MATHEMATICAL DIFFICULTIES ENCOUNTERED BY PHYSICS STUDENTS IN KINEMATICS: A CASE STUDY OF FORM 4 CLASSES IN A HIGH SCHOOL IN BOTSWANA

by

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DECLARATION

I, Ndumiso Michael Moyo, hereby declare that the work in this thesis is my own work, that all the sources that are used or quoted in the study, have been indicated and acknowledged by means of complete references and, that I have not previously, in its entirety or in part, submitted it at any other university for a degree.

Signature of student

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ABSTRACT

The study set to investigate the mathematical difficulties that students encounter when learning kinematics in physics. It examined the nature of mathematical difficulties, their possible sources and the potential impact they might have during construction of their own knowledge. If a teacher knows in detail the difficulties experienced by students during knowledge translation process between mathematics and physics, one can decide how mathematics might be supportive and develop new teaching strategies that can help to overcome their problems. An understanding of the physical science concepts forms the basis upon which new physical science knowledge is constructed. The pragmatic paradigm was useful to gather instruments that would help to answer questions for the study.

A cohort 40 students out of a population of 600 learners doing Physics Pure Award randomly participated in the study. A diagnostic test was useful to establish a baseline knowledge about students' conceptions and misconceptions in Mathematics and Physics. A survey questionnaire administered in which 35 students responded to interrogate the nature of mathematical difficulties encountered by physics students when learning kinematics. Purposive sampling was useful to select six participants for the individual and focus group interviews.

The main findings of the study confirmed existence of a variety of mathematical difficulties that hinder the effective learning of kinematics. Students lack adequate skills related to simplifying equations, determining the square root, making one value a subject of the formula, factorising, solving simple fractions and dividing and subtracting negative and positive numbers as often used in the equations of motion to describe different patterns of movement. The use of symbolic representations instead of numbers unlike in mathematics made students to have difficulties in understanding physics concepts. Symbols were often confusing to the learners as they are interchangeable used in a multiple of representation in physics. Students had problems in understanding graphs because of mathematics used to explain different concepts such as reading coordinates, calculating the gradient and determining the area under the line of a graph.

Recommendations emanating of the study so as to improve teaching and learning in Mathematics and Science education include in-service training workshops to both Physics and Maths teachers to resource them on how to handle maths related concepts in the two

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subjects. It also requires that the teachers from the two subjects where possible should engage team teaching. To the curriculum developers it would be better to find out about the kind of mathematics to promote interdisciplinary learning.

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OPSOMMING

Die studie het ondersoek ingestel na die wiskundige probleme wat studente ondervind tydens die aanleer van kinematika in fisika. Daar is ondersoek ingestel na die aard van wiskundige probleme, hul moontlike bronne en die potensiële impak wat dit op die konstruksie van hul eie kennis kan hê. As 'n onderwyser in detail weet wat die probleme ondervind word tydens die vertaalproses tussen wiskunde en fisika, kan 'n mens besluit hoe wiskunde ondersteunend kan wees en nuwe onderrigstrategieë ontwikkel wat kan help om hul probleme te oorkom. 'N Begrip van die fisiese wetenskaplike konsepte vorm die basis waarop nuwe fisiese wetenskaplike kennis gekonstrueer kan word. Die pragmatiese paradigma is gebruik om instrumente te versamel wat sou help om vrae vir die studie te beantwoord.

Altesaam 40 studente uit 'n bevolking van 600 leerders wat die Fisika Suierstoekenning verwerf het, is lukraak gekies om aan die studie deel te neem. 'N Diagnostiese toets is gebruik om 'n basiese kennis oor studente se opvattings en wanopvattings in Wiskunde en Fisika te vestig. 'N Vraelys vir opnames is uitgevoer waarin 35 studente gereageer het om die aard van wiskundige probleme wat fisika-studente ondervind het tydens die aanleer van kinematika te ondervra. Doelgerigte steekproefneming is gebruik om ses deelnemers vir die individuele en fokusgroeponderhoude te kies.

Die belangrikste bevindings van die studie het bevestig dat daar 'n verskeidenheid wiskundige probleme is wat die effektiewe leer van kinematika belemmer. Studente het nie voldoende vaardighede wat verband hou met die vereenvoudiging van vergelykings, die bepaling van die vierkantswortel, een waarde tot onderwerp van die formule maak nie, faktorisering, die oplos van eenvoudige breuke en die verdeling en aftrekking van negatiewe en positiewe getalle, soos dikwels gebruik in die bewegingsvergelykings om verskillende bewegingspatrone te beskryf. . Die gebruik van simboliese voorstellings in plaas van getalle anders as in wiskunde, het studente moeilik gemaak om fisika-konsepte te verstaan. Die simbole was dikwels verwarrend vir die leerders, aangesien dit dikwels verwissel word in 'n veelvoud van die fisika-voorstelling. Studente het probleme ondervind met die begrip van grafieke as gevolg van wiskunde wat gebruik is om verskillende konsepte te verduidelik, soos die lees van koördinate, die berekening van die gradiënt en die bepaling van die oppervlakte onder die lyn van 'n grafiek.

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Aanbevelings oor die studie ten einde onderrig en leer in Wiskunde- en Wetenskaponderrig te verbeter, sluit in-werkswinkels vir beide Fisika- en Wiskunde-onderwysers om hulle te help om wiskunde-verwante konsepte in die twee vakke te hanteer. Dit vereis ook dat die onderwysers uit die twee vakke, waar moontlik, spanonderrig moet doen. Vir die kurrikulumontwikkelaars is dit beter om uit te vind oor die soort wiskunde om interdissiplinêre leer te bevorder.

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ACRONYMS

1. Statistical Package for Social Sciences (SPSS)
2. General Systems Theory (GST)
3. Extended Semantic Model Theory (ESMT)
4. Test of Understanding Graph Concepts in Kinematics (TUG-K)
5. Force Constant Inventory (FCI)

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CHAPTER 1: INTRODUCTION AND ORIENTATION TO THE STUDY

1.1 Motivation and background

Studies in education have shown that mathematics has a significant role to play in students' learning of physics. (Thompson, Christen, Pollock, & Moutcastle, 2009) posit that specific mathematical skills and concepts are requisite for a complete understanding and appreciation of physics. Physics education research has shown that in understanding physics, conceptualization and problem solving are two key factors. Problem solving is desribed as the heart of the work of physicists (Fuller, 1982). According to (Hestenes D. , Toward a Modeling Theory of Physics Instruction., 1987), problem solving is a process that involves following appropriate reasoning parts to obtain knowledge about physical objects or processes that define them. In a majority of cases, such problem solving involves the use of mathematics as a tool to explore them (Redish, 2005).

(Wigner, 1960) points out that mathematics is a mind game, while natural sciences are empirical in nature, involving the formation of concepts from perceptual experiences. Mathematics helps to explain physical phenomena by providing necessary skills to develop in learners critical thinking as they formulate situations, interpret data in order to reach conclusions and make generilizations about presented situations. Furthermore, (Uhden, R, Pietrocola, & Pospiech, 2012) suggests that physics uses mathematics as a language to explain the natural world, a tool to construct new knowledge about the world. Such is the overwhelming view that the discourse of physics is mathematical in nature. (Bing & Redish, 2009) says that it is almost natural that students intergrate concepts of mathematics with physics to give meaning to the corresponding knowledge of physics.

Most of the studies that have been done in the field of mathematics-in-physics education seem to converge on the view that mathematics is pivotal in understanding physics. The intimacy between the two disciplines is so strong such that separating them when imparting knowledge of physics may be a hindrance to students' cognition of physics concepts. While still searching for answers to explain such a dialectical relationship, it turns out that the main direction of focus amongst most of the researchers shifts more towards use of mathematics in physics as well as understanding of mathematics in physics. (Larkin, 1980; Sherin, 2001; Kuo, Hull,Gupta and Elby, 2013). Not much seem to have been explored in the context of the nature of mathematics relevant to physics learning, the way this

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mathematics should be intergrated and how best it should be integrated to facilitate effective learning. A critical analysis of some these mathematical complexities students encounter, require more interrogation so as to measure the effectiveness of mathematics in handling the physics tasks. A closure of such gaps forms one of the key basis of this study. Taking an assessment of the topics in the physics curriculum, it is quite evident that mathematics is broadly utilised in a number of topics to explore physics ideas. There is need to reflect deeper as educators into the impact of mathematics in learning physics to iron out any possible sources of difficulties they seem to be encontering whilst constructing the knowledge of the subject.

While students are taught mathematics in mathematics lessons, they surprisingly seem to have challenges with the same concepts when they enter physics lessons. According to (Bosson, 2002), mathematics-in-physics is more than just computation of numbers and manipulation of variables and equations. More attachment seem to be attached to the meaning of the variables , their relations to the physical world and above all, the ability of mathematics solve problems of different nature in the natural world. The nature of physics requires special skills to navigate a variety of learning tasks most of which involve many mathematical representations especially in kinematics. Representations such as experiments, formulas and graphs are challenging to most of the learners with a weak mathematical background. Students find it difficult to contend with many representations more especially if they have to be implemented simultaneously, intergrated rapidly as is usually the case with solution of most of the kinematics tasks. As intoned by (Reddish, 2006), " Physics as a discipline requires learners to employ a variety of methods of understanding the ability to use algebra, geometry and trigonometry, going from the specific to the general and back. This makes learning concepts in physics challenging."

Educators agree that students learn best what they find understandable. Besides students' perceptions about a subject influences their understanding and learning of that subject. (Gebbels, Evans, & Murphy, 2010). This suggests that, a major reason underpinning students' participation in learning tasks is their perceptions of it as interesting/boring or easy/difficult, or relevant/abstract. Such is a situation prevalent when students learning physics. Most of the negative perceptions in physics learning, centre around mathematical incompetencies they have. To them studying physics is a routine process just meant to add a number to the science subjects they are expected to have at the end of their high

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school studies. This has serious implications for the building the capacity of physicists that are required to shape the technological development of any society.The premise of this study is based on exploring the underlying mathematical difficulties students are having which impede their understanding physics with a school in Botswana used as the context of such a study. The study will provide an insight into the aspects of physics that students perceive as a challenge that pause difficulties in their understanding of physics.

The choice of kinematics for this study, has been influenced by an introspection into the amount of breadth and depth of mathematics used by students in understanding the physics in this topic. An extensive review of the concepts of motion reveals a lot of mathematical applications such as ; manipulation of quadratic equations and formulas, use of algebraic expressions, use of symbols, handling system of units, numerical computations, manipulation of variables and graphical representations (Blum, Galbraith, Henn, & Niss, 2007). They further suggests that the conducting of experiments in kinematics lessons provide a conducive environment to sharpen students skills in an inquiry-based approach to learning. Experiments are mathematical models by nature and they need special mathematical skills such as observations, data recording, data analysis, prediction and interpretation to perform. All the special skills require some mathematical background to use them effectively in learning. The implementation of algebraic process skills such as factorising, use of indices, scientific notations and derivation of new formulas puts more demand on students to perfect on their mathematical competence to cope with challenges of physics. It is hoped that the study will help to come up with new innovantions to improve on instructional methods as well as learning approaches in pedagogy. This will probably help to eliminate some of the wrong perceptions students have about physics which usually contribute to their fear about it.

1.2 Problem statement

While it is contestable that mathematics plays a pivotal role in the teaching and learning of physics, the paradox is that it is the use of mathematics in physics that is still a major deterrent in students learning of physics (Albe, Venturi, & Lascours, 2001). Most of the reseachers have argued that use of mathematics in physics is to simplify complex physical relationships and principles. However the actual learning by students seems to be potraying a contradictory picture (Redish, 2005).

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Therefore, given such idiosynchrasies, the current study proposed scrutinizing students' mathematical difficulties that could be contributing to their misconceptions about kinematics. The main focus was to embrace unraveling the mathematical difficulties as well as interrogate their impact in learning physics at high school level. The study will be used to unearth the most recurring mathematical difficulties as well as the extent of their effects to smoothen learning physics (kinematics). It is hoped that such an initiative will eliminate the fear students have about physics and consequently avert the high attrition rate currently prevailing in Botswana.

1.3 Rationale for the study

Physics and mathematics are very close disciplines with shared concepts that interact with each other very often in pursuit to bringing meaning to concepts construction by learners. The interaction is historically natural rather than a compulsion. (Kiray et al. 2007b). The studies so far conducted in physics-in-mathematics research provide diverse and at times contrasting views about the origins of mathematical challenges students encounter when implementing mathematics in physics lessons. The teaching of the two subjects separately makes students to view the subjects as unrelated entities and this tends to widen the gap between the two.

When students appear to have trouble with mathematics in physics tasks, we are often quick to judge them as being incompetent in the subject instead of appreciating the need to strengthen their mathematical skills to help out of their physics tasks. Learning mathematics in- mathematics lessons does not necessarily guarantee their competence in using it effectively in physics lessons. The art of doing mathematics in the context of physics is a different ball game altogether. True, the interplay exists in concept usage, but the way the concepts are developed and later on executed in physics learning is not the same. Such a discourse in instructional methods used to handle the concepts of mathematics in the two subjects, require scrutiny by the curriculum developers. Literature is replete with mathematical difficulties that affect learning of physics. However, it does not reveal explicitly the real sources of such difficulties late alone how they impede learning in the context of a topic such as kinematics in physics. Generally mathematical difficulties exist in physics associated with use of formulas, deriving new formulas, use of graphs, interpretation of data from experiments and use of symbols. However, there are gaps in determining the cause and effects of such mathematical difficulties when it comes to use

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of the very mathematics they acquire in maths lessons in a physics classroom setting. The study navigates such complexities and tries to narrow the existing gaps by interrogating the secret behind use of algebraic equations, graphs, symbols, units and formulas in learning physics.

While a number of studies concerning students' use of mathematics has been conducted quite extensively by (Woolnough, 2000) et.al, few to my knowledge have managed to unravel the secret of sources of mathematical complexities associated with mathematics-in-physics and the limitations they have on understanding kinematics. The study of such difficulties will assist to come up with a nuanced view on how best both learners and their instructors should handle the concepts of mathematics-in-physics to improve learning. Apart from loading numerical values into equations, mathematics has many subtle roles it plays in organising intuitions and attaching meaning to concepts developed as observed by (Pillack; 2003, p421). The study will also investigate the impact of use of mathematics models in problem solving. This is perhaps what we mean by ''getting real physics right'' (Redish, 2005).

1.4 Objectives and Research questions

This study has three objectives

- To identify and analyse mathematical difficulties encountered by physics students when learning kinematics, this will establish a baseline of common difficulties displayed by learners whilst using mathematics in physics.
- To identify possible sources of the mathematical difficulties encountered by the students. This is to discern variations and monitor different sources of mathematical difficulties thus providing a solution to poor performance.
- To propose possible solutions to eliminate the mathematical difficulties and hence improve the learning and understanding of concepts in the subject.

1.5 Research questions

The central question of the research:

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- What is the nature of mathematical difficulties encountered by physics student when learning kinematics?
- What are the possible sources of the mathematical difficulties?
- How the mathematical difficulties mitigate in the physics classroom?

1.6 Research methodology

The researcher used the mixed methods approach in attempt to explore the mathematical difficulties experience by learners in understanding the concepts on kinematics. An approach using both the quantitative and the qualitative methods will be best address all the potential difficulties found that impede their understanding of physics concepts.

1.7 Significance of the study

The study aims at identifying the mathematical challenges experienced by learners suggesting ways of improving the teaching and learning of physics. The findings from the study will help in the designing of the relevant syllabi for the science and mathematics education community that should equip learners to construct their knowledge of physics with minimal difficulties. It would also help to produce both teaching and learning resources that should improve the understanding of physics in the classroom and the real world.

1.8 Delimitations of the study

The study focused on investigating the mathematical difficulties experienced by learners in kinematics. The delimitations of the study are that it confines the findings to a single topic yet many physics topics have concepts best expressed through use of mathematics. Secondly, the study confined the findings to a small sample of the large population of student in the school. The findings are peculiarly for students in the Kweneng Region.

1.9 Limitations of the study

The data were collected from a small population making it difficult for generalisations. The interviews used for the qualitative phase has room for biasness and subjectivity from both the respondents and the interviewer. The mixed methods design requires more time to gather enough data as well as analyse. From this study, there was limited time to come up with interventions to improve on the research.

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1.10 Definition of key terms

Constructivism - is an approach to teaching and learning based on the premise that learning is a result of mental construction.

Mixed method methods approach - is a method of collecting data where both the quantitative and qualitative methods are used.

Sampling - is a technique employed by a researcher to select a relatively smaller number of representative individuals from a pre-defined population to serve as data source for observation.

Kinematics – the study of movement of objects

The General Systems Theory- is a framework that prescribes and explains relationships between subjects, content and ideas in both the natural and social sciences.

Epistemology- a branch of philosophy that studies theories of knowledge.

1.11 Thesis outline

Chapter 1 looks at the motivation and background of the study. The chapter lays out the problem statement of the research and then defines the rationale of the study. The objectives of the research and its central research questions are outlined. Chapter 2 discusses the literature review of the study leading to the conceptual framework of the research in chapter 3. Chapter 4 then looks at the research methodology of the study with a view to come up with relevant data collecting tools implemented in chapter 5. Lastly, the chapter looks at the discussion of the main findings in an attempt to answer the research questions. The analysis of data, interpretations and evaluation of findings made it possible to come up with any recommendations for the study.

1.12 Conclusions

The following chapter presents a conceptual framework for analysing the mathematical difficulties encountered by students that impede their understanding of kinematics. The research structure will link concepts, the empirical research and important learning and pedagogical theories to explore the problem of mathematical difficulties. The framework will come up with a way to provide answers to the question under study. An integrated framework was adapted to explore the difficulties encountered.

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CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

The chapter examines the literature concerning the relationship existing between mathematics and physics learning.The relationship between the two subjects is one that has been of great standing for a very long time. Mathematical concepts and physics concepts relate in a number of ways. The literature review looks at nature of concepts that relate the two disciplines. The teaching and learning of concepts from the subjects has always been of value in the way the concepts complement one another in explaining any knowledge constructed by learners from the two disciplines. Mathematics transcends the physical reality that confronts our senses. Therefore, all which is physical is from the physical world, of which in the natural is the heart of physics.

(Feynman, 1992), accentuates mathematics as an integral part of physics; that all the laws of physics are mathematical; and that it is impossible to explain honestly the beauties of the laws of nature (physics) in a way that people can feel them, without them having some deep understanding of mathematics. Mathematics is the language through which physicists communicate to show the relationship between physics concepts, establishing some laws as well as in explaining physics principles. (Reif, 1995), cites Einstein emphasizing the importance of mathematics in physics by proclaiming, “The physicist’s work demands the highest possible standard of precision in the description of relationships such that only the mathematical language can give” (Feynman, 1992). Therefore, the beauty of mathematics-in-physics is that it elevates the scientific accuracy of physics above that of other sciences where less mathematics is used. The dual purpose of mathematics-in-physics is that of language plus logic, (Feynman, 1992). While physics and mathematics have such a deep relationship they are regarded as different forms of knowledge (Friegej, 2006). They classify these types of knowledge as; situational knowledge (knowledge about typical problem situations); conceptual knowledge (facts, concepts, principles of a domain); procedural knowledge (knowledge about important actions for problem solving (p. 440). (Pettersson & Scheja, 2008), concur with such classification of knowledge as either conceptual or procedural. They describe conceptual knowledge as being particularly rich in relationships, thought of in terms of a connected web of knowledge. Procedural knowledge on the other hand refers to knowledge of rules or procedures for solving mathematical problems. Conceptual knowledge is a type of

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understanding that involves knowing both what to do and why. Procedural knowledge involves simply, knowing how to do something.

In addition to classifying as different “types’’ knowledge, they are in terms of quality. (Friegej, 2006) and Krems (1994) classify the “qualities of knowledge” as:

Hierarchical (superficial versus deeply embedded; inner structure (isolated knowledge elements versus structured, interlinked knowledge); level of automation (declarative versus compiled); and level of abstraction (colloquial versus formal), (p440).

Understanding of both knowledge type and qualities of knowledge classifications will help put the contrasting of mathematics and physics into perspective. Generally, physics entails use of mathematics for a number of reasons. These include quantification, abbreviation, symbolic representations and succinct portrayal of physical relationships of different phenomena (Redish, 2005). However, literature is abound that shows how physics and mathematics differ. For example, physics is useful to explain the interactions amongst objects and processes in the natural world, and come up with rules and generalizations that govern these interactions. Whereas mathematics is touted as being about rigor, precision, exactness and accuracy (Hestenes, Wells, & Swackhamer, 1992),physics is about the best approximations (Buffler, Allie, Lubben, & Campbell, 2001).To (Hestenes, Wells, & Swackhamer, 1992), mathematics is sometimes called the science of patterns ; whereas to (Bosson, 2002), mathematics is concerned with quantity, shape ,data, space and structure. The area of measurements and its emphasis on units is one very distinct difference between mathematics and physics. Mathematics involves numerical computations while physics involves both computations and their applications in a natural setting. Numbers in mathematics can stand for anything real or imaginary; they do not have to have units. Numbers in physics quantify physical entities which they measure and therefore must have units. In physics, symbols stand for ideas rather than quantities (Redish, 2005). In most cases, physics theories are about experiments or observations, while mathematical theories exemplify the extent of the ingenious, almost artistic imaginations of man (Feynman, 1992).

According to Hestenes (2010), another interesting debate is about the relationship between physics and mathematics where he quotes one of the renowned Russian Mathematicians (Arnold, 1997), as saying:

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Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap…In the middle of the 21st centuries attempt to divide mathematics and physics was conducted. The consequences turned to be catastrophic (p.14). (McGinnis, 2003), posits that one fundamental difference between mathematics and physics is in the way mathematics compared to physics is pursued. He argues that there is a difference in the process of validation in that mathematics involves congruence of numbers while physics is concerned with congruence of concepts. Reflecting on all the differences alluded to in these two bodies of knowledge, it is quite possible that such differences could be the source of mathematical difficulties encountered by students in learning physics. Students could be failing to transfer the mathematical skills learnt in mathematics in physics lessons since they learnt separately. To this end, the primary objectives of this study is to investigate the mathematical difficulties student encounter when learning physics as well as soliciting for possible causes of such difficulties with a view of averting them so as to improve understanding of physics concepts. Such mathematical difficulties and their influences ought to be subject to scrutiny when teaching physics. (Tuminaro & Redish, 2004) suggests some reasons, why students struggle with mathematics in physics as , students' lack of the requisite mathematical skills needed to solve problems in physics and/or cannot apply the skills acquired in mathematics in the physics context.

Most of the scholars on mathematics-in-physics research single out algebra as one of the key topics that is used in construction as well as sharpening of tools used to handle the bulk of physics tasks at high school level. The knowledge of algebra is useful to develop and handle mathematical symbols, manipulate algebraic expressions, handling of representations such graphs and diagrams, use of vectors as well as constructing and using mathematical models in solving physics tasks.

2.2 Student's conceptions about mathematical symbols and how they stand as barrier in learning (kinematics)

Symbolic algebra is one of the main representations used to understand physical reality. (Sherin, 2001), points out that, algebraic notations plays an important role as the language in which physicists make precise and compact statements about physical laws and their relations. Such expressions entail both the procedural and structural representations of physics concepts and are all defined in symbolic form. The bulk of physics equations and

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formulas are in symbols and such symbolic representations are mathematically inclined. Mathematical symbols are useful for coding and decoding information, shortening sentences, representing variables and analysing data. The way in which mathematics exploits the spatial features of its symbolism and develops manipulation of symbolic expressions is a special property not shared with ordinary languages such as English. Mathematics, more especially algebra, is a language in itself with internationally recognised syntax and vocabulary (Esty, 2011). For students to be efficient in using algebra they have to be competent in the whole process of symbolization and the use of symbols. One “wonders” then if the use of symbols is of help or a hindrance in concept formation more especially in kinematics (Land, 1963, p.54). In symbolic algebra, quantities in physics expressions and formulas are in symbols. Each symbol in these formulas represent an idea/quantity and therefore has a special meaning entailed in it .The symbols represent reality in the physical world. The primary concerns in physics learning and instructions are to understand the relations between constructions of knowledge and symbols usage. The presumptions are that, to understand those relations our efforts must focus on the places where symbols are useful to acquire full knowledge of their applications in essence the two must resonate.

In symbolic representations, students have to know the meaning of a symbol, the relations between symbols, identifying known and unknown variables. Kinematics equations use a variety of symbols all of which have a special meaning when solving problems in physics .During problem solving students have to identify known quantities, keep track of symbol states and relations between them. The transition from a variable in a general equation to a quantity with specific associations is often one challenge that most of learners experience when using symbolic expressions. Units form an important entity of quantities when solving physics problems, as they are useful to check on precision and error margins. Switching from one variable to another also involves a change in units for those respective quantities and this often poses challenges to a number of students. To make calculations in physics easy, students have to learn units and handling them. The system of units through use of algebraic expressions represented in symbolic form mastered a lot better by students. (Ellermeijer & Heck, 2002).The units in physics gives more meaning to the variables or quantities used in algebraic expression to do with physical phenomena. However, the concept of dimensional analysis on its own never gets the importance it deserves when solving physics tasks. (Feynman, 1965 p.40), expresses that the system of units in

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equations used to solve problems in physics, promotes logic when shaping the learner's view of reality about the physical world. Such a view concurs with (Vyotsky, 1992) when he points out that internalization of symbols plays a pivotal role in the development of human thought. According to Vygotsky, symbolic language gives order to initially undifferentiated streams of infant thought. Failure to establish and assign units for quantities/variables when solving problems in different equations is a clear indication that students cannot link concepts into the forbidding territory of the physics behind the equations. To this end, it compounds the problem of students' understanding the proper use of mathematics-in-physics.

2.3 Students 'conceptions of algebraic signs.

One important concept of algebra in physics, particularly in kinematics is the use of algebraic signs. Like in symbols, signs represent a special meaning in physics (kinematics), compared to mathematics. Right from their early stages in learning, throughout their school, students meet the use of algebraic signs 'plus' and 'minus' across different contexts. They understand them procedurally, without an accompanying comprehensive and appropriate conceptual anchoring. Such procedural knowledge has limited sustainability in introductory physics and little value for the study of more advanced physics. Lack of strong conceptual grounding about proper use of sign conventions in physics can easily generate challenges in problem solving. For example, -5m/s may be considered mathematically smaller than -4m/s, yet in physics especially in vector kinematics -5m/s may be considered larger than – 4m/s in the concept of velocity, acceleration and displacement respectively. When dealing with vectors in kinematics, the issue of appropriately understanding signs is often a serious challenge.

Literature in mathematics-in-physics education research reveals that students have different conceptions of sign conventions used for describing the displacement, velocity and acceleration. Students have difficulties in understanding negative velocities and how they link to a situation.(Goldberg & Anderson 1989; Testa, Monroy and Sassi, 2002). These difficulties in understanding the meaning of negative velocity or acceleration emanates from an exposure of students to, vectors in one way. Students cannot link a vector to a physical situation. As Viernot (2004), rightly puts it, a sign defines a special meaning about quantity in vector kinematics. A sign to a quantity awarded upon having considered an appropriate axis of reference for a coordinate system. (Trowbrdge &

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McDermott, 1980), concluded difficulties with vectors and vector notations in physics (kinematics) amongst students because of their poor knowledge of using sign convections.

The challenges that students experience in vector-kinematics are mainly to do with their inconsistency with their use of the algebraic signs. Students are not sure when used in magnitude or in directions representations. Most of the students seem to have challenges in correct use of sign in both velocity and acceleration. They cannot interpret that a decrease in velocity is the same as deceleration implying negative acceleration in sign convention. To sum it all, students do not understand the reasons for using signs. The study seeks to find out more about students' conceptions about algebraic signs and the extent of their influence in understanding of kinematics in physics.

2.4 Student's conception about graphs as mathematical representations

Graphical knowledge, a core component of science (physics) curriculum, is an essential skill as well as a tool for any technological innovations. A lack of such skill is on its own, results in a poor understanding of physics concepts. (Fry, 1984), defines a graph as information displayed or transmitted by the position of a point, a line or an area on a two-dimensional surface or three - dimensional volume. A graph is a meaningful picture that gives powerful visual pattern recognitions to see trends and subtle differences in shape (Beichner, 1994).The shape of any graph represents a specific meaning to a relationship between variables and has a specific bearing to its meaning. The analysis and interpretation of a graph is largely dependent on its shape. Interpretation of kinematics graphs using the variables position, velocity and acceleration appears to be the most problematic area in teaching and learning physics.

Graphs are commonly used in many gate way subjects such as mathematics and science to convey vital information, yet students have difficulties in interpreting graphs (Zucker & Stephanie, 2013).There are four types of graphs used in physics for experimental results analysis. They are the comparison line graph, the compound graph, Cartesian plane graph and the scatter graph. However, the most commonly used graph at high school level physics (kinematics) is a line graph. According to (Bell & Janvier, 1989), line graphs are more difficult for learners to comprehend than other types of graphs because it is a big step for students to realise that a line on a Cartesian graph represents a relationship between two variables. Generally, most literature in education research observes that the misinterpretations of kinematics graphs, presents the largest problem to students at

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different levels of their academic pursuits. (McDermott, Rosenquist, Emily, & Zee, 1987), found that students even at University level struggle with understanding concept of graph interpretation. The common problems that often give students difficulties in interpreting graphs in kinematics highlighted by (Beichner, 1994) are as follows:

- Mix-up between slope and height
- Misconceptions of graphs as a picture representation of an event
- Confusion in determining the slope of a curve that does not pass through the origin
- Inability to interpret area under a given curve
- Slope interpretation
- Area and gradient mix-up
- Graph construction

All the stated difficulties have a large input to students' understanding of kinematics and more striking is that they seem to zero into the terrain of algebra and geometry .Such a large demand of graphical knowledge, which is very algebraic in nature, could be the source of students misunderstanding of kinematics.

2.5 Line graph slope/height confusion in kinematics

Most of the students exhibit slope/height confusion in physics context more than in the context of the mathematics. (Hestenes, Wells, & Swackhamer, 1992) (McDermott, Rosenquist, Emily, & Zee, 1987); (Bell & Janvier, 1989). A slope on graph in kinematics is important since many physical quantities are described using gradients (e.g. velocity, acceleration) and are represented using line graphs. Students study line graphs in mathematics but because of differences in context, they are not able to realise that they are studying the same phenomenon as in physics lessons.

The study of graphs by (McDermott, Rosenquist, Emily, & Zee, 1987) presents a good overview of students' difficulties with graphs. Students have difficulties in discriminating the slope and height of a graph and interpreting changes in height and changes in slope. They cannot differentiate information extracted from the slope to that extracted from the height of the graph. According to (Beichner, 1994), common mistakes made by students in kinematics graphs involve thinking that a graph is a picture of a situation and they confuse the meaning of the slope of the line with the height of a point on the line. They

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cannot realize which feature of the graph to use in a particular situation, and hence tend to use a position criterion instead of a gradient-based criterion when looking at velocities.

(Leinhardt, Zaslavsky, & Stein, 1990), classified student difficulties on line graphs into three categories: interval/point confusion, where students focus on a single point instead of an interval; slope/height confusion, where students mistake the height of the graph for the slope; and iconic confusion, where students incorrectly interpret graphs as pictures.

Understanding a slope and height in kinematic graphs has an important role, especially in transition from one kinematics graph to another. For example, when a velocity- time graph (v-t graph) is given, students need to know the difference between the slope and the height because in a v-t graph; the height is the velocity whereas the slope of a line graph represents the acceleration. More often, students have no problem in using the distance formula in equation of motion ($d=vt$), but they have difficulties in realizing that the area under the curve/line of a (v-t) graph also represents the distance. Students usually do not know when to use slope or area for interpretation of a given graph.

2.6 ''Graph as picture error'' confusion

Students often expect the position graph to be similar to the velocity- time graphs of an object (Nemirovsky & Steven, 1994) , (Brasell & Rowe 2007). According to (Nemirovsk & Rubin, 1992), "resemblance of graphs gives students tools for making sense of a complex situation. Students probably do not adopt resemblance because they have solid reasons to believe the tools are appropriate, but rather because the tools enable them to organize and solve a bewildering domain of problems''. In other words visual features of a graph may give a wrong interpretation of a graph of velocity- time with respect to distance travelled by an object, as illustrated in the sketch of a graph below.

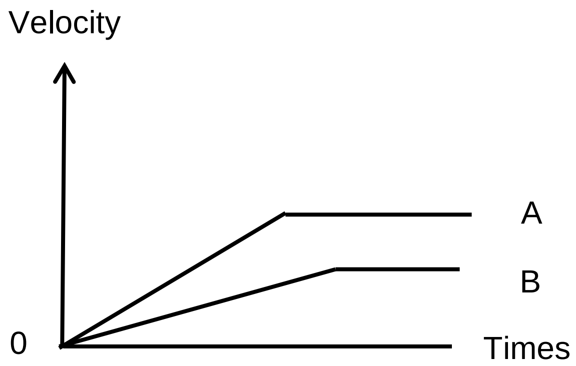

Figure 2. 1: Example of kinematics graph (Adapted from McDermott & Lillian, 1987, p. 55)

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Object A, moves a greater distance than object B, from the area under the graph concept. Students need to know what the slope, height and the area represent in a graph to be able to separate the three from one another. A common problem exhibited in kinematics graphs is seeing graphs as pictures but not a representation of information. This is known as ‘’graph as picture ‘’ error (Berg & Phillips, 1994). When they see a graph as a picture, they do not think about variables on the graph. This often leads to misunderstandings of the relationship between variables in kinematics graphs because what a position-time (p-t) graph represents is not the same as what a velocity-time graph (v-t) graph represents. In other words even though the (p-t) graph and (v-t) graphs have some resembles, the information they depict is different. Graphs in kinematics requires the ability to perceive and remember a pattern of specially arranged visual data as well as the ability to reason about the spatial visual information, (Kozhevnikov, Modes, & Hergarthy, 2007). Most of the studies have revealed that graphs in kinematics are mainly for problem solving. They have both a visual and a spatial imagery of which most of the students seem not to have which consequently hinders that ability to gain conceptual knowledge of physics (Bell & Janvier, 1989).

2.7 Learning physics with and through mathematical models

In recent years, use of models in teaching and learning science has been given serious consideration by science educators (Halloun, 1996) et.al. An extensive research on the role of models and modelling in mathematics education has also surfaced (Confrey & Doerr, 1994) et.al. The view of an existing intimate relationship between mathematics and physics in teaching and learning, presents use of models as alternative approach to developing concepts in kinematics. According to (Blum, Galbraith, Henn, & Niss, 2007) the use of inquiry-based approach to teaching and learning physics, has been explored by many researchers.

Models in science help to connect the mathematical world to the physical world consisting of the abstract “truth”. Physics learning involves understanding the real world, its physical theories and physical models. Human beings understand the world by constructing mental models. The mental models are constructed perceptions and interpretations or acts of imagination through analogical representations of reality (Etkina, Warren, & Gentile, 2006), defines a ‘’model’’ as a surrogate object, a representation or a simplified version of a real object.

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Physics by its very nature involves problem- solving in learning. The concept of problem-solving is a modelling enterprise. According to (Van Heuvelen, 1991), an appropriate order of knowledge construction based on cognitive and epistemological framework is imperative for students' effective learning of physics through problem solving. He indicated that in this knowledge structure, students should be able to see relationships and similarities in diverse pieces of information. The methods of problem solving characterize their algorithmic and heuristic natures. (Prett Naples & Sternberg, 2003; Ormrod, 2004). Algorithm refers to step- by- step procedures which when followed correctly guarantees a correct solution every time. Heuristics refers to general strategies or “rule of thumb” for solving problems (Ormrod, 2004). Problem solving strategies involve use of diagrams, procedural skills and mathematical physics skills. According to Van Heuvelen (1991) and (Reif, 1995), all the strategies have a distinct purpose when solving physics problems. The diagrams spell out the prominent features of the process, procedural skills, steps in to solve the problem and mathematical skills are for analysis and evaluation of the solution to the problem. Such strategies are a true reflection of the modelling process. There is more emphasis put on the procedures or processes in solving problems rather than the result/product.

The concept of modelling is in congruence with (Tuminaro & Redish, 2004), when he emphasizes on ‘’making meaning ’’ in the process of concept development. (Tuminaro & Redish, 2004) in ‘’Mapping mathematics to Meaning’’ epistemic game has modelling explained as a patterns of activities where students working on a physics problem begin with a physics equation, then develop a conceptual story in the process. This concurs with (Gupta & Reddish, 2009) when they present four steps of modelling as; mapping, processing, interpreting and evaluating as critical skills in use of mathematics in physics.

The critical difference in maths as pure mathematics and maths in a physics context is the blending of physical and mathematical knowledge. A simple model by (Redish, 2005), focuses on a flow of the main steps as illustrated in Figure 2.2.

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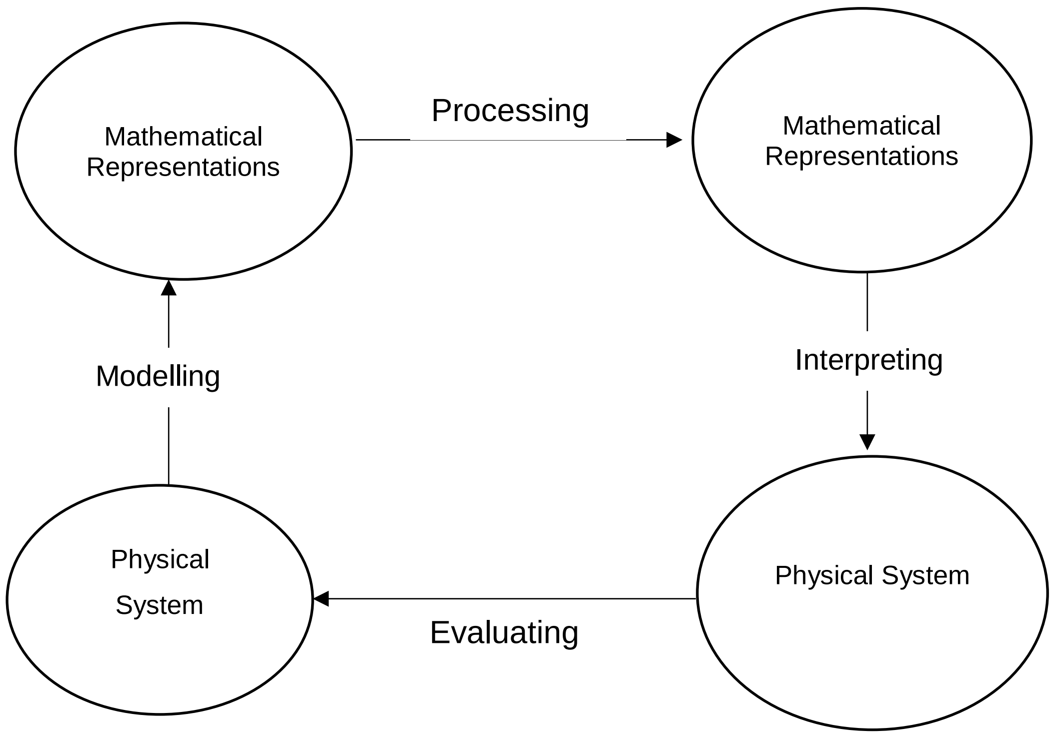

Figure 2. 2: A Model of Mathematical Representations (Adapted from Reddish, 2008, p.148)

A model of mathematical representations in mathematics classes, processing is emphasised than any of the steps. However, in physics mathematics integrates with our physics knowledge and does the work for us. Model construction requires coordination of a set of concepts and such is the efficiency of its applications in learning. Models are student-centred and therefore they construct their own knowledge by actively engaging themselves in activities that help articulate their plans, make their own assumptions, explain their procedures and justify their conclusions. Proper implementation of models should therefore help students to identify holes in their understanding of concepts, promote dialoguing and allowing them to have their arguments to provide answers to problems.

2.8 Context of mathematical modelling in kinematics

Kinematics is one of the topics in physics education that provides a fertile ground for implementing mathematical models for concept development. From a mathematical standpoint, functional reasoning (cognitive reasoning) involving a function concept may involve a complementary between representations. (Otte, 1994) claims that a mathematical concept such as the concept of a function does not exist independently of the totality of its representations pp55. A robust understanding of a function captures three distinct representations such as equations, graphs and data tables and the connections between them (Kaput, 1998). Kinematics, through its reliance on a function concept to

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model; motion, provides an opportunity to function representations and attempts to make connections between them examine the possible limitations present when learners rely on during the modelling process. In kinematics, there is room to create models to describe, observe behaviour during experiments, interpret, analyse, and predict future behaviour and relationships of concepts through use of equations, graphs and data tables.

According to science research, there are three worlds; the physical world, the mental world and the conceptual world. Modelling helps to connect these three worlds in physics, particularly in kinematics to enhance concept development. The linkage of the three worlds is as illustrated in the diagram adapted from Wells and (Hestenes D. , Toward a Modeling Theory of Physics Instruction., 1987).

Figure 2.3 below is an illustration of the importance models to science and pedagogy

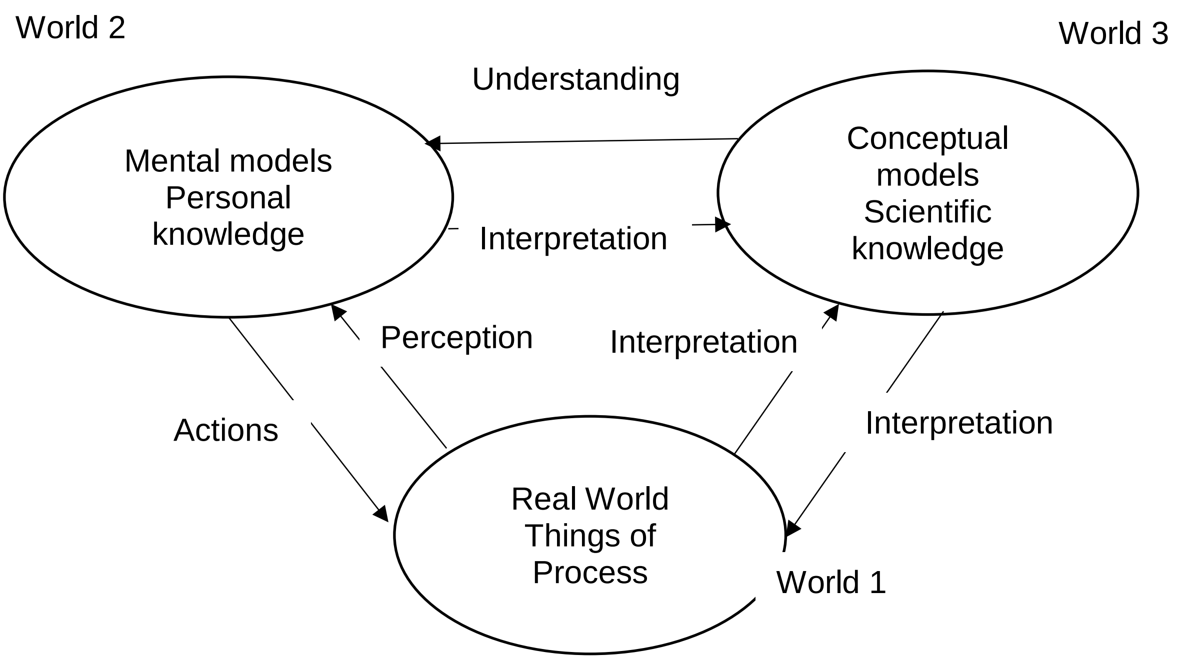

Figure 2.3: Mental and conceptual models (Adapted from Wells & Hestenes, 1987, p. 440-454)

The Modelling Theory puts emphasis on two models; the mental models and the conceptual models. Mental models are private constructions in the mind of an individual. Such can be elevated to conceptual models by encoding model in symbols that activate

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the individual's mental models and corresponding mental models in other minds. Therefore, mental models represent states of the world as conceived not perceived. To know a thing is to make a mental model of it. Modelling instruction provides students with activities that make students have their own explanations for best physical phenomena.

In kinematics, experimentation, manipulation of equations by students, manipulation of dimensions to check errors in equations, graphing and use of data tables involve applications of modelling. Experimental procedures are a typical way of modelling concepts. The derivation of equation is a step- by- step process, involving modelling. Graphing which is key in exploring a number of kinematics concepts, integrates skill of calculation, deduction, interpretation, prediction and analysis. The study ventures into the importance of exploring use of models as alternative in improve learning of kinematics. That is what is meant by "getting the physics right". A model of the relationship in kinematics graph adapted from (Brasell & Rowe, 1993) further illustrates the beauty of exploring concepts at different levels of mental cognition in kinematics graphs.
Modelling is really an "art" of doing physics right. Such was the idea of exploring use of models to meet the objective of improving learning physics concept

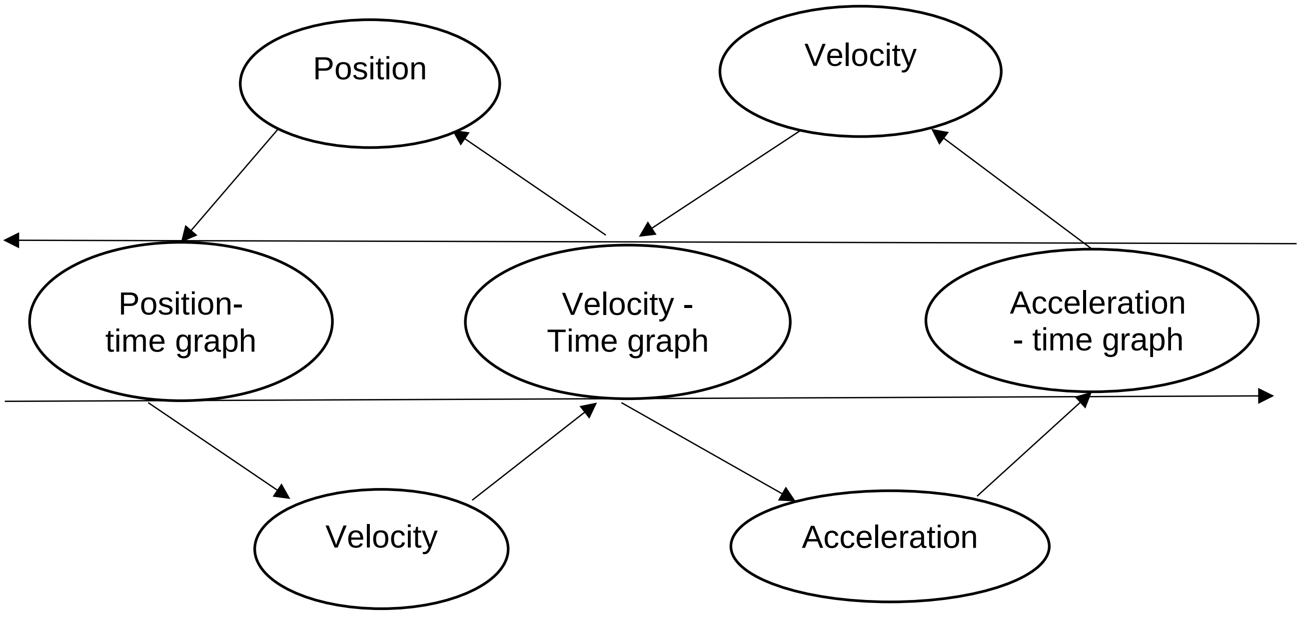

Figure 2. 4: A model of position - time, velocity- time and acceleration- time graphs. Adapted from (Brasell & Rouse, 1993

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2.10 Conclusion

In this Chapter, the literature reviews the relationship between mathematics and physics. The chapter examines the students' conceptions about the importance and role of mathematical symbols in studying physics concepts. The conceptions of algebraic signs in teaching and learning mathematics and literature explaining their impact to teaching and learning physics concepts established. The chapter examined the concept of the effects of graph knowledge in understanding some concepts in physics. The discussion on the importance of graphs as a picture was established and how it affects understanding of physics concepts. To sum it all the chapter also looks at the literature on the importance of using mathematical models in teaching and learning kinematics. Chapter 3 that follow presents a conceptual framework for analysing the mathematical difficulties encountered by students when learning kinematics concepts in physics.

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CHAPTER 3: CONCEPTUAL FRAMEWORK

3.1. Introduction.

This chapter presents a conceptual framework for analysing the mathematical difficulties students often encounter that inhibit their understanding of kinematics at High school level. The research structure will link concepts, empirical research and important learning and pedagogical theories to explore the problem under review. Such an integrated perspective of viewing a problem under study should facilitate a thorough interrogation of the relationship between the main concepts of the study and the ideas developed there in. The framework should come up with a more vivid picture of how a synergy of such ideas provide relevant answers to problems under review. (Liehr & Smith, 1999); (Akintoye, 2015), opine that, a conceptual framework presents assorted remedies to the problem defined consequently providing a firm structure for the study. It helps to define the constructs or variables under investigation and their entire relationships within a given context.

A suitable framework to address possible causes for the mathematical difficulties students experience when learning kinematics, let alone solutions to handle was difficult to find. Two conceptual frameworks widely used by researchers The General Systems Theory (GST) and the Extended Semantic Model Theory (ESMT) were useful for the study.

3.2. The General Systems Theory (GST)

The General Systems Theory is a framework that prescribes and explains relationships between subjects, content and ideas in both the natural and social sciences. (Bertalanffy, 1968), posits that, in the field of education, the GST theory emphasizes is an interdisciplinary study. It seeks to integrate knowledge from different subject disciplines. It thrives at unifying knowledge through integration of different subject disciplines during concept development. This view concurs with the Gestalt theorists as well as Aristotelian dictum which states that, the whole is greater than the sum of its parts. A physics topic like kinematics, constitute of different sub-topics of which all when well integrated during knowledge construction, always produce ‘meaningful learning’. The divergent concepts students often encounter in the process of learning physics topics, more especially kinematics as a case in point, require varying specialised mathematical knowledge and skills. Most of physics topics use mathematics as a language of communication and this often generate mathematical difficulties of varied nature to the learners that inhibits their

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understanding and consequently learning of the subject. (Bertalanffy, 1968), proposes that acquisition of knowledge during learning involves understanding of systems. In his systems theory he observes that there is generally a tendency by educationists to integrate various concepts when constructing knowledge in social and natural sciences. Such an integration is centred in the General Systems Theory (Bertalanffy, 1968). He posits that if a system is to be useful it has to be an open system. An open system is one which is characterised by both inputs and outputs linked together through defined processes. The inputs require specialised skills to process the desired outputs. Such specialised process skills are entrenched in the symbolic language that learners should be well equipped with prior constructing any scientific knowledge in kinematics. (Bertalanffy, 1968), idea behind the systems theory is that isolating a component of a system from its whole does not yield a comprehensive explanation to it. In order to effectively explain and gain a better understanding of something, the system and its holistic properties have to be analysed to find the root of a problem. Exploring the functions of a system as well as its components often times help to increase the awareness of why, when and where a system has such malfunctions and consequently provide a diagnosis of where the system needs attention when need arises. Systems theory takes into account all possible sources of problems identified and examines each one individually and what role they play in the system as a whole. Such a diagnosis helps to establish problems encountered during the construction of concepts.

Exploring kinematics as a system of concepts requires one to identify possible sources of difficulties that often come with teaching and learning the topic. This helps to identify relevant tools in handling the encountered difficulties. By so doing, the theory helps to define interrelationships amongst concepts in kinematics addressing the principal objective for this study, “to identify mathematical difficulties encountered by physics students when learning and solving kinematics problems at high school level.” Kinematics is a system of concepts whose dynamics involves a number of tools to use for the study. The topic consists of an array of connected ideas and each idea forms an important unit of the complex whole. Tools of different types are engaged to offer solutions to problems and each tool has its special function/s it renders to enhance the learning process. Some learners do not have the requisite special skills to handle some of the problems they encounter when constructing knowledge on kinematics and this inhibits their learning. Graphs, equations, vectors and other symbolic tools are useful to explore kinematics and

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these could be potential sources of some mathematical difficulties encountered by learners in handling different concepts of the topic. Therefore competency in use of tools requisite to learn kinematics, the ability to relate concepts developed in a given context, blends well with the second objective of the study; "identifying possible sources of mathematical difficulties encountered by learners in studying kinematics at high school level."

The purpose of the study is to establish a baseline as well as come up with a systematic approach in handling students' mathematical difficulties that may be contributing to their misunderstanding of physics more especially in kinematics. The theory will allow for a more holistic view on the varying mathematical challenges students might have from their Junior School level studies in science with particular reference given to a topic on kinematics. An establishment of a proper baseline on the mathematical difficulties students might be having should help engage relevant tools to minimise or eliminate the difficulties addressing the third objective of this study; "to propose possible solutions to eliminate mathematical difficulties experienced by learners when studying kinematics." Mathematics is a language of learning physics. It is a symbolic language used to express and construct physics concepts and its usage in teaching and learning physics is not just like a walk through the park to some learners. It requires some degree of competence to master as well as execute during the learning process. To this end, any incompetence in proper use of the language is like addressing the symptoms of a problem than its cause. (Bertalanffy, 1968) ,considers symbolic language as an essential implement for increasing acquisition of knowledge and such a view espoused by Vygotsky, is in his theory of constructivism emphasizing on language as an effective tool in construction of any form knowledge by learners. Any language is a vehicle of communicating concepts. Therefore establishing effective communication skills through mastering the art to implement varying symbolic tools is the way to effective and 'meaningful learning' of physics concepts.

To solve problems in kinematics and in life in general requires specialised skills. Using the systems theory helps identify where skills deficiency lies when learning concepts in kinematics and this assists in keeping track of potential sources of difficulties experienced by learners in learning concepts of the topic. The identification of the difficulties and their potential sources makes effective communicators and takes out learners from looking at a problem from a narrowly perspective instead of an expanded view to the whole situation in context. The (GST), demonstrates that instead of addressing just the problems

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encountered we need to review what the whole system looks like to gain an insight of what we are seeing and why? When different parts of a system are working in unison, there is synergy amongst them bringing about intended learning out comes.

(Sergei & Heather, 2002) argue that, the flagship of science education in constructivism (Von Glaserfeld, 1992; Yager, 1995) and conceptual change (Henson & Thorley, 1989; Scott, Asoko & Driver, 1992) if facilitated through science maps or outlines that identify interrelationships, connections and generalities of scientific knowledge in a valid manner can be attainable. One distinct utility of the (GST) is demonstrated in its ability to; identify the system of which the unit in focus is a part; explains the properties or behaviour of the unit in focus as part or function of the system (Skyttner, 2010). In this study, identification of mathematical difficulties, their potential sources and possible solutions to minimize or eliminate them, constitute the three units in focus. The expectation is that a systematic integration of the three should unfold the breadth of the objectives entailed in this study and in the main help solve an array of difficulties associated with learning concepts in kinematics.

From as far back as (Boulding, 1956), (Bertalanffy, 1968) to (Skyttner, 2010) , the GST has evolved through various forms and has been hailed as a useful tool in mapping scientific knowledge by depicting relationships, connections and generalities in different spheres of learning. Through this framework, knowledge fragments across subjects is synchronised. Information from one area of science must fit into other sciences as a whole. The theory advocates for students to have long-term and integrated understanding of science content as well as enabling them to apply their knowledge in solving problems in the world around them. The (GST) opens up thought process about feedback channels and adaptation in learning which serves as a valuable lens in identifying specific difficulties as well as their impact in making the system of learning out of balance.

For the purposes of this study, an effective framework should then be one that is consistently applicable in analysing students' mathematical difficulties in the topic such as kinematics. (GST) is preferred as a guiding tool for the development of the framework and analysis of this study in general. This concurs with (Tuminaro & Redish, 2004), who suggests that, a general knowledge framework offers a whole range of cognitive constructs [that in context influences mathematical difficulties in students' understanding of kinematics].

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3.3 Extended Semantic Model (ESM)

According to (Hestenes D. , Toward a Modeling Theory of Physics Instruction., 1987), a model is a unit of coherently structured knowledge. It is a representative structure in a given system or process. The system itself is the referent of the model. According to (Hestenes, Wells, & Swackhamer, 1992), modelling is the construction, validation and application of models; and for (Tweney, 2011), science rests on the construction and use of appropriate mental models. Mathematics is a science of patterns, conceding that mathematics is pivotal for students understanding of physics. This implies that pattern recognition skills are essential to understanding physics. Models are useful to assist students' use of various mathematical tools to establish meaning in physics learning. The Extended Semantic Model (ESM) is a model of scientific problem solving and reasoning focusing mainly towards conceptual understanding. The essence of developing such a model was to make sense of students' step- by- step progress when solving physics problems.

The extended semantic model advocates for idealized problem solving that includes what it characterizes as the four domains of knowledge. These domains of knowledge are distinct areas of focus when solving physics problems. According to the (ESM), the domains of knowledge identifiable in problem solving are namely; concrete, model, abstract and symbolic respectively. These central areas of focus for students' effective use of divergent mathematical symbols in physics concept development as well as problem solving. The symbolic language used in different domains to explore concepts in physics is a potential source of mathematical difficulties students often experience when learning kinematics.

The concrete domain includes physical objects and events. The model domain constitutes models of reality and abstractions. The abstract domain includes concepts, laws and principles. The symbolic domain is concerned with language and algebra and is seemingly the key source of problems encountered by students when learning concepts in physics. This model was developed by Greeno (1989) and Fig 3.1, below illustrates the respective domains and their linkages as expounded by (Greeno, 1989)

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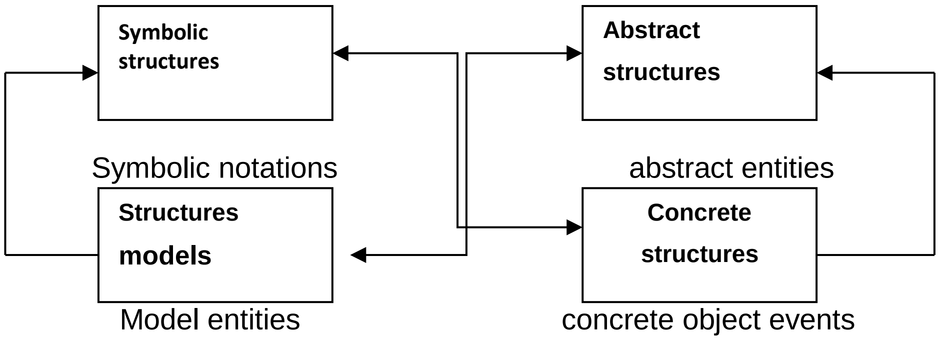

Figure 3. 1: Greeno's Extended Semantic Model (adapted from Cohen, 2000)

According to (Greeno, 1989), the concrete aspect in the model is concerned with that which is physically sensible. Students must develop intuition that helps them make physical meaning from the physics problems they are solving. The model domain involves portraying models of reality and abstractions. In the abstract domain; concepts, laws and principles explain the physical or concrete aspects. Finally, the symbolic domain is concerned with symbolic ways of representing a problem, be it metaphorically in words, or through the mathematics of algebra, or both (p.1093).

Scientific problem solving and reasoning skills are exemplified by correspondences between these domains (Greeno, 1989). Most of the students either do not have such skills or they are incompetent in using them and this inhibits conceptual understanding. Approaches to problem solving should show connections with other domains and this is same approach to knowledge construction espoused in the General Systems Theory. Students should be in a position to realize a connection of various concepts in different domains of knowledge and be able to engage them in solving problems they encounter. The skills that students require are useful to handle problems of a diverse nature that fall within the context of the four domains of knowledge. Thus, a student solving a kinematics question that involves use of mathematics must demonstrate an awareness of all the four domains as different areas of focus, to reflect conceptual understanding. If such awareness is not established, it certainly cripples their understanding of concepts inhibiting learning.

A notable strength of the (ESMT) is that it is able to track students' source of difficulties as they engage their step- by -step approach to solving problems in physics. (Gaigher E. :., 2007) , observed that students have difficulties in switching knowledge across the four domains. Gaigher (2004) quotes Chekuri and Markle (2004, p. 1094) who argue that "although problem solving in physics usually involves algebraic operations, in the symbolic domain, the algebra used should always be linked to the concrete, model and abstract

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domains respectively". Majority of the students lack flexibility and the tenacity in concept development and application. They find it difficult to translate knowledge acquired in mathematics in physics lessons. This is one of the major potential sources of the mathematical difficulties students experience when solving kinematics problems. If students cannot switch between the different (ESMT) knowledge domains, it definitely should influence their understanding of concepts let alone solving problems of different nature in kinematics. The (ESM) should prescribe the awareness of related concepts in different domains as well as sources of mathematical difficulties students often experience. The use of the Extended Semantic Model domain theory should help to analyse and come up with suggestions to solutions to the mathematical difficulties embedded in the domains. Such findings in this regard should help answer the key objectives of the study.

3.4 Mathematical Resources- modelling mathematical thinking

Students' mathematical thinking is described in a framework by (Tuminaro & Redish, 2004). He developed three major theoretical constructs; mathematical resources, epistemic games and frames. There are four types of mathematical resources being namely; intuitive mathematics knowledge, reasoning primitives, symbolic forms and interpretive devices. These mentioned types of mathematical resources were useful in the development of the theoretical framework in the following paragraphs.

3.4.1 Intuitive Mathematics knowledge.

It is the 'basic knowledge' of mathematics students should have like counting and "subitizing" that students acquire at a very early age of their learning. Subitizing is explained as the the ability that humans have to immediately differentiate sets of one, two and three objects from each other without necessarily being able to count (Tuminaro & Redish, 2004). Tuminaro and Redish (2004) give examples of intuitive mathematics knowledge resources and their descriptions as in the Table 3.1 below:

Table 3. 1: List of intuitive resources

Subitizing	The ability to distinguish between sets of one, two and three objects
Counting	The ability to enumerate a series of object
Pairing	The ability to group two object for collective consideration
Ordering	The ability to rank reactive magnitudes of mathematical objects

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3.4.2 Reasoning Primitives

They are abstractions of everyday experiences that involve generalizations of classes of objects and their influences. They are a derivative of DiSessa's (1993) phenomenological prisms (p-prims). They include blocking, overcoming, balancing and more is more. Examples of reasoning primitives and their descriptions are as listed in the table below 3.2 below (Tuminaro & Redish, 2004):

Table 3. 2: List of abstract reasoning primitives

Blocking	The abstraction notion that inanimate objects are not active agents in any physical scenario
Overcoming	The abstract notion that two opposing influences attempt to achieve naturally exclusive results, with one of these influences beating out the other
Balancing	The abstract notion that two opposing influences exactly cancel each other out to produce no apparent results
More is more	The abstract notion that more of the quantity implies more of a related quantity

3.4.3 Symbolic Forms

They are a framework that explains the way physics students view and apply physics equations. They are models that express individuals' understanding of physics equations. Symbolic forms consist of two elements; the symbol template and the conceptual schema. The symbolic template explains the virtual structures through which mathematical expressions are seen, whereas the conceptual schema is the idea to be expressed in the equation. The schema invokes when a student solves a problem; this schema specifies equations and drives the solutions. Symbolic forms describe students' intuitive understanding of physics equations. According to Kuo, Hull, Gopta and Elby (2013), symbol templates blend with conceptual schema to derive the meaning from symbolic forms. Symbolic forms are pivotal to subsequent studies on students' use of mathematics in physics (Tuminaro & Redish, 2004; Jones, 2010). Since students' use of mathematics in physics generally involves the use of equations, this is yet another potential source of mathematical difficulties encountered by students. Not all students are capable of manipulating variables in equations to solve for the unknown quantities. It is in this light that the symbolic form embraced the conceptual framework to illuminate the kind of mathematical difficulties students experience and the role mathematics plays in manipulating different formulae and equations when solving physics problems of different nature.

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One of the eminent points from Sherin's (2001, p. 10) discourse is that, "equations can be understood in terms of more basic and generic intuitions that cut across expert domains". She observes that; a correct use of physics equations may be misleading if only limited to manipulation of variables without grasping the gist behind the manipulations. In other words being able to compute data in equations and coming out with answers is not enough to justify understanding of solving problems in physics. Given such reasoning, not all students are capable of separating variables in algebraic expressions found in physics equations. This tends to generate problems when using equations to solve problems in physics.

3.4.4 Interpretive devices.

Interpretive devices are reasoning strategies used in interpreting physics equations, (Sherin, 2001). (Tuminaro & Redish, 2004) classifies them as formal interpretive devices and intuitive interpretive devices. Formal interpretive devices rely on formal properties of the equations while intuitive interpretive devices, "are abstracted from everyday reasoning and applied to physics equations" (p. 53). A classic example is one given by Kieran (2007) when he cites transposing- a mathematical operation that involves changing signs when changing sides of the equality and carrying out the same operation on both sides of an equation. This is quite common when applying the equations of motion to solve problems in kinematics.

3.5 Design of the Conceptual Framework

A suitable conceptual framework was by combining the GST and the ESMT frameworks. GST guides the sequence of topics in kinematics while the ESMT establishes the pattern of flow of concepts to bring about synergy in ideas.

3.5.1 Kinematics

Starting with the physics topic on kinematics as unit of focus, kinematics is broken down into four sub- topics: Linear motion, Free-fall motion, Motion equations and Motion graphs. The sub-topics constitute different sub-units of a core segment in an evolving framework as illustrated in Fig 2. In this layout, the four sub-topics form the distinct sub-units making the foundation of the conceptual framework.

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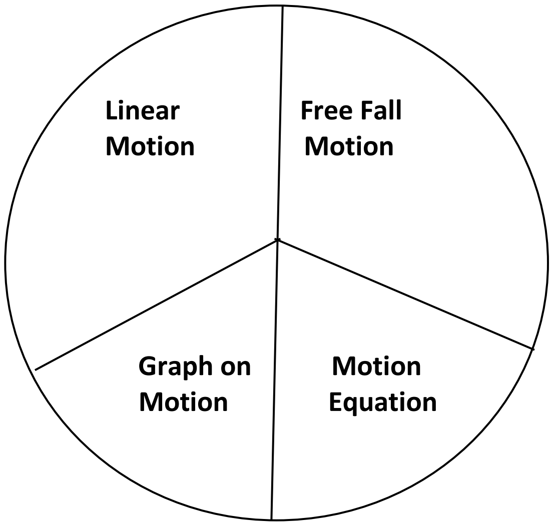

Figure 3. 2: Foundation of conceptual framework.

3.5.2 Mathematical Resources

According to GST classification, the role of mathematics and its symbolism in learning kinematics will form the key unit of focus to be embraced in this framework. The key sub-topics that constitute the topic on kinematics all seem to embrace various algebraic concepts that play a key role in explaining different kinematics concepts. To this end a relevant theory of use and understanding of mathematics is adaptable to the development of the ensuing logic in this study so as to explore the concepts to full capacity. Mathematical resources are proposed to be part of the realm for this study as they seem to form a large aggregate of source of mathematical difficulties students encounter when constructing concepts in kinematics. Mathematical resources represent the wide view of mathematics knowledge elements that should be activated if students are to be successful in solving kinematics problems. These will be adopted to constitute another segment of the evolving framework.

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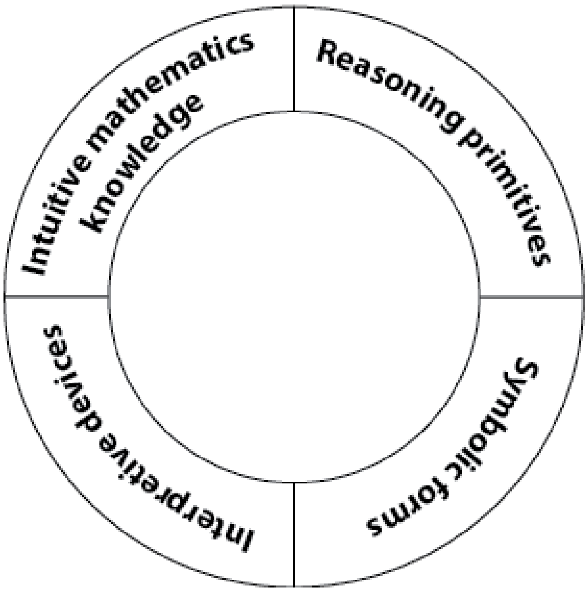

Figure 3. 3: Mathematical Resources

The development of the above two segments and their inherent subsections are in line with the GST's philosophy of units, connections and systems thinking. Segments are a way of presenting the two units of focus being; the physics topic on kinematics and mathematics. The third unit of focus, the "students understanding of physics concepts", will be analysed through (Greeno, 1989) Extended Semantic Model. The ESMT will be used to identify patterns of understanding that are a result of mathematical skills students use in solving problems found in the different kinematics sub-topics (see Fig 3.4). It will be critical for the purposes of this study to be able to discern which mathematical resources lead to what knowledge domain. This observation will help address the research question, "What kind of mathematical difficulties do students encounter when exploring kinematics concepts?" Merging the three segments; kinematics sub-topics, mathematical resources and the ESMT should be able to produce the conceptual framework suitable for this study as illustrated in the Fig 3.4 below:

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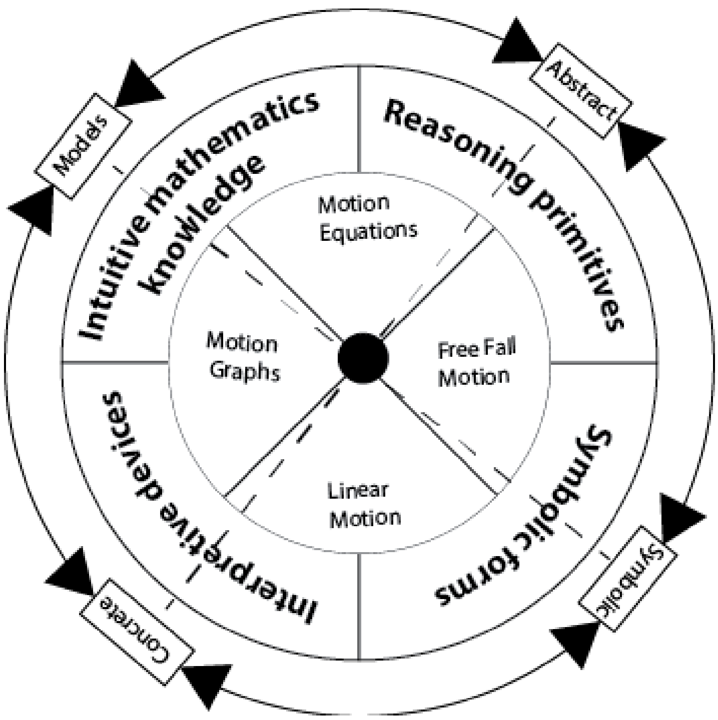

Figure 3. 4: The Kinematics conceptual Framework.

The expectation is that as students implement the conceptual model in solving problems from different sub-topics in kinematics, a particular mathematical resource will be activated. The mathematical resource indicates a suitable mathematical approach to be used in solving problems in kinematics. The pattern of understanding reflects the domain or combination of ESM domains discernible. An activated mathematical resource like Symbolic Forms may link well with the Symbolic domains of the ESM, since both are about symbols. Symbolic forms may also link with the Model Domain as Sherin (2001) puts it, “symbolic forms are simple types of mathematical models”.

The Interpretive Devices are well aligned with Abstract domains since “reason” is an abstraction that could simply be a congruence of concepts, laws and principles. Intuitive mathematical knowledge resource should be activated in any of the four domains when solving physics problems.

It is envisaged that applying the whole framework to the study should reveal the knowledge domains that emerge when specific mathematical resources are activated by a particular problem in kinematics and that should assist handle the difficulties encountered by the learners during construction of physics concepts.

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CHAPTER 4: RESEARCH METHODOLOGY

4.1 Introduction

This chapter discusses the philosophical assumptions, research paradigm, research approaches, and the research design underpinning the study. The pragmatic paradigm was discussed as relevant to the mixed methods approach adopted for the framework of the study. In addition, the chapter discusses the research methods, data collection instruments and data analysis procedures. Ethical considerations are also discussed.

4.2 Research approach

The research design for this study is a pragmatic paradigm using a case study that is analysed through both quantitative and qualitative methods. In this mixed methods design different methods are engaged to maximize the benefits available when applying different approaches to address a research question (Hendrick, 1994; Tashakkori & Teddlie, 1998). Quality and successful research must be attentive and responsive to the key research question(s) and its primary goals. To this end, understanding the kind of data that will be essential to meet the project's goals, developing a well-planned strategy(s) for collecting it, managing and analysing findings was requisite. The choice of the mixed method approach and its strengths and weaknesses was useful for particular approach over a given research question.

The diagnostic tests and questionnaires were used to establish a baseline as well as determine the enduring mathematical difficulties experienced by students when learning physics. Observations, face-to-face interviews and focus group interviews were used as data collection methods. Furthermore, the justification for each data collection method(s) used in the study was discussed. Finally, in order to ensure validity, reliability and trustworthiness of the research, appropriate criteria for both qualitative and quantitative research were discussed, and several methods that included crystallization and triangulation were suggested and later employed.

4.3 Research paradigm

According to (Kuhn, 1962), the word paradigm means a philosophical way of thinking. The word has its aetiology in Greek where it means pattern. Thomas Kuhn used the word to denote a conceptual framework shared by a community of scientists which provided them

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with a convenient model for examining problems and finding solutions. Kuhn defines a paradigm as: “an integrated cluster of substantive concepts variables and problems attached with corresponding methodological approaches and tools….”.He posits that a paradigm refers to a research culture with a set of beliefs, values and assumptions that a community of researchers has in common regarding the nature and conduct of research (Kuhn, 1962). The paradigm is a lens through which the researcher examines the methodological aspects of the research project and how data collected is useful for the study. It is a researcher’s “worldview” (Mackenzie & Knipe, 2006) .This worldview is the perspective, or thinking, or school of thought, or a set of shared beliefs that informs the meaning or interpretation of research data. According to (Guba & Lincoln 1994), the research process has four major dimensions: epistemology, ontology, methodology and the axiology. These elements comprise the assumptions, beliefs, norms and values that each paradigm constitutes.

Ontological and epistemological aspects concern what is commonly referred to as a person’s “world view”, about the nature of knowledge and how knowledge is acquired respectively, which has significant influence on the perceived relative importance of the aspects of reality. Two possible world views are: Objectivistic and constructivism. The different ways of seeing the world are eminent in most academic areas and none of these views is considered superior to the other. According to Lather (1986), research paradigms inherently reflect our beliefs about the world we live in and want to live in. Based on this belief, (Lincoln & Guba, 1994) distinguished between positivist, post-positivist and post-modernist enquiry. (Gephart, 1999), classified research paradigms into three philosophically distinct categories as positivist, interpretivist and critical post-modernism. The three philosophical perspectives view ‘true’ knowledge as based on different perceptions about what characterizes the ‘real world’. Positivists assume that reality is objectively given and is measurable using properties which are independent of the researcher and his or her instruments; in other words, knowledge is objective and quantifiable. However, the interpretivism and post-modernism seem to offer alternative theoretical, methodological and practical approach(s) to research. Objectivity can be replaced by subjectivity in the process of scientific inquiry. In this study, the positivist focus on quantitative methods, were complemented by using qualitative methods to gather broader information outside readily measured variables (Gephart, 1999).

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4.3.1 Interpretivism

The central goal of the interpretivist paradigm used in the study was to understand the subjective world of human experience Guba & Lincoln (1989). This approach makes an effort to get into the head of the participants being studied to understand and interpret what they are thinking as well as establish the meaning they are making of the context. According to Willis (1995), interpretivists believe there is no single research methodology more superior. (Gephart, 1999): argues that interpretivists assume that knowledge and meaning are acts of interpretation, hence there is no objective knowledge which is independent of thinking for reasoning humans. Consequently, the mind interprets experience and events, and constructs, meaning from them. Meaning does not exist outside the mind. Reality should rather be interpreted through the meanings that people give to their life-world. This meaning can be discovered through language, and not exclusively through quantitative analysis (Schwandt, 2007). Interpretivists embrace the view that, the social world cannot be understood by applying research principles adopted from natural sciences. The social science requires a different research philosophy. The three basic principles of interpretivists posits are that social world is constructed and given meaning subjectively by people; human beings are subjects that have consciousness or a mind while behaviour is influenced by knowledge of the social world, which exists only in relation to human beings. The researcher, is part of what is observed driven by interests. (Wisker & Blumberg, 2008).

Knowledge is developed and theory is built through developing ideas from observed and interpreted social constructions. Interpretive paradigm is underpinned by observation and interpretation, thus to observe is to collect information about events, while to interpret is to make meaning of that information by drawing inferences or by judging the match between the information and some abstract patterns (Deetz, 1996). Therefore the interpretivist approach attempts to understand phenomena through the meanings that people attach to a situation (Deetz, 1996)

Reeves and Hedberg (2003), observed that the interpretivist paradigm stresses the need to put analysis in context. They use meaning (versus measurement) oriented methodologies, such as interviewing, participant observations that rely on a subjective relationship between the researcher and subjects. It does not predefine dependent and

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independent variables. Rather, it focuses on the full complexity of human sense making as the situation emerges (Kaplan & Maxwell, 1994)

4.3.2. Positivism

Positivist approach to research is based on measuring variables of a social phenomenon through quantification. According to Guba and Lincoln (1994), they argue that positivism is rooted in the ontological and epistemological assumptions of objective reality. It is concerned with the variables which embrace a number of assumptions about the social world and how it should be investigated. The quantitative methods used in positivist approach concentrates on the confirmatory stage of the research cycle that is the formulation of a hypothesis and the collection of numerical data to test the hypothesis. Quantitative methodology used in this positivist approach aims to measure, quantify or find the extent of a phenomenon, as opposed to qualitative methodology, which is more concerned with describing experiences, emphasizing meaning and exploring the nature of an issue. Positivist approach was considered to provide explanations of different phenomena for realistic outcomes of the study. The large data collected is analysed using statistics to enable generalisation over a social phenomenon investigated.

4.3.3. Pragmatism

Pragmatism is an approach to research that combines both the quantitative and the qualitative aspects of research. It is outcome-oriented and interested in determining the meaning of things (Johnson & Onwuegbuzie, 2004) or focussing on the product of the research (Biesta, 2010). It is characterized by an emphasis on communication and shared meaning-making in order to create practical solutions to social problems. Pragmatist approach places primary importance on the research question (Teddlie, 2003). It is based on the belief that theories can be both contextualised and generalized by analysing them for “transferability” to another situation. To this end, the researcher is able to maintain both subjectivity in their own reflections on research and objectivity in data collection and analysis. According to Morgan (2007), creating “shared meanings and joint action”, points to the underlying belief in complementarity that espouses that qualitative and quantitative can be combined in order to ‘complement’ the advantages and disadvantages present within each approach to research. “Shared-meanings” created by a mixed methods integration promotes inter-subjectivity emphasizing disrupting the dichotomy between “complete objectivity” and “complete subjectivity’ (Morgan, 2007). Pragmatism was

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considered as a paradigm of choice in this study to breakdown the hierarchies between positivist and constructivist ways of knowing in order to look at what is meaningful from both (Biesta, 2010). The use of "abduction" in this approach will promote transferability and such will address the implications of the research study.

4.4 Research Design

A research design can be thought of as the logic or master plan of a research that throws light on how the study is to be conducted. It shows how all of the major parts of the research study- samples or groups, measures, treatments or programs, etc, work together in an attempt to address the research questions. It is synonymous to an architectural outline. The research design can be seen as actualization of logic in a set of procedures that optimizes the validity of data for a given research problem. According to Mouton (1996, p.176), the research design serves to "plan, structure and execute" the research to maximize the "validity of the findings". It gives direction from the underlying philosophical assumptions to research design, and data collection. For the purposes of this study, the researcher used the explanatory sequential mixed methods design. The case study was guided by two philosophical perspectives; the positivist for the quantitative phase of data collection and the constructivist paradigm for the qualitative phase. The researcher collected and analysed quantitative data in the first phase followed by qualitative data in the second phase as follow up to the quantitative results. The data from the two phases was connected by using quantitative results to shape the qualitative research questions, sampled the participants for the qualitative phase of the study as well as collecting relevant and informative data to answer the respective research questions for the study. The justification to choice of the paradigms was two-fold; mainly to help explain quantitative results that needed further exploration as well as using quantitative results to purposefully select best participants for the qualitative study. The key advantages of the explanatory sequential design are that it:

- It increases validity of the research by allowing convergence of different methods of inquiry which promotes triangulation.
- It widens the scope of research consequently increasing both breadth and depth of a study.
- It promotes complementarity of different facets of the inquiry because of the "overlapping" that occurs between the different methods.

In executing the research plan the researcher adopted a summary of phases in data collection as given in the model below by (Creswell & Plano Clark, 2008).

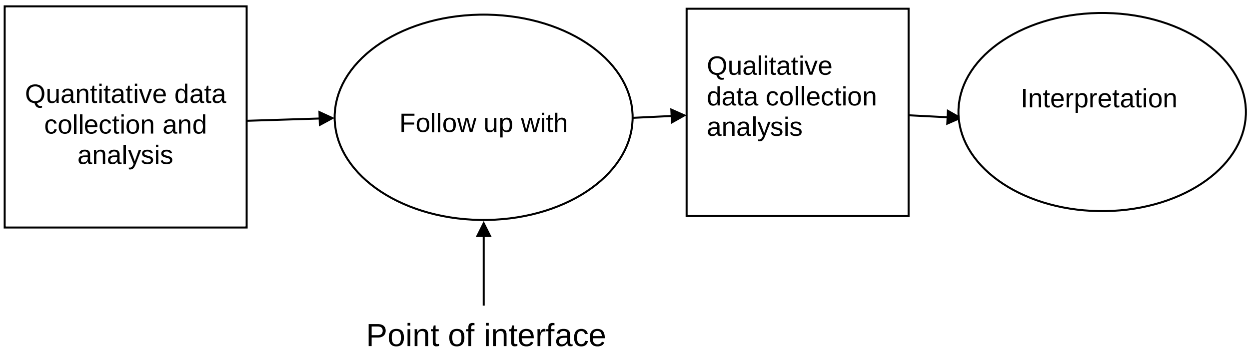

Figure 4. 1: Summary of phases in data collection

4.5 Research Methods

A research method is a strategy of enquiry, which moves from the underlying assumptions to research design, and data collection (Myers, 2009). Although there are other distinctions in the research modes, the most common classifications of research methods is into qualitative and quantitative. At one level qualitative and quantitative refer to distinctions about the nature of knowledge: how one understands the world and ultimate purpose of the research. On another level of discourse, the terms refer to research methods, that is, the way in which data are collected, analysed, and type of generalizations and representations derived from the data.

Quantitative research methods were originally developed in the natural sciences to study natural phenomena. While qualitative research methods were developed in the social sciences to study social and cultural phenomena. Neither of these two methods is intrinsically better than the other; the suitability of each method(s) is/are always decided by the context, purpose and nature of the research questions to be answered.

4.5.1 Quantitative research methods used in this study

Quantitative methodology involves collecting numerical data that can be subjected to statistical analysis. The data collecting methodologies often used involve diagnostic tests, surveys and experiments. For the purposes of this research, the researcher used diagnostic tests. A diagnostic test is an assessment tool given to the subjects to establish any preconceived ideas they are having about the problems being reviewed. The diagnostic test was administered to establish a baseline so as to give a guide to the

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researcher on what level of competence the participants were at, regarding the problems being reviewed in this study. A questionnaire implemented in an attempt answer the sub-questions: What is the nature of mathematical difficulties encountered by students when learning kinematics? Secondly what are the possible sources of the mathematical difficulties? A questionnaire is a tool for securing answers to questions by using a form where the respondents fill in information. They are designed to collect data from a large, diverse and widely scattered group of people. The researcher used the questionnaire for it is less expensive to administer, captures data from a number of people simultaneously, covers a wider area and hence obtains information from more people. Its standardized wording, questions and instructions ensure uniformity from one measurement situation to another. Besides, its anonymity during implementation, gives respondents confidence in responding making them free to express their own views. Lastly, standard questions are asked in the same order which gives reliability and consistency.

According to Maree (2007) and Salkind (2009), a questionnaire is a “economical tool”(i.e. questions are set being the same for all participants) and are treated anonymously. The main objective of a questionnaire is to gather facts and opinions about concepts from people who are informed about a particular issue (Delport et.al, 2005). Even if the questionnaire uses statements of questions, respondents in most cases respond to something written for specific purpose (McMillian & Schumacher, 2006). Quantitative tools such as the questionnaire give precise and measurable data in a quicker way which can be reported in the form of tables and graphs enhancing easy interpretation.

McMillian and Schumacher (2006) opine that, questions and statements used in questionnaires come in different formats such as closed-ended or open- ended formats. Such diversity is data collection effectively measures the respondents’ opinions, attitudes or knowledge (Fraenkel, 2003). In line with the same view, Cohen (2000) and Maree (2007) add that the closed- ended questionnaires have a high user- value as they are easy to complete in a short time. The respondents are kept focused on the topic that appears relatively objective and responses are always easy to table and analyse (Cohen, 2000) ; (Maree, 2007). Closed ended questions that consist of multiple choice questions, allow participants to choose answers from a number of options (Fraenkel, 2003). For the purposes of this study, the researcher used closed-ended questions in the questionnaire for quick access to some information. However, some components of the questionnaire comprised of qualitative questions including such aspects as reading of graph scales,

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calculations on the gradient in both the distance time and a velocity time graph as well as calculations on distance travelled in a velocity- time graph. Reading graphs for quantitative purposes involves various mathematical operations such as: counting, measuring, classifying number concepts and deducing relationships among different variables (Gillians and Lewis, 1994). Linear relationships of variables lid to straight line graphs while curved lines express non- linear relationships. Such connectivity demonstrated the need to understand the concept of ratios in mathematics so as to enable one to relate variables on a graph.

The survey with which data were gathered for this study involved use of a diagnostic Test as in appendix C, appendices D, E and F which constitute the questionnaires were split into two sections: Section A was on graphs in mathematics and Section C was on kinematics graphs physics (kinematics). The constructs tested in the questionnaire as well as the physics and mathematics questions used to investigate the participants' knowledge of these contracts were summarized in Table 4.1. This table shows the aspects tested in terms of categories of physical variables in symbols and their mathematical definition, the aspects assessed as well as the numbers for both physics and mathematics questions of the questionnaire.

Table 4. 1: Constructs tested in the physics and mathematics components of the survey.

Constructs		Mathematical Questions	Physics Questions
Area under graph	Description of area (Qualitative)	MQ11;MQ14	PQ13; PQ7
	Calculations on area (Quantitative)	MQ13;MQ14.	PQ2; PQ5; PQ6
Gradient of graph	Steep/Gentle	MQ1;MQ2;MQ3;MQ4; MQ5	PQ8;PQ9;PQ16;MQ17;18;19;20
	slope.	MQ6;MQ7;MQ8;9	
	Increasing/Decreasing/ Constant.	MQ10	
	(Qualitative)		
	Calculations on slope/gradient.		
Reading coordinates from graph (Quantitative)		MQ13;MQ14;MQ15;MQ1	PQ1;PQ3;PQ4
Reading changes in variables i.e.		MQ13;MQ14;MQ15	PQ7;PQ8;PQ9

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graph height ength (Quantitative)			
Form of graphs/Graph expression. (Qualitative)		MQ13;MQ14;MQ15	PQ5;PQ7;PQ11;PQ12PQ14

Table 4.2, identified the different types of kinematic graphs in physics section of the questionnaire. This also included the graph form and the meaning of the graphs targeted towards testing participants' knowledge of the graph types and graph forms respectively. For instance, participants were tested on graph knowledge from displacement- time *(s –t)* and velocity- time *(v–t)* graphs to determine total distance travelled as well as the velocity and acceleration of objects. They also determined the meaning and the significance of steepness of a slope in relation to either a *(s- t)* or a *(v – t)* graph. Furthermore, they also differentiated uniform from non- uniform velocity or acceleration as well as determining the average velocity.

Table 4. 2: Knowledge of forms of graphs tested in the questionnaire

Graph form	Meaning of graph	Physics Questions
Horizontal line in an *s-t* graph	constant displacement (stationary) zero velocity zero acceleration	PQ1; PQ3; PQ5;7; PQ10; PQ11; PQ12
s-t : slant line	uniformly changing displacement constant velocity zero acceleration	PQ1; PQ2PQ;7;PQ15
s-t: curved line	Changing displacement uniform acceleration. Non-uniformly changing velocity.	PQ3; PQ10; PQ15
v-t: horizontal line	Uniformly changing displacement. constant velocity zero acceleration	PQ3;PQ5;PQ10;
v-t: slant line	Changing displacement uniformly changing velocity zero acceleration	PQ1; PQ2; PQ3; PQ12; PQ15
v-t: curved line	Changing displacement Changing velocity Changing acceleration.	PQ1; PQ7 ;PQ15

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Other concepts tested under kinematics graphs included reading one type of graph to another e.g constant velocity in both a displacement- time (*s-t*) to constant velocity on a *(v –t*) graph, the change in height of slope meaning in both an *(s-t*) graph and a (*v-t*) graph-(slope – height confusion).The matching narrative data with relevant features of a graph, identifying when acceleration is negative or positive in a *(v-t*) graph as well as when velocity is negative in (*s-t*) graphs. The researcher also set questions to measure participants' competency in manipulating equations of motion in solving problems. The skills on manipulating variables, choosing relevant equations as well as deducing meaning of concepts from dimensional analysis was measured in this quantitative phase of the study.

While quantitative methods are used at times in education research, they have their own limitations. Quantitative researchers posit that the quantitative methods have limitations such as being labour intensive, require extra resources to analyse results and are inadequate in understanding some information such as changes in emotional behaviour and feelings by the respondents. They ask for a limited amount of information without explaining which compromises the aspects of validity. In questionnaires there is no way of telling how truthful respondents are. They are not suitable to investigate complex issues. The respondents at times ignore certain questions making the instrument lack detail. Worse off, there is no way of measuring how much thought has been put by the respondent in answering a question. Lastly, from the questionnaire, there is a level of researcher imposition which creates bias in decisions and assumptions as to what is and is not important.

4.5.2. Qualitative methods used in this study

Qualitative research is naturalistic; it attempts to study the everyday life of different groups of people and communities in their natural setting; it is particularly useful to study educational settings and processes. Qualitative research involves an interpretive, naturalistic approach to its subject matter; "it attempts to make sense of, or to interpret phenomena in terms of the meaning people bring to them (Denzin & Lincoln, Handbook of Qualitative Research, 2003)

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. According to Myers (2009), qualitative research is designed to help researchers understand people, and the social and cultural contexts within which they live. Qualitative research reveals people's values, their interpretative schemes, mind maps, belief systems and rules of living so that respondents' reality can be understood (Cavana, Delahaye, & Sekaran, 2001). The qualitative research methods emphasize careful and detailed descriptions of social practices in an attempt to understand how participants experience and explain their own world (Jackson, 1995). It is a constructivist type of research which emphasizes that knowledge is active and creative (Namanji & Ssekyewa, 2012). The goal of qualitative research is to discover the patterns that emerge after close observation, careful documentation and thoughtful analysis. Until these patterns are identified, quantitative proof of the causal nature of the variables cannot be investigated.

Qualitative data sources used in this research included interviews, observations and focus group interviews. Data in qualitative methods can be determined from direct observation of behaviours, from interviews, from written opinions or from documents (Sprinthall, Schmutte, & Sirois, 1991). The basic distinction between qualitative and quantitative research is the form of data collection, analysis and presentation. While qualitative research presents statistical results represented by numerical data, qualitative research presents data in descriptive form (words) attempting to understand phenomena in "natural settings." Quantitative data can be better understood and put into perspective when followed by qualitative data (Tashakkori et al., 2010).The researcher used a phenomenological approach to determine participants" ability to use their mathematical knowledge and skills to interpret and analyse kinematic problems. The qualitative research method used was the focus group interviews. According to Lederman et al. (2002), a focus group interview is " a technique involving the use of in-depth group interview where participants are selected because they are a purposive sampling of a specific population, this group being focused on a given topic" The type of data generated in focus group interviews is often deeper and rich (Thomas et al.,1995). It provides information about a wide range of ideas and feelings that interviews have about certain issues illuminating the differences in perspectives between groups or individuals. It can also generate large amounts of data in a short time and the data is based on the group interactions (Green et al., 2003).

The researcher used one- on -one interviews to solicit for data amongst participants who found it difficult to air their perceptions in a group setting. Both semi-structured and open-

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ended questions were used to gather data. This helped to obtain more informed data about the participants' perceptions on the nature of mathematical difficulties and their possible sources. Open ended questions were used to obtain participants' divergent opinions about the challenges under investigation. A focus group discussion was used with open-ended questions tailor-made to get detailed data on the participants' perceptions under a more relaxed setting of the group. The responses obtained from the open-ended questions were so varied and this provided room for divergent perceptions about the questions under review (Teddlie, 2003). The information collected from the respondents was categorized. The trends identified provided relationships between variables and factors under investigation which created some insights about question(s) of the research (Cresswell, Shope, & Clark, 2006). The focus group interviews centred on the following key constructs:

Basic theoretical Knowledge of kinematics

- Definition and units
- Equations of motion

Basic interpretation of kinematics graphs

- Gradient of displacement- time
- Gradient of velocity-time
- Average velocity

Mathematics knowledge applied in physics (kinematics equations)

- Making available a subject of the formula
- Correct computation of data in the equations e.g squaring of

  variables and determining the square- root.

Mathematics knowledge applied in kinematics graphs

- Equations of a (v- t) graph
- Area of (v-t) graph
- Gradient of a (v-t) graph

To complement the data collected in the focus group interview, a group of (6) participants were also interviewed on one- on- one from the class of 40 learners. This was done in an attempt to gather as more data as possible to broaden the scope of understanding of the

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problems (Marshall & Rossman, 2006). Unlike the in-depth interviews the one-on-one participants were selected randomly from 40 students. The researcher drew a schedule for the interviews which took a period of 4 weeks. Two participants were scheduled for an interview on two days of the week during study time. The focus group interview was also scheduled in the afternoon during the students study periods. Students were able to ask complex questions to explore depth on questions reviewed. The interactions during the interviews gave room to motivation to the participants. There was more room to respond to vague questions making information collected complete.

However, like any other research method, it has its own limitations. The data collected from the instruments are used for a small scale study. The findings from them cannot be generalized to study populations or communities. The data collected has the potential of being inconsistent. It is time consuming. The researcher used the qualitative methods integrated with the quantitative methods in the mixed method design to mitigate limitations inherent in each of the method supposing they were implemented independently. The use of the mixed methods by the researcher helped to understand the contradictions between the quantitative and qualitative findings which enabled collecting rich and comprehensive data to interrogate the research question(s). This helped to give urge to reflecting on participants' point of view allowing them enough voice to air their experiences about the mathematical difficulties they encountered when learning kinematics. The scholarly interactions espoused in the mixed method added breadth and depth in getting into the root cause of the difficulties. Such methodological flexibility by the mixed method approach earned its considerations for use in this research study.

4.6 Sampling

Sampling is a technique employed by a researcher to systematically select a relatively smaller number of representative items or individuals from a pre-defined population to serve subjects (data source) for observation or experimentation as per objectives of his or her study. Simply put, the researcher decides what needs to be known and sets out to find people who can and are willing to provide the information by virtue of their knowledge or experience (Bernard et al.1986). There are two main types of sampling techniques, namely probability also often called random sampling and non- probability sampling also coined as purposive or judgemental sampling. Probability sampling is one where the entire process of having a sample or unit for data collection is done randomly. This is often used

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in quantitative research. Non-probability sampling involves selecting informants with a specific type of knowledge or skill about what is being studied. It is a deliberate choice of informants based on the qualities they possess. Purposive sampling is often used in qualitative research. Probability sampling is easy to assemble and is often considered a fair way of selecting a sample from a given population since every member is given an equal opportunity to be selected. Its unbiased random selection helps the researcher to generalise and draw conclusions from the results of the study collected during quantitative research. Above all the researcher is able to make inferences about a wider population within a short space of time. Purposive sampling on the other hand is useful in providing a wide range of non-probability sampling techniques used in qualitative research designs when a multiple phases of research procedures are involved. It can provide the researchers with justification to make generalisations from the sample that is being studied.

4.6.1 Population

Like in any other research, collecting of data is the heart of any research. Data collection is always a basic necessity to enable the researcher to solicit for the right answers to the questions entailed in the research. A set of predefined procedures was systematically used to facilitate the answering of burning questions of the research. In order to obtain valid findings, sampling was done in this study. The study's objectives and the characteristics of the study population (such as size and diversity) determined which and how many people were selected for the research. According to (Best & Khan, 2003), a proportion of the selected population known as the sample can be used for the both observation and analysis. For the purposes of this study the researcher used both purposive and probability sampling to draw answers for questions reviewed. A class of 40 students doing Pure Physics was drawn from a population of 600 learners also doing the same course at Kgari Sechele Senior Secondary School in Botswana. A diagnostic test and a questionnaire were administered in the quantitative phase of the study in order to identify mathematical difficulties students had which hindered their understanding of physics. This was to find out more about the purported difficulties hypothesised as existing in learning physics as alluded to in the sub-questions of the research study. A sample of 6 participants was then purposively selected from the cohort of 40 students to make the focus group for the qualitative phase of the study. The sampling for this phase of study was based on the students' performance in the diagnostic and the questionnaire tests. A balanced

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performance ranging from least through the average and then best performers were selected to constitute the focus group for the qualitative phase of the research.

4.6.2 Sampling procedures

A class of 40 Form four students all doing Pure Physics was formed using the stratified random sampling method out of six classes doing the same course. In this method, groups of learners sharing same attributes or characteristics made the sample for the study. A random sample from each stratum was taken in a number proportional to the stratum's size when compared to the population of Pure Science learners in the school. The subsets of the strata were then pooled to form a random sample used for the study. Purposive sampling was conducted based on different levels of performance in each sampled questions from the questionnaire. This helped to identify recurring mathematical difficulties demonstrated by learners on solving varying physics problems. The probability sampling used earlier in the quantitative phase helped to establish the target group which positioned the researcher well in blending data collected with problem(s) under investigation (Charles & Mertler, 2008). The sample groups were based on convenience. According to (Anderson, 1998), this meant that the selected participants were easy to monitor, quick and also convenient to access during the research.

4.7 Data collection procedures

The collection of data is of significant importance when any kind of research is conducted. The nature of data must be relevant, adequate, of quality, valid and reliable to answer the proposed research questions. To this end, the procedure of any data collected must unveil the method(s) adopted, tools relevant, the technique used of collecting data, draw out the sample of the study and the steps of administering the tools and their scoring. It also looks at the technique that will be used for data analysis and why such techniques(s) will be of relevance and significant to the study. Precisely, a procedure is the logical progression of events during data collection.

The researcher used the diagnostic test, the questionnaire, in-depth interviews and the focus group as relevant tools to interrogate the study. The researcher collected quantitative data then moved to gather the qualitative data to complement and expand the findings obtained in the quantitative phase. Such an approach helped to cover the aspects of validity and reliability of data collected.

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4.7.1 Quantitative data collection procedures

A group of 40 Form Pure Physics participants was formed from a cohort 600 learners studying Pure Sciences at Kgari Sechele High School in Botswana. Random sampling was implemented to establish a sample used for the study. The researcher drew up a schedule for the afternoon studies and administered the diagnostic test to establish the level of mathematical competence they had prior the research. The diagnostic test sampled questions, helped to elicit difficulties cited as a hindrance to their study of physics. The questionnaire comprised of logically sequenced closed and open-ended questions for the primary phase of the study and also tested constructs associated with, manipulation and application of equations, simplifying algebraic expressions, use of trigonometry as well as computation of data on simple fractions. A summary of the schedule was given as in Table 4.3 below. Table 4.3: Interview questions summary guide.

Table 4. 3: Interview guide schedule

	Type of assessment	Timeline
Phase 1 of data collection (questionnaire)	. Graphs on mathematics constructs . Equations of mathematics constructs	1 week
Phase 2 of data collection (questionnaire)	. Graphs on kinematics . Equations on kinematics	1 week

The researcher took the participants through a questionnaire assessment program which took a fortnight. Some samples of participant responses were video recorded and documented for use in the analysis stage.

4.7.2. Qualitative data collection procedures

A sample of six participants was purposively chosen from a group of 40 students. The group was chosen based on their performance in the questionnaire tasks. Participants' performance in each question was useful as sampling criteria to enhance findings. Onwuegbuzie and Leech (2009), opine that the sample size must be more compatible with the research purpose of the study. Sharing this view, the researcher selected a homogeneous group of participants with the same attributes. The focus group was gender balanced with the group comprised of 3 boys and 3 girls to cater for biasness. The small group was believed would allow participants to interact fully, express themselves freely to

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(Akintoye, 2015)gain insight about problems being explored. The participants from the group were also interviewed on one-on-one to interrogate the constructs under investigation in depth. An interview guide consisting of semi-structured questions was used and data collected through video recording. The interviews were scheduled in the afternoons during their study periods and were run for a period of 4 to 5weeks. After the individual interviews, focus groups discussions were also conducted and recorded. The schedule for the interviews was planned as in Table 4.4 below:

Table 4. 4: Summary of the Participant(s) schedule

	Week 1	Week 2	Week 3	Week 4
Participants	A and B	C and D	E and F	Focus Group

4.8.1 Validity and Reliability for quantitative data.

In quantitative research validity is defined as the extent to which an instrument measures what it is purported to measure. It is measured using empirical and theoretical evidences. The theoretical assessment is done using a panel of experts who rate suitability of each item to measure a particular construct and evaluate its fitness in the definition of a construct. The empirical assessment on another hand involves quantitative analysis that uses statistical techniques. There are two types of validity; internal validity and external validity. Internal validity constitutes two forms, content validity and construct validity while external validity consists of face validity and convergent or discriminant validity. Internal validity is the extent to which the results are attributable to the independent variables. It looks at how the results of changes in the dependant affect the independent variables. In other words, it looks at the extent to which the researcher is able to eliminate any

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confounding evidence in the data collected. External validity looks at the degree in which the results can be generalized to other contexts. Such can be achieved if the data is drawn from a sample that is representative of the population. If the results from a small population can be generalised in a large population then it has a high external validity.

Reliability is the degree to which a research instrument produces stable or consistent results, (Kirk & Miller.M.L, 1986). According to (Joppe) 2000, if the results of a study are able to reproduce under a similar methodology then the instrument is reliable. There are four types of reliability and these are: test-retest reliability, inter-rater reliability, internal consistency reliability and split-half reliability. According to (Drost, 2011)test-retest reliability is a measurement of consistency between measurements of the same construct administered to the same sample at two different points in time. A correlation is done between sets of tests and if significant then observations have not changed substantially and this helps to establish its degree of reliability. The inter-rater reliability, involves rating of observations using a specific measure but by different experts. Reliability in this case is done by correlation of scores from two or more raters on the same construct. Internal consistency reliability is a measure of consistency between different items of the same construct. It measures the consistence within the instrument and questions on how well a set of items measures a particular characteristic of the test. Single items within a test are correlated to estimate the coefficient of reliability. The split-half reliability is another form of reliability and it measures consistency between two halves of construct measure. A correlation between the two halves must be obtained to determine the coefficient of reliability. According to Denzil (1989), error is the main factor affecting reliability. The major sources of error come from the researcher, participants, the social context and the methods of data collection and analysis. The error often brings about bias in data collected, distorted findings and possible elicitation of irrelevant or wrong data.

In this study the diagnostic/baseline test used had items adapted from the Mechanics Baseline Test by Beichner (1994), popularly used by other researchers. The items used in the test required algebraic manipulation which addressed the key objectives of the study. The questions from the standard test helped to validate the nature of items suitable to address the mathematical difficulties and assured high content validity. A panel of experts was engaged to rate the items based on the syllabus objectives. Each item was matched to the objectives to establish the relevant content domain based on the informants' cognitive level which established high content validity. The experts analysed the distractors

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of each question and made requisite modifications where necessary, which assured construct validity was addressed. The wording of each question was analysed and modifications done to ensure clear communication of what was to be tested.

To address reliability, the diagnostic test was administered three times which assured reliability in form of repeated measurements. The refinements effected helped to improve clarity of questions and assured test-retest reliability. Three experts from each department of physics and mathematics subjects were consulted and they made modifications based on the cognitive level of the questions, their wording, the length of the diagnostic test and the distractor items in the questions. The experts marked the scripts of test using the same rubric and where they obtained same scores a measurement of agreement was expressed by calculating the Cohen's kappa interrater coefficient which assured interrater reliability. In cases where the interrater agreement had a k value = 1, they were in complete agreement but if k = 0 then they had no agreement. An establishment of the kappa coefficient helped to minimize bias in the measuring tool and consequently improved reliability.

The questionnaire used in the research was validated by adapting some standard questions from Test of Understanding Graphs in Kinematics (TUG-K). Some of the questions were also obtained from the Force Constant Inventory (FCI) a popular tool also often used by other researchers. The questions adapted strengthened content validity. The items that were added to the (TUG-K) standard questions, covered specific objectives to address the research questions and were aligned to the syllabus which assured construct and content validity. A total of six physics and mathematics experts volunteered in editing the cognitive level of the questions, the wording of questions, distractors of the questions as well as administering the questionnaire which helped to establish content validity.

The questionnaire was administered three times and corrected four times for clarity which assured reliability in the form of repeated measurements. The experts marked the scripts using the same rubric and the extent to which they obtained similar scores using the same rubric was expressed by calculation of the kappa interrater correlation coefficient. A correlation of the scores was determined by using the mean values and a calculation of the Cronbach's alpha coefficient using (SPSS). The correlation analysis increased the objectivity of the instrument which assured internal consistency and its reliability. A

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Cronbach alpha of the range between 0.80 and 1 indicated a high level of consistency while 0.5 and below a low level of internal consistency.

4.8.2 Trustworthiness of qualitative data

In the interpretivist paradigm, the internal and external validity and reliability are replaced by the criteria of trustworthiness and authenticity. The criteria include credibility, dependability, confirmability and transferability. According to Guba (1981), credibility is the extent to which data and data analysis are believable, trustworthy and authentic. It anchors on the researcher's ability to investigate the question to determine how the findings align with reality as constructed by him and the participants engaged in the research. The criteria of dependability, is the ability to observe same outcomes or findings under similar circumstances. It assumes that human behaviour by nature continuously change, contextual and is open or subject to a multiple interpretations of reality. Therefore, cannot reproduce exactly same results. The researcher makes inferences and interpretations influenced by his own construction of meaning from the collected data and ensure that the findings truly emerge from the data gathered. Confirmability on the other hand, refers ensuring that the findings are the result of the informants' true lived experiences. Lastly, transferability refers to the researcher's efforts to ensure they provide enough contextual data about the research so that other readers can relate those findings to their own.

To meet the criteria of trustworthiness, the researcher ensured that credibility of the findings was met through video recording the interviews, transcribed the informants' perceptions and coded them to establish themes about the nature of mathematical difficulties they encountered. Such a rich account of their experiences gave a lot of credence and authenticity to the findings as it captured the finer details of their lived experiences. The interpretations done by the researcher was in contextual detail and helped to identified patterns based on what the informants were reporting as mathematical challenges. For example Mellitah was quoted as saying: “I have a big challenge when it comes to use of the fourth equation of motion where I have to find the square root of a number or make one of the variables the subject of the formular”. Karen also reiterated that: “I really struggle to use equation three which involves squaring numbers and often times the requirement where I have to solve for an unknown variable”. The researcher used probing and iterative questioning to give credibility to the findings. At some stage of

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data collection, the researcher asked students to read transcripts of data collected as a way of checking if what was transcribed was a true reflection of what they had said which also assured credibility in the findings. The examination of previous research findings was also done by the researcher to assess the degree to which the research's findings was congruent with those of the already existing body of knowledge which assured credibility as well. Real students in Form 4 took part in the study and all questions asked were related to the real topics taught in both mathematics and physics syllabus(s) and this established authenticity of the study.

The interview questions were influenced by student responses to the diagnostic test and the questionnaire tool. The questions were shared and discussed with experts in the two subjects, physics and mathematics prior to the interviews to solicit for their perceptions and that assured credibility. A focus group interview was used to collect data for the study. Open-ended questions were used during the interview which allowed participants to openly express their opinions about the mathematical difficulties they encountered from their lived experiences during their studies in the two subjects and this assured dependability. The researcher used inference to give his own interpretations about the students' perceptions concerning the nature of mathematical difficulties which assured dependability. The questions were shared and discussed with experts from the two subjects prior the interviews to minimize bias and which assured confirmability. The interview progressed on what students were saying to ensure that findings were a result of their experiences which assured authenticity.

4.9 Data analysis procedures.

According to Marshall and Rossman (1990), data analysis is the process of bringing order, structure and meaning to mass of data collected. Hitchcock and Hughes (1995), takes it one step further, "…. the ways in which the researcher moves from a description of what is the case to an explanation of what is the case is the case". In any scientific study, comprehensive data must filter what actually exists as a problem and the instruments should measure what they are really intended to measure, Denzin (1997). Sharing the same view, the researcher was mindful in considering the aspects of validity and reliability in data analysis.

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4.9.1 Quantitative data analysis procedures.

All the data collected from the questionnaire was statistically analysed using SPSS a computer software. Graphs on correlation analysis were drawn to interpret the findings from the data collected to facilitate drawing of relevant conclusions.

4.9.2 Qualitative data analysis procedures

The data collected from the interviews and the focus group, was transcribed, thematically coded and responses from the informants categorized to identify recurrent and emergent themes.

4.10 Ethical compliance procedures.

The researcher obtained the ethical clearance from the Regional Office which is a Department of the Ministry of the Skills and Development in Botswana before the research started and permission was granted (see Appendix A). The permission to access data from the institution was granted by the School Headmaster and he signed a consent form granting permission to conduct the research (see Appendix B).

4.10.1 Ethical issues associated with the participants

Ethical issues associated with the participants addressed in this research were informed consent, right of refusal to take part without penalty, right to withdraw without penalty, confidentiality and anonymity, deception and security as well as safety to prevent any emotional or physical harm. Participants were made to sign a consent form (see Appendix C). All participants informed of the nature of the research, how the data would be used and the average length of time that would be spent to complete the investigations. The researcher did not discuss the research with anyone outside the research milieu.

4.10.2 Ethical clearance from the Stellenbosch University

An ethical clearance to do the research was granted by Stellenbosch University (see Appendix D)

4.10.3 Ethical issues regarding data and data analysis and reporting

The data collected was kept secure once collected and stored safely to report the results of the research with integrity and accuracy. No values or information was excluded from

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the analysis. In this whole investigation the researcher attempted to maintain objectivity and integrity and to employ absolute professional judgment.

4.10.4 Triangulating all the analysis

Use of all the three data collecting instruments namely; diagnostic test, questionnaire and interviews gave credence to the findings of the study. The different sources complement and corroborate each other. Depth would be achieved through triangulating the various data sources (students' scripts and interviews). Sampling sites: multiple tests (different kinematics questions); individual and group interviews (multiple views) lead to greater breadth. What the students wrote in the diagnostic test and what they said in the interviews, about the role of mathematics in physics, was corroborated with emerging trends when analysing their mathematical approach to kinematics physics-problems in tests correlating their affective and the cognitive domains.

4.11 Conclusion

In this chapter the researcher discussed the philosophical assumptions, research paradigms and the research approach (es) underpinning the study. The research designs and strategies adopted for the study were discussed in detail and justifications to their choice in this research looked at in depth. The quantitative and qualitative methods were discussed and the adoption of the mixed method design justified for the study. The idea of population and sampling was discussed as means to alert the researcher on ways to improve on data collected during the research. Sampling techniques were described and their effects on data collected highlighted to improve on data collected during the research. Both quantitative and qualitative data collection procedures were implemented for the purposes of this research and appropriate analysis procedures conducted to process the data recorded. Issues on ethical considerations were discussed in this chapter to ensure the correct research procedures were adhered to throughout the research. The chapter that follow looks at the data collection, analysis of results and their interpretation respectively.

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CHAPTER 5: ANALYSIS OF DATA AND INTERPRETATIONS

5.1 Introduction

This chapter presents the implementation of relevant data collecting techniques and tools for the study. Different forms of data analysed in order to arrive at answers to the primary research question, which is:

What mathematical difficulties do physics students encounter when learning kinematics concepts?

Testing of the hypothesis and a thick description of qualitative data and the analysis of the responses of the participants is presented. According to Gay (1996), "Analysis of data is as important as any other component of the research process. Regardless of how well the study is conducted, inappropriate analysis can lead to inappropriate conclusions." In this chapter, both sampling and data collection techniques used in quantitative and qualitative research are the focus. Sampling's primary purpose is for the selection of a suitable population ('elements') so that the focus of the study can be appropriately researched. Effective sample selection is essential because inappropriate procedures may seriously affect findings and outcomes of any study. To this end, ascertaining use of appropriate participants for each research methodology was conducted for the 'right' size(s) of population to yield and meet the purpose(s), context and the richness of data collected. The chapter also looks at the nature of mathematical difficulties encountered by learners, their possible sources and ways or means to mitigate them in a view to address the three sub-questions of the research study which are:

- What is the nature of mathematical difficulties encountered by students when learning kinematics in physics?
- What are the possible sources of the mathematical difficulties?
- How can the mathematical difficulties be mitigated in the physics classroom?

5.2 The sample size

For the purposes of this study, the researcher selected a group of 40 participants from a population of 600 Form 4 learners doing sciences to investigate mathematical difficulties they encountered whilst learning kinematics. A questionnaire was used to collect data for the quantitative phase of the study. For the qualitative phase, 6 students were selected for

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the interviews and focus group discussions were used to collect the data. Students were selected through use of purposive sampling and used as a unit of analysis to identify the kind of mathematical difficulties experienced whilst learning concepts in kinematics. Questions on different constructs were used to explore difficulties in different concepts and why they were a hindrance to learning and understanding (physics) kinematics in particular, as it often contributes to them dropping the subject at Form five before completing the course. Problems encountered by learners in relation to mathematical challenges are multi-dimensional in nature. Therefore, small samples used in interviews and group discussions helped to illuminate the difficulties students encountered.

5.3 Data Analysis

The data on tables and graphs below constitute a summary and analysis of findings gathered to explore the problems on mathematical difficulties encountered by learners during construction of their own knowledge on kinematics concepts. Table 5.1 below illustrates the results for the percentage performance in diagnostic mathematics questions used as baseline to their mathematical background. Table 5.1 shows a summary of the descriptive statistics for the students' performance. The mean was 40.62 % with the lowest learner obtaining 12 % while the highest got 76%. The standard deviation was 18.7.

Table 5. 1: Descriptive statistics for overall performance diagnostic mathematics questions

	N	Minimum	Maximum	Mean	Std. Deviation
Maths_Marks	13	12	76	40.62	18.679
Valid N (listwise)	13				

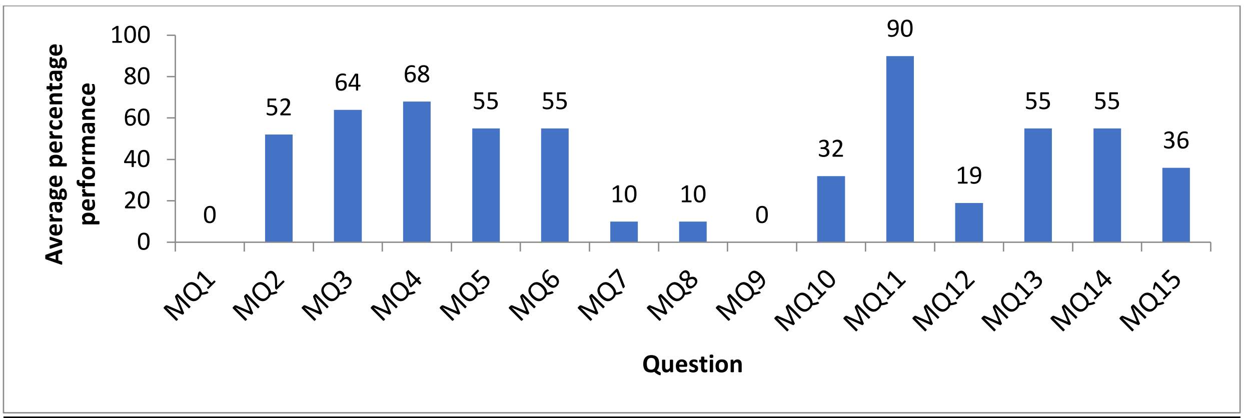

Figure 5. 1: Students' performance on individual questions

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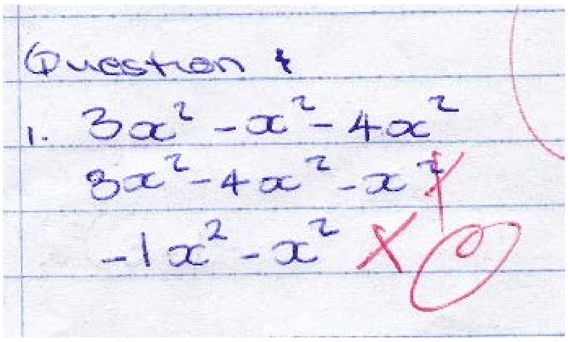
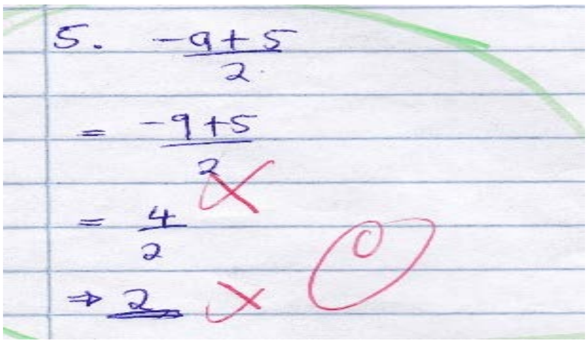
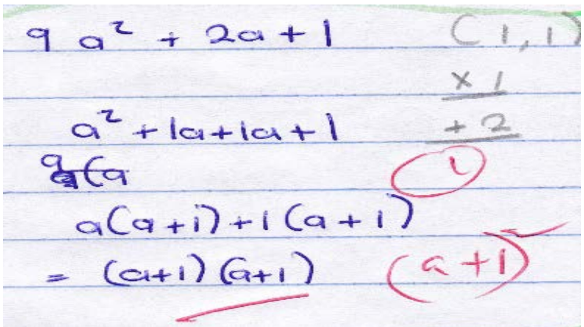
From the data analysed in Figure 5.1, it shows students performed poorly in some questions of the diagnostic test. From the table of results collected, it shows that there was poor performance in question MQ7; MQ8; MQ9. The lowest performance was recorded in question MQ1; MQ9; MQ7 and MQ 8 where the performance was within the range of 0% and 10%. Generally the performance indicates a low performance of students across the different concepts tested. The questions where students performed poorly measured different specific objectives. The Questions MQ1 and MQ9 measured concepts on simplifying equations and evaluating them as illustrated in Figures 5.2a and 5.2c respectively.



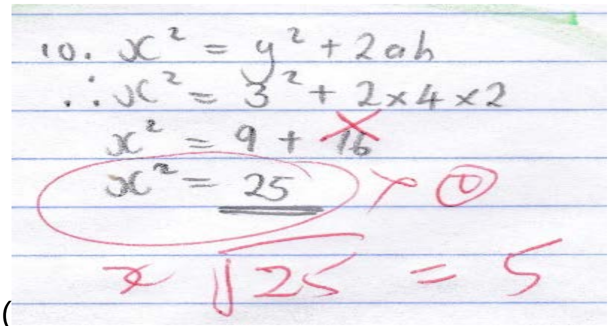
a)



(b



(c)



d)

Figure 5. 2: Simplifying expressions and evaluating equations

The performance by students was also low for question MQ10 where the students were required to find the square root of a number. There was another poor performance recorded questions M12 and M15 which tested the concepts on graph comprehension. Most of the learners could not use mathematics to relate variables on a graph as illustrated in questions MQ12 and MQ15 as illustrated in Figure 5.3 involving the interpretations of the gradient.

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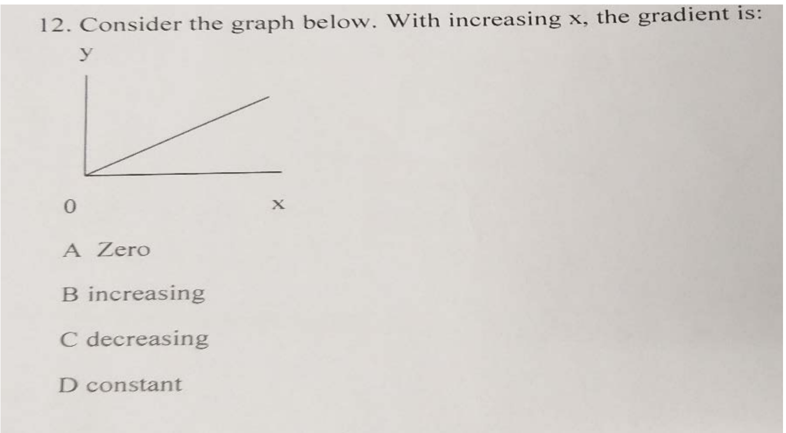

12. Consider the graph below. With increasing x, the gradient is:

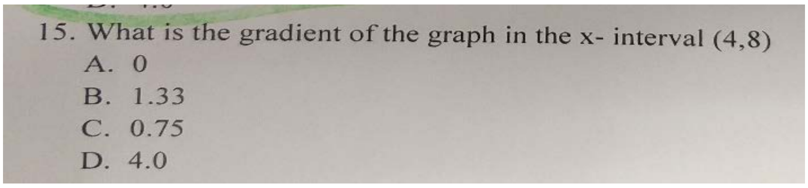
A Zero

B increasing

C decreasing

D constant

a) Difficulties experienced by students in interpreting a zero gradient


15. What is the gradient of the graph in the x- interval (4,8)

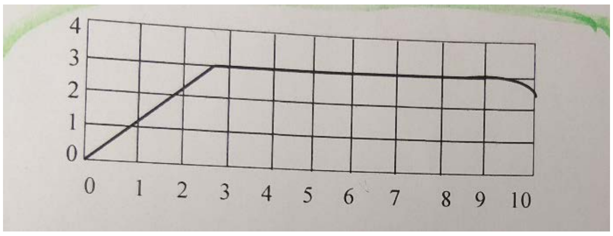
A. 0

B. 1.33

C. 0.75

D. 4.0

b) Students difficulties in describing the relationship between x and y coordinates



c) The calculation of an area under the line of a graph sample **c**

Figure 5. 3: Students difficulties with mathematics concepts

Generally of the misconceptions identified which are also found in physics, it then makes it difficult for students to make sense of physics with some of these misconceptions. Mathematical skills are important in enabling the learners manipulate different equations of motion used in solving different physics problems. Any limitations to that effect tend to risk understanding of different concepts by students in kinematics and physics in general. The other challenges emanated from their limited knowledge of graphs. From the above samples there is an indication that some students did not understand the meaning(of a zero gradient as shown by the poor performance in questions (M12 and M15) above. The caculations of a gradient and that of distance represented by the area under the line of *(a v-t)* graph (Figure 5.3c) were a challenge to some participants.

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The results confirmed that participants had a problem of discriminating meaning amongst position-time, velocity-time and acceleration-time graphs (see attachment on sample question below): This involved difficulties in reading values on given coordinates of a graph, describing the relationship between the x and y - coordinates, calculating gradient of any given line as evidenced in the students written sample exercises in Figure 5.4 below: The majority of the learners gave response C for an answer indicating they could not tell the differences in relationship on varying motion graphs.

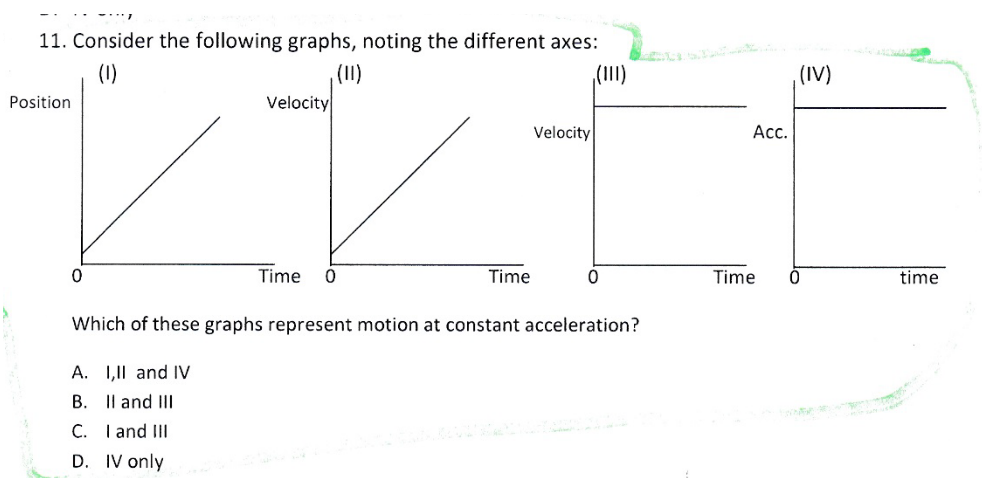

Figure 5. 4: Students' difficulties in differentiate meaning in motion graphs.

In a different context, McDermott, Rosenquist, Emily, and Zee (1987) confirm problems associated with identifying features of graphs like coordinate points, differences between coordinates as well as determining the gradient of any line of a graph. The results of the diagnostic test generally indicated that participants were unable to develop concrete connections between the dependent and independent variables (Connery, 2007). The findings attest the importance of the General Systems Theoretical framework used by the researcher as key in integrating knowledge for meaningful learning. From the mathematics questionnaire maximum standard deviation was approximately 26.4 while the minimum turned out to be 14.3. From this result, depending on how close the standard deviations are and how much less they are than the mean value reflect existence of some problems.

5.4 The contextual knowledge and inferential process

5.4.1 Reading coordinates from a graph

Students performed fairly well in questions MQ.13 and MQ.14 which involved reading values from the coordinates of graph a confirmation that at least more than half the group

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of participants could read coordinates. For a total of 18 students out of the 35 got question MQ.13 right, 8 got it partially correct and 9 missed it completely. For question MQ.14, 19 participants got the question right 7 partially correct and 9 got completely wrong. This attested that at least more than half of the participants could read the coordinates of a point when given one of the coordinates. The results of the constructs given on Table 5.1 are discussed in greater detail in the paragraphs that follow below. Participants' performance in mathematics questions were compared to those of physics as shown in Figure 5.5 and Table 5.2 respectively.

5.4.2 Determining and interpretation of gradient

In order to investigate whether the participants exhibited the mathematical skill of determining the gradient at a point, questions MQ.12 and MQ.13 were given using simply y and x variables Most of the students wrongly calculated gradients as $\frac{x}{y}$ instead of $\frac{\Delta y}{\Delta x}$ and this is in agreement with (Beichner, 1994) findings almost similar to mine where participants had difficulties in calculating the gradient of any line as shown in their poor performance in question MQ.15 where 13 students got the question right while 5 got it partially correct and 17 got it all wrong. There was notably a mixed performance obtained from these results. For straight line graphs passing through the origin, participants were able to find the gradient as in question MQ.13. However, they had challenges with interpreting the meaning of a zero gradient as attested by their failure to answer question MQ.15 as in the diagnostic test. This could be attributed to errors in the *x*-values in a situation where the line is horizontal along the x- axis as cited by (Beichner, 1994).

5.4.4 Area under the graph

About 90.3% of participants were able to determine the geometric shapes with greatest and smallest areas in questions MQ.11. It is evident that the majority of the participants could compare the areas of shapes by inspection. This is an important skill in visual decoding and judgment. Given one of the coordinates of the graph, about 54.5% of the participants were able calculate the area under the graph. Participants' seemed capable of comparing areas of geometric shapes although they had difficulties to calculating the area beneath a line graph.

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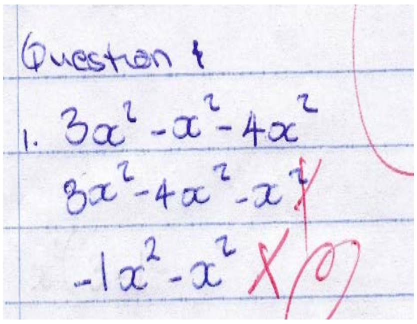
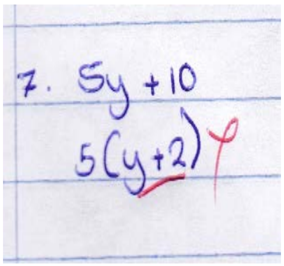
5.5 Solving algebraic equations

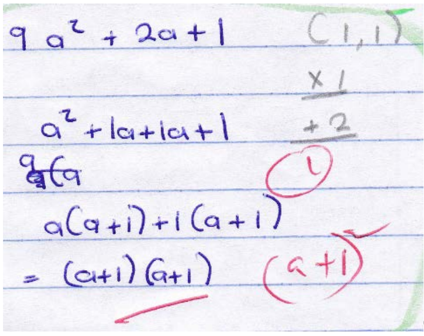
Students had difficulties in simplifying some mathematical expressions where they were to make the unknown variables a subject of the formula as commonly found in most physics equations especially the kinematics equations of the nature; $v^2 = u^2 + 2as$ and $s = ut + \frac{1}{2}at^2$ similar to solving for 'x ' in the expression $x^2 = 2ab + 1s$ given values of b, a and s or solving for(x) in equation: $x^2 = y^2 + 2ah$ given values of y, a and h respectively (see sample solutions in attachment below):



a



b



c

Figure 5. 5: Students' difficulties in solving algebraic equations

From the students' example scripts in Figure 5.4 and 5.5 above could not simplify simple mathematical expressions or just could not solve problems on making a variable a subject of the formula. However a fairly high percentage performance of between 67.7% and 64.5% was recorded for questions MQ.3 and MQ.4 that tested the concept on expanding a bracket commonly encountered in most of the physics equations. This was proof that many students experienced difficulties on some mathematical aspects that seemed prerequisites when handling physics concepts. They, however, ought to be reminded about similar concepts when they get into physics lessons. Overall, few participants passed the diagnostic test administered to test different mathematical concepts. That confirmed existence of some latent mathematical difficulties in solving problems of varied nature linked to different constructs in kinematics. The Figure 5.6 below shows a summary of performance in physics questions administered:

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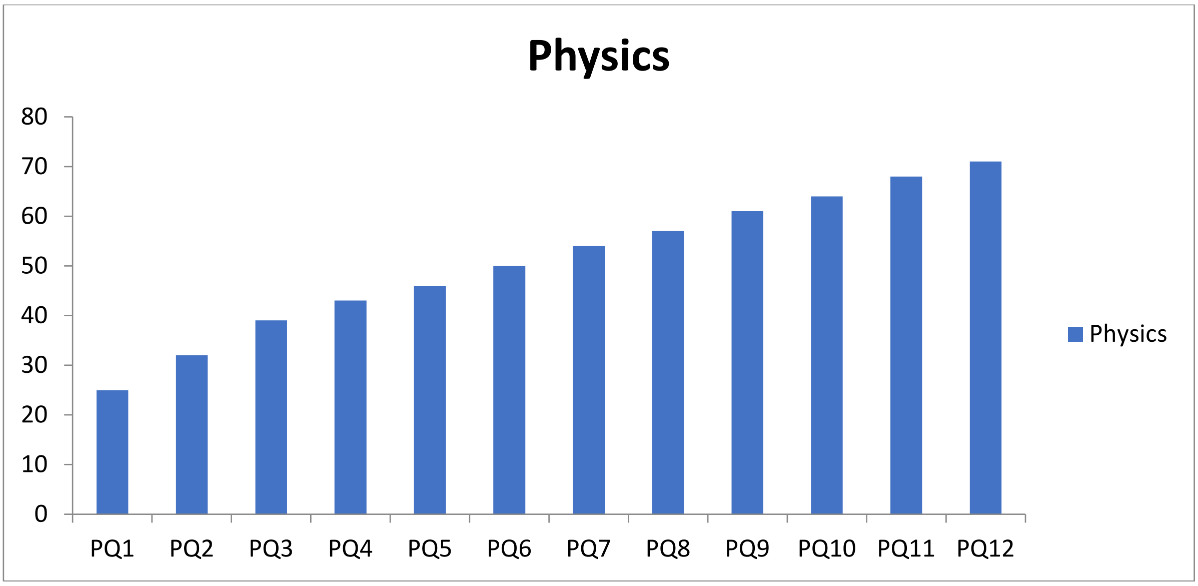

Figure 5. 6: Physics mean percentage performance per question in the class

Table 5.2, below illustrates the performance of students on varied concepts and their frequencies with respect to the scores. The mean performance of about 16% on questions PQ1; PQ.6 and PQ.8 was too low compared to how student faired in other questions testing knowledge on graphs

Table 5. 2: Descriptive statistics for students' performance in the diagnostic test

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Variance
Physics_Marks	12	25.0	71.4	50.875	14.3580	206.153
Valid N (listwise)	12					

.

Participants had difficulties in differentiating distance-time from velocity-time graphs, their gradients and the description of different patterns of movement under such graphs. Nonetheless, some participants scored above 90% in questions PQ.3; PQ.5 and PQ.12 which involved mainly reading information from a graph. Generally, learners seemed to have acquired the skill of reading information from a given graph. However participants still performed poorly on questions P.4; P.6; P.8; PQ.11; PQ.17 and PQ.18 that involved concepts that required use of equations in solving some physics problems. Participants could not use formulae correctly when solving physics problems of a mathematical nature as illustrated in (see Table 5.3: below): Also refer to Appendix E for more examples.

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Table 5. 3: Physics Average Percentage Performance per question.

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2 : Physics_Marks 96.8 Visible: 2 of 2 Variables

	Physics_Question	Physics_Marks	var	var	var	var	var	var	var	var	var	var
1	P1	16.1										
2	P2	96.8										
3	P3	96.8										
4	P4	32.3										
5	P5	90.0										
6	P6	16.1										
7	P7	77.4										
8	P8	16.1										
9	P9	83.9										
10	P10	58.0										
11	P11	38.7										
12	P12	93.5										
13	P13	67.7										
14	P14	64.5										
15	P15	64.5										
16	P16	83.9										

Table 5. 4: Descriptive statistics for students average performance in Physics.

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation	Variance
Physics_Marks	21	16.1	96.8	59.110	26.7325	714.628
Valid N (listwise)	21					

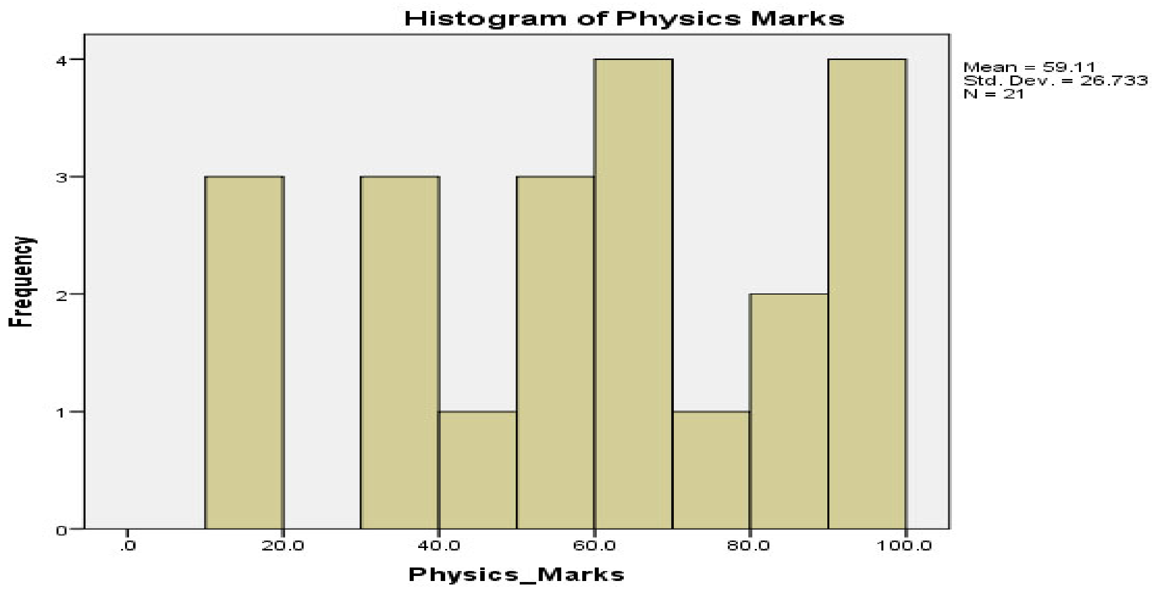
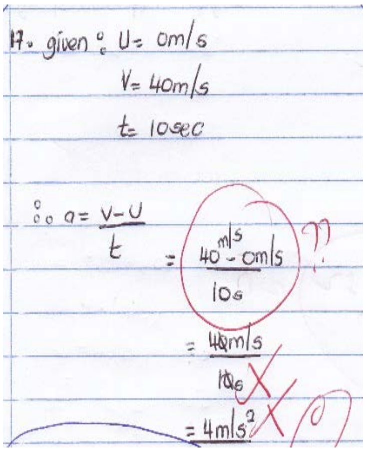
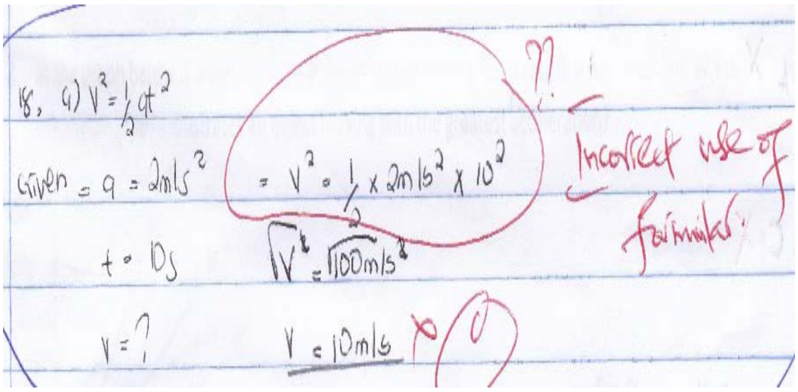


Figure 5. 7: Percentage performance scores per question versus the frequency:



a



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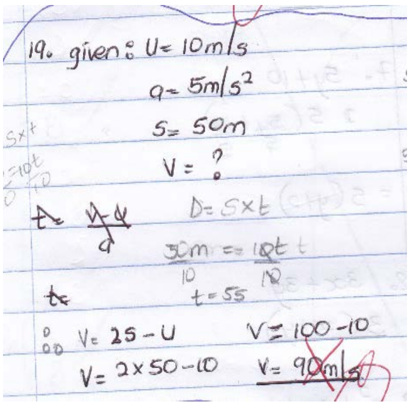

c

Figure 5. 8: Students' difficulties in correctly using the equations of motion

For instance, in Figure 5.8a, the student incorrectly interpreted the motion of the object and subsequently used the wrong formula to find the acceleration. The same occurred in Figures 5.8b and 5.8c respectively. The challenge about graph reading still surfaced in questions PQ.8 and PQ.9, confirming insufficient graph knowledge as a problem among most participants. According to Shah and Hoeffner (2002), students can correctly encode

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a graph's information but fail to map visual features to its physical meaning which results in difficulties in interpreting the graph. Regarding the use of equations, students may recall the relevant formulae to use in solving the problem, but still fail to perform the requisite steps to attain what is required when solving problems. On that note, having passed through mathematics lessons does not necessarily guarantee one adequate competency to handle physics problems in kinematics.

5.6 Comparison of physics and mathematics questions per construct:

The researcher identified constructs that relate corresponding concepts in mathematics and physics in order to come up with a correlation analysis to show that poor performance in kinematics goes hand in hand with poor performance in mathematics. Mathematics and kinematics data was collected to measure different constructs and analysed using the Statistical Package for Social Scientists (SPSS) software. This was carried out to establish if there was any relationship from the sets of data collected in an effort to answer the two sub-questions; What is the nature of mathematical difficulties encountered by students in kinematics that impede the effective learning of the concepts? A correlation does exist between two or more quantitative variables when their respective data plotted, produce a straight-line relationship. The result is a Pearson correlation coefficient denoted as "r" whose values range between +1 and _1. A correlation coefficient of +1 confirms the two values have a perfect relationship while -1 shows that the two are related in a negative manner. A correlation of zero (0) indicates no linear relationship. From the data analysed as illustrated in Tables 5.5 and 5.6 below there seems to exist a positive relationship between performance in mathematics and performance in kinematics.

Table 5. 5: Comparative descriptive statistics for performance in mathematics and physics

	Mean	Std. Deviation	N
Maths_Performance	39.973	27.3367	15
Physics_Performance	50.708	33.8362	13

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Table 5. 6: Correlation between mathematics and physics achievement

		Maths_Performance	Physics_Performance
Maths_Performance	Pearson Correlation	1	.532
	Sig. (2-tailed)		.061
	N	15	13
Physics_Performance	Pearson Correlation	.532	1
	Sig. (2-tailed)	.061	
	N	13	13

The percentage performances in each construct were calculated through expressing the scores obtained by the participants per question over the number of participants tested in that particular construct. The percentage performance per construct in the related disciplines was analysed and descriptive statistics obtained using the SPSS software. The quantitative and qualitative findings from the methodologies in data collected were summarised as in the Figures 5.9 and 5.10 below:

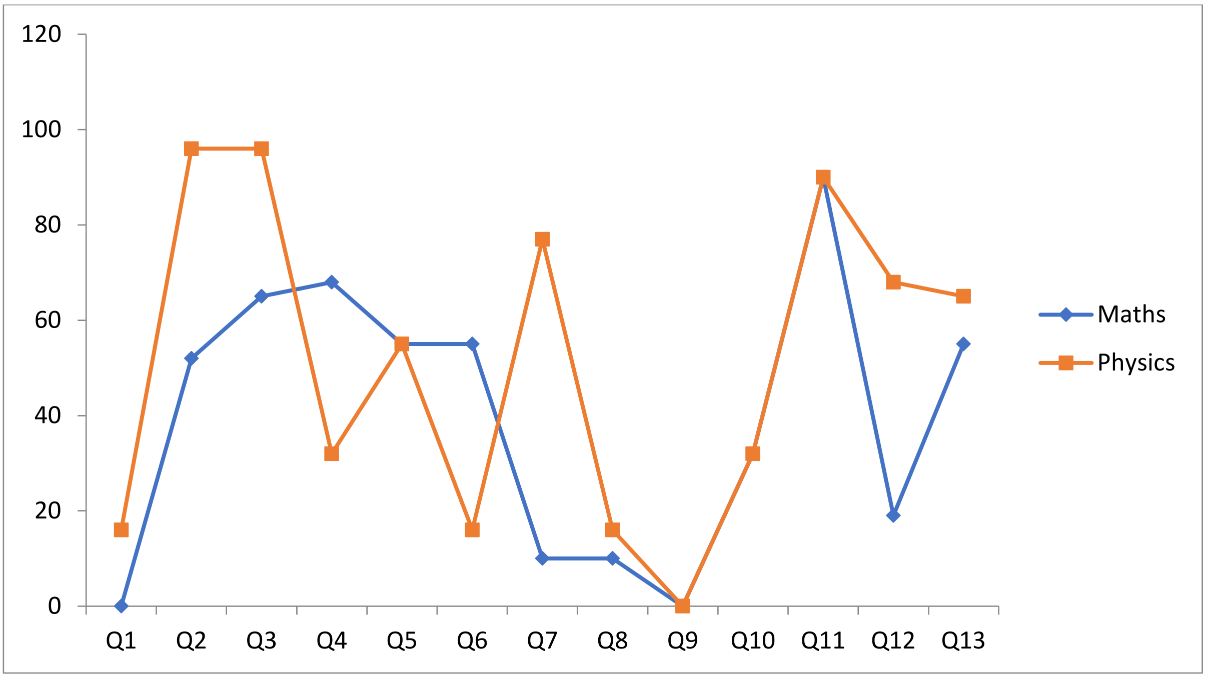

Figure 5. 9: Comparison of Mathematics and Physics marks for related constructs**.**

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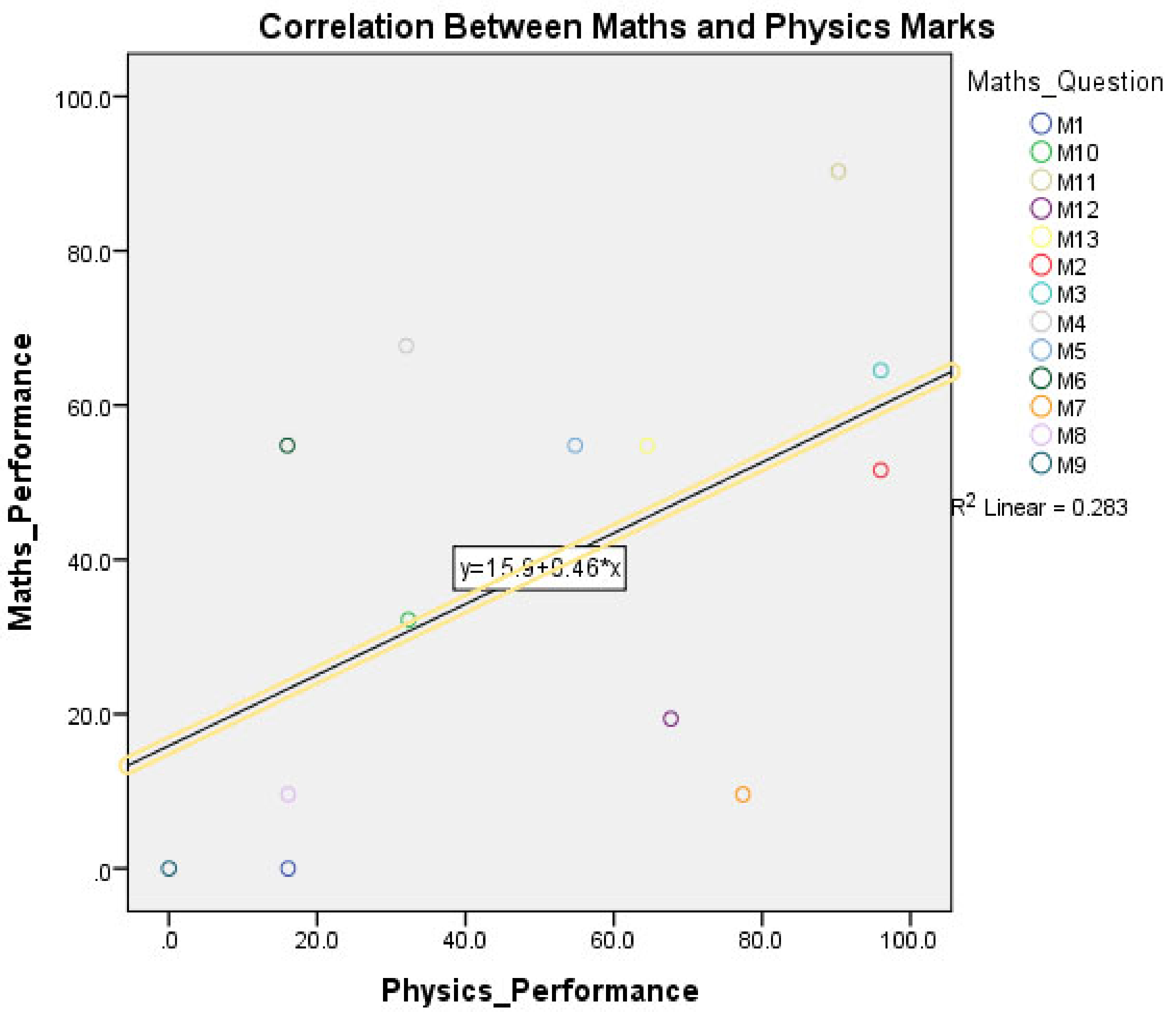

Figure 5. 10: Scatter graph with line of best fit

5.5. Reading of coordinates from a graph

The results indicated some inconsistencies in answering both the mathematics and the physics questions. The highest percentage score was found in question PQ which tested reading of coordinates. The lowest performance was in questions (PQ.6 and PQ.8) which involved calculations of the slope of the graph, determining the distance travelled by an object as represented by the area under the line of a v-t graph and reading of points on a given scale of a graph. Questions PQ.6 and PQ.8 were the lowest in percentage performance by the learners, evidence that they could not read negative and positive values let alone interpret their meaning in a velocity–time graphs. Physics questions on reading coordinates from kinematics graphs had about 28 of the participants getting

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answers right 7 getting them wrong. The participants also did well in physics questions that entailed reading of coordinates as in questions PQ3; PQ4; PQ7; PQ8 and PQ9 which tested more or less similar concepts in the questionnaire. This was evidence that at least most of the learners could read points from a graph which was a requirement in understanding graphs.

5.6 Gradient of a graph

The concept of calculating the gradient by learners was done fairly well in physics compared to mathematics although the related descriptions of the graphs gave the students some challenges. In mathematics students passed well in question MQ.12 which also tested the concept of reading points on a graph. However student lacked confidence in both calculating the slope and interpreting the meaning of the slope evident in question MQ15. Only 13 students got correct answers while 27 out of 40 got them wrong. This is because in physics, variables on different coordinates of the graph always have an attached meaning. For example, more often, participants had difficulties in relating the gradient of the position-time graph to that of velocity-time. Students fail to recognize that a graph is essentially a model of a relationship between variables. A change in y against *x*-variables in physics has a defined meaning. It relates the two variables and gives birth to a new concept. Quite often participants mistake height of a graph to the slope in both an (s-t) ad a *(v-t)* graphs which brings about height-slope confusion as exhibited in questions P1 and P2 as given in Figure 5.11 below:

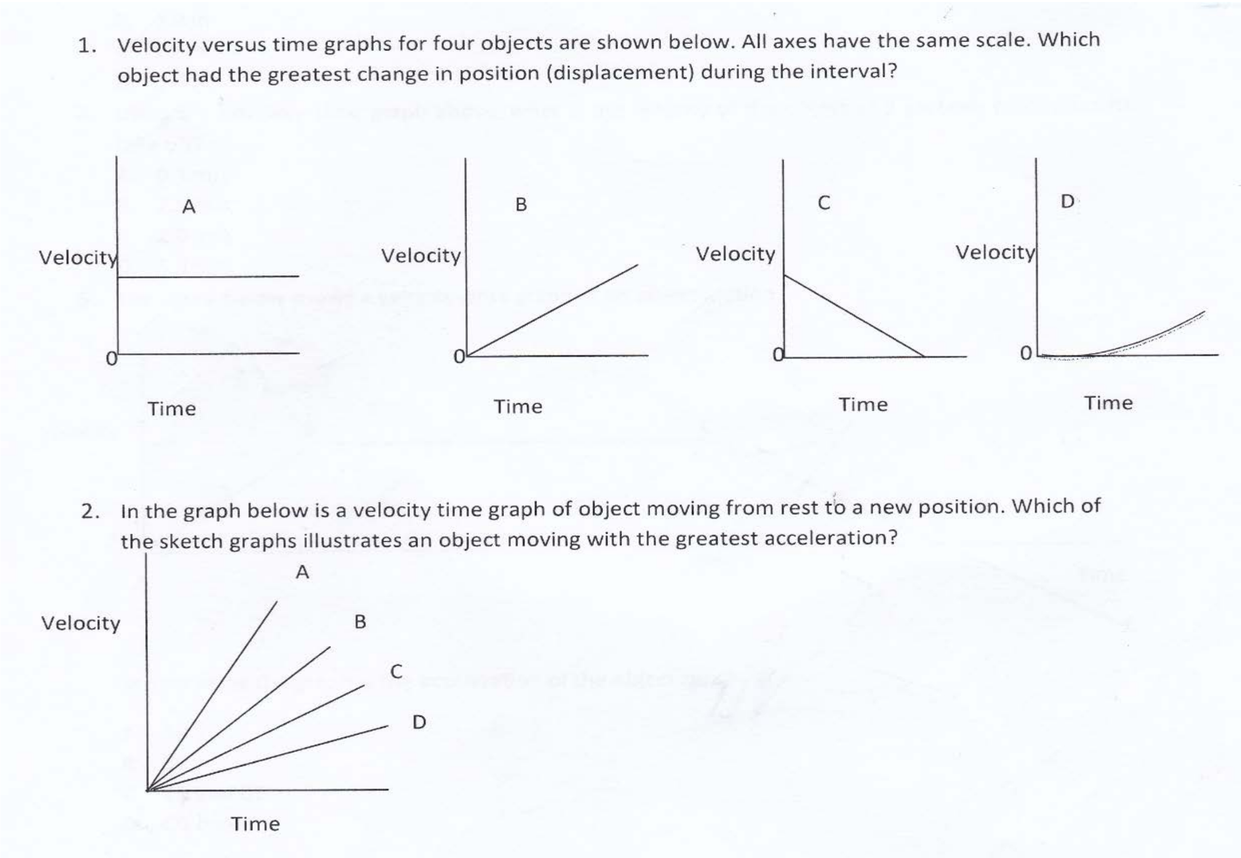

Figure 5. 11: Students' difficulties in differentiating a slope from the height of a graph

From the Figure 5.11, student take it that response B has a greater distance moved because of its steeper height compared to for instance option A. Students forget that the height of the slope has different meanings compared to the area under the slope of any graph. Again in question 2, the slope A is steeper and has a greater height of velocity than all given options, for the distance travelled the area of object A is smaller and therefore the distance it covers is also shorter. This is a concept that often brings about a lot of confusion amongst learners when it comes to calculating the area under a graph in motion concepts.

The learners often calculate gradients as $\frac{y}{x}$ instead of a change in values of values of y to x and this is what brings complexities in the whole concept of understanding the significance difference between height of the gradient to the area under the slope. This is what made participants have difficulties in relating the word constant to any horizontal line given under a velocity-time (v-t) or distance-time graph which is supposed to describe either zero acceleration or object being at rest respectively. The quantitative analysis of gradient in mathematics questions, MQ.13 and MQ.14 shows a mean score of 54.5% for line-graphs passing through the origin. The same score was exhibited in physics questions.

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There was little evidence of understanding how to calculate the gradient of a line passing through the origin in mathematics and physics alike.

5.8 Area under the graph

An analysis of data collected concerning calculations on area under the graph in physics was poorly done. Only 6 out 40 students got the answers right on questions PQ.1 and PQ.8. However question PQ.13 showed a fairly good performance of 67.7% an indication of inconsistencies in display of understanding the same concept by learners. The poor performance in physics questions related to the concept of an area under the graph is in accordance with McDermott et al. (1987) and Beichner (1994) assertion that students generally have problems in interpreting the area under the graph in kinematics. Most of the participants in this study were no exception to this challenge more especially on the qualitative aspect of graph where they had to interpret and describe different patterns of motion as in sample questions PQ.10 and PQ.11 as in the questionnaire (see attachment to qualitative sample questions).

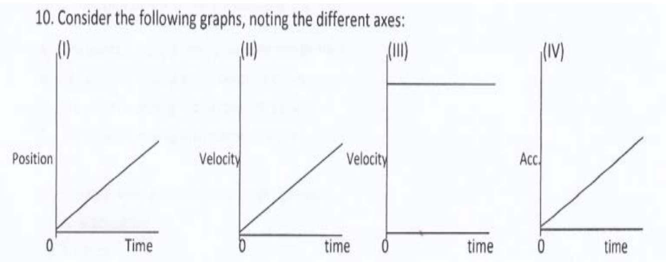


Figure 5. 12: Sample questions testing describing and interpretation of motion graphs.

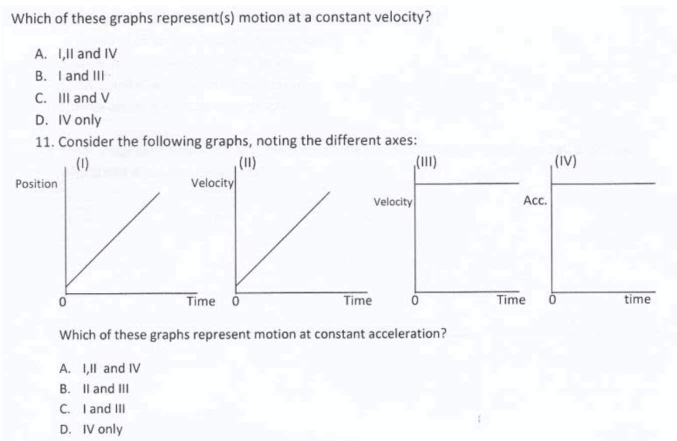
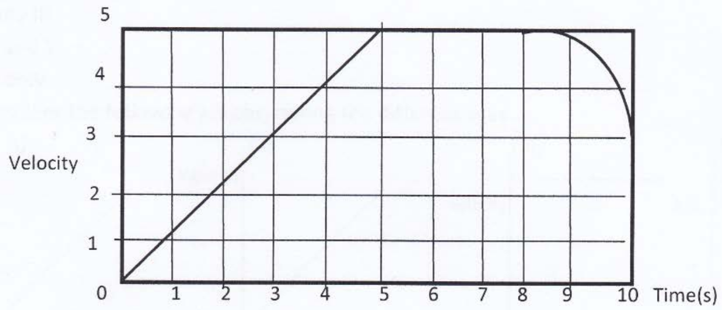


Figure 5. 13: Students' difficulties in desribing different patterns of movement in motion graphs

The quantitative aspects of graphs also gave some students challenges especially on identifying the relevant shapes representing the area under the graph. (see an excerpt for answering questions PQ7; PQ.8 and PQ.9.

7. An elevator at River Walk moves from the basement to the sixth floor of a building. The elevator moves as shown in the velocity –time line graph below,
Use the graph to answer questions 7, 8 and 9 respectively



From 4 to 8 seconds the elevator is :

A. At stand still
B. Moving with a constant velocity
C. Moving with a decreasing velocity
D. Accelerating uniformly

8. How far does it move during the first three seconds of motion?
A. 0.75m
B. 1.32m
C. 4.0 m
D. 6.0m

9. What is the acceleration of the elevator during the first three seconds?
A. $0.75m/s^2$
B. $1.32m/s^2$
C. $3.0m/s^2$
D. $6.0m/s^2$

Figure 5. 14: Sample questions with mathematics requiring calculating of the slope and the distance travelled in motion graphs.

5.8 Form of graph/ algebraic expression.

Participants scored a mean of 67.7% in solving problems related to the quantitative aspects of PQ.4; PQ.5; PQ.7.and PQ.8 which included reading the graph, calculating the velocity, acceleration and the gradient. On the other hand, a mean of 64.5% was scored on questions based on the qualitative aspects of graphing as tested in questions P8;

PQ.10; PQ.11. Some students could not transfer their mathematics knowledge on graph equations such as y = mx+c to a similar physics concept as in equation $v = u + at$ in kinematics (see Figure 5.15). Some of the students could still not use the graph to solve the problem in context but rather came up with some odd formula in attempt to solve such.

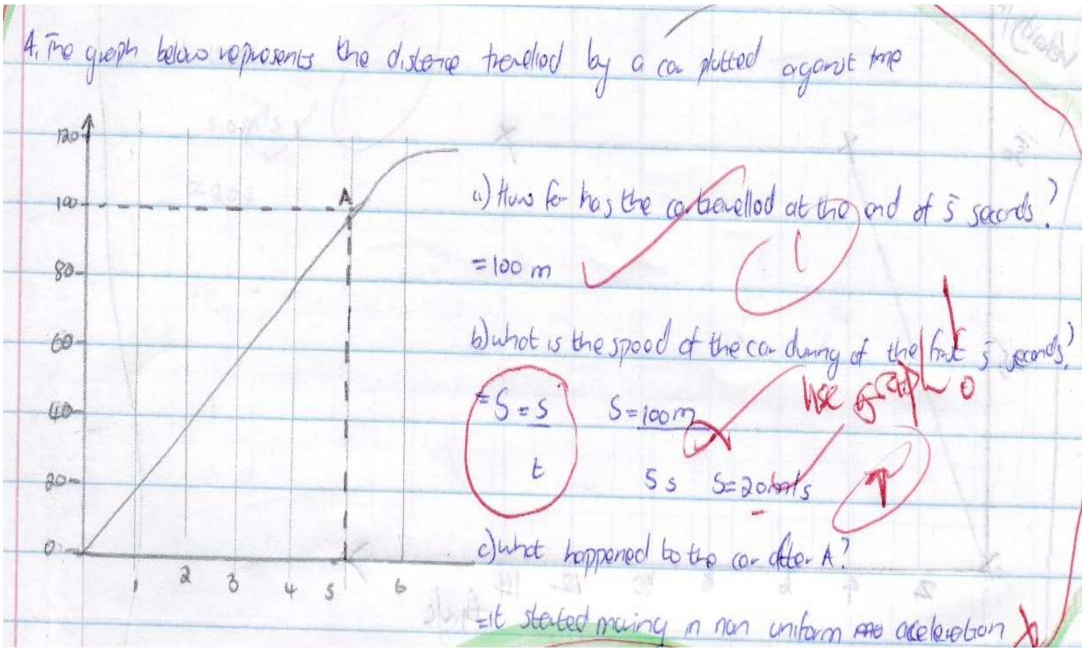


Figure 5. 15: Students' difficulties in solving graph problems

5.9 Solving problems using equations

Participants were able to recall and manipulate the equations well although it seemed they could not attach any meaning during processing of data as illustrated in the figure above. Equations are mathematical models with symbols assigned to variables representing physical quantities of different kinds. The modelling process involves precise procedures tailor-made to assist in providing a solution to a problem. However, it is quite evident from questions PQ.17; PQ.18; PQ.19 and PQ.20 as illustrated on student samples below,

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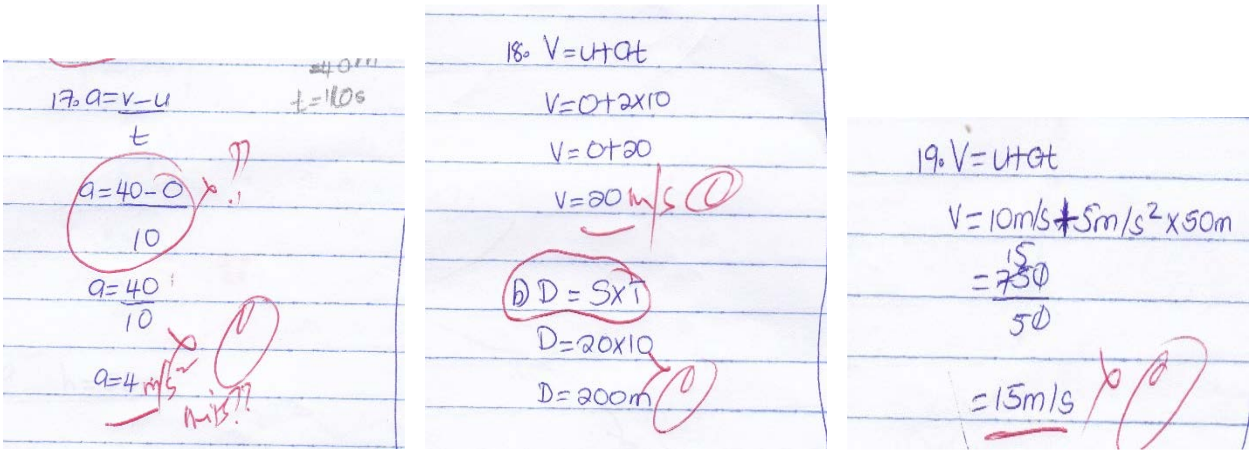

Figure 5. 16: Students' difficulties in correct use of formulae

that majority of learners had difficulties using the formulae to solve problems. They could not identify the parameters on which to use the equations and worse off, cannot follow the requisite order of steps in manipulating the respective equations. They lacked mathematical modelling skills. Students had difficulties in manipulating units as shown in Figure 5.17 below:

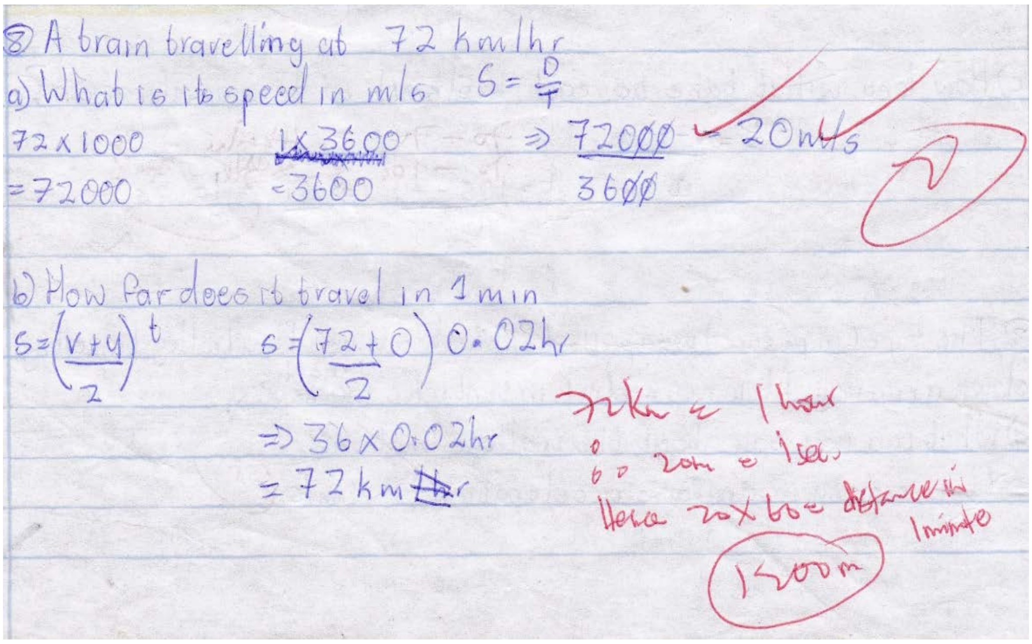

Figure 5. 17: Students difficulties with manipulating units during calculation

Proper handling of units when solving physics problems is an essential skill since units give a guide to meaning of the problems being solved. Participants' performance in problems associated with the use of equations had 14 students getting question PQ.18 right; 16 got question PQ.19 correct and 21 out of 40 participants got question PQ.20 right.

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The highest score was in question PQ.16 with 33 students getting the question right and lowest performance was in question PQ.18 where 14 participants got the question right.

5.10 Comparative analysis of results per construct as summarized in Table 5.5.

Percentage performance in most mathematics and physics constructs varies. Some scores were high while others were very low in general. This pattern could be indicating a difference in weighting levels for the questions addressing each construct. The results show some areas in which participants struggled and others where they did fairly well. The results on testing the construct on area under the curve of a graph and that of the line of graph itself in both a distance –time and a velocity-time graph indicated that some participants could calculate the area under the graph but had a challenge on interpreting the meaning of the area under the graph as evident in questions P.1 and P.8 where the mean score dwindled to as low as 16.1%. Most of the participants were unable to transfer the skills and knowledge gained in mathematics to answer questions of a similar nature in kinematics. The sample work below by student was evidence of the challenges faced by participants when constructing knowledge in kinematics despite having been taught similar concepts in geometry as early as primary school.

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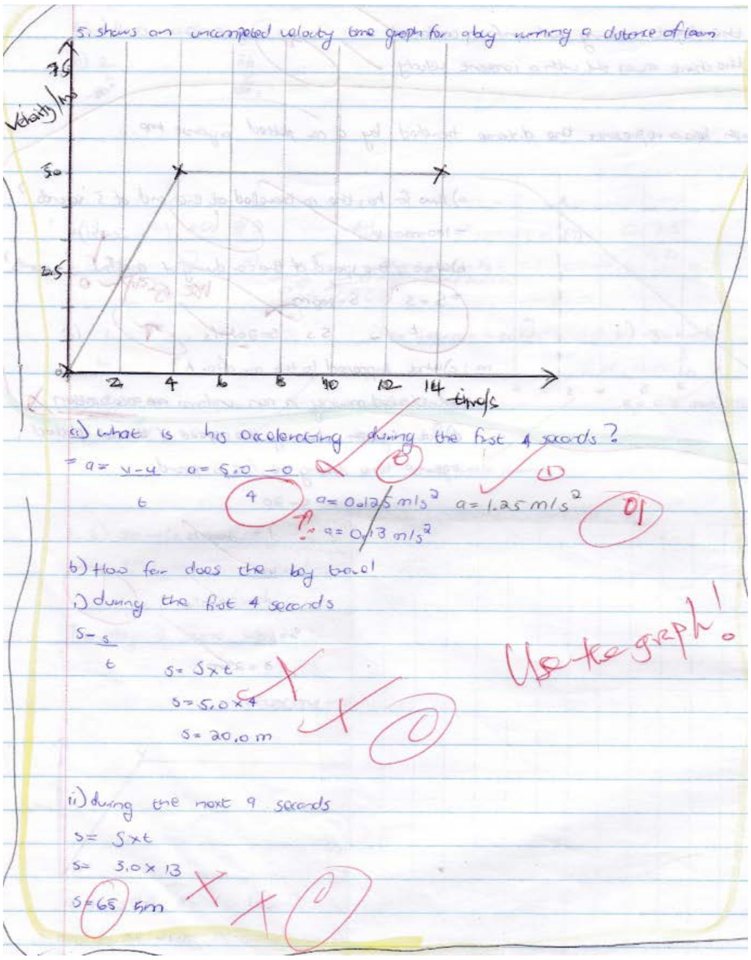

Figure 5. 18: Students differences in calculating area in velocity-time graph.

According to results in Table 5.5, assessment on the construct involving determining the gradient of a line recorded low scores in both mathematics and physics. Participants could not correctly calculate the gradient of a line involving the skills on correctly reading the coordinates points from the graph as well making the necessary computations thereafter. Such evidence confirmed the existence of difficulties among learners in application of correct mathematical skills affirming the hypothesis embraced in two sub-questions concerning the nature of mathematical difficulties affecting students learning of physics and the possible sources of the difficulties. Most of the participants confused the meaning of a horizontal line as implied in stationary objects to be the same in distance-time, velocity-time and acceleration-time graphs. As evident from question P.11 there was an indication that students could not differentiate between a positive and a negative slope of a graph. Participants had no idea of a negative gradient in kinematics judging from their poor performance in this question.

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Participants had no problems reading the coordinates of the graphs in both physics and mathematics save for questions P6 and P8 physics which involved calculations on determining the slope of a line and finding the area under the graph and the same was with question M.12 in mathematics. These questions were more inclined towards interpreting the gradient not the reading of the coordinates. In construct four, about interpreting different graph forms, participants could link different concepts given different variables on a graph. For instance, they could define the concept of velocity in both a position-time and velocity-time graph as evidenced in questions P.10 and P.11 in physics. Participants had no problems in differentiating linear from non-linear forms of graphs as tested in question P.15 in physics.

Overall, results in Table 5.5, indicated that students had difficulties with graphing particularly in kinematics. They did not have all the requisite algebraic skills and knowledge to solve kinematics graphs problems. Students' main challenge appears to lie with their failure to transfer knowledge from mathematics into physics lessons when solving similar graph problems. It's like when they get into physics lessons, the knowledge gained in mathematics always appears new to them because the methods used to teach the concepts in the two disciplines is not the same. From the data recorded in Table 5.5, there is an indication that besides graphs, participants exhibited difficulties in manipulating equations of motion when solving kinematics problems. Literature abound noted in chapter 2 and 3 where The General Systems Theory and the Extended Semantic Models Theory were used as theoretical frameworks for the study, uphold the use of models to solve problems in physics. Models were used in the study to help solve problems associated with applications of formula. David Hestenes, emphasised the importance of using models in developing concepts in science. An appropriate application of conceptual models has been argued as effective in developing concepts in physics and kinematics in particular. Students should be able to derive and manipulate equations if they are to have the capacity to link their mental world to the conceptual world to enhance their understanding of the physical world. The derivation of equations and their use in problem solving demands robust modelling skills which the majority of learners seem not to have. The calculations, interpretations and predictions involved in the use of equations demands that students should have active mental models so as to enable them to construct their own explanations to basic physical phenomena. Unfortunately, that seems not to be the case with most of the learners. From the perspective of the data collected, the researcher found it necessary

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to explore in depth the nature of problems associated with learning kinematics and in the process addressed the two sub-questions concerned with determining the nature of difficulties as well as their sources. To this end, further interrogations were done by means of in-depth interviews whose results are discussed in section 5.11 below.

5.11 Interviews results

After completion of the analysis of the questionnaire, a sample of 6 participants of varying abilities was purposively selected on the basis of their performance in the questionnaire. The criteria for selecting participants considered those who scored below average in the test, those who obtained mean and those who got above mean. The results of the interviews were transcribed and the emergent themes identified and coded to establish trends. Findings from the interviews addressed the sub-questions; “What is the nature and sources of mathematical difficulties students encounter when learning kinematics?”

Table 5. 7: Matrix for assessing level of consensus in interviews

Codes for the constructs.	**Interviewee members**						Level of consensus.	% of consensus
	1	**2**	**3**	**4**	**5**	**6**		
1. use of equations	A	A	A	A	A	A	6/6	100
2.calculationsof slope/Gradient)	A	A	N	A	A	D	4/6	67
3.reading of the coordinates	A	N	A	A	N	D	3/6	50
4.use of symbols	A	N	A	A	A	A	5/6	83
5.calculation of area under the graph	A	A	D	D	A	A	4/6	67
6.correct use of signs	A	A	A	A	A	A	6/6	100
7.reading scales on graphs	A	D	A	A	A	D	4/6	67
8.competency in visual decoding	D	A	A	D	A	A	4/6	67

Key: A = Agreement, N = Neutral D = Disagree.

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Table 5. 8: Themes and codes identified from the transcriptions of the interview.

Theme 1: Use of equations in problem solving. List of identified codes from the text: Codes: • Equations are not just used for calculations but for organizing data. • Putting meaning to mathematics used in physics is not easy. • Processing of data using equations is more complex in physics than in mathematics. • Specifying 'units' of physical quantities is a challenge. • There are many variables in one equations which causes confusion during processing e.g $v^2=u^2+2$ as. • Interpretation of equation associated with physical systems is difficult. • Most of the text in problem solving requires strong computational skills. • Remembering the formulas. • Difficulty in making variable, subject of the formula	**Theme 2: Purpose of symbols and signs.** **Codes**: • Too many which poses confusion. • Some symbols are used to mean difficult things in physics. • Blur difference between constant and variables. • Variation in signs of gravitational constant (g). • (+) and (-) slopes complex to comprehend.
Theme 3: Competency on graphing. **Codes:** • Reading coordinates values from the graph • Scale reading competency • Relationship between variables and graph interpretation skills. • Calculations on area under graph • Calculations on determining gradient • Visual skills – geometrical shapes	

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Interview with Participant 1

Interviewer: Welcome to the interview session concerning the topic on motion. Basically the idea is to find out your perceptions on the topic on kinematics we have been doing. The goal is to gather information with regards to areas that gave you problems during the learning process in this topic.

Interviewer: What is it that you have enjoyed in this topic Karen?

Karen: I enjoyed interpreting graphs on motion

Interviewer: What exactly was exciting you on graphs?

Karen: I could interpret them very well compared to other graphs I have met before.

Interviewer: Which area on motion did you find challenging to you and what really was your main challenge?

Karen: At times I had *difficulties in remembering the relevant formulae* to *use in calculations. Equations on free-fall motion were confusing* [1] at times. I could not tell *when to use (+g) or (-g,) on calculations*[2]. I had also *a challenge of interpreting the significance of (-g) and (+g) in free-fall*[2] calculations.

Interviewer: Which equation of motion was difficult to handle Karen and why?.

Karen: The *equation* $v^2 = u^2 + 2as$. I at times forgot to *square the variables* *v or u when* solving problems. *Determining the square root of a variable was also another problem*. [1] At times I found difficulties in differentiating initial from final velocity.

Interviewer: What else was a major challenge to you in this topic apart from the ones already stated so far?

Karen: My major challenge was that I could not easily *link whatever I learnt in mathematics* to my physics lessons and had challenges especially when *dealing with problems that involved equations.*[1]

Interviewer: If you were to be given an opportunity to revisit concepts on kinematics, which areas of this topic would you like emphasized?

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Karen*:* *Use of equations*[1] of motion and the *ideas on graphs*.[3]

Interpretation.

The participant had a problem of remembering the relevant formulae to use when solving problems. The free-fall motion equations were confusing on deciding when to use positive or negative values of (g). The student did not really understand when to use(+g) or (–g) when doing calculations. There was also a problem of determining the square-root as well as squaring of variables. The student found it difficult to remember and implement what she had learnt in mathematics in a new context in physics.

Interview on participant 2

Welcome to the interview session on kinematics, be free to make your contribution and be assured that no opinion raised in this interview will be regarded as wrong. Rather al your contributions will be highly appreciated for this research.

Interviewer: Which area of motion did you regard as very exciting to you during the study on this topic and why?

Warona: I enjoyed free-fall motion concept as most of the ideas were new to me. The concept of gravity as affecting motion of objects falling was also new and exciting.

Interviewer: What were the challenges you experienced in your encounter with this topic on kinematics?

Warona: *Remembering the equations*[1] wasn't easy for me especially those on free- fall, I did not know *when to use (+g) and (-g) on calculations*.[2]

Interviewer: What other challenges did you encounter in relation to this topic other than those already mentioned and how were they a challenge to you?

Warona*:* *Interpretation of graph was difficult*[3] and I did not understand the meaning of constant velocity or constant acceleration for both objects moving on the ground as well as those on free-fall motion. The concept of terminal velocity on falling bodies was not easy to understand.

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Interviewer: In your own view, which area about graphing did you find difficult to understand?

Warona: On free-fall motion, I at times got confused on *when to use a positive or negative (g)*[2].

Interviewer: What was your experience on processing variables with their respective units during calculations?

Warona: I *hardly process units with variables during problem solving*[2]. I just assign the units to the answers at the end of calculations and this often makes my calculations meaningless. I suggest *memorizing equations of motion*[1] could be of help in solving problems on kinematics.

Interpretation

The participant indicated that she had a problem with remembering formulae used in calculations. The free-fall motion equations were confusing to use. She was not sure when to use (_g) or (+g). The student had problem with graph interpretation problems. She was not confident in interpreting different forms of motion graphs. The idea of processing units when solving problems using equations was not easy for her.

Interview on participant 3

Good afternoon Mellitah. The purpose of this interview is to solicit for your opinion(s) with regards to difficulties you encountered when learning concepts on the topic of kinematics. The interview will be video recorded for the purposes of gathering information in details. Whatever will be discussed in this interview will be treated with strict confidentiality therefore, feel free to make your contributions as much as you can.

Interviewer: Can you kindly share your experiences with regards to learning concepts in this topic on kinematics and why they were of great interest to you?

Mellitah: I enjoyed learning the concept of free-fall motion. However I did not know why objects decelerate when moving upwards and accelerate when falling down.

Interviewer: Which areas on free-fall gave you some challenges and why?

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Mellitah: *Equations of motion were challenging*[1] to use and this was worsened by many calculations involved in manipulating them.

Interviewer: Which equations do you think were more difficult to use during calculations and why?

Melitah: *Equations 3 and 4 were uncomfortable*[1] to me just by inspection. The nature of the equations made me feel insecure especially with my lack of confidence with my mathematics. i.e. *using equations :* $v^2 = u^2 + 2as$ *and* $s = ut + ½ at^2$ *was a challenging*[1].

Interviewer: Why are equations in physics seemingly complex compared to those found in mathematics?

Mellitah: *Symbols found in physics equations are many*[2] and carry a special meaning than in mathematics where the numbers at times have no significant representations. Furthermore, some *equations have constants that have a meaning and this often gives me confusion*[1] when solving problems in physics compared with mathematic problems. Symbols used in physics represent specific quantities unlike in mathematics where (x) may represent anything.

Interviewer: Which other areas besides the use of equations was also a pain to you and why?

Mellitah: *Identifying the shapes on area under the line of a graph and the subsequent calculations involved gave me some challenges*[3].

Interviewer: Would you say mathematics knowledge is essential in physics to understand concepts better?

Mellitah: Yes sir, because *most concepts in physics require mathematical knowledge*[1] to solve them effectively.

Interpretation:

The student mentioned the idea of use of equations as being a challenge. The use equations of motion involved many symbols some of which confuse her and they were difficult to manipulate when solving problems. At times he had difficulties in describing

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motion represented by different patterns of graphs Calculation problems concerning motion graphs were difficult and often confusing at times.

Interview on participant 4

Good afternoon Botlhe. The purpose of the interview is to try and get your perceptions about this topic on kinematics. Your contribution in this interview will never be shared with anyone and be assured of utmost confidentiality in this regard.

Interviewer: What is it that you really enjoyed about this topic on kinematics?

Botlhe : I was excited mostly about the free- fall motion concept. I enjoyed a discussion about the effects of gravity on movement of objects when both falling or when moving upwards. The whole concept was new to me.

Interviewer: What is it that gave you problems on the concept of free-fall motion and why?

Botlhe: I had *difficulties in solving problems that involved calculations*[1] that required *use of (+g) or (-g) on free-fall motion*[2]. I also did not understand *when to use (+) or (-) signs* [2] on the gravitational constant (g).

Interviewer: Which other area(s) on kinematics gave you challenge when learning concept and what were these challenges?

Botlhe: The idea *of reading information from a graph within given coordinates*[3]. I had some challenges on *reading information from a given scale*.[3] Besides, I also had difficulties in *implementing formulae*[1] to *find area under the graph. Calculating the gradient given any slope*[3] of the graph wasn't that easy for me. Calculations as in determining the gradient was difficult to implement. I could *manipulate the equations* well although at times *struggled on making a variable subject of the formulae when solving problems.*[1]

Interpretation:

The student had difficulties which involved use of formulae. She didn't know when to use (_g) and (+g) when doing calculations on free-fall. She also expressed that she had challenges in correctly reading values from the graph. Reading certain scales was difficult for her at times.

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Interview on participant 5

Good afternoon. Emmanuel you are most welcomed to an interview session concerning some ideas about the topic on kinematics. Feel free to make your contributions in this interview and be assured of confidentiality in whatever ideas that are going to crop up from this interview. There is no opinion voiced in this interview that will be regarded as wrong for it is strictly meant to enrich data gathered for my research and will not be disclosed to anyone.

Interviewer: What is it that has been of great interest you in this topic on motion Emmanuel?

Emmanuel: I enjoyed free-fall motion even though I had difficulties *in choosing the right equations*[1] to use when solving some problems. I *was not confident when to use* (+g) *or* (-g) [2]in calculations on free-fall motion. Computation of information, especially one that involved *use of equations 3 and 4* was a struggle for me for instance use of equations: $h = ut + 1/2gt^2$ and $v^2 = u^2 + 2gh$ in particular*. Expressing some variables into a subject of the equation was difficult* let alone *selecting the correct information to use in doing the right calculation*[1] from a given statement of the problem.

Interviewer: What is your take on the concept of *processing units*[2] as you solved problems related to kinematics? Did you have any challenges on analysis of units during problems solving in kinematics?

Emmanuel: I had no difficulties in computation of data although at times I had confusion related to *mixing up of units.*[2] I also found it *difficult to sequence the solutions of calculation*[2] of problems to their respective units. I rather just assigned the units to the answers after doing the required calculations.

Interpretation:

The student had difficulties in choosing the right formula to use when performing some calculation. The equation of motion was more complex to use as he didn't know when he was to use +g or _g. Often times he computes the equation without any understanding of what will be happening. The concept of making one of the variables a subject of formula

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in equation was really a challenge to him. When doing some calculations, he would just mix-up units which made him always get wrong units in his answers.

Interview on participant 6

Good afternoon Pako. I would like to greatly appreciate your coming to this interview. Basically the purpose of the interview is to solicit for information with regards to concepts on a topic on kinematics. The whole essence is to find more on some concepts covered already in this topic with special focus given to areas that were a challenge to you during their development. Kindly be free and open with me as make your valuable contribution(s) in the course of this discussion.

Interview: Which areas did you find challenging during the course of learning some concepts on motions and why?

Pako: I found *use of equations* on motion challenging especially *the equations 3 and 4* [1]respectively.

Interviewer: What exactly was the challenge about using these equations?

Pako: When I am given an equation, I always find *difficulties in performing the substitution* as well as *selecting the right equations* [1]to use in solving the problems.

Interviewer: Did you find it easy to remember the four of the equations. Can you give me some of the equations?

Pako: *The equation:* $v^2 = u^2 + 2gh$ *and* $h = ut + 1/2gt^2$ *from free-fall.*[3]

Interviewer: What was the challenge all about in using these equations?

Pako: The equations *have many symbols*[2] which is often confuses me during solution of problems.

Interviewer: What other challenges did you experience on graphs which were a challenge to you and how was it a challenge to you?

Pako: Had problems *on graph interpretation*[3]. I could not differentiate the meaning of different lines of graphs in both the position-time and the velocity-time.

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Interviewer: Which aspects on graphs did you find so difficult to comprehend?

Pako: *Identifying shapes representing distance travelled was confusing*[3] at times i.e. (visualizing shapes) on area under graph. I had no problems in calculating the area represented by the shape under the line of the graph.

Interviewer: What other challenge did you experience during the learning of kinematics concepts?

Pako: Sometimes I got confused when *reading coordinate values*[3] which in turn affects my *proficiency on my calculations* [1]for determining the gradient of a graph.

Interpretations:

The student had difficulties emanating from use of equations when he was solving problems on kinematics. He also had difficulties in using the right formula when solving some problems in kinematics. At times some scales on graph were difficult to read.

5.5.1 Focus group interview

The purpose of this interview is to help us solicit for your perceptions about the mathematical difficulties often encountered by learners whilst developing some concepts in a topic on motion. Your contributions will be highly appreciated and be assured that whatever the views you will bring along will be very useful for the purposes of this research and will be handled with maximum confidentiality. I have noted that some learners experienced challenges during delivery of content in this topic.

Interviewer: Which area(s) were a challenge to some of you in this topic on kinematics?

Participant A: The *equations used in the topic of motion were a challenge*[1]. I also had problems concerning *graphing and the terminology used in describing the graph patterns*[3]. I had problems related with *reading of the information from the graph.*[3]

Interviewer: What exactly was it that gave you some challenges on the equations of motions?

Participant A: The idea of *remembering the formulas* [1]was a struggle for me.

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Participant C: I had problems associated with the actual *use of the equations in solving problems*[1] on motion.

Participant A: The *use of mathematics is usually a problem to most of us*. The *mathematics involved when doing calculations is demanding*[1] and this contributes to our challenges in solving physics problems.

Participant B: *The use of equation* $v^2 = u^2 + 2as$[1] was challenging for me as often times I *forgot squaring or finding the square-root where there was need be*[1]. I also had problems on *confusing the signs on application of* (g)[2] on using the free-fall equations.

Interviewer: What else seemed to be challenge to you the learners apart from what has been discussed?

Participant B: More often when solving problems *we do not attach units to the numerical data*[2] We just assign the units to the answers without showing the proper *dimensions*[2] on the final data processed, yet units are so essential composite in giving meaning to the product in all physics problems.

Interviewer: Earlier on participant B sighted confusion on signs used in equations of motion during calculations. Where does the issue of sign confusion crop up when using the equations?

Participant D: Whenever I am *switching variables from one side of the equation to another*[1], I *often confuse the signs*[2] in the end it affects the whole calculation involved.

Participant A: *Dealing with quantities and their respective units simultaneously*[2] is a often a challenge to me and I find myself struggling to solve some physics problems.

Participant E: I find it *difficult relating the equations of motion to the world of reality*.[1]To me the equations lack meaning as a result we are just forced to *memorize the formulae*[1] with very little understanding of the meaning behind the equations.

Participant F: I also seem to find very *little meaning attached to use of these equations*[1]. All I often do is, just to plug the data into the formula and work out the required unknown variables. Graphs of motion were a challenge to most of us.

Interviewer: Can you possibly elaborate more on which areas on graphing were difficult

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to most of the learners and why?

Participant B: The meaning attached to *graph patterns*[3] for the distance-time as well as velocity-time graphs have a totally different meaning and above all I found it complex to express a concept on one graph in another graph but having different variables.

Participant A: The other challenge I experienced was about *visualizing the picture or shape that represents distance moved by the object in a velocity - time graph*[3].

Participant F: I had no problems in *identifying the area under the line of a velocity-time graph*[3] but had problems of mixing up coordinates which ultimately affected my calculations.

Participant E: Calculating distance travelled in a velocity time graph is not all about identifying the right shape representing the distance travelled but it is important that one *knows the right formulae to calculate the respective areas*[1] and this is the challenge that most of us do have as students. Background knowledge on calculating areas representing distance travelled as represented by different geometrical shapes such as those *of triangles, rectangles and trapezium*[3] is of great importance.

Participant C: It is essential that one has to be competent in *reading scales to*[3] eliminate problem of mixing up coordinate values which is common, especially where different shapes exist under the same axis.

Participant F: There are times when *I confuse the height of the graph to its slope. I cannot differentiate height of a graph to the nature of a slope /gradient in a velocity-time graph*[3]. I often confuse magnitude of acceleration in a velocity- time graph to the pattern of change in velocity.

Bokang : I found *comparing different slopes of the lines of a graph*[3] a challenge to most of us as it demands knowledge of variation which is confusing to most of us from mathematics.

Interviewer: How do you think we can overcome this gap between linking the pattern of a body's motion to the nature of the slope?

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Participant A: Perhaps *engaging us in more calculations*[1] based on comparing different slopes will make us *appreciate the concept of variation.*[1]

Participant B: Most of us have limited knowledge *on linear and nonlinear equations on graphing*[3]. We cannot appreciate the relevance and applications of the general equation $y = mx + c$ in physics more especially in kinematics. We find it *difficult to link* $y = mx + c$ *to* $v = u + at$[3] equation on motion. When doing graph calculations, most *of us tend use* $\frac{y}{x}$ *in calculating gradient instead of* $\frac{\Delta y}{\Delta x}$ *which makes calculation* [3]on gradient wrong.

Interpretations:

From the focus group interviews, participants seemed to agree that the problems affecting their learning of physics were multidimensional in nature. However, most of their problems are mathematically inclined. Students had mathematical difficulties associated with use of mathematical formulae, use of mathematical symbols to describe concepts and derivation of some quantities which all required algebraic manipulations. The theoretical framework used in this study which is the General Systems Theory emphasizes integration of knowledge from different subjects as the most effective to learning. The use of equations, calculation done on graphing such as determining a slope, area under the line of a graph representing distance travelled by an object all require use of mathematical models to solve such problems. Students have the capacity to construct their own knowledge if equipped with necessary skills to attain the knowledge. In science education, such skills are those entailed in mathematics. For any effective learning of physics students must be able to translate their knowledge of mathematics into their learning of physics concepts.

Conclusion

This chapter gives a summary of data collected from using the mixed method approach design. Different measuring instruments were used in an attempt to provide data to answer the central question of the research study and its related sub-questions as provided on the Tables: 5.1a; 5.2a; 5.3a; 5.4a and 5.5a above. Both the quantitative and qualitative methods of data collection were used for the purposes of this study to investigate on the problems proposed. The diagnostic test conducted was followed up by the questionnaire which helped to obtain quantitative findings. The data collected helped to establish the "nature" of the mathematical difficulties and their possible sources which in process

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attempted to address the central question of the research study as well its subsequent sub-questions. The data on individual interviews and focus group discussions was used as follow-up to quantitative findings from the questionnaire to explore more on the nature of mathematical difficulties that were assumed to be influencing understanding of physics concepts by learners. The interviews and focus group discussions helped to cross-validate student(s) perception(s) and their understanding concerning the mathematical difficulties.

The findings in this chapter confirmed the existence of some mathematical difficulties affecting students' learning of kinematics and physics at large. The mathematical difficulties impact negatively in students' constructing knowledge about kinematics. The next chapter, chapter 6, will give a conclusive summary of the findings from the analysis of the results and provide answers to the research questions. It will also try and suggest solutions to alleviate the students' mathematical difficulties when learning kinematics and physics at large as well as come up with their implications to science curriculum implementation, teaching and learning.

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CHAPTER 6: DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

The chapter begins with a brief summary of all the chapters 1 to 5 of the research followed by a brief discussion of the main findings of the research. A comparison of the findings in literature to those of the research then follows together with an indication of the contribution to knowledge the study makes. Some recommendations for theory, practice, policy makers and suggestions to further research are made by the researcher. Finally limitations of the study are mentioned and recommendations for further research made.

6.2 A summary of chapters 1-5 of the research

The main focus of the study was to determine the mathematical difficulties that affect learning physics, kinematics in particular among the Form 4 learners in Kgari Sechele High School in Botswana. In chapter 1, the researcher gave an overview of the motivation, objectives and research design that helped to reach the aim of this study.

Chapter 2 presented a literature review on the mathematical difficulties often encountered by learners when constructing concepts in physics. The challenges experienced by learners in the use of formulae in equations, symbols in expressing units and the graphical representation was discussed as impeding learning of a multiple of concepts in kinematics and physics at large. The students' conceptions and misconceptions about various mathematical representations were discussed as having an influence in mathematical difficulties they experience when learning.

Chapter 3 presented a conceptual framework for analysing the mathematical difficulties encountered by learners when constructing their own knowledge in kinematics. Different theoretical frameworks were synthesized to provide an analytical lens for the data in the study. The framework helped to define the constructs or variables being investigated and their entire relationship within a given context. The two theoretical frameworks merged in this research were the General Systems Theory (GST) and the Extended Semantic Model (ESM). Both theoretical frameworks espouse integrated learning approach typical of the constructivist philosophy. Students are viewed as having the capacity to construct their own knowledge all they need is to be provided with the opportunity to interact with the

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environment and then construct their own world Vigotsky et.al (1987). From a Vygotskian perspective, the role of a teacher is that of a facilitator of learning. The teacher and the student construct knowledge during their social interaction and the teacher mediates the learning activities.

Chapter 4 looked at the research methodology best suited to interrogate the mathematical difficulties affecting students' understanding kinematics. A suitable research design was adopted and diagnostic tests, survey questionnaires, individual and focus group interview methods identified for use in this study. A population of 600 Form 4 learners was identified as suited to the study and a sample of 40 students selected to generate meaningful data with which to answer the central research question and sub-questions. The data collection and analysis procedures were discussed as well as ethical considerations.

In chapter 5, data collected were analysed to answer the research questions of the study. Relevant data processing procedures were employed for each methodology. Both quantitative and qualitative data were used to assist draw conclusions. Chapter 5 primarily served to provide answers or evidence to the research questions.

6.3 Discussion of findings

From the findings of the study, students had an incorrect interpretation of an equation which resulted in failure to handle what was required by the problems (see figure 5.5a and 5.5b in section 5.4). Students were often uncertain about which equations to use resulting in trying any formula when solving physics problems as long as it had a variable they were looking for (see figure 5.5b in section 5.4). The skill of deriving an equation of the unknown is complex to most of the learners yet it is the very concept they learn in mathematics when they are taught to make one of the variables a subject of the formula. Based from such findings it is quite eminent that interdisciplinary learning can form a very effective strategy to teaching and learning if appropriately implemented. Constructivist theories guiding reforms in science and mathematics education suggest a major shift from learning science and mathematics as an accumulation of rote facts and procedures to learning. The subjects have a number of related concepts in common but in varying contexts that can be developed efficiently through blending the two subjects Greeno et.al (1996).The effective use of formula can be realized if mathematical models are used in teaching concepts that require use of equations. By using models different steps in the modelling process makes the students to understand the functions of the equations better.

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In this research, graphs have been found to play an important role in understanding kinematics. These findings concur with the literature which reveals that there are four aspects of graph operations essential for students' understanding of physics. They involve reading of coordinates, determining the gradient of a line, calculating of area under the graph and connecting representations. Representational competence is an integral part of physics literacy (Glazer, 2011; Linder, Airey, Mayaba & Webb, 2014). Insufficient content knowledge on graphs and graphical skills may retard students' conceptions and lead to misunderstandings of graphs. Findings from the study also seem to concur with the literature as evident in data recorded from the questionnaire, interviews and focus group discussions (see figures 5.3; 5.6; 5.7 and 5.8 in section 5.4) Difficulties encountered by students in calculations of the gradient, reading of the coordinate points, and calculating of the slope of a graph were also a key finding of this study (see figures 5; 5.6; 5.7and 5.8 in section 5.4).

6.4 Results of the study to answering research questions.

The central research question: What mathematical difficulties do physics students encounter when learning kinematics?

The results indicated that there were some mathematical difficulties that hindered students' understanding of kinematics (see tables 5.4a and 5.5a in section 5.4). The responses from the interviews and the focus groups also revealed students' difficulties with calculations on graphs. The results confirm similar findings by other researchers (Trowbridge, 1980) & McDermortt et.al); (Boulding, 1956) (Akintoye, 2015) Basson, 2002 and (Beichner, 1994) that students encounter problems with interpretation and analysis of kinematics graphs.

6.4.1. Results to answer sub-question 1

What is the nature of mathematical difficulties encountered by students in learning kinematics concepts?

The nature of difficulties, fall into different categories namely: deriving of formulae, Identifying and interpreting the formulae, extracting information from a graph and application of mathematics to the physics problems. In both the interviews and focus group discussions, students confirmed had difficulties in using the formulae in equations. From the sample equations given: $v = u+at$ and $s = ut + \frac{1}{2}at^2$ found the equations difficult to manipulate (see figures: 5.5; 5.5b and 5.9 section 5.4).

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6.4.2 Results to answer sub-question 2.

What are the possible sources of the difficulties?

The research agrees with the previous researchers that there are sources of mathematical difficulties encountered by learners when learning kinematics. Most of the difficulties are from mathematical incomprehension. For example, most of the learners do not know that acceleration being the rate of change in velocity equals the gradient of a velocity-time graph. Lack of such understanding of the basic concepts affects their competency in describing various forms of kinematics graphs.

6.4.3 Recommendations for Theory.

Although the theoretical framework used was able to identify the nature of some mathematical difficulties there was need to find out more on how such problems could be mitigated to eliminate them to improve both on the conceptual and pedagogical knowledge.

6.4.4 Recommendations for further research.

Since the study was conducted in one school, it might be better if expanded to more high schools in the country. The study can also be expanded to Universities. Future research should be done on the design and evaluation of teaching sequences to construct and refine students' knowledge and comprehension of kinematics.

6.4.5 Recommendations to curriculum policy

There is need to reflect on the nature of mathematics offered to learners. It is essential to find out if the mathematics offered to students is relevant for the Physics Syllabus. It is also essential to ensure through rigorous in- service programmes, teachers who are the implementers of the curriculum get enough knowledge to deliver the content.

6.5 Limitations of the study

The sample size used in the research was too small for the researcher to generalize findings. The use of the mixed methods was time consuming. The information gathered from interviews and focus groups had some degree of biasness despite the effort to triangulate. The respondents were subjective which gave bias to the findings. Although the responses were video recorded it was difficult to note non- verbal responses at the same time recording what students were saying.

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☎ (267) 5920378

FAX: (267) 5920460

Republic of Botswana

TRAINING & DEVELOPMENT
MOLEPOLOLE EDU.
PO BOX 1293
Molepolole

APPENDIX A

TO: Mr Ndumiso Michael Moyo
Kgari-Sechele Secondary School

Dear Sir

Ref: MEC 1/12/2 I (2) 09 July 2018

PERMISSION TO CONDUCT RESEARCH IN SENIOR SECONDARY SCHOOLS

Reference to your application letter dated 04th June 2018; you are granted permission to conduct research in any secondary schools in Kweneng region as per your stipulated request. Kindly be advised that you can also source for information from any institution that may provide you with any relevant information for your research. You are advised to continue providing an update on your research and further to submit a copy of your completed research document.

Kindly continue adhering to all issues relating to ethical conduct.

Thank you.

Yours Faithfull,

Rapula Kgasudi /For Director- Regional Operations

MOLEPOLOLE
EDUCATION CENTRE
2018 -07- 10
P. O. BOX 1293, MOLEPOLOLE
TEL: 5920378 FAX: 5920460

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APPENDIX B: PERMISSION FROM THE SCHOOL HEADMASTER

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REQUEST FOR PERMISSION FROM SCHOOL HEADMASTER

TO CONDUCT A RESEARCH

Mathematical difficulties encountered by physics students in kinematics: A case study of Form-4 classes in a high school in Botswana.

Your school has been invited to participate in a research study to be conducted by Mr Ndumiso Michael Moyo, an MED student in the Department of Curriculum Studies at Stellenbosch University. This study will contribute to effective use of algebraic skills in teaching and learning concepts physics. This is envisaged to enhance learners' performance and attainment of concepts in various topics of the subject.

1. PURPOSE OF THE STUDY

This research intends to improve students' understanding when learning kinematics and other physics related topics. During the study the researcher intends to improve use of algebraic skills to enhance students' performance in kinematics and other physics related topics at large. The fundamental aim is to minimise misconceptions when students are learning some concepts of the subject that require use of algebraic skills in solving them. Students will be taken through a diagnostic assessment to find their strength in algebraic skills essential to understand physics and requisite algebraic skills will be developed in form of supplementary lessons as part of interventions to strengthen their mathematical competency in handling physics problems of any nature involving use of such. The main focus is to minimise misconceptions and errors that many often encountered during solving kinematics and other physics related topics that require competency in such skills.

2. WHAT WILL BE ASKED OF YOU?

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I would like to ask your permission to allow 40 of your physics students to participate in this research. The following will be done during the study:

Students will participate in the following way:

- They will participate in solving kinematics questions in the pre- and post-tests for research purpose.
- Attend two one-hour lessons four times a week for 6weeks in attaining more algebraic skills as well as solving kinematics problems to improve their understanding of physics concepts.
- Be video-recorded in all their class activities that involve the use algebraic skills in solving kinematics problems.
- Be interviewed for 30 minutes on one-to-one and on a focus group basis about their experiences of the use algebraic skills in solving kinematics concepts.

3. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts for the students to participate in the research at your school.

4. POSSIBLE BENEFITS TO PARTICIPANTS AND/OR TO THE SOCIETY

There are direct possible benefits for the participation of your school in this research. The use of effective algebraic skills for solving kinematics problems will most likely provide opportunities to students to better understand various concepts in kinematics and other physics at large. Students' involvement and engagement in extra class activities during exploration of different algebraic skills in supplementary lessons, might increase their knowledge base of mathematics as such improving their performance and perceptions about physics. In the main, it might help the curriculum designers to come up with effective programmes that may help in delivering the content of physics topics enhancing students learning.

5. PAYMENT FOR PARTICIPATION

Participation of your students will be free of payment and is voluntary. There will be no remuneration for taking part in this research study.

6. PROTECTION OF YOUR INFORMATION, CONFIDENTIALITY AND IDENTITY

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Any information shared by your students during this study will be protected. I will not use any name or anything else that might identify your school, students or teachers in the written work, oral presentations, or publications. The information remains confidential at all times. All data, including field notes and video recordings, will be kept under lock and key and all the electronic versions will be digitally encrypted (password protected). Databases will be destroyed after the research has been presented and/or published which may take up to five years after the data has been collected.

7. PARTICIPATION AND WITHDRAWAL

Participating students will be free to withdraw at any time, even after they have consented to participate. They may decline to answer at any specific questions.

8. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Ndumiso Michael Moyo at 0026771890589 / 0026773676068, [moyo.ndumiso@ymail.com] or the supervisor Professor MC Ndlovu [mcn@sun.ac.za]

9. RIGHTS OF RESEARCH PARTICIPANTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

If you agree for this research to be conducted at your school, please sign below. The second copy is for your records. Thank you very much for your help.

________________________________ ________________

Signature of Principal **Date**

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APPENDIX C: STUDENT ASSENT FORM

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STELLENBOSCH UNIVERSITY
STUDENT CONSENT TO PARTICIPATE IN RESEARCH

Mathematical difficulties encountered by physics students in kinematics: A case study of Form –classes in a high school in Botswana.

Your school has been invited to participate in a research study conducted by Mr. Ndumiso Michael Moyo from the Curriculum Studies Department at Stellenbosch University. The investigation results will contribute to the development of an MED thesis. Your school has been selected as a possible participant in this study because this research will help you to understand and enhance use of algebraic skills to effectively learn physics concepts in various topics such as kinematics.

1. PURPOSE OF THE STUDY

This research intends to improve your use and understanding of algebraic skills requisite o understanding kinematics. The aim is to minimise misconceptions when learning physics concepts through implementation of skills that enhance effective solution of problems in kinematics and other related physics topics.

2. PROCEDURES

If you agree to take part in this study, you will be asked to:

- Engage in solving kinematics questions that require use of algebraic skills in pre-and post-tests for research purpose.
- Attend intensive 2hour-lessons four times a week for 6weeks as supplementary tuition to equip you more on algebraic skills requisite to solving kinematics problems.
- Be interviewed for 30 minutes on one-to-one or focus group basis about the use and importance of algebraic skills in solving kinematics problems and other physics related topics.

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- Be video-recorded in all your class activities that involve the use of algebraic skills in solving kinematics problems.

3. POSSIBLE RISKS AND DISCOMFORTS

The only possible inconvenience might be attending lessons after normal school hours but you will only be involved for 6 weeks. There are no foreseeable risks or discomforts involved in partaking in this research.

4. POSSIBLE BENEFITS TO PARTICIPANTS AND/OR TO THE SOCIETY

Learners participating in this study will be exposed to an opportunity of being given more time to explore more algebraic skills essential to understanding physics. This is envisaged to improve their achievement in solving problems on kinematics and other physics related topics where algebraic skills are of necessity This research will explore how your reasoning, sense making and problem solving skills may be developed to answer kinematics concepts at Form 4 and 5 levels respectively. It will also equip you with additional learning skills needed to understand kinematics better. Such skills have the potential to increase your participation in classroom activities tailor made to effectively comprehend kinematics by linking concepts to realistic contexts.

5. PAYMENT FOR PARTICIPATION

There will be no payment for participation. Participants take part on a voluntarily basis. Learners will be provided with all basic needs throughout the duration of their participation in the study.

6. PROTECTION OF YOUR INFORMATION, CONFIDENTIALITY AND IDENTITY

Any information you share with me during this study and that could possibly identify you as a participant will be protected and will remain confidential and will be disclosed only with your permission or as required by law. Confidentiality will be maintained through:

- Using pseudonyms and special coding of data in the final draft of the research report.
- Storing your personal data on a password-protected desktop and laptop, which include questionnaire and interview results and field notes. Hard copies will be kept under lock and key at the researcher's home.

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Any information gathered from you that includes video-recordings and photographs will be made available to you on request at all times. Your personal data will be accessible to other participants only if prior consent is obtained from you. All materials gathered will be destroyed when no longer needed for the research. This information will only be used for the purpose of this research and publications that may result from the research. The results of the research study will be made available and not the identities of the learners to the school, Department of Education and other researchers on request.

7. PARTICIPATION AND WITHDRAWAL

You can choose whether to be in this study or not. If you agree to take part in this study, you may withdraw at any time without any consequence. You may also refuse to answer any questions you don't want to answer and still remain in the study. The researcher may withdraw you from this study if circumstances arise which warrant doing so to maintain the validity of the data. However, your participation in this study will improve the accuracy of the results because more responses from students will better inform the study about how to continue improving students' understanding and effective use of algebraic skills in solving kinematics and other physics related topics.

8. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Ndumiso Michael Moyo at 0026771890589/0026773676068 [moyo.ndumiso@ymail.com] and/or the supervisor Prof MC Ndlovu [mcn@sun.ac.za] at 021 808 3484.

9. RIGHTS OF RESEARCH PARTICIPANTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

DECLARATION OF CONSENT BY THE PARTICIPANT

As the participant I confirm that:

- I have read the above information and it is written in a language that I am comfortable with.
- I have had a chance to ask questions and all my questions have been answered.

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- All issues related to privacy, and the confidentiality and use of the information I provide, have been explained.

By signing below, I ________________________________ agree to take part in this research study.

____ ______________________

Signature of Participant **Date**

DECLARATION BY THE PRINCIPAL INVESTIGATOR

As the **principal investigator,** I hereby declare that the information contained in this document has been thoroughly explained to the participant. I also declare that the participant has been encouraged (and has been given ample time) to ask any questions. In addition I would like to select the following option:

	The conversation with the participant was conducted in a language in which the participant is fluent.
	The conversation with the participant was conducted with the assistance of a translator (who has signed a non-disclosure agreement), and this "Consent Form" is available to the participant in a language in which the participant is fluent.

____ ______________________

Signature of Principal Investigator **Date**

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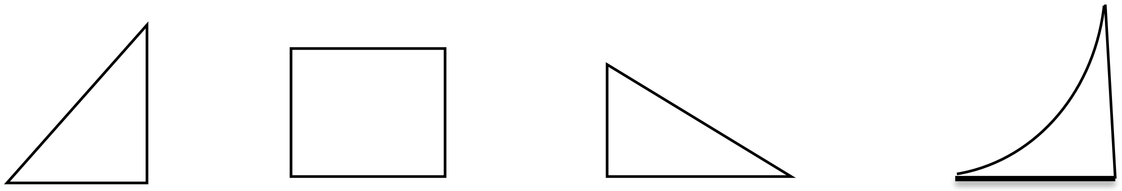
APPENDIX D

Diagnostic mathematical questions for concept applied in mathematics graphs.

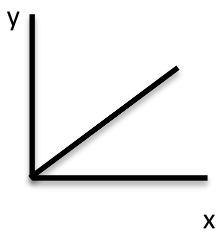
1. Of the geometric figures below, which one has the greatest area? Assume that all the figures have the same width and same height.



A B C D

2. Using the diagram above, which geometric figure has the smallest area? Assume that all figures have the same width and height?

A B C D

3. 

Consider the graph above. With an increase in **x,** the gradient is:

A. Zero

B. Increasing

C. Decreasing

D. Constant

E. Varying

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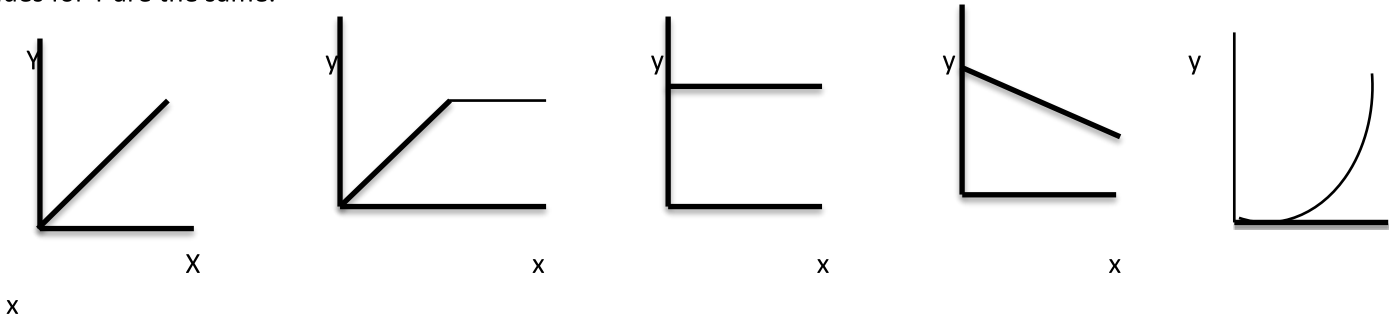
4. Choose the answers of the questions below from the following graphs. In all these graphs the maximum values for Y are the same.

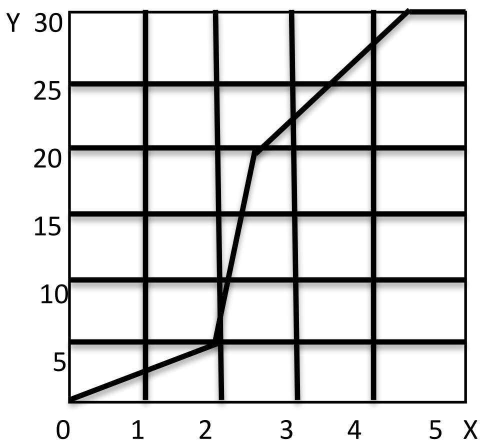


A B C D E

Which one of the graphs has the smallest area under the graph from X=0 to X=5?

5. Which of the graphs has the largest area under the graph from X=0 to X=5?

A B C D E



6. From the above diagram, what is the gradient of the graph where x=2?

A2 B5 C2.5 D4 E10

7. From the above diagram of question 6, what is the gradient of the graph when X=4,5

A5

B 0

C 6

D 8

E 7

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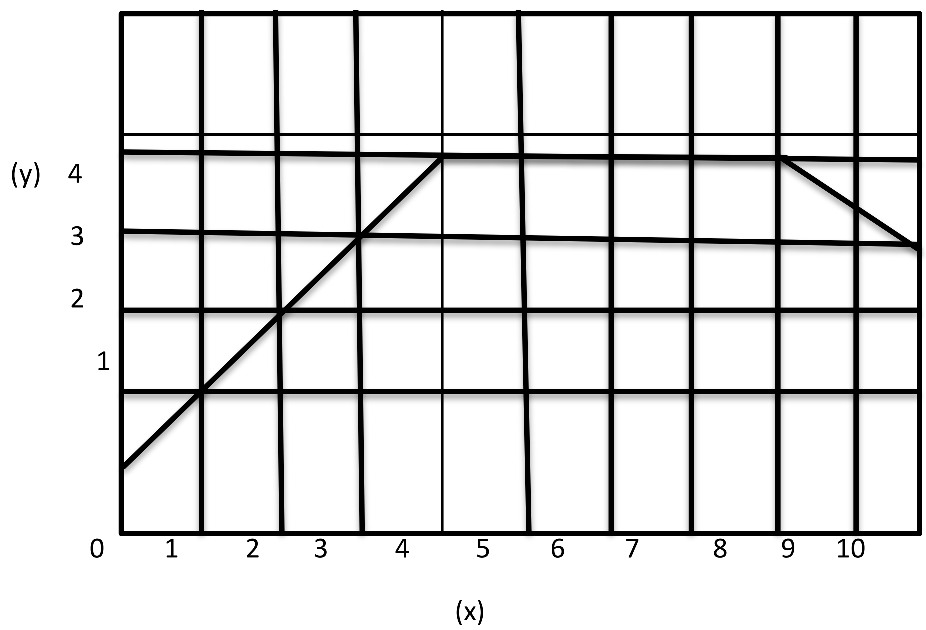
Consider the following graph to answer questions 8 and



8. What is the area under the graph in the X-interval (4,8)?

A. 0

B. 1.33

C.4, 0

D 12.0

E 24.0

9. How does the value of Y change between X=5 and X=7?

A. increases

B. decreases

C. remain constant

D. Zero

E. Not possible to tell

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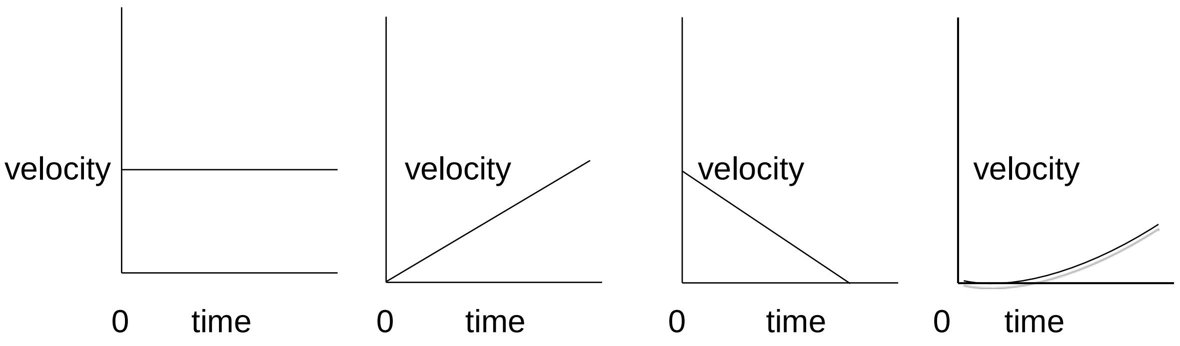
APPENDIX E

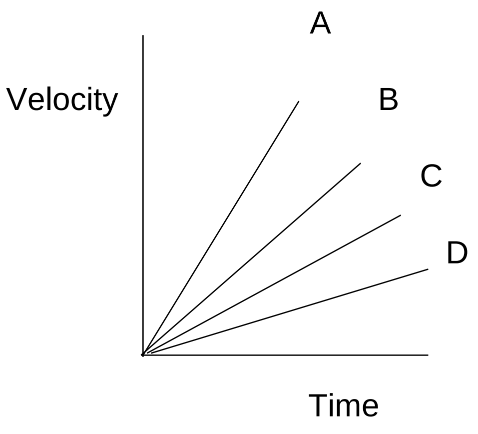
Questionnaire

Four options are given to the multiple choice, items below cross the letter of the best

1. Velocity versus time graphs for four objects are shown below. All axes have the same scale. Which object had the greatest change in position (displacement) during the interval?



2. In the graph below is a velocity time graph of object moving from rest to a new position. Which of the sketch graphs illustrates an object moving with the greatest acceleration?



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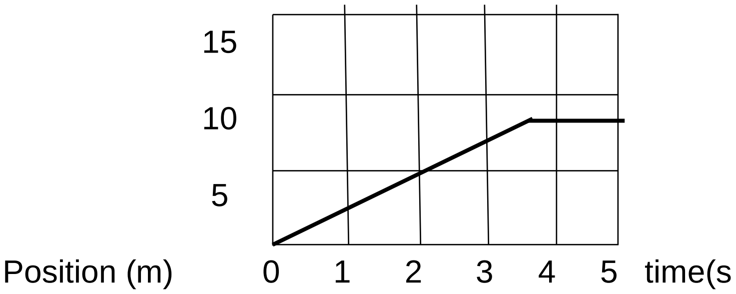
3. The position-time graph below shows the straight line motion of an object. Use the graph to answer questions 3 and 4 that follow.



What is its position at the 2 seconds point after it has taken off?

A. 0.4 m

B. 3.0 m

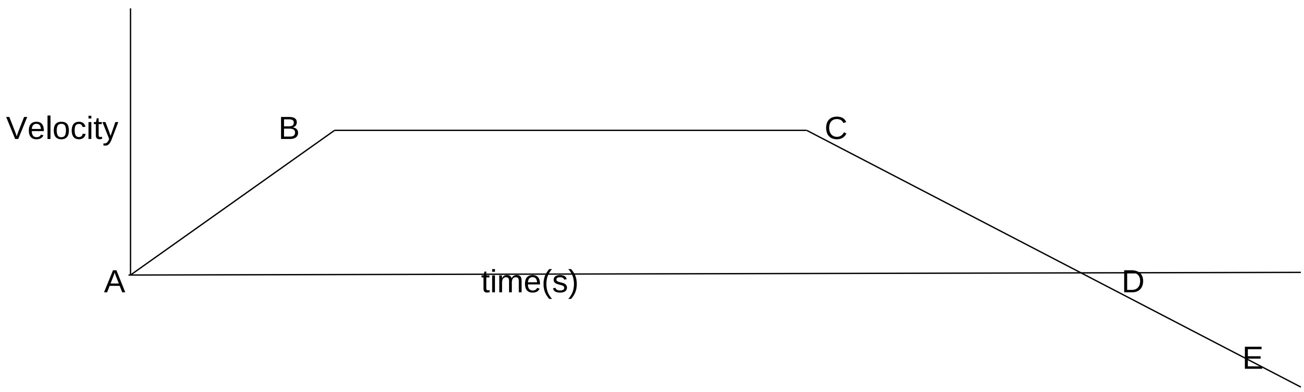
C. 2.5m

D. 5.0 m

4. Using the position- time graph above, what is the velocity of the object at 2 seconds point after its take off?

A. 0.4 m/s
B. 2.5 m/s
C. 2.0 m/s
D. 5.0 m/s

4.1 The figure below shows a velocity-time graph of an object motion



Where along the graph is the acceleration of the object zero?

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A. AB

B. BC

C. CD and DE

D. CD only

4. From the above diagram in question 5, where along the graph is the gradient negative?

   A. AB

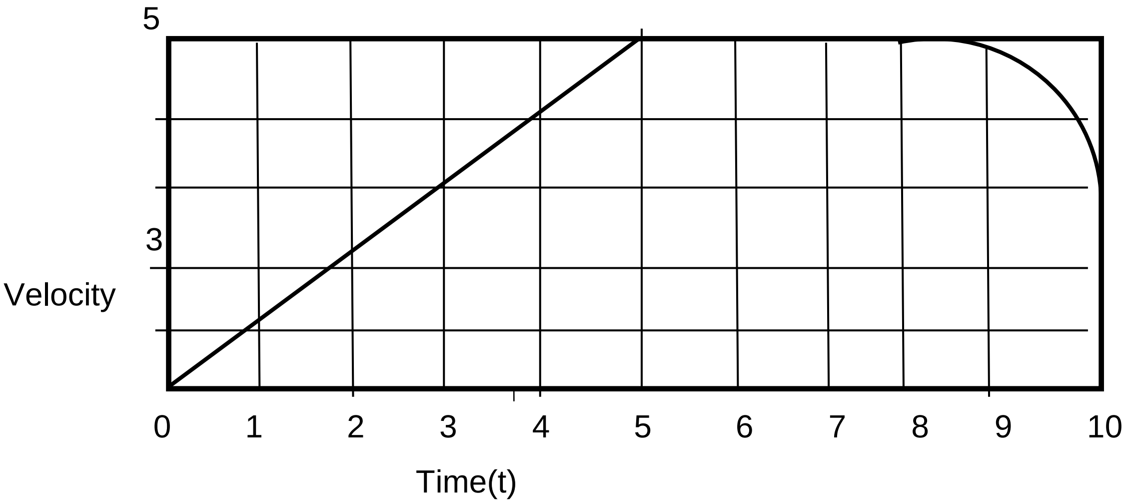
   B. BC

   C. CD and DE

   D. CD

5. An elevator at River Walk moves from the basement to the sixth floor of a building. The elevator moves as shown in the velocity –time line graph below,

   Use the graph to answer questions 7, 8 and 9 respectively



From 5 to 8 seconds the elevator is:

A. At stand still

B. Moving with a constant velocity

C. Moving with a decreasing velocity

D. Accelerating uniformly

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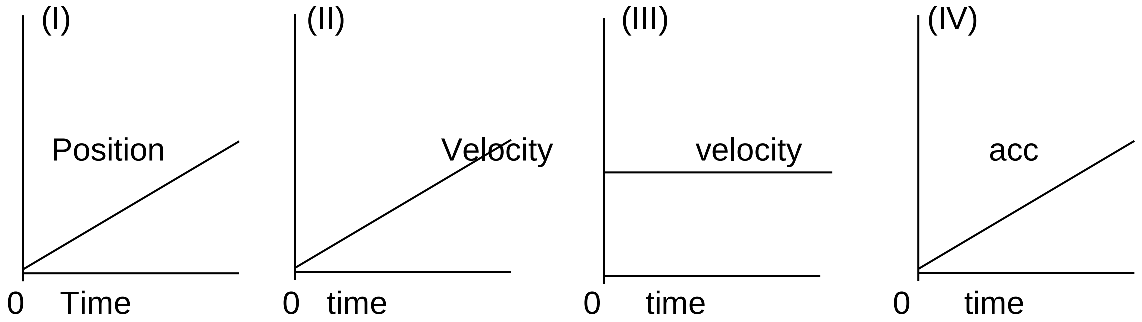
6. How far does it move during the first three seconds of motion?
   A. 0.75m
   B. 1.32m
   C. 4.5 m
   D. 6.0m
7.
8. What is the acceleration of the elevator during the first three seconds?
   A. $0.75 m/s^2$
   B. $1.0 m/s^2$
   C. $3.0 m/s^2$
   D. $6.0 m/s^2$
9. Consider the following graphs, noting the different axes:



Which of these graphs represent(s) motion at a constant velocity?

A. I, II and IV
B. I and III
C. III and V
D. IV only

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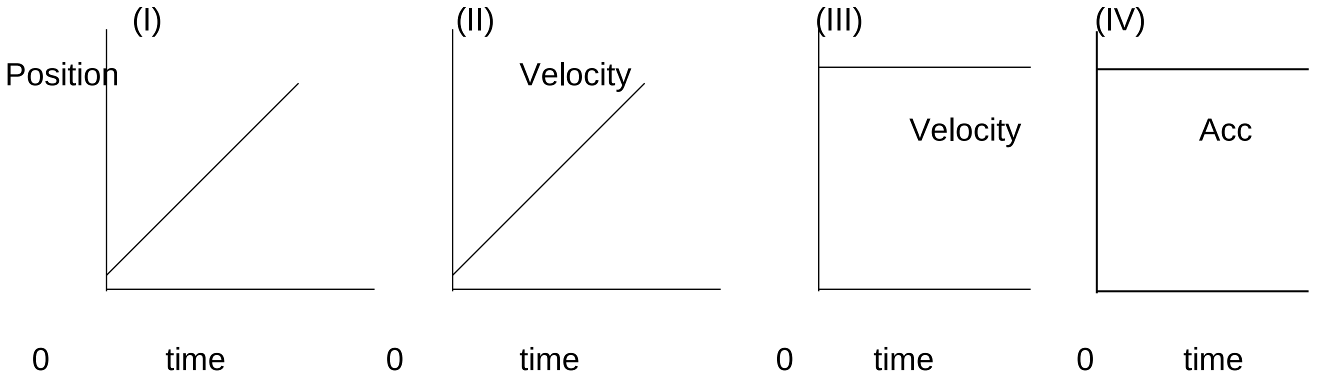
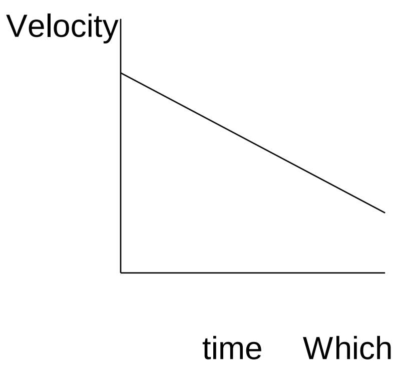
10. Consider the following graphs, noting the different axes:



Which of these graphs represent motion at constant acceleration?

A. I,II and IV
B. II and IV
C. I and III
D. IV only

11. Below is a velocity-time graph of an object motion.



Which sentence best describe the motion of the object?

A. The object is moving with a constant acceleration
B. The object is moving with a constant deceleration
C. The object is moving with a constant velocity
D. The object is moving in a reverse direction

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12. The area under a velocity-time graph represent

    A. acceleration

    B. speed

    C. displacement

    D. change in velocity

13. Displacement can be obtained from the:

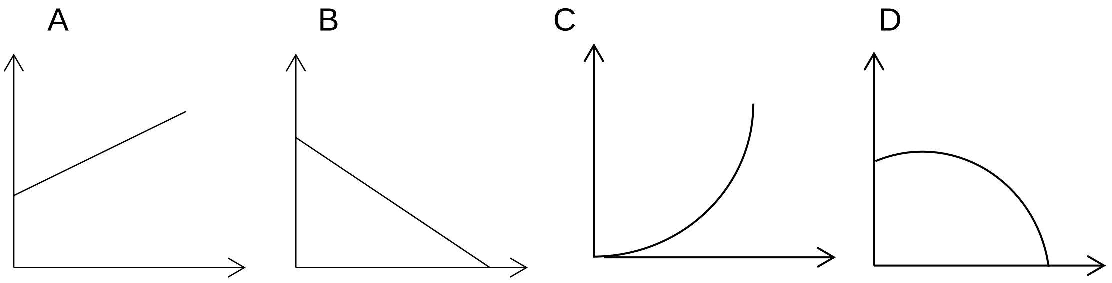
    A. Gradient of an acceleration-time graph

    B. Gradient of a Velocity –time graph

    C. Area under a distance –time graph

    D. Area under a velocity-time graph

14. From the graphs below, what is the form of the v versus time (t)graph if v=u +at is plotted with u and a positive constants?



A. a

B. b

C. c

D. d

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PROBLEMS ON EQUATIONS

1. A car accelerates from 20m/s to 40m/s in 5 seconds. Calculate its acceleration

2. A car moving at 40m/s is applied brakes by the driver when a dog suddenly crosses the road. What is its acceleration if it finally comes to rest in 10seconds
3. A car accelerates at 2m/s^2 in 10 seconds
1. Finds its velocity?
2. Calculate the distance it has travelled in this time
3. A motorbike takes off with a velocity of 10m/s and accelerates to 5m/s^2. Calculate the velocity it will have reached after moving a distance of 50m.
4. A stone falls from the top of a cliff and takes 6s to reach the ground
   a. Calculate the height of the cliff.

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APPENDIX F

OPEN –ENDED INTERVIEW QUESTIONS

1. What is it that you enjoyed most about this topic on kinematics?
2. What is it that you did not enjoy at all about the topic on kinematics?
3. In your view, which were the challenges you encountered in understanding the concepts on kinematics?
4. How did these challenge(s) really affect your understanding of basic concepts in kinematics?
5. Which areas of kinematics were really a pain to you and why?
6. In your view which areas of kinematics were easy to understand and what made them to be easy to understand?
7. Which areas of mathematics are really essential to understand kinematics and what role do these mathematical skills contribute to understanding concept better
8. Which skills of graphing were really a challenge to you and why?
9. What aspects of kinematics equations seemed very difficult to you and why?

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APPENDIX G

INTERVIEW SEMI-STRUCTURE QUESTIONS.

1. What is it that you have really enjoyed during learning in this topic?
2. What was really exciting to you when you were studying concepts of Kinematics?
3. Which area of motion concepts did you find difficult? Which one easy and why?
4. If you were to revisit the topic on kinematics which areas would you say you would more comfortable to handle?
5. In opinion, which areas of mathematics did you find very challenging?
6. Which mathematical skills do you think are very essential for one to comprehend the topic on Kinematics better?

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APPENDIX H

GROUP INTERVIEWS OPEN-ENDED QUESTIONS.

1 Which areas of the topic on Kinematics were difficult to understand?

2 Can you give specific examples of difficulties you encountered during learning of kinematics concepts? Where exactly was your challenge(s) from the example(s) you have given?

3 What are other mathematical challenges have often given you some difficulties when learning physics apart from the ones already given?

4 What is your view about the impact mathematics have in your understanding kinematics better.

5 In your view what do seem to have difficulties in using your mathematics in a physics context?

6 How do you think such gaps between mathematics and physics can be closed to improve understanding of Physics concepts?