A Bandwidth Market in an IP Network

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Declaration

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Signature: ............................  Date:  .................
Abstract

Consider a path-oriented telecommunications network where calls arrive to each route in a Poisson process. Each call brings on average a fixed number of packets that are offered to route. The packet inter-arrival times and the packet lengths are exponentially distributed. Each route can queue a finite number of packets while one packet is being transmitted. Each accepted packet/call generates an amount of revenue for the route manager. At specified time instants a route manager can acquire additional capacity (“interface capacity”) in order to carry more calls and/or the manager can acquire additional buffer space in order to carry more packets, in which cases the manager earns more revenue; alternatively a route manager can earn additional revenue by selling surplus interface capacity and/or by selling surplus buffer space to other route managers that (possibly temporarily) value it more highly. We present a method for efficiently computing the buying and the selling prices of buffer space.

Moreover, we propose a bandwidth reallocation scheme capable of improving the network overall rate of earning revenue at both the call level and the packet level. Our reallocation scheme combines the Erlang price [4] and our proposed buffer space price ($\text{M}/\text{M}/1/K$ prices) to reallocate interface capacity and buffer space among routes. The proposed scheme uses local rules and decides whether or not to adjust the interface capacity and/or the buffer space. Simulation results show that the reallocation scheme achieves good performance when applied to a fictitious network of 30-nodes and 46-links based on the geography of Europe.
Opsomming

Beskou 'n pad-georiënteerde telekommunikasie netwerk waar oproepe by elke roete arriveer volgens 'n Poisson proses. Elke oproep bring gemiddeld 'n vasgestelde aantal pakkies aangebied om te versend. Die inter-aankomstye van pakkies en die pakkietlengtes is eksponensiaal versprei. Elke roete kan 'n eindige aantal pakkies in 'n tou behou terwyl een pakkie versend word. Elke aanvaarde pakkie/oproep genereer 'n hoeveelheid inkomste vir die roetebestuurder. 'n Roetebestuurder kan op vasgestelde tydsteppe addisionele kapasiteit ("koppelvlak kapasiteit") verkry ten einde meer oproepe te hanteer of die bestuurder kan addisionele bufferruimte verkry ten einde meer pakkies te dra, in welke gevalle die bestuurder meer inkomste verdien; andersins kan 'n roetebestuurder addisionele inkomste verdien deur oortollige koppelvlak kapasiteit te verkoop of oortollige bufferruimte te verkoop aan ander roetebestuurders wat (moontlik tydelik) meer waarde daaraan heg. Ons beskryf 'n metode om die koop- en verkoopp pryse van bufferruimte doeltreffend te bereken.

Verder stel ons 'n bandwydteheraanwysingskema voor wat daartoe in staat is om die algehele verdienstekoers van die netwerk te verbeter op beide oproep- en pakkievvlak. Ons heraanwysingskema kombineer die Erlang prys [4] en ons voorgestelde bufferruimteprys (\(M/M/1/K\) prys), om die koppelvlakkapasiteit en bufferruimte tussen roetes te heralokoer. Die voorgestelde skema gebruik lokale reëls, om te besluit hetsy die koppelvlakkapasiteit en/of bufferruimte optimaal te verstel al dan nie. Simulasieresultate toon dat die heraanwysingskema goed werk wanneer dit aangewend tot 'n kunstmatige netwerk met 30 nodusse en 46 skakels gebasseer op die geografie van Europa.
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Chapter 1

Introduction

1.1 Motivation and Objectives

Although the telecommunication technology of today offers network users increasing capacity, the fact is that the users’ bandwidth demands are often higher than the network capacity [17]. This is essentially due to the increasing number of users and the appearance of new bandwidth-consuming services such as multimedia and interactive services. When the users’ bandwidth demands are lower than the network capacity, there is less need for economy, but when the demand exceeds the supply, it becomes important to efficiently manage the bandwidth allocated; especially in situations where parts of the network are under-utilized while other parts are nearly fully or over-utilized. When such a situation occurs, some users’ demands for connection are rejected and revenue is lost, revenue which could be earned if the bandwidth were better managed.

Based on this motivation, this thesis presents a mechanism to minimize lost network revenue by reallocating capacity among the logical paths of a telecommunication network. Each path has two levels of connection, the call level and the packet level. Capacity is assigned to each level: the interface capacity which determines the number of calls the path can accommodate and the effective capacity which expresses the number of packets (in service and queued) the path can simultaneously carry. We propose to employ a scheme in which each logical path places a value on capacity dependent on its current capacity assignment and its current occupancy. Under this scheme, a bandwidth manager is assigned
to each logical path. Each manager calculates both the revenue that the path would gain should the path acquire an extra unit of buffer space and the revenue that the path would gain should the path acquire an extra unit of interface capacity, and also the revenue that the path would lose should the path give up a unit of buffer space and the revenue the path would lose should the path give up a unit of interface capacity. The bandwidth managers then use these revenue measures and the local capacity demand, and determine whether they should re-allocate buffer space and/or interface capacity among themselves in order to maintain the performance of their paths. The buffer space prices and the interface capacity prices form the basis for a mechanism to re-allocate buffer space from paths that place a low value on buffer space to paths that place a high value on buffer space; and a mechanism to re-allocate interface capacity from paths that place a low value on interface capacity to paths that place a high value on interface capacity. The question arises as to how these prices should be calculated.

A previous work on bandwidth prices by Chiera and Taylor [4] provides a way of computing the value of capacity in an Erlang loss model. The authors compute the expected lost revenue over a given time interval due to connections being blocked, conditional on the system starting in a given state. From the expected lost revenue, they derive both the buying and selling prices of a unit of capacity. This work is similar to that of Lanning, Massey, Rider and Wang [24] who study the prices that should be charged to customers in a dynamic loss system. Here the problem is that of Internet billing where the arrival rates to the system are user dependent. Fulp and Reeves [7] study a multimarket scenario where the price of resources is based on the current and future usage. MacKie-Masson and Varian [16] describe the basic economic theory of pricing a congestible resource such as a ftp server, a router and a web site and examine the implications of congestion pricing for capacity under centralized planning. Kelly et al. [8] and Low and Lapsley [30] propose distributed models that optimize different types of aggregate utility as seen by sources.

Other pricing models include WALRAS [28] that computes prices for bandwidth trading in a market-oriented environment by use of a tatonnement process in a competitive equilibrium. This model is set up as a producer-consumer system and requires the simultaneous solution of three constrained linear programming (LP) problems. WALRAS is used in the design of a system where bandwidth is traded at prices computed to reflect current and future requirements.
In our work we use the Erlang prices \[4\] to compute the value of the interface capacity and we present a new pricing function that computes the value of a unit of a buffer space. This pricing function is based on the Chiera and Taylor \[4\] model. However, we consider an $M/M/1/K$ link; that is, a single server with a finite buffer space. The server is characterized by the service rate which represents the amount of bandwidth needed to process packets. The link can queue up to $K - 1$ packets while one is being processed. Revenue is lost when packets are dropped. The lost revenue can be controlled by varying the bandwidth allocated to the path and/or by varying the link buffer space $K - 1$. We view the buying and selling of capacity in terms of the change in the lost revenue when the link buffer space increases or decreases. The rate of earning revenue increases when the manager acquires a unit of buffer space and decreases for each unit of buffer space released.

The two prices (Erlang and $M/M/1/K$ prices) are used to reallocate interface and buffer capacity respectively among the paths of a telecommunication network. The buffer space and interface capacity exchange take place between paths that share the same physical links. The paths are of two kinds: a transit path which uses more than one physical link and a direct path which uses only one physical link. The direct paths on the physical links of the transit path are referred to as its constituent direct paths. Each physical link supports one direct path. Managers of transit paths are responsible for the re-allocation mechanism. The managers send signalling packets to record the buying and the selling prices of the constituent direct paths and, under conditions that will be discussed later, the transit path releases capacity (buffer space or interface capacity) to its constituent direct paths or acquires capacity from its constituent direct paths.

We shall present several models of capacity reallocation schemes. The various models depend on the ways in which the network parameters are set up and on the choice made between different bandwidth prices. Our main objective here is to demonstrate that our re-allocation scheme can work. Discussion on the best way to set up such a scheme is not dealt with in this thesis.

### 1.2 Outline of the Thesis

This thesis is organized into eight chapters with a bibliography at the end.
Chapter 1. Introduction

Chapter 2 is divided into two parts. In the first part we review a simple queueing model with finite buffer size and present some of the performance measures of the model. In the second part, we present formulas for blocking probability in a single link with a single traffic class.

In Chapter 3, we present the context in which this thesis is placed. We briefly review the concept of logical path and logical network, we review some network technologies that use logical paths and introduce the concept of bandwidth management.

In Chapter 4, we present a method to compute the expected lost revenue due to packet blocking on a path. From the expected lost revenue we derive the buying and the selling prices of a unit of buffer space. This chapter also includes some examples of the computation of the expected lost revenue, and examples of the computation of the buying and selling prices.

In Chapter 5, we use the Erlang prices as a basis for bandwidth reallocation in an IP network at a connection level. The $M/M/1/K$ prices are used to move buffer space from paths that place a low value on buffer space to those that place a higher value of buffer space.

In chapter 6, we describe the simulation model we used to compute the performance measures of the reallocation scheme. Chapter 6 also includes the determination of the network’s parameters used for an efficient bandwidth reallocation scheme.

In chapter 7, we present some of the performance of the reallocation scheme on the revenue loss and the loss probability for connections and packets; and also on the loss of packets and connections.

Finally in chapter 8, we conclude and summarize the main contributions.
Chapter 2

Mathematical Background

2.1 Overview

This chapter provides a brief introduction to the simple queueing theory concepts that are used in the thesis. Queueing theory is often used for modeling telecommunication networks and allows the computation of system performance, such as average queue lengths, delays, blocking probabilities, etc., given the system workload and the network equipment processing speeds. In this introduction we focus on the computation of the loss probability in a single link when the link is transporting applications for which timeliness of data is not of high importance.

The chapter is organized as follows. We first present a single server model and some of its performance measures. We then explain the concept of blocking and its computation.

2.2 The Server Model

Consider a stream of packets which arrive at an edge router of a telecommunication network, seeking admittance to the network. Upon arrival the packet is served if the server is idle else it is queued in the buffer if there is buffer space available, otherwise the packet is dropped. This is modeled by the single server queue shown in Figure 2.1 where \( \lambda \) is the mean arrival rate to the server and \( \mu \) is the mean service rate of the server. A “job” here
corresponds to a packet and “servicing the job” corresponds to transmitting the packet. The time to service a packet is the transmission time.

\[
\lambda \quad FCFS \quad \mu
\]

**Figure 2.1: Single Server Queue.**

Throughout our study, we assume that the service process has an exponential distribution with mean \(1/\mu\). That is, the service rate is \(\mu\) and it takes on average \(1/\mu\) seconds to serve a packet; this service rate represents the bandwidth. The inter-arrival process is also assumed to have an exponential distribution with rate \(\lambda\) so that the average time between two successive packet arrivals is \(1/\lambda\).

We consider first-come-first-served (FCFS) scheduling so that the packets are served in the order they arrive in the queue. The queue is constrained in length and an incoming packet will be dropped if there is no place in the queue.

In Kendall notation, the queue is an \(M/M/1/K\) queue, where \(K\) indicates the maximum number of customers the system can hold, 1 denotes a single server and \(M\) denotes the Markovian property of the exponential inter-arrival and service distributions.

### 2.3 Markovian Property

The exponential distribution is often used to model the distribution of the time intervals between events in a continuous-time stochastic processes. This is due to its Markovian property which means that the distribution of time before an event takes place is independent of the instant \(t\) when the previous event took place. More explicitly

\[
P\{X < t + h | X > t\} = P\{X < h\}, \quad \forall t > 0, h > 0
\]
2.4 System States and State Probabilities

For the $M/M/1/K$ queue, the states are described by the number of customers in the system. Let $p_n(t)$ denote the probability that there are $n$ customers in the system at time $t$. The system is assumed to be ergodic so that in the steady state, after the system has been operated for a long period of time, $p_n(t)$ becomes independent of $t$ and is written $p_n$.

2.5 The Steady State Distribution of the System

Let the set of possible states of the system be denoted by $\mathcal{N} = \{0, 1, \ldots, K\}$. The state space of the $M/M/1/K$ queue is a truncation of the $M/M/1$ queue. Since the latter is reversible, the former is also reversible. For each state $n$ of $\mathcal{N}$, the flow out of state $n$ is equal to the flow into that state. The detailed balance equation for state $n$ is therefore:

$$\lambda p_{n-1} = \mu p_n \quad n = 1, 2, \ldots, K.$$

These equation can be solved recursively so that

$$p_n = (\lambda/\mu)^n p_0, \quad n = 0, 1, \ldots, K. \quad (2.1)$$

Since the $\{p_n\}$ are probabilities, they sum to one

$$\sum_{n=0}^{K} p_n = 1. \quad (2.2)$$

Using the normalization Eqn. (2.2) together with Eqn. (2.1) yields

$$p_0 = \frac{1 - \rho}{1 - \rho^{K+1}} \quad \rho \neq 1$$

where $\rho = \lambda/\mu$ denotes the traffic intensity. Note that for this queue $\rho$ can be greater than one while the queueing system remains stable.
A special case is the one in which $\lambda = \mu$, in which we have that each state is equally likely so that

$$p_n = 1/(K + 1) \quad n = 0, \ldots, K.$$ 

### 2.6 Some Performance Measures

The system performance measures are derived from the steady state distribution. This section presents the computation of the average queue length, the average waiting time and the average delay on passing through the system.

#### 2.6.1 The Expected Queue Length

The average number $L$ of packets in the system including the packet in service is

$$L = \sum_{n=0}^{K} np_n = \begin{cases} \frac{\rho}{1-\rho} - \rho^{K+1} \frac{K+1}{1-\rho^{K+1}} & \rho \neq 1 \\ \frac{K}{2} & \rho = 1. \end{cases}$$

#### 2.6.2 The Expected Waiting Time

The mean time $W$ that a packet spends in the system can be evaluated using Little’s formula

$$W = L/\lambda.$$ 

Since packets cannot enter the system when it is full, the rate at which packets enter the system is given by $\lambda(1 - p_K)$. Little’s law yields

$$W = \frac{L}{\lambda(1 - p_K)}.$$
2.6.3 The Expected Queue Delay

The expected delay $W_q$ that a packet experiences is the mean time it spends in the queue waiting for service. This delay can be derived from the mean queue length using Little’s law. The mean queue length $L_q$ itself is derived from the average number of packets in the system

$$L_q = L - L_s$$

where $L_s = 1 - p_0$ denotes the mean number of packets in service. Applying Little’s law yields

$$W_q = \frac{L_q}{\lambda(1 - p_K)}.$$

2.7 Blocking in a Queueing System

Blocking in the $M/M/1/K$ queue occurs when an arrival finds the queue full, whereupon it is blocked and rejected. Blocking is measured by the proportion of packets blocked. Two parameters affect the system and cause blocking: the size $K-1$ of the waiting line and the traffic intensity $\rho$.

An arrival is blocked when the system is full. If $\rho$ denotes the traffic intensity, the fraction of packets blocked is given by

$$P_{loss} = P(K, \rho) = \begin{cases} \rho^K \frac{1-\rho}{1-\rho^{K+1}} & \rho \neq 1 \\ \frac{1}{K+1} & \rho = 1. \end{cases}$$

This formula applies if the link carries one traffic class. The more general case for a link transporting several types of traffic is beyond the scope of this work and further details can be found in [22].
The Recursive Computation of $P(K, \rho)$\[23\]

Eqn. (2.3) cannot be used directly for calculating the link blocking probability due to inexact computations of powers $\rho^K$ for large values of $\rho$ and $K$. The following recursive formula provides an efficient way to compute $P(K, \rho)$ with respect to $K$:

$$P(K, \rho) = \frac{\rho P(K - 1, \rho)}{K + \rho P(K - 1, \rho)}$$

where $P(0, \rho) = 1$. 
Chapter 3

Network Resource Management

3.1 Overview

This chapter presents a brief background concerning the context in which this thesis is located. The main interest is network bandwidth management focusing on the dynamical reconfiguration of bandwidth. This chapter introduces the concept of a logical network, presents two examples of network technologies where logical network can be set up (ATM and MPLS), and introduces a technique for bandwidth management using logical networks. Our proposed scheme for dynamical reconfiguration of bandwidth will be presented in Chapter 5.

3.2 Introduction

Network resource management deals with protocols to reserve resources in order to guarantee a certain quality of service (QoS) in a network [17]. One goal is to allow network providers to efficiently use resources so that the revenue generated from the utilization of resources can be maximized. Several types of network technologies including Asynchronous Transfer Mode (ATM) and Multi-Protocol Label Switching (MPLS) use dynamic resource management capabilities [17]. These capabilities allow the design and implementation of automatic mechanisms to manage network resources. The resources to be managed include bandwidth, buffer space and router processing capacity. A higher layer called the
Figure 3.1: An example of a logical network established with the logical paths $p_1$, $p_2$, etc.

A logical path can be viewed as a reservation of bandwidth between different nodes in order to facilitate the establishment of user connections or flows [17].

The set of logical paths assigned to a physical network is referred to as the logical network (see Fig. 3.1). The logical network acts like a physical network where user connections can be established. However, it has the advantage of being flexible in the sense that its topology (the virtual topology) and the bandwidth assigned to each logical path can be dynamically updated according to user bandwidth demands. Another important advantage of having a logical network over the physical network is in the building of protection mechanisms, where some of the logical paths are established as a set of backup paths to be used in case of the failure of the working paths. The following section describes two connection-oriented network technologies that use such mechanism: MPLS and ATM.
3.4 Network Technologies

3.4.1 Asynchronous Transfer Mode (ATM)

ATM networks are designed to support a wide variety of multimedia applications with diverse services and performance requirements [17]. ATMs have two layers of hierarchy: Virtual Path (VP) and Virtual Channel (VC). They are a form of packet switching network, that is, when user wants to transmit information, he first requests establishment of a virtual connection, i.e., VC through pre-established VPs. The VP connects any two ATM devices including switches and end-points. Once a virtual channel (VC) is deployed the user can generate a stream of cells (packets of fixed length) that flows along the VP. The virtual path layer is used to simplify the establishment and management of new connections (VCs) and constitutes a logical network. This mechanism allows the network to carry out dynamic management of the logical topology and enables its adaptation to improve resource utilization [17].

3.4.2 Multi-Protocol Label Switching (MPLS)

MPLS is a protocol for the management of the core network belonging to a network provider [17], usually in an Internet environment. MPLS groups user transmissions into flows and allows the allocation of bandwidth to aggregates of flows [9, 32]. MPLS is deployed within a domain called the MPLS domain. The routers belonging to a MPLS domain are called Label Switched Routers (LSR). When data packets arrive to an ingress LSR, they are classified into Forwarding Equivalent Classes (FEC) which group the packets according to certain common properties (protocol, size, origin, destination) [17]. A label is assigned to every FEC and all the data packets belonging to the same FEC. Packets inside an MPLS domain are routed from the ingress router to the egress router through pre-established paths called Label Switched Paths [17]. During the transmission process intermediate routers do not make any routing decisions [19]. The set of LSPs constitutes the logical network and it is established using a signalling protocol such as the Label Distribution Protocol (LDP) [17, 19].
3.5 Bandwidth Management

Network resource management is performed automatically on a periodic basis, for example each hour. It includes mechanisms that re-route flows in case of the failure of links. Three functions constitute the key resource management processes: bandwidth management, fault protection and spare capacity planning. Bandwidth management is briefly described below.

Bandwidth management attempts to manage the bandwidth assigned to the logical paths. It often happens that part of the network is under-utilized when another part is nearly congested. When this occurs some of the connections are lost which could have been accepted if the bandwidth were efficiently balanced.

One of the main objectives of bandwidth management is to minimize the blocking probability, i.e., the probability that an offered call or packet is rejected due to insufficient bandwidth or buffers being available for the allocation of the new call or packet. Two actions are usually performed for the bandwidth management system: bandwidth reallocation and logical path re-routing [17].

If on the same link there are over-utilized logical paths and under-utilized logical paths, the bandwidth assigned to each path can be reallocated so that the blocking probability on each logical path is minimized.

The logical path re-routing method deals with links where all the logical paths are congested or nearly congested. In this case it is not possible to move bandwidth between logical paths. A better approach would be to change the topology of the logical network, i.e., the logical paths can be redistributed in order to meet the user bandwidth demand.

The bandwidth reallocation method can be applied to networks that have resource management mechanisms. The logical paths are modified using a distributed (and hence scalable) algorithm which collects and tests local information about logical paths and decides whether or not to adapt the bandwidth assigned to them. The bandwidth pricing functions presented in the following chapter constitute the key information which directs the bandwidth re-allocation process.
Chapter 4

The Bandwidth Pricing Functions

4.1 Overview

This chapter presents a bandwidth pricing function based on an $M/M/1/K$ link model. The model is based on the approach presented in [4]. However, we consider the buying and selling of bandwidth in terms of the variation in the lost revenue when the link buffer space increases or decreases. Revenue is lost when packets are dropped. The lost revenue can be controlled by varying the buffer space allocated to the link. For each unit of buffer space acquired, the buffer space will increase, the packet loss probability will decrease, and the rate of earning revenue will increase. Conversely for each unit of buffer space released the buffer space will decrease, the packet loss probability will increase, and the rate of earning revenue will decrease.

This chapter is organized as follows. We first define a model to compute the expected lost revenue. We next develop a recursive formula for the efficient computation of the lost revenue. Finally we use the expected lost revenue to derive the buying and selling prices of a unit of buffer space.
4.2 Model and Analysis

We consider an $M/M/1/K$ queue system with buffer of size $K-1$. The service time is exponential with parameter $\mu$ and inter-arrival times are exponential with parameter $\lambda$. A packet loss occurs whenever an arriving packet finds the buffer full. Such a system can be modeled \cite{23} by a continuous time Markov chain with state space $\{0, 1, \ldots, K\}$ and transition rates

$$q_{n,n+1} = \begin{cases} \lambda & 0 \leq n < K \\
0 & n = K \end{cases}$$

$$q_{n,n-1} = \begin{cases} \mu & 0 < n \leq K \\
0 & n = 0 \end{cases}$$

Let $\theta$ denote the expected revenue generated per accepted packet. A model to compute the expected loss in revenue, conditional on knowledge of the current number of packets in the system can be set up as follows.

Let $R_n(t)$ denote the expected lost revenue in the interval $[0, t]$ given that there are $n$ packets in the $M/M/1/K$ system at time 0. The quantity $t$ is referred to as the planning horizon. Let $R_n(t|x)$ be the same quantity conditional on the fact that the first time that the $M/M/1/K$ queue departs from the state $n$ is $x$. Since the link is blocked whenever $K$ packets are present, and then loses revenue at rate $\theta\lambda$, we have:

$$R_n(t|x) = \begin{cases} 0 & 0 \leq n < K, t < x \\
\theta\lambda t & n = K, t < x \\
\frac{\mu}{\lambda + \mu}R_{n-1}(t-x) + \frac{\lambda}{\lambda + \mu}R_{n+1}(t-x) & 0 < n < K, t \geq x \\
\theta\lambda x + R_{K-1}(t-x) & n = K, t \geq x \end{cases} \quad (4.1)$$

Let $F_n(x)$ be the distribution of time $x$ until the first transition, when there are $n$ packets in the system. Then,

$$R_n(t) = \int_0^\infty R_n(t|x)dF_n(x). \quad (4.2)$$
Due to the Markovian property of the model, $F_n(x)$ is exponential with parameter $\lambda + \mu$ when $n < K$ and exponential with parameter $\mu$ when $n = K$. Substituting Eqn. (4.1) into Eqn. (4.2) we see that there are three cases to be considered. These are:

**Case 1:** $n = 0$. In this case $dF_0(x) = \lambda e^{-\lambda x}dx$ and

$$R_0(t) = \int_0^t R_0(t|x)dF_0(x) = \int_0^t R_1(t-x)\lambda e^{-\lambda x}dx.$$  

**Case 2:** $0 < n < K$. In this case $dF_n = (\lambda + \mu)e^{-(\lambda+\mu)x}dx$ and

$$R_n(t) = \int_0^t R_n(t|x)dF_n(x) = \int_0^t \left( \frac{\mu}{\lambda + \mu}R_{n-1}(t-x) + \frac{\lambda}{\lambda + \mu}R_{n+1}(t-x) \right) (\lambda + \mu)e^{-(\lambda+\mu)x}dx = \int_0^t (\mu R_{n-1}(t-x) + \lambda R_{n+1}(t-x))e^{-(\lambda+\mu)x}dx.$$  

**Case 3:** $n = K$. In this case $dF_K(t) = \mu e^{-\mu t}dx$ and

$$R_K(t) = \int_0^t R_K(t|x)dF_K(x) + \int_t^\infty R_K(t|x)dF_K(x) = \mu \int_0^t (R_{K-1}(t-x) + \theta \lambda x) e^{-\mu x}dx + \mu \int_t^\infty \theta \lambda te^{-\mu x}dx = \mu \int_0^t R_{K-1}(t-x)e^{-\mu x}dx + \frac{\theta \lambda}{\mu} (1 - e^{-\mu}).$$

Taking the Laplace transform of the above three equations we obtain

$$\tilde{R}_0(s) = \frac{\lambda}{s + \lambda} \tilde{R}_1(s) \quad (4.3)$$

$$\tilde{R}_n(s) = \frac{\lambda}{s + \mu + \lambda} \tilde{R}_{n+1}(s) + \frac{\mu}{s + \mu + \lambda} \tilde{R}_{n-1}(s) \quad 0 < n < K \quad (4.4)$$

$$\tilde{R}_K(s) = \frac{1}{s + \mu} \left( \mu \tilde{R}_{K-1}(s) + \frac{\theta \lambda}{s} \right). \quad (4.5)$$
Given the link parameters \( K, \lambda, \mu \) and \( \theta \), the solution of equations (4.3) through (4.5) and its inversion gives the expected lost revenue \( R_n(t) \) in \([0, t]\) conditional on the number \( n \) of packets present in the system at time \( t = 0 \). The solution of equations (4.3) through (4.5) is obtained in three steps.

First, from Eqn. (4.4) and using the methods presented in [4] we obtain the recurrence relation

\[
P_{n+1}(\xi) = (\xi + \mu/\lambda + 1)P_n(\xi) - (\mu/\lambda)P_{n-1}(\xi)
\]

for \( n \geq 1 \) where \( \xi = s/\lambda \).

Second, we express Eqn. (4.6) in terms of orthogonal polynomials. First substitute

\[
P_n(\xi) = Q_n(\xi + \mu/\lambda + 1)
\]

so that Eqn. (4.6) becomes

\[
Q_{n+1}(\xi + \mu/\lambda + 1) = (\xi + \mu/\lambda + 1)Q(\xi + \mu/\lambda + 1) - (\mu/\lambda)Q_{n-1}(\xi + \mu/\lambda + 1)
\]

which can be written as

\[
Q_{n+1}(\phi) = \phi Q(\phi) - (\mu/\lambda)Q_{n-1}(\phi)
\]

where \( \phi = \xi + \mu/\lambda + 1 \). Next let \( \alpha \) be a constant (to be chosen later) and let

\[
S_n(\phi) = \alpha^n Q_n(\phi).
\]

Eqn. (4.7) becomes

\[
\frac{1}{\alpha^{n+1}}S_{n+1}(\phi) = \frac{\phi}{\alpha^n}S_n(\phi) - \frac{\mu}{\lambda} \frac{1}{\alpha^{n-1}}S_{n-1}(\phi).
\]

Multiplying throughout by \( \alpha^{n+1} \) yields

\[
S_{n+1}(\phi) = \alpha \phi S_n(\phi) - (\mu/\lambda)\alpha^2 S_{n-1}(\phi).
\]

Now choose \( \alpha \) such that \((\mu/\lambda)\alpha^2 = 1\). Then

\[
S_{n+1}(\phi) = \alpha \phi S_n(\phi) - S_{n-1}(\phi)
\]

which can be written as

\[
S_{n+1} \left( \frac{2 \alpha \phi}{\alpha} \right) = 2 \left( \frac{\alpha \phi}{2} S_n \left( \frac{2 \alpha \phi}{\alpha} \right) \right) - S_{n-1} \left( \frac{2 \alpha \phi}{\alpha} \right).
\]
Let $x = \alpha \phi / 2$ and define

$$S_n(2x/\alpha) = C_n(x)$$

so that Eqn. (4.8) becomes

$$C_{n+1}(x) = 2xC_n(x) - C_{n-1}(x)$$

which describes the Chebychev polynomials.

Third, to obtain the explicit form of $C_n(x)$, we express $C_n(x)$ in term of $P_n(x)$

$$C_n(\xi) = S_n(2\xi/\alpha)$$

$$= \alpha^n Q_n(2\xi/\alpha)$$

$$= \alpha^n P_n(2\xi/\alpha - \mu/\lambda - 1)$$

(4.9)

where $n \geq 1$ and $\alpha^2 = \lambda/\mu$. From Eqn. (4.3) and again using the methods presented in [4] we obtain

$$P_1(\xi) = (\xi + 1)P_0(\xi).$$

(4.10)

Taking $P_0(x) = 1$, Eqn. (4.9) yields

$$C_1(\xi) = \alpha P_1(2\xi/\alpha - \mu/\lambda - 1)$$

$$= \alpha (2\xi/\alpha - \mu/\lambda - 1 + 1)P_0(2\xi/\alpha - \mu/\lambda - 1)$$

$$= 2\xi - \alpha \mu/\lambda.$$

Using the initial conditions $C_0(\xi) = P_0(x) = 1$ and $C_1(\xi) = 2\xi - \alpha \mu/\lambda$ it is shown in [31, page 204] that

$$C_n(\xi) = 2T_n(\xi) + U_{n-2}(\xi) - (\alpha \mu/\lambda)U_{n-1}(\xi)$$

where $T_n(\xi)$ and $U_n(\xi)$ are respectively the first and the second kind of Chebychev polynomials. The solution of the recurrence relation is thus

$$P_n(\xi) = \alpha^{-n} C_n(\alpha(\xi + \mu/\lambda + 1)/2)$$

(4.11)

where $n \geq 0$, $\alpha^2 = \lambda/\mu$ and the $C_n(\cdot)$ are Chebychev polynomials.

Using the same arguments present in [4], it follows that the solution of Eqns. (4.3) through (4.5) is given by $\tilde{R}_n(s) = A(s)P_n(s/\lambda)$, where $P_n(s/\lambda)$ are Chebychev polynomials. Using the condition for $n = K$, we obtain

$$A(s) = \left(\frac{1}{s}\right) \left(\frac{\theta \lambda}{(s + \mu)P_K(s/\lambda) - \mu P_{K-1}(s/\lambda)}\right)$$
and so
\[ \tilde{R}_n(s) = \left( \frac{1}{s} \right) \left( \frac{\theta \lambda}{(s + \mu)P_K(s/\lambda) - \mu P_{K-1}(s/\lambda)} \right) P_n(s/\lambda). \]

### 4.3 A Numerically Stable Calculation of $\tilde{R}_n(s)$

The computation of $\tilde{R}_n(s)$ is not straightforward. Eqn. (4.11) cannot be used to compute $P_n(x)$ since $\alpha = \sqrt{\lambda/\mu}$ can be small and $n$ large so that the power $\alpha^n$ can lead to numerical problems. A numerically stable computation of $\tilde{R}_n(s)$ is derived as follows [3]. We first compute $\tilde{R}_K(s)$

\[ \lambda \tilde{R}_K(s) = \theta/x \left\{ x + \varrho - \varrho \frac{P_{K-1}(x)}{P_K(x)} \right\} \]

where $x = s/\lambda$ and $\varrho = \mu/\lambda$. From Eqn. (4.4)

\[ \frac{P_{n-1}(x)}{P_n(x)} = \frac{1}{F - \varrho \frac{P_{n-2}(x)}{P_{n-1}(x)}} \]

for $0 < n < K$ where $F = x + \varrho + 1$. From Eqn. (4.3), the recursion is terminated by

\[ \frac{P_0(x)}{P_1(x)} = \frac{1}{x + 1}. \]

We can now compute $\tilde{R}_n(s)$

\[ \tilde{R}_n(s) = \tilde{R}_K(s) \prod_{i=n+1}^{K} \frac{P_{i-1}(x)}{P_i(x)} \]

where $0 \leq n < K$. The successive terms are bounded $0 < \left| \frac{P_{i-1}(x)}{P_i(x)} \right| < 1$ where $|z|$ denotes the norm of the complex variable $z$.

In order to derive $R_n(t)$, we need to invert $\tilde{R}_n(s)$ which can be done using the Euler method presented in [14].

### 4.4 Numerical Examples

This section presents several examples of the computation of the lost revenue function $R_n(t)$. 
Figure 4.1: The $M/M/1/K$ lost revenue function $R_n(t)$ for $n = 0, \ldots, 6$, $K = 6$ and $\lambda = 1.5$.

4.4.1 The $M/M/1/6$ Queueing System

Fig 4.1(a) presents the lost revenue function $R_n(t)$ for a small $M/M/1/K$ queue where $K = 6$, $\theta = 1$, $\lambda = 1.5$, $\mu = 2$, $n = \{0, \ldots, 6\}$ and the planning horizon $t \in [0, 10]$. The function $R_0(t)$ is the lowest curve and the function $R_6(t)$ is the highest curve. We observe that $R_n(t)$ is increasing with $n$. We also observe that with increasing $t$, $R_n(t)$ is well-approximated by a linear function with a slope equal to $\theta \lambda P(\rho, K)$ where $\rho = \lambda / \mu \neq 1$ and

$$P(\rho, K) = \rho^K \frac{1 - \rho}{1 - \rho^{K+1}}$$

(4.12)

is the equilibrium probability that $K$ packets are in the system. The difference in the height of the linear part of the functions $R_{n+1}(t)$ and $R_n(t)$ reflects the difference in the
expected revenue incurred after equilibrium is reached when the system starts with \( n + 1 \) packets rather than \( n \) packets.

Fig. 4.1(a) presents the lost revenue function for a system with low blocking \( P(\rho, K) = 0.05 \). Fig. 4.1(b) presents the lost revenue function for a system with a larger blocking probability which is achieved by decreasing \( \mu \) to 1. The blocking probability \( P(\rho, K) = 0.35 \).

The load and hence the equilibrium slope of the curves, is greater in Fig. 4.1(b) than in Fig. 4.1(a). However the latter is still given by \( \theta \lambda P(\rho, K) \). The difference in the equilibrium heights of the function \( R_{n+1}(t) \) and \( R_n(t) \) does not vary as much between \( n = 0 \) and \( n = 5 \) as for the low blocking system. This reflects the fact that in the low blocking system, states with high occupancy are unlikely to be visited in the short term if the route does not start with a high occupancy. In the high blocking system, the probability of moving to states with high occupancy in the short term is relatively higher even if the starting state has a low occupancy [4].

### 4.4.2 The M/M/1/100 Queueing System

Fig. 4.2 presents the lost revenue function \( R_n(t) \) for a larger system with \( K = 100, \theta = 1, \lambda = 85, \mu = 80 \) and \( n = \{0, 25, 50, 75, 90, 100\} \). As for the smaller system, we observe that \( R_n(t) \) is increasing with \( n \) and we also observe that after the initial period in which the starting state has an effect, the \( R_n(t) \) increase linearly at the same rate. The \( R_n(t) \) increase with increasing \( n \), with a more pronounced increase as \( n \) becomes large.

### 4.5 The Price of Buffer Space

The expected lost revenue is transformed into a price at which \( u \) units of extra buffer space should be “bought” or “sold”. We assume that the network manager is making buying and selling decisions for a planning horizon of \( t \) time units, and that the choice of \( t \) is the decision of the network manager.

As in [4], once the manager has chosen \( t \), we regard the value of \( u \) extra units of buffer
Figure 4.2: The $M/M/1/K$ lost revenue function $R_n(t)$ for $n = \{0, 25, 50, 75, 90, 100\}$, $K = 100$, $\lambda = 85$ and $\mu = 80$.

space as the difference in the total expected lost revenue over the time interval $[0, t]$ if the system were to increase its buffer space by $u$ units at time zero. Conversely, we calculate the selling price of $u$ units of buffer space as the difference in the total expected lost revenue over time $[0, t]$ if the system were to decrease its buffer space by $u$ units at time zero.

The buying $B_n(t)$ and the selling $S_n(t)$ prices of buffer space when $n$ packets are present at the route (1 packet is in service, $n - 1$ packets are queued), the route waiting line has capacity $K - 1$, the mean packet service rate is $\mu$ and the planning horizon is $t$, can be written as

$$B_n(t) = R_{n,\mu,K}(t) - R_{n,\mu,K+u}(t)$$

$$S_n(t) = \begin{cases} R_{n,\mu,K-u}(t) - R_{n,\mu,K}(t) & 0 \leq n \leq K - u \\ R_{K-u,\mu,K-u}(t) - R_{n,\mu,K}(t) & K - u < n \leq K \end{cases}$$

where the extra subscripts in $R_{n,\mu,K}(t)$ indicates the initial bandwidth and the initial number of packets a link can maximally hold. We expect that for all $n$, $K$ and $t$, $S_n(t) > B_n(t)$. Some examples of $B_n(t)$ and $S_n(t)$ are given in the following section.
# The $M/M/1/7$ Queueing System

Fig. 4.3(a) and (b) present the buying (dotted lines) and selling (continuous lines) prices for a $M/M/1/K$ system with $K = 7$, $\lambda = 3$, $n \in \{3, 4, 5, 6\}$, $\theta = 1$, $u = 1$ in the case of low and high blocking where $\mu = 6$ and $\mu = 3$ respectively. Function $S_3(t)$ is the lowest continuous line and function $S_6(t)$ is the highest continuous line. Conversely, function $B_3(t)$ is the lowest dotted line and function $B_6(t)$ is the highest dotted line.

The figures show that the selling price $S_n(t)$ is greater than the buying price $B_n(t)$ for all $n$ and $t$. As $n$ approaches the capacity $K$ the system places a higher value on the available bandwidth for both the buying and the selling prices.

![Graph](image)

**Figure 4.3:** The $M/M/1/7$ buying and selling prices.
4.5.2 The $M/M/1/100$ Queueing System

Similar observations can be made for a larger system with $K = 100$, $\lambda = 85$, $u = 1$ and $\mu = 100$. The values of $B_n(t)$ and $S_n(t)$ for $n \in \{85, 90, 95, 99\}$ are given in Fig. 4.4. The buying prices remain smaller than the selling prices and both increase as the link is fully occupied.

Figure 4.4: The $M/M/1/100$ buying and selling prices for $n \in \{85, 90, 95, 99\}$, $K = 100$, $u = 1$, $\lambda = 95$ and $\mu = 100$. 
Chapter 5

A Distributed Scheme for Bandwidth Re-Configuration

5.1 Introduction

Arvidsson et al. [2] propose a bandwidth reconfiguration scheme that can be used in a telecommunication network that uses long-lived paths (such networks are referred to as “path-oriented networks”) to provision bandwidth for flows whose average holding times are less than the path lifetimes. The focus is on the improvement of the network’s rate of earning revenue. This is achieved by minimizing the call blocking probabilities on the path. Each path of the network is modeled as an Erlang loss model because the primary concern is efficient resource management in a connection-oriented network.

This chapter discusses a similar reallocation scheme where the paths of the network are modeled as $M/M/1/K$ queues. Our purpose is to set up a bandwidth reallocation scheme that can be used in a packet switched network environment. Our proposed reallocation scheme for a packet network is based on the model of Arvidsson et al. [2]. The main difference is that in addition to the connection level where capacity is moved based on [2], we also consider the packet level where buffer space is moved at the originating nodes of paths based on the packet traffic dynamics and the buffer space prices. The idea is that a user requests a data connection between peer nodes to carry data packets. The data packets are transferred from the source node to the destination node through a long-lived path which acts as a single server with a buffer of finite size at the ingress of the path. For
conformity reasons we adopt the same terminology as in [2]: that is we call a long-lived path a route. Routes can traverse one or more physical links.

A route is characterized by its interface capacity and its effective capacity. The interface capacity represents the number of calls that can be connected simultaneously on that route. The effective capacity consists of the bandwidth allocated (service rate) and the buffer space at the ingress of the route. The effective capacity determines the number of data packets (in service and queued) that can be carried simultaneously on the route. However, if at any point of time only a fraction of capacity (interface or effective) allocated to a route is utilized while the capacity of another route is nearly fully utilized, it may be advantageous, if it is possible, to move the unused capacity from the under-utilized route to the over-utilized route.

We considered a scheme in which each route places a value on capacity dependent on its current capacity and its current occupancy. Capacity is then transferred from routes that place a low value on capacity to routes that place a high value on capacity. Under this scheme, bandwidth managers are assigned to each route. The managers use the knowledge of routes’ current occupancy at the packet level and the routes’ current occupancy at the call level to calculate the value of an extra unit of buffer space (the buffer’s “buying price”) and the value of an extra unit of interface capacity (the interface’s “buying price”) respectively, and also the value that the route would lose should it give up a unit of buffer space (the buffer’s “selling price”) or a unit of the interface capacity (the interface’s “selling price”). The managers then use these prices to determine whether the route should acquire or release a unit of buffer space and/or a unit of interface capacity or do neither.

We view two types of route: a direct route which uses a single physical link and a transit route which uses more than one physical link. We assume that each link supports one direct route. The direct routes on the links of a transit route are referred to as its constituent direct routes; and the constituent direct route attached to the originating node of the transit route is referred to as its first constituent direct route. Bandwidth reallocation is driven by the managers of transit routes and takes place between the transit routes and their constituent direct routes. In this way the managers are autonomous and behave entirely according to local rules [2].

Buying and selling prices are communicated via an in-band signalling mechanism. Specifi-
cally, signals or control packets are sent at random time intervals along each route, recording the buying and the selling prices of the constituent direct routes.

If the transit route buying price of effective capacity is greater than the first constituent direct route selling price, then the transit route acquires a unit of buffer space from its first constituent direct route. Alternatively if the transit route selling price of effective capacity is less than the first constituent direct route buying price, then the transit route releases a unit of buffer space to its first constituent direct route.

On the other hand, if the transit route buying price of interface capacity is greater than the sum of the constituent direct routes selling prices, then the transit route acquires a unit of interface capacity from its constituent direct routes. Alternatively if the transit route selling price of interface capacity is less than the sum of the constituent direct routes buying prices, then the transit route releases a unit of interface capacity to its constituent direct routes.

Such a scheme is expected to reduce the blocking probability at the call and the packet level along each route of the logical network which will increase the average rate of earning revenue. The next sections review the prices of buffer space and our bandwidth reallocation scheme.

## 5.2 The Price of Bandwidth

We will consider the Erlang Price [4] and the $M/M/1/K$ price functions presented in Chapter 4. The Erlang prices will be used to reallocate route interface capacity while the $M/M/1/K$ prices will be used to reallocate route buffer space. We assume that a route manager is making decisions for a planning horizon of $t$ time units. For a route $r$ with bandwidth $\mu_r$ and buffer of size $K_r - 1$, let $R_{n_r, \mu_r, K_r}(t)$ denote the expected revenue lost in the interval $[0, t]$, given that there are $n_r$ packets at time 0. Then the buying and the selling prices $B^{(r)}_{n_r, \mu_r, K_r}(t, u)$ and $S^{(r)}_{n_r, \mu_r, K_r}(t, u)$ of $u$ units of buffer space when the initial state is $n_r$, the current buffer space is $K_r - 1$ and the service rate is $\mu_r$ are given by
\[ B_{n_r, \mu_r, K_r}(t, u) = R_{n_r, \mu_r, K_r}(t) - R_{n_r, \mu_r, K_r+u}(t) \]  
(5.1)

\[ S_{n_r, \mu_r, K_r}(t, u) = \begin{cases} 
R_{n_r, \mu_r, K_r-u}(t) - R_{n_r, \mu_r, K_r}(t) & 0 \leq n_r \leq K_r - u \\
R_{K_r-u, \mu_r, K_r-u}(t) - R_{n_r, \mu_r, K_r}(t) & K_r - u < n_r \leq K_r 
\end{cases} \]  
(5.2)

For all \( n_r, \mu_r \) and \( K_r \), the function \( R_{n_r, \mu_r, K_r}(t) \) is a concave function of \( t \). It is defined only for integer values of \( K_r \), but for all \( n_r \) and \( t \) is a strictly convex function of \( K_r \) in the sense that, for all \( u \),

\[ B_{n_r, \mu_r, K_r}(t, u) < S_{n_r, \mu_r, K_r}(t, u). \]  
(5.3)

## 5.3 A Distributed Bandwidth Reallocation Scheme

### 5.3.1 The Logical Network

We formulate a physical network as a set of nodes and links \((\mathcal{N}, \mathcal{L})\), where link \( \ell \in \mathcal{L} \) has a total transmission rate of \( b_\ell \) bits/sec for packets and an interface of capacity \( B_\ell \) for calls. The physical network supports a set \( \mathcal{R} \) of routes which form the overlay logical network. Node \( o_r \) sends traffic to a destination node \( d_r \) along a fixed route \( r \in \mathcal{R} \).

To provision route \( r \), bandwidth is reserved on the path \( \mathcal{L}_r = \{\ell_1, \ell_2, \ldots, \ell_{k_r}\} \) of physical links that connect \( o_r \) to \( d_r \). If \( o_r \) and \( d_r \) are directly connected by one physical link, this single physical link is used to provision bandwidth to route \( r \), in which case \( k_r = 1 \). Such a route is called a direct route. If nodes \( o_r \) and \( d_r \) are connected via more than one physical link then \( k_r > 1 \) in which case the originating node of \( \ell_1 \) is \( o_r \) and the terminating node of \( \ell_{k_r} \) is \( d_r \). Such a route is called a transit route. We assume that each physical link supports a constituent direct route and denote the set of routes, both direct and transit routes, that pass through link \( \ell \) by

\[ \mathcal{A}_\ell = \{r : \ell \in \mathcal{L}_r\}. \]

For a transit route \( r \), let \( \mathcal{D}_r \) be the set of direct routes that traverse the single links \( \ell \in \mathcal{L}_r \). The routes in \( \mathcal{D}_r \) are called the constituent direct routes corresponding to the transit route.
Each route $r$ is allocated a rate of $\mu_r$ bits/sec such that the physical bandwidth constraints are satisfied. Thus for $\ell \in \mathcal{L}$, we have,

$$\sum_{r \in \mathcal{A}_\ell} \mu_r = b_\ell$$

(5.4)

At the ingress of route $r$ a buffer of space $K_r - 1$ is provisioned. Thus route $r$ can hold a maximum number of $K_r$ packets.

At the call level, route $r$ is allocated an interface capacity $C_r$ such that the physical constraints

$$\sum_{r \in \mathcal{A}_\ell} C_r = B_\ell$$

(5.5)

are satisfied.

Fig. 5.1 illustrates a simple physical network with four nodes $O_1, O_2, O_3$ and $O_4$ and three physical links $\ell_1, \ell_2$ and $\ell_3$. The physical network is overlaid by three direct routes $r_1, r_2$ and $r_3$; and two transit routes $r_4$ and $r_5$. The direct route $r_1$ connects nodes 1 and 2 and has an associated service rate of $\mu_1$ bits/sec and an interface capacity of $C_1$ provisioned by the physical link $\ell_1$. The direct route $r_2$ connects nodes 2 and 3 and has an associated service rate of $\mu_2$ bits/sec and an interface capacity of $C_2$ provisioned by the physical link $\ell_2$. The direct route $r_3$ with a service rate of $\mu_3$ bits/sec and an interface capacity of $C_3$ connects nodes 3 and 4, and it is provisioned by link $\ell_3$. Nodes 1 and 3 are connected by the transit route $r_4$ with an associated service rate of $\mu_4$ bits/sec and an interface capacity of $C_4$ provisioned by links $\ell_1$ and $\ell_2$. Finally nodes 2 and 4 are connected by the transit route $r_5$ with a bandwidth of $\mu_5$ bits/sec and an interface capacity of $C_5$ provisioned by links $\ell_2$ and $\ell_3$. Thus $\mathcal{A}_1 = \{r_1, r_4\}$, $\mathcal{A}_2 = \{r_2, r_4, r_5\}$ and $\mathcal{A}_3 = \{r_3, r_5\}$. The physical bandwidth constraints are given by $b_1 \geq \mu_1 + \mu_4$, $b_2 \geq \mu_2 + \mu_4 + \mu_5$ and $b_3 \geq \mu_3 + \mu_5$ at the packet level; and by $B_1 \geq C_1 + C_4$, $B_2 \geq C_2 + C_4 + C_5$ and $B_3 \geq C_3 + C_5$ at the call level. Each route can queue $K_r - 1, (r \in \{1, 2, 3, 4, 5\})$ packets while one packet is being transmitted. The transit route $r_4$ can buy or sell interface capacity from or to the direct routes $r_1$ and $r_2$; the transit route $r_5$ can buy and sell interface capacity from and to the direct routes $r_2$ and $r_3$. The transit route $r_4$ can buy or sell buffer space from or to the
direct route $r_1$; the transit route $r_5$ can buy and sell buffer space from and to the direct route $r_2$.

![Transit routes and direct routes.](image1)

**Figure 5.1:** Transit routes and direct routes.

### 5.3.2 Bandwidth Reallocation

In our bandwidth reallocation model, we consider the notion of interface capacity and effective capacity. The interface capacity $C_r$ of route $r$ represents the capacity on the connection interface attached to node $o_r$ for users requesting data connections on route $r$. On the other hand, the effective capacity represents the capacity (the service rate $\mu_r$ and the buffer size $K_r - 1$) allocated to route $r$ to process data packets. These two notions are illustrated in Fig. 5.2.

![Data connections and data packets.](image2)

**Figure 5.2:** Data connections and data packets.

Node $o_r$ receives on its interface requests for data connections along route $r$ in a Poisson process of rate $x_r$. The interface at node $o_r$ is constrained in capacity and can accommodate maximally $C_r$ connections on route $r$. If the current number $c_r$ of connections is less than
Chapter 5. A Distributed Bandwidth Reallocation Scheme

$C_r$ then the request is accepted and $c_r$ is increased by one. Otherwise the request is rejected. Successful requests hold the interface capacity for an average time which is exponentially distributed with mean $1/y_r$ [2].

The successful requests on node $o_r$ are pooled together to form a combined stream that offers packets to route $r$ at rate $\lambda_r$. Route $r$ behaves as a single server with a buffer of finite size $K_r - 1$. Packets arrive to route $r$ in a Poisson process and compete for service in a "first-come-first-served" manner. In practice individual packets do not arrive to an IP network in a Poisson process, however the aggregated arrival of the packets from many calls follows an approximate Poisson process.

Once a packet arrives, it is transmitted if the current number $n_r$ of packets in service on route $r$ is zero; else the packet is queued if $n_r$ is less than $K_r$ or it is lost otherwise. An admitted packet holds the bandwidth for a time which is exponentially distributed with mean $1/\mu_r$. The buying and selling prices $B_{n_r,\mu_r,K_r}(t,u)$ and $S_{n_r,\mu_r,K_r}(t,u)$ of effective capacity on route $r$ will thus vary over time because both the number $c_r$ of connections and the number $n_r$ of packets varies over time. Likewise, the buying and the selling prices of interface capacity will also vary due to the change in $c_r$. Therefore there will likely be periods when it will be advantageous to reallocate capacity (buffer space and interface capacity) among routes.

Bandwidth reallocation takes place between transit routes and their constituent direct routes. If only such "local" transfers are permitted, we avoid the need to consider the potentially widespread implications of a particular reallocation [2].

At fixed periods of time, the manager of a transit route $r$ makes a comparison between the buying price of effective capacity on its route and the selling price of the first constituent direct route $\ell$ of route $r$. The manager also compares the buying price of the first constituent direct route and the selling price of effective capacity on its route. If,

$$B_{n_r,\mu_r,K_r}(t_r,u) > S_{n_\ell,\mu_\ell,K_\ell}(t_r,u) \quad (5.6)$$

then the transit route acquires $u$ units of buffer space from the first constituent direct route. Else, if
\[ S_{n_r, \mu_r, K_r}^{(r)}(t_r, u) < B_{n_t, \mu_t, K_{\ell}}^{(\ell)}(t_r, u) \]

then the transit route releases \( u \) units of buffer space to the first constituent direct route. Otherwise no reallocation takes place.

Inequalities (5.6) and (5.7) cannot simultaneously be satisfied (see Eqn. (5.3) ).

Note, a reallocation scheme is also applied on the connection level using the Erlang prices and based on the Arvidsson et al. [2] model.

The algorithm in Fig. 5.3 depicts one way to implement the reallocation scheme in a network at the packet level. The implementation at the call level is described in [2].

- At specified time points, a transit route \( r \) sends out a control packet that records two pieces of information
  
  \[
  \begin{align*}
  \text{ACQUIRE} & := B_{n_r, \mu_r, K_r}^{(r)}(t_r, u), \\
  \text{RELEASE} & := S_{n_r, \mu_r, K_r}^{(r)}(t_r, u).
  \end{align*}
  \]

- At the first constituent direct route \( \ell \) of route \( r \), the information in the control packet is modified according to
  
  \[
  \begin{align*}
  \text{ACQUIRE} &= B_{n_r, \mu_r, K_r}^{(r)}(t_r, u) - S_{n_t, \mu_t, K_{\ell}}^{(\ell)}(t_r, u), \\
  \text{RELEASE} &= S_{n_r, \mu_r, K_r}^{(r)}(t_r, u) - B_{n_t, \mu_t, K_{\ell}}^{(\ell)}(t_r, u).
  \end{align*}
  \]

  A check of the information is performed and

  - If \( \text{ACQUIRE} > 0 \), then \( K_r := K_r + u \) and the first constituent direct route \( \ell \) performs \( K_{\ell} := K_{\ell} - u \).
  
  - If \( \text{RELEASE} < 0 \), then \( K_r := K_r - u \) and the first constituent direct route \( \ell \) performs \( K_{\ell} := K_{\ell} + u \).
  
  - Otherwise no change occurs.

Figure 5.3: An Algorithm to implement the reallocation scheme.
5.3.3 Scalability, Local Information, Global Reach

The bandwidth reallocation scheme presented above is distributed and scalable in the sense that the decision of performing a logical network capacity reconfiguration uses only local information that the transit route possesses. The local information for transit route $r$ consists of its interface capacity buying/selling prices, its effective capacity buying/selling prices and also the interface capacity buying/selling prices and the effective capacity buying/selling prices of its constituent direct routes. Each interface capacity reconfiguration is processed between a transit route and its constituent direct routes. However once the decision is made, the interface capacity reconfiguration processed on the constituent direct routes of the transit route $r$ will affect the future decisions of the transit routes that share the same physical links with transit route $r$. For example, if a transit route $r$ acquires/releases interface capacity, then the constituent direct routes $s \in D_r$ will release/acquire interface capacity. The bandwidth prices for each direct route $s \in D_r$ will therefore change. Consider a direct route $s$ on link $\ell$ whose interface capacity price has changed. This price change has a knock-on effect in the sense that the transit route $r \in A_r$ which passes through link $\ell$ may now decide, given the changed price of interface capacity on direct route $s$, to either acquire or release interface capacity, and this in turn may lead to further knock-on effects. These knock-on effects at the call level have a direct implication at the packet level since the packet traffic dynamic is affected by calls [2].

Finally, the reallocation scheme based on the local information has an implication on the whole logical network and affects non-local variables. As an illustration [2], we consider the logical network in Fig. 5.1, if transit route $r_4$ acquires interface capacity then its constituent direct route $r_1$ and $r_2$ will release interface capacity. The price of interface capacity on $r_1$ and $r_2$ may increase, depending on the current value $n_1$ and $n_2$ of the number of calls that flow on routes $r_1$ and $r_2$. Suppose the price of interface capacity on $r_2$ increases. This may induce the transit route $r_5$ to released interface capacity. Thus although the transit routes $r_4$ and $r_5$ do not communicate directly with each other, they nonetheless influence each other via interface capacity changes to the constituent direct route $r_2$ which they share. Each time that there is a change at the connection level, the buffer prices of a transit route and that of its first constituent direct route will also change. This may result in further knock-on effects at the packet level.
5.3.4 Suitable Values for the Bandwidth Reallocation Parameters

The bandwidth reallocation scheme presented above depends on certain network parameters whose values must be specified. For example we need to decide how often the managers of transit routes send out control packets, to specify the planning horizons used in the calculation of buying and selling prices and to determine the amount of interface or buffer capacity to be transferred in each reallocation in order to achieve improved performance.

Assume that the managers of transit routes are sending out control packets too frequently. This may result in a lot of unsuccessful interface capacity or buffer space reallocation since the occupancy will not change after the previous reallocation. Alternatively if control packets are sent out too infrequently many opportunities for reallocation may be missed. It is important to choose the reallocation rates to balance these competing objectives. Significantly, Arvidsson et al. [2] proposed setting the reallocation rate as a function of the data connection arrival rate, i.e., if \( \eta_r \) denotes the signalling rate on transit route \( r \), \( \eta_r \) is assumed to be proportional to the data connection arrival rate \( x_r \) on route \( r \). Thus,

\[
\eta_r = V x_r
\]  

(5.8)

where \( V \in [0,10] \) is the signalling ratio. If \( V = 0 \) then no reallocation takes place.

We also work [2] with per-route planning horizons \( \tau_r \) that are assumed to be a multiple of the average reallocation interval \( 1/\eta_r \). Thus

\[
\tau_r = P/\eta_r
\]  

(5.9)

where \( P \) is the planning ratio.

The unit \( u \) of buffer space or the unit \( U \) of interface capacity to be traded must be reasonable. If a small unit of buffer space or a small unit of interface capacity is traded, then many trades are required to meet the route’s capacity demands. Also if a large unit of buffer space or interface capacity is traded the buffer distribution or the interface capacity distribution will be coarser. These situations must be avoided.
The way the planning horizons, the reallocation ratio and the buffer size unit values are set up is explained in Chapter 6.
Chapter 6

The Simulation Model

6.1 Introduction

The telecommunication network simulation presented in this thesis is based on the model described in [2], implemented in C++ (the “TRADE Simulator”). The model in [2] contains two basic events: the connection event and the signalling event. The connection event corresponds to connection arrivals and connection completions. The signalling event corresponds to the signalling activities that record prices and reallocate bandwidth [2]. This model was extended to simulate a packet switching network. The packet model is obtained by assuming that each connection offers on average a fixed number of packets to a route of the network; the route is modeled as a single server with a finite buffer size. Two new components namely source and packet are added to TRADE simulator.

This chapter reviews the TRADE simulator and explains how TRADE was extended to simulate packet processing. The chapter also explains the determination of some of the simulation parameter values of the packet model. Most of the work presents here is based on [2].
6.2 The Model Entities

The simulator maintains a schedule of events which are arranged in chronological order. The events represent the next connection to arrive to each route, the next packet to be served, the packets that are queued in the server buffer, the packet that will complete service at the server, the connections that will complete on each route and the next signalling event on each transit route. The current event is modeled by de-queuing and executing the event from the head of the schedule.

A connection arrival is modeled as follows. If the route has insufficient interface capacity the connection is rejected and a counter of lost connections is incremented; else the connection completion time is sampled and the connection completion event is scheduled into the calendar. For each accepted connection, the packet inter-arrival time is computed and a source event is created and scheduled into the calendar. The arrival time of the next connection to this route is computed and the next connection arrival event is scheduled. A connection completion event updates various counters in the simulator.

The source event generates packets at Poisson intervals. If the server is busy and its queue is full, the arriving packet is lost and a counter of lost packets is incremented; else if the server is not busy, the packet is put into service, the completion time is randomly generated and the packet completion event is scheduled into the calendar; otherwise the packet is put into the queue. The arrival time of the next packet to this route is computed; the next source event is scheduled into the calendar if the packet arrival time is less than the connection completion time; otherwise the connection has completed and the source event is removed from the calendar. Various counters are updated as required. All random variables used by the packet simulator are sampled from exponential distributions.

Bandwidth reallocation is driven by the managers of transit routes. The manager of transit route $r$ sends individual control packets along its forward route from the ingress router $o_r$ to the egress router $d_r$ at the instants of a Poisson process with rate $\eta_r$. Prices are computed as the control packet moves from link to link along the forward route. After each price calculation, a link delay is sampled and the signalling event on the next link on the forward route is scheduled into the calendar.
When the control packet reaches the egress router, a calculation is performed to determine if the transit route $r$ can profitably acquire $u$ units of buffer space from its first constituent direct route and/or $U$ units of the interface capacity from its constituent direct routes. Or, if Eqn. (5.7) is satisfied at a packet level or its equivalent in a call level (see [2]), then transit route $r$ releases $u$ units of buffer space to its first constituent direct route and/or $U$ units of interface capacity to its constituent direct routes. In practice, we require that Eqs. (5.6) and (5.7), and their equivalent at a call level (see [2]) be satisfied by some small margin $\epsilon$ which is set arbitrary to $10^{-6}$ to reduce the possibility of oscillatory behavior.

The control packet is returned along the reverse route from the egress router to the ingress router making the necessary reallocations to the interface capacities of the constituent direct routes. After each reallocation, a link delay is sampled and the signalling event on the next link on the reverse route is scheduled into the calendar. The buffer space of the transit route and the buffer space of the first constituent direct route are both adjusted when the control packet reaches the egress router, and the interface capacity of the transit route is adjusted when the control packet reaches the ingress router. The next signalling event is scheduled into the calendar.

6.3 The Efficient Calculation of Bandwidth Prices

The calculation of the bandwidth prices involves the inversion of a Laplace transform which is computationally intensive. The simulator therefore maintains caches, one for each route, of previously calculated bandwidth prices. When computing the price of bandwidth on a route, the simulator first consults the route cache. If the required price is in the cache—a cache hit—the price is returned immediately. If the desired price is not in the cache—a cache miss—the price is computed and stored in the cache for future reference.

The size $T_r$ of the route $r$ cache is set so that routes with large offered loads are assigned large caches. Thus $T_r$ is set equal to the next prime larger than $T \max(1, \lceil \rho_r/\rho \rceil)$ where the average load $\rho = \sum_{r \in R} \rho_r/|R|$ and $\rho_r = \lambda_r/\mu_r$ and $T = 40000$. Each route cache can thus store at least $T = 40000$ prices.

Note that the problem of efficient computation of bandwidth prices exists not only in the
TRADE simulator but also in any real implementation. Caching as described above may be worth employing in such an implementation.

6.4 Determination of the Parameter Values

6.4.1 Parameterizing the Network Model

This section describes a model of a fictitious 30-node 46-link network based on the geography of Europe. This network will be used to test the efficacy of the proposed bandwidth reallocation scheme. The network, which is illustrated in Fig.(6.1), is constructed as follows.

First, the number of networks nodes (cities) in each country is determined by the population of the country. Next the links, which are bi-directional, are placed according to the designs of several European rings. This results in a network with connectivity $2|\mathcal{L}|/(|\mathcal{N}|(|\mathcal{N}| - 1)) = 0.105$. Each $O - D$ pair is connected by a single shortest-path (minimum geometric
distance) route. The average route length of the network model is \( \sum_{r=1}^{|\mathcal{R}|} |L_r|/|\mathcal{R}| = 4.7 \) hops [2].

Consider subscribers in a city \( i \) of population \( M_i \). Let \( Q_{i,j} = M_j / \sum_{k \in \mathcal{N} - \{i\}} M_k \) denote the fraction of the calls originating in city \( i \) that are destined for city \( j \) where \( j \neq i \). We assume that a busy hour subscriber generates calls sampled from a normal distribution truncated at \( O \) with mean \( a \) erlangs and standard deviation \( b \) erlangs. The bi-directional offered load between cities \( i \) and \( j \) is sampled from a normal distribution with mean \( a(M_iQ_{i,j} + M_jQ_{j,i}) \) and variance \( b^2(M_iQ_{i,j} + M_jQ_{j,i}) \). We choose \( a = 0.001 \) erlangs and \( b = 0.0005 \) erlangs [2].

Having estimated the offered loads to each route, the inverse of Erlang’s formula is used to compute the route capacities so that, without bandwidth reallocation, each route has a loss probability of (2%). The capacity \( C_\ell \) of link \( \ell \) is computed according to Eqn. 5.5. This procedure yields a network model that is dimensioned in that it has neither a large oversupply nor a large undersupply of physical capacity on any of its links. The route length distribution for the network model [2] is given in Table. 6.5.

Table 6.5: The route length distribution for the network model

<table>
<thead>
<tr>
<th>length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>all</th>
</tr>
</thead>
<tbody>
<tr>
<td>routes</td>
<td>46</td>
<td>72</td>
<td>87</td>
<td>88</td>
<td>69</td>
<td>43</td>
<td>21</td>
<td>7</td>
<td>2</td>
<td>435</td>
</tr>
</tbody>
</table>

The above method of assigning values to the parameters yields a network model where the largest link is Paris-London with 18663 units of capacities carrying 44 routes. The smallest link is Oslo-Stockholm with 40 units of capacities carrying 2 routes. The average link capacity is 6835 units. The link that carries the most routes is Prague-Berlin with 82 routes. The links Manchester-Dublin and Manchester-Birmingham carry only 1 route each. Each link carries an average of 35 routes [2].

Table (6.5) shows the route length distribution. Table (6.6) lists the top ten links showing the link capacities and the number of routes carried by each link [2].

The packet model parameters are derived for the European network as follows. First, the interface bandwidth \( r_c \) for data connections on route \( r \) is set equal to the route \( r \) capacity. Second, we dimensioned each server \( s \) on the packet model with a service rate \( \mu_s \) and
a queue of length $K_r - 1$. The service rate $\mu_s$ allocated to server $s$ is derived from the interface bandwidth $r_c$. We assumed that each circuit of the model is a $T1$ line. A $T1$ line can carry data at a rate of 1.544 Megabits per second. Thus for a route $r$ with $r_c$ capacity, if $MBPS$ represents $10^6$ bits and $P_{usr}$ the packet size, then $\mu_s$ is given by

$$\mu_s = rc(T1)(MBPS)/8.0/P_{usr}.$$ 

This quantity expresses the number of packets the server $s$ can process per unit time.

The packet arrival rate on route $r$ is set equal to the connection arrival rate on route $r$ times the average number of packets generated per connection. This value over-estimates the packet arrival rate, since some connections are rejected due to insufficient interface bandwidth. We therefore reduce the connection arrival rate by the rate of lost connections due to blocking, and the resulting value is multiplied by the average number of packets generated per connection.

### 6.4.2 Signalling Overhead

The reallocation scheme is mediated by a signalling mechanism. The impact of signalling on the Grade of Service (GOS) is determined by the amount of bandwidth used to transmit the control packets and also by the delays experienced by the control packets which convey the signalling information. The bandwidth used by the control packets is subtracted from the interface bandwidth and the remainder is used to transport subscriber calls. Excessive

<table>
<thead>
<tr>
<th>Link</th>
<th>Capacity</th>
<th>Routes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris-London</td>
<td>18663</td>
<td>44</td>
</tr>
<tr>
<td>Berlin-Hamburg</td>
<td>15265</td>
<td>56</td>
</tr>
<tr>
<td>Berlin-Prague</td>
<td>14003</td>
<td>82</td>
</tr>
<tr>
<td>Milan-Zurich</td>
<td>13975</td>
<td>64</td>
</tr>
<tr>
<td>Paris-Ruhr</td>
<td>13928</td>
<td>39</td>
</tr>
<tr>
<td>Paris-Lyon</td>
<td>13892</td>
<td>42</td>
</tr>
<tr>
<td>Lyon-Zurich</td>
<td>13645</td>
<td>53</td>
</tr>
<tr>
<td>Hamburg-London</td>
<td>13566</td>
<td>60</td>
</tr>
<tr>
<td>Vienna-Prague</td>
<td>12970</td>
<td>80</td>
</tr>
<tr>
<td>Hamburg-Ruhr</td>
<td>10118</td>
<td>57</td>
</tr>
</tbody>
</table>
signalling delays will cause the reallocation scheme to work with out-of-date price information which might give rise to inappropriate bandwidth reallocation and misconfigured routes.

### 6.4.3 Confidence Intervals

Confidence intervals for the expected number of lost packets and calls were determined from 50 independent replications of the simulation. Each replication was started from the same initial conditions and terminated when $10^7$ data connections have been completed. The independence of the replications was accomplished by using different seeds to initialize the random number generators in each replication. The initialization bias was reduced by starting each simulation with $\lfloor n_r \rfloor$ data connections in progress on each route $r$ where $n_r$ is the average occupancy for an $M/M/C_r/C_r$ Erlang loss system with capacity $C_r$ [2].

Let $X_i$ denote the number of lost packets or calls recorded during the $ith$ replication of the simulation. Since the replications are independent, $X_1, X_2, \ldots, X_I$ are independent identically distributed random variables. Denote their mean by $\omega$ and variance by $\sigma^2$. Then

$$\bar{X}(I) = \frac{1}{I} \sum_{i=1}^{I} X_i$$

is an unbiased estimator of $\omega$ and

$$S^2(I) = \frac{1}{I-1} \sum_{i=1}^{I} (X_i - \bar{X}(I))^2$$

is an unbiased estimator of $\sigma^2$. The $1 - \alpha$ confidence interval half-length for $\omega$ is given by

$$\delta(I, \alpha) = t_{I-1,1-\alpha/2} S(I)/\sqrt{I}$$

where $0 < \alpha < 1$ and $t_{I-1,1-\alpha/2}$ is the $1-\alpha/2$ critical point of the $t-$distribution with $I-1$ degrees of freedom.

Let $\epsilon(I) = |\bar{X}(I) - \omega|/\omega$ denote the relative error in the estimate of $\omega$. The estimate
\( \bar{X}(I) \) will have a relative error of at most \( \gamma = \delta(I, \alpha) / \bar{X}(I) - \delta(I, \alpha) \) with a probability of approximately \( 1 - \alpha \) [1].

![Graph showing the European network model: 95% confidence intervals and relative errors for the expected number of lost calls/packets as a function of the number \( I \) of independent replications of the simulation.](image)

Fig. 6.2 shows the estimates \( \bar{X}(I) \), the confidence intervals and the relative errors \( \gamma(I) \) as function of the number \( I \) of replications of the simulation with \( V = 0.1, P = 1, u = 5 \) and \( U = 4 \). The choice of these parameter values is motivated in the next section.

After 50 replications of the simulation the average number of lost packets was \( \bar{X}(50) = 2947 \). The 95% confidence interval half-length \( \delta(50, 0.05) = 128.8 \) and the relative error \( \gamma = 1.8\% \). And the average number of lost calls was \( \bar{X}(50) = 57438 \). The 95% confidence
interval half-length $\delta(50, 0.05) = 347.8$ and the relative error $\gamma = 0.6\%$.

The simulation had to be replicated three times to reduce the relative error to less than 7% for packets and less than 5% for calls; and ten times to reduce the relative error to less than 3% for packets and less than 1% for calls. Each simulation experiment presented in this thesis was therefore replicated fifteen times to reduce the relative error on the average number of lost packets to a few percent. Each replication processed $10^7$ connection completions.

6.5 Assigning Suitable Values to the Simulation Parameters

The simulation model is parameterized by the values of the planning ratio $P$, the signalling ratio $V$, the reallocation units $u$ and $U$ and the buffer space.

6.5.1 The Planning Ratio $P$ and the Signalling Ratio $V$

The first issues to be investigated are the choice of suitable values for the planning ratio $P$ and the signalling ratio $V$ when computing the values of the per-route planning intervals $\tau_r$ and the per-route signalling rates $\eta_r$. In particular we are interested in how frequently bandwidth reallocation must be attempted in order to make a significant improvement to the performance of the network.

Fig. 6.3(b) plots the average number of lost packets versus the planning horizon $P$ and the signalling ratio $V$. The buffer space reallocation unit $u$ and the interface reallocation space $U$ are set to 5 and 4 respectively. The choice of these values is explained below. The scales on the $V$ and $P$ axes are logarithmic. Recall that the network is configured so that some 2% of the $10^7$ offered connections and some $75.10^{-5}\%$ of the $8.10^8$ packets offered (each call brings on average 80 packets) are lost when no reallocation are attempted. The Figure shows that

- the number of lost packets/calls initially decreases as the signalling ratio $V$ increases until it reaches a minimum in the region $1 \leq V \leq 10$, and
Figure 6.3: The European network model: the number of lost connections/packets vs the planning ratio $L$ and the signalling ratio $V$. 
• the number of lost calls decreases as the planning ratio $P$ decreases; while the number of lost packets decreases in the region $0 \leq V \leq 1$ and increases a bit in the region $1 \leq V \leq 10$ as the planning ratio decreases. However we set the planning ratio $P$ to 1, to obtain at the call level the performance obtained by Arvidsson et al. [2].

6.5.2 The Reallocation Units

In [2], Arvidsson et al. investigated the effect of the bandwidth unit $U$ on the average number of lost connections with and without signalling overheads. They found that the reallocation scheme gives the best result when the bandwidth is reallocated with a unit of 4. We therefore use the same unit to reallocate bandwidth in a connection level. At the packet level we consider an arbitrary value of 5 units to move buffer space between routes.

6.5.3 The Server Queue Size $K$

There are some empirical rules for the choice of the server buffer size. The most well known is the bandwidth delay product. This method sets the buffer length as the product of the average Round Trip Time (RTT) in the network and the capacity of the link. However other investigations [20, 12] have shown the drawback of this method and have proposed [20] other ways of computing the buffer size. To keep our experiment simple, none of these methods are considered in our implementation. In our experiment we initially used an arbitrary value of the buffer length such that the proportion of lost packets for the network is less than a small value $\alpha \in [10^{-5}, 10^{-7}]$. Thus we initially set $K = 81$. 
Chapter 7

Experiment and Evaluation

The evaluation of the bandwidth reallocation scheme is done using the simulator presented in Chapter 6. The simulator is applied to the network model presented in Fig. 6.1. The model in Fig. 6.1 represents a fictitious 30-node and 46-link network based on the geography of the Europe. This network was chosen in order to test the impact of the bandwidth reallocation scheme on the system’s overall rate of earning revenue.

The experiment was performed using the parameter values presented in chapter 6. Each simulation was replicated 15 times, each replication used a different random number seed.

The results show that bandwidth reallocation substantially reduces the route loss probability (see Fig. 7.1) and the lost revenue (see Fig. 7.2) for both the connection level and the packet level. For example Fig. 6.3(a) shows that the bandwidth reallocation reduces the average network connection loss probability by a factor of 3 while the average packet loss probability (see Fig. 6.3(b)) is reduced by a factor of 1.7. Explicitly, some 2% of the offered calls and $75.10^{-5}\%$ of the offered packets were lost without the reallocation (Fig. 6.3(a) and (b) $V = 0$ and $P = 1$). After the bandwidth reallocation the loss of calls is reduce to less than 0.7% and the loss of packers to less than 44$10^{-5}\%$ (Fig. 6.3(a) and (b) $V = 10$ and $P = 1$).

We conclude that bandwidth reallocation can be effective in reducing both the proportion of lost connections and the proportion of lost packets. The immediate effect of this reduction is the improvement of the network rate of earning revenue at the connection level and the
Figure 7.1: The European network model: the probability of lost connections/packets vs the route length $L$ and the signalling ratio $V$. 
We also investigated the effect of the bandwidth reallocation as a function of the route length. Let \( N_{r,c} \) and \( L_{r,c} \) denote the number of offered connections and the number of lost connections on route \( r \). We assumed that revenue earned per unit of time on route \( r \) is proportional to the length of the route. Hence the revenue lost on route \( r \) per unit of time per accepted connection is \( R_{r,c} = \theta_{r,c} L_{r,c} / y_r \) where \( \theta_{r,c} = |L_r| \theta_c \) and \( \theta_c \) is the revenue earned per unit of time and per connection on a single link.

Likewise, let denote by \( N_{r,p} \) and \( L_{r,p} \) the number of offered packets and the number of lost packets on route \( r \) respectively. Therefore, the revenue lost per unit time per accepted packet is \( R_{r,p} = \theta_{r,p} L_{r,p} / \mu_r \) where \( \theta_{r,p} = |L_r| \theta_p \) and \( \theta_p \) (not necessarily equal to \( \theta_c \)) denote the revenue earned per unit of time and per packet on a single link.

Let \( R_m = \{ R : |L_r| = m \} \) denote the set of routes of length \( m \) and \( \overline{N}_c(m) = \sum_{r \in R_m} N_{r,c} \), \( \overline{L}_c(m) = \sum_{r \in R_m} L_{r,c} \) and \( \overline{R}_c(m) = \sum_{r \in R_m} R_{r,c} \) denote the total number of offered connections, the total number of lost connections and the total connection revenue lost on routes of length \( m \) respectively. Likewise let \( \overline{N}_p(m) = \sum_{r \in R_m} N_{r,p} \), \( \overline{L}_p(m) = \sum_{r \in R_m} L_{r,p} \) and \( \overline{R}_p(m) = \sum_{r \in R_m} R_{r,p} \) denote the total number of offered packets, the total number of lost packets and the total packet revenue lost on routes of length \( m \) respectively. Then \( \overline{Y}_c(m) = \overline{L}_c(m) / \overline{N}_c(m) \) and \( \overline{Y}_p(m) = \overline{L}_p(m) / \overline{N}_p(m) \) are the estimates of the loss probability experienced by calls and by packets on routes of length \( m \).

Figures 7.1, 7.2 and 7.3 show the loss probability \( \overline{Y}_c(m) \) and \( \overline{Y}_p(m) \), the total number of lost connections \( \overline{L}_c(m) \) and of lost packets \( \overline{L}_p(m) \) and the total lost revenue \( \overline{R}_c(m) \) and \( \overline{R}_p(m) \) for routes of length \( m \) where \( 1 \leq m \leq 9 \). The figures show that the reallocation reduces the loss probabilities, the lost connections/packets and the lost revenue for all but the longest routes.

The connection loss probabilities on routes of length 8 and 9 are high but the number of lost connections and the lost revenue on these route is negligible. At the packet level the above two route lengths offer a low loss probability with a negligible number of lost packets and lost revenue.

The concave shape of the curves (lost connections/packets and lost revenue for connections/packets) are due to the route length distribution and to the fact that the revenue
earned on route $r$ is proportional to route $r$ length [2].

Another issue we investigated is the impact of the reallocation scheme on the packet transmission delay. The transmission delay is the time spent by the packet before reaching the destination node. We did not consider the effect of the propagation delay. Fig. 7.4) shows that the packet transmission delay decreases as the signalling ratio $V$ increases and reaches its minimum in the region $1 \leq V \leq 10$. This shows that the reallocation scheme reduced the time spent by the packets in the network.
Figure 7.2: The European network model: the number of lost connections/packets vs the route length $L$ and the signalling ratio $V$. 
Figure 7.3: The European network model: the connection/packet revenue lost vs the route length $L$ and the signalling ratio $V$. 
Figure 7.4: The European network model: the transmission delay vs the route length $L$ and the planning ratio $P$. 
Chapter 8

Conclusion

In this thesis we studied the problem of pricing and reallocating bandwidth and buffers, with the aim of improving the network’s overall rate of earning revenue. Pricing resources has an important role in the management of telecommunication networks. Apart from being a mechanism of selection and classification of users according to their utility [13], it is also a mechanism for efficiently managing resources in a telecommunication network. The fact that certain routes of a telecommunication network need additional resources to process more data while others have a surplus of resources can be used to define rules for a bandwidth market where the network resources are traded periodically to improve the network performance.

In this thesis, we define a model for the optimal management of bandwidth in an IP network. We view an IP network as collection of $M/M/1/K$ routes that offer to users a connection interface of type $M/M/C/C$. A $M/M/1/K$ queue describes a queueing model where packets are offered to the single server in a Poisson process.

Our scheme for the optimal management of bandwidth and buffers is derived in two steps. First, we consider a model which, given the current state of a $M/M/1/K$ queue, computes the expected lost revenue $R_n(t)$ due to packet loss on a time period $t$. Over a planning horizon $t$ we translate the expected losses into buying and selling prices of $u$ units of buffer space.

The expected lost revenue $R_n(t)$ is expressed as a system of renewal equations. We derived
a system of recurrence relations satisfied by the Laplace Transform of $R_n(t)$. The solution of this system of recurrence relations can be determined in terms of Chebychev polynomials. We invert these Laplace transforms numerically using the Euler method.

We then considered a mechanism that uses local information about routes, computes the price of buffer space (the “$M/M/1/K$ prices”) and the price of the interface capacity (the “Erlang prices”) and decides whether or not to reallocate buffer space and/or interface capacity among routes. The scheme is distributed and scalable. We next incorporate packet modeling into an existing event-oriented simulator. We described a simulator which simulates the actions of the route managers. The simulator computes state-dependent capacity prices which form the basis of a capacity (interface or effective) re-allocation (trading) scheme in a path-oriented network. The simulator uses the Erlang bandwidth prices to adjust the interface capacity of the network paths and the $M/M/1/K$ prices to adjust the buffer space at each ingress of the network paths. The simulator also applies resource- and policy-based Call Admission Control (CAC) to provide a quality guaranteed transport service.

A fictitious network of 30-nodes 46-links based on the geography of the Europe is used to the simulator to evaluate the performance of the reallocation scheme. We investigate the effect of the reallocation scheme on the blocking probabilities along routes of various length. Initial experiments indicate that the resource allocation schemes leads to a substantial improvement in network performance.
Bibliography


