Estimating the efficiency effects of farm mergers: An ex-ante application of Data Envelopment Analysis

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Declaration

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Abstract

The main aim of this thesis is to measure the expected effects of farm level resource pooling mergers on the technical efficiency of farm businesses. The non-parametric efficiency measurement approach of Data Envelopment Analysis (DEA) is used to measure the slack-based efficiency of diversified crop-livestock farms in the North West province of South Africa. Since the non-parametric nature of DEA requires no assumption about the sample distribution, the bootstrap procedure is used to estimate the actual sampling distribution.

Tenure and livestock diversification are identified as variables that may influence the efficiency level of these farms. Regression analysis is used to determine the statistical significance and the extent of the correlation between each of these two factors and farm efficiency.

Three independent merger cases are simulated that each involve pooling the production resources of two independent farm businesses. The merger analysis methodology allows the calculation of the potential efficiency gain or loss that a merger may bring about. It also enables us to distinguish between three effects that, when combined, produce the overall potential efficiency effect. The learning effect represents the effect that learning best practices from each other may have on the efficiency of a merged entity. The harmony effect represents the potential efficiency gain attainable through reallocating production processes to a division of another farm business that have a lower marginal cost associated with its production. The scale effect represents the potential efficiency advantages or dis-advantages that operating at a larger scale may bring to the table of a merged entity.

The significance of this thesis lies in its ability to illustrate the adequacy of the merger analysis methodology to generate quantifiable estimates of the expected efficiency effects of farm level resource pooling mergers without the need to actually take place.

Opsomming

Die hoof doelstelling van die navorsing is om die verwagte invloed van plaasvlak hulpbron samesmeltingsooreenkomste op die effektiwiteit van ‘n boerdery te bepaal. Die nieparametriese benadering tot effektiwiteitsbepaling, “Data Envelopment Analysis (DEA)” word gebruik om die effektiwiteit van gediversifiseerde saai- en vee boerderye in die Noord Wes provinsie van Suid Afrika te bepaal. Omrede DEA nie ‘n aanname vereis oor die steekproefverdeling van die data nie, word die skoenlusmetode gebruik om die werklike steekproefverdeling te benader. Die metode suiwer ook die effektiwiteitsyfers aan vir die inhirente opwaartse bevooroordeling wat in die effektiwiteitsyfers vervat is.

Grondeienaarskap en inkomste diversifikasie is geidentifiseer as faktore wat ‘n uitwerking kan hê op die effektiwiteit van ‘n boerdery. Die invloed van beide die faktore op boerderyeffektiwiteit word bepaal deur regressie analiese.

Drie onafhanklike gevallestudies word gebruik, om elkeen ‘n samesmelting voor te stel tussen twee boerdery eenhede. ‘n Samesmeltingsanalise word gebruik om die potensiële effektiwiteitsvoordele van elke geval te bepaal. Die metode tref onderskeid tussen drie verskillende bonne waaruit voordele kan spruit. Die leereffek verteenwoordig die effektiwiteitsvoordele wat daaruit spruit dat die twee boerderye van mekaar se praktyke leer om die beste praktyke te behou en van swak praktyke ontslae te raak. Die harmonie-effek verteenwoordig die effektiwiteitsvoordele wat spruit uit die hertoedeling van produksieprosesse na afdelings in die boerdery wat dit teen die laagste marginale koste kan handhaaf. Die skaaleffek verteenwoordig die effektiwiteitsvoordele wat daaruit spruit deur die skaal van die twee individuele boerderye aan te pas tot een saamgesmelte boerdery.

Wat die tesis uniek maak is dat dit verder gaan as die gemiddelde effektiwiteitsanaliese navorsing deur die samesmeltingsanalise by te voeg. Dit gee aan die model die vermoë om kwantifiseerbare waardes te heg aan die onderskeie effekte wat ‘n samesmelting tussen twee boerderye op die effektiwiteit van die saamgesmelte boerdery tot gevolg mag hê.

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Thank you to Mr. Jerry Maritz who at that time served as General Manager Grain at the North West Cooperative for granting me access to the farmer study group database after it was made anonymous to protect the identity of the businesses involved.

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Chapter 1

1.1 Introduction

The farming environment is known to be dynamic and volatile. Farmers, well aware of this, navigate toward long term sustainability, profitability and efficiency. During this process, each chooses his/her own strategies and goals. These strategies and goals are highly dependent on the current resources and long term vision. Consequently, there are a variety of options before each farm business as to how it will utilize its strengths, remedy weaknesses, pursue opportunities and mitigate threats. Fortunately, it is in the presence of bountiful options that choices are made powerful. Therefore, it is of utmost importance to understand how individuals and firms evaluate options, set goals and adapt to meet the goals. Enterprise governance plays an integral role in setting the goals and values that are required to guide the firm towards its long term survival and prosperity.

Technological progress in genetics, mechanization and the industrialization of agriculture transforms the way production and processing takes place. It consequently reshapes the relationships between producers and consumers, with a higher concentration of bargaining power in corporate hands (Lauck, 2000). Technological progress enables efficiency increase possibilities that allows for an increase in output, by using the same or less inputs. The reality is that this technology is very expensive and in many instances only justifiable above a certain scale of operation. The disparity in the rate of the price increase of inputs (technology) and that of outputs (commodities) results in the well-known “cost squeeze phenomena”. The producers of commodity agricultural products, such as grain, are intimately acquainted with this reality as it gradually diminishes the margin of their operation.

The diminishing margin has placed renewed emphasis on the way in which the farmer makes short term operational choices and organize itself in its long term business strategy. Industry examples provide innovative approaches that farmers already use to navigate their business through the stormy waters. The worldwide decline in farming entities and the increase in scale of operation is one such observed trend. The farmer who strives to keep his operations relevant during these times has to reconsider the resources at his disposal and the efficiency with which it is utilized. A larger scale of operation does not necessarily mean more profit, but may bring about various other advantages such as economies of scale and decreased fixed cost per hectare. The cost of scaling up, is proportionately higher for a small enterprise than for a larger enterprise, often times to the extent that it is not a feasible option. Farmers seek for innovative ways to reduce the cost of upscaling whilst harnessing the advantages thereof. Partnerships and farm level mergers are examples of such innovative business models that farmers might consider. The impact of it delivering the desired end result however remains difficult to measure. The purpose of this thesis is to develop a framework within which a proposed merger can be assessed with regards to its expected end result.

1.2 Significance and Motivation

Organizational decisions that reshape the total context of a current, functioning organization are sometimes necessary, but due to its extent and the associated uncertainty, are sometimes left as a last resort. The significance of this thesis lies therein that its end result would reduce the uncertainty when considering large organizational changes. It will do so by providing more information upon which such decisions can be based. It by no means claims to predict the success or failure of a proposed merger or partnership, but rather provides quantitative estimates upon which further contractual agreements may be tailored.

From an academic point of view, within the sphere of agricultural production economics, most efficiency studies focus only on the calculation of farm efficiency and testing for its determinants. This thesis is unique in that it applies a merger analysis technique that is primarily developed and used in the banking and public service sector. The end result therefore not only informs the individual farmer of his performance with regard to a reference group and the effect of his tenure structure and diversification strategy on his current efficiency, but also provides a tool to evaluate the potential of organizational changes to increase the efficiency of his business beyond his current resource base.

1.3 Research question and objectives

This thesis ultimately aims to provide an answer to the extent to which a proposed merger or resource pooling agreement is expected to influence the technical efficiency of an individual farm business.

To be successful in this, the following research objectives are set out to assist in providing satisfactory answers to the research question:

- To research and establish a theoretical basis for ex-ante merger analysis.
- To research and develop a mathematical model to identify, distinguish and measure the various effects of a proposed merger.
- To statistically validate the model to ensure its repeatability and universal applicability
- To test the model on a real life dataset and evaluate its ability to estimate the expected effects of a proposed merger between two farm businesses.

This thesis follows the conventional efficiency measurement research methodology to assess the current efficiency level of farms within a study group. It also applies regression analysis in a second stage to determine the effects of external variables on farm efficiency. These estimates then serve as baseline from which three merger cases will be assessed with regards to its expected net effect on efficiency. This effect is decomposed to identify the underlying sources of a possible efficiency gain or loss.

1.4 Thesis delineation

Chapter two provides the theoretical framework of efficiency measurement. It also explores the two predominant efficiency measurement tools and their respective strengths and weaknesses. Data Envelopment Analysis is selected as the measurement tool for this thesis. Some statistical methods are explored that improve the statistical relevance and reliability of the results obtained through this tool. A literature review presents previous findings as it relates to farm efficiency and collaborative action. A theory for the ex-ante estimation of the potential efficiency effects of a merger is presented. A brief theory of case studies as a research tool is also discussed.

Chapter three contains the methodological and mathematical model specifications of efficiency measurements. A numerical example is provided for illustration. The methodology of statistical inference techniques to validate and evaluate efficiency estimates is discussed. A model for estimating the potential efficiency effects of a merger is presented at the hand of an illustrative example. The last section of chapter 3, data validation, presents the dataset that will be used to test the proposed merger analysis methodology.

Chapter four contains a summary of the dataset and the efficiency of the individual farms within the dataset. The proposed merger analysis methodology is then tested at the hand of three independently constructed merger cases.

Chapter five provides the conclusions and summary of this thesis and its findings. Areas for further research is presented with some recommendations based on the shortcomings and opportunities that this thesis uncovered.

Chapter 2: Theoretical framework and previous findings

Chapter two presents a literature study on the theoretical foundation of efficiency studies and the tools that exist to measure it. It also presents previous findings on resource use efficiency as it applies to the agricultural sector. A theory for the ex-ante estimation theory as a basis for estimating the potential efficiency effects that a merger may bring about. The theory of case studies as a research tool is discussed in the last section of the chapter.

2.1 Theory of efficiency and production economics

Micro economic theory considers production processes to be the result of optimization behavior. Producers make decisions concerning what to produce and how to produce it to achieve specific objectives. A technical perspective of these decisions assumes that a producer will seek to maximize physical output for a given endowment of resources. When the aspect of affordability is introduced through the inclusion of input and output prices, an allocative perspective comes into play. Now a producer will seek to minimize the cost of producing a given level of output. By combining these two perspectives, the producer's ultimate objective is to be profit efficient by following the production plan that maximizes benefit, at the prevailing output and input prices. Theoretically it seems easy to grasp the implications of such an objective, yet in practice, very few producers seem to succeed at achieving it (Fare, Grosskopf & Lovell, 1994).

The theory of performance measurement provides a framework within which the disciplines of economic- and operations research seek to assist producers in their optimization endeavor. Performance measurement is what enables a decision maker to visualize the process of moving from where he was to where he is, and ultimately to where he wants to be in the future. The objective of performance measurement is to calculate production-, cost- and profit functions for a given set of producers. These functions are combined to construct a benchmark that enables a producer to calculate the deviation of his technical-, allocative- and profit- efficiency from those firms that define the benchmark practice frontier (Reig-Martínez & Picazo-Tadeo, 2004).

Efficiency is often treated as being the same as effectiveness. Figure 2.1 is adapted from Kumar & Gulati (2010) and illustrates that the performance of an organization can be evaluated both in terms of its resource utilization (efficiency) and the degree to which its pre-determined goals are met (effectiveness). An overall performance measure can then be derived from the product of efficiency and effectiveness measures. Therefore neglecting either of these, provides an incomplete picture of the true performance of an organization. Productivity, although closely related, is fundamentally different to efficiency. Productivity purely refers to an observed input-output ratio and does not combine it, as efficiency does, with the optimal input-output ratio. Efficiency can be calculated using both panel and cross sectional data. Productivity is calculated using only panel data (Ray, 2004).

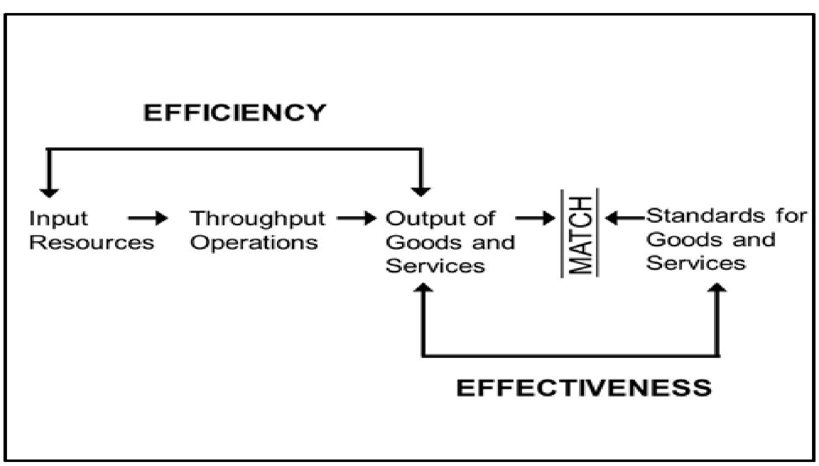


Figure 2.1: Transformation flowchart, Efficiency vs Effectiveness. Source: Kumar & Gulati, 2010

Effectiveness relates to the question of: "Am I doing the right thing?" Am I producing the correct type of output for instance? Efficiency relates to the question of: "Am I doing the thing right?" Am I producing what I am producing, be it the right type or not, in the most resource efficient way?" Ultimately these two measures work hand in hand and firms should strive to do the right thing, in the right way to ensure good performance. This thesis will however only focus on the efficiency aspects of production. Therefore the research question is focused on how producers can produce their current output in a better, i.e. more efficient way.

Agricultural efficiency is a fundamental precondition for sustainable economic development and growth in an economy. The importance of agricultural efficiency becomes clear when we consider the spinoff effects it has on the downstream industries of an economy. An increase in efficiency will enable the release of redundant resources previously used for food production to other non-food industries. Although more complex than to ascribe only one reason to it, the increasing rate of urbanization may very well be largely attributed to increased agricultural efficiency.

2.1.1 Theoretical foundation of efficiency as it relates to agriculture

Central to the production economic theory is the production function, which is essentially a mathematical representation of the relationship between inputs and outputs. It represents a defined input vector (x) that is to be converted into some output vector (y). The production technology, represents the function *f(x)* according to which this conversion will take place. This technology represents a manager's choice of resource combination in an attempt to reach some objectives. Clearly no two firms are identical in their resource selection and combination, yet they may produce an identical output. This provides a just way of determining the most efficient resource combination. Efficiency is therefore internally determined by managerial decision making and the technology used to leverage those decisions into reality. It is important to note that variables external to the decision makers' control such as environmental conditions and regulatory policies may also have a significant influence on efficiency.

The combination of production economic theory and efficiency measurement creates a comprehensive framework within which production processes can be analyzed and recommendations be made in order to improve performance. The first comprehensive analytical approach to measure efficiency in production was developed by Koopmans and Debreu during the 1950's. They introduced a non-radial measure by which we can differentiate, on a relative basis, between efficient and inefficient states of production. This measure however had two short comings. The first was that it lacked the ability to estimate the exact degree of inefficiency. It also could not identify specific efficient producers against which an inefficient producer could be measured. These shortcomings were addressed by the development of a new radial measure of efficiency that identified the maximum feasible equi-proportional reduction (increase) in all inputs (outputs) in order to move from an inefficient state to an efficient one (Debreu, 1951). This measure did however not account for a shortfall in output production or an excess in input usage. Therefore a decision making unit (DMU) considered to be efficient by Debreu's method may be considered inefficient by Koopmans method because it will lie on the boundary of the frontier but not on the efficient subset of the frontier. This concept is referred to as slacks within an efficient state of production. It is evident that a DMU can only be considered efficient if, and only if it satisfies the Koopmans definition of efficiency (Fare *et al.*, 1994).

From a technical perspective of material flow, producers seek to maximize output for a given endowment of resources. A production process would be considered technically efficient if, and only if increasing any output or decreasing any input is only possible by decreasing other outputs or decreasing other inputs (Koopmans, 1951). This means that an increase in the efficient level of outputs can only be obtained through an increase in inputs or a change in production technology. Similarly a decrease of input usage, whilst remaining at full efficiency can only be obtained by a reduction in the current levels of output or a change in production technology.

The presence of several underlying aspects that, when combined, resulted in what was understood to be **technical efficiency** (TE) was only later identified. This led to the re-definition of technical efficiency to be the product of scale efficiency, pure technical efficiency and mix efficiency (Fare *et al.*, 1994).

A production unit is **scale efficient** (SE) when its size of operations is optimal. This means that any changes in the size of the operation, larger or smaller, will render the unit less efficient. SE can be attributed to the presence of fixed costs or overheads in the production function. The reason being that overheads are not as directly linked to the level of output as the variable cost of production. Therefore, the spreading of fixed costs of production across a greater level of output amounts to a competitive advantage known as economies of scale. A firm will however start to operate in a state of diseconomies of scale or negative scale efficiencies once the operational size becomes too large. This will be discussed later in this chapter.

Once SE is isolated, we are left with **pure technical efficiency** (PTE) which is a more accurate measure of the efficiency of input conversion into outputs. Upon further decomposition PTE relates to the use of the optimal combination of inputs as described by **mix efficiency** (ME). ME captures the efficiency of managerial decisions about what to produce and which input-mix to use to achieve specific objectives. Input- and output mix inefficiency is caused by suboptimal input and output mixes resulting from overspecialization, inconsistency in the input mix, and the persistent use of inferior production techniques (Mugera, 2016).

There is another component to production efficiency, which works in combination with TE and its constituent measurements. The **allocative** or economic **efficiency** (AE) of a producer reflects a producers' ability to select the optimal technically efficient input-output vector in the presence of economic circumstances as reflected in the prices of inputs and outputs (Farrell, 1957). Technical efficiency is a necessary, but not sufficient condition for economic or allocative efficiency. A technically efficient firm may not be allocative efficient, but a firm can never be allocative efficient except for first being technically efficient (Kuosmanen, 2001).

When input prices are known, the **cost efficiency** (CE) of a producer can be calculated. Similarly when output prices are known, the producers' **revenue efficiency** (RE) can be calculated. Subsequently in the presence of known input and output prices, the **profit efficiency** (PE) of a producer can be calculated. Profit efficiency illustrates the ability of a producer to construct a production plan that maximizes benefit (profit) within the economic environment at its prevailing input and output prices. Figure 2.2, derived from Fare et al. (1994), Mugera (2016), Farrell (1957) and Kuosmanen (2001), illustrate how the different types of efficiency relate to one another.

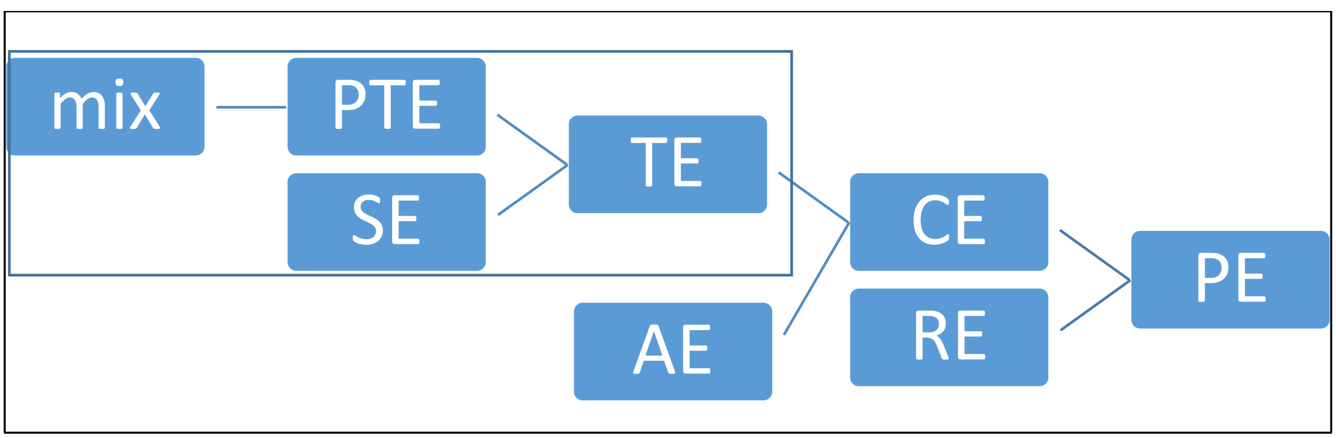


Figure 2.2: Different types of efficiency Source: Fare et al., 1994.

A key concept worth mentioning here is the efficiency-profitability tradeoff. Optimal profit efficiency does not necessarily equate to maximum profit. Often a firm has to choose between maximizing efficiency and maximizing profitability since both are not always jointly attainable. An efficiency maximization decision with regards to fertilizer use may entail the application of quantities that will not necessarily maximize profit due to the prevailing market prices. A rational profit maximizing producer will for instance choose to produce at point C in Figure 2.3 below where the iso-profit line (P) is tangent to the estimated production frontier (ABCD). At this point, both efficiency and profitability is maximized. A firm operating at point B will maximize output from a technical perspective, however it will be inefficient at maximizing profit in conjunction with the quantity of output. The concept of the tradeoff between efficiency and profitability is Illustrated in Figure 2.3 below and thoroughly discussed in Mugera & Langemeier (2013).

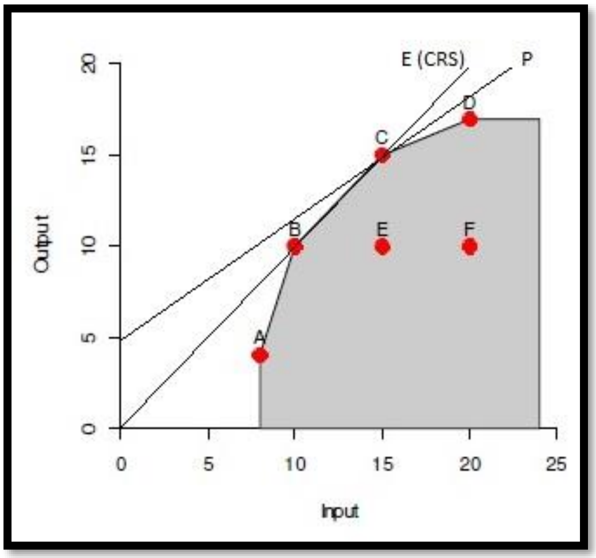


Figure 2.3: Profitability-Efficiency tradeoff; Source: Mugera & Langemeier, 2013

2.1.2 A priori determinants of efficiency

The transformation flowchart in Figure 2.1 illustrates that any factor which affects the inputs, technology or the outputs is likely to influence the efficiency of a specific farm.

The determinants (explanatory variables) of farm efficiency can broadly be classified into three categories: Farm characteristics, Environmental characteristics, and Socioeconomic characteristics (Rouse, Harrison & Chen, 2010).

(1) Farm characteristics include aspects like farm size, debt structure and specialization. Larger farms and higher levels of specialization tend to be more efficient whereas the results on debt structure are mixed. Financial structure may also be taken into account in the efficiency analysis of agricultural production (Davidova & Latruffe, 2003). The effects of debt structure on efficiency can be viewed from two perspectives. The first being that banks prefer to finance more efficient farms and therefore a positive relationship between debt and efficiency can be expected. The second and contrary perspective is that there is a negative relationship between debt and farm efficiency, because farmers with lower debt burden are abler to adjust their operations and therefore are more efficient.
(2) Environmental characteristics include aspects like climate, natural resource endowment, pests and disease, yield potential and political and regulatory aspects.
(3) Socioeconomic characteristics include aspects that pertain to the human capital like education, skills, and experience.

2.1.3 A priori efficiency effects of mergers

Frequent reports of mergers and takeovers in business reflects the important role it has to play in the restructuring of many sectors. Mergers are motivated by reasons that are either internal or external to an organization. Internal organizational reasons include the possibility of exploiting economies of scale and scope, risk sharing, scarce managerial skills, labor specialization, etc. External market oriented reasons on the other hand include the possibility of increased bargaining power or the facilitation of collusive behavior (Bogetoft & Wang, 2005). Important to note is the use of the word "possibility" in the sentences above, indicating the uncertainty of the outcome of such collaborative actions. There are many obstacles to mergers such as incompatibility of organizational culture and public policy directed against the exercise of market power.

The success or failure of collaborative actions can essentially be attributed to two groups of aspects. The first is the so-called "**soft**" or non-quantifiable aspects such as the compatibility of organizational culture and managerial views. The second group include the "**hard**", more quantifiable aspects that are measurable production economic effects such as the ability to decrease the cost of production. For collaborative action to succeed in its goals, it has to have a healthy balance of both of these aspects. Interestingly, more often than not, it is the soft aspects that determine the success or failure of collaborative action.

Agency theory and transaction cost theory provides a comprehensive framework to assess the possible effects of the "soft" aspects on collaborative action. It deals with the problems that may surface in agency relationships between principles and agents due to unaligned goals or different risk aversion levels. Agents (decision makers) acting in line with objectives that are contrary to that of the organization (principle) may totally negate all other positive attributes of collaborative action. If no deliberate provision is made for accountability loops and cross checks within a merged entity, the efficiency of its collaborative actions may be crippled by moral hazard problems. Transaction cost theory deals with the costs of interaction between two economic units or divisions thereof. These costs may be monetary or non-monetary in nature. Larger organizational structures typically have higher transaction costs associated with the execution of decision-making due to their hierarchical structures. Remaining within production economic theory, this section is directed to inform managers on the "hard" production economic aspects of collaborative action.

Based on the concept of the production function, firms that consider to collaborate can do so based on one or more of the following principles (Röller, Stennek & Verboven, 2000):

(1) According to the principle of the <u>rationalization of production</u>, mergers may bring about cost savings from the reallocation of production processes across firms to those that have the lowest marginal cost without the need to increase the joint technological capabilities.
(2) Mergers may also allow for lower average production costs according to the <u>economies of scale</u> principle by spreading fixed costs over a larger output base.
(3) The diffusion of knowledge through closer relationship between the management of two separate firms is a source of <u>technological progress</u>.
(4) <u>Purchasing economies</u> through joint marketing and sales may bring about savings in factor prices such as intermediate goods and the cost of capital.
(5) When two firms decide to pool resources, excess capacity in one resource may supplement a shortfall of the same resource in the other firm and vice versa through a <u>reduction of slacks</u> in the production system.

From a welfare economics perspective, these five production economic effect can either be **real or redistributive** in nature. Typically only real gains are considered beneficial to society (the merged firm in this case) as a whole. Real gains refer to the collaborative action's ability to save productive resources in the economy through increased efficiency. Among the preceding five principles, motivation for collaborative action that lead to real gains are the rationalization of production, economies of scale, technological progress and slack reduction. Redistributive gains from the perspective of producers relates to collaborative action's ability to transfer consumer surplus to producer's surplus. Among the list of motivating principles for collaborative action, purchasing economies is an example that will lead to redistributive gain. The nature of production and the market structure within which a firm operates will determine the ability of collaborative action to yield redistributive gains. Collaborative action may furthermore be considered **defensive or offensive** in nature. When collaboration is born out of the need to correct for market failure (inefficient resource allocation), it is considered to be defensive. Conversely, when collaboration is born out of the need to facilitate monopolistic behavior, it contributes to market failure by distorting the free market mechanism of price setting.

A second consideration may be the effect of merger gains on fixed and variable costs. Savings in fixed cost usually arises from economies of scale, technological progress and purchasing economies. Variable cost savings may come from any of the above mentioned merger effects. Economies of scope may also be a merger motive in a situation where it is technically more efficient to produce an output bundle within a single diversified firm than producing each individual output in separate specialized firms. A merger between two firms in such a case will lead to efficiency gains.

More intensive utilization of machinery resources may reduce the capital cost per unit of area worked. A larger utilization base will also make it possible to invest in improved technology such as precision farming equipment that will improve resource application efficiency. A larger asset base through pooled resources provides more collateral to facilitate access to credit. Coordinated purchasing and marketing may be a source of bargaining power to secure lower factor costs and higher product prices. Larger operations will create the opportunity for labor specialization that will improve the efficiency of labor.

Besides the majority of positive effects, there are also possible negative effects to consider. Moral hazard, the central topic of contract theory poses some threats to the success of resource sharing partnerships. A second possible negative is the timeliness-cost, associated with not being able to perform a specific task at the optimal point in time. Scheduling and provision for excess capacity, even though to the detriment of efficiency, may mitigate the risk of timeliness costs (Larsén, 2010)

2.2 Efficiency measurement

As discussed above, efficiency combines the observed input-output ratio with the optimal input-output ratio. This enables the analyst to benchmark and compare the efficiency of an observed input-output ratio of a given producer with that of others and more importantly to the optimal input-output ratio. The next section will discuss tools to calculate the optimal input-output ratio from empirical data. More specifically, it will elaborate on the calculation of production, cost and profit frontiers to represent the industry best practices. These benchmarks allow us to calculate the degree of departure from technical, economic and profit efficiencies experienced by individual firms (Reig-Martínez & Picazo-Tadeo, 2004).

An extensive literature concerning the measurement of efficiency in production has developed since Debreu (1951), Farrell (1957) and Koopmans (1951) provided basic definitions for technical and allocative efficiency in production. Farrell (1957) suggested that one could analyze technical inefficiency in terms of realized deviation from an idealized frontier isoquant. This frontier estimation approach to efficiency measurement is closely related to the econometric approach of identifying inefficiency with unexplained variation in regression models. Two distinct approaches to efficiency measurement exist. Data Envelopment Analysis (DEA) follows a non-parametric estimation approach whilst Stochastic Frontier Analysis (SFA) follows a parametric calculation approach.

A brief discussion of econometric simulation and regression in Greene (1993) provides a foundation to understand how frontier estimation techniques such as DEA and SFA measure efficiency. Any simulation is a simplification of reality in order for us to understand how reality works. Production econometric models use regression analysis with the objective to use the actual input data of a production system to produce simulated values that describe the actual output as accurately as possible. Such models are usually in the form of mathematical equations known as production functions. A model's ability to reach its objective is measured by the variation of the observed values from the simulated values of the model. Mathematical models, being a simplification of reality, can produce simulated estimates with high levels of precision. Despite the precision of these estimates, their accuracy in explaining the variation in output many a time falls short. Any differences between the observed values and the simulated values are regarded as unexplained variation. In an attempt to increase model accuracy, an error term is included as an estimate of the unexplained variation. Models can only be as accurate as the data we use to specify them and the relevance of the assumptions we impose thereupon. Unexplained variation is caused by three types of errors:

(1) Measurement errors are the result of inaccurate accounting of the actual observed values of variables which gives rise to statistical outliers.
(2) Systematic errors are the result of faulty assumptions or calibration errors that incorporates a constant bias across all observations.
(3) Random errors, referred to as noise or shocks, are the result of variables exogenous to the model with an influence on variables within the model.

The ability of a model to explain the variation in a production process with the least unexplained variation, establishes its superiority among other.

The central tendency of regression analysis fits a line around the mean through the middle of a dataset. The functional form of this line is based on the distributional assumptions about the data. Any deviation from this line, above or below, is considered an error or unexplained variation. In frontier estimation the objective is to find the highest level of efficiency as revealed by the data. Therefore regression analysis is insufficient because some observations, depending on the deviation direction, may fall above the regression line and remain undetected.

Figure 2.4 illustrates how frontier estimation techniques adjusts the central tendency of regression analysis to be frontier oriented so that all data points are enveloped by the frontier function (Clemente, Lirio & Gomez, 2015).

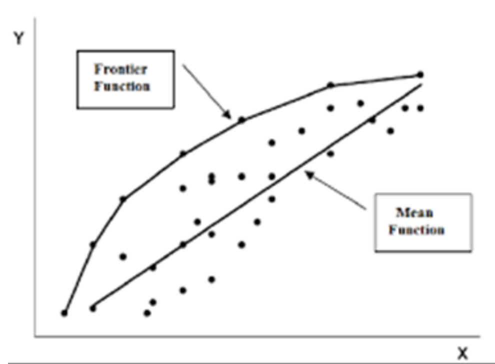


Figure 2.4: Frontier- vs Central tendency estimation. Source: Clemente, Lirio & Gomez, 2015

Any deviation from this frontier is attributed to inefficient managerial decisions with regard to resource utilization. In reality however, some of this deviation is attributable to measurement errors, systematic errors and random noise that are outside the control of management. The deviation from the frontier should therefore be considered the sum of managerial inefficiency and uncontrollable errors.

The main difference between SFA and DEA lies in their approach to construct the efficient reference frontier whilst minimizing the unexplained variation in an attempt to isolate the inefficiency from the error term to determine the deviation attributable solely to managerial inefficiency.

2.2.1 Parametric Stochastic Frontier Analysis (SFA)

Parametric SFA requires an a-priori assumption about the functional form of an observed dataset and its distribution. Many agricultural production functions are assumed to have a Cobb-Douglas functional form. SFA uses various ways to correct the central tendency of conventional econometric ordinary least squares regression analysis. One is the corrected ordinary least squares (COLS) regression which adds the maximum error to all observations to envelop all observations so that no observation lies above the regression line. This is however a source of constant bias across all observations.

The Maximum likelihood approach is a better way to adjust the regression line. It specifies a likelihood function by adding parameters to the input and output data variables. The likelihood principle states that the most likely parameter values are those that make the actual observations as likely as possible. The objective is to select values for the parameters that will maximize the likelihood function, i.e. to maximize the probability of obtaining the sample that was drawn. SFA allows for the separation of a technical inefficiency term from the traditional error term. It is easy to perform statistical testing through confidence intervals to identify and eliminate outliers to correct data for outliers.

Because SFA uses a statistical approach through regression analysis to formulate an econometric frontier model, it has the ability to construct the frontier and calculate the elasticities of variables in one step. Based on the elasticities of the regressed variables we can easily identify the determinants of efficiency. Its ability to estimate an error term makes it useful to separate the inefficiency attributable to management and the variation caused by factors that are outside the control of management. Figure 2.5 illustrates how SFA is particularly well able to separate the noise effect from the inefficiency (Porchelli, 2009). Statistical confidence interval tests can also be used to identify outliers.

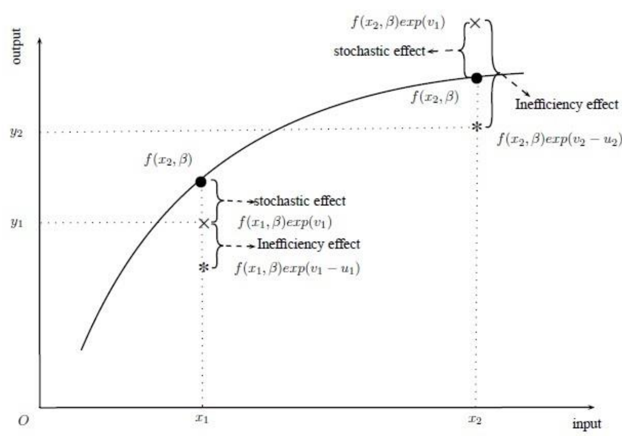


Figure 2.5: Parametric frontier estimation Source: Porchelli, 2009

SFA does not perform well to measure the efficiency of multiple input, multiple output production systems. Aggregation of several variables into one is sometimes used, but it makes it difficult to account for collinearity between variables. SFA also requires that all variables be measured in the same units, requiring standardization. This makes it difficult to incorporate performance ratios that entails the use of variables of different units. Another drawback of SFA is its need for an a-priori assumption about the functional form. This makes it prone to misspecification that may result in an inaccurate model.

2.2.2 Non Parametric Data Envelopment Analysis (DEA)

2.2.2.1 Overview

The non-parametric approach to the efficiency estimation concerns mathematical programming based measures of efficiency. More specifically, linear programming (LP) approaches that rely on convexity assumptions are known as Data Envelopment Analysis (DEA). DEA was introduced in the seminal work of Charnes, Cooper and Rhodes (1978) which was based on the theory of efficiency measurement of Debreu (1951), Koopmans (1951) and Farrell (1957).

DEA is particularly capable of dealing with some of the shortcomings of SFA. It is less prone to miss-specification because it does not require a priori assumptions about the functional form of a data set. Instead, it applies LP to construct a frontier that reveals the functional form of the underlying data. The LP approach does however make the constructed frontier vulnerable to be influenced by outliers since it has no statistical basis to estimate an error term. DEA therefore considers all deviation from the frontier as inefficiency. Provision has been made in literature to enable DEA to deal with outliers in a second stage through bootstrapped regression analysis and confidence interval tests, establishing it as a nonparametric stochastic frontier estimation methodology (Banker & Natarajan, 2008). Because DEA does not rely on regression analysis, but on LP to construct the efficient frontier, the problem of collinearity among variables is eliminated. This makes DEA particularly able to estimate the efficiency of multi-input, multi-output production systems. It is unit invariant, meaning that variables of different measurement units can be used without the need of standardization. DEA can therefore be used to measure efficiency by combining multiple performance measures.

2.2.2.2 Formal description

DEA compares organizations or parts thereof, referred to as decision making units (DMU's) that share common goals and use similar resources to produce similar products by calculating the efficiency with which they convert bundles of inputs into bundles of outputs. It is a practical tool that can be used by academics for research as well as by industry and consultants for improved performance measurement and accountability. Since its introduction, researchers in a number of fields have quickly recognized that it is an excellent and easily used methodology for modeling operational processes for performance evaluation (Cook & Zhu, 2005).

Given a set of observed data, DEA follows a linear programming (LP) approach to obtain empirical estimates of production economic concepts such as production functions and efficient production possibility sets. According to the minimum extrapolation principle, DEA constructs a piecewise linear frontier that envelops the observed data in such a way to represent the minimal production set that satisfies some imposed production assumptions. The use of the minimum extrapolation principle directly connects DEA with optimization principles. DEA optimizes the production objectives of a DMU with reference to the efficient frontier. Firms that exhibit the most efficient production process of converting inputs into outputs define the frontier. Then based on the production objective to either minimize input or maximize output, the efficiency of each DMU is calculated (Kuosmanen, 2001).

Depending on the objectives of management, a DEA model can be directed to either the input or the output side of the production function. An input oriented model would be relevant for a minimization objective such as reducing costs or input consumption. Input oriented efficiency measurements indicate the extent to which a firm can radially reduce its input use without affecting its output levels simply by using the remaining inputs more efficiently. Conversely, an output oriented model would be relevant for a maximization objective such as increasing revenue or output. Output oriented efficiency measurements represent a radial measure of the extent to which a firm can produce more output by using its current level of inputs more efficiently as determined by firms that define the best practice frontier. This formulation is closely related to the neo-classical production function that defines the maximum achievable output for given input quantities (Färe, Grosskopf & Lovell, 1994).

The decision on what orientation to use should be based on information regarding which factors (input or output) management has most control over. An output orientation may be appropriate in instances where inputs such as land are fixed in the short run and therefore non-adjustable by management (Larsén, 2010). One can also argue, if owned land is redefined and included in the DEA model as utilized land (owned or rented), that an input oriented approach may be appropriate since a farmer can adjust the amount of land without the need to sell or buy, but by using the rental market to facilitate the adjustment.

2.2.2.3 Assumptions

Benchmarking studies compare observed performance against a systematic description of possible performances referred to as the technology set (T). The technology set is therefore a description of the input-output combinations that are assumed to be feasible in a given context. Production economic concepts provide a comprehensive framework to help us make reasonable assumptions to construct the technologies from actual observations (Bogetoft, Otto, Gendreau, Michel and Potvin, 2011).

Generally, there are four assumptions that guide the construction of the technology frontier that will envelop the observed data. These assumptions are represented in terms of the following mathematical properties: disposability, convexity, returns to scale and additivity.

<u>Disposability</u>

Disposability is regarded as the first-order curvature condition of the production frontier. It assumes that if a producer is able to produce a certain quantity of outputs with a given quantity of input, then he will also be able produce the same quantity of outputs even if he had more inputs at his disposal. It is therefore assumed that he can freely dispose of surplus inputs, without affecting his current level of outputs. Similarly, if a given level of inputs can produce a given quantity of outputs, then the same input can also be used to produce less output by disposing of surplus output. It is therefore assumed that any production plan within a technology set T can be freely adjusted based purely on the technical relationship between inputs and outputs. Hence the disposal is considered free i.e. at no monetary cost or any restriction (Bogetoft *et al.*, 2011).

Figure 2.6 from Bogetoft *et al.*, (2011) illustrates four firms and the corresponding technology set T represented by any input-output combination below and to the right of the data points (shaded) based on the assumption of free disposability:

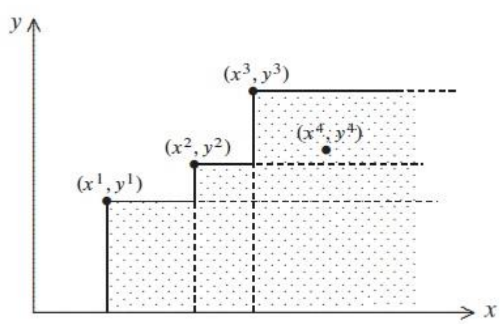


Figure 2.6: Disposability assumption; Source; Bogetoft et al., 2011

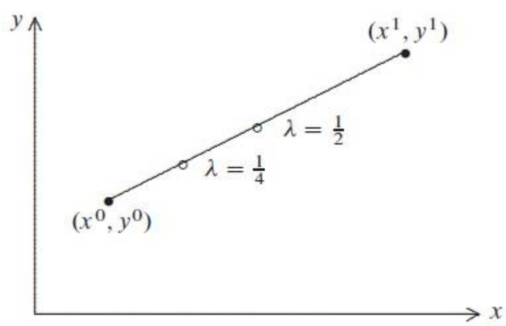
It is evident that T is not dependent on observation $(x^4; y^4)$, because its feasibility is inferred by $(x^3; y^3)$ under the assumption of free disposability.

Generally, it is considered to be a safe assumption because it will be adhered to in most production systems. It is also considered to be weak in its ability to extend the production possibility set. In cases of jointly produced outputs, however this assumption may not be adhered to and therefore will be considered to be a strong assumption, making it perhaps less appropriate. Strong disposability implies that a scaled input vector will not congest (decrease) the output vector. Weak disposability on the other hand implies that a scaled input vector will produce at least the original output vector. It follows that strong disposability implies weak disposability, but weak disposability does not imply strong disposability (Färe & Grosskopf, 2000).

Convexity

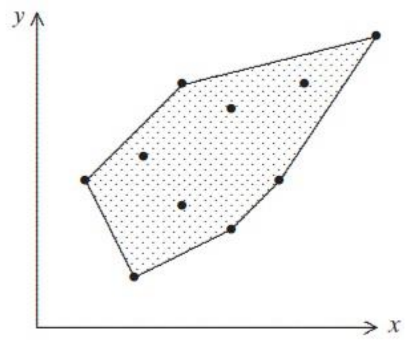
Convexity is regarded as the second-order curvature condition of the production frontier. It assumes that a convex combination of weighted averages (λ) of two feasible production plans (firms) are also feasible as shown in the left pane of Figure 2.7 below (Bogetoft *et al.*, 2011). This implies that for any two points within the convex hull, the production plans on the line that connects them are also within T, implying that they are attainable (RHS).

Convex combinations



2 observed firms and their weighted convex combinations representing two additional firms

Convex Hull



10 firms with all feasible weighted convex combinations

Figure 2.7 Convexity assumption. Source: Bogetoft et al., 2011

Convexity is a relatively strong assumption and therefore needs to be validated by sound microeconomic principles. The adherence of an input-output vector to the convexity assumption is motivated by the law of diminishing marginal rates of substitution that requires marginal products to be non-increasing i.e. decreasing (Petersen, 1990). Assuming global convexity of the technology set is however in some instances not ideal from a theoretical point of view. Convexity requires divisibility of inputs because a convex combination is essentially an addition of down-scaled production plans. Inputs are in reality not always perfectly divisible. From an operational point of view however, it is a convenient, yet harmless assumption as far as the results are concerned. For a given set of aggregate data on the processes of a firm, a convex combination can provide a reasonably accurate estimate of alternative but non-observed aggregations. From a benchmarking perspective, the convexity assumption serves the role of enlarging the technology set compared to that of free disposability. It also creates technologies that are better able to distinguish between average performance and best practice (Bogetoft *et al.*, 2011).

Returns to scale

Returns to scale is viewed as a homogeneity condition because it imposes assumptions that relates to the homogeneity of inputs in order to allow for the rescaling of observed production plans. There are two broad homogeneity (scale) conditions that may be imposed according to the underlying nature of the inputs used in production. Constant returns to scale (CRS) assumes identical input vector units across all firms. The homogeneity of input variables implies that a proportional change in input levels would result in an equal proportionate change in output levels. Variable returns to scale (VRS) on the other hand, assumes that input vector units are not perfectly homogeneous in the sense that a proportional change in input levels will not necessarily lead to a proportionate change in output levels. This disproportionality may be larger when increasing returns to scale (IRS) prevails or smaller when decreasing returns to scale (DRS) prevails. Therefore by assuming VRS, IRS and DRS are implied.

The assumed scale properties of a DEA model and its orientation reveals an interesting dynamic. Under CRS, an input oriented model and an output oriented model will generate the exact same efficiency scores. However, when VRS is assumed, the efficiency estimates of an input oriented and an output oriented model may differ significantly. Therefore, the choice of orientation is only significant when VRS technologies are assumed. Careful attention to the reasoning behind the selection of orientation should therefore be given for a VRS DEA model. Ray OBC in Figure 2.8 for instance represents a CRS efficiency frontier. Ray ABCD represents a VRS efficiency frontier that includes IRS, CRS and DRS each at its own respective portions of the frontier (Seiford & Zhu, 1999).

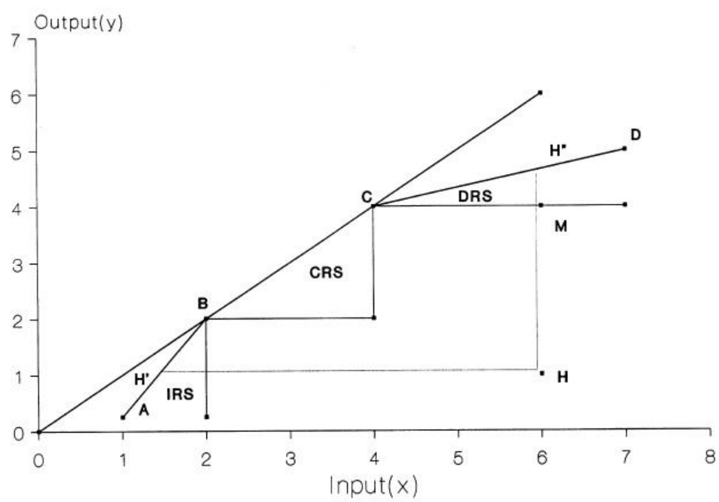


Figure 2.8: Scale assumptions Source: Seiford & Zhu, 1999

Evidently the shape and location of the efficient frontier is heavily dependent on the assumption of the returns to scale (RTS) properties of the efficient frontier. Subsequently, the efficiency scores (the distance of a firm to the frontier) will also be greatly affected by the RTS assumption. It is therefore of utmost importance to make appropriate assumptions about the scale properties of the efficient frontier. A thorough understanding of the nature of agricultural production generally provides some a priori reasons to assume VRS. Literature does however provide tests to determine the exact scale properties of a production technology to ensure the appropriateness of the scale assumptions (Simar & Wilson, 2002).

Additivity

The additivity assumption of technologies has bearing on the way in which the sum of two production plans are assessed. According to this assumption, a combination of two feasible production plans, ceteris paribus, will also be feasible (Bogetoft *et al.*, 2011). Illustratively, if (x, y) is feasible, $2(x, y) = (x, y) + (x, y)$ and $3(x, y) = 2(x, y) + (x, y)$ will also be feasible. In general terms, if (x, y) and (x', y') are possible, so is $h(x, y) + k(x', y')$ for random h and k values. This produces a full grid of feasible production plans from the two observed plans as shown in Figure 2.9 below (Bogetoft *et al.*, 2011).

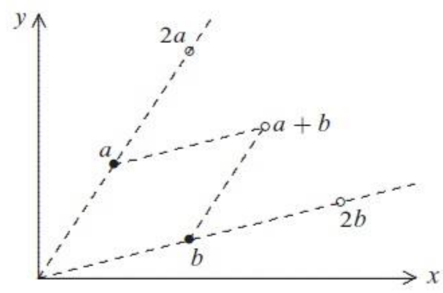


Figure 2.9: Additivity Assumption. Source: Bogetoft, 2011

Essentially, it is assumed that if *(a)* produces y' using x' and *(b)* produces y'' using x'', *(a + b)* should be able to produce at least y' + y'' from x' + x'', since it can simply operate as two independent divisions imitating the original ones. The standard convexity assumption does not have this "proved by way of example" rationale (Bogetoft & Wang, 2005). Conceptually, it is a particularly appealing assumption because it rules out all the externalities between two production plans. In reality however, additivity based models may require complex mathematical ingenuity to construct a technology frontier that properly represents the actual technology.

There is a very important interrelationship between additivity, returns to scale and convexity. The properties of a production possibility set that assumes DRS in combination with an additive technology is the same as if convexity is assumed under CRS. Similarly, assuming convexity of the production function automatically implies CRS (Bogetoft *et al.*, 2011).

From an applied point of view, the additivity assumption has advantages over the scaling and convexity assumptions that are typically adhered to in microeconomic literature (Bogetoft & Wang, 2005). It is important to understand these assumptions, because it affects the plausibility of the benchmarks we derive.

2.2.3 Model specification:

All DEA models estimates the technology from an observed dataset by using the minimal extrapolation approach according to a set of assumptions about the nature of the technology as discussed in the preceding section. Different DEA models can be specified, each based on its own unique set of combined assumptions.

Consider a situation where each of *n* DMU's, $i \in I = \{1, 2, \dots, n\}$ use *p* inputs to produce *q* outputs. For a particular DMU^i, let $x^i = \left(x_1^i, \dots, x_p^i\right)$ *and* $y^i = \left(y_1^i, \dots, y_q^i\right)$ be the inputs consumed and outputs produced. Also, let $T = (x, y)$ be the production possibility set or technology that specify the environment within which inputs are transformed into outputs. As discussed above, Equation 2.1 combines these assumptions about T in various combinations:

$$T(s) = x \geq \sum_i \lambda^i x^i \;\; ; \;\; y \leq \sum_i \lambda^i y^i \;\; ; \;\; \lambda \in \mathrm{K(s)} \tag{2.1}$$

s can be specified as any of the following: $K(crs) = 0,\; K(drs) = \sum_i \lambda^i \leq 1 \; or \; K(vrs) = \sum_i \lambda^i = 1$

1. Disposability: $(x', y') \in T \;\; \& \;\; x'' \geq x' \;\; \& \;\; y'' \leq y' \;\; \rightarrow \;\; (x'', y'') \in T$
2. Convexity: T is convex
3. S-returns to scale: $(x', y') \in T \;\; \rightarrow \;\; k(x', y') \in T \;\; for \;\; k \in K(s)$
4. Additivity: $(x', y') \in T \;\; \& \;\; (x'', y'') \in T \;\; \rightarrow \;\; (x' + x'', y' + y'') \in T$

Table 2.2 is constructed from information presented in (Bogetoft *et al.*, 2011) and presents six classical DEA models, each with its own unique combination of the four assumptions discussed above.

Table 2.1: Six classical DEA model specifications

Model	Disposability	Convexity	Returns to scale	Additivity	Parameter set
Free disposable hull (FDH)	X	-	$k = 1$	-	$\sum \lambda^k = 1\,;\, \lambda^k \in \{0.1\}$
Variable returns to scale	X	X	$k = 1$	-	$\sum \lambda^k = 1$
Decreasing returns to scale	X	X	$k \leq 1$	-	$\sum \lambda^k \leq 1$
Increasing returns to scale	X	X	$k \geq 1$	-	$\sum \lambda^k \geq 1$
Constant returns to scale	X	X	$k \geq 1$	-	$\lambda^k \geq 1$
Free replicable hull (FRH)	X	-	$k = 1$	X	$\lambda^k \in N_0$

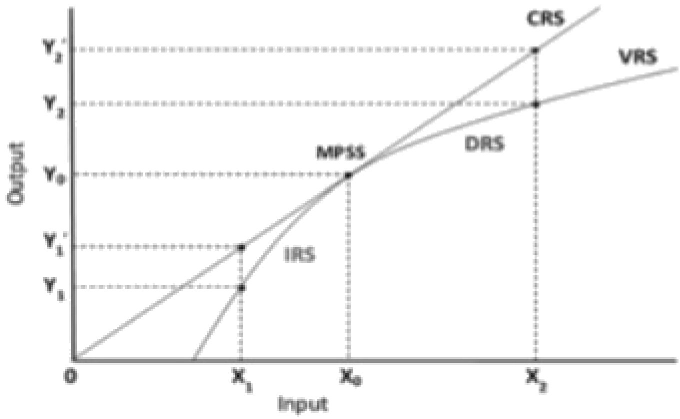
Source: Bogetoft et al.,2011

The size of the technology set will be heavily dependent on the assumptions according to which the envelopment frontier was constructed. FDH has the smallest technology set. VRS is larger because it includes convexity that allows for weighted combinations of the observed firms. When scaling is allowed, the technology set will enlarge once more. DRS enlarges the set for small input values, and IRS that of larger input values. CRS represents the largest technology since it allows for full rescaling and convexity. FRH is less comparable to the others, yet its technology set is larger than FDH but still smaller than that of CRS. A larger technology set will be more optimistic about a firm's improvement potential, causing them to appear less efficient than in smaller technology sets.

2.2.3.1 Specifications that allow the calculation of scale efficiency

The original model proposed by Charnes, Cooper & Rhodes (1978) combined disposability, convexity and constant returns to scale. This formulation was however only relevant when all firms operate at an optimal scale. In the real world of imperfect competition, market failure and government intervention firms more often than not operate at a suboptimal scale. Subsequent models such as Banker, Charnes & Cooper (1984) also assumed disposability and convexity, but assumed variable- and constant returns to scale. This made it possible to exclude the inefficiency effects resulting from operating at a sub-optimal scale from technical inefficiency of a firm. That meant more accurate information to base performance improvement policy on (Coelli, Rao, O'Donnell & Battes, 2005).

Economies of scale within the production economic framework determines that the average cost of production will decrease as output increases, however only up to a certain point where diseconomies of scale ensues. This point is illustrated by point MPSS in Figure 2.10 on the next page (Kelly *et al.*, 2013). This inflection point denotes the optimal output level where average cost per unit will be the lowest. A firm's coordinates in relation to the most productive scale size (MPSS) will indicate its scale efficiency. The MPSS for a given input and output mix is the scale size at which the outputs produced per unit of input is maximized (Banker *et al.*, 1984)



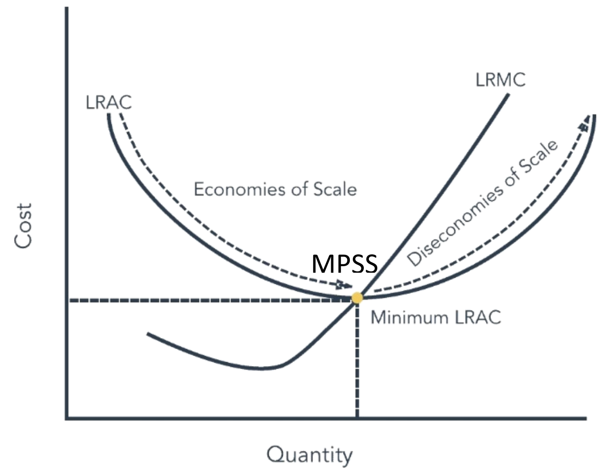


Figure 2.10: Most Productive Scale Size. Source: Banker et al., 1984

The calculation of the scale efficiency of a firm involves solving the same DEA model, first under the assumption of VRS and in a subsequent stage under CRS. The ratio of the two resulting efficiency estimates reveals the scale efficiency of that firm. When $E_{VRS} = E_{CRS}$, the firm would be scale efficient, indicating that it operates at the most productive scale size. If $E_{VRS} \neq E_{CRS}$, the firm is scale inefficient, indicating that an adjustment of the scale of operation would result in increased efficiency. The question of the adjustment direction, larger or smaller, can further be assessed by testing the scale properties of the technology of the frontier in the region surrounding the firm. If a firm is operating in a region of IRS (DRS), increasing (decreasing) the size of operation would move the firm to the MPSS (Banker *et al.*, 1984). Table 2.3 represents the returns to scale test developed by Simar & Wilson (2002) that enable us to determine the scale properties of the technology set surrounding a specific DMU.

Table 2.2: Returns to scale test

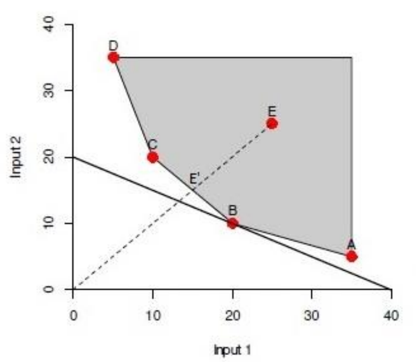
	Test 1	Test 2
Test	Globally CRS vs Globally VRS	Globally NIRS vs Globally VRS
Hypotheses	H_0: technology is globally CRS H_1: Technology id globally VRS	H_0^*: technology is globally NIRS H_1^* : Technology is globally VRS
By	$S^1 = \frac{\sum_{i=1}^{n} E_{CRS}^i}{\sum_{i=1}^{n} E_{VRS}^i}$ Average distances to CRS and VRS frontiers	$S^2 = \frac{\sum_{i=1}^{n} E_{NIRS}^i}{\sum_{i=1}^{n} E_{VRS}^i}$ Average distances to NIRS and VRS frontiers
H_1 rejected	Indicate CRS	
H_0 rejected	Indicate VRS, proceed to test 2	
H_1^* rejected		Indicate DRS
H_0^* rejected		Indicate VRS

Source: Simar & Wilson, 2002

2.2.3.2 Specifications that allow the calculation of allocative efficiency

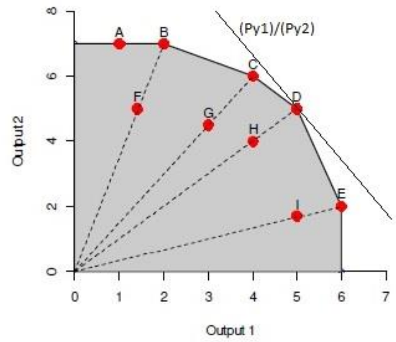
From an input perspective, allocative efficiency is related to a firm's ability to choose the least costly technically efficient resource mix. From an output perspective, it is concerned with a firm's ability to choose the revenue maximizing product mix. This concept is illustrated in Figure 2.11 (Mugera, 2016). DEA use price information to construct a price ratio line in accordance with the concept of marginal rate of substitution. Its tangency to the technology frontier will identify the least cost resource mix or the revenue maximizing product mix. This serves as the reference point according to which all firms will be measured for their allocative efficiency.

Input oriented DEA model:
Minimize input use subject to market prices



Firm A, B, C and D is considered technically efficient, however only B is allocatively efficient

Output oriented DEA model:
Maximize output subject to market prices



Only D is both technically and allocatively efficient

Figure 2.11: DEA model specification. Source: Mugera, 2016

2.2.3.3 Slack Based models

The CCR and BCC models both use the radial Farrell measure of efficiency. Figure 2.12 illustrates the radial reduction (increase) of inputs (outputs) in the direction of the origin that is required for a specific DMU to move toward the frontier (Coelli *et al.*, 2005). It is evident that the radial Farrell projection of Farm C onto the efficient frontier F(y) falls on the section of the frontier that is parallel to the y axis.

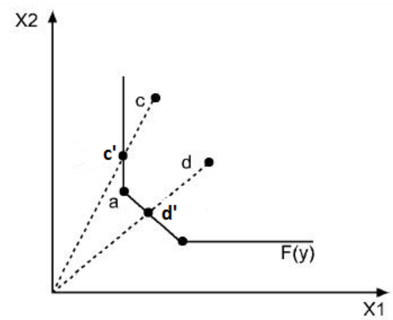


Figure 2.12: Input slack illustration. Source: Coelli et al., 2005

An input oriented CRS CCR DEA model following Farrell’s definition of an efficient firm would deem both the efficient projection of firm (c), at (c’) and (a) as technically efficient, each with an efficiency score of E=1. However, it is evident that (c’) can still reduce, although not radially anymore, its consumption of X2 up to what (a) has proven to be the ultimate efficiency level without affecting its output level according to F(y). This additional non-radial reduction potential of X2 is called input slack. Firm (a), having no slacks is therefore deemed strongly efficient and (c) weakly efficient. It is clear that Farrell’s definition of efficiency fails to distinguish between strong efficiency and weak efficiency. The welfare economic concept of Pareto-Koopmans efficiency provides a non-radial framework to calculate efficiency whilst distinguishing between weak and strong efficiency. According to this definition, a DMU is fully efficiency if, and only if it is not possible to improve any input or output without worsening some other input or output. This definition leads to the development of a DEA model that supplements the Farrell radial adjustment measure of efficiency with non-radial slack adjustments. This model is called the additive slack based DEA model (Charnes, Cooper, Golany, Seiford & Stutz, 1985).

Input slacks identify excess utilization of inputs and output slacks identify shortfall in output production beyond technically efficient levels. This dimension of efficiency analysis implies that all firms that define the frontier are in fact not all equally efficient. Therefore, strictly speaking a firm can only be efficient when it conforms to the Koopmans definition of efficiency (Kuosmanen, 2001).

2.2.3.4 Sub vector models

Classical DEA models such as the one discussed above assume all components of the input vector are perfectly adjustable and transferable. However in reality, due to the time horizon and asset fixity some inputs may be fixed. To enable accurate efficiency estimation in such situations, a sub vector DEA model assumes one or more variables of the input vector as fixed, while optimizing the remainder according to the specified objective (Mugera, 2016). Models that allow for this assumption measure the improvement potential of an inefficient firm only along the axis of the variable inputs. Figure 2.13 illustrates the instance where input x_F is considered to be fixed (Bogetoft *et al.*, 2011).

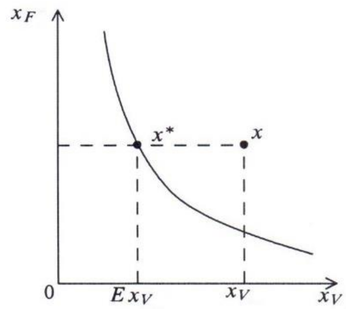


Figure 2.13: Fixed input assumption in DEA. Source: Bogetoft et al., 2011

These various specifications and the evolution thereof has been the topic of many research papers (Petersen, 1990). Ultimately, combining the appropriate set of assumptions relies on a thorough understanding of the industry, the nature of its production and the objective function of a set of firms within that industry.

2.2.4 Extending DEA to a complete methodological framework:

In the introduction of DEA as a tool to measure efficiency, it was mentioned that literature makes provision to deal with some of the shortcomings of DEA to establish it as a worthy methodology to provide sufficiently accurate estimates of efficiency and its determinants (Banker & Natarajan, 2008).

An efficiency study should be conducted in a complete methodological framework where methods for outlier detection, returns to scale identification and bias correction is applied to ensure the accuracy and meaningfulness of the DEA estimates (Flokou, Aletras & Niakas, 2016).

Being directed to frontiers rather than central tendencies, DEA is particularly capable of revealing relationships that otherwise may have remained hidden. From a managerial perspective, this enables DEA to benchmark DMU's against the industry best, rather than the industry average. This highlights the underlying origins of DEA in operations and management science and is therefore not purely statistical. Basic production economic properties like free disposability, economies of scale, convexity and additivity in combination with sound logic of the production structure serves to validate a DEA model in the same way as statistical tests serve to validate a statistical model.

DEA is considered to be deterministic because it relies on the estimation of an unobserved frontier from observed data, which subsequently serves as reference for the measurement of efficiency. The efficiency estimates from DEA models are therefore subject to uncertainty that arises from sampling variation. DEA is furthermore nonparametric, indicating that no assumption is made about the distribution or functional form of the observed data. Since efficiency scores are calculated through linear programming models rather than statistical regression, there exists no basis to apply statistical tests to reduce its vulnerability to sampling variation, leaving measurement errors, systematic errors and random errors to affect the shape and position of the frontier. DEA on its own is therefore unable to provide information on the sensitivity of the efficiency results. It is also unable to separate variation caused by statistical noise from variation that is attributable to managerial inefficiency (Simar & Wilson, 2000).

The realization that the DEA efficiency estimates has the same distributional functional form than the dataset from which it was calculated sets a formal basis that enables statistical testing and validation of models and results. Three statistical testing and validation approaches exist:

For datasets where the underlying distribution is unknown, non-parametric tests like the Kolmogorov-Smirnov and Kruskal-Wallis tests may be relevant. The second approach of parametric tests use asymptotic statistical theory to make assumptions regarding the distribution of inefficiency and noise. Finally, the predominant statistical validation procedure in DEA literature is bootstrapping. It replicates sampling variation by creating repeated samples of the original sample. Bootstrapping involves an iterative random sampling process from the DEA efficiency values. By repeating the process over and over again, it derives pseudo-estimates from the samples. These pseudo estimates define an empirical distribution related to a specific estimator of interest. This distribution is an acceptable approximation of the true underlying sampling distribution (Baležentis, Kriščiukaitiene & Baležentis, 2014). For a detailed discussion on the procedure of bootstrapping, refer to (Olson & Vu, 2007).

Often times these statistical methods are used in a second stage post efficiency analysis to validate the model and to explore the possible causes of the variation in efficiencies. A typical two-stage DEA analysis involves the calculation of the relative efficiency of each DMU based on actual data on input consumption and output production in the first stage. In the second stage the efficiency score of each DMU is regressed on potential secondary factors to identify the factors which impact on efficiency is statistically significant (Banker & Natarajan, 2008). Various regression analysis approaches have been cited, however ordinary least squares (OLS), and Tobit regression is the most prevalent. OLS has been proven to be more consistent than Tobit under very specific assumptions about the data generation process (McDonald, 2009). Tobit regression is however thoroughly validated as the most commonly used approach (Watto & Mugera, 2014).

The two-stage approach is particularly appealing for agricultural applications where production systems are especially subject to sampling variation due to the inherent uncertainty that stems from the impact of environmental factors on efficiency. This approach enables the: (1) Identification of the key determinants that lead to one firm being more efficient than another, (2) Determining if the variation in the estimated efficiency sufficiently reflect the variation in performance or did we leave out important inputs and outputs in the DEA model, and (3) determining if categorical variables may explain some of the variation in efficiency.

DEA and SFA has in many instances been discussed with an attitude of “either or”. The nonparametric and deterministic nature of DEA has often been cited as its Achilles heel. Similarly, the need for a-priori assumptions about the functional form in SFA has been noted to its detriment. Clearly both have theoretical advantages and shortcomings. However, when combined with the statistical foundation in a second stage analysis, DEA has proven to generate efficiency estimates comparable to that of SFA (Banker & Natarajan, 2008). Literature therefore suggests that any DEA study should employ bootstrapping as standard practice, providing that sample sizes are not too large as to make it impractical (Davidova & Latruffe, 2003). For more detail refer to Wilson & Simar (1995) for a comparison of nonparametric DEA and parametric SFA as it pertains to statistical relevance and consistency.

2.3 Empirical applications and findings that apply to research questions

2.3.1 Empirical findings on the determinants of farm efficiency

The most common procedure to examine the determinants of farm efficiency is to assume the efficiency score as the dependent variable regressed against a number of a-priori explanatory variables that are hypothesized to affect efficiency levels (Hassen, Beshir & Hussien, 2016). This section reviews the findings of DEA studies that provide empirical evidence of a priori efficiency determinants.

Lambert & Bayda (2005) assessed the relationship between farm financial structure and efficiency of 54 North Dakota crop farms from four regions between 1995 and 2001. They used an input oriented DEA model to estimate the technical and scale efficiency scores for each region. The input vector is defined by: labor (hrs.), operating expense ($), crop acres and capital ($). The second stage Tobit regression revealed that the use of intermediate debt had a significantly positive influence on technical efficiency. This indicates that bankers may prefer to extend intermediate term capital to more efficient farmers. It also suggests that lenders are more willing to lend to more efficient producers. Short term debt was significantly negatively related to technical efficiency. This results support the existence of agency cost. Technically inefficient farmers may not be able to generate internal financial resources to cover operating expenses so are forced to increase borrowing. Intermediate debt had a significantly positive relationship with scale efficiency, whereas no significant relationship could be identified between short and long term debt on scale efficiency. The relationship between financial structure and scale efficiency may depend on whether farms exhibit decreasing or increasing returns to scale. However it was found that financial structure of larger farms does not affect their scale efficiency. Other variables that were tested for included: dummy variables (farm location and year), nonfarm-to-farm income, farming experience, insurance payments and government subsidies. Only insurance payments had a significantly significant effect on technical efficiency. It had a negative sign indicating that adverse conditions result in lower farm efficiency.

Olson & Vu (2007) assessed the economic efficiency of farms and factors that explain the differences between 400 Minnesota mixed crop-livestock farms between 1993 and 2005. The output oriented DEA model used considered both CRS and VRS assumptions to calculate technical efficiency, allocative efficiency and scale efficiency. The input vector was specified with three labor inputs (family labor, hired labor, and nonfarm labor), three non-labor variable inputs (livestock-related-, crop-related- and operating-related expenditures), and three inputs for land (rented cropland, owned cropland and owned pasture). The output vector was specified with two crops products (corn and soybean), three livestock products (beef, milk and hog), and nonfarm income. Price data were included to enable the calculation of allocative efficiency. The bootstrapped technical efficiency scores revealed a significant upward bias of 14%, proving the necessity for bias correction. Initial technical efficiency estimates suggested that on average, inefficient farms could expand output by 11.5% by moving their operations to full efficiency as defined by the efficiency frontier. The bias-corrected efficiency score however suggested on average, a required output expansion of 29.2% to reach full efficiency. The lower and upper bounds of the 95% confidence interval for the corrected efficiency scores were 0.69 and 0.89. This means that farms within the sample could expand their output by between 12.1% to 44.1% by means of increased technical efficiency. The scale efficiency scores revealed that 61.8% of farms were operating under “too large”, 18.3 “too small”, and 19.9% optimal scale. The variables used in the second stage Tobit regression analysis to determine factors that explain the differences in farm efficiencies include: financial condition (income, assets, leverage, depreciation, current asset share of total assets, capital-labor ratio, land-labor ratio), labor characteristics (number of employees, managerial experience in years, and hired labor ratio), land tenure (hired vs owned land ratio), regional dummy (South or East), organization dummy (partnership or not), and relative importance of different outputs (nonfarm income ratio, and the Herfindahl index of output concentration). Both a standard Tobit and a weighted Tobit analysis were conducted.

The weighted Tobit regression used the information on variances of TE to improve the estimation by "prioritizing" the observations with lower standard errors and "punishing" those with higher standard errors. A higher share of current assets and a lower leverage ratio significantly contributed to higher efficiency. The capital to labor- and land to labor ratio had a positive correlation with efficiency, indicating that increasing capital and land relative to labor can increase technical efficiency. A higher ratio of hired labor to on farm labor showed to have a significantly positive effect on efficiency. Farms with more employees (labor supply) were proven to have higher technical efficiency scores. More specialized farms, identified by the Herfindahl index, proved more efficient than less specialized farms. Interestingly, farm size in terms of income, asset value and investment ratio, had no significant relationship with farm technical efficiency. Business organization, although significant in the standard Tobit regression, proved insignificant in the weighted Tobit analysis. Degree of mechanization, indicated by depreciation ratio, neither had a significant relationship with technical efficiency.

Within the South African context, it was found that farmers view the trend of decreasing farming units and increasing farm size as a rational economic reaction to capture economies of scale. Another aspect cited is the effect of technical change or mechanization. Non-parametric analysis revealed a highly significant negative correlation between scale efficiency and debt burden. Scale efficiency was however positively correlated with managerial ability. The managerial ability of farmers was measured by an index of indicators such as budgeting and record keeping (Van Zyl, Binswanger & Thirtle, 1998).

For more applications of two stage bootstrapped DEA to identify the key drivers to farm efficiency, refer to: Watkins, Hristovska, Mazzanti, Wilson & Schmidt (2014), Davidova & Latruffe (2003), Speelman, Haese, Buysse & Haese (2008), Rouse *et al.* (2010), Latruffe (2009) and Bojnec & Latruffe (2008).

2.3.2 Empirical findings of the efficiency effects of collaborative action

The benchmarking property of DEA is particularly appropriate for the analysis of the efficiency change brought about by collaborative action like mergers or resource sharing agreements. The benchmarks of the individual firms before merger can be compared to the benchmark of the merged firm within the same industry in order to evaluate the efficiency changes that took place.

In a two stage DEA model, Larsén (2010) analyzed the effects of machinery-sharing arrangements on farm efficiency in Sweden. The objective of the study was to analyze the impact of machinery-sharing arrangements among farmers on farm efficiency using an unbalanced panel of Swedish farms over the period 2001 to 2004. The technical efficiency scores of 678 crop farms and 596 livestock farms were obtained using an output oriented DEA model for each specialization under both CRS and VRS assumptions. The input vector consisted of: land (arable and pasture hectares), capital utilization (sum of depreciation, interest, machinery maintenance, fuel cost and hired services), labor hrs, fertilizer cost, electricity cost and other expenses (seed and fodder cost). The output vector was represented by total value of production in Swedish Krona. All monetary values were reported in 2004 terms. The results proved that both crop and livestock partnership farms had, on average, higher technical efficiency scores than non-partnership farms. It was also found that both crop and livestock partnership farms had higher scale efficiency scores than non-partnership farms.

In a second stage, the effect of partnership arrangements on farm efficiency was analyzed while controlling for other farm/farmer characteristics that are expected to influence farm efficiency. Distinction was made between the organizational forms of the sharing arrangements. One form that shared all mechanical resources, and another that only partially does so. The bootstrap procedures of Simar & Wilson (2000) was used in addition to the conventionally used Tobit regression. It was found that both crop and livestock farms that participated in machinery sharing arrangements, had significantly higher levels of technical efficiency that those who did not.

Furthermore, crop farms with joint machinery ownership had on average 18% higher efficiency scores than those who engaged in partial agreements. Other significant efficiency determinants that was analyzed include: biological yield capacity, share hired labor, share owned land, farmer's age, and some dummy variables for geographical location.

Davidova & Latruffe (2003) employed an input oriented DEA model to assess the efficiency differences between corporately and individually managed farms according to their specialization in either crop or livestock production. From a sample of 753 farms, 256 crop and 88 livestock farms were selected for analysis based on a 65% contribution to total value of farm output. Management of crop farms was 86% corporate and that of livestock farms 60%. It is assumed that the production technology for each specialization is different. Therefore, four frontiers were estimated, one for each specialization, crop (256) and livestock (88) and each management form, individual and corporate. Contrary to theoretical expectations related to transaction costs, corporate farms were found to be more technically efficient than individual farms.

In a second stage Tobit regression is used to test the effect of financial variables (leverage, liquidity), size variables (ha), technology proxies (capital/labor and land/labor), integration (share hired labor, share rented land) and dummy variables (two legal form and five regional districts) on farm managerial efficiency. The debt to asset ratio revealed a significantly positive relationship with efficiency for individual livestock and corporate crop farms. Higher current ratios proved beneficial for the technical efficiency of individual farms, whilst lower current ratios was more beneficial to corporate crop farms and no significant influence on the corporate livestock farms. They attribute the negative implications of current liabilities on the efficiency of individual farms of both specializations to higher agency cost for dispersed individual farms than for cooperative farms. Size had a significantly positive relationship with the efficiency of individual livestock farms. The availability of land pet unit of labor had a positive impact on the efficiency of all types of individual and corporate farms.

With the theoretical production economic advantages and disadvantages and the empirical evidence of the actual realized ex-post efficiency effects of collaborative action discussed, we now turn to the ex-ante evaluation of the potential efficiency effects of mergers.

2.4 Ex-ante estimation theory & basis for estimating the potential gains from merger

The quantitative literature on efficiency until now was mainly concerned with the measurement of the efficiency of individual firms and organizations. The efficiency estimates demonstrated the amount of input savings or output saving potential that can be achieved through increased individual firm-level efficiency. However when we want to evaluate the technical efficiency of a merger, we need to go beyond the conventional efficiency estimation of the observed input-output bundles used by each of the firms involved in the merger. Instead, we consider the output producible by a single merged firm from the combined input bundles of the constituent firms and compare it with the total output from the efficient operation of the existing firms operating as separate entities. When the output from the combined input bundle is greater than the combined output from the constituent individual input bundles, the merger will improve efficiency.

Befitting the objective of this thesis Bogetoft & Wang (2005) propose a framework within which ex-ante DEA models can be used to estimate the expected efficiency gains attainable from pooling production resources. The objective of their methodology is to seek a reorganization that maximizes the potential efficiency gains. Ex-ante DEA merger analysis utilize the proven descriptive and analytical ability of ex-post DEA analysis to estimate the maximum attainable efficiency gains of two separate firms through pooling production resources based on their ex-post performance. Since the specification of a DEA model determine its significance, a different application also calls for a modified specification and combination of assumptions. This brings us to the fourth assumption of additivity that was briefly introduced in section 2.2.2.3. A detailed discussion of its necessity follows:

2.4.1 Additivity properties of technology

The additivity or replicability assumption refers to the ability of a merged firm to replicate the technology of its constituent firms. This implies that the potential efficiency gains from a merger is dependent on the additivity properties of the technology of a production function. Consider the additivity test in Equation 2.2 that represents a single input, single output production function:

$$y^* = f(x) \tag{2.2}$$

Where y^* is the optimal output vector producible from the input vector x under the prevailing technology *f()*. The additivity nature of the technology for a production function that has an input vector consisting of three inputs $x_i (i = 1, 2, \dots, n)$ may be tested by using the criteria in Table 2.5 below.

Table 2.3: Properties of the additivity assumption

Nature of Technology	Implication
Locally additive	$f(x_1 + x_2 + x_3) = f(x_1) + f(x_2) + f(x_3)$
Locally super additive	$f(x_1 + x_2 + x_3) > f(x_1) + f(x_2) + f(x_3)$
Locally sub additive	$f(x_1 + x_2 + x_3) < f(x_1) + f(x_2) + f(x_3)$

Consider the merger of two candidate firms (A and B) each facing the same production function defined in Equation 2.3:

$$f(x) = 2\sqrt{x} - 4 \qquad for\ all \quad x \geq 4 \tag{2.3}$$

Given a scenario where firm A initially consumes an input level of $x_A = 6$ that results, according to the production function in an output of $y_A = f(x) = 0.889$ and firm B that consumes $x_B = 18$ that, according to the same production function, results in an output of $y_B = f(x) = 4.4853$. Their combined individual output therefore is $f(x_A) + f(x_B) = 5.3743$. Now consider the possible joint output from a pooled resource base $f(x_A + x_B) = f(24) = 5.7980$.

The joint output exceeds the combined output by 7.88% as a result of higher production efficiency in the merged firm. The technology for this specific scenario is therefore locally super additive implying theoretical potential for collaborative synergy in production.

Given a different scenario where firm A initially consumes an input level of $x_A = 9$ that results, according to the same production function in an output of $y_A = f(x) = 2$ and firm B that consumes $x_B = 25$ that, according to the same production function, results in an output of $y_B = f(x) = 6$. Their combined individual output therefore is $f(x_A) + f(x_B) = 8$. Now consider the possible joint output from a pooled resource base $f(x_A + x_B) = f(34) = 7.6619$. The combined individual output exceeds the joint output by 4.3% as a result of lower production efficiency in the merged firm. The technology for this specific scenario is therefore locally sub additive implying no theoretical potential for collaborative synergy in production.

It is evident that the additivity properties may vary along the same production function. The reason for this can be found in the relationship between the additivity and returns to scale properties of technology. To understand this relationship, consider Equation 2.4 where $g(x_A; x_B)$ represents the theoretical potential of joint production synergy for firm A and B:

$$g(x_A; x_B) = f(x_A + x_B) - [f(x_A) + f(x_B)] \tag{2.4}$$

From the individual input- and output vectors we can define the following:

Average combined input set:	$\bar{x} = \frac{1}{2}(x_A + x_B)$
Average combined output set:	$\bar{f}(x_A; x_B) = \frac{1}{2}[f(x_A) + f(x_B)]$
The difference between the scaled average combined input and output set with a scale factor of 2:	$g(x_A; x_B) = f(2\bar{x}) - 2\bar{f}(x_A; x_B)$

By combining the average combined input set, average combined output set and the difference between the scaled average combined input set, Equation 2.5 calculates the theoretical potential efficiency gain or loss attributable to joint production:

$$g(x_A; x_B) = [f(2\bar{x}) - 2f(\bar{x})] - 2\left[\bar{f}(x_A; x_B) - f(\bar{x})\right] \tag{2.5}$$

Where the first expression in square brackets captures the returns to scale properties at the mean joint input level $\bar{x}$. It will be positive (negative) when increasing (decreasing) returns to scale hold over the input range $\bar{x}\ to\ 2\bar{x}$. The second expression in square brackets after the minus captures the curvature properties of the production function. If it is concave (convex) the expression will be negative (positive) so that it contributes positively (negatively) to the gains from merger. The curvature depends on the second derivative of the production function and the difference between the two input levels cf. Ray (2004) for detailed mathematical explanation.

Usually the returns to scale effect diminishes the potential efficiency gains from mergers. It is however possible that sufficient concavity of the technology, represented by the curvature properties, may secure overall positive synergy gains from merger even when DRS prevails. Referring to the two scenarios described above, the calculation of the overall potential synergy of joint production illustrates this dynamic relationship between scale and additivity in Table 2.6.

Table 2.4: Merger synergy illustrative example

Scenario 1	Scenario 2
Define average input and average output: $\bar{x} = \frac{1}{2}(x_A + x_B) = 12$ $\bar{f}(x_A; x_B) = \frac{1}{2}[f(x_A) + f(x_B)] = 2.687$	Define average input and average output: $\bar{x} = \frac{1}{2}(x_A + x_B) = 17$ $\bar{f}(x_A; x_B) = \frac{1}{2}[f(x_A) + f(x_B)] = 4$
Substitute to calculate contribution to synergy: $g(x_A; x_B) = [f(2\bar{x}) - 2f(\bar{x})] - 2[\bar{f}(x_A; x_B) - f(\bar{x})]$ $g(x_A; x_B) = [f(24) - 2f(12)] - 2[2.687 - f(12)]$ $= [5.798 - 0.482] - 2[2.687 - 2.928]$ $= [5.316] + [0.482]$ $= 5.798$ → Merger synergy	Substitute to calculate contribution to synergy: $g(x_A; x_B) = [f(2\bar{x}) - 2f(\bar{x})] - 2[\bar{f}(x_A; x_B) - f(\bar{x})]$ $g(x_A; x_B) = [f(34) - 2f(17)] - 2[4 - f(17)]$ $= [7.662 - 8.492] - 2[4 - 4.246]$ $= [-0.83] + [0.492]$ $= -0.338$ → No merger synergy

Source: Ray, 2004

The most productive scale size (MPSS) for the given production function $f(x) = 2\sqrt{x} - 4$ will be $x^* = 16$ as calculated where IRS and DRS meet on the production possibility frontier. In scenario one, B operates in a region of DRS $(x_B = 18)$ and A in a region of IRS $(x_A = 6)$. Even though the merged entity AB will clearly operate in a region of DRS $f(24)$, the merger will still yielded potential efficiency synergies due to sufficient concavity at this production level. This example proves that IRS is not a prerequisite for a merger to yield overall efficiency gains. However, when IRS holds for both $x_A \; and \; x_B$, gains from merger would necessarily be positive. Globally increasing (decreasing) returns to scale is therefore a sufficient condition for the super additivity (sub additivity) of the production technology, implying efficiency gains (losses) from merger of smaller firms into a single larger firm.

2.4.2 Previous studies

DEA literature provides a methodological framework within which the overall efficiency gains attainable through merging resources may be calculated and linked to its respective sources, be it the scale or curvature properties of the production technology.

This framework has been applied to assess the potential efficiency gain from merging Danish agricultural extension offices. Offices within a radius of 50 km of each other were identified as candidates to merge. Out of the 458 possible merger combinations that were simulated, 409 showed efficiency improvement potential under a CRS assumption. Furthermore, 100 mergers estimated an improvement potential that ranged between 8% and 10% savings in inputs (academic staff, extension officers, office rent and operational costs). The overall savings were attributed to two sources. The first is the harmony effect which measures how much inputs can be saved by reallocating production across firms to those with the lowest marginal cost. This savings potential is generated by the curvature properties of the production function. The second source of input savings potential is attributed to the scale properties of the production function. It measures to what extent the merger can assist producers to produce at a lower average cost as they move toward a more productive scale size. The study revealed that the larger part of the overall savings should be attributed to the harmony effect, whilst the scale effect in most mergers diminished the input savings potential. Accordingly, the overall potential gains were considerably less under a VRS technology assumption. One specific merger between two technically efficient firms revealed an input savings potential of up to 15.58%. The harmony effect alone contributed 17.3%, whilst the scale effect, being negative, decreased this estimate by 1.72% (Bogetoft & Wang, 2005).

An important note about the controllability (the decision on which variables are discretionary and which are non-discretionary) should be made about their approach. It is assumed that all resources and products can be reallocated, but in reality some reallocations are easier than others. The controllability of variables is related to aspects such as asset specificity and the adjustment time horizon. In the short run, less inputs and outputs can be adjusted, reducing the savings potential from reorganization. This aspect can be modeled using a sub vector DEA model as briefly described in Section 2.2.3.4.

The ex-ante application of DEA is relatively new and not many applications exists in the context of primary agricultural production. Perhaps the reason may be that VRS represents the technology of agricultural production the best and merger analysis requires the assumption of additivity, which under VRS often times result in infeasible solutions. This however does not entirely dismiss its relevance for assisting farmers in evaluating the prospects of a possible merger. A considerable literature exists for the application of merger analysis with DEA in the hospital and banking sector. For more applications of DEA in merger analysis refer to: Bogetoft, Thorsen & Strange (2003) Kristensen, Bogetoft & Pedersen (2010), Flokou *et al.* (2016), Schain & Bonnet (2016), Wanke, Maredza & Gupta (2016)

2.5 The case study approach as a research tool

Organizational contexts are dynamic and complex with multiple, influencing variables. Case studies enable the study and interpretation of these within a given context (Fitzgerald, 2009). Case study research is a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) through data collection and reports a case description. This research method therefore follows an empirical nature of enquiry. In essence, case study research seek to conduct an in-depth analysis of an issue, within its specific context in order to understand the issue from the perspective of participants. A case study typically involves a case which is the object of the study. The case is placed within a bounded system (context) to manage contextual variables that is significant to understanding the case. This thesis follows a case study approach in chapter four to assess the possible effects that farm level mergers may effect on the efficiency of the farming units.

2.6 Conclusion

The focus of this research project is to determine the effect of a merger between two farms in terms of efficiency. Each farm consist of a unique set of resources and also managerial ability. Should two farms merge the efficiency of the “new” entity would be good indication if such a merger would benefit the current farms. For this purpose the various analytical methods that is based on the production function was presented in Chapter 2. Chapter 2 reviewed the theory of efficiency measurement and the tools that exist to calculate efficiency. It singled out empirical applications of efficiency measurement in previous research. The ex-ante estimation theory it provided serves as basis for the merger efficiency analysis that this thesis will apply in the context of a case study approach. In the context of South African farming, this approach may assist farmers to make rational decisions when crafting their long term strategy to stay ahead and relevant in an ever changing environment. The real benefit of this approach lies therein that, based on the results and the understanding it brings, tailored clear cut strategies can be formulated to aid in each of the three aspects of learning, harmony and scale. Full-scale mergers need not be the only option, and based on the results of the DEA merger model, farmers may consider various other options to steer them in the desired direction of improved efficiency.

Chapter 3: Empirical framework, deterministic frontier methods and models

3.1 Introduction

Central to the research aim is the measurement of efficiency and especially the impact of a possible merger of two farms on the efficiency of the "new" entity. The previous chapter presented the typical tools of analysis used form the point of view of the production function. Frontier analysis was identified as the method with possibly the most scope to quantify these potential gains in efficiency through a merger. The methodological framework for efficiency measurement will be discussed in the first section of this chapter. Statistical inference techniques to validate efficiency estimates is discussed in the second section. The third section provides a method of estimating the potential efficiency changes that a merger may bring about. The last section is dedicated to data validation and the description of the data and the assumptions that will be used to test the methodology that is developed throughout the chapter.

3.2 Efficiency measurement using Data Envelopment Analysis

Within a set of *n* DMU's, each indexed by $i = (1, 2, \dots, n)$ use an input set X where each input x^i is indexed by $p = (1, 2, \dots, p)$ to produce an output set Y where each output y^i is indexed by $q = (1, 2, \dots, q)$. Any given DMU^i may for instance use two inputs $\left(x_1^i, x_2^i\right)$ to produce one output $\left(y_1^i\right)$ within a technology set *T*.

Suppose we are interested in calculating the efficiency of a particular firm $DMU^k, k \in n$. The production possibility set will be represented by Equation 3.1:

$$P = \{(x, y) : (x \geq X\lambda), (y \leq Y\lambda), (\lambda \geq 0)\} \tag{3.1}$$

3.2.1 Technical Efficiency

An industry best isoquant (technology) for its given output level Y_k and its corresponding weighted λ_k input and output levels can be calculated by using an input oriented DEA model defined in Equation 3.2.

$$\theta_k{}^* = \min \theta \tag{3.2}$$

Subject to the following assumptions:

$$\sum_{i \in I}^{n} \lambda_i y_q^i \geq y_k \qquad \textit{Output constraint}$$

$$\sum_{i \in I}^{n} \lambda_i x_p^i \leq \theta x_k \qquad \textit{Input constraint}$$

$$\sum_{i \in I}^{n} \lambda_i = 1 \qquad \left\{ \begin{array}{l} \textit{VRS} \rightarrow \sum \lambda_i = 1 \\ \textit{CRS} \rightarrow \textit{exclude} \sum \lambda_i \\ \textit{DRS (NIRS)} \rightarrow \sum \lambda_i \geq 1 \\ \textit{IRS (NDRS)} \rightarrow \sum \lambda_i \leq 1 \end{array} \right.$$

$$\lambda_i > 0 \qquad \textit{Non-negativity constraint}$$

If DMU^k consumes an input level (technical) in a combination (allocative) that is determined to be efficient according to the efficiency frontier that was mathematically constructed from the sample of *n* firms, its outputs will be best produced using all of its own inputs in their current combination $\left(x_p^k = 1 \;\&\; x_p^i = 0\right)$ for all $i \neq k$ with an efficiency score of $\theta = 1$. If however the DMU is deemed to be inefficient, its current outputs will be best produced by a mixture of other DMU's within sample *n* using a fraction θ of all its inputs with $0 < \theta^k < 1$. The technical efficiency score for all DMU's within *n* can be calculated by using the same model by varying *k*.

Two alternative ways to calculate the technical efficiency was discussed in the model formulation section of the literature study. The mathematical formulation of the slack based- and sub vector models is presented below.

3.2.1.1 Slack based technical efficiency DEA model

Input slack represents the amount of non-radial reductions in inputs and expansion of outputs beyond the radially projected Farrell full efficiency level. Therefore, to incorporate these slacks a Farrell technically efficient $DMU^k(\lambda X_k, \lambda Y_k)$ will now be represented as $DMU^k(\lambda X_k + s^-, \lambda Y_k - s^+)$ where s^- indicate the input excess and s^+ the output shortfall (Tone, 2011). The revised slack based input oriented DEA model is defined by Equation 3.3:

$$\theta^* = \min \theta - \varepsilon\left(\sum_{i=p}^{p} s_p^- + \sum_{i=q}^{q} s_q^+\right) \tag{3.3}$$

Subject to the following assumptions:

$$\sum_{i \in I}^{n} \lambda_i y_q^i - s^+ = y_k \qquad \textit{Output constraint}$$

$$\sum_{i \in I}^{n} \lambda_i x_p^i + s^- = \theta x_k \qquad \textit{Input constraint}$$

$$\sum_{i \in I}^{n} \lambda_i = 1 \qquad \left\{ \begin{array}{ll} \textit{VRS} & \rightarrow \sum \lambda_i = 1 \\ \textit{CRS} & \rightarrow \textit{exclude} \sum \lambda_i \\ \textit{DRS (NIRS)} & \rightarrow \sum \lambda_i \geq 1 \\ \textit{IRS (NDRS)} & \rightarrow \sum \lambda_i \leq 1 \end{array} \right.$$

$$\lambda_i, s^+ s^- \geq 0 \qquad \textit{Non-negativity constraint}$$

Notice that the greater- and smaller than signs in the constraints has been replaced by equal sings. This is because pure technical efficiency in itself is not a sufficient condition for a DMU to be technically efficient in a slack based DEA model.

3.2.1.2 Sub vector Model

A DEA model that will allow some variables to be fixed and not adjustable in the short run, is called a sub vector model. It is not very different from the classic DEA model other than the θ in the input constraint's right hand side is omitted for those input variables that are considered to be fixed in the short run.

3.2.2 Scale Efficiency

Now that the technical efficiency of DMU^k is determined, either via the slack based or sub vector model, we can proceed to calculate scale efficiency. To do this, the VRS assumption is omitted and the model run again to obtain the technical efficiency score of each DMU under CRS. Equation 3.4 is used to perform the scale efficiency calculation.

$$E_k^S = \frac{\theta_{VRS}}{\theta_{CRS}} \tag{3.4}$$

When $E_k^S = 1$, DMU^k is calculated to be scale efficient. This implies that $\theta_{CRS} = \theta_{VRS}$, and DMU^k is either on, or projected onto the efficiency frontier where the VRS and CRS technology sections of the frontier intersects. Otherwise, if $E_k^S \neq 1, DMU^k$ is scale inefficient. The next question in mind should be if it is operating at too large a scale or too small a scale. The scale efficiency index method will reveal whether DMU^k is either on or projected onto the DRS (indicating it's too large) or IRS (indicating it's too small) portion of the frontier. When

$(\theta_{VRS} > \theta_{DRS})$ or $(\theta_{CRS} = \theta_{DRS})$, it implies that DMU^k is either on or projected onto the IRS portion of the frontier. When $(\theta_{VRS} = \theta_{DRS})$, it implies that DMU^k is either on or projected onto the DRS portion of the frontier (Seiford & Zhu, 1999). Pure technical efficiency of DMU^k can be calculated by subtracting the portion that is attributable to the scale effect $(\theta_k - E_i^S)$.

3.2.3 Cost efficiency

When input prices are known, the input oriented DEA model can be adjusted to calculate the cost efficiency of each DMU by determining the intersection of the cost function (price ratio) with the DEA constructed iso-cost frontier. This is done by incorporating an additional term w_p^i , $(p = 1{,}2)$ to denote the price of input x_1^i & x_2^i. The cost function of DMU^k will therefore be represented by $x_1^k\, w_1^k + x_2^k w_2^k$. To determine the cost efficiency of DMU^k, we first calculate its actual cost based on the cost function and its actual input levels. Then we determine its technically efficient and slack adjusted level of inputs $\left(\theta_k x_1^k - s_1^-\right)$ & $\left(\theta_k x_2^k - s_2^-\right)$ needed to produce its current slack adjusted output $\left(y_1^k + s_2^+\right)$. Next, we recalculate the total cost according to the technically efficient slack adjusted input levels and compare it to the actual cost. The difference is the cost efficiency of the firm, represented by Equation 3.5:

$$CE^k = \frac{\sum_{i=k}^{n} w_p^k x_p^{*k}}{\sum_{i=k}^{n} w_p^k x_p^k} \tag{3.5}$$

Subsequently Equation 3.6 allows the calculation of $DMU^{k\prime}s$ allocative efficiency since $CE = TE \times AE$

$$AE^k = \frac{CE^k}{\theta^k} \tag{3.6}$$

3.2.4 Numerical example

Consider Table 3.1 that contain a sample of seven farms $(n = 7)$ each producing maize y^i, using capital x_1^i and labor x_2^i as inputs:

Table 3.1: Illustrative dataset of 7 DMU's

DMU	Output (y)	Input 1 (x_1^i)	Input 2 (x_2^i)
1	12	8	9
2	8	6	5
3	17	12	8
4	5	4	6
5	**14**	**11**	**9**
6	11	8	7
7	9	7	10

The efficiency of farm five $(i = 5)$ can be calculated with the DEA model specified in Table 3.2:

Table 3.2: Illustrative Input oriented VRS DEA model

Generic		Farm 5
$min\ \theta$		$min\ \theta$
Subject to:		subject to:
$\sum_{i \in I}^{n} \lambda_i y_q^i \geq y_k$	Output constraint	$12\lambda_1 + 8\lambda_2 + 17\lambda_3 + 5\lambda_4 + 14\lambda_5 + 11\lambda_6 + 9\lambda_7 \geq \mathbf{14}$
$\sum_{i \in I}^{n} \lambda_i x_1^i \leq \theta x_k$	Input one constraint	$8\lambda_1 + 6\lambda_2 + 12\lambda_3 + 4\lambda_4 + 11\lambda_5 + 8\lambda_6 + 7\lambda_7 \leq \mathbf{11}$

$\sum_{i \in I}^{n} \lambda_i x_2^i \leq \theta x_k$	Input two constraint	$9\lambda_1 + 5\lambda_2 + 8\lambda_3 + 6\lambda_4 + 9\lambda_5 + 7\lambda_6 + 10\lambda_7 \leq \mathbf{9}$
$\sum_{i \in I}^{n} \lambda_i = 1$	Scale assumption	VRS → $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 = 1$
$\lambda_i \geq 0$	Non-negativity	$\lambda_i \geq 0$

The technical efficiency scores of all seven farms are listed in Table 3.3. It contains both the slack based and the sub vector model results where input two is assumed to be quasi-fixed (θ omitted from the RHS of the input two constraint):

Table 3.3: Illustrative efficiency results for the slack based and sub-vector model specifications

Farm	$\boldsymbol{\theta_{VRS}}$ (Slack based)	$\boldsymbol{\theta_{VRS}}$ (sub vector)($\overline{\boldsymbol{x_2^i}}$)
1	1.00	1.00
2	1.00	1.00
3	1.00	1.00
4	1.00	1.00
5	**0.89**	**0.87**
6	**0.97**	**0.93**
7	0.90	0.90

Farms five and six are almost insignificantly, but surely less efficient when input two is left out of the efficiency estimation because of its fixed nature in the short run. The remainder of this illustrative example will be based on the efficiency scores of the slack based model. Table 3.4 presents the efficiency measures of all the farms under variable returns to scale (column two) and constant returns to scale (column three). Equation 3.4 is then used to calculate the scale efficiency in column four of Table 3.4.

Table 3.4: Illustrative scale efficiency results

Farm	$\boldsymbol{\theta_{VRS}}$	$\boldsymbol{\theta_{CRS}}$	$\boldsymbol{E^S}$	Specific Scale Property
1	1.00	1.00	1.00	MPSS
2	1.00	0.92	1.09	IRS
3	1.00	1.00	1.00	MPSS
4	1.00	0.83	1.20	IRS
5	0.89	0.88	1.01	IRS
6	0.97	0.95	1.02	IRS
7	0.90	0.86	1.05	IRS

Farm one and three represents the output level at which less (more) output, will result in scale efficiency contributing negatively (positively) toward the overall VRS efficiency score according to the scale efficiency index.

When the same dataset of farms is analyzed using a slack based input oriented DEA model as in section 3.1.1.1, the first four farms were determined to be Pareto-Koopmans efficient or strongly efficient since neither input- nor output sacks were found in their production systems. Despite the fact that none of the last three farms, five, six and seven were even technically efficient, their Farrell projected technically efficient levels revealed the presence of slacks: Output slacks were detected in the output vectors of farms five and six. In order for them to move to a higher level of efficiency, beyond the Farrell technical projection, a shortfall in output of 0.19 and 0.56 respectively needs to be filled in. No farms had input slacks in their use of capital. Input slack was however detected for labor in the last three farms. The model calculated that an input reduction of 0.54, 0.03, and 2.41 beyond what is needed to be technically efficient in labor hours needs to be pursued in order for them to be Pareto-Koopmans efficient.

If the prevailing market prices for Capital and Labor is assumed to be $w_1^i = R10$ and $w_2^i = R5$ then the corresponding cost and allocative efficiency of these seven farms is listed in Table 3.5:

Table 3.5: Illustrative cost- and allocative efficiency results

Farm	Actual cost $\left(x_1^i\, w_1^i + x_2^i w_2^i\right)$	Min Cost $\left(\theta_k x_1^k - s_1^-\right) + \left(\theta_k x_2^k - s_2^-\right)$	CE (Act. C/Min. C)	AE (CE/TE)
1	115	115	1.00	1.00
2	85	85	1.00	1.00
3	160	160	1.00	1.00
4	70	70	1.00	1.00
5	155	135	0.87	0.98
6	115	112	0.97	1.00
7	120	96	0.80	0.89

In summary, farms one to four are determined to be technically efficient according to the Pareto-Koopmans definition, cost efficient and allocative efficient. Farms one and three are also scale efficient. Farms five, six and seven are technically inefficient, cost inefficient and allocative inefficient, except for farm six that is allocative efficient.

3.3 Statistical inference of efficiency estimates

Since the DEA model used to generate the efficiency scores is nonparametric, it provides no information on the sampling properties of the efficiency estimates. The sampling information is important because it allows statistical analysis to validate the accuracy of the efficiency scores. The only way to obtain the sampling properties is to rely on some of the assumptions of inferential statistics. The goal of inferential statistics is to determine the value of statistical parameters of a population.

The approach of traditional statistical inference is to make assumptions about the distribution of the population in order to derive the sampling distribution of a specific statistic. Conversely, the sampling distribution $\hat{f}$ of a randomly drawn sample $X = \{x_1; x_2; \dots; x_n\}$ will mimic the unknown probability distribution f of the population from which it was drawn. Therefore, a certain sample efficiency statistic $\hat{\theta} = \theta(X)$ from sample X will be an estimate of the corresponding population efficiency statistic $\theta = \theta(f)$. This dual relationship between the sample and the population is the basis upon which statistical analysis methods may be applied to nonparametric DEA efficiency estimates.

3.3.1 Bootstrapping efficiency estimates

A nonparametric bootstrap approach allows the estimation of the sampling distribution of a certain statistic empirically without the need to assume the distributional properties of the population. The bootstrap methodology discussed by Wilson & Simar (1995) yields statistical confidence intervals for nonparametric efficiency measures. It also allows correction of the inherent bias which causes the DEA efficiency scores to understate (overstate) the amount of inefficiency (efficiency) in individual firms. The degree of bias depends on the size of the sample. This causes problems when comparing structural inefficiencies derived from different samples of different sizes. Empirical examples indicated that the magnitude of the bias in the first stage DEA efficiency estimates can be substantial, with efficiency estimates overstating efficiency by up to 35% in some instances (Staat, 2002). Bootstrap allows the estimation of bias corrected efficiency scores.

Bootstrapping draws repetitive samples indexed by $B\ (b = 1,2, \dots, B)$ of size *n* from the set of efficiency scores obtained from the DEA model, each time by replacement. The mean and median of each bootstrapped sample $S_b^i = \left(X_1^i, X_2^i, \dots, X_B^i\right)$ is calculated for each repetition. By replacing the bootstrapped sample before drawing the next, each sequential bootstrap sample is expected to yield a slightly different statistic to the previous. By repeating this process multiple times $(B = 2\ 000)$ it is expected that the distribution of the bootstrapped mean and median will be representative of the distribution of the set of efficiency scores, which in turn is expected to be representative of the distribution of the input and output vectors of the sample set of farms. The minimum bootstrap iterations to yield statistical significance of confidence interval estimates is 1 000. If the sole purpose of the bootstrap procedure is bias correction and standard deviation calculation, a smaller number of iterations would be sufficient (Olson & Vu, 2007).

3.3.2 Determinants of efficiency (Tobit)

Studies that use this two-stage approach assume that explanatory variables Z have a significant influence on the managerial choices of inputs X and outputs Y and the resulting efficiency of farms. These explanatory values include environmental and organizational characteristics. The independence of Z and the production function (X, Y) should be tested using the approach of Simar & Wilson (2007) otherwise there would be no motivation for the second-stage regression.

The impact of some of the variables that is hypothesized to influence efficiency in section 2.1.2, will be considered in this section. The specific influence of tenure and income diversification on farm efficiency will be measured. The regression analysis approach is used to calculate the correlation between the efficiency estimates and the two external variables. The efficiency score is defined as the dependent variable and each of the two external variables as an independent variable. This is an important step in the analysis because it allows us to identify key variables that may assist in the merger analysis at a later stage. If efficiency is for instance found to be significant and positively related to the level of livestock diversification, and collaborative action is expected to allow for a higher level of livestock diversification than what is individually possible, then it can be expected that farm-level partnerships will have a significantly positive influence on farm efficiency.

3.4 Estimating potential efficiency effects of a proposed merger

3.4.1 Measurement of potential merger gains

Corporate synergy occurs when firms, through their interactions, are able to produce more outputs with a given set of resources, or to produce a given set of outputs with less resources. The same DEA logic that is applied to evaluate individual entities can be used to evaluate merged entities. The larger the distance to the frontier, the more inefficient the merged firm will be. Being inefficient represents a loss, and indicates that there is room for improvement. Merger synergies can be captured by the increase in improvement potential when we move from independent to joint operations (Bogetoft *et al.*, 2011).

Figure 3.1 illustrates such an integration of two independent production plans, one for each farm A and B into one through a horizontal resource pooling scheme:

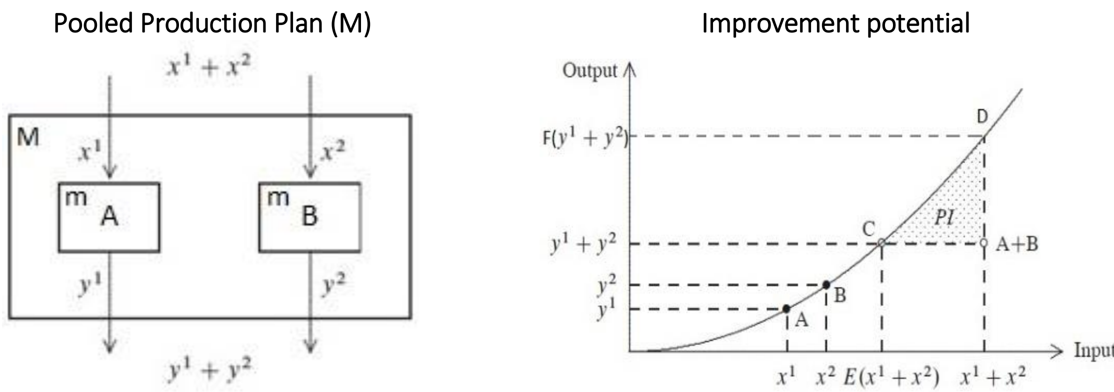


Figure 3.1: Integration of production plans and improvement potential *Source: Bogetoft et al., 2011*

The individual production plans $(A)(x^1, y^1)$ and $(B)(x^2, y^2)$, when combined results in a pooled production plan $(A + B)(x^1 + x^2\ ,\ y^1 + y^2)$. Within a set of n DMU's, each indexed by $i = (1, 2, \ldots, n)$, a subset of m DMU's, each indexed by $i = (1, 2, \ldots, m)$, is selected to form a new merged firm M where $M \in (m_1, m_2, \ldots, m_i)\ and\ m_i \in n$. The resulting input and output vector of M will be $(\sum_{k \in M} x^m)$ & $(\sum_{k \in M} y^m)$ respectively. The pooled production plan $(A + B)$ is however technically inefficient because, according to the efficiency frontier (constructed under IRS in this case), less inputs can produce the same or more outputs. The potential efficiency gains from the merger is based on the efficiency with which the pooled input vector is transformed into the pooled output vector i.e. the efficiency of the merged firm with relation to the technology set. According to the Farrell input measure of efficiency, the aggregate input consumption can be scaled down by a factor *E*. Similarly the aggregate output can be scaled up by a factor F. The shaded potential improvement set *PI* therefore represents the overall potential merger synergy from merging A and B.

More formally: Within a set of n DMU's, each indexed by $i = (1, 2, \ldots, n)$, each use an input set X^i which consists of p inputs, each represented by x_p^i $(p = 1, 2, \ldots, p)$ to produce an output set Y^i which consists of q outputs, each represented by y_q^i $(q = 1, 2, \ldots, q)$. Any given DMU^i may for instance use two inputs, labor (x_1^i) and capital (x_2^i) to produce one output, apples (y_1^i) within a certain technology set *T* with n observed production plans, one for each DMU. In a set of n DMU's, we define a subset $m, (m \in n)$ to represent all the DMU's within n that are considering to merge their production plans into one. Each DMU within subset m is indexed by $i = (1, 2, \ldots, m)$. Subset m is treated as a new DMU M so that $M \in m$. With its individual constituent farms omitted from n, M is now part of a new sample of $n - m + 1$ DMU's.

The pooled production plan of M will therefore be equal to that of the sum of its constituent DMU's, resulting in an input set of $X^m = \sum_{i\in m} x_p^m$ and an output set of $Y^m = \sum_{i\in m} y_q^m$.

The input oriented merger efficiency model defined by Equation 3.7 calculates the maximal proportional reduction in the aggregated input set X^m that allows the production of the aggregated output set Y^m:

$$E^M = min(E.\sum_{i\in m} X^m \;;\; \sum_{i\in m} Y^m) \in T \qquad (3.7)$$

If $E^M < 1$, the merger produces savings, and if $E^M > 1$, the merger is costly. An efficiency score of $E^M = 0.8$ would suggest that 20% of all inputs could be saved by integrating the firms into *M*. Similarly, a score of $E^M = 1.3$ suggests that integration would require 30% more of all the resources.

It must be noted that this model may not always be feasible under classical DEA assumptions. This is because the merged entity may fall outside the initial DRS or VRS technology. CRS or IRS will ensure the inclusion of M in the technology set. A second reason may be because the merged entity may require an input mix that is not "powerful" or an output mix that "difficult" to produce in the case of a FDH model. Since the additivity assumption ensures that M is included in the technology set, it is sufficient to ensure a feasible DEA model (Flokou *et al.*, 2016).

The overall potential efficiency gain from merger E^M, represents a best scenario upper limit of the possible gains. It is however an optimistic and rough estimate that requires further refinements. A proportion of the overall gains may be realized by each individual firm without the need to merge.

A portion of the potential overall efficiency gains may be attributed to gains that does not require a full scale merger per se to be realized. These should therefore be excluded from the overall potential merger gains. The overall potential efficiency gains is a product of the combined efficiency effect of learning from best practices, the scale of operations and the harmony that exist in the product mix of a firm (Bogetoft & Wang, 1999).

3.4.2 Learning effect

In reality, the individual firms involved in a merger may not be fully efficient, implying that there is room for individual improvement. Although these inefficiencies may be reduced through new management and diffusion of knowhow through a merger, it is argued that a merger is not the only way in which these inefficiencies may be addressed. Learning from the practice of peer or reference firms through study groups may be an option to improve individual efficiency without the need to merge. Improved incentive schemes may also help individual firms to improve their efficiency. However, if the individual firm inefficiency is attributed to the scarcity of managerial talent, a full-scale merger may still be necessary to transfer control to the more efficient administrative teams and thereby improve the managerial efficiency. Another effect of a full-scale merger comes from the fact that it is a change event where established rules and processes are re-evaluated and improved that enable the merged firm to reduce slacks that were previously difficult to deal with individually.

The learning effect therefore represents a firm's individual ability to adjust to industry best practices. When estimating the potential efficiency effects of a merger, it is important to avoid compounding the effects of individual inefficiency. The overall merger gain is therefore adjusted for the technical- or learning effect. This is done by using the revealed efficiency scores of each firm in M and project them to the frontier and use their projected production plans as the basis to evaluate the remaining gains from merger. Figure 3.2 on the next page illustrates the efficiency effect of learning for two individual firms A and B and their merged production plan represented by M (Bogetoft *et al.*, 2011):

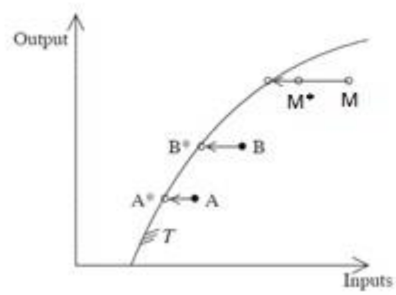


Figure 3.2: Learning effect Source: Bogetoft et al.*, 2011*

Distance A-A* and B-B* represents the efficiency improvement potential of the individual firms through learning from peer- and industry norms. This combined potential is represented by the distance M-M*. Since M* is not projected onto the efficiency frontier after adjusting for the learning effect, there exists an additional efficiency improvement potential that may be realized by other means such as a full scale merger. The potential efficiency gain attributable to the merger itself is then quantified by the distance of M* to the frontier. We use the projected production plans A* and B* in Equation 3.8 below to calculate the pure efficiency effect E^{*M} of a proposed merger.

$$E^{*M} = min\left(E.\sum_{k\in M} E^m x^k \ ; \ \sum_{k\in M} y^k\right) \in T \tag{3.8}$$

The efficiency score of the individual firms prior to the merger is represented by E^m. The gain attributed to the individual firms in M can therefore be calculated as $L^M = \frac{E^M}{E^{*M}}$. The learning effect will always be positive in the sense that $L^M \leq 1$ so that there are potential savings of $1 - L^M$.

Recall how the radial Farrell projections were supplemented by non-radial slack adjustments. When we assume that the individual firms $(m_1, m_2, \dots, M)$ has the ability to reduce slacks within their individual production plans, we may supplement the proportional projections in Equation (3.8) with non-proportional slack adjustments to generate Equation 3.9, the slack adjusted learning effect formula:

$$E^{*M} = min\left(E.\sum_{k\in M} s_k^- E^m x^k \ ; \ \sum_{k\in M} s_k^+ y^k\right) \in T \tag{3.9}$$

Input slacks that are present in the individual input vectors of the m firms prior to merger are accounted for by including the term s_k^-. Assuming that individual inefficiencies and slacks has been dealt with, we continue to decompose the potential overall merger gains into the harmony and scale effect.

3.4.3 Harmony effect

According to the rationalization of production, partners to a merger may decide to reallocate production across firms to those with the lowest marginal cost. This restructuring may lead to new input and output mixes that subsequently result in M having a more efficient production plan. The efficiency gains generated through this restructuring is called the harmony, scope or mix effect.

Figure 3.3 represents two farms, A and B that use the same two inputs to produce the same output represented by isoquant *L(x)* (Bogetoft *et al.*, 2011). The composition of the input use differ, but these two farms produce the same amount of output.

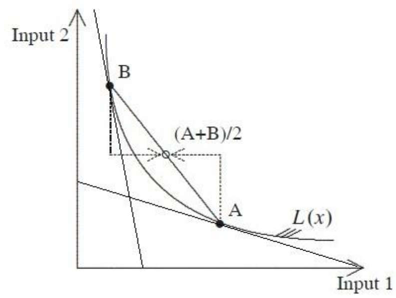


Figure 3.3: Harmony effect *Source:* *Bogetoft* et al.*, 2011*

Input one is used in higher quantities by A and Input two is used in higher quantities by B. A and B therefore has unequal rates of substitution, represented by the tangency point of each's price ratio line at their respective position on the isoquant L(x). According to the rationalization of production, output may therefore be increased through the transfer of resources (represented by dashed lines). Similar possibilities exist on the output side by moving some obligations from A to B and other obligations from B to A. Clearly harmony gains owes its existence to the curvature properties of the production technology. More convex combinations will lead to higher gains while less convex combinations will yield less harmony gains. The largest reallocation gain between firms will be realized in a convex, free disposable technology.

Since the learning effect has been eliminated, the remaining two effects to quantify is the harmony- and scale effect. The assumption of constant returns to scale eliminates any scaling effects, leaving only the effect of resource redistribution (harmonizing) to be measured. Equation 3.10 represents the CRS DEA model formulation that allows for the calculation of the harmony effect.

$$H^M = min(\propto \sum_{k\in M} E^m x^m \; ; \propto \sum_{k\in M} y^m) \in T \qquad (3.10)$$

The symbol $\propto\in [0,1]$ is a scalar that represents the activity level at which the harmony gains is calculated. If the merger is expected to yield harmony gains in terms of input saving, H^M would be larger than 1. Conversely a result of $H^M > 1$ suggest that the harmony effect negatively contributes to the expected overall efficiency gain. A convex technology set will in all situations result in the harmony effect contributing positively toward the expected overall efficiency gain. Figure 3.4 illustrates the harmony effect contributing positively (on the left) and negatively (on the right) to the overall expected efficiency improvement potential of a proposed merger between two farms (Bogetoft, 2005).

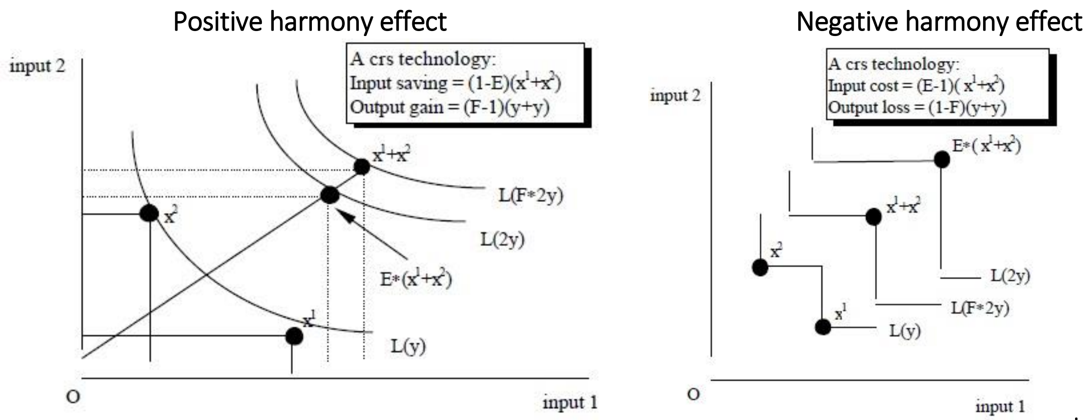


Figure 3.4: Positive vs Negative Harmony effect. Source: Bogetoft, 2005.

Generally, when the farms in M are similar in size, $\propto=\frac{1}{M}$ in Equation (3.10) is assumed so that the harmony gains may be estimated apart from any size effect. Such a formulation captures the harmony gains by examining how much the average input $\left(\frac{x_A+x_B}{2}\right)$ could have been saved in the production of the average output $\left(\frac{y_A+y_B}{2}\right)$ if firm A and B should merge. If there are significant size differences between the firms in M, some scale effect may be included in the estimate of harmony gains. This may be accounted for by varying $\propto$. A low value of $\propto$ in a DRS technology will assign some scale effect to the harmony component. Assuming CRS makes H^M independent of $\propto$. A DRS technology set will result in H^M weakly increasing as the activity level $\propto$ increase. Conversely an IRS technology set result in H^M weakly decreasing as $\propto$ decrease. The VRS assumption results at different levels of $\propto$ will yield non-monotonic results.

An important underlying principle pertaining to the harmony effect is that a convex, free disposable technology set will result in the largest harmony effect possible by reallocation between the merged firms. It is also important to note that harmony gains may be realized through means other than pure merger. Independent firms may cooperate to some degree to improve their pre-merger performance, which will diminish the pure merger gain. Resource reallocation between firms may be facilitated through contracting. However, the importance of timely reaction in combination with increased resource demand in the industry during peak season may limit the attractiveness of such options. Market transactions may be another option, but may be subject to higher transaction costs.

3.4.4 Scale effect

In addition to the learning and harmony effect, a merger will have an impact on the scale of operation. Firms that operate within an industry frontier that exhibits increasing returns to scale find it attractive to operate at a larger scale since it will allow them to produce at a lower average cost. Conversely an industry frontier that exhibits decreasing returns to scale will encourage firms to decrease their scale if they seek to be efficient in their operations. It is therefore important to determine the scale properties of the industry frontier at the portion where the individual firms currently find themselves and also where the merged entity will lie with reference to the frontier. For firms operating under IRS, the scale effect will be positive so that the merger will contribute to the overall potential efficiency gain. However, for firms operating at DRS, the scale effect will diminish the overall potential efficiency gain. The scale effect therefore illustrates the amount of inputs that can be saved by operating at full scale rather than the average scale used to calculate the harmony effect. To evaluate the effect of up scaling the average efficient firm, we solve the DEA model in Equation 3.11:

$$S^M = min(H^M \sum_{h\in M} E^m x^m ;\ \sum_{h\in M} y^m) \in T \tag{3.11}$$

The scale effect resulting from the merger will contribute to the overall potential efficiency gain if $S^M > 1$. Conversely, when $S^M < 1$, the scale effect of the merger will diminish the overall potential efficiency gain. In a convex technology set that satisfies the assumption of constant or increasing returns to scale, the size effect is always positive. Figure 3.5 on the following page graphically illustrates the dynamics of merging two production plans of individual farms into one that yield a positive scale effect on the right and a negative scale effect on the left (Bogetoft, 2005). The x axis represents the input bundle farm 1 and 2 use to produce their respective output bundle measured on the y axis. Factor E represents the input saving or additional cost required to operate at the merged scale. Similarly the factor F represents the output expansion or reduction for operating at the merged scale.

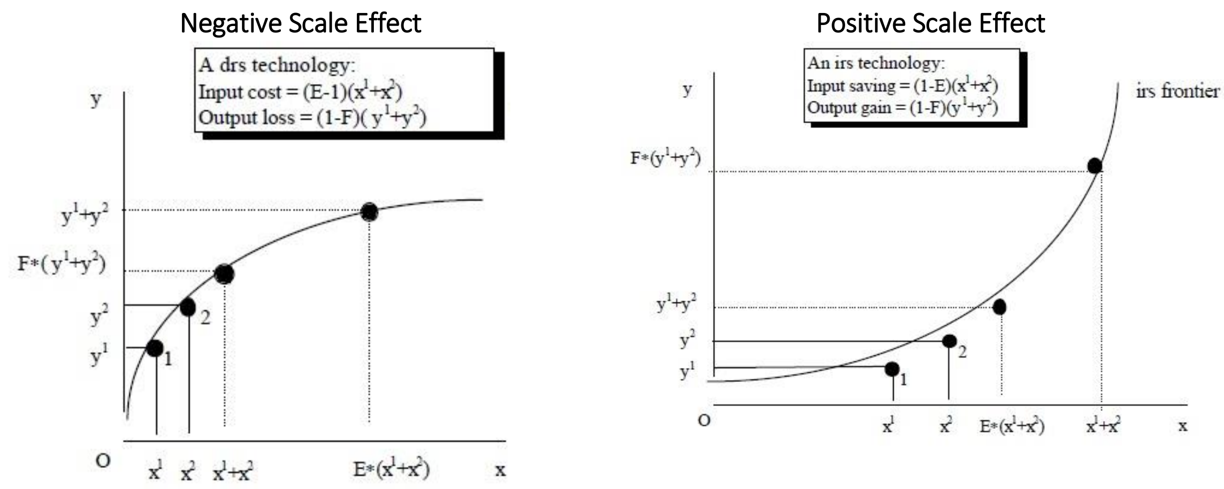


Figure 3.5: Positive and negative scale effect. Source: Bogetoft, 2005

In the event of low scale efficiency, individual farm level attempts to increase its scale of operation may require large capital layouts. This may require substantial short term investment in infrastructure or technology that will only yield returns in the medium and long run. Depending on the financial position of a farm, this investment reality may call for other means of increasing the scale of operation. If the short term investment requirement is too high for the farm to carry until it will yield positive returns, the farm may consider to merge with another in order to achieve its desired result of increased scale efficiency. An alternative to a merger, a farm may consider to outsource some of its present resource consuming activities to free up additional managerial, technological or financial resources to enable the expansion of its current scale. This may in return lead to some risks such as quality control and increased production cost. The nature of production might not allow for third party involvement where pre-arranged contracts are in place. It is evident that each merger case will present its own merits depending on its pre-merger state and the desired outcome. It is evident from the preceding sections that the learning-, harmony- and scale effects jointly affect the potential synergy of a merger. The combined overall expected efficiency improvement potential is therefore represented by Equation 3.12:

$$E^M = L^M \times H^M \times S^M \tag{3.12}$$

3.4.5 Numerical example

Consider a single-output, multi-input dataset within which K farms indicate that they consider merging their production plans. Let the input vector of farm *j be* $x^j = (x_{1j}\,; x_{2j}\,; \dots;\, x_{nj})$ and the output scalar y_j such that $j(j = 1\,; 2\,; \dots; N)$. Let the objective be to estimate the potential efficiency gain from merging K farms. K being the total amount of firms selected from the sample N to become one new merged farm. Table 3.6 presents an excerpt of N_i = 99 farms within which farm 43 and 53 (*K* = 2) are selected to merge into a new farm M. Each farm uses an input vector $x^j = (Labor\,; Capital; Fuel)$ to produce the output scalar $y_j = (KWH)$.

Table 3.6: Numerical merger example dataset

Farm (j)	KWH (y_j)	Labor (x_{1j})	Fuel (x_{2j})	Capital (x_{3j})
1	8	1	297	137
⋮	⋮	⋮	⋮	⋮
43	4 148	27	48 273	4 805
⋮	⋮	⋮	⋮	⋮
53	6 770	50	72 407	14 797
⋮	⋮	⋮	⋮	⋮
99	53 918	383	554 120	56 639

Step 1: Solve the output oriented BCC (VRS) DEA model in Table 3.7 to calculate the efficiency estimates of each of the K farms. Farm 43 is denoted as farm A and farm 53 as farm B:

Table 3.7: Output oriented VRS DEA model

Generic	Substitute for firm A
$Max\ \ \theta_k$	$Max\ \ \theta_A$
s. t.	s. t.
$\sum_{j=1}^{N} \lambda_j y^j \geq \theta_k y_k$ $\sum_{j=1}^{N} \lambda_j x_1^j \leq x_1^k$ $\sum_{j=1}^{N} \lambda_j x_2^j \leq x_2^k$ $\sum_{j=1}^{N} \lambda_j x_3^j \leq x_3^k$ $\sum_{j=1}^{N} \lambda_j x^j = 1$ $\lambda_j \geq 0\,; (j = 1, 2, \ldots, N)\,;\ \theta_k\ free$	$\lambda_A y^A + \lambda_B y^B + \lambda_C y^C + \lambda_D y^D + \lambda_E y^E + \lambda_F y^F \geq \theta_A y_A$ $\lambda_A x_1^A + \lambda_B x_1^B + \lambda_C x_1^C + \lambda_D x_1^D + \lambda_E x_1^E + \lambda_F x_1^F \leq x_1^A$ $\lambda_A x_2^A + \lambda_B x_2^B + \lambda_C x_2^C + \lambda_D x_2^D + \lambda_E x_2^E + \lambda_F x_2^F \leq x_2^A$ $\lambda_A x_3^A + \lambda_B x_3^B + \lambda_C x_3^C + \lambda_D x_3^D + \lambda_E x_3^E + \lambda_F x_3^F \leq x_3^A$ $\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F = 1$ $\lambda_A; \lambda_B; \lambda_C; \lambda_D; \lambda_E; \lambda_F \geq 0$ $y^A = 4\,148 \quad \& \quad y_*^A = 5\,314$ $\theta_A = 1 - \frac{5\,314 - 4\,148}{4\,148} = 0.719$

From the optimal solution for the max output producible: $(y_*^A = 5\,314)$ & $(y_*^B = 8\,975)$ we can calculate the efficiency scores, $(\theta_A = 0.719)$ & $(\theta_B = 0.674)$ for the output produced. Therefore if A produce 5 314 outputs it would lie on the efficiency frontier. Similarly, if B produce 8 975, instead of the current 6 770, it would also produce at a level that will cause it to move to the frontier, i.e. achieving full technical efficiency. With all technical inefficiency removed from an output perspective, the presence of input slacks may still cause a Farrell efficient firm lying on the frontier to operate at a sub efficient level. Therefore to simulate a Koopmans efficient firm, the input bundle should be adjusted for slacks. In this example, included in the technically efficient 14 797 units of Capital used by Firm B is 2 455 units in excess (slack).

Table 3.8 shows the technically efficient output level and the slack adjusted input bundle for each of the candidate farms A and B:

Table 3.8: Technically efficient and slack adjusted inputs for two farms that merge

Farm	Revealed E	Output (y_*^k)	Labor (x_{1k}^*)	Fuel (x_{2k}^*)	Capital (x_{3k}^*)	Adjusted E
A	0.719	5 314	27	48 273	4 805	1.00
B	0.674	8 975	50	72 407	12 342	1.00
Total		14 289	77	120 680	17 148	

Assuming that individual technical inefficiencies have been dealt with, we are left with the two most interesting production economic effects of a merger, the one is the scaling or size effect and the other is the harmony, scope or mixture effects.

Step 2: Construct from the efficiency- and slack adjusted data, the average input and average output bundle for the sum of the candidate farms K as in Table 3.9 below:

Table 3.9: Average input and output bundle for the merged farm

Output (KWH)	Input (Labour)	Input (Fuel)	Input (Capital)
$\bar{y} = \frac{1}{K}\sum_{k=1}^{K} y_*^k$	$\bar{x}_1 = \frac{1}{K}\sum_{k=1}^{K} x_*^k$	$\bar{x}_2 = \frac{1}{K}\sum_{k=1}^{K} x_*^k$	$\bar{x}_3 = \frac{1}{K}\sum_{k=1}^{K} x_*^k$
$\bar{y} = \frac{1}{2}(5\,314 + 8\,975)$	$\bar{x}_1 = \frac{1}{2}(27 + 50)$	$\bar{x}_2 = \frac{1}{2}(48\,273 + 72\,407)$	$\bar{x}_3 = \frac{1}{2}(4\,805 + 12\,342)$
$\bar{y} = 7\,145$	$\bar{x}_1 = 38.8$	$\bar{x}_2 = 60\,340$	$\bar{x}_3 = 8\,574$

Step 3: Solve the **output oriented BCC (VRS) DEA** model in Table 3.10 to calculate the maximum output obtainable by an imaginary farm H using the average slack adjusted input bundle (38.8; 60 340; 8 574) in sample N_i , however this time omitting farms A and B and including farm H:

Table 3.10: Output oriented VRS DEA model using the average slack adjusted input bundle of the simulated merged farm

Generic	Substitute for firm H
$Max\ \theta^H$	$Max\ \theta^H$
s. t.	s. t.
$\sum_{j=1}^{N} \lambda_j y^j \geq \theta^H \bar{y}$ $\sum_{j=1}^{N} \lambda_j x_1^j \leq \bar{x}\,_1^H$	$\lambda_C y^C + \lambda_D y^D + \lambda_E y^E + \lambda_F y^F + \lambda_H y^H \geq \theta^H \bar{y}$ $\lambda_C x_1^C + \lambda_D x_1^D + \lambda_E x_1^E + \lambda_F x_1^F + \lambda_F x_1^H \leq \bar{x}_1^A$ $\lambda_C x_2^C + \lambda_D x_2^D + \lambda_E x_2^E + \lambda_F x_2^F + \lambda_F {x_2}^H \leq \bar{x}_2^A$ $\lambda_C x_3^C + \lambda_D x_3^D + \lambda_E x_3^E + \lambda_F x_3^F + \lambda_F x_3^H \leq \bar{x}_3^A$ $\lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_H = 1$ $\lambda_C; \lambda_D; \lambda_E; \lambda_F; \lambda_H \geq 0$

$\sum_{j=1}^{N} \lambda_j x_2^j \leq \bar{x}\,_2^H$ $\sum_{j=1}^{N} \lambda_j x_3^j \leq \bar{x}\,_3^H$ $\sum_{j=1}^{N} \lambda_j x^j = 1$ $\lambda_j \geq 0\,;\ (j = 1, 2, \ldots, N)\,;\ \theta^H\ free$	
	$y_*^H = 7\ 253$

The optimal solution determines that a farm using the average input set should, according to the revealed industry technology, be able to produce 7 253 units of output.

Step 4: Double the average slack adjusted input and output bundle of the K farms:

$$x_1^T = K(\bar{x}_1) = 2(38.8) = 77.6$$

$$x_2^T = K(\bar{x}_2) = 2(60\ 340) = 120\ 680$$

$$x_3^T = K(\bar{x}_3) = 2(8\ 574) = 17\ 148$$

$$y^T = K(\bar{y}) = 2(7\ 145) = 14\ 290$$

Step 5: Run the output oriented BCC (VRS) DEA model in Table 3.11 to calculate the maximum output obtainable by an imaginary farm T that uses the total slack adjusted input bundle (77.6 ; 120 680 ; 17 148) again omitting A and B and including farm T (do not adjust output bundle for slacks in the multi-output case when defining $\bar{y}$).

Table 3.11: Output oriented VRS DEA model using the total slack adjusted input bundle of the simulated merged farm

Generic	Substitute for firm T
$Max\ \theta^T$	$Max\ \theta^T$
s. t.	s. t.

$\sum_{j=1}^{N} \lambda_j y^j \geq \theta^T y_T$ $\sum_{j=1}^{N} \lambda_j x_1^j \leq \bar{x}\,_1^T$ $\sum_{j=1}^{N} \lambda_j x_2^j \leq \bar{x}\,_2^T$ $\sum_{j=1}^{N} \lambda_j x_3^j \leq \bar{x}\,_3^T$ $\sum_{j=1}^{N} \lambda_j x^j = 1$ $\lambda_j \geq 0\,;\ (j = 1, 2, \ldots, N)\,;\ \theta^T\ free$	$\lambda_C y^C + \lambda_D y^D + \lambda_E y^E + \lambda_F y^F + \lambda_H y^T \geq \theta^T y_T$ $\lambda_C x_1^C + \lambda_D x_1^D + \lambda_E x_1^E + \lambda_F x_1^F + \lambda_F x_1^T \leq \bar{x}_1^T$ $\lambda_C x_2^C + \lambda_D x_2^D + \lambda_E x_2^E + \lambda_F x_2^F + \lambda_F x_2{}^H \leq \bar{x}_2^T$ $\lambda_C x_3^C + \lambda_D x_3^D + \lambda_E x_3^E + \lambda_F x_3^F + \lambda_F x_3^H \leq \bar{x}_3^T$ $\lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_T = 1$ $\lambda_C; \lambda_D; \lambda_E; \lambda_F; \lambda_T \geq 0$ $y_*^T = 14\ 031$

The optimal solution determines that a farm using the total slack adjusted input set should, according to the revealed industry technology be able to produce 14 031 units of output.

Step 6: Calculate the merger efficiency (ME) by calculating the ratio in Table 3.12:

Table 3.12: Merger efficiency calculation

$ME = \theta_*^T = \frac{y_*^T}{y^T} = \frac{14\ 031\ (from\ step\ 5)}{14\ 290\ (from\ step\ 4)} = 0.9819$		
ME < 1	ME = 1	ME > 1
Merger efficiency loss	Merger gains neutral	Merger efficiency gains
A ME of 0.9819 indicates that a merger of A and B would result in an overall efficiency loss of 1.81%. Therefore, it would be more efficient for them to operate separately than combined.		

More insight can be obtained as to exactly what the fundamental sources for the overall loss/gain in efficiency is by decomposing the merger efficiency into the scale- and harmony effects in the next step.

Step 7: Use the formula in Table 3.13 to isolate the harmony effect and the formula in Table 3.14 to isolate the scale effect:

Table 3.13: Harmony effect

$HE = \theta_*^H = \frac{y_*^H}{\bar{y}} = \frac{7\ 253\ (from\ step\ 3)}{7\ 145\ (from\ tep\ 2)} = 1.0152$

HE < 1	HE = 1	HE > 1
Merger gains negative	Merger gains neutral	Merger gains positive, indicating a convex production possibility set
HE = 1.0152 indicates that a merger of A and B would result in an efficiency gain of 1.52%		

Table 3.14: Scale effect

$SE = \theta_*^S = \dfrac{\theta_*^T}{\theta_*^H} = \dfrac{0.9819\ (from\ step\ 6)}{1.0152\ (from\ step\ 7)} = 0.967$		
SE < 1	SE = 1	SE > 1
Merger gains negative	Merger gains neutral	Merger gains positive $f\left(\sum \bar{x}_j\right) > f(\bar{x}_A) + f(\bar{x}_B)$
SE = 0.967 indicates that the merged firm using twice the average input bundle would be 3.27% less efficient than the separate entities each using the average input bundle.		

In this example the negative scale effects overwhelms the positive harmony effect and a full-scale merger would result in a net efficiency loss. However, as indicated by the positive harmony gains, the farms may both improve their efficiency by learning from each other. It is possible that sufficient harmony gains may exceed scale losses to result in an overall merger efficiency gain. Important to note: Unless both input bundles lie on the same isoquant, output of the average bundle will incorporate a portion of scale effect in the harmony effect. For merging firms in a multi-input, multi-output production technology, the observed input-output quantities of the candidate firms needs to be adjusted for technical inefficiency in both the input (output) bundle in an input (output) oriented model. The optimal input and output bundles should be adjusted for slacks. Even when output slacks are present in the optimal solution calculated in step 4, no slack adjustment should be made in the definition of $\bar{y}$ in the DEA model used in step 5. Otherwise, θ_*^H & θ_*^T would not refer to radial expansion of the same output vector, causing the scale factor measure to lose significance.

3.5 Data validation

3.5.1 Sample description & summary statistics

Farm level data of diversified crop-livestock farm businesses in the Delareyville area of the North West province were obtained from a local cooperative. The data was anonymized to protect the identity of the businesses before receiving it from the cooperative. These farms operate within a radius of maximum 80km from one another. Figure 3.6 illustrates the boundary that defined the data collection area. The cooperative provides an annual benchmarking service to farmers based on the financial and managerial data obtained from its Delareyville study group. The data from 2008 to 2016 include on average 15 farm entities per year. Some of the farms took part in the service throughout all the years, and others only took part in some of the years.

Figure 3.6: Geographical source region of the data

The diversified nature of the farm businesses results in no two being identical. Their operations include a diverse set of activities that range from cultivating maize, peanuts, sunflower, sorghum and soybeans on both irrigated and/or dry land, to tending livestock in dairy and feedlot systems. This makes it particularly difficult to draw comparisons or devise a universal assessment method. Efficiency estimation is a very useful tool to use under these circumstances. The majority of farm businesses operate a maize and sunflower division with some degree of extensive cattle division. For the purposes of this thesis the dryland maize and sunflower division and the commercial cattle production divisions of each farm is used to define the farm business regardless of the other divisions that a farm may be involved in. Setting this criteria eliminates a few farms, leaving too little in one particular year to statistically allow efficiency estimation with DEA. The data for 2015 and 2016 is therefore merged to construct a large enough database to ensure a statistically sound analysis with DEA.

3.5.2 Model specification

Small datasets for illustrative purposes can be analyzed by manual mathematical calculation. When the dataset is larger and repetitive calculations are required, statistical computing programs are necessary to solve the optimization models. RStudio is used to solve the optimization model of this thesis. R Studio is a popular open source and enterprise-ready professional software platform for statistical computing (R Core Team, 2016).

1. An input oriented slack adjusted DEA model under different returns to scale assumptions is used to calculate the efficiency of 32 farming entities.
2. Regression analysis is then used to determine if there exists a correlation between the efficiency of a farm and its level of livestock diversification and the percentage of owned vs rented land respectively.
3. Three hypothetical mergers are independently constructed each involving two farming entities in a case study approach

3.5.2.1 Output vector

The output vector of each farm business is defined by the sum of the gross income derived from the sales of dryland produced maize and sunflower. This output vector is strategically chosen to account for the farmer's ability to produce a certain volume of grain within a given year and its environmental conditions. It furthermore represents the farmer's ability to secure the highest prices for his grain, be it through hedging and pre-season contracts. Price and yield are therefore incorporated in the efficiency estimates through revenue. Sales figures are not adjusted for inflation. Another important factor that the output vector accounts for is the farmer's ability to manage his exposure to production risk by diversifying his/her crop portfolio by planting both sunflower and maize.

3.5.2.2 Input Vector

The input vector of each farm business includes the following input variables: cropped hectares (ha), directly allocated variable costs of maize and sunflower (R). The cropped hectares include the total dryland hectares used in the production of maize and/or sunflower. It includes both owned and rented land. It is assumed that the geographical proximity of the farms serve as sufficient grounds to assume homogeneity of soil type and yield potential. Seed, fertilizer and chemicals are included as directly allocated variable costs. It represents a farmer's ability to secure the highest quality inputs at the lowest price. This can be done using various strategies such as early purchase discounts or bulk buy discounts through buyers groups. The producer price index is assumed to be constant for both years 2015 and 2016.

3.5.2.3 Second stage regression variables

Tenure is a measure that represents the proportion of owned vs rented land a farmer uses to produce crops. The importance of this variable is found in the degree of certainty it provides a farmer with. Investing in soil health and conservation practices that yield essential benefits in the long run may be considered from different perspectives depending on the tenure structure according to which the land is held. Owned land brings about more certainty and commitment to the building up of the soil that will greatly improve a farm's efficiency. Contractual provisions attached to rented land such as the degree of utilization of crop residues with cattle may be to the detriment of organic matter buildup, exposing the soil to water and wind erosion, both having a severely negative influence on the efficiency with which crops can be produced.

Livestock income as a portion of crop income represents the extent of diversification of each farm business. This estimate is calculated by dividing the income generated from the livestock enterprise with the combined income generated from the sunflower and maize enterprise of a particular farm business.

3.5.3 Dataset

The complete dataset is attached in appendix A. Table 3.15 is included to present the statistical summary of this data:

Table 3.15: Summary statistics of dataset

Statistic	Output	Ha	DAVC	Tenure	LS share
Mean	8 068 972	1 191	3 415 044	0.70	0.17
St.dev.	6 243 380	591	1 881 650	0.25	0.15
Median	5 986 040	913	2 809 792	0.75	0.17
Min	996 198	548	908 545	0.20	0.00
Max	24 192 108	2 475	8 740 380	1.00	0.66
Nobs	32	32	32	32	32

The output in terms of income derived from the dryland cultivation of sunflower and maize ranges from a minimum of approximately R1 mil. to a maximum of R 24 mil. The standard deviation of R6.2 mil. shows the large variation in output among these farms. It seems that a higher level of directly allocated variable cost and more hectares cultivated is related to having a higher output. This observation will be tested later on in this thesis. There is also considerable variation in the tenure structure of these farms. Some farm businesses only own 20% of the land they use for cultivating crops and other own 100% of the land they cultivate. In terms of diversification, there are farm businesses that tend no livestock at all. This is in contrast to other that derive the equivalent of 66% of their crop income from livestock. The mean contribution of livestock to farm income is estimated at 17% of the value of the crop income. It can therefore be concluded that crop cultivation contributes the largest portion of farm income.

3.5.4 Assumptions

In order for the efficiency model to have sufficient discriminatory power in differentiating between efficient and inefficient entities, the production data for 15 farm businesses in 2015 and 17 farm businesses in 2016 were combined to produce a large enough dataset of 32 entities. Each of these 32 entities are defined as different farming entities within one year. The geographical proximity of the farming entities justify the assumption of homogeneous weather conditions for all entities within one year. The necessity of the combined dataset does however require a further and more unrealistic assumption of homogeneous weather conditions faced by farming entities in both 2015 and 2016. This assumption, although unrealistic, is justified in that it is not used to compare or rank the efficiencies of the farm businesses. It is purely used to generate a frontier that will allow us to simulate the possible efficiency effects that a merger may bring about. It is assumed that hectares and the directly allocated variable costs, seed, fertilizer and chemicals are the only inputs these farmers. In reality these only represent a portion of the variables that is used by farm businesses of this type. The same can be said about the assumption that sunflower and maize are the only crops produced using the abovementioned inputs. Because of the immense variability in soil type and yield potential, each hectare on all farms is assumed homogeneous. Variable returns to scale is assumed as far as possible and where infeasible results were obtained, additivity is assumed because of its universal ability to evaluate all combinations of decision making units within the efficiency frontier. The output orientation is selected in the DEA model because it is argued that the producer has more control over how much inputs is used than how much output is produced.

3.6 Conclusion

The case study approach used in this thesis randomly constructs three possible mergers that are completely independent of each other. The selection of candidate farms to merge is purely hypothetical. It is not based on observed mergers, hence the ex-ante approach of the thesis. Care is taken not to select the same farming entity in 2015 and 2016 as partners to a merger. Each case is evaluated separately in order to test the merger analysis methodology presented in Chapter 3.3. Each case serve to test the methodology with regards to its ability to generate quantifiable results that indicate the extent of the expected efficiency results that a merger would bring about. This thesis acknowledges the importance of the “soft” aspects such as managerial style and philosophical convictions in merger compatibility. It therefore assumes that the candidate farm businesses, are willing and able to manage the “soft” aspects to the benefit of the whole.

Chapter 4: Results

4.1 Introduction

This thesis ultimately aims to provide an answer to the extent to which a proposed merger or resource pooling agreement is expected to influence the technical efficiency of an individual farm business. This thesis follows the conventional efficiency measurement research methodology to assess the current efficiency level of farms within a study group. It also applies regression analysis in a second stage to determine the effects of external variables on farm efficiency. These estimates then serve as baseline from which three merger cases will be assessed with regards to its expected net effect on efficiency. This effect is decomposed to identify the underlying sources of a possible efficiency gain or loss. This chapter focus on the outcome of the method of DEA applied to actual farm situations. The farming area of concern is the Sannieshof Delareyville area and the farms in question can be described as more or less homogeneous.

4.2 Efficiency results

The efficiency results of the slack based input oriented DEA model under different scale assumption is presented in the statistical summary in Table 4.1 below. The complete set of efficiency results is included in Appendix C. The code that was used to generate the results is attached in Appendix E. The specific programming packages utilized are listed and described in Appendix D.

Table 4.1: Statistical summary of efficiency estimates under different scale assumptions

	VRS	CRS	**ADD**	SE
nobs	32	32	**32**	32
Min	0.37	0.11	**0.43**	0.37
1^{st} quartile	0.60	0.36	**0.76**	0.60
3^{rd} quartile	0.86	0.65	**1.00**	0.84
Mean	0.73	0.51	**0.85**	0.71
Median	0.75	0.46	**0.93**	0.72
Stdev	0.18	0.21	**0.18**	0.17

It is evident that different assumptions about the production technology affects the efficiency estimates. This is because different scale assumptions result in different sizes of the enveloped area. Figure 4.1 on the next page illustrates how different returns to scale assumptions results in differences in the relative position of a specific farm with regard to the efficiency frontier. The additivity assumption yields higher efficiency estimates than both the variable returns and constant returns to scale assumption. It is important to keep this in mind when attempting to compare the efficiency results of models that were subject to different scale assumptions. The standard deviation of the efficiency results indicate that the degree of "fit" of the additive frontier and the variable returns to scale frontier are the same. This means that they represent the true efficiency frontier with the same degree of accuracy. The constant returns to scale assumption yields a higher degree of variation of the actual efficiency and the simulated frontier. This is because of its linear functional form that does not adequately represent the true nature of the production technology faced by the farm businesses in this dataset.

Table 4.2 presents the VRS efficiency of three farm businesses. We will focus on farm five and how its efficiency is derived from the two reference farms, four and eleven, that define the VRS efficiency frontier at the region where farm five is located in the production possibility set.

Table 4.2 Illustrative DEA results

Farm	Eff (VRS)	Ref1	Ref2	W(rf1)	W(rf2)
5	0.62	Farm 4	Farm 11	0.46	0.54

The slack based input oriented VRS DEA model identified farms 4,9,11 and 26 as being fully efficient from a technical point of view. These four farms define the complete efficiency frontier. The portion of the frontier where farm five is located is however only defined by farm four and eleven. A piecewise linear combination of the production plans of farms four and eleven serve as the reference against which the efficiency of farm five is calculated. Reference farm four contributes 46% and reference farm 11, 54% of the total efficiency value of 62% for farm number five. This means that farm five should, according to what is revealed by farm four and eleven, be able to generate its current output of R7.1mil with only 62% of its current input vector levels. A further R 148 429 worth of input slack in directly allocated variable costs indicate a further reduction in input consumption without reducing the current level of output.

It is important to note that even though there are four farms that define the efficiency frontier, farm eleven is the only one that operates at the most productive scale size. The scale efficiency of farm five is calculated at 0.84. Since it is less than one it indicates that farm five is scale inefficient. Upon further investigation it is determined that it is operating at a portion of the production possibility frontier where decreasing returns to scale prevails. This means that an increase in the size of its operations alone would not add any efficiency to its production process. To the contrary it will reduce the technical efficiency of farm five.

4.3 Bias corrected technical efficiency results

A statistical summary of the bootstrapped slack adjusted VRS efficiency estimates is presented in Table 4.3. The bootstrap sampling procedure is repeated two thousand times to ensure the estimates converge to a reliable end result.

Table 4.3: Statistical summary of bootstrapped slack adjusted VRS efficiency estimates

	Eff (VRS)	Eff (VRS)(BS)	Bias	Var	2.5% CI	97.5% CI
Min	0.37	0.36	0.02	0.00	0.33	0.37
Max	1.00	0.88	0.22	0.15	0.80	0.99
1st quartile	0.60	0.56	0.04	0.00	0.52	0.60
3rd quartile	0.86	0.78	0.07	0.00	0.70	0.85
mean	0.73	0.66	0.07	0.01	0.60	0.72
median	0.75	0.68	0.05	0.00	0.62	0.74
Stdev	0.18	0.15	0.05	0.03	0.14	0.18

The bootstrap procedure eliminated an average upward bias of 7% that was included in the unadjusted efficiency estimates. The largest bias was 22% and the lowest 2%. Referring back to the illustrative example in Table 4.2 above, the bootstrap procedure converged to a 4% upward bias included in the efficiency estimate of farm five with a variation of 0.1% across all two thousand iterations. This means that the bias corrected efficiency estimate of farm five is 0.58.

4.4 Determinants of efficiency

The truncated regression procedure that tested the influence of tenure and income diversification on farm efficiency yielded the results in Table 4.4.

Table 4.4 Regression results

	Estimate	Std. Error	t-value	Pr(>|t|)
(Intercept)	0.637324	0.076113	8.3733	< 0.00000000000000022 ***
tenure	0.132308	0.110166	1.2010	0.22976
ls_share	-0.369665	0.186877	-1.9781	0.04792 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				
Log-Likelihood: 17.089 on 4 Df				

The influence of tenure on farm efficiency is calculated at a factor of 0.132. However, since the p value of 22.9% is larger than the 5% confidence interval, its influence is not statistically significant. There is therefore no statistical basis to assume that tenure has an influence on farm efficiency. This implies that a farmer renting all his hectares has no technical efficiency advantage in his use thereof and that of directly allocated variable cost to produce an income from cultivating either maize or sunflower or a combination thereof over a farmer who owns all his hectares.

Livestock income as a percentage of crop income has a negative effect on the efficiency of the cropping enterprise of a diversified farm that is statistically significant. A 1% increase in livestock income as a portion of crop income causes a reduction of 0.37 in the efficiency with which the farm business utilize its cropped hectares and directly allocated variable cost to generate an income from crops. This result suggests that diversification in income to have both livestock- and crop income will lead to lower efficiency levels of the cropping enterprise. Conversely, farms with a lower portion of livestock income as a portion of crop income is expected to be more efficient in their cropping enterprise. Stated differently, there exists a statistically significant chance that specializing in crop production will lead to a higher efficiency level that will result in more efficient use of hectares and directly allocated variable costs. This narrative may seem to favor specialization, but in reality a rational farm business may choose to forgo the possible efficiency advantages of specialization and choose to diversify its income to reduce its overall risk profile.

4.5 Case studies

This section offers three independently constructed cases or scenarios, each illustrating a merger simulation of two randomly selected farm businesses. The merger model of Chapter 3.3 is used to assess the merged production plan in each case with regards to its expected efficiency effects. Table 4.5 presents a summary and description of each merger case.

Table 4.5 Case study farms

Case	Partners	Description
1	11, 14	One reference firm, the other inefficient (similar input use level)
2	1, 9	One reference firm, the other inefficient (differ significantly in input use level)
3	7, 8	Both inefficient (differ significantly in input use level)

Case one simulate a merger between farm eleven and fourteen. It serves to represent a merger between two farms that use nearly the same input level. It also aims to capture the fact that both partners to the merger defines the efficiency frontier with the one emulating the MPSS.

Case two simulate a merger between farm one and nine. These farms were selected specifically because both define the frontier, but at different ends of the scale size. The size of operation of farm nine is much larger than that of farm one.

Case three simulate a merger between farm seven and eight. These two farms are both technically inefficient i.e., not one of them embodies the most efficient use of land and directly allocated variable costs. A similarity that they share is that both use nearly the same level of inputs.

Figure 4.1 on the next page illustrate the different production possibility frontiers of the variable returns to scale and the additivity assumptions. Merger one and three fall within the VRS technology set and can therefore be assessed with regard to their efficiency improvement potential by using the initial efficiency frontier. Merger two on the other hand falls outside the VRS technology set and therefore requires a different yet realistic assumption about the production technology. The additivity assumption assumes that any combination of the merger between two production plans are viable and attainable based on the viability and attainability proved by the presence of the observed individual production plans. We use this assumption to measure the efficiency improvement potential of merger two. A graphical representation of the shape of the efficiency frontier under various other technology assumptions is included in Appendix B.

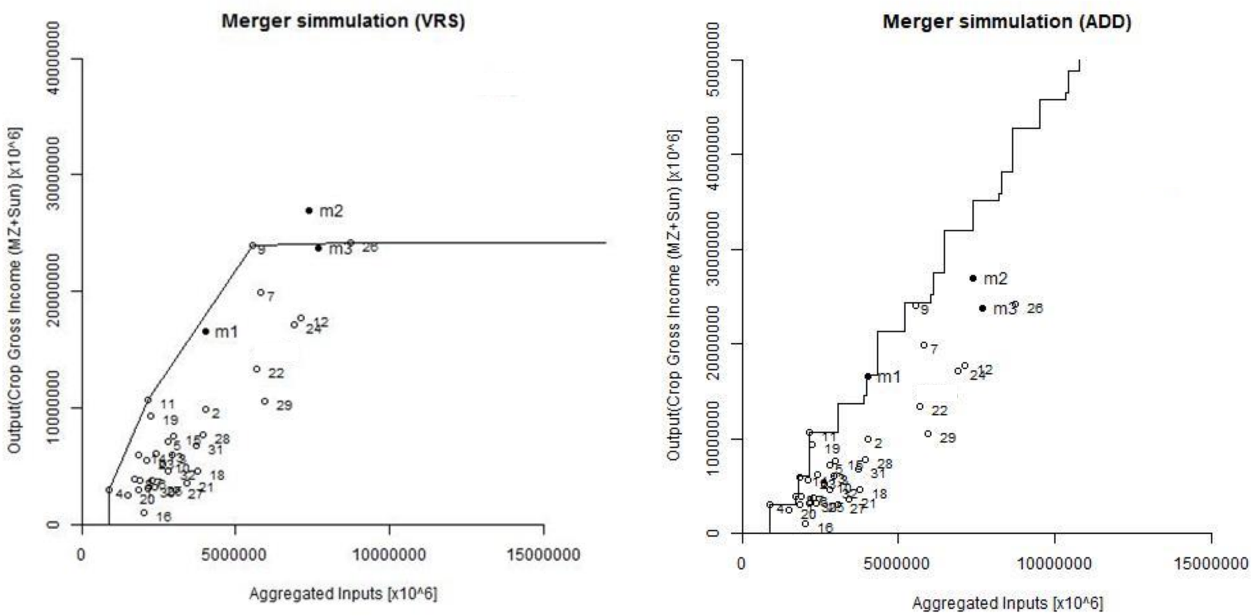


Figure 4.1: VRS frontier (LHS) and ADD frontier (RHS)

We can test the position of the merged farm in relation to the efficiency frontier before we perform the merger analysis. This enables us to choose the scale assumption that will be able to envelop the merged farm enabling it to generate feasible merger results.

Table 4.6 illustrates the feasibility of merger analysis in each case under different scale assumptions.

Table 4.6 Feasibility of merger analysis under different assumptions

Case	ADD	CRS	VRS	IRS	DRS	FDH
1	Feasible	Feasible	Feasible	Feasible	Feasible	Infeasible
2	Feasible	Feasible	Infeasible	Feasible	Infeasible	Infeasible
3	Feasible	Feasible	Feasible	Feasible	Feasible	Feasible

Mathematically, the VRS production frontier includes the CRS, IRS and DRS assumptions. The VRS assumption therefore has the ability to account for both the advantages and disadvantages of operating at different scale sizes. This makes it the preferred assumption to assess the prospects of a proposed merger. In some cases though, such as case two for example, the VRS assumption is not able to envelop the merged production plan. Figure 4.1 illustrates this shortcoming, and how the frontier constructed under the additivity assumption is able to envelop the merged production plan of case two. Even though the additivity-, constant- and increasing returns to scale assumption are able to assess the merger in case two, the additivity assumption is preferred. This is because its non-linear form allows for scaling and also ensures the feasibility of all production plans.

Using different scale assumptions in the three cases is not considered to be a problem since the cases are independent and not to be compared to each other with regard to the magnitude of the estimated efficiency effects. The option to select various model specifications is in fact evidence of the ability of this modeling approach to generate quantitative estimates across a wide range of applications.

4.5.1 Case One

<u>Context before merger</u>

Assuming VRS, farm 11 is the most efficient farm in the dataset in terms of its utilization of land (813ha) and directly allocated variable cost (R 2 153 084) to generate a crop revenue (R 10 669 043). Its technical efficiency is therefore equal to one. The bootstrap procedure identified an upward bias of 12% and thus adjusted the efficiency estimate to 0.88. The size of its operation is also optimal, defining the most productive scale size with a scale efficiency estimate of one. Farm 11 serves as an efficiency reference for 25 out of the 32 farms within the dataset. It owns 55% of the land it utilizes for crop production. Its livestock income is the equivalent of 9% of its crop income.

Farm 14 cultivates 799ha and spends directly allocated variable costs worth R 1 867 182 to generate a crop revenue of R 5 934 945. Again assuming VRS, the efficiency of farm 14 is a weighted combination of 62% of the efficiency of farm four and 38% of the efficiency of farm eleven. The efficiency of farm 14 is 0.81. The bootstrap procedure adjusted this estimate to 0.75 indicating a 6% upward bias. Farm 14 is scale inefficient as indicated by a scale efficiency estimate of 0.79. When assessed for the specific scale property, it is evident that it operates in the region of the production possibility frontier where DRS prevails. The slack based model identified R 134 949 worth of slack (excess) in farm 14's use of directly allocated variable cost. This implies that according to the most efficient use of directly allocated variable cost, as revealed by the dataset, Farm 14 should be able to reduce its DAVC expense by this amount without reducing its output. It is important to note that should farm 14 choose to do so, its efficiency will not increase because slacks imply a further reduction in the input vector along the efficiency frontier. Farm 14 owns 77% of the land it utilizes for crop production. The amount of livestock income it generates is the equivalent of 12% of its crop income.

Upon investigation it is evident that both farms exhibit very similar production plans with little difference in the quantities of land and DAVC utilized. Their output vectors differ remarkably though with Farm 11 generating 1.8 times the revenue of Farm 14, thus confirming the efficiency results mentioned above. This thesis tested the effect of tenure and livestock income as a portion of crop income on the efficiency of the farms in the dataset. Livestock income is the only variable that had a statistically significant influence on farm efficiency. As livestock income increase by 1% as a portion of crop income, efficiency will decrease by 0.37. Farm 11 derives the equivalent of 9% of its crop income from livestock and farm 14, 12%. The proportional difference of 3% is then multiplied by 0.37 to equate to 1.11%. It can therefore be said that 1.11% of the 13% difference in farm efficiency of these two farms can be attributed to the different levels of income diversification.

Since their crop operations are very similar in terms of scale, we may assume that the remaining 11.9% (13%-1.11%) efficiency difference may be attributed to various factors other than scale advantages. One such factor may be under fertilization or insufficient weed control represented by a lower DAVC that could explain the lower crop revenue of farm 14.

Efficiency results of merger

The slack adjusted and bootstrapped input and output vectors of farms 11 and 14 is presented in Table 4.7 on the next page for the evaluation of the possible efficiency effects that a merger in their crop enterprises may bring about:

Table 4.7: Case one, merger between farm 11 and 14

Farm	output	ha	DAVC
11	10 669 043	813	2 153 084
14	5 934 945	799	1 732 233
11+14	16 603 988	1612	3 885 317

Reintroducing the merged entity into the sample, this time omitting farms 11 and 14, and measuring its efficiency against the VRS efficiency frontier results in the efficiency effects presented in Table 4.8 below:

Table 4.8: Case one merger efficiency results

m1	learning	harmony	size	Eff	E*	IP	Pure gain	PHG	PSG
VRS	0.916	1.000	1.028	0.942	1.028	0.058	(0.028)	-	(0.028)

The 0.942 merger efficiency indicates an efficiency improvement potential of 5.5% (1-0.942). This estimate should theoretically be adjusted for the portion of gain that can be attributed to learning best practices from one another because this learning can be facilitated through other means than merging. The pure efficiency improvement potential is therefore -2.8%. This value comprises a harmony effect that has a neutral impact and a scale effect that has the potential to reduce the efficiency of the merged farm with 2.8%.

4.5.2 Case Two

Context before merger

Farm one cultivates 760ha and spends DAVC worth R 1 425 257 to generate a crop revenue of R 3 042 708. Assuming VRS, an efficiency estimate of 0.72 is calculated solely with reference to the efficiency of farm four that defines the efficiency frontier in the region where farm one finds itself in the production possibility frontier. The bootstrap procedure adjusted the efficiency of farm one to 0.66, indicating an upward bias of 6.4%. The scale efficiency of farm one is 0.45, indicating that it is severely scale inefficient. Farm one spends R427 108 worth of excess (slack) DAVC as estimated by the slack based DEA model. There is a furthermore output (revenue) shortfall worth R 75 438 in its output vector. Farm one owns 87% of the land it uses for crop production. Its livestock income equates to 24% of the value of its crop derived income.

Farm nine cultivates 2010ha and utilize DAVC worth R 5 536 358 to generate crop revenue of R 23 999 416. Again assuming VRS, the efficiency of farm nine is equal to 1.00 indicating that it serves as a reference to itself with regards to its efficiency estimate. This estimate is subsequently corrected for a 21.1% upward bias by the bootstrap procedure to yield a more realistic efficiency estimate of 79%. Even though it serves as a reference to itself with regards to technical efficiency, it is scale inefficient with an estimate of 0.91. The model identified no slacks in either of its input and output vectors. Farm nine owns 100% of the hectares it cultivates. The livestock income it earns is equal to 21% of the value of its crop revenue.

We can extrapolate the regression results to determine the contribution of income diversification to the 13% efficiency difference between farm one and nine. Farm one derives the equivalent of 24% of its crop income from livestock and farm nine, 21%. The proportional difference of 3% in income diversification is multiplied by the regression coefficient of -0.37 to equate to -1.11%. It can therefore be assumed that 1.11% of the 13% difference in farm efficiency of these two farms can be attributed to the different levels of income diversification. The remaining 11.9% efficiency difference may be attributed to a range of other factors that include economies of scale such as bulk purchase discount on directly allocated variable cost.

Efficiency results of merger

The individual farm efficiencies of farm one and nine was assessed under the VRS assumption. Their combined production plan falls outside the VRS constructed production possibility set. A different assumption about the technology set should therefore be made to be able to measure the potential efficiency effects that a merger between these two farms may bring about. The additivity assumption is used to ensure a feasible merger analysis.

The slack adjusted and bootstrapped input and output vectors of farms one and nine is presented in Table 4.9 below for the evaluation of the possible efficiency effects that a merger in their crop enterprises may bring about:

Table 4.9: Case two, merger between farm 1 and 9

Farms	output	ha	davc
1	3 042 708	760	1 425 257
9	23 999 416	2 010	5 536 358
1+9	27 042 124	2 770	6 961 615

If we reintroduce the merged firm into the sample and omit farms one and nine, the efficiency effects in Table 4.10 may be brought about by the proposed merger under the additivity assumption:

Table 4.10: Case two merger efficiency results

M2	learning	harmony	size	Eff	E*	IP	Pure gain	PHG	PSG
ADD	0.932	1.064	0.877	0.87	0.933	0.13	0.067	(0.064)	0.123

The 0.87 merger efficiency indicates a merger efficiency improvement potential of 13%. This estimate should theoretically be adjusted for the portion of gain that can be attributed to learning best practices from one another. The pure efficiency improvement potential is therefore 6.7%. This value comprise of a -6.4% harmony

effect and a 12.3% scale effect. It is interesting to note that even though none of the constituent farms were operating at scale efficient levels, their combined production plan produced considerable scale advantages.

It is important to reiterate that the results in case two cannot be directly compared to that of case one because of the different assumptions about the scale properties of the efficiency frontier. The additivity assumption produces a larger production possibility set than the VRS assumption and will accordingly result in larger scale advantages attainable.

4.5.3 Case Three

Context before merger

Farm seven cultivates 2 360ha and utilize DAVC worth R 5 818 039 to generate a crop revenue of R 19 925 195. Assuming VRS, the efficiency of farm seven is calculated at 0.77, a weighted combination of 69% of the efficiency of farm nine and 31% of the efficiency of farm 11. The bootstrap procedure adjusted this estimate to 0.67 by eliminating an upward bias of 10%. The scale efficiency estimate of farm seven is 0.89. The slack based DEA model estimated 182 ha of slack in the production plan of farm seven. This implies that farm seven should according to the revealed production frontier be able to reduce its cultivated hectares by 182ha without affecting its crop related income. Farm seven owns 80% of the land it cultivates. The livestock income it earns equates to 9% of that of its crop enterprise.

Farm eight cultivates 913 ha and utilize DAVC worth R 1 876 801 to generate a crop revenue of R 3 846 495. Again assuming VRS, its efficiency estimate of 0.63 is derived from a weighted combination of 89% of the efficiency estimate of farm four and 11% of the efficiency estimate of farm 11. The bootstrap procedure adjusted this to 0.58, eliminating an upward bias of 5.4%. Its scale efficiency is estimated at 0.66 and thus scale inefficient. The specific scale properties of the efficiency frontier where farm eight is located is determined to be DRS. The slack based model identified a slack in the DAVC of farm eight worth R 144 191. Farm eight owns 69% of the land on which it cultivate crops. Its livestock income is equal to 26% of its crop related income.

These two farming entities differ considerably in their scale of operation and an intuitive comparison of their efficiencies is not as clear as in case one. Again referring to the regression results,

The 17% difference in income diversification should theoretically contribute 5.18% to the 9% efficiency difference between the two farms.

Efficiency results of merger

The slack adjusted and bootstrapped input and output vectors of farms seven and eight is presented in Table 4.11 below for the evaluation of the possible efficiency effects that a merger in their crop enterprises may bring about:

Table 4.11: Case three, merger between farm 7 and 8

Farms	output	ha	Davc
7	19 925 195	2 178	5 818 039
8	3 846 495	913	1 732 610
7+8	23 771 690	3 091	7 550 649

Reintroducing the merged entity into the sample, omitting farms seven and eight, and measuring its efficiency against the efficiency frontier assuming VRS results in the efficiency effects presented in Table 4.12 below:

Table 4.12: Case three merger efficiency results

M3	learning	harmony	size	Eff	E*	IP	Pure gain	PHG	PSG
VRS	0.741	0.880	1.113	0.726	0.979	0.274	0.021	0.12	(0.113)

The 0.726 merger efficiency indicates a merger efficiency improvement potential of 27.4%. This estimate should theoretically be adjusted for the portion of gain that can be attributed to learning beat practices from one another. The pure merger efficiency improvement potential is therefore only 2.1%. This value comprise of a 12% harmony gain and a -11.3% scale effect. This merger shows considerable potential for reallocating production processes between the two farms to those that poses the lowest marginal cost of production. Knowing that the larger scale of the merged farm will cause scale disadvantages, precautionary steps can be taken to reduce these should they decide to merge.

4.6 Conclusion

The merger analysis methodology developed in this thesis have been put to the test by following a case study approach. It proved to be a valuable tool to generate measurable estimates that may inform farmers about the efficiency dynamics of their current operation and how it might be affected by a proposed merger with a neighbor or any other farm business for instance. The model is successful in providing a basis upon which contractual agreements can be tailored to improve the probability of a merger delivering the desired outcome.

The key focus of the research project was to provide an answer to the extent to which a proposed merger or resource pooling agreement is expected to influence the technical efficiency of an individual farm business. This thesis followed the conventional efficiency measurement research methodology to assess the current efficiency level of farms within a study group. It also applied regression analysis in a second stage to determine the effects of external variables on farm efficiency. These estimates then serve as baseline from which three merger cases will be assessed with regards to its expected net effect on efficiency. This effect is decomposed to identify the underlying sources of a possible efficiency gain or loss.

The different scenarios was used to test the method. The farms are all within a geographical area that van be defined as relative homogenous and merging is thus an option. In all three scenarios the method employed showed insightful results regarding the potential impact of merging two farms. The measurement in terms of scale, harmony and efficiency indicated a quantified expected effect. Not only does it provide insight in the potential of merging, but also where the potential gains are most.

In the final Chapter 5 summarize the gist of this thesis is summarized, conclusions are drawn from the findings and identify areas where the shortcomings of this thesis can be addressed. Areas of further research is also suggested.

Chapter 5: Summary, Conclusions and Recommendations

This chapter provides a summary of the thesis and each of its chapters. It also draws a few conclusions about the findings of this thesis. The last section of the chapter is dedicated to suggesting areas for further research that was identified through the course of the research done for this thesis.

5.1 Summary

The purpose of this thesis was to research and develop a framework and build a model that will be able to estimate the benefits and costs of mergers before they actually take place. The theory of efficiency proved to be well able to serve as basis to help answer this research question.

Chapter two presented the theoretical basis of efficiency measurement in the context of production economic theory. The two predominant approaches to measure efficiency were considered after which Data Envelopment Analysis was found to be best suitable for the purposes of this thesis. A thorough literature review presented previous findings on the determinants of farm efficiency. The second section of the literature review probed into previous findings on the efficiency effects of collaborative action. Ex-ante estimation theory is presented as the basis for estimating the potential efficiency effects that farm level mergers may bring about. The additivity assumption is established as sufficient to enable a DEA model to estimate the potential efficiency effects of mergers. Previous studies that assume additivity in DEA efficiency estimation models are summarized.

Chapter three contains the empirical framework and methodology that is used to measure efficiency of individual firms within an industry or study group. Different mathematical model specifications are presented to estimate the different types of efficiencies. Statistical inference of efficiency estimates are discussed to improve the statistical significance of DEA derived efficiency estimates. Regression analysis is shown to be well able to correlate the influence of external variables on efficiency estimates. The merger analysis methodology of (Bogetoft & Wang, 1999) is introduced. Its ability to simulate, distinguish and measure the learning-, harmony- and scale effect that a proposed merger is expected to realize makes it pivotal to this thesis. A numerical example is used to explain the application of the methodology. The last section of the chapter presented the dataset that was used to test the merger analysis methodology. The model input and output variables were discussed together with the assumptions that were made. The case studies upon which the merger analysis methodology is applied is discussed in the last section of this chapter.

Chapter four presented a summary of the efficiency results that were obtained from the sample of 32 farming entities. These were corrected for bias using the iterative bootstrap procedure. The regression results show the statistical significance that tenure and livestock diversification has on the technical efficiency of the farms contained in the dataset. Three independent case studies is used to measure the expected efficiency effect of a merger between two randomly selected farming entities. Each of the partners is thoughroly described with regard to their pre-merger production plan and the efficiency thereof. The level of livestock diversification for each farm is also presented and discussed with reference to the regression results and its contribution toward each farm's efficiency level.

5.2 Conclusion

This thesis followed the conventional efficiency measurement research methodology to assess the current efficiency level of farms within a study group. These efficiency estimates were adjusted for the inherent bias of a DEA model through the bootstrap procedure. It also applied regression analysis in a second stage to determine the effects of external variables on farm efficiency. These estimates serve as baseline from which three merger cases were assessed with regards to their expected net effect on efficiency. This effect is decomposed to identify the underlying sources of a possible efficiency gain or loss.

The first objective of this thesis is addressed in chapter two where the theory of efficiency measurement was identified as the theoretical basis for the ex-ante analysis of a proposed merger. A thorough literature study provides a solid foundation upon which the thesis seeks to meet its second objective. A methodological framework is developed in chapter three that is theoretically and mathematically able to identify, distinguish and measure the effects of a proposed merger. The third objective of statistical relevance is theoretically addressed in chapter two, and mathematically in chapter three. The fourth objective to test the merger analysis methodology on a real life dataset is met in chapter four.

By successfully addressing the objectives set out in the beginning of the thesis, the research question is also successfully addressed in each of the three case studies. The methodology proved adequate even under different assumptions about the scale properties of technology. Even though these case studies cannot directly be compared to one another, they serve as evidence of the ability of the methodology to answer the research question in all three cases.

5.3 Recommendations

The application possibilities of the methodology that this thesis developed is seemingly endless. For each of the assumptions made and approaches followed, multiple options exists. What is important is to know what assumptions and approaches will succeed in answering your research question.

If data existed on the level of production finance that each farmer use, it could be used to determine if there exists a correlation between collateral in terms of owned land and farm efficiency. The inclusion of lime as a farming practice as a variable in the second stage regression analysis may also deliver interesting results. It should however be considered that multi-year data should exist for at least four to five years because of the advantages of lime only realizing in years to come after application.

The research and crafting of this thesis presented endless possibilities with regard to how the methodology could be tested. Different case study designs may be considered in future research. An innovative approach may be to keep one partner to the merger constant in case one and then substituting other partners in subsequent case studies. A sub vector DEA model may also be used to simulate a restriction on the reallocation of resources to the new merged firm. Similarly the sub vector DEA model could be quite capable to determine how specific negotiated resource reallocation terms influence the maximum attainable gains.

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Appendix

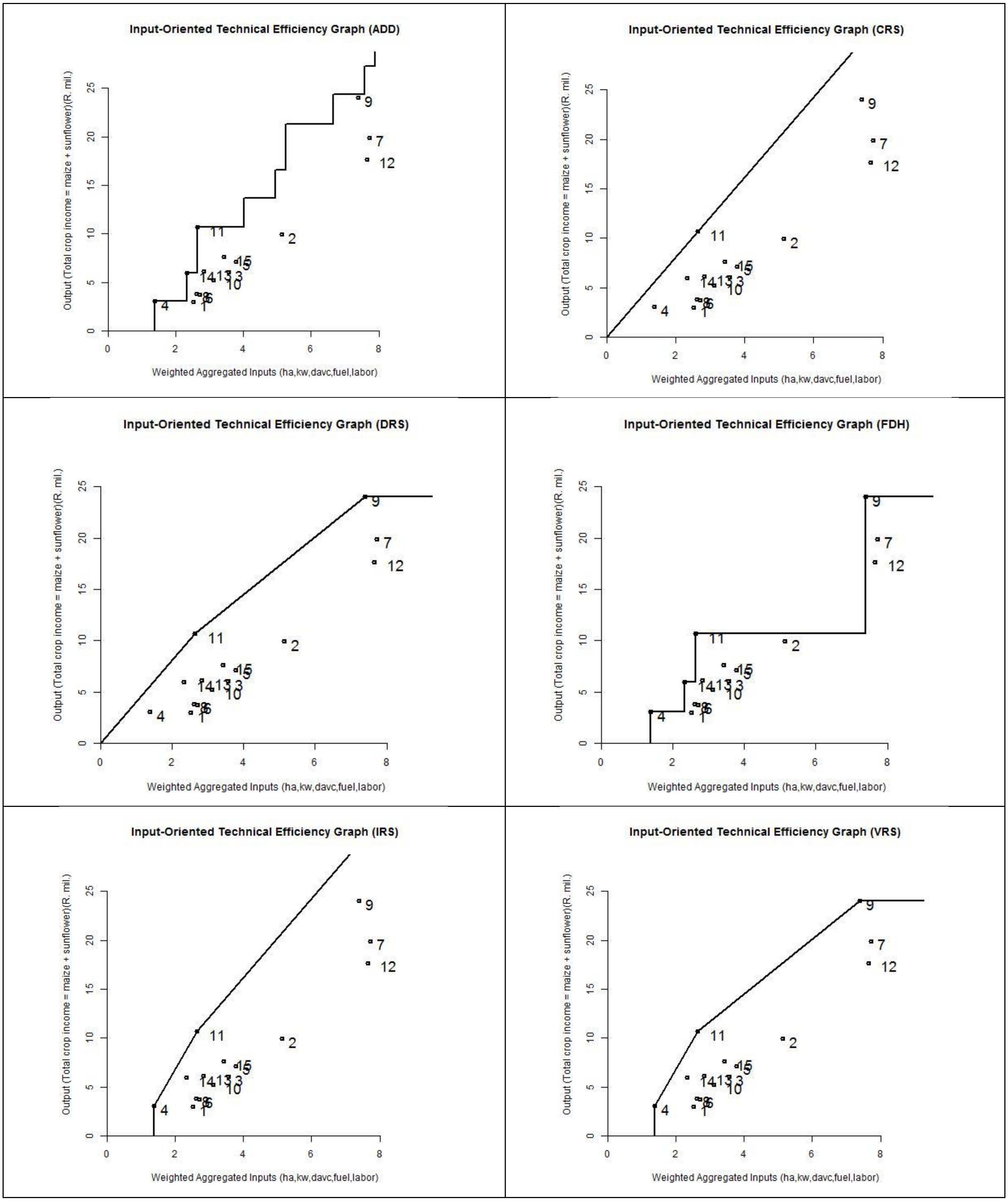
A. Dataset

The complete dataset that was studied in this thesis is listed below. Output is represented by the gross income generated from the dryland crop division of each farm. The two inputs, hectares and directly allocated variable costs represent the input vector. Ownership is included to represent the % of owned land used in the production of crops. Gross income derived from livestock as a percentage of crop income captures the diversification level of each farmer.

Farm	Output (gi_cr)	ha	davc	Ownership	gi_ls_cr
1	2 967 270	760	1 852 365	0.87	0.24
2	9 930 561	1 611	4 033 999	0.51	0.14
3	6 037 135	1 052	2 951 304	0.46	0.06
4	3 042 708	548	908 545	0.95	0.22
5	7 140 445	1 120	2 799 533	1	0.16
6	3 759 526	813	2 290 692	0.83	0.27
7	19 925 195	2 360	5 818 039	0.8	0.09
8	3 846 495	913	1 876 801	0.69	0.26
9	23 999 416	2 010	5 536 358	1	0.21
10	5 209 274	778	2 619 893	0.21	0.03
11	10 669 043	813	2 153 084	0.55	0.09
12	17 696 034	1 790	7 092 388	0.47	0
13	6 126 150	760	2 430 605	0.76	0.2
14	5 934 945	799	1 867 182	0.77	0.12
15	7 596 002	760	2 983 136	0.42	0.03
16	996 198	664	2 014 673	0.63	0.48
17	3 869 368	700	1 734 338	0.92	0.25
18	4 624 560	1 611	3 749 853	0.49	0.66
19	9 314 375	1 014	2 254 186	0.33	0
20	2 522 455	556	1 515 869	1	0.2
21	3 622 250	1 207	3 431 114	1	0.39
22	13 400 358	2 145	5 663 852	0.95	0.27
23	5 562 560	726	2 106 233	0.88	0.12
24	17 145 268	2 475	6 878 401	0.89	0.23
25	3 231 118	913	2 347 834	0.74	0.21
26	24 192 108	2 444	8 740 380	1	0.12
27	2 997 306	879	3 080 929	0.2	0.04
28	7 748 597	1 315	3 913 953	0.55	0.17
29	10 538 832	1 911	5 940 207	0.42	0
30	3 169 287	665	2 166 096	0.66	0.18
31	6 804 210	1 156	3 709 523	0.88	0
32	4 588 061	838	2 820 050	0.42	0.03

B. Different production possibility graphs under the various assumptions about technologies

The following graphs represent the different production possibility sets, each constructed under its respective scale assumption:



C. Complete set of efficiency estimates from the slack based input oriented VRS DEA model

The following table lists the efficiency estimates obtained from the slack based input oriented VRS DEA model. Also listed is the bootstrapped results and the bias corrected efficiency estimates. The last two columns show the slacks present in the production plan of each of the farm business in the dataset:

Farm	eff (VRS)	eff.bc	bias	var	2.5%CI	97.5%CI	Slack (Ha)	Slack (DAVC)
1	0.72	0.67	0.056	0.002	0.599	0.717	-	427 108
2	0.50	0.45	0.051	0.002	0.410	0.499	24	-
3	0.62	0.59	0.026	0.000	0.555	0.617	-	432 072
4	1.00	0.87	0.126	0.011	0.777	0.986	-	-
5	0.62	0.58	0.040	0.001	0.534	0.610	-	148 429
6	0.70	0.67	0.033	0.001	0.620	0.701	-	588 691
7	0.77	0.66	0.112	0.018	0.559	0.763	182	-
8	0.63	0.58	0.047	0.001	0.534	0.625	-	144 191
9	1.00	0.78	0.216	0.116	0.641	0.984	-	-
10	0.80	0.77	0.032	0.001	0.724	0.796	-	836 785
11	1.00	0.88	0.123	0.007	0.797	0.983	-	-
12	0.81	0.72	0.087	0.008	0.620	0.795	-	1 784 863
13	0.86	0.82	0.044	0.001	0.766	0.857	-	683 526
14	0.81	0.75	0.058	0.001	0.703	0.802	-	134 949
15	0.93	0.87	0.061	0.002	0.802	0.921	-	1 120 436
16	0.83	0.76	0.061	0.003	0.690	0.822	-	754 167
17	0.82	0.77	0.056	0.002	0.703	0.819	-	385 461
18	0.37	0.36	0.016	0.000	0.336	0.372	-	236 811
19	0.86	0.78	0.080	0.004	0.705	0.850	103	-
20	0.99	0.87	0.120	0.010	0.770	0.976	-	585 513
21	0.47	0.45	0.021	0.000	0.416	0.469	-	611 914
22	0.50	0.45	0.053	0.002	0.401	0.495	20	-
23	0.88	0.83	0.050	0.001	0.772	0.869	-	524 094
24	0.56	0.50	0.064	0.004	0.429	0.555	-	78 855
25	0.61	0.57	0.035	0.001	0.522	0.605	-	486 759
26	1.00	0.78	0.221	0.170	0.621	0.985	-	-
27	0.62	0.59	0.035	0.001	0.540	0.622	-	1 012 216
28	0.54	0.51	0.032	0.001	0.468	0.537	-	441 265
29	0.42	0.38	0.045	0.001	0.340	0.420	-	381 253
30	0.83	0.78	0.054	0.002	0.704	0.827	-	870 119
31	0.59	0.56	0.028	0.000	0.519	0.584	-	655 533
32	0.72	0.69	0.028	0.000	0.650	0.715	-	864 112

D. R Studio packages used:

Packages are a collection of functions, compiled code and sample data. These are stored in the "library" and needs to be called into the R environment to be used in a specific program. This allows for quicker coding and to ensure the repeatability of the code. The following packages were used to code the model that was developed throughout this thesis:

Benchmarking: Package for performing benchmarking and frontier analysis in R (Bogetoft & Otto, 2018):

Stargazer: Package that allows for well formatted regression and summary statistics tables (Hlavac&Marek,2015).

fBasics: R package for portfolio modelling, optimization and back testing (Rmetrics Association 2014).

dea: Estimates a DEA frontier and calculates efficiency measures a la Farrell (Bogetoft *et al.*, 2011).

dea.plot: Draw a graph of a DEA technology (Bogetoft *et al.*, 2011).

dea.boot: Bootstrap DEA models and returns bootstrap of Farrell efficiencies (Bogetoft *et al.*, 2011).

dea.plot.frontier: Draw a graph of a DEA technology (Bogetoft *et al.*, 2011).

truncreg: Estimation of models for truncated Gaussian variables by maximum likelihood (Croissant, Y. & Zeileis, A., 2018).

make.merge: Make an aggregation matrix to perform mergers of firms (Bogetoft *et al.*, 2011).

dea.merge: Calculate and decompose potential gains from mergers of similar firms (Bogetoft *et al.*, 2011).

E. R Code

The complete R code program that was used to formulate this thesis follows:

```
#=====================================================================================================
# NWK DEA Nominal Crop model 2015/16
# Date: 8 Sept 2019
#=====================================================================================================
# Objective: Input oriented variang returns to scale Slack Based BCC Merger efficiency analysis with two variables regressed.
#=====================================================================================================

# Prepare working space
  # A. Clear memory
  objects()                                   #list all objects in memory
  rm(list=ls())                               #remove all (clean up old stuff)
  print(paste("Following executed on", date()))   #date-time stamp
  citation()

 # B. Load the necessary R packages

  library(Benchmarking)
  library(stargazer)
  library(fBasics)

 # C. Set up and check your working directory
  getwd() # check your working directory
  #setwd("xxx")
  dir()   # check the files in the directory

#=====================================================================================================
# 1. Load data from Microsoft CSV file

  ds = read.csv("2016_data_nom.csv")        # Read data and rename it mydata
  fix(ds)                             # Check the data in R and edit if necessary (bw)
  names(ds)                           # Print the name of variables in data
  head(ds)                            # Print first 6 observations, just to check your data. use (tale) to see last data
```

```

 # Extract output data and rename it
  output = ds$gi_cr                   # ds_o = (original dataset with real values)

 # Extract input data and rename it
  ha     = ds$ha_cr
  davc   = ds$exp_davc

 # Extract other socio-economic data   (for second stage)
  tenure     = ds$ownership
  ls_share   = ds$gi_ls_cr

 # Group data (macros) (so that I do not have to name all each time I write a comand)
  y_var = cbind(output)                 # cbind = combine to use in first stage
  x_var = cbind(ha,davc)                # to use in first stage
  z_var = cbind(tenure,ls_share)        # to use in second stage

#=====================================================================================================
# 2. Explore data and generate summary statistics before analysis

 # Generate summary statistics of DEA data
  ds = cbind(y_var, x_var, z_var)
  options(scipen=999)
  basicStats(ds)                                                                 # summary statistics of production data (using basicStats
  ds_sum = basicStats(ds)[c("Mean", "Stdev", "Median", "Minimum", "Maximum", "nobs"),]  # select type of statistics to reportsubset the data
  t(ds_sum)                                                                      # transpose table
  write.table(t(ds_sum), file = "16_1 data summary stats.csv", sep = ",", col.names = NA)    # write table to .csv file in current directory

```

```
#===================================================================================================
# 3. Efficiency section
#====================================================================================================
# 3.1 Compute input-oriented technical efficiency (ITE) measures under different RTS assumptions
  # ?dea
  ev = dea(x_var,y_var, RTS="vrs", ORIENTATION="in")  # RTS -> varaying returns to scale
  ec = dea(x_var,y_var, RTS="crs", ORIENTATION="in")  # RTS -> constant returns to scale
  ei = dea(x_var,y_var, RTS="irs", ORIENTATION="in")  # RTS -> Non-increasing returns to scale
  ed = dea(x_var,y_var, RTS="drs", ORIENTATION="in")  # RTS -> Non-decreasing returns to scale
  ef = dea(x_var,y_var, RTS="fdh", ORIENTATION="in")  # RTS -> free disposable hull
  ea = dea(x_var,y_var, RTS="add", ORIENTATION="in")  # RTS -> additive

  tev = eff(ev)                                     # convert to numeric argument
  tec = eff(ec)                                     # convert to numeric argument
  tei = eff(ei)                                     # convert to numeric argument
  ted = eff(ed)                                     # convert to numeric argument
  tef = eff(ef)                                     # convert to numeric argument
  tea = eff(ea)                                     # convert to numeric argument

  se =  tec/tev                                       # calculate scale efficiency

  tech_eff = cbind(tev, tec, tei, se, ted, tef, tea)           # bundle the efficieny measures together
  print(tech_eff)
  pairs(tech_eff[,-5])

  basicStats(tech_eff)
  basicStats(tech_eff)[c("Mean", "Stdev", "Median", "Minimum", "Maximum", "nobs"),]  # select type of statistics to reportsubset the data

  isp_tev = (1-tev)*x_var                              # input saving potential under VRS
  #isp_tec = (1-tec)*x_var                             # input saving potential under CRS
  #isp_tei = (1-tei)*x_var                             # input saving potential under IRS
  #isp_ted = (1-ted)*x_var                             # input saving potential under DRS
  #isp_tef = (1-tef)*x_var                             # input saving potential under FDH
  isp_tea = (1-tea)*x_var                             # input saving potential under ADD

```

```

  isp_tev
  isp_tea

  #write.table(tech_eff, file = "16 Input oriented technical efficiency.csv", sep = ",", col.names = NA)                # Write table to
  write.table(basicStats(tech_eff), file = "16_2 Input oriented technical efficiency statistical summary.csv", sep = ",", col.names = NA)    # Wr
  #write.table(isp_tev,file ="16 IOTE_input_saving_potential under VRS.csv", sep = ",", col.names = NA)


  #Plot efficiency density distribution
  plot(density(tev), col="red", lty=1, lwd=2, main="Eff. dens. distr.", bty="l")
  lines(density(tec), col="blue", lty=2, lwd=2)
  lines(density(tei), col="green", lty=3, lwd=2)
  lines(density(ted), col="brown", lty=4, lwd=2)
  lines(density(tef), col="purple", lty=4, lwd=2)
  lines(density(tea), col="pink", lty=4, lwd=2)
  #legend("topright", names=c("ev", "ec","ei","ed"), lty=c(1:4), bty="n")  # Error in this line (unused argument (Names = ...))

   # Export efficiency density distribution plot to jpg
  #jpeg()
  #plot(density(tev), col="red", lty=1, lwd=2, main="Eff. dens. distr.", bty="l")
  #lines(density(tec), col="blue", lty=2, lwd=2)
  #dev.off()

#==========================================================================================
# 3.2 Find the peers and weights for each DMU
 ev_p = peers(ev, NAMES = FALSE)
 ev_w = lambda(ev, KEEPREF = FALSE)
 ev_pw = cbind (ev_p,ev_w)
 ev_pw
 #write.table(ev_pw, file = "16 IO_TE_VRS_Peers_Weights.csv", sep = ",", col.names = NA)

```

```
#ec_p =  peers(ec, NAMES = FALSE)
#ec_w =  lambda(ec, KEEPREF = FALSE)
#ec_pw = cbind(ec_p,ec_w)
#ec_pw
#write.table(ec_pw, file = "IO_TE_CRS_Peers_Weights.csv", sep = ",", col.names = NA)

#ei_p =  peers(ei, NAMES = FALSE)
#ei_w =  lambda(ei, KEEPREF = FALSE)
#ei_pw = cbind(ei_p,ei_w)
#ei_pw
#write.table(ei_pw, file = "IO_TE_IRS_Peers_Weights.csv", sep = ",", col.names = NA)

#ed_p =  peers(ed, NAMES = FALSE)
#ed_w  = lambda(ed, KEEPREF = FALSE)
#ed_pw = cbind(ed_p,ed_w)
#ed_pw
#write.table(ed_pw, file = "IO_TE_DRS_Peers_Weights.csv", sep = ",", col.names = NA)

#ef_p =  peers(ef, NAMES = FALSE)
#ef_w =  lambda(ef, KEEPREF = FALSE)
#ef_pw = cbind(ef_p,ef_w)
#ef_pw
#write.table(ef_pw, file = "IO_TE_FDH_Peers_Weights.csv", sep = ",", col.names = NA)

#ea_p =  peers(ea, NAMES = FALSE)
#ea_w =  lambda(ea, KEEPREF = FALSE)
#ea_pw = cbind(ea_p,ea_w)
#ea_pw
#write.table(ea_pw, file = "IO_TE_ADD_Peers_Weights.csv", sep = ",", col.names = NA)

```

```

#==============================================================================================
# 3.3 Determine nature of production technology for the DMUs (Returns to Scale of each DMU)
#not possable for slack based model

  test_MPSS  = (tev == tec & tev == tei)
  test_IRS  = (tei == tec & tec != tev)
  test_DRS  = (tev == tei & tei != tec)

  RTS = cbind(test_MPSS,test_IRS,test_DRS)
  summary(RTS)
  t(summary(RTS))

  # save results in a csv excell file. Saved to operating directory)
  #write.table(RTS, file = "16 RTS test.csv", sep = ",", col.names = NA)

#==============================================================================================
# DEA results
  DEA_Results = cbind(y_var,x_var,tech_eff,isp_tev,ev_pw,RTS)
  write.table(DEA_Results,file = "16_3 DEA_Results.csv",sep =",",col.names = NA)
#==============================================================================================
 # 3.4 Plots

   dea.plot(x_var/1000000,y_var/1000000,RTS="vrs",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (VRS)",xlab="Wei
   #dea.plot(x_var/1000000,y_var/1000000,RTS="irs",ORIENTATION="in-out",txt=TRUE,fex=0.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (IRS)",xlab="We
   #dea.plot(x_var/1000000,y_var/1000000,RTS="crs",ORIENTATION="in-out",txt=TRUE,fex=0.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (CRS)",xlab="We
   #dea.plot(x_var/1000000,y_var/1000000,RTS="drs",ORIENTATION="in-out",txt=TRUE,fex=0.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (DRS)",xlab="We
   #dea.plot(x_var/1000000,y_var/1000000,RTS="fdh",ORIENTATION="in-out",txt=TRUE,fex=0.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (FDH)",xlab="We
   dea.plot(x_var/1000000,y_var/1000000,RTS="add",ORIENTATION="in-out",txt=TRUE,fex=0.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (ADD)",xlab="Wei

   # export efficiency plot to jpg
  jpeg("16_eff_VRS.jpg")
  dea.plot(x_var/1000000,y_var/1000000,RTS="vrs",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (VRS)",xlab="Weigh
  dev.off()

```

```r

  jpeg("16_eff_IRS.jpg")
  dea.plot(x_var/1000000,y_var/1000000,RTS="irs",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (IRS)",xlab="Weig
  dev.off()

  jpeg("16_eff_CRS.jpg")
  dea.plot(x_var/1000000,y_var/1000000,RTS="crs",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (CRS)",xlab="Weig
  dev.off()

  jpeg("16_eff_DRS.jpg")
  dea.plot(x_var/1000000,y_var/1000000,RTS="drs",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (DRS)",xlab="Weig
  dev.off()

  jpeg("16_eff_FDH.jpg")
  dea.plot(x_var/1000000,y_var/1000000,RTS="fdh",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (FDH)",xlab="Weig
  dev.off()

  jpeg("16_eff_ADD.jpg")
  dea.plot(x_var/1000000,y_var/1000000,RTS="add",ORIENTATION="in-out",txt=TRUE,fex=1.5,lwd=2,main="Input-Oriented Technical Efficiency Graph (ADD)",xlab="Weig
  dev.off()
#==================================================================================================================================================================
# 4. Estimate Slacks

# Calculate input-oriented slackbased technical efficiency (IO_SB_TE)
  sb_ev = dea(x_var,y_var, RTS="vrs", ORIENTATION="in",SLACK=TRUE)  #  calculate eff under RTS -> varaying returns to scale
  #sb_ec = dea(x_var,y_var, RTS="crs", ORIENTATION="in",SLACK=TRUE) #  calculate eff under RTS -> constant returns to scale
  #sb_ei = dea(x_var,y_var, RTS="irs", ORIENTATION="in",SLACK=TRUE) #  calculate eff under RTS -> Non-increasing returns to scale
  #sb_ed = dea(x_var,y_var, RTS="drs", ORIENTATION="in",SLACK=TRUE) #  calculate eff under RTS -> Non-decreasing returns to scale
  #sb_ef = dea(x_var,y_var, RTS="fdh", ORIENTATION="in",SLACK=TRUE) #  calculate eff under RTS -> free disposable hull
  sb_ea = dea(x_var,y_var, RTS="add", ORIENTATION="in",SLACK=TRUE)  #  calculate eff under RTS -> free disposable hull

  sb_ev
  sb_ea
```

```r

  slacks = data.frame(sb_ev$slack,sb_ev$sum,"sx"=sb_ev$sx,"sy"=sb_ev$sy)
  slacks                    # Note: if FALSE = Koopmans efficient (no slacks)

  input_slacks = cbind(slacks$sx.sx1,slacks$sx.sx2,slacks$sx.sx3,slacks$sx.sx4,slacks$sx.sx5)
  output_slacks = cbind(slacks$sy1)

  x_var_sa = x_var - input_slacks
  y_var_sa = y_var + output_slacks


  Input_efficiency =(x_var - input_slacks)/x_var
  Input_mix_efficiency = cbind(rowMeans(Input_efficiency))

  Output_efficiency =(y_var + output_slacks)/y_var
  Output_mix_efficiency = cbind(rowMeans( Output_efficiency))

  phi =  Input_mix_efficiency*Output_mix_efficiency

# 5. Bootstrap slack adjusted eff measures to compute bias corrected slack based efficieny measures

  nrep=2000
  btev_sb <- dea.boot(x_var_sa,y_var_sa, NREP=nrep, ORIENTATION="in",RTS="vrs") #execute procedure

  btev_sb_res = cbind(btev_sb$eff, btev_sb$eff.bc, btev_sb$bias, btev_sb$var, btev_sb$conf.int)
  colnames(btev_sb_res) =c("eff","eff.bc","bias","var","2.5%CI","97.5%CI")
  print(btev_sb_res)
  names(data.frame(btev_sb_res))
  summary(btev_sb_res)


  SB_DEA_Results = cbind(x_var,y_var,btev_sb_res,input_slacks,output_slacks,x_var_sa,y_var_sa)

  write.table(SB_DEA_Results, file = "16_4 Bootstrapped Slack Based DEA Results.csv", sep =",",col.names = NA)
  write.table(btev_sb$boot, file = "16_5 Bootstrap_itterations.csv", sep = ",", col.names = NA)
```

```
#=============================================================================================================
# 6. Second stage regression analysis (needs refinement & add Tobit)

  # Prepare data by combining technical efficiency and z variable data
 library(truncreg)
 library(censReg)

 eff_bc = btev_sb$eff.bc

 Xreg = cbind(eff_bc)
 Yreg = z_var
 reg_data = data.frame(Xreg, Yreg)
 names(reg_data)

#  truncreg1 = truncreg(Yreg ~ Xreg, point=0, direction = "left",data=reg_data)
 ?truncreg

 # Determinants of technical efficiency                    # Here we use trunacted regression
 library(truncreg)
 trunc_reg= truncreg(eff_bc ~ tenure + ls_share, data = reg_data,  point=0 ,direction = "left",scaled = TRUE)
 summary(trunc_reg)

 write.table(summary(trunc_reg), file = "16_5 reg_results.csv", sep = ",", col.names = NA)

  # Determinants of bias-corrected technical efficiency   # Here we use OLS
 ols_reg <- lm(eff_bc ~ tenure + I(tenure^2) + tenure + ls_share, data = reg_data)
 summary(ols_reg)
```

```
#=============================================================================================================
# <<<<Merger section>>>>
#=============================================================================================================
# plot efficiency frontier

  #dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7, xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gross Income (MZ+Sun) [x10^6]",ac
  #dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="drs",add=T,lty="dashed",txt=T, xlab="Agg Inputs", ylab="Output", col="blue")  # plot DMU's defining the l
  #dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="crs",add=T,lty="dotted",txt=T, xlab="Agg Inputs", ylab="Output", col="red")  # plot DMU's defining the bo
  #dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="add",add=T,lty="dashed",txt=T, xlab="Agg Inputs", ylab="Output", col="green")  # plot DMU's defining the

  #dea(x_var,y_var,RTS="vrs")

#=======================================================================================================================
# <<<<Construct multiple independant Merger cases>>>>
  dea.merge?

  m1i1 = 11
  m1i2 = 14
  m2i1 =  1
  m2i2 =  9
  m3i1 =  7
  m3i2 =  8
  m4i1 =  7
  m4i2 = 12
  m5i1 =  5
  m5i2 = 15

  m1 = c(m1i1,m1i2)       # merger 1: One defining the frontier, the other inefficient (Similar input use)
  m2 = c(m2i1,m2i2)         # merger 2: One defining the frontier, the other inefficient (one significantly larger than the other)
  m3 = c(m3i1,m3i2)         # merger 3: Both inefficient (one significantly larger than the other)
  m4 = c(m4i1,m4i2)       # merger 4: Both inefficient (Similar Input use)(Large firms)
  m5 = c(m5i1,m5i2)       # merger 5: Both inefficient (Similar Input use)(Smaller firms)
```

```

  cfarm_index = rbind(m1,m2,m3,m4,m5)       # summarize mergers and constituent firms
  colnames(cfarm_index) = c("f1","f2")      # specify column names
  cfarm_index                               # print merger index

  m1f1 = cbind(y_var,x_var)[m1i1,]
  m1f2 = cbind(y_var,x_var)[m1i2,]
  m2f1 = cbind(y_var,x_var)[m2i1,]
  m2f2 = cbind(y_var,x_var)[m2i2,]
  m3f1 = cbind(y_var,x_var)[m3i1,]
  m3f2 = cbind(y_var,x_var)[m3i2,]
  m4f1 = cbind(y_var,x_var)[m4i1,]
  m4f2 = cbind(y_var,x_var)[m4i2,]
  m5f1 = cbind(y_var,x_var)[m5i1,]
  m5f2 = cbind(y_var,x_var)[m5i2,]

  cfarm_vectors = rbind (m1f1,m1f2,m2f1,m2f2,m3f1,m3f2,m4f1,m4f2,m5f1,m5f2)   # generate unadjusted pre-merger matrix
  cfarm_vectors

  M = make.merge(list(m1,m2,m3,m4,m5), nFirm=32, X=cbind(x_var,y_var))                 #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
  t(M)                                                                                 # show mergers

  # merge vectors
  xmer = M %*% x_var                            # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
  ymer = M %*% y_var                            # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
  m_vectors = cbind(ymer,xmer)                  # construct matrix with merged firms and its input and output vectors
  rownames(m_vectors) = c("m1","m2","m3","m4","m5")
  m_vectors
  m_sim = cbind(cfarm_index,m_vectors)                      # add farm numbers

  plot.new()
  dea.plot.frontier(x_var,y_var,RTS="vrs",txt=T,fex=0.7,xlim=c(0,17000000),ylim= c(0,40000000),main="M1-M5 independant merger cases (VRS)", xlab="Aggregated
  points(rowSums(xmer),ymer,pch=16)                           # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
  text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)      # name points
```

```r
jpeg('Merger simmulation (ADD).jpg')
dea.plot.frontier(x_var,y_var,RTS="add",txt=T,fex=0.8,xlim=c(0,17000000),ylim= c(0,50000000),main="Merger simmulation (ADD)", xlab="Aggregated Inputs [x10^6
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)     # name points
dev.off()

jpeg('Merger simmulation (FDH).jpg')
dea.plot.frontier(x_var,y_var,RTS="fdh",txt=T,fex=0.8,xlim=c(0,17000000),ylim= c(0,40000000),main="Merger simmulation (FDH)", xlab="Aggregated Inputs [x10^6
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)     # name points
dev.off()

jpeg('Merger simmulation (CRS).jpg')
dea.plot.frontier(x_var,y_var,RTS="crs",txt=T,fex=0.8,xlim=c(0,17000000),ylim= c(0,40000000),main="Merger simmulation (CRS)", xlab="Aggregated Inputs [x10^6
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)     # name points
dev.off()

jpeg('Merger simmulation (VRS).jpg')
dea.plot.frontier(x_var,y_var,RTS="vrs",txt=T,fex=0.8,xlim=c(0,17000000),ylim= c(0,40000000),main="Merger simmulation (VRS)", xlab="Aggregated Inputs [x10^6
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)     # name points
dev.off()

jpeg('Merger simmulation (IRS).jpg')
dea.plot.frontier(x_var,y_var,RTS="irs",txt=T,fex=0.8,xlim=c(0,17000000),ylim= c(0,40000000),main="Merger simmulation (IRS)", xlab="Aggregated Inputs [x10^6
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)     # name points
dev.off()

jpeg('Merger simmulation (DRS).jpg')
dea.plot.frontier(x_var,y_var,RTS="drs",txt=T,fex=0.8,xlim=c(0,17000000),ylim= c(0,40000000),main="Merger simmulation (DRS)", xlab="Aggregated Inputs [x10^6
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1","m2","m3","m4","m5"),pos=4)     # name points
dev.off()
```

```r

write.table(m_sim, file = "16_8 mergers & pooled production plans.csv", sep = ",", col.names = NA)
write.table(cfarm_vectors, file = "16_9 Observed individual production plans of candidate farms.csv", sep = ",", col.names = NA)

# calculate merger gains and its decomposition under specified RTS
mgain_vrs = dea.merge(x_var,y_var,M, RTS="vrs")
mgain_crs = dea.merge(x_var,y_var,M, RTS="crs")
mgain_add = dea.merge(x_var,y_var,M, RTS="add")
mgain_drs = dea.merge(x_var,y_var,M, RTS="drs")
mgain_irs = dea.merge(x_var,y_var,M, RTS="irs")
mgain_fdh = dea.merge(x_var,y_var,M, RTS="fdh")

mgain_sum = data.frame(m_sim,mgain_vrs,mgain_crs,mgain_add,mgain_drs,mgain_irs,mgain_fdh)
            # (Eff      -> overall merger efficiency)(1-eff = potential gain)
            # (Estar    -> eff score after adjusting for learning)(1-estar = potential gain after adjusting for learning)
            # (learning -> Pure merger eff = eff score with learning eliminated)(1-elearning = individual learning potential)
            # (harmony  -> learning under VRS - learning under CRS )
            # (Size -> under CRS = 1)
?dea.merge

write.table(mgain_sum, file = "16_10 merger gain summary.csv", sep = ",", col.names = NA)

# Calculate slack adjusted merger gains and its decomposition nder specified RTS
sb_mgain_vrs = dea.merge(x_var_sa,y_var_sa,M, RTS="vrs")
sb_mgain_crs = dea.merge(x_var_sa,y_var_sa,M, RTS="crs")
sb_mgain_add = dea.merge(x_var_sa,y_var_sa,M, RTS="add")
sb_mgain_drs = dea.merge(x_var_sa,y_var_sa,M, RTS="drs")
sb_mgain_irs = dea.merge(x_var_sa,y_var_sa,M, RTS="irs")
sb_mgain_fdh = dea.merge(x_var_sa,y_var_sa,M, RTS="fdh")

sb_mgain_sum = data.frame(m_sim,sb_mgain_vrs,sb_mgain_crs,sb_mgain_add,sb_mgain_drs,sb_mgain_irs,sb_mgain_fdh)
# (Eff      -> overall merger efficiency)(1-eff = potential gain)
# (Estar    -> eff score after adjusting for learning)(1-estar = potential gain after adjusting for learning)
# (learning -> Pure merger eff = eff score with learning eliminated)(1-elearning = individual learning potential)
# (harmony  -> learning under VRS - learning under CRS )
# (Size -> under CRS = 1)

write.table(sb_mgain_sum, file = "16_11 slack adjusted merger gain summary.csv", sep = ",", col.names = NA)

```

```
# Calculate bootstrapped slack adjusted merger gains and its decomposition nder specified RTS
#bs_sb_mgain_vrs = dea.merge(x_var_sa,y_var_sa,M, RTS="vrs")
#bs_sb_mgain_crs = dea.merge(x_var_sa,y_var_sa,M, RTS="crs")
#bs_sb_mgain_add = dea.merge(x_var_sa,y_var_sa,M, RTS="add")

#bs_sb_mgain_sum = data.frame(m_sim,sb_mgain_vrs,sb_mgain_crs,sb_mgain_add)
# (Eff       -> overall merger efficiency)(1-eff = potential gain)
# (Estar     -> eff score after adjusting for learning)(1-estar = potential gain after adjusting for learning)
# (learning -> Pure merger eff = eff score with learning eliminated)(1-elearning = individual learning potential)
# (harmony  -> learning under VRS - learning under CRS )
# (size -> under CRS = 1)

#write.table(bs_sb_mgain_sum, file = "16_12 slack adjusted merger gain summary.csv", sep = ",", col.names = NA)


#=====================================================================================================================
# <<<<Illustrate individual Merger cases>>>>

# Merger 1 unadjusted
m1 = c(m1i1,m1i2)                       # merger 1 and its constituent firms (nr. 29 & 36)
cfarm_index = rbind(m1)                     # summarize mergers and constituent firms
colnames(cfarm_index) = c("f1","f2")        # specify column names
cfarm_index                                 # print merger index
m1_f1 = cbind(y_var,x_var)[m1i1,]
m1_f2 = cbind(y_var,x_var)[m1i2,]
cfarm_vectors = rbind (m1_f1,m1_f2)         # generate unadjusted pre-merger matrix
cfarm_vectors
M = make.merge(list(m1), nFirm=32, X=cbind(x_var,y_var))  #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
t(M)                                                        # show mergers

xmer  = M %*% x_var/1000000                 # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
ymer  = M %*% y_var/1000000                 # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
m_vectors = cbind(ymer,xmer)          # construct matrix with merged firms and its input and output vectors
rownames(m_vectors) = c("m1")         # define row name for merged firm
m_vectors                             # show combined imput and output vector of merged firm
cbind(cfarm_index,m_vectors)          # add constituent farm numbers to vector

plot.new()
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7,main="M1 - One partner constant" , xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gr
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1"),pos=4)                 # name points
```

```
jpeg('M1 VRS.jpg')
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7,main="M1 VRS", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gross Income (MZ+Sun)
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1"),pos=4)                 # name points
dev.off()

#merger 2 slack adjusted
m1 = c(m1i1,m1i2)                       # merger 1 and its constituent firms (nr. 29 & 36)
cfarm_index = rbind(m1)                     # summarize mergers and constituent firms
colnames(cfarm_index) = c("f1","f2")        # specify column names
cfarm_index                                 # print merger index
m1_f1 = cbind(y_var_sa,x_var_sa)[m1i1,]
m1_f2 = cbind(y_var_sa,x_var_sa)[m1i2,]
cfarm_vectors = rbind (m1_f1,m1_f2)         # generate unadjusted pre-merger matrix
cfarm_vectors
M = make.merge(list(m1), nFirm=15, X=cbind(x_var,y_var))  #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
t(M)                                                        # show mergers

xmer  = M %*% x_var/1000000                 # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
ymer  = M %*% y_var/1000000                 # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
m_vectors = cbind(ymer,xmer)          # construct matrix with merged firms and its input and output vectors
rownames(m_vectors) = c("m1")         # define row name for merged firm
m_vectors                             # show combined imput and output vector of merged firm
cbind(cfarm_index,m_vectors)          # add constituent farm numbers to vector

plot.new()
dea.plot.frontier(x_var_sa/1000000,y_var_sa/1000000,RTS="vrs",txt=T,fex=0.7,main="M1 - One partner constant" , xlab="Aggregated Inputs [x10^6]", ylab="Output(C
points(rowSums(xmer),ymer,pch=16)                               # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m1"),pos=4)                 # name points

# Merger 2
m2 = c(dmu_c,dmu_p2)                            # merger 2 and its constituent firms (nr. 27 & 33)
cfarm_index = rbind(m2)                     # summarize mergers and constituent firms
colnames(cfarm_index) = c("f1","f2")        # specify column names
cfarm_index                                 # print merger index
m2_f1 = cbind(y_var,x_var)[dmu_c,]
m2_f2 = cbind(y_var,x_var)[dmu_p2,]
cfarm_vectors = rbind (m2_f1,m2_f2)         # generate unadjusted pre-merger matrix
cfarm_vectors
M = make.merge(list(m2), nFirm=15, X=cbind(x_var,y_var))  #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
t(M)                                                        # show mergers
```

```
xmer = M %*% x_var/1000000                    # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
ymer = M %*% y_var/1000000                    # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
m_vectors = cbind(ymer,xmer)          # construct matrix with merged firms and its input and output vectors
rownames(m_vectors) = c("m2")         # define row name for merged firm
m_vectors                             # show combined imput and output vector of merged firm
cbind(cfarm_index,m_vectors)          # add constituent farm numbers to vector

plot.new()
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="VRS",txt=T,fex=0.7,main="M2 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Grc
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m2"),pos=4)         # name points

jpeg('M2_one partner constant.jpg')
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7,main="M2 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Grc
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m2"),pos=4)         # name points
dev.off()

# Merger 3
m3 = c(dmu_c,dmu_p3)                        # merger 2 and its constituent firms (nr. 27 & 33)
cfarm_index = rbind(m3)                     # summarize mergers and constituent firms
colnames(cfarm_index) = c("f1","f2")        # specify column names
cfarm_index                                 # print merger index
m3_f1 = cbind(y_var,x_var)[dmu_c,]
m3_f2 = cbind(y_var,x_var)[dmu_p3,]
cfarm_vectors = rbind (m3_f1,m3_f2)         # generate unadjusted pre-merger matrix
cfarm_vectors
M = make.merge(list(m3), nFirm=15, X=cbind(x_var,y_var))  #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
t(M)                                                      # show mergers

xmer = M %*% x_var/1000000                  # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
ymer = M %*% y_var/1000000                  # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
m_vectors = cbind(ymer,xmer)          # construct matrix with merged firms and its input and output vectors
rownames(m_vectors) = c("m3")         # define row name for merged firm
m_vectors                             # show combined imput and output vector of merged firm
cbind(cfarm_index,m_vectors)          # add constituent farm numbers to vector

plot.new()
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="VRS",txt=T,fex=0.7,main="M3 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Grc
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m3"),pos=4)         # name points

jpeg('M3_one partner constant.jpg')
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7,main="M3 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gro
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m3"),pos=4)         # name points
dev.off()

# Merger 4
m4 = c(dmu_c,dmu_p4)                        # merger 2 and its constituent firms (nr. 27 & 33)
cfarm_index = rbind(m4)                     # summarize mergers and constituent firms
colnames(cfarm_index) = c("f1","f2")        # specify column names
cfarm_index                                 # print merger index
m4_f1 = cbind(y_var,x_var)[dmu_c,]
m4_f2 = cbind(y_var,x_var)[dmu_p4,]
cfarm_vectors = rbind (m4_f1,m4_f2)         # generate unadjusted pre-merger matrix
cfarm_vectors
M = make.merge(list(m4), nFirm=15, X=cbind(x_var,y_var))  #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
t(M)                                                      # show mergers

xmer = M %*% x_var/1000000                  # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
ymer = M %*% y_var/1000000                  # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
m_vectors = cbind(ymer,xmer)          # construct matrix with merged firms and its input and output vectors
rownames(m_vectors) = c("m4")         # define row name for merged firm
m_vectors                             # show combined imput and output vector of merged firm
cbind(cfarm_index,m_vectors)          # add constituent farm numbers to vector

plot.new()
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="VRS",txt=T,fex=0.7,main="M4 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gro
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m4"),pos=4)         # name points

jpeg('M4_one partner constant.jpg')
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7,main="M4 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gro
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m4"),pos=4)         # name points
dev.off()

# Merger 5
m5 = c(dmu_c,dmu_p5)                        # merger 2 and its constituent firms (nr. 27 & 33)
cfarm_index = rbind(m5)                     # summarize mergers and constituent firms
colnames(cfarm_index) = c("f1","f2")        # specify column names
cfarm_index                                 # print merger index
m5_f1 = cbind(y_var,x_var)[dmu_c,]
m5_f2 = cbind(y_var,x_var)[dmu_p5,]
cfarm_vectors = rbind (m5_f1,m5_f2)         # generate unadjusted pre-merger matrix
cfarm_vectors
M = make.merge(list(m5), nFirm=15, X=cbind(x_var,y_var))  #(1&2) & (3&4) of X=x firms to form (m1)&(m2) respectively
t(M)                                                      # show mergers

xmer = M %*% x_var/1000000                  # merge x vectors of m1 & m2 (x_m1 = x_firm1 + x_firm2)(x_m2 = x_firm3 + x_firm4)
ymer = M %*% y_var/1000000                  # merge y vectors of m1 & m2 (y_m1 = y_firm1 + y_firm2)(y_m2 = y_firm3 + y_firm4)
m_vectors = cbind(ymer,xmer)          # construct matrix with merged firms and its input and output vectors
rownames(m_vectors) = c("m5")         # define row name for merged firm
m_vectors                             # show combined imput and output vector of merged firm
cbind(cfarm_index,m_vectors)          # add constituent farm numbers to vector

plot.new()
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="VRS",txt=T,fex=0.7,main="M5 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gro
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m5"),pos=4)         # name points

jpeg('M5_one partner constant.jpg')
dea.plot.frontier(x_var/1000000,y_var/1000000,RTS="vrs",txt=T,fex=0.7,main="M5 - One partner constant", xlab="Aggregated Inputs [x10^6]", ylab="Output(Crop Gro
points(rowSums(xmer),ymer,pch=16)                         # plot merged firms on original dea plot (xmer vector summed for plotting purposes)
text(rowSums(xmer),ymer,labels=c("m5"),pos=4)         # name points
dev.off()
```