Exploring the Number Sense of Final Year Primary Pre-service Teachers

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DECLARATION

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ABSTRACT

This study explored the number sense of 47 final year primary school pre-service teachers in Namibia and was motivated by the poor performance of Namibian primary school learners in both national and international standardised assessment tests. The literature review revealed that learner performance is linked to teacher subject knowledge (Ball, 1990, Ma, 1999) and that teachers' confidence in doing and teaching mathematics influences the way they teach and their willingness to learn mathematics (Ball, 1990; Graven 2004). Number sense studies of pre-service teachers (Kaminski, 1997; Tsao, 2004; Veloo, 2010; Yang, Reys & Reys, 2009) have indicated that the development of number sense should be a focus of primary pre-service teacher education.

The data in this mixed method research design were obtained from a Number Sense Questionnaire, a Written Computations Questionnaire and a Mental Calculations Questionnaire. These questionnaires were adapted from instruments developed by Professor Der-Ching Yang for 6th and 8th grade learners in Taiwan. Teacher confidence was measured by the McAnallen Confidence in Mathematics and Mathematics Teaching Survey. Six randomly selected pre-service teachers were interviewed to determine their use of number-sensible strategies.

The correlation analysis shows a strong relationship between number sense and mental calculations; between number sense and confidence in both the ability to do and the ability to teach mathematics and between mental and written calculations.

The overall results of this study reveal that the final year primary pre-service teachers demonstrate limited number sense and possess very few of the indicators of number sense that were described by Kalchman, Moss and Case (2001). The findings expose a lack of conceptual understanding of the domain numbers and operations, particularly in the domain of rational numbers and the operations of multiplication and division. The pre-service teachers have little or no access to a variety of flexible number-sensible strategies to solve problems and calculate mentally. They lack the fluency in basic facts and procedures to perform written calculations efficiently and correctly. Unexpectedly, the analysis of the confidence survey shows that they are confident in both their ability to do mathematics and their ability to teach mathematics.

It is recommended that mental calculations and computational estimation should become a focus of primary school mathematics education. Institutions responsible for teacher training should develop the number sense of pre-service teachers and research effective and long-term professional development programmes. The confidence and willingness of the teachers to learn can be used as an important resource.

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OPSOMMING

Hierdie studie ondersoek die getalbegrip van 47 finale jaar primêre skool voordiens-onderwysers in Namibië en is gemotiveer deur die swak prestasie van die Namibiese primêre skool leerlinge in beide nasionale en internasionale gestandaardiseerde assesseringstoetse. Die literatuurstudie het aan die lig gebring dat leerlinge se prestasie gekoppel is aan onderwyservakkennis (Ball, 1990, Ma, 1999) en dat onderwysers se vertroue in hulle vermoë om wiskunde te doen en te onderrig, die manier waarop hulle onderrig en hul bereidwilligheid om wiskunde te leer beïnvloed (Ball, 1990, Graven 2004). Studies van voordiens primêre onderwysers se getalbegrip (Kaminski, 1997; Tsao, 2004; Veloo, 2010; Yang, Reys & Reys, 2009) toon dat die ontwikkeling van getalbegrip 'n fokus van primêre voordiens-onderwyseropleiding behoort te wees.

Die data in hierdie gemengde metode navorsing is verkry uit 'n Getalbegrip, 'n Skriftelike Berekeninge en 'n Hoofrekene Vraelys. Hierdie vraelyste is gebaseer op die instrumente wat ontwikkel is deur Professor Der-Ching Yang vir graad 6 en 8 leerlinge in Taiwan. Onderwyservertroue is gemeet deur die McAnallen Confidence in Mathematics and Mathematics Teaching Survey. Ses ewekansig geselekteerde voordiens-onderwysers is ondervra om te bepaal watter sinvolle strategieë hulle gebruik om vrae oor getalbegrip te beantwoord.

Die korrelasie-analise toon 'n sterk verband tussen getalbegrip en hoofrekene; tussen getalbegrip en vertroue in die vermoë om wiskunde te doen en te leer, en tussen vermoë om hoofrekene en skriftelike bewerkinge te doen.

Die algehele resultate van hierdie studie dui daarop dat die finale jaar primêre voordiens-onderwysers oor beperkte getalbegrip en baie min van die aanwysers van getalbegrip wat deur Kalchman, Moss en Case (2001) beskryf is, beskik. Die bevindinge toon 'n gebrek aan begrip van die domein van getalle en bewerkings, veral in die domein van rasionale getalle en die bewerkings vermenigvuldiging en deling. Die voordiens-onderwysers beskik oor min of geen soepel strategieë om probleme op te los en hoofrekene te doen nie. Hulle beskik nie oor die vlotheit in basiese feite en bewerkings om skriftelike berekeninge doeltreffend en korrek uit te voer nie. Die vertroue wat voordiens-onderwysers uitgespreek het in hulle vermoë om wiskunde te doen en onderrig staan in sterk teenstelling met hierdie bevindige.

Dit word aanbeveel dat hoofrekene en skatting 'n fokus van primêre skool wiskunde-onderwys behoort te wees. Instansies gemoeid met onderwyseropleiding behoort die getalbegrip van voordiens-onderwysers te onwikkel en navorsing te doen oor effektiewe en lang-termyn programme vir professionele ontwikkeling. Onderwysers se vertroue en bereidwilligheid om te leer kan as 'n belangrike hulpbron gebruik word.

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ACRONYMS USED IN THIS STUDY

BEd	Bachelor of Education
BES	Basic Education Support
BETD	Basic Education Teachers Diploma
CCK	Common content knowledge
CPD	Continuous Professional Development
CF	Common Fraction
ETSIP	Education and Training Sector Improvement Plan
LP	Lower primary
MASTEP	Mathematics and Science Teacher Extension Programme
MCMMTS	McAnallen Confidence in Mathematics and Mathematics Teaching Survey
MCQ	Mental Calculation Questionnaire
MoE	Ministry of Education
NCTM	National Council of Teachers of Mathematics
NUNW	National Union of Namibian Workers
NSQ	Number Sense Questionnaire
PSTs	Pre-service Teachers
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SACSA	South Australian Curriculum Standards and Accountability
SCK	Specialised content knowledge
UP	Upper primary
USAID	United States Agency for International Development
WCQ	Written Computation Questionnaire
WN	Whole number

CHAPTER ONE

GENERAL INTRODUCTION

1.1   Introduction

This study explored the number sense of final year lower primary pre-service teachers in Namibia. Chapter 1 provides a background to the study and explains its relevance. It explains the context in which the study took place, gives a rationale for the study, demarcates the goal and the research problem and introduces the methodology. Finally, the chapter provides an overview of the study.

1.2   Context of the Study

This section describes perceptions of the effectiveness of the Namibian education system in general, relates these to the outcomes of mathematics education in particular and highlights connections to the effectiveness of mathematics teaching and mathematics teacher education.

In an address to the 2007 NUNW National Symposium on Productivity and Employment, Mihe Gaomab (2007), a Namibian economist and secretary to the Namibia Competition Commission, listed "poor quality, low efficiency, low economic and functional skills relevance" as some of the characteristics of the Namibian education system. He believes that our education system needs a complete overhaul so that it can produce the skills required to realise Namibia Vision 2030, which sees a diversified and growing manufacturing, service oriented, and knowledge based economy in Namibia.

The same sentiment about education in Namibia was expressed in a 2005 World Bank Study (in Links, 2010a) which stated, "The ineffectiveness of the education system renders most basic education graduates untrainable and unemployable" (p. 24). A skills shortage survey (Links, 2010b) indicated an acute skills deficit in the professional (e.g. engineers and accountants), the technical and the trade fields and viewed the curricula of tertiary institutions, including the university and the vocational training centres, as largely irrelevant and not aligned to international standards. The survey concludes that

> In the absence of scientifically derived projections and predictions, and given the levels of pessimism around the skills issue, in particular around the apparent slowness of key stakeholders to address the issue, it is evident that economic activity has already and is being affected by the existence of skills shortages or scarcities and that this situation

> will in all probability escalate and impact ever more severely in the future, across various economic and social sectors. (p.10)

All identified skills shortage fields require appropriate numeracy skills and/or a high levels of mathematical proficiency. However, educational statistics reveal both low numeracy levels at the end of primary school and a shortage of Grade 12 graduates with higher-level mathematics.

The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) conducts large-scale cross-national educational research projects every five to six years to generate information that can be used by decision makers in its 15 member countries to plan for the improvement of the quality of education. The sample from which English reading and mathematics scores are obtained, is selected from the target population of Grade 6 learners attending registered mainstream primary schools. The eight levels of mathematics competency are labeled: Pre Numeracy, Emergent Numeracy, Basic Numeracy, Beginning Numeracy, Competent Numeracy, Mathematically Skilled, Concrete Problem Solving and Abstract Problem Solving. Level 4, Beginning Numeracy, is defined as "Translates verbal or graphic information into simple arithmetic problems; uses multiple different operations (in the correct order) on whole numbers, fractions and/or decimals." (SACMEQ, 2010, p. 8). The results of SACMEQ III, which was conducted in 2007, showed that 81.7% of Namibian learners performed below Level 4, Beginning Numeracy. Equally disturbing was the fact that some Grade 6 teachers performed at equally low levels. It was estimated that approximately 40% of Grade 8 learners enter secondary school effectively illiterate and innumerate.

This situation leads to an annual failure rate of more than 50% of Grade 10 learners in the external Junior Certificate examinations. Also, in 2009, only 672 (1.86%) of Grade 10 and 415 (2.67%) of Grade 12 learners enrolled for higher level studies in mathematics and only 15% of 17 000 Grade 12 learners qualified for further studies or training in mathematics, science and technology related fields (Directorate of National Examinations and Assessment, 2010).

Over the last ten years, various studies in Namibia investigated the effectiveness of teaching in general and mathematics teaching in particular: Presidential Commission on Education, Culture and Training (Ministry of Basic Education, Sport & Culture, 1999); Mathematics Inquiry (Mathematics and Science Teachers Extension Programme (MASTEP), 2002); Basic Education Support Project (American Institutes for Research, 2006; LeCzel, 2004),; Mathematics Consultancy (Clegg & Courtney-Clarke, 2009). All findings point to serious

shortcomings in the quality of mathematics teaching linked to poor teacher subject knowledge and understanding, especially at primary school level. Concerns were raised about the standards and relevance of primary school teacher education programmes.

Against this background, the Ministry of Education (MoE) has initiated an ambitious Education and Training Sector Improvement Plan (ETSIP) to address the quality of education at all levels. English and mathematics have been singled out as focus areas for improvement. The new Curriculum for Basic Education (MoE, 2008) identifies mathematics as a key learning area, recognises numeracy as one of the core skills and anticipates the implementation of mathematics as a compulsory subject to Grade 12 from 2012.

Until the end of 2010, Namibian primary school teachers were educated at four colleges of education and qualified with a three-year Basic Education Teachers Diploma (BETD). A pass in mathematics was not an admission requirement for lower primary teacher trainees and there is very little evidence in the curriculum and syllabus for Lower Primary Education (Ministry of Education, 2001) that a lack of numeracy skills has been acknowledged or addressed during pre-service training. The BETD programme did not deliver the expected teacher quality and the four colleges of education have been fully incorporated into the university since the beginning of 2011. The last BETD students will graduate in 2012 and the BETD programme is being replaced by a BEd (Primary) degree. This implies higher admission criteria for prospective primary school teachers and a new primary teacher education curriculum with the expectation that the quality of teaching in primary schools will improve in the future.

1.3   Rationale for the Study

This section relates some anecdotal and research evidence of both learner and teacher mathematics competence that has motivated the researcher to investigate the number sense of primary pre-service teachers. It defines number sense and quotes research studies that highlight the importance of number sense and related skills as a topic in primary school mathematics and the relevance of teacher knowledge in this domain.

During the researcher's involvement in a school support programme in 2007/2008, she had the opportunity to observe and reflect upon the ways in which Grade 7 learners solve a variety of simple tasks. Learners still count on their fingers to find the answers to 5×6 and have to revert to count-all to find 6×6. Simple word problems, such as "What is the cost of 10 T-shirts if one T-shirt costs N\$65?" are solved by repeated addition. Situations that

involve estimation are solved by guessing and resolved by consensus without any reasoning, reference to reality or mathematical principles. Grade 7 learners will be satisfied that a person may be 20m – or even 90m - tall, if the majority agrees. The absence of known number facts, the lack of meaning for operations on numbers and the inability of the learners to make sense of situations that involve numbers and operations and to use numbers flexibly and with understanding to solve real life problems and judge the reasonableness of the results at the end of primary school, points to a complete lack of number sense.

A recent consultancy (Clegg & Courtney-Clarke, 2009) identified the poor quality of teaching at Namibian primary schools, which emphasises procedural thinking and rote learning as the most important contributing factor to poor performance of learners in mathematics. They suggest that primary school teachers lack the confidence to do more learner-centred teaching involving problem solving and the exploration of different methods, approaches and applications due to their own poor mathematical knowledge base, conceptual understanding and strategic competence. These findings are corroborated by research done in Namibia by Haufiku (2008) who established a median mark of 60% when he tested lower primary teachers' knowledge of the Grade 4 and 5 mathematics curriculum content.

The National Council of Teachers of Mathematics (NCTM) states that learners with number sense "naturally decompose numbers, use particular numbers as referents, solve problems using the relationships among operations and knowledge about the base-ten system, estimate a reasonable result for a problem, and have a disposition to make sense of numbers, problems, and results" (NCTM, 2000, chapter 3). Number sense may be defined as a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations (Reys & Yang, 1998; Tsao, 2004). The term "numeracy" is strongly associated with number sense as reflected in Askew, Biddy and Brown (2001) who view numeracy as "the possession of an integrated network of knowledge, understanding, techniques, strategies and application skills concerned with numbers, number relationships and operations" (p. 3).

Proficiency in mental calculation and computational estimation is related to number sense. The methods used to calculate mentally involve quantities and number relationships and are variable and highly flexible, often depending entirely on the numbers appearing in the calculation. Recent research into the development of mathematical knowledge (Anghileri, 2006; Carpenter, et al., 1998; Heirdsfield & Cooper, 2004; Reys & Reys, 1998) has provided

strong evidence that mental calculations and number sense develop together and "one cannot exist without the other" (Griffin, 2003, p. 306). Proficiency in computational estimation is regarded as a manifestation of number sense. Veloo (2010, p. 39) summarises that good estimators have a good grasp of basic facts, place value and arithmetic properties; are skilled at mental computation; are self-confident and tolerant of error; and can use a variety of strategies and switch easily between strategies.

McIntosh, Reys and Reys (1992) argue that the acquisition of number sense by all learners should be a major goal of compulsory education, because in the information age and the technological world both adults and children have to deal with a wider range of numbers in a broader context using new tools, such as calculators and computers. Indeed the teaching and learning of number sense is considered a major topic in international mathematics curricula (Anghileri, 2006; Australian Education Council, 1991; NCTM, 2000; Rose, 2009; van den Heuvel-Panhuizen & Wijers, 2005; Yang, Reys & Reys, 2009). The latest Namibian primary school mathematics syllabuses also stress that "emphasis is to be placed on mental arithmetic strategies to develop learners' awareness of number and number sense" (Ministry of Education, 2006, p.2).

Teaching number sense focuses on learners and their solution strategies rather than on the right answers; on thinking rather than the mechanical application of rules; and on learner-generated solutions rather than on teacher-supplied answers (Reys et al., 1992). It is therefore essential for primary school teachers to know and understand how the number system works, to be able to use this understanding in flexible ways to make mathematical judgments and to have a repertoire of strategies for handling numbers and operations (NCTM, 1991). Without this, they will be ill-equipped to make sense and take advantage of children's often unorthodox but very number sensible solution strategies. Askew, Biddy and Brown (2001) believe that it is necessary to raise the awareness among teachers of the nature and development of numeracy and of the detailed achievements and difficulties of their pupils to help them focus their teaching. This crucial role of teacher knowledge was emphasised in Ofsted's 2002 evaluation of the first three years of England's National Numeracy Strategy (in Vollard, et al., 2008) which stated that deeper changes in teaching and learning were hampered by teachers who were "not confident in their teacher knowledge, and did not have a grasp of all relevant mathematical concepts" (p. 22).

In conclusion, the concept "number sense" is not used or promoted in mathematics education in Namibia and it is doubtful whether mathematics educators, at both schools and tertiary

institutions, understand the depth of the concept or the relevance of teaching for number sense to develop the mathematical proficiency of learners. Mathematics teaching in primary school classrooms is still characterised by drill, practice and rote learning despite efforts by policy makers and curriculum developers to implement changes, which are more in line with the world-wide reform movement towards a more constructivist and learner-centred paradigm. Mathematics pre-service teachers are a product of how and what they were taught in schools and will, without an appropriate education at tertiary level, carry the same practices forward to their own teaching at school.

Believing that one cannot develop number sense in others if one does not possess a certain measure of number sense oneself, this study explores the number sense of final year primary pre-service teachers.

1.4   Statement of the Problem

The poor foundation laid at primary school may be the major cause of the small number of learners who are willing and able to enter advanced mathematics courses at secondary level. To improve teaching at lower primary level, the Ministry of Education intends to develop a national numeracy strategy for Namibia. This, however, does not address what may be the root cause of the problem, i.e. the low levels of numeracy and the lack of number sense of primary school mathematics teachers. This study intends to provide some baseline data on the mathematical proficiencies of primary pre-service mathematics teachers.

The aim of this study is to answer the key research question:

Do Namibian final year primary pre-service teachers possess skills, knowledge, strategies and confidence that are related to number sense?

 The following sub-questions will guide the study:
1. How proficient are final year primary pre-service teachers in performing written computations?
2. How proficient are they in mental calculations?
3. Does their performance on a number sense test correlate with their performance on a written computation and a mental calculations test?
4. Do they use any number-sensible strategies to answer number sense test-items?
5. Are they confident in their own ability to do mathematics and teach mathematics?

The study may contribute to the understanding of the importance of developing number sense in pre-service teachers and influence the Primary Education Mathematics Syllabus for the BEd (Primary) degree in Namibia.

1.5   Research Methodology

This study employed a mixed methods approach to data collection and interpretation. According to Mackenzie and Knipe (2006), with the research question 'central', data collection and analysis methods should be chosen as those most likely to provide insights into the question with no philosophical loyalty to any alternative paradigm.

A quantitative research approach was used to answer sub-questions 1, 2, 3 and 5. The instruments were adapted with the permission of the researchers who developed them. The Number Sense Questionnaire (NSQ), Mental Computation Questionnaire (MCQ) and the Written Computation Questionnaire (WCQ) have been adapted from instruments developed by Professor Der-Ching Yang in 1997 (Reys & Yang, 1998) for Grade 6 and Grade 8 learners in Taiwan and used by Tsao (2004) with elementary pre-service teachers. McAnallen (2010) developed the McAnallen Confidence in Mathematics and Mathematics Teaching Survey (MCMMTS) as part of her thesis for the degree of Doctor of Philosophy. The choice of instruments from outside southern Africa was based on the failure to locate similar instruments of southern African origin and the researcher's inexperience in designing similar suitably reliable and valid instruments herself.   Descriptive statistics were calculated for all the test scores and correlation analysis was performed between the data obtained from the various instruments.

To answer sub-question 4 a qualitative approach was used. A semi-structured interview was conducted with a randomly selected sample of 10% of the pre-service teachers on six items of the Number Sense Questionnaire to establish whether students use number sensible strategies to estimate answers, and to triangulate the findings of the quantitative data.

The research was conducted with the 2011 final year primary pre-service teachers at a suitable teacher training campus in Namibia.   The choice of the purposive sample is explained in detail in Chapter 3. To increase the reliability and validity of the instruments and the semi-structured interview a pilot study (Teijlingen & Hundley, 2001) was conducted with local lower primary teachers.

1.6   Definition of terms and concepts

Lower primary pre-service teachers are student teachers preparing to teach Grades 1 to 4 at Namibian primary schools.

Upper primary pre-service teachers are student teachers preparing to teach mathematics in Grades 5 to 7 at Namibian primary schools.

Learners are children or young people attending schools in Namibia.

Number sense may be defined as a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations.

Mental calculations are calculations that do not involve the use of algorithms and are performed using known facts, derived facts and variety of strategies. Mental calculations are performed without the use of pencil and paper or calculators, but may be accompanied by jottings on paper (e.g. the use of the empty number line) to keep track of one's working or to explain one's strategy.

Computational estimation is the process of simplifying an arithmetic problem using some set of rules or procedures to produce an approximate but satisfactory answer through mental calculation.

1.7   Overview of the Study

The study is organised into five chapters.

Chapter 1 provides an overview of the study. In this chapter, the background of the study, the rationale, the research problem and the research methodology are discussed briefly. The chapter concludes with an overview of the whole study.

Chapter 2 reviews the relevant literature and provides the theoretical background to the research. It deals with research done in Namibia; the concept of number sense; the relationship between number sense, mental calculations and computational estimation; and

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primary school teachers' and pre-service teachers' knowledge of mathematics in the content area of Number and their confidence in doing and teaching mathematics.

Chapter 3 outlines the research approach and design. Aspects dealt with in this chapter include the selection of the sample, the design of instruments, the research site, the piloting and refining of instruments. The approach used to analyse and interpret data, limitations and ethical considerations are described and discussed.

Chapter 4 presents, discusses and interprets the research findings in relation to the main research question and the sub-questions.

Chapter 5 gives a summary of the research results, draws conclusions and makes recommendations based on the research findings.

CHAPTER TWO

LITERATURE REVIEW

Chapter 2 reviews the relevant literature and provides the theoretical background to the research. It deals with research done in Namibia, the concept of number sense, the relationship between number sense, mental calculations and computational estimation; primary school teachers' and pre-service teachers' knowledge of mathematics in the content area of Number and their confidence in doing and teaching mathematics.

2.1   Introduction

During the last part of the 20th century, new and important perspectives on mathematics education have emerged. Mathematics is viewed as a sense-making human activity involving problem solving and mathematical modelling of reality as opposed to mathematics as a body of facts and procedures (De Corte, 2004; Kilpatrick, Swafford & Findell, 2001; Schoenfeld, 1992).

This new vision of what mathematics is, has had a profound impact on the goals of mathematics education, as Schoenfeld (1992, p.334) puts it "goals for mathematics instruction depend on one's conceptualization of what mathematics is, and what it means to understand mathematics."

"Mathematics for all" has been a rallying call for decades and is reflected in the urgency to make mathematics a compulsory subject to Grade 12 in South Africa and Namibia. A mathematically literate nation can compete in the increasingly global technological world of work and information. Access to and success in economic activities demands basic skills and understanding of mathematics. Any ordinary citizen in his daily life has to deal mathematically with resources such as time and money.

The achievement of the new goals of mathematics education depends on learning experiences that differ from those provided in the traditional subject-centred classroom. The paradigm of learning based on a socio-constructivist view has become widely accepted as a foundation of learning with understanding and teaching for understanding. Windschitl (2002) sees the challenges for the teachers not only in having to acquire new skills, but also in understanding the true nature of constructivism, changing classroom cultures to be in line with constructivist philosophy and dealing with conservatism in education that works against the efforts to teach

for understanding. Darling-Hammond (1996) argues that the major challenge education faces in this century will be the advancement of teaching and the resolution of that challenge will be to "develop a knowledge for a very different kind of teaching" (p. 7) than has been the norm.

2.2   *The Goal of Mathematics Education*

The focus of mathematics instruction moved from problem solving in the 1980s to the development of mathematical thinking as major goals. Five categories of cognition emerged as important aspects of mathematical thinking, i.e. the knowledge base, problem solving strategies, monitoring and control, beliefs and affects and practices (Schoenfeld, 1992). Since then researchers have refined this focus of mathematics instruction. De Corte (2004) formulates the acquisition of a mathematical disposition as the major goal of mathematics education. Such a disposition requires the mastery of five categories of aptitude (ibid., p. 282): A well-organised and flexibly accessible domain-specific knowledge base that constitutes the contents of mathematics as a subject field; strategies for problem solving; metacognitive knowledge; cognitive self-regulation; mathematics-related beliefs about the self, the social context and about mathematics and mathematics learning.

The framework of "mathematical proficiency", a term that was developed by Kilpatrick et al. (2001) based on a review of research, is widely used in research and textbooks (Baroody, Lai & Mix, 2006; Kaasila, Pehkonen & Hellinin, 2010; van de Walle, Karp & Bay-Williams, 2007) to describe the desired outcomes of mathematics education. According to the Kilpatrick et al. (2001), mathematical proficiency has five strands (ibid., p. 116):

1. *Conceptual understanding* refers to an integrated and functional grasp of mathematics. Conceptual understanding is reflected in the ability to represent mathematical situations in different ways and applying the most useful representations in different situations.
2. *Procedural fluency* refers to the knowledge of procedures, when and how to use them and the skill in performing them flexibly, accurately and efficiently. It also includes the knowledge of ways to estimate the results of a procedure. Rapid, automatic access to facts and procedures enables a learner to pay attention to the interrelationship between concepts and the task of solving a problem by reducing the very heavy load on working memory.
3. *Strategic competence* refers to the ability to formulate*,* represent and solve mathematical problems. A proficient problem solver is flexible in his approach to

solving non-routine problems, forms mental representations of problems, detects mathematical relationships and devises new solution methods where needed.

4. *Adaptive reasoning* refers to the capacity for logical thought, reflection, explanation and justification. "In mathematics, adaptive reasoning is the glue that holds everything together, the lodestar that guides learning" (ibid. p. 129). Adaptive reasoning includes informal explanation and justification and intuitive and inductive reasoning based on pattern, analogy and metaphor.

5. *Productive disposition* is a tendency to see mathematics as sensible, useful and worthwhile and seeing oneself as capable to learn, understand and do mathematics.

These strands are not independent; they represent different aspects of a complex whole and have to be developed together. To become proficient, learners need "to spend sustained periods of time doing mathematics – solving problems, reasoning, developing understanding, practicing skills – and building connections between their previous knowledge and new knowledge" (ibid. p. 133).

Baroody et al. (2006) argue that conceptual understanding is a key basis for the other aspects of mathematical proficiency. Conceptual understanding can be seen as a web of connections. It can support the efficient, flexible and appropriate use of skills and procedures; a web of interconnected knowledge facilitates both strategic thinking and adaptive reasoning and lastly, a productive disposition is more likely to develop if mathematics makes sense to learners.

2.3   Primary School Mathematics Education in Namibia

2.3.1   Background

After independence in 1990, Namibia implemented complex constructivist reforms in teaching and learning in primary education. The approach to teaching and learning is based on the paradigm of learner-centred education with the aim to develop learning with understanding. Learning should take into account learners' prior knowledge and experience; actively involve learners in the learning process through cooperation and collaboration. The ambitious reform program was implemented in the context of rapid expansion of primary school education and severely limited resources. At the same time English, a language over which many primary school teachers and learners had and still have only a limited command, was introduced as the medium of instruction from Grade 4. The apartheid segregation of

schools was lifted and classrooms today include a greater range of learners from diverse cultural, linguistic and socio-economic backgrounds and mathematical competences.

These wide-ranging changes have threatened the quality of teaching and learning as Namibian teachers have found it increasingly difficult to interpret and practice the new education policies (Van Graan & Leu, 2006). Concerned with declining quality, the government has outlined a new reform program, *The Strategic Plan for the Education and Training Sector Improvement Programme* (ETSIP) (Government of the Republic of Namibia, 2005), which appears to shift Namibia towards more standards-based and behaviorist approaches while maintaining the constructivist principles of the past.

2.3.2   Review of Namibian research into the state of mathematics education

This section will briefly review the findings of three general reports on the state of mathematics education in Namibia, three research studies into the subject content knowledge of lower primary teachers and the results of three attainment tests that have been conducted over the last 10 years.

The Presidential Commission on Education, Culture and Training (Ministry of Basic Education Sport & Culture, 1999) found that many teachers feel inadequate to teach mathematics and are unable to develop the skills that children need to succeed in primary school and unless "the foundations are secured, it will be extremely difficult to build mathematical and scientific success at secondary level" (p.112). To address the shortcomings the Commission recommended that "further increments of professional skill and knowledge must be constantly added" (p. 181).

The Mathematics Inquiry (Mathematics and Science Teachers Extension Programme (MASTEP), 2002) raised the same concerns. The structure and curriculum of the Basic Education Teacher Diploma (BETD) offered too little subject content preparation of primary school teachers, assuming that school content is mastered and sufficient for teaching at this level resulting in lack of teacher competence. Teaching in school lacked application and contextualization, created little "feel" for number and emphasised rote procedures (p. 21). Among others, it was recommended that there should be "continuous national upgrading programmes focusing on mathematics teaching for teachers at all phases to strengthen their subject confidence and upgrade their content knowledge and methodology" (p. 24).

The Mathematics Consultancy (Clegg & Courtney-Clarke, 2009) found that none of the recommendations made in these reports had been followed up or implemented, with the result that the situation has not improved over the last 10 years. They found that mathematics teachers tend to be insecure in their subject knowledge; mental mathematics is either taught poorly or not at all; routines, procedures and rules are taught as isolated bits of knowledge; there is very little focus on conceptual understanding and mathematical processes such as reasoning and problem solving; the level of teacher support in mathematics countrywide is inadequate both in terms of the amount and the kind of support given and resources for mathematics teaching are minimal and poorly organised. The pre-service teacher training programmes are largely theoretical and much of the mathematics content that is taught is inappropriate, e.g. upper primary pre-service teachers are taught the content of the Grade 8 – 10 mathematics curriculum, often from the textbooks that are used in schools. The consultancy also established that continuous professional development (CPD) for primary school teachers is still limited to occasional cascade-model facilitator training which rarely filters down to the classroom teachers. It was recommended that numeracy levels of primary school teachers should be tested and a CPD programme for teachers should be developed as a first priority.

Due to the general lack of research into mathematics education and teacher knowledge and competence in Namibia, the three reports by the Presidential Commission (Ministry of Basic Education, Sport and Culture, 1999), the Mathematics Inquiry (Mathematics and Science Teachers Extension Programme, 2002) and the Mathematics Consultancy (Clegg & Courtney-Clarke, 2009) were based on observations, inputs from teachers, advisory teachers, government officials and other stakeholders and anecdotal evidence. Three research studies into the subject content knowledge of Namibian primary school teachers, reviewed below, confirm the findings of these reports.

Makuwa (2005) reported on the outcomes of both teachers and learners in the SACMEQ II survey that was conducted in 2000. He found that the level of subject matter knowledge among Namibian Grade 6 teachers "leaves much to be desired" (p. 183): 49% of the teachers did not reach the highest levels 7 and 8 of learner performance. He recommended remediation of the situation through in-service training courses and an increase in the standards in teacher training colleges. Duthilleul and Allen (2005) investigated the relative impact of teacher factors on the SACMEQ II mathematics achievement of Grade 6 pupils in Namibia after adjusting for the home backgrounds of pupils and school resources. They found that both subject matter knowledge and pedagogical skills of the teachers mattered and recommended

that only individuals with minimum relevant subject matter knowledge should be admitted to teacher education. Alternatively, teacher education should provide opportunities for students to improve their knowledge to the required level.

Haufiku (2008) conducted a study to investigate the subject content knowledge of 30 lower primary teachers at five schools in the Ohangwena region in northern Namibia. His research instrument was adapted from the BES pilot attainment tests for Grade 4 (see below) and were based on the content of the Grade 4 and 5 mathematics syllabuses for learners. He established a mean raw score of 20.2 over 30 test items. When he analysed the results by teaching qualification he found that the teachers with a Namibian BETD qualification had the lowest mean score of 14. The most poorly answered questions were from the topic areas Number Concepts, Measures and Mensuration. As part of his research, he conducted semi-structured interviews with a sample of the teachers, established that most of them were aware of their shortcomings of mathematical content knowledge and admitted that they were not able to assist Grade 7 learners (often their own children) with any problems in mathematics. He also confirmed that teachers had either no or very sporadic opportunities for CPD during the past years.

Poor teacher subject content knowledge may be an important factor contributing to the poor performance in mathematics of Namibian primary school children on standardised attainment tests. The USAID Basic Education Support Project (BES) conducted Grade 4 standardised attainment tests in 2005, 2006 and 2007. The baseline was established in 2005 in 183 schools with 3236 learners in six northern regions of Namibia. They reported a mean raw score of 9.09 over 30 test items. The items were categorised into four performance levels and 84.7% of the learners performed at the lowest two levels.

Based on the BES pilot projects the Directorate of National Examinations developed a National Standardised Achievement Test for Grade 5 learners, which was conducted country-wide for the first time in 2009. The results, shown in Table 2.1, were reported at the Bi-Annual National Conference entitled "Collective Commitment to Quality Education" in September 2010. There was consensus among the conference participants that these "results are disastrous" and action had to be taken to improve the situation.

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Table 2.1 Results for 2009 Standardised Achievement Test Grade 5

	% of learners
Below basic	46
Basic	44
Above basic	8
Excellent	2
Total	100
Below basic: Learner experiences consistent difficulty in dealing with the subject and demonstrates little to no ability of skills and knowledge	
Basic: Learner demonstrates sufficient knowledge but has limited skills in dealing with the subject.	

The data provided by standardised tests give an indication of performance on a sample of individual components of mathematics competence. The test items are based on a matrix mapping syllabus topics against knowledge, comprehension and application as defined by Bloom's Taxonomy of Educational Objectives. The data do not provide an insight into higher order thinking skills, mathematical reasoning or problem solving abilities and have therefore limited scope to inform us of the level of conceptual understanding that a learner has developed.

The National Standardised Achievement Tests will in future be conducted for Grade 5 and Grade 7 learners in alternative years and the fully analysed results distributed to each school. However, it is left up to the teachers and school management to take action at school level to address the shortcomings. The question is whether the limited competencies that the mathematics teachers exhibit, qualify them to devise remedial actions without the support of outside agencies and interventions to address their own lack of subject and pedagogical content knowledge.

Given the diversity of cultural, linguistic and socio-economic backgrounds of the majority of young Namibian learners and the poor performance of primary school learners on regional and local attainment tests, Namibia should take cognizance of research that states that in developing countries, "early interventions must be geared toward entire systems, as these systems are frequently at the same levels as children who in the developed world are seen as needing special help" (Reubens, 2009, p. 2).

Against this background, the development of number sense may be a starting point in the search for action to improve teachers' mathematical proficiency of numbers and operations as the foundation of primary school mathematics.

2.4   Number Sense

This section discusses the concept "number sense", its characteristics and proposed theoretical frameworks and clarifies the similarities and differences between number sense and numeracy. The term "number sense" is relatively new to the language of mathematics but research in this area over the last two decades has refined and defined the understanding of this elusive concept.

2.4.1   Introduction

Many people are very skillful in dealing with numerical situations in their daily lives without relying on calculations using paper-and-pencil or the calculator. Manifestations of number sense can be seen in all walks of life: in the casual worker who realises that she has been underpaid; in a builder who estimates the building cost of a wall; in the engineer who judges the feasibility of a bridge design; in an auditor who spots an irregularity in the accounts of a client; in a person who judges the reasonableness of a restaurant bill.

The origins of the term "number sense" are not clear. It seems that it was first used in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) in the United States which state

> Intuition about number relationships helps children make judgments about the reasonableness of computational results and of proposed solutions to numerical problems. Such intuition requires good number sense. Children with good number sense (1) have well-understood number meanings, (2) have developed multiple relationships among numbers, (3) recognize the relative magnitudes of numbers, (4) know the relative effect of operating on numbers, and (5) develop referents for measures of common objects and situations in their environments (p. 38).

Working with numbers and developing number sense has become the most important goal of primary school mathematics education (Anghileri, 2006; Australian Education Council, 1991; Rose, 2009; NCTM, 2000; van den Heuvel-Panhuizen & Wijers, 2005; Yang, Reys & Reys, 2008). Kilpatrick et al. (2001) argue that the concepts in the domain of number "serve as fundamental building blocks for the entire mathematics curriculum" (p. 106) and form the backbone to understanding measurement, shape and space, algebra and data handling. In this

context, Seeley and Schielack (2008) have stated that for learners to succeed in algebra, one of the best tools they can have is a deep understanding of the number system, its operations, and the properties related to those operations. McIntosh, Reys and Reys (1992) believe that there will be an increased focus on developing and maintaining number sense in the 21st century.

Case (1989) observed that mathematical activity is present in all cultures, even without formal schooling. He hypothesises that number sense is both universal and natural, but that the degree of number sense varies between individuals, groups and cultures and is influenced by the type and amount of mathematical activity in which young children engage. Researchers (Baroody et al., 2006; Chard et al., 2008; Gersten, Jordan & Flojo, 2005) argue that children who do not grow up in an environment that exposes them early to quantitative thinking and analysis will not acquire number sense without formal explicit instruction. These include children from low-income families, minority groups, children from homes where both parents work, non-English speakers and children living in poverty. In this context Masanja (1996, p. 198) observed that the mathematics that Tanzanian children met in school differed tremendously from what they encountered in day-to-day life. He maintains that the cultural gap between formal and informal knowledge coupled with the early use of English as a language of instruction has created serious problems in the conceptualization of mathematics.

Research studies (Cranfield et al., 2005; Hiebert, Carpenter & Moser, 1982; Irwin, 1996; Nagel & Swingen, 1998; Reys, & Yang, 1998; Sowder, 1992) indicate that early number sense develops over time and through sequential levels, starting with counting through to the use of known facts and derived strategies to solve problems involving multi-digit numbers. Later mathematics difficulties can be associated with a lack of a sound foundation for mathematical understanding and become ever more difficult to remediate as time passes (Chard et al., 2008). This has serious implications for the situation in Namibia where all research points to problems with acquisition of number sense in the primary school years.

2.4.2   The characteristics of number sense

Most characterisations of number sense focus on its intuitive nature, its gradual development and the way in which it is manifested. Number sense describes an "intangible quality possessed by successful mathematicians" (McIntosh et al., 1992, p. 3); is "considered a desirable trait to foster" (Hope, 1989, p. 12); relates to a "deep understanding of number" (Griffin, 2003, p. 306); entails "a good intuition about numbers and their relationships"

(Howden, 1989, p.11); and "refers to several important, but elusive capabilities" (Greeno, 1991, p. 170).

Researchers at a number sense conference at San Diego University (Sowder & Schapelle, 1989) agreed that it is easy to detect number sense by looking at behavioural characteristics of people who possess a high degree of it. These people have a disposition to make sense of numerical situations, look at problems holistically, and use numbers flexibly to do mental calculations, produce reasonable estimates of numerical quantities and use figures to support an argument. Resnick (1989, p. 37) links numbers sense to higher order thinking and lists the following key features of number sense: Number sense is non-algorithmic; tends to be complex; often yields multiple solutions, each with costs and benefits, rather than unique solutions; involves nuanced judgements and interpretation and the application of multiple criteria; often involves uncertainty; involves self-regulation of the thinking process and imposing meaning; number sense is effortful.

No universally accepted definition of number sense exists, but various researchers have attempted to define number sense by emphasising what they regard to be the salient features. Markovits and Sowder (1994) define number sense as

> … a well-organised conceptual network of number information that enables one to relate numbers and operations to solve problems in flexible and creative ways" (p. 23).

McIntosh, Reys and Reys (1992) state that number sense is

> …. a propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics has certain regularity (makes sense) (p. 4).

From the behavioural characteristics, the links to higher order thinking and the definitions it can be deduced that number sense fulfils the characteristics of a mathematical proficiency. It requires learners to understand and connect their number knowledge, and this connection is a key factor in whether they can use what they know productively in solving problems. Number sense depends on knowledge about one's own thinking and ability to monitor one's own understanding and problem-solving activity, i.e. strategic competence and adaptive reasoning. Finally, it needs a disposition to make sense of numerical situations.

2.4.3   Theoretical frameworks for number sense

Not much progress has been made towards a theory of number sense that is useful for mathematics education researchers. Lists of components, indicators and attributes of number sense can be found in the literature, but how these fit together has not been described. Greeno (1991) developed a theoretical framework from the perspective of situated cognition, while Dehaene (2001) hypothesises from cognitive neuro-science perspective that number sense is biologically determined. McIntosh, Reys and Reys (1992) developed a framework of the components of number sense, which could and has served as a basis for assessing number sense. Faulkner (2009) developed an instructional model of number sense to help teachers make connections that develop number sense. The contributions by these researchers are reviewed briefly below.

2.4.3.1 Greeno: The conceptual environment of number and quantities

Greeno's (1991) theoretical framework uses the metaphor of a conceptual domain and several analogs to illuminate all components of number sense from the perspective of situated cognition. His framework gives directions for an understanding of what number sense is and how number sense develops through learning and teaching.

Greeno considers the domain of numbers as closely associated with quantities, because numbers form an important part of the description of quantities, such as length, mass and time. He characterises number sense in terms of flexible mental computation, numerical estimation and qualitative judgements. He describes the domain of numbers and quantities as a conceptual domain using the metaphor of an environment and number sense as knowing one's way around in this environment.

> … number sense is an example of cognitive expertise – knowledge that results from extensive activity in a domain through which people learn to interact with the various resources of the domain, including knowing what resources the environment offers, knowing how to find the resources and use them in their activities, perceiving and understanding subtle patterns, solving ordinary problems routinely, and generating new insights (p. 170).

Greeno stresses that the "idea of learning in a domain as coming to know a conceptual environment emphasises the active nature of learning" (p. 211). One can learn about an environment through descriptions and explanations, but to live there one has to be engaged in activities in the environment, which foster positive epistemological beliefs and attitudes. The

activities include working in a social setting with significant use of mathematical notation as a crucial element in learning and reasoning.

2.4.3.2 Dehaene: A cognitive neuroscience perspective

Dehaene (2001) hypothesises from a cognitive neuroscience perspective that number sense qualifies for a biologically determined category of knowledge. His hypothesis is based on studies of the brain as well as experiments with animals and infants to establish basic numerosity.

> I propose that the foundations of arithmetic lie in our ability to mentally represent and manipulate numerosities on a mental 'number line', an analogical representation of number; and that this representation has a long evolutionary history and a specific cerebral substrate as single, analog representation (p. 17).

Through development and education, this central representation becomes connected to other cognitive systems and conceptual structures, which need to be internalised and coordinated. He contends, "The constant dialogue, within the child's own brain, between linguistic, symbolic, and analogical codes for numbers eventually leads to numerate adults" (p. 27). Individual differences in proficiency are due to the time, attention and effort that talented people have invested in their study of mathematics and their passion for numbers and mathematics.

Dehaene's (2001) hypothesis contributes to the debate around "nature and nurture" in the domain of number sense. His theory of an evolutionary, biological base of number sense fits the observations made by others (Gelman & Gallistel, 1978 in Greeno, 1991; Jordan, 2007) about the intuitive understanding of numbers and counting of children under the age of six.

2.4.3.3 McIntosh, Reys and Reys: The components of number sense

McIntosh, Reys and Reys (1992) proposed a framework (Table 2.2) for examining basic number sense in an attempt to identify and organise key components of number sense. The framework extends and clarifies the components of number sense for primary school as contained in the NCTM Standards (1989) quoted in 2.4.1 above.

Table 2.2 Framework for Considering Number Sense

1. Knowledge and facility with NUMBERS	1.1 Sense of orderliness of numbers	1.1.1 Place value 1.1.2 Relationship between number types 1.1.3 Ordering numbers within and among number types
	1.2 Multiple representations for numbers	1.2.1 Graphical/symbolical 1.2.2 Equivalent numerical forms (including decomposition / recomposition) 1.2.3 Comparison to benchmarks
	1.3 Sense of relative and absolute magnitude of number	1.3.1 Comparing to physical referent 1.3.2 Comparing to mathematical referent
	1.4 System of benchmarks	1.4.1 Mathematical 1.4.2 Personal
2. Knowledge of and facility with OPERATIONS	2.1 Understanding the effects of operations	2.1.1 Operating on whole numbers 2.1. Operating on fractions / decimals
	2.2 Understanding mathematical properties	2.2.1 Commutativity 2.2.2 Associativity 2.2.3 Distributivity 2.2.4 Identities 2.2.5 Inverses
	2.3 Understanding the relationship between operations	2.3.1 Addition/multiplication 2.3.2 Subtraction/division 2.3.3 Addition/subtraction 2.3.4 Multiplication/division
3. Applying knowledge of and facility with numbers and operations to COMPUTATIONAL SETTINGS	3.1 Understanding the relationship between problem context and necessary computation	3.1.1 Recognise data as exact or approximate 3.1.2 Awareness that solution may be exact or approximate
	3.2 Awareness that multiple strategies exist	3.2.1 Ability to create and/or invent strategies 3.2.2 Ability to apply different strategies 3.2.3 Ability to select an efficient strategy
	3.3 Inclination to utilize an efficient representation and/or method	3.3.1 Facility with various methods (mental, calculator, paper/pencil) 3.3.2 Facility choosing efficient numbers
	3.4 Inclination to review data and result for sensibility	3.4.1 Recognise reasonableness of results 3.4.2 Recognise reasonableness of calculation

McIntosh, Reys and Reys (1992) (p. 4)

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"Knowledge of and facility with numbers" includes a sense of orderliness of numbers, multiple representations for numbers, a sense of the relative and absolute magnitude of numbers and a system of benchmarks. "Knowledge of and facility with operations" refers to understanding the effects of operations, understanding mathematical properties such as commutativity, identities and inverses and understanding the relationship between operations such as the relationship between multiplication and addition. "Application to computational settings" encompasses the application of the knowledge of and facility with numbers and operations through recognising the connection between the problem context and the applicable computation, the ability to select among different strategies, the inclination to use efficient representations or methods and to check for the reasonableness of the answer.

The three areas of number sense, namely number concepts, operations with numbers and the applications of numbers and operations, are interconnected as a person thinks about and reflects on numbers, operations and results. The connection between the components implies monitoring similar to that used in metacognition and present in problem solving activities.

McIntosh, Reys and Reys (1992) framework for number sense attempts to provide an overview of the components of number sense. It provides a starting point for identifying and assessing number sense. The researchers make no claim to the completeness of the model, but state that number sense should expand and grow throughout secondary school and beyond.

2.4.3.4 Faulkner: An instructional model for number sense

Faulkner (2009) describes the model of number sense shown in Figure 2.1.

This model represents discussions and connections within every topic in the mathematics curriculum, "each wedge is to be connected to each lesson throughout the curriculum" (p. 27). The model is designed to help teachers make connections that develop number sense.
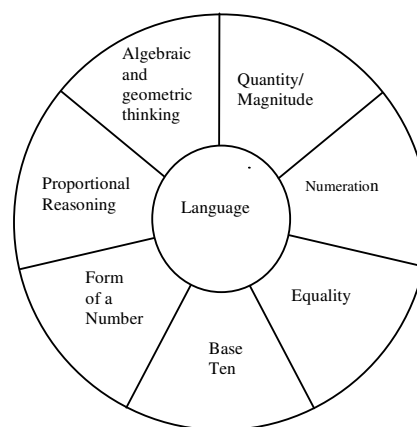
Figure 2.1 The Components of Number Sense
(Created by G. Gain, M. Doggett, V. Faulkner, and G. Hale, 2007). In Faulkner (2009)

She gives an example by considering how the fraction $\frac{3}{4}$ would fit into each compartment.

> Magnitude/Quantity: A place on the number line
> Numeration: Numerator and denominator
> Equality: $\frac{3}{4} = \frac{3}{4}$ only if there is a unit whole; $\frac{3}{4} = \frac{6}{8}$
> Base Ten: Decimals are special fractions
> Form of a Number: $\frac{3}{4} = \frac{6}{8} = 0.75 = 75\%$
> Proportional Reasoning: 3 for every 4
> Algebraic and geometric thinking: Gradient / Ratio of sides

Using this model as framework for the North Carolina Math Foundations training, Faulkner and her colleagues have been able to show teachers' growth in knowledge as measured by the Learning for Mathematics Teaching Measures developed by the University of Michigan.

The perspectives of the four researchers complement each other by highlighting different and important aspects of number sense. Greeno (1991) explains the essence of number sense and its development from the perspective of situated cognition, whereas Dehaene (2001) regards number sense as a biologically determined category of knowledge. Both Dehaene and Greeno point out the crucial role that communication, active engagement and positive beliefs play in the acquisition of number sense. McIntosh, Reys and Reys (1992) have analysed the components of number sense to provide a framework for assessing number sense and Faulkner's (2009) model is a guide for teachers to develop number sense.

This investigation will make use of the framework provided by McIntosh, Reys and Reys (1992) to assess and discuss the number sense of final year primary pre-service teachers.

2.4.4   Number sense and Numeracy

The terms number sense and numeracy are often used synonymously, but there are important differences, which will be clarified in this section.

The definitions and descriptions of number sense emphasise the understanding of the meaning of number, of the relationships between numbers and the contexts in which they are used. The Cockcroft Report (Cockcroft, 1982) coined the term "at homeness with number" to describe the competence and confidence to use numbers in everyday situations. The term "numeracy" was used in England and Wales in the *National Numeracy Strategy* (Department for Education and Employment [DfEE],1999) and includes:

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- An 'at homeness' with numbers
- The ability to make use of mathematical skills to cope with the practical mathematical demands of everyday life
- The ability to estimate and approximate number in a range of situations
- An appreciation and understanding of information presented in mathematical terms (in graphs, charts or tables)

The Rose Report (Rose, 2009) reiterated this view of numeracy. The first three components of "numeracy" reflect what is understood by "number sense" today, i.e. a child's fluidity and flexibility with numbers, the sense of what numbers mean, to look at the world quantitatively and make comparisons using mental calculations and estimation. The term "numeracy" as used in this sense therefore includes the attributes that are ascribed to "number sense".

Today, however, numeracy is viewed in a much wider sense to include everyday mathematics that is needed to understand consumer finance, home management, civic issues, employment conditions and benefits and media reports on a wide range of issues that arise from mathematical and statistical analysis (Westerford, 2008). Numerate adults can manage personal, societal and work related mathematical demands, solve problems in complex settings using technological tools and have the analytical and reasoning skills to draw conclusions, justify how they are reached, and identify errors or inconsistencies. The mathematical knowledge of a numerate person does not only include confidence and flexibility in working with numbers, but also an understanding of algebra, spatial relationships, data and probability and information represented in tables, graphs and diagrams. Numeracy in this wider sense is also called quantitative literacy (US), mathematical literacy (South Africa) or functional mathematics (England).

The Australian and the South African curriculums make a clear distinction between number sense and numeracy. Australia defines numeracy as

> "… using mathematical ideas efficiently to make sense of the world. While it necessarily involves understanding some mathematical ideas, notations and techniques, it also involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use and critically evaluating its use. Each individual's interpretation of the world draws on understandings of number, measurement, probability, data and spatial sense combined with critical mathematical thinking" (New South Wales Department of Education and Training, 2010).

Number sense is defined as

> "… a person's understanding of number concepts, operations, and applications of numbers and operations. It includes the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations" (ibid).

The Curriculum and Assessment Policy Statement (CAPS) for the Foundation Phase published recently (2010) by the Department: Basic Education of the Republic of South Africa refers to the development of number sense as a main content focus. Number sense includes "the meaning of different kinds of numbers; the relationship between different kinds of numbers; the relative size of different numbers; representation of numbers in various ways; and the effect of operating with numbers" (p. 10).

Mathematical literacy in senior secondary mathematics is defined in the wider sense of numeracy:

> "Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems" (Department of Education, 2003).

From this discussion, it can be concluded that number sense is the foundation on which numeracy is built and developed.

The lack of number sense among primary school learners has been demonstrated by various studies in Australia, Sweden, Finland, China, Taiwan, Kuwait and the United States (Alajmi & Reys, 2007; Aunioa et al, 2006; Gersten & Chard, 1999; Markovits & Sowder, 1994; McIntosh, Reys & Reys, 1997; Reys & Yang, 1998; Reys et al., 1999). After a review of research in mathematics education, Kilpatrick, Swafford and Findell (2001) concluded that the "curriculum should provide opportunities for students to develop and use techniques for mental arithmetic and estimation as a means of promoting deeper number sense" (p. 415).

The next section will show the connections between number sense, mental calculations and computational estimation.

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2.5 *Number Sense, Mental Calculations and Computational Estimation*

The ability to quickly and accurately perform mental computations and estimation is a skill often used by adults to judge a numerical magnitude without the aid of pencil and paper or a calculator.

Mclellan (in Heirdsfield & Cooper, 2004) states that number sense is

> …. part of a richly connected web of mental computation and computational estimation for which the child needs a knowledge of number relationships, a facility with basic facts, an understanding of arithmetical operations, the ability to make comparisons between numbers, and possession of base-ten place value concepts. (p. 444)

This definition of number sense highlights the close relationship between number sense, mental calculations and computational estimation.

Research has shown a link between competence in mental calculations, computational estimation and number sense. Markovits and Sowder (1994) implemented an instructional programme with a focus on understanding number magnitude (including rational numbers) and developing invented mental calculation strategies followed by instruction of computational estimation with a group of Grade 7 learners. The post instructional measures showed the learners made significant changes in both the use of strategies that reflected number sense and accompanying explanations that showed a depth of understanding not apparent before instruction.

2.5.1   The connection between number sense and mental calculations

Recent research (Carpenter, et al., 1998; Reys & Reys, 1998; Anghileri, 2006; Heirdsfield & Cooper, 2004) on the development of mathematical knowledge has provided strong evidence that mental calculations and number sense develop together and "one cannot exist without the other" (Griffin, 2003, p. 306). The phrase 'mental calculations' is seen to include two important aspects of mental work, i.e. recall and strategic methods.

Kilpatrick et al. (2001) refer to research evidence that shows a strong correlation between basic-facts fluency and strong mathematics achievement, but at the same time indicates that successful arithmetic skills are based on a combination of memory and strategies. Henry and Brown (2008) studied the effect of accelerating basic-facts memorization, as prescribed by the California standards. They found that instructional events focused on getting the answer

right (mostly by counting) are negatively correlated with Number Sense Proficiency and confirmed that by far the largest correlation "reflects the synergistic effect between memorization and derived-facts strategy" (p.172). They advocate that learners should be given sufficient support and experience with counting strategies to develop foundational understanding of numbers and operations generally, and then move to derived-fact strategies and memorization.

Kamii and Dominick (1997, p. 60) argue that mental calculations build number sense by encouraging children to think in their own ways and construct increasing higher levels of thinking that are rooted in their own knowledge, but proficiency in using standard paper-and-pencil algorithms might be harmful to children's development of numerical thinking. Mental calculations for operations with single and multi-digit numbers teach children how numbers work, how to make decisions about procedures, and how to create different strategies to solve mathematics problems, whereas the use of written algorithms encourages children to follow different steps without thinking (Veloo, 2010). Other researchers, such as Carpenter et al. (1998), Griffin (2003), Heirdsfield and Cooper (2002, 2004), and Reys and Reys (1998), support this argument. Pesek and Kirshner (2000) identify cognitive, attitudinal and metacognitive factors that interfere with meaningful learning after initial rote learning of algorithms. Mental calculations encourage children to reflect about the process and to think about what numbers mean in relation to the problem.

Heirdsfield and Cooper (2004) regard mental calculations as a subset of number sense, if it includes the attributes of 'accurate and flexible' and maintain that proficient mental calculators are mastery-orientated, as opposed to performance orientated, and aim for understanding and flexibility. Sowder (1994, p.144) refers to number sense as "expertise in the domain of number". Adaptive expertise requires an understanding of how and what procedures work and how these can be modified to fit the constraints of a problem. It requires the ability to deal with a problem holistically, to know what to do without the need to articulate a procedure before even beginning and to monitor and regulate the process of problem solving. Plunkett's (in Reys & Barger, 1994, p. 31) characteristics of a mental algorithm as "fleeting, variable, flexible, holistic, constructive and requiring understanding" clearly reflect the requirements of such adaptive expertise.

Quoting various researchers Heirdsfield and Cooper (2004) analyse the factors in the cognitive area of number sense that influence the ability to manipulate numbers mentally.

These factors include an

- understanding of partitioning of numbers in canonical (34 = 30 + 4) and non-canonical form (34 = 20 + 14)
- conceptualizing numbers as entities, rather than symbols side by side
- understanding the effects of operating on numbers, e.g. decomposing and recomposing numbers to suit an operation
- using properties of operations, such as commutative and associative laws

All these factors have been identified by McIntosh, Reys and Reys (1992) in their framework for considering number sense (Table 2.2 above).

2.5.2 The connection between number sense and computational estimation

Estimation has become to be viewed as a significant manifestation of number sense. Computational estimation can be defined as the process of simplifying an arithmetic problem using some set of rules or procedures to produce an approximate but satisfactory answer through mental calculation (LeFevre, Greenham & Waheed, 1993). Apart from the benefits as a real-world skill, it allows learners to check the reasonableness of their answers and helps them to develop a better understanding of place value, mathematical operations and general number sense (Kilpatrick et al., 2001).

In an analysis of the components involved in computational estimation Sowder and Wheeler (1989) identified three types of conceptual knowledge, namely (a) understanding of the role of approximate numbers in estimation, (b) understanding that estimation could involve multiple processes and have multiple answers, and (c) understanding that the appropriateness of an estimate depends on the context. Related concepts and skills require knowledge of place value, basic facts and properties of operations as well as an ability to compare numbers by size, compute mentally and work with powers of ten.

Research into estimation (Alajmi & Reys, 2007; Dowker, 1992; Hanson & Hogan, 2000; LeFevre et al., 1993; Sowder & Wheeler, 1989) has revealed that most children and adults lack the basic and necessary skills, probably because of the limited exposure to estimation in schools. Hanson and Hogan (2000) found that college students had difficulty in estimating answers to fraction and decimal problems and they were better at estimating answers to addition and subtraction of whole numbers than estimating answers to multiplication and division problems. They suggest that this may reflect a lack of a deep understanding of multiplicative reasoning and of rational numbers. Bana and Dolma (2004) came to the same conclusion in a study with Grade 7 learners in Australia. Alajmi and Reys (2007) report that

most Kuwaiti middle school teachers view a reasonable answer to be an exact answer and maintain that the heavy emphasis on procedural rules has impacted both teachers' and learners' development of number sense.

Kilpatrick et al. (2001) argue that computational estimation is a complex activity that should integrate all strands of mathematical proficiency. It requires a flexibility of calculation that emphasises adaptive reasoning and strategic competence, guided by children's conceptual understanding of both the problem situation and the mathematics underlying the calculation and fluency with computational procedures (p. 215). A disposition to make sense of a situation to produce reasonable answers instead of wild guesses is a prerequisite of competent estimators.

Number sense is a way of thinking and not a body of knowledge. Mental calculations promote thinking and reflection about numbers and operations. Good estimators rely on mental calculations to find an approximate but satisfactory answer. They demonstrate a deep understanding of numbers and operations; they are flexible in their thinking and use a variety of strategies when solving problems. Estimation is a process and not a content topic. Mental calculation and estimations skills should be developed continuously throughout the primary school years and beyond for number sense to develop (Kilpatrick et al., 2001).

For number sense to develop, the knowledge and facility with numbers and operations may be a prerequisite, but are not sufficient. It is necessary to apply this knowledge to problems requiring reasoning with numbers (Anghileri, 2006, p.5). This includes an understanding of the relationship between a problem context and appropriate solution strategies; an awareness that multiple strategies exist; an inclination to utilize an efficient representation and/or method and an inclination to review data and results. Reys and Reys (1998) advocate a problem-centred approach to teaching computations. The focus should be on rich mathematical tasks that engage and advance learners' thinking and help them make sense of numerical situations. Learners should be encouraged to invent strategies for computing that are based on their emerging understanding of numbers and operations so that they can build a conceptual foundation of number. However, conceptual and procedural knowledge should develop iteratively. Fluency in basic skills is often the basis for understanding processes that are more complex. It frees up mental energy to focus on the demands of a complicated problem (Wu, 1999; Rittle-Johnson, Siegler & Alibali, 2001). The knowledge that teachers need to develop number sense in children will be the next topic under review.

2.6 Number Sense and Teacher Proficiency

2.6.1 Introduction

Number sense as a mathematical proficiency requires conceptual understanding of numbers, whole numbers and rational numbers, and operations with numbers; the procedural fluency to perform operations on these numbers, both mental and paper-and-pencil; the adaptive reasoning to use different representations and benchmarks to estimate the reasonableness of an answer; the strategic competence to apply the knowledge in different contexts and a disposition to make sense of numerical situations.

It is often assumed that anyone who has completed secondary education can teach primary school mathematics, because it is not more than teaching the arithmetic operations of addition, subtraction, multiplication and division which everyone knows from their own school days. There is also the notion that teachers who have studied mathematics at tertiary institutions are more competent at teaching mathematics at school. Ball (1990) investigated these assumptions in a study of 252 prospective teachers and found that their understandings of elementary mathematics subject content tended to be "rule-bound and thin" (p. 449). They lacked the concepts and principles underlying the many rules that they had learnt and the connections among them. Mewborn (2000) in a review of the research conducted over three decades concluded that the studies showed that primary school teachers in the United States had only a rudimentary and procedural knowledge of mathematics and lacked the conceptual understanding to provide explanations for rules and algorithms. Researchers in other countries have come to the same conclusion (Alajmi & Reys, 2007; Aunioa et al., 2006; Biddulph, 1999; Corcoran, 2005; Kaasila et al., 2010; Morris, 2001; Southwell & Penglase, 2005; Yang et al., 2009).

This study investigated the number sense of pre-service primary school teachers to determine their level of understanding of the number system, its operations and the properties related to those operations. The study therefore dealt with teachers' understanding of and proficiency in primary school mathematics subject content rather than with their pedagogical content knowledge.

2.6.2 Teaching for number sense

Researchers who attended the 1989 Number Sense Conference (see Sowder & Schapelle, 1989) agreed that very few learners develop number sense in a traditional classroom setting where both teachers and learners identify calculations and exactness as the essence of

mathematics. Greeno (1991) argues that the "capabilities that we associate with number sense go beyond knowing facts and procedures; they involve participation in activities" (p.211) and in line with the environmental metaphor, sees teaching as analogous to the help that a resident in the environment can give to newcomers. Several researchers have identified the classroom setting in which number sense can develop.

Sowder and Schappelle (1994) maintain that teaching for number sense requires that all instructional activities focus on sense-making with numbers. Children need to

- make sense of the orderliness and regularity of the number system, starting with a knowledge of relative and absolute size of whole numbers, decimals and common fractions,
- have frequent opportunities over an extended period of time to construct understanding of place value,
- develop methods for computations, both mental and paper-and-pencil, that are useful and meaningful to them,
- estimate, using various methods and invented algorithms and learn to round in flexible ways.

To achieve this, teachers will have to create classrooms that are conducive to sense-making where children are required to explain, elaborate and defend their own positions so that they can integrate and elaborate knowledge in new ways. Trafton (1989) indicates that number sense should not be treated as a new topic, but rather that "number sense needs to be an ongoing, informal emphasis in all work with numbers" (p.75).

Murray, Olivier and Human (1998) promoted a problem-centred approach to teaching mathematics and laying the foundations for number sense in the lower primary grades in South Africa. Their approach is based on an over-simplified model called the three pillars:

- well-planned number concept activities, including activities which promote the building of patterns and relationships,
- well-planned problems, and
- effective discussion.

They maintain that neglect of any of these "pillars" shows in students' behaviour or understanding, even after only a few months.

Hiebert et al. (1997) developed a conceptual framework of five dimensions and core features of classrooms that facilitate learning with understanding: mathematical tasks that are problematic for the child; the teacher who is able to guide and intervene at appropriate

moments; a classroom culture as a learning community; the use of mathematical tools to record, communicate and think with and lastly that mathematics must be accessible to all learners in the classroom,   They conclude that the "essential features are intertwined and work together to create classrooms for understanding. [These features] define a system of instruction rather than a series of individual components. It makes little sense to introduce a few of the features and ignore the rest; their benefits come from working together as a coherent, integrated system." (Hiebert et al., 1997, p. 172).

Burns (2007) lists some practical teaching strategies for building learners' number sense that emphasise the connections between problem solving, reasoning and discussions, the hallmarks of teaching for understanding:

- Model different methods for computing.
- Ask students regularly to calculate mentally.
- Have class discussions about strategies for computing.
- Make estimation an integral part of computing.
- Question students about how they reason numerically.
- Pose numerical problems that have more than one possible answer.

It stands to reason that a teacher who lacks number sense will not have the insight and competence to implement any of these strategies. Developing number sense as a mathematical proficiency implies a shift away from teaching a collection of procedures and rules to developing a mathematical disposition that engages children at an early age in mathematical thinking and on laying the foundation for learning concepts that are more advanced.

2.6.3   Number sense and teachers' subject knowledge

Greeno (1989) states quite emphatically that if someone "is to serve as an effective guide to newcomers in an environment, it is essential that the guide himself or herself should be a comfortable resident of the environment" (p. 55) and this would require developing a different relationship between teachers and the subject matter of mathematics than they may have at the time.

Ten years later Ma (1999) developed the notion of Profound Understanding of Fundamental Mathematics (PUFM) during a study comparing the knowledge of primary school mathematics teachers in China and the United States.   Teachers with PUFM display knowledge that is connected, has multiple perspectives, demonstrates awareness of basic

ideas of mathematics, and has longitudinal coherence. She compares a teacher with PUMF to a taxi driver who knows a road system well and knows how to guide students from their current understandings to further learning and to prepare them for future travel. Teachers should see a knowledge "package" when they are teaching a topic and know the role of the present knowledge in that package. They should know the ideas or procedures, which support the topic they are teaching so that their teaching is going to rely on, reinforce, and elaborate the learning of these ideas (Ma, 1999, p. 18).

Another decade later, Ball (2008) hypothesises that the mathematical knowledge that is demanded by the work that teachers do goes beyond common content knowledge (CCK) which she defines as the mathematical knowledge and skills that are used in settings other than teaching. CCK involves merely calculating an answer or solving a mathematical problem. However, teachers' work involves an "uncanny kind of unpacking of mathematics" (p. 400) that is not needed by others in everyday settings. This second domain of content knowledge, which she termed specialised content knowledge (SCK), as unpacked mathematical knowledge, enables teachers to make features of particular content visible to and learnable by children. She tentatively includes a third category, horizon content knowledge, which describes an awareness of how a mathematical topic develops over the span of the curriculum. Hill (2008) examined the relationship between teacher subject matter knowledge and instruction and came to the "inescapable conclusion […] that there *is* a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction" (p. 497).

The reviewed research indicates that primary school teachers should not only have number sense on a procedural level, but that they should also be able to unpack number sense on a conceptual level to enable them to develop number sense in their learners.

2.6.4   Review of some research into the number sense of teachers

Numerous research studies have been conducted into the number sense of learners (Alajmi & Reys, 2007, Cranfield et al., 2005; Dowker, 1992; Heirdsfield & Cooper, 2004; Hiebert, Carpenter & Moser, 1982; Hanson & Hogan, 2000; Irwin, 1996; LeFevre et al., 1993; Nagel & Swingen, 1998; Reys & Yang, 1998; Reys et al., 1999; Sowder & Wheeler, 1989, Sowder, 1992). After their study assessing number sense of learners in Australia, Sweden, Taiwan and the United States, Reys et al. (1999) concluded that "it was the consistently low performance of students across all countries that reminded us of the common international challenge this topic provides" (p. 68).

Although relatively few studies have investigated pre-service teachers' knowledge in the domain of numbers and number sense, the findings indicate that their performance on number sense tests is equally low and have pointed specifically to deficiencies in multiplicative thinking and their knowledge of fractions, decimals and percentages (Golding, Rowland & Barber, 2002; Hanrahan, 2002; Hill, 2008; Lamb & Booker, 2003).

Kaminski (1997) conducted a small-scale study in Australia to investigate pre-service teachers' use of number sense in the whole number domain. The pre-service teachers completed sets of mathematical exercises exploring aspects of numeration and non-standard computational procedures, including mental computation. He then interviewed the six students to obtain data on their use of number sense. The results showed that the students preferred using written calculations and infrequently utilised estimation; rarely associated numbers with quantities; tended to lack an understanding of multiple relationships in the number and operations domain and had difficulties with mental calculations. Kaminski concluded that the students had an underdeveloped number sense and recommended that the development of number sense should be an integral component of mathematics teacher education.

Tsao (2004) explored the connections between number sense, mental computations and written computations of 155 pre-service elementary school teachers in Taiwan and found a significant correlation between the scores on the number sense test, the mental computation test and the written computation test. A comparison of parallel test items showed that the correct responses on exact computations were higher than those requiring mental computation, estimation or other aspects of number sense. A comparison with performance of 6th and 8th grade learners revealed that the pre-service teachers who demonstrated an underdeveloped number sense, obtained lower scores on the mental calculation and written computation tests than the learners. He recommends that estimation and mental mathematics skills should be developed and practiced and the applications of computation skills to solving problems emphasised in undergraduate mathematics courses of elementary school teachers.

In another study Tsao (2005) conducted interviews with 12 pre-service elementary school teachers, six who scored in the top 10% on a Number Sense Test (NST) and six who scored in the bottom 10% of the NST. He explored five characteristics of number sense: the ability to decompose/recompose numbers; recognising the relative and absolute magnitude of numbers; the use of benchmarks,; understanding the relative effect of operations on numbers; and flexibility of applying the knowledge of numbers and operations to computational

situations (including mental computation and computational estimation). He presented items of the NST and the students' responses were judged according to their answers and explanations. The data indicated that the high ability students achieved twice the frequency of correct responses over the low ability group and some demonstrated the use of the number sense characteristics in their explanations. The low ability students tended to use rule-based methods and preferred the use of standard written algorithms rather than number sense based strategies. He found that both groups of students had difficulties in dealing with items that included fractions; that most low ability students had not established the connection between decimals and fractions; and that most students felt uncomfortable providing estimates. He concluded that approximately 35% of the high ability and 75% of the low ability students displayed little number sense and recommended that the number sense of pre-service teachers should be assessed and addressed in the elementary teacher programme.

These findings were corroborated by Yang, Reys and Reys (2009) in a study examining number sense strategies and misconceptions of 280 Taiwanese pre-service elementary teachers. This study was limited to two components of number sense, namely using benchmarks in recognising the magnitude of numbers and estimation in knowing the relative effects of an operation on various numbers. Participants answered 12 questions and were instructed to use mental computation or estimation to arrive at the answers and briefly explain how they arrived at their answers. Despite the instruction not to use paper-and-pencil procedures, over 60% of responses used rule-based approaches relying on exact computation algorithms. The researchers raised the concern that the failure of pre-service teachers to use attributes of number sense - benchmarks and estimation - to produce answers and explain their thinking, influences their ability to promote number sense when they teach mathematics in primary school.

Johnson (1998, in Tsao, 2005) studied number sense and related misconceptions about selected rational number concepts of prospective elementary teachers in his doctoral research at the University of Florida. He found that students resisted looking at mathematics in creative, non-algorithmic ways. He identified the following common misconceptions in relation to rational numbers: fractions with the larger denominator are always larger; two fractions that are almost equal are equivalent; confusion about decimal place value; use of flawed algorithms such as multiplying fractions by finding a common denominator and multiplying the numerators; the belief that area models must be rectangular or regular in order to find a fractional part.

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The National Council of Teachers of Mathematics states "intuition about number relationships helps children make judgements about the reasonableness of computational results and of proposed solutions of numerical problems. Such intuition requires good number sense." (NCTM, 1989, p. 38). The study by Alajmi and Reys (2007) focused on the perspectives of Kuwaiti middle school teachers on the reasonableness of answers. Their sample included 13 middle school teachers of two schools, including the senior teacher in each school. The interview questions focused on the meaning of the term "reasonable answer"; the ways teachers determine the reasonableness of answers; and how they valued this concept. They found that the common view of the teachers of a reasonable answer was an exact answer and that they would use a computational procedure to determine the reasonableness of an answer. In addition, teachers did not consider determining whether answers were reasonable an important instructional goal. They concluded that the responses of the teachers "raised questions about the number sense of these teachers and their overall understanding of mathematics" (p. 91) and recommended that these shortcomings should be addressed through pre-service and in-service teacher education.

Glenda Lappan (1999), past president of the NCTM, said about the importance of teacher content knowledge:

> Our own content knowledge affects how we interpret the content goals we are expected
> to reach with our students. It affects the way we hear and respond to our students and
> their questions. It affects our ability to explain clearly and to ask good questions. It
> affects our ability to approach a mathematical idea flexibly with our students and to
> make connections. It affects our ability to push each student at that special moment
> when he or she is ready or curious. And it affects our ability to make those moments
> happen more often for our students.

This statement underlines the importance of a profound understanding of relevant mathematical content knowledge as a basis upon which the teacher's ability to develop pedagogical content knowledge and to teach for understanding, grows.

From the research into the number sense of teachers quoted above, it is evident that the majority of primary school teachers have limited number sense and very little understanding of what number sense is. They tend to rely on standard algorithms and avoid mental calculations and estimation; they do not use benchmarks to reason about the effects of operations; they cannot flexibly apply numbers and operations to computational situations; and they hold some of the same misconceptions as their learners, especially in the domain of rational numbers.

Yang et al (2009) concluded that

> Breaking the shackles of rotely applying algorithms and promoting greater development
> and application of number sense components in mathematical problem solving is an
> international challenge (p. 400).

These researchers recommend that if we want to improve learners' number sense, the development of number sense should become a focus of pre-service primary school teacher education.

2.7   Teacher Confidence

Teachers' beliefs and attitudes about mathematics have a powerful impact on the practice of teaching (Uusimaki and Nason, 2004). The NCTM (2003) states that "Candidates' comfort with, and confidence in, their knowledge of mathematics affects both what they teach and how they teach it." (p. 4).

Much research has focused on the connection between teachers' maths anxiety and their confidence to teach mathematics. Teacher's lack of confidence in their own ability to do and teach mathematics is one of the causes of maths anxiety (Bursal & Paznokas, 2004; Swars et al, 2006). Uusimaki and Nason (2004) reported that situations, which caused most anxiety for the participants in their study, included communicating one's mathematical knowledge in either a test situation or in the teaching of mathematics during practice teaching due to insecure feelings of making mistakes or not being able to solve problems correctly. They found that most anxiety was caused by the topics of algebra, space and number operations, especially division. Research (Swars et al, 2006) seems to suggest that in general maths anxiety has a negative impact on teacher efficacy, namely on the confidence in one's own skills and ability to be an effective mathematics teacher who can affect learner performance. When teachers with high mathematics anxiety perceive themselves as effective mathematics teachers, the basis of their belief is learner-orientated: feeling a sense of understanding struggling learners.

Ball (1990) suggests that the pre-service primary school teachers' lack of knowledge in mathematics resulted in their negative attitude towards the subject. In her study of 252 teacher candidates' understanding of mathematics, their feelings about mathematics and teaching mathematics were investigated. She found that the candidates' self-confidence affected the way they approached problems, the connections they were able to make between

related concepts and their repertoires of strategies. Candidates who thought they knew mathematics and believed they could learn mathematics exhibited feelings of confidence and control. In contrast the candidates who disliked and feared mathematics, had doubts about their mathematical ability in general.

Research in England (Morris, 2001) and Scotland (Henderson & Rodrigues, 2008) found that generally pre-service primary school teachers are not competent or confident in their mathematical knowledge, even if their mathematics qualifications would be deemed to provide sufficient subject content knowledge. Morris (ibid) raises concerns about teachers who exhibited confidence in their own incorrect knowledge and reacted with disbelief when they failed an audit of their content knowledge. Students who exhibit strong confidence in their own incorrect knowledge have strongly held misconceptions which are very resistant to change and unless these are rectified, will take their misconceptions into their teaching.

Fast and Hankes (2010) designed an intervention to develop a deeper understanding of mathematical concepts among pre-service elementary teachers and investigated its effect on the students' confidence in mathematically correct knowledge. They found a highly significant gain in directional knowledge confidence, where directional confidence refers to the confidence that a correct answer is correct as opposed to the confidence in an incorrect answer being correct. The positive change in directional knowledge confidence indicated that the intervention of the researchers had the desired effect.

Graven (2004) interprets confidence as part of an individual teacher's ways of "learning through experiencing, doing, being, and belonging" (p. 179) which is deeply interconnected with learning as changing meaning, practice, identity and community. As such, it plays an important role in enabling teachers to face the challenge to their mathematical competence introduced by the demands of the new curriculum. The data of her study of participants in an INSET programme in Johannesburg, South Africa, indicate that the confidence in being able to learn mathematics is a resource that enables teachers with little mathematical training to learn the mathematical competences required for teaching mathematics. This is of particular relevance to the Namibian situation where the reforms that were introduced fifteen years ago have failed to take root in the classroom.

The researchers quoted above advise that attempts to improve pre-service teachers' content knowledge should be coupled with a safe and supportive environment in which these students can develop confidence and positive feelings towards the subject.

2.8   Conclusion

Namibia's progress towards achieving a knowledge-based society as envisioned by Vision 2030 is hampered by the general lack of adult numeracy skills and the small percentage of school leavers with a higher-level mathematics qualification. This is reflected in the official unemployment rate of 51.8% and an acute skills shortage in the professional and technical fields (Links, 2010b).

The poor results of the SACMEQ II and III surveys and standardised assessment of primary school learners in the country indicate that mathematics education at primary school level in Namibia is largely ineffective in developing the basic numeracy skills required for further studies in mathematics. Investigations into the state of mathematics education in Namibia (Courtney-Clark and Clegg, 2009; Mathematics and Science Teachers Extension Programme, 2002; Ministry of Basic Education, Sport and Culture, 1999) are unanimous in their conclusions that the poor performance of learners in secondary school mathematics goes back to the foundation laid in primary schools. They recommend that pre-service teacher education programmes should be overhauled and wide-ranging professional development opportunities for in-service teachers should be implemented as a matter of urgency.

Number sense is increasingly seen as an important outcome of primary school mathematics education. The notion of number sense embodies the most important concepts, skills and attitudes that learners should acquire in primary school as a foundation for further studies in mathematics, mathematical literacy and numeracy. Number sense is a mathematical proficiency - a term widely used in research literature today to describe the desired outcome of mathematics education. It requires the *conceptual understanding* of the meaning of number, the relationships between numbers and the operations on numbers; the *fluency* to use *procedures* efficiently, accurately and flexibly; the *strategic competence* to apply knowledge and skills to solve problems in context; *adaptive reasoning* to explain and justify solutions and make deductions from patterns, analogy and metaphor; and a *productive disposition* to see mathematics as sensible and oneself capable to learn, understand and do it (Kilpatrick et al., 2001).

Number sense, like common sense, is an elusive term and not much progress has been made to portrait number sense in a clear and comprehensive manner. Number sense develops gradually through "exploring numbers, visualizing them in a variety of contexts, and relating them in ways not limited by traditional algorithms" (Howden 1989, p.11).

The skills and disposition to make sense of numerical situations in everyday life are highly valued in a numerate society and bear the hallmarks of mathematical proficiency. They are based on making connections between related concepts, choosing and carrying out appropriate procedures flexibly and reflecting, evaluating and justifying the outcomes. These skills also form the foundation for the development of numeracy as "the ability to understand, critically respond to and use mathematics in different social, cultural and work contexts" (SACSA Framework, 2000)

At best, number sense can be assessed by focusing on indicators such summarised by Kalchman, Moss and Case (2001, p. 2):

    (a) fluency in estimating and judging magnitude,

    (b) ability to recognise unreasonable results,

    (c) flexibility when mentally computing,

    (d) ability to move among different representations and to use the most appropriate representations,

    (e) ability to represent the same number or function in multiple ways depending on the context or purpose of this representation.

The theoretical frameworks that have been proposed indicate that for number sense to develop, the connections have to be established between conceptual and procedural knowledge. According to Hiebert & Lefevre (1986), conceptual knowledge is developed by constructing relationships between pieces of information on a primary, contextual level and a more abstract, reflective level. Procedural knowledge on the other hand can be learnt by rote and consists of the surface knowledge of the symbols and syntax of mathematics as well as the rules, algorithms and procedures that are used to solve a mathematical task.

The focus on procedural knowledge at the expense of conceptual knowledge in mathematics teaching in schools and teacher training institutions may account for the general lack of numeracy skills and number sense in Namibia.

Number sense is a way of thinking and develops gradually from an early age if children grow up in cultures that expose them early to quantitative thinking and analysis. Children from low-income families, children who live in poverty and young children who learn in a second or third language may need formal explicit instruction to acquire the early foundations of number sense (Chard et al., 2008). Number sense develops in classrooms that teach for

understanding. A competent teacher should be able to model different methods for computing; ask learners to calculate mentally and make estimates of the answers; discuss strategies for computing with the learners and ask learners to explain their reasoning; and use problems in context and pose problems that have more than one possible answer (Burns, 2007).

To develop number sense in children, the teacher has to have number sense, as so aptly put by Greeno (1989) who stated that if someone "is to serve as an effective guide to newcomers in an environment, it is essential that the guide himself or herself should be a comfortable resident of the environment" (p. 55). However, it is not enough that a teacher has number sense, he/she also needs to have a profound understanding of fundamental mathematics (Ma, 1999) and be able to unpack number sense on a conceptual level to make the features of number sense visible to and learnable by children (Ball, 2008).

Researchers into the number sense of teachers in various countries, for example Australia (Kaminski, 1997), the United States (Johnson, 1998 in Tsao, 2005), Taiwan (Yang, Reys, & Reys, 2009; Tsao, 2004, 2005), Kuwait (Alajmi and Reys, 2007) and Brunei (Veloo, 2010) have come to the conclusion that the majority of primary school teachers have limited number sense and very little understanding of what number sense is. They see the promotion of a greater development of number sense and application of number sense in problem solving as an international challenge and recommend that it should become the focus of teacher pre-service and in-service education.

However, many primary school teachers exhibit a dislike and fear of mathematics, which will negatively influence the way they approach the practice of teaching (Swars et al., 2006). A lack of confidence in one's ability to do and learn mathematics is one of the causes of maths anxiety (ibid). Teachers' self-confidence affects the way in which they approach problems, the connections they are able to make between related concepts and their repertoires of strategies (Ball, 1990). Graven (2004) hypothesises that the confidence in being able to learn mathematics is a resource that enables teachers with little mathematics training to learn the mathematical competences required for teaching mathematics. Attempts to improve pre-service teachers' content knowledge should therefore be coupled with a safe and supportive environment in which these students can develop confidence and positive feelings towards the subject.

The review of the research literature suggests that Namibia, to reach the ambitious goals of Vision 2030, should critically examine its teacher education policies and practices based on the recommendations made by the various researchers quoted above. Recognising that teacher quality is a central element of overall education quality, assessment of teacher subject and pedagogical content knowledge and interventions to address any shortcomings in a holistic way should be a priority in the way forward in a collective commitment towards quality education. This study intends to contribute to this by providing initial data on the mathematical subject knowledge of pre-service primary school teachers as reflected in their number sense and their confidence in the ability to do and to teach mathematics.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1   Introduction

This chapter describes the research design and methodology used to investigate the Namibian final year pre-service teachers' skills, knowledge, strategies and confidence related to number sense. It first states the goals of the research against the background of the literature study; it explains the choice of paradigm and research design and methods. It then describes the research instruments used in the study, the choice of sample, the data collection and analysis procedures. The pilot study and its outcomes are documented and the issues related to validity, reliability and ethics are considered. The chapter concludes by briefly stating the limitations of the research.

3.2   Literature Study and the Research Goal

The literature study revealed that number sense is the mathematical proficiency that underpins the development of numeracy as the "ability to understand, critically respond to and use mathematics in different social, cultural and work contexts" (SACS Framework, 2000). As such, the development of number sense may be regarded as a critical goal of mathematics education in primary school.

Number sense develops gradually from early childhood and is characterised by a deep understanding of the meaning of number, the relationships between different representations of number and the effects of operations on number as well as the flexible use and application of this knowledge and facility with numbers and operations to contextual settings. Number sense is manifested in the proficiency with mental computation and computational estimation (Heirdsfield & Cooper, 2004; Kilpatrick et al., 2001; Reyes & Barger; 1994).

Ball (2008), Greeno (1991), Kilpatrick et al. (2001) and Ma (1999) have highlighted the importance of mathematics teachers' subject knowledge. Yang et al. (2009) maintain that if "teachers don't understand the mathematics and have a solid knowledge of number sense, it is unlikely they will be able to promote number sense in their students" (p. 386). Research into the number sense of teachers and pre-service teachers (Alajmi & Reys, 2007; Kaminski, 1997; Tsao, 2004, 2005; Yang et al., 2009) has revealed that very few of them are able to use features of number sense to solve problems and researchers advocate that the topic of number sense should be a focus of teacher education programmes.

The goal of this study was to provide a baseline for the degree to which Namibian primary pre-service teachers possess number sense at the completion of their teacher education programme. At the same time the study also provides data on the confidence that these pre-service teachers have in their own ability to learn and teach mathematics, because this confidence may be regarded as a resource that enables teachers with little mathematical understanding to learn the mathematics required for teaching (Graven, 2004).

3.3 Paradigm

This study was conducted within pragmatic knowledge claims. According to Creswell (2003), pragmatic knowledge claims arise out of actions, situations, and consequences. The pragmatists look to the "what" and "how" of research based on its intended consequences. The problem is the focus of the study and researchers use all approaches to understand the problem. This research study arose from the concern over the poor performance of Namibian learners in mathematics and aimed to shed some light on the knowledge and understanding of pre-service teachers in the domain of numbers and operations as manifested in the number sense that they possess. It was real-world practice orientated and the outcomes of this study may influence future pre-service and in-service education of primary school teachers.

3.4 Research Design

This study employed a mixed methods approach to data collection and interpretation. According to Mackenzie and Knipe (2006), with the research question 'central', data collection and analysis methods should be chosen as those most likely to provide insights into the question with no philosophical loyalty to any alternative paradigm.

Mixed methods employ both quantitative (QUAN) and qualitative (QUAL) methods either concurrently or sequentially (Creswell, 2003). In a concurrent mixed methods design (triangulation design) different but complementary data are collected on the same topic in the same phase of the study. The data are integrated in the interpretative phase. This interpretation can either note the convergence of the findings as a way to strengthen the knowledge claims of the study or explain any lack of convergence that may result (ibid). A visual design of the concurrent mixed methods model is shown in Figure 3.1.

In this study a concurrent mixed methods design was used. Questionnaires and tests were administered to collect data on pre-service primary school teachers' number sense, their

proficiency in mental and written calculations and their confidence in their ability to learn and teach mathematics. Concurrent with this data, qualitative data was collected through semi-structured interviews to explore the central phenomenon of number sense of the pre-service primary school teachers in Namibia. The reason for collecting both quantitative and qualitative data was to bring together the strengths of both forms of research to compare and validate results.
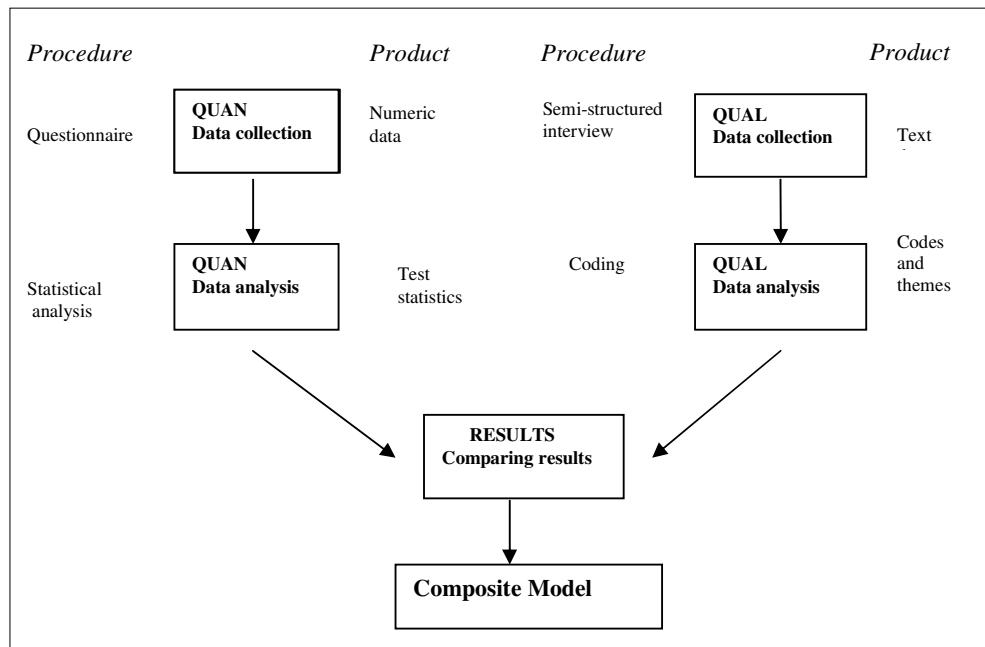
Figure 3.1 Concurrent Triangulation Model Visual Design

In this study priority was given to the quantitative method. The role of the qualitative data was to clarify and understand the quantitative data. The results of the two methods were integrated in the interpretive phase.

3.5 Research Instruments

The quantitative data were gathered by firstly administering proficiency tests adapted from Yang who developed these tests in 1997 to assess the number sense of Grade 6 and Grade 8 learners in Taiwan: the Number Sense Questionnaire (NSQ), the Mental Computation Questionnaire (MCQ) and the Written Computation Questionnaire (WCQ). These tests were labelled as questionnaires rather than tests to prevent any association with formal assessment tests. Secondly, the student teachers' confidence in their ability to learn and teach mathematics was measured by the McAnallen Confidence in Mathematics and Mathematics Teaching Survey (MCMMTS) developed by McAnallen in 2010. Copies of the

questionnaires are attached as Appendix A (Number Sense Questionnaire), B (Written Computation Questionnaire), C (Mental Calculations Questionnaire) and D (McAnallen Confidence in Mathematics and Mathematics Teaching Survey).

To triangulate and validate the findings from the quantitative data, a semi-structured interview schedule was designed to probe into any number-sensible strategies that students used in answering a selection of six questions from the Number Sense Questionnaire. The interview schedule and selected questions are attached as Appendix E.

3.5.1 Quantitative Measures

Professional researchers in Taiwan and the United States designed the instruments used to gather quantitative data. Permission was granted by Professor Der-Ching Yang to use and adapt the Number Sense Test, Written Computation Test and the Mental Computation Test and by Dr Rachel McAnallen to use and adapt the McAnallen Confidence in Mathematics and Mathematics Teaching Survey for the purpose of this research study in Namibia.

The choice of instruments from outside southern Africa was based on the failure to locate similar instruments of southern African origin and the researcher's inexperience in designing similar suitably reliable and valid instruments herself. Number sense, for example, is an elusive construct and the operationalisation of this construct requires the expertise of an experienced professional researcher.

The Number Sense Questionnaire (NSQ) (Appendix A)
The Number Sense Questionnaire (NSQ) consisted of 27 items selected and adapted from the 40 item NST developed by Yang in 1997 for Grade 6 and 8 students in Taiwan (Reys & Yang, 1998). The 27 selected items covered the three main categories of the Framework for Considering Number Sense developed by McIntosh, Reys and Reys (1992), namely "Knowledge of and facility with numbers", "Knowledge of and facility with operations" and "Application to computational settings", as well as the number domains of whole numbers, fractions and decimal fractions as depicted in Table 3.1

Table 3.1 Number Sense Questionnaire Analysis
Based on the Framework for Number Senses developed by McIntosh, Reys and Reys (1992)

	Knowledge of and facility with NUMBERS	Knowledge of and facility with OPERATIONS	Application to COMPUTATIONAL SETTINGS
Whole number	17	1, 3, 4, 15, 23	2, 10, 14, 19
Common fractions	5, 6, 18, 20 24, 25, 21	7, 8,	11, 12,
Decimal fractions	16, 26	22	9, 13, 27

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The questions on the NSQ did not require complex or long calculations, but relied on the respondent's ability to make sense of the numerical situation, to apply his/her knowledge of numbers and operations and to use insight and/or estimation to arrive at the answer. Fifteen questions required respondents to estimate the answers, three questions required an exact answer, which could be obtained by mental computation and the remaining nine questions probed into the understanding of order, properties and representation of numbers.

Some of the selected items were adapted to make them more accessible to the expected competency levels of Namibian primary school teachers. These adaptations included

(a) The reduction of the whole number range to numbers less than 10 000 to enable lower primary teachers with little mathematics background to estimate and calculate mentally. For example, the numbers in Question 14 (Q14) were changed from the original 913 582 ÷ 6183 to 9135 ÷ 61.

(b) Changing decimals to whole numbers to increase the items in the whole number domain in which primary school teachers were expected to perform better. In Q2 the lengths of strings measuring 54,125m and 29,85m in the original question, were changed to 541m and 298m respectively.

(c) Adapting contexts, such as names, to the Namibian situation. For example, in Q2 Wang and Lin were replaced by Sarah and Leonard and in Q27 cubic feet were changed to litres.

Table 3.2 shows three sample items of the NSQ.

Table 3.2 Sample Items of the Number Sense Questionnaire

3. Without calculating an exact answer, circle the expression, which represents the larger amount.	A. 2452×4 B. $2541 + 2457 + 2460 + 2465$ C. Cannot tell without calculating
5. Is $\dfrac{3}{8}$ or $\dfrac{7}{13}$ closer to $\dfrac{1}{2}$?	_____ Why? _____ _____
27. A drum has 125 litres of water in it. A farmer fills more water into the drum and the water level rises at a rate of 1,5 litres per minute. How many litres of water will be in the drum after 4 minutes?	_____

The Written Computation Questionnaire (WCQ) (Appendix B)

The first 12 questions of the WCQ matched the calculations in the first 12 questions of the NSQ. Students were required to find the exact answers by any written methods of their choice. The questions covered the four operations with whole numbers and common

fractions and addition of decimals. Questions 13 to 15 required subtraction, multiplication and division of decimals. Table 3.3 shows the questions and analysis of the WCQ items.

Table 3.3 Written Computation Questionnaire Analysis

	Addition	Subtraction	Multiplication	Division
Whole numbers	**1.** 995 + 872 + 838 + 809 **3.** 2451 + 2457 + 2460 + 2465	**2.** 541 − 298	**3.** 2452 × 4	**4.** 45 × □ = 2700 **10.** 170 ÷ 15
Fractions	**5.** $\frac{10}{11}+\frac{7}{8}$; **12.** $\frac{5}{9}+\frac{8}{15}$	**6.** $\frac{1}{2}-\frac{3}{8}$; $\frac{7}{13}-\frac{1}{2}$ **11.** $2-\frac{1}{3}-\frac{1}{2}$	**7.** $\frac{15}{24}\times\frac{5}{12}$	**8.** $6\frac{2}{5}\div\frac{7}{8}$
Decimals	**9.** 53,7 + 7,8 + 0,9	**14.** 851,5 − 34,45	**13.** 0,46 × 0,5	**15.** 72 ÷ 0,025

Written computations with whole numbers, fractions and decimals feature as one of the main components of the primary mathematics curriculum in Namibia. The main reason for the inclusion of the WCQ was to investigate the relationship between the written computation skills of primary school teachers in Namibia and their number sense and the relationship between written computation skills and mental computation skills.

The Mental Calculation Questionnaire (MCQ) (Appendix C)

The twenty items of the MCQ were taken from the Mental Computation Test (MCT) developed by Yang for Grade 6 learners in Taiwan. The items were designed by Yang (ibid) to elicit the use of mental calculation strategies that students possess. For example, Q3, 232 – 98, should prompt students to use the "count on" strategy. All the items are listed in Table 3.4 below.

Table 3.4 Mental Calculation Questionnaire Analysis

	Addition	Subtraction	Multiplication	Division
Whole number	**1.** 600 + 50 + 250 **2.** 189 + 36 **13.** 264 + 99	**3.** 232 – 98 **4.** 56 + 23 – 16	**5.** 13 × 5 × 2 **7.** 36 × 50	**6.** 512 ÷ 4 **8.** 550 ÷ 25
Fractions	**9.** $\frac{1}{2}+\frac{3}{4}$	**10.** $8-4\frac{1}{2}$ **20.** $5\frac{1}{4}-3\frac{3}{4}$	**11.** $4\times3\frac{1}{2}$ **19.** $\frac{1}{2}\times6\frac{1}{2}$	**12.** $40\div\frac{1}{2}$
Decimals	**14.** 6,5 + 1,9	**15.** 4,5 – 2,6	**16.** 5 × 2,12 **18.** 95 × 0,1	**17.** 6,5 ÷ 0,5

The McAnallen Confidence in Mathematics and Mathematics Teaching Survey (MCMMTS)
(Appendix D)

Rachel McAnallen at the University of Connecticut developed the MCMMTS. The instrument used in this study had 25 items, 12 relating to personal self-efficacy and anxiety and 13 relating to teaching self-efficacy and anxiety. Certain words and phrases were replaced with terminology used in the Namibian context (e.g. the word "student" was replaced with "learner") or with expressions that were judged to be more familiar to Namibian students (e.g. "I cringe" was rephrased as "I feel anxious").

3.5.2   Qualitative Measures: The Semi-structured Interview

Semi-structured interviews are conducted to gain a detailed picture of a participant's perceptions or accounts of a particular topic (De Vos, Strydom, Fouche & Delport, 2002). The open-ended nature of the questions has the advantage that the interviewer may go into more depths and test the limits of the participant's knowledge with a minimum restraint on the answers and their expression (Cohen, Manion & Morrison, 2000).

The purpose of the semi-structured interview in this study was to validate and explain the results obtained by means of the quantitative instruments. The interview schedule (attached as Annexure E) was prepared to serve as a guide to the overall issues to be covered in the interview, namely determining the written, mental or estimation strategies that students used to answer number sense based questions. The six items from the NSQ were selected to represent the three main categories of the Number Sense Framework, two items from each category; as well as the three different number domains, three items from the whole number, two from common fractions and one from the decimal fractions domain, as illustrated in Table 3.5 below. The six items were sequenced from easier to more difficult questions to establish an atmosphere where the participant would not feel threatened and would be encouraged to talk freely (Cohen et al., 2000).

According to De Vos et al. (2002), the interview process is a "much more elusive though powerful component of the interview" (p. 296) than the content of the interview. The behaviour of the interviewee, the pauses in the responses and the use of language all contribute to the essence of the interview. In this study it was anticipated that how the student behaves and talks during the interview would give some information on the student's confidence in his/her ability to do mathematics and the student's ability to explain mathematical concepts and procedures when teaching mathematics to primary school children.

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Table 3.5 Interview Item Analysis

Number Sense Domain	Number sense skill/strategy	Number Domain	Question
Knowledge and facility with numbers	Recongise relative size of numbers	Common fractions	Is $\dfrac{3}{8}$ or $\dfrac{7}{13}$ closer to $\dfrac{1}{2}$?
Knowledge and facility with numbers	Translation between representations	Common fractions	Which letter in the number line names a fraction where the numerator is slightly more than the denominator? (Number line given)
Knowledge and facility with operations	Relationship between operations / estimation	Whole numbers	Which expression represents the larger amount? 2452×4 $2451 + 2457 + 2460 + 2465$?
Knowledge and facility with operations	Estimation	Decimal numbers	Place the decimal comma in the answer to following multiplication problem: $534,6 \times 0,545 = 291357$
Application to computational settings	Recognition of operation / estimation	Whole numbers	Sarah has a string of 541 centimetres. Leonard has a string of 298 centimetres. Estimate how much longer Sarah's string is than Leonard's string.
Application to computational settings	Recognition of operation / mental calculation	Whole numbers	A minibus can transport 15 people. How many minibuses are needed to transport 170 learners?

3.6   Sampling

Primary school teacher education is offered at four satellite campuses of a local university. The rural campuses are situated within clearly demarcated mother-tongue populations and attract mainly local students, whereas the urban campus attracts students from all regions of the country. Based on this information and considerations of factors such as expense, time and accessibility, the researchers decided to employ purposive sampling. A purposive sample is based entirely on the judgement of the researcher, in that the sample contains the most characteristic, representative or typical attributes of a population (Cohen et al., 2000; De Vos et al., 2002). The targeted sample for this study consisted of the 74 final-year lower primary and upper primary mathematics pre-service teachers enrolled at an urban university education campus in Namibia. This campus caters for 22% of the total number of primary pre-service students. This sample was regarded as more representative of the total population than a sample from any of the rural campuses.

Cohen et al. (2000) state that this type of sample is selective and biased and cannot lay claim to represent the wider population. Therefore, the results of such a study cannot be generalised to the whole population. However, in this study the researcher believes that the data gained from this sample would adequately indicate the proficiencies related to number sense and

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dispositions towards mathematics and mathematics teaching that can be expected from the wider population.

In the concurrent mixed method design, both quantitative and qualitative data are gathered at the same time and the same place (Creswell, 2003). In this study, the qualitative data was used to clarify and understand the quantitative data, which formed the central part. It was decided to interview six students representing 8% of the target sample. This sample was selected prior to the study. Numbers were allocated to each pre-service lower primary teacher and each pre-service upper primary teacher. The sample was then selected from each subset by using random number tables. Taking into account that not all participants would be willing to be interviewed, 12 student numbers were selected in this way.

3.7   Data Collection

This section describes the whole process of data collection from piloting, refining of instruments and data collection for the main study.

3.7.1 The Pilot Study

The advantages of a pilot study according to Teijlingen and Hundley (2001) are that it may give advanced warning of where problems in the main study may occur, where research protocols may not be followed or where proposed methods and instruments are inappropriate or too complicated. Piloting is an effective way of ensuring relevance, validity and reliability of designed instruments (De Vos et al., 2002).

The aims of the pilot study in this research, were

(a) to identify problems with language and understanding of the NSQ and the MCMMTS items,

(b)   to determine the appropriateness of the content and level of difficulty of the items of the NSQ, the WCQ and the MCQ,

(c) to establish whether the time allocation for the different instruments was appropriate,

(d)   to practice the semi-structured interview and review the interview schedule.

The pilot study was conducted with seven lower primary and two upper primary mathematics teachers of a local primary school. This sample was chosen because these in-service teachers had an educational background very similar to that of the subjects of the main study and the site was removed enough from the research site to avoid contamination of the final data (Teijlingen & Hundley, 2001).

Each teacher was issued with a number, which would be the only means of identification on the questionnaires. The questionnaires were administered in the same sequence as intended for the main study, i.e. NSQ, WCQ, NCQ and lastly the MCMMST. However, the administration of each questionnaire was immediately followed by a short discussion to identify any shortcomings. Two teachers, one of each subset volunteered to be interviewed.

The pilot study proved to be essential in refining the instruments and the data collection process. Table 3.6 below summarises how shortcomings were addressed in the main study.

Table 3.6 Summary of changes made to the instruments after the Pilot Study

Questionnaire	Final instrument or procedure
NSQ	• The wording of two expressions was changed slightly. • The percentage question was deleted.
WCQ	• No changes were made
MCQ	• A data projector and a PowerPoint presentation were used to get the timing of the questions right. • The time for each question was reduced from 30 seconds to 20 seconds to prevent participants from using written calculations. • The first four questions, which tested basic number facts, were replaced by mental calculation questions. • The layout of the answer sheet was adapted to leave no space for written calculations. • Participants were not allowed to copy down the questions.
MCMMST	• Two practice examples (one positively and one negatively worded) with scales were included on the instruction sheet and done with the students before they answered the survey questions. • The wording "arithmetic calculations" was changed to "mental calculations"
Interview	• One easy question on whole numbers was added at the beginning to ensure that the interviewees could relax. • A focus question on estimation was added.

The lack of insight into the number sense questions relating to fractions and decimals as well as the performance on the parallel written computations showed that the teachers were very insecure when working with rational numbers. Teachers admitted that they could only do the mathematics that they had to teach and since they had been teaching the same grade every year, they could not remember the mathematics prescribed for the other grades. Therefore, for example, a Grade 1 teacher claimed not to know anything about fractions or decimals. The researcher decided not to reduce the number of questions relating to rational numbers, as she expected pre-service teachers who are just completing their training, to have more conceptual and procedural access to the rational number domain.

3.7.2   Quantitative Data Collection

Although the university does not have an official lunch hour in its timetable, the allocated time slot between 11:30 and 15:00 might have prevented some students from turning up. The final sample of 47 students who were willing to participate in the study represented 64% of the total targeted sample. This sample included 18 lower primary and 29 upper primary pre-service teachers.

The participants gathered in the mathematics lecture hall. The researcher explained the purpose of the study and the data collection procedures. The researcher read the consent form which all the participants then signed. To preserve anonymity and protect the participants' identity, each participant was given a unique number that was used throughout the study. This number was used on the cover page of each instrument and in the selection of the interviewees.

The data collection instruments were administered without incident in the following order and time allocation.

1. *Number Sense Questionnaire, 30 minutes.*

The questionnaires were handed out and the instructions on the front page read aloud. The participants were told that they should not work out answers by using written calculations, but use their knowledge about numbers to estimate and/or mentally determine the answers. One participant left after the completion of the NSQ. Her paper was disregarded for analysis purposes.

2. *Written Computation Questionnaire, 25 minutes*

The questionnaires were handed out and the instructions read aloud before the participants started working on the questions.

3. *Mental Computation Questionnaire, 6 minutes 40 seconds*

The answer sheets to the MCQ were handed out and the instructions read aloud. According to McIntosh (2005) a time of 5 seconds should be allowed for short items (the answer should be known immediately) and 15 seconds for long items (time should be given to work out calculations mentally). Using a timed power point presentation, the questions in this research were presented for 20 seconds on screen and read once.

4. *Confidence in Mathematics and Mathematics Teaching Survey, appr. 20 minutes*

The MCMMTS was handed out and the instructions read aloud. Two practice items were completed with the participants to prepare them for positive and negatively phrased questions. They were urged to answer all the questions as truthfully as possible, as this

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questionnaire did not have right or wrong answers. Enough time was allocated for each one to complete the questionnaire.

3.7.3   Qualitative Data Collection

After the completion of the instruments, the pre-selected participants were invited to be interviewed. Six of the randomly pre-selected participants, two lower primary and four upper primary pre-service teachers, volunteered to be interviewed. The interviews started after a short break and were conducted one-on-one in the lecture hall. All interviews were audiotape recorded with the consent of the interviewees. According to Gay, Mills and Airasian (2006) for interviews the data collection of choice is audio- or videotape recording, because tapes provide the researcher with the original data for use at any time. De Vos et al. (2005) maintain that recording of an interview allows the researcher to concentrate on the interview and its progress, without the additional burden of taking notes during the interview.

The interviewee received a written copy of the question and the question was read aloud by the researcher before the interviewee was requested to answer the question. Data from the students' thought processes was obtained by prompting them to explain their answer and probing deeper into background knowledge when they failed to give an explanation or gave a wrong explanation. All responses, even incorrect ones, where acknowledged without judgement.

In conducting the interviews, the researcher kept in mind the advice given by Gay, Mills and Airasian (2006, p. 420): (1) Listen more, talk less; (2) Follow up on what the participant says, ask questions when you don't understand; (3) Don't interrupt; tolerate silences; (4) Keep participants focused and ask for concrete details; (5) Don't be judgmental; stay neutral.

The interviews took longer than anticipated, because the interviewees found it very difficult to express their thoughts in words and were hesitant in answering. It took a lot of probing into incoherent responses and prompting to obtain a reasonable picture of their knowledge and thinking. The data from the interviews did not only yield a perspective on the participants' access to and use of number-sensible strategies, but also revealed their confidence in mathematics and gave an insight into their ability to explain mathematical ideas as future teachers.

3.8   Methods of Data Analysis

Data analysis is the "attempt by the researcher to summarize collected data in a dependable and accurate manner" (Gay, Mills & Airasian, 2006, p. 467). In this mixed-method research design, it was necessary to analyse data obtained by the quantitative measures as well as the qualitative data collected during the interview. Data analysis and interpretation is probably the most intimidating task for an inexperienced researcher (De Vos et al, 2002). Advice and assistance with the data analysis methods and interpretation was obtained from an experienced local social sciences research data analyst.

The data analysis followed the path of the concurrent triangulation model as depicted in Figure 3.1. The qualitative data analysis did not build on the results of the quantitative data analysis, but was conducted independently.   The results of the two sets of data were integrated in the interpretative phase of the research in Chapter 4.

3.8.1   Quantitative Data Analysis Methods

3.8.1.1 Scoring the NSQ, WCQ, MCQ and MCMMTS

The four quantitative instruments were scored as follows.

The Number Sense Questionnaire
Each item on the NSQ was assigned a maximum of 2 points. All items except Questions 5, 16 and 21 were given 2 points for the correct answer. Items 5, 16 and 21 required an answer as well as an explanation or examples. One point was given for the correct answer and one point for the correct explanation or examples. No points were awarded if the explanation was correct, but the answer was incorrect. The total possible score for the NSQ was 54 points.

The Written Computation Questionnaire
Scoring was dichotomous, one point for the correct answer and no points for an incorrect answer. No partial credit was given. The total possible score was 15.

The Mental Computation Questionnaire
Scoring was dichotomous, one point for the correct answer and no points for an incorrect answer. The total possible score was 20.

The McAnallen Confidence in Mathematics and Mathematics Teaching Survey
The MCMMTS used a 5-point Likert scale where the subjects responded on a scale of 1 to 5 according to their agreement with the statement. Twelve items, 6 phrased positively and 6 phrased negatively, related to personal self-efficacy and anxiety and thirteen items, 5 phrased

positively and 7 phrased negatively, related to teaching self-efficacy and anxiety. The scores of both domains were adjusted so that a score of 5 was given to the response which was hypothesised to indicate a positive relation to learning and teaching mathematics. Therefore a high score on each domain scale and on the cumulative score on both domains, represented a positive attitude to personal and teaching self-efficacy.

3.8.1.2 Data Analysis

The data from the four quantitative instruments were analysed using the Predictive Analysis Software, Version 18 (PAWS Statistics 18), which was formerly know as the Statistical Package for Social Sciences.

Descriptive statistics (mean, median, standard deviation and graphical representations) were obtained for the NSQ, the WCQ and the MCQ according to the frameworks in Tables 3.2, 3.3 and 3.4 above for the total sample and for each of the subsets of lower primary and upper primary teachers. Descriptive statistics were also obtained for the MCMMTS on both domain scores and the cumulative score for the total sample and its two subsets.

Correlation analysis was performed on the results of the NSQ with the WCQ, the NSQ with the MCQ, the MCQ with the WCQ and the NSQ with the MCMMTS to establish a possible relationship between the written computation and mental computation skills and number sense and the confidence in mathematics and mathematics teaching and number sense.

The results are discussed in Chapter 4.

3.8.2 Qualitative Data Analysis Methods

All the interviews were transcribed from the audiotape. In a transcription, data are inevitably lost from the original encounter (Cohen et al., 2000). To ensure that the essence of the interview was recorded in the transcription, the researcher indicated non-verbal communication such as a hesitant reply by inserting dots (…), long pauses, laughing and pointing or drawing a diagram. The non-verbal communication enabled the researcher to infer the interviewees command of language as well as his/her confidence in the ability to do mathematics.

Coding entails ascribing a category label to each piece of data with the category label either decided before or after the data collection (Cohen et al., 2000). At first, the researcher used pre-assigned categories to the responses relating to number sense: answers were coded

correct/incorrect and reasoning/explanations/calculations were coded correct/incorrect/none or rule-based/number-sense-based where applicable. The data was then quantitised by counting the incidences and expressing them as a percentage of the total responses. Reflecting on this reduction, the researcher realised that the essence of the interviews and episodes during the interviews would be lost to the reader of the research and this analysis would also not be useful for integrating the results of the quantitative and qualitative data results. Instead, the researcher decided to give a brief summary of the responses to each of the six questions. The results were then coded into four categories: Initial Answers, Strategies (rule-based / number-sense based), Language Use and Confidence.

The findings are discussed in detail in Chapter 4.

3.9 Reliability and Validity

3.9.1 Reliability

Reliability refers to the extent to which "independent administration of the same instrument (or highly similar instruments) consistently yields the same (or similar results) under comparable conditions" (De Vos et al., 2002, p. 168).

The key quantitative instruments used in this study, namely the Number Sense Questionnaire, the Written Computation Questionnaire, the Mental Calculation Questionnaire and the McAnallen Confidence in Mathematics and Mathematics Teaching Survey, were developed and tested by experienced researchers and used in research after their reliability had been established by statistical methods. According to Yang (in Hing, 2007), the split-half reliability of the NST is over 0.80 for both 6th and 8th Grade of students and the Cronbach's alpha reliability coefficients for the NST is 0.80. Yang's NST has been used in research in Taiwan (Reys & Yang, 1998, Tsao, 2004), Kuwait (Alajmi & Reys, 2007), Hong Kong, (Hing, 2007), Australia, Sweden and the United States (Reys et al., 1999). The reliability of the adapted tests was not established.

The reliability analysis of the MCMMTS (McAnallen, 2010) yielded the following: factor number one (teaching self-efficacy and anxiety factor) had a Cronbach's Alpha of 0.923 and factor number two (personal math self-efficacy and anxiety factor) had a Cronbach's Alpha of 0.952.

Reliability in quantitative research is based on the possibility of replication. This is clearly not workable for qualitative research such as interviews. Bakker (2004) maintains the internal reliability of qualitative studies can be interpreted as virtual replicablility, namely the research should be documented in such a way that the reader can track the learning process of the researcher and reconstruct his/her study. The researcher in this study has taken care to describe the all the important steps and decisions in this research process.

3.9.2 Validity

Validity refers to the degree to which a study actually measures the concept under investigation and whether it measures the concept accurately. Internal validity is demonstrated when the explanations and interpretations of the findings are actually sustained by the data which was collected (Cohen et al 2000). In qualitative research, validity can be addressed through "honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher", while in quantitative research, validity is improved by "careful sampling, appropriate instrumentation and appropriate statistical treatments of the data" (ibid, p.105).

The researcher used several ways to improve the internal validity of this study.

(a) *Choice of sample*: Although random sampling techniques were not employed, the sample was purposively chosen after discussions with representatives of the institution to include the widest range of academic abilities and cultural/linguistic backgrounds among the four education campuses.

(b) *Choice of quantitative instruments*: The three questionnaires and the confidence survey were adapted from instruments developed by experts in the field of number sense and teacher confidence. The researcher took care to make the instruments accessible to Namibians by changing unfamiliar terminology and reducing the demands of working with numbers at the top range of the local primary school curriculum without changing the essence of the questions. All instruments were piloted to determine their suitability.

(c) *Statistical analysis of quantitative data:* The appropriate and accurate analysis of the data were ensured by employing the assistance and advice of an experienced data analyst and by the use of appropriate computer software.

(d) *Qualitative data collection*: An interview schedule was drawn up prior to the data collection. The questions were selected to reflect the three number sense domains and included questions that investigated numerical estimation and mental computation skills. All interviewees were asked the same questions in the same sequence. The conversations were tape-recorded and then transcribed verbatim (low-inference descriptors). The interview was practiced with a group of local primary school teachers.

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(e) *Qualitative data analysis*: The responses were analysed according to the predetermined number sense domains and skills. Additional observations were made relating to the language use and confidence of the interviewees.

(f) *Triangulation*: In this mixed-method study, the quantitative data were triangulated with the interview responses of a sample of the participants. If the outcomes of the questionnaires and the confidence survey correspond to those of the interviews, the researcher can be more confident about the findings (Cohen et al, 2000).

External validity refers to the degree to which results can be generalised to a wider population or other cases and situations. Similar quantitative investigations into number sense have been conducted elsewhere (Alajmi & Reys, 2007, Hing, 2007, Reys & Yang, 1998, Reys et al., 1999, Tsao, 2004) and the findings of this study will contribute to the generality of the issues addressed there. This study does not seek to ensure transferability of the interview findings to other settings. The interview data served to corroborate, elaborate or illuminate the research findings of the quantitative data.

3.10 Ethical Considerations

According to De Vos et al (2002) ethics is a set of moral principles that "offer rules and behavioural expectations about the most correct conduct towards experimental subjects and respondents" (p. 63). This study and pilot study were conducted with permission of the relevant institutions and anonymity of the students was assured by assigning a number to each participant. This number was used throughout the study and remains the only way of identification.

Informed consent is a fundamental right of the participants in a research study. It is a procedure in which individuals choose to participate in an investigation after being informed of the purpose of the research, the protection of their privacy, the possible benefits or harm that the investigation could hold for them and any other facts that would be likely to influence their decision (Cohen et al., 2000). An Informed Consent Form was developed and each participant received a copy of the form and the information contained in this form was read aloud to the participants before the study was conducted. Participants then signed the form voluntarily.

This study is compliant with all ethical requirements set by the Stellenbosch University and ethical clearance was obtained before the investigation was completed (Appendix F).

3.11 Limitations of the Study

This investigation was prompted by the poor learning outcomes in mathematics at all levels of education in Namibia and the lack of research into teacher competency and confidence in teaching mathematics as one possible contributing factor to these outcomes. The study by Haufiku (2008) indicated that lower primary teachers lack the basic content knowledge prescribed in the Namibian Primary School Mathematics Syllabus and the pilot study conducted for this investigation showed that in-service teachers' content knowledge stretches no further than the content they have to teach. The researcher's previous work with primary school mathematics teachers seems to confirm these findings. The dilemma was to find the right pitch so that all participants could demonstrate what they know and can do. To address this dilemma, the number range in the NSQ, the WCQ and the MCQ and the level of difficulty were scaled down from the original tests as described in section 3.5.1 above. However, only the results of this study will show whether the dilemma was actually resolved.

The reliability of the adapted instruments was not established due to time limitations and the inexperience of the researcher.

Although the sampling was purposive and not random, the researcher believes that the sample was fit for the aims of this study. However, the researcher acknowledges that the outcomes of this study cannot be generalised to the whole population of pre-service teachers.

The allocated time slot for the data collection over lunch hour from 11:30 to 15:00 may have negatively affected the results of the study. Slow arrival of students delayed the beginning of the data collection even further and the researcher could sense that a few students were getting fidgety towards the end, because they might be late for the afternoon lecture.

The interviews took longer than anticipated, because the interviewees found it difficult to express themselves and the researcher had to spend more time on probing and prompting to find the depth of understanding or the levels of misunderstanding. In the data analysis, it was sometimes necessary to infer understanding from not very clearly stated explanations. The lack of English language skills of some interviewees may have been a contributing factor, which could not be accounted for. In naturalistic research, researchers often cannot anticipate what they will see or what the will look for (Cohen et al., 2000). This study led the researcher into more prompting and probing than expected with the resulting tension of how far to go and when to stop. In the end, not all interviews explored exactly the same issues. For example, only one respondent was asked to do long division and then revealed her

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confusion of the digit behind the decimal comma in the answer with the remainder. In the overall analysis of the data these instances were disregarded.

3.12 Summary

This chapter discussed how the investigation into the number sense of pre-service teachers was conducted within a pragmatic paradigm using a mixed-method research design. The selection and adaptation of instruments was explained. A report was given of the sampling and data collection procedures as well as the data analysis process. Reliability and validity of the study were discussed and ethical issues were addressed. Finally the researcher gave an account of her own position with regard to the limitations of the study.

CHAPTER FOUR

DATA ANALYSIS AND INTERPRETATION

4.1   Introduction

This study explored the number sense of final year lower primary pre-service teachers in Namibia with the key research question: Do Namibian final year primary pre-service teachers possess skills, knowledge, strategies and confidence that are related to number sense?

A mixed-method research design was used in this study. Quantitative data were collected at a teacher training university campus by means of number sense, mental computation and written computation questionnaires and a confidence in mathematics and mathematics teaching survey. To triangulate the quantitative data and shed light on the use of number-sensible strategies, qualitative data were obtained through interviews with a sample of the participants.

The data from the four quantitative instruments were analysed using the Predictive Analysis Software, Version 18 (PAWS Statistics 18), which was formerly known as the Statistical Package for Social Sciences. The tape-recorded interviews were transcribed and then coded into four categories: Initial Answers, Strategies (rule-based / number-sense based), Language Use and Confidence.

In this chapter first the quantitative data and then the qualitative data will be analysed and described. In the interpretation section, the data will be integrated and discussed and the outcomes related to the research question and sub-questions.   The chapter will conclude with a summary of the findings and the answers to the research questions.

4.2   Quantitative Data Results

The descriptive statistics were obtained for the four quantitative instruments and a correlation analysis was performed between the number sense questionnaire (NSQ), the written computation questionnaire (WCQ), the mental calculations questionnaire (MCQ) and the confidence survey, the MCMMTS. The results will be presented in this section together with the analysis of performance on the number domains of whole numbers, common fractions and decimals; the operations of addition, subtraction, multiplication and division; and the

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number sense domains of "Knowledge of and facility with numbers", "Knowledge of and facility with operations" and "Application to computational settings".

4.2.1 Performance on the Number Sense, the Written Computation and the Mental Calculation Questionnaires

Table 4.1 summarises the mean, median and standard deviation by phase of the raw scores for the Numbers Sense Questionnaire (NSQ), the Written Computation Questionnaire (WCQ) and the Mental Calculation Questionnaire (MCQ).

Table 4.1 Descriptive statistics for the NSQ, WCQ and MCQ by phase (raw scores)

Questionnaire	Statistic	Lower Primary N= 18	Upper Primary N= 29	Total N = 47
NSQ Max score: 54	Mean	20.7	20.5	20.6
	Median	21.0	20.0	20.0
	Standard Deviation	5.2	6.2	5.8
WCQ Max score: 15	Mean	5.2	6.4	5.9
	Median	4.0	6.0	6.0
	Standard Deviation	2.7	2.7	2.7
MCQ Max score: 20	Mean	4.2	4.9	4.6
	Median	3.5	4.0	4.0
	Standard Deviation	3.2	2.8	3.0

The results show that there is no significant difference in the mean scores of the lower primary pre-service teachers (LP PSTs) and the upper primary pre-service teachers (UP PSTs) on any of these tests. The LP group's mean score on the NSQ was slightly higher (20.7) than that of the UP group (20.5) and the standard deviation of the LP group (5.2) was slightly lower than that of the UP group (6.2). The UP group performed better on the WCQ with a mean of 6.4 compared to a mean of 5.2 for the LP group. There was no difference in standard deviation for the WCQ. On the MCQ, the UP PSTs had a higher mean (4.9) than the LP PSTs (4.2) and a smaller standard deviation (2.8)

To compare the results of the three questionnaires the means, medians and standard deviations were converted to percentages as shown in Table 4.2. The mean of 39.3% for the total group on the WCQ was only slightly higher than the mean of 38.1% on the NSQ. However, the total group achieved a significantly lower mean of 23% on the MCQ. The standard deviations show that the variation in scores was the lowest on the NSQ (SD = 10.7%) and the highest on the WCQ (SD = 18%).

Table 4.2 Descriptive statistics for the NSQ, WCQ and MCQ by phase (percentages*)*

Questionnaire	Statistic	Lower Primary N= 18	Upper Primary N= 29	Total N = 47
NSQ	Mean	38.3	38.0	38.1
	Median	38.9	37.0	37.0
	Standard Deviation	9.6	11.5	10.7
WCQ	Mean	34.7	42.7	39.3
	Median	26.7	40	40
	Standard Deviation	18	18	18
MCQ	Mean	21.0	24.5	23.0
	Median	17.5	20.0	20.0
	Standard Deviation	16.0	14.0	15.0

Figure 4.1 compares the distribution of the total scores obtained on the NSQ, the WCQ and the MCQ.
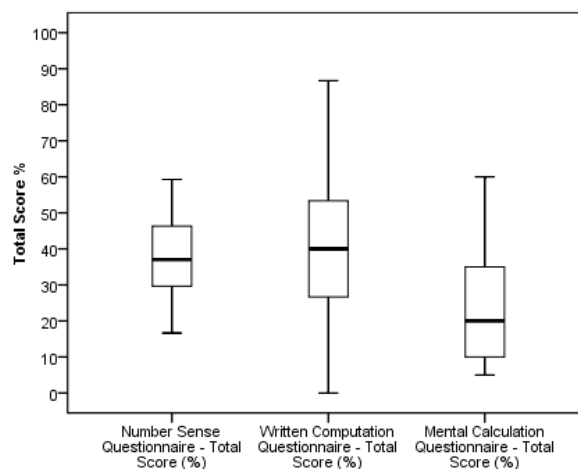
Figure 4.1 Distribution of total scores for the NSQ, WCQ and MCQ

The figure shows that the NSQ scores have the least variation and are approximately symmetrically distributed. The highest variation in scores occurred on the WCQ, which is slightly positively skewed. The distribution of the MCQ is highly positively skewed and has the lowest median of the three distributions.

4.2.2   Performance on the Number Domains

The performance of the pre-service teachers (PSTs) on the number domains of whole numbers, common fractions and decimals, was analysed for each questionnaire. Table 4.3 shows that the questionnaires included fewer of questions on the decimal number domain than on the domains of whole numbers or common fractions. As mentioned in Chapter 3, this was done to accommodate the possible shortcomings of LP PSTs who will not have to teach the concept of decimal fractions in Grades 1 to 4.

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Table 4.3 Distribution of questions over the three number domains

	Whole numbers	Common fractions	Decimals	Total
NSQ	10	11	6	27
WCQ	5	6	4	15
MCQ	9	6	5	20
Total	24	23	15	62

The mean, median and standard deviation for the percentage scores obtained in the domains of whole numbers, common fractions and decimals are shown in Table 4.4.

Table 4.4 Descriptive statistics for the NSQ, WCQ and MCQ by number domain (%)

Questionnaire N = 47	Statistic	Whole numbers	Common fractions	Decimals
NSQ	Mean	49.4	22.3	41.8
	Median	50.0	22.7	41.7
	Standard Deviation	17.5	12.1	17.5
WCQ	Mean	52.8	35.5	28.7
	Median	60.0	33.3	25.0
	Standard Deviation	24.8	29.0	22.1
MCQ	Mean	25.5	19.1	23.8
	Median	22.2	16.7	20.0
	Standard Deviation	19.4	17.0	20.7

The mean for the whole number domain is the highest in all three questionnaires: 49.4% for the NSQ, 52.8% for the WCQ and 25.5% for the MCQ. The mean for the domain of common fractions is the lowest for the NSQ (22.3%) and the MCQ (19.1%). The mean for common fractions in the WCQ is relatively high at 35.5%, but the standard deviation is the highest overall with 29.0% pointing to a greater variation in scores than for the same domain in the results of the other questionnaires. The means obtained in all three domains on the MCQ are lower than the respective means on the NSQ and the WCQ. The best performance in the domain of decimals is on the NSQ with a mean of 41.8%.

Figures 4.2, 4.3 and 4.4 compare the distribution of scores for the three numbers domains on the NSQ, the WCQ and the NSQ respectively.

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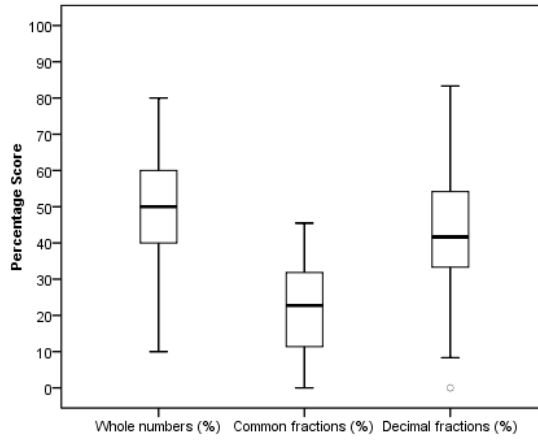
Figure 4.2 Distribution of scores for the three number domains on the NSQ
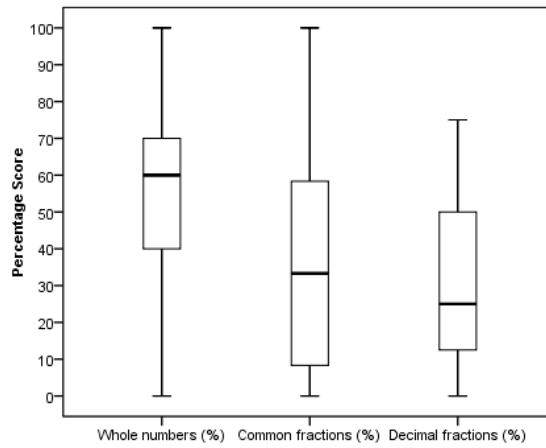
Figure 4.3 Distribution of scores for three number domains on the WCQ
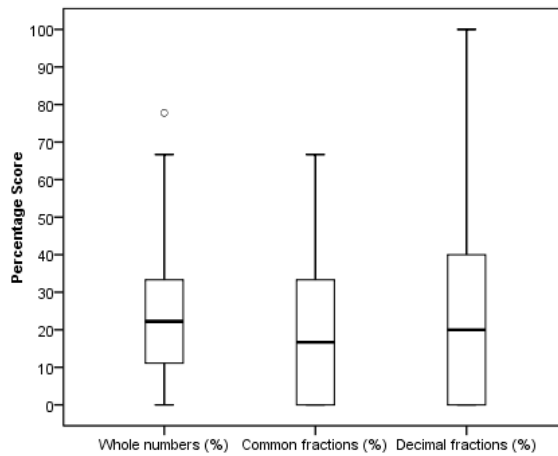
Figure 4.4 Distribution of scores for the three number domains on the MCQ

The diagrams show the great variation in scores on the three domains on all questionnaires, with a range of 100% for the whole number domain on the WCQ and the common fractions domain on the WCQ and the decimal fractions domain on the MCQ. The least variation of 45% occurred on the common fractions domain on the NSQ, but this is also accompanied by the lowest maximum value, indicating a poor overall performance.

The distributions on the whole number domain on the NSQ and the WCQ are moderately negatively skewed. The common fractions domain on the NSQ is approximately symmetrically distributed around the low median of 22.7%. All other distributions are either moderately or highly positively skewed illustrating the poor performance of the PSTs on these domains.

4.2.3   Performance on the four operations (WCQ and MCQ)

The results on the WCQ and the MCQ were analysed according to the performance on the operations of addition, subtraction, multiplication and division. Table 4.5 shows that the questions were distributed almost equally over the four operations.

Table 4.5 Distribution of questions over the four operations

	Addition	Subtraction	Multiplication	Division	Total
WCQ	4	4	3	4	15
MCQ	6	4	6	4	20
Total	10	8	9	8	35

The mean, median and standard deviation for the percentage scores obtained for each of the operations are summarised in Table 4.6.

Table 4.6 Descriptive statistics for the WCQ and MCQ by operation (percentages)

Questionnaire N = 47	Statistic	Addition	Subtraction	Multiplication	Division
WCQ	Mean	52.5	49.5	37.6	18.1
	Median	50.0	50.0	33.3	25.0
	SD	24.9	30.2	24.7	20.7
MCQ	Mean	35.5	29.3	15.2	10.6
	Median	33.3	25.0	16.7	0
	SD	21.0	25.2	15.9	17.9

On both the WCQ and the MCQ the highest means were obtained for addition, 52.5% and 35.5% respectively, followed by subtraction with means of 49.5% and 29.3% respectively. The means for multiplication and division on both questionnaires were significantly lower, i.e. 37.6% and 15.2% for multiplication and 18.1% and 10.6% for division on the WCQ and

the MCQ respectively. The means for all operations on the MCQ were significantly lower than those on the WCQ.

The high standard deviations on all operations in both tests of between 30.2% for subtraction on the WCQ to 15.9% for multiplication on the MCQ show a wide variation in performance of the participants. On both tests for all operations the minimum mark obtained was 0%, with a maximum mark of 100% for all operations except multiplication on the MCQ (66.7%), division on the MCQ (50%) and division on the WCQ (75%).

The Figures 4.5 and 4.6 below illustrate the distribution of the total scores (%) obtained by the participants on the WCQ and the MCQ for each operation.
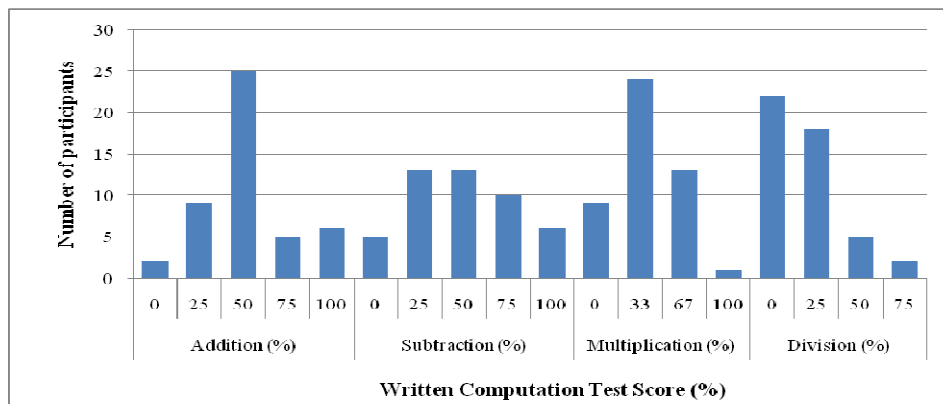
Figure 4.5 Distribution of total scores (percentages) on the WCQ by operation
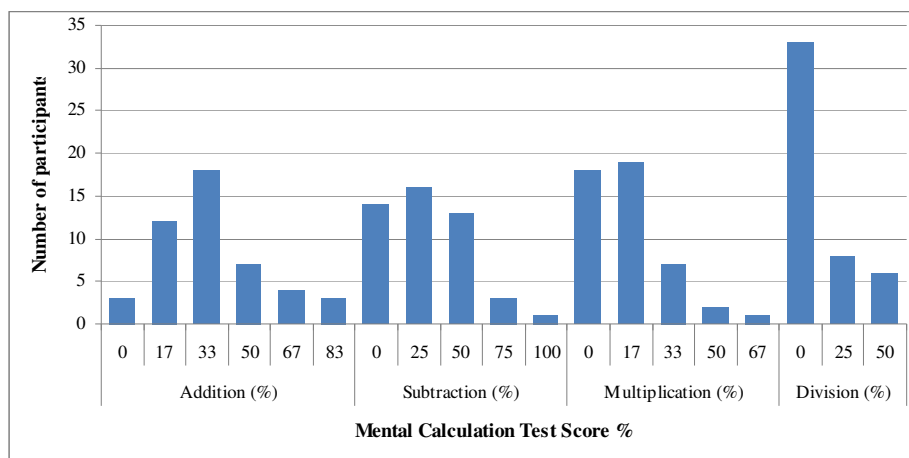
Figure 4.6 Distribution of total scores (percentages) on the MCQ by operation

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The highest mode of 50% was obtained for addition on the WCQ, followed by a mode of 33% for addition on the MCQ and multiplication on the WCQ. The mode for division on both questionnaires was 0%. More than half the participants obtained less than the mean score on all operations on both tests.

4.2.4   Performance on the number sense domains

The results of the NSQ were analysed for the following number sense domains identified in the framework for examining basic number sense by McIntosh, Reys and Reys (1992): "Knowledge of and facility with numbers", "Knowledge of and facility with operations" and "Application to computational settings". "Knowledge of and facility with numbers" includes a sense of orderliness of numbers, multiple representations for numbers, a sense of the relative and absolute magnitude of numbers and a system of benchmarks. "Knowledge of and facility with operations" refers to understanding the effects of operations, understanding mathematical properties such as commutativity, identities and inverses and understanding the relationship between operations such as the relationship between multiplication and addition. "Application to computational settings" encompasses the application of the knowledge of and facility with numbers and operations through recognising the connection between the problem context and the applicable computation, the ability to select among different strategies, the inclination to use efficient representations or methods and to check for the reasonableness of the answer.

The mean, median and standard deviation for the percentage scores obtained for each of the number sense domains are summarised in Table 4.7.

Table 4.7 Descriptive statistics for the number sense domains on the NSQ (percentages)

N = 47	Knowledge of and facility with numbers	Knowledge of and facility with operations	Application to computational settings
No. of questions	10	8	9
Mean	22.7	36.2	52.7
Median	20.0	37.5	55.6
SD	11.3	18.5	14.9

The highest mean of 52.7% was obtained for the number sense domain "Application to computational settings", followed by 36.2% for "Knowledge of and facility with operations" and 22.7% for "Knowledge of and facility with numbers".

Figure 4.7 below illustrates the distribution of the total scores (%) obtained by the participants for each number sense domain on the NSQ.
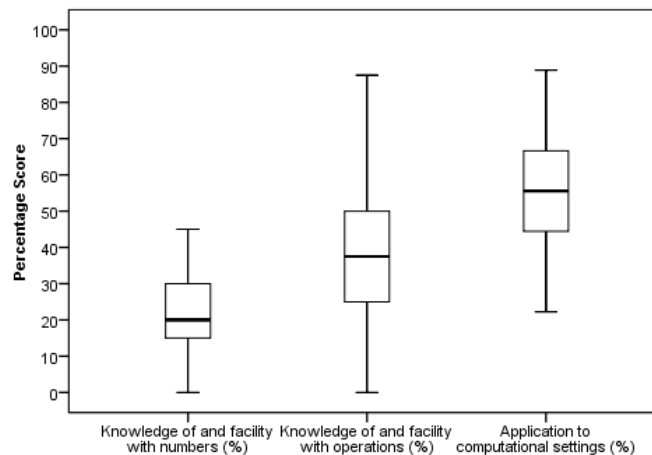
Figure 4.7 Distribution of total scores by number sense domain on the NSQ

The diagram shows that the poorest performance was achieved on the "Knowledge of and facility with numbers" domain with a median of 20% and a variation between a minimum of 0% and a maximum of 45% and an approximately symmetric distribution. The highest variation between 0% and 89% is indicated in the "Knowledge of and facility with operations" domain with a median of 37% and a moderately positive skewness. The distribution of the "Application to computational settings" domain is approximately symmetric and shows a variation of 67% with a minimum of 22% and a maximum of 89% with a median of 55%.

4.2.5 The McAnallen Confidence in Mathematics and Mathematics Teaching Survey

The MCMMTS consisted of 25 statements designed to elicit beliefs, attitudes and feelings regarding the respondent's ability to do and to teach mathematics. The overall domain was split into Factor 1, twelve questions, indicating confidence to do mathematics and Factor 2, thirteen questions, indicating confidence in teaching mathematics. The data was analysed after recoding the negatively phrased statements so that the highest score of 5 would in each case indicate a greater confidence in the ability to do and to teach mathematics.

Table 4.8 below shows mean, median and standard deviation as well as the minimum individual mean score and the maximum individual mean score for the Overall Domain as well as Factor 1 and Factor 2.

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Table 4.8 Descriptive statistics for the MCMMTS

N = 47	Mean	Median	SD	Min	Max
Overall **Max: 5**	3.78	3.88	0.43	2.88	4.40
Factor 1 **Max: 5**	3.57	3.75	0.57	2.17	4.42
Factor 2 **Max: 5**	3.97	4.00	0.41	3.08	4.77

The overall mean was 3.78, with median 3.88 and standard deviation of 0.43. The lowest individual mean score was 2.88 and the highest individual mean score was 4.40. Factor 1, confidence in doing mathematics, results were lower than the overall results with a mean of 3.57, median of 3.75, standard deviation of 0.57 and a minimum individual mean of 2.17 and maximum individual mean of 4.42. The results for Factor 2, confidence in teaching mathematics, showed a higher mean (3.97), median (4.00), and lower standard deviation (0.41) than the overall results. The minimum and maximum individual means were also higher with 3.08 and 4.77 respectively.

Figures 4.8 and 4.9 show the distribution of mean scores for Factor 1 and for Factor 2.
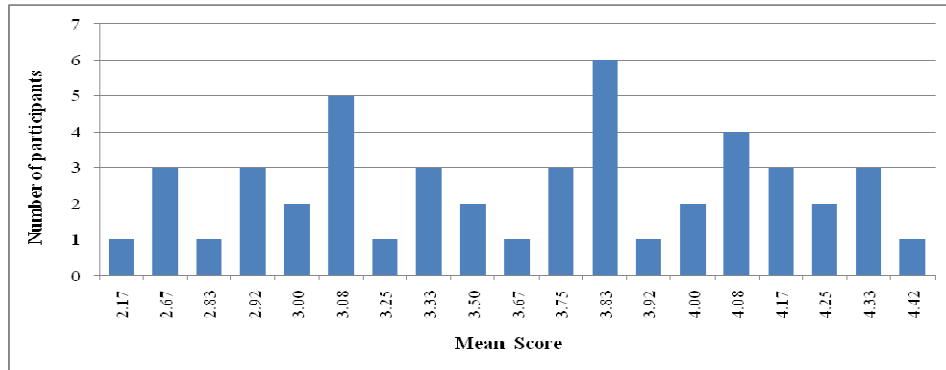
Figure 4.8 Distribution of mean scores for Factor 1
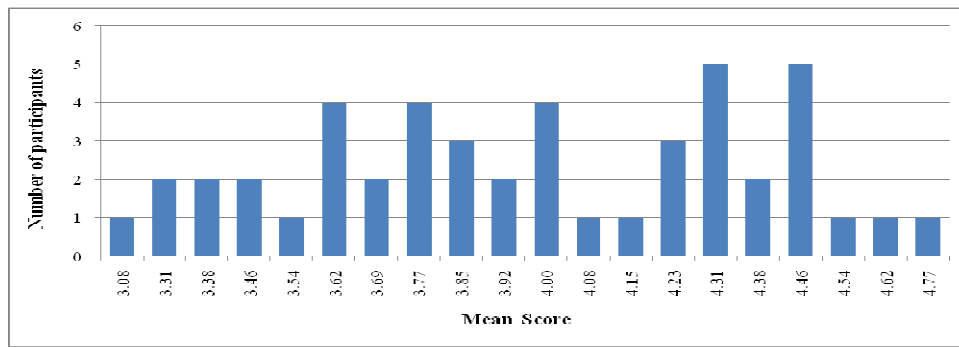
Figure 4.9 Distribution of mean scores for Factor 2

For both factors, the mode lies above the mean. The mode for Factor 1 is 3.83 compared to the mean of 3.57. The distribution for Factor 2 has two modes, namely 4.23 and 4.46 compared to the mean of 3.97. Just more than half the individual mean scores lie above the mean for both factors, i.e. 55% of the scores for Factor 1 and 51% of the scores for Factor 2. It can be deduced that both distributions a moderately negatively skewed.

The mean scores for LP PSTs and UP PSTs on the MCMMTS were analysed and are shown in Table 4.9.

Table 4.9 Descriptive statistics for the MCMMTS by phase

N = 47		Lower Primary N = 18	Upper Primary N = 29	Total N = 47
Overall	Mean	3.69	3.84	3.78
Max: 5	SD	0.47	0.40	0.43
Factor 1	Mean	3.35	3.72	3.57
Max: 5	SD	0.61	0.50	0.57
Factor 2	Mean	4.00	3.95	3.97
Max: 5	SD	0.43	0.40	0.41

The UP PSTs' mean scores on the Overall (3.84) and the Factor 1 (3.72) results are significantly higher than those of the LP PSTs, which are 3.69 and 3.35 respectively. The mean scores on Factor 2 do not differ significantly with the LP PSTs' mean of 4.00 slightly higher than that of the UP PSTs' of 3.95. The standard deviations of all results are greater for the LP PSTs with the greatest standard deviation of 0.61 for Factor 1.

The participants also compared themselves to other primary school mathematics pre-service teachers in terms of their mathematical abilities. The results are shown in Table 4.10.

Table 4.10 Comparison of self to other PSTs in terms of mathematical ability

	Number Lower Primary	Number Upper primary	Number Total
Below average	1	0	1
Average	7	5	12
Above average	7	11	18
Way above average	2	5	7
One of the best	1	8	9
Total	18	29	47

Only one participant rated himself/herself "below average" as compared to others in terms of mathematical abilities. 12 PSTs rated themselves as "average", 18 as "above average", 7 saw their abilities "way above average" and 9 as "one of the best". 44% of the LP PSTs rated themselves as "average" or "below average" compared to only 17% of the UP PSTs who rated themselves in these categories.

In Table 4.11 the answers to the question, "Which maths classes have you successfully completed in school?" are summarised.

Table 4.11 Highest grade successfully completed a school

	Number Lower primary	Number Upper primary	Number Total
Grade 7	1	0	1
Grade 10	5	4	9
Grade 12	12	25	37
Total	18	29	47

According to this information, which was not independently verified, 67% of the LP PSTs and 86% of the UP PSTs indicated that they offered mathematics as a subject and completed it successfully at Grade 12 level.

4.2.6 Correlation Analysis

The Pearson Correlation Coefficient was obtained between the total scores on parallel items of the NSQ and the WCQ, between the total scores on the NSQ and MCQ and between the total scores on the WCQ and the MCQ. Table 4.12 shows the results.

Table 4.12 Pearson's Correlation Coefficients between the NSQ, WCQ and MCQ

N = 47	NSQ	WCQ	MCQ
NSQ	1.00		
WCQ	0.241	1.00	
MCQ	0.540*	0.457*	1.00

* Significant at p < 0.01 (2-tailed)

The correlation of $r = 0.241$, $p > 0.05$ between the NSQ and the WCQ was weak, positive and insignificant. The correlations of $r = 0.540$, $p < 0.01$ between the NSQ and the MCQ and of $r = 0.457$, $p < 0.01$ between the WCQ and the MCQ were substantial, positive and significant.

Table 4.13 shows the correlations obtained between the total scores obtained on the NSQ and the total scores obtained on the MCMMTS Overall, the NSQ and Factor 1 of the MCMMTS and the NSQ and Factor 2 of the MCMMTS.

Table 4.13 Pearson's Correlation Coefficients between the NSQ and the MCMMTS Overall, Factor 1 and Factor 2

N = 47	NSQ
NSQ	1.00
MCMMTS Overall	0.553*
MCMMTS Factor 1	0.478*
MCMMTS Factor 2	0.502*

* Significant at p < 0.01 (2-tailed)

All three Pearson Correlation Coefficients were substantial, positive and significant. The highest correlation of $r = 0.553$, $p < 0.01$ was obtained between the NSQ and the overall total scores on the MCMMTS, followed by a correlation of $r = 0.502$, $p < 0.01$ between the NSQ and the total scores obtained on Factor 2, the confidence in the ability to teach mathematics, and a correlation $r = 0.478$, $p < 0.01$ between the NSQ and Factor 1, the confidence in the ability to do mathematics.

The conclusions that can be drawn from the quantitative analysis are summarised below.

4.2.7   Conclusions based on findings of the quantitative analysis

The analysis of the quantitative data indicated that the primary pre-service teachers in this sample demonstrated an underdeveloped sense of number and identified the areas of particular weakness on the three number sense domains, the three number domains and the four operations. The following conclusions were drawn from this analysis:

A mean of 38.1% on the Number Sense Questionnaire indicated that the PSTs have a poorly developed number sense. The mean of 22.7% and maximum score of 45% on the domain "Knowledge of and facility with numbers" showed a lack of conceptual understanding of the orderliness of numbers, relative magnitude of number and multiple representation of numbers.

Despite the focus in Namibian schools on the application of procedures and standard algorithms to obtain a correct answer, the mean of the Written Computation Questionnaire was only 39.3%. However, a high standard deviation indicated that some PSTs are competent in performing written computations.

The very poor performance on the Mental Calculations Questionnaire with a mean of 23.0% indicated that PSTs demonstrated a low proficiency to calculate mentally at a Grade 6 level.

The analysis of performance on the three number domains showed that the PSTs are more competent and comfortable working with whole numbers than with either common fractions or decimal fractions. Performance on the common fractions domain was particularly poor on the Number Sense and the Mental Calculations Questionnaires.   The slightly better performance on this domain on the Written Computation Questionnaire, indicates that some PSTs are proficient in applying paper-and-pencil methods to calculations with fractions.

PSTs are more competent in performing addition and subtraction than multiplication and division. Performance on division was particularly poor with a mean of 18.1% on the Written Computation Questionnaire and 10.6% on the Mental Calculations Questionnaire.

PSTs are surprisingly confident in both their ability to do mathematics and in their ability to teach mathematics with an overall mean score of 3.78 out of a possible score of 5. Only 25% rated their mathematical abilities as 'average' or 'below average' compared to other PSTs.

The correlation analysis showed a weak, positive but insignificant correlation between the NSQ and the WCQ. However substantial, significant and positive correlations were obtained between the NSQ and both the MCQ and the MCMMTS, as well as between the MCQ and the WCQ. This indicates that a weak relationship exists between number sense and written computation, but that number sense is related to both mental calculation proficiency and confidence and than there is a significant relationship between written and mental calculation proficiency.

These outcomes will be discussed in more detail and related to the research questions and international research studies in section 4.4.

4.3   Qualitative Data Results

The analysis of the qualitative data is divided into two sections. In the first section a summary of the interview process by number sense domain and question is given. Based on the analysis of the interviews the second section describes and summarises the outcomes of the four coded categories: Strategies, Initial Answers, Language and Confidence. A more detailed description of the categories is given in section 4.3.2 below.

Two lower primary pre-service teachers, numbered LP1 and LP2, and four upper primary pre-service teachers, referred to as UP1, UP2, UP3 and UP4, were interviewed. UP4 was the only male respondent.

4.3.1   Analysis of interviews by Number Sense domain and Question

The Number Sense domains are based on the Framework developed by McIntosh, Reys and Reys (1992) as described in detail in section 4.2.4 above. The results of the interviews for each question in these domains will now be discussed.

4.3.1.1 Number Sense Domain: Knowledge of and Facility with Number

Both number sense components that were investigated in this section were selected from the common fractions domain. The first concerned the relative magnitude of numbers and the second the multiple representation of numbers.

(1) *Number sense component: Recognise the relative size of numbers*

This question aimed at establishing whether the respondents had a sense of the magnitude of two fractions compared to $\frac{1}{2}$ as referent.

Question 1 (Interview item 3): Is $\frac{3}{8}$ or $\frac{7}{13}$ closer to $\frac{1}{2}$?

Possible solution strategies:

(a) The rule-based strategy would entail subtraction of fractions by applying the standard algorithm.

(b) A number-sensible approach would use a residual strategy such as arguing that three-eighth is one-eighth away from a half and seven-thirteenth is only half of one-thirteenth or one-twenty-sixth away from a half. As one-twenty-sixth is smaller than one-eighth, seven-thirteenth is closer to a half.

All respondents found this question challenging. The lack of confidence was apparent in their initial answers.

UP1	A very tricky question.
UP3	I think, no, because if you simplify them it does not give me a half. They are not equivalent fractions to one-half.
UP4	Know ... I think ... the first two, I think we need to add them and then find out if the answer is going to be a half ... to think ... (pause)

The three respondents, who actually gave an answer, did so hesitatingly.

LP1	I just guess ... this one three over eight is closer...
LP2	I think seven over thirteen is closer, is closer to a half.
UP2	I would say three-eighth is closer to a half.

When asked to explain his or her answer, no one suggested the rule-based strategy. UP2 came close to a number-sensible explanation by drawing a number line and then

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> UP2   ... four over eight is actually one over two; it is equivalent to one over two. So, that is why I would say, three over eight is closer to a half.

However, she could not produce a similar argument for seven-thirteenth.
LP2 who suggested the correct answer, explained as follows

> LP2   Because ... mmm ... you divide the two ... the one into the seven will be a remainder, but if I take the first expression, then the three can go into the eight like two times ... that will be less than one over eight ... that will be bigger than seven over ...

From this muddled explanation, one could perhaps infer that she had the intuitive feeling of the closeness of the fractions to a half, but that she could not put it into words. However, when she placed the fractions on a number line, it became clear that she had not realised that seven-thirteenth is greater than a half as shown in Figure 4.10.
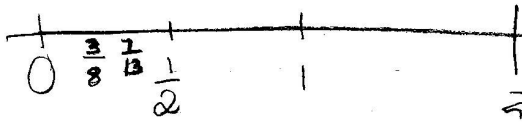
Figure 4.10 LP2: Order of fractions on a number line

Further probing into the respondents' knowledge of the order of fractions requested them to place the fractions on a number line. The only one, who managed to get the order correct, was UP2 who had initiated the use of the number line herself. All respondents had a problem interpreting $\frac{7}{13}$: two placed it on the number line between 0 and $\frac{1}{2}$; one put it on 1 and two inserted it to the right of 1.

From the attempted explanations, as shown in the extract of LP2s response above, it was concluded that the respondents were not able to verbalise their understanding of fractions in clear and simple language suitable for the classroom.

Table 4.14 summarises the analysis of Question 1.

Table 4.14 Summary of analysis of Question 1 (Interview item 3)

N = 6	Analysis
Initial answers	1 correct, 2 incorrect, 3 could not answer the question at all
Strategies: **Reasoning**	All explanations were confused, incoherent or incomplete. <u>Rule-based</u>: none <u>Number sense based</u>: two incomplete attempts to use the residual strategy
Language use	Difficulties in verbalising their understanding
Confidence	A lack of confidence apparent from their initial responses to the question

(2) *Number sense component: Understanding multiple representations of number*

This question investigated whether respondents could translate between written, symbolic and diagrammatic representations of a fraction.

<u>Question 2 (Interview item 6)</u>: Which letter in the number line names a fraction where the numerator is slightly more than the denominator?
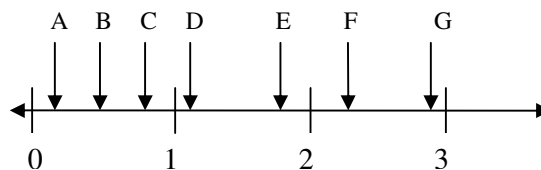
<u>Possible solution strategies</u>: This was a closed question that did not offer different solution strategies. Respondents had to translate the written statement into a symbol, i.e. an improper fraction, and then translate the fraction symbol to a diagrammatic representation.

All respondents expressed a lack of confidence in answering this question by admitting that they could not do it.   Their responses can best be summarised by the statement of UP4: "Aaai…. I didn't have a clue". Two respondents offered an answer, LP1 suggested C, but could not progress from there. Guessing was evident from LP2s' response:

LP2	It's ... aah ... B.
Researcher	Why did you choose B?
LP2	I mean G.
Researcher	G, okay, why did you say G?
LP2	Because ... I feel ... I think ... (pause)

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All six respondents had to be prompted to translate the words into a symbol, but then failed to translate the symbol to a point on the number line. Only those who were prompted to write the improper fraction as a mixed number, managed to solve the problem. UP2 exclaimed at the end, "I never thought of changing it into a mixed fraction!" An excerpt of the interview with UP3 illustrates the lack of understanding of improper fractions.

Researcher	Where on the number line would you put six-fifths?
UP3	Must I put it here or can it be extended?
Researcher	If you think it must be extended, it can be extended.
UP3	(Extends number line and writes 6 as the next unit – see Figure 4.11). It's a half, half and then it's a half (draws halfway marks between units) together with the whole numbers, I think.
Researcher	Okay, where would six-fifths be?
UP3	Maybe put it somewhere over here (puts it on the number line close to her 6).

Figure 4.11 illustrates this misconception which may arise from seeing a fraction as two separate numbers and not understanding the relationship between the numerator and the denominator.
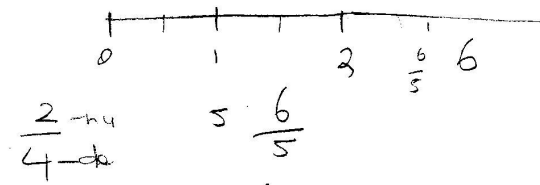
Figure 4.11 Improper fraction misconception

The same misconception may have played a role in the inability of other respondents to place the improper fraction on the number line. It was also noted that all respondents referred to fractions in the form "two over three" or "six over five" – a form of language, which emphasises the two parts of a fraction and is not conducive to the understanding of a fraction as one number.

Table 4.15 summarises the analysis of Question 2.

Table 4.15 Summary of analysis of Question 2 (Interview item 6)

N = 6	Analysis
Initial answers	2 incorrect, 4 did not offer an answer
Strategies:	No one could make any progress without prompting
Reasoning	No evidence of strategies or ability to translate representations
Language use	Misleading expression "... over ..." for fractions
Confidence	All lacked the confidence to answer this question.

4.3.1.2 Number Sense Domain: Knowledge and Facility with Operations

The understanding of the relationship between operations in the whole number domain and the understanding of the effect of an operation in the decimal domain were the number sense components investigated in this section.

(1) *Number sense component: Understand the relationship between operations*

This question established whether the respondents understood the relationship between addition and multiplication of whole numbers.

Question 3 (Interview item 2):   Which expression represents the larger amount?

$$2452 \times 4$$

$$2541 + 2457 + 2460 + 2465?$$

Possible solution strategies:

(a) The rule-based strategy would entail calculating both answers by applying the standard algorithms or any other paper and pencil methods.

(b) A number-sensible approach would use the relationship between multiplication and addition and a comparison of the relative size of the numbers to determine that the answer to the first expression would be smaller.

This question was accessible to all respondents although only LP2 and UP2 expressed confidence in their answers.

LP2   It's the last expression.

UP2   I actually answered this question. I worked out 2541 plus
            2457 plus 2460 plus 2465 and it gave me the bigger answer
            than 2452×4 (referring to the calculation done in the WCQ).

The other four respondents were a bit more hesitant in their replies.

LP1	I think the second one.
UP1	I think it is the first one.
UP3	Mmm ... I think this ... (points to the second expression)
UP4	I think ... eeh ... I think the underneath one (points to the second expression)

Five gave the correct answer and could explain the relationship between multiplication and addition. However, no one offered a good, fluent and complete explanation and it took some probing from the researcher to get a picture of his or her understanding. The researcher was left wondering whether the learners in a classroom would be able to make sense of the explanations.

LP2	It's the last expression.
Researcher	Why?
LP2	Because, first of all the second one has more numbers than the first expression. This expression (points to the first expression) is four of them together which would make it lesser than this one.
Researcher	Hmm. Could you work out 2452 × 4 in another way?
LP2	Yes, I add the 2452 together four times.
Researcher	Yes. And you would think that answer would be smaller than the second answer?
LP2	Yes.
Researcher	Why was that again?
LP2	Because the 2452 is for me one of the smaller numbers than to that of the first expression.

UP1 gave an incorrect answer and insisted on it even after being prompted to compare the numbers and stating that the numbers in the second expression "are bigger than the first one". She had more confidence in the correctness of her calculations on the WCQ than in her sense of numbers and operations.

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Table 4.16 summarises the analysis of Question 3.

Table 4.16 Summary of analysis of Question 3 (Interview item 2)

N = 6	Analysis
Initial answers	5 correct, 1 incorrect
Strategies: **Reasoning**	Correct reasoning was inferred from explanations Rule-based: none Number-sense based: 5 could apply relationship between multiplication and addition
Language	Difficulty in expressing understanding
Confidence	Only two were confident in their answers

(2) *Number sense component: Understand the effect of operations*

This question investigated whether respondents understood the effect that multiplication by a decimal fraction has on a number.

Question 4 (Interview item 5): Place the decimal comma in the answer to following multiplication problem: $534,6 \times 0,545 = 291357$

Possible solution strategies:

(a) Application of the rule: "The sum of the digits behind the decimal comma in the multiplicand and the multiplier equals the number of digits behind the decimal comma in the product." This rule would have led to the incorrect answer of 29,1357, unless a respondent realised that the answer to 5346×545 is 2913570.

(b) A number-sensible approach would use estimation, for example $550 \times 0,5$ is more than 250 but less than 300 and therefore $534,6 \times 0,545 = 291,357$.

All respondents offered an answer to this question, except one, who said that she was "not happy with place value". Three respondents gave the correct answer (291,357). One could not explain the answer and the other two used the rule for addition of decimal numbers by re-writing the question in column form, aligning the decimal commas.

LP2	I put it three numbers after the comma (puts it correctly between 1 and 3)
Researcher	Why did you place it there?
LP2	Because I did it ... I put it in fraction form. Like I wrote the 534,6 (writes it) ... there is one place behind the decimal comma ... then the 0,545 (writes it underneath and aligns the commas) ... and then when I multiply it, I obviously have three numbers after the comma.

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The two incorrect answers (2913,56) were both based on adding the number of decimal places in front of the comma in the multiplier and multiplicand and then counting the same number of places from the beginning of product, as UP4 explained very confidently

UP4   In the sum you have ... three digits in front of the comma in the first one and one before the decimal comma in the second one. So together it gives me four. So in the answer I have four decimals before the comma.

UP4 came across as one of the least confident and articulate interviewees, but he was fairly confident and fluent when he was asked to explain a rule as opposed to explaining his thinking or understanding of a concept.

No one used estimation spontaneously as a strategy. After probing, four of the respondents could verbalise that multiplication by a decimal fraction makes the answer smaller and UP1 realised that her answer of 2913,57 was not reasonable.

Researcher	Is that a reasonable answer?
UP1	I don't think so.
Researcher	Why not?
UP1	I think it is a bit too big.
Researcher	Why do you think it is a bit too big?
UP1	Because it's times zero point something.
Researcher	Okay, and if you multiply by "zero point something" what happens to your answer?
UP1	It would probably get smaller.

Table 4.17 summarises the analysis of Question 4.

Table 4.17 Summary of analysis of Question 4 (Interview item 5)

N = 6	Analysis
Initial answers	3 correct, 2 incorrect, 1 cannot answer the question at all
Strategies: Methods	Rule-based: 4 incorrect – based on incorrect rules Number-sense based: 4 showed an understanding of an estimation strategy after prompting
Language	Fluent and coherent when explaining "their" rule
Confidence	Confidence in "their" rule

4.3.1.3 Number Sense Domain: Application to Computational Settings

The understanding of the relationship between a problem context and the necessary computation in the whole number domain and the ability to select an efficient strategy, including mental calculations and estimation were the number sense components investigated in this section.

(1) *Number sense component: Recognise operation and select efficient strategy*

In this question respondents had to recognise subtraction as the applicable computation. It was investigated what strategies they would use to estimate or mentally calculate the answer.

Question 5 (Interview item 1): Sarah has a string of 541centimetres. Leonard has a string of 298 centimetres. Estimate how much longer Sarah's string is than Leonard's string.

Possible solution strategies:

(a) The rule-based strategy would entail the application of the standard algorithm for subtraction.

(b) A number-sensible approach would make use of rounded numbers to estimate and a count-up strategy for mental subtraction, e.g. "from 300 to 540 makes 240".

This was the first question of the interview and chosen to boost the confidence of the interviewees. All six respondents recognised that subtraction was the applicable operation (Table 4.18) and showed confidence in using the standard algorithm to work it out.

Four respondents gave a reasonable estimate, but UP3 could not explain how she arrived at her answer. She was confused:

Researcher	How did you estimate?
UP3	I think I ... aah ... if Leonard's string is 298 and then the difference between 541 should be in the range of two hundred somewhere.
Researcher	Because...
UP3	Because, because if I … add them … 298 and 541 should give me the … the total of the … of the length of the string.

Three respondents displayed a proficiency in estimating by rounding. The best explanation was offered by UP2, although she made a mental calculation mistake at the end.

UP2	Because when you estimate you say the two hundred and ninety eight is close to three hundred. The other one is five hundred and fourty one. So it's close to ... like ... if you are going to estimate it's close to five hundred and fifty. The five hundred and fifty minus the three hundred, so I make it about or more than one hundred and fifty. Between one hundred and fifty and two hundred.
Researcher	It will not give you more than two hundred?
UP2	Not more than two hundred.

Three respondents were requested to use a written method to work out the answer. All applied the subtraction algorithm and arrived at the correct answer, but it took each of them more than one minute to do this calculation, which a proficient calculator would be able to do in less than 20 seconds. This might be an indication that they were not very fluent in their basic addition and subtraction facts.

Three respondents were asked if they could find the answer mentally. UP3 admitted that because of the big numbers, she would not be able to do the calculation mentally. LP2 and UP4 explained the mental application of the algorithm.

Researcher	Can you work it out mentally?
UP4	Ja, I can ... but you know it will be a little bit difficult.
Researcher	How would you do it?
UP4	Now subtracting I can write the 541 on the top and the 298 underneath. I do the subtraction orally, you know, mentally. Then I get the answer.

Table 4.18 summarises the analysis of Question 5.

Table 4.18 Summary of analysis of Question 5 (Interview item 1)

N = 6	Analysis
Initial answers	4 correct, 2 incorrect – all recognised subtraction as operation
Strategies: **Methods**	<u>Rule-based</u>: 3 used mental calculation based on algorithm <u>Number-sense based</u>: 3 used estimation by rounding
Language use	Mostly good and fluent
Confidence	All, except one, were confident when answering this question

(2) *Number sense component: Recognise operation and select efficient strategy*

In this question respondents had to recognise division as the applicable computation. It was investigated whether they could find the answer mentally and how they would deal with the remainder.

<u>Question 4:</u>   A minibus can transport 15 people. How many minibuses would you need to transport 170 learners?

<u>Possible solution strategies:</u>

(a) The rule-based strategy involves long division or any other paper-and-pencil algorithm.

(b) A number-sensible approach based on the mental decomposition and recomposition of the numbers, for example $170 = 150 + 20 = 10 \times 15 + 1 \times 15 + 5$ or $10 \times 15 = 150$; $150 + 15 = 165$. Therefore $170 \div 15 = 11$ remainder 5. Considering the context of the problem, the answer should be 12 buses.

Although all six respondents recognised this as a division problem (Table 4.19) it soon became clear that division was a surprisingly difficult operation for them. LP1 and UP2 did not offer an answer initially, but they managed to get close to the correct answer by repeated addition of pairs of fifteen. Repeated addition of pairs of 15 was the preferred method of four respondents, who applied it like an algorithm as the following extract illustrates:

Researcher	How could you work it out?
LP1	Like ... but I did not get the correct answer.
Researcher	It does not matter.
LP1	I did like fifteen, fifteen, fifteen, fifteen, fifteen ... then I go thirty, thirty.
Researcher	Okay.
LP1	(Writes 15, 15, 15, 15, 15 underneath each other, then 30, 30 next to it).

The researcher was surprised by guesses of five, six or seven buses by UP3 and UP4 and a guess of sixteen buses by LP2 and realised that not a single respondent made use of the number fact $10 \times 15 = 150$ or could find the answer by mentally decomposing or recomposing numbers. UP3 was asked directly:

Researcher	If you had 150 learners, how many buses would you need?
UP3	Five?
Researcher	So, if you put 15 learners into five buses, you will fit 150 learners into the five buses?
UP3	Yes, I think so.
Researcher	Or otherwise you would do long division? Can you do long division?
UP3	No, I can't do that ... I'm not good at...
Researcher	How would you then divide if you had to find the answer?
UP3	Aah ... uuh ... with a calculator?

The only mental calculation strategy that was evident was repeated addition as UP1 demonstrated.

Researcher	Could you do it mentally?
UP1	I did it mentally. This one was a bit long.
Researcher	How did you do it mentally?
UP1	I calculated 15×2.
Researcher	What is 15×2?
UP1	Thirty.
Researcher	And then?
UP1	And then I continued until it gave me ... until it gave me ... one hundred and fifty ... and really I couldn't do it anymore.

The excerpt above is an example of how much prompting by the researcher was often needed to get some insight into the respondents' thought processes, especially when the respondent did not feel very confident in his or her ability to solve the problem. The lack of confidence was also reflected in long pauses and incoherent language, such as

UP4	I mean ... I think ... to me ... the last one will not ... I think ... the same...

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LP2 showed her proficiency in applying the long division algorithm correctly. She arrived at the answer 11,3 and interpreted her answer as 11 buses and 3 learners remain. UP2 and UP4 were not quite sure what to do with the remaining learners.

UP2   They will have to find a place or they will have to stay.

Overloading vehicles is common in Namibia, so, to the respondent this answer was probably a very sensible one.

Table 4.19 summarises the analysis of Question 6.

Table 4.19 Summary of analysis of Question 6 (Interview item 4)

N = 6	Analysis
Initial answers	4 incorrect, 2 did not give an answer – all recognised the operation of division
Strategies: Methods	<u>Rule-based</u>: 1 used long division, 4 used repeated addition <u>Calculator</u>: 1 would solve the problem with a calculator. <u>Number-sense based</u>: Repeated addition done mentally No use of $10 \times 15 = 150$ as a benchmark or decomposition and recomposition of numbers
Language use	Associated with confidence
Confidence	Lack of confidence inferred from pauses and monosyllabic answers

To summarise, the analysis of the interviews by number sense domain showed that the respondents had a point of entry when dealing with the problems in the "Applications to computational setting" , but found the more abstract questions in the domain of "Knowledge of and facility with numbers" almost inaccessible. There were sporadic demonstrations of number sense: all, except one respondent, could judge the size of the answers in Question 3 by making use of the relationship between multiplication and addition; four respondents recognised that an answer of 2913,57 to $534,6 \times 0,545$ was unreasonable because the multiplier was less than 1 and three displayed their ability to estimate the difference between Sarah's and Leonard's strings by rounding. The probing into the respondents' ability to calculate answers mentally revealed that they did not have access to any strategies except the mental application of an algorithm.

4.3.2   Outcomes of Coded Categories

The main purpose of the interviews was to investigate the methods and reasoning that the respondents had used to obtain their answers. This formed the basis for the category "Strategies". The "Initial answers" were of interest, because all interview questions were familiar to the respondents from the Number Sense Questionnaire and the computations required to find the answers to Questions 1, 3, 5 and 6 were included in the Written

Computation Questionnaire. It became clear during the interviews that there was a variation in the language use of the respondents from fluent and clear to hesitant and confused. The researcher therefore decided to include "Language" as a category. The transcription of the tape-recorded interviews indicated interplay between language and confidence. This initiated the category "Confidence" to triangulate the findings from the quantitative confidence survey.

The first coded category "Initial Answers" reports on the answers that the respondents offered before the researcher probed into further background knowledge. The category "Strategies" records the strategies that the respondents used to determine the answers. Special focus was given to number-sense based strategies, such as estimation. The use of "Language" is analysed briefly. This category was included, because discourse has been identified as an important factor in the development of conceptual knowledge and as classroom teachers, the respondents should be able to engage in and lead the discourse with the learners and among the learners. In the last category, "Confidence" was inferred from utterances made by the respondents and their use of language.

4.3.2.1 Initial Answers

Table 4.20 summarises the number of correct and incorrect answers or no answers that were offered by the respondents before prompting by number domain.

Table 4.20 Initial answers to interview questions by number domain

N = 36	Whole numbers	Common fractions	Decimals	Total
Correct	8	1	3	12
Incorrect	9	7	2	18
None	1	4	1	6
Total	18	12	6	36

Only one third of the initial answers were correct despite the fact that the respondents had been exposed to all the questions in the NSQ and the calculations to four of the six questions had been included in the WCQ. Most of the correct answers came from the whole number domain. The questions involving common fractions were answered very poorly.

In Table 4.21, the initial responses are analysed by number sense domain.

Table 4.21 Initial answers to interview questions by number sense domain

N = 36	Knowledge of and facility with number	Knowledge of and facility with operations	Application to computational settings	Total
Correct	1	8	3	12
Incorrect	7	3	8	18
None	4	1	1	6
Total	12	12	12	36

From this table it can be seen that the questions relating to "Knowledge of and facility with operations" produced the most correct answers. However, it must be noted that all three answers to Question 4, which involved the multiplication of decimals, were based on either a guess or the incorrect application of a rule. The domain "Knowledge of and facility with numbers" posed the greatest challenge, possibly because both questions involved the understanding of common fractions.

4.3.2.2 Strategies

The strategies were coded as rule-based or number-sense based. Rule-based strategies included the use of a standard algorithm. An example of a rule-based strategy, other than the application of a standard algorithm, is the use of repeated addition in the problem, how many minibuses would be needed to transport 170 learners, if one minibus can transport 15 people. This strategy was coded rule-based, because two-thirds of the respondents relied on it as the only method of answering the question and all proceeded with it in the same way, namely adding pairs of 15. Number-sense based strategies included the use of estimation and mental calculations.

In the common fractions domain, out of twelve possible answers, two were based on an incomplete number-sense based strategy and there was no other evidence of any strategy or reasonable approach to answer the questions. No respondent was able to translate, without prompting, between multiple representations of common fractions from written form to a point on the number line in Question 2. From the answers to both questions in this domain, the researcher got the impression that the respondents lacked conceptual understanding of common fractions and were not familiar with the use of the number line.

In the decimal fractions domain, out of six possible answers, four were based on the incorrect application of a rule and one answer was a guess without explanation. One respondent did not attempt to answer the question. Nobody used the number-sense based strategy of

estimation to answer the question, but after probing four respondents showed an awareness of the fact that multiplication by a fraction makes the answer smaller.

In the whole numbers domain respondents demonstrated number-sensible strategies in 8 out of the possible 18 answers: five used the relationship between multiplication and addition to answer Question 3 (Which expression represents the larger amount: 2452×4 or $2541 + 2457 + 2460 + 2465$) and three used rounding and mental subtraction to estimate the answer to Question 5 (Sarah has a string of 541centimetres. Leonard has a string of 298 centimetres. Estimate how much longer Sarah's string is than Leonard's string). Eight answers were obtained by application of a standard algorithm or the rule-based use of repeated addition and twice a respondent did not provide and answer.

The descriptions of subtraction using the standard algorithm in Question 5 (see above) and of division by repeated addition in Question 6 (A minibus can transport 15 people. How many minibuses would you need to transport 170 learners?) were the only evidence of mental calculations. Surprisingly the use of the multiplication fact $10 \times 15 = 150$ and decomposition and recomposition of numbers to find the answer to Question 6 was not demonstrated by any respondent.

The interviews showed that respondents did not possess any noteworthy number-sensible strategies to answer the questions and that where they attempted to apply algorithms to mental calculations, failed to obtain the correct answer. Where respondents demonstrated a paper-and-pencil calculation, as in Question 5 and Question 6, it was noted that it took them a long time. This may point to a lack of basic fact fluency.

Table 4.22 summarises the percentage of correct and incorrect responses for both strategies.

Table 4.22 Summary of analysis of strategies

Reasoning (Q 1, 2, 3)			Method (Q 4, 5, 6)			
Correct	Incorrect	None	Rule based (60%)		Number-sense based (40%)	
			Correct	Incorrect	Correct	Incorrect
22%	28%	50%	30%	30%	30%	10%

4.3.2.3 Language use

English was not the mother tongue of any respondent, but their detailed individual language profiles were not established. However, they were all educated through the medium of English and will have to teach through the medium of English. It was noted that UP2 was the

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most articulate and comfortable in the use of English, whereas it was difficult at times to understand and follow UP4.

For the purpose of this study the correctness of grammar and pronunciation was not analysed and evaluated. The fluent use of language and coherence of explanations were of interest, because these will have a direct impact on the understanding that learners will develop in the classroom.

The respondents used language fluently to explain the application of a rule, for example, when LP2 did long division (Figure 4.12) she said

Figure 4.12 Long division by LP2

LP2 Fifteen goes into seventeen one times. Now one times fifteen is fifteen. Write it here. Then fifteen minus seventeen is two. Bring down a zero, because fifteen does not go into two. Fifteen goes into twenty one times. One times fifteen is fifteen. Then I subtract ... and ... oh ... I have to bring another zero down, because the fifteen cannot go into the five. Then I say fifteen goes into fifty ... like three times. And I put a comma here.

However, LP2 was much more hesitant and confused in her language when she had to explain why ten-ninths is closer to one than to two in Question 2.

LP2 Because of the ... it's the denominator ... I think they count more than the denominator ... they count more than the denominator because numerators are ... because the denominator is bigger than the numerator, let me say that.

Only UP2 showed a good and fluent use of English to explain and reason. She for example reasons

UP2	… if I have 150 learners then there will still be twenty learners who won't have a place … which means I will still need one more bus.
Researcher	But there are only 15 learners in the bus. What happens to the other five?
UP2	(Laughs) They will have to find a place or they will have to stay.

Although none, except UP2, spoke English fluently and well, they were much more inarticulate when they had to give an explanation based on conceptual understanding than when they had to explain a rule.

4.3.2.4 Confidence

The overall impression was that the pre-service teachers lacked confidence in their ability to do mathematics. This was inferred from pauses, hesitations and unfinished sentences, as well as the frequent use of "I think…" "To me…" To illustrate the point

LP2	I think around sixteen.
LP1	Ja, first I think there is zero and ... nothing ... It's like ... (pause)
UP2	I would say it would be.... (laughs)... I would say it would be more than ... more than one hundred.

They were more confident in their ability to deal with questions in the whole number domain and with questions where they could apply a rule, even if the incorrect rule was applied. They all lacked the confidence to tackle the questions in the common fractions domain where out of 12 possible answers, only 2 answers were offered. Comments included

LP1	(Laughs when she sees the question). That was too hard.
LP2	That really got me.
UP3	Mmm … the question like this I cannot …
UP4	Aaai ... I didn't have a clue...

4.3.3 Conclusion based on findings of qualitative analysis

The interview data was analysed in two sections. A short analysis of the interviews by the three number sense domains, "Knowledge of and facility with number", "Knowledge of and facility with operations" and "Applications to computational settings" was followed by a discussion of the outcomes of the four coded categories, "Initial answers", "Strategies", "Language use" and "Confidence". The following conclusions were drawn form the qualitative analysis:

Some number-sensible strategies were evident such as the application of the relationship between multiplication and addition and estimation by rounding on the whole number domain. However, apart from two incomplete attempts, no number-sensible strategies were used to answer questions in the domain of rational numbers and respondents had no access to number-sensible mental calculation strategies to do subtraction or division but relied on the mental application of algorithms. Rule-based strategies were either incorrect (placing the decimal comma in a product), inefficient (division by repeated addition) or time-consuming (subtraction algorithm).

Respondents found it difficult to express their thoughts and understanding in words. The use of language was related to the confidence in their ability to do mathematics. A lack of confidence could be inferred from long pauses, hesitations and uncompleted sentences.

The results of this analysis will be integrated in the discussion in section 4.4 in relation to the research questions and international studies. However, a remark on the use of language is appropriate here, because it did not form part of this research. The teacher's oral fluency, grammatical correctness and clear and correct pronunciation serve as a model for the learners and determine the richness and precision of explanations the teacher can offer. Research conducted in schools in South Africa (Taylor & Vinjevold, 1999; Setati, 2001) found that because of their lack of conceptual knowledge and poor language skills most teachers resorted to drill, practice and rote learning and were unable to engage learners in either procedural or conceptual discourse. The observations made in this study indicate that issues around language and mathematics need to be addressed in Namibia.

4.4   Discussion of the Results

The results will be interpreted to find an answer to the key research question, "Do Namibian final year lower primary pre-service teachers possess skills, strategies and confidence that are related to number sense?"   The findings will be discussed in relation to the following sub-questions that guided the study:

1. How proficient are final year primary pre-service teachers in performing written computations?
2. How proficient are they in mental calculations?
3. Does their performance on a number sense test correlate with their performance on the written computation and mental calculation test?
4. Do they use any number-sensible strategies to answer number sense test items?
5. Are they confident in their own ability to do mathematics and teach mathematics?

4.4.1   Outcomes of the Number Sense Questionnaire

The aim of this study was to gain an overview of the number sense proficiency of primary pre-service teachers in Namibia. The overall performance on the number sense questionnaire (NSQ) was analysed according to the broad number sense domains identified by McIntosh, Reys and Reys (1992), namely "Knowledge of and facility with number", "Knowledge of and facility with operations" and "Applications to computational settings".

The 27 items of the NSQ were selected and adapted from a number sense test developed by Yang for Grade 6 and 8 learners in Taiwan (Reys & Yang, 1998). The adaptations are described in detail in Chapter 3, section 3.5.1 and were aimed at making the items more accessible to the expected competency levels of Namibian primary school teachers.

The mean of 38.1% on the NSQ indicates that the Namibian pre-service teachers displayed a low number sense proficiency compared to that of pre-service primary school teachers in international studies by Kaminski (1997), Tsao (2004, 2005) and Yang, Reys and Reys (2009) quoted in Chapter 1.

There was no significant difference in the performance of the LP and the UP PSTs on the NSQ. The interviews also showed that the four UP PSTs did not demonstrate a deeper insight or understanding of the subject content. For example, none of the four UP PSTs used estimation spontaneously to place the decimal comma in the question $534{,}6 \times 0{,}545 = 291357$. Three applied incorrect rules and one admitted that she did not understand place

value. This indicates that they lack the concepts and principles underlying the many rules that they have learnt and the connections among them (Ball, 1990). It can be concluded that the UP pre-service mathematics curriculum which is based on the mathematics syllabus for junior secondary mathematics learners does not develop the number sense proficiency required for teaching primary school mathematics.

4.4.1.2 Performance on the number sense domains

The analysis of performance on the three number sense domains showed the best performance on "Applications to computational settings" with a mean of 52.7%. The following items received the most correct answers:

2. Sarah has a string of 541 centimetres. Leonard has a string of 298 centimetres.
   How much longer is Sarah's string than Leonard's string?
   Without calculating an exact answer, circle the best estimate
   A. 100 cm   B. 200 cm   C. 240 cm   D. 570 cm   **(42/47 correct)**

9. Use the following picture to answer the question.
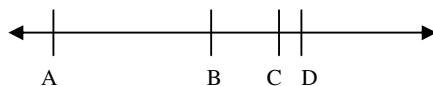
   The distance between A and B is 53,7 km, between B and C is 7,8 km and between C and D is 0,9 km. What is the approximate distance between A and D?
   A. 45 km   B. 50 km   C. 60 km   D. 62 km   **(42/47 correct)**

19. A library had 2590 books. 585 books were lost and 5 books are out on loan.
    How many books are left in the library?
    A. 1500   B. 2000   C. 2010   D. 2500   **(33/47 correct)**

All three questions are based on simple real life addition or subtraction contexts and require a choice between four given answers. Question 2 featured in the interviews and was answered confidently and correctly by 4/6 interviewees and 2/6 interviewees could arrive at a correct estimate. This seemed to be the level of question where the majority of participants could demonstrate competence. As soon as the calculations involved multiplication or division and demanded basic mental computation, performance dropped, as the following examples show:

10. A mini-bus can transport 15 people. If 170 learners need to be transported to the next town, calculate mentally how many mini-buses are needed.

(17/47 correct)

27. A drum has 125 litres of water in it. A farmer fills more water into the drum and the water level rises at a rate of 1,5 litres per minute. How many litres of water will be in the drum after 4 minutes? **(12/47 correct)**

The interview data obtained for Question 10 showed that although all respondents recognised division as the applicable operation, none of them was able to provide the correct answer without either doing long division or repeated addition of 15 or using a calculator. Initial answers included wild guesses of six, seven and sixteen buses. This indicated poor number sense and the absence of flexible mental calculation strategies.

The mean for "Knowledge of and facility with operations" questions was only 36.2%. The questions with the most correct answers once again involved the whole number domain.

3. Without calculating an exact answer, circle the expression, which represents the larger amount.

A. 2452×4 B. $2541 + 2457 + 2460 + 2465$

C. Cannot tell without calculating

(34/47 correct)

15. Circle the best answer:

(a) $3 \times 5 \times 6 \times 8$ (b) 15×48 (c) $24 \times 5 \times 6$

A. $a > b > c$ B. $b > c > a$ C. $a = b > c$ D. $a = b = c$

(34/47 correct)

The majority of the PSTs were able to apply the relationship between multiplication and addition and the interview data for Question 3 corroborated this. This was not surprising because their preferred method for multiplication and division was repeated addition. They were also able to apply the associative and commutative properties of multiplication in the whole number domain in Question 15.

However, Question 7, which tested the knowledge of the application of the associative property to the multiplication of common fractions, was answered correctly by only 3 PSTs.

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7. Without calculating an exact answer, circle the best estimate for: $\dfrac{15}{24} \times \dfrac{5}{12}$

    A. More than $\dfrac{15}{48}$ B. Less than $\dfrac{15}{48}$

    C. Equal to $\dfrac{15}{48}$ D. Cannot tell without calculating   **(3/47 correct)**

The results indicate that the "Knowledge of and facility with operations" was limited to the whole number domain and did not extend to the domain of rational numbers.

A very low mean of 22.7% was obtained for the questions relating to the "Knowledge of and facility with number". This domain tested the PSTs' sense of orderliness of numbers, multiple representations for numbers, a sense of the relative and absolute magnitude of numbers and a system of benchmarks. Perhaps the low mean could be explained by the fact that only one of the ten questions was from the whole number domain. However, even this question was answered correctly by only 15 PSTs.

17. Which of the following number sequences is wrong?
    A. 3000; 3500; 4000; 4500   B. 7600; 7700; 7800; 7900
    C. 6097; 6098; 6099; 7000   D. 8080; 8090; 8100; 8110

                                                (15/47 correct)

During the interviews it became clear that the respondents lacked conceptual knowledge of the relative size of common fractions and multiple representations of common fractions. The interview question "Is $\dfrac{3}{8}$ or $\dfrac{7}{13}$ closer to $\dfrac{1}{2}$?" was answered correctly by 12 PSTs on the NSQ, but only one could give a reasonable explanation of the answer. Not only the representation of fractions on a number line was problematic for the PSTs, but also less than half the PSTs could find the fraction that was represented by a shaded area in a diagram (Question 20, Appendix A).

The question relating to the density of rational numbers obtained the least correct answers.

21. How many different fractions are there between $\dfrac{2}{5}$ and $\dfrac{3}{5}$?

    A. None. Why?   B. One. What is it?
    C. A few. Give two.   D. Lots. Give three.   **(3/47 correct)**

The three PSTs who chose D provided examples such as $\dfrac{2,5}{5}$ instead of $\dfrac{1}{2}$.

The very poor performance on the domain "Knowledge of and facility with numbers" indicates that the PSTs lack the conceptual understanding of numbers and reinforces the impression that their knowledge is limited to surface aspects and does not constitute a web of interconnected knowledge (Ma, 1999).

4.4.1.2 Performance on questions requiring estimation

Estimation is regarded as an indicator of number sense. It requires knowledge of place value, basic facts and properties of operations as well as an ability to compare numbers by size, compute mentally and work with powers of ten (Sowder & Wheeler, 1989). Tsao (2005) found that pre-service primary school teachers in his study felt uncomfortable providing estimates. This may be true for Namibian PSTs as well: the overall mean for the 13 NSQ questions requiring estimation was only 40%.

Table 4.23 shows an analysis of the 13 estimation questions by number domain and operations.

Table 4.23 Analysis of estimation questions on the NSQ

Estimation questions on the NSQ	Number domain and operation	Mean
Q2, Q3, Q9	Addition or subtraction of two whole numbers smaller than 100, or decimals	73%
Q1, Q4, Q13, Q14, Q22	Addition of more than 2 whole numbers greater than 100, multiplication and division of whole numbers or decimals	40%
Q6, Q7, Q8, Q11, Q12	Operations on common fractions	20%

This analysis confirms the findings of Hanson and Hogan (2000). Their study indicated that college students had difficulty in estimating answers to fraction and multiplication and division problems and the researchers suggested that this might reflect a lack of deep understanding of multiplicative reasoning and of rational numbers.

Three respondents demonstrated a proficiency in rounding whole numbers during the interviews, but one did not obtain a reasonable answer, because of a mental calculation error. Sowder and Wheeler (1989) maintain that poor estimators are limited to using only a rounding strategy and that they rarely compensated. This may be the reason why the PSTs found it so much more difficult to estimate the sum of four numbers (995 + 872 + 838 + 809) in NSQ. If rounding of whole numbers is the only strategy that the PSTs have access to,

then it is not surprising that they fail to estimate answers to rational number questions. Evidence of this was found in the interview with UP4:

Researcher	You are multiplying with a decimal. What normal or common fraction is close to 0,545?
UP4	Ohh … I... ahhh … five hundred and fourty five divided by a thousand. Meaning that you multiply this one with 545 divided by 1000.

These results reflect that estimation does not receive much attention in Namibian schools and is restricted mainly to estimation by rounding to find the sum and difference of whole numbers and decimals.

To summarise, Namibian primary pre-service teachers' performance on the Number Sense Questionnaire was poor compared to that found in similar international studies (Tsao, 2004, 2005). The very poor performance on the domain "Knowledge of and facility with numbers" indicates that they lack the conceptual understanding of numbers. The assessment of the domain "Knowledge of and facility with operations" showed that their competencies were limited to the whole number domain and did not extend to the domain of common fractions. On the domain "Applications to computational settings", the analysis showed that they displayed numbers sense only when dealing with simple context questions requiring addition or subtraction of whole numbers or small decimals. Estimation skills, one of the indicators of number sense, were apparent in questions where rounding whole numbers to add or subtract was appropriate, but not in questions involving multiplication and division or rational numbers.

4.4.2   Discussion of findings related to the research sub-questions

The purpose of the sub-questions was to find some explanations for the performance on the Number Sense Questionnaire. In Chapter 1 it was argued that number sense may be regarded as a mathematical proficiency that requires conceptual understanding, procedural fluency, adaptive reasoning, strategic competence and a productive disposition. The results of the NSQ indicated that the PSTs displayed a lack of conceptual understanding of numbers, which is the foundation on which number sense is built.

4.4.2.1 Sub-question 1: How proficient are the primary pre-service teachers in written computations?

The procedural fluency of the PSTs will be discussed based on the results of the written computation questionnaire (WCQ) which tested PSTs' abilities to apply standard algorithms or any other paper-and-pencil methods to find exact answers to fifteen items.

Twelve WCQ items paralleled items on the number sense questionnaire (NSQ). Table 4.24 compares the number of correct answers on the parallel items of the NSQ and the WCQ. It shows that more correct were achieved on the WCQ than on the NSQ on only five items. Four of these items related to operations with fractions. This seems to indicate that the PSTs' intuitive choices among multiple given answers in the whole number domain on the NSQ were correct more often than the answers arrived at by exact computation. In the domain of common fractions, however, the application of procedures resulted in a correct answer more often than their choices of answer based on their intuitive understanding.

Table 4.24 Comparison of correct answers on parallel items on the NSQ and the WCQ

Question	Number of correct answers (N = 47)		Operation	Calculations
	NSQ	WCQ		
1	22	32	WN addition*	$995 + 872 + 838 + 809 =$
2	42	36	WN subtraction	$541 - 298 =$
3	34	31	WN multiplication/ addition	Which one is bigger? Work out and then circle the correct answer. A. $2452 \times 4 =$ B. $2451 + 2457 + 2460 + 2465 =$
4	16	11	WN division	Find $\square$: $45 \times \square = 2700$
5	12	25	CF subtraction*	Which one is bigger? Work out and then circle the correct answer. $\dfrac{1}{2} - \dfrac{3}{8} =$ $\qquad$ $\dfrac{7}{13} - \dfrac{1}{2} =$
6	10	18	CF addition*	$\dfrac{10}{11} + \dfrac{7}{8} =$
7	3	17	CF multiplication*	$\dfrac{15}{24} \times \dfrac{5}{12} =$
8	14	9	CF division	$6\dfrac{2}{5} \div \dfrac{7}{8} =$
9	42	32	Decimal addition	$53,7 + 7,8 + 0,9 =$
10	17	14	WN division	$170 \div 15 =$
11	18	15	CF subtraction	$2 - \dfrac{1}{3} - \dfrac{1}{2} =$
12	11	16	CF addition*	$\dfrac{5}{9} + \dfrac{8}{15} =$
Mean	20.1	21.3		

* Indicates WCQ result better than NSQ result

Previous research (Tsao, 2004) found that pre-service teachers performed substantially better on written computation items than on either number sense or mental computation items and concluded that the PSTs could apply standard algorithms efficiently and correctly. Yang et al. (2009) noted the students' "high performance on tedious computations" (p. 397), and a preference of rule-based methods when dealing with fractions. Other researchers (Reys & Yang, 1998, Veloo 2010) came to the same conclusion in studies with learners at school. This study did not confirm these findings. The mean of 39.3% and the range of scores between 0% and 86.7% on the WCQ indicated that the PSTs' ability to perform written calculations is very poor and that there is a great variation in competence. Considering that these calculations form part of the most basic competencies of the primary school mathematics, even the highest means of 52.8% and 52.5% on the questions relating to whole numbers and addition respectively are unacceptably low.

Although the analysis of methods used and mistakes made by the PSTs on the WCQ did not form part of this research, some inefficient and error-prone methods emerged regularly. Two examples are given in Figure 4.13 and 4.14.

1. The sum of four whole numbers was obtained by calculating partial sums as shown in Figure 4.13.

Figure 4.13 Example: Multiplication and addition of whole numbers (LP 17)

2. Multiplication and division of whole numbers was done by repeated addition as shown in figure 4.13 and 4.14.

Figure 4.14 Example: Division of whole numbers (UP 40)

It may be inferred from the examples that many PSTs were not fluent in their basic facts knowledge of addition and multiplication and weak in their application of standard procedures in the whole number domain.

This is supported by evidence gathered during the interviews. Three interviewees used the standard algorithm for the subtraction required to solve the problem, "Sarah has a string of 541centimetres. Leonard has a string of 298 centimetres. Estimate how much longer Sarah's string is than Leonard's string." They all arrived at the correct answer, but the fact retrieval was very slow and it took each one more than a minute to perform the calculation.

By far the lowest mean of only 18.1% on the WCQ was achieved on the questions involving division. An attempt to divide 2700 by 45 (Question 4) is shown above in Figure 4.14. Only 11 of the 47 participants answered this question correctly. Fourteen correct answers were obtained for $170 \div 15$ (Question 10). This calculation also featured in the interviews and the preferred method was repeated addition of pairs of 15, which was a very slow and laborious process. During the interviews UP3 indicated that she would only be able to solve the problem with a calculator; UP2 admitted that she cannot do long division and LP2 was able to arrive at the answer 11,3 buses by long division, but could not interpret the remainder. Where repeated addition failed and participants had to rely on algorithms, the performance was even poorer: 9 participants managed to divide by a fraction, $6\frac{2}{5} \div \frac{7}{8}$ (Question 8) and not a single participant could do the division by a decimal, $72 \div 0{,}025$ (Question 15).

It is suspected that the reliance on calculators from the beginning of secondary school is, over and above the poor retrieval of basic facts, partly to blame for the lack of procedural skills.

The results of this study indicate that the PSTs are not competent to perform written computations. Although researchers (Reys & Yang, 1998; Veloo, 2010) argue that competence in written computation does not promote number sense, the poor recall of basic facts knowledge and procedural fluency that is indicated by the results of this study will influence the competency in mental calculations as the next section will show.

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4.4.2.2 Sub-question 2: How proficient are the primary pre-service teachers in mental calculations?

Mental calculations are calculations performed without the application of paper-and-pencil methods or the calculator. Mental calculations receive little or no attention in Namibian schools and the researcher did not expect the PSTs to be very competent in this area. Therefore the 20 items of the MCQ were selected from a mental calculations test designed for Grade 6 learners and were shown on screen for 20 seconds and read out aloud as opposed to the usual oral only format of mental calculations tests.

Despite these measures, the performance on the MCQ was extremely weak with a mean of 4.6/20. The best mental calculator in the group answered 12 questions correctly. The weak results were due to incorrect answers and not due time constraints, because 41 PSTs answered at least 15 questions.

To illustrate this poor performance, Figure 4.15 shows the distribution of the scores and Table 4.25 lists the mental calculation questions and the corresponding number of correct answers.
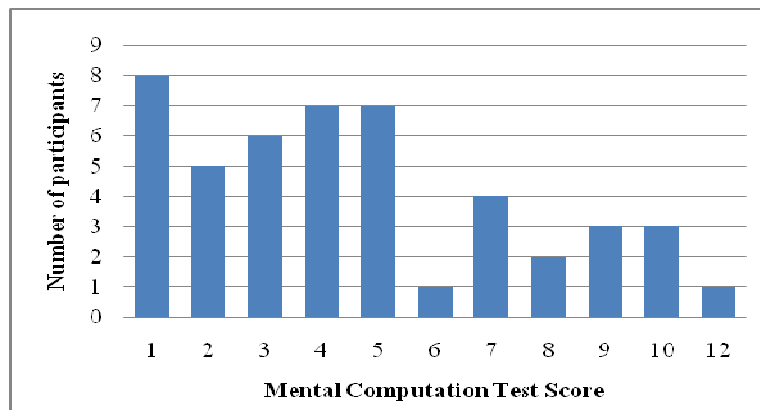

Figure 4.15 Distribution of scores on the Mental Calculations Questionnaire

Figure 4.15 shows a distribution that is highly skewed to the right. The mode of the MCQ is 1 point which was obtained by 8 candidates and 70.2% of the participants obtained a score of 5 points (25%) or lower.

Table 4.25 Mental calculation questions and number of correct answers

N = 47	Question	No of correct answers		Question	No of correct answers
1	600 + 50 + 250	36	11	$4 \times 3\frac{1}{2}$	4
2	189 + 36	8	12	$40 \div \frac{1}{2}$	9
3	232 – 98	6	13	264 + 99	14
4	56 + 23 – 16	14	14	6,5 + 1,9	30
5	13 × 5 × 2	21	15	4,5 – 2,6	9
6	512 ÷ 4	1	16	5 × 2,12	7
7	36 × 50	2	17	6,5 ÷ 0,5	4
8	550 ÷ 25	6	18	95 × 0,1	6
9	$\frac{1}{2} + \frac{3}{4}$	12	19	$\frac{1}{2} \times 6\frac{1}{2}$	3
10	$8 - 4\frac{1}{2}$	26	20	$5\frac{1}{4} - 3\frac{3}{4}$	0

From Table 4.25 it is clear that only questions 1 and 14 were answered correctly by more than 60% of the participants. Similar to the performance on the estimation questions, these relate to addition of powers of ten and small decimal numbers. Awareness of the associative property, which was established in the analysis of the number sense domains, probably contributed to the relative success in answering questions 1 and 5.

The interview data showed that the respondents relied on the mental application of an algorithm to work out the answers to both 541 – 298 (Interview question 5) and 170 ÷ 15 (Interview question 6). UP3 and UP4 admitted that the size of then numbers made it difficult to find the answer to the subtraction question mentally. The only mental strategy available to the respondents to divide 170 by 15 was repeated addition of 15, which "was a bit long" (UP3). Where the respondents demonstrated their skill to carry out paper-and-pencil computations it was inferred from the long time this took that they had difficulty in basic facts retrieval. Considering the interview data it is not surprising that only a few PSTs managed to find the correct answers to the whole number questions 2, 3, 4, 6, 7, 8 and 13 and the decimal questions 15, 16 and 17 within 20 seconds.

Except for Question 10 the common fractions questions 9, 11, 12 19 and 20 were accessible to less than a quarter of the PSTs. These results confirm the findings of the analysis of the

WCQ and the NSQ that the PSTs lack the intuitive understanding and procedural skills to cope with questions involving common fractions.

The researcher was astonished that only 6 PSTs could answer $95 \times 0,1$ (Question 18) correctly. Poor conceptual understanding of multiplication, place value and the relationship between common and decimal fractions may be the reason. This would be in agreement with Veloo (2010) who found strong evidence in his study that the learners' understanding of basic underlying concepts of fractions and decimals "may be weak and limited to surface aspects of the concept" (p.244).

The research reviewed in Chapter 1 (Anghileri, 2006; Carpenter et al., 1998; Griffin, 2003; Heirdsfield & Cooper, 2002, 2004; Reys & Reys, 1998) pointed out that mental calculations and number sense develop together and according to Griffin (2003, p.306) "one cannot exist without the other". Mental calculations require a combination of memory of basic facts and strategies (Kilpatrick et al, 20001), i.e. a combination of procedural fluency and conceptual understanding. Research on mental calculation has shown that there is a relationship between mental calculation and conceptual understanding of numbers (Blöte, Klein & Beishuizen, 1994; McIntosh, 1998 in Veloo, 2010). The lack of conceptual understanding of numbers identified in the analysis of the NSQ may therefore have influenced the performance on the MCQ.

The next section will look at the performance from the perspective of strategic competence and adaptive reasoning.

4.4.2.3 Sub-question3: Do primary pre-service teachers use any number-sensible strategies?

People with number sense have developed useful and efficient strategies for managing numerical situations (Reys & Yang, 1998); they are proficient mental calculators (Griffin, 2003) and good estimators (Sowder & Wheeler, 1989) Both proficient mental calculators and good estimators display a flexible and adaptive use of strategies (Heirdsfield & Cooper, 2002, 2004; Kilpatrick et al., 2001).

In the discussion of the performance on estimation items on the NSQ and in the interviews it was inferred that the PSTs' estimations skills were limited to finding the sum of or difference between at most two whole numbers or small decimal numbers by rounding. The weak

performance on the remaining estimation items on the NSQ may indicate a lack of strategies to deal with estimation of products, quotients and answers involving rational numbers.

The interview data in this study showed no evidence that the respondents were able to use any number-sensible strategies, such as count-up, to mentally calculate the difference between 541 and 298 (Interview question 5), or decomposition and recomposition of numbers to divide 170 by 15 (Interview question 6). The poor results of the MCQ indicate not only poor procedural fluency, but also the lack of a variety of flexible mental calculation strategies. A few examples from the MCQ will illustrate this.

Question 3: 232 – 98, was answered correctly by only 6 of the 47 PSTs. This indicates that the majority was trying work this out by mental application of the algorithm, which is difficult because if involves 'borrowing'. The 'count-up' strategy would have yielded the correct answer very quickly: 98 <u>+ 2 + 132</u> = 232.

Question 6: 512 ÷ 4, received one correct answer. The preferred strategy of repeated addition of 4 would indeed take a very, very long time. Here application of the fact that $4 \times 25 = 100$ would have been one possible efficient strategy.

Question 7: 36 × 50, was answered correctly by 2 PSTs. Perhaps some PSTs tried to add 50 thirty-six times; others might have tried to do long multiplication mentally. The obvious strategy here would have been to multiply by 100 and then divide by 2.

During the interviews respondents had to explain their answers. The researcher was left wondering whether primary school learners would be able to follow the often confused and incoherent explanations. Apart from language problems, the respondents' struggle to explain and reason indicated that they lacked conceptual understanding, which is a prerequisite for number sense. However, they could offer reasonably fluent explanations of rules or algorithms. This concurs with the Taiwanese studies by Tsao (2005) who found that only high ability students demonstrated the use of number sense characteristics in their explanations and Yang et al. (2009) whose study showed that less than one-third of the 280 pre-service teachers used components of number sense to explain their answers.

It may be concluded that the primary pre-service teachers in this study demonstrated neither the strategic competence nor adaptive reasoning characteristic of people with number sense.

4.4.2.4 Sub-question 4: Does the pre-service teachers' performance on the number sense questionnaire correlate with their performance on the written computation and mental calculation questionnaires?

Correlation analysis was conducted to establish whether a relationship existed between number sense and written computation skills, number sense and mental computation skills and written and mental computation skills. In this study the correlation analysis between the scores on the parallel items of the NSQ and WCQ yielded a Pearson Correlation Coefficient of $r = 0.241$, $p > 0.05$ which was positive, weak and insignificant, indicating that no relationship existed between the competence in written computation and number sense.

International studies by Tsao (2004) with primary pre-service teachers and Reys and Yang (1998) with Grade 6 and 8 learners, found a high and significant correlation between number sense and written computation competence indicating that subjects who were competent in written computation also displayed higher levels of number sense. However, they also found, contrary to the finding of this study, that their subjects were highly skilled in applying paper-and-pencil algorithms and their performance on written computation test items was far better than their performance on parallel number sense items. This led them to the conclusion that competence in written computation does not promote number sense and confirms the views of other researchers (Kamii & Dominick, 1997; McIntosh et al., 1992; Pesek & Kirshner, 2000; Veloo, 2010).

The correlation analysis between the WCQ and the MCQ yielded a substantial, positive and significant correlation of $r = 0.457$, $p < 0.01$, indicating a relationship between mental calculation and written computation proficiencies and confirms the findings by Tsao (2004). This relationship is not surprising since both mental and written calculation skills require basic fact fluency, which many of the Namibian PSTs seem to be lacking.

The correlation analysis between the NSQ and the MCQ indicated that a relationship exists between number sense and mental calculation competence with a substantial, positive and significant correlation of $r = 0.540$, $p < 0.01$. This confirms that for the Namibian primary pre-service teachers in this sample numbers sense and mental calculations go hand in hand as shown by many international studies conducted with learners as well as teachers (Anghileri, 2006; Carpenter et al., 1998; Griffin, 2003; Heirdsfield & Cooper, 2002, 2004; McIntosh et al., 1992; Reys & Reys, 1998; Reys & Yang, 1998, Sowder, 1994; Tsao 2004, 2005; Veloo, 2010).

4.4.2.5 Sub-question 5: How confident are the primary pre-service teachers in their ability to do and to teach mathematics?

The Pearson Correlation Coefficients between the total scores on the NSQ and the total scores obtained on the MCMMST Overall, Factor 1 and Factor 2 were all positive, substantial and significant, namely $r = 0.553$, $p < 0.01$; $r = 0.478$, $p < 0.01$ and $r = 0.502$, $p < 0.01$ respectively. This indicates the PSTs' with greater confidence in their ability to do and their ability to teach mathematics also displayed a greater degree of number sense.

These findings are in line with other international studies. Tsao (2004) found in a similar study with primary pre-service teachers a correlation of $r = 0.399$, $p < 0.01$, between number sense test scores and Confidence in Learning Mathematics Scales. Ball (1990) investigated how pre-service primary teachers' self-confidence affected the way they approached problems, the connection they were able to make between related concepts and their repertoire of strategies. She found that those who thought they knew mathematics and could learn mathematics, exhibited a higher level of confidence.

The descriptive analysis of the responses to the McAnallen Confidence in Mathematics and Mathematics Teaching Survey (MCMMTS) showed that the primary pre-service teachers in this study exhibited above average confidence in both Factor 1, their ability to do mathematics, and Factor 2, their ability to teach mathematics. The overall mean on a scale of 5 was 3.78, the mean for Factor 1 was 3.57 and the mean for Factor 2 was 3.97.

The upper primary pre-service teachers were more confident in their ability to do mathematics with a mean score of 3.72 for Factor 1 as compared to the mean score of 3.35 for the lower primary pre-service teachers for the same factor. Only 17% of the UP PSTs and 44% of LP PSTs rated themselves as "average" or "below average" compared to other primary school mathematics teachers in terms of mathematical ability.

The MCMMTS was completed after the three questionnaires and directly after the Mental Calculation Questionnaire (MCQ). The results indicate that overall poor performance on the questionnaires, especially the MCQ, did not affect the confidence of the pre-service teachers in their ability to do mathematics.

This is both surprising and disturbing. The results of the number sense test showed no significant difference between the performance of lower primary and upper primary pre-service teachers and the analysis of the interviews came to the same conclusion. The

respondents did not display a high level of confidence, especially when they had to explain their understanding of concepts and procedures. However, when the interviewees explained a rule or procedure, they were much more confident, even when the rule or procedure was incorrect.

More research is needed to find an explanation for these contradictory finding. However, the researcher would like to put forward two conjectures:

Firstly, the confidence that the UP PSTs displayed in their ability to do mathematics was, in their minds, not linked to the performance on the questionnaires, but to their performance in their university mathematics course. According to the data, 86% of the UP PSTs had successfully completed Grade 12 mathematics and most of them probably cope very well with their university mathematics course, which is based on the Grade 8 – 10 mathematics syllabuses for schools.

Secondly, and more worryingly, these primary pre-service teachers exhibit confidence in their own incorrect knowledge. Morris (2001) found that students who exhibit strong confidence in their own incorrect knowledge have strongly held misconceptions which are very resistant to change and unless these are rectified, will take their misconceptions into their teaching. Evidence of this was found in the interviews in this study where four PSTs explained with great confidence their incorrect rules for placing the decimal comma in the product of two decimals.

The mean for Factor 2, confidence in the ability to teach mathematics, was 3.97 – substantially higher than the mean of 3.57 for Factor 1. On Factor 2, the LP PSTs' mean score of 4.00 was slightly higher that of the UP PSTs of 3.95. Three explanations for this are suggested and further research recommended.

Firstly, the PSTs confidence in their ability to do mathematics might strongly influence their confidence in teaching mathematics, especially at primary school level where it is assumed that anyone who has completed secondary school mathematics is competent to teach.

Secondly, the PSTs' beliefs about the nature of mathematics is probably what they experienced at school, namely mathematics as a set of rules and procedures to be learnt by rote and memorised, and the interview data showed that they are quite confident in their rules and procedures.

Thirdly, they may believe that teaching consists primarily of offering clear explanations to children, underestimating the complexity of teaching and the kind of knowledge, especially subject matter knowledge, that they will need to be successful (Ambrose, 2004). This study has revealed that both the PSTs' language competence and subject matter knowledge is insufficient for effective teaching of mathematical concepts and procedures.

Apart from conceptual understanding, procedural fluency, strategic competence and adaptive reasoning, mathematical proficiency also requires a productive disposition. One question in the MCMMST was, "I am confident that I can learn more difficult mathematics concepts" and 85% of PSTs agreed or strongly agreed with the statement. According to Graven (2004), confidence in being able to learn mathematics is a resource that enables teachers to learn the mathematical competences required to teach mathematics. This indication of a productive disposition of the primary school pre-service teachers may be a way forward to move Namibian teachers from teaching procedures to teaching conceptually and developing number sense, both their own and that of the learners.

4.5   Conclusion

In this mixed methods research study the qualitative data illuminated the findings of the quantitative data. The main findings of the quantitative analysis of the three questionnaires, the Number Sense Questionnaire, the Written Computation Questionnaire and the Mental Calculation Questionnaire were corroborated by the interview data. However, it was found that the high levels of confidence revealed by the McAnallen Confidence in Mathematics and Mathematics Teaching Survey were not displayed in the interviews. Explanations for this discrepancy were suggested and further research recommended.

The answers to the sub-questions of this research study provide the background to answer the main research question: Do Namibian final year primary pre-service teachers possess skills, knowledge, strategies and confidence that are related to number sense?

It was found that the final year primary pre-service teachers in this study lacked the foundations on which number sense is built, namely "a well-organised conceptual network of number information" (Markovits & Sowder, 1994). This was reflected in their inability to represent mathematical situations in different ways and apply the most useful representations in different situations. Their lack of procedural fluency to access facts and procedures rapidly, automatically and correctly influenced their performance in written and mental

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computations. They could not demonstrate the flexible use of a variety of number-sensible strategies to calculate mentally or estimate answers. When requested to justify their answers to a few questions on the Number Sense Questionnaire and in the interviews, they struggled to provide clear and logical explanations. Lastly, although they demonstrated a lack of confidence in answering number sense questions during the interviews, the confidence survey showed that they see themselves as capable to learn, understand, do and teach mathematics.

The next chapter will summarise this study and make recommendations for further action and research.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

There is a perception that the Namibian education system fails to equip young people with the knowledge and skills to fulfil their future roles as informed, competent, content and productive citizens. One worrying trend is that the majority of learners in the country perform below standard in mathematics and English and improvements are at best marginal.

This study was motivated by the researcher's conviction that a focus on the development of number sense in primary school education will result in a long-term and lasting improvement in the numeracy levels of the Namibian people. The main research question of this study was, "Do Namibian final year primary pre-service teachers possess skills, knowledge, strategies and confidence that are related to number sense?"

The orientation in Chapter 1 motivated the research against the background of mathematics education in Namibia, explained the essence of what is described by the term "number sense", stated the main research questions and gave an overview of the study.

The literature study in Chapter 2 reviewed research done into mathematics education in Namibia and explored various international perspectives and investigations relating to number sense and the role of teacher knowledge and confidence in mathematics education.

Chapter 3 described and motivated the use of a mixed methods research methodology and the various instruments employed, it described the data collection and data analysis process and outlined the limitations of the study.

The results of the data analysis were presented and discussed in Chapter 4. The outcomes of this study were verified against the information presented in the literature review.

This concluding chapter will present a summary of findings; state the conclusions, make recommendations, and note limitations of the study.

5.2 Summary of the findings

The overall results of this study revealed that the final year primary pre-service teachers demonstrated limited number sense and possessed very few of the indicators of number sense that were described by Kalchman, Moss and Case (2001). They lacked fluency in estimating and judging magnitude, particularly in the domain of rational numbers and the operations of multiplication and division. They paid little attention to unreasonable results and showed a tendency to guess the answer to a problem. They had little or no access to a variety of flexible number-sensible strategies to solve problems and calculate mentally. They had trouble to move among different representations of numbers and select the most appropriate representation in the context of a problem.

More particularly, the findings to answer the sub-questions of this study indicated that the final year primary pre-service teachers lacked the fluency in basic facts and procedures to perform written calculations efficiently and correctly. The dismal performance on mental calculations and on questions relating to the knowledge of and facility with numbers together with the subsequent analysis of the interview data relating to the use of number-sensible strategies exposed their poor conceptual understanding of the domain of numbers and operations. Unexpectedly, the analysis of the confidence questionnaire showed that they were both confident in their ability to do mathematics and to teach mathematics.

The correlation analysis showed a strong relationship between number sense and mental calculation proficiency; between number sense and confidence in both the ability to do and the ability to teach mathematics and between mental calculation and written calculation proficiencies.

5.3 Conclusions

This study is relevant, important and timely, because it exposes the depth of the root cause of the low standards of performance of Namibian learners in mathematics and the lack of improvement over the last decade or more. It also indicates a way forward.

Mathematics pre-service teachers are a product of how and what they were taught in schools. Despite long-term efforts to implement reforms in the way teaching and learning takes place in Namibian classrooms, the change to a constructivist learner-centred approach has remained elusive. Mathematics teaching in Namibia is still characterised by drill, practice and rote learning (Clegg & Courtney-Clarke, 2009; Mathematics and Science Teachers Extension

Programme (MASTEP), 2000; Ministry of Basic Education, Sport & Culture, 1999). Against this background, the limited number sense displayed by the primary pre-service teachers in this study was not unexpected.

The reasons for this limited number sense go deeper than the lack of understanding of the relatively new concept "number sense" by mathematics educators, both in schools and at tertiary institutions. This study has revealed that the pre-service teachers do not have what Ball (1990) classifies as specialised content knowledge (SCK). Indeed, there were indications that they also lack common content knowledge as their inability to use basic facts and procedures fluently in written computations showed.

The mathematical proficiency characterised by conceptual understanding, procedural fluency, strategic competence and adaptive reasoning in the domain of numbers and operations forms the fundamental building block of the entire mathematics curriculum upon which the understanding of measurement, shape and space, algebra and data handling rests, according to research reviewed by Kilpatrick et al. (2001). This study shows that the primary pre-service teachers are not equipped to lay this foundation.

This sample consisted of *final* year primary pre-service teachers. This leads to the conclusion that the mathematics education curriculum that they have been exposed to over the last three years is to a very large extend irrelevant. It has not developed the profound understanding of fundamental mathematics required to promote number sense in the learners that they will teach.

Finally, a ray of hope: these primary pre-service teachers have confidence in their ability to do, teach and learn mathematics. The researchers' own experience working with primary in-service teachers has shown that most of them are aware of their shortcomings and are desperate to learn and to improve their skills and their understanding.

5.4 Recommendations

Concerns about primary school mathematics education in Namibia have been raised for more than a decade. The lack of action, including the recent stillborn attempt to develop a national numeracy strategy, has led Namibia into a cycle of underachievement and urgent action is needed to break this cycle.

The following recommendations for action are based on the literature review and the results of this study. The recommended actions should be coupled with research to monitor progress and to ensure that the actions have the desired outcomes.

As a first step, there should be a focus on mental calculations and a much-reduced emphasis on teaching the standard algorithms in the primary school years. The literature review showed that there is ample research evidence that mental calculations, if they include both memory and strategies, develop an understanding of the number system, increases the flexibility in working with numbers and promote number sense. The substantial correlations found in this study confirmed that positive and significant relationships exist between mental calculations and number sense as well as mental and written computations.

Computational estimation is a very useful and important real-world skill. It is recommended that building on improved mental calculations of primary school children, computational estimation should feature prominently in the junior secondary years of mathematics education. The proficiency in computational estimation should go beyond rounding and include estimations in the domain of fractions, decimals and percentages.

These changes cannot be achieved by just revising the school mathematics curriculum. That is the easy part. Namibia has learnt an important lesson in its failure to establish learner-centred teaching and learning by issuing policy documents, which the teachers found difficult to interpret and implement. As Greeno (1989) so aptly stated if someone "is to serve as an effective guide to newcomers in an environment, it is essential that the guide himself or herself should be a comfortable resident of the environment" (p.55). This study has shown that the primary pre-service teachers are not ready to serve as such "guides" in the environment of numbers and operations. Teacher-training institutions should devise and implement programmes to develop the number sense of pre-service teachers.

However, there are thousands of in-service teachers who, most likely, do not possess the knowledge, skills and understanding to implement a focus on number sense through mental calculations and computational estimation. Here the responsibility lies with the relevant directorates of the Ministry of Education to research and develop effective and long-term professional development programmes, using the confidence and willingness of the teachers to learn as an important resource to implement these programmes.

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More research is needed in the field of mathematics education in Namibia, possibly in collaboration with other southern African countries that experience similar problems. The mathematics education departments at teacher training institutions should research and identify the mathematics that teachers need to know and the ways in which teacher understanding of subject content needs to be transformed so that they can teach children to develop mathematical proficiency. Academic language proficiencies required for mathematics teaching to ensure that conceptual and procedural discourse can take root in primary school classrooms should form part of this research.

5.5   Limitations of the Study

It is important to note that this study has explored broad issues around number sense and should serve as a starting point for discussion among mathematics educators about the challenges regarding the improvement of general numeracy of Namibian people.

The findings were limited to final year pre-service teachers of one university campus and should not be generalised across all teacher training or similar institutions. However, recommendations in this study made generalisations across the whole population of pre-service and in-service teachers based not only on this study but also on the background and findings of shortcomings in primary mathematics teacher knowledge described and discussed in various research reports reviewed and quoted in this study.

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APPENDIX A   **Number Sense Questionnaire**
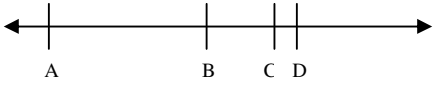
<div style="border">

Number Sense Questionnaire
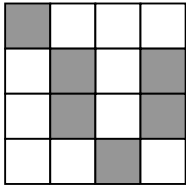
Time: 30 minutes
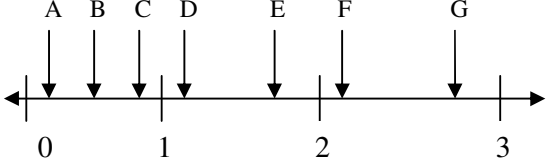
Instructions

1. Do not start until I tell you to do so.

2. Use the pen provided to you to answer the questions.

3. Read each question carefully and then write down your answer or circle the letter in front of the most appropriate answer.

4. Do not attempt to work out answers using written calculations. Where an exact answer is required, it can be calculated mentally.

5. This questionnaire has 27 questions.

6. You have 30 minutes to complete the questionnaire. That is about 1 minute for each question.

7. Do not spend too much time on a question. I will tell you while you are working how much time has passed and how many questions you should have answered. I would like each one of you to complete the questionnaire.

8. Fill in your number at the top of this page now.

9. Wait until I tell you to turn over the page and start.

</div>

1. Without calculating an exact answer, circle the best estimate for: 995 + 872 + 838 + 809	A. 3000 B. 3200 C. 3500 D. 4000
2. Sarah has a string of 541 centimetres. Leonard has a string of 298 centimetres. How much longer is Sarah's string than Leonard's string? Without calculating an exact answer, circle the best estimate.	A. 100 cm B. 200 cm C. 240 cm D. 570 cm
3. Without calculating an exact answer, circle the expression which represents the larger amount.	A. 2452×4 B. $2541 + 2457 + 2460 + 2465$ C. Cannot tell without calculating
4. Make an estimate for the missing number: $45 \times \square = 2700$	$\square = $ _____
5. Is $\frac{3}{8}$ or $\frac{7}{13}$ closer to $\frac{1}{2}$?	_____ Why? _____ _____
6. Without calculating circle the best estimate for: $\frac{10}{11} + \frac{7}{8}$	A. 1 B. 2 C. 17 D. 19 E. I don't know
7. Without calculating an exact answer, circle the best estimate for: $\frac{15}{24} \times \frac{5}{12}$	A. More than $\frac{15}{48}$ B. Less than $\frac{15}{48}$ C. Equal to $\frac{15}{48}$ D. Cannot tell without calculating

8. Without calculating and exact answer, circle the best estimate for: $$6\frac{2}{5} \div \frac{7}{8}$$	A. More than $6\frac{2}{5}$ B. Less than $6\frac{2}{5}$ C. Equal to $6\frac{2}{5}$ D. Impossible to tell without working
9. Use the following picture to answer the question. The distance between A and B is 53,7 km, between B and C is 7,8 km and between C and D is 0,9 km. What is the approximate distance between A and D?	A. 45 km B. 50 km C. 60 km D. 62 km
10. A mini-bus can transport 15 people. If 170 learners need to be transported to the next town, calculate mentally how many mini-buses are needed.	_____
11. John has two pizzas. He gave one third of a pizza to his sister and one half of the other to his brother. About how much pizza is left over altogether?	A. More than 1 whole pizza B. Less than 1 whole pizza C. Exactly 1 whole pizza D. No pizza
12. Which total is more than 1?	A. $\frac{5}{11} + \frac{3}{7}$ B. $\frac{7}{15} + \frac{5}{12}$ C. $\frac{1}{2} + \frac{4}{9}$ D. $\frac{5}{9} + \frac{8}{15}$
13. The following multiplication problem has been carried out except for placing the decimal comma. Place the decimal comma, using estimation.	$534,6 \times 0,545 = \underline{291357}$

14. Without calculating an exact answer, circle the best estimate for: $9135 \div 61$	A. 100 B. 150 C. 30 D. 15
15. Circle the best answer: a. $3 \times 5 \times 6 \times 8$ b. 15×48 c. $24 \times 5 \times 6$	A. $a > b > c$ B. $b > c > a$ C. $a = b > c$ D. $a = b = c$
16. Which is closer to 12: 11,8 or 12,4?	_____ Why? _____ _____
17. Which of the following number sequences is wrong?	A. 3000; 3500; 4000; 4500 B. 7600; 7700; 7800; 7900 C. 6097; 6098; 6099; 7000 D. 8080; 8090; 8100; 8110
18. Which of the following is wrong?	A. $\frac{1}{4} > \frac{1}{5} > \frac{1}{6}$ B. $\frac{8}{9} > \frac{7}{8} > \frac{6}{7}$ C. $\frac{9}{8} > \frac{8}{7} > \frac{7}{6}$ D. $\frac{1}{6} = \frac{5}{30} = \frac{8}{48}$
19. A library had 2590 books. 585 books were lost and 5 books are out on loan. How many books are left in the library?	A. 1500 B. 2000 C. 2010 D. 2500
20. Find the fraction to represent the shaded area. 	A. $\frac{3}{5}$ B. $\frac{4}{12}$ C. $\frac{5}{16}$ D. $\frac{3}{8}$ E. $\frac{6}{15}$

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21. How many different fractions are there between $\frac{2}{5}$ and $\frac{3}{5}$?	A. None. Why? _____ B. One. What is it? _____ C. A few. Give two. _____ and _____ D. Lots. Give three. _____ and _____ and _____
22. Without calculating an exact answer, circle the best estimate for: $36 \times 0,96$	A. more than 36 B. less than 36 C. equal to 36 D. Impossible to tell without working it out.
23. $362\,000 \div 1200$ is	A. equal to $362 \div 12$ B. equal to $3620 \div 120$ C. equal to $3620 \div 12$ D. equal to $362 \div 120$
  24. Which letter in the number line above names a fraction where the numerator is a little bit greater than the denominator?	_____
25. Which letter in the number line above names a fraction where the numerator is nearly double the denominator?	_____

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26. Write these numbers in order starting with the smallest on the top line. $0,595;\ \dfrac{3}{5};\ 61\%;\ \dfrac{5}{8};\ 0,3562$	_____ _____ _____ _____ _____
27. A drum has 125 litres of water in it. A farmer fills more water into the drum and the water level rises at a rate of 1,5 litres per minute. How many litres of water will be in the drum after 4 minutes?	_____

APPENDIX B **Written Computation Questionnaire**

<u>**Written Computation Questionnaire**</u>

Time: 25 minutes

<u>**Instructions**</u>

1. Do not start until I tell you to do so.

2. Use the pen provided to you to answer the question.

3. In this questionnaire you will have to calculate exact answers.

4.   Show all your calculations in the work space provided.

5. Even if you can work out the answer mentally, write down how you did it.

6. This questionnaire has 15 questions.

7. You have 25 minutes to complete the test. That is 1 minute and 40 seconds per question.

8. Fill in your number at the top of this page now.

9. Wait until I tell you to turn over the page and start.

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WORK SPACE

1.

995 + 872 + 838 + 809 = _____

2.

541 – 298 = _____

3.

Which one is bigger? Work out and
then circle the correct answer.

A. $2452 \times 4 =$ _____

B. 2451 + 2457 + 2460 + 2465 =

_____

4.

Find ☐ :

$45 \times$ ☐ $= 2700$

☐ = _____

5.

Which one is bigger? Work out and
then circle the correct answer.

A. $\dfrac{1}{2} - \dfrac{3}{8} =$ _____

B. $\dfrac{7}{13} - \dfrac{1}{2} =$ _____

6.

$\dfrac{10}{11} + \dfrac{7}{8} =$ _____

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7. $\dfrac{15}{24} \times \dfrac{5}{12}$ = _____	
8. $6\dfrac{2}{5} \div \dfrac{7}{8}$ = _____	
9. 53,7 + 7,8 + 0,9 = _____	
10. 170 ÷ 15 = _____	
11. $2 - \dfrac{1}{3} - \dfrac{1}{2}$ = _____	
12. $\dfrac{5}{9} + \dfrac{8}{15}$ = _____	
13. 0,46 × 0,5 = _____	
14. 851,5 – 34,45 = _____	
15. 72 ÷ 0,025 = _____	

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APPENDIX C   **Mental Calculations Questionnaire**

Time: 6 minutes 40 seconds

Instructions

1. In this questionnaire you have to answer all questions
    by mentally calculating the answer.

2. Write down the answer only – no other calculations -
    in the space provided on the answer sheet.

3. I will show each question for 20 seconds on the
    screen and say it once at the beginning. You may
    copy down the question.

4. I will not go back to any question later.

6. Fill in your number on the answer sheet now.

#	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

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This sheet is **not** handed out to the research subjects.
Each question will be presented on a screen for 20 seconds and read out once.

1. $600 + 50 + 250$

2. $189 + 36$

3. $232 - 98$

4. $56 + 23 - 16$

5. $13 \times 5 \times 2$

6. $512 \div 4$

7. 36×50

8. $550 \div 25$

9. $\frac{1}{2} + \frac{3}{4}$

10. $8 - 4\frac{1}{2}$

11. $4 \times 3\frac{1}{2}$

12. $40 \div \frac{1}{2}$

13. $264 + 99$

14. $6,5 + 1,9$

15. $4,5 - 2,6$

16. $5 \times 2,12$

17. $6,5 \div 0,5$

18. 95×0.1

19. $\frac{1}{2} \times 6\frac{1}{2}$

20. $5\frac{1}{4} - 3\frac{3}{4}$

APPENDIX D   McAnallan Confidence in Mathematics and Mathematics Teaching Survey

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Confidence in Mathematics and Mathematics Teaching Survey

Instructions   **Time: 20 minutes**

1. This is not a test. So, there are no correct or incorrect answers.

2. The first part of the survey has 25 statements on your attitudes and feelings towards mathematics and mathematics teaching.

3. You should rate your own response to the statement on a scale from 1 (strongly disagree) to 5 (strongly agree).

4. Do not spend time thinking about the answer, but respond quickly.

5. Circle the number that best describes you agreement with the statement.

6. Fill in your number at the top of this page now.

7. Complete the practice examples below and wait until the answers have been checked.

8. Wait until I tell you to turn over the page and start.

Practice Examples: Circle the answer that best describes your level of agreement with the statement.

	Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
Mathematics has always been my favourite subject.	1	2	3	4	5
I did not enjoy mathematics lessons at school.	1	2	3	4	5

Please circle the number that best describes your level of agreement with the statement.

	Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1. I was one of the best math students when I was in school.	1	2	3	4	5
2. I feel uncomfortable when I have to work with fractions.	1	2	3	4	5
3. I feel confident in my ability to teach mathematics to primary school learners.	1	2	3	4	5
4. I am confident that I can learn more difficult math concepts.	1	2	3	4	5
5. When teaching mathematics, I welcome questions form learners.	1	2	3	4	5
6. I have trouble finding alternative methods for teaching a mathematical concept when a learner is confused.	1	2	3	4	5
7. I can easily do mental calculations in my head.	1	2	3	4	5
8. I find it difficult to teach mathematical concepts to learners.	1	2	3	4	5
9. I feel confident using sources other than the mathematics textbook when I teach.	1	2	3	4	5
10. I don't have the math skills to differentiate instruction for the most talented learners in my math classes.	1	2	3	4	5
11. I will dislike having to teach math every day.	1	2	3	4	5
12. I avoided taking math in Grade 11 and 12 at school.	1	2	3	4	5
13. I have a lot of self-confidence when it comes to mathematics.	1	2	3	4	5
14. I am confident that I can solve math problems on my own.	1	2	3	4	5
15. I become anxious when I have to compute percentages.	1	2	3	4	5
16. I have math anxiety.	1	2	3	4	5
17. It makes me nervous to think about having to do any math problem.	1	2	3	4	5

	Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
18. On the average, other student teachers will probably be much more capable of teaching math than I am.	1	2	3	4	5
19. I get upset when a learner asks me a math question that I can't answer.	1	2	3	4	5
20. I am comfortable working on a problem that involves algebra.	1	2	3	4	5
21. I have strong ability when it comes to math.	1	2	3	4	5
22. I doubt that I will be able to improve my math teaching ability.	1	2	3	4	5
23. If I don't know the answer to a learner's mathematical question, I have the ability to find the answer.	1	2	3	4	5
24. I become anxious when a learner finds a way to solve a problem with which I am not familiar.	1	2	3	4	5
25. I would welcome the chance to have my supervisor evaluate my math teaching.	1	2	3	4	5

26. I am a: ❏ Male ❏ Female

27. Place a check mark in front the following math classes you successfully completed in high school:

_____ **Grade 7** _____**Grade 10** _____ **Grade 12**

28. Compare yourself to other primary pre-service students in terms of your mathematical abilities:

❏ 1	❏ 2	❏ 3	❏ 4	❏ 5	❏ 6	❏ 7
One of the worst	Way below average	Below average	Average	Above average	Way above average	One of the best

APPENDIX E **Interview Schedule**

<u>Semi-Structured Interviews to investigate the number sense strategies that the students use to answer number sense questions</u>

<div style="border:1px solid black; padding:10px;">

<u>Interview Schedule</u>

<u>Instructions</u>

- We will look at 6 questions of the number sense questionnaire one by one.
- I will ask you to answer the question and then explain to me how you worked out the answer.
- If you cannot work out the answer I would like you to tell me why.
- I hope it is okay with you that I tape our conversation? You are welcome to listen to the tape at the end of this interview.

<u>Researcher:</u>   Here is the question:………………….

→→ Give student time to think about the question and to indicate an answer.

(a) If the student indicates that he/she cannot answer the question

<u>Researcher:</u> Why not?

<u>Student:</u> I need to work it out on paper. **Or**   We have not learnt how to do it.

<u>Researcher:</u> Can you do it any other way?

<u>Student:</u> Yes [go onto **(b)]**   <u>Student:</u> No [move onto the next question]

(b) If the student indicates an answer

<u>Researcher:</u> Please tell me how did you do this? **Or**   Please tell me your reasons
Or   Why did you do it this way?

<u>Student:</u> …………..

→→ If the student comes up with another strategy, Researcher asks for explanation, and verifies the explanation – repeat until student has exhausted all strategies.

<u>Researcher:</u> Thank you for your willingness to work with me.

</div>

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APPENDIX F　　　　　　　　　　**Ethical Clearance Letter**

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UNIVERSITEIT·STELLENBOSCH·UNIVERSITY
jou kennisvennoot · your knowledge partner

18 April 2011

Tel.: 021 - 808-9183
Enquiries: Sidney Engelbrecht
Email: sidney@sun.ac.za

Reference No. 452/2010

Ms M.A.E Courtney-Clarke
Department of Curriculum Studies
University of Stellenbosch
STELLENBOSCH
7602

Ms M.A.E Courtney-Clarke

LETTER OF ETHICS CLEARANCE

With regards to your application, I would like to inform you that the project, *Exploring the number sense of final year primary pre-service teachers,* has been approved on condition that:

1. The researcher/s remain within the procedures and protocols indicated in the proposal;
2. The researcher/s stay within the boundaries of applicable national legislation, institutional guidelines, and applicable standards of scientific rigor that are followed within this field of study and that
3. Any substantive changes to this research project should be brought to the attention of the Ethics Committee with a view to obtain ethical clearance for it.

We wish you success with your research activities.

Best regards

MR SF ENGELBRECHT
Secretary: Research Ethics Committee: Human Research (Humanoria)