

# **On the use of spreadsheet algebra programmes in the professional development of teachers from selected township high schools**

Faaiiz Gierdien

Faculty of Education

Research Unit for Mathematics Education,

University of Stellenbosch (RUMEUS)

## **ABSTRACT**

This paper reports on the initial stages of a small-scale project involving the use of ‘spreadsheet algebra programmes’ (SAPs) in the professional development of eight teachers from three township high schools. In terms of the education context, the paper draws on social practice theory. It then details what is meant by spreadsheet algebra. An analysis of teacher conversations and artefacts generated during an in-service workshop is conducted which takes into account the education context and two practices – the teachers’ pencil-paper algebra and the mathematics teacher educator’s spreadsheet algebra. Implications for professional development are considered, especially the teachers’ strategies with respect to the ‘top down’ imposition of the curriculum in terms of patterns and functions.

*Key words: professional development, mathematics teacher education, spreadsheet algebra, function concept, linear inequalities.*

## Introduction

Spreadsheets can be used to represent school algebra content such as linear and other functions and inequalities in the form of multiple representations (symbols, graphs and tables) as dynamic ‘spreadsheet algebra programmes’ (SAPs). In South Africa we do not yet know much about such ‘ready to use’ SAPs for the purposes of professional development. We do, however, know about and utilise Computer Algebra Systems (Mbekwa & Julie, 2003), and dynamic *GeoGebra* (Berger, 2013) for in-service mathematics teachers, and *Sketchpad* (Ndlovu, Wessels & De Villiers, 2013) for pre-service teachers. This paper reports on the theory underpinning the use of ‘spreadsheet algebra’ in a small-scale project that focuses on the professional development of mathematics teachers from selected township high schools outside Cape Town.

Details on the project and its activities are outlined below. Contact with the teachers started with visits to the principals of the three schools involved. They were informed about the goals of the project and its teaching sessions. The principals were amenable to the idea of using spreadsheets as a means to explore particular algebra content in the mathematics curriculum. The teachers were asked to participate in the project on a voluntary basis, meaning that they could withdraw from the project at any point. The necessary ethical clearance was obtained from the Western Cape Education Department, the principals and the teachers. The professional development sessions took place ‘outside the classroom’, i.e. after school. The sessions were conducted at one of the schools in a classroom fitted with a computer for each of the participating teachers. A typical session is centred around an SAP that represents school algebra content. To avoid overload, the teachers are not required to construct any of SAPs they encounter. The SAPs in use have been designed in ways that do not follow the enacted curriculum where, for example, linear equations/functions (and inequalities) and their graphs are usually considered as separate topics. During a teaching session, depending on whether the computers were working properly at the particular school, teachers explore the SAPs individually and/or collectively. Further details on the research methods are provided below.

The education context of the project – three high schools located in a township in the greater Cape Town area – is significant. These schools are located in segregated neighbourhoods referred to as townships that were historically reserved for coloureds, Indians and blacks during the Apartheid era. As high-density residential areas, townships are distinct from the leafy suburbs of Cape Town. These schools differ radically in terms of the availability of resources found in “affluent schools” (Van der Berg, Burger, Burger, Louw & Yu, 2005), located mainly in traditionally white suburbs. According to Christie (2008), township schools face unique challenges in coping with the demands of the academic curriculum. In contrast, using hegemonic beliefs one can describe affluent schools as “fortified sites” (Christie, 2008). Teachers working in the context of township schools have to cope with denigration in the media (Slonimsky & Brodie, 2006) and pervasive deficit discourses (Keitel, 2005), which impact on their mathematics teaching. Every year these teachers have to contend with reports in newspapers (e.g. *Die Burger*, 3 August 2011) that publish matriculation (i.e. Grade 12) results by distributing schools (and learners) into “halls of shame” and “halls of fame” according to the ranking of examination results (Keitel, 2005).

Township (and other high school) teachers understand their practice in an environment dominated by static ‘pencil-paper algebra’ and high-stakes assessment. The teachers do not use spreadsheets in their teaching. When working with these teachers, mathematics teacher educators who design and use SAPs aimed at exploring linear (and other) equations represented as a comparison between two functions have to be aware of the teachers’ pencil-paper

orientation to algebra and the context in which they do their work. The following two questions are of interest to mathematics teacher educators who use information and communication technologies (ICTs) for the professional development of mathematics teachers:

- What is meant by spreadsheet algebra and why would one use SAPs designed to explore linear functions and inequalities?
- What happens to teachers' strategies when a SAP is used in the professional development of teachers from particular township high schools?

The literature review starts with social practice theory, followed by an examination of spreadsheet use with respect to functions and variables in school algebra. A framework is then constructed and used to analyse selected data excerpts showing conversations / interactions between the teachers' static 'pencil-paper algebra' and dynamic 'spreadsheet algebra.' The paper ends with a discussion of the findings and formulates a conclusion on the professional development of teachers.

### **On social practice theory**

Social practice theory (SPT) is useful in terms of understanding how mathematics teachers and mathematics teacher educators go about doing their work either individually or collectively. SPT starts with the notion of practices "which are constituted in communities" and not as "conceptual structures that are constructed in the mind" (Brodie, 2012, p. 3). Thus SPT is not concerned with what happens in people's heads (such as reference to mental schemas), but has to do with what people do together as communities in order to achieve particular goals. Slonimsky and Brodie (2006) trace the roots of SPT to the work of Scribner and Cole (1981) in the area of literacy. Here literacy is conceived as a set of socially organized practices in which individuals engage. Brodie and Shalem (2011, p. 421) define social practices as "patterned, coordinated regularities of action directed towards particular goals." In terms of coordination in the current project, there are two distinct but overlapping practices that come into contact – the teachers' static, pencil-paper algebra and the mathematics teacher educator's use of dynamic spreadsheet algebra.

### **What is meant by spreadsheet algebra?**

Spreadsheet algebra is defined as presenting linear and other functions peculiar to the middle grades and secondary curriculum displayed through multiple representations in a reorganised and dynamic way by means of a spreadsheet. 'Reorganised' (Pea, 1985) here means that (linear) equations or inequalities are conceptualised as a comparison between two functions. 'Dynamic' means the spreadsheet is designed by means of scrollbars as a way to explore algebra as a study of the relationships between variables (A. Olivier, personal communication). The concepts of a variable and a function are central to a definition of 'spreadsheet algebra.' A body of literature underpins this broad definition.

Leung (2006, p. 33) refers to "spreadsheet algebra" as the algebra learned through spreadsheets. This prompts the question – what are spreadsheets and what do we know about their use in algebra learning? Historically "electronic spreadsheets" (Pea, 1985) are an example of a computer and "cognitive technology" (Pea, 1987) that can be used to show values and functional relationships in numerical form in any content domain, e.g. arithmetic, algebra, physics or accounting (Leung, 2006). The most dramatic difference compared to a pencil-paper static spreadsheet is that one can change entries in cells and see the dynamic repercussions of that change recalculated with immediate effect throughout the spreadsheet. We can thus use a spreadsheet to track particular numerical inputs (in cells or columns) and their corresponding numerical outputs. This linking of columns and cells in a spreadsheet gives it the hybrid status of an arithmetico-algebraic tool. Ainley, Bills and Wilson (2005, p. 196) speak of a spreadsheet

as offering a “continuum from arithmetic to algebra.” Several studies at the middle grade levels on spreadsheet use in school algebra teaching and learning (Dettori, Garuti & Lemut, 2001; Haspekian, 2005) exploit this continuum. According to Arzarello, Bazzini and Chiappini (2001) and Rojano and Sutherland (1997), this continuum can also be viewed in terms of the arithmetic-algebra transition. Their studies give spreadsheets a potentially positive role in this transition at the middle school level. In South Africa, at the Research Unit for Mathematics Education University of Stellenbosch (RUMEUS) we find several spreadsheet programmes that have been designed with a focus on middle and high school algebra content. These programmes fall within what Kieran (2007, p. 718) calls “functional approaches,” where there are ample numerical tables and corresponding graphs and expressions.

The pencil-paper early algebra literature on the arithmetic-algebra transition points to the importance of the concept of a variable. Schoenfeld and Arcavi (1988, p. 420) consider this concept as the “basis for transition from arithmetic to algebra” and recommend that learners should have experiences in observing number or arithmetic patterns and making generalisations before they use variables (p. 424), i.e. letters. In algebra letters are symbolic in the sense that they can take on meaning with respect to numbers, i.e. arithmetic. Küchemann (1981) found that children go through several stages towards acquiring the concept of a variable, which include the letter used as a specific unknown, the letter as a generalised number and the letter used as a variable. Awareness of these stages is important, because it can alert teachers to the way that arithmetical processes become compressed into variables (read letters or symbols). Likewise, Leitzel (1989, p. 29) notes that “the concept of variable is more sophisticated than we often recognise and frequently turns out to be the concept that blocks students’ success in algebra.”

The arithmetic-algebra transition calls for a particular conceptualisation of functions. Schwartz and Yerushalmy (1992) and Schwartz (2010) argue that functions in school mathematics can be conceptualised as “number recipes” (and “quantity recipes” such as volume, mass or distance). Schwartz (2010) provides the example of the expression  $x + 13$ , which he considers not as a number but rather as a recipe for computing a number. For various numerical inputs (integers or real numbers) for  $x$ , there are specific outputs which can be represented through a table and a graph. We can think of the  $x$  as a (integer or real) number that can vary, i.e. a variable. To paraphrase Schwartz (2010), making the transition from arithmetic to algebra, implies helping learners move from specific numbers to number recipes that depend on variable numbers. Furthermore, he notes that the concepts of variables and functions are not independent, meaning that the one implies the other, and vice versa.

The pencil-paper concept of a variable in the above becomes clearer when we consider Haspekian’s (2005, p. 120) ‘cell-variable’ and Wilson, Ainley and Bills’ (2005) reference to the action of ‘naming’ a column in the case of a spreadsheet. The cell argument (A1) in a spreadsheet (column A and row 1) can be viewed as a geographic reference, a ‘box’ (compartment of a grid) or an ‘abstract, general reference’ like the ‘formula’ A1. The latter is closest to the pencil-paper view of a variable (Haspekian, 2005). In Figure 1 the linear number recipe ‘ $2n + 4$ ’ via a spreadsheet can be represented in two dynamic ways.

	A	B
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	A	B
	:‘named’	
	$n$	

1	1	=2*A1+4	1	1	=2*n+4
2	2	=2*A2+4	2	2	=2*n+4
3	3	=2*A3+4	3	3	=2*n+4
4	4	=2*A4+4	4	4	=2*n+4

Figure 1:

*Two dynamic ways of representing the linear number recipe  $2n + 4$  through a spreadsheet*

In the left-hand box there are inputs 1, 2, 3, and 4 in column A. In column B there are the ‘opened cells’ showing the equal sign and the computation of the particular outputs from adjacent cells in column A. In spreadsheet syntax the equal sign is an instruction for a computation to occur. In the right-hand box showing columns A and B, column A has been ‘named’  $n$ . The epistemic value of ‘naming’ column A is that the pencil-paper number recipe ‘ $2n+4$ ’ has become mathematically defined through the equal sign ( $=$ ) in column B. This number recipe in column B can be ‘filled down’ through a range of cells in a column. The tables in Figure 1 illustrate the process-object duality (Sfard & Linchevski, 1994) with respect to ‘ $2n+4$ ’, which as a number recipe originates in or from a pencil-paper, arithmetical or computational process.

### **On the design and use of an SAP showing $ax + b = d$**

This section presents the theory and a rationale for the design and use of an SAP showing the equation  $ax + b = d$  used in the teachers’ professional development programme. Also, it will be pointed out how the use of computerised instruments such as any SAP can potentially ‘hide’ the embedded mathematics/algebra when used in teachers’ professional development.

The SAP in Figure 2 shows the equation  $ax + b = d$  as a comparison between two linear functions. The choice of the SAP (for use with the teachers) has to do with illuminating the arithmetic-algebra transition through number recipes (functions). The SAP represents a linear function, a constant function and what is called a difference function in multiple ways, i.e. there are numerical relations between the numbers in the cells and in the columns. In columns A, B, C and D, ‘named’  $x$ ,  $ax + b$ ,  $d$  and  $(ax + b) - d$  respectively, we have various number recipes. Column A has the input value,  $x$ , ranging from -10 through to 7 spaced in intervals of 1. These values were chosen for convenience. In column B there is a tabular numerical representation of  $f(x) = ax + b$ . In column C we have the constant function  $g(x) = d$  and in column D a “difference function” (Schwartz & Yerushalmy, 1992) —  $h(x) = (ax + b) - d$ . The latter can be represented symbolically as  $f(x) - g(x)$ . In spreadsheet syntax the examples are B2 – C2 in the cell D2; B3 – C3 in the cell D3, and so on. In the cells K2, H2 and N2 there are the specific values, 2, 1 and 3 respectively. By ‘leaning’ on the scrollbars located in the cells K3, H3 and N3, one can vary the integer values in these cells from -5 to 5. In Figure 2 we have the specific case of the equation  $2x + 1 = 3$  in other words,  $a = 2$ ,  $b = 1$  and  $d = 3$ . To the right of the four columns there is a graphical representation, which is linked to the columns A, B and C, of two particular functions (number recipes):  $f(x) = 2x + 1$  and the constant function  $g(x) = 3$ .

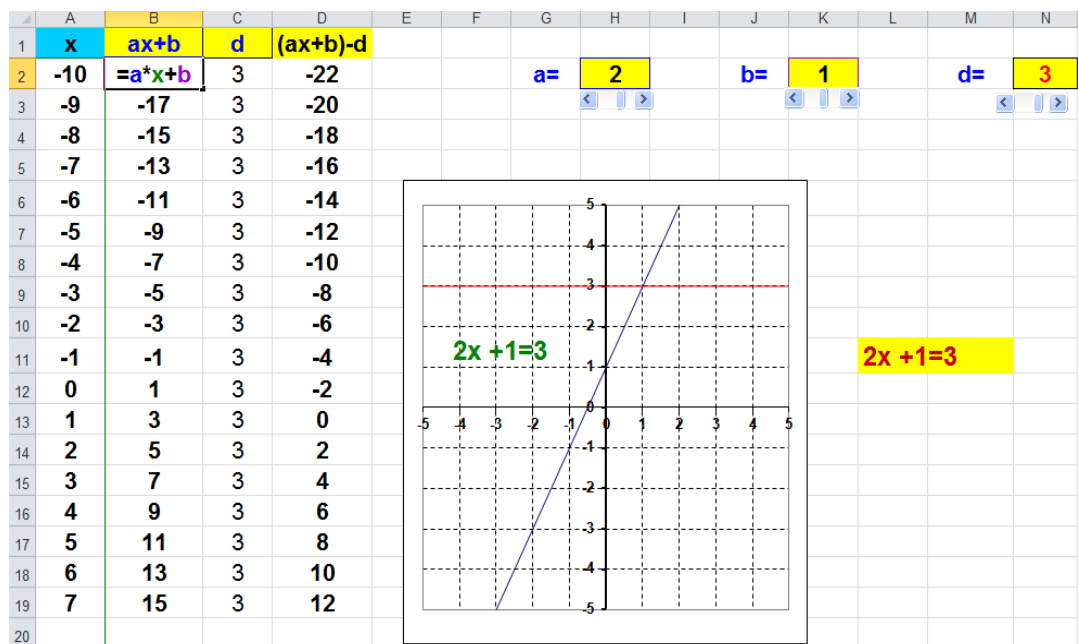


Figure 2:

An SAP showing multiple representations of  $ax + b = d$ .

The equation in the SAP differs from pencil-paper algebra in terms of spreadsheet syntax. First, the equal sign in the spreadsheet syntax (in cell B2) needs clarification. This equal sign is not the same as in an equation in pencil-paper algebra. Every cell in column B is linked to its adjacent cell in column A, as shown in the ‘opened’ cell B2,  $=a*x + b$  which is, in part, a representation of the left-hand side of the (pencil-paper) equation  $ax + b = d$ . The scrollbars – H2, K2 and N2 – were designed as a means to vary  $a$ ,  $b$ , and  $d$ , respectively. ‘Leaning’ on the scrollbars has the dynamic effect of changing the variables  $a$ ,  $b$  and  $d$ . Column A has been ‘named’  $x$ . (See cell A1.) Concealed or hidden in cell B3 we have the specific number recipe computation  $= 2 \times (-9) + 1$ , which gives  $-17$  as an output for the input of  $-9$  in column A, in terms of the scrollbars where  $a = 2$ ,  $b = -9$  and  $d = 1$ . The difference function (see column D) as a comparison of functions is in line with seeing and understanding equations, inequalities and relations as comparisons of functions (Yerushalmy & Schwartz, 1993, p. 42).

A focus on the tabular/numerical representation of the difference function ( $h(x)$ ) in column D can be used to interpret (in)equalities (Kieran, 2004) such as the following general, pencil-paper algebra  $f(x) > g(x)$ ,  $f(x) < g(x)$ , and  $f(x) = g(x)$ . Changes in  $a$ ,  $b$  and  $d$  by means of the scrollbars have repercussions with immediate effect throughout the spreadsheet, i.e. changes in the graphical, tabular/numerical and symbolic representations of the functions. Changing the variables  $a$ ,  $b$  and  $d$  through the scrollbars and posing accompanying questions can also be conceived as graphic heuristics. Verikios and Farmaki (2010) recommend these heuristics when it comes to understanding linear and other (in)equalities. At the Grade 10 level we know about the dominant algorithmic approach and (pencil-paper) algebraic manipulation in teaching inequalities with its focus on rules (Davis & Gripper, 2011).

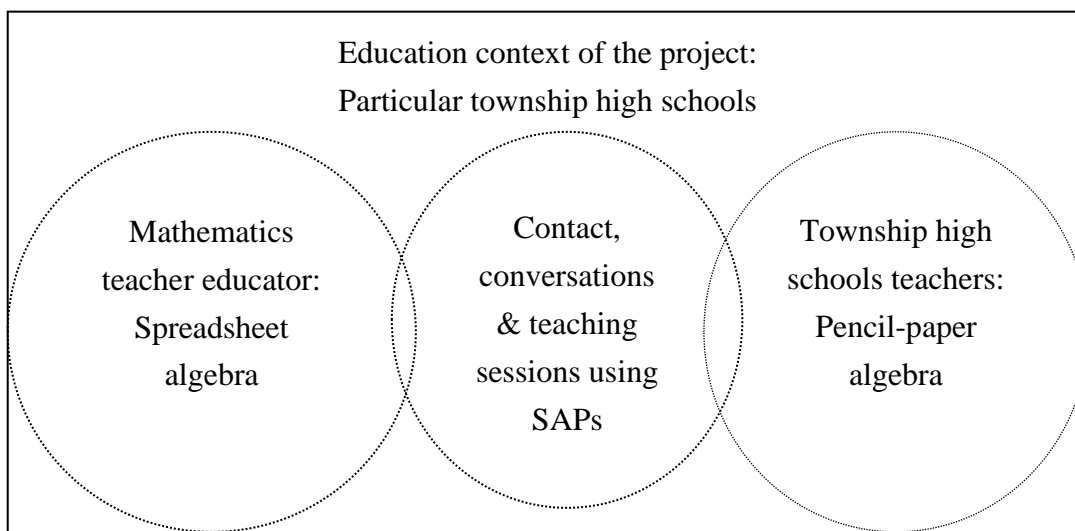
The ‘ready to use’ SAP can be viewed as an “instrument” (Haspekian, 2005) as well as a “black box” (Strässer, 2002; Williams & Wake, 2007). From my perspective as the mathematics teacher educator, the SAP is an instrument because I designed it and I’m familiar with the underlying theory, and I have an idea of how it might be used to point out the epistemic effects of varying the said variables. In the case of the teachers the SAP can be a black box, because they are not familiar with how the SAP was designed and it comes to them ‘ready to use’. The

difference function in column D might thus be a black box which will have to be “de-greied” (Strässer, 2002). De-greying involves explaining to the teachers the design of the SAP, for example, how using the scrollbars illustrates the arithmetic-algebra transition or the process-object duality of the symbols ( $a$ ,  $b$ ,  $d$  and  $x$ ) interpreted as variables. In particular, de-greying entails drawing the teachers’ attention to the dynamic and multiple representations of the (linear) inequalities in the SAP and how these are different from static, pencil-paper algebra version. An example of de-greying is in the ‘opened’ cell B2, showing the expression:  $(ax + b) - d$ .

Why use the SAP in teachers’ professional development? The first reason has to do with raising awareness of the arithmetic-algebra transition and functions as number recipes. As mentioned before, tabular/numerical, symbolic and graphical changes in the number recipes are effected by ‘leaning’ on the scrollbars. In pencil-paper algebra this transition is characterised as ruptures, i.e. discontinuities and false continuities (Bednarz & Janvier, 2001) between algebra and arithmetic. In the pencil-paper algebra there are very few, if any, connections between the letters used in algebra and numbers as found in arithmetic. The SAP in Figure 2 is thus a means of working with the ruptures, i.e. discontinuities. Also, the design and use of the SAP is not a simple adaptation of a static pencil-paper case such as “solve for  $x$ ” in the case of  $2x + 1 = 3$ . In particular, the teachers’ attention can be directed to changes in the tabular/numerical representation of the difference function. Usually in pencil-paper algebra there are references to the independent ( $x$ ) and dependent variables ( $y$ ). The second reason, not unrelated to the first one, is that the particular SAP is a reorganizer (Pea, 1985) of the fundamental nature and arrangement of the mathematics curriculum. The SAP calls for an arrangement or view of (linear) equations as a comparison between two functions; this is not the case in traditional curricula. The third has to do with making the study of an algorithmic approach or rules (see Davis & Grippner, 2011 in the case of inequalities) more important. For instance, a focus on the difference function (in column D) can offer numerical ways of interpreting changes in the inequality signs.

### Analytical framework

Social Practice Theory (SPT) informs the construction of the analytic framework. In this project there are (social) interactions within and between the education context and its assessment system, the particular SAP in use, the teachers’ pencil-paper orientation to algebra and my practice as mathematics teacher educator with an interest in spreadsheet algebra. Figure 3 is an attempt to represent connections between the two practices by way of the SAP used in the teaching and the overarching presence of the education context. The two circles on the left and right indicate the respective practices.



*Figure 3: An analytical framework for studying the teaching of spreadsheet algebra through the use of SAPs*

The circle in the middle is used to show interactions in the form of teaching and conversation by means of the SAP as a representation of spreadsheet algebra. Any analysis has to consider the fact that SAPs per se are absent in the teachers' pencil-paper algebra.

### **Research methods**

In line with SPT, the unit of analysis is the data excerpt revealing interactions between the two practices. Data can thus be collected and generated by making any SAP in use a 'didactic object.' In recent work in mathematics and ICT (Ainley & Pratt, 2002; Bowers, Bezuk & Aguilar, 2011) and on the professional development of mathematics teachers, we see the use of didactic objects (Lima, McClain, Thompson & Castillo-Garsow, 2009). According to Thompson (2002, p. 198), the phrase *didactic object* is used

[t]o refer to "a thing to talk about" that is designed with the intention of supporting reflective mathematical discourse.... [O]bjects cannot be didactic in and of themselves. Rather, they are didactic because of the conversations that are enabled by someone having conceptualized them as such.

For example, excerpts of transcribed conversations during or outside of teaching sessions are a data source. Another source can be artefacts showing the teachers' practice with respect to similar algebra/functions represented in the SAP used during a teaching session.

When using the SAP with the teachers, the notions of "researcher positionality" (Milner, 2007) is useful. Any mathematics teacher educator/researcher with an interest in spreadsheet use or other ICTs, for that matter, has to attend to his/her positionality along two dimensions: one relates to what s/he considers as good algebra teaching (through the use of SAPs) and the other to the ways s/he interprets the pencil-paper algebra teaching of the teachers in the context described earlier.

What can be said about the number of teachers and the teaching sessions? During any particular teaching session the teachers numbered around eight. As mentioned earlier, teacher attendance at the sessions is voluntary, which is a requirement stipulated by the University ethics committee. During the first three sessions teachers became familiar with what spreadsheets look like and how they can be used in general. These sessions were subject to whether the computers were working properly at the particular school. The impression then was that the teachers had become familiar to some degree with spreadsheets and how they might be viewed with respect to their 'pencil-paper algebra.' For example, during one earlier session they explored an SAP showing multiple representations of two linear functions on the same spreadsheet. They were asked to comment on what they noticed about where and when the functions intersect or not. Data excerpts from the fourth session will be presented for analysis because of what they reveal in relation to the education context.

Answers to the second research question – *What happens to teachers' strategies when a SAP is used in the professional development of teachers from particular township high schools?* –



are contained in four excerpts A, B, C and D. These excerpts reflect interactions between the two practices. They come from the fourth teaching session in which I (as the researcher (R)) made the SAP in Figure 1 a didactic object. At the beginning of the session I drew the teachers' attention to the overhead screen where the SAP was projected and had them talk about what they were seeing. I explained what the scrollbars are and how they can be used to vary the values of  $a$ ,  $b$  and  $d$  in the equation  $ax + b = d$ . At one point the teachers could see and read off the solution to  $2x + 1 = 3$  on the graph. I then wrote  $f(x) = 2x + 1$  and  $g(x) = 3$  up on the board. In the following move I asked the teachers to look at the columns B, C and D and asked them what sense they made of those columns as another place to find a solution for  $2x + 1 = 3$ . The analysis will be done by means of a comparison (cf. Corbin & Strauss, 2008), which is a way to answer the second research question.

## Findings

Excerpt A: *Teachers refer to their pencil-paper algebra understanding of difference between functions*

- 1                    R            What do you think is happening there?  
                           (*Pointing towards column D*)  
                           *Silence and pause*  
                           It's just... it's the difference between columns B and C.  
                           What do you think is happening there?  
                           *Silence and pause*  
                           (*Pointing to column D*): The difference here is -22, right. As you go  
                           down, -20, then it's -18, until the difference is zero and that happens  
 8                    at?  
 9                    T1            At 1.  
 10                   R            At 1 yes. Do you follow what was done here?  
 11                   T2            Yes.  
 12                   R            In other words?  
 13                   T2            It's the difference between two functions  
 14                   R            Yes...  
 15                   T2            A very popular question in the exams.  
 16                   R            Say a little bit more.  
 17                   T2            In the case of the straight line graph, they give you factors or they  
                           give you a vertical line that connects the one graph to the other graph,  
                           say  $f(x)$  and  $g(x)$  and they ask you what's the value of  $f(x) - g(x)$ .  
 20                   R            And this is your Grade 11?  
 21                   T2            Grade 11 and 12.  
 22                   R            Grade 11 and 12?  
 23                   T2            Grades 11 and 12 in one of the textbooks.  
 24                   R            If you look at what is up on the screen here, where do you think this is  
                           pitched at?  
 26                   T2            Grade 8, Grade 9. Grade 9, 10. I'd say 10. The same concept, the  
                           same concept  $(ax + b) - d$  is being tested in Grades 11 and 12. The  
                           concept changes but the difference is the same,  $(ax + b) - d$ , the  
                           vertical distance. The question is the same, the concept changes, the  
                           vertical difference between two functions. It is seen as a higher order  
                           question.  
 31                   R            Namely, the differences between two functions?

- 32 T2 Yes, the vertical distance... the vertical difference between two functions.
- 33 R Okay.
- 34 T3 It's seen as a higher-order question.
- 35 R Say a little bit more.
- 36 T2 Some of the questions, when they set the questions, they always  
37 remind us, when we moderate, that a certain percentage of the questions, it's about according to Bloom's taxonomy, 7½% must be higher-order questions. And this question, I always notice it. Say they give you a parabola and an exponential graph. Yes, they will ask you the normal things like where does the graph, cut the y-axis, the x-axis, perhaps, what is the axis of symmetry and lastly, remember the higher order question, where is  $f(x)$  greater than  $g(x)$  or where is  $f(x)$  equal to  $g(x)$ ? Learners normally struggle with such a question.
- 42

The notion of the SAP as a black box and the teachers' pencil-paper algebra are useful in terms of explaining excerpt A. The teachers' silence at the beginning in lines 1 through 8 can be explained in terms of viewing the SAP as a black box. Although the excerpt comes from the fourth teaching session, the teachers did not immediately notice how the SAP was designed in terms of number recipes with inputs and outputs, as I may have wanted it. It is only when I started directing (line 3) them to the number recipe (lines 4–8) that T1 (line 9) notices the numerical inputs and linked outputs and differences shown in column D. Furthermore, the teachers interpret what they see in the SAP by making comparisons with their pencil-paper algebra teaching of (linear) functions. Their comparisons are understandable, because they use their practice as a frame of reference to make sense of what they see in the SAP. They recognised (line 13) and compared the difference function represented numerically e.g. as a number recipe (-22, -20, -18...) in column D with a similar question at the Grades 11 and 12 examination (line 15). T2, however, noted that the difference function in column D is also applicable at the Grade 8 level (Line 28: *The concept changes but the difference is the same*) as in the SAP. T2's observation comes about through conversational turns found in lines 20 and 21. Also, through conversational turns (lines 33 and 35) T2 talks about how the "the difference between two functions" is viewed as a higher-order question (line 34, 39 and 43) at the Grade 12 level. In other words, she provides details on ways that "the difference between two functions" (lines 36-43) appears in the Grades 11 and 12 curriculum.

Her comparison provides us with details on ways pencil-paper algebra is prescribed, i.e. the institutional impact on the teachers' practice. We notice how the teachers' "difference between two functions" is stipulated as a "higher-order question", with Bloom's taxonomy (line 38) as justification. This institutional impact or control is evident in references to "they" (line 36 and line 40), i.e. the education department officials/examiners. Bloom's taxonomy with its 7½% is thus a way to standardise what the teachers can focus on in their practice. The conversational turns in the above excerpt then go in a direction where the teachers start to talk about their learners' difficulty with inequalities (lines 44-58) below.

*Excerpt B: Teachers refer to difference function and inequalities*

- 44 T2 ... $f(x)$  equals to  $g(x)$ ? Learners normally struggle with such a question.
- 45 R Why?
- 46 T2 They fail to interpret the question.
- 47 R Why?

- 48 T2 They fail to see and understand where  $f(x)$  equal to  $g(x)$  say. They don't read the question properly, a common error.
- 50 T3 But generally learners struggle with those kinds of questions.
- 51 T2 Yes.
- 52 T3 When I was teaching the Grade 11s last year, when you have  $f(x) < g(x)$ . When  
53 they see that together, they get lost really.
- 54 R You mean inequalities?
- 55 T2 As soon as they see them in front of them.
- 56 T3 Yes. When they see it in front of them. But then, I think it makes a little more  
58 sense when you separate, when you give them two graphs, that are intersecting and you ask them that question, where is  $f(x) > g(x)$ ?

From the above we see that the teachers' learners – like the teachers in Greece (Verikios & Farmaki, 2010) – struggle with inequalities. One explanation could be the dominant (pencil-paper) algorithmic approach with its focus on rules (Davis & Gripper, 2011). The latter explains the teachers' reference to their learners' "common error" (line 49). Although not guaranteed, the understanding of inequalities is also possible through a conversation around the difference function represented numerically in column D. During the professional development session the teachers saw a numerical representation of the difference function in the SAP as a way to interpret inequalities. Evidence for the latter comes from line 56 (*But then, I think it makes a little more sense when you separate, when you give them two graphs, that are intersecting and you ask them that question, where is  $f(x) > g(x)$* ). The influence of this SAP with its tabular-numeric difference function on the teachers' practice is marginal.

*Excerpt C: High-stakes assessment influence and teachers' approach to functions*

The teachers have to contend with the ways high-stakes assessment moulds algebra content such as (linear) functions. On the other hand, the teachers have ways of working with and using linear functions in some sort of connected way, moulded by the grade levels, in their teaching, as we see next:

- 1 T2 The big question is why they test it separately at the Grade 12 level, in the  
3 National Senior Certificate. They would look at the values for  $ax + b$  they would  
4 look at the values 21, 19, 17... which is a clear linear pattern, and that is tested  
separately in a paper.
- 5 R Separately, compared to?
- 6 T2 Separately, compared to the functions.
- 7 R Are you asking me? I don't know how to answer you, but ...
- 8 T2 Because that is how they guide us. You must teach the *linear pattern* and the  
*quadratic pattern* (emphasis in original) as a separate question, because that is  
how it will be tested. Learners know the linear pattern in Grade 12, they have a  
different name, they call it the arithmetic sequence and the quadratic pattern 6  
marks and the linear pattern, but what I see here is an integration between the  
linear pattern and the graph and the function, and when you get to the function  
14 again, you have the same question, where they ask you the same question that you  
are asking now, where are the two functions equal or what is the difference  
between the two functions.

- 19            What I'm seeing here, in Grade 8 they also have that the number pattern, like  $ax + b$ , we have 21, 19, 17, next and we ask the learners write the  $T_n$  formula for that. Give the  $T_n$  formula for that.
- 20    R        You did that in your question paper? What led you to that insight?
- 21    T2        It's a function,  $T_n$
- 22    R        Okay

Apparently, T2 has become aware of functions, specifically linear and quadratic (“linear pattern,” “quadratic pattern” in lines 8–10) as a key unifying concept in school mathematics to the point where she has her Grade 8 learners do linear patterns in arithmetic sequences with linear functions of the form  $ax + b$ . She is aware of institutional constraints such as different grade levels on ways functions appear in the curriculum (lines 8–10). In the SAP she sees an “an integration between the linear pattern and the graph and the function” (line 13–14). She is frustrated and questions the institutional influence coming from the education department on ways “functions” are “tested separately in a paper” (lines 1–4 and lines 8–19).

*Excerpt D: Teacher artefact on the linear function*

After the session, T2 passed on to me an artefact on their “attempt” to connect the function concept across the grade levels.

Please find enclosed our Patterns and Sequences Questions and own attempt to “connect” it to the Function concept... see Q8 ... June 2015

**Question 8**                      **Patterns & Sequences**

a) The table shows the relationship between  $x$  and  $y$ :

$x$	1	2	3	4	5	6
$y$	1	5	9			

i. Complete the table.

ii. Write down a formula which shows the relationship between  $x$  and  $y$  in the form  $y = \underline{\hspace{2cm}}$ .

iii. Draw a graph of this relationship and label the axes.

*Figure 3: Artefact showing the teachers' approach to linear functions*

She and her colleagues connected “patterns and sequences” with “the function concept” in their Grade 8 question paper. In this question they also asked their learners to “draw a graph” of this linear function. One explanation for the teacher presenting the note to me has to do with finding commonality between their practice and the SAP showing the two linear functions. The teachers are aware of the importance of the function concept (excerpt C). The SAP is thus a kind of confirmation of their practice with respect to the function concept (see line 11 is Excerpt C). Noting the context, another reason could be the teachers’ “attempt” at preparing their learners at the Grade 8 level for the high stakes assessment.

### **Discussion and conclusion**

Three interrelated areas deserve attention. The first concerns the education context. The second is the need to shore up the teachers’ understanding of functions. The third has to do with the use of the SAP with its difference function and the teachers’ teaching of inequalities.

The teachers’ encounter with spreadsheet algebra in the described context is heavily influenced by the ways pencil-paper algebra features in the high-stakes assessment. In their own way the teachers fortify (Christie, 2008) themselves against the academic/mathematical demands of the curriculum with its high-stakes assessment and related life consequences for learners. For example, they do attempt to go against the fragmentation of the grade levels by way of the function concept, as can be seen in excerpt C. Through the function concept they “attempt” to work with the ruptures, i.e. discontinuities, between algebra and arithmetic and across the grade levels. In the words of one of the teachers, these ruptures are evident in ways the pencil-paper algebra is “tested separately” in the “National Senior Certificate.” Ultimately the teachers’ “connecting” the pencil-paper algebra they teach through a focus on “the function concept” should be viewed as a particular attempt to handle the education context in which the education department compares the performance of their schools with those of affluent schools in the high-stakes assessment process.

The teaching of (linear) inequalities through using an SAP with its focus on a difference function is not common to the teachers’ practice. What might highly likely be dominant (at the Grade 10 level) is the static pencil-paper algebra algorithmic approach with its focus on rules. The use of the scrollbars in the SAP offers the possibility of noticing the process-object duality by means of tables associated with the symbols in  $ax + b = d$ . Any move towards viewing (linear) inequalities as a comparison between two number recipes (functions) cannot come solely from the teachers’ encounters with spreadsheet algebra. Such a view is more likely to take root if the high-stakes assessment reflects such ways of seeing the (linear) inequalities. Also, any SAP showing inequalities might in effect be perceived as a black box, which will have to be ‘de-greied.’ Broadly understood, ‘de-greying’ entails drawing the teachers’ attention to the dynamic and multiple representations of the (linear) inequalities in the SAP and how these are different from or similar to a static, pencil-paper algebra version. ‘De-greying’ also has its high-stakes assessment counterpart.

These findings have implications for professional development. It appears that the use of SAPs as bearers of spreadsheet algebra can attain meaning across two practices, viz., township high school mathematics teachers and mathematics teacher educator(s) with an interest in using SAPs for professional development purposes. From interactions in the project it seems that the hidden design features of SAPs can make them black boxes, meaning that connections with the

teachers' pencil-paper algebra have to be attended to in explicit and coordinated ways. Important stakeholders such as developers of the high-stakes assessment also have a role in this coordination. We should note what the teachers are seeing and how they might reorganise their practice. Also, we should note how the teachers are going about their work and how they are talking in animated ways about the function concept. But we should also notice their frustration with ways functions are institutionalised in the high-stakes assessment.

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