Modelling multi-product industries in computable general equilibrium (CGE) models

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Declaration

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Abstract

It is common practice in computable general equilibrium (CGE) models that the output composition of multi-product industries remains constant despite changes in relative prices of products. The results of any scenario will show that products produced by a single industry will still be produced in the same ratio to each other as reflected by the base data. The objective of the study was to develop a CGE model for South Africa in which this assumption of fixed composition of output can be selectively relaxed. In order to allow industries to adjust their output composition in response to changes in relative prices of products a Constant Elasticity of Transformation (CET) function and the related first order condition were incorporated into an existing CGE model. This alternative specification of an output transformation function in the model enables the modeller to allow selected multi-product industries to increase production of products that show greater price increases relative to other products. The first order condition of the CET function determines the optimal combination of products for each industry. With the inclusion of the CET function there is a trade-off between theoretical rigour of the model and realism of the results, therefore an assumption of input-output separability was introduced as a way of recognising that the inclusion of a CET function violates the assumption that prices in the same row of a social accounting matrix (SAM) are equivalent.

The model was calibrated with a SAM for South Africa for 2007 that was developed for purposes of this study. Set controls were included in the model to generalise the model in order that it can be calibrated with data from other countries as well. The SAM for South Africa contains provincial level information in the accounts for agriculture, labour and households. The agricultural industries are defined by geographical area, hence these industries are particularly good examples of multi-product industries that respond to relative price changes when determining production levels of individual products.

The adjusted CGE model was used to analyse four scenarios focusing on selected issues mentioned in the National Development Plan for South Africa released by the National Planning Commission in 2011. The scenarios relate to increases in fruit exports as a result of global positioning, technical efficiency improvements for the agricultural sector through continued research and development, factor productivity growth in government and selected services sectors resulting from fighting corruption and curbing strikes, and augmenting the supply of skilled labour through an improvement in the quality of education. The results of the adjusted model show the desired effect: producers produce *relatively* more of the

products for which they can get a *relatively* higher price and *vice versa*. This holds true regardless of whether the level of industry output increases or decreases.

The impact of the model adjustment and the effects of changes in the levels of elasticities and choice of variables to close the model were analysed as part of the sensitivity analyses. The impact of changes in the functional form, elasticities and model closures on results, are different for each scenario.

Opsomming

Dit is erkende praktyk in berekenbare algemene ewewigsmodelle dat die verhoudings waarin produkte tot mekaar geproduseer word deur multi-produk industrieë konstant gehou word, ongeag veranderings in relatiewe pryse van produkte. Die resultate van enige senario sal dus aandui dat die produkte wat deur ‘n enkele industrie geproduseer word steeds in dieselfde verhouding tot mekaar geproduseer sal word, soos weerspieël in die basis data. Die doel van die studie was om ‘n berekenbare algemene ewewigsmodel vir Suid-Afrika te ontwikkel wat die aanname dat die samestelling van elke industrie se uitset onveranderbaar is, selektief kan verslap. Om toe te laat dat industrieë die samestelling van uitset kan aanpas namate die relatiewe pryse van produkte verander, is ‘n Konstante Elastisiteit van Transformasie funksie en die gepaardgaande eerste orde voorwaarde in ‘n bestaande berekenbare algemene ewewigsmodel ingesluit. Die eerste orde voorwaarde bepaal die optimale verhoudings waarin produkte geproduseer moet word. Met die insluiting van die Konstante Elastisiteit van Transformasie funksie word teoretiese korrektheid van die model ingeboet in ruil vir meer realistiese resultate, dus is die aanname van inset-uitset onafhanklikheid gemaak en daardeur word ook erken dat as gevolg van die insluiting van die Konstante Elastisiteit van Transformasie funksie word daar nie meer voldoen aan die aanname data alle pryse in dieselfde ry van die sosiale rekeninge matriks (SRM) aan mekaar gelyk is nie.

Die model is gekalibreer met ‘n SRM vir Suid-Afrika vir 2007 wat vir doeleindes van die studie ontwikkel is. Deur die insluiting van kontroles vir versamelings is die model veralgemeen sodat die model ook met data van ander lande gekalibreer kan word. Die SRM vir Suid-Afrika se rekeninge vir landbou, arbeid en huishoudings bevat inligting op provinsiale vlak. Die landbou industrieë is volgens geografiese gebiede afgebaken en is dus besonder goeie voorbeelde van multi-produk industrieë wat reageer op relatiewe prys veranderings wanneer die produksievlakke van afsonderlike produkte bepaal word.

Die aangepaste algemene ewewigsmodel is gebruik om vier senarios te ondersoek wat fokus op geselekteerde onderwerpe vervat in die Nasionale Ontwikkelingsplan wat deur die Nasionale Beplanningskommissie van Suid Afrika in 2011 vrygestel is. Die senarios hou verband met ‘n styging in vrugte uitvoere as gevolg van globale posisionering, tegniese produktiwiteitsverhogings vir die landbousektor deur volgehoue navorsing en ontwikkeling, verhoging in die produktiwiteit van produksiefaktore van die regering en geselekteerde dienste sektore deur die aanspreek van korrupsie en vermindering in stakings, en die toename in geskoolde arbeid deur ‘n verbetering in die kwaliteit van onderwys. Resultate van

die aangepaste model toon die gewenste uitwerking: produsente produseer *relatief* meer van die produkte waarvoor hulle ‘n *relatiewe* hoër prys kan kry, en omgekeerd. Dit geld ongeag of daar ‘n verhoging of ‘n verlaging in die vlak van die industrie se uitset is.

Die impak van die modelaanpassing, die effek van veranderings in die vlakke van elastisiteite en die keuse van veranderlikes om die model te sluit, is geanaliseer as deel van die sensitiwiteitsanalises. Die impak van veranderings in die funksionele vorm, elastisiteite en modelsluiting op resultate, is verskillend vir elke senario.

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Table of contents

List of Figures

List of Tables

Abbreviations

AIDS	Almost Ideal Demand System
AIDADS	An Implicitly Directly Additive Demand System
CDE	Constant Difference of Elasticities
CGE	Computable General Equilibrium
CES	Constant Elasticity of Substitution
CET	Constant Elasticity of Transformation
CIF	Cost Insurance Freight
CRESH	Constant Ratio of Elasticities of Substitution Homothetic
CRETH	Constant Ratio of Elasticities of Transformation Homothetic
ELES	Extended Linear Expenditure System
FOB	Free On Board
GCE	Generalised Cross Entropy
GDP	Gross Domestic Product
IES	Income and Expenditure Survey
IFPRI	International Food Policy Research Institute
LES	Linear Expenditure System
LFS	Labour Force Survey
NDP	National Development Plan
NIPA	National Income and Product Accounts
NPC	National Planning Commission
NPISH	Non-Profit Institutions Serving Households
PROVIDE	Provincial Decision-Making Enabling Project
SAM	Social Accounting Matrix
SARB	South African Reserve Bank

SARS	South African Revenue Service
SNA	System of National Accounts
SSA	Statistics South Africa
UN	United Nations
VAT	Value added tax

Descriptions of model variables, parameters and sets are listed in the addendum.

1 Introduction

1.1 Introduction and background

Computable general equilibrium (CGE) models are part of a class of models that captures the effects of changes in relative prices on the economy. These models are widely used in policy and impact analyses. General equilibrium refers to the view that the economy is an interrelated system of markets and that a situation could exist where supply and demand relationships for goods and services in the entire economy are in balance. A CGE model is a system of simultaneous equations and the functions in a CGE model indicate the assumptions with regard to how the different categories of role players, or agents, in an economy are perceived to act. A CGE model is an economy-wide model that captures the linkage effects in an economy through its price and quantity systems. Comparative static CGE models, such as the one used in this study, are used to estimate the impact of a shock or change in the economy by measuring the direction and magnitudes of the changes that occur when the economy moves from the pre-shock equilibrium to the new post-shock equilibrium.

There are examples in the literature of CGE models that allow for industries (firms or activities) to be specified as multi-product industries, i.e. industries can produce more than one product and the same product can be produced by more than one industry. A mixed farming operation that produces field crops, livestock and horticultural crops serves as an example. In many CGE models the output of multi-product industries is characterised by the assumption that the composition of output by each industry stays the same regardless of the total level of output by the industry, and hence regardless the changes in relative prices of products. This assumption would be realistic in the context of industries that produce by-products in fixed proportion to their main output. However, there are industries that are capable of changing the proportions of the products that they produce in response to changes in relative price. Because there exists a degree of price responsiveness amongst certain producers, of which agricultural producers are particularly good examples, the relaxation of the assumption of fixed output composition in favour of a more flexible specification of output composition is expected to contribute to improve the quality of CGE model results.

Literature reveals that the need to get a better understanding of supply response is often linked to the agricultural sector because agricultural producers typically respond to price incentives when taking production decisions in terms of the mix of products that they produce. During the past century the debate and model advancement with regard to the supply response of the agricultural sector was probably stimulated mostly by economists and government officials who wished to understand the response of agricultural producers to policy changes. Although the existence of agricultural output response cannot be denied, the level of responsiveness is variable and influenced by various factors. Various supply response models have therefore been developed over the last century to estimate supply response. Nerlove's model of agricultural supply response (Nerlove and Addison, 1958) provided a significant base for debate and further model development regarding this issue, as indicated by the abundance of literature in this regard.

Supply response models are often estimated econometrically. When these systems are included in CGE models to determine the behaviour of producers, the relevant elasticities are not calculated endogenously, but are supplied exogenously to the CGE model. Supply response systems in the literature that are deemed potentially appropriate for inclusion in a CGE model to cater for price responsive output behaviour, include the CRESH/CRETH and the CES/CET production systems. Examples of such applications in CGE models include, amongst other, Gelan and Schwarz (2008) who investigated the effects of single farm payments on Scottish agriculture. They included a CET function for output transformation and found that despite the fact that reliable supply elasticities were unavailable, their model yielded policy effects that are likely to represent behaviour of a profit maximisation farmers. Another example is found in the ORANI-G model, which is a general equilibrium model for the Australian economy, which contains a CET transformation of aggregate industry output to multiple products (Horridge, 1993).

Within the agricultural sector strategic direction has often been influenced by policy analyses using CGE models. CGE models are widely used in policy analyses and as such it informs policy decisions, with potentially widespread economic implications. Public domain applications for South Africa include investigations in trade liberalization, green trade restrictions, currency devaluations and government expenditure and restructuring. McDonald and Kirsten (1999) used a CGE model to analyse the impact of a drop in the world gold price on the agricultural sector. Thurlow and Van Seventer (2002) presented a comparative static CGE model for South Africa based on the standard static International Food Policy Research

Institute (IFPRI) model, which also provided the basis for Kearney's (2003) extension to assess the implications of different value added tax options. An example of an environmental application can be found in De Wet and Van Heerden (2003). Van Schoor and Burrows (2003) adapted the standard IFPRI CGE model to account for imperfect competition and returns to scale. They used this model to analyse the impacts of unilateral free trade and a reduction in conjectural variations in all sectors of the economy. On a provincial level, a CGE model was developed for the Western Cape (McDonald, 2003) and used to analyse trade liberalisation effects (Chant, McDonald and Punt, 2001) and some welfare implication of a land tax in the Western Cape (McDonald and Punt, 2003a). As part of the working paper series of the PROVIDE project (www.elsenburg.com/provide) various topics were analysed using a CGE model. Scenarios related to the following topics were analysed: the international price of sugar as a result of international deregulation, the oil price, technical change associated with productivity increases, the wheat import tariff rate, excise duties, wine exports, a trade agreement with China, etc. Mabugu and Chitiga (2007) explored poverty and inequality impacts of trade policy reforms in South Africa, whereas Hassan, Thurlow, Roe, Diao, Chumi and Tsur (2008) analysed the macro-micro feedback links of water management in South Africa. This list is not exhaustive, but gives an indication of some of the relevant topics that have been addressed using CGE models for South Africa.

1.2 Problem statement

Price support programs in agriculture, amongst other, insulate producers from market price signals and thereby weaken their response to market price shocks (Burfisher, Robinson and Thierfelder, 2003). Deregulation in the mid 1990's has strengthened the role of organised markets and producer responsiveness to price signals in South Africa (OECD, 2006). Therefore, in this context price responsiveness of agricultural producers is particularly relevant in South Africa.

Despite the fact that CGE modelling has been widely used for various issues, including issues relevant to agriculture, a scan of the general equilibrium literature reveals very few instances of CGE models that include output transformation functions to allow for multi-product industries to be price responsive in their output composition. To the knowledge of the author no such application has been done in the context of South Africa.

Although none of the mentioned applications for South Africa mentions the relaxation of the assumption of fixed composition of output in the CGE model, there are existing examples of SAMs for South Africa with detailed agricultural accounts (McDonald and Punt, 2003b;

PROVIDE, 2006) that could be used to calibrate a revised CGE model to derive benefit from the proposed CGE model revision. Agricultural industries are captured as agronomic regions in the mentioned agricultural SAMs for South Africa, therefore the combinations of output from these industries are deemed more price responsive than output combinations of other industries. Often the level of aggregation of the data in social accounting matrix (SAM) based models are such that the benefits of improved model specification will not be realised, unless accompanied by more detailed data in the SAM. Any model changes as discussed in this study would only have an effect on results when multi-product industries are explicitly catered for in the calibration data contained in the SAM. Although rarely found in practice, single output production processes are often introduced in models to facilitate modelling. CGE models that assume single output production processes (through the underlying SAM data) would however not benefit from an extension such as the one proposed in this study. Thus, there is a need for an updated SAM for South Africa that will be suitable to calibrate the adjusted CGE model.

1.3 Objectives and contributions of the study

The main objective of the study is to develop a CGE model for South Africa in which the assumption of fixed composition of output can be selectively relaxed in order to enhance the quality of CGE model results. This objective is achieved by means of two sub-objectives. The first sub-objective is to improve the specification of the output transformation function of multi-product industries of an existing CGE model to reflect more realistically the supply response to changes in relative prices for those industries that are price responsive in their output composition. The second sub-objective is to develop a SAM for South Africa with agricultural, labour and household detail on provincial level that can be used to calibrate the adjusted CGE model for use in policy analyses.

The main contributions of the study include the following:

- Availability of an updated social accounting matrix (SAM) for South Africa with agricultural detail, which can be used to calibrate the CGE model;
- Description of the development of the SAM for South Africa and the data sources that were used in order to contribute towards future development of SAMs for South Africa;
- Availability of model and calibration code of the model changes that will allow other researchers to incorporate the changes into their models;
- A case study on South Africa using the revised version of the model calibrated with the newly developed SAM as a contribution to the policy debate;

- Comparison of results of two alternative transformation specifications and sensitivity analyses of different parameter values for the selected transformation function.

Frandsen and Jensen (2001:1) indicate that "using more formal analytical approaches such as CGE modelling contributes to a more focused, disciplined and hence a more constructive policy debate". The improved specification of the output transformation function in a CGE model directly impacts on the quality of the results derived with the model and is therefore expected to make a positive contribution to the policy debate.

1.4 Research method

The following specific tasks were carried out in order to meet the objectives:

Task 1: A literature review on aspects of social accounting, CGE models and functional forms was conducted to present the theoretical context for the study.

Task 2: The model specification of the output transformation function for multi-product industries of the existing single country CGE model was enhanced. The GAMS-based CGE model called the STAGE (Static Applied General Equilibrium) model, developed by McDonald (2007), was used as a starting point. In the STAGE model the composition of output by each industry is not responsive to relative price changes, and is therefore dictated by the composition in the base case. This assumption is relaxed by introducing a CET function on industry output to allow for a different simulated output composition compared to the output composition in the base case. According to Hallem (1998) agricultural supply response can imply the change in supply in response to changes in both output and input prices. This study focuses on supply in response to changes in output prices.

Task 3: A SAM for South Africa with detailed agricultural accounts, which is used to calibrate the adjusted CGE model, was developed. The SAM is for the base year 2007. The SAM includes detailed agricultural accounts because agricultural industries classified by agronomic region are particularly good examples of multi-product firms that are price responsive in their output composition. The SAM follows a broadly similar structure as that of the agricultural SAM for South Africa for 2000 developed as part of the PROVIDE Project (PROVIDE, 2006), but there are certain deviations at a more micro level because of changes in data sources and presentation compared to 2000.

Task 4: A case study on employment and the agricultural sector was carried out using the developed SAM for South Africa and the adjusted CGE model. Four scenarios were analysed. The scenarios relate to increases in fruit exports as a result of global positioning, technical efficiency improvements for the agricultural sector through continued research and development, factor productivity growth in government and selected services sectors resulting from fighting corruption and curbing strikes, and augmenting the supply of skilled labour through an improvement in the quality of education.

Task 5: The four scenarios used in the case study were used to test the robustness of the model, as well as to demonstrate the impact of the CGE model adjustments on the results through comparing the results from the adjusted model to the results obtained when using the base model. Sensitivity analyses results with regard to elasticities and closure rules are also reported.

1.5 Dissertation outline

The outline follows the structure of the tasks specified in the research method. Chapter 2 explores the theory of social accounting and touches on the dual function of a SAM as database and as an approach to modelling. The SAM as a database is explained in Chapter 2, while the SAM approach to modelling is discussed in more detail in Chapter 3. Chapter 3 also focuses on the different functional forms most commonly used in CGE models, notably the constant elasticity of substitution (CES) and constant elasticity of transformation (CET) functions, and a discussion of the base model is included. Chapter 4 presents a discussion of the suggested alternative model specification for handling flexible output composition and the adjustments to the base CGE model. The development of an agricultural SAM for South Africa, additional data requirements for the alternative model specification and some findings from the SAM are discussed in Chapter 5. Chapter 6 reports results on a case study on employment creation in agriculture for four different scenarios. Chapter 7 reports the main findings when comparing results from the original and new model specifications, as well as findings from sensitivity analyses with regard to elasticities' values and closure rules. Main findings and recommendations for further research are presented in Chapter 8. The addendum contains the lists of model variables and parameters and their descriptions, GAMS code of the core model equations of the adjusted model, the list of SAM accounts and the municipalities represented by each of the agricultural industries in the SAM.

2 Theory of Social Accounting

2.1 Introduction

Social accounts refer to the accounts of society that are used to describe and understand society. These accounts can be applied to economic, socio-demographic and environmental information (Stone, 1986). Naudé (1993), Miller and Blair (1985) and Stone (1978 and 1986) provide detailed historic overviews of the development of social accounting and general equilibrium modelling. Thus the precursors to current general equilibrium modelling can be traced back to the seventeenth century when William Petty reported the first estimates of national income of Britain and Gregory King compiled what can be considered the first social accounting matrices for England, France and Holland with the purpose of determining the contribution made to wealth by various groups in society. In 1758 Francois Quesnay, French economist and physician of Louis XV, published an Economic Table that was a diagrammatic representation of how expenditures can be traced through an economy in a systematic way. He used this table to warn of the impending danger of revolution in France. By the end of the nineteenth century there were income estimates available for about twenty countries, with Mulhall making a substantial contribution. In the early 1920's Gromon and Popov was involved in the publication of input-output tables for the Soviet Union. Bowley from England and Kutznets from the United States also made contributions with regard to national income estimates.

Colin Clark published *National Income and Outlay* in 1937 which combined estimates of income, output, consumer expenditure, government revenue and expenditure, capital formation, savings, foreign trade and the balance of payments. Up to this point the figures were not yet formally set in an accounting framework, but the work of Clark came close to consistency. Clark was also the teacher at Cambridge of Sir Richard Stone who later played a significant role in the development of an accounting framework through his involvement in the development of the United Nation's System of National Accounts (SNA). Essentially the role of an accounting framework is to impose consistency on the national estimates through standardising definitions of the components of the economic system and how they are valued. The Cambridge Growth Project also contributed largely to the use of a matrix framework for reconciling and presenting data and this led to the introduction of a SAM structure in the 1968 SNA (Drud, Grais and Pyatt, 1986).

According to Robinson (1989) the national income and product accounts (NIPA) provide the statistical foundation for macro models, just as the input-output accounts underlie multi-sector models. However, in order to incorporate issues such as income distribution and structural adjustment in the analyses, income and expenditure flows that are captured in the national income and production accounts should be included. The desire to reconcile the national income and product accounts with the input-output accounts in a unified statistical framework led to the development of SAMs.

The SNA is the accounting framework that is used by National Statistical Agencies to capture the data in a consistent manner. Section 2.2 gives a short overview of the definition and purpose of the SNA. The SNA price definitions are presented, as these are relevant in valuing entries in the SAM. The section on the SNA also gives an overview of the different types of accounts and tables related to the SAM in order to illustrate where the SAM fits into the SNA. The SNA view of satellite accounts and classification systems are also briefly mentioned as these will become relevant in Chapter 5 which addresses the calibration data for the model.

Since the infancy of social accounting there has been a close link between social accounts and policy analyses. The national accounts of an economy present an overview of the status of the economy but they do not constitute a model. When properly arranged, the data can be used as the basis for models. According to Pyatt (1991:316) "...a social accounting matrix is a framework both for models of how the economy works as well as for data which monitor its workings". The SNA recognises the unique contribution of SAMs in presenting a consistent framework to include data and to be used as a basis for modelling. Section 2.3 highlights the role of a SAM as a database by touching on some general theoretical and structural concepts of a SAM, as well as the information content of a SAM. It is shown that a SAM captures the full circular flow in the economy. This section also explores the differences between supply and use SAMs and input-output SAMs, which is particularly relevant in the context of this study. Section 2.3 is concluded by a brief overview of estimation techniques which are used to estimate missing information in order to derive a balanced SAM. The role of a SAM in supporting economic modelling is discussed in the next chapter. Conclusions are presented in section 2.4.

2.2 The System of National Accounts (SNA)

2.2.1 Definition and purpose of the SNA

The United Nations Statistics Office responsible for the co-ordination of the SNA defines the SNA as follows: “The System of National Accounts (SNA) is the internationally agreed standard set of recommendations on how to compile measures of economic activity. The SNA describes a coherent, consistent and integrated set of macroeconomic accounts in the context of a set of internationally agreed concepts, definitions, classifications and accounting rules.” (United Nations, 2011)

The main role of accounting systems is to organise data in the form of accounts in order to clearly show the incomes, expenditures and stock flows for different entities or agents in the economy. These entities can then be analysed as part of a bigger system. An accounting system requires that definitions of the components of the economic system are standardised and it sets out how the different components should be valued. The SNA is one such accounting framework that imposes consistency on national estimates. The three main uses of the SNA are indicated as monitoring the behaviour of the economy, macro-economic analyses and to allow for international comparisons (United Nations, 2009). The SNA can fulfil this role because the framework provides accounts that are (United Nations 2009, par 1.1):

- ‘comprehensive’ because it covers all agents and their actions in the economy;
- ‘consistent’ because of the accounting rules that are imposed; and
- ‘integrated’ because all the consequences of an action by any agent are reflected in the accounts.

Sir Richard Stone played a key role in developing the conceptual framework of national income accounting. This led to the first formalisation of international standards for such accounts in 1953 (Pyatt, 1994). Revisions appeared as the 1968 SNA and the 1993 SNA. The 2008 SNA is the fifth and most recent version of the international statistical standard for the national accounts, adopted by the United Nations Statistical Commission (UNSC). Besides the SNA, there are also ten Handbooks of National Accounting that provide methodological support in the implementation of the 2008 SNA, all of which have been released before 2004. Updates of the handbooks following the revision of the 2008 SNA have therefore not been finalised at the time of writing this.

The 2008 SNA captures the necessary information in three different categories of accounts (United Nations 2009, par 2.83 – 2.85):

- Current accounts that capture production and the generation, distribution and use of income;
- Accumulation accounts that cover changes in assets and liabilities and changes in net worth;
- Balance sheets that include stocks of assets and liabilities and net worth.

The current accounts and the accumulation accounts are collectively referred to as the flow accounts because they capture the flow of funds in the economy. When the balance sheet information is added, it is referred to as the full sequence of accounts.

2.2.2 SNA price definitions

In the 2008 SNA report section on the production account, the SNA discusses basic, producers' and purchasers' prices. Different components of the SAM are valued at different prices, hence the importance of the discussion in the context of this study. Different prices exist because of taxes, subsidies and transport charges. Taxes (or subsidies) can be levied on production or on products; hence output can be measured according to either basic prices or producers' prices. The SNA price definitions are as follows:

"The *basic* price is the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any tax payable, and plus any subsidy receivable, by the producer as a consequence of its production or sale. It excludes any transport charges invoiced separately by the producer." (United Nations 2009, par 6.51)

"The *producer's* price is the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any VAT, or similar deductible tax, invoiced to the purchaser. It excludes any transport charges invoiced separately by the producer." (United Nations 2009, par 6.51)

In comparison to the basic price, the producer price includes tax on products, but excludes subsidies on products. The producer price is therefore the amount, exclusive of value added tax (VAT), paid by the purchaser, whereas the basic price is the amount that the producer retains after paying tax. When transport charges are also taken into account, the purchasers' price is derived, with the following 2008 SNA definition:

"The *purchaser's* price is the amount paid by the purchaser, excluding any VAT or similar tax deductible by the purchaser, in order to take delivery of a unit of a good or service at the time and place required by the purchaser. The purchaser's price of a good includes any transport charges paid separately by the purchaser to take delivery at the required time and place." (United Nations 2009, par 6.64)

Table 1 gives an overview of the main differences between the basic prices, producers' prices and purchasers' prices.

Table 1: Relationship between prices

Basic prices
+
Taxes on products excluding invoiced VAT
-
Subsidies on products
=
Producers' prices
+
VAT not deductible by the purchaser
+
Separately invoiced transport charges
+
Wholesalers' and retailers' margins
=
Purchasers' prices

Source: 2008 SNA (United Nations 2009, par 6.69)

The 2008 SNA recommends that the use of 'market price' is avoided in a system where there is VAT or similar deductible taxes (United Nations 2009, par 6.68), such as the case in South Africa.

Two other price concepts are worth mentioning, namely the free on board (FOB) price for exports and total imports and the cost, insurance and freight (CIF) price for detailed imports, where CIF values include the insurance and freight charges incurred between the exporter's frontier and that of the importer. Imports and exports of goods are recorded in the SNA at border values and these are valued FOB, i.e. at the exporter's customs frontier (United Nations 2009, par 3.149). The CIF price for imports can be likened to the basic price of domestically produced goods and services, whereas the FOB price can be regarded as a purchasers' price that would be "paid by an importer taking delivery of the goods at the exporter's frontier after loading on to a carrier and after payment of any export taxes or the receipt of any tax rebates" (United Nations 1994, par 15.36). FOB values at a detailed product level are not always readily available and the goods are valued at the importer's customs frontier CIF and then supplemented with global adjustments to FOB values (United Nations 2009, par 3.149).

2.2.3 Product balances and T-accounts

The 2008 SNA presents the product balance as follows (United Nations 2009, par 14.4):

output + imports = intermediate consumption + final consumption +
capital formation + exports

The above statement indicates that total supply in an economy can either come from domestic production or imports. For an accounting period demand and supply must equate, and demand or use comprises intermediate consumption, final consumption, capital formation (including changes in inventories) or exports. Taking prices into account, the product balance statement becomes the following (United Nations 2009, par 14.5): "... the sum of output at basic prices plus imports plus trade and transport margins plus taxes on products less subsidies on products is equal to the sum of intermediate consumption, final consumption and capital formation, all expressed at purchasers' prices, plus exports."

The T-account format is the basis for presenting accounts information in the SNA. The goods and services account in Table 2 is an example of information presented in T-account format and shows that for the entire economy the total amount of product available (resources or supply) must be equal to the total amount used. The data is for South Africa for 2005. Given the price definitions in the previous section, it can be mentioned that in the resource column output would be valued at basic prices, imports at CIF prices and total resources at purchasers' prices. In the column of uses, all prices are valued at purchasers' prices. Under ideal circumstances the residual item should go to zero.

The SNA also refers to accounts for the following: production, generation of income, allocation of primary income, redistribution of income, use of income and the capital and financial accounts. These accounts are not shown here, but all of the mentioned accounts in T-account format can be found in SSA (2009d) as a country example for South Africa for 2005.

The T-account format is suitable for presenting aggregate information, but for presenting data at a more disaggregated product level, a matrix format is often desirable. The supply and use tables, input-output tables and SAMs are examples of ways to present information in matrix format.

Table 2: Goods and services account

Resources	R million	Uses	R million
Output	**3 248 151**	**Intermediate consumption**	**1 847 084**
Taxes on products	**175 667**	**Final consumption expenditure**	**1 296 505**
Subsidies on products	**-5 652**	Private consumption expenditure	990 773
Imports of goods and services	**437 559**	Government consumption expenditure	305 732
		Gross capital formation	**282 129**
		Gross fixed capital formation	263 753
		Changes in inventories	18 376
		Exports of goods and services	**430 170**
		Residual item	**-163**
Total resources	**3 855 725**	**Total uses**	**3 855 725**

Source: SSA (2010:9, Table E)

2.2.4 Supply and use tables

When a complete set of product balances are available, a set of detailed supply and use tables can be constructed (United Nations 2009, par. 14.13). The tables are detailed in the sense that the numerous products and industries are identified separately. The supply and use tables must be valued at the same prices, usually purchasers' prices, and both tables must cover the same products and the same industries. Data from different sources are combined in supply and use tables and consistency is ensured because the tables provide a rigorous framework that highlight discrepancies between different flows of goods and services, to facilitate the elimination of these discrepancies. When the product balance holds for every product group included in the supply and use tables, then all the data is consistent with each other. The framework therefore ensures that alternative measures of gross domestic product converge to the same value (United Nations 2009, par. 14.15). The product balances included in the supply and use tables imply that the focus is on the processes of production and consumption respectively.

A framework of a supply table is included in Table 3. The supply table includes the output by every industry, imports and the CIF/FOB adjustments. These three elements add up to give total supply at basic prices. Information on taxes and subsidies are also included. When net taxes are added to total supply at basic price, the total supply at producers' prices can be derived (not typically shown in a supply table, at least not for South Africa). When trade and transport margins are also added, total supply at purchasers' prices are derived. The trade and transport margins were not shown in the goods and services account in the previous section because it sums to zero for the whole economy. Trade and transport margins are therefore only relevant when the product and industry accounts are detailed. As indicated in the table, it is only the production matrix which records the supply at basic prices per industry that is disaggregated into a big sub-matrix containing all the industry groups identified. The row headings are the identified product groups. As mentioned, the respective industry and product groups in the supply table should correspond to those in the use table otherwise the two tables cannot be consistent.

A framework of a use table is included in Table 4. The use table includes intermediate consumption, final consumption and capital formation, all expressed at purchasers' prices, plus exports. The supply and use information can also be captured in one matrix such as the simplified framework set out in Table 5. The shaded areas in Tables 3 to 5 indicate sub matrices that contain no entries.

Table 3: Framework for a supply table

	Industries	Agriculture	Industry	Construction	Trade, hotel, transport	Finance, real estate, business	Other service activities	**Total domestic output at basic prices**	Intra country imports CIF	Extra country imports CIF	Imports CIF	**Total supply at basic prices**	Trade and transport margins	Taxes less subsidies on products	**Total supply at purchasers' prices**
	Products	1	2	3	4	5	6	**7**	8	9	10	**11**	12	13	**14**
1	Products of agriculture	Production matrix						**Domestic output**	Import matrix		Imports CIF	**Total supply at basic prices**	Valuation matrix		**Total supply at purchasers' prices**
2	Products of industry														
3	Construction work														
4	Trade, hotel, transport services														
5	Finance, real estate, business														
6	Other services														
7	**Total**	**Total output of industries at basic prices**							**Imports CIF**				**Total**		
8	CIF/FOB adjustment on imports														
9	Direct purchases abroad by residents														
10	**Total**	**Total output of industries at basic prices**							**Imports FOB**				**Total**		

Source: Eurostat (2008)

Table 4: Framework for a use table

		Input of industries							Final uses									
	Industries	Agriculture	Industry	Construction	Trade, hotel, transport	Finance, real estate, business	Other service activities	**Total**	Final consumption expenditure by households	Final consumption expenditure by non-profit organisations	Final consumption expenditure by government	Gross fixed capital formation	Changes in valuables	Changes in inventories	Exports intra country FOB	Exports extra country FOB	**Total**	**Total uses at purchasers' prices**
	Products	1	2	3	4	5	6	**7**	8	9	10	11	12	13	14	15	**16**	**17**
1	Products of agriculture	Intermediate consumption at purchasers' prices							Final demand at purchasers' prices									
2	Products of industry																	
3	Construction work																	
4	Trade, hotel, transport services																	
5	Finance, real estate, business																	
6	Other services																	
7	**Total**																	
8	CIF/FOB adjustment on imports								Adjustment items									
9	Direct purchases abroad by residents																	
10	Domestic purchases by non-residents																	
11	**Total**																	
12	Compensation of employees	Gross value added at basic prices																
13	Other net taxes on production																	
14	Consumption of fixed capital																	
15	Operating surplus, net																	
16	Gross value added at basic prices																	
17	**Output at basic prices**																	

Source: Eurostat (2008)

Table 5: Simplified supply and use framework

		Products			Industries			Final uses			Total
		Agricultural products	Industrial products	Services	Agriculture	Industry	Service activities	Final consumption expenditure	Gross fixed capital formation	Exports extra country FOB	
Products	Agricultural products				Intermediate consumption by product and by industry			Final uses by product and by category			Total use by product
	Industrial products										
	Services										
Industries	Agriculture	Output of industries by products									Total output by industry
	Industry										
	Service activities										
Value added					Value added by component and by industry						Total value added
Imports		Total imports by product									Total imports
Total		Total supply by product			Total output by industry			Total final uses by category			

Source: Eurostat (2008)

Supply and use tables are the basis from which to develop symmetric input-output tables and SAMs. A supply and use table identifies both products and industries. An input-output table is a reduced form of a supply and use table because it combines the information from both tables into a single table and it loses either the product or the industry dimension. A SAM on the other hand contains even more information than supply and use tables. Input-output tables and SAMs and its relation to supply and use tables are discussed in more depth in the next two sections.

2.2.5 Input-output tables

Input-output tables are derived from use tables and find application in impact analyses based on multipliers. The use table records information by both products and industries, but input-output tables record information by either one of the two, not both. An input-output table will therefore display intermediate consumption as a square matrix, i.e. the rows and columns will have similar headings and information will be shown for either products or industries. Also, the row and column totals of the complete matrix will be equal to each other; hence the matrices are referred to as being symmetric.

Industry classifications typically identify industries according to the main type of goods and services they produce, but there are more products than industries. In some cases similar products can be produced by more than one industry, but it is obviously also possible that some industries can produce more than one product, i.e. there are secondary products.

According to the 2008 SNA there are three types of secondary products (United Nations 2009, par 28.46):

- *Subsidiary products:* "... are technologically unrelated to the primary product. Just a few examples include a large retailer with a fleet of trucks used primarily for its own purposes that may occasionally offer transport services to another unit, a farmer who use part of his land as a caravan site, or a mining company that builds access roads and accommodation for its workers."
- *By-products:* "...products that are produced simultaneously with another product but which can be regarded as secondary to that product, for example gas produced by blast furnaces."
- *Joint products:* "...products that are produced simultaneously with another product that cannot be said to be secondary (for example beef and hides)."

It is the secondary products that need to be dealt with when deriving a symmetric input-output table because theoretically symmetric input-output tables do not allow for secondary products because it assumes every product is produced by only one industry and every industry produces only one product. If this were true, then the supply table would not be necessary, because the main purpose of the supply table is to show secondary production. In the absence of secondary production the supply table would have the same number of rows and columns and the only entries would be on the diagonal. The 2008 SNA proposes two approaches to eliminate secondary products through reallocation. Both approaches apply information from the use matrix to the supply matrix to reduce it to a purely diagonal one. In the process the use matrix is also transformed and it is the transformed use matrix that is referred to as an input-output matrix, while the supply matrix becomes redundant.

With the technology approach a product by product matrix is derived (United Nations 2009, par. 28.48). In this case the matrices that originally had a product dimension (final demand) remain unaltered and the matrices that originally had an industry dimension (intermediate consumption and value added) are adjusted by reallocating entries from one column to another within the given row total. The implicit assumption is that the demand for intermediate consumption and labour and capital inputs are determined by the nature of the products made.

The sales structure approach can be followed to derive an industry by industry matrix (United Nations 2009, par. 28.49). In this case the matrices that originally had an industry dimension (intermediate consumption and value added) remain unaltered and the matrices that originally had a product dimension (final demand) are adjusted by reallocating items of final demand between rows but not between columns. In contrast to the product by product case, the quadrant relating to final demand will change and will show demand related to the industry supplying the products and not to the products themselves. It assumes that as the level of output of an industry changes, the pattern of sales will remain the same.

A product by product use table shows which products are used in the production of which other products. An industry by industry use table shows which industry uses the output of which other industry. The 2008 SNA suggests that there are four options available when compiling input-output tables because for each of the two mentioned approaches there are two possible assumptions that can be made (United Nations 2009, par. 28.62). The SNA acknowledges that hybrid approaches have emerged and literature confirms it, but the discussion here will be restricted to the four options presented in the SNA. Options i and iv may result in negative entries, but not options ii and iii:

i) A product by product approach using a product technology assumption;
ii) A product by product approach using an industry technology assumption;
iii) An industry by industry approach assuming a fixed product sales structure;

iv) An industry by industry approach assuming a fixed industry sales structure.

The 1993 SNA recommended the product by product production structure (United Nations 1994, par 15.150). The 2008 SNA however gives a more detailed discussion of each of the approaches and indicates that "...knowledge of the type of product or industry in question should dictate whether an industry-based conversion procedure or a product-based one is most appropriate". The 2008 SNA acknowledges that the different tables serve different analytical functions, for example a product by product matrix would be preferred when ensuring that price indices are strictly consistent is of particular importance. Industry by industry tables might be preferred if labour market questions are the focus, because the value added information per industry is retained (United Nations 2009, par. 28.63).

Table 6 shows the framework for a product by product input-output table to illustrate the relationship between supply and use tables and input-output tables**.** The shaded areas in indicate sub matrices that contain no entries.

According to the SNA input-output tables are 'analytical constructs' because they cannot be compiled directly from basic data like the supply and use tables (United Nations 2009, par. 28.67). Because the input-output tables are reduced form tables derived with data manipulation, they are associated with loss of information compared to supply and use tables and they do not capture the full circular flow of the economy. Despite these shortcomings input-output tables are still used for multiplier analyses.

Table 6: Framework for a product by product input-output table

		Homogeneous branches							Final uses									
	Industries	Agriculture	Industry	Construction	Trade, hotel, transport	Finance, real estate, business	Other service activities	Total	Final consumption expenditure by households	Final consumption expenditure by non-profit organisations	Final consumption expenditure by government	Gross fixed capital formation	Changes in valuables	Changes in inventories	Exports intra country FOB	Exports extra country FOB	Agriculture	Total uses at basic prices
	Products	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	Products of agriculture	Intermediate consumption at basic prices							Final demand at basic prices									
2	Products of industry																	
3	Construction work																	
4	Trade, hotel, transport services																	
5	Finance, real estate, business																	
6	Other services																	
7	Total at basic prices																	
8	Direct purchases abroad by residents																	
9	Domestic purchases by non-residents																	
10	Taxes less subsidies on products																	
11	Total at purchasers' prices																	

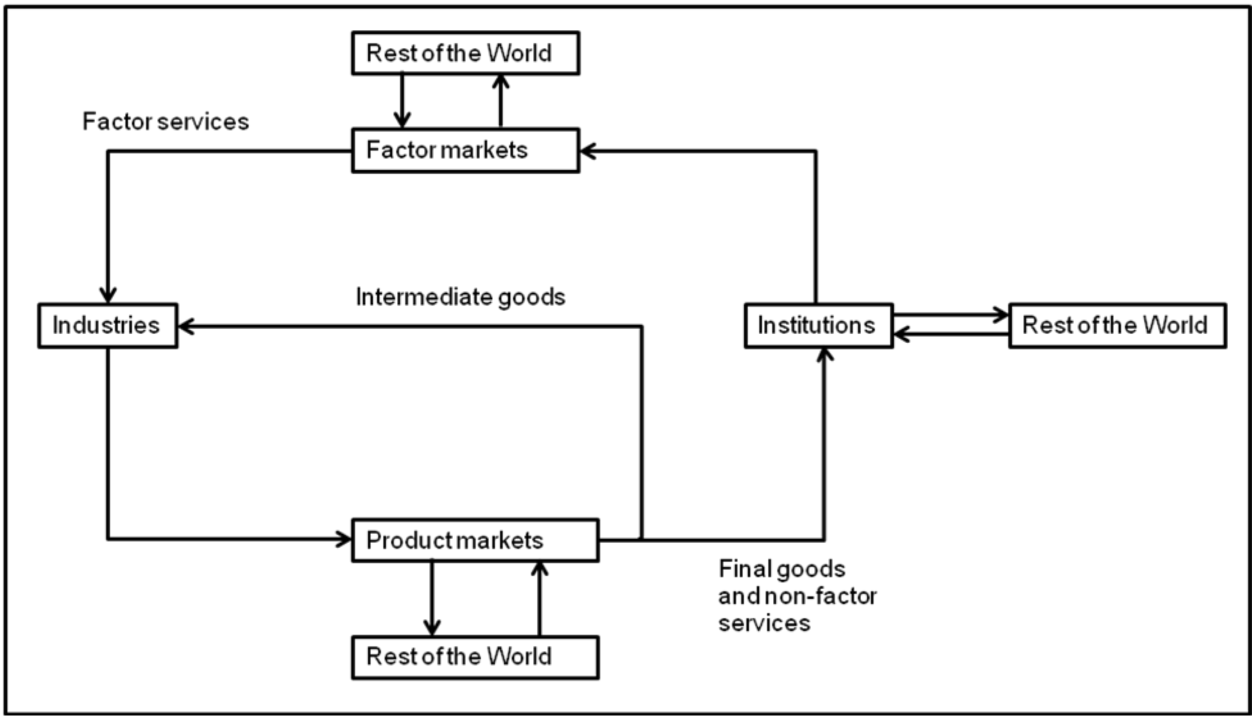
	Industries	Homogeneous branches							Final uses									
		Agriculture	Industry	Construction	Trade, hotel, transport	Finance, real estate, business	Other service activities	Total	Final consumption expenditure by households	Final consumption expenditure by non-profit organisations	Final consumption expenditure by government	Gross fixed capital formation	Changes in valuables	Changes in inventories	Exports intra country FOB	Exports extra country FOB	Agriculture	Total uses at basic prices
12	Compensation of employees	Gross value added at basic prices																
13	Other net taxes on production																	
14	Consumption of fixed capital																	
15	Operating surplus, net																	
16	Gross value added at basic prices																	
17	Output at basic prices																	
18	Imports CIF intra country	Imports CIF																
19	Imports CIF extra country																	
20	Imports CIF																	
21	Supply at basic prices																	

Source: Eurostat (2008)

2.2.6 Social accounting matrices – capturing the full circular flow

Figure 1 indicates the circular flow in the economy. Going one way (in the direction of the arrows), the circular flow represents the flow of goods and services, while going the opposite way the circular flow represents the flow of funds, i.e. the payment for the goods and services, where services include factors services. The SNA in theory contains the accounts for all the components in the diagram below and a SAM is particularly useful in capturing the full circular flow of the economy because it contains all the components in the diagram below in matrix format. According to the 2008 SNA the SAM provides a framework in which to ‘cast the whole of the sequence of accounts’ (United Nations 2009, par 28.4).

Figure 1: Circular flow in the economy



Source: Adapted from McDonald and Punt (2001)

Industries are responsible for production, which entails bringing into being goods and services. In the product markets the industries sell intermediate goods and services to other industries or final consumption goods to institutions. Final goods can either be used or consumed during the accounting period, or the goods can be accumulated for future use. Production, consumption and accumulation are the three basic forms of economic activity in a closed economy. In an open economy local production can be supplemented with imports and final goods can be destined for exports in addition to local consumption and accumulation. Transactions can take place with the rest of the world through both product and factor markets. For products this would be imports and exports, whereas for factor markets these would include citizens of a particular country who are working abroad, or foreigners who are working in the particular country.

Institutions (households, corporations and government) sell factor services in factor markets where industries act as purchasers. Factor services typically include labour, capital and land. Payments to labour, capital and land constitute salaries and wages, returns to capital and returns to land, respectively. The adaptation in Figure 1 compared to the referenced figure is to show that transactions can also take place between institutions and the rest of the world. This would include household remittances and foreign aid/funding.

Supply and use tables give substantial details on processes of production and consumption, but they do not capture the full circular flow in the economy. With reference to Figure 1, the part of the circular flow that is not captured in the supply and use tables is the link between income distribution (factors) and consumption (institutions). In order to capture the full circular flow, Pyatt (1999) identifies three mappings that are required, i.e. the mapping of value added from industries to factors, mapping of factor income to institutions and mapping of income of institutions into demand for goods and non-factor services. In addition, information on transfers between institutions is also necessary. These comprise unrequited transfers, property income transfers of rent, interest and the payment of dividends by the corporate sector to their shareholders. The SNA recognises that supply and use tables do not capture the full circular flow and therefore addresses it in the form of guidelines for the construction of SAMs. The SAM incorporates all the information contained in supply and use tables, but a SAM is an extension of supply and use tables because it explicitly incorporates this missing information. And for the same reason, a SAM can be viewed as an extension of input-output tables.

A SAM is essentially a matrix to present the full sequence of accounts. The SNA presents two versions of the sequence of accounts in matrix format, one displaying only the flow accounts as shown in Table 7 and the other showing all accounts, including the balance sheets. The adaptation in the table compared to the referenced SNA table is that the word ‘account’ has been omitted from all the account headings in Table 7 and each account in the SNA table indicates that it comprise a local and a ‘rest of the world’ component. These two components for each account are not shown in Table 7, but are implied. The matrix presents output, intermediate consumption, value added, national income, disposable income, savings etc. The structure also allows some flexibility as to whether these values are presented as net or gross values, depending on whether consumption of fixed capital is separate or included in these values.

Table 7: The SNA flow accounts in matrix form

	Goods and services	Production	Primary distribution of income	Secondary distribution of income	Use of income	Capital	Financial	Totals
Goods and services	Imports and exports	Intermediate consumption			Final consumption	Gross capital formation		
Production	Output							
Primary distribution of income		Value Added	Property Income					
Secondary distribution of income			Balance of primary income	Current transfers				
Use of income				Disposable income	Change in pension entitlements			
Capital		Consumption of fixed capital			Saving	Capital Transfers		
Financial						Net borrowing or lending		
Totals								

Source: Adapted from 2008 SNA Table 28.10 (United Nations, 2009)

Round (2003) highlights that one can observe the original four blocks of accounts proposed by Stone, namely production, consumption, accumulation and the 'rest of the world'. As Round (2003:7) puts it, the structure also shows that "...the balancing items for the accounts are recorded in a natural step-wise fashion as income 'cascades' from one account to the next in sequence.". The full circular flow of income is captured in the matrix proposed by the SNA and hence the matrix can be regarded as a SAM.

2.2.7 Satellite accounts

The SNA distinguishes two types of satellite accounts, each serving a different purpose (United Nations 2009, par 29.85):

- An internal satellite account takes the full set of accounting rules and conventions of the SNA but focuses on a particular aspect of interest by moving away from the standard classifications and hierarchies. Examples include tourism and environmental protection expenditure.
- An external satellite may add non-economic data or vary some of the accounting conventions or both.

One of the main aims of satellite accounts is to make the SNA flexible for different countries to be able to focus on country specific issues, i.e. issues that all countries are unlikely to be able to address in a uniform manner. All satellite accounts need to be consistent with the main SNA, but they do not have to be consistent with each other. The SNA actually refers to a SAM as a satellite account, because the SAM requires detailed information in addition to the information contained in the supply and use tables such as detailed household information, i.e. it is an extension of the central SNA. In the disaggregation of households the numbers are no longer based on rigorous accounting but often on information contained in a single household survey.

As indicated, external satellite accounts could include non-economic data or physical units of assets, such as area of land, number of people employed, etc. A SAM can thus also be supplemented by data in a satellite accounts. In the context of a SAM the physical quantities, of for example employment by industry, can appear as separate tables or external matrices. The satellite account will be consistent with the SAM insofar as having consistent industry categories as the SAM, but the SAM will capture the values of the corresponding transactions, whereas the satellite account will capture only the quantity information.

2.2.8 Classification systems

The SNA refers to various classification systems but it also allows for countries to use their own classification systems. Functional classifications, which refer to classifications to analyse consumption by different sectors according to the purpose for which the expenditure is undertaken, include (United Nations 2009, par. 29.9):

a. **C**lassification **O**f **I**ndividual **CO**nsumption by **P**urpose (COICOP);

b. **C**lassification **O**f the **F**unctions **O**f **G**overnment (COFOG);

c. **C**lassification **O**f the **P**urposes of **N**on-profit **I**nstitutions serving households (COPNI);

d. **C**lassification of **O**utlays of **P**roducers by **P**urpose (COPP).

The International Standard Industrial Classification of All Economic Activities Revision 4 (ISIC, Rev 4) is used to group together establishments that have the same principal activity into industries (United Nations 2009, par. 2.39). In South Africa the Standard Industrial Classification (SIC, Fifth Edition) (CCS, 1993) is used for industry classification and since 2009 the Central Product Classification (CPC, Version 2) is used for product classification. It should be noted that the SIC and CPC do not link completely as do the ISIC and the CPC (SSA, 2012) The Harmonised System (HS) published by the U.S. International Trade Commission (ITC) and adjusted by the South African Revenue Service (SARS), is used to classify traded goods in South Africa.

2.3 The SAM as a database

Sections 2.3.1 and 2.3.2 present some aspects of the SAM structure from the perspective of using the SAM as a base for modelling. In section 2.3.3 some of the main differences between the SAM structures for purposes of national statistics as opposed to economic modelling are highlighted.

2.3.1 From double to single entry bookkeeping

In simplest terms a SAM is a database that captures the value of transactions between different agents in the economy of a country or region during a specific period in square matrix format. The period of a SAM usually refers to one year. A SAM has a square format because it includes a row account and a corresponding column account for every agent identified. The rows record incomes and the columns record expenditures, representing principles of double entry book keeping (Pyatt 1988, 1991; Robinson, 2003). The main difference compared to traditional accounts is that in the SAM every entry only appears once. Hence the entry t in the i^{th} row and j^{th} column is simultaneously the expenditure by the j^{th} account on the product of the i^{th} account and the income to the i^{th} account from sales of its product to the j^{th} account. If *y* is the account total and by definition for every account the row and column totals must equate, then:

$$y = \sum_j t_{ij} = \sum_i t_{ij} \tag{1}$$

2.3.2 Accounts of the SAM

Pyatt (1991) names institutions, assets and transactions as the three fundamental elements of social accounting. Examples of institutions include households, government and companies. Institutions can a) own assets and incur liabilities and b) engage in transactions. A SAM should have a particular classification system which is “mutually exclusive and exhaustive of all the institutions in the society” (Pyatt, 1991), in other words all institutions in the economy should be included, but included only once, in the SAM. Assets can include real assets (real capital, natural resources, and human resources) and financial claims (mortgages, equity, loans, securities, etc.). It is only the payments towards the use of assets during the accounting period that is captured in the SAM, not the full value of the assets. There are SAMs discussed in the literature that incorporate financial accounts, but financial accounts are not essential in SAMs and the inclusion thereof would be dictated by the purpose for which the SAM was developed. Transactions are captured in a SAM and the SAM indicates between which two institutions the transaction has taken place and the monetary value of the transaction.

Pyatt (1991) indicates that a SAM without accounts for real assets and financial claims provides a sufficient information base and conceptual framework for a wide range of analytical work. Also, Robinson (2003) indicates that there are theoretical tensions between working with flows and working with assets. Hence, Pyatt (1991) derives a SAM with six main account groups. The six types of SAM accounts are discussed here in more detail. The order in which the groups of agents appear in a SAM is irrelevant. The order followed in the discussion represents the order of the SAM used for purposes of the dissertation and it also corresponds in principle to the layout of the SAM derived by Pyatt (1991). Each of the main types of accounts may contain numerous (sub) accounts and the extent to which these accounts are detailed/disaggregated in the SAM depends on the intended focus of the analyses that will be conducted using the SAM; and in practice it also often depends on data availability. The account types discussed here relate to the flow of funds accounts of the SNA, i.e. the current accounts and to a limited extent also the accumulation accounts.

Product accounts

The product or commodity accounts record the goods and services that are supplied during the accounting period (Pyatt, 1991). Goods and services constitute intermediate demand if it is used in another production process in the accounting period. Alternatively it will be regarded as final demand if it is consumed by institutions, exported, or recorded as part of stock changes. The SNA allows for three types of goods: market, own final use and non-market goods and services (United Nations 2009, par 2.36). Market goods are automatically included in the SNA, because they have monetary values. When goods or services are produced for own final use, it implies that no transactions take place with other units. In these cases internal transactions have to be recorded whereby producers allocate the goods or services for their own consumption or capital formation and values have to be estimated for these products (United Nations 2009, par 1.37). Non-market goods and services (bartered or transfers) should also be included although they do not have monetary values determined by the market and in order to do so the value of these goods also have to be estimated (United Nations 2009, par 1.36).

Production/Industry accounts

The production or industry accounts record the cost of production in the columns and the revenues from products in the rows. The costs include the cost of intermediate goods and services, as well as factor services. Purchases that are used up during the production process are recorded as part of intermediate goods, whereas changes in stocks are recorded as part of final demand (Pyatt, 1991). Payments to factor services such as capital, labour and land, are also referred to as value added at basic prices. The information in the columns of the industry accounts can therefore be used in the calculation of the gross domestic product (GDP). The SNA uses a unit of production called an establishment in order to allow for principal production and secondary production (United Nations 2009, par 2.38). In order for goods and services to be regarded as outputs in the SNA these goods and services must be such that they can be sold on markets or transferred between agents in the economy, regardless of whether they are sold or given away. The production boundary, which indicates what is regarded as output for purposes of the SNA, includes all production destined for the market, irrespective of whether it is sold or exchanged, as well as all goods or services provided without charge to individual households or communities (United Nations 2009, par. 1.40).

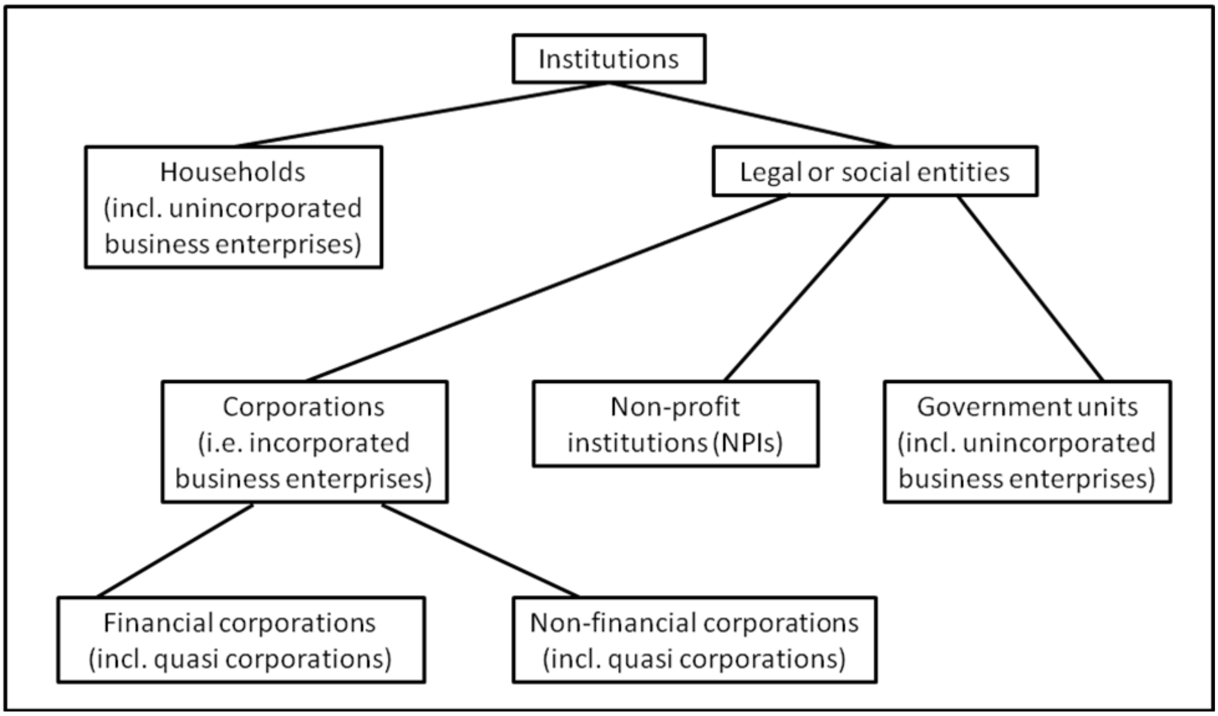
Factor accounts

The factor accounts typically include labour and capital, and sometimes land is also distinguished. These factors represent assets owned by institutions and the accounts capture the payments for the use of these assets, which can be identified as returns to land (use of natural resources), wages for labour (services provided by human capital) and returns to capital e.g. rentals on buildings and machinery (services of real capital) (Pyatt, 1991). When a service is provide by an asset of which the institution is the owner, then the transaction is regarded as an imputed transaction. For example if a farmer owns the land it is farming on, the cost of this should be explicitly taken into account through adding imputed income both as a factor income and payment. The payment will be by the agricultural industry towards the production factor 'land' and the income will accrue to the institution (household or corporation) who owns the land. In the case of capital that is owned, the imputed value of the services of real capital will be given as profits to the household or corporations who own the capital and the payment will flow from the industry which is using the capital towards the production factor 'capital' (Pyatt, 1991).

Current accounts of domestic institutions

According to the 2008 SNA "an institutional unit is an economic entity that is capable, in its own right, of owning assets, incurring liabilities and engaging in economic activities and in transactions with other entities." (United Nations 2009, par. 4.2) Four main institutional sectors are identified, namely corporations, non-profit institutions (NPIs), government and households as shown in Figure 2.

Figure 2: Framework of institutional sectors



Source: Own diagram based on information in the 2008 SNA

The nature of the economic activity undertaken by an institutional unit determines in which sector the institution is grouped. The three basic economic activities recorded in the SNA are production of goods and services, consumption to satisfy human wants or needs and accumulation of various forms of capital. According to the 2008 SNA corporations undertake either production or accumulation (or both) but do not undertake (final) consumption; government undertakes production (but mainly of a different type from corporations), accumulation and final consumption on behalf of the population; all households undertake consumption on their own behalf and may also engage in production and accumulation; whereas NPIs are diverse in nature, with some acting like corporations, some being part of government and some undertaking activities similar to government but independently of it (United Nations 2009, par 4.17). The domestic institutions include households, non-profit institutions (NPI's) serving households (NPISH's), corporations and government. Except for households, all other institutions are legal or social entities.

Legally constituted corporations are also referred to as incorporated enterprises and these are often divided between financial corporations that mainly provide financial services, and non-financial corporations that mainly provide goods and other services.

The 2008 SNA also refers to quasi-corporations, which are unincorporated enterprises owned by institutional units that have sufficient information to compile a complete set of accounts and is operated as if it were a separate corporation and whose relationship to its owner is that of a corporation to its shareholders. Unincorporated enterprises also include unincorporated partnerships or trusts, owned by households that are operated as if they were privately owned corporations (United Nations 2009, par 4.42). Quasi-corporations are treated as separate institutional units from its owners and are not included in the household sector, but are included in either the financial or non-financial corporations sector (United Nations 2009, par 3.104). Many commercial farming operations would be regarded as quasi-corporations in the non-financial corporations sector, whereas subsistence farming would be classified as unincorporated business enterprises under the household sector. As part of the production process an agricultural industry will pay for the use of capital as production factor. These payments will be captured in the gross operating surplus (GOS) row accounts. In the case of unincorporated business enterprises under the household sector, the returns to capital will flow from the GOS account to be redistributed directly to households. In the case of quasi-corporations the returns to capital will flow from the GOS account to the corporations account, upon which it will then be redistributed to households. In both cases the distribution to households will be determined by patterns of capital ownership.

The current account captures all transactions during the relevant period, excluding transactions related to assets (Pyatt 1991). Transfers are also recorded in the current accounts of institutions. Payments recorded in the current accounts therefore refer to consumption.

Capital accounts of domestic institutions

The capital account captures all asset related transactions, including the part of purchases that relates to purchases of products that are not entirely consumed during the relevant period, i.e. transactions involved in goods produced in previous periods, natural resources and financial claims (Pyatt, 1991). Payments recorded in the capital accounts therefore refer to investment and will add to the stocks of physical assets in the economy.

Rest of the world accounts

The ‘rest of the world’ account reflects transactions between the rest of the world and the domestic institutions. It records international trade in goods and services, factor income received or paid abroad, as well as transfers between international and domestic institutions. The domestic economy relates to our geographic economy (Pyatt, 1991). Global CGE models would have more detailed ‘rest of the world’ accounts to distinguish between different trading partners, whereas single country CGE models would typically have only one ‘rest of the world’ account and more detailed household and factor accounts to trace the income distribution effects within the country.

2.3.3 SAM structure

Round (2003) captures the difference between the structure of SAMs typically used for CGE modelling and the SNA SAM structure in Table 7. The SNA SAM framework focuses more on the process of income transmission between institutional sectors, through primary and secondary income distribution accounts and the use of income accounts. In most SAMs for CGE modelling these various elements of income redistribution between sectors are compressed into a single sub-matrix and property income and current transfers are included in one set of cell entries. As Robinson (2003:2) puts it, a SAM is a 'compact' way to present the national accounts and to trace the circular flow from production industries, to factor payments, to income of institutions, back to demand for products produced by industries. The SAM structure generally used for CGE applications is shown and discussed in more detail in this section.

Table 8 is a representation of a SAM which broadly follows the structure proposed by Pyatt (1988) and which is representative of the SAM developed for purposes of the dissertation. The layout proposed by Pyatt more closely follows the structure proposed by the 1968 SNA and is widely used for SAMs that are used for modelling as opposed to only as a statistical database. Pyatt (1994) places it in perspective by indicating that the 1993 SNA incorporates the SAM as the subject of a special chapter, whereas in the 1968 SNA the SAM provided the organising framework within which the system was conceived.

Comparing the accounts from Table 7 and Table 8 it can be seen that the first three accounts are similar (products, industries and factors/value added). In Table 8 the institutional accounts each appear only once (comparable to the use of income account in Table 7); hence there are no accounts for the allocation of primary income or for the distribution of secondary income. In Table 8 the primary income is directly transferred to the institutions as reflected in the rows of the accounts, whereas institution expenditure (use of income) is reflected in the columns of these accounts. This difference between the two frameworks holds significant implications for policy analyses, with the framework in Table 8 being suitable to trace income distribution effects, whereas the framework in Table 7 is not suitable because of the absence of a mapping between the factor and institutional accounts. With reference to domestic institutions in Table 8, the non-profit institutions are captured under the household accounts and the financial and non-financial corporations are collectively referred to as corporations.

The capital, fixed capital formation and financial accounts in Table 7 are combined into a single account in Table 8, and similarly the current and capital accounts for the rest of the world in Table 7 are combined into a single account in Table 8, but a SAM framework suitable for CGE modelling could still maintain the disaggregation of the capital and financial accounts, as well as disaggregation of current and capital 'rest of the world' accounts. These are therefore not fundamental differences with regard to the SAM frameworks proposed in Tables 7 and 8.

Pyatt (1988) identifies the disaggregation of the household account without corresponding disaggregation of the factor account as one of the reasons why many models do not reveal desirable results on the effect of policy on income distribution. The two-stage mapping, first and second stage distribution of income, proposed by the 1993 SNA does not allow for detailed analyses of distributional implications of policies because there is no direct detailed mapping between labour categories and household categories, i.e. it is not clear which factor is owned by which household. This is because the first and second stage distribution of income accounts each contains only a single household account. It is only in the use of income that the household account is further disaggregated. Factor income 'payments' to households are therefore aggregated to a single household in the allocation of primary income account and the information on which factor belongs to which household is not explicit. This is not the case in the framework in Table 8 where the mapping is explicit. By omitting the detailed information the data can be more accurate and hence the more aggregate framework contributes to the objective of national statistical agencies to ensure that data from different sources are consistent, but in the absence of the detailed data the SAM is less suitable for modelling of issues related to redistribution.

Table 8: Schematic of a framework of a SAM to calibrate a model

	Products	Industries	Factors	Household	Corporations	Government	Capital	Rest of World	Totals
Products	Marketing margins	Use matrix		Household consumption		Central government expenditure	Investment expenditure and stock changes	Exports of goods & services	Product demand
Industries	Supply matrix								Production
Factors		Remuneration of factors						Factor income from RoW	Incomes to factors
Households			Distribution of factor incomes	Inter household transfers	Distribution of corporation income	Transfers to households		Remittances from RoW	Household income
Corporations			Distribution of factor incomes			Transfers to corporations		Corporation income from RoW	Corporation income
Government	Taxes less subsidies on products	Taxes less subsidies on production	Factor taxes	Hhold income tax & transfers to government	Ent income tax & transfers to government			Current transfers from RoW	Government income
Capital			Depreciation	Household savings	Corporation savings	Government savings	Total stock changes	Capital account balance	Savings
Rest of World (RoW)	Imports of goods & services		Factor payments to RoW	Remittances to RoW	Corporation payments to RoW	Current transfers to RoW		Re-exports	Imports of g&s from RoW and transfers to RoW
Totals	Product supply	Cost of production	Expenditure on factors	Household expenditure	Corporation expenditure	Government expenditure	Investment expenditure	Exports of g&s to RoW and transfers from RoW	

Source: Adapted from McDonald and Punt (2001)

2.3.4 Supply and use SAM, input-output SAM and prices

The layout of the SAMs in Tables 7 and 8 conform to the class of supply and use SAMs, each with separate product and industry accounts. There are currently many examples of supply and use SAMs in the literature and the model changes suggested in this dissertation are particularly relevant to supply and use SAMs that can explicitly capture the production of multiple products by industries.

The SAM reports values, each of which can be thought of as a price multiplied by a quantity. The prices adopted in a SAM must be consistent with the price system of the SNA in order for the accounting identities to hold, because the SAM is a representation of the SNA. It has been mentioned that the supply table includes the output valued at basic prices, imports, trade and transport margins and net taxes on products. The use table includes intermediate consumption, final consumption and capital formation, all expressed at purchasers' prices, plus exports. Since the SAM uses the information from the supply and use tables, similar prices apply to the relevant sub-matrices in the SAM.

In the supply and use SAMs the structure allows that the supply matrix shows explicitly whether only main product outputs of industries are recorded, or whether multiple or secondary products are recorded. When secondary products are recorded, these reflect as entries on the diagonal and off the diagonal of the supply matrix as shown by the bold figures in the hypothetical aggregated SAM in Table 9. The row entries of the product accounts maintain the law of one price (see section 3.4.2) and all these transactions are valued at purchasers' prices. The outputs captured in the supply matrix are valued at basic prices because product taxes and imports are kept separate in the column for products.

Table 9: Supply and use SAM recording secondary products

		Products		Industries		Factors		Hholds		Govt	Capital	RoW	Total
		Agric	Non	Agric	Ind	Lab	Cap	Rural	Urban				
Products	Agricultural			40	70			50	70	40	15	85	370
	Non-agric			50	150			130	60	65	55	50	560
Industries	Agricultural	**295**	**25**										320
	Industry	**25**	**390**										415
Factors	Labour			120	115								235
	Capital			100	70								170
Households	Rural					145	90						235
	Urban					90	80						170
Government		20	25	10	10			25	15				105
Capital								30	25			15	70
Rest of World		30	120										150
Total		370	560	320	415	235	170	235	170	105	70	150	

Source: Own

It is also possible that supply and use SAMs only capture main products produced in the supply matrix, i.e. only entries on the diagonal as shown by the bold figures in Table 10. The inclusion of multiple products depends on the purpose of the SAM and the format of available data.

Table 10: Supply and use SAM recording main products only

		Products		Industries		Factors		Hholds		Govt	Capital	RoW	Total
		Agric	Non	Agric	Ind	Lab	Cap	Rural	Urban				
Products	Agricultural			40	70			50	70	40	15	85	370
	Non-agric			50	150			130	60	65	55	50	560
Industries	Agricultural	**320**											320
	Industry		**415**										415
Factors	Labour			120	115								235
	Capital			100	70								170
Households	Rural					145	90						235
	Urban					90	80						170
Government		20	25	10	10			25	15				105
Capital								30	25			15	70
Rest of World		30	120										150
Total		370	560	320	415	235	170	235	170	105	70	150	

Source: Own

Analogous to an input-output table that is derived from supply and use tables as discussed in section 2.2.5, an input-output SAM is a reduced form of the supply and use SAM. Deriving a supply matrix with the same number of accounts for products and industries and no off-diagonal entries, such as the SAM shown in Table 10, is the first step towards deriving and input-output SAM. Once the secondary products have been reallocated and the products and industries have the same number of accounts, then the supply matrix becomes redundant and the use matrix becomes the input-output matrix. The resultant SAM will then record production either by product or by industry.

Table 11 shows a schematic representation of the product by product input-output SAM. In the product by product input-output SAM the row entries of the product accounts are still in purchasers' prices, similar to the row entries of the products account of the supply and use SAM. The row totals of the sub-matrices of the product columns of the input-output SAM are still the same as the row totals of the sub-matrices of the industry columns of the supply and use SAM, but the column totals are different, because the production costs of secondary products are allocated to main industries producing those products. This implies that the figures for intermediate expenditure, remuneration of factors and taxes and subsidies on production will adjust, but taxes on subsidies on products and imports will remain the same as in the supply and use SAM.

Table 11: Product by product input-output SAM

	Products	**Factors**	**Institutions**	**Capital**	**Rest of World**	**Totals**
Products	Input-output matrix at **purchasers' prices**		Consumption at **purchasers' prices**	Investment expenditure & stock changes at **purchasers' prices**	Exports at **purchasers' prices**	Product demand at **purchasers' prices**
Factors	Remuneration of factors				Factor income from RoW	Incomes to factors
Institutions	Taxes less subsidies on **products and production**	Distribution of factor incomes	Direct taxes & transfers		Remittances and transfers from RoW	Institution income
Capital		Depreciation	Savings	Total stock changes	Capital account balance	Savings
Rest of World (RoW)	Imports at **purchasers' prices**	Factor payments to RoW	Remittances and transfers to RoW		Re-exports	Imports of g&s from RoW and transfers to RoW
Totals	Product supply at **purchasers' prices**	Expenditure by factors	Expenditure by institution	Investment expenditure	Exports of g&s to RoW and transfers from RoW	

Source: Own

There are some interpretational differences between the product by product SAM in Table 11 and the industry by industry SAM in Table 12, as reflected in the descriptions of the sub-matrices. One of the main differences is that the values in the product row and column of the product by product SAM are recorded at purchasers' prices, whereas the values in the industry row and column of the industry by industry SAM are recorded at basic prices.

In Table 12 the industry row records intermediate demand by industries, final consumption of domestically produced goods and services by institutions, investment expenditure on domestic goods and services, and exports. These figures are now all valued at basic prices and the figures for final demand will further adjust because they are now recorded by industry and no longer by product. Only remuneration of factors remains exactly the same as in the supply and use SAM.

Another main difference is the treatment of imports. The demands for imports are recorded in the row of the 'rest of the world' account, with imports by industries recorded separately from final consumption imports by institutions and investment expenditure on imported goods. This implies that the industry by industry input-output SAM does not record the imports for each product separately, but only the total imports by industries and institutions respectively. From a modelling perspective this is not desirable because this format of presenting the imports information does not allow for substitution between different imported products.

The rows of the tax accounts record all taxes less subsidies on production by industries. This component will remain the same as in the supply and use SAM, because it is still recorded by industry. The treatment of taxes and subsidies on products are however affected. The industries column account will also reflect the part of taxes less subsidies on products which are due on intermediate goods and services used by industries. Taxes less subsidies on final consumption, investment and exports are recorded in the columns for the institution, capital and 'rest of the world' accounts, respectively. The taxes on products therefore appear in different sub-matrices compared to the product by product input-output SAM, where it is all found in the column for products.

Table 12: Industry by industry input-output SAM

	Industries	**Factors**	**Institutions**	**Capital**	**Rest of World**	**Totals**
Industries	Input-output Matrix at **basic prices**		Domestic consumption at **basic prices**	Investment expenditure on domestic goods and stock changes at **basic prices**	Exports at **basic prices**	Total production at **basic prices**
Factors	Remuneration of Factors				Factor Income from RoW	Incomes to Factors
Institu-tions	All taxes less subsidies on production and **taxes less subsidies on intermediate products**	Distribution of Factor Incomes	Direct tax, transfers and **taxes less subsidies on final products consumed**	**Taxes less subsidies on investment products**	Remittances and Transfers from RoW and **taxes less subsidies on products exported**	Institution Income
Capital		Depreciation	Savings	Total Stock Changes	Capital Account Balance	Savings
Rest of World (RoW)	**Imports by industries at basic prices**	Factor Payments to RoW	**Imports by institutions at basic prices** & remittances and transfers to RoW	**Investment expenditure on imported goods at basic prices**	Re-exports	Imports of G&S from RoW and transfers to RoW
Totals	Cost of production at **basic prices**	Expenditure by factors	Expenditure by institution	Investment expenditure	Exports of G&S to RoW and transfers from RoW	

Source: Own

From a perspective of price, the 2008 SNA indicates that product by product input-output matrices are preferred because in the product by product matrices price indices are strictly consistent. However, the industry by industry input-output SAMs have the advantage of recording value added by industry. So the choice between product by product input-output SAMs and industry by industry input-output SAMs depends on the purpose for which the SAM is constructed.

One of the main drawbacks of the input-output SAM is that the details in terms of which industries produce which products are lost in the reduced form input-output SAM; therefore no benefit will be derived from the model extensions discussed in this study if the CGE model is based on an input-output SAM. Reduced form input-output SAMs still find application in multiplier models and CGE models.

2.3.5 SAM estimation

The process of compiling a SAM results in a matrix of first estimates for transaction values in the SAM, but the SAM will most likely not meet the necessary accounting constraints of a SAM, e.g. that each account must balance. This is a common problem in constructing SAMs because of incomplete and inconsistent data sources. Various techniques exist to estimate the missing information in order to obtain a balanced SAM that can be used for modelling. The objective is to determine a SAM that meets the necessary accounting constraints that is as close as possible to the prior data. Round (2003) gives an overview of some of these methods, with the main formal methods used in practice being RAS, Stone-Byron and cross entropy. According to Van Schoor (PROVIDE, 2006) different methods measure the difference between the estimated and the prior data in different ways. Not all methods explicitly measure the difference, but for those that don't, there is always an equivalent method that minimises a difference measure. For example, the common RAS method's implicit difference measure has been shown to be equivalent to a specific formulation that is based on cross entropy.

Round (2003) identifies two types of balancing problems. The first is found when balancing and updating input-output tables when the row and column constraints are known, whereas the other problem arises when balancing SAMs when some of the row and column totals are also unknown. The RAS method is typically more suited in the first case and the Stone-Byron, cross entropy and generalised cross entropy methods are more often used in the second instance.

RAS Method

According to Round (2003) the RAS technique, which involves the application of row and column multipliers, is more suitable for balancing sub-matrices of a SAM than for balancing a SAM. In an effort to address some of the shortcomings of the RAS technique an extension of the method to accommodate uncertainty in the row and column totals and negative elements have been proposed by Gunluk-Senesen and Bates (quoted in Round, 2009:19). Also the Diagonal Similarity Scaling method was developed by Schneider and Zenios (quoted in Round, 2003:20) and it appears to be suitable for SAM scaling, but it relies on scaling adjustments across whole rows and down whole columns which is undesirable because it distorts the technology relationships presented in the SAM. It will be seen in section 3.4.2 that the column coefficients of the SAM capture the interdependence in the economy and the preservation of this information is very important. For a mathematical presentation of the methods mentioned here, see Round (2003).

Stone-Byron Method

According to Round (2003) the earlier versions of the Stone-Byron method was suggested by Stone, Champerdown and Meade in 1942 and later formalised in the SAM context by Stone in 1977. The Stone-Byron method is more flexible than the RAS method because it is suitable in cases where there are linear constraints between the elements of the SAM, which can either be that some of the row and column totals are unavailable, or linear restrictions on sums of subsets of elements, or restrictions on ratios of elements. The Stone-Byron minimand, in cases where all restrictions are linear, is the following:

$$L(X^*:X,V)=\sum_{ij}(x_{ij}^* - x_{ij})^2 / v_{ij} \tag{2}$$

Where X is the initial estimate SAM and X^* is the SAM that satisfies the constraints; X and X* can be expressed as ordered elements of x and x* and then V is the variance-covariance matrix associated with the vector x_{ij}, whereas v_{ij} is analogous to the variances of the elements.

The method has desirable properties such that the coefficients of variation can be chosen based on the perceived reliability of the different data components and the subjectivity enters through the tolerance factors rather than the estimates themselves. The Stone-Byron method can also be extended to cases where non-linear restrictions are imposed.

The Cross Entropy Method

The cross entropy method by Robinson, Cattaneo and El-Said (quoted in Round, 2003:22) has a minimand based on the derivation of a coefficient structure of the SAM (A^*) and it includes the estimation of a set of error weights (w_{ij}) which is part of the generation of error variables (e_i). The initial column coefficients (A) are used rather than the transaction flows (X^*) as shown in the following equation:

$$L(A^*,W:A)=\sum_{i,j} a_{ij}^* \ln\left(a_{ij}^* \big/ a_{ij} \right) + \sum_{i,h} w_{ij} \ln(nw_{ih}) \tag{3}$$

The error variables (e_i) are not included in the minimand, but its purpose is to ensure that corresponding row and column sums balance. The error weights and error variables are part of a constraint set that ensures that accounting relationships between coefficients and flows are maintained in addition to the ordinary accounting constraints (Round, 2003).

The Generalised Cross Entropy Method

The generalised cross entropy method (GCE) by Golan, Judge and Miller (quoted in PROVIDE, 2006:56) was used in the development of the PROVIDE project SAM for South Africa for the base year 2000. The method assumes that values exist for the data that is being estimated, but that the values are subject to various kinds of (unspecified) measurement error. The GCE method uses a prior for each value that is estimated and in addition it also makes use of the characteristics of the measurement errors generated as part of the process. In this way it can compare the measurement errors against the estimates. Error distributions are then estimated that can explain the difference between the measured values and the estimated values that are determined as part of the process. Detailed prior information regarding the measurement process can therefore be incorporated into the estimates. The method also allows that better quality data can be given greater weight than lower quality data and prior information about row and column totals and various ‘macro’ aggregates can be included to improve the accuracy of the estimation.

Method of choice

Round (2003:23) indicates that Robinson, Cattaneo and El-Said favour the cross entropy method, but warns that when carrying out experiments to determine which method is most favourable, the criteria for success are related to the choice of minimand and hence there is a bias built into the experiment. Round (2003:24) indicates that the Stone-Byron method has certain advantages over other methods and mentions as examples the ability to incorporate judgement on the reliability of different date sources and to accommodate initial multiple estimates. The revised version of the generalised cross-entropy method used in the PROVIDE project (PROVIDE, 2006) however also allows for these features. The generalised cross entropy method is therefore used in the development of the SAM for purposes of this study. The choice of method for this study was therefore partly determined by the fact that GAMS code for the generalised cross entropy method was available to the author, and partly because there is enough evidence in the literature to suggest the acceptability of this method for the purpose of balancing a SAM in practice. The author agrees with Fofana, Lemelin and Cockburn (2005:8): “... that all practitioners should maintain a strong element of the judgment approach and not see any SAM-balancing program as a magic solution.” In this study therefore the generalised cross entropy method was not mechanically applied, but iteratively in an attempt to explore initial data inconsistencies and correct those where possible (see Chapter 5).

2.4 Summary and conclusions

This chapter explored the theoretical aspects of social accounting. It showed the relationship between product balances, T-accounts, supply and use matrices, input-output tables, SAMs and satellite accounts as set out in the SNA. The supply and use matrices for South Africa form a core component of the SAM developed for purposes of this study (see Chapter 5). The matrix framework for capturing the sequence of accounts as proposed by the SNA illustrates the suitability of the SAM framework to ensure that data contained in the national accounts are complete and consistent. The SAM is complete in the sense that it is capable of capturing the full circular flow of the economy, which is a distinctive feature of a SAM.

The price definitions proposed by the SNA are also discussed, with the main prices referred to being basic, producers' and purchasers' prices. Adhering to price definitions is important to ensure that the different entries in the SNA tables are valued correctly. Prices are therefore also important in the valuation of the different sub-matrices of the SAMs since these relate to the price definitions and price system supported by the SNA.

The SAM framework as proposed by the SNA ensures that national accounts data are complete and consistent, but the SAM framework can also be used as a basis for economic modelling. It is shown that the SAM structure proposed by the 1993 and 2008 SNA's is not particularly suitable for this purpose. In the section on the SAM as a database, the SAM structure commonly used to calibrate a general equilibrium model is discussed and it is this structure that was used for the SAM for South Africa for 2007. The section highlights the importance of having an explicit mapping between the factor and household accounts if distributive implications of policy options need to be analysed.

The discussion on supply and use SAMs compared to input-output SAMs illustrates that the derivation of a reduced form SAM such as the input-output SAM is accompanied by a loss of information. The implications of this loss of information are for example that an industry by industry input-output SAM is less suitable for modelling trade issues because the treatment of imports does not allow for substitution between different imported products since products are not separately identified. The loss of information in input-output tables specifically relate to information on secondary production by industries, the inclusion of which is of particular importance in the current study. The SAM for calibration of the CGE model in this study therefore needs to be a supply and use SAM in order for the model changes to have any effect on results.

The section concludes with comments on different techniques to estimate missing information to balance the SAM when compiling a SAM from various primary and secondary data sources. The generalised cross entropy method was used in this study. The SAM balancing ensures that the SAM is consistent, i.e. that incomes and expenditures of the respective agents and markets equate. The SAM as basis for a model and the theoretical aspects of CGE models are discussed in more detail in the next chapter.

3 Theory of Computable General Equilibrium (CGE) models

3.1 Introduction

This chapter gives a short overview of the concept of general equilibrium and the different types of CGE models as it relates to coverage and adjustment time frames. The theoretical aspects of SAM based modelling and the functional forms used in CGE models in general are discussed. It is found that the Constant Elasticity of Substitution (CES) and the Constant Elasticity of Transformation (CET) functions are often used and hence the theory of these two functions is presented in more detail. The chapter also provides an overview of the price and quantity relationships and the functional forms used in the base CGE model. The focus of the chapter is such that it sets the basis for the discussion of the proposed adjustments to the base CGE model in the following chapter. By way of introduction, some concepts often encountered in CGE literature are explained.

According to Chiang (1984:37) equilibrium for a specified model is ‘a situation that is characterised by a lack of tendency to change’. With general equilibrium models, economists aim to study market equilibrium in many markets at once (Nicholson and Snyder, 2007). It takes into account that markets are interconnected through the interrelationship of prices and outputs of goods and factors of production (Bilas, 1967). Partial equilibrium models, with their focus on one market at a time, do not address this economic interdependence. *Computable* general equilibrium models are empirical models and essentially comprise a set of simultaneous equations. Various different equations or functional forms are available for inclusion in the models to depict the behaviour of agents in the economy.

CGE models are calibrated with data from SAMs and elasticities. Model calibration describes the process whereby certain of the model function parameters are estimated using the base SAM. Annabi, Cockburn and Decaluwé (2006:3) describe it as ‘determining the numerical values of the various parameters of functions compatible with the equilibrium of the initial SAM’. The aim of calibration is to reproduce the base SAM using the specified functional forms, calibration parameters and base values when the model is run without scenarios. The SAM presents data only as values, whereas the CGE model distinguishes between prices and quantities in its behavioural relationships, which could be either linear or non-linear functions. The product of defined prices and quantities should return the values reflected in a SAM, for example the price of output (*PX*) multiplied by the quantity of output (*QX*) should give the value of output that is captured in the SAM. Certain parameters are determined as part of the calibration process and are dependent on the base values of the SAM, for example the shift parameters of production functions. But there are also parameters used in the model that need to be provided in addition to the values in the SAM, such as elasticities for the functional forms for production, trade and household consumption. Some theoretical aspects of these functional forms and their elasticities are discussed in more detail in this chapter.

Various different types of CGE models exist, but the models focused on in this dissertation are developments of models that emerged in the late 1980's and early 1990's (Robinson et al., Kilkenny and Devarajan et al., quoted in PROVIDE 2005b:3). The model used in this study is a comparative static single country CGE model which assumes perfect competition. The point of departure is the STAGE model developed by Scott McDonald (2007). Although other more sophisticated extensions to CGE models have emerged over time, this model contains all the elements necessary to illustrate the proposed alternative treatment of output transformation by multi-product industries.

General equilibrium is defined in the next section and section 3.3 gives an overview of different types of CGE models. Section 3.4 focuses on the role of a SAM in supporting economic modelling. The SAM approach to modelling as formulated by Pyatt (1988) and Drud, Grais and Pyatt (1986) is explained briefly. This is followed by a more detailed discussion on prices and accounting identities with reference to SAM based modelling. This section touches on issues such as the transaction-value form of a model, closure rules, price interdependence and the law of one price. Section 3.5 provides an overview of the type of equations found in CGE models and the overview proceeds according to the SAM framework. The flexibility, relative ease of computation and regular flexibility of the CES and CET functional forms have contributed to its popularity in CGE modelling. Therefore apart from the Linear Expenditure System (LES) to model household demand, only the CES and CET functional forms are used in the base CGE model. The theory of the general CES and CET functions and the derivations of the first order conditions are presented in sections 3.5.6 and 3.5.7 while the CES, CET and LES functions, as applied in the base model with regard to production, trade and household demand respectively, are discussed in sections 3.7 to

3.9. Section 3.6 presents a short discussion of the CGE model that is used as base for this study. The commonly adopted specification of output transformation for industries in CGE models is discussed as part of section 3.7 on modelling production. The theoretical aspects of dealing with output transformation of industries in CGE models and the extension of the base CGE model to improve the specification of the output transformation function is discussed in more detail in Chapter 4.

3.2 Defining general equilibrium

The economy can be viewed as an interrelated system of markets and when a situation exists where a general set of supply and demand relationships for goods and services holds simultaneously, it is referred to as general equilibrium (Gravelle and Rees, 1992). Theoretically general equilibrium relates to the concept of fixed points, which is a point where a mapping of a set X into itself is its own image. Hence the point remains unchanged under the mapping. Brouwer's Fixed Point Theorem can be used to prove that an equilibrium solution in terms of resource allocation exists and at the same time provides one set of circumstances under which fixed points will exist. Brouwer's Theorem states the following: "...a continuous mapping of a closed, bounded, convex set into itself, always has a fixed point." (Gravelle and Rees, 1992:470). Besides the fact that equilibrium exists, two other issues are of importance. First, the stability of the equilibrium is important to make predictions of the effects of changes in exogenous variables when comparing equilibria, because stability implies that a system will return to a position of equilibrium when it is in disequilibrium. Second, uniqueness of equilibria is important for comparative statics (Gravelle and Rees, 1992), of which the current study is an example.

The Walrasian general equilibrium model assumes that consumers and firms are passive price takers because there are so many of them that the decisions of any one of them do not impact on any of the others. These price takers choose net demands and supplies for a given price vector and when demands and supplies are consistent an equilibrium is achieved. The Edgeworthian equilibrium model assumes that buyers and sellers actively participate in the market until everyone believes they have made the best deal possible. When the number of agents in an economy is relatively small, the Edgeworthian general equilibrium model gives a larger set of equilibrium resource allocations than Walrasian general equilibrium model. As the number of agents increase, the Edgeworthian set of equilibrium resource allocations tend to decrease until it converges with the Walrasian set of equilibrium resource allocations (Gravelle and Rees, 1992). The Walrasian general equilibrium model is therefore typically used in country and global CGE models where there are numerous buyers and sellers and information about all the individuals are not available and not plausible to include even if they were available. Also the Walrasian general equilibrium model gives the outcome for economies according to both the Walrasian and Edgeworthian view of the economy.

In the current study therefore it is assumed that the economy is in equilibrium and when the experiments are introduced, it induces disequilibrium in the economy. The economy however reverts back to a new equilibrium and the generated results measure the difference between the two equilibria.

3.3 Types of computable general equilibrium models

CGE models can differ with respect to time-frame, e.g. comparative static, recursive dynamic, dynamic or intertemporal; and/or with respect to the scope of coverage, e.g. global, multi-region or single country. The aim of a comparative static model is to compare different equilibrium states that are associated with different sets of values of parameters and exogenous variables (Chiang 1984). Comparative static models do not include explicit time dimensions; hence the time dimensions are simply the time the economy takes to adjust from one equilibrium to another. There is however a distinction between short run and long run through the selection of model closures. If it is assumed that capital cannot be reinvested in another sector during the adjustment period it would imply a short run scenario, whereas capital mobility between sectors implies a long run scenario. Recursive dynamic and dynamic or intertemporal models all have explicit time dimensions, i.e. there is information available about the adjustment paths. The dynamic effect is obtained through the updating of values of parameters from one year to the next. These parameters could relate to factor supply, sector productivity growth, nominal wages, investment decisions, etc. (Van Seventer 2002). Recursive dynamic models are solved one period (usually a year) at a time, i.e. sequentially. It is therefore assumed that behaviour is influenced by current and previous conditions in the economy. Fully dynamic or intertemporal models are solved simultaneously for all the periods; hence agent decisions in the current period are based on foresight and expectations of what will happen in future periods in addition to past conditions.

Global CGE models attempt to cover the entire world, whereas multi-region CGE models could have different regional coverage, where regions refer to a number of countries (Van Tongeren, Van Meijl and Surry, 2001). Single county CGE models refer to a model for a nation. The large body of literature on CGE models based on the Global Trade Analysis Project (GTAP) database provide examples of global models. Various extensions of the

GTAP model have followed, for example the agricultural focused GTAP model, GTAP-AGR (Keeney and Hertel, 2005) and the environmental GTAP model, GTAP-E (Burniaux and Truong, 2002; McDougall and Golub, 2007). The Globe model, which is a SAM based global model using GTAP data, was developed by McDonald, Thierfelder and Robinson (2007). Prior to GTAP the global and multi-region models included, amongst other, some of those discussed in Van Tongeren et al. (2001), namely G-Cubed, a multi-region intertemporal CGE model covering eight regions, the OECD's global environmental model called GREEN and Australia's MEGABARE global CGE model.

Single country CGE models have been developed for numerous countries by various individuals and institutions. To single out one institution, the International Food Policy Research Institute (IFPRI) did extensive CGE modelling work in several African countries, amongst other: Tanzania (Wobst, 2001), Malawi (Löfgren, 2001), Zambia (Löfgren, Robinson and Thurlow, 2002), Zimbabwe (Robilliard, Sukume, Yanoma and Löfgren, 2002), Botswana (Thurlow, 2007), Mozambique (Arndt, 2002), and of course South Africa (Thurlow and Van Seventer, 2002). There is no shortage of studies using single country models in the literature.

Micro-simulation models in combination with CGE models have gained popularity in recent years. Micro-simulation models focus on the distributive effects of taxes and transfers, but they often do not model prices, wages and macro variables. On the other hand CGE and macro models generally lack the distributional detail found in micro-simulation models; hence the recent attempts to combine micro-simulation models with CGE models (Davies, 2009).

3.4 The SAM as basis for a model

3.4.1 The SAM approach to modelling

The SAM approach to modelling concerns the close relationship between SAMs (databases) and economic models, and represents the title of the paper by Pyatt (1988) in which he gives his perspective on how a matrix framework of social accounts can be used in the development and understanding of model structures and results. In short, the SAM approach to modelling makes use of the SAM format to present economic theory (PROVIDE, 2003a). The primary objective with Pyatt's paper (1988) to formalise the SAM approach to modelling was to make the relationship between models and their supporting databases more explicit, and in so doing, to promote the use of macro-modelling techniques in economic policy formulation.

According to Keynes every economic model has a corresponding accounting framework and macroeconomic models require an accounting framework that is complete, i.e. every income must have an equal and corresponding expenditure. By definition a SAM is a complete dataset, therefore a SAM can serve as an accounting framework for a macroeconomic model, i.e. the model can be expressed within the SAM framework (Drud, Grais and Pyatt 1986).

The SAM approach to modelling highlights the idea of parallel development of data and theory within a common SAM framework. The data side of the development is to organise the data into a complete and consistent square matrix and thereby resolve any data discrepancies, i.e. the process of developing a balanced SAM. Parallel to the data development is the development of the CGE model.

Pyatt (1988) divides model design into three stages and an overview of these three stages is given here:

- Choice of model focus, i.e. the desired level of aggregation of institutions, assets, factors, industries and products;
- Specification of the transaction-value form of the model;
- Choice of closure rules.

In the *first stage* of the SAM approach to modelling the focus of the model is chosen by deciding the level of detail in the SAM accounts. Important here is that the selection of product groups should influence the choice of industry groups and *vice versa*. Similarly the industries should not be selected independently of the factors and institutions. Within this study the disaggregation of the agricultural industries by province and municipal districts allowed for useful disaggregation of agricultural products; and importantly it also capitalises on the availability of provincial level factor and household accounts when results are generated. Besides the level of aggregation of the SAM accounts, the choice of classification also relates to other concepts that would allow homogeneity of behaviour of groups, specifically for households and factors. In this study, for example, self-employed labour and labour in employment are reflected separately in the labour categories, whereas household categories are separated based on main source of income. For both factors and households, this allows for the distinction of groups with interests in agriculture compared to other industries.

With regard to the *second stage*, the SAM approach to modelling is based on the concept of two versions of the SAM, one containing the values of transactions and the other containing the algebraic expressions for determining the corresponding transactions value (Drud, Grais and Pyatt 1986; Pyatt, 1988). The two versions of the SAM relate to the use of a SAM as a database compared to the use of a SAM as a basis for a model. The SAM that contains the algebraic expressions is referred to as the transaction-value (TV) form of the model, i.e. a model set out according to the sub-matrices of a SAM. It implies that each value in the SAM can be traced back to a specific equation and the system of equations must be solved simultaneously in order to derive a complete and consistent SAM. The novelty of the SAM approach to modelling was that the transaction-value form of the model contained a set of equations to determine transaction values by modelling prices and value flows, whereas the economic convention had been to specify equations to determine prices and quantities (Drud, Grais and Pyatt 1986; Pyatt 1988). The quantity equations can be deduced from the transaction-value form of the model because quantities are implied by value flows and prices.

The advantages of having complementary pairs of SAMs for data and algebra are the following (Drud, Grais and Pyatt, 1986):

- The level of detail in the SAM must take account of both data availability and the issues to be analysed. The parallel development of data and theory within a common SAM framework facilitates this process;
- The SAM framework enhances data quality because of the inherent requirements of a SAM to be complete and consistent;
- When expressing the model within a SAM framework by means of its transaction value form, it aids understanding of the model structure;
- With a complementary pair of SAMs the CGE model can always be calibrated to reproduce the SAM in the base case. This solution can then be used to compare alternative cases.

When presenting a model in transaction-value form, substantial insight can be gained with regard to the underlying price system of a model and the SAM, as well as the accounting identities and other assumptions of the model. The price system and accounting identities are discussed in more detail in the next section.

The *third stage* of model design is choosing the closure rules (Pyatt, 1988). According to Drud, Grais and Pyatt (1986) the system derived from the equations in the transaction-value form of the model in the second stage, is underdetermined. It must therefore be further restricted to derive a fully determined system and the additional restrictions required to derive a fully determined system are referred to as closure rules.

3.4.2 Prices and accounting identities

The general transaction value form

When presenting a model in the transaction-value form, it is necessary to decide how each of the elements of a SAM will be determined, i.e. decide the behavioural assumptions. In the process each of the numbers in the SAM is replaced by an algebraic specification of the way in which the number is determined (Drud, Grais and Pyatt, 1986). Following the notation of Pyatt (1988), given a SAM matrix T, then the single entry in the j^{th} row and k^{th} column of the SAM, t_{jk}, is defined as

$$t_{jk} = t_{jk}\left(y;p,f,\lambda\right) \tag{4}$$

Where y is the income vector, p is a vector of product and non-factor services prices, f is factor prices and λ is the exchange rate, then the value of the element is a function of income and prices. It is notable that no quantities appear in the function, but it is recognised that each quantity q can be determined as y/p. The above equations describe the way in which transaction flows are determined. Based on the condition that the row and column totals must equate, the following equation can be derived (Pyatt, 1988):

$$Ti = y = T'i \tag{5}$$

So if equation (4) is substituted into equation (5), two sets of equations can be derived, namely the column summation equations and the row summation equations.

Column summation equations

The column summation equations for products and industries are presented following the notation of Pyatt (1988):

$$p = p\left(y;p,f,\lambda\right) \tag{6}$$

The equation shows that product and industry prices depend on the scale of product, hence the income levels of industries, the price of other products and industries, factor prices and the exchange rate. The equations are usually linear homogeneous, implying that under the assumption that the scale of production remains constant, then a doubling of input prices will lead to a doubling of output prices.

The equations mentioned thus far largely relate to product and industry accounts. The column summation of the other accounts such as factor services, institutions and ‘rest of the world' accounts, only provide a check on the transaction-value form of the model to ensure adding-up conditions are satisfied, but "no other new information" (Pyatt, 1988:339).

Pyatt (1988) distinguishes three types of transaction-value specification for each cell of the SAM in the context of SAM Leontief multiplier models. If t_{jk} is dependent on income, then the cells are endogenous, whereas if t_{jk} is independent of income, the cells are exogenous. The third type of cell contains a mixture of endogenous and exogenous elements, and constitutes residual balancing cells, the value of which is the difference between the endogenous and exogenous components, which are defined as *N* and *X* respectively. Since a SAM consists of endogenous and exogenous components, the SAM matrix *T*, can be expressed as the following (Pyatt, 1988):

$$T = N + X \tag{7}$$

From this perspective a different column summation equation can be derived (Pyatt, 1988):

$$y' = i'T = i'N + i'X \tag{8}$$

Where *i'X* is zero and *i'N* is equal to *y'* if row and column sums equate. The columns sum equations represent supply or production in the economy because it shows the costs involved in producing different goods and services. This form of the column summation equation is used to derive the row summation equations.

Row summation equations

The row summation equation is the following:

$$y = n + x \tag{9}$$

Where *n* and *x* are the column vectors of the row sums of *N* and *X* respectively. The row sum equations correspond to the demand side of the system, showing that total income derive from endogenous and exogenous demands in SAM Leontief multiplier models (Pyatt, 1988).

Equations for closing the system

As mentioned, the third stage of model design according to Pyatt (1988) is choosing the closure rules, which are the equations necessary to close the system. The column summation equations $p = p(y; p, f, \lambda)$ and the row summation equations $y = n + x$ give

[p]+[y]-1 independent equations, so that a third set of [f]+2 equations are necessary to close the system, and this third set relates to the closure rules.

The equations determining prices (equation 6) and the row-balance equations (equations 9) are all linear homogeneous, therefore at least one of the closure rules must not be linear homogeneous, otherwise the system will not solve. One of the closure rules is usually made not to be linear homogeneous by setting one particular price as numéraire for the entire system (Pyatt, 1988). This implies that all other prices are measured relative to this price.

Interdependence of prices

Pyatt (1988) explains the interdependence between prices in SAM Leontief multiplier models when looking at the column sums of the product and industry columns by stating that if total cost must equal total revenue, the price (average revenue) must equal average cost, which in turn depends on prices. The interdependence between prices is therefore captured by column summation equations for industries and products (equation 6), which shows that product and industry prices are amongst other also dependent on each other.

According to McDonald (2011) all prices in CGE models are derived from accounting identities and this follows economic theory because prices for each row are the average revenues received for each product and the prices in the columns capture the average cost of producing each product. Similar to SAM Leontief multiplier models, the interdependence between prices in CGE models are also captured by the structural information in the columns of the SAM.

The law of one price

As mentioned, the entry in the i^{th} row and j^{th} column is simultaneously the expenditure by the j^{th} account on the product of the i^{th} account and the income to the i^{th} account from sales of its product to the j^{th} account. In order for a SAM to be complete and consistent the total income ($\sum_i T_{ij}$) and total expenditure ($\sum_j T_{ij}$) for every account must equate. Following the notation and logic of McDonald (2011), with p denoting price and q denoting quantity, this can be represented as follows:

$$\sum_i p_{ij} q_{ij} = \sum_i T_{ij} = \sum_j T_{ij} = \sum_j p_{ij} q_{ij} \quad \forall i = j \tag{10}$$

This indicates an important assumption that prices are homogeneous along each row of the SAM, i.e. the price for any transaction in a row is therefore the same irrespective of the agent/account that buys it (Drud, Grais and Pyatt, 1986). This is often referred to as the law of one price because it indicates the requirement that each price in the model is uniquely determined (Pyatt 1994; McDonald 2011). If the price is the same irrespective of the purchaser, then the product is homogenous. This implies that the quantities in any row are measured in the same units, and can be summed so that the row totals are defined as the product of the respective price and the sum of the quantities that are recorded in each transaction in the row. Following the notation of McDonald (2011) this can be represented as follows:

$$T_{ij} = \sum_j p_i q_{ij} = p_i Q_i \qquad and \qquad \sum_j q_{ij} = Q_i \tag{11}$$

Per definition row and column totals must equate in a SAM, but because prices in a column relate to different goods and services the prices are not the same and hence summing quantities down columns (i.e. across rows) is not meaningful because quantities are not in the same units, i.e. $\sum_i q_{ij} \neq Q_j$. Thus, in order for the accounting identities of the SAM to hold, the quantities in each row should be in similar units and should have a common single price. All prices in all CGE models are derived from accounting identities and these identities must hold in the base equilibrium and when an experiment solution is reached (McDonald, 2011).

Some features often found in CGE models make the presence of the law of one price less visible. These relate to the use of the CES and CET functions in the model, where the CES functions apply when aggregating down columns and CET functions apply when aggregating across columns (McDonald, 2011). McDonald (2011) demonstrates how the law of one price either applies or is breached in five different cases, namely a) imports governed by a CES Armington function, b) exports governed by a CET function, c) multi-product industries where output is governed by a CET function, d) aggregation of primary factors governed by a CES function and e) agent specific product taxes. In four of the five examples either CES or CET functions are involved. The law of one price as it relates to CES and CET functions in CGE models is discussed as in sections 3.8.3 and 3.8.5 in the context of imports and exports respectively, and also in section 4.2.3 in the context of output transformation.

3.4.3 Equilibrium conditions and model closures

Besides the core set of non-linear and linear equations the model also requires a set of equilibrium conditions and model closure equations. Robinson (1989) refers to these as the system constraints, which although not taken into account in decision-making by individual agents, need to be satisfied by the decisions of all agents jointly. The equilibrium conditions include market clearing conditions for factor and product markets (i.e. ensuring that demand and supply in the product and factor markets are equal in quantity terms) and income balance conditions for households and government. The income balance conditions are also referred to as macro-economic equilibrium conditions and include the savings-investment balance, the balance of payments and the government surplus/deficit (Thiele, 1998). The equilibrium conditions for the base model can be viewed in the model equations presented in the addendum, since they remain the same in the adjusted model.

Closely related to the equilibrium conditions are model closures. Mathematically model closure refers to the process of ensuring that the numbers of equations and variables in the model are consistent in order for the model to solve as mentioned in section 3.4.2. In practice it involves fixing a number of variables, either to base/original levels, or fixing them to new levels as part of a scenario. From an economic perspective the choice of which variables to fix conveys assumptions about how the economy operates and it has the potential to influence simulation results. The variables which can be fixed as part of model closure are found in the model equations related to foreign exchange, capital (savings-investment), corporations, government and factors of production (labour, capital and land).

There are many different options available when choosing closure rules, but the choice of closure rules will typically determine how the factor and capital markets of the economy are closed (Pyatt, 1988). In the case of the factor market (capital, labour, and land) this will entail a decision about whether labour is assumed to be fully employed, in which case wages adjust to bring demand and supply in balance; or whether there is a measure of unemployment, in which case wages will remain fixed while the level of employment adjusts. In the case of the capital market (investment and savings), there is a choice between the assumptions of investment driven savings versus savings driven investment. Total savings are also determined by the levels of savings by government and the external balance.

The prices equations, excluding price equations that are accounting definitions, in the CGE model are first order differentials of the quantity equations that are linear homogenous of degree one (i.e. production function exhibits constant returns to scale). Thus CGE models are usually homogenous of degree zero in prices, i.e. a doubling of all prices will have no impact on quantities (PROVIDE, 2003b). Since only relative prices are of importance, a particular price, such as the consumer price index (*CPI*), should be set as numéraire (i.e. fixed) as part of model closure (Devarajan, Go, Lewis, Robinson and Sinko, 1997). Price changes in model results are therefore interpreted as relative price changes.

The choice of closure rules is not dictated by the SAM, but extends into the sphere of general macroeconomics. Pyatt (1988) suggests that many of the weakness of CGE models appear at the point where the SAM data need to be complemented with additional data outside of the SAM framework, e.g. with regard to estimation of parameters (elasticities) of some of the functions and the choice of closure rules, because it is separate from the SAM but it can significantly influence the modelling results.

3.5 Behavioural relationships in CGE models

The behavioural relationships determine how the agents in a model will react to exogenous changes in the parameters of the model, where parameters could include variables that have been fixed to a base value as part of model closure. This section presents an overview of some of the most common behavioural assumptions found in literature. Table 13 gives an overview of the main behavioural relationships for different agents in a CGE model, as represented in the SAM framework. The table was taken from McDonald (2007) and adjusted to present a more general overview of behavioural relationships compared to those that were used in the base model for this study. The base model will be discussed in more detail in section 3.6.

CGE models include both linear and non-linear relationships. The main non-linear functional forms included in the CGE model typically relate to only five sub-matrices of the SAM, namely production (intermediate use matrix and factor demands matrix), trade flows (imports matrix and exports matrix) and household consumption, although this can vary depending the focus of a particular study. Various functional forms exist that could potentially be included to estimate the values of the mentioned sub-matrices and each functional form would imply a different behavioural assumption about production, consumption and trade decisions. The discussion commences with some general findings concerning the functional forms used for non-linear relationships and then proceeds with the various commonly used non-linear relationships and concludes with the commonly used linear relationships.

Table 13: Behavioural relationships for the CGE model

	Products	Industries	Factors	Households	Corporations	Government	Capital	RoW	Total
Products	0	Input-Output Coefficients	0	Utility Functions	Fixed in Real Terms	Fixed in Real Terms and Export Taxes	Fixed Shares of Savings	Product Exports	Product Demand
Industries	Domestic Production	0	0	0	0	0	0	0	Constant Elasticity of Substitution Production Functions
Factors	0	Factor Demands	0	0	0	0	0	Factor Income from RoW	Factor Income
Households	0	0	Fixed Shares of Factor Income	Fixed shares of income	Fixed Shares of Dividends	Fixed (Real) Transfers	0	Remittances	Household Income
Corporations	0	0	Fixed Shares of Factor Income	0	0	Fixed (Real) Transfers	0	Transfers	Corporation Income
Government	Tariff Revenue Domestic Product Taxes	Indirect Taxes on Industries	Fixed Shares of Factor Income Direct Taxes on Factor Income	Direct Taxes on Household Income	Fixed Shares of Dividends Direct Taxes on Corporation Income	0	0	Transfers	Government Income
Capital	0	0	Depreciation	Household Savings	Corporation Savings	Government Savings (Residual)	0	Current Account 'Deficit'	Total Savings
Rest of World	Products Imports	0	Fixed Shares of Factor Income	0	0	0	0	0	Total 'Expenditure' Abroad
Total	Product Supply	Industry Input	Factor Expenditure	Household Expenditure	Corporation Expenditure	Government Expenditure	Total Investment	Total 'Income' from Abroad	

Source: Adjusted from McDonald (2007:7)

3.5.1 General notes on functional forms for non-linear relationships

Functional forms can be used for econometric analyses or within a system of equations such as CGE models. When functional forms are used in econometric analyses, the requirements are different compared to when these functional forms are used in general equilibrium (Perroni, and Rutherford, 1998). Functional forms for inclusion in CGE models should be continuous and homogeneous of degree zero (Shoven and Whalley, quoted in Annabi, Cockburn and Decaluwé, 2006:7). In general equilibrium models functional forms should also display global regularity and certain third-order curvature properties (Perroni, and Rutherford, 1998). As a result of this, flexible functional forms have been more widely used in applied econometrics than in general equilibrium. Flexible functional forms such as the Translog (Transcendental Logarithmic), the Generalised Leontief and the Normalized Quadratic functions were found to behave well at the equilibrium point but then can cause numerical solution methods to fail if they lack global regularity and the Translog function was found prone to loss of regularity when the function is calibrated to large cross-elasticity values (Perroni and Rutherford, 1998). The globally regular functions are therefore deemed too flexible and hence Perroni and Rutherford (1995:336.) indicate the need for functional forms with what they term as ‘regular flexibility’. As Turner (2009:652) states: “In practice, however, this argument over whether to use CES or FFF is likely to boil down to a trade off between flexibility and tractability.”, where FFF refers to flexible functional forms.

A solution to the problem of obtaining the desirable properties of the flexible functional forms without the unwanted unstable behaviour in general equilibrium models, is found in the use of CES and CET functional forms as part of a nested production structure; hence the numerous examples of CGE models containing such nested structures in the literature. The two-level multi-factor CES function developed by Sato (1967) is an example of a nested function within itself, because two levels of CES functions are combined into one function. Nested production structures in CGE models are usually not depicted by a single production function, but by a set (or nest) of functions. For example at the top level of production, aggregate output is derived from aggregation of aggregate intermediate goods and aggregate primary inputs (capital, labour and land), using a particular functional form. At the next level another functional form could be used to aggregate intermediate inputs from different individual inputs. The combination of capital, labour and land to form aggregate primary inputs at the second level of production, can also be governed by various functional forms. In some cases more levels are added. The nested CES and CET functions serve as an approximation of the Translog and CRESH/CRETH systems[1] and at the same time the nested CES/CET system appears to give more stable results compared to the more complex Translog and CRESH/CRETH systems. For the same reasons nested utility functions are often used to model household consumption. Strotz (quoted in Keller, 1976:175) introduced the concept of utility trees in the theory of consumer demand, but it was Keller (1976) that first combined the concept of the utility tree with nested CES-type functions. A nested structure for production is used in the base model, but the base model does not contain a nested utility tree for consumption.

3.5.2 Functional forms for modelling domestic production

Various functional forms exist to specify production within CGE models, including Leontief production, Cobb-Douglas functions and CES functions. In some instances the dual cost function of the functional forms are assumed to have a flexible form. Some of these flexible functional forms such as the Translog, Generalised Leontief, CRESH/CRETH systems and the Normalised Quadratic function have also been used in CGE models. Flexible functional forms have less restrictive assumptions about elasticities of substitution, but as indicated in the previous section some of these flexible functions forms are likely to lack global regularity (Peronni and Rutherford, 1995). Producers are assumed to maximize profits subject to technology constraints and the first order conditions related to the production functions are included in the CGE model to determine the optimal level of resource use and combination of factors.

The Leontief production function is often applied to derive intermediate input use because it assumes that demands are in fixed proportions relative to the output of each industry. The Leontief production function and the Cobb-Douglas production function can be regarded as special cases of the generalised CES function. Both the Cobb-Douglas and the CES production functions place a restriction on the elasticity of substitution. The main restriction of the Cobb-Douglas production function is that it assumes an elasticity of substitution between inputs equal to one. The CES function relaxes this assumption to the extent that the elasticity of substitution can take on a value other than one, but the elasticity is still assumed to be constant for all pairs of inputs in the function. Two-level CES functions, and therefore nested production structures, assume strong separability, i.e. that factors can be partitioned into separate bundles (Sato, 1967). In CGE models these separate factor bundles typically distinguish capital, labour (or different labour types) and in some cases land.

[1] See next page for explanation of CRESH/CRETH systems.

The Translog production function is a generalisation of the CES production function and allows for variability of Allen partial elasticities of substitution and for using any number of inputs (Khalil, 2005). The Translog function has been applied to differentiate cross substitution among labour categories in the context of wage subsidies by Go, Kearney, Korman, Robinson and Thierfelder (2009). It was introduced to allow for different degrees of complementarity between higher skilled and lower skilled labour in various sectors, closer but different substitution among lower skilled labour in different sectors, and greater but different degrees of complementarity between high skilled labour and capital in various sectors. The Translog function in this case replaced the traditional nested CES formulation, with consequences related to instability of results as a result of lack of global regularity as mentioned in the previous section.

In order to relax the assumption in CES and CET functions that pair-wise partial substitution and transformation elasticities are equal, Vincent, Dixon and Powell (1980) developed the CRESH/CRETH production systems, where CRESH is the Constant Ratio of Elasticities of Substitution Homothetic function and CRETH is the Constant Ratio of Elasticities of Transformation Homothetic function. The CRESH production functions and CRETH transformation frontiers are therefore generalisations of the CES and CET functions and the main advantage of the CRESH and CRETH functions is that it permits substitution and transformation elasticities to differ among different pairs of factors and products, respectively. The MONASH model, ORANI, used the CRESH production function and the CRETH transformation frontier and the first application of CRESH/CRETH in CGE modelling was with the ORANI model by Dixon et al. (quoted in Dixon and Rimmer, 2002:219). The newer version of the ORANI model, ORANI-G, reverted back to the CES and CET functions in the production structure (Horridge, 1993).

Diewert and Wales (quoted in McKitrick, 1998:561) recommend a semi-flexible version of the Normalized Quadratic function for use in CGE modelling because this version reduces the required number of parameters. Hunt and Evans (2009) indicate that the difficulty in estimating parameters of the Normalised Quadratic function has precluded regular use in CGE models.

As mentioned in the previous section, one way of overcoming unstable results because of flexible functions forms that do not display global regularity, is to incorporate nested production structures. Nested production structures are commonly found in CGE models with an environmental focus because nested production structures allow for more detailed input substitution. O'Ryan, Miller and de Miguel (2003) included a three level nest. In the first level there is a CES aggregation of aggregate non-energy-producing intermediate inputs, aggregate factors (i.e. capital and labour) and energy producing inputs. Aggregate non-energy-producing intermediate input is maintained by assuming a Leontief function. On the factors' side, aggregate factors are a CES aggregate of labour on the one side and capital and energy on the other side. Similarly capital and energy is a CES aggregate of capital and energy respectively. A nested structure is also used in the dynamic California CGE model for energy and environmental policy analyses where the energy bundle is a CES aggregate of different fuel types and a further level allows for each fuel type to be imported from a different region of origin (Roland-Holst, 2008).

3.5.3 Functional forms for modelling international trade

Generally consumption in CGE models can be divided into intermediate consumption and final demand, which include demand by domestic institutions, investment and exports. On the supply side CGE models typically allow for both domestically produced and imported goods. A CES function can be used to combine domestically produced and imported goods into 'composite' goods, which can then be consumed. This follows the Armington assumption that there is product differentiation depending on the country of origin, and therefore there is imperfect substitution between locally produced and imported goods. As a result of the introduction of the CES function it is assumed that elasticities of substitution in each market are constant and that 'the elasticity of substitution between any two products competing in a market is the same as that between any other pair of products competing in the same market' (Armington, 1969:161). In this case markets would imply a product market where similar products from different countries or regions would be competing. The optimal ratios of imported and domestic products are determined by the relative prices of the imported and domestic products through the first order condition of the CES function (McDonald, 2007). De Melo and Robinson (1985) recognised that similar logic applies to the export side and proposed a CET function to allow for imperfect substitution (transformation) between producing for the local market versus the export market. According to De Melo and Robinson (1985) the assumption of imperfect substitution between domestic and traded goods is that the estimates of the impact of trade policy on resource allocation and welfare in CGE models will yield much less dramatic results compared to an assumption of perfect substitution. This is because extreme specialisation and price fluctuations are avoided (McDonald, 2007). The inclusion of the CES function on imports and the CET function on exports is currently standard practice in CGE modelling.

Although the AIDS was initially developed to improve upon flexible functional form approach to consumer demand analyses, Robinson applied the AIDS to aggregate domestically produced goods and imports in the global CGE model called the Western Hemisphere Free

Trade Area (WHIFTA) model in the early 1990's (Pogany, 1996). The WHIFTA model retained the CES aggregation option because by imposing certain restrictions on the AIDS it reverts to a CES function. Pogany (1996) indicates both the advantages and the disadvantages of using the AIDS approach in modelling trade. One of the advantages of the AIDS approach is that the number of behavioural parameters is enlarged compared to using a CES that is not a flexible functional form because the AIDS allows for different elasticities of substitution for imports from different countries. One of the main disadvantages stems from the fact that the AIDS is not necessarily globally regular and hence can lead to mathematical inconsistencies. The problem of irregularity in the cost-curve dual of the utility function can be corrected using the principle of Cholesky factorisation, which guarantees convexity, but in the process an undetermined degree of flexibility is lost again.

3.5.4 Functional forms for modelling household consumption

The homothetic Cobb-Douglas system, Linear Expenditure System (LES), Constant Difference of Elasticities (CDE) demand system, Almost Ideal Demand System (AIDS), the Translog system, the Rotterdam model and An Implicitly Directly Additive Demand System (AIDADS) are the most popular demand systems in 'recent' applied work (Yu, Hertel, Preckel and Eales, 2004). The Extended Linear Expenditure System (ELES) can be added to the list, especially where dynamic models are concerned. Consumers are assumed to maximise utility subject to income constraints.

According to Unnevehr, Eales, Jensen, Lusk, McCluskey and Kinsey (2011:507) one of the most important aspects in demand analyses is to determine how food prices and income elasticities change as incomes grow. This is related to Engel's Law (Unnevehr et al., 2011) which states that the income elasticity of demand for food declines as incomes rise. There is a trade-off in demand systems between simplicity and conformity to Engel's Law.

The Cobb-Douglas function in the context of household demand assumes that household expenditures are allocated in fixed value shares to each consumption product at the consumer price, ensuring that all disposable income is exhausted. With the Cobb-Douglas function it is assumed that the cross price elasticity is zero, and that all price and income elasticities and elasticities of substitution are equal to one. Yu et al. (2004) warns that as a result of the assumption of constant average budget shares the Cobb-Douglas fails to describe changing consumption and trade patterns in the world food market and is in contradiction with Engel's law. Despite the strong, even unrealistic assumptions, the Cobb-Douglas function is often used because of its ease of calibration and the fact that no additional elasticities need to be supplied to the model (Annabi, Cockburn and Decaluwé, 2006). The only unknown parameters that must be calculated as part of calibration are the consumption shares and these can be calculated from the base SAM data.

The LES is a variant of the Cobb-Douglas function. Contrary to the Cobb-Douglas function, the LES does not assume unit income elasticity. The LES utility function allows for subsistence expenditure and is therefore deemed a realistic assumption in a developing country such as South Africa with many poor consumers (McDonald, 2007). The proportions of current expenditure on products defined to constitute subsistence consumption can also be varied. The STAGE model that is used as base model in this study makes use of the LES (McDonald, 2007). The IFPRI (International Food Policy Research Institute) standard single country model also includes an LES for consumption demand (Löfgren, Harris, Robinson, Thomas and El-Said, 2002).

The ELES developed by Lluch (1973) is an extension of the LES by taking savings behaviour into account (Annabi, Cockburn and Decaluwé, 2006). The ELES is therefore most relevant in the context of dynamic models because it allows for income expectations to impact on consumption patterns and total consumption is endogenously determined; hence savings and investment are also endogenously determined (Lluch, 1973). O'Ryan, Miller and de Miguel (2003) applied an ELES utility function in the ECOGEM-Chile model to study the effects of imposing environmental taxes in Chile. Mabugu (2005) applied an ELES in a dynamic CGE model to analyse alternative trade policy simulations relevant to South Africa. An ELES was also used in the dynamic California CGE model for energy and environmental policy analyses (Roland-Holst, 2008).

According to Yu et al. (2004) the Constant Difference of Elasticity function (CDE) was proposed by Hanoch in 1975 and has been widely used in CGE models since the work of Hertel and his co-researchers in 1991. The system is globally regular but while the marginal budget shares are non-constant in the CDE system, it prevents luxury goods from becoming necessities as income grows. A drawback of the CDE is that although the adjustment of the marginal budget shares as households become wealthier are typically in the right direction, the magnitude is often too small compared with econometric results.

The AIDS model was designed by Deaton and Muellbauer (quoted in Unnevehr et al., 2011:508) with 'a more intuitive embedded Engel relationship' and is therefore deemed better suited to food demand estimation. According to Unnevehr et al. (2011:508) the AIDS model is based on the functional form suggested by Working in 1943 and applied by Leser to household food expenditures in 1963. Savard (2010) integrated AIDS in a CGE micro-simulation model in order to obtain greater household heterogeneity. He highlighted application problems due to the nature of the demand system as the possibility of generating negative consumption shares in simulation and the fact that the system behaves well locally but not necessarily globally. Yu et al. (2004) also confirms that the AIDS system does not satisfy the global regularity requirements.

The An Implicit Directly Additive Demand System (AIDADS) has greater flexibility of Engel responses than the LES, i.e. the Engel curves show the desired responses in reaction to changes in income, and the AIDADS demand system remains regular under a very large reduction in income (Powell, McClaren, Pearson and Rimmer, 2002). Powell et al. (2002) replaced the LES in the ORANI-G model with the AIDADS to obtain the ORANAIDAD model. Verma and Hertel (2009) applied the AIDADS in a CGE model for Bangladesh to analyse the impact of volatile product prices on calorie intake. They specifically used AIDADS because the demand system can span the entire population in the country because it can take into account the great differences in per capita income.

The Rotterdam system is a demand system that is typically used in applied econometric work, but there appears to be no reference to the incorporation of the system into a CGE model. The embedded Engel relationships in the Rotterdam model do not conform to Engel's Law (Unnevehr et al., 2011). The Translog demand system by Christensen, Jorgenson, and Lau (quoted in Yu et al., 2004:4) also suffers from the fact that fitted budget shares may become negative and hence the global regularity requirements are not satisfied.

Similar to nested production structures, nested utility functions can be used to achieve greater flexibility without loss of regularity. As mentioned, Keller (1976) developed a utility tree with nested CES-type functions. Another more recent example of a nested utility tree can be found in Löfgren and El-Said (1999), where the AIDS is used at the top level to generate demand for disaggregated food items and an aggregated non-food item, whereas at the lower level aggregated non-food demand is disaggregated using LES demand functions.

3.5.5 Potential linear relationships in CGE models

Generally in CGE models the values in the other sub-matrices (i.e. excluding production, trade and household demand) are derived as fixed shares or linear relationships from the base values in the SAM. For example all tax revenues by government could be calculated as shares of some other value, e.g. income tax as a share of direct income and import duties as a share of the value of imports. Savings by institutions could be derived as fixed shares or residuals. Final demand by corporations and government, and demand for investment, could be fixed in real terms. These linear relationships could be elaborated on depending on the focus of the study, but there are few examples of such extensions in the literature. The set of linear equations also includes the equilibrium conditions referred to in section 3.4.3.

3.5.6 The CES function in more detail

This section reviews some theoretical aspects of the CES function. The CES function was first discussed in detail by Arrow, Chenery, Minhas and Solow (1961). At the time of the development of the CES function, the popular growth models included the Harrod-Domar growth model, with its assumption of fixed proportions of labour and capital, and the neo-classical model, with the Cobb-Douglas function and its assumption of unitary elasticity of substitution between labour and capital. Arrow et al. (1961) therefore aimed to derive a function that fit empirical evidence more closely, so the attributes that they sought to capture in the function, included the following: a) homogeneity, b) constant elasticity of substitution between labour and capital, and c) the possibility of different elasticities for different industries. With the CES function, all these objectives were achieved. They also showed that the Leontief (fixed proportions) and Cobb-Douglas functions were special cases of the CES function. Arrow et al. (1961) presented the CES function for the two-factor case and subsequently Mukerji (1963) generalised the CES function to the *n*-factor case in the format that it is currently applied in CGE models. Sato (1967) proposed a further extension of the multi-factor CES function, namely a two-level CES function. The general form of the CES function for the two-factor and multi-factor cases are shown as well as the derivation of the first order conditions. The CES function was originally expressed in the context of production under the assumption of profit maximisation. It can be similarly expressed from in the context of consumption under the assumption of utility maximisation, as applied in the LES utility function and the CES function for the aggregation of imports and domestically produced goods into a composite good. The two-argument CES function is a special case of the multi-argument CES function.

The constant elasticity of substitution (CES) function for the two-factor case

The CES function for the two-factor case as proposed and explained by Arrow et al. (1961) is the following:

$$V = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho} \tag{12}$$

Where V is value added obtained using two inputs, labour (L) and capital (K). The parameter γ (gamma) is the efficiency parameter, ρ (rho) is the substitution parameter and δ (delta) is the distribution parameter.

The parameter γ *(γ > 0)* is a neutral efficiency parameter because it changes the output for a given set of inputs in the same proportion.

The relationship between the elasticity of substitution (σ) of a linear homogenous CES function and the substitution parameter (ρ) is the following:

$$\sigma = \frac{1}{1+\rho}, \; 0 < \sigma \leq \infty \tag{13}$$

The range of the substitution parameter (ρ) is given as $-1 \leq \rho \leq \infty$, $\rho \neq 0$, which implies that the elasticity of substitution (σ) ranges from 0 to $+\infty$. For ρ = -1 there will be infinite elasticity of substitution, for $-1 < \rho < 0$ elasticity of substitution is greater than unity and for $0 < \rho < \infty$ elasticity of substitution is smaller than unity. If $\rho = 0$ it implies an elasticity of substitution of unity, but the CES function is not defined for $\rho = 0$ due to division by zero. However, using L'Hôpital's Rule it can be shown that as ρ tends towards zero, the linear homogenous CES production function approaches the linear homogenous Cobb-Douglas function. If ρ approaches infinity (∞) the elasticity of substitution tends to zero, then the CES function approaches the case of fixed proportions.

The parameter δ ($0 \leq \delta \leq 1$) is the distribution parameter which determines the functional distribution of income between the two factors.

One of the features of the Arrow et al. (1961) CES function is that it exhibits constant returns to scale. This was challenged by Walters (1963), but it was Ferguson (1965) that presented a more general form of the CES function for Hicks-Neutral technology to allow for non-constant returns to scale:

$$Y = Ae^{\lambda}\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-m/\rho} \tag{14}$$

Where Y represents output, A represents Hicks-neutral technological progress and ρ, δ, λ and *m* are the substitution, distribution, technology and homogeneity parameters respectively. If *m* = 1 then the function is homogenous of degree one, alternatively for any other value assumed by *m*, the function will be homogenous of degree *m*. According to Ferguson (1965) the values of *m* should not be taken as directly related to whether there is diminishing, constant or increasing returns to scale. He proposes that a better statistical fit for a model is obtained by the inclusion of *m* because it allows for non-constant returns to scale, but care should be taken with the economic interpretation thereof.

Following the notation of Nicholson (1992), when multiplying each factor by the technology parameter (λ), where ε determines the degree of homogeneity of the production function, the level of output (Q) changes as follows:

$$
\begin{aligned}
Q' &= A\left[\delta(\lambda.K)^{-\rho} + (1-\delta)(\lambda.L)^{-\rho}\right]^{-\varepsilon/\rho} \\
&= \lambda^{\varepsilon}.A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-\varepsilon/\rho} \\
&= \lambda^{\varepsilon}.Q
\end{aligned}
\tag{15}
$$

In the context of CGE models, which assume constant returns to scale, linear homogenous functions with $\varepsilon = 1$ are implied. All subsequent formula discussed here will therefore show '1' for the case of constant returns to scale instead of the 'ε' for the general case.

The first-order condition for profit-maximisation can be derived directly by substituting the profit maximising condition where the wage of a factor F equals the marginal revenue product of the factor ($W_F = MRP_F$) into the equation for the marginal physical product of the factor (MPP_F), hence $MPP_F{*}P = MRP_F = W_F$, where P is the price of the output. The marginal physical product (MPP_F) is the first derivative of factor F. The equilibrium factor demand equation for factor K is as follows:

$$
\begin{aligned}
MPP_K &= \left(-\frac{1}{\rho}\right)A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho-1}\left[\delta(-\rho)K^{-\rho-1}\right] \\
\frac{MRP_K}{P} &= A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho-1}\left[\delta.K^{-\rho-1}\right] \\
W_K &= P.A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho-1}\left[\delta.K^{-\rho-1}\right]
\end{aligned}
\tag{16}
$$

The equilibrium factor demand equation for factor L can be derived in the same way:

$$
\begin{aligned}
MPP_L &= \left(-\frac{1}{\rho}\right)A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho-1}\left[(1-\delta)(-\rho)L^{-\rho-1}\right] \\
\frac{MRP_L}{P} &= A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho-1}\left[(1-\delta)L^{-\rho-1}\right] \\
W_L &= P.A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho-1}\left[(1-\delta)L^{-\rho-1}\right]
\end{aligned}
\tag{17}
$$

These equations satisfy the first-order condition for profit maximisation that can be derived by defining a profit function (Π) as total revenue minus total cost:

$$\Pi = P.A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho} - W_K K - W_L L \tag{18}$$

By taking the first order partial differentials of the profit function with respect to *K* and *L* respectively and solving these simultaneously, the profit equilibrium condition can be obtained. Hence the first order partial differentials of the profit function with respect K and L are set to zero:

$$\frac{\partial \Pi}{\partial K} = \left(-\frac{1}{\rho}\right) P.A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho - 1}\left[-\rho\delta K^{-\rho-1}\right] - W_K = 0 \tag{19}$$

$$\frac{\partial \Pi}{\partial L} = \left(-\frac{1}{\rho}\right) P.A\left[\delta.K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho - 1}\left[-\rho(1-\delta)L^{-\rho-1}\right] - W_L = 0 \tag{20}$$

Then, using the first order partial differentials of the profit function, the optimal ratio of each factor is determined by the ratio of the prices according to the following profit equilibrium condition:

$$\frac{W_K}{W_L} = \frac{\delta}{1-\delta}\left(\frac{L}{K}\right)^{\rho+1} \tag{21}$$

Or, by rearranging to have the ratio of factors as dependent variable, the following profit equilibrium condition is obtained:

$$\frac{L}{K} = \left(\frac{W_K}{W_L} * \frac{1-\delta}{\delta}\right)^{\frac{1}{\rho+1}} \tag{22}$$

Generalisation of the two-factor CES function

The CES function was extended to include *n* factors by Mukerji (1963). The generalised multi-factor CES function, $Q = f(x_1,....,x_n)$, takes on the following form[2]:

$$\begin{aligned} Q &= A\left[\delta_1 x_1{}^{-\rho} + \delta_2 x_2{}^{-\rho} + \ldots + \delta_n x_n{}^{-\rho}\right]^{-\varepsilon/\rho} \\ &= A\left[\sum_{i=1}^{n} \delta_i x_i{}^{-\rho}\right]^{-\varepsilon/\rho} \end{aligned} \tag{23}$$

---

[2] The function proposed by Mukerji (1963) did not contain the homogeneity parameter (ε). It was introduced by Sato (1967).

The above function was extended by Sato (1967) to form a two-level CES function, based on the concept of nested production structures. The two-level CES function is a special case of a class of strongly separable functions, which implies that the allocation of factors within each factor category (e.g. capital or labour) is determined solely by relative factor prices of that category only (Sato, 1967).

Similar to the two-factor case, multiplication of each of the factors x_i by a factor λ will increase output by a factor λ^{ε}. Thus, this is a linear homogenous function for $\varepsilon = 1$. The first order condition for profit maximisation for factor x_i is derived in the same way as those for the two-factor case above. By setting the wage of factor x_i (W_i) equal to the marginal revenue product of x_i (MRP_i), i.e. the marginal physical product of x_i (MPP_i), multiplied by the price of the output (P), the first order condition can be derived. The first order condition for a linear homogenous function ($\varepsilon = 1$) is derived as follows:

$$
\begin{aligned}
MPP_i &= \left(-\tfrac{1}{\rho}\right) A\left[\delta_1 x_1^{-\rho} + \delta_2 x_2^{-\rho} + \ldots + \delta_n x_n^{-\rho}\right]^{-\frac{1}{\rho}-1}\left[(-\rho)\delta_i x_i^{-\rho-1}\right] \\
&= A\left[\delta_1 x_1^{-\rho} + \delta_2 x_2^{-\rho} + \ldots + \delta_n x_n^{-\rho}\right]^{-\frac{1}{\rho}-1}\left[\delta_i x_i^{-\rho-1}\right] \\
MRP_i &= P.A\left[\delta_1 x_1^{-\rho} + \delta_2 x_2^{-\rho} + \ldots + \delta_n x_n^{-\rho}\right]^{-\frac{1}{\rho}-1}\left[\delta_i x_i^{-\rho-1}\right] \\
W_i &= P.A\left[\delta_1 x_1^{-\rho} + \delta_2 x_2^{-\rho} + \ldots + \delta_n x_n^{-\rho}\right]^{-\frac{1}{\rho}-1}\left[\delta_i x_i^{-\rho-1}\right] \\
&= P.A\left[\sum_{i=1}^{n}\delta_i x_i^{-\rho}\right]^{-\frac{1}{\rho}-1}\left[\delta_i x_i^{-\rho-1}\right] \\
&= P.Q\left[\sum_{i=1}^{n}\delta_i x_i^{-\rho}\right]^{-1}\left[\delta_i x_i^{-\rho-1}\right]
\end{aligned}
\tag{24}
$$

Given the following:

$$
\begin{aligned}
Q &= A\left[\sum_{i=1}^{n}\gamma_i x_i^{-\rho}\right]^{-\frac{1}{\rho}} \\
\left[\sum_{i=1}^{n}\gamma_i x_i^{-\rho}\right] &= \left[\frac{Q}{A}\right]^{-\rho}
\end{aligned}
\tag{25}
$$

Then equation 24 can be rewritten in terms of the quantity ratio:

$$W_i = P.Q\left[\sum_{i=1}^{n}\gamma_i x_i^{-\rho}\right]^{-1}\left[\gamma_i x_i^{-\rho-1}\right]$$

$$\frac{W_i}{P} = Q\left[\left[\frac{Q}{A}\right]^{-\rho}\right]^{-1}\left[\gamma_i x_i^{-\rho-1}\right]$$

$$= Q\left[\frac{Q}{A}\right]^{\rho}\left[\gamma_i x_i^{-\rho-1}\right]$$

$$= \frac{Q}{Q^{-\rho}A^{\rho}}\left[\gamma_i x_i^{-\rho-1}\right]$$

$$= \frac{\left[\gamma_i x_i^{-\rho-1}\right]}{Q^{-\rho-1}A^{\rho}}$$

$$= \frac{\gamma_i}{A^{\rho}} * \frac{x_i^{-\rho-1}}{Q^{-\rho-1}}$$

$$= \frac{\gamma_i}{A^{\rho}} * \left[\frac{x_i}{Q}\right]^{-\rho-1}$$

$$\left[\frac{x_i}{Q}\right]^{-\rho-1} = \left[\frac{W_i}{P} * \frac{A^{\rho}}{\gamma_i}\right]$$

$$\frac{x_i}{Q} = \left[\frac{P_i}{P} * \frac{A^{\rho}}{\gamma_i}\right]^{\frac{1}{-\rho-1}} \tag{26}$$

CES functions in the base model are linear homogenous, implying that if all inputs are multiplied by the same scalar, the output will also increase by that same factor. Linear homogenous production functions have the following properties (Gravell and Reese, 1992:187):

- constant returns to scale at all input combinations;
- marginal products which are independent of scale;
- the slopes of the isoquants depend only on the input proportions and are independent of the scale of production;
- output is equal to the sum of the marginal products of the inputs multiplied by their level of use.

3.5.7 The CET function in more detail

The CET function was initially derived by Powell and Gruen (1968) and later one of the less obvious restrictions in the generalisation to the multi-output case was made explicit in Shumway and Powell (1984). The first application of the CET appears to be in the context of econometric analyses of the responsiveness of output composition to changing relative prices (Powell and Gruen, 1967). The CET function is less prominent in the literature compared to the CES function. The CES function, the general form of the CET function for the two-product and multi-product cases are shown, as well as the derivation of the first order conditions. The two-argument CET function is a special case of the multi-argument CET function.

The constant elasticity of transformation (CET) function for the two product case

The CET production function is algebraically identical to the CES except that the sign of the substitution parameter is negative for the CES function and positive for the CET function (Powell and Gruen, 1968; Shumway and Powell, 1984). A CET function is therefore concave with respect to the origin (in the two-argument case), whereas the CES is convex. For the two-argument case the CET function is the following:

$$Q = A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho} \tag{27}$$

Where Q is a composite (aggregate) output from two products, x_1 and x_2, that are produced from the same 'conglomerate' of resources. The parameter A $(A > 0)$ is an efficiency or shift parameter. The parameter $\gamma\,(0 \le \gamma \le 1)$ is a distribution or share parameter, which distributes the composite output into two products. Constant returns to scale is assumed in the context of the CGE model, hence the homogeneity parameter is set to one. The range of the function exponent ρ is $(-\infty \le \rho \le 1,\ \rho \ne 0)$. For $\rho < 1$ the function is concave to the origin, but for $\rho > 1$ the function reverts to a standard CES function. As ρ approaches 1, elasticity of transformation (Ω) goes to infinity (De Melo and Robinson, 1985). The relationship between the function exponent and the elasticity of transformation (Ω) is the following:

$$\Omega = \frac{1}{\rho - 1},\ 0 \le \Omega \le \infty \tag{28}$$

The first order condition defines optimum ratios of products in relation to relative prices. The derived supply equations for products included in the CET function can be determined by setting the price of an individual product (P_i) equal to the aggregate product price (P) multiplied by the first derivative of output (Q in equation 27) with respect to a particular product. The derived supply equation for product x_1 is the following:

$$
\begin{aligned}
P_1 &= P\frac{\partial Q}{\partial x_1} \\
&= P\left(\frac{1}{\rho}\right)A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho - 1}\left[\gamma(\rho)x_1^{\rho-1}\right] \\
&= P.A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho - 1}\left[\gamma x_1^{\rho-1}\right] \\
&= P.Q\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{-1}\left[\gamma x_1^{\rho-1}\right]
\end{aligned}
\tag{29}
$$

Similarly, the derived supply equation for product x_2 is the following:

$$
\begin{aligned}
P_2 &= P\frac{\partial Q}{\partial x_2} \\
&= P\left(\frac{1}{\rho}\right)A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho - 1}\left[(1-\gamma)(\rho)x_2^{\rho-1}\right] \\
&= P.A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho - 1}\left[(1-\gamma)x_2^{\rho-1}\right] \\
&= P.Q\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{-1}\left[(1-\gamma)x_2^{\rho-1}\right]
\end{aligned}
\tag{30}
$$

These equations satisfy the first order condition for profit maximisation that can be derived by defining a net profit function as total income less total expenses:

$$
\Pi = P.A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho} - p_1 x_1 - p_2 x_2 \tag{31}
$$

The first order condition can be obtained by taking the first order partial differentials of the net profit function with respect to x_1 and x_2 respectively and solving these simultaneously. Similarly, by setting the marginal rate of transformation between the products equal to their price ratio under the assumption of profit maximisation, the following first order condition, with the price ratio as dependent variable, is obtained:

$$\begin{aligned}
\frac{P_2}{P_1} &= \frac{\partial Q/\partial x_2}{\partial Q/\partial x_1} \\
&= \frac{\left(\frac{1}{\rho}\right) A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho - 1}\left[(1-\gamma)(\rho)x_2^{\rho-1}\right]}{\left(\frac{1}{\rho}\right) A\left[\gamma x_1^{\rho} + (1-\gamma)x_2^{\rho}\right]^{1/\rho - 1}\left[\gamma(\rho)x_1^{\rho-1}\right]} \\
&= \frac{(1-\gamma)\, x_2^{\rho-1}}{\gamma x_1^{\rho-1}} \\
&= \frac{(1-\gamma)}{\gamma} * \left(\frac{x_2}{x_1}\right)^{\rho-1}
\end{aligned} \tag{32}$$

Or, by rearranging, the first order condition with the quantity ratio as dependent variable is:

$$\begin{aligned}
\frac{\gamma x_1^{\rho-1}}{(1-\gamma)\, x_2^{\rho-1}} &= \frac{P_1}{P_2} \\
\frac{x_1^{\rho-1}}{x_2^{\rho-1}} &= \frac{P_1}{P_2} * \frac{(1-\gamma)}{\gamma} \\
\frac{x_1}{x_2} &= \left[\frac{P_1}{P_2} * \frac{(1-\gamma)}{\gamma}\right]^{\frac{1}{\rho-1}}
\end{aligned} \tag{33}$$

Generalisation of the two-product CET function

The CET function can be extended to include multiple products. The generalised multi-output CET function, $Q = f(x_1,....,x_n)$, takes on the following form:

$$\begin{aligned}
Q &= A\left[\gamma_1 x_1^{\rho} + \gamma_2 x_2^{\rho} + \ldots + \gamma_n x_n^{\rho}\right]^{1/\rho} \\
&= A\left[\sum_{i=1}^{n} \gamma_i x_i^{\rho}\right]^{1/\rho}
\end{aligned} \tag{34}$$

For a linear homogenous function, the first order condition for profit maximisation is derived in similar way as that for the two-product function above. In the multi-product CET function by setting the product price (P_i) equal to the aggregate output price (P) multiplied by the first derivative of output (Q in equation 34) with respect to each product, as follows:

$$\begin{aligned} P_i &= P\frac{\partial Q}{\partial x_i} \\ &= P\left(\tfrac{1}{\rho}\right)A\left[\gamma_1 x_1^{\rho} + \gamma_2 x_2^{\rho} + \ldots + \gamma_n x_n^{\rho}\right]^{1/\rho - 1}\left[(\rho)\gamma_i x_i^{\rho-1}\right] \\ &= P.A\left[\sum_{i=1}^{n}\gamma_i x_i^{\rho}\right]^{1/\rho - 1}\left[\gamma_i x_i^{\rho-1}\right] \\ &= P.Q\left[\sum_{i=1}^{n}\gamma_i x_i^{\rho}\right]^{-1}\left[\gamma_i x_i^{\rho-1}\right] \end{aligned} \tag{35}$$

Given the following:

$$\begin{aligned} Q &= A\left[\sum_{i=1}^{n}\gamma_i x_i^{\rho}\right]^{1/\rho} \\ \left[\sum_{i=1}^{n}\gamma_i x_i^{\rho}\right] &= \left[\frac{Q}{A}\right]^{\rho} \end{aligned} \tag{36}$$

Then equation 35 can be rewritten in terms of the quantity ratio, i.e. the share of product x_i in output Q:

$$\begin{aligned} P_i &= P.Q\left[\sum_{i=1}^{n}\gamma_i x_i^{\rho}\right]^{-1}\left[\gamma_i x_i^{\rho-1}\right] \\ \frac{P_i}{P} &= Q\left[\left[\frac{Q}{A}\right]^{\rho}\right]^{-1}\left[\gamma_i x_i^{\rho-1}\right] \\ &= Q\left[\frac{Q}{A}\right]^{-\rho}\left[\gamma_i x_i^{\rho-1}\right] \\ &= \frac{Q}{Q^{\rho}A^{-\rho}}\left[\gamma_i x_i^{\rho-1}\right] \\ &= \frac{\left[\gamma_i x_i^{\rho-1}\right]}{Q^{\rho-1}A^{-\rho}} \\ &= \frac{\gamma_i}{A^{-\rho}} * \frac{x_i^{\rho-1}}{Q^{\rho-1}} \\ &= \frac{\gamma_i}{A^{-\rho}} * \left[\frac{x_i}{Q}\right]^{\rho-1} \\ \left[\frac{x_i}{Q}\right]^{\rho-1} &= \left[\frac{P_i}{P} * \frac{A^{-\rho}}{\gamma_i}\right] \\ \frac{x_i}{Q} &= \left[\frac{P_i}{P} * \frac{A^{-\rho}}{\gamma_i}\right]^{\frac{1}{\rho-1}} \end{aligned} \tag{37}$$

3.6 Overview of the quantity and price relationships of the base CGE model

The STAGE model by McDonald (2007) was used as base CGE model for this study and is discussed here with reference to price and quantity diagrams. The discussion in each case commences with the general price and quantity relationships found in the model and then proceeds to production relationships in the model. The discussion draws from McDonald (2007). The model equations of the base model are not repeated here; however the model equations for the adjusted model can be viewed in the addendum.

3.6.1 Quantity relationships

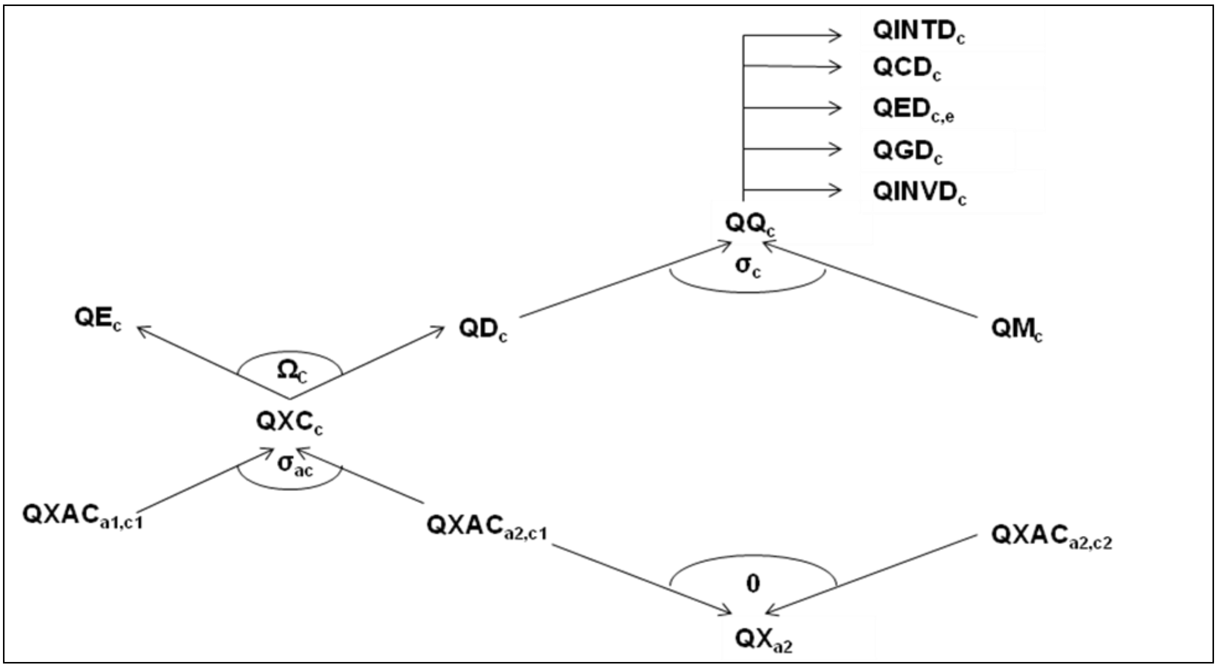
Figure 3 shows the relationships between the quantity variables in the base model (McDonald, 2007). Total demand consists of demand for intermediate inputs ($QINTD_c$) as well as final demand, which include consumption by households (QCD_c), corporations ($QED_{c,e}$) and government (QGD_c), gross fixed capital formation ($QINVD_c$) and stock changes ($dstocconst_c$). The products demanded are referred to as composite products (QQ_c) because these products include domestically produced goods (QD_c) and imports (QM_c), the aggregation of which is governed by a CES function (elasticity of substitution σ_c) and the relevant first order condition. The domestically produced goods sold on the local market (QD_c) represent only a share of the total domestic supply (QXC_c) because the rest of the domestically produced products are destined for the export market (QE_c). A CET function (elasticity of transformation Ω_c) and its first order condition govern the distribution of goods to the domestic and export market. Therefore the relative share of products sold on each market depends on the relative prices that can be obtained on each market. Product equilibrium conditions ensure that the domestic market clears, i.e. total demand and total supply of composite products (QQ_c) equate and all of the total domestic supply (QXC_c) is either exported or consumed domestically.

The model makes provision for multi-product industries. If one industry can produce more than one product it also implies that each type of product can be produced by more than one industry. The products produced by different industries ($QXAC_{a,c}$) can be aggregated across industries to obtain the total domestic production of a product (QXC_c). The aggregation takes place by means of a CES function (elasticity of substitution $\sigma_{a,c}$) and the optimal combinations are driven by industry prices and are determined by the first order condition. Each industry produces its combination of products in fixed proportions, i.e. the output of $QXAC_{a,c}$ is a fixed proportions (Leontief) aggregate of the output of each industry (QX_a). The assumption of fixed proportions (which is relaxed in Chapter 4) allows input use to be determined jointly with output levels, because inputs would increase in the same proportions as output. Domestic production (QXC_c), production for the domestic market (QD_c) and imports (QM_c) are all valued at basic prices because it is exclusive of product taxes, subsidies and margins. Exports (QE_c) are valued at basic prices in this study because there are no export taxes. Intermediate demand ($QINTD_c$) and final demand products (QCD_c, $QED_{c,e}$, QGD_c and $QINVD_c$) are valued at purchasers' prices because it is inclusive of product taxes and subsidies, and trade and transport margins.

The LES utility function is applied to determine consumption by households (QCD_c). It allows for subsistence expenditure and is therefore deemed a realistic assumption in a developing country such as South Africa with many poor consumers. The proportions of current expenditure on products defined to constitute subsistence consumption can also be varied in the model (McDonald, 2007).

Final demand by corporations ($QED_{c,e}$) and government (QGD_c), and demand for investment ($QINVD_c$), are fixed in real terms because demand for these purposes are not deemed to be price driven when compared to decision made by households and industries. This is also illustrated by the fact that the corporations, government and investment accounts in a SAM multiplier model would often be selected as the exogenous accounts. These final demand values can however be scaled uniformly by adjusters for purposes of experiment scenarios. Thus the impact of changes in real terms, as opposed to nominal terms, can be simulated. The SAM for purposes of this study does not include final demand for corporations, but corporations are included in the set 'e' together with other legal or social entities (excluding government) as depicted in Figure 2. It should be noted however that in the production, distribution and accumulation accounts of the South African Reserve Bank (SARB), the non-profit institutions serving households (NPISH's) are included with household data and therefore captured as part of the household accounts and not the corporations account.

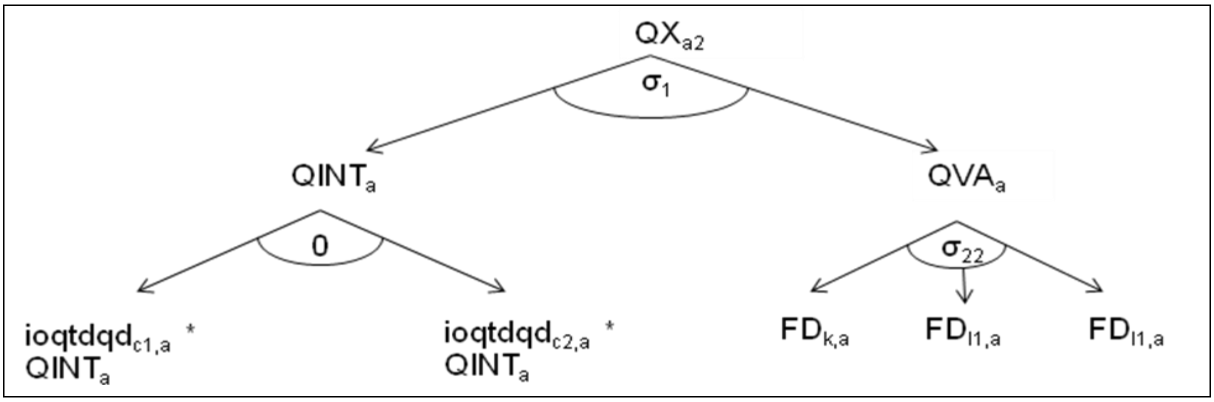
Figure 3: Quantity relationships for the base CGE model: Other



Source: McDonald (2007, Figure 3)

Production relationships are captured in a two-level nested production structure and the production relationships in terms of quantities are shown in Figure 4. At the top level of the nest industry output (QX_a) is a CES aggregate derived from aggregate intermediate inputs ($QINT_a$) and value added (QVA_a). The elasticity of substitution (σ_1) between aggregate intermediate inputs and value added can be different for each industry. Aggregate intermediate inputs are a Leontief aggregate of the (individual) intermediate inputs, which is indicated in the diagram for simplicity as only two inputs. Leontief aggregation implies zero substitution between individual intermediate inputs, i.e. inputs are combined in fixed proportions ($ioqtdqd_{c,a}$) per unit of aggregate intermediate input ($QINT_a$) and hence output. Aggregate value added (QVA_a) is a CES aggregate of the quantities of primary inputs ($FD_{f,a}$) demanded by each industry, with only three primary inputs indicated for simplicity. The elasticity of substitution (σ_{22}) between primary inputs ($FD_{f,a}$) can be different for each industry, but it remains fixed between any pair of factors. The first order conditions for the CES function is used to allocate the total supply of factors (FS_f) between competing industries and this allocation depends on the relative factor prices.

Figure 4: Quantity relationships for the base CGE model: Production



Source: McDonald (2007, Figure 4)

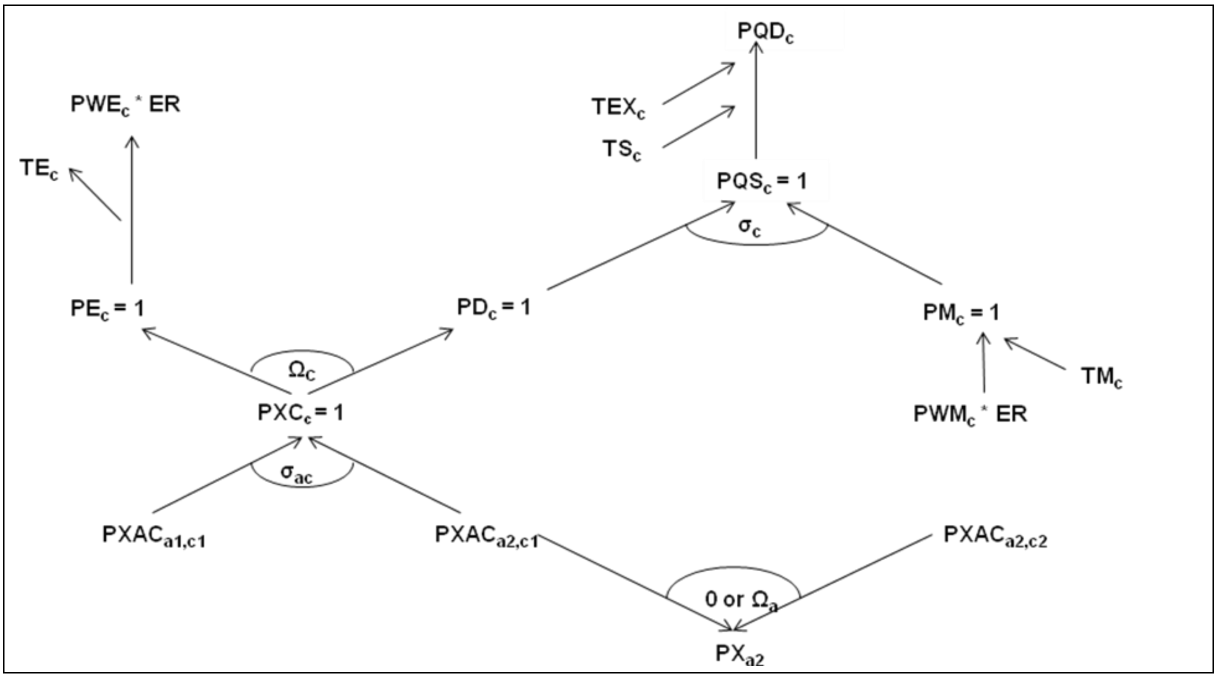
3.6.2 Price relationships

Figure 5 is the price diagram for the base model (McDonald, 2007). The supply prices of the composite products (PQS_c) are basic prices and are defined as the weighted averages of the domestically produced products that are domestically sold (PD_c) and the domestic prices of imported products (PM_c) because the composite products are CES aggregates of the domestically produced and imported products as discussed in the previous section. The weights are updated in the model through first order conditions for optima. It is shown in section 3.8.3 that the inclusion of the CES function for imports ensures that the law of one price is still adhered to. The inclusion of the CES function allows for substitution between domestically produced products and imported products, and it takes into account that prices of these products can change relative to each other. The prices of imported products (PM_c) are obtained by multiplying the world prices of products (PWM_c) and the exchange rate (ER) and then adding any *ad valorem* import duties (TM_c). The average prices exclude sales taxes, therefore when (*ad valorem*) sales taxes (TS_c) and excise duties (TEX_c) are included it reflects the composite consumer price (PQD_c). The composite consumer price (PQD_c) is valued at purchasers' prices (see Table 1) because it is inclusive of taxes, VAT, transport charges and margins. The explicit treatment of different taxes makes the model particularly useful for fiscal policy analyses.

The producer prices of products (PXC_c) are basic prices defined as the weighted averages of the prices received for domestically produced products sold on domestic market (PD_c) and the export market (PE_c). These weights are updated in the model through first order conditions for optima for the CET function to determine the quantity of production destined for the export market as discussed in the previous section. The domestic export prices (PE_c) are derived by multiplying the world price of exports (PWE_c) and the exchange rate (ER) less any exports duties due, which are defined by *ad valorem* export duty rates (TE_c). It is recognised that the price formation on exports does not adhere to the law of one price (see section 3.8.5) but this treatment allows for substitution between producing for the domestic and export markets, which greatly enhances the realism of the model results by avoiding extreme specialisation. There are no export duties included in the SAM used for this study. The average price per unit of output received by an industry (PX_a) is defined as the weighted average of the producer prices of each product produced by each industry ($PXAC_c$), where the weight of each product in the output mix remains constant. It is this assumption that is relaxed in the following chapter.

Tax rates are declared as variables in the model and have associated scaling factors which are declared as variables. This allows for equiproportionate and additive varying of a set of tax rates in scenarios for policy analyses. Similar scaling factors are also available for household and enterprise savings rates and inter-institutional transfers (McDonald, 2007).

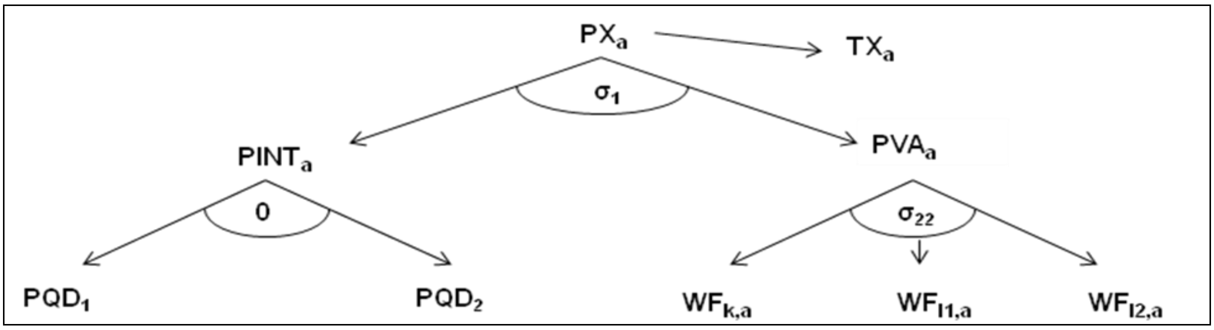
Figure 5: Price relationships for the base CGE model: Other



Source: Adjusted form McDonald (2007, Figure 1) to include *PXAC* in line with model code

Figure 6 shows the corresponding price relationships for the production nest. After paying indirect/production/output taxes (TX_a), the average price per unit of output received by an industry (PX_a) is divided between the price of aggregate value added (PVA_a) and the price of aggregate intermediate inputs ($PINT_a$). Price of value added (PVA_a) is the amount available to pay primary inputs ($FD_{f,a}$), whereas the price of intermediate inputs per unit of aggregate intermediate input ($PINT_a$) is defined as the weighted sums of the prices of the inputs (PQD_c) where the weights are the volume of each product in total intermediate input. The prices paid for intermediate inputs (*PQD_c* in Figure 6) are the same as that paid for final demands (*PQD_c* in Figure 5). This indicates that the law of one price is adhered to in domestic demand because both intermediate demand and final demand are valued at purchasers' prices. The factor prices (*$WF_{f,a}$*) are factor and industry specific, i.e. each industry could potentially pay a different rate to capital, different labour types and land.

Figure 6: Price relationships for the base CGE model: Production



Source: McDonald (2007, Figure 5)

3.6.3 Model closures

The choice of variables to fix as part of model closure are found in the model equations related to factors of production (labour, capital and land), capital (savings-investment), corporations, government and foreign exchange. The model closures can be changed relatively easily in the base model. The main alternatives for closures in the base model are the following:

- Full employment, where the supply of labour is fixed and the wage rate is flexible, vs. allowing for unemployment and keeping the wage rate fixed;
- Mobility of capital, where the total supply of capital is fixed but the supply at industry level is flexible, vs. immobility of capital achieved when fixing the supply at industry level as well;
- Mobility of land vs. immobility of land, where mobility of land would make economic sense if agricultural accounts were categorised by enterprise instead of agronomic region;
- Savings driven investment vs. investment driven savings;
- The government accounts closure can be a combination of fixing or flexing the following: tax rates, the volume of government expenditure or government expenditure as share of GDP, and the government deficit;
- A floating exchange rate and fixed current account deficit/surplus, vs. fixed exchange rate and flexible current account deficit/surplus;
- The consumer price index (*CPI*) is set as numéraire, vs. any other price or price index.

The closures selected for purposes of the sensitivity analyses and the case study are discussed in Chapters 6 and 7.

3.7 Modelling production in the base model

3.7.1 Overview

The discussion is based on the quantity relationships for production shown in Figure 4 and shows how the CES function is applied to the first and second level of the nest and how the Leontief production function is used in the second level of the nest. These functional forms have corresponding first order conditions which are also included in the CGE model to determine the optimal level of use and combination of factors. The optimal combinations of factors depend on relative prices, hence it relates to the price relationships for production shown in Figure 6. This discussion of production deals with both the input specification and the output specification of industries. The general theory of CES functions and the derivation of the first order conditions were presented in section 3.5.6. The discussion is based on McDonald (2007).

3.7.2 The CES function at the top level of production

In the CGE model a two-argument CES function is found at the top level of the production nest. Output by an industry *a* (QX_a) is determined by the aggregate value added (QVA_a), i.e. quantities of factors used, and aggregate intermediate input ($QINT_a$), where δ $(0 \leq \delta \leq 1)$ is the share parameter, ρ $(-1 \leq \rho \leq \infty)$ is the substitution parameter and ad $(ad > 0)$ is the efficiency or shift parameter.

$$QX_a = adx_a \left(\delta_a QVA_a{}^{-\rho_a} + \left(1-\delta_a\right) QINT_a{}^{-\rho_a} \right)^{-\frac{1}{\rho_a}} \tag{38}$$

The first order conditions defines the optimum ratios of value added to intermediate inputs and can be expressed in terms of the relative prices of value added (PVA_a) and intermediate inputs ($PINT_a$) (see section 3.5.6 for the derivation of the first order condition for the two-factor case):

$$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a}{\left(1-\delta_a\right)} \right]^{\frac{1}{\left(1+\rho_a\right)}} \tag{39}$$

3.7.3 The CES function at the second level of production

At the second level of the production nest is a multi-factor CES function that combines factors of production into aggregate value added (QVA_a):

$$QVA_a = adva_a * \left[\sum_f \delta_{f,a} * FD_{f,a}^{-\rho_a} \right]^{-\frac{1}{\rho_a}} \tag{40}$$

The first order condition for profit maximisation determines the wage rate of factors (WF_f) during optimal use of the available factors (FD_a), as shown in the following equation:

$$WF_f * WFDIST_{f,a} = PVA_a * QVA_a * \left[\sum_f \delta_{f,a} * FD_{f,a}^{-\rho_a} \right]^{-1} * \delta_{f,a} * FD_{f,a}^{(-\rho_a - 1)} \tag{41}$$

$WFDIST_{f,a}$ is the ratio of factor payments to factor from f industry a in cases where the same factor earns different wages in different industries (i.e. non-homogenous factors), PVA_a is the price of value added, i.e. the amount available to pay primary inputs.

3.7.4 Leontief production function at second level of production

In the base CGE model the Leontief technology assumption is used at the second level of the production nest for intermediate inputs, implying that intermediate inputs combine in fixed proportions ($ioqintqx_a$) to form aggregate intermediate input ($QINT_a$) for a particular industry a. This is therefore an example of multiple intermediate inputs each of which is used in fixed proportion relative to the level of output (QX_a). The Leontief production function assumes zero substitutability between inputs as indicated in the following equation:

$$QINT_a = ioqintqx_a * QX_a \tag{42}$$

Because aggregate intermediate input ($QINT_a$) is given by industry a, there is a need to transform this to intermediate input by product c ($QINTD_c$) using fixed ratios ($ioqtdqd_{c,a}$), as indicated in the following equation:

$$QINTD_c = \sum_a ioqtdqd_{c,a} * QINT_a \tag{43}$$

3.7.5 Transformation of output and intermediate inputs

In the production functions mentioned above the output is the output of an industry a (QX_a), i.e. the variable is defined over a. In CGE models that assume that each firm produces a single product c, each output by industry (QX_a) would equal the product output (QXC_c) by that industry. However in order to allow for multi-product firms, an assumption needs to be made with regard to the output composition of the firm, i.e. the ratio in which different products are produced by each industry. In the base model it is assumed that the product output ratio stays what it is in the base case regardless the output level of the industry, i.e. the Leontief assumption of fixed proportions. If the output ratio in the base case is represented by the parameter $ioqxacqx_{a,c}$, and if the composition of inputs stays the same when the level of output (QX_a) changes, then the quantity of each product produced by each industry ($QXAC_{a,c}$) is calculated as follows:

$$QXAC_{a,c} = ioqxacqx_{a,c} * QX_a \tag{44}$$

The Leontief assumption related to output composition is relaxed in Chapter 4.

3.8 Modelling trade in the base model

3.8.1 Overview

Models that allow for international trade in goods and services are referred to as open economy models. On the demand side imported goods therefore compete with domestically produced goods and on the supply side the domestically produced goods can be sold either domestically or exported. Armington (1969) suggested imperfect substitutability between imported vs. domestic goods. De Melo and Robinson (1985) proposed the use of the CET function in single country CGE models to similarly allow for imperfect substitutability between exported vs. domestic goods on the supply side. The CES function to determine substitutability on the demand side and the CET function to determine substitutability on the supply side are used in the base model.

3.8.2 The CES function for imports vs. domestic goods

The CES function used in trade in a single country CGE model is an example of a two-argument function, which allows for substitution between imports and domestically produced goods. Domestic demand by product (QQ_c) is determined by the quantities of imports (QM_c) and domestically produced goods (QD_c), the distribution or share parameter δ $(0 \leq \delta \leq 1)$, the substitution parameter ρ $(-1 \leq \rho \leq \infty)$ and the efficiency or shift parameter ad $(ad > 0)$. The Armington function assumes constant returns to scale and is presented by the following equation:

$$QQ_c = ac_c \left(\delta_c QM_c^{-\rho_c} + (1-\delta_c) QD_c^{-\rho_c} \right)^{-\frac{1}{\rho_c}} \tag{45}$$

The first order condition determines the optimal ratios of imports (QM_c) to domestic demand (QD_c) and is dependent on the relative prices of imported (PM_c) and domestically supplied (PD_c) products as well as the distribution or share parameter (δ) and the substitution parameter (ρ):

$$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{(1-\delta_c)} \right]^{\frac{1}{(1+\rho_c)}} \tag{46}$$

In global CGE models the Armington function is extended to incorporate multiple trading regions, hence all the variables have an extra dimension for the trading country or region. A similar extension can be incorporated in a single country CGE model if the SAM records trade with different trading partners explicitly. The extension of such a model is achieved through adding a multi-argument CES function to aggregate imports from different regions denoted by w ($QM_{w,c}$) into aggregated imports (QM_c). Aggregated imports then feed into the original two-argument CES function that distinguishes between aggregate imports and domestically produced goods. The extension of the model to allow for multiple trade partners also requires some modifications to the treatment of import duties. McDonald (2006) gives a description about the extension of a single country model to incorporate multiple trade partners.

3.8.3 The CES import function and the law of one price

One important assumption of a SAM was introduced in section 3.4.2, namely that the price of any factor or product is independent of the account that buys it, i.e. prices are homogeneous along each row of the SAM (Drud, Grais and Pyatt, 1986). This is also referred to as the law of one price (Pyatt 1994:10). McDonald (2011) demonstrates how the law of one price applies to imports governed by an Armington function (i.e. CES function). He shows that CES functions are not necessarily in violation of the law of one price because the price formation information is contained in the columns. The price definitions follow the law of one price if there is unique price formation for the individual components of the aggregate. Following the notation of the price diagram in Figure 5, the price of a composite product (PQS_c) is the weighted average of the domestic supply price (PD_c) and the import price (PM_c) (McDonald, 2011):

$$PQS_c = PD_c * {QD_c}/{QQ_c} + PM_c * {QM_c}/{QQ_c} \tag{47}$$

The equation confirms that the price of the composite product is determined by its individual cost components and because both the domestic supply price (PD_c) and the import price (PM_c) are basic prices, the price of a composite product (PQS_c) is also a basic price. The basic prices in the base CGE model are equal to one and all other prices are initialised relative to the basic prices. Price normalisation can take place in different places in the model, but then the model prices will not be consistent with the price relationships of the underlying SAM in terms of whether it is basic or purchasers' prices. In the case of the import price, the domestic import price (PM_c) is the basic price; hence the world import price (*pwm*) must be less than one because of the presence of an import tariff (TM_c) (McDonald, 2011):

$$PM_c = pwm_c * (1 + TM_c) * ER \tag{48}$$

Similarly if the price of a composite product (PQS_c) is the basic price, then the purchaser price (PQD_c) must be larger than one because of the presence of sales tax (or any other product taxes) (McDonald, 2011):

$$PQD_c = PQS_c * (1 + TS_c) \tag{49}$$

The base CGE model used for this study developed by McDonald (2007) subsumes the trade and transport margins into the products supplied to the economy. This implies that the law of one price is no longer adhered to because when the marketing margins are included in the domestic supply price (PD_c), then domestic supply is no longer valued at basic prices.

3.8.4 The CET function for export vs. domestic market

The CET function used in trade in a single country CGE model proposed by De Melo and Robinson (1985) is an example of a two-argument CET function, where the two arguments are the goods sold on the export market and goods sold on the domestic market. The CET function and first order condition determine the share of goods exported vs. the share of goods sold on the domestic market.

Domestic production by product (QQ_c) is determined by the quantities of exports (QE_c) and quantities sold on the domestic market (QDS_c), the distribution or share parameter γ ($0 \le \gamma \le 1$), the substitution parameter ρ ($-1 \le \rho \le \infty$) and the efficiency or shift parameter at ($at > 0$). The CET function for exports in the CGE model implies constant returns to scale and is presented by the following function:

$$QX_c = at_c \left(\gamma_c QE_c^{\rho_c} + (1-\gamma_c) QD_c^{\rho_c} \right)^{\frac{1}{\rho_c}} \tag{50}$$

The first order condition defines the optimum ratio of products to be sold on the export market vs. products to be sold on the domestic market, in relation to the relative prices of export (PE_c) and domestic prices (PD_c) (see section 3.5.7 for the derivation of the first order condition for the two-argument case):

$$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1-\gamma_c)}{(\gamma_c)} \right]^{\frac{1}{(\rho_c - 1)}} \tag{51}$$

Similar to the Armington import function, the export function can also be extended to incorporate multiple trading regions in global models by using nested functions CET functions. The first CET function distributes production between the domestic and export market and the second CET function distributes the products for the export market between different export destinations (McDonald, 2006).

3.8.5 The CET export function and the law of one price

McDonald (2011) demonstrates how the law of one price relates to exports that are governed by a CET function. For the law of one price to hold, all products in the same row account should have the same price. This suggests that a product should be sold for the same price when sold on the export market compared to when it is sold on the domestic market, because exports and domestic demand appears in the same row of the SAM. This could also be portrayed by including an additional product row account, with the only entry being exports as shown in Table 14. In this case heterogeneity of products for the domestic market and products for the export market is assumed. This is mirrored by two different columns of product accounts, where the new export products' column captures supply of export products by industries and any marketing margins and export taxes on exports payable to government. The inclusion of only one industry account implies an assumption of single technology, i.e. that the basic price of products for the domestic market and the export market are the same. Differentiation is therefore only based on the marketing margins and taxes that are different for the two product groups.

Table 14: SAM with separate export product account

	Products Domestic	Products Export	Indust	Factors	Hholds	Govt	Capital	RoW	Total
Products domestic			310		310	105	70		795
Product export								135	135
Industries	610	125							735
Factors			405						405
Households				405					405
Government	35	10	20		40				105
Capital					55			15	70
Rest of World	150								150
Total	795	135	735	405	405	105	70	150	

Source: Own

Export and domestically consumed products are actually not modelled as if they appear in the same row because of the distinctly different behavioural assumptions for each. They are indeed modelled as reflected in the SAM layout above. Each industry produces a composite product (QXC_c) and this is then distributed to either the domestic market (QD_c) or the export market (QE_c). The price of the composite good (PXC_c) is dependent on the distribution ratio between the two markets:

$$PXC_c = \left(PD_c * {QD_c}/{QXC_c}\right) + \left(PE_c * {QE_c}/{QXC_c}\right) \tag{52}$$

Although the above equation would hold in the base case, there appears to be a violation of the law of one price in the event of simulations which lead to relative price changes (McDonald, 2011). For example if the world price of exports (pwe_c) is changed in a simulation, for a given export tax (TE_c) the domestic price of exports (PE_c) will change relative to the price of domestic products (PD_c):

$$PE_c = pwe_c * (1 + TE_c) * ER \tag{53}$$

This indicates a violation of the law of one price because although the domestic price of exports (PE_c) is regarded as a basic price in the model, this basic price is not determined by the underlying cost structure of the industry producing the good as in the case of the domestic price (PD_c), which is also a basic price. The purchasers' prices of both will be different because of the different marketing margins and product taxes as reflected in the SAM above.

McDonald (2011) indicates that when introducing a CET function on exports, the law of one price will hold under the assumptions that the input structure of production for the domestic market is exactly the same as the input structure of production for the export market and that the input composition will not change as the output composition changes. Under these assumptions the basic price of products for both markets will be the same and only one industry account is necessary. But in reality the purchasers' prices of the same product sold on the domestic market is likely to differ from the purchasers' prices of the same product sold on the export market due to different trade margins and taxes. McDonald (2011) therefore concludes that when a model includes a CET function to model export supply, it violates the law of one price because the product produced for the domestic market is differentiated from the same product produced for the export market, even if only because of the margins, yet the implicit assumption in the CGE model is that both use the same technology and hence both have the same cost structure. The base model also contains a CET function for exports and therefore also suffers from a violation of the law of one price, but with the benefit of avoiding extreme specialisation and obtaining more realistic results.

3.9 Modelling household consumption in the base model

3.9.1 Overview

Single country CGE models are typically calibrated with SAMs that contain a number of household categories. This is different from global CGE models, for example the GTAP (Global Trade Analysis Project) model, which contain a single institutional account per country/region, but data on numerous countries/regions to capture detail on international trade. Single country CGE models are often designed to measure the implications of policies on redistribution, hence the detail on households.

The base model makes use of a LES functional form to estimate household demand. The LES functional form for household demand is the only non-linear functional form in the CGE model used in this study that is neither a CES nor a CET function. It is briefly explained here in the context of a CGE model.

3.9.2 Linear Expenditure System (LES)

The LES function is a variant of the Cobb-Douglas and CES function. Contrary to the Cobb-Douglas function, the LES function does not assume unit income elasticity. For a LES utility function the household consumption demand (QCD_c) consists of two components, minimal or subsistence demand (*qcdconst*) and discretionary demand (McDonald, 2007):

$$QCD_c = \frac{\left(\sum_h \left(PQD_c * qcdconst_{c,h} + \sum_h beta_{c,h} * \left(HEXP_h - \sum_c \left(PQD_c * qcdconst_{c,h}\right)\right)\right)\right)}{PQD_c} \tag{54}$$

Where discretionary demand is the marginal budget shares (*beta*) spent on each product out of residual income, i.e. household consumption expenditure ($HEXP_h$) less total expenditure on subsistence demand.

Following Annabi, Cockburn and Decaluwé (2006), but using the current symbols, the income elasticities for the LES, which are no longer unitary, are calculated as follows:

$$\varepsilon_{h,c} = \frac{beta_{c,h} * HEXP_h}{PQD_c * QCD_c} \tag{55}$$

And the own price elasticities are:

$$\varepsilon_{p,c} = \frac{(1 - beta_{c,h}) * \sum_c \left(PQ_c * qcdconst_{c,h}\right)}{QCD_c} - 1 \tag{56}$$

The calibration of the LES is more complicated than the Cobb-Douglas and CES functions because minimal consumption levels must also be determined. Annabi, Cockburn and Decaluwé (2006) present two methods to estimate minimal consumption levels. The first method entails the estimation of the discretionary consumption budget shares and the substitution of these shares into the price elasticity function; whereas the second method entails the estimation of Frisch parameter (ratio of total to discretionary consumption) for inclusion in the demand function to obtain the minimal consumption level. The second method is used in the base model.

3.10 Summary and conclusions

This chapter presented some theoretical background about CGE models, model types and functional forms to place the model adjustments discussed in the next chapter in perspective. The section on the SAM as an approach to modelling focuses on the role of a SAM in presenting an economic model, specifically the transaction-value form of the model. Although the transaction-value of the model is not explicitly given in the current study, the behavioural assumptions of the CGE model are set in a SAM framework. The SAM approach to modelling is particularly useful in the discussion on prices and accounting identities in order to derive the column and row summation equations and closure rule equations, and to illustrate the interdependence of prices and the law of one price.

A short literature review highlights some of the main functional forms that are applied in CGE modelling. The main areas in which non-linear functional forms are used include production, trade and household demand. Although flexible functional forms such as the Translog, AIDS, CRESH, etc. are less restrictive in their assumptions about elasticities of substitution compare to the functional forms such as the Cobb-Douglas, LES and CES functions, there is a trade-off in terms of global regularity. Some of the more recent literature indicates developments to try to impose regularity, but this once again comes at the expense of flexibility. The LES and nested CES (and CET) functions are therefore popular because they are less complex than flexible functional forms and at the same time more stable. The CES and CET functions are discussed in more detail to show the derivation of the first order conditions which are used to determine optimal ratios of products or factors, given relative prices. The CES and CET functions and their first order conditions, as they appear in the base model, are presented.

The quantity and price relationships in the base model used for this study are also discussed. The model contains nested Leontief and CES aggregation functions for production, CES and CET functions for international trade and a LES function for household demand. The production relationships are represented by a two level nested structure. In the discussion on the law of one price it was shown that in some instances the CET function for trade violates the law of one price, but the CES function does not violate the law of one price regardless of whether it is used in the context of import substitution or production.

In models with multi-product industries the transformation of aggregate industry output into different products is often governed by fixed proportions, i.e. the level of output of secondary products is always in the same proportion to the level of output of the main product that is produced by an industry. Therefore, if the products produced are not necessarily related as part of the production process the assumption of fixed proportions is restrictive. When agricultural production is defined by region, the expectation is that the composition of agricultural products produced in each region will be flexible and dependent on prices. The agricultural sector therefore provides a good example of price responsive multi-product industries. The base model assumes that the proportionate contribution of each product in total industry output remains constant, but this assumption is relaxed in Chapter 4.

4 Modelling price responsive output composition

4.1 Introduction

Consider an agricultural unit that produces maize and sheep. If the world export price (*pwe*) for maize increases, then it is expected that the world import price (*pwm*) of maize will also increase as a result of an appreciation of the exchange rate. This implies that the domestic export price (*PE*) and import price (*PM*) of maize will increase *ceteris paribus*; and hence domestic prices such as the producer price (*PXC*) and the domestic supply price (*PD*) of maize will also increase because producers will favour the export market rather than the domestic market. It is anticipated that this increase in the domestic price of maize relative to that of sheep will cause a *marginal* shift away from sheep production towards maize production. The domestic maize price increase could also have the indirect impact of causing an increase in the price of animal feed. Since animal feed is an intermediate input into the production of sheep, a decrease in sheep production can be expected. The base model would however show an increase in sheep production proportionate to the increase in maize production given an increase in the world price of maize for that particular agricultural unit. This is an unrealistic result and one that is achieved because the relative quantities of maize and sheep that are produced are assumed to remain fixed.

The technical consideration of the described situation is that aggregate output of an industry (QX_a) is defined by industry and not by product (e.g. QXC_c). In CGE models which assume that each industry produces a single product, the value of each industry's output would be equal to the value of the single product produced by that industry. However in order to allow for multi-product industries, an assumption needs to be made with regard to the output composition of the industry, i.e. the ratio in which the different products are produced by each industry. The base model (McDonald, 2007) and the widely used standard IFPRI model (Löfgren et al., 2002) provide examples of models that assume that product ratios in total output stay constant regardless the output level of the industry, i.e. the Leontief assumption of fixed proportions.

The next section commences with a short overview of supply response models and some evidence from the literature of models that do not assume fixed output proportions. It also discusses some theoretical arguments concerning the handling of transformation, specifically related to input-output separability and the law of one price. In section 4.3 the assumption of fixed output composition in the base model is relaxed through the inclusion of a CET function on industry output and the first order condition for determining optimal product ratios given relative price changes. The discussion starts with a brief overview of the adjusted model with reference to the price and quantity diagrams, focusing on adjustments from the base model that was discussed in detail in the previous chapter. The algebraic statement of the main model adjustments are presented, followed by a discussion of the need for and implementation of set controls for making the new equations operational in the model. The section concludes with a discussion of the calibration of model parameters that becomes necessary as a result of the adjustments to the core equations of the model. The model code for the equations of the adjusted CGE model is included in the addendum, with adjustments compared to the base model clearly marked. Section 4.4 concludes the chapter.

4.2 Transformation of output and intermediate inputs re-examined

4.2.1 Early supply response models

Agricultural producers respond to price incentives when taking production decisions, but at the same time it is recognised that the magnitudes of these responses are influenced by numerous factors (Yu, Lui and You, 2010; Hueth and Ligon, 1999). CGE models are economy-wide models, but they are underpinned by micro-economic theory. Agents optimise behaviour subject to constraints, hence the models contain, amongst other, demand systems for households and production or supply systems for firms. These systems are often developed in a partial equilibrium framework and the supply function can be estimated econometrically or by simulation models by generating input and output quantity data under the assumption of a profit or utility maximisation objective of producers subject to certain constraints (Griffith, I'Anson, Hill and Vere, 2001).

Some of the earliest analyses on supply response by the agricultural sector are referenced to Cassels in 1933 (Taylor and Shonkwiler, 1985) who looked at the nature of statistical supply curves. Another main contribution in this field appeared about twenty years later in the form of Nerlove's model of agricultural supply response (Taylor and Shonkwiler, 1985; Nerlove and Addison, 1958). Nerlove's model provided a significant base for debate and further model development regarding this issue, as indicated by the literature in this regard. Subsequent developments in supply side models moved from focusing on individual products to multi-product systems. Powell and Gruen (1968) derived the CET function and derived the elasticity of transformation analogously with the elasticity of substitution in the CES function. They derived a CET transform of a linear supply system and applied it in an econometric model to estimate supply response for multi-product agricultural industries in Australia that produce mainly wool, wheat and coarse grain.

Colman (1983) warns against confusing supply response and production response, where supply would indicate total supply of a product on a market, inclusive of imports and stock changes, and where production would be actual output of producers. According to Hallem (1998) agricultural supply response can imply the change in supply in response to changes in both output and input prices, and the output composition of agricultural industries. The focus in this study is on changes in output composition in response to changes in output prices.

Griffith et al. (2001) indicate that supply response can take two forms: a) an expansion effect when there is a net increase in output in one or more products, or b) a transformation effect, which represents change in the output composition of the products along the production frontier. The latter is the result of greater relative profitability of the product whose price has increased relative to the prices of other products.

4.2.2 Examples of output transformation in applied CGE modelling

Goldin and Knudsen (1990) indicate that they followed the multi-market model approach which views agricultural production as a sectoral level strategic decision where output composition and input demand are jointly decided. They do not however give any details about how they achieved this. Lee (2002) specifies a CET function to govern the revenue-maximising output composition for all industries, but confirms that most industries in her database are single product industries. Peerlings (1993) uses a CET function to disaggregate the aggregate output by the dairy industry into milk and cattle respectively.

The ORANI-G model, which is a general equilibrium model for the Australian economy, contains a CET transformation of aggregate industry output to multiple products (Horridge, 1993). This is a simplification of the original ORANI model which contains a nested two level Leontief-CRETH transformation system for domestic agricultural supply. In the original ORANI model agricultural industries are defined by region and each region produces a number of composite goods, e.g. wool and sheep. The products in the composite good is produced in fixed proportions (Leontief assumption), but the composition of the composite goods produced by each agricultural industry can change as determined by CRETH elasticities of transformation (Dixon, Parmenter, Sutton and Vincent, 1982). An assumption in both models (using CET and CRETH) is input-output separability. Adams (1987:214) explains input-output separability in the ORANI model as follows: “Each industry is viewed as purchasing a level of activity (or a production possibility frontier). Inputs are regarded as non-specific to outputs, since the former merely produce a capacity for production that can be transformed into a variety of products. Thus an industry’s input and output decisions are treated separately...”. Dixon et al. (1982) place this in perspective by indicating that an agricultural producer will have a certain level of inputs available, e.g. tractors, fertiliser, etc. giving him the ability to change output composition over one year in response to changes in expected product prices. Following Oktaviani, Hakim, Siregar and Sahara (2004) and Pyatt (1994) the production function can be defined as the following:

$$0 = F(x, z) \tag{57}$$

Where x represents output and z represents inputs. Then the simplifying assumption of separability is the following:

$$q(x) = X1TOT = h(z) \tag{58}$$

Where *X1TOT* is an index of industry activity. The assumption of separability implies that the decision as to what combination of products to produce is separate from the decision as to what combination of inputs to use. Hence input prices do not affect output combinations and output prices do not affect input combination, but both affect the level of output. The implication is that demand functions and supply functions, although dependent on activity level, contain only input prices or product prices, but not both. The assumption of separability therefore simplifies the model (Oktaviani et al., 2004). Given the assumption of separability, then profit maximising behaviour will be the selection of the output composition that maximises total revenue (Pyatt, 1994). If a product transformation function is assumed to be completely strongly separable, the marginal rate of technical transformation between any two outputs is independent of the supply of another output and complete strong separability implies that the product transformation function is the aggregate quantity index of the input (Berndt and Christensen 1973, in Peerlings 1993:28). Separability between inputs and outputs in the agricultural sector can be tested with econometric analyses if time series data are available (Shaik and Helmers, 1998). In two instances the assumption of separability becomes potentially problematic. First, when transformation takes place between substantially different products that require notably different inputs. Second, when the elasticity of transformation is relatively high and large changes in the shares of individual products in aggregate output are observed.

Gohin and Gautier (2003) indicate the use of regular-flexible functional forms for the specification of production technologies, but although they give no details about the model in this regard, they refer the reader to the Gohin and Meyers (2002) for details about the model. Gohin and Meyers (2002) apply a CET to allocate the aggregate industry output to different products. Their database contains agricultural sectors classified according to main products and some of these sectors produce a single product, while others produce multiple products. They also make adjustments on the input side by not assuming complete separability between inputs and outputs. Gohin and Meyers (2002) use as starting point fixed coefficients for intermediate inputs, but in the alternative specification they make use of the Nonseparable N-Stage Constant Elasticity of Substitution (NNCES) functional form to allow for substitution between feed ingredients for nine livestock sectors and the compound feed industry.

Gelan and Schwarz (2008) investigate the effects of single farm payments on Scottish agriculture. They include a CET function for output transformation following Gohin and Gautier (2003) and find that despite the fact that reliable supply elasticities were unavailable, their model yields policy effects that are likely to represent behaviour of profit maximisation farmers. The model results were however found to be sensitive to the choice of supply elasticities. Besides the CET function, they also replace the CES function, which combines products from different industries to form an aggregated domestic product, with a Leontief function. Their argument is that the composite product comes from different industries, i.e. different decision units. It has been shown however by McDonald (2011) that the CES does not violate the law of one price because the price of the composite good is merely a weighted average of the cost of separate products (see section 3.8.3). Another difference between the base model in this study and the model by Gelan and Schwarz (2008) is that they use a Cobb-Douglas function to derive aggregate value added (at the second level of the production nest), as well as to aggregate intermediate inputs with value added (at the top level of the production nest). In the model used in this study, a CES aggregation was used in both instances. Their reported first order condition for output transformation is written in terms of price ratios, whereas the first order condition for output transformation for this study is written in terms of quantity ratios. This is just a matter of preference, with no impact on the results as indicated in section 4.3.2.

Cattaneo (2001) provides an example of a Translog transformation function between aggregate industry output and multiple products, specifically for multi-product agricultural industries. On the production side a nested structure is adopted. Aggregate real value added is a CES aggregate of primary factors of production, whereas real value added and intermediate inputs are aggregated using fixed-coefficients (Leontief assumption) to form aggregate output by industry. Aggregate output of the multi-product agricultural industry is then transformed into different products using a Translog function under the assumption of separability.

4.2.3 CET output transformation functions and the law of one price

McDonald (2011) demonstrates how the law of one price applies to multi-product industries where output is governed by a CET function. He finds that the CET function does not adhere to the law of one price. The discussion starts from the single product case.

Following McDonald (2011), in the case of every industry producing only a single product and every product is produced by only one industry, i.e. a diagonal square supply matrix, then the basic price of the product is the average revenue and the average cost of the product is the weighted sum of the costs of producing the product. The first line of the following equation indicates the accounting identity that the value of industry output at basic prices is equal to the cost of producing the output:

$$\begin{aligned} PX_a * QX_a &= \sum_c SAM_{c,a} + \sum_f SAM_{f,a} + SAM_{"prodtax",a} \\ &= \sum_c \left(PQD_c * ioqx_{c,a} * QX_a\right) + \sum_f \left(W_{f,a} * FD_{f,a}\right) + \left(PX_a * QX_a * tx_a\right) \\ &= \sum_c \left(PQD_c * ioqx_{c,a} * QX_a\right) + \left(PVA_a * QX_a\right) + \left(PX_a * QX_a * tx_a\right) \\ PVA_a &= PX_a * \left(1 - tx_a\right) - \sum_c \left(PQD_c * ioqx_{c,a}\right) \end{aligned} \tag{59}$$

Where the cost components comprise intermediate inputs ($\sum_c SAM_{c,a}$), factor payments ($\sum_f SAM_{f,a}$) and net production taxes ($SAM_{"prodtax",a}$). The behavioural assumptions, as represented by the production function, indicate the assumptions about the weights in determining the average cost. In the above equation it is assumed that intermediate inputs are employed in fixed quantities per unit of output according to a Leontief technology assumption. In this case relative quantities of intermediate input use are insensitive to price changes and the average revenue (PX_a) can be written in terms of the price of value added (PVA_a) as indicated in the last line of the equation, where PX_a is the industry output price, QX_a is the quantity produced, $ioqx_{c,a}$ is a matrix of intermediate input (Leontief) coefficients, $W_{f,a}$ is the price of factor *f* used by industry *a*, $FD_{f,a}$ is the amount of factor *f* employed by industry *a* and *tx* the *ad valorem* production tax rate. It is desirable to write the industry output price (PX_a) in terms of the price of value added (PVA_a), since the latter can be used in the first order condition for profit maximisation.

If the production function is a constant returns to scale Cobb Douglas function, then the first order condition for profit maximisation is the following:

$$W_{f,a} * FD_{f,a} = PVA_a * QX_a * alpha_{f,a} \tag{60}$$

Where $alpha_{f,a}$ represents the share of factor f in industry a. Equations 59 and 60 together ensure that all costs are accounted for in the calculation of the average revenue for each industry (McDonald, 2011). Similarly, all costs are accounted for if the production function is a constant elasticity of substitution (CES) function with the following first order condition:

$$WF_{f,a} * FD_{f,a}^{(\rho_a+1)} = PVA_a * QVA_a * \left[\sum_f \delta_{f,a} * FD_{f,a}^{-\rho_a} \right]^{-1} * \delta_{f,a} \tag{61}$$

Where $\delta_{f,a}$ is the distribution or share parameter, ρ_a is the elasticity parameter, QVA_a is the quantity of value added and the interpretation of the other variables are similar as for equation 60.

In the case of secondary production, then similar products produced by different industries can be aggregated using a CES function under the assumption that similar products are differentiated by the industries that produce them (as indicated by the difference in average prices). McDonald (2011) shows by means of a CES function on imports that CES functions do not violate the law of one price. He also confirms that the law of one price is maintained in the case of aggregating similar products produced by different industries. Although the basic price of the composite product is the weighted average of the basic prices of its components, it is still a unique price, because the basic prices of its components are uniquely determined.

McDonald (2011) extends the discussion on secondary production to focus on the fact that one industry produces various products. The products can include any type of secondary product as indicated in section 2.2.5 in addition to the main product. Since these products are produced by one industry there is only one production function and hence one cost structure to determine a single price (average revenue) that will be relevant to all the products produced by the particular industry. A fixed input structure would imply that the output composition should also remain constant and, as mentioned, models such as the standard IFPRI model (Löfgren et al., 2002) and the PROVIDE model (PROVIDE, 2005b) make the assumption that products are produced in fixed proportions. In the presence of the assumption of fixed proportions output mix, the accounting identity in the *ex post* equilibrium is therefore maintained because the average revenue adjusts in response to input price changes.

Although the assumption of producing products in fixed proportions follows from the assumption of a fixed input structure, a constant output composition is not always realistic, for example for agricultural industries with mixed farming practices, where different agricultural products produced are neither by-products nor joint products. In mixed farming operation the decision to produce a particular product is often driven by price (McDonald, 2011). If industry output is aggregated by a CET function rather than assuming fixed output proportions to derive more realistic results by allowing producers to respond to changes in relative prices, then the inclusion of a CET function will violate the law of one price because the output composition can vary but this is not accompanied by a change in input structure (McDonald, 2011). This points towards an assumption of separability because the decisions as to what combination of products to produce are made separately from the decision as to what combination of inputs to use, implying that output prices do not affect input combinations. Besides the direct impact of changing the output composition of individual industries, the inclusion of a CET function on industry output also impacts on the indirect results because when the output composition of an industry changes the domestic demand for the outputs of other industries will be affected (McDonald, 2011). Although the CET on output places ‘strain’ on the specification of production functions, their use may be justified because of what is gained through more realistic results. McDonald (2011) warns that the seeming solution of using an input-output table rather than a supply and use table does not solve the problem, but only hides it. This is because input-output tables are merely linear transformations of supply and use tables and a transformation cannot address unresolved issues in the original data as indicated in section 2.3.4.

4.3 An adjusted CGE model with flexible output composition

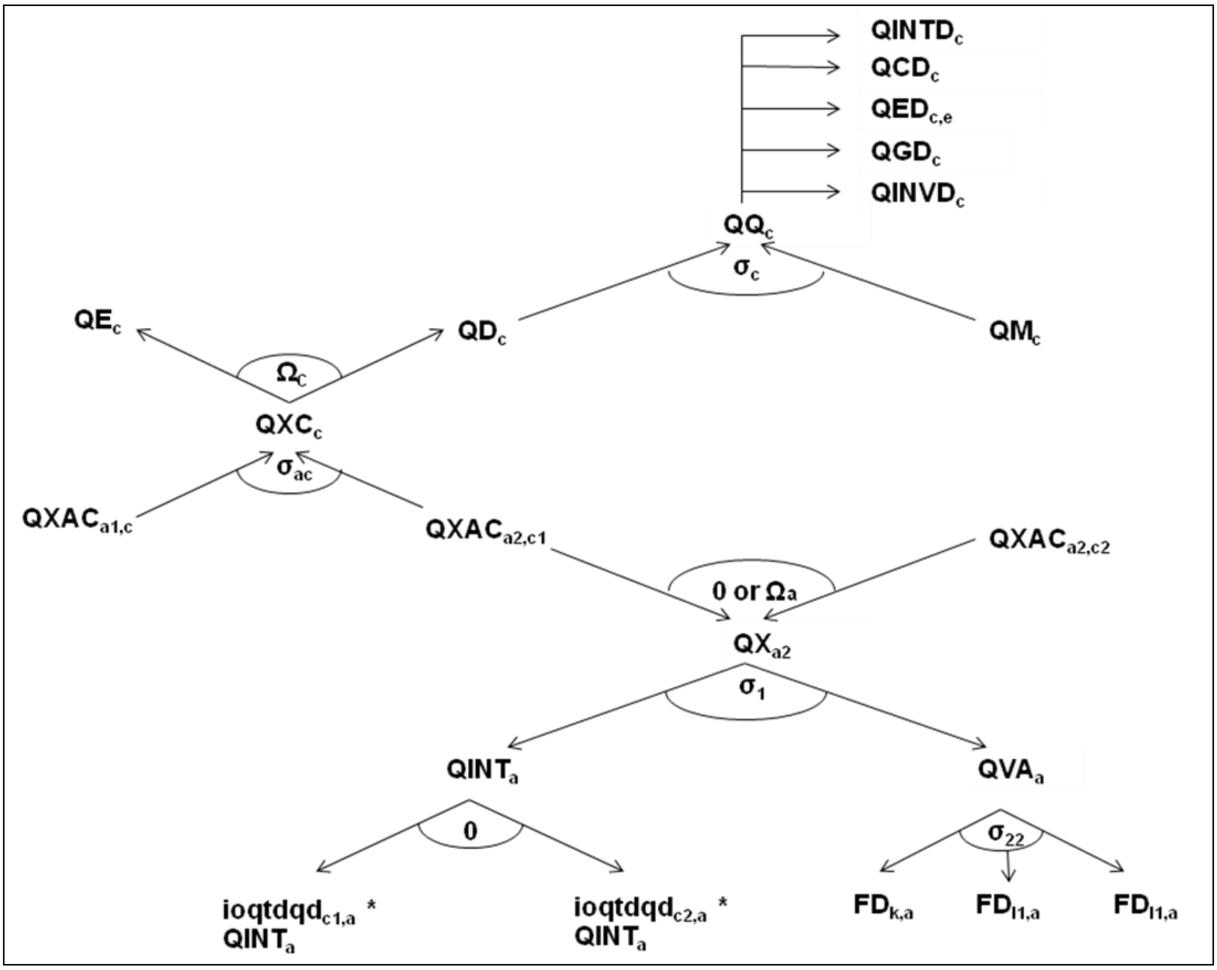
In the previous section it was indicated that the literature provides limited evidence of instances where the assumption of fixed output composition in CGE models have been relaxed. The use of a CET function in this regard is most common. In all instances however the authors have mentioned the assumption of separability between inputs and outputs. This is a model simplifying assumption and although seemingly commonly accepted in the literature, it does not negate the fact that the CET function violates the law of one price.

The model used in this application uses a CET function to relax assumption of fixed output composition under the assumption of separability between inputs and outputs. The model adjustments are discussed in more detail in this section.

4.3.1 Behavioural relationships

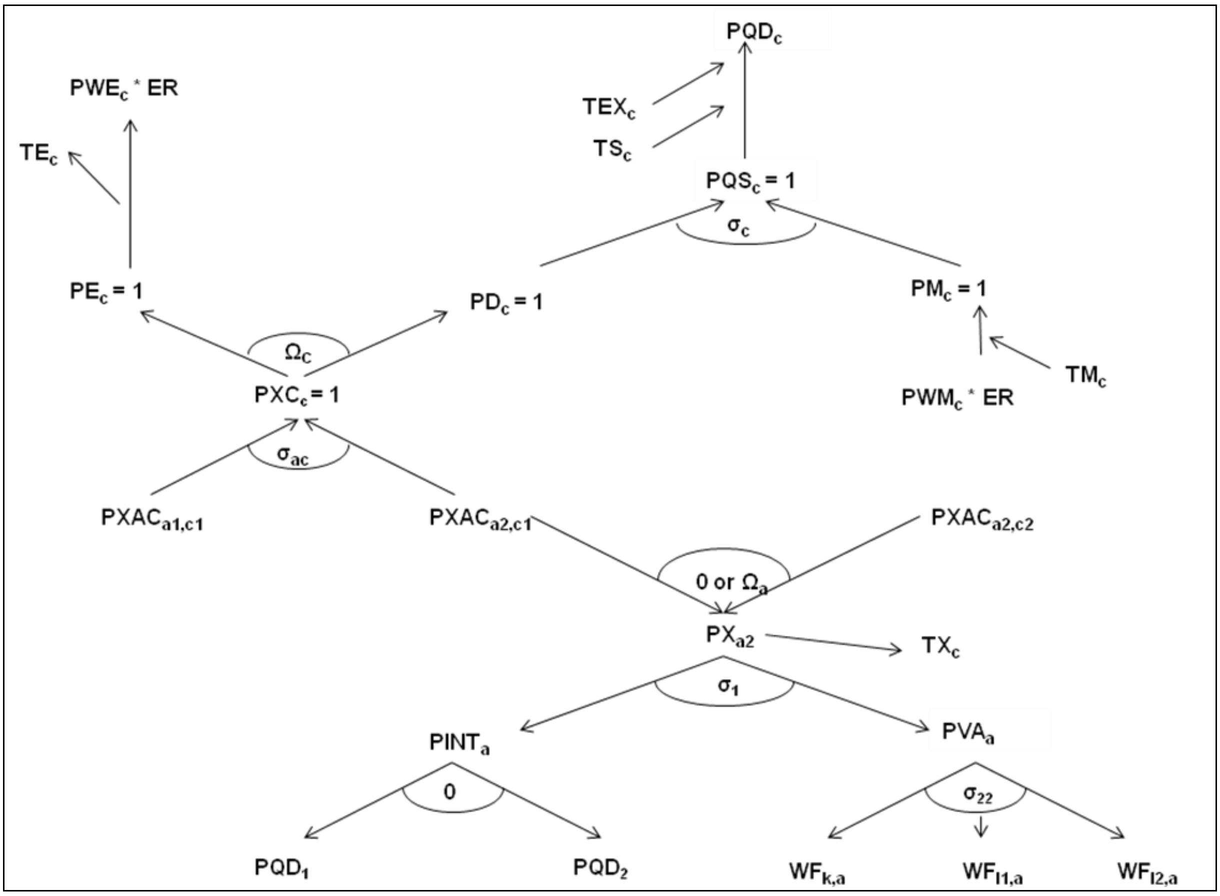
The quantity relationships for the adjusted model are shown in Figure 7. The inclusion of the *omega* sign (Ω_a) above QX_a (industry output) is the only difference in the diagram compared to Figure 3, but it has numerous implications for the model. It shows the possibility of a CET aggregation as an alternative to the Leontief aggregation for product output by industries ($QXAC_{a,c}$). When *omega* is set to zero the model reverts to the Leontief aggregation. Certain industries will produce secondary products, but these will be assumed to be in fixed proportions to each other, so the option of Leontief production in terms of output will still be relevant and realistic in this case. However, in the case of industries that can change their output composition in response to changes in relative prices the CET function can be implemented. It should be noted that one underlying assumption is that, regardless the price incentives as a result of a particular shock to the economy, industries will not start producing a product which it did not produce in the base data. The other assumption is separability between inputs and outputs. Aggregate industry output (QX_a) is produced by combining aggregate intermediate inputs ($QINT_a$) and aggregate value added (QVA_a) per industry according to a CES function as indicated by the elasticity of substitution (σ_1).

Figure 7: Quantity relationships for the adjusted CGE model



The price relationships are shown in Figure 8. The same elasticities that govern the underlying quantity relationships are shown in this figure. The average price per unit of output received by an industry (PX_a) is defined as the weighted average price of all the products produced by the industry ($PXAC_{a,c}$) where the weight of each product is the quantity share of the product in the industry's output. For industries whose output composition is governed by a CET function, the weights will be variable whereas it will be constant for those industries whose output is governed by a fixed Leontief function as indicated by the respective *omega* sign (Ω_a) and zero above PX_a in the figure.

Figure 8: Price relationships for the adjusted CGE model



4.3.2 Main equations to capture flexible output composition

In the base model the output by industry (QX_a) is transformed to indicate output of each product by each industry ($QXAC_{a,c}$) by means of fixed proportions of products for each industry ($ioqxacqx_{a,c}$) as determined by the base SAM, as shown in the following equation:

$$QXAC_{a,c} = ioqxacqx_{a,c} * QX_a \tag{62}$$

If the assumption of fixed proportions is relaxed to allow multi-product industries a degree of substitution between which products they produce in response to price changes, the transformation of industry output (QX_a) into different products for each industry ($QXAC_{a,c}$) can be governed by a CET function as opposed to the fixed shares, as indicated in the following equation:

$$QX_a = ati_a * (\sum_c gammai_{a,c} * QXAC_{a,c}^{rhoti_a})^{1/rhoti_a} \tag{63}$$

Where ati_a is the shift parameter, $gammai_{a,c}$ is the share parameter and $rhoti_a$ is the elasticity parameter for the CET function. The elasticity of transformation indicates the ease with which an industry can switch production between different products. The relationship between the elasticity parameter (*$rhoti_a$*) and the elasticity of transformation (*omegaout*) is the following:

$$rhoti_a = (1/omegaout_a) + 1 \tag{64}$$

The CET function is included in the model for clarity, but the model does not require the inclusion of the actual CET function in equation 63 because the level of output (QX_a) is already determined by the CES aggregation of aggregate value added and aggregate intermediate inputs (not shown here). However, the model does require the first order condition to determine the optimal levels of production of each product by each industry, given relative product prices. The first order condition ensures that the industry price of an individual product ($PXAC_{a,c}$) is equal to the (aggregate) industry price (PX_a) multiplied by the derivative of the production function with respect to that particular product ($QXAC_{a,c}$). The first order condition expressed in terms of the price ratio of each individual product ($PXAC_{a,c}$) to aggregate industry price (PX_a), is the following:

$$\begin{aligned}
PXAC_{a,c} &= PX_a * \frac{\partial QX_q}{\partial QXAC_{a,c}} \\
&= PX_a * \left(1/rhoti_a\right) * ati_a * (\sum_c gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a})^{1/rhoti_a - 1} \\
&\quad * \left[\left(rhoti_a\right) gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a - 1}\right] \\
&= PX_a * ati_a * (\sum_c gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a})^{1/rhoti_a - 1} \left[gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a - 1}\right] \\
&= PX_a * ati_a * (\sum_c gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a})^{1/rhoti_a} (\sum_c gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a})^{-1} \\
&\quad * \left[gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a - 1}\right] \\
&= PX_a * QX_a * (\sum_c gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a})^{-1} \left[gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a - 1}\right]
\end{aligned} \tag{65}$$

Given the following:

$$\begin{gathered}
QX_a = ati_a \left[\sum_c gammai_{a,c} * QXAC_{a,c}^{rhoti_a}\right]^{1/rhoti_a} \\
\left[\sum_c gammai_{a,c} * QXAC_{a,c}^{rhoti_a}\right] = \left[\frac{QX_a}{ati_a}\right]^{rhoti_a}
\end{gathered} \tag{66}$$

Then equation 65 can be rewritten in terms of the quantity ratio of each individual product ($QXAC_{a,c}$) relative to the aggregate output (QX_a):

$$
\begin{aligned}
PXAC_{a,c} &= PX_a * QX_a * \left[\sum_c gammai_{a,c} * QXAC_{a,c}^{rhoti_a}\right]^{-1} * \left[gammai_{a,c} * QXAC_{a,c}^{rhoti_a-1}\right] \\
\frac{PXAC_{a,c}}{PX_a} &= QX_a * \left[\left[\frac{QX_a}{ati_a}\right]^{rhoti_a}\right]^{-1} * \left[gammai_{a,c} * QXAC_{a,c}^{rhoti_a-1}\right] \\
&= QX_a * \left[\frac{QX_a}{ati_a}\right]^{-rhoti_a} * \left[gammai_{a,c} * QXAC_{a,c}^{rhoti_a-1}\right] \\
&= \frac{QX_a}{QX_a^{\rho} * ati_a^{-rhoti_a}} * \left[gammai_{a,c} * QXAC_{a,c}^{rhoti_a-1}\right] \\
&= \frac{\left[gammai_{a,c} * QXAC_{a,c}^{rhoti_a-1}\right]}{QX_a^{rhoti_a-1} * ati_a^{-rhoti_a}} \\
&= \frac{gammai_{a,c}}{ati_a^{-rhoti_a}} * \frac{QXAC_{a,c}^{rhoti_a-1}}{QX_a^{rhoti_a-1}} \\
&= \frac{gammai_{a,c}}{ati_a^{-rhoti_a}} * \left[\frac{QXAC_{a,c}}{QX_a}\right]^{rhoti_a-1} \\
\left[\frac{QXAC_{a,c}}{QX_a}\right]^{rhoti_a-1} &= \left[\frac{PXAC_{a,c}}{PX_a} * \frac{ati_a^{-rhoti_a}}{gammai_{a,c}}\right] \\
QXAC_{a,c} &= QX_a * \left[\frac{PXAC_{a,c}}{PX_a} * \frac{ati_a^{-rhoti_a}}{gammai_{a,c}}\right]^{\frac{1}{rhoti_a-1}}
\end{aligned}
\tag{67}
$$

The first order condition in terms of the quantity ratio (equation 67) is included in the adjusted CGE model, but the model also returns the same results when the first order condition in terms of the price ratio (equation 65) is included instead. It should be noted however that in order to run the model with the first order condition in terms of the price ratio, the second last line of equation 65 should be used, not the last line. When the first order condition contains QX_a (industry output) on the right hand side of the equation a calibration problem arises even though the two lines of code are mathematically consistent. This is because QX_a, which appears in the last line, is a variable that is determined elsewhere in the model, whereas the share parameter (ati_a), which appears in the second last line, is a parameter that is fixed during calibration.

Allowing for supply response implies that the product composition of output by an industry is no longer fixed, but becomes variable, and the model variable $IOQXACQXV_{a,c}$ is introduced to capture the changes in the composition of products. To determine producer prices (PX_a) the variable ($IOQXACQXV_{a,c}$) is used instead of the parameter ($ioqxacqx_{a,c}$) to reflect changing shares of products in output as shown by the following function:

$$
PX_a = \sum_c \left(IOQXACQXV_{a,c} * PXAC_{a,c}\right) \tag{68}
$$

The replacement of parameter $ioqxacqx_{a,c}$ by variable $IOQXACQXV_{a,c}$ causes the model variable and equation counts to be unequal, therefore as part of model closure an additional balancing equation was introduced:

$$QXAC_{a,c} = IOQXACQXV_{a,c} * QX_a \qquad (69)$$

4.3.3 Introducing set controls

It is not deemed realistic to assume that all industries will be flexible in their output composition. Hence when this issue is introduced in the model the original assumption of fixed output composition should be retained as an alternative to flexible output composition. The choice between whether an industry's output transformation is governed by Leontief assumption or a CET function, is controlled by implementing the equations in the previous section for different subsets of industries. The subsets are defined in the model. The logic of how the subsets were formed and the set controls to implement it in the model is discussed in this section.

The characteristics of the industries in question should be taken into consideration when deciding whether the CET output function, or a fixed shares transformation function, should be applied to a particular industry. The characteristics of the industries can be derived from the underlying output data contained in the SAM, but also requires insight from the policy analyst. For purposes of this discussion industries can be categorised into three groups depending on the nature of their output: a) single product, b) by-product and c) multi-product industries. The nature of each industry's output determines whether it can be suitably modelled using a CET function. In the model categorisation is done by defining subsets and using these subsets to specify for which subset of industries certain equations should be implemented.

Single product industries are by their nature not suitable for a CET function, because no substitution between products supplied in the base case is possible. These industries can be easily identified from the base data contained in the SAM. Single product industries are subject to the fixed proportions transformation function, and will continue to only produce a single product. Single product industries are captured in an industry subset called '$acetsp_a$', whereas multi-product industries (including by-product industries) are captured in subset '$acetmp_a$'.

By-product industries can be regarded as a special kind of multi-product industry. By-product industries produce more than one product, but the level of production of the main product determines the production levels of the by-products, because these are always produced in fixed proportions. These industries should be identified by the policy analyst. For purposes of this model any other industries that the policy analyst deems not to have substitution possibilities, are handled similarly to by-product industries. Any multi-product industry that is assigned a CET output elasticity of zero is treated by the model as a by-product industry with no substitution possibility.

Multi-product industries that are capable of changing the product composition of their output in response to product price changes can be modelled with a CET output function. The CET specification is particularly useful when secondary products comprise a substantial share of total output, because the Leontief assumption is likely to render unrealistic results in this instance. Similarly, but from a technical perspective in the context of a general equilibrium model, the price changes required to fulfil the market clearance conditions might lead to distortive price changes in cases where one or more products contribute a relatively small share to the total output by a particular industry. Therefore, in order to avoid the risk of capturing unrealistic market power effects in the results when using a CET function, a set control was included to ensure that the CET function will not be implemented when the share contribution of a product as a percentage of the total output by that industry is below a user defined cut-off value. Multi-product industries with products with output share contributions smaller than a specified cut-off are assigned to a subset '$acets_a$', whereas multi-product industries with products with sufficiently large shares are included in the subset '$acetl_a$'.

From the discussion above it follows that industries have to fulfil certain criteria before the CET output function can be implemented for these industries. Industries that meet the criteria are assigned to the subset '$acet_a$', while all other industries are assigned to subset '$acetn_a$' and the latter set of industries will be governed by the fixed proportions transformation function. In order to belong to the subset of industries governed by the CET function ($acet_a$) an industry must have been assigned a non-zero elasticity of substitution **and** it must be a multi-product industry **and** the share contributions of the products produced must be above a user defined cut-off value. All other industries are assigned to subset '$acetn_a$' and are assumed to maintain the original product composition in their output regardless of relative price changes. For industries belonging to predefined subset '$acetn_a$' the transformation function will be the same as before (fixed proportions), but with a new the set constraint indicated after the dollar sign as shown in the equation below.

In the model equation *ACTIVOUT*, the first set constraint for $QXAC_{a,c}$ is $ioqxacqx_{a,c}$ and implies that the calculation for the quantity of each product produced by each industry ($QXAC_{a,c}$) is only performed if such a product is produced by the industry in the base case. The second set constraint for $QXAC_{a,c}$ is '$acetn_a$' and implies that the calculation for the quantity of each product produced by each industry is only performed for the subset of industries whose output is not governed by the CET function:

$$ACTIVOUT_{a,c}\$(ioqxacqx_{a,c}\_and\_acetn_{a})............QXAC_{a,c} = ioqxacqx_{a,c} * QX_{a} \tag{70}$$

For multi-product industries with non-zero elasticity of substitution and appropriately large product shares, the transformation is governed by a CET function. The CET equation (*ACTIVOUT1*) is introduced and implemented for industries in subset '$acet_a$' and the calculation is only performed when a product is produced by an industry in the base case (i.e. if $ioqxacqx_{a,c}$ is non-zero):

$$\begin{aligned} &ACTIVOUT1_{a,c}\$(ioqxacqx_{a,c}\_and\_acet_{a}).....\\ &...................QX_{a} = ati_{a} * (\sum_{c} gammai_{a,c} * QXAC_{a,c}{}^{rhoti_{a}})^{1/rhoti_{a}} \end{aligned} \tag{71}$$

The first order condition to determine the optimal levels of production of each product by each industry, given relative product prices, should only be implemented for multi-product industries with non-zero elasticities of substitution and appropriately large product shares, as indicated by the set control in the function *ACTIVOUT2*:

$$\begin{aligned} &ACTIVOUT2_{a,c}\$(ioqxacqx_{a,c}\_and\_acet_{a})...\\ &....QXAC_{a,c} = QX_{a} * (PXAC_{a,c} / (PX_{a} * gammai_{a,c} * ati_{a} ** rhoti_{a})) ** (1/(rhoti_{a} - 1)) \end{aligned} \tag{72}$$

4.3.4 Calibration statements for parameters

In order to run a model in GAMS, each parameter and variable should be given a starting value, i.e. the parameters and variables should be initialised. When a variable assumes its initial value it is denoted in the model by adding the suffix "0", e.g. $PXAC_{a,c}$ becomes $PXAC\mathbf{0}_{a,c}$. The latter is treated as a parameter in the model because the base value will remain fixed regardless of future scenario values of the variables. The base values are assigned and initialised and then they are used to calculate the values of some of the functional form parameters in the model. Once the values of all the parameters have been determined or assigned, the parameters can be included in the main equations of the model and the model can be run. This process is called calibration of the model. The aim of calibration is therefore to establish the values that parameters should adopt that will return the base values in the SAM when the model is run. The equations to calibrate the parameters are shown in this section.

The elasticity parameter ($rhoti_a$) in the CET function $ACTIVOUT1$ (equation 71) is calibrated only for industries in the set $acet_a$. The elasticity of transformation ($omegaout_a$) is supplied exogenously to the model and cannot be determined from the data contained in the SAM. The relationship between the elasticity parameter (*$rhoti_a$*) and the elasticity of transformation (*$omegaout_a$*) is shown in the following equation:

$$rhoti_a \$ (acet_a) = (1 / omegaout_a) + 1 \tag{73}$$

The other two parameters in the CET function $ACTIVOUT1$ (equation 71) are calibrated using base values of variables, as denoted by the suffix ‘0’ to each variable. The share parameter (*$gammai_{a,c}$*) is calibrated only if products are produced by an industry in the base case and only for industries that will use the CET function as indicated by the set control (*$\$QXAC0_{a,c}\_and\_acet_a$*), as follows, where set ‘cp’ is a GAMS set alias for set ‘c’:

$$gammai_{a,c} \$ \left(QXAC0_{a,c}\_and\_acet_a\right) = (PXAC0_{a,c} * QXAC0_{a,c} **(1 - rhoti_a)) / \sum_{cp\$QXAC0_{a,cp}} (PXAC0_{a,cp} * QXAC0_{a,cp} **(1 - rhoti_a)) \tag{74}$$

The shift parameter (*ati_a*) is calibrated only for industries that are governed by the CET as indicated by the set control (*$acet_a$*), as follows:

$$ati_a \$ acet_a = QX0_a / \sum_c \left(gammai_{a,c} * QXAC0_{a,c} **(rhoti_a)\right) **(1 / (rhoti_a)) \tag{75}$$

The base value of the variable (*$IOQXACQXV_{a,c}$*) is initialised using the base value of the variable *$QXAC_{a,c}$* (i.e. the supply matrix) instead of the SAM values as in the case of the parameter (*$ioqxacqx_{a,c}$*), as follows:

$$IOQXACQXV0_{a,c} = QXAC0_{a,c} / \sum_{cp} QXAC0_{a,cp} \tag{76}$$

As the level of *$QXAC_{a,c}$* (*$QXAC.L_{a,c}$*) changes, so the level of the share variable (*$IOQXACQXV.L_{a,c}$*) should also change whereas previously it remained fixed. The level (.L) of the variable (*$IOQXACQXV_{a,c}$*) is initialised as follows:

$$IOQXACQXV.L_{a,c} = QXAC.L_{a,c} / \sum_{cp} QXAX.L_{a,cp} \tag{77}$$

The full set of model equations is included in the addendum. All adjustments to the base model are marked.

4.4 Summary and conclusions

Several CGE models assume that products will continue to be produced in the same proportion as in the base case. This is a restrictive assumption especially in the case of agricultural industries that are modelled as mixed farming enterprises that are deemed to be price responsive in their output composition. A scan of the literature reveals that efforts to relax the assumption of fixed output proportions are usually related to studies focused on agriculture and that in the majority of the few cases available in the literature, a CET function was implemented to replace the Leontief production function. In most cases the CET function was specified in the context of assumptions of strong input-output separability. The assumption of separability in the context of modelling flexible output composition in CGE models is in line with Pyatt's (1994) generalisation of Stone's proposed model of product balances. It is recognised however that the introduction of a CET function for output transformation violates the law of one price because the necessary assumption of similar production techniques for different products is not realistic.

The model for purposes of this study was adjusted to include a CET function for selected multi-product industries. The adjustment of the CGE model entailed the revision of the function for industry prices, the addition of the CET output function and the corresponding first order condition and the addition of a product equilibrium function for purposes of model closure. The GAMS code for calibration of the model parameters were done for the shift, share and elasticity parameters for the CET function. All new variables were declared and initialised. Selection of industries governed by the CET function occurs by means of set controls in the model. The set controls make the model more general to ensure that the model will run with different aggregations of the South African data as well as data for different countries.

Because of the input-output separability assumption there was no attempt to adjust the intermediate input structure or composition of value added according to changes in the output composition. This is a potential shortcoming of the model but the improvement in realism of results on the supply side is deemed to exceed the potential errors on the input side as will be illustrated in Chapters 6 and 7.

It should be noted that in order to relax the assumption of fixed output composition only the first order condition of the CET function needs to be included. The results of the model are not affected by the inclusion of the CET function to determine aggregate industry output, because it is already determined by the CES aggregation of aggregate intermediate inputs and value added. It is the first order condition of the CET function that determines the optimal composition of products in the total output of an industry and this will be dependent on the relative prices of products.

5 Calibration data for the CGE model

5.1 Introduction

The adjusted CGE model that was discussed in the previous chapter is calibrated with an agricultural social accounting matrix (SAM) for South Africa for 2007. The agricultural SAM was developed as part of this study. The theoretical background to SAMs was described in Chapter 2. The point of departure for the development of the detailed SAM is an aggregated SAM, or macro SAM. From the national accounts published by the South African Reserve Bank (SARB), the relevant data is used to derive a macro SAM. The national accounts for South Africa are balanced except for a relatively small residual published by SARB, which is absorbed into the institutional accounts. The row and column totals of the macro SAM are therefore equal and the macro SAM is a balanced matrix. To derive a detailed SAM the aggregated accounts of the macro SAM are disaggregated into numerous accounts for each main account category. Different data sources provide the information for disaggregation, hence the detailed SAM is unbalanced after all the data from different data sources have been incorporated; hence a balancing process is followed to estimate missing information in order to balance the SAM.

Three sets of data are used for this study: a social accounting matrix (SAM) that records all transactions in the economy, a factor use matrix that identifies the quantities of each factor used by each industry, and elasticities that control the operation of the model’s behavioural functions. The discussion commences in the next section with an overview of existing SAMs for South Africa. Section 5.3 gives an overview of the SAM developed for this study. The structure of the most detailed version of the SAM (543 accounts) is presented and then the main data sources used are reported. The methods used to develop the macro SAM and the detailed SAM are discussed, followed by a short discussion of the balancing process that was implemented to estimate the missing information in order to balance the SAM. A short overview of the SAM aggregation (137 accounts) that was used for purposes of the case study is then presented followed by a description of the South African economy as portrayed by the aggregated version of the SAM. Section 5.4 presents an overview of the factor use matrix that identifies the quantities of each factor used by each industry while section 5.5

reports on the elasticities and other parameters that are required in addition to the SAM data to calibrate the CGE model. A summary is presented at the end of the chapter.

5.2 Overview of existing SAMs for South Africa

SAMs for South Africa that have been developed over the past 15 years include, amongst other, the following:

- The SAMs published by the South African national statistical agency, Statistics South Africa (SSA) for 1998 (SSA, 2002c), 2002 (SSA, 2006a) and 2005 (SSA, 2009d);
- SAMs used by international organisations, for example the International Food Policy Research Institute (IFPRI) SAMs for 1998 (Thurlow and Van Seventer, 2002) and 2000 (Thurlow, 2003), and the PEP network SAMs for 2005 (Mabugu, Maisonnave, Robichaud and Chitiga, 2011) and versions of SAMs based on these SAMs;
- SAMs developed by government organisations, notably the PROVIDE project SAM for 2000 (PROVIDE, 2006) of the national and provincial Departments of Agriculture;
- SAMs developed by Quantec and by Conningarth Consultants, which are used in consultancy work and hence not always available in the public domain.

Statistics South Africa has closely followed the structure for a SAM as proposed by the 1993 SNA in the layout of their National Accounting Matrices[3] (SSA 2008a:19) and also in their SAMs. All current SAMs published by Statistics South Africa are still based on the 1993 SNA, because the 2008 SNA was only released in 2009 and Statistics South Africa has not released a SAM since then. The SAM structure proposed by the 2008 SNA (United Nations 2009:521) closely follows that proposed by the 1993 SNA, so no major revisions in the layout of future SAMs produced by Statistics South Africa are therefore anticipated. As indicated in section 2.3.4 the SAMs that follow the layout of the 1993 and 2008 SNAs are not ideal for policy analyses concerned with redistribution of income. Also, the SAMs by Statistics South Africa have only a single account for agriculture, forestry and fisheries.

The IFPRI, PEP network and PROVIDE project SAMs all follow the SAM structure similar to the one proposed by Pyatt (1988) discussed in section 2.3.3, which makes it suitable for calibration of CGE models for policy analyses. The IFPRI and PEP network SAMs do not contain detailed agricultural accounts. The PROVIDE project SAM was developed with the intent to include detailed agricultural accounts as well as a regional focus (PROVIDE, 2006). It is the use of the PROVIDE project SAM to calibrate a single country CGE model that led to unrealistic results with regard to output composition that inspired the current model

[3] See SAM Table 20.4 in the 1993 SNA.

adjustments. What was intended as an update of the PROVIDE project SAM to a more recent base year however turned out to be a complete redevelopment of the SAM because of the numerous changes in the base data sources compared to those used for the 2000 PROVIDE project SAM.

5.3 The 2007 agricultural SAM for South Africa

5.3.1 Overview of the developed agricultural SAM for South Africa for 2007

The detailed SAM that was developed as part of this study is a national SAM for 2007. Despite being a national SAM, the SAM contains provincial detail for agricultural industries, households and labour, which makes it particularly useful for analyses at provincial level. The full version of the SAM contains 543 accounts: 115 products (of which 19 are agricultural products), trade and transport margins, 143 industries (of which 48 are agricultural industries), 1 capital account, 1 land account, 142 labour accounts, 126 household accounts, 1 corporation account, 8 tax accounts, 1 general government account, an investment/savings account, an account for stock changes and an international trade and transfers account. In order to focus the results on the agricultural industry, an aggregated version of the SAM (137 accounts) was used for the applications in Chapters 6 and 7 and the list of accounts used in this aggregated SAM appears in the addendum.

Agricultural industries are distinguished by region, meaning that a given agricultural industry represents all farming industries within that region and each region has a fixed total supply of land, but the enterprise mix within that region can vary. There are 48 agricultural industries/regions identified, with between three and ten agricultural industries per province. The regions within provinces, which determine the agricultural industries, are based on district municipalities. In certain instances two or more district municipalities were combined because of limited agricultural activity in some of these municipalities. Agricultural industries by region are useful because it allows for regional analyses, which often are of interest to politicians in the context of spatial development. The disadvantage of recording agricultural industries as mixed product firms is that it does not allow for clear implementation of scenarios such as for example subsidies to dairy farmers (because many dairy producers also engage in other enterprises) or the analyses of technological progress in wheat production (because the production structure of wheat is not identified separately in the data). The income and expenditure data on agriculture that is used to compile the SAM is collected on a regional basis and therefore the approach that is followed requires less adjustment and assumptions than if agricultural accounts presented what in fact is just one enterprise of a farm, i.e. dairy, or wheat, etc. The provincial SAMs for 2006 developed by

Conningarth Economists provide examples of SAMs with agricultural accounts representing separate farming enterprises, such as dairy and wheat, as opposed to regions (Conningarth, 2009). It was indicated in section 2.3.4 that such a representation of the data leads to a loss of information.

Agricultural products are differentiated by type of product and the choice of products was informed by the products included in the census of agriculture for 2002 (SSA, 2005) because of consistent data availability for these products at municipal level. The aggregation of the detailed products into categories was also influenced by the products that appear in the Global Trade Analysis Project (GTAP) database. This allows for the results of the GTAP global CGE model trade simulation scenarios to be used as inputs into the single country model to determine redistribution and employment effects on South Africa for a given global trade scenario. The SAM is rich in agricultural detail in order to allow for policy analyses of issues of interest to the agricultural sector.

Agriculture in South Africa comprise only about 85% of the agricultural, forestry and fisheries sector in terms of GDP and data for this larger sector are often combined. Agriculture, forestry and fisheries are therefore each recorded separately in the most disaggregated version of the agricultural SAM in order not to overestimate the contribution of agriculture. Forestry and fishery industries are not found in all provinces, hence the importance of disaggregating agriculture, forestry and fisheries if agriculture is to be recorded separately per province.

It is only the agricultural industry accounts that are identified by province and region, all other product and industry accounts reflect national values. A national product market is therefore assumed for agricultural products despite the fact that agricultural production is recorded per region. Products and industries for the rest of the economy are classified in the agricultural SAM according to the classification used by Statistics South Africa in the use table for 2002 (SSA, 2006b). The use table contains 95 products and 94 industries (as opposed to the supply table that contains 153 products and 94 industries). In the supply and use tables published by Statistics South Africa there is only a single product account and a single industry account for agriculture, forestry and fisheries combined. The disaggregation of the agricultural account takes place at a later stage.

Trade and transport margins are included in the full version of the SAM, but these are absorbed into the supply table for the version of the SAM that is used to calibrate the model for analyses.

Both household and labour (factor) categories are distinguished in the first instance according to their province of residence or work respectively. Households are further disaggregated according to race (Blacks, Indians and Asians in one group and Whites in a second group), main source of income (from working in the agricultural sector, from working in non-agricultural sectors, asset income, or social transfers), income level (poor or non-poor), and for those employed in agriculture there is the additional distinction as to whether the income is labour income or from self-employment. The poor and non-poor cut-off for households was carried out at a national level by ranking all households according to per capita income and taking half of the households on the lower income end as poor households and the other half as non-poor households. This ensures that the poor vs. non-poor distinction can be retained when aggregating households across location, income source or race. Besides the geographical dimension, labour factors are further disaggregated on the basis of race (Black and White), gender (male and female), industry (agriculture vs. non-agriculture) and skill level (skilled, semi-skilled or unskilled).

In addition to the 142 labour accounts, the factor accounts also include one capital account and one land account. The land account records the returns to land for each agricultural industry separately. Because of the single land account the total returns to land is captured and then distributed or mapped to the appropriate households, as opposed to explicit redistribution of returns to land for each specific agricultural industry if there were nine land accounts. One of the assumptions in the distribution of returns to land to households is that land owners are located in the same province as the land that they own. The available land per province is constrained to agricultural industries in that province by means of set controls in the CGE model.

The government tax accounts include accounts for value added tax, import duties, excise duties, other product taxes, product subsidies, production taxes, production subsidies and direct income taxes. There is also a main government account that receives the tax revenue and transfer incomes from various source accounts and pays subsidies and transfers to other recipient accounts.

The single international trade and transfer account, referred to as the ‘rest of the world’ account, captures all transactions between South Africa and the rest of the world.

5.3.2 Main data sources

The main data sources are presented in Table 15.

Table 15: Data sources for each type of account in the SAM

Type of account	Statistical sources
Products and industries	2002 SU-tables (SSA, 2006b) SARB Quarterly Bulletin (SARB, September 2009a) 2002 Census of Commercial Agriculture (SSA, 2005) 2007 Census of Commercial Agriculture (SSA, 2009a) 2007 27 Account Supply and Use Table (SSA, 2009b) 2008 Forestry Information (Godsmark, 2008)
Factors and employment satellite account	2002 SU-tables (SSA, 2006b) 2000 Income and Expenditure Survey (SSA, 2002a)[4] 2000 Labour Force Survey (SSA, 2002b) SARB Quarterly Bulletin (SARB, September 2009a) 2007 27 Account Supply and Use Table (SSA, 2009b)
Households	2000 Income and Expenditure Survey (SSA, 2002a) 2000 Labour Force Survey (SSA, 2002b) SARB Quarterly Bulletin (SARB, September 2009a) 2007 27 Account Supply and Use Table (SSA, 2009b)
Corporations	2000 Income and Expenditure Survey (SSA, 2002a) SARB Quarterly Bulletin (SARB, September 2009a)
Government	SARB Quarterly Bulletin (SARB, September 2009a) 2002 SU-tables (SSA, 2006b) 2007 Unpublished National Accounts data (SARB, 2009b) 2007 27 Account Supply and Use Table (SSA, 2009b)
Capital	2002 SU-tables (SSA, 2006b) SARB Quarterly Bulletin (SARB, September 2009a) 2007 27 Account Supply and Use Table (SSA, 2009b)
International trade/'rest of the world'	2002 SU-tables (SSA, 2006b) 2007 Import and export data (SARS, 2008) SARB Quarterly Bulletin (SARB, September 2009a) 2007 27 Account Supply and Use Table (SSA, 2009b)

5.3.3 Deriving the macro SAM

The macro SAM for South Africa (see Table 16) provides the control totals for each of the sub-matrices of the detailed national SAM for South Africa. The base year for the macro SAM is 2007. The data source for the macro SAM is the Quarterly Bulletin of September 2009 published by the South African Reserve Bank (SARB, 2009a). Inter-household

[4] The 2000 Income and Expenditure Survey data was used because the data can be linked to the September 2000 Labour Force Survey data, which is important to establish the labour payments to households. This direct link is no longer possible with the 2005 Income and Expenditure Survey data.

transfers are recorded in the detailed SAM, as derived from data in the Income and Expenditure Survey. Only net inter-household transfers are typically recorded in a SAM, therefore inter-household transfers in the macro SAM are zero. Details on margins are obtained from the SU-tables and do not enter into the macro SAM, but are recorded in the detailed supply and use SAM.

Each row of the macro SAM is discussed separately. The rows and columns in Table 16 are numbered for the reader to identify the relevant sub-matrix that is referred to, e.g. household consumption in the sub-matrix in the first row and fourth column will be referred to as SM1:4. If more than one description appears, the first corresponds to the sub-matrix descriptions in Table 16 and the second to the descriptions of the totals in the SARB Quarterly Bulletin. The numbers at the end indicate the code used in the SARB Quarterly Bulletin followed by the page number.

Product row

SM1:1 Marketing margins: Net margins as recorded in this sub-matrix equal zero and no control total is reported by SARB, therefore derived as part of detailed SAM;

SM1:2 Intermediate consumption at purchasers' prices 6871J S-129;

SM1:4 Final consumption expenditure by households at purchasers' prices 6007J S-104;

SM1:6b Final consumption expenditure by general government at purchasers' prices 6008J S-104 and S-127;

SM1:7a Investment: Gross fixed capital formation 6009J S-104, plus the reported residual 6011J S-104;

SM1:7b Stock changes: Change in inventories 6010J S-104 and S-129;

SM1:8 Exports: Exports of goods and services 6013J S-104 and S130.

Table 16: A macro SAM for South Africa for 2007 (R million), based on SARB data

		1	2	3a	3b	3c	4	5	6a	6b	7a	7b	8
		Products	**Industries**		**Factors**		**Households**	**Corporations**	**Government**		**Capital**		**Rest of World**
				Capital	*Labour*	*Land*			*Taxes*	*Expend*	*Invest-ment*	*Stock changes*	
1	**Products**		2 311 972				1 227 816			394 653	422 300	16 046	629 092
2	**Industries**	4 086 944											
	Factors												
3a	*Capital*		883 774										42 873
3b	*Labour*		855 845										5 575
3c	*Land*		8 977										
4	**Households**			420 433	853 182	8 843		207 298		101 428			732
5	**Corporations**			185 342			204 864	18 573					
	Government												
6a	*Taxes*	224 114	26 376				161 167	156 232					
6b	*Income*			-39 130			4 286	11 189	567 889				1 107
	Capital												
7a	*Savings*			256 373			-6 827	14 914		27 810			146 076
7b	*Stock changes*										16 046		
8	**Rest of World**	690 821		103 628	8 238	135	610	573		21 450			-61 729
9	**Totals**	**5 001 879**	**4 086 944**	**926 647**	**861 420**	**8 977**	**1 591 916**	**408 779**	**567 889**	**545 341**	**438 346**	**16 046**	**763 726**

Source: Own calculations from SARB Quarterly Bulletin for September 2009 (SARB, 2009a)

Industry row

SM2:1 Supply of products by industries at basic prices 6870J S-129.

Factor rows

SM3a:2 GOS income: Gross operating surplus/mixed income 6212J S-129, or net operating surplus 6001J S-104 plus consumption of fixed capital 6002J S-104;

SM3a:8 GOS income from the rest of the world: Direct investment 5704Y S-82, and non-direct investment 5705Y S-82, less income to land in SM3c:2;

SM3b:2 Labour income: Compensation of employees 6000J S-104;

SM3b:8 Labour income from the rest of the world: Compensation of employees 5703J S-82;

SM3c:2 Land income: Calculated as 7.5% of gross income from agriculture as reflected in the supply table for 2007 (SSA, 2009b);

SM3c:8 Assumed to be zero.

Household row

SM4:3a Income to households from non-corporate business enterprises: Gross operating surplus 6826J S-128, plus property income received 6827J S-128, less property income paid 6832J S-128, less consumption of fixed capital 6849J S-128, less income from land in SM4:3c;

SM4:3b Income to households from labour: Compensation of employees 6240J S-129;

SM4:3c Income to households from land: Calculated as 7.5% of gross income from agriculture as reflected in the supply table for 2007 (SSA, 2009b), of which 1.5% is assumed to go to foreigners as reflected in SM8:3c;

SM4:4 Inter-household transfers: Net transfers are zero in a macro SAM;

SM4:5 Household income from incorporated business enterprises: Current transfers from incorporated business enterprises (SARB (2009b), plus adjustment for the change in net equity of households in pension funds reserves 6723J S-125, plus the residuals of financial corporations 6724J S-125 and non-financial corporations 6764J S-126;

SM4:6b Government transfers to households: Social benefits paid 6798J S-127 plus miscellaneous current transfers 6801J S-127;

SM4:8 Remittances from the rest of the world: Current transfer receipts - other sectors 5708Y S-82.

Corporation row

SM5:3a Distribution of factor income: Gross operating surplus 6706J S-125, plus property income received 6707J S-125, less property income paid 6710J S-125, less consumption of fixed capital 6727J S-125 for financial corporations, plus gross operating surplus 6746J S-126, plus property income received 6747J S-126, less property income paid 6752J S-126, less consumption of fixed capital 6766J S-126 for non-financial corporations;

SM5:4 Transfers to corporations: Current transfers receivable from households (SARB, 2009b);

SM5:5 Transfers to corporations: Internal current transfers (SARB, 2009b).

Government rows

SM6a:1 Net product taxes: Taxes on products 6603J S-104 less subsidies on products 6604J S-104;

SM6a:2 Net production (industry) taxes: Other taxes on production 6600J S-104 less other subsidies on production 6601J S-104;

SM6a:4 Household tax: Current taxes on income and wealth 62451J S-128;

SM6a:5 Corporation tax: Current taxes on income and wealth from financial corporations 6717J S-125 and non-financial corporations 6758 S-126;

SM6b:4 Transfers from households: Current transfers receivable from households (SARB, 2009b);

SM6b:5 Transfers from corporations: Income from financial and non-financial corporations (SARB, 2009b);

SM6b:6a Transfer from tax accounts to general government: taxes on production 6603J S-104, plus other taxes on production 6600J S-104, less subsidies on products 6604J S-104, less subsidies on production 6601J S-104, plus current taxes on income and wealth 6251J S-127;

SM6b:8 Transfers from the rest of the world 5707Y S-82.

Capital and stock changes rows

SM7a:3a GOS savings: Consumption of fixed capital 6002J S-104;

SM7a:4 Household savings: Savings by households 6200J S-124;

SM7a:5 Savings by corporations: Corporate savings 6201J S-124, plus residual 6011J S-104;

SM7a:6b Government savings: Saving of general government 6202J S-124;

SM7a:8 Capital account balance: Foreign investment 6206J S-124;

SM7a:9 Total savings: Gross capital formation 6180J S-124 plus residual 6011J S-104;

SM7b:7a Stock changes: Change in inventories 6010J S-104.

Rest of the World row

SM8:1 Imports: Imports of goods and services 6014J S-104;

SM8:3a Factor payments to the rest of the world: Direct investment income 5724Y, plus non-direct investment income 5725Y S-82;

SM8:3b Factor payments to the rest of the world: Income payments - compensation of employment 5723Y S-82;

SM8:3c Factor payments to the rest of the world: Rent from land. Assumed as 1.5% of total rent from land;

SM8:4 Household remittances: Transfers to the rest of the world (SARB, 2009b);

SM8:5 Corporations payments to the rest of the world: Transfers to the rest of the world (SARB, 2009b);

SM8:6b Government transfers to the rest of the world: Current transfer payments - central government 5727Y S-82;

SM8:8 Current account balance: Exports of goods and services 6013J S-104, less imports of goods and services 6014J S-104.

5.3.4 Deriving an unbalanced detailed SAM by disaggregation

A best estimate SAM refers to a SAM that is compiled from different, usually inconsistent, data sources that lead to a SAM of which the row and column totals usually do not equate as they should do. The process of deriving the unbalanced SAM is discussed here and the estimation of the missing data to derive a complete and consistent SAM is discussed in the next section. The first four steps focus on estimating very detailed supply and use tables for 2007, with full disaggregation of products, industries and product taxes. In steps five and six the aim is to develop a SAM out of the supply and use tables and these two sections discuss the data disaggregation related to factors and households. Data for other accounts are not disaggregated further but taken directly from the macro SAM and are therefore not mentioned here. In step seven the disaggregation of agriculture, forestry and fisheries into three separate industry accounts is discussed and step eight gives information on the disaggregation of the agricultural product and industry accounts into detailed agricultural accounts. The eight steps are discussed in the next eight sub-sections.

Step 1: Deriving detailed supply and use tables for 2007

The 27 account supply and use tables for 2007 (SSA, 2009b) were used as base to derive more detailed supply and use tables for 2007 with 94 industries and 95 products (single agricultural industry and product accounts). The 27 account supply and use tables for 2007 were used for control totals and the more detailed structural information (shares/ coefficients) were obtained from the 2002 supply and use tables[5] (SSA, 2006b).

Unpublished data for the 27 account supply table include a breakdown of taxes of products and subsidies on products, and trade and transport margins for 27 product groups. Unpublished data for the 27 account use table include a breakdown of components of final demand, value added, and production taxes and subsidies for 27 product and industry groups (SSA, 2009c).

The 95 account supply and use tables derived for 2007 were then further extended to include more detail on factors, households, government and agriculture, as discussed in the next few sections.

Step 2: Disaggregating final demand in the use table

Final consumption expenditure by households is aggregated to a single household in the derived use table for 2007. In order to disaggregate this column vector into a full matrix showing consumption expenditure by all 126 identified household groups, the consumption expenditure data reported in the Income and Expenditure Survey (IES) for 2000, (SSA, 2002a) were used. For each product the share of expenditure by the different household groups were calculated and then multiplied with the control total for household consumption per product from the use table for 2007 in order to derive the full matrix of household final consumption expenditure.

The other components of final demand such as government expenditure, investment, exports and changes in inventories are obtained directly from the derived 2007 use table.

Step 3: Disaggregating compensation of employees in the use table

The compensation of employees reported in the use table for 2007 provides the control total for salaries and wages per industry, but only for aggregate labour. For each industry these salaries and wages are distributed across the different labour categories using the data contained in the September Labour Force Survey (LFS) for 2000 (SSA, 2002b) and the IES 2000.

[5] The 2005 supply and use tables were not available at the time of developing the SAM.

Step 4: Disaggregating product taxes in the supply table

Control totals for four product tax groups for 2007 were obtained from unpublished data from SARB (SARB, 2009b). The four taxes that were disaggregated include value added tax, import tariffs, excise duties and other taxes (which consist of provincial taxes and transfer duties). The distribution of these taxes across the 27 accounts included in the 27 account supply table (SSA, 2009c) were based on the external product tax account data released as part of the 2005 SAM developed by Statistics SA (SSA, 2009d), combined with the 27 account supply table product tax totals. Shares on the more detailed product accounts were based on share of total product taxes in 2002.

Step 5: Labour income payments to households and abroad

The payments of salaries and wages to households were also derived from data contained in the LFS 2000 and the IES 2000.

The distribution of factor payments from abroad to each of the 144 factor accounts are assumed to be proportionate to each factor's share in total domestic employment income.

Step 6: Household income sources

Besides labour income mentioned above, households also derive income from owning capital (GOS) and land, as well as receiving transfers from other households, corporations, government and abroad. The IES 2000 contains data on household income from all of the mentioned sources, except transfers from abroad. Shares per household for all of these income sources were derived from data in the IES 2000, except for remittances from abroad, which was assumed to follow the same distribution pattern as domestic transfers.

Step 7: Disaggregating the agriculture, forestry and fisheries industries

Information on the agricultural sector as reported by Statistics South Africa typically includes agriculture, forestry and fisheries, although not always explicitly mentioned. As mentioned, agriculture in South Africa comprises only about 85% of the agricultural, forestry and fisheries sector. Forestry and fisheries industries are not found in all provinces and because there is an interest in agriculture at provincial and regional level it becomes important to separate these three industries.

The official value added figures for agriculture, forestry and fisheries at a national level (SSA, 2008a) provides some control totals for the three industries. The LFS 2000 gives factor payment information for each of these industries, which is used to separate compensation of employees. Some adjustments to this data were done where more recent industry information (Godsmark, 2008) contradicted some of the LFS 2000 data.

Although forestry and fisheries are not shown per province in the SAM, these were initially estimated per province in order to subtract the appropriate amounts from each province's combined figures for agriculture, forestry and fisheries in order to get a more accurate reflection of agriculture per province. The factor returns from land are distributed between agriculture and forestry based on their relative value added.

Step 8: Disaggregating agricultural products and industries

The single agricultural industry is disaggregated on a regional base according to district municipalities across South Africa. There are nine provinces in South Africa and each of these nine provinces are divided into between three and ten district municipalities, with a total of 48 agricultural industries defined for purposes of the SAM. Each of these districts is regarded as a production unit and these production units in the agricultural sector provide good examples of multi-product firms. Each of the agricultural industries can produce one or more of the 19 identified agricultural products.

The first stage entailed separating agricultural industries into the nine provinces. This was done using the preliminary agricultural census data for 2007 (SSA, 2009a). At the time of developing the SAM the agricultural census results per district for 2007 were not yet available, hence the data on district level were taken from the agricultural census for 2002 (SSA, 2005). The provincial results of the agricultural census for 2007 provided estimates for provincial gross income of three main agricultural product groups, i.e. field crops, horticulture, and animal and animal products. The 2007 information provided more recent control totals for further disaggregation using more dated information. The process of disaggregating the agricultural sector into the nine provinces was useful insofar as it influenced the prior disaggregation of the agriculture, forestry and fisheries and these two processes were somewhat iterative till value added for agriculture per province and for forestry and fisheries were giving realistic results.

Marketing margins: No detailed information were available therefore the respective trade and transport margins were distributed across the agricultural products based on each product's share of total supply as obtained from the agricultural census data for 2002.

Intermediate consumption: This is the only sub-matrix in which both industries and products had to be disaggregated. The disaggregation was done first for industries and then for products. Intermediate consumption by the 48 agricultural industries was estimated first. The estimates on current expenditure per province for 2007 were used to update some of the regional figures that appeared in the agricultural census for 2002. The derived use table for South Africa for 2007 was used to provide information on which products (including a single agricultural product) were used as intermediate inputs by the agricultural industry. The distribution of this expenditure across the different agricultural industries was based on information on current expenditure reported in the census' of agriculture for 2002 and 2007. There are three current expenditure items reported in the census of agriculture for 2007 that were not included in the calculation of intermediate consumption, namely depreciation, licence fees and rates paid. These three items are not defined as intermediate consumption in the SAM framework.

The second stage of disaggregation was to estimate the intermediate consumption of agricultural products by the non-agricultural industries. The single agricultural product account provided the control total for intermediate use of agricultural products by non-agricultural industries. The agricultural products that are used by each of the non-agricultural industries were first identified. First estimates of the distributions of the value of intermediate use by each non-agricultural industry across the relevant agricultural products were based on the share of total income for that product. These initial estimates were then adjusted where better information existed. And the last stage of adjustment was carried out during various rounds of estimation (SAM balancing) as discussed in section 5.3.5. Results on errors on micro and macro targets generated as part of the estimation process were used to identify peculiarities in the priors. The largest of these errors were then individually and iteratively addressed by adjusting the share of intermediate consumption by each non-agricultural industry related to each agricultural product.

The last stage of the disaggregation was to estimate the intermediate consumption of agricultural products by the agricultural industries. Total expenditures on agricultural products by each agricultural industry were determined in the first stage of the disaggregation process for this sub-matrix. During this final stage the expenditures by each agricultural industry were distributed across each of the individual agricultural products. The ratios of distribution were based on the share of each agricultural product in total income and adjusted where better information existed.

Final consumption expenditure by households: Final consumption expenditures were derived from the IES 2000 data. All expenditure items in the questionnaire that related to agricultural products were mapped to one of the 19 agricultural products.

Stock changes: No consistent data on stock changes per product were available. Stock changes for winter cereals, summer cereals and oil seeds were derived from unpublished data obtained from the Bureau for Food and Agricultural Policy (BFAP, 2009). The remaining values of agricultural stock changes were distributed across livestock, animal fibres and game, which were subsequently adjusted as part of the estimation process.

Exports of goods and services: Exports of agricultural products were derived from the 2007 export data for South Africa from the South African Revenue Service (SARS, 2008). For more details on the handling of trade data refer to PROVIDE (2004).

Supply: Supplies of products by the single agricultural industry were distributed across the 48 agricultural industries (regions) based on information obtained from the 2002 Census of Agriculture on each region's total income from each agricultural product. The supplies of agricultural products by non-agricultural industries were equally distributed across products for each industry, as this supply accounts for a relatively small amount of total supply. It was however assumed that forestry only produces forestry products and fisheries produces only fish products.

Gross operating surplus (GOS) income: The GOS income for agriculture was distributed across all agricultural industries according to each industry's share in total income.

Labour income: The income data were derived from data contained in the IES 2000 and the LFS 2000. Only a few manual adjustments were made in instances where the distribution of labour income between agriculture on a provincial basis, and labour income for forestry and fisheries appeared to be inconsistent with the 2000 data.

Income from land: Income accrues to land as a production factor. Land as a production factor was included for agricultural industries and forestry. The rental values for agricultural land were estimated using an extension officer's 'rule of thumb' of three percent of gross income per hectare times 2.5. The rental income is thus proportionate to the gross income of a region. The rental value for forestry land was based on the share of area of forest land compared to agricultural land.

Product taxes: Import duties on agricultural products are based on tariff revenue data obtained from SARS. For lack of better information VAT is distributed across agricultural products based on each product's share of income.

Product subsidies: Due to lack of information subsidies are distributed across agricultural products based on each product's share of income.

Production (industry) taxes: The agricultural census for 2002 provides information on tax paid to local authorities per agricultural region. These data were used to derive a first estimate of production taxes per agricultural industry.

Production (industry) subsidies: Subsidies were distributed across agricultural products based on each regions share of non-specified current expenditure.

Imports of goods and services: Imports of agricultural products were derived from 2007 imports data from SARS.

5.3.5 The GCE estimation method to estimate missing information

The process of compiling the prior SAM results in a matrix of priors for transaction values in the national SAM, but the SAM does not meet the necessary accounting constraints of a SAM, e.g. that each account must balance, that is receipts and expenditures must equal. This is a common problem in SAM database building because of incomplete and inconsistent data sources. Various techniques exist to estimate the missing information in order to obtain a balanced SAM that can be used for modelling, an overview of which was presented in section 2.3.5. For this study the same method was applied as for the 2000 SAM for South Africa namely the generalised cross entropy (GCE) method, which is discussed in detail in the relevant PROVIDE Technical Paper (PROVIDE, 2006). The approach makes use of an information metric termed *entropy* to numerically estimate the best fitting SAM given the prior data and the necessary constraints. The main difference compared to the 2000 SAM was that a more iterative and less mechanical approach was followed to ensure that more effort went into adjusting the least reliable priors.

Sequential disaggregation

As part of the development of the prior SAM, the starting point was a balanced 16 account macro SAM and then sub-matrices of this macro SAM were disaggregated in various phases. In this process the total value of the disaggregated sub-matrix is always constrained to be equal to the total value of the sub-matrix before disaggregation. This implies that higher levels of accuracy are assumed for more aggregated versions of the SAM. Thus the balancing process starts at the more aggregated level to impose the more reliable macro level constraints upon the SAM before further disaggregation. The semi-formalised method of top-down SAM estimation and accompanying software for implementing it, that was developed as part of the PROVIDE Project (PROVIDE, 2006), were followed. The problem is divided into phases, with each phase adding more disaggregation. The end point of each phase is assumed fixed and is used as the starting point in the next phase, hence the additional prior data from subsequent phases cannot affect the estimates from previous phases. The outcome of the sequential disaggregation in the PROVIDE project was a balanced fully disaggregated agricultural SAM, using an unbalanced fully disaggregated agricultural SAM compiled in GAMS as prior. In this study the outcome of the sequential disaggregation and balancing in GAMS was a disaggregated SAM with a single agricultural product and industry account. The disaggregation of the product tax account and the agricultural product and industry accounts was then carried out by hand in Excel in order to have greater flexibility in adding additional prior information for individual cells where additional more reliable information existed. The unbalanced agricultural SAM was then used as the prior in a next round of balancing through sequential disaggregation.

The sequential disaggregation approach starts by delineating the structure of a series of consecutive SAMs, each of which is more disaggregated than the previous one, with the last one being the target SAM to be estimated. The structure then also includes the mappings between the accounts of these SAMs. This data structure is used during the estimation process as a basis for sequential disaggregation, but at the same time it provides a useful means of organising prior data at differing levels of aggregation. The phases into which the process is to be split are also specified, each consisting of two SAMs that are two adjacent steps in the series of defined SAMs, a “macro” and a “micro” SAM at each level. After specifying the aggregations of the SAMs for the different phases, then balancing constraints are specified as well as the targets and their characteristics. A target is a generalised unit of prior data to be reflected as part of the problem’s objective function, the entropy divergence function. When configuration is complete the program can be run to solve the constrained maximisation problems set up in each phase. Following successful completion, a balanced SAM will have been produced along with additional information about the estimation process that can help to evaluate the resultant SAM.

Prior Data

The prior data used in the estimation of the SAM consists of:

- a macro SAM derived using SARB data;
- a 27 account SAM derived using data from Statistics South Africa;
- the 472 account national SAM with single agricultural product and industry accounts;
- the detailed 543 account agricultural SAM.

SAM and phase configuration

This sub-section describes the structure of the SAMs and phases in their relation to the 'system of SAMs' data structure used in the estimation process. The final bottom-level SAM in the first round is the full supply and use SAM with single agricultural product and industry accounts. At the top of the system is the 16 account macro SAM, followed by an expanded version of this macro SAM based on 9 sectors in the next phase (32 accounts). Following this, various intermediate account SAMs are estimated until the last phase with the final 543 account SAM is reached. A summary of the different phases according to which the SAM balancing took place is given in Table 17.

Table 17: Phase and SAM configuration for estimation of the SAM for South Africa

Phase	Macro SAM	Micro SAM	Macro Constraints	Description
Phases 1 to 5 done consecutively in GAMS				
Phase 1	16 account macro SAM, 1 product, 1 industry	35 accounts, 9 products, 9 industries	Maximum Std Error 0.1% of priors	Estimates a 9 sector SAM with 1 labour, 1 capital, 1 land, 1 household and 5 government accounts using a combination of data from the 9 sector SU data and aggregated data *from 472 account detailed prior non-agricultural SAM* as the micro SAM, and the 16 account macro SAM
Phase 2	35 accounts, 9 products, 9 industries	71 accounts, 27 products, 27 industries	No Error	Estimates a SAM with 27 sectors, with 1 labour, 1 capital, 1 land, 1 household and 5 government accounts
Phase 3	71 accounts, 27 products, 27 industries	206 accounts, 95 products, 94 industries	No Error	Disaggregates product and industry accounts, but retains single agricultural product and industry account
Phase 4	206 accounts, 95 products, 94 industries	222 accounts, 95 products, 94 industries	No Error	Disaggregates labour factors and households per province
Phase 5	222 accounts, 95 products, 94 industries	472 accounts, 95 products, 94 industries	No Error	Disaggregates labour factors and households fully, arriving at the final full non-agricultural SAM
Phases 6 to 10 done consecutively by hand in Excel				
Phase 6	472 accounts, 95 products, 94 industries	474 accounts, 95 products, 96 industries	No Error	Disaggregates agriculture, forestry and fisheries industries
Phase 7	474 accounts, 95 products, 96 industries	477 accounts, 95 products, 96 industries	Aim to minimise errors in prior	Disaggregates product tax account into VAT, import taxes, excise duties and other product taxes
Phase 8	477 accounts, 95 products, 96 industries	485 accounts, 95 products, 104 industries	Aim to minimise errors in prior	Disaggregates single agricultural industry into 9 agricultural industries (per province)
Phase 9	485 accounts, 95 products, 104 industries	489 accounts, 99 products, 104 industries	Aim to minimise errors in prior	Partially disaggregates agricultural products into crops, horticulture and animals (only income estimates) for each of the 9 agricultural industries

Phase	Macro SAM	Micro SAM	Macro Constraints	Description
Phase 10	489 accounts, 99 products, 104 industries	543 accounts, 115 products, 143 industries	Aim to minimise errors in prior	Fully disaggregates agricultural products to derive full unbalanced agricultural SAM
Phase 11 to 19 done consecutively in GAMS				
Phase 11	16 account macro SAM, 1 product, 1 industry	35 accounts, 9 products, 9 industries	Std Error 0.1% of priors	Estimates a 9 sector SAM with 1 labour, 1 capital, 1 land, 1 household and 5 government accounts using a combination of data from the 9 sector SU data and aggregated data *from 544 account detailed prior agricultural SAM* as the micro SAM, and the 16 account macro SAM
Phase 12	35 accounts, 9 products, 9 industries	71 accounts, 27 products, 27 industries	No Error	Estimates a SAM with 27 sectors, with 1 labour, 1 capital, 1 land, 1 household and 5 government accounts
Phase 13	71 accounts, 27 products, 27 industries	206 accounts, 95 products, 94 industries	No Error	Disaggregates product and industry accounts, but retains single agricultural product and industry account
Phase 14	206 accounts, 95 products, 94 industries	222 accounts, 95 products, 94 industries	No Error	Disaggregates labour factors and households per province
Phase 15	222 accounts, 95 products, 94 industries	355 accounts, 95 products, 94 industries	No Error	Disaggregates only labour factors fully
Phase 16	355 accounts, 95 products, 94 industries	370 accounts, 97 products, 104 industries	No Error	Disaggregates single agricultural product into agriculture, forestry and fishery products respectively and single agricultural industry into nine provincial agricultural industries and forestry and fisheries. Disaggregation of the product tax account into VAT, import taxes, excise duties and other product taxes
Phase 17	370 accounts, 97 products, 104 industries	372 accounts, 99 products, 104 industries	No Error	Disaggregates single agricultural product into 3 main agricultural product groups, retaining the 1 forestry and 1 fishery product
Phase 18	372 accounts, 99 products, 104 industries	388 accounts, 115 products, 104 industries	No Error	Disaggregates three agricultural products into 19 agricultural products, retaining the 1 forestry and 1 fishery product
Phase 19	388 accounts, 115 products, 104 industries	427 accounts, 115 products, 143 industries	No Error	Disaggregates nine provincial agricultural industries into 48 agricultural industries
Phase 20	427 accounts, 115 products, 143 industries	543 accounts, 115 products, 143 industries	No Error	Disaggregates households fully, arriving at a balanced fully detailed agricultural SAM

During the development of the best estimate SAM with 472 accounts (i.e. unbalanced SAM with single product and industry accounts for agriculture) an aggregated SAM consisting of 41 accounts (amongst other 12 product accounts, 12 industry accounts and single household and factor accounts) was derived. Initially these discrepancies between row and column totals on individual account level reached a maximum of 14%. Through various adjustments to the prior data, mainly the detailed supply and use table data, these discrepancies were largely removed with the maximum discrepancy remaining being less than 1%. It should be remembered that the reported 94 sector supply and use tables for 2002 and the 27 sector supply and use tables for 2006 both contain discrepancies (residuals) of up to 5% for individual accounts, and since both these datasets were used, these discrepancies affected the priors.

In the disaggregated SAM (472 accounts), the product and industry accounts presented few problems because of the adjustments to the priors while targeting small discrepancies at the aggregated level (41 accounts). Because of the method followed in disaggregating the labour accounts, these accounts showed negligible (maximum 1.5%) discrepancies. The result was that the bulk of the adjustment was placed on the household account data, which was deemed the least reliable because the household income and expenditures reported in the IES 2000 showed large discrepancies which were carried into the priors. Of the 16 590 households who reported greater income than expenditure in the IES, the incomes exceeded expenditure on average by 36.2%, whereas for the 9608 household who under-reported income, the expenditure exceeded incomes on average by 33.6% (PROVIDE, 2005a). In nine out of 126 household accounts either the row or column totals was more than double the other total, with the most extreme discrepancy being a household in North West with a row total of about four times the column total. Because there was no obvious systematic way of dealing with these discrepancies in the household accounts and because of the small proportion of such cases, it was decided not to attempt to adjust these household priors but to address it as part of the computerised estimation process.

5.3.6 Overview of the aggregation of the SAM used in the case study

The aggregation of the SAM used in the next section and in Chapters 6 and 7 is briefly discussed here. It is deemed appropriate to use an aggregated version of the SAM for case studies in order to focus the results towards the purpose of the study. At the same time it avoids the inclusion in the model of accounts with relatively small values in the detailed SAM. The availability of a highly disaggregated SAM allows flexibility in choosing the appropriate aggregation depending on the nature of the topic investigated. The level of aggregation for purposes of this dissertation was chosen partly such that detailed results can be presented in figures and tables, but retaining enough detail in the data to allow for sensible price transmissions in the model. The aggregation of product accounts was also aimed at highlighting the impact of the model adjustments.

The SAM used in the case study contains 137 accounts: 21 products (of which six agricultural and four food), 31 industries (of which 16 agricultural and four food), 38 factors (of which one GOS, one land and four labour accounts for each of the nine provinces, i.e. 36 labour accounts), 36 household accounts (four for each of the nine provinces), one enterprise account, five tax accounts and one account for each of the following: general government, capital, stock changes and the 'rest of the world'. See section 10.3 in the addendum for the list of accounts.

There are one or two agricultural industry accounts for every province. In the most disaggregated SAM the agricultural industries are defined per district municipality. This SAM combines the district municipalities within a province that produce more than 50% of either crops, or horticultural products, or animals and animal related products into one account. The remaining district municipalities are grouped together into a second account. The principle is that areas with similar production technologies should be grouped together. If the second account is too small in terms of value of output to justify a separate account, then the agricultural industry in the province is represented by only one account. For Gauteng and the Eastern Cape there is only one agricultural account for each province. The account name indicates the predominant crop. See section 10.4 in the addendum for the categorisation of the district municipalities into agricultural industry accounts in the SAM.

The labour accounts for the aggregated SAM are categorised by a) province; b) whether the person works in the agricultural or non-agricultural sectors; and c) whether they are skilled or unskilled (inclusive of semi-skilled). Household accounts are grouped according to a) province of residence; b) whether the main source of income is derived from work in the agricultural sector or from other sources, which include work in the non-agricultural sector, welfare grants and asset income; and c) whether they are poor or non-poor, where poor households constitute the bottom 50% of all households ranked according to per capita income.

5.3.7 The South African economy as portrayed by the SAM used in the case study

A bird's eye view of the South African economy

Table 18 presents the macro SAM for South Africa in 2007 derived as an aggregation from the detailed balanced SAM. It therefore differs slightly from the SARB totals in Table 16. Gross domestic product at purchasers' prices was R1 999 billion in 2007, with 42.8% (R856bn) accruing to labour services, 44.2% (R883bn) accruing to capital, 0.45% (R9bn) accruing to land and the remaining 12.5% accounted for by net taxes on products (R224bn) and production (R26bn).

Imports accounted for 13.8% (R691bn) of total supply of R5 002bn, with the remaining 86.2% (R4 311bn) of supply produced domestically (measured at purchasers' prices). Total domestic production cost at basic prices (plus trade and transport margins) of R4 087bn, comprises payments to primary inputs of 42.8% (R1 740bn), intermediate inputs of 56.6% (R2 312bn) and production taxes of 0.6% (R26bn).

The demand for products as intermediate inputs accounted for 46.2% (R2 312bn) of total demand for products (R5 002bn); domestic final demand accounted for 41.2% (R2 061bn) and exports for 12.6% (R629bn). South Africa was a net importer of goods and services from the rest of the world in 2007 (R62bn), and together with the net factor and institutional expenditures to the rest of the world (R84bn) it caused a net deficit on the current account of R146bn.

Sources of household income (R1 626bn), as captured in the household row account, were reasonably diversified. Income from labour services provided 52.5% of household incomes, with the remainder coming from returns to capital (25.8%), returns from land ownership (0.5%), inter household transfers (2.1%), payments from corporations (12.8%) and transfers from government (6.2%). Household expenditures, as captured in the household column account, were dominated by current consumption (75.5%), payments to corporations (12.6%), with direct taxes taking a further 9.9% and savings being negative at -0.4%. Transfers to other households are 2.1% of total household expenditure.

Table 18: A macro SAM for South Africa for 2007 (R million), aggregated final SAM

		1	2	3a	3b	3c	4	5	6a	6b	7a	7b	8
		Products	**Industries**	**Factors**			**Households**	**Corporations**	**Government**		**Capital**		**Rest of World**
				Capital	*Labour*	*Land*			*Taxes*	*Expend*	*Invest-ment*	*Stock changes*	
1	**Products**		2 311 956				1 227 874			394 654	422 300	16 051	629 092
2	**Industries**	4 086 994											
	Factors												
3a	*Capital*		883 393										42 873
3b	*Labour*		856 296										5 575
3c	*Land*		8 973										
4	**Households**			420 046	853 632	8 838	33 661	207 298		101 428			732
5	**Corporations**			185 343			204 864	18 573					
	Government												
6a	*Taxes*	224 114	26 376				161 168	156 233					
6b	*Income*			-39 130			4 286	11 189	567 890				1107
	Capital												
7a	*Savings*			256 378			-6 827	14 914		27 810			146 076
7b	*Stock changes*										16 051		
8	**Rest of World**	690 820		103 629	8 238	135	610	573		21 450			-61 729
9	**Totals**	**5 001 928**	**4 086 994**	**926 266**	**861 870**	**8 973**	**1 625 635**	**408 780**	**567 890**	**545 342**	**438 351**	**16 051**	**763 726**

Source: Own calculations as discussed

Total government income (gross) was R584bn, with 97.2% from production, product and income taxes; 2.6% from transfers from institutions and 0.2% from transfers from the rest of the world. There was a R39bn loss with regard to returns to capital, giving net government income of R545bn. Total government expenditure of R545bn comprised consumption expenditure (72.4%), transfers to institutions (18.6%) and transfers to the rest of the world (3.9%). The government savings of R21bn accounts for the remaining 5.1% of government income.

Gross domestic investment was R422bn (21.1% of GDP), and this was complemented by a small increase in stocks (R16bn), giving total investments of R438bn. Domestic savings come from dissavings by households (R7bn), the allowance for depreciation (R256bn) and savings out of corporations' income (R15bn).

Looking at the South African economy at a more detailed level

This section presents selected results from the SAM aggregation discussed in the previous section. Table 19 shows the share of the total value of demand of intermediate use, household consumption, government consumption, investment and stock changes, and exports for the product groups included in the SAM. Cereals, which include wheat, maize and other cereals, are mostly used as intermediate demand (96.6% of demand) and the remainder is exported (3.3%). Other crops such as sugar and cotton are predominantly used as intermediate products (60.9%), but a large share is also exported (39.3%). Vegetables are mostly consumed locally (95.6%), whereas a large share of fruit and other horticultural products are exported (25.9% and 36.2% respectively). Processed food, beverage and tobacco products are the products mostly consumed directly by households (between 63.2% and 77.0%).

Mining exports accounts for 57.8% of the demand for mining products, whereas intermediate use is the most dominant demand for forestry and fishing products (82.2%), utilities (electricity and water) (61.2%) and trade and transport services (61.4%). 64.4% of construction is demanded from investment. The share of demand accounted for by intermediate input use of other services (commercial and government services) is 43.6%, whereas 27.3% of demand for other services is accounted for by household consumption and 23.2% by government consumption.

Table 19: Components of total demand by product (row percentages)

	Intermediate demand	Household consumption	Government consumption	Investment and stock changes	Exports	Total
			%			(R million)
Cereals	96.6	0.0	0.0	0.1	3.3	14 163
Other crops	60.9	0.0	0.0	-0.2	39.3	11 215
Vegetables	3.0	95.8	0.0	0.0	1.2	25 716
Fruit	23.6	50.6	0.0	0.0	25.9	22 754
Other horticulture	41.9	21.9	0.0	0.0	36.2	4 103
Animals and animal products	53.6	33.5	0.0	0.8	12.1	50 693
Forestry and fishing products	82.2	7.2	0.0	0.0	10.6	19 412
Mineral products	42.7	0.0	0.0	-0.5	57.8	332 513
Food high crop content products	29.6	63.2	0.0	0.6	6.7	101 950
Food high horticultural content products	12.2	64.8	0.0	0.3	22.7	22 115
Food high animal content products	24.9	67.4	0.0	1.5	6.3	131 127
Food beverage tobacco and other products	15.5	77.0	0.0	1.1	6.4	138 498
Textile and clothing products	38.8	46.2	0.0	0.0	15.1	137 071
Wood paper and media products	79.1	5.8	0.0	2.1	13.0	132 255
Petroleum and other non-metallic products	66.7	22.0	0.0	1.9	9.4	521 332
Metallic and machinery products	46.7	6.8	0.0	25.1	21.5	610 877
Transport and other manufacturing products	41.6	23.8	0.0	20.6	14.0	429 851
Utilities	61.2	37.0	0.0	0.0	1.8	86 986
Construction and building	34.2	0.0	0.0	64.4	1.3	237 311
Trade and transport services	61.4	26.6	0.0	0.0	12.0	267 760
Other services	43.6	27.3	23.2	1.6	4.3	1 704 225
Total	46.2	24.5	7.9	8.8	12.6	5 001 928

Source: Own calculations based on developed SAM

Table 20 shows the share of the total value of output of payments to intermediate inputs, gross operating surplus (GOS), land, labour, and net taxes for product groups in the SAM. For agriculture the returns to land are between 6.8% and 8.3% of total output and intermediate inputs up to 60.9% (Western Cape other agriculture). The payments to capital (GOS) are relatively high for agricultural industries because these payments also contain mixed income for producers who are land owners. A share of this income could be regarded as labour income, but the information to separate this from payments to capital is unavailable. The forestry and fishing industry, followed by the minerals industry, are the most capital intensive industries, with GOS comprising 44.1% and 34.8% of their total output, respectively.

Table 20: Components of total output by industry (row percentages)

	Interme-diate use	GOS	Land	Labour	Net production taxes	Total
			%			(R million)
Western Cape horticulture >50%	51.1	26.5	7.7	16.8	-2.1	12 806
Western Cape other agriculture	60.9	22.3	7.0	11.5	-1.7	9 033
Northern Cape animals >50%	46.6	30.3	7.5	17.5	-1.9	1 990
Northern Cape other agriculture	49.8	32.0	7.3	13.0	-2.1	4 382
Eastern Cape animals >50%	52.3	29.3	7.4	13.2	-2.2	7 138
KwaZulu-Natal crops >50%	54.6	24.3	7.8	15.7	-2.4	5 073
KwaZulu-Natal other agriculture	53.6	30.3	7.2	10.7	-1.8	8 221
North West crops >50%	59.2	23.2	8.3	11.7	-2.5	3 847
North West other agriculture	51.8	31.5	7.0	11.7	-2.0	7 694
Gauteng animals >50%	56.1	26.9	7.4	11.8	-2.2	9 734
Free State crops >50%	58.2	25.8	7.5	10.3	-1.8	8 484
Free State other agriculture	50.8	29.7	7.3	14.6	-2.5	7 185
Mpumalanga horticulture >50%	44.0	39.4	7.2	10.7	-1.4	5 160
Mpumalanga other agriculture	65.5	15.3	7.6	13.9	-2.2	6 928
Limpopo horticulture >50%	58.7	23.9	7.9	11.6	-2.1	3 946
Limpopo other agriculture	63.1	16.6	6.8	15.2	-1.7	3 275
Forestry and fishing	40.8	44.1	7.6	10.0	-2.4	15 985
Minerals	43.1	34.8	0.0	21.5	0.5	260 375
Food high crop content	73.7	13.2	0.0	12.7	0.3	65 601
Food high horticultural content	74.5	11.6	0.0	13.8	0.1	11 415
Food high animal content	88.1	6.7	0.0	5.1	0.1	76 542
Food beverage tobacco and other	65.2	22.6	0.0	11.7	0.5	77 979
Textile and clothing	73.7	9.8	0.0	16.8	-0.3	42 351
Wood paper and media	74.6	9.5	0.0	15.6	0.3	98 902
Petroleum and other non-metallic	74.1	15.6	0.0	10.2	0.1	335 937
Metallic and machinery	70.8	15.1	0.0	13.9	0.2	307 440
Transport and other manufacturing	77.3	13.1	0.0	9.7	-0.1	245 917
Utilities	51.8	29.6	0.0	18.5	0.1	84 365
Construction and building	76.3	9.5	0.0	13.9	0.3	218 799
Trade and transport services	49.6	26.2	0.0	23.4	0.9	615 798
Other services	44.8	24.5	0.0	29.4	1.3	1 524 692
Total	56.6	21.6	0.2	21.0	0.6	4 086 994

Source: Own calculations based on developed SAM

Table 21 shows the distribution of labour payments to labour categories within each industry. The labour groups are identified on the basis of whether more than 50% of income is earned from agricultural industries or non-agricultural industries and the skill level of labour. Semiskilled labour is mostly grouped with unskilled labour, except for certain instances in the agricultural industries where there were too few observations in the original data to keep skill levels separate. For example at the most disaggregated level of the SAM all white females in agriculture would be categorised as skilled labour because there are too few unskilled white females to justify a separate account. Of the total labour income of R856bn, only 1% goes to skilled labour in agriculture, whereas 0.6% goes to unskilled labour in agriculture. Skilled and unskilled labour in non-agricultural industries earn 48.0% and 50.4% of total income respectively.

Table 21: Distribution of labour payments by industry (row percentages)

	Agric Skilled	Agric Unskilled	Non-agric Skilled	Non-agric Unskilled	Total
	%				(R million)
Western Cape horticulture >50%	46.3	53.7	0.0	0.0	2 156
Western Cape other agriculture	56.3	43.7	0.0	0.0	1 037
Northern Cape animals >50%	87.5	12.5	0.0	0.0	349
Northern Cape other agriculture	87.0	13.0	0.0	0.0	569
Eastern Cape animals >50%	52.9	47.1	0.0	0.0	941
KwaZulu-Natal crops >50%	40.1	59.9	0.0	0.0	799
KwaZulu-Natal other agriculture	57.0	43.0	0.0	0.0	884
North West crops >50%	68.9	31.1	0.0	0.0	452
North West other agriculture	80.0	20.0	0.0	0.0	900
Gauteng animals >50%	73.4	26.6	0.0	0.0	1 146
Free State crops >50%	87.9	12.1	0.0	0.0	876
Free State other agriculture	82.5	17.5	0.0	0.0	1 051
Mpumalanga horticulture >50%	44.9	55.1	0.0	0.0	554
Mpumalanga other agriculture	66.8	33.2	0.0	0.0	961
Limpopo horticulture >50%	64.5	35.5	0.0	0.0	456
Limpopo other agriculture	62.6	37.4	0.0	0.0	496
Forestry and fishing	0.0	0.0	23.7	76.3	1 591
Minerals	0.0	0.0	23.5	76.5	56 101
Food high crop content	0.0	0.0	27.8	72.2	8 349
Food high horticultural content	0.0	0.0	28.1	71.9	1 573
Food high animal content	0.0	0.0	23.3	76.7	3 913
Food beverage tobacco and other	0.0	0.0	33.5	66.5	9 095
Textile and clothing	0.0	0.0	21.5	78.5	7 108
Wood paper and media	0.0	0.0	35.5	64.5	15 428
Petroleum and other non-metallic	0.0	0.0	50.6	49.4	34 153

	Agric Skilled	Agric Unskilled	Non-agric Skilled	Non-agric Unskilled	Total
	%				(R million)
Metallic and machinery	0.0	0.0	36.4	63.6	42 708
Transport and other manufacturing	0.0	0.0	30.3	69.7	23 956
Utilities	0.0	0.0	46.3	53.7	15 602
Construction and building	0.0	0.0	12.8	87.2	30 367
Trade and transport services	0.0	0.0	34.0	66.0	143 997
Other services	0.0	0.0	63.3	36.7	448 730
Total	1.0	0.6	48.0	50.4	856 296

Source: Own calculations based on developed SAM

Figure 9 presents the different sources of income for the different household categories. Incomes from corporations and international remittances account for less than 0.2% of total income and have been omitted from the figure because it is not visible. As mentioned, the GOS income for non-poor households receiving more than 50% of the income from agricultural industries (62%) also include some payments to entrepreneurship. The non-agricultural poor households receive 22% of their income in the form of government grants, followed by the agricultural poor households with 10%.

Figure 10 indicates expenditure shares by households. Household are grouped based on the main income source of the head of household and are classified as poor or non-poor depending on whether the household is in the bottom or top 50% of all South African households sorted by per capita income. On average non-poor non-agricultural households spend a larger proportion of their disposable income on services and non-food items (77%) compared to any of the other household groups. In contrast the agricultural poor households spend 57% of their disposable income on agricultural and processed food items.

Figure 9: Household income sources

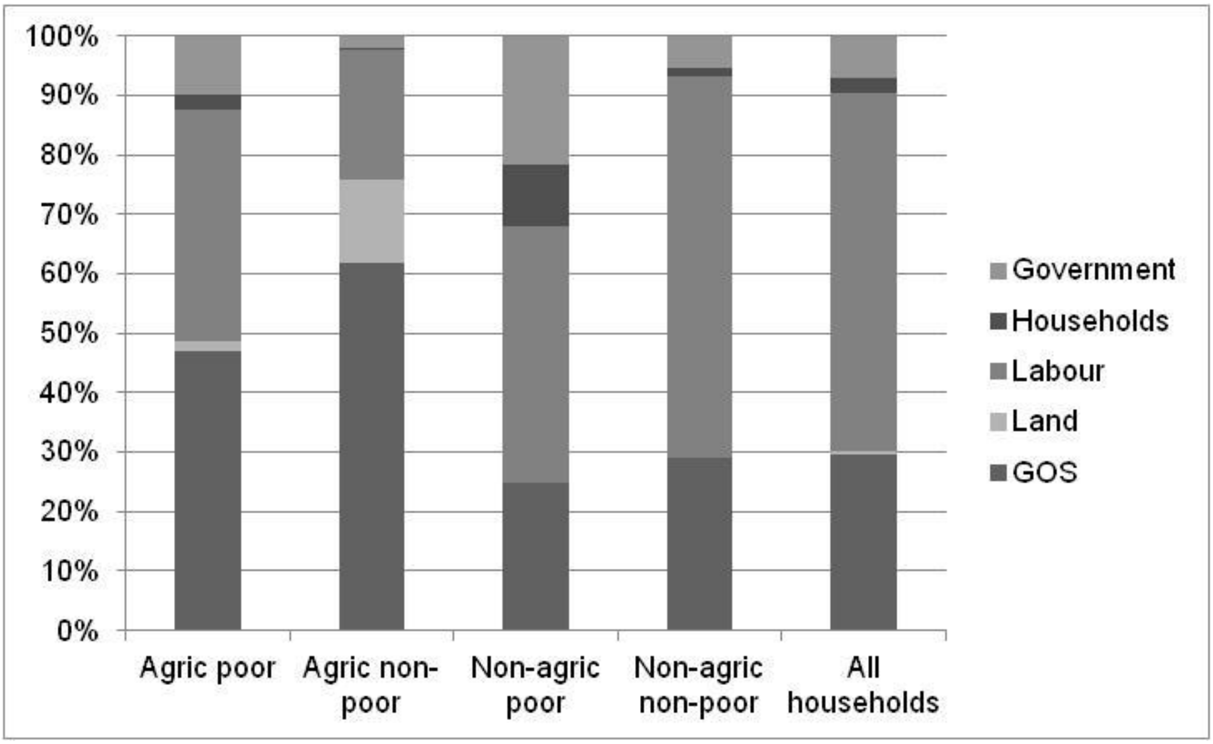
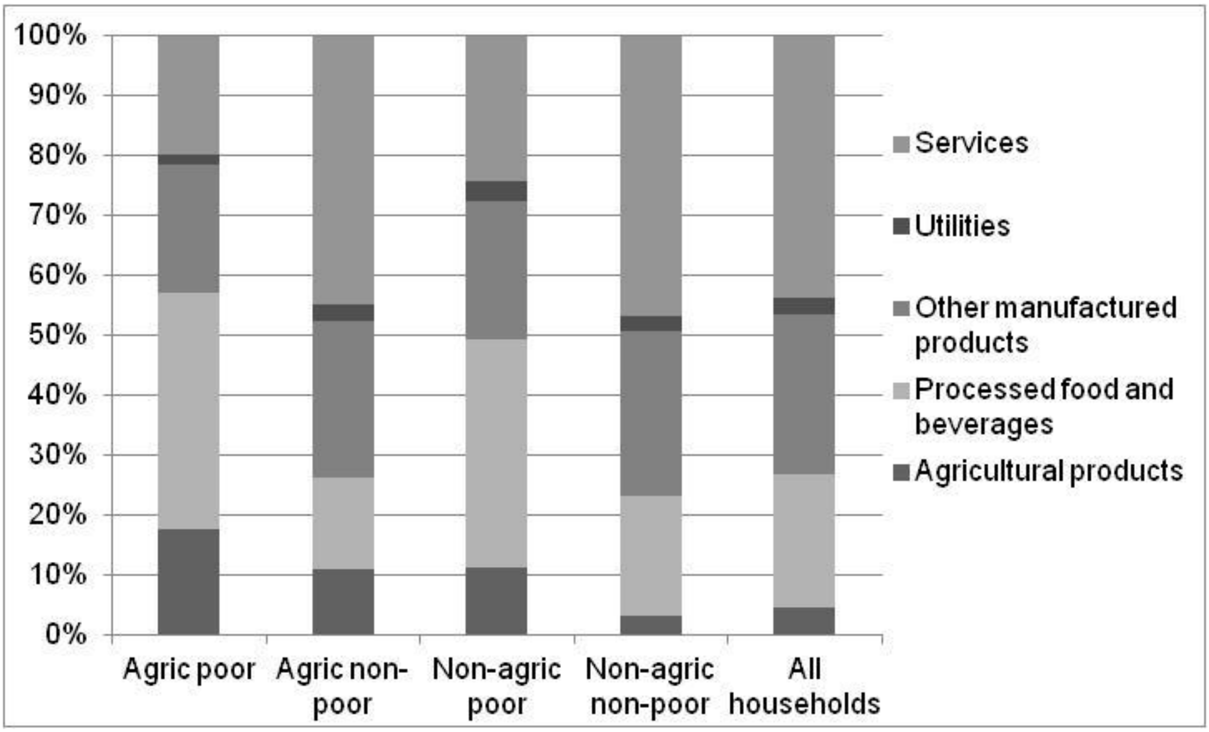


Figure 10: Expenditure shares of households



5.4 Employment data

Employment data is treated as a satellite (external) account to the SAM and provides quantity information on the number of employees from each labour category employed in each of the identified industry categories. Although it is not essential, it is useful to have employment data, because it allows employment results to be expressed not only in terms of values, but also in terms of physical units such as the number of employees. Also, it allows for information on wage rates that can be derived from dividing the SAM value by the number of employees. It is because of the way in which wage rates are deduced that it is important that employment data should correspond with the data used for the SAM sub-matrix that depicts the payments from labour to households. If the same set of data is not used, then the wage rates will be distorted because the wage and salary information will no longer pertain to the original number of wage and salary earners.

The data contained in the SAM are based on IES and LFS for 2000. The initial intention was to use the IES and LFS for 2005, but when the data was released in 2008 it was found that the respondents in the survey samples of the two surveys were no longer linked. The 2005 surveys therefore did not contain sufficient information to include it in the SAM, because it was unclear to which household category each of the members of the labour categories belonged and in order to analyse income redistribution it is important to trace which households are the recipients of factor incomes.

The employment data were taken from the LFS 2000, but the data needed some adaptation. Two main issues arose because of the older employment data that were used. First, the total number of employees in 2007 was different to the number of employees in 2000. Second, during the balancing of the SAM, some of the values of wages and salaries are changed; hence the implicit wage rates are changed if the employment data are not proportionately adjusted. Hence the employment data for the entire sub-matrix was first scaled to the 2007 value of salaries and wages, then the implicit wage rates were calculated and the SAM values were divided by the wage rates to determine the employment numbers. The last step was to scale all the figures to add up to the total number of employees in 2007, i.e. 12.5 million (SSA, 2008b).

Table 22 shows the distribution of labour across different labour groups for each of the industry groups included in the SAM satellite account. There were 12.5 million people employed in 2007, of which 6.7% were earning more than 50% of their income from agriculture (2.1% skilled and 4.4% unskilled). In the non-agricultural sectors there were 21.2% skilled and 72.4% unskilled workers. These employment figures do not represent full years worked, but rather number of people that performed work for pay during 2007, as captured in the LFS. The LFS 2000 did not collect the necessary information to calculate the employment periods of the persons who performed work during the year.

Table 22: Employment by labour group and industry (row percentages)

	Skilled agriculture	Unskilled agriculture	Skilled non-agriculture	Unskilled non-agriculture	Total
	%				(Number)
Western Cape horticulture >50%	14.8	85.2	0.0	0.0	96 967
Western Cape other agriculture	34.6	65.4	0.0	0.0	48 735
Northern Cape animals >50%	38.2	61.8	0.0	0.0	12 282
Northern Cape other agriculture	31.0	69.0	0.0	0.0	19 594
Eastern Cape animals >50%	41.5	58.5	0.0	0.0	114 868
KwaZulu-Natal crops >50%	14.9	85.1	0.0	0.0	106 521
KwaZulu-Natal other agriculture	31.1	68.9	0.0	0.0	94 650
North West crops >50%	35.2	64.8	0.0	0.0	34 272
North West other agriculture	47.6	52.4	0.0	0.0	23 205
Gauteng animals >50%	47.4	52.6	0.0	0.0	40 555
Free State crops >50%	60.7	39.3	0.0	0.0	32 938
Free State other agriculture	48.5	51.5	0.0	0.0	46 208
Mpumalanga horticulture >50%	20.0	80.0	0.0	0.0	35 259
Mpumalanga other agriculture	38.3	61.7	0.0	0.0	44 309
Limpopo horticulture >50%	30.8	69.2	0.0	0.0	37 872
Limpopo other agriculture	26.7	73.3	0.0	0.0	24 507
Forestry and fishing	0.0	0.0	2.1	97.9	80 120
Minerals	0.0	0.0	7.7	92.3	749 533
Food high crop content	0.0	0.0	8.7	91.3	189 882
Food high horticultural content	0.0	0.0	14.1	85.9	32 141
Food high animal content	0.0	0.0	11.0	89.0	65 105
Food beverage tobacco and other	0.0	0.0	11.7	88.3	178 767
Textile and clothing	0.0	0.0	8.3	91.7	193 602
Wood paper and media	0.0	0.0	11.8	88.2	257 773
Petroleum and other non-metallic	0.0	0.0	18.3	81.7	417 966
Metallic and machinery	0.0	0.0	15.4	84.6	590 871
Transport and other manufacturing	0.0	0.0	11.8	88.2	386 245
Utilities	0.0	0.0	22.2	77.8	152 234
Construction and building	0.0	0.0	2.9	97.1	696 983
Trade and transport services	0.0	0.0	11.6	88.4	2 815 398
Other services	0.0	0.0	38.7	61.3	4 925 613
Total	2.1	4.4	21.2	72.4	12 544 975

Source: Own calculations based on developed SAM

Table 23 shows the average annual wages earned per person within each of the four labour groups for each of the identified industries. As indicated above, employment numbers do not necessarily represent a full year of work; hence the average annual earnings are not a reflection of wage rates because it does not take the number of days per year worked into account. The expectation is that skilled labour will be less prone to have periods of unemployment than unskilled labour. This aggravates the discrepancy in average earnings between skilled and unskilled labour compared to the discrepancy in average earnings based on actual wage rates if all labour were fully employed during the year. The numbers below will capture the differences in actual wage rates as well as differences between temporary and permanent jobs, where a person that worked in a temporary might be unemployed for a part of the year. The discrepancies between skilled and unskilled average annual earnings are noteworthy in both the agricultural (R33 245 vs. R8 928) and the non-agricultural sectors (R154 935 vs. R47 517).

Table 23: Average annual earnings per person by industry and factor group (Rand)

	Skilled agriculture	Unskilled agriculture	Skilled non-agriculture	Unskilled non-agriculture	Weighted Average
	(Rand)				
Western Cape horticulture >50%	69 266	14 031			22 232
Western Cape other agriculture	34 639	14 230			21 286
Northern Cape animals >50%	65 076	5 757			28 403
Northern Cape other agriculture	81 412	5 453			29 035
Eastern Cape animals >50%	10 435	6 605			8 195
KwaZulu-Natal crops >50%	20 164	5 278			7 497
KwaZulu-Natal other agriculture	17 128	5 820			9 335
North West crops >50%	25 782	6 335			13 182
North West other agriculture	65 239	14 771			38 782
Gauteng animals >50%	43 775	14 260			28 250
Free State crops >50%	38 517	8 179			26 591
Free State other agriculture	38 688	7 736			22 751
Mpumalanga horticulture >50%	35 333	10 813			15 710
Mpumalanga other agriculture	37 809	11 667			21 685
Limpopo horticulture >50%	25 262	6 177			12 054
Limpopo other agriculture	47 567	10 323			20 256
Forestry and fishing			229 034	15 474	19 862
Minerals			229 482	61 984	74 848
Food high crop content			140 892	34 748	43 968
Food high horticultural content			97 910	40 932	48 943
Food high animal content			126 551	51 841	60 095
Food beverage tobacco and other			146 046	38 288	50 875
Textile and clothing			94 791	31 439	36 714
Wood paper and media			179 424	43 789	59 851
Petroleum and other non-metallic			226 192	49 410	81 712
Metallic and machinery			170 765	54 371	72 280

	Skilled agriculture	Unskilled agriculture	Skilled non-agriculture	Unskilled non-agriculture	Weighted Average
	(Rand)				
Transport and other manufacturing			158 973	49 020	62 023
Utilities			213 748	70 725	102 484
Construction and building			189 298	39 160	43 570
Trade and transport services			149 378	38 219	51 146
Other services			149 031	54 555	91 101
Average (weighted)	33 245	8 928	154 935	47 517	68 258

Source: Own calculations based on developed SAM

5.5 Parameters of functional forms

Elasticity parameters describe the responsiveness of producers and consumers to relative price changes and are essential for the calibration of the CGE model. Elasticities are therefore an important data requirement in addition to the SAM. Several sets of elasticities are required by CGE models. In practice some authors estimate elasticities econometrically, but in many cases the elasticities are borrowed from other studies or based on some general norms. See Annabi, Cockburn and Decaluwé (2006) for an overview of elasticities used in various studies. There exists a trade-off between model complexity and data availability, but, despite the lack of estimated elasticities, the model results tend to appear more realistic when including CES and CET functions compared to using Leontief fixed shares or Cobb-Douglas specifications.

The linear expenditure system (LES) functional form for household consumption is included in the CGE model and is deemed appropriate in the sense that it allows for a subsistence expenditure component, which is relevant for a developing country such as South Africa. The LES function does however require parameters outside of what is supplied by the SAM, e.g. Frisch parameters and income elasticities, and these are difficult to estimate because of lack of appropriate data for South Africa. Income elasticities for all *rich* households were assumed 0.4 for agricultural products, 0.6 for mining products, 0.7 for processed food products and 1.2 for other industries. Income elasticities for all *poor* households were assumed 0.6 for agricultural products, 0.8 for mining products, 0.9 for processed food products and 1.4 for other industries. The general trend is consistent with literature that poor households exhibit higher income elasticities than richer households, reflecting the larger consumption share of subsistence expenditure (Philippidis, Bourne and Childs, 2011). Frisch parameters were assumed to be between -3.1 and -4.5 for poorer households and between -1 and -3 for richer households, which is consistent with literature that the absolute value of the Frisch parameter should be larger for poorer households because a greater share of their income is used for subsistence expenditure compared to richer households (Philippidis, Bourne and Childs, 2011).

In trade relationships there are the elasticities of substitution for traded products relative to domestic products. The Armington elasticities for substitution between imports and domestically produced goods were informed by elasticities for South Africa reported by Gibson (2003) for non-agricultural products and Ogundeji (2007) for selected agricultural products, with primary and secondary products being mainly inelastic and tertiary products being elastic. Table 24 lists the Armington elasticities for substitution between imports and domestic products that were used in the case study in Chapter 6. For the CET function on exports and domestic products the elasticity for all products were set to 2.

Table 24: Armington elasticities used in the case study

Agricultural and food products	Armington Elasticity	Other products	Armington Elasticity
Cereals	1.273	Forestry and fishing products	1.273
Other crops	1.273	Mineral products	2.771
Vegetables	1.273	Textile and clothing products	1.262
Fruit	1.273	Wood paper and media products	1.205
Other horticulture	1.273	Petroleum and other non-metallic products	0.730
Animals and animal products	1.273	Metallic and machinery products	0.747
Food high crop content products	0.937	Transport and other manufacturing products	0.932
Food high horticultural content products	0.937	Utilities	1.437
Food high animal content products	0.937	Construction and building	2.100
Food beverage tobacco and other products	1.570	Trade and transport services	0.861
		Other services	1.066

With regard to production there are four elasticities of substitution for the CES production functions. Industry output is a CES aggregate of the quantities of aggregate intermediate inputs and value added (elasticity equals 1.2 for all industries). Aggregate value added is a CES aggregate of the quantities of primary inputs demanded by each industry (elasticity equals 0.5 for all industries), allowing for substitution between all factor types. In the case of differentiated products, as opposed to homogenous products, domestic product output is a CES aggregate of differentiated products by various industries for which the elasticity equals 4 for all products.

There are also the elasticities of transformation for the CET function on output to reflect price responsiveness of producers in their output composition. The elasticity for the CET function on output equals 0.9 for multi-product industries, notably for the agricultural industries because they do not have a clearly identifiable principal product. A summary of all the elasticities used in the adjusted CGE model is presented in Table 25.

Table 25: Elasticity values for the adjusted CGE model

	Functional form	Set	Value
Trade	CES on domestic production and imports	Products	Various
	CET on domestic production and exports	Products	2
Production	CES on value added and intermediate inputs	Industries	1.2
	CES on factors of production	Industries	0.5
	CES on product aggregation from different industries	Products	4
	CET for transformation of industry output to different products	Industries (agricultural)	0.9
Consumption	LES income elasticities	Industries*poor households	0.6 to 1.4
	LES income elasticities	Industries*rich households	0.4 to 1.2
	LES Frisch parameter	Poor Households	-3.1 to -4.5
	LES Frisch parameter	Rich Households	-1 to -3

Pyatt (1988:328) mentions that elasticities are one of the weaknesses of CGE models because elasticities are derived outside of the SAM framework. The weakness also lies in the fact that the responses to shocks can vary significantly when different values for elasticity parameters are used. Sensitivity analyses are ways to gauge how sensitive model results are for different values for elasticity parameters. Since the dissertation focuses on a change in a functional form, the sensitivity of model results to the inclusion of the CET function on output and the elasticity of transformation are reported in section 7.2. Results of the sensitivity analyses with regard to different values for all other elasticities and different model closures are presented in sections 7.3 and 7.4 respectively.

5.6 Summary and conclusions

This chapter gives an overview of existing SAMs for South Africa and data required for the calibration of the CGE model. Three sets of data are required, the main one being the SAM that captures the flows of funds between agents in the economy for the base year. The other two sets are the employment numbers consistent with the SAM and the elasticities for the calibration of the functional forms.

An agricultural SAM for South Africa for the base year 2007 was developed. The aim of the SAM is to have a SAM that can be used to calibrate the adjusted CGE model to be able to trace the distributional implications of policy changes on different household groups, specifically where regional policies are concerned. The SAM therefore includes numerous agricultural, household and labour accounts with provincial detail. The full version of the SAM contains 543 accounts, allowing for different aggregations depending on the focus of scenarios. A 137 account aggregation of the SAM was used for the case study and sensitivity analyses.

The main data sources of the 2007 SAM was the macro SAM developed from the national accounts data from the South African Reserve Bank, the 27 account supply and use tables for 2007 and the supply and use tables for 2002 from Statistics SA. Agricultural data for the detailed agricultural accounts were based on the censuses of agriculture for 2002 and 2007. Trade data for 2007 from the South African Revenue Service were also used for the international trade accounts for the disaggregation of the agricultural product accounts. Household and labour account details were obtained from the IES and LFS for 2000.

The two main challenges of the development of a SAM stem from its very nature, i.e. the requirement to be complete and consistent. Scarcity of data poses challenges to get information for the required levels of disaggregation for each of the sub-matrices, while the inconsistency of data from different data sources poses challenges for estimation of missing information to ensure that the row and column totals of the SAM are consistent for all accounts. Specific challenges in the South African context include the following:

- Lack of an official macro SAM according to the 1968 SNA structure, which is more appropriate for use in modelling developing countries than the structure proposed by the 1993 SNA that is currently used by Statistics SA.
- Detailed data on the agricultural sector at a regional level for South Africa are often dated by the time the agricultural surveys are released and the level of detail covered in the agricultural surveys in South Africa are decreasing over time, making it difficult to obtain all the data required for the development of a SAM. Agricultural data from other surveys are therefore required, but it is common that unless agriculture is the focus of a survey the agricultural data from other surveys are often not very reliable because of small sample size compared to for example the manufacturing industry.
- The IES for 2005 could not be directly linked to the LFS of 2005 in the similar way as could be done with the data for 2000. The 2005 surveys therefore fail to supply some of the core data for the SAM. The linking of persons in both surveys is necessary to get the information on household ownership of labour to complete the SAM sub-matrix on factor payments to households which is crucial in any income distribution policy analyses.

- In 2008 the biannual LFS were replaced by the Quarterly Labour Force Survey (QLFS) and the QLFS no longer contains income information that was previously available for the compilation of SAMs, implying even more challenges for future updates of SAMs for CGE model calibration.
- Official economic statistics on provincial level remains scarce, impacting on the accuracy of provincial level information in the SAM.
- Official agricultural census statistics still only apply to the commercial sector. Little reliable information on subsistence agriculture is available and data on products are scarce and inconsistent. Some of these issues are addressed as part of the SAM estimation process, but the confidence levels for the final figures in the SAM could be greatly improved if the base data for agriculture were more reliable.
- The absence of accurate elasticities for South Africa makes the CGE model results less reliable. Sensitivity analyses in Chapter 7 indicate sensitivity of model results for choice of elasticities, but even then the true elasticities are still unknown. Model results could be more sensitive for relative changes in elasticities between set elements than the actual level of the elasticities. Model results become more realistic when functional forms such as the CES, CET and LES are included, but it raises questions over the accuracy of the elasticities which remain unanswered.

In the extension of the CGE model and the development of the SAM for 2007, the SAM approach to modelling was followed insofar as the data and the model were developed in parallel, the one influencing the other. The agricultural industries by region, which produce various agricultural products, provide good examples of multi-product industries that are price responsive in their output composition. The CGE model was adapted to allow for price responsiveness in output composition of any multi-product industry in order to improve model results. The application of the model and data in a case study is presented in Chapter 6. The impact of the change in functional form in the CGE model and sensitivity analyses results with regard to different elasticities and model closures are presented in Chapter 7.

6 Case study: Employment and the agricultural sector

6.1 Introduction

The application of the adjusted model and developed SAM in a case study is reported in this chapter. In November 2011 the National Planning Commission (NPC) of South Africa released its National Development Plan (NDP) (NPC, 2011). The document first gives a short assessment of the current socio-economic situation in the country and then proceeds to identify the key issues that need to be addressed and some broad indications as to how these issues can be addressed. The scenarios developed for the case study relate to issues identified in the NDP.

Section 6.2 presents a short overview of selected topics from the National Development Plan. In section 6.3 the version of the SAM and the elasticities used are discussed as well as the scenarios that were analysed and the model closures that were implemented. The data and assumptions about elasticity values were discussed in the previous chapter, hence the focus here is on the scenarios and model closures. Results are discussed in section 6.4, focusing on a comparison of the impacts of the different scenarios. The chapter closes with a summary and conclusions. The validation of the model and sensitivity analyses with regard to elasticity values and model closures are presented in Chapter 7.

6.2 The National Development Plan

An integrated and inclusive rural economy is stated as one of the ten key objectives in the NDP (NPC, 2011). The aim is therefore to improve integration of South Africa's rural communities through greater opportunities to participate fully in the economy. This also implies greater access to basic services. The expected outcome of the objective according to the NDP is to create employment and reduce poverty in the rural areas and hence to address to some extent the huge inequalities in South Africa because poverty and inequality are perceived to be more severe in rural areas than in urban areas (NPC, 2011:195). The NDP mentions that the agricultural sector is viewed as one of the key drivers for improving livelihoods in the rural areas of South Africa, mainly because of its potential to absorb relatively large numbers of unskilled labour. The agricultural sector in South Africa has shed labour in recent years despite attempts to increase employment in agriculture, but the trend of shedding labour already started in the 1960's. From 1968 to 2003 the agricultural sector has shed 800 000 jobs and another 200 000 since 2003 (Sandrey, Punt, Jensen and Vink, 2011). Important also is that regular jobs are increasingly being replaced by casual labour, causing further vulnerability for the rural poor (Aliber and Simbi, 2000). According to the NDP the agricultural sector has the potential to create one million new job opportunities by 2030 in the agricultural sector and related industries (NPC, 2011:197). This goal presents a substantial challenge because it is not only a question of speeding up the rate of job creation but of reversing the current downward trend in employment in agriculture. The NDP recognises that rural economies will not be dependent only on agriculture, but also on mining, tourism, agro-processing and fisheries, where possible (NPC, 2011:196).

Besides improved integration of South Africa's rural communities into the mainstream economy, the NDP refers to various other objectives that relate either directly or indirectly to the agricultural sector, for example the positioning of South Africa in the world, the improvement of the quality of education and health care and fighting corruption (NPC, 2011). The labour shedding that has occurred in the agricultural sector during the past 10 to 15 years makes it obvious that it is not only economic determinants such as supply and demand that affect production decisions although those are also very important. The scenarios analysed in this chapter focuses on economic aspects such as export prices and other more general aspects such as technical efficiency through research and development, factor productivity in government and services and quality of education to address the skills shortage. The aim is to determine to what extent improvements in the conditions within which the agricultural sector operates have an impact on employment in the agricultural sector. In the agricultural sector a balance needs to be struck between encouraging high productivity employment and the expansion of lower wage jobs in order to reduce unemployment. Also, there is often a trade-off when targeting one sector instead of another. It is therefore important to determine the national impacts, because unemployment is not just a sectoral problem but a national problem. CGE models are particularly useful in determining the national impacts because it takes the interdependence in the economy into account.

6.3 Modelling employment and the agricultural sector

6.3.1 The SAM and elasticities

The SAM used in this application was discussed in section 5.3.6. Elasticity values used in this chapter are those listed in Table 25 in section 5.5. See section 10.3 in the addendum for the list of accounts of the SAM and section 10.4 for the categorisation of the district municipalities into agricultural industry accounts in the SAM.

There are one or two agricultural industry accounts for every province. This SAM combines the district municipalities within a province that produce more than 50% of either crops, or horticultural products or animals and animal related products into one account. The remaining district municipalities are grouped together into a second account. Because of the way the aggregation was carried out, all the agricultural industries produce at least one product that comprises a share of less than 2% of total output because fixed improvements (construction) for all of the agricultural industries are assumed to be around 1% of total output. The implication is that if the cut-off value for the CET function on product transformation is set above 1% the CET function will not be implemented for any of the agricultural industries. The CET cut-off was set at 0.01% so according to this criterion all industries with a non-zero elasticity of transformation will be price responsive in their output composition. All agricultural industries were assigned an elasticity of transformation of 0.9 whereas all non-agricultural industries were assigned an elasticity of transformation of 0. All agricultural industries were therefore assumed to be price responsive in their output composition. The implications of this assumption are considered as part of the sensitivity analyses in section 7.2 when results derived with the new model are compared to results from the base model. See section 4.3.3 for a discussion of the CET cut-off value.

6.3.2 Scenarios

Four different scenarios related to the NDP discussion are analysed using the CGE model.

Global positioning that leads to export growth: The first scenario relates to global positioning and assumes that through negotiation additional markets are opened up for fruit products. The NDP refers to the global positioning of South Africa and mentions as a goal the increase in exports (NPC, 2011:12). This could be interpreted to imply that new markets should be opened up for products, but certainly it also implies that current markets should be maintained. In other words the objective is market creation and not market diversion. The scenario is implemented in the model through changes in the world export price (*PWE*) because a CGE model is a price driven model; hence the variable for export quantities cannot be increased exogenously. BFAP (2011) identifies 19 agricultural products that are produced with labour intensive practices and that show high growth potential. 11 of these 19 products are fruit, with citrus fruit, deciduous fruit and table grapes ranking among the top 11 exporting products. The scenario therefore focuses on fruit and assumes a 5% increase in the world export price (*PWE*) of fruit and a 2% increase in the world export price of food products with a high horticultural content. The inclusion of the specific food products supports the assumption that the value chain is addressed when looking at market access. The primary agricultural producers respond favourably to both increased export demand and increased domestic demand by food manufacturers producing for either the domestic market or the export market. The international market is represented by a single account in the SAM, namely ‘the rest of the world’. The CGE model does not distinguish between different trading partners, hence no particular trading partner is targeted and the export price referred to in the current study is an aggregate export price faced by exporters. Although the model allows for assumptions of small country (price taker) or large country (influence international prices), where the large country assumption is introduced with downward sloping demand curves, the large country assumption was not implemented.

Improved technical efficiency of agricultural industries: The second scenario assumes a 1% increase in technical efficiency of agricultural industries relative to all other industries in the economy. Multi-factor productivity for South Africa between 1947 and 2008 is estimated at 1.49% (Liebenberg and Pardey, 2011:401). An increase in technical efficiency would require research and development, but there are already institutions in place to ensure this and the level of increase is assumed to be low enough to ensure that this can take place within current budgets allocated for this purpose. No additional investment is therefore simulated as part of the scenario. The scenario is implemented on the top level of the production nest with a CES aggregation of aggregate intermediate inputs (*QINT*) and aggregate value added (*QVA*) (primary factors) to form total industry output (*QX*). The value of the shift parameter (adx_a) of the top level CES function for agricultural industries is increased by 1% in the model (see equation 38 in section 3.7.2). The shift parameter (adx_a) is thus treated as a variable (ADX_a) in the model because it can assume new values. The way the scenario is implemented assumes that there is an improvement in the way that primary factors and intermediate inputs are combined and that the productivity levels of both are increased by the same proportion (Pauw, McDonald and Punt, 2007). This is in line with multi-factor productivity growth studies that include intermediate inputs in the input index in addition to factors of production (Liebenberg and Pardey, 2010).

Productive use of available resources: The third scenario assumes that corruption and strikes are minimised in the economy and therefore relates to a productivity increase as a result of more efficient and appropriate use of capital and labour as production factors. The NDP specifically refers to the inefficiencies in the government sector and widespread corruption (NCP, 2011:24). The existence of corruption implies that capital is not fully utilised for the purposes it is intended for and this is captured in the closure assumption that capital is not fully employed. South Africa ranks 144th out of 144 countries for labour-employer relations according to the Global Competitiveness Report (Schwab, 2012:325). In 2010 about 20 million work days were lost due to strikes (NCP, 2011:111). It is recognised that various industries suffer from strikes, but only strikes in government, municipalities and other service industries are taken into account in the way that the scenario was run. The scenario is implemented at the bottom level of the production nest with a CES aggregation of different factors of production to form value added (QVA). In the context of the services industry this implies an increase in the productivity of capital and labour. The value of the shift parameter ($adva_a$) for the second level CES function for 'other services' is increased by 2% in the model (see equation 40 in section 3.7.3). The shift parameter ($adva_a$) is thus treated as a variable ($ADVA_a$) in the model because it can assume new values. The scenario is implemented for 'other services' because it includes the general government, municipal services, and health and social work. It also includes financial services and other business services, but excludes trade and transport services. The efficiency with which all factors in 'other services' are used, is captured. This results in a biased form of technical change at the level of the production in the sense that producers of 'other services' will use more primary factors relative to intermediate inputs due to the relative improvement in the productivity of factors compared to intermediate inputs (Pauw, McDonald and Punt, 2007).

Improvement in the quality of education: The fourth scenario looks at the issue of quality of education, which is also mentioned in the NDP. South Africa has a relatively high share of total budget going towards education, accounting for 6.1% of GDP in 2009/2010 (World Bank, 2011:16), yet the results in terms of producing skilled workers are unsatisfactory. According to the Global Competitiveness Report (Schwab, 2012:325) South Africa ranked 140th out of 144 countries with regard to the quality of the educational system of higher education and training and 132nd for primary education. The inequalities from the apartheid era in terms of public spending on poor versus rich communities have been largely eliminated, yet the outcomes for poor students are lagging behind (World Bank, 2011:3). The World Bank suggests that this is caused by, amongst other, teachers that devote too little classroom time to teaching, deficiencies in management, lack of local accountability and perverse salary structures that attract and retain lower performing teachers. According to the

World Bank (2011:15): "... due to the fiscal space created and the successful targeting of fiscal resources to the poor, South Africa finds itself in an enviable position: fiscal resources have not been the binding constraint to improved service coverage and quality. Instead, the challenge is to ensure that public transfers result in better outcomes - current outcomes are disappointing." And as a specific example the report mentions the poor performance of South African learners in international education evaluations in the areas of maths, science and reading. It is therefore deemed possible to increase the number of skilled people in the country by 2% without additional budget but through improved management and accountability. The scenario is implemented by increasing the number of skilled labour (skilled labour subset for the factor supply variable – FS_s) by 2%, i.e. by 58 500. One of the reasons to implement the scenario across all economic sectors and not only for labour in the agricultural sectors is to illustrate the impact of increased effective demand in the economy on food consumption and agricultural production. This scenario is implemented together with the model closure that skilled labour is fully employed, which is deemed realistic in the context of South Africa which is characterised by a shortage of skilled labour (Deloitte, 2009). Unskilled labour is not assumed to be fully employed, hence the actual level of employment of unskilled labour cannot be fixed in the model, but is endogenously determined and this is not influenced by the supply of unskilled labour.

Combined effects: The fifth scenario assumes that all the prior scenarios can be achieved at the same time and constitutes the combination of the first four scenarios. It will be seen that the impact of the fifth scenario is greater than the sum of the impacts of the individual scenarios, confirming the additional multiplier effects in the economy. Although this scenario was run the results are not presented in most of the discussion due to limited space in graphs. In most cases the combined results are just slightly greater than the sum of the results for the different scenarios.

One of the explicit assumptions for all of these scenarios is that the results can be achieved within current budgets, but not as part of business as usual. The low productivity and widespread corruption in South Africa suggest that it is not additional budget that is required but making better use of available resources. Therefore no additional external funding is assumed in the scenarios in order to achieve these outcomes. The scenarios are summarised in Table 26. The first column shows the variables and accounts of the model through which the shocks are implemented.

Table 26: Scenarios

	Global positioning	Technical efficiency	Productivity of factor use	Quality education	All
World export prices (*PWE*) of:					
Fruit products	+5%	--	--	--	+5%
Food products with high horticultural content	+2%	--	--	--	+2%
Technical efficiency (*ADX*):					
All agricultural industries	--	+1%		--	+1%
Productivity (*ADVA*):					
Other services industry	--		+2%	--	+2%
Supply of labour (*FS*):					
All skilled labour	--	--	--	+2%	+2%

6.3.3 Model closure

From a modelling point of view the number of equations and variables need to be equal in order to close the model to ensure a solution. From an economic point of view the choice of variables to fix is indicative about how the economy is assumed to operate. The model closures are:

- For the current account the exchange rate remains flexible and the balance on the current account, which was a surplus of R146 billion in 2007, remains fixed. The flexible exchange rate is representative of the current situation for South Africa.
- Government savings, tax rates and transfer rates are fixed to base levels. The government account is therefore brought into equilibrium through flexible government consumption.
- For corporations the consumption volume as a share of final demand remains fixed and corporations' transfers to households are assumed to remain fixed.
- In the savings-investment closure the following identity must be balanced: investment = government balance + external balance + savings. The economy is assumed to be investment driven (as opposed to savings driven). The value of investment as a share of total final domestic demand is fixed; hence the savings rate of households and corporations is flexible and adjusts to maintain equilibrium. It was mentioned that the government and external balances remain fixed.
- Capital is assumed to be unemployed but mobile. The assumption that capital is not fully employed stems from the findings of the National Planning Commission that there is substantial corruption and mismanagement of funds in South Africa (NPC, 2011:401). The assumption of capital mobility reflects a longer term view within the CGE modelling context because it is assumed that capital can be withdrawn from one sector and reinvested in another within the period of adjustment between base equilibrium and the new (after shock) equilibrium.

- Land is assumed to be fully employed but immobile. Each agricultural industry is defined as a geographic region that produces a number of different products, and because land cannot be added or taken away from a predefined region the assumption of immobility of land is deemed realistic. The assumption that land is fully employed is a matter of subjectivity, but since none of the scenarios focus specifically on land saving technology, it is deemed a realistic assumption in the current set of scenarios.
- Skilled labour is assumed to be fully employed and mobile; unskilled labour is assumed to be unemployed and mobile. This is deemed to reflect the current reality in South Africa.
- The consumer price index (CPI) is used as numéraire, therefore all prices are expressed relative to the CPI, which is fixed.

6.4 Results

Results for selected macro indicators for the five scenarios are shown in Table 27. The global positioning scenario, which assumes a 5% increase in the world export price of fruit and 2% for food products with a high horticultural content, leads to an increase in the gross domestic product (GDP) of South Africa of 0.06% and a similar increase for employment. There is a 0.08% increase in the nominal value of total exports and a 0.06% increase in imports. The direct and indirect impacts of the world export price increases are also captured in the appreciation of the Rand of 0.02%.

The results in the table can be similarly interpreted for the other scenarios. The 1% increase in technical efficiency of agricultural industries leads to a slightly greater impact on GDP (0.27%) compared to the global positioning scenario. Results show that the increase in factor productivity of the government and services sector leads to a notable 2.32% increase in GDP and a greater increase in imports (2.38%) and exports (2.87%) than the global positioning and technical efficiency scenarios. The exchange rate depreciation is also notably larger than for the other scenarios. This is because the productivity growth scenario is implemented on the services sector which comprises a large share of the economy. Also, the nature of the impact is that changes for most of the industries are in the same direction; hence the macro results are larger. For the first two scenarios the changes for different industries often move in opposite directions, hence the net effects captured by the macro results are smaller. The exchange rate depreciates for all the scenarios except the global positioning scenario because these scenarios have a positive income effect, which leads to increased imports, which in turn put pressure on the exchange rate.

The 2% increase in the number of skilled labour in the economy (education scenario), has a positive impact on the economy with GDP increasing by 1.96%, bearing in mind that skilled labour is assumed to be fully employed. The last scenario, which combines all the effects, gives an increase of 4.67% in GDP, whereas the sum of increases for the individual scenarios amounts to 4.61%. This indicates the additional multiplier effects in the economy.

Table 27: Change in selected macro indicators (%)

	Global	Tech change	Productivity	Education	All
GDP	0.06	0.27	2.32	1.96	4.67
Employment (SA)	0.06	0.16	1.45	1.92	3.63
Exports (nominal)	0.08	0.30	2.87	2.36	5.68
Imports (nominal)	0.06	0.25	2.38	1.86	4.61
Exchange rate (domestic per world unit)	-0.02	0.07	0.31	0.08	0.44

On a more detailed level, Figure 11 shows that industry output (*QX*) increases for all agricultural industries for all the scenarios, except for the global positioning scenario where some of the agricultural industries with predominant animal and crop production are negatively affected. This is because it is only in the global positioning scenario that affects the relative prices of agricultural products directly. Agricultural industries are therefore responding differently to these price signals. Results for the global positioning scenario clearly show that the industries that engage in substantial fruit production will increase output relatively more than other agricultural industries. The agricultural industries with fruit as the main output include the industries in the Western Cape (horticulture >50%), Northern Cape (other agriculture), Mpumalanga (horticulture >50%) and Limpopo (horticulture >50%). Fruit production also comprise more than 20% of the output of the Western Cape (other agriculture) and Eastern Cape (animals >50%), hence the positive impact on output for these six industries.

The increases in industry output for the technical change scenario are between 0.4% and 1.4%, and between 0.7% and 1.1% for the productivity growth scenario. The education scenario leads to increases in output of agricultural industries between 0.7% and 1.3%. These positive impacts on industry output are according to expectation.

Figure 11: Change in industry output for agricultural industries (*QX*)

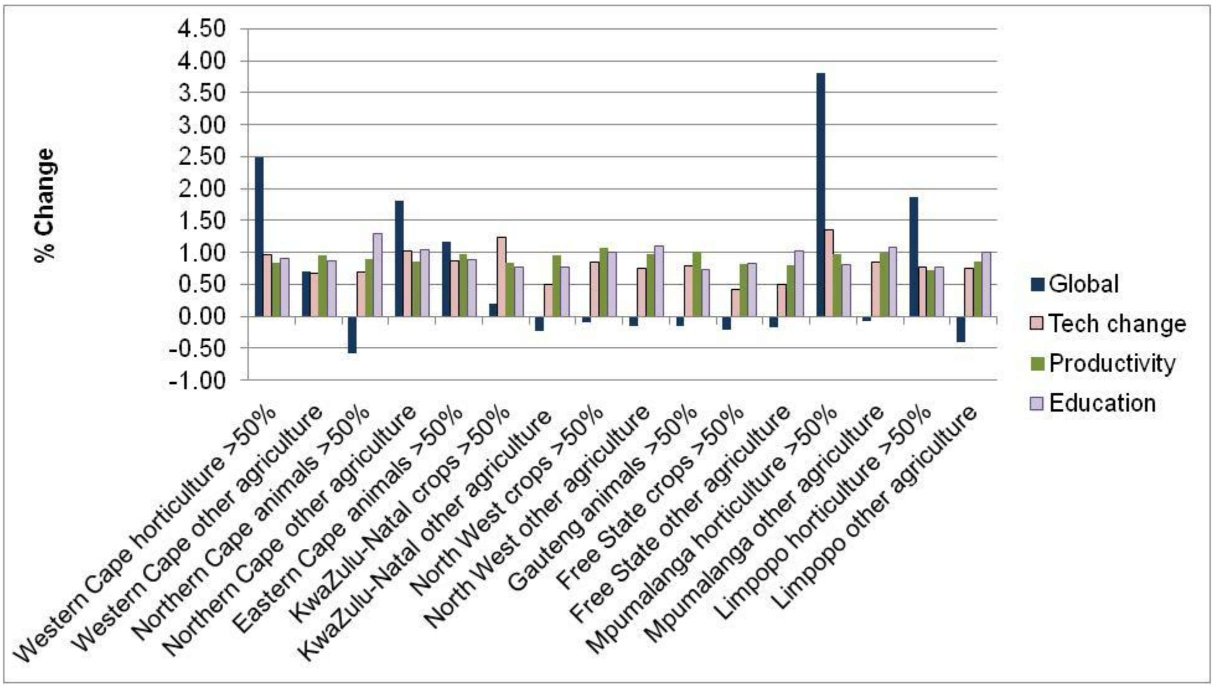


Figure 12 shows the changes in output (*QX*) of non-agricultural industries. The output of the industries that produce food with a relatively high horticultural content shows a greater increase in output than other non-agricultural industries in the global positioning scenario because the high horticultural content food industries are directly impacted in the shock introduced in the model. Although most industries show an increase in output, one exception is found in the global positioning scenario, namely for the minerals industry which is a relatively prominent exporter. The appreciation of the Rand in the global positioning scenario causes a small decline in output by the minerals industry due to declining competitiveness.

The technical efficiency improvements in the agricultural industries have a relatively small but positive impact on non-agricultural industries as a result of the indirect effects of the expansion of the agricultural industries that lead to increased incomes and expenditure in the economy. The productivity growth and education scenarios give fairly similar results for the non-agricultural industries, with the impact of the productivity growth scenario being the greater of the two. The productivity growth in the services sector increases output in the non-agricultural industries between 1.1% and 2.9% and these changes are larger compared to the changes for the agricultural industries shown in the previous figure (between 0.7% and 1.1%). Similarly, for the education scenario the non-agricultural industries increase output relatively more (between 1.0% and 2.7%) in reaction to increasing skilled labour by 2%, compared to the increases for the agricultural industries (between 0.7% and 1.3%). This is because the flexible wage rate for land leads to larger price effects as opposed to quantity effects for the agricultural industries. When capital is assumed to be fully employed then the change in the rate of return to capital has a similar effect on non-agricultural industries and the magnitudes of the changes are more similar across all the industries.

Figure 12: Change in industry output for non-agricultural industries (*QX*)

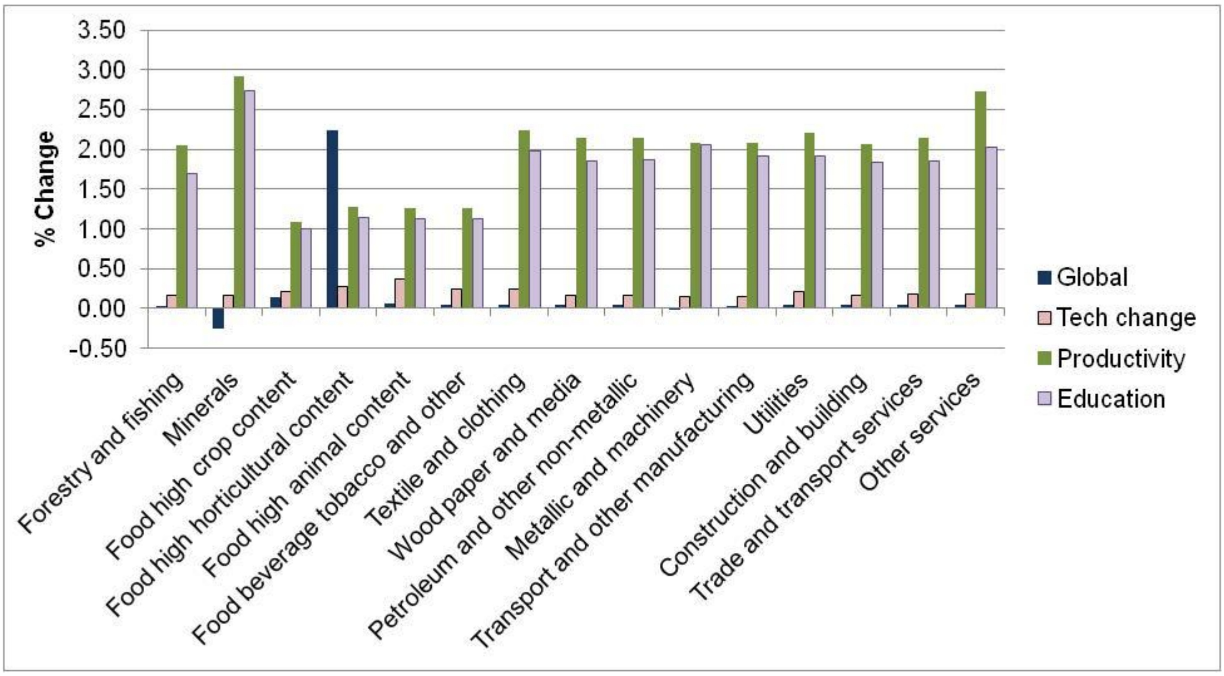


Table 28 shows the percentage change in output composition (*IOQXACQXV*) for the global positioning scenario for four agricultural industries, i.e. the change in the volume of each product as a share of total volume produced by a specific industry. This set of results is the direct outcome of the adaptation to the model which allows for changes in the output composition of industries. This was implemented for all the agricultural industries in the case study. The five agricultural industries for which results are reported were selected as the agricultural industries from the two top fruit producing provinces, the Western Cape and Mpumalanga; and the Eastern Cape, because it has a reasonable level of output of fruit products in the base situation (30.2% of total output from this industry, as shown in the bottom row of the table).

The world export prices of fruit are increased in the global positioning scenario. There is a substantial amount of fruit exported, hence the share of fruit in total output increases relative to the share of output of other products. Results show similar trends for all the agricultural industries. The share of fruit in total output in Mpumalanga (horticulture >50%) increases by 0.41%, whereas in the Western Cape (horticulture >50%) it increases by 0.69%. Bear in mind that the increase in the Western Cape is from a smaller share of agricultural output in the base case (55.9% vs. 73.0%). The changes in volume shares are relatively small because of the realtively inelastic elasticitiy of transformation (*omega* equal to 0.9) that was used. The assumption that land is immobile, as introduced through the closure for the land account, does not restrict expansion of fruit relative to other products but causes greater changes in output composition than would be the case if land was assumed to be mobile (as explained with reference to Figure 81 in Chapter 7). Detailed results for changes in output composition are only presented for the global positioning scenario because this scenario is the only one that directly affects relative prices of agricultural products. The changes in composition all appear realistic.

Table 28: Global positioning: change in output composition (*IOQXACQXV*) (%)

	Western Cape horticulture >50%	Western Cape other agriculture	Mpumalanga horticulture >50%	Mpumalanga other agriculture	Eastern Cape animals >50%
Cereals	-0.80	-0.28	-1.08	-0.07	-0.40
Other crops	-0.79	-0.27	-1.07	-0.06	-0.39
Vegetables	-0.95	-0.43	-1.22	-0.22	-0.55
Fruit	0.69	1.22	0.41	1.44	1.10
Other horticulture	-0.78	-0.26	-1.06	-0.05	-0.38
Animals and animal products	-0.89	-0.37	-1.16	-0.16	-0.49
Fruit base level share of output	**55.9**	**22.9**	**73.0**	**8.3**	**30.2**

The following three graphs show the minimum, maximum and average changes in output (*QXAC*), output composition (*IOQXACQXV*) and output prices (*PXAC*) for all agricultural industries for all four scenarios. Thereafter the results for each of these are shown for each of the agricultural industries separately, but only for the global positioioning scenario because it has the greatest impact on these results.

Figure 13 shows the minimum, maximum and average changes in quantity of each product produced by each agricultural industry (*QXAC*). It is only for the global positioning scenario that there is an actual decrease in production of products other than fruit. For all three the other scenarios production of all products increase, but it is only the next figure that reveals changes in the composition of the total output (*IOQXACQXV*). The longer the line, the more variation there is in the magnitudes of the changes that take place amongst the different agricultural industries. If agricultural industries tend to respond similarly to price changes, then the lines in the figures are shorter.

Figure 14 shows the minimum, maximum and average changes in the shares of products in the output per industry for all agricultural industries (*IOQXACQXV*). The figure indicates the relatively larger impact of the global positioning scenario on results compared to the other scenarios. This is because the price of fruit is changed as part of the scenario and the price change is relatively larger than the endogenously determined price changes obtained in any of the other scenarios. For the global positioning scenario the changes in shares range between -1.22% and 1.69%, with the share of fruit increasing on average by 1.3% and the shares of other products decrease on average with around 0.2%.

For the technical change scenario the changes in shares range between -0.51% and 0.96%, between -0.25% and 0.23% for the productivity scenario and between -0.21% and 0.22% for the education scenario and for all three scenarios the results are different in terms of which products increase in production share. Because no particular product is targeted in any of these three scenarios, the changes in output composition is merely the net effect of all product price changes in the economy.

The changes in composition are driven by changes in relative product prices faced by different industries (*PXAC*), which are shown in Figure 15. For the global positioning scenario the prices of fruit increase between 1.03% and 1.84%, with an average increase of 1.59%. For the technical efficiency scenario all industry product prices decrease (between -0.07% and -1.48%), but for the productivity growth scenario all the industry product prices increase (between 0.19% and 0.62%). Therefore the relative price changes impact in the same manner on various industries regardless of their product composition in the base case. There are mixed results for the education scenario, but the changes are relatively small (between -0.28% and +0.18%), hence the relatively small impact on the agricultural industries compared to the global positioning scenario.

Figure 13: **Change in product output by industry (*QXAC*) - summary by product**

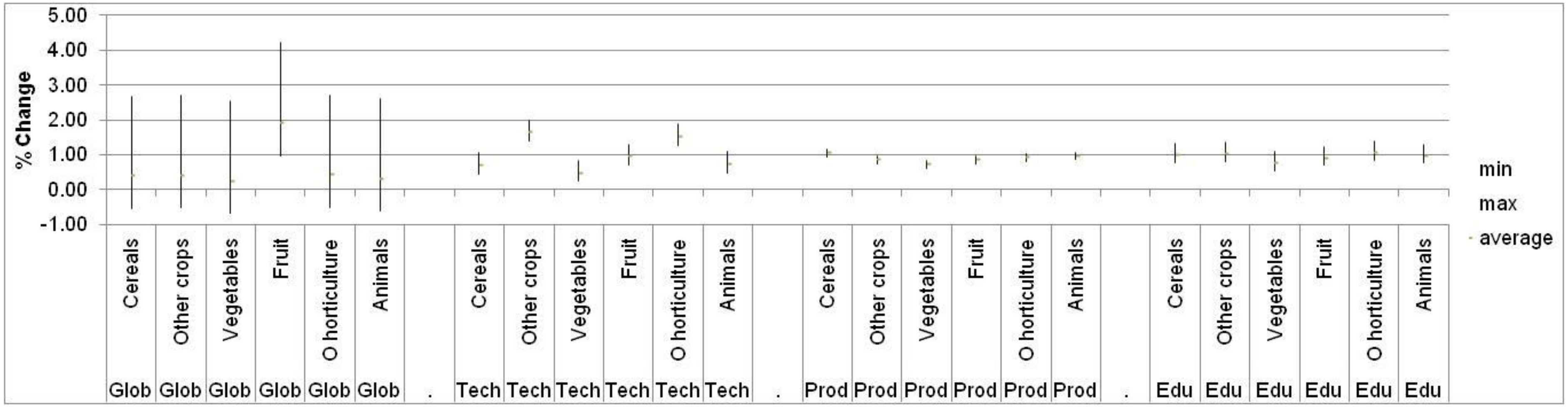


Figure 14: **Change in output composition (*IOQXACQXV*) - summary by product**

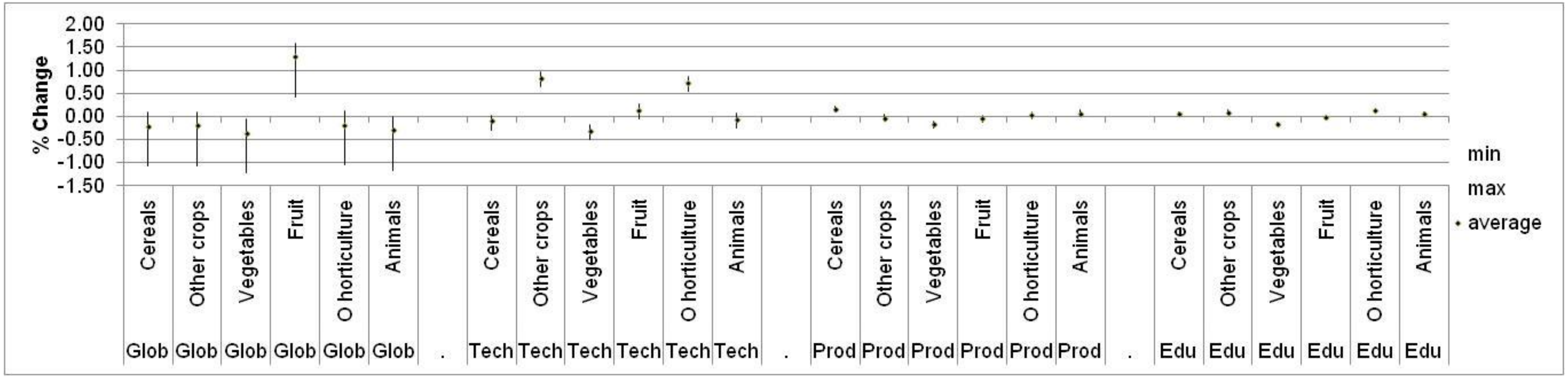
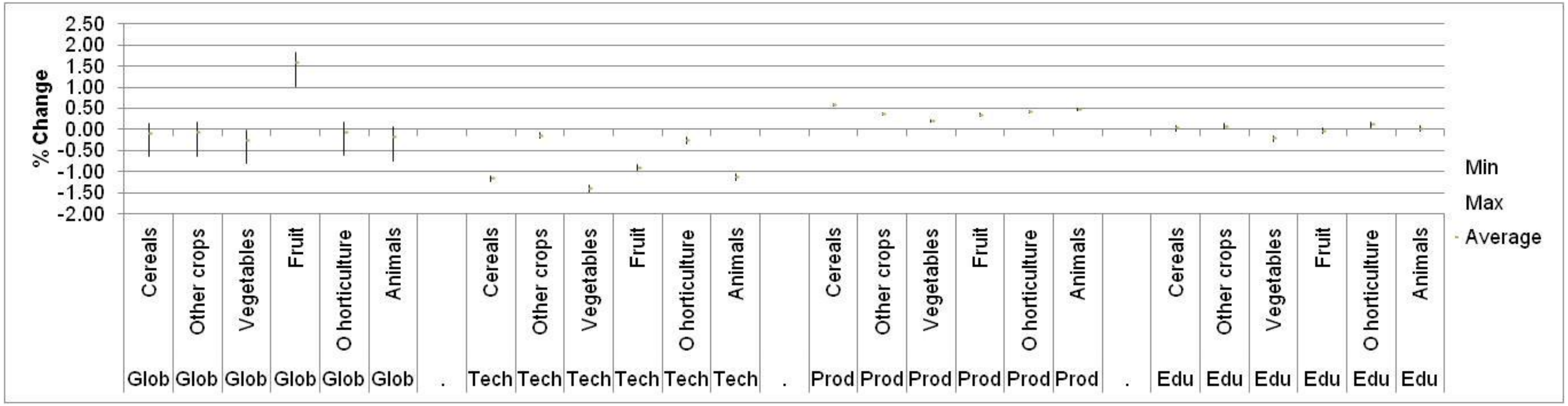


Figure 15: Change in industry output prices (*PXAC*) - summary by product



The following three graphs show the changes in quantity produced (*QXAC*), output composition (*IOQXACQXV*) and industry product prices (*PXAC*) only for the global positioning scenario in order to show the agricultural industry detail which was hidden in the previous three figures. The results for the other scenarios are not shown here because the changes are generally small as shown in the previous three figures.

Figure 16 indicates the changes in production of each product for each agricultural industry (*QXAC*). The main fruit producing areas (Western Cape horticulture >50%, Western Cape other agriculture, Northern Cape other agriculture, Eastern Cape, Mpumalanga horticulture >50% and Limpopo horticulture >50%) increase their production of fruit relative to other products, but because of limited substitutability this also causes pressure to increase production of all other products. This is not true for areas, as can be seen by the decrease in production of products other than fruit.

Figure 16: Global positioning: change in product output by industry (*QXAC*)

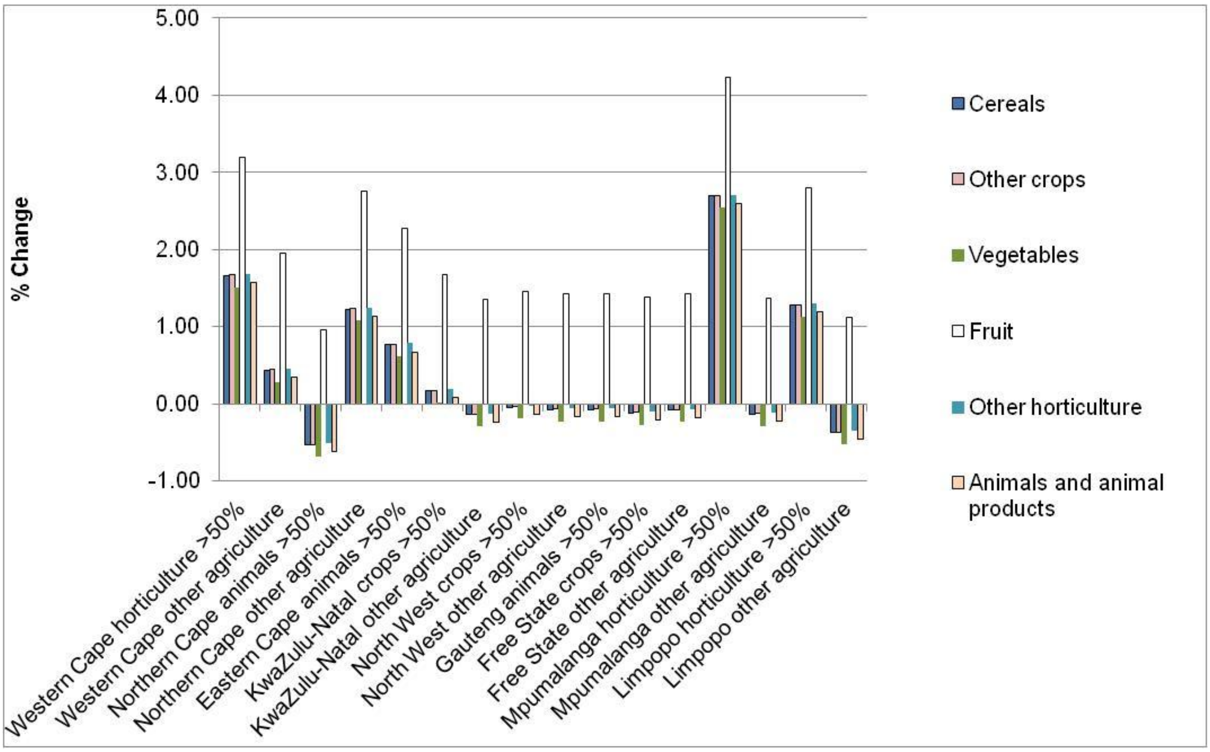


Figure 17 shows the changes in output composition of agricultural industries (*IOQXACQXV*) for the global positioning scenario. This figure reveals that as the share of fruit in total composition increases then the shares of some of the other products have to decrease. This effect is especially pronounced in the main fruit producing areas. Taking into account all the agricultural industries it was found that the share of fruit increases between 0.41% and 1.59%, while the share of other products change between -1.22% and +0.11%

Figure 17: Global positioning: change in output composition (*IOQXACQXV*)

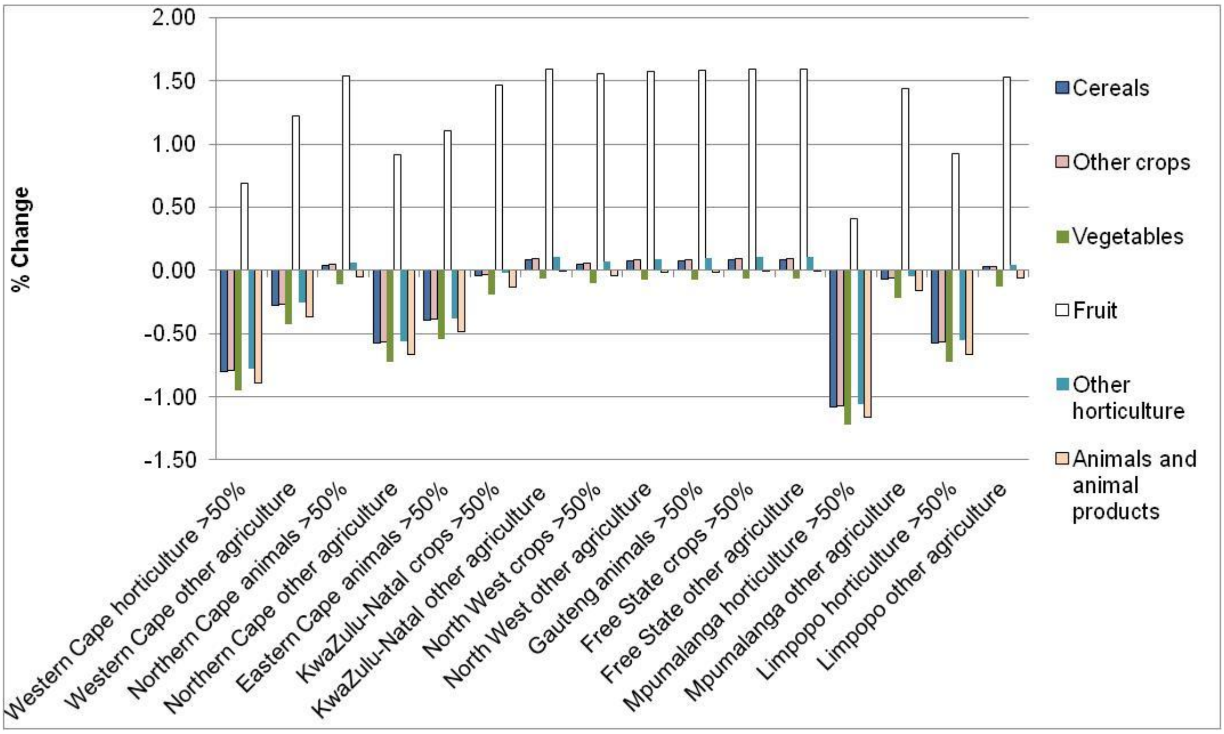


Figure 18 shows the changes in product prices for each agricultural industry (*PXAC*) for the global positioning scenario. The prices of fruit increase between 1.03% for Mpumalanga (horticulture >50%) and 1.84% for Northern Cape (animals >50%). In the main fruit producing areas the price increases for fruit are relatively smaller than in other areas and the price decreases of other products are greater in magnitude than in other areas, e.g. the price for animals and animal products in Mpumalanga (horticulture >50%) decreases by 0.73%, whereas it increases by 0.06% in Northern Cape (animals > 50%). The quantity of fruit produced in the base case in these areas is relatively larger and changes in quantities are therefore relative greater, hence the relatively smaller changes in prices.

Figure 18: Global positioning: change in industry product prices (*PXAC*)

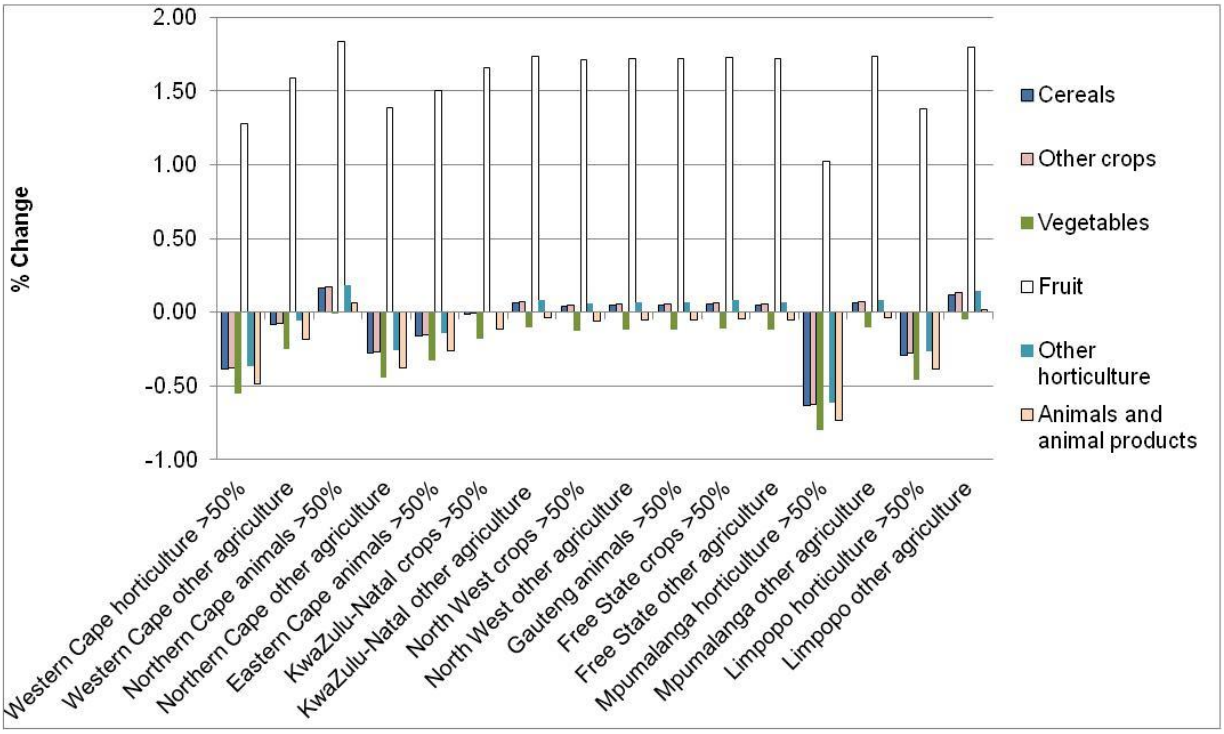


Figure 19 shows the producer prices (*PXC*) which are the weighted averages of the industry product prices (*PXAC*) which were shown in the previous figure. The quantities of products from each industry serve as weights. The greatest price increase is for fruit in the global positioning scenario (1.34%) because the price change is a direct result of the way the scenario was implemented. Results for the manufacturing products omitted from the figure look similar to that for 'textile and clothing products'.

All agricultural and food prices decrease for the technical efficiency scenario (up to -1.37%) because of increased competitiveness of agricultural industries. The food sectors benefit because they use the cheaper agricultural products as intermediate inputs.

All agricultural and food prices increase for the productivity scenario (up to 0.58%). The results for 'other services' are also included in the graph because this sector is directly affected by the productivity scenario. The producer price for 'other services' decreases whereas the producer prices for all other products increase. This is because the price of value added of 'other services' decreases as a direct impact of the productivity increase in the use of factors for 'other services' because 'other services' increases its use of unskilled labour and capital significantly, but decreases its use of skilled labour which is assumed to be fully employed. The net impact is that other industries have access to more skilled labour relative to unskilled labour and hence the increase in the price of value added of the other industries and this transmits as increases in industry and product prices.

Results are mixed and small for the education scenario (between -0.19% and 0.24%) because the scenario was implemented for all skilled labour the impact on *relative* product prices are limited.

Figure 19: Change in producer price of composite output (*PXC*)

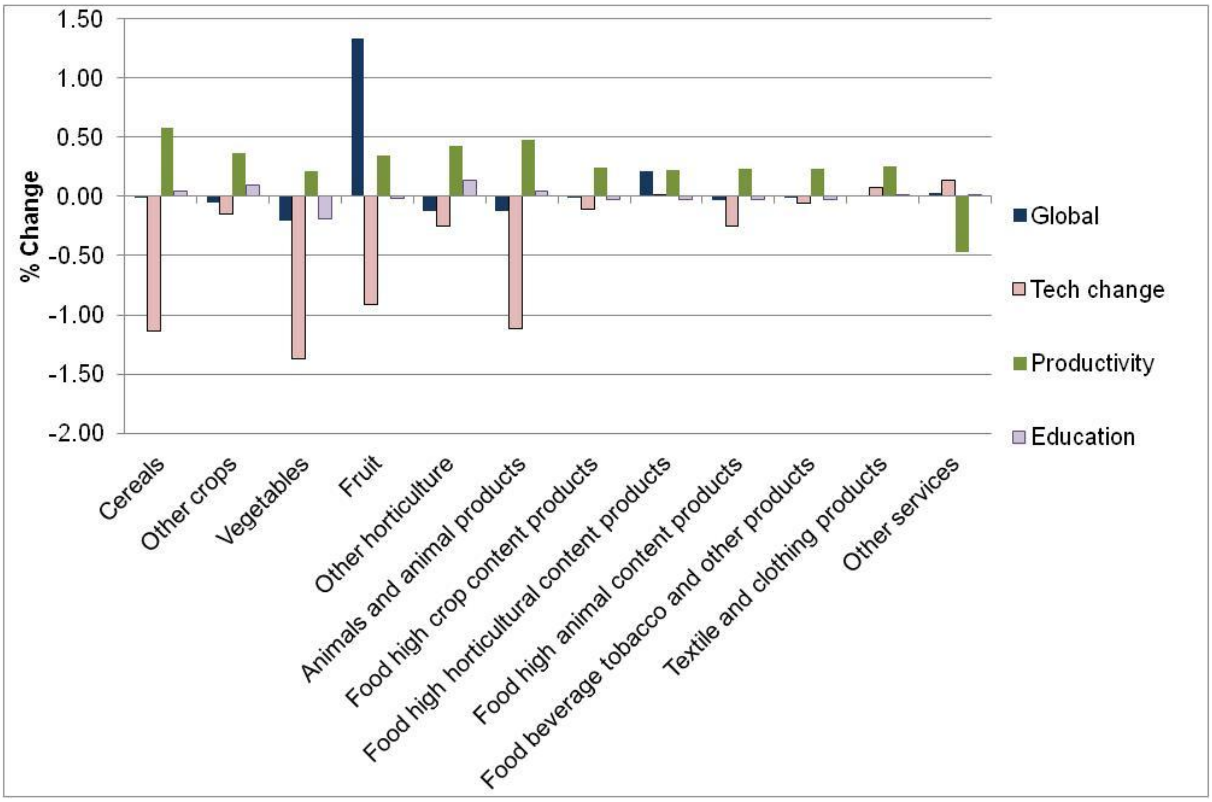


Figure 20 shows purchasers' prices (*PQD*) (i.e. after exports, imports and taxes are taken into consideration) for agricultural and food prices. Results for other services are also included in the graph because it is directly affected by the productivity scenario and the results for this product look different to those for other non-agriculture and non-food products. The results for non-agriculture and non-food products follow similar patterns and magnitude as those for 'textile and clothing products' shows in the figure. For the global positioning scenario agricultural and food products all show a decrease in purchasers' prices whereas non-agricultural products show positive but negligibly small increases (smaller in magnitude to that for 'other services' shown in the figure). The appreciation of the Rand causes the import prices of all products to decrease and this has the downward effect on purchaser prices of agricultural and food products.

The technical efficiency scenario causes purchasers' prices of food and agricultural products to decrease (-1.4% and -0.01%) because of increased competitiveness of agricultural industries. The food products show price decreases because food industries use agricultural products as intermediate inputs. Productivity growth in the services sector causes these prices to rise (0.2% to 0.5%) as explained for the previous figure. The education scenario gives mixed results and relatively small changes (-0.2% and 0.1%), which are the net effects of expansion in the economy.

Figure 20: Change in purchasers' prices for agricultural and food products (*PQD*)

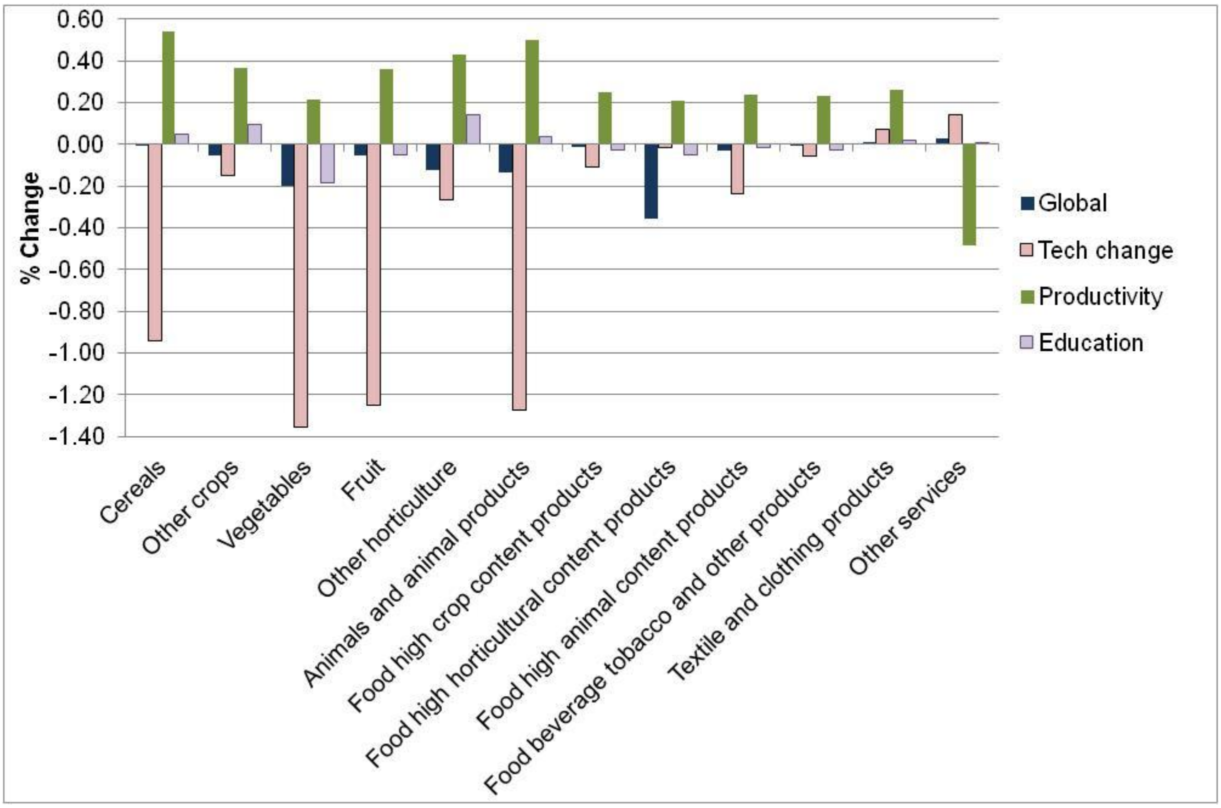


Figure 21 shows the weighted average price of intermediate inputs (*PINT*) paid by agricultural industries, where the weights are the volume shares of individual products in the aggregate intermediate product. The price changes for the global positioning and education scenarios are negligible (between -0.02% and 0.04%). The expectation was that the price would change most for the technical change scenario because the agricultural industries are directly affected by it as a result of the way the scenario is implemented. However, it is the productivity growth scenario that impacts most on the price of intermediate inputs of agricultural industries because of the biased productivity change implemented in this scenario which causes relative factor related prices to change more. The ‘other services’ is a fairly large industry and around 10% of the agricultural industries’ intermediate inputs come from this industry. The productivity increase in ‘other services’ puts upward pressure on the price of intermediate inputs of all agricultural industries (between 0.22% and 0.25%). The increase in intermediate input prices is the result of increases in purchasers’ prices shown in the previous figure.

For the technical efficiency scenario the impact on the price of intermediate inputs of the different agricultural industries are small and mixed (between -0.06% and 0.05%). The largest decreases are however for the predominant crop and animal areas producing areas. The price of intermediate inputs for each industry depends on the production structure of each industry (which serves as weights) and the prices of the intermediate products, which are all differently influenced by the technical efficiency scenario (see Figure 20), hence the mixed results for price of intermediate inputs.

Figure 21: Change in price of intermediate inputs for agricultural industries (*PINT*)

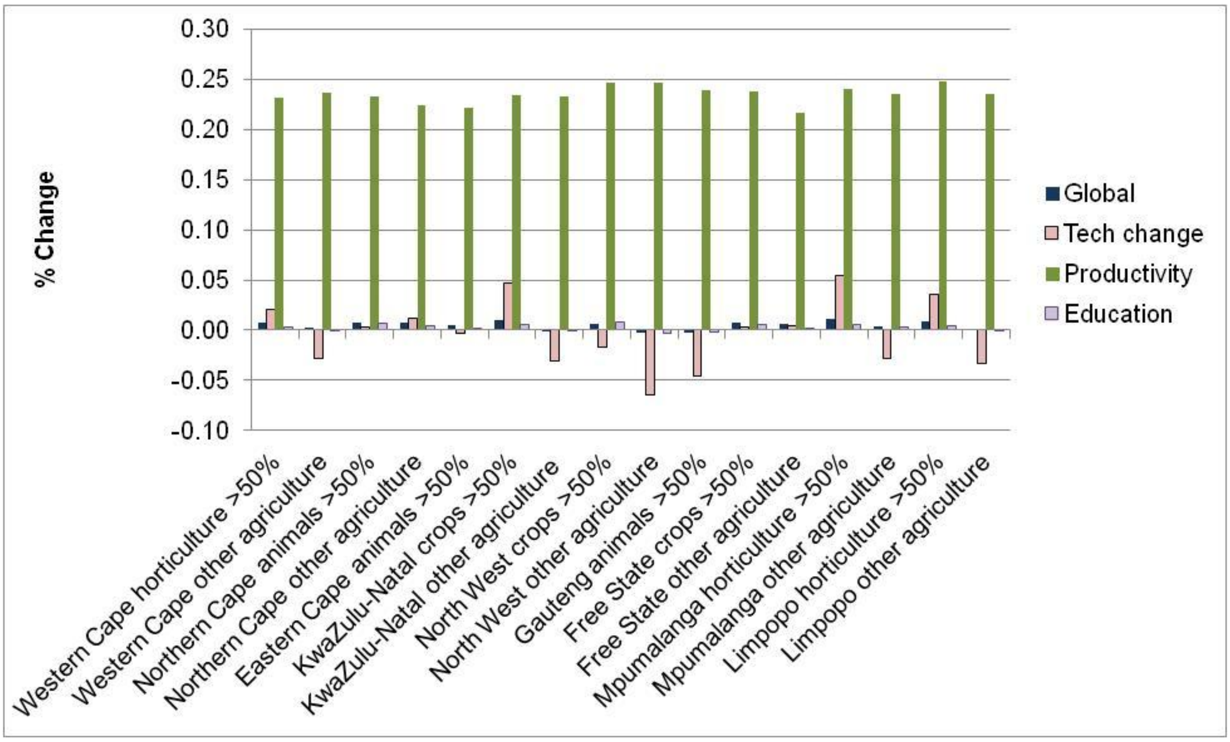


Figure 22 shows the price of intermediate inputs (*PINT*) for non-agricultural industries. The global positioning and education scenarios have a negligible impact. The technical efficiency gains in the agricultural sector causes the purchasers' prices of these products to decrease as was shown in Figure 20 and since these products are a main input into the production of food products, it leads to a decrease in the aggregate intermediate input prices of the food producing industries (between -0.02% and -0.26%). The food industries that produce food with high animal and crop content benefit most from lower intermediate input prices because of the lower intermediate input prices of the animal and crop producing regions as shown in the previous graph. The intermediate input prices of the non-food industries increase because these industries are uncompetitive relative to the agricultural industries and do not benefit directly from lower priced inputs produced by the agricultural industries, as do food industries.

The productivity increase in 'other services' leads to intermediate input price increases in all non-agricultural industries (between 0.13% and 0.28%), except for other services (-0.15%) and trade and transport services (-0.11%). For the productivity growth scenario the price of intermediate inputs for trade and transport services shows a decrease because of the decrease in the purchasers' price (*PQD*) of 'other services' (see Figure 20), which comprise more than 50% of intermediate inputs of trade and transport services.

Figure 22: Change in price of intermediate inputs for non-agricultural industries (*PINT*)

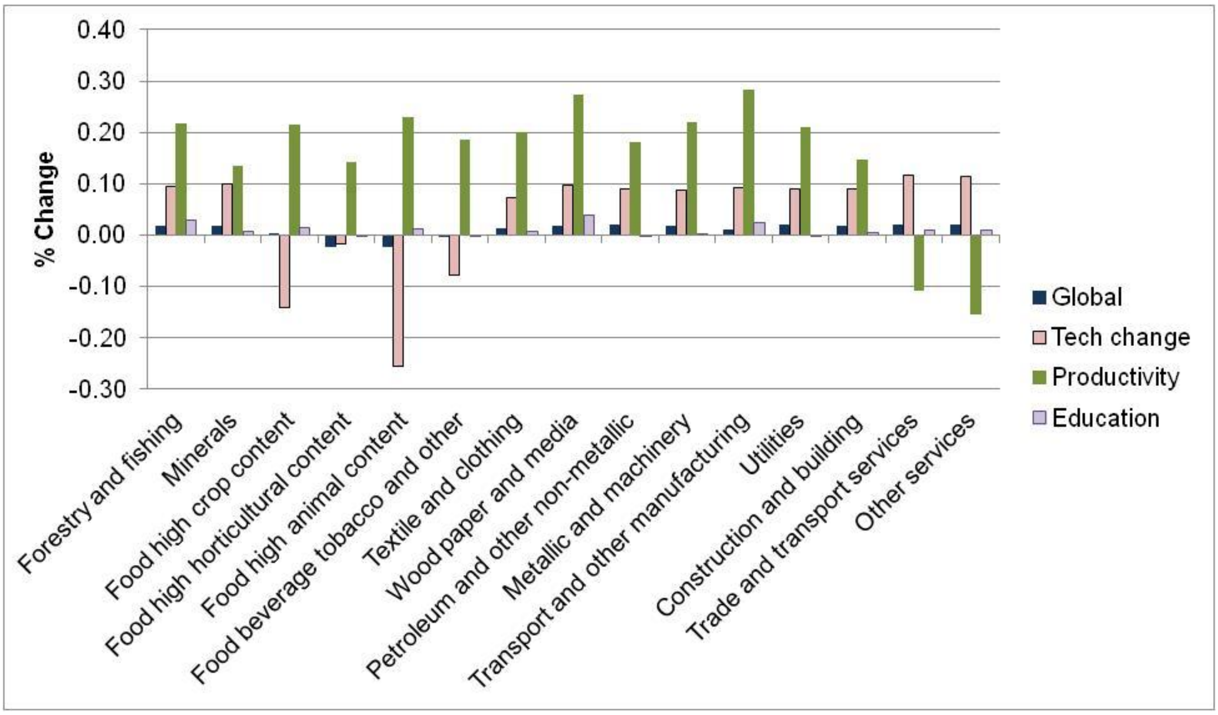


Figure 23 shows the industry output price (*PX*) for agricultural industries. The industry output price is the weighted average of the price of intermediate inputs (*PINT*) and the price of value added (*PVA*) that are discussed in the previous figure and the next figure, respectively. Results for the global positioning scenario indicate that when export prices of fruit and food products with high horticultural content increase the industry output price of the main fruit producing agricultural industries increase (up to 0.57%). There is a slight decrease for industries producing very little fruit (up to 0.05%).

Although not shown here, the results for the technical change scenario show that in quantity terms the industry outputs for all agricultural industries increase relatively to the use of intermediate inputs and value added, according to theory. The changes in the price of value added (*PVA*) and the price of intermediate inputs (*PINT*) are very similar to the changes in the quantity of value added (*QVA*) and quantity of intermediate inputs (*QINT*) for the technical change scenario. In contrast, the industry output price (*PX*) decreases for all the agricultural industries (see below) and the quantity of output (*QX*) increases for the technical efficiency scenario (Figure 11). The decrease in the industry output price for agricultural industries is the result of becoming more competitive relative to other industries and this allows the agricultural industries to expand relative to other industries.

Industry output prices (*PX*) for agricultural industries increase for the productivity growth scenario by up to 0.5%. The increase in prices is the results of increases in wage rates of skilled labour combined with an expansionary effect on the economy. Results are negligibly small and negative for the education scenario.

Figure 23: Change in output price for agricultural industries (*PX*)

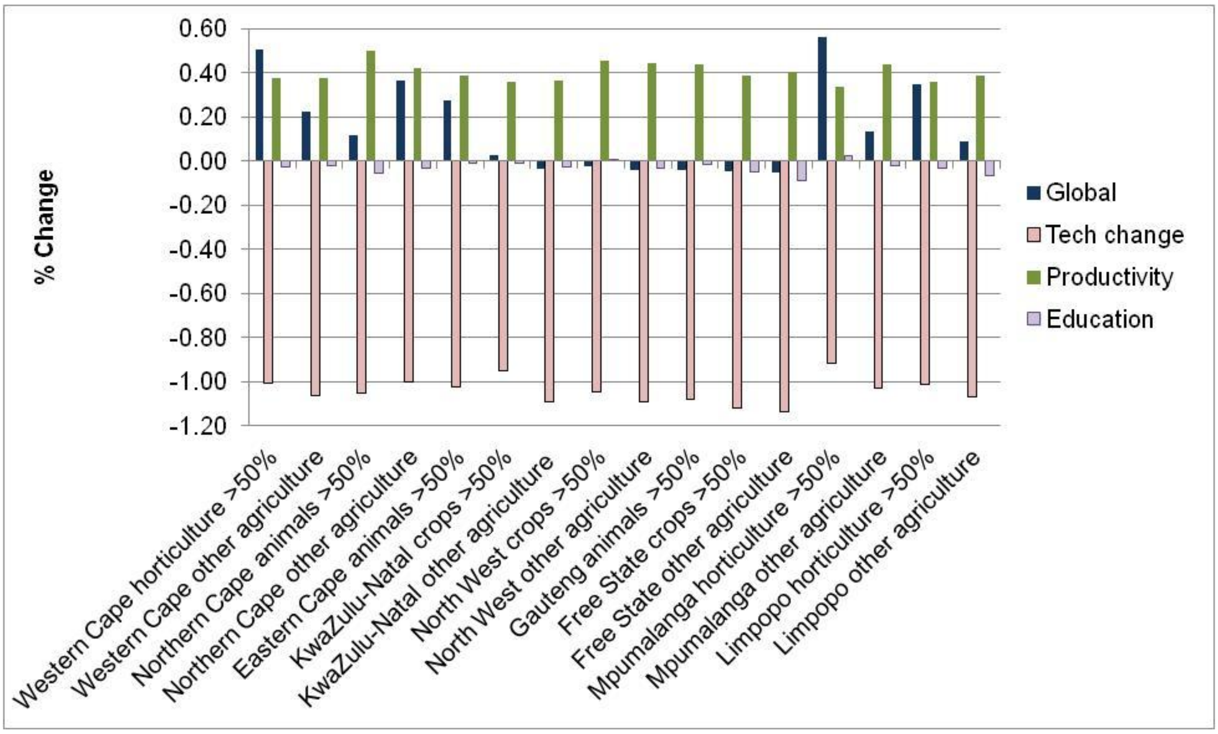
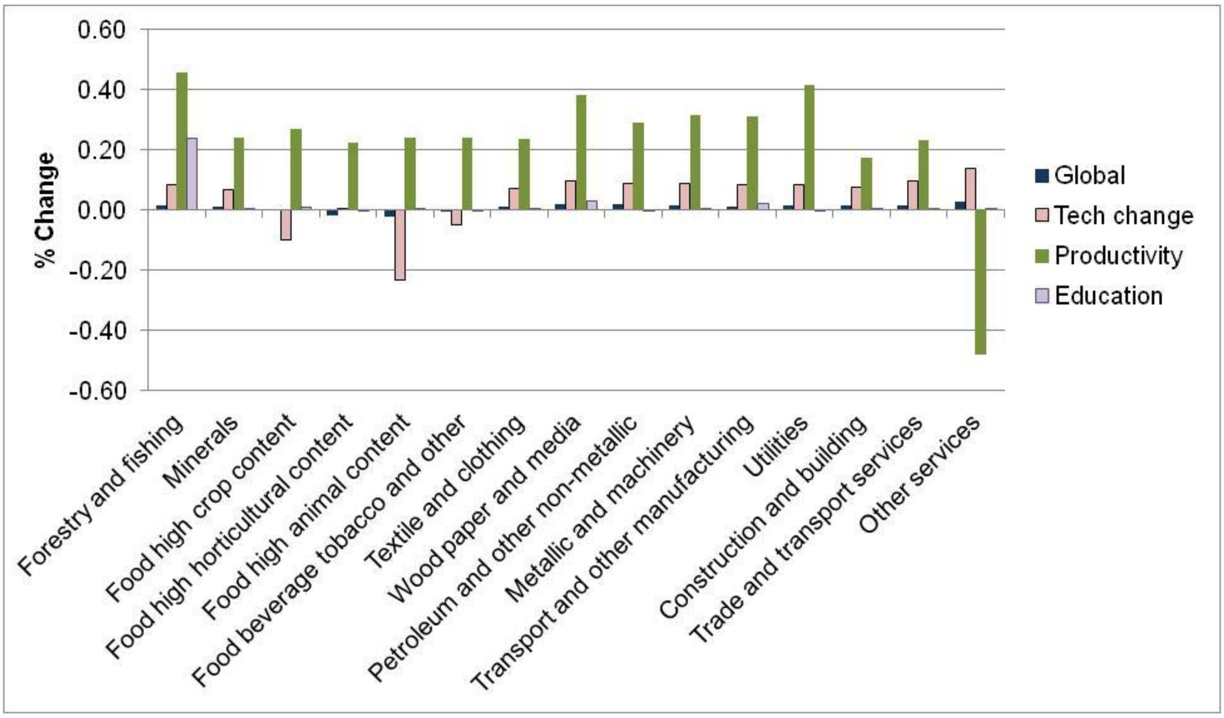


Figure 24 shows the changes in industry output price (*PX*) for non-agricultural industries. The impact of the global positioning scenario is negligible. For the technical efficiency scenario the food industries (especially those producing food with high crop and animal content) show decreases in output prices because of their use of agricultural products as intermediate inputs. Non-food industries show increases in their output prices of up to 0.14% because they have become relatively uncompetitive compared to the agricultural industries.

As with most of the other price results for the productivity scenario, the output price of 'other services' decreases (-0.48%) and the output prices for other non-agricultural industries increase (up to 0.46%). As mentioned this is because the 'other service' industry sheds skilled labour, which is absorbed by other industries with the effect of increasing output prices.

The results for the education scenario are negligibly small except for the output price of forestry products which is driven up by the increase in the rate of returns to land (*WF*). The output prices of the agricultural industries in the previous graph decrease slightly. This is in spite of the increase in the rate of returns to land because there are proportionately larger decreases in the wage rates of skilled agricultural labour which offsets the increase in the rate of returns to land. The forestry (and fishing) sector is the only non-agricultural sector that is influenced by the increase in the rate of returns to land and this is in combination with increases in wage rates for skilled non-agricultural labour; hence the relatively large increase in the price of output for this industry.

Figure 24: Change in output price for non-agricultural industries (*PX*)



The price of value added (*PVA*) in Figure 25 is the weighted average price that every industry pays for all the factors of production that it uses. It is therefore influenced by the wage rates of labour and the rate of returns to capital and land (*WF*). The scenarios for global positioning for fruit and selected food exports and the productivity increases in the services sectors exert the greatest upward pressure on the price of value added. As expected, the increase in the price of value added is greatest for the main fruit producing industries (up to 1.11%) that expand relatively more than industries with limited fruit production, whose price of value added decreases (up to -0.13%).

The technical efficiency scenario has a relatively small and generally negative impact on the price of value added of agricultural industries (up to -0.36%). This is driven by the decreases in wage rates of skilled agricultural labour (up to -0.91% - see Figure 27) because of the assumption of full employment of skilled agricultural labour. The rate of returns to land is almost unaffected and therefore does not greatly influence the price of value added for the agricultural industries.

For the productivity scenario there is an increase in the price of value added for all agricultural industries (between 0.43% and 0.87%). As explained, this is because the price of value added (*PVA*) of 'other services' decreases as a direct impact of the productivity increase in the use of factors for 'other services' because 'other services' increases its use of unskilled labour and capital significantly, but decreases its use of skilled labour which is assumed to be fully employed. The net impact is that other industries have access to more skilled labour relative to unskilled labour and hence the increase in the price of value added of the other industries.

The education scenario has a small negative impact on the price of value added of agricultural industries because the net effect of the decrease in wage rates (*WF*) of skilled agricultural worker outweighs the increase in the returns to land (see Figure 27).

Figure 25: Change in price of value added for agricultural industries (*PVA*)

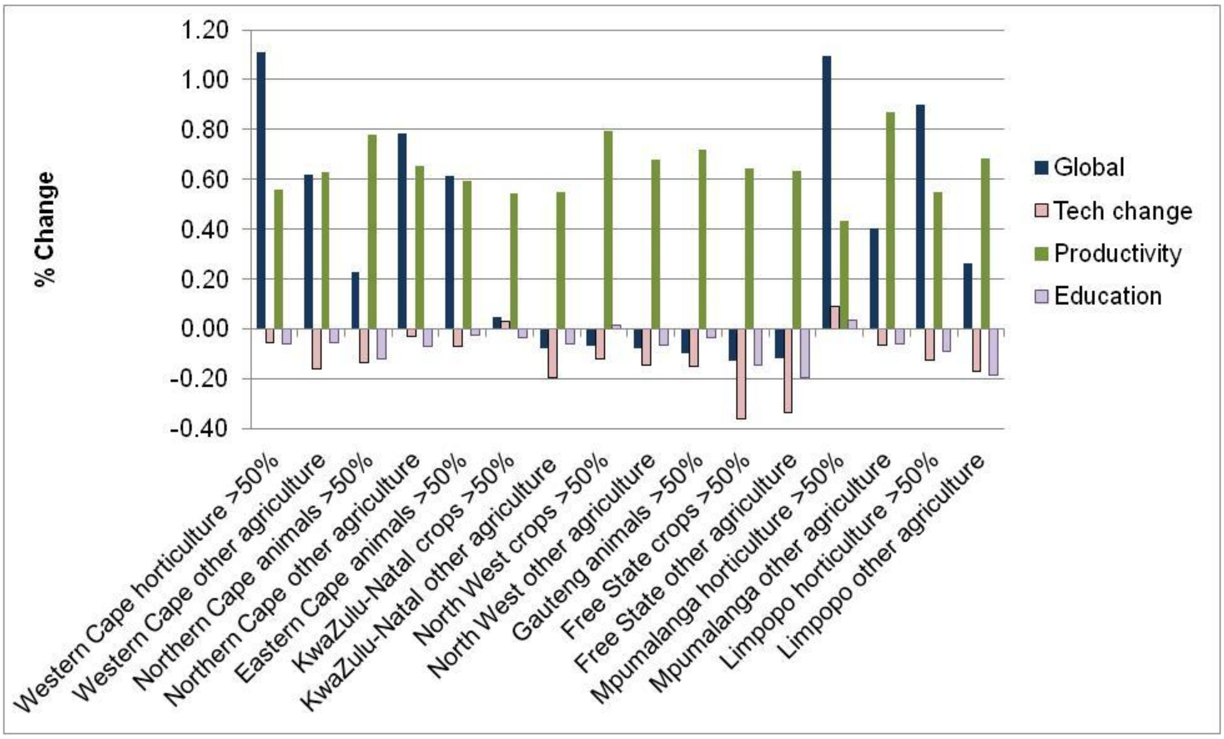


Figure 26 shows that the results for price of value added (*PVA*) are quite different for non-agricultural industries compared to agricultural industries. For these industries the productivity scenario is the only one that has a noteworthy impact on this variable. The increase in the productivity in the 'other services' industry exerts downward pressure on the price of value added of the industries directly affected as part of the simulation (-0.75%), but exerts upward pressure on the price of value added of the other non-agricultural industries that are indirectly affected (0.3% to 0.8%). The fact that the largest change in price of value added is achieved with the productivity scenario is not surprising because the scenario is implemented by means of the shift parameter of the CES function that aggregates the factors of production.

The technical efficiency scenario has a small positive impact on non-agricultural industries (between 0.04% and 0.16%). This reflects the fact that the non-agricultural industries have become relatively uncompetitive compared to their agricultural counterparts.

For the education scenario the relatively greater impact on the agricultural industries (see previous graph) compared to the non-agricultural industries is the result of the selected land model closure. If land is assumed to be fully employed and mobile then the results for the price of value added for the agricultural and non-agricultural industries are similar in magnitude because there are no increases in the rate of returns to land which puts upward pressure on the price of value added for the agricultural industries compared to that of non-agricultural industries. As mentioned, the forestry and fishing industry experiences an increase in the price of value added because of increases in both the rate of returns to land and the wage rates of skilled non-agricultural labour.

Figure 26: Change in price of value added for non-agricultural industries (*PVA*)

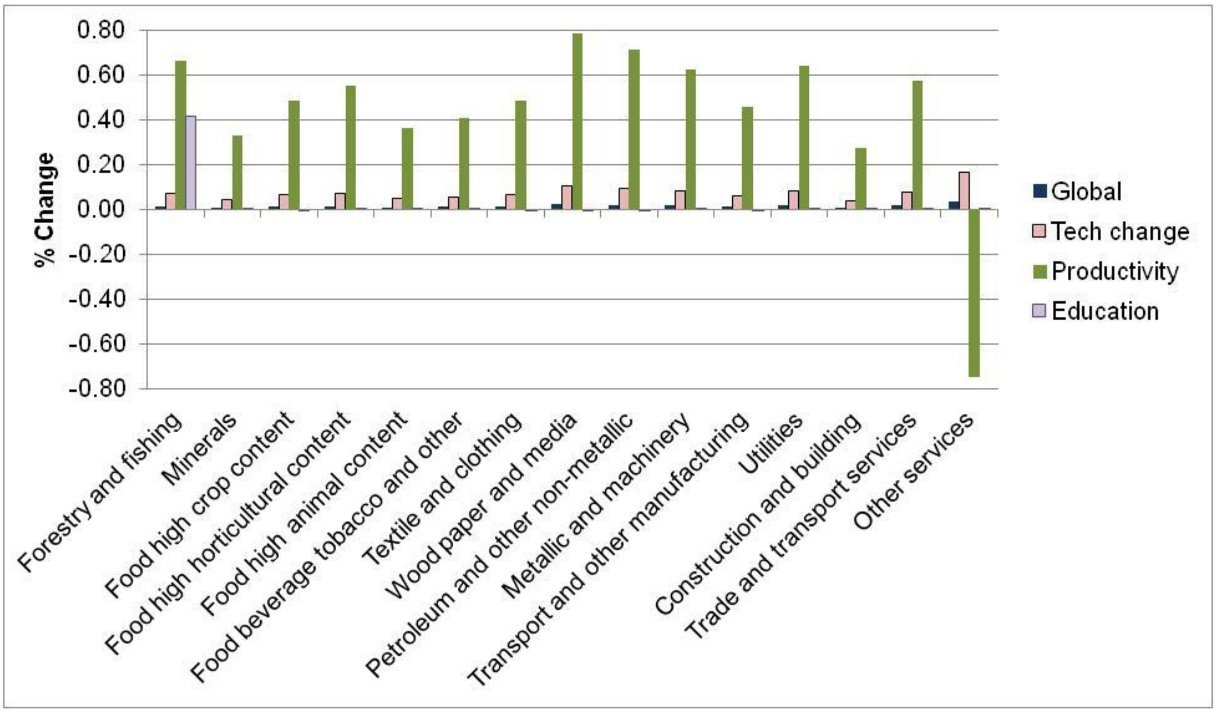


Figure 27 shows the changes in wage rates (*WF*) for land and skilled labour, which determines the price of value added. Capital and unskilled labour are assumed to be not fully employed; hence the rate of returns to capital and the wage rate of unskilled labour remain unchanged for all the scenarios. For the global positioning scenario the wage rates of skilled labour increases most (up to 2.74%) for labour employed in the main fruit producing industries. This is because of the expansion of output in these industries (see Figure 11). The general expansion of these industries also put upward pressure on the rate of returns to land (4.67%).

For the technical change scenario the wage rates of skilled labour employed in the agricultural sector declines (-0.003% to -0.91%), but it increases in the non-agricultural sector (0.47% to 0.48%). This is because of a ‘decrease in demand’ for skilled labour in the agricultural sector as a result of improving efficiency, i.e. fewer resources needed to produce the same output. Under the assumption of full employment a decrease in demand translates into lower wage rates. For unskilled labour their wage rates stay the same, but actual demand decreases on average by -0.14% (not shown here).

For the productivity growth scenario the wage rates for skilled agricultural labour increase (between 1.59% and 2.10%) and the increases are even greater for skilled non-agricultural labour (between 3.51% and 3.86%). This is because the expansion of agricultural production (see Figure 11) is smaller than that of non-agricultural production (see Figure 12) for the productivity growth scenario because ‘other services’ comprises a greater proportion of intermediate inputs for non-agricultural industries than for agricultural industries (28.1% and 7.5% respectively) and thus the non-agricultural industries benefit more from the lower purchasers’ price (*PQD*) of ‘other services’ and absorb more of the skilled labour shed by the ‘other services’ industry.

For the education scenario the wage rates for all skilled labour in agriculture decreases (between -0.2 and -2.16%), whereas results for non-agriculture are generally positive but small (between -0.03 and 0.2). The rate of returns to land increases because of the increase in agricultural production (between 0.74% and 1.30% - see Figure 11). The wage rates of skilled labour in agriculture decrease because the increase in production is relatively smaller than the increase in skilled labour (2%), and since skilled labour is assumed to be fully employed, there is a depression of wage rates paid by the agricultural sector. Production by non-agricultural industries increases on average by 1.74% (see Figure 12) for the education scenario, which is sufficient to absorb the additional skilled labour without significant changes in wage rates.

Figure 27: Change in wage rates (*WF*)

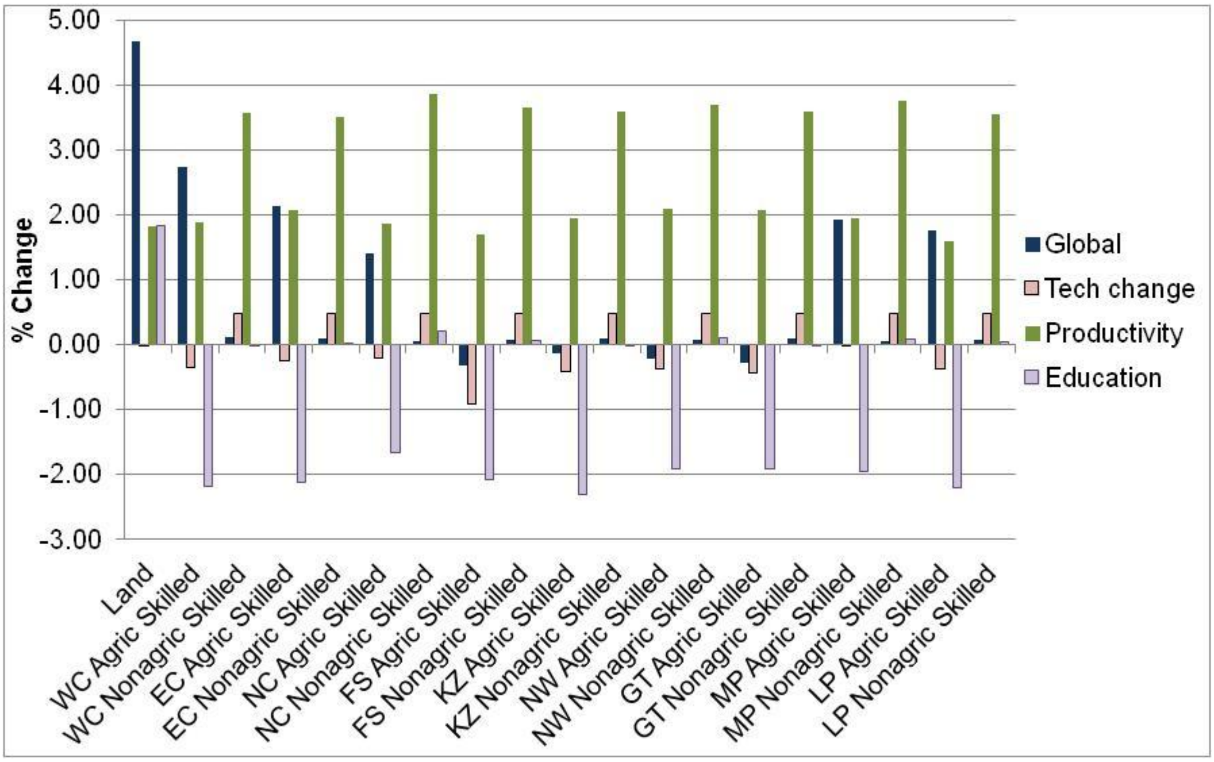


Figure 28 shows the changes in factor demands (*FDF*) for unskilled labour. The assumption of full employment for skilled labour implies that an increase in demand for labour will result in an increase in the wage rate of skilled labour because no increase in the number of employees is possible. Only for the education scenario does factor demand for skilled labour increase because of the assumption that the number of skilled labour increases and that they will all find employment. In the case of unskilled labour the number of employees changes because of the unemployment assumption, but the wage rate will remain unchanged. Figure 28 shows the changes in demand for unskilled factors. Agricultural sector workers benefit from the world exports price changes in the global positioning scenario (average 0.7%), compared to non-agricultural workers (average 0.02%).

The technical change scenario generally has a small negative impact on unskilled agricultural labour (average -0.14%) and a small positive impact on unskilled non-agricultural labour (0.23%). This is because of the fact that the shock was introduced on the agricultural industries only; hence the agricultural industries can use less unskilled labour and still increase output (see Figure 11). The demand for skilled labour and capital are fixed because of the closure assumptions.

When there are productivity increases in the services industry the demand for non-agricultural labour increases relatively more (average 2.0%) than the demand for agricultural labour (average 0.95%). In the education scenario the non-agricultural sector labour benefits relatively more than their agricultural sector counterparts (average 2.0% vs. 0.92%). This result is comparable to the productivity scenario. For both scenarios the increase in factor demands for unskilled labour for the agricultural industries (non-agricultural industries) are similar in magnitude to the increase in output (*QX*) of the agricultural industries (non-agricultural industries) – see Figure 11.

Figure 28: Change in factor demand of unskilled labour (*FDF*)

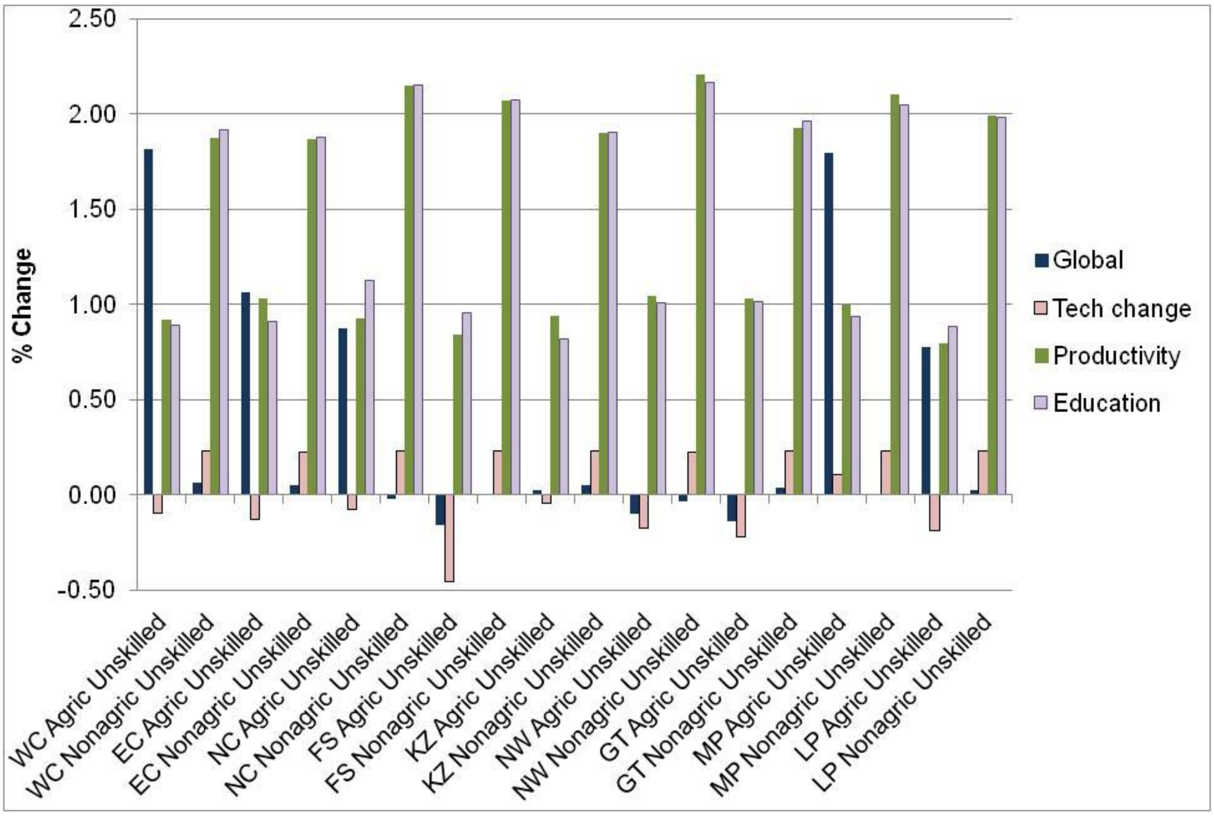


Table 29 shows the number of employment opportunities (*FS*) created in the agricultural and non-agricultural industries of South Africa for the five different scenarios. In the last scenario when all the impacts take place simultaneously, then 457 530 employment opportunities are created, of which 19 529 (4.3%) in the agricultural sector. The scenarios that cause the greatest increase in employment in the agricultural sector are not the scenarios that impact directly on agriculture (global positioning and technical change), but those that impact indirectly (productivity change and education). It should be noted that the 'other services' industry is quite large, hence the relatively large impact on employment. Also, in the education scenario 58 500 employment opportunities were directly imposed because of the way the scenario was set up. The remaining 183 783 employment opportunities were however created through indirect effects in the economy stimulated by the availability of additional skilled labour in the economy. Demand for agricultural and food products are dependent on effective demand by all South African households, thus any impact on the economy that increases household income is likely to have a positive indirect effect on agricultural production and hence employment creation. The increase in the last scenario that combines all the scenarios leads to an increase of 457 530 employment opportunities, which is 4 652 greater than the sum of the increases if the scenarios are run separately. This shows the additional indirect effect when all the scenarios are run simultaneously.

Table 29: Increase in employment (*FS*) (numbers)

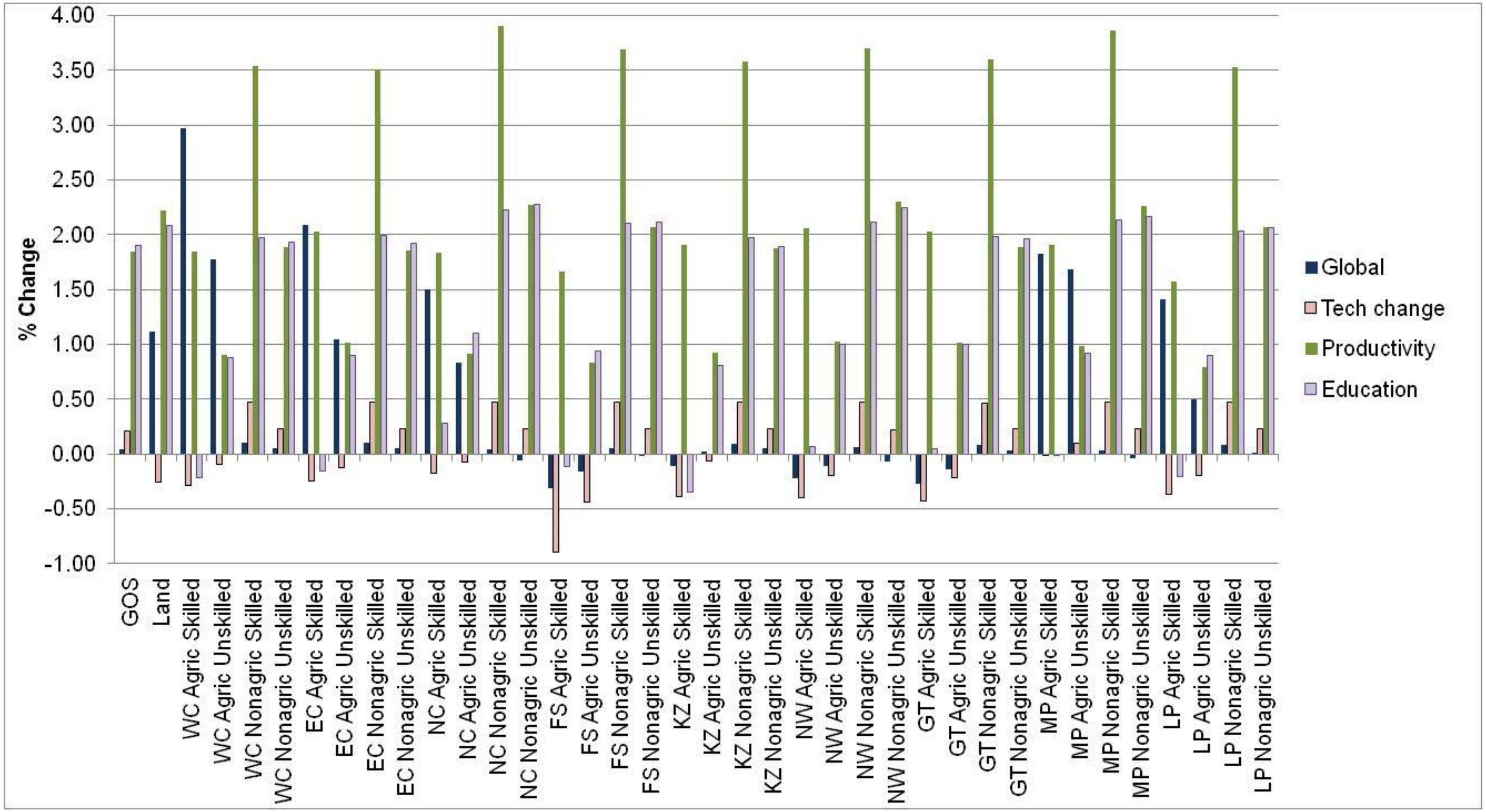
	Global	Tech change	Productivity	Education	All
Agriculture	4 244	-572	5 203	10 303	19 529
Non-agriculture	3 148	20 866	177 706	231 981	438 001
South Africa	7 392	20 293	182 909	242 283	457 530

The changes in factor demands (*FDF*) for unskilled labour and wage rates (*WF*) for skilled labour are reflected as changes in factor incomes (*YF*) in Figure 29. The global positioning scenario favours labour in the agricultural sector, especially in provinces with predominant fruit production such as the Western Cape, Eastern Cape, Northern Cape, Mpumalanga and Limpopo.

The technical efficiency scenario gives the smallest impact on factor incomes (between -0.9% and 0.48%) and the results are quite mixed for the different provinces. But generally the increase in income of skilled non-agricultural labour is the greatest (between 0.47% and 0.48%), followed by unskilled non-agricultural labour (between 0.22% and 0.23%). Generally incomes of agricultural skilled labour show the largest decreases (between -0.89% and -0.01%), followed by incomes of agricultural unskilled labour (between -0.44% and +0.10%). The labour incomes in the agricultural industries decrease because of the increase in technical efficiency which is accompanied by reduced inputs relative to output.

The productivity growth scenario shows the greatest increases in incomes for skilled non-agricultural labour (between 3.51% and 3.9%), followed by unskilled non-agricultural labour (between 1.85% and 2.30%), skilled agricultural labour (between 1.57% and 2.06%) and lastly unskilled agricultural labour (between 0.79% and 1.03%). The relative increases for each of the four mentioned categories reflect the differences in average annual earnings between these four categories as captured in the base data. The scenario therefore does not significantly change the relative earnings potential of the four categories.

Figure 29: **Change in factor incomes (*YF*)**



The main increases in factor incomes for the education scenario are for labour employed in the non-agricultural sectors, with incomes of skilled labour increasing between 1.97% and 2.23% and incomes of unskilled labour increasing between 1.89% and 2.28%. The increases in incomes for skilled and unskilled labour in the non-agricultural sectors are therefore comparable. For the agricultural industries the changes in incomes of skilled agricultural labour are lower (between -0.35% and 0.28%) compared to the increases in incomes of unskilled labour (between 0.80% and 1.11%). This is because of the decreases in the wage rates of skilled labour as a result of the increase in supply of skilled labour in this scenario.

Table 30 shows that household incomes (*YH*) come from different sources and derive mainly from factor incomes (*YF*) because of factor ownership, i.e. wages and salaries and returns to land and capital (between 90.78% and 91.52%, depending on the scenario). Other income sources include transfers from government (6.71% to 6.88%), transfers from other households (around 2.17%), net enterprise income (-0.36% to 0.17%) and remittances from the rest of the world (around 0.01%). For individual households the distribution of additional income is largely dependent on the distribution of factor ownership because incomes to different factors do not necessarily change proportionately for different scenarios.

Table 30: Sources of national household income (shares of total income)

	Base	Global	Tech change	Productivity	Education
Factor payments	90.78%	90.80%	90.85%	91.42%	91.22%
Government transfers	6.88%	6.87%	6.86%	6.76%	6.78%
Inter-household transfers	2.17%	2.17%	2.17%	2.17%	2.17%
Corporations (net)	0.17%	0.15%	0.11%	-0.36%	-0.18%
Rest of the World	0.01%	0.01%	0.01%	0.01%	0.01%
Total	100.0%	100.0%	100.0%	100.0%	100.0%

Figure 30 shows the impacts of the scenarios on household incomes (*YH*). Agricultural households are those that derive the majority of the household income from agriculture. The way the factor accounts are grouped illuminates differences in wage earnings for different categories as seen in the discussion of the previous graph. Each factor is linked to a particular household in the base data and factor incomes are allocated as household incomes to different household categories. The household categories contain a mix of the different factors and hence the results are less distinct per household category. For example for the global positioning scenario the changes in household income for the non-agricultural poor households vary between -0.03% and +0.07%, whereas for the non-agricultural non-poor households it varies between -0.03% and +0.04%. So it is partly the way the household categories were determined and partly the fact that household incomes comprise factor incomes and incomes from various other sources that causes fewer households to experience decreases in household incomes compared to the number of factors that experiences decreases in factor incomes.

In the global positioning scenario the agricultural households in the main horticultural producing areas, i.e. Western Cape, Mpumalanga, Northern Cape and Limpopo, derive the greatest benefit (up to 1.2%), compared to agricultural households from other provinces (up to 0.3%). Changes in household income for the global positioning scenario for all households are between -0.09% and 1.15%.

The technical change scenario changes household incomes of individual household categories between -0.26% and 0.25%, which appears to be a smaller impact compared to the global positioning scenario. The overall increase in *total* household income is however larger for the technical change scenario (0.21%) than for the global positioning scenario (0.04%) (not shown here). This is because the technical change scenario leads to greater increases for the non-agricultural households, which represents a larger share of income than the agricultural households which experience the notable percentage increases as a result of the global positioning scenario.

For the productivity scenario the increases are slightly larger (between 1.12% and 2.17%) indicating the positive effect of the productivity increases for households. For the education scenario, not only does the number of skilled labour increase, but employees also receive relatively greater wages compared to unskilled labour, hence the increases in household incomes of between 1.03% and 1.92%.

Figure 30: **Change in household incomes (*YH*)**

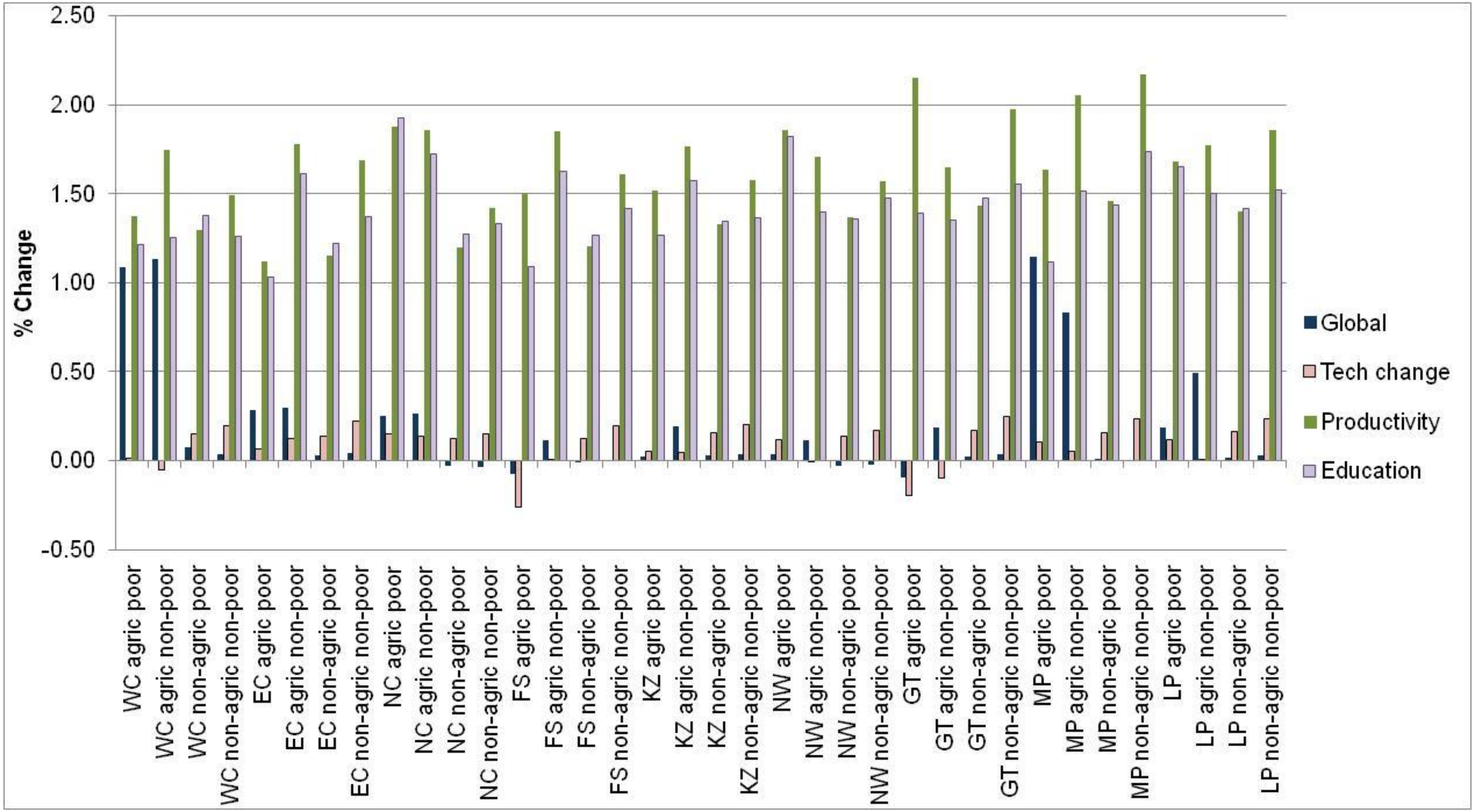


Figure 31 shows the direct impact of the global positioning scenario on the exports (*QE*) of fruit (10.5%) and food products with a high horticultural content (5.1%). The indirect impact on other agricultural products is relatively small and on other non-food products it is almost negligible. The results for the other manufacturing products are not shown here because they are all similar in magnitude to the results for 'textile and clothing products' for all scenarios.

When the agricultural industries become more efficient in the technical change scenario, exports of all agricultural products increase between 2.2% and 3.4% because of an increase in competitiveness. The increase in competitiveness has a small but positive indirect impact on non-agricultural products (increase between 0.04% and 1.0%), especially food products that benefit from cheaper agricultural products as intermediate inputs.

The improvement in factor productivity of the 'other services' industry in the third scenario leads to an increase in 'other services' exports of 4.3% due to increased competitiveness. The export orientated mineral industry also benefits from the depreciation of the Rand as seen through the increase in exports of mineral products of 3.0%. The education scenario has an overall positive impact on exports (>0.9%) due to the economic expansion as a result of the increase in the number of skilled labour. Mineral exports increase by 2.9%.

Figure 31: Change in quantity of exports (*QE*) for selected products

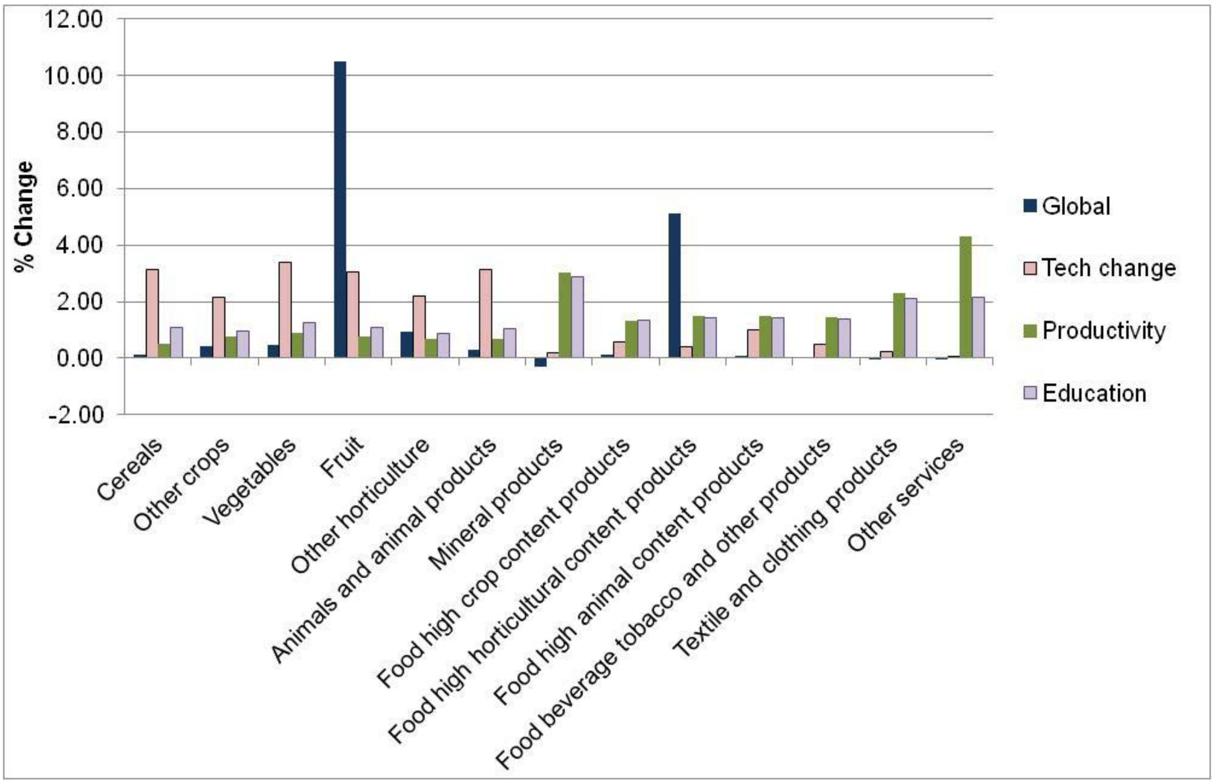


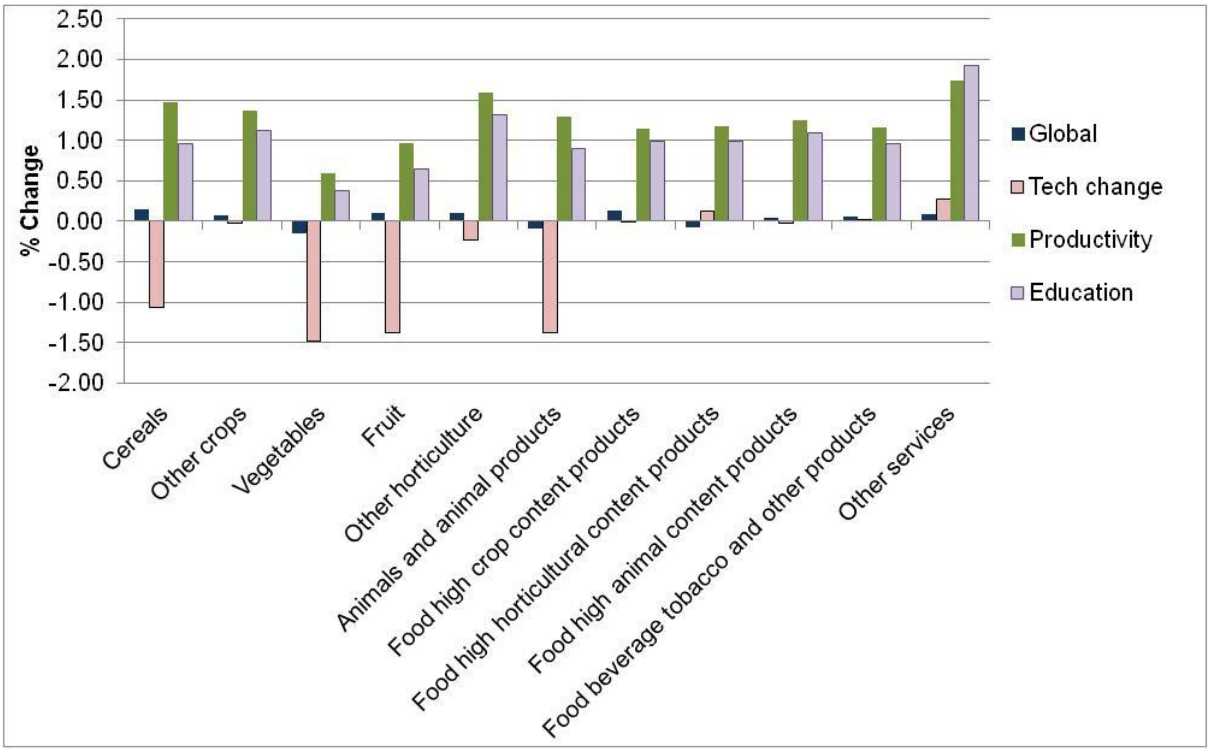
Figure 32 shows that for the global positioning scenario the impact on imports (*QM*) are small but generally positive (up to 0.2% increase in imports) as a result of the 0.06% depreciation of the Rand (see Table 27). Results for other products are not shown here, but are very similar in magnitude to that for 'other services', which has been included in the figure.

For the technical efficiency scenario the volume of imports declines for all agricultural products (between -0.02% and -1.48%). This indicates the increased competitiveness of the domestic agricultural producers relative to the foreign competitors. The impact on imports of non-agricultural products tends to be positive due to the 0.07% depreciation of the Rand.

The productivity scenario causes imports to increase between 0.6% and 2.4% for all products. This is mainly because of the increase in effective demand as a result of increased household incomes (see Figure 30) due to increased factor incomes (see Figure 29). The positive impact on imports is also driven by the 0.31% depreciation of the Rand.

For the education scenario there is an increase in imports of agricultural and food products (0.9% to 2.2%), and even more so for non-agricultural products (between 2.6% and 3.3%) (not shown in the figure). Similar to the productivity scenario, the general increase in imports for the education scenario is the result of a positive income effect.

Figure 32: Change in quantity of imports (*QM*) for agricultural and food products



6.5 Summary and conclusions

In this chapter the adjusted model was applied to analyse the implications of four different scenarios on employment creation in South Africa and its implications for the agricultural sector. The scenarios stem from issues raised in the National Development Plan for South Africa that was released by the National Planning Commission in 2011. The scenarios relate to a) global positioning of the fruit industry in terms of exports, b) technology efficiency improvements in agriculture as a result of research and development, c) productivity growth in the government and services sector as a result of curbing strikes and corruption, and d) addressing skills shortages through improving the quality of education.

In the first scenario on global positioning it is assumed that current markets are maintained and that new markets for fruit and food products with a high horticultural content are opened up. The simulation is implemented by increasing the world export price of fruit by 5% and that of food with high horticultural content with 2%. The direct impact of increased world export price changes for fruit and food products with a high horticultural content is an appreciation of the Rand. The indirect impact is a general dampening effect on exports and imports of all other products. The exports of the directly affected products obviously increase significantly in response to the increase in the world price of those products. Exports of other products respond mainly to the exchange rate changes and changes in exports of other products are much smaller compared to those for fruit. The increase in exports of fruit and food products is brought about by an increase in output by the related industries; hence it is not only a diversion of products from the domestic market to the export market, but an increase in total output. The expansion of the fruit and food industries thus also leads to increased factor demands by these industries, but there is little impact on factor demands by other non-agricultural industries. The purchasers' prices of fruit and food products with high horticultural content decrease, whereas the price of value added for all the agricultural industries increases because of the increase in the rate of returns to land. The agricultural producers are assumed to be price responsive hence the industries will adjust their output composition in response to changes in the product prices faced by each industry. In this scenario the production of fruit increases relative to other agricultural products as expected. This is the outcome of the model adjustment and it can therefore be confirmed that the adjusted model provides more realistic results for the multi-product agricultural industries compared to the base model. Relative price changes were smaller in the other three scenarios, therefore these scenarios had a smaller impact on changes in output composition.

The second scenario assumes that agricultural research and development continue and that this leads to technical efficiency gains of 1% across all the agricultural industries. The results for the technical change in agricultural industries show that the output of agricultural industries increases whereas output prices decrease. There is little impact on output volumes and prices of non-agricultural industries. Agricultural exports increase and imports decrease as a result of increased competitiveness, whereas trade of non-agricultural products are only affected indirectly through the relatively small change in the exchange rate. The results for value added and use of intermediate inputs are mixed for the agricultural industries. The fact that employment and income effects are more pronounced for non-agricultural workers and households confirms the implicit assumption of technology improvements that resource use decreases relative to output, i.e. there is a relative shift in labour away from the agricultural sector towards the non-agricultural sector. Agricultural producers are still assumed to be price responsive but in this scenario the relative changes in product prices faced by each of the agricultural industries are smaller compared to the previous scenario, hence it was found that the changes in the output composition are also less pronounced.

The third scenario assumes a decline in corruption and strikes and is implemented as a 2% increase in factor productivity in the government and services sectors as captured in the SAM account for ‘other services’. The results for the increase in factor productivity of the ‘other services' industry follow a relatively simple pattern. Factor demands of unemployed factors by the ‘other services’ industry increase but a proportion of skilled labour is released to other industries. The net effect is an increase in the quantity of value added for the ‘other services’ industry because of the relatively greater productivity of the factors. Imports of ‘other services’ decrease because of increased competitiveness, but otherwise all other quantity variables in the quantity system increase for all products and industries. The quantity of exports and quantity of value added for ‘other services’ increase, but the magnitude of the increase is substantially larger than for the other products and industries. Turning to the price system, the wage rates of fully employed factors, skilled labour and land increase and industries employ relatively more skilled labour; hence the price of value added increases for all industries with the exception of the ‘other services’ industry. All other prices in the system increase for all products and industries except for ‘other services’. Similar to the technical change scenario, the impact on changes in output composition of the agricultural industries are relatively small.

The final scenario on education assumes a 2% increase in the number of skilled workers in the economy, and that all these workers will find employment. The scenario is set within the context of skills shortages in South Africa. In general, the price effects are almost negligible, except for the wage rate effects. The increase in skilled labour leads to increases in output. For the agricultural industries this puts upward pressure on the rate of returns to land. At the same time the increase in skilled agricultural labour leads to decreases in wage rates of skilled agricultural labour. The changes in the rate of returns to land and wage rates lead to a slightly negative net impact on the price of value added of the agricultural industries. It however puts upward pressure on the price of value added for the forestry industry which now faces the increase in the rate of returns to land, but no substantial decrease in the wage rate of skilled non-agricultural labour. Factor demand in the non-agricultural sector increase more compared to the agricultural sector, because the agricultural sector faces a constraint with regard to land that is fully employed. Capital is not assumed to be fully employed therefore there is no production factor other than skilled labour that puts a similar constraint on the non-agricultural industries. The number of employment opportunities increases by 242 000, of which 183 000 are created through indirect effects in the economy. The increases in factor incomes throughout the economy translate into increases in household incomes and demand for food products. Imports also increase because of the income effect. Similar to the technical change and productivity growth scenarios, the impact on changes in output composition of the agricultural industries are relatively small.

When comparing the four scenarios, the scenarios for productivity growth in services and improving education have the greatest positive impact throughout the economy in terms of GDP and employment respectively. The results for these scenarios indicate that policies that affect marginal changes in the economy without requiring substantial additional budget or investment can have a notable impact. If all four objectives can be achieved simultaneously then the number of employment opportunities increases by 457 000, of which only 19 500 in the agricultural sector. But these changes require more efficient use of resources. If the objective as set out in the NDP is to create five million new job opportunities, of which one million in the agricultural sector, the target will not be reached through marginal changes. Business as usual and ‘normal’ price and demand effects in the economy will not be sufficient to create one million new job opportunities in the agricultural sector.

Sensitivity analyses with regard to the model adjustment, elasticities and model closures for each of the scenarios are presented in the next chapter.

7 Model validation and sensitivity analyses

7.1 Introduction

This chapter reports model results from sensitivity analyses to validate the adjusted model. Sensitivity analyses were conducted with regard to a) choice of the functional form for output transformation, b) the chosen values of all elasticities included in the model, and c) the selected model closures. The same version of the SAM and the same four scenarios were used as in Chapter 6.

In order to validate the adjusted CGE model, i.e. to best highlight the effect of changing the model's production structure, the global positioning scenario that assumes world export price increases of specific products best illustrates the responsiveness of producers to change their product output composition when faced with *relative* changes in product prices. World export price increases are particularly effective in stimulating domestic production and when the price of only one product is increased there are notable changes in relative product prices faced by producers. The implications of the model changes in the context of the other three scenarios were also determined in order to illustrate the difference in the magnitudes of the changes and the nature of the results. All four scenarios were therefore used as part of the sensitivity analyses.

For each of the scenarios a range of changes were also run to determine the range of changes for which the model can solve and to ensure that the model behaves normally over the entire range. Results were then analysed for world export price changes from -20% to +20% for the global positioning scenario. For the other three scenarios the shift parameters, *adx* and *adva,* and the labour supply of skilled labour (*FS*) were each changed from -40% to +40%. It was found that the model behaves normally over the entire range of changes for each of the scenarios. Detailed results are not presented here, only selected results that aid the explanation of some of the results in Chapter 6.

The elasticity values as indicated in Table 25 apply for the results in sections 7.2 and 7.4. In section 7.3 the values of the various elasticities are adjusted to test the sensitivity of model results for changes in the values of elasticities. A set of closure rules that is deemed representative of the conditions in South Africa was selected for the case study in Chapter 6 and presented in section 6.3.3. Similar closures were used in sections 7.2 and 7.3. In section 7.4 results are presented that were obtained using altered model closures in order to illustrate the sensitivity of model results to the different closures.

Section 7.2 presents the comparison of the results from the base model and the adjusted model as well as the impact of different levels of the elasticity of transformation. In sections 7.3 and 7.4 the sensitivity analyses of different values for elasticity parameters and different model closures are presented. Selected results for a range of changes in variables used in each of the scenarios are presented in section 7.5. The chapter closes with a summary and conclusions.

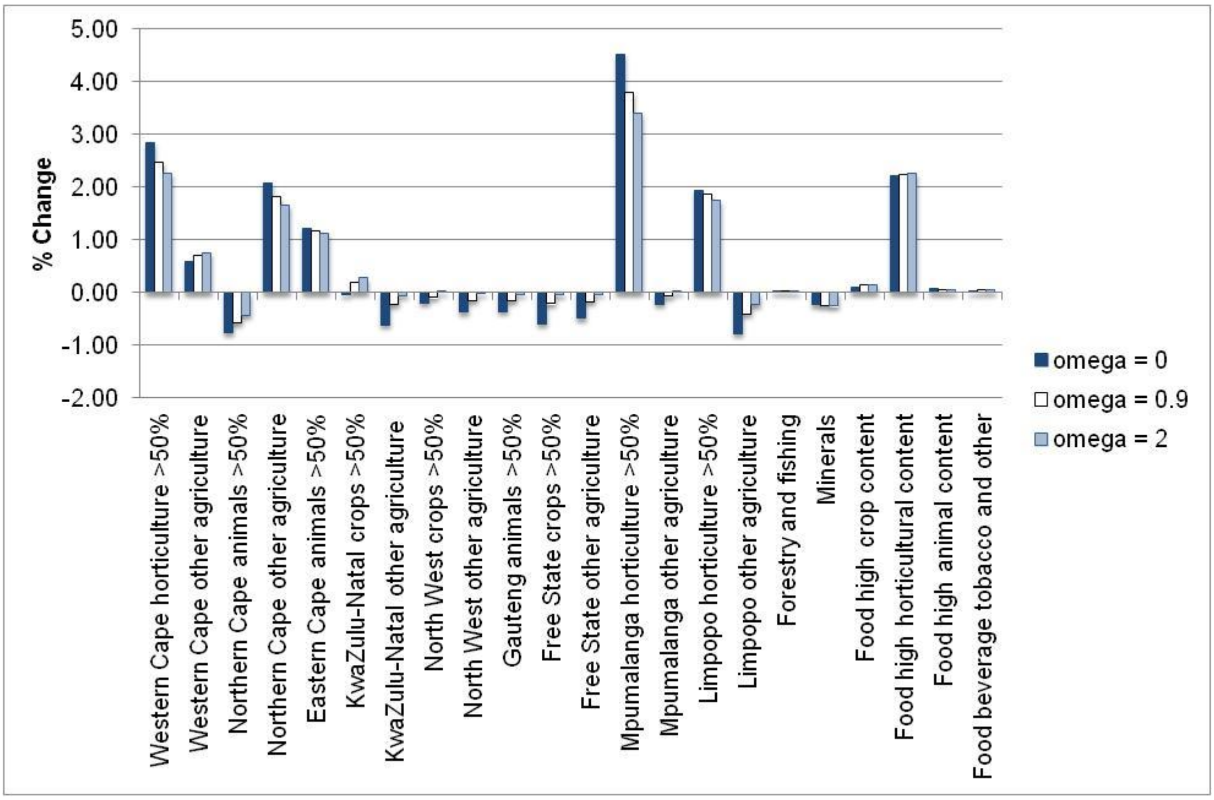
7.2 Results: Functional form sensitivity analyses

This section reports the differences in results obtained with the base model and the adjusted model, which includes a CET function to allow for changes in the output composition of multi-product industries. Results for all four scenarios were analysed. The impact of changes in the level of the elasticity of transformation (*omega*) on results, is also analysed. The adjusted model includes the option of a CET output transformation function as opposed to a Leontief function, and as applied in the previous chapter, the CET output transformation function was used for all the agricultural industries included in the model. There are different ways to activate the output transformation function in the model. First, the industry should be included in the set of multi-product industries that show price responsive behaviour and second, the industry should have been assigned a non-zero elasticity of transformation.

Three values for the elasticity of transformation have been selected and results derived with each are compared. In the first instance the value of the elasticity of transformation is set to zero and the model reverts to the base model where all industries are assumed to retain their original output composition. This first series of results in each figure therefore reports the results for the base model. The second series of results is obtained with an elasticity of transformation set equal to 0.9, which corresponds to the results reported in Chapter 6. For the third series of results the elasticity of transformation is set to 2. The comparison is therefore between results derived with the base model (*omega* equal to 0), the adjusted model used for the case study (*omega* equal to 0.9) and the adjusted model that allows for a relatively greater degree of price responsiveness (*omega* equal to 2).

Figure 33 shows changes in the quantity of output for each industry (*QX*) for the global positioning scenario. The agricultural industries with a share of fruit of less than 30% all show a decline in production in the base model, but when industries have the ability to adjust output composition it enables the industries to maintain or even increase production. In the adjusted model (*omega* > 0) the negative impact on industries with a smaller share of fruit is therefore less severe, but at the same time the positive production impact on industries with a relatively greater share of fruit is also smaller compared to the base model. This is because these industries will increase the production of fruit, but the production of other products will not be increased by the same percentage as fruit, as would be the case in the base model (*omega* equal to 0).

Figure 33: Global positioning: change in industry output (*QX*)



The output of each industry is the aggregate of the production of each product by each industry (*QXAC*). See Figure 16 for the changes in production of each product by each industry (*QXAC*) for the global positioning scenario for *omega* equal to 0.9. From the detailed results for *QXAC* the minimum and maximum changes in production of a product by any of the industries as well as the average change in production of the same product by any of the industries are calculated and presented in the following four figures for the respective scenarios.

Figure 34 shows the changes in production for different products, summarised over industries, for the global positioning scenario. If *omega* is set to 0, then the changes in production for all the products will be the same, because although production between different industries will change by different amounts (between -0.78% and +4.52%, with an average of +0.55%), the change in production of different products by individual industries will be the same. For example the industry showing the greatest increase, Mpumalanga (horticulture >50%), will increase the production of all its products by 4.52%. When *omega* is increased to 0.9, the changes in production of different products is different, with fruit production increasing between 0.96% and 4.23% (average 1.92%), whereas the changes in production of the other products are in the range between a decrease of 0.5% and an increase of 2.6%, with an average increase of about 0.4%. For *omega* of 2, producers are assumed to be even more responsive and fruit production increases between 2.0% and 4.02% (average 2.7%), while the changes in production of the other products are in the range between a decrease of 0.5% and an increase of 1.6%, with an average increase of about 0.2%.

The results for *omega* of 0 highlight two perverse results of the base model in the context of multi-product industries. First, it is unrealistic that a single producer will increase production of a product for which it faces a decreasing price and *vice versa*. Second, it is unrealistic that there will be such a relatively large discrepancy between different industries in the change in output given the initial shock. This is because the change in output of an industry will follow the change in the product price of its main output.

Figures 35 to 37 show the results for changes in production for different products (*QXAC*), summarised over industries, for the technical efficiency, productivity growth and education scenarios. Results show increases in production of all products in all three scenarios.

Figure 34: **Global positioning: change in product output by industry (*QXAC*) – summary by product**

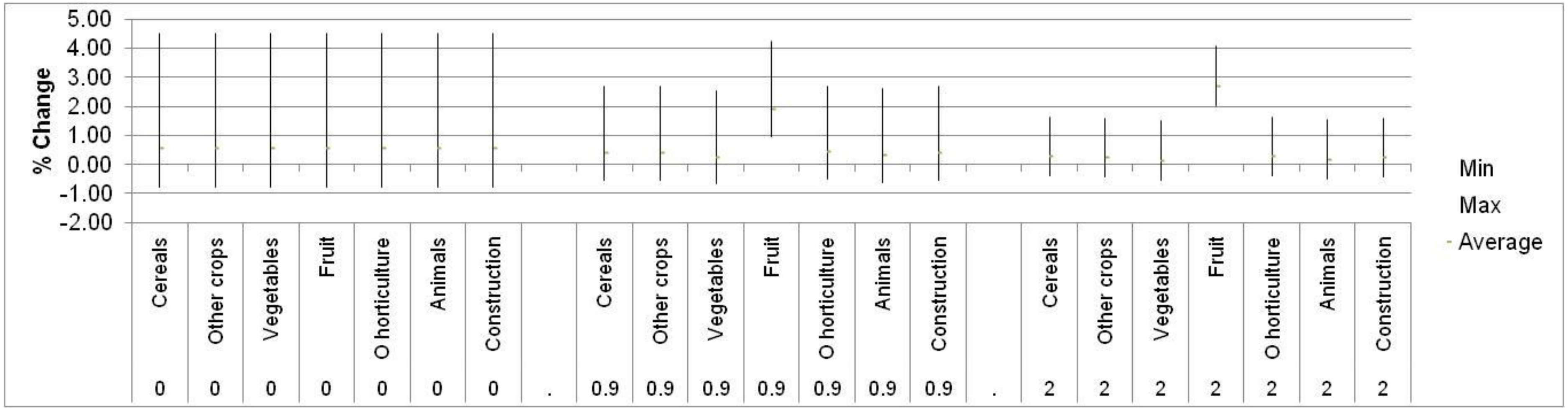


Figure 35: **Technical efficiency: change in product output by industry (*QXAC*) - summary by product**

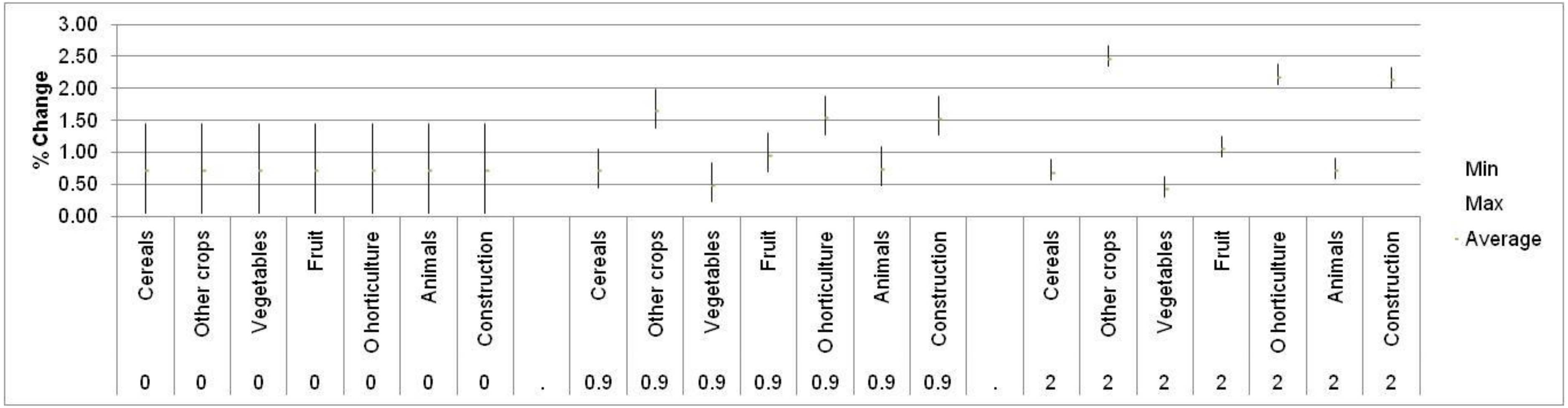


Figure 36: Productivity growth: change in product output by industry (*QXAC*) - summary by product

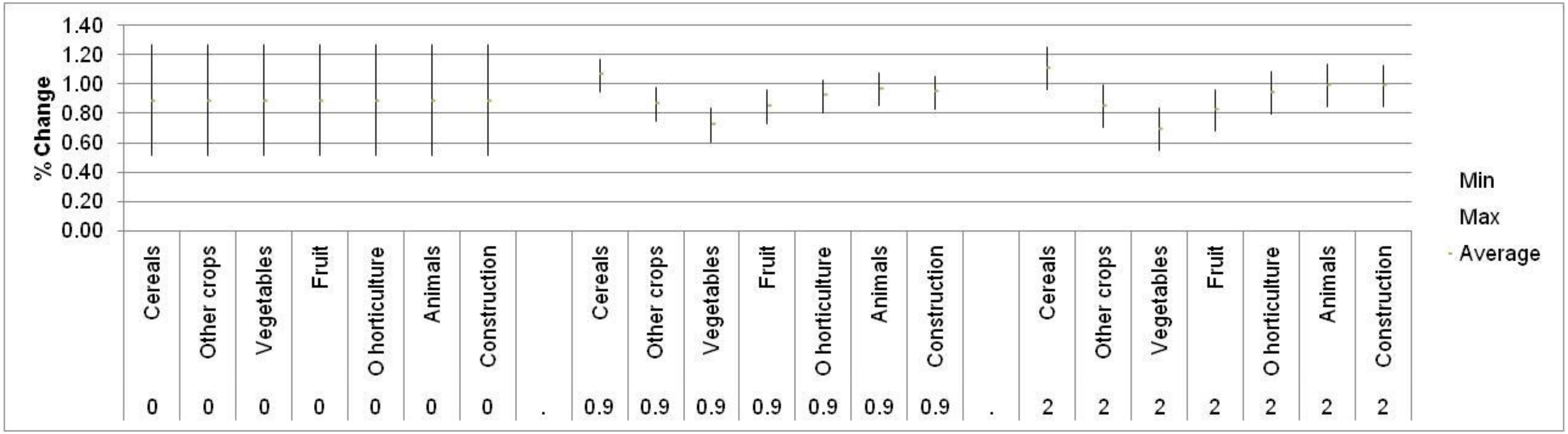
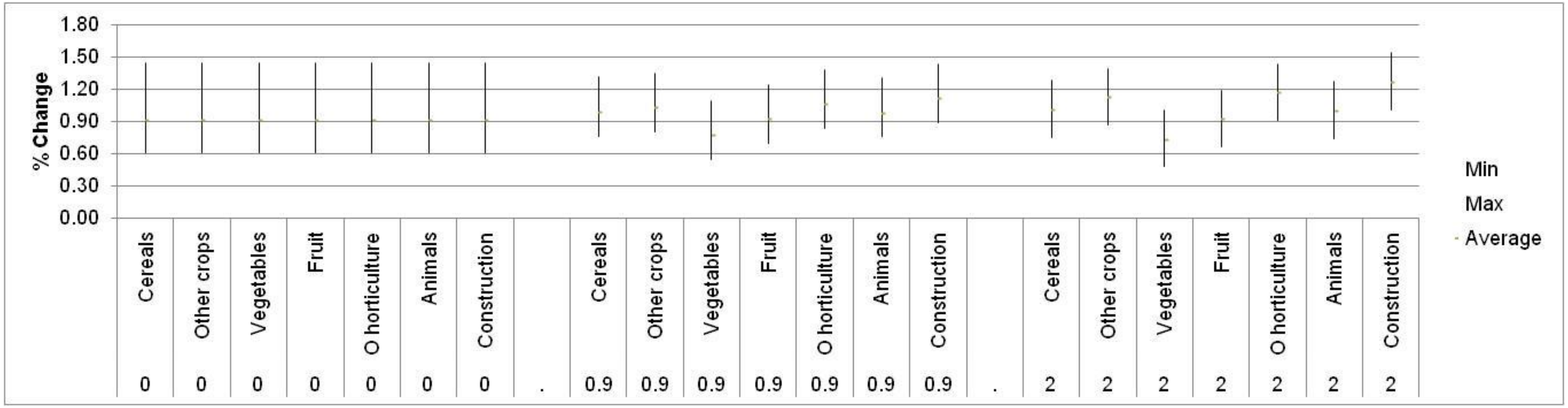


Figure 37: Education: change in product output by industry (*QXAC*) - summary by product



The changes in quantities of products produced are driven by changes in relative product prices faced by each industry (*PXAC*). The following four figures show the minimum, maximum and average changes in the product prices faced by each industry for each of the four scenarios. At higher values of *omega* smaller price changes are necessary to obtain equilibrium because producers are more responsive in adjusting the volumes of production of different products. This is illustrated by the shorter length of the lines for higher values of *omega* in all four scenarios.

The largest changes in the industry product prices, which are obtained for the global positioning scenario, are shown in Figure 38. For the base model the industry prices of fruit increase between 1.36% and 2.69%, with an average increase of 2.35%. Industry prices of the other products decrease on average between 0.04% and 0.70%. The changes are smaller when producers are price responsive (*omega* equal to 2), with the industry prices of fruit increasing between 0.84% and 1.35%, and an average increase of only 1.18%. Industry prices of the other products decrease on average between 0.02% and 0.09%.

Results for the other three scenarios (Figures 39 to 41) reflect the trends in industry prices (*PX*) previously discussed, i.e. a general decrease in industry product prices (*PXAC*) for the technical efficiency scenario, but general increases in industry product prices for the productivity growth and education scenarios. The net impacts on relative prices for different products are different for each of the scenarios.

Figure 38: Global positioning: change in industry product prices (*PXAC*) – summary by product

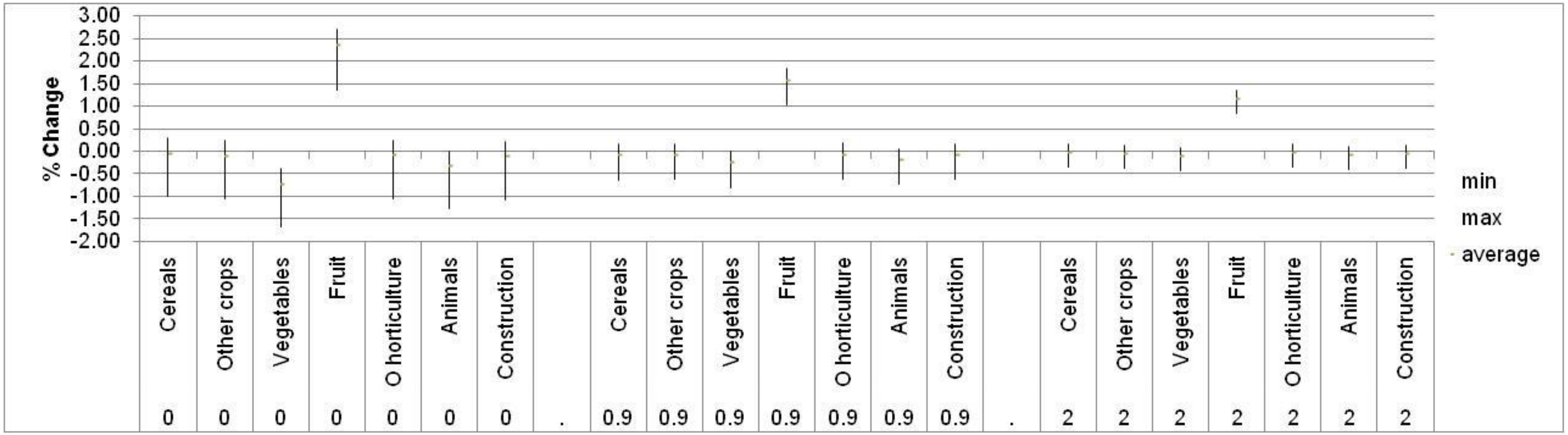


Figure 39: Technical efficiency: change in industry product prices (*PXAC*) – summary by product

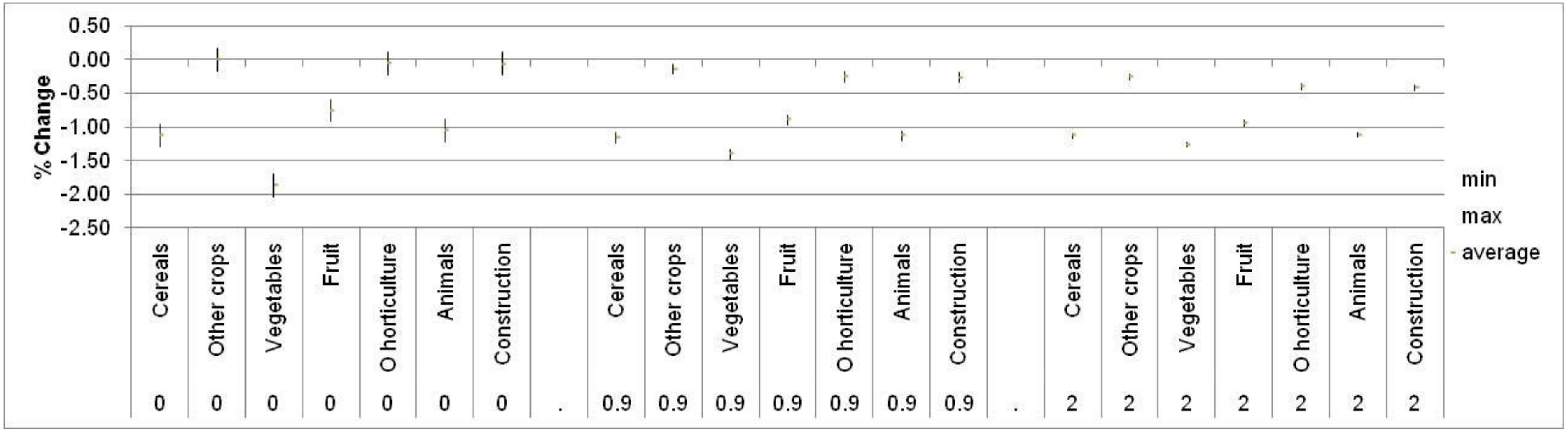


Figure 40: **Productivity growth: change in industry product prices (*PXAC*) – summary by product**

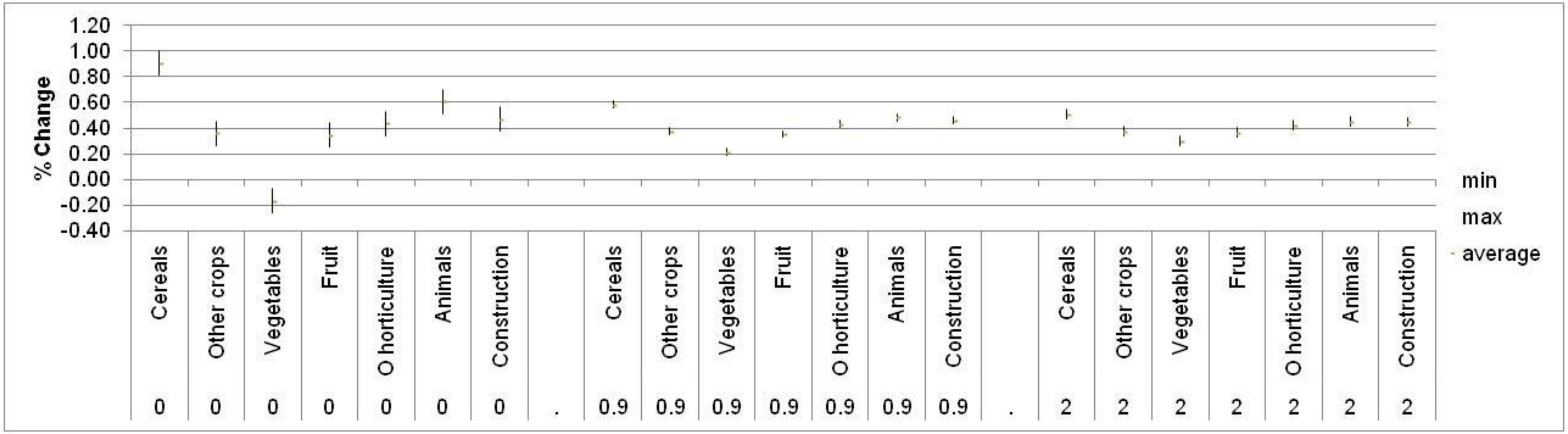
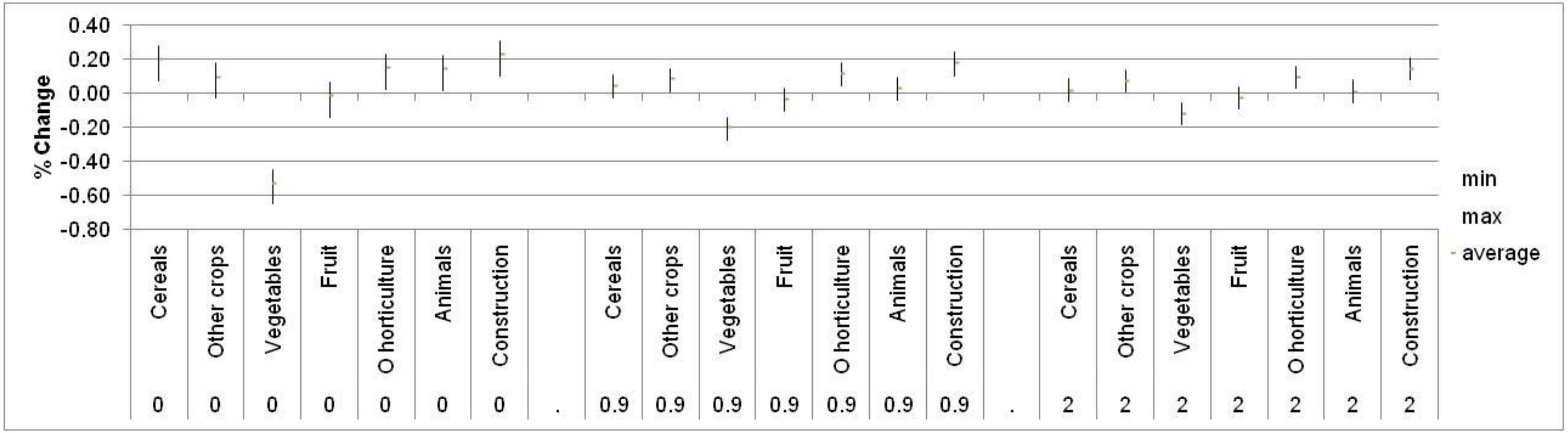


Figure 41: **Education: change in industry product prices (*PXAC*) – summary by product**



The changes in production of different products by industries (*QXAC*) do not reveal whether the composition of output remains the same, hence it is necessary to look at the share of each product in each industry's output (*IOQXACQXV*). This variable is directly affected by the model adaptation. The following four figures show the summary information for changes in output composition for each of the four scenarios.

Figure 42 shows the minimum, maximum and average changes in the shares that each product comprises in total output of a particular industry for the global positioning scenario. If *omega* is 0, then products are still produced in the same ratios as indicated by the zero average changes for all products. For *omega* of 0.9, the share of fruit in total output increases between 0.41% and 1.59% depending on the agricultural industry, with an average share increase of 1.30%. The share of the other products decrease on average by about 0.2%. For *omega* of 2, the share of fruit in total output increases between 0.66% and 2.50% depending on the agricultural industry, with an average share increase of 2.04%. The shares of the other products decrease on average by about 0.3%. It can be concluded that when producers are price responsive (adjusted model) the volume shares of fruit increase when the world export prices of fruit and food products with high horticultural content increase and the shares of all other products decrease.

The results for changes in output composition (*IOQXACQXV*) show that the variation between industries in the changes in shares of products tends to increase as *omega* increases. This is indicated by the longer lines for each of the three sets of results obtained with higher values of *omega* for all four scenarios.

Results for the other three scenarios (Figures 43 to 45) show varied results for different products, but the changes in composition clearly follow the changes in relative industry prices for different products (*PXAC*) shown in Figures 39 to 41.

Figure 42: Global positioning: change in output composition (*IOQXACQXV*) – summary by product

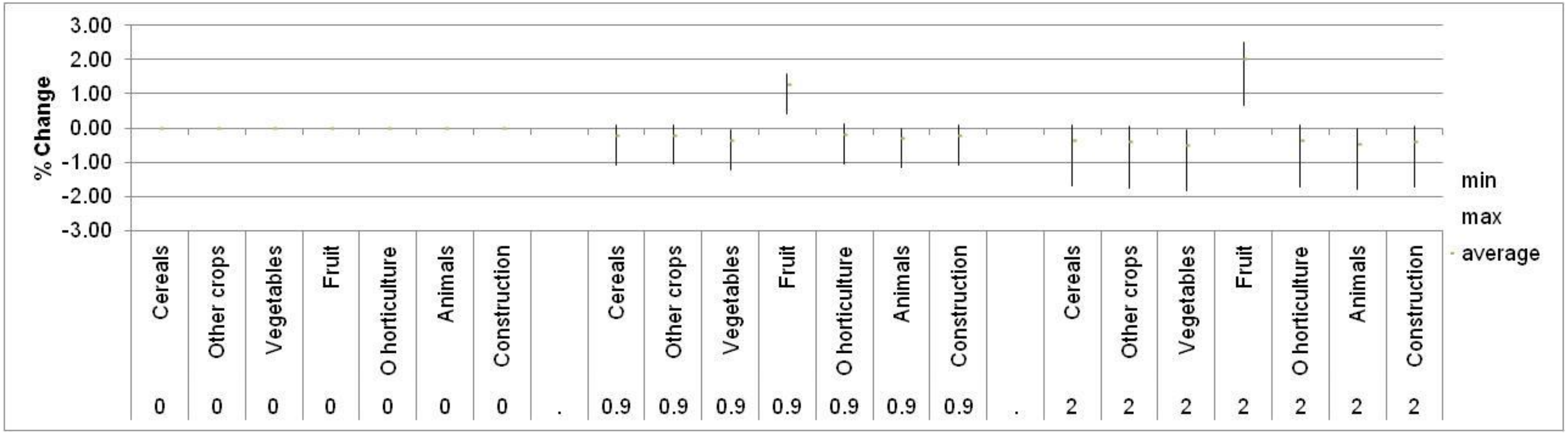


Figure 43: Technical efficiency: change in output composition (*IOQXACQXV*) – summary by product

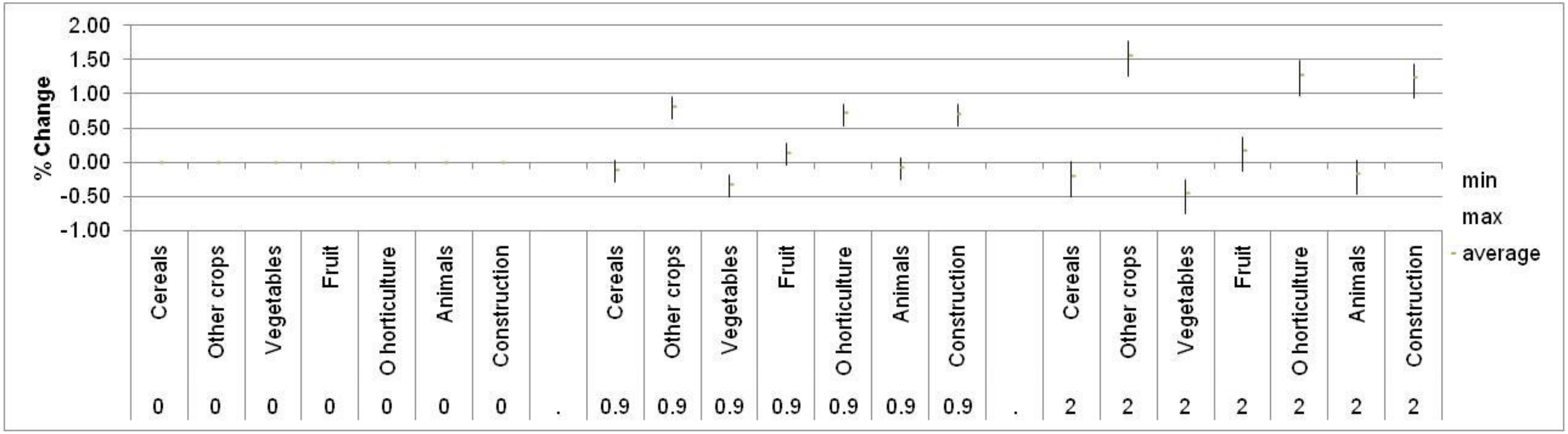


Figure 44: Productivity growth: change in output composition (*IOQXACQXV*) – summary by product

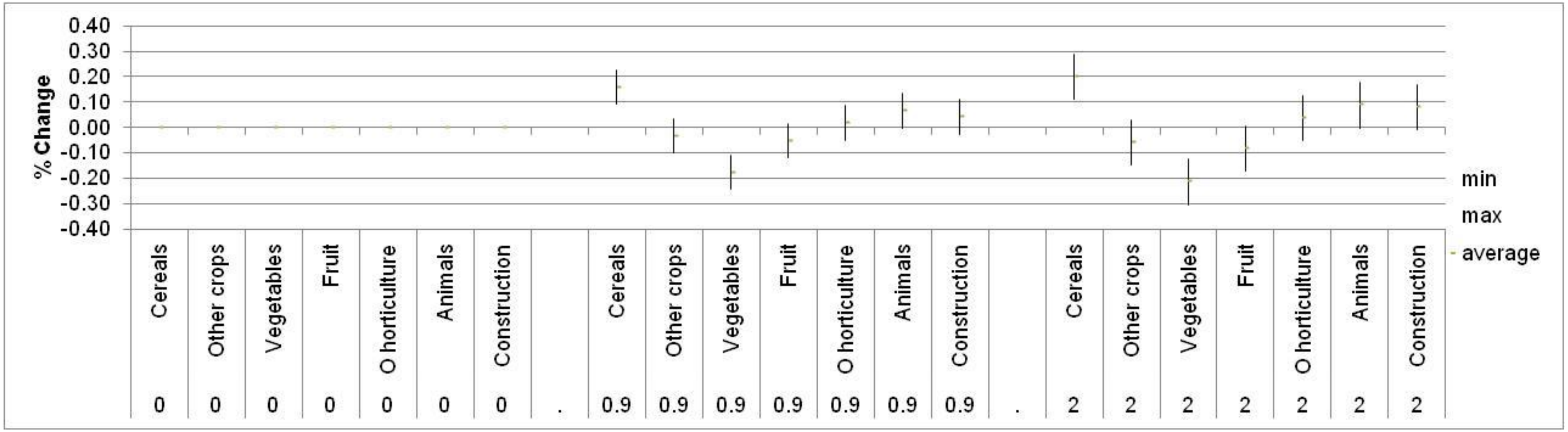
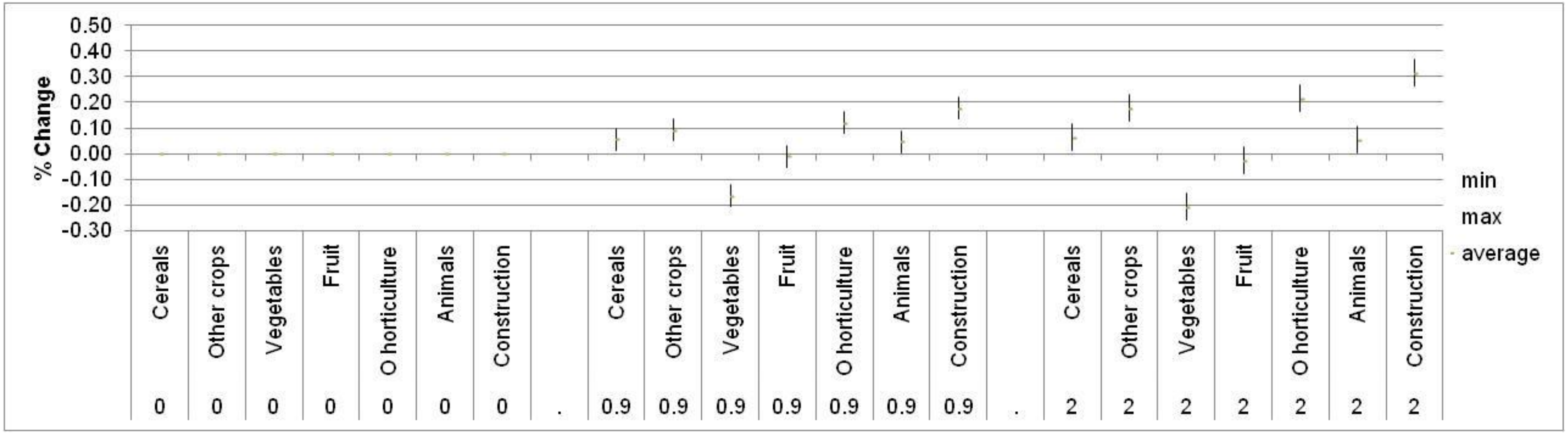
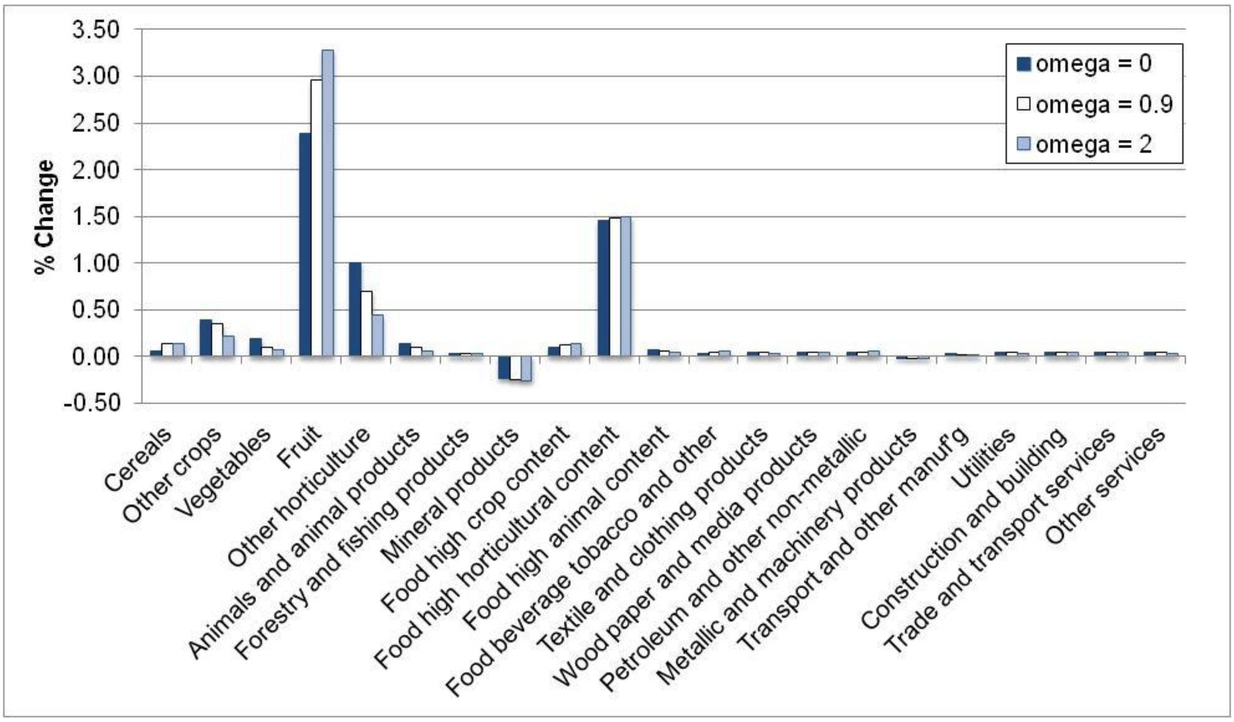


Figure 45: Education: change in output composition (*IOQXACQXV*) – summary by product



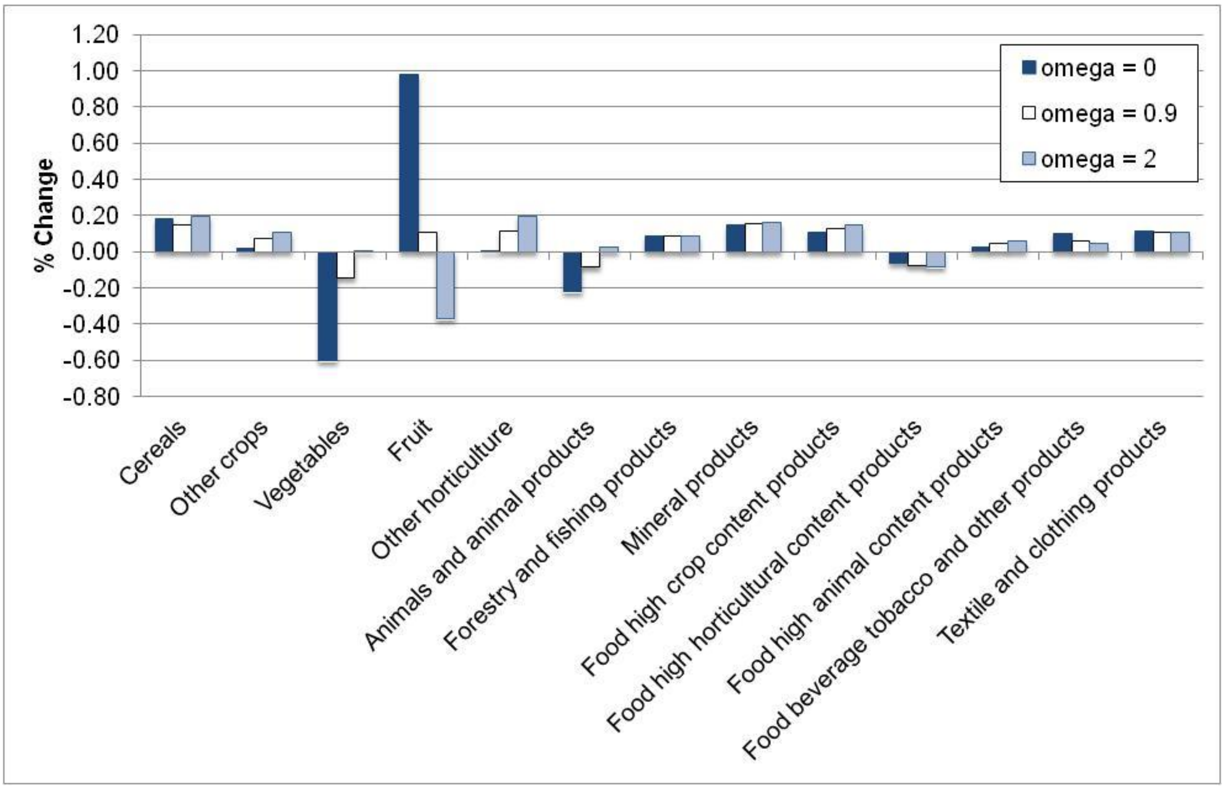
The results in Figure 46 show the percentage change in quantity of products produced (*QXC*) for the global positioning scenario. If producers are price responsive and changes the composition of their output (*omega* = 2), the aggregate quantity of fruit produced increases by 3.3%, compared to the increase of 2.4% for the base model. If agricultural industries can respond to price changes, the quantity of 'other horticultural' products produced would increase, but to a lesser extent compared to the case of non-responsiveness (0.4% vs. 1.0%). This is because the industries that produce fruit also produce other horticultural products and if fruit production increases and industries are not price responsive, then the production of 'other horticulture' will also increase. The results for fruit and food products with high horticultural content are directly affected by the model changes, whereas the impact on mineral products captures the indirect effect of the exchange rate appreciation. South Africa is a net exporter of mineral products; hence the exchange rate appreciation leads to a contraction of domestic production and exports in favour of imports.

Figure 46: Global positioning: change in quantity of products produced (*QXC*)



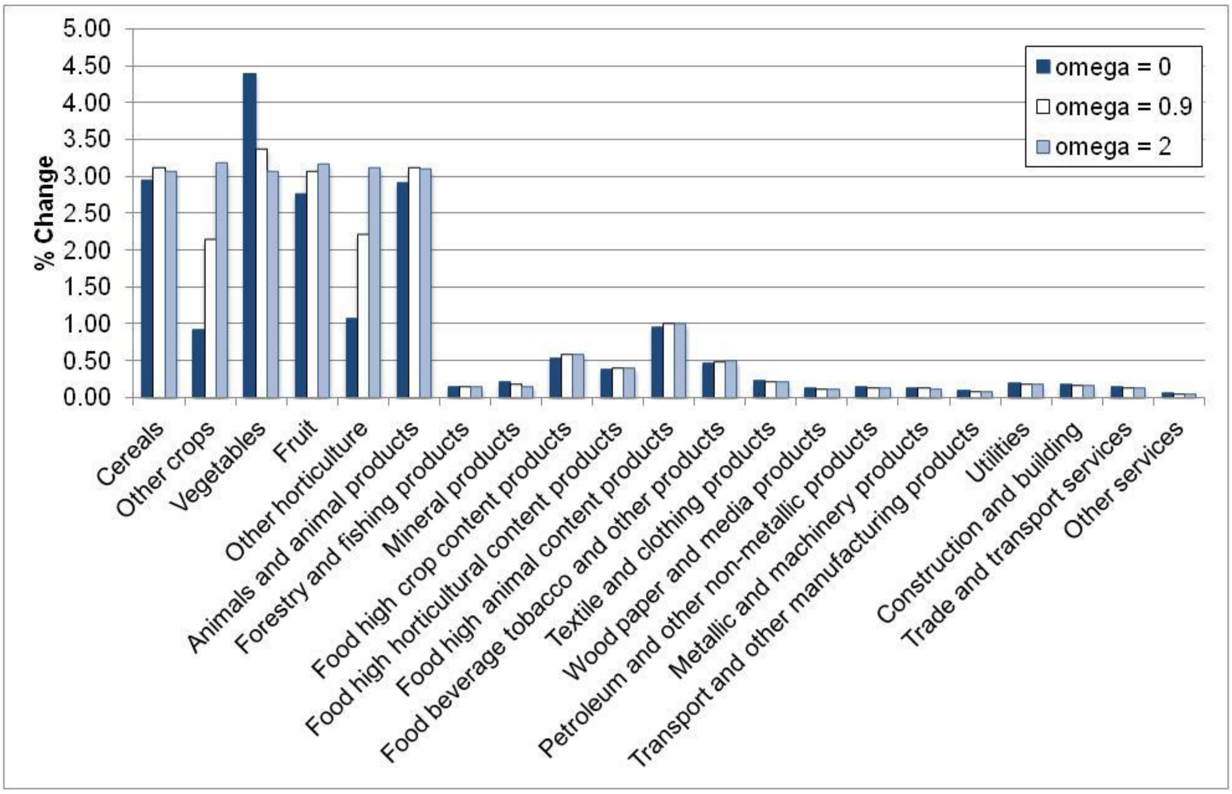
Changes in quantity imported (*QM*) for the global positioning scenario are reported in Figure 47. There are sign changes in the results when comparing the results from the two models on changes in imports. In the base model the quantity imported of vegetable and animal related products decreases notably but in the adjusted model it is the import quantity of fruit that shows a notable decrease. It was shown in the previous figure that for *omega* equal to zero there is a smaller increase in domestic production of fruit compared to when *omega* is set to two; hence the greater need for fruit imports. As the elasticity of transformation increases the domestic production of fruit can increase to the point where imports decrease. The same principle holds for the other products. Production of vegetables will increase relatively more when producers are not price responsive, because the production of all products increases equiproportionately. So fewer imports of vegetables will be required compared to when producers are price responsive.

Figure 47: Global positioning: change in quantity of imports (*QM*)



Changes in quantity exported (*QE*) for the technical efficiency scenario are shown in Figure 48. No sign changes occur between the two models. Fruit exports show an increase of 2.76% in the base model (*omega* equal to 0) and an increase of 3.17% in the adjusted model with *omega* equal to 2. Generally there is a greater increase in exports of agricultural products if producers can be more responsive in changing their output composition. This implies that producers can take better advantage of becoming more competitive when they are price responsive.

Figure 48: Technical efficiency: change in quantity of exports (*QE*)



The selected results of this section show that the level of *omega* influences results in all four scenarios, but the impacts for the global positioning scenario are the greatest. The results of changes in output composition show that the adapted model gives more realistic results than the base model. The choice of an appropriate level of elasticity of transformation is an area for future research.

7.3 Results: Elasticity sensitivity analyses

7.3.1 <u>Elasticities for comparison</u>

The previous section reports sensitivity analyses with regard to the change in the first order condition on transformation of aggregate industry output to different products, where the adjusted model assumed a value of 0.9 for the elasticity of transformation. Sensitivity of model results to different levels of the elasticity of transformation was also handled in the previous section, because it relates to the model changes that are the focus of the study. This section presents selected results to illustrate the sensitivity of model results to changes in the levels of all the other elasticities used in the model. For every elasticity in the model, two additional elasticity values are compared, one low value and one high value as depicted in Table 31. All results are generated with the adjusted model and the middle value is the value of the elasticity used in the adjusted model reported in section 6.4. Only one parameter value is changed at a time, therefore all the other elasticities assume the middle value as part of the *ceteris paribus* assumption. Results are reported for the four different scenarios because it is recognised that each scenario could lead to different results concerning the sensitivity analyses. The parameter related to the variable that is changed in each scenario generally has a greater impact on results compared to the other parameters.

Table 31: Compared elasticity values for the adjusted CGE model

	Functional form	**Low value**	**Middle value**	**High value**
Trade	CES on domestic production and imports	0.2	various	4
	CET on domestic production and exports	0.3	2	4
Production	CES on value added and intermediates	0.2	1.2	3
	CES on factors of production	0.2	0.5	1.5
	CES on product aggregation from different industries	0.5	4	6
	CET for transformation of industry output to different products	0.5	0.9	2
Consumption	LES income elasticities (poor hholds)	0.4 to 1.2	0.6 to 1.4	1.2 to 2.
	LES income elasticities (rich hholds)	0.2 to 1	0.4 to 1.2	1 to 1.8
	LES Frisch parameter (poor hholds)	-3.1 (all)	-3.1 to -4.5	-4.5 (all)
	LES Frisch parameter (rich hholds)	-1 (all)	-1 to -3	-3 (all)

7.3.2 Trade elasticities

Elasticity of substitution: Armington import elasticity

There are two trade elasticities included in the model. The first is the elasticity of substitution for the CES aggregation of imports (*QM*) and domestically produced products (*QD*) to form a composite good (*QQ*) (see Figure 7). The elasticity values for different products that were used in the model are recorded in Table 24. These values are adjusted here to 0.2 and then to 4 for all products and the results reported are for these three sets of values of the elasticity of substitution, with all other parameters assuming the middle values as indicated in the table above.

As expected, the macro results indicate that the change in the Armington elasticity impacts directly on the quantity of imports, the import tariff revenue and government savings (not shown here). There are also differences in magnitudes of the changes in the exchange rate. All differences are however very small (less than 0.05%). The level of *sigma* has negligible impact on other macro indicators in all the scenarios.

Figure 49 shows that for a relatively low value for *sigma* of 0.2 there will be a slight increase in imports of vegetables (0.14%), animals and animal products (0.04%) and food products with high horticultural content (0.18%), whereas for higher values of *sigma* there will be decreases in imports for these products of up to 8.4%. Food products with high horticultural content are affected more than other products because the global positioning scenario directly affects the world export prices of these food products. The point to note is that for a low *sigma* none of the products show a decrease in imports. If consumers cannot easily substitute between products produced domestically and those imported, then the positive income effect from the increased exports translates into increases in both imports and domestically produced products. When there is a greater degree of substitution, then there is a relatively greater increase in demand for all domestically produced agricultural and food products (not shown here), which is sufficient to dampen the demand for imports of most of these products.

Figure 49: Global positioning: change in quantity of imports (*QM*)

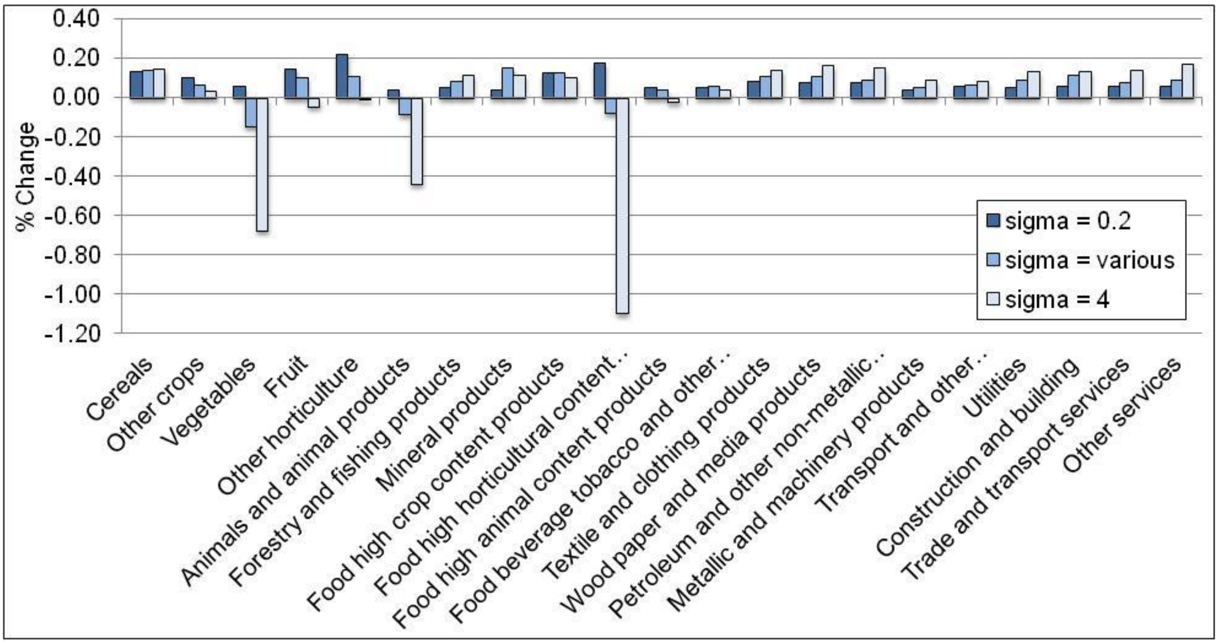
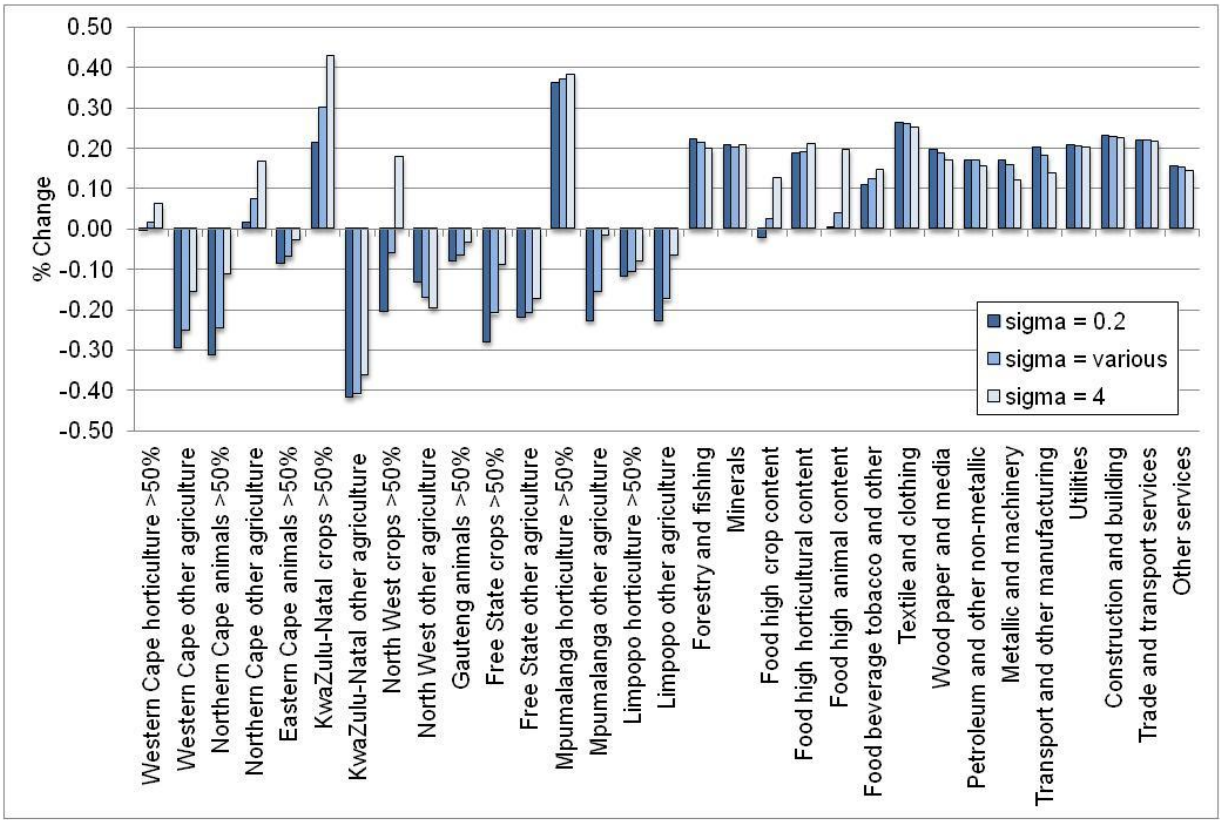


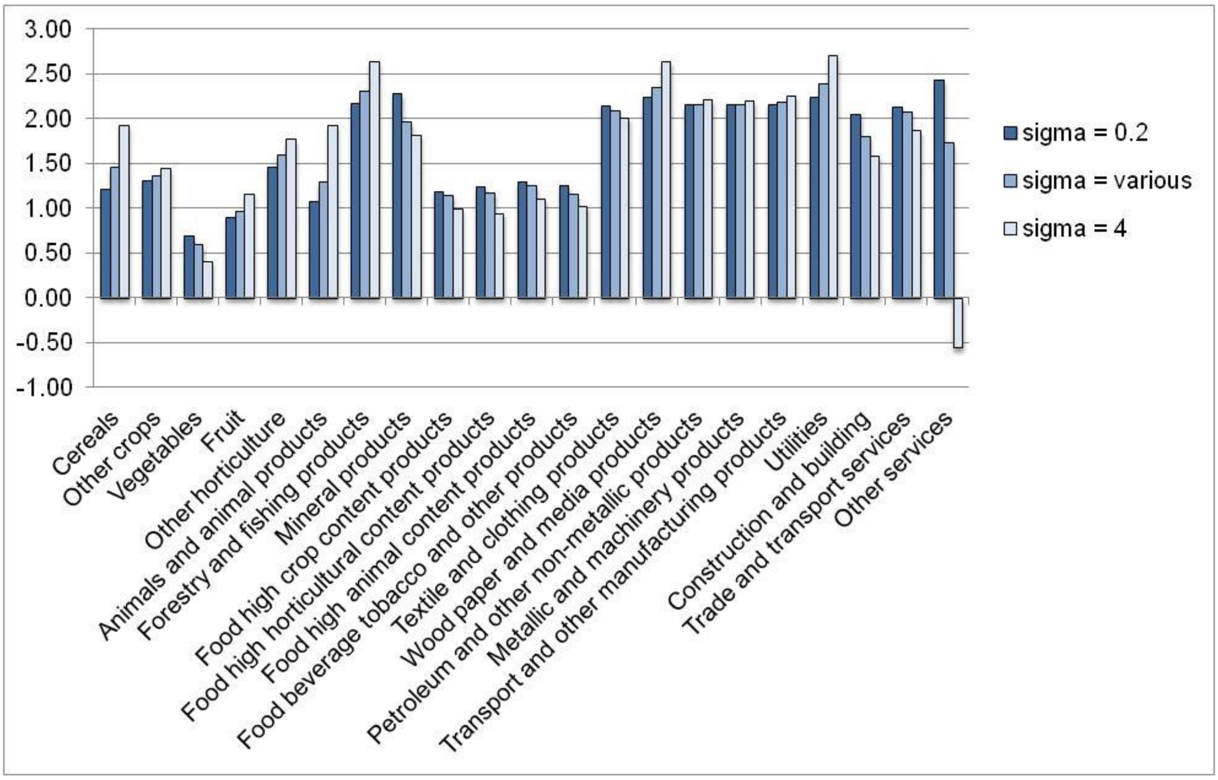
Figure 50 shows that the changes in factor demands (*FDA*) for the technical efficiency scenario where the shift parameter (*adx*) of the top level CES function for all agricultural industries are changed. Agricultural industries become more competitive in this scenario, therefore when there is greater substitutability between imports and domestic products (i.e. at higher levels of *sigma*), then domestic products will be in greater demand than imports (not shown here). Industry output of agricultural industries will therefore tend to be relatively greater for higher levels of *sigma*; hence the relatively greater demand for factors as shown in the figure. The level of *sigma* therefore also affects results for other variables such as wage rates, factor income and household incomes and expenditures (not shown here).

Figure 50: **Technical efficiency: change in factor demand (*FDA*)**



The only notable impact of the change in level of *sigma* on results from the productivity scenario, is on the changes in quantities imported (*QM*) as shown in Figure 51. The level of *sigma* determines whether there will be an increase or a decrease in the imports of 'other services', with subsequent impacts on the magnitudes of the changes of imports of other products, but without causing sign changes. The 'other services' industry becomes relatively more productive in this scenario, thus when greater levels of substitution become possible (*sigma* = 4) the industry gains from greater competitiveness and imports of 'other services' decline. Other results for this scenario are largely unaffected by different levels of *sigma* for the import aggregation function.

Figure 51: Productivity growth: change in quantity of imports (*QM*)



The effects of changes in the level of *sigma* on the results for the education scenario are negligible.

The impact of changes in the level of *sigma* on the results for the share of each product in each industry's output (*IOQXACQXV*) is negligible for the global positioning scenario. There are some small changes in shares for different levels of *sigma* for the other three scenarios, but nothing noteworthy.

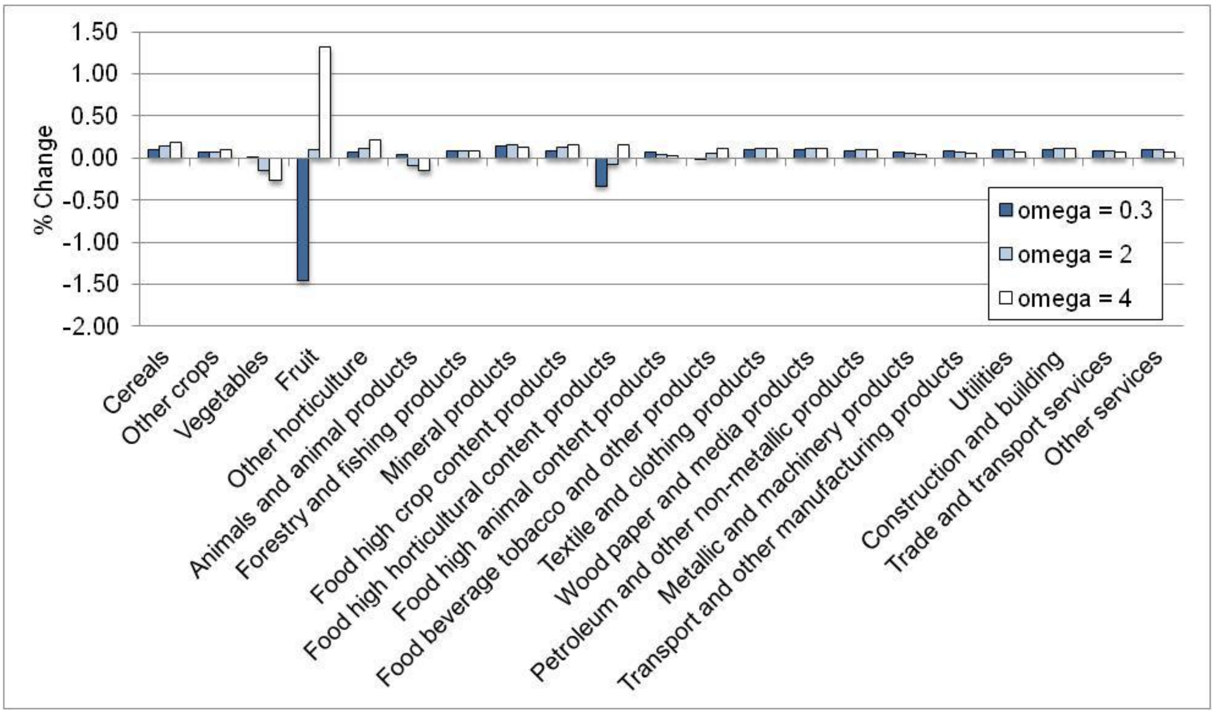
Elasticity of transformation: export elasticity

The second trade elasticity is the elasticity of transformation of the domestically produced output (*QXC*) to determine the relative shares of exports (*QE*) versus products produced for the domestic market (*QD*) (see Figure 7). The value used in the model is 2 for all products. This value is adjusted here to 0.3 and then to 4 for all products.

The results from the productivity growth and education scenarios are not greatly affected by different levels of *omega*. There are no sign changes, except for the price of value added and price of intermediate inputs for the education scenario but the changes are less than 0.04%. Although there are some differences in the magnitudes of the changes, they are relatively small compared to the other two scenarios and not shown here.

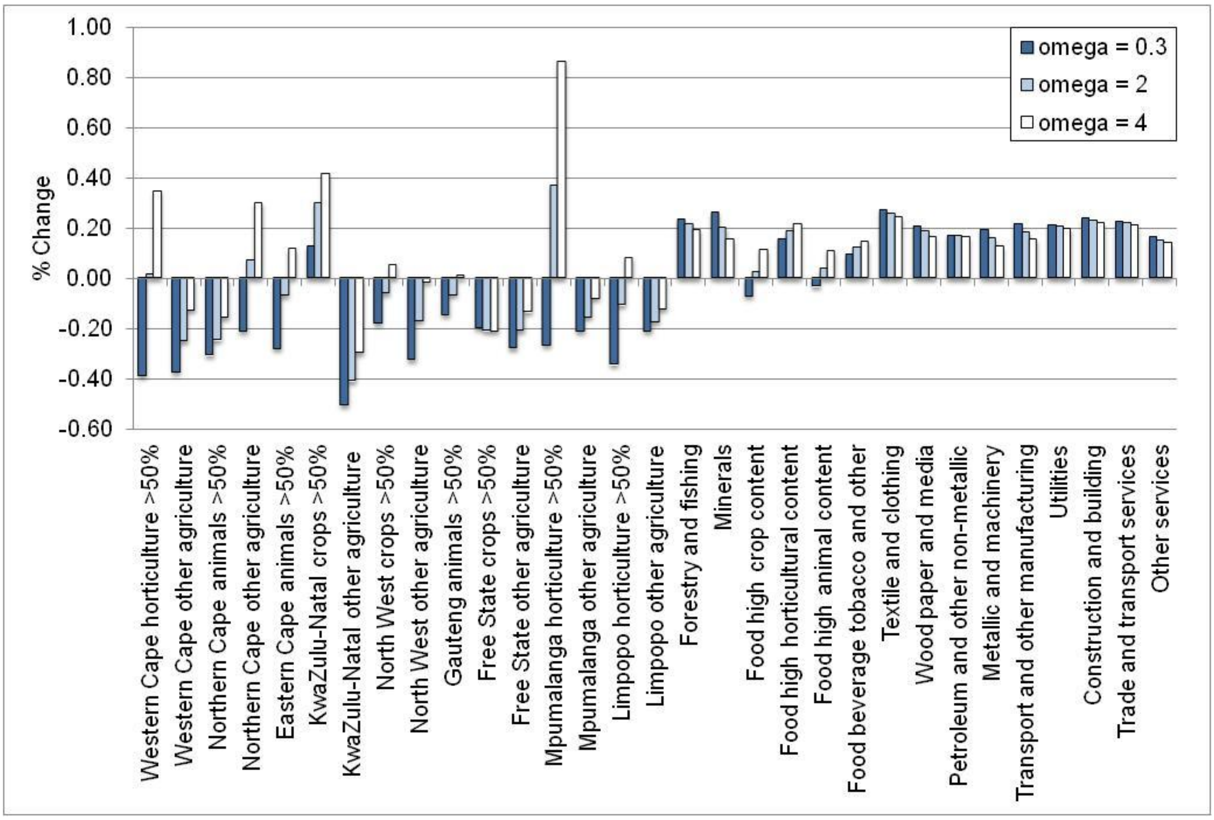
The ease of transformation between producing for the export market as opposed to the domestic market has a notable impact on the results of the global positioning scenario because this scenario has a direct impact on exports. For the global positioning scenario the exchange rate shows a slight increase (Rand depreciates) of 0.001% for *omega* equal to 0.3 and a decrease of 0.02% when *omega* is 4. This implies that the indirect effect on quantities traded of other products can vary depending on the value of *omega*. The quantities exported, which are directly affected by the level of *omega* do not show sign changes, only differences in magnitude. The quantities imported (*QM*) for the two products affected by the global positioning scenario, i.e. fruit and food products with high horticultural content, both show sign changes as shown in Figure 52. For *omega* equal to 4 there is greater flexibility to export rather than produce for the domestic market. The exports of both fruit and food with high horticultural content increase more for higher levels of *omega*; hence the production for the domestic market must be supplemented with imports as seen in the figure below. Most of the other results for the global positioning scenario show slight differences in magnitude rather than sign changes.

Figure 52: Global positioning: change in quantity of imports (*QM*)



For the technical efficiency scenario the results for value added, and therefore factor demands (*FDA*), wage rates (*WF*) and factor incomes (*YF*), are sensitive to the level of *omega*. Figure 53 shows that for several of the agricultural industries the level of *omega* determines whether factor demand increases or decreases. For *omega* equal to 4 there is greater flexibility to export rather than produce for the domestic market. All agricultural industries become more competitive in this scenario, but the industries that demand more factors of production are those that produce relatively more export orientated products.

Figure 53: Technical change: change in factor demand (*FDA*)



The impact of changes in the level of *omega* on the results for the share of each product in each industry's output (*IOQXACQXV*) is the largest for the global positioning and technical change scenarios and almost negligible for the productivity growth and education scenarios. This is as expected, because the global positioning and technical change scenarios have a focus on international competitiveness. Figure 54 and Figure 55 show the minimum, maximum and average changes in the shares that each product comprises in total output of a particular industry (*IOQXACQXV*) for the global positioning and technical efficiency scenarios respectively. In both scenarios the variation in response between different industries become greater for higher values of *omega*, as reflected by the longer lines. The average changes in shares also become greater for higher values of *omega*.

Figure 54: **Global positioning: change in output composition (*IOQXACQXV*) – summary by product**

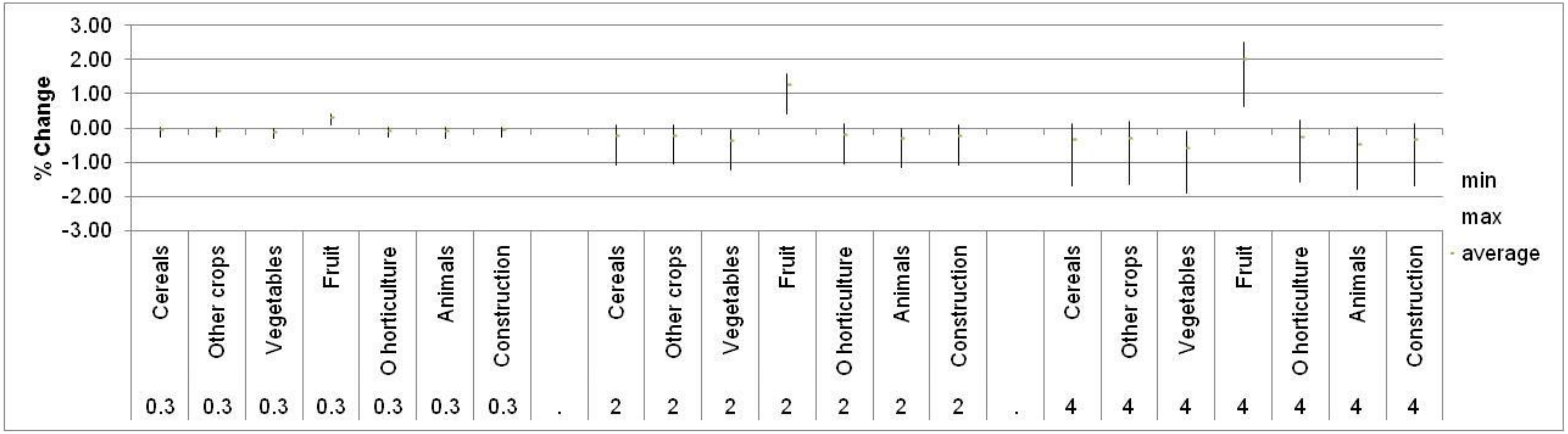
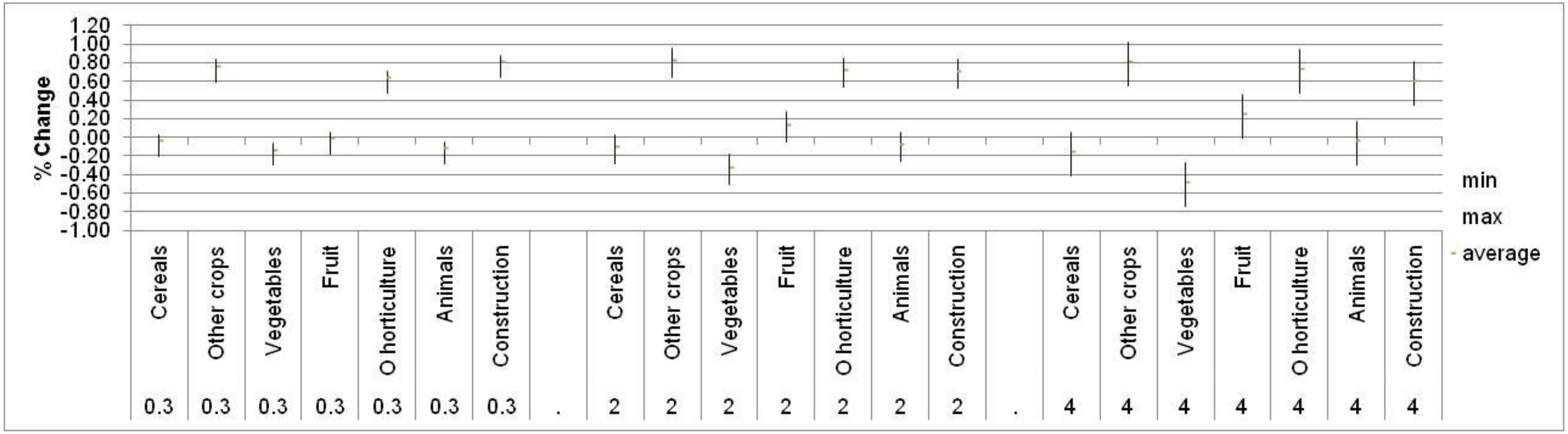


Figure 55: **Technical efficiency: change in output composition (*IOQXACQXV*) – summary by product**



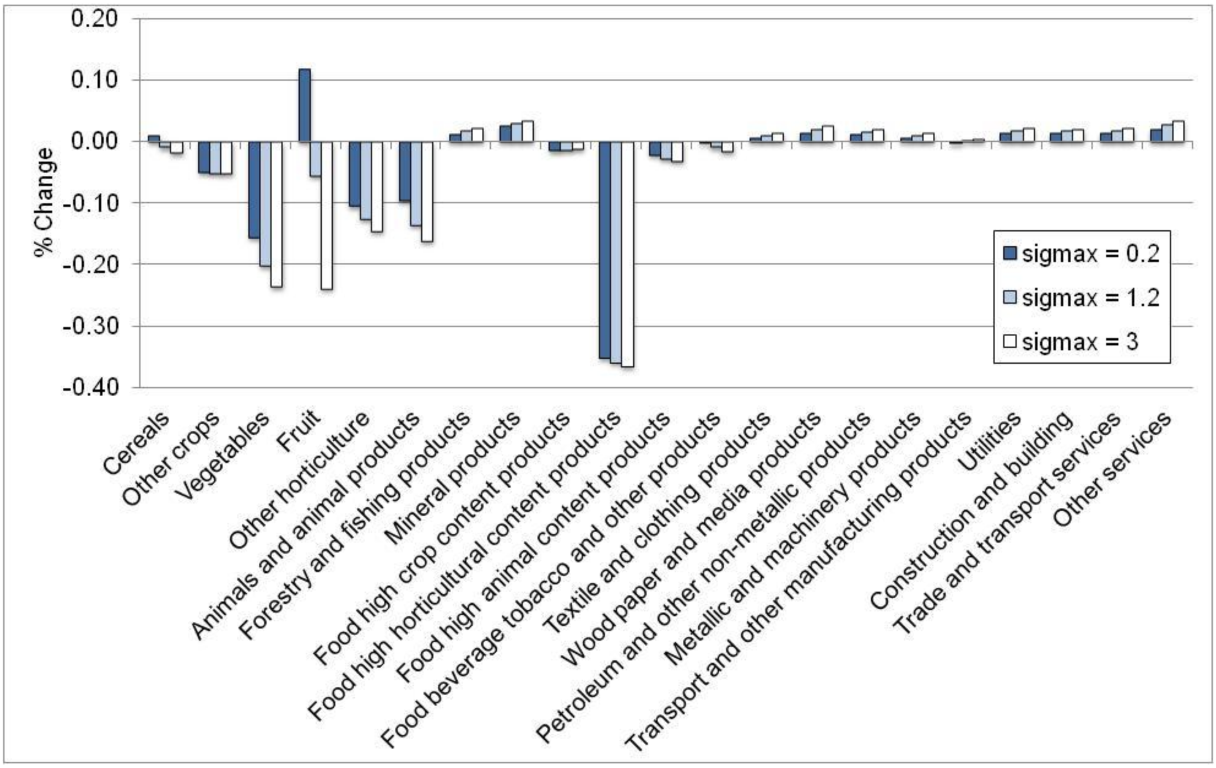
7.3.3 Production elasticities

Elasticity of substitution: value added and intermediate inputs

There are four production elasticities included in the model. The first one to be discussed is the elasticity of substitution for the CES aggregation of value added (*QVA*) and intermediate inputs (*QINT*) to form industry output (*QX*) (see Figure 7). The value for the elasticity of substitution (*sigmax)* used in the adjusted model is 1.2 for all industries. This value is adjusted here to 0.2 and then to 3 for all industries. Results indicate very few sign changes.

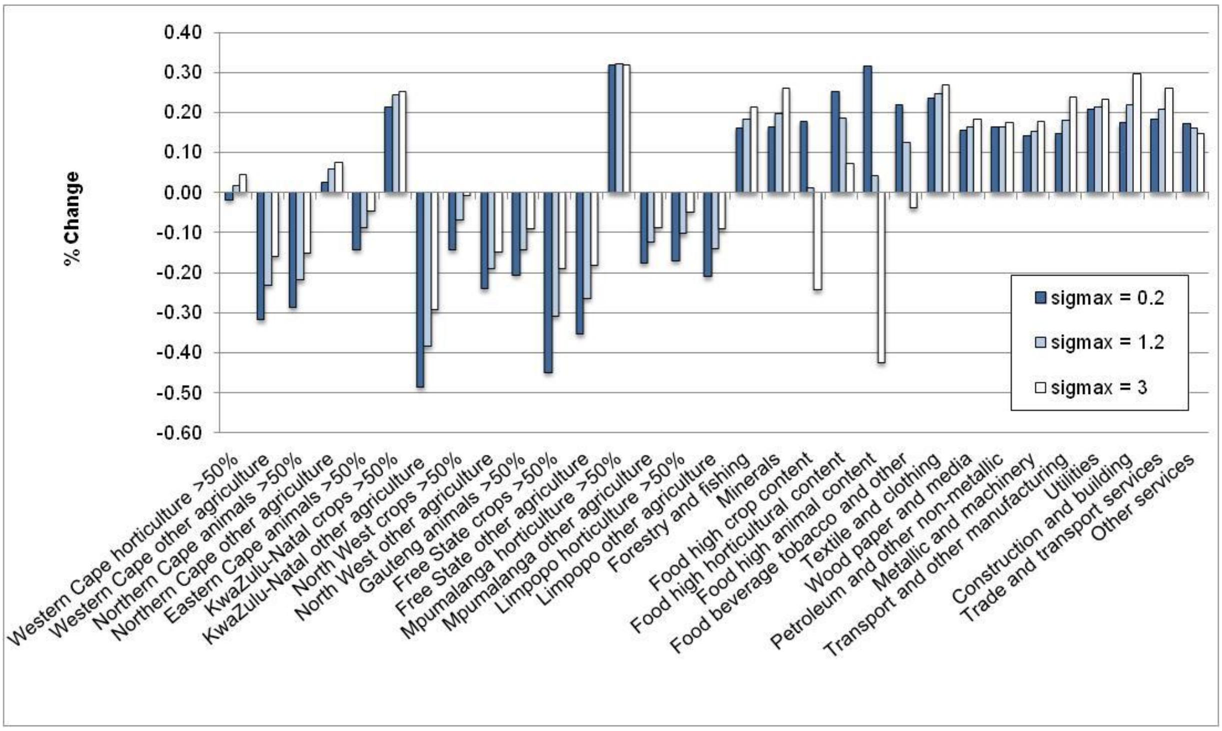
In the global positioning scenario the increase in world prices and exports have an expansionary effect on fruit production, which puts upward pressure on wage rates of skilled agricultural labour and land because of the greater demand for factors for agricultural production in fruit producing areas. This causes the price of value added (*PVA*) to increase relative to the price of intermediate inputs (*PINT*) for agricultural industries. For lower values of *sigmax*, i.e. when industries are less flexible in substituting the relatively more expensive factors of production for intermediate price, the upward pressure on wage rates is greater than for smaller values of *sigmax*. Results from the global positioning scenario show that as the value of *sigmax* increases, agricultural industries producing fruit will tend to increase the use of production factors (*QVA*) relative to intermediate inputs (*QINT*) because of these price effects. This leads to a relatively greater increase in production (*QX*) than would be the case for lower values of *sigmax* because industries have greater flexibility in substituting between value added and intermediate inputs. Domestic output (*QD*) and exports (*QE*) increase relatively more for higher values of *sigmax*, to the extent that fruit imports (*QM*) of fruit decrease, whereas it would increase for lower levels of *sigmax*. There is still a net positive effect on composite supply (*QQ*) for higher values of *sigmax*, i.e. the decrease in imports is smaller than the increase in domestic supply. The relatively greater increase in composite supply (*QQ*) for higher values of *sigmax* is sufficiently large to cause a decrease in the supply price of the composite product (*PQS*) as shown in Figure 56.

Figure 56: Global positioning: change in supply price of composite product (*PQS*)



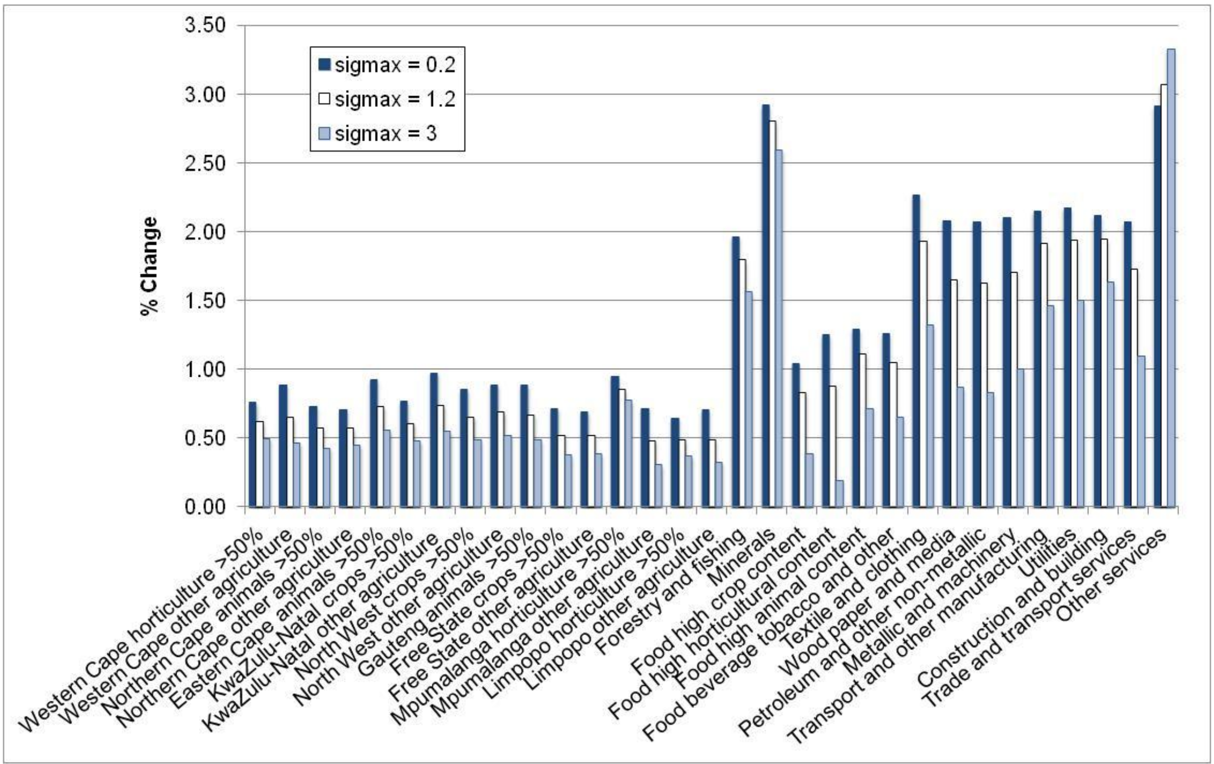
The direct impact of changing *sigmax* is on the quantity of value added (*QVA*) and quantity of intermediate inputs (*QINT*) in the top level of the nested CES production function and it is the shift parameter (*adx*) of this same function that is increased as part of the technical efficiency scenario. The difference in changes for the quantity of value added (*QVA*) for the technical efficiency scenario is shown in Figure 57. Smaller differences and no sign changes are found in the changes in intermediate inputs (*QINT*) and industry output (*QX*). It is the users of intermediate products from the more competitive agricultural industries that are most affected by the level of *sigmax.* When the agricultural industries become more competitive the agricultural products used as intermediate inputs (*QINT*) by the food industries become cheaper relative to value added (*QVA*), i.e. the purchasers' prices (*PQD*) of all agricultural products decrease and the price of intermediate products (*PINT*) of all food industries decrease. The relatively cheaper intermediate inputs (*QINT*) will lead to increases in production (*QX*) of the food industries. If there are limited substitution possibilities between intermediate inputs and value added (*sigmax* equal to 0.2), there is an increase in value added more comparable to the increase in intermediate input use. However if the food industries have greater substitution possibilities between intermediate inputs and value added (*sigmax* equal to 3), they will use relatively more intermediate inputs and relatively less value added, as reflected in the figure below. Although not shown here, the results for intermediate inputs and output indicate greater increases for higher values of *sigmax* compared to lower values for the food industries, as expected.

Figure 57: Technical efficiency: change in value added (*QVA*)



For the productivity growth scenario the shift parameter (*adva*) of aggregate value added of the 'other services' industry was increased, leading to 'other services' increasing the use of its more productive input, namely value added (*QVA*) (as opposed to intermediate inputs (*QINT*)). If the substitution possibility at the top level of the production function is also increased through higher values of *sigmax*, then further substitution between value added and intermediate inputs are possible and 'other services' tend to use even more value added. Figure 58 shows that the increase in value added (*QVA*) for other services is greater for higher values of *sigmax*. The 'other services' industry comprises more than a third of the economy, therefore if there is a relatively larger increase in the use of value added by 'other services' then there is a relatively smaller increase in the use of value added by all the other sectors because factors move from the other industries to 'other services'. A similar principle applies to the use of intermediate inputs (*QINT*), which indicates the opposite trend for different values of *sigmax* (not shown here). At higher levels of *sigmax* the 'other services' industry decreases its use of intermediate inputs relative to value added compared to lower levels of *sigmax*, whereas all other industries increase the use of intermediate inputs relative to value added for higher levels of *sigmax* compared to lower levels of *sigmax*.

Figure 58: Productivity growth: change in value added (*QVA*)



Results for the education scenario indicate limited influence for different levels of *sigmax* and are not reported here.

The impact of changes in the level of *sigmax* on the results for the share of each product in each industry's output (*IOQXACQXV*) is negligible for the global positioning, technical change and education scenarios. There are only some small changes in shares for different levels of *sigmax* for the productivity growth scenario.

Elasticity of substitution: factor inputs (value added)

The second production elasticity is the elasticity of substitution for the CES aggregation of different factors of production (*FD*), i.e. capital, labour and land, to form aggregate value added (*QVA*) at the second level of the production nest (see Figure 7). The value of the elasticity of substitution (*sigmava*) used in the adjusted model is 0.5 for all industries. This value is adjusted here to 0.2 and then to 1.5 for all industries.

The direct impact of changing *sigmava* is on the quantity of value added (*QVA*) in the bottom level of the nested CES production function and it is the shift parameter (*adva*) of this same function that is increased as part of the productivity growth scenario. Figure 59 shows the changes in value added obtained with different levels of *sigmava*. The difference between these results and the ones reported in the previous figure is clear. As the level of *sigmava* is increased, industries are able to substitute more between different factors in order to use more of the relatively cheaper factors. The result is that all industries will increase aggregate value added (*QVA*), which leads to increases in output (*QX*) and use of aggregate intermediate inputs (*QINT*) for higher levels of *sigmava* compared to lower levels of *sigmava*. As expected, there is no notable substitution between value added and intermediate inputs for different levels of *sigmava* as was the case for changes in *sigmax* as discussed with regard to the previous figure.

Figure 59: Productivity growth: change in value added (*QVA*)

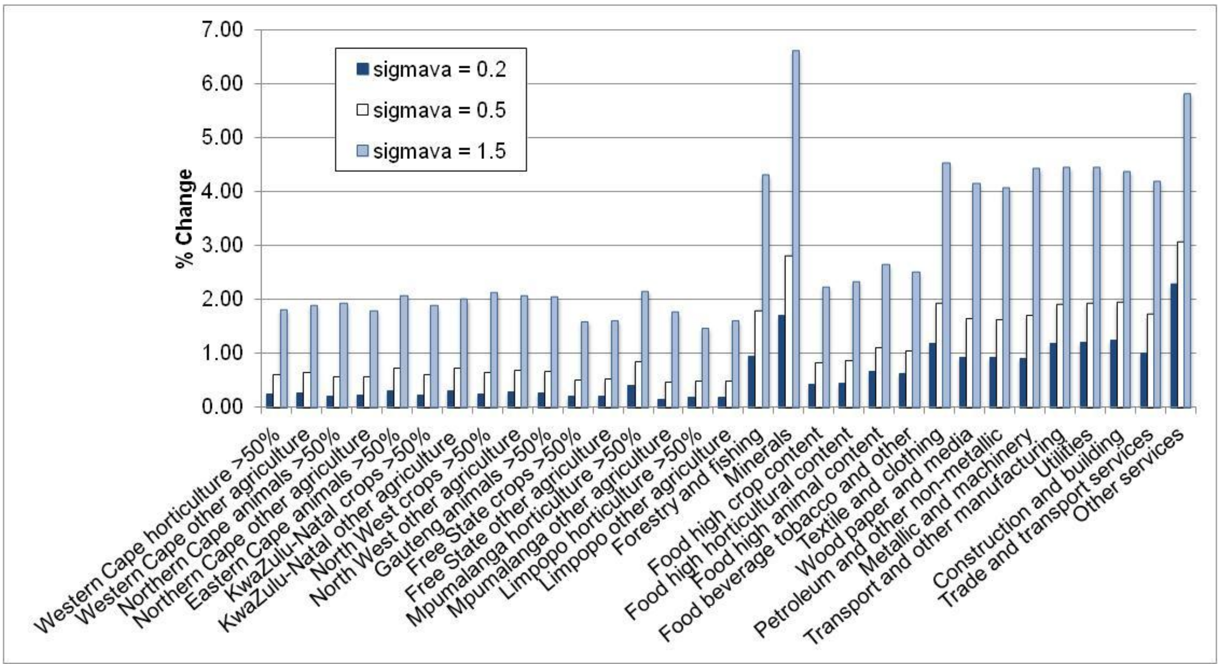
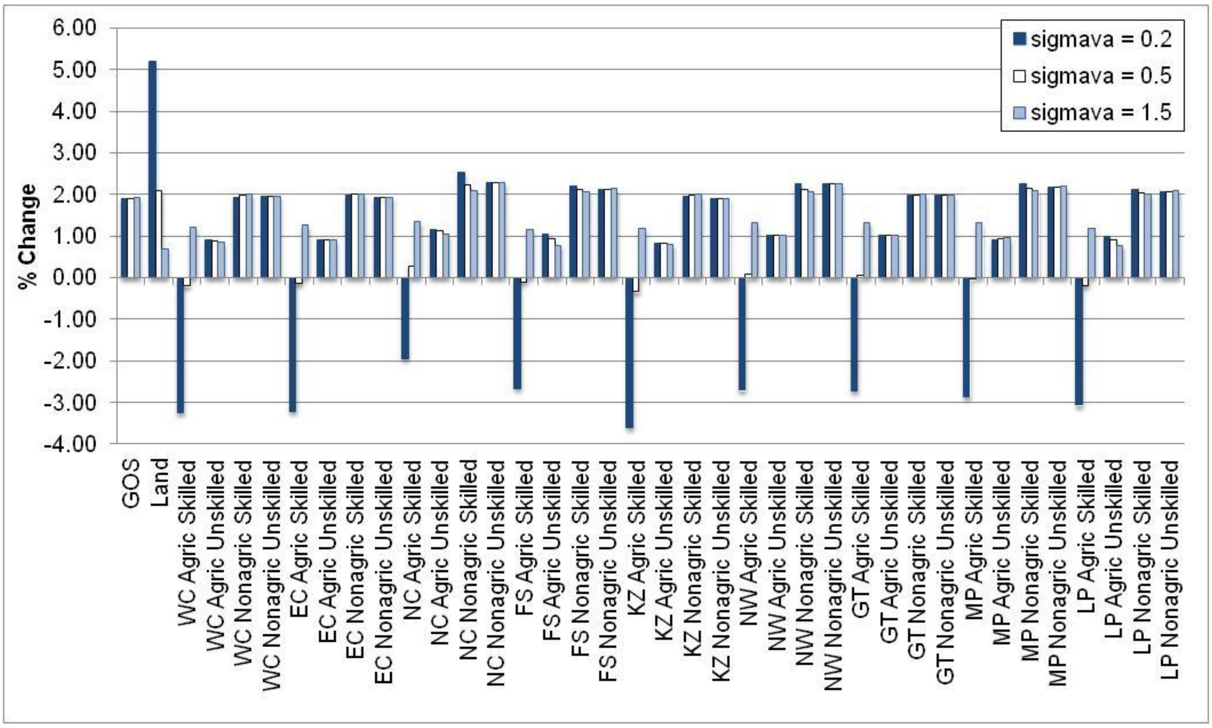


Figure 60 shows the changes in factor incomes (*YF*) obtained with different levels of *sigmava* for the education scenario. A low *sigmava* value (0.2) causes a relatively greater decrease in wage rates of skilled agricultural workers compared to higher values of *sigmava* (0.5 and 1.5) (not shown here). In the education scenario there is an increase in skilled labour and at lower levels of substitution skilled labour cannot be substituted for other factors. The wage rates of skilled agricultural labour therefore decrease while the rate of returns to land increases, under the assumption that both are fully employed. For higher levels of *sigmava* (1.5) the changes in wage rates of skilled agricultural labour and the rate of returns to land are almost negligible. As a result of the changes in the wage rates, factor incomes of skilled agricultural workers show a net decrease for a *sigmava* value of 0.2, but factor incomes of skilled agricultural workers increase for *sigmava* values of 1.5. Household incomes and expenditures are also affected because of the impacts on factor incomes, but only in terms of magnitude.

Figure 60: **Education: change in factor incomes (*YF*)**



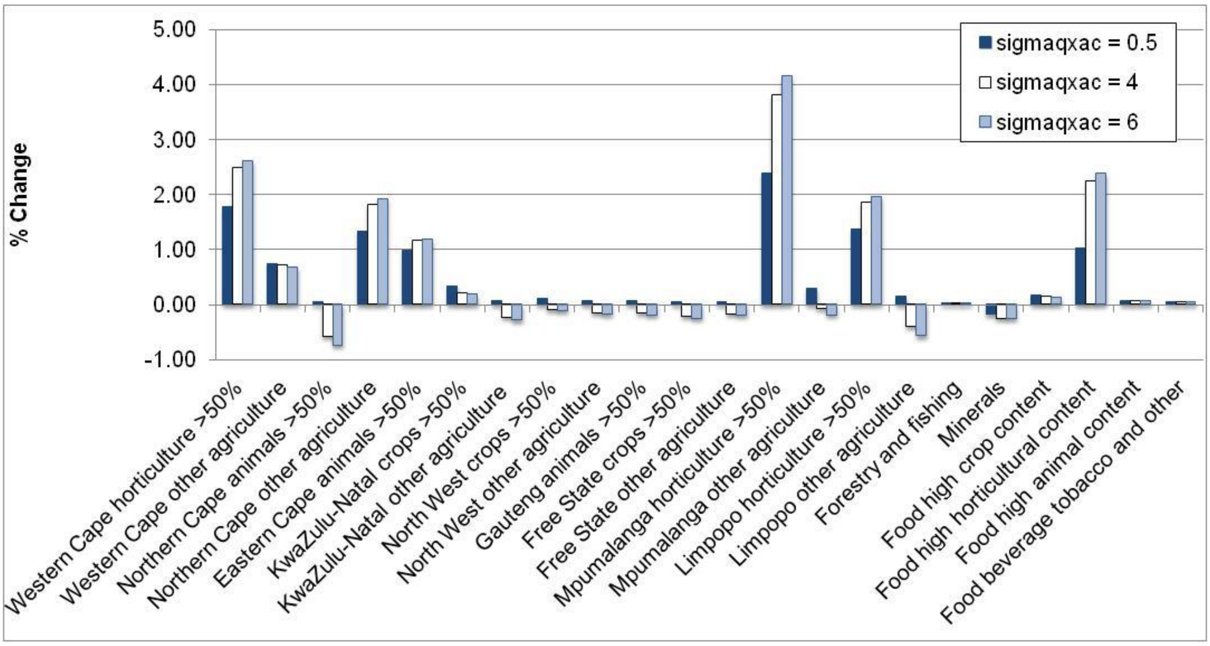
The impact of changes in the level of *sigmava* on the results for the share of each product in each industry's output (*IOQXACQXV*) is negligible for the global positioning, technical change and education scenarios. There are some small changes in shares for different levels of *sigmava* for the productivity growth scenario, but nothing noteworthy.

Elasticity of substitution: similar products from different industries

The third production elasticity is the elasticity of substitution to determine the relative shares of similar products produced by different industries (*QXAC*) to make up the domestic good (*QXC*) (see Figure 7). The value of the elasticity of substitution (*sigmaqxac*) used in the model is 4 for all products. This value is adjusted here to 0.5 and then to 6 for all products. The results indicate that a *sigmaqxac* of 4 is already a relatively high elasticity and therefore the results do not differ much from a *sigmaqxac* equal to 6. The global positioning scenario illustrates the impact of different levels of *sigmaqxac* most clearly because the scenario impacts directly on the price of one particular agricultural product.

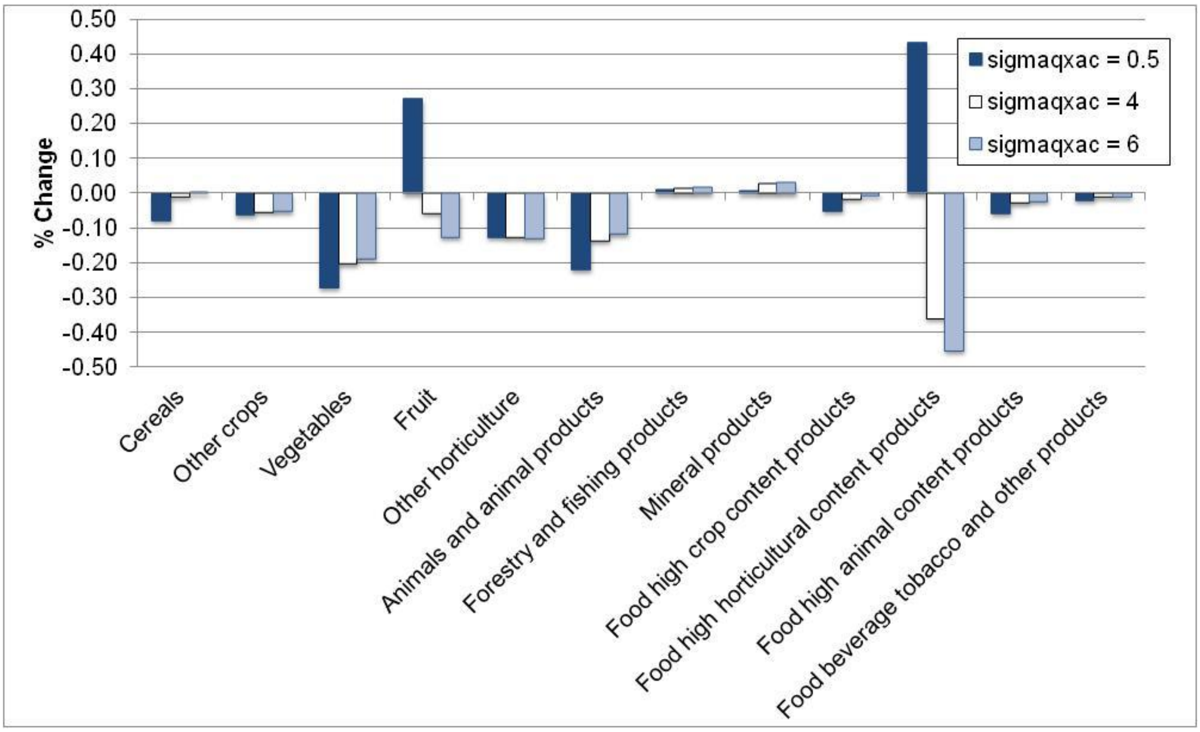
Figure 61 shows the changes in quantity of industry output (*QX*) for different levels of *sigmaqxac* for the global positioning scenario. The main fruit producing industries increase their fruit production relatively more for higher values of *sigmaqxac*, i.e. if there is greater flexibility in the share of each industry in total domestic supply of a particular product. This flexibility leads to a greater supply of intermediate products (*QINT*) to the food producing industries (not shown here), thereby enabling an increase in production (*QX*) of food products with high horticultural content. The impact of changes in the elasticity on the output of non-agricultural and non-food industries is negligible.

Figure 61: Global positioning: change in industry output (*QX*)



For the global positioning scenario when fruit production increases for higher values of *sigmaqxac* the composite supply (*QQ*) of fruit and food products with high horticultural content also increases (not shown here) and this exerts downward pressure on the purchasers' prices of the composite product (*PQD*) (i.e. domestic plus imported) to the extent that the prices decrease for high values of *sigmaqxac*, whereas it increases for low values of *sigmaqxac*, as shown in Figure 62.

Figure 62: Global positioning: change in purchasers' price of composite product (*PQD*)



There are no sign changes for the results of the other scenarios, only differences in magnitude and the impacts on agricultural industries are mixed because of the way the scenarios were implemented.

The impact of changes in the level of *sigmaqxac* on the results for the share of each product in each industry's output (*IOQXACQXV*) is negligible for the technical change scenario. There are some small changes in shares for different levels of *sigmaqxac* for the global positioning, productivity growth and education scenarios. Figure 63 and Figure 64 show the minimum, maximum and average changes in the shares of products in total output of a particular industry (*IOQXACQXV*) for the productivity growth and education scenarios respectively. In both scenarios the variation in response between different industries becomes smaller for higher values of *sigmaqxac*, as reflected by the shorter lines. The average changes in shares also become smaller for higher values of *sigmaqxac.* Industries that can respond more productively to the production of products that face higher prices will increase production relatively more than other industries for lower levels of substitution.

Figure 63: **Productivity growth: change in output composition (*IOQXACQXV*) – summary by product**

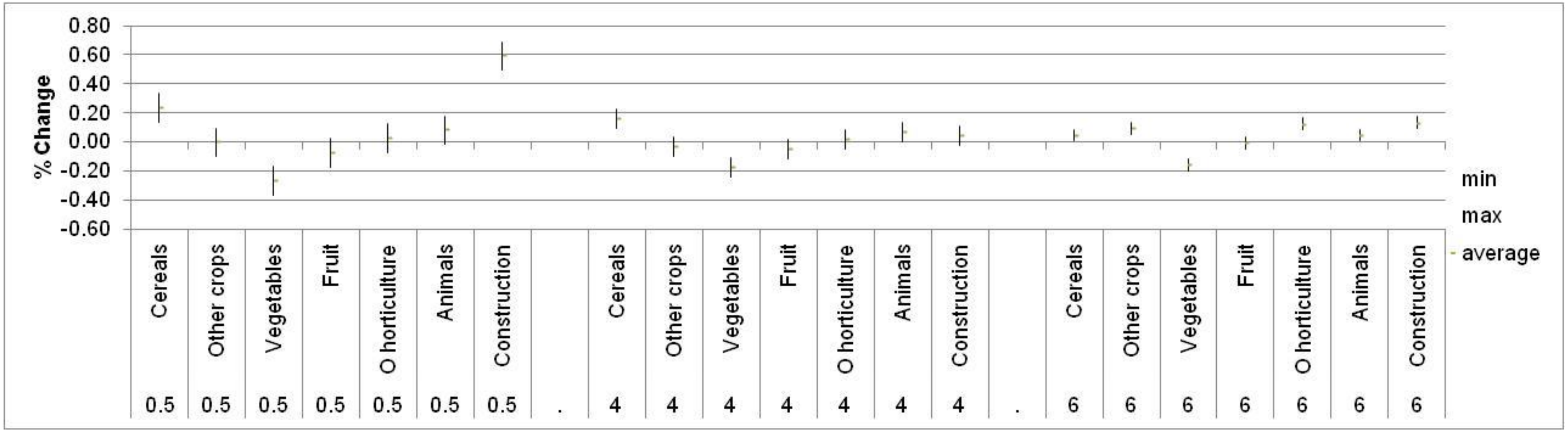
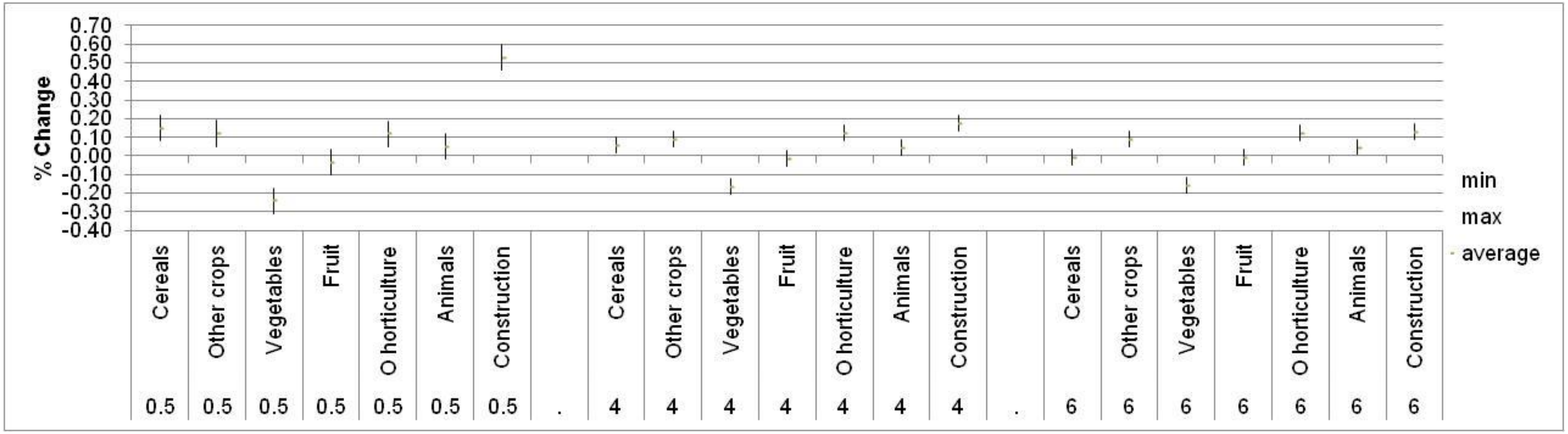


Figure 64: **Education: change in output composition (*IOQXACQXV*) – summary by product**



Elasticity of transformation: different products from same industry

The results for changes in this elasticity were discussed in section 7.2.

7.3.4 Consumption elasticities

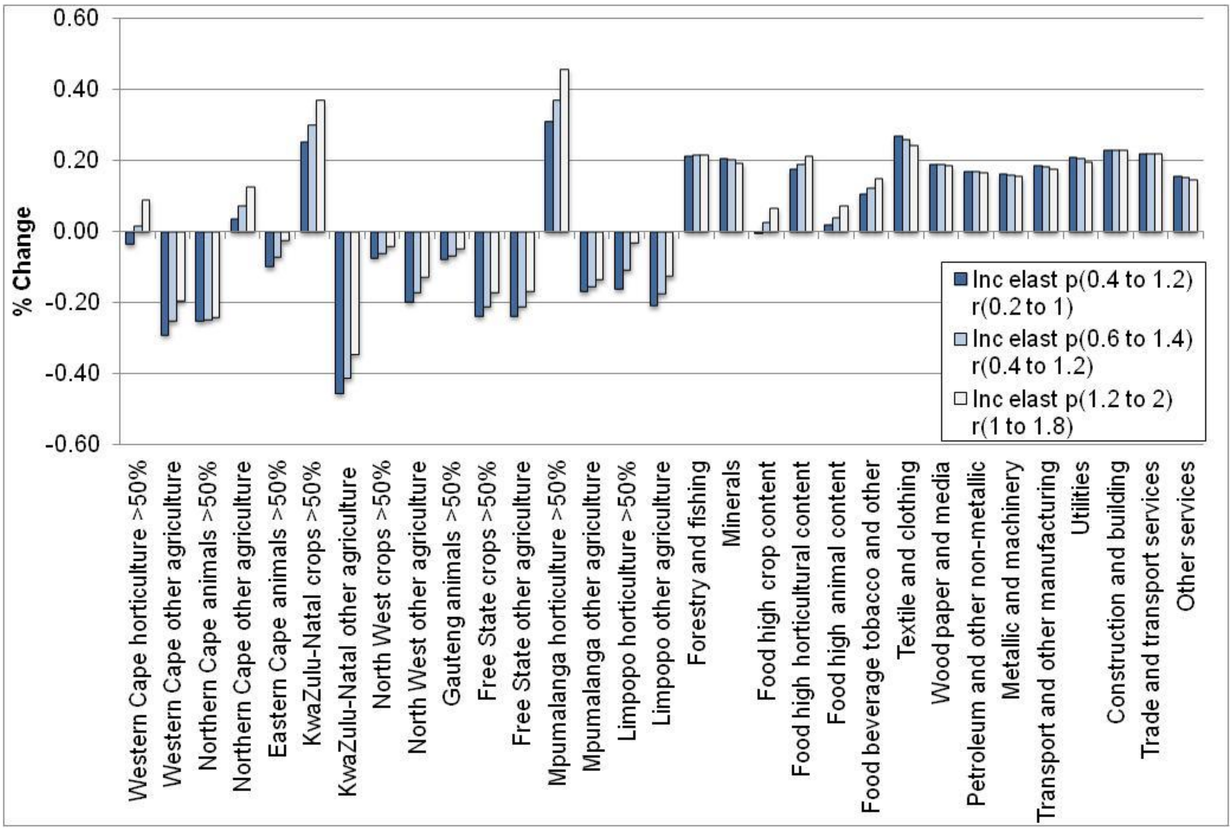
LES income elasticity

There are two sets of parameters required for the LES function for household consumption demand. The first set is the income elasticities for the LES function. The model assumes different values in the base case, ranging between 0.4 and 1.2 for poor households and between 0.6 and 1.4 for rich households, depending on the product. The elasticities are all decreased by 0.2 for the 'low values' compared to the 'middle values' and increased by 0.6 for the 'high values' compared to the 'middle values'.

Results for the global positioning scenario indicate that the differences in the magnitudes of the changes are negligibly small when comparing the results with the low, middle and high sets of income elasticities for poor and rich households respectively. Also, there are no sign changes in any of the results for the different sets of income elasticities. The model results therefore appear to be largely unaffected by the level of the income elasticities within the mentioned boundary values.

Results for the technical efficiency scenario are most affected by different sets of income elasticities for variables related to value added (*PVA* and *QVA*), intermediate inputs (*QINT*), factor demands (*FDF* and *FDA*), wage rates (*WF*) and income (*YH* and *YF*). The results show a few sign changes and differences in magnitude, especially for the agricultural industries. Figure 65 serves as example of the results for the technical efficiency scenario. Factor demands for Western Cape agriculture (horticulture >50%) show a sign change for different levels of income elasticities, whereas the other agricultural and food industries show relatively larger differences compared to non-agricultural and non-food industries. Greater income elasticities imply that a greater proportion of income can be spent on agricultural and food products as incomes increase. There is therefore relatively greater demand for agricultural industries, leading to higher levels of production and demand for factors when income elasticities are higher, compared to the non-agricultural industries.

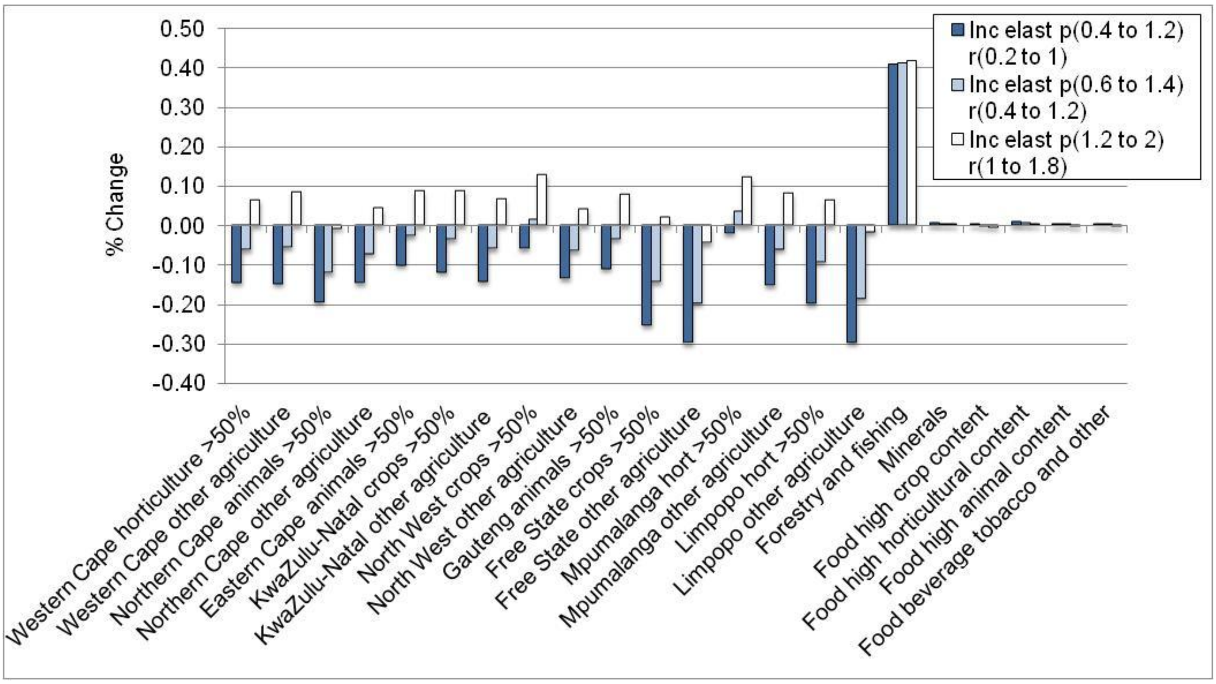
Figure 65: Technical efficiency: change in factor demand (*FDA*)



Results for the productivity growth scenario show no sign changes in any of the results for the different sets of income elasticities. The levels of income elasticities impact most on the agricultural and food sectors. Higher income elasticities lead to greater changes for most variables. This result is achieved because greater income elasticity implies a greater share of expenditure on food consumption, which includes both primary agricultural and processed food products. The income elasticity therefore impacts along the same principle as for the technical efficiency scenario, therefore no figures are presented.

Results for the education scenario show differences for different sets of income elasticities for most of the price variables. There is limited impact on quantity and income results. Figure 66 shows the different results obtained for changes in price of value added (*PVA*) for the different sets of income elasticities. Results are only noteworthy for the agricultural and forestry (and fishing) industries. This is because the results are driven by the changes in the rate of returns to land and labour (*WF*). The higher levels of income elasticities causes a relatively greater demand for agricultural and food products and hence increases in agricultural and food production. The increase in agricultural production leads to a relatively larger increase in the rate of returns to land and because more of the additional skilled labour in the economy can be employed there is also a smaller decrease in wage rates of skilled agricultural labour compared to low levels of income elasticities. The net effect of the upward pressure on wage rates for both land and skilled labour causes the price of value added (*PVA*) for agricultural industries to move from a decrease to an increase if the demand for agricultural and food products increase sufficiently when income elasticities are high. The increase in price of value added for the forestry industry is the result of the increase in the rate of returns to land in the absence of a decrease in the wage rate of skilled non-agricultural labour because the non-agricultural sectors can expand when the number of skilled labour increases because capital is not assumed to be fully employed.

Figure 66: Education: change in price of value added (*PVA*)



The impact of changes in the level of the income elasticities on the results for the share of each product in each industry's output (*IOQXACQXV*) is negligible for the global positioning and technical change scenarios. For the productivity growth and education scenarios the variation in changes in shares becomes smaller as the income elasticities increase. Prices in the productivity growth and education scenarios are affected relatively more by changes in income elasticities compared to the other two scenarios. The outcome, however, is that for higher income elasticities the changes in prices of different products are more similar in magnitude compared to lower income elasticities; hence smaller changes in the output composition are obtained.

LES Frisch parameter

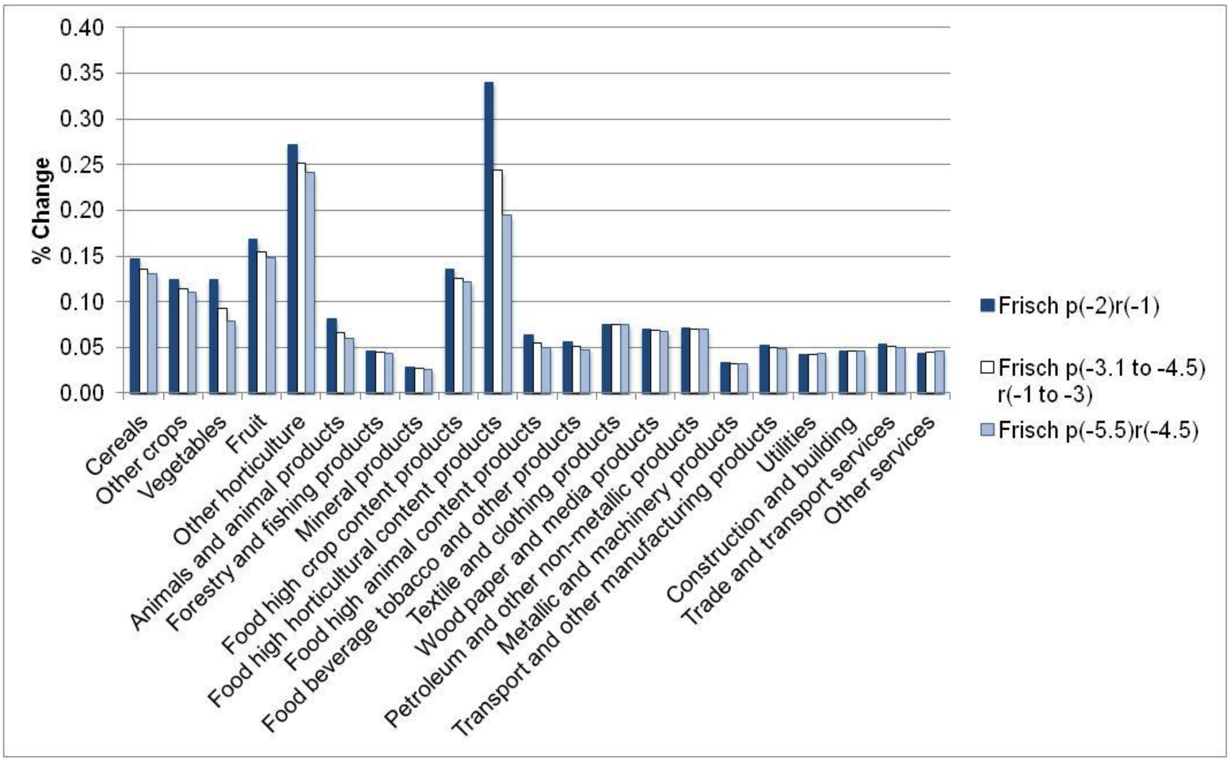
The second set of parameters for modelling consumption is the Frisch parameter for the LES function. The values used in the model range between -3.1 and -4.5 for poor households and between -1 and -3 for rich households. The 'low values' assumed here are

-2 for all poor households and -1 for all rich households; and the 'high values' assume the top of the range in absolute terms, i.e. -5.5 for all the poor households and -4.5 for all the rich households.

Results for the global positioning, productivity growth and education scenarios indicate that the differences in the magnitudes of the changes are relatively small when comparing the results with the low, middle and high sets of Frisch values for poor and rich households respectively. The model results therefore appear to be largely unaffected by the detailed level of the Frisch parameter within the mentioned boundary values. Higher (absolute) Frisch values imply greater subsistence expenditure by households for the same level of total household income.

Results for the technical efficiency scenario are most affected by different sets of Frisch values for variables related to value added (*PVA* and *QVA*), intermediate inputs (*QINT*), factor demands (*FDF* and *FDA*), wage rates (*WF*) and income (*YH* and *YF*) and expenditure (*HEXP*). It is mostly the agricultural industries that are affected by the different Frisch values because the Frisch values affect the demand for agricultural and food products directly. Figure 67 shows the results for the technical efficiency scenario for the supply of composite products (*QQ*). For higher Frisch values the demand for agricultural and food products are lower compared to lower Frisch values. This is because food already comprises a greater proportion of subsistence expenditure.

Figure 67: Technical change: change in composite supply (*QQ*)



The impact of changes in the level of the Frisch parameter on the results for the share of each product in each industry's output (*IOQXACQXV*) is negligible for the global positioning, technical change and education scenarios. For the productivity growth scenario there are small changes in shares as the Frisch parameter increases.

7.4 Results: Model closure sensitivity analyses

7.4.1 Closures for comparison

The macro closure rules that were discussed in section 6.3.3, referred to as the base closures, are changed in this section to illustrate the sensitivity of model results to different closures. The base and new closures are shown in Table 32. It is recognised that different combinations of closures could have very different impacts on the results, but only one closure is changed at a time in order to isolate the impact of a specific closure and only selected results are presented. All results are generated with the adjusted model, using the middle values of the parameters as shown in Table 31.

Table 32: Compared closures for the adjusted CGE model

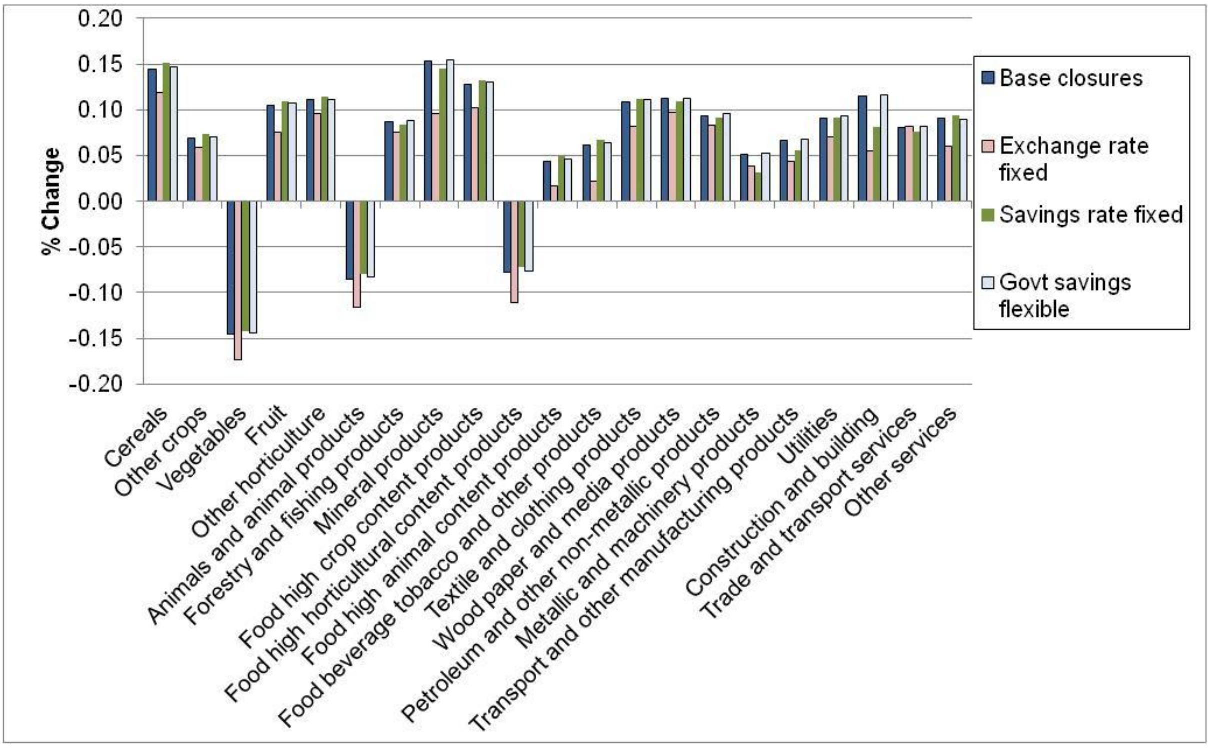
Accounts	Variables	Base closures	New closures
Current account	Exchange rate	Flexible	Fixed
	External balance	Fixed	Flexible
Investment-savings	Value of investment as share of total final domestic demand	Fixed	Flexible
	Savings rate of households and corporations	Flexible	Fixed
Government	Government consumption as a share of final demand	Flexible	Fixed
	Government savings	Fixed	Flexible
Factors of production	Capital	Unemployed and mobile	Fully employed and mobile
	Land	Fully employed, industry specific	Fully employed and mobile
	Labour	Only skilled is fully employed, all are mobile	Fully employed and mobile

7.4.2 Current account, investments – savings and government accounts

This section reports scenario results when using different closures for the macro accounts, namely the current account or external balance account, the investment-savings account and the government account. In each instance the closure of only one account is changed compared to the closures that were used to generate the main results in Chapter 6. Selected results for all four scenarios are reported. In the discussion the results generated with the base model are compared to results generated with each of the new closures respectively.

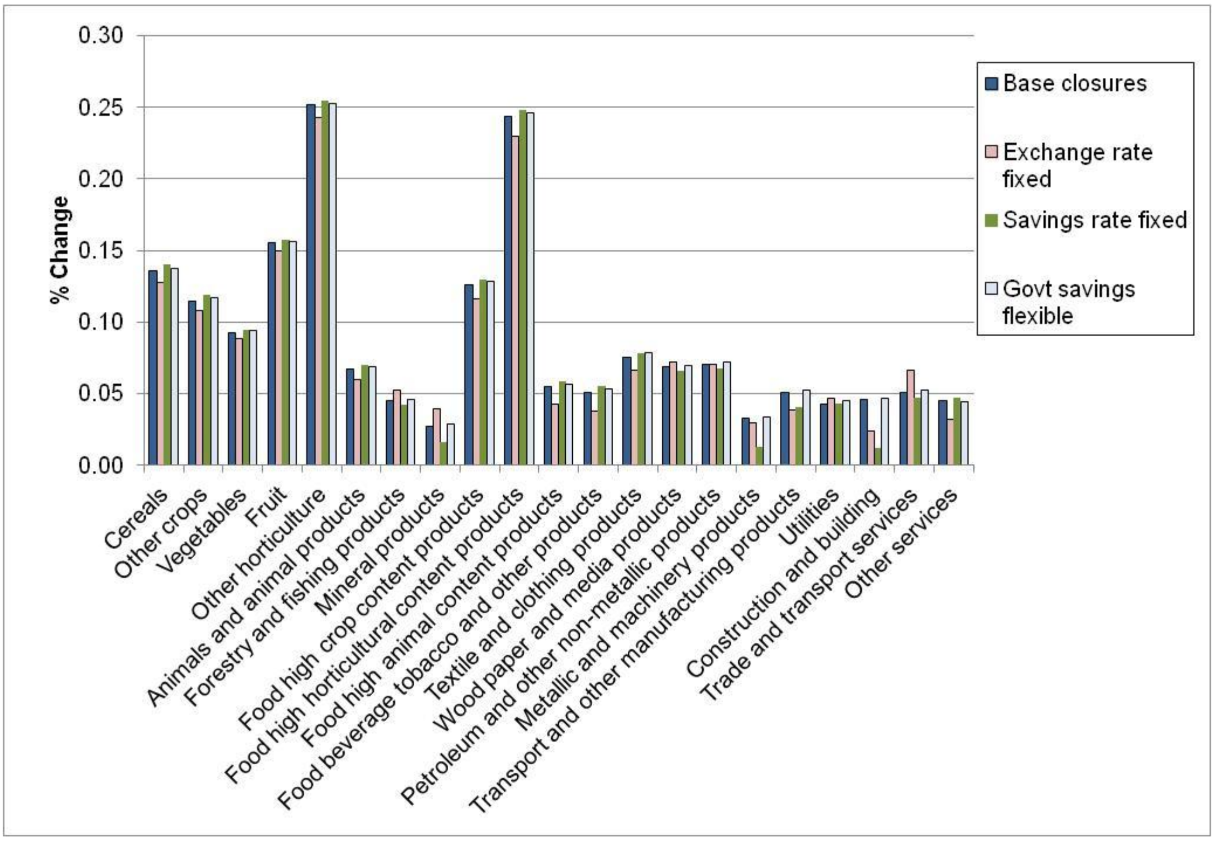
In general for the global positioning scenario the results are not greatly affected for changes in the macro closures. The greatest impact on results is found when the exchange rate closure is changed. This is because the scenario assumes world export price increases, so the direct impact is an appreciation of the exchange rate, which then impacts on imports of all products. When the exchange rate is assumed to be fixed, instead of flexible, the changes in imports (*QM*) are most affected as shown in Figure 68. Since the global positioning scenario under the assumption of a flexible exchange rate causes an appreciation, the fixed exchange rate implies a weaker Rand against foreign currencies, when the exchange rate is fixed. The result is that there is a smaller increase and a greater decrease in imports for all products compared to when the base closure of a flexible exchange rate is assumed because imports are now more expensive relative to domestically produced goods.

Figure 68: Global positioning: change in quantity of imports (*QM*)



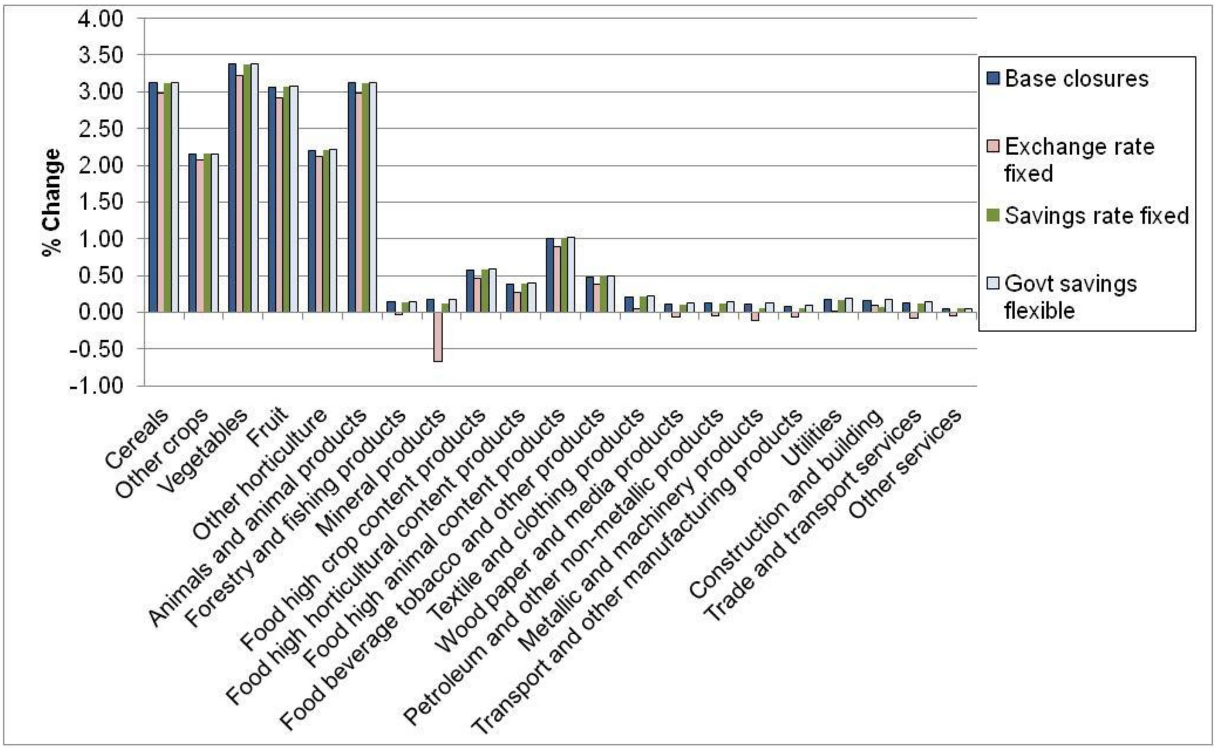
For the global positioning scenario the changes in composite supply (*QQ*), i.e. imports and domestically produced goods combined, are affected most by the changes in the investment-savings closure. Figure 69 shows that capital goods, i.e. metallic and machinery products and construction and building services, show smaller increases in composite supply when the value share of investment in total final demand is flexible compared to when it is fixed. This is because there is a decrease in investment share when it is flexible and the savings rate of households and corporations are fixed.

Figure 69: Global positioning: change in composite supply (*QQ*)



In general for the technical efficiency scenario the greatest differences in results are obtained when the exchange rate is assumed to be fixed compared to when it is assumed to be flexible. This is because the scenario assumes that agricultural industries are becoming more competitive compared to the international counterparts. Changes in exports (*QE*) are thus influenced by the exchange rate closure as can be seen in Figure 70. The greatest impact is on the mining industry because it is one of the main exporters.

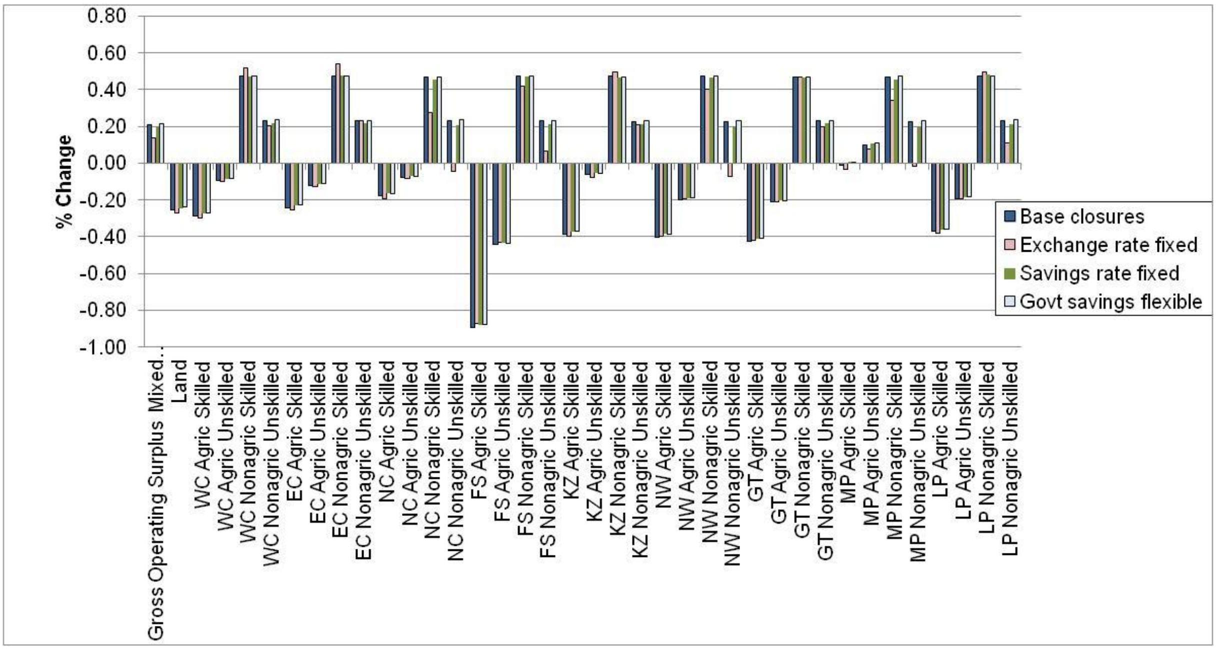
Figure 70: Technical efficiency: change in quantity of exports (*QE*)



Because of the impact of the exchange rate closure on exports, it implies that it also affects factor demands (*FD*), wage rates (*WF*) and factor incomes (*YF*) which can be seen in the detailed results. Only the results for factor incomes are shown here. Figure 71 indicates that those non-agricultural unskilled labour in provinces that engage substantially in mining activities (i.e. North West, Northern Cape, Free State, Mpumalanga and Limpopo) receive lower incomes when the exchange rate is fixed and mining exports decline, compared to when the exchange rate is flexible.

Results for the technical efficiency scenario show similar trends for the composite product (*QQ*) as with the global positioning scenario insofar as the capital goods, i.e. metallic and machinery products and construction and building services show smaller increases in composite supply when the value share of investment in total final demand is flexible compared to when it is fixed (not shown here).

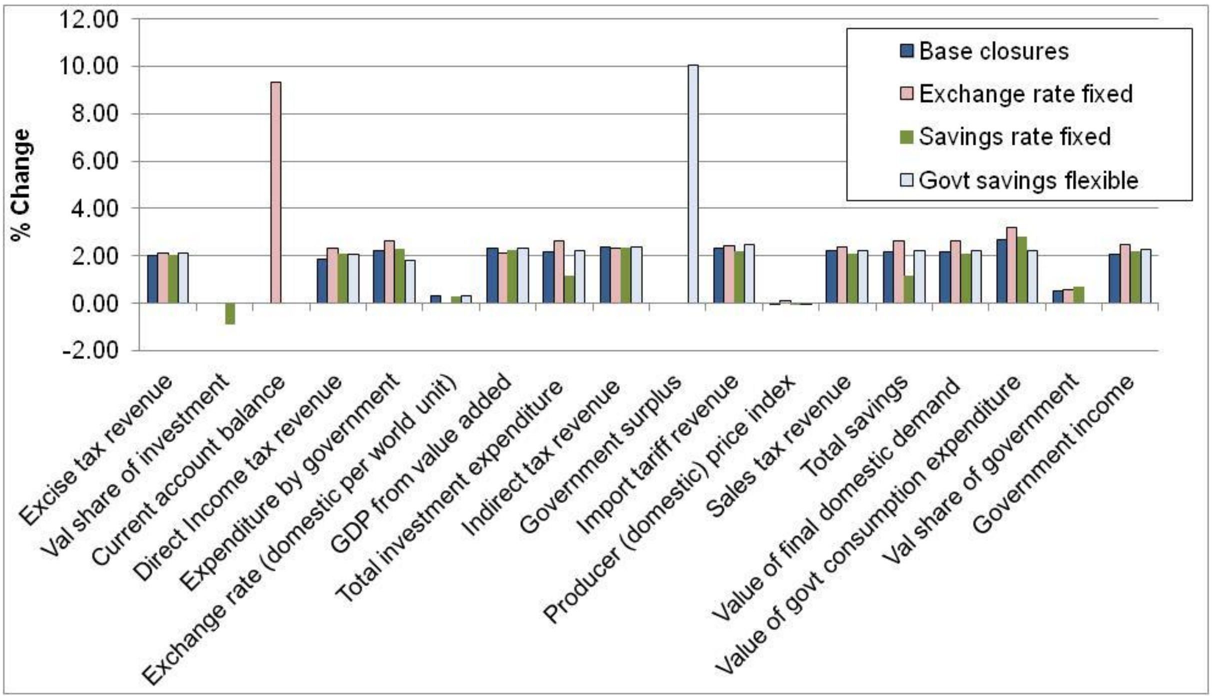
Figure 71: Technical efficiency: change in factor incomes (*YF*)



In general for the productivity growth and education scenarios it is found that the changes in macro account closures have limited impact on the results. The main impact is through the exchange rate and hence the mineral sector is affected, as well as the non-agricultural factors (and households) in North West province, where mining industries are predominant. The outcomes for these two scenarios are similar in nature to that for the previous two scenarios and the results are not shown here. For the education scenario the exchange rate and other macro closures have a smaller impact on results compared to the other three scenarios.

Figure 72 shows the impact on macro variables of changing the current account closure, the investment-savings closure and the government closure respectively, compared the base closure. Results are for the productivity scenario which shows slightly greater changes in macro variables compared to the other scenarios. There are no notable differences between the results for the base closure and the investment-savings closure, except for total savings and total investment expenditure. The results for the change in the current account closure are more notable but differ very little from the results for the base closure. The differences between the results for the base closure and the government closure are limited.

Figure 72: Productivity growth: change in macro indicators



The changes in the output composition (*IOQXACQXV*) of agricultural industries are not greatly affected by different macro closures for the two scenarios that impact directly on either agricultural products or agricultural industries, i.e. the global positioning and technical efficiency scenarios. Different macro closures do however affect the changes in the output composition (*IOQXACQXV*) of agricultural industries for the productivity growth and education scenarios because the impacts on relative agricultural product prices are not primarily driven by the scenarios, but by the combined effects on all products and industries from the agricultural and non-agricultural sectors. Results for the technical efficiency and education scenarios are shown for the different macro account closures in this section and results for the global positioning and productivity growth scenarios are shown for the different factor account closures in the next section.

Figure 73 shows the changes in the output composition (*IOQXACQXV*) of the Western Cape (horticulture >50%) for the technical efficiency scenario. Similar results are obtained for the other agricultural industries in the sense that the impact of changing the macro closures has a negligible impact on this variable. The impact for the global positioning scenario is even smaller and results are therefore not shown here.

Figure 73: Technical efficiency: change in output composition (*IOQXACQXV*)

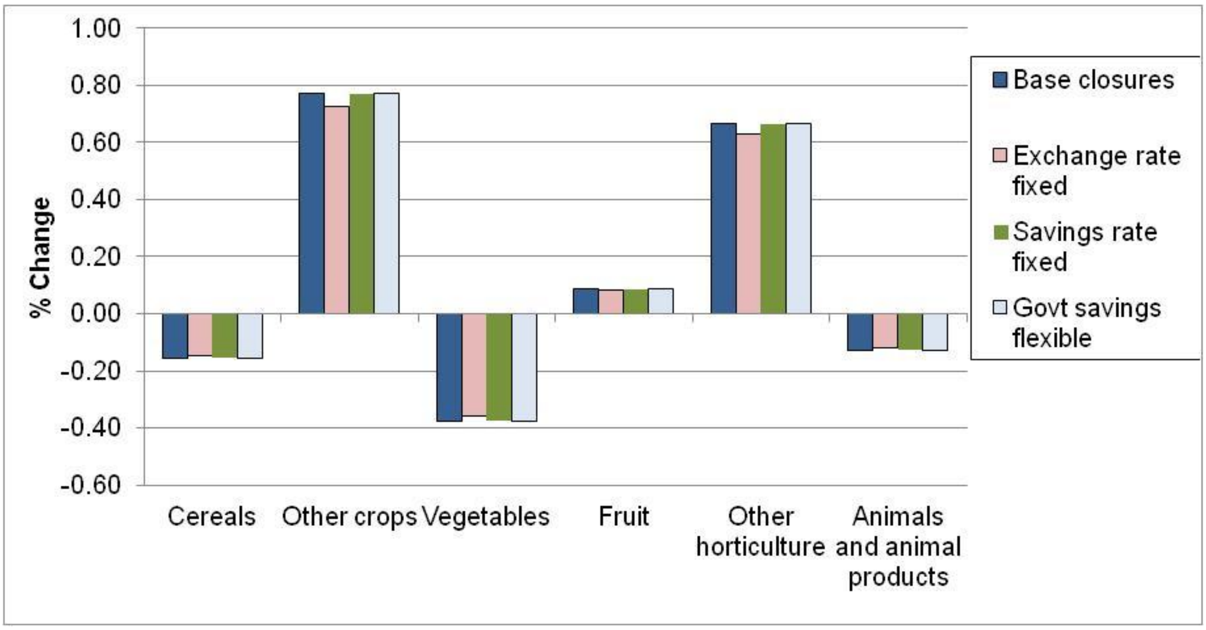
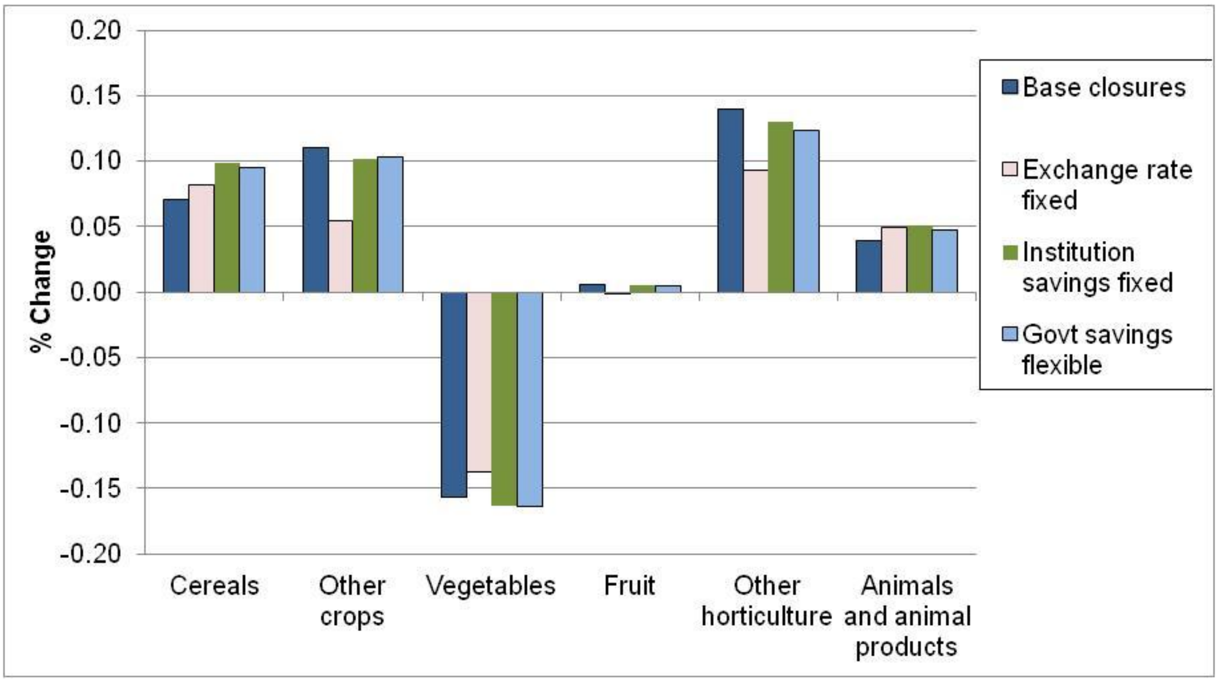


Figure 74 shows the changes in the output composition (*IOQXACQXV*) of the Western Cape (horticulture >50%) for the education scenario. Similar results are obtained for the other agricultural industries in the sense that the impact of changing the macro closures has a notable impact on this variable, especially when the exchange rate is assumed to be fixed.

Figure 74: Education: change in output composition (*IOQXACQXV*)



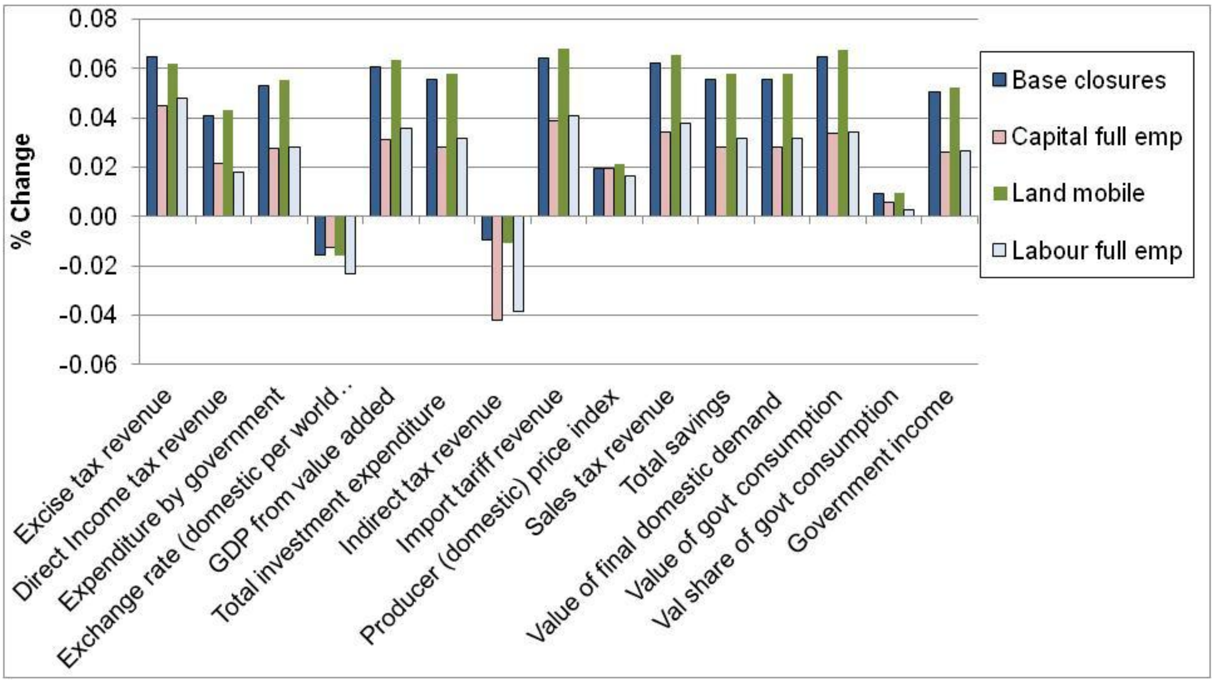
7.4.3 Factor accounts

Changes in the factor account closures have a more pronounced effect on detailed results than changes in the macro account closures. The basic neo classical closure of full employment and mobility of capital, land and labour respectively are each in turn compared to the factor account closures used in the previous chapter.

In the base closure capital is assumed to be not fully employed, but mobile. In the new closure capital is assumed to be fully employed and mobile. Both closures indicate a long term view where capital can be pulled out of one sector and invested in another within the adjustment timeframe. In the base closure land is assumed to be fully employed but immobile (fixed in each of the agricultural industries). In the land closure land is no longer immobile but mobile, which is not realistic given the fact that the agricultural industries are determined by provincial boundaries. In Chapter 6 only skilled labour is assumed to be fully employed whereas unskilled labour is assumed to experience some extent of unemployment. In this section all labour is assumed to be fully employed and mobile.

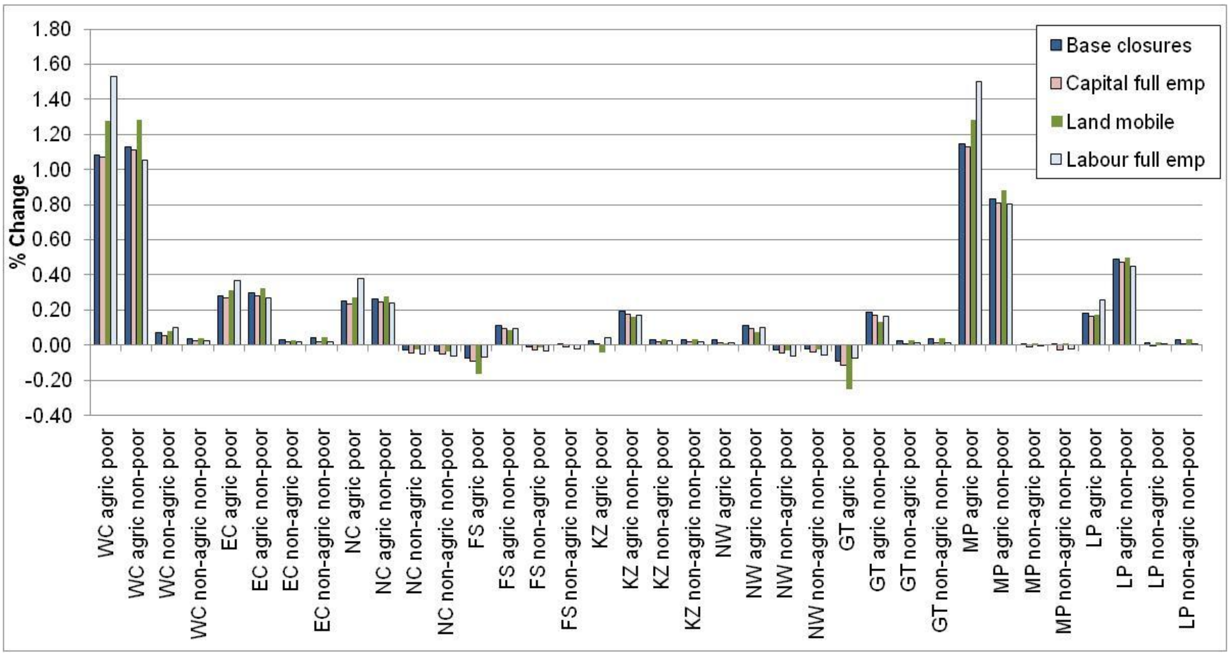
For all four scenarios it was found that changes in macro variables are not greatly affected by whether land is mobile or immobile. The changes in macro variables are generally smaller when capital and labour are assumed to be fully employed compared to when it is assumed to be not fully employed. As an example the changes in macro variables are shown for the global positioning scenario (Figure 75), but there is a similar trend for all four scenarios although the magnitudes of the changes differ. When capital and labour are assumed to be fully employed there is upward pressure on wage rates that acts as a constraint on economic expansion, hence the smaller increases in the macro variables.

Figure 75: Global positioning: change in macro indicators



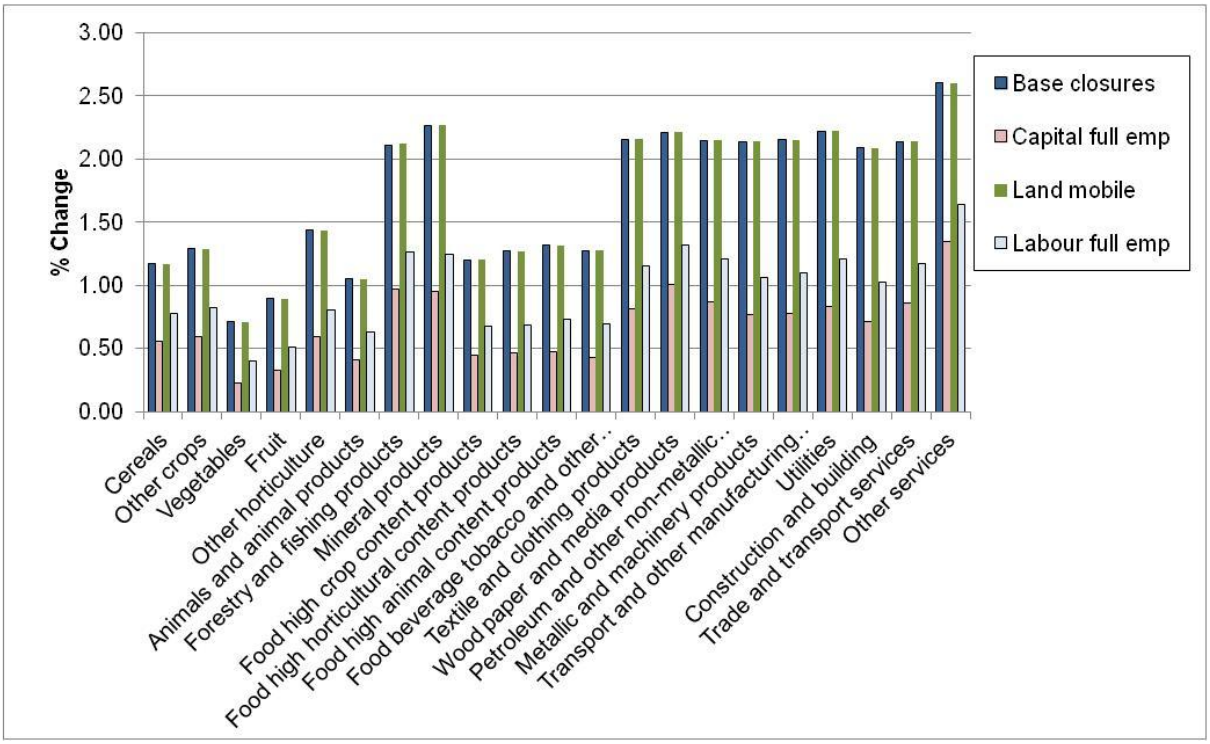
When looking at the more detailed results for the global positioning scenario, it was found that under the assumption of full employment the factor incomes (*YF*) for agricultural unskilled workers are higher compared to the assumption of unemployment for unskilled workers. This implies that the increase in the wage (full employment assumption) rate adds more to factor incomes than additional employment (unemployment assumption). The higher factor incomes for unskilled agricultural workers with the full employment closure translate into higher household incomes (*YH*) and expenditure (*HEXP*) for poor household working in the agricultural sector. This is especially so for the main fruit producing areas such as the Western Cape and Mpumalanga, which experience relatively greater expansion than other industries, as shown in Figure 76.

Figure 76: Global positioning: change in household incomes (*YH*)



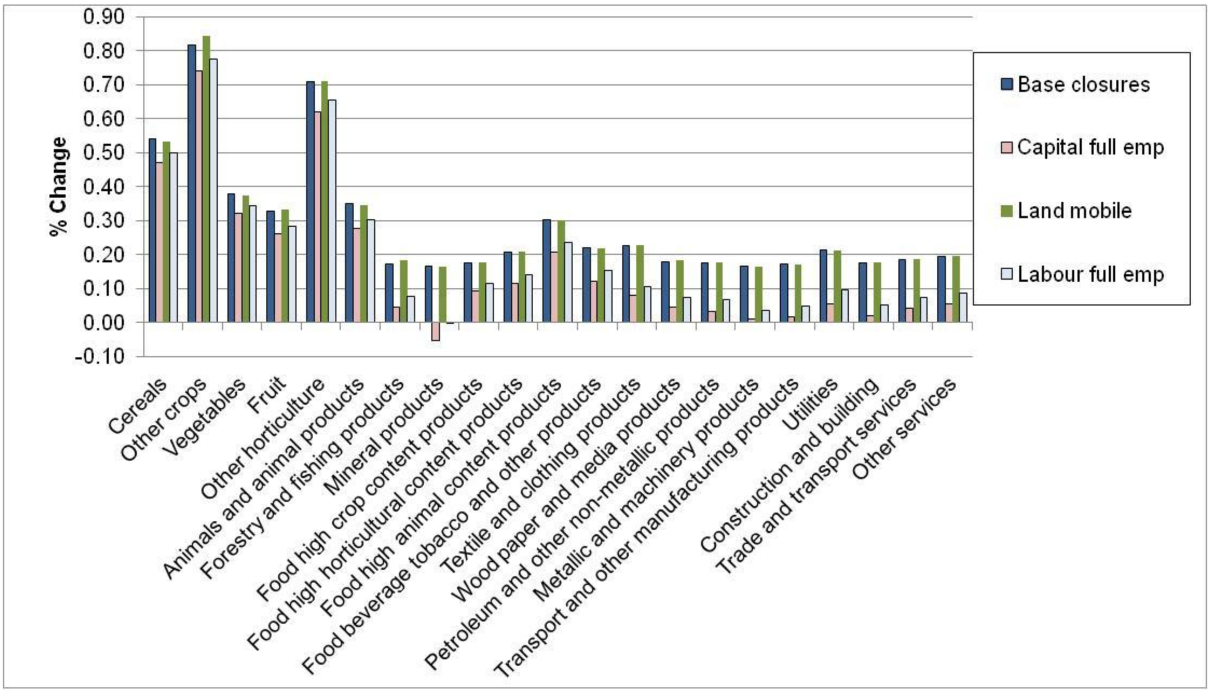
For the global positioning scenario the assumptions of full employment for capital and unskilled labour have similar dampening effects on agriculture and non-agriculture compared to the base closures for most of the quantity variables. Figure 77, which shows the changes in composite product supply (*QQ*) for different closures, serves as an example.

Figure 77: Global positioning: change in composite supply (*QQ*)



For the technical efficiency scenario the labour and capital closures have a notable impact on results pertaining to wage rates (*WF*), factor demand and supply (*FDA*, *FDF* and *FS*), and factor and household incomes and expenditure (*YF*, *YH* and *HEXP*). Results for the price of value added (*PVA*) show some differences for different closures, but the impact on other prices are relatively insignificant. Quantity variables that are most affected are domestic supply (*QD*) and exports (*QE*), but generally only for the agricultural industries. The impact on non-agricultural industries is relatively small. Results for intermediate input use (*QINT*) and domestic supply (*QXC*) are comparable for the base and land closures, whereas results for value added (*QVA*) and output (*QX*) are comparable for the labour and capital closures. Figure 78 shows the change in domestic demand (*QD*) for different closures. When land is assumed to be mobile the results do not differ much compared to when land is assumed to be immobile (base closures). The increases in domestic demand when capital is assumed to be fully employed and when labour is assumed to be fully employed are smaller compared to those obtained with the base closures. Under the assumption of full employment for capital and labour respectively, the increase in wage rates acts as a constraint on expansion. The results for mineral products are driven by differences in the change in exchange rate for the different closures.

Figure 78: Technical efficiency: change in domestic demand (*QD*)

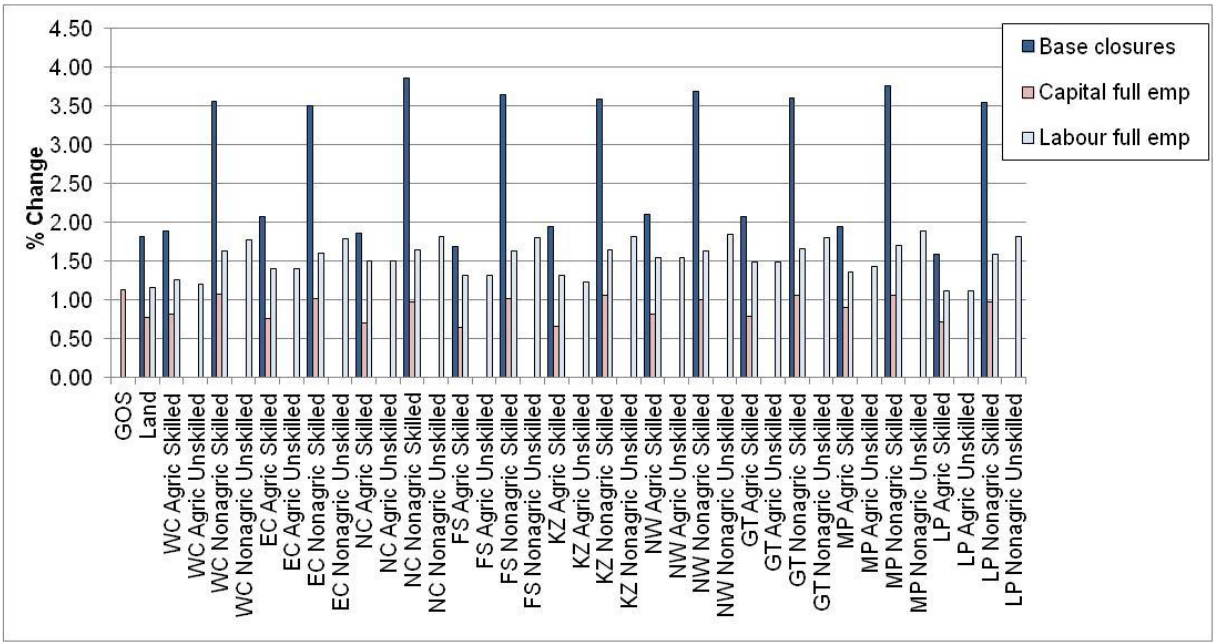


For the detailed results the assumption of mobile land provides similar results to that obtained with the closure of immobile land for all variables in the productivity and education scenarios, i.e. the two scenarios which do not impact directly on agricultural products or industries.

For the productivity growth scenario the labour and capital closures have a notable impact on results pertaining to wage rates (*WF*), factor demand and supply (*FDA*, *FDF* and *FS*), and factor and household incomes (*YF*, *YH*) and household expenditure (*HEXP*). Price changes obtained when capital is assumed to be fully employed are larger for the price of value added (*PVA*), the output price (*PX*) and the supply price (*PXC*). For all the quantity variables the increases are smaller under the assumptions of full employment for capital and labour. Capital appears to be a more scarce resource, because when capital is fully employed the increases in quantity variables are even smaller compared to when unskilled labour is assumed to be fully employed.

Figure 79 shows the differences in changes in wage rates (*WF*) for the productivity growth scenario for the different factor closures. The results for the mobile land closures are omitted because they are near identical to that of the base closure. When capital is assumed to be fully employed the rate of returns to capital increases to 1.13%, because the expansion of the economy implies an increase in demand for capital but supply (*FS*) is fixed. For the other closures the rate of returns to capital is fixed and the demand for capital (*FD*) increases. When all labour is assumed to be fully employed the increase in the wage rate for non-agricultural labour is still larger than that of agricultural labour, but less so when compared to the base closure. In the base closure capital supply does not place a direct constraint on expansion, but land does. Thus the non-agricultural industries expand notably more than the agricultural industries, leading to a relatively larger increase in demand for non-agricultural labour, as indicated by the increases in their wage rates.

Figure 79: Productivity growth: change in wage rates (*WF*)



For the education scenario the increase in supply of skilled labour causes the economy to expand. There is therefore an increase in demand for both skilled and unskilled labour (and other factors). The additional supply of skilled labour causes the wage rate (*WF*) of skilled labour to decrease, but the assumption that unskilled labour is also fully employed causes the wage rates (*WF*) of unskilled labour to increase (see Figure 80). In the base closure the wage rates (*WF*) of unskilled labour were fixed because of the unemployment assumption for unskilled labour. The labour closure in the education scenario therefore causes sign changes for various factor, household and price variables. Results for quantity variables generally do not show sign changes and it follows the general pattern that changes are smaller for the labour and capital closures compared to the base and land closures.

Figure 80: Education: change in wage rates (*WF*)

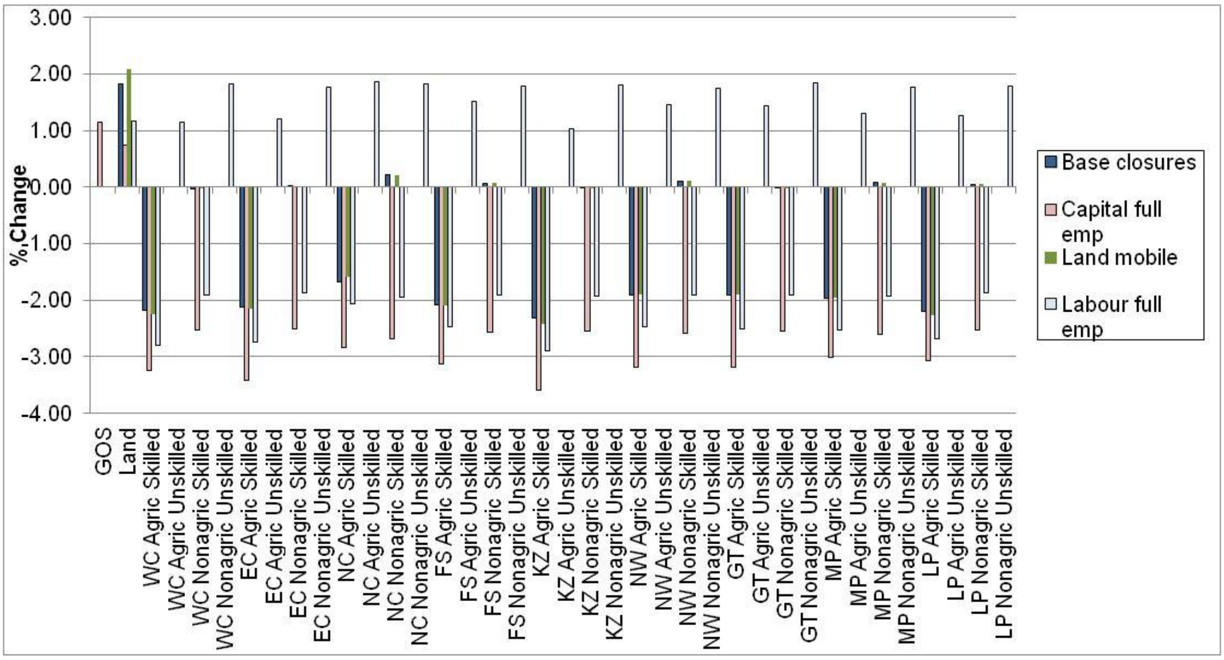


Figure 81 shows the changes in the output composition (*IOQXACQXV*) of the Western Cape (horticulture >50%) for the global positioning scenario. Similar results are obtained for the other agricultural industries in the sense that the impact of changing the macro closures has a negligible impact on this variable. When land is assumed to be mobile the rate of returns to land increases less compared to when land is industry (area) specific, and hence the main fruit producing areas, for example the Western Cape (horticulture >50%), show a smaller expansion in fruit production. The price increase of fruit relative to other products is therefore smaller and therefore even the agricultural industries that do not produce a large share of fruit show a smaller increase in fruit production then would be the case if land was immobile. The capital and labour full employment assumptions have little impact on the changes in composition of output. The impact for the technical efficiency scenario is even smaller and results are therefore not shown here.

Figure 81: Global positioning: change in output composition (*IOQXACQXV*)

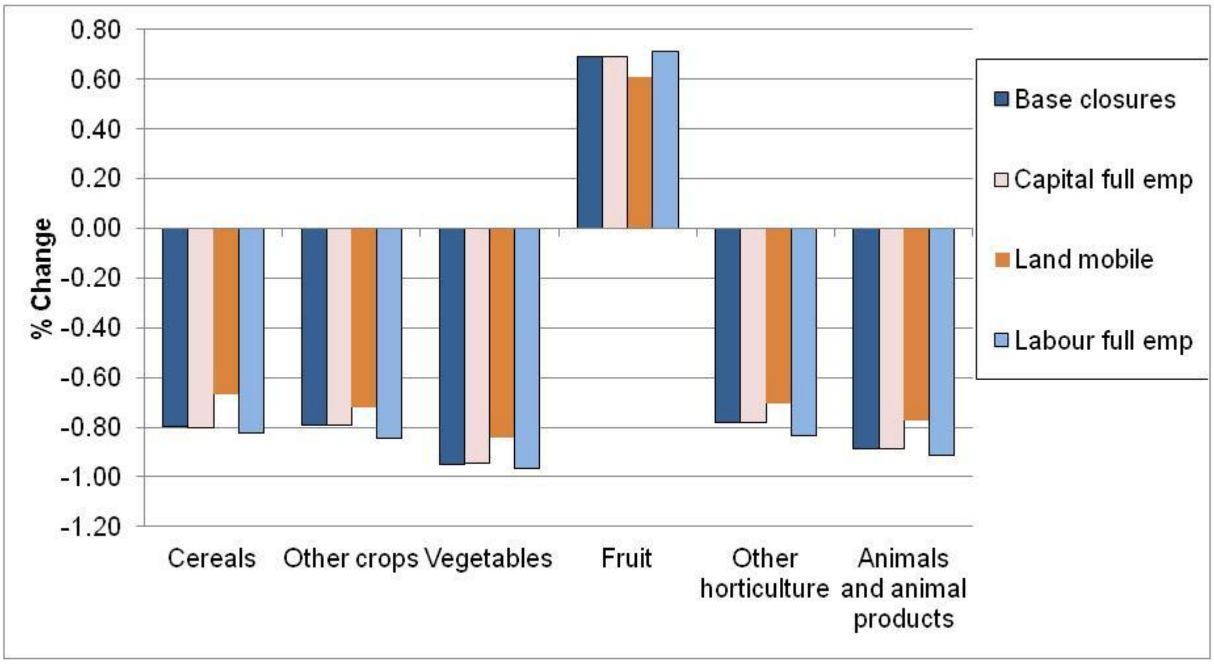
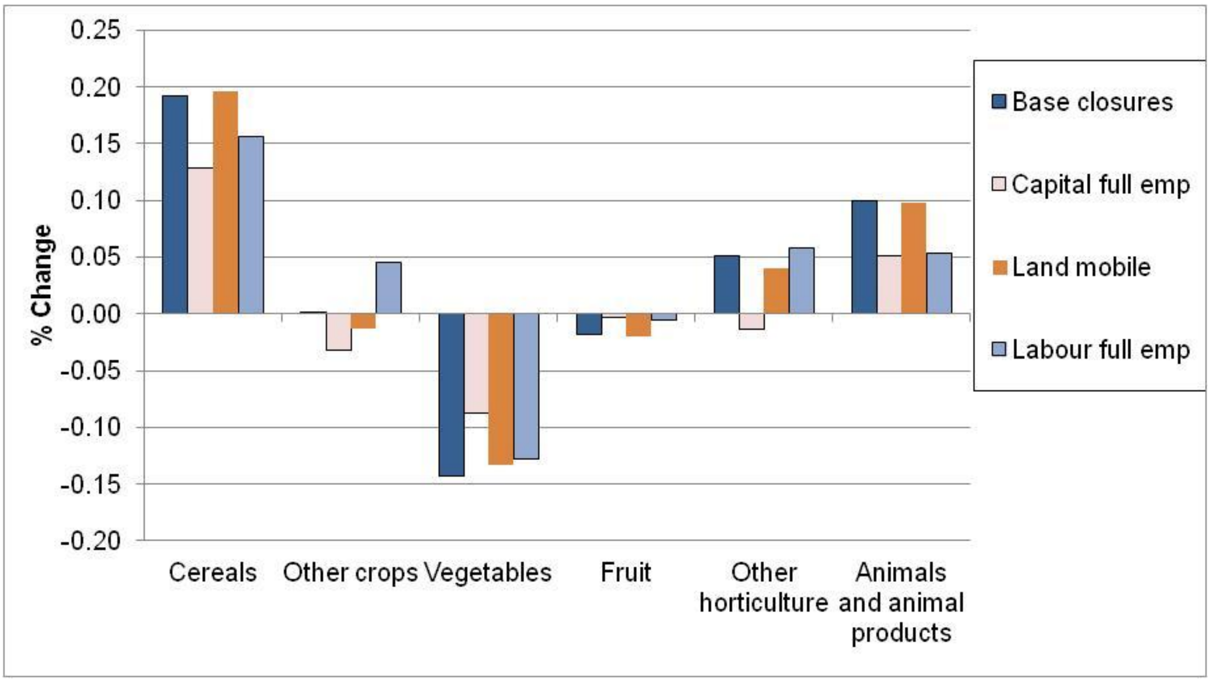


Figure 82 shows the changes in the output composition (*IOQXACQXV*) of the Western Cape (horticulture >50%) for the productivity growth scenario. Similar results are obtained for the other agricultural industries in the sense that different macro closures have a notable impact on this variable. The assumption about whether or not land is mobile has the least impact on the results. The results are fairly mixed because this scenario does not directly impact on a particular agricultural product or industry, so the relative price differences that drive this result are merely the net effect of the changes in the economy.

Figure 82: Productivity growth: change in output composition (*IOQXACQXV*)



7.5 Results: robustness of the adjusted model

Results were also generated separately for each of the four scenarios over a range of changes of the relevant variables. The aim was to establish whether there were no unexplained curvatures in the results over the range of changes and that the model behaved normally. Results are presented for the variables most affected by each of the scenarios.

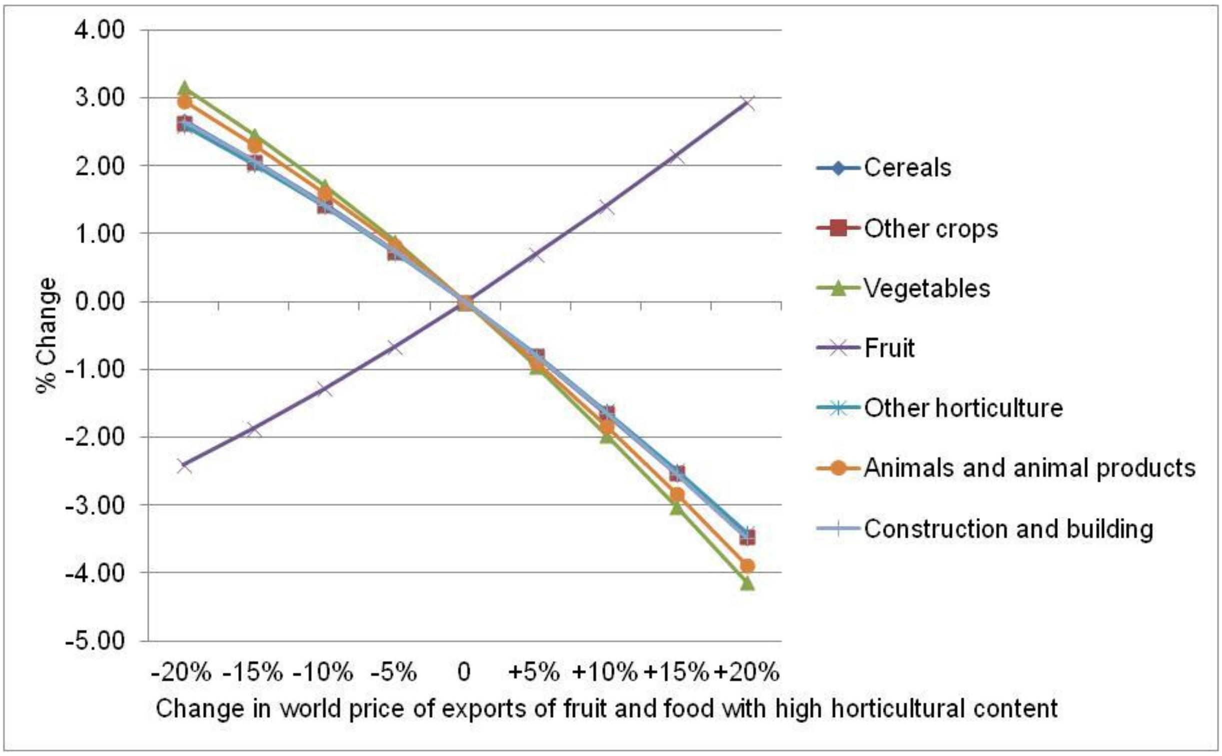
The model solves for quite wide ranges of changes in variables. The model solved when world export price (*pwe*) changes in the range of -85% to +450% were implemented simultaneously for fruit and food products with high horticultural content. Similarly the model solved for changes in the shift parameter of the top level CES function (*adx*) from -50% to +50%; changes in the shift parameter of the bottom level CES function (*adva*) for ‘other services’ from -45% to +450% and for skilled labour supply from -50% to +500%.

The detailed results with regard to these widest ranges are not reported here, because the greatest possible decreases and increases in the variables are not symmetrical and therefore results appear inconsistent when in fact they are not. The variable changes for each of the scenarios that were used in the range results are the following: -20% to +20% for fruit and simultaneously -8% to +8% for food products with high horticultural content for the global positioning scenario and -40% to +40% for the variables in the other three scenarios. Selected results over the range of changes are presented for each scenario in the next sections. The aim is *not* to present detailed numbers and explanations for individual products or industries, but rather to present a lot of information visually as confirmation that the results generated with the model behave normally. All the figures show all the products, industries or factors, but only the ones that show the greatest changes are included in the legends.

7.5.1 <u>Range results for the global positioning scenario</u>

Range results for the global positioning scenario presented no peculiarities, therefore only one result of interest is reported. Figure 83 shows the changes in the share that each product comprises in total output of an industry (*IOQXACQXV*). The results are shown for the Western Cape (horticulture >50%) only, but are similarly linear for other agricultural industries. These changes are the direct results of the model change to allow for price responsiveness in output composition. The results are according to expectation, i.e. if the world export price of fruit increases relative to that of other agricultural products, then the share of fruit production increases relative to that of other agricultural products.

Figure 83: Global positioning: change in output composition (*IOQXACQXV*)



7.5.2 <u>Range results for the technical efficiency scenario</u>

The majority of results for the technical efficiency scenario do not appear linear over the range of changes of the shift parameter of the top level CES function. As an example see Figure 84 which shows value added (*QVA*). The quantity of value added for Mpumalanga (horticulture >50%) increases by 30.20% for an increase in *adx* of 40%. It decreases by 1.9% for a decrease in *adx* of 10%, but again increases by 8.32% for a decrease in *adx* of 40%. The reason for the results not being linear is that the scenario is implemented over all of the agricultural industries at the same time, each of which is a multi-product industry. There are therefore numerous price and quantity effects that take place simultaneously and the final results are merely the net effects of all the changes taking place, which is difficult to disentangle. When the shift parameter is only implemented for one agricultural industry the results appear linear over the range. This is shown in Figure 85.

Figure 84: Technical efficiency: change in value added (*QVA*)

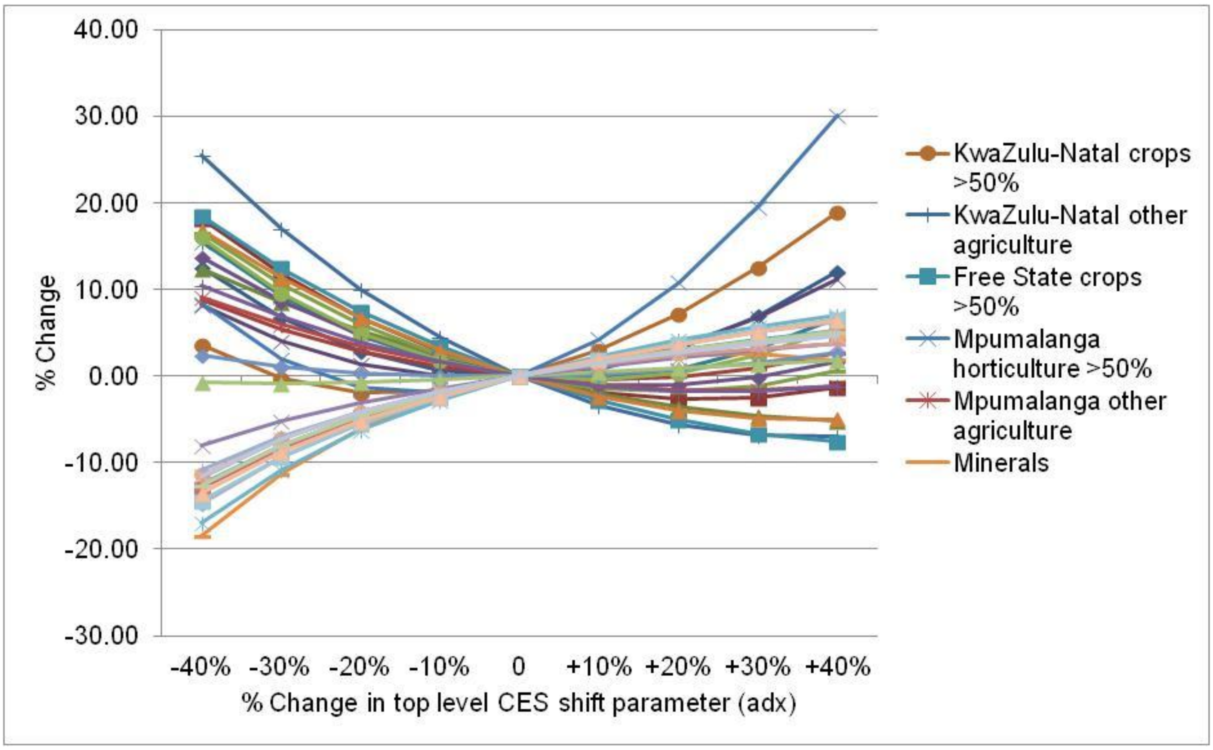
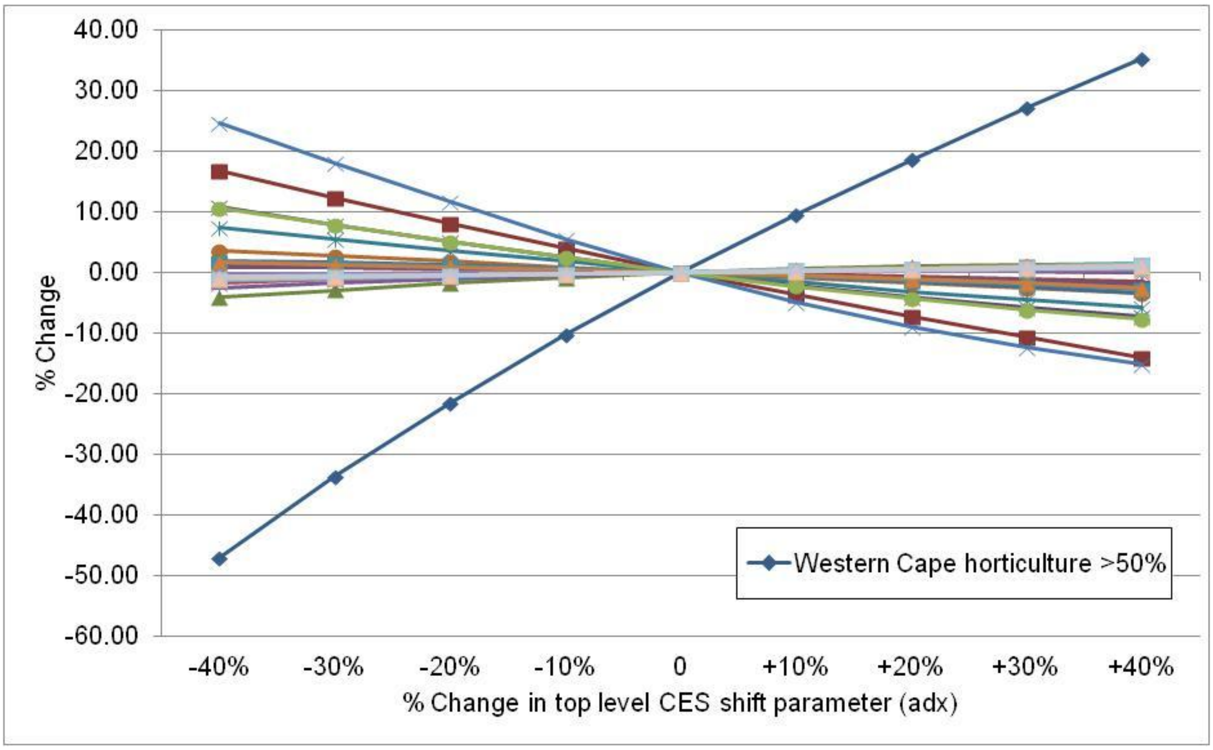


Figure 85 shows that when the shift parameter for only one industry, e.g. the Western Cape (horticulture >50%), is changed over the same range as before, then the results behave normally and are fairly linear over the range of changes in the shift parameter.

Figure 85: Technical efficiency for one industry only: change in value added (*QVA*)



7.5.3 Range results for the productivity growth scenario

All results for the productivity scenario appear to behave normally and are therefore not reported here.

7.5.4 Range results for the education scenario

With regard to the education scenario all the results for quantities appear near linear over the range of decreases (from -40%) and increases (up to +40%) in skilled labour and are not shown here. All the price results are generally non-linear, with the changes in the price of value added (*PVA*) in Figure 86 as an example. The price of value added is affected by the changes in the number of skilled labour and in turn, it affects all the wage rates and all other prices in the system, which then also show non-linear trends. A 40% decrease in skilled labour puts upward pressure on wage rates for skilled labour (all fully employed) and hence the price of value added increases. As the supply of skilled labour increases, the economy will be faced by other constraints; hence relatively smaller impacts are found when the number of skilled labour increases compared to when it decreases. The agricultural industries are affected more severely by the change in number of skilled labour than the non-agricultural industries. This is because agricultural industries have a restricted production factor (land fully employed) but non-agricultural industries do not (capital not fully employed). See Figure 87 when the closure is changed to assume capital is fully employed.

The relatively large change in the price of value added (*PVA*) of the ‘forestry and fishing’ industry in Figure 86 is the result of the change in the rate of returns (*WF*) to land, because land is assumed to be fully employed and industry specific. The additional skilled labour available to agriculture pushes up the price of land, and ‘forestry and fishing’ is the only non-agricultural industry that uses land. When the closure is changed to assume that land is mobile, then the change in price of value added for the ‘forestry and fishing’ industry is similar to that of other non-agricultural industries (not shown here).

Figure 86: Education: change in price of value added (*PVA*)

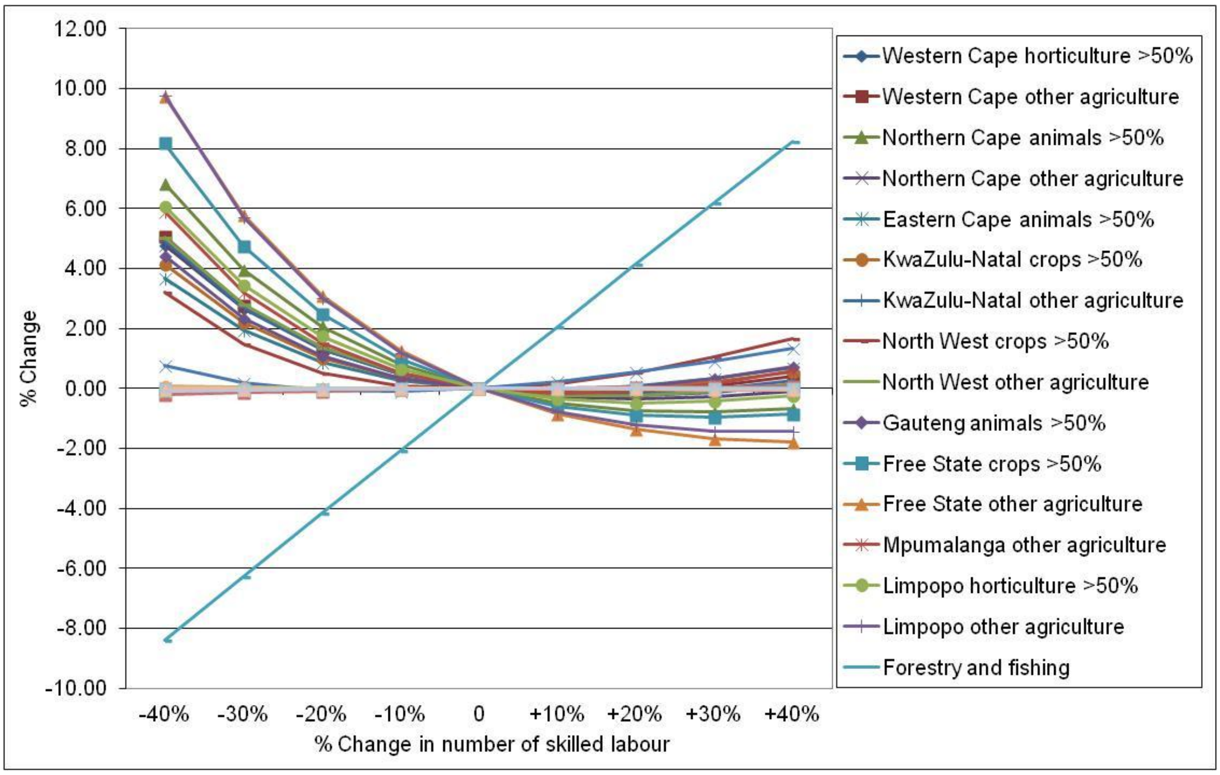
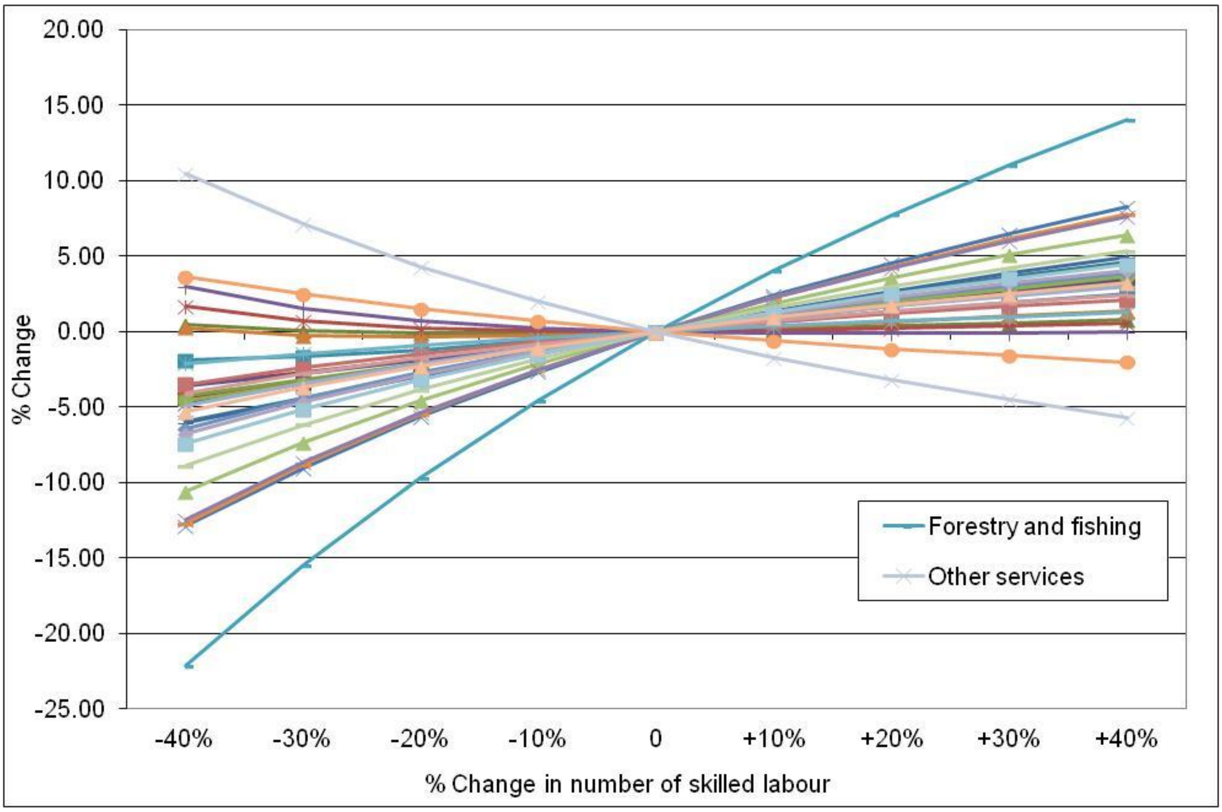


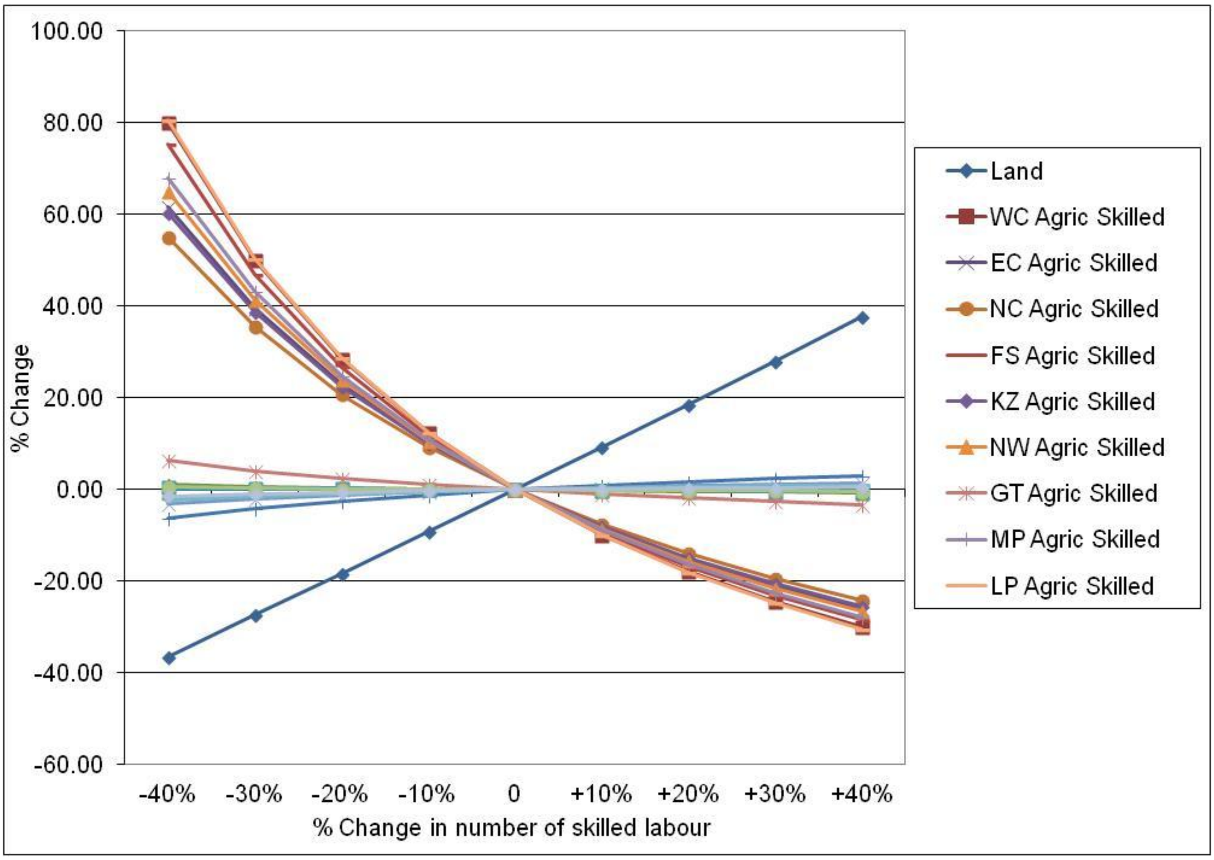
Figure 87 shows the price of value added (*PVA*) when the closure is changed to assume capital is fully employed. Under this new closure the changes in price of value added of agricultural and non-agricultural industries are comparable and not distinctly different as is the case in Figure 86.

Figure 87: Education: change in price of value added (*PVA*) – capital fully employed



The changes in price of value added (*PVA*) in Figure 86 are mainly driven by the changes in wage rates reflected in Figure 88. As the supply of skilled labour decreases, there is upward pressure on wage rates. The upward pressure generally causes an increase in the price of value added. This leads to lower levels of production, i.e. a decrease in production (*QX*) (not shown here), which leads to a decrease in the rate of returns to land shown in Figure 88. The lines that form a band around the horizontal axis are the wage rates for non-agricultural skilled labour, which are much less affected mainly because capital is assumed not to be fully employed.

Figure 88: Education: change in wage rates (*WF*)



7.6 Summary and conclusions

This chapter constitutes sensitivity analyses with regard to the model adjustment as well as elasticity parameters and model closures. The scenarios that were used are the same as in the previous chapter and relate to the global positioning of the fruit industry, technical efficiency gains of the agricultural industries, productivity growth of the government and services industries, and improved education that increases the number of skilled workers in the economy. The global positioning scenario is particularly useful to illustrate the impact of the change in the model's functional form for the determination of industry output composition because it impacts directly on relative product prices, which is the driver of the changes in the composition of industry output.

The first set of results comprises a comparison of the results from the adjusted model with the base model in terms of the functional form and value of the elasticity contained in the functional form. The adjusted model contains a CET first order condition to determine the transformation of aggregate industry output into different products. When producers are price responsive (adjusted model) it is found that for all the agricultural industries the volume shares of fruit products increase whereas the volume shares of crops and animal related products decline when world export prices of fruit and food products with high horticultural content increase relative to those of other products. This result is realistic compared to the results from the base model which indicate fixed output composition. The results are smaller and mixed for the other scenarios because none of the other scenarios directly impact on the price of a particular agricultural product. Amongst the production elasticities the elasticity of transformation for output composition of industries is of particular relevance in this study because it is the additional parameter added in the adjusted model. The output elasticity of transformation (*omega*) was set to 0, 0.9 and 2, where *omega* equal to 0 implies that the model reverts to the base model, i.e. fixed output composition. Higher values of *omega* show greater changes in output composition, as expected. Agricultural industries are therefore better able to increase production of the products for which they face higher prices. Even with the global positioning scenario the changes in output composition are not large, but this is realistic in the context of an assumption of input-output separability.

In the second set of comparisons, low, middle and high values for seven different elasticities or parameters are assumed and results generated with the different values of elasticities are compared for each of the individual elasticities for each scenario. The parameters for which sensitivity analyses were conducted include the two elasticities for the CES and CET trade functions, three elasticities related to CES production functions and the income elasticity and the Frisch parameter for the LES function for household consumption. Results confirmed that the outcomes of the sensitivity analyses are influenced by the combinations of elasticities and scenarios because each elasticity and scenario directly affects a different part of the price or quantity system. Every scenario impacts directly on a specific functional form and when the parameter value of that particular functional form is changed it generally has a relatively greater impact on results than when the value of any other parameter's value is changed. There were instances, however, where changes in some of the other elasticities also had quite a significant impact on the results, revealing the interdependence in the economy as captured by the CGE model.

The trade elasticities have a notable impact on the results of the global positioning and technical efficiency scenarios because both of these scenarios cause changes in the exchange rate, which influences trade. Compared to the other production elasticities the changes in the value of the elasticity of substitution at the second level of the production nest had the most significant impact on results for all four scenarios, especially for the productivity and education scenarios, which are not significantly influenced by changes in values of other elasticities. This is because the elasticity of substitution that impacts on the factors of production directly influences factor demands and hence wage rates and factor payments.

When wage rates change, there is an impact on production structures and prices. Although results are generally not particularly sensitive to the values of the Frisch parameter or income elasticities of the linear expenditure system within the range of values tested, it was found that the levels of income elasticities impact most on the agricultural and food sectors because greater income elasticities imply a greater share of expenditure on food consumption, which includes both primary agricultural and processed food products. The sensitivity analyses results were largely according to expectation and served to validate the choice of elasticity values used in the adjusted model.

For the scenario on global positioning (increase in world export prices), the model results are most sensitive to the choice of the elasticity of transformation (*omega*) of the CET export function. There are no sign changes in the results, except for quantities of imports, where the quantity of imports of directly affected products (fruit and food with high horticultural content) decreases for *omega* equal to 0.3, whereas it increases for *omega* equal to 4. Other changes are mostly in magnitude (as opposed to sign changes) and these changes are seen throughout the results. The average changes in composition of output also become greater for higher values of the elasticity of transformation between products destined for the export market as opposed to the domestic market. With regard to changes in the value of the (Armington) elasticity of substitution between imports and domestic goods the only noteworthy impact on results are for quantity of imports of the directly affected products. For a low value of *sigma* (0.2) imports of fruit and food products with high horticultural content show an increase, but for higher values of *sigma* (2 and 4) it shows a decrease, because with greater substitutability users can switch to domestically produced goods more readily. Changes in the values of elasticities of substitution at the first and second levels of the production nest impacts mostly on the magnitudes of the results, with limited sign changes. Changes in the value of the elasticity of substitution between similar products from different industries generally have limited impact on results. The only noteworthy impact for this elasticity is that it caused sign changes for imports and purchasers' price of the composite good of food products with high horticultural content, i.e. an increase for a low value of *sigma* (0.5) and a decrease for high values of *sigma* (4 and 6). Different levels of the elasticity of transformation to allow for changes in the output composition of industries lead to sign changes in results for some of the agricultural industries with relatively low shares of fruit production. This relates to factor demands, wage rates and factor and household incomes, intermediate demand and value added. Imports of fruit products and the resultant purchasers' prices also show sign changes for different levels of the elasticity of transformation. The results indicate that if industries cannot adjust their output composition, there would be greater import demand for fruit because the industries will not be able to increase output sufficiently to meet the additional export and domestic demand for fruit. Some of the industries with relatively low shares of fruit production would decrease industry output if they cannot adjust their output composition, whereas they would increase output if they can adjust their output composition in favour of the products facing higher export prices.

For the scenario on technical change for agricultural industries, the changes in the values of elasticities of substitution at the first and second levels of the production nest impact on the magnitudes of most of the results, but there are no noteworthy sign changes in results. The results were also sensitive to changes in the elasticity of transformation (*omega*) between products for the export market and the domestic market. Low versus high values of the elasticity of transformation caused sign changes on the results related to factor demand, and therefore on factor and household incomes and household expenditures. The average changes in composition of output also become greater for higher values of the elasticity of transformation between products for the export market and the domestic market. Different levels of the elasticity of transformation to allow for changes in the product composition of industries lead to sign changes in some of the agricultural industries for results on wage rates, factor demands and incomes and these differences are also visible in the results for price of value added and intermediate demand. Hardly any sign changes occur in the quantity results, but these are affected in terms of the magnitudes of the changes.

For the scenario on productivity growth for the ‘other services’ industry, the model results are most sensitive to the values of the elasticities of substitution at both levels of the production nest. Changes in the elasticity of substitution between the different factors of production at the bottom level of the production nest impact mostly on factor and quantity results. Price results were largely unaffected. Changes are mostly in magnitude, with few sign changes. For higher values of the elasticity of substitution between intermediate inputs and value added at the top level of the production nest, intermediate products are substituted for value added for the ‘other services’ industry and the opposite is true for other industries. This is because the ‘other services’ industry becomes more productive in its use of value added relative to intermediate inputs and would therefore want to use relatively more value added. Impacts of changes in the elasticity of substitution between intermediate inputs and value added at the top level of the production nest are limited to changes in magnitude. Different levels of the elasticity of transformation to allow for changes in the product composition of industries hardly have any impact on results from the productivity scenario.

The results of the education scenario (increasing number of skilled labour in the economy) are least sensitive to changes in parameter values compared to other scenarios. The model results for the education scenario are most sensitive to the choice of the elasticity of substitution between the different factors of production at the bottom level of the production nest. The main impacts are on wage rates and factor incomes of skilled agricultural workers, and hence on household incomes and household expenditures. A low *sigma* value (0.2) causes a relatively greater decrease in wage rates of skilled agricultural workers compared to higher values of *sigma* (0.5 and 1.5) because of the lower substitution possibilities between skilled labour and land. Factor incomes of skilled agricultural workers show a net decrease for a *sigma* value of 0.2, but factor incomes increase for *sigma* values of 1.5. Different levels of the elasticity of transformation to allow for changes in the product composition of industries lead to sign changes in the price of intermediate inputs for some of the agricultural and food industries, but had limited impact on the other results. For none of the other elasticities did changes in the values of the elasticities have noteworthy impacts on results of the education scenario.

In the third set of comparisons, the closures used in the case study in Chapter 6 are adjusted and the results achieved under different assumptions are compared. Changes in the closures of the investment-savings and government accounts have a limited impact on the results. Fixing of the exchange rate has a bigger impact because an exchange rate change feeds through the entire price system and directly or indirectly affects all the prices. It is however the changes in the factor account closures that give some interesting results because these closures impact directly on the production structure and wage rates. When capital is assumed to be fully employed it leads to an increase in the rate of returns to capital because there is economic expansion for all four scenarios. The fixed supply of capital serves as a constraint to economic expansion and generally the changes in all the variables for all four scenarios are smaller compared to when capital is assumed to be not fully employed. When land is assumed to be mobile the increase in the rate of returns to land is smaller compared to when land is assumed to be immobile. Although land mobility in the context of agricultural industries denoted by area is not a realistic assumption, the closure shows that there are very limited impacts on results, especially results for macro indicators. When land is mobile the wage rate increases relatively less because land can 'move' to the most productive industry, i.e. land is a relatively less scarce resource for expanding industries.

The last set of results aimed to establish whether the model is robust in the sense that it provides 'stable' results over a wide range of changes for a particular variable. It was found that for each of the scenarios the model solves for decreases of between 15% and 85% of the original value of the parameter changed as part of each scenario and for increases of between 50% and 450% of the original value. Results appear to behave normally over this range. At first glance the range results for the technical efficiency scenario appear irregular over the range but when the changes are implemented for only one agricultural industry at a time the results appear near linear and can be explained. The range results for the technical efficiency scenario therefore merely display the net effects of all these changes.

The conclusion reached at the end of the sensitivity analyses is that the chosen values for parameters for the CGE model and the chosen model closures provide a plausible set of results.

8 Summary, conclusions and recommendations

Industries that produce a number of products are expected to adjust their production patterns over time if there is a sustained change in relative prices of these products. Rational behaviour would be to increase production of those products that experience price increases, while production will be decreased of products that experience price decreases. This behaviour is not generally captured in computable general equilibrium (CGE) models. There are CGE models that assume that every industry only produces one product, i.e. the underlying SAM data display off-diagonal entries of zero for the supply matrices. In these instances industries would respond to price changes by increasing the level of the single output, but the fact that the models are calibrated with reduced form SAMs and the associated loss of information present a drawback. In cases where the models assume that industries are multi-product firms, the majority of CGE models still assume that industries will continue to produce products in the same ratios to each other as dictated by the base data in a particular year. This implies that if scenarios are run and relative prices of products change then the composition of output of industries will not adjust in line with relative price changes. In a CGE model the choice of the output transformation function determines the composition of output for each industry.

The main objective of the study was to develop a CGE model for South Africa in which the assumption of fixed composition of output can be selectively relaxed in order to enhance the quality of CGE model results. The first sub-objective of the study was to improve the specification of the output transformation function of multi-product industries to reflect more realistically the supply response to changes in relative prices for those industries that are price responsive in their output composition. The objective of the research was achieved through modifying an existing CGE model to incorporate a CET function and the related first order condition into the base model. Generally CGE models incorporate the Leontief assumption of fixed shares for output composition. The Leontief fixed shares option is still retained in the adjusted model. This allows the modeller a choice between the two functions for each of the industries.

It is the first order condition of the CGE function that determines the optimal combination of products produced by an industry, given the relative prices of the different products. In more technical terms, the first order condition ensures that the industry price of an individual product as a share of the (aggregate) industry price is equal to the derivative of the production function with respect to that particular product. It was found that the inclusion of the actual CET function had no impact on results because the level of aggregate industry output is already determined by the nested production structure in the CGE model. Only the first order condition therefore needs to be included in the model to determine the output composition.

Set controls were included to make the model more general in order for it to run with data from different countries. The set controls allow for the implementation of the CET first order condition for industries which are assumed to be price responsive in their output composition. The Leontief fixed share function is implemented for the other industries. If the underlying SAM data only reflects single product industries, then the set of industries using the CET function will be empty and the CET first order condition will not be activated. With the Leontief specification perverse results might be obtained if the secondary products produced by an industry comprise a large share of output. For example if a farmer that produces both sheep and maize is faced with an increase in the world export price of maize, then the expectation would be a marginal shift in production away from sheep towards maize. Under the Leontief assumption of fixed composition of output, the increase in the export price of maize will cause an increase in production of both maize and sheep in the same proportion, which is unrealistic. Therefore it is useful to use the CET function for output transformation in cases where secondary production comprises a large share of total output by an industry.

It was expected that if a secondary product comprises a relatively small proportion of industry output then unrealistic price changes for this product might be generated if flexible output composition is an option through the inclusion of the CET function. To prevent the use of the CET function if there are secondary products that comprise a relatively small share of industry output, a minimum cut-off value for the share of secondary production was introduced into the model to treat industries which have secondary production below this cut-off, as industries whose output composition will remain fixed. The SAM data that was used in this study preclude the sensible use of the cut-off because the potentially price responsive agricultural industries all have construction as an output that comprise about 1% of individual industry output. But these industries still have significantly large shares (e.g. between 20% and 50%) of other agricultural products that are produced. The cut-off mechanism was therefore included in the model, but the value was set below 1% in order to ensure that all agricultural industries are handled as price responsive industries. The price changes of construction by agricultural industries, which is the product with a relatively small share in agricultural industries, however did not show any of unrealistic price effects as anticipated. It is unclear whether this outcome is general or specific to the SAM for South Africa; hence the impact of the CET cut-off requires further analyses using different data sets.

This study followed a SAM approach to modelling insofar as the CGE model can be explained within the SAM framework, i.e. following the sub-matrices of the SAM. Also, industry and product prices are determined by information contained in the columns of the SAM. An implicit assumption of the SAM is that the law of one price holds, i.e. all buyers of a product pays the same price for that product, i.e. the same price is relevant across every row of the SAM. Literature reveals that CES aggregation functions adhere to the law of one price, but that the CET function only adheres to the law of one price under restrictive assumptions, which usually do not hold. When implementing the CET function to transform the domestically produced products into products for the domestic market and products for the export market the restrictive assumption that should hold is that products are sold at the same price on the domestic market and the export market.

The CET function to transform aggregate industry output into different products, of which the first order condition is implemented to allow for flexible output composition, does not adhere to the law of one price. The industry output price is determined by the input coefficients, but if the output composition changes it is reasonable to expect that the input composition will also be affected. No provision is made in the CGE model to adjust input composition and therefore an assumption of input-output separability was introduced. This remains a weakness of the model and an area for further research.

The second sub-objective of the study was to develop a detailed SAM for South Africa for 2007 with which to calibrate the adjusted CGE model. The SAM was developed using a wide range of South African national account and industry data. The generalised cross entropy method was used for the estimation of missing information in order to derive a consistent SAM. The SAM is a supply and use SAM, with different number of industries and products where the industries are multi-product industries. The full version of the SAM contains 543 accounts but the version used in the case study contains only 137 accounts in order to focus on the agricultural industries. The factor, household and agricultural accounts include provincial detail to allow for the tracing of income transmission from industries to factors to households, all on a provincial level. The developed SAM therefore is the only supply and use SAM for South Africa (excluding the PROVIDE project SAM, which is its dated precursor) that contains agricultural and provincial detail within a national context. This makes the SAM useful for detailed agricultural and income distribution analysis at a regional level. The SAM is also suitable to benefit from the model adjustments that were carried out because the SAM has a supply and use framework and contains multi-product industries.

The labour accounts in the 137 account version of the SAM are categorised by province of residence (one of nine provinces), main sector of work (agricultural or non-agricultural) and skill level (skilled or unskilled). Household accounts are grouped according to province of residence, main source of income (agriculture or non-agriculture) and income level (poor or non-poor). The SAM includes detailed agricultural accounts with six agricultural products and 16 agricultural industries identified. There are one or two agricultural industry accounts for every province, where the agricultural industry data is an aggregation of data from various district municipalities according to main products produced in each of the district municipalities. Agricultural industries classified by agronomic region are particularly good examples of multi-product firms that are price responsive in their output composition.

The SAM is based on the principles of national accounting and follows the System of National Accounts (SNA) for 1968 in terms of the layout of the SAM. The subsequent revisions of the SNA for 1993 and 2008 do not capture sufficiently the functional distribution of income because the mapping of factors to households, as the owners of these factors, is not fully articulated in the more recent revisions of the SNA. The SNA classifies the SAM as a satellite account because it requires detailed household data in addition to data contained in countries' national accounts data. Other satellite accounts can however in turn be used to augment the data in the SAM. In this study employment data (as opposed to payments to labour) were used together with the SAM data to derive estimates of changes in employment opportunities in the economy. Theoretically the inclusion of the labour and household detail according to the 1968 SNA SAM layout is one of the strengths of the SAM because of its suitability for modelling income distribution. Unfortunately at the same time it is one of the weaknesses of the SAM because of the dated surveys that were used. The fact that the more recent household and labour surveys conducted by Statistics South Africa are no longer linked to each other creates challenges for future updating of SAMs. Another weakness of the SAM is that the different censuses of agriculture are still used as the main data source for the agricultural industries; hence the small scale agricultural sector is not explicitly captured as a separate industry with potentially different production technologies. Also, no distinction is made between dry-land and irrigated agriculture which could be useful especially because of its different structures with regard to the use of capital versus labour.

The adjusted CGE model was calibrated with the developed SAM for South Africa for 2007 and applied in a case study that analysed four different scenarios. The scenarios were selected based on the issues mentioned in the National Development Plan for South Africa released by the National Planning Commission in 2011. The scenarios relate to global positioning for the fruit industry, technical efficiency increases for the agricultural sector, factor productivity increases in the government and services sector through fighting corruption and fewer strikes and an increase in the supply of skilled labour through an improvement in the quality of education. The global positioning scenario was implemented as a 5% increase in the world export price of fruit and a 2% increase in the world export price of food products with a high horticultural content. Technical efficiency for agricultural industries was implemented as a 1% increase in the shift parameter of the CES function that aggregates intermediate inputs and value added. Factor productivity growth for 'other services' (including government) was implemented as a 2% increase in the shift parameter for the CES function that aggregates capital, skilled labour and unskilled labour to form aggregate value added. For the education scenario the factor supply of skilled labour was increased by 2% under the assumption of full employment for skilled labour.

The case study results show that agricultural producers respond to the changes in relative prices of products. This is most pronounced in the global positioning scenario where the export price of fruit increases relative to other products. For all the agricultural industries there is an increase in the share of fruit produced relative to other products, and this effect is more prominent in the main fruit producing areas in provinces such as the Western Cape, Northern Cape, Eastern Cape, Mpumalanga and Limpopo. The results with the adjusted model that includes the CET function for output transformation therefore appear more realistic compared to results generated with the base model that assumed fixed composition of output. As part of the sensitivity analyses the output elasticity of transformation (*omega*) was set to 0, 0.9 and 2, where *omega* equal to 0 implies that the model reverts to the base model, i.e. fixed output composition. Higher values of *omega* show greater changes in output composition as expected, because agricultural industries are better able to increase production of the products for which it faces higher prices. Even with the global positioning scenario the changes in output composition are not large, but this seems to be realistic in the context of a static model. Changes in the quantity of output for the global positioning scenario indicate that the agricultural industries with a share of fruit of less than 30% all show a decline in production in the base model, but when industries have the ability to adjust output composition it enables these industries to maintain or even increase production. In the adjusted model (*omega* > 0) the negative impact on industries with a smaller share of fruit is therefore less severe, but at the same time the positive production impact on industries with a relatively greater share of fruit is also smaller compared to the base model. This is because these industries will increase the production of fruit, but the production of other products will not be increased by the same percentage as fruit, as would be the case in the base model (*omega* equal to 0).

The two scenarios that impact directly on the agricultural sector, namely the global positioning and the technical efficiency scenarios have a notable impact on the agricultural industries, but a relative small impact on the non-agricultural industries. The increased factor productivity of the government and services sector has a substantial influence throughout the economy because this sector is a relatively large sector. It has an expansionary effect on the economy and causes an increase in effective demand in the economy for primary agricultural products and other food products. The increase in supply of skilled labour in the education scenario has a substantial impact on production throughout the economy, but less so for the agricultural industries because under the assumption of full employment of land, the unavailability of additional land serves as a constraint to expand agricultural production. When the scenarios are jointly implemented in the model, it is estimated that employment opportunities will increase by 457 530, of which 19 530 in the agricultural sector. The results indicate that the National Planning Commission's target for 2030 of an increase of five million employment opportunities in the economy, of which one million in the agricultural supply chain, is ambitious.

A number of different elasticity parameters are used in the functional forms included in the CGE model and the values for these parameters are exogenously assigned and not estimated. Sensitivity analyses were carried out for all of these elasticities in the context of all four scenarios. The elasticities include the two trade elasticities, four production elasticities and two household demand parameters. This process served as validation for the model and data. Sensitivity analyses with regard to the trade elasticities indicate that changes in the level of the elasticity of substitution between domestically produced goods and imports have the greatest impact. On the imports side low values of the elasticity of substitution between imports and domestically produced goods imply that consumers are not able to take advantage of 'cheaper' imports if the exchange rate declines (appreciation of the Rand). For relatively higher values of the elasticity of transformation, i.e. greater flexibility to export rather than produce for the domestic market, the exports of products facing a higher export price will increase relatively more. Therefore the production for the domestic market must be supplemented with imports and the increase in imports tends to be greater than would be the case for lower values of the elasticity of transformation.

Sensitivity analyses with regard to the production elasticities revealed that the results are most sensitive for the value of the elasticity of substitution between different factors of production at the bottom level of the production nest. Higher levels of substitution have the impact of augmenting the factor supply because factors are used more productively. Although results are generally not particularly sensitive to the values of the Frisch parameter or income elasticities of the linear expenditure system within the range of values tested, it was found that the levels of income elasticities impact most on the agricultural and food sectors because greater income elasticity implies a greater share of expenditure on food consumption, which includes both primary agricultural and processed food products. The sensitivity analyses confirm that results are affected by the choice of the value of the elasticities, but the extent of the impact and which variables are most impacted, will depend on the variable changed within each scenario.

Sensitivity analyses with regard to different model closures were also conducted. It was found that the capital and labour closures of full employment versus unemployment have the greatest impact on detailed industry results. This is because under the assumptions of full employment the expansionary effect of all four scenarios causes increases in wage rates, which in turn affect industry prices and factor and household incomes. The implication is that if corruption in South Africa can be addressed it has the same impact as removing the constraint imposed by a fixed amount of capital that is fully utilised (employed). If at the same time factor productivity growth can be achieved, the positive impact on the economy is magnified compared to when the effects of factor productivity growth is curbed by capital constraints. Similarly, when the human capital constraints in South Africa can be relieved through improved education the benefits from economic expansion are significant. Individual agricultural industries will respond to relative price changes by changing their output composition, but the extent to which they can change the overall level of output still depends on the extent to which factor constraints (related to land, skilled labour and capital) can be removed.

To summarise, the study has contributed towards the following:

- A SAM for South Africa for 2007, with agricultural, household and factor detail at provincial level, is available for calibration of the adjusted CGE model;
- A description of the development of the SAM for South Africa and the data sources that were used, is available;
- The model and calibration code pertaining to the model changes is available to allow for other researchers to incorporate these changes into their models;

- Results from a case study on South Africa, using the revised version of the model and calibrated with the newly developed SAM, have been reported as a contribution to the policy debate;
- A comparison of results of the two alternative transformation specifications and sensitivity analyses of different parameter values for the selected transformation function has been reported for the benefit of conducting future case studies;
- Results on a wide range of general sensitivity analyses for four different scenarios have been reported to inform the selection of elasticity values and model closures for future studies.

Recommendations for future research include the following:

- Although the model adjustments were generalised through set controls the model should still be tested with other data sets;
- The inclusion of the CET first order condition on output transformation can be included in a dynamic model with agricultural detail to investigate the timing effect of investment decisions implied by changes in the composition of output;
- The agricultural industries in the SAM can be further refined to capture small scale farming explicitly;
- There is room for improvement with regard to the sensitivity analyses because more sophisticated techniques have become available in the literature;
- Improved sensitivity analyses are likely to assist with selecting appropriate elasticities for model equations. Empirical estimation of elasticities is often not viable because of lack of data to estimate elasticities consistent with all product or industry groups in the SAM database. Data required for the estimation of the elasticity of transformation for the output function is particularly complex due to the different possible product combinations of the various agricultural industries. The product and industry combinations will also change depending on the specific aggregation of the SAM selected for a particular study;
- The structure of the single country model for South Africa that was developed is such that it can be embedded in an existing global model in order to allow for policy analysis of the agro-food complex in a global context;
- The level of detail in the factor and household accounts makes the model suitable for use with a microsimulation model to enable more detailed household level welfare analyses;
- Literature shows advances in the sophistication with which the labour markets in CGE models are handled, but there is scope for further research. This relates to areas such as market clearing conditions of migration functions when allowing for sluggish movement

of labour between different sectors of the economy. Also, the productivity effects of relocating labour to different sectors of the economy needs some improvement. Improved modelling of the labour market is expected to have a positive impact on policy analysis related to topical issues such as minimum wage and wage subsidies;

- The CGE model and data can also be developed to include more detailed information and handling of environmental issues such as climate change, competing land use (e.g. agriculture vs. mining), water and other natural resources. Similarly nested energy structures can be incorporated to look at issues such as electricity tariffs, biomass from agriculture, etc.

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10 Addendum

10.1 Descriptions of model variables, parameters and sets

Variable and parameter names and set descriptions were taken from McDonald (2007) and adjusted in line with the model adjustment.

10.1.1 Model variables

CAPGOV	Government savings
CAPWOR	Current account balance
CPI	Consumer price index
DS	Partial household and enterprise savings rate scaling factor
DTAX	Direct income tax revenue
DTE	Partial export tax rate scaling factor
DTEX	Partial excise tax rate scaling factor
DTF	Partial fuel tax rate scaling factor
DTM	Partial tariff rate scaling factor
DTS	Partial sales tax rate scaling factor
DTX	Partial indirect tax rate scaling factor
DTYE	Partial direct tax on enterprise rate scaling factor
DTYF	Partial direct tax on factor rate scaling factor
DTYH	Partial direct tax on household rate scaling factor
EG	Expenditure by government
EGADJ	Transfers to corporations by government scaling factor
ER	Exchange rate (domestic per world unit)
ETAX	Export tax revenue
EXTAX	Excise tax revenue
$FD_{f,a}$	Demand for factor f by industry a
FS_f	Supply of factor f
FTAX	Fuel tax revenue

FYTAX	Factor income tax revenue
$GOVENT_e$	Government income from enterprise e
$HEXP_h$	Household consumption expenditure
HGADJ	Scaling factor for government transfers to households
$HOENT_{h,e}$	Household Income from enterprise e
$HOHO_{h,hp}$	Inter household transfer
IADJ	Investment scaling factor
INVEST	Total investment expenditure
INVESTSH	Value share of investment in total final domestic demand
$IOQXACQXV_{a,c}$	Share of product c in output by industry a
ITAX	Indirect tax revenue
MTAX	Tariff revenue
PD_c	Consumer price for domestic supply of product c
PE_c	Domestic price of exports by industry a
$PINT_a$	Price of aggregate intermediate input
PM_c	Domestic price of competitive imports of product c
PPI	Producer (domestic) price index
PQD_c	Purchaser price of composite product c
PQS_c	Supply price of composite product c
PVA_a	Value added price for industry a
PWE_c	World price of exports in dollars
PWM_c	World price of imports in dollars
PX_a	Composite price of output by industry a
$PXAC_{a,c}$	Industry product prices
PXC_c	Producer price of composite domestic output
$QCD_{c,h}$	Household consumption by product c
QD_c	Domestic demand for product c
QE_c	Domestic output exported by product c
$QED_{c,e}$	Enterprise consumption by product c
QEDADJ	Enterprise demand volume scaling factor
QGD_c	Government consumption demand by product c
QGDADJ	Government consumption demand scaling factor
$QINT_a$	Aggregate quantity of intermediates used by industry a
$QINTD_c$	Demand for intermediate inputs by product c
$QINVD_c$	Investment demand by product c
QM_c	Imports of product c
QQ_c	Supply of composite product c

QVA_a	Quantity of aggregate value added for level 1 production
QX_a	Domestic production by industry a
$QXAC_{a,c}$	Domestic product output by each industry
QXC_c	Domestic production by product c
SADJ	Savings rate scaling factor for BOTH households and corporations
SEADJ	Savings rate scaling factor for corporations
SEN_e	Corporation savings rates
SHADJ	Savings rate scaling factor for households
SHH_h	Household savings rates
STAX	Sales tax revenue
TE_c	Export taxes on exported product c
TEADJ	Export subsidy scaling factor
TEX_c	Excise tax rate
TEXADJ	Excise tax rate scaling factor
TF_c	Tax rate on factor use
TFADJ	Tax rate on factor use scaling factor
TM_c	Tariff rates on imported product c
TMADJ	Tariff rate scaling factor
TOTSAV	Total savings
TS_c	Sales tax rate
TSADJ	Sales tax rate scaling factor
TX_a	Indirect tax rate
TXADJ	Indirect tax scaling factor
TYE_e	Direct tax rate on corporations
TYEADJ	Enterprise income tax scaling factor
TYF_f	Direct tax rate on factor income
TYFADJ	Factor tax scaling factor
TYH_h	Direct tax rate on households
TYHADJ	Household income tax scaling factor
VED_e	Value of enterprise e consumption expenditure
$VEDSH_e$	Value share of enterprise consumption in total final domestic demand
VFDOMD	Value of final domestic demand
VGD	Value of government consumption expenditure
VGDSH	Value share of govt consumption in total final domestic demand
WALRAS	Slack variable for Walras's Law
WF_f	Price of factor f
$WFDIST_{f,a}$	Sectoral proportion for factor prices

YE_e	Enterprise incomes
YF_f	Income to factor f
$YFDISP_f$	Factor income for distribution after depreciation
$YFWOR_f$	Foreign factor income
YG	Government income
YH_h	Income to household h

10.1.2 Model parameters[6]

ac_c	Shift parameter for Armington CES function
$adva_a$	Shift parameter for CES production functions for QVA
adx_a	Shift parameter for CES production functions for QX
$adxc_c$	Shift parameter for product output CES aggregation
$alphah_{c,h}$	Expenditure share by product c for household h
at_c	Shift parameter for export CET function
$beta_{c,h}$	Marginal budget shares
$caphosh_h$	Shares of household income saved (after taxes)
$comactactco_{c,a}$	Intermediate input output coefficients
$comactco_{c,a}$	Use matrix coefficients
$comgovconst_c$	Government demand volume
$comhoav_{c,h}$	Household consumption shares
$comtotsh_c$	Share of product c in total product demand
$dabshh_h$	Change in base household saving rates
$dabsen_e$	Change in base enterprise saving rates
$dabte_c$	Change in base export taxes on product c imported from region w
$dabtex_c$	Change in base excise tax rate
$dabtfue_c$	Change in base fuel tax rate
$dabtm_c$	Change in base tariff rates on product c imported from region w
$dabts_c$	Change in base sales tax rate
$dabtx_a$	Change in base indirect tax rate
$dabtye_e$	Change in base direct tax rate on corporations
$dabtyf_f$	Change in base direct tax rate on factors
$dabtyh_h$	Change in base direct tax rate on households
$delta_c$	Share parameter for Armington CES function

[6] This list includes parameters that appear in calibration statements

$deltava_{f,a}$	Share parameters for CES production functions for QVA
$deltax_{a}$	Share parameter for CES production functions for QX
$deltaxc_{a,c}$	Share parameters for product output CES aggregation
$deprec_{f}$	Depreciation rate by factor f
$dstocconst_{c}$	Stock change demand volume
$econ_{c}$	Constant for export demand equations
$entgovconst_{e}$	Government transfers to enterprise e
$entvash_{e,f}$	Share of income from factor f to enterprise e
$entwor_{e}$	Transfers to enterprise e from world (constant in foreign currency)
eta_{c}	Export demand elasticity
$factwor_{f}$	Factor payments from RoW (constant in foreign currency)
$frisch_{h}$	Elasticity of the marginal utility of income
$gamma_{c}$	Share parameter for export CET function
$gammai_{c}$	Share parameter for output CET function
$goventsh_{e}$	Share of enterprise income after tax, savings and consumption to govt
$govvash_{f}$	Share of income from factor f to government
govwor	Transfers to government from world (constant in foreign currency)
$hexps_{h}$	Subsistence consumption expenditure
$hoentconst_{h,e}$	Transfers to household h from enterprise e (nominal)
$hoentsh_{h,e}$	Share of enterprise income after tax, savings and consumption to household h
$hogovconst_{h}$	Transfers to household h from government (nominal but scalable)
$hohoconst_{h,hp}$	Interhousehold transfers
$hohosh_{h,hp}$	Share of household h after tax and saving income transferred to hp
$hovash_{h,f}$	Share of income from factor f to household h
$howor_{h}$	Transfers to household from world (constant in foreign currency)
$invconst_{c}$	Investment demand volume
$ioqtdqd_{c,a}$	Intermediate input output coefficients
$ioqintqx_{a}$	Agg intermediate quantity per unit QX for Level 1 Leontief agg
$ioqvaqx_{a}$	Agg value added quant per unit QX for Level 1 Leontief agg
$ioqxacqx_{a,c}$	Share of product c in output by industry a
$kapentsh_{e}$	Average savings rate for enterprise e out of after tax income
$predeltax_{a}$	Dummy used to estimated deltax
$pwse_{c}$	World price of export substitutes
$qcdconst_{c,h}$	Volume of subsistence consumption
$qedconst_{c,e}$	Enterprise demand volume
$rhoc_{c}$	Elasticity parameter for Armington CES function

$rhocva_a$	Elasticity parameter for CES production function for QVA
$rhocx_a$	Elasticity parameter for CES production function for QX
$rhocxc_c$	Elasticity parameter for product output CES aggregation
$rhot_c$	Elasticity parameter for export CET function
$rhoti_a$	Elasticity parameter for output CET function
$sumelast_h$	Weighted sum of income elasticities
$te01_c$	0-1 par for potential flexing of export taxes on products
$tex01_c$	0-1 par for potential flexing of excise tax rates
$tfue01_c$	0-1 par for potential flexing of fuel tax rates
$tm01_c$	0-1 par for potential flexing of tariff rates on products
$ts01_c$	0-1 par for potential flexing of sales tax rates
$tx01_a$	0-1 par for potential flexing of indirect tax rates
$tye01_e$	0-1 par for potential flexing of direct tax rates on corporations
$tyf01_f$	0-1 par for potential flexing of direct tax rates on factors
$tyh01_h$	0-1 par for potential flexing of direct tax rates on households
$use_{c,a}$	Use matrix transactions
$vddtotsh_c$	Share of value of domestic output for the domestic market
$worvash_f$	Share of income from factor f to rest of world
$yhelast_{c,h}$	(Normalised) household income elasticities

10.1.3 Model set descriptions

$$c = \{\text{products}\}$$
$$a = \{\text{industries}\}$$
$$f = \{\text{factors}\}$$
$$h = \{\text{households}\}$$
$$g = \{\text{government}\}$$
$$e = \{\text{enterprises}\}$$
$$i = \{\text{investment}\}$$
$$w = \{\text{rest of the world}\}$$

$$
\begin{aligned}
ce(c) &= \{\text{export products}\} \\
cen(c) &= \{\text{non-export products}\} \\
ced(c) &= \{\text{export products with export demand functions}\} \\
cedn(c) &= \{\text{export products without export demand functions}\} \\
cm(c) &= \{\text{imported products}\} \\
cmn(c) &= \{\text{non-imported products}\} \\
cx(c) &= \{\text{products produced domestically}\} \\
cxn(c) &= \{\text{products NOT produced domestically AND imported}\} \\
cd(c) &= \{\text{products produced AND demanded domestically}\} \\
cxac(c) &= \{\text{differentiated products produced domestically}\} \\
cxacn(c) &= \{\text{UNdifferentiated products produced domestically}\} \\
aqx(a) &= \{\text{industries with CES aggregation at Level 1}\} \\
aqxn(a) &= \{\text{industries with Leontief aggregation at Level 1}\} \\
acet(a) &= \{\text{multi-product industries with CET function on output}\} \\
acetn(a) &= \{\text{industries with fixed composition output shares}\}
\end{aligned}
$$

10.2 Model equations

The model equations listed here are for the adjusted model. The model equations for the base model are taken from McDonald (2007) and all adjustments to the base model are marked.

Table 33: Equation and variable counts for the model

Name	Equation and set control		Number of Equations	Variable	Number of Variables
	EXPORTS BLOCK				
$PEDEF_c$	$PE_c = PWE_c * ER * (1 - TE_c)$	$\forall ce$	ce	PE_c	ce
CET_c	$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + (1-\gamma_c) * QD_c^{rhot_c}\right)^{\frac{1}{rhot_c}}$	$\forall ce$ **AND** cd	c	QD_c	c
$ESUPPLY_a$	$\dfrac{QE_c}{QD_c} = \left[\dfrac{PE_c}{PD_c} * \dfrac{(1-\gamma_c)}{\gamma_c}\right]^{\frac{1}{(rhot_c - 1)}}$	$\forall ce$ **AND** cd	c	QE_c	c
$EDEMAND_c$	$QE_c = econ_c * \left(\dfrac{PWE_c}{pwse_c}\right)^{-eta_c}$	$\forall ced$			
$CETALT_c$	$QXC_c = QD_c + QE_c$	$\forall (cen$ **AND** $cd)$ **OR** $(ce$ **AND** $cdn)$			

Name	Equation and set control	Number of Equations	Variable	Number of Variables
	IMPORTS BLOCK			
$PMDEF_c$	$PM_c = PWM_c * ER * (1+TM_c) \ \forall cm$	cm	PM_c	cm
$ARMINGTON_c$	$QQ_c = ac_c \left(\delta_c QM_c^{-rhoc_c} + (1-\delta_c) QD_c^{-rhoc_c}\right)^{-\frac{1}{rhoc_c}} \quad \forall cm$ **AND** cx	c	QQ_c	c
$COSTMIN_c$	$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{(1-\delta_c)}\right]^{\frac{1}{(1+rhoc_c)}} \quad \forall cm$ **AND** cx	c	QM_c	c
$ARMALT_c$	$QQ_c = QD_c + QM_c \quad \forall (cmn$ **AND** $cx)$ **OR** $(cm$ **AND** $cxn)$			
	PRODUCT PRICE BLOCK			
$PQDDEF_c$	$PQD_c = PQS_c * (1+TS_c + TEX_c) \quad \forall cd$ **OR** cm	c	PQD_c	c
$PQSDEF_c$	$PQS_c = \frac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \quad \forall cd$ **OR** cm	c	PQS_c	c
$PXCDEF_c$	$PXC_c = \frac{PD_c * QD_c + (PE_c * QE_c)\$ce_c}{QXC_c} \quad \forall cx$	cx	PXC_c	cx
	NUMERAIRE BLOCK			
$CPIDEF$	$CPI = \sum_c comtotsh_c * PQD_c$	1	CPI	1
$PPIDEF$	$PPI = \sum_c vddtotsh_c * PD_c$	1	PPI	1

Name	Equation and set control	Number of Equations	Variable	Number of Variables
	PRODUCTION BLOCK			
~~$PXDEF_a$~~	$\cancel{PX_a = \sum_c ioqxacqx_{a,c} * PXAC_{a,c}}$	~~a~~	$\cancel{PX_a}$	~~a~~
$PXDEF_a$	$PX_a = \sum_c IOQXACQXV_{a,c} * PXAC_{a,c}$	a	PX_a	a
$PVADEF_a$	$PX_a * (1 - TX_a) * QX_a = (PVA_a * QVA_a) + (PINT_a * QINT_a)$	a	PV_a	a
$PINTDEF_a$	$PINT_a = \sum_c (ioqtdqd_{c,a} * PQD)_c$	a	$PINT_a$	a
$ADXEQ_a$	$ADX_a = \left[(adxb_a + dabadx_a) * ADXADJ \right] + (DADX * adx01_a)$	a	ADX_a	a
$QXPRODFN_a$	$QX_a = AD_a^x \left(\delta_a^x QVA_a^{-rhoc_a^x} + (1 - \delta_a^x) QINT_a^{-rhoc_a^x} \right)^{-\frac{1}{rhoc_a^x}} \quad \forall aqx_a$	a	QX_a	a
$QXFOC_a$	$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{(1 - \delta_a^x)} \right]^{\frac{1}{(1+rhoc_a^x)}} \quad \forall aqx_a$	a	$QINT_a$	a
$QVADEF$	$QVA_a = ioqvaqx_a * QX_a \quad \forall aqxn_a$			
$QINTDEF$	$QINT_a = \mathrm{ioqintqx}_a * QX_a \quad \forall aqxn_a$			
$ADVAEQ_a$	$ADVA_a = \left((advab_a + dabadva_a) * ADVAADJ \right) + (DADVA * adva01_a)$	a	$ADVA_a$	a
$QVAPRODFN_a$	$QVA_a = AD_a^{va} * \left[\sum_{f \$ \delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}} \right]^{-1/\rho_a^{va}}$	a	QVA_a	a
$QVAFOC_{f,a}$	$WF_f * WFDIST_{f,a} * (1 + TF_{f,a})$ $= PVA_a * QVA_a * AD_a^{va} * \left[\sum_{f \$ \delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}} \right]^{-1}$ $* \delta_{f,a}^{va} * ADFD_{f,a}^{-\rho_a^{va}} * \delta_{f,a}^{va} * FD_{f,a}^{(-\rho_a^{va} - 1)} \quad \forall \delta_{f,a}^{va}$	(f*a)	$FD_{f,a}$	(f*a)
$QINTDEQ_c$	$QINTD_c = \sum_a ioqtdqd_{c,a} * QINT_a$	c	$QINTD_c$	c

Name	Equation and set control	Number of Equations	Variable	Number of Variables
$COMOUT_c$	$QXC_c = adxc_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-1/\rho^{xc}_c}$ $\forall cx_c$ and $cxac_c$	c	QXC_c	c
	$QXC_c = \sum_a QXAC_{a,c}$ $\forall cx_c$ and $cxacn_c$			
$COMOUTFOC_{a,c}$	$PXAC_{a,c} = PXC_c * QXC_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)}$ $* \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)}$ $\forall cxac_c$	$(a*c)$	$PXAC_{a,c}$	$(a*c)$
	$PXAC_{a,c} = PXC$ $\forall cxacn_c$			
~~$ACTIVOUT_{a,c}$~~	~~$QXAC_{a,c} = ioqxacqx_{a,c} * QX_a$~~	~~$(a*c)$~~	~~$QXAC_{a,c}$~~	~~$(a*c)$~~
$ACTIVOUT_{a,c}$	$QXAC_{a,c} = ioqxacqx_{a,c} * QX_a$ $\forall ioqxacqx_{a,c}$ and $acetn_a$	$(a*c)$	$QXAC_{a,c}$	$(a*c)$
$ACTIVOUT1_{a,c}$	$QX_a = ati_a * (\sum_c gammai_{a,c} * QXAC_{a,c}{}^{rhoti_a})^{1/rhoti_a}$ $\forall ioqxacqx_{a,c}$ and $acet_a$	$(a*c)$	$QXAC_{a,c}$	$(a*c)$
$ACTIVOUT2_{a,c}$	$QXAC_{a,c} = QX_a * (PXAC_{a,c} / (PX_a * gammai_{a,c} * ati_a ** rhoti_a))$ $** (1/(rhoti_a - 1))$ $\forall ioqxacqx_{a,c}$ and $acet_a$			
	FACTOR BLOCK			
$YFEQ_f$	$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right)$	f	YF_f	f
$YFDISPEQ_f$	$YFDISP_f = \left(YF_f * \left(1 - deprec_f \right) \right) * \left(1 - TYF_f \right)$	f	$YFDIST_f$	f

Name	Equation and set control	Number of Equations	Variable	Number of Variables
	HOUSEHOLD BLOCK			
$YHEQ_h$	$YH_h = \left(\sum_f hovash_{h,f} * YFDISP_f\right) + \left(\sum_{hp} HOHO_{h,hp}\right) + HOENT_h + \left(hogovconst_h * HGADJ * CPI\right) + \left(howor_h * ER\right)$	*h*	YH_h	*h*
$HOHOEQ_{h,hp}$	$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right)$	*h*hp*	$HOHO_{h,hp}$	*h*hp*
$HEXPEQ_h$	$HEXP_h = \left(\left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right)\right) - \left(\sum_{hp} HOHO_{hp,h}\right)$	*h*	$HEXP_h$	*h*
$QCDEQ_{c,h}$	$QCD_{c,h} * PQD_c = \left(PQD_c * qcdconst_{c,h}\right) + beta_{c,h} * \left(HEXP_h - \sum_c \left(PQD_c * qcdconst_{c,h}\right)\right)$	*c*h*	$QCD_{c,h}$	*c*h*
	CORPORATION / ENTERPRISE BLOCK			
$YEEQ_e$	$YE_e = \left(\sum_f entvash_{e,f} * YFDISP_f\right) + \left(entgovconst_e * EGADJ * CPI\right) + \left(entwor_e * ER\right)$	e	YE_e	e
$QEDEQ_{c,e}$	$QED_{c,e} = qedconst_{c,e} * QEDADJ$	*c*	$QED_{c,e}$	*c*
$VEDEQ_e$	$VED_e = \left(\sum_c QED_{c,e} * PQD_c\right)$	e	VED_e	E
$HOENTEQ_{h,e}$	$HOENT_{h,e} = hoentsh_{h,e} * \left(\begin{array}{l}\left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ -\sum_c \left(QED_{c,e} * PQD_c\right)\end{array}\right)$	*h*e*	$HOENT_{h,e}$	*h*e*
$GOVENTEQ_e$	$GOVENT_e = goventsh_e * \left(\begin{array}{l}\left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ -\sum_c \left(QED_{c,e} * PQD_c\right)\end{array}\right)$	e	$GOVENT_e$	e

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX RATE BLOCK			
$TMDEF_c$	$TM_c = \left(\left(tmb_c + dabtm_c\right) * TMADJ\right) + \left(DTM * tm01_c\right)$	*cm*	*TM*	*cm*
$TEDEF_c$	$TE_c = \left(\left(teb_c + dabte_c\right) * TEADJ\right) + \left(DTE * te01_c\right)$	*ce*	*TE*	*ce*
$TSDEF_c$	$TS_c = \left(\left(tsb_c + dabts_c\right) * TSADJ\right) + \left(DTS * ts01_c\right) \quad \forall cd$ **OR** cm	*c*	*TS*	*c*
$TEXDEF_c$	$TEX_c = \left(\left(texb_c + dabtex_c\right) * TEXADJ\right) + \left(DTEX * tex01_c\right) \forall cd$ **OR** cm	*c*	*TEX*	*c*
$TXDEF_a$	$TX_a = \left(\left(txb_a + dabtx_a\right) * TXADJ\right) + \left(DTX * tx01_a\right)$	*a*	*TX*	*a*
$TFDEF_{f,a}$	$TF_{f,a} = \left(\left(tfb_{f,a} + dabtf_{f,a}\right) * TFADJ\right) + \left(DTF * tf01_{f,a}\right)$	*f*a*	*TF*	*f*a*
$TYFDEF_f$	$TYF_f = \left(\left(tyfb_f + dabtyf_f\right) * TYFADJ\right) + \left(DTYF * tyf01_f\right)$	*f*	*TYF*	*f*
$TYHDEF_h$	$TYH_h = \left(\left(tyhb_h + dabtyh_h\right) * TYHADJ\right) + \left(DTYH * tyh01_h\right)$	*h*	*TYH*	*h*
$TYEDEF_e$	$TYE_e = \left(\left(tyeb_e + dabtye_e\right) * TYEADJ\right) + \left(DTYE * tye01_e\right)$	*e*	*TYE*	*e*

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX REVENUE BLOCK			
MTAXEQ	$MTAX = \sum_{c}\left(TM_{c} * PWM_{c} * ER * QM_{c}\right)$	1	*MTAX*	1
ETAXEQ	$ETAX = \sum_{c}\left(TE_{c} * PWE_{c} * ER * QE_{c}\right)$	1	*ETAX*	1
STAXEQ	$STAX = \sum_{c}\begin{pmatrix} TS_{c} * PQS_{c} * \\ \left(QINTD_{c} + QCD_{c} + QED_{c} + QGD_{c} + QINVD_{c} + dstocconst_{c}\right)\end{pmatrix}$ $= \sum_{c}\left(TS_{c} * PQS_{c} * QQ_{c}\right)$	1	*STAX*	1
EXTAXEQ	$EXTAX = \sum_{c}\left(TEX_{c} * PQS_{c} * QQ_{c}\right)$	1	*EXTAX*	1
ITAXEQ	$ITAX = \sum_{a}\left(TX_{a} * PX_{a} * QX_{a}\right)$	1	*ITAX*	1
FTAXEQ	$FTAX = \sum_{f,a}\left(TF_{f,a} * WF_{f} * WFDIST_{f,a} * FD_{f,a}\right)$	1	*FTAX*	1
FYTAXEQ	$FYTAX = \sum_{f}\left(TYF_{f} * \left(YF_{f} * \left(1 - deprec_{f}\right)\right)\right)$	1	*FYTAX*	1
DTAXEQ	$DTAX = \sum_{h}\left(TYH_{h} * YH_{h}\right) + \sum_{e}\left(TYE_{e} * YE\right)$	1	*DTAX*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	GOVERNMENT BLOCK			
$YGEQ$	$YG = MTAX + ETAX + STAX + EXTAX + FTAX + ITAX + FYTAX + DTAX + \left(\sum_{f} govvash_{f} * YFDISP_{f}\right) + \sum_{e} GOVENT_{e} + (govwor * ER)$	1	YG	1
$QGDEQ_c$	$QGD_{c} = comgovconst_{c} * QGDADJ$	c	QGD_c	c
$VGDEQ$	$VGD = \left(\sum_{c} QGD_{c} * PQD_{c}\right)$	1	$VQGD$	1
$EGEQ$	$EG = \left(\sum_{c} QGD_{c} * PQD_{c}\right) + \left(\sum_{h} hogovconst_{h} * HGADJ * CPI\right) + \left(\sum_{e} entgovconst_{e} * EGADJ * CPI\right)$	1	EG	1

Name	Equation	Number of Equations	Variable	Number of Variables
	INVESTMENT BLOCK			
$SHHDEF_h$	$SHH_h = \left(\left(shhb_h + dabshh_h\right) * SHADJ * SADJ\right) + \left(DSHH * DS * shh01_h\right)$	h	SHH	H
$SENDEF_e$	$SEN_e = \left(\left(sen_e + dabsen_e\right) * SEADJ * SADJ\right) + \left(DSEN * DS * sen01_e\right)$	e	SEN	e
$TOTSAVEQ$	$TOTSAV = \sum_h \left(\left(YH_h * \left(1 - TYH_h\right)\right) * SHH_h\right) + \sum_e \left(\left(YE * \left(1 - TYE_e\right)\right) * SEN_e\right) + \sum_f \left(YF_f * deprec_f\right) + CAPGOV + \left(CAPWOR * ER\right)$	1	$TOTSAV$	1
$QINVDEQ_c$	$QINVD_c = \left(IADJ * invconst_c\right)$	c	$QINVD_c$	c
$INVEST$	$INVEST = \sum_c \left(PQD_c * \left(QINVD_c + dstocconst_c\right)\right)$	1	$INVEST$	1
	FOREIGN INSTITUTIONS BLOCK			
$YFWOREQ_f$	$YFWOR_f = worvash_f * YFDISP_f$	f	$YFWOR_f$	f

Name	Equation	Number of Equations	Variable	Number of Variables
	MARKET CLEARING BLOCK			
$PRODEQUIL_{a,c}$	$QXAC_{a,c} = IOQXACQXV_{a,c} * QX_a$			
$FMEQUIL_f$	$FS_f = \sum_a FD_{f,a}$	f	FS_f	f
$QEQUIL_c$	$QQ_c = QINTD_c + \sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + QINVD_c + dstocconst_c$	c		
$GOVEQ$	$CAPGOV = YG - EG$	1	$CAPGOV$	1
$CAEQUIL$	$CAPWOR = \left(\sum_{cm} PWM_{cm} * QM_{cm}\right) + \left(\sum_f \frac{YFWOR_f}{ER}\right) - \left(\sum_{ce} PWE_{ce} * QE_{ce}\right)$ $- \left(\sum_f factwor_f\right) - \left(\sum_h howor_h\right) - \sum_e entwor_e - govwor$	1	$CAPWOR$	1
$VFDOMDEQ$	$VFDOMD = \sum_c PQD_c * \left(\sum_h QCD_{c,h} + \sum_e QED_{c,e} + QGD_c + QINVD_c + dstocconst_c\right)$	1	$VFDOMD$	1
$VEDSHEQ$	$VEDSH_e = VED_e / VFDOMD$	1	$VEDSH$	1
$VGDSHEQ$	$VGDSH = VGD / VFDOMD$	1	$VGDSH$	1
$INVESTSHEQ$	$INVESTSH = INVEST / VFDOMD$	1	$INVESTSH$	1
$GDPVAEQ$	$GDPVA = \left(\sum_{f,a} WF_f * WFDIST_{f,a} * FD_{f,a}\right) + MTAX$ $+ETAX + STAX + EXTAX + ITAX + FTAX$	1	$GDPVA$	1
$WALRASEQ$	$TOTSAV = INVEST + WALRAS$	1	$WALRAS$	1

Name	Equation	Number of Equations	Variable	Number of Variables
	MODEL CLOSURE			
			$\overline{ER}$ or $\overline{CAPWOR}$	1
			$\overline{PWM_c}$ and $\overline{PWE_c}$ or $\overline{PWE_{cedn}}$	2*c*
			$\overline{SADJ}$,$\overline{SHADJ}$,$\overline{SEADJ}$ or $\overline{IADJ}$ or $\overline{INVEST}$ or $\overline{INVESTSH}$	1
			$\overline{QEDADJ}$ or $\overline{VED}$ or $\overline{VEDSH}$	1
			At least **one** of $\overline{TMADJ}$, $\overline{TEADJ}$, $\overline{TSADJ}$, $\overline{TEXADJ}$, $\overline{TFADJ}$, $\overline{TXADJ}$, $\overline{TFADJ}$, $\overline{TYHADJ}$, $\overline{TYEADJ}$ $\overline{DTM}$,$\overline{DTE}$,$\overline{DTS}$,$\overline{DTEX}$,$\overline{DTF}$,$\overline{DTX}$,$\overline{DTYF}$,$\overline{DTYH}$,$\overline{DTYE}$, and $\overline{CAPGOV}$	7
			at least **two** of $\overline{QGDADJ}$, $\overline{HGADJ}$,$\overline{EGADJ}$,$\overline{VGD}$ and $\overline{VGDSH}$	3
			$\overline{FS_f}$ and $\overline{WFDIST_{f,a}}$	$(f*(a+1))$
			$\overline{CPI}$ or $\overline{PPI}$	1

10.3 List of accounts in the social accounting matrix (SAM)

Products
Cereals
Other crops
Vegetables
Fruit
Other horticulture
Animals and animal products
Forestry and fishing products
Mineral products
Food high crop content products
Food high horticultural content products
Food high animal content products
Food beverage tobacco and other products
Textile and clothing products
Wood paper and media products
Petroleum and other non-metallic products
Metallic and machinery products
Transport and other manufacturing products
Utilities
Construction and building
Trade and transport services
Other services
Industries
Western Cape horticulture >50%
Western Cape other agriculture
Northern Cape animals >50%
Northern Cape other agriculture
Eastern Cape animals >50%
KwaZulu-Natal crops >50%
KwaZulu-Natal other agriculture
North West crops >50%
North West other agriculture
Gauteng animals >50%
Free State crops >50%
Free State other agriculture
Mpumalanga horticulture >50%
(Industries cont)
Mpumalanga other agriculture
Limpopo horticulture >50%
Limpopo other agriculture
Forestry and fishing
Minerals
Food high crop content
Food high horticultural content
Food high animal content
Food beverage tobacco and other
Textile and clothing
Wood paper and media
Petroleum and other non-metallic
Metallic and machinery
Transport and other manufacturing
Utilities
Construction and building
Trade and transport services
Other services
Factors
Gross Operating Surplus Mixed Income
Land
WC Agric Skilled
WC Agric Unskilled
WC Nonagric Skilled
WC Nonagric Unskilled
EC Agric Skilled
EC Agric Unskilled
EC Nonagric Skilled
EC Nonagric Unskilled
NC Agric Skilled
NC Agric Unskilled
NC Nonagric Skilled
NC Nonagric Unskilled
FS Agric Skilled
FS Agric Unskilled
(Labour cont)
FS Nonagric Skilled
FS Nonagric Unskilled
KZ Agric Skilled
KZ Agric Unskilled
KZ Nonagric Skilled
KZ Nonagric Unskilled
NW Agric Skilled
NW Agric Unskilled
NW Nonagric Skilled
NW Nonagric Unskilled
GT Agric Skilled
GT Agric Unskilled
GT Nonagric Skilled
GT Nonagric Unskilled
MP Agric Skilled
MP Agric Unskilled
MP Nonagric Skilled
MP Nonagric Unskilled
LP Agric Skilled
LP Agric Unskilled
LP Nonagric Skilled
LP Nonagric Unskilled
Households
WC agric poor
WC agric non-poor
WC non-agric poor
WC non-agric non-poor
EC agric poor
EC agric non-poor
EC non-agric poor
EC non-agric non-poor
NC agric poor
NC agric non-poor
NC non-agric poor
NC non-agric non-poor
(Households cont)
FS agric poor
FS agric non-poor
FS non-agric poor
FS non-agric non-poor
KZ agric poor
KZ agric non-poor
KZ non-agric poor
KZ non-agric non-poor
NW agric poor
NW agric non-poor
NW non-agric poor
NW non-agric non-poor
GT agric poor
GT agric non-poor
GT non-agric poor
GT non-agric non-poor
MP agric poor
MP agric non-poor
MP non-agric poor
MP non-agric non-poor
LP agric poor
LP agric non-poor
LP non-agric poor
LP non-agric non-poor
Other
Business corporations
Import duties
Excise duty
Other sales taxes (net)
Net production taxes
Direct income taxes
Government
Savings
Stock Changes
Rest of the World

10.4 District municipalities per agricultural industry account in the SAM

SAM account	District Municipality
Western Cape horticulture >50%	Cape Winelands City of Cape Town Overberg
Western Cape other agriculture	Central Karoo Eden West Coast
Northern Cape animals >50%	Kgalagadi Namakwa Pixley Ka Seme
Northern Cape other agriculture	Frances Baard Siyanda
Eastern Cape animals >50%	Amathole Cacadu Chris Hani Thambo Nzo Ukhahlamba
KwaZulu-Natal crops >50%	Ilembe Ethwekwini Ugu District Umkhanyakude Uthungulu Uthukela
KwaZulu-Natal other agriculture	Amajuba Sisonke District Umgunundlovu Umzunyathi Zululand

SAM account	District Municipality
North West crops >50%	Bophirima Central
North West other agriculture	Bojanala Platinum Southern
Gauteng animals >50%	City Of Johannesburg City Of Tshwane Ekurhuleni Metropolitan Metsweding Sedibeng West Rand
Free State crops >50%	Fezile Dabi District Lejweleputswa Xhariep
Free State other agriculture	Motheo Thabo Mofutsanyane
Mpumalanga horticulture >50% Mpumalanga other agriculture	Ehlanzeni Gert Sibande Nkangala
Limpopo horticulture >50%	Mopani Vhembe
Limpopo other agriculture	Capricorn Waterberg