UNIVERSITY OF STELLENBOSCH
DEPARTMENT OF MECHANICAL ENGINEERING

FINITE ELEMENT ANALYSIS OF A WING TYPE
STRUCTURE WITH EXPERIMENTAL VERIFICATION OF RESULTS

PART A
THEORY AND COMPUTER PROGRAM

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A. THEORY

A1 INTRODUCTION

The advent of the high speed computer has revolutionized structural design in all spheres of Engineering. Up till then structural stress analysis was limited to over-simplification of the structure in question to comply with derived classical mathematical solutions.

In practice however the picture is very different, the structure usually being complex and highly redundant in nature. The techniques involving Energy Methods to solve such structures have been known for a long time. However they required weeks of hand calculations to solve only a small number of redundancies in a structure. Neville Shute mentions this in his book Slide Rule.

The development of Matrix algebra and the finite element method has made it possible to analyse, say, a complete aircraft structure in a matter of days, using a large capacity high speed computer.

Experimental results have shown that finite element stress analysis comes much closer to reality than the dated classical methods.
A2. THE FINITE ELEMENT METHOD

A2.1 Introduction

Let us look at a very simple example to start off with.

Consider a spring fastened at one end and a tensile force applied to its other end as in Figure 1.

![Figure 1](image)

The spring will stretch a distance $u$ and we have the well known spring equation

$$ku = F \quad \quad \quad \quad \quad \quad \quad a2.1.1$$

If the situation were complicated slightly more and three springs were arranged as in Figure 2.

![Figure 2](image)

it would be necessary to determine the overall stiffness of the structure in the $y$-direction then apply the force to determine the deflection $u_y$.

$$\therefore \quad (k_1 + k_2 + k_3)u_y = F$$
If the roller supports were removed from the point 4 a deflection in the x-direction would also be experienced and the deflection $u_y$ would not be the same as in the first case. The deflections $u_x$ and $u_y$ can now be split up into components parallel to the springs and the forces in the springs can be calculated.

This is in effect what the finite element method does.

A structure can be split up into elements e.g. Rods, panels, plates and beams all connected at their ends or corners. These connection points are called nodal points.

Each element has a certain stiffness which can be expressed, in terms of its geometry and elastic properties, at each of its nodal points.

In the structure shown in Figure 3 there are 5 nodal points. Each element has a certain stiffness in each of the 3 global axes directions and the sum of the stiffnesses, in a specific direction at a nodal point, then represents the stiffness of that nodal point in that direction. In Figure 3 there are thus $5 \times 3$ unknown deflections i.e. 15 simultaneous linear equations are needed to solve for the unknowns.

Written in matrix form the equations for the entire structure looks similar to equation a2.1.1

$$\begin{bmatrix} K & \cdots & \cdots \\ \cdots & \ddots & \cdots \\ \cdots & \cdots & K \end{bmatrix} \begin{bmatrix} U_1 \\ \vdots \\ U_5 \end{bmatrix} = \begin{bmatrix} P_1 \\ \vdots \\ P_5 \end{bmatrix}$$

.................... a2.1.2
Where $K$ is a square symmetric matrix of nodal point stiffness coefficients, $\mathbf{U}$ is a column vector of the unknown displacements and $\mathbf{F}$ is the column vector of the nodal point loads.

The set of equations a2.1.2 are solved by inversion of the $K$ matrix

$$\mathbf{U} = K^{-1} \mathbf{F}$$

and then the solved deflections back substituted into the stiffness matrices of the individual elements to determine the stresses and strains in them.

This method is known as the displacement stiffness method, and is now widely used in many large finite element programs such as ASKA and NASTRAN.

The reciprocal of the displacement method is the force or flexibility method in which the forces are the unknowns. This method is more complicated to apply and is thus not as popular as the displacement method although hybrid elements incorporating both the methods are in use.

A2.2 Formulation of the Displacement Method

The theory of finite elements can be extended to incorporate special material properties such as non-isotropy and non-linearity, however this discussion will be based on the assumption that the materials used have a linear elastic region. This assumption is made in most books on Strength of Materials and Elasticity Theory.

A2.2.1 Basic elasticity equations

It can be shown (Ref. 6) that if a body is deformed in such a way so that the deformation in the $x$, $y$ and $z$ directions can be expressed as functions $u$, $v$, and $w$ respectively, then the strains can be expressed as follows in the kinematic equations.
\[
\begin{align*}
\varepsilon_{xx} & = \frac{\partial u}{\partial x} \\
\varepsilon_{yy} & = \frac{\partial v}{\partial y} \\
\varepsilon_{zz} & = \frac{\partial w}{\partial z} \\
\varepsilon_{xy} & = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\
\varepsilon_{xz} & = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\
\varepsilon_{yz} & = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}
\end{align*}
\]

In the one-dimensional case the stress and the strain are related as follows

\[
\varepsilon = \frac{1}{E} \cdot \sigma
\]

where \( E \) is the Modulus of Elasticity and \( \sigma \) the stress in the direction of the chosen strain. Poisson discovered that in the three-dimensional strain case the relationship is as follows.

\[
\begin{align*}
\varepsilon_{xx} & = \frac{1}{E} \left[ \sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz}) \right] \\
\varepsilon_{yy} & = \frac{1}{E} \left[ \sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz}) \right] \\
\varepsilon_{zz} & = \frac{1}{E} \left[ \sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy}) \right]
\end{align*}
\]

where \( \nu \) is called Poisson's ratio.
For shear strain we have

\[
\begin{align*}
\varepsilon_{xy} &= \frac{\sigma_{xy}}{G} \\
\varepsilon_{xz} &= \frac{\sigma_{xz}}{G} \\
\varepsilon_{yz} &= \frac{\sigma_{yz}}{G}
\end{align*}
\]

\[\text{................................. a2.2.5}\]

where \( G \) is called the Modulus of Elasticity in Shear and is found to be related to \( E \) as follows

\[
G = \frac{E}{2(1 + v)}
\]

\[\text{................................. a2.2.6}\]

"Special" cases of the general theory are frequently encountered, such as Plane Stress which occurs approximately in thin sheet panels and which obviously has application in an aircraft structure.

To derive the plane stress equations the terms \( \sigma_z \), \( \sigma_x \), and \( \sigma_y \) are set to zero and the state of stress is then specified by \( \sigma_x \), \( \sigma_y \) and \( \sigma_{xy} \) only. After manipulation equations a2.2.4 and a2.2.5 can be written

\[
\begin{align*}
\sigma_{xx} &= \frac{E}{1 - v^2} \varepsilon_{xx} + \frac{vE}{1 - v^2} \varepsilon_{yy} \\
\sigma_{yy} &= \frac{vE}{1 - v^2} \varepsilon_{xx} + \frac{E}{1 - v^2} \varepsilon_{yy} \\
\sigma_{xy} &= \frac{E}{2(1 + v)} \varepsilon_{xy}
\end{align*}
\]

\[\text{................................. a2.2.7}\]

which can be expressed in matrix form as follows

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} = \begin{bmatrix}
1 & v & 0 \\
v & 1 & 0 \\
0 & 0 & \frac{1-v}{2}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{xy}
\end{bmatrix}
\]

\[\text{................................. a2.2.8}\]
A2.2.2 Work and Energy

Work is defined as a force multiplied by the distance through which it moves,

\[ W = \vec{F} \cdot \vec{s} \] .......................... a2.2.9

or in differential form

\[ dW = \vec{F} \cdot d\vec{s} \] .......................... a2.2.10

Thus if a linearly elastic system (e.g. a structure) undergoes a deflection \( du \) then the work done on the system is

\[ dW = P \, du \] .......................... a2.2.11

\( P \) is the applied load.

The work done by the structure in resisting the applied forces is

\[ dU = \vec{F} \cdot du \] .......................... a2.2.12

where \( \vec{F} \) are the internal forces of the system (material Stresses)

![Figure 4](http://scholar.sun.ac.za)

At this stage the principle of virtual or imaginary displacements is introduced which states that:

For a system which is in equilibrium, the sum of the internal and external work done during a virtual displacement is equal to zero, on condition that the kinematic equations and the geometric boundary conditions are satisfied.
Mathematically the virtual displacement $\delta u$ behaves the same as the differential quantity $du$.

For virtual displacements the equations of work now become

$$\delta W = P \delta u \quad \text{.................... a2.2.13}$$

$$\delta U = F \cdot \delta u \quad \text{.................... a2.2.14}$$

and from the above stated Principle of Virtual Displacements

$$\delta U + \delta W = 0 \quad \text{.................... a2.2.15}$$

The next step is to relate the internal forces to the external ones by means of the structures elastic properties.

Consider now the well known linear stress-strain diagram of a material Figure 5.

The expression $\sigma \delta \varepsilon$ has as its dimensions $\frac{N}{m^2}$, $\frac{m}{m}$ which is Work/unit volume. Thus if the integral is taken over the volume of the system, or a single element, the variation of the strain energy of the system can be found.

$$\delta U = \int_{V} \sigma \delta \varepsilon \, dV \quad \text{.................... a2.2.16}$$

The system is acted upon by external forces and body forces (gravitational, initial strains, temperature effects) and these are also given per unit area (external) and per unit volume (body) and integrated over the area of application and volume of the system respectively.
Substituting a2.2.16 and a2.2.17 into a2.2.15 gives

\[ \delta W = \int_V \bar{g} \delta \bar{u} \, dV + \int_A \bar{q} \delta \bar{u} \, dA \quad \text{.......... a2.2.17} \]

Substituting a2.2.16 and a2.2.17 into a2.2.15 gives

\[ -\int_V \sigma \delta \bar{e} \, dV + \int_V \bar{g} \delta \bar{u} \, dV + \int_A \bar{q} \delta \bar{u} \, dA = 0 \quad \text{a2.2.18} \]

The minus sign in front of the \( \delta U \) term arises from the fact that the stresses oppose the applied forces. Equation a2.2.18 is subject to the conditions that the kinematic equations and the prescribed boundary conditions are satisfied.

As was seen previously, a structure is divided into a finite number of elements, connected at their nodal points, the next step thus, is to establish functions for the displacements in the x, y and z directions and express the displacements within the system in terms of the nodal point deflections which arise out of the solution of equation a2.1.3.

To obtain the strains \( \bar{e} \) in the system it follows that the chosen displacement functions must be continuous differentiable at least once. If they still satisfy the geometric boundary conditions then they are allowable functions (Ref. 4).

It can be shown that if the displacement functions

1. include all possible rigid body displacements
2. include uniform strain conditions i.e. constant stress conditions
3. satisfy the geometric interelement and boundary conditions

Then the finite element method converges monotonously to the correct solution as the number of elements is increased.

If conditions 1. and 2. only are satisfied then convergence still takes place but not monotonously.

It has been seen from equations a2.2.1 and a2.2.2 that the displacement functions have to be partially differentiated in one form or another to
produce the strains. These partial differentiations will be signified by the comma.

\[ \bar{\varepsilon} = \bar{u}, \]

where \( \bar{\varepsilon} \) takes into account all the strains in the system and \( \bar{u} \) the displacements.

\[ \mathcal{I} \delta \bar{u} + \mathcal{I} \bar{g} \delta \bar{u} + \mathcal{I} \bar{q} \delta \bar{u} \mathcal{d}A = 0 \quad \text{...... a2.2.19} \]

Let the displacement functions be

\[ \bar{u}(x, y, z) = \Phi(x, y, z) \hat{u} \]

\[ \therefore \delta \bar{u} = \Phi \delta \hat{u} \]

\[ \therefore \delta \bar{u}, = \Phi, \delta \hat{u} \]

where \( \hat{u} \) are the nodal point deflections. Substituting a2.2.20 into a2.2.19

\[ \delta \bar{u} \left\{ -\mathcal{I} \int_{\mathcal{V}} \Phi, \sigma \mathcal{d}V + \int_{\mathcal{V}} \bar{g} \Phi \mathcal{d}V + \int_{\mathcal{A}} \Phi \bar{q} \mathcal{d}A \right\} = 0 \]

\[ \therefore -\mathcal{I} \int_{\mathcal{V}} \Phi, \sigma \mathcal{d}V + \int_{\mathcal{V}} \bar{g} \Phi \mathcal{d}V + \int_{\mathcal{A}} \Phi \bar{q} \mathcal{d}A = 0 \quad \text{.. a2.2.21} \]

From equation a2.2.8 it can be seen that

\[ \bar{\sigma} = \bar{E} \bar{\varepsilon} \]

also

\[ \bar{\varepsilon} = \bar{u}, \]

\[ = \Phi, \hat{u} \]

\[ \therefore \bar{\sigma} = \bar{E} \Phi, \hat{u} \quad \text{.................. a2.2.22} \]
Substituting into a2.2.21

\[- \int \phi, E \phi, \dot{u} \, dv + \int \theta, g \phi, dv + \int q \phi, dA = 0\]

But the terms \(\phi, E, \dot{u}, \phi\) are matrices, thus from the laws of matrix algebra where

\[(A)^2 = A^T A\]

we get

\[- \int \phi^T, E \phi, \dot{u} \, dv + \int \theta^T, g \phi, dv + \int q^T, q, dA = 0 \quad \text{a2.2.23}\]

From the first term in equation a2.2.23 must come a stiffness multiplied by a deflection thus

\[\int \phi^T, E \phi, \, dv \text{ is the stiffness matrix of the system and}\]
\[\dot{u} \text{ the nodal point deflections. The stiffness matrix is signified by the symbol } K \text{ for an element and } K \text{ for the complete structure.}\]

The last two terms of the equation represent the loads on the system. The \(g\) vector can incorporate initial strains due to lack of fit, temperature strains and gravitational forces due to the system's own weight. The \(q\) vector represents the external loads on the system. Now as mentioned previously the system is divided up into a finite number of elements connected at a finite number of nodal points thus the external and internal forces must be applied at the nodal points. If point loads are applied to the system then there should be no problem, the system can be split up in such a way that the point load coincides with a nodal point. If however the system is loaded with a distributed load then equivalent nodal point forces must be determined using the integral expression in equation a2.2.23.

Thus taking the force terms to the RHS of the equation and combining them gives for the whole structure.
\[ \mathbf{K} \mathbf{U} = \mathbf{P} \]  
\[ \text{........................... a2.2.24} \]

which is the same as the original simple spring equation a2.1.1 and for a single element

\[ \mathbf{k} \mathbf{u} = \mathbf{p} \]  
\[ \text{........................... a2.2.25} \]

A2.3 Some Common Element Stiffness Matrices

A2.3.1 Endload or Rod Element

This is the simplest element used in finite element analysis. It is generally used in the modeling of a structure where a component is expected to be in pure tension or compression i.e. it has no bending stiffness. It is used to model pin jointed truss structures, beam flanges and stiffeners in aircraft structures.

\[ \text{Area} = A \]
\[ \text{Elasticity Modulus} = E \]

\[ u_i, p_i \]
\[ u_j, p_j \]

\[ L \]

\[ \text{Figure 6} \]

Figure 6 shows the rod element lying along the \( x \)-axis with its nodal point deflections and loads given by \( u_i, u_j \) and \( p_i, p_j \) respectively.

For a uniform rod the strain is constant, therefore we can assume that the displacement function \( u \) is a linear function of \( x \)

\[ \mathbf{u} = \mathbf{\phi}(x) \]
\( \ddot{u} = A_1 x + A_2 \) ........................ a2.3.1

\( \varepsilon_{xx} = \frac{\ddot{u}}{\ddot{x}} = A_1 \) ........................ a2.3.2

For the rod in question the following boundary conditions exist

at \( x = 0 \)
\( u = u_1 \)

\( \therefore A_2 = u_1 \)

and at \( x = L \)
\( u = u_j \)

\( u_j = A_1 L + u_1 \)

\( \therefore \frac{(u_j - u_1)}{L} = A_1 \)

\( \therefore \ddot{u} = \frac{(u_j - u_1)}{L} x + u_1 \) ........................ a2.3.3

\( = u_1 (1 - \frac{x}{L}) + \frac{u_j}{L} x \)

\( = \begin{bmatrix} (1 - \frac{x}{L}) & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_j \end{bmatrix} \)

\( \ddot{u} = \ddot{\phi} \hat{u} \)

\( \therefore \ddot{u}, = \ddot{\phi}, \hat{u} \)

\( = \begin{bmatrix} -1 \\ L \end{bmatrix} \begin{bmatrix} u_1 \\ u_j \end{bmatrix} \) ........................ a2.3.4

From equation a2.2.23 the stiffness matrix is

\( \bar{k} = \int_V \bar{\phi}^T \bar{E} \bar{\phi}, dV \) ........................ a2.3.5
For the one dimensional case

\[ E = E \]

\[ \therefore \quad \bar{k} = E \int_{V} \begin{bmatrix} \frac{1}{L} & 1 \\ \frac{1}{L} & \frac{1}{L} \end{bmatrix} \, dV \]

\[ \therefore \quad \bar{k} = E \int_{V} \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ \frac{1}{L^2} & \frac{1}{L^2} \end{bmatrix} \, dV \]

and if the area is constant

\[ \bar{k} = EA \int_{0}^{L} \begin{bmatrix} \frac{1}{L^2} & \frac{1}{L^2} \\ \frac{1}{L^2} & \frac{1}{L^2} \end{bmatrix} \, dx \]

\[ \bar{k} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

which is the stiffness matrix for the rod element in local co-ordinates.

It is customary when analysing a structure to analyse it according to some reference set of axes. Let's call them the global axes. Now not all the elements in the structure are going to be orientated such that they lie in the planes of the global axes so the element has to be derived in its own set of axes, called local axes, and then transformed in such a way that its stiffnesses are then in the planes of the global axes. The deflections will then be in the directions of the global axes.

The transformation matrices will be dealt with in the next section.

A2.3.2 The Plane Stress Triangle

This element is more commonly known as the C.S.T. element (Constant
Strain Triangle) because its displacement functions are chosen such that the strains $\varepsilon_{xx}$, $\varepsilon_{yy}$ and $\varepsilon_{xy}$ are constant throughout the element.

Figure 7 shows the element in its local coordinate system.

![Figure 7](image)

It can be seen that if the displacement functions are given by

\[
\begin{align*}
\text{u} &= A_1 x + A_2 y + A_3 \\
\text{v} &= B_1 y + B_2 x + B_3
\end{align*}
\]

then

\[
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} = A_1 \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} = B_1 \\
\varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = A_2 + B_2
\end{align*}
\]

which gives the constant strains required.

Now by substituting the three boundary conditions into equations a2.3.7 i.e.

\[
\begin{align*}
\text{u}(0,0) &= u_1 & \text{u}(x_2,0) &= u_j & \text{u}(x_3, y_3) &= u_k \\
\text{v}(0,0) &= v_1 & \text{v}(x_2,0) &= v_j & \text{v}(x_3, y_3) &= v_k
\end{align*}
\]

gives
\[
A_1 = \frac{(u_j - u_1)}{x_2}
\]
\[
A_2 = \left[ u_k x_2 + (x_3 - x_2) u_1 - u_j x_3 \right]/x_2 y_3
\]
\[
A_3 = u_1
\]
\[
B_1 = \left[ v_k x_2 + (x_3 - x_2) v_1 - v_j x_3 \right]/x_2 y_3
\]
\[
B_2 = \frac{(v_j - v_1)}{x_2}
\]
\[
B_3 = v_1
\]
\[
u = (u_j - u_1)y_3/x_2 y_3 \cdot x + \left[ u_k x_2 + (x_3 - x_2) u_1 - u_j x_3 \right]/x_2 y_3 \cdot y + u_1 \cdot \frac{x_2 y_3}{x_2 y_3}
\]

We can see that
\[
\text{Area}_\Delta = \frac{1}{2} x_2 y_3 = \Delta
\]
\[
u = \frac{1}{2\Delta} \left[ \left( -y_3 x + (x_3 - x_2) y + x_2 y_3 \right) u_1 + (y_3 x - x_3 y) u_j + x_2 y u_k \right] \quad \cdots \cdots \cdots \ \text{a2.3.8}
\]
\[
v = \frac{1}{2\Delta} \left[ \left( -y_3 x + (x_3 - x_2) y + x_2 y_3 \right) v_1 + (y_3 x - x_3 y) v_j + x_2 y v_k \right] \quad \cdots \cdots \cdots \ \text{a2.3.9}
\]

From equations a2.3.8/9 we can write
\[
\begin{bmatrix}
u
v
\end{bmatrix} = \frac{1}{2\Delta} \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_4 & 0 & N_5 & 0 & N_6 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix}
\]
\[
\overline{(u)} = (\Phi) \hat{(u)}
\]
Where $N_i, i = 1-6$, are functions of $x$ and $y$.

\[
\begin{bmatrix}
N_1, x & 0 & N_2, x & 0 & N_3, x & 0 \\
0 & N_4, y & 0 & N_5, y & 0 & N_6, y \\
N_1, y & N_4, x & N_2, y & N_5, x & N_3, y & N_6, x
\end{bmatrix}
\begin{bmatrix}
u_1 \\
v_1 \\
u_j \\
v_j \\
u_k \\
v_k
\end{bmatrix}
\]

\[
= \frac{1}{2A}
\begin{bmatrix}
-y_3 & 0 & y_3 & 0 & 0 & 0 \\
0 & (x_3-x_2) & 0 & -x_3 & 0 & 0 \\
(x_3-x_2) & -y_3 & -x_3 & y_3 & x_2 & 0
\end{bmatrix}
\begin{bmatrix}
u_k \\
v_k
\end{bmatrix}
\]

\[
\therefore \bar{k} = \int_V \bar{\Phi}^T \bar{E} \bar{\Phi} \, dV
\]

where $\bar{E}$ is the matrix given in equation 2.2.6. It can be seen however that the terms in $\bar{\Phi}$ are all constants, and if the thickness of the element is constant then

\[
\bar{k} = \Delta \bar{\Phi}^T \bar{E} \bar{\Phi}, \quad t
\]

which is the stiffness matrix for the plane stress $\text{CSI}$ element in local co-ordinates.

A2.3.3 Quadrilateral Elements

There are various methods used in assembling a four sided plane stress element.

Firstly it can be built up out of four $\text{CSI}$ elements and the internal nodal point can be eliminated from the set of equations

\[
\bar{k} \hat{\vec{u}} = \bar{p}
\]

by setting the $\bar{x}$ and $\bar{y}$ forces at the internal point equal to zero.

A computer condensation routine is given in Ref. 3.
Secondly displacement fields for the quadrilateral can be found and the normal procedure followed, this however is a long and complex business and the isoparametric formulation has become necessary for speedy assembly of the stiffness matrix. This is explained fully in Refs. 2, 3 and 4. A computer routine for the assembly of the plane stress quadrilateral stiffness matrix, using isoparametric formulation can be found in Ref. 3.

A2.3.4 Approximate elements

Shear Triangle

For those who are familiar with the classical analysis of aircraft structures it will be remembered that the structure is usually idealized into a series of booms and shear carrying panel members.

The boom is the normal endload element derived in section A2.3.1. The triangular shear shear panel can be derived in the same way as the CST element of section A2.3.2 but the $E$ matrix becomes

$$
\begin{bmatrix} 
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & G \\
\end{bmatrix}
$$

where $G = \frac{E}{2(1 + \nu)}$

Multiplying out

$$
\bar{k} = \int_V \frac{T}{\phi} \bar{E} \frac{T}{\phi}\,dV
$$

$$
\begin{bmatrix} 
(x_3 - x_2)^2 & -y_3(x_3 - x_2) & y_3^2 \\
-x_3(x_3 - x_2) & x_3 y_3 & x_3^2 \\
y_3(x_3 - x_2) & -y_3^2 & -x_3 y_3 & y_3^2 \\
x_2(x_3 - x_2) & -x_2 y_3 & -x_2 x_3 & x_2 y_3 & x_2^2 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$
It can be seen that this element has no stiffness in the $v_k$ direction, this is overcome in the structure however because each side of the panel will be stiffened by a boom.

Quadrilateral Shear Panel (Ref. 1)

The assumption is made that the shear strain is constant throughout the panel thus

\[
\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = C_1 \tag{a2.3.10}
\]

and

\[
\begin{align*}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} = C_2 + C_3 y \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} = C_4 + C_5 x
\end{align*}
\]

Integration of equation a2.3.11 yields

\[
\begin{align*}
\mu &= C_2 x + C_3 xy + F_1(y) \\
\nu &= C_4 y + C_5 xy + F_2(x)
\end{align*} \tag{a2.3.12}
\]

Substituting these into a2.3.10 yields

\[
\begin{align*}
C_3 \frac{\partial F_2}{\partial x} + C_5 y + \frac{\partial F_1}{\partial y} &= C_1 \\
\text{or} \\
G_1(x) + G_2(y) &= C_1
\end{align*}
\]

Thus $G_1(x)$ and $G_2(y)$ must be constants

\[
\begin{align*}
\frac{\partial F_1}{\partial y} + C_5 y &= C_6 \\
\frac{\partial F_2}{\partial x} + C_3 x &= C_9
\end{align*}
\]
Integrating yields

\[ F_1 = -\frac{C_5}{2} y^2 + C_6 y + C_7 \]
\[ F_2 = -\frac{C_3}{2} x^2 + C_9 x + C_8 \]

Substituting 2.3.13 into 2.3.12 yields

\[ u = C_2 x + C_3 xy + C_6 y + C_7 - C_5 \frac{y^2}{2} \]
\[ v = C_4 y + C_5 xy + C_9 x + C_8 - C_3 \frac{x^2}{2} \]

and \[ C_9 = C_1 - C_6 \]

By substituting the boundary conditions, at the four nodal points, into equations we get the relationship

\[ \bar{u} = \Phi \ddot{u} \]

and from \[ \bar{k} \left\{ \begin{array}{c} \Phi^T \bar{E} \Phi \end{array} \right\} dV \quad \text{with} \quad \bar{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
yields (Ref. 1)

\[ \begin{bmatrix} x^2 n_j \\ -y_n i x_n j \\ -x_m i x_n j \\ y_m i x_n j \\ -x^2 n_j \\ -y^2 n_i \\ x_m i x_n j \\ -y_n i x_n j \\ y_n i x_n j \\ -y^2 n_i \\ x_m i x_n j \\ -y_n i x_n j \\ y_n i x_n j \\ -y^2 n_i \\ x_m i x_n j \\ -y_n i x_n j \\ y_n i x_n j \\ -y^2 n_i \\ x_m i x_n j \\ -y_n i x_n j \\ y_n i x_n j \\ -y^2 n_i \end{bmatrix} \]

where \[ x_{n j} = x_n - x_j \] etc.

\[ A = (x_{j l} y_{m i} + x_{m i} y_{n j} - x_{n i} y_{m i}) \]

\[ t = \text{panel thickness} \]
In its local co-ordinates the panel is arranged as in Figure 8.

It must be pointed out here that the shear panel and boom idealization of an aircraft structure is a rough approximation of the real thing, and the plane stress elements should be used rather than the shear panels. A comparison of the results obtained from both configurations will be given in the Experimental Section B of this thesis.

A2.4 The Transformation Matrices

A2.4.1 Transformation Matrix for the End-load Element

Consider Figure 9 where the rod-element is in three dimensional space with the set of axes at the "i" end of the rod parallel to the global axes. Only two angles of rotation define the elements orientation relative to the global axes.
If the element is first rotated about the y-axis through an angle $\alpha$ then the relationship between the new co-ordinates $x_1, y_1, z_1$ and $x, y, z$ is

$$
\begin{bmatrix}
  x_1 \\
  y_1 \\
  z_1
\end{bmatrix} =
\begin{bmatrix}
  \cos \alpha & 0 & \sin \alpha \\
  0 & 1 & 0 \\
  -\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
$$

Further rotation about the $z_1$ axis through an angle $\beta$ yields the relationship between $x_2, y_2, z_2$ and $x_1, y_1, z_1$.

$$
\begin{bmatrix}
  x_2 \\
  y_2 \\
  z_2
\end{bmatrix} =
\begin{bmatrix}
  \cos \beta & \sin \beta & 0 \\
  -\sin \beta & \cos \beta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  y_1 \\
  z_1
\end{bmatrix}
$$

Substituting (2.4.1) into (2.4.2) yields

$$
\begin{bmatrix}
  x_2 \\
  y_2 \\
  z_2
\end{bmatrix} =
\begin{bmatrix}
  \cos \alpha \cos \beta & \cos \beta \sin \alpha & \sin \alpha \\
  -\cos \alpha \sin \beta & \cos \beta \sin \beta & \cos \alpha \\
  -\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
$$

or in matrix notation

$$(\mathbf{T}) = (T)(\mathbf{T})$$

where $(T)$ is called the transformation matrix. Now if we let barred quantities be local deflections and forces then

$$(\ddot{u}) = (T)(\dot{u})$$

and

$$(\ddot{p}) = (K)(\ddot{u})$$

also

$$(\ddot{p}) = (T)(\ddot{p})$$
Then combining these relationships yields:

\[(T)(p) = (k)(T)(u)\]

or

\[(p) = (T)^{-1}(k)(T)(u) \quad \text{............... a2.4.7}\]

The expression \((T)^{-1}(k)(T)\) is called the global stiffness matrix and \((p)\) and \((u)\) are in global axis directions. Thus we have once again the familiar equation

\[\{p\} = \{k\}\{u\}\]

The fortunate situation exists, in that, the transformation matrix does not have to be inverted but only transposed because it is orthogonal i.e.

\[|T| = 1\]

thus

\[\{k\} = (T)^T\{k\}\{T\} \quad \text{............... a2.4.8}\]

Now considering Figure 9 once more, expressions for the trigonometric functions of the angles can be found from the co-ordinates of the two nodal points.

Let \(z_{ij} = z_j - z_i\) etc.

\[p = (z_{ij}^2 + x_{ij}^2)^{\frac{1}{2}}\]

and

\[L = (x_{ij}^2 + y_{ij}^2 + z_{ij}^2)^{\frac{1}{2}}\]

then

\[\sin \alpha = \frac{z_{ij}}{p} ; \quad \cos \alpha = \frac{x_{ij}}{p} \]

\[\sin \beta = \frac{y_{ij}}{L} ; \quad \cos \beta = \frac{p}{L} \]

and the transformation matrix becomes

\[
\begin{bmatrix}
\frac{x_{ij}}{L} & \frac{y_{ij}}{L} & \frac{z_{ij}}{L} \\
\frac{x_{ij}}{L} & \frac{y_{ij}}{L} & \frac{z_{ij}}{L} \\
\frac{z_{ij}}{p} & 0 & \frac{x_{ij}}{p}
\end{bmatrix}
\]
Now in order that the matrix multiplication of equation a2.4.8 may take place the stiffness matrix of the end load element must be made three dimensional. The endload element is one dimensional i.e. it only has stiffness in the $x$ direction thus the stiffnesses in the $y$ and $z$ directions are zero, therefore equation a2.3.6 becomes

$$
(k) = \begin{bmatrix}
1 & 0 & 0 & \text{sym.} \\
0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

and in this form the multiplication can take place.

$$
(k) = \begin{bmatrix}
(T) & 0 \\
0 & (T)
\end{bmatrix}^T \begin{bmatrix}
(T) & 0 \\
0 & (T)
\end{bmatrix}
$$

A2.4.2 Transformation Matrix for a Plane Element

Any three points on a plane surface are sufficient to define its orientation in 3-D space, thus only 3 corners of a quadrilateral need be used to define its co-ordinate transformation matrix.
Consider in Figure 10 a triangular plane $ijn$ with its local $\overrightarrow{x}$ axis lying along the $ij$ side and the $\overrightarrow{y}$ and $\overrightarrow{z}$ axes at right angles to it as shown. By obtaining the direction cosines of the unit vectors $\overrightarrow{v}_x, \overrightarrow{v}_y$ and $\overrightarrow{v}_z$ the transformation of the co-ordinates is complete.

Let 

$$y_{ji} = y_j - y_i$$

$$z_{ni} = z_n - z_i$$ etc.

then the vector representing side $ij$ is

$$\overrightarrow{v}_i = \overrightarrow{i} x_{ji} + \overrightarrow{j} y_{ji} + \overrightarrow{k} z_{ji}$$

from vector theory and its unit vector

$$\frac{\overrightarrow{v}_i}{L} = \frac{\overrightarrow{i} x_{ji}}{L} + \frac{\overrightarrow{j} y_{ji}}{L} + \frac{\overrightarrow{k} z_{ji}}{L}$$

where 

$$L = (x_{ji}^2 + y_{ji}^2 + z_{ji}^2)^{\frac{1}{2}}$$

The vector representing side $in$ is

$$\overrightarrow{v}_{in} = \overrightarrow{i} x_{ni} + \overrightarrow{j} y_{ni} + \overrightarrow{k} z_{ni}$$

and its unit vector

$$\frac{\overrightarrow{v}_{in}}{K} = \frac{\overrightarrow{i} x_{ni}}{K} + \frac{\overrightarrow{j} y_{ni}}{K} + \frac{\overrightarrow{k} z_{ni}}{K}$$

where 

$$K = (x_{ni}^2 + y_{ni}^2 + z_{ni}^2)^{\frac{1}{2}}$$

Now the cross product of two vectors which have an angle $\theta$ between them is given by

$$\overrightarrow{A} \times \overrightarrow{B} = |A| \cdot |B| \sin \theta \overrightarrow{u}$$

where $\overrightarrow{u}$ is a unit vector perpendicular to the plane made by vectors $\overrightarrow{A}$ and $\overrightarrow{B}$.

Now cross multiplication of $\frac{\overrightarrow{v}_i}{L} \times \frac{\overrightarrow{v}_{in}}{K}$ and deviding by $\sin \theta$ yields $\overrightarrow{v}_x$.
From Figure 10 it can be seen that

$$\sin \theta = \frac{h}{k}$$

thus multiplying out equation e2.4.9, yields

$$\mathbf{v}_z = \frac{y_{ji} z_{ni} - y_{ni} z_{ji}}{L h} i + \frac{x_{ji} x_{ni} - x_{ji} z_{ni}}{L h} j + \frac{x_{ji} y_{ni} - y_{ji} x_{ni}}{L h} k$$

and similarly

$$\mathbf{v}_y = \mathbf{v}_z \times \mathbf{v}_x$$

Now we can write

$$\mathbf{v}_x = i \lambda_{xx} + j \lambda_{xy} + k \lambda_{xz}$$
$$\mathbf{v}_y = i \lambda_{yx} + j \lambda_{yy} + k \lambda_{yz}$$
$$\mathbf{v}_z = i \lambda_{zx} + j \lambda_{zy} + k \lambda_{zz}$$

thus the transformation matrix for a plane element is

$$\mathbf{(T)} = \begin{bmatrix} \lambda_{xx} & \lambda_{xy} & \lambda_{xz} \\ \lambda_{yx} & \lambda_{yy} & \lambda_{yz} \\ \lambda_{zx} & \lambda_{zy} & \lambda_{zz} \end{bmatrix}$$

The plane element only has stiffness in its own plane so the local co-ordinate matrix must also be "prepared" by adding rows and columns of zeros in the $z$ direction before multiplication with the $(T)$ matrix takes place.
A3. THE AIRSTR COMPUTER PROGRAM

A3.1 Why write another one?

There is no better way of mastering a subject other than applying the theory oneself. The program was started, and built up on, as the lectured course in finite elements progressed. Many problems were encountered along the way and solved in one way or another.

The program is by no means a highly efficient piece of programming, however, for the smaller type of exercise used say by undergraduate students in aircraft structures it is faster than using a large system like ASKA and requires less pre-use reading of manuals etc.

A3.2 Description of Method

The entire program is executed in core i.e. no data is read to or from files, thus eliminating almost entirely Input/Output time which is very costly.

The program has a main driver routine which calls all the subroutines necessary, in order. A main driver card tells the program how many nodal points are used and how many of each element is used. The nodal point co-ordinates are read in first and stored in column arrays. Each element is read in on a single card and its stiffness matrix, in local co-ordinates, assembled, prepared (zeros added), transformed and then packed into the (K) matrix in banded form (Ref. 3). The known deflections and nodal point loads are added and the set of equations solved. The solved deflections are correctly associated with the nodal points at which they occur and stored. Each element card is then re-read and the deflections used to calculate the stresses and strains in each element.

There is a full "data echo" of all the input data. The output consists of a title heading for the job, the nodal point deflections and the stresses in each element, all given in clearly legible format.

A complete listing of the program can be found in Appendix A1.
A3.3 **Elements Available in AIRSTR**

A3.3.1 **End-load or Rod element**

Can be arbitrarily orientated in space.

![Diagram of a rod element between points i and j.](image)

**Input**
- Nodal point numbers $i$ and $j$
- Crosssectional area $A_f$
- $E$ - Young's Modulus.

**Output**
- $\sigma_{xx}$: Axial stress
- $F_x$: Axial force

For a linearly tapered element the mean cross-sectional area may be used

$$A_f = \frac{A_{f1} + A_{fj}}{2}$$

A3.3.2 **C.S.T. Element** (Constant Strain Triangle)

This element can be arbitrarily orientated in space, and can either be used for plane stress or plane strain analysis.

![Diagram of a C.S.T. element.](image)
Input  Nodal point numbers \( i, j \) and \( n \)
t - panel thickness
\( v \) - Poissons ratio
E - Youngs Modulus.

Output \( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}, \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \)
with \( i - j \) as \( x \)-axis

If a variable thickness panel is used a mean value for the thickness may
be chosen.
\[
t = \frac{t_1 + t_2 + t_3}{3}
\]

A3.3.3 4- C.S.T. Quadrilateral Element

A four sided plane stress membrane element arbitrarily orientated in
space.

It is made up of 4-C.S.T. elements with the dummy point lying at the
intersection of the diagonals. The centre point is eliminated by putting
the forces there equal to zero.

Input  Nodal point numbers \( i, j, m \) and \( n \)
t - panel thickness
\( v \) - Poissons ratio
E - Youngs Modulus.

Output \( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}, \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \)
for each triangle with the \( x \)-axis parallel to side \( i-j \). The mean
stresses of the 4 triangles are also given.
A3.3.4 Quadrilateral Plane Stress Element

Basically the same as the previous element. Only stress output at the centre of the element

\[ \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \]

A3.3.5 Shear Quadrilateral Panel Element

The panel is assumed to carry shear stresses only which are constant throughout the panel. Arbitrary orientation in space.

Input

Nodal point numbers: i, j, m and n

\( t \) - panel thickness

\( v \) - Poisson's Ratio

\( E \) - Youngs Modulus

Output \( \varepsilon_{xy} \) and \( \sigma_{xy} \) constant throughout panel.

Depending on the variation in thickness in a quadrilateral panel discretion must be used in choosing a representative thickness (\( t \)).

A3.3.6 Idealized Panel Element

The program automatically generates an idealized panel as outlined in Ref. 5, in which, the following approximation is made:

![Diagram of idealized panel](Stellenbosch University http://scholar.sun.ac.za)
Moment of inertia about the x-x axis for both cross-sections must be equal

\[ \frac{1}{12} th^3 = 2A_e \left( \frac{h}{2} \right)^2 \]

\[ A_e = \frac{1}{6} th \]

Now considering the quadrilateral element in Figure 12

\[ A_e = \frac{1}{6} t \left( h_1 + h_2 \right) \]

and \[ i-n \] and \[ j-m \]

\[ A_e = \frac{1}{6} t \left( L + K \right) \]

This idealization satisfies the special case where the panel is a rectangle.
Input  Nodal point numbers  i, j, m, n
      t - panel thickness
      v - Poisson's ratio
      E - Young's Modulus.

Output  Same as for Shear Panel and End-load elements.

The option of having Shear Panels and End-load elements separately available gives the program flexibility in that the programmer may wish to make his own idealization of a structure.

A3.3.7  Shear Triangle Panel Element

This is an approximate element as described in section A2.3.4 and only carries shear stresses. It must be accompanied by rod elements to give it, in-plane direct stress stiffness.

Input  Nodal point Numbers  i, j and n
      t - panel thickness
      v - Poisson's Ratio
      E - Young's Modulus

Output  Same as for C.S.T. element. Ignore direct stresses and strains.

The program will reject badly shaped triangular elements (C.S.T. and Shear triangles), i.e. if the perpendicular from nodal point n to line i-j does not lie between the points i and j. The programmer should remember this when drawing up his finite element grid.
A3.3.8 Idealized Triangular Panel Element

There is no suggested method for idealizing a triangular panel element so an attempt has been made by the writer.

Consider the triangular panel shown in Figure 13.

Let $nb = bj$

$nd = di$

and $ia = aj$

The moment of inertia of the triangle is taken about the point C and a mean height $h'$ taken.

$$h' = \frac{(oj + mi + pn)}{3}$$

Then making the rod elements i-n, n-j and i-j all of the same equivalent cross-sectional area yields

$$\frac{1}{12} t (h')^3 = A_e \left[ (ac)^2 + (bc)^2 + (dc)^2 \right]$$

$$\therefore A_e = \frac{t (h')^3}{12(ac^2 + bc^2 + dc^2)}$$

Thus the idealized triangular panel element consists of a triangular shear panel stiffened by rod elements of area $A_e$ between the three nodal points.

Input Same as for Shear Triangle Element.

Output Same as for Shear Triangle and three rod elements.
A3.4 Use of the AIRSTR program

One run of the program will do a single static load case on a semi-monocoque type structure. The dimensioned stiffness matrix limits the number of nodal points to 100 and a bandwidth of 190 i.e. in the numbering of the nodal points no two connected nodal points may differ in value by more than 63. The program was written and developed on a UNIVAC 1110 computer using the EXEC 8 system. All the data cards are written in free format i.e. the values are separated by commas on each card. Floating point values must have a decimal point and integers not.

The data deck of cards is given below. The element cards must be arranged in the order specified. To comply with their stiffness derivation, elements must be numbered in an anticlockwise direction as indicated in section A3.3

The deck is headed by the identification of the user and his account number, called the @RUN card.

@RUN ...

The elements are read twice, so they are stored in a temporary file called in EXEC 8 an ELEMENT; So the element cards are headed by say

@ELT,IELMNTS

Then follows the data cards for the elements. Each element type is grouped together and regardless of the types of elements used the order of sequence in which they appear in the data deck must be as follows. One card for each element

i, j, A_f, E-modulus (End-load or Rod Elements).
i, j, n, t, v, E (Plane Stress C.S.T. elements)
i, j, n, t, v, E (Plane Strain C.S.T. elements)
i, j, m, n, t, v, E (4-C.S.T. Plane Stress Quadrilaterals)
i, j, m, n, t, v, E (Plane Stress Quad. element)
i, j, m, n, t, v, E (Shear Quad. Panel elements)
i, j, m, n, t, v, E (Idealized Quad. Panel elements)
i, j, n, t, v, E (Shear Triangle Panel elements)
i, j, n, t, v, E (Idealized Triangle Panel elements)
That is the end of the elements in the structure. Next comes the execution card which starts the run of the program which is stored on file.

@XQT..EDDY.AIRSTR

The first card the program reads is the title of the job submitted. The complete card, i.e. 80 spaces, is available for the title, e.g. STARBOARD.WING.LANDING-LOAD.CASE-1

The Main Driver card follows next. This card tells the program how many nodal point co-ordinates, and elements of each type to read. All the numbers on this card are integer values. Once again the order of the numbers must be as shown, and if an element of a certain type is not to be used, a zero must be inserted in the correct position.

N,NRD3,NCST,NTRIM,NQAT,NQUA4,NSHT,NIPAN,NTRIS,NIDTS

Where

N - No. of Nodal points
NRD3 - " End-load elements
NCST - " Plane Stress C.S.T. elements
NTRIM - " Plane Strain C.S.T. elements
NQAT - " 4-C.S.T. Plane Stress Quadrilaterals
NQUA4 - " Plane Stress Quadrilateral Elements
NSHT - " Shear Quadrilateral Panel Elements
NIPAN - " Idealized Quadrilateral Panel Elements
NTRIS - " Shear Triangle Panel Elements
NIDTS - " Idealized Triangle Panel Elements.

The nodal point co-ordinates are read next.

I , XX(I) , YY(I) , ZZ(I)
Nod. pt. number, x-coord, y-coord, z-coord.

The nodal point number is an integer and the co-ordinates floating point. They do not have to be read in in order i.e. ↑ to N. If a two dimensional structure is being analysed let all the z-co-ordinates be zero (0,0)

The structure stiffness matrix is now assembled so the element data must be read in, this is simply done with the following card.

@ADD..ELMNTS
A secondary driver card is inserted next to tell the program how many nodal point loads and displacements it must read. The card description is self explanatory.

**ILoad, IDISP**

It now follows that the loads are defined. Each nodal point load is given on a separate card as follows.

Nodal Point No., Direction, Magnitude  
INOD, ID, ALoad

The load magnitudes are positive in the direction of the chosen global axes. Should the load at a particular nodal point not lie parallel with any of the global axes, then that load must be split up into its components parallel with global axes and each component read in on a separate card. The direction identification is as follows:

ID = 1 - x-axis (global)  
ID = 2 - y-axis (global)  
ID = 3 - z-axis (global)

For a structure to be in static equilibrium either all the forces and moments on it must balance or it must have supports to balance the applied loads. The supports can either be assumed to be rigid i.e. they deflect very little in comparison with the rest of the structure or they can deflect a known amount. These then are the known displacements, which are, like the loads, each read in on separate cards.

Nodal Point No., Direction, Magnitude  
INOD, ID, DISP

If a two dimensional structure is being analysed (x-y plane) then all the nodal point deflections in the z-direction must be set to zero.

The program now solves the set of equations, and to complete the back substitution to determine the stresses in each element, they are read again by inserting the card

@ADD..ELMNTS

and the run is terminated by the

@FIN card.
An example structure with its data deck listing and print out can be found in Appendix A2.

Footnote:

The load vector is zeroed in the program before the applied loads are read, so nodal points which do not have loads applied to them, do not need zero load cards.

If at a particular point in a structure the deflection is known, but the applied loads unknown, then the no of loads (ILOAD) may be made zero and the known deflection inserted. The stresses in the structure will be output for such a condition.
REFERENCES


4. R.J. du Preez  "Eindige Element Metodes van die Strukturele Meganika"  Notes from the University of Stellenbosch, Civil Engineering Department


Listing of the AIRSTR computer program
**PROGRAMMER E. M. E. BAUMGARTNER**

**ENGINEERING MASTER'S THESIS**

**STATIC FINITE ELEMENT ANALYSIS OF SEMI-MONOCOQUE AIRCRAFT STRUCTURES**

---

**COMPILER**(xx=1)

**DIMENSION A(300,190),B(300)**

**DIMENSION U(5,100),XX(100),YY(100),ZZ(100)**

**COMMON A**

**NODAL = 100**

**NBAND = 190**

**IMATK = NODAL * 3**

**READ IN TITLE CARD**

**WRITE(6,15)**

**WRITE(6,5)**

**READ MAIN DRIVER CARD**

**WRITE(6,11)**

**READ IN NODAL POINT COORDINATES**

**READ(5,10)(J,XX(J),YY(J),ZZ(J),I=1,N)**

**WRITE(6,13)**

**WRITE(6,12)(J,XX(J),YY(J),ZZ(J),J=1,N)**

**VACATE STIFFNESS AND LOAD MATRIX**

**DO 100 I=1,NN**

**END**

**CONTINUE**

**ASSEMBLY OF STRUCTURE STIFFNESS MATRIX**

**CALL RODPAK(NROD3,IMATK,NODAL,A,XX,YY,ZZ,NBAND)**

**JSTRS = 1**

**CALL TRIM3(NCST,IMATK,NODAL,A,XX,YY,ZZ,NBAND,JSTRS)**

**JSTRN = 0**

**CALL TRIM3(NTRIM,IMATK,NODAL,A,XX,YY,ZZ,NBAND,JSTRN)**

**CALL QUAPAC(NQAT,IMATK,NODAL,A,XX,YY,ZZ,NBAND)**

**CALL QUAP3(NQUA4,IMATK,NODAL,A,XX,YY,ZZ,NBAND)**

**CALL SHTRAPC(NSHTR,IMATK,NODAL,A,XX,YY,ZZ,NBAND)**

**CALL SHTRAPC(NSHTR,IMATK,NODAL,A,XX,YY,ZZ,NBAND)**

**CALL TRISH(NIPAN,IMATK,NODAL,A,XX,YY,ZZ,NIPAN,NBAND)**

**CALL TRISH(NIDTS,IMATK,NODAL,A,XX,YY,ZZ,NIDTS,NBAND)**

**SECONDARY DRIVER CARD. NO. OF LOADS AND DISPLACEMENTS**

Stellenbosch University http://scholar.sun.ac.za
330 READ(5,40) ILOAD,IDISP
C** LOAUS
C*
   IF(ILOAD.EQ.0) GO TO 705
   WRITE(6,500)
   DO 700 ILO=1,ILOAD
      READ(5,50) INOD,ID,ALOAD
      WRITE(6,505)INOD,ID,ALOAD
      IF(INOD.LE.N.AND.ID.LE.3) GO TO 690
      WRITE(6,680)
      GO TO 1000
   690 IACOL=(INOD-1)*3+ID
   B(IACOL)=ALOAD
   700 CONTINUE
C*
C** DEFLECTIONS OR BOUNDARY CONDITIONS
C*
   705 WRITE(6,510)
      CALL SUPORT(A,B,IMATK,NBAND,IDISP)
C** SOLVE SET OF EQUATIONS
   CALL SOLVE(A,B,NN,IMATK,NBAND)
   WRITE(6,60)
   WRITE(6,5)
   NFP = NN-IDISP
C** SET THE DEFLECTIONS TO THE CORRECT NODAL POINTS
C*
   920 DO 925 JNOD=1,N
      DO 925 JDIR=1,3
      ND= (JNOD-1)*3+JDIR
      U(JDIR,JNOD)=B(ND)
   925 CONTINUE
   WRITE(6,520)
   WRITE(6,521)
   WRITE(6,525)(J,(U(I,J),I=1,3),J=1,N)
C*
C** CALCULATION OF THE LOADS IN THE ELEMENTS
C** ***************
C** RE-READ THE DATA FOR EACH ELEMENT AND DETERMINE THE STRESSES
C*
   WRITE(6,60)
   WRITE(6,5)
   WRITE(6,28)
C** ENLOAD ELEMENTS
   951 IF(NROD3.EQ.0) GO TO 960
   WRITE(6,21)
      CALL RODES3(NROD3,U,NODAL,XX,YY,ZZ)
C** CALCULATION OF STRESSES IN C.S.T. ELEMENTS (PLANE STRESS)
   960 IF(NCST.EQ.0) GO TO 966
      IPS = 1
      CALL CONTES(NCST,U,NODAL,XX,YY,ZZ,IPS)
   966 IF(NTRIM.EQ.0) GO TO 968
      IPS = 0
      CALL CONTES(NTRIM,U,NODAL,XX,YY,ZZ,IPST)
C** 4-CST QUADRILATERALS (PLANE STRESS)
   968 IF(NGAT.EQ.0) GO TO 970
      CALL QUASTR(NGAT,U,NODAL,XX,YY,ZZ)
C** CALCULATION OF STRESSES IN QUA4 ELEMENT
   970 IF(NQUA4.EQ.0) GO TO 975
CALL Q4ST(NQUAD,U,NODAL,XX,YY,ZZ)

** CALCULATION OF STRESSES IN SHEAR QUADRILATERAL ELEMENTS **
975 IF(NSHTR.EQ.0) GO TO 980
ISH = 0
WRITE(6,60)
WRITE(6,22)
CALL PANSTR(NSHTR,U,NODAL,XX,YY,ZZ,ISH)

** IDEALIZED QUADRILATERAL PANELS **
980 IF(NIPAN.EQ.0) GO TO 985
IDP = 1
WRITE(6,22)
CALL PANSTR(NIPAN,U,NODAL,XX,YY,ZZ,IDP)

** SHEAR TRIANGLES **
985 IF(NITRI.S.EQ.0) GO TO 990
KIO = 0
WRITE(6,965)

** IDEALIZED TRIANGLES **
990 IF(NIDTS.EQ.0) GO TO 1000
KIDTS = 2
WRITE(6,965)

FORMAT(80H)
5 FORMAT(10X)
10 FORMAT(10X,10I5)
12 FORMAT(10X,4E13.4)
13 FORMAT(10X,'NODAL POINT COORDINATES'/)
15 FORMAT(2X,10F18.8)
21 FORMAT(2X,'ROD NOD.PTS',3X,'AREA ',4X,'E-VALUE ',4X,'LEN
CGTH',4X,'AXIAL LOAD',4X,'STRESS '/)
22 FORMAT(2X,'SHEAR PANEL NOD.PTS',4X,'B-LENGTH ',4X,'HEIGHT ',7X
C', 'G-VALUE',2X,'THICKNESS',2X,'EPSXY ',2X,'SHEAR STRESS '/)
28 FORMAT(46X,'SEMI-MONOCOCQUE STRUCTURAL ANALYSIS',36(''))
40 FORMAT(2X)
50 FORMAT(10X)
50 FORMAT(10X,2I6,5E10.4)
510 FORMAT(2X,'KNOWN DEFLECTIONS'/)
520 FORMAT(2X,10F18.8)
521 FORMAT(2X,'NODAL POINT DEFLECTIONS',3X,'X-OEFL ',7X,'Y-DEFL ',7X
Z-DEFL'/)
680 FORMAT(30X,'ERROR- CHECK YOUR LOAD INPUT DATA'/)
965 FORMAT(10X,'THESE PANELS ONLY CARRY SHEAR. IGNORE OTHER OUTPUT. ')
1000 CALL EXIT

ND(1) = IC
ND(2) = JC
ND(3) = NC

** SET UP LOCAL COORDINATE SYSTEM **

SUBROUTINE CONTES(NCST,U,NODAL,XX,YY,ZZ,JID)

C** DETERMINATION OF STRESS AND STRAIN IN C.S.T. ELEMENTS, SHEAR
C** TRIANGLES AND IDEALIZED TRIANGLES **

DIMENSION NOD(3),U(3,NODAL),V(12,12),VT(12,12),T(12,12)
DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
WRITE(6,16)
WRITE(6,17)
DO 50 K = 1,NCST
READ(15,25) IC,JC,NC,TH,ANU,EE
NODENT(1) = IC
NODENT(2) = JC
NODENT(3) = NC

CLEANUP
XJI = XX(JC) - XX(IC)
XNI = XX(NC) - XX(IC)
XNJ = YY(NC) - YY(JC)
YJI = YY(JC) - YY(IC)
YNJ = YY(NC) - YY(JC)
ZJI = ZZ(JC) - ZZ(IC)
ZNI = ZZ(NC) - ZZ(IC)
EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
EK = SQRT(XNI**2 + YNI**2 + ZNI**2)
EH = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
WHY = (EK**2 + EL**2 - EH**2)/(2.0*EL)
H1 = SQRT(EK**2 - WHY**2)

AREA = 0.5*EL*H1

C** LOCAL COORDINATE DIMENSIONS

XBI = 0
YBI = 0
XBJ = EL
YBJ = 0
XBN = WHY
YBN = H1
XBJJ = XBJ - XBI
XBNJ = XBN - XBJ
XNON = XBN = XBI
YNON = YBN = YBI

DO 10 JC = 1:3

NK = NO(JC)
DO 10 K2 = 1,3
V(K2,JC) = U(K2,NK)
10 CONTINUE

CALL PLATRA(IC,JC,NC,XX,YY,ZZ,NODAL,T)
CALL AMTMUL(3,3,TV,VT,12)
EPSXX = (VT(1,2) - VT(1,1))/XBJJ
EPSYY = (VT(2,3)*XBJJ + VT(2,1)*XBNJ - VT(2,2)*XBNJ)/(XBJJ*YBNJ)
EPSXY = (VT(1,3)*XBJJ + VT(1,1)*XBNJ - VT(1,2)*XBNJ)/(XBJJ*YBNJ)

C(VT(2,2) - VT(2,1))/XBJJ
IF(JID.EQ.1) GO TO 40

C** PLANE STRAIN ANALYSIS

A = (1.0 - ANU)*EE/((1.0 + ANU) *(1.0 - 2.0*ANU))
SIGX = A*(EPSXX + ANU/EPSYY)
SIGY = A*(EPSYY + ANU/EPSXX)

GO TO 45

C** PLANE STRESS ANALYSIS

40 A = EE/(1.0-ANU**2)
SIGX = A*(EPSXX + ANU*EPSYY)
SIGY = A*(EPSYY + ANU*EPSXX)

45 SHEAR = EE/(2.0*(1.0 + ANU)*EPSXY)
WRITE(6,15) (NOD(I),I=1:3),EPSXX,EPSYY,EPSXY,SIGX,SIGY,SHEAR
IF(JID.LT.2) GO TO 50

C** IDEALIZED TRIANGLE STRESS RETRIEVAL

WRITE(6,27)
WRITE(6,21)
COST = WHY/EL
COTT = WHY/H1
TANT = 1.0/COTT
SINT = H1/EL
TANPH = H1/(EL-WHY)
COSPH = (EL-WHY)/EH
SINPH = H1/EH
AC = 0.5*(EK/COST - EL)*COTT
DC = 0.5*(EK*TAN - (EK/COST - EL)/SINT)
BC = 0.5*(EH*TANPH - (EH/COSPH - EL)/SINPH)

HDASH = 2.*3.*AREA*(1./EL + 1./EK + 1./EH)

RA = TH*HDASH**3/(12.*0.*(AC**2+DC**2+BC**2))

C** IDEALIZED TRIANGLE STIFFENER LOADS

C** ROD NO. 1
Q2 = (U(1,JC)*XJI + U(2,JC)*YJI + U(3,JC)*ZJI)/EL
Q1 = (U(1,IC)*XJI + U(2,IC)*YJI + U(3,IC)*ZJI)/EL
EPSXX = (Q2 - Q1)/EL
SIGMA = EPSXX*EE
FAX = SIGMA*RA
WRITE(6,26)IC,JC,RA,EE,EL,FAX,SIGMA

C** ROD NO. 2
Q2 = (U(1,NC)*XNJ + U(2,NC)*YNJ + U(3,NC)*ZNJ)/EH
Q1 = (U(1,JC)*XNJ + U(2,JC)*YNJ + U(3,JC)*ZNJ)/EH
EPSXX = (Q2 - Q1)/EH
SIGMA = EPSXX*EE
FAX = SIGMA*RA
WRITE(6,26)JC,NC,RA,EE,EH,FAX,SIGMA

C** ROD NO. 3
Q2 = (U(1,NC)*XNI + U(2,NC)*YNI + U(3,NC)*ZNJ)/EK
Q1 = (U(1,IC)*XNI + U(2,IC)*YNI + U(3,IC)*ZNJ)/EK
EPSXX = (Q2 - Q1)/EK
SIGMA = EPSXX*EE
FAX = SIGMA*RA
WRITE(6,26)IC,NC,RA,EE,Ek,FAX,SIGMA

50 CONTINUE
15 FORMAT(10X,3I4,4X,6(E10.4,4X))
16 FORMAT(10X,'C.S.T. ELEMENT STRESSES AND STRAINS')
17 FORMAT(10X,'NODAL PT.NO. \5X', 'STRAIN-XX', '5X', 'STRAIN-YY', '5X', 'STRAIN-XY', '5X', 'STRESS-XX', '5X', 'STRESS-YY', '5X', 'SHEAR-XY', '5X')
21 FORMAT(2X,'ROD NOD-PTS', '3X', 'AREA', '4X', 'E-VALUE', '4X', 'LEN')
25 FORMAT(10X,'AXIAL LOAD', '4X', 'STRESS')
26 FORMAT(10X,4X,2I4,4X,5(E10.4,4X))
27 FORMAT(10X,'IDEALIZED TRIANGLE STIFFENER LOADS')
RETURN
END
WRITE(6,5)
GO TO 14
10 WRITE(6,6)
14 DO 50 ISTR = 1,NSHTR
READ(5,15)IS,JS,MS,NS,T,ANU,EE
WRITE(6,25)IS,JS,MS,NS,T,ANU,EE
NOD(1) = IS
NOD(2) = JS
NOD(3) = MS
NOD(4) = NS
C** TRANSFORM THE ELEMENT INTO A LOCAL COORDINATE SYSTEM
XJI = XX(JS) - XX(IS)
XNI = XX(NS) - XX(IS)
XMJ = XX(MS) - XX(JS)
XMI = XX(MS) - XX(IS)
XNJ = XX(NS) - XX(JS)
YNJ = YY(NS) - YY(JS)
YMI = YY(MS) - YY(IS)
YMJ = YY(MS) - YY(JS)
YNI = YY(NS) - YY(IS)
YUJ = YY(JS) - YY(IS)
ZUJ = ZZ(JS) - ZZ(IS)
ZNI = ZZ(NS) - ZZ(IS)
ZMJ = ZZ(MS) - ZZ(JS)
ZMI = ZZ(MS) - ZZ(IS)
ZNJ = ZZ(NS) - ZZ(JS)
C** CALCULATE THE LENGTHS OF THE SIDES + DIAGONALS OF THE QUADRILATERAL
EL = SQRT(XJI**2 + YJI**2 + ZUJ**2)
EH = SQRT(XNI**2 + YNI**2 + ZNI**2)
EI = SQRT(XMJ**2 + YMJ**2 + ZMJ**2)
IF (EL.GT.0.0 .AND.EH.GT.0.0 .AND.EI.GT.0.0) GO TO 7
WRITE(6,35) ISTR
CALL EXIT
7 EK = SQRT(XMI**2 + YMI**2 + ZMI**2)
EM = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
EX = (EL**2 + EI**2 - EK**2)/(2.0*EL)
WHY = (EH**2 + EL**2-EM**2)/(2.0*EL)
H1 = SQRT(EH**2 - WHY**2)
H2 = SQRT(EI**2 - EX**2)
C** DIMENSIONS IN LOCAL COORDINATES
XBI = 0
YBI = 0
XBJ = EL
YBJ = 0
XBM = EL - EX
YBM = H2
XBN = WHY
YBN = H1
XBNJ = XBN - XBI
XBJ = XBN - XBJ
XPM = XBM - XBI
YBN = YBN - YBI
YBI = YBM - YBI
XBJ = XBJ - XBI
YBNJ = YBN - YBJ
AREA = XBJI*YBMI + XBMI*YBNJ -XBI*YBMI
G = EE/(2.0*1.0 + ANU)
Z = T*G/(2.0*AREA)
AA = (XBNJ**2)*Z
B = YBNI*XBNJ*Z
C = XBMIXUNJ*Z
D = YBMIXUNJ*Z
E = (YBMIX**2)*Z
F = YBMIX*YBMIX*Z
GE = YBMIX*YBMIX*Z
A1 = (XBMIX**2)*Z
AJ = YBMIX*YBMIX*Z
AL = (YBMIX**2)*Z

S(1,1) = AA
S(5,1) = -AA
S(5,5) = AA
S(2,1) = -B
S(6,1) = B
S(5,2) = B
S(6,5) = -B
S(3,1) = -C
S(7,1) = C
S(5,3) = C
S(7,5) = -C
S(4,1) = D
S(8,1) = -D
S(5,4) = -D
S(8,5) = D
S(2,2) = E
S(6,6) = E
S(6,2) = -E
S(3,2) = F
S(7,2) = -F
S(6,3) = -F
S(7,6) = F
S(4,2) = -GE
S(8,2) = GE
S(6,4) = GE
S(8,6) = -GE
S(3,3) = AI
S(7,7) = AI
S(7,3) = -AI
S(4,3) = -AJ
S(8,3) = AJ
S(7,4) = AJ
S(8,7) = -AJ
S(4,4) = AL
S(8,8) = AL
S(8,4) = -AL

NN = N+1
DO 20 J = 1, NN
IB = J + 1
DO 20 I = IB, N
S(J, I) = S(I, J)

20 CONTINUE
CALL PREPES(8, S, ST)
CALL PLATRA(IS, JS, NS, XX, YY, ZZ, NODAL, TR)
CALL TRPOSE(TR, TRT, 12, 12)
CALL AMTMUL(12, 12, STR, TR, STF, 12)
CALL ELEPAC(12, STF, A, NOAL, IMATK, NBBAND)
IF(J(JD, EQO) /GO TO 50

C** STIFFENERS FOR IDEALIZED PANEL
A1 = H1*T/6.0
A2 = H2*T/6.0
A3 = EL*T/6.0
A4 = (EL-EX-WHY)*T/6.0
AREA(1) = (A1 + A2)/2.0
AREA(2) = (A3 + A4)/2.0

** C**
ROD NO. 1
RA = AREA(1)
CALL RODEL3(RA,EE,XX,YY,ZZ,IS,JS,STF,NODAL)
NOD(1) = IS
NOD(2) = JS
CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)

** C**
ROD NO. 2
CALL RODEL3(RA,EE,XX,YY,ZZ,MS,NS,STF,NODAL)
NOD(1) = MS
NOD(2) = NS
CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)

** C**
ROD NO. 3
RA = AREA(2)
CALL RODEL3(RA,EE,XX,YY,ZZ,IS,NS,STF,NODAL)
NOD(1) = IS
NOD(2) = NS
CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)

** C**
ROD NO. 4
CALL RODEL3(RA,EE,XX,YY,ZZ,JS,MS,STF,NODAL)
NOD(1) = JS
NOD(2) = MS
CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)

50 CONTINUE
5 FORMAT(10X,'SHEAR PANEL DATA',/)
6 FORMAT(10X,'IDEALIZED PANEL DATA',/)
15 FORMAT( )
25 FORMAT(10X,'4I6,3E13.4')
35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDINATES OF SHEAR OR IDEALIZED PANEL - CARD NO.',16/)RETURN
END

** SUBROUTINE TRPOSE(A,ATR,IR,IC)
C**
SUBROUTINE TRANPOSES THE A-MATRIX TO THE ATR-MATRIX
DIMENSION A(IR,IC),ATR(IC,IR)
DO 10 I = 1,IR
DO 10 J = 1,IC
ATR(J,I) = A(I,J)
10 CONTINUE
RETURN
END

** SUBROUTINE PREPEL(ORDER,S,SA)
C**
SUBROUTINE FILLS IN ROWS AND COLUMNS OF ZERO'S IN 2D MATRIX
C**
TO PREPARE IT FOR MULTIPLICATION WITH THE TRANSFORMATION MATRIX.
INTEGER ORDER
DIMENSION SA(12,12), TEMP(64), S(ORDER,ORDER)
NONDER = ORDER/2 + ORDER

** VACATE SA MATRIX

DO 10 I = 1, NONDER
DO 10 J = 1, NONDER
SA(I, J) = 0
10 CONTINUE

IT = 0
DO 20 J = 1, NONDER
DO 20 I = 1, NONDER
IT = IT + 1
TEMP(IT) = S(I, J)
20 CONTINUE

IT = 0
DO 30 J = 1, NONDER
IF(MOD(J, 3) .EQ. 0)
25 DO 40 I = 1, NONDER
IF(MOD(I, 3) .EQ. 0)
30 IT = IT + 1
SA(I, J) = TEMP(IT)
40 CONTINUE
50 CONTINUE
RETURN
END

SUBROUTINE ELEPAC(ORDER, STF, A, NOD, IMATK, NBAND)
INTEGER ORDER
DIMENSION A(IMATK, NBAND), STF(12, 12), NOD(4)
NORDER = ORDER/3

C** ESTABLISH ROWS
DO 350 JROW = 1, NORDER
IR = NOD(JROW)
NROWK = (IR-1)*3
IF(NROWK .LT. 305) NROWK = 305
305 DO 350 JDI = 1, 3
NROWK = NROWK + 1
IRSTF = (JROW-1)*3 + JDI
C** ESTABLISH COLUMNS
DO 320 KCOL = 1, NORDER
IC = NOD(KCOL)
NCOLK = (IC-1)*3
DO 320 KDI = 1, 3
ICSTF = (KCOL - 1)*3 + KDI
NCOLK = NCOLK + KDI + 1 - NROWK
C** SKIP STORING IF BELOW BAND
IF (NCOLK .LT. 310)
310 A(NROWK, NCOLK) = A(NROWK, NCOLK) + STF(IRSTF, ICSTF)
320 CONTINUE
330 CONTINUE
350 CONTINUE
RETURN
END

SUBROUTINE PANSTR(NSHTR, U, NODAL, XX, YY, ZZ, JID)
C** CALCULATION OF STRESS AND STRAIN IN A SHEAR QUADRILATERAL AND
C** IDEALIZED PANEL ELEMENT
DIMENSION NOD(4), U(3, NODAL), V(12, 12), VT(12, 12), T(12, 12)
DIMENSION XX(NODAL), YY(NODAL), ZZ(NODAL)
DIMENSION A(5, 5), B(5), AA(5, 5), AREA(2)
DO 50 KS = 1, NSHTR
READ(5, 25) IS, JS, MS, NS, TH, ANU, EE
NOD(1) = IS
NOD(2) = JS
NOD(3) = MS
NOD(4) = NS
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C** SET UP LOCAL COORDINATE SYSTEM

\[
\begin{align*}
G &= \frac{EE}{(2.0 \times (1.0 + ANU))} \\
\text{XX(MS)} &= \text{XX(NS)} \\
\text{YY(MS)} &= \text{YY(NS)} \\
\text{ZZ(MS)} &= \text{ZZ(NS)} \\
\text{XX(JS)} &= \text{XX(IS)} \\
\text{YY(JS)} &= \text{YY(IS)} \\
\text{ZZ(JS)} &= \text{ZZ(IS)} \\
\end{align*}
\]

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\[
\begin{align*}
\text{XX(IS)} &= \text{XX(MS)} - \text{XX(JS)} \\
\text{YY(IS)} &= \text{YY(MS)} - \text{YY(JS)} \\
\text{ZZ(IS)} &= \text{ZZ(MS)} - \text{ZZ(JS)} \\
\text{XNI} &= \text{XX(NS)} - \text{XX(IS)} \\
\text{YNI} &= \text{YY(NS)} - \text{YY(IS)} \\
\text{ZNI} &= \text{ZZ(NS)} - \text{ZZ(IS)} \\
\end{align*}
\]

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\[
\begin{align*}
\text{XYM} &= \text{XX(MS)} - \text{XX(JS)} \\
\text{YYM} &= \text{YY(MS)} - \text{YY(JS)} \\
\text{ZYM} &= \text{ZZ(MS)} - \text{ZZ(JS)} \\
\end{align*}
\]

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\[
\begin{align*}
\text{XX(JS)} &= \text{XX(MS)} - \text{XX(JS)} \\
\text{YY(JS)} &= \text{YY(MS)} - \text{YY(JS)} \\
\text{ZZ(JS)} &= \text{ZZ(MS)} - \text{ZZ(JS)} \\
\end{align*}
\]

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\[
\begin{align*}
\text{EL} &= \sqrt{\text{XX(IS)}^2 + \text{YY(IS)}^2 + \text{ZZ(IS)}^2} \\
\text{EH} &= \sqrt{\text{XX(IS)}^2 + \text{YY(IS)}^2 + \text{ZZ(IS)}^2} \\
\text{EJ} &= \sqrt{\text{XX(IS)}^2 + \text{YY(IS)}^2 + \text{ZZ(IS)}^2} \\
\text{EK} &= \sqrt{\text{XX(IS)}^2 + \text{YY(IS)}^2 + \text{ZZ(IS)}^2} \\
\text{EM} &= \sqrt{\text{XX(IS)}^2 + \text{YY(IS)}^2 + \text{ZZ(IS)}^2} \\
\text{EJ} &= \sqrt{\text{XX(IS)}^2 + \text{YY(IS)}^2 + \text{ZZ(IS)}^2} \\
\text{EX} &= \frac{\text{EL}^2 + \text{EJ}^2 - \text{EK}^2}{2.0} \\
\text{WHY} &= \frac{\text{EH}^2 + \text{EL}^2 - \text{EM}^2}{2.0} \\
\text{H1} &= \sqrt{\text{EH}^2 - \text{WHY}^2} \\
\text{H2} &= \sqrt{\text{EL}^2 - \text{WHY}^2} \\
\text{H3} &= \frac{\text{H2} + \text{H1}}{2.0} \\
\end{align*}
\]

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\[
\begin{align*}
\text{DO} 10 & \text{ JC } = 1 \times 4 \\
\text{NK} & = \text{NOD} (\text{JC}) \\
\text{DO} 10 & \text{ K2 } = 1 \times 3 \\
\text{V}(\text{K2,JC}) & = \text{U} (\text{K2}, \text{NK}) \\
\text{10} & \text{ CONTINUE} \\
\text{CALL} & \text{ PLATRA (IS,JS,NS,XX,YY,ZZ,NODAL,T)} \\
\text{CALL} & \text{ AMTMUL (3,4,T,V,vT,12)} \\
\text{DO} 20 & \text{ I } = 1 \times 5 \\
\text{DO} 20 & \text{ J } = 1 \times 5 \\
\text{A}(1,J) & = 0.0 \\
\text{20} & \text{ CONTINUE} \\
\text{B}(5) & = \text{VT}(2,2) - \text{VT}(2,1) \\
\text{B}(2) & = \text{VT}(1,3) - \text{VT}(1,1) - (\text{EL} - \text{EX})/\text{EL} \times (\text{VT}(1,2) - \text{VT}(1,1)) \\
\text{B}(3) & = \text{VT}(2,3) - \text{VT}(2,1) \\
\text{B}(4) & = \text{VT}(1,4) - \text{VT}(1,1) - \text{WHY}/\text{EL} \times (\text{VT}(1,2) - \text{VT}(1,1)) \\
\text{B}(1) & = \text{VT}(2,4) - \text{VT}(2,1) \\
\text{A}(5,2) & = -0.5 \times (\text{EL}^2) \\
\text{A}(2,2) & = \text{H2} \times (\text{EL} - \text{EX}) \\
\text{A}(3,2) & = -0.5 \times (\text{EL} - \text{EX})^2 \\
\text{A}(4,2) & = \text{WHY} \times \text{H1} \\
\text{A}(1,2) & = -0.5 \times (\text{WHY}^2) \\
\text{A}(3,1) & = \text{H2} \\
\text{A}(1,1) & = \text{H1} \\
\text{A}(2,3) & = -0.5 \times (\text{H2}^2) \\
\text{A}(3,3) & = (\text{EL} - \text{EX}) \times \text{H2} \\
\text{A}(4,3) & = -0.5 \times (\text{H1}^2) \\
\text{A}(1,3) & = \text{WHY} \times \text{H1} \\
\text{A}(2,4) & = \text{H2} \\
\text{A}(4,4) & = \text{H1} \\
\end{align*}
\]
A(5,5) = EL
A(3,5) = (EL-EX)
A(1,5) = WHY
CALL STMEQ(5,A,AA,B,X,5)
EPSXY = X(4) + X(5)
TAU = EPSXY*G
WRITE (6,15) (NOD(I), I = 1:4), EL, H3, G, TH, EPSXY, TAU
IF (JID,E9.0) GO TO 50
A1 = H1*TH/6.0
A2 = H2*TH/6.0
A3 = EL*TH/6.0
A4 = (EL-EX-WHY)*TH/6.0
AREA(1) = (A1 + A2)/2.0
AREA(2) = (A3 + A4)/2.0
WRITE (6,35)
WRITE (6,21)
C** ROD NO. 1
Q2 = (U(1,JS)*XJI + U(2,JS)*YJI + U(3,JS)*ZJI)/EL
Q1 = (U(1,IS)*XJI + U(2,IS)*YJI + U(3,IS)*ZJI)/EL
EPSXX = (Q2 - Q1)/EL
SIGMA = EPSXX*EE
FAX = SIGMA*AREA(1)
WRITE (6,26) IS, JS, AREA(1), EE, EL, FAX, SIGMA
C** ROD NO. 2
Q2 = (U(1,MS)*XMN + U(2,MS)*YMN + U(3,MS)*ZMN)/EI
Q1 = (U(1,NS)*XMN + U(2,NS)*YMN + U(3,NS)*ZMN)/EI
EPSXX = (Q2 - Q1)/EI
SIGMA = EPSXX*EE
FAX = SIGMA*AREA(1)
WRITE (6,26) NS, MS, AREA(1), EE, EI, FAX, SIGMA
C** ROD NO. 3
Q2 = (U(1,JS)*XMJ + U(2,JS)*YMJ + U(3,JS)*ZMJ)/EI
Q1 = (U(1,IS)*XMJ + U(2,IS)*YMJ + U(3,IS)*ZMJ)/EI
EPSXX = (Q2 - Q1)/EI
SIGMA = EPSXX*EE
FAX = SIGMA*AREA(2)
WRITE (6,26) JS, MS, AREA(2), EE, EI, FAX, SIGMA
C** ROD NO. 4
Q2 = (U(1,NS)*XNI + U(2,NS)*YNI + U(3,NS)*ZNI)/EH
Q1 = (U(1,IS)*XNI + U(2,IS)*YNI + U(3,IS)*ZNI)/EH
EPSXX = (Q2 - Q1)/EH
SIGMA = EPSXX*EE
FAX = SIGMA*AREA(2)
WRITE (6,26) NS, IS, AREA(2), EE, EH, FAX, SIGMA
WRITE (6,25)
END
50 CONTINUE
15 FORMAT (2X,4I5,6E13.4)
21 FORMAT (2X, ROD NOD, PTS,3X, AREA ,4X, E-VALUE ,4X, LEN
CGTH ,4X, AXIAL LOAD,4X, STRESS ,4X)
25 FORMAT (2X)
26 FORMAT (/4X,2I4,4X,5(E10,4X))
35 FORMAT (2X, 'IDEALIZED PANEL STIFFENER LOADS'2/)
45 FORMAT (/)
SUBROUTINE SIMQ(NFN,ADUP,A,B,X,IMATK)
DIMENSION ADUP(IMATK,IMATK),A(IMATK,IMATK),B(IMATK)
DIMENSION X(IMATK),Y(250)

C*  GAUSS ELIMINATION SOLUTION OF EQUATIONS
C*

DO 820 K=1,NFN
A(1,K)=ADUP(1,K)
IF(K.EQ.1) GO TO 820
A(K+1) = ADUP(K,1)/A(1,1)
820 CONTINUE

DO 850 J=2,NFN
DO 850 K=J,NFN
TOT1=0
MJ=J-1
DO 830 IS=1,MJ
IF(J.NE.IS) GO TO 825
TOT1 = TOT1 + A(IS,K)
825 CONTINUE

A(J,K)= ADUP(J,K)-TOT1
TOT2=0
DO 840 IS=1,MJ
IF(K.NE.IS) GO TO 835
TOT2 = TOT2 + A(IS,J)
835 CONTINUE

IF (K.EQ.J) GO TO 850
A(K,J) = (ADUP(K,J)-TOT2)/A(J,J)
850 CONTINUE

Y(1) = B(1)
DO 860 K=2,NFN
TOT3=0
NK=K-1
DO 870 IS=1,NK
IF (K.NE.IS) GO TO 860
TOT3 = TOT3 + Y(IS)
860 CONTINUE

Y(K) = B(K)-TOT3
870 CONTINUE

X(NFN)=Y(NFN)/A(NFN,NFN)
DO 900 K=2,NFN
KK=(NFN+1)-K
TOT4=0
NKK=KK+1
DO 890 IS=KK,NFN
TOT4=TOT4+ A(KK,IS)*X(IS)
890 CONTINUE

X(KK)= (Y(KK)-TOT4)/A(KK,KK)
900 CONTINUE
RETURN
END
SUBROUTINE AMTMUL(IMR,IMC,F,B,C,ISIZE)
DIMENSION F(ISIZE,ISIZE),B(ISIZE,ISIZE),C(ISIZE,ISIZE)

C** TO MULTIPLY TWO MATRICES (F-COLS = B-ROWS)
C*
DO 20 IC=1,IMR
  DO 20 J=1,IMC
    TOTROW=0
    DO 10 I=1,IMR
      TOTROW=TOTROW+F(IC,I)*B(I,J)
    10 CONTINUE
    C(IC,J)=TOTROW
  20 CONTINUE
RETURN
END

SUBROUTINE TRISH(NTRIS,IMATK,NODAL,A,XX,YY,ZZ,JID,NBAND)
C** STIFFNESS MATRIX OF SHEAR OR IDEALIZED TRIANGLE PANEL
C**outines-SK(6,6),SKP(12,12),TR(12,12) ,SKTR(12,12)
C** DIMENSION NOD(:3) , A(IMATK,NBAND), SKF(12,12)
DIMENSION NOD(:3), A(IMATK,NBAND), SKF(12,12)
N = 6
IF(JID.EQ.1) GO TO 8
WRITE(6,5)
GO TO 14
8 WRITE(6,9)
14 DO 50 IST = 1,NTRIS
  READ(5,15) IS,JS,NS,TH,ANU,EE
  WRITE(6,25) IS,JS,NS,TH,ANU,EE
  NOD(1) = IS
  NOD(2) = JS
  NOD(3) = NS
  G = EE/(2.0*(1.0 + ANU))
C** SET UP LOCAL COORDINATE SYSTEM
  XJI = XX(JS) - XX(IS)
  YJI = YY(JS) - YY(IS)
  ZJI = ZZ(JS) - ZZ(IS)
  XNI = XX(NS) - XX(IS)
  YNI = YY(NS) - YY(IS)
  ZNI = ZZ(NS) - ZZ(IS)
  XNJ = XX(JS) - XX(NS)
  YNJ = YY(JS) - YY(NS)
  ZNJ = ZZ(JS) - ZZ(NS)
  EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
  EK = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
  EH = SQRT(XNI**2 + YNI**2 + ZNI**2)
  IF(EL.GT.0.0. AND.EK.GT.0.0. AND.EH.GT.0.0.) GO TO 7
  WRITE(6,35) IST
  CALL EXIT
  7 WHY = (EK**2 + EL**2 - EH**2)/(2.0*EL)
  IF(WHY.GE.0.0 AND.WHY.LE.EL) GO TO 16
  WRITE(6,17) IST
  CALL EXIT
  16 H1 = SQRT(EK**2 - WHY**2)
  AREA = 0.5*EL*H1
  D = G*TH*AREA/(EL**2)
  SK(1,1) = ((WHY-EL)/H1)**2*D
  SK(2,1) = -(WHY-EL)/H1*D
  SK(3,1) = -(WHY*(WHY-EL)/(H1*H1))*D
  SK(4,1) = (WHY-EL)/H1*D
  SK(5,1) = EL*(WHY-EL)/(H1*H1)*D
  SK(6,1) = 0.0
  SK(2,2) = D
\[ SK(5,2) = - \text{WHY}/H1 \times D \]
\[ SK(5,3) = - \text{WHY} \times \text{EL}/(H1 \times H1) \times D \]
\[ SK(5,4) = 0.0 \]
\[ SK(6,2) = 0.0 \]
\[ SK(6,3) = 0.0 \]
\[ NN = N-1 \]

DO 10 J = 1, NN
IB = J + 1
DO 10 I = IB, N
SK(I, J) = SK(I, J)
10 CONTINUE

CALL PREPEL(6, SK, SKP)
CALL PLATRA(IS, JS, NS, XX, YY, ZZ, NODAL, TR)
CALL TRPOSE(TR, TRT, 12, 12)
CALL AMTMUL(9, 9, TRT, SKP, SKTR, 12)
CALL AMTMUL(9, 9, SKTR, TR, SKF, 12)
CALL ELEPAC(9, SKF, A, NOD, IMATK, NBAND)

IF (JID .GE. 0) GO TO 50

C** IDEALIZED TRIANGLE STIFFENERS ARE ADDED

COST = \text{WHY}/EK
COTT = \text{WHY}/H1
TANT = 1.0/COTT
SINT = H1/EK
TANPH = H1/(EL - \text{WHY})
COSPH = (EL - \text{WHY})/EH
SINPH = H1/EH
AC = 0.5*(EK*COST - EL*COTT)
DC = 0.5*(EK*TANT - (EK/COST - EL)/SINT)
BC = 0.5*(EH*TANPH - (EH/COSPH - EL)/SINPH)

HDASH = 2./3.*AREA*(1./EL + 1./EK + 1./EH)
RA = TH*HDASH**3/(12.0*(AC**2+DC**2+BC**2))

C** ROD NO. 1
CALL RODEL3(RA, EE, XX, YY, ZZ, IS, JS, SKF, NODAL)
NOD(1) = IS
NOD(2) = JS
CALL ELEPAC(6, SKF, A, NOD, IMATK, NBAND)

C** ROD NO. 2
CALL RODEL3(RA, EE, XX, YY, ZZ, JS, NS, SKF, NODAL)
NOD(1) = JS
NOD(2) = NS
CALL ELEPAC(6, SKF, A, NOD, IMATK, NBAND)

C** ROD NO. 3
CALL RODEL3(RA, EE, XX, YY, ZZ, IS, NS, SKF, NODAL)
NOD(1) = IS
NOD(2) = NS
CALL ELEPAC(6, SKF, A, NOD, IMATK, NBAND)

50 CONTINUE
5 FORMAT(10X, 'SHEAR TRIANGLE DATA',/)
9 FORMAT(10X, 'IDEALIZED TRIANGLE DATA',/)
15 FORMAT(
17 FORMAT(10X, '*** ERROR *** BADLY SHAPED SHEAR TRIANGLE. CARD NO.',/
C16,11)
25 FORMAT(10X, '*** ERROR *** INCORRECT NUMBERING OR NODAL POINT COORDINATES OF SHEAR TRIANGLE. CARD NO.',16$/)
RETURN
END
SUBROUTINE RODPAK(NROD3,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
C** SUBROUTINE READS DATA AND PACKS ENLOAD ELEMENTS INTO STRUCTURE
C** K-MATRIX
C*
DIMENSION NOD(2),RODS(12,12),A(IMATK,NBAND)
DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
WRITE(6,14)
DO 320 JROD = 1,NROD3
READ(5,10)JR,KR,RA,RE
WRITE(6,15)JR,KR,RA,RE
CALL RODEL3(RA,RE,XX,YY,ZZ,JR,KR,RODS,NODAL)
NOD(1) = JR
NOD(2) = KR
CALL ELEPAC(6,RODS,A,NOD,IMATK,NBAND)
320 CONTINUE
10 FORMAT( )
14 FORMAT(10X,*FLANGE ELEMENT DATA*/)
15 FORMAT(10X,216,2*(E10.4,2X))
RETURN
END

SUBROUTINE RODE33(NROD3,U,NODAL,XX,YY,ZZ)
C** CALCULATION OF LOADS IN ENLOAD ELEMENTS.
C*
DIMENSION U(3,NODAL),XX(NODAL),YY(NODAL),ZZ(NODAL)
DO 960 JROD = 1,NROD3
READ(5,10)JR,KR,RA,RE
DIFX = XX(KR) - XX(JR)
DIFF = YY(KR) - YY(JR)
DIFZ = ZZ(KR) - ZZ(JR)
EL = SQRT(DIFX**2 + DIRY**2 + DIFZ**2)
Q2 = (U(1,KR)*DIFX + U(2,KR)*DIFY + U(3,KR)*DIFZ)/EL
Q1 = (U(1,JR)*DIFX + U(2,JR)*DIFY + U(3,JR)*DIFZ)/EL
EPSXX = (Q2-Q1)/EL
SIGMA = EPSXX*RE
FAX = SIGMA*RA
WRITE(6,25)JR,KR,RA,RE,EL,FAX,SIGMA
960 CONTINUE
10 FORMAT( )
25 FORMAT(/4X,214,4X,*E10.4,4X))
RETURN
END

SUBROUTINE QUATRI(XX,YY,ZZ,TH,EE,ANU,NODAL,SK,N60)
C** STIFFNESS MATRIX OF QUADRILATERAL ELEMENT MADE UP OF 4 CST ELEMENTS
C*
DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
DIMENSION SK(10,10),EMAT(6,6)
DIMENSION B(6,6),NOD(4),X(5),Y(5),LNOD(12)
DIMENSION ITNOD(3),BTR(6,6),BE(6,6),S(36)
I = NOD(1)
J = NOD(2)
M = NOD(3)
N = NOD(4)
C** SET UP LOCAL COORDINATE SYSTEM
XJI = XX(IJ) - XX(I)
YJI = YY(J) - YY(I)

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ZJI = ZZ(J) - ZZ(I)
XNI = XX(N) - XX(I)
YNI = YY(N) - YY(I)
ZNI = ZZ(N) - ZZ(I)
XMI = XX(M) - XX(I)
YMI = YY(M) - YY(I)
ZMI = ZZ(M) - ZZ(I)
XNJ = XX(N) - XX(J)
YNJ = YY(N) - YY(J)
ZNJ = ZZ(N) - ZZ(J)
XMJ = XX(M) - XX(J)
YMJ = YY(M) - YY(J)
ZMJ = ZZ(M) - ZZ(J)
EL = SQRT(XJI**2+YJI**2+ZJI**2)
EH = SQRT(XNI**2+YNI**2+ZNI**2)
EI = SQRT(XMJ**2+YMJ**2+ZMJ**2)
IF(EL.GT.0.0.AND.EH.GT.0.0.AND.EI.GT.0.0) GO TO 4
WRITE(6,35)
CALL EXIT
4 EK = SQRT(XMI**2+YMI**2+ZMI**2)
EM = SQRT(XNJ**2+YNJ**2+ZNJ**2).
EX = (EL**2 + EI**2 - EK**2)/(2.0*EL)
WHY = (EH**2 + EL**2 - EM**2)/(2.0*EL)
H1 = SQRT(EH**2 - WHY**2)
H2 = SQRT(EI**2 - EX**2)
X(1) = 0.0
Y(1) = 0.0
X(2) = EL
Y(2) = 0.0
X(3) = EL-EX
Y(3) = H2
X(4) = WHY
Y(4) = H1
X(5) = H1*EL*(EL-EX)/(H2*(EL-WHY) + H1*(EL-EX))
Y(5) = H2*X(5)/(EL-EX)
EMULT = EE/(1.0 - ANU**2)
DO 5 IR = 1,10
DO 5 JR = 1,10
5-SK(IR,JR) = 0.0
DO 10 IR = 1,6
DO 10 JR = 1,6
10 EMAT(IR,JR) = 0.0
C** PLANE STRESS E-MATRIX
EMAT(1,1) = EMULT
EMAT(2,2) = EMULT
EMAT(2,1) = EMULT*ANU
EMAT(1,2) = EMULT*ANU
EMAT(3,3) = (1.0-ANU)/2.0*EMULT
LNOD(1) = -1
LNOD(2) = 2
LNOD(3) = 5
LNOD(4) = 5
LNOD(5) = 2
LNOD(6) = 3
LNOD(7) = 4
LNOD(8) = 5
LNOD(9) = 3
LNOD(10) = 1
LNOD(11) = 5
LNOD(12) = 4
ICOUNT = 0
DO 200 ITR = 1,4
IF(ITR .NE. 1) GO TO 20
ICOUNT = 1
IEND = 3
GO TO 30
20 ICOUNT = ICOUNT + 3
IEND = ICOUNT + 2
30 JNOD = 0
DO 40 ILD = ICOUNT, IEND
JNOD = JNOD + 1
40 ITNOD(JNOD) = LNOOD(ILD)
IS = ITNOD(1)
JS = ITNOD(2)
NS = ITNOD(3)
A1 = X(JS)*Y(NS) + X(IS)*Y(JS) + Y(IS)*X(NS)
A2 = X(IS)*Y(JS) + X(NS)*Y(JS) + Y(NS)*X(IS)
AREA = 0.5*ABS(A1-A2)
DO 50 IST = 1,6
DO 50 JST = 1,6
B(1,1) = Y(JS) - Y(NS)
B(1,3) = Y(NS) - Y(IS)
B(1,5) = Y(IS) - Y(JS)
B(2,2) = X(NS) - X(JS)
B(2,4) = X(IS) - X(NS)
B(2,6) = X(JS) - X(IS)
B(3,1) = X(NS) - X(JS)
B(3,2) = Y(JS) - Y(NS)
B(3,3) = B(2,5)
B(3,5) = B(3,3)
B(3,6) = B(3,4)
B(3,4) = B(3,6)
B(2,5) = B(2,2)
B(2,6) = B(2,4)
B(1,5) = B(1,3)
B(1,3) = B(1,1)
B(1,2) = Y(JS) - Y(IS)
B(1,4) = Y(IS) - Y(JS)
B(1,6) = Y(NS) - Y(JS)
B(1,5) = B(1,1)
B(1,2) = Y(JS) - Y(NS)
B(1,4) = Y(NS) - Y(IS)
B(1,6) = B(1,3)
B(1,3) = B(1,5)
DO 60 IST = 1,6
DO 60 JST = 1,6
B(IST,JST) = 0.0
B(1,1) = Y(JST) - Y(NS)
B(1,3) = Y(NS) - Y(IS)
B(1,5) = Y(IS) - Y(JS)
B(2,2) = X(NS) - X(JS)
B(2,4) = X(IS) - X(NS)
B(2,6) = X(JS) - X(IS)
B(3,1) = X(NS) - X(JS)
B(3,2) = Y(JS) - Y(NS)
B(3,3) = B(2,5)
B(3,5) = B(3,3)
B(3,6) = B(3,4)
B(3,4) = B(3,6)
B(2,5) = B(2,2)
B(2,6) = B(2,4)
B(1,5) = B(1,3)
B(1,3) = B(1,1)
B(1,2) = Y(JS) - Y(NS)
B(1,4) = Y(NS) - Y(IS)
B(1,6) = Y(NS) - Y(JS)
B(1,5) = B(1,1)
B(1,2) = Y(JS) - Y(NS)
B(1,4) = Y(NS) - Y(IS)
B(1,6) = B(1,3)
B(1,3) = B(1,5)
DO 60 IST = 1,6
DO 60 JST = 1,6
B(IST,JST) = B(IST,JST)/(2.0*AREA)
CALL TRPOSE(B,BTR,6,6)
CALL AMTMUL(6,3,BTR,EMAT,BE,6)
CALL AMTMUL(6,6,BE,B,BTR,6)
DO 70 KTR = 1,6
DO 70 JTR = 1,6
B(KTR,JTR) = BTR(KTR,JTR)*TH*AREA
C** MAKE LOCAL MATRIX INTO A COLUMN
IT = 0
DO 80 J = 1,6
DO 80 I = 1,6
IT = IT + 1
80 S(IT) = B(I,J)
C** PACK LOCAL MATRIX INTO QUAD MATRIX
IT = 0
DO90 ICOL = 1,3
IK = ITNOD(ICOL)
DO 90 JDI = 1,2
IACOL = (IK-1)*2 + JDI
DO 90 IROW = 1,3
IR = ITNOD(IROW)
DO 90 IDI = 1,2
IAROW = (IR-1)*2 + IDI
IT = IT + 1
SK(IAROW,IACOL) = SK(IAROW,IACOL) + S(IT)
CONTINUE

35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDINATES OF 4-CS1 PANEL.***')
RETURN
END

SUBROUTINE QUA4(NQUA4*IMATK,NODAL,A,XX,YY,ZZ,NBAND)

C** STIFFNESS MATRIX OF 4-NODAL POINT QUADRILATERAL

C** PLANE STRESS ELEMENT OF CONSTANT THICKNESS

C** THE SIDES REMAIN STRAIGHT AFTER DEFORMATION

DIMENSION E(3,3) , SE(8,8) , ST(12,12) , TR(12,12) , TRT(12,12)
DIMENSION XL(4) , YL(4) , B(3,8) , NOD(4) , SA(12,12)
DIMENSION A(IMATK,NBAND) , XX(NODAL), YY(NODAL) , ZZ(NODAL)

WRITE(6,5)
DO 200 IQ = 1,NQUA4
READ(5,15) IS,JS,MS,NS,TH,ANU,EE
WRITE(6,25) IS,JS,MS,NS,TH,ANU,EE
NOD(1) = IS
NOD(2) = JS
NOD(3) = MS
NOD(4) = NS
I = IS
J = JS
M = MS
N = NS

C** SET UP LOCAL COORDINATE SYSTEM.

XJI = XX(J) - XX(I)
YJI = YY(J) - YY(I)
ZJI = ZZ(J) - ZZ(I)
XNI = XX(N) - XX(I)
YNI = YY(N) - YY(I)
ZNI = ZZ(N) - ZZ(I)
XMI = XX(M) - XX(I)
YMI = YY(M) - YY(I)
ZMI = ZZ(M) - ZZ(I)
XNJ = XX(N) - XX(J)
YNJ = YY(N) - YY(J)
ZNJ = ZZ(N) - ZZ(J)
XMJ = XX(M) - XX(J)
YMJ = YY(M) - YY(J)
ZMJ = ZZ(M) - ZZ(J)

EL = SQRT(XJI**2+YJI**2+ZJI**2)
EH = SQRT(XNI**2+YNI**2+ZNI**2)
EI = SQRT(XMI**2+YMI**2+ZMI**2)
EX = (EL**2 + EI**2 - EH**2)/(2.0*EL)
WHY = (EI**2 + EX**2 - EH**2)/(2.0*EL)
H1 = SQRT(EH**2 - WHY**2)
H2 = SQRT(EI**2 - EX**2)

XL(1) = 0.0
YL(1) = 0.0
XL(2) = EL
YL(2) = 0.0
XL(3) = EL*EX
YL(3) = H2
XL(4) = WHY
YL(4) = H1

CALL EXIT

RETURN
END
DO 10 IE = 1,3
DO 10 JE = 1,3
10 E(IE*JE) = 0.0
C** PLANE STRESS E-MATRIX
EMULT = EE/(1.0-ANU**2)
E(1,1) = EMULT
E(2,2) = EMULT
E(1,2) = EMULT*ANU
E(2,1) = EMULT*ANU
E(3,3) = (1.0 - ANU)/2.0*EMULT
C** CLEAR ARRAYS. OVERWRITE MATRIX E BY UPPER TRIANGLE U.
DO 3 K = 1,8
DO 3 L = K,8
3 SE(KL) = 0.0
E(1,1) = SQRT(E(1,1))
E(1,2) = E(1,2)/E(1,1)
E(1,3) = E(1,3)/E(1,1)
E(2,2) = SQRT(E(2,2) - E(1,2)*E(1,2))
E(2,3) = (E(2,3) - E(1,2)*E(1,3))/E(2,2)
E(3,3) = SQRT(E(3,3) - E(1,3)*E(1,3)/E(2,3))
C** START GAUSS QUADRATURE LOOP.
DO 20 II = 1,2
DO 20 JJ = 1,2
CALL SHAPE(II, JJ, XL, YL, B, DETJAC)
C** OVERWRITE MATRIX B BY U*B.
DO 9 K = 1,3
DO 9 L = K,8
9 DUM1 = 0.0
9 DUM2 = 0.0
9 B(KL) = DUM1 + E(K,M)*B(M,L)
9 B(KL) = DUM1
C** ADD IN CONTRIBUTION TO ELEMENT MATRICES.
DUM1 = DETJAC
DO 20 NROW = 1,8
DO 20 NCOL = NROW,8
20 DUM2 = 0.0
20 DUM2 = DUM2 + B(L,NROW)*B(L,NCOL)
C** QUADRATURE ENDED. COMPLETE STIFFNESS MATRIX BY
C** SYMMETRY AND MULTIPLY BY THE PANEL THICKNESS
DO 30 K = 2,8
DO 30 L = K,8
30 SE(KL) = SE(L,K)
30 SE(KL) = SE(L,K)
C** PREPARE, ROTATE AND PACK ELEMENT INTO STRUCTURE
C** STIFFNESS MATRIX
CALL PREPEL(8, SE, ST)
CALL PLATRA(15, JS, NS, XX, YY, ZZ, NODAL, TR)
CALL TPOSE(12, TRT, 12)
CALL AMTMUL(12, 12, TRT, ST, SA, 12)
CALL AMTMUL(12, 12, SA, TR, ST, 12)
CALL ELEPAC(12, ST, A, NOD, IMATK, NBAND)
200 CONTINUE
5 FORMAT(10X, 'QUAT PANEL DATA')
15 FORMAT( )
25 FORMAT(10X, 4I6, 3E13.4)
35 FORMAT(10X, '***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDIN')
IATES OF QUA4 ELEMENT, CARD NO.*16//)
RETURN
END

SUBROUTINE RODEL3(RA, RE, XX, YY, ZZ, JR, KR, RODS, NODAL)
DIMENSION RODS(12), XX(NODAL), YY(NODAL), ZZ(NODAL)

C** CALCULATION OF END-LOAD ELEMENT STIFFNESSES IN 3-DIM.

N = 6
DIFX = XX(KK) - XX(JR)
DIFF = YY(KK) - YY(JR)
DIFZ = ZZ(KK) - ZZ(JR)
EL = SQRT((DIFX**2 + DIFF**2 + DIFZ**2))
IF (EL GT .0 .0) GO TO 7
WRITE(6,35)
CALL EXIT
7 RO = SORT((DIFX**2 + DIFF**2))
C** TRANSFORMATION COEFFICIENTS
Z = RA*HE/EL
T11 = DIFX/EL
T12 = DIFF/EL
T13 = DIFZ/EL
A = T11*T11*Z
B = T11*T12*Z
C = T11*T13*Z
D = T12*T12*Z
E = T12*T13*Z
F = T13*T13*Z
RODS(1,1) = A
RODS(2,2) = B
RODS(3,3) = C
RODS(4,4) = -A
RODS(5,5) = -B
RODS(6,6) = -C
RODS(7,7) = -D
RODS(8,8) = -E
RODS(9,9) = -F
RODS(10,10) = A
RODS(11,11) = B
RODS(12,12) = C
RODS(13,13) = D
RODS(14,14) = E
RODS(15,15) = F
RODS(16,16) = C

M = N+1
DO 20 J = 1,M
20 CONTINUE
35 FORMAT(10X,*ERROR*** AN ENDLOAD ELEMENT HAS NO LENGTH, CHECK NOCO-ORDINATES*,//)
RETURN
END
SUBROUTINE SUPPORT(A,B,IMATK,NBAND,DISP)
C** INSERTION OF BOUNDARY CONDITIONS
DIMENSION A(IMATK,NBAND),B(IMATK)
DO 500 IDISP = 1,IDISP
READ(5,50)INOD,ID,DISP
WRITE(6,515)INOD,ID,DISP
NROWK = (INOD - 1)*3 + ID
A(NROWK,1) = 1.0
B(NROWK) = DISP
DO 430 J = 2,NBAND
NR = NROWK + 1 - J
NC = NROWK-1+J
B(NR) = B(NR) - DISP*A(NR,J)
B(NC) = B(NC) - DISP*A(NROWK,J)
A(NROWK,J) = 0.0
IF(NR)430,430,425
A(NR,J) = 0.0
425 CONTINUE
430 CONTINUE
500 CONTINUE
50 FORMAT( )
515 FORMAT(10X,2I6,E10.4)
RETURN -...
END

SUBROUTINE SOLVE(A,B,NN,IMATK,NBAND)
C** GAUSS ELIMINATION SOLUTION OF EQUATIONS (BANDED COEFFICIENT MATRIX)
DIMENSION A(IMATK,NBAND),B(IMATK)
C** REDUCE MATRIX
DO 300 N = 1,NN
I = N
DO 290 L = 2,NBAND
I = I + 1
IF(A(N,L))240,290,240
240 C = A(N,L)/A(N,1)
J = 0
DO 270 K = L,NBAND
J = J + 1
IF(A(N,K))260,270,260
260 A(I,J) = A(I,J) - C*A(N,K)
270 CONTINUE
280 A(N,L) = C
C** AND LOAD VECTOR
C** FOR EACH EQUATION
B(1) = B(1) - C*B(N)
290 CONTINUE
300 B(N) = B(N)/A(N,1)
C** BACK SUBSTITUTION
N = NN
350 N = N-1
IF(N)500,500,360
360 L = N
DO 400 K = 2,NBAND
L = L + 1
IF(A(N,K))370,400,370
370 B(N) = B(N) - A(N,K)*B(L)
400 CONTINUE
GO TO 350
500 CONTINUE
RETURN
END
SUBROUTINE QUAPAC(NGAT, IMATK, NODAL, A, XX, YY, ZZ, NBAND)

C** STIFFNESS MATRIX FOR 4-CST QUADRILATERAL (PLANE STRESS)

DIMENSION SK(10,10), S(8,8), ST(12,12), TR(12,12)

DIMENSION TRT(12,12), STR(12,12), STF(12,12), NOD(4)

DIMENSION A(IMATK,NBAND), XX(NODAL), YY(NODAL), ZZ(NODAL)

DIMENSION Q10(6), Q9(8)

WRITE(6,5)

DO 100 IOT = 1, NGAT

READ(5,15) IS, JS, MS, NS, TH, ANU, EE

NOD(1) = IS
NOD(2) = JS
NOD(3) = MS
NOD(4) = NS

CALL QUATRI(XX, YY, ZZ, TH, EE, ANU, NODAL, SK, NOD)

C** REDUCE SK MATRIX TO AN 8X8

D1 = SK(10,9)/SK(10,10) - SK(9,9)/SK(9,10)
D2 = SK(10,10)/SK(10,9) - SK(9,10)/SK(9,9)

DO 50 I3 = 1, 8

Q10(I3) = SK(9, I3)/(SK(9, 9)*D2) - SK(10, I3)/(SK(10, 9)*D2)
Q9(I3) = SK(9, I3)/(SK(9, 10)*D1) - SK(10, I3)/(SK(10, 10)*D1)

50 CONTINUE

WRITE(6,25)

DO 70 ICOL = 1, 8

S(IROW, ICOL) = SK(IROW, ICOL) + Q9(ICOL)*SK(IROW, 9) + O10(ICOL)*CSK(IROW, 10)

70 CONTINUE

CALL PREPEL(8, S, ST)

CALL PLATRA(IS, JS, NS, XX, YY, ZZ, NODAL, TR)

CALL TRPOSE(TR, TRT, 12, 12)

CALL AMTMUL(12, 12, TRT, ST, STR, 12)

CALL AMTMUL(12, 12, STR, TR, STF, 12)

CALL ELEPAC(12, STF, A, NOD, IMATK, NBAND)

CONTINUE

5 FORMAT(10X, '4-CST PANEL DATA',/) 15 FORMAT(10X)

25 FORMAT(10X, 416, 3E13.4)

RETURN

END

SUBROUTINE PLATRA(IS, JS, NS, XX, YY, ZZ, NODAL, T)

C** PLANE ELEMENT TRANSFORMATION MATRIX (VECTOR METHOD)

C** LOCAL COORDINATE SYSTEM SET UP WITH I AND J ON THE LOCAL X-AXIS

DIMENSION T(12,12), XX(NODAL), YY(NODAL), ZZ(NODAL)

XNI = XX(NS) - XX(IS)
YNI = YY(NS) - YY(IS)
ZNI = ZZ(NS) - ZZ(IS)
XJI = XX(JS) - XX(IS)
YJI = YY(JS) - YY(IS)
ZJI = ZZ(JS) - ZZ(IS)
XNJ = XX(NS) - XX(JS)
YNJ = YY(NS) - YY(JS)
ZNJ = ZZ(NS) - ZZ(JS)

EL = SORT(XJI**2 + YJI**2 + ZJI**2)
EK = SORT(XNI**2 + YNI**2 + ZNI**2)
EH = SORT(XNJ**2 + YNJ**2 + ZNJ**2)

WHY = (EK*2 + EL*2 - EH*2)/(2.0*EL)
H1 = SORT(EK**2 - WHY**2)

AREA = 0.5*EL*H1
A = XJI/EL
B = YJI/EL
C = ZJI/EL

G = (ZJI*YNI - ZJI*YN1)/(2.0*AREA)
H = (ZJI*XNI - XJI*ZNI)/(2.0*AREA)
P = (XJI*YN1 - YJI*XNI)/(2.0*AREA)
D = H*C - P*B
E = P*A - G*C
F = G*B - A*H

DO 10 I = 1,12
DO 10 J = 1,12

T(I,J) = 0
10 CONTINUE

T(1,1) = A
T(1,2) = B
T(1,3) = C
T(2,1) = D
T(2,2) = E
T(2,3) = F
T(3,1) = G
T(3,2) = H
T(3,3) = P
T(4,4) = A
T(4,5) = B
T(4,6) = C
T(5,4) = D
T(5,5) = E
T(5,6) = F
T(6,4) = G
T(6,5) = H
T(6,6) = P
T(7,7) = A
T(7,8) = B
T(7,9) = C
T(8,7) = D
T(8,8) = E
T(8,9) = F
T(9,7) = G
T(9,8) = H
T(9,9) = P
T(10,10) = A
T(10,11) = B
T(10,12) = C
T(11,10) = D
T(11,11) = E
T(11,12) = F
T(12,10) = G
T(12,11) = H
T(12,12) = P

RETURN
END
SUBROUTINE MASTR(NGAT, U, NODAL, XX, YY, ZZ)
C** STRESSES IN THE 4-CST PANEL (PLANE STRESS)
DIMENSION NOD(4), U(3, NODAL), V(12, 12), Vt(12, 12)
DIMENSION T(12, 12), XX(NODAL), YY(NODAL), ZZ(NODAL)
DIMENSION EPSX(4), EPSY(4), EPSXY(4), SK(10, 10), DUP(8)
DIMENSION SIGX(4), SIGY(4), SIGXY(4)
WRITE(6, 60)
DO 100 I = 1, NGAT
READ(5, 15) IS, JS, MS, NS, TH, ANU, EE
NOD(1) = IS
NOD(2) = JS
NOD(3) = MS
NOD(4) = NS
C** SET-UP LOCAL COORDINATE SYSTEM
XXN = XX(NS) - XX(NS)
YYN = YY(NS) - YY(NS)
ZZN = ZZ(NS) - ZZ(NS)
XXJ = XX(JS) - XX(IS)
YYJ = YY(JS) - YY(IS)
ZZJ = ZZ(JS) - ZZ(IS)
XXI = XX(NS) - XX(IS)
YYI = YY(NS) - YY(IS)
ZZI = ZZ(NS) - ZZ(IS)
XXU = XX(VS) - XX(JS)
YYU = YY(VS) - YY(JS)

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C** SECONDTRIANGLE
A1 = EL+H2 + Y0*(EL-EX)
A2 = EL+Y0 + (H2-Y0)*DUP(3)+Y0*DUP(5))/2.0*AREA
A3 = EL+H2 + Y0*DUP(3)+Y0*DUP(5))/2.0*AREA
C = DUP(5) + Y0*DUP(6))/2.0*AREA
C   SUBROUTINE TR
   
   A1 = X1+H1
   A2 = X2+H2
   L1A = H1*H1
   L2A = H2*H2
   C1 = (A1-A2)*A1
   
   C = (L1A+L2A)*(H1+H2)*0.5
   
   IF (JTR) GO TO 7
   
   DO 50
   
   DO 50 I = 1, NTRIM
   
   WRITE(6,8)
   7 WRITE(6,8)
   14 DO 50 ICST = 1,NTRIM
   
   STIFFNESS OF CST ELEMENT (PLANE STRESS OR PLANE STRAIN)
   
   WRITE(6,5)
   
   IF(JID,EQ.1) GO TO 7
   
   7 WRITE(6,8)
   
   C** STIFFNESS OF CST ELEMENT (PLANE STRESS OR PLANE STRAIN)
   
   DIMENSION EMAT(6,6),B(6,6),BTR(6,6),BE(6,6)
   
   DIMENSION CSP(12,12),TR(12,12),TRT(12,12),CSTR(12,12)
   
   DIMENSION NODA(NOD)+A(IMATK,NBAND),CSF(12,12)
   
   IF(JID.EQ.1) GO TO 7
   
   WRITE(6,5)
   
   GO TO 14
   
   WRITE(6,8)
   
   DO 50 ICST = 1,NTRIM
   
   READ(5,15)IC,JC,NC,TH,ANU,EE
   
   WRITE(6,25)IC,JC,NC,TH,ANU,EE
   
   NOD(1) = IC
C** SET UP LOCAL COORDINATE SYSTEM

\[ \begin{align*}
X_{JI} &= X_J - X_I \\
X_{NI} &= X_N - X_I \\
X_{NJ} &= X_N - X_J \\
Y_{JI} &= Y_J - Y_I \\
Y_{NI} &= Y_N - Y_I \\
Y_{NJ} &= Y_N - Y_J \\
Z_{JI} &= Z_J - Z_I \\
Z_{NI} &= Z_N - Z_I \\
Z_{NJ} &= Z_N - Z_J \\
E_L &= \sqrt{(X_{JI}^2 + Y_{JI}^2 + Z_{JI}^2)} \\
E_K &= \sqrt{(X_{NI}^2 + Y_{NI}^2 + Z_{NI}^2)} \\
E_Z &= \sqrt{(X_{NJ}^2 + Y_{NJ}^2 + Z_{NJ}^2)} \\
\end{align*} \]

IF \((E_L \geq E_K \text{ AND } E_K \geq E_Z)\) GO TO 13

WRITE (6,35) ICST
CALL EXIT

13 WHY = \((E_K + E_L - E_Z) / (2.0 \times E_L)\)

IF \((WHY \leq 0.0 \text{ AND } WHY \geq 0.0)\) GO TO 16

WRITE (6,17) ICST
CALL EXIT

16 \(H_1 = \sqrt{(E_K^2 - WHY^2)}\)

AREA = \(0.5 \times E_L \times H_1\)

EMAT(IR,IC) = 0.0

10 B(IR,IC) = 0.0

IF \((JID \neq 1)\) GO TO 20

C** PLANE STRAIN E-MATRIX

\[ \begin{align*}
EMULT &= E/((1.0 - ANU/ANU) - 2.0 \times ANU)) \\
EMAT(1,1) &= EMULT \\
EMAT(2,1) &= EMULT \times ANU \times (1.0 - ANU) \\
EMAT(1,2) &= EMULT \times ANU \times (1.0 - ANU) \\
EMAT(2,2) &= EMULT \\
EMAT(3,3) &= EMULT \times (1.0 - 2.0 \times ANU) / (2.0 \times (1.0 - ANU)) \\
\end{align*} \]

GO TO 24

C** PLANE STRESS E-MATRIX

20 EMULT = E/((1.0 - ANU^2))

EMAT(1,1) = EMULT \\
EMAT(2,2) = EMULT \\
EMAT(1,2) = EMULT \times ANU \\
EMAT(2,1) = EMULT \times ANU \\
EMAT(3,3) = (1.0 - ANU) / 2.0 \times EMULT \\

24 B(1,1) = -H_1 \\
B(1,3) = H_1 \\
B(2,2) = WHY - EL \\
B(2,4) = -WHY \\
B(2,6) = EL \\
B(3,1) = WHY - EL \\
B(3,2) = -H_1 \\
B(3,3) = -WHY \\
B(3,4) = H_1 \\
B(3,5) = EL \\

DO 30 IR = 1,6

30 B(IR,IC) = B(IR,IC) / (2.0 \times AREA)

CALL TRPOSE(B,BTR,6,6)

CALL AMTMUL(6,3,BTR,EMAT,BE,6)

CALL AMTMUL(6,6,BE,B,BTR,6)
DO 40 ITR = 1,6
DO 40 JTR = 1,6

40 B(ITR, JTR) = BTR(ITR, JTR) * TH * AREA
CALL PREPEL(6, B, CSP)
CALL PLATRA(1C, J, NC, XX, YY, ZZ, NODAL, TR)
CALL TRPOSE(TR, TRT, 12, 12)
CALL AMTMUL(9, 9, TRT, CSP, CSTR, 12)
CALL AMTMUL(9, 9, CSTR, TR, CSF, 12)
CALL ELEPAC(9, CSF, A, NOD, IMATK, NBAND)

50 CONTINUE

5 FORMAT(10X, 'C.S.T. DATA (PLANE STRAIN)',/)
8 FORMAT(10X, 'C.S.T. DATA (PLANE STRESS)',/)
15 FORMAT(10X, '***ERROR*** BADLY SHAPED CST ELEMENT, CARD NO.', I6//)
25 FORMAT(10X, '***ERROR*** INCORRECT NUMBERING OR NODAL-POINT COORDINATES OF CST ELEMENT, CARD NO.', I6//)
RETURN

SUBROUTINE Q4ST(NQUA4, U, NODAL, XX, YY, ZZ)
C** STRESS CALCULATION IN THE QUA4 ELEMENT
DIMENSION NOD(4), U(3, NODAL), V(12, 12), VT(12, 12)
DIMENSION T(12, 12), XX(NODAL), YY(NODAL), ZZ(NODAL)
DIMENSION XL(4), YL(4), DUP(8)
WRITE(6, 60)
WRITE(6, 5)
DO 200 IQ = 1, NQUA4
READ(5, 15) I, J, M, N, TH, ANU, EE
NOD(1) = I
NOD(2) = J
NOD(3) = M
NOD(4) = N
C** SET UP LOCAL COORDINATE SYSTEM
XJI = XX(J) - XX(I)
YJ1 = YY(J) - YY(I)
ZJ1 = ZZ(J) - ZZ(I)
XN1 = XX(N) - XX(I)
YN1 = YY(N) - YY(I)
ZNI = ZZ(N) - ZZ(I)
XMI = XX(M) - XX(I)
YMI = YY(M) - YY(I)
ZNI = ZZ(N) - ZZ(I)
XNJ = XX(N) - XX(J)
YNJ = YY(N) - YY(J)
ZNJ = ZZ(N) - ZZ(J)
XMJ = XX(M) - XX(J)
YMJ = YY(M) - YY(J)
ZMJ = ZZ(M) - ZZ(J)
EL = SQRT(XJ1**2 + YJ1**2 + ZJ1**2)
EH = SQRT(XNI**2 + YNI**2 + ZNI**2)
EI = SQRT(XMJ**2 + YMJ**2 + ZMJ**2)
EK = SQRT(XMI**2 + YMI**2 + ZMI**2)
EM = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
EX = (EL**2 + EI**2 - EK**2)/(2.0*EL)
WHY = (EH**2 + EL**2 - EM**2)/(2.0*EL)
H1 = SQRT(EH**2 - WHY**2)
H2 = SQRT(EI**2 - EX**2)
XL(1) = 0.0
YL(1) = 0.0
XL(2) = EL
YL(2) = 0.0
XL(3) = EL = EX
YL(3) = H2
XL(4) = WHY
YL(4) = H1
DO 10 JC = 1,4
NK = NOD(JC)
DO 10 K2 = 1,3
C** TRANSFORM DEFLECTIONS-GLOBAL TO LOCAL
CALL PLATRAC(I,J,N,XX,YY,ZZ,NODAL,T)
CALL AMTMUL(3,4,T,V,T,V,12)
DO 20 IC = 1,4
DO 20 IR = 1,2
IT = 0
20 DUTP (IT) = VT(IR, IC)
AJI1 = 0.25*(-XJ1 + XL(2) + XL(3) - XL(4))
AJI2 = 0.25*(-XL(2) + XL(3) + XL(4))
AJI12 = 0.25*(-YL(3) - YL(4))
AJI22 = 0.25*(-YL(3) + YL(4))
DETJ = AJ11*AJI22 - AJI1*AJ22
AJS11 = AJI2/DETJ
AJS12 = -AJI1/DETJ
AJS21 = -AJI2/DETJ
AJS22 = AJI1/DETJ
UXI = 0.25*(-DUP(1) + DUP(3) + DUP(5) - DUP(7))
UET = 0.25*(-DUP(1) - DUP(3) + DUP(5) + DUP(7))
VXI = 0.25*(-DUP(2) + DUP(4) + DUP(6) - DUP(8))
VET = 0.25*(-DUP(2) - DUP(4) + DUP(6) + DUP(8))
UX = AJS11*UXI + AJS12*UET
UY = AJS21*UXI + AJS22*UET
VX = AJS11*VXI + AJS12*VET
VY = AJS21*VXI + AJS22*VET
SUBROUTINE SHAPE(II, JJ, XL, YL, B, OETJAC)

** FIND SHAPE FUNCTIONS AND THEIR DERIVATIVES. **

REAL XI(N), ETI(J), JAC(3,3)

DIMENSION XI(4), ETI(4), JAC(2,2)

AA(1) = -1./SGRT(3.0)

AA(2) = 4.*ETI(1)*AA(1)

XI(1) = 0.0 *

XI(2) = 1.0 *

XI(3) = 1.0 *

XI(4) = 0.0 *

ETI(1) = -1.0 *

ETI(2) = -1.0 *

ETI(3) = 1.0 *

ETI(4) = 1.0 *

DO 10 I = 1, 4

DUM1 = (1.0 + XI(I)*AA(I))**0.25

DUM2 = (1.0 + ETI(I)*AA(J))**0.25

XI(I) = (1.0 + X(NI(I))*AA(I))**0.25

ETI(I) = (1.0 + ETI(I)*AA(J))**0.25

10 JAC(I,1) = 0.0

JAC(I,2) = JAC(I,1) + NXI(I)*XL(I)

JAC(I,3) = JAC(I,2) + YL(I)

JAC(2,1) = JAC(2,1) + NET(I)*XL(I)

JAC(2,2) = JAC(2,2) + NET(I)*YL(I)

DETJAC = JAC(1,1)*JAC(2,2) - JAC(2,1)*JAC(1,2)

DUM1 = JAC(1,1)/DETJAC

JAC(1,1) = JAC(2,2)/DETJAC

JAC(1,2) = -JAC(1,2)/DETJAC

JAC(2,1) = -JAC(2,1)/DETJAC

JAC(2,2) = DUM1

C** FORM THE STRAIN-DISPLACEMENT MATRIX B.*

DO 50 L = 1, 4

J = 2*L

I = J-1

B(I,1) = JAC(I,1)*NXI(L) + JAC(I,2)*NET(L)

B(I,J) = 0.0

B(2,I) = 0.0

B(2,J) = JAC(2,1)*NXI(L) + JAC(2,2)*NET(L)

B(3,I) = B(2,I)

50 B(3,J) = B(1,I)

RETURN

END
APPENDIX A2

An example illustrating the use of AIRSTR

Consider a cantilever box beam as illustrated below in Figure 1.

![Diagram of a cantilever box beam with dimensions and forces applied at the corners.]

**Figure 1**

The beam is made up of 1 mm thick steel sheeting on the top and sides and 3 ribs which are folded over and riveted to the skin. The fold has a contact length of 10 mm with the skin. The 4 corners of the beam are stiffened by 15 x 15 x 4 mm steel angle irons.

Two loads of 100 N each are applied at the corners as shown.

**Computer model**
The angle stiffeners are represented by end-load elements of cross-sectional area \( = 104 \text{ mm}^2 \) and an \( E \) value of \( 210 \times 10^9 \text{ N/mm}^2 \). They connect nodal points 1,5 , 7,11 etc.

The rib folds are also represented by end-load elements of cross-sectional area \( = 10 \text{ mm}^2 \) and they connect points 7,8 , 5,7 , 9,11 etc.

The rectangular panels are steel with a thickness of 1 mm, \( E = 210 \times 10^9 \text{ N/mm}^2 \) and \( v = 0.3 \).

Three runs were made on this structure.

(a) Using end-load and 4-C.S.T. panel elements
(b) Using end-load and Idealized panel elements
(c) Using end-load and Idealized panel elements, no loads but deflections of 1 mm in the neg. y direction at nodal points 15 and 16.

A sample of the data card deck used and the complete print out from the three cases follows.
<table>
<thead>
<tr>
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**BOX TEST BEAM (RODS + U-TEST PANELS)**

**ROD NO.**

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| 7 | 1.0400+03 |
| 11 | 1.0400+03 |
| 2 | 1.0400+03 |
| 6 | 1.0400+03 |
| 10 | 1.0400+03 |
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| 12 | 1.0400+03 |
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| 8 | 1.0400+03 |
| 12 | 1.0400+03 |
| 14 | 1.0400+03 |
| 6 | 1.0400+03 |
| 8 | 1.0400+03 |
| 12 | 1.0400+03 |
| 14 | 1.0400+03 |

**LENGTH**

| 5 | 2.100+06 |
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| 6 | 2.100+06 |
| 8 | 2.100+06 |
| 12 | 2.100+06 |
| 14 | 2.100+06 |

**STRESS**

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| 11 | -2.907+02 |
| 2 | -2.907+02 |
| 6 | -2.907+02 |
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**AVSIGX = 0.3043-02 AVSIGY = -0.1058-01 AVSIGXY = -0.6607+00**
### Table 1: Element Connectivity and Stress Values

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**BOX TEST BEAM (RODS + IDEALIZED QUAD PAIRS)**

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UNIVERSITY OF STELLENBOSCH

DEPARTMENT OF MECHANICAL ENGINEERING

FINITE ELEMENT ANALYSIS OF A WING TYPE
STRUCTURE WITH EXPERIMENTAL VERIFICATION OF RESULTS

PART B

EXPERIMENTAL

By

E.M.E. BAUMGARTNER

Promoted by

Mr E. FOURIE
Prof. R.J. DU PREEZ

Second part of the thesis presented for the
Degree of Master of Engineering

STELLENBOSCH JUNE 1976
ACKNOWLEDGEMENTS

I wish to thank the Mechanical Engineering Department's Technical Staff for their invaluable help in the construction of the structure and supports, especially to Mr H. Foot who did the lion's share of the work.
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B. EXPERIMENTAL

B1 INTRODUCTION

The aim of the experimental work is to determine the degree of accuracy of the Finite Element Displacement Method applied to semi-monocoque structures.

A light three spar wing type structure is loaded under statically determinate and indeterminate external conditions. Strains are measured on the structure and a graphical representation of the results are made for direct comparison between theory and experiment.
B2 DESCRIPTION OF THE STRUCTURE

§ B2.1 Construction

A symmetrically tapered three spar box shaped structure was chosen for ease of construction in our workshops and for ease of geometrical design.

Figure B2.1.1 shows the general layout and shape of the structure.

![Figure B2.1.1 General layout of Structure](image)

It is supported at points 1, 2 and 3 and loaded at various points near the tip. A centre spar runs straight down the centre line while the front and rear spars close the structure to form a typical 2 cell wing type section. The top and bottom is covered with a 0.7 mm thick aluminium skin (2S - \( \frac{1}{2} \)H (H4)). The centre spar is an assembled I-beam having a 0.9 mm thick aluminium web (2S - \( \frac{1}{2} \)H (H4)), with the top and bottom flanges made up of two \( \frac{1}{2} \)" x \( \frac{1}{2} \)" x \( \frac{1}{16} \)" aluminium angles (50S- TF) placed back to back and glued and pop rivetted to the web. The front and rear spars are of similar construction except that only one angle is used top and bottom. Evenly spaced ribs maintain the structure shape between the tip and the root. They are made of 0.9 mm aluminium sheeting (2S - \( \frac{1}{2} \)H (H4)) and have two 50 mm \( \phi \) holes punched and dished.
into them. The ribs are folded over on three sides, i.e. top, bottom and next to the front or rear spars and there glued and pop riveted to the adjacent structure. The top and bottom skin panels, between the spars, are stiffened by a $\frac{1}{4}'' \times \frac{1}{4}'' \times \frac{1}{16}''$ angle running along a line bisecting the distance between the spars. The spar webs are stiffened by the same size angles directly opposite the rib attachment points and act as the rib attachment to the centre spar.

A typical section through the structure is shown in Figure B2.1.2

![General View of Section through Structure](image)

The glue used was Midbond 10 Epoxy Adhesive with an approximate cured shear strength of 29 M.Pa. After each stage of the construction was completed the adhesive was allowed to cure for 12 to 15 hours at 35°C.

Photographs of both the completed structure and without the top skin can be seen in Appendix B1. The workshop drawings submitted are also included in the same appendix.

§ B2.2 Computer Model and Experimental Procedure

For analysis with the AIRSTR program, the spar caps, web stiffeners and rib folds were assumed to be endload or flange elements. The skin and rib panels were assumed to be plane stress membrane elements while the spar webs were assumed to be idealized panel elements.
Best results are obtained if the actual (experimental) strain readings are taken in the middle of the elements chosen. This eliminates to a great extent, the chance of stress concentrations affecting the readings. Rosette strain gauges (45°) were attached to both sides of the centres of 5 top skin panels, 5 bottom skin panels and the 5 web panels between the root and the tip. Linear gauges were attached to the spar caps of the rear spar, centre spar and to the front skin stiffeners both top and bottom.

The strain gauges were split up into batches of 30 as the number of ballancing units was restricted to 30.

The top skin, bottom skin and main spar web, each having 5 panels with 3 component Rosettes on either side of the panels, were lumped together to give 30 gauges each, while the linear gauges were analysed together.

The instrumentation for the experimental readings was set up as shown in Figure 52.2.1. The linear gauges used on the structure were aluminium temperature compensating KYOWA KFD-5-C1-23 of resistance 120 Ω (nominal) and gauge factor 2.14. The rosettes used were Aluminium temperature compensating KYOWA KFC-5-017-23 of resistance 120 Ω (nominal) and a gauge factor of 2.12. A single linear gauge of the first type was used as the dummy gauge. This was fixed to a piece of aluminium sheet of the type used for the webs and skin of the structure.

Before commencement of each set of readings the Hottinger signal amplifier was calibrated with the calibration box to give a reading of $2V = 200 \mu (1 \mu = 10^{-6} \text{m/m})$ for a gauge factor of 2. Recorded readings were thus adjusted by a factor of 2/gauge factor to give the actual strain in the structure.

It was discovered that the Digital Recording unit, when switched on, induced a voltage of 14 mV into the bridge circuit. Thus when calibrating and zeroing the strain gauges all components of the equipment
Figure B2.2.1

Balancing Units

20 of Hottinger UM3301

10 of Hottinger US 10

Scanning Unit

Hottinger US100 ST

Amplifier Unit

Hottinger KWS 6A-5
with TVE 01A Ampl. Units

DVM; A/D Converter

Hewlett-Packard 5326B
Timer-Counter - DVM

Digital Recorder

Hewlett-Packard 5055A
Digital Recorder

Dummy Strain Gauge

Hottinger K3603 S6
350 n

Calibration Unit
were allowed to warm up for 30 minutes and left on for the duration of the experiment.

The D.V.M.'s purpose was two-fold. Firstly to act as an analogue to digital converter of the bridge signal and secondly to act as an accurate means of reading off the zero point when ballancing the strain gauges, because the Hottinger Amplyfier Monitoring Gauge could only be read off to an accuracy of 20 mV.

Each batch of 30 gauges were ballanced and zeroed and then the readings for the 4 loading cases taken. Because of the tendency for the bridges to drift and the gauges to creep, a zero reading was taken before each loading case was applied to the structure. The difference between the readings then represented the strain in the structure.

The loads were applied to the structure by means of two "G" shaped frames which hung horizontally when a 4,54 kg mass was put onto the mass-pan. A drawing of the loading G can be seen in Figure B2.2.2.

![Figure B2.2.2 Loading "G"](image)

The mass hung underneath the structure while the rubber ball pressed on the structure at the point required, giving an accurate point load. The loads were kept small so as to avoid buckling in the underside
skin panels as much as possible.

To accomplish the theoretical analysis of the structure it was broken up into elements corresponding, as near as possible, to that of the actual structure. The division of the structure and the nodal point numbering is shown in Figure B2.2.3

The elements singled out for comparison were

<table>
<thead>
<tr>
<th>(i) Top skin panels:</th>
<th>12</th>
<th>10</th>
<th>40</th>
<th>42</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>8</td>
<td>38</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>6</td>
<td>36</td>
<td>38</td>
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<td>2</td>
<td>32</td>
<td>34</td>
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<table>
<thead>
<tr>
<th>(ii) Bottom skin panels:</th>
<th>11</th>
<th>9</th>
<th>39</th>
<th>41</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9</td>
<td>7</td>
<td>37</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>5</td>
<td>35</td>
<td>37</td>
</tr>
<tr>
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<td>3</td>
<td>33</td>
<td>35</td>
</tr>
<tr>
<td></td>
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<td>1</td>
<td>31</td>
<td>33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(iii) Main spar web panels:</th>
<th>69</th>
<th>67</th>
<th>68</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>67</td>
<td>65</td>
<td>66</td>
<td>68</td>
</tr>
<tr>
<td></td>
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<td>63</td>
<td>64</td>
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<td>63</td>
<td>61</td>
<td>62</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>61</td>
<td>59</td>
<td>60</td>
<td>62</td>
</tr>
</tbody>
</table>

| (iv) Spar caps and stiffeners |
| Rear spar top flange:        | 20  | 22  |
|                             | 22  | 24  |
|                             | 24  | 26  |
|                             | 26  | 28  |
|                             | 28  | 30  |

| Rear spar bottom flange:    | 19  | 21  |
|                            | 21  | 23  |
|                            | 23  | 25  |
|                            | 25  | 27  |
|                            | 27  | 29  |
Figure B2.2.3  Nodal point numbering
| Main spar top flange: | 60 62 |
|                       | 62 64 |
|                       | 64 66 |
|                       | 66 68 |
|                       | 68 70 |
| Main spar bottom flange: | 59 61 |
|                        | 61 63 |
|                        | 63 65 |
|                        | 65 67 |
|                        | 67 69 |
| Top skin stiffener:   | 32 34 |
|                       | 34 36 |
|                       | 36 38 |
|                       | 38 40 |
|                       | 40 42 |
| Bottom skin stiffener: | 31 33 |
|                         | 33 35 |
|                         | 35 37 |
|                         | 37 39 |
|                         | 39 41 |

Two basic loading cases were used, one symmetrical and the other asymmetrical. Both were applied to the structure for two different support restriction cases, i.e.

(i) Free and
(ii) Fixed.

Free implies that the supports at nodal points 11 and 19 were free to move in the z-direction while 16 was free in the x-direction.

Fixed implies that the supports 11 and 19 were restricted in all 3 global co-ordinate directions while at 16, because of the bending effect of the rear support, it was not absolutely rigid in the x-direction. An equivalent endload element was inserted aft of nodal point 16 (16 - 73) and
given such dimensions as to be equivalent to the bending effect of the rear supporting structure.

The structure was thus tested for the following 4 conditions:

<table>
<thead>
<tr>
<th>Loading Case 1</th>
<th>(1) Free (LC = 1 Free)</th>
<th>(2) Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading Case 2</td>
<td>(3) Free</td>
<td>(4) Fixed</td>
</tr>
</tbody>
</table>

For Loading Case 1 Weights of
67.21 N at nodal point 58
and 66.92 N at nodal point 32 were applied to the structure (neg. y-direction)

For Loading Case 2 Weights of
67.21 N at nodal point 2
and 66.92 N at nodal point 34 were applied to the structure (neg. y-direction)

The actual dimensions of the elements were used wherever they coincided with the nodal points. The nodal points did not however coincide with the centres of gravity of the spar caps and stiffeners so a correction to the areas was made to ensure that the moment of inertia of the section remained correct. An effective thickness was given to the ribs to allow for the lightening holes cut into them.

A graphical comparison of the results is given in Appendix B2.
B3 DISCUSSION OF RESULTS

From the computer model predicted stresses, it was seen that the maximum strains expected were in the order of 30 μ (30 x 10^-6 m/m). This meant that a high amplification setting had to be used on the bridge measuring equipment, i.e. 200 μ = 2 V. This implies an extremely sensitive installation in that 10 mV of drift on the bridge means 1 μ error in the reading which means a 3% error for every 1 μ of drift.

Initially using the QUA4 membrane element (linear displacement field, straight sided quadrilateral) for all the quadrilateral panels and a uniform Elasticity Modulus for the Aluminium (panels and flanges), a discrepancy of 25% and worse in the flanges was recorded between expected and actual strains. It was suspected that localized bending effects were causing the additional strain in the experimental results.

Using the ASKA program, eccentrically loaded beam elements were compared with flange elements in a similar type of structure. The difference in results was negligible and the error sought elsewhere.

Enquiries directed at the manufacturers of the material revealed that the Elasticity Modulus of the sheet material was some 7% higher than that of the angles used and the Poisson's ratio 13% higher than initially assumed. An experiment carried out on the flange material confirmed the manufacturers value, using these values a new computer prediction for the strains was obtained. These predicted values can be seen in the graphs of Appendix B2 labeled EXACT.

It was still felt however that the correlation should be better.

In Reference 3 (Part A), mention is made of the bad behaviour of the linear plane stress quadrilateral (QUA4) under pure bending. In the derivation thereof, the assumption is made that the panel sides remain straight and do not curve, as would be expected under pure in plane
bending. This lead to the conclusion that because the element is stiffer in bending than it should be, the predicted loads in the spar flanges and stiffeners would be lower than in reality. Now under pure bending the Idealized Panel Element conforms exactly to the Elementary Theory of Bending (Reference 1, Part A), so these elements were inserted in place of the QUA4 elements in the webs of the 3 spars. The strain prediction in the spar caps and stiffeners was greatly improved as can be seen in Appendix B2, Graphs 1 to 6, 12 to 17, 23 to 28 and 34 to 39 curves labeled MIXED. The upper prediction line labeled ETB, on the fore mentioned graphs, was obtained by using Rod elements and computer generated idealized quadrilaterals and triangles. It is evident that this model is wholly inadequate and more careful modelling is required, by including the whole skin cross sectional area into the shear panel stiffeners.

Comparing the MIXED prediction and Experimental results of the lower skin stiffener, (Graphs 4, 15, 26, 37) it can be seen that the stringer loads between the front supports (Nodal points 11 and 19) and 70% of the span, tend to be higher than predicted. This was to be expected because the strain gauge readings on the bottom skin panels indicated that buckling had taken place and the skin was thus carrying less load than expected. This is born out by inspection of Graphs 8, 19, 30 and 41 in which the direct strains \( E_{xx} \) are lower than those predicted by the MIXED model. The extreme deviation from the predicted strains near the supports (10% span) can be attributed firstly to greater buckling and secondly to the coarseness of the F.E. grid near the supports which are in effect a discontinuity in the structure and in which localized effects play a major role.

The main spar flange loads, both top and bottom, Graphs 1, 2, 12, 13, 23, 24, 34 and 35, follow the predicted strains closely in the region 30 to 100% of the span. The deviation at the 10% span reading on the lower flanges can be attributed once again to the proximity of the supports and the probable buckling of the skin panels in that region. The good correlation can be attributed to the fact that the flanges are securely
held by the surrounding structure, i.e. the spar web and the skin, and they thus follow very closely the approximation that they only carry axial stresses and remain straight.

Inspection of the top skin stiffener (stringer) results Graphs 3, 14, 25 and 36 indicate that the assumptions made in the computer model follow reality very closely. The maximum deviation being in the order of 2 μ which represents a 7.5% error. Considering the sensitivity of the data acquisition apparatus this is negligible.

The direct strain readings in the top skin panels, Graphs 7, 18, 29 and 40 show a maximum deviation of 12.5% in the results between 30% and 100% of the span. Support proximity effects appear to play a role in the panel nearest to it. The 12.5% maximum deviation in the results, can be attributed to the fact that the QUA4 element sides remain straight and do not curve and can therefore not follow the exact deformation of the structure. This deformation difference however, cannot be too great, as the results converge, towards the tip of the structure.

The rear spar flange results, Graphs 5, 6, 16, 17, 27, 28, 38 and 39 suggest that the Idealized Panel assumption is too severe and that an element somewhere between the QUA4 and the Idealized Panel should be used in modelling the spar webs. This could be accomplished by using a higher order element, i.e. one which would curve in bending as well as maintain it's plane stress membrane characteristics. The correlation is, however, still good over 70% of the span with the now characteristic rise in the experimental results near the supports.

The shear strain components of the top and bottom skin panels are plotted on Graphs 9, 10, 20, 21, 31, 32, 42 and 43. The large deviations in the bottom panels nearest the support in the fixed loading case conditions can be attributed to the fact that, in the theoretical model, a single panel cannot deform sufficiently to give a realistic prediction of the strains expected throughout the panel.
In a 45° three component rosette all three direct strains are needed to predict the shear strain. From Mohrs strain circle we have

\[ \varepsilon_{xy} = \varepsilon_{xx} + \varepsilon_{yy} - 2 \varepsilon_{45} \]

It is clear that should there be an error in all three of the strain components and should they all be in the "wrong" direction, then the error is four times as large as for a single strain, using the same sensitive bridge setting.

It should be noted at this stage that the sign of the shear strain component given by the finite element analysis is opposite to that of the experimental results.

Graphs 11, 22, 33 and 44 show the results of the shear strains in the main spar web panels. The EXACT prediction in the fixed loading case conditions on the panel nearest the web is way off the mark; even the sign is incorrect. This is once again due to the limitations of the QUA4 element. The error between the MIXED prediction and the experimental results can be accounted for by the same reasoning as for the skin shear strains. The trends however are correct as is very clearly evident in Graphs 33 and 44 for the unsymmetrical loading case.
The results show that the finite element method can be used with confidence in predicting accurately the stresses in a complex structure. However, care and a knowledge of the limitations of the elements to be used, is of great importance.

More sophisticated elements have been developed and are available in the large Finite Element Systems which are on the market. These higher order elements do however, take more computer time to assemble and result in larger sets of equations to solve, so it is ultimately up to the stress analyst to weigh up the advantages of greater accuracy against cost of analysis. It appears though that careful use of the simpler elements can give adequate accuracy for design purposes as the predicted strains are slightly higher than those actually measured.
APPENDIX B1

Photographs and Workshop drawings of test structure.
INSTRUCTIONS
The row shown are as seen from the view (top). The adjacent row to the right is the top view. The row shown are the top view for a load case of 4 toeing from the foot and 10° from the side. The dimensions on this drawing are given in millimeters for all the rows.

SHEET DIACTION TALE

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<td>125</td>
<td>61.5</td>
</tr>
</tbody>
</table>

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APPENDIX B2

Graphical representation of predicted and experimental strains

ETB Prediction curve using computer generated Idealized Panels and Flange Elements.

EXACT Prediction curve using Plane Stress Quadrilaterals, C.S.T. and Flange Elements.

MIXED Prediction curve using Plane Stress Quadrilaterals and C.S.T. Elements for the skin and ribs, computer generated Idealized Panels for the spar webs, and Flange Elements.

Experimental results.

\[ 1 \mu = 1 \times 10^{-6} \text{m/m (strain)} \]

Note: The % span is taken from a line through the front supports to the free end of the structure.
Graph 20
Top skin panels
LC = 1 Fixed
Graph 21
LC=1, Fixed
Bottom skin panels

$\epsilon_{xy}, [\alpha] -ve$
Graph 22
Main spar web
LC=1 Fixed
Graph 30
Bottom skin panels
LC=2 Free
strain $[u]$ +ve

Graph 34

Main span top flange

LC = 2 Fixed
Graph 40
Top skin panels
LC=2 Fixed