THE PRODUCTION OF HYPERNUCLEI VIA THE WEAK INTERACTION

By

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DECLARATION

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Signature

Date

ABSTRACT

In this thesis we present a relativistic formalism for the description of hypernuclei production via the weak interaction. It will be shown that the cross section can be written as the contraction of a leptonic and hadronic tensor if we model the interaction as a quasifree process. The hadronic tensor is written in a model-independent way in terms of thirteen nuclear structure functions. A Born term model is used to describe the underlying elementary hyperon production process. The bound state wave functions of the hyperon and nucleon are calculated within a relativistic mean-field approximation. Together with the relativistic kinematics a fully relativistic framework for experimental predictions is constructed and a specific cross section calculation is discussed.

SAMEVATTING

In hierdie tesis word 'n formalisme daargestel waarmee die vorming van hiperkerne weens die swak wisselwerking beskryf kan word. Daar sal aangetoon word dat die reaksie kansvlak geskryf kan word as 'n kontraksie tussen 'n leptontensor en 'n hadrontensor indien the interaksie as quasi-vry gemodelleer word. Die hadrontensor word geskryf as 'n uitbreiding in terme van dertien struktuurfunksies wat dit model-onafhanklik maak. 'n Born-term model word gebruik om die onderliggende elementere hiperon produksie proses te beskryf. Die gebonde-toestand golffunksies van die hiperon en die nukleon word bereken deur gebruik te maak van 'n relatiwistiese gemiddelde-veld benadering. Tesame met die relatiwistiese kinematika word 'n raamwerk wat ten volle relatiwisties is gekonstrueer vir eksperimentele voorspellings. 'n Spesifieke kansvlak berekening word ook bespreek.

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CONTENTS

LIST OF FIGURES

Introduction

The main motivation for this work is the recent large-scale interest in neutrino physics stemming from advances in technology and scientific expertise enabling the experimental community to produce neutrinos and conduct neutrino experiments. This is because the extremely small mass and electric neutrality of the neutrino makes it extremely difficult to detect and to direct in a beamline. At Fermilab in the United States the MiniBooNE experiment is currently running in which the flavour-changing behaviour of neutrinos, first observed at Los Alamos National Laboratory in 1995, is investigated. This experiment utilises a 1 GeV neutrino beam and a single 800-ton mineral oil detector. If the previously unpredicted behaviour of neutrinos is confirmed, the full BooNE experiment will be conducted with a 2-detector arrangement allowing the precise measurement of the oscillation parameters and illuminating questions on CP and CPT violation[1].

Since neutrinos only interact weakly with matter (i.e. via the weak interaction) nuclei containing hyperons (i.e. hypernuclei) are very likely products of neutrino-nucleus reactions. Associated kaon production is also important since kaons produced by the interactions of atmospheric neutrinos with nuclei can mimic the signals of kaons arising from proton-decay predicted by supersymmetric theories [Da02].

Our model is the first attempt at describing the weak production of hypernuclei. It is a fully relativistic treatment of this problem in that both relativistic kinematics and dynamics are used. The general development of theoretical models for processes in which hyperons and kaons are formed have been hampered by the lack of experimental results [Da02]. [Sh75] and [De81] give descriptions of the elementary neutrino-nucleon scattering process in terms of Born s, t and u-channels and [Me78] discusses a general formalism for the description of neutrino-induced associated production. A description of quasielastic neutrino-nucleus scattering with nucleon knockout is presented in [Va04]. The effects of strange quarks on the cross sections in neutrino-nucleus scattering is investigated in [Va06]. Our work is based on a general scattering formalism developed in the latter two papers.

The work presented is divided into three parts. Chapter 1 contains the discussion of our model starting with an identification of the types of elementary processes that could serve as underlying processes in the production of hypernuclei. The general cross section for neutrino-induced reactions is then derived with a look at the normalisation and the transition matrix element written

[1] Ref. *http://www-boone.fnal.gov/*

as the contraction of leptonic and hadronic tensors. The construction of the leptonic tensor is straightforward and we adopt the approach of [Va04] and [Va06]. The complicated nuclear forces (strong interaction effects), however, necessitate a much more complicated hadronic current. As a first approximation to this process we make use of a Born term model for the quasifree process to construct the hadronic tensor. This approach has been followed for the elementary process presented in [Sh75] and [De81] and neutrino-nucleus scattering presented in [Va04] and [Va06].

Chapter 2 presents an application of our model to the description of neutrino-induced hypernuclei production from a ^{12}C target. The reason for this choice is mainly the high concentration of ^{12}C in the mineral oil detectors used in the BooNe experiment. Our results can easily be extended to other target nuclei (within the relativistic mean-field theories). In the second part of this chapter the results of our calculations are shown and discussed.

Appendix A contains a more detailed discussion on the weak interaction and the derivation of the current operators for leptonic and hadronic vertices. The first part sketches the phenomenological picture whereas the second part is an overview of the Glashow-Salam-Weinberg theory for electroweak unification, a well-known part of the Standard Model.

Appendix B illustrates our approach to the form factor decomposition of the hadronic current operator and discusses the origin of the various form factors of the vertices in the Born diagrams.

CHAPTER 1
The Model

1.1 Overview

As stated in the introduction, our model is a fully relativistic treatment of the weak interaction between neutrinos and nuclei. The kinematics follow from a standard geometric diagram of the scattering process. The cross section is written in terms of a kinematic factor, a momentum-conserving δ-function and the transition matrix element squared, which we write as the contraction of a leptonic and a hadronic tensor. These tensors describe the relativistic dynamics of the projectile and the nucleus. The leptonic tensor is derived using a conventional relativistic approach of Dirac plane wave spinors and a leptonic current operator derived in Appendix A. The hadronic tensor is parametrised in terms of a basis and 13 structure functions. Its form is model-independent. A Born term model (tree diagrams) is used to construct the current of the underlying elementary hadronic process leading to the desired hypernucleus. Its form is extended to the quasifree case by integrating over the momenta of the bound nucleon and hyperon. The bound state wave functions we use are calculated using a relativistic mean-field formalism. The structure functions are then extracted from the newly constructed model-dependent hadronic tensor.

1.2 Relevant reactions

Neutrino-nucleon elementary reactions with produced strangeness can be divided into three main types [Ma03]. These are identified by

- the charge of the ejectile lepton i.e. the type of ejectile lepton which can be either a neutrino (in the case of the so-called neutral current or NC reactions) or a muon (in the case of the charged current or CC reactions) and
- the nett change in strangeness (ΔS). Since there are no strange quarks present in the incident channel, the change in strangeness is solely determined by the reaction products which are
  - a kaon (K^+, K^0 has strangeness +1; K^-, $\bar{K}$ has strangeness -1),
  - a hyperon (strangeness -1) or a nucleon (strangeness 0).

This leads to the classification scheme shown in table 1.1. Clearly the CC $\Delta S = 1$ reactions are not of interest to our work since no hyperons and therefore no hypernuclei are formed. Also note

CC ($\Delta S = 0$)	CC ($\Delta S = 1$)	NC ($\Delta S = 0$)
$\nu + n \to \mu^- + K^0 + \Sigma^+$	$\nu + p \to \mu^- + K^+ + p$	$\nu + p \to \nu + K^0 + \Sigma^+$
$\nu + n \to \mu^- + K^+ + \Sigma^0$	$\nu + n \to \mu^- + K^0 + p$	$\nu + p \to \nu + K^+ + \Sigma^0$
$\nu + p \to \mu^- + K^+ + \Sigma^+$	$\nu + n \to \mu^- + \pi^+ + K^0 + n$	$\nu + p \to \nu + K^+ + \Lambda$
$\nu + n \to \mu^- + K^+ + \Lambda$		$\nu + n \to \nu + K^+ + \Sigma^-$
		$\nu + n \to \nu + K^0 + \Sigma^0$
		$\nu + n \to \nu + K^0 + \Lambda$
		$\nu + n \to \nu + K^- + \Sigma^+$

Table 1.1: Examples of the different types of weak processes.

that no NC $\Delta S = 1$ reactions have thus far been observed experimentally. This behaviour is the reason for the incorporation of the Glashow-Iliopoulos-Maiani mechanism which lead to the prediction of the existence of the charmed quark. The observation of these strangeness-changing neutral current processes at rates comparable to the other weak current processes would imply new physics beyond the Standard Model [Ma03].

1.3 Formalism

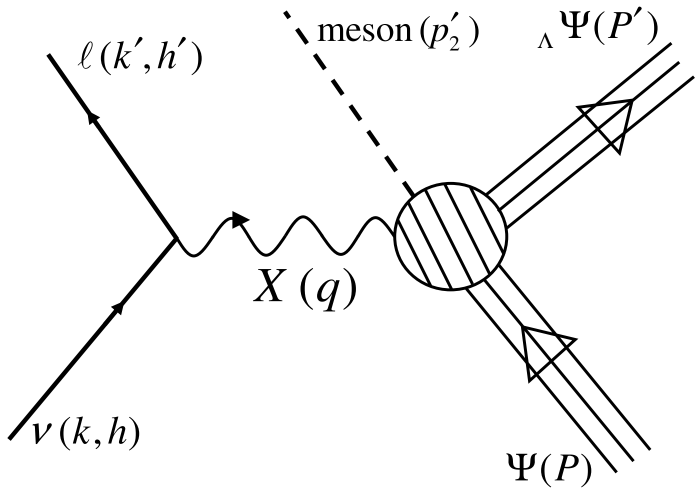


Figure 1.1: Neutrino-nucleus weak interaction

Figure 1.1 shows the diagram of the typical process we describe. The interaction is mediated by a virtual weak vector boson with four-momentum q. For the two types of processes, CC and NC, the ejectile lepton and vector boson are shown in table 1.2.

	CC	NC
ℓ	μ^-	ν
X	W^+	Z^0

Table 1.2: Ejectile leptons and vector bosons of the charged and neutral current reactions.

If we assign the four-momenta of the particles as

$$\begin{aligned} k &= \text{projectile neutrino,} \\ k &= \text{ejectile neutrino or muon,} \\ p_1 &= \text{nucleon,} \\ p_1' &= \text{hyperon,} \\ p_2' &= \text{kaon,} \\ P &= \text{nucleus} \\ P' &= \text{residual nucleus,} \end{aligned} \tag{1.1}$$

we can start by deriving the cross section for the elementary process shown in figure 1.2.

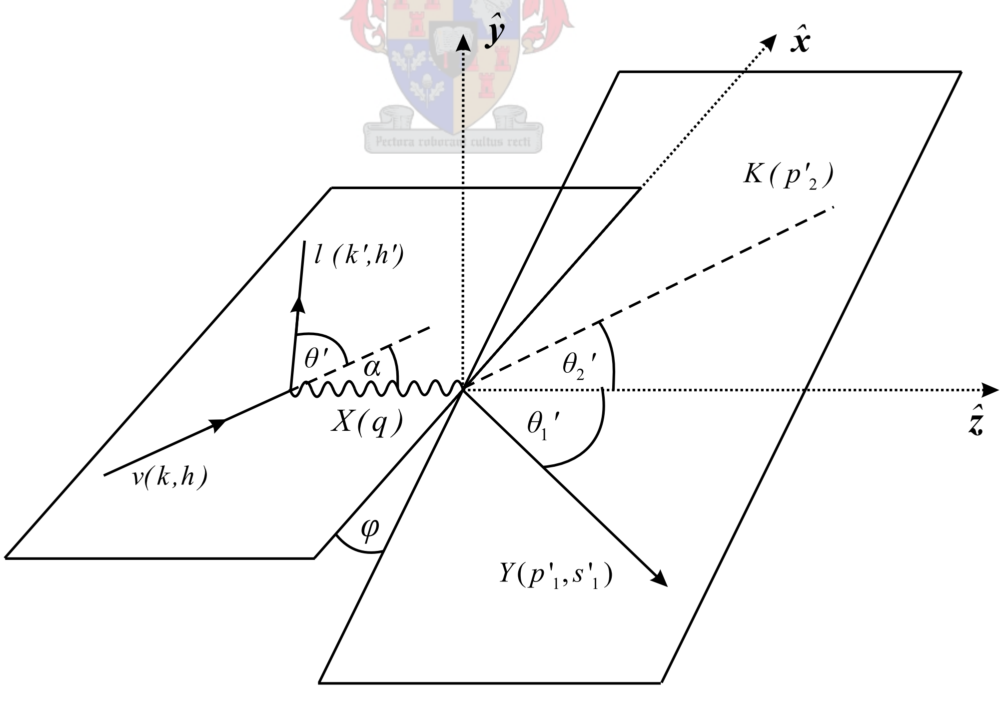


Figure 1.2: The elementary process. Refer to table 1.2 for the definitions of ℓ and X

1.3.1 Kinematics and the general form of the cross section

The general derivation of the cross section follows directly from the S-matrix theory presented in [Pe95], [Bj64] and [Gr94]. The transition rate per unit volume from the initial to the final state, W_{if}, in a reaction, can be written in terms of the transition probability per unit time (T) per unit volume (V)

$$W_{if} = \frac{|T_{if}|^2}{TV}, \tag{1.2}$$

where

$$T_{if} = -i \quad \overbrace{N_{V_1} N_{V_2} N_{V_3} N_{V_4} N_{V_5}}^{\text{normalisation factor for each particle}} \quad (2\pi)^4 \;\; \delta^4 \left(k + p_1 - k' - p_2' - p_1'\right) \; \mathcal{M}. \tag{1.3}$$

In general the position space wave function for a boson is taken to be the solution of the relativistic Klein Gordon equation. The density (the number of bosons per unit volume) follows straightforwardly from a construction of the four-current density and the continuity equation as is done in [Gr87]

$$\phi(x) \;=\; N_V e^{-ip\cdot x}, \tag{1.4}$$

$$\rho \;=\; \frac{n}{V} = 2E\,|N_V|^2\,. \tag{1.5}$$

The wave functions of the fermions are taken to be solutions of the free-space relativistic Dirac equation

$$\psi(x) \;=\; N_V u(\mathbf{p}, s) e^{-ip\cdot x}, \tag{1.6}$$

$$\rho \;=\; \psi^\dagger \psi = |N_V|^2\, u^\dagger u. \tag{1.7}$$

We normalise the Dirac spinors $u(\mathbf{p}, s)$ non-covariantly

$$u^\dagger(\mathbf{p}, s') u(\mathbf{p}, s) = \delta_{ss'}, \tag{1.8}$$

as is done in [Va04] since this is also the standard normalisation employed for bound state wave functions (see section 1.5) where

$$\int d\mathbf{r}\; \mathcal{U}_\alpha^\dagger(\mathbf{r}) \mathcal{U}_\alpha(\mathbf{r}) = 1. \tag{1.9}$$

With this normalisation the fermion density is

$$\rho = \frac{n}{V} = N_V^2, \tag{1.10}$$

where n is the number of particles and V the volume.

The transition rate per unit volume is now given by

$$\begin{aligned} W_{if} &= \frac{\left(N_{V_1}^2 N_{V_2}^2 N_{V_3}^2 N_{V_4}^2 N_{V_5}^2\right)\left[(2\pi)^4\,\delta^4\left(k+p_1-k'-p_2'-p_1'\right)\right]^2|\mathcal{M}|^2}{TV} \\ &= \left(N_{V_1}^2 N_{V_2}^2 N_{V_3}^2 N_{V_4}^2 N_{V_5}^2\right)(2\pi)^4\,\delta^4\left(k+p_1-k'-p_2'-p_1'\right)|\mathcal{M}|^2\,. \end{aligned} \tag{1.11}$$

The cross section is defined as

$$d\sigma = \frac{W_{if}}{\text{initial flux} \times (n/V)_{\text{target}}} \times \text{number of final states}, \tag{1.12}$$

where

$$\text{initial flux} = \frac{|\mathbf{v}_1 - \mathbf{v}_2|}{V} = \frac{4\left[(k\cdot p_1)^2 - m_\nu^2 M^2\right]^{1/2}}{(2E_k)\,(2E_{p_1})\,V}. \tag{1.13}$$

Here $\mathbf{v}_1$ and $\mathbf{v}_2$ refer to the velocities of the neutrino and nucleon respectively.

The particle normalisation is chosen as one particle per unit volume (i.e. $n = 1$ in eqs 1.5, 1.10 and 1.12). The number of final states in a momentum interval of the reaction products is derived in a similar fashion as is done for electron-proton scattering treated in [Gr94] and is given by

$$\left(\frac{V}{(2\pi)^3}\,d^3\mathbf{k}'\right)\left(\frac{V}{(2\pi)^3}\,d^3\mathbf{p}_1'\right)\left(\frac{V}{(2\pi)^3}\,d^3\mathbf{p}_2'\right). \tag{1.14}$$

The normalisation factors N_V are

$$\begin{aligned} N_V^2 &= \frac{1}{V} \qquad \text{fermions (neutrino, muon, hyperon, nucleon)}, \\ N_V^2 &= \frac{1}{2EV} \qquad \text{bosons (kaon)}. \end{aligned} \tag{1.15}$$

With eqs 1.13 (where $m_\nu = 0$), 1.14 and 1.15 inserted into the cross section for the elementary process it becomes

$$d\sigma = \frac{(2E_k)(2E_{p_1})}{8\,(2\pi)^5\,(k\cdot p_1)\,E_{p_2'}}\,\delta^4\left(k+p_1-k'-p_2'-p_1'\right)d^3\mathbf{k}'\,d^3\mathbf{p}_2'\,d^3\mathbf{p}_1'\,|\mathcal{M}|^2\,. \tag{1.16}$$

If we generalise this expression to the quasifree case we obtain

$$d\sigma = \frac{(2E_k)(2E_P)}{8\,(2\pi)^5\,(k\cdot P)\,E_{p_2'}}\,\delta^4\left(k+P-k'-p_2'-P'\right)d^3\mathbf{k}'\,d^3\mathbf{p}_2'\,d^3\mathbf{P}'\,|\mathcal{M}|^2\,. \tag{1.17}$$

In the lab frame $\mathbf{P} = 0$ and the cross section simplifies to

$$d\sigma = \frac{1}{2\,(2\pi)^5\,E_{p_2'}}\,\delta^4\left(k + P - k' - p_2' - P'\right) d^3\mathbf{k}'\,d^3\mathbf{p}_2'\,d^3\mathbf{P}'\,|\mathcal{M}|^2\,. \tag{1.18}$$

The integrals over the kaon and ejectile lepton momentum can by simplified by noting that the chosen coordinate system (see figure 1.2) allows for the substitutions

$$\begin{aligned} d^3\mathbf{k}' &= 2\pi E_{k'}^2\,dE_{k'}\,d(\cos\theta'), & (1.19)\\ d^3\mathbf{p}_2' &= \sqrt{E_{p_2'}^2 - M_{\text{kaon}}^2}\,E_{p_2'}\,dE_{p_2'}d\Omega_2'. & (1.20)\end{aligned}$$

1.3.1.1 The δ-function

If the δ-function is written as

$$\delta^4\left(k + P - k' - p_2' - P'\right) = \delta(E_k + E_P - E_{k'} - E_{p_2'} - E_{P'})\;\;\delta^3\left(\mathbf{k} + \mathbf{P} - \mathbf{k}' - \mathbf{p}_2' - \mathbf{P}'\right) \tag{1.21}$$

and the integral over the spatial component of the nucleus momentum ($\mathbf{P}$) is performed, the cross section is obtained in the form

$$d\sigma = \frac{1}{2\,(2\pi)^4}\;\;E_{k'}^2\sqrt{E_{p_2'}^2 - M_{\text{kaon}}^2}\,\delta(E_k + E_P - E_{k'} - E_{p_2'} - E_{P'})\,dE_{k'}\,d(\cos\theta')\,dE_{p_2'}\,d\Omega_2'\;\,|\mathcal{M}|^2\,. \tag{1.22}$$

To simplify the energy-conserving δ-function further, it can be written as a function of the kaon energy $E_{p_2'}$. To accomplish this, note the following

$$E_P = M_A \quad \text{lab frame}, \tag{1.23}$$

where M_A is the mass of the target nucleus. The energy of the residual nucleus can be written as

$$\begin{aligned} E_{P'} &= \sqrt{\mathbf{P}'^{\,2} + M_{A-1}^2}\\ &= \sqrt{(\mathbf{q} - \mathbf{p}_2')^2 + M_{A-1}^2}, \quad \text{from momentum conservation}. & (1.24)\end{aligned}$$

The residual mass is computed using

$$M_{A-1} = M_A - (M_{\text{nucleon}} - E_{\text{bound nucleon}}) + (M_{\text{hyperon}} - E_{\text{bound hyperon}})\,. \tag{1.25}$$

The momentum term in eq. 1.24 can be written in terms of the kaon's energy and mass as well as the momenta and energies of the neutrino and ejectile lepton

$$
\begin{aligned}
(\mathbf{q}-\mathbf{p}_2')^2 &= \mathbf{q}^2 + E_{p_2'}^2 - M_{\mathrm{kaon}}^2 - 2\,|\mathbf{q}|\,\sqrt{E_{p_2'}^2 - M_{\mathrm{kaon}}^2}\,\cos\theta_2', && (1.26)\\
|\mathbf{q}| &= \left[E_k^2 - E_{k'}^2 - M_{\mathrm{lepton}}^2 - 2E_k\sqrt{E_{k'}^2 - M_{\mathrm{lepton}}^2}\,\cos\theta'\right]^{1/2}. && (1.27)
\end{aligned}
$$

With this expression the δ-function in eq. 1.22 is completely defined in terms of known quantities and the kaon energy. The integral over the kaon energy, which appears via the substitution of eq. 1.20 into the cross section, can now be performed by making use of the identity

$$
\delta\left(f(z)\right) = \sum_{z_i\,\in\,\text{roots of }f(z)} \frac{1}{|f'(z_i)|}\,\delta(z - z_i), \tag{1.28}
$$

where

$$
f(z) = a - z - \left[b + z^2 - c\sqrt{z^2 - M_{\mathrm{kaon}}^2}\right]^{1/2}, \tag{1.29}
$$

with

$$
\begin{aligned}
a &= E_k - E_{k'} + M_A,\\
b &= |\mathbf{q}|^2 - M_{\mathrm{kaon}}^2 + M_{A-1}^2,\\
c &= 2\,|\mathbf{q}|\cos\theta_2'. && (1.30)
\end{aligned}
$$

1.3.1.2 The cross section

After the integration discussed above the cross section becomes

$$
d\sigma = \frac{1}{2(2\pi)^4} E_{k'}^2 \sqrt{E_{p_2'}^2 - M_{\mathrm{kaon}}^2}\; dE_{k'}\; d(\cos\theta')\; d\Omega_2'\; \frac{1}{\left|f'(E_{p_2'})\right|}\,|\mathcal{M}|^2\,. \tag{1.31}
$$

Since the CC reactions allow for the detection of the ejectile muon, the differential cross section can be used in the form

$$
\frac{d^3\sigma}{dE_{k'}\; d(\cos\theta')\; d\Omega_2'} = \frac{1}{2(2\pi)^4} E_{k'}^2 \sqrt{E_{p_2'}^2 - M_{\mathrm{kaon}}^2}\; \frac{1}{\left|f'(E_{p_2'})\right|}\,|\mathcal{M}|^2\,. \tag{1.32}
$$

The differential cross section for a NC reaction can be calculated by integrating over the energy and momentum of the undetected ejectile neutrino

$$\frac{d\sigma}{d\Omega_2'} = \frac{1}{2(2\pi)^4}\int_{E_{k'}^{\min}}^{E_{k'}^{\max}} dE_{k'}\int_0^{\pi} d\theta' \sin\theta' E_{k'}^2 \sqrt{E_{p_2'}^2 - M_{\mathrm{kaon}}^2}\,\frac{1}{\left|f'(E_{p_2'})\right|}\,|\mathcal{M}|^2\,, \tag{1.33}$$

where the limits of the energy integral are determined by the input kinematics.

1.4 The transition matrix element

The transition matrix contains the dynamics of the reaction process. For a general discussion of the weak interaction and the derivation of the associated transition matrix element, refer to Appendix A. Due to the short range of the weak interaction, it is assumed to be propagated by a massive vector boson (one-boson exchange approximation [Va04]). For small momentum transfer reactions the transition matrix element is (see eq. A.8)

$$-i\mathcal{M} = [\eta_\ell \ell_\mu][\frac{i}{M_{\mathrm{boson}}^2}][\eta_h h^\mu], \tag{1.34}$$

where ℓ_μ and h_μ are the leptonic and hadronic currents respectively. The couplings of the weak vector boson (η_ℓ and η_h) can be obtained phenomenologically (section A.1.1) or from the Glashow-Salam-Weinberg Theory (section A.2). If we combine them into a single coupling parameter η, the transition matrix element can be written as

$$\mathcal{M} = \eta G_F \ell_\mu h^\mu, \tag{1.35}$$

where

$$\eta = \begin{cases} \frac{1}{2\sqrt{2}} & \text{NC} \\ \frac{1}{\sqrt{2}}\cos\theta_C & \text{CC},\ \ \Delta S = 0 \\ \frac{1}{\sqrt{2}}\sin\theta_C & \text{CC},\ \ \Delta S = 1 \end{cases} \tag{1.36}$$

and G_F is the Fermi constant for beta decay ($G_F \approx 1.66 \times 10^{-5}$ GeV^{-2}) and θ_C the Cabibbo angle ($\theta_C \approx 0.13°$) used to account for the experimentally observed asymmetry between the two types of CC processes [Ai82].

Calculation of the cross section requires (from eqs 1.32 and 1.33) the quantity $|\mathcal{M}|^2$ which, using eqs 1.35 and 1.36, results in

$$|\mathcal{M}|^2 = \eta^2 G_F^2 \ell_{\mu\nu} W_{\mu\nu}. \tag{1.37}$$

1.4.1 The leptonic vertex

In the dicussion of the leptonic vertex and the derivation of the leptonic tensor we follow the method of [Va06].

The leptonic current can in general be written as

$$\ell_\mu = \bar{u}(\mathbf{k}', s')\, L_\mu\, u(\mathbf{k}, s). \tag{1.38}$$

For the leptonic wave functions we make use of Dirac plane wave spinors which we normalise non-covariantly

$$u^\dagger(\mathbf{k}, h)\, u(\mathbf{k}, h) = 1. \tag{1.39}$$

Since only left-handed neutrinos (those with their spins antiparallel to their momenta) take part in weak processes, the helicity representation is a natural form to use for the lepton spinors

$$u(\mathbf{k}, h) = \sqrt{\frac{E_k + m}{2E_k}} \begin{pmatrix} \phi_h(\hat{\mathbf{k}}) \\ \frac{h|\mathbf{k}|}{E_k + m}\, \phi_h(\hat{\mathbf{k}}) \end{pmatrix}, \quad \left(E_k = \sqrt{\mathbf{k}^2 + m^2}\right). \tag{1.40}$$

This form of the spinor also has the property

$$u(\mathbf{k})\, \bar{u}(\mathbf{k}) = \frac{\not{k} + m}{4E_k} \left[(1 + h\gamma_5 \not{s})\right], \tag{1.41}$$

where $\not{k} = k_\mu \gamma^\mu$. Here s is a four-component spin-vector

$$s^\mu(\mathbf{k}) = \frac{1}{m} \left(|\mathbf{k}|\,, E_k \hat{\mathbf{k}}\right). \tag{1.42}$$

For massless left-handed neutrinos $m = 0$ and $h = -1$ giving

$$\nu(\mathbf{k})\, \bar{\nu}(\mathbf{k}) = \frac{\not{k}}{4E_k} \left[(1 + \gamma_5)\right]. \tag{1.43}$$

For the massive muon the general eq. 1.41 will hold.

The form of the leptonic current operator is discussed in Appendix A, sections A.1.1 and A.1.2. It is

$$L_\mu = \gamma_\mu (1 - \gamma_5). \tag{1.44}$$

Note that the different coupling strengths for CC and NC reactions are already taken into account in the general coupling constant η defined in eq. 1.36.

The total leptonic current (without the coupling) is therefore

$$\ell_\mu = \bar{u}(\mathbf{k}', h')\, \gamma_\mu (1-\gamma_5)\, u(\mathbf{k}, h). \tag{1.45}$$

The leptonic tensor is now defined by

$$\ell_{\mu\nu} = \ell_\mu \, (\ell_\nu)^* \,. \tag{1.46}$$

By means of Feynman trace techniques the general leptonic tensor for neutrino induced reactions can be written as

$$\ell_{\mu\nu} = \mathrm{Tr}\left[(\gamma_\mu - \gamma_\mu\gamma_5)\, (\nu(\mathbf{k},h)\bar{\nu}(\mathbf{k},h))\, (\gamma_\nu - \gamma_\nu\gamma_5) \left(u(\mathbf{k}',h')\bar{u}(\mathbf{k}',h') \right) \right], \tag{1.47}$$

where u can either refer to a neutrino spinor (NC reactions) or a muon spinor (CC reactions).

By means of the identities

$$\begin{aligned} \mathrm{Tr}\,[\gamma_5\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta] &= -4i\epsilon_{\mu\alpha\nu\beta}, \quad (\text{i.e. } \epsilon_{0123} = -1) && (1.48) \\ \mathrm{Tr}\,[\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta] &= 4\,(g_{\mu\alpha}g_{\nu\beta} - g_{\alpha\beta}g_{\mu\nu} + g_{\nu\alpha}g_{\mu\beta}), && (1.49) \end{aligned}$$

as well as the properties of the lepton spinors given in eqs 1.41 and 1.43 this tensor may be written as the sum of a symmetric and an antisymmetric part

$$\ell_{\mu\nu} = \overbrace{\frac{2}{E_k E_{k'}} \left(k_\mu K'_\nu + K'_\mu k_\nu - g_{\mu\nu} k\cdot K' \right)}^{\ell^S_{\mu\nu}} + \overbrace{\frac{2i}{E_k E_{k'}} \left(\epsilon_{\mu\nu\alpha\beta} k^\alpha K'^\beta \right)}^{\ell^A_{\mu\nu}}, \tag{1.50}$$

where

$$K' \equiv \frac{1}{2}(k' - h' m_{k'} s) \xrightarrow[m_{k'}=0]{} k' \delta_{h',-1} \,. \tag{1.51}$$

For the NC reactions ($m_{k'} = 0$) this expression simplifies to

$$\ell^{\mathrm{NC}}_{\mu\nu} = \frac{2}{E_k E_{k'}} \left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' + i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \right). \tag{1.52}$$

1.4.2 The hadronic vertex

The hadronic current is defined in terms of the transition amplitude

$$h^\mu = \langle p_2'; \Psi_f(P') \mid \eta_h J^\nu(q) \mid \Psi_i(P) \rangle \,, \tag{1.53}$$

where $\psi_{i/f}$ refers to the initial/final nuclear states. As explicitly shown in this equation, the final state also contains a meson (kaon) characterised by the four-momentum p_2'. As was done for the leptonic tensor, the hadronic tensor as a also written as a sum of symmetric and antisymmetric parts

$$W^{\mu\nu} = W_S^{\mu\nu} + W_A^{\mu\nu}. \tag{1.54}$$

This may be done by means of a model-independent parametrisation in terms of the independent four-momenta at the hadronic vertex in figure 1.1 (q^μ, P^μ and $p_2'^\mu$), the Levi-Civita tensor ($\epsilon^{\mu\nu\alpha\beta}$) and the metric tensor ($g^{\mu\nu}$), making use of 13 structure functions [Va06]

$$\begin{aligned} W_S^{\mu\nu} &= W_1 g^{\mu\nu} + W_2 q^\mu q^\nu + W_3 P^\mu P^\nu + W_4 p_2'^\mu p_2'^\nu \\ &\quad + W_5(q^\mu P^\nu + P^\mu q^\nu) + W_6(q^\mu p_2'^\nu + p_2'^\mu q^\nu) + W_7(P^\mu p_2'^\nu + p_2'^\mu P^\nu), \end{aligned} \tag{1.55}$$

$$\begin{aligned} W_A^{\mu\nu} &= W_8(q^\mu P^\nu - P^\mu q^\nu) + W_9(q^\mu p_2'^\nu - p_2'^\mu q^\nu) + W_{10}(P^\mu p_2'^\nu - p_2'^\mu P^\nu) \\ &\quad + W_{11}\epsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta + W_{12}\epsilon^{\mu\nu\alpha\beta} q_\alpha p'_{2,\,\beta} + W_{13}\epsilon^{\mu\nu\alpha\beta} P_\alpha p'_{2,\,\beta}, \end{aligned} \tag{1.56}$$

where the structure functions are functions of $q^2,\ P^2,\ p_2'^2,\ q\cdot P,\ q\cdot p_2'$ and $P\cdot p_2'$.

The same parametrisation may be adopted since the hadronic part of the diagram in fig. 1.1 is identical to the one in [Va06] if the following identifications are made:

$$\begin{aligned} \text{meson} &\longleftrightarrow \text{nucleon}, \\ \text{residual hypernucleus} &\longleftrightarrow \text{residual nucleus}. \end{aligned}$$

In addition, the spin of the outgoing nucleon was summed over in [Va06] leading to the same number of independent four-momenta as in this work.

The structure functions can be obtained by contracting each of the basis elements in turn with the hadronic tensor. This can be done for the symmetric and antisymmetric elements separately since the contraction of a symmetric and an antisymmetric tensor is zero. For the symmetric basis elements this results in

$$\begin{bmatrix} g_{\mu\nu}W^{\mu\nu} \\ q_\mu q_\nu W^{\mu\nu} \\ P_\mu P_\nu W^{\mu\nu} \\ p'_{2,\,\mu}p'_{2,\,\nu}W^{\mu\nu} \\ (q_\mu P_\nu + P_\mu q_\nu)W^{\mu\nu} \\ (q_\mu p'_{2,\,\nu} + p'_{2,\,\mu}q_\nu)W^{\mu\nu} \\ (P_\mu p'_{2,\,\nu} + p'_{2,\,\mu}P_\nu)W^{\mu\nu} \end{bmatrix} = X_S \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \end{bmatrix}, \tag{1.57}$$

where

$$X_S = \begin{bmatrix} 4 & q^2 & P^2 & p_2'^2 & 2A & 2B & 2C \\ q^2 & q^4 & A^2 & B^2 & 2q^2A & 2q^2B & 2AB \\ P^2 & A^2 & P^4 & C^2 & 2P^2A & 2AC & 2P^2C \\ p_2'^2 & B^2 & C^2 & p_2'^4 & 2BC & 2p_2'^2B & 2p_2'^2C \\ 2A & 2q^2A & 2P^2A & 2BC & 2A^2+2q^2P^2 & 2AB+2q^2C & 2P^2B+2AC \\ 2B & 2q^2B & 2AC & 2p_2'^2B & 2AB+2q^2C & 2B^2+2q^2p_2'^2 & 2BC+2p_2'^2A \\ 2C & 2AB & 2P^2C & 2p_2'^2C & 2P^2B+2AC & 2BC+2p_2'^2A & 2C^2+2P^2p_2'^2 \end{bmatrix}, \tag{1.58}$$

if we define $A \equiv q \cdot P$, $B \equiv q \cdot p'_2$ and $C \equiv P \cdot p'_2$. The antisymmetric basis elements yield

$$\begin{bmatrix} (q_\mu P_\nu - P_\mu q_\nu)W^{\mu\nu} \\ (q_\mu p'_{2,\,\nu} - p'_{2,\,\mu}q_\nu)W^{\mu\nu} \\ (P_\mu p'_{2,\,\nu} - P_\mu p'_{2,\,\nu})W^{\mu\nu} \\ \epsilon_{\mu\nu\alpha\beta}q^\alpha P^\beta W^{\mu\nu} \\ \epsilon_{\mu\nu\alpha\beta}q^\alpha p_2'^\beta W^{\mu\nu} \\ \epsilon_{\mu\nu\alpha\beta}P^\alpha p_2'^\beta W^{\mu\nu} \end{bmatrix} = X_A \begin{bmatrix} W_8 \\ W_9 \\ W_{10} \\ W_{11} \\ W_{12} \\ W_{13} \end{bmatrix}, \tag{1.59}$$

where

$$X_A = \begin{bmatrix} 2q^2P^2 - 2A^2 & 2q^2C - 2AB & 2AC - 2P^2B & 0 & 0 & 0 \\ 2q^2C - 2AB & 2q^2p_2'^2 - 2B^2 & 2p_2'^2A - 2BC & 0 & 0 & 0 \\ 2AC - 2P^2B & 2p_2'^2A - 2BC & 2P^2p_2'^2 - 2C^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2A^2 - 2q^2P^2 & 2AB - 2q^2C & 2P^2B - 2AC \\ 0 & 0 & 0 & 2AB - 2q^2C & 2B^2 - 2q^2p_2'^2 & 2BC - 2p_2'^2A \\ 0 & 0 & 0 & 2P^2B - 2AC & 2BC - 2p_2'^2A & 2C^2 - 2P^2p_2'^2 \end{bmatrix}, \tag{1.60}$$

if we define A, B and C as above.

Inverting the matrix X_S and multiplying by the left-hand side of eq. 1.57 gives the symmetric structure functions in a column vector

$$\begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_7 \end{bmatrix} = X_S^{-1} \begin{bmatrix} g_{\mu\nu}W^{\mu\nu} \\ q_\mu q_\nu W^{\mu\nu} \\ \vdots \\ (P_\mu p'_{2,\nu} + p'_{2,\mu}P_\nu)W^{\mu\nu} \end{bmatrix}. \tag{1.61}$$

The same operation is applied to the matrix X_A and eq. 1.59 to obtain the antisymmetric structure functions.

1.4.3 General form of the transition matrix element

The leptonic and hadronic tensors written in terms of sums of symmetric and antisymmetric parts lead to the general form found in [Va06]

$$\begin{aligned} |\mathcal{M}|^2 &= \eta^2 G_F^2 \ell_{\mu\nu} W^{\mu\nu} \\ &= \eta^2 G_F^2 \left[\ell^S_{\mu\nu} W_S^{\mu\nu} + \ell^A_{\mu\nu} W_A^{\mu\nu}\right]. \end{aligned} \tag{1.62}$$

The leptonic tensor in the form of eq. 1.50 and the hadronic tensor in its expanded form (eqs 1.55 and 1.56) inserted into 1.62 give

$$\begin{aligned}
\ell^S_{\mu\nu} W^{\mu\nu}_S &= \left(\frac{4}{E_k E_{k'}}\right)\left[-W_1(k\cdot K') + W_2 f_1(q) + W_3 f_1(P) + W_4 f_1(p'_2)\right.\\
&\quad \left. + W_5 f_2(P,q) + W_6 f_2(q,p'_2) + W_7 f_2(P,p'_2)\right], \qquad (1.63)
\end{aligned}$$

$$\begin{aligned}
\ell^A_{\mu\nu} W^{\mu\nu}_A &= \left(\frac{4i}{E_k E_{k'}}\right)\left[W_{10}\epsilon_{\mu\nu\alpha\beta} k^\mu K'^\nu P^\alpha {p'_2}^\beta + W_{11} f_3(q,P)\right.\\
&\quad \left. + W_{12} f_3(q,p'_2) + W_{13} f_3(P,p'_2)\right]. \qquad (1.64)
\end{aligned}$$

where the definitions of the functions $f_{1,2,3}(x)$ are given by [Va06]

$$\begin{aligned}
f_1(x) &= (k\cdot x)(K'\cdot x) - \frac{x^2}{2}(k\cdot K'),\\
f_2(x,y) &= (k\cdot x)(K'\cdot y) + (k\cdot y)(K'\cdot x) - (x\cdot y)(k\cdot K'),\\
f_3(x,y) &= (k\cdot y)(K'\cdot x) - (k\cdot x)(K'\cdot y). \qquad (1.65)
\end{aligned}$$

For the NC case, $K' = k'$ (i.e. $m_{k'} = 0$) which results in $f_1(q) = f_2(P,q) = f_2(q,p'_2) = 0$.

Note that the formalism up to this point has been completely independent of the model used to describe the hadronic current. The use of the structure function expansion, in fact, facilitates the use of any such model in that the structure functions can be easily extracted from the model-dependent hadronic tensor using eq. 1.61. The following section describes our specific choice of model for the hadronic process.

1.5 Model-dependent evaluation of the hadronic vertex

For calculations to proceed, some model is needed to describe the process at the hadronic vertex. This model not only has to specify a hadronic current operator but also needs to incorporate the nuclear structure.

In our model the interaction at the hadronic vertex is modelled as a quasifree process. We therefore assume that the vector boson couples to a single bound nucleon as shown in figure 1.3. We invoke the impulse approximation by assuming that the current operator retains its free-space (elementary) form in the nuclear medium. Nuclear structure effects are included by means of bound state wave functions for the nucleon and hyperon. These wave functions are calculated in a relativistic mean-field formalism. Note that we are now limited to simple particle-hole hypernuclei configurations. In this paradigm the quasifree hadronic current is calculated in

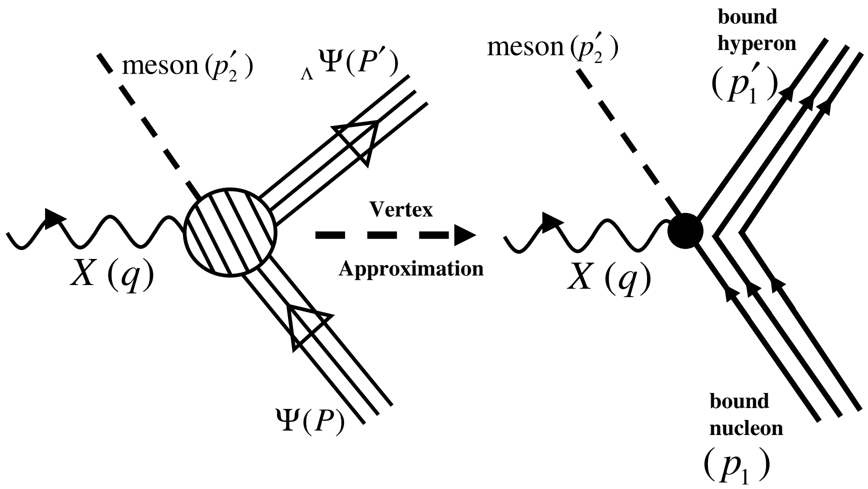


Figure 1.3: Quasifree approximation

momentum space as

$$h^\mu = \int d^3\mathbf{p}_1 \int d^3\mathbf{p}_1'\, \bar{\mathcal{U}}_Y(\mathbf{p}_1')\ J^\mu\, \mathcal{U}(\mathbf{p}_1)\ \delta(\mathbf{p}_1' + \mathbf{p}_2' - \mathbf{q} - \mathbf{p}_1). \tag{1.66}$$

Here $\mathcal{U}_Y$ and $\mathcal{U}$ are the bound state wave functions of the hyperon and nucleon respectively. From momentum conservation it follows that

$$h^\mu = \int d^3\mathbf{p}_1\, \bar{\mathcal{U}}_Y(\mathbf{q} + \mathbf{p}_1 - \mathbf{p}_2')\ J^\mu\, \mathcal{U}(\mathbf{p_1}). \tag{1.67}$$

The general hadronic current operator for a weak elementary process is derived in Appendix B, section B.1. The specific forms applicable to CC and NC reactions will be discussed in section 1.5.2. The total hadronic current operator is constructed by summing the hadronic current operators of the s, t and u Born channels

$$J^\mu_{\text{total}} = J^\mu_s + J^\mu_t + J^\mu_u. \tag{1.68}$$

The model-dependent hadronic tensor is obtained by

$$(W^{\mu\nu})_{\text{model}} = \sum_{m_N, m_Y} h^{\mu}_{m_N, m_Y} \left(h^{\nu}_{m_N, m_Y}\right)^*, \tag{1.69}$$

where m_N and m_Y refer to the projections of the total angular momentum of the nucleon and hyperon respectively.

1.5.1 Nuclear structure and the bound state wave functions

The bound state wave functions of the nucleon and hyperon used in the momentum integral of eq. 1.67 have been obtained using a relativistic mean-field formalism ([To05] and [Lu03]). The position-space wave functions are given by [Va04]

$$\mathcal{U}_{E\kappa m} = \frac{1}{x} \begin{bmatrix} g_{E\kappa}(x)\mathcal{Y}_{\kappa m}(\hat{x}) \\ i f_{E\kappa}(x)\mathcal{Y}_{-\kappa m}(\hat{x}) \end{bmatrix}, \tag{1.70}$$

where $\mathcal{Y}$ refers to the spin-spherical harmonics and the total angular momentum j has been redefined in terms of κ

$$j = |\kappa| - \frac{1}{2} \quad \text{and} \quad \ell = \begin{cases} \kappa, & \kappa > 0 \\ -1-\kappa, & \kappa < 0 \end{cases}. \tag{1.71}$$

We obtain the momentum-space wave function by taking the Fourier transform of eq. 1.70

$$\begin{aligned} \mathcal{U}_{E\kappa m}(\mathbf{p}) &= \int d\mathbf{x}\, e^{-i\mathbf{p}\cdot\mathbf{x}}\, \mathcal{U}_{E\kappa m}(\mathbf{x}) \\ &= 4\pi(-i)^* \begin{bmatrix} g_{E\kappa}(p) \\ i f_{E\kappa}(p)\, \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}} \end{bmatrix} \mathcal{Y}_{\kappa m}(\hat{\boldsymbol{p}}), \end{aligned} \tag{1.72}$$

where

$$\begin{aligned} g_{E\kappa}(p) &= \int dx\, g_{E\kappa}(x)\, j_\ell(px), \\ f_{E\kappa}(p) &= \operatorname{sgn}(\kappa) \int dx\, x\, f_{E\kappa}(x)\, j_{2j-\ell}(px) \end{aligned} \tag{1.73}$$

and j_ℓ is the Bessel function for orbital angular momentum ℓ.

1.5.2 The currents

The elementary currents are constructed by means of a Born term model of s, t and u-channels. Higher order effects are absorbed in the form factors. This method is also employed in [Sh75] and [De81]. It is the standard methodology adopted in so-called effective field-theory models of hadronic interactions. Other examples of this approach for photon/electron-nucleus interactions can be found in [Wi92] and [Be95]. To construct the elementary current expressions the following is needed:

- **Propagators**
  - spin-$\frac{1}{2}$: $\dfrac{\not{p}+m}{p^2-m^2}$
  - spin-0: $\dfrac{1}{p^2-m^2}$
- **Vertex factors** [Me78]
  - **Type I**: Strong coupling vertices; two baryons, one meson

    These vertices occur in the s, t and u-channel diagrams. In our calculations they are approximated by a pseudoscalar strong coupling as is done in [Me78].
  - **Type II**: Weak coupling vertices; two mesons

    This type of vertex is found in the t-channels. Phenomenological kaon form factors [Me78] are used.
  - **Type III**: Weak coupling vertices; two baryons

    These vertices occur in the s and u-channels. They can be described by means of weak form factors as is done in [Sh75] and [De81].

Type I vertices

These vertices are of the form

$$\left\langle KB' \mid B \right\rangle = g_{KB',B} \;\; \bar{u}_{B'} \, \gamma_5 \, u_B. \tag{1.74}$$

The dimensionless strong coupling constants needed for our reactions fall into 2 categories. Their values are taken from [Sh75]

$$g_{K\Lambda,N} = -10, \tag{1.75}$$

$$g_{K\Sigma,N} = 1.3. \tag{1.76}$$

Type II vertices

Vertices of this kind are described by the vertex factors found in [Me78]. For the **CC** reactions they are

$$\left\langle K^0(p_2') \mid J^\mu_{\mathrm{CC}}(q) \mid K^+ \right\rangle = (2p_2' - q)^\mu \, F_{K^{0,+}}(q^2), \tag{1.77}$$
$$\left\langle K^+ \mid J^\mu_{\mathrm{CC}} \mid K^0 \right\rangle = (2p_2' - q)^\mu \, F_{K^{+,0}}(q^2), \tag{1.78}$$

where

$$F_{K^{+,0}} = -F_{K^{0,+}} = 2F_{K,\rho}. \tag{1.79}$$

For the **NC** reactions the vertex factors are

$$\left\langle K^+ \mid J^\mu_{\mathrm{NC}}(q) \mid K^+ \right\rangle = (2p_2' - q)^\mu F_{K^+}(q^2), \tag{1.80}$$
$$\left\langle K^0 \mid J^\mu_{\mathrm{NC}} \mid K^0 \right\rangle = (2p_2' - q)^\mu F_{K^0}(q^2), \tag{1.81}$$

where

$$F_{K^+} = F_{K,\rho} + F_{K,\omega} + F_{K,\phi}, \tag{1.82}$$
$$F_{K^0} = -F_{K,\rho} + F_{K,\omega} + F_{K,\phi}. \tag{1.83}$$

The F form factors are functions of the vector boson momentum and the vector meson masses

$$F_{K,\rho} = \frac{f_{\rho K\bar{K}}}{f_\rho} \frac{m_\rho^2}{m_\rho^2 - q^2}, \tag{1.84}$$
$$F_{K,\omega} = \frac{f_{\omega K\bar{K}}}{f_\omega} \frac{m_\omega^2}{m_\omega^2 - q^2}, \tag{1.85}$$
$$F_{K,\phi} = \frac{f_{\phi K\bar{K}}}{f_\phi} \frac{m_\phi^2}{m_\phi^2 - q^2}. \tag{1.86}$$

Experimentally it has been determined that

$$\frac{f_{\rho K\bar{K}}}{f_\rho} = 0.5, \tag{1.87}$$
$$\frac{f_{\omega K\bar{K}}}{f_\omega} = 0.17, \tag{1.88}$$
$$\frac{f_{\phi K\bar{K}}}{f_\phi} = 0.33. \tag{1.89}$$

The masses of the mesons are taken to be $m_\rho = 770$ MeV, $m_\omega = 782$ MeV and $m_\phi = 1020$ MeV.

Type III vertices - charged current

These vertices are treated by means of the form factor description of the weak charged current operator. If second-class currents and terms proportional to the lepton mass are omitted, as is done in [Va06] (refer to Appendix B), the general hadronic current operator is

$$J^{\mu}_{\mathrm{CC}} = \left[J^{\mu}_{\mathrm{CC}}\right]_V - \left[J^{\mu}_{\mathrm{CC}}\right]_A = f_1(q^2)\gamma^{\mu} + \frac{i}{2m} f_2(q^2)\sigma^{\mu\nu}q_{\nu} - g_{A,\mathrm{CC}}(q^2)\gamma^{\mu}. \tag{1.90}$$

The **Type III** vertices in the **CC *s*-channels** are all of the form

$$\left\langle p \mid J^{\mu}_{\mathrm{CC},+} \mid n \right\rangle . \tag{1.91}$$

The weak vector form factors (f_1 and f_2) are related to the isovector electromagnetic Dirac and Pauli form factors of the proton and neutron through the Conserved Vector Current hypothesis. For a more complete discussion refer to Appendix B, section B.2.1. The vector form factors for this variant of the Type III vertex are

$$f_1(q^2) = F_1^{IV} = F_1^{(p)} - F_1^{(n)}, \tag{1.92}$$

$$f_2(q^2) = F_2^{IV} = F_2^{(p)} - F_2^{(n)}. \tag{1.93}$$

The axial form factor has been determined phenomenologically. It is briefly discussed in Appendix B, section B.2.2, and is usually written as

$$g_{A,\mathrm{CC}}(q^2) = g_A G_D^A(q^2). \tag{1.94}$$

Type III vertices in the **CC *u*-channels** can however not be treated similarly because of the presence of strange quarks (in the hyperons) in the transitions

$$\left\langle Y' \mid J^{\mu}_{\mathrm{CC},+} \mid Y \right\rangle . \tag{1.95}$$

We therefore resort to SU(3) current algebra to derive a general set of form factors for the baryon octet. A more complete discussion can be found in Appendix B, section B.2.3. The weak charge-raising current is written as

$$\begin{aligned} \left\langle B_i \mid J^{\mu}_{\mathrm{CC},+} \mid B_k \right\rangle = \bar{u}_i \Big[& i\left(f_{i1k} + i f_{i2k}\right)\left(f_1^F \gamma^{\mu} + \frac{i}{2m} f_2^F \sigma^{\mu\nu} q_{\nu} - g^F \gamma^{\mu}\gamma_5\right) \\ & + \left(d_{i1k} + i d_{i2k}\right)\left(f_1^D \gamma^{\mu} + \frac{i}{2m} f_2^D \sigma^{\mu\nu} q_{\nu} - g^D \gamma^{\mu}\gamma_5\right)\Big] u_k, \end{aligned} \tag{1.96}$$

where d_{ijk} and f_{ijk} are the symmetric and antisymmetric structure constants of SU(3) given in Appendix B, section B.2.3.

The vector form factors of the baryon octet are

$$f_1^D(q^2) = -\frac{3}{2}F_1^{(n)}(q^2), \tag{1.97}$$

$$f_2^D(q^2) = -\frac{3}{2}F_2^{(n)}(q^2), \tag{1.98}$$

$$f_1^F(q^2) = F_1^{(p)}(q^2) + \frac{1}{2}F_1^{(n)}(q^2), \tag{1.99}$$

$$f_2^F(q^2) = F_2^{(p)}(q^2) + \frac{1}{2}F_2^{(n)}(q^2). \tag{1.100}$$

The axial form factor is discussed in section B.2.3.2 and is parametrised in the form

$$g_{A,\mathrm{CC}}(q^2) = g^D(q^2) + g^F(q^2) = g_A G_D^A(q^2) = \left(g^D(0) + g^F(0)\right) G_D^A(q^2), \tag{1.101}$$

where [Sh75]

$$g^D(0) = 0.78 \pm 0.02, \tag{1.102}$$

$$g^F(0) = 0.45 \pm 0.02. \tag{1.103}$$

Type III vertices - neutral current

The general form of the weak neutral current operator is derived in Appendix B, sections B.1 and B.3. It is

$$J^\mu_{\mathrm{NC}} = \tilde{f}_1(q^2)\gamma^\mu + \frac{i}{2m}\tilde{f}_2(q^2)\sigma^{\mu\nu}q_\nu - \tilde{g}_A(q^2)\gamma^\mu\gamma_5. \tag{1.104}$$

The **Type III** vertices in the **NC *s*-channels** are all of the form

$$\left\langle n \mid J^\mu_{\mathrm{NC}} \mid n \right\rangle, \tag{1.105}$$

or

$$\left\langle p \mid J^\mu_{\mathrm{NC}} \mid p \right\rangle. \tag{1.106}$$

The form factors for this type of vertices are given in eqs B.75 and B.76. They are written in terms of the Dirac and Pauli form factors of the neutron and proton as

$$\tilde{f}_i(q^2) = (1 - 4\sin^2\theta_W)F_i^{(\mathrm{p})}(q^2) - F_i^{(\mathrm{n})}(q^2) \tag{1.107}$$

for protons and

$$\tilde{f}_i(q^2) = (1 - 4\sin^2\theta_W)F_i^{(\mathrm{n})}(q^2) - F_i^{(\mathrm{p})}(q^2) \tag{1.108}$$

for neutrons.

The axial form factor is obtained in a similar fashion to the CC s-channel axial form factor

$$\tilde{g}_A(q^2) = g_A G_D^A(q^2). \tag{1.109}$$

Type III vertices in the **NC $\boldsymbol{u}$-channels** are also of the hyperon-hyperon type

$$\left\langle Y' \mid J^\mu_{\mathrm{NC}} \mid Y \right\rangle. \tag{1.110}$$

The form factors for this vertex are obtained by writing the neutral current operator in SU(3) current form and comparing the F and D form factors of this SU(3) neutral current operator between proton and neutron states with the well-known proton and neutron weak form factors of eqs B.75 and B.76. This gives the general weak form factors of the neutral current in the baryon octet as a ratios of the proton and neutron weak form factors as can be seen in eq. B.88.

CHAPTER 2
Example

As an application of our model, we consider the reaction

$$\nu(k) \ + \ ^{12}\mathrm{C}(P) \ \longrightarrow \ \mu^-(k') \ + \ K^+(p_2') \ + \ ^{12}_{\Lambda}\mathrm{C}(P'), \tag{2.1}$$

of which the underlying elementary process is

$$\nu(k) \ + \ n(p_1) \ \longrightarrow \ \mu^-(k') \ + \ K^+(p_2') \ + \ \Lambda(p_1'), \tag{2.2}$$

since it is known to have the largest cross section of the CC reactions [Ma03].

2.1 Model application

2.1.1 Kinematics

The kinematics for this process is simply the CC kinematics discussed in section 1.3.1. The general form of the cross section is given by eq. 1.32.

2.1.2 Dynamics

- The leptonic tensor is given by the CC tensor found in eq. 1.50 for the massive ejectile muon.
- The hadronic tensor is parametrised in terms of structure functions as discussed in section 1.4.2. The contraction of the leptonic and hadronic tensors gives the transition matrix element in the form of eq. 1.62
- The hadronic current of the elementary process is constructed by summing the Born terms of the elementary process as dicussed in section 1.5 of Chapter 1. A complete discussion of each channel is presented in the following sections. The hadronic current of the quasifree process is obtained by integrating the elementary current with respect to the momentum of the bound neutron as shown in eq. 1.67.
- The model-dependent hadronic tensor is then constructed according to eq. 1.69.
- As discussed in section 1.4.2, the symmetric and antisymmetric structure functions can be obtained seperately and can then be substituted into eqs 1.63 and 1.64 after which the cross section can be calculated.

2.1.2.1 s-channel

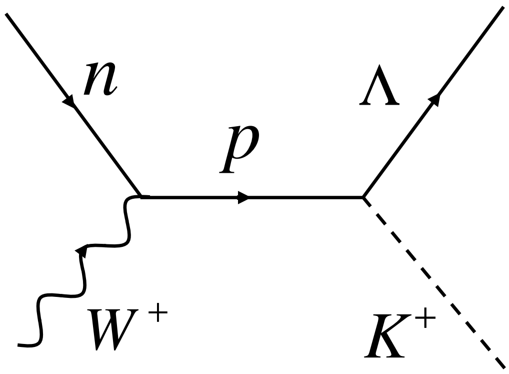


Figure 2.1: s-channel diagram of elementary process

- **Propagator**

  The propagator is a proton (momentum $p = p_1 + q$)

$$\frac{\not{p} + m_p}{p^2 - m^2}. \tag{2.3}$$

- **Type I vertex**

  This vertex is described by the pseudoscalar strong-coupling

$$\gamma_5 G_{K\Lambda,N}. \tag{2.4}$$

- **Type III vertex**

  This weak-coupling vertex is described by the form factor charged current expression of eq. B.13 with the form factors given by eqs. B.29 and B.30

$$\left\langle p \mid J^{\mu}_{\mathrm{CC},+} \mid n \right\rangle = \bar{u}_p \left[f_1(q^2)\gamma^{\mu} + \frac{i}{2m} f_2(q^2)\sigma^{\mu\nu} q_{\nu} - g_{A,\mathrm{CC}}(q^2)\gamma^{\mu}\gamma_5 \right] u_n, \tag{2.5}$$

  where

$$f_1(q^2) = F_1^{\mathrm{IV}} = F_1^{(\mathrm{p})} - F_1^{(\mathrm{n})}, \tag{2.6}$$
$$f_2(q^2) = F_2^{\mathrm{IV}} = F_2^{(\mathrm{p})} - F_2^{(\mathrm{n})}, \tag{2.7}$$
$$g_{A,\mathrm{CC}}(q^2) = g_A G_D^A(q^2). \tag{2.8}$$

The s-channel of the **quasifree process** is therefore described by the current

$$h_s^\mu = \int d\mathbf{p}_1\, \bar{\mathcal{U}}_\Lambda(p_1') \left[\gamma_5\, G_{K\Lambda,N}\, \frac{\not{p} + m_p}{p^2 - m_p^2} \left(f_1(q^2)\gamma^\mu + \frac{i}{2m_n} f_2(q^2)\sigma^{\mu\nu}q_\nu - g_{A,\mathrm{CC}}(q^2)\gamma^\mu\gamma_5\right)\right] \mathcal{U}_n(p_1). \tag{2.9}$$

2.1.2.2 t-channel

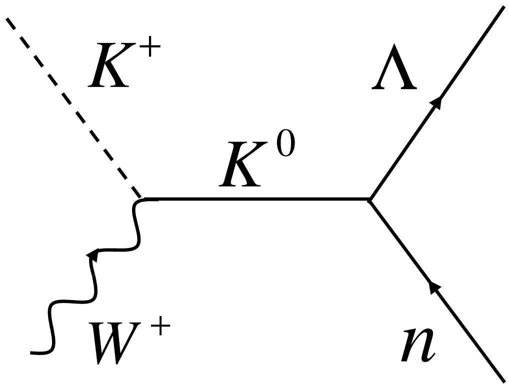


Figure 2.2: t-channel diagram of elementary process

- **Propagator**

  The propagator is a kaon (momentum $p = p_2' - q$)

$$\frac{1}{p^2 - m_{K^0}^2}. \tag{2.10}$$

- **Type I vertex**

  This vertex is described by the pseudoscalar strong-coupling

$$\gamma_5 G_{K\Lambda,N}. \tag{2.11}$$

- **Type II vertex**

  This weak-coupling meson vertex is described by the phenomenological form factor expression of eq. 1.78

$$\left\langle K^+ \mid J^\mu_{\mathrm{CC},+} \mid K^0 \right\rangle = (2p_2' - q)^\mu F_{K^+,0}(q^2). \tag{2.12}$$

The t-channel of the **quasifree process** is therefore described by the current

$$h_t^\mu = \int d\mathbf{p}_1 \, \bar{\mathcal{U}}_\Lambda(p_1') \left[\gamma_5 \, G_{K\Lambda,N} \, \frac{(2p_2' - q)^\mu}{p^2 - m_K^2} \, F_{K^+,0}(q^2) \right] \mathcal{U}_n(p_1). \tag{2.13}$$

2.1.2.3 u-channel

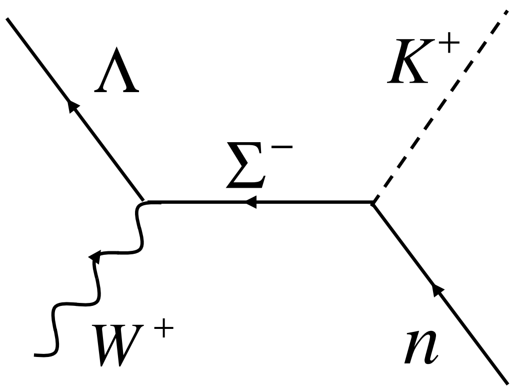


Figure 2.3: u-channel diagram of elementary process

- **Propagator**

  The propagator is a Σ^- hyperon (momentum $p = q - p_1'$)

$$\frac{\not{p} + m_{\Sigma^-}}{p^2 - m_{\Sigma^-}^2}. \tag{2.14}$$

- **Type I vertex**

  This vertex is described by the pseudoscalar strong-coupling

$$\gamma_5 G_{K\Sigma,N}. \tag{2.15}$$

- **Type III vertex**

  This weak-coupling vertex is described by the form factor charged current expression of eq. B.63 with the form factors derived via the SU(3) current algebra method

$$\begin{aligned} \left\langle \Lambda^0 \mid J^\mu_{\mathrm{CC},+} \mid \Sigma^- \right\rangle &= \bar{u}_{\Lambda^0} \Big[i\,(f_{i1k} + i f_{i2k}) \left(f_1^F \gamma^\mu + \frac{i}{2m_{\Sigma^-}} f_2^F \sigma^{\mu\nu} q_\nu - g^F \gamma^\mu \gamma_5 \right) \\ &\quad + (d_{i1k} + i d_{i2k}) \left(f_1^D \gamma^\mu + \frac{i}{2m_{\Sigma^-}} f_2^D \sigma^{\mu\nu} q_\nu - g^D \gamma^\mu \gamma_5 \right) \Big] u_{\Sigma^-} \\ &= \bar{u}_{\Lambda^0} \Gamma^\mu u_{\Sigma^-}, \end{aligned} \tag{2.16}$$

where

$$f_1^D(q^2) = -\frac{3}{2}F_1^{(\mathrm{n})}(q^2), \tag{2.17}$$

$$f_2^D(q^2) = -\frac{3}{2}F_2^{(\mathrm{n})}(q^2), \tag{2.18}$$

$$f_1^F(q^2) = F_1^{(\mathrm{p})}(q^2) + \frac{1}{2}F_1^{(\mathrm{n})}(q^2), \tag{2.19}$$

$$f_2^F(q^2) = F_2^{(\mathrm{p})}(q^2) + \frac{1}{2}F_2^{(\mathrm{n})}(q^2). \tag{2.20}$$

The octet classification of Λ^0 and Σ^- can be found in B.46 and the SU(3) structure constants f_{ijk} and d_{ijk} in B.39 and B.37. The axial form factors are obtained from a phenomenological parametrisation shown in B.67

$$g^D(q^2) = g^D(0)G_D^A(q^2). \tag{2.21}$$

The u-channel of the **quasifree process** is therefore described by the current

$$h_u^\mu = \int d\mathbf{p}_1\, \bar{\mathcal{U}}_\Lambda(p_1')\left[\Gamma^\mu \frac{\not{p} + m_{\Sigma^-}}{p^2 - m_{\Sigma^-}^2}\, \gamma_5\, G_{K\Sigma,N}\right]\mathcal{U}_n(p_1), \tag{2.22}$$

with

$$\Gamma^\mu = \frac{2}{\sqrt{6}}\left(f_1^D(q^2)\gamma^\mu + \frac{i}{2m_{\Sigma^-}} f_2^D(q^2)\sigma^{\mu\nu}q_\nu - g^D(q^2)\gamma^\mu\gamma_5\right). \tag{2.23}$$

2.2 Input

2.2.1 Particle masses

The masses of the baryons and mesons relevant to the Born channels of the reaction in eq. 2.2 are shown in tables 2.1 and 2.2. The muon mass is taken as 0.106 GeV and the neutrino is assumed to be massless.

The ^{12}C nucleus has a mass of 11.268 GeV. The mass of the hypernucleus is determined using

$$M_{\mathrm{hyper}} = M_{^{12}\mathrm{C}} - \left(m_n - E_n^{\mathrm{bound}}\right) + \left(m_\Lambda - E_\Lambda^{\mathrm{bound}}\right). \tag{2.24}$$

p	n	Λ	Σ^-
0.9383	0.9383	1.11563	1.19743

Table 2.1: The masses in GeV of the baryons in the s, t and u Born channels.

ρ	ω	ϕ	K^0	K^+
0.770	0.782	1.020	0.49767	0.49364

Table 2.2: The masses of the mesons in GeV.

2.2.2 Wave functions

The position space radial wave functions used as input are shown in figure 2.4. The neutron wave function has been calculated using the FSUGold model presented in [To05]. The hyperon wave function has been calculated with the NLSH model [Lu03]. The momentum space radial wave functions are shown in figure 2.5. They have been calculated using eq. 1.73.

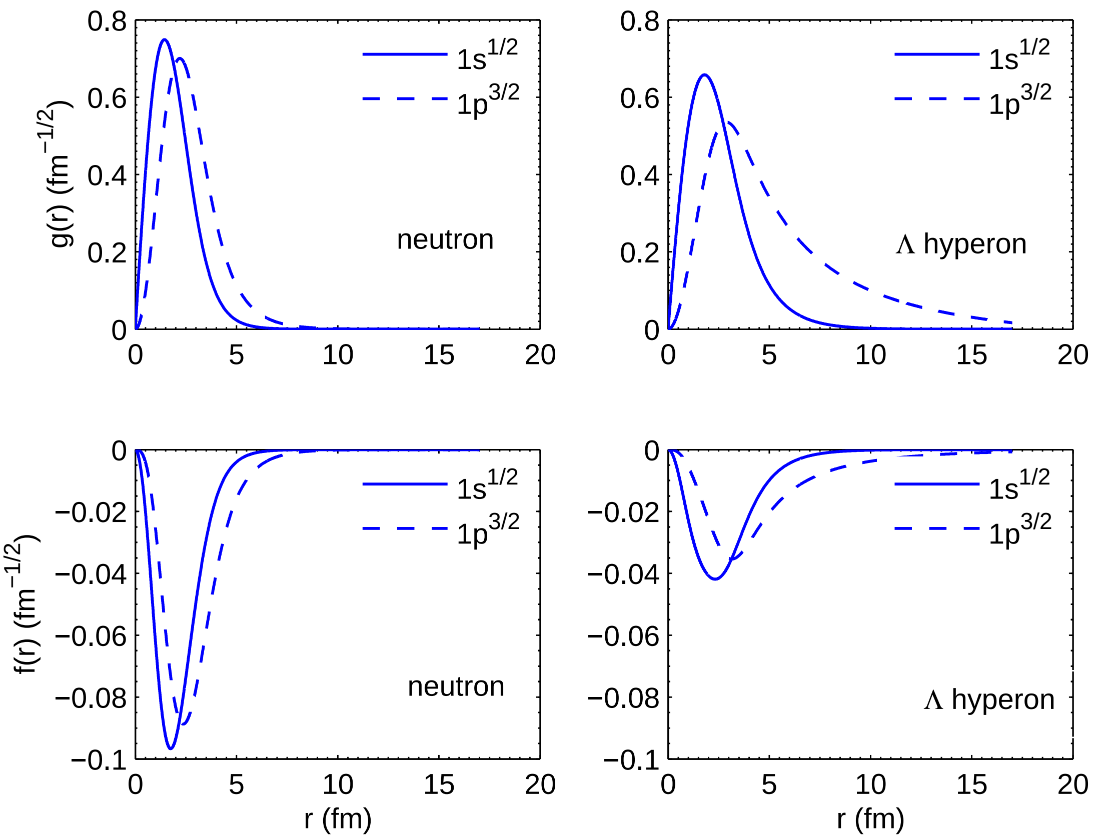


Figure 2.4: Upper (g(r)) and lower (f(r)) radial wave functions in position space of the $1s^{1/2}$ and $1p^{3/2}$ neutron and hyperon orbitals of ^{12}C

The binding energies derived from the FSUGold (neutron) and NLSH (hyperon) models are shown in table 2.3.

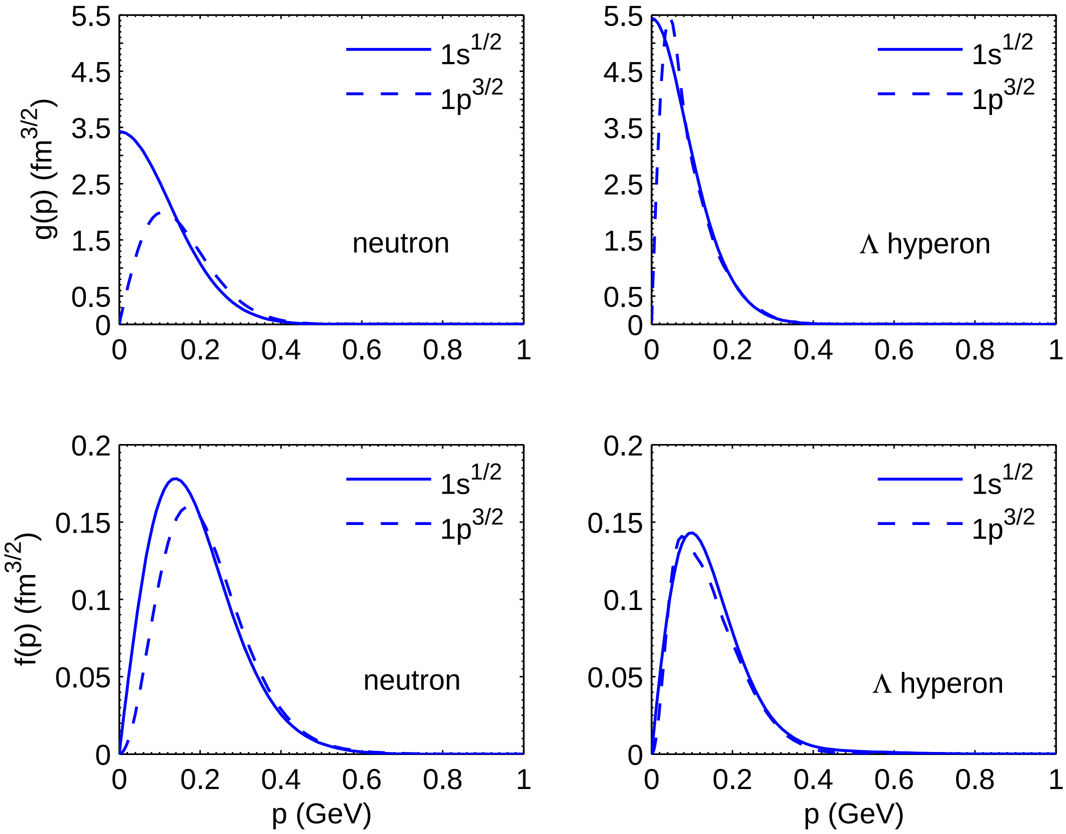


Figure 2.5: Upper (g(p)) and lower (f(p)) radial wave functions in momentum space of the $1s^{1/2}$ and $1p^{3/2}$ neutron and hyperon orbitals of ^{12}C

	$1s^{1/2}$	$1p^{3/2}$
neutron	42.80	16.87
hyperon	10.815	0.605

Table 2.3: Binding energies (in MeV) for the different nucleon and hyperon orbitals in a ^{12}C nucleus.

2.2.3 Kinematics

The particle masses also play a role in the threshold energy of the reaction. For an elementary process this energy is given by

$$E_{\mathrm{th}}^{\mathrm{el}} = \frac{(m_\mu + m_K + m_\Lambda)^2 - m_\nu^2 - m_N^2}{2m_N}. \tag{2.25}$$

For a quasifree reaction this equation is modified by the binding energies of the nucleon and hyperon

$$E_{\mathrm{th}}^{\mathrm{qfree}} = \frac{\left(m_\mu + m_K + m_\Lambda - E_\Lambda^{\mathrm{bound}}\right)^2 - m_\nu^2 - \left(m_N - E_N^{\mathrm{bound}}\right)^2}{2\left(m_N - E_N^{\mathrm{bound}}\right)} \tag{2.26}$$

The threshold energies for the different neutron to hyperon transitions are shown in table 2.4.

neutron \ hyperon	$1s^{1/2}$	$1p^{3/2}$
$1s^{1/2}$	1.174	1.194
$1p^{3/2}$	1.116	1.135

Table 2.4: Threshold energies (in GeV) for the different nucleon to hyperon transitions in a ^{12}C nucleus.

Three input data sets were selected to investigate the effect of different incident neutrino energies, transfer momenta and kaon angles on the cross section. The muon angle (θ') was kept fixed at 5°. The kaon angle θ'_2 was varied and the angle ϕ between the leptonic and hadronic plane was kept at 0°. The sets are shown in table 2.5. Cross section calculations were done for these different energy sets for all the different combinations of transitions between the $1s^{1/2}$ and $1p^{3/2}$ neutron and $1s^{1/2}$ and $1p^{3/2}$ hyperon orbitals.

	E_k (GeV)	$E_{k'}$ (GeV)	θ' (deg)
Set 1	2.0	1.0	5
Set 2	3.0	1.5	5
Set 3	4.0	2.0	5

Table 2.5: The input kinematics for which the cross section was calculated.

2.2.4 Coupling constants

The values of the strong coupling constants used for the baryon-baryon-meson vertices are taken from [Sh75]. They are $G_{K\Lambda,N} = -10$ and $G_{K\Sigma,N} = 1.3$.

2.3 Results

The differential cross sections have been plotted as functions of the kaon angle θ_2'. In the figures the following conventions have been adopted:

- *total* refers to the differential cross section calculated with the complete hadronic current operator i.e. the sum of the operators for the s, t and u-channels.
- h' refers to the helicity of the outgoing muon

As expected, the cross sections peak for small kaon angles. They then fall off smoothly to zero as the kaon angle increases.

Higher incident neutrino energies and larger momentum transfer result in larger differential cross sections (at small kaon angles) for all the neutron to hyperon transitions. The results also show that higher incident neutrino energies and momentum transfer cause the cross sections to drop to zero more rapidly (as the kaon angle increases) for a specific neutron to hyperon transition

Figure 2.18 shows the differential cross sections for specific neutrino/muon energy sets plotted as funtions of the kaon angle θ_2'. The different curves resemble the different neutron to hyperon transitions. The cross sections nicely display the nuclear structure. The two highest cross sections are for the $1p^{3/2}$ (valence) neutron orbital. Of these two, the cross section for the transition to the $1s^{1/2}$ Λ state is the highest. Similar behaviour is seen for the transitions from the $1s^{1/2}$ neutron state.

The figures showing the contribution of the different helicity states of the outgoing muon to the total differential cross section show that the contribution of negative helicity muon ($h' = -1$) far exceeds that of the positive helicity one for all transitions and all energy sets. This can be ascribed to the fact the the muon mass is so small in comparison with its energy that the helicity of the neutrino ($h = -1$) is more or less conserved in the reaction.

The figures also show that the largest channel-contribution to the differential cross sections come from the s and t-channels for which the form factors are exact in the sense that we use phenomenological parametrisations. The u-channels, for which we assumed an exact SU(3) symmetry which lead to approximate form factors in the baryon octet, deliver only a small contribution for all transitions and energy sets.

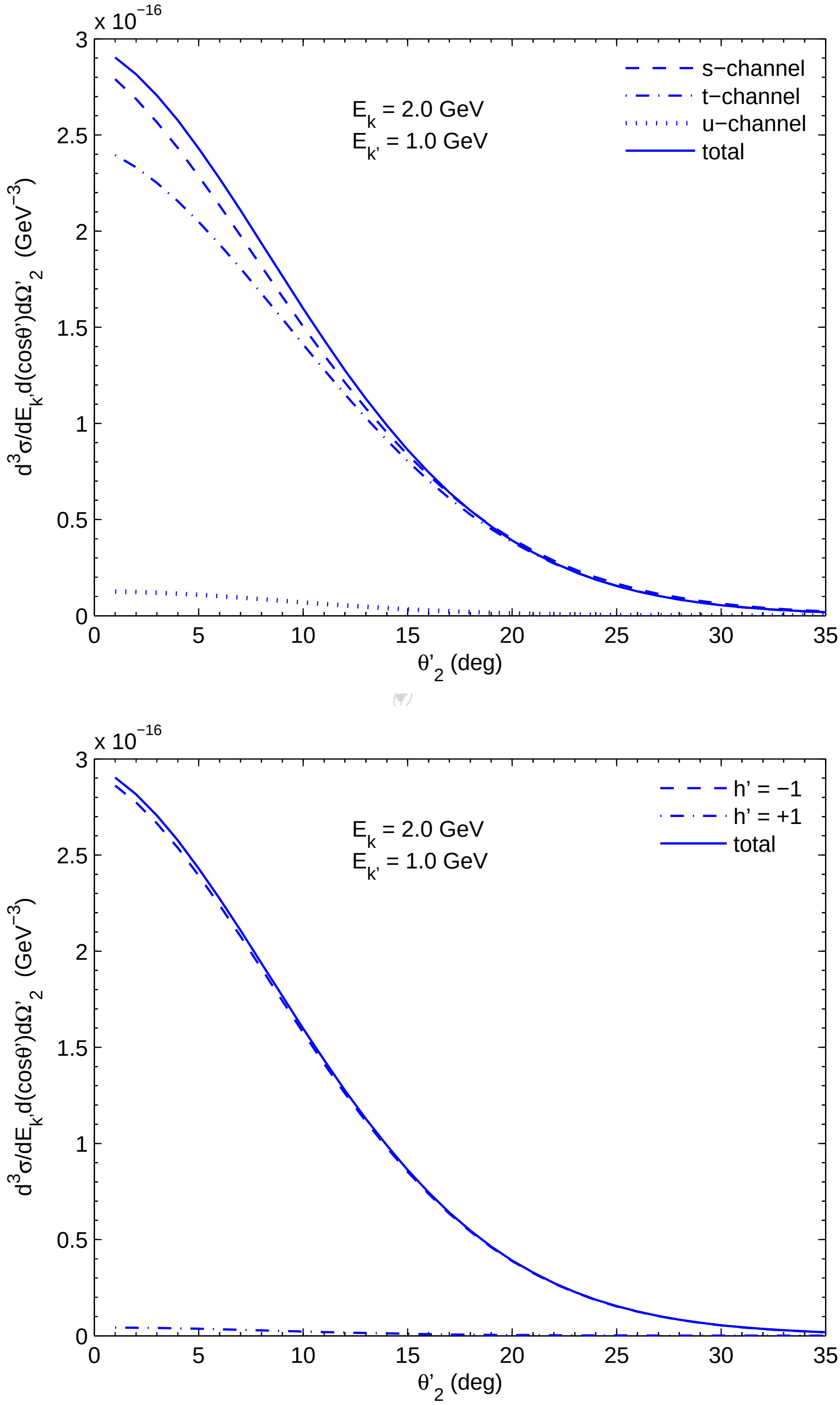


Figure 2.6: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1s^{1/2}$ neutron and $1s^{1/2}$ hyperon orbitals.

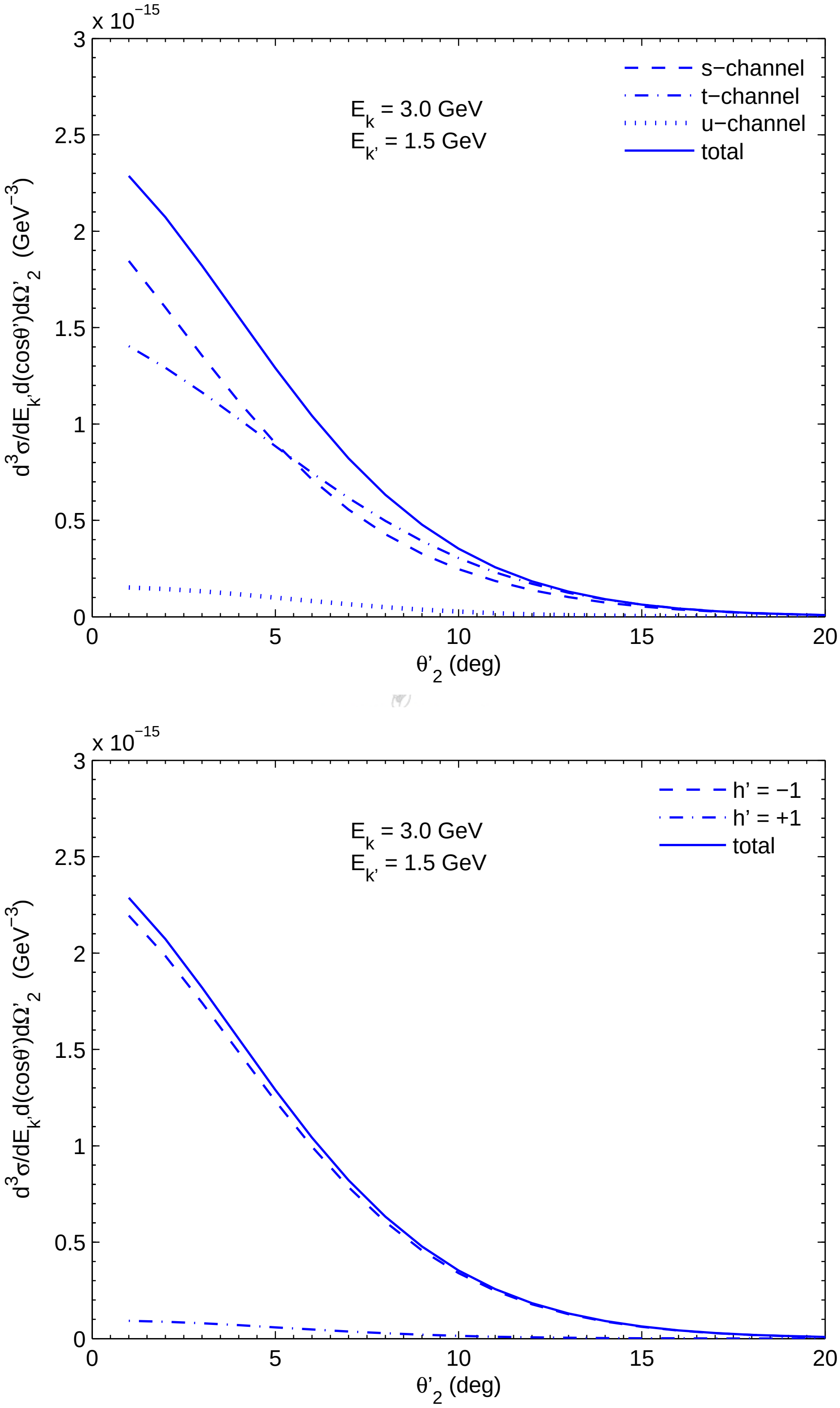


Figure 2.7: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1s^{1/2}$ neutron and $1s^{1/2}$ hyperon orbitals.

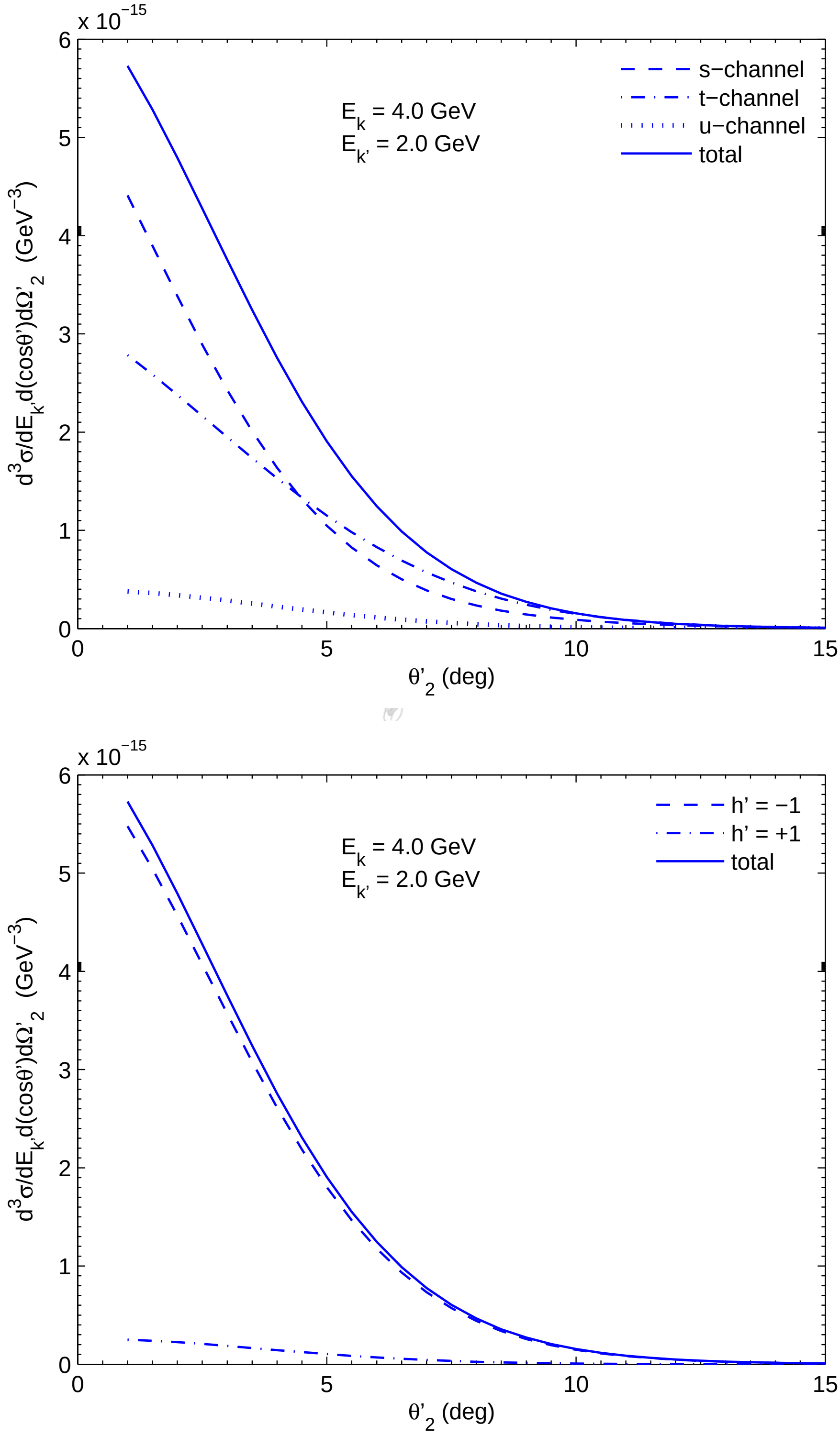


Figure 2.8: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1s^{1/2}$ neutron and $1s^{1/2}$ hyperon orbitals.

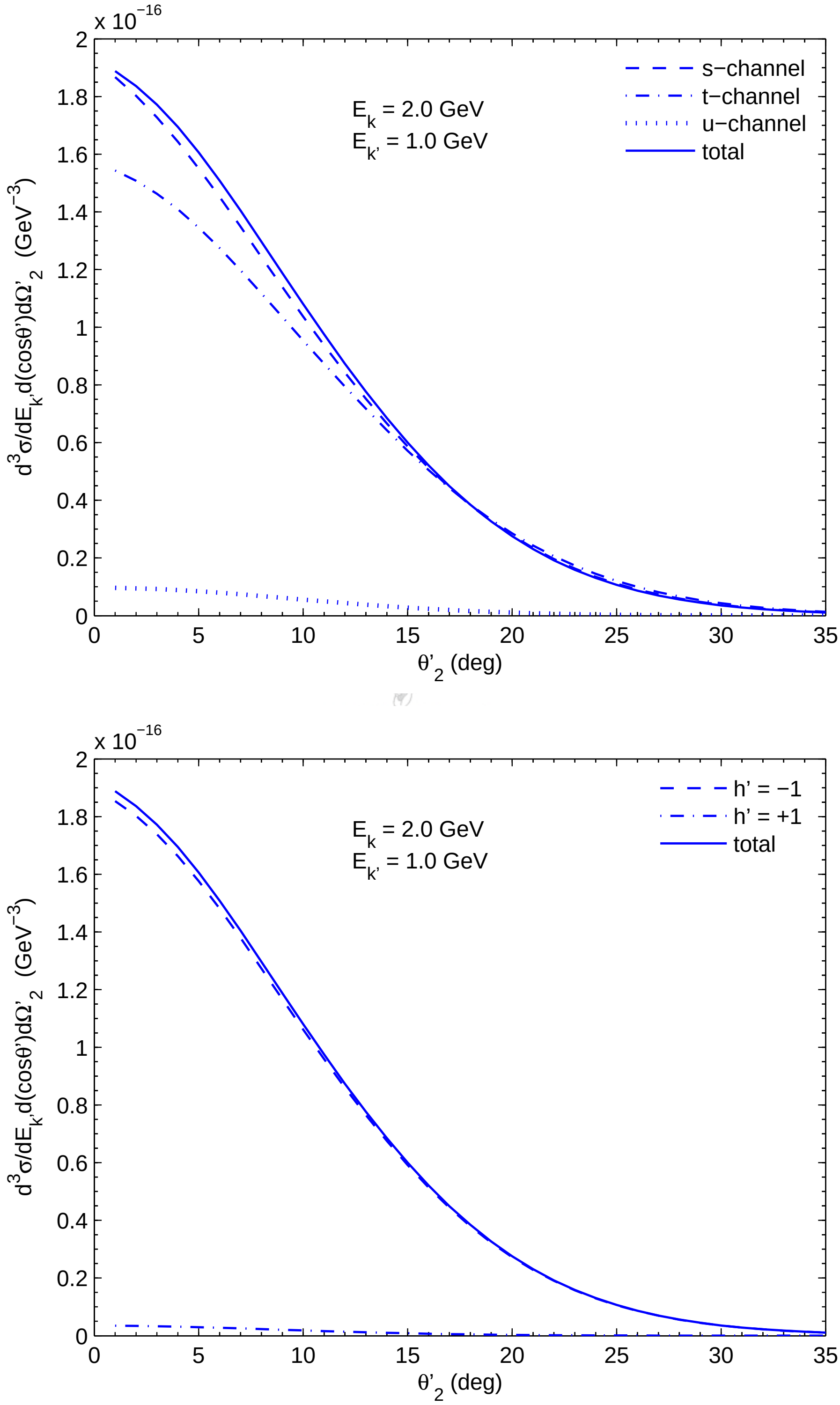


Figure 2.9: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1s^{1/2}$ neutron and $1p^{3/2}$ hyperon orbitals.

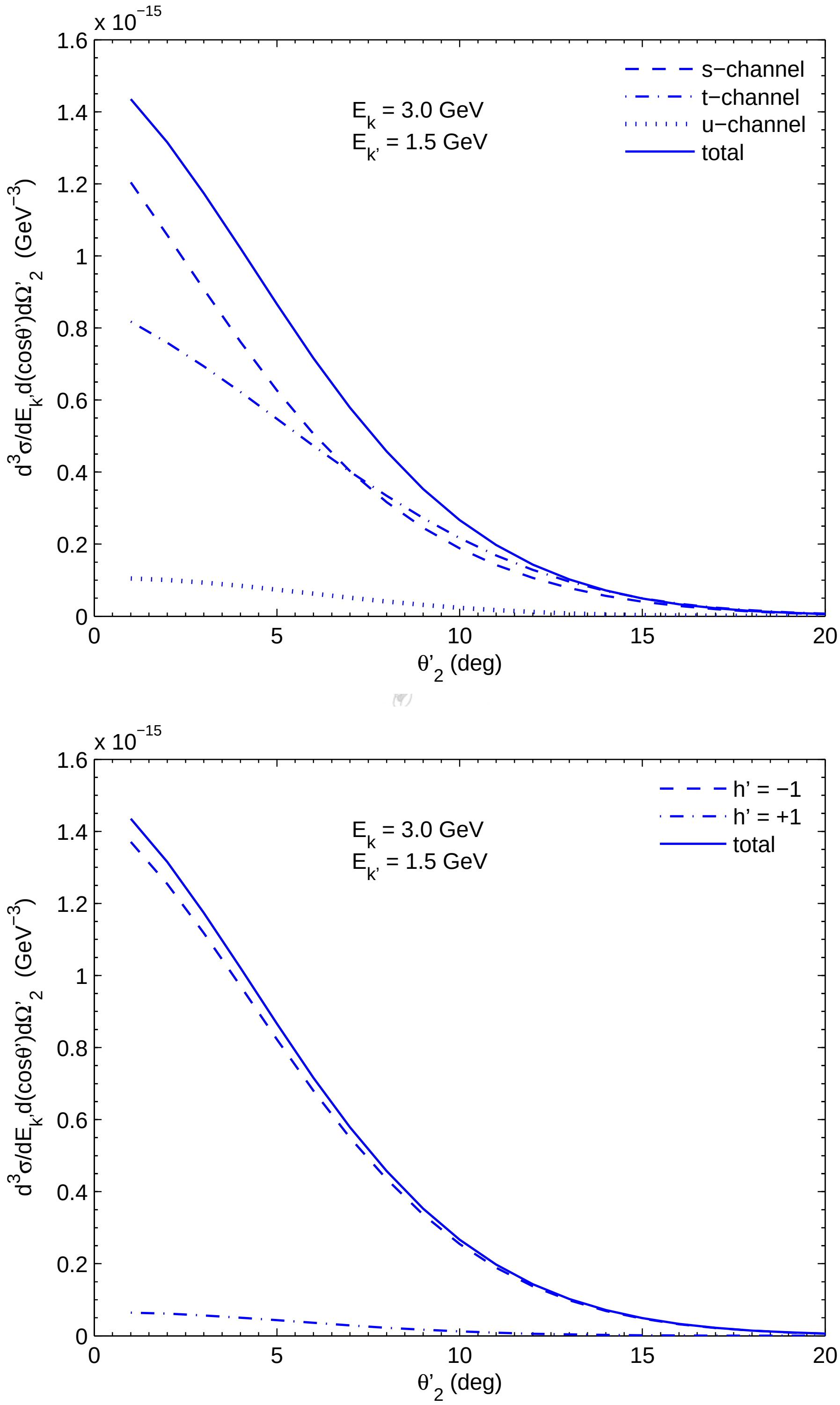


Figure 2.10: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1s^{1/2}$ neutron and $1p^{3/2}$ hyperon orbitals.

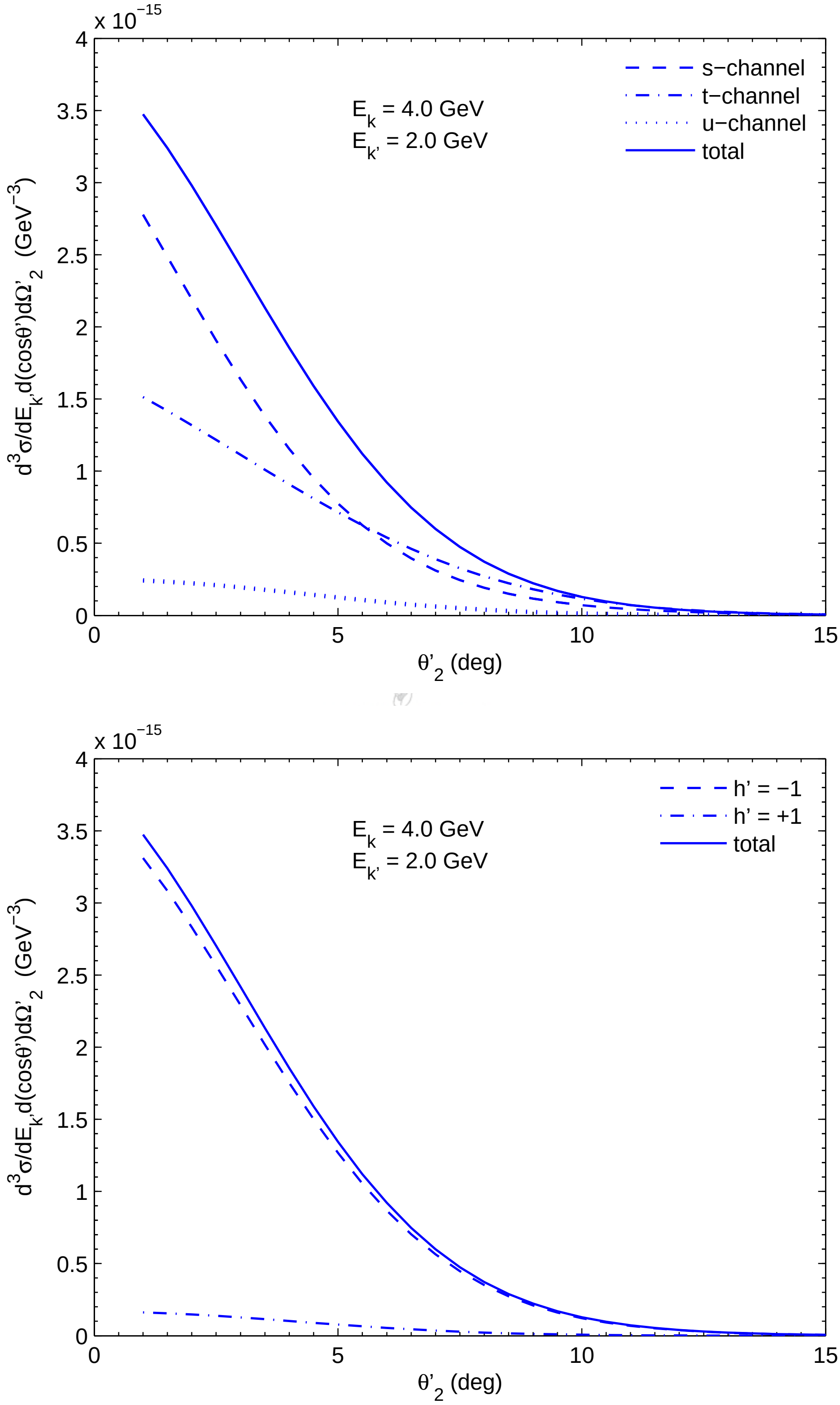
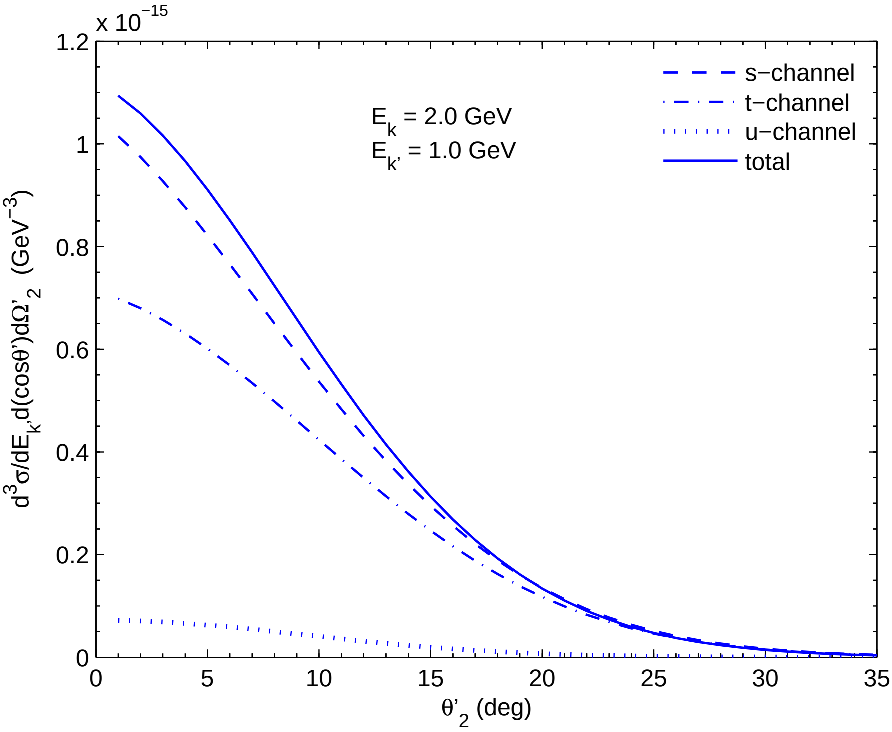


Figure 2.11: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1s^{1/2}$ neutron and $1p^{3/2}$ hyperon orbitals.



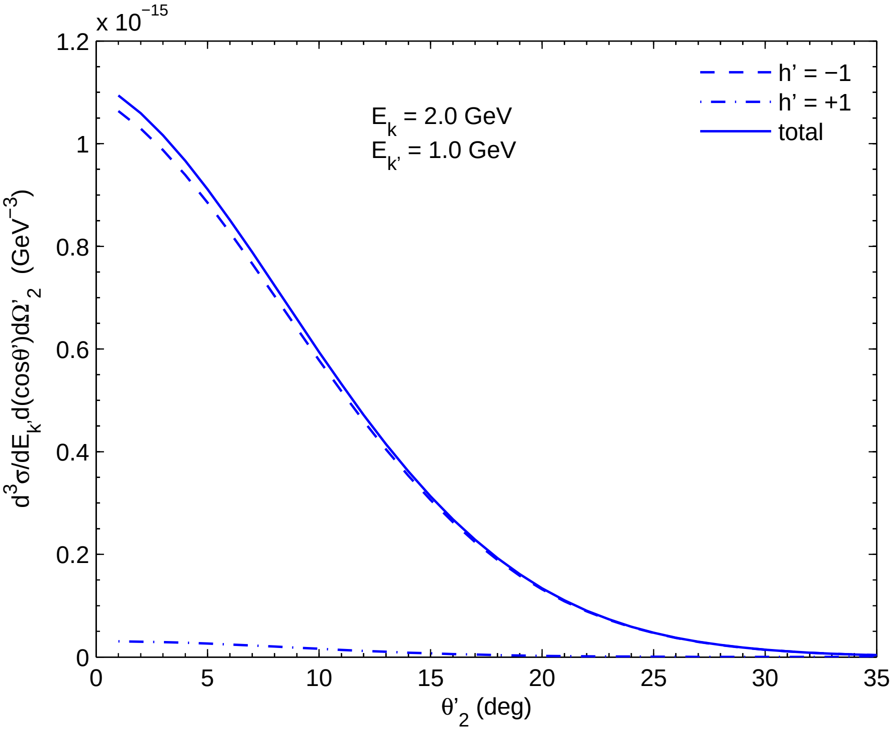
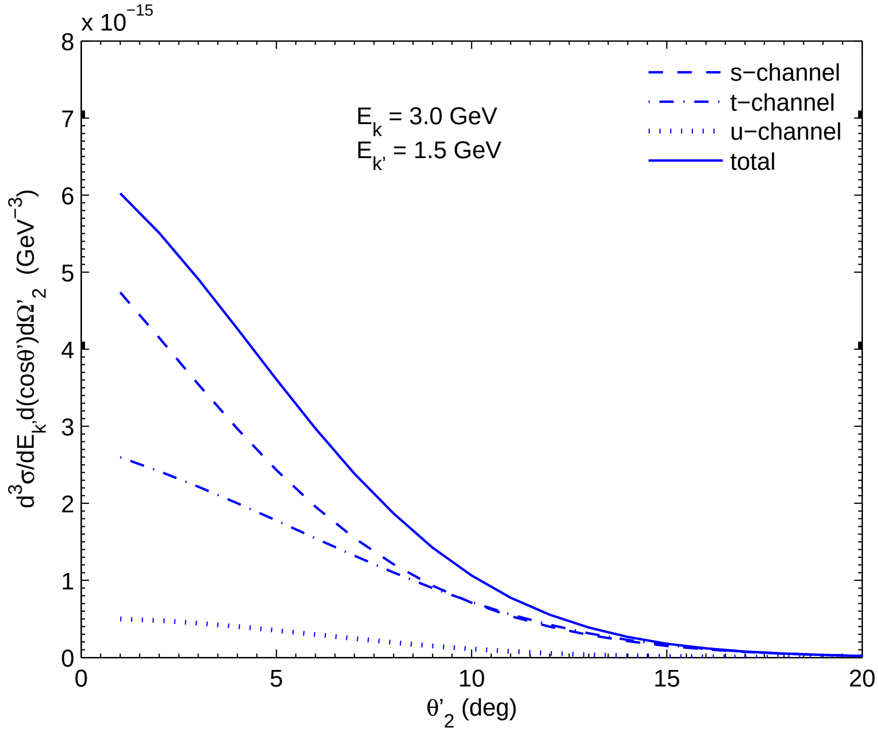


Figure 2.12: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1p^{3/2}$ neutron and $1s^{1/2}$ hyperon orbitals.



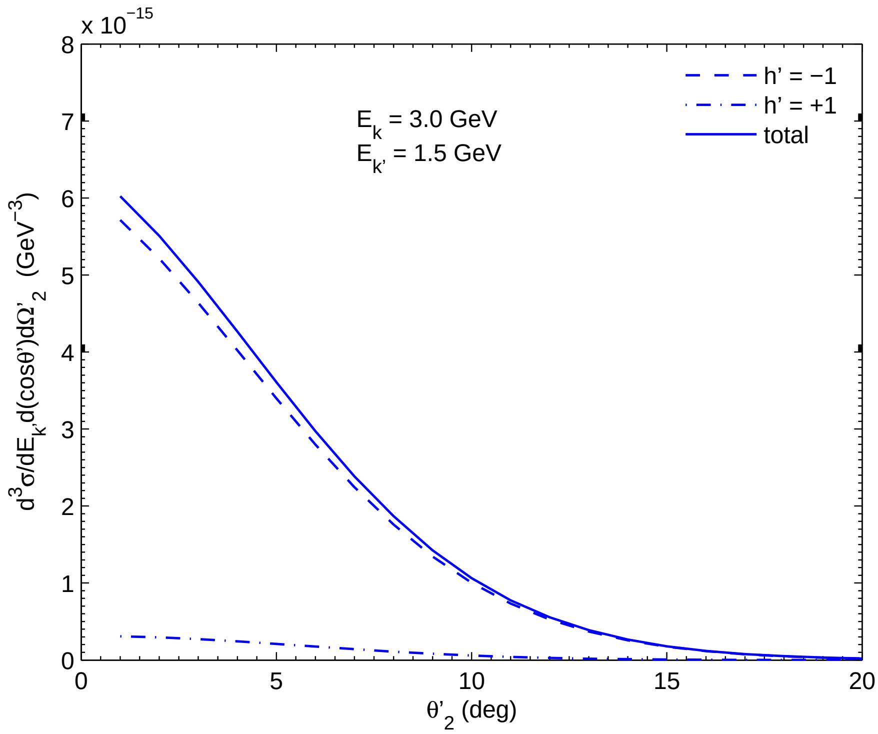


Figure 2.13: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1p^{3/2}$ neutron and $1s^{1/2}$ hyperon orbitals.

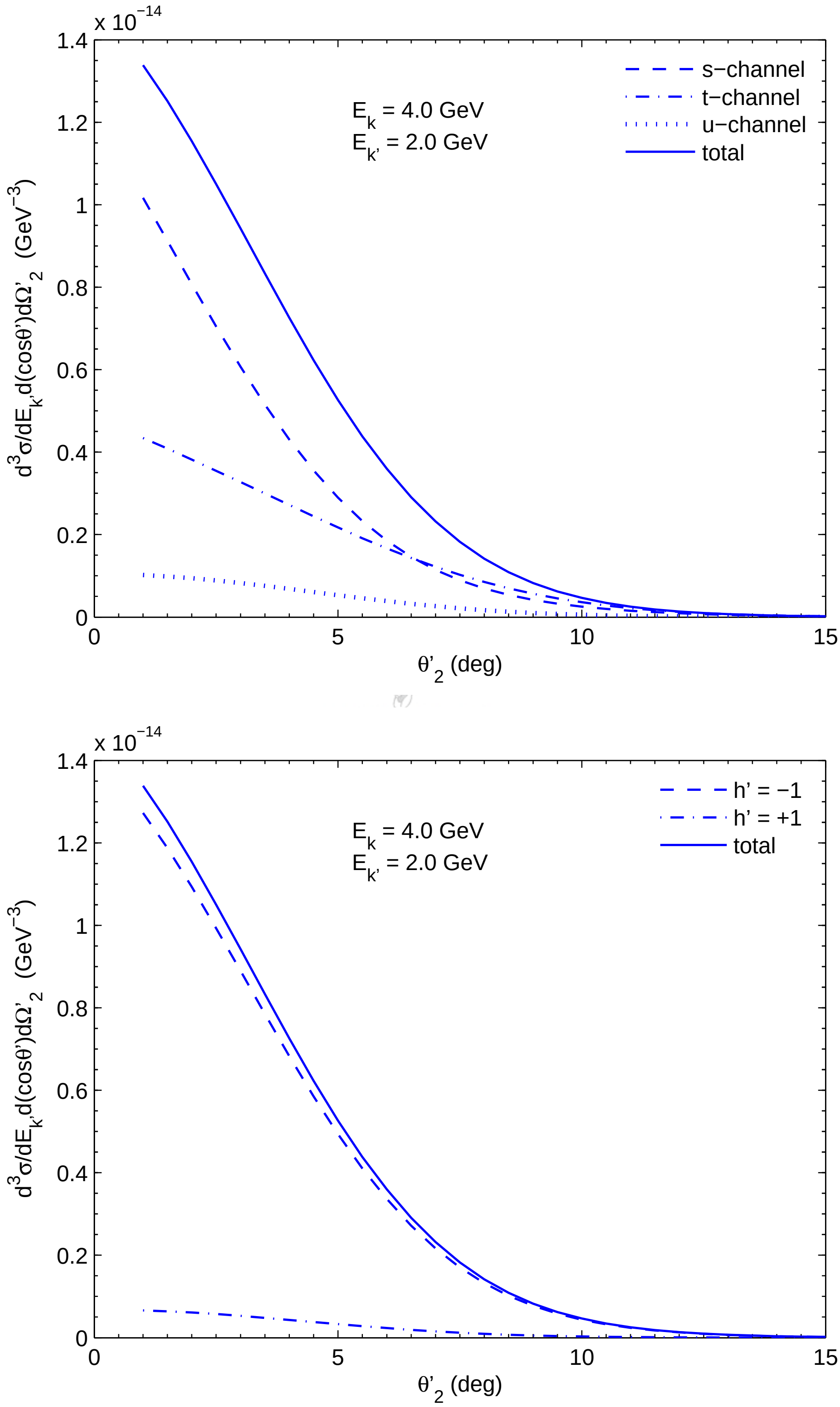


Figure 2.14: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1p^{3/2}$ neutron and $1s^{1/2}$ hyperon orbitals.

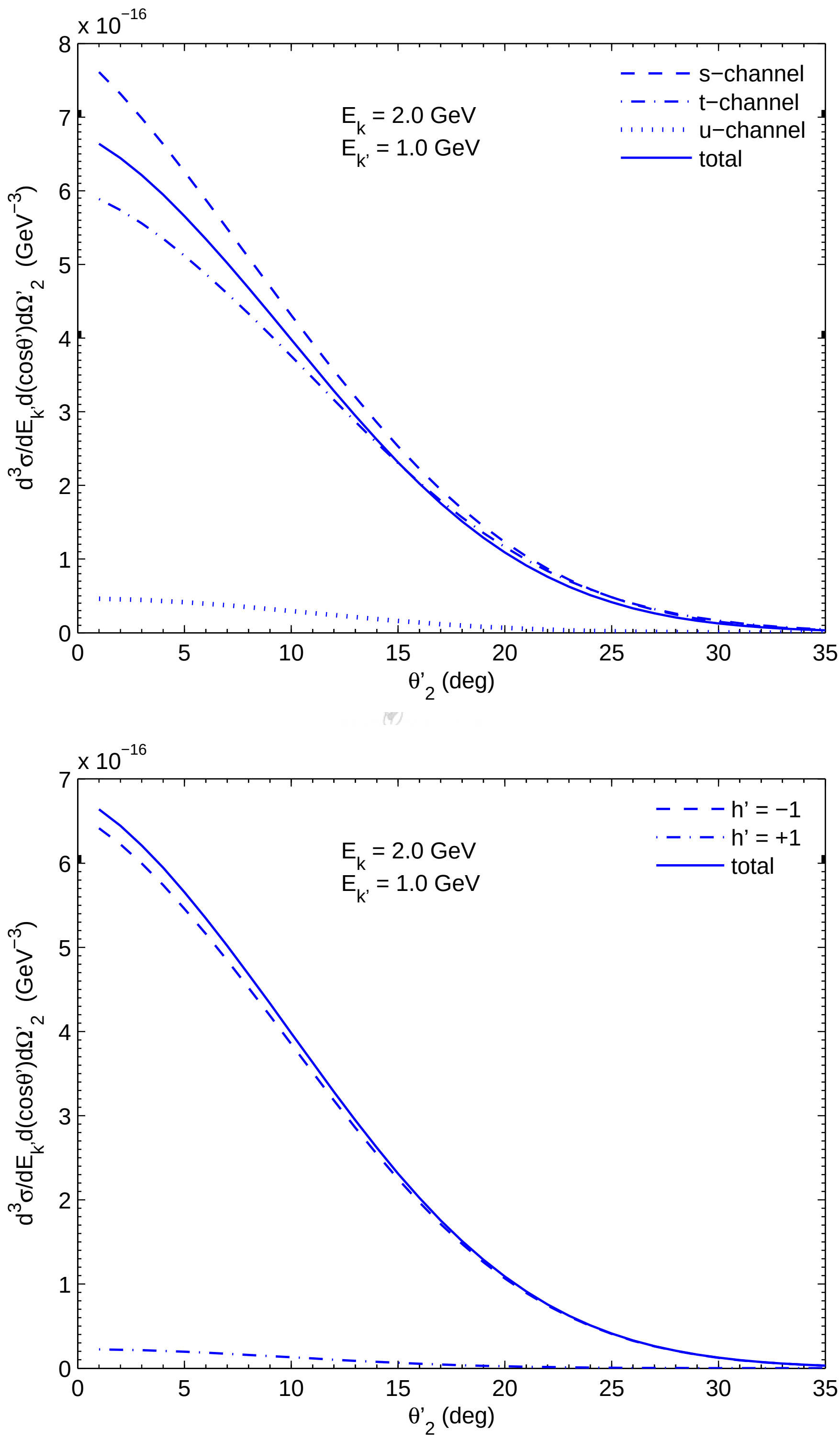
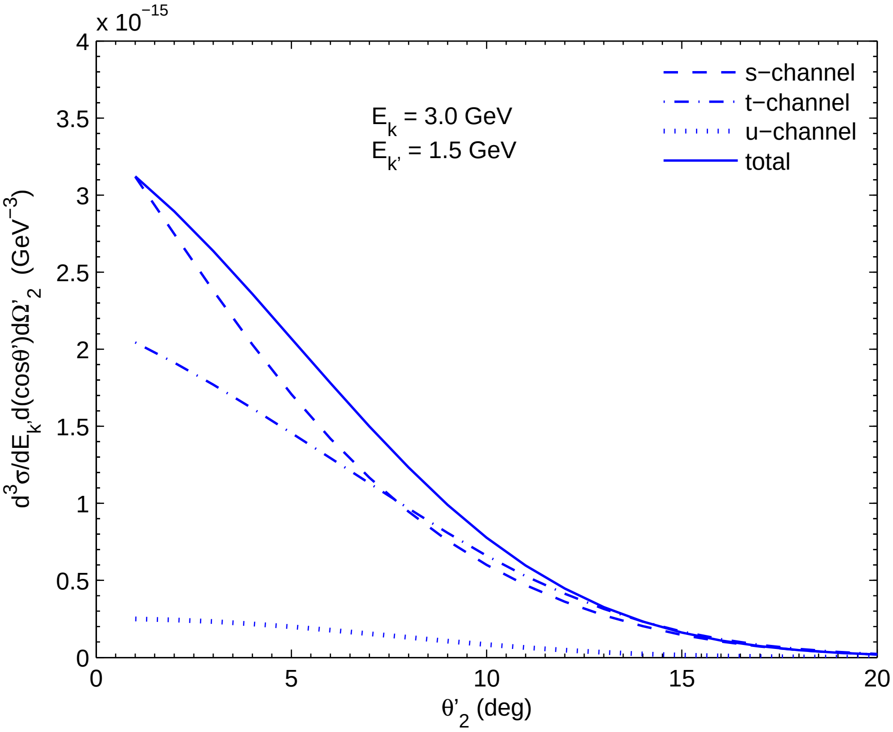


Figure 2.15: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1p^{3/2}$ neutron and $1p^{3/2}$ hyperon orbitals.



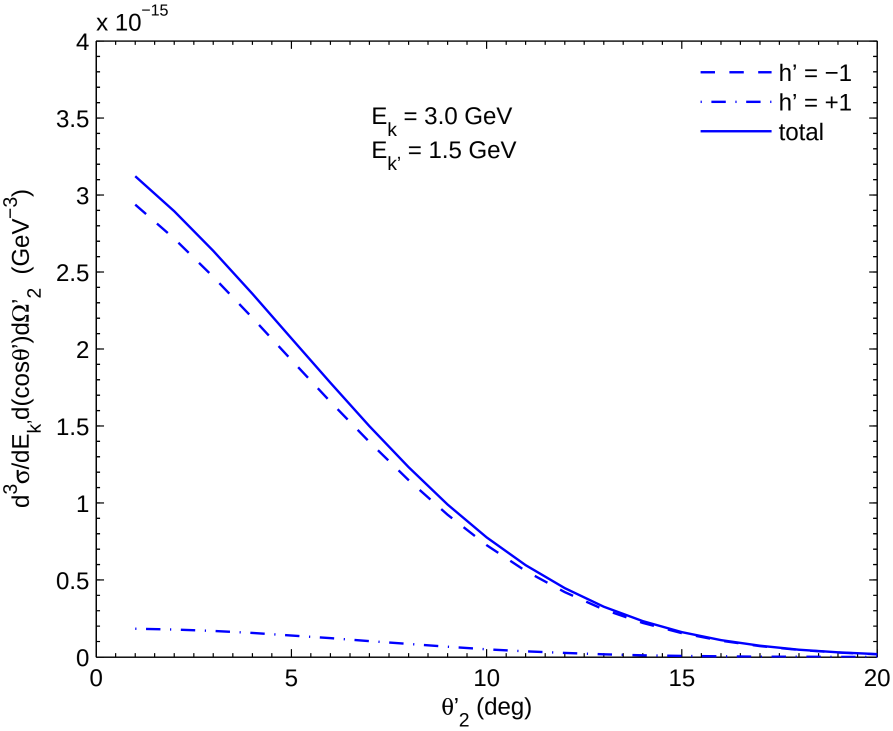


Figure 2.16: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1p^{3/2}$ neutron and $1p^{3/2}$ hyperon orbitals.

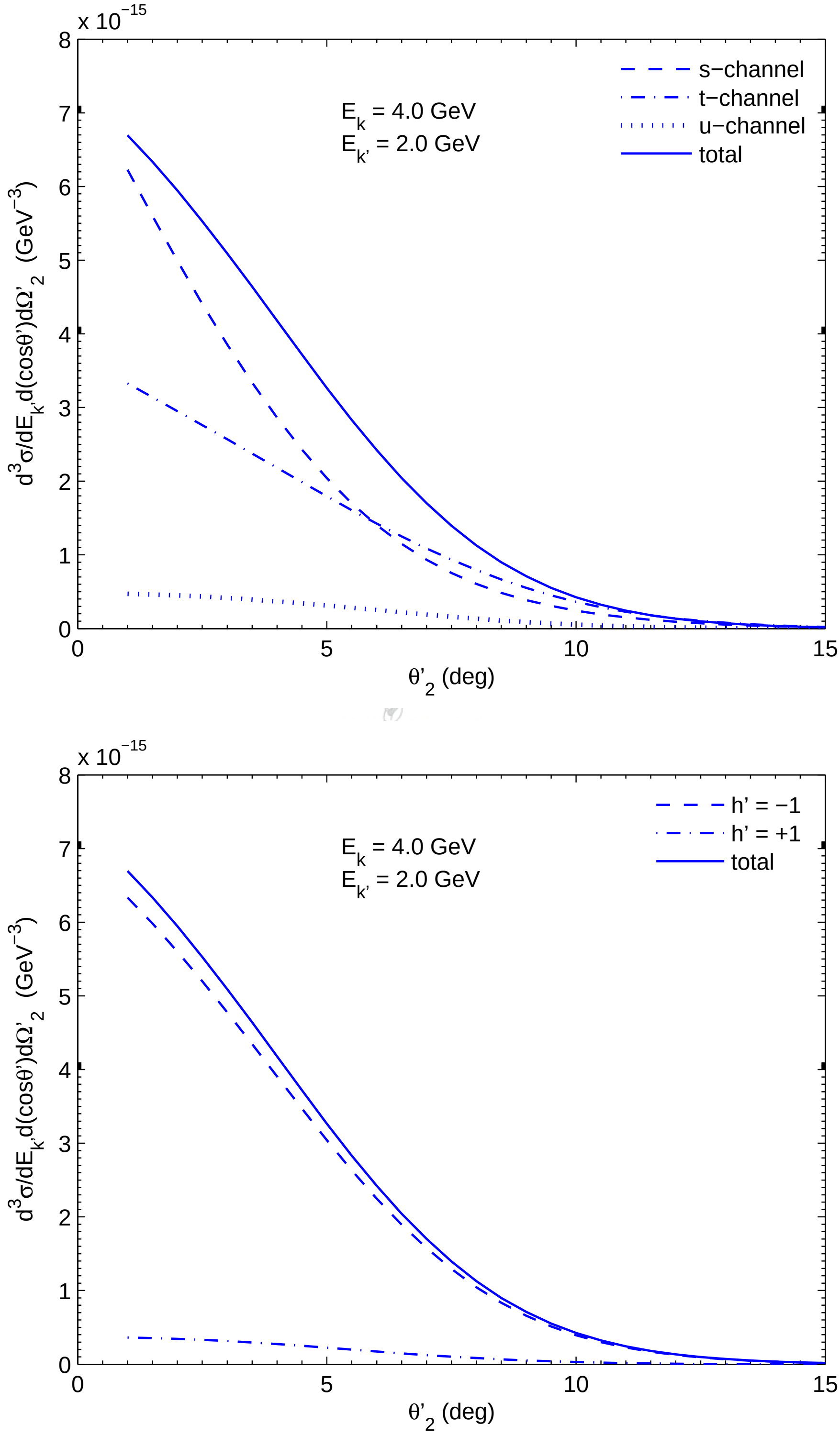


Figure 2.17: Differential cross section (top) and contributions of the different helicity states of the outgoing muon to the total differential cross section (bottom) for the $1p^{3/2}$ neutron and $1p^{3/2}$ hyperon orbitals.

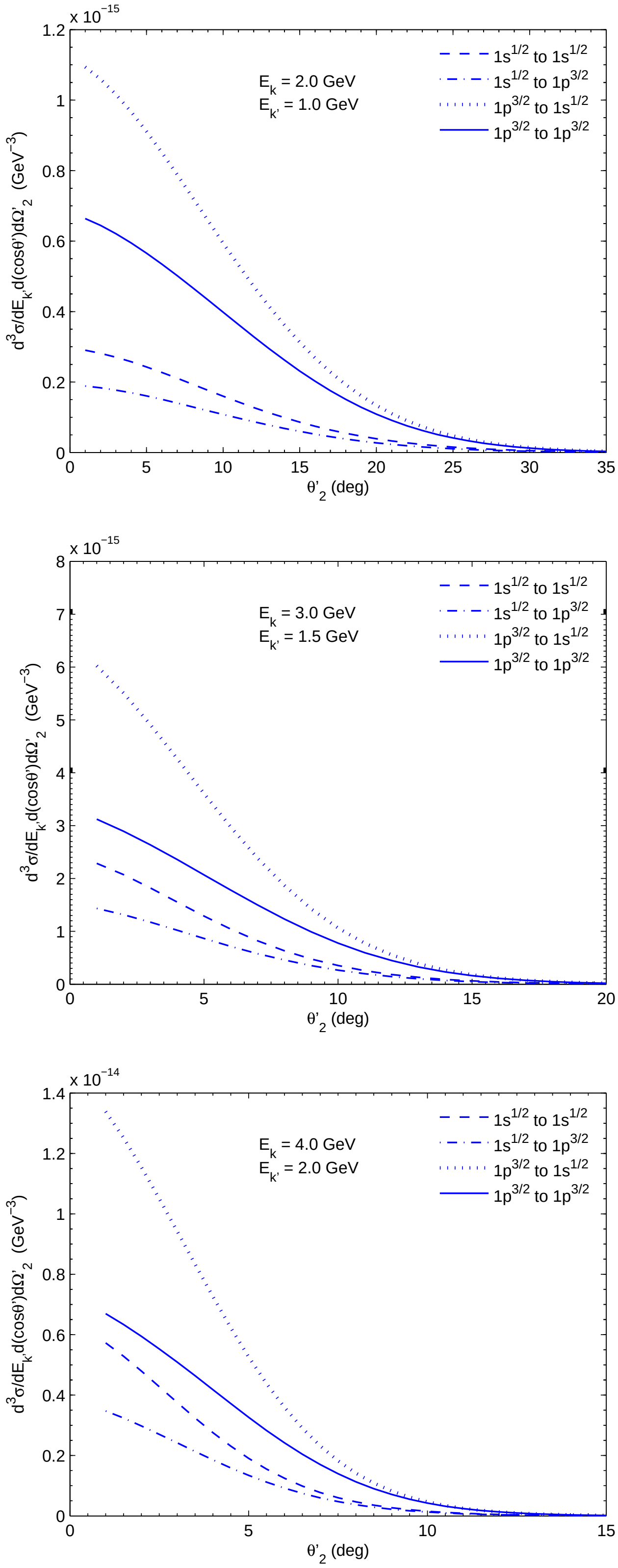


Figure 2.18: Comparison of the differential cross sections of the different neutron to hyperon transitions.

Conclusion and outlook

We have shown that using a relativistic approach to kinematics and the dynamics of the weak interaction one can construct a fully relativistic model for the description of the production of hypernuclei via the weak interaction. This is done by a standard treatment of the leptonic vertex and a model-independent parametrisation of the hadronic vertex. The strength of our model lies in the fact that up until a choice has been made for the hadronic current operator and the wave functions, it is fully model-independent and a large part of the calculations can be treated without considering the explicit form of the hadronic current.

In this work a simple model for the underlying elementary process was adopted since the primary aim was to develop a general relativistic formalism for weak hypernuclei production. However, since the elementary amplitudes form one of the primary components (the other being the wave functions) of our fomalism, an urgent need exists for experimental work on the weak production of strange particles. This will further quantify the model used for the elementary process. Note, however, since the formalism was developed in terms of nuclear structure functions, improvements in the model for the elementary process would only impact the hadronic tensor. The rest of the formalism will be left unchanged.

We believe that our model and its predictions will serve as a solid baseline for further investigation of this interaction process and the nuclear structure effects as well as for the interpretation of results coming from experiments in the nearby future. These experiments could have important consequences for the Standard Model and we hope that we have contributed to the understanding of observations of the weak interaction between neutrinos and nuclei.

APPENDIX A
The Weak Interaction and Glashow-Salam-Weinberg Theory

A.1 Phenomenological picture

[2]At first the weak interaction was thought to be of the vector nature, analogous to the electromagnetic interaction. Fermi postuled a four-fermion point interaction consisting of the contraction of vector leptonic and hadronic currents

$$\mathcal{L}_{\mathrm{wk}} = -\frac{G_F}{\sqrt{2}}(\ell_\mu^\dagger h^\mu + \ell^\mu h_\mu^\dagger), \tag{A.1}$$

with $\ell^\mu = \bar{u}\gamma^\mu u$ and G_F the Fermi constant for beta decay. In order to include all the possible leptonic, hadronic and semi-leptonic processes a general Hamiltonian function for all weak processes consisting of leptonic and hadronic currents can be written as [Wa95]

$$H_{\mathrm{wk}} = \frac{G_F}{\sqrt{2}} J^\mu_{\mathrm{wk}} J_\mu^{\dagger\,\mathrm{wk}}, \tag{A.2}$$

with

$$J_\mu^{\mathrm{wk}} = \ell_\mu + h_\mu. \tag{A.3}$$

The Fermi point-like form of the matrix element for a semileptonic process will therefore be

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\ell_\mu h^\mu. \tag{A.4}$$

To account for the short range of the weak interaction as well as a first step in trying to unify weak and electromagnetic forces, a massive weak analog to the photon was postulated. These so-called intermediate vector bosons could be charged, unlike the photon, to account for the possible change in charge involved in weak processes. This lead to the introduction of the propagator for massive vector bosons

$$D^{\mu\nu} = \frac{-g^{\mu\nu} + q^\mu q^\nu / M^2_{\mathrm{boson}}}{q^2 - M^2_{\mathrm{boson}}}, \tag{A.5}$$

where q is the momentum transfer. This leads to the general weak matrix element for a semi-leptonic process

$$-i\mathcal{M} = [\eta_\ell \ell^\nu][iD_{\mu\nu}][\eta_h h^\mu], \tag{A.6}$$

[2]The reference in section A.1 is [Ai82] unless stated otherwise.

with η_l and η_h containing the couplings of the vector boson To the leptons and hadrons.

For $q^2 \ll M^2_{\text{boson}}$ therefore

$$D^{\mu\nu} = \frac{g_{\mu\nu}}{M^2_{\text{boson}}} \tag{A.7}$$

and

$$-i\mathcal{M} = [\eta_\ell \ell_\mu][\frac{i}{M^2_{\text{boson}}}][\eta_h h^\mu]. \tag{A.8}$$

A.1.1 Charged currents

A.1.1.1 Leptons

From experimental results it is known that only the left-handed neutrinos (spin antiparallel to the direction of motion i.e. *negative helicity*) take part in the weak interaction. This arises because of the fact that the weak interaction does not conserve parity. The *helicity* of a particle is a measure of the alignment of its spin and momentum

$$h = \frac{\boldsymbol{p} \cdot \boldsymbol{s}}{|\boldsymbol{p}|}. \tag{A.9}$$

Since helicity is a pseudoscalar, it changes sign under the parity transformation. Any interaction in which only one helicity state of a particle participates therefore violates parity. The weak interaction is therefore one such interaction. To take this in account into the leptonic weak current operator we first define the lepton spinor as

$$u = \begin{bmatrix} \phi \\ \chi \end{bmatrix} \tag{A.10}$$

and observe the Dirac equation in the form $Eu = (\boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m)u$. If we make use of the gamma matrices in the representation

$$\gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{A.11}$$

the Dirac equation becomes

$$\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \phi = \frac{E}{|\boldsymbol{p}|}\, \phi - \frac{m}{|\boldsymbol{p}|}\, \chi, \tag{A.12}$$

$$\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}\, \chi = \frac{-E}{|\boldsymbol{p}|}\, \chi + \frac{m}{|\boldsymbol{p}|}\, \phi. \tag{A.13}$$

In the case of massless leptons (like neutrinos) this simplifies to

$$\begin{aligned} \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\,\phi &= \phi \quad \mathrm{right-handed}, && \text{(A.14)} \\ \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\,\chi &= -\chi \quad \mathrm{left-handed}. && \text{(A.15)} \end{aligned}$$

The lower component of the spinor is thus the one with negative helicity and if we define a projection operator

$$P_L = \frac{1}{2}(1-\gamma_5) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{A.16}$$

we can see that it is exactly this operator that selects the left-handed component of the neutrino spinor. If we write the general lepton spinor as

$$\begin{aligned} u &= \left(\frac{1+\gamma_5}{2}\right)u + \left(\frac{1-\gamma_5}{2}\right)u \\ &= P_R u + P_L u \\ &= u_R + u_L, \end{aligned} \tag{A.17}$$

where the subscripts R and L now refer to the right- and left-handed components respectively, we can write the vector interaction as

$$(u_R^\dagger + u_L^\dagger)(\gamma_0\gamma_\mu)(u_R + u_L). \tag{A.18}$$

Since both projection operators contain γ_5 they commute with γ_μ and the interaction can be written as

$$u_R^\dagger(\gamma_0\gamma_\mu)u_R + u_L^\dagger(\gamma_0\gamma_\mu)u_L. \tag{A.19}$$

This implies that the vector interaction (V) conserves helicity. If we insert a γ_5 into the current operator (making it an axial vector interaction A), helicity is still conserved. The total leptonic charged current operator is therefore a combination of V and A

$$L_\mu \propto \gamma_\mu(1-\gamma_5). \tag{A.20}$$

A general leptonic current can be writen in the form

$$\ell_\mu = \frac{g}{2\sqrt{2}}\,\bar{u}_2\gamma_\mu(1-\gamma_5)u_1, \tag{A.21}$$

where g is the coupling strength of the W boson. If we define

$$\eta_l = -\frac{ig}{2\sqrt{2}}, \tag{A.22}$$

we can use A.8 to write the matrix element of a purely leptonic process for the case $q^2 \ll M_W^2$

$$\begin{aligned} -i\mathcal{M} &= \frac{i\eta_l^2}{M_W^2}\,\ell_\mu \ell^\mu \\ &= \frac{-ig^2}{8M_W^2}\,\bar{u}_3\gamma_\mu(1-\gamma_5)u_1\,\bar{u}_4\gamma^\mu(1-\gamma_5)u_2. \end{aligned} \tag{A.23}$$

Comparison to the Fermi point-like matrix element A.4 gives the relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}. \tag{A.24}$$

A.1.1.2 Quarks and Hadrons

Experimental results show that hadronic charge-changing weak currents are V-A in nature as is the case for the leptonic currents. They also show that the strangeness-changing interactions are suppressed relative to the strangeness-conserving ones. This lead Cabibbo to postulate that the strength of the hadronic weak interaction is shared between the two types of processes according to

$$h \propto h_{\Delta S=0}\cos\theta_C + h_{\Delta S=1}\sin\theta_C, \tag{A.25}$$

where θ_C is the Cabibbo angle ($\theta_C \approx 0.13$). This simply means that the two types of hadronic charge-changing currents can be written as

$$\begin{aligned} j^\mu(d\to u) &= \frac{g}{2\sqrt{2}}\cos\theta_C\;\bar{u}\gamma^\mu(1-\gamma_5)d, \quad \Delta S = 0, \\ j^\mu(s\to u) &= \frac{g}{2\sqrt{2}}\sin\theta_C\;\bar{u}\gamma^\mu(1-\gamma_5)s, \quad \Delta S = 1, \end{aligned}$$

where u, d and s refer to the up, down and strange quarks respectively.

If we define

$$\eta_h = \frac{-ig}{2\sqrt{2}}\cos\theta_C, \tag{A.26}$$

we can use A.8 to write for a $\Delta S = 0$ weak charge-changing process where $q^2 \ll M_W^2$

$$\begin{aligned} -i\mathcal{M}_{\rm CC} &= \frac{i\eta_\ell \eta_h}{M_W^2}\,\ell_\mu h^\mu \\ &= -\frac{ig^2}{8M_W^2}\cos\theta_C\;\ell_\mu h^\mu \\ &= -\frac{iG_F}{\sqrt{2}}\cos\theta_C\;\ell_\mu h^\mu, \\ \mathcal{M}_{\rm CC} &= \frac{G_F}{\sqrt{2}}\cos\theta_C\;\ell_\mu h^\mu. \end{aligned} \tag{A.27}$$

A.1.2 Neutral currents

The existence of a neutral weak current can be explained by the presence of a heavy neutral boson (Z^0). The neutral currents are assumed to be a mixture of vector and axial vector currents and can in general be parametrised in terms of left- and right-handed components of particles. For leptons in general

$$\ell_\mu^{NC} = g_N\;\bar{u}\gamma_\mu\left(c_L^\ell\frac{(1-\gamma_5)}{2} + c_R^\ell\frac{(1+\gamma_5)}{2}\right)u. \tag{A.28}$$

For the neutrino this simply implies

$$\ell_\mu^{NC} = g_N c^\nu\;\bar{u}_\nu\gamma_\mu\left(\frac{(1-\gamma_5)}{2}\right)u_\nu. \tag{A.29}$$

The factor g_N determines the overall strength of the NC in relation to the CC. The coupling c^ν and the factor g_N can be determined from experiment

$$g_N = \frac{g}{\cos\theta_W}, \tag{A.30}$$

$$c^\nu = \frac{1}{2}, \tag{A.31}$$

with θ_W known as the Weinberg angle ($\sin\theta_W \approx 0.23$). In section A.2 it will be shown that $\cos\theta_W = \frac{M_W}{M_Z}$.

The quark neutral current can also be written as

$$j^\mu_{NC} = g_N\;\bar{q}\gamma^\mu\left(c_L^q\frac{1-\gamma_5}{2} + c_R^q\frac{1+\gamma_5}{2}\right)q, \tag{A.32}$$

where q can now refer to u, d and s quarks. Again from experiment it follows that

$$c_L^u = \frac{1}{2} - \frac{2}{3}a, \tag{A.33}$$
$$c_R^u = -\frac{2}{3}a, \tag{A.34}$$
$$c_L^d = -\frac{1}{2} + \frac{1}{3}a, \tag{A.35}$$
$$c_R^d = \frac{1}{3}a, \tag{A.36}$$

where $a = \sin\theta_W$. Note that the absence of hadronic strangeness-changing neutral currents ($\Delta S = 1$, $\Delta Q = 0$) lead to the prediction of the charmed quark.

A.2 Glashow-Salam-Weinberg Theory

[3]The Glashow-Salam-Weinberg electroweak theory follows from the realisation that an underlying SU(2) symmetry is present in the lepton as well as the quark family with regard to the weak interaction.

A.2.1 Leptons

Since the weak interaction induces transitions between specific leptons, it is natural to group these leptons in doublets. Since we know that the charged current is left-handed, we can assume a $\mathrm{SU(2)_L}$ symmetry and define the left-handed doublets

$$L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L, \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix}_L \quad \ldots \tag{A.37}$$

where

$$L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L = \frac{1}{2}(1-\gamma_5)\begin{bmatrix} \nu_e \\ e \end{bmatrix}. \tag{A.38}$$

If we write the interaction Lagrangian for CC reactions involving the electron and electron neutrino (the same equation holds for the muon and muon neutrino, and the tau lepton and tau neutrino) as

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}\gamma_\mu(1-\gamma_5)e\, W_+^\mu + \bar{e}\gamma_\mu(1-\gamma_5)\nu\, W_-^\mu\right], \tag{A.39}$$

[3]The reference in section A.2 is [Re90] unless stated otherwise.

we can use the doublet structure to write it as

$$\mathcal{L}_{CC} = -\frac{g}{2}\left[\bar{L}\gamma_\mu\tau_+ L\, W_+^\mu + \bar{L}\gamma_\mu\tau_- L\, W_-^\mu\right], \tag{A.40}$$

with $\tau_+ = (\tau_1 \pm \tau_2)/2$ and $\tau_i,\ \ i = 1,2,3$ are the Pauli spin matrices. If we further define

$$W_\pm^\mu = \left(W_1^\mu \mp W_2^\mu\right)/\sqrt{2}, \tag{A.41}$$

we can write

$$\mathcal{L}_{CC} = -\frac{g}{2}\left[\bar{L}\gamma_\mu\tau_1 L W_1^\mu + \bar{L}\gamma_\mu\tau_2 L W_2^\mu\right]. \tag{A.42}$$

This form of the leptonic currents suggests the existence of a third current

$$\begin{aligned}(j_3^t)_\mu &= \bar{L}\gamma_\mu\frac{\tau_3}{2}L \\ &= [t_3^{\nu_L}\bar{\nu}_L\gamma_\mu\nu_L + t_3^{e_L}\bar{e}_L\gamma_\mu e_L] \\ &= \frac{1}{2}\left[\bar{\nu}_L\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu e_L\right].\end{aligned} \tag{A.43}$$

The quantum number t_3 is the weak isospin eigenvalue and has the value $\frac{1}{2}$ for left-handed neutrinos and $-\frac{1}{2}$ for electrons. The right-handed components are singlets with weak isospin value $t_3 = 0$.

The electromagnetic current can be written in terms of left- and right-handed parts if we make use of the projection operators defined in A.17

$$\begin{aligned}j_\mu^e/e &= -\left[\bar{e}_L\gamma_\mu e_L + \bar{e}_R\gamma_\mu e_R\right] \\ &= (j_3^t)_\mu + \frac{1}{2}(j^y)_\mu,\end{aligned} \tag{A.44}$$

where $(j^y)_\mu$ is called the weak hypercharge (y) current and is of the form

$$(j^y)_\mu = -(\bar{L}\gamma_\mu L) - 2(\bar{R}\gamma_\mu R), \tag{A.45}$$

using A.43. From this it follows that $y = -1$ for the doublet and $y = -2$ for the singlet.

In an effort to attain electroweak unification we can assume that the hypercharge current has U(1) symmetry and couples to a gauge boson B^μ with coupling constant g'. If the neutral current and the hypercharge current are added to the Lagrangian A.39

$$\begin{aligned}\mathcal{L} &= -\left[g\sum_{i=1}^{3}\left(\bar{L}\gamma_\mu\frac{\tau_i}{2}L\right)\,W_i^\mu + \frac{g'}{2}(j^y)_\mu\,B^\mu\right]\\ &= \underbrace{-\frac{g}{2}\left[\bar{L}\gamma_\mu\tau_1 L\,W_1^\mu + \bar{L}\gamma_\mu\tau_2 L\,W_2^\mu\right]}_{\mathcal{L}_{\mathrm{CC}}} - \underbrace{\left[g\left(\bar{L}\gamma_\mu\frac{\tau_3}{2}L\right)\,W_3^\mu + \frac{g'}{2}(j^y)_\mu\,B^\mu\right]}_{\mathcal{L}_{\mathrm{NC}}}. \qquad \text{(A.46)}\end{aligned}$$

Since we have identified the physical gauge bosons for the neutral current and the electromagnetic current as Z^0 and the photon respectively, we can suppose that the fields W_3^μ and B^μ are linear combinations of the fields associated with these physical particles

$$W_3^\mu = \cos\theta_W Z^\mu + \sin\theta_W A^\mu, \qquad \text{(A.47)}$$

$$B^\mu = -\sin\theta_W Z^\mu + \cos\theta_W A^\mu. \qquad \text{(A.48)}$$

This leads to the NC interaction Lagrangian in the form

$$\begin{aligned}\mathcal{L}_{\mathrm{NC}} &= -\left[g\sin\theta_W(j_3^t)_\mu + g'\cos\theta_W\frac{1}{2}(j^y)_\mu\right]A^\mu\\ &\quad -\left[g\cos\theta_W(j_3^t)_\mu - g'\sin\theta_W\frac{1}{2}(j^y)_\mu\right]Z^\mu \qquad \text{(A.49)}\end{aligned}$$

and since we know the coupling of A^μ to the electromagnetic current we can immediately write

$$g\sin\theta_W = g'\cos\theta_W = e \qquad \text{(A.50)}$$

and therefore

$$\begin{aligned}\mathcal{L}_{\mathrm{NC}} &= -\left[e(j_3^t)_\mu + e\frac{1}{2}(j^y)_\mu\right]A^\mu\\ &\quad -\left[g\cos\theta_W(j_3^t)_\mu - g'\sin\theta_W\frac{1}{2}(j^y)_\mu\right]Z^\mu\\ &= -j_\mu^e A^\mu - \frac{g}{\cos\theta_W}\left[(j_3^t)_\mu - \sin^2\theta_W\left(\frac{j_\mu^e}{e}\right)\right]Z^\mu\\ &= -j_\mu^e A^\mu - j_\mu^Z Z^\mu, \qquad \text{(A.51)}\end{aligned}$$

where

$$\begin{aligned}j_\mu^Z &= \frac{g}{2\cos\theta_W}\bar{f}\gamma_\mu\left[(t_3^f - Q_f\sin^2\theta_W)(1-\gamma_5) - Q_f\sin^2\theta_W(1+\gamma_5)\right]f\\ &= \frac{g}{2\cos\theta_W}\bar{f}\gamma_\mu\left[C_L^f(1-\gamma_5) + C_R^f(1+\gamma_5)\right]f. \qquad \text{(A.52)}\end{aligned}$$

This shows that the electroweak neutral current is not in general only left-handed for all fermions. The parameters C_L^f and C_R^f determine the relative strengths of the left- and right-handed couplings. A.52 can also be written as [Ai82]

$$\begin{aligned} j_\mu^Z &= \frac{e}{\sin\theta_W \cos\theta_W}\bar{f}\gamma_\mu\left(t_3^f\frac{(1-\gamma_5)}{2} - \sin^2\theta_W Q\right)f \\ &= \frac{e}{2\sin\theta_W\cos\theta_W}\bar{f}\left[\left(t_3^f - 2Q\sin^2\theta_W\right)\gamma_\mu - \left(t_3^f\right)\gamma_\mu\gamma_5\right]f. \end{aligned} \tag{A.53}$$

If the free terms for the leptons are included in the general weak Lagrangian we obtain

$$\begin{aligned} \mathcal{L} &= i\bar{L}\gamma^\mu\left[\partial_\mu + i\frac{g}{2}\,\boldsymbol{\tau}\cdot\boldsymbol{W}_\mu + i\frac{g'}{2}\,y_L B_\mu\right]L \\ &\quad + i\bar{R}\gamma^\mu\left[\partial_\mu + i\frac{g'}{2}\,y_R B_\mu\right]R, \end{aligned} \tag{A.54}$$

where the possibility of a right-handed term has now been included. Note the fact that the weak charged vector bosons only couple to the left-handed doublets whereas the neutral boson couples to the left-handed doublets as well as the right-handed singlets.

If we require that this Lagrangian is fully SU(2)⊗U(1) invariant, the gauge fields have to transform as

$$\begin{aligned} \boldsymbol{W}'^\mu &= \boldsymbol{W}^\mu + g\,\boldsymbol{\alpha}\times\boldsymbol{W}^\mu + \partial^\mu\boldsymbol{\alpha}, \\ B'^\mu &= B^\mu + \partial^\mu\theta. \end{aligned} \tag{A.55}$$

Terms for the free fields B^μ and W^μ are also needed. From the gauge transformation properties of the fields, it follows that these terms are

$$\mathcal{L}_B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

with

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \tag{A.56}$$

$$\tag{A.57}$$

and

$$\mathcal{L}_W = -\frac{1}{4}\boldsymbol{W}_{\mu\nu}\cdot\boldsymbol{W}^{\mu\nu},$$

with

$$\boldsymbol{W}^{\mu\nu} = \partial^\mu\boldsymbol{W}^\nu - \partial^\nu\boldsymbol{W}^\mu - g\,\boldsymbol{W}^\mu\times\boldsymbol{W}^\nu. \tag{A.58}$$

Mass terms for the various particles still need to be added to the Lagrangian. It can be shown that terms of the following form

$$m\bar{e}e = m(\bar{e}_L + \bar{e}_R)(e_L + e_R) = m(\bar{e}_L e_R + \bar{e}_R e_L) \tag{A.59}$$

and

$$\frac{1}{2}M_W^2\, \boldsymbol{W}_\mu \cdot \boldsymbol{W}^\mu \tag{A.60}$$

are however not invariant under the SU(2)⊗U(1) transformation. The SU(2) symmetry must also be broken since the particles in the doublets have different masses. We therefore assume that the symmetry is spontaneously broken in such a way that the photon remains massless while the three remaining gauge bosons have non-zero mass.

Making use of the weak isospin Higgs doublet of complex scalar fields

$$\phi(x) = \begin{bmatrix} \phi^\dagger(x) \\ \phi^0(x) \end{bmatrix}, \tag{A.61}$$

we can write down a general SU(2)⊗U(1) Lagrangian containing this doublet

$$\mathcal{L}_H = [D_\mu\phi(x)]^\dagger[D^\mu\phi(x)] - \underbrace{\mu^2\phi^\dagger(x)\phi(x)}_{\text{mass term}} - \underbrace{\lambda[\phi^\dagger(x)\phi(x)]^2}_{\text{self}-\text{interaction term}}, \tag{A.62}$$

where

$$D^\mu = \partial^\mu + i\frac{g}{2}\,\boldsymbol{\tau}\cdot\boldsymbol{W}^\mu + i\frac{g'}{2}yB^\mu. \tag{A.63}$$

If we now break the ground state symmetry by choosing

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \sqrt{\frac{\mu^2}{-\lambda}} \end{bmatrix}, \tag{A.64}$$

we can write the Higgs doublet as

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{bmatrix} \sigma_1(x) + i\sigma_2(x) \\ f + \eta_1(x) + i\eta_2(x) \end{bmatrix}, \tag{A.65}$$

consisting of four real fields (where $f = \sqrt{-\frac{\mu^2}{\lambda}}$). We can make use of the SU(2)⊗U(1) invariance of the Higgs doublet and apply the transformation

$$\phi'(x) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ f + H(x) \end{bmatrix} = U\phi(x) \tag{A.66}$$

(where $H(x)$ is known as the Higgs scalar) to gauge away the unwanted fields of the massless Goldstone bosons $(\sigma_1, \sigma_2, \eta_2)$. This leaves us with the required transformation properties of the four gauge bosons and now an added scalar Higgs particle (associated with the field $\eta_1(x)$).

Considering interactions between the lepton doublet or singlet and the Higgs doublet the Yukawa coupling is assumed and for the electron case the SU(2)⊗U(1) invariant Lagrangian is written as

$$\begin{aligned} \mathcal{L}_{eH} &= -g_e \left[(\bar{L}\phi)R + \bar{R}(\phi^\dagger L) \right] \\ &= -\frac{g_e}{\sqrt{2}} \left[f(\bar{e}_L e_R + \bar{e}_R e_L) + \bar{e_L} H e_R + \bar{e}_R H e_L \right], \end{aligned} \tag{A.67}$$

with g_e a dimensionless constant for the electron coupling. Comparison with A.59 shows that through this coupling the electron has aquired a mass $m_e = \frac{g_e f}{\sqrt{2}}$.

From eq. A.62 it also follows that the $W^\pm$ and Z^0 bosons aquire mass through their coupling to the Higgs field, while the photon remains massless

$$\begin{aligned} M_W &= \frac{gf}{2}, \\ M_Z &= \sqrt{(g^2 + g'^2)}\frac{f}{2}, \\ \text{i.e. } M_W &= M_Z \cos\theta_W. \end{aligned} \tag{A.68}$$

From the Fermi constant's connection to the boson mass in low-energy phenomonology (eq. A.24) we can determine f.

A.2.2 Quarks and Hadrons

The hadronic weak current can be either a strangeness-conserving (transition between up and down quarks in nucleons) or a strangeness-changing current (transition between up and strange quarks). It is thus natural to construct quark doublets where the weak interaction induces transitions between the two components of a doublet. For a complete description of the weak interaction the following two doublets are needed

$$\begin{aligned} q &= \begin{bmatrix} u \\ d_c \end{bmatrix} = \begin{bmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{bmatrix}_L , \\ q' &= \begin{bmatrix} c \\ s_c \end{bmatrix} = \begin{bmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{bmatrix}_L . \end{aligned} \tag{A.69}$$

The weak isospin and charge values for these doublets are $t_3 = \frac{1}{2}$ and $Q = \frac{2}{3}$ for the upper and $t_3 = -\frac{1}{2}$ and $Q = -\frac{1}{3}$ for the lower components. This form of the doublets was proposed by Cabibbo to account for the decay of strange particles. The value of the Cabibbo angle can in fact be deduced from the ratio of $\Delta S = 0$ to $\Delta S = 1$ cross sections. This gives $\theta_C \approx 0.13°$.

With the left-handed components of the quarks associated with the doublets as defined in eq. A.69 and the right-handed components with singlets the CC quark Lagrangian can be written as

$$\begin{aligned} \mathcal{L}^q_{\mathrm{CC}} &= -\frac{g}{2}\left[\bar{q}\gamma_\mu\tau_1 q\; W_1^\mu + \bar{q}\gamma_\mu\tau_2 q\; W_2^\mu\right] - \frac{g}{2}\left[\bar{q}'\gamma_\mu\tau_1 q'\; W_1^\mu + \bar{q}'\gamma_\mu\tau_2 q'\; W_2^\mu\right] \\ &= -\frac{g}{2\sqrt{2}}\left[\bar{u}\gamma_\mu(1-\gamma_5)d_c\; W_+^\mu + \bar{d}_c\gamma_\mu(1-\gamma_5)u\; W_-^\mu\right] + \ldots \end{aligned} \tag{A.70}$$

In terms of quark currents the third component of the weak isospin current and the electromagnetic current can be written as

$$\begin{aligned} (j_3^t)^\mu &= \bar{q}\gamma^\mu\frac{\tau_3}{2}q = \frac{1}{2}(\bar{u}_L\gamma^\mu u_L - \bar{d}^c_L\gamma^\mu d^c_L), \\ j_q^\mu/e &= \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}^c\gamma^\mu d^c. \end{aligned} \tag{A.71}$$

This leads to the Lagrangian for the neutral current

$$\mathcal{L}^q_{\mathrm{NC}} = -e\sum_{i=u,d,s,c} e_i\;\bar{q}_i\gamma_\mu q_i\; A^\mu - \frac{g}{2\cos\theta_W}\sum_i \bar{q}_i\gamma_\mu(g_V^i - g_A^i\gamma_5)q_i\; Z^\mu. \tag{A.72}$$

Comparison with A.53 leads to the identification

q	g_V	g_A
u	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
d	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

Table A.1: The vector and axial-vector couplings of the weak neutral current to the quarks.

APPENDIX B
The Weak Form Factors

B.1 The hadronic current operator

If we define the hadronic current in terms of plane wave Dirac nucleon spinors as

$$j^\mu = \bar{u}(\mathbf{b}, m_b)\; J^\mu \; u(\mathbf{a}, m_a), \tag{B.1}$$

the hadronic current operator J^μ is a 4×4 matrix and can be written as

$$J^\mu = A^\mu I_4 + B^\mu \gamma_5 + C^{\mu\nu}\gamma_\nu + D^{\mu\nu}\gamma^5\gamma_\nu + E^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}. \tag{B.2}$$

The expansion coefficients A^μ, B^μ can be written in terms of further expansions using the independent four-momenta a^μ and b^ν at the hadronic vertex (following from momentum conservation), i.e.

$$A^\mu = A_1 a^\mu + A_2 b^\mu. \tag{B.3}$$

The second-rank coefficients $C^{\mu\nu}$ and $D^{\mu\nu}$ can be expanded using these independent four-momenta and the metric and levi-civita tensors

$$C^{\mu\nu} = C_1 g^{\mu\nu} + C_2 a^\mu a^\nu + C_3 b^\mu b^\nu + C_4\left(a^\mu b^\nu + b^\mu a^\nu\right) + C_5\left(a^\mu b^\nu - b^\mu a^\nu\right) + C_6 \epsilon^{\mu\nu\alpha\beta} a_\alpha b_\beta. \tag{B.4}$$

Keeping in mind the fact that the third-rank coefficient has to be antisymmetric in α and β since $\sigma_{\alpha\beta}$ is antisymmetric, it follows that

$$\begin{aligned} E^{\mu\alpha\beta} \;&=\; E_1 a^\mu\left(a^\alpha b^\beta - b^\alpha a^\beta\right) + E_2 a^\mu \epsilon^{\alpha\beta\eta\lambda} a_\eta b_\lambda + E_3 b^\mu\left(a^\alpha b^\beta - b^\alpha a^\beta\right) \\ &\quad + E_4 b^\mu \epsilon^{\alpha\beta\eta\lambda} a_\eta b_\lambda + E_5 \epsilon^{\mu\alpha\beta\nu} a_\nu + E_6 \epsilon^{\mu\alpha\beta\nu} b_\nu. \end{aligned} \tag{B.5}$$

If we now use the on-shell relation $\left(\not{p} - m\right) u = 0$ for the nucleons it follows that

$$J^\mu = A_1 a^\mu + A_2 b^\mu + B_1 a^\mu \gamma_5 + B_2 b^\mu \gamma_5 + C_1 \gamma^\mu + D_1 \gamma_5 \gamma^\mu. \tag{B.6}$$

With the four-vectors

$$q \;=\; b - a, \tag{B.7}$$

$$l \;=\; b + a, \tag{B.8}$$

we can write the hadronic current operator as

$$J^\mu = A_1 q^\mu + A_2 l^\mu + B_1 q^\mu \gamma_5 + B_2 l^\mu \gamma_5 + C_1 \gamma^\mu + D_1 \gamma_5 \gamma^\mu. \tag{B.9}$$

If we assume that $m_a = m_b = M$ (which effectively implies an exact SU(3) symmetry of u, d and s quarks) in B.1 the Gordon-like indentities [No05] can be used to show that

$$J^\mu = f_1 \gamma^\mu + f_2 q^\mu \gamma_5 + f_3 \gamma^\mu + f_4 \gamma_5 \gamma^\mu + f_5 \sigma^{\mu\nu} q_\nu + f_6 \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta} q_\nu. \tag{B.10}$$

This is usually [Re90] written in the form

$$\begin{aligned} J^\mu &= J_V^\mu - J_A^\mu, \\ J_V^\mu &= f_1 \gamma^\mu + \frac{i}{2M} f_2 \sigma^{\mu\nu} q_\nu + f_3 q^\mu, \\ J_A^\mu &= g_1 \gamma^\mu \gamma_5 + \frac{i}{2M} g_2 \sigma^{\mu\nu} \gamma_5 q_\nu + g_3 \gamma_5 q^\mu. \end{aligned} \tag{B.11}$$

The form factors are known as

$$\begin{aligned} f_1 &= \text{vector}, \\ f_2 &= \text{weak magnetism}, \\ f_3 &= \text{induced scalar}, \\ g_1 &= \text{axial vector}, \\ g_2 &= \text{pseudotensor}, \\ g_3 &= \text{induced pseudoscalar}. \end{aligned}$$

The issue of the so-called second-class currents is also addressed in [Re90] where it is shown that the terms f_2, f_3, g_2 and g_3 arise because of strong interaction effects. These terms have to obey the symmetry laws of QCD like G-parity ($G = CR_y$, i.e. charge conjugation and rotation about the isospin axis). If this transformation is applied to J^μ, it is found that f_3 and g_2 transform with signs opposite to the other terms. They are dubbed *second-class currents* and from above argument their contributions are expected to be small. They are therefore omitted leaving the weak hadronic current operator in the general form

$$J^\mu = f_1 \gamma^\mu + \frac{i}{2M} f_2 \sigma^{\mu\nu} q_\nu - g_1 \gamma^\mu \gamma_5 - g_3 \gamma_5 q^\mu. \tag{B.12}$$

B.2 Form factors of the weak charged current

[4]All of the charged current reactions we investigate have an ejectile muon at the leptonic vertex. Since the pseudoscalar term (g_3) in the hadronic current operator gives a contribution proportional to the muon mass (which is extremely small) we will omit it as is done in [Sh75] and [Va06]. The most general form factor description of the weak charged current operator is then

$$\begin{aligned} J^\mu_{\mathrm{CC}} &= \left[J^\mu_{\mathrm{CC}}\right]_V - \left[J^\mu_{\mathrm{CC}}\right]_A \\ &= f_1(q^2)\gamma^\mu + \frac{i}{2m} f_2(q^2)\sigma^{\mu\nu}q_\nu - g_{A,\mathrm{CC}}(q^2)\gamma^\mu\gamma_5. \end{aligned} \tag{B.13}$$

B.2.1 The Conserved Vector Current hypothesis

For an exact SU(2) symmetry of protons and neutrons we can write a nucleon spinor as

$$U = \begin{bmatrix} u_p \\ u_n \end{bmatrix}. \tag{B.14}$$

The electromagnetic current operator can be written as the sum of isoscalar and isovector parts

$$\begin{aligned} J^\mu_{\mathrm{EM}} &= (J^\mu_{\mathrm{EM}})_{\mathrm{IS}} + (J^\mu_{\mathrm{EM}})_{\mathrm{IV}} \\ &= I_2 \otimes \left[F_1^{\mathrm{IS}}\gamma_\mu + \frac{i}{2m}F_2^{\mathrm{IS}}\sigma^{\mu\nu}q_\nu\right] + \frac{\tau_3}{2} \otimes \left[F_1^{\mathrm{IV}}\gamma_\mu + \frac{i}{2m}F_2^{\mathrm{IV}}\sigma^{\mu\nu}q_\nu\right] \\ &= I_2 \otimes j^\mu_{\mathrm{IS}}(q^2) + \frac{\tau_3}{2} \otimes j^\mu_{\mathrm{IV}}(q^2), \end{aligned} \tag{B.15}$$

where τ_3 is the Pauli matrix. For the current $\langle N' \mid J^\mu_{EM} \mid N\rangle$ the form factors are

$$F_1^{\mathrm{IS}} = \frac{1}{2}(F_1^{(\mathrm{p})} + F_1^{(\mathrm{n})}), \tag{B.16}$$

$$F_2^{\mathrm{IS}} = \frac{1}{2}(F_2^{(\mathrm{p})} + F_2^{(\mathrm{n})}), \tag{B.17}$$

$$F_1^{\mathrm{IV}} = F_1^{(\mathrm{p})} - F_1^{(\mathrm{n})}, \tag{B.18}$$

$$F_2^{\mathrm{IV}} = F_2^{(\mathrm{p})} - F_2^{(\mathrm{n})}, \tag{B.19}$$

[4]The reference in section B.2 is [Re90] unless stated otherwise.

where $F_1^{(\mathrm{p,n})}$ and $F_2^{(\mathrm{p,n})}$ are the Dirac and Pauli form factors of the proton and neutron respectively. For small values of q^2 it is known that

$$\begin{aligned} F_1^{(\mathrm{p})}(0) = 1, \quad & F_2^{(\mathrm{p})}(0) = \lambda_p = 1.79, \\ F_1^{(\mathrm{n})}(0) = 0, \quad & F_2^{(\mathrm{n})}(0) = \lambda_n = -1.91. \end{aligned} \tag{B.20}$$

In general these form factors are parametrised as [Va06]

$$F_1^{(\mathrm{p})}(q^2) = \left(\frac{1+\tau(1+\lambda_p)}{1+\tau}\right) G_D^V(q^2), \tag{B.21}$$

$$F_2^{(\mathrm{p})}(q^2) = \left(\frac{\lambda_p}{1+\tau}\right) G_D^V(q^2), \tag{B.22}$$

$$F_1^{(\mathrm{n})}(q^2) = \left(\frac{\lambda_n(1-\eta)}{1+\tau}\right) G_D^V(q^2), \tag{B.23}$$

$$F_2^{(\mathrm{n})}(q^2) = \left(\frac{\lambda_n(1+\tau\eta)}{1+\tau}\right) G_D^V(q^2), \tag{B.24}$$

where

$$\tau = \frac{-q^2}{4m^2}, \tag{B.25}$$

$$\eta = (1+5.6\tau)^{-1}, \tag{B.26}$$

$$G_D^V(q^2) = \left(1-\frac{q^2}{m^2}\right)^{-2}. \tag{B.27}$$

The vector part of the weak isospin-raising $n \to p$ current can be written in terms of the nucleon spinor U as

$$\langle p \mid [J^\mu_{\mathrm{CC}}]_V \mid n\rangle = \bar{U}(\mathbf{p}', s')\left[f_1(q^2)\gamma^\mu + \frac{i}{2m} f_2(q^2)\sigma^{\mu\nu}q_\nu\right]\tau_+ U(\mathbf{p}, s). \tag{B.28}$$

Similarly the weak transition $p \to n$ can be written in terms of τ_-. This leads to the conclusion that the weak vector currents and the isovector electromagnetic current form part of an isotriplet of currents through the Pauli matrices $\tau_\pm$ and $\frac{\tau_3}{2}$. Feynman and Gell-Mann made the further assumption that since the electromagnetic current is conserved, the weak vector current is as well (the Conserved Vector Current hypothesis or CVC). We can therefore obtain the weak vector form factors in terms of the isovector electromagnetic form factors

$$\begin{aligned} f_1(q^2) &= F_1^{\mathrm{IV}} = F_1^{(\mathrm{p})} - F_1^{(\mathrm{n})}, \\ f_2(q^2) &= F_2^{\mathrm{IV}} = F_2^{(\mathrm{p})} - F_2^{(\mathrm{n})}. \end{aligned} \tag{B.29}$$

B.2.2 The axial form factor

A phenomenological value is used for $g_{A,\mathrm{CC}}$. It is usually parametrised in terms of a dipole form factor [Va06]

$$g_{A,\mathrm{CC}}(q^2) = g_A G_D^A(q^2), \tag{B.30}$$

where

$$G_D^A(q^2) = \left(\frac{1-q^2}{M_A^2}\right)^{-2} \tag{B.31}$$

and

$$g_{A,\mathrm{CC}}(0) = g_A = 1.26. \tag{B.32}$$

The axial mass M_A is taken as 0.95 GeV [Sh75].

B.2.3 SU(3) and the baryon octet

If we assume a system is invariant under the interchange of u, d and s quarks (i.e. *mass degenerate*), we can combine them into a SU(3) triplet

$$q' = \begin{bmatrix} u' \\ d' \\ s' \end{bmatrix} = U \begin{bmatrix} u \\ d \\ s \end{bmatrix} = Uq, \tag{B.33}$$

where

$$U = \exp\left(\sum_{j=1}^{8} i\alpha_j \lambda_j/2\right), \tag{B.34}$$

with $\lambda_j,\ j = 1, 2, \ldots 8$ being the eight generators of SU(3)

$$\begin{aligned}
\lambda_1 &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\lambda_4 &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}, \quad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \\
\lambda_7 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.
\end{aligned} \tag{B.35}$$

These operators satisfy the SU(3) commutation relation

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k, \tag{B.36}$$

with f_{ijk} the (antisymmetric) structure constants of SU(3). They are

ijk	f_{ijk}
123	1
147	$\frac{1}{2}$
156	$-\frac{1}{2}$
246	$\frac{1}{2}$
257	$\frac{1}{2}$
345	$\frac{1}{2}$
367	$-\frac{1}{2}$
458	$\frac{\sqrt{3}}{2}$
678	$\frac{\sqrt{3}}{2}$

. (B.37)

This representation of SU(3) also has the property

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\delta_{ij} + 2d_{ijk}\lambda_k, \tag{B.38}$$

where the constants d_{ijk} are totally symmetric

ijk	d_{ijk}	ijk	d_{ijk}
118	$\frac{1}{\sqrt{3}}$	355	$\frac{1}{2}$
146	$\frac{1}{2}$	366	$-\frac{1}{2}$
157	$\frac{1}{2}$	377	$-\frac{1}{2}$
228	$\frac{1}{\sqrt{3}}$	448	$-\frac{1}{2\sqrt{3}}$
247	$-\frac{1}{2}$	558	$-\frac{1}{2\sqrt{3}}$
256	$\frac{1}{2}$	668	$-\frac{1}{2\sqrt{3}}$
338	$\frac{1}{\sqrt{3}}$,	778	$-\frac{1}{2\sqrt{3}}$
344	$\frac{1}{2}$	888	$-\frac{1}{\sqrt{3}}$

. (B.39)

Since the system will also be invariant under interchange of any two of the quarks, there are three SU(2) subgroups corresponding to the interchanges u↔d, d↔s and u↔s. For each of these we can define shift operators. Let $I_\pm$ be these operators for u↔d, $U_\pm$ for d↔s and $V_\pm$ for u↔s. In terms of the SU(3) generators these operators are

$$\begin{aligned}
I_\pm &= \frac{1}{2}(\lambda_1 \pm i\lambda_2) = F_1 \pm iF_2,\\
U_\pm &= \frac{1}{2}(\lambda_6 \pm i\lambda_7) = F_6 \pm iF_7,\\
V_\pm &= \frac{1}{2}(\lambda_4 \pm i\lambda_5) = F_4 \pm iF_5,\\
I_3 &= F_3,\\
Y &= \frac{2}{\sqrt{3}}F_8.
\end{aligned} \tag{B.40}$$

Clearly the operators $I_\pm,\ I_3$ can be associated with SU(2) isospin (they contain the Pauli matrices $\tau_1,\ \tau_2$ and τ_3). The $U_\pm$ and $V_\pm$ are the so-callled U-spin and V-spin operators. The generators F_i satisfy the SU(3) commutation relation

$$[F_i, F_j] = if_{ijk}F_k, \tag{B.41}$$

which leads to the relations

$$\begin{aligned}
[Y, I_3] &= 0, \\
\\
[Y, I_\pm] &= 0, \\
[Y, U_\pm] &= \pm U_\pm, \\
[Y, V_\pm] &= \pm V_\pm, \\
\\
[I_3, I_\pm] &= \pm I_\pm, \\
[I_3, U_\pm] &= \mp\frac{1}{2} U_\pm, \\
[I_3, V_\pm] &= \pm\frac{1}{2} V_\pm.
\end{aligned} \tag{B.42}$$

Since the operators I_3 and Y commute, they can be used to label the states in a given SU(3) multiplet. From B.35 and B.40 these operators are

$$I_3 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad Y = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{bmatrix}. \tag{B.43}$$

The fundamental triplet are the eigenvectors of these matrices

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad s = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{B.44}$$

Their eigenvalues, written as (I_3, Y) are $(\frac{1}{2}, \frac{1}{3})$, $(-\frac{1}{2}, \frac{1}{3})$ and $(0, -\frac{2}{3})$. This so-called 3 representation of SU(3) is plotted in figure B.1. From the commutation relations in B.42 we can immediately conclude

$$\begin{aligned}
&I_\pm \quad \text{causes } \Delta Y = 0 \;\; \Delta I_3 = \pm 1, \\
&U_\pm \quad \text{causes } \Delta Y = \pm 1 \;\; \Delta I_3 = \mp\frac{1}{2}, \\
&V_\pm \quad \text{causes } \Delta Y = \pm 1 \;\; \Delta I_3 = \pm\frac{1}{2}.
\end{aligned} \tag{B.45}$$

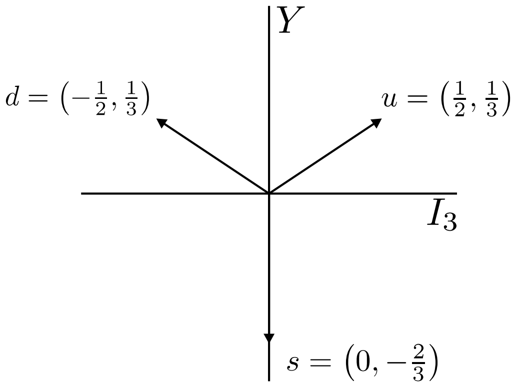


Figure B.1: The fundamental triplet of SU(3).

The action of these operators on the SU(3) triplet is shown in figure B.2. From the 3 represen-

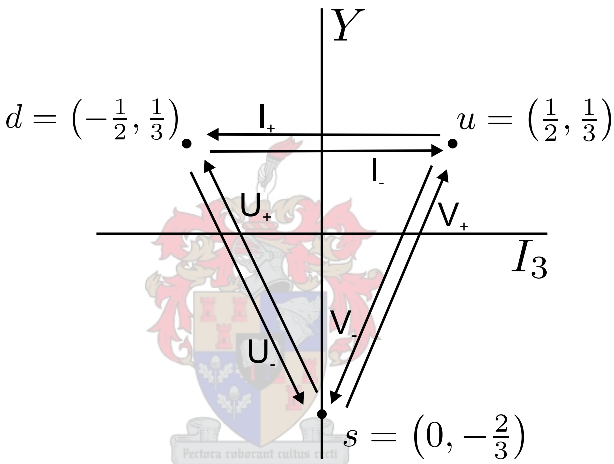


Figure B.2: The ladder-operators acting on the fundamental SU(3) triplet.

tation we can obtain the $3 \otimes 3 \otimes 3$ baryon multiplet. The quark field of eq. B.33 implies that this is the Kronecker product $q \otimes q \otimes q$. The result can be reduced and one of the irreps that emerge is the baryon octet in figure B.3. The baryon octet can be labelled by the indices of the F_i operators that act on the general quark triplet. This results in the classification of [Re90]

$$\begin{aligned}
p &= \tfrac{1}{\sqrt{2}}(B_4 + iB_5), & n &= \tfrac{1}{\sqrt{2}}(B_6 + iB_7), & \Sigma^{\pm} &= \tfrac{1}{\sqrt{2}}(B_1 \pm iB_2), \\
\Xi^{-} &= \tfrac{1}{\sqrt{2}}(B_4 - iB_5), & \Xi^{0} &= \tfrac{1}{\sqrt{2}}(B_6 - iB_7), & & \\
\Sigma^{0} &= B_3, & \Lambda^{0} &= B_8. & &
\end{aligned} \tag{B.46}$$

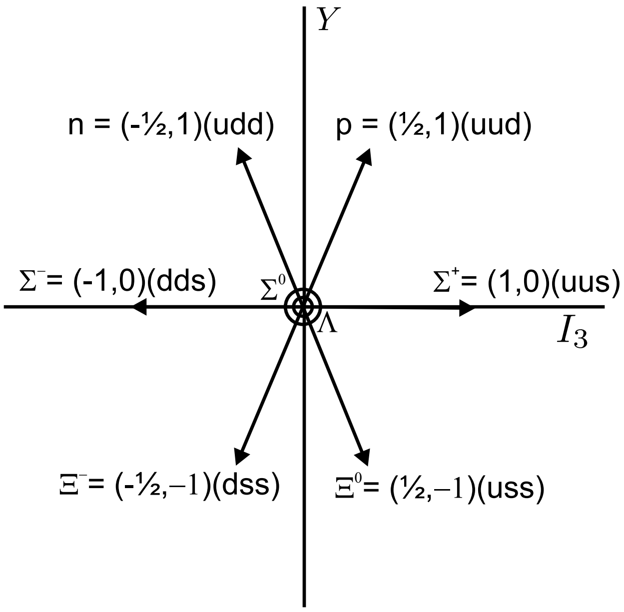


Figure B.3: The baryon octet

B.2.3.1 SU(3) currents

We can write the quark field of equation B.33 as

$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}_{12\times 1} . \tag{B.47}$$

The SU(3) generators F_j, $j+1,2\ldots 8$ satisfy the commutation relation given in B.41. These 3×3 matrices can be expanded to 12×12 matrices by the Kronecker product

$$T_j = F_j \otimes I_4. \tag{B.48}$$

We can now introduce vector and axial vector current operators

$$V_i^\mu = \frac{i}{2}\bar{q}(x)(T_i \otimes \gamma^\mu)q(x), \tag{B.49}$$

$$A_i^\mu = \frac{i}{2}\bar{q}(x)(T_i \otimes \gamma_5\gamma^\mu)q(x). \tag{B.50}$$

The vector part of the weak transition d→u can now be written in terms of quark fields as

$$
\begin{aligned}
\bar{u}\gamma^\mu d = \bar{q}(T_+ \otimes \gamma^\mu) q \quad &= \quad \begin{bmatrix} \bar{u} & \bar{d} & \bar{s} \end{bmatrix} \begin{bmatrix} 0 & I_{4\times 4} \otimes \gamma^\mu & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ d \\ s \end{bmatrix} \\
&= \quad \bar{q}\left[(T_1 + iT_2) \otimes \gamma^\mu\right] q = V^\mu_{1+i2}.
\end{aligned} \tag{B.51}
$$

The transition u→d (isospin lowering) is given by V^μ_{1-i2}.

In terms of SU(3) current algebra (for u, d and s quarks), the EM current operator is

$$
\begin{aligned}
J^\mu_{\mathrm{EM}} \quad &= \quad V^\mu_3 + \frac{1}{\sqrt{3}} V^\mu_8 \\
&= \quad \frac{i}{2}\bar{q}(x)(T_3 \otimes \gamma^\mu) q(x) + \frac{1}{\sqrt{3}}\frac{i}{2}\bar{q}(x)(T_8 \otimes \gamma^\mu) q(x).
\end{aligned} \tag{B.52}
$$

The total hadronic CC operator can be written as

$$
J^\mu_{\mathrm{CC}} = \underbrace{\cos\theta_C \left(V^\mu_{1\pm i2} - A^\mu_{1\pm i2}\right)}_{\Delta S=0} + \underbrace{\sin\theta_C \left(V^\mu_{4\pm i5} - A^\mu_{4\pm i5}\right)}_{\Delta S=1}. \tag{B.53}
$$

The $\cos\theta_C$ and $\sin\theta_C$ are the Cabibbo factors introduced to account for the asymmetry between the two types of processes (refer to Appendix A, section A.1.1.2).

B.2.3.2 Form factors of the baryon octet

[Re90] writes the general matrix element of an octet current operator between octet baryon states as

$$
\langle B_i \mid O_j \mid B_k \rangle = i f_{ijk}\ \bar{u}(B_i) F u(B_k) + d_{ijk}\ \bar{u}(B_i) D u(B_k). \tag{B.54}
$$

Since the EM current operator contains only SU(3) vector currents (V_3 and V_8), its form factors can only be of the vector nature

$$
F \quad = \quad F^F_1 \gamma^\mu + \frac{i}{2m} F^F_2 \sigma^{\mu\nu} q_\nu, \tag{B.55}
$$

$$
D \quad = \quad F^D_1 \gamma^\mu + \frac{i}{2m} F^D_2 \sigma^{\mu\nu} q_\nu. \tag{B.56}
$$

If O_j is taken to be the EM current operator and the octet baryons to be the neutron and proton respectively, we obtain

$$\begin{aligned}\langle n \mid J^{\mu}_{\mathrm{EM}} \mid n\rangle &= -\frac{2}{3}\,\bar{u}Du \\ &= \bar{u}\left[F_1^{(\mathrm{n})}\gamma^{\mu} + \frac{i}{2m}F_2^{(\mathrm{n})}\sigma^{\mu\nu}q_{\nu}\right]u \end{aligned} \tag{B.57}$$

and

$$\begin{aligned}\langle p \mid J^{\mu}_{\mathrm{EM}} \mid p\rangle &= \bar{u}\left[\frac{D}{3} + F\right]u \\ &= \bar{u}\left[F_1^{(\mathrm{p})}\gamma^{\mu} + \frac{i}{2m}F_2^{(\mathrm{p})}\sigma^{\mu\nu}q_{\nu}\right]u. \end{aligned} \tag{B.58}$$

If we compare this to eqs. B.55 and B.56 we can immediately identify

$$F_1^D(q^2) = -\frac{3}{2}F_1^{(\mathrm{n})}(q^2), \tag{B.59}$$

$$F_2^D(q^2) = -\frac{3}{2}F_2^{(\mathrm{n})}(q^2), \tag{B.60}$$

$$F_1^F(q^2) = F_1^{(\mathrm{p})}(q^2) + \frac{1}{2}F_1^{(\mathrm{n})}(q^2), \tag{B.61}$$

$$F_2^F(q^2) = F_2^{(\mathrm{p})}(q^2) + \frac{1}{2}F_2^{(\mathrm{n})}(q^2). \tag{B.62}$$

The weak charged current can be written in terms of SU(3) currents (eq. B.53) and by making use of eq. B.54 we can write it as

$$\begin{aligned}\langle B_i \mid J^{\mu}_{\mathrm{CC}} \mid B_k\rangle &= \bar{u}_i\left(J_1^{\mu} \pm iJ_2^{\mu}\right)u_k \\ &= \bar{u}_i\Big[i\left(f_{i1k} \pm if_{i2k}\right)\left(f_1^F\gamma^{\mu} + \frac{i}{2m}f_2^F\sigma^{\mu\nu}q_{\nu} - g^F\gamma^{\mu}\gamma_5\right) \\ &\quad + \left(d_{i1k} \pm id_{i2k}\right)\left(f_1^D\gamma^{\mu} + \frac{i}{2m}f_2^D\sigma^{\mu\nu}q_{\nu} - g^D\gamma^{\mu}\gamma_5\right)\Big]u_k, \end{aligned} \tag{B.63}$$

where the current operators have been defined as

$$J_i^{\mu} = V_i^{\mu} - A_i^{\mu}. \tag{B.64}$$

Because the weak charged current and the electromagnetic current are connected via CVC and the SU(3) current operators, their form factors are directly related

$$\begin{aligned} f_1^D(q^2) &= F_1^D(q^2), \\ f_2^D(q^2) &= F_2^D(q^2), \\ f_1^F(q^2) &= F_1^F(q^2), \\ f_2^F(q^2) &= F_2^F(q^2). \end{aligned} \tag{B.65}$$

The axial form factors are obtained by a comparison

$$\begin{aligned} \langle p \mid \left[J^\mu_{\mathrm{CC}}\right]_A \mid n\rangle &= \bar{u}_p \left(-g_{A,\mathrm{CC}}\gamma^\mu\gamma_5\right) u_n \\ &= \bar{u}_p \left(-(g^D + g^F)\gamma^\mu\gamma_5\right) u_n. \end{aligned} \tag{B.66}$$

The axial form factor $g_{A,\mathrm{CC}}$ is therefore usually parametrised as

$$\begin{aligned} g_{A,\mathrm{CC}}(q^2) &= g^D(q^2) + g^F(q^2) \\ &= g_A G_D^A(q^2) \\ &= \left(g^D(0) + g^F(0)\right) G_D^A(q^2) \\ &= 1.26 G_D^A(q^2). \end{aligned} \tag{B.67}$$

The values of g^D and g^F are determined from experiment. [De81] gives them as

$$g^D(0) = 0.78 \pm 0.02, \tag{B.68}$$

$$g^F(0) = 0.45 \pm 0.02. \tag{B.69}$$

B.3 Form factors of the weak neutral current

If eq. B.12 is taken as the starting point in the treatment of the current operator, we can again neglect the pseudoscalar term since it gives a contribution proportional to the neutrino mass which we assume to be zero [Va04]. The weak neutral current operator can be written as

$$\begin{aligned} J^{\mu}_{\mathrm{NC}} &= \left[J^{\mu}_{\mathrm{NC}}\right]_V - \left[J^{\mu}_{\mathrm{NC}}\right]_A \\ &= \tilde{f}_1(q^2)\gamma^{\mu} + \frac{i}{2m}\tilde{f}_2(q^2)\sigma^{\mu\nu}q_{\nu} - \tilde{g}_A(q^2)\gamma^{\mu}\gamma_5. \end{aligned} \tag{B.70}$$

B.3.1 Protons and neutrons

Using the nucleon spinor defined in eq. B.14 we can therefore write

$$\left\langle N' \mid \left[J^{\mu}_{\mathrm{NC}}\right]_V \mid N \right\rangle = \bar{U}(\mathbf{p}', s')\left[\tilde{f}_1(q^2)\gamma^{\mu} + \frac{i}{2m}\tilde{f}_2(q^2)\sigma^{\mu\nu}q_{\nu}\right]U(\mathbf{p}, s), \tag{B.71}$$

$$\left\langle N' \mid \left[J^{\mu}_{\mathrm{NC}}\right]_A \mid N \right\rangle = \bar{U}(\mathbf{p}', s')\left[\tilde{g}_A(q^2)\gamma^{\mu}\gamma_5\right]U(\mathbf{p}, s). \tag{B.72}$$

The vector and axial vector current operators can be written in terms of the EM current operator (without the strange quark contribution appearing in [Va04])

$$\left[J^{\mu}_{\mathrm{NC}}\right]_V = (2 - 4\sin^2\theta_W)\overbrace{J^{\mu}_{EM}(T=1)}^{\text{isovector}} - 4\sin^2\theta_W \overbrace{J^{\mu}_{EM}(T=0)}^{\text{isoscalar}}, \tag{B.73}$$

$$\left[J^{\mu}_{\mathrm{NC}}\right]_A = -2A^{\mu}(T=1). \tag{B.74}$$

This follows directly from GSW theory discussed in Appendix A, section A.2. With this form it follows from eqs. B.71 and B.72 that the vector form factors are

$$\tilde{f}_i(q^2) = (1 - 4\sin^2\theta_W)F_i^{(\mathrm{p})}(q^2) - F_i^{(\mathrm{n})}(q^2) \tag{B.75}$$

for protons and

$$\tilde{f}_i(q^2) = (1 - 4\sin^2\theta_W)F_i^{(\mathrm{n})}(q^2) - F_i^{(\mathrm{p})}(q^2) \tag{B.76}$$

for neutrons.

The axial form factor is determined in a similar phenomenological fashion to the axial form factor of the charged current [Va04]

$$\tilde{g}_A(q^2) = g_A G_D^A(q^2). \tag{B.77}$$

B.3.2 Form factors of the baryon octet

In terms of the octet currents the weak neutral current is [Um95] (without strange quark contributions)

$$J^\mu_{\mathrm{NC}} = V_3^\mu - A_3^\mu - 2\sin^2\theta_W \left(V_3^\mu + \frac{1}{\sqrt{3}} V_8^\mu \right). \tag{B.78}$$

Using this SU(3) form of the neutral current we obtain

$$\begin{aligned} \left\langle B_i \mid J^\mu_{\mathrm{NC}} \mid B_k \right\rangle &= \bar{u}_i \left[J_3^\mu - 2\sin^2\theta_W J^\mu_{\mathrm{EM}} \right] u_k \\ &= \bar{u}_i \left[J_3^\mu \right] u_k - 2\sin^2\theta_W \bar{u}_i \left[J^\mu_{\mathrm{EM}} \right] u_k, \end{aligned} \tag{B.79}$$

where

$$\begin{aligned} \bar{u}_i \left[J_3^\mu \right] u_k &= \bar{u}_i \Bigg[i f_{i3k} \left(\tilde{f}_1^F \gamma^\mu + \frac{i}{2m} \tilde{f}_2^F \sigma^{\mu\nu} q_\nu - \tilde{g}^F \gamma^\mu \gamma_5 \right) \\ &\quad + d_{i3k} \left(\tilde{f}_1^D \gamma^\mu + \frac{i}{2m} \tilde{f}_2^D \sigma^{\mu\nu} q_\nu - \tilde{g}^D \gamma^\mu \gamma_5 \right) \Bigg] u_k, \end{aligned} \tag{B.80}$$

$$\begin{aligned} \bar{u}_i \left[J^\mu_{\mathrm{EM}} \right] u_k &= \bar{u}_i \Bigg[i f_{i3k} \left(\tilde{f}_1^F \gamma^\mu + \frac{i}{2m} \tilde{f}_2^F \sigma^{\mu\nu} q_\nu \right) \\ &\quad + d_{i3k} \left(\tilde{f}_1^D \gamma^\mu + \frac{i}{2m} \tilde{f}_2^D \sigma^{\mu\nu} q_\nu \right) \\ &\quad + \frac{i f_{i8k}}{\sqrt{3}} \left(\tilde{f}_1^F \gamma^\mu + \frac{i}{2m} \tilde{f}_2^F \sigma^{\mu\nu} q_\nu \right) \\ &\quad + \frac{d_{i8k}}{\sqrt{3}} \left(\tilde{f}_1^D \gamma^\mu + \frac{i}{2m} \tilde{f}_2^D \sigma^{\mu\nu} q_\nu \right) \Bigg] u_k. \end{aligned} \tag{B.81}$$

If the initial and final baryons are now taken to be the proton and neutron, we obtain

$$\begin{aligned} \left\langle p \mid J^\mu_{\mathrm{NC}} \mid p \right\rangle &= \bar{u}_p \Bigg[\left(\left(\frac{1}{2} - 2\sin^2\theta_W \right) \tilde{f}_1^F + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \tilde{f}_1^D \right) \gamma^\mu \\ &\quad + \left(\left(\frac{1}{2} - 2\sin^2\theta_W \right) \tilde{f}_2^F + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) \tilde{f}_2^D \right) \frac{i}{2m} \sigma^{\mu\nu} q_\nu \\ &\quad - \frac{1}{2} \left(\tilde{g}^F \gamma^\mu \gamma_5 + \tilde{g}^D \gamma^\mu \gamma_5 \right) \Bigg] u_p \end{aligned} \tag{B.82}$$

and

$$\begin{aligned} \left\langle n \mid J^\mu_{\mathrm{NC}} \mid n \right\rangle &= \bar{u}_n \Bigg[\left(\left(-\frac{1}{2} \right) \tilde{f}_1^F + \left(-\frac{1}{2} - \frac{4}{3}\sin^2\theta_W \right) \tilde{f}_1^D \right) \gamma^\mu \\ &\quad + \left(\left(-\frac{1}{2} \right) \tilde{f}_2^F + \left(-\frac{1}{2} - \frac{4}{3}\sin^2\theta_W \right) \tilde{f}_2^D \right) \frac{i}{2m} \sigma^{\mu\nu} q_\nu \\ &\quad + \frac{1}{2} \left(\tilde{g}^F \gamma^\mu \gamma_5 + \tilde{g}^D \gamma^\mu \gamma_5 \right) \Bigg] u_n. \end{aligned} \tag{B.83}$$

Comparison to eqs B.75 and B.76 gives the relations

$$(1-4\sin^2\theta_W)F_i^{(\mathrm{p})}(q^2)-F_i^{(\mathrm{n})}(q^2) = \left(\frac{1}{2}-2\sin^2\theta_W\right)\tilde{f}_i^F+\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)\tilde{f}_i^D, \quad (\mathrm{B}.84)$$

$$(1-4\sin^2\theta_W)F_i^{(\mathrm{n})}(q^2)-F_i^{(\mathrm{p})}(q^2) = \left(-\frac{1}{2}\right)\tilde{f}_i^F+\left(-\frac{1}{2}-\frac{4}{3}\sin^2\theta_W\right)\tilde{f}_i^D. \quad (\mathrm{B}.85)$$

If this is written in the form

$$c = a\tilde{f}_i^F+b\tilde{f}_i^D, \quad (\mathrm{B}.86)$$

$$z = x\tilde{f}_i^F+y\tilde{f}_i^D, \quad (\mathrm{B}.87)$$

it is straightforward to show that the vector form factors of the neutral current in the baryon octet are

$$\tilde{f}_i^F = \frac{-cy+bz}{-bx+ay},$$
$$\tilde{f}_i^D = \frac{-cx+az}{bx-ay}. \quad (\mathrm{B}.88)$$

The axial form factors $\tilde{g}^F$ and $\tilde{g}^D$ can in principle be obtained by a similar parametrisation as for the charged current form factors. No experimental values are however available for $\tilde{g}^D(0)$ and $\tilde{g}^F(0)$.

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