

# **Quantitative Modelling Methods for the Incorporation of Uncertainty into Construction Project Estimates**

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Assignment presented in partial fulfilment of the requirements for the degree of  
Master of Engineering at the University of Stellenbosch



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## **Declaration**

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I, the undersigned, hereby declare that the work contained in this assignment is my own original work and has not previously in its entirety or in part been submitted at any university for a degree.



**Signature**

**Date**

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## **Abstract**

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Most construction projects do not complete exactly as scheduled or exactly as priced. During the implementation of a project there is almost certainly some deviation from the original estimate. The implementation of a majority of projects has actually been shown to cost more and take longer than originally estimated. However, the duration and cost performance of a project's implementation is measured against the initial estimate produced. Thus if a project is considered to have completed late or over budget then essentially the duration or cost estimated was insufficient.

Due to the fact that estimates are produced in a present day environment for inherently unique projects that occur in uncertain future environments, the estimates produced will need to incorporate uncertainty to increase their likelihood of achievability.

This study aims to derive methods to incorporate future uncertainty into project estimates. This uncertainty is incorporated, analysed and manipulated through the use of Probabilistic models and First Order Second Moment Reliability methods. The derived methods provide project management professionals with tools that enable them to design estimates that incorporate future uncertainty and are reliable to a specified degree. Further methods are then derived to probabilistically assess the commercial feasibility of a project in an uncertain future environment.

These derived methods then provide project managers and decision makers with more reliable procedures and information which in turn should assist them in making correct, project orientated decisions and ultimately increase profit reliability and client satisfaction.

## Opsomming

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Die meeste konstruksie projekte word nie volgens skedule of begroting voltooi nie. Afwykings van die oorspronklike beplanning kom met redelike sekerheid gedurende uitvoering van 'n projek. Die meerderheid van projekte toon 'n oorskryding van koste en tydsduur van die oorspronklike beraming. Die oorskryding word egter waargeneem as uitvoering van die projek gemeet word teen die oorspronklike beraming wat opgestel is. Die laat voltooiing of oorbesteding van 'n projek kan inderwaarheid toegeskryf word aan onvoldoende beraming.

Aangesien beramings opgestel word binne heersende toestande vir redelik unieke projekte wat binne onsekere toekomstige omgewings uitgevoer staan te word, moet onsekerhede in ag geneem te word om die verwagte haalbaarheid daarvan te verhoog.

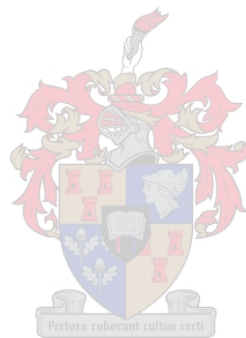
Hierdie studie beoog om metodes te ontwikkel waarmee toekomstige onsekerhede in projekberamings geïnkorporeer kan word. Sodanige onsekerhede word ge-analiseer en gemanipuleer deur gebruik van probabilistiese en Eerste Orde Tweede Moment betroubaarheids-metodes. Hiermee word gereedskap verskaf wat professionele projekbestuurders in staat stel om beramings te ontwerp wat voorsiening maak vir toekomstige onsekerhede tot 'n gespesifiseerde vlak van betroubaarheid. Verdere metodes word dan ontwikkel wat die kommersiële uitvoerbaarheid van 'n projek in 'n onsekere toekomstige omgewing kan bepaal.

Hierdie afgeleide metodes bied aan projekbestuurders en besluitnemers meer betroubare prosedures en inligting wat hulle dan help om korrekte projek gerigte besluite te neem wat uiteindelik lei tot verhoogde betroubaarheid van winste asook verbeterde kliënte bevrediging.

## **Acknowledgments**

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## List of Symbols and Acronyms

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$ADE$	=	Anticipated Duration Units Estimate
$ACE$	=	Anticipated Cost Estimate
$C$	=	Total cost model of all individual cost variables
$CPU$	=	Cost Per Unit of material
$D_c$	=	Represents random variable, Cumulative Critical Duration Units.
$d_i$	=	Triangularly distributed random variables which represent the durations of the individual critical tasks
$\xi_i$	=	The cost design partial factor to apply to task $i$
$\Phi(\beta)$	=	The cumulative Normal Distribution Function
$\gamma_i$	=	The duration design partial factor to apply to task $i$
$I_0$	=	The initial costs of the project (capital costs of a project)
$\kappa_i$	=	The consequence of the occurrence of the global factor on the task
$k$	=	Management specified Profit Margin
$P$	=	Task's Rate of Production
$Pr_i$	=	The designed task cost estimate representing a desired level of reliability
$Q$	=	Quantity of item required to complete task.
$r$	=	The annual discount rate in %.
$R_i$	=	Normally distributed Revenue variable generated at time $i$
$RPU$	=	Rate Per Unit of duration (assumed fixed)
$T_i$	=	The designed task duration estimate representing a desired level of reliability
$UDR$	=	Uncertainty Driver Ratio
$v_i$	=	The Probability of the occurrence of the global factor within the project
$\psi_i$	=	The Expected value of disruption to a task's duration resulting from the occurrence of the global factor.
$w$	=	Risk Free variable in %
$z$	=	Risk Premium in %

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## 1. Introduction

The Project Management (PM) profession is required to plan and estimate future events and activities that need to occur in order for a project to complete. These future events and activities occur in a highly dynamic, ever changing environment where numerous influencing factors, many unknown, can affect these events and activities to such an extent that the original estimates and plans become redundant and obsolete. The number and type of such influencing factors is great and varied respectively. The nature of these various influencing factors make it practically impossible to predict with any confidence what they will do in the future and thus it is even more difficult to predict the effects that changes in these influencing factors will have on the plans and estimates produced.

This practically impossible task is however precisely what PM professionals are required to do. It is thus a major challenge to the PM profession to deal with these influencing factors in the virtual world of the project estimate in a manner that best identifies and mitigates for their effects on the project in the physical world. This is an incredibly difficult task compounded by the fact that the plans and estimates PM professionals produce are those against which projects are not only awarded but also evaluated against after completion.

The difficulty of this task is evident especially in the construction industry where plans and estimates form the basis of a project tender and thus the eventual revenue stream of a construction company as well as the magnitude of client's investment. The construction industry has consistently shown that it is ill equipped to produce plans and estimates that are realistically achievable. This might appear to be incongruous as most PM professionals will vehemently defend their estimates. They are mostly entitled to do so because the estimates they have produced have been compiled using all the relevant information available to them.

Realistically though the information available to them is normally insufficient to provide an estimate of either time or cost that is accurate and in any sense reliable. It is practically even more difficult to predict the changes and hence effects of known influencing factors on a project and then impossible to do the same for unknown influencing factors which often drastically effect the actual project execution or cost.

Evidence for this notion that PM professionals are ill equipped to produce realistic estimates is the fact that a recent analysis of the construction industry showed that 50 percent of projects went over budget and 58 percent were completed late. (DTI - Construction Industry KPI's Industry Progress Report – 2002)

Conventionally, the above figures are blamed on the lack of performance on the management side of the projects. This is true to a certain extent; managerial incompetence can contribute largely to these failures. However, given the nature of the environment within which the estimates are produced and the inherent lack of reliable information, a large percentage of the failures is the result of the production of unrealistic estimates and not managerial incompetence. Another important point to consider when reading the above analysis by the DTI is the fact that the eventual actual project duration and cost are evaluated against the original estimate produced by the PM professional. Thus the figures of 50% and 58% were measured against the original estimate compiled by a PM professional with insufficient information. Thus considering the statistical result of the analysis that 50% of projects have overspent, the semantics of the analysis should be rewritten to read, "50% of projects cost more than estimated" or in other words, "50% of cost estimates were too low".

The important consideration for doing this is to focus on the fact that any percentage deviation, positive or negative, in either cost or duration is measured against an estimated value. This estimated value is calculated by PM professional and estimators from the industry and is based on their knowledge, experience, historical data and in all cases incomplete information.

The irony is that these estimates are then assumed to be correct approximations of the required time and cost required to complete a project. However, if 50% of projects actually cost more than their cost estimates and 58% actually complete above their duration estimates does it not indicate that the estimates of both cost and time are wrong at least 50% of the time?

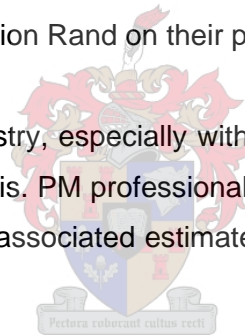
The assumption that the estimates are correct and achievable coupled with the subsequent defence of this assumption provides the rope with which the industry often hangs itself. Therefore instead of focussing all the blame and hence the new techniques,

procurement methods and innovations on improving the management and execution of projects should the question not be asked, “How can we improve the reliability of the estimates against which projects are measured?”

This is not a simple task as is clearly evident from the statistic that at least half of the current estimates produced are incorrect. The PM professionals compiling these estimates must, as stated earlier, attempt to predict the future with incomplete information. These professionals must then quantify the effects of their predictions on a project. This is an exceptionally difficult proposition not unique to the PM industry. For example, it would take an exceptionally brave economist, analyst or stockbroker to firstly;

- put their name to an exact prediction of future events including the exact effects these events will have and then secondly;
- to stake a few hundred million Rand on their predictions and estimates.

Yet the reality within the PM industry, especially within construction projects, is that this practically happens on a daily basis. PM professionals are asked to produce an, as near as possible, exact prediction and associated estimate upon which the company or client stakes large sums of money.



Another problem that contributes and compounds the problem highlighted by the above statistics is the fact that, especially in the tendering environment, the lowest prices are often more readily chosen by a client. Thus those tenders that have potentially underestimated the project are more likely to be awarded the contract for the project and thus potentially more likely to fail to meet their estimates. This situation does not however form part of this study but must be duly noted.

As a result of the obvious importance of an estimate, lots of time and effort should be and is expended to facilitate their production. PM professionals questioned from the industry mostly strongly believe their estimates to be as accurate as practically possible and more importantly they believe them to be achievable. They try to protect their estimates by nearly always inserting extra float, akin to a safety factor, to ensure that they further believe themselves not to have underestimated. Unfortunately such

judgement is subjective and therefore unmanageable and further the statistics prove that they are still more often than not incorrect.

The PM professionals therefore cannot be blamed at this juncture because they are forced to act on incomplete information, information that is not readily obtainable but that has a serious impact on the actual performance of the project. This information is not solely in the realm of known influencing factors such as weather and erroneous design assumptions; they also include influencing factors like exchange rate volatility, conceptual problem solving durations, scope changes, labour availability, skill, morale and numerous others. Although these mentioned influencing factors can be identified, it is an exceptionally difficult task during the estimating phase of a project to ascertain which factors will occur, what their effect will be and how to quantify this effect in order to incorporate it into the estimate. In addition there are also then those influencing factors that cannot be identified during estimation notwithstanding the unknown consequences of interaction between the various influencing factors that can either neutralise or amplify their effects on a project.

Unfortunately in the modern project there are far too many unquantifiable influencing factors for the PM professional to identify and accurately predict the effect that they will have on the project. Some of these influencing factors are in fact fields of extensive study in their own right. Thus one cannot feasibly expect the PM professional to predict anywhere near accurately. One is thus forced to accept the belief of the PM professional that based on their experience and track record the estimate produced is accurate and reliable. This status quo is through no fault of PM professionals for they do not have the tools available to them to predict with any level of reliability. For example, a planner does not have the advantage of the decades of research into variables, resultant formulas and software applications that are available to the structural engineer. Developing these tools took a long time and required major effort from structural engineers. Yet these tools are still not perfect and are continuously being improved. It is however as a result of such tools one would not accept, as one does with a PM professional's estimate, a structural design that the engineer simply believed to be sufficient. On the contrary, the structural engineer has the tools available to estimate the performance of a structure based on presently acceptable structural design codes and must prove any structural estimation against currently accepted structural reliability levels. These tools and such practices

have resulted in structural failures occurring with a frequency far less than 50% or 58% of all structural designs produced.

Similar tools are unfortunately not available to a PM professional and will probably not come into existence in the future as the influencing factors concerned, for the most part, cannot be studied to near exhaustion like the structural engineers have done with concrete and steel. As a result the PM industry must simply accept this fact with its associated risks. It must attempt to manage this uncertainty in a manner that maintains profitability but does not err too far on the side of caution that it prices projects and itself out of existence. Therefore if a company is able to take cognisance of uncertainty within a project, is able to incorporate it into a project estimate and is able to quantify it in a reliable manner then that company will price such uncertainty in a consistent manner which should in turn enable it to outperform its competition over the longer term.

### **1.1 Purpose of this Study**

The purpose of this study is to create tools that a PM professional can utilise to;

- Systematically incorporate uncertain influencing factors into a project estimate.
- Systematically quantify the influence these factors have on the estimate
- Design estimates that mitigate for the effects of these factors to specified levels of mitigation
- Consistently quantify the level of project risk carried within the estimate in terms of duration and cost.
- Calculate tender values in an uncertain environment
- Analyse the feasibility of a project in an uncertain environment.



## 1.2 Application of These Tools

The results generated by using these tools can be used by project managers, construction managers, clients, business managers, investors and shareholders to assess and act accordingly to proposed projects at the planning phase of the project lifecycle.

This study aims to achieve answers to questions like;

- What is the most likely return of this project?
- How much risk is being carried or priced for in the project?
- How reliable is the estimated duration of this project?
- How reliable is the estimated cost of this project?
- How reliable is profit given the calculated tender value?
- What will the duration and cost estimate be if the decision maker wants them to be reliable to a certain percentage?

## 1.3 Limitations

The future cannot be predicted and there will be no attempt to do so in this study. Current project estimates are estimations of future events. The methods described later in this study will assist in incorporating uncertainty caused by the effects on the project of various influencing factors in to these estimations. These resultant effects on the project estimate are only as accurate as the information inputted by the PM professional. The methods described in this study cannot thus predict the future; they can only assist in producing more plausible or probabilistically reliable estimates by explicitly modelling uncertainty into the estimate.

This study does not attempt to analyse the accuracy of the precedence network established by a PM professional. The methods and calculations along with their resultant conclusions assume that the original logical order which defines the precedence network is correct. The analysis of the project estimates actually uses the original precedence network as an input.

## **2. The Estimating Process**

The purpose of this study is to introduce uncertainty into project estimates. The methods used to achieve this will be described in the next few chapters. In order to do this though, it is necessary to understand when during a project estimates are made and used and how these estimates are made.

Every project will have a duration and will incur cost. When a project is proposed its cost and duration are not usually known. It is the task of the PM professional to provide a calculated estimation of both the duration and cost of the project. Projects are also dynamic and constantly evolving. The level of information available during a project also thus evolves and increases with time. Estimates are made based on this evolving information and thus those made at the beginning of a project will differ from those made later into a project. It is therefore important to know when estimates are made and consequently the level of information that is included in those estimates. This chapter will very briefly discuss when estimates are made and summarise the process used to compile an estimate.

### **2.1 The Project Life Cycle**

In order to describe when an estimate is made, a brief overview of the project life-cycle is necessary. The project life-cycle is a simple management tool that fragments a project into different phases. This fragmentation helps a PM professional to break the project down into smaller phases or packages that can be more effectively managed. The phases are then often viewed as projects themselves and can be fragmented into even smaller packages should this be required. The project life-cycle and the resulting phases are, as their names suggest, indications that a project has a birth or beginning, it passes through different phases of its life and then a death or completion. As a result the phases of the life cycle occur in a sequence mapped against time.

Commonly a project is broken down into four main life cycle phases (Burke, 2003, 28). These phases are often given different names depending on the PM professional. For the purpose of this study the following phases, in consecutive order, will be used;

1. Concept and Initiation Phase
  - A need or opportunity is identified; the feasibility of this need or opportunity is investigated. A basic estimate and financial evaluation are performed.
2. Design and Development Phase
  - The concept from the first phase is used to develop a design, detail the build or implementation method, develop schedules and plans to implement the project. Detailed estimates are produced and a detailed financial model is created.
3. Implementation or Construction Phase
  - The project is implemented or constructed as detailed in the design and development phase.
4. Commissioning or Handover Phase.
  - The last phase acknowledges that the project has been implemented as per design and terminates the project.

A fifth phase could be added to a project life-cycle, Operate and Maintain. This phase however would not be common to most projects and as a result is often not referred to in the literature. It is however an important phase if the generation of income or profit is considered part of the project. Such situations arise in Design, Build, Finance and Operate (DBFO) projects common in the construction environment and also in concession type projects where the initial capital investment is used to generate future incomes normally over a period of years or decades. In such projects the operation and maintenance of a project is crucial for its financial success and thus this phase should be included into the life-cycle.

## **2.2 Estimating in a Project Environment**

During the typical project there are two levels of estimates produced. There are those estimates produced at the early stages of a project that are often gross approximations due to the lack of available information and unwillingness to spend too much money.

These estimates are typically used to evaluate whether or not to proceed with a project or not. Then there are those that are produced at the later stages of a project lifecycle that are far more detailed and accurate as a result of an increase in the available information. Typically estimates are only made in the first two phases of the project lifecycle. They differ in detail and accuracy yet are both crucially important to the successful management of a project. The different estimates produced will be detailed below.

### **2.2.1 Concept and Initiation Phase**

The first phase of a project is a phase where estimating plays a vital role. In this phase the project is essentially created and evaluated. At this stage the project is merely a concept or an identified need or market opportunity. This concept or opportunity must be assessed and evaluated to determine if the project is feasible and will be able to meet the objectives of its initiator. The project itself is not yet tangible; it is rather collection of ideas and possibilities. For these ideas and possibilities to result in something tangible they need to be brought together in a logical manner and analysed. This is usually done by evaluating the feasibility of the project. To do this the PM professional must identify what the objectives of the project are and develop means to meet these objectives. Often more than one alternative means is proposed.

The alternative means of meeting the objectives generally take time and cost money. The PM professional must provide estimates of such time and cost for each of the alternatives proposed. The PM professional must also then estimate the level to which the different alternatives meet the desired objectives of the project. Using these estimates, the PM professional or project initiator must choose the alternative that best suits the project and evaluate its potential for success.

The evaluation of the eventual success of the project is done using the estimates, produced either by the PM professional or commissioned by the PM professional. These estimates are normally fairly crude as they cost money to produce and at this stage, do not guarantee that the project will proceed and produce the necessary return to justify their cost. If the estimates show the project not to be feasible then all the money spent to produce them is wasted. As a result little money is spent to produce these initial estimates and as a result they are normally very rough. Unfortunately, even though the

estimates are crude at this stage, they are the deciding factors used by the project initiator to decide whether to proceed with a project or not.

At this phase of a project the following initial estimates are typically required;

- Design duration and cost
- Implementation duration and cost
- Operational and maintenance costs (when applicable)
- Future revenue streams
- Return on investment
- Lifespan (total duration) of the project

These initial estimates are produced through various methods but generally rely on quick ad hoc calculations based on historical data and expert's experience. These estimates are intended to provide a crude evaluation of whether to proceed with the project or not. Thus during the Concept and Initiation Phase of a project, the initial estimates, although normally crude, are vital for a project's continuation.

### **2.2.2 Design and Development Phase**

Once a project has been evaluated to be feasible it will pass on to the next phase in the life cycle, Design and Development. The PM professional will usually outsource this phase of the project to a suitable expert who will produce the design or development plan. During this phase the means to meet the project's objectives are devised in great detail. For instance, in a construction project a detailed construction design is produced or in an IT project a detailed software package is designed. These designs are then used in the next project phase the, Implementation or Construction Phase where the designs are turned into reality, the design is built or the software package is programmed.

At the Design and Development phase of the project the level of detail or amount of information available increases greatly as does the cost of its acquisition. The design duration and cost estimated in the Concept or Initiation phase now becomes an actual duration and cost. Further, the detailed design produced enables a PM professional to

produce a far more detailed and accurate estimate of the cost and duration required to implement the project. The PM professional will normally achieve this by approaching the expert person or company, the contractor, which is envisaged to be doing the implementation works required and request an estimated price and duration for doing those works based on the completed design. A common method of achieving this price and duration is through requesting a quotation or tender from the contractor. The contractor will then compile an estimated duration and cost. The following process is typically used;

- Step 1 PM professional will issue the design and request a quotation or tender
- Step 2 The contractor will review the design and devise methods to execute the works and establish all the activities or tasks required and their sequence.
- Step 3 The contractor will develop a programme or schedule of the required tasks. This programme will detail the estimated duration of the works.
- Step 4 Using the programme as a basis, the contractor will then estimate the project costs by calculating the direct costs, based predominantly on rates, and indirect costs such as Preliminary and Generals and Overheads.
- Step 5 The estimated costs will be summed, the contractor will add on their mark-up and risk premiums and the final cost estimate is produced.

The PM professional will then often compare these more accurate estimates with the initial estimates and the feasibility of the project is often reassessed if required.

During a project there are thus essentially two types of estimates made. There are the less costly, Initial Estimates which are generally cruder but are applicable to the feasibility and continuation of a project as a whole and then there are the more expensive, Detailed Estimates that generally pertain the works required during the implementation phase of the project. Both types of estimates are important to the

success of a project as gross inaccuracies in either of them can mean that a project can fail.

In the following chapters this study will derive methods whereby uncertainty in the form of variability can be incorporated into both the Detailed estimates and the coarser Initial estimates. This will be done by creating probabilistic models of these estimates. Using these probabilistic models the initial feasibility of a project and the detailed estimates of its implementation can then be evaluated probabilistically.

### **2.3 Reliability based Project Estimating**

Ker-Fox investigated the effects of uncertainty on project performance and ways in which this can be modelled analytically (Ker-Fox). He applied the approximate methods of reliability analysis to represent the performance of a project as determined by the uncertainty of the various activities of the project in terms of an estimated cost model for the project. Since the uncertainty in cost estimates for individual tasks are not independent, Ker-Fox developed analytical techniques for the treatment of correlated variables as applied in approximate reliability analyses. Although he demonstrated the application of these techniques in representing the performance of a project in terms of such cost models, he indicated that a project can also be modelled in terms of its duration and cash flow.

In this investigation, the reliability performance of a project is modelled in terms of cost and time. This is done using approximate reliability techniques similar to the way in which it was done by Ker-Fox, but with differences in the way in which they are applied. A different approach is taken in particular in the way in which external influences and sources of uncertainty are treated by considering such factors explicitly and quantitatively, as opposed to the treatment as correlated variables used by Ker-Fox.

Although cash flow is clearly also an important performance requirement for projects, it is not included as such in this investigation. Whilst some special procedures may need to be developed to model the cash flow of a project in reliability terms, it is judged that the general approach taken in this investigation would still be applicable.

### **3. Creating a Probabilistic Model**

In Chapter 2 the different types of estimates were described. This chapter will focus on the detailed estimates that a contractor must produce in order to provide a quotation or tender. It is assumed that the contractor will have a project management professional employed who will be responsible for producing these estimates. Thus the term PM professional in this chapter relates to the person who will be compiling the estimates.

The methods derived in this chapter also require that all the relevant tasks required to complete the project have been identified and specified. The estimated costs and durations of these tasks will be calculated during the process of the methods described later. Secondly, the precedence network has also been established by the PM professional. Importantly it is assumed that this precedence network is correctly established by the PM professional and no attempt will be made in this study to verify or analyse its reliability. Thus the methods derived require all tasks and their interrelationships as inputs.

This chapter will then derive a method to incorporate variability into the estimates compiled by the PM professional. The derived methods will also provide the PM professional with tools to design project estimates that take into account the incorporated variability. In order to do this it is necessary to create a probabilistic model of the project estimates.

#### **3.1 Incorporating the Effects of Various Influencing Factors into the Project Estimate**

A project consists of its constituent activities or tasks. These tasks are detailed, sequenced and combined to estimate project duration and then also the project cost. These individual task durations and costs are currently estimated individually in a discrete manner based on production rates, plant required and many other factors. These factors are however not easily determinable values and are often guessed or based on previous experience and historical data of similar tasks executed in past projects. These calculated guesses then combine to produce the overall project estimates.



This method of estimating individual tasks by calculated guess work is probably the greatest cause of project estimate inaccuracy as they are often underestimates. This situation occurs because the individual estimates regularly represent a desired estimated duration or cost which generally errs on the side of optimism. Another major problem faced by PM professionals when compiling an estimate has to do with the fact that conservative estimates usually require longer durations and higher costs both of which are not favourably viewed by many. As a result there is normally added pressure on the PM professional to be optimistic. The important point to note here is that the individual task estimates are exactly that, estimates. The eventual actual duration or cost of a task might be higher or lower than the estimate. If the task were repeated numerous times its duration and cost will fluctuate above and below the estimated values in a random fashion. This is evident when historical data has been captured for a fairly common, similar and reoccurring task. Such data can be analysed statistically and a random variable can be produced for the task with a suitable distribution and statistical parameters. This random variable is then representative of the task in either cost or duration depending on the units of the historical data.

If this representation of the task attributes as a random variable was calculated using actual task durations or costs through many repetitions of the same task over time, then the random variable would incorporate the effects that most of the influencing factors had on that task. The resulting random variable would then reflect most possible outcomes, their frequency and their probability of occurrence.

Such data for each task in a project and the resultant random variables would allow a PM professional to construct a probabilistic model of the project. If the data was extensive and available for every task then this probabilistic model would incorporate most of the possible outcomes for each task and thus also most of the effects that influencing factors have on each task. All that would then be required to make a more plausible estimate for the project are the tools to analyse such a model to obtain useful information.

There are tools available to perform such analyses; probably the most prevalent of these are Monte Carlo simulation and PERT analysis. Both of these have their advantages but both remain limited in the amount of useful information they return. This study will

present a new method to analyse and also design project estimates to achieve desired results.

### **3.2 Creating Random Variables for Individual Tasks.**

In order to build a probabilistic model of a project, it is first required to create random variables of each task that represent the task's cost and duration. The best way of doing this is to obtain as much historical data relating to the task and then through analysis of this data create random variables that are representative of this data. In the project environment this is unfortunately extremely rare for two main reasons.

Firstly, tasks within projects are almost always unique to that project. A project can be loosely defined as a relatively short term endeavour with a specified beginning and end that is undertaken to achieve a defined objective. The objectives are normally always different in some manner or the other and must also be met within the influences of different dynamic external environments. This inevitably results in the creation of specific tasks unique to the project which they form part of. Thus it is difficult to obtain long term task data that traverses numerous projects because common tasks are not the norm.

Secondly, when similar projects with similar objectives do occur there will inevitably be similar common tasks. These tasks are very seldom the same though as they occur in different projects subject to different external environments. Thus capturing their data as if they are repetitions of a common task would produce statistically incorrect variables. For example, in a construction project a common load and haul operation will achieve vastly different production rates and cost if it were to operate in a wet, rocky, mountainous environment as opposed to a dry, sandy, flat environment. In this example the discrepancies will be great but they will also be clearly identifiable and thus an appropriate data sort will no doubt result in two sets of data reflecting the different external environments from which two random variables can be created. The more common situation, where these discrepancies are not easily identifiable, would result in the creation of random variables that are not necessarily representative of the actual task required in a new project.

Although random variables created from historical data are not commonly available in practice and inherently inaccurate they still represent actual events pertaining to specific tasks. They should thus be used where possible and plausible.

A second option to create random variables of task duration or cost would be to do trials of the required task within an external environment similar to the one in which it will be eventually executed. Time, motion and cost studies of these trials can then be converted into random variables which could then be used to generate the probabilistic model of the project. Such trials however cost money, require resources and take time, all items that are usually scarce. As a result these random variables will be crude as the size of the sample taken will normally be small and unable to capture all the effects of all of the influencing factors.

A third option to create a random variable is to estimate a value range for the task which it is to represent. This estimation can then be converted into a random variable by means of approximation. This method is merely an extension of a current estimating process where the PM professional will estimate a duration or cost based on experience or on a calculation using other estimated variables. This option converts this estimate into an approximate random variable that can be used in the probabilistic model.

### **3.2.1 Converting an Estimate into a Random Variable.**

To begin with this process, estimated data are required. The estimated data can be generated in many different ways based on the knowledge of the PM professional. The ultimate aim here is to obtain as much information from as little estimated data as possible. This will simplify the process and make it more understandable to a PM professional and thus achieve more accurate random variables.

A rather simple method that can be modified to estimate these random variables is one found in the Program, Evaluation, and Review Technique (PERT). In this method the estimated value of a task's duration is generated by only three simple but well understood estimated values. The PM professional must estimate, based on experience and calculations using other variable factors, the following;

- Optimistic duration (Best Case)
- Pessimistic duration (Worst Case)
- Anticipated duration (Anticipated case)

These three estimates can be applied in relation to costs too, by simply altering the semantics to read; lowest possible cost, highest possible cost and anticipated cost. The method found in PERT then weights the values and averages them out in order to achieve a weighted estimate. Unfortunately this is extremely limited in the fact that it only returns a Best, Worst and Anticipated case. The Anticipated case is merely the result of using all the weighted averages for all the tasks.

PERT does not result in a random variable that represents the task's duration or cost. It does however supply a simple method for obtaining relevant information in a simple, practical manner. The PM professional simply specifies three estimates or parameters from which a potentially more reliable resultant estimate is calculated.

In statistical mathematics there are a variety of distributions that can represent random variables and of these the triangular distribution appears to be the most suited to creating the random variable based on the three simple estimates used in PERT.

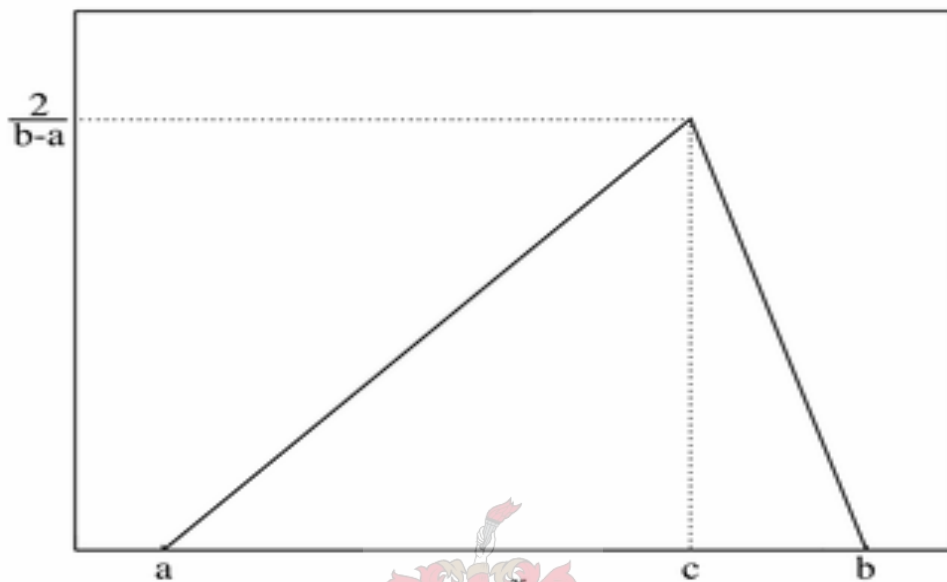
It must be noted here that when a PM professional makes the three estimates, Best, Worst and Anticipated, the PM professional must be very clear as to under what conditions these estimates are made. This is important when considering the effects of identifiable global factors that will be described in Chapter 5. For instance, if the PM professional includes the potential effects of identifiable global factors, such as rainfall, in the Best, Worst and Anticipated estimates then the modelling described in Chapter 5, particular to incorporating the effects of this identifiable global factor into the estimate, is unnecessary and will lead to an overly conservative estimate if it is performed. Thus the PM professional must be very clear in terms of what is and what is not considered and included in the estimates of Best, Worst and Anticipated cases at this stage.

### **3.2.1.1 The Triangular Distribution**

The Triangular distribution also requires three input parameters namely a lower limit  $a$ , mode  $c$  and upper limit  $b$  as shown in Figure 3.1. The lower limit  $a$  is similar to the Optimistic Case from PERT, the mode  $c$  is similar to the Anticipated Case from PERT and the upper limit  $b$  is similar to the Pessimistic Case from PERT. From these three estimates the values of  $a$ ,  $b$  and  $c$  can be plotted onto the  $x$  axis. The area of the triangle created represents the total cumulative probability and therefore must be equal to 1. As

a result the height of the triangle can be calculated by means of simple arithmetic and plotted as shown.

Figure 3.1 Triangular Distribution Probability Density Function



The advantages of using the triangular distribution are that the distribution is continuous and either the estimated cost or duration can be determined with various probability values. Similarly the likelihood of completing or costing less than a certain value can be estimated.

The Triangular distribution is slightly crude in its approximation of the representative variable but it is still most probably the most suitable distribution to be used here because of its simplicity and the fact that the input parameters themselves are crude.

The use of the Triangular distribution easily highlights how PM professionals can unknowingly underestimate their estimates.

As an example a PM professional compiling an estimate of a typical load and haul operation could say;

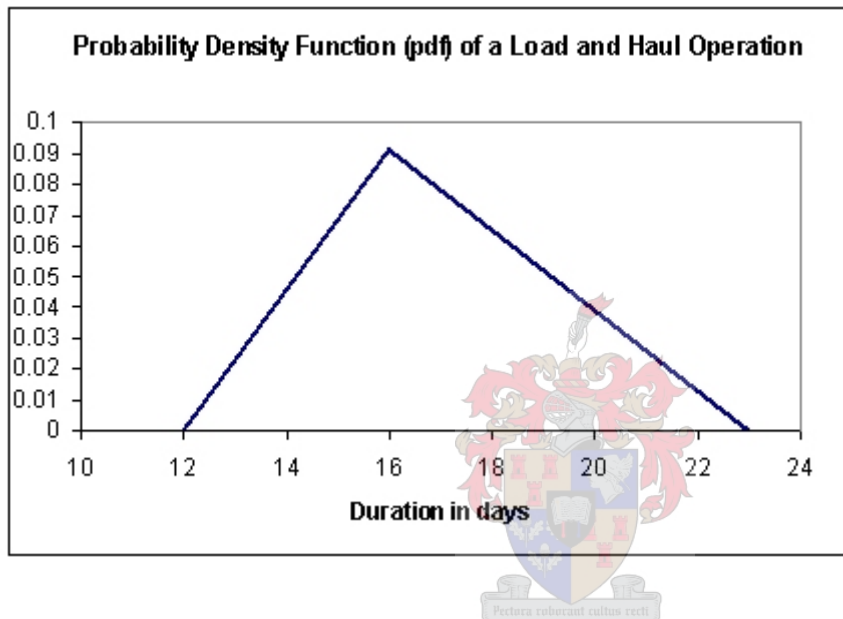
- “If all went perfectly well the task could complete in 12 days.” (Optimistic).
- “If all went terribly wrong the task could complete in 23 days.” (Pessimistic).
- “It will probably take about 16 days though to complete” (Anticipated)

From the above estimations the parameters for the Triangular Distribution would be;

$$a = 12, b = 23, c = 16$$

The resultant Probability Density Plot of the above Triangular Distribution parameters would be as shown in Figure 3.2.

Figure 3.2



From the distribution the mean, standard deviation and skewness of the variable can be established. (See Appendix B for the calculations)

The Statistics of the variable are calculated to be:

- |                           |                      |
|---------------------------|----------------------|
| Mean:                     | $\mu = 17$ days      |
| Standard Deviation:       | $\sigma = 2.27$ days |
| Skewness:                 | $\gamma = 0.25545$   |
| Coefficient of Variation: | $\Omega = 0.1337$    |

These results clearly show that the estimated Anticipated value was an underestimation. The calculated mean value of 17 days is greater than the Anticipated value that the PM

professional estimated and would have entered in to the compiled estimate had no probabilistic estimation been used.

### **3.3 The Probabilistic Model**

All of the tasks in the project must be converted into random variables. A random variable representing cost and a random variable representing duration must be created for each task. Those tasks for which historical data is available must have this data used in order to create representative random variables.

Once all the tasks have been converted into random variables a probabilistic model can be created. A separate model can be created for both duration and cost.

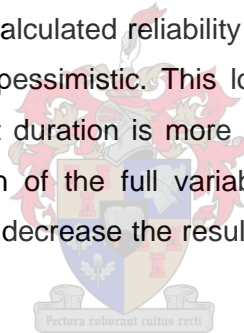
#### **3.3.1 Duration Model**

The duration of a project is dependent on the durations of all of the individual tasks that constitute the project and the interrelations between these tasks. The precedence network describes the interrelations between the tasks and based on these interrelations the critical path method then schedules the tasks in such a manner as to return the shortest possible project duration. The critical path method is however dependent on the individual task durations and can only be calculated after these have all been estimated. Thus for the purposes of creating a probabilistic duration model the individual task durations will form the basis of that model. The reliability of the duration of the entire project is also therefore dependant on the reliability of the durations of each task's estimated duration.

A simple model can therefore be created to test the reliability of the project's duration by summing the individual task duration variables. This will result in a model representing the Total Cumulative Duration Units required to complete the project. The Total Cumulative Duration Units is calculated without consideration of the interrelations between the tasks and is thus not necessarily equal to the total project duration. This probabilistic model can then be used to analyse the reliability of the Total Cumulative Duration Units and hence the actual project duration too. However, using the Total Cumulative Duration Units model to determine the reliability of the project's duration will often result in an unrealistic and pessimistic value.

The reason for this pessimistic calculation of reliability is the fact that many tasks do in fact occur concurrently. Intuitively, in a project consisting of only tasks A and B in series, the reliability of the project's total duration is independent of the positioning of task A in relation to task B. In other words in terms of the reliability of the total duration, it does not matter whether task A precedes B or B precedes A. In the case of a project consisting of only tasks A and B in parallel however, the reliability of the project duration becomes more dependent on the task with the greatest duration, say task A, as its duration increases in relation to the shorter task, task B. In fact, the reliability of the project duration will, for all practical purposes, eventually become solely dependent on the duration of longer task. This will occur when task A's duration has reached a point where for all values of both A and B, there is no chance that the duration of B will be greater than A.

Therefore, in such a case, to include the variability of Task B into the calculation of the project's reliability will result in a calculated reliability value lower than the actual project reliability and this will hence be pessimistic. This logic can be extended to all cases where the reliability of the project duration is more dependent, in varying degrees, on some of the tasks. The inclusion of the full variability of those tasks on which the reliability is less dependent would decrease the resultant calculated reliability and would result in a pessimistic figure.



To overcome the likely pessimism encountered when calculating reliability using the Total Cumulative Duration Units model one must examine the critical path.

The critical path calculation arranges the tasks in the project in such a manner as to reduce the project duration to a minimum based on the logical sequence of tasks. It uses the precedence network and individual task durations to calculate the minimum project duration. The Critical Path calculation is a simple arithmetic calculation based on the precedence network. It is widely used and its description does not form part of the purpose of this study and thus shall not be detailed in this study. An example calculation is however shown in Appendix C.

In rearranging the tasks the critical path method not only calculates the shortest possible project duration but it also identifies those tasks which are critical to complete the project



in accordance to the calculated duration. These critical tasks are so named because any change in any one of their durations will result in a change in the calculated project duration. These critical tasks have no float and thus a change in duration in any critical task, irrespective of its position within the programme or in relation to all other tasks, will change the duration of the project.

The duration of the project is thus dependant on the critical path. This in turn means that the duration of the project is dependant on the durations of the individual tasks on the critical path. Therefore the reliability of the duration of a project is dependant on the tasks on the critical path.

Further, a change in duration of any critical task will cause all its successor critical tasks to shift in position but it will not affect their durations. For instance, in a project consisting of three critical tasks, an increase in duration of the first task will cause the second and third to both occur later but it will not alter their durations. Similarly a change in duration for the second task will cause the third to occur later but it will not alter its duration. Lastly, if the duration of the third task changes then neither of the first two task's durations are altered. In all three cases though the project's total duration is altered.

### 3.3.1.1 Probabilistic Model of the Critical Tasks

In terms of creating a probabilistic model to calculate the reliability of the project's duration, the fact that the variation of any one of these critical tasks will result in a variation of the project's duration and that a variation in an individual critical task's duration does not alter the durations of the other critical task, the model can be constructed by viewing the critical tasks in series. Thus a probabilistic model can be created to calculate reliability of a project's estimate duration by simply summing all of the critical task duration variables. The model is then given by;

$$Dc = \sum_{i=1}^n d_i \quad (3.1)$$

- Where  $d_i$  = Triangularly distributed random variables which represents the durations of the individual critical tasks and
- $n$  = represents the number of critical tasks in the project.
- $Dc$ = Represents a new random variable, Cumulative Critical Duration Units.

Unfortunately the model given in Equation 3.1 requires that the critical tasks have been identified. The critical path calculation must therefore already have been done. The problem here is that the critical path calculation can only be done with discrete values of the task durations.

The critical path calculation and hence identification of the tasks required for inclusion in Equation 3.1, is dependent on the durations of the individual tasks. The individual task durations are variable and thus one must estimate a discrete value within the range of values of each task's duration variable in order to calculate the critical path. The critical path thus calculated will then only represent the critical path of the project based on those discretely chosen estimations of task durations. Therefore if one were calculate the critical path a second time using different discrete values then it there is a probability that a different critical path will result and hence the tasks identified for inclusion in Equation 3.1 could to be different too.

It is therefore necessary to determine the critical path in such a manner that the variability of all of the tasks is incorporated.

### **3.3.1.2 Monte Carlo and Critical Task Determination**

The method that is best able to determine the critical path whilst incorporating the variability of all of the project tasks is Monte Carlo simulation. To do this simulation all of the task duration variables must be utilised. In this simulation process, after each iteration, the critical path must be calculated and then the relative frequency of a task's presence on the critical path can be measured.

Thus after each iteration the randomly estimated durations must be substituted into the precedence network and the critical path must be calculated. This will identify the critical tasks given the randomly estimated task durations. This process must then be repeated a number of times. The tasks that then have the greatest relative frequency on the critical path will be the tasks most likely to be critical. These tasks are then the tasks that should be incorporated into Equation 3.1 in order to analyse and later design the duration estimate.

By using a Monte Carlo simulation the most likely critical tasks are identified. These likely critical tasks then also determine the reliability of the project's duration.

Through the use of the Monte Carlo simulation there is the possibility that all of the project tasks are on the critical path at least once during the iteration process. In this case substituting all of the tasks that are likely to be critical into Equation 3.1 would require the inclusion of all of the tasks. This result would negate the use of the Monte Carlo simulation in the first place as all of the tasks could have been used initially. In such cases it will be beneficial to use the most likely critical tasks. In other words use those tasks whose relative frequency is greater than a designated percentage frequency. The value of this designated frequency must be determined by the PM professional.

The Monte Carlo simulation method is however computationally demanding and cannot be done without the use of a random number generator and a computer processor. There is currently software available that can perform Monte Carlo simulations in a project environment but due to the cumbersome calculations required the simulations can take many hours and therefore, although accurate, the Monte Carlo simulation is clearly not practical.

### **3.3.1.3 Reliability and Critical Path Determination**

A second method that can be used to determine the most likely critical tasks is to use First Order Second Moment (FOSM) reliability analysis to systematically calculate the critical path for different levels of reliability. This method is much less demanding computationally and can be done by hand if required. A description of FOSM follows in the next chapter.

This method calculates the critical path at different levels of reliability. In a similar manner to the Monte Carlo method it calculates the number of times a task appears on the critical path and generates a critical relative frequency for that task. This then identifies those tasks most likely to be critical.

The benefits of this approach are that it is computationally simpler and that it can greatly reduce the required number of calculations. The limitations of this method are that the

critical path calculated at a set level of reliability represents the critical path if every task's duration has been adjusted to the set level of reliability. In other words, it calculates the critical path as if all the tasks are set to their 'Best Case' durations or all the tasks are set to their 'Worst Case' durations. Thus, unlike the Monte Carlo simulation, this method does not incorporate the more realistic situation where some tasks will actually finish as per their best case and others as per their worst case.

Such situations will thus result in critical paths that are not necessarily the same as either the best case critical paths or the worst case critical paths as calculated using FOSM. Thus the critical tasks identified in such a situation by the Monte Carlo simulation method will not be identified by the FOSM process and will thus not be included in the calculation of the relative frequencies of the critical tasks. There is therefore the potential for some tasks not to be identified as critical tasks by means of this method.

The likelihood of this happening decreases as the number of times the critical path is calculated at different reliability levels increases. Unfortunately irrespective of the number of times the critical path is calculated at different reliability levels the possibility of not identifying some of the potentially critical task possibility cannot be totally eradicated.

It is however important to understand that this method is only being used to identify those tasks that are potentially on the critical path. The likelihood that a task is potentially only on the critical path once is very small. The reason for this is that for such a situation to arise there must be a totally unique configuration of all the other variable tasks so as to make the said task critical only once. The fact that the configuration must be unique means that the likelihood that that task is critical is small. Due to the fact that the likelihood of such a situation happening is small, the uniquely critical task has a small relative frequency of occurrence. Therefore the resultant effect of this infrequently critical task on the reliability of the project's duration is minimal to negligible. Therefore should the FOSM method not identify such tasks their omission from Equation 3.1 will have a small to negligible effect on the project's duration reliability.

The identification of critical tasks by means of FOSM in comparison to Monte Carlo simulation was tested as presented in an experiment, Example 3.1 shown in Appendix A

This example was performed to assess the accuracy of the FOSM method in identifying critical tasks given the above limitation. The FOSM method was used to identify the critical tasks at 11 different levels of reliability ranging between 0% and 100%. A Monte Carlo simulation was then used to identify the critical tasks using firstly 200 iterations and then 5000 iterations. The results of the two methods were then compared.

In order to incorporate a greater range of possible outcomes the example was conducted three times for three different cases namely;

Case 1: Small Standard Deviations for the variables and a simple precedence network

Case 2: Large Standard Deviations for the variables and a simple precedence network

Case 3: Large Standard Deviations for the variables and complex precedence network.

The relative frequency of a task's appearance on the critical path was calculated. If the relative frequency was greater than zero then the task appears on the critical path at least once. Hence the identification of the most likely critical tasks is achieved. For each of the three cases the relative frequency of appearing on the critical path is mapped in the following figures;

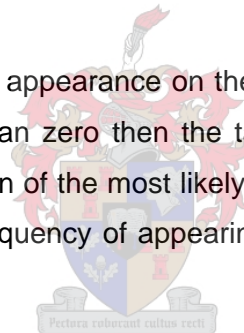


Figure 3.4.1: Frequency of a task's appearance on the Critical Path.  
 Case 1-Small Standard Deviations, Simple Precedence Network

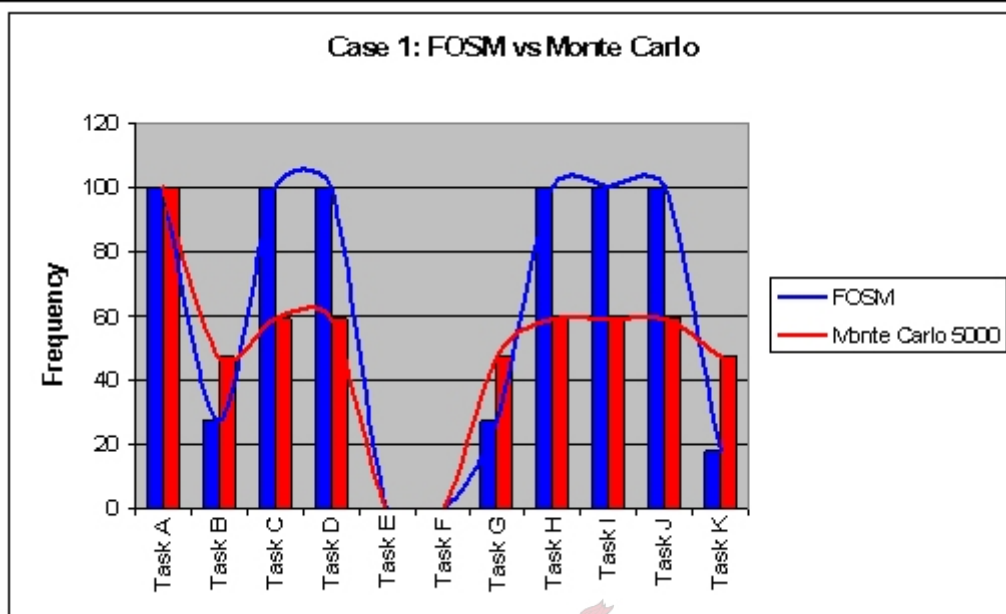


Figure 3.4.2: Frequency of a task's appearance on the Critical Path.  
 Case 2- Large Standard Deviations, Simple Precedence Network

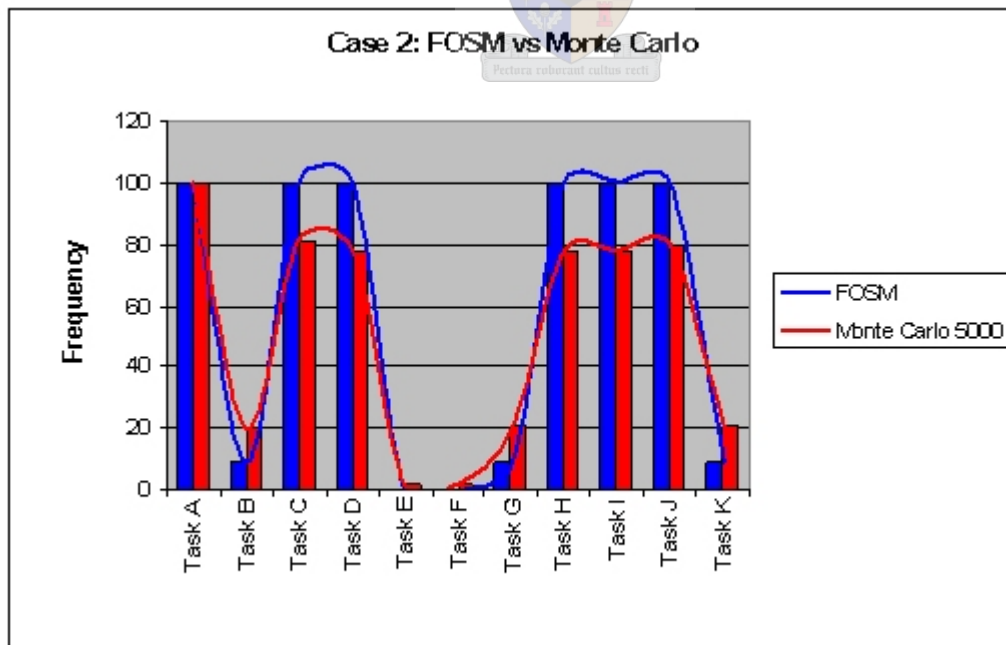
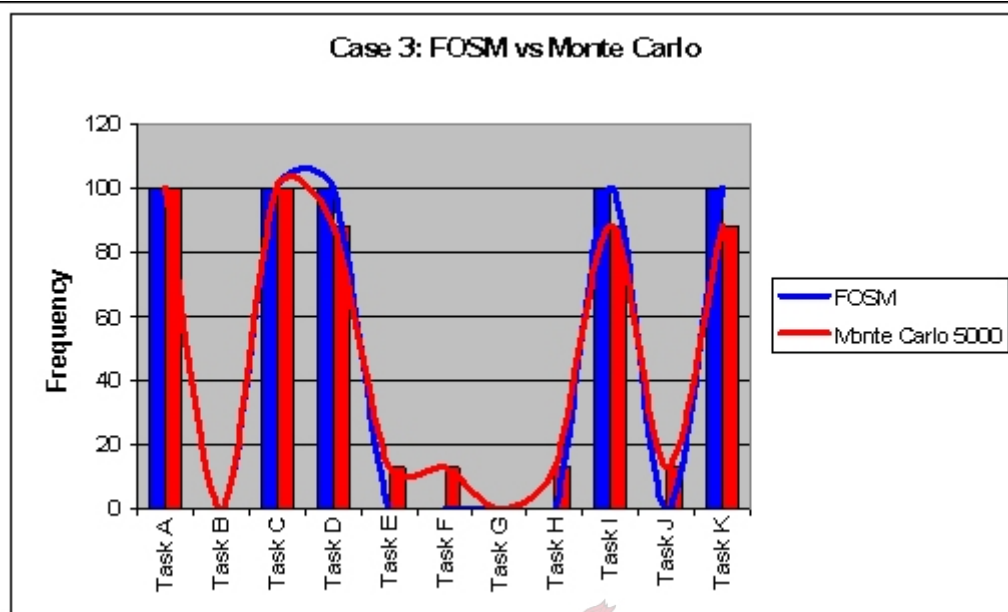


Figure 3.4.3: Frequency of a task's appearance on the Critical Path.  
Case 3- Large Standard Deviations, Complex Precedence Network



From the above figures it is evident that under all three cases the FOSM correctly predicted which tasks would most likely be on the critical path. It did not however identify some tasks to have occurred on the critical path when the Monte Carlo Simulation did. As predicted though the relative frequencies of appearance on the critical path of these missed tasks was low and their resultant effect on the reliability of the project duration will be negligible.

To test this, a second experiment was performed, Example 3.2. This example focussed on Case 3 where FOSM failed to identify four tasks identified as critical by a Monte Carlo Simulation. In this experiment the duration reliability, returned as a percentage, was calculated for the project firstly where all of the critical tasks as identified by Monte Carlo were included as randomly distributed variables and secondly where only the critical tasks as identified by FOSM were included as randomly distributed variables. Spot dates were extracted from the simulation data and compared for reliability.

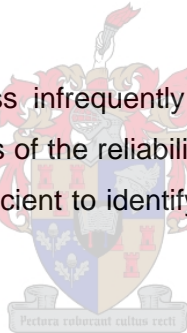
The example made use of the Monte Carlo Simulation method to calculate the reliability and is shown in Appendix A.

The results were as follows:

Table 3.1: Duration Reliability Difference between Critical Tasks Identified

Date	Reliability in % Calculated with Monte Carlo Identified Critical Tasks	Reliability in % Calculated with FOSM Identified Critical Tasks	Difference in %
13-Jun-07	10.9	9.3	-1.6
21-Jun-07	17.2	19.1	1.9
29-Jun-07	29.0	30.6	1.6
10-Jul-07	44.0	44.7	0.7
20-Jul-07	61.7	61.8	0.1
31-Jul-07	74.1	74.9	0.8
16-Aug-07	90.0	89.6	-0.4
		Average of the Absolute Values of Differences.	1%

Thus although the potential to miss infrequently critical tasks is there, the effect of missing these tasks is small in terms of the reliability of the project duration and thus the FOSM determination method is sufficient to identify the tasks that must be included into Equation 3.1.



#### 3.3.1.4. Selection of Probabilistic Duration Model.

For the purpose of this study the method of determining the critical tasks to be included into the critical duration units model will be the FOSM method. A description of the methodology follows in the next chapter.

Further, to summarise, the probabilistic duration model that will be utilised to analyse and manipulate the duration estimate consists of all the representative variables of those tasks that are potentially critical. These critical variables are summed in series and their durations are uncorrelated. The probabilistic model to analyse and design durations based on different levels of reliability is thus given by Equation 3.1.



### 3.3.1.5 Interrelations Between Tasks

Note that in section 3.3.1 it was described and again in 3.3.1.4 it was stated that the task durations were uncorrelated as a variation in one task only shifts other tasks but does not alter their durations. In reality however and especially in a project environment some of the external influencing factors that occur are calendar dependant. A simple example is rainfall. Thus if a task is shifted in time by a variation in a preceding task it may be shifted into a time zone that is affected by an influencing factor from a time zone that was not affected by that factor or vice versa. If the shifted task is then susceptible to the influence of this factor then its duration will change. Thus the effects of tasks calendar position cannot be ignored and as such the modelling of these effects will be incorporated in Chapter 5.

### 3.3.2 Cost Model

In a similar fashion a cost model can be produced. The individual task costs must first have been estimated as random variables. All tasks in a project incur a cost of sort and, unlike duration, all tasks will contribute to the entire cost of the project. The total cost of all the tasks in the project can therefore be modelled by simply summing the individual cost variables of each task. The resultant probabilistic model will thus be:


$$C = \sum_{j=1}^n c_j \quad (3.2)$$

- Where  $c_j$  = a random variable which represents the cost of an individual task  
 $n$  = the number of tasks in the project.  
 $C$  = A probabilistic model, Total Cost of Project Tasks.

Importantly task costs are often dependant on the task duration. Other task costs are also attributable to materials or consumables used to execute the task. These consumables cost money, the amount of money is dependant on the quantity of the material required and not time. For such tasks the estimated cost will be calculated by;

Task cost = (Duration)x(Rate per duration unit)+(Material Quantity)x(Cost per unit)

In the above formulation the task Duration will be represented by  $d_i$ , the estimated duration variable. The Cost per unit of duration could be assumed to be discrete although its value could change as a result of calendar position in the form of overtime for instance. This cost per unit of duration includes resources, labour and other time-dependant costs applicable to the task. The Material Quantity will also need to be represented by a random variable and the cost per unit of material could easily be assumed as a discrete value at time of estimation. Such a general formulation could be used to estimate all individual task costs by eliminating the irrelevant cost by simply setting equal to zero the irrelevant cost component, either material quantity or task duration.

Thus the total cost of all the project activities can be modelled by;

$$C = \sum_{i=1}^n (d_i \cdot RPU + q_i \cdot CPU) \quad (3.3)$$

where;

$C$	=	Total cost model of all individual cost variables
$d_i$	=	Estimated Task duration variable
$RPU$	=	Rate Per Unit of duration (assumed fixed)
$q_i$	=	Estimated Task material quantity variable
$CPU$	=	Cost Per Unit of material (assumed fixed)

The next chapter will describe and detail methods to use these models to analyse and design estimates to achieve specified desired outcomes and assist in decision making.

#### 4. First Order Second Moment Reliability Method Analysis.

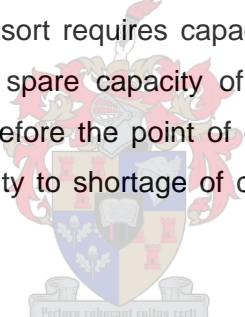
In First Order Second Moment (FOSM) analysis a first order approximate solution of a limit state function is obtained, where the limit state function is expressed in terms of probabilistic variables which are themselves approximated by second order (Normal) distributions. FOSM analysis then estimates reliability in terms of the probability that the limit state function is less than zero.

The limit state function is based on the basic performance function of Capacity  $C$  minus Demand  $D$  or;

$$f(x) = C - D \quad (4.1)$$

Where  $C$  and  $D$  are functions of random variables

This function can be representative of many different properties of many different systems where a demand of any sort requires capacity to appease that demand. The function  $f(x)$  then represents the spare capacity of a given system if  $C > D$  and the shortage of capacity if  $C < D$ . Therefore the point of equilibrium or the point where the function moves from spare capacity to shortage of capacity is represented by  $C = D$  or simply when;


$$0 = C - D \quad (4.2)$$

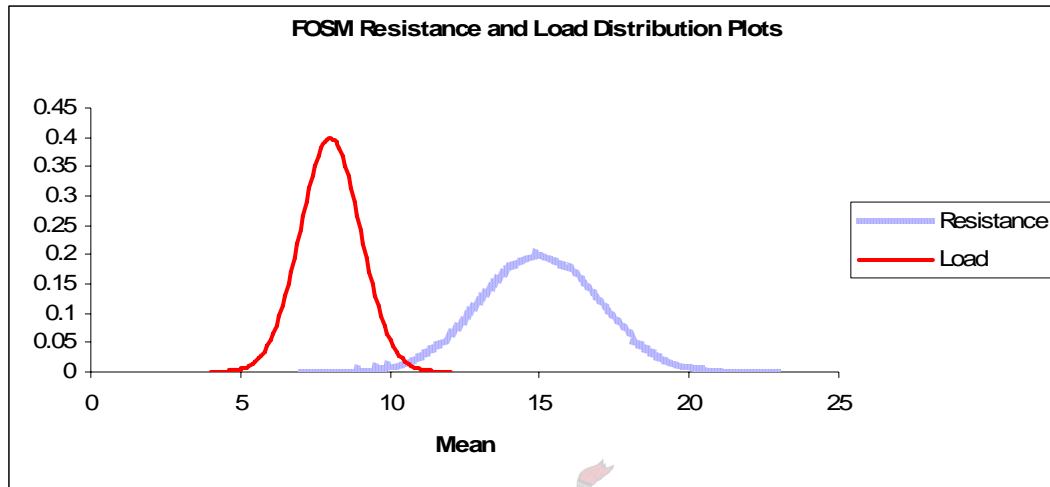
Equation 4.2 represents the exact configuration where the demand is at the limit of the capacity or in other words, where the performance function will fail and the point where the system represented by the performance function, Equation 4.1, moves from spare capacity to shortage of capacity. The performance function at this configuration is called the limit state function.

The limit state function (LSF) is then used by FOSM to determine the reliability of the function, it calculates the probability that  $C > D$  given that  $C$  and  $D$  are functions of random variables.

Graphically FOSM calculates the probability that the Capacity distribution is greater than the Demand Distribution. As shown in Figure 4.1 the Capacity distribution, notated

Resistance, is greater than the Demand distribution, notated Load, for most values of both Capacity and Demand and thus the probability that Capacity is greater than Demand, calculated by FOSM as reliability, is high.

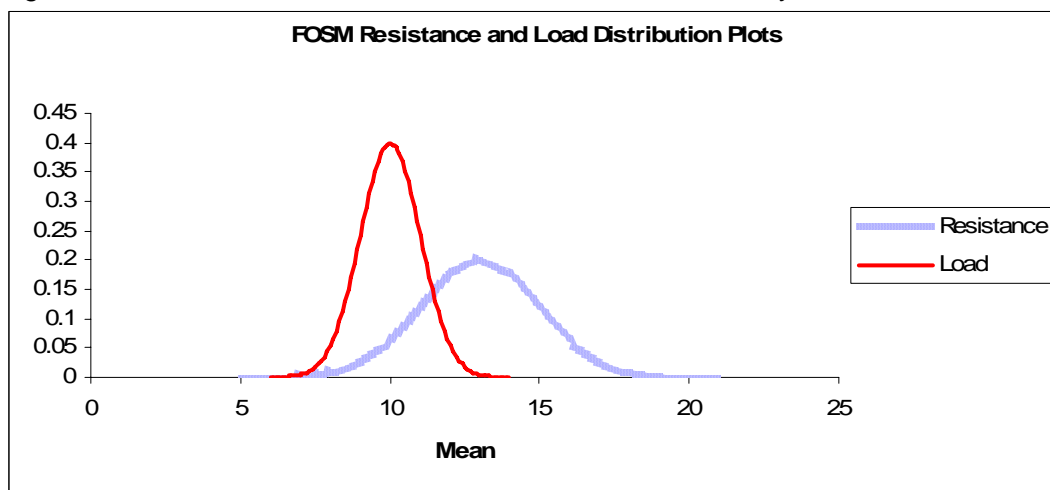
Figure 4.1: Performance function with HIGH level of reliability



In figure 4.2 the Capacity values are greater than the Demand values to a much lesser extent than as shown in Figure 4.1. The probability that Capacity is greater than demand, once again calculated as reliability, is thus far less in this case than as shown in Figure 4.1.

The FOSM is then a mathematical method that calculates the actual probabilities of the above graphical representation.

Figure 4.2: Performance function with LOW level of reliability



FOSM is currently used in structural engineering to determine the level of safety of a design and also during the establishment of the design codes. It is particularly useful because the load on a structure, i.e. demand and its designed strength, i.e. capacity, can readily be modelled and inserted into the LSF and analysed.

During analysis FOSM calculates a reliability factor denoted by the symbol  $\beta$  (beta). This factor details the level of reliability present in the LSF by being related to the probability of the capacity being sufficient to meet the demand, or stated otherwise, the probability of the capacity being greater than demand in the LSF by;

$$P(C > D) = \Phi(\beta)$$

Where  $P(C > D)$  = The probability that the performance function will not fail and can be considered reliable  
 $\Phi(\beta)$  = The cumulative Normal Distribution Function

In other words  $\beta$  details the level of probability that the Capacity  $C$  is greater than the Demand  $D$  or simply the probability that the LSF is 'safe'. FOSM analysis also calculates, for each variable in the LSF sensitivity factor values called; Directional Cosine or Alpha values, denoted by the symbol  $\alpha$ . The calculated alpha values detail the relative contribution of each variable to the total variability of the LSF. This intuitively highlights the variables whose variability affects the reliability of the LSF the most.

Most structural design codes stipulate a  $\beta$  value representative of an acceptable level of reliability or safety. This  $\beta$  value is decided upon on a country or regional basis and thus differs for various countries. The code writers must then ensure that the expressions they stipulate are in accordance to the desired level of reliability. Traditionally this level of reliability is assured by multiplying the design variables by Safety Factors in order to ensure that the resistance is, in most to all, cases greater than the load by a sufficient margin and hence safe. FOSM is currently used to calculate these factors by means of the results of the probabilistic modelling of structural performance.

The calculated safety factors for the respective design variables are then called; Partial Factors. The value of these partial factors is determined by the statistics of each variable in the LSF, the calculated Alpha value and the desired  $\beta$  value. The  $\beta$  value used to determine the Partial Factors is the  $\beta$  value that is prescribed in the respective country. The Partial factors calculated then are used in a similar manner as safety factors except that they now result in values for a variable that represent the desired level of reliability.

The Partial factors are thus used as design tools to assist a designer in ensuring the design or expression produced is safe to a desired level of probability. It is this property of the FOSM that holds the most potential when viewed from a Project Management perspective.

#### **4.1 FOSM in Project Analysis and Design.**

As already seen, FOSM analyses a probabilistic model of a situation and can detail the level of reliability within that model. It is also able to detail the relative contribution of the variables that constitute the model to the overall reliability of the model. Based on this analysis, Partial factors can then be calculated which can be incorporated into the model to ensure that the, now designed, model meets a desired level of reliability. In the context of Project estimation, FOSM and the resulting partial factors that are calculated can be used to represent the reliability of an estimate or in other terms, the level of probability that an estimate is achievable.

The process of using the FOSM to analyse and determine the reliability of an estimate and also the methods of then designing the estimate to achieve a desired level of reliability will be described below. The resultant conclusions that can be drawn will also be detailed. To begin, the duration estimate will be studied and the relevant processes detailed.

The probabilistic model of the Critical Duration Units estimation is given by Equation 3.1. In this model each critical variable  $d_i$  is described by a Triangular distribution with an estimated Mean and Standard deviation. The Triangular distributions will often have a Skewness value highlighting the weighting of the individual distributions.

FOSM approximates the variables to be Normally distributed. Techniques are available to create Normalised approximations of other distribution types but these are computationally complicated. In view of the crudeness of the original Triangular distribution estimation it is rather unnecessary to apply these techniques here. Instead, simple Normalised approximations will be created here by assuming the same statistical parameters calculated with the Triangular distribution for the approximated Normal distribution.

This will not incorporate the skewness value of the original Triangularly distributed variable. However, the fact that the mean of the Triangular distribution will always shift away from its mode into the direction of the skewness means that by assuming this mean for a Normalised approximation the probability density will shift in the same direction. This will thus over-estimate the calculated probability values of the approximated Normal distribution in accordance to the original skewness. Although this approximation is not exact, it can be considered an acceptable approximation for the purpose at hand.

This suitability of the above approximation is limited in the fact that as the magnitude of the skewness of the Triangularly distributed variable increases, the margin for error of the above approximation increases. The larger the approximation error, the greater the likelihood of erroneous conclusions regarding probability of the estimated duration or cost values. In highly skewed distributions the margin of error can be reduced by approximating the Triangularly distributed variable as a 3-Point Lognormal distribution with matching skewness. The 3-Point Lognormally distributed variables can be inserted into the model and Equivalent Normal approximation techniques can be applied to facilitate FOSM analysis and design. The techniques are however outside the scope of this study and as such have not been presented in this study.

Furthermore, the magnitude of the skewness in a Triangularly distributed variable that necessitates the utilisation of the 3-Point Lognormal distribution will be an important parameter that will need to be determined by further study. This parameter is also outside of the scope of this study and is as a result not covered here.

## 4.2 FOSM Analysis and FOSM Based Design Estimates

The FOSM methods used to calculate the reliability of the duration and cost models and secondly to design the duration and cost models will be described here in a stepwise fashion. To begin, the linear case will be described, analysed and designed. Thereafter the non-linear case will be described. The Critical Duration Units estimate model will be discussed first followed by the Total Cost estimate Model.

### 4.2.1 Determining the Critical Path using FOSM

As described in Chapter 3 the reliability of the project duration is dependent on the critical tasks and in a variable project environment these tasks can vary. Thus it is important to determine those tasks that are potentially on the critical path. The process used to calculate this using FOSM will be detailed here in a stepwise fashion.

#### Step 1: Approximate all the variables as Normal Distributions.

The individual duration estimates, represented by a Triangularly distributed variables must be approximated as a Normally distributed variables for all of the tasks in the project. All of these variables must be summed and the **Total** Cumulative Duration Units model  $Dt$  can then be created.


$$Dt = \sum_{i=1}^n d_i \quad (4.2a)$$

where  $Dt$  is  $N(\mu_{Dt}, \sigma_{Dt})$  and,

$d_i$  is  $N(\mu_{d,i}, \sigma_{d,i})$

Due to the fact the expression of the Total Cumulative Duration Units model is linear and all of the constituent variables are Normally distributed the following steps can be executed by the following simple calculations.



**Step 2: Calculate the Mean of the model  $\mu_{Dt}$**

The mean of the model is simply the sum of all the individual task means.

$$\therefore \mu_{Dt} = \sum_{i=1}^n \mu_{d,i} \quad (4.2b)$$

**Step 3: Calculate the Alpha Values.**

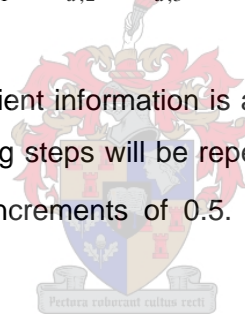
The Alpha values are calculated for each task with the following formula;

$$\alpha_i = \frac{\sigma_i}{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2}} \quad (4.2c)$$

Thus,

$$\alpha_{d,i} = \frac{\sigma_{d,i}}{\sqrt{\sigma_{d,1}^2 + \sigma_{d,2}^2 + \sigma_{d,3}^2 + \dots + \sigma_{d,n}^2}}$$

At this stage of the process sufficient information is available to begin the calculation of the critical path. Thus the following steps will be repeated for values of  $\beta$  ranging from negative 2.5 to positive 2.5 in increments of 0.5. Thus a total of 11 calculations is required if  $\beta=0$  is included.



**Step 4a: Calculate the Partial Factors Corresponding to a Beta Value.**

Using the  $\alpha$  values and the  $\beta$  value it is possible to calculate the amount by which a variable must be adjusted in order to achieve a reliability level that corresponds to the specific  $\beta$  value. This amount is calculated as a partial factor using the following formula;

$$\gamma_i = \mu_{d,i} \left( \frac{1 + \alpha_{d,i} \cdot \beta \left( \frac{\sigma_{d,i}}{\mu_{d,i}} \right)}{dx_i} \right) \quad (4.2d)$$

where;

$\gamma_i$  = the partial factor to apply to task  $i$

$\mu_{d,i}$  = the mean duration value of task  $i$

$dx_i$  = the characteristic value of the duration (equals the mean value in this case)

In this case if it assumed that the characteristic value of the task duration equals its mean value then the equation for  $\gamma_i$  can be reduced to

$$\gamma_i = 1 + \left( \frac{\alpha_{d,i} \cdot \beta \cdot \sigma_{d,i}}{\mu_{d,i}} \right) \quad (4.2e)$$

Thus a partial factor, the value of which is in accordance to the specific beta value of the iteration, must be calculated for each variable

**Step 4b: Multiply the Mean values by the Partial Factors.**

The calculated Partial factors for each task's variable must be multiplied by the respective task's mean value. This will calculate a value for the duration of the task that corresponds to the specific  $\beta$  value for the calculation. Thus for all task variables calculate the duration that corresponds to the desired level of reliability by;


$$T_i = \gamma_i \cdot \mu_{d,i} \quad (4.2f)$$

Where:  $T_i$  = the task duration representing the specific  $\beta$

and  $\mu_{d,i}$  = The mean of the individual task's duration units estimation

**Step 4c: Calculate the Critical Path Using the Adjusted Durations**

The  $T_i$  must be substituted for each task's duration into the precedence network. The Critical Path must then be calculated using these adjusted task durations and for each task that is found to be critical, one success is recorded.

**Step 4d: Repeat Steps 4a to 4c for the different Beta Values.**

Repeat these steps for the values of  $\beta$  (-2.5 - 2.5) recording the successes after each iteration.

**Step 5: Determine the most likely Critical Tasks**

For each task, sum the number of times a task is on the critical path, the successes, and divide this number by the number of iterations (11). This value then represents the relative frequency of the task on the critical path. All those tasks that have a relative frequency of greater than zero are likely to be critical at least once and their representative variables must be included into the Critical task duration units model given by Equation 3.1.

Thus all of the tasks that are on the critical path at least once are identified and incorporated into the critical task duration model. This model can then be used to analyse and design the respective duration estimates.

**4.2.2 Duration Units Estimation Analysis and Design**

Now that the critical tasks have been identified and the critical duration units model has been created, the model can be analysed and manipulated to design project duration estimates to specified levels of reliability. This process is similar to the process described above to determine the critical tasks except that the model now only consists of those critical tasks. Thus, again in a step wise fashion, the process required to analyse and design project duration estimates is detailed below.

#### 4.2.2.1 Duration Units Estimation Analysis

**Step 1: Analysis: Calculate the Mean and Standard deviation of the Model  $D_c$**

The mean of the model is simply the sum of all the individual critical task means.

$$\therefore \mu_{D_c} = \sum_{i=1}^n \mu_{d,i} \quad (4.3)$$

where  $\mu_{d,i}$  = the mean of the critical task variable  
 $n$  = the number of critical tasks

The Standard Deviation is given by the formula:

$$\sigma_{D_c} = \sqrt{\sum_{i=1}^n \sigma_{d,i}^2} \quad (4.4)$$

where  $\sigma_i$  = the standard deviation of the critical task's duration variable  
 $n$  = the number of critical tasks

The Critical Duration units model  $D_c$  is now equal to a normally distributed variable  $D_c = N(\mu_{D_c}, \sigma_{D_c})$  which represents the distribution of the Critical Duration units estimate. The variable  $D_c$  can be plotted with the function;

$$f_{D_c}(x) = \frac{1}{\sigma_{D_c} \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2} \left( \frac{x - \mu_{D_c}}{\sigma_{D_c}} \right)^2} \quad (4.5)$$

**Step 2: Analysis: Calculate the Original, Discretely Estimated Duration Units**

The Original, discretely estimated duration units is simply the sum of the mode duration unit values of the original Triangular distributions of the critical tasks. These values are simply the values that the PM professional originally estimated with the semantics, "Anticipated Duration" from section 3.2.1.

Importantly, only the mode values of the critical tasks must be summed.

This value denoted by  $ADE$ , Anticipated Duration Units Estimate, is simply equal to;

$$ADE = \sum_{i=1}^n ade_i \quad (4.6)$$

Where  $ade_i$  = the anticipated duration unit estimate for a critical task made by the PM professional.

By ignoring the Best and Worst case estimates, the resulting task's  $ade_i$  value is a discrete value with no variability and thus  $ADE$  is also a discrete value with no variability.

The  $ADE$  value is a cumulative duration units value and is thus not equal to the anticipated duration. To calculate the anticipated project duration the  $ade_i$  values must be substituted into the precedence network and the critical path calculated. The calculated duration is then the Anticipated project duration.

**Step 3: Analysis: Calculate the Probability of the Project Critical Duration Units being Less than or Equal to the Anticipated Duration**

The probability that the critical project duration units will less than or equal to the Anticipated duration units is given by the probability that the variable  $D_c$ , is less than the Anticipated duration units estimate or  $P(D_c < ADE)$ . The calculation of this probability is given by the expression;

$$P(D_c < ADE) = \int_{-\infty}^{ADE} f_{D_c}(x) dx \quad (4.7)$$

The resulting value calculated in this step represents the probability that the project would complete by or on the finish date as originally Anticipated by the PM professional.

This calculated probability value is an indication of the estimating bias of the PM professional. This is an interesting observation to make in that the resulting project

duration if the  $ade_i$  values were substituted into the precedence network is the duration that the PM professional would have used in practice had no probabilistic analysis and design been done. Thus the probability of this duration being achieved or bettered, based on estimated variability indicates to the estimator or decision maker the bias of the PM professional to over or underestimate.

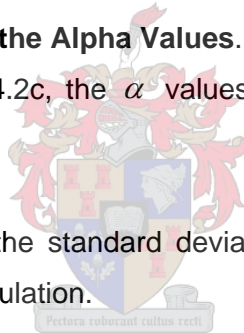
Up until this point of the process the model has merely been analysed. The next steps detail how the project duration can be designed to meet specified decision criteria. Thus from this point on, the level of reliability desired by the decision maker of the designed project duration must be specified. This must be specified as a target or desired  $\beta$  value or simply a desired reliability.

#### 4.2.2.2 Duration Units Estimation Design

**Step 4: Design: Calculate the Alpha Values.**

Utilising Equation 4.2c, the  $\alpha$  values must be calculated for all critical tasks;

**Importantly**, only the standard deviations of the critical tasks must be included in this calculation.



**Step 5: Design: Calculate the Partial Factors Corresponding to a Desired Level of Reliability.**

Using these calculated  $\alpha$  values and the specified desired reliability, denoted here as  $\beta$ , it is possible to calculate the amount by which a variable must be adjusted by in order to achieve the desired reliability. This amount is calculated as a partial factor  $\gamma_i$  using Equation 4.2d.

if it assumed that the characteristic value of the task duration equals its mean, the equation for  $\gamma_i$  can be reduced to Equation 4.2e

Therefore for **each critical task** only a partial factor  $\gamma_i$  is calculated.

**Step 6: Design: Multiply the Mean values by the Partial Factors.**

The Partial factors have been calculated to a certain desired level of reliability. For example Partial factors calculated with a  $\beta$  value of 1.65 represent partial factors that when multiplied by the mean values of the respective task variables will result in duration values that represent 95% reliability. Thus for all critical task variables calculate the duration that corresponds to the desired level of reliability by;

$$T_i = \gamma_i \cdot \mu_{d,i} \quad (4.2f)$$

Where:  $T_i$  = the designed task duration representing the desired level of reliability  
and  $\mu_{d,i}$  = The mean of a critical task's duration estimate

**Step 7: Design: Calculate the Critical Path and Project Duration to the Desired Level of Reliability**

The calculated values  $T_i$  must be substituted for their respective task's duration into the original precedence network. The calculated Critical Path and resulting Project Duration are then representative of the desired level of reliability used to calculate  $T_i$ .

Thus the project Gantt chart and hence duration calculated in this manner are reliable to the specified level of reliability. This means that the probability that the physical project will complete before the designed completion date is that probability value represented by the specified reliability level.

**4.2.3 The Non-linear Case**

Up until this stage the PM professionals initial estimated duration values have been assumed to apply to duration as if duration is a single variable. This single variable representing duration has been approximated to a normal variable and the resulting critical duration units model was in this manner intentionally kept linear.

This was done to reduce the amount of calculations required and also ease the description of the method. It is however common in the project estimating environment to calculate task durations as a function of a task's variable quantity and production rate as given in Equation 4.2.3a, which implies that duration is not estimated as simply a single variable as assumed above.

$$d_i = \frac{Q_i}{P_i} \quad (4.2.3a)$$

$P_i$  = Tasks Rate of Production

$Q_i$  = Quantity of item required to complete task.

If both Quantity and Production rate described in Equation 4.2.3a are random variables then in terms of FOSM, Equation 4.2.3a is non-linear. Equation 4.2.3a can also be broken down further into more detail as the production rate is itself a result of a combination of various other easily identifiable and estimated random variables. However, for the purposes of this study and the description of the methods used to deal with the non-linear estimations of duration, Equation 4.2.3a will be sufficient and thus be utilised.

In practice, calculations to estimate the duration of a task are often made using non-linear equations similar to Equation 4.2.3a which cannot be dealt with by the methods described in the in sections 4.2.1 and 4.2.2 as these sections deal with the linear case only.

The analysis and design of the duration estimate using FOSM in this case, relies on an iterative process. It is however possible that this iterative process can be avoided in the project environment by making a plausible approximation, as will be described later in this section. In this section a method will be derived to design duration estimates of non-linear task duration functions, simply described by Equation 4.2.3a, by using an approximated simplification of FOSM.

In order to do this the non-linear expression of the task's estimated duration will be reduced to a single duration variable with an approximated mean and deviation. This



reduced variable can then be used in Section 4.2.1 and 4.2.2 as the estimated duration variable  $d_i$  in Equation 4.2a and if critical in Equation 3.1.

All of the non-linear task expressions are reduced to approximated linear variables. Then the methods described in Section 4.2.1 and 4.2.2 can be used to design task durations that meet a desired level of model reliability.

The process utilised to reduce the non-linear expressions into single linear variables will be explained through the use of the non-linear task duration equation, Equation 4.2.3a. Both of the variables, quantity and production are randomly distributed variables either estimated by the PM professional or obtained from historical data or both. Thus Equation 4.2.3a in effect represents a probabilistic model of an individual task's duration.

#### **4.2.3.1 Designing the Duration to a Desired Level of Reliability.**

The use of FOSM for the above, non-linear case, is far more intricate than the method described in Sections 4.2.1 and 4.2.2. The method relies on the Limit State Function and utilises an iterative process to determine the design point at which reliability is determined. In the project estimating environment this requirement can be avoided by making the approximation that the design point for each variable is represented by the mean value of each variable. This approximation is credible because firstly, the non-linearity of Equation 4.2.3a is not severe and secondly because the loss of accuracy in terms of the calculation is minor and negligible when considered in terms of the crudeness of the basic variable estimations.

This approximation however negates the need to use an iterative process and greatly simplifies the method described in this section.

Thus if the design point utilised in FOSM for the non-linear case was approximated to be the estimated mean value of the variable it pertains to, then the following method can be used to design estimates of non-linear duration estimation expressions.

##### **Step 1: Reduce the Non-linear Function into a Single Variable.**

This step is the most important in the process as in this step the non-linearity of the cost expression is removed reducing the expression into a single variable. The single variable

of importance here is the duration of the task  $d_i$ . Thus the production rate and quantity variables will be combined and reduced to result in the single variable  $d_i$ . The statistics of the variable  $d_i$  will incorporate and represent the statistics of its constituent variables.

In order to calculate this approximate single variable  $d_i$ 's statistics, the mean and standard deviation of Equation 4.2.3a must be calculated. Instead of using the iterative process required to calculate the design point utilised in FOSM, this need is removed through approximating the design point by using the mean values of the individual variables.

Then the approximated Mean of the reduced task duration variable  $d_i$ , is given by;

$$\mu_{d,i} \approx \frac{\mu_{Q,i}}{\mu_{P,i}} \quad (4.2.3b)$$

And the Standard Deviation of the reduced tasks duration variable is given by;

$$\sigma_{d,i} = \sqrt{\left( \sigma_{Q,i} \cdot \left( \frac{\partial d_i}{\partial Q_i} \right) \right)^2 + \left( \sigma_{P,i} \cdot \left( \frac{\partial d_i}{\partial P_i} \right) \right)^2} \quad (4.2.3c)$$

(Ang & Tang, 346)

Where the partial derivatives are calculated with the following formulas.

$$\frac{\partial d_i}{\partial Q_i} = \frac{1}{P_i} \approx \frac{1}{\mu_{P,i}}$$

$$\frac{\partial d_i}{\partial P_i} = -\frac{1}{P_i^2} \cdot Q_i \approx -\frac{\mu_{Q,i}}{\mu_{P,i}^2}$$

Thus the non-linear expression of  $d_i$ , detailed by Equation 4.2.3a, can now be represented by a single variable;

$$d_i = N(\mu_{d,i}, \sigma_{d,i}) \quad (4.2.3d)$$

The reduced non-linear duration variables given by Equation 4.2.3d then representative of the variables  $d_i$  in Equation 4.2a and if determined to be critical of  $d_i$  into in Equation 3.1.

The process described in Section 4.2.1 can then be utilised as if the task durations were estimated as single variables and the critical tasks can be determined.

The process described in Section 4.2.2 can then be utilised to analyse and design the duration estimate.

**Step 1: Analysis: Calculate the Mean and Standard deviation of the Model  $D_c$**   
Remains unchanged

**Step 2: Analysis: Calculate the Original, Discretely Estimated Duration Units.**

In the non-linear case the  $ade_i$  values required in this step must be calculated using Equation 4.2.3a. The originally estimated mode values of Equation 4.2.3a's constituent variables must be substituted into Equation 4.2.3a and the equation must be solved for  $d_i$ . Thus the values representing the Anticipated values estimated by the PM professional for the variables, Quantity and Production must be substituted into Equation 4.2.3a and the resultant duration value will be equal to the anticipated duration estimate value discussed in section 4.2.2.

This value denoted by  $ade_i$  Anticipated Duration Estimate, for each task non-linearly estimated by Equation 4.2.3a is therefore simply equal to;

$$ade_i = \frac{q_i}{p_i} \quad (4.2.3e)$$

Where  $ade_i$  = the discrete anticipated duration estimate, equal to a function of the mode values of the original Triangular distributions of its constituent variables as estimated by the PM professional.

$q_i$  = the anticipated quantity as estimated by the PM professional

$p_i$  = the Anticipated Production Rate as anticipated by the PM professional

These calculated  $ade_i$  values must then be substituted into Equation 4.6 and the  $ADE$  value is calculated

**Step 3: Analysis: Calculate the Probability of the Project Critical Duration Units being Less than or Equal to the Anticipated Duration**

Remains unchanged

**Step 4: Design: Calculate the Alpha Values.**

Remains unchanged

**Step 5: Design: Calculate the Partial Factors Corresponding to a Desired Level of Reliability.**

Remains unchanged

**Step 6: Design: Multiply the Mean values by the Partial Factors.**

Remains unchanged

**Step 7: Design: Calculate the Critical Path and Project Duration to the Desired Level of Reliability ( $\beta$  Values)**

Remains unchanged



#### 4.2.4 Information Acquired

Using this process the PM professional will be able to;

- Identify the most likely critical tasks quickly and simply.
- Determine the Probability of success of the originally anticipated project duration.
- Identify those tasks that are not only critical but that have the greatest effect on the uncertainty of the project duration
- Design task durations to incorporate a desired level of reliability.
- Design a project duration to a specified level of reliability.

### 4.3 Total Cost Estimation Analysis.

#### 4.3.1 The Linear Case

The method to analyse and then design the total cost estimate is similar to the method described to analyse and design the duration units estimate, resultant critical path and project duration except that it is not necessary to differentiate between critical tasks and ordinary tasks as all tasks must be considered to estimate the cost. The other differences in the methodology will be described below.

Firstly, the equation to be used to analyse and design a total cost estimate will be the one given by Equation 3.3 shown again for reference below.

$$C = \sum_{i=1}^n (d_i \cdot RPU + q_i \cdot CPU) \quad (3.3)$$

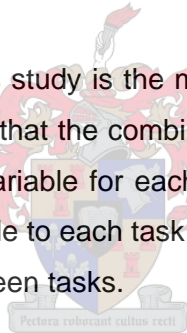
where;

$C$	=	Total cost model of all individual cost variables
$d_i$	=	Estimated Task duration variable
$RPU$	=	Rate Per Unit of duration (assumed fixed)
$q_i$	=	Estimated Task material quantity variable
$CPU$	=	Cost Per Unit of material (assumed fixed)

There are three different ways to analyse this that will all result in the same answers. These methods simply pertain to the manner in which the statistics are manipulated. The three methods are;

- |          |   |
|----------|---|
| Method A | Spilt equation 3.3 into two models; the first relating to duration based costs and the second relating to quantity based costs. These two models could then be analysed and designed separately.              |
| Method B | Combine the duration based cost variable with the quantity based cost variable creating a single cost variable per task. Analyse and design the total cost estimate based on this one cost variable per task. |
| Method C | Leave equation 3.3 as it is and analyse and design the total cost estimate without any manipulation of equation 3.3.  |

The method that will be used in this study is the method described briefly as Method B above. The reason for this choice is that the combination of the two cost constituents for each task results in a single cost variable for each task. This is beneficial because the analysis and design is then applicable to each task as a whole. This simplifies the results and also aides in comparisons between tasks.



The other methods cannot be ignored as they too have their benefits and will be more suited to specific situations. For instance the method described by Method A above will be particularly useful when a PM professional would like to assign different levels of reliability to the different cost components in a project. Such a situation could arise if a project being planned was to be executed in a country where labour costs were substantially lower than material costs and the variability of quantity based costs was more of a priority than duration based costs. Method C is also useful if a PM professional wants to investigate which of duration based or quantity based costs are a bigger threat to an individual task's estimated cost.

#### 4.3.1.1 Total Cost Estimate Analysis and Design Method B.

Method B requires the combination of the Duration and Quantity based costs into one variable. This single variable is then used to analyse and design the project total cost estimate. Method B uses Equation 3.3 as a starting point. Once again this process is best described in a step wise fashion.

**Step 1: Analysis: Approximate all the variables as Normal Distributions.**

The individual duration and quantity based estimates, represented by a Triangularly distributed variables must be approximated as a Normally distributed variables. The Cost model  $C$  will then be comprised of the sum of all of the individual duration estimates in the form of Normally distributed variables.

$$C = \sum_{i=1}^n (d_i \cdot RPU + q_i \cdot CPU)$$

where  $C$  is  $N(\mu_C, \sigma_C)$   
(meaning  $C$  is a normally distributed variable with a mean equal to  $\mu_C$  and a standard deviation equal to  $\sigma_C$ )

and,  $d_i$  is  $N(\mu_{d,i}, \sigma_{d,i})$

$q_i$  is  $N(\mu_{q,i}, \sigma_{q,i})$

Due to the fact the  $RPU$  and  $CPU$  values have been chosen to be deterministic the expression of the Cost model is linear. Further, all of the constituent variables are Normally distributed and thus the following steps can be executed by the calculations presented below. Should the  $RPU$  and  $CPU$  values not be deterministic then the expression would not be linear and a more complicated analysis and design procedure is required. FOSM is readily applied in such a case. The amended process applicable to the non-linear case is detailed below in Section 4.3.2.

**Step 2: Analysis: Combine the Duration and Quantity Based Costs.**

For simplification purposes a new variable Task Cost or  $tc_i$  is created. This variable will be equal to the combination of the variables  $d_i$  and  $q_i$ . Variable  $tc_i$  will, like  $d_i$  and  $q_i$  have a mean and standard deviation value. These values are required to perform the analysis and design and will be calculated from the statistics of the variables found in the expression given below.

$$d_i \cdot RPU + q_i \cdot CPU$$

Thus

$$\mu_{tc,i} = RPU \cdot \mu_{d,i} + CPU \cdot \mu_{q,i} \quad (4.8)$$

and

$$\sigma_{tc,i} = \sqrt{(RPU \cdot \sigma_{d,i})^2 + (CPU \cdot \sigma_{q,i})^2} \quad (4.9)$$

Thus variable  $tc_i$  is  $N(\mu_{tc,i}, \sigma_{tc,i})$

And the Total Cost Estimate Model is now given by;

$$C = \sum_{i=1}^n (tc_i) \quad (4.10)$$

**Step 3: Analysis: Calculate the Mean and Standard deviation of the Model given by Equation 4.10**

The mean of the model is simply the sum of all the individual task means.

$$\therefore \mu_C = \sum_{i=1}^n \mu_{tc,i} \quad (4.11)$$

The Standard Deviation is given by the formula:

$$\sigma_C = \sqrt{\sum_{i=1}^n \sigma_{tc,i}^2} \quad (4.12)$$



The model  $C$  is now equal to a normally distributed variable

$$C = N(\mu_c, \sigma_c) \quad (4.13)$$

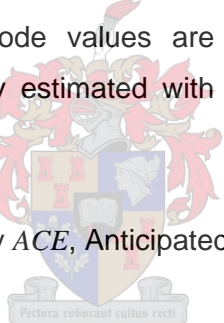
which represents the distribution of the Total Cost estimate. The variable  $C$  can be plotted with the function;

$$f_c(x) = \frac{1}{\sigma_c \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2} \left( \frac{x - \mu_c}{\sigma_c} \right)^2} \quad (4.14)$$

**Step 4: Analysis: Calculate the Original, Discretely Estimated Total Cost**

The Original, discretely estimated Total Cost is simply the sum of the modes of all the individual cost estimates from the Triangular distribution estimates. These mode values are simply the values that the PM professional originally estimated with the semantics, “Anticipated Cost” from Section 3.2.1.

This value denoted by  $ACE$ , Anticipated Cost Estimate, is simply equal to;



$$ACE = \sum_{i=1}^n ace_i \quad (4.15)$$

Where  $ace_i$  =the anticipated cost estimates, equal to the mode values of the original Triangular distributions as estimated by the PM professional.

By ignoring the best and worst case estimates, the  $ace_i$  value is a discrete value with no variability and thus  $ACE$  is also a discrete value with no variability.

**Step 5: Analysis: Calculate the Probability of the Project Cost being Less than or Equal to the Anticipated Cost**

The probability that the Total project cost will less than or equal to the Anticipated cost is given by the probability that the variable  $C$ , is less than

the Anticipated cost or  $P(C < ACE)$ . The calculation of this probability is given by the expression;

$$P(C < ACE) = \int_{-\infty}^{ACE} f_C(x) dx \quad (4.16)$$

In addition to analysing the probability that the anticipated cost will be greater than the eventual project cost, a PM professional is able to test the probability of the success of various different cost values that might be of interest to the PM professional. For example if a PM professional has a stipulated budget to work within then the PM professional can calculate the probability of meeting that budget by substituting the value of the budget in the place of  $ACE$  in Equation 4.16.

**Step 6: Design: Calculate the Alpha Values.**

The  $\alpha_{tc,i}$  values are calculated using Equation 4.2c. Importantly, unlike the duration estimate design that only takes into account the critical tasks, the cost model includes all tasks. The calculation of the  $\alpha_{tc,i}$  values here must therefore incorporate all tasks. Thus the denominator of Equation 4.2c includes the standard deviations of all tasks.

**Step 7: Design: Calculate the Partial Factors Corresponding to a Desired Level of Reliability.**

Using the Alpha values and the desired reliability it is possible to calculate the amount by which a variable must be increased by in order to achieve a desired reliability. This amount is calculated as a partial factor using the following formula;

$$\xi_i = \mu_{tc,i} \left( \frac{1 + \alpha_{tc,i} \beta_i \left( \frac{\sigma_{tc,i}}{\mu_{tc,i}} \right)}{d\chi} \right) \quad (4.17)$$

where;

$\xi_i =$  the partial factor to apply to task  $i$

$\mu_{tc,i} =$  the mean cost value of task  $i$

$dx_i =$  the characteristic value of the cost (equals the mean value in this case)

Thus in this case if it assumed that the characteristic value of the task cost equals its mean then the equation for  $\xi_i$  can be reduced to

$$\xi_i = 1 + \left( \frac{\alpha_{tc,i} \cdot \beta \cdot \sigma_{tc,i}}{\mu_{tc,i}} \right) \quad (4.18)$$

**Step 8: Design: Multiply the Mean values by the Partial Factors.**

The Partial factors are calculated to a certain desired level of reliability. Thus for all variables calculate the cost estimate that corresponds to the desired level of reliability by;

$$Pr_i = \xi_i \cdot \mu_{tc,i} \quad (4.19)$$

Where:  $Pr_i =$  the estimated task cost representing a desired level of reliability

and  $\mu_{tc,i} =$  Individual task's cost estimate, mean value

**Step 9: Design: Calculate the Total Cost Estimate with a Desired Level of Reliability (Beta Values)**

The total cost estimate calculated to represent a desired level of reliability denoted by  $C_\beta$  is simply the sum of the estimated task costs representing a desired level of reliability;

$$C_\beta = \sum_{i=1}^n Pr_i \quad (4.20)$$

### 4.3.2 The Non-linear Case

In the above section the design of the project estimates was confined to the linear normal case of FOSM. As in the case of duration this was done to reduce the amount of calculations required and also ease the description of the method. It is however common in the project estimating environment to calculate task costs as a function of a variable quantity, production rate, cost per unit and cost per unit of time. This is often more accurate as the estimators produce an estimate based on calculations of variables whose behaviour they are familiar with. A corollary of this situation is that the task cost estimates produced in such a way are sometimes easier to comprehend than the simple duration or cost directly estimated as in the linear case.

The calculations to estimate the cost of a task are thus often made using non-linear equations which cannot be dealt with by the method described in the above section as the above method is only applicable to the linear case of FOSM. The non-linear case of FOSM relies on an iterative process. In the same manner as with non-linear duration estimates the iterative process can be avoided in the project environment by making the same approximation that the design point values utilised by FOSM are equal to the mean values of the constituent variables. This process will be described later in this section.

In order to do this a simple non-linear expression for a single task's cost will be described. This non-linear expression of the task's estimated cost will then be reduced to a single cost variable with an approximated mean and deviation. This reduced variable can then be used in Section 4.3.1.1 as an estimated cost variable  $tc_i$  in Equation 4.10. Thus if all of the non-linear task expressions were reduced to approximated linear variables then the method further described in Section 4.3.1.1 can be used to design task costs to meet a desired level of model reliability.

The process will be explained through the use of a non-linear task cost function. The expression that will be used is;

$$tc_{nl} = \left( \frac{CT}{P} + Mc \right) \cdot Q \quad (4.21)$$

where

$t_{C_{nl}}$	=	Non-linear Estimated Task Cost
$CT$	=	Time Cost or cost per unit of time
$P$	=	Rate of Production
$Mc$	=	Material Cost
$Q$	=	Quantity of item required to complete task.

All of the above variables are randomly distributed variables either estimated by the PM professional or obtained from historical data or both. Equation 4.21 therefore represents a probabilistic model of an individual task's cost.

#### **4.3.2.1 Designing the Cost to a Desired Level of Reliability.**

As with the non-linear duration estimates the use of FOSM for the above, non-linear cost estimate is also more intricate than the method described in Section 4.3.1.1 the method relies on the Limit State Function and utilises an iterative process to determine the design point at which reliability is determined. In the project estimating environment this requirement can be avoided by making the approximation that the design point for each variable is represented by the mean value of each variable. This approximation is again credible because firstly, the non-linearity of Equation 4.21 is also not severe and secondly because the loss of accuracy in terms of the calculation is minor and negligible when considered in terms of the crudeness of the basic variable estimations.

Thus if the design point utilised in FOSM was approximated to be the estimated mean value of the variable it pertains to, then the following method can be used to design estimates of non-linear estimations.

#### **Step 1: Reduce the Non-linear Function into a Single Variable.**

In order to calculate this approximate variable the mean and standard deviation of Equation 4.21 must be calculated. Instead of using the iterative process required to calculate the design point utilised in FOSM this need is removed through approximating the design point by using the mean values of the individual variables.

Then the Mean of the reduced cost variable is given by;

$$\mu_{tc,nl} \approx \left( \frac{\mu_{CT}}{\mu_p} + \mu_{Mc} \right) \cdot \mu_Q \quad (4.22)$$

The Standard Deviation of the reduced cost variable is given by;

$$\sigma_{tc,nl} \approx \sqrt{\left( \sigma_{CT} \cdot \left( \frac{\partial tc_{nl}}{\partial CT} \right) \right)^2 + \left( \sigma_p \cdot \left( \frac{\partial tc_{nl}}{\partial P} \right) \right)^2 + \left( \sigma_{Mc} \cdot \left( \frac{\partial tc_{nl}}{\partial Mc} \right) \right)^2 + \left( \sigma_Q \cdot \left( \frac{\partial tc_{nl}}{\partial Q} \right) \right)^2} \quad (4.23)$$

(Ang & Tang, 346)

Where the partial derivatives are calculated with the following formulas.

$$\frac{\partial tc_{nl}}{\partial CT} = \frac{Q}{P} \approx \frac{\mu_Q}{\mu_p}$$

$$\frac{\partial tc_{nl}}{\partial P} = -\frac{1}{P^2} \cdot Q \approx -\frac{1}{\mu_p^2} \cdot \mu_Q$$

$$\frac{\partial tc_{nl}}{\partial Mc} = Q \approx \mu_Q$$

$$\frac{\partial tc_{nl}}{\partial Q} = \left( \frac{CT}{P} + Mc \right) \approx \left( \frac{\mu_{CT}}{\mu_p} + \mu_{Mc} \right)$$

Thus the non-linear expression of  $tc_{nl}$ , detailed by Equation 4.21, can now be represented by a single variable;

$$tc_{nl} = N(\mu_{tc,nl}, \sigma_{tc,nl}) \quad (4.24)$$

## Step 2: Substitute the Reduced Variables into the Cost Model

The approximated mean and standard deviation of Equation 4.21 must be calculated using the above formulation for all non-linear task estimates. Once this has been done

reduced variables must be substituted into the cost model. Therefore the calculated  $tc_{nl}$  Variables must then substituted into Equation 4.10 as the  $tc_i$  variables where;

$$\mu_{tc} = \mu_{tc,nl} \quad (4.25)$$

and

$$\sigma_{tc} = \sigma_{tc,nl} \quad (4.26)$$

Therefore the variables  $tc$  of Equation 4.10 are for each non-linear task now given by;

$$tc = N(\mu_{tc,nl}, \sigma_{tc,nl}) \quad (4.27)$$

Then Steps 4 to 9 of Section 4.3.1.1 can be executed and the cost estimate can be designed using the same method described there, once again with the one notable change found in Step 4, Analysis: Calculate the Original, Discretely Estimated Total Cost.

In the non-linear case the Original, discretely estimated cost values,  $ace_i$  are simply the resulting values calculated by Equation 4.21 if the modes of all of the constituent variables are used. These modes are the values that were originally estimated with the semantics, "Anticipated" by PM professional when estimating the Triangular distribution for each of the respective variables.

This value denoted by  $ace_i$ , Anticipated Cost Estimate for each non-linear task, is simply equal to;

$$ace_i = \left( \frac{ct_i}{p_i} + mc_i \right) \cdot q_i \quad (4.28)$$

Where

- $ace_i$  = the anticipated cost estimates .
- $ct_i$  = the anticipated time based cost value as estimated by the PM professional
- $q_i$  = the anticipated quantity as estimated by the PM

- professional
- $p_i$  = the Anticipated Production Rate as anticipated by the  
PM professional
- $mc_i$  = the Anticipated Material Cost per unit as anticipated by the  
PM professional

The calculated  $ace_i$  values must then be substituted into Equation 4.15 and the resulting  $ACE$  value is calculated and the method described in Section 4.3.1.1 can continue as described there.

#### 4.3.3. Derived Information

Using this process the PM professional will be able to;

- Determine the Probability of completing the project within the originally anticipated cost or budget.
- Identify those tasks that have the greatest effect on the uncertainty of the project cost.
- Design task cost estimates to incorporate a desired level of reliability.
- Design a project cost estimate to a specified level of reliability.

#### 4.4 Designing for Groups of Tasks

A major benefit of using the FOSM method is that it enables a PM professional to design either, task durations or costs on an individual basis. This is done in Step 6 of the duration design process and Step 8 of the cost design process where the calculated partial factor is multiplied by the task's mean duration or mean cost. The resultant duration or cost then represents the duration or cost that is representative of the specified reliability level.

In the above steps describing the duration and cost estimate design process the desired level of reliability used to calculate these partial factors was assumed constant for the project. This implies that the level of reliability required for each task is constant for all tasks.



In the project environment however the situation could arise whereby the consequence of task variability is not constant for all tasks. A situation might arise in project estimating where the variability of some of the tasks could carry far greater consequences for the project than other tasks. These situations are ad hoc and project specific. For instance a project might include a specific milestone that carries payment bonuses or penalties depending on it being met or not. The meeting of this milestone could then have a greater priority for the PM professional than project completion by the estimated completion date.

An example of such a situation in the construction environment is found in projects that involve operational service delivery infrastructure such as roads or railways. In these projects, the inoperability or closure of the road or railway is normally of great concern to the client. As such large penalties are imposed on the contractor if the opening of the road or railway for trafficking is delayed. As a result, tasks required to ensure the road or rail opens for traffic as estimated and planned are considered priority. Their timely completion is often considered more important to the PM professional than the completion of the whole project which could include non priority tasks like the erection of new signboards or landscaping.

In such a situation the design of the estimated duration of the priority tasks could warrant a higher level of reliability.

Irrespective of the reason for priority, FOSM provides the PM professional with the option of designing a project estimate with different levels of reliability for different tasks. For the higher priority tasks, such as those required to meet the example milestone, the PM professional could design their durations with a higher level of reliability.

To do this the PM professional must simply specify the required level of reliability when calculating the partial factor for a task. To simplify this procedure the PM professional can group the priority tasks together and assign a specific reliability level to the group. Then through the FOSM process these priority tasks will have partial factors calculated for them that represents their specific reliability level. This can be extended further to other priority groups each with their own specific level of reliability.

FOSM therefore presents the PM professional with the option of creating priority task groups and designing durations or costs for these groups at different specific levels of reliability. This option gives a PM professional greater control over the designed estimate produced. It facilitates very specific variability mitigation and gives the PM professional greater flexibility to design an estimate as per the requirements of the decision maker.

#### 4.5. Uncertainty Driver Ratio

One interesting and potentially useful observation can be drawn from comparing the designed task duration estimates and the designed task cost estimates. This observation comes in the form of the Uncertainty Driver Ratio or UDR. If the designed level of reliability is the same for both the Duration and cost estimate then the ratio of the partial factors applicable to a task is an indication of the driver of uncertainty. The ratio given by;

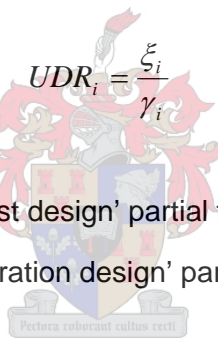
$$UDR_i = \frac{\xi_i}{\gamma_i} \quad (4.32)$$

$\xi_i =$

the 'cost design' partial factor to apply to task  $i$

$\gamma_i =$

the 'duration design' partial factor to apply to task  $i$



indicates the level by which cost uncertainty is greater or less than duration uncertainty. This information is particularly useful to a PM professional when the client has specified that either cost or duration in the form of milestones is important. The UDR ratio can deliver an indication of the important areas requiring extra managerial input and focus in the form of cost control or production control. The UDR ratio is also calculated for each task so the PM professional is then able to exercise this extra managerial input and control on a task by task basis to meet the desired needs of the client. This should simplify the process and lead to a greater probability of success when it comes to meeting the desires of the client.

## **4.6. Assessment of the Above Method**

### **4.6.1 Benefits**

The main benefits of the procedures presented in this section are that PM professionals now have tools at their disposal whereby they can incorporate uncertainty into a project estimate and manipulate that uncertainty to achieve a desired result. They now have tools available to design estimates that incorporate different levels of uncertainty or risk into them by increasing or decreasing the level of reliability of their estimates. They are also able to do this in a consistent mathematical manner that can be used across different projects for comparison and decision making purposes. PM professionals can now present and defend their estimates calculated to an explicit rational, not subjective level of reliability and risk. The estimates produced incorporate inherent uncertainty through the known, easily estimated, variability of either the duration or cost of the constituent tasks. This variability is then actively utilised to design estimates that can mitigate for the inherent uncertainty that results. Designing estimates that accommodate this uncertainty should result in estimates that are more realistic and more importantly, achieved on a more regular basis.

### **4.6.2 Hazards**

The designed estimates are still reliant on estimated variables. The occasion where historical data is available and can be incorporated into a basic variable will obviously result in as accurate estimation of reliability as possible. This will however not always be the case and it is predicted that most variables used in this process will be experience based guessed estimates. Thus the reliability calculations will often result in designed estimates that are designed using approximated input data. This fact results in the possibility of there remaining some unaccounted for, or over accounted for, uncertainty in the estimate as the individual variables may have been optimistically or pessimistically estimated by the PM professional. The process of estimating the variability of individual tasks should be an easier proposition than estimating the variability of the entire project but the fact remains that whichever method is chosen the result is still an estimate.

## **5. Influencing Factors.**

Projects invariably take place in dynamic environments where external factors can influence the performance of the project. These factors also tend to affect more than one task in a project and sometimes can affect all the tasks within the project. These factors can be in the form of project external global factors such as exchange rates, the business cycle, price fluctuations, weather, political disturbances or project internal global factors such as contractor or sub-contractor inexperience, labour unrest or productivity issues, fuel shortages and others. The factors all have the potential to affect the project in at least one way. For instance, weather conditions can affect certain project durations and cost. Further, exchange rates can affect multinational projects in any business sector. Unfortunately, the problem for PM professionals is that these types of influencing factors are very difficult to predict and thus present significant risks to a project. Quantifying this risk is a real challenge for companies and the field of project management in general. The focus of this section is thus to illustrate a method whereby the effects of global factors influencing more than one task can be incorporated into a project model and quantified during the planning and estimation phase.

### **5.1 Global Factors**

The term Global Factor is used for these influencing factors affecting a number of tasks because of the umbrella like nature of their influence and hence effects too. These factors form part of a wider, more encompassing environment than the localised environment the PM professional or project participants are consciously aware of. Global factors are also generally not within the control of these project participants. Further these factors have the ability to affect more than one project concurrently. As a result a single global factor can affect all projects in a company's portfolio from one end of the world to the other or simply across the street. This property of these global factors however requires higher level of management input and is beyond the scope of this study.

An important property of these global factors is that they tend to affect each vulnerable individual project in a manner that is unique to that project.

Global factors are often unpredictable in their magnitude or volatility to such an extent that some have lead to fields of study and extensive research in their own right. From a

project perspective, as a result of the above considerations, these factors are generally well known and easily identifiable. Thus when a risk assessment is compiled they usually feature. Unfortunately for the PM professional they are unpredictable and their resultant effects on projects are not usually clear or cannot confidently be quantified.

Project managers and those compiling a project estimate must however quantify these unpredictable factors often over a period of years, and sometimes even decades. This is, at this present time and with the currently available technology a near impossible task. Yet PM professionals must attempt to tackle it in order to raise the level of rational treatment of uncertainty intrinsic to the project estimating environment.

PM professionals have taken on this task in a pragmatic or possibly defensive way by estimating the effect that the occurrence of one of these global factors has on a project. By focussing on the effects on the project or in other words, the project's vulnerability to the global factors, the PM professionals are able to concentrate on an area where they have knowledge and experience and therefore the potential for accurate quantification.

PM professionals can therefore fairly accurately estimate the effects on a project should a global factor exercise its influence. They cannot however use such estimations directly because the global factors are not guaranteed to exercise their influence on a project. If these estimated effects were simply incorporated into a project estimate it would lead to unrealistic estimates. The PM professional therefore does not include these estimated effects directly into the estimate but instead creates allowances that are collectively intended to mitigate for the somewhat random actions, occurrence and influence of these global factors on the project. These allowances are then used to inflate the original estimate to a level such that the difference in magnitude in terms of cost or duration between the original estimate and the inflated estimate is supposed to mitigate for the effects of the influencing factors should they occur.

The unpredictable occurrence of these influencing factors is not guaranteed and thus these allowances are in effect quantifications of the risk of the occurrence of an influencing factor. These allowances are therefore often removed from the estimate and grouped as risk items, the ownership of which is normally then negotiated.

## 5.2 Quantifying Global Factor Risk

In order to obtain a more realistic quantification of the potential effect of the global factor on the project, in other words the risk associated with the particular global factor, the PM professional must firstly identify or estimate the occurrence of the global factor's influence on the project and then the magnitude of the influence of such an occurrence.

This introduces a common statistical property of risk namely, the Expected Value. The expected value is loosely defined as the product of the consequence of an event occurring and the likelihood or probability of that event occurring. The quantification of the risk presented by the global factor to the project can therefore be estimated using the expected value as it incorporates the probability of the occurrence of the global factor's influence and the magnitude of that influence. The expected value of the occurrence of a global factor's influence can thus be viewed as a credible approximation of the potential effect of a global factor on a project.

Mathematically the expected value can be defined for a discrete random variable as follows; if  $X$  is a discrete random variable with values  $x_1, x_2, \dots, x_n$  each with corresponding probabilities  $v_1, v_2, \dots, v_n$  then the expected value  $E(X)$  can be computed as the sum or series,


$$E(X) = \sum_{i=1}^n v_i \cdot x_i \quad (5.1)$$

Therefore to quantify the risk associated with a global factor on a project by means of the expected value, the PM professional must determine or estimate the probability of the occurrence of the global factor influencing the project and the consequence or effect of such an occurrence.

## 5.3 The Consequence of the Influence of the Global Factor.

The consequence of the occurrence of the global factor on a project's duration or cost must be determined by the PM professional. It is possible to determine this by means of using historical data. Historical data is however often unavailable or unsuitable to the unique conditions applicable to the project being estimated. For this reason the consequence of the global factor will often be estimated by the PM professional based

on experience or expert opinion or both. The estimated consequence can be in the form of a discrete value of either duration or cost, whichever is under consideration, or some other form such as a percentage disruption or percentage increase in cost.

Importantly, PM professionals often make allowances for the effects of a global factor on a task's cost or duration whilst they are estimating the Best, Worst and Anticipated cases as discussed in Section 3.2.1.1.

If a PM professional compiling a project estimate desires to analyse the effects of a specific global factor on the project then the incorporation of these effects in the initial estimation must be explicitly avoided. If the PM professional does not avoid this then the resultant analyses will be overly conservative.

#### **5.4 The Probability of Global Factor Occurrence**

The probability that the global factor will occur, in other words, the probability that it will exercise its influence on the project must also be determined by the PM professional. The determination of the probability value will differ depending on the type of global factor under consideration.

##### **5.4.1 Project External**

Due to the fact that many of this type of global factors are identifiable, common and generally influential, many have resulted in fields of study in their own right. Therefore statistical data regarding their nature is quite often available. For instance weather records are generally kept worldwide and can be used to estimate probability of occurrence. Historical price data and exchange rate data is also available and analysis of this data can result in probabilities being estimated for certain events. Where such information is unavailable the probability of the occurrence of the global factor could be estimated by the PM professional.

Importantly the probability value determined is unaffected by or independent of the project itself, in other words it is not influenced by the project. It is also common and constant to all tasks within a project with the exception of the calendar predictable probabilities described later in Section 5.9.

### 5.4.2 Project Internal

Project internal global factors are more localised in nature because they are generally inherent or only affect the project. As a result determining the probability of their occurrence is generally an ad hoc exercise specific to the project. This also means that statistical data is not only generally unavailable it is also difficult to rationally compile. Therefore to determine the occurrence of such global factors will most often need to be estimated by the PM professional based on experience or expert opinion.

### 5.5 Incorporating Global Factors into Project Estimates

After the Probability of occurrence of a global factor and the consequence of that occurrence has been determined or estimated for all tasks, these values can be incorporated into a project estimate in order to quantify their effect on the project. The rest of this section focuses on creating a method to incorporate the effects of these global factors into the estimate and to calculate risk allowances that mitigate for the influence these factors can have on the project.

### 5.6 The Global Factor Incorporated into a Duration Estimate.

This section will describe a method to incorporate the effect of a global factor on a project's duration

In Chapter 4 the duration model was created and the total project duration was designed to meet a desired level of reliability. The individual estimated task durations were increased through multiplication by partial factors that were calculated at a specified level of reliability. This ensured that the now designed durations, estimated for the specific tasks, were representative of durations that were achievable in accordance with the specified level of reliability. It is important to note that the original triangular duration estimates of Best, Worst and Anticipated for each task were devoid of any variability as a result of an identifiable global factor.

The designed task durations, given by Equation 4.2f in the form;

$$T_i = \gamma_i \cdot \mu_{d,i} \quad (4.2f)$$

Where:  $T_i$  = the task duration representing the specific  $\beta$



and  $\mu_{d,i}$  = The mean of the individual task's duration units estimation

and the resultant Critical path and Project duration were then calculated to the desired level of reliability, independent of the influence of the global factor.

These designed task durations and costs as well as the resulting project duration are required as an input to the process described here to incorporate the effects of identifiable global influencing factors.

Once again a step wise fashion best describes the process.

**Step 1: Duration Equation**

A new model must be created. This new model will be constructed to represent the effects of a global factor on a project. The new model will be based on the basic expression of the original critical duration units model given by Equation 3.1, except that the  $d_i$  variables are replaced with the calculated  $T_i$  values and  $D_c$  is now simply represented by  $D$ . The model will be based on the expression;

$$D = \sum_{i=1}^n T_i \tag{5.3}$$

Where  $D$  = The Total Designed Critical Duration Units  
 $T_i$  = The discrete durations of the designed critical tasks  
 $n$  = The number of critical tasks

Equation 5.3 now represents the total cumulative duration units of the tasks previously identified as critical tasks. These tasks are therefore those tasks that were on the critical path at least once when the most likely critical path was determined and thus include most of the possible critical tasks under most conditions. However, when considering the effects of the global factor on project duration, all of the tasks, irrespective of their criticality need to be considered. The reason for this is based on the assumption that the original task duration variable estimates were estimated devoid of the influence of the

global factor under consideration. Thus the critical path determination was devoid of the influence of the global factor and as such some of the tasks previously identified as non-critical could become critical when the effect of the global factor is included. To incorporate this possibility all the tasks must therefore be included when quantifying the effects of a global factor on a project's duration.

Therefore if;

$h$  = the total number of tasks in the project

$n$  = the total number of previously critical tasks identified (devoid of the influence of the global factor)

Then let;

$m = h-n$ , the number of remaining previously non-critical tasks.

Equation 5.3 does not however incorporate the previously non-critical tasks. These task's durations must be incorporated into Equation 5.3. To do this the original  $d_i$  variables estimated by the PM professional for these non-critical tasks will be utilised for this purpose. Due to the fact that the  $T_i$  values have been designed to incorporate the desired level of duration reliability devoid of the influence of this global factor, the variability of these previously non-critical tasks can be ignored and their durations can be assumed as their calculated mean values. Therefore the total duration units can be represented by the following equation.

$$D = \sum_{i=1}^n T_i + \sum_{j=1}^m \mu_{d,j} \quad (5.4)$$

Where

$T_i$	=	The duration of the designed critical tasks
$\mu_{d,j}$	=	The mean durations of the non critical tasks.
$m$	=	The number of non critical tasks

The  $T_i$  values of Equation 5.4 are discrete values representative of a level of reliability designed in Chapter 4. The mean values of the previously non-critical tasks are also discrete values, Therefore Equation 5.4, at this stage of the modelling process, includes no variability at all. Equation 5.4 is now a representation of the total designed project cumulative duration units. Equation 5.4 incorporates the designed task durations of previously identified critical tasks and the mean durations of the previously identified non critical tasks. Equation 5.4 is thus devoid of the influence of the global factor.

## **Step 2: Calculate the Expected Value**

The Expected Value of the influence the global factor must now be calculated for each task. Thus the probability of the occurrence of the global factor and the consequence of such an occurrence must therefore be determined for each task.

The consequence of the occurrence of the global factor as described in Section 5.3 must be determined by the PM professional.

A percentage disruption value is suited to the project environment as it can be positive or negative and applied to either cost or duration. Therefore, to generalise the process of incorporating the effect of a global factor into an estimate the consequence of the occurrence of a global factor will be estimated or represented as a percentage disruption and is represented by  $\kappa_i$ .

The probability of the global factor occurring must also be determined and is given by  $v_i$ . Then the expected value of the disruption caused to an individual task by the influence of the global factor is calculated by;

$$\psi_i = \kappa_i \cdot v_i \quad (5.5)$$

- Where  $\psi_i$  = The Expected value of disruption to a task's duration resulting from the occurrence of the global factor.
- $v_i$  = The Probability of the occurrence of the global factor within the project
- $\kappa_i$  = The consequence of the occurrence of the global factor on the task

The probability  $v_i$  has no units and thus the expected value of the disruption to the task duration is given in the units of the consequence, percentage units. The expected value thus represents the expected percentage disruption that a task will experience as a result of the occurrence of the global factor.

### Step 3 Incorporate the Expected Value into the Estimate

The expected value of the disruption to each task's duration is given in percentage units. At this stage it therefore does not equate to a tangible value that can be considered a duration risk allowance. In order to calculate a duration risk allowance the expected value must be incorporated into Equation 5.4.

Due to the fact that the expected disruption variable returns a percentage value, the total task duration including the disruption can be calculated by multiplying each tasks duration by the following factor, Factor 5.6;

$$\left(1 + \frac{\psi_i}{100}\right) \tag{5.6}$$

Then the total task duration units required to incorporate the effects of the global factor on the duration of the project can then be calculated by incorporating the Factor 5.6 for each task into Equation 5.4 as follows.

$$D_g = \sum_{i=1}^n \left(1 + \frac{\psi_i}{100}\right) \cdot T_i + \sum_{j=1}^m \left(1 + \frac{\psi_j}{100}\right) \cdot \mu_{d,j} \tag{5.7}$$

- Where
- $D_g$  = The discrete cumulative duration units including the effects of the global factor of the project
  - $\psi_i$  = The expected values calculated for all previously critical tasks
  - $\psi_j$  = The expected values calculated for all previously non-critical tasks

### 5.6.1 Incorporating Uncertainty

Up and until this point the estimations of the consequence of a global factors occurrence, albeit a percentage value, have been kept as discrete values.

In line with the unpredictable nature of the global factors and the desired incorporation of uncertainty into project estimates, this study proposes that the consequence of the occurrence of the global factor is estimated as a randomly distributed variable by using the Triangular distribution methodology described in the previous chapters.

In other words the PM professional will have to estimate the Best Case, the Worst Case and the Anticipated Case in terms of the consequence of the global factor as an effect on the project's performance. These triangularly distributed estimations must then be converted into Normally distributed variables as done previously.

The estimates will take the form of a percentage disruption that the occurrence of the global factor can have on the task's duration.

Thus the PM professional must estimate the percentage change that the task's duration will experience as a result of the occurrence of the global factor. Certain tasks are however more susceptible or vulnerable to the occurrence of the global factor and thus the consequence of this occurrence will differ from task to task. As a result the disruption resulting from the occurrence of the global factor will differ for each task and task will be unique to that task.

Therefore for each task the PM professional thus estimates the following;

- The least potential Percentage Disruption (Optimistic Case)
- The greatest potential Percentage Disruption (Pessimistic Case)
- The anticipated potential Percentage Disruption (Anticipated Case)

The three estimations are then firstly approximated as a Triangularly distributed variable as described in Section 3.2.1.1 and then secondly as a Normally distributed variable as described in Section 4.1. This Normally distributed variable is then called the Disruption variable and is denoted by  $\phi_i$ .

For each task the disruption variable is thus given by:

$$\phi_i = N(\mu_{\phi_i}, \sigma_{\phi_i}) \quad (5.8)$$

Where:  $\phi_i$  represents a disruption variable of a given global factor.

At this stage  $\phi_i$  is only representative of the *conditional* amount of disruption to a task's duration or cost that could result from the occurrence of the global factor.

All tasks within the project are not necessarily susceptible to the influence of certain global factors, for instance a task that occurs indoors would generally be unaffected by the occurrence of the global factor rainfall. In such cases the estimated percentage disruption would be zero for all three estimated cases; best, worst and anticipated.

The fact that the consequence of the global factors occurrence is estimated as a Normally distributed variable with units of percentage means that the PM is able to incorporate uncertainty into the estimate in a generalised manner that can be used for different types of global factors.

Equation 5.5 can then be converted into a probabilistic model by substituting the discrete consequence  $\kappa_i$  with the randomly distributed disruption variable  $\phi_i$  as follows;

$$\Psi_i = v_i \cdot \phi_i \quad (5.9)$$

Where  $\Psi_i$  is now a probabilistic model of the expected value of the global factor.

By replacing the discrete expected values of Equation 5.7 with the expected value model from Equation 5.9, Equation 5.7 is converted into a probabilistic model and is given below,

$$D_G = \sum_{i=1}^n \left(1 + \frac{\Psi_i}{100}\right) \cdot T_i + \sum_{j=1}^m \left(1 + \frac{\Psi_j}{100}\right) \cdot \mu_{d,j} \quad (5.10)$$

By expanding the expected value models into their constituent components, as shown in Equation 5.10a below, it is clear that the equation now forms a probabilistic model where the variability is only found in the randomly distributed disruption variables  $\phi$ .

$$D_G = \sum_{i=1}^n \left( 1 + \frac{v_i \cdot \phi_i}{100} \right) \cdot T_i + \sum_{j=1}^m \left( 1 + \frac{v_j \cdot \phi_j}{100} \right) \cdot \mu_{d,j} \quad (5.10a)$$

Equation 5.10a is thus a probabilistic model that is representative of the effect of a global factor on the total project duration units. Here the task durations are all discrete values and the model's inherent uncertainty is **solely attributed** to the Global factor by means of the disruption variable  $\phi$ .

The effects of these global factors on the project can now be analysed and designed probabilistically. Thus the PM professional can incorporate the effects of a global factor to a desired level of reliability. This means that the PM professional can choose the level of risk mitigation desired when calculating the allowances attributed to the risk associated with a global factor.

The calculation of the risk mitigating allowances probabilistically is done in the following steps, numbered as a continuation of the above process.

**Step 4: Calculate the Alpha Values.**

Using the model  $D_G$ , The Alpha values must be calculated for all disruption variables by means of Equation 4.2c. The standard deviations of all disruption variables must be included in the denominator of Equation 4.2c.

**Step 5: Calculate the Partial Factors Corresponding to a Desired Level of Mitigation.**

Partial Factors must be calculated for each disruption variable to the specified level of reliability with the following equation;

$$\lambda_i = 1 + \left( \frac{\alpha_{\phi,i} \cdot \beta \cdot \sigma_{\phi,i}}{\mu_{\phi,i}} \right) \quad (5.11)$$

where;

$\lambda_i$  = the partial factor to apply to the disruption variable i

$\mu_{\phi,i}$  = the mean value of the disruption variable i

$\sigma_{\phi,i}$  = the standard deviation of the disruption variable i

$\beta$  = the Beta value representative of the desired level of mitigation

The  $\beta$  value chosen thus determines the level of the mitigation cover. In other words, in the case of a risk, the higher the specified  $\beta$  the greater the calculated risk allowance and thus the greater the level of risk mitigation.

A logical way of deciding on what level of  $\beta$  is appropriate is to consider the magnitude of the consequence of the occurrence of the global factor. The greater the magnitude of the consequence, the greater the level of risk mitigation desired and hence the higher the  $\beta$  value specified.

**NOTE:** Step 5 must also be executed for all of the non-critical tasks denoted by the subscript  $j$  in Equation 5.10a returning the partial factors

$\lambda_j$

**Step 6: Calculate the Risk Mitigating Total Duration Units.**

The Partial factors have been calculated to a certain desired level of mitigation. For all disruption variables the risk mitigating disruption percentage that corresponds to the desired level of mitigation is calculated by multiplying each disruption's mean value by its respective partial factor. This is shown in Equation 5.10b.



$$D_R = \sum_{i=1}^n \left( 1 + \frac{v_i \cdot \lambda_i \mu_{\phi,i}}{100} \right) \cdot T_i + \sum_{j=1}^m \left( 1 + \frac{v_j \cdot \lambda_j \mu_{\phi,j}}{100} \right) \cdot \mu_{d,j} \quad (5.10b)$$

Where  $D_R$  = The Total Mitigating Duration units

Solving Equation 5.10b results in the total duration units required to mitigate for the effects of the global factor on the duration units of the project to a specified level of mitigation. The mitigating duration units required to mitigate for the effects of the global factor is then given by the difference between the solution of Equation 5.10b and Equation 5.4 given by;

$$Allowance = D_R - \left( \sum_{i=1}^n T_i + \sum_{j=1}^m \mu_{d,j} \right) \quad (5.10c)$$

As the model given by Equation 5.10a is representative of the total duration units, it does not incorporate the interrelations between the tasks and it does not represent the actual increase in duration of the project estimate that is required to mitigate for the global factor. In order to calculate this, the critical path must be re-calculated using the mitigating durations of each individual task calculated with the formulas.

For all previously critical tasks;

$$D_{r,i} = \left( 1 + \frac{v_i \cdot \lambda_i \mu_{\phi,i}}{100} \right) \cdot T_i \quad (5.10d)$$

Where  $D_{r,i}$  = Mitigating duration for previously critical task  $i$

and for all previously non-critical tasks;

$$D_{r,j} = \left( 1 + \frac{v_j \cdot \lambda_j \mu_{\phi,j}}{100} \right) \cdot \mu_{d,j} \quad (5.10e)$$

Where  $D_{r,j}$  = Mitigating duration for previously non-critical task  $j$

The respective  $D_{r,i}$  and  $D_{r,j}$  values must then be substituted into the precedence network as the duration values for their corresponding tasks. The critical path must be calculated and the resulting project duration is then representative of the actual duration that includes mitigation for the effects of the global factor.

### 5.7 The Global Factor Model applied to Cost

A similar process can be applied to the Project cost model. In the cost model the disruption variables calculated to evaluate the effects of the global factor on the duration would need to be calculated to represent the influence of the global factor on the project cost, also as a percentage.

The process to calculate the financial risk allowances attributed to a global factor follows the same process as described for duration with the notable differences described below;

#### Step 1: Cost Equation

The calculation of the mitigating cost allowance must consider all tasks as each task cost will contribute to the total project cost. The Equation that forms the basis of the probabilistic model to be created is given by;

$$C = \sum_{i=1}^h Pr_i \quad (5.12)$$

Where  $Pr_i =$  The designed task cost to the specified level of reliability  
 $h =$  The total number of project tasks.

**Step 2:** Is unchanged apart from the fact that the consequence represents the effect on the cost of the task and not its duration

#### Step 3 Incorporate the Expected Value into the Estimate

The total project cost required to incorporate the financial effects of the global factor on the project can then be calculated by incorporating the Factor 5.6 variables for each task into Equation 5.12 as follows.

$$C_g = \sum_{i=1}^h \left( 1 + \frac{\psi_i}{100} \right) \cdot Pr_i \quad (5.13)$$

Where  $C_g$  = The discrete cost including the effects of the global factor on the project

$\psi_i$  = The expected values calculated for all tasks

### 5.7.1 Incorporating Uncertainty

To incorporate uncertainty the disruption percentages must be estimated as randomly distributed variables as done Section 5.6.1. The variables are then used to create the probabilistic expected value model which is incorporated into Equation 5.13 in place of the discrete expected values  $\psi_i$ . This then means that Equation 5.13 becomes a probabilistic model and is given by 5.13a below;

$$C_G = \sum_{i=1}^h \left( 1 + \frac{\Psi_i}{100} \right) \cdot Pr_i \quad 5.13a$$

$$C_G = \sum_{i=1}^h \left( 1 + \frac{v_i \cdot \phi_i}{100} \right) \cdot Pr_i \quad (5.13b)$$

Equation 5.13b is thus a probabilistic model that is representative of the effect of a global factor on the total project cost. The model's inherent cost uncertainty is **solely attributed** to the Global factor.

#### Step 4 Calculate the Alpha Values.

Using the model  $C_G$ , The Alpha values must be calculated for all disruption variables by means of Equation 4.2c

**Step 5:** Is unchanged

**Step 6: Calculate the Risk Mitigating Cost Allowance.**

The Partial factors are calculated to a desired level of mitigation therefore, for all disruption variables calculate the risk mitigating disruption percentage that corresponds to the desired level of mitigation by multiplying each disruption variable's mean value by its respective partial factor. This is shown in Equation 5.13c.

$$C_R = \sum_{i=1}^h \left( 1 + \frac{v_i \cdot \lambda_i \mu_{\phi,i}}{100} \right) \cdot Pr_i \quad (5.13c)$$

Where  $C_R$  = the total mitigating cost

The mitigating cost allowance is thus given by the difference between Equations 5.13c and Equation 5.12 or;

$$Allowance = C_R - \sum_{i=1}^h Pr_i \quad (5.14)$$

This cost allowance would then represent the amount of money that is needed to accommodate the financial risk of a Global Factor's disruption.

The cost allowance is independent of the calendar position of the tasks concerned and thus the cumulative allowance given by Equation 5.14 is a sufficient representation of the cost allowance required to mitigate for the effects of the respective global factor. On larger and longer projects however, inflation and escalation plays a role on task costs and the cost of a task becomes dependant on its calendar position. In such projects the mitigating allowances will need to be calculated for each individual task with the Equation 5.15.

$$c_{r,i} = \left( 1 + \frac{v_i \cdot \lambda_i \mu_{\phi,i}}{100} \right) \cdot Pr_i \quad (5.15)$$

Where  $c_{r,i}$  = the mitigating cost for an individual task

These individual mitigating task costs will then need to be inflated by an appropriate escalation factor based on the task's calendar position. The final mitigating cost allowance will then be the sum of the escalated individual allowances.

### **5.8 Calendar Predictability Global Factors.**

Importantly, in the above methods the probability of the occurrence of the global factor within the project was kept constant. This was deliberately done so as to illustrate the methods that can be applied to any global factor. The above methods can be used to model the influence of other global factors such as other weather conditions, supply chain or resource unavailability, exchange rate volatility and others.

In order to calculate mitigating allowances for other global factors the disruption variable  $\phi$ , estimated as done in Equation 5.2 as a percentage, and the probability of the global factor occurrence will need to be determined for the specific global factors under consideration.

In sections 5.7 and 5.8 a method was illustrated that can be used to design allowances in terms of both duration and cost to accommodate and mitigate for the effects of global factors. The probability of the occurrence of the global factor within the project was kept constant throughout duration of the project. This will not always be the case though as some global factors can be considered calendar predictable. Global factors that are calendar predictable are weather, inflation or the common lack of resources over the Easter holiday period and more.

As a result of this calendar predictability the PM professional is able to glean extra information for these calendar predictable global factors, incorporate this into the modelling and hence achieve greater estimate reliability. For instance a weather global factor such as rainfall is seasonally predictable therefore the probability of rainfall occurring will be different for different seasons of the year.

This highlights the fact that the calculation of mitigating risk allowances for both cost and duration must be performed after the initial duration model has been designed to the required level of reliability. The completion of the design of the reliable duration estimates will include the calculation of the critical path and as a result the tasks will be

arranged into their final designed calendar positions. Thus for each task an execution date is also known and as a result when analysing and designing the global factor allowances for calendar predictable factors an appropriate probability value can be used.

For the purpose of illustrating this point, the calendar predictable global factor Rainfall will be examined. Data for the global factor rainfall can be sourced from the local weather service and probabilities can be calculated according to an appropriate calendar. As an example this has been done for the City of Johannesburg with data sourced from the South African Weather Service Archives on their website or through a data request at,

<http://www.weathersa.co.za/Climat/Climstats/JohannesburgStats.jsp>

As the sourced rainfall data is presented on a monthly basis, the calendar predictability will also be calculated on a monthly basis.

Table 5.1: Average Rainfall Data for Johannesburg from 1970-2005

Month	Days in Month	Monthly		Daily	
		Average Monthly (mm)	Standard Deviation (mm)	Mean Daily Rainfall	Standard Deviation (mm)
January	31	134.65	62.74	4.34	2.02
February	28	107.32	67.76	3.83	2.42
March	31	97.84	57.01	3.16	1.84
April	30	43.41	33.06	1.45	1.10
May	31	16.41	24.92	0.53	0.80
June	30	7.42	11.71	0.25	0.39
July	31	3.19	5.19	0.10	0.17
August	31	6.85	9.61	0.22	0.31
September	30	30.33	38.07	1.01	1.27
October	31	79.08	45.71	2.55	1.47
November	30	108.97	52.47	3.63	1.75
December	31	111.82	43.50	3.61	1.40

From Table 5.1 a simple calculation can be made to estimate the Probability that Rainfall  $P(R)$  will disrupt the project in the month.

In order to calculate the probability that rainfall will disrupt the project, the PM professional will need to ascertain what level of rainfall will cause a disruption to the project. This can be done through investigation of historical data or can be experience based. In this **illustrative** example it was assumed that a rainfall of 6mm or greater in any one day (24hr period) will cause a disruption to the project. The magnitude of this disruption is estimated by the disruption variables.

By using simple statistical calculations of the data in Table 5.1 the probability that more rain will fall in a 24 hour period for at least one day in the month than the disrupting rainfall level of 6 mm can be determined. The results of this calculation are shown in Table 5.2. See Appendix G for calculations.

Table 5.2: Probability of Rainfall > 6mm in 24 hours for at least 1 Day of Each Month

Month	Probability that Daily Rainfall > 6 mm
January	0.26030
February	0.22741
March	0.06496
April	0.00002
May	0.00000
June	0.00000
July	0.00000
August	0.00000
September	0.00004
October	0.00976
November	0.09636
December	0.04610

Graphically represented in Figure 5.1, it is clear that during the respective months of the year the probability that rainfall has the potential to disrupt the project changes and thus the calendar position of a task becomes important. For instance, all tasks scheduled in the month of January would have a probability of being affected by Rainfall of 0.26 whereas if those same tasks were scheduled in July this would be near to zero. These probability values must then be used in Equation 5.5 to calculate the expected value of the factor Rainfall that is then used in Factor 5.6. Therefore it is clear that the mitigating

risk allowances calculated for the project would be dependent on the calendar position of the variable tasks when the influencing global factor is calendar predictable.

Figure 5.1: Probability of Rainfall > 6mm in 24 hours for at least 1 Day of Each Month

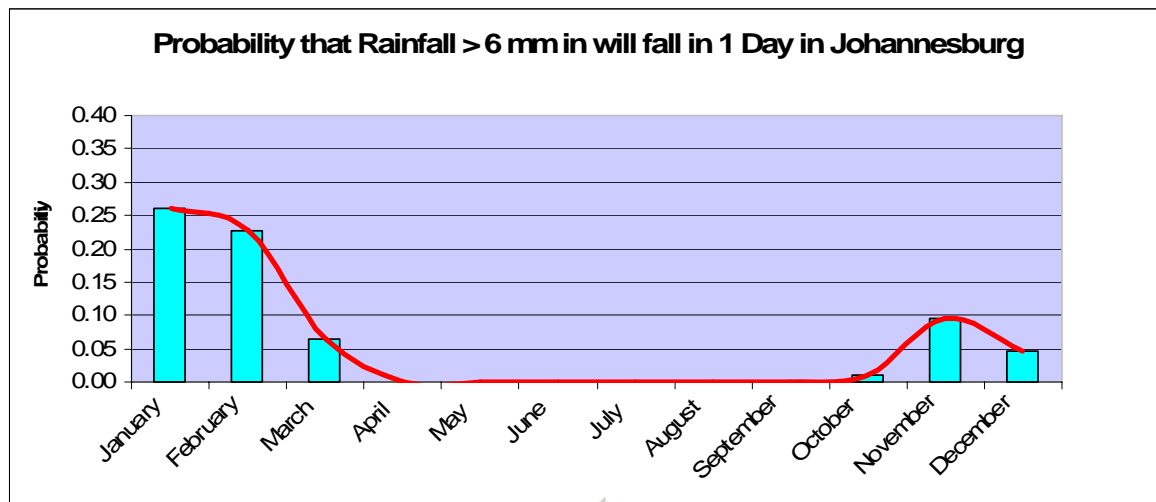


Figure 5.1 represents the probability that rainfall of 6mm or more will fall in a single day in Johannesburg as calculated on a monthly basis. The probability values thus represent the probability of rainfall > 6mm falling in 1 day or 2 days or 3 days and so on for each month. See Appendix G for the calculation.

The allowances that result from the calculations using the appropriate calendar determined probability value and the respective estimated disruption variable will be more realistic and consequently the monetary value required to accommodate these allowances will also be more realistic. As this monetary value can be viewed as the monetary value of the risks associated with the particular global factor, in this case Rainfall, this method should be exceptionally useful to a PM professional, company or client.

### 5.9 General features of the application of global factors

The above method of including the effects of a global factor on a project's cost and duration estimate, in the form of factors that are themselves variables, presents the PM professional with ways to probabilistically analyse and design risk allowances that accommodate for the influence that such global factors have on a project. By applying



these variables into a discrete model already representative of a designed level of reliability they can be analysed independent of other factors and thus the risk allowances for each global factor can be isolated and calculated separately. In this manner the PM professional is able to quantify the risk solely associated with the global factor under consideration.

The process of scheduling the tasks onto a calendar and applying the appropriately calculated global factors to the scheduled tasks presents a method whereby risks can be calculated on a consistent and specific basis. The risk values calculated will be far more detailed and realistic than many methods common today.

The method does not make any attempt to calculate the effects on the project of the combined effects of two or more global factors occurring within the project simultaneously. *On the contrary* the purpose of the method is to isolate the global factor to establish risk allowances attributable to the global factor. Such allowances can then be incorporated into a project cost or duration estimate or ownership thereof can be negotiated.

The method is still, as are all the current methods, just an approximation of risk as it still utilises human assumptions that will most often not be verifiable by any reputable data. This method is however able to supply detailed risk allowance calculation based on a desired level of mitigation cover calculated by probabilistic methods. This gives a PM professional, company or client the ability to incorporate uncertainty and provides the flexibility to make better decisions where risk is concerned and is therefore a potentially useful method.

## 6. Project Profit Potential

At this stage of the overall process a PM professional is able to mathematically incorporate variability or uncertainty into a project estimate. Using this uncertainty the PM professional is then able to analyse the estimate and design a project's duration and cost that reflects a desired level of reliability. This calculated duration and cost then represents to the PM professional, given the desired level of reliability, what the project will cost and how long it will take. The PM professional is then further able to calculate risk allowances based on the estimated effects of identifiable global influencing factors. The focus thus far has been on analysing and designing estimates of costs and duration and the potential costs of associated risks. There has so far been no mention of income and potential profitability.

Ultimately profit is the driving factor behind projects. The project initiator wants to see a return on investment in order to make some profit out of the overall project. A contractor doing works during the execution phase of the project also wants to make a profit out of executing those works. There are rare occasions where a project is not seen as a profit generating engine. Such projects are found in the public sector where social or economic benefits are often seen as the motivating factor behind some projects. In most cases though, a project is often eventually evaluated for the profit it generated.

Company CEOs and shareholders are aware of the brand and status potential certain projects can bring to a company but if those projects do not produce a profit they will be seen as failures. The construction or IT firm that produces a brilliant product at a significant loss might cease to exist because of its financial failure even though it could be regarded by its clients as the best in the industry. In short, "*The bottom line is the bottom line*". Profit is the dominant deciding factor when decisions about projects have to be made.

This chapter will investigate profitability probabilistically and look at ways to evaluate projects' profitability potential and thus enable decision makers to make informed strategic decisions. Two methods will be discussed here; the first applies to the detailed estimates a contractor makes in the Design and Development phase of the project. The second method focuses on the initial estimates and project feasibility assessment done in the Concept or Initiation phase.

## 6.1 Profit Evaluation of Detailed Estimates Designed by Contractors.

During the Design and Development phase of a project, contractors are often used to implement or execute the tasks required. This means that they essentially construct a building, perform a survey, develop a software program or create whatever revenue generating asset the project requires. The construction, development or creation of such assets is often undertaken by a specialist contractor in the respective field. These contractors are normally separately trading companies from the ones who own the project. As a result the contractors who perform these works consider, from their point of view, their section of the work to be an entire project. Depending on the procurement method utilised by the PM professional they will normally need to tender a bid to win the contract to do the works required. This section will describe a method of how a contractor can use the methods described in the previous chapters to design estimates to create a tender bid with a desired level of profit reliability.

Simplistically, the profit generated by a contractor is equal to the payments received from the client or owner of the project less the costs of generating those payments. The payments received from the client are traditionally calculated on the basis of the tendered price submitted by the contractor. Thus assuming there are no scope changes, the contractor has a perfect design or brief then actual contractor profit is given by;


$$P = T - C \quad (6.1)$$

Where  $P$  = Profit  
 $T$  = Tender Value  
 $C$  = Project Cost

Projects however rarely ever cost exactly the amount estimated. The estimated final project cost is itself variable. This is shown in Chapter 4 where the project cost is represented by a probabilistic model given in Equation 4.10. As a result the Project cost calculated in Equation 6.1 is not a discrete number but instead variable. Thus the estimated profit is also represented by a probabilistic model. Equation 6.1 is therefore a probabilistic model of the profit estimate.

The value of the tender,  $T$  of Equation 6.1, is a calculated value based on the desired profitability and the value of the risk owned by the contractor. This value is normally decided upon by the contractor's senior management. The value chosen by the contractor's senior management determines firstly whether or not the contractor will win the bid for work and secondly the amount of potential profit the project can make. This value is calculated based on the contractors cost estimate.

### 6.1.1. Calculating a Contractor's Tender Value based on the Contractors Cost Estimate

The tender value decided upon by the contractor is dependent on many factors. The contractor's need for work, the general business environment, the level of tender competition and the nature of the client all play a part. Importantly the value decided upon is based on the estimated cost of executing the works, as estimated by the contractor, but the final value chosen is essentially the result of a management decision.

### 6.1.2 Tender Value

#### 6.1.2.1 Margin Based Tender Value Calculation

The tender value can be calculated using two different approaches dependent on the desires of management. The first approach is for management to decide on a profit margin in terms of a percentage. This will result in a tender value for which the level of reliability that its value is greater than the estimated project cost can be determined. Consequently the resultant margin based profit value will represent the same calculated level of reliability.

The tender value thus equals;

$$T_k = \left(1 + \frac{k}{100}\right) \cdot \left(\sum_{i=1}^n Pr_i + IC(\text{calculated at } \beta_{Design})\right) \quad (6.2)$$

where  $k$  = Management specified Profit Margin  
 $Pr_i$  = Task cost values designed to a desired level of reliability  
calculated by Equation 4.19 in Section 4.3.1.1

- $T_k$  = The Tender value calculated with a specified profit margin
- $IC$  = The indirect costs such as preliminaries and generals, overheads and insurance (commonly priced as a percentage of the estimated cost)

Then using Equation 4.16, the calculated reliability that the margin based tender value will be greater than the actual project cost or simply the reliability that the project will make at least R1 profit, is equal to

$$P(C < T_k) = \int_{-\infty}^{T_k} f_C(x) dx \quad (6.3)$$

The contractor then knows probability of at least breaking even if a tendered profit margin of  $k$  percent is submitted.

#### 6.1.2.2 Reliability Based Tender Value Calculation

The second approach is for management to decide the level of reliability that the tender value must be greater than the eventual project cost. More simply stated the reliability of making a profit of at least R1 must be chosen. This approach will calculate a profit value that reflects the specified level of reliability and not a desired profit margin.

This approach uses the method described in Chapter 4 to design a cost estimate. The difference is now that the cost estimate is calculated with a new, higher specified level of reliability. The new  $\beta$  value,  $\beta_{profit}$  represents the desired level of reliability that the actual project cost will be less than the tendered value and the project will make a profit.

This new designed cost value then represents the tender value to be submitted. The level of the reliability that this tender value was calculated with is reflective of the level of reliability that the decision maker wishes to have that the project will make a profit.

In other words the PM professional uses the cost estimate model from Chapter 4 to calculate a new cost estimate. This time to the level of reliability specified by the decision reflects the reliability that the project will make a profit. This level of profit reliability,  $\beta_{profit}$  should be higher than the level of reliability specified to calculate the original cost

estimate,  $\beta_{cost}$ . All insurances, calculated risk allowances and overheads must then be added to this cost and the resultant value is the Tender value. The tender value reliable to the specific level of profit reliability is then given by;

$$T_{Profit} = \sum_{i=1}^n Pr_i(\text{calculated at } \beta_{profit}) + IC(\text{calculated at } \beta_{profit})$$

The resultant profit margin can then be calculated by;

$$M = \frac{T_{Profit} - \sum_{i=1}^n Pr_i(\text{calculated at } \beta_{cost}) + IC(\text{calculated at } \beta_{cost})}{\sum_{i=1}^n Pr_i(\text{calculated at } \beta_{cost}) + IC(\text{calculated at } \beta_{cost})} \quad (6.4)$$

where  $M$  = The calculated Profit margin at the desired level of reliability



## 6.2 Profit Evaluation of Initial Estimates

### 6.2.1 Project Feasibility Calculation by Project Initiators or Owners

At the Concept or Initiation phase of the project the PM professional must estimate numerous factors that will enable the basic feasibility of the project to be evaluated. The feasibility of a project can be evaluated in various forms such as environmental feasibility and market feasibility amongst others. This section deals with profit and hence the financial feasibility of a project will be investigated. For such purposes financial models are usually constructed to assess the project's potential financial performance and hence feasibility. Numerous variables are estimated over the lifespan of the project and these variables are inserted into the model and the results analysed. The purpose of this section of the study is to introduce the inherent uncertainty in projects, by means of probabilistic methods, into the standard financial modelling tools.

### 6.2.2 Net Present Value

Net Present Value or NPV is a commonly used financial modelling tool that evaluates project monetary return and thus feasibility. NPV is normally used by project initiators in order to evaluate the potential financial rewards that a project could generate. It takes an overall view of the entire project life-cycle into account and approximates the revenue stream a project can produce. In short the method estimates all costs associated with a project and maps these onto a calendar. All revenues are also estimated and also mapped onto the same calendar. All costs and revenues are then discounted by an interest rate to a common initial date and then compared. If the resultant Present cost value is greater than the resultant Present Revenue Value the project should make a loss and if the Present Revenue value is greater than the Present cost value then the project should make a profit, hence the name Net Present Value.

Net Present Value is calculated from the formula;

$$NPV = \sum_{i=1}^n \frac{(R_i - C_i)}{(1+r)^i} - I_0 \quad (6.5)$$

Where	$R_i$	= Revenue generated from project at time $i$
	$C_i$	= Cost incurred at time $i$
	$r$	= the annual discount rate in %.
	$I_0$	= the initial costs of the project (capital costs of a project)
	$i$	= the time, usually in years,
	$n$	= the total number of time units of project duration, (years)

If the basic cost variables and revenue variables are common throughout the lifespan of the project i.e. the same cost variable and the same revenue variable are used for each year of the project lifespan, then Equation 6.5 can be simplified, using a geometric series, to read;

$$NPV = (R_i - C_i) \cdot \left( \frac{1 - (1+r)^{-n}}{r} \right) - I_0 \quad (6.5a)$$

### **6.2.2.1 Uncertainty and Sensitivity Analysis.**

In a project environment during the Concept and Initiation phase usually all of the above variables are uncertain and are thus estimated. The estimates are normally discretely done and the result calculated. To assume that the estimates of the future revenues and costs are discretely determinable is, for most projects incorrect, and as a result sensitivity analysis is usually done. The sensitivity analysis is performed on the discrete financial model by substituting different values for the uncertain variables and calculating the resultant financial situation. Such analysis will provide the project initiator with an indication of the reliability of the results of the financial modelling. This approach is limited in the fact that the resultant sensitivity range is dependent on the variable that is being analysed. The financial modeller will therefore usually only test the sensitivity of the model against the most uncertain variables. The sensitivity analysis is further limited because the combined effect of various variables is not incorporated very well. The modeller is generally limited to analysing a maximum combined effect of two variables simultaneously. The sensitivity analyses thus generally results in a number of tables being produced that highlight the effects on the financial model of variables discretely and independently and leaves the decision maker to guess what the combined effects of more than two variables will be.

### **6.2.2.2 FOSM and Sensitivity Analysis.**

FOSM can be used to demonstrate financial sensitivity far more efficiently and accurately by calculating the reliability of the financial model. FOSM is also able to do this for all of the uncertain variables simultaneously and produces one result as opposed to a table of results. FOSM also allows a modeller to individually manipulate individual variables that might hold some importance for whatever reason for the modeller.

The facilitation of this process and its results are achieved by estimating randomly distributed variables for the discrete variables found in the NPV Equation 6.5. For the purpose of simplifying the explanation the common variable situation described by Equation 6,5a will be used.

With all of the estimated variables now represented by randomly distributed variables, FOSM then views Equation 6.5a as a probabilistic model of the financial feasibility of the project.



FOSM analysis is based on the limit state condition of capacity less demand as given in Equation 4.1

FOSM then calculates, as a result of the variable nature of both demand and capacity, the reliability that the capacity will be greater than the demand. It returns the value  $\beta$  which in turn can be transformed into a probability or percentage value. The greater the  $\beta$  value the greater the probability that the capacity is greater than the demand and the greater the reliability that the function represents a safe, in this case profitable, situation.

Examining the NPV formulation it is clear that a FOSM type capacity less demand function is also apparent. In the NPV formulation, which calculates the profit magnitude, all incomes represent a capacity and all costs represent a demand. Thus in the NPV formulation

- $C_i$  = Cost incurred at time  $i$
- $I_0$  = the initial costs of the project (capital costs of a project)

Are both demand variables and

- $R_i$  = Revenue generated from project at time  $i$

Is a capacity variable

Thus by setting Equation 6.5a equal to zero would result in a limit state situation where the costs and revenues would be equal and thus it represents the situation where a project would change from being safe, thus making a profit to unsafe and making a loss. The reliability details the probability that the capacity is greater than the demand which here translates to the probability that the revenue is greater than the cost, hence profit. FOSM could therefore be used to analyse the limit state function to determine the reliability of making a profit.

### 6.2.2.3 Creating a Probabilistic Model of NPV

To create the model the randomly distributed variables must be created. This will be done for each variable as follows;

- The Revenue  $R_i$  generated by a project at time  $i$

The estimated revenue generated by a project during a time period can be estimated using the Triangular distribution method discussed in Chapter 3. Thus the lowest, highest and anticipated revenue values can be estimated. These values are transformed into a basic variable with a Triangular distribution and then into a Normally distributed variable as done before. If the true values are known as is sometimes the case then these exact values must be used as discrete values in the model.

- The costs  $C_i$  incurred at time  $i$

The estimated costs incurred during the generation of revenue can be transformed into a randomly distributed variable in the same manner as the revenue values by using the Triangular distribution. In this case the lowest highest and anticipated costs must be estimated. The Triangularly distributed variable must then be transformed into a normal variable as done before.

- The Discount rate  $r$

The interest rate or discount rate used to calculate the NPV is usually a combination of the predicted risk free rate  $w$  over the lifespan of the project and a risk premium  $z$ . To predict the risk free rate for a long period of time is very difficult and beyond the scope of this study. Common practice is to use the government lending rate in the form of the yield paid on government bonds such as the South African R153

Given the long term nature of some projects, the inclusion of uncertainty into the risk free rate may be desired. If for instance it was assumed that the bond yield rate will be between 6% and 10% for 85% of the lifespan of the project then this uncertainty can be modelled into a random variable quite easily.

The mean value would simply be the average of 6% and 10 % which is 8% and the standard deviation is calculated by either of the following formulas;

$$\sigma = \frac{x_{85\%} - \mu}{\beta_{85\%}} \quad (6.6a)$$

$$\sigma = \frac{\mu - x_{15\%}}{\beta_{85\%}} \quad (6.6b)$$

The resultant variable,  $w = N(\mu, \sigma)$ , would predominantly fluctuate around the 8% region and remain between 6% and 10%, 85% of the time. If economic conditions lead a decision maker into believing that the bond yield outlook will still remain between the 6% and 10% margins 85% of the time except that it will be more likely to fluctuate about the 9% region then this can also be modelled by incorporating skewness. For the purpose of this study the Normally distributed approximation will be used with skewness equal to zero. The discount rate then equals risk free rate plus the risk premium which is assumed discrete.



- The Initial or Capital costs of the project  $I_0$

The initial or capital cost of the project is simply the initial project cost estimate. At the initial phase of the project this value can be estimated as a randomly distributed variable as has been done by using the Triangular distribution to eventually estimate a Normally distributed variable.

Often the financial feasibility of the project is reassessed near the end of the design and development phase of the project. This is done because a far more accurate estimate of the implementation phase of the project is available. If the financial feasibility of the project is to be assessed at this phase then the Initial or Capital cost is equal to the randomly distributed project cost model given by,

$$C = N(\mu_C, \sigma_C) \quad (4.13)$$

with the mean and standard deviation calculated by the formulas 4.14 and 4.15 respectively

$$\therefore \mu_C = \sum_{i=1}^n \mu_{tc,i} \quad (4.11)$$

$$\sigma_C = \sqrt{\sum_{i=1}^n \sigma_{tc,i}^2} \quad (4.12)$$

Thus the NPV formula is transformed into a financial feasibility probabilistic model as follows

$$NPV = \sum_{i=1}^n \frac{(R_i - C_i)}{(1 + w + z)^i} - I_0 \quad (6.7)$$

This, by means of a geometric series, reduces to;

$$NPV = (R_i - C_i) \cdot \left( \frac{1 - (1 + (w + z))^{-n}}{w + z} \right) - I_0 \quad (6.7a)$$

- Where
- $R_i$  = Normally distributed Revenue variable generated at time  $i$
  - $C_i$  = Normally distributed Cost variable incurred at time  $i$
  - $w$  = risk free rate variable in %
  - $z$  = risk Premium in %
  - $I_0$  = the initial cost estimate **or**
  - $I_0$  =  $C, N(\mu_C, \sigma_C)$  the detailed cost estimate
  - $n$  = The number of years of the project lifespan (assumed discrete)

Then setting Equation 6.7a equal to zero and applying FOSM will result in a value  $\beta$  that represents the level of reliability that the project will make a profit or not. The greater the value of  $\beta$ , the greater the reliability of profit generation and the greater the magnitude of profit.

### 6.2.3 The Internal Rate of Return.

Another common financial modelling tool is the Internal Rate of Return or IRR. This tool is merely a mathematical manipulation of the NPV formula to return the discount rate that results in a NPV equal to zero. The IRR method estimates the initial cost, revenues at time  $i$  and costs at time  $i$  in the same manner as the NPV method. The NPV is then set equal to zero and the NPV function solved for the discount rate  $r$ . The resulting calculated discount rate then represents the percentage return that a project generates given the estimated costs and revenues. This calculated discount rate is thus called the Internal Rate of Return.

IRR is particularly useful to project initiators who are seeking investment funding for a project or for managers deciding on whether or not to proceed with a project. The IRR identifies the percentage return that can be expected from investing in a project. Investors will decide whether or not to invest in a project depending on the magnitude of the IRR given the project specific risks. IRR is further useful because it returns a percentage value that has no units and thus, as a result, many projects can be compared to each other in an un-weighted manner that is easily understood.

The formula used to calculate the IRR is given by;

$$\sum_{i=1}^n \frac{(R_i - C_i)}{(1 + IRR)^i} - I_o = 0 \quad (6.8)$$

Similar to NPV, if the basic cost variables and revenue variables are common throughout the lifespan of the project i.e. the same cost variable and the same revenue variable are used for each year of the project lifespan, then Equation 6.8 can be simplified, using a geometric series, to read;

$$(R_i - C_i) \cdot \left( \frac{1 - (1 + IRR)^{-n}}{IRR} \right) - I_0 = 0 \quad (6.8a)$$

This equation is similar to the limit state situation except that the value of the variable  $IRR$  is not known. The equation is such that it is usually solved by estimating a value for the  $IRR$ , solving the equation and repeating this in an iterative fashion until the solution to the equation equals zero.

By substituting the random variables estimated in Section 6.2.2.3 into Equation 6.8a changes Equation 6.8a into a probabilistic model with one unknown namely,  $IRR$ . The value of this variable is an important value to an investor or manager who is evaluating a project and as result it is this value that will be calculated probabilistically. A randomly distributed variable for the  $IRR$  will be calculated using the probabilistic model of Equation 6.8a and  $\beta$  values.

### 6.2.3.1 Beta $\beta$ and the Limit State Function

In order to explain how the distribution of the value of  $IRR$  is calculated it is necessary to return to the basic limit state function  $f(x) = C - D$  of FOSM. To explain the concept of  $\beta$  and how it will be used to estimate the  $IRR$ , a basic structural engineering example will be used. In this example the limit state equation is given by;

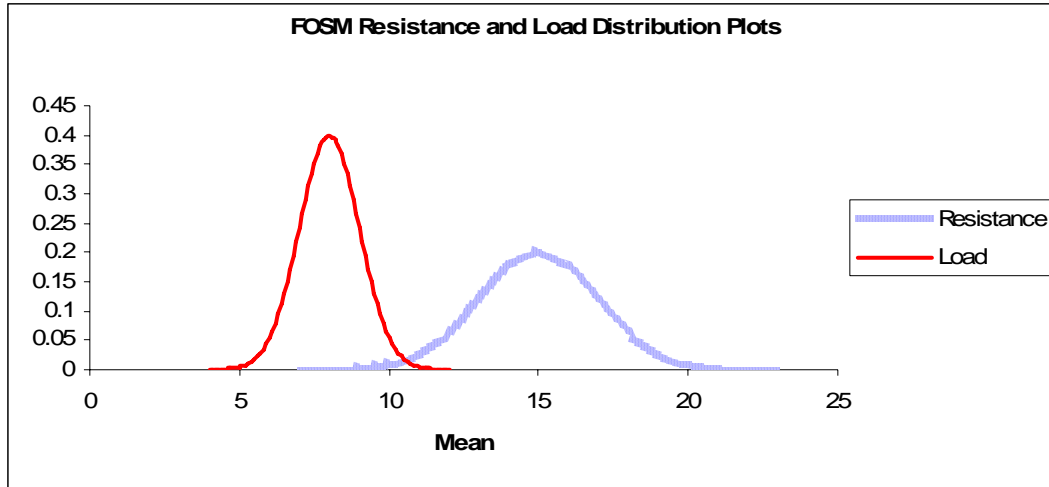
$$f(x) = R - L \quad (6.9)$$

In this limit state equation  $R$  for Resistance represents the Capacity and  $L$  for Load represents the Demand.

If both Resistance and Load are randomly distributed variables, each with their own mean and standard deviation then graphically, in two dimensions, their probability density functions or  $pdf$  can be plotted as shown by Figure 6.1a for an example where;

$$R=N(15,2) \quad \text{and} \quad L=N(8,1)$$

Figure 6.1a: Resistance and Load Probability Density Plot:  $R=N(15,2)$  and  $L=N(8,1)$



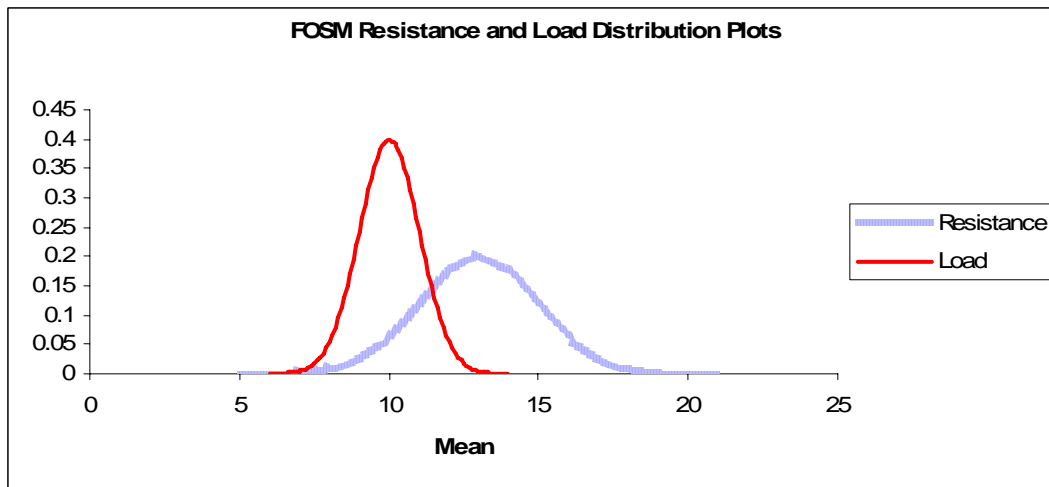
From the plot in Figure 6.1a it can be seen that there is a relatively small chance that the load values will be greater than the resistance values and hence the  $\beta$  value, or reliability that Resistance > Load, calculated for the limit state function of these two variables will be fairly high. For this example the calculated reliability is  $\beta = 3.13$

If one were to change the mean values of the two variables to the following;

$$R=N(13,2) \text{ and } L=N(10,1)$$

The two *pdf* plots would be as shown in Figure 6.1b.

Figure 6.1b: Resistance and Load Probability Density Plot:  $R=N(13,2)$  and  $L=N(10,1)$



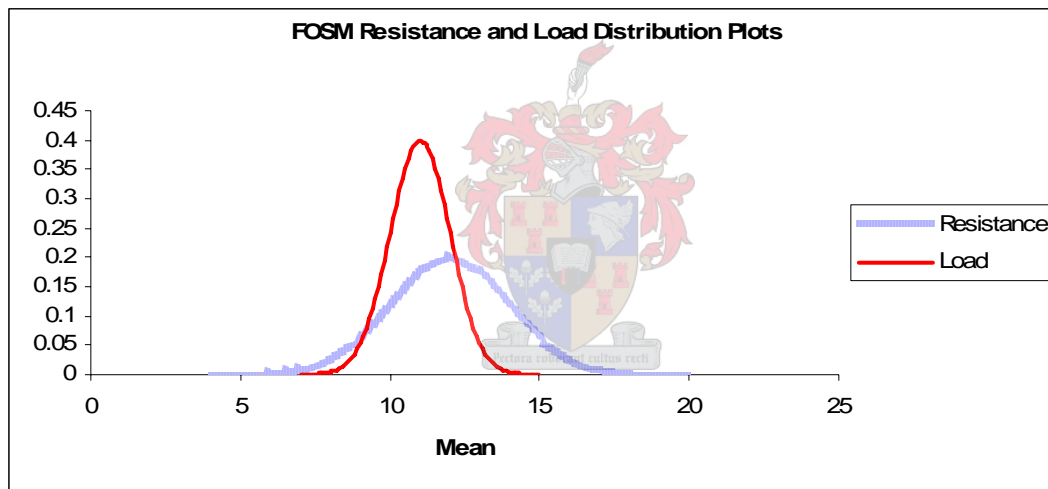
Clearly the chance of the load being greater than the resistance has increased and as a result the reliability will decrease. The reliability is now calculated to be given by  $\beta = 1.34$

Repeating this process one more time with;

$$R=N(12,2) \text{ and } L=N(11,1)$$

Results in the Probability Density Function (pdf) plot shown in Figure 6.1c where the chance of failure, Load > Resistance, has increased further and consequently the reliability has further decreased to  $\beta = 0.44$

Figure 6.1c: Resistance and Load Probability Density Plot:  $R=N(12,2)$  and  $L=N(11,1)$



Thus it can be seen that as the magnitudes of the mean values of the two variables are brought closer in value; the *pdf* plots shift towards each other, the chance of the load being greater than the resistance increases and the reliability, or  $\beta$  value, decreases.

It can be summarised by stating that as the magnitudes of the two mean values in the LSF given by Equation 6.9 tend towards each other, the  $\beta$  value tends towards zero. If the two mean values were equal in magnitude and skewness was zero for both variables then the  $\beta$  value will equal zero.



Conversely, in the model given by Equation 6.9 if  $\beta$  equals zero and the skewness of both variables is zero then the mean values,  $\mu_R$  and  $\mu_L$  are equal in magnitude.


Therefore given the above LSF, at a reliability of zero, the likelihood that a randomly selected discrete solution of Equation 6.9 being **equal to zero** is at its highest.

Now, if it is assumed that  $R$  is a known variable  $N(\mu_R, \sigma_R)$  and  $L$  is unknown, it can be stated that the most probable magnitude of the value of the variable  $L$  that will result in a discrete solution to the function  $f(x) = R - L$  being equal to zero, is that value of  $L$  that results in the calculated reliability of  $f(x) = R - L$  being equal to zero.

This situation is similar to the situation found in Equation 6.8a where a function is defined where all of the variables are known except for the one, the  $IRR$  variable. Secondly the magnitude of the  $IRR$  variable is equal to the value that results in the solution of the function being equal zero.

### 6.2.3.2 $IRR$ and the $\beta$ Value.

Revisiting Equation 6.8a



$$(R_i - C_i) \cdot \left( \frac{1 - (1 + IRR)^{-n}}{IRR} \right) - I_0 = 0 \quad (6.8a)$$

Rewriting Equation 6.8a as the discrete function  $f(x)$  results in;

$$f(x) = (R_i - C_i) \cdot \left( \frac{1 - (1 + IRR)^{-n}}{IRR} \right) - I_0 \quad (6.8b)$$

If one was to view Equation 6.8b as a function of discrete variables of which all are given except  $IRR$ , then the value of the  $IRR$  is defined as that percentage value that results in;

$$f(x) = 0$$

Now, if one were to substitute the randomly distributed variables estimated in Section 6.2.2.3 into Equation 6.8b, Equation 6.8b would be converted into a probabilistic model given by;

$$f(X) = (R_i - C_i) \cdot \left( \frac{1 - (1 + IRR)^{-n}}{IRR} \right) - I_0 \quad (6.8c)$$

Then it can be stated that the most probable magnitude of the value of the variable *IRR* that will result in a discrete solution to the function given by Equation 6.8c being equal to zero, is that *IRR* value that will result in the calculated reliability of the function given by Equation 6.8c being equal to zero.

Therefore the most probable *IRR* value can be calculated by the following systematic process;

- Choose a discrete value for *IRR*,
- Insert that value into the probabilistic model described by Equation 6.8c
- Set Equation 6.8c equal to zero, thus creating the limit state function
- Calculate the resulting reliability.
- Repeat the process with a new discrete chosen value of *IRR* until the calculated reliability value returned equals zero.

The *IRR* value that results in the reliability equalling zero is then the most probable *IRR* value of the project given the uncertainty in the estimated variables of Equation 6.8c.

Therefore the decision maker now knows the most probable *IRR* value of the project given the variability of all of Equation 6.8c's constituent variables and across all of these constituent variables' value ranges. Thus this *IRR* value incorporates most of the estimated uncertainty within the project. This *IRR* value therefore provides the decision maker with a far better indication of the probable return to be made from the project than the current sensitivity analysis tables.

Another interesting and useful observation that can be made during the calculation of the *IRR* value is the resulting calculated reliability value. After the calculation of the reliability

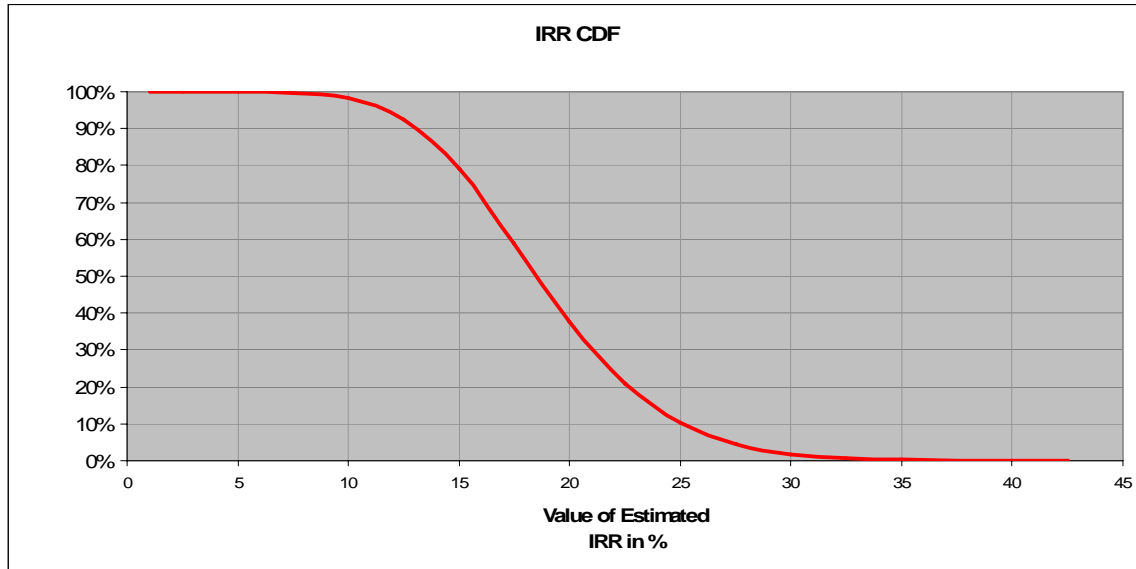
of Equation 6.8c for a chosen *IRR* percentage value the reliability level calculated indicates the level of spare capacity in the LSF. Thus the calculated  $\beta$  value details, at the chosen *IRR* value, the probability of spare capacity, or in this case the probability of spare revenue available in the LSF. Spare revenue implies that the actual project *IRR* will be higher than the chosen *IRR*. Therefore the calculated  $\beta$  value, when converted to a probability, details the probability that the actual project *IRR* value will be greater than the chosen value of the *IRR*.

This is thus useful because if the selection of the discrete *IRR* values is done systematically then it will result in a range of  $\beta$  values, each calculated for different *IRR* values. These  $\beta$  values can then be converted in to probabilities representing the probability that the actual *IRR* will be greater than the chosen *IRR*. By plotting the chosen *IRR* values against their resulting probability values a Cumulative Density Function plot or CDF can be produced for the *IRR*.

This has been done for Example 7.1 of Section 7 and is presented in full in Appendix D. In Example 7.1 the value of the *IRR* was systematically chosen at incremental intervals of 2.5% from zero up to a value of 42.5%. For each *IRR* value the reliability of Equation 6.8c was calculated. The reliability values were converted into probability values and plotted against their respective *IRR* values

The resulting plot represents a CDF where the percentage values on the ordinate represent the percentage chance that the actual *IRR* realised will be greater than the chosen *IRR*. The most probable *IRR* value is then that value that returns a reliability of zero or in percentage terms, 50%. The CDF is shown in Figure 6.1d below

Figure 6.1d: IRR CDF



Through the use of the CDF the decision maker is able to determine the reliability level of different *IRR* values. Figure 6.1d displays the probability that the actually achieved *IRR* value will be greater than the *IRR* value displayed on the abscissa of the plot. These reliability levels are calculated whilst incorporating all of the random variables in Equation 6.8c and thus, based on this given estimated information, incorporates most possible outcomes of the configurations of all of these variables. The use of the CDF plot provides the decision maker with additional information in that the probabilities of a range of *IRR* values are returned and hence the probability density of the *IRR* can also be determined from the plot or through approximation techniques if so desired. The approximation techniques are not part of this study and are thus not presented here.

This result is an immensely powerful result as it far outperforms the current sensitivity analysis methods which normally return, in tabular format, with no indication of probability or reliability, the potential *IRR* values for different combinations of only two random variables at a time. The current methods can be extended three variables at a time if the sensitivity analysis is done in 3-Dimensions but this is not common. Using the current sensitivity analysis methods the project's calculated *IRR* value is thus chosen based on a discrete combination of the variables that has no indication of the probability

of being achieved and that does not incorporate most of the possible combinations of all of the variables nor their variability.

In the face of such uncertainty it is understandable that many projects fail to meet their 'predicted' returns.

Thus the use of the probabilistic methods found in the FOSM methods derived here to determine the CDF plot for the value of *IRR* is a great improvement on the current methods as it;

- Incorporates the variability of all of the constituent variables
- Returns values with associated probability values
- Returns one simple, easily understood result, the CDF plot of the *IRR*, upon which to base a decision.



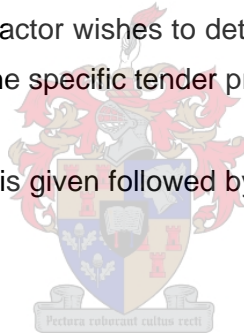
## 7. Method Summations

In this section of the study the methods used to analyse and design the project duration and cost estimates will be illustrated by means of an example. The further methods utilised to calculate risk allowances and profit potential will then also be demonstrated. The purpose of this chapter is to detail the methods used in a stepwise fashion in order to run through the entire estimating process from start to finish.

This chapter will illustrate the methods derived using two examples. The first will illustrate the analysis of initial project feasibility, then the creation, analysis and design of the project duration and cost and the calculation of risk allowance factors for the global factor rainfall for the particular project.

The second example is taken from the perspective of the contractor that is tendering for the works. The contractor has already calculated a cost estimate based on the project and the FOSM process. The contractor wishes to determine a tender price and evaluate the reliability of making a profit if the specific tender price is chosen.

A brief description of the example is given followed by the description of the method and results.



### 7.1 Example 7.1

#### 7.1.1 Description of Example.

The director of a private corporation believes that there is money to be made in the municipal waste business. The director has approached their internal Project Management division (PMD) to manage a project that entails the feasibility, design, construction, ownership and operation of a landfill site in South Africa.

The brief to the PMD reads as follows:

“We believe there to be money available in the waste business as waste disposal becomes more undesirable from a government policy perspective. The business case is based on the fact that legislation will soon be approved that increases the price of waste disposal in South Africa and decreases the number of operational landfill sites. The hope

is that this will defer waste disposal to recycling. The fact that waste recycling facilities will not be widely available on a commercial scale in the next 5 -10 years means that the value to the rights of the limited cubic metres of landfill void will increase over the next few years. The corporation wishes to construct and own the rights to this void. The corporation wishes to operate the landfill site for a period of 10 years after its construction completion.

Your project is to investigate the commercial feasibility of investment into the creation of an entirely new landfill site in South Africa. And if feasible proceed with the project.

Part 1. You must perform a basic commercial feasibility investigation to assess whether or not the project is commercially viable. For this purpose the feasibility criterion are:

- **A positive NPV with a reliability level of 90%**
- **An IRR value of 11% with a probability of being achieved of 90%**

You are to Assume

- The cost of operating a landfill site is between R900,000 and R1.6million but anticipated at R1.2 million per annum
- The risk free rate will remain within the range of 3%-6%, 85% of the time over the lifetime of the project.
- The void will be fully exhausted at the end of the 10 year period and as such there is no residual value.
- A risk premium of 8% for discounting purposes.
- Assume a constant volume of waste disposed per annum into the landfill site of between 290,000m<sup>3</sup> and 333 000m<sup>3</sup> per annum with an anticipated volume of 310,000 m<sup>3</sup>.
- The average price per m<sup>3</sup> of waste disposed over the ten year period is anticipated to be R9.50 fluctuating by 15 % either side.

If the basic feasibility investigation identifies the project to be viable then you must provide;

- Part 2. A detailed programme reliable to a level of 85%
- Part 3. A detailed cost estimate reliable to a level of 85%
- Part 4. A Financial Risk Allowance to mitigate for 80% of the deleterious effects of rainfall on the project if it is assumed that Rainfall Greater than 6mm in one day is required to disrupt the performance of a task.”

### **7.1.2 Part 1: Commercial Feasibility**

The commercial feasibility of the project will be evaluated against the NPV and IRR values as given by the director’s brief. The fact that reliability values are attached to the desired NPV and IRR values thus requires that FOSM is utilised to perform the feasibility determination probabilistically.

The basic Commercial Feasibility of the project is thus evaluated by using Equation 6.7a for the NPV and Equation 6.8c for the IRR and the methods described in Section 6.2 of this study.

From the description of the task within the brief to the PMD it is clear that most of the initial estimates of the variables required in Equations 6.7a and 6.8c are given. The PMD must however estimate the capital required to construct the new landfill site, i.e. the Basic Cost estimate.

For the purpose of this example the Basic Cost estimate is estimated by the PMD to be equal to;

- Optimistic Cost - R 5,500,000
- Pessimistic Cost - R 10,000,000
- Anticipated Cost - R 7,000,000

The rest of the estimates are tabled below in Table 7.1.



Table 7.1: Initial Estimates

Item	Given Estimated Data		
	Optimistic	Pessimistic	Anticipated
<b>Cost Stream</b>			
Basic Cost Estimate $I_o$	R 5,500,000	R 10,000,000	R 7,000,000
Operating Cost $C_i$	R 900,000	R 1,600,000	R 1,200,000
<b>Revenue Stream</b>			
Tariff	333,000	290,000	310,000
Revenue $R_i$	R 2,688,975	R 3,168,250	R 2,945,000
n (years)	10		
Risk Premium	8%		
Risk Free Rate	3%	6%	4.50%

The estimates are then approximated as a Normally distributed variables by means of the methods described in Section 3.2.1.1 and Appendix B and the results are detailed in Table 7.2.

Table 7.2: Normally Distributed Variables

Item	Calculated Statistics of Data		
	Mean	Std Deviation	COV
<b>Cost Stream</b>			
Basic Cost Estimate $I_o$	R 7,500,000	R 935,414	0.12472
Operating Cost $C_i$	R 1,233,333	R 143,372	0.1162
<b>Revenue Stream</b>			
Revenue $R_i$	R 2,934,075	R 97,907	0.0333
n (years)	10		
Risk Premium	8%		
Risk Free Rate	4.50%	1.43%	0.31746

These approximated variables are then substituted into Equation 6.7a converting it into a probabilistic model. Thus the basic commercial feasibility of the project will be evaluated with the following probabilistic model;

$$NPV = (R_i - C_i) \cdot \left( \frac{1 - (1 + (w + 8\%))^{-10}}{w + 8\%} \right) - I_o \quad (7.1)$$

The above model is non-linear and thus an iterative process is required. Secondly the degree of non-linearity is high and thus an approximation of this non-linear case as done in Chapter 4.2.3 would not be suitable. Therefore a MATLAB application was written to calculate the reliability of the above model at its limit state or when the NPV is equal to zero. This process is demonstrated in Appendix D Section 1 in more detail.

The iterative process returns a  $\beta$  value equal to 1.344 Then the probability in this case that the NPV will be greater than zero, is equal to;

$$\begin{aligned}
 P(NPV > 0) &= \Phi(\beta) & (7.2) \\
 P(NPV > 0) &= \Phi(1.344) \\
 P(NPV > 0) &= 91.05\%
 \end{aligned}$$

where  $\Phi( )$  = the Cumulative Normal Distribution function  
 $P(NPV > 0)$  = the probability that the NPV is greater than zero.

Thus the reliability that the NPV will be positive and the project will make at least R1 of profit is 91.05%. This is greater than the 90% value requested as per the Directors brief and is thus acceptable.

The next value to calculate is the IRR. To do this the variables from Table 7.2 are substituted into Equation 6.8c and the methods described in Section 6.2.3.2 are utilised. Thus a probabilistic model is created. The model is set equal to zero to create the limit state function as given below.

$$(R_i - C_i) \cdot \left( \frac{1 - (1 + IRR)^{-10}}{IRR} \right) - I_0 = 0 \quad (7.3)$$

In accordance with the methods in Section 6.2.3.2 the PM professional must systematically choose different IRR values, substitute these values into Equation 7.3 and calculate the resulting reliabilities.

Once again the non-linearity of Equation 7.3 is high and an approximation method is not suitable. Therefore an iterative process is required to calculate the reliability of the model at the estimated IRR value. The calculation of the reliability for an IRR value equal to 10% is shown in Appendix D Section 2.

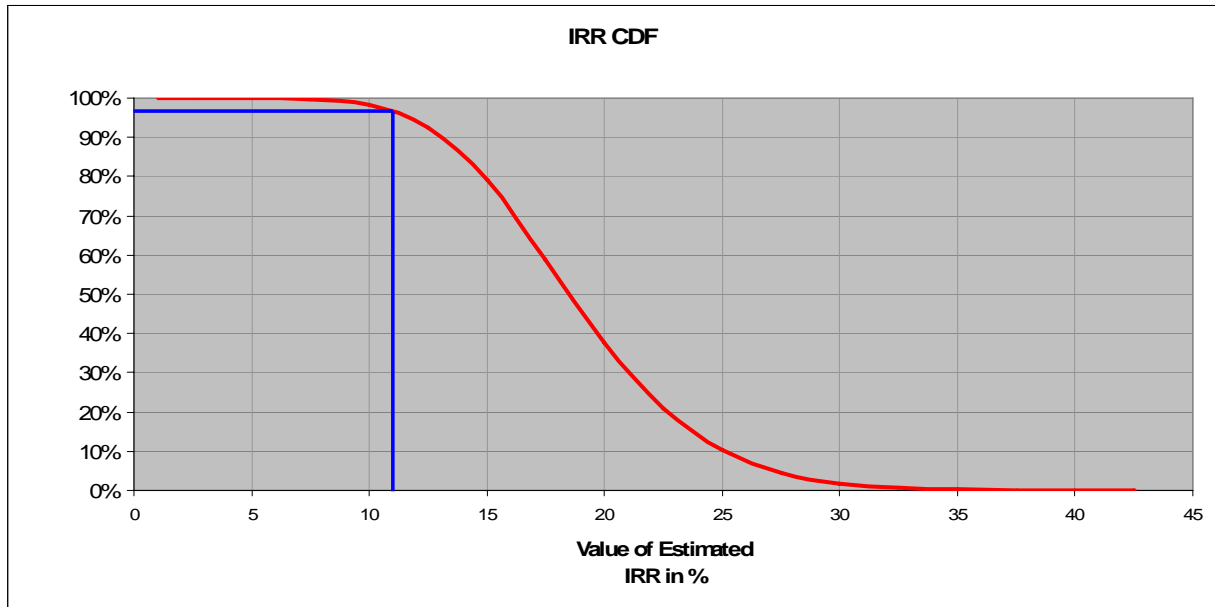
Thus if the PM professional systematically begins by calculating the reliability of Equation 7.3 with an IRR value of 1% and then repeats this calculation each time increasing the IRR value by 2.5%, the resulting reliability for the range of IRR values is calculated. The results of this process are shown in tabular format in Table 7.3.

Table 7.3: Reliability at different IRR values.

Estimated IRR %	$\beta$	P(Actual IRR > Estimated IRR)
1	4.5505	1.0000
2.5	4.139	1.0000
5	3.4459	0.9997
7.5	2.7554	0.9971
10	2.0796	0.9812
12.5	1.4287	0.9235
15	0.8102	0.7911
17.5	0.2291	0.5906
20	-0.3119	0.3776
22.5	-0.8122	0.2083
25	-1.2722	0.1017
27.5	-1.6938	0.0452
30	-2.0792	0.0188
32.5	-2.4311	0.0075
35	-2.7522	0.0030
37.5	-3.0452	0.0012
40	-3.3129	0.0005
42.5	-3.5576	0.0002
42.5	-3.7817	0.0001

The range of IRR values and the resulting Probability values are then plotted below in Figure 7.1.

Figure 7.1: IRR and Probability of Achievement



From Figure 7.1 it can be seen that the probability that the Actual IRR value will be greater than 11% is equal to approximately 97%. This value is greater than the value specified in the Directors brief and is thus acceptable.

The high values of reliability chosen by the Director for both the NPV calculation and an IRR of 11% reflect the Directors aversion to risk. These high values of reliability should, based on the variability of the input variables, ensure that for this project to proceed the probability of making a profit of at least R1 is high. The director thus wants to ensure that only projects highly likely to be profitable will be executed. The director is thus risk averse in this instance.

Therefore in terms of the Directors brief the project meets both the commercial feasibility criteria and the PM professional can thus proceed with the project.

### 7.1.3 Part 2: A detailed programme reliable to a level of 85%

#### 7.1.3.1 Analysis

In order to create the detailed programme the PM professional must first identify all of the tasks required to execute the project. The interrelations between the tasks must be established and the precedence network must then be created. As this requirement does not form part of this study it will not be demonstrated here.

In order to create the programme the PM professional must however estimate the durations of all of the constituent tasks. The PM professional thus estimates for each task the following;

1. The shortest possible duration (Optimistic Case)
2. The longest possible duration (Pessimistic Case)
3. The anticipated duration (Anticipated Case)

The three duration estimates for each task have been made and are presented in Table 1 of Appendix E.

The PM professional then inserts the discrete, Anticipated Duration Estimates values ( $ade_i$ ), for **all** of the tasks into the precedence network. This facilitates the creation of the Anticipated Duration Gantt Chart. This Gantt Chart is constructed using the Anticipated values as estimated by the PM professional for all tasks and is thus representative of the Anticipated duration of the project. Therefore if no uncertainty was included in the estimate then this Gantt Chart would be utilised to estimate duration and would form the basis of the cost estimation.

Therefore, after incorporating the uncertainty into the estimate it is of particular interest to analyse the likelihood of this Anticipated discrete estimate being achieved as it gives an indication as to the bias of the PM professional to over or underestimate.

This Gantt Chart is shown in Appendix E as, *Anticipated Duration Gantt Chart* From this Gantt Chart or Programme the Anticipated Duration of the project, as initially anticipated by the PM professional is;

- 943 days long.
- Starting on the 1<sup>st</sup> of November 2005
- Finishing on the 6<sup>th</sup> of June 2009

In order to calculate the probability of completing the project prior to the 6<sup>th</sup> of June, as initially anticipated, uncertainty must be incorporated into the estimate. To do this the probabilistic model, Critical Duration Units as described by Equation 3.1, must be constructed. In order to construct this model the PM professional firstly needs to determine which tasks are most likely to be critical.

The process of identification of the critical tasks has been highlighted in Appendix E Section 2. The Critical Duration Unit model  $D_c$  is then constructed by summing all of the critical tasks' duration variables.

This model is not linked to a calendar because it is devoid of the interrelations between tasks, in other word the durations of two critical tasks running concurrently are summed. The model therefore represents the cumulative sum of the durations of all of the critical tasks. It does not represent an actual duration but rather a cumulative duration and hence is called Critical **Duration Units** model.

The model mean and standard deviation is then calculated to be;

$$\mu_{D_c} = 1083 \text{ duration units, in this case days.}$$

$$\sigma_{D_c} = 62.63 \text{ duration units, in this case days.}$$

Thus  $D_c$  can now be represented by a single normally distributed variable,

$$D_c = N(1083, 62.63)$$

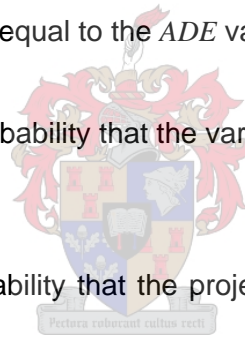
The next value that must be calculated to determine the probability of completing the project before the Anticipated completion date of 6 June 2009 is the value, Anticipated Duration Estimate Units or *ADE*.

The *ADE* value is simply equal to the sum of the initially estimated Anticipated duration values of all the **critical** tasks. This *ADE* value, calculated by means of Equation 4.6 is also not bound to the calendar and thus its value represents the cumulative sum of the Anticipated durations of all of the critical tasks. The *ADE* value of the example is calculated to be equal to;

$$ADE = 1000 \text{ duration units, in this case days}$$

Now, the probability that the project would thus complete before the anticipated completion date of 6 June 2009 is equal to the probability that the project's Critical Duration Units will be less than or equal to the *ADE* value.

This probability is given by the probability that the variable  $D_c$ , is less than the *ADE* value or  $P(D_c < ADE)$ .



Thus from Equation 4.7 the probability that the project will complete by the 6<sup>th</sup> of June 2009 is calculated by;

$$P(D_c < 1000) = \int_{-\infty}^{1000} f_{D_c}(x) dx \tag{7.4}$$

$$P(D_c < 1000) = 0.093$$

Thus the probability that the project will complete before the 6<sup>th</sup> of June 2009 is only equal to 9.3%.

Thus there is approximately a 90% chance that the project would have completed late if the project's actual duration was equal to the 943 days as programmed using the Anticipated durations of the tasks. This highlights the underestimation bias of the PM

professional's Anticipated duration and probably assists in explaining why a majority of projects are completed late.

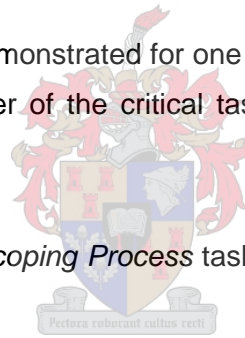
This calculation is shown in Appendix E, Section 3, Table 4

### 7.1.3.2 Design.

The duration of the project must be designed to meet the desired level of reliability of 85% as specified by the director. Thus the specified  $\beta$  is equal to 1.04.

In order to design the duration of the programme to meet the desired level of reliability the PM professional must design the durations of all of the individual critical tasks. This is done by multiplying their duration mean values by calculated the partial factors  $\gamma_i$  for all of the critical tasks.

The calculation process will be demonstrated for one critical task from the example here. The calculations for the remainder of the critical tasks are shown in tabular format in Appendix E, Section 3, Table 4



The critical task selected is the *Scoping Process* task, Task 39 as shown in Appendix E Anticipated Duration Programme.

The Tasks duration variable is given in days by;

$$d_{Task39} = N(61.7, 7.2)$$

Now using Equation 4.2c the  $\alpha_i$  values of Task 39 must be calculated. This is equal to

$$\alpha_{Task39} = \frac{7.2}{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2}}$$

$$\alpha_{Task39} = \frac{7.2}{\sqrt{3922.5}}$$

$$\alpha_{Task39} = 0.114$$



The value  $\alpha_{Task39}$  is then used in Equation 4.2e to calculate the partial factor to design Task 39's duration to the specified level of reliability. The partial factor is given by;

$$\gamma_{Task39} = 1 + \left( \frac{\alpha_{Task39} \cdot \beta \cdot \sigma_{Task39}}{\mu_{Task39}} \right)$$

$$\gamma_{Task39} = 1 + \left( \frac{0.114 \cdot 1.04 \cdot 7.2}{61.7} \right)$$

$$\gamma_{Task39} = 1.014$$

The designed duration is thus now equal to the Mean duration of Task 39 multiplied by its calculated partial factor as given by Equation 4.2f.

Thus the Designed duration reliable to a level of 85% is then equal to;

$$T_{Task39} = \gamma_{Task39} \cdot \mu_{Task39}$$

$$T_{Task39} = 1.014 \cdot 61.7$$

$$T_{Task39} = 62.52$$

In the example the durations were rounded to the nearest day for demonstration purposes and thus the designed duration value highlighted in Appendix E, Section 3, in Table 4 is equal to 63 days.

This process is repeated for all of the critical tasks and new durations for all of the critical tasks are designed. The results of this process are shown in Appendix E, Section 3, in Table 4

The last step in the process is to calculate the actual designed duration. This is done by substituting the designed durations for all critical tasks into the precedence network and recalculating the critical path and thus project duration. This is done and is shown in Appendix E, Designed Duration Programme.

From this result the designed project duration is now;

- 1074 days long.
- Starting on the 1<sup>st</sup> of November 2005
- Finishing on the 11<sup>th</sup> of December 2009

This duration is thus now designed to meet the desired level of reliability of 85% which means that the probability that the actual physical project will complete before the 11<sup>th</sup> of December 2009 is 0.85 or 85%.

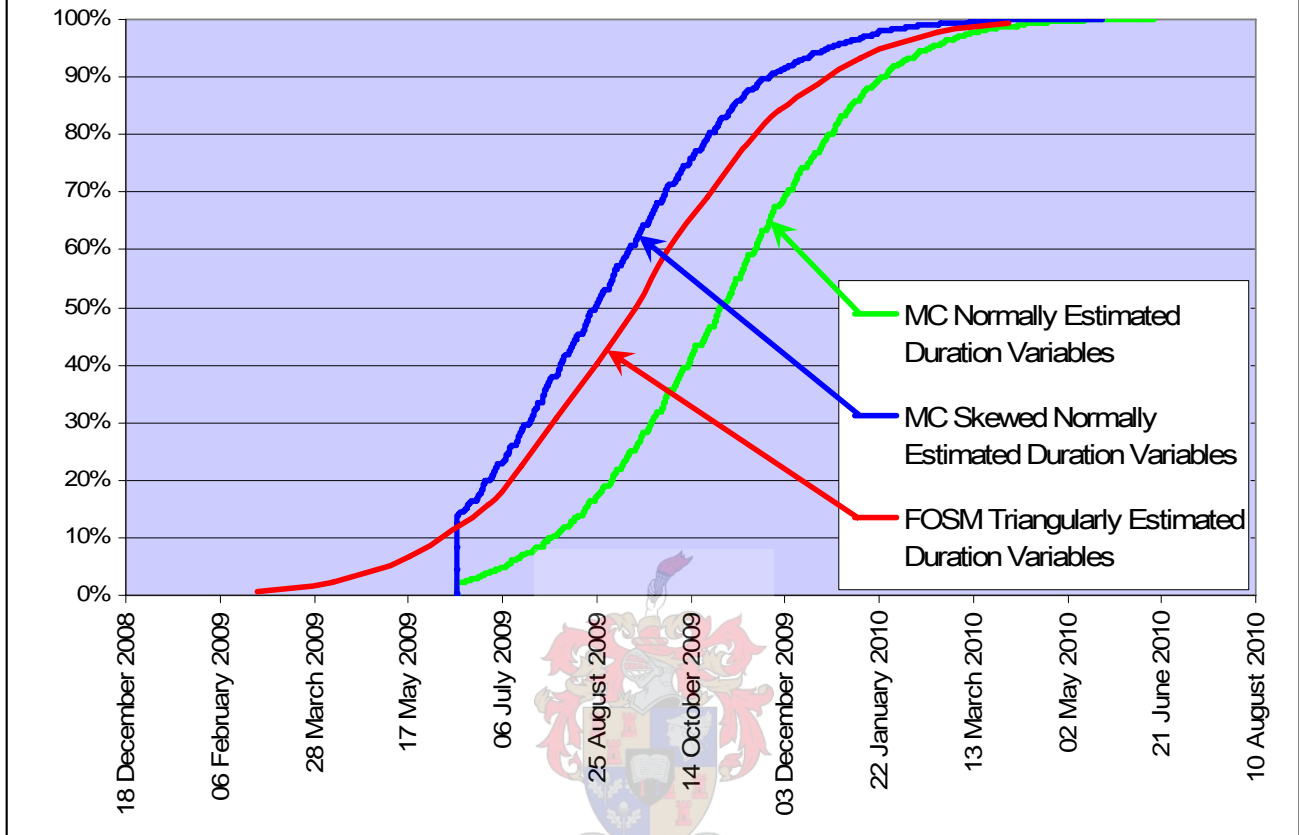
Additionally, unessential to the example, the designed duration results that are achieved by a Monte Carlo Simulation and those achieved by the derived FOSM method were compared.

The software utilised to perform the Monte Carlo simulation offers the user the option of choosing the type of approximation to use when converting the three estimates; best, worst and anticipated into a randomly distributed variable. Unfortunately this software does not offer the user the option of approximating the variables as triangularly distributed variables as done in this study. The options it does offer are approximating the variables as Normally distributed variables and Skewed Normally distributed variables. The Monte Carlo simulation was firstly run for both cases performing 5000 iterations each.

Secondly, the FOSM methods derived in this study were utilised to design the duration estimate for a range of reliability from near zero to near 100%. Thus the duration was designed using a range of  $\beta \in (-2.5; 2.5)$ . The resulting completion dates were then plotted against the reliability percentages.

The results of this exercise are shown in Figure 7.2 below.

Figure 7.2: CDF Plot of Muchawaste Monte Carlo Simulation Results vs FOSM Design Results



From the plots of the two Monte Carlo cases it can be seen that the results are sensitive to the type of distribution chosen to approximate the individual task durations. It can also be seen that the FOSM design method is within the bounds of the two cases and thus the Triangular distribution approximation methods utilised are suitably accurate. Further the shapes of the three curves are similar and thus, apart from the different mean values due to the different approximation methods, the distributions of the three cases are similar which further backs up the suitability of the derived FOSM methods to design project duration estimates.

#### 7.1.4. Part 3: A detailed cost estimate reliable to a level of 85%

For the purpose of this example the Method B approach described in Section 4.3.1 will be utilised for this example. For the purpose of description the cost estimates; Optimistic, Pessimistic and Anticipated shown in Table 1 of Appendix F include both the duration based costs and the quantity based costs.

The steps required to calculate the respective values displayed in Appendix F Table 1 are detailed in Section 4.3.1.1.

Costs and figures have been rounded off for descriptive purposes here. The un-rounded values are given in detail in Appendix F.

From Table 1 Appendix F the Calculated *ACE* value given by Equation 4.15 is;

$$ACE = R\ 7,053,000$$

The probability that the actual project cost will finish less than this value is then calculated by substituting the *ACE* value into Equation 4.16 so that you get;

$$P(C < ACE) = \int_{-\infty}^{R7,053,000} f_C(x) dx$$
$$P(C < ACE) = 0.201$$

The probability that the actual cost would be less than the initially Anticipated cost is low and the project would have a 79.9% chance of running over budget. This once again highlights the PM professional's bias to underestimating.

The project cost estimate must be designed to meet a reliability level of 85% or in other words must represent a cost value for which there is an 85% chance that the actual project cost will be less than that cost.

Task 39: *Scoping Process* is again utilised to demonstrate the methods derived in Section 4.3.1.1.

From the three estimates;

Optimistic	-	R 63,000
Pessimistic	-	R 90,000
Anticipated	-	R 75,000

the *Scoping Process* cost variable is given by;

$$c_{Task39} = N(R76000, R5523)$$

Now using Equation 4.2c the  $\alpha_i$  values of Task 39 must be calculated. This is equal to

$$\alpha_{Task39} = \frac{R5,523}{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2}}$$

$$\alpha_{Task39} = \frac{R5,523}{R105,048}$$

$$\alpha_{Task39} = 0.053$$

The value  $\alpha_{Task39}$  is then used in Equation 4.18 to calculate the partial factor to design Task 39's duration to the specified level of reliability. The partial factor is given by;

$$\xi_{Task39} = 1 + \left( \frac{\alpha_{Task39} \cdot \beta \cdot \sigma_{Task39}}{\mu_{Task39}} \right)$$

$$\xi_{Task39} = 1 + \left( \frac{0.053 \cdot 1.04 \cdot R5,523}{R76,000} \right)$$

$$\xi_{Task39} = 1.004$$

The designed cost of Task 39 is thus now equal to the Mean duration of Task 39 multiplied by its calculated partial factor as given by Equation 4.19.

Thus the designed cost of Task 39 reliable to a level of 85% is then equal to;

$$\begin{aligned}Pr_{Task39} &= \xi_{Task39} \cdot \mu_{Task39} \\Pr_{Task39} &= 1.004 \cdot R76,000 \\Pr_{Task39} &\approx R76,300\end{aligned}$$

Now the Designed Project cost is the sum of all of the individual tasks' designed costs and is given by Equation 4.20. The Designed Project cost reliable to 85% is thus equal to (rounded off);

$$C_{85\%} = R7,250,000$$

#### 7.1.5 Part 4. A Financial Risk Allowance

The Directors brief details that the effects of a project external factor, rainfall must be modelled. The director's brief further defines that, in terms of the project, rainfall greater than 6mm in a 24 hour period is classified as a rainfall occurrence. The Directors brief also dictates that the rainfall mitigation must be in the form of a financial allowance that must be able mitigate for 80% of the deleterious effects of the rainfall on the project. Due to the fact that the director wishes to have a financial allowance calculated the example will be utilising the methods derived in Section 5.7.

The PM professional must therefore estimate or determine the probability of the occurrence of the global factor and the consequence of such occurrence. This financial allowance will be modelled probabilistically to incorporate uncertainty and also design the mitigation allowance to the desired level of 80% therefore  $\beta = 0.878$ .

The model detailed by Equation 5.13b will thus be utilised to design this risk mitigation allowance.

The PM professional must determine the probability of the occurrence of disruptive rainfall and estimate the as a percentage value the;

- Least possible percentage disruption (Optimistic Case)
- Greatest possible percentage disruption (Pessimistic Case)
- Anticipated percentage disruption (Anticipated case)

of the occurrence of that disruptive rainfall.

Once again this is detailed for one task with the remainder shown in Appendix G. For this purpose Task 69: *Lay and Weld Geomembrane Liner Bay 1* was chosen instead of Task 39 as Task 39 is unaffected by the global factor rainfall.

#### 7.1.5.1 The Probability of Occurrence

Probability that rain greater than 6mm will fall in a 24 hour period for at least one day in the month is given in the below table taken from Table 5 Appendix G;

Table 5.2: Disruptive Rainfall by Month

Month	Probability of disruptive rainfall by month
January	0.2603
February	0.2274
March	0.0650
April	0.0000
May	0.0000
June	0.0000
July	0.0000
August	0.0000
September	0.0000
October	0.0098
November	0.0964
December	0.0461

Task 69 occurs in January and February of 2009 and thus the probability of it being affected by a disruptive rainfall event is equal to the probability of a disruptive rainfall

event in January or February. The probability is thus equal to the sum of the probabilities of a disruptive rainfall event in January and February.

This equals 0.488.

See Table 6 Appendix G for the remainder of the tasks' probabilities.

### 7.1.5.2 The Disruption Variable

For Task 69, the potential disruptions, as a result of the disruptive rainfall event, are estimated by the PM professional as a percentage value. From Table 7 of Appendix G the three estimates

- Optimistic - 10 % Disruption
- Pessimistic - 30 % Disruption
- Anticipated - 15 % Disruption

are approximated as a normally distributed variable. For Task 69 this variable, the Disruption Variable is given by;

$$\phi_{Task\ 69} = N(18.3, 4.43)$$

### 7.1.5.3 Design

Now using Equation 4.2c the  $\alpha_i$  values of Task 69 must be calculated. This is equal to

$$\alpha_{Task\ 69} = \frac{4.43}{\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2}}$$

$$\alpha_{Task\ 69} = \frac{4.43}{10.173}$$

$$\alpha_{Task\ 69} = 0.418$$



The value  $\alpha_{Task69}$  is then used in Equation 5.11 to calculate the partial factor to design Task 69's financial risk mitigation allowance to the specified level of mitigation. The partial factor is given by;

$$\lambda_{Task69} = 1 + \left( \frac{\alpha_{Task69} \cdot \beta \cdot \sigma_{Task69}}{\mu_{Task69}} \right)$$

$$\lambda_{Task69} = 1 + \left( \frac{0.418 \cdot 0.878 \cdot 4.43}{18.3} \right)$$

$$\lambda_{Task69} = 1.082$$

The calculated partial factor, the probability of disruption and the disruption variable's mean value are then substituted into Equation 5.15 and the cost of Task 69 including mitigation is then calculated as done below;

$$C_{Task69} = \left( 1 + \frac{q_{Task69} \cdot \lambda_{Task69} \cdot \mu_{\phi_{Task69}}}{100} \right) \cdot Pr_{Task69}$$

$$C_{Task69} = \left( 1 + \frac{0.488 \cdot 10.82 \cdot 18.3}{100} \right) \cdot R561,800$$

$$C_{Task69} = R616,000.$$



and the Financial risk mitigation allowance for Task 69 is given by;

$$\begin{aligned} \text{Task 69 Allowance} &= R 616,000 - R 561,800 \\ &= R 54,200 \end{aligned}$$

The mitigation allowances are calculated for all of the other tasks and summed and utilising Equation 5.13c the total risk mitigation allowance for the global factor rainfall is calculated to be equal to;

$$\text{Rainfall Risk Mitigation Allowance} = R 259,700$$

The above design calculations are shown in Table 8 of Appendix G.

Thus a mitigation allowance of R 259,700 mitigates for 80% of the deleterious effects of the global factor rainfall for the example project.

An important observation can be made by considering the risk allowance calculated for Task 74: *Lay and Weld Geomembrane Liner Bay 2* as shown in Table 8 of Appendix G.

This task is identical to Task 69 in that is essentially the same task merely occurring at a later stage in the project. The Disruption variable for Task 74 is thus equal to the Disruption variable of Task 69.

Interestingly though the risk allowance calculated for Task 74 is only R 7,300 as opposed to the R 54,400 of Task 69. This is due to the fact that Task 74 occurs in March and April of 2009 when the likelihood of a disruptive rainfall event is greatly reduced.

This highlights the value of incorporating the calendar predictability of the global factor into the methods derived in Section 5 as the resulting mitigation allowances are calculated to be far more realistic than current subjective methods.

## 7.2 Example 7.2

### 7.2.1 Description of Example

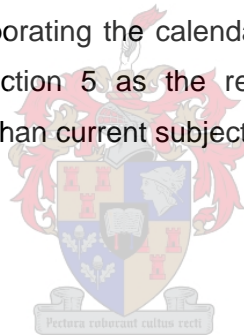
You are a contractor preparing a bid for an engineering, procurement and construct (EPC) contract to perform a said project. For the purpose of this example assume that the programme and cost estimate of Example 7.1 is applicable to your project and that the statistics of the cost estimate done there are the same as those calculated whilst preparing your programme and cost estimate. Thus after designing your project duration and cost estimates to desired levels of reliability your estimates are as follows.

Designed completion date 85% Reliable:

- Monday 11<sup>th</sup> of December 2009

Cost reliable to a level of 85%

- R 7,250,000



As the profit consideration is only applicable to the project cost, the only statistics required to design a margin based profit value are the project cost statistics of the original cost model before any adjustments had been made. The statistics of the cost model are:

- Mean                                    R 7,141,000
- Standard Deviation                R 105,000

Your task now as PM professional is to;

**a)** Calculate the final tender value to be submitted to the directors of the firm if the tender value must be prepared for a required profit margin of 2%. The directors also wish to know the probability of making at least R1 profit if this calculated tender value is chosen.

**b)** Calculate the final tender value to be submitted in the tender if the profit value must be greater than the actual cost of the project with a reliability of 98%. In other words the decision maker wishes to be highly risk averse and ensure that the probability of a profit of at least R1 will be made on this project is very high.

For both of the above calculations assume that the Indirect Costs (*IC*) have been absorbed into the rates and thus  $IC = 0$  for all cases.

The PM professional will then be able to calculate the tender values as shown below:

**7.2.2 Part A: Margin Based Tender Value Calculation**

Part A requires that the profit is calculated on a margin based method as described by Equation 6.2

$$T_k = \left(1 + \frac{k}{100}\right) \cdot \left(\sum_{i=1}^n Pr_i + IC(\text{calculated at } \beta_{Design})\right) \tag{6.2}$$

In this case the variable *k* equals 2%.

$\sum_{i=1}^n Pr_i$                     =            the value of the designed cost estimate as desired by the decision maker from example 7.1

Thus in this case it is equal to the value of R 7,250,000 designed as per Example 7.1

$$IC = 0$$

Substituting into Equation 6.2,  $T_k$  is then equal to

$$\begin{aligned} T_k &= \left(1 + \frac{2}{100}\right) \cdot (7,250,000 + 0) \\ &= R7,395,000 \end{aligned}$$

The reliability that the actual cost is less than the calculated profit value is given by Equation 6.3;

$$P(C < T_k) = \int_{-\infty}^{T_k} f_C(x) dx \quad (6.3)$$

Thus substituting the value of  $T_k$  into Equation 6.3 results in

$$P(C < R7,395,000) = \int_{-\infty}^{R7,395,000} f_C(x) dx$$

which calculates that the actual project cost will be less than R7,395,000 with a probability value of 0.99 which is high. Thus a profit margin of 2% would be a safe value to choose if a profit of at least R1 was desired from the project.

### 7.2.3 Part B: Reliability Based Tender Value Calculation

Requires that the profit is calculated on a reliability based method as described by Equation 6.4

$$M = \frac{T_{Profit} - \sum_{i=1}^n Pr_i(\text{calculated at } \beta_{cost}) + IC(\text{calculated at } \beta_{cost})}{\sum_{i=1}^n Pr_i(\text{calculated at } \beta_{cost}) + IC(\text{calculated at } \beta_{cost})} \quad (6.4)$$

The desired level of profit reliability is 98%. This translates to a Beta value of approximately  $\beta_{profit} = 2.12$ .

Using the cost model and the procedures described in Chapter 4 the cost must be designed to this level of  $\beta_{profit}$ . In this example insurances, risk allowances and overheads are assumed zero.

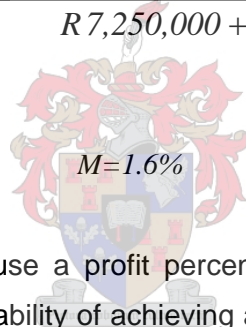
The result of this process leads to a cost calculation to the value equal to;

$$T_{profit} = R\ 7,363,000$$

Substituting these values into Equation 6.4

$$M = \frac{R7,363,000 - R7,250,000 + 0}{R7,250,000 + 0}$$

This results in a profit margin of;



Thus the decision maker could use a profit percentage value of 1.6% in the tender submission and know that the reliability of achieving at least R1 profit from the project is 98%.

## 8. Summary and Conclusions

### 8.1 Results

The methods presented in this study provide the tools with which to incorporate and manage uncertainty in a project estimating environment. These methods are simple and can easily be incorporated into the estimating process. These tools essentially provide a PM professional the ability to;

- Analyse the probability of meeting a desired cost or estimated duration
- Quantify the risk incorporated into a project estimate and the risk excluded from a project estimate.
- Design a project cost or duration to a desired and known level of reliability
- Determine risk allowances required to mitigate for known uncertain influencing factors
- Determine the commercial feasibility of a project probabilistically;

In practice estimates will be produced by PM professionals for different role players in a project each with different objectives. The methods presented in this study can however be used in all cases according to the role players requirements and needs thus;

A PM professionals working for a client can;

- Calculate the probability of realising a profit from a project through the use of the NPV probabilistic model.
- Calculate the probability of achieving different IRR magnitudes.
- Request the specification of the level of reliability of both the duration and cost estimates submitted by a tenderer.
- Specify to tenderers the level of reliability desired in either duration or cost of submitted tenders.

A PM professionals working for a tenderer can;

- Calculate the probability of achieving the estimated calculated cost or duration
- Design an estimate of either cost or duration to a desired level of reliability
- Determine the level of either duration risk or cost risk included in or excluded from an estimate probabilistically
- Calculate values for mitigating risk allowances for known global factors probabilistically
- Calculate the level of reliability of profit realisation for a tender price.
- Design a tender to a desired level of profit reliability

## 8.2 Limitations

The designed estimates are still reliant on estimated variables. As observed in Figure 7.2, the final results are sensitive to the approximation of the random variables and the distribution type chosen. Thus the occasion where historical data is available, and this data can be incorporated into a basic variable, will result in as accurate estimation of reliability as possible. Historical data is limited by the uniqueness of projects and thus will not always be suitable or available and the author predicts that most variables used in this process will be knowledge and experience based guessed estimates. Thus the reliability calculations will result in designed estimates that are designed using approximated input data.

This fact results in the possibility of there remaining some unaccounted for, or over accounted for, uncertainty in the estimate as the individual variables may have been optimistically or pessimistically estimated by the PM professional. This situation is unavoidable and the magnitude of the error indefinable. The process of estimating the variability of individual tasks as opposed to the variability of the entire project should reduce the prevalence of this situation. The fact however remains that this inherent uncertainty is unavoidable. This uncertainty is also based on human assumptions and the designed results produced are still therefore only estimates that will most often not be verifiable by any reputable data.

The estimates produced are based on current assumptions made with incomplete information in for projects that will occur in a future uncertain environment and thus the estimates produced using these methods are only as accurate as the data that is inputted into them. The methods described in this study cannot thus predict the future; they can only assist in producing more plausible or probabilistically reliable estimates by explicitly modelling uncertainty into the estimate.

This study does not attempt to analyse the accuracy of the precedence network established by a PM professional. The methods and calculations along with their resultant conclusions assume that the original precedence network is correct. Any uncertainty regarding the precedence network is not incorporated into the methods described in this study thus when estimating using the methods described in this study this fact must be considered.

### **8.3 Conclusions**

This study highlights the fact that PM professionals must produce estimates for projects that will occur in unpredictable and highly dynamic environments. PM professionals are tasked to quantify these estimates of future events in this uncertain environment with insufficient information. This is an incredibly difficult task as is shown by the 2002 survey that highlighted that 50% of projects were completed over budget and 58% of projects were completed late. This study argues that, given the difficult conditions under which the estimates are produced and the fact that project performance is measured against these estimates, the semantics, "*Over Budget*" or "*Late*" should be changed to, "*Underestimated*" with the intention of shifting the focus from the performance of PM professionals executing the project onto improving the reliability of the estimates produced by PM professionals

In the face of such uncertainty it is understandable that the estimates produced are often incorrect and many projects fail to meet their 'predicted' performance. This is also then compounded by the tendency of Clients to select the submitted tenders that have the lowest price and shortest duration which have potentially underestimated the project by the biggest margin.



The fact that most projects are unique in some way and generally not repetitive in nature means that the unique estimates produced by a PM professional will be the basis upon which many decisions regarding the project are made. The uniqueness and unrepetitive nature also means that the estimates will always be produced with incomplete uncertain information for projects that will occur in a future uncertain environment.

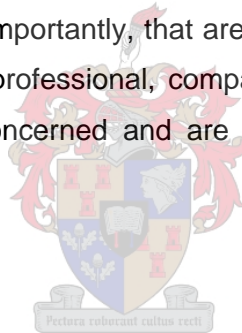
The uniqueness of a project, the varying sources of uncertainty across projects and the unpredictable nature of those sources of uncertainty has meant that few generic tools have been developed to assist the PM professional to incorporate this uncertainty into the estimates produced. As a result most project estimates are produced in a discrete manner with the incorporation of uncertainty left to the discretion of the individual PM professional. Secondly, the level of uncertainty actually then incorporated and the resulting exposure to uncertainty not incorporated is practically unknown or unquantifiable.

This study derives methods for the incorporation of uncertainty into project estimates by means of a simple, systematic, generic and quantifiable process. This is achieved by deriving methods to incorporate uncertainty within the well established, accepted and understood estimating tool, the Gantt chart with its inherent precedence network and critical path functionality. The estimates produced incorporate inherent uncertainty through the known, easily estimated, variability of either the duration or cost of the constituent tasks within a project. This variability is then actively utilised to design estimates that can mitigate the inherent uncertainty that results. The methods can also thus easily be adopted and incorporated into the existing PM estimating environment.

The method whereby the effects of a known global factor on a project's cost and duration estimate is incorporated into the estimate, in the form of percentage disruption variables, presents the PM professional with ways to probabilistically analyse and design risk allowances that accommodate for the influence that such global factors have on a project. By applying these variables into a discrete model already representative of a designed level of reliability they can be analysed independent of other factors and thus the risk allowances for each global factor can be isolated and calculated separately. In this manner the PM professional is able to quantify the risk solely attributed to the global factor under consideration. The process of scheduling the tasks onto a calendar and

applying the appropriately calculated global factors to the scheduled tasks presents a further method whereby risks can be quantified in a consistent and specific manner. The risk allowance values calculated in this manner include a greater level of detail and potentially reflect more realism than many methods common today.

The overall conclusion of the value of these methods is that PM professionals are now able to incorporate uncertainty into a project estimate and manipulate that uncertainty to achieve a desired result. They now have tools available to design estimates that incorporate different levels of uncertainty or risk by increasing or decreasing the level of reliability of their estimates. They are also able to do this in a consistent mathematical manner that can be used across different projects and which provides the framework for comparison and decision making purposes. PM professionals can now also present and defend their estimates calculated to a determinable, not guessed, level of reliability and risk. Designing estimates that accommodate this uncertainty should result in estimates that are more realistic and more importantly, that are achieved on a more regular basis. These methods thus give a PM professional, company or client the flexibility to make better decisions where risk is concerned and are therefore potentially a very useful methods.



## 9. References:

### Chapter

- 1 DTI - Construction Industry KPI's Industry Progress Report – 2002 (United Kingdom)  
Also referenced at: <http://www.kpmg.co.uk/services/f/ew/ec.cfm>
  - 2 Rory Burke, 2003, *Project Management: Planning & Control Techniques*, Fourth Edition, page 28, Burke Publishing, ISBN 0958239150  
  
Ker-Fox, G.M. and Retief J.V., A general first order second moment reliability method, September 2005  
  
Ker-Fox, G.M. and Retief, J.V., UTILITY OF THE GENERAL CORRELATED FORM OF THE DIRECTION COSINE, September 2005
  - 3 Creating a Probabilistic Model
    - Figure 3.1 [http://en.wikipedia.org/wiki/Triangular\\_distribution](http://en.wikipedia.org/wiki/Triangular_distribution)
    - Monte Carlo Simulations performed by Asta Powerproject, Version 8, Risk Analysis Software Add on. (See Reference: Appendix A, below)
  4. First Order Second Moment Reliability Analysis Method.  

- Equation 4.1: Based on: **Jörg Schneider**, 1997, *Introduction to Safety and Reliability of Structures*, Structural Engineering Document Series number 5, Section 4.33, Page 78, Zürich, Switzerland, International Association for Bridge and Structural Engineering IABSE
- Equation 4.2b: Based on: **Milan Holický**, 2005, Reliability Analysis For Structural Design. Lecture notes prepared for the intensive one-week course at the University of Stellenbosch, South Africa. Annex 3- Statistical parameters of functions of random variables. Row 5, Page 159. Unpublished work produced at the Klokner Institute, CTU, Prague.
- Equation 4.2c **Alfred H-S. Ang & Wilson H. Tang**. Probability Concepts in Engineering Planning and Design, Equation 6.16b, For the linear Case, Bottom of Page 350
- Equation 4.2.3c: Based on: **Alfred H-S. Ang & Wilson H. Tang**. Probability Concepts in Engineering Planning and Design, Equation 6.18, Page 346.

- Equation 4.5: Based on: **Jörg Schneider**, 1997, *Introduction to Safety and Reliability of Structures*, Structural Engineering Document Series number 5, Equation 2.18, Page 40, Zürich, Switzerland, International Association for Bridge and Structural Engineering IABSE.
- Equation 4.8: Based on: **Milan Holický**, 2005, Reliability Analysis For Structural Design. Lecture notes prepared for the intensive one-week course at the University of Stellenbosch, South Africa. Annex 3- Statistical parameters of functions of random variables. Row 4, Page 159. Unpublished work produced at the Klokner Institute, CTU, Prague.
- Equation 4.9: Based on: **Milan Holický**, 2005, Reliability Analysis For Structural Design. Lecture notes prepared for the intensive one-week course at the University of Stellenbosch, South Africa. Annex 3- Statistical parameters of functions of random variables. Row 4, Page 159. Unpublished work produced at the Klokner Institute, CTU, Prague.
- Equation 4.11 Based on: **Milan Holický**, 2005, Reliability Analysis For Structural Design. Lecture notes prepared for the intensive one-week course at the University of Stellenbosch, South Africa. Annex 3- Statistical parameters of functions of random variables. Row 5, Page 159. Unpublished work produced at the Klokner Institute, CTU, Prague.
- Equation 4.12 Based on: **Milan Holický**, 2005, Reliability Analysis For Structural Design. Lecture notes prepared for the intensive one-week course at the University of Stellenbosch, South Africa. Annex 3- Statistical parameters of functions of random variables. Row 5, Page 159. Unpublished work produced at the Klokner Institute, CTU, Prague.
- Equation 4.17: Based on: **Jörg Schneider**, 1997, *Introduction to Safety and Reliability of Structures*, Structural Engineering Document Series number 5, Page 93, Equation 4.38, Zürich, Switzerland, International Association for Bridge and Structural Engineering IABSE

Chapter 5:

<http://www.weathersa.co.za/Climat/Climstats/JohannesburgStats.jsp>

Chapter 6

Equation 6.5:  
<http://www.investopedia.com/terms/n/npv.asp>

Equation 6.8:  
[http://en.wikipedia.org/wiki/Internal\\_rate\\_of\\_return](http://en.wikipedia.org/wiki/Internal_rate_of_return)

## Appendix A

Information available regarding Asta Powerproject Software utilised is available at:

<http://www.astadev.com/software/powerproject/index.asp>

## Appendix B

Equations(1),(2)&(3)Sourced from:

[http://en.wikipedia.org/wiki/Triangular\\_distribution](http://en.wikipedia.org/wiki/Triangular_distribution)

Equations (4) Sourced from

<http://mathworld.wolfram.com/VariationCoefficient.html>

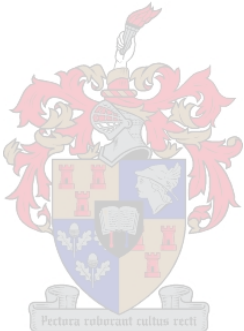
## Appendix C

Adams, 2005, Analysis of an Existing Project Management Software Tool,  
Unpublished, 2005



Appendix A: Example 3.1 and 3.2

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## Appendix A: Example 3.1

### Case 1

The purpose of this example is to determine which of the tasks are potentially on the critical path.

A Generic Project was constructed and variability estimated. Then using a Monte Carlo simulation and the 'Criticality Index' function found in the Risk Analysis add on to the Asta Powerproject package the critical tasks are identified as shown below.

FOSM was then used to calculate the critical path at different levels of reliability. The task durations were adjusted accordingly and the critical path was calculated after each reliability calculation. Then, as shown in the table below the relative frequency of a task on the critical path was calculated.

Any task found to be on the critical path at least once is then deemed critical.

The results of the two methods are then compared.

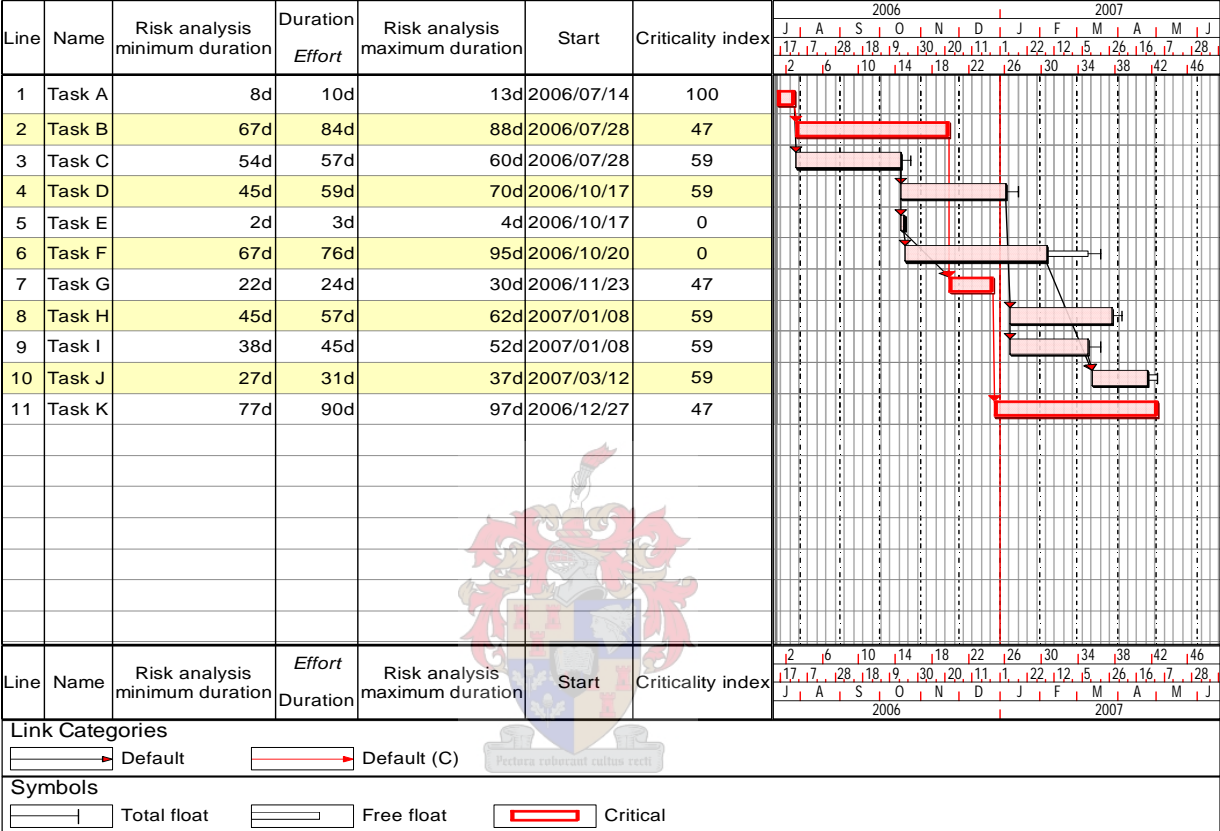
The above process was repeated three times under the different conditions to confirm the accuracy of the results. The three cases were;

- Case 1: Small Standard Deviations for the variables and a simple precedence network
- Case 2: Large Standard Deviations for the variables and a simple precedence network
- Case 3: Large Standard Deviations for the variables and complex precedence network.

Screenshots of the respective programme Gantt Charts are shown below as well as the Tabular results of the comparisons.

**Case 1**

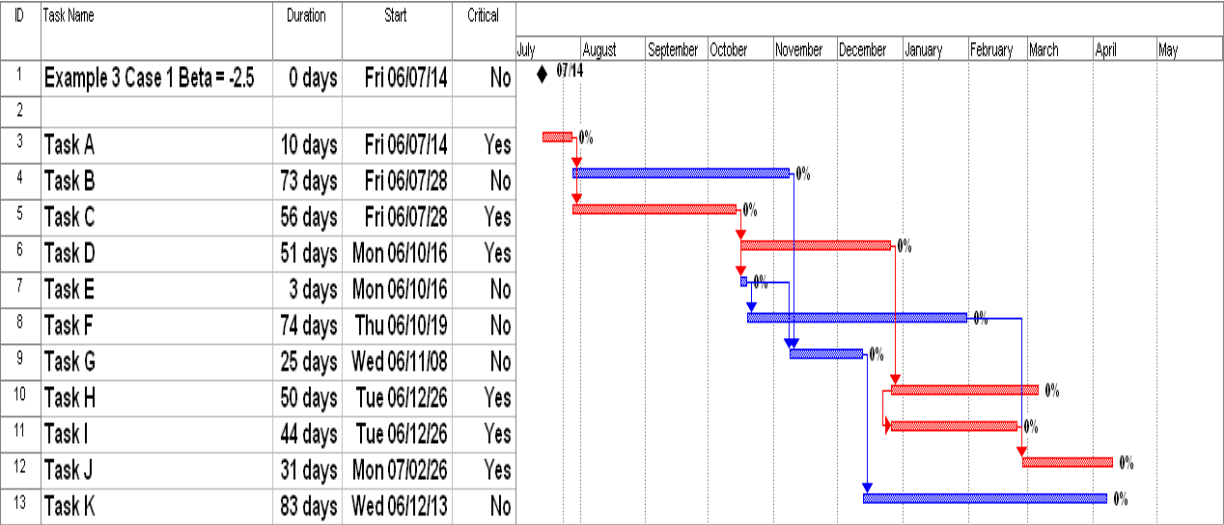
**Monte Carlo Simulation Results**



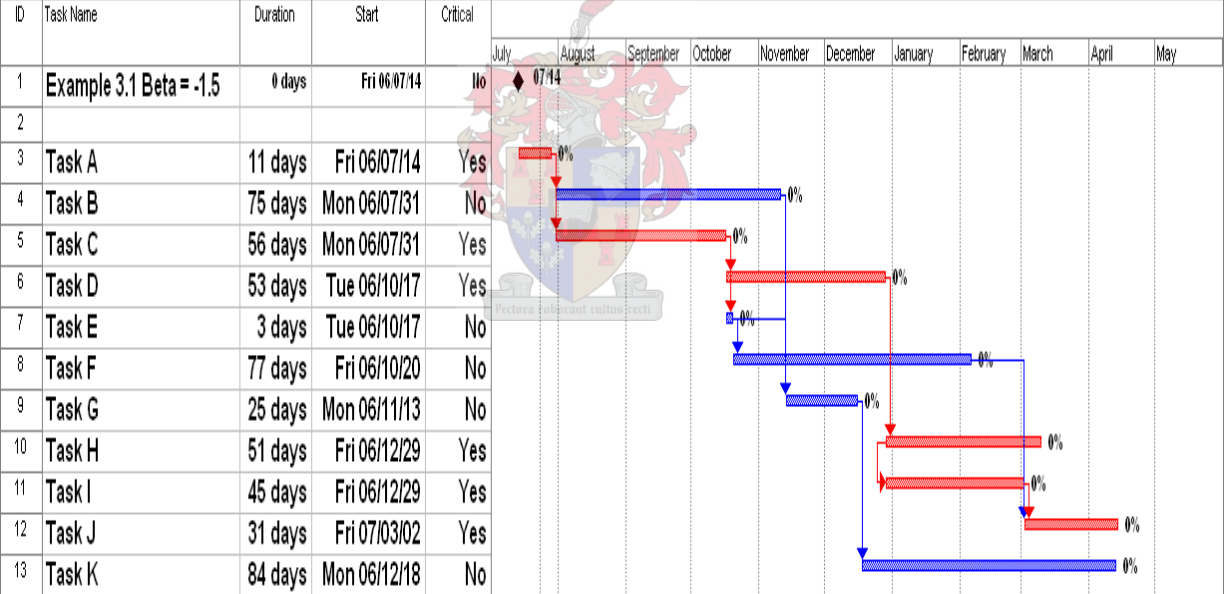


**FOSM analysis**

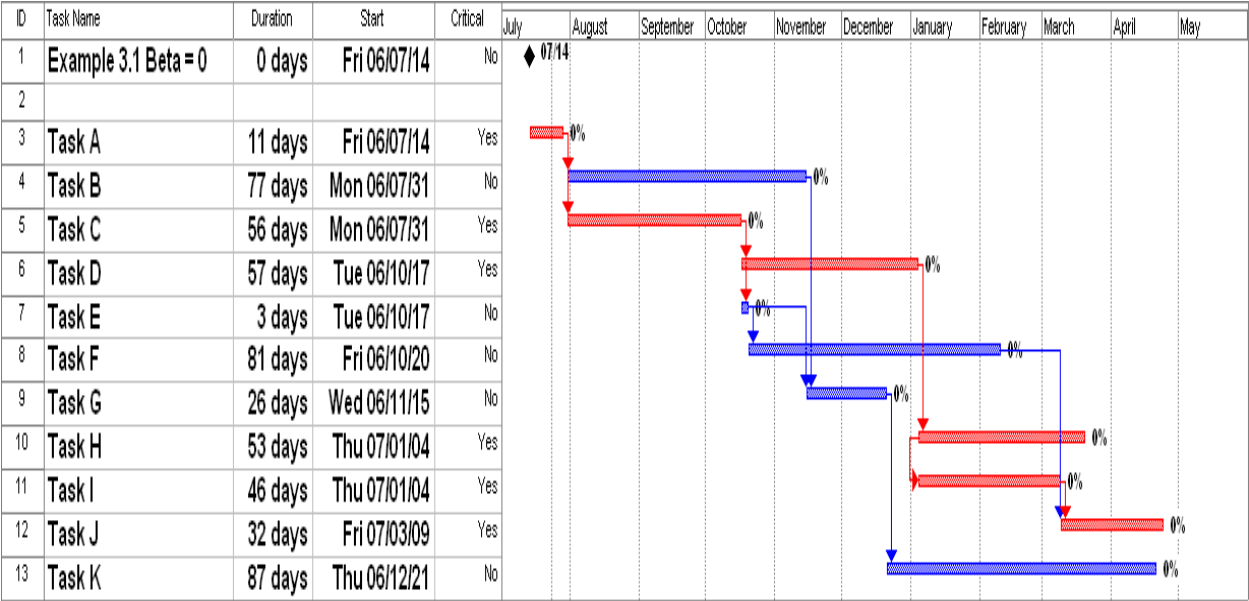
**Beta = -2.5**



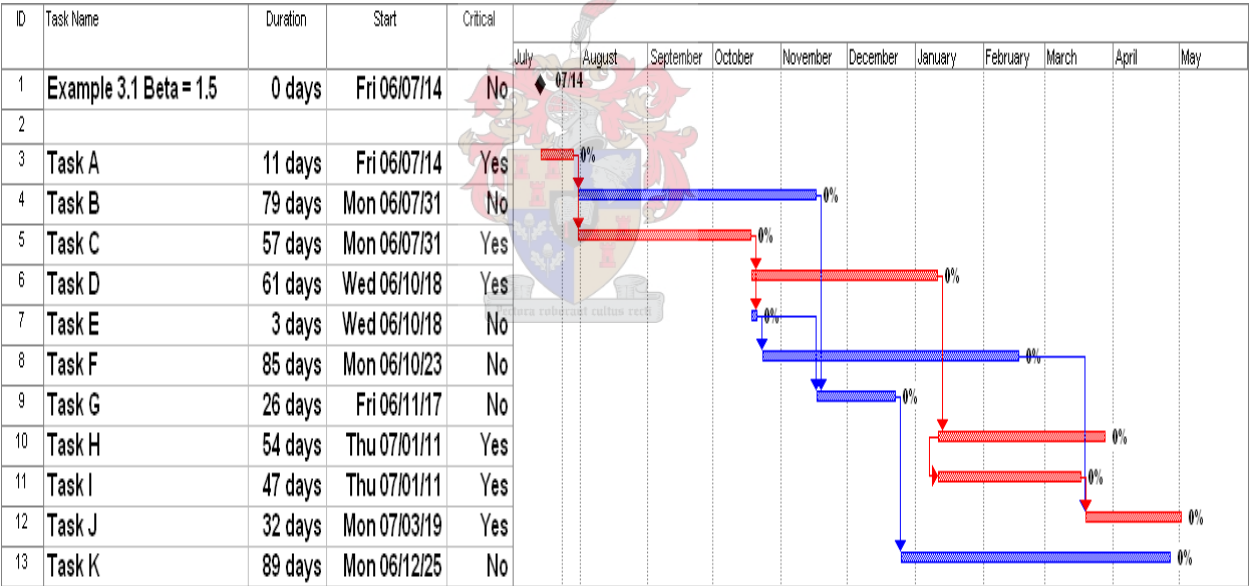
**Beta = -1.5**



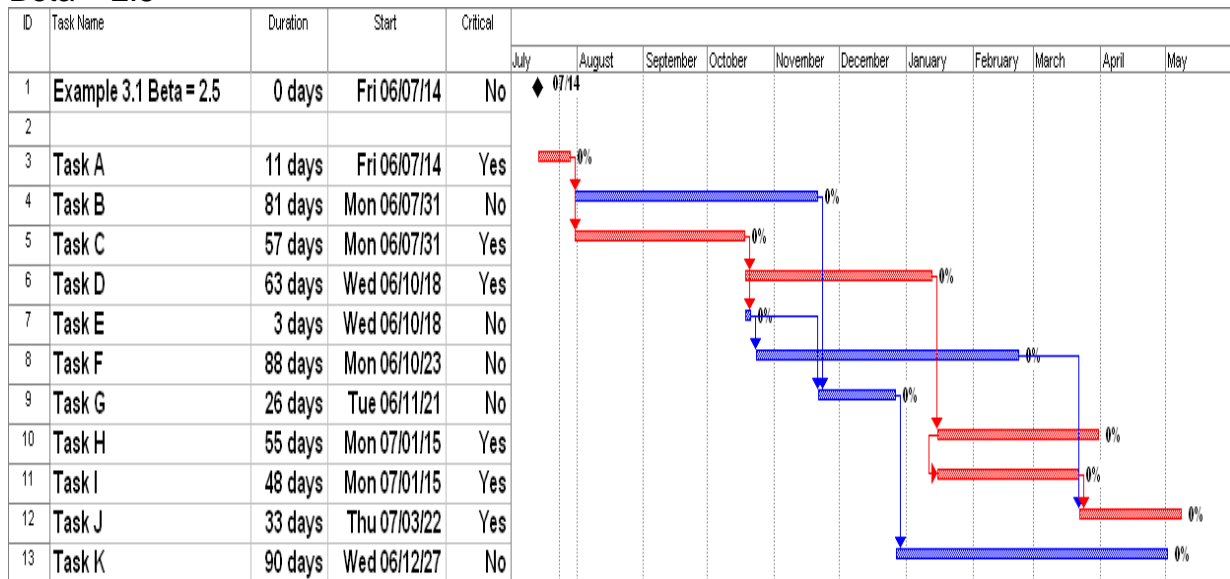
Beta = 0



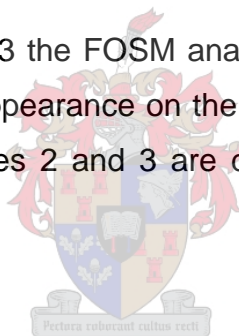
Beta = 1.5



Beta = 2.5

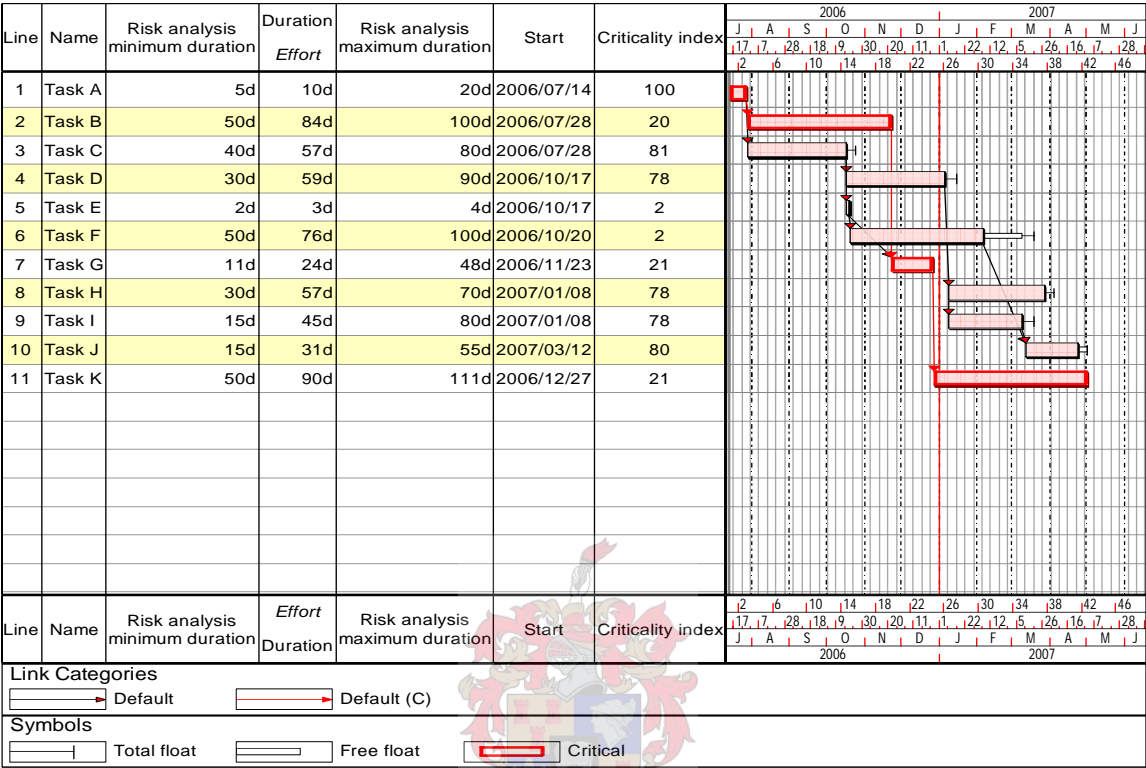


Similarly for Case 2 and Case 3 the FOSM analysis was used to determine the relative frequency of a tasks appearance on the critical path. The tabular results of the FOSM analysis for Cases 2 and 3 are detailed in Table 2 and Table 3 respectively.

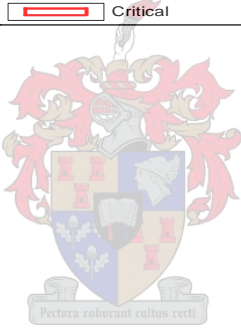
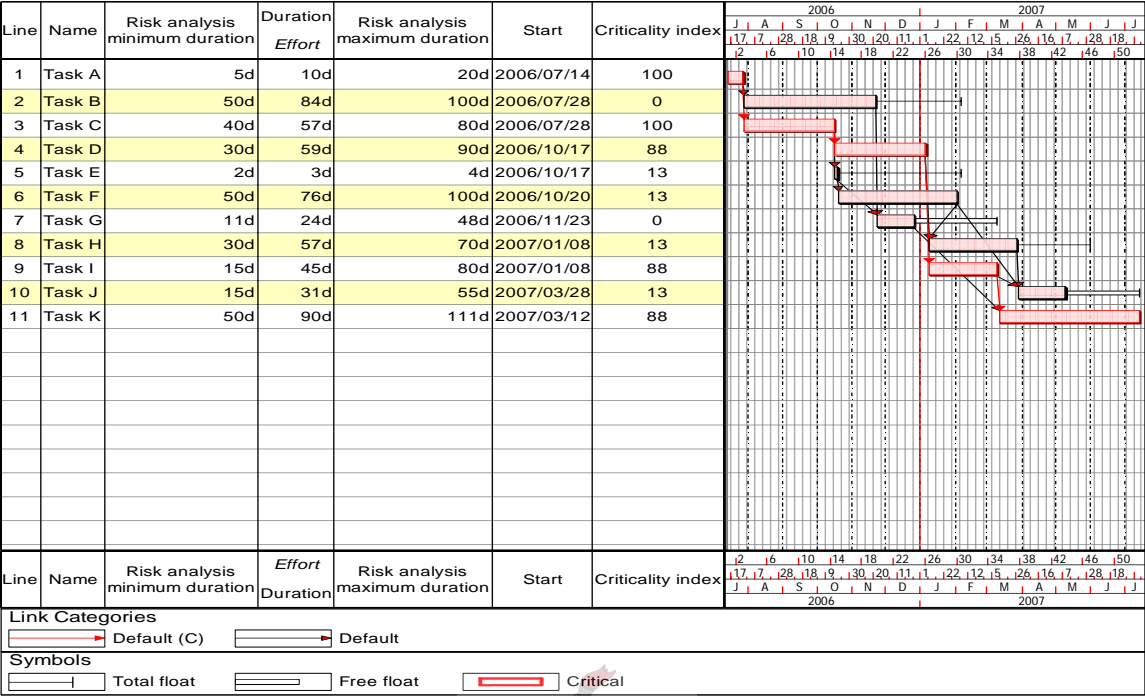


The results of the Monte Carlo Simulations for Cases 2 and 3 are shown here.

Case 2: Monte Carlo Simulation Results



Case 3: Monte Carlo Simulation Results



The Results of the approaches for all three cases is summarised in the following Tables.

Table 1: Case 1 Critical Tasks Identified by the Two Methods

<b>CASE 1</b>															
	Iterations by Beta Values											FOSM	Monte Carlo Iterations		
	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5		200	1000	5000
												Relative Frequency	Relative Frequency		
Task A	1	1	1	1	1	1	1	1	1	1	1	100	100	100	100
Task B		1			1			1				27	51	48	47
Task C	1	1	1	1	1	1	1	1	1	1	1	100	56	58	59
Task D	1	1	1	1	1	1	1	1	1	1	1	100	56	58	59
Task E												0	0	0	0
Task F												0	0	0	0
Task G		1			1			1				27	51	48	47
Task H	1	1	1	1	1	1	1	1	1	1	1	100	56	58	59
Task I	1	1	1	1	1	1	1	1	1	1	1	100	56	58	59
Task J	1	1	1	1	1	1	1	1	1	1	1	100	56	58	59
Task K		1			1							18	51	48	47

The Two methods identify the same critical tasks as shown in the next figure. Here the Relative Frequency of the Task's appearance on the Critical Path as identified by the two different methods, FOSM and Monte Carlo, is mapped against.

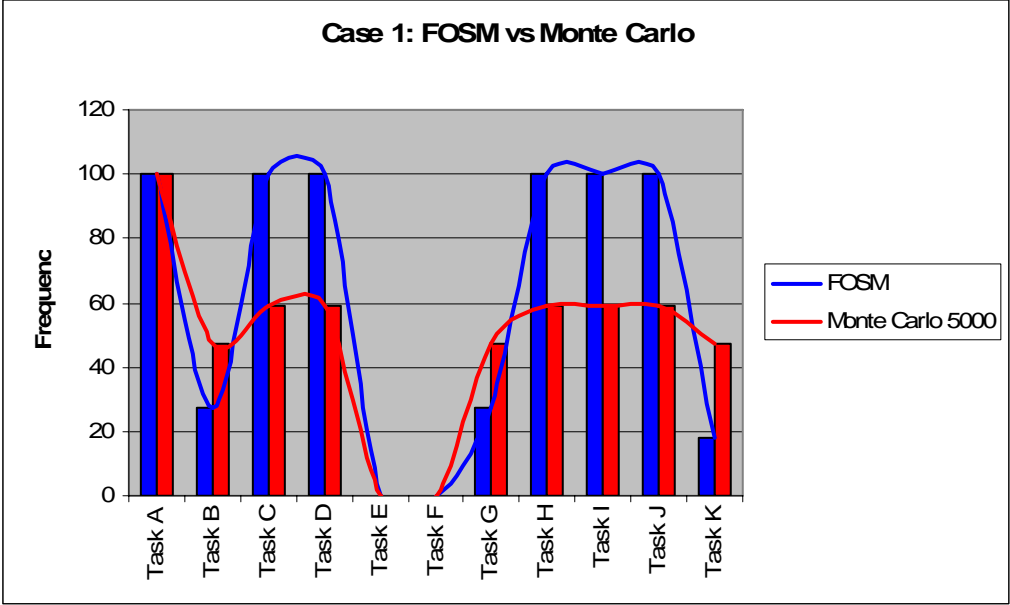


Table 2: Case 2 Critical Tasks Identified by the Two Methods

CASE 2															
	Iterations by Beta Values											FOSM	Monte Carlo Iterations		
	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	Relative Frequency	200	1000	5000
Task A	1	1	1	1	1	1	1	1	1	1	1	100	100	100	100
Task B	1											9	20	19	20
Task C	1	1	1	1	1	1	1	1	1	1	1	100	82	82	81
Task D	1	1	1	1	1	1	1	1	1	1	1	100	80	79	78
Task E												0	2	3	2
Task F												0	1	2	2
Task G	1											9	20	20	21
Task H	1	1	1	1	1	1	1	1	1	1	1	100	80	79	78
Task I	1	1	1	1	1	1	1	1	1	1	1	100	80	79	78
Task J	1	1	1	1	1	1	1	1	1	1	1	100	80	81	80
Task K	1											9	20	20	21

The Two methods again identify the same critical tasks as shown in the next figure

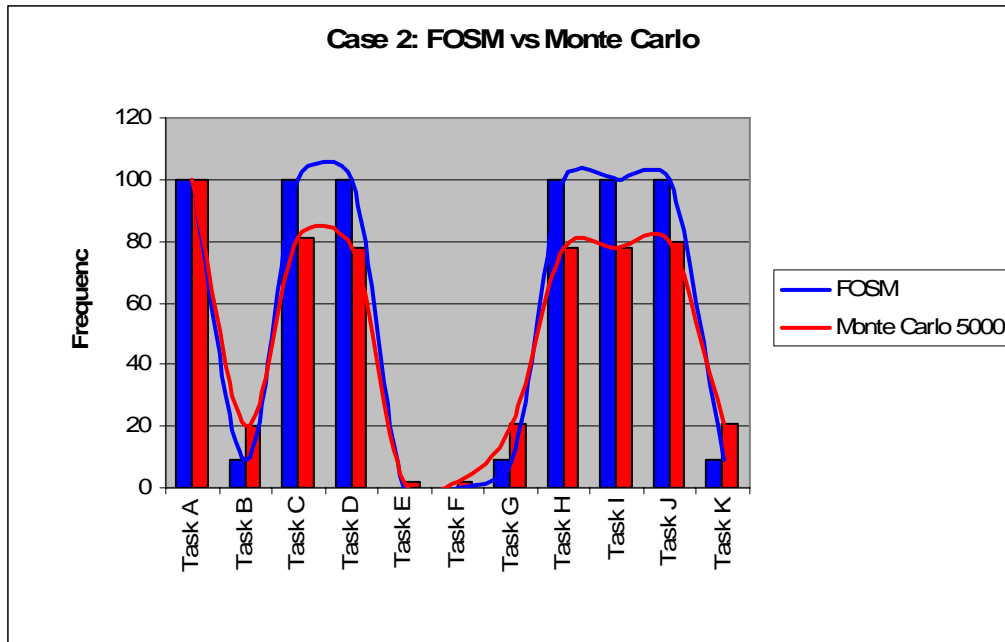


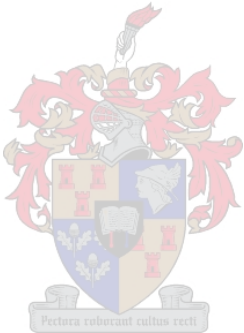
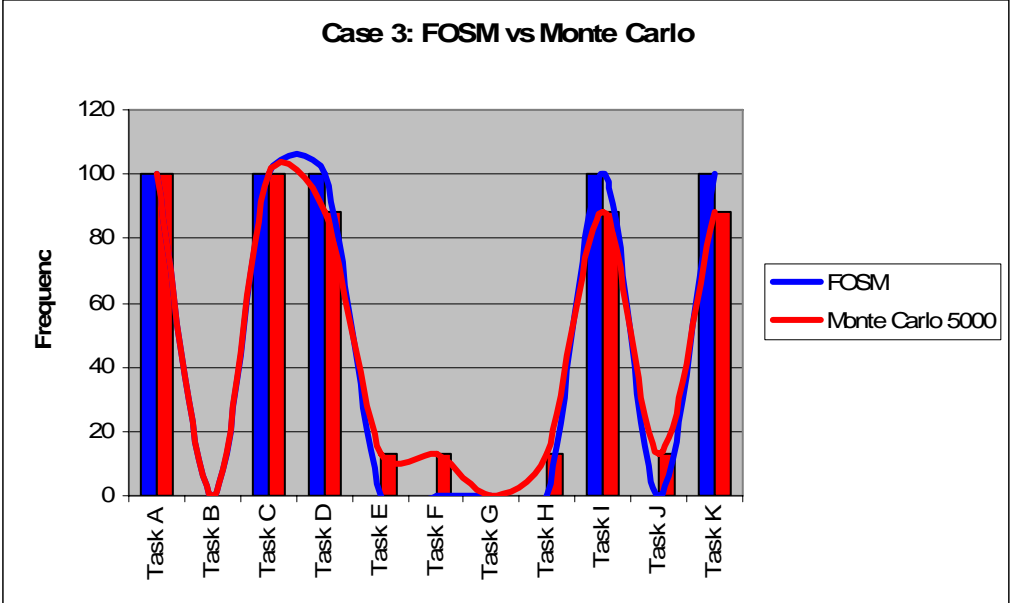
Table 3: Case 3 Critical Tasks Identified by the Two Methods

CASE 3																
	Iterations by Beta Values											FOSM		Monte Carlo Iterations		
	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	Relative Frequency	200	1000	5000	
												Relative Frequency	Relative Frequency			
Task A	1	1	1	1	1	1	1	1	1	1	1	100	100	100	100	
Task B												0	0	0	0	
Task C	1	1	1	1	1	1	1	1	1	1	1	100	100	100	100	
Task D	1	1	1	1	1	1	1	1	1	1	1	100	86	88	88	
Task E												0	14	13	13	
Task F												0	14	13	13	
Task G												0	0	0	0	
Task H												0	14	13	13	
Task I	1	1	1	1	1	1	1	1	1	1	1	100	86	88	88	
Task J												0	14	13	13	
Task K	1	1	1	1	1	1	1	1	1	1	1	100	86	88	88	

The Two methods again identify the same critical tasks as shown in the next figure.

However, FOSM failed to identify 4 potentially critical tasks. Tasks, E,F,H and J. Each of these tasks does have a low relative frequency as predicted.





### Example 3.2

In this example the effect on the project duration reliability was calculated if the critical tasks not identified by the FOSM methods of Chapter 3 were omitted from the reliability calculation.

Case 3 from Example 3.1 was utilised to calculate this as FOSM failed to identify 4 potentially critical tasks in this case.

The effect on the project duration reliability was calculated by using Monte Carlo simulations. The software used to perform the Simulations was again Asta Powerproject, Version 8 with the Risk Analysis Add-on installed.

The Method was to first calculate the reliability of the project duration using the duration variables of those tasks identified as potentially critical by the Monte Carlo Method. All tasks not identified as critical by the Monte Carlo method were assumed to be discrete values and their duration magnitudes were set to their Anticipated duration values, as estimated by the PM professional.

The Monte Carlo Simulation was run, iterating 1000 times, and the reliability of the different project durations was determined. The result of this simulation is shown in Figure A below by the blue curve.

Secondly the Monte Carlo simulation was run again, this time with only the durations of the tasks identified as critical by the FOSM identification method being inputted as randomly distributed variables. Once again all tasks not identified as critical were assumed to be discrete and their duration magnitudes were set to their Anticipated duration values, as estimated by the PM professional.

The results of this, FOSM critical tasks, simulation were plotted against the first, Monte Carlo critical tasks, simulation results and are shown in Figure A by the yellow curve.

The curve shows that the assumption that the tasks not identified as critical by the FOSM method will not have a major impact on the reliability of the duration to be correct.

To get an idea of the magnitude of the effect on the project's duration reliability of omitting the unidentified critical tasks, a few spot dates were extracted from the simulation data of both simulations and compared. This is shown in Table 4 below.

The difference was calculated for each spot date. The average of the absolute values of these differences calculates to 1%.

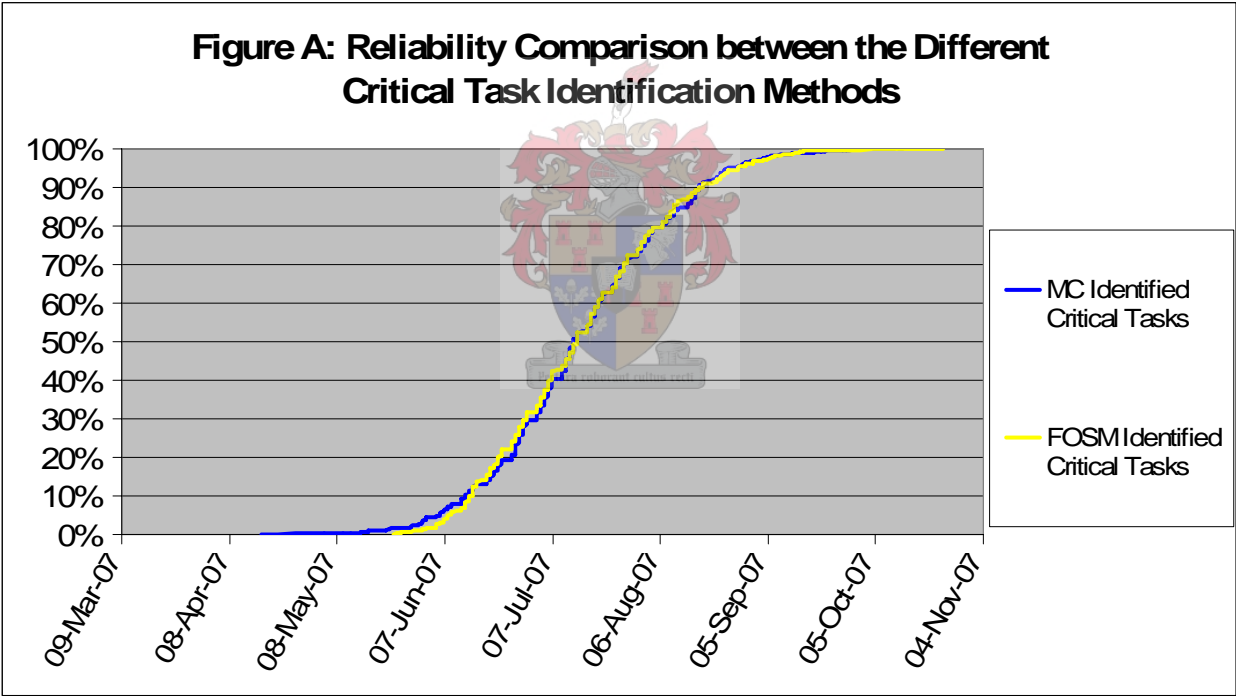
From Table 3 above it can be seen that FOSM failed to identify 4 of the tasks identified by Monte Carlo simulation to be critical. Each of these critical tasks does however have a low relative frequency.

The Monte Carlo simulation identified a total of 9 critical tasks. Thus the FOSM method failed to identify 44% of the potentially critical tasks in the example project of Case 3.

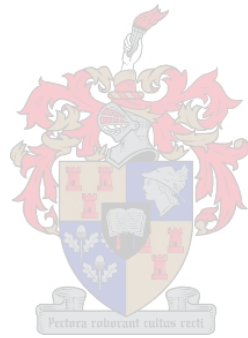
The fact that the average difference in reliability is only 1%, given 44% of the Critical tasks were not included in the FOSM method further illustrates the fact those tasks not identified as critical by the FOSM method will have little effect on the overall project's duration reliability and therefore the use of the FOSM method described in the main body is suitable.

**Table 4:** Difference in Reliability on Different Dates

Date	Reliability in % Calculated with MC Identified Critical Tasks	Reliability in % Calculated with FOSM Identified Critical Tasks	Difference in %
13-Jun-07	10.9	9.3	-1.6
21-Jun-07	17.2	19.1	1.9
29-Jun-07	29.0	30.6	1.6
10-Jul-07	44.0	44.7	0.7
20-Jul-07	61.7	61.8	0.1
31-Jul-07	74.1	74.9	0.8
16-Aug-07	90.0	89.6	-0.4
		Averages	1%



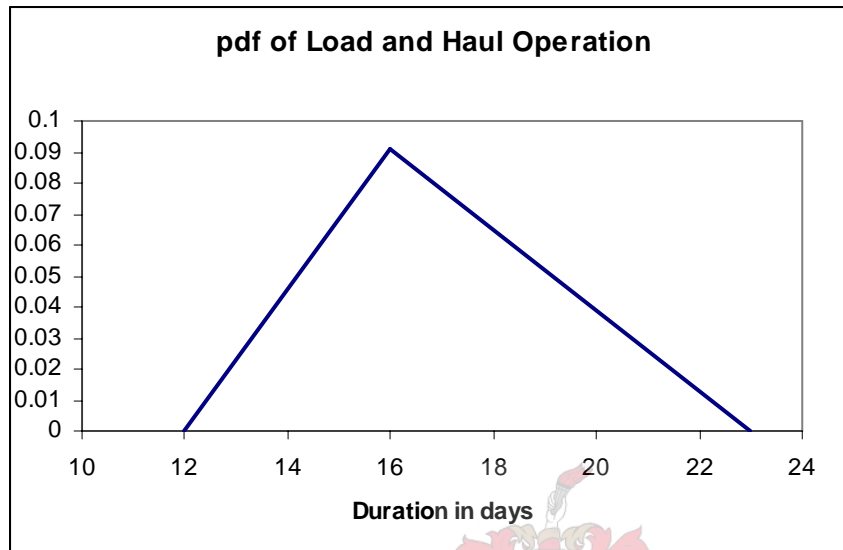
## Appendix B: The Triangular Distribution



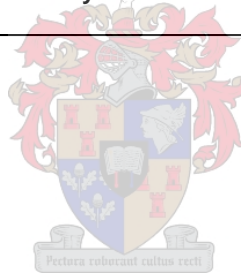
# Appendix B

## Chapter 3.2.1.1 The Triangular Distribution

Calculation of  $\mu, \sigma, \gamma$



a=12  
b=23  
c=16



**Mean  $\mu$  :**

$$\mu = \frac{a + b + c}{3} \quad (1)$$

*Therefore*

$$\mu = \frac{12 + 23 + 16}{3}$$

$$\mu = 17$$

**Standard deviation  $\sigma$  :**

$$\sigma = \sqrt{\frac{a^2 + b^2 + c^2 - a \cdot b - a \cdot c - b \cdot c}{18}} \quad (2)$$

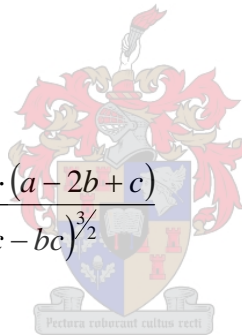
Therefore

$$\sigma = \sqrt{\frac{12^2 + 23^2 + 16^2 - 12 \cdot 23 - 12 \cdot 16 - 23 \cdot 16}{18}}$$

$$\sigma = 2.27303$$

**Skewness  $\gamma$  :**

$$\gamma = \frac{\sqrt{2} \cdot (a + b - 2c) \cdot (2a - b - c) \cdot (a - 2b + c)}{5 \cdot (a^2 + b^2 + c^2 - ab - ac - bc)^{3/2}} \quad (3)$$



Therefore

$$\gamma = \frac{\sqrt{2} \cdot (12 + 23 - 2 \cdot 16) \cdot (2 \cdot 12 - 23 - 16) \cdot (12 - 2 \cdot 23 + 16)}{5 \cdot (12^2 + 23^2 + 16^2 - 12 \cdot 23 - 12 \cdot 16 - 23 \cdot 16)^{3/2}}$$

$$\gamma = 0.25545$$

**Coefficient of Variation  $\Omega$** 

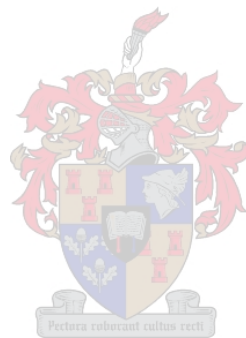
$$\Omega = \frac{\sigma}{\mu} \quad (4)$$

$$\Omega = \frac{2.27303}{17}$$

$$\Omega = 0.133708$$

Equations (1),(2) & (3) Sourced from : [http://en.wikipedia.org/wiki/Triangular\\_distribution](http://en.wikipedia.org/wiki/Triangular_distribution)

Equations (4) Sourced from <http://mathworld.wolfram.com/VariationCoefficient.html>





## Appendix C: The Critical Path Method

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## 1. The Critical Path Calculation

The Critical Path Calculation is a simple arithmetic calculation based on two *passes* namely a *forward pass* and a *backward pass*. The following user input is required (this will be detailed in the example);

- Start Date or Point in time
- Tasks
- Duration of Tasks
- Relevant links between Tasks.(This intrinsically details the sequence of tasks)

All of this data is to be inputted by the user who will need to have some knowledge of the process being scheduled to ensure the correct links are established.

The method is best described with an example.

Assuming the following data is supplied by the user;

The user defined start point is time unit 0

Table 1 Example Project

TASKS	SUCCEEDED BY	TYPE OF LINK	DURATION
Task 1	Task 2	Finish to Start	7
Task 2	Task 3 Task 5 Task 7	Finish to Start Finish to Start Start to Start	5
Task 3	Task 4	Finish to Start	10
Task 4	Task 6	Finish to Start	3
Task 5	Task 6	Finish to Start	12
Task 6	Task 10	Finish to Start	5
Task 7	Task 8	Finish to Start	13
Task 8	Task 9	Finish to Start	6
Task 9	Task 10	Finish to Start	9
Task 10	none	Finish to Start	5

The semantics;

'Finish to Start' implies that the end of the preceding task, its *finish*, is linked to the start of the succeeding task.

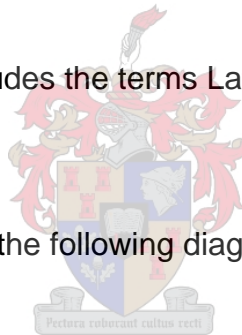
'Start to Start' implies that the start of the preceding task is linked to the start of the succeeding task.

**The method will then use the following processes,**

The *forward pass*, which includes the terms Early Start (ES) and Early Finish (EF)

The *backward pass*, which includes the terms Late Start (LS) and Late Finish (LF)

The process will be shown use the following diagram to describe each task.



TASK	
DURATION	
ES	EF
LS	LF

The algorithm then calculates the Early Start and Early Finish in the forward pass and the Late Start and Late Finish in the backward pass. These results are then compared and the Tasks on the critical path are identified.

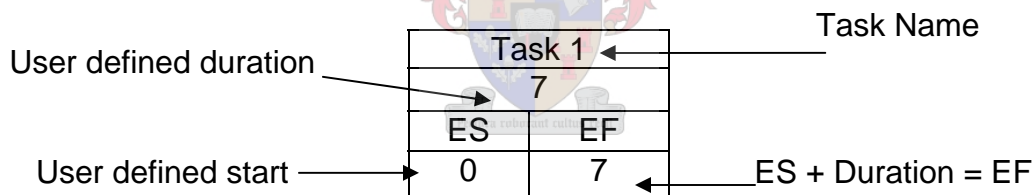
## Step1

**The Forward Pass**

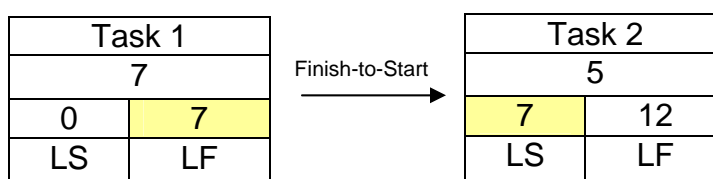
In the forward pass the user defined start is used as the first task's ES. The task's Duration is then added to the ES and the EF is calculated. The links between the first task and its succeeding tasks are then identified. Using these links, the ES of the succeeding tasks is calculated. This calculated ES from the succeeding task is based on the semantics of the link itself, for instance for a 'start to start' link the succeeding task's ES is equal to its preceding task's ES and for a 'finish to start' link the succeeding task's ES is equal to the preceding task's EF.

Where 2 or more tasks precede a task it's ES is then the earliest possible start governed by the preceding tasks. In other words its earliest possible start will be the latest EF of its preceding tasks.

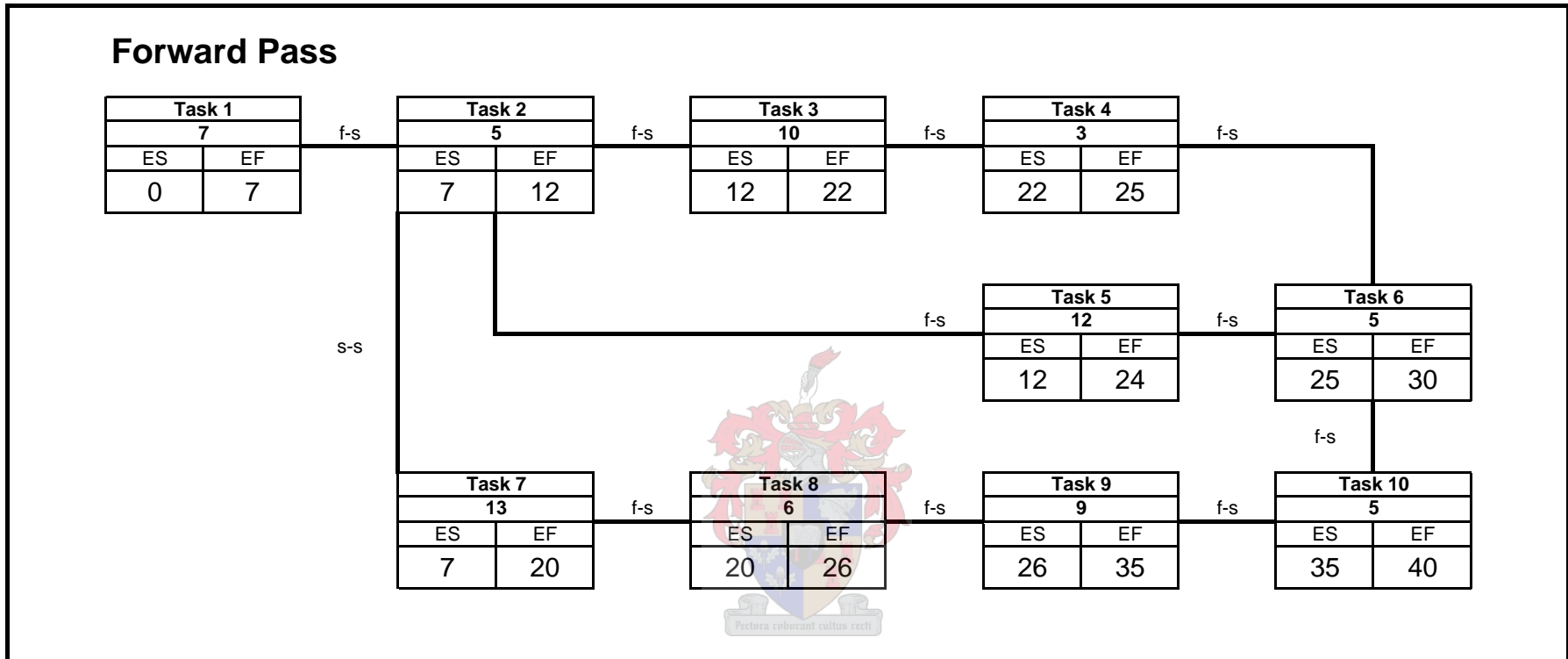
Therefore for Task 1 the diagram will look like this



Now the links are identified. There is a *finish-to-start* link to Task 2. This then results in the calculation of the following variables for Tasks 2 as follows.



This process is continued until all the ES and EF are calculated for all the tasks in the process. This is shown graphically on the next page.



## Step 2

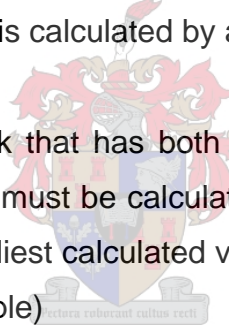
### The Backward Pass

The next step is to calculate the Backward Pass. This is done by beginning at the last scheduled task. The EF of this task is then used as the beginning point for the backward pass. This is then called the Late Finish. The process is then reversed, worked backwards. The duration of the last task is subtracted from the LF to give the Late Start (LS). The links are then identified and the LS of the last task becomes the LF of its preceding tasks. This process is continued until all tasks LS and LF have been calculated.

Where a task is succeeded by two or more tasks then its LF is calculated from the latest LS of its successors.

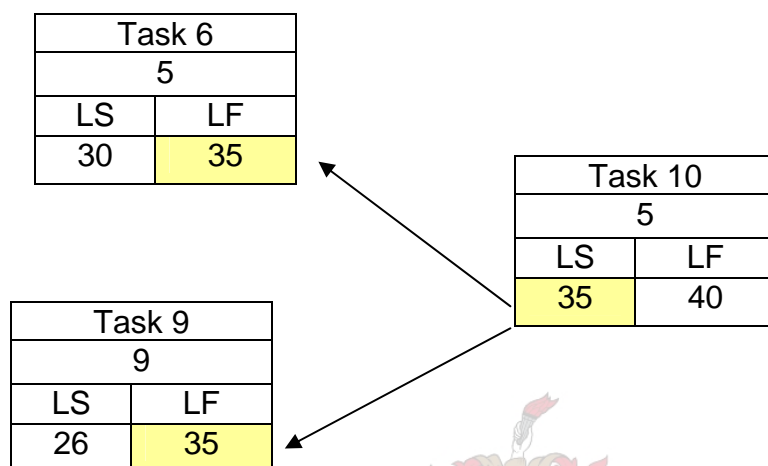
Where a Start-to-Start Link is present the succeeding task's LS becomes the preceding task's LS and the LF is calculated by adding the task duration.

When there is a preceding task that has both Start-to-Start and Finish-to-Start links then its LS and LF values must be calculated for both the Start-to-Start link and Finish-to-Start link. The earliest calculated value for the LS is then chosen as the LS (see Task 2 in the example)



So for the example;

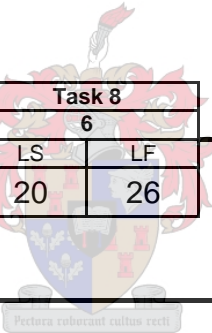
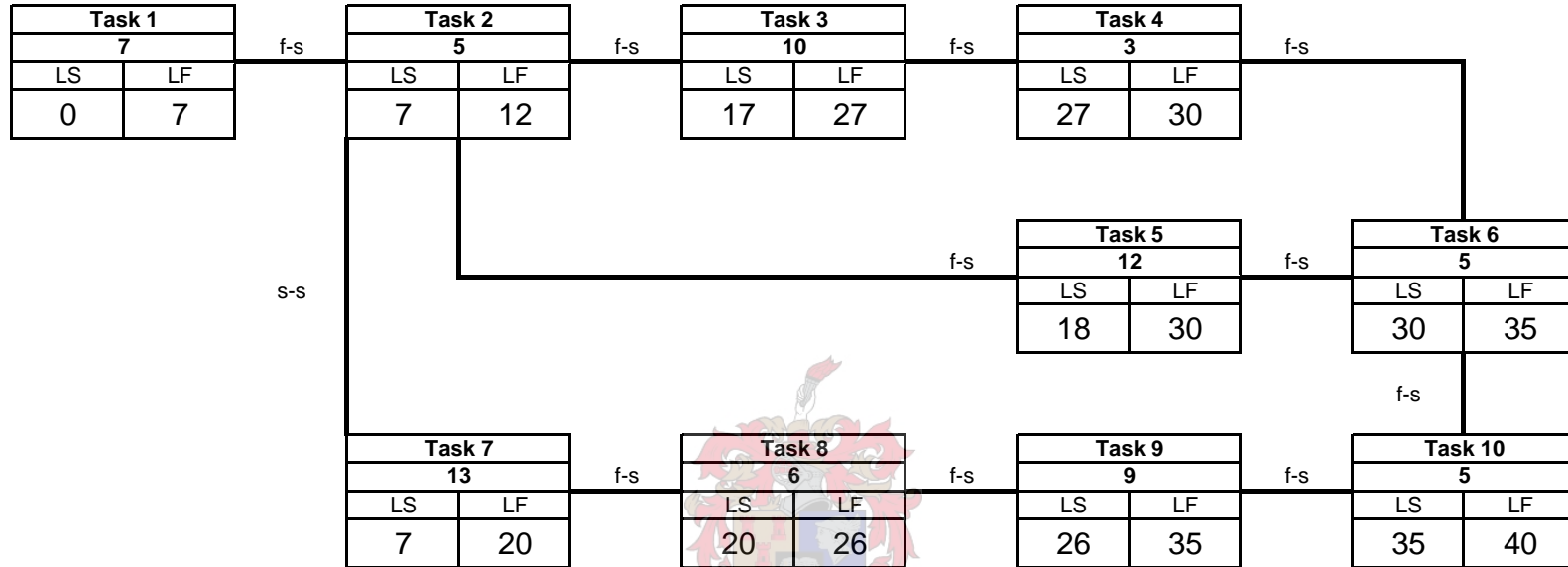
The LS and LF of Task 6 and Task 9 are;



The results of the completed backwards pass are shown on the next page.



## Backward Pass



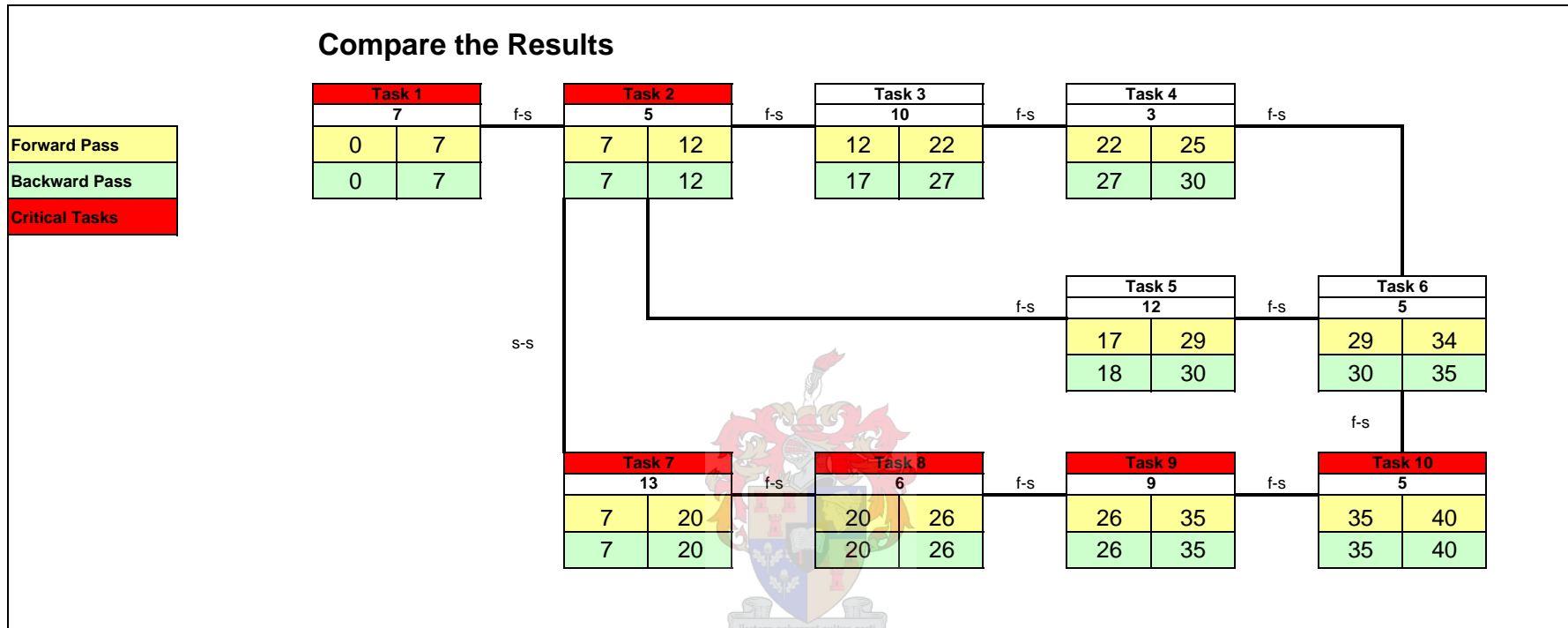


### Step 3

#### **Compare the Results**

The final step to the process is to compare the results of the forward and backward pass. All tasks that have equal ES and LS and equal EF and LF are then critical tasks. This is logical as the earliest possible start for the task is equal to the latest possible start for a task and therefore there is no allowance for adjustments to its position in the program schedule with regards to time. This also shows the planner that should such a task's duration increase or decrease then the overall duration of the project will be affected. The project path that follows these critical tasks is known as the Critical Path. The results of this investigation are shown on the following page.





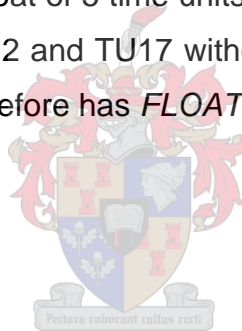
Where a task's ES and LS results are not the same then the task is said to have *FLOAT*.

This implies that the tasks position in time can be moved to the extent of the float.

For example

Task 3	
10	
12	22
17	27

Task 3 has an ES of 12 and a EF of 17. This then logically says Task 3's earliest possible start is on time unit 12 (TU12) and its latest possible start is on time unit 17 (TU17). Therefore it has a float of 5 time units. This means that Task 3 can be started anywhere between TU12 and TU17 without affecting the overall duration of the program schedule. It therefore has *FLOAT* and is not on the critical path.



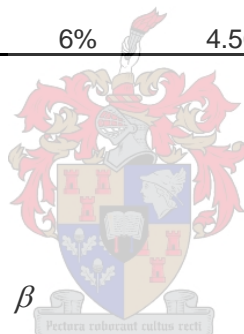
## Appendix D: Commercial Feasibility Calculations



## Appendix D: Commercial Feasibility

### Input Data

Input Data						
Name	Estimations			Statistics (Calculated as described in Appendix B)		
	Optimistic	Pessimistic	Anticipated	Mean	Std Deviation	COV
<b>Muchawaste Landfill Project</b>						
Basic Cost Estimate $I_0$	R 5,500,000	R 10,000,000	R 7,000,000	R 7,500,000	R 935,414	0.12472191
Revenue Stream						
Tariff	R 8.08	R 10.93	R 9.50			
Volume	333,000	290,000	310,000			
Revenue $R_i$	R 2,688,975	R 3,168,250	R 2,945,000	R 2,934,075	R 97,907	0.03336923
Operating Cost $C_i$	R 900,000	R 1,600,000	R 1,200,000	R 1,233,333	R 143,372	0.11624764
$n = 10$ years				10		
Risk Premium = 8%				8%		
Inflation	3%	6%	4.50%	4.50%	1.43%	0.31746032



### Section 1: NPV Calculation of $\beta$

Below is a copy of the programming code utilized to calculate the Reliability value,  $\beta$  for the highly non-linear NPV Equation 6.5a for Example 7.1.

The coding of all the methods called in the Main Program is detailed below the Main program in this Appendix

All text in green is comment describing the calculation process following it.

The Matlab Files are provided on the attached CD in the folder NPV calculation.

## i).Main Program

This Main Program calculates the Beta value for the NPV using an iterative process, the Modified Matrix Procedure of Rackwitz-Fiessler

Using assumed values [R,C,W,I] of Equation 6.5a create a vector Despt for the Design point(the values are assumed as the mean values for all of the variables for the first iteration) The design point refers to the values of the variables on the failure surface.

Create vectors for the Mean, Mew, and coefficient of variation, cov, for all variables.

```
Despt = [R, C , Inflation , Io]
```

```
Despt = [2934075, 1233333.33 , 0.045,7500000]
Mew = [2934075, 1233333.33 , 0.045,7500000]
cov = [0.033369, 0.116247, 0.31746,0.1247]
```

```
n = length(Despt)
```

```
BetaEstimate = 100
```

Calculate the standard deviations of all of the variables.

```
sigR = Mew(1) * cov(1)
sigC = Mew(2) * cov(2)
sigW = Mew(3) * cov(3)
sigI = Mew(4) * cov(4)
```



Create a Standard deviation vector, Sigma

```
Sigma = [sigR, sigC, sigW, sigI]
```

Using n-1 of the variables in the Despt vector and calculate the nth variable using the CalcRnominal method also attached with this Appendix.

The calculated variable is called NewI. NewI is on the failure surface

```
newI = CalcRnominal(Despt)
```

%Substitute the NewI into the Design point variable

```
Despt1 = [Despt(1), Despt(2), Despt(3), newI]
```

```
% create a new design point vector
```

```

Despt = Despt1

%Reduce the variables to standard normalised space

for i =1:n

DEE = Despt(i)
MUU = Mew(i)
SSIG = Sigma(i)
RedVAR(i) = reduce(DEE,MUU,SSIG)
ZD(i) = RedVAR(i)
end

ZED = ZD'

% Calculate the Partial Derivatives for each variable with
respect to the
% LSE at the design point.

for i=1:n
d=Despt(i)
h=0.0001*Despt(i)
varMinus=Despt
varMinus(i)=varMinus(i)-h
varPlus=Despt
varPlus(i)=varPlus(i)+h
deltaY=LSE(varPlus)-LSE(varMinus)
deltaX=2*h
D(i)=deltaY/deltaX
end

%Create a vector of the partial derivatives

for i=1:n

Gee(i)= D(i)*Sigma(i)
Gg(i)= -1*Gee(i)

end

%Transpose this vector

G=Gg'

Calculate Beta at the assumed design point

Beta = (G'*ZED)/(sqrt(G'*G))

Calculate alpha at the assumed design point

Alpha = G / (sqrt(G'*G))
count = 1

```

Iterate until beta converges

```
while abs(1-BetaEstimate/Beta)>0.00001
    BetaEstimate = Beta
```

```
for i=count
    BTA(i)=Beta
end
```

Calculate the new design point values for n-1 of the variables

```
for i=1:(n-1)
    newZED(i) = Alpha(i)*Beta
    DEE = Despt(i)
    MUU = Mew(i)
    SSIG = Sigma(i)
    newDespt(i) = MUU + newZED(i)*SSIG
```

```
end
```

Substitutue these new design point values into the Despt vector

```
for i=1:(n-1)
```

```
Despt(i) = newDespt(i)
end
```

Recalculate a NewI (now as NewM) using the CalcRnominal method and repeat the process described above to calculate beta.

```
NeWM = CalcRnominal(Despt)
```

```
Despt2 = [Despt(1),Despt(2),Despt(3),NeWM]
Despt = Despt2
```

Reduce the Design Points

```
for i =1:n
```

```
DEE = Despt(i)
MUU = Mew(i)
SSIG = Sigma(i)
RedVAR(i) = reduce(DEE,MUU,SSIG)
ZD(i) = RedVAR(i)
end
```

```
ZED = ZD'
```

```
for i=1:n
    d=Despt(i)
    h=0.0001*Despt(i)
```



```

varMinus=Despt
varMinus(i)=varMinus(i)-h
varPlus=Despt
varPlus(i)=varPlus(i)+h
deltaY=LSE(varPlus)-LSE(varMinus)
deltaX=2*h
D(i)=deltaY/deltaX

end

for i=1:n
Gee(i)= D(i)*Sigma(i)
Gg(i)= -1*Gee(i)
end

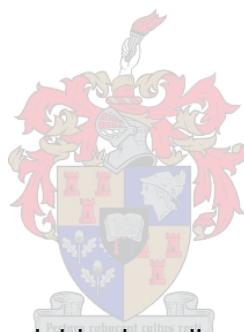
G=Gg'

Beta = (G'*ZED)/(sqrt(G'*G))
Alpha = G /(sqrt(G'*G))

count = count+1

end
BTA'
Beta

```



## ii). CalcRnominal Method

This method solves for the nth variable described in the main program. The limit state equation is set equal to zero and the nth variable is made the subject of the formula. RCalc then solve the nth variable thus ensuring that the value calculated for the nth variable is on the failure surface.

```

function RCalc = CalcRnominal(Despt)

R=Despt(1)
C=Despt(2)
W=Despt(3)
I=Despt(4)

RCalc = (R-C)*((1-(1+(W+0.08))^-10))/(W+0.08)

```

### iii). Reduce Method

This method simply reduces the variable to standardised normal space.

```
function z = reduce(DEE,MUU,SSIG)
```

```
z = (DEE-MUU)/SSIG
```

### iv). Limit State Equation

This is merely Equation 6.5a encoded and is used to calculate the partial derivatives described in the main program.

```
function g = LSE(Despt)
```

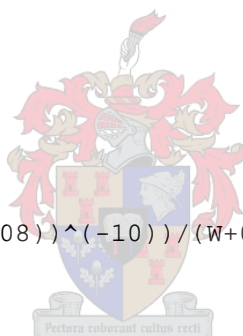
```
R=Despt(1)
```

```
C=Despt(2)
```

```
W=Despt(3)
```

```
I=Despt(4)
```

```
g = (R-C)*((1-(1+(W+0.08))^-10)/(W+0.08))-I
```



### v) Results

Running the main program results in three iterations before the value of  $\beta$  converges to a margin of difference of less than 0.0001.

This can be tested by running the Main program in Matlab as supplied.

The  $\beta$  values for the three iterations are;

1. 1.3239
2. 1.3439
3. 1.3439

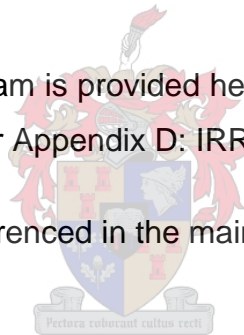
## Section 2: $\beta$ Calculated at a IRR value of 10%

In this section the reliability calculation for an IRR value of 10% is detailed. The process followed is simple in that the discrete IRR is systematically chosen at intervals, substituted into Equation 6.8a and the reliability is then calculated of the equation. This reliability value represents the probability that Equation 6.8a will return a positive value given the chosen IRR.

The IRR value is however by definition the percentage value at which Equation 6.8a returns a zero value and thus from the perspective of the IRR value the reliability calculated and its corresponding probability value represent the probability that the *actual IRR* realised will be greater than the value of the systematically chosen IRR value.

The coding of the Matlab program is provided here. The Matlab files are included on the CD provided in the folder Appendix D: IRR Calculations

The coding of the methods referenced in the main program are detailed below the main program.



The results of the calculations are then also detailed below.

### i) Main Program

This Main Program also calculates the Beta value for the chosen IRR value using an iterative process, the Modified Matrix Procedure of Rackwitz-Fiessler

Note that the mean of the value of IRR is equal to 0.1 or 10% as chosen for this illustrative calculation. This value is chosen as a discrete value and thus the Coefficient of variation of the IRR variable is practically set to zero at the value 0.000000000001.

Using assumed values [R,C,W] of Equation 6.8a create a vector Despt for the Design point.(the values are assumed as the mean values for all of the variables for the first iteration) The design point refers to the values of the variables on the failure surface.

Create vectors for the Mean, Mew, and coefficient of variation, cov, for all variables.

```
Despt = [R, C , IRR , Io]
```

```
Despt = [2934075, 1233333.33 , 0.1,7500000]
Mew = [2934075, 1233333.33 , 0.1,7500000]
cov = [0.033369, 0.116247, 0.000000000001,0.1247]
```

```
n = length(Despt)
```

```
BetaEstimate = 100
```

Calculate the standard deviations of all of the variables.

```
sigR = Mew(1) * cov(1)
sigC = Mew(2) * cov(2)
sigW = Mew(3) * cov(3)
sigI = Mew(4) * cov(4)
```

```
%Create a Standard deviation vector, Sigma
```

```
Sigma = [sigR, sigC, sigW, sigI]
```

```
%Using n-1 of the variables in the Despt vector and calculate the
nth
%variable using the CalcRnominal method also attached with this
appendix.
% The calculated variable is called newI. NewI is on the failure
% surface
```

```
newI = CalcRnominal(Despt)
```

```
%Substitute the NewI into the Design point variable
```

```
Despt1 = [Despt(1), Despt(2), Despt(3), newI]
```

```
% create a new design point vector
```

```
Despt = Despt1
```

```
%Reduce the variables to standard normalised space
```

```

for i =1:n

DEE = Despt(i)
MUU = Mew(i)
SSIG = Sigma(i)
RedVAR(i) = reduce(DEE,MUU,SSIG)
ZD(i) = RedVAR(i)
end

ZED = ZD'

% Calculate the Partial Derivatives for each variable with
respect to the
% LSE at the design point.

for i=1:n
d=Despt(i)
h=0.0001*Despt(i)
varMinus=Despt
varMinus(i)=varMinus(i)-h
varPlus=Despt
varPlus(i)=varPlus(i)+h
deltaY=LSE(varPlus)-LSE(varMinus)
deltaX=2*h
D(i)=deltaY/deltaX

end

%Create a vector of the partial derivatives

for i=1:n

Gee(i)= D(i)*Sigma(i)
Gg(i)= -1*Gee(i)

end

%Transpose this vector

G=Gg'

% Calculate Beta at the assumed design point

Beta = (G'*ZED)/(sqrt(G'*G))

%Calculate alpha at the assumed design point

Alpha = G /(sqrt(G'*G))
count = 1

%Iterate until beta converges

while abs(1-BetaEstimate/Beta)>0.0001

```

```

BetaEstimate = Beta

for i=count
    BTA(i)=Beta
end

%Calculate the new design point values for n-1 of the variables

for i=1:(n-1)
    newZED(i) = Alpha(i)*Beta
    DEE = Despt(i)
    MUU = Mew(i)
    SSIG = Sigma(i)
    newDespt(i) = MUU + newZED(i)*SSIG

end

%Substitue these new design point values into the Despt vector
for i=1:(n-1)

Despt(i) = newDespt(i)
end

%Recalculate a NewI (now as NewM) using the CalcRnominal method
and repeat
%the process described above to calculate beta.

NeWM = CalcRnominal(Despt)

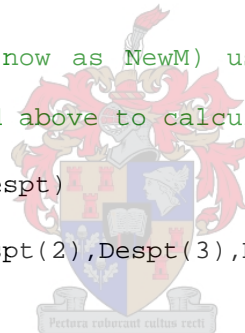
Despt2 = [Despt(1),Despt(2),Despt(3),NeWM]
Despt = Despt2

%Reduce the Design Points

for i =1:n

DEE = Despt(i)
MUU = Mew(i)
SSIG = Sigma(i)
RedVAR(i) = reduce(DEE,MUU,SSIG)
ZD(i) = RedVAR(i)
end

ZED = ZD'
```



```

for i=1:n
    d=Despt(i)
    h=0.0001*Despt(i)
    varMinus=Despt
    varMinus(i)=varMinus(i)-h
    varPlus=Despt
    varPlus(i)=varPlus(i)+h
    deltaY=LSE(varPlus)-LSE(varMinus)
    deltaX=2*h
    D(i)=deltaY/deltaX

end

for i=1:n
    Gee(i)= D(i)*Sigma(i)
    Gg(i)= -1*Gee(i)
end

G=Gg'

Beta = (G'*ZED)/(sqrt(G'*G))
Alpha = G / (sqrt(G'*G))

count = count+1

end
BTA'
Beta

```



## ii). CalcRnominal Method

This method solves for the nth variable described in the main program. The limit state equation is set equal to zero and the nth variable is made the subject of the formula. RCalc then solve the nth variable thus ensuring that the value calculated for the nth variable is on the failure surface.

```

function RCalc = CalcRnominal(Despt)

R=Despt(1)
C=Despt(2)
W=Despt(3)
I=Despt(4)

RCalc = (R-C)*((1-(1+W)^(-10))/W)

```

### iii). Reduce Method

This method simply reduces the variable to standardised normal space.

```
function z = reduce(DEE,MUU,SSIG)
```

```
z = (DEE-MUU)/SSIG
```

### iv). Limit State Equation

```
function g = LSE(Despt)
```

```
R=Despt(1)
```

```
C=Despt(2)
```

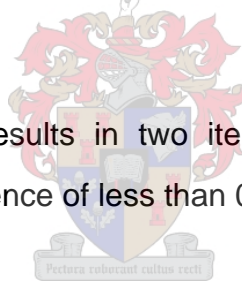
```
W=Despt(3)
```

```
I=Despt(4)
```

```
g = (R-C)*((1-(1+(W))(-10)))/W)-I
```

### v) Results

Running the main program results in two iterations before the value of  $\beta$  converges to a margin of difference of less than 0.0001.



This can be tested by running the Main program in Matlab as supplied.

The  $\beta$  values for the two iterations are;

1. 2.0796

2. 2.0796

Thus at a Discrete IRR value of 10%  $\beta = 2.0796$  and the probability that the actual realised IRR value will be greater than 10% is 0.9812 or 98%.

The calculations can be checked by running the Main program file in Matlab.

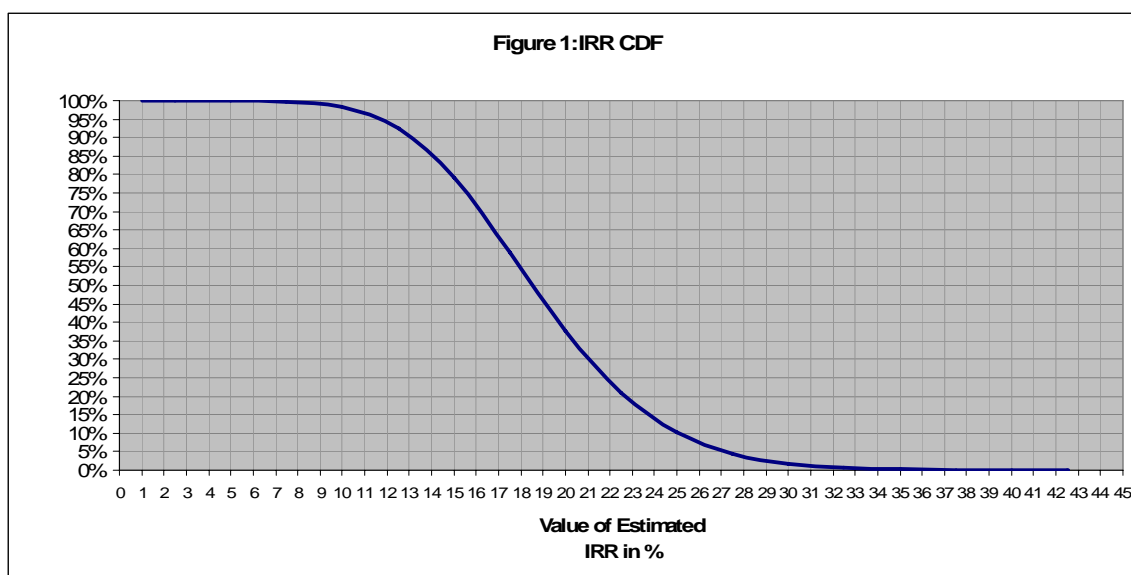


This process was repeated for different IRR values as the results are shown in the below Table 2 and is then plotted in Figure 1.

Table 2: Reliability at Different IRR values.

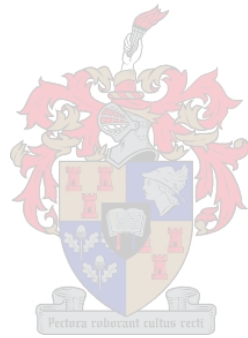
Estimated IRR	$\beta$	P(Actual IRR > Estimated IRR)
1	4.5505	1.0000
2.5	4.139	1.0000
5	3.4459	0.9997
7.5	2.7554	0.9971
<b>10</b>	<b>2.0796</b>	<b>0.9812</b>
12.5	1.4287	0.9235
15	0.8102	0.7911
17.5	0.2291	0.5906
20	-0.3119	0.3776
22.5	-0.8122	0.2083
25	-1.2722	0.1017
27.5	-1.6938	0.0452
30	-2.0792	0.0188
32.5	-2.4311	0.0075
35	-2.7522	0.0030
37.5	-3.0452	0.0012
40	-3.3129	0.0005
42.5	-3.5576	0.0002
42.5	-3.7817	0.0001

Figure 1: IRR CDF



## Appendix E: Duration Design Estimate

---



## Appendix E: Estimated Task Durations

### Section 1

Table 1: Estimated Task Durations

TASK Name	Best Case	Anticipated	Worst Case
<b>Muchawaste Landfill Project</b>			
<b>SECURE BRIDGING FINANCE</b>			
Prepare Briefing Document	5	10	20
<b>Prepare Basic Business Case</b>			
Basic Engineering Solution	12	15	25
Basic Cost Estimate	10	15	16
Financial Model [Bridging Draft]	3	5	10
Assess Basic Feasibility	3	5	6
Positive Feasibility Report	1	1	1
Apply for a Conditional Operating Licence	60	80	120
Receive Conditional Operating Licence	1	1	1
Draft Offtake Agreement with local Council	15	20	25
Negotiate Offtake Agreement with local council	45	60	90
Sign Offtake Agreement	1	1	1
Solicit expressions of Interest and Terms of Bridging Finance	18	20	23
Acquire Bridging Funding	9	10	20
Company Decides to Proceed with the Project	1	1	5
Sign Bridging Finance Document	1	1	1
<b>Obtain Unconditional Operating Licence</b>			
<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>			
Preliminary Geotechnical Investigation	15	20	30
Geotechnical Permeability Study	18	20	21
Basic Volumetric Study	16	20	23
Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	30	40	55
Revised Cost Estimate	5	10	13
<b>FUNDING</b>			
Prepare Finance Briefing document	7	10	15
Submit Briefing document to potential funders	1	1	1
Acquire and evaluate funding prosals	18	20	25
Finalise Funding Requirements	1	1	2
Appointment, Equity & Debt Providers	40	60	70
Sign Funding Agreement	1	1	1
<b>FINANCIAL CLOSURE</b>			
<b>Environmental approval Process South Africa</b>			
Produce basic Project Information Memorandum	5	10	15
Prepare bidding document to solicit environmental consultants	10	15	17
Environmental consultants respond to Bid	22	30	35
Consortium evaluates bids and appoints consultant	7	10	20
Scoping Process	45	60	80
Applicant conducts EIA			
Specialist Studies	200	270	400
Appeal Period and Process	0	90	200
<b>Authority issues a positive ROD</b>			
<b>EPC CONTRACT</b>			
Shortlist potential EPC contractors	35	40	45
Submit Level 2 Basic Design	1	1	1
Contractors Prepare and Submit Bid	50	60	60
Evaluate bids	35	45	60
APPOINT EPC subject to Financial Close	35	40	50
<b>Detailed Design</b>			
Detailed Geotechnical Investigation	26	40	49
Detailed Volumetric Study	30	2	50
Geotechnical Design	50	60	73
Design Synthetic Lining System	17	20	25
Produce Detailed Design	18	1	30
<b>Landfill Construction Phase</b>			
<b>Construction Programme</b>			
Establishment	3	5	6
Remove topsoil to Stockpile	6	10	15
Excavate to Stockpile Bay 1 to Formation Level	26	30	40
Survey Foundation	1	1	1
Prepare Bay 1 Formation	2	3	3
Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	35	50	76
Survey Foundation	1	1	1
Survey Top of Clay Bay 1	1	1	1
Prepare Clay for Liner on Bay 1	3	4	5
Prepare Bay 2 Formation	2	3	3
Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	35	40	60
Lay and Weld Geomembrane Liner Bay 1	12	15	22
Lay and Weld Geotextile Protection Bay 1	12	15	22
Lay Drainage Blanket Bay 1	20	25	30
Survey Top of Clay Bay 1	1	1	1
Prepare Clay for Liner on Bay 1	3	4	5
Lay and Weld Geomembrane Liner Bay 2	12	15	22
Lay and Weld Geotextile Protection Bay 2	12	15	22
Lay Drainage Blanket Bay 2	20	25	30
Survey Top of Drainage Blanket	1	1	2
Place Waste Protection Layer	6	10	13
Commission Landfill	5	5	10
Receive first Public Waste	0	0	0

# Appendix E: Anticipated Duration Gantt Chart.

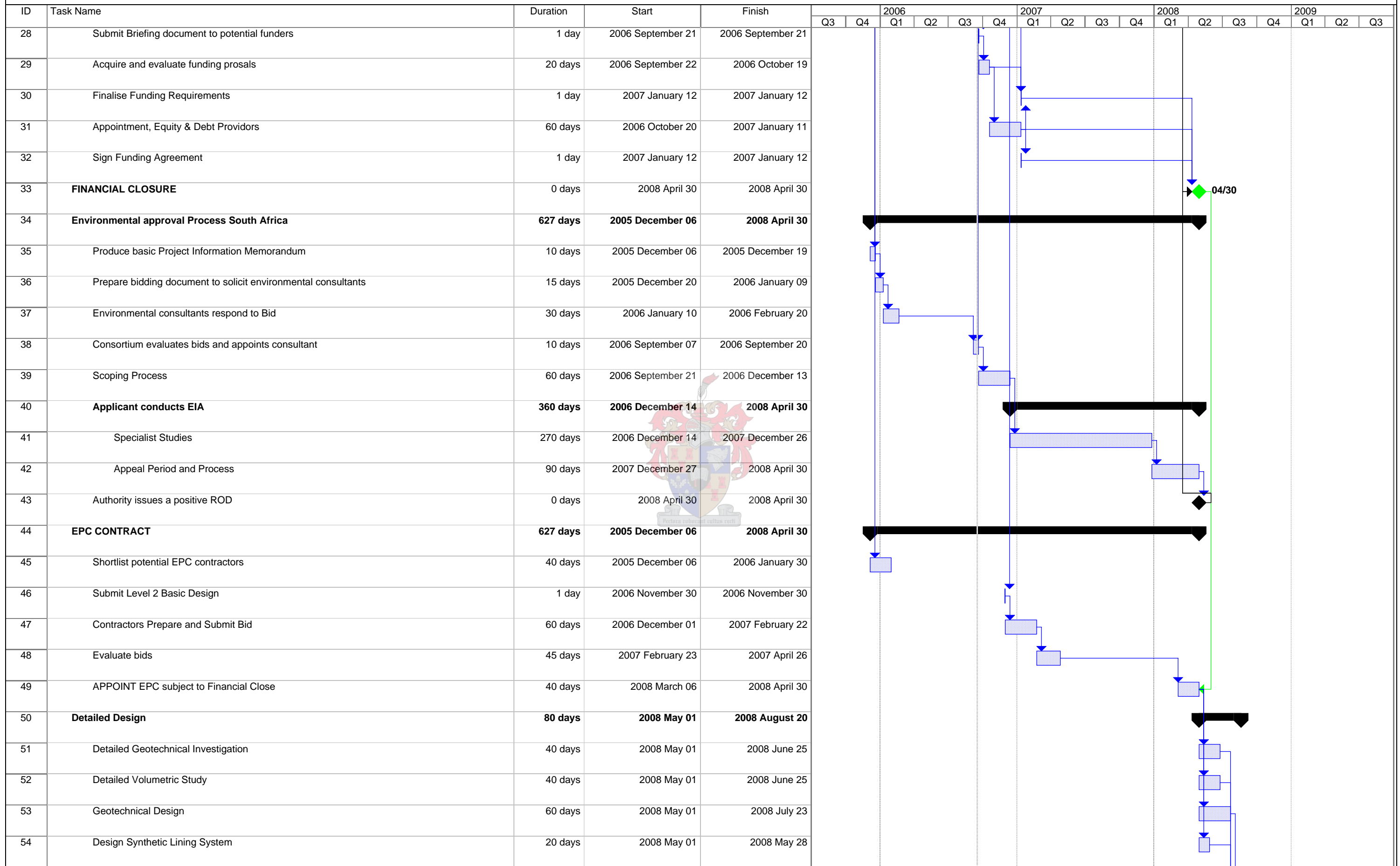
ID	Task Name	Duration	Start	Finish	2006				2007				2008				2009						
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3		
1	<b>Muchawaste Landfill Project</b>	<b>943 days</b>	<b>2005 November 01</b>	<b>2009 June 11</b>																			
2	<b>SECURE BRIDGING FINANCE</b>	<b>652 days</b>	<b>2005 November 01</b>	<b>2008 April 30</b>																			
3	Prepare Briefing Document	10 days	2005 November 01	2005 November 14																			
4	<b>Prepare Basic Business Case</b>	<b>40 days</b>	<b>2005 November 15</b>	<b>2006 January 09</b>																			
5	Basic Engineering Solution	15 days	2005 November 15	2005 December 05																			
6	Basic Cost Estimate	15 days	2005 December 06	2005 December 26																			
7	Financial Model [Bridging Draft]	5 days	2005 December 27	2006 January 02																			
8	Assess Basic Feasibility	5 days	2006 January 03	2006 January 09																			
9	Positive Feasibility Report	0 days	2006 January 09	2006 January 09																			
10	Apply for a Conditional Operating Licence	80 days	2006 January 10	2006 May 01																			
11	Receive Conditional Operating Licence	0 days	2006 May 01	2006 May 01																			
12	Draft Offtake Agreement with local Council	20 days	2006 May 02	2006 May 29																			
13	Negotiate Offtake Agreement with local council	60 days	2006 May 30	2006 August 21																			
14	Sign Offtake Agreement	0 days	2006 August 21	2006 August 21																			
15	Solicit expressions of Interest and Terms of Bridging Finance	20 days	2006 July 25	2006 August 21																			
16	Acquire Bridging Funding	10 days	2006 August 22	2006 September 04																			
17	Company Decides to Proceed with the Project	1 day	2006 September 05	2006 September 05																			
18	Sign Bridging Finance Document	1 day	2006 September 06	2006 September 06																			
19	Obtain Unconditional Operating Licence	0 days	2008 April 30	2008 April 30																			
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>	<b>70 days</b>	<b>2006 September 07</b>	<b>2006 December 13</b>																			
21	Preliminary Geotechnical Investigation	20 days	2006 September 07	2006 October 04																			
22	Geotechnical Permeability Study	20 days	2006 September 07	2006 October 04																			
23	Basic Volumetric Study	20 days	2006 September 07	2006 October 04																			
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	40 days	2006 October 05	2006 November 29																			
25	Revised Cost Estimate	10 days	2006 November 30	2006 December 13																			
26	<b>FUNDING</b>	<b>92 days</b>	<b>2006 September 07</b>	<b>2007 January 12</b>																			
27	Prepare Finance Briefing document	10 days	2006 September 07	2006 September 20																			

Appendix E: AD Gantt Chart

Appendix E: Duration Design Estimate

Task: Progress: Summary: External Tasks: Deadline: Split: Milestone: Project Summary: External Milestone:

## Appendix E: Anticipated Duration Gantt Chart.



<p>Appendix E: AD Gantt Chart</p> <p>Appendix E: Duration Design Estimate</p>	<p>Task</p> <p>Split</p>	<p>Progress</p> <p>Milestone</p>	<p>Summary</p> <p>Project Summary</p>	<p>External Tasks</p> <p>External Milestone</p>	<p>Deadline</p>	
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## Appendix E: Anticipated Duration Gantt Chart.

ID	Task Name	Duration	Start	Finish	2006				2007				2008				2009							
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4						
55	Produce Detailed Design	20 days	2008 July 24	2008 August 20																				
56	<b>Landfill Construction Phase</b>	<b>211 days</b>	<b>2008 August 21</b>	<b>2009 June 11</b>																				
57	<b>Construction Programme</b>	<b>211 days</b>	<b>2008 August 21</b>	<b>2009 June 11</b>																				
58	Establishment	5 days	2008 August 21	2008 August 27																				
59	Remove topsoil to Stockpile	10 days	2008 August 28	2008 September 10																				
60	Excavate to Stockpile Bay 1 to Formation Level	30 days	2008 September 11	2008 October 22																				
61	Survey Foundation	1 day	2008 October 23	2008 October 23																				
62	Prepare Bay 1 Formation	3 days	2008 October 24	2008 October 28																				
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	50 days	2008 October 29	2009 January 06																				
64	Survey Foundation	1 day	2009 January 07	2009 January 07																				
65	Survey Top of Clay Bay 1	1 day	2009 January 08	2009 January 08																				
66	Prepare Clay for Liner on Bay 1	4 days	2009 January 09	2009 January 14																				
67	Prepare Bay 2 Formation	3 days	2009 January 15	2009 January 19																				
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	40 days	2009 January 20	2009 March 16																				
69	Lay and Weld Geomembrane Liner Bay 1	15 days	2009 January 15	2009 February 04																				
70	Lay and Weld Geotextile Protection Bay 1	15 days	2009 January 19	2009 February 06																				
71	Lay Drainage Blanket Bay 1	25 days	2009 February 09	2009 March 13																				
72	Survey Top of Clay Bay 1	1 day	2009 March 17	2009 March 17																				
73	Prepare Clay for Liner on Bay 1	4 days	2009 March 18	2009 March 23																				
74	Lay and Weld Geomembrane Liner Bay 2	15 days	2009 March 24	2009 April 13																				
75	Lay and Weld Geotextile Protection Bay 2	15 days	2009 March 26	2009 April 15																				
76	Lay Drainage Blanket Bay 2	25 days	2009 April 16	2009 May 20																				
77	Survey Top of Drainage Blanket	1 day	2009 May 21	2009 May 21																				
78	Place Waste Protection Layer	10 days	2009 May 22	2009 June 04																				
79	Commission Landfill	5 days	2009 June 05	2009 June 11																				
80	Receive first Public Waste	0 days	2009 June 11	2009 June 11																				

Appendix E: AD Gantt Chart

Appendix E: Duration Design Estimate

Task		Progress		Summary		External Tasks		Deadline	↓
Split		Milestone	◆	Project Summary		External Milestone	◆		

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Section 2

Table 2: Calculation of the task Durations at Beta =2 in order to Determine the Critical Tasks

ID	Name	Estimated Variables					FORM Calculations			Design Durations
		Optimistic	Pessimistic	Anticipated	Mean	Std Deviation	Step 3	Step 4a	Step 4b	
							Alpha	Gamma	Designed Duration	
1	Muchawaste Landfill Project									
2	SECURE BRIDGING FINANCE									
3	Prepare Briefing Document	5	20	10	11.67	3.118	0.04862462	1.02599	12	
4	Prepare Basic Business Case								0	
5	Basic Engineering Solution	12	25	15	17.33	2.779	0.04333558	1.01390	18	
6	Basic Cost Estimate	10	16	15	13.67	1.312	0.0204653	1.00393	14	
7	Financial Model [Bridging Draft]	3	10	5	6.00	1.472	0.02295459	1.01126	6	
8	Assess Basic Feasibility	3	6	5	4.67	0.624	0.00972492	1.00260	5	
9	Positive Feasibility Report	1	1	1	1.00	0.000	0	1.00000	1	
10	Apply for a Conditional Operating Licence	60	120	80	86.67	12.472	0.19449848	1.05598	92	
11	Receive Conditional Operating Licence	1	1	1	1.00	0.000	0	1.00000	1	
12	Draft Offtake Agreement with local Council	15	25	20	20.00	2.041	0.03183229	1.00650	20	
13	Negotiate Offtake Agreement with local council	45	90	60	65.00	9.354	0.14587386	1.04199	68	
14	Sign Offtake Agreement	1	1	1	1.00	0.000	0	1.00000	1	
15	Solicit expressions of Interest and Terms of Bridging Finance	18	23	20	20.33	1.027	0.0160219	1.00162	20	
16	Acquire Bridging Funding	9	20	10	13.00	2.453	0.03872565	1.01479	13	
17	Company Decides to Proceed with the Project	1	5	1	2.33	0.943	0.0147027	1.01188	2	
18	Sign Bridging Finance Document	1	1	1	1.00	0.000	0	1.00000	1	
19	Obtain Unconditional Operating Licence				0.00					
20	LEVEL 2 BASIC DESIGN AND ENGINEERING				0.00					
21	Preliminary Geotechnical Investigation	15	30	20	21.67	3.118	0.04862462	1.01400	22	
22	Geotechnical Permeability Study	18	21	20	19.67	0.624	0.00972492	1.00662	20	
23	Basic Volumetric Study	16	23	20	19.67	1.434	0.02235826	1.00326	20	
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	30	55	40	41.67	5.137	0.0801095	1.01975	42	
25	Revised Cost Estimate	5	13	10	9.33	1.650	0.02572973	1.00910	9	
26	FUNDING									
27	Prepare Finance Briefing document	7	15	10	10.67	1.650	0.02572973	1.00796	11	
28	Submit Briefing document to potential funders	1	1	1	1.00	0.000	0	1.00000	1	
29	Acquire and evaluate funding proposals	18	25	20	21.00	1.472	0.02295459	1.00322	21	
30	Finalise Funding Requirements	1	2	1	1.33	0.236	0.00367568	1.00130	1	
31	Appointment, Equity & Debt Providers	40	70	60	56.67	6.236	0.09724924	1.02140	58	
32	Sign Funding Agreement	1	1	1	1.00	0.000	0	1.00000	1	
33	FINANCIAL CLOSURE									
34	Environmental approval Process South Africa									
35	Produce basic Project Information Memorandum	5	15	10	10.00	2.041	0.03183229	1.01300	10	
36	Prepare bidding document to solicit environmental consultants	10	17	15	14.00	1.472	0.02295459	1.00483	14	
37	Environmental consultants respond to Bid	22	35	30	29.00	2.677	0.04174765	1.00771	29	
38	Consortium evaluates bids and appoints consultant	7	20	10	12.33	2.779	0.04333558	1.01953	13	
39	Scoping Process	45	80	60	61.67	7.169	0.11179131	1.02599	63	
40	Applicant conducts EIA									
41	Specialist Studies	200	400	270	290.00	41.433	0.64612483	1.18463	344	
42	Appeal Period and Process	0	200	90	96.67	40.893	0.63770591	1.53954	149	
43	Authority issues a positive ROD									
44	EPC CONTRACT									
45	Shortlist potential EPC contractors	35	45	40	40.00	2.041	0.03183229	1.00325	40	
46	Submit Level 2 Basic Design	1	1	1	1.00	0.000	0	1.00000	1	
47	Contractors Prepare and Submit Bid	50	60	60	56.67	2.357	0.03675676	1.00306	57	
48	Evaluate bids	35	60	45	46.67	5.137	0.0801095	1.01764	47	
49	APPOINT EPC subject to Financial Close	35	50	40	41.67	3.118	0.04862462	1.00728	42	
50										
51	Detailed Geotechnical Investigation	26	49	40	38.33	4.732	0.07378668	1.01822	39	
52	Detailed Volumetric Study	30	50	40	40.00	4.082	0.06366457	1.01300	41	
53	Geotechnical Design	50	73	60	61.00	4.708	0.07342157	1.01133	62	
54	Design Synthetic Lining System	17	25	20	20.67	1.650	0.02572973	1.00411	21	
55	Produce Detailed Design	18	30	1	16.33	5.949	0.09276986	1.06758	17	
56	Landfill Construction Phase									
57	Construction Programme									
58	Establishment	3	6	5	4.67	0.624	0.00972492	1.00260	5	
59	Remove topsoil to Stockpile	6	15	10	10.33	1.841	0.02870795	1.01023	10	
60	Excavate to Stockpile Bay 1 to Formation Level	26	40	30	32.00	2.944	0.04590918	1.00845	32	
61	Survey Foundation	1	1	1	1.00	0.000	0	1.00000	1	
62	Prepare Bay 1 Formation	2	3	3	2.67	0.236	0.00367568	1.00065	3	
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	35	76	50	53.67	8.469	0.13206883	1.04168	56	
64	Survey Foundation	1	1	1	1.00	0.000	0	1.00000	1	
65	Survey Top of Clay Bay 1	1	1	1	1.00	0.000	0	1.00000	1	
66	Prepare Clay for Liner on Bay 1	3	5	4	4.00	0.408	0.00636646	1.00130	4	
67	Prepare Bay 2 Formation	2	3	3	2.67	0.236	0.00367568	1.00065	3	
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	35	60	40	45.00	5.401	0.08422031	1.02022	46	
69	Lay and Weld Geomembrane Liner Bay 1	12	22	15	16.33	2.095	0.03267012	1.00838	16	
70	Lay and Weld Geotextile Protection Bay 1	12	22	15	16.33	2.095	0.03267012	1.00838	16	
71	Lay Drainage Blanket Bay 1	20	30	25	25.00	2.041	0.03183229	1.00520	25	
72	Survey Top of Clay Bay 1	1	1	1	1.00	0.000	0	1.00000	1	
73	Prepare Clay for Liner on Bay 1	3	5	4	4.00	0.408	0.00636646	1.00130	4	
74	Lay and Weld Geomembrane Liner Bay 2	12	22	15	16.33	2.095	0.03267012	1.00838	16	
75	Lay and Weld Geotextile Protection Bay 2	12	22	15	16.33	2.095	0.03267012	1.00838	16	
76	Lay Drainage Blanket Bay 2	20	30	25	25.00	2.041	0.03183229	1.00520	25	
77	Survey Top of Drainage Blanket	1	2	1	1.33	0.236	0.00367568	1.00130	1	
78	Place Waste Protection Layer	6	13	10	9.67	1.434	0.02235826	1.00663	10	
79	Commission Landfill	5	10	5	6.67	1.179	0.01837838	1.00650	7	

The Cells highlighted in Red Refer to the respective Steps being followed as described in Section 4.2.1 of the Main Document.

STEP 2  
Model Mean 1640

Table 3: Critical Task Identification

ID	Name	Beta																				Critical Tasks	
		-2.5	-2.25	-2	-1.75	-1.5	-1.25	-1	-0.75	-0.5	-0.25	0	0.12	0.1	0.25	0.38	0.53	0.83	1.3	1.05	1.65		2.5
1	Muchawaste Landfill Project	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
2	<b>SECURE BRIDGING FINANCE</b>																						
3	Prepare Briefing Document	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
4	<b>Prepare Basic Business Case</b>																						
5	Basic Engineering Solution	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
6	Basic Cost Estimate	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
7	Financial Model [Bridging Draft]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
8	Assess Basic Feasibility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
9	Positive Feasibility Report	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
10	Apply for a Conditional Operating Licence	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
11	Receive Conditional Operating Licence	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
12	Draft Offtake Agreement with local Council	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
13	Negotiate Offtake Agreement with local council	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
14	Sign Offtake Agreement	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
15	Solicit expressions of Interest and Terms of Bridging Finance	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
16	Acquire Bridging Funding	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
17	Company Decides to Proceed with the Project	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
18	Sign Bridging Finance Document	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
19	<b>Obtain Unconditional Operating Licence</b>																						
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>																						
21	Preliminary Geotechnical Investigation	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
22	Geotechnical Permeability Study	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
23	Basic Volumetric Study	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
25	Revised Cost Estimate	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
26	<b>FUNDING</b>	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
27	Prepare Finance Briefing document	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
28	Submit Briefing document to potential funders	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
29	Acquire and evaluate funding proposals	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
30	Finalise Funding Requirements	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
31	Appointment, Equity & Debt Providers	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
32	Sign Funding Agreement	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
33	<b>FINANCIAL CLOSURE</b>																						
34	<b>Environmental approval Process South Africa</b>																						
35	Produce basic Project Information Memorandum	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
36	Prepare bidding document to solicit environmental consultants	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
37	Environmental consultants respond to Bid	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	
38	Consortium evaluates bids and appoints consultant	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
39	Scoping Process	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
40	Applicant conducts EIA	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
41	Specialist Studies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
42	Appeal Period and Process	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task



43 Authority issues a positive ROD																					
44 EPC CONTRACT																					
45 Shortlist potential EPC contractors	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
46 Submit Level 2 Basic Design	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
47 Contractors Prepare and Submit Bid	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
48 Evaluate bids	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
49 APPOINT EPC subject to Financial Close	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
50 Detailed Design	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
51 Detailed Geotechnical Investigation	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
52 Detailed Volumetric Study	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
53 Geotechnical Design	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
54 Design Synthetic Lining System	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
55 Produce Detailed Design	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
56 Landfill Construction Phase																					
57 Construction Programme																					
58 Establishment	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
59 Remove topsoil to Stockpile	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
60 Excavate to Stockpile Bay 1 to Formation Level	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
61 Survey Foundation	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
62 Prepare Bay 1 Formation	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
63 Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with ex	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
64 Survey Foundation	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
65 Survey Top of Clay Bay 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
66 Prepare Clay for Liner on Bay 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
67 Prepare Bay 2 Formation	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
68 Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excav	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
69 Lay and Weld Geomembrane Liner Bay 1	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
70 Lay and Weld Geotextile Protection Bay 1	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
71 Lay Drainage Blanket Bay 1	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
72 Survey Top of Clay Bay 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
73 Prepare Clay for Liner on Bay 1	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
74 Lay and Weld Geomembrane Liner Bay 2	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
75 Lay and Weld Geotextile Protection Bay 2	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
76 Lay Drainage Blanket Bay 2	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
77 Survey Top of Drainage Blanket	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
78 Place Waste Protection Layer	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
79 Commission Landfill	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task
80 Receive first Public Waste	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Critical Task



ID	Task Name	Duration	Start	Finish	2006		2007				2008				2009				2010		
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	
1	<b>Muchawaste Landfill Project</b>	<b>1133 days</b>	<b>2005 November 01</b>	<b>2010 March 04</b>																	0%
2	<b>SECURE BRIDGING FINANCE</b>	<b>825 days</b>	<b>2005 November 01</b>	<b>2008 December 29</b>																	0%
3	Prepare Briefing Document	12 days	2005 November 01	2005 November 16																	
4	<b>Prepare Basic Business Case</b>	<b>43 days</b>	<b>2005 November 17</b>	<b>2006 January 16</b>																	
5	Basic Engineering Solution	18 days	2005 November 17	2005 December 12																	
6	Basic Cost Estimate	14 days	2005 December 13	2005 December 30																	
7	Financial Model [Bridging Draft]	6 days	2006 January 02	2006 January 09																	
8	Assess Basic Feasibility	5 days	2006 January 10	2006 January 16																	
9	Positive Feasibility Report	1 day	2006 January 17	2006 January 17																	
10	Apply for a Conditional Operating Licence	92 days	2006 January 18	2006 May 25																	
11	Receive Conditional Operating Licence	1 day	2006 May 26	2006 May 26																	
12	Draft Offtake Agreement with local Council	20 days	2006 May 29	2006 June 23																	
13	Negotiate Offtake Agreement with local council	68 days	2006 June 26	2006 September 27																	
14	Sign Offtake Agreement	1 day	2006 September 28	2006 September 28																	
15	Solicit expressions of Interest and Terms of Bridging Finance	20 days	2006 September 01	2006 September 28																	
16	Acquire Bridging Funding	13 days	2006 September 29	2006 October 17																	
17	Company Decides to Proceed with the Project	2 days	2006 October 18	2006 October 19																	
18	Sign Bridging Finance Document	1 day	2006 October 20	2006 October 20																	
19	Obtain Unconditional Operating Licence	1 day	2008 December 29	2008 December 29																	
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>	<b>71 days</b>	<b>2006 October 23</b>	<b>2007 January 29</b>																	
21	Preliminary Geotechnical Investigation	22 days	2006 October 23	2006 November 21																	
22	Geotechnical Permeability Study	20 days	2006 October 23	2006 November 17																	
23	Basic Volumetric Study	20 days	2006 October 23	2006 November 17																	
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	42 days	2006 November 20	2007 January 16																	
25	Revised Cost Estimate	9 days	2007 January 17	2007 January 29																	
26	<b>FUNDING</b>	<b>92 days</b>	<b>2006 October 23</b>	<b>2007 February 27</b>																	
27	Prepare Finance Briefing document	11 days	2006 October 23	2006 November 06																	

Appendix E: Critical Tasks at Beta =2	Critical		Task		Baseline		Milestone		Project Summary		Deadline	
	Critical Split		Split		Baseline Split		Summary Progress		External Tasks			
	Critical Progress		Task Progress		Baseline Milestone		Summary		External Milestone			

ID	Task Name	Duration	Start	Finish	2006		2007				2008				2009				2010			
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2		
28	Submit Briefing document to potential funders	1 day	2006 November 07	2006 November 07																		
29	Acquire and evaluate funding prosals	21 days	2006 November 08	2006 December 06																		
30	Finalise Funding Requirements	1 day	2007 February 27	2007 February 27																		
31	Appointment, Equity & Debt Providers	58 days	2006 December 07	2007 February 26																		
32	Sign Funding Agreement	1 day	2007 February 27	2007 February 27																		
33	<b>FINANCIAL CLOSURE</b>	1 day	2008 December 30	2008 December 30																		
34	<b>Environmental approval Process South Africa</b>	<b>794 days</b>	<b>2005 December 13</b>	<b>2008 December 26</b>																		
35	Produce basic Project Information Memorandum	10 days	2005 December 13	2005 December 26																		
36	Prepare bidding document to solicit environmental consultants	14 days	2005 December 27	2006 January 13																		
37	Environmental consultants respond to Bid	29 days	2006 January 16	2006 February 23																		
38	Consortium evaluates bids and appoints consultant	13 days	2006 October 23	2006 November 08																		
39	Scoping Process	63 days	2006 November 09	2007 February 05																		
40	<b>Applicant conducts EIA</b>	<b>493 days</b>	<b>2007 February 06</b>	<b>2008 December 25</b>																		
41	Specialist Studies	344 days	2007 February 06	2008 May 30																		
42	Appeal Period and Process	149 days	2008 June 02	2008 December 25																		
43	Authority issues a positive ROD	1 day	2008 December 26	2008 December 26																		
44	<b>EPC CONTRACT</b>	<b>796 days</b>	<b>2005 December 13</b>	<b>2008 December 30</b>																		
45	Shortlist potential EPC contractors	40 days	2005 December 13	2006 February 06																		
46	Submit Level 2 Basic Design	1 day	2007 January 17	2007 January 17																		
47	Contractors Prepare and Submit Bid	57 days	2007 January 18	2007 April 06																		
48	Evaluate bids	47 days	2007 April 09	2007 June 12																		
49	APPOINT EPC subject to Financial Close	42 days	2008 November 03	2008 December 30																		
50	<b>Detailed Design</b>	<b>79 days</b>	<b>2008 December 31</b>	<b>2009 April 20</b>																		
51	Detailed Geotechnical Investigation	39 days	2008 December 31	2009 February 23																		
52	Detailed Volumetric Study	41 days	2008 December 31	2009 February 25																		
53	Geotechnical Design	62 days	2008 December 31	2009 March 26																		
54	Design Synthetic Lining System	21 days	2008 December 31	2009 January 28																		

Appendix E: Critical Tasks at Beta =2	Critical		Task		Baseline		Milestone		Project Summary		Deadline	
	Critical Split		Split		Baseline Split		Summary Progress		External Tasks			
	Critical Progress		Task Progress		Baseline Milestone		Summary		External Milestone			

ID	Task Name	Duration	Start	Finish	2006				2007				2008				2009				2010	
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2		
55	Produce Detailed Design	17 days	2009 March 27	2009 April 20															0%			
56	<b>Landfill Construction Phase</b>	<b>228 days</b>	<b>2009 April 21</b>	<b>2010 March 04</b>																0%		
57	<b>Construction Programme</b>	<b>228 days</b>	<b>2009 April 21</b>	<b>2010 March 04</b>																0%		
58	Establishment	5 days	2009 April 21	2009 April 27																		
59	Remove topsoil to Stockpile	10 days	2009 April 28	2009 May 11																		
60	Excavate to Stockpile Bay 1 to Formation Level	32 days	2009 May 12	2009 June 24																		
61	Survey Foundation	1 day	2009 June 25	2009 June 25																		
62	Prepare Bay 1 Formation	3 days	2009 June 26	2009 June 30																		
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	56 days	2009 July 01	2009 September 16																		
64	Survey Foundation	1 day	2009 September 17	2009 September 17																		
65	Survey Top of Clay Bay 1	1 day	2009 September 18	2009 September 18																		
66	Prepare Clay for Liner on Bay 1	4 days	2009 September 21	2009 September 24																		
67	Prepare Bay 2 Formation	3 days	2009 September 25	2009 September 29																		
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	46 days	2009 September 30	2009 December 02																		
69	Lay and Weld Geomembrane Liner Bay 1	16 days	2009 September 25	2009 October 16																		
70	Lay and Weld Geotextile Protection Bay 1	16 days	2009 September 29	2009 October 20																		
71	Lay Drainage Blanket Bay 1	25 days	2009 October 21	2009 November 24																		
72	Survey Top of Clay Bay 1	1 day	2009 December 03	2009 December 03																		
73	Prepare Clay for Liner on Bay 1	4 days	2009 December 04	2009 December 09																		
74	Lay and Weld Geomembrane Liner Bay 2	16 days	2009 December 10	2009 December 31																		
75	Lay and Weld Geotextile Protection Bay 2	16 days	2009 December 14	2010 January 04																		
76	Lay Drainage Blanket Bay 2	25 days	2010 January 05	2010 February 08																		
77	Survey Top of Drainage Blanket	1 day	2010 February 09	2010 February 09																		
78	Place Waste Protection Layer	10 days	2010 February 10	2010 February 23																		
79	Commission Landfill	7 days	2010 February 24	2010 March 04																		
80	Receive first Public Waste	0 days	2010 March 04	2010 March 04																03/04		

Appendix E: Critical Tasks at Beta =2	Critical		Task		Baseline		Milestone		Project Summary		Deadline	
	Critical Split		Split		Baseline Split		Summary Progress		External Tasks			
	Critical Progress		Task Progress		Baseline Milestone		Summary		External Milestone			

Section 3: Design the Duration

Table 4: Critical Task Duration Design.

ID	Name	Estimated Variables					FORM Calculations		Design Durations
		Optimistic	Pessimistic	Anticipated	Mean	Std Deviation	Step 4	Step 5b	Step 6
							Alpha	Gamma	Designed Duration
1	<b>Muchawaste Landfill Project</b>								
2	<b>SECURE BRIDGING FINANCE</b>								
3	Prepare Briefing Document	5	20	10	11.67	3.118	0.04978497	1.01384	12
4	<b>Prepare Basic Business Case</b>								0
5	Basic Engineering Solution	12	25	15	17.33	2.779	0.04436971	1.00740	17
6	Basic Cost Estimate	10	16	15	13.67	1.312	0.02095367	1.00209	14
7	Financial Model [Bridging Draft]	3	10	5	6.00	1.472	0.02350236	1.00600	6
8	Assess Basic Feasibility	3	6	5	4.67	0.624	0.00995699	1.00138	5
9	Positive Feasibility Report	1	1	1	1.00	0.000	0	1.00000	1
10	Apply for a Conditional Operating Licence	60	120	80	86.67	12.472	0.19913987	1.02980	89
11	Receive Conditional Operating Licence	1	1	1	1.00	0.000	0	1.00000	1
12	Draft Offtake Agreement with local Council	15	25	20	20.00	2.041	0.03259191	1.00346	20
13	Negotiate Offtake Agreement with local council	45	90	60	65.00	9.354	0.1493549	1.02235	66
14	Sign Offtake Agreement	1	1	1	1.00	0.000	0	1.00000	1
15	Solicit expressions of Interest and Terms of Bridging Finance	18	23	20	20.33	1.027	0.01640424	1.00086	20
16	Acquire Bridging Funding	9	20	10	13.00	2.483	0.03964977	1.00788	13
17	Company Decides to Proceed with the Project	1	5	1	2.33	0.943	0.01505356	1.00633	2
18	Sign Bridging Finance Document	1	1	1	1.00	0.000	0	1.00000	1
19	<b>Obtain Unconditional Operating Licence</b>								
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>								
21	Preliminary Geotechnical Investigation	15	30	20	0.00	0.000	0	1.00000	20
22	Geotechnical Permeability Study	18	21	20	0.00	0.000	0	1.00000	20
23	Basic Volumetric Study	16	23	20	0.00	0.000	0	1.00000	20
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	30	55	40	0.00	0.000	0	1.00000	40
25	Revised Cost Estimate	5	13	10	0.00	0.000	0	1.00000	10
26									
27	Prepare Finance Briefing document	7	15	10	0.00	0.000	0	1.00000	10
28	Submit Briefing document to potential funders	1	1	1	0.00	0.000	0	1.00000	1
29	Acquire and evaluate funding proposals	18	25	20	0.00	0.000	0	1.00000	20
30	Finalise Funding Requirements	1	2	1	0.00	0.000	0	1.00000	1
31	Appointment, Equity & Debt Providers	40	70	60	0.00	0.000	0	1.00000	60
32	Sign Funding Agreement	1	1	1	0.00	0.000	0	1.00000	1
33	<b>FINANCIAL CLOSURE</b>								
34	<b>Environmental approval Process South Africa</b>								
35	Produce basic Project Information Memorandum	5	15	10	0.00	0.000	0	1.00000	10
36	Prepare bidding document to solicit environmental consultants	10	17	15	0.00	0.000	0	1.00000	15
37	Environmental consultants respond to Bid	22	35	30	0.00	0.000	0	1.00000	30
38	Consortium evaluates bids and appoints consultant	7	20	10	12.33	2.779	0.04436971	1.01040	12
39	Scoping Process	45	80	60	61.67	7.169	0.11445903	1.01384	63
40	Applicant conducts EIA								
41	Specialist Studies	200	400	270	290.00	41.433	0.66154354	1.09830	319
42	Appeal Period and Process	0	200	90	96.67	40.893	0.65292372	1.28725	124
43	<b>Authority issues a positive ROD</b>								
44	<b>EPC CONTRACT</b>								
45	Shortlist potential EPC contractors	35	45	40	0.00	0.000	0	1.00000	40
46	Submit Level 2 Basic Design	1	1	1	0.00	0.000	0	1.00000	1
47	Contractors Prepare and Submit Bid	50	60	60	0.00	0.000	0	1.00000	60
48	Evaluate bids	35	60	45	0.00	0.000	0	1.00000	45
49	APPOINT EPC subject to Financial Close	35	50	40	41.67	3.118	0.04978497	1.00387	42
50	Detailed Design				0.00	0.000	0	1.00000	0
51	Detailed Geotechnical Investigation	26	49	40	0.00	0.000	0	1.00000	40
52	Detailed Volumetric Study	30	50	40	0.00	0.000	0	1.00000	40
53	Geotechnical Design	50	73	60	61.00	4.708	0.07517365	1.00603	61
54	Design Synthetic Lining System	17	25	20	0.00	0.000	0	1.00000	20
55	Produce Detailed Design	18	30	1	16.33	5.949	0.09498366	1.03598	17
56	<b>Landfill Construction Phase</b>								
57	<b>Construction Programme</b>								
58	Establishment	3	6	5	4.67	0.624	0.00995699	1.00138	5
59	Remove topsoil to Stockpile	6	15	10	10.33	1.841	0.02939301	1.00545	10
60	Excavate to Stockpile Bay 1 to Formation Level	26	40	30	32.00	2.944	0.04700472	1.00450	32
61	Survey Foundation	1	1	1	1.00	0.000	0	1.00000	1
62	Prepare Bay 1 Formation	2	3	3	2.67	0.236	0.00376339	1.00035	3
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	35	76	50	53.67	8.469	0.13522043	1.02219	55
64	Survey Foundation	1	1	1	1.00	0.000	0	1.00000	1
65	Survey Top of Clay Bay 1	1	1	1	1.00	0.000	0	1.00000	1
66	Prepare Clay for Liner on Bay 1	3	5	4	4.00	0.408	0.00651838	1.00069	4
67	Prepare Bay 2 Formation	2	3	3	2.67	0.236	0.00376339	1.00035	3
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	35	60	40	45.00	5.401	0.08623009	1.01076	45
69	Lay and Weld Geomembrane Liner Bay 1	12	22	15	0.00	0.000	0	1.00000	15
70	Lay and Weld Geotextile Protection Bay 1	12	22	15	0.00	0.000	0	1.00000	15
71	Lay Drainage Blanket Bay 1	20	30	25	0.00	0.000	0	1.00000	25
72	Survey Top of Clay Bay 1	1	1	1	1.00	0.000	0	1.00000	1
73	Prepare Clay for Liner on Bay 1	3	5	4	4.00	0.408	0.00651838	1.00069	4
74	Lay and Weld Geomembrane Liner Bay 2	12	22	15	16.33	2.095	0.03344974	1.00446	16
75	Lay and Weld Geotextile Protection Bay 2	12	22	15	16.33	2.095	0.03344974	1.00446	16
76	Lay Drainage Blanket Bay 2	20	30	25	25.00	2.041	0.03259191	1.00277	25
77	Survey Top of Drainage Blanket	1	2	1	1.33	0.236	0.00376339	1.00069	1
78	Place Waste Protection Layer	6	13	10	9.67	1.434	0.02289181	1.00353	10
79	Commission Landfill	5	10	5	6.67	1.179	0.01881695	1.00346	7

The Cells highlighted in Red Refer to the respective Steps being followed as described in Section 4.2.2 of the Main Document.

STEP 1		STEP 1	
Model Mean	1083	Model Std Dev	62.63

STEP 2 ADE	STEP 3
1000	P(Dc < ADE)= 9.34%

Step 5a	
Reliability	85%
Beta	1.04

# Appendix E: Designed Duration Gantt Chart.

ID	Task Name	Duration	Start	Finish	2006				2007				2008				2009				2010			
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	
1	<b>Muchawaste Landfill Project</b>	<b>1074 days</b>	<b>2005 November 01</b>	<b>2009 December 11</b>																				
2	<b>SECURE BRIDGING FINANCE</b>	<b>769 days</b>	<b>2005 November 01</b>	<b>2008 October 10</b>																				
3	Prepare Briefing Document	12 days	2005 November 01	2005 November 16																				
4	<b>Prepare Basic Business Case</b>	<b>42 days</b>	<b>2005 November 17</b>	<b>2006 January 13</b>																				
5	Basic Engineering Solution	17 days	2005 November 17	2005 December 09																				
6	Basic Cost Estimate	14 days	2005 December 12	2005 December 29																				
7	Financial Model [Bridging Draft]	6 days	2005 December 30	2006 January 06																				
8	Assess Basic Feasibility	5 days	2006 January 09	2006 January 13																				
9	Positive Feasibility Report	1 day	2006 January 16	2006 January 16																				
10	Apply for a Conditional Operating Licence	89 days	2006 January 17	2006 May 19																				
11	Receive Conditional Operating Licence	1 day	2006 May 22	2006 May 22																				
12	Draft Offtake Agreement with local Council	20 days	2006 May 23	2006 June 19																				
13	Negotiate Offtake Agreement with local council	66 days	2006 June 20	2006 September 19																				
14	Sign Offtake Agreement	1 day	2006 September 20	2006 September 20																				
15	Solicit expressions of Interest and Terms of Bridging Finance	20 days	2006 August 24	2006 September 20																				
16	Acquire Bridging Funding	13 days	2006 September 21	2006 October 09																				
17	Company Decides to Proceed with the Project	2 days	2006 October 10	2006 October 11																				
18	Sign Bridging Finance Document	1 day	2006 October 12	2006 October 12																				
19	Obtain Unconditional Operating Licence	1 day	2008 October 10	2008 October 10																				
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>	<b>70 days</b>	<b>2006 October 13</b>	<b>2007 January 18</b>																				
21	Preliminary Geotechnical Investigation	20 days	2006 October 13	2006 November 09																				
22	Geotechnical Permeability Study	20 days	2006 October 13	2006 November 09																				
23	Basic Volumetric Study	20 days	2006 October 13	2006 November 09																				
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	40 days	2006 November 10	2007 January 04																				
25	Revised Cost Estimate	10 days	2007 January 05	2007 January 18																				
26	<b>FUNDING</b>	<b>92 days</b>	<b>2006 October 13</b>	<b>2007 February 19</b>																				
27	Prepare Finance Briefing document	10 days	2006 October 13	2006 October 26																				

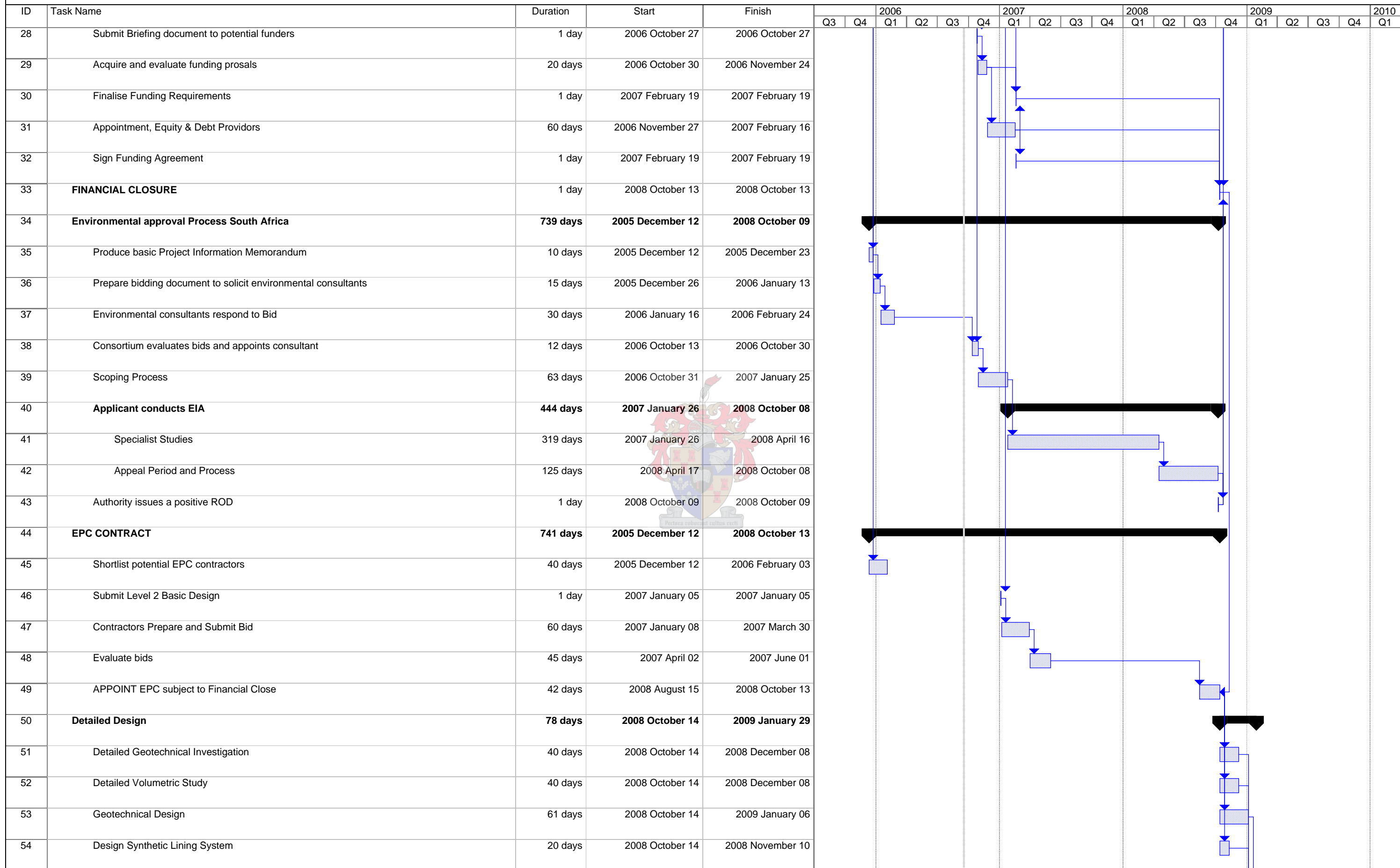
Appendix E: Designed Duration Gantt

Appendix E: Duration Design Estimate

Task Progress Summary External Tasks Deadline

Split Milestone Project Summary External Milestone

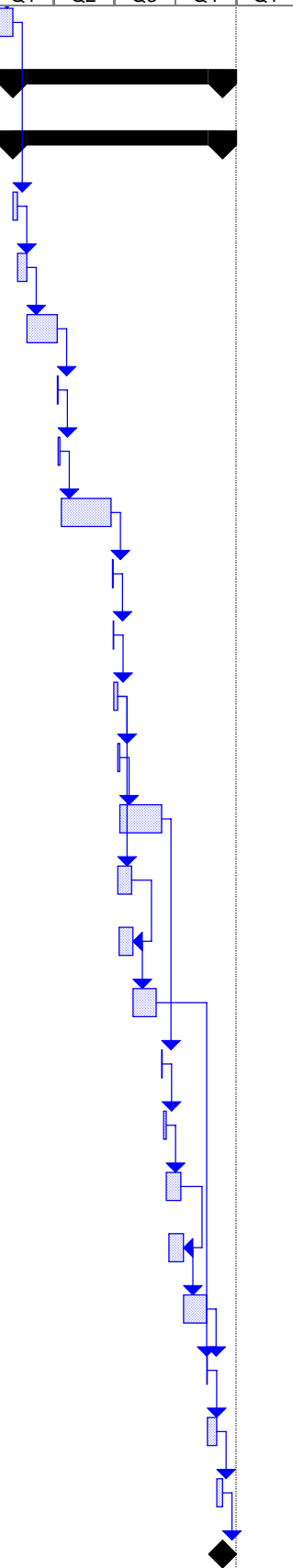
## Appendix E: Designed Duration Gantt Chart.



Appendix E: Designed Duration Gantt Appendix E: Duration Design Estimate	Task Split	Progress Milestone	Summary Project Summary	External Tasks External Milestone	Deadline 
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### Appendix E: Designed Duration Gantt Chart.

ID	Task Name	Duration	Start	Finish	2006				2007				2008				2009				2010
					Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1		
55	Produce Detailed Design	17 days	2009 January 07	2009 January 29																	
56	<b>Landfill Construction Phase</b>	<b>226 days</b>	<b>2009 January 30</b>	<b>2009 December 11</b>																	
57	<b>Construction Programme</b>	<b>226 days</b>	<b>2009 January 30</b>	<b>2009 December 11</b>																	
58	Establishment	5 days	2009 January 30	2009 February 05																	
59	Remove topsoil to Stockpile	10 days	2009 February 06	2009 February 19																	
60	Excavate to Stockpile Bay 1 to Formation Level	32 days	2009 February 20	2009 April 06																	
61	Survey Foundation	1 day	2009 April 07	2009 April 07																	
62	Prepare Bay 1 Formation	3 days	2009 April 08	2009 April 10																	
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	55 days	2009 April 13	2009 June 26																	
64	Survey Foundation	1 day	2009 June 29	2009 June 29																	
65	Survey Top of Clay Bay 1	1 day	2009 June 30	2009 June 30																	
66	Prepare Clay for Liner on Bay 1	4 days	2009 July 01	2009 July 06																	
67	Prepare Bay 2 Formation	3 days	2009 July 07	2009 July 09																	
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	45 days	2009 July 10	2009 September 10																	
69	Lay and Weld Geomembrane Liner Bay 1	15 days	2009 July 07	2009 July 27																	
70	Lay and Weld Geotextile Protection Bay 1	15 days	2009 July 09	2009 July 29																	
71	Lay Drainage Blanket Bay 1	25 days	2009 July 30	2009 September 02																	
72	Survey Top of Clay Bay 1	1 day	2009 September 11	2009 September 11																	
73	Prepare Clay for Liner on Bay 1	4 days	2009 September 14	2009 September 17																	
74	Lay and Weld Geomembrane Liner Bay 2	16 days	2009 September 18	2009 October 09																	
75	Lay and Weld Geotextile Protection Bay 2	16 days	2009 September 22	2009 October 13																	
76	Lay Drainage Blanket Bay 2	25 days	2009 October 14	2009 November 17																	
77	Survey Top of Drainage Blanket	1 day	2009 November 18	2009 November 18																	
78	Place Waste Protection Layer	10 days	2009 November 19	2009 December 02																	
79	Commission Landfill	7 days	2009 December 03	2009 December 11																	
80	Receive first Public Waste	0 days	2009 December 11	2009 December 11																	



Appendix E: Designed Duration Gantt

Appendix E: Duration Design Estimate

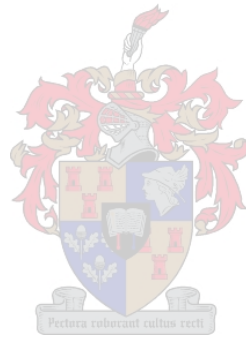
Task
Progress
Summary
External Tasks
Deadline
  
Split
Milestone
Project Summary
External Milestone

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## Appendix F: Cost Design Estimate

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Appendix F: Cost Estimate Design

ID	Name	Statistics					FOSM Calculations		
		STEP 1 AND 2			Step 3		STEP 6	STEP 7b	STEP 8
		Optimistic	Pessimistic	Anticipated	Mean	Std Deviation	Alpha	Epsilon	New Designed Cost
1	Muchawaste Landfill Project								
2	SECURE BRIDGING FINANCE								
3	Prepare Briefing Document	R 6,000.00	R 9,000.00	R 7,000.00	R 7,333.33	R 623.61	0.005936406	1.00053	R 7,337.18
4	Prepare Basic Business Case								
5	Basic Engineering Solution	R 130,000.00	R 165,000.00	R 150,000.00	R 148,333.33	R 7,168.60	0.068241007	1.00343	R 148,842.09
6	Basic Cost Estimate	R 7,000.00	R 8,000.00	R 7,500.00	R 7,500.00	R 204.12	0.001943145	1.00006	R 7,500.41
7	Financial Model (Bridging Draft)	R 20,000.00	R 30,000.00	R 25,000.00	R 25,000.00	R 2,041.24	0.019431449	1.00165	R 25,041.25
8	Assess Basic Feasibility	R 2,500.00	R 2,500.00	R 2,500.00	R 2,500.00	R 0.00	0	1.00000	R 2,500.00
9	Positive Feasibility Report	R 500.00	R 500.00	R 500.00	R 500.00	R 0.00	0	1.00000	R 500.00
10	Apply for a Conditional Operating Licence	R 9,000.00	R 11,000.00	R 10,000.00	R 10,000.00	R 408.25	0.00388629	1.00017	R 10,001.65
11	Receive Conditional Operating Licence	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
12	Draft Offtake Agreement with local Council	R 10,000.00	R 12,000.00	R 11,000.00	R 11,000.00	R 408.25	0.00388629	1.00015	R 11,001.65
13	Negotiate Offtake Agreement with local council	R 33,000.00	R 33,000.00	R 33,000.00	R 33,000.00	R 0.00	0	1.00000	R 33,000.00
14	Sign Offtake Agreement	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
15	Solicit expressions of Interest and Terms of Bridging Finance	R 10,000.00	R 12,000.00	R 11,000.00	R 11,000.00	R 408.25	0.00388629	1.00015	R 11,001.65
16	Acquire Bridging Funding	R 7,000.00	R 8,000.00	R 7,500.00	R 7,500.00	R 204.12	0.001943145	1.00006	R 7,500.41
17	Company Decides to Proceed with the Project	R 1,000.00	R 2,000.00	R 1,500.00	R 1,500.00	R 204.12	0.001943145	1.00028	R 1,500.41
18	Sign Bridging Finance Document	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
19	Obtain Unconditional Operating Licence								
20	LEVEL 2 BASIC DESIGN AND ENGINEERING								
21	Preliminary Geotechnical Investigation	R 100,000.00	R 130,000.00	R 120,000.00	R 116,666.67	R 6,236.10	0.059364058	1.00330	R 117,051.67
22	Geotechnical Permeability Study	R 19,500.00	R 20,500.00	R 20,000.00	R 20,000.00	R 204.12	0.001943145	1.00002	R 20,000.41
23	Basic Volumetric Study	R 40,000.00	R 46,000.00	R 45,000.00	R 43,666.67	R 1,312.33	0.012492674	1.00039	R 43,683.72
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	R 160,000.00	R 210,000.00	R 180,000.00	R 183,333.33	R 10,274.02	0.097802816	1.00570	R 184,378.35
25	Revised Cost Estimate	R 5,000.00	R 5,000.00	R 5,000.00	R 5,000.00	R 0.00	0	1.00000	R 5,000.00
26	FUNDING								
27	Prepare Finance Briefing document	R 5,000.00	R 5,250.00	R 5,500.00	R 5,250.00	R 102.06	0.000971572	1.00002	R 5,250.10
28	Submit Briefing document to potential funders	R 500.00	R 500.00	R 500.00	R 500.00	R 0.00	0	1.00000	R 500.00
29	Acquire and evaluate funding proposals	R 10,000.00	R 12,000.00	R 11,000.00	R 11,000.00	R 408.25	0.00388629	1.00015	R 11,001.65
30	Finalise Funding Requirements	R 500.00	R 500.00	R 500.00	R 500.00	R 0.00	0	1.00000	R 500.00
31	Appointment, Equity & Debt Providers	R 30,000.00	R 35,000.00	R 32,000.00	R 32,333.33	R 1,027.40	0.009780282	1.00032	R 32,343.78
32	Sign Funding Agreement	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
33	FINANCIAL CLOSURE								
34	Environmental approval Process South Africa								
35	Produce basic Project Information Memorandum	R 10,000.00	R 12,000.00	R 11,000.00	R 11,000.00	R 408.25	0.00388629	1.00015	R 11,001.65
36	Prepare bidding document to solicit environmental consultants	R 7,500.00	R 10,000.00	R 8,000.00	R 8,500.00	R 540.06	0.005141078	1.00034	R 8,502.89
37	Environmental consultants respond to Bid	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
38	Consortium evaluates bids and appoints consultant	R 6,000.00	R 6,000.00	R 6,000.00	R 6,000.00	R 0.00	0	1.00000	R 6,000.00
39	Scoping Process	R 63,000.00	R 90,000.00	R 75,000.00	R 76,000.00	R 5,522.68	0.052572754	1.00397	R 76,301.96
40	Applicant conducts EIA	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
41	Specialist Studies	R 200,000.00	R 350,000.00	R 250,000.00	R 266,666.67	R 31,180.46	0.296820286	1.03609	R 276,291.87
42	Appeal Period and Process	R 25,000.00	R 100,000.00	R 50,000.00	R 58,333.33	R 15,590.24	0.148410145	1.04125	R 60,739.63
43	Authority issues a positive ROD								
44	EPC CONTRACT								
45	Shortlist potential EPC contractors	R 20,000.00	R 30,000.00	R 25,000.00	R 25,000.00	R 2,041.24	0.019431449	1.00165	R 25,041.25
46	Submit Level 2 Basic Design	R 500.00	R 500.00	R 500.00	R 500.00	R 0.00	0	1.00000	R 500.00
47	Contractors Prepare and Submit Bid	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
48	Evaluate bids	R 22,500.00	R 30,000.00	R 25,000.00	R 25,833.33	R 1,559.02	0.014841014	1.00093	R 25,857.40
49	APPOINT EPC subject to Financial Close	R 21,000.00	R 21,000.00	R 21,000.00	R 21,000.00	R 0.00	0	1.00000	R 21,000.00
50	Detailed Design	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
51	Detailed Geotechnical Investigation	R 90,000.00	R 115,000.00	R 100,000.00	R 101,666.67	R 5,137.01	0.048901408	1.00257	R 101,927.92
52	Detailed Volumetric Study	R 28,000.00	R 33,000.00	R 30,000.00	R 30,333.33	R 1,027.40	0.009780282	1.00034	R 30,343.78
53	Geotechnical Design	R 55,000.00	R 65,000.00	R 60,000.00	R 60,000.00	R 2,041.24	0.019431449	1.00069	R 60,041.25
54	Design Synthetic Lining System	R 30,000.00	R 36,000.00	R 35,000.00	R 33,666.67	R 1,312.33	0.012492674	1.00051	R 33,683.72
55	Produce Detailed Design	R 20,000.00	R 23,000.00	R 20,000.00	R 21,000.00	R 707.11	0.006731251	1.00024	R 21,004.95
56	Landfill Construction Phase								
57	Construction Programme								
58	Establishment	R 15,000.00	R 34,000.00	R 20,000.00	R 23,000.00	R 4,020.78	0.038275516	1.00696	R 23,160.05
59	Remove topsoil to Stockpile	R 80,000.00	R 110,000.00	R 90,000.00	R 93,333.33	R 6,236.10	0.059364058	1.00413	R 93,718.34
60	Excavate to Stockpile Bay 1 to Formation Level	R 272,000.00	R 368,000.00	R 320,000.00	R 320,000.00	R 19,595.92	0.186541912	1.01188	R 323,801.88
61	Survey Foundation	R 850.00	R 1,150.00	R 1,000.00	R 1,000.00	R 61.24	0.000582943	1.00004	R 1,000.04
62	Prepare Bay 1 Formation	R 17,850.00	R 24,150.00	R 21,000.00	R 21,000.00	R 1,285.98	0.012241813	1.00078	R 21,016.37
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	R 701,250.00	R 948,750.00	R 825,000.00	R 825,000.00	R 50,520.73	0.480928367	1.03063	R 850,268.72
64	Survey Foundation	R 850.00	R 1,150.00	R 1,000.00	R 1,000.00	R 61.24	0.000582943	1.00004	R 1,000.04
65	Survey Top of Clay Bay 1	R 850.00	R 1,150.00	R 1,000.00	R 1,000.00	R 61.24	0.000582943	1.00004	R 1,000.04
66	Prepare Clay for Liner on Bay 1	R 23,800.00	R 32,200.00	R 28,000.00	R 28,000.00	R 1,714.64	0.016322417	1.00104	R 28,029.11
67	Prepare Bay 2 Formation	R 17,850.00	R 24,150.00	R 21,000.00	R 21,000.00	R 1,285.98	0.012241813	1.00078	R 21,016.37
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	R 573,750.00	R 776,250.00	R 675,000.00	R 675,000.00	R 41,335.14	0.393486846	1.02506	R 691,915.43
69	Lay and Weld Geomembrane Liner Bay 1	R 540,000.00	R 583,000.00	R 560,000.00	R 561,000.00	R 8,784.46	0.083622988	1.00136	R 561,763.97
70	Lay and Weld Geotextile Protection Bay 1	R 380,000.00	R 460,000.00	R 410,000.00	R 416,666.67	R 16,499.16	0.157062534	1.00647	R 419,361.72
71	Lay Drainage Blanket Bay 1	R 745,000.00	R 965,000.00	R 825,000.00	R 845,000.00	R 45,460.61	0.432758921	1.02421	R 865,460.42
72	Survey Top of Clay Bay 1	R 850.00	R 1,150.00	R 1,000.00	R 1,000.00	R 61.24	0.000582943	1.00004	R 1,000.04
73	Prepare Clay for Liner on Bay 1	R 3,400.00	R 4,600.00	R 4,000.00	R 4,000.00	R 244.95	0.002331774	1.00015	R 4,000.59
74	Lay and Weld Geomembrane Liner Bay 2	R 544,000.00	R 587,000.00	R 564,000.00	R 565,000.00	R 8,784.46	0.083622988	1.00135	R 565,763.97
75	Lay and Weld Geotextile Protection Bay 2	R 384,000.00	R 464,000.00	R 414,000.00	R 420,666.67	R 16,499.16	0.157062534	1.00641	R 423,361.72
76	Lay Drainage Blanket Bay 2	R 745,000.00	R 965,000.00	R 825,000.00	R 845,000.00	R 45,460.61	0.432758921	1.02421	R 865,460.42
77	Survey Top of Drainage Blanket	R 850.00	R 1,150.00	R 1,000.00	R 1,000.00	R 61.24	0.000582943	1.00004	R 1,000.04
78	Place Waste Protection Layer	R 34,000.00	R 46,000.00	R 40,000.00	R 40,000.00	R 2,449.49	0.023317739	1.00149	R 40,059.40
79	Commission Landfill	R 2,975.00	R 4,025.00	R 3,500.00	R 3,500.00	R 214.33	0.002040302	1.00013	R 3,500.45
80	Receive first Public Waste	R 0.00	R 0.00	R 0.00	R 0.00	R 0.00	0	1.00000	R 0.00
		R 6,289,125.00	R 8,081,125.00	R 7,053,000.00	R 7,141,083.33	R 105,048.34			R 7,249,833.61
					Model Std Dev	R 105,048.34			
					Model Mean	R 7,141,083.33			

The Cells highlighted in Red Refer to the respective Steps being followed as described in Section 4.3.1.1 of the Main Document.

STEP 4  
ACE = R 7,053,000.00

STEP 5  
P(C < ACE) 20.09%

STEP 7a  
Desired Reliability 85%  
Beta 1.04

## Appendix G: Risk Mitigation Allowance Calculation



## Appendix G: Rainfall Risk Mitigating Allowances

Rainfall Data Supplied by the South African Weather Service for Johannesburg

Table 1: Rainfall Data

Cumulative Monthly Rainfall in mm 1970-2005												
Year	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1970	82.0	112.2	25.7	61.7	16.8	6.0	17.0	6.0	16.2	70.7	94.8	98.8
1971	107.2	86.9	71.0	130.1	18.7	8.8	0.0	0.2	43.6	74.0	152.2	105.0
1972	338.1	38.9	109.3	16.9	8.3	2.3	0.0	2.7	5.2	93.7	126.7	75.8
1973	73.4	30.9	97.3	62.4	1.6	0.0	0.0	16.8	45.7	82.1	110.5	108.3
1974	55.8	91.0	100.7	107.2	7.7	2.3	0.8	0.8	15.9	42.0	163.4	106.9
1975	326.1	113.8	44.7	100.0	10.0	9.6	1.1	0.3	0.2	17.0	147.0	158.2
1976	95.2	190.7	119.9	66.1	79.8	0.0	3.6	0.0	15.4	151.4	69.4	94.0
1977	117.2	24.7	125.0	18.8	7.9	0.0	0.0	0.7	72.3	89.4	106.3	103.2
1978	231.4	141.8	99.0	35.2	6.9	0.8	0.0	12.3	21.9	60.6	76.3	128.1
1979	93.3	38.8	84.2	42.7	13.8	0.1	15.4	46.6	30.2	68.6	188.5	67.7
1980	209.1	173.6	80.4	13.9	2.2	0.0	0.0	0.0	80.2	11.7	147.6	37.5
1981	139.8	121.3	66.5	44.9	0.0	8.3	0.0	12.5	29.3	93.9	79.8	96.9
1982	160.8	59.7	100.0	42.1	0.0	0.5	10.9	0.3	9.7	64.0	45.6	105.6
1983	97.2	95.6	56.2	11.8	14.2	36.0	3.7	5.5	17.7	74.4	130.9	167.3
1984	60.5	13.1	77.6	12.2	1.1	10.5	14.7	0.4	11.9	74.2	117.8	48.6
1985	108.7	146.9	90.0	3.6	6.5	0.0	5.8	11.7	28.4	189.5	99.9	112.1
1986	69.6	91.1	93.2	37.1	0.0	24.7	7.3	2.8	124.4	184.5	181.3	90.1
1987	134.2	84.3	219.2	9.2	0.0	0.1	0.0	19.2	175.0	69.5	200.2	107.8
1988	126.5	70.5	130.1	48.0	1.5	26.3	4.8	0.1	53.8	79.3	59.1	127.8
1989	128.0	172.7	45.6	51.1	22.3	53.1	0.0	1.2	0.0	63.6	175.8	57.4
1990	84.5	131.7	98.9	109.6	11.8	0.4	2.5	0.6	7.7	44.5	38.5	103.9
1991	138.0	92.5	126.0	2.5	5.1	6.1	0.0	2.0	13.7	66.9	54.4	88.3
1992	109.2	91.7	21.7	20.0	0.0	5.4	0.0	15.2	14.7	25.1	114.6	91.8
1993	145.7	64.8	137.9	31.6	0.8	0.0	0.0	0.4	29.8	148.7	111.3	97.5
1994	105.8	190.3	84.2	22.1	0.0	0.0	0.0	0.0	4.0	103.8	50.9	166.4
1995	86.9	70.6	138.7	74.7	4.2	0.0	0.0	7.9	2.6	114.0	148.2	220.4
1996	168.7	322.5	59.0	63.4	24.0	2.2	0.0	13.7	0.4	46.3	86.2	114.0
1997	172.5	35.4	333.9	30.0	99.2	7.6	7.3	3.3	28.4	34.7	146.2	105.4
1998	154.1	78.2	26.6	12.2	0.0	0.0	0.0	0.0	32.2	75.2	208.2	130.1
1999	125.3	25.3	55.4	41.6	56.0	5.3	0.8	3.4	8.0	15.0	0.0	170.0
2000	153.6	263.0	164.5	33.8	25.1	2.0	0.0	4.5	39.4	117.0	114.8	170.8
2001	72.8	102.9	49.4	31.1	67.6	5.6	1.6	19.9	99.4	177.3	110.4	76.6
2002	125.5	93.0	83.4	25.2	63.5	18.7	0.0	23.8	4.1	65.4	22.6	141.0
2003	129.3	105.6	91.0	3.3	0.0	20.2	0.0	8.1	8.2	84.4	43.1	45.7
2004	163.8	211.5	109.5	46.2	13.0	4.2	14.5	3.8	0.5	26.9	52.4	214.4
2005	157.5	86.1	106.7	100.3	1.0	0.0	0.0	0.0	1.6	47.7	148.0	70.4

Table 2: Statistical Indicators

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Mean	134.6	107.3	97.8	43.4	16.4	7.4	3.2	6.9	30.3	79.1	109.0	111.8
Standard Deviation	62.7	67.8	57.0	33.1	24.9	11.7	5.2	9.6	38.1	45.7	52.5	43.5
Sample Variance	3936.4	4591.7	3249.6	1093.1	620.8	137.1	26.9	92.3	1449.3	2089.0	2752.7	1892.2
Range	282.3	309.4	312.2	127.6	99.2	53.1	17.0	46.6	175.0	177.8	208.2	182.9
Minimum	55.8	13.1	21.7	2.5	0.0	0.0	0.0	0.0	0.0	11.7	0.0	37.5
Maximum	338.1	322.5	333.9	130.1	99.2	53.1	17.0	46.6	175.0	189.5	208.2	220.4
Sum	4847.3	3863.6	3522.4	1562.6	590.6	267.1	111.8	246.7	1091.7	2847.0	3922.9	3913.7
Count	36.0	36.0	36.0	36.0	36.0	36.0	35.0	36.0	36.0	36.0	36.0	35.0

Table 3: Calculation of the Probability of Rainfall Greater than 6mm in one day

Month	Days in Month	Monthly		Daily		Z at X=6mm	Probability that Daily Rainfall >6mm in any one Day
		Average Monthly (mm)	Standard Deviation (mm)	Mean Daily Rainfall	Standard Deviation (mm)		
January	31	134.65	62.74	4.34	2.02	-2.05	0.2065
February	28	107.32	67.76	3.83	2.42	-1.50	0.1853
March	31	97.84	57.01	3.16	1.84	-1.61	0.0610
April	30	43.41	33.06	1.45	1.10	-1.13	0.0000
May	31	16.41	24.92	0.53	0.80	-0.42	0.0000
June	30	7.42	11.71	0.25	0.39	-0.12	0.0000
July	31	3.19	5.19	0.10	0.17	0.54	0.0000
August	31	6.85	9.61	0.22	0.31	-0.09	0.0000
September	30	30.33	38.07	1.01	1.27	-0.64	0.0000
October	31	79.08	45.71	2.55	1.47	-1.60	0.0097
November	30	108.97	52.47	3.63	1.75	-1.96	0.0879
December	31	111.82	43.50	3.61	1.40	-2.43	0.0441

The Probability that rainfall greater than 6mm will fall in one day only is thus given by the Table 3. This however does not incorporate the possibility that it could rain more than 6mm for 2 consecutive days or 3 consecutive days and so on. The probability of it raining for 2 or more consecutive days must therefore be accounted for.

The probability that it rains (>6mm) for 2 consecutive days is equal to the probability that it rains (>6mm) on Day 1 **and** the probability that it rains (>6mm) on Day 2.

Therefore the probability of it raining more than 6mm for 2 consecutive days is given by;

$$P(2Days) = P(Day1) \cap P(Day2)$$

Thus if  $n$  equals the number of days in the month then the probability of rainfall greater than 6mm falling in  $n$  consecutive days is given by;

$$P(n Days) = P(Day1) \cap P(Day 2) \cap \dots \cap P(Day n)$$

but the probability of rainfall (>6mm) on Day 1 of the month is equal to the probability of rainfall (>6mm) Day 2 therefore;

$$P(n Days) = P(Day 1)^n$$

Using the probability values calculated in Table 3 this has been calculated for each of the respective months and is presented below in Table 4.

Table 4: Probability of Rainfall &gt; 6mm falling on n Consecutive Days by Month

Month	Days in Month	Probability that Daily Rainfall >6mm will fall for the below Number of Consecutive Days								
		1	2	3	4	...	28	29	30	31
January	31	0.2065	0.0427	0.0088	0.0018		7E-20	1E-20	3E-21	6E-22
February	28	0.1853	0.0343	0.0064	0.0012		3E-21			
March	31	0.0610	0.0037	0.0002	1E-05		1E-34	6E-36	4E-37	2E-38
April	30	0.0000	3E-10	6E-15	1E-19		1E-133	3E-138	5E-143	
May	31	0.0000	3E-23	1E-34	6E-46		0	0	0	0
June	30	0.0000	0	0	0		0	0	0	
July	31	0.0000	0	0	0		0	0	0	0
August	31	0.0000	0	0	0		0	0	0	0
September	30	0.0000	2E-09	8E-14	3E-18		3E-123	1E-127	6E-132	
October	31	0.0097	9E-05	9E-07	9E-09		4E-57	4E-59	4E-61	3E-63
November	30	0.0879	0.0077	0.0007	6E-05		3E-30	2E-31	2E-32	
December	31	0.0441	0.0019	9E-05	4E-06		1E-38	5E-40	2E-41	9E-43

The probability that rainfall, greater than 6mm will fall in a 24 hr period for at least one day in the month is then equal to the Probability that it rains more than 6mm for 1 day **or** 2 consecutive days **or** 3 consecutive days and so on.

Thus;

$$P(\text{Rain} > 6\text{mm in at least 1 day}) = P(1 \text{ Day}) \cup P(2 \text{ Days}) \cup \dots \cup P(n \text{ Days})$$

This is then equal to the sum of each row in Table 4 and thus for each month is as presented in Table 5 below.

Table 5: Probability that rain greater than 6mm will fall in a 24 hr period for at least one day in the month

Month	Probability of disruptive rainfall by month
January	0.2603
February	0.2274
March	0.0650
April	0.0000
May	0.0000
June	0.0000
July	0.0000
August	0.0000
September	0.0000
October	0.0098
November	0.0964
December	0.0461

Further if a task was executed over more than one month then the probability of that task being affected by disruptive rainfall is equal to the probability that disruptive rainfall falls in the first month **or** the second month **or** the third month in which the task is executed.

Thus if a task was to be executed over January and February the probability of that task being affected by rainfall is equal to;

$$P(\text{Disruptive Rain}) = P(\text{Disruptive Rain January}) \cup P(\text{Disruptive Rain February})$$

$$P(\text{Disruptive Rain}) = 0.2603 + 0.2274$$

$$P(\text{Disruptive Rain}) = 0.4877$$

Table 6: Probability that Rainfall will Disrupt a Task

ID	Name	Start	Finish	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	P(Rainfall) Affecting Task	
1	Muchawaste Landfill Project	01 November 2005	11 June 2009														
2	SECURE BRIDGING FINANCE																
3	Prepare Briefing Document	01 November 2005	14 November 2005											1		0.096	
4	Prepare Basic Business Case																
5	Basic Engineering Solution	15 November 2005	05 December 2005											1	1	0.142	
6	Basic Cost Estimate	06 December 2005	26 December 2005												1	0.046	
7	Financial Model [Bridging Draft]	27 December 2005	02 January 2006	1												0.306	
8	Assess Basic Feasibility	03 January 2006	09 January 2006	1												0.260	
9	Positive Feasibility Report	09 January 2006	09 January 2006	1												0.260	
10	Apply for a Conditional Operating Licence	10 January 2006	01 May 2006	1	1	1	1	1								0.553	
11	Receive Conditional Operating Licence	01 May 2006	01 May 2006					1								0.000	
12	Draft Offtake Agreement with local Council	02 May 2006	29 May 2006					1								0.000	
13	Negotiate Offtake Agreement with local council	30 May 2006	21 August 2006					1	1	1	1					0.000	
14	Sign Offtake Agreement	21 August 2006	21 August 2006								1	1				0.000	
15	Solicit expressions of Interest and Terms of Bridging Finance	25 July 2006	21 August 2006							1	1					0.000	
16	Acquire Bridging Funding	22 August 2006	04 September 2006								1	1				0.000	
17	Company Decides to Proceed with the Project	05 September 2006	05 September 2006									1				0.000	
18	Sign Bridging Finance Document	06 September 2006	06 September 2006													0.000	
19	Obtain Unconditional Operating Licence																
20	LEVEL 2 BASIC DESIGN AND ENGINEERING																
21	Preliminary Geotechnical Investigation	07 September 2006	04 October 2006									1	1			0.010	
22	Geotechnical Permeability Study	07 September 2006	04 October 2006									1	1			0.010	
23	Basic Volumetric Study	07 September 2006	04 October 2006										1	1		0.010	
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	05 October 2006	29 November 2006											1	1	0.106	
25	Revised Cost Estimate	30 November 2006	13 December 2006												1	1	0.142
26	FUNDING																
27	Prepare Finance Briefing document	07 September 2006	20 September 2006									1				0.000	
28	Submit Briefing document to potential funders	21 September 2006	21 September 2006									1				0.000	
29	Acquire and evaluate funding proposals	22 September 2006	19 October 2006										1	1		0.010	
30	Finalise Funding Requirements	12 January 2007	12 January 2007	1												0.260	
31	Appointment, Equity & Debt Providers	20 October 2006	11 January 2007	1										1	1	0.413	
32	Sign Funding Agreement	12 January 2007	12 January 2007	1												0.260	
33	FINANCIAL CLOSURE																
34	Environmental approval Process South Africa																
35	Produce basic Project Information Memorandum	06 December 2005	19 December 2005												1	0.046	
36	Prepare bidding document to solicit environmental consultants	20 December 2005	09 January 2006	1											1	0.306	
37	Environmental consultants respond to Bid	10 January 2006	20 February 2006	1	1											0.488	
38	Consortium evaluates bids and appoints consultant	07 September 2006	20 September 2006									1				0.000	
39	Scoping Process	21 September 2006	13 December 2006									1	1			0.010	
40	Applicant conducts EIA	14 December 2006	30 April 2008	1	1	1	1								1	0.599	
41	Specialist Studies	14 December 2006	26 December 2007												1	0.046	
42	Appeal Period and Process	27 December 2007	30 April 2008	1	1	1	1								1	0.599	
43	Authority issues a positive ROD																
44	EPC CONTRACT																
45	Shortlist potential EPC contractors	06 December 2005	30 January 2006	1												0.260	
46	Submit Level 2 Basic Design	30 November 2006	30 November 2006											1		0.096	
47	Contractors Prepare and Submit Bid	01 December 2006	22 February 2007	1	1										1	0.534	
48	Evaluate bids	23 February 2007	26 April 2007			1	1									0.292	
49	APPOINT EPC subject to Financial Close	06 March 2008	30 April 2008				1	1								0.065	
50	Detailed Design	01 May 2008	20 August 2008					1	1	1	1					0.000	
51	Detailed Geotechnical Investigation	01 May 2008	25 June 2008					1	1							0.000	
52	Detailed Volumetric Study	01 May 2008	25 June 2008					1	1							0.000	
53	Geotechnical Design	01 May 2008	23 July 2008					1	1	1						0.000	
54	Design Synthetic Lining System	01 May 2008	28 May 2008					1								0.000	
55	Produce Detailed Design	24 July 2008	20 August 2008							1	1					0.000	
56	Landfill Construction Phase																
57	Construction Programme																
58	Establishment	21 August 2008	27 August 2008								1					0.000	
59	Remove topsoil to Stockpile	28 August 2008	10 September 2008								1	1				0.000	
60	Excavate to Stockpile Bay 1 to Formation Level	11 September 2008	22 October 2008									1	1			0.010	
61	Survey Foundation	23 October 2008	23 October 2008										1			0.010	
62	Prepare Bay 1 Formation	24 October 2008	28 October 2008											1		0.010	
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	29 October 2008	06 January 2009	1									1	1	1	0.413	
64	Survey Foundation	07 January 2009	07 January 2009	1												0.260	
65	Survey Top of Clay Bay 1	08 January 2009	08 January 2009	1												0.260	
66	Prepare Clay for Liner on Bay 1	09 January 2009	14 January 2009	1												0.260	
67	Prepare Bay 2 Formation	15 January 2009	19 January 2009	1												0.260	
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	20 January 2009	16 March 2009	1	1	1										0.553	
69	Lay and Weld Geomembrane Liner Bay 1	15 January 2009	04 February 2009	1	1											0.488	
70	Lay and Weld Geotextile Protection Bay 1	19 January 2009	06 February 2009	1	1											0.488	
71	Lay Drainage Blanket Bay 1	09 February 2009	13 March 2009		1	1										0.292	
72	Survey Top of Clay Bay 1	17 March 2009	17 March 2009					1								0.065	
73	Prepare Clay for Liner on Bay 1	18 March 2009	23 March 2009					1								0.065	
74	Lay and Weld Geomembrane Liner Bay 2	24 March 2009	13 April 2009					1	1							0.065	
75	Lay and Weld Geotextile Protection Bay 2	26 March 2009	15 April 2009					1	1							0.065	
76	Lay Drainage Blanket Bay 2	16 April 2009	20 May 2009					1	1							0.000	
77	Survey Top of Drainage Blanket	21 May 2009	21 May 2009						1	1						0.000	
78	Place Waste Protection Layer	22 May 2009	04 June 2009					1	1							0.000	
79	Commission Landfill	05 June 2009	11 June 2009							1						0.000	
80	Receive first Public Waste	11 June 2009	11 June 2009							1						0.000	



Table 7: Cost Disruption Variables

ID	Name	Percentage Disruption Estimation			Mean %	Standard Deviation %
		Optimistic	Pessimistic	Anticipated		
1	<b>Muchawaste Landfill Project</b>					
2	<b>SECURE BRIDGING FINANCE</b>					
3	Prepare Briefing Document	0	0	0	0.000	0.000
4	<b>Prepare Basic Business Case</b>					
5	Basic Engineering Solution	0	0	0	0.000	0.000
6	Basic Cost Estimate	0	0	0	0.000	0.000
7	Financial Model [Bridging Draft]	0	0	0	0.000	0.000
8	Assess Basic Feasibility	0	0	0	0.000	0.000
9	Positive Feasibility Report	0	0	0	0.000	0.000
10	Apply for a Conditional Operating Licence	0	0	0	0.000	0.000
11	Receive Conditional Operating Licence	0	0	0	0.000	0.000
12	Draft Offtake Agreement with local Council	0	0	0	0.000	0.000
13	Negotiate Offtake Agreement with local council	0	0	0	0.000	0.000
14	Sign Offtake Agreement	0	0	0	0.000	0.000
15	Solicit expressions of Interest and Terms of Bridging Finance	0	0	0	0.000	0.000
16	Acquire Bridging Funding	0	0	0	0.000	0.000
17	Company Decides to Proceed with the Project	0	0	0	0.000	0.000
18	Sign Bridging Finance Document	0	0	0	0.000	0.000
19	<b>Obtain Unconditional Operating Licence</b>					
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>					
21	Preliminary Geotechnical Investigation	0	0	0	0.000	0.000
22	Geotechnical Permeability Study	0	0	0	0.000	0.000
23	Basic Volumetric Study	0	0	0	0.000	0.000
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	0	0	0	0.000	0.000
25	Revised Cost Estimate	0	0	0	0.000	0.000
26	<b>FUNDING</b>					
27	Prepare Finance Briefing document	0	0	0	0.000	0.000
28	Submit Briefing document to potential funders	0	0	0	0.000	0.000
29	Acquire and evaluate funding proposals	0	0	0	0.000	0.000
30	Finalise Funding Requirements	0	0	0	0.000	0.000
31	Appointment, Equity & Debt Providers	0	0	0	0.000	0.000
32	Sign Funding Agreement	0	0	0	0.000	0.000
33	<b>FINANCIAL CLOSURE</b>					
34	<b>Environmental approval Process South Africa</b>					
35	Produce basic Project Information Memorandum	0	0	0	0.000	0.000
36	Prepare bidding document to solicit environmental consultants	0	0	0	0.000	0.000
37	Environmental consultants respond to Bid	0	0	0	0.000	0.000
38	Consortium evaluates bids and appoints consultant	0	0	0	0.000	0.000
39	Scoping Process	0	0	0	0.000	0.000
40	Applicant conducts EIA	0	0	0	0.000	0.000
41	Specialist Studies	0	0	0	0.000	0.000
42	Appeal Period and Process	0	0	0	0.000	0.000
43	<b>Authority issues a positive ROD</b>					
44	<b>EPC CONTRACT</b>					
45	Shortlist potential EPC contractors	0	0	0	0.000	0.000
46	Submit Level 2 Basic Design	0	0	0	0.000	0.000
47	Contractors Prepare and Submit Bid	0	0	0	0.000	0.000
48	Evaluate bids	0	0	0	0.000	0.000
49	APPOINT EPC subject to Financial Close	0	0	0	0.000	0.000
50	Detailed Design	0	0	0	0.000	0.000
51	Detailed Geotechnical Investigation	0	0	0	0.000	0.000
52	Detailed Volumetric Study	0	0	0	0.000	0.000
53	Geotechnical Design	0	0	0	0.000	0.000
54	Design Synthetic Lining System	0	0	0	0.000	0.000
55	Produce Detailed Design	0	0	0	0.000	0.000
56	<b>Landfill Construction Phase</b>					
57	<b>Construction Programme</b>					
58	Establishment	2	8	5	5.000	1.225
59	Remove topsoil to Stockpile	2	10	5	5.667	1.650
60	Excavate to Stockpile Bay 1 to Formation Level	10	20	15	15.000	2.041
61	Survey Foundation	2	10	6	6.000	1.633
62	Prepare Bay 1 Formation	5	10	8	7.667	1.027
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	15	30	20	21.667	3.118
64	Survey Foundation	2	10	6	6.000	1.633
65	Survey Top of Clay Bay 1	2	10	6	6.000	1.633
66	Prepare Clay for Liner on Bay 1	5	10	8	7.667	1.027
67	Prepare Bay 2 Formation	5	10	8	7.667	1.027
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	10	20	15	15.000	2.041
69	Lay and Weld Geomembrane Liner Bay 1	10	30	15	18.333	4.249
70	Lay and Weld Geotextile Protection Bay 1	10	25	15	16.667	3.118
71	Lay Drainage Blanket Bay 1	2	10	5	5.667	1.650
72	Survey Top of Clay Bay 1	2	10	6	6.000	1.633
73	Prepare Clay for Liner on Bay 1	5	10	8	7.667	1.027
74	Lay and Weld Geomembrane Liner Bay 2	10	30	15	18.333	4.249
75	Lay and Weld Geotextile Protection Bay 2	10	25	15	16.667	3.118
76	Lay Drainage Blanket Bay 2	2	10	5	5.667	1.650
77	Survey Top of Drainage Blanket	2	10	6	6.000	1.633
78	Place Waste Protection Layer	2	10	6	6.000	1.633
79	Commission Landfill	0	0	0	0.000	0.000
80	Receive first Public Waste	0	0	0	0.000	0.000

Table 8: Calculated Financial Risk Mitigating Allowances

ID	Name	Step 2			Step 4	Step 5	Step 6		
		P(Rainfall Affecting Task)	Mean %	Standard Deviation %	Alpha	Lamda	Previously Designed Cost Estimate	Designed Cost Including Mitigation	Mitigating Allowance
1	<b>Muchawaste Landfill Project</b>								
2	<b>SECURE BRIDGING FINANCE</b>								
3	Prepare Briefing Document	0.096	0.00000	0.00000	0.000	1.0000	R 7,337.18	R 7,337.18	R 0.00
4	<b>Prepare Basic Business Case</b>								
5	Basic Engineering Solution	0.142	0.00000	0.00000	0.000	1.0000	R 148,842.09	R 148,842.09	R 0.00
6	Basic Cost Estimate	0.046	0.00000	0.00000	0.000	1.0000	R 7,500.41	R 7,500.41	R 0.00
7	Financial Model [Bridging Draft]	0.306	0.00000	0.00000	0.000	1.0000	R 25,041.25	R 25,041.25	R 0.00
8	Assess Basic Feasibility	0.260	0.00000	0.00000	0.000	1.0000	R 2,500.00	R 2,500.00	R 0.00
9	Positive Feasibility Report	0.260	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
10	Apply for a Conditional Operating Licence	0.553	0.00000	0.00000	0.000	1.0000	R 10,001.65	R 10,001.65	R 0.00
11	Receive Conditional Operating Licence	0.000	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
12	Draft Offtake Agreement with local Council	0.000	0.00000	0.00000	0.000	1.0000	R 11,001.65	R 11,001.65	R 0.00
13	Negotiate Offtake Agreement with local council	0.000	0.00000	0.00000	0.000	1.0000	R 33,000.00	R 33,000.00	R 0.00
14	Sign Offtake Agreement	0.000	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
15	Solicit expressions of Interest and Terms of Bridging Finance	0.000	0.00000	0.00000	0.000	1.0000	R 11,001.65	R 11,001.65	R 0.00
16	Acquire Bridging Funding	0.000	0.00000	0.00000	0.000	1.0000	R 7,500.41	R 7,500.41	R 0.00
17	Company Decides to Proceed with the Project	0.000	0.00000	0.00000	0.000	1.0000	R 1,500.41	R 1,500.41	R 0.00
18	Sign Bridging Finance Document	0.000	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
19	<b>Obtain Unconditional Operating Licence</b>								
20	<b>LEVEL 2 BASIC DESIGN AND ENGINEERING</b>								
21	Preliminary Geotechnical Investigation	0.010	0.00000	0.00000	0.000	1.0000	R 117,051.67	R 117,051.67	R 0.00
22	Geotechnical Permeability Study	0.010	0.00000	0.00000	0.000	1.0000	R 20,000.41	R 20,000.41	R 0.00
23	Basic Volumetric Study	0.010	0.00000	0.00000	0.000	1.0000	R 43,683.72	R 43,683.72	R 0.00
24	Basic Stage 1 Design: Sufficient to Produce EPC Lump Sum Price	0.106	0.00000	0.00000	0.000	1.0000	R 184,378.35	R 184,378.35	R 0.00
25	Revised Cost Estimate	0.142	0.00000	0.00000	0.000	1.0000	R 5,000.00	R 5,000.00	R 0.00
26	<b>FUNDING</b>								
27	Prepare Finance Briefing document	0.000	0.00000	0.00000	0.000	1.0000	R 5,250.10	R 5,250.10	R 0.00
28	Submit Briefing document to potential funders	0.000	0.00000	0.00000	0.000	1.0000	R 500.00	R 500.00	R 0.00
29	Acquire and evaluate funding proposals	0.010	0.00000	0.00000	0.000	1.0000	R 11,001.65	R 11,001.65	R 0.00
30	Finalise Funding Requirements	0.260	0.00000	0.00000	0.000	1.0000	R 500.00	R 500.00	R 0.00
31	Appointment, Equity & Debt Providers	0.413	0.00000	0.00000	0.000	1.0000	R 32,343.78	R 32,343.78	R 0.00
32	Sign Funding Agreement	0.260	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
33	<b>FINANCIAL CLOSURE</b>								
34	<b>Environmental approval Process South Africa</b>								
35	Produce basic Project Information Memorandum	0.046	0.00000	0.00000	0.000	1.0000	R 11,001.65	R 11,001.65	R 0.00
36	Prepare bidding document to solicit environmental consultants	0.306	0.00000	0.00000	0.000	1.0000	R 8,502.89	R 8,502.89	R 0.00
37	Environmental consultants respond to Bid	0.488	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
38	Consortium evaluates bids and appoints consultant	0.000	0.00000	0.00000	0.000	1.0000	R 6,000.00	R 6,000.00	R 0.00
39	Scoping Process	0.010	0.00000	0.00000	0.000	1.0000	R 76,301.96	R 76,301.96	R 0.00
40	Applicant conducts EIA	0.599	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
41	Specialist Studies	0.046	0.00000	0.00000	0.000	1.0000	R 276,291.87	R 276,291.87	R 0.00
42	Appeal Period and Process	0.599	0.00000	0.00000	0.000	1.0000	R 60,739.63	R 60,739.63	R 0.00
43	<b>Authority issues a positive ROD</b>								
44	<b>EPC CONTRACT</b>								
45	Shortlist potential EPC contractors	0.260	0.00000	0.00000	0.000	1.0000	R 25,041.25	R 25,041.25	R 0.00
46	Submit Level 2 Basic Design	0.096	0.00000	0.00000	0.000	1.0000	R 500.00	R 500.00	R 0.00
47	Contractors Prepare and Submit Bid	0.534	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
48	Evaluate bids	0.292	0.00000	0.00000	0.000	1.0000	R 25,857.40	R 25,857.40	R 0.00
49	APPOINT EPC subject to Financial Close	0.065	0.00000	0.00000	0.000	1.0000	R 21,000.00	R 21,000.00	R 0.00
50	Detailed Design	0.000	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
51	Detailed Geotechnical Investigation	0.000	0.00000	0.00000	0.000	1.0000	R 101,927.92	R 101,927.92	R 0.00
52	Detailed Volumetric Study	0.000	0.00000	0.00000	0.000	1.0000	R 30,343.78	R 30,343.78	R 0.00
53	Geotechnical Design	0.000	0.00000	0.00000	0.000	1.0000	R 60,041.25	R 60,041.25	R 0.00
54	Design Synthetic Lining System	0.000	0.00000	0.00000	0.000	1.0000	R 33,683.72	R 33,683.72	R 0.00
55	Produce Detailed Design	0.000	0.00000	0.00000	0.000	1.0000	R 21,004.95	R 21,004.95	R 0.00
56	<b>Landfill Construction Phase</b>								
57	<b>Construction Programme</b>								
58	Establishment	0.000	5.00000	1.22474	0.120	1.0250	R 23,160.05	R 23,160.05	R 0.00
59	Remove topsoil to Stockpile	0.000	5.66667	1.64992	0.162	1.0400	R 93,718.34	R 93,718.57	R 0.23
60	Excavate to Stockpile Bay 1 to Formation Level	0.010	15.00000	2.04124	0.201	1.0232	R 323,801.68	R 324,288.57	R 486.89
61	Survey Foundation	0.010	6.00000	1.63299	0.161	1.0370	R 1,000.04	R 1,000.64	R 0.61
62	Prepare Bay 1 Formation	0.010	7.66667	1.02740	0.101	1.0115	R 21,016.37	R 21,032.27	R 15.90
63	Excavate Bay 2 and Engineer Impermeable Clay Layer Bay 1 with excavations	0.413	21.66667	3.11805	0.306	1.0374	R 850,268.72	R 929,107.49	R 78,838.76
64	Survey Foundation	0.260	6.00000	1.63299	0.161	1.0370	R 1,000.04	R 1,016.23	R 16.20
65	Survey Top of Clay Bay 1	0.260	6.00000	1.63299	0.161	1.0370	R 1,000.04	R 1,016.23	R 16.20
66	Prepare Clay for Liner on Bay 1	0.260	7.66667	1.02740	0.101	1.0115	R 28,029.11	R 28,594.89	R 565.78
67	Prepare Bay 2 Formation	0.260	7.66667	1.02740	0.101	1.0115	R 21,016.37	R 21,440.60	R 424.23
68	Excavate From Stockpile and Engineer Clay Layer Bay 2 with Excavations	0.553	15.00000	2.04124	0.201	1.0232	R 691,915.43	R 750,603.53	R 58,688.11
69	Lay and Weld Geomembrane Liner Bay 1	0.488	18.33333	4.24918	0.418	1.0821	R 561,763.97	R 616,116.35	R 54,352.38
70	Lay and Weld Geotextile Protection Bay 1	0.488	16.66667	3.11805	0.306	1.0486	R 419,361.72	R 455,106.82	R 35,745.09
71	Lay Drainage Blanket Bay 1	0.292	5.66667	1.64992	0.162	1.0400	R 865,460.42	R 880,373.06	R 14,912.64
72	Survey Top of Clay Bay 1	0.065	6.00000	1.63299	0.161	1.0370	R 1,000.04	R 1,004.08	R 4.04
73	Prepare Clay for Liner on Bay 1	0.065	7.66667	1.02740	0.101	1.0115	R 4,000.59	R 4,020.75	R 20.15
74	Lay and Weld Geomembrane Liner Bay 2	0.065	18.33333	4.24918	0.418	1.0821	R 565,763.97	R 573,057.10	R 7,293.14
75	Lay and Weld Geotextile Protection Bay 2	0.065	16.66667	3.11805	0.306	1.0486	R 423,361.72	R 428,169.60	R 4,807.88
76	Lay Drainage Blanket Bay 2	0.000	5.66667	1.64992	0.162	1.0400	R 865,460.42	R 865,461.34	R 0.92
77	Survey Top of Drainage Blanket	0.000	6.00000	1.63299	0.161	1.0370	R 1,000.04	R 1,000.04	R 0.00
78	Place Waste Protection Layer	0.000	6.00000	1.63299	0.161	1.0370	R 40,059.40	R 40,059.40	R 0.00
79	Commission Landfill	0.000	0.00000	0.00000	0.000	1.0000	R 3,500.45	R 3,500.45	R 0.00
80	Receive first Public Waste	0.000	0.00000	0.00000	0.000	1.0000	R 0.00	R 0.00	R 0.00
							R 7,246,333.15	R 7,506,022.75	R 259,689.60

The Cells highlighted in Red Refer to the respective Steps being followed as described in Section 5.7 of the Main Document.

Level of Mitigation Cover Requested = **80%**  
 Thus required Beta Value = **0.848**

**Results**  
 Total Calculated Rainfall Cost Allowance at the Desired Risk Mitigation level of 80%  
**R 259,689.60**