RESEARCH FOR THE LEARNING AND TEACHING OF MATHEMATICS: AN EMERGENT DESIGN

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requirements for the degree of
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DECLARATION

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I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Signature: …………………………………... Date: …………………………

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ABSTRACT

This thesis deals with my practice as a mathematics teacher at a post-matric programme at the University of Stellenbosch. I use aspects of three different approaches to social science research, namely *phenomenology*, *narrative inquiry* and *the discipline of noticing*, to research my personal experiences. These experiences include *learning* mathematics at school and university as well as *teaching* mathematics in a post-matric programme. These experiences are presented by means of brief-but-vivid descriptions, journal entries and records of classroom conversations and are reflected on in the light of relevant literature.

The reflections and readings lead to alternative ways of thinking about *learning*, *teaching* and *researching* as found in the cognitive theory of *enactivism*. These new ways of thinking are used to reflect on my current practice by focusing particularly on *knowing*, *listening* and *noticing* and are used to imagine what my practice *could* look like in the future.

My approach, framework, focus areas and imagined practice all emerge during the research process. This process is presented through 'behind-the-scene' reflections on my own experiences of doing research. By presenting these experiences, this thesis also deals with the process of doing research – in particular it deals with the process of using an *emergent design*.

OPSOMMING

Hierdie tesis handel oor my praktyk as wiskunde-onderwyseres by 'n post-matriekprogram aan die Universiteit van Stellenbosch. As deel van die navorsing oor my personlike ervarings, gebruik ek aspekte van drie verskillende navorsingsbenaderings binne die sosiale wetenskappe, naamlik *fenomenologie*, *narratiewe ondersoek* en *waarneeming as dissipline*. Hierdie ervarings gaan veral oor my eie *leer* van wiskunde op skool en universiteit asook my *onderrig* van wiskunde as deel van die post-matriekprogram. My ervarings word geboekstaaf as kort-maar-duidelike beskrywings, joernaalinskrywings en afskrifte van klaskamergesprekke. Ek reflekteer oor hierdie ervarings op grond van van toepaslike literatuur.

Die refleksie en literatuurstudie lei tot alternatiewe maniere om oor *leer*, *onderwys* en *navorsing* te dink, soos dit aangetref word in die kognitiewe teorie van "*enactivism*". Hierdie nuwe denkwyse word gebruik om oor my eie praktyk te reflekteer deur veral te fokus op maniere van *ken*, *luister* en *waarneem* en word ook gebruik om voorstellings te maak oor hoe my praktyk in die toekoms *sou kon* lyk.

My benadering, raamwerk, fokusareas en voorgestelde praktyk het tydens die navorsingsproses vorm aangeneem. Hoe hierdie proses verloop het, word aan die hand van 'agter-die-skerms'-refleksies oor my eie navorsingservarings beskryf. Deur hierdie ervarings met die leser te deel word hierdie tesis ook een wat oor die navorsingsproses handel – veral oor hoe die navorsingsontwerp tydens die navorsingsproses vorm kan aanneem.

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CONTENTS

OVERVIEW

This is the story of the construction of my thesis. It is an attempt to represent the "complex and complicated" (Davis & Sumara, 1997) nature of research where what I read influences what I write which influences what I see which influences what I hear which influences who I am which influences what I read which influences what I write.

Like Eddy (2003: 6), I believe that "it makes no sense for me to present a standard format of introduction, literature review, methodology, results, discussion and conclusion, when these concepts have become so mutually intertwined, continually feeding back on one another. To separate them would destroy even further the glorious cyclic and chaotic nature of research". Therefore, I have chosen instead to structure my thesis using the analogy of a tapestry. It is a tapestry which "shows images on its front side and displays the underlying construction of the back" (Merz, 2002: 148); a tapestry which tells its own story; a tapestry depicting its own conception and ongoing construction.

In my analogy the finished tapestry represents the finished thesis. The weaving of the tapestry refers to the writing of the thesis. The loom that the tapestry is woven on is the framework used for the research. The tapestry viewed from the front shows images which represent the significant events I describe and reflect on. The threads used to weave the tapestry signify the themes found in the significant events, some of which are extracted and looked at in more detail. The tapestry viewed from the back shows its messy construction representing my reflections on the messy process of writing this thesis, looking in particular at my experience of using an emergent design. The unravelled threads, trailing off the loom, knotted through use and contact with other materials represents how my thesis became tied up in my practice, embedded in my everyday life.

These then are the different sections:

Section 1: "Preparing the loom", looks at how I came to start constructing this tapestry.

Section 2: "Weaving the tapestry", looks at the different images on the tapestry.

Section 3: "Following threads", planned to look at recurring themes that were used in the weaving of different images.

Section 3*: "Knotting threads", looks at the threads which became knotted into (un)imagined images.

SECTION 1
OF THE CONSTRUCTION OF MY TAPESTRY:

PREPARING THE LOOM

A section in which I reflect on how I came to write this thesis. I reflect on the different approaches to research which I explored while searching for the "right" one to frame my research in and how, finally, this search led me to using an emergent design.

"Research, like almost everything else in life, has autobiographical roots."
(Seidman, 1991)

"As with any research project the question of appropriate methodology evolved simultaneously as I defined and redefined the research question."
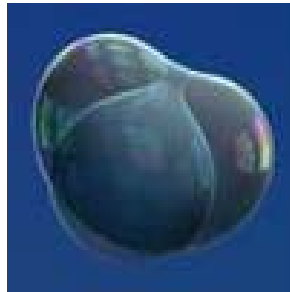(Campbell, 1997)

SECTION 1.1

OF THE PREPARATION OF MY LOOM:

Deciding to construct a tapestry

A section in which I share and reflect on events that, in retrospect, influenced my decision to do a Master's degree in education.

"I write because I want to find something out. I write in order to learn something that I did not know before I wrote it."
(Richardson, 2000: 924)

Overview

When I first enrolled for a Master's in Education I was unable to express why I had chosen to do so. I remember the first session of our methodologies course – each student was asked to introduce themselves and briefly tell the class why they had decided to enrol for a Master's in Education. All I could come up with was, "Well, um, I'm just doing my masters because, um, I just want to". I felt threatened by this need for an explanation. Wasn't it good enough to do something just because one wanted too?

Many of my views regarding being aware of what motivates me have changed since that first session. They have changed as I have continued with my (re)search. In fact, they have been changed through writing this particular section.

In this section I will share a particular series of events that, in retrospect, played a significant role in my decision to attempt this research. By doing this I am taking part in the human practice of telling stories to myself and to others to make sense of the worlds I inhabit (Swindler, 2000: 553 cited in Andrews & Hatch, 2002: 188).

Reflecting

During my fourth year of studying mathematics I was given permission to replace one of my mathematics courses with two didactics courses, namely Didactics for Mathematics and Advanced Didactics for Mathematics. I thoroughly enjoyed the courses. For the first time I discovered a whole world I had been unaware of, a world of people who thought about how to teach mathematics, people who had put into words things I hadn't even realised I knew. John Mason (1998) writes about work "resonating" with the reader – that is exactly what happened to me in these courses. So much of what I read resonated with my own experiences. For the first time I was given the words to describe what I had only sensed before.

Although I found our class discussion meaningful and I can look back and see how they have influenced me, what influenced me most was my experience of writing up our term project. Below I have attempted to reconstruct the thoughts I had on the day "everything fell into place".

Wow. I still can't believe that it happened – that everything came together. I mean I knew it had to happen but still, I didn't really believe it would. I have just finished my project. Well it's not totally finished but I know what I want to write – I have the big picture, I have a structure, filling in the details is no problem.

Our instructions were "make a summary of everything that we have discussed this term and what it means to you". Right from the beginning I found the idea appealing, there was so much new stuff that I had learnt and I was keen to go through it all together and to "process" it in a way. Before today I had spent some time rereading the articles we had been given but had been busy with other work. Since my project has to be in next Friday I put aside today (Saturday) to work on it. I started off by taking a green highlighter and going through everything again, highlighting the sections that were meaningful to me. Most of these referred to things that reminded me of my own experiences – in and out of the classroom.

I then summarised all the different theories, making a list of the important things that I wanted to put in my project. But I didn't have a structure, a connection, something to bring it all together. I kept on reading and highlighting and summarising and then, suddenly, an image that I had had since I first heard about constructivism and schemas started to grow. At first it was just my personal way of thinking about schemas but the more I thought about it the more I saw how I could use it to discuss everything that I had on all my summaries. It was so easy; it just "fell out" after all the hours with no results.

Towards the end of my fourth year I started investigating options for the following year. I had always wondered what it would be like to teach mathematics and what it would be like to live in a small town so when a school in a small town offered me a post to teach mathematics and science (even though I didn't have a teaching diploma), I decided to go and find out.

I took the job but after a year of teaching I found out about a post at the university where I had studied. The post was for a mathematics lecturer for a post-matric programme. I would be teaching students who had completed their schooling but would be rewriting two of the subjects (one of them being mathematics) for their National Senior Certificate at the end of the year. It sounded more in the line of what I wanted to do, so I applied and was appointed. I accepted the job and, since I had a lot of time

available and two of my colleagues were also busy with postgraduate studies, I started investigating doing a Master's in Education. Somewhere during the application process found that I had decided to give it a try.

Underview

But why *did* I actually decide to start making this tapestry, to start constructing this thesis?

Although I started this section, and my studies, believing that it was enough to say that I was doing research "because I wanted to", I have subsequently come to realise that it is not enough. I need to move beyond this answer if I am to make meaning of my decisions. I need to reconsider my original answer, "Because I wanted to".

Why did I *want* to do a Master's in Education?

Through writing this section I have come to realise that my main motivation was closely linked to the pleasure I experience when everything comes together, when I manage to make some sense out of the chaos, when I make connections between theory and my own experiences, when I make meaning or when I create something that 'feels right' – all the things I experienced on that Saturday in my fourth year. I realise now that the desire to again experience the feeling of "Yes, I found the right representation!" not only motivated my initial decision but has motivated me subsequently.

Looking back at this section I realise that perhaps what is more important than sharing why I chose to do my Master's, is sharing how, as I wrote, I came to know something I didn't know before I started to write – how I came to know the extent to which the desire to construct something that "feels right" has influenced me. Now I too can join Richardson (2000: 924) when she says, "I write to know things that I didn't know before I started to write."

SECTION 1.2
OF THE PREPARATION OF MY LOOM:
Trying to design a loom

A section in which I reflect on my "stumblings" as I try to find a suitable methodology for my research. I begin by investigating phenomenology, move on to narrative research and then to research from the inside.

"All research will be a product of human choice and influence. The claim that research 'discovers' something is arguably fallacious; researchers produce and construct research reports according to their own values and outlook. The idea of reflexive research is that we are aware of our part in articulating both the research questions and research methods. We should at least acknowledge that the choice has been ours and we should be ready to defend our position."
(Carson & Fairbairn, 2002: 26)

"On the other hand, if an adjustment is made because a path is not being fruitful, the need for an alteration may be described as a 'stumbling/misstep' (Ely et al., 1997).
Whether it is an emergent design or a stumbling, the process needs to be reflected upon and documented for the reader (Berg, 2001)."
(Merz, 2002: 143)
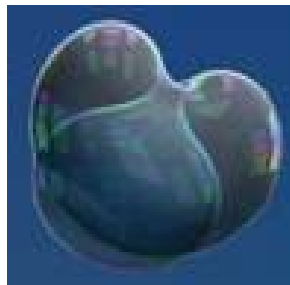
Overview

Before turning to consider phenomenology, narrative inquiry and research from the inside, I share two events that influenced my understanding of paradigms and methodologies as a science student entering the social sciences for the first time. The first one is a conversation and the second one is a lecture.

The conversation

The thought of being able to explain why things work the way they do has always appealed to me. Before my second year at university I believed that physics was a means by which one could do this. I believed physics was really explaining what was happening in the world; that the things that were being described were the things themselves. But as I was exposed to more and more physics, I gradually began to wonder whether this was indeed the case. What bothered me the most was that we kept on being told that the reason why we did particular things during our mathematical calculations (things that weren't always strictly mathematical) was because they "worked". A particular conversation[1] in my second year confirmed my doubts.

Ingrid: Hey Thomas, um, have you seen Clinton anywhere?

Thomas: No. I think he went out for coffee or something. Is it urgent?

Ingrid: Uh no, just wanted to say hi. How's your work going?

Thomas: Slowly. How you enjoying second year?

Ingrid: Ag, it's okay. I just get a bit frustrated sometimes with our lecturers who don't seem to care about why we do particular things

Thomas: Like?

Ingrid: Um, like in the proofs and so on. It's as though it doesn't matter what you do as long as you get the right answer or an answer that works. It just feels wrong. I mean we are trying to explain how things work, you can't just do steps because they give you the answer that you want.

Thomas: What do you mean "trying to explain how things work"?

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[1] Throughout my thesis I have changed people's names to protect their anonymity.

Ingrid: Well, I mean, physics is supposed to tell you why things are the way they are. It's supposed to explain the way the world works. Or at least that's what I always thought. But now it seems like we just try find equations and make them fit what we see. It doesn't seem right.

Thomas: Well, we won't ever be able to explain how "the world" works perfectly. Whatever you use to try and explain how something works will always just be a model of the thing. And so it'll always be a simplification of the real thing. I mean the only way you can model something perfectly is to use the thing itself – and then you aren't modelling it any more.

Ingrid: Ja, but still… *[so does that mean… the only model that will describe the world perfectly is …the world itself…everything else is just going to be an imperfect attempt of describing the real thing. That means everything is just…our made up explanation…]* ag, never mind. How's your PhD going?

I later read a quote about research that addresses this issue:

> A completely full and faithful account would require real time, just as a map with scale 1 to 1 would be indistinguishable from what is mapped … the whole point about a novel, or a map, or research, is to summarise, condense, and distil, by making distinctions and being selective, without losing contact with what you consider to be the essence of the original.

(Mason, 2002: 245)

On the way to my next class I considered, for the first time, that "science" might just be a human-made creation in an attempt to explain the natural world. That afternoon, while sitting under a tree talking to Gus, a fellow student, I came the closest to quitting my studies I have ever come. However, I didn't quit and two years later, while still studying, I attended the following lesson.

The Lesson

Jacques and I are walking to the engineering buildings. Jacques is also doing his Honours in Mathematics. We are on our way to our first "Philosophy of Science" lecture which we will be attending together with the Applied Maths and Computer Science Honours students. We manage to find the classroom on the fifth floor of the second building. For the first time since the end of the

previous year I see classmates from undergraduate years. I suddenly realise how much I miss being part of a class that's bigger than just 5 people.

I find a seat next to Mercia but we don't get much chance to catch up since the lecturer starts straight away. The first thing he does is to give us a quick overview of all the power point slides that he will discuss with us. He spends some time talking about the first slide, which mentions that before "science" people used myths to explain things. He then mentions that the climax of the course is the last slide – but that he wants to keep it till the end, that he doesn't want to spoil the course for us. I immediately guess what the last slide says:

>   *Science is a myth.*

My guess is made based on the way he emphasised the first slide and the way he makes it sound as though the last slide has something very 'unconventional' written on it. The rest of the lesson is uneventful. I struggle to stay awake in the classroom which has been darkened for the PowerPoint presentation.

It's only later, as Jacques and I walk back to the mathematics building, that I start to consider the implications of the statement: *If science is a myth or is like a myth then ... well that just confirms my revelation in second year – it's just something humans came up with to explain things. There is nothing inherently special about it. Not that it isn't useful or meaningful – most definitely not; it's just that, like a myth, it is only people's attempt to explain what they don't understand.*

Paradigms and Paradigm shifts

I first read about Thomas Kuhn and his views on science in the above-mentioned philosophy of science course. Kuhn's notion of a paradigm that guides research and of science advancing, not through the accumulation of knowledge but rather, through paradigm shifts made sense to me. I had already realised that physics was attempting to model the natural world and I easily made the jump from "working on a model to explain the natural world" to "being guided by the 'accepted' model to explain the natural world". I also knew enough about the history of science to appreciate Kuhn's examples of normal science, crisis period and paradigm shifts. From second-year physics practical sessions I knew about "theory guiding research" as opposed to "research guiding theory" or, as Einstein put it: "It is wrong to assume that theories are based on observation. The opposite is the case. The theory

determines what we can observe" (Hoyt, 1998 cited in Prinsloo, 2004: 38). And most significantly, I could relate to the idea of struggling with a problem for a long time and coming up with a solution in a "flash of intuition" (Kuhn, 1962: 122).

When I started reading literature on the philosophy of education I felt lost. My understanding of paradigms and paradigm shifts which I had constructed in my science course did not "fit" with the way in which these concepts were being referred to in the social sciences.

Although the paradigm shifts Kuhn was referring to described different ways of seeing the world, the different paradigms were all still "scientific". In education, on the other hand, different paradigms were referred to but only one of these was a 'scientific' paradigm. How could there be a scientific paradigm, if there were different paradigms within the scientific paradigm?

Added to this, Kuhn wrote that different paradigms were incommensurable (Connole, 1993: 15), but it seemed that social scientists moved between paradigms (Le Grange, 2000: 194; Lincoln & Guba, 2000: 164). How could they be paradigms, if they were not incommensurable?

I finally made my own meaning by creating two levels of paradigms. I viewed the new paradigms I was learning about as being different to Kuhn's original paradigms in that they were on a higher level, containing Kuhn's paradigms on a lower level. Thus they could be different (by being commensurable) but similar (by also guiding research).
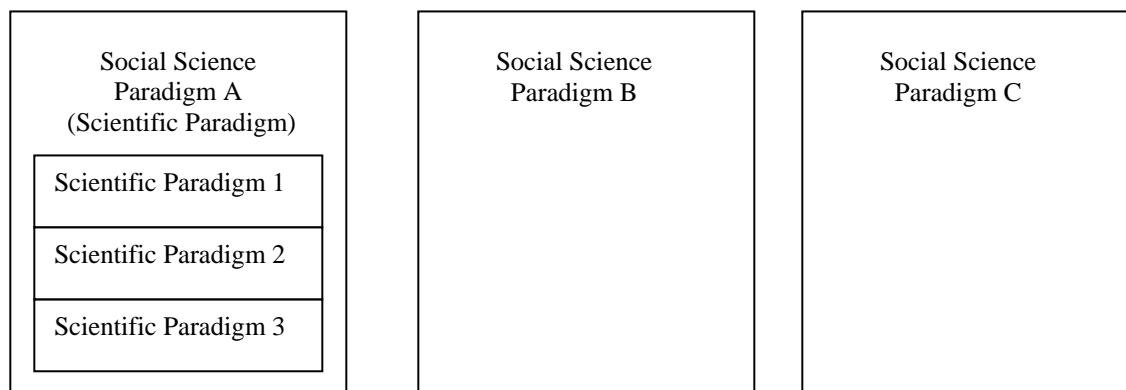

Figure 1: An initial understanding of the relationship between paradigms in the social sciences and in the natural sciences

This original understanding changed after reading more about research. A pivotal moment occurred when I realised that I was now working in the social sciences as opposed to the natural sciences and, more importantly, that the social sciences had originally been based on the natural sciences (Connole, 1993: 10; Cantrell, 1993: 82). I no longer needed to try and incorporate the paradigms identified by Kuhn in the natural sciences in my growing understanding of the paradigms identified in the social sciences. The scientific or positivistic paradigm of the social sciences was not *equivalent* to a paradigm in the natural sciences but only *based* on such paradigms?

I realised that the natural and social sciences differ with regard to what they research. Where the natural sciences research physical objects and events (which have not been produced by humans), the social sciences research social objects and events (which have been constructed by human minds) (Cantrell, 1993: 82; Connole, 1993: 19-20).

And yet, once I had made meaning of the idea of a paradigm in the social sciences I had to start making meaning of the various ways researchers classify different paradigms. In each paper I read on research paradigms they were classified slightly (or not so slightly) differently and were compared on the basis of different features.

In the end there were a few conceptualisations that I found meaningful and that I was able to combine into Table 1. The table is based on Patti Lather's (1992: 89) extension of Habermas's (1972) categorisation, but has been further extended by drawing on the work of Lincoln and Guba (2000, 170-173); Cantrell (1993: 83-84); Connole (1993: 22-23) and Gough (2000: 5).

PARADIGM	Positivist	Interpretive	Critical	Poststructuralist/ Deconstructionist
Purpose	Predict and Control	Understand	Emancipate	Deconstruct
Ontology Nature of Reality	Single stable law like external reality	Reality is internal subjective experience, i.e. there are multiple realities which require multiple methods for understanding them	Reality is internal subjective experience, i.e. there are multiple realities but they are problematic through distorted communication	Reality is constituted in and through language, i.e. there is no real world accessible to us beyond language
Epistemology Nature of Knowledge how we are able to know	Events explained based on knowable facts or real causes; law-like realities exist	Events understood through mental process of interpretation, which is influenced by and interacts with social context	Events understood within social and economic context with emphasis on ideological critique and praxis	There is no fixed meaning. Meaning shifts according to its contexts and the motivation of speakers/writers and listeners/readers
Relationship between Knowledge and Researcher	Independent Dualism	Interrelated Dialogic	Interrelated Influenced by society and commitment to emancipation	Interrelated
Methodologies	Scientific methods Quantitative methods	Constructive Phenomenological Hermeneutic Interpretive Naturalistic	Neo-Marxist Feminist Freirean Participatory	Discourse Analysis Deconstruction Post-modern Post-structural Textual Analysis

Table 1: Research Paradigms

(Adapted from Patti Lather (1992); Habermas (1972); Lincoln and Guba (2000); Cantrell (1993); Connole (1993) and Gough (2000))

As soon as I had made sense of different paradigms and different ways of comparing them, I had to make meaning of the difference between methods, methodology and research design. By this time I was getting tired of continually having to sift through pages of conflicting ideas. Coming from a field where unambiguous definitions are essential, I could not understand why there weren't fixed definitions in the social sciences. I later accepted that this was something I was going to have to "put up with" and that I would simply have to form my own understanding of the different concepts and be able to share it with others. It was only when I read the following quote by Morwenna Griffiths (1998 cited in Gough, 2000: 1; emphasis added), that I finally began to consider that not having fixed definitions might in fact be a positive thing:

> The exact meanings of terms like 'methodology', 'method' and 'technique' are inherently unstable, precisely *because of the depth of argument about them*. This situation can be confusing to anyone new to the field. If you, the reader, are feeling it is somehow your fault that you can't find one clear definition that works for everything you read, then you need to know that you can abandon the search. Instead, you need to develop an understanding of the range of use, and to be clear about your own understanding, as a result.

Quantitative and Qualitative Research

A final distinction that I would like to mention is that of quantitative and qualitative research. When I began doing research this distinction was not very meaningful for me. In fact I avoided it because it didn't fit into the table I had constructed to explain paradigms to myself. I later read papers discussing the value of using these labels to classify research and whether qualitative and quantitative should be used to describe different paradigms, methodologies, methods or types of data collected (Le Grange, 2000; Lather cited in Campbell, 1997). Although the distinction wasn't meaningful for me, I knew that my research *was definitely not* quantitative. It was only when I read the following quote by Denzin and Lincoln (2000: 3) that I was willing to classify my research as qualitative and not simply as "not quantitative":

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings attempting to make sense of, or to interpret, phenomena in terms of the meanings people bring to them.

1.2.1 Phenomenology

I first encountered phenomenology in a philosophy of education module which I was attending as part of my Masters. Embree (1997: n.p.) writes that "for many people, the word 'phenomenology' is difficult to pronounce and those who hear the word for the first time often ask what it means". This was indeed the case in our philosophy course. I remember the laughter that arose from the attempts at pronouncing 'phenomenology' and I remember how I struggled to understand what it meant.

When I finally made meaning of phenomenology, it was because I related it to two experiences I had had previously. The first was the experience of repeating a word over and over. I discovered that when repeating a word aloud, after a while the word starts "breaking up". What used to be a single word becomes separate meaningless sounds. It is almost as though I begin to hear the word as someone who has never heard it before – it has no meaning.

As I understand it, this is what Husserl (1917) meant when he called for the return to the things themselves - that things should initially be viewed as they are before we attach meaning to them. This enables us to become aware of the meanings/connotations that we (or others) *do* attach to words and experiences. There are those who argue that, although we can never achieve this completely (Van Manen, 2003), this shouldn't stop us from trying.

The second experience was suddenly seeing the road that I was walking on as "itself". One moment it was the road that I walked home on everyday and the next moment I saw it for what it really was – a layer of tar lying on a piece of earth. Just lying there. It was a strange feeling. In one instant all the connotations that the road had were stripped away. For the first time I saw it "out of context". I didn't see it as "the road that we use to drive on" or "part of our means of transport" or "the thing that it is easier to ride a bike on than grass" or as any other "meaning" that I had previously attached to the road – consciously or unconsciously. It just became what it was. And I could join with Van Manen (2003: n.p.) to exclaim that,

> When we are struck with wonder, our minds are suddenly cleared of the clutter of everyday concerns that otherwise constantly occupy us. We are confronted by the thing, the phenomenon in all of its strangeness and uniqueness. The wonder of that thing takes us in, and renders us momentarily speechless.

Wanting to use phenomenology

Originally I wanted to use phenomenology to research the lived experience of coming to know mathematics. After being introduced to phenomenology, struggling to make meaning of it and finally having a breakthrough when I did make my own meaning, I decided that I wanted to use it as my research methodology. I read about phenomenology (Moustaksa, 1994) as well as examples of research which had made use of phenomenology (Van der Mescht, 1996). I found (and still find) the idea of researching something (an experience in this case) by "extracting the essence" extremely appealing. In fact, the search for the essence or what Devenish (2002: 14) refers to as a "process of *distillation*" is what attracted me to mathematics, particularly category theory.

And yet in the end I chose not to use phenomenology as my only research approach. I realised that phenomenology aims to research a particular experience – for example, learning to drive a car – and that what I wanted to research was a process – how peoples' understanding changes. I decided that a narrative approach would be more suitable.

I moved on to investigating narrative theory and later research from the inside. Later I decided on a combination of all three and, although I didn't know how I was going to do this yet, I started writing about each of the three approaches (see subsection 1.3.3).

1.2.2 Narrative theory

My original motivation for using narrative theory was that it would enable me to capture the *process* of 'coming to know mathematics' that people experienced as they studied mathematics at tertiary level – something I felt phenomenology would not be able to do since it focused only on *particular* experiences.

Narrative inquiry would also allow me to use a range of data, as pointed out by Connelly and Clandinin (1990: 5), who write that in narrative theory "Data can be in the form of field notes of the shared experience, journal records, interview transcripts, others' observations, story telling, letter writing, autobiographical writing, documents such as class plans and newsletters, and writing such as rules, principles, pictures, metaphors, and personal philosophies".

Originally there were three ideas in narrative theory that particularly appealed to me. These were

- the idea that people are grabbed by 'the particular' (Connelly & Clandinin, 1990: 8);
- the idea that "human's tell stories to themselves and to others to make sense of the worlds they inhabit" (Swindler, 2000 cited in Andrews & Hatch, 2002: 188); and
- the idea that "there is a sense in which all research is concerned with telling stories about ourselves and about the world" (Carson & Fairbairn, 2002: 16).[1]

These ideas appealed to me because I had personal experiences that confirmed each of them.

As I started considering using narrative theory as a methodology, I realised that in fact it was my own story, my own process of 'coming to know mathematics' that I wanted to write about. I wanted to write what Thomas (1993) refers to as 'autonomous writings' as opposed to 'collaborative accounts', which I would have written together with others. But at this stage I had no idea that it was possible to research one's own experiences.

Personal narratives

I then came across the work of Ellis and Bochner (2000). After reading their chapter "Autoethnography, Personal Narrative, Reflexivity: Researcher as Subject" in the *Handbook of Qualitative Research*, I felt as though I had been given permission to do what I had been wanting to do: to write my own story.

I subsequently wrote a proposal for researching my own lived experience of coming to know mathematics using narrative theory as my research methodology. Once I had written this proposal I continued to read about narrative theory. I discovered more aspects of narrative theory that appealed to me, namely to accept that it is impossible to be totally objective (Prinsloo, 2004; Ellis & Bochner, 2000 and Carson & Fairbairn, 2002) and to rather embrace the complexity of research and life in general (O'Dea, 1994; Kvernbekk, 2003 and Connelly & Clandinin, 1990).

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[1] Although there are those who see all attempts at meaning making, including all research, as a form of story telling, there are others (such as Polkinghorne cited in Phillips, 1994: 15) who differentiate between narrative meaning making and, for example 'logico-mathematical reasoning'.

Presenting a paper

After completing my proposal I had to present a paper at a student conference. I was to talk about my research thus far. I had written an abstract entitled "The use of narrative theory in mathematics education" but when I came to writing it, things didn't turn out as I had expected. While preparing for the conference I read Robert Nash's (2004) book *Liberating Scholarly Writing* and started questioning whether a narrative approach was the most suitable option for my research.

I found Nash's book inspirational and moving, but as I read I started to question my own motives for wanting to use narrative theory. Here were stories of people who had been through hardships, whose stories were moving, stories that touched me. Stories of "characters embedded in the complexities of lived moments of struggle, resisting the intrusions of chaos, disconnection, fragmentation, marginalisation, and incoherence, trying to preserve or restore the continuity and coherence of life's unity in the face of unexpected blows of fate that call one's meanings and values into question" (Ellis & Bochner, 2000: 744).

When I reflected on my story of coming to know mathematics, I found it difficult to identify the "complexities of lived moments of struggle". I had experienced such moments, but felt that they were not relevant to my experiences of mathematics and, even if they had been, I would not have been willing to write about them.

Not only was I unwilling to write about such personal struggles, but I realised that what I wanted to achieve through narrative theory did not 'fit' with the goals of narrative theory that were identified by Ellis and Bochner (2000: 747), namely to "encourage compassion and promote dialogue." I realised that I was not writing to understand others through understanding myself. I was not attempting to do "something meaningful for (my)self and the world" – another aim of narrative theory identified by Ellis and Bochner (2000: 738). I wanted to write my story simply because it meant a lot to me.

Because of this my paper ended up being two stories – one 'neat' or front-view story and one 'messy' or back-view story. The first, neat story told, in a very 'cause and effect' way, how I came to the point of deciding to write an abstract entitled 'The value of doing personal narrative research in mathematics education'. The second, messy story focused on my reservations regarding narrative theory and why I finally chose to use only aspects of narrative research.

It is important, at this stage, to mention that the views of narrative theory held by the authors/researchers I have been focusing on (such as Ellis & Bochner, 2000; Muchmore, 1999; Connelly & Clandinin, 1990; Carson & Fairbairn, 2002; Powel, 2004 and O'Dea, 1994), are not the only views. Many other researchers understand narrative theory in a more positivistic way, collecting and analysing the narratives of teachers and students using strategies such as discourse analysis, grounded theory and even statistics (see, for example, Andrews & Hatch, 2002; Assude, Sackur & Maurel, 1999; Atkinson, 2004; Bohanek, Fivush & Walker, 2005; Churchill, 2000; Clements, 1999; Hopper & Rossi, 2001; McVee, 2004 and Pomson, 2004).

This more traditional approach to using narratives was in fact what I had originally intended to do – to analyse the narratives of other postgraduate mathematics students. But as I read more about narrative research, it was the personal stories of trying to make meaning of the discontinuities of life that simultaneously appealed to me and scared me.

1.2.3 Research from the Inside

Although 'research from the inside' was the last of the three approaches that I considered using, it was in fact the first approach I was interested in. I originally came across it in a chapter by John Mason in Sierpinska and Kilpatrick's book "*Mathematics Education as a Research Domain: A Search for identity*". Mason (1998) writes about research in general and about what he calls 'research from the inside'. The idea that struck me most – as I could validate it through my own experiences – was that the main products of research are the transformation in the being of the researcher and the stimuli to other researchers and teachers to test out conjectures for themselves in their own context (*ibid*.: 357).

I was also drawn to the idea of 'inner research', which Mason (1998: 362) describes as "developing sensitivity, whether to mathematical ideas, to pedagogical possibilities, or to the thinking of other people". He predicts that "disciplined *inner* research by teachers of their own experience (including their practice, their awareness, and their emotional-energy, as well as the stories they construct to make sense of that experience) will form the core of the significant developments in mathematics teaching and learning in the future" (*ibid*.: 371). It was here that I first realised that I was not alone in wanting to research my own experiences.

I continued reading and came across numerous examples of research from the inside. In fact, Muchmore (1999: 2) argues that "there has been an increasing interest among educational researchers in understanding the lives of teachers – including the way they think about their subject matter and curriculum in general" and Adda (1998: 52) notes the increase in researchers examining the mathematics classroom as "ethnographers" – i.e. "from the inside".

But even as I looked at *different* variations of research from the inside I was aware of *similarities* between them. Ball (2000: 365), who refers to first-person research as opposed to research from the inside, expresses this same idea: "Although first-person approaches to inquiry vary, they overlap and share many similar aims, methods, questions, standards, and perspectives. They focus on issues of practice. They seek to probe beneath the surface of the obvious and taken for granted."

The similarities in the approaches which I identified were with respect to the *focus* of the research and the *goals* of the research. All approaches focused on the researchers' own lived experiences. And all aimed to improve the researchers' practice (Ball, 2000: 366; Garcia, Sanchez & Escudero, 2006 and Mason, 1998) as well as to stimulate others to research their own practice (Dolk, Den Hertog & Gravemeijer, 2002: 176; Magidson, 2005: 137; Pinar, Reynolds, Slattery & Taubman, 1995: 56, 57 and Mason, 2002).

Wanting to use research from the inside
Even though research from the inside appealed to me from the beginning, because of its strong focus on researching one's own *teaching* practice as opposed to one's own experience of *learning*, and because I could find no examples of what research from the inside would look like in a thesis, I did not consider using it at this stage.

Instead, as has been indicated in subsections 1.2.1 and 1.2.2, I first considered phenomenology and then narrative theory. After presenting my paper at the student conference and consequently deciding not to use narrative theory exclusively, I was again unsure of how to proceed. I therefore decided to carry on reading and summarising and reflecting and writing in my research journal – trusting that something would emerge.

While doing this I managed to get hold of a copy of Dave Hewitt's (1994) thesis, which was based on Mason's work. It was inspiring – I wrote in an email, "I didn't think that it was possible that someone could write more than 200 pages of research that I actually *wanted* to read".

After reading his thesis I decided that I too wanted to use the discipline of noticing as my approach. I decided that I wanted to extract themes from my own experiences and so I went back reread everything I had written in my research journal. During this reading I suddenly noticed a recurring theme – the recurring theme of 'noticing'. Soon after noticing this theme, I drew the following Venn diagram:
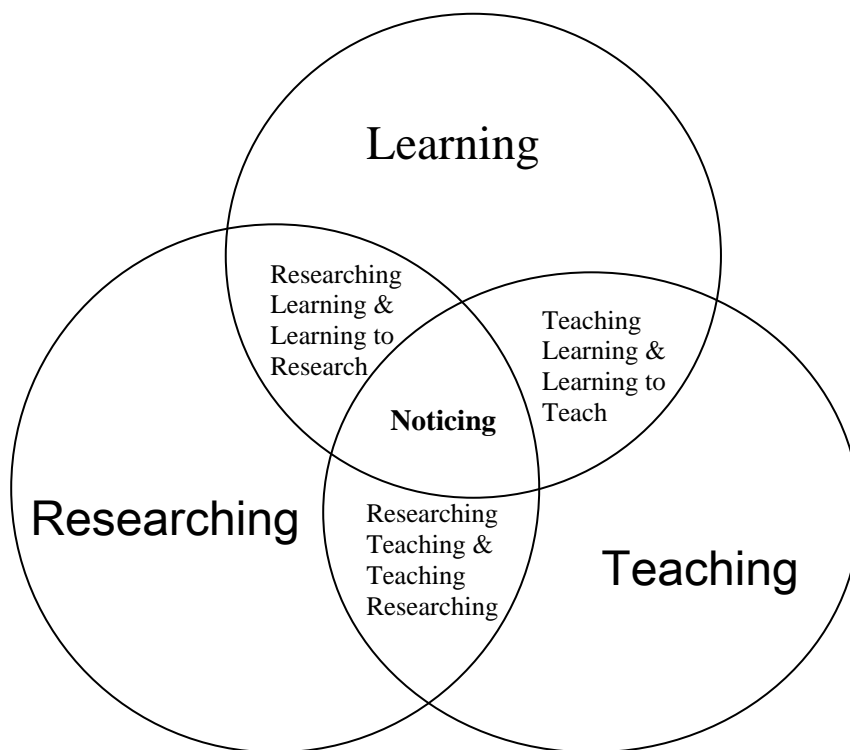
Figure 2: An intersection between Learning, Teaching and Researching

After this 'revelation' I start working on a proposal entitled "In search of an intersection: Reflecting on learning, teaching and researching". I planned to use my reflections to show the extent to which noticing plays a fundamental role in learning, teaching and researching.

I wrote the following regarding my methodology:

6.1   I will write a narrative sharing how I came to be interested in this research (which will be divided into a learning narrative, a teaching narrative and a researching narrative)

6.2   I will then reflect on my experiences by using ideas from the discipline of noticing to give "accounts of" and "accounts for" significant events in the regions of overlap (in the Venn diagram) to identify important themes

6.3   I will then use phenomenological ideas to describe what I think the "essence" of learning/teaching/researching is, namely noticing/becoming aware

But I soon realised that I had once again been blind to the fact that I would not be able to use phenomenology as a research method, since I didn't know enough about it – I had included it because having *three* methodologies felt right[1] and because it seemed a waste to have investigated it and not use what I thought I knew about it.

It was at this stage that the final 'tapestry analogy' emerged (see section 1.3), which in turn led me to writing this section.

Underview

Writing this section has been extremely difficult and at the same time very rewarding. What I found most difficult was cutting down on the length of this section. Looking back at my reflections in my research journal written during the time I was writing this section, I realise again how difficult it was to write without knowing what was going to emerge.

24 April 2006

I'm scared. Scared to write, scared of what I might find out, scared of all these things inside me. Scared of all the contradictions.

At the same time I also realise again that the most rewarding part was experiencing the design emerge as I wrote without knowing what I was going to do.

4 May 2006

It feels so strange. I feel like I am about to have a breakthrough or as though I am busy having a breakthrough – like I wrote yesterday, I think my methodology just emerged. I am incredibly tired but don't want to stop working cause I feel I might miss out on this roll that I am on. It feels good. But it's hard work.

But it's fun too – writing and seeing what happens, what emerges.

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[1] See Tatha (1996: 20) for a discussion of the appeal of the number 3.

SECTION 1.3

OF THE CONSTRUCTION OF A TAPESTRY:

Starting to build a loom

A section in which I share how I came to realise that I would have to start building my loom without having a final plan; that the plan would evolve as I started to build it. I also share what happened when I finally started 'without a plan'.

"The design work does not proceed linearly; instead, the design is iteratively adjusted in the course of the research. The adjustments are themselves objects of study, and contribute to theory building."

(Ball, 2000: 387)

Overview

One of the challenges of using a narrative approach is the complexity of the "ongoing stories being told and retold in the inquiry" (Connelly & Clandinin, 1990: 9). Writing about the story one is busy writing causes many complications as it is difficult to write in chronological order. This section, although it has been written after section 1.1 and 1.2, shares how I came to write them and also what emerged from writing them. Chronologically, what I describe in this section occurred after what I have described in the previous section.

At the end of the previous section I shared how, after considering all three approaches, I was still not sure about which one to use. Even though I didn't know which approach to use, I did know that I wanted to use the idea of extracting themes from different images. I therefore decided to look back at all the notes that I had made about my own learning, teaching and researching and to start identifying recurring themes in the hope that these themes would help me move closer to what I wanted to do.

1.3.1 Waiting

What bothered me most as I looked back at the notes I had made about my experiences was that, although it was possible to write my learning and teaching experiences in terms of descriptions of separate events, my research experiences formed a story and they therefore couldn't be chopped up into smaller pieces and looked at independently.

I considered different options but always felt as though I was having to separate my work into different sections using different approaches…

I wrote more possible chapter divisions. I looked at all the proposals that I had written. I was torn between doing something easy and doing something that was honest.

Then one day I decided to go through all the articles I had read and to identify themes in the articles – themes that would hopefully tie into the themes I had identified in my own writings.

And so I read and summarised and corrected page numbers and looked for proper references and was amazed at all the articles I summarised badly and all the page numbers I had forgotten to fill in. All the time waiting for the design to emerge.

1.3.2 That which came before

I had come across the idea of an emergent design soon after deciding not to use phenomenology as my only research methodology. Initially the idea of an emergent design was more of an excuse not to have to decide on a methodology. I didn't know which methodology I wanted to use, so it seemed easier to say that I would figure it out as I went along. I soon found out that this was not the case.

The use of emergent designs has become more frequent as more researchers have opted for a post-positivistic approach (Campbell, 1997). Campbell explains that within the post-positivistic paradigm "the research project is frequently one of emergent design, where the question under investigation may be designed and re-defined several times in the life of the project and where the research does not draw final conclusions until the interpretation has been completed." (*ibid*.: n.p.)

I encountered the idea of using an emergent design as a research approach in three different 'contexts', namely while reading about using a 'bricoleur' approach to research design, while reading about writing as a form of research and while reading about research which attempted to represent the 'mess' of the world we live in. In the following sections I share arguments and writings that influenced my decision to use an emergent design and that encouraged me during my period of reading and writing and waiting without a concrete plan.

The bricoleur approach

In their introduction to their handbook on qualitative research Denzin and Lincoln (2000) consider the changes that qualitative research has undergone in the last few years. They point out that one of the things that has changed is the role of the researcher. In fact, they suggest that the researcher may now be seen as *bricoleur*, as a "jack of all trades" or a kind of professional do-it-yourself person. Such a qualitative researcher uses whatever 'strategies, methods, or empirical materials' are available. The researcher may go so far as to develop or piece together new tools if needs be.

By the time I came across Denzin and Lincoln's work I had already investigated phenomenology, narrative theory and the discipline of noticing, and was wanting to combine them in an attempt to represent my experiences of learning and teaching mathematics and of doing qualitative research. I wanted to produce the bricolage they described: "a reflexive collage or montage – a set of fluid,

interconnected images and representations, … a quilt, a performance text, a sequence of representations connecting the parts to the whole" (Denzin & Lincoln, 2000: 6).

Denzin and Lincoln (2000) are not the only ones to write about this change in approach. Young (2001, cited in Merz, 2002: 142) calls for "researchers to use multiple methodologies and to work from different perspectives". Merz goes on to make the link I mentioned previously between emergent designs and a bricoleur approach. She writes that "one implication is that we are likely to see an increase in *emergent* designs as researchers explore incorporating multiple perspectives into single research projects." (Merz, 2002: 142)

In fact, Denzin and Lincoln (2000: 4) make a similar claim, namely that when a bricoleur approach is used, the "choices as to which interpretive practices to employ are not necessarily set in advance". This explains why "the solution [bricolage] which is the result of the bricoleur's method is an [emergent] construction that changes and takes on new forms as different tools, methods, and techniques of representation and interpretation are added to the puzzle." (*ibid*.)

Writing as method

While investigating narrative research as a possible methodology, I came across the writing of Laurel Richardson, who writes about writing. She explores using writing as "a means of inquiry" which "departs from standard social science practices and offers an additional – or alternative – research practice" (Richardson, 2000: 923).

She challenges the idea of writing merely being a mode of "telling" about the social world, a "mopping up exercise" at the end of a research project. Instead she argues that writing is also a way of "knowing" – a method of discovery and analysis. Therefore writing is seen as part of the research. The *writing process* and the *writing product* are deeply intertwined. The product cannot be separated from the producer or the mode of production or the method of knowing.

As I have mentioned in section 1.1, Richardson (2000: 924) writes,

> I write because I want to find something out. I write in order to learn something that I did not know before I wrote it. I was taught, however, as perhaps you were, not to write until I knew what I wanted to say, until my points were organised and outlined.

Thus her approach also necessitates an emergent design.

Once again my own experiences confirmed Richardson's claims and provided another motivation for using an emergent design.

Research as Mess

Bechhofer (1974 cited in Bryman & Burgess, 1994: 2) writes that the research process is not a clear-cut sequence of procedures following a neat pattern, but a messy interaction between the conceptual and empirical world, deduction and induction occurring at the same time. Bryman and Burgess (1994: 2) go on to describe this research process as 'messy', writing that "research seldom involves the use of a straightforward set of procedures. Instead the researcher has to move backwards and forwards between different sequences in the research process."

Law (2003, in Gough, 2005: 2) makes the same claim: "in practice research needs to be messy and heterogeneous. It needs to be messy and heterogeneous because that is the way it, research, actually is."

But Law takes this point further. Not only does he argue that research is and should be messy but that the world being researched is messy and that the research attempting to represent this real messy world must therefore be messy. He asks the question: "if this is an awful mess…then would something less messy make a mess of describing it?" (*ibid.*). Gough takes this to mean that Law is arguing that *simplification does not help us to understand mess*. Whatever else simplification might help us with, it does not help us with understanding mess.

Law (2003), also in Gough (2005: 2), writes that "contemporary social science methods are hopelessly bad at knowing mess" and this introduces a motivation for emergent designs. Using an emergent design recognises that the research process is not linear but evolves as the research continues and, I feel, by documenting the way in which the research has evolved/emerged moves closer to representing the mess that research really entails. Which is what I have attempted to do in this thesis.

My desire to capture the messiness of my own research and of the world being researched (namely my own experiences) provided a final motivation for me to use an emergent design.

Encouragement

While I was busy rereading the literature that I had collected and was waiting for 'something' to emerge, I was encouraged by examples of other people who had used emergent designs.

I could identify with Alice Merz (2002) in her article "A journey through an emergent design and its path for understanding", where she discusses the use of emergent designs through reflecting on her own experiences of using such a design. She describes her own "missteps" and how she realised that they were perhaps not missteps after she read Berg (2001 in Merz, 2002), who writes that "whether it is an emergent design or a stumbling, the process needs to be reflected upon and documented for the reader."

Perhaps the most reassuring writing I read on the topic of "emerging research" was by Annie Oberg (2004). She writes about reflecting through writing and this leads her to the phenomena of things emerging as they are done. "My writing, as well as my teaching practice, always proceeds, usually in spite of me, in the manner of creative activity, emerging in the course of taking action" (Oberg, 2004: 239).

Writing about research methodology in particular she says,

> I tell my students in my research methodology courses that it is possible to proceed without topic or method being explicitly formulated. I tell them that when they allow themselves to inquire into something that interests them deeply, they are already researching, and that as they undertake to articulate precisely the topic of their already ongoing research, the research process is continuing to unfold. I tell them that contained in this unfolding is a rudimentary method, which, like the topic, comes into being as it is articulated.

(Oberg, 2004: 240)

1.3.3 That which emerged

The emergence of the structure

One day, after sorting out my summaries, I decided I needed to look at my structure again. I began by looking at two examples of dissertations I had been sent. One of them used the idea of working between different spaces. I wanted to use a similar idea.

17th March 2006

I will share 3 different spaces. The space between these spaces is where I find myself and my thesis.

1st tapestry   my learning

2nd tapestry   my teaching

3rd tapestry   my researching

This is my tapestry. All along I will tell you how I came to construct it; I will show you pieces of the front and pieces of the back. In the end I will follow common threads.

I wrote down the contents pages of the two dissertations I was working from and started playing around with Venn diagrams showing the different areas I wanted to look at and how they overlapped. I was still toying with the idea of working between spaces.

And then suddenly the three sections appeared: a section in which I would construct images of three different parts of my experiences (learning, teaching and researching); a section where I would write about the process I went through to get to this point; and a section where I collected the important threads from the images and wrote about them in more detail. I wrote down my first version of my contents page which has changed only slightly since then.

> Overview
>
> Setting up the loom
>
> Weaving the tapestry
>
> Following the threads
>
> "Underview"

I still don't know how it happened. It just did. Like the day when I was writing my project for Maths Didactics – only this time it took almost a year to happen and not just a week.

With the emergence of the structure came the emergence of the analogy. Looking back I am able to identify all the different references to tapestries and weaving that I had come across during my readings and which have contributed, to a greater or a lesser degree, to my final tapestry analogy.

The emergence of the analogy

The very first reference to a tapestry I came across was in the form of 'following threads'. In one of the very first articles that I read, namely Handa's (2003) article "A phenomenological exploration of

mathematical engagement", he uses the analogy of following different threads to refer to following different themes that he identifies in the lived experience of "mathematical engagement". Although he didn't extend this metaphor himself, reading his article and specifically his reference to threads, brought to my mind the image of weaving with threads. I imagined following significant threads and then using them to construct a new tapestry.

Soon after this I read Mason's (1998) article "Research from the inside in mathematics education" and came across his extended metaphor of weaving and threading for the first time. What I also found interesting was the connection he made between research from the inside and narratives. Mason (1998: 367) suggests that "our sense of an event is the story we *weave* using the salient fragments that are readily recalled, which Bruner (1991) described by saying that we organise our experience and our memories of human events mainly in the form of narrations" (emphasis added). Thus he sees our narratives as tapestries woven from threads which consist of 'salient fragments' – those parts of an event that we remember most readily.

Campbell makes a similar point in her article focusing on her own experience of doing research. She writes, "This paper is an account, the *interweaving*, of the narrative of the writing of my own thesis" (Campbell, 1997: n.p.; emphasis added). Although she doesn't explicitly mention a tapestry, the word *interweaving* once again brought the idea of a tapestry to mind.

While reading Mason's book (2002) "Researching your own practice: The discipline of noticing", I again came across his tapestry or threading metaphor on several different occasions. He begins by referring to George Eliot's (1872 in Mason, 2002: 64) observation, which he paraphrases as follows: "although we have numerous strands of experience, many of us never lay those strands alongside each other". I took this reference to strands to be similar to other references to threads. This was confirmed when Mason went on to argue that to be able to learn from our experiences it is necessary for us to take part in "some sort of mental laying of strands alongside each other, *weaving* them into a *tapestry-story* even, or using them to sensitise myself for noticing in the future" (*ibid.*).

Later in the book he again refers to this metaphor and particularly to how a tapestry-story is constructed – this time in terms of *threading*. In his metaphor *threading* refers to connecting accounts of experiences like threading beads on a string which, he writes, is an almost inescapable activity. Accordingly "a 'thread' in a collection of accounts is a theme, issue, or tension which emerges in the

mind of a (th)reader usually by reference to their experience" (Mason, 2002: 119). He again refers to the idea that our experiences are similar to a tapestry: "The tapestry which is our mental lived experience works on several different levels simultaneously" (*ibid*.). Both ideas – of *a reader of accounts* being *a threader of beads* and of our *lived experiences* being a *tapestry* – resonated with what I had read previously, and more importantly, with my own lived experiences.

Another significant influence was a thesis I read by Dave Hewitt (1994: 25). He describes his approach as follows:

> I will summarise in a short sentence, or even a single word, key factors which have pedagogic significance for me, which stem from my reflections. These will be put in curly brackets - **{...}** - and will be picked up and explored further in later chapters.

Although he also doesn't explicitly mention a tapestry metaphor, his 'summary of key factors' once again brought to my mind the image of 'extracting threads' and the 'picking up and development of these key factors' brought the image of 'weaving this threads'. I now had an example of how the analogy that was slowly growing in my mind could be implemented in the context of a thesis.

Of all the references that I came across the one that impacted on me the most – in the sense that it brought everything together and gave me the idea for the final structure – was by Merz (2002) in her previously mentioned article "A journey through emergent design and its path for understanding". She refers to the following quote by Smith (1994): "'The figure under the carpet' … [is] like a tapestry, which shows images on its front side and displays the underlying construction on the back."

Rereading her work I realised that Merz interpreted Smith's tapestry differently to the way in which I did. She took his analogy as referring to capturing the beauty of the essence of her data as opposed to merely trying to 'capture' bits of reality of the data. I took it as referring to presenting those parts of research that are usually hidden, i.e. as referring to being honest about the construction of a thesis as opposed to merely showing the neat finished version.

The emergence of the loom
After the structure and the analogy emerged I started writing section 1.1 and then section 1.2. At this stage I had no intention of writing section 1.4 …

6th June 2006

...and yet as I wrote & reflected on my experiences with each different approach (thus weaving the part of my tapestry that told of the planning of the tapestry[1] and the designing of the loom[2], ending with the section that shared how I stared writing without an explicit framework[3]), my framework[4] emerged. I know well enough not to assume that this is my final say regarding my approach but just as there came a point where I felt my tapestry analogy was stable enough to work with, I feel that this approach is stable enough to work with for now.

And so I decided to include an extra section where I described the loom that emerged.

Underview

This section was a description of how my structure, my analogy and my loom emerged. I started writing my first section having no idea what would emerge – all the while holding on to the following quote:

> This way of proceeding feels risky. Setting out, there is always the risk that nothing will happen. When something does happen, there is the risk that the result will not be viewed as credible: proceeding without method is suspect in an academic environment where adherence to method is the only legitimate source of legitimacy. And yet I feel compelled to proceed in this way, not only in teaching, researching, and writing, but also in relating with colleagues, acquaintances, friends, and family members. Being open, paying attention, and not knowing, that is, presuming as little as possible about others, is a deeply respectful way of relating.
>
> (Oberg, 2004: 242)

What finally emerged was my loom – a framework linking phenomenology, narrative theory and research from the inside. I present this framework in the next section.

Although this section has been easier to write than section 1.2 or 1.4, writing it has forced me to go back and look at the process that I have been through. It amazes me how long it has taken me to get this far. It has been almost a year since my final structure emerged for the first time and I am still busy with the first section.

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[1] Section 1.1
[2] Section 1.2
[3] Section 1.3 thus far
[4] Section 1.4

SECTION 1.4

OF THE CONSTRUCTION OF A TAPESTRY:

The loom that emerged

A section in which I make connections between the three different methodologies through which I journeyed.

"Seeing 'the underlying pattern beneath appearances' (Nachmanovitch, 1990: 31) opens up a new way of seeing and hence a new world of possibilities."
(Oberg, 2004: 242)
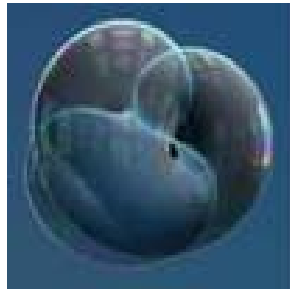
Overview

In the previous three sections I looked at what influenced my decision to start this research project, my investigation of three different approaches and my experience of coming to use an emergent design.

In this section I share in more detail 'that which emerged' through writing the previous sections. Originally I hadn't planned to include such a section and yet as I wrote about each of the approaches the similarities emerged. At first I was only vaguely aware of them and then, as I once again started to write without knowing what I wanted to write, the structure crystallised.

In this case it began with an image (similar in nature to the image I had while trying to sum up the terms work in my fourth-year didactics course): a circle consisting of three threads made of words, each thread a different colour. At some places the threads overlap. In others they are separate. The circle represents the continuous circular nature of research and the relationship between researcher, research and reader.

By constructing this mental image on paper I managed to get a structure for this section. The challenge was to represent a circular three-stranded structure in a thesis that only allows one to proceed linearly from one starting point to one end point.

To overcome this challenge I have chosen to include three consecutive representations: The first one is the closest to the mental image and contains the least written information. The second one is a linear version of the first one. It also attempts to represent the logic of the circular argument. The third representation contains the most written information and is where I justify my claims. It is entirely linear and is to be read in conjunction with the other two representations.

1.4.1 Three Representations

A First Representation

reflecting on your own & others experiences

re-enter the world with a deeper understanding of your past and think by reflecting on your own experiences and improving your awareness ... reflecting on your own experiences ... make meaning of your past & act differently in the future ... paying attention and allowing themes to emerge, choosing not to another, selecting threads, developing sensitivity

so that you can make meaning of your lived experiences ... narrative phenomenology inquiry ... research from ... of their own & other lived experiences

and so that they can construct new phenomenological knowledge ... be co-participant in the dialogue [and use] ... interpret texts for themselves

relate with their pre-reflective experiences ... speaks to them on an emotional level so that they can use that own experiences to ... resonate with their experiences

which ... can make sense of their past and act differently in the future by reading descriptions ... for accounts ... can start communicating with others ... can be made more aware of other's experiences so that others ... there are no perfect or complete answers ... remembering that there are ...

by writing & re-reading narratives, describing ... accounts of the ...

A Second Representation

use phenomenology to study your own and other's lived experiences
use narrative inquiry to study your own and other's lived experiences
use research from the inside to study own lived experiences

so that you

       can re-enter the world with a deeper understanding
       can make meaning of your past and future
       can make meaning of your past and have different options for your future actions

           by

           reflecting on your own and others' experiences
           reflecting on your own experiences
           reflecting on your own experiences

           and

           improving your attentiveness
           paying attention
           developing your sensitivity

           and

           allowing themes to emerge
           (choosing not to analyse)
           seeking threads

                by

                writing and rewriting descriptions
                writing and rewriting stories
                writing and rewriting accounts of and accounts for

                remembering that

                no text is ever perfect
                no text is ever complete
                there are no final answers

and

so that others

       can be made more aware of others' experiences
       can be made more aware of others' experiences & can start communicating with others
       can make sense of their past and have different options for acting in the future

by

reading descriptions
reading stories
reading accounts of and accounts for

which

resonate pre-reflective experiences
speak to people on an emotional level
resonate with their experiences

so that they

(can use their own experiences to validate claims)
can use their own experiences to test generalizability
can use their own experiences to validate claims

and so that they can

construct new phenomenological knowledge
be co-participants in the dialogue
interpret texts for themselves

and possibly…

use phenomenology to study their own and other's lived experiences
use narrative inquiry to study their own and other's lived experiences
use research from the inside to study their own lived experiences

Third Representation

As mentioned previously, this representation is entirely linear and contains the most written information. Here the three questions (*What* is being researched? *Why* is it being researched? and *How* is it being researched?) that guided the first representation are stated explicitly and are answered with regards to each of the three approaches. Notice that the second question has two answers in all three cases, i.e. two main reasons why the research is being done and that each reason has its own corresponding proposed method to achieve this aim.

What is being researched? (Focus)

Overview

Although all three questions are linked and are equally important, generally the question of *what* is being researched is answered first. Once this has been established it is easier to turn to questions of motivation and method.[1] Each of the three approaches to be discussed researches lived experiences. This is possibly the most fundamental way in which they overlap. It is also the main reason why I was initially drawn to each of them.

Phenomenology

Of the three approaches phenomenology is the only one that focuses entirely on lived experiences. In fact, Van Manen (1990: 57) has described it as "the study of lived experiences". Smith (2005: n.p.) elaborates this description and writes that phenomenology is "the study of the structure of various types of experience ranging from perception, thought, memory, imagination, emotion, desire, and volition to bodily awareness, embodied action, and social activity, including linguistic activity."

Although phenomenology focuses specifically on lived experiences, I agree with Van Manen (2003: n.p.) that it is possible to differentiate between "phenomenological research performed by professional philosophers and phenomenological research conducted by professional practitioners". Devenish (2002: 2-3) makes a similar distinction between phenomenology and *applied* phenomenology. The theory of phenomenology, to a large extent, considers the structure of lived experiences in general but says little regarding how to research *specific* lived experiences. My feeling is that the theory of phenomenology was based on the lived experiences of the philosophers who developed the theory. I am

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[1] It would be possible to start with a motivating factor, which then guides one's focus and method, or to start with a particular method one wants to apply and then choose one's focus.

suggesting that to theorise about the nature of lived experiences one needs to reflect on the nature of one's *own* lived experiences.[1]

In contrast to 'theoretical' phenomenology, 'applied' phenomenology is practised by professional practitioners in "the applied domains of the human sciences such as education, clinical psychology, nursing, medicine, and specializations such as psychiatry or midwifery" (Van Manen, 2003: n.p.). Here practitioners use phenomenological methods to research the lived experiences of *others*.

I see myself as falling into neither of these two categories – or perhaps as transcending the distinction between them. I am not a professional philosopher interested in "philosophical topics, themes, and issues" (Van Manen, 2003: n.p.) and yet, even though I am a professional practitioner working in the applied domain of human science, namely education, I am not interested in researching the lived experiences of others. It is my own lived experiences that I wish to research.

Narrative Inquiry

Like phenomenology, narrative inquiry focuses on researching lived experiences. According to Connelly and Clandinin (1990: 2), narrative inquiry is "the study of the ways humans experience the world" and that "because of its focus on *experiences* and the qualities of life and education, narrative is situated in a matrix of qualitative research" (*ibid*.: 3).

As in phenomenology, it is possible to make a distinction regarding the focus of research in narrative inquiry. Thomas (1993) differentiates between autonomous and collaborative narratives. Autonomous narratives are written by individuals about their own experiences, while collaborative narratives are written by individuals with the help of another – be it a mentor or a researcher.

Being interested in researching my own experiences, I am therefore interested in writing autonomous or personal narratives. Ellis and Bochner (2000: 744) write that "Usually the author of an evocative (or personal) narrative writes in the first person, making herself the object of research" and, similarly, that such narratives "make the researcher's own experience a topic of investigation in its own right" (*ibid*.: 733). In the same way the Personal Narratives group in the USA (1989: 14) argues that "your own experience is a valid part of your own knowledge, as long as it is subject to public critical appraisal."

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[1] Much in the same way that Mason's *Research from the Inside* aims to research one's own experiences

Research from the Inside

Like the other two approaches, research from the inside attempts to research lived experiences, specifically lived experiences relating to a "personal practice". Mason (2003a: 1) proposes that research from the inside is a "legitimate form of research for practitioners wishing to study their own practice". Ball (2000: 365) writes that research from the inside where "the teacher is also the principal investigator of the research" focuses on "issues of practice" and seeks to "probe beneath the surface of the obvious and taken for granted."

Unlike the other two approaches, it is not possible to make a distinction based on the focus of such research – research from the inside, per definition, focuses on your own experiences. It is, however, possible to differentiate between the aims of such research as mentioned in subsection 1.2.3.

Of the three approaches, research from the inside is most suited to my research question, aims and methodology, as my interests lie in researching my own experiences. This should not detract from the impact that the other two approaches have had on my research.

Underview

While my research has always focused on my own lived experiences, the emphasis has changed during the research process. Originally I planned to research my experiences of coming to know mathematics. Later I began to research my experiences of learning and teaching mathematics. At this stage I have changed my emphasis so as to include researching my experiences of coming to use an emergent design to do qualitative research.

Why is the research being done? (Primary reason)

Now that it has been established that all three approaches focus on lived experiences, the question of motivation can be considered. As mentioned previously, the aims of all three approaches can be divided into two main aims. These are: to change the researcher and to change the reader. Each aim has a corresponding method with which to achieve the aim.

Researching to change the researcher

Overview

A review of the literature focusing on motivations for research shows that 'changes in the researcher' is not often listed as one of the major aims of research projects. As seen in section 1.2 research projects

traditionally aim to discover something or to prove or disprove a hypothesis. Others aim to understand or to empower or to deconstruct. As mentioned above, I first came across the idea that research in fact primarily changes the researcher in John Mason's work. Although he is referring particularly to research in mathematics education, I feel that this can be extended to all research.

Phenomenology

Campbell (1997: n.p.) writes that "Phenomenology … offer[s] ways of understanding not offered by other research methodologies. In contrast to the scientific method it is both poetic and interpretive but those working from an emancipatory view of the role of research express dissatisfaction that it does not go beyond interpretation; it does not become emancipatory. Research needs to do more than offer understanding about human experience."

Although I agree that phenomenology aims to give the researcher a better understanding of the world, or as Van Manen (1990: 57) puts it, "it aims for a deeper understanding of the nature or meaning of our everyday lived experiences", I would argue that in as much as it succeeds in changing the researcher, it does go beyond interpretation. Even if it does not necessarily become emancipatory, then at least it makes a difference in the researcher. This is echoed in the following quote regarding reduction (the primary method used in phenomenological research): "Therefore, the reduction is not an end in itself – rather, the reduction is a means to an end: [allowing the researcher] to be able to return to the world as lived in an enriched and deepened fashion" (Van Manen, 2003: n.p.).

The idea that phenomenology aims to change the researcher is also seen in the way that the information that it provides is viewed. Van Manen (2003: n.p.) writes that "Phenomenology does not provide us with 'information' in the usual sense of the term. Instead, the practical significance of phenomenological knowledge is formative in nature: It enhances our perceptiveness, it contributes to our sense of tact in human relations, and it provides us with pathic forms of understanding that are embodied, situational, relational and enactive."

Narrative Inquiry

The most common motivation for using narrative inquiry as a research approach is that humans make meaning of their past experiences through telling and retelling stories (Blake, Blake & Tinsley, 2001; Dhunpath, 2000; Ponte, 2001; Muchmore, 1999; Connelly & Clandinin, 1990 and Phillips, 1994). Or

as Swindler (2000 cited in Andrews & Hatch, 2002: 188) put it, "humans tell stories to themselves and to others to make sense of the worlds they inhabit."

Even though most research employing narrative inquiry aims either to offer understanding about human experience or to make sense of the past of the individuals writing the narratives (be they the researcher themselves or individuals approached by the researcher), there are those who argue that it is possible to go beyond this. Such research aims to influence the future of those writing about their past experiences.

Abbs (1974 cited in Clements, 1999: 2) captures this idea in his definition of autobiography when he writes that it is "an act of writing perched in the present, gazing backwards into the past while poised ready for flight into the future."

Assude, Sackur and Maurel (1999: n.p.) go so far as to make a connection between learning and writing narratives. They hypothesise that learning includes "developing the ability to recognise in the present an 'echo' of a *past* story which lies in our memory. This permits us in the *present* to anticipate, to know what our expectations are, to understand and thus act for the *future*." Since narratives allow for this "triple representation of *past, present* and *future*" they allow us to learn how to act in the future. They conclude that "The *present* is both the time of *memory* and the time of *anticipation*, that is, of attention for a project concerning the future" (emphasis added).

But it is not only learners that can benefit from narratives in this way but also practitioners, particularly teachers. De Vries (2000: 177) and Dhunpath (2000: 544) both argue for the use of narratives in education precisely because they provide the opportunity for teachers to improve their practice.

This shift in emphasis from exclusively looking at the past to looking at the past in order to influence the future brings with it a shift in the kind of questions asked about narratives. Bochner (in Ellis & Bochner, 2000: 746) addresses this when he writes:

> So the question is not, 'Does my story reflect my past accurately?' as if I were holding a mirror to my past. Rather I must ask, "What are the consequences my story produces? What kind of person does it shape me into? What new possibilities does it introduce for living my life?

Research from the Inside

Research from the inside shares narrative inquiry's aims of making meaning of the past and informing the future actions of the researcher. In fact, as mentioned above, John Mason (1998: 357) suggests that "the most significant products are the transformations in the being of the researchers". He later argues that within research from the inside it is possible to find assertions to "help make sense of *past* experiences, to 'fit' with *current* experiences and to inform *future* practice." (*ibid*.: 361; emphasis added)

In the research approach developed by John Mason, referred to as the Discipline of Noticing, a strong emphasis is placed on the future. Mason (2002: 249) writes that "the discipline of noticing is firmly directed to the future" and that "the only virtue in making distinctions is to inform future practice and to facilitate communication" (Mason, 2004: 12).

Mason (2002: 88) summarises the aims of the Discipline of Noticing as follows:

> The Discipline of Noticing is a personal and collective enquiry into how to sharpen moments
> of noticing so that they shift
>> from retrospective 'I can have …' or 'I should have…'
>> to presently spective 'I could…' so that can participate in making a choice,
> by means of
>> descriptive but non-judgemental postspective review: 'I did…',
>>> and
>> prospective preparation of 'I will …' using the power of mental imagery.

Ball (2000), while discussing the use of research from the inside in education, also refers to the aim of influencing future actions. She writes that "reflection in and on teaching has as its primary goal to heighten deliberation in and about practice, to consider alternative interpretations, to seek information for next steps as a teacher of the class or child at hand" (Ball, 2000: 366).

What I found most interesting was another link mentioned by Mason (1998: 371) between narrative inquiry and research from the inside:

> I predict that disciplined inner research by teachers of their own experiences (including their
> practice, their awareness, and their emotional-energy, as well as the *stories* they construct to

make sense of that experience) will form the core of the significant developments in mathematics teaching and learning in the future, augmented and informed by sensitivities located through other more traditional outer research styles" (emphasis added).

Underview

Research aiming to inform future actions initially didn't appeal to me at all. I wanted to research my own experiences merely for the sake of researching them. As I began constructing this tapestry, I came to appreciate that by telling my stories I was making meaning of my past. But I still didn't see why I should be concerned about my future actions.

It has taken a long time for me to come to the place where I now consider research that does *not* have a focus on the future less meaningful than research that does. We cannot change the past, we cannot change others, but we can work on changing the way we act in the future (Mason, 2002).

Even though I feel that I have changed, I still often find myself once again with a mindset of doing research for the sake of doing research.

How are the (primary) aims achieved?

As mentioned above, each of the three approaches has two major aims – to change the researcher and to change the reader. In this section five different means by which the first aim of 'changing the researcher' can be achieved (namely by reflecting, by improving awareness, by extracting themes, by writing and by accepting that no work is ever finished) are considered.

Reflecting

Overview

Reflection can refer to a wide variety of activities. In this section I am using it in the most general sense, namely any form of writing, talking or thinking about one's experiences.

All three approaches make use of reflection although to different extents. Because the focus of this section is the transformations in the researcher, the reflection referred to is necessarily reflection done by the researcher on her own experiences.

Phenomenology

As mentioned previously, both theoretical and applied phenomenology include an aspect of researching personal experiences. In applied phenomenology this is seen in particular research approaches where the researcher not only asks the participants to write about the experience being researched, but writes about her own experiences. As such the researcher needs to *reflect* on her personal experiences of the particular lived experience she is researching. Van Manen (2003: n.p.) goes so far as to write that "the method of phenomenology is radical reflection."

Narrative Inquiry

In their discussion of the use of collaborative narratives in pre-service teacher training Dolk *et al*. (2002: 165) state that "*reflection* is seen as a precondition for learning, especially for learning from one's own activities" (emphasis added). In education in general, collaborative narratives can be used to encourage, assist and evoke *reflection* on perceptions of personal experiences and on teaching practices (O'Dea, 1994; Dolk *et al*., 2002; Dhunpath, 2000; Thomas, 1993; Blake, Blake & Tinsley, 2001).

Similarly, McCammon and Smigiel (2004: n.p.), writing about using narratives in in-service teacher training, note that "Writing, editing, discussing the narrative enables teachers to reflect (Schön, 1983) on their lives in classrooms. It is primarily in *reflection* … that new discoveries about teaching and learning are made" (emphasis added).

Although mainly referring to collaborative narratives, I feel that these comments are just as applicable to autonomous narratives.

Research from the Inside

Both Ball (2000) and Mason (2002) note that reflection is an important aspect of research from the inside. More specifically, in the book written by Mason on the Discipline of Noticing he emphasises the fundamental role played by reflection: "Reflection is widely acknowledged as an important component of intentional learning, and it is the aim of this book to go beyond mere word, and to offer detailed, structured, systematic practices which can serve to develop an inner witness" (Mason, 2002: 25).

Underview

Reflection is important in all three approaches although all three have different focuses, namely: reflecting on one's past experiences to understand the structure of the experience, reflecting on one's past experiences to make meaning of them and to make meaning of the future, and reflecting on past and present experiences to make meaning of them and to change future actions.

For my thesis I have reflected on my past and present experiences of learning and teaching mathematics as well as on my present experiences of doing research so that I can approach my learning, teaching and researching differently in the future.

Improving Awareness

Overview

While reflection focuses more on past experiences, awareness or attentiveness can refer to the past or to the present. One can reflect on *past* experiences attentively and one can work on being more attentive in *present* situations, i.e. work on being more aware in the present moment. The three approaches each call for a certain awareness or attentiveness.

Phenomenology

Phenomenology, with its focus on past experiences, uses awareness or rather attentiveness to improve reflection. Van Manen (2003: n.p.) puts it as follows: "the reduction refers to a certain attentiveness. If we want to come to an understanding of the unique meaning and significance of something we need to reflect on it by practicing a thoughtful attentiveness."

Narrative Inquiry

Although the importance of 'being attentive' or 'improving one's awareness' is not often explicitly mentioned in literature on narrative research, I feel that it is definitely an important aspect of writing autonomous narratives. I suggest that it is not only important to be aware of present experiences in order to reflect on them later, but that the extent to which one is transformed through the process of writing is often related to the extent to which one is aware of, or attentive to, the process of transformation.

An exception to the lack of focus on attentiveness in narrative inquiry is found in Oberg's work. Oberg, while reflecting on reflecting, writes that she has realised the importance of attention, not in the sense

of paying attention to a particular event or person but in the sense of being open. She writes that "attention must be open and without presumption of knowing." She goes on to write that "Opening, paying attention, and not knowing have become key in my theories about my teaching. It makes sense that they would also be key in my writing" (Oberg, 2004: 240).

Research from the Inside

Of the three approaches, research from the inside in general and Mason' version of it in particular, focuses the most on the idea of attention. There is, however, a slight difference in terminology in that, rather than referring to awareness or attentiveness, Mason refers to sensitivity and noticing.

Mason (2002: 90) writes that "the product of enquiry is not a collection of assertions of uncertain generality, but rather a way of working which may enhance *sensitivity to notice*, or, in other words, educate *awareness*."

Mason's (2004: 6-7) argument for developing sensitivity is based on the belief that a lived experience can be of "simultaneous immersion and separation". He posits that "it is possible to develop an inner witness or monitor which observes without involvement while the predicated 'I' is immersed, fully involved and caught up in action. This is much more than post-event introspection or reflection."

Underview

In my own research process, the notions of being aware, being sensitive, paying attention and noticing appealed to me immediately. Reflecting back on this initial attraction and taking into consideration the argument that we are only able to see what we are already aware of, I can now see how the fact that I have always been interested in noticing details and of being attentive to my own experiences made me more likely to be interested in this type of research.

What I have found a challenge is to move from noticing things in the past or present to reflecting on them with the goal of changing future actions. And, even more importantly, moving from such noticing to doing research. Mason addresses this exact point in the following extract:

> [T]here are four levels of awareness. The first is not being aware at all; the second is *noticing*
> but only becoming aware of it later as a result of someone else saying something which brings

it to mind; the third is *marking*, that is, being able to make a subsequent re-mark about it; the fourth is being sufficiently energised to make a *recording* of what is noticed, in some way.

These three observable states are associated with energy: *noticing-marking-recording.* Each takes more energy than the preceding. It is easy to notice, harder to mark, and much harder to record, even though it seems it should be the other way around.

These states of noticing-marking-recording are perfectly natural and ordinary. Discipline is only required if you want to become more systematic, to probe more deeply, and to enhance your opportunities to make choices in the future.

(Mason, 2003a: 5)

As in most situations, it is precisely the fact that discipline is required which makes the task challenging and, at the same time, worthwhile.

Extracting themes

Overview

A third approach is that of analysing data by extracting themes. In this aspect, more than any other, the approaches differ. In fact, there are those who argue that research that employs autonomous narratives should not make use of the extraction or identification of themes at all.

Phenomenology

Van Manen (1990: 57) refers to the use of themes in phenomenology when he writes that "from the lived experience description the overall thematic quality of the description is extracted". Devenish (2002: 14) describes this process as a process of distillation, where "on the one hand topics and texts which are un-related to the research question are removed from the research, and on the other statements and core-themes which fall within the selected parameters of the research question are increasingly brought into view."

Although phenomenology makes use of the extraction of themes, the process of identifying these themes is slightly different to the usual sense in which it is used. This is seen in the following quotes:

Phenomenologists do not use coding, but assume that through continued readings of the source material the 'essence' of an account will be revealed. This approach does not lead to uncovering laws, but rather to a 'practical understanding' of meanings and actions.

<div align="right">(Miles & Huberman, 1994 cited in De Vries, 2000: 166-167)</div>

So the eidetic reduction is not a simplification, fixation, or contraction of the world into a system of fully resolved concepts – rather it is the exact opposite: the eidetic reduction makes the world appear as it precedes every cognitive construction: in its full ambiguity, irreducibility, contingency, mystery, and ultimate indeterminacy.

<div align="right">(Van Manen, 2003: n.p.)</div>

Narrative Inquiry

In narrative theory, especially when use is made of collaborative narratives, there is a tendency for narratives to be analysed using themes. Often researchers compare narratives written by different individuals who have participated in the researcher by means of themes extracted from each narrative. The extraction of themes is also sometimes used in autonomous narratives. For example, the first two steps in the research carried out by De Vries (2000: 166) consisted of, firstly, writing an autobiographical narrative and, secondly, analysing the autobiographical narrative, drawing the essential themes from it.

In contrast to this there are narratives which are used without any form of analysis. Pomson (2004: 647) refers to a piece by Riseborough (1988) where "Tommy, an embittered and psychologically battered teacher, tells the story of his career entirely in his own words. Riseborough, the researcher, offers no analysis. The story speaks for itself."

Similarly, Ellis and Bochner argue that autonomous narratives can be meaningful without analysing them. They write that, in personal narratives, "the author privileges stories over analysis, allowing and encouraging alternative readings and multiple interpretations" (Ellis & Bochner, 2000: 745). Bochner (*ibid*.) goes on to address the fact that in most cases researchers are expected to analyse the narratives they use in their research. He points out that often, unless the researcher subjects the narrative to some form of analysis, the research is not considered to be social science. Instead, Bochner would prefer to see the writing of narratives as social science research in its own right.

Research from the Inside

Unlike narrative inquiry, all versions of research from the inside that I have encountered have made use of themes in some or other form. In the discipline of noticing in particular, themes are extracted but with a slightly different focus. Where phenomenology researches a particular experience and themes are thus extracted from descriptions of this particular experience; in the discipline of noticing the researcher is researching her *practice* and therefore reflects on a certain time period and extracts themes based on significant experiences with in this time period.

Mason (2002: 95) refers to this process as "systematic reflection", where accounts are kept to facilitate reflection and the identification of "common threads, themes, and issues". Like phenomenology, themes are not extracted through content analysis or similar means. Rather the process of seeking threads (or themes) is described as one of "re-entering a succession of moments and allowing a 'flavour' to emerge" (*ibid*.: 56).

Underview

In this thesis I have chosen to use both approaches, namely of analysing on the one hand *and* of choosing not to analyse on the other. When researching my own experiences of learning and teaching mathematics, I will attempt to extract themes as understood by Mason. When researching my own experiences of researching within the social sciences, I have chosen rather to "privilege stories over analysis, allowing and encouraging alternative readings and multiple interpretations" (Ellis & Bochner, 2000: 745).

Writing

Overview

Each of the three approaches employs writing as a means of research, although 'research from the inside' less so than the other two. I feel that writing, more than anything else, has the potential to transform the researcher. It is through writing that the other means of achieving this aim are executed – we reflect through writing, we become more aware as we write, and we extract themes through writing.

Phenomenology

Writing plays a fundamental role in phenomenology. In fact, Van Manen (2003: n.p.) describes it as the way that phenomenology is practised, not in the sense that phenomenological research merely involves writing but that "research is the work of writing – writing is at the very heart of the process."

Van Manen (2003: n.p.) begins his discussion of the use of writing in phenomenology by saying what phenomenological writing is *not*. "Writing is not just externalizing internal knowledge, it is not simply writing up one's conclusions, it is not composing the final research report, it is not something that comes at the end of phenomenological inquiry, as if it were a mere stage in the complex set of procedures of the research process." Rather, he writes, something happens through the act of phenomenological writing. "Phenomenological writing is the very act of making contact with the things of our world. It is in this sense that to do research is to write and that the insights achieved depend on the right words and phrases, on styles and traditions, on metaphor and figures of speech, on argument and poetic image" (Van Manen, 2003: n.p.).

Narrative Inquiry

In narrative inquiry, as in phenomenology, writing is seen as more than "simply writing up one's conclusions" or as a "mopping up activity at the end of a research project", but rather as "a method of discovery and analysis" (Richardson, 2000: 923). Narrative inquiry views writing not as something *different* from research but rather as an *integral part* of the research, as a "way of knowing" (*ibid*.: 932); as a method of discovery (Richardson, 2001: 35).

Research from the inside

Unlike phenomenology and narrative inquiry, research from the inside views writing as a necessary part of research, but *not* as a method of discovery and analysis or as being at the heart of the process. Instead, *noticing* and *being aware* are at the heart of the process.

Even so, writing plays an important role - it is the means by which personal experiences are shared. Although these personal experiences are often shared using a narrative style – an example of the overlap between these approaches (see Magidson, 2005), within the discipline of noticing, personal experiences are shared in another way. Mason (2002: 95) explains that rather than using a narrative style where factual accounts are merged and interwoven with justification and theorising, the discipline of noticing uses brief-but-vivid accounts which, as far as possible, remove the "inferred, the interpreted and the explanatory, as well as justifications, judgements, and investments".

Although narrative inquiry and the discipline of noticing differ in terms of what is written (narratives as opposed to brief-but-vivid descriptions), both go through similar processes. Mason (2002: 174) cites Fenstermacher (1997), who suggests that "in order to be useful, narratives go through a sequence of

transformations: of telling and constructing; retelling and reconstruction; and deconstruction and critique." Mason goes on to say that these phases parallel the phases that brief-but-vivid descriptions in the discipline of noticing go through.

Underview

I have realised from personal experience that whether writing is used as a means of doing research or merely as one aspect of doing research, it has the potential to transform the researcher.

Accepting that no work is ever finished

Overview

This is less a 'means by which the researcher can be transformed' than a view shared by the three approaches regarding the nature of the research done in an attempt to transform the researcher. Thus, while one reflects, becomes more aware and extracts themes through writing, it is important to remember that research is always an ongoing process with no final stage or conclusions to be drawn.

Phenomenology

Within phenomenology the view that research and the meaning that is determined are never complete but are to be continuously reviewed is seen in the following two quotes:

> It is also helpful to be reminded that phenomenological inquiry-writing is based on the idea that no text is ever perfect, no interpretation is ever complete, no explication of meaning is ever final, no insight is beyond challenge. (Van Manen, 2003: n.p.)

> Also, it is important to remember that the phenomenological determination of meaning is itself always indeterminate, always tentative, always incomplete, always inclined to question assumptions by returning again and again to lived experience itself, the beginnings of phenomenological inquiry. (Van Manen, 2003: n.p.)

Narrative Inquiry

The view that the product of narrative inquiry, namely the text that is produced, is never complete is referred to by various people. Muchmore (1999: 16), in his discussion of narrative inquiry (both autonomous and collaborative), reminds us that "no reading or writing of a life is ever complete or final…there can only be multiple versions of a biography or autobiography."

Similarly Richardson (2000: 923), referring to Rose (1992), argues that when writing is viewed as a method of research, we 'word the world' into existence. She goes on to say that we 'reword' the world by retyping work, by choosing different words, by moving paragraphs again and again, and that this 'worded world' never "accurately, precisely, completely captures the studied world, yet we persist in trying."

Later on in her article, Richardson (2000: 929) considers the implications of post-structuralism for qualitative writers stating that one such implication is that as writers we are freed from "trying to write a single text in which we say everything at once to everyone."

Research from the Inside

Within research from the inside, it is the continuous nature of the research process that is considered (as opposed to the view that a particular text can never be complete). Mason (2002: 234) writes that "one of the features of the Discipline of Noticing is that there are no final answers, only on going development and enquiry." Eddy (2003: 90) demonstrates this when, in the concluding remarks of his thesis, he writes that although he has finished his research as such, he still has questions he needs to answer and questions he needs to ask.

Underview

I have found the view that no piece of work and no enquiry are ever complete to be encouraging and liberating. As soon as one is no longer trying to "say everything at once to everyone" (Richardson, 2000: 923), it is possible to focus on a specific context and to make claims that are relevant to that context without having to generalise to other contexts.

It has also helped me to start writing. What I wrote no longer had to address everyone about everything or to provide final answers to questions. It meant that I could start writing about what I knew at the time, while being aware that the way I would see things would change in the future.

On the other hand, this view makes doing research more challenging. If no text is ever complete, when does one stop working on the text? How do you decide what should be part of your research at a specific time and what should be left for the future or be left in the past?

Why is the research being done? (Secondary reason)

This question has been asked and answered before. It was mentioned that the aims of all three approaches can be divided into two main aims: to change the researcher and to change the reader. It was also mentioned that each aim has corresponding methods with which to achieve this aim. I have discussed the first aim and the corresponding methods and will now consider the second aim and its methods.

Researching to change the reader

Overview

Just as one does not often come across research projects listing 'the transformation of the researcher' as a major aim of research, 'the transformation of the reader' is also rarely explicitly stated as a desired outcome. Interestingly enough I came across both ideas in the same article by John Mason. He not only writes that "the most significant products are the transformations in the being of the researcher" but that "the second most significant products are stimuli to other researchers and teachers to test out conjectures for themselves in their own context", which as I understand it, leads in turn to transformations in their being (Mason, 1998: 357).

I will begin by considering the particular way in which each approach aims to transform the reader and will then move onto the methods suggested to achieve these transformations.

Phenomenology

Within phenomenology there is a close link between the aim of transforming the researcher and the aim of transforming the reader. In the first part of "Why is the research being done?" I interpreted Van Manen's point that "the reduction is a means to an end: to be able to return to the world as lived in an enriched and deepened fashion" as referring to the researcher. But in retrospect and taking into consideration his quote regarding the significance of knowledge – namely that "it enhances *our* perceptiveness, it contributes to *our* sense of tact in human relations, and it provides *us* with pathic forms of understanding that are embodied, situational, relational and enactive" – I suggest that Van Manen is referring to the researcher as much as to the reader. In this sense there is a close link between the two aims.

Elsewhere Van Manen (2003: n.p.) makes specific mention of the effect a text can have on the *reader*. He reminds the researcher that, while writing a text, one must be aware of the possible effects that the

text can have on different readers. He also mentions that when "a moment of wonder" stimulates a question about "the meaning of some aspect of lived experience", the researcher should be challenged to write a text that stimulates a similar "sense of wondering attentiveness to the topic under investigation" – and therefore a change – in the reader.

Narrative Inquiry

In the context of narrative inquiry the researcher aims to transform the reader by allowing her to experience a phenomenon from a different perspective (Muchmore, 1999: 24). Bochner (in Ellis & Bochner, 2000: 747) extends this aim by putting it in the context of promoting dialogue and encouraging compassion. He writes: "Actually, I would be pleased if we understood our whole endeavour as a search for better conversation in the face of all the barriers and boundaries that make conversation difficult." I understand him to mean that the aim of sharing narratives is to transform people through allowing them to see things from a different perspective and thus to understand others better.

Research from the Inside

As mentioned in the overview, the second most important product of the discipline of noticing is to transform the reader. In this case 'transforming the reader' refers to enabling the reader to "act freshly in the future" (Mason, 2002: 161) and to therefore improve their practice (Mason, 2004: 12). Rather than trying to persuade the reader to change, the research aims to stimulate the reader to change using various methods discussed in the following sections.

Underview

A common thread I have noticed in the three approaches, and that I now realise always appealed to me, is the indirect way in which each approach aims to change the reader. There is no attempt to persuade the reader through logical arguments. Instead the reader is given suggestions, stimulated, given alternative options and points of view, and new experiences.

The aim of this section was to show that the approaches have this second focus. In the following sections I will consider the means by which each approach intends to achieve this aim.

How are the (secondary) aims achieved?

In the most general sense the reader is transformed through reading the text produced by the researcher. A more meaningful question is thus perhaps: "How does each approach make use of the text produced to transform the reader?" I have already discussed the effect that *writing* the text has on the *researcher*. Now I will focus on the effect that *reading* the text has on the *reader* and more importantly what the different texts entail.

Writing texts that resonate

Overview

An object resonates when it is comes into contact with something vibrating at the object's natural frequency. The object is then stimulated to vibrate at its natural frequency and this vibration feeds on itself, growing in strength. Similarly a reader, through contact with particular ideas or descriptions can be stimulated to think of past (natural) experiences or thoughts which, through the stimulation, grow in strength and meaning.

Each of the three approaches uses this 'resonating' ability of text to reach the reader.

Phenomenology

Phenomenology places a great deal of emphasis on the ability of text to resonate with the reader. Van Manen (2003: n.p.) writes that "the intent of writing is to produce textual portrayals that *resonate* the kinds of meanings that we seem to recognize in prereflective experience" (emphasis added). He suggests that the following questions need to be asked "Does this piece of text bring the experience into view? Does this phrase resonate with our prereflective sensibilities? Are these portrayals of lived meaning recognizable?"

Essentially this method is based on the assertion that a description or anecdote can go beyond the words used to construct it and can bring an experience vividly into the present – what Van Manen (2003: n.p.) refers to as bringing something "into nearness through the vocative power of language."

Narrative Inquiry

Ellis and Bochner address the role played by 'resonance' in narrative inquiry. Their emphasis is slightly different as they place the responsibility on the reader writing that "It is important to think *with* a story,

not just about a story. Thinking with a story means allowing yourself to *resonate* with the story, reflect on it, become part of it" (Ellis & Bochner, 2000: 753, emphasis added).

As mentioned previously, Connelly and Clandinin (1990: 8) suggest that "it is the particular and not the general that triggers emotion and moves people" and therefore allows the narrative to resonate with the reader.

This emphasis on 'the particular' does not occur in phenomenology where the emphasis is on 'iconic' texts (Van Manen, 2003), but does occur in the discipline of noticing in the form of brief-but-vivid descriptions.

Research from the inside

Hewitt (1994: 14), discussing the approach he adopted in his thesis, writes that "each particular incident is … described in a manner designed to resonate with the reader's experience." Within the discipline of noticing these are referred to as brief-but-vivid descriptions.

Mason suggests an additional means by which the reader's experiences can be brought to mind, namely through task exercises. Task exercises are simply a task given to the reader allowing them have a particular experience which then brings to mind other similar experiences, i.e. experiences that resonate with the experience of the task exercise (Mason, 2004: 6).

Underview

The idea that words or descriptions or explanations could and did resonate with me was in itself an example of an idea that resonated with me. I first read about it in Mason's article "Researching from the inside in mathematics education". Looking back, I see that there are ideas, words, descriptions or explanations that resonate with my own experiences in each of the approaches I have investigated.

Writing texts that allow for validation

Overview

By referring to 'texts that allow for validation' I am referring more to the researcher's view of validation than to the actual content of the text. Both narrative inquiry and research from the inside adopt a post-structuralist view of validation. Rather than using triangulation to validate results, both views suggest that validation depends on the individual.

Phenomenology

Within phenomenology, although use is made of approaches such as triangulation, Shepard, Jensen, Schmoll, Hack and Gwyer (1993: 93) explain that, when the research aims to understand a certain group of individuals (and their experiences), validity can be addressed through allowing "the reader [to] judge whether the findings re-ported for individuals studied be considered applicable to groups of individuals the reader knows". In this sense phenomenology also uses the previous experiences of individuals to judge the validity of the findings.

Narrative Inquiry

Ward (2001) discusses the notion of validity in narrative inquiry as opposed to validity in 'paradigmatic thinking'. Within narrative inquiry, a valid story – or in Bruner's terms "a good story" – convinces the reader of "the likeliness of the events occurring or the characters existing". This is opposed to validity in paradigmatic thinking where the reader is convinced of the validity of the statement through a "well-formed argument" (Bruner, 1988 in Ward, 2001).

Ellis and Bochner (2000: 751) in turn consider what it is that convinces one of "the likeliness of the events occurring or the characters existing" in a narrative. They suggest that "a story's generalizability (or validity) is constantly being tested by readers as they determine if it speaks to them about their experience or about the lives of others they know."

A valid or generalisable narrative is therefore one that convinces the reader that it could have happened by reminding the reader of their own life or of the lives of others. Mason (2002: 175) addresses exactly this point but from a different perspective. He suggests that the reason why it is important for narratives to be shared is to avoid solipsism. Researchers therefore need to look for "resonance and recognition in others reading the narrative". He concludes that "validity depends on the narrative speaking to the experience of others."

In his article on narrative inquiry – or what he refers to as the narradigm – Dhunpath (2000: 546) also addresses the question of validity. He cites Eisner (1981 in Dhunpath, 2000: 546), who makes the same point made by Mason, Ellis and Bochner, and Ward – namely that "validity in the arts is the product of the persuasiveness of the personal vision; its utility is determined by the extent to which it informs". By 'personal vision' I understand the 'pictures' or memories and associations of past experiences that are brought to mind by the work of art (i.e. the narrative in this case) and which then convince us of the validity of the work.

Research from the Inside

As in narrative inquiry, validity in research from the inside is understood as being a personal construction. The researcher does not attempt to persuade the reader through logical arguments. Instead, the researcher presents the reader with brief-but-vivid descriptions of events as well as with possible reasons for the events occurring and/or suggestions for alternative actions in similar situations. The brief-but-vivid descriptions are intended to bring to mind past experiences of the reader (her 'personal vision'), which she can then use to test the conjectures made by the researcher.

In fact Mason (1998: 358) states that this notion of validity is not restricted to research from the inside but is used by readers of any educational reports who have to "interpret what they construe from what they read within their own context. They decide whether it seems relevant, informative, or productive to pursue".

Underview

The issue of validity originally didn't seem that important to me. It is interesting to see how my view of validity has changed during the research process. I now consider it to be very important and at one stage wanted to go into much more detail about it.

Writing texts that allow for co-construction

Overview

When referring to 'texts that allow for co-construction' I am not so much referring to the manner in which the text is written, but rather to the view of knowledge and meaning held by the researcher. Rather than viewing knowledge and meaning 'transferable', the three approaches each view knowledge and meaning as being personally constructed by the reader.

Phenomenology

Within phenomenology, Van Manen (2003: n.p.) points out that "phenomenological meaning and understanding have to be produced constantly anew by the writers and the *readers* of phenomenological texts." (emphasis added)

Narrative Inquiry

Muchmore (1999: 4, 5) describes the stories she shares of her own experiences as "fictional narrative productions". She argues that every narrative is fictional in the sense that no telling of an experience can ever capture the experiences in its entirety. More importantly, in the context of this section, she concludes that instead she *and the reader* are "*jointly* constructing a unique fictional version of [her] lived experience" (emphasis added). The reader is thus repositioned to play an active role in the construction of the narrative.

Ellis and Bochner (2000: 744) also mention the repositioning of the reader in narrative inquiry. They write that "the accessibility and readability of the text (i.e. the narrative) repositions the reader as a co-participant in dialogue and this rejects the orthodox view of the reader as a passive receiver of knowledge."

Research from the Inside

Mason (1998: 371) writes that within research from the inside "Insight is a personal construct that is produced through participation in an action, supported and influenced by a collegial conjecturing environment. It is *co-produced*, not instigated. It evolves" (emphasis added).

Underview

The idea that the meaning of a text depends on the reader and not on the writer (or that "[t]he text …does not determine interpretation; rather the interpretation determines the text" (Muchmore, 1999: 5)) bothered me a lot initially. Not because I wanted to control what other people thought about what I wrote, but because I heard people interpreting texts in a way that seemed illogical to me. It felt wrong that someone should make something meaningful by means of a 'non-meaningful' method. But I have come to realise that a more meaningful focus is what on someone's interpretation reveals about that person and, more importantly, what my interpretations or the things that I notice reveal about me.

Writing texts that provoke

Overview

Researchers who accept that it is not possible to force others to change, but only to work on changing oneself (and that such transformations are possible through adopting a particular approach to research) can still hope to stimulate or provoke others to work on transforming themselves by adopting a similar approach.

Pinar *et al*. (1995: 56, 57), discussing research on curricula in general, write that "the point of contemporary curriculum research is to stimulate self-reflection, self-understanding, and social change. Simply put, practical or theoretical research is intended as much to provoke questions as it is to answer questions."

In this section I consider how each approach does (or does not) hope to provoke the reader to continue with their own research. By research I do not necessarily mean 'academic' research but any kind of enquiry.

Phenomenology

In the literature I have read about phenomenology I have not yet come across any reference to research aiming to provoke others to do their own phenomenological research. Mention *is* made of writing a text which provokes (in the reader) a sense of wonder at the experience. But nowhere is this taken a step further, encouraging the reader to research her own situation actively using a phenomenological approach.

Narrative Inquiry

Although readers of narrative inquiries are not explicitly encouraged to start researching their own situations or experiences by means of narrative inquiry, according to Ellis and Bochner (2000: 747), the researcher hopes to provoke them to "broaden their horizons, reflect critically on their own experiences, enter emphatically into worlds of experience different from their own, and actively engage in dialogue regarding the social and moral implications of the different perspectives and standpoints encountered."

McVee (2004: 881) similarly argues that narratives have the ability to transform individuals. Thomas (1993) takes a slightly different stand, claiming that narratives allow the reader to generate questions

for themselves, while Muchmore (1999: 24) points out that "we are more likely to learn from vicarious experiences (such as reading someone else's narrative) because they tend to be less threatening than direct encounters."

Magidson (2005), Maxwell (1998) and Powell (2004) have all written autonomous narratives in which they state what they hope their narratives will achieve with regards to the reader. Magidson (2005: 137) hopes to provoke the reader to reflect on similar situations; Maxwell (1998: 5) hopes to help the reader to explore their own beliefs and behaviours, while Powel (2004) aims to change others through "'agitat[ing]' individuals sufficiently so that they reflect upon their own instances and develop their own questions and approaches".

Research from the Inside

As mentioned previously, Mason (1998: 357) considers the "stimuli to others to researchers and teachers to test out conjectures for themselves in their own context" to be the second most significant product in research from the inside. His main motivation for this view point is that "the track record of solutions prepared by some for others is not a happy one" (*ibid*.: 358).

Dolk *et al*. (2002:176) express a similar sentiment (namely that research should stimulate others) in their article about research on multimedia case studies – research that they conducted 'from the inside'. They write that if the account of their research process inspires other teacher educators and researchers to take their framework as a point of departure for "more formal research or more informal experiments on the use of multimedia case studies", then their research would have achieved what they intended it to achieve.

Underview

Having read research based on narrative inquiry (especially personal narratives) and research done 'from the inside', I have experienced the stimulus to research my own experiences in my own context. Indeed this thesis is as a result of such stimulations.

1.4.2 My Research

I will use phenomenology, narrative inquiry and research from the inside to study my own experiences of learning and teaching mathematics as well as of using an emergent design in social science research…

so that I, the researcher …

can make meaning of my past experiences of learning and teaching mathematics and can use these to develop alternative possibilities for my teaching practice…

by…

reflecting on my own experiences…

and…

improving my attentiveness and developing my sensitivity…

and…

seeking threads which I will share with others (my students[1] and my colleagues[2]) in an attempt to develop alternative possibilities for my teaching practice[3]…

by…

writing and rewriting accounts-of and accounts-for significant experiences in my learning of mathematics,[4] writing and reflecting on the journal entries I made while taking part in a mathematics course[5] and writing and rewriting a description of a lesson I taught[6]…

remembering that…

no text is ever perfect and that there are no final answers…

and…

so that you, the reader…

can be made more aware of the experiences of others – my own,[7] my students',[8]

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[1] Section 3.2
[2] Section 3.3
[3] Section 3.4
[4] Section 2.1
[5] Section 2.2
[6] Section 2.3
[7] Section 2
[8] Section 3.2

my colleagues'[1] and my author friends'[2]...

by…

reading my accounts-of and accounts-for, journal entries and descriptions…

which…

will hopefully resonate with your own experiences…

so that you can…

use your own experiences to validate my claims…

and so that you can…

interpret my texts for yourself and be co-participants in the dialogue…

and possibly…

use phenomenology, narrative inquiry or research from the inside to study your own and others' lived experiences.

1.4.3 Writing about Research

Although the main focus of my research is my own experiences of learning and teaching mathematics and the implications they have for my teaching practice, I have also focused on my research process – as mentioned in my initial overview and as described in sections 1.2 and 1.3 thus far. The different reasons for this are discussed in the following paragraphs.

Mason (1998), Jessop and Penny (1999) and Muchmore (1999) all argue that traditional research reports do not give a true reflection of the research process. Muchmore (1999: 20) writes that "readers of traditional research reports are often provided with such scant information about the way that the study was actually conducted – receiving instead 'an idealised version of *the* scientific method' – that they usually have little or no basis for judging the author's knowledge claims, other than to accept his or her word that they are indeed valid."

Jessop and Penny (1999: 214) point out that conventional academic research tends to "'neaten' up the rough edges of process in favour of delivering a 'rhetoric of conclusions'." They argue that in narrative

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[1] Section 3.3
[2] Section 3.1

research in particular (but I would extend this to all social science research) this 'neatening' up of edges "lowers the debate about data analysis and representation, to a set of techniques, viewed in exclusion from the dilemmas and contradictions which inevitably accompany making the choices which constitute 'doing' research" (*ibid*.). The implications for the readers of such reports are that they are "faced with the challenge of accepting a privileged discourse, having not been privy to the original exchange, while having to trust the rigour of the analysis and representation, without knowing enough about how the research was 'done'" (*ibid*.).

Mason (1998: 370) addresses the issue of research reports by first pointing out that the "insight and effect of having participated in carrying out research is transformed into something quite different when it is presented to others". He goes on to compare research reports to the description of a car journey from a helicopter: "major turns and traffic conditions may be reported, but not the views seen and moods experienced by those inside the car. Researchers very rarely report on, much less communicate, transformations in themselves" (*ibid*.).

Those who argue that traditional research reports do not give a true reflection of the research process also argue that alternative representations need to be developed. Some suggest that researchers need to acknowledge that they made certain choices and should be ready to defend these choices (Carson & Fairbairn, 2002: 26). Others call for the researcher's judgements to be made explicit through documenting such decisions in a research journal (Byrne, 1998). There are those who recommend that the research process (including the decisions made) should be reflected on or 'brought to the surface' (Mason, 1998: 371 and 2002: 156), while others would have the researcher share these decisions with the reader (see, for example, Mason, 2002: 156 and Jessop & Penny, 1999: 214).

The two major reasons given for reflecting on and sharing 'what really happens in research' is to allow the reader to understand what happened (Jessop & Penny, 1999: 214) so that she is in a position to judge the author's knowledge claims (Muchmore, 1999: 20) and to allow the researcher, once the research has been completed, to be in a better position to support others in their research endeavours (Mason, 1998: 371).

Richardson (2000) suggests an alternative representation – namely the "writing story" – a story that tells about the writing itself. Writing stories are narratives about the writing process itself, about "contexts in which the writing is produced. They situate the author's writing in other parts of the author's life, such

as disciplinary constraints, academic debates, departmental politics, social movements, community structures, research interests, familial ties, and personal history. They offer critical reflexivity about the writing-self in different contexts as a valuable creative analytic practice" (Richardson, 2000: 931). Mason (2002: 156) takes this a step further, suggesting that there are those for whom the process of making their assumptions (i.e. the reasons for their choices) explicit and keeping track of the changes in their being becomes the topic of research itself.

A survey of the literature reveals that there are indeed researchers who have developed alternative representations in an attempt to report on the research process more 'truthfully'. Campbell (1997: n.p.) shares the "narrative of the writing of [her] own thesis" and the "personal transformation" she experienced. Magidson (2005: 140) sets out to focus explicitly on her concept of self as a teacher, a researcher and a designer, and to trace how these identities developed and merged during her research process. Jessop and Penny (1999: 213), in their article "A story behind a story: developing strategies for making sense of teacher narratives", take the reader "through the often hidden steps involved in doing research, and unveil some of the problematics of narrative and voice" by sharing the different representations of the narratives they collected and considered using.

It is within this context of searching for a more 'truthful' representation of the research process that I have been through that I have chosen to use the analogy of a tapestry that tells of its own construction - a tapestry that allows one to see the 'neat' story on the front and the 'mess' of construction on the back. I have shared my story in the hope that you as the reader will find yourself in a position to judge for yourself whether the suggestions that I have made are 'valid' and whether they will be useful in your own research journey. I have attempted to share the 'dilemmas and contradictions' that I have encountered as well as the different representations I considered using at various stages in my research.

Underview

I really enjoyed writing this section. It was the piece that "fell out" after all the hard work. After writing such a piece one realises how much work goes into writing something solid. I wrote pages and pages of messing around, categorising, trying different combinations, collecting like terms before it all came together. It reminds me so much of solving mathematical problems, in particular of the quote, "And then you just spit it out", by a mathematician describing the final step in solving a problem (Handa, 2003: 25).

SECTION 2
OF THE CONSTRUCTION OF MY TAPESTRY:

THE WEAVING

A section in which I continue weaving my tapestry, panel by panel, sharing images from my learning and teaching as well as the underside of the tapestry. I approach each panel differently using accounts-of and accounts-for, journal entries made during a mathematics course I attended and a description of a lesson I taught.

"These stories we tell are part of the fabric which constitutes our being, the floor-coverings, the wall-hangings and the ceilings of our personal worlds of significance, occasioned by experience."
(Mason, 2002: 246)
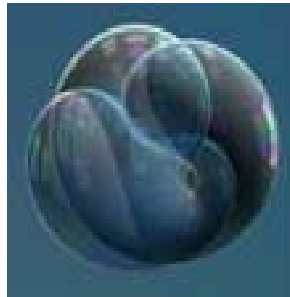
SECTION 2.1

OF THE WEAVING OF MY TAPESTRY:

Learning

A section in which I share and reflect on significant events in my own learning of mathematics at school and at university.

"The sense that I am a different person, that I have a different way of looking at things, that I have a different amount of control over things, is very exciting and is a result of educating my awareness. I believe this sense of personal growth to be the greatest and most effective form of motivation in the classroom, and yet I fear that many students experience it too rarely within their mathematics lessons."

(Hewitt, 2001: 39)
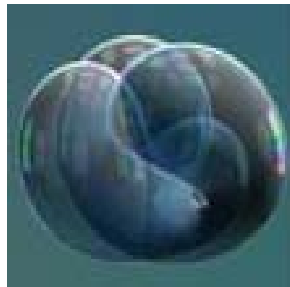
Overview

In the following three panels I share significant moments from my own experiences. During the reflections on these experiences I make use of Hewitt's (1994) method of identify threads by writing a few words in curly brackets - **{…}**. These threads are collected and arranged in section 2.4 showing how I intend to discuss them in section 3.

In this first panel I make use of John Mason's (2002) method of writing an account-of and an account-for in an attempt to separate what happened from what the observer thinks was the reason for what happened. The main motivation behind this is to allow you, the reader, to (possibly) recognise the experience being described and therefore to be in a position to judge whether you find my reflections and propositions regarding the event to be valid.

Images

2.1.1   Equalities

I was standing in the middle of the classroom while everybody else was sitting at their desks working. I was looking into a glass cabinet which had several shelves inside. On one of the shelves lay a card with $1 + \underline{\quad} = 4$ written on it. The cabinet was locked preventing me from opening it to touch the paper. My teacher, Mrs Kopke had tried to explain the difference between $1 + 3 = \underline{\quad}$ and $1 + \underline{\quad} = 4$ but I had been unable to make meaning of her explanation and was still confused. I felt as though there was something wrong with me because I couldn't understand what she was saying.

Reflections on Equalities

In one sense the memory I have just shared does not reflect the 'truth'. I had lived with this memory for many years before it struck me that it was very unlikely that we had a glass cabinet in the middle of the classroom with the problems inside, or that I should have been standing alone working on a problem.

On the other hand, the memory does reflect a certain 'truth'. I felt separated from the problem even though I could see the problem – as though looking at it through glass. The problem and its solution were locked away, unreachable. I felt totally helpless. **{helplessness}**

Reflecting on this memory now, I realise that what I was struggling with was a concept many people struggle with namely the "dual interpretation of the equality sign: '=' can be regarded as a symbol of

identity, or as a 'command' for executing the operations appearing at its right side" (Sfard, 1991: 6). I was only able to interpret the equality sign in the latter manner and could therefore not make meaning of the new type of question. **{equal signs}**

Looking back after having taught mathematics, I can now imagine how Mrs Kopke might have felt in this situation. By now, I too have been in situations where the concept that I have been trying to explain has become so obvious to me that I struggle to explain it. **{too obvious to explain}** Having reflected on my experience, I now realise that I need to be careful to let my students know that *my* inability to explain a concept is not a reflection on *them*. I don't want students who feel helpless and cut off from a problem to feel it is their fault that I can't remember what it was like not to understand the concept.

One approach that I have tried in such situations is to ask another student in the class to explain the concept. I choose someone who has recently grasped the concept and therefore can still remember what it was that they didn't understand and what shifted when they finally did make sense of it. **{asking someone who has recently made meaning}**

2.1.2 Limits

In my first year at university we were introduced to the epsilon-delta definition of limits. I learnt the definition off by heart, but wasn't able to use it to prove that functions were continuous. It was only in my third year that I finally understood the definition. Suddenly 'the lights went on' and I realised that Jacques, who had always been able to use the limit definition to prove statements (proofs that I had always somehow learnt off by heart), was not some kind of super-genius – he just understood the definition. I no longer had to write things down from memory, I wrote things down because they made sense.

Reflections on Limits

Paul Halmos, a well-known mathematician, had a similar experience. He writes: "… one afternoon something happened. I remember standing at the blackboard in Room 213 of the mathematics building talking with Warren Ambrose and suddenly I understood epsilon. I understood what limits were, and all of that stuff that people were drilling in me became clear. I sat down that afternoon with the calculus textbook by Granville, Smith, and Longley. All of that stuff that previously had not made any sense became obvious…" (Albers & Alexanderson cited in Sfard, 1994: 44).

What implications do these revelations have for teaching and learning? I believe that one of the most important implications is that often there has to be a time of confusion before clarity is gained **{availability of things to fall into place}**. But what does this mean in practice? How does one move away from wanting everything to be crystal clear in the students' heads and move toward accepting that often clarity only comes with time and effort? In an article on the nature of mathematical conceptions, Sfard (1991: 33) addresses this important part of learning. She writes that "insight cannot always be expected as an immediate reward for a person's direct attempts to fathom a new idea. The reification, which brings relational understanding, is difficult to achieve, it requires much effort, and it may come when least expected, sometimes in a sudden flash." **{reification}**

2.1.3   Algebra

By the time I was halfway through my third year at university I had realised that being prepared for a class or a course was a big advantage. Before the midyear break I asked one of my lecturers for his notes for the algebra course he would be presenting the following semester. I spent many hours during break reading the notes he had given me, trying to make sense of the work. I think I only managed to make sense of the first half a page and then I was lost. I carried on reading, but couldn't understand the work because of the previous sections which I hadn't understood. And yet I kept on reading the first few pages over and over again. As time went by more things started to make sense – initial definitions made sense after seeing examples of the structures being defined; theorems made sense after seeing how they were used. But it was very frustrating. I had wanted to work through the whole set of notes by the time the term started and instead all I managed was to read through everything once without understanding much past page 2.

But when the term started, being in algebra class was incredible. Because I had a multitude of questions – questions I had wanted to ask for weeks – when we finally discussed the work in class and my questions were answered, the work suddenly made sense.

Reflections on Algebra

I can still remember what those first two weeks (while we were doing the work I had wrestled the most with during the holidays) felt like. I felt as though I was building a firm structure. So often I feel that my understanding of sections of work is 'shaky'; that it has been built too quickly; that it isn't stable. Often when I understand something quickly, I soon forget the work. In contrast, the work that I came to

understand in this algebra course had taken time and effort and has remained with me longer than many other sections of work. **{learning deeply}**

The idea that a new concept only makes sense when it answers a question, i.e. when there is a *need* for the new concept, has been recognised by many people. Davis (1992: 229) writes that "'understanding' occurs when a new idea can be fitted into a larger framework of previously-assembled ideas. This is particularly important when a new idea is recognised as the 'answer' to a question that had already been of interest."

Halmos (1994 in Mason, 2004: 1) suggests that "the way to begin all teaching is with a question". Henderson (2001: n.p.) makes a similar claim: "In order for me to be satisfied by a proof the proof must answer my why-question". Although he refers only to making meaning of a proof, I feel that his point can be generalised to any new work that needs to be understood.

Once again, how does one practically find the balance between exposing students to work in such a way that they are able to formulate questions on their own which can then be answered during a class discussion **{need question before answer makes sense}** and between exposing them to work which is too difficult and results in their wanting to give up and feeling as though they will never understand any of the work? **{helplessness}**

2.1.4 Real Analysis Homework

Another course I took in my third year was real analysis, a course where we were given homework each day. But this homework was different to what I had been used to. We were no longer expected to do calculations but to prove statements, find examples of special cases and exceptions. On the first day that we received homework I went home and tried every problem and couldn't do a single one of them. I wanted to cry. I felt incredibly helpless. I knew I couldn't go and ask my lecturer to explain every single sum to me. Fortunately one of my classmates, Jacques, agreed to help me. We went to the library and sat in one of the study rooms and started to work through the problems. Jacques could do everything. I was amazed and kept on asking him how he knew what to do. I could follow what he did, but at each step there seemed to be hundreds of options yet Jacques just seemed to know which was the right one. I became convinced that he had been born with this intuition and that I hadn't and would therefore never be able to solve such problems. He, on the other hand, tried to persuade me that the

reason why he could do the work was because he had been exposed to similar kind of questions before, especially while doing Mathematics Olympiads. I wasn't convinced.

I later started working with Gawie, who would also work through all the problems with me, explaining as he went along. Gawie, too, had been involved in Mathematics Olympiads while still at school. I was also convinced that he had been born able to do maths, while I would never be able to. And yet as the term progressed I slowly realised that I was now able to do some of the problems on my own, that I could follow and sometimes anticipate what Gawie would try and do. By the end of the semester I realised that what Jacques had said at the beginning was true – it *is* possible to learn to think in a particular way, to *develop* ones intuition. People *aren't* just born like that.

Reflections on Real Analysis Homework

This shift to a new type of problem that I faced in my real analysis course is what David Tall refers to as "the step from 'elementary mathematics' of calculation and manipulation to 'advanced mathematics' of defining and proving" (Tall, Michael, Davis, Gray & Simpson, 2000: 15). Although we had been doing 'advanced mathematics' since my first year, until this course we had never been expected to prove statements or construct examples on our own. We *had* been expected to learn proofs that had been discussed in class, but never to construct them ourselves. Tall (in Selden & Selden, 1996: n.p.) addresses this exact issue, namely that unlike school, "by mid-level university … teachers no longer ask students for answers about, or even have them think about familiar mathematical objects. Students' previous, often successful, approaches to learning mathematics no longer work when they are confronted with formal definitions and axioms which specify properties of (unknown) objects. In this situation, their task becomes the construction of mathematical objects (examples) – something quite alien to their previous experiences." **{students constructing own proofs and examples}**

My shift from believing I would never be able to 'know what to do' when trying to solve a problem or prove a statement to realising that I could learn how to 'know what to do' has had a great influence on me and, looking back, is probably one of the main reasons why I ended up in teaching. This 'knowing what to do' has also been described as intuition and much has been written about the use, or lack thereof, of intuition in education.

My initial belief about 'knowing what to do' or intuition is shared by other students. In an interview in John-Steiner (1997: 183-184), John Howarth was asked "whether it was possible to help students to rely upon intuition as part of their preparation for becoming physicists". He replied:

> Intuitive solution of problems is important. Essentially it is finding the answer to a problem before you have solved it. Students are tempted to believe that physical intuition is something that you either have or don't have. We certainly all have different talents, but the process can certainly be encouraged – that's one of the things that teaching is about.

My experience of realising that it is possible to learn intuition is echoed by Burton (1999b), who interviewed 70 practising mathematics on their practices. She notes that "Far from believing that you simply do, or do not, 'have' the intuitive process, it seems to me that some of the participants were reporting that they had learned, over time, to recognise and trust their intuitions and that, with such acknowledgement, the quality of this process was improved. Otherwise, how can their reliance on experience and knowledge as this source be explained?" (*ibid*.: 31). I too had to *learn* to recognise and *trust* my intuition and I too found that the quality of this process (of using intuition to solve problems) improved. **{developing intuition} {learning to trust ones own abilities}**

But what implications does this have for teaching? How do I, as a teacher, help my students to realise that they can solve problems which seem impossible? One solution is to teach them by example, i.e. by demonstrating how I would approach the problem (as Jacques and Gawie did). But this brings its own problems, since students who believe they cannot do the work are often willing simply to listen to others explaining the work and never move to doing the work on their own[1] **{choosing to let others do the thinking}**. How do I move from showing them how I would do something to letting them do it on their own?

2.1.5   Topology

Another of my third year courses was Topology. There were only four people in our Topology class – David, Scott, Jacques and I. It was one of the courses I enjoyed the most. Dr Peters, our lecturer, presented the class in such a way that we, as students, had a great deal of say with regards to what we did in class. It was the first time that he was presenting the class, so he would hand out the notes as he compiled them – four pages one day, three the following week and so on. Unlike all the others we had

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[1] See subsection 2.2.4

ever received, Dr Peters' notes didn't include the proofs to the theorems we discussed. We either worked out the proofs together in class or we were given them to do for homework.

Reflections on **Topology**

In this course I experienced what a difference it makes when, as a student, one is able to have an input regarding the work to be covered. **{students directing class}**

I also experienced a lecturer doing mathematics in front of us. **{demonstrating mathematics}** I remember one of my didactics lecturers telling us that, when she was still teaching mathematics at school, she would sometimes solve Euclidean geometry questions 'unprepared' so that she could demonstrate to the students how she did mathematics. Dr Peters, in fact, did not only stand in front of the class and demonstrate to us how he would have gone about proving a particular theorem, but made us part of the process. The five of us proved the theorems together and thus learnt not just the mathematics, but the processes behind the mathematics as well. **{working together in class}**

2.1.6   Cryptology

As part of my undergraduate degree I took a course in Cryptology. Towards the end of the course we were given a project which involved designing our own cryptosystem – a system used to encode information. I spent many days trying to come up with my own system, playing around with different options. One afternoon, as I was walking across the pedestrian bridge on our campus, I suddenly came up with an idea. Although it still took a reasonable amount of effort to sort out the details of my idea and to write it up, the 'hard work' happened in an instant.

Reflections on **Cryptology**

This phenomenon of suddenly coming up with a solution to a problem (be it an example one is looking for or a theorem that one is trying to prove) after struggling with it without any progress for a long time, has been noted by many mathematicians. It must be pointed out that what I am referring to here is not the experience of suddenly making meaning of a new concept that one has been struggling to understand – although these two experiences have both been referred to as an 'aha-moment'. **{aha moments}**

Liljedahl (2001: 83) writes about this experience noting that "As mathematicians, we *know* that, given the right circumstances, the solution will come to us; there will be an illumination. The 'AHA moment'

is both an accepted and expected part of the problem solving process. We go through private rituals to bring it on, and then we wait patiently for it to come" (emphasis added).

I have found that this *knowing* that the solution will come plays a very big role in solving problems. I often share that the reason why I did well in mathematics at school was because I knew that the problems we were given had solutions and were therefore not impossible and consequently there was no reason why I shouldn't be able to solve them – I trusted that the answers would 'come to me' (Liljedahl, 2001: 83). But so often I see students with no faith in this process. They do not believe that the answers will come to them – and then, unfortunately, the answers don't come. What can I do to allow my students to experience this process so that they too can have faith in the 'aha-moment'? **{learning to trust one's own abilities}**

2.1.7 Computational Physics

The most challenging subject I had at university was a six month course called Computational Physics. In the beginning I hated it. We were taught some algorithms and were then told to investigate them, keeping record of our research in a "black book" which had to be handed in every second Friday.

I remember lying curled up on the floor under a table in the physics labs during a lunch hour trying not to cry (although, in retrospect I realise that this memory is similar to the one of the glass cabinet – it couldn't have happened as I remember, but it reflects how I felt at the time – like hiding away) because I didn't know what to do. For the first time in my life I didn't know what was expected of us. In every other course I was able to do well by producing that which was expected. Now I didn't know where to start.

In the end I gave up trying to figure out what was expected and started off with the very basic suggestions that our lecturer had made - still having no idea where I was heading. Fortunately I was able to keep a good record of all the different analyses I ran which I could show as 'proof' of the work I had done. Even so I dreaded getting my book back each time as I never knew whether what I had done was 'acceptable'.

As the term continued I slowly gained confidence and started deciding for myself what parameters to change and why and by the end of the course I felt that I would now be able to tentatively do research on my own.

Reflections on Computational Physics

Why did I find this course so challenging in the beginning and so rewarding in the end? I think it was because, in the beginning, I was unable to work entirely on my own, and because, in the end, I had started to do exactly that – to work independently.

I realise now that before I took this course I had never been expected to hand in 'original work' for a mathematics or science course. Although I had done projects and had made my own meaning of others' work, the projects were still only a summary of work that had already been done by others. But in this computational physics *I* had to decide what to do. *I* had to choose which parameters to investigate and, more than that, I had to *motivate* these choices. **{doing original work}**

Why was this so difficult for me? I think the reason was two fold. Firstly, I had always been able to base my decisions regarding what to do for a project on what I felt the teacher/lecturer wanted. I knew this either because they had specified it or because I 'just knew'. In this case I didn't know what was expected of us. All we were told was to play around and see what happens. Because I didn't know what was expected and because I had no way of finding out, I felt powerless. Secondly, I had never been expected to reason on my own. For the first time I had to hand in work that exposed my own thinking and reasoning. I use the work 'expose' on purpose here because this is what it felt like to me. Until I took this course I had been able to hide behind the reasoning of other people or behind my ability to solve problems using the tools we had been given. Here there was no problem to solve. There was no other work to base my work on (or rather there wasn't much of it). I had to write down on paper (with no chance of explaining that I had been misunderstood and meant something else as is the case when one has a verbal conversation with someone) how I had reasoned. I felt very exposed **{feeling exposed}**.

In the end I found great value in the course but this, I believe, was because of the struggle that I went through and the perseverance I displayed. Do we always have to struggle to feel a sense of achievement? How does one ensure that the balance between struggling and success is kept? (Thom & Pirie, 2002: 2) How does one teach one's students to persevere? Is it even possible to teach them to persevere? **{perseverance}** How does one balance internal and external sources of motivation? **{sources of motivation}**

2.1.8   Not aha-moment

Although I have often had aha-moments in mathematics there have been times where I came to know things in a different kind of way. In particular I remember the concept of a statement being vacuously true. I first came across it in my third year when one of our lecturers said that the statement 'I own all the original Van Goghs hanging in this room' was (vacuously) true. I remember feeling as though I could almost touch the 'sense' that this meaning had but that it was just out of reach, like seeing something through a haze. Like seeing a star constellation that looks bright when you don't look at it directly and looks blurry when you do. As though my brain knew on one level that the statement made sense, but just couldn't grasp it yet. And then as time went by I realised that my brain had somehow become used to the idea and that it now made sense to me.

Reflections on **Not aha-moment**

I find this a very interesting way of 'coming to know', possibly because I have found nothing written about it and so much written about the 'aha-moment'. It almost happens without one noticing it. Slowly you become familiar with the new concept until it becomes difficult to remember how uncomfortable it felt at first. **{gradually coming to know}**

2.1.9   Failing Complex

In the June exams of my fourth year I wrote an exam that I studied very hard for. I failed it. After writing the exam I was ready to stop studying. I couldn't see the point in carrying on if I had put in so much effort and reaped no reward.

Reflections on **Failing complex**

Although I initially felt like giving up because I felt that what had happened was unfair **{much effort, little reward}**, I soon realised that I was in fact to blame. I had not been honest with myself – I had convinced myself that I knew how to prove the main theorem of the course when in fact, as I found out in the exam, I didn't. I still find it difficult to be honest with myself about whether I really understand a concept or whether I just feel that I have spent enough time working on it.

How does one learn to be honest with oneself about the quality of work that one is doing? **{being honest with self}** I have found that situations (such as writing an exam or presenting a paper) where I am forced to explain myself to other people are useful in establishing whether I really understand something. **{being forced to explain to others}**

2.1.10 Shift in understanding

Today I experienced a shift in attention in the way I think about graphs. I have always seen graphs as an effective way of communicating information, most often as a graphical representation of a function, showing how the elements in the domain of the function are connected to the elements in the range of the function.

But today I suddenly became aware of the idea of a graph being a collection of co-ordinates. In retrospect, I realise that I have come across this definition before, that I have probably even used it as in the following definition:

$$F = \{(x,y): y = 4x + 3\}.$$

Even though I had known about this definition, I now realise that it had become something I knew on my own. If I had been asked to give a definition of a graph, this definition would not have come to mind. I don't know what happened today to make me suddenly see it. It just happened.

Reflections on **Shift in understanding**

Anna Sfard (1991) distinguishes between operational and structural conceptions of mathematical objects. An operational conception "speaks about *processes, algorithms and actions*", while a structural conception treats mathematical notions "as if they referred to some abstract *objects*". As an example Sfard (1991: 11) refers to the fact that "a ratio of two integers was initially regarded as a short description of a measuring process (operational conception) rather than as a number (procedural conception)".

The shift in understanding that I experienced today is an example of a shift from a procedural conception of a function (a well-defined method of getting from one system to another) to a structural conception (a set of ordered pairs) (Sfard, 1991: 5). What is perhaps more interesting than the fact that I was able to make this shift is the fact that I have been aware of the structural conception of a function for a long time but never realised it.

As mentioned previously Mason (2003a: 5)[1] identifies 4 levels of awareness: not being aware at all, noticing, marking and recording. The fact that I could remember seeing the definition in question

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[1] Page 49

before this day and had used it when prompted by others indicates that I had *noticed* it, but the fact that I never used it of my own accord suggests that I hadn't *marked* it or made a *recording* of it (as I have now).

Having experienced this shift in attention makes me, once again, more sensitive to the shifts in attention that my students need to make to understand new concepts. It also highlights how easy it is to be able to use a concept (when told to) without understanding it properly (whatever this might be defined as). **{importance of shifts in attention}**

Threads

Here is a list of threads that I have identified in this section as well as the section in which I will look at each thread in the light of other peoples' reflections. Not all threads identified will be used in later sections.

> **{helplessness} – 3.2**
>
> **{equal signs}**
>
> **{too obvious to explain}**
>
> **{asking someone who has recently made meaning} – 3.3**
>
> **{availability of things to fall into place} – 3.1**
>
> **{reification} – 3.1**
>
> **{learning deeply} – 3.2**
>
> **{need question before answer makes sense} – 3.1**
>
> **{students constructing own proofs and examples}**
>
> **{developing intuition} – 3.1**
>
> **{learning to trust ones own abilities} – 3.2**
>
> **{choosing to let others do the thinking} – 3.2**
>
> **{students directing class} – 3.3**
>
> **{demonstrating mathematics} – 3.3**
>
> **{working together in class} – 3.3**
>
> **{aha moments} – 3.1**
>
> **{doing original work} – 3.2**
>
> **{feeling exposed} – 3.2**
>
> **{perseverance} – 3.1**

{sources of motivation} – 3.2

{gradually coming to know} – 3.1

{much effort, little reward} – 3.2

{being honest with self} – 3.2

{being forced to explain to others} – 3.2

{importance of shifts in attention} – 3.1

Underview

Connelly and Clandinin (1990: 10) warn that "It is too easy to become committed to the whole, the narrative plot, and to one's own role in the inquiry and to lose sight of the various fine lines that one treads in the writing of a narrative." Looking back I think I have often missed the fine lines I should have been treading – moving either too far to the one side or to the other. Most significantly, has been my attempt to walk between wanting to write about my own experiences merely for the sake of writing about them and not wanting to write about them at all.

SECTION 2.2

OF THE WEAVING OF MY TAPESTRY:

Teaching while Learning

A section in which I share and reflect on significant events in a mathematics course I attended while simultaneously teaching a mathematics course.

"The best way to resonate with other people's experiences is to try to keep awake to your own experience, to put yourself in situations in your own discipline deliberately which parallel, if not duplicate, experiences of you students."
(Mason, 2002: 207)

Overview

As mentioned previously, in this section my panel is woven out of journal entries and reflections on these journal entries as opposed to the brief-but-vivid images that were used in section 2.1. The entries were made while I attended a course presented to fourth-year Mathematics students. I was unable to take this course in my fourth year as the lecturer who presented the course was overseas at that stage. I attended this course, firstly, because I had wanted to take it in my fourth year and, secondly, because I agreed with Mason (2002: 207), who writes that one way of researching one's own teaching is to put oneself in the position of one's students by attending a math course (obviously one more advanced than the course one is teaching). So for one semester I attended class once a week and spent time trying to understand new concepts and, at the same time, trying to keep record of how I experienced learning new concepts.

In the following subsections I use three different writing styles. Some paragraphs are accounts-of what happened in the classes (normal font). Others describe my thoughts during the classes (italics). And others are my attempt to refer to literature to account-for what happened (in text boxes).

2.2.1 Week 1 (20 July 2005)

Today I started a maths course as part of my research for my Master's. The module is about category theory and is the direction that I would have gone into if I had carried on with pure maths and had not changed to mathematics education.

There are six of us taking the course: Sunette, Karen, Wendy, Ann, Karel and myself. Our lecturer is Dr Peters.

At the beginning of the lesson Dr Peters told us a bit about the history of category theory… *It felt as though everyone was listening more keenly during those first few minutes than during the rest of the class, although someone looking from the outside probably wouldn't have noticed a difference in the way we listened at the beginning compared to the way we listened during the rest of the lesson. It is of course possible that I was the only one that experienced this intenseness at the beginning because I find it very important to have an idea of how something fits into the bigger picture.*

Today in the maths class I taught I introduced infinite series and I had the same experience. It felt as though the whole class was listening more intently while I was talking about the "history" or background of infinity – all the things that aren't in the syllabus – than during the rest of the lesson.

While Dr Peters was telling us about the background of category theory, he also tried to give us an idea of what the "fundamental" idea in category theory is. To do this he first talked about set theory and mentioned that it could be seen as a language used to describe concepts/structures in maths. He said that category theory was similar in the sense that it was also a language used to describe certain concepts. He then asked us what we thought the fundamental idea in set theory was.

Suddenly I was on the other end of the "leading question". I could see both sides. The students not knowing the answer but feeling they should actually know and therefore being willing to guess. And the teacher not wanting to give the answer because s/he feels the students really should think about the question and should be able to figure it out themselves but slowly realising that the students aren't going to figure it out. So many times I have asked similar questions and waited in anticipation which slowly gave way to frustration. Surely there are other ways of emphasising that something is very important? **{not asking leading questions}**

When no-one could give Dr Peters the answer he was looking for, he told us that the fundamental concept in set theory is the idea of belonging. As opposed to this, the fundamental concept in category theory is the idea of relationships. He mentioned how this could be connected quite nicely with different philosophies – are we who we are because of where we *belong* or because of our *relationship* with "others".

Dr Peters is using his own notes and so he writes them on the board and then we copy them down. I was sitting next to one of the other girls in the class and I felt the urge to cover what I was writing so that she couldn't see what I was writing – not because I didn't want to help her, but because I wasn't sure about what she would think about the way that I wrote down the work. **{feeling exposed}** But then I remembered noticing that one of the girls in my class covered her work so that I couldn't see it. I realised that she was probably scared that I would judge her by her work, when in fact I was only interested in seeing what she was doing. Being on 'the other side' helped me feel free to expose my work to other people.

2.2.2 Week 2 (27 July 2005)

We started by discussing the homework we had been given for today. Dr Peters first spoke about generators and relations on groups. I have taken a course on groups, but I couldn't remember much of the work we had covered. It felt as though my brain was rusted. I could almost feel the answers to the questions that he was asking, but just couldn't reach them. **{just out of reach}**

As Dr Peters mentioned the concepts, it was as though I was simultaneously remembering the work and understanding it (or rather appreciating it) for the first time. I think appreciating it is a much better description. I had understood the work in my Honours year but it hadn't meant anything to me then. Now it suddenly had meaning as I saw the connections that I hadn't been able to see before. **{availability of things to fall into place}**

After he had explained the notion of generators and relations with regards to groups, he then looked at the case for categories. Because I had spent so much time thinking about the first problem, after he had shown us what a category would look like I was able to give a specific example. *As I gave the example I felt the classes 'energy' leaking out because I knew that I was taking away other people's chance to think about the problem.*

What I was referring to here was the chance to have what Barnes refers to as a 'magical moment' and which has also been referred to as an 'aha-moment'. Barnes (2000: 41) suggests that there is a direct relationship between the frequent occurrence of 'magical moment' in a particular class she was studying and the class rule: "Don't spoil things for the others." ***{spoiling things for others}***

It is such a strange position to be in – teacher and student. I know what it feels like to have a student that answers all the questions. I know what it feels like to be the student that is so excited about what they have just discovered that they want to desperately share it with everyone. I know the feeling of losing energy or motivation when you are working on a problem and suddenly someone else shouts out that they have the answer. If this is a regular occurrence, it just becomes so much easier to wait for someone to explain the problem than to try and struggle through it yourself. ***{choosing to let others do the thinking}***

When I answered this question I felt so 'big' in the class; so involved with the mathematics that there wasn't any space for the other students in the class to "be involved". I often feel like this in the classes that I teach. I take up so much of the space because I enjoy the mathematics so much, that I don't leave any room for my students to enjoy or do the mathematics. {excluding others from mathematics} I need to remember to step back and allow them to experience the mathematics.

After he had told us what a category would look like, I also realised what the fundamental thing that I had been missing when I was trying to do today's homework. In fact Dr Peters had mentioned it in the first lecture – category theory is about relations. We don't worry about what goes on inside the sets.

As soon as I understood this, I wanted him to stop explaining so that I could figure out the rest of the problems on my own and so that he didn't give the answers away. Luckily we only looked at one of the other problems and I managed to figure it out before he told us the answer (or simultaneously – as he told us the answer). I realised how important it is to give time in class to let the students digest the work. I notice this in class so often and I don't know how to deal with it.

In today's Honours class I wanted to say "Can't you just wait a second while I write down this thing that I have just realised before it goes out of my brain again and before you start with something new which means I will miss the beginning and thus probably not know what you are talking about?" It was a feeling of having to be in two places at once – trying to capture a thought before it vanished and trying to listen to new work. {keeping up in class}

How do I deal with this as a student? What is it that I am trying to achieve? If it is good marks then I might be willing to sacrifice understanding for marks. If it is understanding I might be willing to sacrifice marks for understanding. Although I always wanted to understand the work we were doing, when faced with a choice I almost always did what would guarantee better marks than what would ensure a deeper understanding. When I started studying I met people who were more concerned about understanding than about getting the best marks possible. This meant they were willing to fall behind with new work to ensure that they understood the old work {learning deeply}. At the time I couldn't understand their reasoning but now I often wish I had such a hunger for understanding, for learning deeply.

2.2.3 Week 3 (3 August 2005)

I am aware of the feeling of being lost when Dr Peters talks about things that I can't remember doing because I haven't done maths for so long. I feel like I am part of the communal space or "common place location" (Sumara & Davis, 1997: 117) that the class operates in until he mentions something I don't understand and then I am excluded from the space. {communal space}

We had some discussion about the homework that we had to hand in and one of the girls, Sunette, had a problem with a proof. She had an alternate proof. After trying to explain it "in the air", she went to the black board and wrote it down. Then Dr Peters asked us what we thought about what she had written. There ensued a long discussion between Sunette, Karen and myself. Karen and I tried to explain to Sunette why what she had written was incorrect.

I remember sitting and being aware that Dr Peters was choosing not to be involved in our discussion. **{teacher choosing not to be involved}** *Maybe it's because I have taught or because of the work that I have read but I know the feeling of choosing to let students explain things to each other even though you might be able to "do it better". He was sitting behind me and I could feel him involved in the discussion even though he didn't say anything. Finally Sunette understood.* **{being silently involved}**

Dr Peters then asked her what other 'naughty' thing she had written in her version of the proof – something Karen and I had not noticed at all. From this it was clear that he had been following our discussion even though he had not been involved.

We then looked at some more examples of categories as well as at the definition of an isomorphism. We had all seen the idea of an isomorphism before and so there weren't any problems in that regard.

But then we were introduced to a concept that was totally new – that of a dual space.[1] It was something we had to 'get our minds around'. Trying to understand a totally new concept always feels to me like having to contort my body into some strange position. I can see that other people can do it and that I should, theoretically, be able to do it, but I just can't.

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[1] See 2.2.8 Week 12 for further thoughts on getting used to a new idea (that of a Dual)

As Dr Peters explained what a Dual was, I went through phases of understanding what we were doing and then, when someone asked a question, of being confused again by the answer. **{just out of reach}** At one stage I "had a revelation" of what I thought was going on and wanted to tell everybody in the class, to let them see it in my way. But I knew that this would once again steal the energy away from others, making it more difficult for them to come up with their own understanding. **{spoiling things for others}**

2.2.4 Week 5 (17 August 2005)

It's difficult to believe that this was our fifth lesson. Well, it wasn't really a lesson. I had to rush through the rain to get to class on time because my previous class ended late. When I got there Sunette, Karen and Wendy were working on a problem and Karel was explaining something to Ann.

I sat for a while and caught my breath and when Karel had finished explaining to Ann I asked him to help me with the work I had struggled with the evening before.

Karel is someone that I consider 'cleverer' than I am and I noticed two things about working with him. The first was the feeling of reassurance I felt when I discovered that he was also still unsure about a certain concept/definition that we had been given to investigate for homework. I felt 'okay, at least I am not alone – even Karel doesn't understand it totally'. **{comparing self to others}** The other thing that I noticed was the 'switch off' mode I went into when he was thinking about something I didn't understand. This is something that I recognise from working with people whom I consider 'cleverer' than me. When there is a particularly difficult problem that I have struggled with and I bring it to them for help, as soon as they start thinking about it, I switch off and leave them to do the work. **{choosing to let others do the thinking}**

Because I have become aware that I switch off in such situations, I was able to choose not to be passive but to think about the problem as well. In the end I was able to contribute to finding the solution by repeatedly trying to make sense of the definition we were using until we saw how we could use the definition to solve the problem.

Dr Peters didn't arrive, so we just carried on working on our own. It turned out that he was sick at home and the secretary had forgotten to let us know.

2.2.5 Week 6 (24 August 2005)

I saw myself in the way Dr Peters was doing the new work today. It is all work that he has seen before and that he enjoys and so he finds it easy to see what is going on in a problem straight away. I do the same thing when I teach. I enjoy solving the problems my students are solving, especially the ones that are more challenging, and so I get carried away with solving the problem and forget that I am supposed focus on the didactics (making sure they understand) and not on the mathematics (making sure I understand). **{getting too involved in the mathematics as teacher}**

2.2.6 Week 8 (4 September 2005)

I hate doing maths. They say that the line between loving something and hating something is very fine.

I have work that I was supposed to hand in today. Last night I finally got around to looking at it. And then I suddenly realised that I couldn't do half of it. I was SO frustrated that I just wanted to chuck everything. Here is some of my thinking.

> *What on earth am I doing? I don't even have to do this course and here I am sitting, spending my whole evening looking at these problems getting more and more upset – why? Um so that you can write about this later??? Aaaaaaaaaaaaaaaaaaahhhhhhhhhhhh*

I was really upset. I felt as though everything was so wound up inside of me that I just wanted to scream. **{frustration}** I suppose it is really good to go through the experience – to have a renewed understanding of what my students go through. Or what they choose to avoid by giving up. Where do you draw the line between persevering and realising that you are just wasting your time and that you just aren't going to figure the thing out on your own?

2.2.7 Week 10 (26 September 2005)

Last week I volunteered to present the weekly problem to the class this week. On Sunday I decided to look at the problem so that I could ask Dr Peters if I had any problems. I didn't get very far because I didn't even know what the third word in the problem (natural isomorphism) meant. I started off by writing down what an isomorphism between two functors would be (using the definition of an isomorphism) but then I couldn't figure out what to work with because natural transformations aren't really "functions" but families of functions. So I stopped. On Monday morning I emailed Dr Peters and

then an amazing thing happened – between one paragraph and the next I suddenly had a revelation of what a natural isomorphism might mean.

Dear Dr Peters
About the work that I have to do for Wednesday: I just wanted to check what exactly "naturally isomorphic" means. I don't know if I somehow missed some discussion in class or whether it is obvious what it means. I would think that two functors, F and G (both from A -> B), are naturally isomorphic if there is a natural transformation b: F -> G and a natural transformation a: G -> F so that ab = idF and ba = idG. But this is where I start getting confused because I struggle to see natural transformations as functions (in the sense of you giving them one thing and they giving you something back) because, as far as I understand, natural transformations are a family of B-arrows that satisfy a certain condition. I can see that one could define the "inverse" of b: F -> G (what i have called a: G -> F) as a family of B-arrows that satisfy the "inverse" condition. My problem is still what do I mean by ab = idF and ba = idG?

Perhaps I am totally on the wrong track. Any comments would be most welcome.

mmm. I just had a thought. Perhaps the 'inverseness' of a and b happens on the level of the B arrows (or in the category B) so that when I say that ab = idF I mean that for every B arrow that satisfies my condition there is an inverse B arrow that satisfies the inverse condition.

It's really a pain trying to do category theory on microsoft outlook web access.

maybe I should just come and gate crash your tutorial again.

Ingrid

In an instant something clicked and, although I didn't have the full definition yet, I had an idea of where to start. I experienced what Thomas Harris (2000 in Liljedahl, 2001: 83) refers to when he writes that "In that moment when the connection is made, in that synaptic spasm of completion when the thought drives through the red fuse, is our keenest pleasure." ***{aha moments}***

In his reply Dr Peters wrote:

Hi Ingrid

Gatecrashing will not help as I am at home sick today. If you get stuck then you are welcome to phone (6740043).

But, you really should trust your instinct, which is correct. Namely that there is a natural transformation so that each of the arrows (at B-level) are isomorphisms in the category B.

Cheers,
Roger

I looked at the work again and, although I had to sit a while and just look at it and then wrote down what I already had, suddenly it was obvious how to define the functions.

> *The process I went through was very similar to the one described by Duane (a mathematician interviewed by Handa (2003)). Commenting on notes he has made while solving problems Duane explains, "These notes give a sense of what I'm thinking. Essentially, first lay out the play of attack [...] It happens in different ways. So what is it I'm looking for, what is it I need to show. So, there will be things there such as 'so now it remains to show,' much like writing a proof. And then you just spit out the work." (Handa, 2003: 25) **{and then you just spit out the work}** I have found that this is a very common experience – by writing down what it is that one wants to do and what one has already done, the answer often just 'falls out'. I have not only experienced it in mathematics but also while writing this thesis.*

Man, what a kick - to figure something out on my own. To have an idea of what might work and then to actually prove that it does work. This rocks.

> *Looking back at what I wrote after solving this problem, I recognise that this was an instance (even though it wasn't such a big one) where, as Handa (2003: 22) put it, "'doing mathematics' became for me more than just 'doing mathematics', becoming what Joseph Campbell (1991: 182) has referred to as a "moving power of your life" or what Max van Manen (1990: 14) terms a "self-forming process"."*

2.2.8 Week 12 (12 October 2005)

The best thing, once again, about today's lesson was how much we laughed. To work hard and to laugh a lot – what a pleasure.

Things I noticed about myself: there were times when I felt like I was in a different space – times when I could see why we were doing things. It was as though my body was still in class but my mind was in this empty space – or rather this space where there were only the necessary things – the links that I was making. **{communal space}** I was in the space of understanding, a space one can only enter if one understands the work. I felt as though Dr Peters and I were the only ones in this space, because the

others didn't understand. I felt as though I was taking up all the room in this space, as though I was excluding the others. It was like week two. **{excluding others from mathematics}**

It's so weird sitting here with the concept of dual and being aware that I don't understand it. I feel as though I almost understand it. If I try too hard, the understanding slips away, but if I don't try it somehow it settles close by. I sit here and I know that I will come to understand this concept. A few years ago I would have felt frustrated and wanted to give up. Now I know that if I persevere, **{perseverance}** if I keep on trying to understand, keep on looking at examples and problems about duals then I will eventually get used to the idea. It's as though I know that this is a concept which I won't suddenly come to understand in an aha-moment but that it will slowly grow inside me. **{gradually coming to know}**

2.2.9 Week 14 (26 October 2005)

While working on a problem in class today, Dr Peters said, "I feel in my stomach that we should have a map from the smaller one to the bigger one". The only problem with this was that I "felt in my stomach" that it should be the other way round.

> *Burton (1999b), writing on the practices of mathematicians, addresses this point, namely of one's intuition not always being correct. She writes that one mathematician who "described an insight as a 'short cut' went on to point out that: Often you can't trust it!" (Burton, 1999b: 29)* ***{dangers of intuition}.*** *Other mathematicians have also commented on the need to be careful of one's intuition (Barnes, 2000: 39, Burton, 1999a: 132).*

Threads

Here is a list of threads that I have identified in this section as well as the section in which I plan to look at each thread in the light of other peoples' reflections.

{not asking leading questions} – 3.3

{feeling exposed} – 3.2

{availability of things to fall into place} – 3.1

{spoiling things for others} – 3.2

{choosing to let others do the thinking} – 3.2

{excluding others from mathematics} – 3.2

{keeping up in class} – 3.2

{learning deeply} – 3.2

{communal space} – 3.2

{teacher choosing not to be involved} – 3.3

{being silently involved} – 3.3

{comparing self to others} – 3.2

{getting too involved in the mathematics as teacher} – 3.3

{frustration} – 3.1

{aha moments} – 3.1

{and then you just spit out the work} – 3.1

{perseverance} – 3.1

{gradually coming to know} – 3.1

{dangers of intuition} – 3.1

Underview

From this section emerged themes that I will look at in more detail in the next section. I have enjoyed writing this section as it brought back memories of a course I enjoyed attending – most of the time. I learnt so much about teaching and in particular my own style of teaching from attending this course.

It scared me having to ask those in the class to read my reflections and say whether they agreed with what I had written, whether they were happy to let me use their names or whether I should change them. Writing for a faceless audience is much easier than writing for people you know.

SECTION 2.3

OF THE WEAVING OF MY TAPESTRY:

Learning while Teaching

A section in which I share and reflect on one lesson in a mathematics course I taught and what I learnt through this lesson.

"If we do not rely on general background theories to tell us how to teach, how then might we approach the question posed? I am enough of an empiricist to suggest that we experiment in classrooms, that we look at the conversations that do arise, that we focus in particular on the nature of the students' participation in those conversations, and that we investigate what students might actually be learning in the course of that participation."

(Cobb in Sfard, Nesher, Streefland, Cobb & Mason, 1998: 46)

"The apparent need for mathematics to be infallible and objective requires the removal of the evidence of the very human beings who created and worked with the mathematical objects, missing 'entirely the stumbling human process that created those results in the first place.' (Smith and Hungwe, 1998: 46)."

(Burton, 1999b: 30)

"It's how you *are,* you*, with your class, in contact with the force of mathematics, manifesting it in your every act. That's what we need to work on – on Being; on Mathematical Being; on Being Mathematical with pupils."*
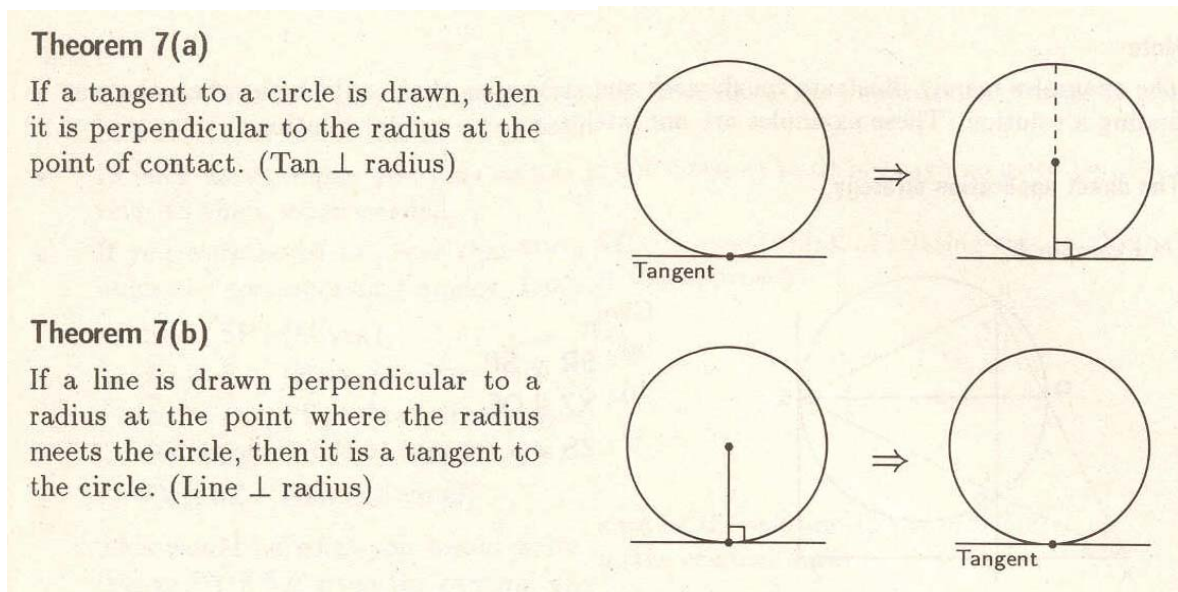
(Mason, 1986: 43)

Overview

In this, the last of my three panels, I describe a part of a lesson that I taught. Although I have written in the form of a dialogue as though from a transcript, I didn't record the lesson on video or audio tape. At the time I was not planning on using the transcript of a lesson in my research. And yet, after the lesson, I felt something had happened that I wanted to keep a record of so I sat down immediately and wrote down what I could remember. I was surprised at the amount of detail that I could remember.

Originally I chose this image out of all my 'learning while teaching images', because when I reflected on it I became aware of many aspects of my teaching style that I would like to change. It was only when I started looking in more detail at the mathematics that we discussed that I realised how shallow my understanding of the two theorems we worked with had been. I used these theorems when I was at school and I have taught them to my students for the last 3 years, but on this particular day I still wasn't able to recognise a situation where they could be used. I now have a two-fold reason for using this image – to highlight aspects of my teaching practice that I would like to change and to highlight a part of the process of making meaning in mathematics.

The two theorems mentioned in the image (note that the first theorem's converse is also true, but that no mention is made of the second theorem's converse) are:

Theorem 7(a)

If a tangent to a circle is drawn, then it is perpendicular to the radius at the point of contact. (Tan ⊥ radius)

Tangent

Theorem 7(b)

If a line is drawn perpendicular to a radius at the point where the radius meets the circle, then it is a tangent to the circle. (Line ⊥ radius)

Tangent

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Theorem 8

If two tangents are drawn from the same point outside a circle, then they are equal in length. (Tan. from same pt)
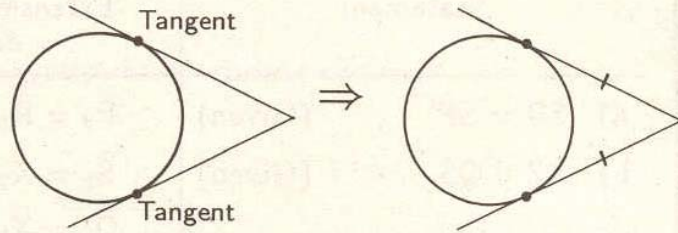

This then is my construction of what happened on this day and my reflections (in italics) on it.

A day in my practice

I had promised my students that today we would discuss the second examination paper that they had written a few weeks ago. We had spent the previous two days working through the interesting questions in the first paper. It had taken much longer than planned because I chose to follow issues that came up and, as much as possible, to allow the students to work on the problems on their own in the class rather than doing them on the overhead projector. (During the holidays I had sat in a class where students presented problems that they had worked out beforehand. I realised how little one benefits from watching someone else do a pre-worked out problem on the board and decided to try and minimise this in my classes **{learning from reverse positions}**).

We worked through the first few questions. The description starts when we got to the following question:

2.1 The circle with centre C (2 ; – 3) passes through point A (6 ; – 1) and through point B, which lies on the *y*-axis.
P (4 ; k) is a point such that PA is a tangent to the circle.

2.1.1 Determine the equation of the circle
2.1.2 Determine the equation of tangent PA
2.1.3 Determine the value of *k*
2.1.4 Hence, prove analytically that PB is a tangent to the circle.

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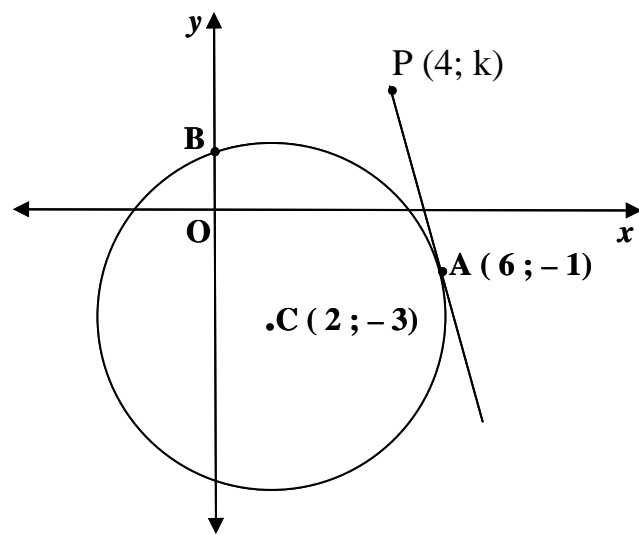
We quickly answered any questions regarding the first three questions and then looked at the fourth question. When I first tried to solve this problem, I only made use of the fact that a tangent is a line which only touches a circle once. I did this by solving the equation of the circle and the equation of the line simultaneously and showing that there is only one solution. During the school holidays I had taught some 'extra' classes in which I had given the students this same problem to work on. Two students had suggested alternative methods. One had suggested that we prove that the length of the line was equal to the length of the other line, which was also a tangent. Another student suggested that one show that the radius and the line were perpendicular and thus show that the line was a tangent. I had been impressed with these alternative methods, but we had not discussed them in detail.

Ingrid: Okay guys – talk to the person next to you and see if you can come up with some way of doing 2.1.4.

*From Mason (1986: 44) I have adopted the practice of asking a question and then allowing my students to 'talk to the person next to them'. Often I expect hardly any discussion as I assume that answer is very obvious and am then surprised at the amount of discussion that does takes place. {**talk to the person next to you**} I am then forced to wonder what would have happened had I answered the question and carried on with the work.*

In back right-hand corner we have Temba, Siya and Bethwel.
Back left-hand corner are Steven and Aldane. In front of them is Xolisa. Right in front is Donavon. Behind him sits Sarah.

Donavon: It's nature of the roots, Miss.
Ingrid: Nature of the roots? What do you mean? Nature of what roots? And anyway - don't tell me – talk to Sarah.

*Here Donavon is telling me what he thinks the problem involves because he wants me to tell him whether he is correct or not. {**choosing to let others do the thinking**}*

I move on, listening to other conversations and thinking about the problem myself. I start to wonder about whether the proofs suggested by the holiday school class were mathematically correct.

Julia: Ingrid, is this right? I want to prove that this line here is equal to this line here.

Ingrid: Sounds good to me. What does Jackie think?

I try to throw the question back to the students. I want them to convince each other, I do not want to be the source of approval {seeking external approval}. But I am aware that my casual 'Sounds good to me' probably gave Julia the approval she was looking for. She, like other students, was probably interested in investing as little energy as possible in solving the problem and was therefore looking for cues in what I was saying and doing (Mason, 1989: 7). But students are not the only ones trying to invest as little energy as possible to get through events. Am I not doing the same thing when I choose to tell a student that what they are doing is right or wrong rather than to sit with them and, through asking questions and listening, help them realise it for themselves?

I move on again. This time Donavon is playing with his phone.

Ingrid: Donavon, your phone is not going to give you the answer.

             Sarah what have *you* decided?

Sarah: Ja…what Donavon said…ja…

I have chosen not to explore the challenge of teaching third-language English speakers. It remains to be researched in the future.

Ingrid: But what did Donavon say? How are you going to do it?

             You don't have to do the whole sum but I want a detailed plan of action.

In an attempt to encourage meta-cognitive thinking – thinking about thinking – I have introduced the notion of a 'plan of action'. {developing a plan of action} I hope to encourage the students to think about what they are going to do and whether it will work rather than simply doing something for the sake of doing something. I first came across the idea of a plan of action in my third year. One of our lecturers would ask us to write down the plan of action of the proof we had studied – in other words, not to do the proof but to say what was being done in each step. I hope that by encouraging my students (and myself) to step back and think about what has been done or what is going to be done, we can learn more about what works in maths and what doesn't.

(To the whole class):

> Guys, I think we all agree that the first thing we have to do is to decide how we are going to prove that line is a tangent and then we have to decide how we are going to get the things that we need to prove that it is a tangent *(aarrg what a bad sentence).*

Here I suggested what the first step in my *plan of action would be. I would have preferred to give the students a chance to make their own suggestions. {spoiling it for others}*

I go stand in front of the class again. I am aware of the time pressure we have today and so I call everyone's attention, even though I can see there are still groups discussing their 'plans of action'.

Ingrid: Okay guys. Let's try bringing this all together – there have been some interesting things coming out. Siya, Bethwel, are you guys still busy discussing?

Bethwel: Ja, we haven't got anywhere – only lots of dead ends.

Ingrid: Well, why don't you tell us what you've tried so far and maybe we can do something with it.

Bethwel: Okay, we want to use the distance formula.

Ingrid: The distance formula? Why?

Unlike some of my previous questions which I ask even though I know the answer (questions referred to as "pedagogical questions" by Gadamer (1990)), this is what Gadamer refers to as a hermeneutic question, because I do not know the answer but desire to know it – I have no idea why they want to use the distance formula.

Bethwel: Well, we said that if we can prove that PB is equal to PA then we…

Bethwel has suggested the first alternative method of solving the problem. But I interrupt…

Ingrid: Oh, I see. Do you guys agree with what Bethwel has said?

A few people: Ja.

Ingrid: So… *(looking at Bethwel again)*

Bethwel: Well, we don't know what the co-ordinates of B are.

Ingrid: Well… any suggestions for how we can get the co-ordinates of B?

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Steven: Well Miss, if you take 0 and put it in the equation of the circle then you can get the y-co-ordinate and then …

Bethwel: Ah!

Bethwel starts talking to Temba and Siya

Steven: …you can get the gradient of PB and you can get the gradient of OB and then you can show that the two lines are perpendicular.

Steven has answered my question and has gone on to suggest the second alternative method of proving that the line is a tangent.

Ingrid: Okay, so Steven has suggested another way of proving that the line is a tangent. Do you guys agree that if we can prove that this line is perpendicular to this line then this line must be a tangent?

General agreement

Ingrid: And that both of the methods suggested would be pretty easy if we just knew what the co-ordinates of B were. But we know the y-co-ordinate of B – I mean the *x*-co-ordinate of B is zero and so we can substitute it into the equation of the circle. Does someone have the equation of the circle? $(x-2)^2 + (y+3)^2 = \sqrt{20}$? No? Just 20? Okay. *(I write it on the overhead projector)* There we have the equation of the circle. Can someone quickly work out what *y* is if we let *x* = 0? Julia? 1? Thanks. Okay, so now we know what the co-ordinates of B are and we can easily work out the lengths of these lines or the gradients of these lines.

I start wondering about the two methods that have been suggested. In retrospect I realise that I was doubting whether the converse of two theorems which I knew to be true were in fact true. What is interesting is that (as I have mentioned) I had taught students the proof to the converse of one of the theorems. And even so I hadn't fully grasped it.

Ingrid: Okay. Now I have to tell you guys that I am doubting myself. I am no longer convinced that these two methods that seem so nice are actually valid. I really don't know, so please tell me if you think I'm wrong. What I am wondering about is whether showing that these two lengths are equal is enough to force this second line to be a tangent. And whether showing that these lines are perpendicular is enough. I mean can't we have a line that starts here and touches the circle and that is the same length as a tangent line. I mean what about this drawing?
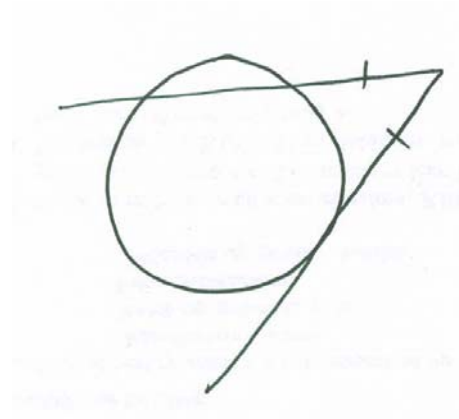
I make the drawing and move away so that I can see it on the overhead projector screen. As I look at it from this perspective, I suddenly realise that it is not possible. The only way that the second line could have the same length is if it is a tangent. I see this and am so excited that I carry on without allowing my students to see it on their own **{getting too involved in the mathematics as teacher} {spoiling things for others}**.

Ingrid: Oh, but look guys – is it possible that this could ever happen? Can any of you draw a picture where these lines are the same length? 'Cause I mean – all we need is one… *teen voorbeeld, what is teen voorbeeld in English? ...* one opposite… one… *(and suddenly it comes to me)* counter-example. Like if I say "All women in the world think that Siya is good looking" then all I need to say that this is not true, to disprove my statement, is to find one girl that doesn't think that he is good looking.

Donavon: Sarah…

Sarah: Haai!

Logic and things like counter-examples are some of the things that I think are extremely important in mathematics but that are taught on their own in the curriculum. Whenever I come across a situation

where I want to use these ideas, I always feel that suddenly talking about logic would detract from what we are busy with, it would stop the 'flow'. And so I find myself giving examples that, when I am honest with myself, probably only make sense to people who already have an understanding for such concepts. How can I change this?

Ingrid: Okay, so can anyone get a drawing like that?

John: Yes miss, I got one. Look here – this line is the same length as that one.

Ingrid: Yes, but John you have to draw it with a ruler…

John: I did – here's then my ruler!

Ingrid: … and a compass.

John: What difference is that compass going to make?

Ingrid: You need the compass, John, or else it won't work.

John: But Miss…

Ingrid: Okay, so are you all happy that if this line is the same length as that line then it *forces* this line to be a tangent – it doesn't have any other option?

Again I avoid a situation that I feel would prevent the flow of what is happening (especially as I am aware of the time pressure) even though I know that it is imperative that students realise the difference between getting a feeling for something from a drawing and using a drawing as proof. I realise now that the drawings that I was making were not done with a compass or a ruler. It wasn't just the drawing that helped me but my experience of knowing how circles and lines work. How can I deal with such a situation better in the future? How does one deal with time pressure?

Ingrid: So it looks like Bethwel and Siya's idea will work.

What about what Steven was saying? You see the thing that I am not sure about – and really guys, I don't know if I'm talking rubbish or not – these are things I just started wondering about now in class, so I could be totally wrong – is whether showing that these two lines are perpendicular is *enough*. I mean – if a line is perpendicular to a radius, does that force the line to be a tangent? I mean – what about this situation?

*Here I am thinking aloud as I begin to doubt the suggestions that I have previously said were valid {**demonstrating mathematics**}. In retrospect I wonder whether this kind of doubting is a good thing for*

students to see – there is a possibility that they might become so confused that they give up trying to follow the discussion? The problem with this situation was that I was so focused on trying to figure out what was going on that I was not paying attention to ensuring that all my students understood the process I was going through.
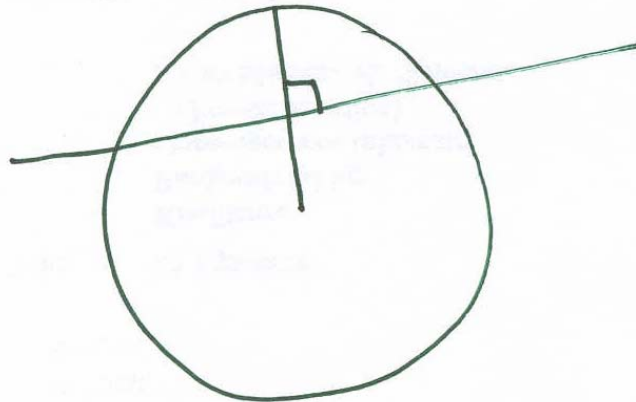
Isn't this an example of a line that is perpendicular to a radius but that is not a tangent?

General consensus.

Ingrid: So actually the difference between the two situations is that for Siya's suggestion we had the theorem "if a line is a tangent then these lengths will be equal" and the converse was also true[1] "if these lengths are equal then the line is a tangent" but in this situation although we know that the theorem is true "a tangent is perpendicular to the radius", the converse isn't true.[2]

I was playing around with different drawings, I was looking for what was the same and what was different in the two situations – all the kind of thinking processes that I want my students to be able to

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[1] As mentioned earlier, the textbook in fact makes no mention of the converse and it was only when I was re-reading this section that I realized that I had simply assumed that the converse was true. How often do I use intuition like this in my problem solving and then get frustrated when my students make similar (sometimes incorrect) assumptions?
[2] In fact the converse is true as I soon 're-discovered'.

do. {demonstrating mathematics} Although Barnes (2000: 41) encourages teachers to model this process of reflecting I worry that I might have 'lost' my students along the way.

By this time I was once again standing away from the overhead projector and looking up at the screen.

Ingrid: I mean – what's the problem with this line – it cuts the circle twice.

Julia: But Ingrid – we are working with the line PB, not that line. So then we can see it is the tangent.

Ingrid: You mean because it only touches it once – but you see if we knew already that it only touches it once, then we would know that is was a tangent and we wouldn't have to use the perpendicular-ness.

Julia: Yes, but… uuum… no, it's not right…

Ingrid: So, although we know that if a line is a tangent, then it is perpendicular to the radius, we can't say if a line is perpendicular to a radius then it is a tangent – unless we say that it only touches the circle at one point then we are using what we are trying to say *(aagg another horrible sentence)*. Do you guys get what I am trying to say?

Some: Yes…

Others: No…

Here things start getting messy when it comes to the definition of a tangent. The line looks like a tangent so why isn't it a tangent? How do we separate what we know to be true from what seems obvious from the drawing?

I again look at my drawings and suddenly I see a difference between the two.

Ingrid: Hey guys – isn't there a difference between the two? I mean, okay let's draw them again next to each other (I redraw them on a new page). What do you guys think is the difference between them?
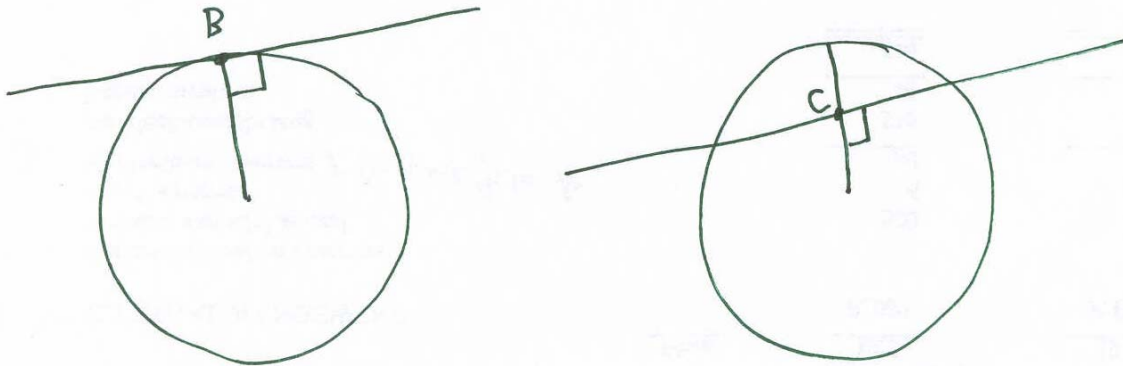
Aldane: The one is a tangent and the other isn't.

Ingrid: Yes, yes, okay but I mean in the way that I have drawn the situations?

I brush over an answer that was actually a chance to go deeper into what we were busy doing. I forge ahead because I have now seen how to solve the problem and want them to share in my excitement. **{getting too involved in the mathematics as a teacher}**

Ingrid: Look over here – here the place where they are at 90 degrees is in the middle and here it is at B. Do you guys see that that is a big difference? This one is perpendicular at the point of intersection. This one is perpendicular over here. Doesn't that make a difference?

I know, let's try write down the theorems properly.
What would the theorem be?

I write on the overhead:

"If a line is a tangent to a circle,

then the line is perpendicular to the radius drawn to the point of contact"

So what would the converse be?

If a line…

Um, okay, well let's start with the easy part. What is the "then" part going to be?

"then the line is a tangent".

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Ingrid: Okay. So what should the "if" part be? Does someone want to try? Sarah? And remember it has to have a verb in it.

Thandi: Yes (nodding her head)

After two years of marking sentences that didn't make sense, I finally realised that my second- and third-language English students usually didn't know any better. So I started explicitly mentioning the structure of sentences and especially the necessity of a verb in a sentence.

Ingrid: If…

Sarah (joined by others):

If a line is perpendicular to a radius at the point of contact with a circle.

Ingrid: Okay. What do the rest of you guys think? Do you agree with what Sarah has said? Let's look at it again. If a line is perpendicular to a radius at the point of contact with a circle then the line is a tangent. Okay, do you guys see how important this little bit is "*at the point of contact*"? That is the piece I was missing out on earlier.

This is in fact the first time that I *ever realised how important 'this little bit' is. By important I mean necessary. This little piece 'at the point of contact' is necessary but only at this point did I* experience *its necessity. **{experiencing the necessity of mathematical statements}***

Drouhard, Sackur, Maurel, Paquelier & Assude (1999: 320) point out that most statements in mathematics are necessary but that little research has been done on "when and how students do become aware of the necessity of such statements". They go on to list a set of questions including: "Is it possible to experience the necessity of necessary mathematical statements? If so, what is the very nature of such an experience? And, are some students actually experiencing it, and how?" (Drouhard et al.*, 1999 : 321).*

From my own experiences, in particular the one described thus far, I would suggest that the answers to the first and third questions posed by Drouhard et al. *(1999: 321) are yes. But although I would agree that some students are experiencing the necessity of statements, I would still find it difficult to explain* how *they experience it.*

If I was able to do well in my mathematics at school and was able to teach mathematics to school children without being aware of the necessity of this part of the theorem, how many of the students that I teach have appreciated its necessity? What can I do as a teacher to help students experience the necessity of mathematical statements?

Ingrid: So, it seems as though Steven was right all along.

Donavon: But he didn't know why he was right, Miss. The credits go flying.

I have no idea what he means by this. It's obviously some class joke, since everyone else starts laughing.

Ingrid: So it works *because* the place where the lines are perpendicular is at B and not somewhere else. But what I wanted you guys to learn from this was …*(100 things go through my mind but all that comes out is)*… how important each part of a theorem is.

Xolisa: Can I make a suggestion? Because of the time factor, can we just go through all the questions and then, if there are people who still have questions about the marks, then they can ask you afterwards?

Ingrid: Okay. Sometimes we do get off the topic unnecessarily, but I wanted you guys to learn something from this – this is important stuff, guys.

*Was this detour a waste of time as far as my students were concerned? Was I the only one who learnt an incredible amount about mathematics from this lesson? Xolisa's suggestion makes me think it wasn't as meaningful for them as it was for me. Or perhaps Xolisa was trying to survive in the system by working to get as many marks as possible rather than to learn deeply {**learning deeply**}*

*And yet even as I doubt whether sharing my own confusion and 'stumbling' with my students was the best option, I am reminded of what Davis (1985: 40) wrote: "Probably, no other change would improve mathematics education more than to begin actually teaching mathematics – teaching the processes that are involved in doing mathematics, instead of merely showing some of the results that have been achieved by others." The doubt and confusion and subsequent clarity and excitement that I tried to share with my students during this lesson are all part of the processes of doing mathematics – as is the subsequent reflection on what happened in this class. {**demonstrating mathematics**}*

Threads

Here is a list of threads that I have identified in this section as well as the section in which I plan to look at each thread in the light of other peoples' reflections. Not all threads identified will be used in later sections.

> **{learning from reverse positions} – 3.3**
>
> **{talk to the person next to you} – 3.3**
>
> **{choosing to let others do the thinking} – 3.2**
>
> **{seeking external approval} – 3.2**
>
> **{developing a plan of action}**
>
> **{spoiling it for others} – 3.2**
>
> **{getting too involved in the mathematics as teacher} – 3.3**
>
> **{demonstrating mathematics} – 3.3**
>
> **{experiencing the necessity of mathematical statements}**
>
> **{learning deeply} – 3.2**

Underview

This has been an amazing section to write. I didn't expect so much to come out of this one day. Once again in the writing I have discovered many things I didn't know before I started writing.

SECTION 2.4

OF THE WEAVING OF MY TAPESTRY:

The Threads that Emerged

A section in which I collect the threads that have been used to weave the images I have shared and reflected on and in which I identify those threads which I will look at in more detail in section 3 and will finally use to weave an image of what my practice could look like.

"When you have a collection of accounts, you can work at locating some common threads, some features of sameness, some issues for you that you want to explore. Sometimes you can be surprised by the themes which emerge, sometimes they are what you expected."
(Mason, 2002: 69)

Overview

I have extracted and arranged the threads from each of the three panels into three sections. Each section has a different focus and the threads I plan to consider in each section in view of the reflections of others – namely other mathematics education researchers, other students and other teachers.

2.4.1 Threads for Theory of Mathematics Education

Paradox of learning

> {aha moments}

> {availability of things to fall into place}

> {need question before answer makes sense}

> {experiencing the necessity of mathematical statements}

> {gradually coming to know}

Working through frustration

> {frustration}

> {perseverance}

Intuition

> {developing intuition}

> {dangers of intuition}

> {and then you just spit out the work}

Structural to Procedural

> {importance of shifts in attention}

> {reification}

2.4.2 Threads for being a Mathematics Student

Being honest

> {being forced to explain to others}

> {being honest with self}

Self-Perceptions

> {doing original work}

> {feeling exposed}

> {seeking external approval}

> {learning to trust one's own abilities}

{comparing self to others}

Coping with work

{helplessness}

{choosing to let others do the thinking}

{keeping up in class}

{learning deeply}

{much effort, little reward}

{sources of motivation}

Relating in class

{communal space}

{spoiling it for others}

{excluding others from mathematics}

2.4.3 Threads for being a Mathematics Teacher

Relationship to Mathematics

{getting too involved in the mathematics as teacher}

{demonstrating mathematics}

{working together in class}

{teacher choosing not to be involved}

{being silently involved}

Relating to students in class

{asking someone who has recently made meaning}

{talk to the person next to you}

{not asking leading questions}

{students directing class}

{learning from reverse positions}

Underview

The first time I went through my writing about my own experiences I identified 182 threads in 80 images. The first time I went though all the articles I had read about mathematics education I identified 137 threads. It has been a long process trying to choose which threads and in which images to use. It

has been a process of co-emergence[1] – I would identify what I felt were important threads and would then go through the images in which these (and other) threads were mentioned and would decide which were meaningful and which weren't and would then look at the threads that were left and then at the images until I ended up with the threads and images I have used in this section.

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[1] See Underview of section 3.2*.

SECTION 3
OF THE CONSTRUCTION OF MY TAPESTRY:
THE PLANNED THREADING

A section in which I *planned* to thread the themes that were extracted from the images shared in the previous section to create an image of what my practice *could* look like in the future.

"The avenues along which this research may lead depend not only on the data, but also on the interests and interpretations of the researcher. The tendency of such hermeneutic research to use progressive focusing rather than pre-ordinate design (Hartnett, 1982) makes this kind of research as interesting as a mystery story, even if the mystery is to some extent self-created. In this respect, mathematics education research may have elements in common with mathematics research."
(Presmeg, 1998: 58)

"What we inquire into is not predetermined, but emerges through our activity… This emergent process is necessarily full of surprises, since we cannot know in advance what will become the key issues for inquiry and exploration."
(Jaworski, 2007: 15)

SECTION 3.1
OF THE PLANNED THREADING OF MY TAPESTRY:
Theory of Mathematics Education

A section in which I *planned* to examine certain threads in the light of what my author friends have written.

"When we work now, we don't know where we're going to end up. We start with something, and end up with something very different. We begin with an idea that interests us, but very quickly it becomes many ideas. It splits apart into pieces. We split apart into pieces. Our one idea becomes a kind of jumble."
(Jipson & Paley, 1997)

John Mason (2002: 99), writing about the Discipline of Noticing, suggests that after one has identified significant threads from the brief-but-vivid descriptions of one's practice, one option is to "make use of the literature to connect or distinguish your experiences with those recorded by others." This is what I planned to do in this section. In particular I wanted to look at the threads I had collected in the subsection entitled 'Theory of Mathematics Education'.[1]

So I wrote…

Different ways of coming to know mathematics

In an attempt to kick-start the process of writing this section I once again reflected on the images that I had woven and selected in section 2. Among other things, I noticed the different ways in which I described 'coming to know' mathematics[2] and for the first time I realised the importance of considering how we actually come to know something and the implications it has for teaching. At this stage I had been teaching for three and a half years and had never stopped to think about how I believed people learn something new and how my answer to this question influences the way I teach. I had read about the idea that teachers' implicit views about learning influence their teaching (see Hopper & Rossi, 2001; Atkinson, 2004 as well as Davis, Sumara & Luce-Kapler, 2000: 51), but had not appreciated what they were talking about. Mason (2003b) mentions that sometimes we hear something but that it doesn't mean anything to us. It is only later, when we experience a "dissatisfaction" that it comes back to us and we are then able to make sense of it. This was one such an example.

So I started reading about different people's ideas on how we come to know something new.

At this stage I still planned to move on and look in more detail at what it means to come to know something in mathematics – i.e. to explore the nature of *mathematical* knowledge? And so I wrote…

Different ways of seeing mathematical structures

Closely linked to the ideas of coming to know something in general are the ideas about coming to know concepts in mathematics. As with the nature of knowledge in general, much has been written regarding the nature of mathematical knowledge but I have found Anna Sfard's (1991, 1994)

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[1] Subsection 2.4.1

[2] {aha moments} – 2.1.6, 2.2.7
  {gradually coming to know} – 2.1.8, 2.2.8
  {availability of things to fall into place}- 2.1.2 & 2.2.2

notions of "operational and structural conceptions" and of "reification" most meaningful. I first read about the idea that an important part of being able to 'do' mathematics is being able to shift from seeing a concept as an operation to seeing it as a structure (and visa versa). This idea appealed to me immensely. In fact, in a way, it captures what it is that I enjoy about mathematics – taking a process and considering it as an object. This explains why I enjoyed Category Theory, which is entirely dedicated to this. My description of the day on which I saw functions in a different light[1] is an example of making this shift.

Shifts in attention

Closely linked to the idea of reification or the shift between an operational and a structural conception (Sfard, 1994: 53) is the question regarding what it is that changes when we make this transition, when we 'abstract'. Mason (1989: 2) writes that "the uses of the word abstract *in mathematics ... refers to a common root experience: an extremely brief moment which happens in a twinkling of an eye:* a delicate shift of attention *from seeing an expression* as *an expression of generality, to seeing the expression* as *an object or property". Mason (1989: 7) goes on to suggest that one of the problems in teaching is that often teachers are not aware that they have made this delicate shift in attention – they are not aware of "what they are attending to ... nor that their students may not be stressing the same features."*

But as I read more about different ways of coming to know and particularly about co-emergence, an enactivist notion, it felt as though what I had written thus far was no longer meaningful. I started reading about ways of coming-to-know and ended up "with something very different" (Jipson & Paley, 1997). The more I read, the more the threads that I had woven unravelled, until I wrote:

Everything has become a mess. I feel as though the tapestry that I was weaving has begun to unravel even before I finish weaving it. In the beginning I imaged the front being neat but having a messy back to represent the neat version of research which we present to 'the world' while hiding the messy process we followed to get there. Now I am no longer willing to create a neat 'front view' and just be happy with providing a 'behind the scenes' glimpse of my research. I want the research itself to show the messiness of the research. I want to show how the ideas emerged, how they bumped unruly into each other, how they fermented, how everything came about. I want to do

---

[1] {shift in attention} – 2.1.10

daredevil research – but do I dare to do it? Do I dare to unravel the last section of my tapestry? To knot my threads? To write my poetry? To reveal myself? Do I dare?

I got to the point where I couldn't look at what I had written in the same way any more. Not only did reading about enactivism and other similar theories for learning and teaching change the way I looked at the images I had woven but, more importantly, it changed the way that I looked at my own practice. It is almost as though, as I continue on the journey of writing this thesis, I keep on being moved closer and closer to my own practice. I started off only wanting to research my own experiences of learning mathematics, but have ended up using these merely as an entry point into researching my own experiences of *teaching* mathematics.

Unravelled threads

So instead of reading about the different threads I had identified in subsection 2.4.1, I started looking at what was happening in my practice from a different perspective and I started writing about the things I was noticing. In the end I decided to redo this section. I have given it a new name, 'Coming to Know',[1] and have structured it around a moment in my practice. It is now the knotted messy result of, simultaneously, reading about enactivism and other related topics, teaching, and being aware of the threads that were being unravelled by this whole process.

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[1] Section 3.1*

SECTION 3.2

OF THE PLANNED THREADING OF MY TAPESTRY:

Being a Mathematics Student

A section in which I *planned* to consider certain threads in the light of my students' thoughts and comments on these threads

"When we work now, no top, no bottom. No back, no front. No under, no over, no symmetry. No yard we own. Heap ideas on top of ideas until they give us their own shape anywhere. Sometimes this can be startling – sometimes a mess. Sometimes this results in a hunger for something we can't name."

(Jipson & Paley, 1997)

As mentioned in section 3.1, John Mason (2002: 99), writing about the Discipline of Noticing, suggests that after one has identified significant threads from the brief-but-vivid descriptions of one's practice, one option is to "make use of the literature to connect or distinguish your experiences with those recorded by others". Another option is to share one's brief-but-vivid descriptions with colleagues to see if they have had similar experiences and, if these are then described, to start locating and sharing similarities and differences (Mason, 2002: 106). Because the collection of threads in the subsection 'Being a Mathematics Student' were necessarily about being a student and not a teacher, I planned to share my descriptions with my 'student colleagues' – the learners in my class – to see whether my themes resonated with them.

So I chose four of my own lived experiences and wrote brief-but-vivid descriptions of them as stimulus to the students and asked them to write about similar experiences.

Hey guys
I need to ask you a favour. Some of you might know that I am busy with my Master's in Mathematics Education. This means that I have to do one huge "taal en denke" project. Part of it includes doing research on learning and teaching mathematics. I have chosen to research my own learning and teaching of mathematics. During this process of looking at the ways in which I learn maths and the things I struggled with I came up with 4 "themes" or "issues that kept coming up". What I'd like to do is to see if other people also struggle with these things. Seeing as you guys are busy with learning maths every day (except maybe Sundays) I figured you would be a group to ask. So, what I've done is to try and describe each of the "issues" that I have noticed in my own learning. I'd like you to read each of these descriptions and to write down any similar situations that come to mind. You can also write down how you felt in the situation and what you did to cope in the situation. It's possible that my description doesn't ring any bells in your experience – well then that's fine too.

"The need for confusion"
When I was studying at university we learnt about something called bijective and injective functions in my first year. I didn't understand them. For two years I carried this confusion round in my head and finally in my third year someone explained the work to me again and it all made sense. It was exactly the same explanation, the difference was that this time round I had lived with the things that didn't make sense to me, I had questions that I wanted someone to answer. The first time round I didn't know anything about the work so I didn't have any questions.
I have often found that when one learns new work at the beginning it doesn't make sense. It is as though one first needs to have some confusion before things can make sense.

"Being honest with myself"
Another time while I was studying I had an exam I was learning for. We had about 20 theorems we had to learn. While I was studying I was convinced that I could prove all of the theorems. But when I got to the exam, I realised that I had been kidding myself. I couldn't prove some of them. It was easier to convince myself that I could do the work than to be honest with myself and admit that I didn't understand it because then I actually had to go and do something about it.

"Being embarrassed about my work"
Something that is similar to the issue I mentioned above is the whole issue of being embarrassed about my work. When I am busy doing something that I actually know I don't know what I'm doing then I never want anyone to see what I'm doing. Often when I was at school I'd like use my textbook to cover my work or something. It's a weird feeling – feeling like you have to pretend you know what's going on when actually you don't. And then your teacher wants to come round and check if you understand and you don't even want them to find out that you don't know what you are doing even though it's their job to help you understand.
While thinking about this "issue" I've realised that most of the time when I don't want others to realise that I don't understand it's because I feel that it is my own fault that I don't understand (like I didn't do the homework or I haven't tried hard enough). When I feel like it isn't my fault that I don't understand (like if the teacher isn't explaining well) then I don't mind asking questions or showing people where I got stuck.

"Coping in class"
The last thing I've noticed is how often in a class I have to choose between trying to figure out something that I almost understand (by talking to the person next to me or by finishing an example or just thinking about the work) and trying to follow what the teacher is saying about the next piece of work. Most of the time I choose to listen to the teacher because I'm scared I'll miss something important. And it's usually easier to listen to someone else than to try figure something out on my own. But I wonder whether it isn't just as important to figure things out for myself – but to do that I need time in class.
So, guys, these are the four things I'd like you to think about and to write about. There are no right or wrong answers, just your own experiences.

Although I gave them this exercise to do with the idea of letting them write whatever came to mind when they read my prompts, when I read their responses I realised that I actually had preconceived ideas of what I wanted them to write. In a sense I was looking for validation of my own experiences.

Some students wrote responses which suggest that they 'missed' what I was trying to share in my brief-but-vivid descriptions. This is one of the reasons why one checks one's descriptions with others – so that one can refine them. Others seemed to recognise the descriptions and their resulting descriptions resonated with my own experiences. These descriptions helped to clarify the experiences that I was trying to describe. Here are some written responses which "missed the point", my reflections on these responses and some responses which "got the point".

The need for confusion

Nomsa

With me also, when I learn something for a first time sometimes it doesn't make a sense. I can not even ask because a teacher used to say which part of this you don't understand and find that you can't say what you don't understand because it does not make a sense from the start. But I discovered that the confusion must not discourage me or let me give up but must make me curious and going to different people who understands better. If something do not make a sense to me does not mean that it does not have a sense. It does; but the problem is with my understanding and its not the end of the world, I still gonna understand.

Although this description mentions an important observation, namely that it is often difficult to understand a new concept the first time one is exposed to it, it seems as though Nomsa has missed the point that I was trying to make – that this initial confusion is not a waste of time but is in fact often necessary for meaningful understanding. Or as Nicole, one of my students, put it: "It goes without saying – in order to understand something you have to be confused first". The following response also captures this point.

Mmabatho

I share the same sentiments, it's like the confusion creates more pieces to the puzzle and finally when you get someone to explain it again, you have all the pieces and now you just have to fit them in. The confusion creates elements of understanding.

Being honest with myself

Nicole

On another occasion, when I was grade 11 we did Euclidean Geometry and had to prove theorems about 12 of them. Only 3 came in the June exam. I guessed that if I studied the most famous ones/theorems like midpt and quads; I needn't study all of them. So I studied the famous ones and to my amazement only one of which I studied came in. So undoubtedly I tried to fool myself into believing I could do it my way.

Once again this descriptions brings up the valid point that 'spotting' for tests doesn't always work. But the experience that I was trying to describe is one where we convince ourselves that we know something when in fact we don't. The following two responses, I feel, do capture this.

Desiree

Oh yes, about the theorems, when I was in matric, I would be convinced that I understand *the theorems when the teacher explains them but come test, I'll get them wrong.*

Mmabatho

One tries to be confident about maths so as to help you cope but we also tend to shut out the fact that we cannot do some things *and what we cannot do we cannot be confident about, so telling yourself that you understand while you do not is a way of staying positive but while the intentions are good, it still does not address your problem and sometimes I was to lazy to do something about it.*

Being embarrassed about my work

Interestingly enough for this thread/theme all the descriptions captured the idea of being embarrassed about one's own work. Something that ties in with feeling embarrassed and which I didn't put into my description although I have experienced it, is the fear of being exposed.[1] The following description captures this fear.

Mmabatho

This is major problem for me which occurs in all my subjects but especially in maths, I was so scared that my classmates and teacher would find out *that I hid my work almost all the time. I felt as if there was something wrong for me or that the teacher would think maths wasn't for me and then I had to switch subjects but when it is a general problem when most or all people have a problem it's easier to come clean and say you don't understand as well.*

Unexpected responses

And then there were responses that were unexpected, responses that reminded me once again how important it is to share our experiences with each other.

Jenny

I presently feel like I don't belong here because I always miss the point and I get horrible marks with my poor background and I feel like I could get a course that does not require maths.

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[1] {feeling exposed} – 2.1.7, 2.2.1

Claudia

I am so seriously relieved, after reading about your experiences, I feel SO much better and less of an idiot.

After extracting themes from my images and reading my students' responses, I searched for literature describing students' experiences of studying mathematics. I struggled to find anything which I considered to be meaningful. Mason (2002: 157), referring to the process of phrasing a research question, suggests that "asking yourself what you would do with an answer [to the research question] were someone to walk in with one often shows up that the question as posed either has no answer, or that an answer would not serve any useful purpose." Although I wasn't busy trying to phrase a research question, asking myself what it was that I was looking for in the literature helped me realise that even if I found it, it wouldn't be meaningful. I realised that I was looking for students' (phenomenological) descriptions of the lived experience of coming to know mathematics. The problem is that most students aren't aware of their experiences of coming-to-know mathematics, so descriptions of this experience are almost always written from the point of view of mathematicians (see, for example, Sfard, 1994).

At the same time I was reading enactivist literature and realising that I didn't want to separate my research from my practice. I wanted my research to be a means of improving my practice and that meant listening to *my* students. Although other students' descriptions might give me more entries into my own students' experiences, in the end I would still have to come alongside my students and listen to them.

Unravelled threads

So in this section, too, the threads unravelled as I realised that descriptions of the practices of others wouldn't help me understand *my* practice and that if I wanted to improve *my* practice I would have to start *listening* to my learners' experiences and not only to their responses to my experiences. In the end I decided to redo this section as well. I have given it a new name, 'Listening',[1] and have structured it around a moment in my practice. It is now the knotted messy result of, simultaneously, reading about enactivism and other related topics, teaching and being aware of the threads that were being unravelled by this whole process.

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[1] Section 3.2*

SECTION 3.3

OF THE PLANNED THREADING OF MY TAPESTRY:

Being a Mathematics Teacher

A section in which I *planned* to look again at certain threads, this time in the light of my colleagues' comments on them.

"When we work now, everything changes without letting us know. It gets harder and harder to hear. It gets harder and harder to hear. It gets harder and harder to stop. It gets harder and harder to know."
(Jipson & Paley, 1997)

"To live between the opposites means that we not only recognise opposites, but rejoice that they exist. To live between we stretch out our arms and push the opposites as far apart as we can, and then live in the resonating space between them."
(Bly, 1990: 175)
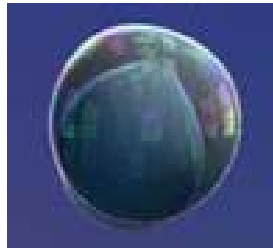
As mentioned in sections 3.1 and 3.2, John Mason (2002: 99), writing about the Discipline of Noticing, suggests that after one has identified significant threads from the brief-but-vivid descriptions of one's practice, one option is to "make use of the literature to connect or distinguish your experiences with those recorded by others". Another option is to share one's brief-but-vivid descriptions with colleagues to see if they have had similar experiences and, if these are then described, to start locating and sharing similarities and differences (Mason, 2002: 106). This is what I planned to do in this section using the threads collected in 'Being a Mathematics Teacher'.[1]

Once again I started by reflecting on these themes. I played around with different ideas, continually writing down my thoughts.

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5 May 2007

So really, what were the things that really made a difference to me in my practice?
I don't just want to write a bunch of words just for the sake of writing them.
I want to write about the things/ theories that made a <u>difference</u> in the way <u>I</u> see things.

I don't know what to ask my colleagues.

Are there things that bother me about my teaching?

Is this thesis about my practice or about my learning or what?

I suppose it's about my practice which I am investigating via my own learning experiences and via reflecting on my teaching.

So I need to keep in mind that it's about my *practice*.

Because in the end I'm going to ask "What possibilities emerged?" And those possibilities are possibilities for my *practice*.

There are things that bother me about my practice:
     that I ask leading questions and
     that I don't use the homework effectively and
     that I talk too much about the things <u>I've</u> realised

What would section 3 look like if I were only to focus on my practice when writing it?

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[1] Subsection 2.4.3

After rereading Breen's (1993) article on the tensions in teaching mathematics I rephrased my ideas for this section in terms of the tensions in my practice – some which I have managed to work on and others which remain with me.

I noticed the following tensions in my teaching; tensions which flowed from the issues mentioned above; tensions between: asking and telling;

getting students to think and coaching them for exams and

showing them how I think and encouraging them think for themselves.

And so I wrote…

Asking vs. Telling

As a teacher I have experienced a tension between, on the one hand, wanting to ask students questions so that they can think about a concept and come to their own conclusions and, on the other hand, not wanting my questions to turn into guessing games.

As mentioned in section 2.2.1 such questions can be referred to as leading questions. According to Gadamer (1990), these are "pedagogic" questions because the person asking the question already has an answer in mind and therefore the answer given won't make a difference to what happens next. In the case of leading questions, the question simply gets asked again or rephrased until the 'right' answer is given.

Because of this I now try treating "leading questions" differently. When I find myself asking a leading question and not getting the response I was looking for, instead of trying to backtrack and ask more leading questions leading to the original question and thus creating a guessing game I try to rephrase what I wanted to say. For example instead of asking "What do you guys think is the most important thing we learnt this week?" I will say "For me, the most important thing we learnt this week is that …"

Even so I still often catch myself waiting for my class to answer a leading question. How can I avoid asking such questions? Are there more meaningful questions I could be asking? What would they look like?

Getting students to think vs. Coaching them for exams

The other thing I really have a problem with (something which didn't come out in the images in section 2) is my relationship to homework and tests. I struggle to deal with homework and feedback of tests in a manner that I feel is meaningful.

I am torn between wanting to develop my students' intuition and, wanting them to do better in their external exam at the end of the year.

Because of this I find myself teaching one way in class and then using the homework as "preparation for the exam". Ideally I want the homework to be part of what is happening in class. Not just exercises to practise.

I'm thinking about the way in which research would help me with this dilemma. And I don't think it would. Although there was this article about how research from the inside could possibly help a teacher…

The article I was thinking of is by Kieren (1997) and he argues that a teacher consulting "constructivist, interactionist and enactivist theorizing and research" (what I remembered as being 'research from the inside') must realise that it "will not tell her how her students must behave but will provide her with insights that she can use in observing and listening differently to the mathematical actions and languaging of her students, and in entering into a different form of conversation with them (one which will focus less on 'right answers' per se and more on her students' mathematical actions, re-presentations and explanations)" Kieren (1997: 32).

But this means, again, that I have to change the way I look at my practice, at the way I understand what it means to learn and what it means to be a teacher.

Feedback from tests is just as bad. I feel that writing a test and not giving students an opportunity to learn from the test is a big waste of time. How do I give meaningful feedback?

Showing them how I think vs Encouraging them think for themselves

One thing I really struggle with is what my relationship to mathematics should be in the classroom. I don't believe that I am in class just to tell the students what to do, to teach them recipes. So what is my role? Good question. Never actually asked myself this before. To nudge them in the right direction?

Before I say what others have said, let me clarify my dilemma. I have read and have experienced, on the one hand, the fact that mathematics textbooks give you the idea that maths is perfect and that there was no stumbling involved in when people do maths. As Sfard (1998a: 507) put it:

> On a closer inspection, mathematicians and their written works may not be the best source of inspiration. As Gauss remarked, mathematicians like to 'make the scaffolding invisible' once the product of their efforts is rendered in its final form. Mathematicians do not choose to reveal the ways their mathematical work is being done.

On the other hand, I have experienced the absolute pleasure of being in a class doing messy mathematics with the lecturer.[1] So I have this desire to demonstrate and to do mathematics together with my students. I love being stuck and having a student figure out how we can get unstuck. But I am pulled apart by the times when I get carried away – when I get so caught up in the mathematics that I take up all the space. Or when I 'connect' with only a few of the students and we take up all the 'space' and exclude the others.[2]

Because of this I try as often as possible to move towards a situation where the students are helping each other. Then I am able to come around and "be alongside" them (Brown & Coles, 2007: 9) and we can do it together. Such moments are filled with lots of "Um…I don't know, why don't we try this?" "I don't know if this is going to work but let's just try and see what happens" "Um, okay, can I show you what I would do?"

So what is my role? I can't believe I've been teaching for three and half years and I have never stopped to ask myself what exactly my role in the classroom is.

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[1] subsection 2.1.5
[2] subsection 2.2.2, 2.2.8

Unravelled Threads

Unlike sections 3.1 and 3.2, the threads that had been collected to form this section were to do with my practice. So as I read the literature proposing a new way of thinking about theory and practice, the unravelling that had taken place in sections 3.1 and 3.2 didn't take place. Instead, the different ways of thinking about what it means to learn and what it means to be a teacher as well as the fact that I didn't know what to ask my colleagues, caused this section's threads to unravel and tangle with the other threads. In the end I decided to redo this section too. I have given it a new name, 'Coming to Notice',[1] and have also structured it around a moment in my practice. It too is now the knotted messy result of reading about enactivism and other related topics while teaching and being aware of the threads that were being unravelled by this whole process.

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[1] Section 3.3*

SECTION 3.4

OF THE PLANNED THREADING OF MY TAPESTRY:

The Possibilities that Emerged

A section in which I *planned* to consider what my practice *could* look like.

"When we work now, similarities never compete. We scrounge. We descend. We get turned around and when we put something down, we wonder: OK, this something next to that something next to that something. What do we see? What is that image telling us? We try to fill our little beach pails with all the sea and sand we could never quite get in them so many summers ago. Sometimes it's so easy now, and sometimes it's hard. Sand and sea. Scrounging for a way on our way. All you have to do is choose. When we work now."

(Jipson & Paley, 1997)

As will be obvious from reading the previous three sections, at this stage I am not in a position to discuss the possibilities for my *practice* which emerged during the writing of this section. Instead I am able to consider the possibilities which did emerge, namely the possibilities for my *research*.

I have already mentioned that the main reason why my threads started to 'unravel' was because of the enactivist literature I was reading. Although enactivism is a theory of cognition (which I will elaborate on in section 3*), it can also be used as a research methodology since, as Reid (1996: 203) points out, because "research is learning, theories for learning and research methodologies in mathematics education overlap."

I found that I was not only forced to look at my *practice* differently, but that I could no longer look at my *research* in the same way either. Where I had started off wanting to research my own experiences simply for the sake of researching them, I now saw them rather as entry points into the experiences of my students and colleagues.

Although I have always been aware of the emerging nature of my research, even going so far as to explicitly adopt an 'emerging design', enactivism and the notion of co-emergence[1] finally provided me with the vocabulary to describe what I have been experiencing – that my theory and my data influence each other[2] and that everyone involved in my research process learns from it, especially me (Reid, 1996: 203, 206).

By adopting an enactivist approach to research I have also shifted my emphasis. Kieren (1997: 33) explains that 'constructivist, interactionist, enactivist' research practices "point to alternative effective teaching practices" and to the following new emphases:

- on listening to rather than simply listening for;

- on acting with students in doing mathematics rather than simply showing students how to do mathematics;

- on establishing effective discourses of mathematical argument or mathematical conversations rather than simply the discourse of telling, interrogating, and evaluating;

- on the mechanisms of students' mathematical thinking rather than simply on student's answers;

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[1] See section 3.2*
[2] See underview of section 2.4.

- on the teacher and students as fully implicated by their actions each in the learning of the other; and

- on the teacher as co-developer of a lived mathematics curriculum not just a recipient of or a conduit for a pre-decided curriculum

This shift to an emphasis on teaching practice is evident in my research in the way in which I have structured my second version of section 3. Each section is focused on a moment in my practice and on how I make sense of what happens in these moments. I focus on how a different way of thinking about learning has influenced the way I think about my practice and my role as a teacher and how this has influenced what actually happens in my classes.

SECTION 3*
OF THE CONSTRUCTION OF MY TAPESTRY:

THE KNOTTING

A section in which I consider the tangled and knotted threads from section 3 and the new, unplanned images that emerges from the tangled mess of knots. In each subsection I discuss certain aspects of enactivism, how these answer my previous questions and how these influenced my practice.

"There is a cyclic nature to our journeys in life. Like a circle, we travel around and seem to arrive back at the place we began; however, the circle is also a wheel, and as we have circumnavigated its circumference, the wheel has also journeyed forward on its axis, carrying us to that place that is both known and new, familiar and strange."
(Lewis, 2001: 123)

"Inner research also involves a shift from dependency on chains of cause-and-effect reasoning to a perspective in which there is mutual-interaction and co-determination (Varela, Thompson & Rosch, 1991). The whole changes not because of the action of particular parts, but through the presence of a multiplicity of factors that all participate in that change. Cause-and-effect reasoning produces simplification, which is attractive and effective when appropriate; mutuality permits transformation while maintaining an in-dwelling complexity."
(Mason, 1998: 363)

SECTION 3.1*

OF THE KNOTTING OF MY TAPESTRY:

Coming to Know

A section in which I consider a moment in my practice *inspired by* different ways of thinking about learning but *demonstrating* a different way of thinking about structures.

"[The] perturbations of the environment do not determine what happens to the living being; rather it is the structure of the living being that determines what change occurs in it. This interaction is not instructive, for it does not determine what its effects are going to be … [The] changes that result from the interaction between the living being and its environment are brought about by the disturbing agent but determined by the structure of the disturbed system*."*
(Maturana & Varela, 1987: 26; emphasis in original)

"Mathematics education research which does not aim to take into account the complexity of the worlds of teachers and learners of mathematics is of limited value in those worlds."
(Presmeg, 1998: 58)

As mentioned in section 3.1, it was the collections of threads I called 'Different ways of Coming to Know' that first prompted me to start reading again. And it was the memory of the word 'co-emergence' (which I had come across in my previous readings) and the fact that it resonated with my own experiences of coming to know that led me to read about enactivism (a theory of cognition based on complexity theory and ecology (Davis, 2004: 185)) in particular.

In his book, "Inventions of Teaching: A Genealogy", Brent Davis (2004) considers different Western worldviews and their corresponding conceptions of teaching. He uses the following Genealogical Tree (p. 185) to organise the different ways of understanding the nature of the universe, the source of knowledge, the means by which we come to know and the corresponding conceptions of teaching.

CONCEPTIONS OF TEACHING

schooling — facilitating
instructing — inculcating — mediating — emancipating
informing — conditioning — mentoring — liberating
edifying — training — modeling — empowering
directing — remediating — initiating — giving voice
disciplining — lecturing — pedagogy — improvising
indoctrinating — occasioning
inducting — THE MEANS BY — structuring
training — WHICH WE COME TO KNOW — framing
guiding — participating

Empiricism — Structuralism
Rationalism — Poststructuralism
Religion
educating — Complexity Science
nurturing — Mysticism — THE SOURCE — Ecology — conversing
fostering — OF KNOWLEDGE — listening
tutoring — minding
caring

Episteme — Intersubjectivity
Gnosis — Interobjectivity

THE NATURE
OF THE UNIVERSE
The Metaphysical — The Physical

The Metaphysical

The universe is seen to be complete and unchanging—and, hence, understood in terms of other-worldly ideals and essences.

Collective knowledge and personal learning tend to be framed in terms of convergences onto ultimate truths. They are thus understood as acquisitions and accumulations of absolute facts. Thought is seen to be uniquely human.

WESTERN WORLDVIEWS

The Physical

The universe is understood to be emergent—and, hence, described in terms of transformations and diversifications.

Collective knowledge and personal learning tend to be framed in terms of divergences—that is, as opening up of new possibilities. They are understood as processes that occur constantly, on and across many levels of organization.

Figure 3: A Genealogical Tree of Contemporary Conceptions of Teaching (Davis, 2004: 185)

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Although not indicated on Davis's tree, certain branches can be seen as supporting specific theories of cognition. For example, cognitivism is a theory of cognition based on rationalism, behaviourism is based on empiricism and constructivism (both radical and social) are based on structuralism. It is in this sense that enactivism is a theory of cognition based on complexity theory and ecology which share the common root of interobjectivity.

I have found Davis's discussion of different conceptions of teaching very useful, not only for understanding the ideas that enactivism is based on and how it differs from other theories, but also for understanding the conceptions of teaching that I have encountered as a student and a teacher in the "conventional culture of schooling" (Davis & Sumara, 1997: 107).

Davis *et al*. (2000: 58, 61) suggest that the conventional culture of schooling is based on rationalist and empiricist (which are together referred to as 'modernist') conceptualisations of teaching. Rationalist and empiricist notions are based on the belief that there is an objective external world and that it is possible to discover this world (either through logical reasoning or through extensive experience) and thus to make more accurate representations of reality[1] (Davis, 2004: 73). Because of this, the modern or 'commonsense' orientation to cognition (referred to as 'representationism' by cognitive theorists) "defines understanding in terms of the level of correspondence between subjective (inner) representations and an objective (external) world" (Davis & Sumara, 1997: 107).

Another consequence of the modernist perspective is the desire to predict what will happen. This can be seen to be a result of the linear mathematical relationships that were used to describe properties in nature such as trajectories, orbits and collisions. Using such linear relationships (especially Newtonian mechanics) it was possible to predict simple systems (Davis, 2004: 94).

Closely linked to the desire to predict is the desire to control. Davis (1996: 6) points out that in the modernist perspective,

> method came to be seen in increasingly mechanical and technological terms; the universe, correspondingly, was reduced to a similarly technical form. Today, machine metaphors frame and reveal Western perspectives on the universe, the earth, nature, our bodies, and – ultimately, with the development of the computer – our minds. With this technical mind-set,

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[1] See "A conversation" on p. 9.

the aim of inquiry has grown beyond the desire to better our understandings. The primary goal is now to control the objects of our inquiry (emphasis added).

Although public schools and teacher education are still mainly framed in terms of modernist sensibilities (Davis, 2004: 74), alternative worldviews and theories of cognition have developed. Davis (2004: 92) suggests that the major break with the modernist tradition was prompted by Darwin's theory of evolution. If physical forms are changing, then it is not possible to discover and thus represent a complete knowledge of the universe. He goes on to frame this shift in terms of a move from seeing the universe as being *metaphysical* in nature to seeing it as being *physical* in nature (*ibid*.: 95).

Intersubjectivism

This shift gave rise to structuralist and later post-structuralist discourses. Perhaps the biggest difference between empiricist and rationalist notions and structuralist and post-structuralist notions is that the latter see knowledge as something that is created, while the former see it as something that is discovered (Davis, 2004: 117). It is important to note that the word *structure,* as in structuralism, is not used in the sense of architecture[1] but rather in the sense of biology (*ibid*.: 117). As Davis (2004: 118) points out, structure "points to the complex histories of organic forms. Structure in this sense is both caused and accidental, both familiar and unique, both complete and in process."

Just as there was a shift from rationalism and empiricism to structuralism there was a corresponding shift in cognitive theories from mentalism and behaviourism to constructivism and social constructivism. Davis and Sumara (1997: 109) explain that during this shift, the "mechanistic metaphors of representationism" were rejected and "more organically based interpretations" were taken up. They go on to say that

This shift from the language of physics to the language of biology is an important one, as the images of forces, trajectories, and direct cases are replaced with thinking about thinking in terms of constant change and complex interdependencies. Cognition is thus understood as a process of organizing and reorganizing one's own subjective world of experience, involving the simultaneous revision, reorganization, and reinterpretation of past, present, and projected actions and conceptions. (Davis & Sumara, 1997: 109)

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[1] See example of this metaphor on p. 72.

Structuralism and post-structuralism are both rooted in intersubjective ideas – namely the "matters of immediate human interest and interactivity" (Davis, 2004: 100). Simply put, for the intersubjectivist, although it *is* possible to differentiate between the world and descriptions of the world, only the latter can be studied. The reason for this is that our perceptions are flawed and so "the world we see is not the world as it is, but the world as we have learnt to see it" (*ibid*.). Instead of asking how well our representations fit the 'real' world, intersubjectivists ask how well the representations of different people fit each other (*ibid*.).

Interobjectivism

At the same time that a shift in the humanities (from the subjective to the intersubjective) occurred, a shift in the sciences occurred from a "desire for objectivity" to the "phenomenon of interobjectivity" (Davis, 2004: 101). The notion of interobjectivity is difficult to understand, especially because it "relies on a rejection of some of the most deeply engrained assumptions about the nature of the universe – like, for example, the separation of descriptions of phenomena from actual phenomena" (Davis, 2004: 102). These ideas developed from experiments in science in which it became clear that the observer of the experiment influences the experiment. For example, in the double split experiment light acts either as a particle or as a wave - depending on where the observer chooses to observe (Zohar & Marshall, 1994: 20). Put differently - every measurement is taken by a measurer and every observation is made by an observer (Maturana, 1988). We are in the world and as we change the world changes (Davis, 2004: 101).

One example of a field based on interobjectivist notions is ecology. Unlike environmentalism, ecology (which is often used as a term having the same meaning) does not make a distinction between *humans* and *the environment*. Rather everything is seen as being part of everything else. Ecology is therefore concerned with "interrelationships and interconnections" (Davis, 2004: 103).

Another example of a field based on interobjectivist notions is complexity theory. Complexity theory developed because it became clear that phenomena exist which can't be described using Newtonian or statistical ideas. Such phenomena cannot be predicted or controlled and are described as "adaptive, self-organising systems" (Davis, 2004: 104). Complexity theory is the study of such systems.

Enactivism

As mentioned above, enactivism is a theory of cognition based on both complexity theory and ecology (Davis, 2004: 185) and therefore on interobjectivist notions. Although enactivist notions can be found in the work of Bateson (1979) and Merleau-Ponty (1962), it was Maturana and Varela (1987) and Varela, Thompson and Rosch (1991) who first brought the ideas together into a theory of cognition. Davis (in Davis, 1996; Davis & Simmt, 2006 and in Davis *et al*., 2000) extended these ideas to mathematics education in particular.

Because enactivism developed from ideas in ecology and complexity theory, both of which are based on interobjectivist notions, enactivism can be described as a theory of cognition incorporating interobjectivist ideas. This is clear in Breen's (2002: 28) description of enactivism:

> [Enactivism] attempts to find a middle way between the mental and the physical (inner and outer) by suggesting that the body is that which renders the mind and the world inseparable… It involves both becoming part of an ongoing existing world and the shaping of a new one. …[I]t asserts that our knowledge and our identities are dependent on being in the world, which is inseparable from our bodies, our language and our social history. The basis of cognition is not to be found in the Rationalist 'I think' or in the Empiricist 'I observe' but in the Enactivist 'I act', and acting demands re-unions of mind and body, and subject and object.

It is also evident in the following quote from Varela, Thompson and Rosch (1991: 9):

> [Enactivism is based on] the growing conviction that cognition is not the representation of a pregiven world by a pregiven mind but is rather the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs.

Another 'intersubjectivist' idea in enactivism is the rejection of a "core, essential, inner self" which is regarded as a product. Instead, "self" is seen as "product, producer, and process" (Davis, 1996: 9) and as "a dynamic and fluid structure that is constantly changing and that is always re-configuring itself" (Davis, Sumara & Kieren., 1996: 154).

Enactivism and Complexity Theory

Enactivism makes use of numerous ideas found in complexity theory. As mentioned above, complexity theory is the study of complex systems, that is systems which are "adaptive" and "self-organising"

(Davis, 2004: 104). Such complex systems differ from complicated systems in that the latter can be described in terms of their constituent parts. For example, to explain (predict) how a clock works, it is enough to understand how each of the components of the clock work. On the other hand, complex systems such as societies or the climate cannot be understood through the 'examination of their components, largely because their components are similarly alive or dynamic' (Davis *et al*., 2000: 174). Therefore, unlike a complicated system, a complex system is not the sum of its parts but the "product of its parts and their interactions" (Davis, 2004: 151).

The fact that the components of a complex system are 'similarly alive or dynamic' necessitates another important concept in complexity theory and enactivism, namely that the entities making up a complex system are, simultaneously, made up of wholes, part of a whole and a whole (Davis & Sumara, 1997: 118). Or, put differently, "each [component] forms a whole with respect to its parts while simultaneously being part of a larger whole" (Capra, 1997:28). Davis and Sumara (1997: 118) use the human 'self' as an example.

> [Our selves] are comprised of subsystems that are intertwined and have their own integrities (e.g. organs and cells) while they are themselves subsystems of larger systems, with their own particular integrities (e.g., a family, a community, a society). Each of us is, all-at-once, a collective of wholes, a whole, and a part of a whole.

Not only is complexity used to re-cognise students, teachers, classrooms and society as complex systems consisting of complex systems, but it is also used to redefine cognition as follows:

> Cognition is understood as ongoing processes of adaptive activity. As with complex systems, the cognizing agent can be seen as an autonomous form and/or as an agent that is coupled to other agents and, hence, part of a grander form. An implication is that cognition is not seen to occur strictly inside an agent. Rather, cognition is used to refer to all active processes – internal and external to the cognizing agent – that are part of its ongoing adaptive actions. (Davis, 2004: 153)

Just as it is difficult to become accustomed to interobjectivist ideas, it is difficult to become accustomed to the way cognition is defined in enactivism. Davis and Sumara (1997: 109) write that:

> the difficulty here amounts to a deeply embedded cultural inability to rid ourselves of the notion that knowledge, in itself, has some sort of corporeal existence. Knowledge tends to be discussed as if it were an object – some *third thing* – to be grasped, held, stored, manipulated, and wielded, rather than being associated with out acting and existing in a biologically and phenomenologically

constituted world. Depending on the theoretical stance, such *objective* knowledge is assigned a particular location.

Enactivism not only redefines cognition but reframes teaching as a *complex* process rather than a *complicated* process – unlike most teacher manuals which, by simply providing lists of skills that need to be mastered to ensure that one is a 'good teacher', imply that teaching is merely a complicated process (see Davis *et al*., 2000: 62).

Enactivism and Ecology

Enactivism takes the ecological idea that we are not *in* the world but *part of* the world and applies it to learning and teaching. If we are part of the world then when something as little as a thought changes, the world changes (Davis *et al*, 2000: 64). Therefore, by educating we don't only change our students but the situations surrounding our students – just as a change in the situations surrounding our students changes the students themselves. Also, rather than seeing teaching as preparing students *for* the 'real world', enactivism sees teaching and learning as processes that take place *in* the real world. (Davis & Sumara, 1997: 108; Davis *et al*., 1996: 157-158).

An alternative understanding of the idea that it is not meaningful (or possible) to separate us (humans) from the world is the idea that everything is linked to everything else. Therefore, ecologically, it is not possible to explain the things that happen in the world (or in teaching and learning) in terms of cause and effect, but in terms of emergence and co-emergence. As Davis and Sumara (1997: 110) put it:

> learning (and similarly, teaching) cannot be understood in monological terms: there is no direct causal, linear, fixable relationship among the various components of any community of practice. Rather, all the contributing factors in any teaching/learning situation are intricately, ecologically and complexly related. Both the cognizing agent and everything with which it is associated are in constant flux, each adapting to the other in the same way that the environment evolves simultaneously with the species that inhabit it. In simple terms, ecological thinking understands that the boundaries we perceive between different objects and different events are mere heuristic conveniences. Everything is inextricably intertwined with everything else."

A moment in my practice

I started reading about enactivism and the ideas that I have discussed in this section towards the end of the second term in 2007. These ideas, particularly that teaching and learning can be framed in terms of the *structural determinism* of an *autopoetic system* and that a class can be seen as a *complex system*, together with the fact that I wanted to finish off the term with some kind of reflective exercise, made me start to consider different 'class exercises'.

An *autopoetic system* is "an emergent phenomenon arising from the interaction of components which, by way of these interactions, give rise to new interactions and new components, while preserving the system's autopoetic character" (Reid, 1996: 204). This notion is best described by referring to an example of an autopoetic system – namely a cell. A cell is an autopoetic system, because the components that it consists of such as nucleic acids and proteins, form structures such as the cell nucleus, various organelles, a cell membrane and cytoskeleton which, in turn, produce the components from which they are constructed.

Reid (1996: 204) goes on to define *structural determinism* as the fact that what "an autopoetic entity does is determined by its own structure, not by an external stimulus", even though this external stimulus might "trigger some action the structure was determined to do". Varela (1992: 50) illustrates this notion with an example explaining how a chime made of pieces of glass will tinkle when a gust of wind blows, but that the sound it makes is not determined or instructed by the wind but by the structural configuration of the chime. He points out that "in order to understand the sound patterns we hear, we turn to the nature of the chimes and not to the wind that hits them."

Although in Varela's example the structure is a simple one, the idea that its action depends on its structure and not on the perturbation can be extended to more complex structures, including humans. But, as Breen (2001) points out, when considering more complex structures, it is important to remember that, since each structure is "uniquely determined and evolving", each structure is triggered in a unique way by an external stimulus or "perturbation to the environment".

In enactivism humans are taken to be autopoetic systems or, as Davis *et al*. (1996: 154) put it, "we are highly complex structures arising from the continuing interplay of biological constitution and socially

and historically framed experience". Therefore we "do not 'pick up information' from the environment; rather, [our] structure specifies which environmental patterns will trigger action" (Davis, 1996: 10).

Because of this, Davis and Sumara (1997: 115) suggest that, rather than thinking of teaching in terms of *prescribing* what is to be learnt, teaching should be framed as "occasioning". In this sense "the teacher participates in, but does not determine, student learning" (*ibid*.: 115).

Breen (2005) captures the difference that this makes to teaching:

> The concept that, as teacher, I can become 'a disturbing agent' in the classroom but that the changes (which I take to be the learning) will be determined by the structure of the learner is extremely liberating. In the end, I have a limited influence on what actually gets taken up and my control over the outcomes is limited. This leaves me free to pursue my interest in becoming a better perturbator with relish!

Davis, Sumara and Luce-Kapler explore these ideas in their book, *Engaging Minds* (2000). Besides discussing enactivist ideas, they give examples of lessons based on these ideas. One such series of lessons involved students using shoes as prompts for writing descriptions which were later combined to write short stories about the places the shoes had travelled to. The idea of using prompts and then using the resulting descriptions as further prompts appealed to me and I decided to try a similar approach in my 'reflective session'.

But I found myself incredibly busy on the days leading up to the reflective lesson I had planned and only made time to sit and think on the day of the lesson…

I plan the following prompts:

Think back to this semester and write down
- 5 words describing your feelings
- 5 topics we covered
- as many things you realised that you didn't realise before

Listen to quote 1 (and 2)

Have you had a similar experience in maths?

Then I remember I haven't organised poster paper and have left all my coloured paper and pens and string at home and that I haven't arranged the tables in groups. So I stop my planning and rush to organise these materials and end up arriving in class a few minutes late. Fortunately the students are late too, so they aren't all waiting for me yet.

Because of this rush I end up doing a very spur-of-the-moment session. Students help arrange the tables, while I hand out poster paper and start cutting coloured paper into little blocks (far too many it turns out). Each group receives a pile of green papers and I ask them to write down 5 topics that we have covered this year so far. A suggestion from one of the students makes me update the prompt to: write down any 5 topics for any reason, but write down your reason as well.

While they are writing, I cut pink blocks. When I feel enough people are finished, I give them the next instruction: write down 5 words describing how maths had made you feel during the past 5 months.

All this takes longer than I anticipated. Next I read the following two quotes by students describing their experience of mathematics and ask them to write down any similar experiences that this quote makes them think of. I had wanted to do three quotes but ended up only doing two.

Quote 1
"I attempted to visualise it in my mind at first but I struggled with that so I decided that I would build just the one face of the cube and then see if I can work anything out from that. I took the blocks and put them together and just before I finished putting up one face, Michael already had the answer. He gave the answer to us… He went on with the other blocks – the blocks that would have only two sides painted, one side painted and no sides painted. He just gave us like the answers to all that.

I tried to ignore him because I was still looking at my face trying to figure out things but it was difficult for me to ignore the answers that he gave to us. So I don't know if that put me under pressure or what. I don't know if I should call it pressure, but basically I just couldn't figure out things any more. I couldn't think. I had the face in my hand. I tried to move my fingers across it, trying to visualise things and make a few calculations. I couldn't, because he had already given us the answer and what he was doing now as I was aware or trying to figure what was going on, he was sort of moving on to a 4 x 4 x4 cube and that put me under even more pressure.

So I found myself asking him 'How did you figure that out? Can you just explain that to me how you got the answers?' And then he went on like this, OK listen Prince, on the corners you have 4 and between them we have one blah blah blah and that makes... It didn't make any sense to me."
(Nofemela 2003 cited in Breen, 2005)

"Okay guys, now you get to make connections between your experiences on the white pages, your feelings on the pink pages and the topics on the green pages. And you can arrange them in any way on the poster paper."

I walk around from group to group observing what they are doing. Some make mind maps. Others do linking collages. Some start with the topics, arranging them. Others start by reading each other's experiences. Some put lots of effort into the presentation. Others don't.

Varela (1999: 53) points out that a complex system "behave[s] as a unit and as if there were a coordinating agent present at its center…. [A coherent global pattern] emerges from the activity of simple local components, which seems to be centrally located, but is nowhere to be found, and yet is essential as a level of interaction for the behaviour of the whole". As the students work on their posters, it appears as though there is a co-ordinating agent present as each group independently (the students didn't talk to each other during the initial part of the activity) comes up with a slogan for their representation.

The slogans are:
- The confessions of a mathematics student
- Mathematics – it's all relative
- Mathematics is like weed; when you understand you feel high, when you don't you wish you were
- It's difficult if you don't have any actual plan

As the groups start finishing, I ask them to stick up their posters in the front of the class and they start looking at each other's poster. Here follows a time of happy confusion as people read and laugh and comment on each other's experiences. We experience the posters together.

After a while I feel we need to have some way of finishing the session – something I hadn't anticipated at all. So I ask them to write a last piece where they are to comment on the experience we have just shared. To say what they think about it.

And here, right at the end, during that part of the activity that I hadn't planned at all, something totally unexpected emerges – the overwhelming appreciation of the fact that we are all one, that we are all together in the struggle, that we are not alone.

Because of this session I gained a new appreciation for the enactivist idea of seeing a class as a body, made up of other 'smaller' bodies (the students and the teacher) and part of a larger body (the school or university). Our class feels like more of a unit. The students have become more aware of each other and the fact that they are responsible for each other as well.

Underview

Although the moment in my practice that I described in this section was prompted by a new way of thinking about 'coming to know', namely co-emergence, when I started writing it was the ideas which need to be in place before one can discuss co-emergence (namely, autopoesis and structural determinism) which I noticed in my practice and consequently wrote about.
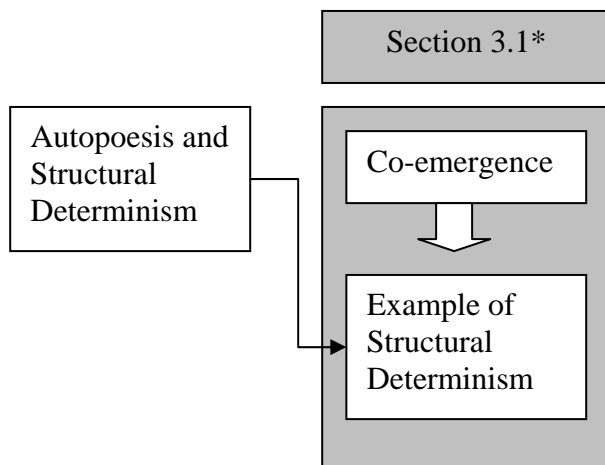
It is interesting to know that we are one. We are together in the struggle. When you don't understand something, you feel like stupid when looking around and everyone seem to understand.
Doing this activity (mind map) was helpful, it made me to have more confidence and now that I know that it is normal to feel lost sometimes I have found faith in me again.

The lesson was great. You realise how human and alike everyone is. Also light bulb moments are bits of understanding come together. The more you study and work through a chapter you understand the better it is

I have realised that sometimes we only look at things when blocks along that way, and we forget the moments we had great fun doing mathematics.

Discovering that other also share a common feeling with me made me feel so much better. When I do practice maths I'm going to know that my fellow group a classmates feels my frustrations too. It will give me a sense of hope and courage. What was fun was to find out that my fellow classmates also struggle. Makes me feel: "we are all in the struggle 2getha" Een mag dra krag! One thing still miss mostert is very funny! You laughed with us, not at us like other people do.

I enjoyed it. It made me release my stress about things I felt stupid with just because I don't understand them. Now I know it's not me only

Maths is a universal struggle! That's what I learnt from compiling my own 'colauge'. I realise whatever problems I have chances are everybody has exactly the same problems and maths is fun with all it's challenges, obstacles and theories I can do it with lots of help and practise. When I enjoy maths, I really enjoy life coz it applies to everyday life.

Very nice to explore how other people feel about maths and give inspiration that I can steel do it, I'm not the only one struggling.

I've bonded with some of my classmates after having realised that we are in the same struggle.

I enjoyed this activity because I think that it was worthwhile, and its such a great relief to know that there are other people who had the same experiences and feelings as I had, with regards to some of the topics. I hope that you will do something about our feelings and experiences in many of the topics in maths.

It's nice to know that I'm not the only one who finds certain maths topics difficult – what do you know, I'm not alone after all.

This activity was interesting because I realised that some people feel like I feel. I am not alone in the struggle. I also realised that there are people who see the stuff I think its difficult easy for them so if its manageable for them why not me, I have to look at it again.

It was so great, miss mostert, a big ups to you! This was a great experience yeah only now I can get words, sometimes one feels so alone when you trying to achieve your best in maths but ultimately we are all in this together, together to make a success of our lives. That's the card I deal...and doing my best is the card I'll play with...thanks a lot you've really outdone yourself...!

I realised that I wasn't the only struggling with certain things in fact all most the whole class struggles with some things.

I saw that different people had different outlooks on th maths language and that many accept it by means of
-doing
-understanding
-interpretation
-applied knowledge
-and working with associations

I realised that I'm not alone, who has the same problem in maths

i.e. many people think differently and have different answers. Even to life itself.

When reading other students reports was quite interesting to see that we share one class and few of us experience the same things without knowing about it, and thought we were the only ones that feel like that. Today was interesting to see the links of feelings to the different sections of maths. And I found it exciting and interesting and fun!!

SECTION 3.2*

OF THE KNOTTING OF MY TAPESTRY:

Coming to Listening

A section in which I consider a moment in my practice *inspired by* different ways of listening but *demonstrating* a different way of thinking about learning and teaching.

"Learning something new depends upon knowing something; at the same time new knowledge helps to re-shape old knowledge. It is a circle of experience of understanding which has indiscernible beginning- or end-points."
(Davis *et al*., 1996: 157)

Overview

The following section consists of three different types of writing. The first (in the blocks) is the notes I made immediately after a particular moment in my practice. The second (in italics in blocks) is the reflections on the notes I made and were written at a later stage. The third (outside the blocks) was also written at a later stage and is my attempt to clarify my understanding of enactivism. It is possible to read this section linearly or to read 'in the blocks' first and then 'out of the blocks'.

A moment in my practice

> **31 May 2007**
>
> Today I saw my class at 10:00 instead of at 8:00. The way the last two lessons had gone was really bothering me and so I set aside the two hours from 8:00 – 10:00 to prepare for the lesson (besides the thinking I did in bed last night before I fell asleep). We are scheduled to cover exponential equations this week. I've been reading Brent Davis's and Chris Breen's work about co-emergence and the idea that learning and teaching could be conceptualised as a conversation rather than a monologue.

In my discussion of enactivism in section 3.1* I considered the notions of *autopoesis* and *structural determinism*. The other key idea in enactivism developed in response to the following question:

> How is it that individual humans, especially if our actions are determined by our own particular structures, are capable of *joint action* that is not merely coordinated, but rhythmic and attuned – action that leads to the putting aside of selves, action out of which ideas and behaviours, which certainly could not have been independently achieved, emerge? (Davis *et al.,* 1996: 164, emphasis added)

This 'joint action' has been described as a "fusing of horizons" by Gadamer (1990), as "coupling" by Merleau-Ponty (1962), as "structural coupling" by Maturana and Varela (1987), and as "co-emergence" or "mutual specification" by Varela, Thompson and Rosch (1991).

Enactivism draws from complexity theory to answer this question. Within complexity theory the question can be phrased in terms of how it is that individual components are able to arrange themselves and interact in such a way as to form a complex system. Due to the nature of complex systems, rather

than explaining (predicting) how this process happens, complexity theorists attempt to *describe* what happens when this joint action occurs.

Reid (1996: 204) explains that this joint action occurs when "autopoetic entities organise themselves into networks of interaction". When they are in this state they are said to be *structurally coupled* or *co-emerging*. Although each entity acts according to its own structures, these structures are such that the actions of the entities become co-ordinated (*ibid.*: 205).

Davis *et al.* (1996: 164) also point out that "As these sub-systems come together, structures arise and complex behaviours emerge that transcend each of the component parts. The emergent character of these higher order structures cannot be understood by examining the sub-systems; we must attend to the actual structure that arises, treating it as a unity in and of itself."

> The thing that caught my attention was the suggestion that a conversation is not determined beforehand - that the people taking part in the conversation are not in control, that they are in fact led by the conversation, and the implications that this has for teaching seen in terms of a conversation. A lesson seen as a conversation can't have predetermined outcomes. A lesson seen as a conversation must allow for the contributions that the students will make and the influence that these contributions have on what is discussed next.[1]

In his book *Teaching Mathematics: Towards a Sound Alternative*, Davis (1996) extends Gadamer's and Merleau-Ponty's ideas of what happens during a conversation in order to explore the implications of the notion of co-emergence for teaching and learning.

Gadamer and Merleau-Ponty both discuss the nature of 'the conversation' and the way in which those involved in the conversation are joined. Davis (1996: 40) points out that Merleau-Ponty spoke of a "new transcendent unity" which "arises when two or more persons come together in conversation or in any shared action". Merleau-Ponty (1962: 354) also describes how,

> [As I listen to another, my body] discovers in that other body a miraculous prolongation of my intentions, a familiar way of dealing with the world. Henceforth, as the parts of my body together

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[1] {students directing class} – 2.1.5

As mentioned in my earlier discussion of co-emergence, when separate entities are structurally coupled, the actions of the resultant complex structure are more than what would have been possible by the entities on their own. Similarly, in a conversation, Davis (1996: 41) points out that "we set aside our illusions of autonomy, allowing a collective consciousness to emerge; in this conversational unity, we become capable of greater insight and deeper understanding, capable even of cutting beneath the conscious intent of the speaker."

It was with these ideas that I sat preparing for class this morning – going through the chapter that I had given exercises from, trying to decide what was important, what I wanted to cover, whether I wanted to do specific examples on the board. I was scribbling ideas down as I went along. The most important thing I noticed/remembered/realised was that we were once again busy with solving **equations** and that once again we could use the work we had done on **functions** (exponential functions in this case) in the previous term.

Even so, my reflecting and reading and scribbling weren't achieving the results I wanted. The work wasn't falling into 'nice sections'. I couldn't decide in what order I wanted to look at the different ideas. It just didn't feel right. So I just kept going through it and tried to be aware of the things that I was noticing. In the end, because of the things I have been reading lately, I felt okay to go down to class and to 'do' this lesson **together** with my students. I felt okay not to have a pre-planned order of doing things but to wait and see what would happen. This didn't mean I didn't prepare – I knew what work I wanted to cover and that I wanted them to see how exponential equations fit into the bigger picture and to make connections to similar work we had done with other functions. I just didn't know how it was going to happen.

In a recent edition of "For the Learning of Mathematics", a conversation about 'being alongside' was published. Laurinda Brown and Alf Coles (2007) discuss the idea of coming alongside a student by striving to know what the student knows and, through this striving, temporarily 'forgetting' one's own mathematical knowledge. Dick Tahta (Tahta & Williams, 2007: 11) goes on to consider the problem of

preparing for an 'open situation' – one where the students are free to explore a particular problem. If one wants to come alongside students – to strive to know what they know – is it better to 'have explored the situation thoroughly oneself or rather to remain as ignorant as possible.'

In my own classes I mostly opt to remain as ignorant as possible when it comes to situations where I want the students to explore a problem on their own. In this case though, although I wasn't planning on giving the students a problem to work on, the idea of "deliberate over-preparation in order to be able to 'let go'" resonated with me. By preparing all the different concepts I wanted to cover in the lesson, I was able to 'let go' and see what happened.

So I went down to today's class without really knowing what would happen. Of course I never know what will happen, but today I had made a conscious decision to "be open".

I am reminded of Oberg's (2004: 240) reflection on reflecting and her acknowledgement of the importance of "opening, paying attention, and not knowing" in her theories about teaching. I too strive to be more open, to pay more attention and to be alongside my students through 'not knowing'.

I was amazed by what emerged.

I started off by asking them to all sit closer, because there were some things I wanted us to talk about.

In other words things I wanted us to have a conversation *about, as opposed to me just* telling *them, things which they had to write down and 'know'.*

Gadamer, on the other hand, points out that the outcome of a real conversation can never be predicted and that it is more appropriate to think of the people having the conversation as being led by the conversation than visa versa. He writes:

> We say that we 'conduct' a conversation, but the more genuine a conversation is, the less its conduct lies within the will of either partner. Thus a genuine conversation is never the one that we wanted to conduct. ... No one knows in advance what will 'come out' of a conversation. (Gadamer, 1990: 383)

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In this sense Gadamer argues that it is possible to conceive of the conversation as a "triad involving you, me and the topic or subject matter which exists neither in you nor in me but between us" (Davis, 1996: 40). This in turn ties in with the intersubjectivist (and therefore enactivist) notion of knowledge not being a 'third thing', but rather something which exists in the interactions between people and between people and the world. Or as Davis and Sumara (1997: 105) put it, "in the possibility for shared action".

This made such a huge difference – having all 15 of us sitting round two tables as opposed to being spread across the whole classroom. I told them that today I wanted us to stop and take a step back and to see how what we were doing fitted into the bigger picture. We spoke about the fact that we had focused on graphs in the previous term and that this term we were focusing on solving equations and more importantly on using what we had learnt about graphs to solve equations. We revised exponential functions, drew graphs and looked at the increasing and decreasing situations. We spoke about the domain and the range of the functions. The fact that the function values of an exponential function of the form $y = a^x$ are all strictly greater than 0 led to me asking about the equations of the form $3^x = -1$.

I didn't plan this link, although I knew I wanted to show how one could use graphs to solve equations.
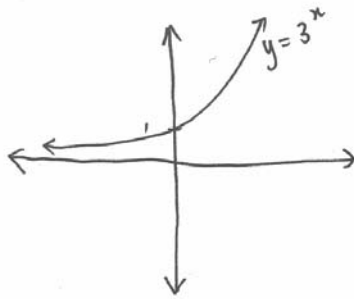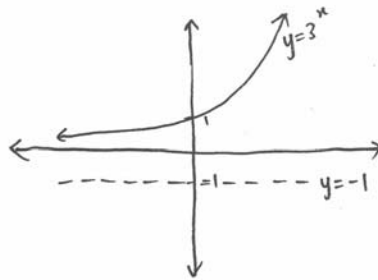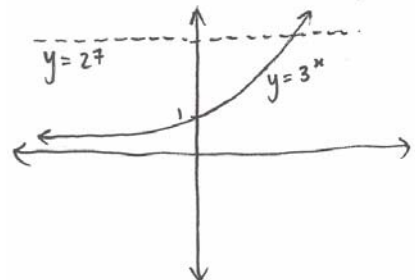
| Figure 4.1 | Figure 4.2 | Figure 4.3 |

I had drawn the graph of $y = 3^x$ (see Figure 4.1) and it appeared as though most students didn't have a problem seeing that $3^x = -1$ doesn't have a real solution (Figure 4.2). After this I wrote down a simple equation $3^x = 27$ and asked how many solutions this equation would have. *"Remember we can see this as representing an attempt to solve two equations simultaneously where the one is $y = 3^x$ and the other one is $y = 27$ ".* Once again most students had no problem seeing that there would only be one solution (Figure 4.3).

Next we spoke about the situations we had encountered when working with parabolas. We had considered the solutions of the equation as being the x-co-ordinates of the points of intersection of a parabola and straight line. There had been three different cases – two (unequal) solutions (Figure 5.1), two equal solutions (Figure 5.2) and no real solutions (Figure 5.3).
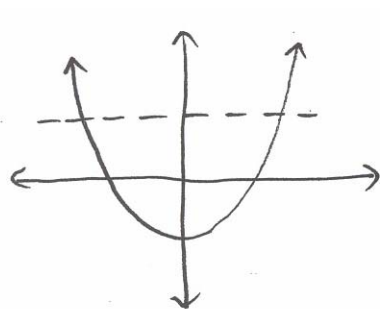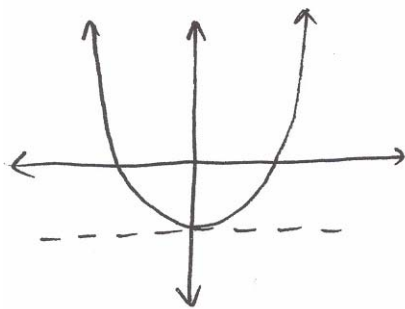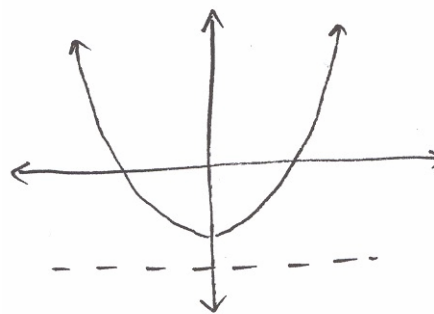

| Figure 5.1 | Figure 5.2 | Figure 5.3 |

For exponential functions we had seen two cases – namely no real solutions (Figure 4.2) and one real solution (Figure 4.3).

"So guys, can you think of any other cases that might happen when working with an exponential function and a straight line?"

As I asked this question I realised that the cases which we looked with the parabola only included horizontal lines so I revised my question to: "*...an exponential function and a **horizontal** line*", but the students had started talking to each other and some didn't hear what I said. After a time of discussion I asked for suggestions. Most said we would only have the two cases we had discussed so far.

Davis (1994) takes the idea of a conversation and considers what a class that can be described as 'conversational' might look like as opposed to a 'monological' class. He does this by describing two different classes and goes on to suggest that "it is the nature of the communication that occurs among the members of the two communities" (the students and the teacher) that primarily distinguishes the two classes from each other. In Wendy's (monological) class "there seems to be almost no *inter* action. The actions of the students and the teacher appear to be coordinated, but independent; they lack a common rhythm. There is a clear sense that Wendy has taught this lesson before and will teach it again.

The particular students in the class have little effect on the lesson's course. Their backgrounds, their varied interpretations are unimportant. The 'correct understanding,' one that is independent of them, is presented for mastery. The mathematics in Wendy's classroom is thus an object that stands between the teacher and the learners… It is an object that does not facilitate their relationships, but holds them apart." (Davis, 1994)

Samkelo: **"What if it's a skew line?"**

Ingrid: "Well then there are more cases. But that's why I said it had to be a horizontal line."

But my response to Samkelo's question didn't satisfy me and I tried something I hadn't done before.

In a certain sense, Samkelo's question can be seen as a perturbation in my environment which presented "an occasion for [me] to act according to [my] structures" (Davis, 1996: 10).

I drew two graphs (an exponential graph and a straight line with a positive gradient) which intersected each other twice and asked the students to work in pairs and to come up with an equation that could possibly describe the situation (see Figure 6). And so a new type of question emerged.
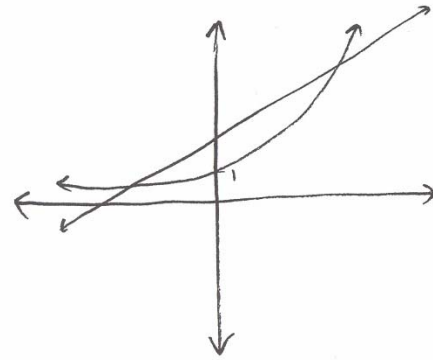
Figure 6

It was so cool!!!! They were so good. Eliza, who often struggles to make sense of the work, very quickly suggested $2^x = 2x + 2$. I walked around to see what the different pairs did. When I felt that everyone could see what I wanted them to see I prepared to move on.
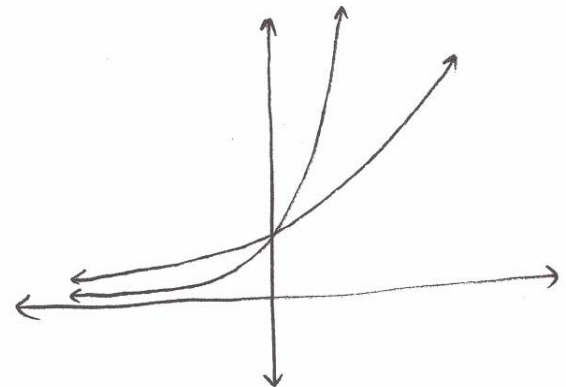
But instead someone (it could have been me, I can't remember) asked what would happen if you had 2 exponential graphs (as opposed to an exponential graph and a straight line graph). Some students said they would only intersect once, others said it they could

Figure 7.1

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intersect twice. I knew intuitively that they would only intersect once, but I didn't know how to go about proving it. I drew two graphs on the overhead where it was clear that they would only intercept at one point (which would be the y-intercept if the graphs were in the form $y = a^x$) (Figure 7.1).
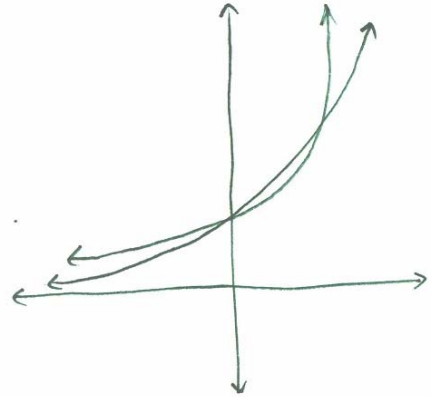
But I thought I had an idea of what the people who had answered that they could cut twice were imagining. So I drew this version on the overhead (Figure 7.2) and asked the class why this wasn't possible.

Figure 7.2

We didn't really have an answer. I knew that they have been doing logical arguments in their thinking skills course, so I decided to see if I could bring in the ideas of a logical argument to somehow "prove" that there could only ever be one point of intersection.

Figure 8

I started off with a general case $a^x = b^x$ and asked what x must be for this to be true. Suggestions were shouted out but I told the students to specify the laws they were using. Someone suggested that we subtract b^x from both sides so I wrote that down on one side (Figure 8) but asked if this would help us with any of our laws. No. Someone else suggested we divide both sides by b^x so I wrote this down as another option (Figure 8). As I walked around and looked at what the different pairs were doing, I saw students writing down $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$. After a while I wrote down $\left(\frac{a}{b}\right)^x = 1$ and asked what we could say about x in this case.

Someone suggested that x had to be zero. So we concluded that in this case there would only be one point of intersection (occurring when $x = 0$) and therefore the equation would only have one solution.

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Towards the end of this discussion one of the students from the other class, Musa, came in. Their teacher was away for a few days and she left them work to do and instructions to come and ask me if they got stuck.

And then a "hap" happened.

In his discussion of enactivist teaching, Davis (1996: 257) introduces the idea of a "hap" – a word used by Weinsheimer to refer to 'watershed moments'. Davis (1996: 257) argues that "the hap is the center-point of enactivist teaching. Such teaching is an attendance to the unexpected consequence, to the sudden insight, to the inexplicable interest that is conditioned or occasioned by the teacher's actions. The hap may be anticipated (and, because of this possibility, the teacher has a responsibility to consider what might happen in a given setting), but will more likely be a matter of happenstance."

By foregrounding the notion of the 'hap' Davis and Sumara (1997: 122) remind us that "such notions as controlling learners and achieving pre-set outcomes must be set aside in favour of more holistic, all-at-once co-emergent curricula that are as much defined by circumstance, serendipity, and happenstance as they are by predetermined learning objectives."

Her question was exactly the type of question that the previous discussion was about:

Solve for x if $3^{x+1} = 5^{x+1}$.

So I decided to go in for some 'class identity' building and announced to my class that Musa had come with a 'request' from the other class, namely a problem that they couldn't solve. I told them that we would be able to solve it and that I wanted us to do this together and then to explain to Musa what we had done so that she could take the message back to the other class.

In retrospect I would suggest that this idea of class identity also emerged from my reading of enactivism. The idea that a class can be seen as a body with its own way of functioning and knowing appealed to me and I wanted to develop this idea of us being one body, and therefore being separate from the other bodies (classes).

So I wrote the question on the overhead and pointed out how similar it was to the one we had just done and asked how we could solve it.

The first suggestion was to follow the same procedure as with the general case – to divide both sides by 5^{x+1}. Instead of following the suggestion, I interrupted:

Ingrid:	Wait guys, what was the only solution that we could get over here? *(Referring to the general case)* That $x = 0$. So what has to happen over here?
Geraldine:	x must equal zero.
Desiree:	No, $x + 1$ must be zero, not x. So $x = -1$.
Geraldine:	Oh yes, that's what I meant.

I felt confident that most of the students in my class followed what had happened, but I suspected that Musa might not have. I checked with her and when she said she didn't understand, I asked the class to explain to her. Samkelo volunteered. What I found interesting about his explanation was that he applied the method we had applied in the general case.

Why didn't he use the idea of two exponential graphs only cutting once at the y-intercept? Was it too time-intensive to explain? Or did the reason why we did the proof get lost during the actual proving? Was he attending to something different to what I was attending to? I suppose I could have asked him.

Musa said she understood and I sent greetings to her class on behalf of our class.

In contrast to this the mathematics in Tom's class offers "a space to foster relationships and to model listening" (Davis, 1994). Whereas Wendy seems to be sceptical that her students will be able to make a contribution to the lesson, Tom tries to structure his lesson around the contributions of his students. By doing this he demonstrates his belief that "the learner is justified in what he is doing or saying, that there is *reason* (whether mathematical or otherwise) in his actions" (Davis, 1994). Coles (in Brown & Coles: 2007: 10) points out that by doing this – i.e. by allowing students to drive some of the "key moments" in a lesson – students are given a sense of power.[1]
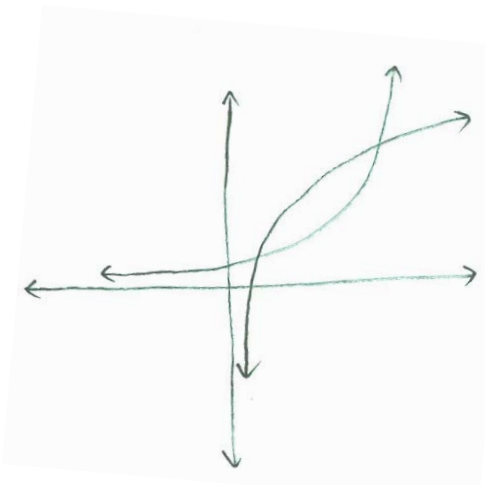
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[1] {students directing class} – 2.1.5

Figure 9

At this stage I wanted to carry on (I didn't know yet to what) **but then Shannon came and showed me what he had drawn** – two exponential type graphs intersecting each other. He mentioned the inverse of the exponential graph to me when he was trying to explain what he had done and because I knew we were doing log equations next I decided to use this as an opportunity to revise log graphs.

Once again I hadn't planned *to look at this but the possibility had been 'available', it had been 'imagined'.*

So I drew Shannon's graphs on the overhead (Figure 9) and, once again, I asked the students to set up an equation to describe what was happening. And, once again, things that I hadn't thought of at all "emerged".

At one of the tables the students saw an exponential graph and the inverse of an exponential graph. They "swapped the *x* and the *y*" to get the new equation, but got stuck when they wanted to set the two equations ($y = 3^x$ and $x = 3^y$) equal to each other. After a while someone suggested that we could use logs. But no one could remember how. After again emphasising that logs were introduced precisely so that we could write $x = 3^y$ in the form $y = \ldots$, I wrote down the general relationship between an exponential function and a logarithmic function (if $x = a^y$ then $y = \log_a x$) and left them to finish on their own.

At the other table something entirely different was happening. A pair of students had decided that the other graph was in fact one arm of a hyperbola. Wow. So we went through the steps (unfortunately I did most of them) to get an equation for a graph that would look like the one in the drawing.

Start with: $$y = \frac{4}{x}$$

Reflect it around the *x*-axis: then $y = \dfrac{4}{x}$ becomes $y = \dfrac{4}{-x}$.

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Hey, we can also reflect it around the *y*-axis:

$$\text{then } y = \frac{4}{x} \text{ becomes } -y = \frac{4}{x} \text{ which is the same as } y = \frac{4}{-x}.$$

Shift it up maybe… 5 units: then $y = \frac{4}{-x}$ becomes $y = \frac{4}{-x} + 5$.

Take the other equation to be $y = 3^x$.

Solve the two equations simultaneously to get $3^x = \frac{4}{-x} + 5$.

So in the end we had two possible equations: $3^x = \log_2 x + 4$ and $3^x = \frac{4}{-x} + 5$.

At this stage I could have exposed each of the different ideas to all the groups and asked them to work on them or to ask each group to present their own work. Instead, pressed by time, I simply ran through the steps to derive the two possible equations on the over head projector.
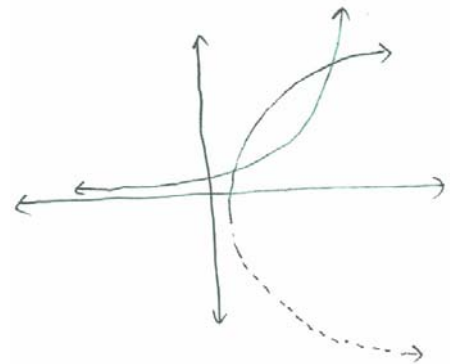
Once again I thought we were ready to move on when **Samkelo said that he and Shannon had another option** (I hadn't looked at what they were doing). They suggested that the other graph was one arm of the inverse of a parabola. Double wow. So now we had something else I hadn't thought of at all (See Figure 10).

Figure 10

We discussed what equation we could get to describe the situation in this case, namely: $3^x = \sqrt{x-2}$

Because of the different responses I started wondering about the difference between a hyperbola and an exponential (or log) graph and asked the class what they thought. (See Figures 11.1 and 11.2)
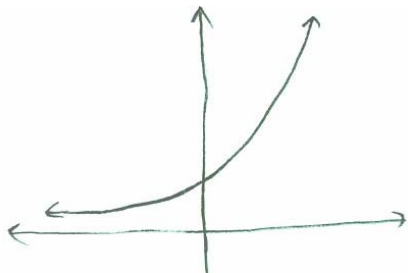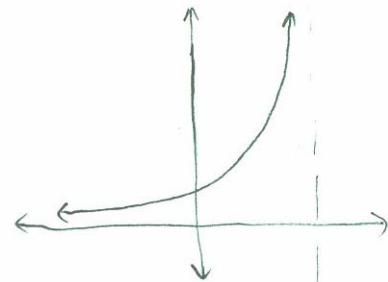
Figure 11.1

Figure 11.2

At first this didn't generate too much discussion but then Amanda volunteered to come up and try and draw the difference. To me it was clear that she had an intuitive idea of the difference but was struggling to explain it. She even struggled to draw the two graphs we were comparing. So I decided to see if I could explain what I saw (or, in terms of Mason's vocabulary, where I was placing my attention) – namely that the hyperbola has two asymptotes and the log and exponential graphs only have one.

How could I have done this in a better way? I feel as though I just told them and didn't do any listening. Why did I do it like this? I think once again it has to do with the fact that it was the first time that I noticed this difference and I wanted to tell someone about it. In this way I was in fact "spoiling it for the others"[1] and "getting carried away with the mathematics".[2]

At this stage our allocated time for the lesson was almost up and because of the term planner which my colleague and I try to stick to, I gave them exercises to practice solving exponential equations even though we hadn't had a chance to discuss different ways of approaching such problems.[3]

Underview

As is clear from the above work, rather than attempt to analyse this lesson by cutting it up into pieces or by classifying certain moments or shifts, I have decided to simply share the account which I wrote down straight after this lesson, interspersed with my retrospective thoughts which have been shaped by the reading and writing I've done since.

What happened in this lesson can be seen as an example of co-emergence between my 'theory' and my 'practice' and how the two can't be separated. It was the 'theory' that I was reading that provided the impetus to approach this lesson differently and to reflect on it afterwards and it was this record of my

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[1] {spoiling it for other} – 2.2.2, 2.3
[2] {getting too involved in mathematics as teacher} – 2.2.5, 2.3
[3] Once again I'm faced with the tension of class work and home work – see section 3.3

'practice' which enabled me to gain a deeper understanding of the 'theory' I had read. As Reid (1996: 206) explains, "The analysis of data in enactivist research can also be seen as a process of co-evolution of ideas. *Theory and data coemerge in the medium of the researcher*. The necessity of theory to account for data results in a dialogue between theory and data, with each one affecting the other" (emphasis added).

At the same time this lesson can be seen as an example of co-emergence between my understanding as the teacher and the understanding of the students. The ideas that emerged wouldn't have emerged if I had been working on my own or if my students had been working on their own. Davis and Sumara (1997: 110-111), writing about co-emergence and structural coupling, make this point.

> While there may be an experience of "self forgetting" in the midst of shared action, it is in the fusing of consciousnesses, in the "coupling" of identities, in the "mutual specification" of forms and actions, that there arises a possibility for actions/understandings to emerge that likely could not have been achieved by either participant independently.

As in section 3.1*, what I planned to look at in this section – the ideas that triggered the moment – is not what I ended up writing about. I was triggered by the ideas of conversations and listening and ended up discussing co-emergence in learning and teaching (which is what I had planned to do in section 3.1*).
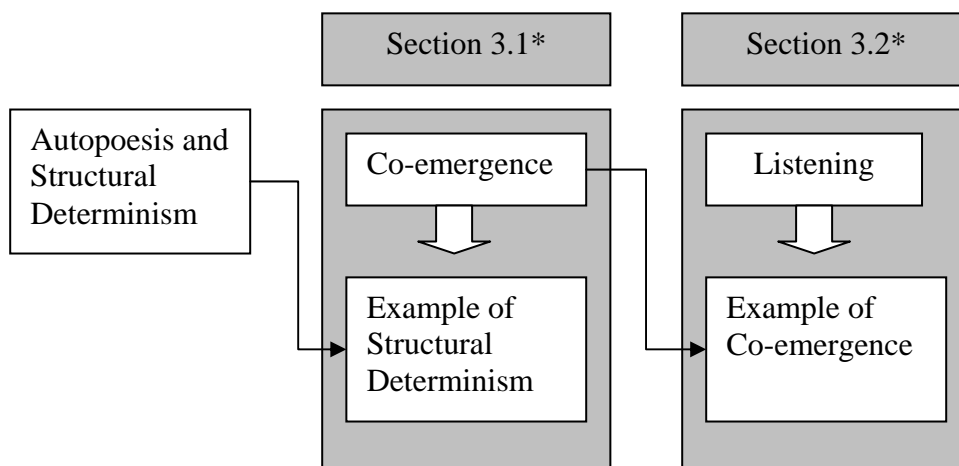
SECTION 3.3*

OF THE KNOTTING OF MY TAPESTRY:

Noticing

A section in which I consider a moment in my practice and how being sensitive to what I notice as a teacher shaped this moment.

"It is important to note that the theories and models of enactivist research are not models of. *That is to say they do not purport to be representations of an existing reality. Rather they are theories* for*; they have a purpose, clarifying our understanding of the learning of mathematics for example, and it is their usefulness in terms of that purpose which determines their value."*

(Reid, 1996: 208)

"Being aware, attending to that awareness, and being explicitly articulate about sensitivities is no mean task. Yet surely I need to be sensitive to the structure, importance, and techniques of a topic if I am to assist others to alter the structure of their attention."

(Mason, 1998: 363)

"[T]he mental structures required by mathematicians are the endowments of seeing, hearing, perceptive people becoming aware of their own functionings."

(Gattegno, 1982: 29)

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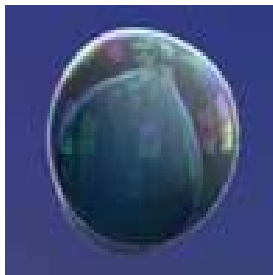
Since the beginning of this process of constructing my tapestry, the idea of noticing has been a central and recurring theme. As I mention in subsection 1.2.3, there was a stage where I wanted to focus on the idea that noticing was the common factor between learning, teaching and researching. As I wove and threaded and as my threads later began to unravel, 'noticing' shifted out of focus as ideas from enactivism shifted into focus. Even so, I was still aware of the significance of noticing – even planning to discuss the role that noticing has played in my practice in this section.

Returning to the ideas of John Mason in his theory of the Discipline of Noticing after interacting with the ideas I encountered in enactivism sensitised me to notice things I hadn't noticed previously. It also reminded me that it was here that I had first encountered what has been referred to as 'the learning paradox' (Sfard, 1998b: 7). Mason (2002: 229) expresses this paradox as follows:

> What we perceive is what we are prepared to perceive, and what we are prepared to perceive is what we have perceived in the past. How then does one ever come to perceive anything new?"

Other versions include:

> If I have to construct my knowledge for myself, how do I come to know something new? (Wheeler, 1986 in Mason, 2002: 230)

> Learning something new depends upon knowing something; at the same time new knowledge helps to re-shape old knowledge. It is a circle of experience of understanding which has indiscernible beginning- or end-points. (Davis *et al*., 1996: 157)

> In order to learn something, the human subject needs to know something. However, in the process of learning something new, old understandings are revised. (Sumara & Davis, 1997: 412)

> It is a sort of paradox: one learns something which is not yet defined and learning it, this person learns what is the nature of what s/he is learning." (Drouhard *et al*., 1999: 328)

> How can we want to acquire a knowledge of something that is not yet known to us? Indeed, if this something does not yet belong to the repertoire of the things we know, then, being unaware of its existence, we cannot possibly inquire about it. Or, to put it differently, if we can only become cognizant of something by recognising it on the basis of the knowledge we already possess, then

nothing that does not yet belong to the assortment of the things we know can ever become one of them. Conclusion: Learning new things is inherently impossible. (Sfard, 1998b: 7)

Mason (2002: 231) goes on to point out that "every philosopher has elucidated their own take on this question, precisely because it lies at the heart of questions about how we know what we think we know, whether what we think we know is valid or true, and how we manage to learn something new." In fact, this was also one of the aims of Davis's (2004) book *Inventions of Teaching*, namely to consider the ways in which different worldviews approach this question.

As I understand it, there are essentially two questions regarding 'coming to know'. "What does it mean to come to know something new?" and "How does one come to know something new?"

What does it mean to come to know something new?

Regarding the first question, I have found the following ideas meaningful:

My view is that learning consists of becoming more sensitive to making distinctions, developing awareness of connections and inter-relations amongst those distinctions, broadening the range of resources one calls upon and the tasks one undertakes in order to pursue aims and goals which are more precisely articulated, and increasing the scope and nature of possibilities and potential we recognise, because our present moment (Bennett, 1964a) is extended. (Mason, 2002: 231)

The discussion of distinction-making reminds me that learning and teaching are not so much about coming to know what you don't know as they are about coming to notice what you haven't noticed. That, perhaps, has been the most significant of Bateson's many prompts to my thinking – the realization that all education is education in perception…which, of course, entails a transformation of knower, not simply what the knower knows. (Davis in Brown & Davis, 2004: 3)

What actually happens when something is learned? Something changes, but what? I shall invite participants to explore the notion that learning consists of shifts in the structure of attention: in what is attended to and how, and how this influences the practice of teaching of mathematics. (Mason, 2003a: 9)

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I take this to mean that I have come to know something new when I am aware of a new distinction, when I have noticed something new or when my attention has shifted.

How does one come to know something new?

Regarding the second question and the paradox or vicious circle that it leads to, I have found the notions of 'co-emergence' and the 'participation metaphor' (as opposed to the 'acquisition metaphor') to be most meaningful. As Mason (2002: 229) puts it:

> One approach is to try to break the circle of cause-and-effect and to declare one place as the initiator. But another approach is to embrace the circularity and to invoke a hermeneutic perspective such as that of Maturana (1998, von Glasersveld website) in which instead of seeing a cycle of cause-and-effect we see co-emergence. Perception and preparedness to be able to perceive emerge together as the result of perceiving and preparing to perceive.

And as Sfard (1998b: 6) explains,

> Just as different organs combine to form a living body, so do learners contribute to the existence and functioning of a community of practitioners… [The participation metaphor] makes salient the dialectic nature of the learning interaction: The whole and the parts affect and inform each other. On the one hand, the very existence of the whole is fully dependent on the parts. On the other hand,…the identity of an individual, like the identity of a living organ, is a function of his or her being (or becoming) a part of a greater entity.

Implications for teaching

So what does it mean to make a new distinction, to develop my sensitivity, to become more sensitive to making distinctions? And if this is what it means to learn, what does it mean to teach, especially if, as Mason (2002: 148) puts it (and as is implied by the notion of a perturbation), "Noticing is triggered by a disturbance" and as Davis (in Brown & Davis, 2004: 2) remembers, "The notion that information was about perturbation rather than objects or substances – that it wasn't something that was passed along in packets or that flowed like liquid – changed everything."

What does it mean if perception (or the ability to make distinctions) and preparedness to be able to perceive (or preparedness to be able to make distinctions) co-emerge? And if this is the case, what implications does it have for me as a teacher?

I have found the responses of both Mason and Davis to the question of the implications of this view of learning to be helpful and meaningful.

Davis (in Brown & Davis, 2004: 3) approaches the question by focusing on "the manner in which teachers attend to students". He wonders how "teachers might prompt awarenesses of the 'patterns that connect' by listening to learners and endeavouring to present differences that might make a difference" (*ibid*.) and suggests we consider reframing teaching as listening … "which is a recursively elaborative participation in the evolution of the student's understanding." (*ibid*.).

Thus the idea of learning and teaching being a conversation as opposed to a monologue (which I mentioned in section 3.2* and which was framed in terms of co-emergence) is extended/refined to looking at ways in which teachers *attend* to students or, put slightly differently, ways in which teachers *listen* to students. One way in which Davis attempts to answer this is by classifying different types of listening.

Mason (2003a: 7) has a different approach:

> I am interested in the lived experience of mathematical thinking. Reflection has led me from the important question of what learners are attending to, to becoming aware of what I am attending to, and through that, to the question of how I am attending, that is, to the form and structure of my attention. This leads me back to the form and structure of learners' attention.

After becoming aware of what he was attending to as a teacher and then becoming aware of what learners were attending to, Mason (2004: 16) suggests that one of the reasons why learners and teachers misunderstand each other is because they are attending to different things. He looks in particular at mathematical abstraction and suggests that it is in fact a "delicate shift in attention" which "seems the most natural and obvious movement imaginable" to a mathematician but not necessarily to a student (Mason, 1989: 2).

In fact Davis (in Brown & Davis, 2004: 3) makes a similar comment when considering one of the things that teachers notice/attend to and which their students don't – namely "the patterns which connect" (Bateson, 1979). He writes, "I realized that most (if not all) of my most frustrating moments as a teacher were around learners' inability to notice a pattern – and, of course, my failure at the time to notice the pattern that they weren't noticing."

So I started doing both:

> trying to **listen** differently (which is what I *planned* to illustrate in section 3.2*)

> > and

> trying to attending to what I was attending to – to **notice** what I was noticing as a teacher

Listening

Looking back at my reflections over the last two years, I see how often I wished to listen to my students in a more meaningful way. Sometimes I felt I got it right. Most of the time it was something I wanted to do better. What surprises me is the fact that I made all these notes to myself, but was still unaware of them in a certain sense. It is as though I was able to notice and mark this desire, but it is only now that I've written this section that I am fully aware of what I want to change and why I want to change it and how I could change it.

Davis's (1996: 51-53) three modes of listening have been helpful in becoming aware of the way in which I listen and the way in which I could listen. The first mode is *evaluative* listening which, in a classroom context, aims to *"assess* what has been learned". The second is *interpretive* listening which aims to "*access* subjective sense" or, in the context of a classroom, to "get at what learners are thinking". The third mode is *hermeneutic* listening which is "more negotiatory, engaging, and messy, involving the hearer and the heard in a shared project" and, in that it seeks to problematise the notion that we are "essentially subjective, autonomous, isolated, and insulated", is essentially an interobjective notion.

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13 June 2007

I just realise more and more how important it is to listen to my students. I can't stand in front and teach to myself. I don't see the mistakes that they see. I can't begin to notice the difference between what I am noticing and what they are noticing unless I stop talking so much and start listening. But it's so great when I actually manage to listen. I had a session with 4 of my students today where we discussed a test they had written. It was amazing. I realised all these things that I just took for granted aren't so obvious. To listen, to listen, to listen.

Although I have been exposed to alternative ways of thinking about mathematics – as something that exists not on its own (either 'out there' or 'in here') but in the inter-actions between me and my students (Davis, 1996: 79) – I still find myself acting as though it is something existing on its own with correct answers that I know and my students don't. And this means that most of my listening is either evaluative or interpretive. In fact, on reflection, even the "better listening" that I have wished to do (as in the journal entry from 13 June 2007) has been either evaluative or interpretive. I have wished I would remember to ask my students what they already know about certain topics (evaluative listening). I have wished I would remember to ask them what their understanding of a concept is – not so that I can understand the concept in a different way, but so that I can understand them better (interpretive listening). But even though these are both important types of questions to ask, they are not hermeneutic questions. They still assume the separation between me and my students, that we are autonomous selves. What could hermeneutic questions look like?

Noticing

After becoming aware that my students and I might be attending to different things and that I might be making shifts in attention that I wasn't consciously aware of any more, I started paying more attention to what I was attending to – to what I was noticing. By doing this I was able to "focus attention on, to stress explicitly, the act of stressing" while teaching (Mason, 1989: 7).

I became aware of the following:

- When working with angles of elevation and depression, I noticed that there is a particular way in which the angles are referred to: we always talk about the angle of elevation *of* A *from* B. Before I became aware of this grammatical indicator, I had been unable to help my students identify which angle was which. I could read the sentence and understand what was going on, but was unable to explain how I knew.

- A similar 'grammatical' distinction I noticed was the difference between the words 'a' and 'the'. For a long time I wasn't consciously aware of the difference that an article makes. It was only when I started teaching second-language English students that I became consciously aware of the difference and started mentioning it especially when dealing with the wording of theorems.

- When drawing the *sin* and *cos* graphs for trigonometry, I was never aware that I automatically plotted the points every $\left(\dfrac{period}{4}\right)^\circ$. For example when plotting $y = sin2x$ for $0° \leq x \leq 180°$, I

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would plot the points for *x = 0°, 45°, 90°, 135° and 180°* and then 'join the dots'. I could never understand why many of my students struggled to draw trigonometric graphs and I could never 'explain' how I drew them – until I made an effort to attend to what I was attending to when I drew them.

- When applying trigonometry formula's such as

$$\cos(w + t) = \cos w.\cos t - \sin w.\sin t$$

I was never aware of the things that I was attending to in order to be able to determine the value of something like

$$\cos(30° + 45°)$$

until I could no longer ignore the fact that many of my students were not able to apply such formulas. Only then did I become aware of all the things I was stressing and ignoring (Pimm, 1993: 35).

- When solving geometry problems, I wasn't aware that I would begin by finding the 'connecting side' – i.e. the side which formed part of both the polygon about which information was given and the polygon about which information was to be calculated. Once again it was only when I was faced with a seemingly unending number of students who were unable to solve these problems that I began to pay attention to what I was attending to when I was solving them.

- After becoming frustrated by the many students who would write things like:

$$sin2x = 2sinx \quad \text{or} \quad cos\ a - cos\ b = cos\ (a - b)$$

I realised that I was seeing *sin* as the name of a function, while they were seeing it as a factor in a product. This confusion is understandable because of the ambiguity of the notion. By referring back to function notation that my students were more familiar with I was able to point out what I was attending to and what they were attending to. When I come across this kind of notation confusion now, I ask the following questions:

If I gave you *f(2x)* would you want to write it as *2f(x)*?

If I gave you *f(x) – f(y)* would you want to take out *f* as a common factor?

By then referring back to the trigonometric functions I am able to explain why

$$sin2x = 2sinx \quad \text{and} \quad cos\ a - cos\ b = cos\ (a - b)$$

aren't necessarily true.

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A moment in my practice - Listening and Noticing

At the end of the second term in 2007 I was busy writing section 3.3 and was reminded again about those aspects of my practice that I was unhappy about, especially the way I gave feedback after tests. At about the same time my students wrote a class test and, while discussing the test, I realised that I had never told any of my classes what I meant by certain "marking notation" which I always used such as:

↓?

I asked my class and, although one of the students answered that it meant "How did you get from one line to the next line?", I began to wonder whether the reason why my students didn't seem to be 'learning' from their tests was because they weren't aware of what to be aware of when going through their marked test. I realised that *I* was learning more and noticing more by marking than my *students* were.

So, for the June exam I decided to approach my marking and feedback differently in an attempt to point out what I was attending to as the teacher and what they too should be attending to. Rather than simply circling the mistake in a particular line – in which case they would not be forced to wonder what was wrong – I decided to mark a line wrong without specifying where the mistake was and then to ask the student to identify the mistake.

As I started writing the questions, I became aware that some of my questions were just the verbal version of circling the mistake and putting a question mark next to it. For example, asking "What happened to the square root?" is not a meaningful question because it identifies the mistake (that the square root "vanished") and because most of the answers one can provide aren't very meaningful (for example, "I don't know, I just forgot about it"). I therefore started changing these questions to ones which didn't identify the mistake (since I wanted the student to do this) and which actually had meaningful answers. For example, "What mistake did you make when you went from line 6 to line 7?" can be answered with "I forgot about the square root", or "I applied the wrong formula", etc.

Another type of question that I ended up asking often was "What law/rule/formula did you use to get from line 4 to line 5 and why can't you use it in this situation?" I asked this question because I wanted my students to become sensitive to when a particular law, rule or formula could be used. I was trying to

give them an idea of the kinds of questions that they could ask themselves about every test of theirs that had been marked.

After the holidays we spent several afternoon sessions working through the papers in smaller groups. I asked the students to answer each of the questions I had asked them in their booklets and to ask me if they didn't understand the question.

Three weeks into the term the students wrote a test on the work we had covered thus far. After considering different options for feedback after the test, I decided to use the opportunity to see if the questions I had asked them to answer after their exam had made a difference in the way that they now treated feedback from tests. I therefore photocopied memorandums of the test and helped them mark their own tests.

The thing that surprised me the most was the number of students who weren't able to see from the memorandum that what they had written down was wrong. In other words, they were not attending to the things that I as a teacher was attending to – such as the correct notation for differential calculus. This raises the question of whether, if I had marked the tests and given them back with a memorandum, these students would have known why certain steps had been marked wrong. And whether they would have asked me if they didn't know why it had been marked wrong.

Two days after they marked their own tests I gave them a questionnaire to fill in.

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1. Describe how you dealt with tests that you got back when you were still at school.

2. I marked your June exams differently to the way I usually mark your tests. What was it like having specific questions to answer?

3. For the test you wrote on Friday you had to play my role and mark it yourself. What was it like 'playing the teacher'?

4. In what way have these two exercises (having to answer specific questions that I set up and having to mark your own test and ask yourself these questions) changed the way that you think about tests and feedback from tests?

I asked my students to complete this questionnaire for two reasons. I wanted to have some indication of whether my alternative feedback methods had been successful and I wanted to use it to discuss the idea of teaching as listening. But as I started writing this section I began to realise that the questions I had asked them were all either evaluative (question 1) or interpretive (question 2, 3 and 4). In fact I realised that although it might be possible to ask evaluative and interpretive questions in a questionnaire, due to the nature of hermeneutic listening, it is not possible to ask hermeneutic questions. As mentioned previously, hermeneutic listening is "negotiatory, engaging, and messy, involving the hearer and the heard in a shared project" and seeks to problematise the notion that we are "essentially subjective, autonomous, isolated, and insulated" (Davis, 1996: 53).

I had originally planned two more activities but because most students didn't have their question books with them, I decided against doing so. In retrospect, these two activities were the most hermeneutic – and also the most unsuitable for a questionnaire.

> 5. Go through your 'questions booklet' and for each exam question, ask yourself whether you learnt something. If you did, circle the question and make a note for yourself (and me) about what you learnt.
>
> 6. Go through your 'question booklet' and put a star next to the questions I asked that you found helpful.

They are tasks which could lead to hermeneutic questions (Gadamer, 1990)), questions for which I don't have answers in my head already. They are questions that could lead to hermeneutic listening. But for this I need to come alongside each student individually so that we can have a conversation. Time constraints do not allow for this.

So I'm left with the responses to the other questions. In particular, some responses to question 4:

> *Vuyiseka*
> *It has changed my whole attitude about my tests, I am/will be able to reflect back on my test, now I know it is the best thing, rather than looking for the right answers on the memo.*

> *Mmabatho*
>
> *It has shown me that writing test/exam is not as important as the feedback because it is there that I learn how to improve and apply my knowledge more efficiently the next time and I gain confidence in my knowledge and grow as a person.*
>
> *Samkelo*
>
> *It made me realise the importance of writing test and learning from the mistake you made as opposed to writing and never looking back (it forced me to revise and relook the test)*
>
> *Eliza*
>
> *Tests are not as hideous and horrible as I used to see it. I can see now that I've wasted time in stressing and cramming and that the test is only there to test your understanding and critical skills (thinking)*

Even though these quotes suggest a change in the way my students view their test, I am left with a feeling of dissatisfaction. I feel I have missed an opportunity to listen to them hermeneutically.

And then I remember the conversation I had with Eliza after class:

It's the end of the lesson – the last lesson before the beginning of a long weekend. I'm on my way out when Eliza comes hurrying back.

S: Ingrid, I wanted to ask you some questions about the proportionality problems so that I can sort them out before I go home.

Earlier today when we were doing proofs for geometry theorems, Eliza was almost in tears because she couldn't understand why we were doing what we were doing. She was able to follow each step logically but still wasn't satisfied. Henderson (2001) suggests that this might have been because it didn't answer her "why-question" – in other words she didn't know why each of the steps had been done. Because of this she felt she wouldn't have been able to come up with the proof on her own – she felt she wouldn't be able work "backwards" to figure out where to start.

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We are now looking at problems where we need to prove certain statements using the proportionality theorems we did a few days ago. Eliza once again doesn't know where to start.

So I demonstrate my thinking to her[1] by showing her how I would prove it. Again and again I refer back to the things I am now aware of attending to: "What is it that we know to be true?", "What is it that they are asking us to prove?", "What would we need to be able to prove this thing that they are asking us to prove?"

Once again she can follow my steps, but doesn't know how I knew where to start.

And so I tell her the story of my "real analysis homework".[2] Of how I tried each question and couldn't do a single one and almost ended up in tears. Of how I couldn't go ask my lecturer to explain EVERYTHING to me. Of how Jacques and later Gawie helped me. Of how I was convinced that they were born being able to do maths, that I wasn't and that I was therefore doomed never to "know where to start". Of how I slowly began to see that this wasn't the case.

I: So there really is hope. Like with this stuff that you feel you can't do now. You can learn to do it with experience.

E: So what are they doing now – your friends?

I: Well, Jacques is doing his PhD in mathematics and Gawie has finished his M and is looking for a job. He did financial mathematics.

E: Wow. And you're busy with your masters?

I: Yes, but in mathematics education. Not mathematics.

E: So what do you actually have to do when you do your masters?

I: Well, that depends on what you are doing it in. You see if you are doing it in mathematics…

And so I tell her everything that I have learnt about paradigms and research approaches, about shifts from positivism to interpretivism, about critical theory and the discipline of noticing.

---

[1] {demonstrating mathematics} – 2.1.5
[2] subsection 2.1.4

And as I share my understanding with her, I make new connections, I see things differently, my world emerges through the telling of my understanding and her listening, her acceptance of who I am. I am reminded of a quote by Levin (1989: 88):

> In listening to others, accepting them in their irreducible difference, we help them listen to themselves, to heed the speech of their own body of experience, and to become, each one, the human being he or she most deeply wants to be.

I tell her about the problems with each of the approaches to research. I tell her how the discipline of noticing rests on the maxim that 'I can't change others, I can work on changing myself'. I tell her how I want to listen more to the students in the class.

I: Because it doesn't help if I explain things so that I understand them but I don't know whether anyone else in the class understands them.

E: Ja, 'cause I've seen the type of questions that you ask us in those worksheets and I've wondered about them.

I: So what do you want to study – I can't remember if I've asked you before?

E: Physics. I look around and I want to understand everything I see.

And I recognise, in her desire, the desire I used to have. And choose not to share the disillusions I experienced regarding physics.[1]

E: I have this memory of walking to school one morning. Walking across a bridge and hearing this lady's shoe making a noise as she walked and trying to imagine/figure out how the sound waves were working.

I imagine a completely different bridge, a different morning, a different shoe – all based on my 'history'. From Davis et al. (*1996: 157) I understand that while Eliza and I "have different histories, and hence bring forth and reflect the world differently, enactivism suggests that [we] are working on a common project – the simultaneous bringing forth of [our]selves and the world – even if [our] respective interpretations of [our] actions and experiences differ."*

E: And Einstein is my hero.

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[1] See "A conversation" p. 9.

I: You know he didn't do well at school.

E: Ja. But it's not just that. The way he figured things out.

And some of the ladies in my community. There is this one lady who has her own salon now. Who were your heroes?

My heroes. I don't know. I never had heroes in the sense of people I wanted to be like. I once made a list of people I'd classify as my heroes though.

I: The headmaster of my primary school. And Sir Thomas Moore *(and Joan of Arc but for some reason I don't mention this).*

E: And why did you look up to them?

I: Um…I suppose because the headmaster guy was so passionate about what he did and he was a people's person. And Thomas Moore because he was willing to stand up for what he believed in – even though he was beheaded for it.

E: That's hectic.

I: Ja.

E: I'm trying really hard to stay on the right road. To be the best person I can be.

I: You know what, it's important to try and stay on the right road but I've realised it's also important to remember that you *will* go off the road. Because you're human. And then what's important is to learn from those times. It's like with the test feedback *(I suddenly realise)* – if you got everything right in the test then there wouldn't be anything to learn. It's the times when we screw up that we have the opportunity to learn. But its not easy – to go and look at those things you did wrong. Most people just settle for not being the best they can be because it's just easier.

Suddenly Breen's (1993: 9) discussion of the 'dark shadow' within mathematics takes on a new meaning. Breen describes how one of his students, at the end of year, "told the class how the experience of looking at her fear of mathematics, re-engaging with the subject in a different way in which she had some power and control, and of coming to terms with mathematics sufficiently to allow her to pass an examination had contributed enormously to her ability to find the strength to cope with her father's death." I realise how my engagements with mathematics – learning to accept failure and still to continue, learning to trust that the answer will 'fall out' and learning to trust my own intuition – have helped me through difficult times in my life.
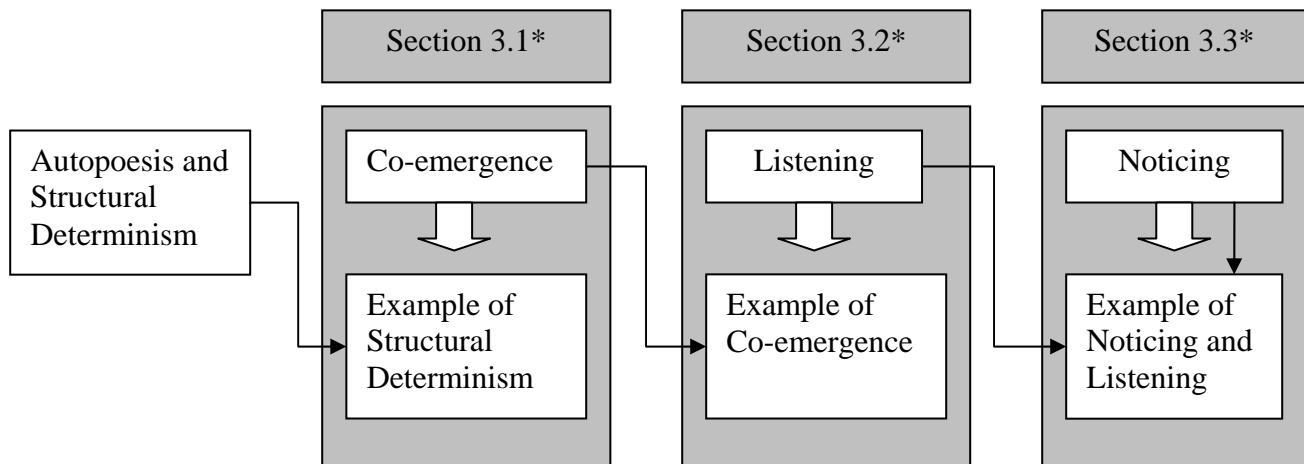
178

E: Ja, hey. I didn't think about that – about the making mistakes bit.

I didn't plan to discuss the nature of research or the need to keep on trying when attempting to 'do the right thing' with your life. I didn't plan to talk to her about her (or my) heroes. And yet we did. We spoke about what mathematics meant and means to me – a possibility for action between me and the people I previously studied with and between me and the students I currently teach.

Underview

Writing this section has once again been an example of writing and through the writing coming to know something I didn't know before (Richardson, 2000: 924). Even though I had been reading enactivist literature extensively, it was only through writing this section that I came to appreciate the differences between the different types of listening that Davis identifies. By coming to appreciate what hermeneutic listening might look like I now have a deeper understanding of notions such as a fluid self (Davis *et al.*, 1996), what it means to conceive of mathematics as something that exists in the interactions between us (Davis & Sumara, 1997:100, 116; Davis, 1994) and what is meant by the idea that "the whole unfolds from and is enfolded in each of the parts" (Davis & Sumara, 1997: 119).

Unlike section 3.1* and 3.2*, in this section I did in fact end up discussing that which had originally triggered the described moment in my practice – namely the idea of noticing. What I didn't plan was to look at what I had intended to look at in section 3.2* - namely the different modes of listening.

Section 3.1*	Section 3.2*	Section 3.3*
Co-emergence	Listening	Noticing
Example of Structural Determinism	Example of Co-emergence	Example of Noticing and Listening

Autopoesis and Structural Determinism

SECTION 3.4*

OF THE CONSTRUCTION OF MY TAPESTRY:

The Possibilities that Emerged

A section in which I consider what my practice *could* look like.

"'Transformational research' means research that does not focus on 'what is' but that deals broadly with 'what ought to be'."
(Gravemeijer, 1998: 277)

"Research into teaching is not about how to teach better. It's about broadening; about extending awareness and options. It's about awakening to alternative ways of seeing the pupils, of hearing and appreciating what they way and do."
(Mason, 1986: 43)

"[T]eachers with more reflective and discursive identities (i.e. those who focus more on mathematics as a process of coming to know), may, through an ongoing 'conversation' with their practice, adopt stances that respond to their students' difficulties, seek to collaborate with students and colleagues in resolving these, look for ways forward in professional guidance and research but only as starting points for their own enquiry, and adopt complex measures of success."
(Kelly, Berry & Battersby, 2007: 44)

In an article discussing research designs, Gravemeijer (1998) identifies three approaches to research: 'explanation', 'understanding' and 'transformational research'. Interestingly, as represented in the table below, it is possible to identify parallels between these three approaches to research and Davis's three modes of listening. Although, if one views research as a form of listening, then it is in fact not surprising that these parallels exist.

Mode of listening	Evaluative: Aims to judge	Interpretive: Aims to understand	Hermeneutic: Aims to participate in the unfolding of possibilities
Approach to Research	Explanation: Focuses on causal relations between dependant and independent variables	Understanding: Focuses on making sense of what is going on	Transformational: Focuses on 'what ought to be' instead of 'what is'

Table 2: Comparison between modes of listening and approaches to research

My own research, which has combined ideas from phenomenology, narrative theory, the discipline of noticing and enactivism, can be located between the 'understanding' and the 'transformational' approach. I want to make sense of my own experiences of coming to know maths and those of my students but I want to understand them *so that* I can begin to imagine what 'ought to be' rather than simply describing 'what is'. Pimm (1993: 28) frames this shift from 'what is' to 'what ought to be' in terms of moving from viewing research as something which tells us what we *should* do, to something which tells us what we *could* do.

So in this section I turn from describing what my practice *does* look like to imagining what my practice *could* look like.

I imagine a practice with different possibilities for the way I act, for the way my students act and for the way we inter-act.

I imagine a practice where

- I catch myself asking leading questions and am able to change my questions into statements and to ask questions which I don't have answers for;
- I catch myself getting carried away by the excitement of solving a problem and am able to slow down and come alongside my students rather than rushing on ahead;
- I catch myself thinking about homework merely as 'practice' for an exam and am able think about it as a means of creating a shared experience which can be used as a point of departure for class discussions;
- I catch myself telling my students how to do something and am able to choose instead to listen to what they plan to do and why they want to do it;
- I am sensitive to the shifts in my attention and am able to share what I stress and what I ignore with my students;
- I am able to ask my students for their suggestions about the way in which our lessons are structured remembering that in the end they determine whether a lesson has been meaningful or not (Davis, 1996: 235; Breen, 1990: 39);
- I regularly record what I notice in my practice and reflect on it with the aim of changing the way I act in the future.

I imagine a practice where

- my students shift from being scared to ask questions in class for fear of appearing stupid to feeling safe enough to stop a discussion when it no longer makes sense to them;
- my students shift from seeing mathematics as a bunch of rules to learn off-by-heart to something which is supposed to make sense;
- my students shift from seeing tests as something to fear to seeing tests as a learning opportunity;
- my students shift from believing intuition is something one is born with to realising that it is something one can learn;
- my students shift from seeing me as the authority on the validity of their solutions to seeing that they must determine the validity of their solutions on their own;
- my students regularly reflect on what they have noticed in their learning and how this can help them change the way they act in the future.

I imagine a practice where

- we develop a conjecturing environment (Sfard *et al*., 1998: 48);
- we come to appreciate that we form a body and are therefore all connected and each of our actions affects all the other members of the body;
- we come to realise what it means if learning is not something that can be caused but something that can be occasioned;
- we sometimes experience mathematics as something more than 'just doing mathematics' (Handa, 2003: 22) but as a 'moving power' in our lives (Campbell, 1991 in Handa, 2003: 22);
- we experience mathematics as a possibility for interaction between us;
- we are able to "move beyond what is already known or what has already been established" and, through working together, are able to generate new knowledge (Davis *et al*., 1996: 167);
- we regularly reflect on what we have learnt as a body and how this can help us act differently in the future.

Underview

It is so easy to make a list of the possibilities of what my practice could look like. And so difficult to change. Davis and Sumara (1997: 13) capture this tension when reflecting on work they did in a school: "One of the most frustrating aspects of these experiences was the tendency each of us had to fall back on teaching behaviours that we felt were incompatible with (and even contradictory to) the enactivist theory of learning and understanding." My experiences confirm what Breen (2003: 537) writes about the process of teacher development, namely that it "takes an enormous amount of focus, energy and time".

Even though my understanding of teaching and learning has changed, I too find myself acting in ways contradictory to these new understandings. As Whitehead (1989: 45) puts it, we hold "educational values while at the same time negating them." But, as he goes on to point out, it is precisely this contradiction which "moves us to imagine alternative ways of improving our situation." (*ibid*.).

And even so, amongst all the tension and contradictions, there are moments when I catch a glimpse of what I imagine my practice could look like.

6 June 2007

Today was another one of those lessons that just went well. I had this moment of 'Man, if all our lessons were like this, it would be so great'. It happened when I noticed my students 'really doing maths'. What made me identify this as doing maths as opposed to the other things we do? We were looking at problems where they had to prove things and not just solve equations. Everyone in the class was working. Some people were moving around to find someone to help them figure out what was going on. Others were working even without me there to tell them to look at the next problem. It just felt so right. Which isn't enough to count as research.

I need to move from 'gut feeling' to 'disciplined conversation' (Breen, 2002).

Yesterday we already started with this atmosphere of proving things on our own. I didn't do any of the proofs on the overhead projector. I started the first one and then moved from grouping to grouping to keep an eye on everyone – nudging here, explaining there, taking ideas from one group to another group, seeing different ways of proving the same thing. Seeing ways of proving theorems that I hadn't seen before – very cool. Vuyiseka came up with a way that followed what we had done in the previous sections.

What was it? This feeling of people working – not just of applying rules but of figuring something out. And me running round from group to group. Of the groups being able to work at their own pace because as they finished a question they were able to go on to the next question.

I don't know if it was just because of what I have been reading about co-emergence that this feeling of us all working together and not individually was so strong. There was no-one working alone. Would I have noticed it otherwise? Probably not.

UNDERVIEW
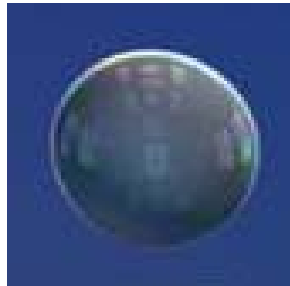
It's the first day of spring. I'm exhausted. But contented. The sky is covered in wisps of white cloud. During my lunch break I noticed new leaves on an oak tree for the first time this year.

Constructing my loom, weaving my tapestry, following threads and having them knotted has not been easy. Learning to trust that a pattern will emerge, learning to trust my own intuition, learning to trust. In the end those things that I was expected to decide on before I started researching did emerge. I have a research paradigm or methodology - enactivism. I have a research design – an emergent design. And I have a method – a combination of phenomenology, narrative inquiry and the discipline of noticing.

But now I come to the point in my weaving where I must choose to stop. It is time to stand back and view the whole tapestry. For too long I have been focusing on one panel or one thread at a time. It is time to stand back and make peace with the fact that my tapestry will never be perfect. That there will always be loose threads trailing on the floor and knotted threads hanging from the sides. I could spend forever tweaking a thread here, reweaving a section there. But there are other tapestries to be woven, other stories to be lived and told.

It's time to take my tapestry off the loom.

I am faced with a choice now. I can either take my tapestry and frame it and hang it on my wall where it can be admired from a distance. Where I can congratulate myself on the representation that I have made. Where it will stay, immovable, gathering dust.

Or I can take it and use it. Make it part of my life. Transform it into a rug, a coat, something that can be used - not from a distance. Transport it to new places, sharing it with new people.

Because constructing this tapestry has changed me. And I would share this change with others.

I am no longer the same person who started writing this thesis. I no longer desire to construct a final perfect representation.[1] Yet I have deliberately chosen not to go back and reweave the sections which contradict what I have woven later. They are evidence of the change that has occurred in me.

A change which has lead me closer to understanding that "all knowing is doing is being" (Davis, 2004: 154).

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[1] Underview p. 6.

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