

## WEIGHING EVIDENCE

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Arriving at fair and impartial judgements is an important objective of legal proceedings. Often different pieces of evidence of different degrees of credibility and certainty are submitted. Then a court “weighs” the evidence and eventually arrives at a conclusion. We consider the following fundamental question: Exactly *how* should evidence be weighed? Merely mentioning evidence does not constitute weighing evidence. Standard textbooks concerned with the Law of Evidence do not address this particular topic. The use of probability theory comes to mind. The well-known “Blue and green taxicabs” example (see [1]) will show us how a situation may be analysed using probability theory.

### Example 1

Two taxicab companies operate in a certain city after sunset. The Blue Cab Company deploys 15 blue cabs, and the Green Cab Company deploys 85 green cabs. On a rainy night in an empty street with orange street lighting a parked car was damaged by a cab as it carelessly scraped past the car. The cab then drove off. An eye witness who observed the incident from some distance away testified that a blue cab was involved. The witness was subsequently tested under similar conditions in an effort to establish the credibility of his testimony. It was found that the witness could correctly identify the true colour of a cab with probability 80% (see [2]). Which taxicab company should be held accountable for damages?

In this example there are two pieces of evidence pointing in opposite directions. On the one hand there were many more green cabs on the streets than blue cabs. This is motivation for the point of view that more likely a green cab was involved in the accident. However, on the other hand there was fairly reliable evidence by an eye witness who testified that a blue cab was involved. So how should available evidence be weighed to arrive at the best conclusion?

### Solution 1 (Counting cabs)

Under the prevailing conditions the eye witness would have identified about 80% of the 15 blue cabs (i.e. 12 blue cabs) as being “blue”, and he would have identified about 20% of the 85 green cabs (i.e. 17 green cabs) as being “blue”. So he would have identified altogether about  $12 + 17 = 29$  of the 100 cabs as being “blue”.

The probability that a blue cab was involved in the accident is therefore

$$\frac{12}{12 + 17} = 0.413 (= 41.3\%).$$

Since 41.3% is below 50%, the correct inference is that a blue cab was probably not involved in the accident. The Green Cab Company should therefore be held accountable for damages.

### Solution 2 (Using probabilities)

This is a more general approach which also leads to the outcome obtained in Solution 1.

Let  $A_1$ ,  $B$  and  $\sim B$  be the following events:

- $A_1$  : the eye witness testifies that a blue cab was involved in the accident,
- $B$  : a blue cab was involved in the accident,
- $\sim B$  : a green cab was involved in the accident.

We are given the following probabilities:  $P(B) = 0.15$ ,  $P(\sim B) = 0.85$ ,  $P(A_1|B) = 0.8$  and  $P(A_1|\sim B) = 0.2$ .

$P(A_1|B)$  is the conditional probability that the eye witness testifies that a blue cab was involved in the accident, given the event that the cab was blue. Similarly,  $P(A_1|\sim B)$  is the conditional probability that the eye witness testifies that a blue cab was involved in the accident, given the event that the cab was green. But we are more concerned with  $P(B|A_1)$ , the conditional probability that the cab was indeed blue, given the event that the witness testified that a blue cab was involved in the accident.

By Bayes's theorem,

$$\begin{aligned} P(B|A_1) &= \frac{P(B)P(A_1|B)}{P(B)P(A_1|B) + P(\sim B)P(A_1|\sim B)} \\ &= \frac{(0.15)(0.8)}{(0.15)(0.8) + (0.85)(0.2)} \\ &= 0.413 \quad (= 41.3\%). \end{aligned}$$

Bayes's theorem gives us the same conclusion as the one arrived at in Solution 1. The Green Cab Company should therefore be held accountable for damages.

### Remark

Many people might feel uneasy that the probability that a blue cab was involved in the accident turns out to be less than 50%. They typically think "A reliable witness testified that he saw a blue cab. Surely the probability  $P(B|A_1)$  should then be above 50%?" This type of reasoning is seriously flawed. If someone mistakenly thinks that  $P(B|A_1)$  should be above 50%, an incorrect conclusion is drawn because an assessment of the probability of a proposition is confused with the strength of the evidence for the proposition. This error is known as the *prosecutor's fallacy*. Solution 2 shows that  $P(B|A_1) = 0.413$  (= 41.3%), but  $P(A_1|B) = 0.8$  (= 80%).

We tend to forget that the witness's testimony is only 80% reliable. The further mistake made in such reasoning is that the respective numbers of green and blue cabs on the streets are not taken into consideration. Using Bayes's theorem avoids making these mistakes.

### Other ways of weighing evidence

In more complicated examples we may also use methods based on Bayes's theorem which can be formulated in terms of odds and likelihood ratios (see [3]), or methods involving Bayesian networks (see [4]). Computer software is available to facilitate matters, especially if we have to weigh interconnected pieces of evidence. By all the above methods at our disposal we would have arrived at the same conclusion to Example 1.

### Example 2

This example is an extension of Example 1 and we assume everything that was given in Example 1. At a later stage an independent *second* witness comes forward. Suppose that he

also testifies that he saw a blue cab involved in the accident. And under the prevailing conditions this second eye witness also correctly identifies the true colour of a cab with probability 80%. Which taxicab company should *then* be held accountable for damages?

### Solution of Example 2

In Example 1 (with one eye witness) it was shown that the probability that a blue cab was involved in the accident is 41.3%. Now there is an independent second eye witness testifying the same as the first eye witness. Of course the effect will be that the posterior probability that a blue cab was involved will now be above 41.3%. But how far above?

Let  $A_2$  be the event that the second witness testifies that a blue cab was involved in the accident. As in Solution 2 of Example 1, we know that  $P(A_2|B) = 0.8$  and  $P(A_2|\sim B) = 0.2$ .

By Bayes's theorem the posterior probability that the cab that was involved in the accident was indeed a blue cab is

$$\frac{(0.413)(0.8)}{(0.413)(0.8) + (0.587)(0.2)} = 0.737 \text{ (= 73.7\%).}$$

The correct conclusion is that probably a blue cab was involved in the accident. Under the circumstances of Example 2 the Blue Cab Company should be held accountable for damages. Many people might believe that with the testimony of *two* witnesses in Example 2 the assertion that a blue cab was involved must be true "beyond reasonable doubt". Surprisingly this is not the case. In fact, the assertion is only true "on preponderance of probability".

### Remarks

- (a) The methods used in [4] gave a conclusion to the case *R v Blom (1939) AD188* that is different from the judgement in that case.
- (b) Not only should the man-in-the-street be informed of the outcome of legal proceedings, he should also be able to obtain a detailed exposition explaining exactly *how* evidence was weighed.

### References

1. D. Kahneman & A. Tversky "On prediction and judgment" (1972) *Oregon Research Institute Bulletin* 12(4)
2. More explicitly, we mean the following: If the eye witness is exposed 100 times to a green cab under similar circumstances he would say about 80 times that the cab is green, and he would say about 20 times that the cab is blue. Similarly, if the eye witness is exposed 100 times to a blue cab under similar circumstances he would say about 80 times that the cab is blue, and he would say about 20 times that the cab is green.
3. M.A. Muller "Handling uncertainty in a court of law" (2012) *Stellenbosch Law Review* 23(3) 599 - 609 section 6, <http://scholar.sun.ac.za/handle/10019.1/98245>
4. M.A. Muller "Combining uncertainties in a court of law using Bayesian networks" (2017) *Obiter* 8(3) 505 - 516, <http://scholar.sun.ac.za/handle/10019.1/103226>

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