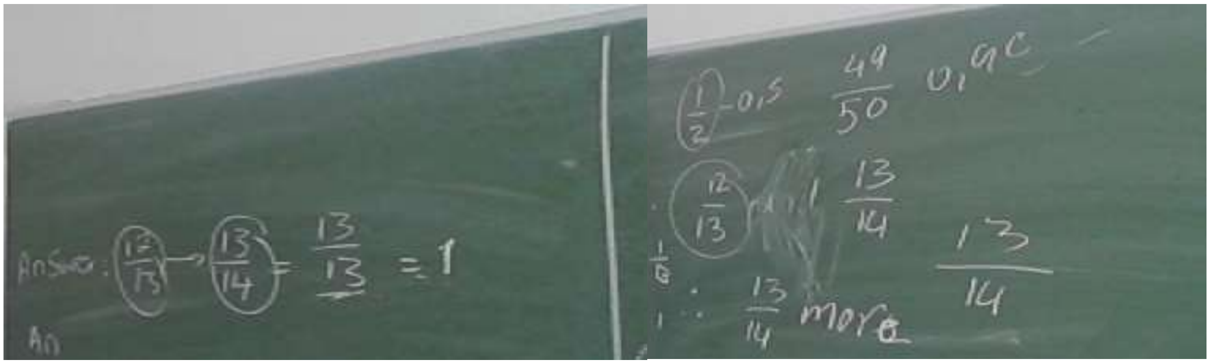

Figure 6.12: Sample of responses to the number sense activities at the beginning of the intervention

Two students responded to this activity, the first student whose work is on the left end attempted to equate 13 fourteenths to 13 thirteenths and concluded that it is equal to 1. The other student worked out the decimal representation and added that when the difference between the numerator and denominator is closer to zero the fraction tends to be closer to 1 and therefore 13 fourteenths is closer to 1. The results presented by the first student lacked number sense reasoning (WAA). The work of the second student seems to suggest some number sense reasoning (CCU).

Figure 6.12 shows the responses of two students to the question that required them to indicate how far the fence of a home sted had gone if it covered three quarteers in an anticlockwise direction from the starting point indicated in the diagram. In this activity students were presented with a traditional fence for enclosing the cattle as illustrated in Figure 6.13. By presenting the diagram of the kraal where the cows are kept and also taking into consideration that the culture of keeping cows is practised throughout all regions of the country, therefore the researcher anticipated that this brought the mathematical concept within the ethnocontext of the students.

The diagram shows a traditional fence of a kraal. Indicate how far the fence of a home sted had gone if it covered three quarteers in an anticlockwise direction from the starting point indicated in the diagram.

Figure 6.13: Activity 3, comparing two fractions with different denominators

The responses of the students are summarised in figure 6.14.

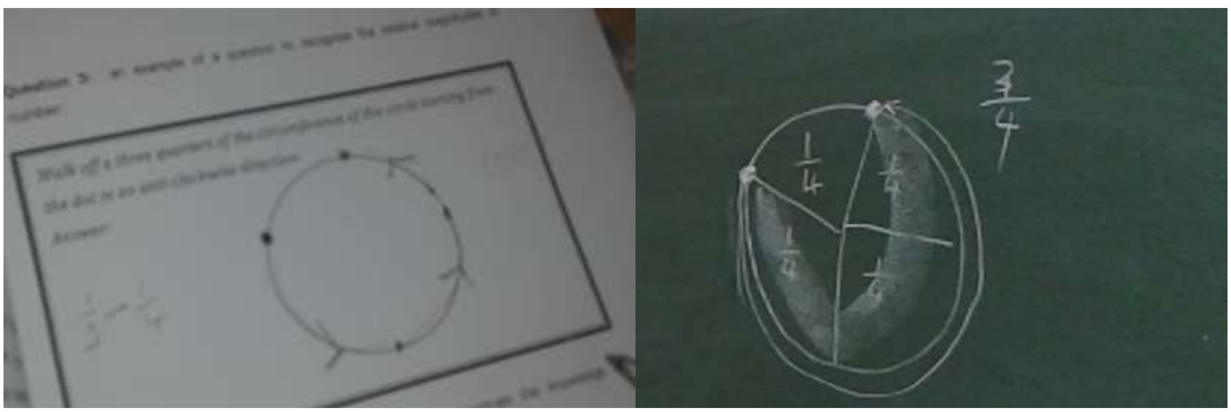

Figure 6.14: A sample of students' responses to the number sense intervention activity

Figure 6.14 shows that the both students could recognise the anti-clockwise direction, hence the choice of a wrong direction did not affect their interpretations of the numbers given. I addition both students divided the circle into four quarters and therefore marked that the fence ended up at the third dot which shows some conceptual understanidng. The stduent whose work in on the left handside (in pencil) the seemed to suggest the presence of number sense as the student could accuretely estimate quarters that divided the circle into four equal parts. However the second student whose work is shown on the right did not pay attention to the fact that a fraction divides parts of a whole into equal parts.

The fact that the students were asked to explain their answers to their classmates on the chalkboard also explains that there was some learner centrerd teaching. The notion of ASEI-PDSI was also accounted for in the sense that students had to work on the activities themselves with a hope that they practised, did, saw, and ultimately it was anticipated for them to improve.

The other activity diagram shows a traditional pottery used to store the water. This was also considered part of the ethnomathematics for the reason that such pottery were derived from the cultural setup of the students.

Figure 6.15: Activity 4, a traditional pottery artefact used to store water

By using the pottery the students were asked to shade a fraction of the rectangle when the clay pot illustrated in Figure 6.15 is 20% full. The fact that students were required to move from reality to abstract as they relate 20% of the clay pot to the length of a rectangle was perceived to initiate some critical thinking. The fact that the clay pot was also presented to the students it shows the realistic part of mathematics, i.e. mathematics in a practical set up.

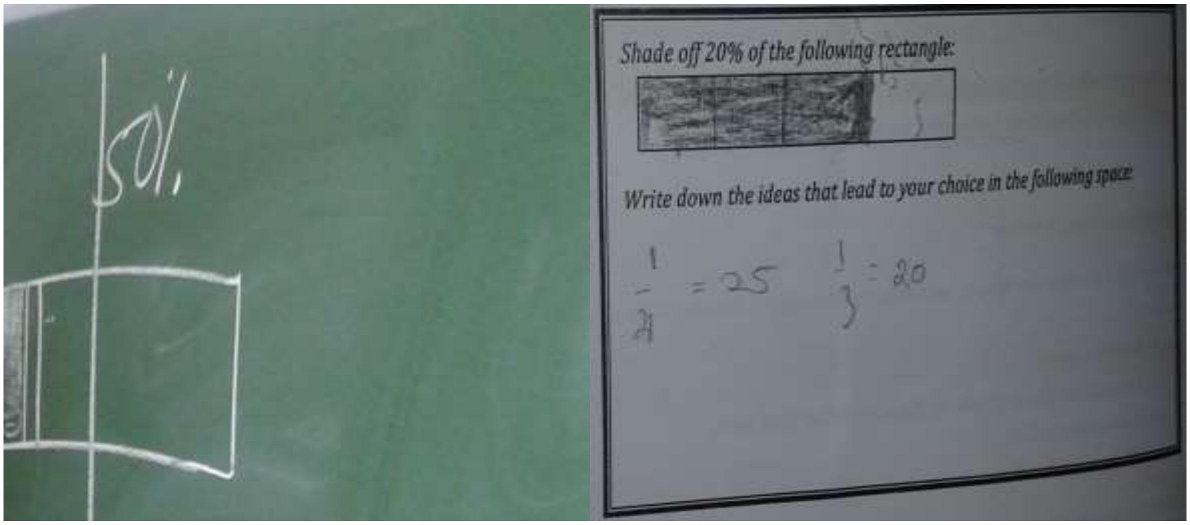

Figure 6.16: A sample of students' responses to the number sense intervention activity

The first student (whose work is illustrated on the left handside) approached this activity by dividing the rectangle into two equal halves, with a long vertica line, which according to the student is 20%. His could be equated to halving the clay pot.The student divided the first half into two equal halves each of which represented 25%. Then from the first quarter the studtent estimated that 20% should be slightly less than 25%. This reasoning shows very good characteristics of number sense. It seems that as the students went through their number sense training activities they developed number sense reasoning also (CCU).

The second student recognised that a quarter equals 25%, however the student erroneously took a third to be less than a quarter, hence the conclusion that a third equals 20%. Lead by this error the student divided the rectangle into four parts. Each part according to this student was regarded as a quarter, which makes numerical sense. The student went on to conclude that once 3 parts out of a total of four parts are shaded off it will give a third. This student lacks the understanding that a fraction represents a number of equal parts out of the total parts in a whole. More over the student lacks the understanding of relative sizes of different representations of the same numbers since the (s)he failled to recognise that a third is different from 20 % (WAA).

Figure 6.17 indicates the activty that required the students to fill a third of 6 calabashes with milk, by shading .

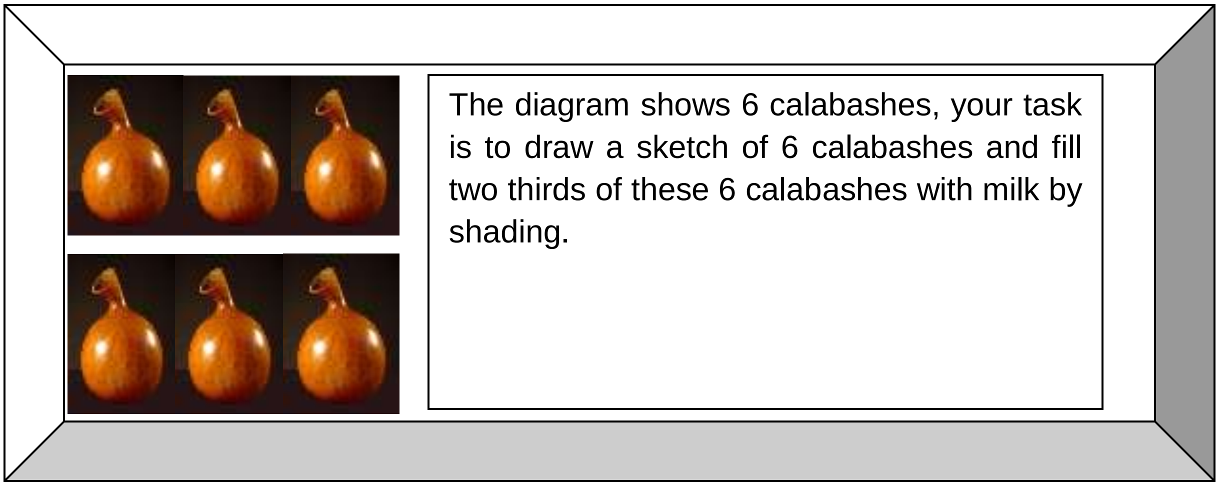

Figure 6.17: Activity 5, filling two thirds of 6 calabashes with milk

The responses of the students to the fore going acitvity are summarised in figure 6.18.

Figure 6.18: A sample of students' responses to the number sense intervention activity

The researcher believes that this activity bears features of the CRENS in that the calabashes are realistic to the students. Namibia is a cattle rearing country, due to its semi-arid nature, milk is stored culturally in the calabashes and some of the students at the university level must have gone through the practice of processing and storing milk in the calabash. Most of the communities use calabashes to store water or to fetch water from rivers, dams etc. Calabash making is also a culture of the Namibian society and the traditional calabashes are also culturally used to collect and store water. Therefore this activity acknowledges the ethno and the realistic part. Working out a third is also believed to bear a probing element to promote critical thinking among the students.

The first student (whose work is represented on top) worked out what a third of 6 is and obtained an answer of 2. Therefore for this particular student the task was completed by simply shading two calabashes out of 6 (a typical of CCU). The second student (whose work is represented at the bottom of the board with circles) confused the meaning of a third that it means the number that appears at the third position. The student basically shaded the third calabash in the row. This is a clear indication that the meaning of a concept of a third as a fraction did not click in this student's mind. This suggests very poor or no number sense (this was considered a WAA typical).

The other activity in Figure 6.19 students were given a condition that the mat of leather covers seven fifteenths of the floor shown and that the mat owner wants to buy another mat that will cover three fifths of the floor. Their task was to decide whether or not this mat will be bigger than the one shown in the diagram.

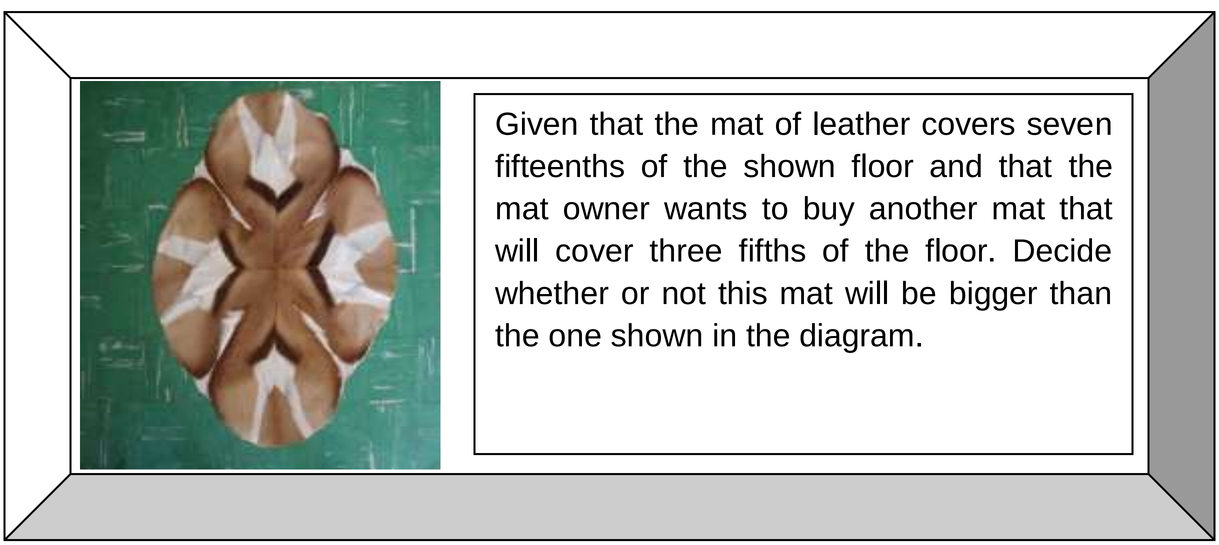

Figure 6.19: Activity 6, comparing two fractions using a mat on the floor

Figure 6.20 represents the responses of students to the activity.

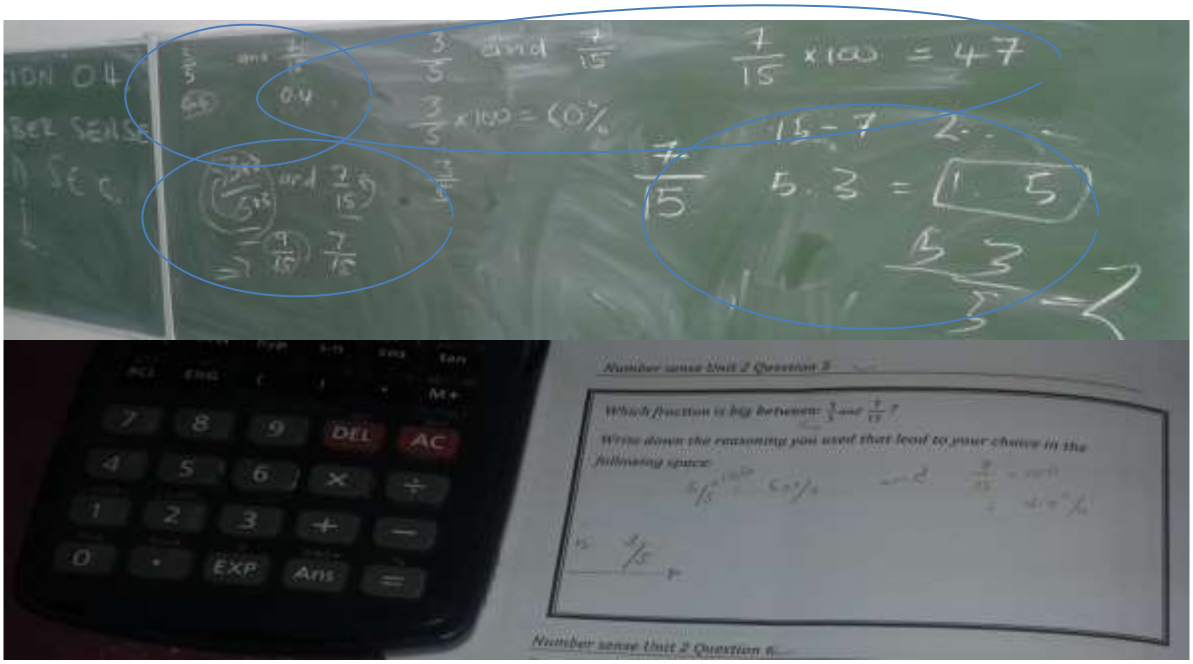

Figure 6.20: A sample of students' responses to the number sense intervention activity

The first student, whose work is represented on the top left hand corner, converted the fractions to decimal to obtain 0.6 and 0.4 for three fifths and seven fifteenths respectively, the student then based the decision on these decimal representations of the two fractions (CAA). The second student whose work is illustrated in the top middle part of the chalkboard as well as the other student whose work is illustrated on the piece of paper converted each of the fractions to percentage to obtain 47% and 60% and therefore concluded that three fifths is larger than seven fifteenths these students' work was also considered to be based on mere algorithms hence the response was classifies as (CAA).

The fourth student whose work is illustrated at the bottom right of the chalkboard attempted to divide each numerator into the denominator to get 2.142 for 15 divided by 7 and 1.666 for 5 divided by 3. The student went on to conclude therefore that since for seven fifteenth the resulting quotient is larger than for three fifths seven fifteenths is larger (WAA).

A fifth student whose work is illustrated at the bottom left hand corner of the chalkboard converted three fifths to a denominator of 15 to make the comparison clearer. This student obtained 9 fifteenths and concluded that three fifths which gives 9 fifteenths is larger. The student obtained the correct answer however it was merely based on the algorithm (CAA).

Looking at the responses of the students there seemed to be some number sense but very limited within the scope of this question. Most of the students whose work is presented in Figure 6.27 approached the question by merely performing the algorithms without attempting to understand the number sense which is not a good characteristic of number sense as indicated in section 3.3.

Other students lacked basic understanding of simple mathematical and numerical reasoning. For instance for the student to reason that the reciprocal of the fraction does not alter the size of a fraction it exhibits poor conceptions of numerical principles. The fact that other students converted to decimal, percentages or common denominators also indicates a lack of numerical comprehension (CAA). Students were expected to reason that three fifths is larger than half but seven fifteenths is smaller than half therefore three fifths is the larger fraction.

This section presented the qualitative data regarding the level of number sense of preservice secondary mathematics teachers. In most cases, similar to the qualitative data presented in section 5.1. The number sense of the preservice mathematics secondary teachers was low up to early stages of the CRENS intervention according to these activities. Therefore in responding to the question of what the level of number sense comprehension of preservice secondary mathematics teachers in Namibia is, it can be confirmed from the qualitative data that at the time of the study there was a very low level of number sense comprehension among the preservice secondary mathematics teachers in Namibia up to the early stages of the study.

The next section presents the qualitative data regarding the relationship between the number sense of preservice secondary mathematics teachers and their academic performance in mathematics (Basic Mathematics for Teachers). As in chapter 5, the term academic performance in mathematics should thus for the purpose of this study understood as the performance of preservice secondary mathematics teachers in their first year core semester 1 module (Basic Mathematics for Teachers). Based on the analysis on students' responses and also considering that the students have shown competencies that are below basic in most of the sections from the data it could thus be inferred that the level of number sense of students is slightly above the below basic level but lower than the basic level.

6.3 The relationship between the number sense and academic performance in mathematics

This section presents the responses of the preservice secondary mathematics teachers on how the number sense intervention impacted their performance in mathematics and/or comprehension. This was thus considered as qualitative data as argued by Loraine (1998) that qualitative data deals with reflections from the personal experiences of the participants and that this should then be used to aid the quantitative data to enhance the data that is reliable with a considerable level of depth. The researcher thus taking in consideration of the argument presented by Loraine (1998) offered an ideal opportunity to triangulate this qualitative data set from the responses of the participants to the questionnaire with the quantitative data presented in section 5.2.

As stated in section 4.5.3, the questionnaire issued to the students was administered with the purpose of finding out the mathematical experiences of the respondents in the experimental group after taking the number sense course. The responses of students to this questionnaire regarding how the number sense course influenced their understanding of the concepts they learnt in their mathematics module (Basic Mathematics for Teachers) are summarised here.

Respondents were asked based on what they have learned in the number sense course that they took; what they thought was the relationship between number sense and academic performance in mathematics (Basic mathematics for teachers). The "QP" refers to the fact that they were respondents to a questionnaire.

QP 1: *I think there is a POSITIVE RELATIONSHIP, in number sense we deal with numbers and in Basic Mathematics for Teachers we deal with mathematics.*

QP 3: *Of course number sense is needed in life but ONE DOES NOT NEED NUMBER SENSE to PASS THE BMT EXAMINATIONS.*

QP 4: *I think NUMBER SENSE IS RELATED to the mathematics we do in Basic mathematics for teachers, I mean number sense and Basic Mathematics for Teachers ARE CO-EXISTING; MOST OF THE THINGS WE LEARNED ARE SIMILAR to the ones we learned in Basic mathematics for teachers.*

QP 5: *I believe the relationship between number sense and academic performance in mathematics IS NEUTRAL. What you learn in number sense is what you learn in Basic Mathematics for Teachers.*

QP 9: *I think in number sense we deal with numbers and in Basic Mathematics for Teachers WE DEAL WITH THE SAME.*

QP 10: *I DO NOT SEE ANY RELATIONSHIP because in number Basic Mathematics for Teachers we do not use the methods that we use IN MATHEMATICS EXAMINATION.*

The students seemed to suggest that there was a lack of deep understanding on relation the number sense and academic performance. Despite the fact that most of the students saw a positive relationship between number sense and the academic performance in mathematics, their explanations were merely based on shallow understandings such as, they all have to DO WITH NUMBERS or there IS A NEUTRAL RELATIONSHIP since what is DONE IN MATHEMATICS IS ALSO DONE IN NUMBER SENSE.

Based on what they had learned in the number sense course, other students on the other hand did not seem to see a strong relationship between the academic performance in mathematics and number sense and these described this kind of no relationship as a neutral relationship. This weak correlation or relationship was also confirmed by the correlation coefficients obtained in the quantitative data presented in section 5.3, where the statistically significant values of correlations between the number sense proficiency and number sense reasoning were 0.486 and 0.374 were obtained.

In addition, the students were asked to explain what mathematical skills they thought the number sense course equipped them with in order to understand mathematics better.

QP 3: *In most cases we just use calculators, it helps us to assess and reflect whether the ANSWERS we are getting are REASONABLE or not, it means ONE CAN PASS MATHEMATICS EASILY.*

QP 4: *It introduces us to some steps that allow us to SAVE TIME in the, i.e. with number sense one need NOT CARRY OUT A CALCULATION up to the last step. You can predict your final solution.*

QP 6: *I acquired mathematical skills on concepts taught in Basic Mathematics for Teachers such as FRACTIONS, INDICES, and ALGEBRAIC MANIPULATION which could be relevant to PASSING the EXAMINATION for Basic Mathematics for Teachers.*

QP 9: *If you have number sense you can PASS mathematics EXAMINATION EASILY, things we are taught in number sense are the SAME as the ones we learn in mathematics.*

QP 10: *Number sense can make one to THINK FASTER and this can help in the EXAMINATION.*

The responses of students suggested that they did not identify concrete concepts that they learn in mathematics and how they link directly to high school mathematics. From this it could be argued that the relationship between number sense and the academic performance in mathematics did not seem to be very obvious to these students. It also appeared from the responses that most of their responses were driven by the anxiety of examination as some of them kept on referring to the examination. Many of the respondents had a hope that by being fluent in number sense one could pass their

Basic Mathematics for Teachers examination.

Additionally, the respondents were asked to state with a reason whether or not they need number sense as a secondary mathematics teacher to be. Their responses are summarised here.

QP 1: *Yes, from what I learned I think ONE NEEDS to understand number sense, to be able to EXPLAIN TO THE STUDENTS HOW ONE GOT THOSE ANSWERS.*

QP 2: *Yes as a teacher trainee I need number sense to be able to GET THE MATHEMATICAL ANSWERS IN DIFFERENT WAYS.*

QP 4: *No, one NEED NOT have number sense, because the learners must know by themselves.*

QP 8: *Yes, as a teacher trainee I NEED number sense to be able to deal with numbers and to help me THINK FASTER when dealing with numbers.*

QP 9: *Yes, as a preservice mathematics teacher I need number sense knowledge so that I can PASS IT TO OTHERS.*

Many of the students felt that number sense was necessary for a mathematics teacher. Also, the students elaborated adequately how the number sense would improve their pedagogical practices as mathematics teachers. Responses such as to PASS IT TO OTHERS, to be able TO DEAL WITH NUMBERS, to be able to GET THE ANSWERS IN DIFFERENT WAYS and many others were vaguely given. Their responses seemed to suggest that they recognised the influence of number sense on their pedagogical practices as mathematics teachers once they become teachers.

Furthermore, the students were tasked to reflect on their experiences and explain in their own opinion; having gone through both courses (Number sense Programme and Basic Mathematics for Teachers) how relevant the number sense training was to the Basic Mathematics for Teachers course. The following sample of responses was gathered:

QP 2: *NOT THAT RELEVANT since it does not mean knowing number sense will imply you will pass.*

QP 3: *RELEVANT, but, one has to figure out which part is and is not related to the*

Basic Mathematics for Teachers. It is not clear which part is to be used for Basic Mathematics for teachers.

QP 6: *It APPEARS TO BE RELEVANT but is NOT DIRECTLY RELEVANT since the questions in Basic Mathematics for teachers are not the same as the ones in the number sense course that we took.*

QP 8: *One has to still work hard because the CONTENT OF NUMBER SENSE is NOT THE SAME as the contents of the basic mathematics for teachers that we are learning.*

The perceptions of the students based on their experiences to this question varied. Most of the students felt number sense was relevant to the module they took in mathematics while others felt that it was not relevant. Among the respondents who felt that the number sense they gained from the number sense course was relevant to their mathematics indicated that the relevance was not an obvious one had figure out what part of number sense was relevant for academic performance in mathematics. Additionally other students felt that the relevance of number sense did not appear to be direct.

Many of the students did not observe any relationship between number sense and their academic performances, basing their arguments on the contents of both courses that they took. Reasons given by these include the fact that being competent at number sense does not automatically guarantee a pass in mathematics, in addition one has to work hard to still pass despite the number sense proficiency, and that the questions that appeared in the number sense items were NOT THE SAME AS the ones in the examination for mathematics.

Based on these responses the researcher can conclude that the students based their decisions on the similarity of the number sense test items to the examination questions. Therefore, due to the mere fact that these were not exactly the same, it appeared therefore that the number sense was not directly linked to Basic Mathematics.

More than half of the students felt that the effect of number sense on their Basic Mathematics for teachers had had no significance, but one could use number sense to understand mathematics. This could be interpreted to imply if the relevance at all existed it did not really significantly contribute to their marks in mathematics. This observation was however contrary to the tests of statistical significance between the number sense and the academic performance that were presented section 5.2, which concluded that there was a statistically significant association between number sense and the academic performance in mathematics.

The researcher attempted to gain a clearer picture on how the number sense knowledge could have possibly affected the performance of preservice secondary mathematics teachers' academic performance in mathematics. The respondents were asked to indicate based on their experiences acquired on the number sense training how helpful was the number sense course that they took in boosting up their academic performance in mathematics performance in mathematics.

QP1: *This course of number sense MIGHT HAVE HELPED me in passing mathematics because we learned how numbers make sense to us. It somehow helped me to do math at my level best.*

QP3: *They are all working with numbers SO THIS WAS EASIER FOR ME TO COPE with the module I took having been a number sense student. It helped me because I WAS ABLE TO DO MATHEMATICS OR CALCULATIONS IN THE EXAMINATION QUICKLY WITHOUT USING CALCULATORS.*

QP4: *It IMPROVED MY COMMON CAPABILITY in approaching difficult questions in mathematics.*

QP5: *This really helped because it SAVED TIME INSTEAD OF ALWAYS BEING BUSY PRESSING calculator; I already get the right answer. Sometimes we fail due to the limited time.*

QP8: *I think it HELPED ME because sometimes when I did not understand something at mathematics number sense helped me.*

QP 10: *This helped me in passing math as I will know and I am now able to TELL WHETHER THE CALCULATION I USE WILL LET ME TO GIVE A CORRECT*

ANSWER even not using a calculator.

Based on the foregoing responses it could be noted that students still believed that the impact of number sense on academic performance in mathematics could be perceived by how much of number sense appeared in their examination paper. In other, but many cases, students kept on referring to the examination that they took as opposed to actually assessing the impact of number sense on their academic performance from a holistic point of view throughout their learning of mathematics in their Basic Mathematics module.

About a more than two thirds (7 out of 10) students indicated that the number sense impacted their performance in mathematics as they could now SPEED UP (IP3 and 5) their process of answering examination questions to work faster in the examination. Yet others indicated that their CONFIDENCE in approaching the difficult questions WAS BOOSTED up; while 7 out of 10 students felt that they were empowered by the number sense course they took to face examination questions even without a calculator.

This section presented how the learnt number sense experiences of the students related to their academic performance in mathematics. Despite the fact that some students described their number sense experiences to be relevant to their academic performance in mathematics, there were still few (3 out of 10) who felt that their number sense was not relevant and did not impact their academic performance. In most cases respondents concluded that they could not establish a firm relationship between the number sense intervention and their academic performance in mathematics due to differences between the number sense test items examination questions for the Basic Mathematics for Teachers module from which the data regarding their academic performance in mathematics were drawn.

The quantitative data presented in section 5.3 which concluded that there was a statistically significant relationship between the number sense and the academic performance of preservice secondary mathematics teachers in mathematics. Also, based on the responses presented by the students in this section, it was clear that the number sense of preservice secondary mathematics teachers to some extent impacted their academic performance in mathematics.

The results presented in chapter 5 suggested that the impact of number sense on the academic performance of preservice secondary mathematics teachers could only be explained up to 20% which could be classified as a medium effect. Reasoning on the basis of the qualitative data presented in this section; it appears also that there is a relationship between the number sense and academic performance of preservice secondary mathematics teachers.

6.4 The impact of a Critical Theory intervention

This subsection presents the data analysis related to the impact of the critical theory intervention that was offered to the experimental group. This was done to seek the answer to the question: What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers? The responses of the pre-service secondary mathematics teachers during the pre-test were compared with those of the post-test for some number sense test questions. The researcher paid attention to the comparison of written work as well as the numerical sense presented in the written work of the pre-test versus the post-test.

The responses of the pre-service secondary mathematics teachers to the interview questions as well as the questionnaire were also studied to analyse how they experienced the number sense during the intervention. Therefore this analysis focused on the comparison of the experiences and or skills which the pre-service secondary mathematics teachers had prior to the intervention and the acquired skills after the intervention.

In this section, the researcher also compared the methods of responding to the number sense test items in the post test with the pre-test. This was then used to establish whether or not there was any improvement of number sense strategies as opposed to the method of merely following average scores. Moreover the study matched and compared the confidence level of preservice secondary mathematics teachers before and after the intervention.

It was deemed necessary to compare the responses of the respondents in the experimental group in the pre- and post-test. Consequently, these were organised according to the number sense sections as illustrated in the following sections.

6.4.1 Number sense test data regarding the impact of critical theory

This section presents the number sense test qualitative data regarding the impact of a Critical Theory intervention. The data is presented according to post-test responses on the number sense test.

6.4.1.1 Post-test responses to the meaning and size of numbers component

Generally for this question the post-test responses of students did not seem to differ with the pre-test. It can thus be argued that for this particular item the students did not perform differently in their pre- and post-test results. Nonetheless, there was still a need for a deep understanding and comparison of the responses provided by the respondents as illustrated in Figure 6.15.

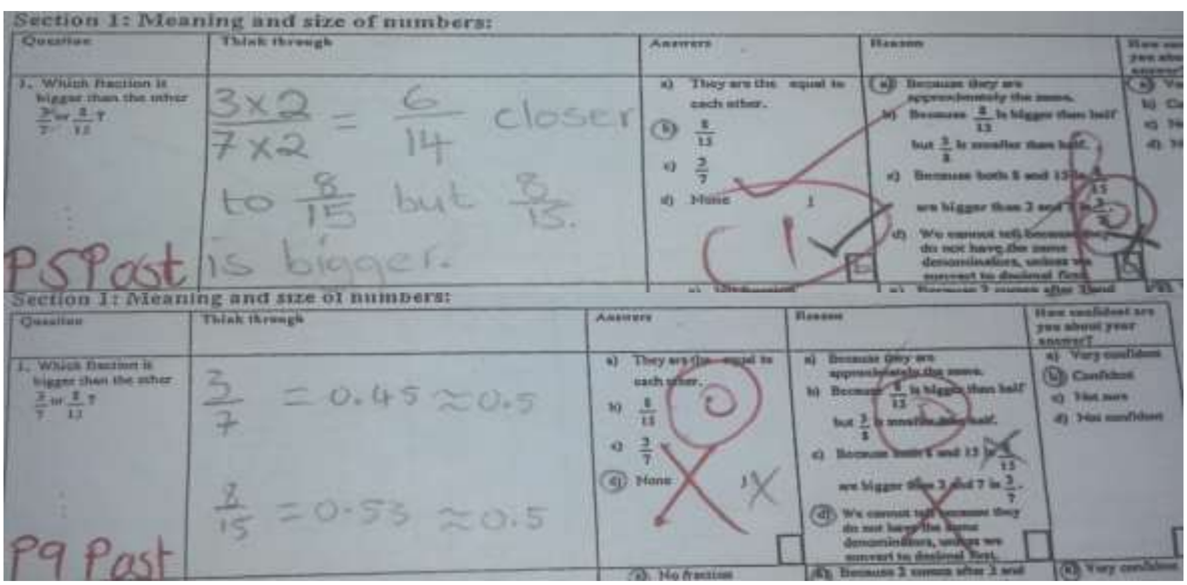

Figure 6.21: Samples of post-test responses on the meaning and size of numbers

Figure 6.15 further indicates that in the post-test IP 5 utilised the reasoning that 3 out of 7 is equivalent to 6 out of 14 which is closer to 8 out of 15, but 8 out of 15 is still larger (CCU). However IP 5 in the pre-test used a wrong reasoning that 8 out of 15 is larger than 3 out of 7 since both the denominator and numerator of 8 out of 15 are big (WAA). For this respondent it can be concluded that the respondent improved from WAA to CCU.

Also, the post-test response of this student as described above shows characteristics of number sense. Since the student, could not utilise these characteristics of number sense to arrive at the desired outcome, it could be argued that the student did not have a full grasp of the number sense around the question. This is one of the indicators of the impact of the CRENS intervention. The fact that the student utilised the correct number sense reasoning after the intervention as opposed to before shows a positive development of number sense.

The post-test response of P 9 shown in Figure 6.15 was compared with the pre-test responses in Figure 6.2. For the post-test the same respondent recognised the magnitude of the numbers represented in the question, and never illustrated the work using a diagram as was the case in the pre-test (WAA). The fact that the respondent recognised that both fractions were not that far from half and that they were closer to each other it implies that the participant has demonstrated some better understanding of the size of numbers (WCU), regardless of the fact that this understanding was not utilised to produce the correct answer. It can be said that the respondent slightly improved from WAA to WCU.

6.4.1.2 Post-test responses to the equivalence of numbers component

For this item 13 respondents gave a correct answer with the correct reason in the post-test as compared to only 9 in the pre-test. Eight respondents presented correct answers without correct reason compared to 7 in the pre-test. Nine respondents had both a wrong answer and reason as compares to 16 in the pre-test. Although the performance of students was in favour of the post-test; the trend in the student responses was still similar. From this it can thus be argued that for this item the students did performed much better in the post-test as compared to the pre-test.

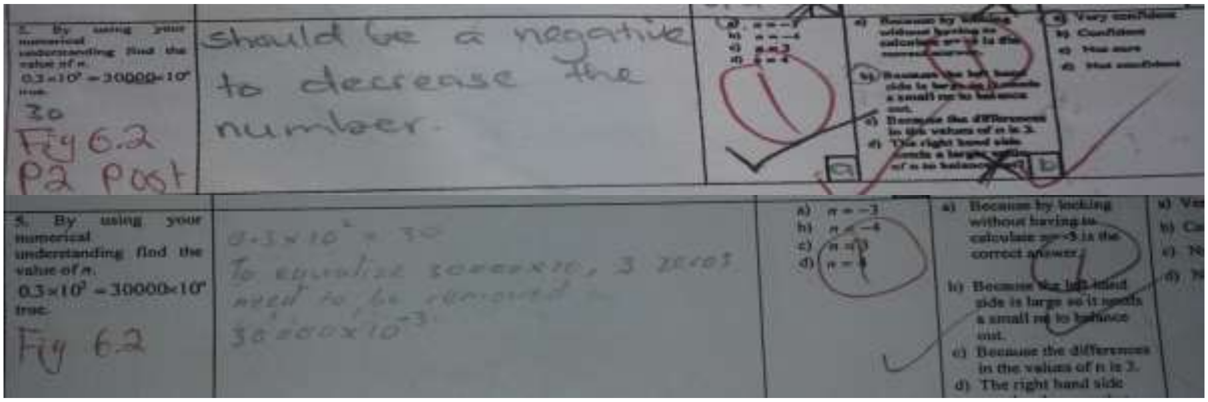

Figure 6.22: Samples of post-test responses to the equivalence of numbers

The post-test results of both P2 and P3 indicated that the respondents had improved their number sense comprehension. The reasoning that 0.3×10^2 on the left hand side was a thousand times smaller than 30000on the right hand side suggests that the student has grasped the relative sizes of numbers which is a predictor of number sense as presented by both participants. The students also then introduced the need of a factor of a thousandth for the two sides to be equal if one side is reduced by a factor of a thousand which also shows some number sense development among these respondents (CCU). A comparison of pre and post-test response for these particular respondents suggests that there is some positive development of number sense and number sense reasoning in the equivalence of numbers. The participants improved from WAA to CCU (see also Figure 6.4 for the pretest performance).

6.4.1.3 Post-test responses to the effects of operations on numbers component

For this number sense question this item 17 post-test respondents presented a correct answer with the correct reason compared to only four in the pre-test. Six post-test respondents presented a correct answer with a wrong reason compared to only three in the pre-test. Seven post-test respondents compared to 23 pre-test respondents presented both the wrong answer and reason. It can therefore be concluded that the performance of the participants in the post-test was qualitatively better than that of pre-test for this particular item.

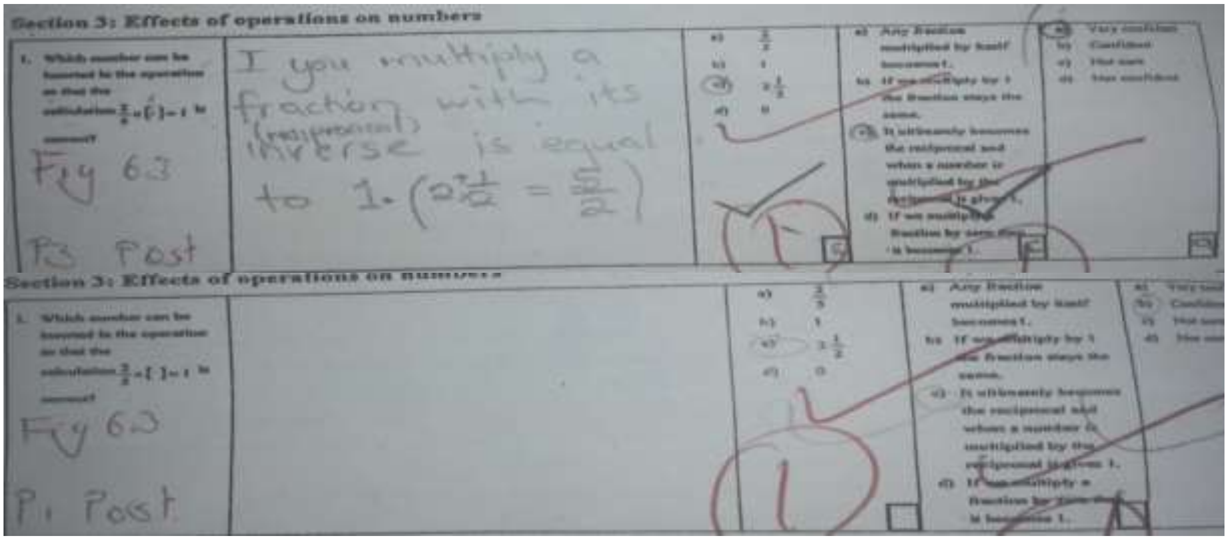

Figure 6.23: Samples of post-test responses to the effects of operations

The fact that in the post-test response P1 attempted to identify the correct answer without having to do some rough calculations and provided a reason for the answer suggests that the student had improved number sense regarding the effects of operations after the intervention (CCU). It could also be a result of guessing the answer correctly (CGA). It can however be observed in Figure 6.5 P1 had exactly the same answer. For this particular respondent in this item it could also therefore be argued that the student performed the same either CCU or CGA as before in the pre-test.

For P3 the post-test result was different, the students clearly explained that the number that needs to multiply 2 out of 5 is the reciprocal of that fraction. The students also portrayed number sense comprehension by acknowledging that once a number is multiplied by its reciprocal it yields the identity of 1 (CCU). For this particular student, in this particular item, it could be argued that there was some number sense in the post. However comparing the post-test response of this student with the pre-test it could be noted that the student did not show an improvement and therefore for this student the number sense transformation remained at CCU.

6.4.1.4 Post-test responses the counting and computational strategies component

For this question 12 post-test respondents compared to ten in the pre-test gave a correct answer with the correct reason. Also, 12 post-test respondents as compared to 7 in pre-test presented a correct answer without a correct reason. Six post-test respondents compared to 13 for the pre-test presented the wrong answer as well as a wrong reason. Based on these figures it can be said that the students performed better in the post-test compared to pre-test.

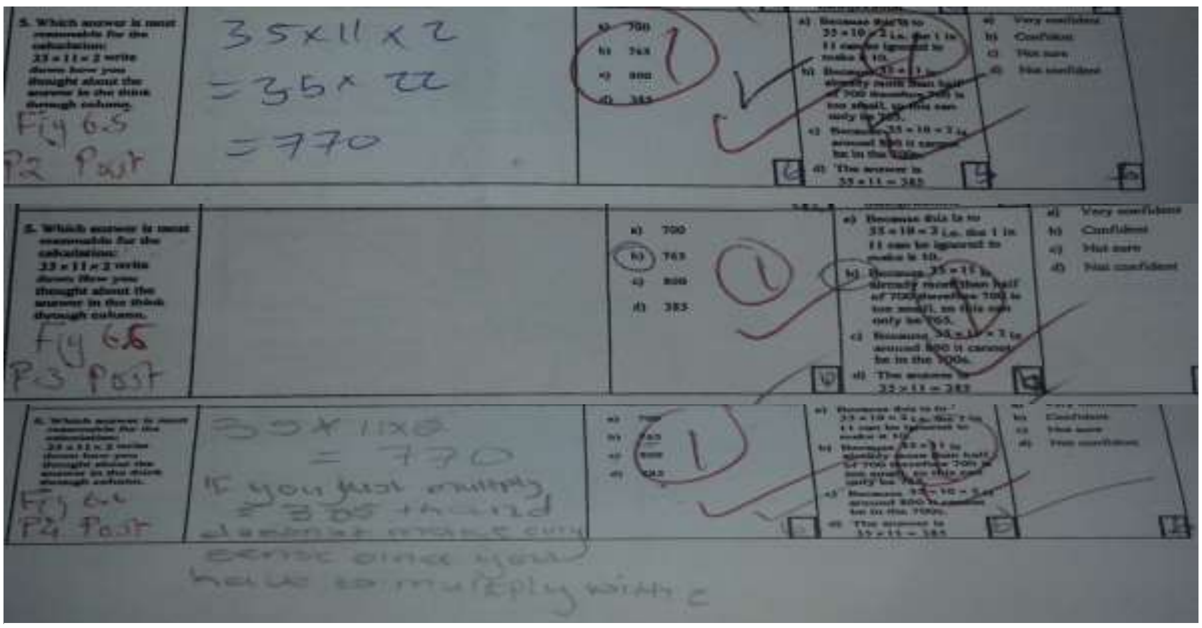

Figure 6.24: Samples of post-test responses to the counting and computational strategies

The post-test result of P2 indicates an algorithm and therefore a choice of a correct answer based on this algorithm (CAA). A comparison of the pre- and post-test results of this participant shows that there was no improvement related to the number sense regarding the computational strategies, since the student maintained the CAA in the post-test as compared to the pre-test. Despite the fact that the participant obtained the correct answer in both the pre and post-tests the pre-test portrayed weaker attributes (see section 3.3) of number sense as compared to the post-test.

The post-test results on the other hand show that participant P3 attempted to identify the correct answer among the alternative answers provided with a correct reason it could be argued that there is a positive development of number sense (CCU). However the fact that the student showed no working in the think through section could also mean there was some guess work there (CGA). The fact that the student obtained the correct answer in the post test is an indication of some improvement in numerical comprehension regarding the counting and computational strategies. Despite the fact that the student obtained the correct answer, the student did not show full numerical understanding and reasoning in counting and computational strategies. For this student it can be concluded that the student improved from WAA to either CCU or at least CGA.

When responding to the post-test participant P4 recognised after performing the algorithm that it did not make much sense and therefore doubled the answer so that it was reasonable. The idea of assessing the reasonableness of the obtained solution is classified as an attribute of number sense (see section 3.3) regarding the counting and computational strategies. By attempting to assess the reasonableness of the solution it could be argued that the respondent possesses some number sense. Therefore this was a typical of CCU. The pretest result of the same respondent shows that the respondent in the pre-test was at WAA. It can thus be concluded that the student improved from WAA to CCU.

6.4.1.5 Post-test responses to the estimation component

The post-test responses show that two post-test respondents compared to one in the pre-test gave a correct answer with the correct reason. Also, three post-test respondents as compared to two in pre-test presented a correct answer without a correct reason. 25 post-test respondents compared to 27 for the pre-test presented the wrong answer as well as a wrong reason. Based on these figures it can be said that the students performed poorly in this section for both the pre- and post-tests.

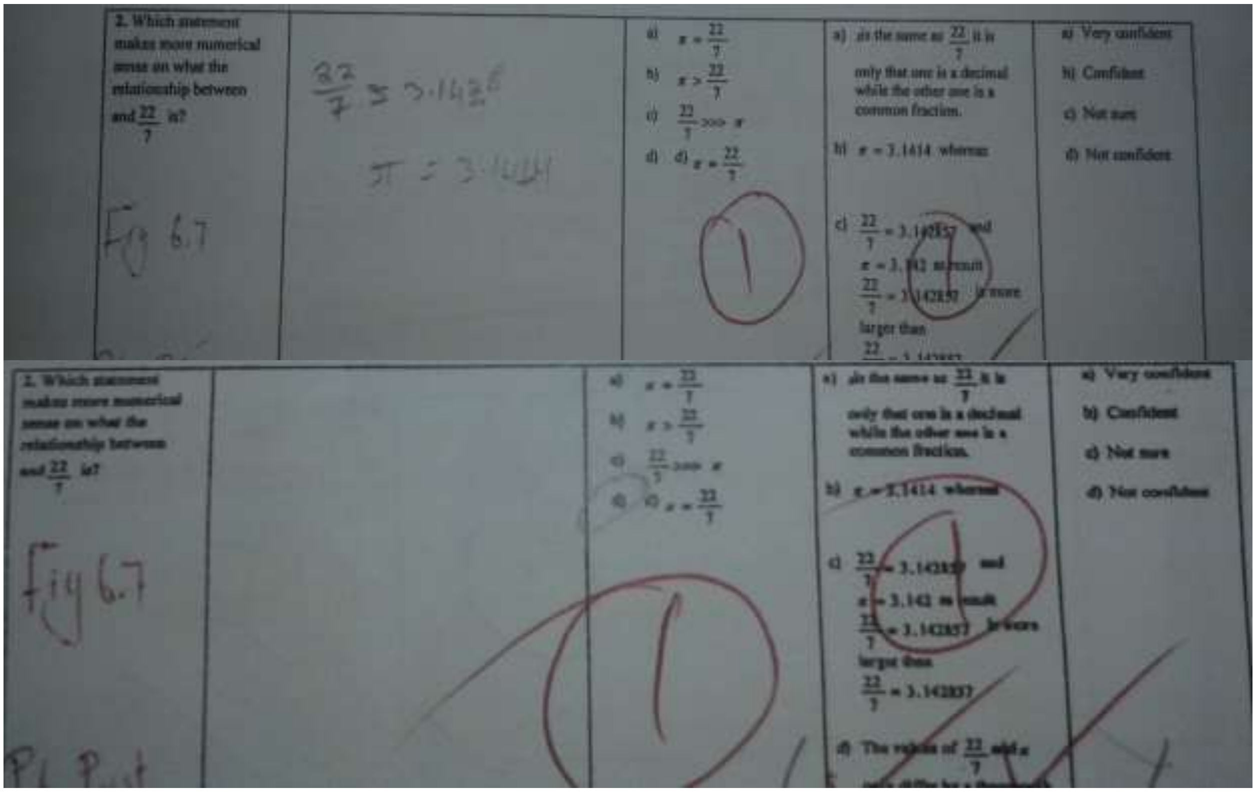

Figure 6.25: Samples of post-test responses to estimation

In the post-test P1 took the same approach but the student recognised that pi actually differs from 22 out of seven at the third decimal place and the student concluded that the pi never equal to 22 out of seven but rather approximates to that. From this difference of the answers presented by the student in the pre and post-test it appears that the student utilised some number sense in the post-test result as opposed to the pre-test (CCU). Nonetheless, the fact that the student did not support both the chosen answer and the reason with the thinking through it could be argued that the student lacks numerical fluency. It can thus be said that the student improved from WAA to CCU; hence a positive development of number sense was observed.

P1 whose work is under P7 the student opted to choose the correct answer before and after the test. The fact that the reasoning was not correct in the pre-test could be interpreted as a CGA. Also, the fact that the student obtained the correct answer in the post-test without a correct reason it can be interpreted that the student guessed the answer (CGA).

This subsection compared the sample of post- and pre-test responses of preservice secondary mathematics teachers for the five sections of number sense. The results presented give an impression that the students performed slightly better in their post-test as compared to their pre-test. The results thus concur with the quantitative data presented in section 5.3 that the students who undertook the number sense intervention programme had a statistically significant improvement in their post-test results as compared to the pre-test results. The next section compares the pre- and post-test interview responses of the students from the experimental group. It should also at be noted this juncture that the interview, and the questionnaires in this study were only administered to the experimental group respondents only as they merely aimed at assessing the experiences of the students over the number sense intervention programme. Given that the students have shown slight improvements in the number sense test items, it appeared that they could have moved from the below basic level to the level in this section. Thus it could be inferred that they only moved one level up from the lowest level of number sense competencies to the next level.

6.4.2 Post-test focus group interview results of the experimental group

This section presents a comparison of the focus group interview responses of experimental group participants before and after the number sense intervention. The data were presented to seek the answer to the question: What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers? The responses were a follow-up on the number sense questions derived from the five components of the number sense test.

6.4.2.1 Focus group responses to the meaning and size of numbers

Based on the number sense item, from section 1 the experimental group students were asked to indicate their preferred answer and their reason to the question: Which fraction is larger than the other between 8 fifteenths and 3 sevenths? Their responses after the intervention are summarised below:

IP 1: *I preferred 3 over 7; we use larger boxes to represent it as fraction as* *compared to the boxes we would use for 8 over 15.* (WAA)

IP2: *I preferred 8 over 15, because 3 over 7 is 0.4 and 8 over 15 is 0.5.* (CCU)

IP3: *I preferred to choose 8 over 15, it is more than a half but 3 over 7 is less than a half, for 3 over 7 a half could have been 3.5 over 7, so 3 did not yet reach a half.* (CCU)

IP4: *The preferred answer is 8 over 15.* (CGA)

IP5: *The answer is 8 over 15. This is because if 7 is divided by 2 it gives 3.5 and 15 divided by 2 gives 7.5.* (CAA)

For this item out of all ten interviewed participants four provided correct responses in the post-test compared only 2 in the pre-test. The reasoning of the respondents indicates some improvement and alignment to the number sense fluency. For instance IP2 in the post improved to CCU from WAA, IP 1 remained at WAA in both post- and pre-test. IP 3 improved from WGA to CCU, IP 4 moved from WGA to CGA and IP 5 from CGA to CCU. At glance the responses of the students after the intervention seem to have improved as compared to their responses prior to the intervention.

6.4.2.2 Focus group results for the equivalence of numbers component

In this section the students were asked to work out the value of *n* in the calculation: $0.3 \times 10^2 = 30000 \times 10^n$; what answer they preferred and the reason they used.

IP1: *n =-3 because 300 times ten to the power of -3 is 30.* (CCU)

IP 2: *I prefer n = -3, 0.3 times ten to the power of 2 equals 30. If we take 30 000 this needs a smaller value to balance out.* (CCU)

IP3: *n = -3, we need a smaller power on the right hand side to balance out.* (CCU)

IP4: *n = - 4, and that is it.* (WGA)

IP5: *n = -3, that is just the only answer that makes sense.* (CGA)

For this particular item is can be observed that the respondents IPP 1, 2, 3 and 5 altered their answers and presented correct answers after the intervention programme. The results further suggest that the number sense reasoning of these suggest positive development of number sense. For instance each of the respondents

IPP 1, 2 and 3 improved to CCU in their post-test results from WAA. IP 4 remained at WGA while IP5 moved to CGA from WAA. It can therefore be noted on the bases of these responses that the students presented more numerically sensible responses after the intervention as compared to before the intervention, despite a few that did not show an improvement such as IP 4 and 5.

6.4.2.3 Focus group responses to the effects of operation on numbers

The respondents for this question were asked to identify the number that goes in the box to make the calculation: $\frac{2}{5}\times[\ \ ]=1$ true. Similar to other items students were to indicate their preferred answers as well as the reason that caused them to prefer such an answer.

IP1: *I prefer em… I choose c) 2 over five times 5 over 2 equals 1. (CCU)*

IP2: *The answer is 1 and 1 over 2, because 1 and 1 over 2 times 2 over 5 gives 1. (WGA)*

IP3: *2 and a half. No reason…I just chose it sir, it is the one that makes sense. (CGA)*

IP4: *Two and half sir, it becomes the reciprocal once converted to fraction sir. (CCU)*

IP5: *The answer is 2 and a half, 5 over 2 times 2 over 5 gives 1. (CCU)*

For the effect of operations on numbers component the responses of respondents IP 3, 4 and 5 improved to CGA, CCU and CCU after the intervention all from WGA, WAA and CAA before the intervention. IP 1 and 5 also improved to CCU after the intervention from WAA and CAA before the intervention. The results suggested an improvement in the number sense reasoning of participants regarding the effect of operation on numbers. Based on the responses above, it can still be concluded that there was an improvement among the responses provided in this section.

6.4.2.4 Focus group results to the computational skills component

For this particular task the emphasis was on finding the most reasonable answer for the calculation: $35\times11\times2$. The respondents were asked to indicate what their preferred answer was as well as the reasons why they opted for that particular answer. The following responses were given before and after the intervention:

IPP1: *The answer I preferred here is 700. Yes just 700 no reason.* (CGA)

IP2: *I prefer 765, since 35 times 11 times 2 gives 770.* (CCU)

IP3: *The answer is 765, it is just that.* (CGA)

IP4: *The answer is, 765, 35 times 10 gives 350, 35 times 2 is 70, so we have a total of 770, therefore I chose 765.* (CCU)

IP5: *The answer is 765, I.e. 35 times 11 gives 385 and 385 times 2 gives 770.* (CCU)

The comparison of post-test results with pre-test shows that the participants improved to some extent. For instance IP 2, 4 and 5 improved to CCU from CAA, WAA and WAA respectively. IP 1 also improved to CCU from CAA while IP 3 moved to CGA from WAA. For this item it could also be concluded that the students have improved their responses. The students also exhibited some abilities to assess the reasonableness of their solutions which is a good characteristic (CCU) of number sense regarding the computational skills.

6.4.2.5 Focus group results on the estimation using relevant benchmarks component

To the question that asked the respondents to state the relationship between the number π and $\frac{22}{7}$.

IPP1: *Always pi Equal 22 over 7.* (WGA)

IPP 2: *Pi approximately equals to 22 over 7. They are not exactly equal.* (CCU)

IPP3: *Pi is much, much larger than 22 over 7.* (WGA)

IPP4: *Pi equals 22 over 7.* (WGA)

IPP5: *Pi equals 22 over 7.* (WGA)

The responses to this question did not change much during the pre and posttest. Respondents IP1, 3, 4 and 5 remained at WGA after and before the intervention. IP 2 was the only participant that improved to CCU from WGA.

The analysis of responses suggests that most of the students did not grasp the concept of rational and irrational numbers. The students seemed to just have the impression that pi is still 22 out of seven as it is equated to in in the circle geometry of the high school mathematics curriculum. It is thus envisaged that if these misconceptions are not rectified they will still be part of the high school mathematics curriculum teaching a situation that fuels the vicious cycle of innumeracy as illustrated in section 1.3.

Additionally, to the interview questions above, students were asked to indicate the materials they needed to work out the items in the number sense test. Their responses before and after the intervention, following are their responses.

IP1*: CALCULATOR.*
IP2: *A PENCIL and PAPER, calculator.*
IP3: *Jus to FIGURE OUT, one must USE THE BRAIN.*
IP4: *I need a CALCULATOR of course.*
IP5: *Just to work out the questions WITH OR WITHOUT ANY MATERIAL.*

All pre-test responses indicated a need for a calculator; however for the post-test some of the responses indicated other methods such as mental arithmetic, pen and paper as opposed to the calculator. For the pre-test many students required a calculator to do mathematical calculations.

Nonetheless, after the intervention some of the students did not require a calculator and could use other methods to work out number sense problems. The characteristics of number sense as alluded to in 3.3 indicate that the number sense requires an individual to be able to explore other alternative avenues of arriving at the desired outcome.

6.5 Summary

This chapter presented the qualitative data for the study. The data presented in this unit was triangulated with the quantitative data presented in chapter 5. The chapter presented the qualitative data according to three sections. Each of these sections targeted to provide an answer to the research questions posed earlier in chapter 1.

The first section presented the qualitative data regarding the level of number sense comprehension of among the preservice secondary mathematics teachers. This section targeted to present results regarding the research question: What is the level of number sense comprehension of first year pre-service secondary mathematics teachers before the intervention? The qualitative data presented in this section triangulated with the quantitative data in section 5.1 this triangulation gave the impression that participants from both the control and the experimental groups possessed very little comprehension of number sense proficiency as well as number sense reasoning. Hence in responding to the question on the level of number sense comprehension the study established that there was a very low level of number sense comprehension (below basic) among the pre-service secondary mathematics teachers before the Critical Theory intervention.

The second section of this chapter presented the data on the subject of the relationship between the number sense and academic performance in mathematics of pre-service secondary mathematics teachers. The section gathered the data aimed at answering the second sub-question: What is the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics? The data presented in this section found that there was some relationship between number sense and academic performance in mathematics. This however does not imply that the students who had number sense automatically passed the Basic Mathematics for Teachers from which the academic performance is derived.

The last section assessed the impact of a Critical Theory intervention in the number sense training of preservice secondary mathematics teachers. The thrust in this section was to provide the answer to the question: What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics? The results show that the intervention based on

Critical Theory improved the number sense of pre-service secondary mathematics teachers to some extend and in some aspects. However there were aspects of number sense that the students did not improve, e.g. the concept of pi. The findings of both chapter 5 and 6 suggest that the role of a Critical Theory intervention was effective in developing some aspects of number sense but not all.

In conclusion the overall data collected in this chapter together with the quantitative data presented in section 5.4 were used to assess the impact of Critical Theory on the number sense training of pre-service secondary mathematics teachers. Thus based on the results collected it could be said that the use of Critical Theory, in the number sense training of preservice secondary mathematics teachers could complement the development of number sense. The next chapter presents the summary, discussion of main findings, conclusions, and recommendations of the study.

CHAPTER SEVEN

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

7.1. Introduction

This chapter presents the discussion of the results, summary, conclusions and recommendations of this study. The chapter first gives a brief summary of each of the chapters, then discusses the main findings of the study as well as provide a summary regarding the answers to the research questions posed earlier in chapter 1 of this study. The chapter then presents the conclusions that were drawn from both the qualitative and quantitative data chapters. The recommendations, research gap identified within this study, the justification and contribution of this study to the already existing body of knowledge as well as the possible area of focus for further research are also presented in this chapter.

7.2 Summary of chapters

This section presents a summary of the 7 chapters in this study:

Chapter 1 presented a framework of the orientation to the study *A Critical Theory enquiry in the development of number sense in Namibian first year pre-service secondary mathematics teachers."* The chapter discussed the nature of the Namibian education system before independence and its reforms and transformation after independence. The problem statement, purpose and research questions of the study, significance of the study, assumptions of the study and research questions were also discussed in this chapter. Closer to its end, chapter 1 gave more details about, delimitations, limitations, the definitions of key concepts and the outline of the thesis.

Chapter 2 reviewed the main tenets of Critical Theory. The chapter further discussed Critical Theory in itself, its manifestations in CME, Ethnomathematics, RME, as well as Hypothetical Learning Trajectories, and ultimately addressed the role of the learner, the educator and the curriculum. The chapter also discussed the research paradigms aligned to Critical Theory and concluded by proposing an eclectic CRENS conceptual framework.

Chapter 3 discussed the literature on number sense training of both pre- and in-service mathematics teachers. The chapter commenced by articulating the meaning of number sense, misconceptions about number sense and the characteristics of number sense. This chapter further discussed ways of measuring and evaluating number sense, the development of number sense in Namibia and elsewhere, the relationship between number sense and academic mathematical proficiency as well as teaching to enhance the development of number sense. Chapter 3 further identified the relative position of number sense on the training of preservice mathematics teachers in Namibia and other parts of the world. The chapter further presented some research findings from the related literature regarding the development of the number sense of preservice teachers. The chapter also discussed issues related to the development of a number sense curriculum and hence a proposed number sense curriculum for pre-service teachers. Chapter 3 concluded with the unique context of this study in the existing literature regarding the proposed CRENS model derived from Critical Theory as well as the key issues from the literature that triggered the researcher's interest.

Chapter 4 presented the methodology used to collect the data in this study. It presented the research design, its justification, population and sampling procedures utilised and research instruments that were utilised to gather the data from participants. Finally, the chapter presents the ethical issues, the validity as and the reliability.

Chapter 5 presented the quantitative data for the study. The quantitative data were presented in chapter 5 according to three sections the data presented about the level of number sense comprehension of preservice secondary mathematics teachers, the relationship between the number sense of preservice secondary mathematics teachers and the role of Critical Theory on the development of number sense of preservice secondary mathematics teachers.

Chapter 6 presented the data in three themes. The data in chapter 6 were presented in three sections: firstly qualitative results regarding the level of number sense comprehension, secondly the relationship between the number sense of preservice secondary mathematics teachers and thirdly the impact of a Critical Theory intervention on the number sense training of secondary mathematics teachers in Namibia.

7.3 Summary of the main findings

The study sought answers to the following main question:

How might a Critical Theory intervention inform the enhancement of Namibian first year pre-service secondary teachers' competencies in number sense?

In order to address the main question the study investigated the following sub-questions:

a. What is the level of number sense comprehension of first year pre-service secondary mathematics teachers?
b. What is the relationship between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics?
c. What is the impact of a Critical Theory intervention programme on the development of number sense of first year pre-service secondary mathematics teachers?

The summary of findings is also presented in three sections the same way in which both the qualitative and quantitative data presentation and analyses were arranged. Thus the following sections present the summary of findings.

7.3.1 The level of number sense comprehension prior to the intervention

This section discusses both the quantitative and qualitative results of the pre-test regarding the number sense proficiency and reasoning of preservice secondary mathematics teachers. Generally, both the quantitative and qualitative results presented in this section indicated that the number sense proficiency as well as reasoning scores of preservice secondary mathematics teachers were Below Basic prior the intervention (see sections 5.2 and 6.2).

The quantitative data presented in section 5.2 showed that most of the participants scored overall number sense proficiency and reasoning scores that were Below Basic. The quantitative data also indicated that very few numbers of respondents scored number sense averages in the proficient and advanced levels of number sense.

From the Qualitative data in section 6.2 it can also be inferred that the participants approached the number sense problems by performing mathematical calculations (CAA or WAA) as opposed to conceptual understanding of number sense for most of the participants. The qualitative data also showed that the level of number sense each of the five individual components of number sense was either below average or at average. An overview of the results indicated that there was weak number sense comprehension among the participants. It was also found that participants performed better on number sense proficiency compared to number sense reasoning

A study by Courtney-Clarke and Wessels (2014) also revealed that there is a poor comprehension of number sense among the preservice primary mathematics teachers. The findings that students had a misconception of taking number sense to be equivalent to the algorithms and computations without necessarily assessing the reasoning behind the calculation has been acknowledged by some literature (Berch 2005; Markovits & Sowder 2004; McIntosh, NCTM 2000; Verschaffel 2007 and Yang & Li 2008) as presented in section 3.9. Since the literature above also acknowledge that the role of number sense in helping students to understand their mathematical content. It could be noted that the pre-service secondary mathematics teachers' poor number sense competencies observed in this study could compromise their academic performance in mathematics.

Since both the quantitative and qualitative data revealed that the number sense of the preservice mathematics secondary teachers was low. In responding to the question of what the level of number sense comprehension of preservice secondary mathematics teachers in Namibia was, before the intervention it could be deduced that there was a very low level of number sense (below basic) among the preservice secondary mathematics teachers in Namibia.

7.3.2 The relationship between number sense and the academic performance

The quantitative data presented in section 5.3 showed a moderate positive association between the number sense proficiency of pre-service teachers and their academic performance in mathematics from both Pearson (r = 0.486) and Spearman's ranking (ρ = 0.552) correlation coefficients. Both the Pearson's and Spearman's' correlation coefficients were found to be significant at 95% level of significance. It was therefore concluded that there is a moderate correlation between the number sense of pre-service secondary mathematics teachers and their academic performance in mathematics. Also, the linear regression analysis model for the number sense proficiency echoed a moderately positive correlation as indicated r = 0.486. The adjusted coefficient of determination of $r^2 = 0.223$ showed that from the effect size it can be deduced that about 22% of the variations in academic performance in mathematics could be explained the number sense proficiency and vice-versa if all other contributing variables are held constant.

A weak positive association for both the Pearson (r = 0.374) and the Spearman's ranking correlation (ρ = 0.329) coefficients regarding the association between the number sense reasoning and the academic performance in mathematics was noted (see section 5.3.) From the coefficient of determination (r^2=0.14) it was inferred that about 14% of the variation in the academic performance in mathematics could be accounted for by number sense reasoning given that the other variables held constant.

The combined effort of the association between the independent variables number sense proficiency and number sense reasoning showed multiple correlation coefficient 0.484. This was interpreted as a moderate positive association. The coefficient of determination showed that about 23% of academic performance in mathematics could be explained by the combination of independent variables number sense proficiency and reasoning.

The study found out that number sense proficiency was the predictor (independent) variable of the dependent variable academic performance in mathematics number sense reasoning whose contribution was not statistically significant. A further analysis of the association between the components of number sense proficiency and academic performance shows that the proficiency in meaning and size of numbers, meaning and effects of operations as well as estimation were the best predictor variables to the dependent variable academic performance, while the variables counting and computational strategies as well as equivalence of numbers were not regarded as best predictor variables to the academic performance in mathematics.

From the qualitative data presented in section 6.3 it was inferred that majority of the students described their number sense experiences to be relevant to their academic performance in mathematics. However a few felt the number sense was not relevant and did not impact their academic performance in mathematics. This reason given by these respondents was that the number sense items were not the same as their examination questions for the Basic Mathematics for Teachers module from which the data regarding their academic performance in mathematics was drawn. It was therefore understood that the students interpreted the information to be relevant primarily if it was similar to the examination that they were taking.

Literature (e.g. Markovits &Sowder, 2004; Verschaffel, 2007) suggests the teaching of mathematics which is examination driven inhibits the inclusion of number sense in the formal mathematics curriculum due to the fact that the connection of number sense to mathematics appears to be concealed. This could be interpreted to imply that these students did not see a direct link between their question papers and the number sense intervention they took hence the assumption that it was not directly relevant to their examination. The position of mathematics education on quantifying the association between number sense and the academic performance in mathematics seems to be silent (Anghileri, 2006; Heirdsfield & Cooper, 2004). It can be seen that the association between number sense and academic performance in mathematics as presented in the foregoing literature is arguable.

Some studies (e.g. Anghileri, 2006; Heirdsfield & Cooper, 2004) acknowledge the existence of the association between number sense and academic performance in mathematics while others do not (e.g. Berch 2005; Yang & Li 200). What is interesting is the fact that none of these based their findings about quantifying the association between number sense and academic performance in mathematics on statistically significant tests results. Additionally these studies were carried out using either elementary or primary school teachers. The fact that the foregoing authors based their decisions on the relationship between number sense and academic performance in mathematics on descriptive statistics suggests a need to carry out an in-depth inferential critique that quantifies the association between number sense and academic performance at secondary school.

By utilising a full bouquet of regression analysis it could be settled that this study filled a very important gap in the literature as it unveiled magnitude of the relationship between number sense and academic performance of preservice teachers in mathematics. Therefore by producing results based on tests for statistical significance, it was considered to be a unique approach on assessing the association between number sense and academic performance. It was for that reason that the results in this study could add to the body knowledge in terms of data to make comparisons with the methodology that was utilised to gather and analyse the data to replicate the studies with similar or other populations, more especially on quantifying and pronouncing the impact of number sense on academic performance. The researcher learned that the impact of the number sense on academic performance cannot be under estimates as number sense can explain about 23% of variations in academic performance in mathematics.

7.3.3 The impact of a Critical Theory intervention

Based on the quantitative data presented in section 5.4 the main findings are that the level of level comprehension for the number sense proficiency of preservice secondary mathematics teachers for the experimental group was below basic (41.17%) prior to the intervention and after the intervention it improved to the basic level (53.57%), (see section 5.4). For the number sense reasoning the pre- and post-test means were 35.1 and 47.4% from this it could be interpreted that the number sense reasoning level of students remained below basic before and after the intervention despite the fact that the mean increased by 12.3%.

The statistical tests for significance of the pre-test-post-test on the experimental group indicate statistically significant differences in the means of the pre- and post-test for the experimental group for both number sense reasoning and proficiency. The differences in the means of the control group remained the same before and after the intervention. The components meaning and size of numbers, counting and computational strategies and estimation had statistically significant differences in their respective means before and after the intervention for both proficiency and reasoning.

The qualitative data presented in section 6.4 by the comparisons of pre- and post-test responses of the students indicated an improvement after the intervention indicated an improvement in terms of the responses of students. In a considerable number of cases the students improved from Wrong Answers based on Algorithms (WAA) to correct answers with Conceptual Understanding (CCU). All pre-intervention respondents indicated a need for a calculator; however for the post-test some of the responses indicated other methods such as mental arithmetic, pen and paper as opposed to the calculator, this was also regarded as evidence for the positive development of number sense. The confidence of the students after the intervention has also shown an improvement as indicated by the results of their self-reflections in section 6.4.

Also, a comparison of the responses given by the participants in the experimental group indicated an improvement in their number sense comprehension. The students responses gave evidence that learning took place (see section 6.4). The students also raised some shortcomings of the intervention such as time constraints for the whole training as well as the fact that they had to attend number sense lessons during their normal semester.

The researcher referred to the limited availability of research on number sense particularly at secondary school level (section 3.1). Despite that other authors (e.g. Johnson, 2008; Post et.al., 2001; Courtney-Clarke & Wessels, 2014) indicated poor number sense among pre-service teachers but did not attempt to assess any intervention to remedy the situation. In addition to finding a low level number sense comprehension among the preservice secondary mathematics teachers this study designed and tested an intervention. This also clearly spells out the contribution of this study to the body of knowledge and the research gap it fills.

Both the qualitative and quantitative data presented in this study supported the finding that the intervention based on the model of CRENS had a positive impact on the development of number sense among the preservice secondary mathematic teachers. It was also inferred that the role of Critical Theory in the development of number sense could not be taken lightly. Therefore to answer the question of what role Critical Theory play in the development of number sense could, the study concluded that Critical theory could improve some of the number sense proficiency and reasoning components significantly.

Using the Cohen's d effect size the study found out that the overall effect size was very large for both number sense proficiency and reasoning components. Therefore the study concluded that the RENS based intervention was effective on the development of number sense.

7.4 Recommendations of the study

By presenting the recommendations of this study in this section it is hoped that these findings will be shared with the preservice secondary mathematics teacher educators of the University of Namibia. The researcher also expects that the findings could be availed to pre-service mathematics teachers themselves, school textbook authors and curriculum planners so that the development of number sense and its inclusion in the teaching of secondary school mathematics could be harmonised.

The impact of critical theory was found to be statistically significant, therefore the researcher recommends the implementation of number sense training that utilises Critical Theory and in particular the CRENS model. As supported by Critical Theory the study recommends that the development of number sense and mathematics teaching should be realistic to the students. In addition the teacher assumes the role of a facilitator but not the source of knowledge, the learning content should be within the experiences or contexts of the students and the student should be an active participant not merely and empty vessel to be filled with information.

It was concluded that the number sense of preservice secondary mathematics teacher had a moderate positive contribution to their academic performance in mathematics. Therefore, it is equally recommended that the development of number sense at secondary school level and teacher training should under no circumstances be perceived as a trivial phenomenon. It is recommended that the concepts of number sense should be fostered at all stages of mathematics education; i.e., primary, secondary and tertiary as far as the education of mathematics is concerned. The study also recommends that there should be a compulsory number sense course for all preservice mathematics teachers regardless of the school level/phase they endeavour to teach mathematics at.

Education policy makers should engage with the University's Faculty of Education particularly the department that trains mathematics teachers to be able to develop informed policies that would ensure numerically proficient teachers that are capable of teaching mathematics effectively once they graduate. Thus this study recommends that the policies regarding the curriculum development of preservice teachers' programmes include components of number sense since its role in the teaching of mathematics could not be underestimated.

The findings of this study reported a poor number sense level (below average) of comprehension that could negatively impact the performance of preservice secondary mathematics teachers' academic performance in mathematics. It is therefore recommended that the development of number sense should be meticulously included in teacher education programmes to ensure that the lack of number sense found in this study is remedied to avoid compromising the quality of mathematics teaching and learning.

It was inferred that the preservice secondary mathematics teachers performed better at number sense proficiency than number sense reasoning. This was merely due to the fact that some number sense proficiency answers could be obtained by merely performing algorithms without necessarily the correct reasoning. The researcher therefore recommends that the development of number sense should equally focus on both the number sense proficiency as well as number sense reasoning to ensure that the students of number sense acquire the number sense.

Some mathematical misconceptions directly related to the high school curriculum were also observed among the majority of students, for instance most if not all of the students had an understanding that *pi* equals 22 out of 7 because when converted to fraction *pi* will give 22 out of 7. This study recommends that the development of number sense should identify and address these misconceptions otherwise such misconceptions could keep on recurring if not corrected before the preservice teachers graduate. The study recommends that by addressing these misconceptions through the integration of number sense development into the school mathematics curriculum in Namibia could potentially break the vicious cycle described in section 1.3.

Among the shortcomings of this study students raised concern over the duration of the intervention. In response to this the researcher it could be better when the number sense course is developed under the same conditions as a normal semester module of 14 weeks per semester to allow the teacher educators to interact with the pre-service teachers. The study proposes that the development of number sense should be practical as opposed to being theoretical; that is, during the number sense training the preservice teachers should do practical activities as opposed to theoretical number sense.

This study after carefully analysing the responses of preservice secondary mathematics teachers to the number sense test items recommends that the opinions of preservice teachers should be taken into account in order to understand their levels of number sense comprehension. Therefore the assessment of number sense should not be merely a qualitative aspect but should also be quantitative.

Textbooks and other teaching and learning material developers should take up the benefits of CRENS into consideration and utilise these as a tool to bring the number sense components to the contexts of the students.

7.5 The research gap

Taking into consideration the limited availability of number sense literature review at secondary school level, could be argued that this discrepancy is also a gap in the literature as number sense teaching is probably assumed to predominantly occur in primary rather than secondary schools. Consequently the study proposes that most of the researchers base their studies on the understanding that number sense the development of number sense is only a primary school level phenomenon but could be nurtured from primary right to tertiary levels of education.

Against this background the study bridged a gap on the development of number sense at secondary school level; therefore the development of number sense should be focused on secondary school mathematics as well. It is envisaged that the findings of this study will help fill an important gap of exploring the development of number sense at secondary school level.

7.6 Contribution of this study to the body of knowledge

From the effect size of 23% and a moderate relationship between number sense and academic performance in mathematics, it could be concluded that this association cannot be perceived as a petty, but to some extend a considerable impact. This study therefore considers the foregoing background as an important contribution that it is made in an African context where the quality of both primary and secondary mathematics education constantly falls short of international benchmark standards such as TIMSS and PISA. Hence this offers a further justification for incorporating number sense training in the curricula for preservice teachers should that is should not just be at primary school only but also even at secondary school level.

By developing the multiple linear regression analysis model presented in the analysis of data, it can also be argued that the study makes a methodological contribution to the research on number sense. This relationship can be used to give guidance on the relationship between number sense and academic performance.

Echoing the ideas presented in section 3.9 that many if not all studies carried out followed a constructivist paradigm based on one of the main constructivist tenets with a major claim that individuals learn by interacting with their environment. It can be observed that most of the studies cited in this chapter assume that number sense can develop by employing the constructivist theory. The researcher therefore argues that by taking a Critical Theory enquiry which allows the students to play an active role, which is the uniqueness of this study, an eclectic approach to the development of number sense becomes feasible.

By applying Critical Theory and therefore introducing the CRENS model which as a unique characteristic of this study, the study fills a gap doing away with a monotonous conceptual frame work that seems to be existing in the development of number sense. A Critical Theory intervention carried out in this study had an impact on the development of number sense of preservice secondary mathematics teachers. Also the average number sense proficiency of preservice secondary mathematics teachers improved from below basic to the basic level. Therefore, it can be claimed that the study contributes to the empowerment of preservice secondary mathematics teachers by improving their number sense.

The nature of resources utilised in the study were suitable for the level of the participants, as a result, these could always be utilised in offering guidance on the number sense training by the other teacher training institutions or the University of Namibia in developing a practical number sense course as per recommendations of this study. As well, the researcher referred to the limited availability of literature in the number sense at secondary school level section 3.10. Therefore by availing this literature, the study makes a crucial contribution by filling a very important knowledge gap of limited availability of literature on the development of number sense at secondary school level.

7.7 Possible focus for further research

This study focused on the development of number sense based on Critical Theory and the CRENS model at pre-service secondary mathematics teachers. The study could be used to inform other studies that could focus on the development of number sense at secondary school level to its possible impact on the number sense development of secondary school preservice mathematics teachers.

The study suggests that further research on number sense could be conducted to train pre-service teachers on how to help them develop the number sense of their learners. Studies therefore should be conducted to find ways and means of how the preservice teachers can plan and design mathematics lessons that incorporate the development of number sense. In this case research will aim at identifying ways and means that could be utilised by preservice secondary mathematics teachers to make number sense an integral part of their teaching when they join the job market.

Further research could also carry out a tracer study to see how these teachers are developing the number sense of their learners once they joined the job market. It could be of interest to assess how these preservice teachers would incorporate the development of number sense among their mathematics learners that they teach when they join the job market. Therefore there is a need to ensure that further studies emerge from this one attempting to assess how the number sense forms a pedagogical practice of these students when they join the job market.

Since the development of number sense was found to have a moderate positive impact on the academic performance of preservice secondary mathematics teachers, this study can be replicated with high school learners to find out the impact of number sense high school mathematics. Therefore for future, research could also inquire into the impact of Critical Theory and CRENS-like models on the development high school mathematics as it has been applied to the development of number sense in this study. This could be done to see if the use of teaching theories supported by Critical Theory, that is, the CRENS model could have a positive impact in the performance of high school learners in mathematics.

Research could also explore the possibility of running in-service number sense training programmes based on critical theory for the practicing teachers. Since the practicing teachers came from environments that fall short number sense it can be assumed that these still lack the number sense. It could therefore be interesting to carry out a study to investigate the levels of number sense comprehension of practicing teachers to identify the means of helping the practicing teachers to implement the number sense of their learners.

References

Adorno, T. (1940). *A Critical Introduction.* New York: Routledge.

Adorno, T. (1998). *Critical Models Interventions*. Catch Words Trans W. Pickford: New York.

Amkugo, E.M. (1993). *Education and Politics in Namibia.* Windhoek: Gamsberg MacMillan Publishers.

Amkugo, E.M. (1993). *Education and Politics in Namibia before and after independence: A comparative perspective.* Windhoek: Gamsberg MacMillan Publishers.

Amutenya, M.N. (2002). *Active Learner Participation: A study of the BETD INSERT teachers' classroom practices in the Ondangwa east & west regions.* A Thesis submitted in the partial fulfilment of the award of a master degree. University of Namibia: Windhoek.

Anghileri, J. (2006). *Developing number sense: Progression in the middle years.* London: Continuum.

Apple, W.M. (1980). *Official knowledge, democratic education in a conserving age.* New York and London: Loutledge.

Apple, W.M. (1982). *Power versus ideology in education Oxford.* United Kingdom: Oxford University Press.

Attidge, C,V., Chard, D.J., Baker, S.K., Clarke, B., Jungjohann, K., & Davis, K. (2010). Learning to teach number sense with technology: A survey of professional development needs, experiences and impacts. *Mathematics Education Research Journal,* 22(1), 31-56.

Ball, D. L., & McDiarmid, D.G. (2000). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 15 (3), 389 – 407.

Ball, D.L. (2000). The mathematical understandings that prospective teachers bring to teacher education. *The Middle School Journal of Mathematics*, 90(4), 449 – 466.

Ball, D.L., & Bell, P. (2004). Developing the estimation and computation abilities students. *Arithmetic teacher*, 22(3), 16 - 25.

Bana, J. (2009). The relationship between the estimation and computation abilities of Year 7 students. In I. Putt, R. Faragher & M. McLean (Eds.), *Proceedings of the 27th annual conference of the Mathematic Education Research Group of Australasia* (Vol. 1, pp. 63-70). Townsville: MERGA.

Bandeira C.F., & Lucena, H. (2004). *Ethnomathematics: Is it the mathematics around us?* Greenwich, CT: Information Age.

Barton, N. (1996). *Ethnomathematics Everywhere. An Exploration of Ideas Across Cultures.* Princeton NJ: Princeton University Press.

Behr, N., Lesh, C.D., & Post, S. (2001). Bridging practices: number sense training of adult learners. *Journal of Teacher Education,* 51, 241-247.

Bell, C. (2004). *Qualitative and quantitative research in statistics education: on symbolizing and computer tools.* Utrecht: CD- Press

Berch, A. (2005). Turning student intuitions into investigations in probability and number sense. *Mathematics Teacher,* 93(3), 120-128.

Berry, B., & Otley, G. (2004). A longitudinal study of invention and understanding in children's multi-digit addition and subtraction. *Mathematical Science and Education, 15*(4), 121-132.

Bobbit, F.V. (2000). The intermediate school mathematics curriculum: Enhancing its potential for developing young children's number sense in the early years at school. *Irish Educational Studies*, 26(1), 5–25.

Bobis, J. (2004). *Number sense and the professional development of teachers.* Perth, Western Australia: Mathematics, Science & Technology Education Centre, Edith Cowan University.

Borba, C.T. (1997). The interactions of mathematics education with culture. *Cultural Dynamics,* 1(2):145–157.

Brink, P. J., & Wood, M. J. (2001). *Basic steps in planning research from question to proposal 5th edition.* Singapore: Jones & Bartlett.

Burn, J. (2004).*Teaching for understanding: A focus on misconceptions about number sense.* Trinidad and Tobago: TRN Research publishers.

Callewaert, S.T. (1999). Critical race theory connects to critical pedagogy. *Educational Philosophy and Theory,* 36, 167–183.

Carpenter, T.P. (1998). *Conceptual Knowledge as a Foundation for Procedural Knowledge.* New York: Lawrence Erlbaum Associates.

Carpenter, T.P. (2008). *Addition and subtraction: A cognitive approach*. Hillsdale, NJ: Erlbaum.

Case, R., & Okamoto, Y. (2006). The role of central conceptual structures in the development of children's thought. *A Journal of research in mathematics education,* 61(5), 134-141.

Case, R., and Sowder, J. (2001). Developing number sense of learners, *For the teaching of mathematics,* 9(1) 33 - 42

Charles, N., & Lester, B. (1984). A model of developing number sense competencies in middle school learners. *Educational Studies in Mathematics*, 19, 333-355.

Chrysostomou M., Pitta-Pantazi D., Tsingi C., Cleanthous E., & Christou, C. (2012). Examining number sense an algebraic reasoning through cognitive styles. *Educational Study Mathematics,* 83, 205-223.

Clandinin, M., & Connelly, K. (2002). *Mathematics Curriculum Review and the development of number sense: A reflection on the New Jersey Mathematics Curriculum and Number sense.* New Jersey: Curriculum Review.

Clegg, A. (2008). *A comparison between the Namibian and the English Mathematics curricula: A paper presented at the annual teachers mathematics congress.* Swakopmund: Mathematics congress, 12-16 May 2008.

Clements, D., & Sarama, J. (2004). Learning trajectories in Mathematics Education: *Mathematical thinking and learning,* 6(2), 81-89.

Clements, D., & Sarama, J. (2007). *The use of trajectories in the learning and teaching early mathematics: The learning trajectory approach.* New York: Routledge.

Confrey, M., Maloney, W., & Nguyen, B. (2011). Learning Trajectories. *Educational Researcher,* 28(7), 4 -14.

Courtney-Clarke, M. (2012). *Exploring the number sense of final year preservice teachers.* A Thesis submitted in the partial fulfilment of the award of a master degree. Stellenbosch University: Stellenbosch.

Courtney-Clarke, M., & Wessels, H.A (2014). Number sense of final year pre-service primary school teachers. *Pythagoras,* 35(1), Art.#244, 9 pages. http://dx.doi.org/10.4102/pythagoras.v35i1.244.

Craig, D. (2006). *Thinking Mathematically.* London: Addison Wesley.

Creswell, S. (2009). Mathematical Abstraction Seen as a Delicate Shift of Attention, A special journal. F*or the Learning of Mathematics,* 9 (2), 2-8.

D'Ambrosio, U. (1985). *Socio-cultural bases for mathematics education*. Sao Paulo: UNICAMP, Campinas.

D'Ambrosio, U. (1990). The history of mathematics and ethnomathematics how a native culture intervenes in the process of learning science. *Impact of Science on Society,* 40(4), 369–377.

D'Ambrosio, U. (1993). *A reflection on Ethnomathematics: Why teach Mathematics*? Sao Paulo, Brazil: Editora Atica.

D'Ambrosio, U. (2006). *Ethnomathematics: Link between traditions and modernity.* Rollerdam the Netherlands: Sense Publications.

D'Ambrosio, U. (2006). Mathematical education in a cultural setting, *International Journal of Mathematics Education in Science and Technology,* 16(4), 469-477.

Dallmayr, L. (1990). Critical theory in mathematics education research. *Educational Studies in Mathematics,* 80(2), 41–55.

Daniel, R. (2005). *The Nature of Mathematical Knowledge.* Oxford University Press: Oxford.

Dehaene, S. (2007). A précis of number sense*. Mind & Language,* 16 (1), 16–36.

Dewey, G.T. (1999). *Democracy of Education.* London: Collier-Macmillan.

Dipoy, E., & Gitlin, L. (1998).*Introduction to research understanding and applying multiple strategies.* Philadelphia: Mosby.

Directorate of National Examinations and Assessment (DNEA), (2011). *Examiner's report: Mathematics NSSC core and extended examinations. Windhoek:* DNEA.

Directorate of National Examinations and Assessment (DNEA), (2012). *Examiner's report: Mathematics NSSC core and extended examinations. Windhoek:* DNEA.

Directorate of National Examinations and Assessment (DNEA), (1998). *Examiner's report: Mathematics IGCSE core and extended examinations.* Windhoek: DNEA.

Directorate of National Examinations and Assessment (DNEA), (2002). *Examiner's report: Mathematics IGCSE core and extended examinations. Windhoek:* DNEA.

Directorate of National Examinations and Assessment (DNEA), (2006). *Examiner's report: Mathematics IGCSE core and extended examinations.* Windhoek: DNEA.

Directorate of National Examinations and Assessment (DNEA), (2008). *Examiner's report: Mathematics NSSC core and extended examinations.* Windhoek: DNEA.

Directorate of National Examinations and Assessment (DNEA), (2009). *Examiner's report: Mathematics NSSC core and extended examinations. Windhoek:* DNEA.

Directorate of National Examinations and Assessment (DNEA), (2010). *Examiner's report: Mathematics NSSC core and extended examinations. Windhoek:* DNEA.

Directorate of National Examinations and Assessment (DNEA), (2013). *Examiner's report: Mathematics NSSC core and extended examinations. Windhoek:* DNEA.

DNEA (1996) *Examiner's report: Mathematics IGCSE core and extended examinations.* Windhoek: DNEA.

Doorman, M. (1999). Exploring Situated Insights into Realistic Mathematics Education, *Journal for Research in Mathematics Education,* 31(1), 13-21.

Dossey, W. (1992). Western mathematics. The secret weapon of cultural imperialism. *Social Studies of Science,* 17, 337–358.

Dowker, P. (2002). Young children's number sense in Finland, Hong Kong and Singapore. *International Journal of Early Years Education, 12*(3), 195-216.

Drijvers, P. H. (2003). *Experiencing School Mathematics: RME approach.* Buckingham: Open University Press.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The Teacher and the Tool; Instrumental Orchestrations in the Technology-Rich Mathematics Classroom. *Educational Studies in Mathematics,* 75(2), 213-234.

Duarte, D. (2004). The algebraic nature of students' numerical manipulation in the New Zealand numeracy project. *Educational Studies in Mathematics,* 58(2), 169-188.

Dyyadova, V., Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2013). Exact and approximate arithmetic in an Amazonian indigene group. *Science,* 30(15), 499–503.

Emmanuelsson, F., & Johansson, S. (1996). *Being numerate: What counts?* Hawthorn, Australia: Australian Council for Educational Research.

Ernest, K., & Skovsmose, O. (1994). Mathematics and sex. *American Mathematical Monthly,* 1976, 83 (5).

Ernest, P. (2001). *Critical Mathematics Education.* In P. Gates (Ed.), Issues In Mathematics Teaching (pp. 277 - 293). London: Routledge Falmer.

Eugene, F.P. (2002). *Promoting mathematical understanding: Number sense in action.* Australian Catholic University: McCauley.

Farrell, L. (2007). What's your number? Developing number sense. *Arithmetic Teacher,* 36(6), 53-55.

Fennell, L., & Landis, M. (2004). Defining, assessing and teaching number sense. In J. Sowder & B. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference* (pp. 35-39). San Diego, CA: San Diego State University.

Fennell, L., & Landis, M. (2004). Why mathematics? Insights from post-structural topologies. In M. Walshaw (Ed.), *Mathematics education within the postmodern,* (pp. 201–218). Greenwich, CT: Information Age

Fishebein, C.L. (2007). *Mathematics Elsewhere. An Exploration of Ideas Across Cultures.* Princeton New Jersey: Princeton University Press.

Foddy, N. (1994). *Handbook of International Research in Mathematics Education.* Mahwah (USA): Lawrence Erlbaum.

Foddy, N. (2004). *Doing research/reading research: A Mode of interrogation for education.* London: Falmer.

Forester, J. (2010). *Critical insights and interventions into mathematics education.* New York: Springer.

Frankenstein, M. (1998). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice,* 22(1), 36–46.

Frankenstein, M. (2000). *Relearning mathematics: A different third R - Radical maths.* London: Free Association Books.

Frankenstein, M., Skovsmose, O., & Ernest, K. (2001). Critical mathematics education: An application of Paulo Freire's epistemology. *Journal of Education,* 165(4), 315-339.

Freedom, V. (1941). Truth and power (C. Gordon, L. Marshall, J. Mepham & K. Soper, Trans.). In C. Gordon (Ed.), *Power/knowledge: Selected interviews and other writings,* 1972–1977 by Michel Foucault (pp. 109–133). New York: Pantheon Books.

Freire, P. (1970). *Cultural action for freedom.* Cambridge, MA: Harvard Educational Review.

Freire, P. (1973). *Education for critical consciousness.* New York: Seabury Press.

Freire, P. (1976). *Pedagogy of the oppressed.* New York: Seabury Press.

Freire, P. (1978). *Pedagogy in process.* New York: Seabury Press.

Freire, P. (1979). The people speak their word: Learning to read and write in Sao Tome and Principe. *Harvard Educational Review,* 51(9), 27-30.

Freire, P. (1993). The importance of the act of reading. *Journal of Education,* 16(5), 5-11.

Freudenthal, H. (1968). Major problems of mathematics education. *Educational Studies in Mathematics,* 12(4), 133-150.

Freudenthal, H. (1977). Why to teach mathematics? *Journal of Mathematics Education,* 11(3), 4–11.

Freudenthal, H. (1983). *Mathematics as an Educational Task.* Dordrecht, Netherlands: Riedel Publishing Company.

Freudenthal, H. (1991). *Revisiting Mathematics Education.* Dordrecht, Netherlands: Kluwer Academic Publishers.

Galbraith, T. (1995). *Investigating Number Sense in Mathematics Teaching: a constructivist enquiry.* London: Falmer Press.

Geary, F. (1993). Predicting Mathematical Achievement and Mathematical Learning Disability With a Simple Screening Tool: The Number Sets Test. *Journal of Psychology of Education and Assessment,* 27(3), 265–279.

Gersten, R. (2009). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities,* 38(4), 293–304.

Geuss, G. (1981). *Developing Mathematical Thinking.* Addison Wesley: London.

Gillham, K. (2008). *Literacy, numeracy and students with disabilities.* Canberra: McMillan.

Gliner, L. (2001). *The Common Sense of Teaching Numbers. Educational solutions.* New York.

Gravemeijer, K. (1994). *Developing Realistic Mathematics Education,* Utrecht, Freudenthal Institute.

Gravemeijer, K., & Doorman, F. (1994) *Developing Realistic Mathematics Education,* Freudenthal Institute: Utrecht.

Greeno, J.G. (2001). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education,* 22 (3), 170 – 218.

Greenough, S., Black, I., & Wallace, J. (1987). Duality, ambiguity and flexibility: Learning Trajectories. *Journal for Research in Mathematics Education,* 25 (2), 115-141.

Griffin, S. (2003). Teaching number sense. *Educational Leadership,* 61(5), 39.

Griffin, S., & Case, R. (1997). Rethinking the primary school math curriculum: An approach based on cognitive science. *Issues in Education,* 3(1), 1–49.

Griffin, S., Case, R., & Siegler, D.F. (1994). *Number worlds.* Durham, NH: Number Worlds Alliance.

Guba,G., & Lincoln, I. J. (1994). *The Politics of Mathematics Education,* Dordrecht: Reidel.

Hambermas, J. (1971). *Understanding Critical Theory. New York: Sage*.

Hambermas, J. (1973).*The Philosophical Discourse of Modernity: Twelve Lectures,* trans. F. Lawrence, Cambridge, Mass: MIT Press.

Hanson, D., & Hogan H. (2000). Designing of number sense lessons for pre-service teachers. *Journal of Mathematics Education,* 48(6), 79-85.

Harç, G.S. (2010). Laying the Foundation for Computational Fluency in Early Childhood. *Teaching Children Mathematics.* 9 (6): 306 – 311.

Harel, M., Behr, L., & Lesh, J. (2001). The Effects of the Cooperative Learning Method Supported by Multiple Intelligence Theory on Turkish Students' Mathematics Achievement. *Asia Pacific Education Review.* 10(4), 465-474.

Heirdsfield, A.M., & Cooper, T.J. (2002). Flexibility and inflexibility in accurate mental addition and subtraction: two case studies. *Journal of Mathematical Behaviour,* 21, 57 – 74.

Heirdsfield, A.M., & Cooper, T.J. (2004). Factors affecting the process of proficient mental addition and subtraction: case studies of flexible and inflexible computers.

Journal of Mathematical Behavior, 23: 443 – 463.

Held, P.K. (1980). Towards a critical theory in education. *Teachers College Record,* 97, 47–68.

Heuvel-Panhuizen, M. (2002). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education,* 33(1), 18–28.

Heuvel-Panhuizen, M. (2003). A study of sixth grade students' development of number sense through well-designed number sense activities. *Chinese Journal of Science Education,* 10(3), 233-259.

Heuvel-Panhuizen, M. (2010). Critical mathematics education: An application of Paulo Freire's epistemology. *Journal of Education,* 165(4), 315-339.

Higgs, P., and Smith, J. (2002). *Rethinking the truth.* Lansdowne: Juta & CO.

Hilbert, J. (2001). *Conceptual and Procedural Knowledge in Mathematics: An Introductory Analysis. In Conceptual and Procedural Knowledge: The Case of Mathematics.* Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Hope, J.A. (2008). Promoting number sense in school. *Arithmetic Teacher,* 36 (6): 12 – 16.

Horkheimer, M. (1937). *Traditional and critical theory.* New York: Herder and Herder.

Horkheimer, M. (1940). *Dialectic of Enlightenment.* New York: Seabury Press.

Howden, H. (2009). The teaching of number sense. *Arithmetic Teacher,* 36(6), 6-11.

Hulley, S.B., Cummings, S.R., Browner, W.S., Grady, D., Hearst, N., & Newman, T. (2001).*Designing clinical research*. Philadelphia: Lippincot, Williams & Wilkins.

Hungerford, C.Y. (2004). Skill in estimation problems of extent and numerosity. *Journal for Research in Mathematics Education,* 13 (3), 211-232.

Illukena, A. (2011). *Presidential commission on education, culture and training report.* Windhoek: Gamsberg Macmillan.

Jackson, P. (1986). *The practice of teaching.* New York: Teachers College Press.

Japan International Cooperation Agency (JICA), (2013*). Strategies for implementing ASEI-PDSI.* Tokyo: JAICA.

Jessop, P. (2012). *Critical race theory: An introduction.* New York, NY: New York University Press.

Jett, C.C. (2012). Critical Race Theory (CRT) Interwoven with Mathematics Education Research: *Journal of Urban Mathematics Education*, 7(3); 88-102.

Johnson, J. (1994). Promoting the understanding of number sense concepts. *Journal of mathematics education.* 21(3), 12-20.

Johnson, J. (2008). *The Cambridge companion to critical theory.* Cambridge, United Kingdom: Cambridge University Press

Jonassen, H. (2004). Teaching Number sense in interactions. *Journal of mathematics education,* 4 (2), 18-19.

Jonker, H., & Pennink, F. (2010). Effects of a problem-solving-based mathematics course on number sense of preservice teachers. *Journal of College Teaching & Learning,* 1(2), 33-49.

Joyner, G.H. (2004). Developing students' number sense*, Journal of mathematics education,* 12 (4), 22-32.

Kalchman, M., Moss, J., & Case R. (2001). Psychological models for the mathematical understanding: Rational numbers and functions. In Carvers, S.M. & Klahr, D. (Eds.) *Cognition and Instruction Twenty-five Years of Progress.* Mahwah, NJ : Lawrence Erlbaum Associates, Inc.

Kamii, C., & Dominick, A. (1997). To teach or not to teach algorithms. *Journal of*

Mathematical Behavior, 16 (1): 51 – 61.

Karmiloff-Smith, E. (1992). Promoting mathematical understanding: A Hypothetical Learning Trajectory Approach. *Mathematics Education Research Journal,* 14(2), 133-149.

Kasanda, C.D. (2010). The role of Continuous Professional Development (CPD) in mathematics education in Namibia. *Journal of Teacher Professional Development,* 5(2). 23-30.

Kasanda, C.D., & Kapenda, H.M. (2011). A study to investigate the implementation of learner centred teaching in selected schools in Namibia. *A paper presented at the annual teachers mathematics congress.* Swakopmund: Mathematics congress, 20-2 May 2011.

Kayhan, D., & Altay, G. (2010). Multiplication with Fractions: A Piagetian Constructivist Approach. *Mathematics Teaching in the Middle School*, 3(5), 339-343.

Kilpatrick, J. (2001). *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press.

Kincheloe, B. (1994). A critical Theory today: A reflective interpretation of critical theorists. *Journal of Urban Mathematics Education,* 3 (1), 9–18.

Kincheloe, J. L., & McLaren, P. (1994). Rethinking critical theory and qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 139–157). Thousand Oaks, CA: Sage.

Kitzinger, V. (2004). *Handbook of research on mathematics teaching and learning*. New York: Macmillan.

Knijnik, Z. (1993). *The position of critical mathematics education.* In D. A. Grouws (Ed.).

Knijnik, Z. (2002). *Ethnomathematics and political struggles.* Zentralblatt für Didaktik der Mathematik 98(6), 188-194.

Krueger, G. (2008). Divisibility and multiplicative structure of natural numbers: Preservice teacher's understanding. *Journal for Research in Mathematics Education,* 27(5), 540–563.

Kunzig, H. (1997). Developing theories of mathematics education research: The ESM story. *Educational Studies in Mathematics,* 51, 23–40.

Kwamboka, I. (2013). *An assessment of implementation of ASEI-PDSI in selected schools in Kenya.* The Mathematics Educator, 13(2), 7–21.

Lankshear, T. (2003). A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics,* 46, 87–113.

Lean, M. (1994). The native culture interventions in mathematics education. *Impact of Science and Mathematics on Society,* 40(4):369–377.

LeFevre, J.A. (2003). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science,* 41(2), 55–66.

LeFevre, JA., Greenham, G.L., & Waheed S.L. (2003). Pathways to mathematics: longitudinal predictors of performance. *Child Development Journal.* 81(6): 1753–1767.

Leinhardt, D., & Smith, R. (2005). Pupils' multiple number and counting strategies. *Journal of Mathematical Behavior,* 13(7), 50–57.

Leung, G. (2001). Divisibility and multiplicative structure of natural numbers: Preservice teacher's understanding. *Journal for Research in Mathematics Education,* 27(5), 540–563.

Lloyd, J. (2006). Rational versus irrational number. *Mathematics Teaching*, 77(12), 1–7.

Loraine, B. (1998). *Handbook of qualitative research*. Oaks, CA: Sage.

Loraine, B. (2004). *How to research.* Birmingham: Open University Press.

MacIntosh, A. (2011). A refining a frame work for examining basic number sense. *For the Learning of Mathematics,* 12, 2 –8.

Mack, H. (2000). Visualizing as a mathematical way of knowing: Understanding figural generalization. *Mathematics Teacher,* 101(1), 69–75.

Mack, H. (2005). Images of numbers: A reflective practice among preservice teachers. *Cognition,* 44, 159–196.

Mack, H. (2010). What I hear I forget, What I see I remember a little what I hear, see and do I will explain to others. *Cognition,* 36, 59–63.

MacMillan, S., & Schumacher, V. (2006). *Educational Research methodology: Qualitative and Quantitative approaches.* Boston: Allyn and Bacon.

Makari, (2009). An assessment of Contextual teaching in Mathematics among selected schools in Khomas Region. *A paper presented at the Annual mathematics teachers conference. Swakopmund*: 16-19 October 2013.

Maloney, N., & Nguyen, T. (2011). *Africa counts: Number and pattern in African cultures (3rded.).* Chicago: Lawrence Hill.

Markovits, Z., & Sowder, J. (1994) Developing number sense: A coparative intervention study in Grade 7 and 8. *Journal for Research in Mathematics Education,* 25 (1), 4-29.

Markovits, Z., & Sowder, J. (2004). Developing number sense: an intervention study in grade 7. *Journal for Research in Mathematics Education,* 25(1),16 – 32.

Markovits, Z., & Sowder, J. (2001). Knowledge and teaching: Mathematics for the future. *Harvard Educational Review,* 57(1), 1–22.

Matos, G., & Alves, V. (2007). The role of critical mathematics education in the mathematics classroom. *For the Learning of Mathematics,* 17(3), 39-43.

MBESC, (1993). *Education in Pre-and post-independent Namibia.* Ministry of Basic Education Sport and Culture MBESC: Windhoek.

MBESC, (1993). *Towards Education for all.* Windhoek: Gamsberg Macmillan.

MBESC, (2012). Special *Report on the NSSCO Results:* Ministry of Education and Culture. Windhoek Namibia.

McCloskey, F., & Macaruso, O. (2005). *One-dimensional man.* Boston: Beacon Press.

McCloskey, F., & Macaruso, O. (1995). Developing algebraic thinking in early grades: Case study of Korean elementary school mathematics*. The Mathematics Educator,* 8(1), 88–106.

McIntosh, A. (1992). A qualitative assessment of the proposed framework for examining basic number sense. F*or the Learning of Mathematics*, 12, 2 –8.

McIntosh, A. (1977). When Will They Ever Learn? 'Arithmogons', *Mathematics Teaching,* 19 (3) 92-95.

McIntosh, A., & Dole, C.J. (2000). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal***,** 13 (84), 496-513.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). Strategies for assessing number sense characteristics. *For the Learning of Mathematics,* 22 (4), 7-15.

McIntosh, A., Reys, B. J., Reys, R. E., Bana, J., & Farrel, B. (1997). *Number sense in school mathematics: Student performance in four countries.* Perth, Australia: Edith Cowan University.

McIntosh, A., Reys, B., & Reys, R. (1992). A proposed framework for examining basic number sense. for the Learning of Mathematics*, Mathematics Research Journal,* 12(2) 8 – 18.

McIntosh, A., Reys, B.J., & Reys, R.E. (1997). *Teaching Mathematics Through Problem Centred Approach.* New York: New York Teachers College Research Publication.

McKenzie, L. (1990). Difficulties of introducing number sense reasoning in middle grades. *Mathematical Thinking and Learning,* 7(3), 231–258.

McMillan, J.H., & Schumacher, S. (2006). *Research in Education: Evidence-based Enquiry.* New York: Pearson.

Mellenberg, W. (2008). Cognitive styles, dynamic geometry and measurement performance. *Educational Studies in Mathematics,* 60 (2), 15 –23.

Menon, R. (2004). Pre-service teachers' number sense. *Focus on Learning Problems in Mathematics,* 26(2), 49–61.

Merton, K., & Kendall, P.C. (2007). Controversies around the Role of Statistical Tests in Experimental Research. *Mathematical Thinking and Learning,* 2(1), 75–98.

Ministry of Education & Culture (MEC), (2012). *Towards mathematics for all: Astudy to investigate the possibility of mathematics as a compulsory subject at grade 12 level in Namibia.* Windhoek: Ministry of Education.

Morgan, F. (2008). Why *the Professor Can't Teach. Mathematics and the Dilemma of University Education.* New York: St Martin's Press.

Morgan, F. (2004). Lesson planning: The key to developing numbers ense. *Arithmetic Teacher,* 36(6), 18-21.

Morgan, F., & Krueger (2003). *Ethno methodological Foundations of Mathematics.* London: Routledge & Kegan Paul.

Morris, K. (2001). *Developing number sense in the middle grades.* Reston, VA: National Council of Teachers of Mathematics.

Mullis, I.V.S., Martin, M.O., Pierre, F., & Alka, A. (2012*). TIMSS 2011 International Results in Mathematics.* Boston: TIMSS & PIRLS International Study Centre.

Munn, M., & Drever, I. (2004). Thinking ahead of Numeracy. Mathematical Illiteracy and Its Consequences. Penguin, London.

Murata, B., & Fuson, F. (2006). Incorporating course content in one document: Learning Trajectories in Mathematics Education. *College Teaching,* 53(2): 88-96.

National Council of Mathematics teachers (NCTM), (2000). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Naukushu, S.T. (2012). *An assessment of number sense of Oshana Education Region learners.* Oshakati: University of Namibia

Naukushu, S.T. (2013). *Preparing students for Vision 2030: A paper presented at the annual research conference of 2013.* Oshakati: 16-19 October 2013.

Naukushu, ST. (2011). *Factors affecting the development of Number Sense of Grade 12 learners in Oshana Education Region and its influence on Academic performance.* A thesis submitted in partial fulfilment of the Degree of Master of Education. University of Namibia: Oshakati Campus.

Ndlovu, M. (2014). The effectiveness of a teacher professional learning programme: The perception and performance of mathematics teachers, *Pythagoras,* 35(2), Art # 237, 10pages.http://dx.doi.org/10.4102/Pythagoras. v35i2#37,

Neuman, W. L. (2003). *Social research methods, 5th edition.* Boston: Pearson Education, Inc.

Ngololo, E.N., & Kapenda, C.D. (2014). *An assessment of inclusion of SMASE concept in the high school teaching of mathematics: A study on few selected schools in the Khomas Region. A paper presented at the Annual EDUCON conference. Windhoek.* 16-19 October 2013.

Nguyen, G. (2011). A study of the Implementation of a Problem-Based Learning Approach in University Classes in Vietnam. *School of Education, College of*

Design and social Context RMIT University, Journal of Mathematics and science education 2(3) 19-55.

Nielson, B. (1996). The integration of mathematics education in the mainstream curriculum for the Learning of Mathematics*, South African Journal of education* 17(3),39-43.

Nkomo, M. (1990). *Pedagogy of Domination.* New Jersey: Africa world press Inc.

Nohda, C., & Emori, V. (2005). Exploring the challenges and opportunities of promoting number sense in the middle grades. *Mathematics Teaching in the Middle School,* 1(3), 114-120.

North West University, (2011). *Advanced Certificate in Mathematics Education Curriculum.* Mafikeng: North West University.

North West University, (2014). *Advanced Certificate in Mathematics Education Curriculum.* Mafikeng: North West University.

NPC, (2003). *National Development plan 2 and 3.* Windhoek**:** National planning commission.

Number Sense Research Group, (1995). Number sense development strategies. *Journal of Elementary Education,* 48(6), 79-85.

Number Sense Research Group, (2005). *Assessment and evaluation standards for number sense.* NCTM.

Ojelanki, O. (2010). What could be more practical than good theory*, Educational Studies in Mathematics*, 58(3), 393– 413.

Okamoto, Y., & Case, R. (1996,). A psychological model of number sense and its development. *Paper presented at the annual meeting of the American Education Research Association,* San Diego. 04-06 April 1996.

Oppenheim, K. (2000). An assessment of estimation proficiency of students. *Mathematics Education,* 21 (7) 60-69.

Orey, L. (2000). On culture, numerical thinking and mathematics education. *American Anthropologist,* 108(2), 347–362.

Ornstein, B., Allan, J., Francis, H.G., & Hunkins, M. (2009). The dynamics curriculum of mathematics education. *American Mathematical,* 107(1) 1–14.

Otieno, F. (2012). The Ethnomathematics of People who live in round houses. *Anerican Research Journal,* 37(1) 18–21.

Oyedele, V. (2003). *Learning and Curriculum studies.* Windhoek: Gamsberg Macmillan.

Pais, A., Fernandes, E., Matos, J., & Alves, A.S. (2007). *Methodology in Critical Mathematics Education: A case analysis.* Lisbon: University of Lisbon.

Patton, M. Q. (1990). *Qualitative evaluation and research methods (2nd ed.).* Thousand Oaks: The University of California: Sage.

Payne, V., & Payne, W. (2004). *Qualitative research for education: An introduction to theory and methods.* Boston: Allyn and Bacon.

Peck, C., & Connell. D. (2011). *Where mathematics comes from.* New York: Basic Books.

Pesek, F., & Kirshner, B. (2000). Interference of instrumental instruction in subsequent relational learning, *Journal of research in mathematics education,* 31, 542-540.

Platt, V. (2001). *Grounded theory methodology,* Boston: South End Press.

Polit , G., & Hungler, D.F. (2007). *Knowledge and values in social and educational research.* Philadelphia: Temple University Press.

Polit, D.F., & Hungler, B.P. (1997). *Nursing research, principles and methods* 5th edition. London: Lippincott Company.

Polit, G., & Hungler, D.F. (1997). *Two in one: Handbook of quantitative and qualitative research,* Philadelphia: Temple University Press.

Post, T. R., Harel, F.G., Behr, T. P, & Lesh, S. (2001). *Teachers' knowledge of rational and irrational number concepts.* Albany, NY: SUNY Press.

Potgieter, P. (2014). *Exploring number sense teaching strategies of Primary school teachers.* Windhoek: University of Namibia.

Powell, A. B., & Frankenstein, M. (2006). Ethno mathematical praxis in the curriculum, *Educational Studies in Mathematics,* 9(3), 31–36.

Powell, A. B., Single, M., & Lloyd, F. (2006). *Ethnomathematics, Challenging Eurocentrism in Mathematics Education.* New York: University of New York Press, Albany NY.

Rasch, G. (2002). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics,* 64(3), 345–371.

Raune, J. ((1988). The application of Critical Theory. *Political studies, 36* (3), 397-590.

Reys, B. (1991). *Developing number sense in the middle grades.* Addenda Series VA: National Council of Teachers of Mathematics.

Reys, B. (1994). *Conference on number sense: Establishing foundations for research on number sense and related topics.* San Diego, CA: San Diego State University.

Reys, P., & McIntosh, A. (1992). The study of developing the fifth grade students' number sense through teaching activities. *Research and Development in Science Educational Quarterly,* 1, 78-119.

Reys, P., & McIntosh, A. (2002). Number sense strategy instruction for students with learning disabilities. *Remedial and Special Education,* 23, 268–278.

Reys, R. (2004). Reasonable and reasonableness of answers: Kuwaiti middle school teachers' perspectives. *Educational Studies in Mathematics,* 65 (7)77 – 94.

Reys, R. E., & Yang, D. C. (2008). Relationship between computational performance and number sense among sixth- and eighth-grade students in Taiwan. *Journal for Research in Mathematics Education,* 29(2), 225–237.

Reys, R. E., & Yang, D. C. (1999). Learning number sense and learning to teach: Learning cycles in mathematics teacher education. *Education Studies in Mathematics,* 26(1), 71-94.

Reys, R. E., & Yang, D. C. (2009). Number sense strategies used by pre-service teachers in Taiwan. *International Journal of Science and Mathematics Education,* 7(2), 383-403.

Reys, R. E., Nohda, O., & Emori, G.H. (2014). Promoting number sense in the middle grades. *Journal of mathematics education,* 1(3), 114-120.

Reys, R., & Reys, B. (1998). Developing mathematics understanding and abstraction: The case of equivalence in the middle school years. Mathematics *Education Research Journal,* 21(2), 76–95.

Reys, R., & Reys, B. (2009). Effects of a problem-solving-based mathematics course on number sense of preservice teachers. *Journal of College Teaching & Learning,* 1(2), 33-49.

Rittle-Johnson, O., Siegler, L., & Alibali, G. (2001). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education,* 34, 37–73.

Robert, G.H. (2002). The design of number sense with fraction teaching. *Journal of Elementary Education*, 48(6), 79-85.

Rosa, M., & Orey, D. C. (2008). Ethnomathematics and cultural representations: Teaching in highly diverse contexts. *Acta Scientiae - ULBRA,* 10, 27-46.

Rosa, M., & Orey, B. (2011). *Ethnomathematics: Challenging euro centrism in mathematics education.* Albany, USA: SUNY Press.

Rosa, M., & Orey, B. (2003). Cultural assertions and challenges towards pedagogical action of an ethnomathematics program. *For the Learning of Mathematics,* 27(1), 10-16.

Rosa, M., & Orey, B. (2009). Challenges faced by multicultural and multilingual schools in the United States: The case of mathematics. *La Salle -Revista de Educação, Ciência e Cultura,* 14(1), 29-44.

Rosa, M.; & Orey, B. (2010). Ethnomodeling: A Pedagogical Action for Uncovering Ethnomathematical Practices. *Journal of Mathematical Modelling and Application,* 1(3), 58-67.

Sarama, J., Clements, D., Barrett, T., Van Dine, F., & McDonel, C. (2011). Five-year growth trajectories of children with learning difficulties in mathematics. *Journal of Learning Disabilities,* 42, 306-321.

Saunders, C.T., Lewis, B., & Thornhill, F. (2009). Preservice teachers' knowledge of difficulties in decimal numeration. *Journal of Mathematics Teacher Education,* 4(3), 205-225.

Schoenfeld, A. H. (2006). Making sense of "out loud" problem-solving protocols. *Journal of Mathematical Behavior.* 4, 171-191.

Schwab, K. (2015). *The global competitiveness Report (2014 – 2015).* Geneva: World Economic Forum.

Shawer, C.J. (2010). *Curriculum in Education Theory and Practice.* New York: Macmillan.

Shirikov, M. (1937). *A textbook of Marxist Philosophy*, London:Gollancz.

Simon M. &Tzur G. (2004). The Role of HLT in Problem Solving. *Harvard Educational Review,* 57 (1), 23-48.

Simon, M. (1995). Learning mathematics the H:T Approach. *Paper presented at the annual meeting of the American Educational research Association,* San Francisco.

Simon, V. & Blume, M. (1992). 'The Surprise' Number sense in action. *For the Learning of Mathematics,* 8 (3) 34–40.

Simson, D.W. (2012). *Critical Constructivist in Mathematics Education Research.* Georgina: College of Education, Georgina State University.

Simson, D.W., & Bullock, E.C. (2012). *Critical postmodern theory in Mathematics Education Research: A praxis of uncertainty.* Georgina: College of Education, Georgina State University.

Skovsmose, O. (2002). Skovsmose, O. & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 383-407). Mahwah, USA: Lawrence Erlbaum.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education.* Dordrecht, The Netherlands: Kluwer.

Skovsmose, O. (2002). *Dialogue and learning in Mathematics Education: Intention, Reflection, Critique.* Dordrecht, the Netherlands: Kluwer.

Skovsmose, O. (2005). Breaking political neutrality: The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz & B. Nebres (Eds.). *Sociocultural research on mathematics education: An international perspective* (pp. 37-55). Mahwah, USA: Lawrence Erlbaum.

Skovsmose, O. (2010). *Critical Mathematics Education for future.* Aalborg University: Denmark.

Skovsmose, O. (1998). Linking mathematics education and democracy: Citizenship, mathematical archaeology, mathemacy and deliberative interaction. *Zentralblatt für Didaktik der Mathematik,* 98(6), 195-203.

Skovsmose, O., & Nielson, L. (1996). Critical mathematics education. In A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 1257-1288). Dordrecht, The Netherlands: Kluwer.

Sowder J., & Case, R. (2000). *Fostering the development of children's number sense.* San Diego, CA: San Diego State University.

Sowder, J. (1990). Mental computation and number sense. *Arithmetic Teacher.* 37(7), 18-20.

Sowder, J. (1992). In J. Sowder & B. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference (pp. 1-5)*. San Diego, CA: San Diego State University.

Sowder, J. (2002). *Estimation and number sense: Handbook of research on mathematics teaching and learning.* New York: Macmillan.

Sowder, J., & Wheeler, C. (2009). Making sense of numbers in school mathematics. In G. Leinhardt, & R. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching.* Hillsdale, N. J.: Erlbaum.

Sowder, J., & Wheeler, H., (2009). Teaching computation in ways that promote number sense. *Paper presented at the conference on Challenging Children to Think When They Compute: Developing Number Sense, Mental Computation and Computational Estimation.* Brisbane: Queensland University of Technology.

Steffe, L. (2003). *The role of Hypothetical learning trajectories in the teaching and learning of Mathematical.* Erlbaum: Mahwah.

Stein, R. (2008). Strategy use and estimation ability of first year college students. *Journal for Research in Mathematics Education, British Journal of mathematics Education.*13 (5) 350-359.

Stigler, O. and Baraness, B. (1988). Multiculturalism a mathematical problems for the future. *Mathematical Intelligencer,* 20(2) 7–15.

Streefland, M. (1985). A meta-analysis of Contextualised Mathematics Education instructional components. *Review of Educational Research,* 79, 1202-1242.

Streefland, M. (1985). *The Real World of Mathematics.* Blackie: Glasgow

Streefland, M. (1993). Aspects of Experience with Concrete and Realistic mathematics activities. *Educational Studies in Mathematics.* 12(2)151–169.

Streefland, M. (1996). Manipulatives, Realistic Mathematics Education in Primary School. *Journal of Mathematical Behaviour,* 21 (5) 44-45.

Streefland, M., Yang, B., & Li (2008). *Chinese Mathematics: a concise history.* Clarendon Press: Oxford.

Sulak, G. (2008). Visualisation in high school mathematics, *For the Learning of Mathematics,* 6 (3), 42-46.

Tansheng, J. (2007). Learning Algebra in a Computer Algebra Environment: Design Research on the Understanding of the Concept of number sense applied to Algebra, *Singaporean Journal for Mathematics Education.* 21 (2)115–121..

Treffers, A. (1978). Wiskobas and Freudenthal Realistic Mathematics Education. *Educational Studies in Mathematics.* 25(2), 89-108.

Treffers, K. (1987). *Intuition in Realistic Mathematics Education.* Reide: Dordecht.

Treffers, K. (1991). *Challenges of developing and implementig Realistic Mathematics Education*. Utrecht: CD-ß Press / Freudenthal Institute.

Troutman, Y. L. (2004). Exploring the connections among number sense, mental computation performance, and the written computation performance of elementary preservice school teachers. *Journal of College Teaching & Learning,* 1(12), 71-90.

Tyle, P., & McKenzie, M. (1990). *Modern Approaches to Learning Trajectories i Mathematics Education.* New Humpshire: Summerset.

Tyler, P. (1949). *Education and Power The curriculum, problems and possibilities.* New York: State University New York Press.

Tyson, B. (2006). *Critical theory and education today: Where did we come from?* London: Routledge.

University of Namibia, (2014). *Faculty Prospectus, Faculty of Education.* Windhoek: University of Namibia.

University of South Africa, (2012). *The mathematics curriculum:* Manual for BEd (Hons). Pretoria: University of South Africa.

Van den Heuvel-Panhuizen, M. (1996). *Realistic Mathematics Education, work in progress. NORMA-lecture held in Kristians Norway on 5-9 June 1998*. Last accessed on 9 January 2015 http://www.did.stu.mmu.ac.uk/msm/rme.html.

Van den Heuvel-Panhuizen, M. (2001). *Mathematics education in the Netherlands: A guided tour. Freudenthal Institute*. Utrecht: Utrecht University. Last accessed on 14 July 2014. http://www.fisme.science.uu.nl/en/rme/TOURdef+ref.pdf.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage*. Educational Studies in Mathematics,* 54(9), 9-35.

Van den Heuvel-Panhuizen, M. (2010). *Freudenthal's work continues.* Paper presented at 12th International Congress on Mathematical Education held 8 -15 July 2012, COEX, Seoul, Korea. Last accessed on 1 July 2014 http://www.icme12.org/upload/submission/2048_F.pdf.

Veloo, C. (2010). Locating negative decimals on the number line: Insights into the thinking of pre-service primary teachers. *The Journal of Mathematical Behavior*, 30(1), 80-91.

Venkat, H. (2013). Theorizing temporal range in early number teaching in South Africa. *For the Learning of Mathematics,* 33(2), 14-22.

Venter, E., Higgs, L., Jeevavanthan, L., Letseka, M., & Mays, T. (2007).*Theory and Research in Education*. Pretoria: University.

Verschaffel, D., Greer, L., & DeCorte, W. (2007). *Mapping the territory, primary students with learning difficulties: Literacy and numeracy,* Canberra: DETYA

Verschaffel, J. (2007). Assessing Computer aided number sense lessons. *School Science and Mathematics,* 110(4), 220-230.

Verschaffel, J., & DeCorte, W. (1996). Number sense. *The Mathematics Teacher,* 81(6), 437– 440

Verschaffel, J., & DeCorte, W. (2006). Building a Knowledge Base for Teachers and Teaching in Mathematics Education. *The Lower Secondary School Journal*. 10(5), 510-524.

Vithal, B. (2003). The third national mathematics assessment: Results and implications for elementary and middle schools. *Arithmetic Teacher,* 31, 14-19.

Waheed, C. (2003). Skill in estimation problems of extent and numerosity. *Journal for Research in Mathematics Education,* 13 (3) 211-232

Wessels, D.C. (2008). *The Mathematics Curriculum.* Pretoria: University of South Africa.

White, K., & Thomson, J. (2005). Metrics and mappings: a framework for understanding real world quantitative estimation. *Psychological Review.* 100 (3) 511-534.

White, M. and Thomson, J. (2009). Mathematisation of Number Sense Matters. *For the Learning of Mathematics,* 3 (1), 45-47.

Wiggershaus, I. (1986). Does Practice Make Perfect? *For The Learning of Mathematics*, (19 (3), 33-35.

Witwatersrand University, (2013). *Educational Studies: Teacher Development Programmes.* Wits Univeristy.

Wolfaardt, F. (2003). The role of English of determining access to tertiary education in Namibia. *Journal of professional Development,* 5(2), 13-17.

Yang, D. C. (1997). *Number sense performance and strategies possessed by sixth-and eighth grade students in Taiwan.* Unpublished doctoral dissertation, University of Missouri, Columbia, USA.

Yang, D. C. (2003). A study of teaching and learning number sense for seventh grade students in Taiwan. *International Journal of Science and Mathematics Education,* 2(3), 407-430.

Yang, D. C. (2003). Teaching and learning number sense–an intervention study of fifth grade students in Taiwan. *International Journal of Science and Mathematics Education,* 1(1), 115-134.

Yang, D. C. (2005). The study a of sixth grade students' development of number sense through well-designed number sense activities. *Chinese Journal of Science Education.* 10(3), 233-259.

Yang, D. C., & Li, M. F. (2008). An investigation of 3rd grade Taiwanese students' performance in number sense. *Educational Studies,* 34(5), 443-455.

Yang, D. C., & Reys, R. E. (2008). Fractional number sense strategies possessed by sixth grade students in Taiwan. *Hiroshima Journal of Mathematics Education,* 10 (2), 53-70.

Yang, D.C. (2007). Investigating the strategies used by pre-service teachers in Taiwan when responding to number sense questions, *School Science and Mathematics,* 107(7), 293-301.

Zanzali, P.L. (2005). The Importance of Number Sense to Mathematics Achievement in middle school children. *A journal of mathematics education,* 20 (2), 82–88.

Zanzali, P.L., and Ghazali, F.E. (1999). The study of number sense: Realistic activities integrated into seventh-grade math classes in Taiwan. *The Journal of Educational Research,* 103(6), 379-392.

APPENDIX A: Approval University of Namibia

UNIVERSITY OF NAMIBIA

MAIN CAMPUS, WINDHOEK

Private Bag 13301, Windhoek, Namibia, Telephone: (++264) (61) 2063111,

02 December 2014

To: Mr Naukushu Shiwana T.
Student number: 17425123-2012

REF: REQUEST TO DO RESEARCH AT UNAM MAIN CAMPUS (BED. 1, SECONDARY) FOR THE PERIOD OF JANUARY TO JULY 2015.

Dear Sir

We hereby grant you permission to conduct research at the main campus of the University of Namibia on the study titled "A CRITICAL THEORY ENQUIRY IN THE DEVELOPMENT OF NUMBER SENSE IN NAMIBIAN FIRST YEAR PRE-SERVICE SECONDARY MATHEMATICS TEACHERS".

We however would like to plea that the normal student activities do not get disrupted to compromise their academic performance. Please note that we at the University of Namibia embrace a culture of research and innovation. Therefore, kind-heartedly share with us a copy of the final report when it is done.

We also urge you to always carry this letter when conducting your studies.

The University of Namibia and the Faculty of Education in particular would like to wish you all the bests in this academic journey.

Should there be any thing that we can assist to ensure that your research runs smoothly, do not hesitate to call on us.

Yours Truly

....

A. Van Wyk.
Chairperson: Research Committee
Faculty of Education
University of Namibia

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2014 -12- 02
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APPENDIX B: Permission Stellenbosch University

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Approved with Stipulations
New Application

22-Aug-2015
Naukushu, Shiwana ST

Proposal #: DESC/Naukushu/Jul2015/4

Title: **A Critical Theory enquiry into the development of number sense in Namibian first year pre-service secondary mathematics teachers**

Dear Mr Shiwana Naukushu,

Your **New Application** received on **15-Jun-2015**, was reviewed
Please note the following information about your approved research proposal:

Proposal Approval Period: **19-Jul-2015 -18-Jul-2016**

The following stipulations are relevant to the approval of your project and must be adhered to:
The application has been approved with the stipulation that the researcher improves the informed consent form by explaining the study in more detail and without the use of technical terminology in the section "Purpose of the study".

Please provide a letter of response to all the points raised IN ADDITION to HIGHLIGHTING or using the TRACK CHANGES function to indicate ALL the corrections/amendments of ALL DOCUMENTS clearly in order to allow rapid scrutiny and appraisal.

Please take note of the general Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

Please remember to use your **proposal number** (**DESC/Naukushu/Jul2015/4**) on any documents or correspondence with the REC concerning your research proposal.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Also note that a progress report should be submitted to the Committee before the approval period has expired if a continuation is required. The Committee will then consider the continuation of the project for a further year (if necessary).

This committee abides by the ethical norms and principles for research, established by the Declaration of Helsinki and the Guidelines for Ethical Research: Principles Structures and Processes 2004 (Department of Health). Annually a number of projects may be selected randomly for an external audit.

National Health Research Ethics Committee (NHREC) registration number REC-050411-032.

We wish you the best as you conduct your research.

If you have any questions or need further help, please contact the REC office at 218089183.

Included Documents:
DESC Checklist form
Research Proposal
Request for institutional permission

APPENDIX C: Consent to participate in research

UNIVERSITEIT·STELLENBOSCH·UNIVERSITY
jou kennisvennoot • your knowledge partner

STELLENBOSCH UNIVERSITY
CONSENT TO PARTICIPATE IN RESEARCH

A Critical Theory enquiry into the development of number sense in Namibian first year pre-service secondary mathematics teachers.

You are asked to participate in a research study conducted by Mr. Naukushu Shiwana Teeleleni, from the Institute for Curriculum Studies at Stellenbosch University. The results of this study will be used to compile a thesis for PHD. You were selected as a possible participant in this study because the study targets to use the First Year Secondary Pre-service Teachers of Mathematics and you are part of them.

1. PURPOSE OF THE STUDY

This study purports to assess the levels of number sense comprehension of first year pre-service secondary teachers of mathematics and evaluate the impact of Critical Theory intervention on their number sense training.

2. PROCEDURES

If you volunteer to participate in this study, we would ask you to do the following things:

1. Completing the questionnaire before the number sense training.
2. Complete the number sense training before the number sense intervention training.
3. Complete the focus group interview discussion before the intervention.
4. Participate in the number sense training intervention programme as a student.
5. Completing the questionnaire before the number sense training.
6. Complete the number sense training before the number sense intervention training.
7. Complete the focus group interview discussion before the intervention.

The total length of this study shall be one (1) semester and we will be meeting for one hour two times a week.

3. POTENTIAL RISKS AND DISCOMFORTS

There are no envisaged potential risks in this study apart from the fact that you are a student and will be using up at least about three hours of every week in the next semester.

There are no psychological risks in this study however if any crop up you will be referred to a counselor should they arise.

4. POTENTIAL BENEFITS TO SUBJECTS AND/OR TO SOCIETY

The potential benefit of participating in this study is that you will be able to reason better numerically. Your numerical understanding might improve and this will help you to understand mathematics better. This might also help you to become a better reflective practitioner once you join the job marked as a teacher.

The potential benefit to the society is that it is envisaged that this study might contribute to the training of secondary teachers of Mathematics with a better numerical understanding. It is also envisaged that the teacher training session will be informed of the number sense training of teachers.

5. PAYMENT FOR PARTICIPATION

There is not payment of monetary values in this study, however the researcher wishes to help participants with their mathematics understanding which is related to number sense.

6. CONFIDENTIALITY

Any information that is obtained in connection with this study and that can be identified with you will remain confidential and will be disclosed only with your permission or as required by law. Confidentiality will be maintained by means of using your University of Namibia student numbers.

The information collected in this study might only be made available to the University of Namibia in its capacity as the hosting institution and the Stellenbosch University being the institution of study by the researcher. This will be done should it be deemed necessary by any of the two institutions from the fore going.

The lessons will be audio taped and all participants will have the rights to review tapes. The tapes will be erased for as long as the Stellenbosch University does not deem it necessary to keep them anymore.

Confidentiality will be maintained during the study and all the possible ways of identification will be removed by means of using codes as prescribed in paragraph 1 of this section.

7. PARTICIPATION AND WITHDRAWAL

You can choose whether to be in this study or not. If you volunteer to be in this study, you may withdraw at any time without consequences of any kind. You may also refuse to answer any questions you don't want to answer and still remain in the study. The investigator may withdraw you from this research if circumstances arise which warrant doing so.

8. IDENTIFICATION OF INVESTIGATORS

If you have any questions or concerns about the research, please feel free to contact Mr. Naukushu Shiwana Teeleleni at 065-2232285 or 0812614233.

9. RIGHTS OF RESEARCH SUBJECTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research subject, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

SIGNATURE OF RESEARCH SUBJECT OR LEGAL REPRESENTATIVE

The information above was described to ……………………………………………………………….. by Naukushu Shiwana Teeleleni in English and I am in command of this language or it was satisfactorily translated to me. I was given the opportunity to ask questions and these questions were answered to my satisfaction.

I hereby consent voluntarily to participate in this study I have been given a copy of this form.

____

Name of Subject/Participant

____

Name of Legal Representative (if applicable)

______________________________________ ______________

Signature of Subject/Participant or Legal Representative Date

SIGNATURE OF INVESTIGATOR

I declare that I explained the information given in this document to __________________ ____________________*. He*/she was encouraged and given ample time to ask me any questions. This conversation was conducted in [*Afrikaans/*English/*Xhosa/*Other*] and [*no translator was used/this conversation was translated into* ___________ by ________________________]*.*

______________________________________ ______________

Signature of Investigator Date

APPENDIX D: Focus Group Interview Guide

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A Focus Group interview guide for the pre-service Mathematics teachers on their development of number sense.

Introduction:

I am Mr. Naukushu Shiwana Teeleleni, a PHD student at Stellenbosch University. I am doing my studies on a title: *A Critical Theory enquiry into the development of number sense in Namibian first year pre-service secondary mathematics teachers.* I am kindly asking you to actively participate in the discourse that will take place soon. Remember, there is no wrong or correct answer to the questions; we are just trying to explore ideas on how to best develop the number sense to aid the teaching and learning of mathematics in schools.

Kindly note that the information that you will give here will be treated with very strict confidentiality and will only used for the purpose of this study. Feel free to ask questions where you do not understand so that I can clarify. Please note that you have the right to withdraw from this discussion at any point in time should it be deemed necessary by you.

Instructions:

- This is a follow-up to the test to understand how you answered the questions in the test.
- We will consider a question from each of the sections in the number sense test that you have written.
- I will present a question to the whole group, and ask each of you what picture you developed in your mind, whether there was another answer equally good as the one you preferred, how you judged the best among the two.
- If you get stuck I will kindly ask you to tell me what the problem is and what you possibly needed in order to be able to work out the question.
- I will also ask you to give me reasons for your choice why you chose them and what guided your confidence.

- These questions will guide us but I will also ask follow-up questions in the process should it be deemed necessary.
- After that process I will open up a discussion floor so that you can interact with your classmates. Please feel free to discuss with them.
- Please ask me if there is anything that you may want to ask, otherwise we can start now if you are ready.

Questions:

1. **Researcher:** What is your preferred answer to this question?
   Respondent:

   ………………………………………………………………………………
   ………………………………………………………………………………
   ……………………………………………………………..………………………
2. R**esearcher:** How did you know it was the correct answer?
   Respondent:

   ………………………………………………………………………………
   ……………………………………………………….………………………………
   ……………………………………………………………………………...….
   .
3. R**esearcher:** How confident were you when you of the answer you have chosen?
   Respondent:

   ………………………………………………………………………………
   ………………………………………………………………………………
   ……………………………………………………………………………
4. **Researcher:** What material did you need in order to work out this question correctly?
   Respondent:
   ………………………………………………………………………………
   ………………………………………………………………………………

Section 1: Meaning and size of numbers

1. Which fraction is bigger than the other $\frac{3}{7}$ or $\frac{8}{15}$?	a) They are the equal to each other. b) $\frac{8}{15}$ c) $\frac{3}{7}$ d) None	a) Because they are approximately the same. b) Because $\frac{8}{15}$ is bigger than half but $\frac{3}{8}$ is smaller than half. c) Because both 8 and 15 in $\frac{8}{15}$ are bigger than 3 and 7 in $\frac{3}{7}$. d) We cannot tell because they do not have the same denominators, unless we convert to decimal first.	a) Very confident b) Confident c) Not sure d) Not confident

Section 2: Equivalence of numbers

1. Which number represents 0.0025?	a) $\frac{1}{4000}$ b) 0.25% c) 0.25×10^{3} d) 2.5×10^{-3}	a) Because $\frac{1}{4000}$ is a fraction whose denominator is not a multiple of 10 we can only compare factions and decimals if the denominator is a multiple of 10. b) 0.25% is far larger than 0.0025% so the two cannot be equivalent. c) Because when 0.25×10^{3} is worked out it gives 0.0025. d) Because 2.5×10^{-3} is a standard form notation equivalent to 0.0025.	a) Very confident b) Confident c) Not sure d) Not confident

Section 3: Effects of operations on numbers

1. Which operation makes much sense: $\frac{2}{5}\times[\ ]=1$	a) $\frac{2}{5}$ b) 1 c) $1\frac{1}{2}$ d) 0	a) Any fraction multiplied by itself becomes1. b) If we multiply by 1 the fraction stays the same. c) It ultimately becomes the reciprocal and when a number is multiplied by the reciprocal it gives 1. If we multiply a fraction by zero then it becomes 1.	a) Very confident b) Confident c) Not sure d) Not confident

Section 4: Computational skills

1. Which answer is most reasonable for the calculation: $35\times11\times2$ write down how you thought about the answer in the think through column.	d) 700 e) 765 f) 800 g) 385	a) Because this is to $35\times10\times2$ i.e. the 1 in 11 can be ignored to make it 10. b) Because 35×11 is already more than half of 700 therefore 700 is too small, so this can only be 765. c) Because $35\times10\times2$ is around 800 it cannot be in the 700s. d) The answer is $35\times11=385$	a) Very confident b) Confident c) Not sure d) Not confident

Section 5: Ability to estimate accurately using reasonable benchmarks

3. Without calculating the exact answer which statement makes sense about the answer to the calculation: 72×0.49?	a) Much larger than 36 b) Much less than 36 c) Slightly less than 36 d) Slightly more than 36	a) Because 72 is more than 36 and 0.49 can be neglected. b) Because when a number is multiplied by 0.something it becomes much less. c) Because it is multiplied with a value slightly lower than half. d) Because it is multiplied with a value that is slightly more than half.	a) Very confident b) Confident c) Not sure d) Not confident

End of interview thank you!

APPENDIX E: Questionnaire

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A questionnaire on general perceptions and views of preservice Mathematics teachers on their development of number sense after taking the number sense course.

Introduction:

Thank you for going through to the final stage of this study and I trust that you will remain up to the end. Congratulations, you have reached the end of this study *(A Critical Theory enquiry into the development of number sense in Namibian first year pre-service secondary mathematics teachers.)* I would like to collect general information on your perceptions, views and attitudes on your own development of number sense after taking the number sense training. I am kindly asking you to actively participate in the process of answering questions as you have always endeavoured to. Kindly note that there are no wrong or correct answers to the questions. I am just trying to explore ideas that you possibly developed about number sense to aid the teaching and learning of mathematics in schools.

Kindly note that the information that you will give here will still be treated with strictest confidentiality and will only be used for the purpose of this study. Please note that you have the right to withdraw from this discussion at any point in time should it be deemed necessary by you. You are just expected to answer the questions according to your own understanding.

Student code:-___________________________

Section 1 (To be answered before the intervention): Biographic information

1.1 Gender

Male [ ] Female [ ]

1.2 Indicate your age in the space provided?

…………………………………………………………………………………………………

Section 2 (To be answered after the intervention): Preservice secondary mathematics teachers reflections on the relationship between number sense and academic performance in mathematics.

2.1 Based on what you have learned in the number sense course that you took, what do you think is the relationship between number sense and academic performance in mathematics (Basic Mathematics for Teachers)?

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.2 What mathematical skills do you think the number sense course equipped you with in order to understand mathematics better?

……………………………………………………………………………………………

……………………………………………………………………………………………

2.3 State with a reason whether or not they need number sense as a secondary mathematics teacher to be. Their responses are summarised here.

Yes [ ] No [ ]

Reason:

……………………………………………………………………………………………

……………………………………………………………………………………………

Do you think the number sense course you took aided you to grasp a better numerical and mathematics understanding?

Yes [ ] No [ ]

Explain:

……………………………………………………………………………………………

……………………………………………………………………………………………

2.5 How relevant was the number sense training was to the Basic Mathematics for Teachers course?

……………………………………………………………………………………………

2.6 How helpful was the number sense course that you took in boosting up your academic performance in mathematics performance in mathematics.

……………………………………………………………………………………………

……………………………………………………………………………………………

Section 3(To be answered before and after the intervention): Pre-service teachers' reflections on the number sense pre-test post-test.

3.1 To the following statements indicate on your attitudes, views or perceptions rating your response to each of the statement on a scale. 1=strongly disagree 5= strongly agree.

3.2 I understand very well the meaning of number sense.

[1] [2] [3] [4] [5]

3.3 I find it difficult to recognise the effects of operations on other numbers.

[1] [2] [3] [4] [5]

3.4 I feel uncomfortable to recognise the equivalence of numbers.

[1] [2] [3] [4] [5]

3.5 I can perform calculations using fractions, decimals, percentages and whole numbers.

[1] [2] [3] [4] [5]

3.6 1.5 I can make reasonable estimations given relevant information to draw benchmarks from.

[1] [2] [3] [4] [5]

3.7 1.6 I am not confident that I can predict the reasonableness of my solution looking at sizes of numbers that I am working with.

[1] [2] [3] [4] [5]

3.8 I am not confident to explain calculations without necessarily using a calculator.

[1] [2] [3] [4] [5]

3.9 I have trouble using alternative methods for explaining a mathematical calculation.

[1] [2] [3] [4] [5]

3.10 I cannot easily do mental calculations in my head.

[1] [2] [3] [4] [5]

3.11 I feel confident using a calculator when doing computations.

[1] [2] [3] [4] [5]

3.12 I don't have the math skills to work out percentages in my head.

[1] [2] [3] [4] [5]

3.13 When given calculations I can only do them with a pen and paper if there is no calculator.

[1] [2] [3] [4] [5]

3.14 I cannot figure out the relative size of my final answer before I finish calculating the whole sum.

[1] [2] [3] [4] [5]

3.15 I hate mental arithmetic.

[1] [2] [3] [4] [5]

3.16 I feel very tired in my mind after doing few sums in my head.

[1] [2] [3] [4] [5]

3.17 I have a lot of self-confidence when it comes to number sense.

[1] [2] [3] [4] [5]

3.18 I have confidence that in doing number sense calculations on my own.

[1] [2] [3] [4] [5]

3.19 I become uncomfortable when I have to work out percentages without a calculator.

[1] [2] [3] [4] [5]

3.20 I have fear for the number sense course that I am about to take.

[1] [2] [3] [4] [5]

3.21 I will not learn new things from this number sense that I am about to take.

[1] [2] [3] [4] [5]

End of questionnaire thank you!

APPENDIX F: The Five Tier Number Sense Test

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jou kennisvennoot · your knowledge partner

A Five Tier Number Sense (FTNST): pre-test

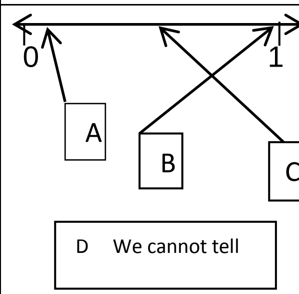
Student number:……………………………….

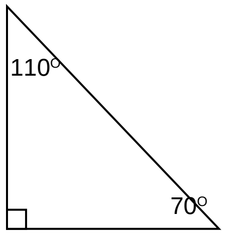
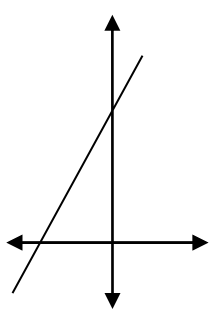
Instructions:

1. You are **not allowed to use an ink pen** in this test, use only the pencil provided to you by the examiner.
2. **No calculators** are allowed in this test.
3. Each question has four parts: The main question, the first three are reason and the confidence; for each question choose the correct answer, the correct reason and the letter that explains the how confident you are by writing the letter of your choice in a black box under each set of alternatives.
4. The fourth part is for you to write down any thinking or rough work that led you to the answer of your choice in the thinking through section; remember, the researcher is more interested in the way you arrived at the answer of your choice.
5. This test consists of 30 questions; each question gives you 1 point in each of the four tiers.
6. Read the questions carefully before you attempt them.
7. You have 1 hour 30 minutes to complete the test, use your time wisely. You will be reminded after every thirty minutes, so that you keep track of your progress. If you have any question you may raise up your hand before the test commences.

Section 1: Meaning and size of numbers:

Question	Think through	Answers	Reason	How confident are you about your answer?
1. Which fraction is bigger than the other $\frac{3}{7}$ or $\frac{8}{15}$?		a) They are the equal to each other. b) $\frac{8}{15}$ c) $\frac{3}{7}$ d) None	e) Because they are approximately the same. f) Because $\frac{8}{15}$ is bigger than half but $\frac{3}{8}$ is smaller than half. g) Because both 8 and 15 in $\frac{8}{15}$ are bigger than 3 and 7 in $\frac{3}{7}$. h) We cannot tell because they do not have the same denominators, unless we convert to decimal first.	e) Very confident f) Confident g) Not sure h) Not confident

2. How many fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$?		a) No fraction b) I do not know c) Infinite d) Two fractions only	a) Because 2 comes after 3 and there is nothing between them. b) Because $\frac{2}{5}$ is almost equal to $\frac{3}{5}$ so it is hard to tell. c) Because there can still be many fractions such as $\frac{2.1}{5}$ or $\frac{2.2}{5}$. d) Two fractions only namely $\frac{2}{5}$ and $\frac{3}{5}$?	a) Very confident b) Confident c) Not sure d) Not confident
3. Which letter in the number line shows a fraction whose denominator is nearly the same but slightly larger than the numerator?			a) When a numerator is slightly greater than denominator the fraction is almost equal to 0. b) When a numerator is slightly larger than the denominator the fraction is almost 1. c) When a denominator is slightly greater than the numerator a fraction is almost equal to $\frac{1}{2}$. d) Because fractions are not related to number lines only decimals are related to number lines.	a) Very confident b) Confident c) Not sure d) Not confident
4. For the function $f(x)=\frac{1}{x}$, as x approaches 0, $f(x)$ …		a) approaches 0. b) approaches 1 c) approaches ∞ d) is undefined	a) Because 1 out of 0 is the same thing as 0. b) Because 1 out of 0 means 1 out of nothing which is still just 1. c) Because 1 out of a number closer to zero means infinity. d) Because 1 out of zero is undefined.	a) Very confident b) Confident c) Not sure d) Not confident

5. In the following triangle which angles are correct using number sense? 		a) 90° and 110° b) 110° , 70° and 90° c) 70° and 110° d) 90° and 70°.	a) Because 90° + 110° does not 180° and angles inside a triangle add up to 180°. b) Because a triangle must have 3 angles. c) Because 70° +110° is already 180° there is no need for 90°since the total must always be 180°. d) Because 110 does not make sense since it is outside the margins of obtuse angles.	a) Very confident b) Confident c) Not sure d) Not confident
6. Which *x* and *y* intercepts of the following function could make more numerical sense if ***the scales on both x and y axes are the same***? 		a) *x* = - *2* and *y* = - *4* b) *x* = - *4* and *y* = *4* c) *x* = - *2* and *y* = - *2* d) *x* = - *2* and *y* = *4*	a) Both the *x* and *y* intercepts are negative. b) *x*- Intercept is negative while *y* intercept is positive. c) Both the x and y intercepts are negative and are equal to each other. d) x – Intercept is negative while y is positive and is twice the size of x.	a) Very confident b) Confident c) Not sure d) Not confident

Section 2: Equivalence of numbers.

1. Which number is not equivalent to $\frac{2}{3}$?		a) $\frac{1}{\left(\frac{3}{2}\right)}$ b) 12:18 c) $\frac{1}{\left(1\frac{1}{2}\right)}$ d) 0.667	a) Because when worked out it yields $\frac{1}{\left(\frac{3}{2}\right)} = \frac{3}{2}$ b) Because this is a ratio and ratios are different from fractions. c) Because when it is worked out it does not give $\frac{2}{3}$. d) Because 0.667 is a decimal approximation to $\frac{2}{3}$.	a) Very confident b) Confident c) Not sure d) Not confident
2. Which number is not equal to 30%?		a) $\frac{3}{10}$ b) $\frac{1}{3}$ c) $\frac{30}{100}$ d) 0.3	a) Because $\frac{3}{10}$ is a fraction and 30% is a percentage, these are two different quantities. b) When $\frac{1}{3}$ is converted to % it gives 33.3%. c) Because $\frac{30}{100}$ is equivalent to $\frac{3}{10}$ as stipulated in a) and this is not equivalent to 30%. d) We cannot compare 0.3 as it is a decimal and 30% is a percentage.	a) Very confident b) Confident c) Not sure d) Not confident
3. Which number is not equivalent to 48?		a) $2\times2\times2\times2\times3$ b) $3\times2^2\times4$ c) 48×10^0 d) $3^2\times2^2$	a) $2\times2\times2\times2\times3$ is prime factor decomposition it can never be equal to 48. b) Because by looking without calculating, $3\times2^2\times4$ is equivalent to 48. c) Because anything to the power of zero is 1 but not 48. d) Because by looking $3^2\times2^2$ is smaller than 48.	a) Very confident b) Confident c) Not sure d) Not confident

4. Which number represents 0.0025?		a) $\frac{1}{4000}$ b) 0.25% c) 0.25×10^{3} d) 2.5×10^{-3}	e) Because $\frac{1}{4000}$ is a fraction whose denominator is not a multiple of 10 we can only compare factions and decimals if the denominator is a multiple of 10. f) 0.25% is far larger than 0.0025% so the two cannot be equivalent. g) Because when 0.25×10^{3} is worked out it gives 0.0025. h) Because 2.5×10^{-3} is a standard form notation equivalent to 0.0025.	a) Very confident e) Confident f) Not sure g) Not confident
5. By using your numerical understanding find the value of *n*. $0.3\times10^{2}=30000\times10^{n}$ true.		a) $n=-3$ b) $n=-4$ c) $n=3$ d) $n=4$	a) Because by looking without having to calculate n= -3 is the correct answer. b) Because the left hand side is large so it needs a small ne to balance out. c) Because the differences in the values of n is 3. d) The right hand side needs a larger value of n to balance out?	a) Very confident b) Confident c) Not sure d) Not confident
6. By using your numerical understanding find the value of *n*. $0.03\div10^{2}=30000\times10^{n}$ true.		a) $n=-6$ b) $n=-4$ c) $n=6$ d) $n=4$	a) Because without necessarily having to calculate n = -4 is the correct answer. b) Because the left hand side is smaller so it needs a larger value of n to balance out. c) Because the differences in the values of n is 6. d) The right hand side needs a larger value of n = 4 to balance out?	a) Very confident b) Confident c) Not sure d) Not confident

Section 3: Effects of operations on numbers

1. Which number can be inserted in the operation so that the calculation $\frac{2}{5}\times[\ ]=1$ is correct?		e) $\frac{2}{5}$ f) 1 g) $2\frac{1}{2}$ h) 0	d) Any fraction multiplied by itself becomes1. e) If we multiply by 1 the fraction stays the same. f) It ultimately becomes the reciprocal and when a number is multiplied by the reciprocal it gives 1. g) If we multiply a fraction by zero then it becomes 1.	e) Very confident f) Confident g) Not sure h) Not confident
2. Insert a number that makes sense: $54-45=45-[\ ]$		a) 54 b) 45 c) 9 d) 36	a) Because the digit that begin were reversed so the answer should also be reversed. b) You simply reverse the digits to make 45 no need to calculate. c) The number 9 is the difference on the left hand side so it is the correct answer. d) The number 36 is the one that gives the common difference.	a) Very confident b) Confident c) Not sure d) Not confident
				e)
3. For the numbers in $\frac{3}{4};\frac{1}{2};\frac{1}{20}$ *and* $\frac{3}{40}$ pick two numbers will give the largest difference?		a) $\frac{3}{4}-\frac{1}{2}$ b) $\frac{1}{20}-\frac{1}{2}$ c) $\frac{3}{40}-\frac{1}{2}$ d) $\frac{3}{4}-\frac{1}{20}$	a) Because this gives: $\frac{3}{4}-\frac{1}{2}=\frac{3-1}{4-2}=\frac{2}{2}=1$ b) Because this gives: $\frac{1-1}{20}=\frac{0}{18}=0$ c) Because this gives: $\frac{3}{40}-\frac{1}{2}=\frac{2}{38}=\frac{1}{19}$ d) Because this gives is the largest minus the smallest it will always give us the greatest difference.	a) Very confident b) Confident c) Not sure d) Not confident

4. Which operation can be done on a third and a fifth to get the minimum positive answer:		a) $+$ b) $\times$ a) $-$ d) $\div$	a) Because this gives: $\frac{1}{3}+\frac{1}{5}$ $=\frac{3+5}{15}$ $=\frac{8}{15}$ And that is the minimum answer. b) A both a third and a fifth are smaller numbers than 1 if we multiply them they even get smaller. c) Minimum answer is obtained from subtracting a big number minus a small number. d) Dividing involves taking a reciprocal which minimizes everything.	a) Very confident b) Confident c) Not sure d) Not confident
5. Which combination of operations make sense in the calculation: $2.5 \ldots\ldots\ldots \frac{1}{100} = 0.025 \ldots\ldots 1$		a) $\div$ $-$ b) $\times$ $\times$ a) $\div$ $\div$ d) $\times$ $+$	a) Division the opposite of subtraction so the two sides will be equal. b) Because a hundredth is included in 0.025. c) If we divide LHS becomes smaller but dividing RHS with one does not change the two sides balance. d) Multiplying the LHS makes it smaller but adding 1 means no major change made to the calculation so the two sides end up the same.	a) Very confident b) Confident c) Not sure d) Not confident
6. Determine the value of [ ] in the calculation: $4\times[\ ]=[\ ]$		a) $\frac{1}{2}$ b) 1 a) 0 d) 4	a) If we multiply by a half reduces to a half so it balances with RHS. b) If we multiply with 1 the numbers always end up balancing. c) If we multiply by zero the two sides end up being zero and they balance off. d) If we multiply by 4 then we get 4 on both sides, therefore the two sides balance off.	a) Very confident b) Confident c) Not sure d) Not confident

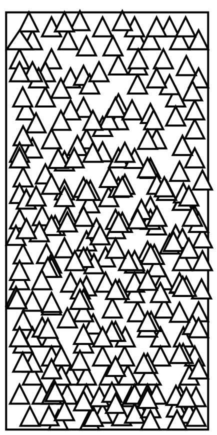
Section 4: Computational skills

1. The letters a-d represent each of the student's first step attempt to a calculation $\frac{2}{3}+\frac{3}{5}-\frac{1}{2}$, which letter carries the correct attempt:		a) $\frac{5}{8}-\frac{1}{2}$ b) $\frac{2}{3}+\frac{2}{3}$ c) $\frac{15+18-20}{30}$ d) $\frac{20}{30}+\frac{18}{30}-\frac{15}{30}$	a) Addition is always the first step before multiplication. b) Subtraction can be started before adding if they are the only ones in the calculation. c) We always make the number out of the lowest common denominator. d) Adding is made easier if all fractions are of the same denominator then parts of a whole can be added or subtracted.	a) Very confident b) Confident c) Not sure d) Not confident
2. What mistake did your learner make to the calculation: $\left(\frac{2}{3}+\frac{1}{6}-\frac{1}{4}\right)$ in order to get the answer $\frac{2}{5}$?		a) Did not follow the BODMAS rule. b) Just added and subtracted the numerator throughout. c) Did not find the common denominator. d) Addition and subtraction of fractions can only be made if the denominator are the same.	a) Because there are mixed operations. b) This is only done in multiplication of fractions but not with addition and subtractions of fractions. c) Because if we do not know the lowest common denominator the answer will always be wrong. d) If the denominators are different we cannot add and subtract the fractions.	a) Very confident b) Confident c) Not sure d) Not confident

3. The correct answer to the calculation $\sqrt{2\frac{7}{9}}$ is:		a) $\frac{25}{9}$. b) $\sqrt{2}+\frac{\sqrt{7}}{3}$. c) $1\frac{2}{3}$. d) We only work out the exact answer with a calculator	a) If we convert to improper fraction we get $\frac{25}{9}$. b) $\sqrt{2\frac{7}{9}}$ equals, $\sqrt{2+\frac{7}{9}}$ which can be written as $\sqrt{2}+\frac{\sqrt{7}}{3}$ c) The square root of $\frac{25}{9}$ is $1\frac{2}{3}$. d) Because 2 and 7 are not perfect squares therefore we cannot work out their exact values of square roots without a calculator.	a) Very confident b) Confident c) Not sure d) Not confident
4. Which one of these is the correct step in the calculation of $3\frac{3}{4}\div 1\frac{1}{2}$?		a) $\frac{15}{4}\times\frac{3}{2}$ b) $3\frac{3}{4}\div\frac{2}{3}$ c) $\frac{4}{15}\times\frac{2}{3}$ d) $\frac{15}{4}\times\frac{2}{3}$	a) The first step is to convert to improper fraction. b) We only convert to improper fraction for the fraction on the RHS since it is the one we are going to get the reciprocal of. c) We convert to improper fraction and take the reciprocal of both fractions to change to multiplication. d) We take the improper fraction of the RHS and change to multiplication.	a) Very confident b) Confident c) Not sure d) Not confident
5. Which answer is most reasonable for the calculation: $35\times11\times2$ write down how you thought about the answer in the think through column.		h) 700 i) 765 j) 800 k) 385	e) Because this is to $35\times10\times2$ i.e. the 1 in 11 can be ignored to make it 10. f) Because 35×11 is already more than half of 700 therefore 700 is too small, so this can only be 765. g) Because $35\times10\times2$ is around 800 it cannot be in the 700s. h) The answer is $35\times11=385$	e) Very confident f) Confident g) Not sure h) Not confident

6. A boy is to receive 15% of N$260 from the father, which calculation can he use to determine how much money he will get?		a) 0.15×100 b) $\frac{15}{100} \times 260$ c) $10\% \rightarrow N\$26-00$ $5\% \rightarrow N\$13-00$ $15\% \rightarrow N\$39-00$ d) All three answers in a) b) and c) are correct.	a) Because % means out of 100 and 15 out of 100 is 0.15. b) To calculate we always make out of 100 and multiply by the total. c) By inspection one should have a feel of numbers to work out without necessarily referring to the algorithm. d) All of the three previous answers the same.	a) Very confident b) Confident c) Not sure d) Not confident

Section 5: Ability to estimate accurately using reasonable benchmarks

1. Approximately how many triangles are enclosed in the following diagram? 		a) 250 b) 2 500 c) 800 d) 500	a) If you multiply the row by column you get 250. b) By looking at them one can tell they are about 2 500. c) By looking they appear to be about 800. d) They look 500 "ish".	a) Very confident b) Confident c) Not sure d) Not confident

2. Which statement makes more numerical sense on what the relationship between and $\frac{22}{7}$ is?		a) $\pi = \frac{22}{7}$ b) $\pi > \frac{22}{7}$ c) $\frac{22}{7} >>> \pi$ d) d) $\pi \approx \frac{22}{7}$	a) π is the same as $\frac{22}{7}$ it is only that one is a decimal while the other one is a common fraction. b) $\pi = 3.1414$ whereas c) $\frac{22}{7} = 3.142857$ and $\pi = 3.142$ as result $\frac{22}{7} = 3.142857$ is more larger than $\frac{22}{7} = 3.142857$ d) The values of $\frac{22}{7}$ and π only differ by a thousandth therefore $\frac{22}{7} \approx \pi$.	a) Very confident b) Confident c) Not sure d) Not confident
3. Without calculating the exact answer which statement makes sense about the answer to the calculation: 72×0.49?		e) Much larger than 36 f) Much less than 36 g) Slightly less than 36 h) Slightly more than 36	e) Because 72 is more than 36 and 0.49 can be neglected. f) Because when a number is multiplied by 0.something it becomes much less. g) Because it is multiplied with a value slightly lower than half. h) Because it is multiplied with a value that is slightly more than half.	a) Very confident b) Confident c) Not sure d) Not confident
4. By using number sense which one is the most reasonable estimate for the answer to the calculation: $\frac{2}{5} \times \frac{11}{12}$?		a) $\frac{13}{17}$ b) 1 c) $\frac{22}{60}$ d) $\frac{11}{30}$	a) Because once worked out we . b) Because when a number is multiplied by 0.something it becomes much less. c) Because it is multiplied with a value slightly lower than half. d) Because it is multiplied with a value that is slightly more than half.	a) Very confident b) Confident c) Not sure d) Not confident

5. Without calculating the exact answer which total is greater than $\frac{1}{2}$ **?**		a) $\frac{2}{9}+\frac{5}{21}$ b) $\frac{6}{25}+\frac{7}{29}$ c) $\frac{8}{35}+\frac{9}{36}$ d) We cannot tell without calculating the exact answer	a) Because if you work out you get $\frac{7}{30}$ which is greater than $\frac{1}{2}$. b) Because when a you work out you get $\frac{349}{725}$ which is greater than $\frac{1}{2}$. c) Because it one of the fractions is already a quarter. d) Because only when you have worked out the final answer can you compare it to half.	a) Very confident b) Confident c) Not sure d) Not confident
6. How does the answer to the calculation: $2\frac{1}{5}\div\frac{12}{13}$ compare with $2\frac{1}{5}$?		a) Much larger than $2\frac{1}{5}$ b) Much less than $2\frac{1}{5}$ c) Slightly larger than 36 d) We cannot tell without knowing the exact answer.	a) Because 72 is above 36 and 0.49 can be neglected. b) Because when a number is multiplied by 0.something it becomes much less. c) Because it is multiplied with a value slightly lower than half. d) Because it is multiplied with a value.	a) Very confident b) Confident c) Not sure d) Not confident

End of test Thank you!

Appendix G: The rubric for interpreting the levels of number sense

THLT Domain	**Below Basic** *(0 - 49 %)*	**Basic** *(50-59 %)*	**Proficient** *(60–74 %)*	**Advanced** *(75% +)*
the meaning and size of numbers both rational and irrational numbers	Possesses no or very minimum understanding of the meaning and size of numbers.	Possesses minimum understanding of the meaning and size of numbers.	Possesses a good/reasonable understanding of the meaning and size of numbers.	Possesses a very good/ exceptional understanding of the meaning and size of numbers.
equivalence of numbers both rational and irrational numbers	Experiences difficulties and has very poor ability to and cannot recognise different forms of representing numbers.	Experiences difficulties or struggles to recognise different forms of representing numbers.	Experiences minimum or no difficulties to recognise numbers when represented different forms.	Holds very good/ excellent ability to recognise different forms of representing numbers.
meaning and effects of operations;	Demonstrated very minimum/no understanding and/struggles to recognise and understand the meaning and effect of operations.	Demonstrated limited grasp and often struggles to recognise and understand the meaning and effect of operations.	Demonstrated a reasonable grasp and often recognises and apprehends the meaning and effect of operations.	Demonstrated an outstanding grasp and regularly recognises and apprehends the meaning and effect of operations with minimum or no problems.
counting and computational strategies	Holds little or lack of knowledge in counting and computing tactics and relied on a calculator heavily to do most if not all the sums.	Holds little competency in counting and computational strategies and sometimes relied on a calculator to do many of the sums.	Holds reasonable competency in counting and computational strategies and sometimes used a calculator only when necessary.	Holds counts proficiently , is fast paced and totally in command of the count
estimation using relevant benchmark without calculating	Cannot make any reasonable estimation and cannot identify a benchmark as a reference point of estimation.	Finds it difficult to make reasonable estimations and does not find it easy to identify a reasonable benchmark as a reference point of estimation.	Can often make reasonable estimations and does not find it difficult to identify a reasonable benchmark as a reference point of estimation.	Make very reasonable and accurate estimations and identifies very reasonable and accurate benchmark as a reference point of estimation.