A DESIGN BASED RESEARCH ON STUDENTS' UNDERSTANDING OF QUADRATIC INEQUALITIES IN A GRAPHING CALCULATOR ENHANCED ENVIRONMENT

by

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DECLARATION

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ABSTRACT

The purpose of this design based research (DBR) study was to investigate the grade 11 students' understanding of quadratic inequalities in a graphing calculator (GC) enhanced mathematics classroom. The study was framed within the pragmatic paradigm which is committed to multiple world-realities. This pragmatic paradigm embraces mixed methods to collect both quantitative and qualitative data on students' understanding of quadratic inequalities and to generate evidence that would guide educational practice. This study consisted of three main research cycles of the teaching experiments i.e., three high schools in Gauteng province and were conducted in phases. A hypothetical learning trajectory (HLT) was developed in the first phase and used for monitoring the hypotheses, assessing the starting point of students' understanding and formulating the end goals. The instructional activities were created using the heuristics from the guided reinvention, didactical phenomenology and emergent models. The feed-forwards from the first two research cycles helped to improve the HLT leading to a coherent local instructional theory for quadratic inequalities in a GC environment.

The findings of the three research cycles were that the use of an integrated approach (graphic and algebraic) proved to be an effective learning strategy for solving quadratic inequalities in a GC mediated classroom. Students were able to visualise and interpret the graphs and their properties (e.g., zeros, intervals, axis of symmetry, concavity and domain) displayed on the screens of the GCs. Students used instrumented action schemes of graphing and tabulating values to develop and reify the concept of quadratic inequalities. Students also led to meaningfully written solution sets of quadratic inequalities using correct interval notations. The results of the pre- and post-tests showed that there was a significant difference in the mean scores, suggesting an improved performance.

The effectiveness of the GC use on students' performance was practically justified by the Cohen's d effect sizes, which were large in all the three cycles. Secondly the use of real-life mathematical situations involving linear inequalities as the starting points supported the students' conceptual understanding of quadratic inequalities. The students' understanding of real-life mathematical situations moved from the referential level to the general level. The use of the GC also enhanced the students'

reasoning and problem solving skills in quadratic inequalities. These skills enabled students to represent real world problems mathematically (horizontal mathematization), solve the problem using the initiated strategies, interpret the model solutions and look back at the adequacy of their solutions. However, a cognitive obstacle for many learners was to help them to develop metacognitive or executive control skill of self-monitoring during problem solving in all three cycles. The use of the GC also afforded the students an opportunity to move from the informal reasoning (horizontal mathematising) to formal reasoning (vertical mathematising). The findings support previous studies in the domain that the use of the GC improves students' understanding in learning mathematics.

The findings of the three cycles permitted to produce evidence-based heuristics such as design principles that might inform the future decisions for learning quadratic inequalities in a flexible GC environment. The main design principle of this study was: *Graphically representing quadratic inequalities in a flexible graphing calculator environment.* To this end, the focus was to help students become flexible in dealing with quadratic inequalities in the form of symbols, graphs, or contextual problems. Other essential design principles that emerged in these three cycles were a) the training students to use the GC fluently to reduce chances of the limited viewing window for becoming a source for students' misconceptions and b) using the GC cannot address all learning styles, and must be complemented by other traditional methods.

It is hoped that the findings of this study will contribute to the research literature on how to effectively teach the topic of quadratic inequalities. Similarly, professional development programmes and workshops for teachers can be conducted at cluster or district level starting with piecemeal group. Furthermore, the findings might be recommended to the textbook or curriculum developers for designing more explorative learning activities with graphing calculators. The results of the three DBR cycles might be added to the likelihood of transferability to other algebraic concepts.

OPSOMMING

Die doel van hierdie ontwerpgebaseerde navorsing (DBR) was om die graad 11-studente se begrip van kwadratiese ongelykhede in 'n grafiese sakrekenaar-verbeterde wiskundeklaskamer te ondersoek. Die studie is geraam binne die pragmatiese paradigma wat verbind is tot veelvuldige wêreldrealiteite. Hierdie pragmatiese paradigma bevat gemengde metodes om sowel kwantitatiewe as kwalitatiewe gegewens te versamel oor studente se begrip van kwadratiese ongelykhede en om bewyse te genereer wat die onderwyspraktyk kan lei. Hierdie studie het bestaan uit drie hoofnavorsingsiklusse van die onderrigeksperimente, dit wil sê drie hoërskole in die provinsie Gauteng en is in fases uitgevoer. In die eerste fase is 'n hipotetiese leerbaan (HLT) ontwikkel en gebruik vir die monitering van die hipoteses, die beoordeling van die beginpunt van studente se begrip en die formulering van die einddoelwitte. Die onderrigaktiwiteite is geskep deur gebruik te maak van die heuristiek uit die geleide heruitvinding, didaktiese fenomenologie en ontluikende modelle. Die aanvoerders vanaf die eerste twee navorsingsiklusse het gehelp om die HLT te verbeter, wat gelei het tot 'n samehangende plaaslike onderrigteorie vir kwadratiese ongelykhede in 'n GC-omgewing.Die bevindinge van die drie navorsingsiklusse was dat die gebruik van 'n geïntegreerde benadering (grafies en algebraïes) 'n effektiewe leerstrategie was om kwadratiese ongelykhede in 'n GC-bemiddelende klaskamer op te los. Studente kon die grafieke en hul eienskappe (byvoorbeeld nulle, intervalle, simmetrie-as, konkawiteit en domein) wat op die skerms van die GC's verskyn, visualiseer en interpreteer. Studente het instrumentale aksieskemas gebruik om grafieke en tabelleerwaardes te gebruik om die konsep van kwadratiese ongelykhede te ontwikkel en te vernuwe. Studente het ook gelei tot sinvol geskrewe oplossings vir kwadratiese ongelykhede met korrekte intervalnotasies.

Die resultate van die voor- en na-toetse het getoon dat daar 'n beduidende verskil in die gemiddelde tellings was, wat dui op 'n verbeterde prestasie. Die doeltreffendheid van die GC-gebruik op studente se prestasie is markties geregverdig deur die Cohen se d-effekgroottes, wat in al die drie siklusse groot was. Tweedens het die gebruik van wiskundige situasies uit die werklike lewe wat lineêre ongelykhede betrek as vertrekpunte die studente se konseptuele begrip van kwadratiese ongelykhede ondersteun. Die studente se begrip van wiskundige situasies in die werklike lewe het van die referensiële vlak na die algemene vlak beweeg. Die gebruik van die GC het ook die studente se redenasie- en probleemoplossingsvaardighede in kwadratiese ongelykhede verbeter. Hierdie vaardighede het studente in staat gestel om regte wêreldprobleme wiskundig voor te stel (horisontale wiskunde), die probleem op te los met behulp van die geïnisieerde strategieë, die modeloplossings te interpreteer en terug te kyk na die toereikendheid van hul oplossings. 'N Kognitiewe struikelblok vir baie leerders was egter om hulle te help om metakognitiewe of uitvoerende beheersvaardighede van selfmonitering tydens probleemoplossing in al drie die

siklusse te ontwikkel. Die gebruik van die GC het ook aan die studente die geleentheid gebied om van die informele redenering (horisontale wiskunde) na formele redenering (vertikale wiskunde) oor te gaan. Die bevindings ondersteun vorige studies op die gebied dat die gebruik van die GC studente se begrip in die leer van wiskunde verbeter. Die bevindings van die drie siklusse is toegelaat om bewysgebaseerde heuristieke te produseer, soos ontwerpbeginsels wat die toekomstige besluite oor kwadratiese ongelykhede in 'n buigsame GC-omgewing kan inlig.

Die belangrikste ontwerpbeginsel van hierdie studie was: grafiese voorstelling van kwadratiese ongelykhede in 'n buigsame grafiese sakrekenaaromgewing. Met die oog daarop was die fokus om studente te help om buigsaam te raak in die hantering van kwadratiese ongelykhede in die vorm van simbole, grafieke of kontekstuele probleme. Ander noodsaaklike ontwerpbeginsels wat in hierdie drie siklusse na vore gekom het, was: a) die opleiding van studente om die GC vlot te gebruik om die kanse te verminder dat die beperkte kykvenster 'n bron word vir studente se wanopvattings en b) die gebruik van die GC kan nie alle leerstyle aanspreek nie, en moet aangevul word met ander tradisionele metodes.Die bevindings van hierdie studie kan gebruik word om kennis uit te brei en 'n bydrae te lewer tot die navorsingsliteratuur oor hoe om die onderwerp van kwadratiese ongelykhede effektief te onderrig. Op soortgelyke wyse kan professionele ontwikkelingsprogramme en werkswinkels vir onderwysers op groeps- of distriksvlak aangebied word vanaf 'n groepsverband. Verder kan die bevindings aanbeveel word aan die handboek of kurrikulumontwikkelaars om meer ontdekkende leeraktiwiteite met grafiese sakrekenaars te ontwerp. Die resultate van die drie DBR-siklusse kan moontlik bygevoeg word tot die waarskynlikheid van oordraagbaarheid na ander algebraïese konsepte.

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ACRONYMS

CAD	Computer Added Design
CAPS	Curriculum Assessment Policy Statement
CAS	Computer Algebra System
DBE	Department of Basic Education (from 2009)
DBR	Design Based Research
DoE	Department of Education of South Africa (before 2009)
FET	Further Education and Training
GC	Graphing Calculator
GET	General Education and Training
HLT	Hypothetical Learning Trajectory
ICT	Information and Communication Technology
IEA	International Association for the Evaluation of Educational Achievement
ISTE	International Society for Technology in Education
NCS	National Curriculum Statement
NCTM	National Council of Teacher Mathematics
NSC	National Senior Certificate
OBE	Outcome Based Education
PCK	Pedagogical Content Knowledge
PK	Pedagogical Knowledge
PME	Psychology of Mathematics Education
QIPST	Quadratic Inequality Problem Solving Test
RME	Realistic Mathematics Education
SPSS	Statistical Package for the Social Sciences
TCK	Technological Content Knowledge
TELE	Technology-Enhanced Learning Environment
TIG	Theory of Instrumental Genesis
TIMSS	Trends in International Mathematics and Science Study
TPACK	Technological, Pedagogical and Content Knowledge
ZPD	Zone of Proximal Development

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CHAPTER 1: INTRODUCTION AND ORIENTATION OF THE STUDY

1.1 Introduction

This chapter explains the background of the problem to lend relevance to the study. A brief description of technology use in mathematics education in the South African context was dealt with. In this study, the notion of technology was understood as the use of graphing calculators (GCs). The statement of the problem, the purpose of the study, the research questions and significance of the study provided the reason for conducting this research. Design-based research (DBR) is introduced as the methodology suitable for technology-enhanced learning environments [TELEs]. The limitations and delimitations of the study are discussed. Definitions of key terms and terminology used in this study are made. The chapter then concludes by giving an overview of the thesis.

1.2 Background of the study

The nature and quality of learning mathematics consistently seems to be a concern in secondary schools in South Africa. There is growing disappointment that education does not achieve the national goals as reflected in the National Senior Certificate (NSC) Mathematics results. Grade 12 results which are the benchmark of the country's level of performance in mathematics do not match up with the government's effort towards improving quality education. As a nation we underperform in mathematics; this is more pronounced in disadvantaged secondary schools. Bennet and Carre (1993) stated that this is not only a South African problem but a worldwide concern. They further express the view that it is essential for students to receive quality education resulting from their teachers' comprehension of a broad curriculum and deeper knowledge of some specialised aspects of it. A question arises as to whether students in South Africa receive quality learning supported by the productive use of technology in mathematics classrooms.

1.2.1. Government policies on education system

Government policies have an impact on the education system of any given country. In the context of the South African history, an apartheid regime had educational policies which disadvantaged the education of the majority. The Bantu Education Act (1953) legitimised the downgrading of the quality and level of education for black people so that their academic certificates became irrelevant to the labour market (Hlatshwayo, 2000). In addition, the apartheid education policies were characterised by teacher-centred teaching, rote learning, and an obsession with content and punitive formal examinations designed to achieve high levels of failure (Edwards, 2016). Educational resources were unevenly distributed in schools and it has proved to be difficult to redress the situation by just donating resources to those schools. Since the advent of democracy in 1994, South Africa has been investing much in education to close the gap created by the apartheid government by the provision of quality education to disadvantaged communities. It should be noted that the apartheid regime left a legacy of unequal distribution of resources in schools populated by the black majority students (Mooketsi, 2016; Hlatshwayo, 2000). This has also affected the distribution of ICT resources in schools, hence impacting on the provision of quality education.

Major socio-economic reform initiatives have taken place to replace the apartheid policies with policies that would promote democratic principles and be relevant for a multicultural society (Sayed & Kanjee, 2013; Edwards, 2016). The democratic government of South Africa gazetted the White Paper on e-Education policy (DoE, 2004), intended to transform and reconstruct the education system (Mooketsi, 2016). The policy states, in some of its key clauses that South African teachers and learners ought to: 1) use available information and communication technologies to actively participate and contribute to the knowledge society, and 2) become efficient in communication and collaboration skills with or without use of ICTs (DoE, 2004). This suggests that this policy on e- Education is meant to facilitate and provide proper guidance on the integration and advancement of digital technology in teaching and learning. The e- Education policy further states that teachers and learners have to acquire and master ICT skills in order to be able to interact meaningfully with ICT (DoE, 2004). Such skills may help both teachers and learners to actively interact with graphing calculators in mathematics classrooms as the available technology for information and communication.

In pursuit of the e-education policy, there have been reforms of curriculum structure to suit the implementation of the ICT education policy, which requires the integration of technology in mathematics classrooms, in particular. This has seen major curriculum reform initiatives taking place to replace the apartheid curriculum such as Outcome Based Education (OBE), National Curriculum Statement (NCS) and Curriculum Assessment Policy and Statement (CAPS). The new curriculum aimed to promote democratic principles and be relevant for a multicultural society (Sayed & Kanjee, 2013; Edwards, 2016). The main focus of the reformed curriculum structure was to improve and incorporate technology in the teaching and learning process in secondary school classrooms and administrative practices. This has been on how to add value to the provision of education through the pronouncement of pedagogically integrated technology in the learning.

1.2.2. South African students' international performance in mathematics

South Africa is one of the low performing countries in mathematics compared to other participating countries. The Trends in International Mathematics and Science Study (TIMSS) is an assessment of the mathematics and science knowledge of fourth and eighth grade students around the world. TIMSS was developed by the International Association for the Evaluation of Educational Achievement (IEA) to provide participating nations opportunity to benchmark the students' educational achievement across borders in mathematics and science (Mc Tighe & Seif, 2003). In the case of this current study, the researcher is interested in the performance of the secondary school learners. The earlier South African data showed that a high number of Grade 8 learners did not attempt to answer many of the mathematics items, which made estimating achievement scores extremely difficult (Reddy, Visser, Winnaar, Arends, Juan and Prinsloo, 2016). To provide better estimates, in 2003 South Africa assessed Grade 8 and 9 learners, and in TIMSS 2011 and 2015 only Grade 9 learners were assessed.

A sample of 300 schools for The TIMSS 2015 was drawn from 10 009 schools in South Africa, that offered Grade 9 classes. A total of 12 514 learners, 334 mathematics teachers participated in the study conducted by the Human Sciences Research Council. Thirty-six countries participated at the Grade 8 level and three countries at the Grade 9 level (Norway, Botswana and South Africa). Of the 39 participating countries, South Africa was ranked one of the five lowest performing countries with average scale score of 372, which included Botswana (391), Jordan (386), Morocco (384) and Saudi Arabia (368) (Reddy, et al., 2016). This means the

South African learners achieved a mathematics score below the international benchmark of 400 points, a score denoting the minimum level of competence. The TIMSS curriculum and assessment frameworks are organised around the mathematics content domains of number, algebra, geometry, data and chance. These results are indicative that students had misconceptions in these content domains at Grade 9, which can be transferred to the next grades if not properly resolved. Other researchers had different perspectives towards the poor learner achievement of mathematics in South Africa. Reasons cited were that learners struggle to understand mathematics and are very good at recalling facts or answering questions involving procedural knowledge in TIMSS, 2011(Reddy, et al., 2013) but lowly ranked in problem solving and higher- level cognitive abilities (Spaull, 2013). Another possible reason for learners' poor understanding is ineffective and poor teaching (Stols, 2013) which does not develop learners' ability to solve problems, critical thinking, transfer and application of knowledge in new settings (Mc Tighe & Seif, 2003). This study presumed that in the context of resolving the students' poor understanding of mathematics, the graphing calculator may serve to mediate the teaching and learning processes.

1.2.3. Grade 12 students' national performance in mathematics

The Grade 12 students' performance in public mathematics examinations in South Africa was mediocre from 2014 to 2017. The results are summarised in Table 1.1, below. As shown in the table, only 51.1% of students (127 197) who wrote the NSC Mathematics examination achieved 30% and above, and 35.1% of these students (86 096) achieved 40% and more in 2017 (DBE, 2017). This means 64, 9% achieved below 40% in mathematics, thus a huge percentage of students who did not meet the university requirements. For those students who wrote mathematics examinations in 2015, only 49.1% achieved 30% and more, and 31.9% of them achieved 40% and above. This means students performed badly as 50.9% of those who wrote achieved less than 30% in 2015.

Table 1.1: NSC Mathematics results: 2014-2017

	No. Wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2014	225 458	120 523	53,5	79 050	35,1
2015	263 903	129 481	49,1	84 297	31,9
2016	265 912	136 011	51,1	89 119	33,5
2017	245 103	127 197	51,9	86 096	35,1

These improved results may have been attributed to the increase in the number of candidates who answered the knowledge and routine questions correctly (DBE, 2017). This means students had difficulties with those questions that required non-routine and problem solving skills across all topics in the curriculum. The report from the department of education however indicated that students' algebraic skills are poor (DBE, 2017). It further revealed that most candidates lacked fundamental and basic algebraic competencies, which could have been acquired in the lower grades. In particular, many students were able to factorise the expression but could not solve the inequality (DBE, 2017). Students treated the inequality as an equation and this led to them writing answers that did not make sense. Additionally, candidates also showed little or no understanding of the set builder or interval notation (DBE, 2014; 2015; 2016; 2017). The use of graphing calculator may foster the development of such skills as it combines the algebraic and graphical representations.

The report recommends that when teaching quadratic inequalities, teachers should integrate algebra with functions so that learners have a visual understanding of inequalities (DBE, 2017). It emphasises the need of stressing the meaning of the inequality signs in the teaching of both algebra and functions. It further suggests the use of different methods to solve quadratic inequality problems so that learners can choose the method they understand best. The DBE (2017) realised that students lacked proper understanding of the words "and" and "or" in the context of inequalities as they used them interchangeably. In that context teachers were encouraged to explain the difference between "and" and "or" as they are very different in meaning. The report recommends the use of the graphical representation of the different scenarios to explain the meaning of roots of an equation and the meaning of solution of the inequality (DBE, 2017). The use of the graphing calculator and hence this study was motivated by the suggestions emanating from this DBE report on the solution of quadratic inequalities.

1.2.4. The use of the graphing calculators in Mathematics Education

The use of GCs in mathematics has grown rapidly among students and teachers of developing and developed economies. Different types of GCs, more sophisticated ones have been produced by different companies which include Texas Instruments, Casio and Sharp to mention a few. With the increasingly rapid development of technology, GCs have begun to assume more and more computer-type capabilities (Muhundan, 2005) in mathematics classrooms of many countries. Initially computers and/or Computer Algebra Systems were largely used as instructional tools that can be used to make concepts more accessible, and easier to learn and understand. Because of their low cost, portability, and capability, GCs have been widely accepted as the appropriate technology-tools to be integrated into the teaching and learning of mathematics, in particular (Graham, 2005; Spinato, 2011). In particular, the speed, accuracy, and capabilities of current graphing calculators have led many teachers to believe that more emphasis should be placed on their use in mathematics classrooms (Muhundan, 2005; Spinato, 2011). These affordances of the GCs have assisted in introducing new ways for teaching and learning mathematics through graphical and symbolic representations. These representations may enhance students' understanding of quadratic inequalities at the eleventh grade, which is the focus of this study.

The initial reactions to GC technology in mathematics education were generally positive (Dunham & Dick, 1994; Muhundan, 2005). Students who used GCs experienced a rich mathematics curriculum that allowed them to focus on realistic applications. Muhundan (2005) further stated that the full use of GC could deepen students' understanding of mathematics concepts. The large screen display, graphics capability, exploratory functions of graphing and multiline display calculators have afforded students better opportunities to explore concepts and problem situations of mathematics. With this regard, the use of GC enabled students with a supportive learning environment that may promote growth their mathematical knowledge.

GCs have become more popular among students and teachers for several reasons. They perceive GCs as mini-computers with standard processors, display screens, and built-in software which offer interactive graphics, and on-screen programming and other built-in features, such as zoom-in, zoom-out, trace, and table. Many of these capabilities were previously available only on a mainframe or a microcomputer. These powerful capabilities, together with the decreasing cost and size, have made the use of GCs to be the best alternative technology for use in mathematics classrooms (Muhundan, 2005; Averbeck, 2000). It has been noted that the use of GCs promotes exploration and generalization of mathematical concepts. Today, the use of GCs in school mathematics has increased in many parts of the world, including in Australia, Canada, and many countries in Europe and allowed in many standard mathematics exams (Muhundan, 2005). However, there is limited information about the use of GC in South African schools. In this context, students are deprived of significant range of opportunities benefitted by using this potentially powerful teaching and learning tool.

Several questions and concerns are increasingly raised about the proliferation of GC in mathematics education, despite the recommendations made by renowned experts and researchers in relation to the use of GC. Foley in Muhundan (2005), on the one hand, raises some important questions relating to the presence and affordability of this technology about 1) how can this calculator affect mathematics education; 2) how can this calculator influence what is taught and how it is taught, and 3) how can this calculator improve students' understanding of mathematics?. These are crucial questions that this study sought to answer in the South African context. On the other hand, Harvey in Graham (2005) raises similar concerns to be addressed through the use of GC: 1) we need to analyse carefully the content that we presently teach and that we would like to teach, 2) we need to determine the ways that GCs can help us teach that content, and 3) we must not cling to our present ways of teaching. Muhundan (2005) argues that the question to the mathematics community is not whether a GC is allowed in mathematics classrooms but how it is and should be used in students' learning. In her meta-analysis study, Ellington (2003) provided the answers to the questions raised about the benefits of the use of GC to the students' understanding of mathematics. She summarised that the greatest student gains were found when calculators assumed a pedagogical role in the classroom, beyond being available for checking work. She found that the GC use is correlated with improvements in students' conceptual and problem solving skills, operational skills and positive attitudes towards mathematics. This means Ellington's findings support the use of GC in improving students' understanding of quadratic inequalities.

In addition, Dunham (1999) suggested that curriculum development, assessment, the method of instruction, and required instructional materials for instructors need to be addressed in the use of the GCs in mathematics education. In this case, the mathematics education community has a responsibility to react positively to the available technology and carefully conduct the research studies that should respond to the raised questions and concerns. The AMATYC (1995) recommends adapting to this reality and helping students to use GC appropriately so that they can be competitive in the workforce and adequately prepared for future study.

A significant number of studies have shown the potential benefits associated with appropriate use of the graphing calculators on students' understanding of mathematics. They have specifically indicated that graphing calculators may improve student understanding of various algebraic concepts (Drijvers & Doorman, 1996; Ellington, 2003; Penglase & Arnold, 1996) and student problem solving and reasoning and also enable them to demonstrate greater ability to connect multiple representations of algebraic concepts (Ellington, 2006; Spinato, 2011). This seems to suggest that the students' understanding of quadratic inequalities as part of algebra can be enhanced in a graphing calculator environment. The question is whether these benefits also apply to the teaching and learning of quadratic inequalities in typically under-resourced township schools in the South African context.

Using graphing calculators efficiently provides an opportunity for teachers to create a supportive environment to help their students enhance their mathematical knowledge and understanding (Lee & McDougall, 2010). It is further indicated that the pedagogical affordances of the graphing calculator are closely related to improving learning of mathematics (Choi-Koh, 2003; Leng, 2011; Roschelle & Singleton, 2008). However, one of the important general principles included in the South African CAPS document for Mathematics states that: "No calculators with programmable functions, graphical facilities or symbolic facilities (for example, to factorise or to find roots of equations) should be allowed. Calculators should only be used to perform standard numerical computations and to verify calculations by hand (DoE, 2012, p.8)." It is against this background that the GC is used in this study as the available technology (DBE, 2015) to provide supportive environment to enhance students' understanding of quadratic inequalities. Graphing calculators are used the same way computers are integrated in classrooms. Similarly computers are also not allowed in the assessment but can be used to make concepts more accessible, and easier to learn and understand. For this reason, a careful thought has be given in designing instructional activities of quadratic inequalities for the students to be mediated by the graphing calculator in order to develop students' reasoning and problem solving skills.

This study is in line with the recommendations of National Mathematics Advisory Panel (2008) that research be conducted to determine the effects of the GC use on students' problem solving, conceptual understanding, and computation skills. It also seeks to fill the perceived gap that there is little literature on the GC use at high schools in the South Africa. The idea is to provide students with the length of time with graphing calculators in order to master some of the functions at their own time. This could help to realize the DBE's vision that every learner will be able to enjoy doing mathematics in South Africa.

1.3 Problem statement

A problem is something that challenges the mind and makes a person bewildered (Merriam, 1998). This study examines a problem that is bewildering majority of the schools in South Africa which is a concern in the community of mathematics educators, i.e. learning for understanding quadratic inequalities in Grade 11. In more than two decades of teaching mathematics, I have observed that Grade 12 students often give inconsistent solutions to quadratic inequality problems in the algebra general section of the National Senior Certificate (NSC) Mathematics Examinations. This means students often have many misconceptions, conceive an erroneous inequality representation, which makes them difficult to understand this topic in the classroom. The topic of quadratic inequalities is introduced immediately after quadratic equations in Grade 11 according to the Curriculum and Assessment Policy Statement (CAPS) for Mathematics. According to Bagni (2005), this could influence the misunderstanding of quadratic inequalities because students then easily confuse an inequality with an equation. For example, in their study of the 27 Grade 11 learners' errors and misconceptions on solving quadratic inequalities conducted in Gauteng Province, Makonye and Shingirayi (2014) revealed that "In doing so the inequality signs vanish and are then replaced by equal signs. In the end learners come up with roots to an equation instead of the solution to an inequality" (p. 717). I have also observed that students use commutative multiplication in solving inequalities, and/or fail to change the direction of the inequality sign when multiplying by a negative number as well as misinterpret the interval that is bounded in inequality problems.

The Department of Basic Education (DBE)'s Diagnostic Reports of the National Senior Certificate (NSC) examination have similarly indicated that grade 12 students have little or no understanding of a quadratic inequality and many of them treat an inequality as an equation (DBE, 2014; 2015; 2016). This has led them to write answers that do not make sense. The 2016 report, for example, states that "[t]he inequality signs < and > mean very little to the candidates and they could not use them to describe domain, range and certain restricted values on graphs" (DBE, 2016, p. 154**).** Also**,** most candidates could obtain the critical values but were unable to provide the meaningful solution for the quadratic inequality (DBE, 2014). In addition, Makonye and Shingirayi (2014) found that reading the solution from the diagram or the number line tended to be a common problem among students

The international literature confirms that both students and teachers are frustrated with the difficulties encountered when dealing with inequalities in the mathematics classroom (Tsamir & Bazzini, 2002) and there are two primary reasons highlighted by Blanco & Garrote (2007) as difficulties. These include lack of arithmetic skills or knowledge, and the absence of semantic and symbolic meanings of inequalities. It is further stated that students' difficulties in solving quadratic inequalities even persist at university level if not adequately resolved in high schools. The inclusion of inequalities in the algebra curriculum has been criticised, when it has been openly recognized that inequalities belong to the study of many aspects of mathematics (Burn, 2005; Tall, 2004; Boero & Bazzini, 2004). This placement of inequalities invites the learning of inequalities through memorized, routine procedures. According to Halmaghi (2011), students may fail to make important connections and to solve inequalities that look different from the model they have commonly encountered. This suggests that the use of GC may help to develop activities that can benefit students from the connection between equation and inequalities.

The research problem explored in this study is an educational problem that is directly related to the gaps of knowledge observed in the mathematics classroom. If students fail to have a better understanding of quadratic inequalities, they are likely to meet challenges in other related concepts. Many past studies have used GC to examine students' understanding in quadratic functions (Hollar & Norwood, 1999), but little is said about its use in quadratic inequalities. It is therefore important to determine how the students' understanding of quadratic inequalities can be supported in graphing calculator-enhanced environments at the 11th grade in South Africa.

1.4 Purpose and objectives of the study

This study was conducted in pursuit of the most consistent recommendations from the mathematics researchers who encouraged more algebra teachers to take full advantage of the potentially powerful teaching aid (i.e., graphing calculators) (AMATYC, 1995) to investigate the role of the GC in developing students' reasoning and problem solving abilities (Ellington, 2003; Spinato, 2005). This study therefore explored Grade 11 students' understanding of quadratic inequalities in a graphing calculator enriched environment, with specific reference to how their reasoning and problem solving skills were developed in the South African context.

The following research objectives guided the study:

1. To explore how the pedagogical use of GCs impacted on students' performance in solving quadratic inequalities
2. To explore how students perceived the pedagogical use of the GC towards improving their quadratic inequality problem solving abilities
3. To explore how students perceived the pedagogical use of the GC to be supportive of their mathematical reasoning when solving quadratic inequalities

1.5 Research questions

The following overarching research question guided the study: To what extent does the graphing calculator environment provide students with the opportunity to develop an understanding of quadratic inequality and to engage in mathematical reasoning and problem solving?

The following sub-questions intended to address the overarching research question:

1. To what extent can the pedagogical use of graphing calculator influence high school students' performance in solving quadratic inequalities?
2. In what ways (how) can the pedagogical use of the graphing calculator support the high school students' problem solving ability in relation to quadratic inequalities?

3. In what ways (how) can the pedagogical use of the graphing calculator enhance students' mathematical reasoning ability when solving quadratic inequalities?
4. What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?

1.6 Null hypotheses

The first research question as the basis for this study leads to the following null hypothesis for the quantitative aspects of the data analysis:

H_0 There is no difference between pre-test (before) and post-test (after using graphing calculators) in achievement scores of grade 11 students on quadratic inequalities after each DBR cycle.

H_1 There is a difference between pre-test (before) and post-test (after using graphing calculators) achievement scores of grade 11 students on quadratic inequalities after each DBR cycle.

1.7 Significance of the study

The present study incorporated the teaching of quadratic inequality with graphing calculator and examined its use on student understanding in South African schools. In that respect, its findings aimed to:

- ✓ Provide information for the research community and for FET mathematics teachers on how to successfully use graphing calculators to bring about conceptual understanding of quadratic inequalities by students.
- ✓ Assist in setting education reform policies and develop technological strategies that can be used to improve the teaching and learning of the quadratic inequalities.
- ✓ Contribute to the body of knowledge in Mathematics Education by adding another dimension to the existing empirical evidence about GCs on students' understanding of quadratic inequalities.
- ✓ Support educational planners and policy makers in choosing the appropriate methods of managing changes associated with ICT use in the educational system in South Africa.

- ✓ Provide a robust conceptual framework for analysing the students' understanding in a more visible manner, not only during the reflection phase but also during the planning and implementation. This framework intended to assist in breaking down the complex processes of teaching and learning, particularly quadratic inequalities, making it more understandable and also to afford researchers greater insight into the intricacies of the practices.

1.8 Methodology of the study

This study employed design-based research (DBR) as a research methodology for graphing calculator-enhanced learning environments. As Wang and Hannafin (2005) noted the DBR is an ideal alternative research methodology suitable to both research and design of technology-enhanced learning environments (TELEs). This means that as a new educational research paradigm, design-based research has great potential to change the disconnect between educational research and design practice. In this context, the research focused on how the instructional use of graphing calculator in the mathematics classroom effectively intervened in the teaching and learning process. Literature has indicated that DBR affects teaching practices and/or educational policies as it makes learning research more relevant for classroom practices (Reimann 2011). This, for example, may concern the alignment with curriculum, standards and assessment requirements. In Wang and Hannafin's (2005) opinion, researchers in DBR processes collaborate intimately with participants to achieve theoretical and pragmatic goals and these goals can ultimately change educational practices in a maximum extent. In this regard, the implementation of this research approach is aligned with CAPS of Further Education Training (FET) mathematics when teaching and learning quadratic inequalities in South African schools.

Design-based research has been conceptualized as a research methodology in educational contexts (Anderson & Shattuck, 2012) which allows the researchers to bridge the gap between educational theory and practices (Brown, 1992; Collins, 1992). This implies that researchers of DBR studies usually team up with practitioners to work together over an extended period of time so as to provide a solution(s) to a practical problem that faces a specific educational context. Literature indicates that DBR moves beyond simply observing to involve systematically engineering learning contexts (Barab and Squire, 2004) using systematic design and instructional strategies and tools (The Design-Based Research Collaborative, 2003) which allow researchers to improve and generate evidence-based claims about learning (Van den Akker et al., 2006). Within this context, researchers can improve educational practices (i.e., the pedagogy of quadratic inequalities) conducted in the real, complex learning/teaching environments -in the graphing calculator-enhanced classroom. In this study DBR approach used a sequential mixed methods design, which embraces both the quantitative and qualitative data. More specifically, the pre-test and post-test design constituted the quantitative data while the individual and focus group interviews together with the students'scripts constituted mostly the qualitative data.

DBR studies use the term intervention to denote the object, activity, or process that is designed as a possible solution to address the identified problem. McKenney and Reeves (2012) describe intervention as a broad term used "to encompass the different kinds of solutions that are designed" (p. 14); these solutions include educational products, processes, programs, and policies. This current study identified the graphing calculator artefact as the intervention with a potential solution to the perceived problem of the topic of quadratic inequalities. These studies normally span many years with multiple research cycles that focus on the iterative stages of the intervention analysis, design, development, implementation, and evaluation phases. In order to clearly explain the students' inner processes of thinking and understanding of quadratic inequalities, a model of integrated GC in DBR phases of figure 4.3 in Chapter 4 was implemented. The three main phases are (i) analysis and exploration, (ii) design and construction, and (iii) evaluation and reflection (Reeves, 2006; Shattuck & Anderson, 2013), that lead to the outputs of increased theoretical understanding and effective intervention. Thus, information about the intervention is disseminated and diffused to a wider audience. This means in the reflective phase, the design principles are reflected, shared and published to inform future development and implementation decisions.

The adoption of DBR is consistent with the contemporary approaches to research in mathematics education that employ mixed methods design research to address instructional problems related to teaching and learning mathematics (Bakker 2004; Gravemeijer & Bakker, 2006; Gravemeijer 1994). According to Gravemeijer & Bakker, (2006) such design research projects are iterative and theory based attempts aimed to simultaneously understand and improve educational processes. This suggests that the product of this DBR is usually a theory-driven and empirically-based instruction.

In addition, in the preparatory and design phase, the instructional goals are defined, the hypothetical learning trajectory (HLT) is delineated and the theoretical context of the design outlined. The purpose of the HLT is to frame a possible path which the student can take to master the reasoning and understanding required to comprehend the mathematical concepts involved. In developing the HLT, the researcher has to anticipate and refine the course map along which students' mathematical reasoning evolves in the context of the learning activities (Bakker, 2004). Through a series of design experiments the HLT is tested and refined during each iterative cycle of the teaching experimental phase. The notion of these experiments is to improve the learning process under scrutiny and the means by which it is supported (e.g. graphing calculator-enhanced classroom). Finally, a reflective analysis is carried out to establish if the intended research goal has been achieved.

The conjecture of this design based study was that the use of a graphing calculator strategy within the social constructivist and technological learning environment could promote the development of students' understanding in the domain of quadratic inequality concept. The use of GC as a mediating tool was expected to define and shape inner processes of students' thinking and understanding of quadratic inequalities; hence empowering students to make connections between the algebraic and geometric representation.

1.9 Delimitations and limitations of the study

The study intended to limit its scope to grade 11 students who were doing mathematics from three disadvantaged high schools in Gauteng Province of South Africa. This implies that the results are not generalizable to all high schools in the province. The study mainly focused on the learning of students in the graphing calculator-supported environment in which the graphing calculator was used as an artefact to influence a better understanding of quadratic inequalities. The study did not attempt to produce a fine-grained analysis of students' understanding of quadratic inequalities but rather to assess the extent to which graphing calculator intervened to enhance their mathematical understanding. The study used the results of one school to develop and improve the student activities to be done by the following school.

Some of the limitations of the study that must be considered when interpreting the results and that can provide direction for the future research were:

- This study did not have control groups and hence the researcher cannot make bold conclusions about cause-effect relationships. In fact, the researcher used the Cohen's d effect size to interpret the effect of the GC on the students' academic achievement. In that regard, a combination of the two analyses could have been interpreted differently. Additionally, this study avoided the difficulties addressed by Hiebert (1999) and Slavit (1994), of comparing classes with different objectives when graphing calculators are introduced.
- The financial constraints affected the selection of number of provinces as this could not allow more than one province to be selected. The province of Gauteng was purposefully and conveniently sampled. This discourages the generalisation of the results as these results were from one of the nine provinces of South Africa. Subsequent research should strive to expand the data set to include at least three provinces.
- Another limitation concerns the timeframe which was arguably still short relative to students' prior experiences with the use of the GC. The use of GC is not commonplace in South African schools, so it is not entirely possible that some students may have had previous experience using the GC. Survey data about students' prior use of the GC indicated that the majority had never used such tools (Sections 5.6.5; 6.6.5; 7.6.5). Although this study was effectively conducted for 9 weeks in each research cycle, it was unreasonable to assume that students had adequate experience of using the graphing calculators.
- Another constraint is the nature and design of the experiment itself (i.e., DBR). In an attempt to make the comparison of the GC use and traditional method as straightforward as possible, the subsequent research should consider to include a control group.

1.10 Definitions of key terms

The following key terms are defined and/or explained in order to have a common understanding of the use of these terms in this study.

Technology integration is the appropriate and consistent use of any technological tool that produces changes in educational practices and student learning.

Graphing calculator is a handheld technological tool currently used in mathematics classroom capable of producing graphs of functions, finding roots of functions, solving systems of equations and inequalities, as well as visualising symbolic and geometric representations.

Conceptual understanding is regarded as the student's ability to instrumentally use specific mathematical procedures and symbols, and to relationally develop the mathematical connections between the existing and constructed knowledge in the context of solving (or interpreting solutions of) quadratic inequalities.

Reasoning is a process whereby a student analytically formulates the conjectures, draws logical evidence from a set of assumptions and justifies their results.

Problem solving is a process whereby a student mathematically interprets the situations, selects the appropriate strategy, implements correctly the strategy and reflects on the solutions.

Technology enhanced classroom is one that houses collaborative technology or equipment which provides learning opportunities for students to build meaningful connections between concepts and procedures.

A quadratic inequality is a mathematical statement that relates a quadratic expression as either less than or greater than another, for an example $\pm ax^2 + bx + c \leq 0$ or$\pm ax^2 + bx + c \geq 0$. Solutions to that quadratic inequality are two real numbers: p and q, which produce true statements when substituted for the variable, x. The real values, p and q in the domain of the function are called critical numbers or roots (p and q), and can be obtained by factorizing or sketching the graph of the inequality (Ali & Wilmot, 2016). The following will be the expected solution set of each type of the quadratic inequalities in terms p and q. In the given case, let p and q be the roots of the quadratic inequality,$\pm ax^2 + bx + c \neq 0$, where p<q, then:

1. $ax^2 + bx + c \leq 0$ ➡$\{p \leq x \leq q\}$, because the continuous operator is used.

2. $ax^2 + bx + c \geq 0$ ➡$\{x \leq p \cup x \geq q\}$, because the discontinuous operator is used.

3. $-ax^2 + bx + c \geq 0$➡ $\{x \leq p \cup x \geq q\}$, because the discontinuous operator is used.

4. $-ax^2 + bx + c \leq 0$ ➡$\{p \leq x \leq q\}$, because the continuous operator is used (Ali & Wilmot, 2016)

1.11 Organization of the dissertation

This study is organized into eight chapters, with references and appendices. The appendices consist of examples of the semi-structured interview protocols, letters and communication with others involved in the research study.

Chapter 1 discussed the background and presented the problem to be investigated. Also, the chapter provided the purpose and objectives of the study, and the research questions to be answered.

Chapter 2 reviews relevant literature on the history of inequalities, quadratic inequalities, graphing calculators and, reasoning and problem solving as key components of conceptual understanding that guided the teaching and learning of quadratic inequalities.

Chapter 3 discusses the theoretical frameworks which supported and guided the teaching and learning for mathematical understanding. It additionally established the kind of teaching and learning practices that promoted the development of students' mathematical reasoning and problem solving skills and also presented some key challenges to enact these practices in a technologically enhanced mathematics classroom.

Chapter 4 discusses the design based research as a methodology that was used to extract the results of this study. This chapter included discussions on the population, instruments and sampling techniques, issues of reliability and validity, and also ethical considerations.

Chapters 5, 6 and 7 analyses the results obtained from students' interactions with graphing calculators in solving quadratic inequality problems. These are the chapters that answered the research questions posed in this study.

Finally, chapter 8 completes the study by presenting the discussions, conclusions and recommendations.

1.12 Chapter summary

This chapter started by providing the background to the study and stating the problem to be investigated. The purpose, research questions and significance of this study and definition of terms were included in this chapter. Additionally, some important decisions were made related to the use of graphing calculator as an effective means for improving the Grade 11 students' conceptual understanding of quadratic inequalities. Students' difficulties and misconceptions in learning quadratic inequalities in high schools were discussed. A design based research was adopted as the appropriate methodology of this study which can contribute to developing a useful learning instructional theory of quadratic inequalities in a GC enhanced environment. This methodology assisted to collect both quantitative and qualitative data from the students in three research cycles of teaching experiments.

It has been noted that using graphing calculator could have a positive influence on students' understanding of algebraic concepts, including quadratic inequalities. Students can benefit from the use of this powerful technology-tool, as quadratic inequalities are perceived to be difficult in South African high schools. Researchers (e.g., Ellington, 2003; Graham, 2005; Spinato, 2011) stated that designed mathematical activities (lessons) which integrate the GC-based explorations can enhance student reasoning and problem solving skills. It is the researcher's hope that teaching and learning of quadratic inequalities with the use of GCs can increase students' conceptual understanding.

The next chapter discusses some of the relevant literature that surround this study in particular the opportunities and challenges of using graphing calculators to develop students' understanding of quadratic inequalities.

CHAPTER 2: REVIEW OF THE LITERATURE

2.1 Introduction

This chapter provides a review of the literature relating to the main topics of this study. The first section presented the history of quadratic inequalities in mathematics, research on quadratic inequalities and the methods of solving inequalities. The second section followed with a discussion of the general overview of research on technology in mathematics, research on graphing calculator in mathematics, graphing calculator usage in South African curriculum and problem solving strategies, the role of graphing calculator in mathematics and misconceptions with the use of the graphing. In the third section, conceptual understanding in quadratic inequalities, and reasoning and problem solving as key aspects of conceptual understanding were discussed in relation to the reviewed literature. The fourth and last section presented a summary of the literature review. Experiences of mathematics education researchers, globally and locally were considered to gain insights on how GCs have been used to enhance the understanding of quadratic inequalities in mathematics. The inclusion of Curriculum and Assessment Policy Statement (CAPS) review on the use of technology in education intended to ascertain if the curriculum has been specifically designed to take advantage of the technology. The main thrust was on how graphing calculators have been and could be effectively used to improve student reasoning and problem solving skills in the topic of quadratic inequalities, so that ultimately conceptual understanding of students is achieved.

2.2 The Historical Approach in Mathematics Education

The historical approach can play a valuable role in mathematics teaching and learning as a major issue of the research in mathematics education, with reference to all school levels (Baghi, 2005). Accordingly, Cornu, (1991) argues that when the teaching, learning, or understanding of a concept encounters problems, it is a tradition to search for the answer to the problem from the history of the concept. The history of mathematical may assist to locate periods of slow development. Furthermore, the inclusion of some history is to show mathematics as a human creation and still developing (DBE, 2012). Therefore, this section presented the historical approach of the inequalities, in particular.

The use of the historical incidents usually informs educators, mathematicians and researchers about the developmental stages and epistemological obstacles associated with a mathematical concept. Researchers indicate that the approach creates opportunities for the mathematics community to link psychological learning processes with historical-epistemological issues (Radford, Boero & Vasco, 2000). In the context of the developmental stages of the concept, the researcher may find information about periods of slow development of the concept (Halmaghi, 2011). According to Burn (2005), the historical development of a mathematical concept can reveal actual steps of success in learning. This exploration could be applied when the research in education indicates that the understanding of a concept is not consonant with students' intuitions (Burn, 2005), which seems to be the case with inequalities (Bazzini & Tsamir, 2003). Inequalities have been disregarded for almost two millennia before being considered worth of special attention. Hardy, Littlewood and Polya, (1934) expressed that the historical and bibliographical accounts are difficult in inequalities, which have applications in every part of mathematics but have never been developed systematically. This can be an indication that there are some gaps which are required to be filled in with respect to concept development. Such gaps can slower the concept development hence creating problems for students' understanding of the concept. This has similarly and negatively affected the development of quadratic inequalities.

This approach can help the researchers and educators to identify the difficulties and misconceptions which may hinder the students' understanding of the concept. These are seen as the epistemological obstacles that Radford (1997) interprets as recurrent mistakes. Precisely, students make repetitive and consistent mistakes when they learn a specific topic. Radford further categorises epistemological obstacles into three groups of sources such as (1) an *ontogenetic source* (related to the students' own cognitive capacities, according to their development); (2) a *didactic source* (related to the teaching choices); and (3) an *epistemological source* (related to knowledge itself). Radford's classification signifies that the conceptual mistakes made by students originate from different sources. The implications of historical facts may indicate the real sources of learning and teaching obstacles associated with the topic of quadratic inequalities.

Once the sources of the epistemological obstacles have been identified for the concept under study, then the obstacles in the historical texts can be confronted with better teaching and learning approaches. The teacher can adopt instructional strategies that may reduce the deep-rooted and recurring mistakes made by students associated with the learning of quadratic inequalities. Halmaghi, (2011) states that teaching a concept linked to epistemological obstacles and being aware of that, the teacher can better plan when and how it would be more appropriate to introduce that concept to the students to avoid unnecessary misconceptions. On the other hand, Sfard, (1995) argues that the teacher can work on training the students to practise that concept even when the meaning is obscured and manipulation mistakes are persistent. This study, for example, aims to integrate the graphing calculator as a teaching and learning tool that can minimise the epistemological obstacles associated with quadratic inequalities. The suggestions made by Halmaghi, (2011) and Sfard, (1995) have helped in the preparations and delivery of quadratic inequality sessions for this study.

Studies have further indicated that the historical approach in mathematics is valued not only for detecting epistemological obstacles, but also for informing teachers and researchers in mathematics education in various other ways (Bagni, 2005; Burn, 2005). Historical anecdotes can be used by teachers as motivators to develop a concept and this is viewed as a powerful approach to didactics (Radford, 1997). Recapitulation (presenting topics through their historical development) is another way of using the historical facts in class, since the method essentially sets the stage for the students to recreate the concept (Radford, 1997). This study is not only using the history of inequalities to improve teaching and learning of quadratic inequalities, but also to expose students to the power of inequalities in a broader intuitive and, at the same time, rigorous way. In this regard, the intuitive and rigorous way of learning quadratic inequalities was aimed at improving students' understanding of the concept in question. For the purpose of this study, research findings (e.g., Radford, 1997, Halmaghi, 2011, Burn, 2005) motivate the researcher's engagement with the historical facts of quadratic inequalities to inform teaching and to provide leads for further research on students' understanding.

2.3 The Historical Developments of Inequalities

In this section the study explores the history of inequalities from Antiquity to the beginning of the 20th Century. This may help to better unpack the problems that arise specifically from teaching and learning quadratic inequalities as well as suggest possible ways of solving them. Previous studies have noted that mathematics begins with inequality (Tanner, 1962) and inequalities therefore are the building blocks for many mathematical domains (Halmaghi, 2011). In this context, an understanding of inequalities may improve the learning of other mathematical concepts as well. However, it has emerged that inequalities do not have a long standing history (Halmaghi, 2011) and was scarcely considered till now by researchers in mathematics education (Boero & Bazzini, 2004). As already noted in section 2.2, it took two millennia to change the status of inequalities from mere support for some mathematics to a discipline of study (Fink, 2000). This would suggest that some of the teaching and learning problems are associated with the slow development of the concept of inequalities.

Inequalities are not unfamiliar at all to ancient mathematicians, including the Greeks (Bagni, 2005) and, the Hindus and Chinese (Fink, 2000). Inequalities were first encountered in Classic Geometry, where they were used to express factual relationships between quantities. For example, the ancients knew about the triangle inequality as a geometric fact and the arithmetic-geometric mean inequality, as well as the isoperimetric inequality in words 'alike exceed', 'alike fall short' or 'alike are in excess of' to compare magnitudes. This means that the knowledge of inequalities helped the ancient mathematicians to compare magnitudes and to explain mathematical discoveries in the context of inequalities.

Greek mathematicians were profoundly aware of the power of inequalities to obtain equality (Burn, 2005). Burn employs a metaphor – called "the vice" – to describe the properties of inequalities that Ancient mathematicians used to help produce equality. It consists of the following argument: when a number A is squeezed in between two small quantities, $-\varepsilon<A<\varepsilon$, for all positive numbers ε, then the number $A=0$. The inequality $-\varepsilon<A<\varepsilon$ works as a carpenter's vice, compressing the inner quantity so much as to leave room for only one number in between $\pm\varepsilon$, and that number is zero. The vice could be used to squeeze a difference of two numbers: $-\varepsilon<A-c<\varepsilon$, and to prove the equality $c = A$ (Burn, 2005). Although they seemed to have used the vice, Greek mathematicians were not aware of the existence of negative numbers. Here is an account of the vice in Euclid's Elements:

> *Two unequal magnitudes being set out, if from the greater there is subtracted a magnitude greater than its half, and from that which is left a magnitude greater than its half, and if this process is repeated continually, then there will be left some magnitude less than the lesser magnitude set out. And the theorem can similarly be proven even if the parts subtracted are halves (Katz, 2009, p.82).*

The proposition states that by taking a small quantity, compared to a bigger one (ε<B), one can get a smaller quantity than the smaller of the two initial quantities by successively subtracting halves from the big one. Thus, the inequality becomes $B/2^n < ε$. This can be turned into $B < 2^n ε$, an inequality which signifies that a multiple of ε exceeds B (Burn, 2005). Classical Greeks as well as Archimedes used the potential of this inequality to calculate the volume of a pyramid as one third of the area of the base times the height (Burn, 2005), and many other similar results (Halmaghi, 2011).

According to Halmaghi (2011), Euclid's Elements abounds in propositions that express inequality relationships between angles, sides, perimeters, or areas. However, there is no account of using inequalities in arithmetic or numbers' manipulation (Fink, 2000). The Euclid's words are translated using the inequality symbols in order to help the reader understand and interpret the old text, but those symbols did not exist. This means a great deal of geometry and algebra was expressed verbally in those times. The modern reader needs the symbols alongside with the text to fully understand inequalities in Euclid's work (Halmaghi, 2011) and for example, the symbols are used to write geometric inequalities. This therefore demonstrates that inequalities were used as tools to serve geometry in the ancient times.

Burn's (2005) account summarizes the old history of inequalities and projects the importance of classical work on inequalities for the further development of mathematics. Inequalities were used to measure awkward quantities dating back to Euclid and beyond. Archimedes in particular was skilled in using inequalities to deduce equalities, and after translating his method into algebra, such proofs were used by Fermat (1636). Arabic mathematicians understood the work of the Greeks and proved similar results on the volume of solids (Katz, 2009). They were also skilful in manipulating inequalities in approximations using continued fractions (Fink, 2000). It should be noted that geometry, arithmetic, and number theory were well established mathematics disciplines in Antiquity. However, inequalities were not recognized as sole mathematics concepts. They were only considered as peculiar tools used to develop other theories in mathematics.

In the development of algebra three stages were identified such as rhetorical algebra, syncopated algebra, and symbolic algebra (Radford, 1997). Rhetorical algebra is the algebra of words. Syncopated algebra is the algebra that uses a mixture of words and symbols to express generalities. This is the algebra of Pacioli, Cardan, and Diophantus. It is Francois Viete who made the distinction between a constant and a variable, both being represented by letters. Viete's contributions transformed algebra from an operational level towards process level (Bagni, 2005). As a result equations became the objects of higher-order processes (Sfard, 1995). He purified algebra from all the clutter of words and presented it in abstract form, the encapsulation of a pure mathematics idea (Radford, 1997). From Viete onwards, structural algebra cemented its place in the history of mathematics. This means that the structure in algebra helped geometry capture generality and express operational ideas. In this context, geometry needed algebra for new reifications and new development (Sfard, 1995) than in its early years when algebra used geometry. The new developments in algebra signify that inequalities were no longer under algebra. Algebra metamorphosed from words to symbols. Equations or identities were transformed from heavy paragraphs to delicate formulae. Inequalities, however, seem to have been left behind, forgotten, abandoned, and seemingly having no real use in the development of algebra (Halmaghi, 2011). This suggests that The Middle Ages were a period of great accomplishments for algebra as algebra had more influence on geometry.

The development in algebra might have influenced the mathematicians to produce the symbols that would permit inequalities to come along and evolve. Eves (1969) documents that the symbols < and > were first introduced in mathematics-related texts by Thomas Harriot in North Carolina who got inspired by the symbol on the arm of a Native American in coining the symbols for inequalities (Johnson, 1994, p.144). The account states that Harriot decomposed the Native symbol into the two well-known symbols < and >. Tanner (1962) argues that the origin of the symbols is less mystical than that. She argues that the inequality symbols are modifications of the equal sign, a symbol which was coined by Recorde as two horizontal, parallel and equal lines, to represent that what is on one side of the sign is exactly the same as what is on the left side of it. Tanner (1962) further indicates that, when producing the inequality signs, Harriot "took the equality in Recorde's sign to reside not in the two lengths, but in the unvarying distance between the two parallels" (p.166). According to Tanner (1962), Harriot modified the distance between the two lines of the equal sign, to show that the bigger quantity lies on the side of the longer distance between the lines.

Harriot used < to represent that the first quantity is less than the second quantity and > to represent that the first quantity is greater than the second quantity (Johnson, 1994). "The symbol for greater than is > so that $a > b$ signifies that a is greater than b. The symbol for less than is < so that $a < b$ signifies that a is less than b (Seltman & Goulding, 2007, p.33). Harriot was familiar with the symbolical reasoning introduced by Viete and, moreover, he transformed Viete's algebra into a modern form (Katz, 2009). In addition, Harriot simplified Viete's notations to the point that even a novice in the history of mathematics could understand his formulae. Harriot first used the symbol of inequality to transcribe the well-known inequalities of the means and then, he used the inequalities in his work to solve equations.

Here are some excerpts from Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas, (The Analytical Arts Applied to Solving Algebraic Equations) Harriot's posthumously published work in lemmas and propositions. The propositions use inequalities to solve equations. For example:

Lemma 1

If a quantity be divided into two unequal parts, the square of half the total is greater than the product of the two unequal parts. If p and q are two unequal parts of the magnitude, then it is true that

$$\left.\begin{array}{r}\frac{p+q}{2}\\[2ex]\frac{p+q}{2}\end{array}\right| > pq$$

(Seltman & Goulding, 2007, p.96)

Transcribed, the above inequality reads: $(\frac{p+q}{2})^2 > \boldsymbol{pq}$

Lemma 2

If three quantities are in continued proportion, the sum of the extremes is greater than twice the middle. Suppose b, c and d are in continued proportion; then it is true that $\boldsymbol{b} + \boldsymbol{d} > \mathbf{2}\boldsymbol{c}$. (Seltman & Goulding, 2007, p.96)

Proposition 5

The ordinary equation $\boldsymbol{aaa} - \mathbf{3}\boldsymbol{bba} = +\mathbf{2}\boldsymbol{ccc}$ in which $\boldsymbol{c} > \boldsymbol{b}$, is explicable by a single root. (Seltman & Goulding, 2007, p.100)

The sample of Harriot's work shown above may lead to a simple conclusion – which is that once they were coined and it was shown how they work, the inequality symbols became well established and were easily adopted. However, history shows that the mathematics community did not adopt Harriot's symbols immediately, possibly because Harriot did not publish his work. In the 18th century, the < and > signs finally made their way into Continental Europe (Cajori, 1928-29). Moreover, in 1734, the French geodesist Pierre Bouguer invented the symbols ≤ and ≥, to represent less than/greater than or equal to, respectively. These new symbols were used to "represent inequalities on the continent" (Smith, 1958, p.413). More precisely, the < symbol is used to represent quantities that are different, the first one being less than the second one. The ≤ symbol incorporates the equality as well; it allows the first magnitude to be equal to the second one. This therefore means that the inequality symbols that are now universally accepted in mathematics literature are: < for less than, > for greater than, ≤ for less than or equal to, ≥ for greater than or equal to, and ≠ for not equal to.

The appearance of symbolic algebra helped in the understanding of mathematical texts. Mathematical arguments were in the past presented in longhand. There were no symbols to represent the unknowns and no symbols to represent the relationship between unknowns. That was before Diophantus, during the rhetorical algebra stage (Harper, 1987). Presenting mathematical statements in plain language could mean writing several pages to describe those facts, while expressing the same statement in mathematical symbols could even take a single line. “It is amazing how much the Greek mathematicians could accomplish by using rhetorical means of expressing inequalities and geometrical embodiments” (Halmaghi, 2011; p. 48). In this context, symbolic algebra produced the much-needed tools for the embodiment of geometrical ideas and for the representation of inequalities which were more abstract and specific. Moreover, the use of symbols in mathematics has shortened the time to be spent on expressing mathematical facts, hence more output in terms of performance. Radford (2006) describes algebraic symbolism as a metaphoric machine that is encompassed by a new general abstract form of representation and by the Renaissance technological concept of efficiency. Symbolism therefore helped algebra prosper, while Harriot’s inequality signs stimulated the proliferation of inequalities (Tanner, 1962). This suggests that many presentations of old inequalities may be successfully compressed using the inequality symbols and this symbolism has spurred the development of an inequality concept from a mere peculiarity.

The rise of Algebra and the adoption of mathematical symbols, however, allowed inequalities to become more easily noticed in the bigger picture of mathematics. The circumstances became more favourable for the inequalities to flourish into a discipline. Mathematical statements involving inequalities were expressed in inequality symbols. Inequalities migrated to Algebra to get the power of symbols from there, and then they settled for good into the theory of functions where they were enriched with new structures and philosophy (Hamalghi, 2011). With the rise of the theory of functions, inequalities seemed to have gained greater relevance. Embedded in functions, they became omnipresent in many mathematical branches, from geometry to algebra, to statistics, to numerical analysis, to game theory. Mathematicians began working on proving the famous Antique inequalities (e.g., Cauchy), creating extensions (e.g., Schwarz) or developing new ones (e.g., Newton, Maclaurin, & Bernoulli). Inequalities have been developed inside and through “interactions between different branches of mathematics” (Kjeldsen, 2002, p.2), like the theory of functions, linear algebra, mechanics, calculus, statistics and probability, to name only a few. This included the epsilon-delta proofs in math analysis which is one of the most significant applications of the metaphor of the 'vice'.

The big production of inequalities started with the appearance of the Journal of the London Mathematics Society. Also, the first history of inequalities book was written by Hardy et al. in 1934 when edited the book Inequalities (Fink, 2000). It seems that since Hardy, the development of inequalities has been remarkable. Thus, the table without a border at the end infers that the history of inequalities continues and that the production of inequalities is unbounded. Hardy's work has been much more significant in the development of inequalities. Hardy is the founder of the Journal of the London Mathematical Society, a publication for many papers on inequalities. Linear or quadratic equations, for example, were studied as independent concepts by Babylonian mathematicians. However, before Hardy, quadratic inequalities did not get special attention from mathematicians – nobody took the pains to introduce them to the mathematics community as a mathematical concept rather than as a simple tool used to serve other concepts. Hardy (1934) himself attested, in his Presidential Address to the London Mathematical Society in 1928, that even though inequalities have been intensively used by analysts, there was no coherent reference to the concept. As noted, the inequalities had a sigh of relief after the development of symbolic algebra in the 18th century. This means that for seventeen centuries inequalities were not recognised as independent mathematical concepts.

Long before Hardy, mathematicians knew the power and importance of inequalities, since they used inequalities as tools in developing Geometry and Calculus. One could suggest that there are some contributions related to inequalities which are not yet published. Hardy, Littlewood, & Polya (1934) confessed that the historical and bibliographical accounts were not readily available for the subject like inequalities which had applications in every part of mathematics. In this regard, their contribution was to track down, document, solve and carefully present a volume comprising of inequalities, and to officially write the first page of the history of inequalities. The availability of this information may lead to much better exploration of students' misconceptions and to what instructional tool can be used by students in the classroom to potentially gain better understanding of quadratic inequalities.

2.4 Inequalities in the South African Mathematics Curriculum

The topic of quadratic inequalities is included in the section of algebra in the South African Mathematics Curriculum (Makonye & Shingirayi, 2014). The topic of quadratic inequalities in one variable is primarily introduced in grade 11 immediately after revisiting quadratic equations in the CAPS document (DBE, 2012; p. 11). In addition, this topic is examined in the sections of algebra and functions in the NSC mathematics examinations. This indicates that quadratic inequalities are taught in algebra courses in South Africa, in conjunction with equations or as a section of the chapter on equations. This seems to suggest that quadratic inequalities are still considered as an algebra concept or as an additional section of the equations. Thus this has a bearing in determining the solution of quadratic inequalities as students can just use the same procedures of solving quadratic equations. A study by Makonye & Shingirayi (2014) conducted with 27 learners of grade 11 in Soweto, established that learners committed the procedural and conceptual errors linked to algebraic processes and confused equations and inequalities. In this context, the quadratic inequalities were solved using algebraic methods, disconnected from the concept of quadratic functions. The CAPS document states that student should "solve quadratic inequalities in one variable and interpret the solutions graphically" (DBE, 2012; p.11). However, the recommendations also encourage the use of other available approaches. The recommended textbooks (Bradley, Campbell & McPetrie, 2014; Phillips, Basson & Botha, 2014) for Grade 11 have suggested three different approaches to be used which include functional graphs.

It has been further noted that there are a few studies where inequalities were connected to the study of functions (Boero et al., 2001; Garuti et al., 2001; Sackur, 2004). The concept of function has been widely recognized as being foundational to mathematics education, in particular in solving quadratic inequalities. The NSC Diagnostic reports recommend that teachers should integrate the algebra with functions so that learners have a visual understanding of inequalities (DBE, 2014; 2015; 2016). This means embedding functional approaches in solving quadratic inequalities in the South African curriculum of Grade 11 Mathematics is recommended. This recommendation creates room for infusing the GC technology in teaching and learning quadratic inequalities as it provides both algebraic representations and quality visual images of functional graphs.

The inclusion of inequalities in the algebra curriculum however has been criticised, when it was recognized that inequalities belong to the study of many aspects of mathematics (Burn, 2005; Tall, 2004; Boero & Bazzini, 2004). This placement of inequalities is seen as unfortunate as it invites the learning of inequalities through memorized, routine procedures. According to Halmaghi (2011), students may fail to make important connections and to solve inequalities that look different from the model they have commonly encountered. There is a didactical challenge to develop activities that will help students benefit from the connection between equation and inequalities, while making them aware of the pitfalls of applying the transformational techniques used in solving equations to solving inequalities (Kieran, 2004, p.146). In this context, manipulating inequalities as equations means being able to replicate what can be repeated while changing the parts that need to be changed (Sfard, 1998). This implies that teaching and learning of quadratic inequalities should be not be viewed as an extension of quadratic equations as students mistake one for another. This therefore calls for the use of graphing calculators as a didactical solution to the learning of quadratic inequalities.

2.5 Research on Inequalities in Mathematics Education

In the literature, researchers have witnessed students' and teachers' frustrations with the difficulties encountered when dealing with inequalities (Tsamir & Bazzini, 2002, p.2). Inequalities are perceived by some students and teachers alike as a misfortune (Burn, 2005), rather than an important brick for a solid foundation in mathematics (Halmaghi, 2011). This has motivated the researchers to be engaged in studies about teaching and learning inequalities that can bring about conceptual understanding, and positive perceptions of students.

Accordingly, teachers were the first people to be engaged in writing about teaching and learning inequalities reflecting on classwork activities and presenting their observations in the Mathematics Teacher (McLaurin, 1985; Piez & Voxman, 1997). Their writings focused on various topics which included the students' difficulties in the algebraic manipulation of inequalities, the methods of teaching inequalities, and the preferred methods of introducing inequalities to students (Halmaghi, 2011). Such activities have paved the way for this study to use graphing calculators in teaching and learning quadratic inequalities as the preferred instructional method that can provide students with the opportunity to develop visual thinking and understanding.

A series of conferences were held by the International Group for the Psychology of Mathematics Education (PME), in which the teaching, learning, and understanding of algebra have been fundamental streams of research (Kieran, 2006). The history of research on inequalities was the theme in 1998, at the 22nd PME Conference, when the discussion on inequalities was initiated by the presentation of research conducted by Tsamir, Almog and Tirosh (Bazzini & Tsamir, 2004). Inequalities are viewed as an important subject from the mathematical point of view but a difficult subject for students and a subject scarcely considered till now by researchers in mathematics education (Boero & Bazzini, 2004). Consequently, in 1999, the Project Group of mathematics educators called for research papers on inequalities at the 23rd Psychology of Mathematics Education Conference. The conference required the research papers to focus on some of the following key questions:

What are common errors in inequalities? What are possible sources of students' incorrect solutions in inequalities? What theoretical frameworks could be used for analysing students' reasoning about algebraic inequalities? What is the role of the teacher, the context, different modes of representation, and technology in promoting students' understanding of inequalities? Is there a global theory that may encompass the local theory of inequalities (Bazzini & Tsamir, 2004)? This study therefore fills a perceived gap in the literature as identified by the PME to globally encompass the local theory of quadratic inequalities mediated by the use of the graphing calculator.

The 23rd and 28th PME papers coverage ranged from error patterns in students' solutions of inequalities to didactical perspectives on students' errors; from traditional teaching to teaching with technology; from the didactical aspects of classifying inequalities under algebra to the metaphors of using functions to present inequalities (Halmaghi, 2011). The findings of these papers act as a guide for this study to explore the patterns of students' misconceptions induced by the traditional teaching methods in the quadratic inequality classroom. In this study, the graphing calculator was used as the possible alternative instructional method for improving students' understanding of quadratic inequalities.

The diversity in the conference's theme refers to the approaches used by researchers which include the theoretical frameworks for interpreting students' understanding. Diversity may be found in the different lines of research, different theories to account for the findings and the educational implications that have been put forward by the researchers (Bazzini & Tsamir, 2004). This implies that research papers should have variations in their approaches and presentations. The theoretical frameworks included a Vygotskian perspective, Nunez's metaphor construct, Duval's semiotic registers in mathematics and Frege's theory of denotation, as well as Fischbein's theory of intuitive, formal and algorithmic knowledge (Bazzini & Tsamir, 2004; Halmaghi, 2011). This study similarly seeks to use graphing calculator technology to enhance students' understanding of quadratic inequalities through selecting diverse, theoretical frameworks: RME theory, Vygotsky's socio-cultural learning theory and the theory of instrumental genesis in Chapter 3 to analyse their reasoning and problem solving skills in a supported mathematics classroom.

Kieran (2004) observes that problem solving activities directed toward generating the symbolic form of inequalities were absent from the research on inequalities. Thus, for Kieran, inequalities, as well as algebraic equations, could be meaningfully introduced to young students through contextual problems. In the CAPS document contextual problems are defined as those mathematical problems that should include issues relating to health, social, economic, cultural, scientific, political and environmental issues (DBE, 2012). She observes that students enjoy the process of coming to a generalization by working on recursive aspects of a concept. Kieran (2004) however, has no answer to the question: What instructional support do students need in order to grow from the context to meaningful symbolic manipulations of inequalities? The present study has included contextual problems in its planned students' activities in which graphing calculators are used as an instructional tool to move students to meaningfully symbolic manipulations of quadratic inequalities. The activities are expected to provide opportunities for students to model and interpret a situation mathematically, and promote students' higher order thinking skills.

2.6 Methods of solving quadratic inequalities

This section presents the most common methods of solving quadratic inequalities, which include the graphical method, the sign-chart (the improved sign-chart) method, and the logical connectives method (Halmaghi, 2011; Tsamir & Reshef, 2006; Bradley, Campbell & McPetrie, 2014; Phillips, Basson & Botha, 2014). The methods are explained in detail below, showing the merits and demerits of each.

The graphic method usually consists of creating a function associated with the inequality, graphing the function, comparing the y with the x-axis (or another y in some cases), reading the x values for the appropriate y, and giving the solution (Sackur, 2004). Students sketch the quadratic graph and determine the values of x for which the graph lies either above or below the x-axis as the solutions of quadratic inequalities (Phillips, Basson & Botha, 2014). This means students need to understand how to determine the x-intercepts and the shape of the parabola. The x-intercepts are the critical values which represent the interval limits of the solution set.

The sign-chart method usually consists of finding the intervals where the evaluated expression in the composition of the inequality is either greater than or less than zero (Sackur, 2004). The intervals are bounded by all the zeros of the associated equations aligned on a number line. Test values are taken from each interval determined by locating the zeros on the number line. The values of the function are then calculated for every test value. For the sign chart method (table approach), we keep only the sign of the value of the function at each of the test values (Sackur, 2004; Phillips, Basson & Botha, 2014). This means the interval that makes the inequality true is the correct solution for either a positive or negative quadratic expression.

The logical connectives approach to solving an inequality – for example consists in using the structure of the fraction and logically analysing all the possibilities of getting numerical values that are less than or equal to zero for the given fraction (Sackur, 2004). This approach is not common for the quadratic inequalities but it can be used to enrich students' knowledge. However, Phillips, Basson & Botha (2014) have suggested the use of number line approach to solve quadratic inequalities. This approach uses the critical values (x –intercepts) to divide the line into three regions. Test the x values taken from the left, between and right of the critical values to determine the final solution. The final solution is in the interval where the quadratic expression is true.

In the previous texts, researchers presented papers on the instructional approaches, with specific references to the solution strategies like using the sign-chart method (Dobbs & Peterson, 1991) or graphical methods (Dreyfus & Eisenberg, 1985) to solve quadratic inequalities. On the one hand, McLaurin (1985) and Dobbs and Peterson (1991) have suggested that the sign-chart is the best method for learning quadratic inequalities. In their argument, a good understanding of the sign-chart method can empower students with the necessary skills for solving more complicated quadratic inequalities for which no other method is available. For example, the quadratic inequalities resulting from the transformed rational inequalities are solved using the sign-chart. McLaurin (1985) also pleaded for a unified method to teaching students how to solve absolute value, quadratic, rational and irrational inequalities – by following the sign-chart method.

Tsamir and Almog (2001), on the other hand, investigated students' solution strategies and difficulties when faced with various types of inequalities (linear, quadratic, rational, and square root). Their results showed that using graphical representations usually lead to correct solutions, while difficulties with approaches based on algebraic manipulations arose when students failed to recognise the difference between inequalities and equations. In another Tsamir and Reshef's (2006) experiment conducted with twenty high school students who were split into three groups where each group was first introduced to one of the three methods – graphic, sign-chart, logical connectives. Students interactively familiarised with their methods, and then they presented their methods to the other students who learned the other two methods as well. At the end of the study, the students were tested on solving quadratic inequalities following a method of their choice. As expected, students mostly used and gave preference to their first method that they were introduced to, except those that first learned the logical connectives method. However, there were students who used more than one method in solving different inequalities. In that context, the graphic method was most frequently employed and preferred by students.

Sackur, (2004), however, has not completely favoured the use of graphical representations to solve quadratic inequalities as demanding for students, and that is not free from students' errors and misconceptions. Knuth (2000) also observed that students often have a limited understanding of the relationship between algebraic and graphical representations, and that they strongly prefer to solve tasks algebraically rather than utilising graphical information - even in cases where a graphical solution would be much easier. Similarly, Linchevski and Sfard (1991) identified the logical connectives method as problematic, since it requires more abstract thinking than the other two methods.

Tsamir and Reshef (2006) recommended presenting students with multiple methods when teaching quadratic inequalities. They argue that since inequalities serve different purposes in mathematics education; therefore each representation (approach) is applicable to a specific situation. The logical connectives approach was further criticised for developing pseudo-structural conceptions rather than operational ones which promote relational understanding. Similarly, Piez and Voxman (1997) point out that the students must become familiar with multiple methods and representations when dealing with inequalities. It is essential, therefore, to note that flexibility in manipulating algebraic structures would not only allow students to solve other types of inequalities, but would also improve students' problem-solving and reasoning skills.

Tsamir et al. (2004) also suggested that teaching inequalities through functions or technology could minimize the construction of misconceptions. In their views, a function approach to teaching inequalities means using the graphing calculator to graph functions and then reading the solution of an inequality associated with the graph. In the same vein, Boero and Bazzini, (2004) indicated that this approach can improve students' use of metaphors in understanding inequalities and their capacity to make connections far beyond mathematics. They further state that the "visual enactive activity" with dynamic entities – the functions – "can give a powerful embodied sense of global relationship" between functions and inequalities (Boero & Bazzini, 2004, p.141). This means that the function approach can improve students' visualisation as they interact with the graphs in determining the region defined by the quadratic inequalities.

Abramovich and Ehrlich (2007) were in favour of using a graphic tool- the graphing calculator - in the study of inequalities as that could graph relations, and therefore, inequalities as well. They argued that the solution of the proposed inequality is identified by the machine itself and visually represented on the graph. This suggests that the graphical approach has an advantage over the previous mode of using functions to solve inequalities. They further point out that graphing calculators could be successfully used in solving all types of inequalities, and they believe that students should experience graphing the given inequality as well as the equivalent forms they get by algebraic transformations. The solutions of the new inequalities can inform students about the correctness of their algebraic approach (Abramovich & Ehrlich, 2007). In other words, the graphing calculator can be used as a means for solving quadratic inequalities and, at the same time, a tool for validating the algebraic manipulation of the referred inequalities. Thus, there is a need for encouraging students to evolve from algebraic procedures to manipulating mental entities, in particular graphing calculators which allow for the acquisition of meaningful higher order thinking.

Sackur, (2004) states that that it is not easy at all for students to shift from the dynamism of a graph where y is the moving entity to reading the solution on the x-axis. Also, it is not easy for students to switch from seeing the dynamic solution as a point moving along the x-axis between some boundaries to giving the solution as a static entity – an interval. However, Halmaghi, (2011) states that flexibility in manipulating algebraic structures would not only allow students to solve inequalities, but would improve students' problem-solving skills as well. In this context, the use of graphing calculator as one of the modern methods can assist students to solve quadratic inequalities and to validate their solutions. This switch from algebraic to visual graphical representations helps students conceptualize the symbolic form of inequalities as they can see where the graph is either above or below the x-axis.

2.7 Technology in Mathematics Education

Developments in technology have been affecting every aspect of education, in particular, what and how to teach and learn in the mathematics classroom. Recommendations have been made that technology is essential in teaching and learning mathematics (NCTM, 2009; ISTE Standards for Students, n.d.). The NCTM's Principles and Standards for School Mathematics Technology Principle state that technology influences the mathematics that is taught and enhances students' learning (NCTM, 2009). Similarly, The International Society for Technology in Education (ISTE) standard for students mentions that students need appropriate digital tools and resources for critical thinking skills and conducting research, managing projects, solving problems, and making informed decisions (ISTE Standards for Students, n.d). This implies that technology plays a big role in the life of a student. Furthermore, ISTE standards require teachers to engage students in exploring real-world issues and solving authentic problems using digital tools and resources (ISTE Standards for Teachers, n.d.). This suggests that schools should embrace new instructional methods such as technology, which may help to explore mathematical problems. The e-Education policy of South Africa articulates that every learner in the general and further education and training (GET and FET) bands should manipulate ICTs confidently and creatively (i.e., by 2013) in order to develop the skills and knowledge needed to be a full participant in the global community (DoE, 2004). Admittedly, this is an indication that the presence of the technology is felt in the classrooms of South Africa. Therefore, these three sources emphasise the significance, availability and appropriate use of technology in education, specifically for mathematics. In particular, a graphing calculator technology will be used in this study to bring about new instructional ways for improving students' understanding of mathematics in the South African schools and to develop students for global participation.

Ruthven and Hennessy (2003) successfully studied uses of technology, including GC technology, in mathematics teaching with a group of secondary school teachers, who indicated that technology could, among others, foster student independence and peer support, help students with special needs to overcome difficulties in writing, drawing, and graphing, eliminate error-ridden calculations and save time for higher-order learning activities. Similarly, Wertheimer (1990) highlights that the presence of technology in the classroom frees the teacher for more individualized support of student learning in addition to providing opportunities for students to collaborate and create exciting and nurturing classroom environments. It is further suggested that technology can improve mathematical learning specifically in the areas of problem solving, concept and skill development, reasoning, and communication (Kimmins & Bouldin, 1996; Spinato, 2011). This implies that the benefits derived from the use of technology can be experienced from the use of the GC as an available technology. This suggests that through the use of graphing calculator, students are provided opportunities for graphing, visualizing, producing multiple representations, and computing in the mathematics classroom.

In the literature, many researchers have indicated that technology allows for improvements in the teaching and learning of mathematics. For instance, although technology has a negative effect on teaching lower-order thinking skills, it was found to have positive effects on teaching higher-order thinking skills (Karadeniz, 2015). This means that teachers, who aim to develop or improve higher order thinking skills of their students, should use technology in their mathematics classrooms, thus the use of graphing calculator as the available technology. It is therefore, important to provide students with the opportunities to use graphing calculator in their mathematics classrooms so that they can experience both the benefits and disadvantages of the technology. In addition, it is possible to motivate students especially of lower ability (Ruthven & Hennessy, 2002) because of increased student engagement with technology (Karadeniz, 2015). This therefore potentially leads to explore students' understanding of mathematical concepts, in particular quadratic inequalities, with graphing calculator.

2.8 The use of technology in South African classrooms

The e-Education policy has practically enforced the technology integration in South African classrooms. In 2004 the Department of Education drafted The White Paper on e-Education policy for guiding the ideal implementation of pedagogical technology integration. This policy spells out the framework, objectives, funding, resources and implementation strategies for ICT integration in the classroom (Padayachee, 2017). The main thrust was to build digital and information literacy so that all learners become confident and competent in using technology to contribute to an innovative and developing South African economy (DBE, 2004). This means the policy dictates the government's strategy to improve the quality of learning and teaching across the education and training system. For that reason, this policy on ICT facilitated the integration of digital technology in teaching and learning, including handheld graphing technology.

The policy further indicates that e-Learning may involve the use of the "Internet, CD-ROM, software, other media and telecommunications" (DoE, 2004, p. 15). This means the policy is supportive of the use of graphing calculator in mathematics classroom since this tool has (computer) graphing software. Additionally, the policy recommends the use of the Internet and associated web-based applications as the delivery medium for the learning experience (DoE, 2004). This also justifies and upholds the use of graphing calculator in mathematics classrooms because from the internet one can access software like GeoGebra, Sketchpad and Desmos. These have capabilities similar to those of the GCs. In this context, the use of such media can assist students to develop lifelong skills and knowledge through understanding subtle mathematical concepts. Thus, the graphing calculator can be used as a delivery medium for the learning of quadratic inequalities at the 11th grade. The GCs are portable and do not need internet to be accessed by the users, which means learners can use them any time.

The policy additionally proposes different approaches for the use of ICT in classrooms such as the use of multimedia applications, to create contexts for problem-solving and the creation of knowledge in the productive learning environment. The implications for the teachers are to effectively use technology in their classes in order to develop and empower students with the basic knowledge, skills and attitudes. Such technological knowledge and skills can enable students to access, analyse, evaluate, integrate, present and communicate information (DBE, 2004). This means through the use of technology, including the GC students are provided with a range of cognitive activities that can enhance their understanding of quadratic inequalities. This is in line with the requirements of the CAPS FET mathematics (DBE, 2012) which focus on active participation of the learners and how teachers use the available technology in their mathematics classrooms. As explained by Ertmer et al. (1999), the use of technology in the existing curriculum can support, reinforce, enhance, and enrich student learning. This implies that the GC will be used as the available technology for productive learning.

The CAPS document calls for the need to integrate the available technology in teaching and learning of mathematics. In reference to learning of functions and their graphs in the same document, it is stated that students should

> "*Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test*

conjectures and hence generalise the effects of the parameter which results in a horizontal shift and that which results in a horizontal stretch and/or reflection about the y axis, (DBE, 2012, p.9)".

The implication of this statement is that the GC can be used as an available technology to generate the quality and visual graphs that support the students to make and test conjectures. However, it is further stated in the same document that only scientific and non-programmable calculators are allowed to be used in the formal assessment. This suggests that graphing calculators as programmable calculators will not be used in the assessment activities but allowed as a tool for instruction, visualisation, and checking. In addition, GCs have fewer propensities for learner distraction than computers, and are more portable, and affordable in terms of cost, therefore their pedagogical use in mathematics classrooms can be encouraged where available.

2.9 Research on graphing calculators in mathematics

Several researchers have supported the use of graphing calculators (GCs) in teaching and learning of mathematics (Dunham & Dick, 1994; Heid, 1997; Husna, Munawir & Suraiya, 2005; Penglase & Arnold, 1996). They have indicated that student achievement, understanding, reasoning and problem-solving skills improve in a graphing calculator enhanced classroom. However, there are other studies which have shown mixed results in mathematics achievement between those students who use the GC (experimental) and those who do not (control).

One of the earliest reviews on the literature about the use of the GCs in mathematics education was by Beckmann et al. (1999) who acknowledged that the proliferation of graphing calculators since the late 1980s has profoundly impacted mathematics curricula and instructional practices. This suggests that contemporary approaches of teaching and learning mathematics concepts in a technologically incorporated setting should be adopted to support the use of graphing calculators. In a similar study review, Dunham and Dick, (1994) reported that graphing calculators have the potential to dramatically facilitate changes in teaching and learning mathematics in a more interactive and exploratory environments. In this scenario, students interactively employed graphing calculators to explore connections across different representations which include symbolic algebra, functions and graphs. In their argument, Beckmann et al. (1999) further indicated that graphing calculators provide access to multiple types of representations and allow students to see how one can switch between algebraic, numerical, and graphical forms of functions. The implications for this current study are that the students can use the multiple representations provided by the GCs to solve problems, explain their reasoning, and the meaning of the solution in the context of the problem.

The impact of graphing calculators on learning mathematics has been the focus of several studies. Penglase and Arnold (1996) conducted a critical review on published studies during 1990 to 1995 that examined the effects of GCs in high school and college mathematics. The majority of the reviewed studies focused on two major areas: a) testing the effects of the use of GCs within specific areas of mathematical study; and b) making judgments regarding the effectiveness of such use. They concluded that using GCs in mathematics education had mixed results and that the GC research failed to provide clear direction to mathematics education. They additionally found that most studies that favoured the use of GCs did not show significant differences between GC and non-GC groups. Penglase and Arnold's review found that students' understanding of the connection between functions and their graphs, capabilities with spatial visualization skills, and attitudes toward mathematics were the areas that provided positive results. They, however, questioned the GC usage and testing procedures in several studies and suggested a need for new methods to evaluate students who have been exposed to GC technology.

Burrill et al. (2002) on the other hand, conducted a meta-analysis of 43 studies investigating the use of handheld graphing technology in mathematics instruction with a Computer Algebra Software (CAS). They found that students who used calculators with a CAS were better at applying calculus concepts. However, the calculators seemed to be more effective for lower-achieving students in accuracy but not in conceptual development. Similarly, research of meta-analyses of 53 calculus-based studies from 1984 through 2000 (Ellington, 2003) and of 42 studies (Ellington 2006) were also conducted to investigate the effects of hand-held calculators on the precollege students in mathematics classes. However, her research studies differed from that of Burrill et al. in the sense that they only included graphing calculators that did not include a CAS. In addition, Ellington only considered those studies that had control and experimental students, where the treatment group used the graphing calculator. Her research investigated the effects of non-CAS graphing calculators on procedural skills, conceptual skills, overall achievement and attitude of the students in the middle school, high school and college. Ninety three percent of the studies in these meta-analyses included algebra and pre-calculus concepts, and this suggests that inequalities were included. Ellington found that, when calculators are included in instruction but not testing, there was no benefit for students' ability to apply formulas and procedures, but the calculator was beneficial for student understanding. In addition, she found no effect on overall mathematics achievement. This means that the difference between the experimental and control students in terms of post-test achievement was not statistically significant. She also found that calculators are most helpful to improve students' problem solving skills, understanding of mathematical concepts and overall achievement when used in both instruction and testing. Results also showed a positive impact on students' attitudes toward mathematics. This scenario is yet to be proven in this current study where GCs are used as instructional media but prohibited for the assessment in the CAPS FET Mathematics. This concurs with the National Council of Teachers Mathematics (NCTM, 2009), which reviewed nearly 200 research studies conducted between 1976 and 2009 and found that graphing calculator usage in teaching and learning enhanced the understanding of mathematics concepts and student orientation toward mathematics. This means that exposing students to the use of GCs may help them achieve better understanding of the quadratic inequalities in the current study as well as develop the appropriate abilities for reasoning and problem solving.

2.10 Graphing calculator usage in pre- concepts of quadratic inequality

A large number of research studies has been conducted into the effectiveness of the GC into students' learning about pre-concepts of quadratic inequalities in high schools. For example, Carter (1995) investigated the effects of GCs on student achievement and understanding of the function concept in which the treatment group used the GC and the control group was taught in a traditional manner. Carter concluded that there was a significant gain between pre- and post-test scores for the both groups. However, the GC instruction provided a favourable influence on student achievement and improvement for the treatment group. The difference in the outcomes for the two groups was not statistically significant. Similarly, Armah and

Osafo-Apeanti (2012) conducted a study that investigated the effects of graphing software on students' conceptual understanding of quadratic functions at senior secondary level in Ghana. The aim was to determine the extent to which the effective use of graphing software as an instructional technology could improve the performance of forty three Form 2 students in mathematics. Using a t-test analysis, the results revealed a significantly higher performance in the post-test than the pre-test. Thus, the use of graphing software can effectively improve students' performance in quadratic functions. This means the anecdotal evidence from the studies has provided an idea for a pedagogical model for constructivist and situated learning approaches in conjunction through the effective use of graphing technology.

The research of Heller et al. (2005) found that incorporating GCs into learning mathematics, and increasing their use in particular, in algebra instruction, significantly improved the students' algebra scores, even if the test was taken without the use of the graphing calculators. They further emphasised that significant results are achieved in the algebra curricula that take advantage of the calculators' capabilities. This current study is inspired by these results as it is administered in an environment where GCs are used as instructional media- prohibited in the assessment. Other researchers however, note that only prolonged use of the graphing calculator may lead to enrichment of students' solution repertoires and a better understanding of algebraic concepts such as functions (van Streun, Harskamp, & Suhre, 2000). This means that the significant achievement of students' understanding in a GC-enhanced mathematics classroom is also a function of teacher preparedness and consistent use of GC in a highly supportive curriculum. The implication for this study is that the prolonged use of GCs may lead to the development of better solution reservoirs by students in algebraic concepts, including quadratic inequalities.

On the other hand, Rivera (2007) conducted a qualitative study of pre-calculus students on developing a graphical process for solving polynomial inequalities using the TI-89 graphing calculator. The main goal was for students to make sense of solving polynomial inequalities without being given an algebraic procedure for doing so. Certain limitations of the graphing calculator, such as the capability to receive only explicit equations as opposed to implicit equations, required the students to rely on some algebraic procedures in addition to graphing. However, students used the calculator to explore the ideas of zeros, intercepts, and inequalities, and exploration through the graphing calculator led the class to draw conclusions about how to solve inequalities. Students used features of the graphing calculator such as the trace feature, the table, the math function, and the algebra function. In addition to using the calculator to solve problems, students referred to their experience using the calculator to graph functions and solve problems without the calculator. Rivera concluded that students used the graphing calculator as a psychological tool and that they were able to extend the possibilities for solving problems using this tool. In this sense, students used the tool to assess the reasonableness of solutions provided on the graphing calculator and to make sense of what is provided to them through the calculator. This means that the same algebraic procedures and GC features will be used in the current study to learn the quadratic inequalities in a GC mediated classroom.

In the same study, Carter (1995) examined the effects of GCs on students' difficulties and misconceptions with the function concept. From the questionnaires and interviews, she found that the students who used GCs understood function transformations, understood the connections between the graphical and algebraic representations, were able to make connections between a point on the graph and the two distinct values that the point encodes, were able to solve non-routine problems, and were more active than students not using a GC. The GC environment allows students the freedom to spend more time on problem solving. With graphing calculators, students can switch between graphical and numerical representation of data (Waits & Demana, 2000). Within the use of this medium, it is possible for students to visualize data in more than one way, hence algebraic and graphical connections.

2.11 Graphing calculator usage in problem solving and reasoning

A case study with Grade 10 conducted by Choi-Koh (2003) demonstrated that using graphing calculators promoted students' mathematical thinking. She observed the student's thinking through the learning process for trigonometry tasks. The study included three stages for student understanding; namely, intuitive, operative, and applicative. The student graphed functions with the calculator and observed relations with graphs in the intuitive stage. The operative stage consisted of explaining the reason why the effects occur, abstracting and comprehending the trigonometric algebraic equations, and systematizing (predicting and conjecturing the composite functions). Inductive generalizing by giving detailed examples, making formulas for the given graphs, and reflecting by constructing statements based on discovered properties formed the applicative stage. The student's thinking process was observed advancing from the intuitive to the operative and, then, to the applicative stage with the usage of the graphing calculator. Choi-Koh reported that at the beginning of the operative stage, graphing calculators affected the student's explanations but visual data promoted the students' motivation to explain the graphs and functions. She concluded that the use of GC was beneficial for students to develop their thinking process. This investigative approach was adopted and modified in this current study as it assisted in describing the intuitive, operative and applicative thinking of the 11^{th} grade students in quadratic inequalities.

Spinato (2011) conducted a mixed-method study on the impact of graphing calculator use on high school students' reasoning skills through calculus problems. The results of the study indicate that (1) graphing calculators had a positive impact upon students' reasoning skills (2) graphing calculators were most effective in the areas of initiating a strategy and monitoring progress (3) students' reasoning skills were most improved when graphing calculators were used together with the analytic approach during both instruction and testing and (4) students who used the graphing calculator performed equally as well in all elements of reasoning as those who used pencil and paper to solve problems. In addition, graphing calculators may help students to assess the reasonableness of their answers, to justify their answers, and then to form conclusions, inferences, and generalizations based upon their solution to the problem. The only areas of reasoning for which there was no significant difference when graphing calculators were used were analysing a problem and seeking and using connections. This present study requires such prerequisite skills of students to be developed in order to solve quadratic inequalities in a GC enriched environment.

Ruthven (1990) investigated students' abilities to translate between graphic and symbolic forms of functions in a graphing calculator classroom. The study was conducted in four classes from two high schools involved in a two-year project for the introduction of graphing calculators. Observation methods provided her with information about students' approaches for translating between representations of functions. She observed that when translating functions from graphical to symbolic form, students from the treatment group having access to graphing calculators used graphical techniques to check answers and students from the control group used numerical techniques. Ruthven also identified two stages that students used to translate functions from graphical to symbolic form: identification and refinement. Within the identification stage, students classified graphs with respect to which family of functions the graph belonged, such as linear, quadratic, cubic, or exponential. During the refinement stage, the parameters of the symbolic expression were adjusted to conform to the given graph. Ruthven observed that the students from the treatment group used graphical methods to refine their functions and students from the control group used numerical methods. Her observations indicated a difference in students' approaches used to translate functions from graphical to symbolic form. However, the impact of the graphing calculator on students' learning was not readily determined because no classroom observations. The difference in approaches for the groups could have been attributed to the techniques taught in class. The control group may have been taught only the numerical method for refining parameters of functions. Additionally, the success that was attributed to the treatment group may have been due to the efficiency of the GC rather than ability of the students. Students could check answers easily with the graphing calculator than with a scientific calculator. To check their refinements, students in the control group using a scientific calculator would have to spend more time than the treatment group. Thus, the success attributed to the treatment group and the technique that they used to refine their answers could have been attributed to the tool rather than an increase in understanding or ability. The results of this translation approach may be considered in preparing the instructional materials for the current study and also guide in observing the students' actions when solving quadratic inequalities. This technique may help in checking and interpreting the solution of inequalities using the graphical representations enhanced by the use of the GC. It was noted that there was scant literature about the research on teaching and learning quadratic inequalities using the GCs. For that reason, this study fills that gap.

2.12 The roles of the graphing calculator in mathematics

This section discusses the role of graphing calculator in mathematics classroom as supported by a large portion of research studies (Doerr and Zangor, 2000; Averbeck, 2000; Leng, 2011; Karadeniz, 2015). They have identified at least five different roles of the graphing calculator use in mathematics classroom such as computational tool, transformational tool, data collection and analysis tool, visualization tool, checking tool. Through their findings, they have precisely elaborated how students use graphing calculators for each category in mathematics classroom.

The graphing calculator technology can be used as a computational tool particularly to evaluate numerical expressions and perform calculations. The studies revealed that two issues appeared in the use of graphing calculators as a computational tool: entering symbols and parentheses correctly, and the precision of computational results (Doerr & Zangor, 2000). Students use mathematical reasoning to explain the errors appearing on their graphing calculator screen for the first issue. The second issue arises particularly in rounding off the answers as students' interpretations of rounding off affect their problem solving strategies. They observed that students rounded off the numbers for every step of the problem rather than at the end of the problem.

As a transformational tool, the integration of graphing calculators in mathematics teaching and learning has impacted on the teachers' strategies to change the forms of some questions asked to their students and students' gains from the questioning techniques (Karadeniz, 2015). For instance, the teacher can use both paper-pencil and graphing calculator strategies for asking a question about the inequality solution. With paper and pencil method, students are only able to focus on a specific strategy to solve the question such as factorisation or quadratic formula and then decide for the inequality solution set. However, in the calculator-based form, students are able to determine graphically by showing the inequality graph with the required solution set being within or outside the critical values. Therefore, students have improved their global views of inequality solutions with calculators as "their computational skills are transformed to interpretative skills" (Doerr & Zangor, 2000).

Graphing calculators are also be used as a data collection and analysis tool in mathematics learning. Some of the mathematics activities involve gathering data, controlling phenomena, and finding patterns (Karadeniz, 2015). For instance, students are required to understand the context of the activity through formulating a desired pattern of numbers. In addition, students need to conjecture, refine, negotiate and decide what constituted a satisfactory set of data (Doerr & Zangor, 2000). In a given contextual problem as an activity, students will formulate and solve a quadratic inequality using a graphing calculator as well as decide on the satisfactory solution intervals.

Students can use graphing calculators as a visualization tool in four different ways: (1) to develop visual parameter matching strategies to find equations that fit data sets, (2) to find appropriate views of the graph and determine the nature of the underlying structure of the function, (3) to link the visual representation to the physical phenomena, and (4) to solve equations (Doerr & Zangor, 2000). In the context of quadratic inequalities, these suggested ways are possibly relevant and applicable. Using graphing calculators as a visualization tool also reflects students' ways of solving equations and inequalities (Karadeniz, 2015). For instance, the teacher explains all three available methods for solving an equation; including paper and pencil solutions, use of the calculator's solve button and graphical solution. Doerr & Zangor (2000) observed many students used the graphical approach, which was less computational but provided a more meaningful interpretation for the solution. In the case of this current study, the use of both graphical and graphing calculator approaches will provide more visualization to meaningfully solve quadratic inequalities.

Students use graphing calculators as a checking tool in mathematical tasks to check the conjecture for the problems. They generally pose a conjecture about a possible function as fitting a data set. Then, students checked how well it fits by using their graphing calculators. They also used graphing calculators to understand multiple symbolic forms (Doerr & Zangor, 2000). In this regard students will check their inequality solutions using calculators to confirm the correctness and reasonableness.

Leng (2011) however, identified six different ways of using graphing calculator (TI-Nspire) in a calculus class such as "an exploratory tool, as a confirmatory tool, as a problem solving tool, as a visualization tool, as a calculation tool, as a graphing tool (p. 935).” Three of these types namely, confirmatory tool, visualization tool and calculation tool were similar to the checking tool, visualization tool, and computational tool respectively in Doerr and Zangor’s (2000) category, in terms of their definitions. Students use graphing calculator (TI-Nspire) as an exploratory tool in mathematical tasks; as a problem-solving tool to attempt diverse approaches when solving mathematics problems; and as a graphing tool. The roles of the GC, above were considered in this current study as meaningful guidelines to observe the ways students use the tool in learning quadratic inequalities.

These roles of graphing calculators do affect the students’ learning in mathematics classroom. Students’ decisions whether to use graphing calculators are normally affected by their perspectives on external effects, such as teachers' perspectives on using graphing calculators (McCulloch, 2011). As explained by Ertmer, Addison, Lane, et al., (1999), the way teachers perceive the role of technology is related to how they use technology in the mathematics instruction. In other words, these roles are driven by the teachers’ perspectives on the use of graphing calculators in their classroom. For that reason, awareness of these roles can influence the effective use of graphing calculators in student learning of quadratic inequalities and in designing instructional materials. To effectively achieve integration of the GC, the researcher as a teacher cautiously considered the obstacles or dilemmas faced by teachers when they use technology in their teaching. Using graphing calculators efficiently in mathematics classroom created a supportive environment to help the students enhance their mathematical knowledge and understanding (Lee & McDougall, 2010). This means that the classroom environment enabled students to gainfully engage with graphing calculators as they learned quadratic inequalities. However, there are errors and misconceptions associated with the use of graphing calculators.

2.13 Misconceptions associated with the graphing calculator use

Some research studies have indicated the potential drawbacks of using the GC both technically and conceptually. These are students’ difficulties and misconceptions which are related to the use of the GC in mathematics (Mitchelmore and Cavanagh 2000). For example, students experience new types of errors and misconceptions as they learn mathematics, which are introduced by GCs. These errors are referred to as calculator-induced errors and effectively affect students' mathematics learning (Muhundan, 2005).

In her study of the students' general errors in GC-based classes, Tuska (1992) has identified eight GC associated misconceptions. These misconceptions fall into four categories namely: misunderstanding of the domain of a function, misunderstanding of the end behaviour and asymptotic behaviour of functions, misconception of the solution of inequalities, apparent misconception that every number is rational. In this regard, students need to observe and be informed about the GC-based errors and misconceptions in their use of GCs.

Mitchelmore and Cavanagh (2000) point out that, although there is adequate evidence that students occasionally misinterpret the graphic image, no systematic research have been conducted on the types of' GC- induced misconceptions. In their response, they investigated students' difficulties in using a GC. This particular study investigated how grades 10 and 11 students interpret linear and quadratic graphs on a Casio fx-7400G through clinical interviews. Their findings showed that students' errors in using a GC were attributable to four main causes: a tendency to accept the graphic image uncritically without attempting to relate it to other symbolic or numerical representation; a poor understanding of the concept of scale; an inadequate grasp of accuracy and approximation; and a limited grasp of the processes used by the calculator to display graphs.

Issues associated to scale may be due to students' lack of experience with graphs where the axes are not scaled equally (Williams, 1993, Thomas, 2016). Thomas (2016) pointed out that curricular materials often favour examples and exercises that are easily viewable in the calculator-standard 10x10 viewing window. In this case, students can encounter problems when non-standard window settings are required. This is also related to a poor understanding of using the calculator's built-in zoom operations and finding reasonable window setting manually. All GCs have a *zoom out* feature that increases the range of both axes equally, which is usually useful for adjusting the drawn graphs. However, in many circumstances simply zooming-out results in losing important features of the graph (e.g., the local extrema of a polynomial function). The physical design of the GCs usually contributes to this misconception. Normally, GCs have the common design features of rectangular screens, yet they are to display graphs using square windows. This results in some oddities such as perpendicular lines appearing as though they do not intersect at a right angle. Thomas (2016) emphasised that the error may look like a relatively minor quibble but it could conceivably create misunderstandings to some students.

Mitchelmore and Cavanagh (2000) observed misconceptions and misunderstandings related to accuracy and approximation among the participants in their study. For example, students tended to correlate a greater number of decimal places with increased accuracy when making approximations but at the same time showed a marked preference for integer values. This may also be a by-product of the curriculum as examples and exercises tend to favour problems with integer solutions. A better understanding of how calculator and computer algorithms work and the associated limitations could help to alleviate some related to the accuracy of calculator approximations.

Linking different representations of functions is another potential source of difficulty for students. Students have a tendency of accepting the graph displayed by the calculator without necessarily thinking critically about the results. For instance, students may fail to recognise inconsistencies between the symbolic form of a given function and the graph generated by the GC (Mitchelmore and Cavanagh, 2000). For example, the graph of a negative function can look different in the GC without a negative or the graph of a rational function can look different in the GC particularly near discontinuities. In situations like this, students may not think about their knowledge of the symbolic representations of the functions, leading them to misinterpret their GC output (Ruthven, 1990).

The final category of student difficulties in using the GC observed by Mitchelmore and Cavanagh, (2000) is in the representation of graphs by a finite number of pixels. It was noted that students commented on the jagged appearance of graphs caused by the GC’s low screen resolution. This low resolution and the irregular graphs that it caused can also lead to misunderstanding (Hector, 1992). For example, the asymptotic decay toward the horizontal axis of an exponential function disappears when the calculator can no longer display the curve as separate from the axis itself. This also could be associated with difficulties of linking the symbolic and graphic representations. It is conceivable that more accurate graphical depictions could be used by students to develop more coherent concepts of function types.

According to Thomas (2016), many of the difficulties experienced by students when using the GC are more to do with mathematics than technology. Such misconceptions can be traced to an incomplete or inadequate understanding of the underlying mathematical concepts. On the other hand, these difficulties may be compounded by the design and shortcomings of the calculator as in the case of the scale issues. It is further stated that the representations of graphs can be aggravated if the user does not have a firm understanding of the associated function properties (Thomas, 2016). In addition, Williams (1993) found that the GC can be confusing for some students even after instruction and experimentation and teachers also mistakenly assume that their students have shared understandings. The implications of this study are that when teachers design the instructional activities, they should consider these misconceptions of GC use.

2.14 Conceptual understanding in quadratic inequalities

In Skemp's (1976) views mathematical understanding comprises relational understanding and instrumental understanding. Ideally, the former refers to a situation where the student can know both what to do and why in the mathematics classroom. The latter refers to just being able to know or apply the rules without reasons (i.e., making sense of what these concepts mean and how they are interrelated). This means that in the familiarity phase the students develop an instrumental understanding and then later grows into relational understanding (i.e., at general level). With relational understanding students are able to explain the mathematical ideas (why) and how they are related. For instance, a student who understands quadratic inequalities can explain why the graphical representations are correct and how the solution sets can be meaningfully written.

Similarly, Kilpatrick, Swafford, and Findell (2001) describe conceptual understanding in mathematics as an integrated and functional grasp of mathematical ideas, which enables students to learn new ideas by connecting those ideas to what they already know. This means that students can understand the mathematical concepts when the new ones are linked systematically with the prior ideas and contexts so that they are able to transfer the constructed knowledge into solving the non-routine situations. In an attempt to gain a more complete understanding of the cognitive processes involved in quadratic inequalities, the topic should be integrated with quadratic functions and graphs. A careful analysis of behaviour on mathematical tasks certainly should come into play, suggested Williams, (1993). For example, students may or may not recognize a function and they may not have a complete understanding of all of the elements or be able to transfer the function between different representations of it – ordered pairs, table, equation, graph, etc. This kind of behaviour may compel the student to unconsciously accept the particular form as the definition and is unknowingly blind to the other forms possible of function (Parent, 2015). In this way, the student only understands a particular form of function; due to that being the only one used hence the student can only retain that particular form. Parent (2015) further pointed out that an important ingredient for understanding something is to know where it belongs in a larger scheme and to become familiar with its parts. This means students' understanding of quadratic inequalities can be developed through learning the history of inequalities, including their misconceptions. Gaining insight on what hinders students from forming their own schema of a concept can enrich the current pool of knowledge surrounding quadratic inequalities.

It is important that students should have adequate original information for the background knowledge in order to enhance their higher order thinking to solve quadratic inequalities. This can enable students to effectively transfer the previous knowledge so as to develop new mathematical concepts. The transfer of knowledge occurs when previous learning and experience is used in order to more quickly and efficiently learn a new skill, or mathematics content. According to Willingham (2006), students with a rich base of factual knowledge (i.e., coherent framework of knowledge) find it easier to learn more. In other words, when the student's previous procedural knowledge is sound, it allows him/her to concentrate on constructing new knowledge (ideas) of mathematics. For example, if students know their multiplication tables, they can learn the concept of factoring quadratic expressions to obtain the roots of a function easier. In order to retain deep understanding of the new mathematical concept, students must be engaged in meaningful and contextualized learning experiences. Again, the use of the GC as a potential learning tool may help the students construct new knowledge that can enable them to completely understand quadratic inequalities.

Understanding involves the connections of prior mathematical concepts by students that can be explored when they solve quadratic inequalities in this study. As defined by the NRC (2001), conceptual understanding is to comprehend mathematical concepts, operations and relations. This actually means students should possess proper knowledge of the main concepts and operations involved and how these concepts interrelate. In addition, students can only be able to make sense of the mathematics if they understand the concepts and operations that are involved (NCTM, 2009). In other words, students who understand the concept can identify and use the relationships that exist between the concepts in a given problem. For this reason, engaging students in mathematical tasks involving multiple representations is essential for understanding quadratic inequalities. This can be a meaningful way of facilitating the connections between the different representations of the quadratic inequalities. The National Council of Teachers of Mathematics recommends that high school students should be able to create and use tabular, symbolic, graphical, and verbal representations and to analyse and understand patterns, relations and functions (NCTM, 2000). In this way, students can understand the operations of quadratic inequalities using tabular, symbolic and graphical representations to explain the relationship. In addition, multiple representations in teaching and learning of the quadratic inequalities can enable students to conceptually understand the patterns and relations existing within the concepts. Working with representations allows students to move back and forth between the symbolic and graphic approaches as they solve quadratic inequalities. In this context, the use of the GC may provide visual representations of table, symbolic function and graph, which can make students develop coherent conceptual knowledge or understanding.

The Knuth's (2000) work was considered relevant to this current study as it used the analytical approach (graphical) to improve students' understanding. The study conducted with 284 high school students ranging from first year algebra through Advanced Placement calculus, concluded that although students often appear to understand connections between equations and graphs, their actual understanding of the connections is often superficial. Knuth found that: 1) students relied heavily on algebraic solution methods even if the graphical would have been quicker; 2) students seemed to have developed a ritualistic procedure for solving problems similar to those in the study; and that 3) students may have difficulties dealing with the graph-to-equation direction of solving problems. These observations indicate that students are dependent on rote procedural understanding, and then this conception may similarly be transferred to solving quadratic inequalities. This means that the use of the GC is needed to boost students' understanding of algebraic concepts in order to minimize the use of algorithms and memorization. In the case of this current study, the use of the GC may reduce the level of reliance on algebraic methods and increase the application of graphical methods when students solve quadratic inequalities. Notably, students who have a greater conceptual knowledge are able to apply and adjust their procedures to fit the problem at hand. Such students are capable of solving non-routine problems that they have never come across before.

Learning mathematics, therefore, is not just about finding answers but about the ability to interpret solutions and reflect upon these solutions. For Spinato (2011), this understanding is different from the learning that focuses on manipulative techniques and algebraic methods of problem solving. The desired learning should stimulate cognitive domains that allow students to systematically organise their knowledge into coherent framework which enables them to use reasoning and problem solving skills to unfamiliar problems. In the next section, these students' skills are explained as the constituents of conceptual understanding in quadratic inequalities.

2.15 Aspects of understanding mathematics

In this part of the study the researcher explains students' understanding of mathematics through its constituents: reasoning and problem-solving. Mathematics serves as a tool for problem-solving, communication, reasoning patterns of thinking and connectedness with other aspects (NoprianiLubis, et al., 2017). Similarly, the National Councils for Teachers of Mathematics point out that reasoning, problem-solving and sense making are the cornerstones of mathematics and should be present in every mathematics classroom (NCTM, 2009). In this context, reasoning and problem solving skills are used in this present study to explore the students' understanding of quadratic inequalities in a graphing calculator mediated classroom.

2.15.1 Mathematical reasoning skills

Student reasoning is an important cognitive skill of doing mathematics that has been recommended for inclusion in the curriculum. Reasoning is defined as the knowledge processing which uses evidence to draw conclusions or to produce assumptions (NCTM, 2009; Lithner, 2008). In simple terms, reasoning involves the ways of thinking in trying to solve mathematics problems and reaching to conclusions based on initial assertions. In addition, through reasoning processes, students make use of analytical skills to generalize and apply mathematics to unfamiliar or complex contexts (Garden et al., 2006). This means students formulate conjectures, draw logical deductions from a set of assumptions and justify their results. In this regard, students are engaged in a higher level of thinking that is informed by logical conclusions based on assumptions, reflections, explanations, and justifications. The National Research Council (2001) suggests that students who are not involved in mathematical activities that demands reasoning are cut off from the whole realms of human endeavour. In this perspective, mathematics is a human activity which is not just about finding answers but about the ability to interpret solutions logically and reflect upon them intuitively.

Reasoning is also viewed as the ability to think coherently and logically, and draw inferences from facts known or assumed (Mansi, 2003; Gunhan, 2014). In order to achieve this, students must be able to formulate and represent the mathematical facts adequately and further explain and justify the solution consistently. This line of thought allows students to continually refine and evaluate their own conjectures and assertions (Mansi, 2003), and to develop and maintain their reasoning skills (Gunhan, 2014). Regularly engaging in reasoning, according to Kilpatrick, Swafford and Findell, (2001), makes students build a productive disposition that can help mathematics make more sense to them. They, additionally suggest that asking students questions of “what” and “why” help to develop deeper thinking about the mathematical concept. This means incorporating such higher-level thinking questions into the daily mathematical instruction elevates the learning from procedural to conceptual understanding. Such exploring questions (how and why) help students to create conjectures of mathematics concepts. Along this vein, the students increasingly develop capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalizing.

Researchers have identified and explained the five main domains of reasoning: analysing a problem, initiating a strategy, monitoring one’s progress, seeking and using connections, and reflecting on one's solution (NCTM 2009; Spinato, 2011). Analysing a problem consists of identifying hidden structures, patterns and relationships, and making connections and deductions, and determining whether a solution is appropriate. Initiating a strategy as the second component involves selecting the appropriate concepts, representations, and procedures, making purposeful use of procedures, organizing solutions, and making logical deductions. The third domain, monitoring one's progress consists of reviewing the strategy that one has chosen to make further analysis of a problem and to modify selected strategies if necessary. The fourth domain, seeking and using connections, includes connecting different mathematical domains and connecting different contexts and representations. Reflecting on one's solution involves interpreting and justifying solutions, assessing the reasonableness of solutions, considering alternative ways of solving problems and generalizing solutions. In order to measure and understand students' reasoning, the five main domains of reasoning would be considered in designing the interview questions for this study.

Garden et al. (2006) has further indicated that reasoning is used for solving both routine and non-routine problems. Routine problems are those that students are familiar with and encounter often in a mathematics classroom. Non-routine problems, on the other hand, are those that students do not often encounter and are not familiar with solving. As explained by Schoenfeld (1992), non-routine problem solving can occur after routine problem solving. These types of problems require a level of thinking beyond that of routine problems. Although reasoning skills are used to solve both types of problems, it is the non-routine problems for which reasoning skills are most useful. Students apply their existing knowledge to a new situation that they encounter when solving problems of mathematics.

Reasoning is at the heart of learning mathematics conceptually, and has recently received greater attention in the high school setting (Graham et al., 2010; Nebesniak, 2012). Incorporating thinking skills into learning mathematics is essential to students' understanding and success. This implies that students need to experience mathematics and create meaning for them by reasoning about the mathematics and deduce the sense of what is happening within the mathematics (Graham et al., 2010). In this context, teaching that is focused on creating reasoning increases students' understanding of mathematics. In essence, teachers are expected to infuse reasoning into the curriculum by choosing specific mathematical tasks, facilitating math discussions (Graham et al., 2010), and posing questions that encourage students to think mathematically and explain their thinking (Nebesniak, 2012). An emphasis on reasoning can help students appreciate algebra conceptually (Graham et al., 2010) and facilitate a greater conceptual understanding of mathematical concepts (Nebesniak, 2012). In this context, the use of the GC in the mathematics classroom may assist students to develop both reasoning and sense making.

Teaching reasoning helps lay a strong foundation on which students can build an understanding of concepts (Graham et al., 2010). This means students can accurately carry out mathematical procedures, understand why those procedures work, and know how they might be used. This can help those struggling students suddenly experience mathematics for themselves. The use of GCs can help students retain what they have learnt by connecting to prior knowledge. Thus through the use of GCs students may link new concepts to other mathematical understandings.

2.15.2 Mathematical problem solving skills

Problem solving is a learning process that students normally get involved in school mathematics. When students learn mathematics, they develop important life skills such as analysing, interpreting, predicting, evaluating and reflecting (Anderson, 2009). This means that problem solving skills are students' life skills which can be continually applied when students search for solutions and connections in real-life situations. Kuzle (2013) defines mathematical problem solving as the process which usually involves several iterative cycles of expressing, testing, and revising mathematical interpretation and of sorting out, integrating, modifying, and refining clusters of mathematical concepts. In this context, problem solving provides an important context for learning mathematics as students are provided with a chance to analyse, interpret, predict and reflect on their solutions. Thus problem solving is central to understanding mathematics as students have opportunities to explore problems themselves and to monitor those problem-solving processes.

According to the NCTM (2010), problem solving is an integral part of all mathematics curricula as it involves tasks that have the potential to provide intellectual challenge and improve the understanding of students' mathematical development. Similarly, Karatas and Baki (2013) view problem solving as an intellectual, logical and systematic ability which helps students when dealing with mathematical problems, to search for multiple solutions then, select the best solution with regard to the conditions. Mahmoud Radi (2006) contend that teaching problem solving skills improves communication quality, increases assertiveness skills, and arouses self-efficacy and self-discipline of students. Similarly, McMoran et al. (2007) note that teaching problem-solving skills improved student understanding and personal skills as well as enriched students' mathematical conceptions. This means problem solving enables students to have deep mathematics knowledge and gives them the opportunity of pursuing their own learning enthusiasm of knowing mathematics. This further suggests that problem solving is an effective learning process with a positive drive for promoting students' thinking and understanding in mathematics learning.

Problem solving is the heart of mathematics education (NoprianiLubis, et al., 2017; Dede & Yaman, 2005). For that reason, researchers have suggested several strategies to be adopted for solving mathematical problems. One of the most commonly known problem-solving strategies were introduced by Polya (1945; 1957; 1971), who cited four broad steps to successfully solve a mathematical problem. The heuristic steps of problem solving process in his book *How to solve it* are: understanding the problem (stating the problem in your own words); devising a plan (formulating an inequality or strategy, or examining related problems); executing the plan (implementing a strategy or solving an inequality or checking operations and links); and looking back (checking or reflecting on the results). These heuristic steps enable students to solve the mathematical problems that they created competently as would have gained the required experience. For this reason, this research study draws upon Polya's ideas and modified them such that they can be applied to developing the students' abilities of solving quadratic inequality problems.

Problem-solving is a framework within which creative thinking and learning takes place in an attempt to overcome difficulties that appear to interfere with the goal attainment (Perveen, 2010). This process in mathematics is used to solve problems that do not have obvious solutions (Polya, 1945; Perveen, 2010). Polya (1957) further claimed that not introducing non-routine problems to students was an unforgivable mistake and added that it was a necessity to include these problems in mathematics. The routine problems, as noted by Polya, cannot improve the imagination of students because of its mechanical solutions. This implies that students should develop heuristics for solving the non-routine problems. This process actually provides opportunities for students to create their own approaches and to gain the requisite experience from their own discoveries (Polya, 1957; Perveen, 2010). In this context, students use their experiences and knowledge to solve the mathematical problems even when they do not explicitly know the solution. However, students who cannot understand a problem may not be able to find and use suitable strategies and may not be able to explain what they are doing.

Research reports have put an emphasis on student reasoning and problem solving activities in mathematics (Karatas, & Baki, 2013). In CAPS mathematics curriculum, (DBE, 2011), problem solving is described as a cognitive skill that should be along with mathematics teaching and learning. The mathematics curriculum as explained by Karatas and Baki (2013), bases on the principle that every child should learn mathematics with the basic skills such as problem solving, communication and reasoning. In reference to CAPS FET Mathematics document, four levels of cognitive skills are stated to be developed in students namely procedural knowledge, routine and complex reasoning, and problem solving (DBE, 2012). This implies that problem solving skill is essential for students doing mathematics and for comprehending mathematics meaningfully.

In the context of using technologically enhanced learning environments, students can seek for additional solutions for the problems posed on them for deeper understanding. For example, one of these tools is the GC which provides students a chance to visualize modelling variations (Idris, 2006). The use of the GC, as Goldenberg (2000) stated, offers students a richer and deeper understanding of mathematical topics and also help students improve their problem solving skills. Therefore, this study attempts to use the GC in anticipation to improve and retain students' problem solving strategies and their underlying reasoning skills in quadratic inequalities. Moreover, students' understanding of quadratic inequalities is provided by making sense of a problem, seeking to justify why the solution is true and applying a carefully thought-out strategy (e.g. GC) to ill-defined problems for the inequality solutions.

Many of the reviewed studies indicated and suggested that the use of the GCs improved student achievement and understanding in mathematics. In addition, the use of the GC significantly improved students' problem solving, reasoning, communication, and representation skills. These studies have laid a strong foundation for the current study and it is hoped that the current study would provide encouraging preliminary results about students' reasoning and problem solving skills of quadratic inequalities in the graphing calculator-enriched classroom.

2.16 Chapter summary

The purpose of this research was to investigate the students' understanding of quadratic inequalities in a graphing calculator mediated environment. This chapter began with a discussion of the historical developments of inequalities, inequalities in the South African curriculum and research on inequalities. Through the reviews of multiple studies the researcher noted that inequalities started with mathematics and still remains the pillars of many mathematical contents. However, not many studies have been conducted on inequalities and there is limited amount of literature available specifically to the quadratic inequalities. It was noted that inequalities do not have a long history and makes it difficult to explore the actual steps of success and misconceptions in learning this particular topic. It is essential to understand the historical developments of inequalities so that the educators can concentrate on those misconceptions when designing the instructional materials. Seemingly learning of quadratic inequalities is difficult to be understood by students of high schools. For this reason, this study explored the ways of teaching quadratic inequalities, including the graphing calculator.

A comprehensive review of the related literature on the use of the graphing calculator technology, its role in secondary mathematics instruction, and misconceptions of the GC was performed. In reviewing the literature, the researcher noted that making the graphing calculator available for the students to use as an instructional medium, had an added value to their understanding of quadratic inequalities. Graphing calculators are used to improve classroom dynamics, boost students' confidence levels, and promote students' thinking and problem-solving abilities in quadratic inequalities. The reviewed research studies revealed that the use of the GC had been widely investigated in many areas of mathematics, in particular their effects on students' understanding of concepts. However these studies have not focused much on the understanding of quadratic inequality concept. This means that the use of graphing calculators in quadratic inequalities is sparsely studied and remains unresolved issue. The FET mathematics curriculum (CAPS) is specifically designed to take advantage of the use of the GC as a potentially available technology that can engage students in exploring and solving real-world problems. A large amount of research supporting the use of the GC has expressed the significant gains that may assist students to understand mathematical concepts. The use of the GC as a possible alternative method can connect algebraic method with functions and this allows the students to switch freely between algebraic and visual graphical representations. The use of graphic calculator reduces the drudgery of applying arithmetic and algebraic procedures when solving quadratic inequalities. Students are free to spend more time on problem solving and visualize data in more than one way.

Evidence from the reviewed literature has shown that preparedness of teachers and consistent use of the GC can improve student conceptual understanding in mathematical concepts. Thus, there is a need for the research on the impact of the uses of graphing calculators on the students' understanding of quadratic inequalities. This study provides a rare opportunity to investigate students' reasoning, problem solving and sense making skills of quadratic inequalities in a graphing calculator environment as key aspects of students' understanding.

The next chapter discusses the theoretical frameworks supporting this study on students' understanding in a graphing calculator mediated mathematics classroom.

CHAPTER 3: THEORETICAL FRAMEWORKS

3.1. Introduction

This chapter discussed theoretical perspectives related to the research questions that helped in designing instructional activities and interpreting students' understanding of quadratic inequalities in a graphic calculator enhanced classroom. To this end, the following theories of learning mathematics were discussed in this chapter: sociocultural constructivist (Vygotskian) theory, Realistic Mathematics Education (RME) theory and instrumentation theory (IT), and are compatible with one another. These theories assist in explaining the kinds of productive classroom practices that enhance students' reasoning, the heuristic methods used to solve mathematical problems and the ways of interactions between teacher, students and mathematics concepts in a GC enhanced classroom. The use of these compatible perspectives was aimed to provide a rich spectrum of what constitutes students' understanding, and how mathematics teachers can enact practices that enhance student thinking in a graphing calculator-enhanced classroom.

These theoretical perspectives generally place students at the centre of learning rather than as recipients of direct instruction. In student-centred learning, as suggested by White Clark, DiCarlo & Gilchriest, (2008), students are involved in discovering information while the teacher serves more as a guide and facilitator of learning. The RME and instrumentation frameworks are influenced and associated with constructivist teaching. As pointed out by Amineh and Asl, (2015), the constructivist perspective influences many areas of thought on mathematics education, including particular teaching strategies, conceptual understanding, reasoning and the use of technology. In this way, these perspectives provide students with opportunities to work in a social setting of groups (i.e., mathematics classroom).

3.2. Sociocultural constructivist theory

The purpose of this section is to explore and understand how the key concepts of Vygotskian theory (such as cultural, historical and social) ca n contribute to the cognitive development and learning of the students in the mathematics classroom technologically supported. This section discusses the socio-cultural interaction and zone of proximal development (ZPD) as the main concepts of the theory (Vygotsky, 1978; 1987). The implications of this theory on learning mathematics in a technologically enhanced classroom are also included for discussion. Researchers (e.g. Kravtsova, 2009; Vygotsky, 1997; Ruthven, 2013) distinguish a socio-cultural interaction as fundamental for the full cognitive development of the students in the mathematics classroom, while a ZPD explains the construction of student's knowledge and understanding with the aid of adult or peer who is regarded as a more knowledgeable other. These themes of the theory are expected to provide cognitive power to the teacher-researcher on how to integrate the use of the GC in the teaching of quadratic inequalities..

3.2.1 Socio-cultural environment and discourse

Social constructivism, in Ross' (2006) views, is a theory of knowledge in sociology and communication theory that examines the knowledge and understandings of the world that are developed jointly by individuals. This means students develop their mathematical understanding, significance and meaning of the concepts through interacting with other human beings and their environments. Social constructivism, strongly influenced by Vygotsky's (1978) work, suggests that knowledge is first constructed in a social context and then internalized and used by individuals (Eggen & Kauchak, 2004). Vygotsky considers cognitive development primarily as a function of external factors such as cultural, historical, and social interaction rather than of individual construction. This means culture plays an important role in the construction of knowledge and does model the behaviour of the students.

Learning is a social process that is affected by student's culture (Panhwar, Ansari, & Ansari, 2016) and culture is a critical construct for the cognitive development of students (Vygotsky, 1986). It is further noted that student's education equally relies on both the outside sociocultural forces and the inner stimuli (Panhwar, Ansari, & Ansari, 2016). In this context, the student's intellectual development is initiated by social and cultural influences and interactions which lead to higher and deeper mental development and functions. It is of great importance to note that the human mind rarely works solitarily, but in social contexts. For this reason, socio-cultural environment is a necessary factor in learning and is very critical for intellectual development and understanding of mathematics.

A critical review of literature (e.g., Wertsch, Rio & Alvarez, 1995) indicates that the theory has divided the student's intellectual development into the inter-psychological plane and intra-psychological plane. The inter-psychological development ensues when the student communicates with other people and intra-psychological development takes place when innovative efforts are used by the student to strengthen his/her learning after having acquired from other individuals and society. This specifically suggests that student's cultural development occurs firstly at the social level (inter-psychological) and later, at the individual level (intra-psychological) (Wertsch, Rio & Alvarez, 1995). The student initially acquires knowledge through contacts and interactions with people and then later assimilates and internalises this knowledge adding his personal value to it (Panhwar, Ansari, & Ansari, 2016).This means a meaningful learning occurs when individuals (students) are engaged in social activities and then integrate this into the individual's mental structure (inside the mind of the student).

This transition from social to personal property is not a mere copy, but a transformation of what had been learnt through interaction, into personal values (Panhwar, Ansari, & Ansari, 2016). The idea is when two or more students interact on a psychological plane the student may internalize the information, structures and functions. In the social learning context, the social functions, speech and discourse usually prepare the student for the higher mental activities and synthesis (Wertsch, Rio & Alvarez, 1995). Within the context of educational practices, students do not merely copy their teachers' capabilities but rather they transform what teachers offer them during the processes of appropriation –that is using graphing calculator. To this end, knowledge is not derived directly from the reality but from different perspectives. This implies that the teacher-researcher should consider the social and cultural context and the CAPS policies when delivering sessions of quadratic inequalities in a graphing calculator-enhanced classroom.

Language is critical in the learning process for students as it is used as a means of communication between a student and any person in the environment. Communication is one of the advocacies of the socio-cultural theory in the learning environment and has assisted students to acquire knowledge from one another and from more able peers, parents and teachers (Panhwar, Ansari, & Ansari, 2016). This means, in the socio-cultural contexts, the student internalises and processes the information and knowledge gained through communicating with other people. Thus, the processed information improves the student's development of knowledge and true understanding (Kohler, 2010). The reviewed literature also reveals that social interactions shape the child's mental development and the child does not shape society (Panhwar, Ansari, & Ansari, 2016). This suggests that socio-cultural experiences have a greater influence on the way students learn and build their concepts about the surrounding objects. This implies that in a technologically rich classroom students can be adequately prepared for the higher mental functions and synthesis to internalize the information and structures.

Language is the first stage of development as a vocal means of learning in the social process (i.e. classroom) (Kohler, 2010). Through the appropriate use of language, students can be moved beyond the initial stage of development to internalise and organise the thought processes (Obukhova & Korepanova, 2009). This therefore suggests that a meaningful classroom language can facilitate the meaning construction out of student's thought processes. Students need to make meaning out of thought processes by communicating the results. Meaning can function in terms of mind and behaviour within the social context. Meaning exists in two places: between two people and in the thinking process; —the transactional space between the individual and the world of objects and events. The student firstly constructs the meaning out of his/her environment and then moves to a higher level when it is internalized. The use of appropriate language by the teachers in a GC mediated mathematics classroom may stimulate students to construct their meanings and concepts correctly even to move to the higher levels.

Tajuddin, Tarmizi, Konting and Ali (2009) argue that the *e*xploratory activity in mathematics may facilitate an active approach to learning as opposed to a passive approach where students just listen to the teacher. This type of learning environment can assist students to internalise accurate information and construct correct meaning in the mathematics classroom. The same researchers indicated that the use of graphing calculators provides various kinds of guided explorations that should be undertaken in functions, graphs, equations and inequalities. This means students may use GC to explore the effects of changing parameters of a function on the shape of its graph and also explore the relationships between the graphs and inequalities. In such instances, the use of GC has provided an exploratory learning environment.

3.2.2 Vygotsky's Zone of Proximal Development

The second theme of Vygotsky's theory has the potential for cognitive development which is limited to a zone of proximal development (ZPD). This zone is defined as the distance between a child's "actual developmental level as determined by independent problem solving" and the higher level of "potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky 1978, p. 86). This is the area of exploration and potential learning for which the student is cognitively prepared, but requires assistance and social interaction to fully develop (Sibawu, 2013). Students are allowed to mediate, internalize and develop new concepts, knowledge and skills through interacting with competent people in his learning environment. In this sense, a teacher or more experienced peer is required to provide the learner with necessary support in the ZPD.

In his work, Vygotsky has described student's learning as movement through a series of levels of development with the aid of someone who has higher knowledge. This movement occurs in the student's zone of proximal development. Researchers (e.g., Bozhovich, 2009; Verenikina, 2003) explained the difference between the student's levels of cognitive development in the ZPD. The student's actual level of ZPD refers to what a student can demonstrate alone or perform independently (Bozhovich, 2009) and that is a "yesterday of development" (Verenikina, 2003). The potential level of development refers to the next level attainable through the use of mediating semiotic and environmental tools and capable adult or peer-student facilitation (Bozhovich, 2009) and that is a "tomorrow of development" (Verenikina, 2003). Thus, the ZPD is the distance between what a person can do with and without help of knowledgeable adult or peer. The term proximal (nearby) indicates that the assistance provided goes just slightly beyond the learner's current competence complementing and building on their existing abilities. In this way, an instructor's teaching of a student is regarded not just as a source of information to be assimilated but as a lever with which the student's thought is shifted from a lower level to the higher one.

Roosevelt (2008) argued that in order to keep students in their own ZPDs teachers should give them interesting and culturally meaningful learning and problem-solving tasks that are slightly more difficult than what they do alone. Such tasks can push students to work together either with another, more competent peer or with a teacher or competent adult to finish the task. The idea is that after completing the task jointly, the student will possibly be able to complete the same task individually next time, and through that process, the student's ZPD for that particular task will have been raised. According to Campbell (2008), this process is then repeated at the higher level of task difficulty that the learner's new ZPD requires. Thus through such collaborative endeavours with more skilled persons, students learn and internalize new concepts, psychological tools and skills. Learning mathematics in the ZPD means a student can perform a range of mathematical tasks that he cannot yet handle alone but with the help of instructors or more capable peers. The idea is that student is engaged in cooperative dialogues with more capable partners to discuss the assigned tasks. Consequently, a student may acquire the appropriate language and make it part of his/her private speech, and use this speech to organise his/her independent performance in the same way. In other words, he/she acquires the correct methods and skills, and then uses them in his/her independent performance later.

The socio-cultural learning theory advocates scaffolding in the classrooms. Scaffolding (ZPD) is the assistance given by the more knowledgeable adults, which enables a student or an inexperienced person to solve problems, perform activities or accomplish targets which he/she could not achieve without help (Panhwar, Ansari, & Ansari, 2016). With Daniels (2001), scaffolding is a form of assistance that enables a novice to solve a problem, carry out a task, or achieve a goal which would be beyond his/her unassisted efforts. For this reason, scaffolding in mathematics learning is central as students can solve problems which they could not have succeeded without teacher's or peer's help. The theory underscores student-student interaction and student-teacher interaction, and rejects a teacher-centred approach. It further emphasises the significance of the active and participatory relationship between a student and a supportive teacher in any form as people, peer, social norms and values, ritual, and customs (Daniels, 2001). Thus, a Vygotsky's ZPD model might be appropriate to provide an all-encompassing structure to involve teachers and learners in significant and fruitful collaborative strategies for mathematics learning, in particular, integrating the use of graphing calculators to solve quadratic inequalities.

Vygotsky's theory also emphasises the significance of externally mediated activity that involves the use of external means to reach the student's goal (Verenikina, 2003). This suggests that mediation is central to the sociocultural learning theory. Mediated action is a process that helps human consciousness to develop through interaction with artefacts, tools, and social others in an environment and can result in individuals to find new meanings in their world (Panhwar, Ansari, & Ansari, 2016). This means people who have different levels of skills and knowledge (e.g., teachers) select and shape the students' learning experiences (Verenikina, 2003; Stetsenko & Arievitch, 2004). This process is connected with the application of tools and signs to reconstruct the meanings of the world. The other significant people adopt these cultural tools, having both psychological and social functions, to perform an activity for them and with them (Stetsenko & Arievitch, 2004). Artefacts, or cultural products, are those things which are manufactured and created by people in the culture which include a pen, spoon, table, language, traditions, beliefs, arts, science and so on (Panhwar, Ansari, & Ansari, 2016). For example, graphing calculators may be considered as external means and used as artefacts/tools to solve quadratic inequalities in the mathematics classroom. The idea is that students interact between and with the cultural tools to reconstruct their meanings and functions.

Researchers have argued that student's action is mediated and cannot be separated from the milieu in which it is carried out (Verenikina, 2003). The idea is that learning usually takes place where the use of psychological tools is available to a learner in his or her environment. Learning is about actions of the students and it should be studied in context where the actions take place. For example, a student's speech, thinking, and utterance in the learning process are considered as actions, so they are carefully mediated. The idea is that the students' actions in the mathematics learning process need to be mediated in order to develop their cognitive thinking and true understanding. Psychological tools provide educational activities and materials in a child's learning which are generally considered tools. This means that computers or graphing calculators are Vygotsky's psychological tools referred to and hence a means of mediation. These tools particularly in learning of mathematics provide students with a diverse mode of thinking.

Mediation done through symbols causes not only quantifiable progress in terms of competence and fluency, but it also results in cognitive improvement in terms of critical reasoning and thinking (Panhwar, Ansari, & Ansari, 2016). Thus, the process of the theory suggests that social tools and signs transform and improve the overall flow and organisation of psychological functions of students. This shows that the emergence of new cultural tools (e.g., graphing calculator, GeoGebra, and computers) can transform power and authority from teachers to the students. This means that interacting with GCs can assist the students to transform the learning opportunities in the mathematical classroom and move them into and through the next layer of thinking or understanding.

Another notable aspect of Vygotsky's theory is instruction that is received by students in classrooms. According to Verenikina, (2003), instruction should be geared towards the zone of proximal development that is beyond the learner's actual development level. The idea is that the higher mental functions of students –e.g., mediated perception, logical thinking, deliberate attention and memory- are acquired through learning and teaching. However, teachers should provide opportunities for students to mediate and assist each other in the creation of ZPD in which each student learns and develops. In this context, students can discover the whole meanings by themselves following teacher's instructions in classroom settings.

The theory further regards teacher's instruction as crucial to students' development in the classroom. The theory suggests that the most efficient instruction that can appropriately engage students in learning activities within a supportive learning environment should be mediated by tools (Verenikina, 2003). The instructional tools can be cognitive strategies, a mentor, peers, computers, printed materials, or any instrument that organises and provides information for the student (Verenikina, 2003). In that respect, these instructional tools provide students with dynamic support that can help them complete a given task near the upper end of their ZPD.

However, the mediated support is systematically withdrawn as the students move to higher levels of confidence (Panhwar, Ansari, & Ansari, 2016; Sibawu, 2013). This means the mediated instructions in the students' ZPD should provide appropriate assistance that enables students to increase their potential for future participation. In this study GCs function as tools to mediate, or influence, human activity and learning. This implies that the use of graphing calculator as an instructional tool to solve quadratic inequalities may raise the potentially cognitive development of the grade 11 students in South Africa.

3.3 Realistic Mathematics Education (RME)

RME is a domain-specific instruction theory for mathematics education that has its origin in the Netherlands (Van den Heuvel-Panhuizen, 2013). In this theory, guidelines for instructions are offered that support learners to construct or reinvent mathematics interactively (Gravemeijer, 1994). The idea is to promote the construction of mathematical knowledge and skills of students through the use of models. In this context, the learning of mathematics is a constructive activity in which students are not spoon-fed with factual knowledge (i.e. reproductive learning). Students do not learn through memorizing knowledge but through constructing their own conceptual knowledge. Similarly, a student should always have a learning tool (model) at his/her disposal to help bridge the gap between the concrete and abstract. This theory emphasises the solution of contextual problems and focuses on developing students to start thinking mathematically.

Zulkardi (2014) is of the view that students should be actively involved in the learning process in order to be able to create their own meaning and solutions of real life mathematical problems. This suggests that students work out contextual problems using tools or models, (e.g. graphing calculators) to enhance their thinking and problem solving skills. Such models can allow collaborative learning among students as they meaningfully engage in solving problems in context. Mathematics learning activities cannot simply rely on reception, imitation and memorization (Peters, 2006) rather it should be a process that is lively animating and participatory. Learners therefore should be given room to engage in mathematical activities that are relevant and meaningful to their lives (Barnes, 2005; Gravemeijer, 1994). It should be noted that as a neo-constructivist approach (Ndlovu, 2014), RME provides a possibility for involving students in purposeful activities in a meaningful context. RME theory stresses that mathematics learning should, in Freudenthal's (1977) view, be connected to reality, maintained closer to learner's experience and be relevant to society, in order to be of human value. For this reason, this domain-specific instruction theory may provide the much needed ideas on how to develop a meaningful framework of mathematical relations and objects in this current study.

Researchers proposed the similar (but different number of) learning and teaching principles of the RME approach (Treffers, 1991; Van den Heuvel-Panhuizen, 2010). These include the activity principle (learners use their own productions); level principle (use of models); reality principle (use of context); intertwinement principle (use of various learning strands); interaction principle (interactivity in the teaching/learning process) and the guidance principle (guided re-invention). However, these learning and teaching principles are parallel to the de Lange's (1987) tenets of the RME. The tenets are: (1) *the use of real-life contexts*; (2) *the use of models*; (3) *student's free production*; (4) *interaction*; (5) *intertwining*. In the present study the instructional activities for students are designed and conducted in accordance with Van den Heuvel-Panhuizen's (2010) six principles underpinning RME pedagogy.

3.3.1 The Activity Principle: The use of students' own productions

This first principle of RME considers learning mathematics as a constructive activity (Treffers, 1991), which could best be learned by doing it (Van den Heuvel-Panhuizen, 2013). This principle actually (in Freudenthal's (1991) views) interprets mathematics as a human activity, in which students are treated as active participants. The transfer of ready-made knowledge directly to students is considered to be an 'anti-didactic inversion' (Freudenthal, 1973, 1983, 1991). Rather, students should be confronted with problem situations so as to develop all sorts of mathematical tools and insights, formal or informal, by themselves (Cheung & Huang, 2005). In this context, students are actively involved in the classroom and mathematics becomes meaningful. Consequently, students have the opportunity to produce more concrete objects by themselves in order to develop their own informal problem solving strategies. This can assist students to discover relationships and learn to use their knowledge to develop mathematical concepts by themselves. In addition, the activity principle implies that learners should be confronted with problem-situations in which they can gradually develop their own strategies based on an informal way of working. It should be noted that student's own constructions play a pivotal role towards their understanding of the mathematical concepts. The role of the teacher-researcher in this study, however, should be to facilitate learning in such a way that each individual student can develop his/her own algorithmic way of solving the quadratic inequality problems. As such in this present study rather than being recipients, students were given activities (tasks) of quadratic inequalities to work on by themselves using GCs (see Appendix C).

3.3.2 The Level Principle: The use of models

In this principle, the learning of a mathematical concept or skill is viewed as a process which progressively stretches out over the long term and various levels of abstraction. This principle advocates the use of models and symbols to develop or scaffold students' understanding of mathematical concepts from intuitive, informal and context-bound towards more formal notions (Bakker, 2004). This means the use of a variety of models, schemes, diagrams, and symbols should support this developmental process and these instruments should be meaningful for the students and should have the potential for generalization and abstraction. The use of these instruments can enable students to move through different levels of understanding in the process of learning mathematics. In this present study the use of graphing calculator may help students to move through the different levels as it has the potential for producing visual models, diagrams and symbols.

The level principle emphasises the learning of mathematics using models that can help students make progress from informal to more formal mathematical activity. Gravemeijer (2000) describes how initial models that students create of a contextual situation change and become an entity of their own. In this instance, the intuitive model of a certain (contextual) situation later functions as a model for more formal mathematical reasoning. This means that the use of models moves student thinking away from the contextual situation to the mathematical relationships. In the case of quadratic inequalities, the notion of a graphical representation that displays quadratic graph was envisioned to become a model of numerical points and later a model for more formal quadratic inequality reasoning.

Gravemeijer (1994) describes the four levels of emergent modelling in more general terms: *situation* level, *referential* level, *general* level and *formal* level (Drijvers, 2003; Bakker, 2004; Cheung & Huang, 2005; Van den Heuvel-Panhuizen, 2013; Ndlovu, 2014). This suggests that the realistic mathematics learning starts with modelling the real problem and then ends with modelling situations that give rise to formal knowledge. Gravemeijer (1994) explains the four hierarchical levels as: 1) a *situational level* where students work within the context of the problem. 2) a *referential level* where their models refer to the situation. 3) a *general level* where the mathematical focus is on strategies that govern the reference to the context, and 4) a *formal level* where students work with conventional procedures and algorithms.

In this level principle student activities should first start from the (informal) situation level closely bound to problem contexts so that domain-specific situational knowledge and strategies can be used (Cheung & Huang, 2005). The second or referential level encompasses the use of concrete mathematical models representing mathematical objects. This is a level of 'model of' in reference to the concrete models' close connection to the situations described in the problem (Van den Heuvel-Panhuizen, 2003). The third or general level is a transitional level in which relationships are analysed through general mathematical models that can be dissociated from the problem contexts. The dissolute models are referred to as 'models for' where the focus is more on paradigmatic (or typical examples of) solution procedures that can be used to solve new problem situations (Van den Heuvel-Panhuizen, 2003). The fourth or formal level allows pure cognitive thinking or higher level of formal mathematical reasoning, reflection and appreciation (Cheung & Huang, 2005). This level principle can be used to structure an entire learning sequence of quadratic inequalities. These varying levels may assist in designing the instructional materials of the students, in particular where graphing calculator is used as an instructional instrument. For this reason, the levels of emergent modelling can be adapted to a larger variety of mathematical learning activities.

In this context, a student understanding of the mathematical concept starts from lower level (familiarising with concept) through devising informal context-connected solutions to the higher level (relationships/schematisation). My understanding, in relation to this study, is that students can gain insight into how concepts (quadratic inequalities) and (graphing calculator-enhanced solution) strategies can be related by reflecting on the activities in which they have purposefully participated. This reflection can be elicited by interaction (Van den Heuvel-Panhuizen, 2013). Students' levels of understanding the concept can be enhanced through the use of models which serve as an important device to bridge the ZPD between informal, context-related mathematics and more formal mathematics. Firstly, learners develop strategies closely connected to the context. Later, certain aspects of the context situation can become more general, which means that the context acquires the character of a model. These models give students support for solving other related problems and access to more formal mathematical knowledge. The literature reveals that in order to bridge the gap between the formal and informal level, models shift from a model of a particular situation to a model for all kinds of other equivalent situations (Cheung & Huang, 2010; Van den Heuvel-Panhuizen, 2013). It could be noted that models are rooted in concrete situations and they are useful for higher levels of mathematical activities. This means that the models will enable learners to access the formal mathematical knowledge organized as a discipline. In this study students will use GCs to develop strategies that enable them perform high levels of quadratic inequality situations.

This principle underscores the importance of growth in mathematical understanding (Peters, 2016), thus from the concrete or enactive, to the iconic and, ultimately, to the symbolic representational forms (Ndlovu, 2014). Thus, learning from one level of understanding to another requires scaffolding by more knowledgeable others or the sequencing of instruction in such a way that new learning carefully builds on previous knowledge (cf: Scaffolding in Section 3.2.2). For example, the instructional activities of quadratic inequalities in this study did not end with concrete models, but extended to the use of graphs to determine the regions of the solutions and ultimately to the determination of the symbolic representations (quadratic inequalities) from contextual problems, thus signifying some progression from everyday experiences to 'models of' (horizontal mathematisation) to 'models for' and to higher levels of mathematical reasoning or proof (vertical mathematisation).

3.3.2.1 Horizontal and vertical mathematization

Within the RME, horizontal and vertical mathematization has stood out to be one of key concepts for students' understanding in mathematics. Horizontal mathematization refers to moving from the world of life to the world of symbols (Freudenthal, 1991), schematising the transformed problem mathematically by mathematical means (Treffers, 1987) and mathematizing reality (Drijvers, 2003). This means students can organize, translate, and transform the realistic contextual problems into mathematical terms (i.e., mathematizing reality). The horizontal mathematisation process deals with ordering, schematizing and building models of real situations so that they become open to mathematics (Peters, 2016). On the other hand, Gravemeijer (1994) views this horizontal mathematisation process as describing the contextual problem in order to identify the central relations and to understand the problem better. He further states that this description of the problem does not automatically answer the question, but simplifies the problem by identifying major and minor aspects of the problem. In this context, horizontal mathematisation can be inferred as an understanding of the problem, the language and the intention of the question. It involves an understanding of the context of the problem and an attempt to make mathematical statement. Peters (2016) further clarifies the real contexts in mathematical problems should provide students with a need to engage in the problem and bring forward important mathematical ideas.

Vertical mathematisation is described as the progression of shortening (Freudenthal,1991) and a shift to a solution method that is more sophisticated, better organized and more mathematical (Gravemeijer and Terwel, 2000) and reflection (Gravemeijer, 1994). This can be referred to as when students start shortening their path to a mathematical result but use a more sophisticated path. This could mean that if a student explicitly replaces his or her solution method by one on a higher level. Gravemeijer (1994) explains that students start their mathematical activity by mathematising from reality (informal descriptions of problem situations) and then analysing their own mathematical activity (shaping algorithms and interpreting solutions with mathematical language). Vertical mathematization can be described as the activities that are related to the mathematical process, the solution of the problem, the generalization of the solution and the further formalization (Drijvers, 2003). The models, schemes, symbols and diagrams are instruments that can be used for vertical mathematization (Treffers, 1987). This means progressive mathematizing involves the formalizing and schematizing of informal problem-solving strategies. An example given by Freudenthal (1991), when counting sets of eyes in a group a student starts counting in twos instead of ones. Simply put based on the critical shifts in young students' view of numbers, students move from "adjectives" (eight beads) to nouns (eight) or move from "referents" (1/2 a bar) to "entities" (1/2). In such scenario, it is related to the shift from model of to the model for in mathematics, thus a vertical mathematisation shift. The use of models enables student to build their presentations on each other to develop powerful mathematical ideas. The use of GC may move the student from informal understanding of quadratic inequalities to formal level. At this formal level, students can explain the solution of the inequality by mere looking at the graphical representation.

3.3.3 The Interactivity Principle

This principle states that mathematics education is by nature interactive. This signifies that the learning of mathematics is not only a human activity but also a social activity (Van den Heuvel-Panhuizen, 2010, 2013). Similarly, Treffers (1991), viewed mathematics learning as not an isolated activity but it occurs in a society and is directed and stimulated by the socio-cultural context. To that end, learners should be afforded opportunities to share their experiences, strategies and inventions with each other. For this reason, this principle is linked to social constructivist theory of Vygotsky (See Section 3.2.1). By discussing each other's findings, students can get ideas for improving their strategies (Van den Heuvel-Panhuizen, 2010). This means in their working groups for example, students exchange their ideas and arguments so that they can learn from others.

In interactive instruction, discussion and collaboration enhance reflection on the work (Cheung & Huang, 2010; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 2013).This suggests that keeping students involved in interactive activities can result in them being reflective and ultimately leading them to achieve the higher level of understanding. In the context of this present study, students were divided into manageable groups to answer given tasks using GCs as an interactive tool. The use of the GCs can provide students with a range of opportunities, which makes them easily to refer to the visual representations. Moreover, interaction can evoke both individual and collective reflection, which can scaffold students to higher levels of mathematical understanding (Ndlovu, 2014). In addition, through purposeful interaction students can acquire meaningful ideas for improving their learning strategies. By solving the mathematical problems the students can possibly experience that these problems can be solved differently and at different levels of understanding. The emphasis of this principle is that productive learning in mathematics results from interaction among students accentuated with opportunities for improving their problem solving strategies. Therefore, in this study the emphasis is placed on the fact that GCs may create opportunities for students to actively interact in their learning of quadratic inequalities.

3.3.4 The Reality Principle: The use of context

The reality principle embraces the RME aims of having students who are capable of applying mathematics (Van den Heuvel-Panhuizen, 2010) in the real context. To that end, students should start with rich and meaningful contexts demanding mathematical organisation (Freudenthal, 1991; Cheung & Huang, 2010) that can enable horizontal and vertical mathematisation (Ndlovu, 2014). This principle discourages teachers who begin with abstractions or definitions when teaching mathematical topics as these contexts cannot be mathematised by the students. This means students can develop mathematical tools and understanding when exposed to realistic contexts that can be mathematised. The application of mathematical knowledge is not only considered as something that is used at the end of a learning process but also at the beginning (Van den Heuvel-Panhuizen, 2010). The realistic problem situations in learning activities, in Drijvers' (2015) views, are experientially real to students and meaningful, authentic as starting points, so that students experience the activity as making sense. In this principle, the importance of using real contexts that are meaningful and natural to learners as a starting point for their learning cannot be overemphasised (Cheung & Huang, 2005; Van den Heuvel-Panhuizen, 2010; De Villiers, 2012). For example, in this current study Moses Mabhida Stadium was used as contextual example to introduce the concepts quadratic function and quadratic inequality. This activity allowed students to make conjectures with regard to solutions of quadratic inequalities. (see Activity 8 in Appendix C).

Thus, like any progressive approach, this reality principle strives to enable students to use their tools to solve experientially real problems. In this case, learners should use their mathematical understanding and tools to solve realistic problems (Peters, 2016). The use of GC is indirectly recommended by the CAPS FET Mathematics (DBE, 2011) as an instructional tool that may support this realistic approach when solving quadratic inequality contextual problems. With the aid of GCs students may create visual or mental situations matching mathematical tasks assigned to them which are informed by their own ideas, experiences and imagination. The instructional materials in this study were designed in a manner that allowed participants to mathematise everyday experiences of quadratic inequalities. The emphasis of this principle is that productive learning in mathematics starts with rich and meaningful contexts in an organised manner horizontally and vertically. Therefore, in this study the emphasis is placed on the fact that starting lessons of quadratic inequalities with experientially real and meaningful contexts may increase the students' understanding in a graphing calculator mediated classroom.

3.3.5 The Intertwinement Principle

This principle emphasises the importance of coherent learning in mathematics and a mutual relationship between different mathematical concepts. The notion is that learning mathematics involves the construction of specific knowledge and skills within a connected body of (mathematical) knowledge (Treffers, 1991). The advocacy of this principle is to integrate various mathematical topics and develop an integrated approach to solve mathematics. For example, a better understanding of quadratic inequalities requires the students to have the knowledge of functions, algebra and geometry. Along this line of thought, a concept/topic becomes the part of the solution to the other concept/topic. This integrated approach provides students with flexibility to link topics to different sub-domains and to other disciplines (Van den Heuvel-Panhuizen, 2010). For example, topics like factorisation, arithmetic, functions, and linear inequality are closely related and can be used to solve quadratic inequalities. This means that mathematical domains are not considered as isolated curriculum topics but as heavily integrated (Ndlovu, 2014). In line with this principle, students were given tasks involving rich problems in which they could use or link with different mathematical knowledge both within and across concepts in a subject. This principle aligns with Shulman's (1986) knowledge of the curriculum

(KC). For example, in this study solving quadratic inequalities was integrated with quadratic equations when critical values were involved, quadratic functions when determining the zeros as the boundaries for the regions and were also linked to interval (set builder) notations for expressing solutions.

This principle enables the students to view mathematics as part of a real life solution as it can be implemented within different parts of a prescribed mathematics topic and beyond. This principle of RME intertwines the concepts, topics and approaches in order to develop students' understanding. Van den Heuvel-Panhuizen (2013) mentions that the strength of this principle is that it renders coherency to the curriculum in terms of the school curriculum. In essence, the learning strands in mathematics remain intertwined with each other.

3.3.6 The principle of guided re-invention

In this line of thought, students progressively mathematize their own mathematical activity when they learn mathematics (Treffers, 1987) and they can reinvent mathematics under the guidance of the teacher and the instructional design (Bakker, 2004). The guided reinvention principle states that students should experience the learning of mathematics as a process similar to the process by which mathematics was invented (Gravemeijer, 1994). This suggests that teachers can steer the learning process by providing a powerful learning environment in which the process of mathematical knowledge construction can emerge. The learning process may be meaningfully steered even in a GC-enhanced classroom, where students can be purposefully engaged in mathematical activities such as solving quadratic inequalities.

This principle puts emphasis on mathematics as a process in which learners learn mathematics in activities guided by their teachers or their peers (Sembiring et al., 2008). The idea is that students are provided with a 'guided' opportunity to 'reinvent' mathematics by 'striking a delicate balance between the force of teaching and the freedom of learning' (Freudenthal, 1991, p. 55). In this context, students can apply their mathematical knowledge when they are taught with methods that can create learning opportunities. In this study the GC is used as the delivery mode which can potentially create opportunities for the students in learning quadratic inequalities in the guidance of the teacher-researcher. In this context, the instructional materials, including the designed activities of quadratic inequalities guided the student participants to work in small groups and to present their work to others. The groups were also individually accountable for the completion of their own homework and assessment tasks as well as post-test. The students were additionally encouraged to seek assistance within them first and foremost and consult the teacher-researcher as a last resort. The intention was to create adequate room for the students to re-invent mathematics and to design possible strategies. The teacher-researcher remained available all the time to anticipate participants' difficulties and to help groups in meaning negotiation and collective self-reflection on the effectiveness of their problem-solving strategies in quadratic inequalities. The significance of the guidance principle is that teachers must be able to foresee where and how they can anticipate the students' understandings and skills that are just coming into view in the distance (Van den Heuvel-Panhuizen, 2003; Ndlovu, 2014). This implies that with proper planning of students' activities teachers may create opportunities for them to re-invent mathematics.

It is therefore important for teachers to provide learners with learning environments in which guided re-invention can be possible. It is important to note that this principle, on the learning side, aims at allowing learners to regard the knowledge they acquired as their own personal mathematical knowledge for which they have been responsible. On the teaching side, teachers should provide learners with opportunities to develop their own mathematical knowledge.

3.4 Instrumental approaches with graphing calculator

This section presents the main elements of the instrumentation theory: instrument, instrumental genesis and instrumental orchestration. The theory of instrumentation was developed in the context of educational research on the effective integration of technology into mathematics education by the French researchers, who preferred to call it the 'instrumental approach' initially rather than the theory of instrumentation (Trouche, 2004; Ndlovu, et al., 2013). The instrumental approach is a means of analysing technology-mediated teaching and learning in mathematics (Artigue, 2002; Guin, Ruthven & Trouche, 2005). This approach allows teachers to explain how students appropriate technological tools such as graphing calculators to represent and develop mathematical concepts (instrumental genesis) and use them as instruments to solve mathematical problems (schemes of instrumented action). Ultimately this study describes the teachers' competencies related to didactic management in the technology-enhanced environment (instrumental orchestration).

3.4.1 Instrument and artefact

The theory of instrumentation concerns handling tools and develops on the notions of tool use. A tool or artefact is not automatically a mediating instrument. This means the tool becomes an instrument when its need has been felt and turned out to be valuable instrument for performing a specific task successfully. The following is a simple example used by Drijvers,

> *A hammer may initially be a meaningless thing to a prospective user, unless he/she has used it before, or has seen somebody else using it. Only after the need to have something like a hammer is felt, and after the novice user has acquired some experience in using it, does the hammer gradually develop into a valuable and useful instrument that mediates the activity* (2003, p. 96).

This implies that a hummer becomes an acceptable and useful instrument when the user has acquired the rightful skills to use it and knows exactly in what circumstances to use it. A similar distinction between tool and instrument can be made for other artefacts, such as graphing calculators. For Rabardel (1995), an instrument mediates the activity when there is a meaningful relationship between the artefact, the user and a type of tasks, in this case solving quadratic inequalities. Other researchers stated that the instrument consists of both the artefact and the accompanying mental schemes developed by the user (Drijvers, 2003; Drijvers and Trouche, 2008). This means that for an artefact to become an instrument for a user (mental schemes) there must be a meaningful relationship between them.

Verillon and Rabardel (1995) attest that an artefact can be any physical, electronic, or symbolic tool or technology that influences the activity and thinking of the user. As often referred to in mathematics education literature, an artefact can be a computer algebra system (Drijvers, 2003; Guin & Trouche, 2002), graphing calculators (Artigue, 2002), dynamic geometry (Cayton, 2012), spreadsheet software (Haspekian, 2005), new forms of smart handheld devices (Trouche & Drijvers, 2010). Fahlgren (2015) describes an artefact as an object, material or abstract, available to the user and aimed at performing a certain type of task. Kratky (2016) relates the notion of artefact to the technologies that can perform mathematical tasks and/or respond to the user's actions in mathematically defined ways. In this study an artefact refers to actions of the GC being appropriated by the students to solve the quadratic inequalities. In other words, artefacts can transform the learning opportunities in the classroom as they function as tools that may mediate, or influence, human activity and learning. This means that an instrument is a function of artefact (the graphing calculator), mathematical tasks (quadratic inequalities) and mental scheme (student).

3.4.2 Theory of instrumental genesis

This section discusses how the available tool or artefact can lead to the development of a useful and meaningful instrument through the process of the instrumental genesis. The process of instrumental genesis is used to reflect a relationship that develops between the user and the artefact by which an artefact becomes an instrument for the user (Fahlgren, 2015; Artigue, 2002; Drijvers & Gravemeijer, 2005; Trouche, 2004). Similarly, Kratky, (2016) used this theory to describe the relationship that developed between a mathematical user and the technological tool in his study. Rabardel (2002) describes instrumental genesis as the iterative process by which the user and the artefact influence each other during activity when the user interacts with the tool. This implies that when the user (teacher or student) engages with the artefact in the mathematics activity, they both influence each other. This further suggests that students' understanding of a mathematical concept can be developed or not depending on the appropriateness of the use of technological tool. In her opinion, Drijvers (2003) indicated that students' conceptual and technical (or artefact-oriented) development shows a close relation to each other. In this context, students constantly transform the artefact when they engage in activity with it, thus forming mutual mediation between the student and artefact.

The instrumental genesis, *a*s explained by Trouche, (2004), consists of two relational directions: one towards the artefact (instrumentalisation) and another towards the user (instrumentation). In reference to the former, an artefact affects and shapes the user's actions and the character of the knowledge constructed by the artefact's constraints and potentialities (affordances and enablements). Expressed differently, instrumentalisation is an instrument mastery process that can go through various stages including discovery and selection of relevant functions, personalisation and transformation of the artefact itself (Trouche, 2004; Ndlovu, et al., 2013). During this instrumentalisation process the user attempts to get to know the instrument, to master the instrument and to adopt the instrument to one’s own personal specific needs. For example, various potentialities of the artefact are progressively discovered, or possibly transformed in personal ways, if there is consistent use of the instrument.

By contrast, instrumentation means that a person, who uses the instrument, is affected and shaped by the artefact. Differently stated, instrumentation is the process by which the user is mastered by his/her artefact or by which the artefact prints its mark on the user by allowing him/her to develop activities within some limits (Ndlovu, et al., 2013). In this case, the artefact shapes and transforms the person (user) to enable him to do something that the tool was not originally designed for by adapting to its constraints and possibilities (affordances). For example, using a graphing calculator to represent a quadratic function may affect student’s conceptualisations of the notion of quadratic inequalities in mathematics classrooms.

The focus of the instrumental genesis theory is on developing student's ability to construct and understand knowledge. For this reason, the theory is developmental and psychological in the teaching and learning process. Instrumentalization is a psychological process because it is related to the way of shaping the cognitive activity i.e. the artefact in use. This process develops ways of using, manipulating, and shaping the artefact in use, an organization of usage schemes, a personalization and sometimes transformation of the tool, and a differentiation between the complex processes that constitute instrumental genesis and those which are critical for teachers to master (Guin & Trouche, 2002). On the other hand, instrumentation is a developmental process because mental schemes or instrumented action schemes emerge as users execute a task (Drijvers & Trouche, 2017). When the task is completed the uses of a certain tool become internalized and implemented. This implies that the tool is used to enhance concept development and understanding. In this bi-directional relationship students perform trivial routine classroom activities guided by the teacher (i.e. the level of instrumentation) and are often engaged in non-routine mathematical problems in their free time (i.e. the level of instrumentalisation).

In this process of instrumental genesis the tool is developed in an appropriate and sensible way (Drijvers, 2003) and its results are condensed in the form of utilization schemes. Differently stated, the instrumental genesis consists of building up utilization schemes. Researchers (e.g., Guin & Trouche, 2002; Rabardel, 1995; Trouche, 2000; Drijvers, 2003; Ndlovu, et al., 2013) identified two kinds of utilization schemes of instrumental genesis such as the utility schemes and schemes of instrumented action. The utility schemes (schèmes d'usage), as the first category involve adapting an artefact for specific purposes by changing or extending its functionality. For example, a PC can be upgraded with a new version of a word processing software package and a calculator menu can be customized by downloading additional programs and applications into the graphing calculator. These schemes concern the instrumentalization of the artefact (refers to the user adapting the tool). The schemes of instrumented action (schèmes d'action instrumentée) as the second kind are viewed as coherent and meaningful mental schemes for using the technological tool to solve a specific type of problems. For example, the use of graphing calculator to solve quadratic inequalities using the 'SOLVE', 'TABLE' and 'GRAPH' schemes. An experienced user (student) quickly and accurately applies these schemes by means of a sequence of key strokes and/or mouse clicks. A novice user, however, has to deal with the technical and conceptual aspects. These schemes of instrumented action are concerned with instrumentation of the artefact (refers to using the tool for solving specific cognitive tasks). The instrumentation process leads to the building up of schemes of instrumented action that are useful for fulfilling a specific kind of task.

The utility schemes and schemes of instrumented action − the instrumentalization and instrumentation − in some cases are related, and it is not easy to decide what kind of schemes are developed (Drijvers, 2003). However, this study restricts its focus to concentrate on the algebra-related instrumentation techniques in order to guarantee the coherence with the purpose and research questions on the students' understanding of quadratic inequalities. In particular, the study uses simple algebraic instrumented action schemes for solving quadratic inequalities and, tabular and graphical solving as well as combining these simple instrumentation schemes into more complex instrumentation schemes. In this context, the graphing calculator is used as the available technological tool to achieve specific tasks that can contribute to an improved understanding of quadratic inequalities.

An instrumentation scheme (scheme of instrumented action) has an external, visible and technical part, which concerns the machine actions (Drijvers, 2003) and the mental, cognitive part, which is the most important aspect of the scheme for conceptual development and understanding. For example, suppose a student wants to solve the equation: $-2x + 8 = -2x^2 + 8$ with a graphing calculator. The graphical solving scheme involves the mental step of seeing both sides of the equation as functions: $y_1 = -2x^2 + 8$ and $y_2 = -2x + 8$, which can be drawn. Furthermore, applying the scheme involves the conception of a solution as the x coordinates of the intersection points of the two graphs. These are mental activities that give meaning to the technical actions such as entering functions, drawing graphs and calculating intersection points.

The instrumentation scheme that illustrates the relation between conceptual and technical aspects involves the scaling of the viewing window of a graphing calculator. Students usually have the difficulties with scaling the viewing window of a graphing calculator (Goldenberg, 1988). Students need to be developed the technical skills for setting the viewing window dimensions and the mental image of the calculator screen where the position and the dimensions of a relevant rectangle need to be chosen. These examples illustrate that an instrumentation scheme integrates the machine techniques and mental concepts by interplaying between acting and thinking. It is therefore conjectured that the conceptual part of this scheme can cause the difficulties for students to understand mathematical concepts. Technical skills and conceptual insights are inextricably bound up with each other within the instrumented action scheme. As suggested by Drijvers (2003), the mental part consists of the mathematical objects involved, and of a mental image of the problem-solving process and the machine actions in the case of technological tools. This means the conceptual part of instrumentation schemes includes both mathematical objects and insight into the 'mathematics of the machine'.

According to Haapasalo, (2012), instrumentation and instrumentalisation appear when using CAS, dynamic geometry, dynamic statistics, CAD, online databases, supported learning environments, virtual environments, digital portfolios, etc. The same processes can be performed with hands-on technology-the graphing calculators to solve mathematical problems. Knowledge gained through these processes is not sterile without any transfer, but socially generated, viable knowledge that has both cognitive and pragmatic relevance. Using hands-on technology can potentially promote sustainable mental activities (competences) in which the students make their own interpretation (consequences) against the standard view: e. g. deciding whether the solution of the quadratic inequality is within or outside critical values.

3.4.3 Theory of Instrumental orchestration

Integrating technology meaningfully in the mathematics classroom comes with didactical challenges for the teachers. This has been acknowledged upfront by the researchers (e.g. Trouche, 2004; Artigue, 2002) who stated that there is a complexity of competencies or skills required of teachers when using technology within mathematics classrooms. Similarly, Lagrange and Monaghan (2009) articulated that the availability of technology amplifies the complexity and, as a consequence, challenges the stability of teaching practices. This means teachers are expected to manage new pedagogical situations and at the same time develop a new repertoire of appropriate teaching practices for these technology-rich settings (Drijvers et al., 2014; Ndlovu, et al., 2013). The use of theory of instrumental orchestration, in Trouche's (2004) views, helps teachers how to organize and support students' learning of mathematics in a technology-enhanced classroom. The idea is that the teachers need to influence and steer students' instrumental genesis through skilful use of instructional technology in the mathematics classrooms.

An instrumental orchestration is defined as the teacher's intentional and systematic organisation and use of the available artefacts in computerised learning environment in a given mathematical task situation, in order to guide students' instrumental genesis (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010), including GCs. Stated differently, the skilful use of ICT for instructional purposes with the goal of facilitating instrumental genesis in learners is referred to as instrumental orchestration (Ndlovu, et al., 2013). This means that teachers use a repertoire of their competencies to address the didactical challenges of teaching mathematics in a technology-rich classroom. Such competencies may help to guide students on how to use and interact with instruments to solve mathematical problems in a technology-enhanced classroom. In simple terms, an instrumental orchestration describes the teacher's role in guiding and shaping students' use of technology and their opportunities to engage in instrumental genesis (Kratky, 2016). Within this context, teachers above all should help students to actualise the epistemic function of their schemes– to develop mathematical understandings and knowledge using the artefact cum instrument. The notion is the teacher steers students' learning experiences as (s)he has the power to influence which artefacts the students may access and how they may use those artefacts.

3.4.4 The constructs of instrumental orchestration

Drijvers et al. (2010) identified three constructs of instrumental orchestration within a teacher's instructional activity in a computerised learning environment: a didactic configuration, an exploitation mode and a didactic performance.

A didactic configuration is a teacher competence that refers to the arrangement of artefacts/ instruments in the classroom environment (Drijvers et al., 2010) such as tools, materials, and seating (Kratky, 2016; Ndlovu, et al., 2014) to induce a sound mathematical discourse. Put differently, in the process of didactic configuration the teacher gets into plans of how to select the artefacts that students can use and how to arrange them in the classroom (Trouche, 2004). This means that GCs can be arranged in a way that favours individual work, working in small groups and/or whole class setting by the teacher. This may include any artefacts that students can use on their own or in groups and any artefacts or presentation technologies (such as an overhead projector or smartboards) that may be used during the learning activity. In the teaching experiments students sat in groups of four so that they can interact among themselves.

An exploitation mode is a teacher competence that refers to the way the teacher decides to exploit a didactical configuration (Ndlovu, et al., 2014). Drijvers, et al. (2010) viewed this mode as the way in which the teacher decides to use a didactical setting or configuration for the benefit of his or her instructional intentions and how a mathematical task can be worked through. For example, the tasks can be performed in whole-class, groups or individually. This means a teacher must make a decision as to whether to let learners work in small groups, or individually, with worksheets or with instructions on the board or projected on a screen. This competency allows teachers to plan and exploit ways of how to engage students with mathematical tasks and tools in a technology-rich classroom. With the exploitation mode, in Trouche's (2004) view, the teacher decides how to leverage the didactic configuration with respect to his or her goals for the orchestration. (S)he may demonstrate a particular artefact technique, establish a link between work done with the artefact and work done with paper and pencil, and have a student to present his or her work with the artefact, or initiate a discussion related to a representation generated by the artefact. Each of these examples represents different exploitation modes that the teacher may use to facilitate learning in a particular manner.

Drijvers et al. (2010) further note that the teacher may plan for and design the didactical configuration and exploitation modes for specific instrumental orchestrations, but cannot fully plan his or her didactical performance, since it includes actions that a teacher makes in response to the students and activity within a particular lesson. A didactical performance, as suggested by Ndlovu, et al. (2014) is a teacher competence that refers to ad hoc decisions taken during the teaching process itself. This didactical competence may involve issues such as what question to pose, what interruption(s) to make to draw learners' attention to unexpected technological tool behaviour during task performance (Drijvers et al., 2010) and how to respond to a particular student's response, and how to deal with the unexpected under changing circumstances (Ndlovu, et al., 2011; Kratky, 2016). This means a teacher cannot completely anticipate students' experiences, struggles and successes when using artefacts to solve mathematical activities. Thus, a teacher must be competent to make in-the-moment decisions during a lesson, which make up his or her didactical performance. This- making on-the-spot instructional decision- is achievable when teachers competently use multiple pedagogical moves during their didactical performances in a technologically learning environment.

3.5 Chapter summary

This chapter critically reviewed and discussed the theoretical frameworks related with the research questions of this study and those that were perceived to be helpful in designing instructional activities and interpreting students' understanding of quadratic inequalities in a graphing calculator enhanced classroom. To this end, sociocultural constructivist (Vygotskian) theory, Realistic Mathematics Education (RME) theory and, instrumental genesis and orchestration theory (TIG) were considered as the theories of learning mathematics and were compatible with one another. These frameworks generally place students at the centre of learning as there are influenced and associated with the constructivist teaching and learning. In this way, students are provided with opportunities to work in a social setting of groups (i.e., graphing calculator mediated mathematics classroom).

The researcher noted that the success of the mathematical concept development of students depends on the potentiality of the educational technology (artefact), students' knowledge in the instrumental genesis and the teacher's instrumental orchestrations as teacher's competences for integrating graphing calculator into his/her pedagogy. The aforementioned theoretical frameworks provided the pedagogical strategies and opportunities for teaching and learning the quadratic inequalities in a graphing calculator rich classroom.

The next chapter is Chapter 4 on methodology.

CHAPTER 4: RESEARCH METHODOLOGY

4.1 Introduction

This chapter discusses the research philosophical assumptions (paradigm), research design and research methods of the study. In this way, the study endeavoured to generate new knowledge that can make mathematics teaching and learning more purposeful and sustainable in a graphing calculator environment. The design-based research (DBR) is presented as a paradigmatic methodology within the real context of educational enquiry that embraces mixed-method research framework. This research methodology, DBR is developed and employed as the main research paradigm informing this design study. Thus, it involves a flexible three-stage research framework such as the preliminary phase, teaching experiment and reflective phase. With the use of DBR, this chapter provides a rationale for developing learning theories and improving mathematics learning as well as generating hypotheses. In this sense, the importance of hypothetical learning trajectory (HLT) is also explained in this part of the study.

This chapter further discusses the sequential mixed methods research of exploratory design as data collection and analysis structure. The mixed methods approach, which combines the quantitative and qualitative research methods for use in a single research project, is foregrounded in DBR methodology. Furthermore, the justification for selection of the participants, sampling procedures and the research instruments in the study is discussed. The chapter includes a diagrammatic representation of the major facets of the envisaged framework for the research design and development of the study, and a discussion on the schematic model envisaged for this study. Finally, in order to ensure reliability, validity and trustworthiness of the research, appropriate criteria for pragmatic research methodology are presented, including triangulation. This chapter concludes with a summary.

4.2 Research paradigms in education

Researchers perceive world differently and have assigned different meanings to the concept of paradigm (Creswell, 2009; Livesey, 2011). This has accordingly affected the manner in which they describe and interpret data in their philosophical thinking. The term *paradigm* refers to a research culture with a set of beliefs, values, and assumptions that a community of researchers has in common regarding the nature and conduct of research (Kuhn, 1977). This aspect concerns the conceptual framework shared by a community of scientists and philosophers which provides them with a convenient model for examining problems and finding solutions using appropriate methodological approaches and tools. This means a paradigm plays a fundamental role in the social sciences and research as it best describes the basic set of the researcher's beliefs, values and assumptions.

Renowned researchers have described research paradigm as researcher's worldview (Mackenzie & Knipe, 2006; Creswell, 2009) and as researcher's beliefs, shared assumptions, concepts, values and practices (Johnson & Christensen, 2004) that shape the way s/he perceives and interprets the world around him/her. Others conceived the research paradigm as the researcher's basic set of beliefs that guide the development of the research (Guba and Lincoln. 1994; Denzin & Lincoln, 2005; Alghamdi, 2015). More specifically, a research paradigm would include the accepted theories, traditions, approaches, models, frame of reference, body of research and methodologies; and it could be seen as a model or framework for observation and understanding (Creswell, 2007; Babbie, 2010; Rubin & Babbie, 2010; Babbie, 2011**).** Viewed differently, research paradigms constitutes the perspectives, thinking, school of thought or set of shared beliefs, that informs the meaning or interpretation of research data (Kivunja & Kuyini, 2017). This means the researcher uses the paradigm to determine the methodology of his/her research project, the research methods and how the data can be analysed. As suggested by Alghamdi (2015) and Morris (2006), any particular study should state the research questions or aim at the very beginning followed by the appropriate research paradigm in order to be compatible with the methodology chosen. Poni (2014) adds that research paradigms represent a critical element in the study as they influence both the strategy and the way the researchers construct and interpret the meaning of the reality. In essence, research paradigms influence what should be studied, how it should be studied, and how the results of the study should be interpreted based on the researcher's individual experiences.

4.2.1 Positivist paradigm

The *positivist* paradigm believes that the truth exists independently of the observer (Alreshidi, 2016; Gall et al., 2003) and this truth can be sought by applying the scientific methods of investigation (Kivunja & Kuyini, 2017). Muijs (2011) emphasised that the positivistic researcher attempts to understand the truth about how the world works. The researcher involves the scientific method in the search for cause and effect relationships in nature (Kivunja & Kuyini, 2017) and that tries to interpret observations in terms of facts or measurable entities (Schunk, 2008). Researchers pointed out that the positivists use quantitative methods to investigate the truth about the phenomena (Mackenzie and Knipe, 2006; Barbie & Mouton, 2009; Mouton, 2011; Muijs, 2011). In this sense, the researcher may use experimentation, observation and reason based on experience to understand and explain the observable and measurable human behaviour through unbiased means.

Positivists also believe that an objective reality exists outside personal experiences with its own cause-and-effect relationships (Neuman, 2006; Babbie & Mouton, 2008; Saunders et al., 2009; Muijs, 2011). This implies that positivism entails a belief that valid knowledge can only be produced on the basis of direct observation by the senses; and this would include the ability to measure and record what would be seen as knowledge. Observation in this sense means accepting only empirical evidence as valid evidence. In the context of the positivist paradigm, these assumptions have enabled the researcher to collect, analyse and interpret, and understand relationships embedded in the data (Kivunja & Kuyini, 2017). In this paradigm the researcher is an outsider and the research is not dependent on the researcher (Alreshidi, 2016; Mouton, 2011; Williams, 2007). Therefore, the role of the positivist researcher is to precisely describe and understand the parameters and coefficients in the data of the research study using the prescribed scientific knowledge. For this current study the positivist paradigm played a big role as pre- and post- tests and pre- and post- survey questionnaires were used to collect, analyse and interpret the quantitative data of students' performance in a graphing calculator mediated classroom.

The ontological position of the positivists is that there is only one truth and an objective reality that exists independent of human perception. Epistemologically, the investigator and investigated are independent entities. Therefore, the investigator is capable of studying a phenomenon without influencing it or being influenced by it; “inquiry takes place as through a one way mirror” (Guba and Lincoln, 1994, p.110). The goal is to measure and analyse cause-and-effect relationships between variables within a value-free framework (Denzin and Lincoln, 1994). The scientific theories can be tested by statistical and controlled variables through using surveys or experiments (Hammersley and Atkinson, 2007). This implies that the positivist researchers adopt objective ways of discovering the truth. Therefore, the obtained knowledge is objectively determined which can limit feelings or any subjective experiences.

Sample sizes are much larger than those used in qualitative research so that statistical methods to ensure that samples are representative can be used (Kivunja & Kuyini, 2017). The results of large sample sizes allow the researcher produce generalisations. This paradigm relies on deductive reasoning (Creswell, 2009; Harwell, 2011; McMillan & Schumacher, 2001), formulating and testing hypotheses (Mack, 2010) and offering mathematical calculations and extrapolations (Kivunja & Kuyini, 2017). Through these activities the researcher can make better explanations and predictions based on measurable outcomes. This means the positivist researcher uses the descriptive and inferential statistics to organise, test, interpret and infer collected quantitative data to produce and to generalise valid evidence. The numerical nature of positivist paradigm (i.e., quantitative nature) influences the researchers to use statistical instruments. These statistical instruments help researchers to organise and simplify the measurements, scores or number (Johnson & Christensen, 2008). In this current study the researcher performed the calculations of mean, frequencies, range, standard deviation, effect size, Cohen’s d; draw histogram and also performed hypothesis testing, t-tests and dependent paired t-tests to describe the instrument reliability and to report data quantitatively.

4.2.2 The interpretivist (constructivist) paradigm

Interpretivist paradigm focuses on exploring the complexity of social phenomena with a view of gaining better understanding. The interpretivist claim is that reality is constructed in a subjectively social manner (Tuli, 2011) for understanding individuals’ behaviours (Guba & Lincoln, 1989), investigated within their own social environment (Parahoo, 2006; Alreshidi, 2016). It means that interpretivists study the social events within its subjective contexts without controlling any variables which differs from positivists' views. The purpose here is to understand and interpret everyday happenings (events), experiences and social structures – as well as the values people attached to these phenomena (Collis & Hussey, 2009; Rubin & Babbie, 2010). Interpretivists believe that social reality is subjective and nuanced, because it is constructed by the perceptions of the participants, as well as the values and aims of the researcher. This approach depends on the interaction of investigators with the subjects under study (Alreshidi, 2016). This might lead to different people's perspectives as people perceive different meanings to the same events or phenomena. This means that the interpretivist researcher always attempts to interpret the participants' experiences and events, and then constructs meanings and understanding from these social interactions.

This paradigm attempts to get into the head of the subjects being studied (Kivunja & Kuyini, 2017) in order to understand and interpret the subject's thinking and the meaning s/he has of the context (Wang and Zhu, 2016). Similarly, researchers have indicated that both knowledge and the researcher cannot be separated since the researcher is the only source of the reality (Mackenzie & Knipe, 2006; Muijs, 2011; Wang & Zhu, 2016). This means the researcher cannot be an outsider but should be part of the world that is being investigated. In this context, every action/gesture made by the subject is observed and interpreted by the researcher. Moreover, emphasis is placed on the interpretation of the participants' point of view of the case being investigated (Creswell, 2003). Therefore, the key tenet of this paradigm is that the reality is socially constructed (Mertens, 2005; Bogdan & Biklen, 1998). For this reason, this paradigm can be called the constructivist paradigm because they are contextually related. In this paradigm, Kivunja and Kuyini (2017) point out that theory is grounded in the data generated by the research activity. This means theory does not precede research. The idea is data are gathered and analysed in a manner consistent with grounded theory (Strauss & Corbin, 1990). This paradigm therefore relies on subjectivity and inductive reasoning. For the reasons advanced, the interpretivist paradigm (Barbie & Mouton, 2009; Mackenzie & Knipe, 2006) advocates the use of qualitative research methods to precisely understand and interpret the viewpoints of the subject observed in the study. Interviews, observation, document reviews and visual data analysis are typical examples of qualitative data collection tools used in this paradigm (Mackenzie and Knipe, 2006).

Interpretivists claim that an objective observation of the social world is impossible, as it has meaning for humans only, and is constructed by intentional behaviour and actions. Livesey (2011) explains interpretivism as a method that sees the social world as something that can only be produced and reproduced on a daily basis by people. Something that holds true for the moment (now) might not necessarily hold true tomorrow, or in another society (social environment). Knowledge is developed and theory is built through developing ideas from observed and interpreted social constructions. As such, the researcher seeks to make sense of what is happening. This can even generate findings beyond the common scientific knowledge (Rubin & Babbie, 2010). So, interpretivists attempt to understand subjective realities and to offer explanations, which are meaningful for the participants in the research.

Ontologically, interpretivist researchers believe that there are multiple realities or truths which exist based on one's construction of reality; this leads to variety of meanings for various people. Reality is socially constructed and so is constantly changing (Mackenzie & Knipe, 2006). On the epistemological level, they stated that there is no access to reality independent of our minds and no external referent by which to compare claims of truth. The investigator and the object of study are interactively linked so that findings are mutually created within the context of the situation which shapes the inquiry (Guba and Lincoln, 1994; Denzin and Lincoln, 1994). This suggests that reality has no existence prior to the activity of investigation, and reality ceases to exist when the researcher no longer focuses on it (Smith, 1983). The emphasis of qualitative research is on process and meanings.

In so far as research methodology is concerned, Henning et al. (2004) hold that the interpretive understanding is grounded in an interactive, field-based inductive methodology and is intertwined in the practice within a specific context. Livesey (2011) proposes that the best methods within the interpretive research paradigm are those of observation and interpretation. This means that the researcher would understand how human beings experience and interpret their world through the use of in-depth and focus group interviews and participant observation. Transcripts, conversations and video-tapes may be studied, in order to gain a sense of subtle non-verbal communication or to understand the interaction in its real context (Neuman, 2011). Samples are not meant to represent large populations, rather, small, purposeful samples of articulate respondents. This implies that small, purposeful samples can provide important information, not because they are representative of a larger group (Reid, 1996). This, in addition, allows the interpretivist researcher to engage in active collaboration with the participants so as to address real-life problems in a specific context. The tenets of qualitative research method are integrated in this current study to effectively understand how students experience their learning in a graphing calculator enhanced classroom.

4.2.3 The Pragmatic Paradigm

This pragmatic paradigm embraces the philosophical views that it was not possible to access the truth about the real world solely by virtue of a single research approach, hence discourages the use of one system of philosophy and reality. This paradigm advocates the integration of positivist and interpretivist paradigms. Philosophers (such as Alise & Teddlie, 2010; Biesta, 2010; Tashakkori and Teddlie, 2003) push for the worldview that would provide integrative scientific methods of research. This implies that the combined research approaches would provide the most appropriate benchmarks for studying the phenomenon at hand. As Kivunja and Kuyini (2017) opine, the pragmatists advocate for approaches to research that could be more practical and pluralistic. These approaches are expected to shed light on the actual behaviour of participants, the beliefs that stand behind those behaviours and the consequences that are likely to follow from different behaviours (Kivunja & Kuyini, 2017). Precisely put, pragmatic researchers test hypotheses and provide various views, i.e. inside and outside perspectives (Creswell and Plano Clark, 2011; Johnson & Christensen, 2008). This therefore gave rise to the pragmatic paradigm that advocates the use of mixed methods to understand human behaviour (Creswell & Plano Clark, 2011; Denscombe, 2008; Johnson & Christensen, 2008). The pragmatic paradigm implies that the overall approach to research is that of mixing data collection methods and data analysis procedures within the research.

Pragmatism is the research paradigm for the mixed-method approach (Mackenzie and Knipe, 2006). It mixes the vision of an ordered and understandable world with a passing glance to plurality and social constructivism. The pragmatic approach focuses on providing insight and has no philosophical loyalty (Mackenzie and Knipe, 2006). The ontological view of pragmatists argues that there are different perspectives about social reality and the researcher sees reality or truth based on their own standards and beliefs. This implies that there is no single reality and all individuals have their own and unique interpretations of reality. Epistemologically, the pragmatic paradigm is either objective or subjective, based on the research focus and inquiry (Creswell and Clark, 2011; Teddlie & Tashakkori, 2009). For example, investigators can use the quantitative approach as a primary approach to data collection, while qualitative methods involve a secondary approach to collecting data (QUAN + qual). This means that the relationships in research are best determined by what the researcher deems appropriate to that particular study (i.e., a relational epistemology). The reason for conducting qualitative methods is often to describe quantitative data (Onwuegbuzie & Leech, 2005). It seems that research design and methods can be identified based on research questions.

In order to answer the research questions it is important to choose the right research design and use appropriate methods to collect and analyse the data (Muijs, 2010). It could require more than one research method to address the research questions. Additionally, this approach leads to rich descriptions being obtained (Alreshidi, 2016); and this makes researchers to adopt instruments that permit them to collect intensive data from subjects by giving participants the freedom to talk about their own experiences (Tuli, 2011). For that reason, researchers use appropriate strategies for the purpose of collecting data (Alreshidi, 2016). For the reasons given above, the researcher in this study has adopted the pragmatic approach which combines the positivist and interpretivist worldviews when investigating students' behaviour in a GC environment. Therefore, this study has used mixed methods approaches to collect the data of diversity and the relationships among students.

4.2.3.1 Mixed methods approach

In this study, mixed methods were used to gather data from the selected grade 11 students of the three selected secondary schools in Gauteng Province. A significant number of studies have indicated that mixed methods research is effective in teaching and learning. This is an approach to inquiry in which the researcher integrates both quantitative and qualitative data to provide a unified understanding of a research problem (Creswell & Plano Clark, 2007). Simply stated, it mixes data collection methods and data analysis procedures in the research process. In Ponce and Pagán-Maldonado's (2015) views, mixed methods approaches are used only when we address research problems which have objective and subjective elements in its manifestation. In this context, mixed methods research is blending qualitative and quantitative methods of research to produce a final product which can highlight the significant contributions of both. The notion is that in a single study the researcher combines quantitative and qualitative techniques and methods.

Saunders, Lewis and Thornhill (2003, 2009) state that there are two major advantages of employing multi methods in the same study. Firstly, different methods can be used for different purposes in the same study, hence giving the researcher confidence in addressing the most important issues. The second advantage of using multi-methods approach is that it enables triangulation to take place. Thus, offering strengths for offsetting the weaknesses of separately applied quantitative and qualitative research methods, Ponce, et al. (2015) elaborated. This blending of quantitative and qualitative research methods offers the advantage of the respective qualities of both approaches.

These mixed methods provide a breadth and depth of understanding (Creswell, 2007) and greater confidence in conclusions (Ponce, et al., 2015). Similarly, they argue that mixed methods allow producing more comprehensive internally consistent and valid findings (Johnson, Onwuegbuzie & Turner, 2007) on the research problems. Creswell, (2007) argues that mixed methods enhance the collection of more comprehensive evidence for study problems, help to answer questions that quantitative or qualitative method alone cannot answer, and reduce adversarial relationships among researchers and promote collaboration. Similarly, Ponce et al., (2015) claim that mixed methods research always encourages the use of multiple worldviews and is a practical and natural approach to research. Additionally, Scott and Morrison (2007) argue the advantages of employing mixed method research in a single study as: 1) enhancing triangulation; 2) integrating both outsider and insider perspectives 3) increasing an understanding of the relationship between variables; and 4) allowing appropriate emphases at different stages of the research process.

Creswell (2009) identifies three strategies for mixing qualitative and quantitative methods, namely merging, embedding and connecting the datasets. For the current study the researcher made use of the mixing strategy proposed by Creswell and Plano Clarke (2011) to connect the qualitative data, in order to build or develop the subsequent quantitative data. More specifically, the data are connected in that the qualitative results were used in collaboration with the literature review to design a measurement instrument, namely a questionnaire for quantitative data. For this reason, this study pursued the mixed methods exploratory research design or exploratory sequential research design as the structure (or way) of systematically connecting quantitative and qualitative approaches in mixed methods research (Ponce, 2011, 2014: Caruth, 2013; Creswell & Plano Clark, 2011; Teddlie & Tashakkori, 2009). The exploratory nature of this sequential mixed method research approach is grounded in discovering how the graphing calculator enhances students' understanding of quadratic inequalities through their experiences of using this technology. A sequential mixed methods approach was employed for this study due to the complexity of the research problem and the absence of foundational literature to guide the specific dimensions of this issue. The exploratory design permitted the researcher to interact with the participants through interviews aimed to uncover the relationship between students' understanding and GC (Creswell, 2008). Its exploratory nature, as Creswell (2008) explains, also allows the researcher to collect quantitative data and then collect qualitative data to help explain or elaborate on quantitative results. The idea is the researcher sequentially collects and analyses the quantitative data in the first phase and then uses the findings of the quantitative data to design a second qualitative phase, but using another research approach (Medina, 2012; Perez, 2012; Medina, 2014; Ponce et al., 2015). The established theoretical framework subsequently presented the researcher with the opportunity to identify topic-specific themes and variables for further investigation. In this case, the study began with a quantitative phase and then qualitative approach using the previous findings to design another phase. The idea is that the research problem is sequentially studied by using the findings of the first phase to design the second phase in order to decisively conclude about the attributes of the phenomenon under the study. This sequential structure has much relevance to the DBR.

4.3 Research methodology

A design based research (DBR) is described as a systematic but flexible methodology aimed to improve educational practices through iterative analysis, design, development, and implementation, based on collaboration among researchers and practitioners in real-world settings, and leading to design principles and theories (Wang and Hannafin, 2005, p.6). From another perspective, Bannan-Ritland (personal communication, 2009) viewed DBR as a meta-methodology that combines different methods at different points in the innovation cycle. Similarly, Barab and Squire (2004) noted that DBR as a methodological toolkit for deriving evidence-based claims from naturalistic learning contexts that are engineered in ways that allow for generating and improving these claims with the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching. This means DBR can be conceptualised as a pragmatic research methodology grounded in theories (i.e., strongly based on prior research and theory) that integrates different methods intended to improve educational practices in collaboration among researchers and practitioners through evidenced-based claims in order to produce new theories, artefacts, and practices and design principles. Differently expressed, DBR is a type of research carried out in the real, complex, and messy learning/teaching (naturalistic) contexts (i.e., classrooms and schools) but in the iterative cycles so as to build new theories of teaching and learning, and produce instructional tools that survive challenges found in everyday practice (Shavelson et al., 2003) and improve educational practices mediated by some interventions through continuously refining the end products (Abdallah, 2011). This implies that the problem of students' understanding of quadratic inequalities may be effectively and collaboratively identified through the use of DBR approach, and fully addressed in a GC facilitated classroom in order to produce learning instructional theories.

Other scholars have noted that design-based research has lent itself to the field of educational research as its underlying premise is to develop the design of artefacts, technological tools, and curriculum and to further an existing theory or develop new theories in naturalistic settings that can support and lead to an deepened understanding of learning (Barab, Dodge, Thomas, Jackson, & Tuzun, 2007; Barab & Squire, 2004; Fishman, et al., 2004). There is a natural alignment between design research and research in education (Lesh, 2003) and its cyclic and iterative processes are aligned with the authentic design of educational environments **(**Kennedy-Clark, 2013). In this respect, the methodological approach of DBR supports the investigation of a learning design. Schoenfeld (2009) explained that the products of well-conducted design experiments are improved interventions and improved understandings of the processes that result in their productiveness. Thus, the products/outputs of DBR are design principles, learning theories, interventions, curricular products, instructional tools, and/or practical solutions/prescriptions. In the context of this current study, the DBR is used with mixed methods as other methods that can help to determine the students' understanding processes of quadratic inequalities in which both qualitative and quantitative data are collected and analysed in a GC enriched learning environment.

The main characteristics of design-based research proposed by Wang and Hannafin (2005) were used as guidelines as follows:

- The design study is pragmatic because its goals are addressing current real-world problems (i.e., flexible learning environment for understanding quadratic inequalities) by designing/enacting students' activities in a GC mediated classroom as well as extending local theories and refining design principles.
- The design study is grounded in both theory and the real-world context and conducted in collaboration with mathematics teachers, head of departments and student participants, and is much more likely to lead to effective use of GC to solve quadratic inequalities.
- The design study, in terms of research process, is interactive, iterative and flexible. The researcher interactively worked together with practitioners and participants throughout the design processes to address the emerging local issues in a timely manner. The iterative outcomes of designs provided explanatory platforms and the focus of investigation in the subsequent cycle of inquiry. Ultimately the design processes were flexible as they permitted changes to be implemented and new emerging patterns to be developed.
- The design study is integrative because the researcher integrated a variety of research methods and approaches from both qualitative and quantitative research paradigms. This integrative data from multiple sources served to confirm and enhance the credibility and adaptability of research findings.

- The design study is contextualized and evidence-based because research results were connected with both the design process through which results are generated (teaching experiments) and the setting (schools) where the research is conducted.

This study of the students' understanding of quadratic inequalities in a graphing calculator mediated classroom falls within the parameters of a design based research study because it is pragmatic, grounded, integrative, interactive, iterative and flexible, and contextual.

Confrey, (2006) indicates that even though the design study has evolved from various terms such as design research, design experiment, teaching experiment and design-based research methods, it still enjoys the same research approach. The strengths of design studies are noted by Shavelson, Phillips, Towne and Feuer (2003, p.25), "lie in testing theories in the crucible of practice; in working collegially with practitioners, co-constructing knowledge; in confronting everyday classroom, school, and community problems that influence teaching and learning and adapting instruction to these conditions". This means DBR has survived in the field of educational research because it provides opportunities to the researcher to work closely with the participants and collaborate with practitioners to develop theories. In addition, DBR involves not simply observing but rather systematically engineering learning contexts (Barab and Squire, 2004) for addressing classroom, school, and community problems. Therefore, the use of design and instructional strategies (graphing calculator) as intervention allows the teacher-researcher to improve educational practice, conducted in the iterative cycles of teaching contexts. In this context, the design-based approach has been adopted to enhance the teaching and learning of quadratic inequalities in a technologically mediated setting.

DBR is a theory-guided design and procedure which is implemented in iterative cycles of data collection and analysis. DBR is not a specific data collection and analysis method (Reimann, 2011), but rather uses integrative methods. The integrative methods include survey, expert review, case study, inquiry methods, video recording, semi-structured interviews, questionnaires and statistical analysis (Cobb and Gravemeijer, 2008). Such methods provide opportunities for researchers to maximise the credibility and adaptability of their findings. Easterday, Lewis and

Gerber (2014) indicate that DBR methods allow researchers to generate useful educational interventions and effective theory for solving individual and collective problems of education. In addition, DBR provides ways and procedures for designing specific tasks, materials, tools, patterns of communication and interaction, and instructional sequences (hypothetical learning trajectories) in learning environments (Reimann, 2011). For this reason, using this pragmatic approach in this research may help students to become flexible in dealing with quadratic inequalities, whether as numeric, symbolic or graphic representation as well as increase the relevance of findings about how to use the graphing calculator in mathematics classroom. This may also lead to the articulation of design principles by the researcher that are relevant to other educators and district subject facilitators and that are transferable to similar contexts.

4.4. Phases of design based research

Reeves' (2006) study has identified four interconnected phases of the design research process as shown in Figure 4.1, below to be followed in this study.

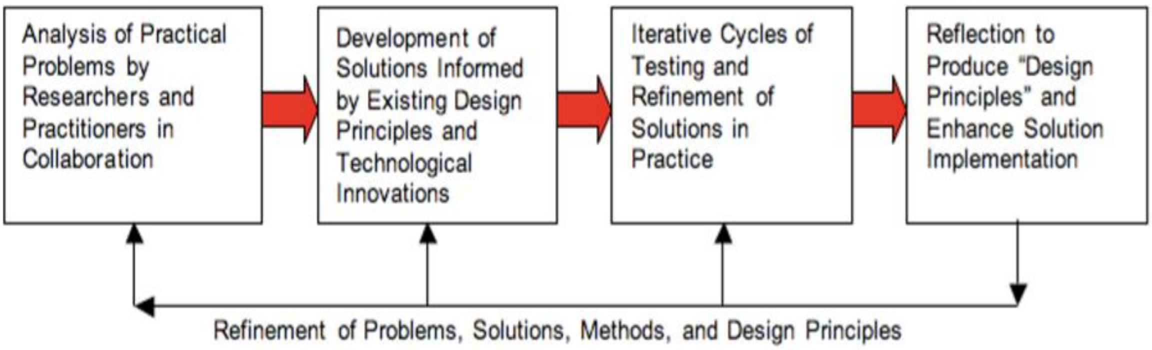


Figure 4. 1: Design-based research phases in educational technology (adapted from Reeves, 2006)

In Reeves' (2006) views, the DBR approach is a process which starts from the identification and analysis of problems by researchers and practitioners in collaboration; and then goes through the development of prototyping solutions informed by theories, existing design principles, and technological innovations; then involves iterative cycles of testing and refinement of solutions in practice; and finally, results in reflection to produce design principles and enhance solution implementation in practice.

4.4.1 Analysis of practical problems- Phase 1

In design based research, the analysis and articulation of the students' problem is fundamental for the ultimate success of the overall study. The purpose of this *preliminary phase* involved identifying and formulating a significant educational problem of the study through: interactions with student participants in secondary school intervention programmes (SSIP); a review of relevant empirical studies to identify the pedagogical-content knowledge (PCK) gap including NSC diagnostic reports (DBE, 2014-2017); and real interactions with mathematics practitioners. Finally, it assisted to collect preliminary empirical data that might bring about the plausible solution to the entire students' misconceptions.

Design-based research places much value on the input of practitioners and researchers working in, or investigating, the problem area (Herrington, Mc Kenney, Reeves, & Oliver, 2015). In this respect, the researcher further consulted most experienced mathematics practitioners such as FET teachers, district subject specialists and university lecturers including the supervisors. These practitioners were regarded as having insights that were based upon their experiences and practical understanding of the mathematical issues. The idea is that conversations with these individuals might provide the researcher with a better appreciation of the multiple educational contexts that comprise students' misconceptions in quadratic inequalities. For this reason, the researcher was consistently involved in long-term, meaningful engagement with practitioners (a long-term process that started already a few years ago) as they are the reservoirs of knowledge about the research problem.

An intensive and systematic preliminary investigation of tasks, problems, and context is crucial in design based research (van den Akker, 1999, Herrington, McKenney, Reeves and Oliver, 2007) as it helps to discover more accurate and explicit connections of the problem. Doing a comprehensive review of literature serves two main purposes: (a) clarifying the key research terms (e.g., graphing calculators in mathematics, quadratic inequalities, and the use of graphing calculators in the curriculum); and (b) providing theoretical frameworks for conceptual understanding. With adequate literature the researcher can formulate the research aim and the general research question, and develop a hypothetical learning trajectory (HLT) (van

Eerde, 2013). In addition, the researcher can identify the disclosure of the relevant studies, the students' difficulties related to the topic and the knowledge gap about the learning of the topic. By so doing, the researcher gains an insight about the research problem (i.e. learners' inability to solve quadratic inequalities) and connects the designed instructional activities with students' current knowledge.

As a result of consultation of literature, students and practitioners as well as Herrington et al.'s (2007) suggestion, the researcher identified the problem and concluded that students were not exposed to the constructivist learning environment. In a constructivist learning environment, "students are engaged in a cognitively rich and authentic environment that lends itself to knowledge construction" (Steketee & Bate, 2013; p. 273). In doing so, the researcher avoided the risk of only partially addressing the problem or missing it altogether. According to Herrington et al (2007), many researchers begin by thinking of a solution – such as a technology-based intervention, an educational game, or a technology tool – before they consider the educational problem to be solved. For this study, relevant research questions were formulated to address potential problem of students in quadratic inequalities. This is in line with the view that research questions should emerge from the stated problem rather than from the stages of design-based research (Herrington, McKenney, Reeves and Oliver, 2007). The research questions were exploratory and open in nature as they sought to improve existing inadequate practices. For example, the underlying questions were framed in the form of "what", "how" and "in what ways" so as to drive innovative design research.

To this end, literature review process, in Herrington, et al.'s (2007) opinion, is critical in design-based research as it normally facilitates the creation of draft design guidelines that inform the design and development of the intervention for the identified research problem. Findings from an iteration of review often help to fine-tune the principles guiding the design. Finally, this phase involves collecting preliminary empirical data through pre-test and semi-structured interviews. The preliminary results are necessary for informing the preliminary design framework that should guide the next stage of this design study (i.e. the prototyping phase).

4.4.2 Development of solutions Informed by design principles- Phase 2

The second phase is where the researcher attempts to develop a feasible solution immediately after firmly conceiving the nature and extent of the research problem. A wide consultation provided a theoretical foundation upon which the problem can be better articulated and its informed solution would be grounded in scholarly principles. The Design-Based Research Collective (2003, p. 6) wrote, "[Solutions] embody specific theoretical claims about teaching and learning, and reflect a commitment to understanding the relationships among theory, designed artefacts and practice". As noted by Barab and Squire (2004, p.6), "design-based research suggests a pragmatic philosophical underpinning, one in which in its ability to produce changes in the world". This means a well-described theoretical framework derived from a powerful literature review can inform a sound basis for the problem solution of the study. The notion is that a DBR researcher is able to produce changes in the world if s/he understands the pragmatic philosophical underpinnings where the value of a theory lies (Barab and Squire, 2004). In this study, different theories of learning mathematics (cf: chapter 3) helped to develop design principles for guiding the design of the intervention (the graphing calculator-enhanced solution). Furthermore, the solutions of quadratic inequalities in the graphing calculator-facilitated environment are developed from the effective consultation and collaboration with practitioners.

Through a review of the literature on mathematics learning theories (cf: chapter 3), their pragmatic underpinnings were that the learning activities needed to embody certain features that would support the construction of meaningful learning outcomes. The solution of these activities should entail the development of cognitive aspects by encouraging students to not only engage in learning, but come to value this process as an integral aspect of their cognitive growth. In this context, the students would be more inclined to engage in the understanding of the concepts if activities were purposeful and relevant to their specific learning needs and contexts. They would be more motivated to learn too if the teachers could link new and existing knowledge and experiences, and more so if activities were learner-centred rather than teacher-centred. Additionally, their learning would be enriched if they had opportunities to collaborate with other students such that they could share perspectives and co-construct their own understandings. The theories further made assumptions about students' learning in the technology-mediated settings. Students' activities in a technology-enriched classroom should be meaningful and engage learners in complex, authentic problem solving and reflection. These authentic activities essentially are the vehicle through which learners engage with content and attempt to make sense by relating them to their own contexts and learning needs. Technological learning environments could act as a mirror to their own understanding by helping students to see their solutions differently and/or with more clarity. Importantly, learning should be scaffolding, primarily by a teacher but also via collaboration with the other students and learning resources.

The reviewed literature assisted to identify the research studies that had listed principles and relevant criteria to address the educational problem. Using this grounded approach a number of critical elements were determined and ultimately used to construct the initial conceptual framework (cf. figure 3.2). Models, frameworks and principles and guidelines (i.e., hypothetical learning trajectories) from the reviewed studies and papers were listed and grouped. From these groups, a list of the suggested HLTs was developed to form a guiding framework for the current study.

The review of proposed design principles and guidelines (i.e., HLTs) was collaboratively done with experts and practitioners in education fraternity. By means of this iterative process, the original list of critical elements (principles and guidelines) was determined and evolved. As indicated earlier, expert reviewers were selected on the basis of their extensive experience of working in the educational fields of ICT and authentic assessment. They were carefully and purposefully chosen from both representations within the literature, as well as by recommendation. In their opinion the practitioners were asked to evaluate the critical elements whether they made sense as a framework. They were also asked to adequately evaluate if the elements were reflective of what should be considered the determination of authenticity within graphing calculator learning environment. Furthermore the experts were asked to provide any feedback or information that might be used to enhance the suitability or applicability of any of the critical elements in addressing research problem.

4.4.2.1 Conceptual design framework

With a clear understanding of the nature and extent of the research problem and wide consultation of literature (Reeves, 2006; Van Wyk & de Villiers, 2014; Vygotsky, 1978; Trouche, 2004; Freudenthal, 1991), a conceptual design framework was synthesised and modelled aiming to improve students' understanding of quadratic inequalities in grade 11. This design model in Figure 4.2 below is anticipated to provide opportunities for conducting the DBR in graphing calculator mediated environment so as to address the students' misconceptions of quadratic inequalities. The components of the model are explained below:

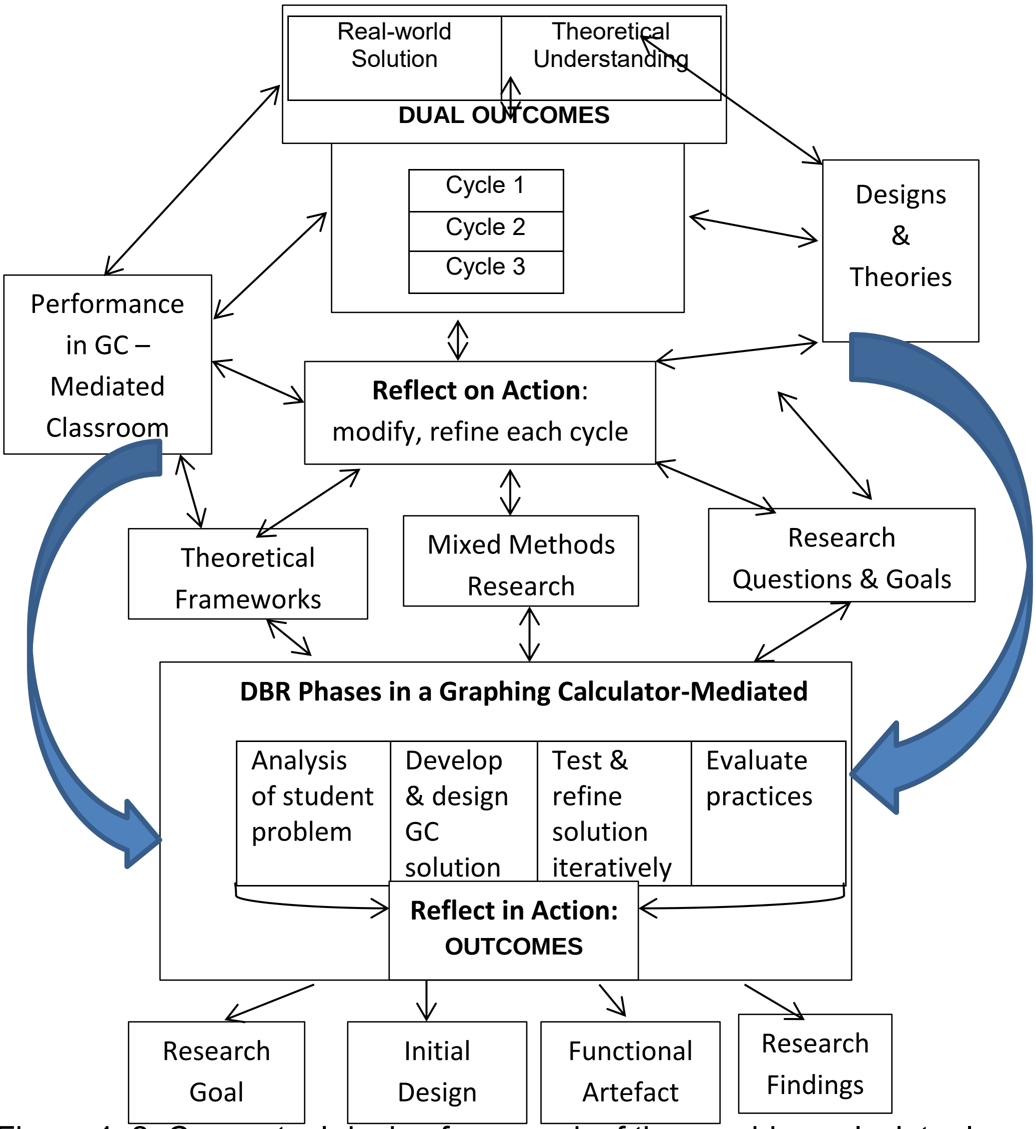


Figure 4. 2: Conceptual design framework of the graphing calculator learning environment

DBR phases in a GC-mediated classroom

This framework incorporated the phases of the design-based research in a GC-mediated classroom and the research problem was analysed using these interrelated and iterative phases as set out in Figure 4.2. The phases represented namely, analysing students' difficulties with quadratic inequalities, developing and designing a GC mediated solution, testing and refining the solution in iterative cycles and, evaluating and reflecting on effectiveness in resolving students' quadratic inequality problem solving difficulties.

Reflection in action and on action

Reflection was explained in two forms in the works of Schon (1983) and, Van Wyk and de Villiers (2014) as reflection in action and reflection on action and these were applied in this framework. *Reflection in action* involves reflecting during an action in each phase of the cycle without interfering to its outcome such as concrete research goals as a result of problem analysis; detailed design of the solution; development of a functional artefact and research findings to answer research question(s). On the other hand, *reflection on action* involves looking back to understand what has happened, thus learning from past experiences (e.g., previous DBR cycle) and also looking forward to meet challenges and then modify and refine subsequent cycle(s).

Iterative cycles

These cycles embedded in this framework were iterative and finite. The iterative cycles were based on specific, critical decisions and informed judgment to enhance, improve and re-design an artefact (e.g., the use of graphing calculator). In this study it was anticipated that three iterative cycles might help to document the desired local instructional theory for quadratic inequalities.

Theoretical frameworks

The theories of sociocultural constructivism, theory of instrumental genesis and Realistic Mathematics Education were used as lenses to analyse students' understanding in the GC-enhanced classroom. The construct of instrumental orchestration helped to use his/her didactic competences in a graphing calculator enhanced mathematics classroom. Hypothetical learning trajectory was critical for designing learning materials and activities of quadratic inequalities.

Learner achievement

Using the graphing and table instrumented schemes students had the opportunities to improve their problem solving and reasoning skills and then deepened understanding of quadratic inequality concepts. Consequently, both reflection in action and reflection on action might have contributed towards enhancing students' performance achievement in mathematics.

Dual outputs

The completion of the final DBR cycle results in dual outputs namely, the implementation of a real-world solution and the formal documentation of theoretical understanding of the phenomenon under study (Van Wyk & de Villiers, 2014). The results of the design model would be used to develop the local instructional theory for quadratic inequalities and design principles.

4.4.3 Iterative cycles of testing and refinement- Phase 3

The third phase of design-based research (i.e., the teaching experiment) focuses on the implementation and evaluation of the proposed solution in a designed and developed learning environment or intervention (Herrington, McKenney, Reeves and Oliver, 2015; Ashford-Rowe, 2008). The goal was to gain an in-depth insight into the effectiveness of the intervention. The designed learning environment would ensure that three research cycles of the teaching experiments, thus in schools A, B and C were occurring to generate and refine evidence-based outcomes.

The researchers, in collaboration with the participants, learn whether the solution is effective in practice or if it needs to be improved in some way. The idea is that the researchers and participants work closely together to interpret and make sense of the data that emerge during the implementations of the proposed solution. The limitations and constraints of the anticipated solution were identified and used by the researcher in the subsequent cycle. Particularly, all parties (e.g., the designers, practitioners, researchers and participants) are instrumental in advancing the pragmatic and theoretical aims of the design study (Wang & Hannafin, 2005). In the context of this study, the teacher was the researcher, students were the participants and the practitioners were the collaborators who had stake in the research outcomes.

As suggested by Gravemeijer and Cobb (2006), the purpose of the teaching experiment is to test and improve the hypothetical learning trajectory (HLT) and to develop an understanding of how it works in the actual learning process. Design-based research is iterative in nature meaning that a single implementation is never sufficient to gather adequate evidence about the success of the intervention (i.e., graphing calculator) and its effect on the students' understanding of quadratic inequalities. Within these cycles changes are consistently implemented and evaluated to adjust the HLT and develop the prototype local instructional theory to further improve its ability to address the research problem. This is in keeping with the focus suggested by Reeves (1999) and Herrington, et al. (2015). The idea here is to use the DBR approach in a GC setting to gain an understanding which should have meaning beyond the immediate setting.

The research cycles of teaching experiments

In the case of this current study there were three research cycles involved and were implemented in three different schools (See Table 4.2). The cycles were conducted as the actual teaching processes in which the sequences of instructional activities were implemented in a naturalistic (graphing calculator) classroom environment. The mathematical content of the teaching experiment for these three cycles remained essentially the same. The developed HLT about the quadratic inequality concept was implemented in a graphing calculator-enhanced classroom environment. The limitations on the cycle were used as the feed-forward of the subsequent cycle.

4.4.4 Reflections to produce design principles- Phase 4

The fourth and final phase was that of developing a set of critical elements into a revised framework based upon the data received at the conclusion of phase three. Because of the collaborative nature of DBR, design principles are generated in an evaluation/reflection phase. Reflection "involves active and thoughtful consideration of what has come together in both research and development (including theoretical inputs, empirical findings, and subjective reactions) with the aim of producing new (theoretical) understanding" (Mc Kenney & Reeves, 2012, p. 151). As part of an ongoing DBR process, design principles are empirically and richly developed in order to eventually lead to theoretical understanding of educational contexts. As described by Amiel & Reeves (2008), the design principles or guidelines derived can be implemented by others interested in studying similar settings and concerns.

4.5.4.1 Design principles in design based research study

A design principle is described as a prediction (Greeno, 2016), a criterion that needs to be fulfilled (Collins, 1990; Edelson, 2002), a value (something that is valued as important in itself), heuristic advice (Van den Akker, 1999), a design methodology (Edelson, 2002, perhaps a combination of such meanings (Bakker, 2004). This means design researchers at the end of their studies provide their advice to future researchers/readers by means of design principles. These design principles may include conjecture maps (Sandoval, 2014), local instruction theories (Gravemeijer, 1998) and hypothetical learning trajectories (Simon, 1995). The design principles help to link the educational goals to an idea on how this could be achieved through design experiments (Herrington, et al., 2007). They additionally proposed three desired outputs (scientific, practical and societal) in the form of both knowledge and products. These were intended to move students from their current levels of understanding to specific learning goals (of quadratic inequalities).

In the scientific design principles, evidence-based heuristics can inform future development and implementation decisions (Linn, Davis & Bell, 2004). In this current study the design principles were used to close the pedagogical gap in the teaching and learning of quadratic inequalities in the GC environment. *In the practical design principles,* learning and teaching artefact(s) and activities are designed by the researcher, programmers and resource specialists to intervene in mathematics education. In this current study the researcher designed a learning environment (learning instructional trajectory) and the instructional activities for improving students' reasoning and problem solving skills of quadratic inequalities. in the *societal design principles*, the researcher and practitioners designed new heuristic approach for solving quadratic inequalities in a GC environment as the local instruction theories to inform future development and implementation decisions.

Similarly, Reeves (2006) identified three fundamental principles of design-based research such as:

1) Addressing complex problems in real contexts in collaboration with practitioners;

2) Integrating known and hypothetical design principles with technological advances to render plausible solutions to these complex problems; and,

3) Conducting rigorous and reflective inquiry to test and refine innovative learning environments as well as to define new design principles.

To this end, design principles can help the design based researchers to develop and refine their artefacts, technological tools and curriculum, and to advance an existing theory or develop new theories that can support and lead to a deeper understanding of mathematics (Kennedy-Clark, 2013; Barab, Dodge, Thomas, Jackson, & Tuzun, 2007; Barab & Squire, 2004). This further means that the principles link the findings of the DBR to theories well-known in mathematics education, which includes a domain-specific instructional theory of RME, socio-cultural constructivist learning theory and theory of instrumental genesis.

4.5. Hypothetical learning trajectories (HLT)

A hypothetical learning trajectory (HLT) is a design and research instrument in all phases of design research (Bakker, 2003; Bakker & van Eerde, 2004). Simon (1995) states that student learning is hypothetical because the actual learning trajectory is not knowable in advance and it characterizes an expected tendency. In other words, these are educational predictions made by teachers on student learning and then testing them in practice. This means the learning pathways (trajectories) are in the hypothetical nature. The HLT is the link between an instruction theory and a concrete teaching experiment (Bakker, 2003) and a bridge linking the theory of constructivism to practice (Uygun, 2016). It is informed by general domain-specific and conjectured instruction theories. In the teaching process, teachers have the opportunity to test the designed hypothetical learning trajectories (HLT) and make modifications based on the experiences obtained in this process. This makes it possible to explain the HLT as a construct for teaching and learning process. In addition, the HLT can be accepted as a cognitive tool for improving mental processes and mathematical learning actions constructed in a learning environment- the graphing calculator.

In the present study, the HLT based on the graphing calculator learning environment was used since it provided the teacher-researcher a framework for supporting the understanding of students' thinking and learning of quadratic inequalities. In the HLT, the teachers make predictions about the progress in the teaching and learning sequence. In simple terms, the HLT explains the usage of the teachers' predictions made with regard to the teachers' knowledge and assessments about student knowledge and their history about how the learning process may happen. As shown in Simon's (1995) model (Figure 4.3) below, learning trajectories make the link between teachers' knowledge and their students' actions around three elements such as learning goals, learning activities and hypothetical learning process. The HLT is defined in terms of the learning goal, the learning activities and the hypothetical learning process (Simon, 1995) in a designed classroom based on the predictions of the teachers (Uygun, 2016). The teachers make the predictions by examining the student's learning and reasoning carefully considering their actions in the classroom, the results of assessments about them and their history. In this way, the HLT helps the teachers understand their students' learning and thinking processes. Learning trajectories are identified as a useful attempt for assessment (Battista, 2004) and teacher education (Wilson, Mojica, & Confrey, 2013). As Simon suggested, the learning goal defines the direction and the hypothetical learning process is an educational prediction made by the teacher of how the students' thinking and understanding may evolve in the context of the learning activities.

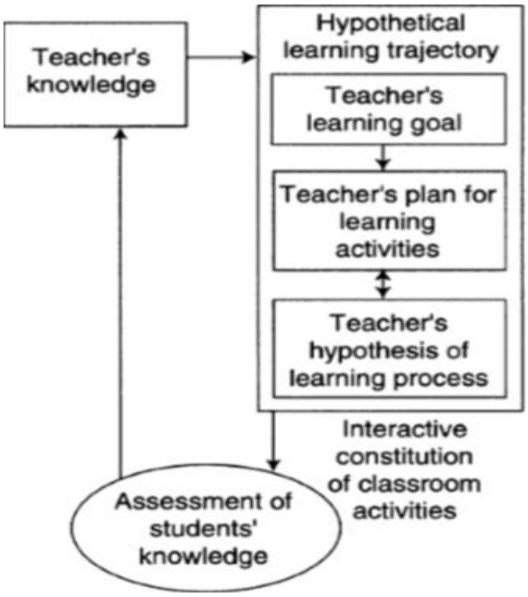


Figure 4. 3: Mathematics teaching cycle (Simon, 1995)

Simon used the HLT for designing and planning the Mathematics teaching cycle, mostly for one or two lessons, but in the present study, it is used as an instrument in the DBR for the instructional sequences of quadratic inequalities in a GC mediated classroom. This implies that HLTs were developed for teaching experiments that constituted of 8-9 sessions per cycle. As a consequence, the HLT in this study comes close to the local instruction theory of the learning of quadratic inequalities. The HLT can be seen as a concrete embodiment of this local instruction theory (Gravemeijer, 1995). Furthermore, this study differs from Simon's approach of a teacher's perspective, by assuming a teacher-researcher's perspective. In light of this, the researcher found it very critical to design the sessions for the teaching of quadratic inequalities in a GC environment. These sessions were designed with the help of the HLT and were expected to be beneficial for assessing students' understanding of quadratic inequalities. Then, necessary modifications of the HLT could be made based on the experiences in the cyclic teaching experiments. In this respect, students could effectively understand quadratic inequalities in a GC environment designed with respect to the social constructivist theory, RME principles and theory of instrumental genesis. The social learning environment was encouraged by the use of GC in which analysing, discussing and demonstrating the algebraic and geometrical ideas were made in order to solve quadratic inequalities.

Researchers have highlighted that the construct of the HLT plays different roles in teaching mathematics (e.g. Bakker & van Eerde, 2004; van Eerde, 2013). Gravemeijer (2000) indicates that the HLT embraces conjectures about student learning processes and how they are supported in the classroom. The HLT also provides the researcher with a mechanism for defining and refining a study map along which students' reasoning evolves in the context of the learning activities. In line with this argument, the instructional sequences are also modelled along the HLTs. The conjectures of students' thinking and reasoning are described in the context of the proposed actions of the teacher. The teacher's hypotheses assist to generate the developmental ideas from the learning activities. This means the teacher-researcher continually adjusts the HLT that he has hypothesized to better reflect his enhanced knowledge.

The construct of the HLT also plays different roles in the DBR phases (Drijvers, 2003; Bakker, 2004; Gravemeijer, 1994). This means that an HLT, after it has been formulated, has different functions depending on the phase of the design based research and continually develops through the different phases. Actually the HLT guides and informs researchers and teachers how to carry out a particular design experiment in a designed classroom. Ultimately, the interplay between the HLT and empirical results forms the basis for theory development, as follows:

1. In the design phase, the HLT guides the design of instructional materials that have to be developed or adapted. With adequate consultation of practitioners and confrontation of concrete activities, the researcher develops a more specific HLT during the design phase (Drijvers, 2003).

2. In the teaching experiment, the HLT functions as a guideline for the teacher and researcher what to focus on in teaching, interviewing, and observing (Drijvers, 2003; Bakker, 2004). It can happen that the teacher-researcher feels the need to adjust the HLT or instructional activity for the next lesson. This is consistent with Freudenthal's (1991) views that the cyclic alternation of research and development can be more efficient if the cycle is shorter. The idea is that minor changes in the HLT can be made because of incidents in the classroom such as anticipations that have not become true, strategies that have not been foreseen, activities that were too difficult, and so on. In such cases, a micro-cycle of design, experiment, and analysis occurs within a macro-cycle of design research (Bakker, 2004). In the DBR such micro-cycles are accepted and provide optimal conditions for changes in the HLT. This means that these changes are successful when supported by theoretical considerations. The HLT can thus also change during the teaching experiment phase.

3. In the reflective analysis, the HLT functions as a guideline determining what the researcher should focus on in the analysis of the instructional interventions in the GC classroom. Because predictions are made about students' learning, the researcher can contrast those anticipations with the observations made during the teaching experiment. Such an analysis of the interplay between the evolving HLT and empirical observations forms the basis for developing an instruction theory. After this analysis, the HLT can be reformulated, in a more drastic way than during the teaching experiment and the new HLT can guide the next cycle.

Simon (1995) has identified four main components that define the HLT: starting points, learning goals, learning activities and hypotheses on students' thinking process. In regard to these components of HLT, this present study used them to elaborate how the students' understanding of quadratic inequalities evolved in a GC mediated learning environment.

Starting points

The starting points are established to connect the planned instructional activities with students' current or relevant knowledge. In this study, starting points are prior mathematical concepts, ideas and representations connected to the quadratic inequalities that can be used for introducing the topic. For example, the use of set builder notation, which identifies the solutions of unknown variable, drawing a number line for the given builder set notation and sketching quadratic functions.

Learning goals

The learning goals are defined to provide direction to the planned instructional activities in solving quadratic inequality. In this study, students are expected to use a graphing calculator as an instructional tool consistent with the CAPS requirements. Students are also expected to understand the zeros of the functional graphs as critical values that define the solution region of quadratic inequalities and to express their solutions in set builder or interval notations.

Learning activities

The context that is embedded in the instructional activities is mostly about solution of quadratic inequality. In this study, students were engaged in learning sessions that involve them to solve inequalities, express the solution in set builder system and use quadratic graphs. The teacher introduced students to the use of the GC in determining solutions of quadratic inequalities from sketched graphs. In this case, the role of the teacher is certainly to support students in finding and reasoning about a better solution for the quadratic inequality.

Hypothetical learning process

In this study the hypothetical learning process involved the teacher's predictions of how the students' thinking and understanding of quadratic inequalities can evolve in the context of answering the learning activities. The proposed broad sequence of the HLT: Solving symbolic, routine, complex and concrete quadratic inequality problems in the graphing calculator environment. In addition, the teacher-researcher made predictions about students' learning and understanding in particular, in a GC facilitated classroom.

Table 4. 1: The HLT for teaching and learning of quadratic inequalities

Session	***Learning Objectives (Purpose)***	***Learning Activities***	***Supporting Learning Theories***	***Data Instruments***	***Researcher's Activities***
1	To assess the prior knowledge of students in solving quadratic inequalities	Students independently determine the solution sets of the quadratic inequalities using traditional method.	The dominating learning theories: RME	Pre-test (Quan & Qual) Observation (Qual)	The researcher assesses the students' prior knowledge in quadratic inequalities.
2	To sketch the graphs and determine the zeros of quadratic functions using the graphing calculator	Students sketch the graphs and read-off the zeros of quadratic functions using graphing calculators.	Principles of RME: Level, Interaction and Guided re-invention, Vygotsky social learning and Theory of Instrumental Genesis.	Questionnaires (Quan) In-depth interviews (Qual) Focus group interviews (Qual)	Teacher uses interactive teaching with graphing calculators as instructional tools.
3	To determine the solution sets of quadratic inequalities using graph sketches and graphing calculators.	Students are engaged in determining the solution sets of the quadratic inequalities using the graphing calculator as an instructional tool.	Principles of RME: Level, Interaction and Guided re-invention, Vygotsky social learning and Theory of Instrumental Genesis.	Questionnaires (Quan), In-depth interviews (Qual) Focus group interviews (Qual)	The researcher works out all of the examples with the students, while asking for student input.
4	Using GCs to solve symbolic Quadratic inequalities.	Students are fully engaged in determining the solution sets of the quadratic inequalities using the GC	Principles of RME, Vygotsky social learning and Theory of Instrumentation.	Questionnaires (Quan), In-depth interviews (Qual) Focus group interviews (Qual) Observation (Qual)	The teacher demonstrates how to use the GC to solve quadratic inequalities with few examples.

5	Solving symbolic quadratic inequalities using the GC as a checking tool	Students are fully engaged in solving the quadratic inequalities verifying their answers with GCs.	Principles of RME: Level, Interaction and Guided re-invention and Theory of Instrumental Genesis.	Questionnaires (Quan), In-depth interviews (Qual) Focus group interviews (Qual) Observation (Qual)	The teacher engages the students to work out the solutions of quadratic inequalities and then uses the GC as a checking tool.
6	To use the GC to answer questions involving the real-life application of quadratic inequalities.	Students are fully engaged in solving quadratic inequalities using the GCs.	Principles of emerging models and guided re-invention, Vygotsky social learning and theory of Instrumentation.	Questionnaires (Quan), In-depth interviews (Qual) Focus group interviews (Qual) Observation (Qual)	The teacher engages the students to apply quadratic inequalities to solve problems using GCs as instructional tools.
7	To apply quadratic inequalities in solving problems with use of GCs.	Students are fully engaged in solving problems with quadratic inequalities using GCs.	Principles of emerging models and Guided re-invention and Theory of Instrumental Genesis.	Questionnaires (Quan), In-depth interviews (Qual) Focus group interviews (Qual) Observation (Qual)	The teacher continues engaging the students in solving problems using GCs as instructional tools.
8	To evaluate the success of the design and the effeccts of the GC- enhanced learning in quadratic inequalities.	Students independently answer problems of quadratic inequalities using conventional methods.	Principles of RME Theory of instrumental genesis Social constructivist theory	Post-test In-depth interviews (Qual) Focus group interviews (Qual) Observation (Qual)	The researcher assesses the students' reasoning and problem solving abilities of quadratic inequalities.

4.6 The proposed teaching experimental cycles of this study

The three research cycles of the teaching experiments in this study are preceded by a preliminary phase which was conducted with grade 11 students over a period of one week. The teaching experiment was enacted over the three cycles of the DBR in three experimental secondary schools of Gauteng province within 2018 (see Table 4.2). The experiences gained from a previous iterative cycle was used as a template for the next iteration and assisted in the learning and teaching of quadratic inequalities in the graphing calculator-supported classroom.

Table 4. 2: A design-based research cycle

1 WEEK	PRELIMINARY PHASE
Grade 11 students at school C	• Examine literature and pedagogical strategies. • Outline key concepts & conceptual trajectory. • Consult practitioners. • Implement assessment strategies.

Overall reflections after preliminary phase

15 WEEKS	CYCLE 1: SCHOOL A 5 weeks	CYCLE 2: SCHOOL B 5 weeks	CYCLE 3: SCHOOL C 5 weeks	RETROSPECTIVE ANALYSIS
PHASE 1	Analysis of practical problems –literature review for conceptual Underpinning	NEW STUDENTS	NEW STUDENTS	Retrospective analysis of the data set seeks to place participants' learning and the means by which it was supported in a broad theoretical context.
PHASE 2	Development of solutions informed by the literature	Refinement of solutions based on reflection	Refinement of solution based on reflection	
PHASE 3	Intervention (first iteration) – data collection and analysis	Intervention (second iteration) – data collection and analysis	Intervention (third iteration) – data collection and analysis	The development of a domain-specific pedagogical strategy for Grade 11 students to develop their conceptual understanding of quadratic inequality concept.
PHASE 4	Reflection to produce design principles and re-design HLT	Reflection to produce design principles and re-design HLT.	Reflection to produce design principles and local theories.	

4.7 Population and sampling methods

Grade 11 students of mathematics from Gauteng province were the core participants in the study. A combination of convenience, purposive and stratified random sampling strategies was used to select the provinces, districts, schools and student participants for the current study. A sample of a total of one hundred and twenty students (N=120) were randomly selected from three schools. Simply put, they were thirty five student participants (n=35) from School A, forty (n=40) from School B and forty five (n=45) from School C. It is also important to indicate that three practising teachers, one district mathematics facilitator and two university lecturers participated in the designing of the learning activities of the DBR study in the GC-mediated learning environment.

Specifically, the researcher used purposive sampling to identify Gauteng province, Ekurhuleni North district and the three public high schools in the underperforming category. Purposeful selection ensured maximal variation as determined by student mathematics achievement (as measured by 2016 NSC results of the schools in mathematics) and diverse school demographics. Purposive sampling is a non-random method of sampling where the researcher selects “information-rich” cases for study in depth (Patton, 2002) and a sample from which the most can be learned (Merriam, 2009). The benefit of purposive sampling is that, as Patton (2002) puts it, “Any common patterns that emerge from great variation are of particular interest and value in capturing the core experience and central, shared dimensions of a setting or phenomenon”. This sampling strategy befits the qualitative aspect as this approach seeks to understand the behaviour of the phenomenon. The other advantage of the use of this judgmental sampling strategy is that particular settings, persons or events are selected deliberately to provide important information that cannot be obtained from other choices. This strategy was ideal for exploratory mixed methods research design and appropriate for the design-based study as there was a limited number of educators who had expertise in the area being researched- use of graphing calculator in mathematics. This was also expected to generate the best case scenarios of the graphing calculator-enhanced mathematics classrooms. However, as a non-random sampling strategy, it is deliberately selective and biased (Cohen, et al., 2007). Another disadvantage of this sample is that the selected schools do not represent the wider population of South African high schools.

The researcher identified the province, district and schools using convenience sampling in order to collect survey data for the research problem and sub-questions. Scott and Usher (2011, p.79) stated that “convenience sampling comprises choosing an unrepresentative sample by selecting respondents because it is convenient for the researcher”. A sample of three high schools of this district in Gauteng province were a convenient sample because they were willing and readily available (Patton, 2002; Lodico, et al. 2010; Creswell, 2011) and became the captive audience (Cohen et al., 2007). The students conveniently indicated their willingness to participate in quantitative surveys and qualitative interviews. The researcher chose this non-probabilistic strategy because the selected participants are often readily available and willing to be studied (Creswell, 2008), is less expensive to administer and often helps to overcome many limitations associated with research. For example, challenges associated with conducting an exploratory qualitative study include extensive time constraints and the significant amount of financial resources involved in traveling to the participants' locations. According to Creswell (2008), the most time-consuming and costly approach of this sampling technique is to conduct individual interviews. However, there are limitations to this sampling type as the researcher cannot attempt to generalize the results beyond the given population (Mertens, 2009; Creswell, 2011) as the population was self-identified as the willing participants by the researcher and the parameters in this type of sample are negligible (Cohen, Manion and Morrison, 2007).

Stratified random sampling procedure was also used to mainly reduce the population heterogeneity and to increase the efficiency of the estimates. The population, as Johnson, et al. (2007) and Cohen, et al. (2007) noted that, is divided into homogenous subgroups/strata, with each group containing subjects with similar characteristics. This means from each stratum, a simple random sample is selected and there are combined together to form the required sample from the population. In this regard, the researcher divided learners into three groups of 3 learners who performed relatively higher than their pre- test average (referred to as “Higher”), 3 learners who performed about their pre-test average (referred to as “Expected”), and 3 learners who performed relatively lower than their pre-test average (referred to as “Lower”) in each school, thus in line with Collins' (2010) suggestions. In this study an equal number of male and female students were randomly selected in order to have a proper representation of each gender, where it was possible, to use graphing calculators in solving quadratic inequalities. A random sample drawn from these subgroups was qualitatively interviewed (i.e., in-depth, semi-structured and focus group interviewed) so as to explore the students' behaviours in a GC facilitated learning environment. A stratified random sample is, a useful blend of randomization and categorization, thereby enabling both a quantitative and qualitative nature of research to be undertaken (Cohen, et al., 2007).

4.8. Research methods

4.8.1. Quantitative research methods

The quantitative section deals with those methods that provide quantitative information and objectively evaluate the data using statistical instruments.

4.8.1.1 Pre-post testing method

The pre-post testing (PPT) collects the quantitative data across two observations: baseline (beginning of program) and at a later point, at the end of the educational program that had been in operation for the possible change to occur (Delucchi, 2014; Schalich, 2015). Therefore, the main purpose of pre-post testing is to assess the value-added learning outcomes of an intervention in classroom (Delucchi, 2014). In this way this testing method is used to measure change in student competencies of critical skills in learning quadratic inequalities.

Pre-testing involves measuring growth in students' academic preparedness or progress, skills, knowledge, attitudes, or behaviours prior to participating in the intervention (Schalich, 2015). This method helped to assess participants when they first enter a program to establish a firm benchmark against which to measure growth or value-added. On the other hand, post-test is student's achievement measured after completing an intervened educational program. In this context, this method is used so that the performance and progress of the students can be easily monitored. Researchers argue that if an intervention is having an impact on its participants, the effect should be reflected as a positive change between participants' scores on the pre-test and the post-test (Delucchi, 2014). This suggests that the pre-post testing method is used to better quantify the learners' baseline knowledge and what they gained from their intervention participation. This method also provides instructor with several opportunities such as feedback on the success about intervention, a better gauge of the time needed for the program, a measure of' participants' confidence in answers and participants' perceptions about incorrect answers. In this study, this PPT method was administered to provide evidence if the use of graphing calculator as an intervention for teaching and learning quadratic inequalities impacted positively on students' thinking and problem solving abilities.

The method (pre-post testing) has some limitations. For example, a positive change in students' performance cannot necessarily be attributed to the effectiveness of the intervention. It is hard to discern if the positive change chartered in the academic improvements was due to the intervention in the classroom (i.e. graphing calculator). If students drop out, the post-test results may be higher because those who remain are more concerned or persistent. This may not mean, on the other hand, that the students who scored so low had little improvement in the post-test scores. Using the same test for both the pre- and post-test can influence post-test scores upwards as students would have absorbed knowledge just from taking the pre-test and concentrate more on that content. McMillan and Schumacher (2006) point out that maturation is a threat to internal validity of the pre-post testing design when the dependent variable is unstable, because of maturational changes. They further state that the threat is more serious if the time between the pre-test and post-test is too long or increasing (Behar-Horenstein & Niu, 2011). This suggests that the convenient time for minimal threat of maturation should be relatively short (i.e. 2-3 weeks) between the pre-test and post-test. Researchers should be cautious about controlling threats such as maturation, which become more apparent with a longer period of time between pre-test and post-test (Behar-Horenstein & Niu, 2011). The idea is the influence of the intervention cannot be effective with a longer period of time between pre- and post-test. In this context, the researcher was cautious of the maturation that threatened the positive results of intervention during the administration of pre-test and post-test.

The instruction time of using the graphing calculator between pre-test and post-test was within three weeks for each school. However, there is one school that was affected by school holidays. For example three sessions including pre-test were done before closing and the remaining sessions were done after re-opening.the GC was used as an intervention tool to support students in the learning of quadratic inequalities. Bell (2010) points out that the longer the time lapse between the pre – and post-test, the more difficult it is to rule out alternative explanations for any observed differences. In this regards, the results of the affected school generated from the pre- and post-tests were not much affected since five sessions were performed after. However, the researcher was cautious when he analysed the results of this school and made claims about the effects of the GC intervention on students' understanding of quadratic inequalities.

In this study the pre-post tests were self-developed by the researcher to be used as a research instrument. The tests were developed after reviewing the related literature and consultation with experts, namely teachers and heads of mathematics department, subject facilitators and university lecturers. The validity of the items was assessed by the three heads of mathematics of the selected schools. The tests contained 8-9 sessions including multiple choice questions intended to measure students' performance skills and conceptual understanding. All the items in the tests were taken from previous examination question papers and 11th grade mathematics textbooks about quadratic inequalities. The tests were constructed in such a manner that they included knowledge, comprehension and application levels (CAPS-DBE, 2012). The same questions for quadratic inequalities were used for the pre-test and post-test but the order or sequence of numbering questions was changed in the post-test for each participating school. Using the same instrument as a pre-test and post-test or using two properly equated tests such as comparable forms of the same test safeguards against instrumentation concerns. See Appendix A. The data (scores) were quantitatively analysed using statistical instruments such as mean, standard deviations, t-distribution and Cohen d effect size. Information collected and analysed did not include student names or other individual identifiable information.

4.8.1.2 Pre- and post-questionnaires

The questionnaires were distributed to all participating students seeking for their perceptions on the use of the GCs in solving quadratic inequalities, their experiences, supporting their problem solving and reasoning skills and helping them do homework independents. A questionnaire has the advantage of taking it to a wider audience compared to interviews, but has a disadvantage of not being possible to customise it to individuals as it is possible with other methods of data collection. The pre-post surveys (PPS) collected the data across two observations: baseline (beginning of program) and at the end of the educational program that had been in operation for the possible change to occur.

4.8.2 Qualitative research methods

The qualitative methods were used to provide subjective information (e.g., focus group and semi-structured interviews, participant observation and video recording) for enriching and understanding the context and actual practice of the study (Alreshidi, et al., 2016) and elucidate the mental processes underlying behaviours as well as to identify the reoccurring patterns that characterise the data (Merriam, 2009).

4.8.2.1 Focus group interview

Focus groups are increasingly being used as a research tool in the social sciences (Wilkinson, 2008; Hopkins, 2007), frequently employed in qualitative research on perceptions (Widdowson, 2012; Freitas, et al., 1998) and in almost any research environment that combines with other qualitative or quantitative methods (Kress & Shoffner, 2007). As a qualitative research tool, focus group always seeks to understand better how people consider an experience, idea, or event (Freitas, et al., 1998). Simply put, it provokes a discussion about what people think, or how they feel, or on the way they act. Its relevance in this study is upheld as it would explore the students' perceptions and experience about the use of graphing calculators in solving quadratic inequalities.

Some researchers view focus groups as group interviews (Hughes and DuMont, 1993), as group discussions (Krueger 1998) or informal discussions among selected individuals (Beck et al. 1986). Focus group is a type of in-depth interview accomplished in a group whose focus (object) is the interaction inside the group (Freitas, et al., 1998). Bedford and Burgess (2001) provide a detailed definition of a focus group as a one-off meeting of between four and eight individuals who are brought together to discuss a particular topic chosen by the researcher(s) who moderate or structure the discussion. This means discussions are carefully planned to obtain perceptions (Krueger 1998) of relatively homogenous groups (Hughes and DuMont, 1993) in a defined environment. The participants influence each other through their answers to the ideas and contributions during the discussion. In most cases, the moderator stimulates discussion with comments or subjects. The fundamental data produced by this technique are the transcripts of the group discussions and the moderator's reflections and annotations.

Focus groups consist of two research methods components such as a group interview and focused interview (Bryman, 2001). The members of a focus group are invited for the interview to share their experiences from a particular situation, which in this case, using GCs to solve quadratic inequalities. Being a focused interview, open questions are asked to the group about a specific situation. Krueger (1994) argues that focus group interviews are useful in obtaining information which is difficult or impossible to obtain by using other methods. As De Vos, Strydom, Fouchẻ and Delport (2002) suggested, a focus group interview is the research technique that collects data through group interaction on a topic determined by the researcher. This means a group of people are brought together in a room to engage in a guided discussion (Babbie & Mouton 2001; Edenborough 2002) in order to collect data of students' thinking and reasoning. Thus using focus groups the researcher can intervene into the conversation and pose questions to probe what members had just said. For example, participants' perspectives are revealed through discussion or participants' questions and arguments.

Focus groups are effective in assessing participants' attitudes, opinions and experiences relative to a specific context (Bryman, 2008; Freitas, et al., 1998), in this case, students' understanding in the GC mediated classroom. Researchers have recommended that group interviews are convenient to conduct and produce immediate results (Krueger, 1988) and are relatively cost effective to reach the same number of participants at a speedy time (Krueger & Casey, 2000). The qualitative data is generated through discussion focused on a topic, which is determined by the research purpose. Freitas, et al. (1998) have recommended the use of focus group research method for generating hypotheses based on the perceptions of the participants and to evaluate different research situations or study populations. This implies that the focus group interview would be useful to explore students' perceptions, opinions and experiences about the use of GC in learning quadratic inequalities. This would help to get an insight into students' beliefs through what the students say, think and write in the classes supported by GC. In this study, a focus group interview protocol is used which constituted of both open-ended and structured questions to examine the students' perceptions and attitudes about the GC's use to learn quadratic inequalities. See Appendix F.

However, there are potential disadvantages of focus group interviews such as increased difficulty with transcribing (Krueger, 1994) especially as some participants normally dominate others in discussion (Wilkinson, 2008). The higher number of participants in a focus group increases the risk of members breaching confidentiality agreements (McParland & Flowers, 2012), may result in less opportunity to participate and could elevate the likelihood of conformity (Wibeck, Dahlgren & Öberg, 2007). To off-set some of these limitations of focus group, this study minimised the number of participants to four. In terms of confidentiality, the researcher stuck to the conditions expressed in the student consent form, see Appendix J.

4.8.2.2 The in-depth interviews

The interview is a social relationship designed to exchange information between the participant and the researcher. In-depth interviews were one of the typical interviews employed in this study and were open to allow the participants to raise their views (Kajornboon, 2005; Babbie and Mouton, 2009). These are organised approaches that collect data with regards to participants' perceptions and interpretation of a given situation (Kajornboon, 2005; McMillan & Schumacher, 2006) and participants' attitudes, thoughts and actions (Kendall, 2008). In addition, interviews afford participants a chance to clarify and elaborate their ideas in their own words. The in-depth face-to-face interviews are selected because of the advantages of the social and gestural cues. Another advantage is that the interviewer can immediately react to the interviewee reactions or responses to seek more information or clarification. The responses of participants are also more spontaneous, without an extended reflection. The participant therefore reacts immediately to the question and it becomes more spontaneous or naturalistic.

4.8.2.3 Semi-structured Interviews

The semi-structured interviews are the most appropriate instrument to answer the research questions. In a semi-structured interview, a researcher initially uses an interview guide (prepared question prompts), but the researcher allows for flexibility in the way they are answered in order to draw as much data from the interviewees as possible (Merriam, 1998). Creswell (2003) recommends that the questions should be few to facilitate interviewees' views and opinions and also to create space for the interviewer to elicit deeper meanings of the responses. This could allow the researcher to ask other questions emanating from what the interviewee said for more clarity on the issue discussed. Mouton (2013) provides a number of factors that might influence response bias; however in this study interviewees were given authority and confidence by making them aware that the researcher was going to learn from their practices before carrying out the interviews.

Semi-structured interviews are non-standardised and are frequently used in qualitative analysis and the interviewer does not do the research to test a specific hypothesis (David & Sutton, 2004). The researcher has a list of key themes, issues, and questions to be covered. In this type of interview the order of the questions can be changed depending on the direction of the interview. An interview schedule is also used, but additional questions can be asked. Kajornboon (2005) indicates that semi-structured interviews are more of an open-ended questions' nature and lend themselves to probing. Kajornboon (2005) points out that if the respondent is uncertain about the question, the researcher can explain or rephrase the question. The semi-structured interviews were most beneficial because probing was possible to understand students' experiences when working in a GC enhanced classroom. They also helped with regards to students' perspective on the effectiveness of GC in teaching and learning so that their marks are improved in quadratic inequalities. The semi-structured interview schedule was validated and moderated by the participating practitioners. In this way the researcher ensured that the semi-structured interview schedule measured what it was supposed to measure.

The interviews were needed to investigate the thinking and reasoning of the students and to obtain a more complete picture of students' understanding of quadratic inequalities. The interviews were conducted within two weeks after the post-test was administered. Pre- and post- tests alone do not provide a complete picture of students' understanding of quadratic inequalities. In addition, data from the interviews provided explanations for students' responses on the post-test. This is within the goals of the interviews which aimed to gather information about students' responses on the post-test, students' use of graphing calculators, and students' problem solving, reasoning and sense making abilities in quadratic inequalities. The first goal was to clarify students' confusing statements or omitted responses, to explore students' preferred representations of quadratic inequalities, and to explain factors for changes in responses from pre-test to post-test. Student misconceptions of quadratic inequalities were not a primary concern of the interviews but were explored when they arose. The second goal of the interviews was to explore students' use of graphing calculators in the classroom and for homework, but was prohibited for examination assessment. The third and last goal of the interviews was to explore whether students were able to apply their understanding of quadratic inequalities to solve the contextual problems with GC. The protocol for the interviews (see Appendix E) was developed.

4.8.2.4 Classroom observations

The purpose of the classroom observations was to collect data about the GC mediated classroom environment in which students encounter quadratic inequalities. The focus of the observations in this study was the students' use and instructor's use of graphing calculators. As Marshall and Rossman (1999) suggested, observation is a systematic noting and recording of events, behaviours and objects in a social setting chosen for the study. Furthermore, they emphasised that observation is a fundamental and highly important method in all qualitative inquiry. Angrasino and de Pèrez (2000) argued that observations need to be conducted in naturalistic settings so that they do not interfere with participants or planned activities. In the case of this study, the smart board classroom of each experimental school was used for the research purpose and provided the naturalistic environment for observation. The information gained from the observations was descriptive in nature and provided a more complete picture of how the created environment influenced students' thinking, behaviours and attitudes in solving quadratic inequalities. The other focus was what and how materials were presented to students in class. Skemp (1971) stated that the mathematical topics presented in class were the experiences in which students construct their understanding due to the abstract nature of mathematics.

In order to capture the complexity of integrating graphing calculators into teaching and learning, the session presentations and student participation were video recorded. The video was also used to capture students' verbal and non-verbal communication which reveals students' mathematical understanding. This facilitated the collection of comprehensive data needed to give detailed descriptions of the teaching and learning process. The video recorded classes were viewed and transcribed by the researcher to evaluate the enablement of GCs during actual sessions. The actions of teacher-researcher (instrumental orchestrations) and learners are captured and the artefacts (e.g. written work and test scripts) used are also included in the synthesis of the data.

The sessions were divided into five main task activities such as quadratic inequality concept, graphic representation, symbolic manipulation skills, numeric manipulation skills and contextual problems. These session activities were spread into three weeks for each DBR cycle and classroom observations began the first week in order for the students to get acquainted with being observed. Daily observations continued until the accomplishment of the sessions and the administration of post-test. The sessions were videotaped and notes on the material presented were taken. After each classroom observation, any research materials handed out by the teacher-researcher were collected and the researcher completed an observation and document summary shown in Appendix G. These summaries were short forms that were used to develop questions for the student interviews and to help with data analysis. The handouts given to students were homework assignments, worksheets and quizzes.

4.9 Validity and reliability in mixed methods research

Criticism has been on the subjectivity, or sample sizes, or unscientific methods used to collect data of qualitative studies. These are obviously the questions of validity and reliability, which address trustworthiness of the research study (Merriam, 1995). This section discusses reliability and validity of the research results. Although terms of validity and reliability engage with positivism and are related to the quantitative approach, they have also been used for interpretive research with the qualitative approach (Alreshidi, et al., 2016; Lincoln & Guba, 1985). Lincoln and Guba (1985) proposed the following appraising criteria for studies adopting mixed methods approaches described in Table 4.3 below.

Table 4. 3: Appraising criteria for studies adopting mixed methods approaches

Concept	Quantitative Methods	Qualitative Methods
Truth value	Internal Validity	Credibility
Applicability	External Validity	Transferability
Consistency	Reliability	Dependability
Neutrality	Objectivity	Confirmability

Reliability refers to the extent to which test scores are free of measurement error in a quantitative research. As noted by Muijs (2011), whenever researchers want to measure something, there is some element of error in the research measurements. However, this study employed various procedures to build the trustworthiness in the research instruments and research results.

Internal reliability refers to the reliability within a research project. It can be improved with several procedures, including a pilot study and the Cronbach alpha coefficient to measure the reliability of instrument. In the previous sections, the data collection methods discussed how the reliability could be increased. Internal reliability further refers to the reasonableness of inferences and assertions (Bakker, 2004). This was improved by discussing the students' misconceptions and errors with practitioners.

External reliability usually denotes replicability or confirmability of the research. This means that the conclusions of the study depended on the subjects and conditions, and not on the researcher (Bakker, 2003). Differently viewed, Lincoln and Guba (1985) refer confirmability to the degree to which the researcher can demonstrate the neutrality of the research interpretations, through an audit trail. In qualitative research, replicability is mostly interpreted as virtual replicability. Bakker & van Eerde (2013) interpreted trackability (transparency) in the research as what the readers/users are able to track in the whole process of the study. In this context, the data should reflect the views of the participants accurately rather than the researcher's views. For this reason, the research must be sequentially documented in such a way that it is clear how the research has been carried out and how conclusions have been drawn from the data.

The readers/users of this present DBR study would be able to track the learning process of the participants and to reconstruct their study as failures and successes, procedures followed, the conceptual framework used, and the reasons to make certain choices were all reported as they manifested in the study. The users would experience the cyclic process of development of students' understanding of quadratic inequalities and this experience can be transmitted to others to become like their own experiences. This study would further provide raw data that can be traced to original sources (i.e., the participants of the three schools). In this sense, students' views are accurately presented through the use of interviews, video observations and written work as multiple data sources. Bakker & van Eerde, (2013) further emphasize the degree of independence of the researcher about how the data was collected and analysed. In this study the researcher's independence was improved by discussing the critical fragments with colleagues, called inter-subjectivity, about the interpretations and conclusions during the retrospective analysis. With graphing calculator strategies as the goal in mind, each response given by the student participants and each action observed was coded using action verbs that describe the steps to solving quadratic inequality problem.

Internal validity refers to the quality of the data collections quantitatively and the soundness of the reasoning that has led to the conclusions (i.e., credibility) qualitatively. Several research instruments were used to improve the internal validity of this study such as improving HLTs in the teaching experiments and testing conjectures that were generated. This study also used video recordings of classroom observations and of focus group discussions, field notes, students' written work, and student interview recording to collect data. The use of mixed instruments of data collection allowed the researcher to perform data triangulation and contributed to the trustworthiness and credibility of data. The credibility of the findings was maintained through prolonged engagement with the participants during interviews and in classrooms, as an attempt to establish an effective vehicle for obtaining and processing reliable information. Furthermore, credibility was maintained through an on-going dialogue with participants, numerous observations, and by the use of member checks. The researcher was fully engaged in observing student participants as they interacted with graphing calculators in solving inequality problems (prolonged engagement) over a nine-week period in order to understand the actions of the students. The data sources helped to search for counterexamples of the conjectures of this study. During this period, the researcher established trust and mutual understanding with the participants, hence granting them autonomy to perform instructional activities with graphing calculators confidently. The activities of every participating member were strictly checked including attendance in class to ensure correct and truthful interpretations to the data collected. At the end, participants were allowed to review the researcher's findings so as to verify if the data collected matched with their experiences and also to confirm the accuracy of interpretations.

External validity is mostly interpreted quantitatively as the generalizability of the research results. Transferability is interpreted differently in both quantitative and qualitative paradigms. Transferability in quantitative paradigm refers to the ability of the researcher to extend the findings of a study beyond the specific individuals and setting in which that study occurred (Merriam, 1995). While in qualitative research, it refers to the question of how we can generalize the results from certain contexts to be functional for other contexts (Bakker & van Eerde, 2013). The question is how the results can be generalized from these specific contexts as to be useful for other contexts. An important way to do so is by framing issues as instances of something more general (Cobb, Confrey, et al., 2003; Gravemeijer & Cobb, 2001). Merriam, (1995) has considered transferability (generalisation) in three concepts such as working hypotheses, concrete universals and reader/user, as the basis for interpreting generalizability of findings of qualitative research. Small sample size and non-random sampling (mostly purposeful) in a qualitative study leads to a notion of non-generalizability across different settings. However, the purpose of qualitative research is to understand the particular in depth, rather than finding out what is generally true of many (Merriam, 1995). In essence, there is no reason not to generalize (or transfer) research findings in other types of similar situations. More often than not, general conclusions are made from similar particulars. Merriam's (1995) contribution is that, "the general lies in the particular" (, p. 58). This suggests that it is up to the reader to decide the extent of generalizability of the research findings, not up the researcher.

To ensure transferability of the findings of this study to other schools, districts, provinces and other departments different from education, the researcher used a mixed methods approach inscribed in DBR to explore and understand the experiences, opinions and perceptions of participants. A detailed, practical description of the findings of this DBR study was presented so that the audience would understand the contextual variables operating within this setting and have a solid framework for comparison (Merriam, 2009). The challenge however was to present the results of instruction theory, HLT, and instructional activities in such a way that others can adjust them to their local contingencies (Barab & Kirshner, 2002). In the conclusions the researcher produced a list of issues and patterns similar that occurred in classes of three schools in the teaching experiments (Chapters 5, 6 & 7). By using the theory of instrumental genesis, the graphs were drawn with the help of graphing calculator environment to provide more abstract explanations connected to the solutions of quadratic inequalities. The graphs were successfully applied in learning quadratic inequalities in all the teaching experiments and can be generalised that their use improved students' understanding in a GC mediated classroom.

4.10. Ethical considerations

Prior to data collection, an ethical clearance letter was obtained from the university granting permission to pursue data collection. Secondly permission was also sought for conducting the study at the high schools in Gauteng Province from the Gauteng Department of Education (GDE) but at district level, school governing boards, and the principals of the participating schools. Thirdly, consent forms were sent to parents of the participating learners to be signed granting permission for their children to be interviewed, observed, tested and taught during the investigating period. Participating students were informed of their rights to withdraw at any time without any consequences and were given options not to answer any question that they were not comfortable with. The researcher emphatically informed the participants that the study was not evaluative and that the findings and conclusions were not be used against them. To protect the rights of participants, the data collected were kept confidentially so there was no link to any student, and codes were used to capture data on an individual participant basis. Upon successful completion of this study, the data were deleted from the researcher's laptop. The researcher had no personal interest in the participating schools, and as such the bias in data collection were minimised. The letter for the school principal was presented in Appendix A and a letter to the teachers was in Appendix B. The letter to the school governing board to inform parents about the researcher's intention was presented in Appendix C. The student consent form was presented in Appendix J.

4.11. Summary

This chapter has presented pragmatism as the research paradigm and the DBR as the research methodology. A pragmatic paradigm was adapted for its constructivist approach that the truth is socially constructed and is independent of the researcher. As a pragmatic approach, DBR integrated different worldviews and assumptions which were used as the bases for mixed approaches and procedures of data collection and analysis. Mixed methods were used to explore and understand students' behaviours in learning quadratic inequalities in a GC mediated classroom. The use of mixed methods assisted the researcher to arrive to the conclusions with more confidence and to provide a holistic view of the findings. In-depth and semi-interviews, focus group interview, classroom observation, video recording, pre-post testing and pre- and post-questionnaires were administered as data collection instruments. The use of mixed instruments of data collection allowed the researcher to perform data triangulation and to achieve the trustworthiness, validity and reliability of data. In addition, issues of population and sampling procedures were also elaborated in this chapter. This sampling strategy helped the researcher to select the three secondary schools and student participants (N=120) learning Pure Mathematics as one of the core subjects from Gauteng province. The HLT for teaching/learning quadratic inequalities was developed in the first phases and implemented in the three cycles in a total of 15-week instructional sequence. The instructional materials and activities were designed with the help of HLT. Permission was sought for conducting the study at the high schools from Gauteng Province Department of Education. Consent forms were signed by the students granting permission for their children to participate in the research. Participating students were informed of their rights to withdraw at any time without any consequences and confidentiality of their information was at maximum level.

The next chapter 5 presented the analysis of the data collected from the various data collection procedures of the study in School A.

CHAPTER 5: RESULTS OF THE FIRST CYCLE OF THE TEACHING EXPERIMENT

5.1. Introduction

The purpose of this present study was to investigate the grade 11 students' understanding of quadratic inequalities in a graphing calculator enhanced mathematics classroom. In addition, the study examined whether the problem solving and reasoning skills of the students were supported by the pedagogical use of the graphing calculator. The data described in this chapter were collected in School A to address the following research questions:

1. To what extent can the pedagogical use of graphing calculator influence high school students' performance in solving quadratic inequalities?

2. In what ways (how) can the pedagogical use of the graphing calculator support the high school students' problem solving ability in quadratic inequalities?

3. In what ways (how) does the pedagogical use of the graphing calculator support the high school students' reasoning ability when solving quadratic inequalities?

4. What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?

This chapter is the first part of the empirical results of the study and addressed the first DBR cycle at the eleventh grade. In attempt to answer the research questions, the results of the first DBR cycle of the teaching experiment in School A are presented in the following sections. First, Section 5.2 describes the starting points of the HLT including the learning outcomes. Second, the educational setting of the cycle is explained in Section 5.3. Third, the results of the pre- and post-tests are analysed in Section 5.4. Four, the results of the problem solving are analysed in Section 5.5. Fifth, Section 5.6 presents the results of the focus group interviews with students. Sixth, the results of pre- and post-surveys are analysed in Section 5.7. The results of the in-depth interviews with students are discussed in Section 5.8. Finally, the study concludes with the reflections, feed forwards and design principles in Section 5.9.

5.2 Starting points of the HLT and learning outcomes

The starting points for the HLTs for quadratic inequalities were the use of set builder notation, which defines the solutions of unknown variable, drawing a number line for the given builder set notation and graphing quadratic functions and the effect of parameters on quadratic functions and quadratic equations. The graphing and symbolic processes of quadratic inequalities were identified as the ground level, and the complex and concrete processes represented the higher level of the concept of quadratic inequalities. The middle level embraced the routine processes of quadratic inequalities. The understanding of the properties of quadratic graphic representations was considered to be important in the reification of symbolic expressions and formulas in the development of algebraic concepts (Drijvers, 2003). In conjunction with the starting points, the following learning outcomes were derived from the prepared sessions that inform the design of the HLT for quadratic inequalities:

Learning outcomes of the instructional activities

The first learning outcome in session two was that students would develop the notions of interval notations, parameters, x-intercepts and finally quadratic graphs in a flexible graphing calculator environment. This means the GC use was expected to facilitate the transition from the graphical representation and interval notation to quadratic inequality representation.

The second learning outcome in session three was that students would develop the notions of solving quadratic inequalities in a flexible graphing calculator environment. This means that the use of graph and tabular instrumented schemes was expected to facilitate the transition from *the quadratic graphs and interval notations to symbolic quadratic inequalities*

The third learning outcome in session four was that students would develop the reasoning skills to solve routine problems in symbolic quadratic inequalities in a flexible graphing calculator environment. This means that the GC use was expected to support the transition from the routine procedures to complex procedures of solving quadratic inequalities.

The fourth learning outcome in sessions 5 and 6 was that students would develop the higher order problem solving and reasoning skills in contextual quadratic inequality situations in a flexible graphing calculator environment. This means that the GC use was expected to support the transition from the symbolic to the contextual quadratic inequality problems.

5.3. Participants and research procedures of the study

The teaching experiment took place in a public high school with eleventh-grade class, thereafter the school is referred to as School A. A total of 42 students participated in this study and were randomly chosen from 130 eleventh graders. The participants were asked not to identify themselves on the pre-tests and post-tests, but rather to label their scripts with symbols given randomly by their teacher, such as A1, A2, A3,..., A42, where A represented the school. However, 5 students decided to withdraw their participation and were not considered during the quantitative analysis of the results. Four students also participated in the interviews. The sample included below average, average and above average students and a top student.

To avoid research bias, the teaching was conducted by the researcher and a pre-test was administered on the first day before the GC intervention. The test consisted of 6 questions to be solved traditionally. Additionally, students were tested on their reasoning and problem-solving abilities in the contextual quadratic inequality problems. Immediately after the pre-test assessment, they answered the pre-intervention questionnaires. Thereafter students received instructions on how to use the GC and were allotted time to explore the graphing of quadratic functions. The same tool- the GC was also used to solve quadratic inequalities and to interpret or determine their solution sets. The students were given more activities to perform using the GC in order to understand and visualise the graphical representations.

The GC enabled students to create and use tabular, symbolic and graphical representations and also to analyse patterns and relations of graphs. An observation of student interaction with the graphing calculators was made as they performed their class activities. Students were given adequate time to familiarise themselves with the use of graphing calculators as they attempted the designed activities of eight sessions. Not deviating from the CAPS document requirements, the students used the GC as a visualisation tool that can make students intuitively understand the properties of algebraic processes, which are missed during the learning process and the reasons of the errors that they make (Stavy, et al., 2006). It was also used, as suggested by Averbeck (2000), as an alternative method for solving mathematical problems, a guide for planning the solution process and a reference or resource for checking accuracy of their graphs and their solutions during the monitoring phase. Ultimately, the post-test was administered which was identical to the pre-test so that the improvements in understanding could be monitored during the teaching experiment. The post-test was administered to measure learning gains attributable to the use of the GC. In answering the questions of the post-test in quadratic inequalities, students were not allowed to use the GC. This is in compliance with the CAPS document to use the GC as an alternative technology. After the post-test assessment, the students completed the post-intervention survey and some were selected for further interviewing. The post survey was used to measure changes in students' opinions after the use of graphing calculator to solve quadratic inequalities. The data collected from this survey were then analysed using SPSS 16.0.

5.4. Analysis of the students' results of the pre- and post-tests

This section presents and discusses the results of the pre- and post- tests and the written tasks which sought to answer the first research question:

> *To what extent does the pedagogical use of GCs impact on students' performance in solving quadratic inequalities?*

The results were also used to test the null hypothesis: H_0: There is no difference between the pre-test mean and the post-test mean of quadratic inequalities for the students in the study (H_0: μ_1 (pre−test) = μ_2 (post−test). Alternatively, H_1: There is a difference between the pre-test mean and post-test mean of quadratic inequalities for the different domains for the students in the study (H_1: μ_1 (pre−test) ≠ μ_2 (post−test). The results of 35 students obtained in School A are presented in the form of descriptive statistics (Table 5.1) and paired samples test (Table 5.2) below.

Table 5.1 below, presents the descriptive statistics analysis of School A showing that students performed much better in the post test (M=35.9143; SD=19.76237) than in the pre-test (M=21.0857; SD=15.98965). The results of the post-test reflected better scores in the median mark of 36% and highest score of 87% from 77%. This suggests that there was a reasonable improvement in the post-test towards the understanding of quadratic inequalities. This could be attributable to effective intervention of the GC in the learning of quadratic inequalities.

Table 5. 1: Descriptive Statistics Analysis of School A

	N	Minimum	Maximum	Mean	Std. Deviation
Pre-Test	35	3.00	77.00	21.0857	15.98965
Post-Test	35	7.00	87.00	35.9143	19.76237
Valid N (listwise)	35				

In Table 5.2 below, the results of the dependent (paired) samples t-test show that $t(34) = -11.384$ and $p = 0.000$. This means the actual probability value is 0.000 and it is substantially smaller than the specified alpha value of 0.05. These t-test results indicate that the null hypothesis was rejected at 5% significant level in favour of the alternative hypothesis. Therefore, there was a statistically significant difference between the students' means of the pre- and post-test scores. In that context, there was a statistically significant improvement of the students' results after the use of the GC in the learning of quadratic inequalities.

Table 5. 2: Paired Samples Test of School A

Pair 1	Paired Differences							
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		T	Df	Sig(2-tailed)
				Lower	Upper			
Pre-Test & Post-test	-1.48286E1	7.70594	1.30254	-17.47565	-12.18149	-11.384	34	.000

Although the results presented above indicated the statistically significant improvement in the test scores of students, they do not tell much about the magnitude of the GC intervention's effect in solving quadratic inequalities. Because there was no control group for this particular task and limitation of statistical significance, the researcher proceeded to calculate the Cohen's d effect size statistic using the pre-test and post-test means in order to determine the magnitude or practical significance of the difference in scores. The effect size was 1.1, indicating that the post-test mean is at 86% of the pre-test mean. This means that there was a large effect impacted by the use of the GC on learning quadratic inequalities (i.e., using Cohen's, (1988) interpretation: 0.2=50%=small effect, 0.5=58%=medium effect and 0.8=79%≥ large effect). The researcher therefore concluded that there was a practically significant improvement of about 0.8 standard deviations in the mean scores from pre-test (M=21.09, SD=16.00) to post-test [M=35.91, SD=19.76, t(34)=-11.384, p=0.000<0.005]. This implies that the pedagogical use of the GC impacted positively on the students' performance in solving quadratic inequalities. This study did not investigate if the teaching and learning of quadratic inequalities with the GC is better than an approach without it, but to show that it can help with the understanding of the topic or concept. The t-test results showed that there were learning gains not only for the purposeful sample of students but also for all the students that were exposed to the teaching intervention with the GC and instructional material. The findings are consistent with the theoretical frameworks as they were used in designing the learning activities.

5.4.1 Students' results in written tasks of symbolic quadratic inequalities

The improvement of students' performance in the post-test could be attributed to the written tasks during the teaching experiment. Students wrote a task with questions related to sessions 3 and 4 and was considered for analysis. These questions were meant to monitor the progress of the students in each session. The results of the students were analysed to determine what percentage of those who answered the symbolic quadratic inequalities correctly, incorrectly, blankly or incompletely and also used graphic approach (see Table 5.3), below.

Table 5. 3: Results of the written task about symbolic quadratic inequalities

Questions of the Written Task	**Correct %**	**Incorrect %**	**Blank %**	**Incomplete %**	***Method used***		
					Graph %	***Others %***	***None %***
4.1 $\mathbf{4x + x^2 \leq 0}$	40	26	9	26	54	20	26
4.2 $\mathbf{(x+2)(3x-7) \geq 0}$	51	14	6	29	66	17	17
4.3 $\mathbf{x^2 - x - 12 < 0}$	54	20	6	23	57	23	20
4.4 $\mathbf{-(x-4)(x+5) < 0}$	43	17	3	37	72	17	11
4.5 $\mathbf{2x^2 - 7x \geq 4}$	37	26	14	23	48	23	29

The analysis of the data in Table 5.3 above revealed that each of the symbolic quadratic inequality questions was solved correctly by more than 40% of the students with the exception of question 4.5. This may suggest that at least 40% (>14 of the students) had acquired adequate knowledge and skills for solving symbolic quadratic inequalities. The percentage of students who had incorrect or blank solutions was between 20 and 35, which indicate an improvement in the understanding of quadratic inequalities. It was also observed that between 20% and 37% of students had incomplete answers. Question 4.4 had the greatest percentage (37%) of incomplete answers. An incomplete solution means that the student used the correct strategy but abandoned the strategy before arriving to the solution. One of the observations made by the researcher was that students were able to determine the critical values and sketch the graph but could not go beyond that. On the other hand, students had difficulties with identifying the solution set of the quadratic inequality as the coefficient of x-squared was negative. This means student had difficulties in determining the domain of inequality function.

Students' answers were supposed to be linked to the use of graphs in order to see the influence of GC. In this vein, students' abilities of using graphs were assessed in solving symbolic quadratic inequalities. It was observed that more than 50% of the students used graphs to determine the solutions of the inequalities, except in question 4.5 with 48%. In this question students attempted to solve before expressing in standard form. Not more than eight students (≤ 23%) used other methods such as line graph and sign chart or table to determine the solutions of quadratic inequalities. A line graph is also shown on the graphing calculator when solving quadratic inequalities. In this case, the influence of the GC was very strong in developing such skills among the students. In the table, only 10 or less did not use any of the three methods linked to the GC.

A sample of answers of the four students in the written task is shown in Figures 5.1 and 5.2 below. The four students were purposefully chosen because their written work represented all types of possible answers presented in School A.

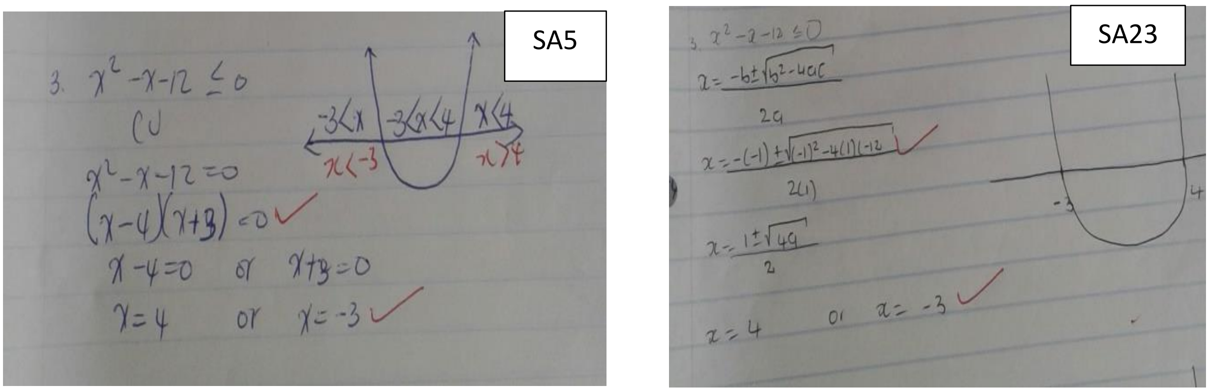


Figure 5. 1: Students' incomplete answers on symbolic quadratic inequality

Figure 5.1, above shows the incomplete solutions of two students SA5 and SA23 who used both the algebraic and graphical methods. The first student correctly determined the critical values using the factorisation method and also correctly sketched the graph but failed to identify the region representing the solution of the inequality. On the other hand, the second student also failed to determine the solution of the quadratic inequality after using the quadratic formula and sketching the graph correctly. The graph of student SA23 lacked details compared to the student SA5's graph. Figure 5.2 below shows the solutions of other two students who used different graphical methods to determine the region of the inequality.

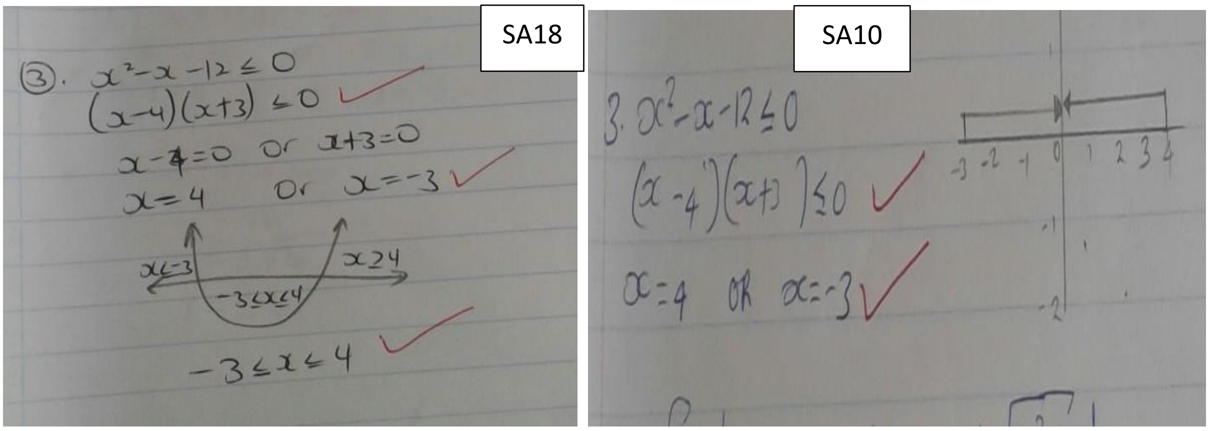


Figure 5. 2: Students' solutions with different graphs

Student SA18 correctly used the sketched graph to determine the region representing the solution of the quadratic inequality. This means the solution was correctly written. Student SA10, on the other hand, correctly used the line graph to indicate the region of the solution set. The two graphs drawn by the students are commonly seen on the graphing calculators; as a result students acquired these from the consistent use of the GC. Based on the students' work, the use of the GC helped them to produce visual models and diagrams as well as integrated meaningfully the graphical and algebraic representations that supported their reasoning and problem solving. This is consistent with the ideas of the level and intertwinement principles of the RME and instrumented action schemes of the instrumental approach theory.

5.4.2 Students' results of written tasks in applications of quadratic inequalities

The improvement of students' performance noted in the post-test could be attributed to the written tasks during the teaching experiment, related to questions in session 5 and was considered for analysis. The written task assisted to monitor the progress of the students. The results of the students were analysed to determine what percentage of those who answered the application problems of quadratic inequalities correctly, incorrectly, blankly or incompletely (see Table 5.4), below.

Table 5. 4: Students' results in application of quadratic inequalities

Question	Written Task on Applications	Correct %	Incorrect %	Blank %	Incomplete %
5.1	**For what values of x will $\sqrt{x^2 - 25}$ be real?**	37	29	14	20
5.2	**For which values of x will Q=$\sqrt{x^2 - 8x + 12}$ be non-real?**	49	20	9	23
5.3	**Given $g(x) = -x^2 + 7x + 6$ For which values of x will g(x)>0?**	40	23	11	26

The analysis of the data in Table 5.4 revealed that 37%, 49% and 40% of the students respectively applied correctly quadratic inequalities to solve questions 5.1, 5.2 and 5.3. This indicates that at least 63% of the students needed to be re-skilled in these problem areas of applications. The proportion of incomplete solutions (i.e., 20%, 23% and 26% respectively) was very high for all the three questions. This implies that students were able to identify the problem and select the right strategy (i.e., forming quadratic inequality) of the application problem. The incomplete solution might indicate that further use of the GC is needed to develop students' cognitive skills and confidence. This therefore lays foundation for the emphasis in the revision for the post- test. However, a number of students still experienced difficulties in determining the solutions of the quadratic inequalities. This included those who had incorrect or blank solutions, which are 46% in Q5.1, 29% in Q5.2 and 34% in Q5.3. The consistent use of the GC may reduce the percentage of students who had incorrect or blank solutions. The level principle of RME theory helped the researcher to interpret how students used the models and graphs to move their informal thinking (horizontal mathematization) away from the application to the formal reasoning (vertical mathematization). This is in line with the principle of intertwinement as this topic involves the connections of many concepts and activity principle was at play as students were confronted with situational problems as well as the socio-cultural learning of Vygotsky's theory where appropriate language was to be used when solving quadratic inequalities.

5.5. Students' results of problem solving in quadratic inequalities

5.5.1 Analysis of the student's problem solving strategies in the post-test

This section intended to answer the second research question, "*In what ways (how) can the pedagogical use of the graphing calculator support the high school students' problem solving ability in quadratic inequalities?*", through discussing the processes used by the students in applying problem solving strategies in Question 5 of the post-test. This question required students to determine the values of x for which $\sqrt{25 - x^2}$ will be non-real. The students applied quadratic inequalities to determine the values of x. The students were purposefully chosen because their written work represented the different problem solving approaches. A rubric for quadratic inequality problem solving test (QIPST) in Appendix H was used to score this question. A sample of answers of the three students is shown in Figure 5.3 below.

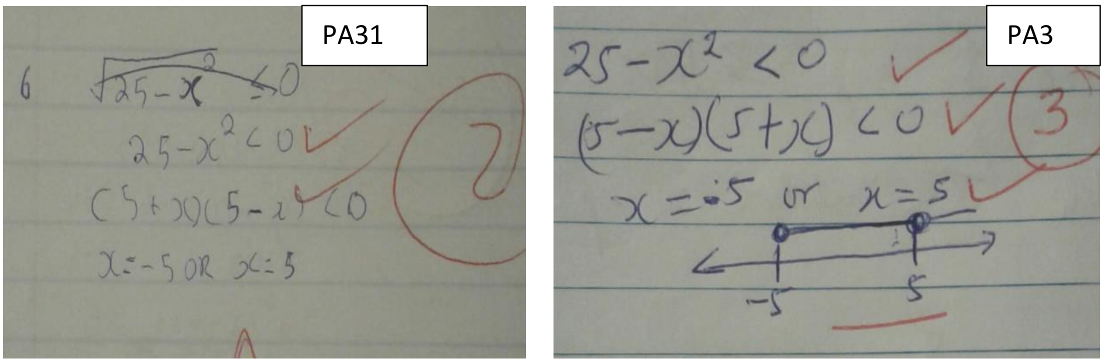


Figure 5. 3: Students' answers on problem solving

Using the rubric QIPST, student PA31 understood the problem and correctly identified the strategy of solving the problem but was not able to apply it. She did not try hard enough to determine the solution. For that reason, she scored a 2. The student used an algebraic approach throughout the entire process of problem solving, including finding roots, setting up the inequality and using the correct procedures of removing the squares. Her procedures did not lead her to the correct roots.

Using the rubric QIPST, student SA3 understood the problem and correctly identified the strategy of solving the problem. He was able to apply the strategy but the solution set was incorrect due to misconceptions. He did not try hard enough to reflect on his solution. In that reason, he scored a 3. The student used an algebraic approach throughout the entire process of problem solving, including setting up the inequality and finding roots. He was able to identify the difference of squares as the correct procedure of determining the roots. A line graph was correctly drawn by the student to illustrate his wrong solution. His mistake, however, was not in understanding of the problem but in an inaccurate application of the quadratic inequalities.

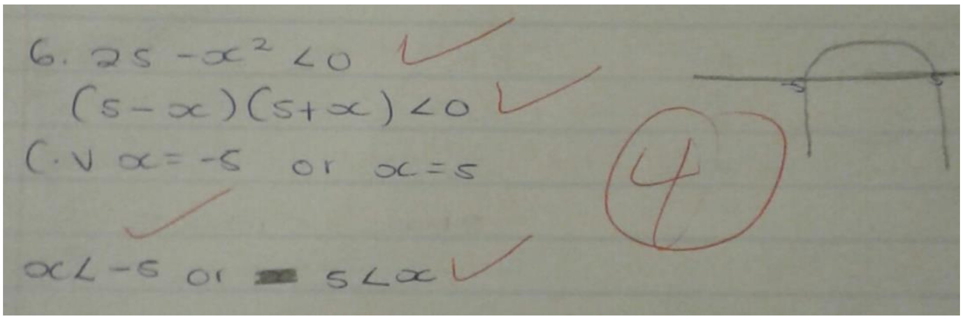


Figure 5. 4: Student's answer on problem solving

Using the rubric QIPST, student SA26 understood the problem and appropriately applied the strategy of solving the problem. This means an inequality was correctly constructed and appropriate solutions arrived at using the correct procedures. She tried hard to reflect on her solution by checking its reasonableness. In that regard, she scored a 5. The student used both graphical and algebraic approaches to help solve the problem. The algebraic approach seemed to be the most useful in setting up the inequality and finding the critical values by using the difference of squares procedure. A graphical approach also helped her to solve the problem by correctly sketching the graph and indicating the zeros of the function. The student appropriately understood the effect of the negative parameter 'a' in the quadratic function. She used the graph to determine more than one interval that would be needed to solve this application problem and realised that one of these intervals was positive and one was negative. The teacher-researcher cautiously concluded that the

GC use supported the students' performance and problem solving strategies. This is aligned with the ideas of the level principle of RME theory as designing and sequencing of the instructional materials were structured using the levels of mathematical reasoning and problem solving.

5.5.2 Student's perceptions of how GC use supported the quadratic inequality problem solving abilities

This section intended to answer the second research sub-question on how the students perceived about the use of the GC towards supporting their problem solving abilities of quadratic inequalities. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 5.5 below.

Table 5. 5: Students' results of how GC use supports problem solving

Students' perception of how GC use supports problem solving	**SD %**	**D %**	**NS %**	**A %**	**SA %**
GC use enabled me to interpret the problem of quadratic inequality	6	11	20	46	17
GC use guided me to sketch the graphs for solving quadratic inequalities	3	6	14	46	31
GC use helped me to use correct methods and procedures to solve quadratic inequalities	9	9	17	40	25
GC use allowed me to check for mistakes and correctness of my quadratic inequality solutions	6	9	14	48	23
Key: SD-Strongly Disagree; D-Disagree; NS-Not Sure; A-Agree; SA-Strongly Agree					

In Table 5.5, sixty-three percent of the students agreed or strongly agreed that the GC use enabled them to interpret the quadratic inequality problems. However, there were 7 of the 35 students (20%) who were not sure. Twenty seven students (77%) affirmed that the GC use guided them to sketch the graphs for solving quadratic inequalities. Sixty six percent of the students agreed or strongly agreed that the GC use developed them to use correct methods and procedures in solving quadratic inequalities. Seventy one percent of the students felt that the GC use allowed them to check for errors, mistakes and correctness of their solutions. Of those students who were not sure in their decisions, the researcher suggested that they needed an expanded opportunity with the use of the GC on how to solve quadratic inequalities. The researcher therefore cautiously concluded that the majority of the students perceived that the use of the GC supported them in problem solving of quadratic inequalities as all the Polya's four-steps of problem-solving processes had high percentages.

The teacher-researcher observed that the students actively participated in solving quadratic inequalities. Students were able to construct meaning of solving quadratic inequalities with the use of GC as they worked with these tools and further explored ways to use the tools. Students became familiar with the tools after working with them and were used to help solve problems. This means students used the graphing calculator to see the connections between a solution and meaning of solution in terms of the graph (White-Clark et al., 2008; Amineh & Asl, 2015). In this case, the GC was used for exploration, an idea more consistent with cognitive constructivism. This is consistent with the Vygotsky's theory of socio-cultural learning where the GC was used as mediator to develop cognitive understanding.

5.6 Students' results from the focus group interviews

This section described the qualitative results of students' focus group interviews on both problem-solving (Section 5.6.1) and reasoning (Section 5.6.2). A question was selected from the post-test for critically exploring the students' problem-solving abilities and students' reasoning skills in quadratic inequalities. Students were expected to use the graphing calculator only to verify the reasonableness of their solutions. According to Lunenberg (1998), learning in a constructivist manner involves asking students to analyse a problem, interpret results, classify terms or concepts, and to make predictions. These cognitive activities are strongly connected to the processes of students' understanding.

A sample of three students was selected from School A for focus group interviews. There were two female students (AF1 and AF3) and one male student (AF2). The interviews took place in their math classroom on a regular school day after school hours. Three students who obtained marks below average, average and above average in the post-test were purposefully selected to participate in the focus group interviews. The participants were asked to solve a contextual quadratic inequality problem that was in the post-test and were also asked several questions relating to the reasoning processes involved in solving that problem. The students were asked to explain their thoughts throughout the interviews in order to understand their thinking processes. Throughout the interview, participants were observed how they used algebraic approaches and, sketches and graphs in their thinking processes through the problem. At the end of the interviews they submitted their interview scripts to further analyse how they reasoned their way throughout this problem. Each interview lasted for approximately thirty minutes. The interviews were audio recorded and then transcribed.

5.6.1. Students' results of the focus group interview on problem solving

This section attempted to address the second research question of the study:

In what ways can the pedagogical uses of the GC enhance students' problem solving abilities when solving quadratic inequalities?

The selected students were presented with a contextual problem that they first saw in the post-test. The problem stated, "*A small manufacturer's weekly profit is given by* $P(x) = -2x^2 + 220x$, *in which* x *is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R6000".* The students were given ten minutes preparation time to read, formulate and solve the problem. They were supposed to explain their thinking processes clearly and not to erase their working. In this case, students answered questions 1.6 and 1.7 (Appendix D) which were central to solving the contextual quadratic inequality problem.

The codes used in the interviews are: TR for the teacher-researcher and AF for the focus group students from School A. Student participants were interviewed as a group and their responses were presented and transcribed below:

TR: (The teacher-researcher handed out the problem to the students). *Please read this problem attentively and then formulate the required mathematical statement.*

Students AF1: $6000 \leq -2x^2 + 220x$
Students AF2: $6000 \geq -2x^2 + 220x$
Students AF3: $-2x^2 + 220x \geq 6000$

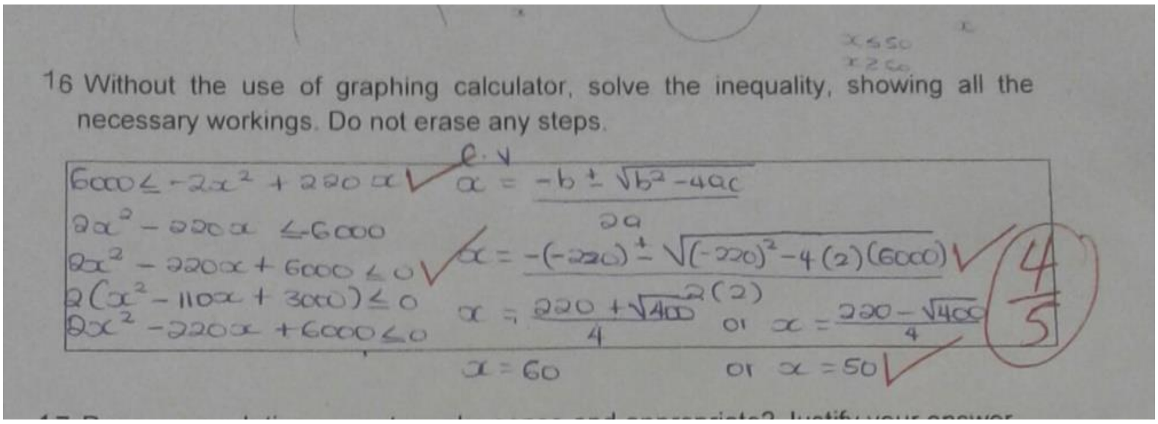
TR: And then solve it without the use of the GC, showing all the necessary working. Please do not erase any step you have written.

Students (AF1, AF2 & AF3): After ten minutes the students handed in their scripts for marking.

Interpretation of the individual students' interview results

Students' attempts are shown below and were analysed in order to determine whether the use of graphing calculator had enhanced their problem solving competencies at School A:

Student AF1 was able to formulate the required quadratic inequality from the contextual problem as $6000 \leq -2x^2 + 220x$. She used an algebraic approach that is a quadratic formula throughout the entire process of problem solving; including finding the critical values (see Figure 5.5). Despite the fact that she was not supposed to erase any attempt, she went on to cancel the work with inequality as faintly seen on top left of her script. This student seemed to confuse the two concepts of equation and inequality, although she knew that the algebraic statement formed was a quadratic inequality. This shows a lack of true understanding by this student.


16 Without the use of graphing calculator, solve the inequality, showing all the necessary workings. Do not erase any steps.

$6000 \leq -2x^2 + 220x$
$2x^2 - 220x \leq -6000$
$2x^2 - 220x + 6000 \leq 0$
$2(x^2 - 110x + 3000) \leq 0$
$2x^2 - 220x + 6000 \leq 0$

C.V
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-220) \pm \sqrt{(-220)^2 - 4(2)(6000)}}{2(2)}$
$x = \frac{220 + \sqrt{400}}{4}$ or $x = \frac{220 - \sqrt{400}}{4}$
$x = 60$ or $x = 50$

$\frac{4}{5}$

Figure 5. 5: Student AF1's solution

Throughout the process, she did not question her results. After finding the x-values, the student did not try another approach. However, at the end of the problem when she was asked to verify her solution using the GC, she realized that the solution was incorrect. She pointed out that, *"I think, I am definitely wrong because it should not be zero. This means the profit is zero"* Her mistake, however, was not in understanding but in confusing the equations and inequalities. The student, then, confused the topics of quadratic inequality and equation in solving this problem.

Student AF2 was not able to model the required quadratic inequality from the contextual problem: $6000 \geq -2x^2 + 220x$. He mistakenly wrote the wrong inequality sign, pointing in the opposite direction. He also used both the algebraic and graphical approaches; however, the algebraic approach (quadratic formula) seemed to be the most useful in helping him to solve the quadratic inequality. After writing the inequality in standard form, the student started graphing the function on the paper and then sketched the region representing the solution. Unfortunately, he did not indicate on his submitted script the required region; see Figure 5.6 below.

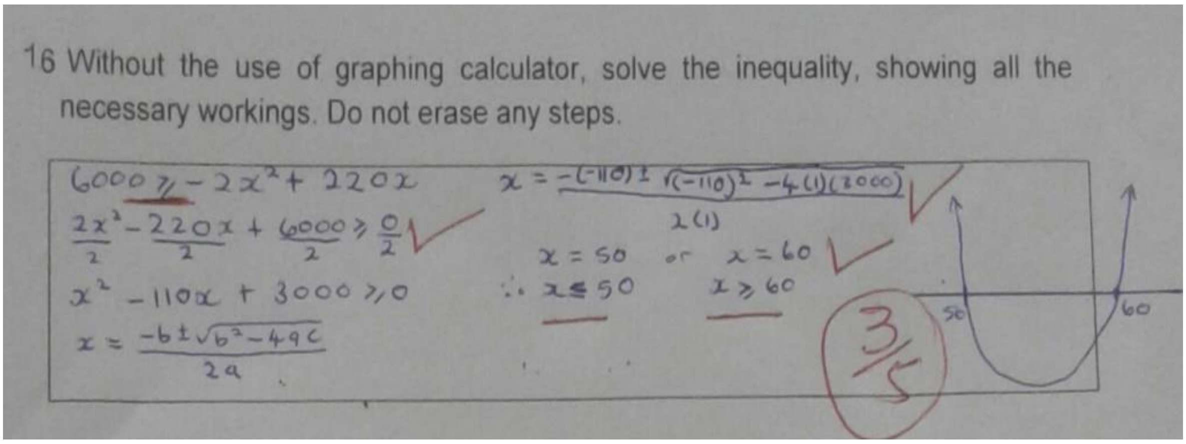


Figure 5. 6: Student AF2's solution

In addition, the student was not able to write a correct solution set of the quadratic inequality. Interestingly, the student was able to graph the correct shape for the function given and he correctly set up the x-intercepts (i.e., the critical values). Throughout solving the contextual problem, he did not question his manipulative skills. This means the student did not try other approaches to validate the determined solution. However, at the end of the problem when he was asked to verify his solution using the GC, he discovered that the solution was incorrect. His problem was partial misunderstanding as he wrote an incorrect inequality sign.

Student AF3 correctly formulated the required inequality from the contextual problem. She used a graphical approach to pave the way for the solution of the problem. She started the problem by sketching the graph of the function onto her script. After looking at the graph, she indicated the region that had the solution of the quadratic inequality. She then switched to an algebraic approach, which is factoring, to find the zeros of the function; see Figure 5.7, below.

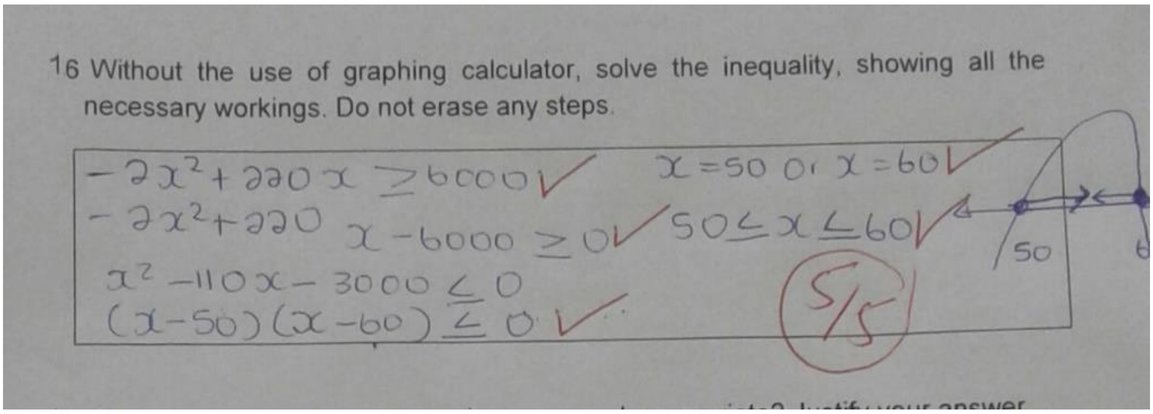


Figure 5. 7: Student AF3’s solution

She made observations throughout when solving this problem and moved freely between algebraic approach and graphical approach, thus switching back and forth from the two approaches. This helped her to reflect on the strategies used to solve the contextual problem correctly. She verified her solution using the values of x and then found that her solution was justified. Consequently, student AF3 wrote the correct interval notation for the solution. Her solution was also verified using the graphing calculator.

Comments on the students’ problem solving abilities

The three students had different levels of proficiency in relation to problem solving processes. For that reason, the students’ problem solving processes were scored in terms of a) forming an inequality, b) using algebraic approach, c) using graphical approach, d) using the graphing calculator for verifying solution and e) obtaining a correct solution set. The scoring ranged from 0 to 5. A 5 represented the highest score of executing all the listed steps correctly. The use of an algebraic approach is mandatory in the CAPS curriculum and should be complemented with the use of the graphical approach. In this regard, Student AF1, an average learner scored a 3 as he did not use the graphical approach and failed to write the solution set correctly. Student AF2, a below average learner scored a 2 as he did not form a correct inequality, did not sketch the graph and did not write correctly the solution set. Student AF3, an above average learner scored a 5 for performing all the steps correctly. This means that Student AF3 used all relevant information to solve the problem and was able to translate the problem into appropriate mathematical language. Based on these results, the teacher-researcher concluded that the use of

GC supported the students' problem solving abilities of the quadratic inequalities. This is consistent with the findings by the earlier researchers (e.g., Spinato, 2011; Karadeniz, 2015; Idris, 2009).

5.6.2 Students' results of the focus group interviews on reasoning

This section addresses the third research question:

> *How does the pedagogical use of the GC support students' reasoning abilities when solving quadratic inequalities?*

The qualitative results of the focus group interviews with students on their reasoning abilities in the contextual quadratic inequality problems were analysed and described in this section. In order to successfully examine the students' reasoning (using evidence to draw conclusions) in solving the contextual quadratic inequality problem, student participants of School A were asked questions during the focus group interviews relating to analysing a problem (Questions 1.1 & 1.3), initiating a strategy (Questions 1.4 & 1.5) and reflecting on one's solution (Questions 1.7 and 1.9). See Appendix D for more details about these interview items. Participants were also observed throughout the process of solving the problem in order to assess how they monitor their progress and how they seek and use connections of concepts. In this context, the three students who participated in the focus group interviews on problem-solving were further assessed their reasoning abilities at School A.

5.6.2.1 Students' results from focus group interview on analytical reasoning

The first reasoning question (Q1.1) of the focus group interviews required the students to identify the main concept involved in the contextual problem. The teacher-researcher observed that all the three students were able to identify the main concept involved as quadratic inequality. The second question (Q1.2) wanted students to analyse the problem whether there were any other concepts used in the problem.

TR: Are there any other mathematical concepts or relationships between concepts that are used in this problem?

Student AF1: Yes, quadratic equations and inequalities

Student AF2: Yes, quadratic equations and graphs.

Student AF3: Yes, quadratic equations, quadratic graphs and domains

All the students were able to list at least two mathematical concepts that were used in solving the problem. However, Student AF1 did not include graphs in her list; therefore this even affected her in solving the contextual problem (see Figure 5.6, above). Students AF2 and AF3 were aware that in order to understand quadratic inequalities they needed the background of the quadratic equations, graphs and domain of the function.

The next reasoning question (Q1.3) requested the students to draw conclusions from their solutions of the contextual problem. In that regard, the students drew the following conclusions about their solutions.

TR: Is there any relevant conclusion that you can make about the solution to the problem? If so, what can you say?

Student AF1: Yes. I think I can say the values of x should be two and both positive. These values must give us profit greater than zero.

Student AF2: No. But I think the solution of the problem should be the values of x because they represent the number of items sold.

Student AF3: Yes. Looking from the graph I have sketched, I assumed that there will be more than one solution and the solution should be between the critical values in order to have a profit of more than R6000.

Interpretation of the students' responses

All the three students were able to state at least one reasonable conclusion. Student AF1's conclusion- "that the solutions would be two and positive", affected the way she determined the solution of the quadratic inequality (see Figure 5.6, above). Student AF2 attempted to provide a valid conclusion but he contradicted himself. This suggests that this student lacked confidence in contextual problems. Student AF3 made two analytical conclusions by stating that *"there would be more than one solution and the solution would be between the critical values."* The use of graphical approach assisted this student to locate the wanted region of the inequality solution. This means student AF3 knew that the region of the quadratic inequality solution in the graph becomes the solution of the problem. The teacher-researcher observed that student AF3 possessed a strong analytical reasoning which is a pre-requisite for understanding inequalities. On the other hand, student AF2 had a weak analysis and this even affected him to write the solution of the inequality correctly. This is linked up with the ideas on the theory of instrumental genesis which include the usage schemes and instrumented action schemes.

5.6.2.2 Students' results from focus group interview on initiative reasoning

In this area of reasoning- Initiating a Strategy, students were assessed on how they purposefully selected the appropriate concepts, representations and procedures when solving the contextual problem. The first reasoning question (Q1.4) of the focus group interview required the students to identify the approaches which were most helpful to solve the contextual problem. The following is how the students responded:

TR: *Which approaches do you think were most helpful in solving this problem- algebraic and/or graphic?*

Student AF1: I think a quadratic formula helped me most to solve the problem.

Student AF2: To me they were two approaches, quadratic formula and graphic, that seemed to be helpful to solve the problem.

Student AF3: Algebraic (factorisation) and graphic approaches were the most helpful to solve the problem.

TR: Briefly explain why you selected these approaches.

Student AF1: I used this quadratic formula approach because I am comfortable with it. I normally get the values of x correctly when I use the quadratic formula.

Student AF2: I used first the quadratic formula approach so that I get the critical values correctly and then I used graphical approach for determining the shape of the graph.

Student AF3: *I used factorisation because it saves time. After getting the values of x, I used them to draw the graph representing the inequality in order to indicate the portion of the solution.*

Interpretation of the students' responses

All the students were able to state and explain the approaches or representations that they executed to solve the contextual quadratic inequality problem. The approaches used by the students were almost similar but differed in the way they were used. Student AF1 correctly responded that she used the algebraic approach- quadratic formula to solve the problem. She used algebraic approach only to find the critical values and did not make any attempt of using another approach (cf: Figure 5.5). Student AF2 responded correctly that he used two approaches- algebraic and graphic to solve the problem. However, the algebraic approach, quadratic formula, dominated throughout the process of solving the problem. His graphical sketch lacked some details although it was related to his modelled inequality (cf: Figure 5.6). Student AF3 also responded correctly that she used two approaches- algebraic and graphic to solve the problem. However, the graphical approach seemed to be the most useful in helping her to solve the problem. The teacher-researcher observed that the student began the problem by sketching the graph representing the function and then went on to indicate the portion where the solution lied. The graph drawn by the student was correct in shape. The algebraic approach-factorisation was only used to determine the zeros of the function which were later on indicated on the graph for the decision. The consistent use of two strategies earned her good results (cf: Figure 5.7). This means student AF3 initiated the correct strategies for solving the contextual problem. Therefore, she had a strong initiative reasoning ability of solving quadratic inequalities. This may suggest that the graphing calculator was a resource of information for students to develop solution strategies of the problem. The use of the GC allowed students to focus on strategies and interpretations of answers. Students were provided with immediate and accurate feedbacks and this contributed towards developing their strategies. This means students received reliable information about the use of graphs to solve quadratic inequalities.

The next reasoning question 1.5 of the focus group interview required students to explain how the use of the graphical approach helped them solve the quadratic inequality. The responses of the students were:

TR: *Explain how helpful the use of graphical approach was in solving the contextual problem.*

Student AF1: *The graph was helpful because with a graph I can tell whether the solution lies inside or outside the critical values.*

Student AF2: *The graphical approach helps to interpret the inequality in the form of a graph. A graph shows the critical values as the x-intercepts and the required region of the inequality defined by the inequality sign.*

Student AF3: *After seeing the shape of the graph, I indicated the x-intercepts which are critical values and these determine the required region of inequality. The inequality sign tells me where the solution lies on the graph.*

Interpretation of the students' responses

The responses given by the students were similar and demonstrated that they understood the role of the graphical sketches when solving quadratic inequalities.

The use of GC provided students with visual representation of quadratic inequality solutions in the form of graphs and this might have influenced them to use graphs too. In fact this was the main purpose of using the GC as an instructional tool. However, the explanation given by Student AF1 lacked specificity. The same student did not sketch the graph to aid him and his critical values were given as the solution of the inequality (cf: Figure 5.5). This means she had a weak initiated reasoning strategy. This is consistent with the level principle of the RME theory where students use models, graphs and diagrams to solve quadratic inequalities.

5.6.2.3 Students' results from focus group interview on reflective reasoning

In this area of reasoning- reflecting on one's solution, students were assessed on how they interpreted their solutions (Q1.6), justified the reasonableness of their solutions (Q1.7), and how they considered alternative ways of solving problems (Q1.9). The questions intended to make students reflect on their solutions of the contextual quadratic inequality problem.

The following reasoning questions (Q1.6 & Q1.7) were asked to find out how the students interpreted and justified their solutions of the contextual problem. The following is how the students responded:

TR: With the values of x that you have obtained, do you think you have solved this problem completely and correctly? Justify your reasoning.
Student AF1: I believe my answer is complete and reasonable.
Student AF2: No. (Scratching his head …), I am struggling to complete the problem.
Student AF3: Yes, there are correct.
TR: Can you justify why you say your answer is reasonable?
Student AF1: Because when I substitute my values of x, it gives me a zero. But this means there were no profits made. (She reasoned …) I think my solution is wrong.
Student AF2: My values of x give me negative profits, which are less than zero. I now doubt my solution because there won't be any profit.
Student AF3: Because when I used my values of x within the interval, the answer is more than 6000. This is reasonable to me.

Interpretation of the students' responses

All the three students were able to decide whether their solutions were reasonable and justified their decisions. Student AF1, who ended after finding the critical values, realised that the answer was not reasonable. She stated, "*No, because when I substitute my values of x, it gives me a zero, which means there were no profits made*." This means that she realised that there should be some profits for the items sold. Student AF2 also realised that his solution was unreasonable and was giving him "*negative"* profits. This was different with Student AF3 as her solution was complete and reasonable. She was able to provide plausible reasons. Her values of x taken from within the solution set provided the profits which were more than 6000. This student was helped by the use of the GC to combine both algebraic and graphic strategies. In this sense, student AF3 was able to reflect on her own solution effectively and this suggests that she displayed strong reflective or metacognitive reasoning skills in respect of this item. This is in line with the interactivity principle which encourages the reflective thinking and also supports the socio-cultural learning theory.

The next reasoning question (Q1.9) of the focused group interview addressed the issue of the reasonableness of the students' solutions using an alternative way. In this context, students were asked, "When using the graphing calculator, do you still get the same solution*?"* Responses of the students to this question were almost similar as students expected their solutions to be reasonably correct. This is the time when students were allowed to use the GC to verify the correctness of their solutions. The students punched the inequality into the graphing calculator to validate their solutions. They used the graphic feature of the GC which validated and interpreted their solution graphically. Students AF1 and AF2 found that they had different solutions and were incorrect. Student AF1 responded, "*I have realised that I did not complete finding the solution of the problem because I left my answer at the critical values.*" Student AF2, on the other hand, indicated that his solution was outside the required region. Student AF3 noted that her solution found earlier using the pencil and paper methods was reasonable and correct. This means she was very correct to state that a profit greater than 6000 was within the critical values of the graph. It is evident that the use of the GC did not develop the students' reflective reasoning completely. This suggests that in the next cycle the emphasis of the teacher-researcher has to focus on the development of students' metacognitive reasoning skills.

5.6.2.4 Students' results of the observed monitoring progress

This section described the students' reasoning skills as they were observed solving the contextual quadratic inequality problem during the focus group interviews. The observation mainly focused on how the students monitored their progress. This section sought answers for the third research sub-question. Students were observed on how they reviewed and/or modified their selected strategies in particular when they encountered difficulties. The observed results of the three purposefully selected students were as follows:

Student AF1 converted the problem into right quadratic inequality and correct procedure (algebraic) was followed through. However, she did not reference any method of graphing to solve the problem. She only ended at finding the solution of the equation and did not realize her mistakes. This means she did not make adequate attempt to monitor her progress; thus why she did not realise the incompleteness of her solutions. Her low level of self-monitoring affected her to attempt other avenues or make any reasonable assumptions that could lead her to review her selected strategy. Throughout solving the problem the student relied on using quadratic formula. When the teacher-researcher asked her to use the GC to solve the problem she was able to visualise the correct solution displayed on the GC.

Student AF2 converted the problem into a quadratic inequality but with wrong inequality sign and correct procedure (algebraic) was followed through. He used mainly algebraic (symbolic) representations and did not use a graph at the beginning of the problem. He seemed to get confused when he realised that he needed to write down the solution of the inequality. An attempt of another approach was made as he struggled to sketch the graph, which was left incomplete (cf: Figure 5.6). This means when he found the x-intercepts correctly, these did not match the sketched graph. Even though he did monitor his progress, he did not verify his assumptions correctly. When the teacher-researcher asked him to verify his solutions with graph produced by the graphing calculator, he realised that the solutions were incorrect.

Student AF3 used two different approaches- algebraic and graphic, to solve the quadratic inequality. A look at the graph of this quadratic function assisted in the monitoring of her progress in the reasoning process. However, she did not rely on one reasoning procedure as she kept on switching from the algebraic to graphical approaches and vice versa. The graph seemed to be very beneficial in the reasoning processes used by this student. Throughout her problem solving process, she continued monitoring her progress and verifying her assumptions. The teacher-researcher observed that every move that she could take, she questioned it. She was also able to use the GC to confirm the shape of the graph, find the zeros of the function, make assumptions and verify her solutions found using the pencil and paper methods. It is evident that the use of the GC supported the students' reasoning domain of monitoring progress as they solved quadratic inequalities.

5.6.2.5 Students' results of the observed seeking and using connections

In this area of reasoning- seeking and using connections, students were observed on how they sought and used connections of different concepts, contexts and representations when solving the contextual quadratic inequalities. This is also about when the students make references to mathematical concepts used earlier in the topic of quadratic inequalities, in other mathematics areas, or in any other subject areas. In this context, the findings were as follows:

Student AF1 was only able to make connections between the solution of quadratic equation and the quadratic inequality (cf: Figure 5.5). However, she did not use this relationship to determine the solution of the inequality. Student AF2 made a similar connection but further used the critical values to determine the solution of the quadratic inequality in interval notations (cf: Figure 5.6). Student AF3 was able to seek and use connections between concepts and representations when solving the quadratic inequality. She made links between the algebraic (symbolic) and graphic representations when she attempted to solve the contextual quadratic inequality problem (cf: Figure 5.7). In this case, she used quadratic graphing (geometry) and solving quadratic equations (algebra) both as viable ways to find a quadratic inequality solution. The student realised that the solution of the equation (x-values) was the x-intercepts of the graph which determine the solution of quadratic inequality. This suggests that the algebraic reasoning (i.e., algebraic symbols and functions) has helped student AF3 to use the connections effectively in solving quadratic inequalities. This means that the GC use supported students' reasoning domain of the using connections and this findings are in accordance of the intertwinement principle of the RME theory.

The teacher-researcher cautiously concluded that the GC use supported the students' reasoning skills in learning quadratic inequalities and this conclusion is linked up with the findings made by the earlier researchers (e.g., Spinato, 2011; Karadeniz, 2015; Idris, 2009; Armah & Osafo-Apeanti, 2012). This further indicates the potentiality of the GC use as a mediated tool in developing critical reasoning as aligned to the ideas of the Vygotsky's ZPD theory. This is consistent with the theory of instrumental approach for instrumented action schemes.

5.6.3 Student's perceptions of how GC enhanced reasoning skills

This section intended to answer the third research sub-question on how the students perceived the use of the GC towards enhancing their reasoning abilities in learning quadratic inequalities. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 5.6 below.

Table 5. 6: Student's perceptions on how GC use enhanced reasoning

Student's perceptions on how GC use enhanced reasoning	**SD**	**D**	**NS**	**A**	**SA**
The graphing calculator helped me to analyse adequately the quadratic inequality problems	6	9	17	40	28
The graphing calculator enabled me to use many approaches when solving quadratic inequalities	3	9	14	46	28
The graphing calculator assisted me check my progress when solving quadratic inequalities	3	11	23	34	29
The graphing calculator helped me to use other concepts to solve quadratic inequalities	9	6	17	46	22
The graphing calculator allowed me to think more about my quadratic inequality solutions	6	3	23	51	17
Key: SD-Strongly Disagree; D-Disagree; NS-Not Sure; A-Agree; SA-Strongly Agree					

In Table 5.6, 69% of the students agreed or strongly agreed that the use of the GC helped them to analyse correctly the quadratic inequality questions. Only 26% of the students, including those who were not sure, denied that the GC enabled them to use new strategies when solving inequalities. Twenty two of the students (63%) agreed or strongly agreed that the GC assisted them monitor or check their progress when solving inequalities. A large proportion of the students (69%) affirmed that the graphing calculator guided them to use other mathematical concepts to solve inequalities. Only three students (8.6%) denied that the GC helped them to think or reason more about their inequality solutions. This implies that the majority of students (69%) felt that the GC allowed them to evaluate the reasonableness of their solutions. Based on these results, the researcher partially concluded that the majority of the students felt that the use of the GC supported them in reasoning skills of solving quadratic inequalities.

5.7 Results of students' responses in the pre-and post-surveys

5.7.1 Comparative results of students' responses in the pre-and post-surveys

This section intended to answer the fourth sub-question by comparing the results of the pre- and post- surveys means on how the students perceived about the GC use in learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

The post-survey intended to gather the perceptions of the students on whether the use of GC in learning quadratic inequalities assisted them to understand the topic. The positive changes of students' perceptions would be attributable to the effective intervention of the GC use as an artefact in learning quadratic inequalities. The eleven items of the pre-survey were similar to the ones on the post-intervention survey to see if students changed their perceptions on how they learned quadratic inequalities after GC intervention as an instructional tool. Students' responses were biased towards the understanding of and lessening the difficulties of learning quadratic inequalities. In this context, an increased confidence in their ability to understand and learn quadratic inequalities is measured by students' option of "disagree" or "strongly disagree" and increased mean. A comparison of the students' perceptions is given in Table 5.7 below, where M_0=post-survey and M_R=pre-survey mean.

Table 5. 7: Results of students' pre- and post- intervention surveys (n=35)

ITEM	*1 = Strongly Agree, 2 = Agree, 3=Not Sure, 4 = Disagree, and 5 = Strongly Disagree*	Pre-survey		Post-survey	
		M_R	SD	M_O	SD
SPQI 1	Quadratic inequalities are difficult to learn and understand	2.66	1.11	3.37	1.09
SPQI 2	I do not see the difference between the equation and inequality	2.69	1.30	3.69	1.37
SPQI 3	It's difficult to determine the solution sets of quadratic inequalities after finding the critical values.	2.49	1.01	3.60	1.31
SPQI 4	I have difficulties with determining factors of quadratic expressions (inequalities)	3.31	1.16	3.37	1.21
SPQI 5	I don't know the difference between critical values and x-intercepts of the graphs	2.43	1.12	3.87	1.11
SPQI 6	In order to understand the quadratic inequality topic I usually memorise it	2.71	1.25	4.20	2.20
SPQI 7	Of all the topics I have done so far I don't enjoy learning quadratic inequalities	2.57	1.29	3.60	1.09
SPQI 8	It's difficult to use graphical sketches to determine the solutions of quadratic inequalities	2.57	1.17	3.71	1.15
SPQI 9	Given an opportunity of not to learn quadratic inequalities I was going to do so	2.60	1.17	3.34	1.24
SPQI 10	Technology (e.g., computers) cannot help me to understand quadratic inequalities	2.49	1.17	3.57	1.29

The results of the first question in the Table 6.7 above show that students had difficulties in learning quadratic inequalities traditionally. This is reflected in the post-survey mean greater than the pre-test survey mean (M_0=3.37>M_R=2.66). It was noted that students did not see the difference between the quadratic equations and inequalities with a pre- survey mean less than the post-survey mean (M_R=2.69< M_O=3.69). Thirdly students felt that it was difficult to determine the solution sets of the quadratic inequalities after they had calculated the critical values when conventionally taught. This is reflected in the post-survey mean greater than the pre-test survey mean (M_0=3.60>M_R=2.49).

Fourthly some students felt that they had difficulties in factoring quadratic expressions. However there was no much difference in the means of the pre- and post- surveys (M_0=3.37>M_R=3.31). Fifthly the students felt that they did not know the difference between critical values and x-intercepts of the graphs using traditional methods. This is shown by the pre-survey mean less that the post-survey mean (M_R=2.43<M_O=3.87). Sixthly students felt that they usually memorise the procedures of solving quadratic inequalities in order to understand the topic when learning traditionally. The pre-survey mean is less than the post-survey mean (M_R=2.71<M_O=4.20), explaining that students learned quadratic inequalities through memorisation. In the seventh question, students felts that they had difficulties in using the graphical sketches to determine the solution sets of quadratic inequalities when traditionally taught. This is reflected by the post-survey mean ($M_O = 3.71$) that is greater than the pre-survey mean (M_R=2.57).

Eighthly students felt that they did not enjoy learning quadratic inequalities when traditionally taught. The results revealed that the pre-survey mean was less than the post-survey mean (e.g., M_R=2.57 and$M_O = 3.60$). In the next question, students felt that if they had an option, they were not going to learn quadratic inequalities. The results revealed that the pre-survey mean was less than the post-survey mean (e.g., M_R=2.60<$M_O = 3.34$). Lastly students felt that the use of technology (e.g., computers) could not help them to understand quadratic inequalities. The results shows that the post-survey mean was bigger that the pre-survey mean (e.g., M_R=2.49<$M_O = 3.57$), thus accepting its importance in learning quadratic inequalities after having used the GC.

Generally, the results from the Table 5.7 show that the use of the GC brought new development on the students' perceptions towards the learning of quadratic inequalities. This was supported by the post-survey means that were greater than the pre-survey means of all the items. The overall student responses showed that the use of the GC supported the students' learning of quadratic inequalities. Initially students indicated that given an option of not learning quadratic inequalities they were going to do so because they were neither enjoying nor understanding the topic. This is consistent with the findings revealed by the earlier researchers who indicated that the use of the GC would develop visual images which can help students to construct their understanding (Spinato, 2011; Karadeniz, 2015). In addition, the GC use provided students with a more meaningful interpretation for the solution (Doerr & Zangor, 2000). Other researchers have criticised rote learning as it leads to surface level understanding and makes students experience challenges in solving problems (McTighe & Self, 2003; Snyder & Snyder, 2008). The use of the GC boosted students' understanding of algebraic concepts i.e., quadratic inequalities in order to minimize the use of algorithms and memorization (Knuth, 2000).

5.7.2 Student's perceptions on how the GC supported the learning sessions

This section intended to answer the fourth research sub-question by analysing the results of students' responses on how they perceived about the GC use in the designed sessions of learning quadratic inequalities.

What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?

In that context students were issued with an eight item post-intervention survey to answer. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 5.8 below.

Table 5. 8: Student's perceptions on how the GC supported the sessions

ITEM	**Students' perceptions of the effects of the graphing calculator sessions on learning quadratic inequalities**	**SA (%)**	**A (%)**	**N (%)**	**D (%)**	**SD (%)**
SPGC 1	The use of the GC in learning sessions assisted me to solve symbolic (algebraic) quadratic inequalities	14	55	11	11	9
SPGC 2	The use of the GC in learning sessions assisted me to understand the difference between critical values and zeros of the graph	26	40	14	9	11
SPGC 3	The use of the GC in learning sessions assisted me to identify correctly the region of the inequality solution	14	52	14	14	6
SPGC 4	The use of the GC in learning sessions assisted me to transform contextual problems into quadratic inequalities	20	43	20	6	11
SPGC 5	The use of the GC in learning sessions assisted me to use graphical sketches when solving quadratic inequalities	23	46	17	6	8
SPGC 6	The use of the GC in learning sessions assisted me to understand the effect of the parameter 'a' in the quadratic inequality	20	49	17	8	6
SPGC 7	The use of the GC in learning sessions assisted me to note that the effect of the parameter 'a' of quadratic function had the same effect on quadratic inequality	23	46	17	8	6
SPGC 8	The use of the GC in learning sessions assisted me to learn and understand much better quadratic inequalities	25	49	11	9	6
1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree						

Table 5.8 shows that 69% percent of the students strongly agreed or agreed that the use of graphing calculator in learning sessions assisted them to solve symbolic (algebraic) quadratic inequalities. Sixty six percent of the students admitted that the use of graphing calculator in learning sessions assisted them to see the difference between critical values and zeros of the graphs. Students affirmed (strongly agreed or agreed) that the use of graphing calculator in learning sessions assisted them to use graphs when solving quadratic inequalities (69%) and to identify correctly the region of the inequality solution (66%). Sixty three percent of students strongly agreed or agreed that the use of graphing calculator in learning sessions assisted them to transform contextual (application) problems into quadratic inequalities. However 20% of them were not sure whether the use of the GC helped in transforming contextual problems. Only 14% percent of the students strongly disagreed or disagreed that the use of graphing calculator in learning sessions

assisted them to understand the effect of the parameter ‘a’ in the quadratic inequality. Seventy four percent of the students strongly agreed or agreed that the use of the GC in the designed sessions assisted them to learn and understand the topic of quadratic inequalities better.

The researcher concluded that the use of the GC in the planned sessions helped students to understand the quadratic inequalities as they were able to identify quadratic inequality with the shapes of the quadratic graphs, to see the effect of the parameter “a” on the different graphs, to use the graphs to solve quadratic inequalities, to transform contextual problems into symbolic quadratic inequalities and to determine the region of the solution. These are considered as the main procedures that can lead the students to solve quadratic inequality correctly. The results are in line with the roles of the GC identified by the previous researchers (Averbeck, 2000; Karadeniz, 2015; Lee & McDougall, 2010). They also perceived that the effect of the different parameters were the same for all the quadratic functions and inequalities expressed in the form of $a(x+p)^2+q \geq\leq 0 \; or \; ax^2+bx+c \leq\geq 0$. The use of the GC created a supportive environment in the sessions (Lee & McDougall, 2010) and provided students with a more meaningful interpretation for the solution (Doerr & Zangor, 2000). During the sessions students used both graphical and graphing calculator approaches which provided them with more visualization to meaningfully solve quadratic inequalities (Karadeniz, 2015). The consistently use of the GC in their quadratic inequality sessions helped students enhance their knowledge and understanding (Lee & McDougall, 2010).

5.8 Results from the in-depth interviews about graphing calculator use in quadratic inequalities

This section of in-depth interview with students mainly attempted to address the fourth research sub-questions about the students’ perceptions on the use of the GC in learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

An in-depth interview was conducted with the three students who were purposefully sampled from those who had obtained marks below average, average and above average from the post–test. They were individually interviewed after school on a regular school day in their classroom. Each interviewee lasted for approximately thirty minutes. The in-depth interview was recorded and then transcribed. The interview consisted of ten questions which were mainly about the use of the GC on students' understanding of quadratic inequalities.

The codes used in the interview are TR for the teacher-researcher and AD for School A student in the in-depth interview (D). They were interviewed separately, but for convenience their responses are given together below. The responses of the in-depth interview questions are discussed in the following sub-sections:

5.8.1. Students' responses to how the use of the GC made their learning of quadratic inequalities easier

The question asked by the teacher-researcher below sought to find out the students' opinions about whether the use of the GC made their learning of quadratic inequalities easier. Students' responses were almost similar as shown below:

TR: Does the use of the GC make your learning of quadratic inequalities easier to understand? Please explain your answer.

Interview with Student AD1

Student AD1: Yes it does. The graphing calculator shows the solutions and the graph which makes it simpler for me to understand the inequality. It illustrates the graph with the critical values. I don't have to calculate the critical values.

TR: Can you please elaborate what you mean, "It illustrates the graph with the critical values."

Student AD1: The displayed graph on the GC screen has x-intercepts; these x-intercepts are the critical values, which normally determine the solution set of the inequality.

Interview with Student AD2

Student AD2: Yes, it is because the GC shows the coordinates and how the graph goes. You don't need long calculations.

TR: What do you mean, "the GC shows the coordinates and how the graph goes?"

Student AD2: Coordinates are the x-intercepts where the graph cuts the x-axis. The x-intercepts help to determine the solution of the quadratic inequality.

TR: Does it mean you have problems of determining the critical values?

Student AD2: No, I can use the quadratic formula to find them but in the process you can make errors.

Interview with Student AD3

Student AD3: Yes, because it gives us the critical values that we need and shows us that the solution on the graph is below or above the x-axis.
TR: If the graph is below the x-axis, what happens?
Student AD3: This means the solution of the inequality is within or outside the graph depending on the coefficient of x-squared.
TR: Ok. Thank you.

Interpretation of students' responses

Students' responses were positive and almost similar as they affirmed that their learning of quadratic inequalities had been made easier after the use of GC. Students did explain that "the x-intercepts and the region of the solutions of the quadratic inequality are always shown on the GC." This means the use of the GC enabled the students to see how the quadratic inequality was solved graphically as displayed on its screen as shown in Figure 5.8, below.

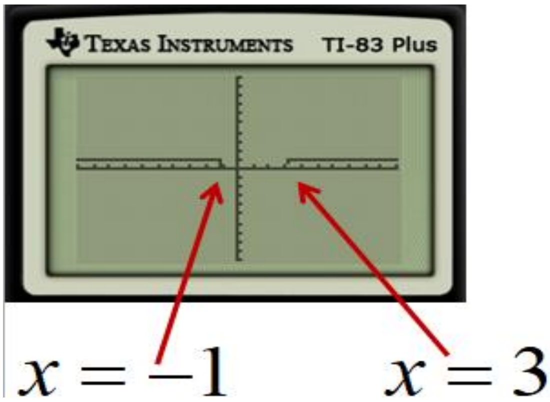


Figure 5. 8: Solution of the quadratic inequality displayed on the GC

In this case, the graph tells us the solution set of quadratic inequality is a disjunction of $x = -1$ and $x = 3$. The use of the GC, according to students, demonstrates the shape of graph with "its x-intercepts which are the critical values" for the inequality. This means that through the use of the GC, students were able to visualise the graphs with critical values which made it easy to determine the solution of inequalities. Students benefitted from the visualisation capabilities of the GC which made them understand easily the relationship/connections between the graphs and solutions of quadratic inequalities. The visual images of the graphical representations contributed to the improved learning of quadratic inequalities as shown by reference to '*shows us if the solution on the graph is below or above the x-*

axis'. This is consistent with the level principle where the GC potentially produced the visual models, symbols and diagrams to help students to move from informal to formal reasoning. The findings by students were in line with the schemes of instrumented actions as students used the GC capabilities to produce the schemes. However, not all the students made use of the graphical models to illustrate their solutions, although they appreciated the graphical meaning on the algebraic solution. They apparently could not link the two representations.

5.8.2 Students' responses to how the use of the GC helped them to feel comfortable with quadratic inequalities

The question asked by the teacher-researcher below sought to find out the students' opinions about whether the use of the GC helped them feel more comfortable with quadratic inequalities. This question is part of in-depth interview which attempts to answer the first research question. Students' responses were almost similar as shown below:

TR: Does the use of the GC help you to feel more comfortable with quadratic inequalities?

Interview with Student AD1

Student AD1: *Yes, I am comfortable. I used to have fear with quadratic inequalities and this topic was always very difficult. I never did well in quadratic inequalities.*

TR: *Explain how the use of the GC helped you to minimise this fear.*

Student AD1: *The use of the GC helped me to gain confidence in learning quadratic inequalities as it use graphs to show the solutions of quadratic inequalities.*

Interview with Student AD2

Student AD2: *Yes, I am comfortable and enjoying solving quadratic inequalities after the use of the GC.*

TR: Explain how the use of the GC contributed to your comfortability.

Student AD2: *With the use of the GC, I can check the values of x that I calculated using quadratic formula if they are correct.*

Interview with Student AD3

Student AD3: *Yes, it does. I am comfortable with solving quadratic inequalities*.

TR: *Please explain your answer.*

Student AD3: *The GC gives me information that I will always remember. The graphs are* clearly illustrated and it is easy to interpret the values from the GC. *This boosts my confidence in learning quadratic inequalities.*

TR: *Ok. Thank you very much.*

Interpretation of students' responses

Students' responses were positive and almost similar as they have expressed enjoyment and confidence in the learning of quadratic inequalities with the use of the GC. This implies that the use of the graphing calculator provided an enabling environment for learning quadratic inequalities. Student AD1 gave an affective answer of a reduced fear of the mathematics content of quadratic inequalities and expressed relief that she has now managed to pass the topic which makes her more comfortable. Student AD2 affirmed that GC use made him not only more '*comfortable*' with the topic but also '*enjoys*' it. Student AD3 affirms that GC use '*boosted*' her confidence and made her '*comfortable with*' the topic because the use of GC '*gives … information*' she will '*always remember*'. When you can always remember that means your confidence is high in that concept. Using the GC fostered the development of the targeted mathematical domain. For example, the GC has stimulated the use of graphical sketches as objects/models that could help students to determine the solutions of quadratic inequalities. Student AD3 emphasised that the '*clearly illustrated'* graphical sketches helped to '*interpret'* the solutions of the quadratic inequalities (see Figure 6.12). This means the use of the GC motivated students' learning of inequalities and students were able to construct the conceptual knowledge. These findings link up well with the ideas on the theory of instrumental genesis: schemes of the instrumented actions.

5.8.3 Students' responses to whether the GC should be used in learning quadratic inequalities

The question sought students' opinions on whether the GC should be used in learning quadratic inequalities or not. The students' opinions were positive and quite similar as shown below:

TR: *Should graphing calculators be used in learning quadratic inequalities at the eleventh grade?*

Student AD1: *Yes, students should use the GC when they learn quadratic inequalities. It should always be supported by the teacher's voice.*

TR: Does it mean you prefer your teacher to the use of the GC?

Student AD1: No it's not that I prefer a teacher to the use of the GC but a teacher is always needed for explaining where I am not clear.

Interview with Student AD2

Student AD2: *Yes, it must be used instead of having the teacher doing all the steps for the learners.*

TR: Does it mean the teacher must be eliminated from classroom and give space to the use of GC?

Student AD2: *No. GC should be used when introducing a topic of quadratic inequalities and when verifying the solution of quadratic inequalities including the quadratic graphs.*

Interview with Student AD3

Student AD3: *Definitely, it must be accepted in the mathematics classroom.*

TR: *Please explain your opinion*.

Student AD3: *The use of the GC provides learners with opportunities to answer more questions. But the teacher must always be there to explain where I don't understand.*

Interpretation of students' responses

Most of the students responded similarly to this question as they supported the use of the GC in learning of quadratic inequalities. This clearly means that with its capabilities, the GC afforded students opportunities to learn better quadratic inequalities. Student AD3 emphatically affirms by saying '*definitely*' and also gives additional reason as giving learners opportunities to '*answer more questions*'. This means the GC was used as a psychological tool to produce enjoyment through its use. This is linked up with the socio-cultural theory of Vygotsky. However, the students felt that there can be more effective learning of inequalities when complemented by the use of traditional methods (e.g., teacher talking). The responses of Students AD1 and AD3 emphasised the need for the teacher to be there to "support" and "explain" to them where they don't understand. They viewed the teacher as an additional resource to the GC who can attend their individual differences in the classroom. This is aligned to the findings made by Ndlovu (2014) that technology cannot orchestrate itself to articulate mathematical understandings to learners. Student AD2 argued, for example, that the GC must be incorporated in the learning of quadratic inequalities to reduce teacher domination (or workload) of lessons. In addition, she emphasised that the GC must be used when '*introducing'* the topic *and 'verifying'* the solution of quadratic inequalities. This suggests the GC appeared to be instrumental in the learning of quadratic inequalities with its capabilities. This is aligned to the schemes of instrumented actions in the theory of instrumental approach.

5.8.4 Students' responses to how the use of the GC improved their understanding of quadratic inequalities

This question sought students' views whether the use of the GC improved their understanding of quadratic inequalities in mathematics classroom. The views of the three students were very similar as shown below:

Interview with Student AD1

TR: Does the use of the *graphing calculator improve your understanding when learning quadratic inequalities?*
Student AD1*: Yes it does.*
TR: *Please explain your answer in detail.*
Student AD1: *After the use of the GC my comprehension has increased and I can interpret the solutions of quadratic inequalities using quadratic graphs. The visualisation of graphs by the GC made me to understand most this topic.*
TR: *Please explain how GC use helped you to solve quadratic inequalities graphically.*
Student AD1: *The use of the GC helped me to consider x-intercepts of the graphs as critical values of the quadratic inequalities, which determine the region of the solution sets.*
TR: Thank you very much for your time.

Interview with Student AD2

TR: Does the use of *graphing calculator improve your understanding when learning quadratic inequalities?*
Student AD2: *Yes, there is a great improvement of understanding quadratic inequalities even if I might not get all the marks in a test.*
TR: *Please explain how your understanding has been improved.*
Student AD2: *Given a quadratic inequality question I know that I must use a graphical sketch to determine the solution set. And the solution set are determine by the zeros of the function which are x-intercepts.*
TR: *Explain how the use of the GC helped you to use the graphic sketches to solve quadratic inequalities.*
Student AD2: *Actually the solutions of quadratic inequalities are displayed from the GC in drawn graphs.*
TR: *Thank you very much for your time.*

Interview with Student AD3

TR: Does the use of the *graphing calculator improve your understanding when learning quadratic inequalities?*

Student AD3: *Yes. I see myself at a better level of understanding than before, because I can explain to my classmates who have difficulties with this topic. The use of the GC helped me to determine the solution of quadratic inequalities by using graphs.*

TR: *How were you helped to use the graph?*

Student AD3: If you use the GC to solve the quadratic inequalities the solutions are shown on the graphs. I realised that I must sketch and use the graphs to determine solutions.

TR: *Thank you very much for your time.*

Interpretation of students' responses

The responses of the students clearly explain that GC use was instrumental towards understanding quadratic inequalities. All the three students emphasised that GC use aided them to understand the topic of quadratic inequalities. Student AD2 emphatically elaborated, "…*there is a great improvement of understanding quadratic inequalities even if I might not get all the marks in a test".* When students press GRAPH button, the punched quadratic inequality is transformed into the quadratic graph with a region of the solution (see Figure 5.8). This means the GC visualises the graphical images which are the solution of the quadratic inequalities. Students indicated that graphical sketches assisted them to determine the regions of the solution sets of quadratic inequalities. The use of the GC influenced students to link algebraic methods with graphs i.e., graphs being drawn as aid for solving quadratic inequalities. This means by looking at the inequality students can picture what the shape of the graph will look like. The linking of the graphical and algebraic representations is one of the recommendations of the CAPS for FET Mathematics document (DBE, 2011), which states that students must solve quadratic inequalities by integrating both methods. This is in accordance with the intertwinement principle of RME theory. This further indicates that the students interacted with GC as a new cultural tool to develop their cognitive thinking and true understanding. This is aligned with the Vygotsky's ZPD theory. The use of GRAPH instrumented action scheme also assisted to reconstruct students' meanings of quadratic inequalities.

5.8.5 Students' experiences of using the GC in learning quadratic inequalities

This question solicited students' experiences after using the GC to solve quadratic inequalities in a mathematics classroom. The student interviewees were asked to relate their experiences. Students expressed exciting experiences, as the responses given below show:

TR: *How can you explain your experiences of using the GC in learning quadratic inequalities?*

Student AD1: *It was an interesting experience because I had never used a GC before in learning mathematics. It made me a little more excited to realise that quadratic inequalities are not difficult to understand. One could see the solution of the quadratic inequality represented on the graphing calculator. This technology combines both algebra and graphs to solve quadratic inequalities.*

Student AD2: It was a wonderful experience because I learnt how to use a GC to solve quadratic inequalities. I now know how to sketch quadratic graphs and how to use such graphs to determine the solutions of quadratic inequalities. I now understand quadratic inequalities more than before.

Student AD3: *It was an educative experience because I now have confidence of solving quadratic inequalities, which used to be so difficult. I explored many questions of quadratic inequalities using the GC. I realised that the solution of the quadratic inequalities can be better determined when I combine algebraic and graphic approaches.*

Interpretation of the students' responses

The responses of the three students revealed that they had 'interesting', 'wonderful' and 'educative' experiences with the use of the GC in solving quadratic inequalities. The use of the GC made students to become less anxious about learning quadratic inequalities and they were inspired to use sketches and graphs. All the three students indicated that the GC use reduced the levels of their difficulty with quadratic inequality problem solving. Student AD1 indicated that "one could see the solution of the quadratic inequality displayed as drawn graph on the screen" when using a GC in Figure 5.9, below.

Solving quadratic inequalities graphically

$$x^2 + 2x - 8 \geq 0$$

$$(x-2)(x+4) \geq 0$$

$$x = -4 \ or \ x - 2$$

$$(-\infty, -4] \cup [2, \infty)$$

Figure 5. 9: Solving quadratic inequalities graphically

This means that the GC acted as a visual aid tool (visualisation tool) and provided the students with opportunity to see how the solutions are presented graphically. This is in line with the fact that what a child has seen it is hard to forget. Students AD1 and AD3 mentioned that the use of GC allowed them to make connections between the algebraic methods (factoring or quadratic formula) and graphs when solving quadratic inequalities. This means that the sketched graphs were used as visual objects to aid his conceptual understanding of quadratic inequalities. Student AD3 also indicated that she used the GC for investigating more quadratic inequality problems and to solve them independently. This was evident when she stated that the use of the GC gave her room for the exploration of several questions of quadratic inequalities. In this case, this student used the GC as an educational and exploration tool for her to solve and to understand a range of quadratic inequality examples. All the three students indicated that they used the GC to solve and graph quadratic inequalities in order to understand the topic better. This is consistent with the theory of instrumental genesis for instrumented action schemes (Trouche, 2004). The findings by the students are also in line with the intertwinement principle of the RME theory which emphasises an integrated approach to solve a mathematical problem.

5.8.6 Students' responses to how the use of the GC helped them to score better marks on quadratic inequalities

This question asked if the use of the GC helped students to score better marks on quadratic inequalities and to rate themselves between 0 and 5, a 5 being the highest. All the students confidently affirmed better marks after the use of graphing calculator.

Interview with Student AD1

TR: *Did the use of the GC make you score better results in quadratic inequalities?*
Student AD1: *Yes, my marks are much better.*
TR: *Explain what made them to improve after using the GC.*
Student AD1: *Through the use of the GC, I learnt to solve quadratic inequalities by using two methods (i.e., the algebraic and graphical methods).*
TR: *In that case, how can you rate your level of understanding in quadratic inequalities after the use of the GC?*
Student AD1: 4 out of 5.
Student AD2: *Yes, for the first time, I was always failing quadratic inequalities.*
TR: *Explain what made you to score better after using the GC.*

Student AD2: The use of the *GC helped me to know the basics of the quadratic inequalities. And I was able to practise on my own.*

TR: *In that case, how can you rate your level of understanding in quadratic inequalities after the use of the GC?*

Student AD2: *I rate myself at 4.*

Student AD3: *Yes, my marks are better as the topic was no longer a challenge.*

TR: *Explain what made you to improve after using the GC.*

Student AD3: *I explored many questions of quadratic inequalities using the GC and practised solving inequalities with graphs.*

TR: *In this case, how can you rate your level of understanding?*

Student AD3: *I rate myself with a definite 5*.

Interpretation of the students' responses

The responses of the three students were almost similar and they rated themselves with at least a 4. The students felt that the use of the GC made their results better in quadratic inequalities. Student AD2 argued that the GC helped him to know the basics of the quadratic inequalities. Student AD3 attributed his better marks to being able to explore many questions of quadratic inequalities using the GC and linking the algebraic and graphical methods. The teacher-researcher noted that Student AD2 had overrated himself; he still needed more sessions of the quadratic inequalities. Actually the use of the GC helped him to gain self-confidence in solving quadratic inequalities and "(…) to know the basics of the quadratic inequalities". This is consistent with the Vygotsky's ZPD theory for scaffolding students to achieve the impossible (Panhwar, Ansari, & Ansari, 2016) and with the activity principle of RME.

5.9. Reflections, design principles and feed-forward of the research cycle

This final section of the chapter presents the reflection and design principles of the research cycle at School A and the way forward for the next cycle. The researcher looked back at the teaching experiment and designed the principles of the first cycle and then concluded by formulating *the feed-forward for the second research cycle to* be implemented at School B.

5.9.1. Reflecting on the starting points and learning outcomes of the HLT

In Section 5.2, the researcher set out the starting points of the HLT for this study concerning the opportunities that the use of the GC would offer for the students in order to achieve the higher level of understanding of the quadratic inequalities. In this current section, the researcher evaluates those learning outcomes (as expectations) individually to find out if they were confirmed in School A.

The first learning outcome in session two was that students would develop the notions of interval notations, parameters, x-intercepts and quadratic graphs in a flexible graphing calculator environment. During the teaching experiment at School A, students were confronted with questions that were mathematically similar but with different coefficients of x^2 (see Session 2 in Appendix D). Students used similar reasoning and problem-solving procedures but different processes as they attempted the quadratic functions and inequalities. The use of the GC helped students to visualise the graphs displayed on the screens and enabled them to understand the properties of quadratic graphs (e.g., zeros, intervals, axis of symmetry, concavity). Students were able to make repetitions of graphs using the GC and this helped them develop and reify the key pre-concepts of quadratic inequalities. As it was anticipated that the use of the GC would provide a flexible environment for students to understand quadratic graphs and their properties, this was not adequately achieved. Some students had difficulties to recognise the concavity of quadratic functions with respect to the coefficients of x^2 and did not make meaningful generalisation. Consequently, these students failed to transfer the correct prior knowledge and they needed more activities for further practice. In this regard, the use of the GC partially supported the transition from the graphical representation to quadratic inequality representation.

The second learning outcome for sessions 3 and 4 was that students would develop the notions of solving quadratic inequalities in a flexible graphing calculator environment. This means that students were supposed to use their knowledge of quadratic functions and graphical properties towards solving symbolic quadratic inequalities, which demand routine reasoning skills. During the teaching experiment at School A, students were confronted with questions that were mathematically similar but with different levels of difficulties (see Sessions 3 and 4 in Appendix D). Students were expected to use the GC as an instructional artefact, in particular the graphic and tabular instrumented action schemes to solve the symbolic quadratic inequalities. These instrumented action schemes allowed students to repeat the processes of graphing and tabling the values and supported them develop and reify the concept of quadratic inequalities. The repetition of the processes made the students to realize that changing the parameter values affected the complete quadratic graphs and inequality solution sets. The graphical visualization of this effect (i.e., changing the parameters) created a strong mental image for the students. The impression is that most of the students started to perceive graphs and inequalities as entities that could symbolize objects. The use of graphical representations (models) made students to extend their graphical conception of the quadratic functions towards the view of understanding the quadratic inequalities. The graphical models mediated very well between the quadratic equations and quadratic inequalities. The graphical schemes of the graphing calculators were helpful for visualizing the effects of parameters and the properties of graphs. Furthermore, students used the tabular instrumented action scheme to check for the solutions in the table of values displayed on the GC. However, the understanding and interpretation of such graphs and inequalities still remained a hard issue for the grade 11 students. The researcher partially concluded that the use of graph and tabular instrumented action schemes facilitated the transition from the graphical representations to symbolic quadratic inequalities.

The third learning outcome in sessions 5 and 6 was that students would develop the higher order problem solving and reasoning skills in contextual quadratic inequality situations in a flexible graphing calculator environment. This means that the transition from *the symbolic quadratic inequalities* to *the contextual quadratic inequality situations* was to be brought about by the use of the GC. In that regard, students were supposed to use their routine reasoning skills of solving symbolic quadratic inequalities into solving contextual quadratic inequality situations. During the teaching experiment at School A, students were confronted with questions that were mathematically similar but with different levels of difficulties (see Sessions 5 and 6 in Appendix D). Students were engaged in the use of the GC as an instructional artefact to solve the contextual quadratic inequality problems. The consistent use of the GC enabled students to extend their routine reasoning skills towards the non-routine reasoning and problem-solving skills in contextual quadratic inequality situations. Students were able to transform the contextual situations into quadratic inequalities with one variable as required by the CAPS FET Mathematics document and solved them similarly as symbolic quadratic inequalities. With the use of the graphical representations displayed on the GC screens, students were able to extend their understanding of the symbolic towards the view of understanding contextual situations. The graphical models successfully mediated between the symbolic and contextual situations as it was further observed that the students attempted well question 4 and fairly question 5 in the post-test. However, the understanding and interpretation of such graphs and inequalities remained a hard issue for the grade 11 students. For that reason, these difficulties continued to prevail in both sessions and post-test with similar questions that demanded the same reasoning and problem solving processes. The researcher therefore concluded that the use of the GC partially supported the transition from the symbolic quadratic inequalities (the routine skills) towards the contextual quadratic inequality situations (non-routine skills) of solving quadratic inequalities.

5.9.2. Reflections on the in-depth interviews with students

The responses of the students from the in-depth interviews affirmed that the use of the graphing calculator provided an enabling environment for learning quadratic inequalities. In their arguments, they revealed that their learning of quadratic inequalities had been made easier after the use of the GC and they were able to see how the quadratic inequalities were solved graphically. Students further contributed that the use of the graphical sketches assisted them to determine the regions of the solution sets of quadratic inequalities and were enabled to visualise the solutions of quadratic inequalities on the GC screens. Through the process of visualisation students were able to establish the relationship between the quadratic graphs and quadratic inequality solutions. As revealed in the interviews, the visual images of the graphic representations improved the learning of quadratic inequalities. Additionally, they affirmed that the use of the GC influenced them to link algebraic methods with graphs i.e., graphs being drawn as aid for solving quadratic inequalities. This was affirmed by student DA3, “by looking at the inequality I can picture what the shape of the graph will look like and where its solution set will be”. The linking of the graphical and algebraic representations is one of the recommendations of the CAPS for FET Mathematics document (DBE, 2011), expressed emphatically in the NSC Examination Diagnostic Reports that students must solve quadratic inequalities by integrating both methods (DBE, 2014; 2015; 2016; 2017). Ultimately, students expressed enjoyment and confidence in the learning of quadratic inequalities with the use of the GC and indicated that it was instrumental towards their understanding of quadratic inequalities. However, not all the students made use of the graphical models to illustrate their solutions in answering the post-test. This means that although these students appreciated the graphical meaning of an algebraic solution they apparently could not connect the two representations. Despite the fact that students had ‘interesting’, ‘wonderful’ and ‘educative’ experiences with the use of the GC in solving quadratic inequalities, they still valued the presence of the teacher as an additional resource to the GC who can attend their individual differences in the classroom.

5.9.3. Reflecting on the focus group interviews

The qualitative results of three students who were engaged in focus group interviews showed different levels of proficiency in relation to problem solving processes. The student’s problem solving processes were scored in terms of a) modelling an inequality, b) using algebraic approach, c) using graphical approach, d) verifying their solution including the use of the graphing calculator and e) obtaining a correct solution set (interval notation). The students’ work revealed that two of them did not confidently use the graphical approach and did not obtain correct solutions of contextual quadratic inequalities. However, all of the three were in a position to use the GC to verify their solutions when asked to do so (cf. Section 5.7.2). Evidently, this means on average the three problem-solving processes were performed. With these results the teacher-researcher concluded that the second research question was achieved partially.

The qualitative results of the focus group interviews with students on their reasoning skills in the contextual quadratic inequality problems revealed that students possessed average analytical reasoning which is a pre-requisite for understanding quadratic inequalities. Two thirds of students were able to identify at least two pre-concepts of quadratic inequalities and all of them stated at least one reasonable conclusion. It was further revealed that the students had average initiative reasoning skill of solving quadratic inequalities. Two of three students were able to use and explain the strategies (approaches) that they executed to solve the contextual quadratic inequality problem, which included algebraic, graphical and graphing calculator. Similarly, two of three students displayed weak reflective or metacognitive reasoning skills of solving the contextual quadratic inequality problem. These students were not able to reflect on their solutions of the contextual quadratic inequality problem through interpreting, justifying and checking and using alternative ways (using the graphic feature of the GC). This suggests that in the next cycle the emphasis of the teacher-researcher has to focus on the development of students' metacognitive reasoning skills. The researcher observed that the reasoning domain of monitoring progress by students was partially supported by the use of the GC. It was noted that only one of three students was able to monitor her progress and verify her assumptions. In the next cycle the teacher-researcher's focus should be on how to improve students' reasoning skills on monitoring their progress. It was further observed that two of three students were able to seek and use connections between concepts and representations in the reasoning process of the contextual quadratic inequality situation. The use of both algebraic and graphing reasoning helped students to solve the problem. The researcher partially concluded that the use of the GC supported the students' reasoning skills on seeking and using connections.

The teacher-researcher observed that during the activity sessions the majority of the students were able to make inferences, draw conclusions and reflect on the reasonableness of their solutions of quadratic inequalities. The use of the GC as an instructional tool provided opportunities for the students to analyse the inequality problems, interpret the solutions and to make predictions about the solution sets. This is in line with the constructivist teaching and learning which requires teachers to focus on the use of physical actions (graphing calculator) to promote the use of senses to construct the underlying meaning of concepts (Vygotsky, 1978), and students' independent thinking and the control of their own learning situation (von Glasersfeld, 1996; Amineh & Asl, 2015). These actions are strongly connected to the processes of reasoning and problem solving.

5.9.4. Feed-forward for the second DBR cycle

The findings of the first research cycle at School A informed the feed-forwards for the next research cycle. The feed-forwards of this cycle concerned the hypothetical learning trajectory, the instructional activities and the research methodology.

The feed-forward concerning the HLT addressed the broad outline: Solving symbolic, routine and contextual quadratic inequality situations in a flexible graphing calculator environment. The results from the teaching experiment suggested that the use of graphical approach was most helpful to students in solving quadratic inequalities because of its dynamic character and visual image. Therefore, the properties of the quadratic graph including the interval notation were addressed earlier in the learning trajectory. However, the use of the GC did not support completely the conception of the graphical approach. Therefore, an emphasis on the conception of quadratic graphical properties should be considered in the next cycle. A second issue concerning the HLT was the use of the GC to support the transition of symbolic quadratic inequalities towards the contextual quadratic inequalities which did not come across in a satisfying way. This can be attributed at least partially to difficulties with solving symbolic and routine quadratic inequalities, and the use of graphical strategy in particular. It is important to master this graphical technique, so that it does not hinder the generalization and visualisation processes. Thirdly on the HLT was the need for linking the algebraic and graphical approaches to holistically develop students' reasoning skills and problem-solving abilities in solving quadratic inequalities. Students incompletely solved the contextual quadratic inequalities because they had partially developed their reasoning domains of the metacognitive (reflective) and monitoring skills.

The feed-forward concerning the instructional activities focused on those teaching materials that needed more time for better understanding. A first point was that students needed more practice in using graphs to solve quadratic inequalities. Additional sessions were designed for students to work in groups using the GC so the visual images can be retained. This approach must be combined with the algebraic representations to holistically solve quadratic inequalities. Students would further be engaged with the graphic and tabular instrumented schemes in separated and integrated practice. As far as the role of the teacher-researcher is concerned, the teacher must use his experience in the next cycle for orchestrating the learning process in group discussions and regard himself as an additional resource of information for the students.

The feed-forward concerning the research methodology addressed the teaching experiments. The first point was to establish a better match between the pre-test and post-test, so that the improvements in understanding could be monitored during the teaching experiment. The second point was to conduct mini-interviews or mini-written tasks during the sessions on selected questions to provide appropriate and meaningful data. The third point was to organise students into groups and give them task to solve in order to observe the students' interaction and thinking; this would provide platform for those students who have many questions to be assisted.

5.9.5 Design principles of the study

This current design based study was driven by the pedagogical gap in the teaching and learning of quadratic inequalities. The researcher identified five fundamental principles of design-based research such as:

1. Starting the topic of quadratic inequalities with the problems of real-life situations
2. Integrating algebraic and graphical representations when teaching quadratic inequalities helps students to develop visual images that lead them to formal reasoning.
3. Using the GC approach as teaching strategy is not effective all the time but it should be complemented with traditional styles to develop the student's understanding of quadratic inequalities.
4. Learning activities should be prepared well ahead of the first contact session in collaboration with practitioners.

The next research cycle is discussed in a similar manner in chapter 6 but incorporating the limitations of the previous cycle.

CHAPTER 6: THE SECOND CYCLE OF TEACHING EXPERIMENT

6.1. Introduction

The purpose of this present study was to investigate the grade 11 students' understanding of quadratic inequalities in a graphing calculator enhanced mathematics classroom. In addition, the study examined whether the problem solving and reasoning skills of the students were supported by the pedagogical use of the graphing calculator. The data described in this chapter were collected in School B to address the following research questions:

1. To what extent can the pedagogical use of graphing calculator influence high school students' performance in solving quadratic inequalities?

2. In what ways (how) can the pedagogical use of the graphing calculator support the high school students' problem solving ability in quadratic inequalities?

3. In what ways (how) does the pedagogical use of the graphing calculator support the high school students' reasoning ability when solving quadratic inequalities?

4. What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?

This chapter is a follow up of the first research cycle at Grade 11 which focused on the hypothetical learning trajectory (HLT) and the experiences during the first cycle of the BDR teaching experiment. In a similar manner, the current chapter addresses the second research cycle in the same grade but at a different school. The teaching experiment sought to investigate how the designed HLTs played out in the classroom and whether the use of the GC provided learning opportunities for improving the students' understanding of the quadratic inequalities in the way it was expected to.

In the current chapter the re-designed HLT is described by the starting points including the learning outcomes in Section 6.2. Second, the participants and research procedures of the cycle are explained in Section 6.3. Third, the results of the pre- and post-tests are analysed in Section 6.4. Four, the results of the problem solving are analysed in Section 6.5. Fifth, Section 6.6 presents the results of the focus group interviews with students. Sixth, the results of pre- and post-surveys are analysed in Section 6.7. The results of the in-depth interviews with students are discussed in Section 6.8. Finally, the study concludes with the reflections, feed forwards and design principles in Section 6.9.

6.2 Starting points of the HLT and learning activities

In this section, the starting points for the HLTs for quadratic inequalities were the use of set builder notation which defines the solutions of unknown variable, drawing a number line for the given set builder notation and graphing quadratic functions and the effect of parameters in quadratic functions and quadratic equations were part of HLT. The understanding of the properties of quadratic graphic representations was considered to be important in the reification of symbolic expressions and formulas in the development of algebraic concepts (Drijvers, 2003). In the first cycle a broad outline of HLT was developed: Solving symbolic, routine and concrete quadratic inequality problems in the graphing calculator environment. This was viewed as a broad trajectory of how to achieve higher level understanding of quadratic inequalities by using the opportunities offered by GC use. Furthermore, the experiences and the feed-forward from the first teaching experiment in School A informed the HLT of the second cycle.

The teaching and learning experiences derived from the first cycle were:

1. Using the GC consistently helps to reduce the percentage of learners who have difficulties in learning quadratic inequalities.

2. Linking graphic and algebraic representations helps learners to understand quadratic inequalities better.

3. Using models, graphs and diagrams in the teaching and learning of quadratic inequalities moves away the learners' informal thinking to formal reasoning and problem solving.

However, there were no major changes in designing the HLT for the second cycle. The feed forwards from the first teaching experiment which guided the development of the HLT for the second cycle in School B were stated as follows:

1. More emphasis to the use of quadratic graphs and their properties at the starting point can develop an insight into the higher levels of learning quadratic inequalities.

2. More time to be created with the GC use can allow for gradual visualisation of quadratic graphs that helps to perceive the effects of the parameters.

3. More emphasis to the integration of the algebraic and graphic representations can lead to a better understanding of quadratic inequalities.

4. More practise with the instrumented action schemes for graphing and tabling may lead to meaningful presentations of the solution sets of quadratic inequalities.

The use of the quadratic graphical representations was considered to be important in the reification of symbolic (algebraic) expressions and formulas and in the conceptual development of quadratic inequalities. The assumption was that students who have a conception of the quadratic inequality pre-concepts were likely to extend towards the higher levels (i.e. complex and problem solving) of quadratic inequalities. The learning in the GC environment was assumed to allow for algebraic exploration and visual geometric representations in solving quadratic inequalities. This was expected to lead towards achieving the higher level understanding of quadratic inequalities with the use of the GC.

The transitions for the learning activities in quadratic inequality concept

The transition from *the quadratic graphs and interval notations to symbolic quadratic inequalities* was to be brought about by confronting the students with several questions that were mathematically similar but with different parameter values (see Session 2 in Appendix D). The expectation here was that the students would perceive the similarity of the reasoning and problem-solving procedures in spite of the different parameter values.

The transition from *the quadratic functions and equations to the quadratic inequalities* was to be brought about by a graphical approach, in which the parameter value was changed gradually and systematically (see Sessions 3 & 4 in Appendix D). In that regard, students were asked to analyse the effect of the changing parameter value on the graph. Mentally, the student realized that changing the parameter value affected the complete graph and quadratic inequality solution sets. The GC was used in this transition for graphing a sequence of 'shifting' graphs and visualisation of the effects of the parameters on the quadratic graphs and their zeros.

The transition from *the symbolic quadratic inequalities to the contextual (complex) quadratic inequality problems* was to be fostered by applying additional algebraic properties to the concrete situations and using the graphical representations (see Sessions 5 & 6 in Appendix D). Solving the contextual quadratic inequalities involves the use and filtering of appropriate graphical properties (intersection points, tangent points, vertices, roots). Such problems were expected to lead to a mental shift of students, so that they could apply reasoning and problem solving procedures. The use of the GC was to allow for this shift as the graphical and tabular representations were expected to make students reify the concept of quadratic inequalities. The tabular instrumented action scheme was to elicit the students to use the concept of interval to reify the concept of quadratic inequality solutions.

6.3. Participants and research procedures of the second DBR cycle

The teaching experiment took place in a public high school with eleventh-grade class, thereafter the school is referred to as School B. A total of 37 students participated in this study and were randomly chosen from 70 eleventh graders. The participants were asked not to identify themselves on the pre-tests and post-tests, but rather to label their scripts with symbols given randomly by their teacher, such as B1, B2, B3,..., B37, where B represented the school. However, two students decided to withdraw their participation and were not considered during the quantitative analysis of the results. Four students also participated in the interviews. The sample included below average, average and above average students and a top student. To collect the data for this second DBR cycle, the same procedures described in Section 5.3 were followed.

6.4. Comparative analysis of the students' results of the pre- and post-tests

This section presented and discussed the results of the pre- and post- tests and the written tasks which sought to answer the first research question:

> *To what extent does the pedagogical use of GCs impact on students' performance in solving quadratic inequalities?*

The results were also used to test the null hypothesis: H_0: There is no difference between the pre-test mean and the post-test mean of quadratic inequalities for the students in the study (H_0: μ_1 (pre−test) = μ_2 (post−test). Alternatively, H_1: There is a difference between the pre-test mean and post-test mean of quadratic inequalities for the different domains for the students in the study (H_1: μ_1 (pre−test) ≠ μ_2 (post−test). The results of 35 students obtained in School A are presented in the form of descriptive statistics (Table 6.1) and paired samples test (Table 6.2) below.

Table 6.1 below presents the descriptive statistics analysis of school B showing that students performed much better in the post test (M=45.1429; SD=21.41762) than in the pre-test (M=22.3429; SD=13.50151). It is further observed that the majority scored better marks in post-test with a lowest mark of 17% and highest mark of 90% compared to pre-test with a lowest mark of 3% and highest mark of 63%. This could be attributed to effective use of the GC in solving designed activities of quadratic inequalities. The median mark (47%) of the post-test was also much better than that of the pre-test (20%). This suggests there was a reasonable improvement in the post-test and this means that half of the students scored above 47%.

Table 6. 1: Descriptive Statistics of School B

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre-Test	22.3429	35	13.50151	2.28217
	Post-Test	45.1429	35	21.41762	3.62024

In the next table, a paired-samples t-test was conducted to test whether the improvement noted in the post-test was significant. The use of the paired-samples t-test helped to test the null hypothesis, which stated that there was no difference between the pre-test mean and the post-test mean of quadratic inequalities for the students in the study (H_0: μ_1 (pre−test) = μ_2 (post−test). In that regard, Table 6.2 below presents the dependent (paired) samples *t*-test results of School B with paired differences of means of the pre- and post-tests of 35 students.

Table 6. 2: Paired Samples Test of School B

Pair 1	Paired Differences							
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		T	Df	Sig(2-tailed)
				Lower	Upper			
Pre-Test & Post-test	-2.280E1	23.57067	3.98417	-30.89681	-14.70319	-5.723	34	.000

Table 6.2 above, presents the dependent (paired) samples t-test of school B with paired differences of the means of the pre- and post-tests of 35 students. The t-test results in the table show that t(34) = -5.723 and p = 0.000. This means the actual probability value is 0.000 and it is substantially smaller than the specified alpha value of 0.05. These t-test results indicated that the null hypothesis was rejected at 5% significant level in favour of the alternative hypothesis. This means that there was a statistically significant difference between the students' means of the pre- and post-test scores. In that context, there was a statistically significant improvement of the students' results after the use of the GC in the learning of quadratic inequalities. Although the results presented above indicated the statistically significant improvement in the test scores of students, they do not tell much about the magnitude of the GC intervention's effect in solving quadratic inequalities. Because there was no control group for this particular task and limitation of statistical significance, the researcher proceeded to calculate the Cohen's d effect size statistic using the pre-test and post-test means in order to determine the magnitude or practical significance of the difference in scores. The effect size was 0.83, indicating that the post-test mean is at 79% of the pre-test mean. This means that there was a large effect impacted by the use of the GC on learning quadratic inequalities (i.e., using Cohen's, (1988) interpretation: 0.2=50%=small effect, 0.5=58%=medium effect and 0.8=79%= large effect). The researcher then concluded that there was a practically significant improvement of about 0.5 standard deviations in the mean scores from pre-test (M=22.3429, SD=13.50151) to post-test [M=45.1429, SD=21.41762, t(34)=-5.723, p=0.000<0.005]. This implies that the pedagogical use of the GC impacted positively on the students' performance in solving quadratic inequalities. This study did not investigate if the teaching and learning of quadratic inequalities with the GC is better than an approach without it, but to show that it can help with the understanding of the topic or concept. The t-test results showed that there were learnings gains not only for the purposeful sample of students but also for all the students that were exposed to the teaching intervention with the GC and instructional material. The findings are consistent with the theoretical frameworks as they were used in designing the learning activities.

6.4.1 Students' results in written tasks of symbolic quadratic inequalities

The improvement of students' performance in the post-test could be attributed to the written tasks during the teaching experiment. Students wrote a task with questions related to sessions 3 and 4 and was considered for analysis. These questions were meant to monitor the progress of the students in each session. The results of the students were analysed to determine what percentage of those who answered the symbolic quadratic inequalities correctly, incorrectly, blankly or incompletely and also used graphic approach (see Table 6.3), below.

Table 6. 3: Students' results of the written task about symbolic quadratic inequalities

Inequality	**Correct %**	**Incorrect %**	**Blank %**	**Incomplete %**	***Method used***		
					Graph %	***Others %***	***None %***
4.1 $\mathbf{4x + x^2 \leq 0}$	57	23	9	14	57	20	23
4.2 $\mathbf{(x+2)(3x-7) \geq 0}$	66	14	6	14	54	20	26
4.3 $\mathbf{x^2 - x - 12 < 0}$	72	17	6	6	66	14	20
4.4 $\mathbf{-(x-4)(x+5) < 0}$	51	26	14	9	54	17	29
4.5 $\mathbf{2x^2 - 7x \geq 4}$	54	20	14	11	51	17	32

The analysis of the data in Table 6.3, below revealed that each of the symbolic quadratic inequality questions was solved correctly by more than 50% of the students. This may suggest that at least 18 of the 35 students had acquired adequate knowledge and routine skills for solving symbolic quadratic inequalities. The percentage of students who had incorrect or blank solutions was between 20 and 40; this can be viewed as an improvement in the understanding of quadratic inequalities. It was observed that some students, in question 4.4 attempted to solve but had some difficulties, thus the reason why it had the greatest percentage (40%) of incorrect or blank answers. It was also observed that less than 15% of students had incomplete answers. An incomplete solution means that the student used the correct strategy but abandoned the strategy before arriving to the solution. These students were able to determine the critical values and/or sketch the graph but could not go beyond that.

Students' answers were supposed to be linked to the use of graphs in order to see the influence of the GC. In this vein, students' abilities of using graphs were assessed in solving symbolic quadratic inequalities. It was observed that more than 50% of the students used graphs to determine the solutions of the inequalities, except in question 4.5 with 48%. In this question students attempted to solve before expressing in standard form. Not more than eight students (≤ 23%) used other methods such as line graph and sign chart or table to determine the solutions of quadratic inequalities. A line graph is also shown on the graphing calculator when solving quadratic inequalities. In this case, the influence of the GC was very strong in developing such skills among the students. In the table, only 10 or less did not use any of the three methods linked to the GC.

Students' answers were supposed to be linked to the use of graphs in order to see the influence of the GC. In this vein, it was observed that more than 50% of the students used graphs to determine the solutions of the inequalities. Not more than 7 students (≤ 20%) used other methods such as line graph and sign chart or table to determine the solutions of quadratic inequalities. A line graph dominated in other methods; this could be that a line graph is also shown on the graphing calculator when solving quadratic inequalities. This means the use of the GC supported the students' abilities of using graphs when solving symbolic quadratic inequalities.

Samples of students' written tasks on symbolic quadratic inequalities

The written tasks of the four students were purposefully sampled to show the different approaches used to answer the problematic question 4.4 in Figures 6.1 and 6.2 below. It was evident that the use of the GC had great impact on students' learning of quadratic inequalities in School B.

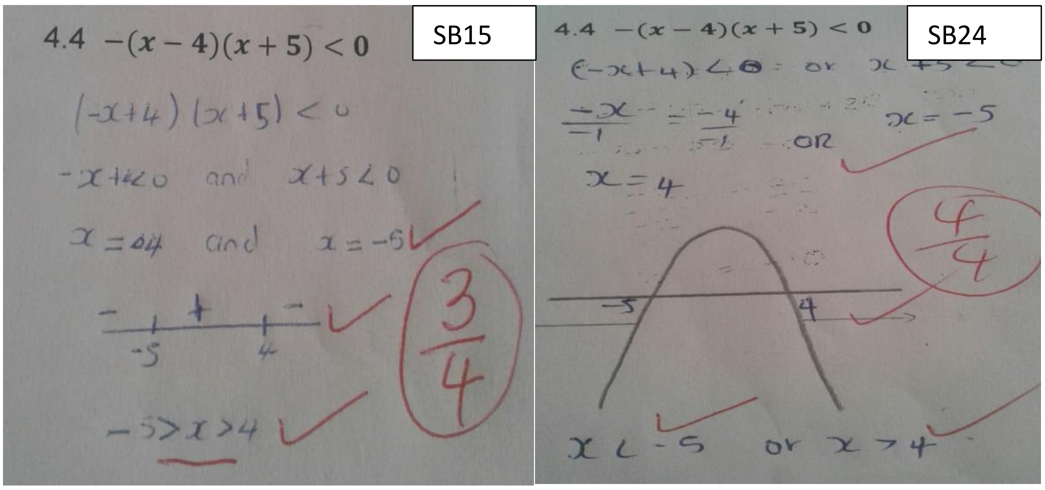


Figure 6. 1: Students' solutions of symbolic inequalities

In figure 6.1 student SB15 used both algebraic and sign chart approaches to solve the symbolic quadratic inequalities. She perfectly combined the two strategies and she knew the solutions represented the negative values. However the solutions were incorrectly written, thus using wrong interval notations. This means the student does not understand the difference between "and" and "or" in quadratic inequality solutions (DBE, 2017). Student SB24 used both algebraic and geometric representations to solve the symbolic quadratic inequality. He perfectly linked the two strategies and they led him to the correct solutions of the inequality. The graph indicated where the solutions were found and he rightly expressed the solutions in interval notations.

In figure 6.2, below student SB7 correctly used the line graph and algebra when solving this question. The regions of the solutions were correctly indicated however the interval notations as solutions were meaningless. She did not apply logical thinking to check the appropriateness of her solutions and it was like she gave her solutions procedurally. It was also noted that the student lacked the algebraic structure sense in quadratic expressions. She did not realise that the quadratic expression and its factorised equivalent were of the same structure but differently interpreted. In that sense, she chose to expand and then re-factorised before applying the null factor law.

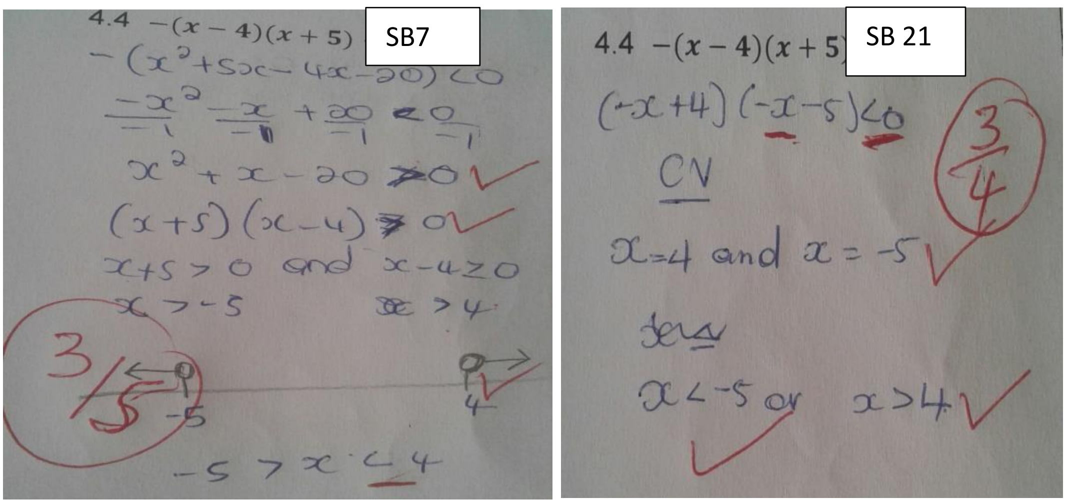


Figure 6. 2: Students' solutions of symbolic quadratic inequalities

In figure 6.2, above student SB21 did not apply the correct procedure of removing the negative sign; as a result the direction of the inequality was not changed. However, he was able to determine the correct critical values and the required solution sets of the inequality. The student did not struggle expressing the solution set he seemed to be having a sound understanding of the effect of a negative parameter (the coefficient of x^2) on quadratic inequality solutions. This means the student used visualisation to write down the solutions. This student strongly relied on algebra and visualisation.

The sampled written tasks of four students showed that the use of the GC had influence on the learning of quadratic inequalities as these graphs are commonly displayed on the GC. Students attempted very well to link algebraic (symbolic) and geometric (graphics, diagrams, lines) representations when solving quadratic inequalities. These models helped students with the visual representations which were expected to move their informal thinking away from the applications to the formal reasoning. This is consistent with the ideas of the level principle of RME theory that reflects the levels of reasoning development from horizontal to vertical mathematization. This was attainable through the consistent use of the GC and repetition of the processes. This is also in line with instrumented action schemes of the instrumental approach theory as students were able to integrate meaningfully the graphical and algebraic representations for supporting their reasoning and problem solving.

6.4.2 Students' results of written tasks in applications of quadratic inequalities

The improvement of students' performance noted in the post-test could be attributed to the written tasks during the teaching experiment, related to questions in session 5 in Appendix D and was considered for analysis. The written task assisted to monitor the progress of the students. The results of the students were analysed to determine what percentage of those who answered the application problems of quadratic inequalities correctly, incorrectly, blankly or incompletely (see Table 6.4), below.

Table 6. 4: Students' responses in application of quadratic inequalities

Question	Application	Correct	Incorrect	Blank	Incomplete
5.1	**For what values of x will $\sqrt{x^2-25}$ be real?**	57	23	11	9
5.2	**For which values of x will Q=$\sqrt{x^2-8x+12}$ be non-real?**	65	20	9	6
5.3	**Given $g(x)=-x^2+7x+6$ For which values of x will g(x)>0?**	51	29	9	11

The analysis of the data in Table 6.4 revealed that 57%, 65% and 51% of the students respectively applied correctly quadratic inequalities to solve questions 5.1, 5.2 and 5.3. This implies that students were able to understand the problems and transform them into quadratic inequalities and then selected the right strategies to solve them. The percentage of students who had incorrect or blank solutions was between 29 and 38; this can be viewed as an improvement in understanding the problems involving quadratic inequalities. These students need expanded opportunities to be adequately re-skilled in these problem areas of applications. Although the proportion (i.e., 9%, 6% and 11% respectively) of students who had incomplete solutions was very low for all the three questions, a further use of the GC would help to develop students' cognitive skills and confidence. This therefore lays foundation for the next cycle. However, it was noted that a large portion of students had experienced difficulties in solving quadratic inequalities. The consistent use of the GC as visual artefact supported with teacher's voice was expected to reduce the percentage of students who had incorrect, blank or incomplete solutions.

A sample of two students' answers was selected to show how they attempted the application problems involving quadratic inequalities in Figures 6.3 and 6.4, below. Student SB10 was able to link the nature of roots with quadratic inequalities. In that regard, she correctly applied the understanding processes of quadratic inequalities to solve the problem. With the aid of both algebraic and graphic approaches the student was able to solve correctly the modelled quadratic inequality.

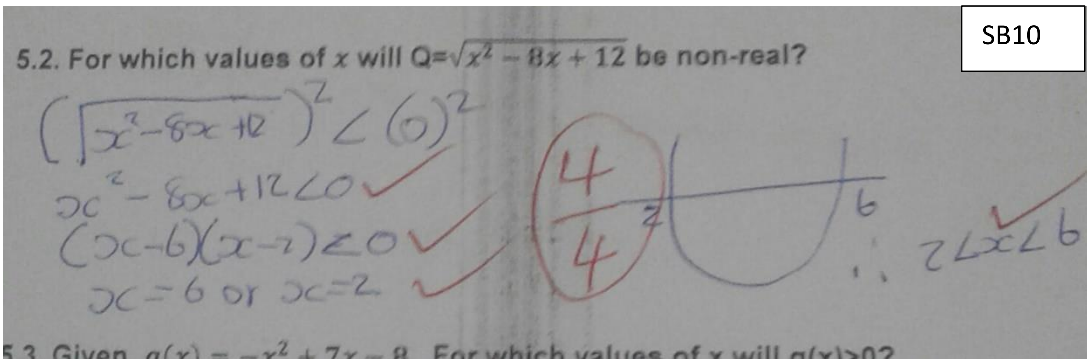


Figure 6. 3: A sampled student's correct solution

Student SB33 correctly converted the quadratic function into the quadratic inequality. The whole inequality was multiplied by a negative sign, converting it to a non-positive expression. However, the student failed to factorise correctly and also wrote the meaningless solution set of quadratic inequality. This means that the solution set was procedurally written without proper understanding of the interval notations.

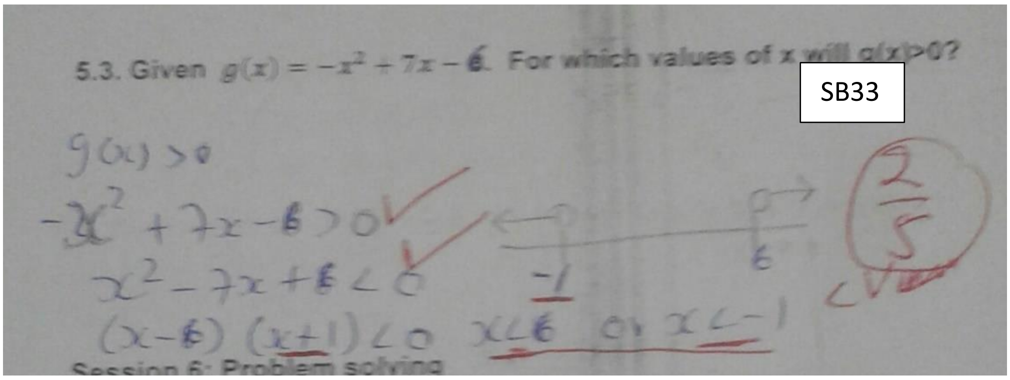


Figure 6. 4: A sampled student's incorrect solution

The work of the two students demonstrates the positive influence of the GC use on solving quadratic inequalities. The students used visual representations: graphic and line graph which are both displayed on the GC. The graphs (line and quadratic) are used as psychological tools to develop cognitive significance and meaning of the solutions of quadratic inequalities. Through the GC use students were able to internalise the structure of writing the solution set and to use the appropriate terms when solving quadratic inequalities. These are the affirmations of the Vygotsky's socio-cultural theory of learning mathematics.

6.5. Students' results of problem solving in quadratic inequalities

This section presents the students' results on problem solving strategies used when answering the post-test and the results *of how the GC use supported their problem solving abilities when learning quadratic inequalities.*

6.5.1 Analysis of the student's problem solving strategies in the post-test

This section intended to answer the second research question, "*In what ways (how) can the pedagogical use of the graphing calculator support the high school students' problem solving ability in quadratic inequalities?*", through discussing the processes used by the students in applying problem solving strategies in Question 5 of the post-test. This question required students to determine the values of x for which $\sqrt{25 - x^2}$ will be non-real. The students applied quadratic inequalities to determine the values of x. The students were purposefully chosen because their written work represented the different problem solving approaches. A rubric for quadratic inequality problem solving test (QIPST) in Appendix E was used to score this question. A sample of answers of the three students is shown in Figures 6.5 and 6.6 below.

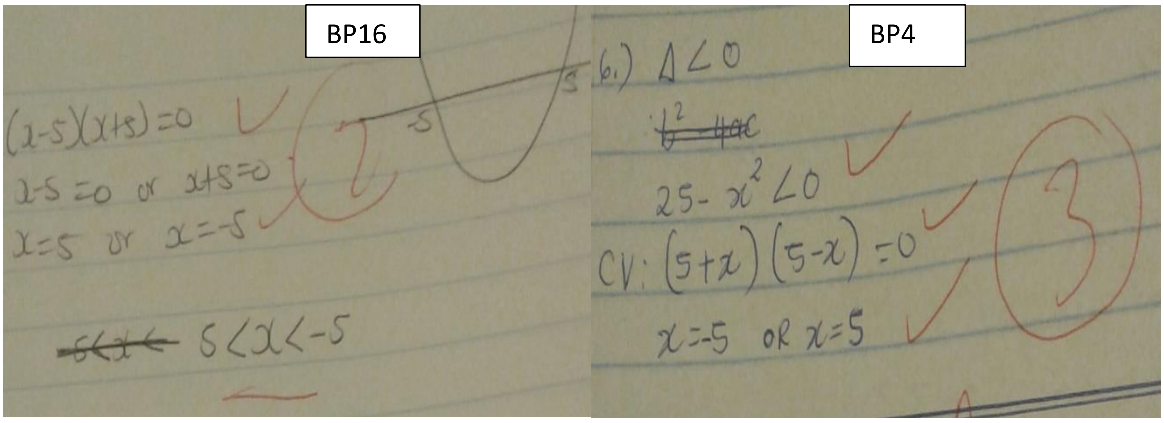


Figure 6. 5: Students' answers on problem solving of quadratic inequality

In figure 6.5, above, student BP16 correctly interpreted the problem and correctly identified the strategy of solving the problem but was not able to apply it. She did not monitor her progress as she worked the problem. For that reason, she scored a 2. The student used an algebraic approach throughout the entire process of problem solving, including finding roots, setting up the inequality and using the correct procedures of removing the squares. Her procedures did not lead her to the correct roots. Student BP4 correctly interpreted the problem and correctly identified the strategy of solving the problem. He was able to apply the strategy but the solution set was incorrect due to misconceptions. He treated inequalities as equations, thus solutions were left at critical values. In that reason, he scored a 3. The student used an algebraic approach throughout the entire process of problem solving, including setting up the inequality and finding roots. He was able to identify the difference of squares as the correct procedure of determining the roots.

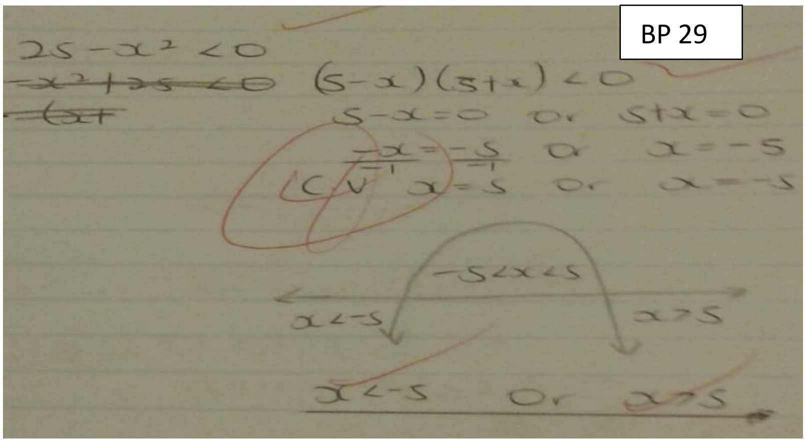


Figure 6. 6: Student BP 29's answer on problem solving

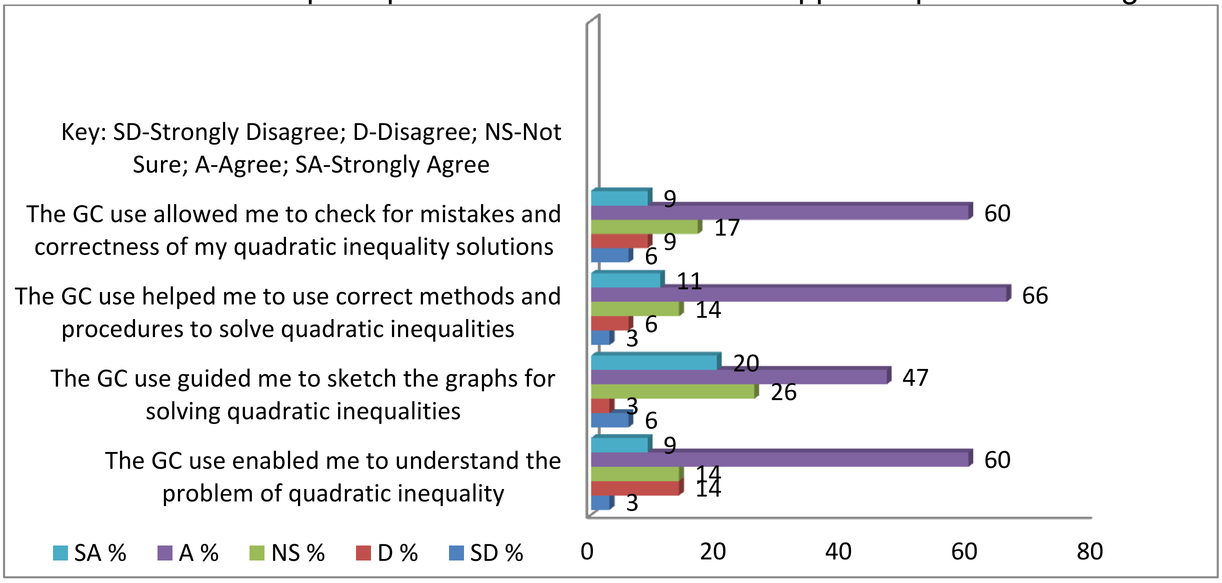
Using the rubric QIPST, student BP29 perfectly understood the problem and successfully applied the strategy of solving the problem. This means an inequality was correctly constructed and appropriate solutions arrived at using the correct procedures. She tried hard to reflect on her solution by checking its reasonableness. In that regard, she scored all the marks thus scooping a 4. The student used both graphical and algebraic approaches to help solve the problem. The algebraic approach seemed to be the most useful in setting up the inequality and finding the critical values by using the difference of squares procedure. A graphical approach also helped her to solve the problem by correctly sketching the graph and indicating the zeros of the function. The student appropriately understood the effect of the negative parameter 'a' in the quadratic function. She used the graph to determine more than one interval that would be needed to solve this application problem and realised that one of these intervals was positive and one was negative.

Based on the students' answers of problem solving, the teacher-researcher cautiously concluded that the GC use supported the students' performance and problem solving strategies. This is aligned with the ideas of the level principle of RME theory as designing and sequencing of the instructional materials were structured using the levels of mathematical reasoning and problem solving.

6.5.2 Student's perceptions of how the GC use supported the quadratic inequality problem solving abilities

This section intended to answer the second research sub-question on how the students perceived about the use of the GC towards supporting their problem solving abilities of quadratic inequalities. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 6.5 below.

Table 6. 5: Student's perceptions on how the GC use supported problem solving



In Table 6.5, sixty-nine percent of the students agreed or strongly agreed that the GC use enabled them to understand the quadratic inequality problems. However, there were 5 of the 35 students (14%) who were not sure. Twenty three students (67%) affirmed that the GC use guided them to sketch the graphs for solving quadratic inequalities. Seventy seven percent of the students agreed or strongly agreed that the use of the GC developed them to use correct methods and procedures in solving quadratic inequalities. Sixty nine percent of the students felt that the GC use allowed them to check for errors, mistakes and correctness of their solutions. Of those students who were not sure in their decisions, the researcher suggested that they needed an expanded opportunity with the use of the GC on how to solve quadratic inequalities. The researcher noted that the results were overwhelmingly impressive on how the students perceived on the use of the GC to support them in problem solving of quadratic inequalities as all the Polya's four-steps of problem-solving processes had high percentages.

This shows that graphing calculator played undoubtedly significant roles in students' learning of quadratic inequalities. The GC was used as a tool by students for problem solving processes in which they graphed functions in order to familiarize themselves with the problem. In the planning phase, students developed a strategy for determining the solution and were used like a compass to point in the right direction for solving the problem. The use of graphing calculators assisted students in their problem solving by helping them develop and confirm their solution strategies. In the monitoring phase the GC was used as a resource for verifying solutions and then students were able to develop a symbolic approach. This is consistent with the ideas of the earlier researchers (e.g., Averbeck, 2000). This means students used the graphing calculator to see the connections between a solution and meaning of solution in terms of the graph (White-Clark et al., 2008; Amineh & Asl, 2015). In this case, the GC was used for exploration, an idea more consistent with cognitive constructivism. This is consistent with the Vygotsky's theory of socio-cultural learning where the GC was used as mediator to develop cognitive understanding.

6.6 Students' results from the focus group interviews

This section described the qualitative results of students' focus group interviews on both problem-solving (Section 6.6.1) and reasoning (Section 6.6.2). A question was selected from the post-test for critically exploring the students' problem-solving abilities and students' reasoning skills in quadratic inequalities. Students were expected to use the graphing calculator only to verify the reasonableness of their solutions. According to Lunenberg (1998), learning in a constructivist manner involves asking students to analyse a problem, interpret results, classify terms or concepts, and to make predictions. These cognitive activities are strongly connected to the processes of students' understanding.

A sample of three students was selected from School B for focus group interviews. There were two female students (BF2 and BF3) and one male student (BF1). The interviews took place in their math classroom on a regular school day after school hours. Three students who obtained marks below average, average and above average in the post-test were purposefully selected to participate in the focus group interviews. The participants were asked to solve a contextual quadratic inequality problem that was in the post-test and were also asked several questions relating to the problem solving processes involved in that problem. The students were asked to explain their thoughts throughout the interviews in order to understand their thinking processes. Throughout the interview, participants were observed how they used algebraic approaches and, sketches and graphs in their thinking processes through the problem. At the end of the interviews they submitted their interview scripts to further analyse how they reasoned their way throughout this problem. Each interview lasted for approximately thirty minutes. The interviews were audio recorded and then transcribed. A full transcription of each interview is included in Appendix G.

6.6.1 Students' results of the focus group interviews on problem solving

This section attempted to address the second research question of the study:

> *In what ways can the pedagogical uses of the GC enhance students' problem solving abilities when solving quadratic inequalities?*

The results of the focus group interviews with students on their problem solving abilities of the contextual quadratic inequalities were analysed and described in this section. In that context, the selected students were presented with a contextual problem that they first saw in the post-test. The problem stated, "*A small manufacturer's weekly profit is given by* $P(x) = -2x^2 + 220x$*, in which* x *is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R6000".* The students were given ten minutes preparation time to read, formulate and solve the problem. They were supposed to explain their thinking processes clearly and not to erase their working. In this case, students answered questions 1.6 and 1.7 (Appendix D) which were central to solving the contextual quadratic inequality problem.

The codes used in the interviews are: TR for the teacher-researcher and BF for the focus group students from School B. Student participants were interviewed as a group and their responses were presented and transcribed below:

TR: (The teacher-researcher hands out the problem to the students). *Please read this problem attentively and then formulate the required mathematical statement.*

Students BF1: $-2x^2 + 220x \geq 6000$

Students BF2: $6000 \leq -2x^2 + 220x$

Students BF3: $-2x^2 + 220x \geq 6000$

TR: And then solve it without the use of the GC, showing all the necessary working. Please do not erase any step you have written.

Students (BF1, BF2 & BF3): After ten minutes the students handed in their scripts for marking.

Interpretation of the individual students' interview results

Students' attempts are shown below and were analysed in order to determine whether the use of graphing calculator had enhanced their problem solving competencies at School B:

Student BF1 was able to formulate the required quadratic inequality from the contextual problem as $-2x^2 + 220x \geq 6000$. He used both algebraic and graphic approaches but factorisation dominated throughout the entire process of problem solving; including finding the critical values (see Figure 6.7). A wrong quadratic graph was just drawn but was not fully utilised to guide him in making decision. For that reason he wrote meaningless intervals: $x \geq 50$ or $x \leq 60$, as his final solution. He was misled by his graph.

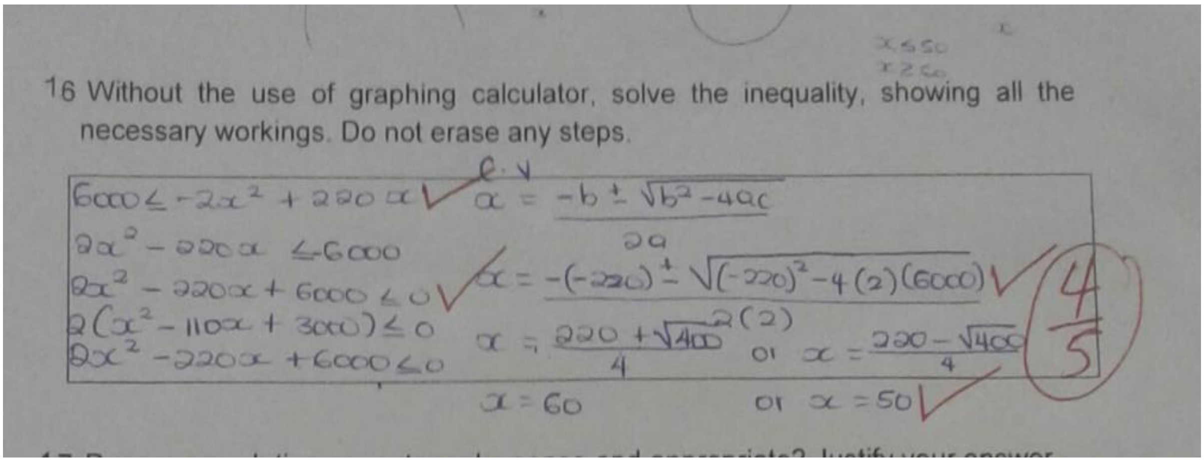


Figure 6. 7: Student BF1's problem solving

Throughout the process of problem-solving, he did question his results. His first solutions which were similar to the final one were scratched out. However, at the end of the problem when he was asked to verify his solutions using the GC, he realized that the solutions were incorrectly written. His problem was partial misunderstanding as he wrote incorrect interval notations.

Student BF2 was able to model the required quadratic inequality from the contextual problem: $6000 \leq -2x^2 + 220x$. She used both the algebraic and graphical approaches; however, the algebraic approach (quadratic formula) seemed to be the most useful in helping her to solve the quadratic inequality. The graph was correctly drawn with critical values but was never used. After writing the inequality in standard form, the student solved the quadratic equation only (i.e., $x = 50$ or $x = 60$). This student seemed to confuse the two concepts of equation and inequality, although she knew that the algebraic statement formed was a quadratic inequality. This shows a lack of true understanding by this student, see Figure 6.8 below.

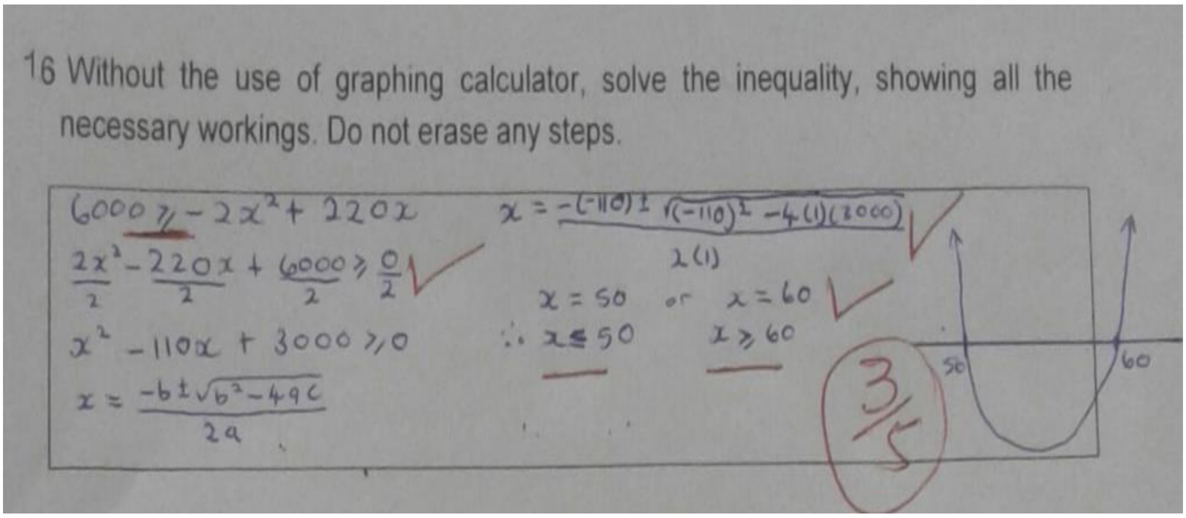
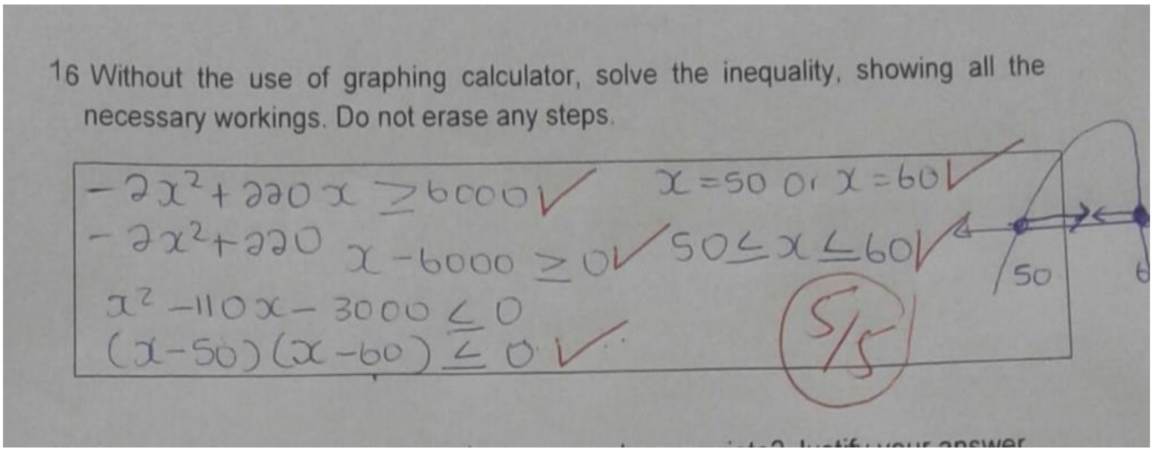


Figure 6. 8: Student BF2's problem solving

In addition, the student was able to graph the correct shape for the function given and she correctly set up the x-intercepts (i.e., the critical values). It was observed that she did not question her solutions thus lack of monitoring one's progress. This means the student did not try other approaches to validate her solutions. However, at the end of the problem when she was asked to verify her solution using the GC, she discovered that her solution did not make any sense. She pointed out that, the x-values should be in inequality form in order to get a profit greater or equal to zero.

Student BF3 correctly formulated the required inequality from the contextual problem. She used a graphical approach to pave the way for the solution of the problem. She started the problem by sketching the graph of the function onto her script. After looking at the graph, she indicated the region that had the solution of the quadratic inequality. She then switched to an algebraic approach, by using quadratic formula, to find the zeros of the function; see Figure 6.9, below.


16 Without the use of graphing calculator, solve the inequality, showing all the necessary workings. Do not erase any steps.

$-2x^2+220x \geq 6000$

$-2x^2+220x-6000 \geq 0$

$x^2-110x-3000 \leq 0$

$(x-50)(x-60) \leq 0$

$x=50$ or $x=60$

$50 \leq x \leq 60$

50

5/5

Figure 6. 9: Student BF3's problem solving

She monitored her progress throughout when solving this problem and moved freely between algebraic approach and graphical approach, thus switching back and forth from the two approaches. This helped her to reflect on the strategies used to solve the contextual problem correctly. She verified her solution using the values of x and then found that her solution was justified. Consequently, student BF3 wrote the correct interval notation for the solution (i.e., $50 \leq x \leq 60$). Her solution was also verified using the graphing calculator.

Comments on the students' problem solving abilities

The three students had different levels of proficiency in relation to problem solving processes. For that reason, the students' problem solving processes were scored in terms of a) forming an inequality, b) using algebraic approach, c) using graphical approach, d) using the graphing calculator for verifying solution and e) obtaining a correct solution set. The scoring ranged from 0 to 5. A 5 represented the highest score of executing all the listed steps correctly. The use of an algebraic approach is mandatory in the CAPS curriculum and should be complemented with the use of the graphical approach or any other relevant approach. In this regard, Student BF1, an average learner scored a 3 as he used an incorrect graph and wrote incorrect solution sets. Student BF2, a below-average learner scored a 3 as she did not use her graph correctly and wrote wrong solutions. Student BF3, an above average learner scored a 5 for performing all the steps correctly. This means that Student BF3 used all relevant processes of problem-solving and was able to translate the problem into appropriate mathematical language. Based on these results, the teacher-researcher concluded that the use of the GC supported the students' problem solving abilities of the quadratic inequalities. This is consistent with the findings by the earlier researchers (e.g., Spinato, 2011; Karadeniz, 2015; Idris, 2009).

6.6.2 Students' results of the focus group interviews on reasoning

This section attempted to address the third research question:

> *How does the pedagogical use of the GC support students' reasoning abilities when solving quadratic inequalities?*

The results of the focus group interviews with students on their reasoning abilities in the contextual quadratic inequality problems were analysed and described in this section. In order to successfully examine the students' reasoning in solving the contextual quadratic inequality problem, student participants of School B were asked questions during the focus group interviews relating to analysing a problem (Questions 1.1 & 1.3), initiating a strategy (Questions 1.4 & 1.5) and reflecting on one's solution (Questions 1.7 and 1.9) in Appendix D.

6.6.2.1 Students' results from focus group interview on analytical reasoning

The first reasoning question (Q1.3) of the focus group interviews required the students to identify the main concept involved in the contextual problem.

TR: State and explain the main concept of this problem (1.3)
Student BF1: It is a quadratic inequality because the profit of items sold must be greater than 6000.
Student BF2: It's a quadratic inequality because the problem talks about greater than or equal to.
Student BF3: The inequality I formed is a quadratic inequality. P(x) representing a quadratic function is greater than 6000, which is the profit.

Interpretation of the students' responses

All the three students were able to identify the main concept involved as quadratic inequality and gave appropriate justification. The key phrase "*greater than*" was used as supporting evidence by the students. However, student BF3 gave a detailed explanation which included "*quadratic function greater than representing profit",* thus a good ability of analytical reasoning.

The next reasoning question (Q1.3) requested the students to draw conclusions from their solutions of the contextual problem. In that regard, the students drew the following conclusions about their solutions.

TR: Is there any relevant conclusion that you can make about the solution to the problem? If so, what can you say?
Student AF1: Yes. I think the solution of the inequality is above the x-axis
Student AF2: The solution of the quadratic inequality lies inside the critical values. The graph drawn from this inequality is facing down
Student AF3: Yes. The profit and number of sold items must be more than zero and my solution is between the critical values

Interpretation of the students' responses

All the three students were able to state at least one reasonable conclusion. Student BF1's conclusion- "that the solutions would be above the x-axis", affected the way he determined the solution of the quadratic inequality (see Figure 6.9). Student BF2 provided a valid conclusion but she gave an incomplete solution (see Figure 6.10). This suggests that this student lacked self-confidence in contextual problems. Student BF3 made two analytical conclusions by stating that "t*he profit and number of sold items must be more than zero and the solution is between the critical values"* This means student BF3 was able to visualise the quadratic inequality solution in the graph. The teacher-researcher observed that student BF3 possessed a strong analytical reasoning which is a pre-requisite for understanding inequalities. On the other hand, student BF1 had a weak analysis and this even affected him to write the solution of the inequality correctly (see Figure 6.11).

6.6.2.2 Students' results from focus group interview on initiative reasoning

In this area of reasoning- Initiating a Strategy, students were assessed on how they purposefully selected the appropriate concepts, representations and procedures when solving the contextual problem. The first initiative reasoning question (Q1.5) of the focus group interview required the students to identify the approaches which were most helpful to solve the contextual problem. The following is how the students responded:

TR: *Which approaches do you think were most helpful in solving this problem- algebraic and/or graphic?*
Student BF1: I think a factorisation helped me most to solve the problem.
Student BF2: To me quadratic formula seemed to be helpful to solve the problem.

Student BF3: Algebraic (quadratic formula) and graphic approaches were the most helpful to solve the problem.
TR: Briefly explain why you selected these approaches.
Student BF1: I used factorisation because I am comfortable with it and easy to use. After getting the values of x, I used them to draw the graph to indicate the portion of the solution.
Student BF2: I used first the quadratic formula approach because it's the quickest approach and saves time. I normally get the values of x correctly when I use the quadratic formula.
Student BF3: *I used first the quadratic formula approach so that I get the critical values correctly and then I used graphical approach for determining the shape of the graph.*

Interpretation of the students' responses

All the students were able to state and explain the approaches that they executed to solve the contextual quadratic inequality problem. The approaches used by the students were almost similar but differed in the way they were used. Student BF1 responded correctly that he used two approaches- algebraic and graphic to solve the problem. However, the algebraic approach, factorisation, dominated throughout the process of solving the problem. A wrong graph was drawn (i.e., concave up) and x-intercepts were indicated but lacked other details (cf: Figure 6.9). Student BF2 correctly responded that she used the algebraic approach-quadratic formula to solve the problem. Although she had attempted to draw a graph but she did not use it to solve the inequality (cf: Figure 6.10). Student BF3 also responded correctly that she used two approaches- algebraic and graphic to solve the problem. However, the graphical approach seemed to be the most useful in helping her to solve the problem. The graph drawn by the student was correct in shape. The algebraic approach- quadratic formula was only used to determine the zeros of the function which were later on indicated on the graph for the decision. The consistent use of two strategies earned her good results (cf: Figure 6.11). All the three students initiated their strategies for solving the contextual problem very well but students BF1 and BF2 were not consistent with their strategies. The influence of the use of the GC is visible on students' work as all of them had drawn a graph to help them solve the inequality. This may suggest that the graphing calculator was a resource of information for students to develop solution strategies of the problem. For that reason, all the three students had developed initiative reasoning ability of solving quadratic inequalities.

The second initiative reasoning question (Q 1.6) of the focus group interview required students to explain how the use of the graphical approach helped them solve the quadratic inequality. The responses of the students were:

TR: *Explain how helpful the use of graphical approach was in solving the contextual problem.*

Student BF1: *Yes it was. The graphical approach helps to interpret or link the inequality in the form of a graph and helps me to see the required region of the inequality using the inequality sign.*

Student BF2: *Yes. Even though algebraic methods are reliable the graph is even more. The graph tells you if the solution lies inside or outside the critical values.*

Student BF3: *Yes* it does. *The graph helps to figure out the x-intercepts and the shape, and then you know where the solution lies. My x-intercepts are the critical values*.

Interpretation of the students' responses

The responses given by the students were similar and demonstrated that they understood the role of the graphical sketches when solving quadratic inequalities. Students responded that the graphical approach helps to *"interpret or link the inequality"* and *"figure out'* and *"is even more reliable".* This is even noted in their work where each student attempted to use a graphic approach to solve the inequality. This means the use of the GC provided students with visual representation of quadratic inequality solutions in the form of graphs and this might have influenced them to use graphs too. In fact this was the main purpose of using the GC as an instructional tool. The responses of the students show that the solution of the inequality is always within or outside the critical values. This means the use of the GC helped to develop students' initiative reasoning strategy.

6.6.2.3 Students' results from focus group interview on reflective reasoning

In this area of reasoning- reflecting on one's solution, students were assessed on how they interpreted their solutions (Q1.8), justified the reasonableness of their solutions (Q1.9), and how they considered alternative ways of solving problems (Q1.10). The questions intended to make students reflect on their solutions of the contextual quadratic inequality problem. The following reasoning questions (Q1.8 & Q1.9) were asked to find out how the students interpreted and justified their solutions of the contextual problem. The following is how the students responded:

TR: With the values of x that you have obtained, do you think you have solved this problem completely and correctly? Justify your reasoning.
Student BF1: Yes. I believe my answer is complete and reasonable.
Student BF2: No. (Scratching his head …), I am struggling to complete the problem.
Student BF3: Yes, there are correct.
TR: Can you justify why you say your answer is reasonable?
Student BF1: Because both my values of x are valid answers.
Student BF2: Yes. My values of x give me profits.
Student BF3: Yes, it does. *Because when I used x=50 I got a positive value*.
TR: Which values of x really represent the profit?
Student BF1: When I substitute my values of x, it gives me a zero. (He reasoned …) I think my solution is wrong.
Student BF2: I now doubt my solution because there won't be any profit.
Student BF3: The values of x that are within the interval give the profits more than 6000.

Interpretation of the students' responses

All the three students were able to decide whether their solutions were reasonable and justified their choices. Student BF1 also realised that his solution was unreasonable and was giving him *negative or no* profits. Student BF2, who ended at the critical values, realised that the answer was not reasonable. She indicated that "*my values of x give me a zero, which means there were no profits made*." She later on realised that there should be some profits for the items sold. This was different with Student BF3 as her solution was complete and reasonable. She was able to provide plausible reasons. Her values of x taken from within the solution set provided the profits which were more than 6000. This student was helped by linking both algebraic and graphic strategies. In this sense, student BF3 was able to reflect on her own solution effectively and this suggests that she displayed strong reflective or metacognitive reasoning skills in respect of this item. This may mean that the use of the GC has not provided adequate opportunities for students to develop their metacognitive reasoning skills of solving the contextual quadratic inequality problems.

The next reasoning questions (Q1.10 and Q1.11) were combined in analysing students' responses since they focused on the use of the GC. Question 1.10 asked about any other relevant information that could be used to justify their solutions. Students' responses were as follows:

TR: Is there any other relevant information that can be used to justify their solutions?

Student BF1: No.
Student BF2: Yes using a graphing calculator.
Student BF3: Yes, I can use a graphing calculator.
TR: When using the graphing calculator, do you still get the same solution? Explain.
Student BF1: Yes the critical values are the same but the solution is within them.
Student BF2: No. I get the same critical values. I did not write the solution of the quadratic inequality.
Student BF3: Yes, I got the same answers and there are within the critical values.

Interpretation of the students' responses

Student BF1 did not realise that the GC could be used as an alternative to verify his solutions. In that regard, the teacher-researcher requested the students to take out their GCs to verify the accuracy of their solutions. After punching the inequality into their graphing calculators students were able to see if their solutions were accurate. Only the solution of student BF3 was accurate. Students BF1 and BF2 found that they had different solutions and were incorrect. Student BF1 responded that, *"the critical values were the same but the solution was within them".* On the other hand, student BF2 noted that *"I did not write the solution of the quadratic inequality because I left my answer at the critical values".* Interestingly, all the students were able to solve the inequality and interpret the results accurately from the GC. This is consistent with idea of the instrumented action scheme in the TIG. It is evident that the use of the GC did not develop the students' reflective reasoning completely. This suggests that in the next cycle the emphasis of the teacher-researcher has to focus on the development of students' metacognitive reasoning skills.

6.6.2.4 Students' results of the observed monitoring progress

This section described the students' reasoning skills of monitoring progress as they were observed solving the contextual quadratic inequality problem. The main focus was to assess how the students reviewed and/or modified their selected strategies in particular when they encountered difficulties (see Rubric in Appendix E). The observed results of the three purposefully selected students were as follows:

Student BF1 converted the problem into right quadratic inequality and correct procedures (algebraic) were followed through (cf: Figure 6.7). However, he did make reference to the incorrect graph as another approach to solve the problem. He was then misled and got wrong solutions. This means he did not make adequate attempt to monitor his progress; thus why he did not realise that he had written meaningless solutions. His low level of self-monitoring affected him to attempt other avenues or make any reasonable assumptions that could lead him to review his selected strategy. When the teacher-researcher asked him to use the GC to solve the problem he was able to visualise the correct solutions displayed on the GC.

Student BF2 converted the problem into a right quadratic inequality and correct procedures (algebraic) were followed through. She used mainly algebraic (symbolic) representations and did not use a graph at the beginning of the problem. She seemed to get confused when she realised that she needed to write down the solution of the inequality. An attempt of another approach was made by sketching a right graph to help in determining the solutions, which was not effectively exploited (see Figure 6.8). Her low level of self-monitoring affected her attempt and could not go beyond critical values. This means that she was able to indicate the x-intercepts correctly. When the teacher-researcher asked her to verify her solutions with graph produced by the graphing calculator, she realised that she had incomplete solutions.

Student BF3 used two different approaches- algebraic and graphic, to solve the quadratic inequality (cf: Figure 6.9). A look at the graph of this quadratic function assisted in the monitoring of her progress in the reasoning process. However, she did not rely on one reasoning procedure as she kept on switching from the algebraic to graphical approaches and vice versa. The graph seemed to be very beneficial in the reasoning processes used by this student. Throughout her problem solving processes, she continued monitoring her progress and verifying her assumptions. The teacher-researcher observed that every move that she could take, she questioned it. She was also able to use the GC to confirm the shape of the graph, find the zeros of the function, make assumptions and verify her solutions found using the pencil and paper methods.

6.6.2.5 Students' results of the observed seeking and using connections

In this domain of reasoning- seeking and using connections, students were observed on how they sought and used connections of different concepts, contexts and representations when solving the contextual quadratic inequalities. This is also about when the students make references to mathematical concepts used earlier in the topic of quadratic inequalities, in other mathematics areas, or in any other subject areas. In this context, the findings were as follows:

Student BF2 was only able to make connections between the solution of quadratic equation and the quadratic inequality (see Figure 6.8). However, she did not use this relationship to determine the solution of the inequality. Students BF1 and BF3 were able to seek and use connections between concepts and representations when solving the quadratic inequality. In their attempts to solve the contextual quadratic inequality problem, they made links between the algebraic (symbolic) and graphic representations (see Figure 6.7 and 6.9). However, student BF1 failed to link correctly the critical values to express in right interval notations. In this case, student BF3 used quadratic graphing (geometry) and solving quadratic equations (algebra) both as viable ways to find a quadratic inequality solution. The student realised that the solution of the equation (x-values) was the x-intercepts of the graph which determine the solution of quadratic inequality. This suggests that the algebraic reasoning (i.e., algebraic symbols and functions) has helped student BF3 to use the connections effectively in solving quadratic inequalities. This means that the GC use supported students' reasoning domain of using connections and these findings are in accordance of the intertwinement principle of the RME theory. This principle underlies the importance of integrating concepts, contexts and subjects in order to develop better understanding of quadratic inequalities.

With regard to these results, the teacher-researcher cautiously concluded that the GC use supported the students' reasoning skills in learning quadratic inequalities. This conclusion is linked up with the findings made by the earlier researchers (e.g., Spinato, 2011; Karadeniz, 2015; Idris, 2009; Armah & Osafo-Apeanti, 2012). This further indicates the potentiality of the GC use as a mediating tool in developing critical reasoning as aligned to the ideas of the Vygotsky's ZPD theory. This is consistent with the theory of instrumental approach for instrumented action schemes.

6.6.3 Student's perceptions of how GC enhanced reasoning skills

This section intended to answer the third research sub-question on how the students perceived the use of the GC towards enhancing their reasoning skills in learning quadratic inequalities. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 6.6 below.

Table 6. 6: Student's perceptions on how the GC enhanced reasoning

Student's perceptions on how the GC enhanced reasoning	**SD %**	**D %**	**NS %**	**A %**	**SA %**
The GC use helped me to analyse adequately the quadratic inequality problems	6	9	11	69	6
The GC use enabled me to use many approaches when solving quadratic inequalities	0	6	23	49	23
The GC use assisted me check my progress when solving quadratic inequalities	3	3	17	60	17
The GC use helped me to use other concepts to solve quadratic inequalities	9	6	17	46	23
The GC use allowed me to think more about my quadratic inequality solutions	0	6	17	66	11
Key: **SD-Strongly Disagree; D-Disagree; NS-Not Sure; A-Agree; SA-Strongly Agree**					

In Table 6.6, 75% of the students agreed or strongly agreed that the use of the GC helped them to analyse correctly the quadratic inequality questions. Only 29% of the students, including those who were not sure, denied that the GC enabled them to use new strategies when solving inequalities. Twenty seven of the students (77%) agreed or strongly agreed that the GC assisted them monitor or check their progress when solving inequalities. A large proportion of the students (69%) affirmed that the graphing calculator guided them to use other mathematical concepts to solve inequalities. Only two students (5.7%) denied that the GC helped them to think or reason more about their inequality solutions. This implies that the majority of students (77%) felt that the GC allowed them to evaluate the reasonableness of their solutions. Based on these results, the researcher partially concluded that the majority of the students felt that the use of the GC supported them in reasoning skills of solving quadratic inequalities.

6.7 Results of students' responses in the pre-and post-surveys

This section presents the results of the students' responses in the pre- and post-surveys of how they perceived about the GC use in learning quadratic inequalities. Their perceptions are presented in the following subsections.

6.7.1 Comparative results of students' responses in the pre-and post-surveys

This section intended to answer the fourth sub-question by comparing the results of the pre- and post- surveys means on how the students perceived about the GC use in learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

The post-survey intended to gather the perceptions of the students on whether the use of the GC in learning quadratic inequalities assisted them to understand the topic. The positive changes of students' perceptions were attributable to the effective intervention of GC use as an artefact in learning quadratic inequalities. The eleven items of the pre-survey were similar to the ones on the post-intervention survey to see if students changed their perceptions on how they learned quadratic inequalities after the GC intervention as an instructional tool. Students' responses were biased towards the understanding of and lessening the difficulties of learning quadratic inequalities. In this context, an increased confidence in their ability to understand and learn quadratic inequalities is measured by students' option of "disagree" or "strongly disagree" and increased mean. A comparison of the students' perceptions is given in Table 6.7 below, where M_0=post-survey mean and M_R=pre-survey mean.

Table 6. 7: Results of students' pre- and post- intervention surveys (n=35)

ITEM	***1 = Strongly Disagree, 2 = Disagree, 3=Not Sure, 4 = Agree, and 5 = Strongly Agree***	**Pre-survey**		**Post-survey**	
		M_R	**SD**	M_O	**SD**
SPQI 1	Quadratic inequalities are difficult to learn and understand	2.46	1.29	3.67	1.11
SPQI 2	I do not see the difference between the equation and inequality	2.57	1.17	3.87	1.17
SPQI 3	It's difficult to determine the solution sets of quadratic inequalities after finding the critical values.	2.49	1.09	4.57	1.37
SPQI 4	I have difficulties with determining factors of quadratic expressions (inequalities)	2.84	1.30	3.09	1.29
SPQI 5	I don't know the difference between critical values and x-intercepts of the graphs	2.61	1.16	3.57	1.09
SPQI 6	In order to understand the quadratic inequality topic I usually memorise it	2.67	1.29	4.10	1.20
SPQI 7	Of all the topics I have done so far I don't enjoy learning quadratic inequalities	2.49	1.11	3.84	1.16
SPQI 8	It's difficult to use graphical sketches to determine the solutions of quadratic inequalities	2.77	1.30	3.71	1.15
SPQI 9	Given an opportunity of not to learn quadratic inequalities I was going to do so	3.21	1.17	3.30	1.25
SPQI 10	Technology (e.g., computers) cannot help me to understand quadratic inequalities	2.69	1.12	3.50	1.17

Table 6.7 shows that the learning of quadratic inequalities was no longer difficult to understand after using the GC. This is reflected by the post-survey mean which is

greater than the pre-survey mean (M_0=3.67 >M_R=2.46), Secondly, the students initially indicated that they did not see the difference between the equation and inequality but after the GC use this does not hold. This is supported by the large value of the post-survey mean ($M_0 = 3.67 > M_R$=2.57). The students further indicated that determining the solution sets of quadratic inequalities after finding the critical values was no longer an issue. This is justified by the pre-survey mean (M_R=2.49) which is less than the post-survey mean (M_0=4.57). Next, students revealed that they had difficulties in determining factors of quadratic expressions (inequalities) before the GC use. This reflected by the post-survey mean which increased after the intervention (M_0=3.09 > M_R=2.84). Fifth, students indicated that initially they did not know the difference between critical values and x-intercepts of the graphs but after the GC use they can. The large post-survey mean supports this claim (M_0=3.57 > M_R=2.61). Sixth, students further revealed that before the GC intervention they used memorise quadratic inequalities in order to understand. The mean of the post-survey shows that the learning improved after the GC use (M_0=4.10 > M_R=2.67). Seventh, the large post-survey mean reveals that the learning of quadratic inequalities was enjoyable after the GC use (M_0=3.84 > M_R=2.24). Eighth, students indicated that they had difficulties to use graphical sketches to determine the solutions of quadratic inequalities before the GC use. This is reflected in survey means where the pre-survey is smaller than the post-survey (M_R=2.77< M_O=3.71). Ninth, students indicated that given an opportunity of not to learn quadratic inequalities they were going to do so before the GC use. The small pre-survey mean (M_R=2.21) compared to post-survey mean (M_0=3.71) reflected that the GC use has brought confidence in learning quadratic inequalities. Before the GC use students fewer students did not believe that the use of technology (e.g., computers) could help them to understand quadratic inequalities. The large post-survey mean indicates that more students appreciate the role of the GC in learning.

Generally, the results from the Table 6.7 show that the use of the GC brought new development on the students' perceptions towards the learning of quadratic inequalities. This was supported by the post-survey means that were greater than the pre-survey means in all the items. The overall student responses showed that the use of the GC supported the students' learning of quadratic inequalities. Initially students indicated that given an option of not learning quadratic inequalities they were going to do so because they were neither enjoying nor understanding the topic. This is consistent with the findings of the earlier researchers who indicated that the use of the GC would develop visual images which can help students to construct their understanding (Spinato, 2011; Karadeniz, 2015) and provide a more meaningful interpretation for the solution (Doerr & Zangor, 2000). This shows that the GC use minimizes the use of algorithms and memorization (Knuth, 2000) which leads to surface level understanding and makes students experience challenges in solving problems (McTighe & Self, 2003; Snyder & Snyder, 2008; Kohler, 2010).

6.7.2 Student's perceptions on how the GC supported the learning sessions

This section intended to answer the fourth research sub-question by analysing the results of students' responses on how they perceived about the GC use in the designed sessions of learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

In that context students were issued with an eight item post-intervention survey to answer. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 6.8 below.

Table 6. 8: Student's perceptions on how the GC supported the learning sessions

ITEM	**Students' perceptions of the effects of graphing calculator on the designed sessions of learning quadratic inequalities**	**SA (%)**	**A (%)**	**N (%)**	**D (%)**	**SD (%)**
SPGC 1	The use of the in learning sessions assisted me to solve symbolic (algebraic) quadratic inequalities	20	46	15	11	8
SPGC 2	The use of the GC in learning sessions assisted me to understand the difference between critical values and zeros of the graph	25	49	11	9	6
SPGC 3	The use of the GC in learning sessions assisted me to identify correctly the region of the inequality solution	14	57	9	14	6
SPGC 4	The use of the GC in learning sessions assisted me to understand contextual (application) problems of quadratic inequalities	20	54	11	6	9
SPGC 5	The use of the GC in learning sessions assisted me to use graphical sketches when solving quadratic inequalities	23	60	8	6	3
SPGC 6	The use of the GC in learning sessions assisted me to understand the effect of the parameter 'a' in the quadratic inequality	28	46	11	9	6
SPGC 7	The use of the GC in learning sessions assisted me to note that the effect of the parameter 'a' of quadratic function had the same effect on quadratic inequality	22	49	17	6	6
SPGC 8	The use of the GC in learning sessions assisted me to learn and understand much better quadratic inequalities	31	46	11	9	3

1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree

Table 6.8 shows that 66% of the students strongly agreed or agreed that the use of the GC in the learning sessions assisted them to solve symbolic (algebraic) quadratic inequalities. Seventy five percent of the students admitted that the use of the GC in learning sessions assisted them to see the difference between critical values and zeros of the graphs. Students affirmed (strongly agreed or agreed) that the use of the GC in learning sessions assisted them to identify correctly the region of the inequality solution (71%). Seventy four percent of students strongly agreed or agreed that the use of the GC in learning sessions assisted them to understand contextual (application) problems of quadratic inequalities. However 11% of them were not sure whether the use of the GC helped to understand contextual problems. Eighty three percent of students strongly agreed or agreed that the use of the GC in learning sessions assisted them to use graphical sketches when solving quadratic inequalities. Only 15% percent of the students strongly disagreed or disagreed that the use of graphing calculator in learning sessions assisted them to understand the effect of the parameter ‘a’ in the quadratic inequality. Only four students (12%) strongly disagreed or disagreed that the use of the GC in learning sessions assisted them to notice the effect of the parameter ‘a’ of quadratic function had the same effect on quadratic inequality. Seventy seven percent of the students strongly agreed or agreed that the use of the GC in the learning sessions assisted them to learn and understand the topic of quadratic inequalities better. This means that the students overwhelmingly perceived that the use of the GC assisted them to learn quadratic inequalities effectively.

The researcher partially concluded that the use of the GC in the planned sessions helped the students to understand the quadratic inequalities as they were able to identify quadratic inequality with the shapes of the quadratic graphs, to see the effect of the parameter “a” on the different graphs, to use the graphs to solve quadratic inequalities, to transform contextual problems into symbolic quadratic inequalities and to determine the region of the solution. These are considered as the main procedures that can lead the students to learn quadratic inequalities effectively. The results are in line with the roles of the GC identified by the previous researchers (Averbeck, 2000; Karadeniz, 2015; Lee & McDougall, 2010). They also perceived that the effect of the different parameters were the same for all the quadratic functions and inequalities expressed in the form of $a(x+p)^2 + q \geq\leq 0 \; or \; ax^2 + bx + c \leq\geq 0$. The use of the GC created a supportive environment in the sessions (Lee & McDougall, 2010) and provided students with a more meaningful interpretation for the solution (Doerr & Zangor, 2000). During the sessions students used both graphical and graphing calculator approaches which provided them with more visualization to meaningfully solve quadratic inequalities (Karadeniz, 2015). The consistently use of the GC in their quadratic inequality sessions helped students enhance their knowledge and understanding (Lee & McDougall, 2010). These results were triangulated with their responses from the in-depth and focus group interviews on how the GC use helped them to understand quadratic inequalities.

6.8 Results from the in-depth interviews about the GC use in quadratic inequalities

This section of in-depth interview with students mainly attempted to address the fourth research sub-questions about the students' perceptions on the use of the GC in learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

An in-depth interview was conducted with the three students who were purposefully sampled from those who had obtained marks below average, average and above average from the post–test. They were individually interviewed after school on a regular school day in their classroom. Each interviewee lasted for approximately thirty minutes. The in-depth interview was recorded and then transcribed. The interview consisted of ten questions which were mainly about the use of the GC on students' understanding of quadratic inequalities.

The codes used in the interview are TR for the teacher-researcher and BD for School B student in the in-depth interview (D). They were interviewed separately, but for convenience their responses are given together below. The responses of the in-depth interview questions are discussed in the following sub-sections:

6.8.1 Students' responses on how the use of the GC made easier students' learning of quadratic inequalities

The question asked by the teacher-researcher below sought to find out the students' opinions about whether the use of the GC made their learning of quadratic inequalities easier. Students' responses were almost similar as shown below:

TR: Does the use of GC make your learning of quadratic inequalities easier to understand? Please explain your answer.

Interview with Student AD1

Student BD1: Yes because I get the x-values at the same time without doing long steps. These x-values help me to find the solutions of the quadratic inequalities.
TR: Getting the x-values at the same time doesn't affect your understanding?
Student BD1: No, I don't have to calculate the critical values as they are displayed on the GC screen as x-intercepts and show the solution set of the inequality.

Interview with Student BD2

Student BD2: Yes it does, because the GC shows the critical values and how the graph goes.
TR: What do you mean, "the GC shows the critical values and how the graph goes?"
Student BD2: The critical values help to determine the solution of the quadratic inequality and I can see where the graph cuts the x-axis.

Interview with Student BD3

Student BD3: Yes it does, because the graphing calculator gives us the critical values that we need and shows us if the graph is below or above the x-axis.
TR: Can you give clarity what you mean by "it shows us that the graph is below or above this x-axis."
Student BD3: The GC indicates whether the solution is above or below the x-axis. This means the solution can be within or outside the critical values.
TR: *Ok. Thank you very much.*

Interpretation of students' responses

Students' responses were positive and almost similar as they affirmed that their learning of quadratic inequalities had been made easier after the use of GC. Students did explain that "the x-intercepts, critical values and the region of the solutions of the quadratic inequality are always shown on the GC." The displayed information on the GC enabled the students to see how the quadratic inequality was solved graphically as shown in Figure 6.10, below.

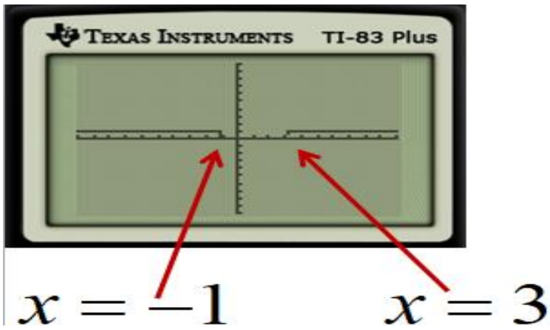


Figure 6. 10: Solution of the quadratic inequality displayed on the GC screen

In this case, the graph indicates that the solution set of quadratic inequality is a disjunction of $x = -1$ and $x = 3$. The use of the GC, according to students, demonstrates that the solution is within or outside the critical values. Using the displayed information in Figure 6.12 the solution is outside the critical values. However, there are instances when the displayed information can be presented in the form of quadratic graph. This means that through the use of the GC, students were able to visualise the graphs with critical values which made it easy to determine the solution of inequalities. The visual images of the graphical representations contributed to the improved learning of quadratic inequalities as shown by reference to '*shows us if the solution on the graph is below or above the x-axis*'. Students benefitted from the visualisation capabilities of the GC which made them understand easily the relationship/connections between the graphs and solutions of quadratic inequalities.

6.8.2 Students' responses to how the use of the GC helped them to feel comfortable with quadratic inequalities

The question below sought to find out the students' opinions about whether the use of the GC helped them feel more comfortable with quadratic inequalities. This question is part of in-depth interview which attempts to answer the first research question. Students' responses were almost similar as shown below:

TR: *Does the use of the GC help you to feel more comfortable with quadratic inequalities?*

Interview with Student BD1

Student BD1: *Yes, I am comfortable. I used to face some challenges with quadratic inequalities before I used graphing calculator.*
TR: Do you still have those challenges?
Student BD1: *Ah no.* Using his hand to demonstrate sometimes, *sometimes but I am ok.*

Interview with Student BD2

Student BD2: *Yes it does. I used to be afraid of learning quadratic inequalities but after the use of the GC I am different.*
TR: Explain how the use of the GC contributed to your comfortability.
Student BD2: *With the use of the GC, I get the needed information about quadratic inequalities that I will always remember.*

Interview with Student BD3

Student BD3: *Yes, it does. I am comfortable with solving quadratic inequalities*.
TR: *Please explain your answer.*
Student BD3: *You can use the graph displayed on the GC to determine the solution of the quadratic inequality. You practise solving many problems and this boosted my confidence in learning quadratic inequalities.*
TR: *Ok. Thank you very much.*

Interpretation of students' responses

Students' responses were positive and almost similar as they have expressed confidence in the learning of quadratic inequalities with the use of the GC. This implies that the use of the graphing calculator provided an enabling environment for learning quadratic inequalities. Student BD1 and BD2 gave an affective answer of a reduced mathematics anxiety thus "challenge" and "afraid" of learning quadratic inequalities. Student BD2 emphatically indicated that the use of the GC '*gives … information*' she will '*always remember*'. Student BD3 affirms that GC use '*boosted*' her confidence as she was able to see graphic representations. This means the use of the GC stimulated the use of graphical sketches as objects/models that could help students to determine the solutions of quadratic inequalities.

6.8.3 Students' responses to whether the GC should be used in learning quadratic inequalities

The question sought students' opinions on whether GC should be used in learning quadratic inequalities or not. The students' opinions were positive and quite similar as shown below:

TR: Should graphing calculators be used in learning quadratic inequalities at the eleventh grade?
Student BD1: Yes it should be used it is easier to understand the GC than the teacher working on the chalk board.
TR: Is the use of the GC not affecting your thinking?
Student BD1: Ah no. With the use of the GC I can substitute the values of x to check my solutions.
TR: Does it mean you prefer the use of the GC to your teacher?
Student BD1: No it's not that. The teacher is always needed for explaining where I am not clear.

Interview with Student BD2

Student AD2: *Yes, it must be used instead of having the teacher doing all the steps for the learners. The GC helped me to answer many questions on my own.*
TR: Does it mean the teacher must be eliminated from classroom and give space to the use of the GC?
Student BD2: *No. But the GC helped me to understand more methods including using the graphs.*

Interview with Student BD3

Student BD3: *Definitely, it must be accepted in the mathematics classroom.*
TR: *Please explain your opinion*.
Student BD3: *The use of the GC simplifies the difficulties that learners have in quadratic inequalities. The GC helps to figure out how the graph would look like and make it easy to decide the region of the solutions.*

Interpretation of students' responses

Most of the students responded similarly to this question as they supported the use of the GC in learning of quadratic inequalities. This clearly means that with its capabilities, the GC afforded students opportunities to learn better quadratic inequalities. Student BD3 emphatically affirms by saying '*definitely*' and also gives additional reason as giving learners opportunities to '*figure out how the graph looks like'* and *'the region of the solutions*'. This means the GC was used as a psychological tool to produce enjoyment through its use. This is linked up with the socio-cultural theory of Vygotsky. Student BD1 viewed the importance of a GC as an instructional tool and checking tool in his learning that should be complemented by his teacher's voice. Student BD2 indicated that the use of the GC provides students with opportunities for *"understanding more methods including using the graphs"* and it reduces *"teacher domination*" during lessons. However, Students BD1 and BD2 emphasised the need for the teacher to be there to "support" and "explain" to them where they don't understand. They viewed the teacher as an additional resource to the GC who can attend their individual differences in the classroom. This is aligned to the findings made by Ndlovu (2014) that technology cannot orchestrate itself to articulate mathematical understandings to learners. This is aligned to the schemes of instrumented actions in the theory of instrumental approach.

6.8.4 Students' responses on how the use of the GC helped them to do homework and other activities of quadratic inequalities

The question asked if the use of the GC helped students to do homework and class activities of quadratic inequalities independently. Students' responses were positive and almost similar.

TR: Was the use of the GC helpful in doing homework and other class activities of quadratic inequalities independently?

Interview with Student BD1

Student BD1: *"Yes it was helpful.*
TR: Please justify your answer
Student BD1: *The use of the GC helped me to trust my answers as it was easy to check if I'm right and it helped me to practise answering many of these. My homework was easy to do because I had done many similar questions of quadratic inequalities on my own using the GC.*

Interview with Student BD2

Student BD2: I did not use the GC to write homework because I didn't have it.
TR: Suppose you had one was it going to be helpful?
Student BD2: Yes it does help.
TR: Please explain your answer
Student BD2: *The GC makes it easier for me to solve inequalities. It helped me to gain the knowledge of drawing graphs and interpret them. With more practice that I had on the use of the GC I would be able to do homework independently of solving inequalities.*

Interview with Student BD3

Student BD3: Yes it does
TR: Please explain your answer
Student BD3: The GC shows graphs with critical values and where the solutions could be. Then the final solution becomes much simpler to determine. Again the GC can correct my mistakes and always did the homework by myself.

The responses from the students seemed to be similar and confirmed their independence on writing any activity involving quadratic inequality after the use of the GC. For example, I did the homework "*on my own", "independently*" and "*by myself*". Student BD1 indicated that the GC helped him to "*trust my answers"* and student BD2, on the other hand gained *"the knowledge of drawing and interpreting graphs*". Student BD3 felt the use of the GC simplified her homework as she was able to interpret the displayed the graphs and critical values to determine the solutions of quadratic inequalities.

6.8.5 Students' experiences of using the GC in learning quadratic inequalities

This question solicited students' experiences after using the GC to solve quadratic inequalities in a mathematics classroom. The student interviewees were asked to relate their experiences. Students expressed exciting experiences, as the responses given below show:

Interview with Student BD1

TR: How can you explain your experiences of using the GC in learning quadratic inequalities?

Student BD1: *It was interesting*

TR: Please explain your experience

Student BD1: *As it was a first time to use a programmable GC in learning quadratic inequality it brought joy and unforgettable experience. It gave me more knowledge of solving quadratic inequalities using graphs. It reduced the difficulties I had on quadratic inequalities.*

Interview with Student BD2

TR: How can you explain your experiences of using the GC in learning quadratic inequalities?

Student BD2: It was great and awesome

TR: Please relate your experience

Student BD2: *I learnt new ways of solving inequalities and it was my first time to use a graphing calculator. I learnt to draw a quadratic graph on the GC and also to solve quadratic inequalities using the drawn graph. The critical values are indicated and as well as where the solutions lies. Now I know the importance of the parameter value in solving quadratic inequalities.*

Interview with Student BD3

TR: How can you explain your experiences of using the GC in learning quadratic inequalities?

Student BD3: It was exciting and very interesting
TR: Tell us how exciting was?
Student BD3: It was the first time to use the GC and it enabled me to solve quadratic inequalities which the topic used to be so difficult for me. It was very interesting to learn quadratic inequalities using graphs.
TR: Didn't the use of the GC affect your thinking?
Student BD3: Definitely no. Instead it inspired me to use sketches and graphs when solving quadratic inequalities. It helped to figure out how the graph will look like always.

Interpretation of the students' responses

The responses of the three students revealed that they had 'interesting', 'great' and 'exciting' experiences with the use of the GC in solving quadratic inequalities. The use of the GC in quadratic inequalities brought *'joy and unforgettable'* experience *to* student BD1 as he was no longer facing difficulties. Similarly, the *'first time'* experience of other two students (BD2 & BD3) to use a graphing calculator brought new approaches such as graphs to solve quadratic inequalities. Students were able see the solutions of the quadratic inequalities displayed as drawn graphs on the screen when using a GC (see Figure 6.13, below). Student BD2 further stated that the use of the GC helped her to understand the effect of the parameter of a quadratic function when solving quadratic inequalities. On the other hand student BD3 emphasised how she was inspired to use sketches and graphs. All the three students indicated that the GC use reduced the levels of their difficulty with quadratic inequality problem solving.

Solving quadratic inequalities graphically

$$x^2 + 2x - 8 \geq 0$$

$$(x - 2)(x + 4) \geq 0$$

$$x = -4 \ or \ x - 2$$

$$(-\infty, -4] \cup [2, \infty)$$

Figure 6. 11: Solving quadratic inequalities graphically

This means that the GC acted as a visual aid tool (visualisation tool) and provided the students with opportunity to see how the solutions are presented graphically. This is in line with the fact that what a child has seen it's hard to forget. This means that the sketched graphs were used as visual objects to aid their conceptual understanding of quadratic inequalities (i.e., *to figure out how the graph will look like always*). All the three students indicated that they used the GC as to solve and graph quadratic inequalities in order to understand better the topic (i.e. the instrumentation process). This is consistent with the theory of instrumental genesis (Trouche, 2004).

6.8.6 Students' responses on how the use of the GC helped them to score better marks on quadratic inequalities

This question asked if the use of the GC helped students to score better marks on quadratic inequalities and to rate themselves between 0 and 5, a 5 being the highest. All the students confidently affirmed better marks after the use of graphing calculator.

Interview with Student BD1

TR: Did the use of the GC make you score better results in quadratic inequalities?
Student BD1: Yes, it was.
TR: Explain what made them to improve after using the GC.
Student BD1: My marks are much better. I used to face challenges with quadratic inequalities but now I have improved a lot.
TR: In that case, how can you rate your level of understanding?
Student BD1: I would rate my level of understanding between 31/2 and 4
TR: Why rate yourself with a 31/2?
Student BD1: Ok a 4

Interview with Student BD2

TR: Did the use of the GC make you score better results in quadratic inequalities?
Student BD2: Yes, it does.
TR: Explain what made you to score better after using GC.
Student BD2: With the use of the GC I have gained more about solving quadratic inequalities.
TR: In that case, how can you rate your level of understanding in quadratic inequalities after the use of the GC?
Student BD2: I rate myself at 4 because there has been an improvement since.

Interview with Student BD3

TR: Did the use of the GC make you score better results in quadratic inequalities?
Student BD3: Yes, I can confirm that.
TR: Explain what made you to improve after using the GC.
Student BD3: Because when you use a GC the chances of you knowing the basics of the quadratic inequalities are high.
TR: In this case, how can you rate your level of understanding?
Student BD3: I rate myself slightly above a 3.

TR: *Thank you very much.*

Interpretation of the students' responses

The responses of the three students were almost similar and they rated themselves with a 4. The students felt that the use of the GC made their results better in solving quadratic inequalities. Student BD3 argued that the GC helped her to know the basics of the quadratic inequalities. The teacher-researcher noted that Student BD1 was not certain with his rating but he justified that his challenges were then over. Conversely, student BD2 gained self-confidence in solving quadratic inequalities and had been active in classes of the quadratic inequalities.

6.9. Reflections, design principles and feed-forward of the research cycle

This final section of the chapter presented the reflection and design principles of the research cycle at School A and the way forward for the next cycle. The researcher looked back at the teaching experiment and designed the principles of the first cycle and then concluded by formula the feed-forward for the second research cycle to be implemented at School C.

6.9.1. Reflecting on the starting points and learning outcomes of the HLT

In Section 6.2, the researcher set out the expectations as starting points of this design study concerning the opportunities that graphing calculators (GC) would offer for the students in order to achieve the higher level of understanding of the quadratic inequalities. In this current section, the researcher evaluated those expectations one by one to find out if they were confirmed in School B.

The first learning outcome in session two was that students would develop the notions of interval notations, parameters, x-intercepts and quadratic graphs in a flexible graphing calculator environment, including the additional expectations (cf: section 6.3). During the teaching experiment at School B, students were confronted with questions that were mathematically similar but with different coefficients of x^2 (see Session 2 in Appendix D). More emphasis was given to the use of quadratic graphs which was viewed as a challenge in the first cycle. Students were given additional activities on quadratic graphic properties and were engaged in groups. The use of the GC indeed proved to be an appropriate instructional artefact for helping students to generate a family of graphs in the same system of axes. In addition, the use of the GC helped students to visualise the graphs displayed on the screens and enabled them to understand their properties (e.g., zeros, intervals, axis of symmetry, concavity and domain). On the other hand, students were able to make repetitions of graphing quadratic functions using the GC and this made them develop and reify the key pre-concepts of quadratic inequalities. As it was anticipated that the use of GC would provide a flexible environment for students to understand quadratic graphs and their properties, this was moderately achieved. Almost all the students' attempts and discussions on quadratic inequalities had graphs with x-intercepts and students were able to recognise the concavity of quadratic functions with respect to the coefficients of x^2. This is a positive sign towards understanding quadratic inequalities. In this regard, the use of GC effectively supported the transition from the graphical representation to quadratic inequality representation.

The second learning outcome blended with the third in sessions 3 and 4 was that students would develop the notions of solving quadratic inequalities in a flexible graphing calculator environment. This means students were supposed to use their knowledge of quadratic functions and equations towards solving symbolic quadratic inequalities, which demand routine reasoning skills. During the teaching experiment at School B, students were confronted with questions that were mathematically similar but with different levels of difficulties (see Sessions 3 and 4 in Appendix D). Students were supposed to use the GC as an instructional artefact, in particular the graphic and tabular instrumentations to solve the symbolic quadratic inequalities. Students were involved in the instrumented action schemes for graphing and tabling that could lead to a better instrumentation (i.e., the fourth additional expectation). These instrumented action schemes allowed students to repeat the processes of graphing and tabling the values and supported them develop and reify the concept of quadratic inequalities. Through the repetition of the processes the students realized that changing the parameter values affected the complete quadratic graphs and inequality solution sets. The graphical visualization of this effect (i.e., changing the parameters) created a strong mental image for the students. The impression is that most of the students started to perceive graphs and inequalities as entities that could symbolize objects. The use of graphical representations (models) made students to extend their graphical conception of the quadratic functions towards the view of understanding the symbolic quadratic inequalities. In particular, the x-intercepts of the graphs were used as the boundaries for deciding whether the quadratic inequality solutions are within or outside them. The graphical models mediated very well between the quadratic equations and quadratic inequalities. Notably, the graphical schemes of the graphing calculators were helpful for visualizing the effects of parameters and the properties of graphs. Furthermore, students used the tabular instrumented action scheme to check for the solutions in the table of values displayed on the GC. The understanding and interpretation of graphs and inequalities has improved and the students' performance in the post-test questions similar to those in sessions 3 and 4 was good. The researcher, for this reason, concluded that the use of graph and tabular instrumented action schemes moderately facilitated the transition from the graphical and tabular representations to quadratic inequality representations.

The fourth expectation inscribed in sessions 5 and 6 was that students would develop the higher order problem solving and reasoning skills in contextual quadratic inequality situations in a flexible graphing calculator environment. This means that the transition from *the symbolic quadratic inequalities* to *the contextual quadratic inequality situations* was to be brought about by the use of the GC. In that regard, students were supposed to use their routine reasoning skills of solving symbolic quadratic inequalities into solving contextual quadratic inequality situations. During the teaching experiment at School B, students were confronted with questions that were mathematically similar but with different levels of difficulties (see Sessions 5 and 6 in Appendix D). Students were engaged in the use of the GC as an instructional artefact to solve the contextual quadratic inequality problems. The consistent use of the GC enabled students to extend their routine reasoning skills towards the non-routine reasoning and problem-solving skills in contextual quadratic inequality situations. In these sessions students interacted with questions that demanded the previous routine reasoning processes in developing non-routine skills which were mediated by the use of the GC. In that regard, students were able to convert the contextual situations into quadratic inequalities with one variable as required by the CAPS FET Mathematics document and solved them similarly as symbolic quadratic inequalities. With the use of the graphical representations displayed on the GC screens, students were able to extend their understanding of the symbolic towards the view of understanding contextual situations. The graphical models successfully mediated between the symbolic and contextual situations as it was further observed that the students attempted well question 4 and fairly question 5 in the post-test. However, the understanding and interpretation of such contextual quadratic inequality problems remained a hard issue for some students. For that reason, these difficulties continued to prevail in both sessions and post-test with similar questions that demanded the same reasoning and problem solving processes. The researcher therefore concluded that the use of the GC did not adequately support the transition from the symbolic quadratic inequalities (the routine skills) towards the contextual quadratic inequality situations (non-routine skills) of solving quadratic inequalities.

6.9.2. Reflections on the in-depth interviews with students

The responses of the students from the in-depth interviews affirmed that the use of the graphing calculator provided an enabling environment for learning quadratic inequalities. In their arguments, they revealed that their learning of quadratic inequalities had been made easier after the use of GC and they were able to see how the quadratic inequalities were solved graphically. Students further contributed that the use of the graphical sketches assisted them to determine the regions of the solution sets of quadratic inequalities and were enabled to visualise the solutions of quadratic inequalities on the GC screens. Through the process of visualisation students were able to establish the relationship between the quadratic graphs and quadratic inequality solutions. As revealed in the interviews, the visual images of the graphic representations improved the learning of quadratic inequalities. Additionally, they affirmed that the use of the GC influenced them to link algebraic methods with graphs i.e., graphs being drawn as aid for solving quadratic inequalities. This was affirmed by student BD3, “by looking at the inequality I can figure out how the graph will look like and where its solution set will be”. The linking of the graphical and algebraic representations is one of the recommendations of the CAPS for FET Mathematics document (DBE, 2011), expressed emphatically in the NSC Examination Diagnostic Reports that students must solve quadratic inequalities by integrating both methods (DBE, 2014; 2015; 2016; 2017). Ultimately, students expressed enjoyment and confidence in the learning of quadratic inequalities with the use of the GC and indicated that it was instrumental towards their understanding of quadratic inequalities. A significant number of students presented their work with graphs and some of graphs were not relevant. This is a sign that the use of the GC had "*inspired the use of the graph*" as indicated by student DB3. However, not all the students made use of the graphical models to illustrate their solutions in answering the post-test. This means that although these students appreciated the graphical meaning of an algebraic solution they apparently could not connect the two representations as expected. Despite the fact that students had 'interesting', 'great' and 'awesome' experiences with the use of the GC in solving quadratic inequalities, they still valued the presence of the teacher as an additional resource to the GC who can attend their individual differences in the classroom.

6.9.3. Reflecting on the focus group interviews

The qualitative results of three students who were engaged in focus group interviews showed different levels of proficiency in relation to problem solving processes. The student's problem solving processes were scored in terms of a) modelling an inequality, b) using algebraic approach, c) using graphical approach, d) verifying their solution including the use of the graphing calculator and e) obtaining a correct solution set (interval notation). The students' work revealed that one of them did not confidently use the graphical approach even if the right graph was drawn and did not obtain correct solutions of contextual quadratic inequalities. However, all of the three were in a position to use the GC to verify their solutions when asked to do so. Evidently, this means on average the three problem-solving processes were performed well. With these results the teacher-researcher concluded that the second research question was moderately achieved.

The results of the focus group interviews with students on their reasoning abilities in the contextual quadratic inequality problems revealed that students possessed average analytical reasoning which is a pre-requisite for understanding quadratic inequalities. Two thirds of students were able to identify at least two pre-concepts of quadratic inequalities and all of them stated at least one reasonable conclusion. It was further revealed that the students had average initiative reasoning skill of solving quadratic inequalities. Two of three students confidently used and explained the strategies (approaches) that they executed to solve the contextual quadratic inequality problem, which included algebraic, graphical and graphing calculator. Similarly, one of three students displayed moderate reflective or metacognitive reasoning skills of solving the contextual quadratic inequality problem. This student was not able to reflect on their solutions of the contextual quadratic inequality problem through interpreting, justifying and checking and using alternative ways (using the graphic feature of the GC). It is evident that the use of the GC did not adequately develop the students' reflective reasoning. This suggests that in the next cycle the emphasis of the teacher-researcher has to focus on the development of students' metacognitive reasoning skills.

It was noted that only one of three students was able to monitor her progress and verify her assumptions. In the next cycle the teacher-researcher's focus should be on how to improve students' reasoning skills on monitoring their progress. It was further observed that two of three students were able to seek and use connections between concepts and representations in the reasoning process of the contextual quadratic inequality situation. The use of both algebraic and graphing reasoning helped students to solve the problem. The researcher partially concluded that the use of the GC supported the students' reasoning skills on seeking and using connections.

The teacher-researcher observed that during the activity sessions the majority of the students were able to make inferences, draw conclusions and reflect on the reasonableness of their solutions of quadratic inequalities. The use of GC as an instructional tool provided opportunities for the students to analyse the inequality problems, interpret the solutions and to make predictions about the solution sets. This is in line with the constructivist teaching and learning which requires teachers to focus on the use of physical actions (graphing calculator) to promote the use of senses to construct the underlying meaning of concepts (Vygotsky, 1978), and students' independent thinking and the control of their own learning situation (von Glasersfeld, 1996; Amineh & Asl, 2015). These actions are strongly connected to the processes of reasoning and problem solving.

6.9.4. Feed-forward for the third cycle

The findings of the second research cycle at School B informed the feed-forwards for the third research cycle. The feed-forwards of this cycle concerned the hypothetical learning trajectory, the instructional activities and the research methodology.

The feed-forward concerning the HLT addressed the broad outline: Solving symbolic, routine and contextual quadratic inequality situations in a flexible graphing calculator environment. The results from the teaching experiment suggested that the use of graphical approach was most helpful to students in solving quadratic inequalities because of its dynamic character and visual image. However, the use of GC did not support completely the conception of graphical approach as its properties including the interval notation, concavity and domain remained a challenge to the students. A second issue concerning the HLT was the use of GC to support the transition of symbolic quadratic inequalities towards the contextual quadratic inequalities which did not come across in a satisfying way. This can be attributed at least partially to difficulties with solving symbolic and routine quadratic inequalities, and the use of graphical strategy in particular. It is important to master this graphical technique, so that it does not hinder the generalization and visualisation processes. Thirdly on the HLT was the need for linking the algebraic and graphical approaches to holistically develop students' reasoning skills and problem-solving abilities in solving quadratic inequalities. Students incompletely solved the contextual quadratic inequalities because they had moderately understood symbolic quadratic inequalities and partially developed their metacognitive and monitoring- reasoning skills.

The feed-forward concerning the instructional activities focused on those teaching materials that needed more time for better understanding. A first point was that students needed more practice in using graphs to solve quadratic inequalities. Additional sessions were designed for students to work in groups using GC so the visual images can be retained. This approach was combined with the algebra in order to holistically solve quadratic inequalities. Students would further be engaged with the graphic and tabular instrumented action schemes in separated and integrated practice. As far as the role of the teacher-researcher is concerned, it is hoped that he would avail himself every time to explore his experience in the third cycle for orchestrating the learning process in class discussions and as an additional resource of information for the students.

The feed-forward concerning the research methodology addressed the teaching experiments. The first point was to establish a better match between the pre-test and post-test, so that the improvements in understanding could be monitored during the teaching experiment. The second point was to conduct mini-interviews during the sessions on selected questions in order to provide appropriate and meaningful data. The third point was to organise students into groups and give them task to solve in order to observe the students' interaction and thinking; this would provide platform for those students who have many questions to be assisted.

6.9.5 Design principles of the study

This current design based study was driven by the pedagogical gap in the teaching and learning of quadratic inequalities. The researcher identified five fundamental principles of design-based research such as:

1. Starting the topic of quadratic inequalities with the problems of real-life situations
2. Considering small number of learners when learning quadratic inequalities experimentally in a flexible graphing calculator environments
3. Using models, graphs and visual images in the teaching and learning of quadratic inequalities moves students' thinking to formal reasoning and problem solving.
4. Using the GC approach only cannot address all learning styles and must be complemented by other methods.
5. Discourage discussions that are not related to classroom objectives as they can distract the effective learning.

CHAPTER 7: THE THIRD CYCLE OF DBR TEACHING EXPERIMENT

7.1. Introduction

This chapter reports on the results of the DBR cycle at School C and focuses on the hypothetical learning trajectory, the learning experiences in the teaching experiment and, final reflections and a feed-forward to the future research. The teaching experiment sought to investigate how the designed HLT played out a grade 11 classroom and whether the use of graphing calculator provided learning opportunities for improving the students' understanding of the quadratic inequalities in an envisaged way.

The re-designed HLT is described by the starting points and main activities included in the teaching materials and their expected learning outcomes in Section 7.2. Second, the participants and research procedures of the cycle are explained in Section 7.3. Third, the results of the pre- and post-tests are analysed in Section 7.4. Four, the results of the problem solving are analysed in Section 7.5. Fifth, Section 7.6 presents the results of the focus group interviews with students. Sixth, the results of pre- and post-surveys are analysed in Section 7.7. The results of the in-depth interviews with students are discussed in Section 7.8. Finally, the chapter concludes with the reflections on the development of the HLT, feed forwards and design principles in Section 7.9.

7.2 The HLT for the quadratic inequality concept in the third DBR cycle

This section presents the HLT of the last DBR cycle in terms of the starting points and the expectations that were explored in this final teaching experiment in School C. In this context, the broad learning trajectory was motivated through attempting quadratic inequalities of the different cognitive levels. Then the activities that were supposed to bring about the transitions between the different learning processes of quadratic inequalities were later described. This could lead to re-affirm the broad HLT in quadratic inequalities: *Solving routine, symbolic and concrete quadratic inequality problems in the graphing calculator environment enhances students' understanding*. Finally, graphic and tabular instrumentations are addressed in this broad learning trajectory.

7.2.1 Starting points and expectations for the concept of quadratic inequalities

As with first and second research cycles, the starting point for the development of the HLT was the symbolic structure of quadratic inequalities for defining the concrete situations and the suggestions of how to achieve higher level of students' understanding in the GC mediated environment. Furthermore, this last research cycle was built upon the experiences and the reflections (feed-forward) from the previous two research cycles. In essence, the approaches used in previous two research cycles were revisited. However, the reflections (feed-forward) of the second research cycle led to the following learning outcomes that were intended to improve the results and effectively inform the re-designed HLT for this last research cycle:

1. In the second research cycle, the properties of the quadratic graphs including interval notations (domain), concavity and the effects of parameter α were identified as the bottlenecks. Also students did not see the true meaning of interval notations; hence they gave meaningless solution sets. Specific attention to this would improve the students' visualisation and presentation of the solution sets of quadratic inequalities in a flexible graphing calculator environment.

2. In the second research cycle, the transition from the *quadratic functions and equations to the symbolic quadratic inequalities* was problematic, which was supposed to be fostered by using the graphic and tabular instrumented action schemes of the graphing calculator. For instance, the use of the graphic and tabular instrumented action schemes would support the students' visualisation of graphs and reification of interval notations and concavity (the effects of parameter α) of quadratic functions. Specific attention was to improve students' skills of quadratic functions and domains towards solving symbolic quadratic inequalities, which demand routine reasoning skills.

3. In the second research cycle, the students did not understand the contextual problems of quadratic inequalities, which hindered their conceptual development. Therefore, being guided by the reality principle of RME theory, the use of real-life mathematical situations including linear inequalities as the starting point was supposed to help the conceptual development of quadratic inequalities at a referential level and foster meaningful generalization at the general level.

4. In the second research cycle, the transition from the *routine problems to the contextual (complex) problems of quadratic inequalities* was a hard issue for some students, which was supposed to be addressed by an earlier intertwinement of the algebraic and graphic representations. Therefore, the effective use of the GC on intertwinement would develop into an integrated approach that can stir up the students' understanding of the quadratic inequalities and would diminish students' difficulties. The learning in the GC environment was expected to allow for algebraic exploration and visual geometric representations in solving quadratic inequalities.

7.2.2. The transitions for the learning activities in quadratic inequality concept

The suggested steps toward solving the design problems are to broadly describe the learning activities that foster the transition to the higher understanding of quadratic inequalities. The transition from *the quadratic graphs and interval notations to symbolic quadratic inequalities* is expected to occur by confronting the students with several questions that are mathematically similar but have different parameter values (Session 2). The expectation here was that the students would perceive the similarity of the reasoning and problem-solving procedures in spite of the different parameter values.

The transition from *the quadratic functions and equations to the quadratic inequalities* is expected to be brought about by a graphical approach, in which the parameter value changes gradually and systematically (Sessions 3 & 4). The student is asked to study the effect of the changing parameter value on the graph. Mentally, the student realizes that changing the parameter value affects the complete graph and quadratic inequality solution sets. The GC can be used in this transition for graphing a sequence of 'shifting' graphs and visualisation of the effects of the parameters on the quadratic graphs and their zeros for expressing in intervals.

The transition from *the symbolic quadratic inequalities to the contextual (complex) quadratic inequality problems* is expected to be fostered by applying additional algebraic properties to the concrete situations and using the graphical representations (Sessions 5 & 6). The solving of the contextual quadratic inequalities involves the use and filtering of appropriate graphical properties (intersection points, tangent points, vertices, roots). Such problems are expected to lead to a mental shift of students, so that they can apply reasoning and problem solving procedures. The use of the GC allows for this shift, because the graph and tabular instrumentations are expected to make students reify the concept of quadratic inequalities. The tabular instrumented scheme elicits to stimulate the students to use the concept of interval to reify the concept of quadratic inequality solutions.

7.3. Participants and research procedures of the study

The teaching experiment took place in a public high school with eleventh-grade class, thereafter the school is referred to as School C. A total of 45 students participated in this study and were randomly chosen from 92 eleventh graders. The participants were asked not to identify themselves on the pre-tests and post-tests, but rather to label their scripts with symbols given randomly by their teacher, such as C1, C2, C3,...,C45, where C represented the school. However, 10 students decided to withdraw their participation citing lack of commitment and transport reasons. Their results were not considered for the quantitative analysis.

To collect the data for this third DBR cycle, the same procedures described in Section 5.3 were followed. Measuring which school did better than the other was not the intention of this study. This study concerned itself on measuring the improvement of students' understanding of quadratic inequalities when the GC as a tool had been intervened. The improvement of the subsequent cycle solely depended on the limitations of the previous cycle, which were "the feed-forwards" (Drijvers, 2003).

7.4. Comparative analysis of the students' results of the pre- and post-tests

This section presented and discussed the results of the pre- and post- tests and the written tasks which sought to answer the first research question:

> *To what extent does the pedagogical use of GCs impact on students' performance in solving quadratic inequalities?*

The results were also used to test the null hypothesis: H_0: There is no difference between the pre-test mean and the post-test mean of quadratic inequalities for the students in the study (H_0: μ_1 (pre−test) = μ_2 (post−test). Alternatively, H_1: There is a difference between the pre-test mean and post-test mean of quadratic inequalities for the different domains for the students in the study (H_1: μ_1 (pre−test) ≠ μ_2

(post−test). The results of 35 students obtained in School C were compared and presented in paired samples t-test (Table 7.1) below.

The analysis of results of School C showed that students performed much better in the post test (M=47.6571; SD=24.06846) than in the pre-test (M=27.6857; SD=15.92218). The results of the post-test reflected highest score of 90% from 63%. This could be attributable to effective intervention of the GC in the learning of quadratic inequalities. However, 7% was the lowest score for both the pre-test and post-test. The median mark (43%) of the post-test was also much better than that of the pre-test (23%). It is further noted that 14 out of the 35 students (40%) scored more than 50% in the post-test while only 3 (i.e., 8.57%) had done so in the pre-test. This suggests that there was a reasonable improvement in the post-test towards the understanding of quadratic inequalities. It was noted that the majority of the students used the graphs very well to solve the quadratic inequalities. From the results of the post-test the researcher concluded that the HLT was adjusted well enough to the level of the students as they incorporated the graphical sketches in solving quadratic inequalities. The idea of using real life situations as the starting point of the HLT could have also assisted to improve the students' understanding of quadratic inequalities. This suggests that the students with competence of graphs were likely to express correctly the solution of the quadratic inequality.

In the next table, a paired samples t-test was conducted to test the significance of the GC use on the students' performance in solving quadratic inequalities. The use of the paired-samples t-test helped to test the null hypothesis, which stated that there was no difference between the pre-test mean and the post-test mean of quadratic inequalities for the students in the study (H_0: μ_1 (pre−test) = μ_2 (post−test). In that regard, Table 7.1 below presents the dependent (paired) samples t-test results of School C with paired differences of means of the pre- and post-tests of 35 students. The t-test results in the table show that t(34) = -3.755 and p = 0.001. This means the actual probability value is 0.001 and it is substantially smaller than the specified alpha value of 0.05. These t-test results indicated that the null hypothesis was rejected at 5% significant level in favour of the alternative hypothesis. This means that there was a statistically significant difference between the students' means of the pre- and post-test scores. In that context, there was a statistically significant improvement of the students' results after the use of the GC in the learning of quadratic inequalities.

Table 7. 1: Dependent (paired) samples t-test of school C

Pair 1	Paired Differences							
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		T	Df	Sig(2-tailed)
				Lower	Upper			
Pre-Test & Post-test	-1.99714E1	31.46659	5.31882	-30.78058	-9.16228	-3.755	34	.001

Although the results presented above indicated the statistically significant improvement in the test scores of students, they do not tell much about the magnitude of the GC intervention's effect in solving quadratic inequalities. Because there was no control group for this particular task and limitation of statistical significance, the researcher proceeded to calculate the Cohen's d effect size statistic using the pre-test and post-test means in order to determine the magnitude or practical significance of the difference in scores. The effect size was 0.8, indicating that the post-test mean is at 79% of the pre-test mean. This means that there was a large effect impacted by the use of the GC on learning quadratic inequalities (i.e., using Cohen's, (1988) interpretation: 0.2=50%=small effect, 0.5=58%=medium effect and 0.8=79%= large effect). The researcher then concluded that there was a practically significant improvement of about 0.8 standard deviations in the mean scores from pre-test (M=27.69, SD=15.92) to post-test [M=47.66, SD=24.07, t(34)=-3.755, p=0.001<0.005]. This study concludes that the pedagogical use of the GC impacted positively on the students' performance in solving quadratic inequalities. This study did not investigate if the teaching and learning of quadratic inequalities with the GC is better than any approach without it, but to show that it can help with the understanding of the topic. The t-test results showed that there were learning gains not only for the purposeful sample of students but also for all the students that were exposed to the teaching intervention with GC and instructional materials. In reference to first and second cycles, the effects of the GC use were practically significant and results significantly improved at 5% level. The performance of School

C was the best with the highest post-test mean. The presentation of the solutions by students improved a lot, thus using correct interval notations and use of graphical approaches increased, in comparison with other cycles. However, School B had the largest Cohen d effect size, signifying the greatest improvement in the post-test (cf: Sections 5.4 & 6.4).

7.4.1 Students' results in written tasks of symbolic quadratic inequalities

The improvement of students' performance in the post-test could be attributed to the written tasks during the teaching experiment. Students wrote a task with questions related to sessions 3 and 4 and was considered for analysis. These questions were meant to monitor the progress of the students in each session. The results of the students were analysed to determine what percentage of those who answered the symbolic quadratic inequalities correctly, incorrectly, blankly or incompletely and also used graphic approach (see Table 7.2), below.

Table 7. 2: Students' results of the written task about symbolic quadratic inequalities

Inequality	Correct %	Incorrect %	Blank %	Incomplete %	Graph Use (%)	*Others %*	*None %*
4.1 $\mathbf{4x + x^2 \leq 0}$	54	26	9	11	71	20	9
4.2 $\mathbf{(x+2)(3x-7) \geq 0}$	60	17	9	14	60	23	17
4.3 $\mathbf{x^2 - x - 12 < 0}$	63	14	6	17	69	20	11
4.4 $\mathbf{-(x-4)(x+5) < 0}$	57	17	9	17	69	17	20
4.5 $\mathbf{2x^2 - 7x \geq 4}$	51	29	14	06	57	14	29

In Table 7.2, above each of the symbolic quadratic inequality questions was solved correctly by more than 50% of the students. This may suggest that at least 18 of the 35 students had acquired adequate knowledge and routine skills for solving symbolic quadratic inequalities. However, Q4.3 had the highest percentage (63%) of correct solutions. The percentage of students who had incorrect or blank solutions was between 20 and 43. This can be viewed as an improvement in the understanding of quadratic inequalities, except in Q4.5. It was further observed that students had some difficulties in this question, thus the reason why it had the greatest percentages of incorrect (29%) and blank (14%). It was also observed that the same question had the least percentage students (6%) who had incomplete answers. An incomplete solution means that the student used the correct strategy but abandoned it before arriving to the solution. Precisely, it means the students were able to determine the critical values and/or sketch the graph but could not go beyond that.

Students' answers were supposed to be linked to the use of graphs in order to see the influence of GC. In this vein, it was observed that more than 55% of the students used graphs to determine the solutions of the inequalities. Between 14% and 23% of the students used other methods such as line graph and sign chart or table to determine the solutions of quadratic inequalities. A line graph dominated in other methods; this could be that a line graph is also shown on the graphing calculator when solving quadratic inequalities. This means the use of the GC supported the students' abilities of using graphs when solving symbolic quadratic inequalities. It was observed that the highest percentage (29%) of students used none method to answer to question 4.5. See the purposefully sampled work of the two students with different approaches they used to answer the problematic question 4.5 in Figure 7.1 below. It was evident that the use of the GC had great impact on students' learning of quadratic inequalities in School C.

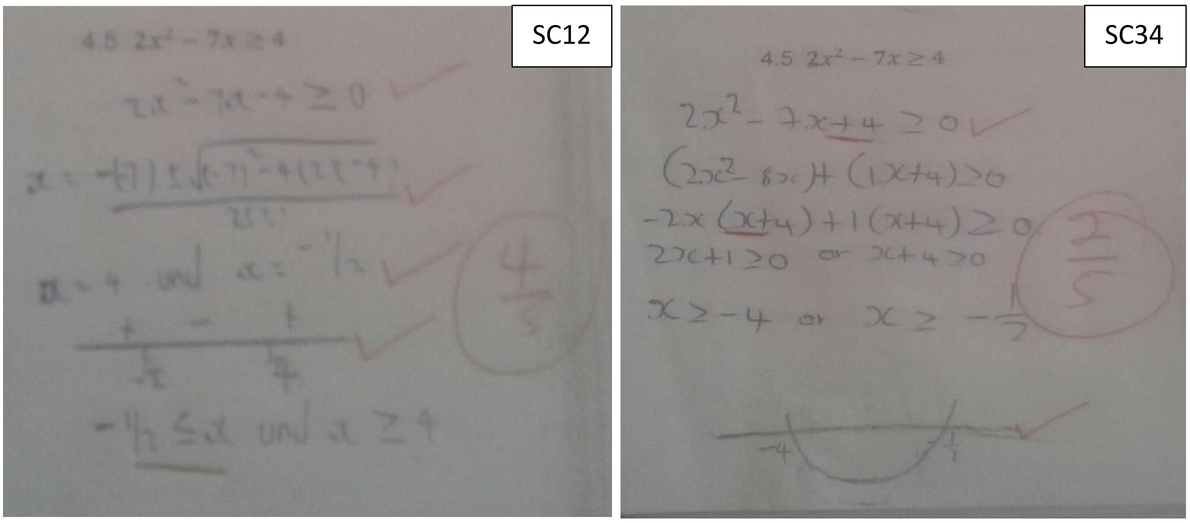


Figure 7. 1: Students' solutions of symbolic quadratic inequality questions

Student SC12 used both algebraic and sign chart approaches to solve the symbolic quadratic inequalities. He perfectly combined the two strategies in such a way that he indicated the regions where the solutions represented the positive values. However the solutions were incorrectly written, thus using wrong interval notations and term 'and'. This means the student does not understand the difference between "and" and "or" in quadratic inequality solutions (DBE, 2017).

Student SC34 committed lot of misconceptions which greatly affected her ability to consistently produce the right answer. She did not apply the correct procedure of transposing a positive term as a result this affected the factorisation method. The linear factors $(i.e., 2x + 1 \geq 0\ or\ x + 4 \geq 0)$ were expressed with inequality signs logically following the quadratic inequality sign. Furthermore, she probably mixed up with the basic simultaneous linear inequalities (with one variable) in Grade 10. She gave the final answer for $x \leq -4$ or $x \geq -\frac{1}{2}$ as $x \geq -4$ or $x \geq -\frac{1}{2}$. Linking and understanding of algebraic and graphic approaches did not help her to solve this question correctly. She wrote the interval notations as solutions which were meaningless. She did not apply logical thinking to check the appropriateness of her solutions and it was like she gave her solutions procedurally.

The sampled work of two students showed that the use of the GC had influence on the learning of quadratic inequalities. Students attempted to link algebraic (symbolic) and geometric (graphics, diagrams, lines) representations when solving quadratic inequalities. The use of the GC helped these students to use visual representations such as graphics, lines and diagrams to solve and understand quadratic inequalities. These models helped students with the visual representations which were expected to move their informal thinking away from the applications to the formal reasoning. This is consistent with the ideas of the level principle of RME theory that reflects the levels of reasoning development from horizontal to vertical mathematization. This was attainable through the consistent use of the GC and repetition of the processes. This is also in line with instrumented action schemes of the instrumental approach theory as students were able to integrate meaningfully the graphical and algebraic representations for supporting their reasoning and problem solving.

7.4.2 Students' results of written tasks in applications of quadratic inequalities

The improvement of students' performance noted in the post-test could be attributed to the written tasks during the teaching experiment, related to questions in session 5 in Appendix D and was considered for analysis. The written task assisted to monitor the progress of the students. The results of the students were analysed to determine what percentage of those who answered the application problems of quadratic inequalities correctly, incorrectly, blankly or incompletely (see Table 7.3), below. .

Table 7. 3: Students' answers in application of quadratic inequalities

Question	Application	Correct %	Incorrect %	Blank %	Incomplete %
5.1	**For what values of x will $\sqrt{x^2 - 25}$ be real?**	63	17	11	9
5.2	**For which values of x will Q=$\sqrt{x^2 - 8x + 12}$ be non-real?**	57	20	9	14
5.3	**Given $g(x) = -x^2 + 7x + 8$ For which values of x will g(x)>0?**	54	26	14	6

The analysis of the data in Table 7.3 revealed that 63%, 57% and 54% of the students respectively applied correctly quadratic inequalities to solve questions 5.1, 5.2 and 5.3. This implies that students were able to understand the problems and transform them into quadratic inequalities and then selected the right strategies to solve them. The percentage of students who had incorrect or blank solutions was between 28 and 40; this can be viewed as an improvement in the understanding of problems involving the applications of the quadratic inequalities. However, question 5.3 had the highest percentage (40%) of incorrect or blank solutions. These students need expanded opportunities to be adequately re-skilled in these problem areas of applications. Although the proportion (i.e., 9%, 14% and 6% respectively) of students who had incomplete solutions was very low for all the three questions, a repetition of similar problems supported by the use of the GC would help to develop students' cognitive skills and confidence. The consistent use of the GC as visual artefact supported with teacher's voice would reduce the percentage of students who had incorrect, blank or incomplete solutions. Students learn best when they construct their own mathematical understanding and they share their learning experience with others.

A sample of two students' answers was selected to show how they attempted question 5.3 an application problem involving quadratic inequalities in Figure 7.2, below. Student SC17 substituted $g(x)$ and expressed the quadratic inequality in standard form. The whole inequality was multiplied by a negative sign, converting it to a non-positive expression. In that regard, she correctly applied the strategy of quadratic inequality to solve the question concerned. The student did not factorise correctly but the wrong critical values were correctly indicated on a right graph. However, she wrote a meaningless solution (i.e., $-1 < x > 6$). This means that the solution set was procedurally written without proper understanding of the interval notations. A right graph used as a model helped the student to figure out the correct region of the solution but she wrongly placed the inequality signs without logical thinking. The meaningless answers given by students helped the teacher-researcher to teach logical thinking to students so that they can analyse and access their students logically (Smith, 2002). As corrective measure, students were taught to use the line graph to reflect on their solutions. This means a solution that makes sense can be represented on a line graph.

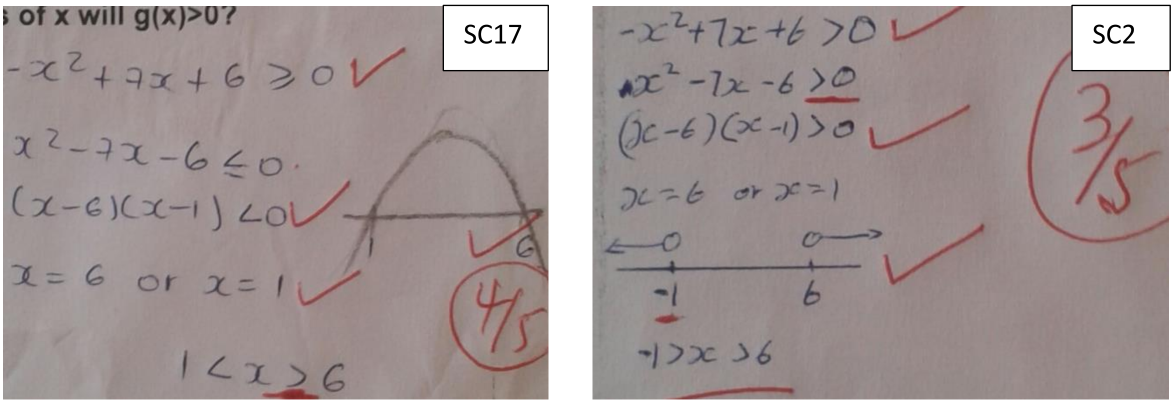


Figure 7. 2: Student’s answers in application of quadratic inequalities

In Figure 7.2 above, student SC2 correctly converted the quadratic function into the quadratic inequality. He failed to convert this inequality to a non-positive expression after multiplying by a negative sign. He was able to factorise correctly and also drew a correct line graph. Using his positive quadratic inequality he correctly indicated the region of the solution sets. However, he wrote a meaningless solution (i.e., $-1 > x > 6$). This means that the solution set was procedurally written without proper understanding of the interval notations. The use of the GC to answer many similar questions was viewed as the best alternative of bringing out solution to this question. Students learn best when they construct their own mathematical understanding and they share their learning experiences with others. This is in line with the socio-cultural learning theory by Vygotsky. Students used the correct language or terms in their application which is a tenet of this theory.

The answers of the two students demonstrate the influence of the use of the GC on solving quadratic inequalities. The students used visual representations: graphic and line graph which are both displayed on the GC. This is consistent with the findings of earlier researchers that graphing calculators provide multiple approaches to studying functions, including graphical, numerical, algebraic approaches, and visualization of concepts (Karadeniz, 2015; Leng, 2011; Committee on the Undergraduate Program in Mathematics, 2004). This is an indication that the pedagogical affordances of the graphing calculator are closely related to the improved learning of mathematics (Choi-Koh, 2003; Leng, 2011; Roschelle & Singleton, 2008).

7.4.3. Student answers in problem-solving questions

In this section students were required to formulate and solve a real-life situation in quadratic inequality (see Session 6 in Appendix D). Immediately after receiving adequate teaching and learning of problem-solving involving quadratic inequality students were given a related real-life mathematical question below:

> *The height of a ball above the ground after it is thrown upwards at 18 metres per second can be modelled by the function* $h(x) = 18x - 3x^2$*, where the height* $h(x)$ *is given in metres and the time* x *is in seconds. At what time in its flight is the ball within 15 metres of the ground?*

The researcher used the rubric for scoring the students' problem-solving abilities (QIPST) in Appendix E. Students were scored on how they understood the problem, devised a plan, carried out the plan and looked back after finding the solution. The results are shown in Figure 7.3, below.

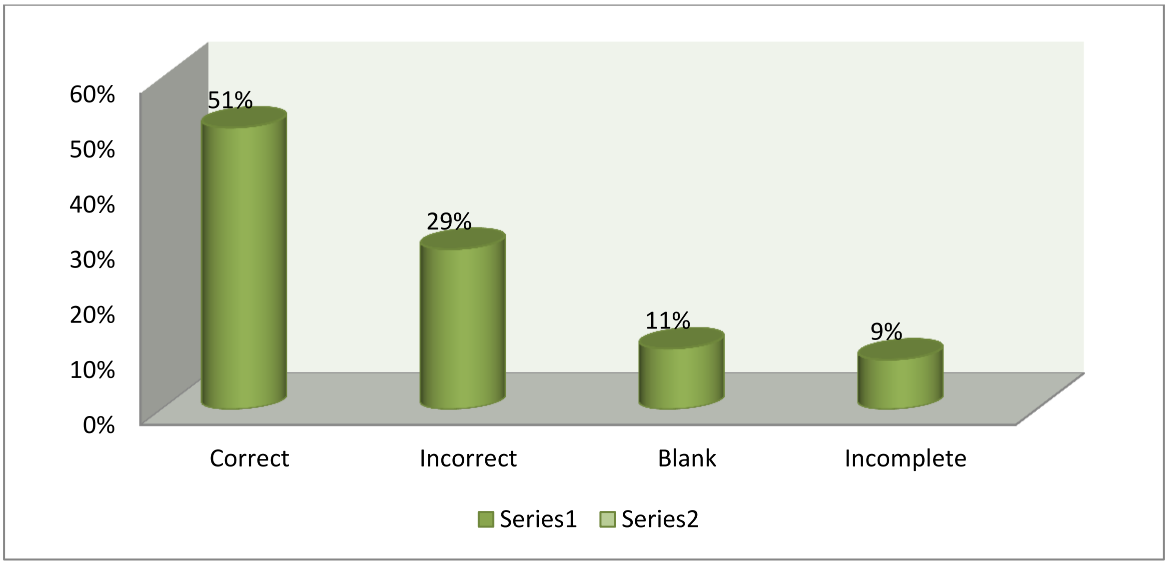


Figure 7. 3: Results of students' problem-solving abilities in quadratic inequalities

Fifty-one percent of the students correctly interpreted the problem, modelled the correct quadratic inequality, used the correct procedures that led to the solution and evaluated their solution correctly. A small proportion of students (9%) had incomplete solutions, which was classified in two categories by the researcher. The analysis revealed that (a) the students were able to identify the problem and set up the correct quadratic inequality but did not solve the problem completely and (b) the students understood the problem, modelled correctly the quadratic inequality and used correct procedures to solve it but was not completed.

On the other hand, a significant proportion of the students (29%) had incorrect solutions. The researcher made three analytical observations related to students' incorrect solutions about their problem-solving abilities. The researcher concluded that (a) some students did not fully comprehend the problem; (b) some students understood the problem but did not know how to formulate a quadratic inequality from the context; (c) other students understood the problem and formulated the quadratic inequality but had difficulty in solving and checking the accuracy of their solutions.

A substantial proportion (11%) of students left blank implying that no attempt was made at all to answer this problem. This is an indication that students had challenges in understanding the problem in its context. Forty percent of students who had incorrect or left blank did not comprehend the conceptual problem involving quadratic inequalities. This information might help in the planning for intervention tasks and for the future research.

7.5. Students' results of problem solving in quadratic inequalities

This section presents the students' results on problem solving strategies used when answering the post-test and the results of how the GC use supported their problem solving abilities when learning quadratic inequalities.

7.5.1 Analysis of the student's problem solving strategies in the post-test
This section intended to answer the second research question, "*In what ways (how) can the pedagogical use of the graphing calculator support the high school students' problem solving ability in quadratic inequalities?*", through discussing the processes used by the students in applying problem solving strategies in Questions 5 and 6 of the post-test.

This post-test question (Question 5) required students to determine the values of x for which $\sqrt{25 - x^2}$ will be non-real. The students applied quadratic inequalities to determine the values of x. The students were purposefully chosen because their written work represented the different problem solving approaches. A rubric for quadratic inequality problem solving test (QIPST) in Appendix E was used to score this question. A sample of answers of the three students is shown in Figures 7.4 and 7.5 below.

Using the rubric QIPST, student SC4 understood the problem and applied inequality to the nature of roots. He correctly identified the strategy of solving the problem i.e., quadratic inequality and used the null factor law to solve it. He did not try hard enough to reflect on his solution. For that reason, he scored a 2. The student used both algebraic and graphic approaches to solve problem, but his wrong graph led him to give incorrect solutions. His work reflects the influence of the use of GC in the classroom.

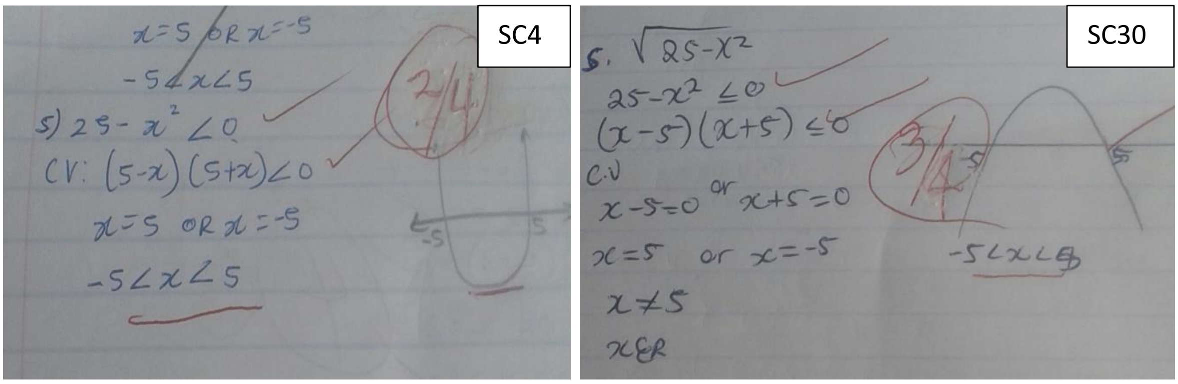


Figure 7. 4: Student's answer on application of quadratic inequalities

Using the rubric QIPST, student SC30 understood the problem and correctly identified the strategy (i.e., quadratic inequality) of solving the problem. She was able to apply the strategy to the nature of roots and identified the difference of squares as the correct procedure of determining the roots. The student used both algebraic and graphic approaches to solve the problem. However, she did not try hard enough to reflect on her solution. In that reason, she set up wrong inequality solutions and she scored a 3. The impact of the use of the GC in solving quadratic inequalities is noted here as the student used the graph as an object to solve mathematical problem and correctly visualised the effect of the negative parameter.

Using the rubric QIPST, student SC19 understood the problem and appropriately applied the strategy of solving the problem in Figure 7.5, below. This means an inequality was correctly constructed and appropriate solutions arrived at using the correct procedures. She tried hard to reflect on her solution by checking its reasonableness. In that regard, she scored a total mark i.e., 4.

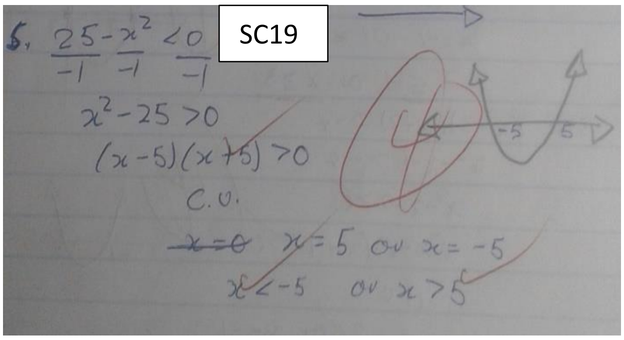


Figure 7. 5: Student’s answer in applications of quadratic inequalities

Student SC19 used both graphical and algebraic approaches to help solve the problem. The algebraic approach seemed to be the most useful in setting up the inequality and finding the critical values by using the difference of squares procedure. A graphical approach also helped her to solve the problem by correctly sketching the graph and indicating the zeros of the function. The student appropriately understood the effect of the negative parameter ‘a’ in the quadratic function. She used the graph to determine more than one interval that would be needed to solve this application problem and realised that one of these intervals was positive and one was negative. The second post-test question (Question 6) stated that: A small manufacturer’s weekly profit is given by $P(x) = -2x^2 + 70x$, in which x is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R600. This problem solving question required students to apply the concept of quadratic inequalities when determining the number of sold items, x that could make the profit to be greater than or equal to R600. The purposefully selected students’ written work represented the different problem solving approaches used by them after the GC intervention. A rubric with Polya’s four-step processes of problem-solving (understanding the problem, devising a plan, carrying out the plan, and looking back) was used to assess the students’ abilities (SPSA) for this question in Appendix E. A sample of answers of the three students is shown in Figures 7.6 and 7.7 below.

Student PC7 partially understood the problem but completely made a wrong quadratic inequality. She implemented correct procedures to solve a wrong inequality but did not make an attempt to check for accuracy of her solutions. Using the rubric SPSA, she was scored 5 out of 12. The student used an algebraic approach throughout the entire process of problem solving, including finding roots, and setting up the inequality. Her procedures did not lead her to the correct roots.

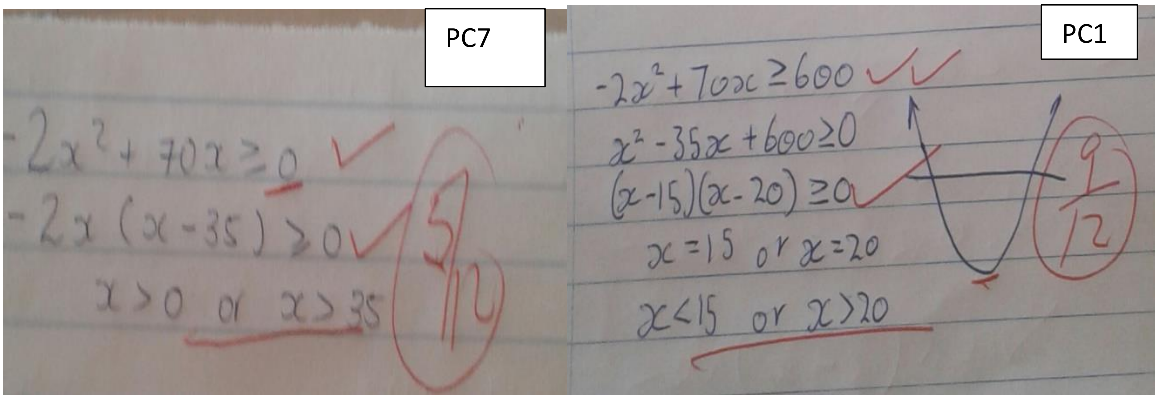


Figure 7. 6: Students' answers in contextual quadratic inequality problem

In Figure 7.6 above, student PC18 completely understood and interpreted the problem and correctly modelled the quadratic inequality. He substantially implemented the correct procedures of solving inequality but did not lead him to correct solution. He incompletely checked for the accuracy of his solution and decided to choose a positive interval. Using the rubric SPSA, she was scored 9 out of 12. The student used an algebraic approach throughout the entire process of problem solving, including setting up the inequality and finding roots. When the student divided the inequality by a negative he did not change the direction of the inequality sign. His mistake, however, was not in understanding of the problem but in an inaccurate application of the quadratic inequalities.

Figure 7.7 below, student PC26 completely understood and interpreted the problem and correctly modelled the quadratic inequality. She completely implemented the correct procedures of solving inequality and led her to correct solution. Subsequently she made an attempt to check for accuracy of her solution in a convincing and appropriate manner using the sketched graph which indicated the region of the solution set.

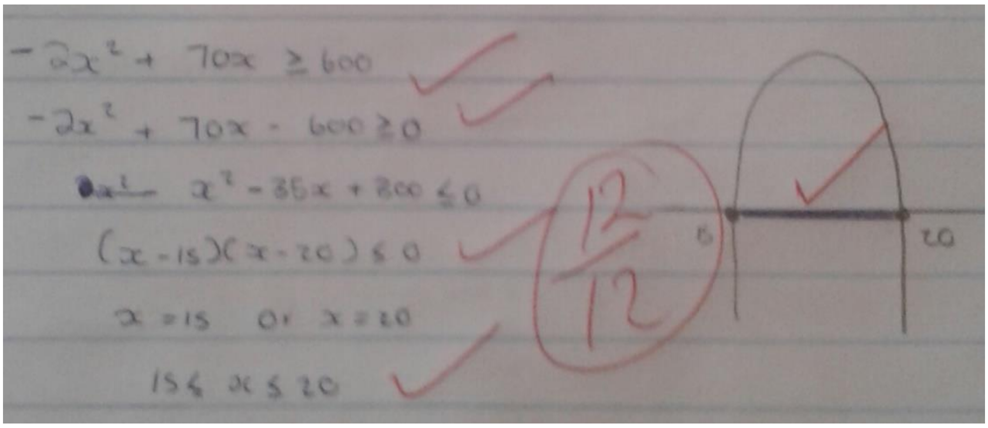


Figure 7. 7: Student’s answer in contextual quadratic inequality problem

Using the rubric SPSA, she was scored 12 out of 12. This means student SA26 used effectively the four-step problem-solving processes to solve question 6 of the post-test. The student used both graphical and algebraic approaches to help solve the problem. The algebraic approach seemed to be the most useful in setting up the inequality and finding the critical values by using the factoring procedure. A graphical approach also helped her to solve the problem by correctly sketching the graph and indicating the zeros of the function. The student appropriately understood the effect of the negative parameter ‘a’ in the quadratic function. She used the graph to determine the interval that would be needed to solve this problem.

The students’ answers indicate the pedagogical affordances of the graphing calculator in improving the learning of quadratic inequalities. The results reflect that the use of the GC contributed effectively towards improving problem-solving strategies and reasoning skills of students. This is consistent with the findings of Roschelle and Singleton (2008), who noted that the GC contributed towards displaying multiple representations, engaging with interactive real world problems, checking their work, and justifying their solutions and providing a supportive context for productive mathematical thinking.

7.5.2 Student’s perceptions of how the GC use supported the quadratic inequality problem solving abilities

This section intended to answer the second research sub-question on how the students perceived about the use of the GC towards supporting their problem solving abilities of quadratic inequalities. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 7.4 below.

Table 7. 4: Student's perceptions on how GC supported problem solving

Student's perceptions on how GC supported problem solving	**SD %**	**D %**	**NS %**	**A %**	**SA %**
The GC enabled me to understand the quadratic inequality problem (applications)	6	9	14	49	23
The GC guided me to sketch the graphs for solving quadratic inequalities	6	3	11	46	34
The GC helped me to use correct methods and procedures to solve quadratic inequalities	9	6	11	54	20
The GC allowed me to check for mistakes and correctness of my quadratic inequality solutions	6	11	11	51	14
Key: SD-Strongly Disagree; D-Disagree; NS-Not Sure; A-Agree; SA-Strongly Agree					

In Table 7.4, seventy-two percent of the students agreed or strongly agreed that the GC use enabled them to understand the contextual problems of quadratic inequalities. However, there were 5 of the 35 students (14%) who were not sure. Twenty-eight students (80%) affirmed that the GC use guided them to sketch the graphs (i.e., parabola and line graph) for solving quadratic inequalities. Seventy four percent of the students agreed or strongly agreed that the use of the GC helped them to use the best alternative methods and procedures in solving quadratic inequalities. However, 26% of them perceived differently either denied or were not certain. Sixty six percent of the students felt that the GC use assisted them to check for errors, mistakes and correctness of their solutions. Of those students who were not sure (17%) in their decisions, the researcher suggested that they needed an expanded opportunity with the use of the GC on how to solve quadratic inequalities. With the overwhelmingly impressive results in Table 7.4 above, the researcher partially concluded that the pedagogical use of the GC supported them in problem solving of quadratic inequalities as all the Polya's main steps of problem-solving processes had high percentages.

Students' responses show that graphing calculator played undoubtedly significant roles in students' learning of quadratic inequalities. The GC was used as a tool by students for problem solving processes in which they graphed functions in order to familiarize themselves with the problem. In the planning phase, students developed a strategy for determining the solution and were used like a compass to point in the right direction for solving the problem. The use of graphing calculators assisted students in their problem solving by helping them develop and confirm their solution strategies. In the monitoring phase the GC was used as a resource for verifying solutions and then students were able to develop a symbolic approach. This is consistent with the ideas of the earlier researchers (e.g., Averbeck, 2000). This means the GC use assisted students to move well through the three stages for conceptual understanding namely, intuitive, operative, and applicative (Choi-Koh, 2003). This means students used the graphing calculator to see the connections between a solution and meaning of solution in terms of the graph (White-Clark et al., 2008; Amineh & Asl, 2015). This is consistent with the Vygotsky's theory of socio-cultural learning where the GC was used as mediator to develop cognitive understanding.

7.6 Students' results from the focus group interviews

This section described the qualitative results of students' focus group interviews on both problem-solving (Section 7.6.1) and reasoning (Section 7.6.2). A question was selected from the post-test for critically exploring the students' problem-solving abilities and students' reasoning skills in quadratic inequalities. Students were expected to use the graphing calculator only to verify the reasonableness of their solutions. According to Lunenberg (1998), learning in a constructivist manner involves asking students to analyse a problem, interpret results, classify terms or concepts, and to make predictions. These cognitive activities are strongly connected to the processes of students' understanding.

A sample of three students (CF1, CF2 and CF3) was selected from School C for focus group interviews. The interviews took place in their math classroom on a regular school day after school hours. Three students who obtained marks below average, average and above average in the post-test were purposefully selected to participate in the focus group interviews. The participants were asked to solve a contextual quadratic inequality problem that was in the post-test and were also asked several questions relating to the problem solving processes involved in that problem. The students were asked to explain their thoughts throughout the interviews in order to understand their thinking processes. Throughout the interview, participants were observed how they used algebraic approaches and, sketches and graphs in their thinking processes through the problem. At the end of the interviews they submitted their interview scripts to further analyse how they reasoned their way throughout this problem. Each interview lasted for approximately thirty minutes. The interviews were audio recorded and then transcribed.

7.6.1 Students' results of the focus group interviews on problem solving

This section attempted to address the second research question of the study:

> *In what ways can the pedagogical uses of the GC enhance students' problem solving abilities when solving quadratic inequalities?*

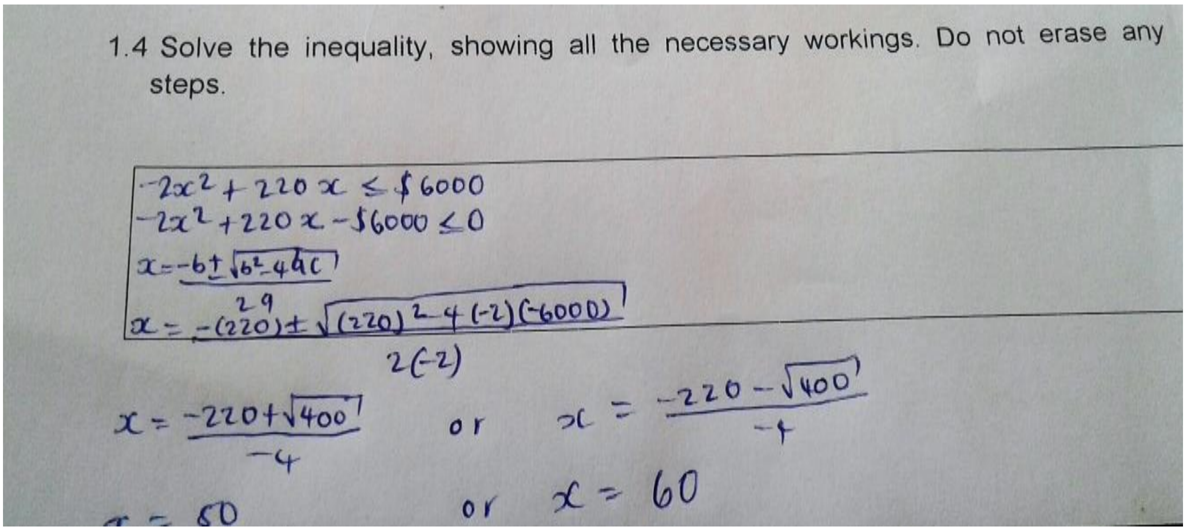
The qualitative results of the focus group interviews with students on their problem solving abilities of the contextual quadratic inequalities were analysed and described in this section. Within that context, the selected students were presented with a contextual problem that they first saw in the post-test. The problem stated, "*A small manufacturer's weekly profit is given by* $P(x) = -2x^2 + 220x$, *in which* x *is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R6000".* The students were given ten minutes preparation time to read, formulate and solve the problem. They were supposed to explain their thinking processes clearly and not to erase their working. In this case, students answered questions 1.6 and 1.7 (Appendix D) which were central to solving the contextual quadratic inequality problem.

The codes used in the interviews are: TR for the teacher-researcher and CF for the focus group students from School C. Student participants were interviewed as a group and their responses were presented and transcribed below:

TR: (The teacher-researcher hands out the problem to the students). *Please read this problem attentively and then formulate the required mathematical statement.*
Students CF1: $-2x^2 + 220x \leq 6000$
Students CF2: $6000 \leq -2x^2 + 220x$
Students CF3: $-2x^2 + 220x > 0$
TR: And then solve it without the use of the GC, showing all the necessary working. Please do not erase any step you have written.
Students (CF1, CF2 & CF3): After ten minutes the students handed in their scripts for marking.

Interpretations of the individual students' interview results

Students' attempts are shown below and were analysed in order to determine whether the use of graphing calculator had support their problem solving competencies at School C: Student CF1 was not able to interpret the problem correctly as she formulated the quadratic inequality with a wrong inequality sign (i.e., less than) from the contextual problem (see Figure 7.8). She used algebraic approach mostly quadratic formula throughout the entire process of problem solving; including finding the critical values. This student seemed to confuse the two concepts of equation and inequality, although she knew that the algebraic statement formed was a quadratic inequality. This shows a lack of true understanding by this student.


1.4 Solve the inequality, showing all the necessary workings. Do not erase any steps.

$-2x^2 + 220x \le \$6000$

$-2x^2 + 220x - \$6000 \le 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(220) \pm \sqrt{(220)^2 - 4(-2)(-6000)}}{2(-2)}$

$x = \frac{-220 + \sqrt{400}}{-4}$ or $x = \frac{-220 - \sqrt{400}}{-4}$

$x = 50$ or $x = 60$

Figure 7. 8: Student CF1's problem solving

Throughout the process, she did not question her modelled inequality and the final solution. It seemed the x-values had no meaning to the student as she did not try another approach. However, at the end of the problem when she was asked to verify her solution, she realized that the solution was not supposed to be $x = 50$ or$x = 60$. These values when substituted gave zero. She pointed out that, *"I think, I am definitely wrong because profit should not be zero".* Her misconception however, was in confusing the equations and inequalities.

Student CF2 was able to convert the contextual problem into the right quadratic inequality: $6000 \le -2x^2 + 220x$. He used both the algebraic and graphical approaches; however, the algebraic approach (quadratic formula) seemed to be the most useful in helping him to solve the quadratic inequality. The graph was correctly drawn with critical values and indicated the portion where the solution of the quadratic inequality was found. He then switched to an algebraic approach, by using quadratic formula to find the zeros of the function see Figure 7.9, below.

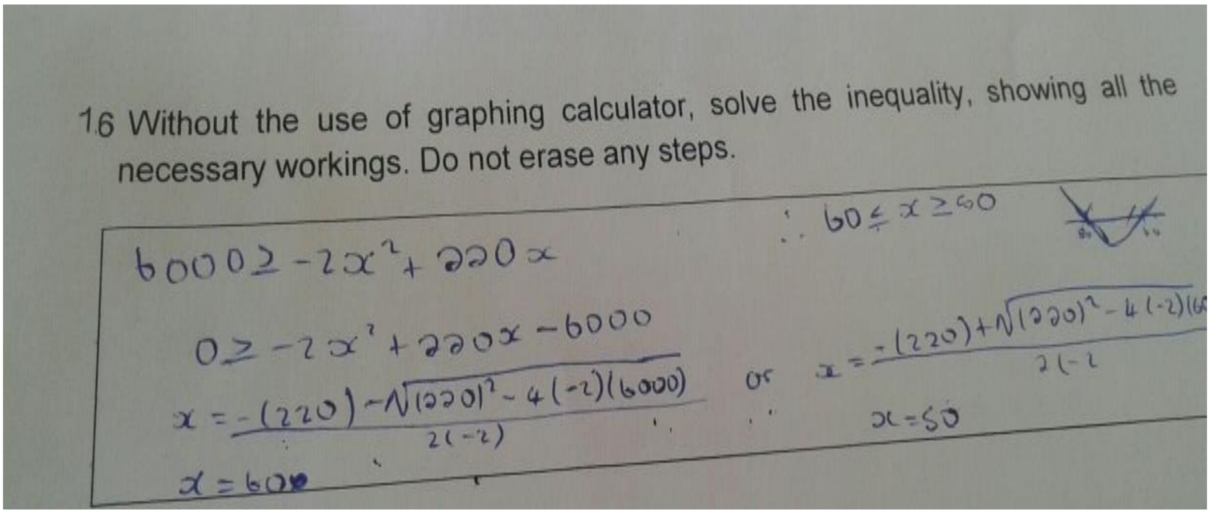
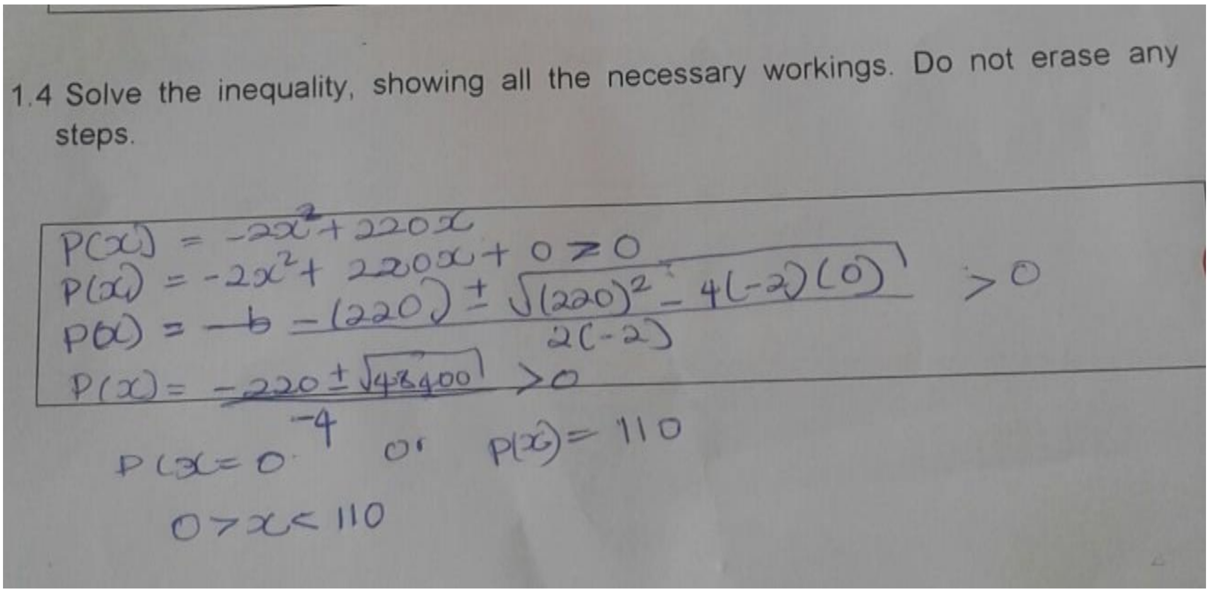


Figure 7. 9: Student CF2's problem solving

Throughout the process of problem-solving, he consistently monitored his progress and questioned any move he took. Notably, he moved freely between algebraic approach and graphical approach, thus switching back and forth from the two approaches. This helped him to reflect on the strategies used to solve the contextual problem with accuracy. He verified his solution using the values of x within the critical values in order to justify his solution. Consequently, student CBF2 wrote the correct interval notation for his solution (i.e.,$\mathbf{50} \leq \boldsymbol{x} \leq \mathbf{60}$). He was also able to verify and confirm his solution when the teacher-researcher asked him to use the graphing calculator.

Student CF3 did not interpret the contextual problem correctly and this led her to model a wrong inequality:$-\mathbf{2}\boldsymbol{x}^{\mathbf{2}} + \mathbf{22}\boldsymbol{x} \geq \mathbf{0}$ without 6000. The inequality sign was correctly written except that she had omitted 6000. This means her solution was different from others. She used the quadratic formula to determine the critical values. However, she never used a graphical approach to support her reasoning. The algebraic approach seemed to be the most useful in helping this student to solve the problem.



1.4 Solve the inequality, showing all the necessary workings. Do not erase any steps.

$P(x) = -2x^2 + 220x$
$P(x) = -2x^2 + 220x + 0 \geq 0$
$P(x) = \frac{-b - (220) \pm \sqrt{(220)^2 - 4(-2)(0)}}{2(-2)} > 0$
$P(x) = \frac{-220 \pm \sqrt{48400}}{-4} > 0$
$P(x = 0$ or $P(x) = 110$
$0 > x < 110$

Figure 7. 10: Student CF3's problem solving

It was observed that she did not question her solutions thus lack of monitoring one's progress. This affected her not to realise that she did not include 6000 in her formulated quadratic inequalities. This means the student did not try other approaches to validate her solutions. She had different critical values from other learners and thus when she realised she did not use 6000. The researcher asked her:

TR: *Why didn't you use 6000 since in the contextual problem was given?*
Student CF3: I did not know what to do with it.
TR: $P(x)$ *is given as a profit function and must be greater than 6000. In your inequality replace the zero with 6000 and then solve for* x.

The student was not able to solve the quadratic inequality correctly. She had a problem of expressing interval notation. In her attempt she used quadratic formula and wrote her solution as $\mathbf{50} > x < \mathbf{60}$. This solution was meaningless. The researcher asked about the appropriateness of her solution:

TR: *Do you think the solution you have given is appropriate?*
Student CF3: Yes because it is inside the graph that is increasing and decreasing.
TR: *Ok.* Use the graphing calculator to check the accuracy of your solution.

The student was able to visualise the shape of a graph but failed to write the solution set. She had problems of adjusting the GC so that the axes are increased to accommodate large values of x and y. After getting assistance from her classmates, she discovered that her solution did not make any sense.

Comments on the students' problem solving abilities

The three students had different levels of proficiency in relation to problem solving processes. For that reason, the students' problem solving processes were scored in terms of a) formulating an inequality, b) using algebraic approach, c) using graphical approach, d) using the graphing calculator for verifying solution and e) obtaining a correct solution set. The scoring ranged from 0 to 5. A 5 represented the highest score of executing all the listed steps correctly. The use of an algebraic approach is mandatory in the CAPS curriculum and should be complemented with the use of the graphical approach or any other relevant approach (DBE, 2015, 2016, 2017). In this regard, Student CF1, an average learner scored a 3 as she did not use a graph and wrote no solution sets. Student CF2, an above-average learner scored a 5 as he did well all the processes of solving the problem. Student CF3, a below-average learner scored a 2 for not executing all the steps correctly. This means that Student CF3 was not able to formulate an inequality, use the graphing calculator for verifying solution and express correct solution set. The use of the graphical approach was implied in solution because she had a visual image of quadratic functions. In respect of these results, the teacher-researcher partially concluded that the pedagogical use of GC supported the students' problem solving abilities of the quadratic inequalities.

7.6.2 Students' results of the focus group interviews on reasoning skills

This section attempted to address the third research question:

> *How does the pedagogical use of the GC support students' reasoning abilities when solving quadratic inequalities?*

The qualitative results of the focus group interviews with students on their reasoning abilities in the contextual quadratic inequality problems were analysed and described in this section. In order to successfully examine the students' reasoning in solving the contextual quadratic inequality problem, student participants of School C were asked questions during the focus group interviews relating to analysing a problem (Questions 1.1 & 1.3), initiating a strategy (Questions 1.4 & 1.5) and reflecting on one's solution (Questions 1.7 and 1.9) in Appendix D.

7.6.2.1 Students' results from focus group interviews on analytical reasoning

The first reasoning question (Q1.3) of the focus group interviews required the students to identify the main concept involved in the contextual problem.

TR: State and explain the main concept of this problem (1.3)
Student CF1: It is a quadratic inequality because sold items representing x-values must give profit more than 6000.
Student CF2: The profit should be greater than or equal to, that is a quadratic inequality.
Student CF3: A quadratic inequality because it's about greater than 6000.

Interpretation of the students' responses

All the three students were able to identify the main concept involved as quadratic inequality and gave satisfactory justifications. The key phrase "*greater than*" guided the students in identifying the underlying concept and was used as supporting evidence. However, student CF1 gave a detailed explanation that, "*the sold items representing x-values must give profit more than 6000",* thus a good ability of analytical reasoning.

The next reasoning question (Q1.3) requested the students to draw conclusions from their solutions of the contextual problem. In that regard, the students drew the following conclusions about their solutions.

TR: Is there any relevant conclusion that you can make about the solution to the problem? If so, what can you say?
Student CF1: Yes. I think the solution of the inequality must be the x-value that when substituted in the inequality the answer is more than 6000.
Student CF2: Yes. The number of sold items must be more than zero for the profit to be more than 6000 and solutions should be between the critical values.
Student CF3: I think the values of x must lie inside the critical values of the graph that is increasing and decreasing.

Interpretation of the students' responses

All the three students were able to state at least one reasonable conclusion. Student CF1's conclusion that- "*the solution must be the x-value*", affected the way she determined the solution of the quadratic inequality (see Figure 7.68). Student CF2 made two analytical conclusions that "*the number of sold items must be more than zero and the solution was between the critical values".* This means student CF2 was able to visualise the quadratic inequality solution in the graph using the effects of parameter value (i.e., the coefficient of x^2). Student CF3 provided a valid conclusion but gave meaningless solution from a wrong quadratic inequality (see Figure 7.10). This suggests that this student lacked self-confidence in contextual problems. It was noted that student CF2 possessed a strong analytical reasoning which is a pre-requisite for understanding inequalities. On the other hand, students CF1 and CF3 had moderate analysis and this even affected them to write the solution of the quadratic inequality correctly.

7.6.2.2 Students' results from focus group interview on initiative reasoning

In this area of reasoning- initiating a strategy, students were assessed on how they purposefully selected the appropriate concepts, representations and procedures when solving the contextual problem. The first initiative reasoning question (Q1.5) of the focus group interview required the students to identify the approaches which were most helpful to solve the contextual problem. The following is how the students responded:

TR: *Which approaches do you think were most helpful in solving this problem- algebraic and/or graphic?*

Student CF1: Algebraic and graphic approaches helped me to solve the problem.

Student CF2: To me quadratic formula and graphs were helpful to solve the problem.

Student CF3: factorisation, quadratic formula and graphic approaches were the most helpful to solve the problem.

TR: Please state and explain those *approaches that you used to solve this problem.*

Student CF1: I used quadratic formula to solve the problem. It is easiest and fastest method to use when solving for x. I use a calculator to calculate the x values.

Student CF2: Quadratic formula and graph were helpful in solving this problem. I used the quadratic formula to find critical values and then used the graph to show the position where the solution is found.

Student CF3: Quadratic formula because it is easy to use.

Interpretation of the students' responses

All the students were able to state and explain the approaches that they executed to solve the contextual quadratic inequality problem. All the students used quadratic formula to determine the critical values. Student CF1 consistently used algebraic approach- quadratic formula throughout the process of solving the problem. Student CF2 also responded correctly that he used two approaches- algebraic and graphic to solve the problem. However, the graphical approach seemed to be the most useful in helping him to solve the problem. The graph drawn by the student was correct in shape. The algebraic approach- quadratic formula was only used to determine the zeros of the function which were later on indicated on the graph for the decision. The consistent use of two strategies earned her good results (cf: Figure 7.8).

Student CF3 correctly responded that she used the algebraic approach-quadratic formula to solve the problem. She used visualisation to decide the region of the solution developed by the use of the GC. Although she had a strong visual image of the solution, she did not write the interval notation correctly. All the three students initiated their strategies differently for solving the contextual problem. This suggests that they possessed different levels of the initiative reasoning skills of solving quadratic inequalities.

The second initiative reasoning question (Q 1.6) of the focus group interview required students to explain how the use of the graphic approach helped them solve the quadratic inequality. The responses of the students were:

TR: *Explain how helpful the use of graphical approach was in solving the contextual problem.*

Student CF1: *No I did not use it but it is helpful in a way that I use it to decide if the solution is within or outside the critical values.*

Student CF2: *The graphical approach helped me to interpret the inequality in the form of a graph and helped me to see the region of the inequality.*

Student CF3: *The graph can help me to figure out the critical values and the position of the solution*.

Interpretation of the students' responses

The responses given by the students were similar and demonstrated that they understood perfectly the role of the graphic approaches when solving quadratic inequalities. Students responded that the graphical approach helped them to *"interpret the inequality"* and *"figure out solution"* and *"to decide for the region".* This is even noted in the work of student CF2 (cf: Figure 7.9) showing how the graphic approach was used to solve the inequality. It is evident that the use of the GC provided the student with visual representation of quadratic inequality solutions in the form of graphs and how it has influenced them to use graphs too. The responses of the students showed that the use of the GC helped to develop students' initiative reasoning strategy. Within the context, student CF2 had a strong initiative reasoning strategy for solving quadratic inequalities.

7.6.2.3 Students' results from focus group interview on reflective reasoning

In this area of reasoning- reflecting on one's solution, students were assessed on how they interpreted their solutions (Q1.8), justified the accuracy of their solutions (Q1.9), and how they considered the alternative way of solving problems (Q1.10). The questions intended to make students reflect on their solutions of the contextual quadratic inequality problem. The following reasoning questions (Q1.8 & Q1.9) were asked to find out how the students interpreted and justified their solutions of the contextual problem. The following is how the students responded:

TR: With the values of x that you have obtained, do you think you have solved this problem completely and correctly? Justify your reasoning.
Student CF1: Yes. I believe my answer is complete and reasonable.
Student CF2: Yes I think there are correct.
Student CF3: (Smiling …), I think so.
TR: Please justify why your answer is reasonable?
Student CF1: Because when I substitute the values of x I got zeros. But I am expecting the answers more than 6000.
Student CF2: Using the graph the values of x are between the critical values and give me profits greater than 6000.
Student CF3: (Smiling …), it is reasonable.

Interpretation of the students' responses

Not all the three students were able to decide whether their solutions were reasonable and justified their choices. Student CF1 realised that her solution was unreasonable and the answers were zeros. She indicated that "*I am expecting the answers more than 6000*." Student CF3 completely failed to justify reasonableness of her solutions. This was different with student CF2 as his solution was complete and reasonable. He was able to provide plausible reasons supported by the use of the graph. His values of x taken from within the critical values provided the profits which were more than 6000. In this sense, student CF2 was able to reflect on his own solution effectively and this suggests that he displayed strong reflective or metacognitive reasoning skills in respect of this item. This may mean that the use of the GC has not provided adequate opportunities for students to develop their metacognitive reasoning skills of solving the contextual quadratic inequality problems.

The next reasoning questions (Q1.10 and Q1.11) were combined in analysing students' responses since they focused on the use of the GC. Question 1.10 asked about any other relevant information that could be used to justify their solutions. Students' responses were as follows:

TR: Is there any other relevant information that can be used to justify their solutions?
Student CF1: There could be.
Student CF2: Yes using a graphing calculator.
Student CF3: No.
TR: When using the graphing calculator, do you still get the same solution? Explain.
Student CF1: No. I did not write the solution of the quadratic inequality. I only wrote critical values.
Student CF2: Yes the solution is within the critical values.
Student CF3: (Smiling …), I don't know how to use the GC.

Interpretation of the students' responses

Students CF1 and CF3 did not realise that the GC could be used as an alternative approach to verify their solutions. In that regard, the teacher-researcher requested the students to take out their GCs to verify the accuracy of their solutions. After punching the inequality into their graphing calculators students were able to see if their solutions were accurate or not. Only the solution of student CF2 was accurate and reasonable. Students CF1 found that she did not complete finding the solution, thus her solution was incorrect. Student CF1 responded that, *"I only wrote critical values".* Interestingly, all the students were able to solve the inequality and interpret the results accurately from the GC. This is consistent with idea of the instrumented action scheme in the TIG. It is evident that the use of the GC did not develop the students' reflective reasoning completely. The future research should focus on the development of students' meta-cognitive reasoning skills.

7.6.2.4 Students' results of the observed monitoring progress

This section described the students' reasoning skills of monitoring progress as they were observed solving the contextual quadratic inequality problem. The main focus was to assess how the students reviewed and/or modified their selected strategies in particular when they encountered difficulties (see Rubric in Appendix E). The observed results of the three purposefully selected students were as follows:

Student CF1 converted the problem into right quadratic inequality and correct procedures (algebraic) were followed through. She used mainly algebraic (symbolic) representations and did not make any reference to the graph as another approach to solve the problem. The use of one approach misled her and left solution at critical values. This means she did not make adequate attempt to monitor her progress; thus why she did not realise that she had not completely answered the problem. Her low level of self-monitoring affected her to attempt other avenues or make any reasonable assumptions that could lead her to review her selected strategy. When the teacher-researcher asked her to use the GC to solve the problem she was able to visualise the correct solutions displayed on the GC.

Student CF2 used two different approaches- algebraic and graphic, to solve the quadratic inequality. A look at the graph of this quadratic function assisted in the monitoring of his progress in the reasoning process. However, he did not rely on one reasoning procedure as he kept on switching from the algebraic to graphical approaches and vice versa. The graph seemed to be very beneficial in the reasoning processes used by this student. Throughout his problem solving processes, he continued monitoring his progress and verifying his assumptions (i.e., the solutions must be within the critical values to produce greater profits). The teacher-researcher observed that every move that he could take, he questioned it. He was also able to use the GC to confirm the shape of the graph, find the zeros of the function, make assumptions and verify his solutions found using the pencil and paper methods. For this reason, student CF2 has a high level of reasoning skills on monitoring his progress.

Student CF3 converted the problem into a wrong quadratic inequality but correct algebraic procedures were followed through. She used mainly algebraic (symbolic) representations and never used a graph. She seemed to get confused when she realised that she needed to write down the solution of the inequality. However, she had developed strong visual images of the graphic representations that she was able to use in finding the solution set of quadratic inequalities. Using the ruling of consistent accuracy, she was able to solve her wrong inequality but wrote meaningless solution set. She could not take the advantage of her strong visualisation to monitor her progress. When the teacher-researcher asked her to verify her solutions using the graphing calculator, she was not able adjust the axes of the GC to accommodate large values. Based on her actions on this problem, she has weak reasoning skills on monitoring her progress.

7.6.2.5. Students' results of the observed seeking and using connections

In this domain of reasoning- seeking and using connections, students were observed on how they sought and used connections of different concepts, contexts and representations when solving the contextual quadratic inequalities. This is also about when the students make references to mathematical concepts used earlier in the topic of quadratic inequalities, in other mathematics areas, or in any other subject areas. In this context, the findings were as follows:

Student CF1 was only able to make connections between the solution of quadratic equation and the quadratic inequality (see Figure 6.7). However, she did not use this relationship to determine the solution of the inequality. For this reason, she has weak reasoning skills on seeking connections. Students CF2 and CF3 were able to seek and use connections between concepts and representations when solving the quadratic inequality. Students CF2, in his attempts to solve the contextual quadratic inequality problem, made links between the algebraic (symbolic) and graphic representations (cf: Figure 7.7). In this case, student CF2 used quadratic graphing (geometry) and solving quadratic equations (algebra) both as viable ways to find a quadratic inequality solution. The student realised that the solution of the equation (x-values) was the x-intercepts of the graph which determine the solution of quadratic inequality using them as critical values. This suggests that the algebraic reasoning (i.e., algebraic symbols and functions) has helped student CF2 to use and seek the connections effectively in solving quadratic inequalities. Student CF3 did link correctly the x-values of the quadratic equations to the critical values of the quadratic inequalities and then used to express the solution of inequality in interval notations. However, she had difficulties to write correctly the solution set of her wrong quadratic inequality. In that regard, student CF3 had moderate reasoning skills of seeking and using connections between concepts and representations when solving the quadratic inequality.

7.6.3 Student's perceptions of how the GC enhanced reasoning skills

This section intended to answer the third research sub-question on how the students perceived the use of the GC towards enhancing their reasoning skills in learning quadratic inequalities. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 7.5 below.

Table 7. 5: Student's perceptions on how the GC enhanced reasoning

Student's perceptions on how the GC enhanced reasoning	**SD %**	**D %**	**NS %**	**A %**	**SA %**
The graphing calculator helped me to analyse completely the quadratic inequality problems	6	11	14	46	23
The graphing calculator enabled me to use many approaches when solving quadratic inequalities	9	6	17	54	14
The graphing calculator assisted me check my progress when solving quadratic inequalities	3	6	17	57	17
The graphing calculator helped me to link/mix concepts when solving quadratic inequalities	9	6	20	49	17
The graphing calculator allowed me to think more about my quadratic inequality solutions	3	9	11	51	26
Key: **SD-Strongly Disagree; D-Disagree; NS-Not Sure; A-Agree; SA-Strongly Agree**					

In Table 7.5, 69% of the students agreed or strongly agreed that the use of the GC helped them to analyse completely the quadratic inequality problems. Only 31% of the students, including those who were not sure, denied that the GC enabled them to use new strategies when solving inequalities. This means a large proportion of students (69%) had perceived positively. Twenty six of the students (74%) agreed or strongly agreed that the GC assisted them monitor or check their progress when solving inequalities. A significant number of the students (66%) affirmed that the graphing calculator helped them to link other mathematical concepts to solve inequalities. Only four students (11.4%) denied that the GC helped them to think or reason more about their inequality solutions. This implies that the majority of students (77%) felt that the GC provided them with opportunities to reflect on (i.e., validate) their solutions. Based on these results, the researcher partially concluded that the majority of the students perceived that the pedagogical use of the GC supported their reasoning skills when solving quadratic inequalities.

7.7 Results of students' responses in the pre-and post-surveys

This section presents the results of the students' responses in the pre- and post-surveys of how they perceived about the GC use in learning quadratic inequalities. Their perceptions are presented in the following subsections.

7.7.1 Comparative results of students' responses in the pre-and post-surveys

This section intended to answer the fourth sub-question by comparing the results of the pre- and post- surveys means on how the students perceived about the GC use in learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

The post-survey intended to gather the perceptions of the students on whether the use of GC in learning quadratic inequalities assisted them to understand the topic. The positive changes of students' perceptions were attributable to the effective intervention of the GC use as an artefact in learning quadratic inequalities. The eleven items of the pre-survey were similar to the ones on the post-intervention survey to see if students changed their perceptions on how they learned quadratic inequalities after GC intervention as an instructional tool. Students' responses were biased towards the understanding of and lessening the difficulties of learning quadratic inequalities. In this context, an increased confidence in their ability to understand and learn quadratic inequalities is measured by students' option of "disagree" or "strongly disagree" and increased mean. A comparison of the students' perceptions is given in Table 7.6 below, where M_0=post-survey mean and M_R=pre-survey mean.

Table 7. 6: Results of students' pre- and post- intervention surveys (n=35)

ITEM	***1 = Strongly Agree, 2 = Agree, 3=Not Sure, 4 = Disagree, and 5 = Strongly Disagree***	**Pre-survey**		**Post-survey**	
		M_R	**SD**	M_O	**SD**
SPQI 1	Quadratic inequalities are difficult to learn and understand	2.71	1.21	4.20	1.31
SPQI 2	I do not see the difference between the equation and inequality	2.49	1.30	4.67	1.24
SPQI 3	It's difficult to determine the solution sets of quadratic inequalities after finding the critical values.	2.21	1.15	3.60	1.11
SPQI 4	I have difficulties with determining factors of quadratic expressions (inequalities)	2.49	1.16	3.71	1.29
SPQI 5	I don't know the difference between critical values and x-intercepts of the graphs	2.43	1.12	3.57	1.11
SPQI 6	In order to understand the quadratic inequality topic I usually memorise it	2.57	1.17	3.77	1.15
SPQI 7	Of all the topics I have done so far I don't enjoy learning quadratic inequalities	2.66	1.11	3.37	1.09
SPQI 8	It's difficult to use graphical sketches to determine the solutions of quadratic inequalities	2.46	1.12	3.68	1.15
SPQI 9	Given an opportunity of not to learn quadratic inequalities I was going to do so	2.43	1.11	3.84	1.20
SPQI 10	Technology (e.g., computers) cannot help me to understand quadratic inequalities	2.43	1.20	3.71	1.09

Before the GC intervention, students indicated that given an option of not learning quadratic inequalities they were going to do so (M_R=2.43) because they were neither enjoying (M_R=2.66) nor understanding (M_R=2.71) the topic (see Table 7.6). However, the students' responses show that the use of the graphing calculator improved their understanding (M_0=4.20) and enjoyment (M_0=3.37) of learning quadratic inequalities. This is reflected in the post-survey means which are respectively greater than the pre-test survey means. The results further reveal that initially the students did not see the difference between the equation and inequality (M_R=2.49), and the difference between critical values and x-intercepts of the graphs (M_R=2.43); but after the GC use they were able to see the difference. This is reflected by the increased post-survey means of M_O=4.67 and M_O=3.57 respectively. The post-survey means exhibited an increased use of the graphs ($M_O > M_R$=3.68 >2.46) and reduced level of memorising procedures ($M_O > M_R$=3.77 >2.57) when learning quadratic inequalities in a GC mediated classroom. The overall results show that the GC use brought good behaviour in students' learning of quadratic inequalities. They further indicated that determining the solution sets of quadratic inequalities was no longer difficult. This is reflected by the pre- survey mean (M_R =2.21) which is less than the post-survey mean (M_O =3.60).

7.7.2 Students' perceptions on how the GC supported the learning sessions

This section intended to answer the fourth research sub-question by analysing the results of students' responses on how they perceived about the GC use in the designed sessions of learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

In that context students were issued with an eight item post-intervention survey to answer. Student perceptions were measured using a Likert scale in which students marked 1 if they strongly disagreed, 2 if they disagreed, 3 if they were not sure, 4 if they agreed, and 5 if they strongly agreed. Students' responses were captured in Table 7.7 below.

Table 7. 7: Student's perceptions on how GC use supported their sessions

ITEM	Students' perceptions of the effects of graphing calculator on the designed sessions of learning quadratic inequalities	SA (%)	A (%)	N (%)	D (%)	SD (%)
SPGC 1	The use of graphing calculator in learning sessions assisted me to solve symbolic (algebraic) quadratic inequalities	23	43	14	3	14
SPGC 2	The use of graphing calculator in learning sessions assisted me to understand the difference between critical values and zeros of the graph	17	54	9	14	6
SPGC 3	The use of graphing calculator in learning sessions assisted me to identify correctly the region of the inequality solution	20	51	17	9	3
SPGC 4	The use of graphing calculator in learning sessions assisted me to understand contextual (application) problems of quadratic inequalities	45	40	14	6	9
SPGC 5	The use of graphing calculator in learning sessions assisted me to use graphical sketches when solving quadratic inequalities	34	46	11	6	3
SPGC 6	The use of graphing calculator in learning sessions assisted me to understand the effect of the parameter 'a' in the quadratic inequality	23	40	17	9	11
SPGC 7	The use of graphing calculator in learning sessions assisted me to note that the effect of the parameter 'a' of quadratic function had the same effect on quadratic inequality	20	43)	20	11	9
SPGC 8	The use of graphing calculator in learning sessions assisted me to learn and understand much better quadratic inequalities	37	49	6	3	6
1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree						

Table 7.7 shows that 66% of the students strongly agreed or agreed that the use of graphing calculator in learning sessions assisted them to solve symbolic (algebraic) quadratic inequalities. Seventy-one percent of the students perceived that the use of graphing calculator in learning sessions assisted them to see the difference between critical values and zeros of the graphs. Students affirmed (strongly agreed or agreed) that the use of graphing calculator in learning sessions assisted them to use graphs when solving quadratic inequalities (80%) and to identify correctly the region of the inequality solution (71%). Fifty-seven percent of students strongly agreed or agreed that the use of graphing calculator in learning sessions assisted them to understand contextual (application) problems of quadratic inequalities. However 14% of them were not sure whether the use of the GC helped to understand contextual problems. Eighty percent of students strongly agreed or agreed that the use of graphing calculator in learning sessions assisted them to use graphical sketches when solving quadratic inequalities. Only 20% percent of the students strongly disagreed or disagreed that the use of graphing calculator in learning sessions assisted them to understand the effect of the parameter 'a' in the quadratic inequality. However, 17% of the students were neutral about their understanding of the effect of the parameter 'a' in the quadratic inequality. Only 3 students (9%) strongly disagreed or disagreed

that the use of graphing calculator in the sessions assisted them to learn and understand much better quadratic inequalities. This means that the students (86%) overwhelmingly perceived that the use of the GC assisted them to learn and understand the topic of quadratic inequalities better.

Based on the results in Table 7.7 above, the researcher partially concluded that students perceived that the pedagogical use of the GC strongly supported their learning sessions of quadratic inequalities. Within this regard, students felt that the use of the GC in the anticipated sessions helped them to understand the quadratic inequalities as they were able to identify quadratic inequality with the shapes of the quadratic graphs, to see the effect of the parameter “a” on the different graphs, to use the graphs to solve quadratic inequalities, to transform contextual problems into symbolic quadratic inequalities and to determine the region of the solution using correct interval notations informed by the calculated critical values. These are considered as the main concepts that can lead the students to solve quadratic inequality confidently. They also perceived that the effect of the different parameters were the same for all the quadratic functions and inequalities expressed in the form of $\boldsymbol{a(x+p)^2+q \geq\leq 0 \; or \; ax^2+bx+c \leq\geq 0}$.

7.8 Results from the in-depth interviews about the GC use in quadratic inequalities

This section of in-depth interview with students mainly attempted to address the fourth research sub-questions about the students’ perceptions on the use of the GC in learning quadratic inequalities.

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

An in-depth interview was conducted with the three students who were purposefully sampled from those who had obtained marks below average, average and above average from the post–test. They were individually interviewed after school on a regular school day in their classroom. Each interviewee lasted for approximately thirty minutes. The in-depth interview was recorded and then transcribed. The interview consisted of ten questions which were mainly about the use of the GC on students’ understanding of quadratic inequalities.

The codes used in the interview are TR for the teacher-researcher and BD for School B student in the in-depth interview (D). They were interviewed separately, but for convenience their responses are given together below. The responses of the in-depth interview questions are discussed in the following sub-sections:

7.8.1. Students' responses on how the use of GC made easier students' learning of quadratic inequalities

The question asked by the teacher-researcher below sought to find out the students' opinions about whether the use of the GC made their learning of quadratic inequalities easier. Students' responses were almost similar as shown below:

TR: Does the use of GC make your learning of quadratic inequalities easier to understand? Please explain your answer.

Interview with Student CD1

Student CD1: Yes it does because I can see how to sketch the graph and the position of the critical values.
TR: Then how does it make your learning easier?
Student CD1: I don't have to calculate the critical values as they are displayed on the GC screen and it shows where the solution lies.

Interview with Student CD2

Student CD2: Yes, because the GC shows the critical values and how to determine the graph.
TR: What do you mean, "the GC shows the critical values and the graph?"
Student CD2: The critical values help to determine the solution of the quadratic inequality and the graph shows where solution lies.

Interview with Student CD3

Student CD3: Yes it does, because the graphing calculator puts the graph in a simpler way to understand.
TR: What do you mean by "it puts the graph in a simpler way?"
Student CD3: With graph it is easier to tell if the solution is above or below the x-axis. The solution can be within or outside the critical values.
TR: *Ok. Thank you very much.*

Interpretation of students' responses

Students' responses affirmed that their learning of quadratic inequalities had been made easier after the use of GC. Students emphasised that the graph and critical values are always displayed on the GC and these are helpful in determining the solutions of the quadratic inequalities. Student CD3 indicated that with the graph it is easier to tell if the solution is within or outside the critical values. The displayed information on the GC made it easier to understand the solutions of the quadratic inequalities (see Figure 7.10).

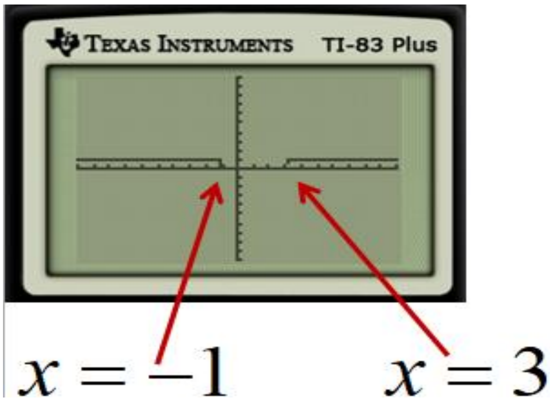


Figure 7. 11: Solution of the quadratic inequality displayed on the GC screen

In this case, the graph indicates that the solution set of quadratic inequality is a disjunction of $x = -1$ and $x = 3$. The use of the GC, according to students, demonstrates that the solution is outside the critical values in Figure 7.10. However, there are instances when the displayed information can be presented in the form of quadratic graph. This means that through the use of the GC, students were able to visualise the graphs with critical values which made it easy to determine the solution of inequalities. The visual images of the graphical representations contributed to the improved learning of quadratic inequalities as shown by reference to '*shows us if the solution on the graph is below or above the x-axis*'. For this reason, students benefitted from the visual capabilities of the GC which made them understand easily the connections between the graph and critical values, and solutions of quadratic inequalities.

7.8.2 Students' responses on how the use of the GC helped them to feel comfortable with quadratic inequalities

The next question of in-depth interview sought to find out the students' opinions about whether the use of the GC helped them feel more comfortable with quadratic inequalities, which attempted to answer the first research question. Students' responses were almost similar as shown below:

TR: *Does the use of the GC help you to feel more comfortable with quadratic inequalities?*

Interview with Student CD1

Student CD1: *Yes, I am comfortable because the GC helped me to do try many problems on my own. I used to have difficulties with quadratic inequalities before I used graphing calculator.*
TR: *Do still have those difficulties with quadratic inequalities*?
Student CD1*: If I have I put the inequality into the GC to solve for me*.

Interview with Student CD2

Student CD2: *Yes I am. I am more excited of learning quadratic inequalities after the use of the GC.*
TR: Explain "I am more excited"
Student CD2: *After the use of the GC, quadratic inequalities are no longer difficult. The GC has made it easy to understand and follow the steps.*

Interview with Student CD3

Student CD3: *I am comfortable with learning quadratic inequalities*.
TR: *Please explain your answer.*
Student CD3: *The use of the GC has made it easy to understand and analyse the graph, which is useful to determine the solution of the quadratic inequality.*
TR: *Ok. Thank you very much.*

Interpretation of students' responses

Students' responses were almost similar and they felt comfortable with the learning of quadratic inequalities in the GC environment. Student CD1 and CD2 gave an affective answer of a minimised mathematics anxiety such as "difficulties" of learning quadratic inequalities. Student CD2 emphatically indicated that *"I am more excited,"* meaning the use of the GC eased the pressure of learning quadratic inequalities. Student CD3 added that the use of the GC stimulated the use of graphical sketches to determine the solutions of quadratic inequalities.

7.8.3 Students' responses on whether the GC should be used in learning quadratic inequalities

Still attempting to answer the first research question, the next question of the in-depth sought students' opinions on whether the GC should be used in learning quadratic inequalities or not. The students' opinions were positive and quite similar as shown below:

TR: Should graphing calculators be used in learning quadratic inequalities at the eleventh grade?

Interview with Student CD1

Student CD1: In my opinion it should be used.
TR: Please explain
Student CD1: It is easier to understand the quadratic inequality because the GC shows where the solution is found.

Interview with Student CD2

Student CD2: *Yes, it must be used to help those learners who have difficulties with quadratic inequalities. The GC helped me to answer many questions on my own.*

Interview with Student CD3

Student CD3: *Sometimes*
TR: *Please explain why sometimes*.
Student CD3: *If the use of the GC is always students might not be able to solve quadratic inequalities on their own. Through linking different methods helps to understand mathematical concepts.*

Interpretation of students' responses

All the students responded supporting the use of the GC in learning of quadratic inequalities. The GC afforded students opportunities to learn better quadratic inequalities. Students indicated that the use of the GC showed "*where the solution is found"* and *"helped me to answer many questions on my own*". Student CD3 emphatically indicated "*sometimes*" and wanted students to link algebraic and GC approaches in order for them to master the procedural steps of solving quadratic inequalities.

7.8.4 Students' responses on how the use of GC helped them to do homework and other activities of quadratic inequalities

The next question of the in-depth interview asked if the use of the GC helped students to do homework and class activities of quadratic inequalities independently. Students' responses were as follows:

TR: Was the use of the GC helpful in doing homework and other class activities of quadratic inequalities independently?

Interview with Student CD1

Student CD1: *"Yes it was helpful.*
TR: Please justify your answer

Student CD1: *The use of the GC was helpful because I can be surer with my solutions. If I get wrong answer it helped me to keep on trying until I understand. I always tried to answer my homework on my own.*

Interview with Student CD2

Student CD2: Yes because there were no longer any difficulties of calculating critical values using quadratic formula. Critical values are always displayed.
TR: Do you have difficulties of calculating critical values?
Student CD2: Ah, no. With the use of the GC you spend more time gaining knowledge of solving the inequalities on your own.

Interview with Student CD3

Student CD3: Yes it was.
TR: Please explain your answer
Student CD3: The GC displays the graphs with critical values and where the solutions could be. It helped me how to use the graph to determine the solution and correct my mistakes. I always did the homework by myself.

Interpretation of the students' responses

The responses from the students seemed to be very similar and confirmed their independence on writing any activity involving quadratic inequality after the use of the GC. For example, I did the homework "*on my own"* and "*by myself*". Student CD1 indicated that the GC helped her to be "surer of *my answers"* and student CD2, on the other hand "to spend more time gaining knowledge". Student CD3 felt the use of the GC helped her "*how to use the graph to determine the solution".* In particular, the use of the GC developed students' confidence in answering any activity on quadratic inequalities.

7.8.5 Students' experiences of using the GC in learning quadratic inequalities

This next question of the in-depth interview solicited students' experiences after using the GC to solve quadratic inequalities in a mathematics classroom. The student interviewees were asked to relate their experiences. The responses given below show:

Interview with Student CD1

TR: How can you explain your experiences of using the GC in learning quadratic inequalities?
Student CD1: It was quite good
TR: Please explain your experience

Student CD1: *At first it was difficult to use a GC in learning quadratic inequalities but later it helped me to improve my understanding. With the GC you can practise answer many questions and it corrects your work.*
TR: *Is it not affecting your thinking?*
Student CD1: Ah no. I answer similar questions on my own then I check if my answers are correct with the GC.

Interview with Student CD2

TR: *How can you explain your experiences of using the GC in learning quadratic inequalities?*
Student CD2: It was very nice
TR: *Please relate your experience*
Student CD2: *I learnt new ideas of solving inequalities and it was my first time to use a graphing calculator. I learnt to draw a quadratic graph on the GC and also to solve quadratic inequalities using the graph. It helped me to decide if the solution lies between the critical values or not. It gave me more time for practising.*

Interview with Student CD3

TR: *How can you explain your experiences of using the GC in learning quadratic inequalities?*
Student CD3: It was so great
TR: *Tell us how great it was?*
Student CD3: It was the first time to use GC and it enabled me to understand quadratic inequalities more than before. I learnt how to solve quadratic inequalities using graphs.
TR: *What about writing inequality solution?*
Student CD3: The use of the GC helped me to decide correctly if the solution was within the critical values or outside. It helped me to figure out how the graph will look like always and the position of the solution.

Interpretation of the students' responses

Students expressed "good", "very nice" and "so great" experiences with the use of the GC in solving quadratic inequalities. The *'first time'* experience of the students to use a graphing calculator brought new *"ideas"* such as graphs to solve quadratic inequalities. Students were able to solve quadratic inequalities graphically (i.e., using quadratic graph or line graph) (see Figure 7.11, below). Students CD1 and CD3 further stated that the use of the GC helped them to understand more than before the quadratic inequalities. In addition, student CD3 indicated that the use of the GC helped her *"to figure out how the graph will look like always and the position of the solution".* All the three students indicated that the GC use reduced the levels of their difficulty with quadratic inequality.

Solving quadratic inequalities graphically

$$x^2 + 2x - 8 \geq 0$$

$$(x - 2)(x + 4) \geq 0$$

$$x = -4 \; or \; x - 2$$

$$(-\infty, -4] \cup [2, \infty)$$

Figure 7. 12 Solving quadratic inequalities graphically

This means that the GC acted as a visual aid tool (visualisation tool) and checking tool as it provided the students with opportunity to see how the solutions are presented graphically. This means that the sketched graphs were used as visual objects to aid their conceptual understanding of quadratic inequalities (i.e., *to figure out how the graph will look like always*). All the three students indicated that they used the GC as to solve and graph quadratic inequalities in order to understand better the topic (i.e. the instrumentation process). This is consistent with the theory of instrumental genesis (Trouche, 2004).

7.8.6 Students' responses on how the use of the GC helped them to score better marks on quadratic inequalities

This last question of the in-depth interview asked if the use of GC helped students to score better marks on quadratic inequalities and to rate themselves between 0 and 5, a 5 being the highest. All the students confidently affirmed better marks after the use of graphing calculator.

Interview with Student CD1

TR: Did the use of the GC make you score better results in quadratic inequalities?
Student BD1: Yes, it did.
TR: Explain what made you to improve after using the GC.
Student CD1: Before the use of the GC I was always struggling with quadratic inequalities but now I have improved a lot. The use of the GC always shows me where I am wrong when I practise solving the quadratic inequalities.
TR: In this case, how can you rate your level of understanding?
Student CD1: I would rate my level of understanding at 5

Interview with Student CD2

TR: Did the use of the GC make you score better results in quadratic inequalities?

Student CD2: Yes, it did.

TR: Explain what made you to score better after using the GC.

Student CD2: With the use of the GC I have gained more knowledge about solving quadratic inequalities as it helped me to understand the steps involved in. it provided me with opportunities to practise answering more problems on quadratic inequalities.

TR: In that scenario, how can you rate your level of understanding in quadratic inequalities after the use of the GC?

Student CD2: I rate myself at 4 because there has been an improvement since.

Interview with Student CD3

TR: Did the use of the GC make you score better results in quadratic inequalities?

Student CD3: Yes, I can confirm that.

TR: Explain what made you to improve after using GC.

Student CD3: Because the GC has everything, meaning it has the graph and critical values that are helpful for understanding the basics of the quadratic inequalities. The GC also provides more opportunities for more practices on your own and for checking if my solution is correct.

TR: In this case, how can you rate your level of understanding?

Student CD3: I rate myself with a 5.

TR: Thank you very much.

Interpretation of the students' responses

The responses of the three students indicated that the use of the GC made them achieve better score marks in solving quadratic inequalities. The use of the GC helped students to know the basics of the quadratic inequalities (CD3) and to gain more knowledge (CD1). On the other hand, student CD1 indicated that she was no longer struggling with quadratic inequalities. They further indicated that the use of the GC also provided them with more opportunities for practicing on their own and for verifying their solutions. The teacher-researcher noted that students CD1 and CD3 had overrated their levels of understanding as these were below- average and average learners.

7.9. Reflection and conclusion of the last DBR cycle

This final section of the chapter presented the reflection of the last DBR cycle at School C. The researcher looked back to the starting points and expected learning outcomes for the third teaching experiments. Then the conclusion was drawn from the hypothetical learning trajectory and the results of the three research cycles in teaching experiments.

7.9.1. Reflecting on the expectations of the HLT

The researcher set out the expectations, based on the feedforwards from the second research cycle in Section 7.2, as starting points of this design study in relation to the opportunities that graphing calculators (GC) would offer for the students to achieve the higher level of understanding of the quadratic inequalities. In this current section, the researcher evaluated those expectations one by one to find out if they were confirmed in School C.

The first expectation in this last cycle was that specific attention to the properties of the quadratic graphs such as domain (interval notations), concavity (the effects of α) would improve the students' visualisation and presentation of the solution sets of quadratic inequalities in a flexible graphing calculator environment. During the teaching experiment at School C, students were confronted with activities that were mathematically similar but with different coefficients of x^2 (see Session 2 in Appendix D). More emphasis was given to the use of quadratic graphs, interval notations (domain), concavity and the effects of parameter α which were identified as the bottlenecks in the second research cycle. Students were given two additional activities on quadratic graphic properties and were engaged in pairs. The use of the GC indeed proved to be an appropriate instructional artefact for helping students to generate a family of graphs in the same systems of axes. In addition, the use of the GC helped students to visualise the graphs displayed on the screens and enabled them to understand their properties (e.g., zeros, intervals, axis of symmetry, concavity and domain). On the other hand, students were able to make repetitions of graphing quadratic functions using the GC and this made them develop and reify the key pre-concepts of quadratic inequalities. As it was conjectured, the findings confirmed that the emphasis on the properties of the quadratic graphs improved the students' visualisation and presentation of the solution sets of quadratic inequalities in the GC environment. Evidence for this is the way the students answered quadratic inequalities in written tasks (cf: Section 7.4.1). Almost all the students' attempts and discussions about quadratic inequalities were supported with graphs and students were able to recognise the concavity of quadratic functions with respect to the coefficients of x^2. This was a positive sign towards understanding quadratic inequalities. In this regard, the use of GC supported the transition from the graphic representation to quadratic inequality representation in the last DBR cycle.

The second expectation in this last DBR cycle was that the graphic and tabular instrumented action schemes would support the students' visualisation and reification of interval notations and concavity (the effects of parameter α) of quadratic functions. The notion is that parameters have a graphical meaning, which was expected to prepare the students for generalization for the solution sets of quadratic inequalities. This means students were supposed to use their prior skills of quadratic functions and equations towards solving symbolic quadratic inequalities, which demand routine reasoning skills. During the teaching experiment at School C, students were confronted with questions that were mathematically similar but with different levels of difficulties (see Sessions 3 and 4 in Appendix D). Students were supposed to use the GC as an instructional artefact, in particular the graphic and tabular instrumentations to solve the symbolic quadratic inequalities. These instrumented action schemes allowed students to repeat the processes of graphing and tabling the values and supported them develop and reify the concept of quadratic inequalities. Through the repetition of the processes the students realized that changing the parameter values affected the complete quadratic graphs and inequality solution sets. The graphical visualization of this effect (i.e., changing the parameters) created a strong mental image for the students. The impression is that most of the students started to perceive graphs and inequalities as entities that could symbolize objects. The use of graphical representations (models) made students to extend their graphical conception of the quadratic functions towards the view of understanding the symbolic quadratic inequalities. In particular, the x-intercepts of the graphs were used as the limits for deciding whether the quadratic inequality solutions are within or outside them. The graphical models mediated very well between the quadratic equations and quadratic inequalities (Sections 7.7.1 & 7.7.2). The notion is that the graphical schemes of the graphing calculators were helpful for visualizing the effects of parameters and the properties of graphs. Evidence is shown in the students' answers in the written tasks (cf: Section 7.4.2). As it was conjectured, the graphic and tabular schemes of the instrumented actions improved the students' understanding and interpretation of graphs in solving quadratic inequalities. The use of graph and tabular instrumented action schemes thus facilitated the transition from the graphical and tabular representations to quadratic inequality solutions.

The third expectation of the last DBR cycle was that introducing a topic of quadratic inequalities with real-life mathematical situations particularly linear inequalities would intrinsically motivate students. The idea was to draw a specific focus on the linear inequality problems of the real-life situations which meant to foster the concept and its significance in life (see Session 7 in Appendix D). The mastery of these real-life linear inequality situations would increase the students' confidence in quadratic inequality problems. Evidence provided by the students' work showed that the use of real-life linear inequality contexts indeed fostered better interpretation of mathematical situations and generalization (see Sections 7.5.1 and 7.6.1). In the referred sections, students' understanding of real-life mathematical situations moved from the referential level to the general level. These findings are embraced in the reality principle of RME theory. In this principle, the importance of using real contexts that are meaningful and natural to learners as a starting point for their learning can develop mathematical tools and understanding when exposed to realistic contexts that can be mathematised (Cheung & Huang, 2005; Van den Heuvel-Panhuizen, 2010; De Villiers, 2012; Drijvers, 2015). This shows that the realistic problem situations in learning activities were experientially real to students and meaningful, authentic as starting points (Drijvers, 2015). In that regard, the linear inequality contexts helped students to develop true meaning to quadratic inequalities. Also, the fact that students were not required solving quadratic inequalities; they developed interests in interpreting the contextual situations. However, in complex situations the students still had difficulties keeping track of the problem-solving strategy.

The fourth expectation was that an earlier intertwinement of the algebraic and graphic representations would stimulate the understanding of the contextual (complex) problems of quadratic inequalities in the GC environment. This is consistent with the principle of intertwinement which advocates for an integrated approach of various mathematical topics (Van den Heuvel-Panhuizen, 2010; Widjaja & Heck, 2003). The use of the integrated approach supported the students' visualisation and enhanced the reification of the graphs as objects that were used to solve quadratic inequalities. During the teaching experiment at School C, students were confronted with questions that were mathematically similar but with different levels of difficulties (see Sessions 5 and 6 in Appendix D). Students were engaged in the use of the GC as an instructional, visualisation, guiding and checking artefact to solve the contextual quadratic inequality problems. Students were able to convert the contextual situations into algebraic expressions -quadratic inequalities and then solved them algebraically and graphically. The integrated approach successfully mediated between the symbolic and contextual situations as evidenced form the students' work in questions 4 and 5 of the post-test (see Figure 7.8). This means the expectation was justified. The use of the GC brought about the appealing visualizations of the quadratic graphs and students found them helpful in solving any type of quadratic inequalities. This earlier intertwinement stirred up the students' understanding of the quadratic inequalities and diminished students' misconceptions. The researcher is tempted to conclude that the use of the GC for the learning of the quadratic inequalities was appropriate for the eleventh grade.

7.9.2. Conclusions drawn from the three DBR cycles

This section described the conclusion drawn from the three DBR cycles about the HLT and the teaching experiment experiences. The cycles were presented in Chapters 5, 6 and 7 for three different schools. In this section the researcher briefly looked back at these three chapters and summarized the main issues that would be considered in more detail in the next chapter.

The development of the HLT of three DBR cycles

The three research cycles in Chapters 5, 6 and 7 led to the development of an HLT for the three teaching experiments in three different schools. The broad outline of HLT: Solving routine symbolic and contextual problems of quadratic inequalities in a flexible graphing calculator environment. This was viewed as a broad learning trajectory of how to achieve higher level understanding of quadratic inequalities by using the opportunities offered by the use of the GC. Student activities were developed to foster the transitions in a natural way of learning quadratic inequalities. These activities included manipulations in the GC environment and, of course, the related mental activities. Furthermore, the experiences and reflections (the feed-forwards) from the first two teaching experiments informed the HLT of the last two DBR cycles. In this way, optimization of the HLT was achieved. However, there were no major changes in designing the HLT for the second and third cycles except that real-life inequality situations should be incorporated as starting points. Thus, the developed HLT remained the same for the three DBR cycles and was supported by the starting points and expected learning outcomes.

A summary of the experiences of the teaching experiments

The results from the teaching experiments suggested that the use of graphical approach was most helpful to students in solving quadratic inequalities because of its dynamic character and visual images. However, the use of the GC did not adequately address the properties of quadratic graph such as domain, the interval notation and concavity without the use of teacher's voice. For that reason, the teacher-researcher had to avail himself every time to explore his experience in the second cycle for orchestrating the learning process in whole-class discussions and also as an additional resource of information for the students. The use of the GC supported the transition of symbolic quadratic inequalities towards the contextual quadratic inequalities however this did not come across in a satisfying way. This was attributed at least partially to algebraic misconceptions of solving routine symbolic quadratic inequalities in general and the use of graphic strategy in particular. It is important to master this graphical technique, so that it does not hinder the generalization and visualisation processes. Linking the algebraic and graphical representations in the GC environment was expected to holistically develop students' reasoning skills and problem-solving abilities in the broad learning trajectories of solving quadratic inequalities. Students incompletely solved the contextual quadratic inequalities because they had partially understood symbolic quadratic inequalities and partially developed the reasoning skills of metacognition and monitoring their progress.

A null hypothesis was stated and tested if there was no difference between the pre-test mean and the post-test mean of quadratic inequalities for the students in the study. A dependent paired statistics t-test and Cohen's d effect-size were conducted to test the significance of the GC use on the students' performance in solving quadratic inequalities. The *t*-test results indicated that the null hypothesis was rejected at 5% significant level in favour of the alternative hypothesis of the three research cycles (cf: Sections 5.4; 6.4; 7.4) and the GC use had practically improved.

The students' results of the in-depth interviews in the three cycles affirmed that the use of the graphing calculator provided an enabling environment for learning quadratic inequalities. Students revealed that the use of the GC made their learning of quadratic inequalities easier and enabled to see how the quadratic inequalities were solved graphically. They also explained that a graph with critical values was always displayed on the GC which helped them to write accurate solutions. They further argued that the use of the GC afforded opportunities to figure out how the graph would look like and to see how the solutions could be presented graphically. This means that the graphs were used as visual objects that aided to understand quadratic inequalities. Students further revealed that the use of the GC helped them to link algebraic methods with graphs when solving quadratic inequalities. This was within the recommendations of the CAPS for FET Mathematics document (DBE, 2011), expressed emphatically in the NSC Examination Diagnostic Reports that students must solve quadratic inequalities by integrating both methods (DBE, 2014; 2015; 2016; 2017). Additionally, students expressed joy and comfort of using the GC, which brought about the appealing visual images for solving quadratic inequalities. For those underlying reasons, the students attributed their high score marks in and minimised anxieties of solving quadratic inequalities to the use of the GC.

The results of three students of each cycle who were engaged in focus group interviews showed different levels of proficiency in relation to problem solving processes. The students, who scored high scores in solving quadratic inequality problem, used an integrated approach (i.e., the algebraic and graphical approaches) and also verified their solution with the graphing calculator when required to do so. The students' work revealed that the graphical approach was used by almost every student (cf: Sections 5.6.1; 6.6.1; 7.6.1). Evidently, the results indicated that the use of the GC supported the students' problem-solving abilities. The results of the focus group interviews also revealed that the use of GC was beneficial in helping students to identify the main concept involved, make convincing conclusions before solving it and make appropriate choice of the strategy to execute. Through the use of the graphic approach students were able to *"interpret the inequality"* and *"figure out solution"* and *"to decide for the region".* Notably, the use of GC did not completely support all the reasoning domains in all the three cycles. Results indicated that the use of the GC supported students' reasoning skills such as analysing, initiating a strategy and seeking and using connections. However, the use of the GC did not support completely the reasoning skills of reflecting on one's solution (metacognitive) and monitoring one's progress when solving quadratic inequalities. It was a hard issue for some students to reflect on their own solutions without the use of the GC. It was observed that students who used multiple approaches –algebraic and graphic- successfully monitored their progress and verified their assumptions as they were able to switch between approaches and to question every action they take.

Design principles of the DBR study

It is well known that students typically struggle with learning of quadratic inequalities which could be facilitated by correctly interpreting graphs in the forms in which they are presented. In this regard, the design principles were formulated to help students become flexible in dealing with quadratic inequalities using functional graphs, whether as symbolic, graphic, or numeric. The use of the graphs or visual graphs from the GC helped to formulate and refine the following principles in the three cycles of design based research. The main design principle of this design based study: *Graphically interpreting the quadratic inequalities in a flexible graphing calculator environment.* The three cycles of DBR also assisted to identify other minor design principles which helped to close the learning gap of quadratic inequalities, as listed below:

1. Helping students deal with quadratic inequalities flexibly, stress the graphical representations of functions
2. Training students to use the GC fluently reduces chances of the limited viewing window of the tool becoming a source for students' misconceptions.
3. Starting the topic of quadratic inequalities with the problems of real-life situations enhances students' understanding in a GC environment.
4. Integrating algebraic and graphical representations when teaching quadratic inequalities
5. Provide feedback to the learners about their solutions in a flexible GC environment

Chapter 8 is the next and final chapter that discusses the main findings of the three DBR cycles presented in Chapters 5, 6 and 7 in a broader didactic spectrum.

CHAPTER 8: DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

8.1. Introduction

This final chapter discusses the main findings of the three cycles presented in Chapters 5, 6 and 7 in a broader didactic spectrum. The purpose was to make a synthesis of these cyclic results and then to reflect on the main findings of the three research cycles. Secondly, the chapter discusses the findings of the four research sub-questions and then derives the answer to the main research question (8.2). Thirdly, it discusses the contributions of the theoretical frameworks and the methodology to these findings (8.3). The fourth it draws conclusions from the findings (8.4) and lastly recommendations for the study are made (8.5).

8.2. Discussions of the findings

In this section the researcher first discusses the findings of this study focusing on the three research sub-questions presented in Chapters 5, 6 and 7. Second, the findings are compared with the initial expectations, and then I look back on the possible alterations in these issues. Third, the researcher reflects on the critical role that the theoretical frameworks played in the study. Fourth, the suitability of the research methodology is also discussed.

8.2.1. First research question about the students' performance

The first research sub-question was,

> *To what extent can the pedagogical use of graphing calculator influence high school students' performance in solving quadratic inequalities?*

The results from the three chapters addressed the first sub-question in the three research cycles (cf: Sections 5.4; 6.4; 7.4) through the quantitative analysis of the pre- and post-tests. Results of the dependent paired statistics *t*-test showed that there were significant differences between the students' means of the pre- and post-test in their performance of quadratic inequalities. This difference is significant for overall performance among all students and specifically in three participating schools. The results showed that the students' mean scores of the post-test in the three research cycles had improved and there was sufficient evidence to conclude that this change from the GC use was statistically significant at 5 % (p=0.05) level and had large Cohen's d effect sizes ($d_1 = 0.8$; $d_2 = 1.1$; $d_3 = 0.8$). With these significantly higher means between the pre- and post- test results in the three research cycles, this indicates that the use of the GC was beneficial in improving the students' performance in quadratic inequalities. It is interesting to note that the largest gains of students' scores of the post- test occurred in the third cycle at School B. This could be attributed to the written tasks that consolidated the learning activities (Sections 5.4.1-2; 6.4.1-2; 7.4.1-2) and well-developed HLT. Furthermore, the use of feedback and the exposure to graphic representations provided by the GC could have influenced this wide difference of students' performance between pre-test and post-test. For this reason, a conclusion can be drawn that supports the alternative hypothesis of this study that the use of the GC enhanced students' understanding of quadratic inequalities. The results showed that there was a significant gain between pre- and post-test score achievement in the GC enhanced environment. These findings were consistent with the earlier studies such as those of Carter (1995); Ellington (2006) and Armah and Osafo-Apeanti (2012), in the GC learning environment. This shows that the GC was a support mechanism for students' learning (Karadeniz, 2015) as provided them with multiple representations to enhance their understanding of the concept (Averbeck, 2000; Spinato, 2011).

8.2.2 Second research question about the students' problem solving

The second research question was,

> *In what ways can the pedagogical use of the graphing calculator support the high school students' problem solving ability in quadratic inequalities?*

In order to answer this question, the results of the post-test (cf: Sections 5.5.1; 6.5.1; 7.5.1) were used as means of analysing the students' problem solving abilities of quadratic inequalities in a graphing calculator learning environment. Question five was selected from the post-test (see Appendix D) for exploring the students' problem solving abilities in the application of quadratic inequalities. The students were not supposed to use the graphing calculator as an additional tool to help solve the problem but for verifying the accuracy of their solutions. A sample of three students' answers was selected and analysed from each DBR cycle. Using the quadratic inequality problem solving test (QIPST) rubric in Appendix D, students' answers were scored and analysed. The results revealed that students understood the problem and appropriately applied the strategy of solving the problem. This means an inequality was correctly constructed and appropriate solutions were arrived at using the correct procedures. However, not all students reflected on their solutions by checking its reasonableness and accuracy. The majority of students scored more than 50%, thus an indication that the pedagogical use of the GC supported students' problem-solving abilities. The use of both graphical and algebraic approaches helped students to solve the problems.

Another quantitative portion of the study sought to answer the same research question through the post-survey results (cf: Sections 5.5.2; 6.5.2; 7.5.2). The post-survey data of the three DBR cycles revealed that the students felt that the use of the graphing calculator supported them in problem solving of quadratic inequalities. In their supporting evidence, students indicated that the use of the GC enabled them to understand the contextual problems, guided them to sketch the graphs, use the best alternative methods and procedures and check for errors, mistakes and correctness of their solutions. Students' perceptions were assessed using the Polya's main steps of problem-solving processes. For that reason, students in all the three cycles perceived that the use of the GC helped them to play out these activities when solving contextual quadratic inequalities. The use of multiple representations (graphical, tabular, and algebraic) of quadratic inequalities offered by the use of the GC allowed students to view more than one representation. The use of the GC thus supported them in problem solving of quadratic inequalities.

The qualitative portion of the study also supported this conclusion. The qualitative results of the focus group interview on problem solving (cf. Sections 5.6.1; 6.6.1; 7.6.1) revealed that students, who scored high scores in solving quadratic inequality problem, used an integrated approach (i.e., the algebraic and graphical approaches) and also verified their solutions correctly with the graphing calculator when required to do so. The graphical approach seemed to be the most useful in solving the quadratic inequality as it helped students to identify the correct region of the solution and to write the meaningful interval notations. It was noted that students who used the integrated approach were also able to move back and forth between the multiple representations. However, there were cases where the students could not confidently use the graph even if the right graph was drawn. Two-thirds of the students of each DBR cycle were able to reflect on their solutions and strategies used to solve the contextual problem correctly and were in a position to use the GC to verify their solutions when asked to do so. Evidently, the results indicated that the use of the GC partially supported the students' problem-solving abilities.

On the quantitative portion of the focus group interviews (Sections 5.6.1; 6.6.1; 7.6.1) students' problem solving abilities were scored in terms of a) formulating an inequality from the contextual problem, b) using the algebraic approach, c) using the graphical approach, d) using the graphing calculator for verifying solutions and e) obtaining a correct solution set. The results also revealed that the graphing calculators were useful in terms of helping to improve students' problem solving and thinking skills. These findings from the results of the post-test, focus group interviews and post surveys were compatible with the results obtained in the previous studies (e.g., Spinato, 2011; Averbeck, 2000; Allison, 2000; Bitter & Hatfield, 1991; Carter, 1995). The exposure of students to different strategies, including the use of the GC supports the development of their problem solving skills in quadratic inequalities. The International Society for Technology in Education standard suggested that having digital tools in the classroom and using them appropriately in mathematics, helps students improve critical thinking skills and, solve and make informed decisions in real-world problems (ISTE Standards for Teachers and Students, n.d.).

8.2.3 Third research question about the students' reasoning

This section reflects on the third research question,

> *In what ways (how) does the pedagogical use of the graphing calculator support the high school students' reasoning ability when solving quadratic inequalities?*

In order to answer this research question, focus group interviews combined with observations (Sections 5.6.2 ; 6.6.2; 7.6.2.) were used as means of exploring the students' reasoning skills (e.g., analysis, initiative, reflection, monitoring and seeking connection) when solving quadratic inequalities. The qualitative results from the focus group interviews with students on their reasoning skills in the contextual quadratic inequality problems revealed that the use of the GC was beneficial in developing different domains of the students' reasoning skills in all the three DBR cycles of teaching experiments. Results indicated that the use of the GC differentially supported the domains of students' reasoning skills in all the three research cycles. In the first two research cycles (Sections 5.6.2; 6.6.2) the researcher observed that the use of the GC mostly improved and developed the domains of the students' reasoning skills such as analysing a problem, initiating a strategy and, seeking and using connections. This means the students were able to reason about solving quadratic inequalities by identifying hidden structures, choosing the appropriate strategies and representations as they connected different mathematical domains and contexts. Students used both algebraic and graphic representations as a way of making meaningful connections to understand quadratic inequalities better. However, the use of the GC did not support completely the development of the reasoning domains of reflecting on one's solution (metacognitive) and monitoring progress when solving quadratic inequalities. It was a hard issue for some students to reflect on their own solutions and monitor their progress without the use of the GC. It was observed that students who used more than one approach –algebraic and graphic-successfully justified their solutions and verified their assumptions as they were able to look back and forth between these approaches and to question every action they take.

In the third DBR cycle (Section 7.6.2) the researcher observed that the domains of reasoning that improved in the GC environment were analysing, initiating a strategy and seeking and using connections and reflecting on their solutions. The researcher attributed the improved reasoning domain of reflecting on the solution to the increased attention to the feed-forwards in Chapter 6. These domains of reasoning require analysing the mathematical problem to determine the possible approaches and strategies that can be used throughout solving the problem and reflecting on the reasonableness of the solutions through justifying their solutions, and connecting mathematical concepts. This means the students were able to reason about solving quadratic inequalities by identifying patterns, choosing the appropriate strategies, and verifying the accuracy of their solutions using connections to understand concepts. Students used both algebraic and graphic representations as a way of seeking meaningful connections and understanding quadratic inequalities. However, students did not seem to improve much in metacognitive reasoning domain of monitoring progress which includes verifying their assumptions as they could not review and modify their selected approaches when solving quadratic inequalities.

The quantitative portion of the study sought to answer this research question through the post-survey results (cf: Sections 5.6.3; 6.6.3; 7.6.3.). The post-survey data of the three research cycles revealed that the students perceived that the use of the graphing calculator supported their reasoning skills when solving quadratic inequalities. In their supporting evidence, students indicated that the use of the GC helped them to analyse completely, use multiple strategies, monitor or check their progress, link with other mathematical concepts and reflect on (i.e., look back) their solutions of the contextual quadratic inequality problems. The pedagogical use of the GC thus supported their reasoning skills when solving quadratic inequalities.

Another quantitative portion of this research also supported this conclusion. The data drawn from the written task indicated that the use of the GC seemed to be very helpful in improving the reasoning skills among student participants of the three DBR cycles in solving quadratic inequalities. However, there were some variations in terms of reasoning skills developed for each cycle of the teaching experiments. For example, students who began with lower reasoning abilities had improved after the use of the GC. It was also noted that the use of GC did not seem to have much effect on those students, who had higher reasoning skills. These findings were consistent with the research literature, which state that graphing calculators are more effective for lower-achieving students (Burrill et al., 2002; Spinato, 2011). The results from the written tasks of the current study support this conclusion. The researcher observed that the higher achieving students maintained higher marks in the written tasks. In most cases, these students used both an algebraic approach and a graphic approach to earn the highest reasoning scores. This means the use of the GC was effective in supporting students' domains of reasoning skills. However, students' reasoning domains did not seem to develop at the same level. This means the development of students' reasoning domains were affected by their limited background of concepts related to quadratic inequalities and the use of the GC syntaxes.

Students' responses from the interviews were directly related to particular domains of reasoning skills, which shows that they had developed these skills. For example, the process of checking their solutions and assumptions throughout solving relates directly to the reasoning domain of monitoring progress. Students were able to identify concepts and relationships, develop strategies, make conclusions, draw connections to other contexts and reflect upon their solutions, effectively monitored their progress and successfully solved the contextual quadratic inequality problem. By contrast, students who did not perform well in all of the reasoning domains struggled to solve the problem, which is consistent with the findings of Karadeniz (2015); Spinato (2011); Graham (2005) and Averbeck (2000).

8.2.4 Fourth research question about the students' perceptions of the pedagogical use of the GC on quadratic inequalities

This section reflects on the fourth research question,

> *What perceptions do students have on the pedagogical use of the graphing calculators in learning quadratic inequalities?*

In order to answer this research question, the results of post-surveys were used (Sections 5.7; 6.7; 7.7) as means of exploring the students' perceptions quantitatively. The comparative results of the pre- and post-surveys showed that the GC use impacted good behaviour in students' learning of quadratic inequalities. This was revealed by the students' responses that favoured the post surveys. The post survey means of the items were greater than the pre-survey means (cf: Tables 5.7; 6.7; 7.7). Students indicated that they understood and enjoyed learning quadratic inequalities and had minimised memorising procedures. Additionally, they indicated that they had no difficulties in determining the solution sets of quadratic inequalities and were able to see the difference between the equation and inequality. With the consistent use of the GC they were able to show the difference between critical values and x-intercepts of the graphs.

In sections (5.7.2; 6.7.2 and 7.7.2), the post-survey data of the three research cycles revealed that the students felt that the use of the graphing calculator supported them in answering quadratic inequality activities. In their supporting evidence, students indicated that the use of the GC enabled them to identify quadratic inequalities by the shapes of the quadratic graphs, to see the effects of the parameter "a" on the different graphs, to use the graphs to solve quadratic inequalities and to use correct interval notations informed by the critical values. All these were considered to be the main concepts that can lead the students to solve quadratic inequality confidently. The pedagogical use of the GC thus supported students' reasoning and problem solving skills of quadratic inequalities.

As a qualitative portion of this section, the in-depth interview (cf: Sections 5.8; 6.8; 7.8) was conducted with the three students who had obtained marks below average, average and above average from the post–test. The interview sought to find out if the use of the GC had influenced students' understanding of quadratic inequalities. The responses of the students from the in-depth interviews affirmed that the use of the graphing calculator provided an enabling environment for learning quadratic inequalities. In their arguments, they revealed that their learning of quadratic inequalities had been made easier after the use of the GC and they were able to see how the quadratic inequalities were solved graphically. Students further contributed that the use of the graphical sketches assisted them to determine the regions of the solution sets of quadratic inequalities and were enabled to visualise the solutions of quadratic inequalities on the GC screens. Through the process of visualisation students were able to establish the relationship between the quadratic graphs and quadratic inequality solutions. This is supported by the findings in the literature that visual data promoted students' motivation to explain the graphs and functions (Choi-Koh, 2003) and also that with visual images students can develop their thinking process (Doerr & Zangor, 2000). The use of the calculators enabled them to visualize the problems and concepts as also found by (Leng, 2011). Using graphing calculators as a visualization tool also reflects students' ways of solving equations and inequalities (Karadeniz, 2015).

As revealed in the interviews, the visual images of the graphic representations improved the learning of quadratic inequalities. Additionally, they affirmed that the use of the GC influenced them to link algebraic methods with graphs i.e., graphs being drawn as aid for solving quadratic inequalities. Ultimately, students expressed enjoyment and confidence in the learning of quadratic inequalities with the use of the GC and indicated that it was instrumental towards their understanding of quadratic inequalities. A significant number of students presented their work with graphs and some of graphs were not relevant. This is a sign that the use of the GC had "*inspired the use of the graph*" as indicated by student DB3. However, not all the students made use of the graphical models to illustrate their solutions in answering the post-test. This means that although these students appreciated the graphical meaning of an algebraic solution they apparently could not connect the two representations as expected. Despite the fact that students had 'interesting', 'great' and 'awesome' experiences with the use of the GC in solving quadratic inequalities, they suggested for a balanced approach with regard to its use and the use of symbolic approaches (Averbeck, 2000). Students still valued the presence of the teacher as an additional resource to the use of the GC who can attend to their individual differences in the classroom.

8.3. Contributions of the theoretical frameworks and methodology

The role of the theoretical frameworks and methodology cannot be over emphasised as they contributed enormously to the improvement of students' understanding of quadratic inequalities in a graphing calculator-assisted classroom.

8.3.1 The theory of Realistic Mathematics Education

The guided re-invention principle of RME was instrumental in developing the hypothetical learning trajectories (HLTs) and designing instructional activities for the three research cycles of teaching experiments. The results of the study revealed that students were able to develop their own informal problem-solving strategies and ideas as they were engaged with the use of the GC and learning activities. The use of the GC helped them to experience an abstract mathematical problem (i.e., solving quadratic inequalities algebraically) as real and meaningful to them. Students learnt with their own authority and greater freedom and learning was always meaningful to them (i.e., linking up of algebraic and graphic representations). It was further noted that students progressively formalised their informal strategies of solving quadratic inequalities (i.e, using graphic approach) and this is in line with the ideas of previous researchers (Drijvers, 2003; Bakker, 2004; Ndlovu et al., 2013). The use of the GC provided the students with a powerful learning environment as students were able to develop their own mathematical domains, ideas and representations, and had the opportunity to experience a process similar to that of solving quadratic inequalities. This concurs with the findings of the earlier researchers (Van den Heuvel-Panhuizen, 2003; Ndlovu, 2014) in a technology learning environment which have similar symbolic algebra capabilities. It was noted that this learning process, however, needed a focused teacher's guidance to develop sensible directions and to leave 'dead-end streets' within the mathematical community. The researcher further observed that it was beneficial to let students graph their own diagrams and compare them with the ones on the GC as this allowed them to formalise their informal reasoning and to make their abstract notions to become more general.

The findings of the study also revealed that the use of the GC as an instructional tool afforded students opportunities to move through different levels (emergent modelling, abstraction, generalization, formalization) of understanding quadratic inequalities as they were able to produce visual models, diagrams and symbols in their attempts. Specifically, the use of the GC helped students to use their models and diagrams to build up their presentations on each other to develop powerful mathematical ideas. In reference to realistic quadratic inequality problems, students were able to organise, translate and transform into quadratic inequalities. At the same time students used their models, diagrams and symbols for formal reasoning in quadratic inequality situations. This was an indication that students were moved from horizontal mathematisation to vertical mathematisation, where they could explain the solution of the inequality by mere looking at the graphical representation. This is consistent with the findings of the previous researchers (Freudenthal, 1991; Gravemeijer, 1994; Drijvers, 2003; Menon, 2012). Drijvers (2003) further suggests that if the realistic problem situation is meaningful to the students then the technology serves as a means for vertical mathematisation. For this study, the concrete and realistic problem situations were meaningful to the students and provided better starting points. Again, the use of graphical approach became the generalised procedure for solving quadratic inequalities.

The RME heuristics (historical and didactic) also contributed to the success of this study. Historically, the most apparent success was the idea to start the HLT with real-life linear inequality situations to support students' understanding of quadratic inequalities. The historical accounts functioned as evolving HLTs which inspired the development of the activities related to real life applications. The design of instructional activities and materials were preceded by a historical study of the relevant pre-concepts of quadratic inequalities such as linear inequalities, quadratic equations and quadratic graphs. The pre-concepts were the basic ingredients of the intended instructional sequence and minimised the epistemological obstacles. This historic principle contributed to the structuring of the learning activities from lower level (familiarity) through informal context-connected solutions to the formal mathematical reasoning. This is consistent with the findings of prior researchers (Radford, 1997, Halmaghi, 2011; Bagni, 2005; Burn, 2005).

8.3.2. Contributions of the theory of instrumentation

The theory of instrumentation proved to be a fruitful framework in this study, although it was applied only to learning and teaching of quadratic inequalities. The findings of this study showed that the use of the GC (instrumental approach) contributed to the building the students' conceptual understanding (knowledge) of quadratic inequalities. The instrumental approach to the use of the GC effectively influenced students to easily construct the graphs (an enablement of the GC) and dynamically manipulated the GC by creating unavailable opportunities in a paper and pencil environment (a potentiality of the GC). Through the teacher-researcher's exploitation mode of didactic performance, students were prompted to effectively use the GC (i.e., teacher's instrumental efforts). In addition, it was through the utilisation schemes of the GC that the students were able to perceive the relation between algebra and graphic visualisation. This relationship helped students to connect the graphic properties and quadratic inequality solutions. The use of the GC provided opportunities for students to use visual models, diagrams and symbols as they were able to figure out the solutions of quadratic inequalities. The qualitative results revealed that students were satisfied with the graphic-numeric approach and only considered algebra when determining the critical values. In addition, the table approach was often utilised after questions from the teacher-researcher. These results are linked to the findings of the previous studies (Drijvers, 2003; Jupri, Drijvers and Van den Heuvel-Panhuizen, 2015) that the use of the GC provided students with visual and representational opportunities..

The researcher further observed that the students were able to develop additional schemes for solving contextual problems of quadratic inequalities which were in line with the conjectured schemes. The usage schemes enabled the students to discover and select the relevant functions, and adjust the screen as they entered the inequalities and put the GC into action. However, in solving the contextual real-life problems, the researcher observed that student difficulties in setting up quadratic inequalities were not caused by their inability to understand each word or phrase in the problem, but by their inability to represent them in an appropriate algebraic expression- quadratic inequality. Students displayed a limited understanding of the contextual quadratic inequality problems, which concerns the process of transforming the problem situation into the world of mathematics (horizontal mathematization). This is consistent with the findings of the earlier studies such as those of Ndlovu et al (2013), Drivjers et al (2010), Trouche (2004) and Van den Heuvel-Panhuizen (2003).

The results revealed that there was a close relation between machine techniques and conceptual understanding of students when using the graphing calculator (the instrumentation process). A technical and a conceptual aspect could be distinguished in many of the problems that the students encountered. This means the understanding of quadratic inequalities and the GC techniques are closely related to one another. It was observed that, on the one hand, students who had difficulties in carrying out GC techniques had limitations in the insight of quadratic inequalities. On the other hand, the development of students' reasoning and problem solving skills of solving quadratic inequalities as mental conceptions were fostered by the GC techniques. This means the limitations in the conceptual aspect hindered the instrumental approach to learning quadratic inequalities in the GC environment. As a consequence, the students' errors that occurred while using the GC revealed a lack of congruence between machine technique and mathematical conception, or indicated limitations in the conceptualization of the mathematics involved. These findings were linked to the results of prior studies such as those of Drijvers (2003), Ndlovu, et al. (2011) and Jupri, Drijvers and Van den Heuvel-Panhuizen (2015) for the instrumented action scheme with related conceptual and technical aspects in a GC environment.

The instrumentation difficulties did play a role in this study and were persistent. A well-known example concerns the scaling of the viewing window of a graphing calculator. The researcher observed that students needed to develop an instrumented action scheme which involves the technical skills of setting the viewing window dimensions of the GC. Some students were hindered to transform the quadratic inequality from algebraic form into the graphic visualization on the graphic interface (the instrumented action schemes). The researcher felt that it was the incompleteness of the conceptual part of such a scheme that caused the difficulties of setting appropriate viewing screens of GC. This means the instrumental genesis sometimes was hindered by conceptual barriers of the students. It was noted that students made syntactic errors in entering negative numbers with a different minus sign than is used for subtraction. In this regard, the instrumentation difficulties were technical as the students were not able to operate the GC in the intended way. This is an indication that some students were still in the process of mastering the GC-utilisation schemes- to accomplish their instrumental genesis. Thus, the instrumentation difficulties were more than it had been expected beforehand. Although the graphic instrumented action scheme facilitated the expected transitions within the topic of quadratic inequalities, the graphing calculator was more an exploratory tool. Students used the tool to explore the conventional symbol system, but not to express themselves by means of their own symbols. This is consistent with the previous findings which revealed that the integration of the GC into teaching and learning often turns out to be more complex than expected (Doerr & Zangor, 2000; Drijvers, 2003; Averbeck, 2000 Guin & Trouche, 2002).

8.3.3 The Vygotsky's socio-cultural theory

The Vygotsky's socio-cultural theory proved to be an adequate framework in this study as it provided opportunities for the teacher--researcher to observe, interpret and understand how the students constructed their knowledge in a GC mediated classroom. This theory also contributed to selecting learning strategies and designing instructional activities for a GC mediated classroom.

The ZPD contributed to the success of this study, more specifically on the organisations of concepts and objects. The teacher-researcher applied in its principles to design learning activities and materials in a graphing calculator classroom. The activities were more experimental and provided students with opportunities to construct their own understanding, significance and meaning through the use of the GC. The designed activities took into consideration of the students' knowledge and difficulties and the GC was used as a planning tool. Taking into cognisance of the students' ZPD, the teacher-researcher started with what the students could do independently based on the prior knowledge in order to link with knowledge that they performed under his assistance. This means the learning activities of quadratic inequalities incorporated real-life problems which were solved with the researcher's help. This is affirmed by the ZPD's principle that providing the appropriate assistance should scaffold the student to perform the task successfully (Vygotsky, 1978; Siyepu, 2013).

An application of the socio-cultural theory's principles helped in modelling sound instructional practices and using the appropriate language. The use of the GC did not substitute the teacher's role instead he assumed his role to guide the students to gain meaningful understanding of quadratic inequalities. Students were viewed as active constructors of knowledge and negotiators in a GC mediated classroom. The well-planned tasks provided opportunity for students to interact, discuss and present their own work in classroom. In their interaction as groups and pairs, students received help from other learners (peers) who were more capable. The theory helped to view learning of mathematics as a human activity, within a socio-cultural setting where students can exchange ideas among themselves using an appropriate language. The teacher as the most knowledgeable made comments on their problem solving efforts in oral and written reflections. This showed that the theory contributed to the promotion of the students' independent thinking and the control of their own learning situation. This theory was also helpful in preventing students from simply memorizing information but to use graphic instrumented action scheme to promote the use of senses to obtain underlying meaning of concepts. This is linked to the findings of the researchers like Noddings (1990), who suggested that students needed building materials, tools, patterns, and sound work habits in order to construct mathematical objects and relationships.

8.3.4. The suitability of the methodology of the study

The design based research (DBR) as a pragmatic methodology played a critical role for attempting to understand and explore how the use of the GC improved the learning of quadratic inequalities at the eleventh grade. This methodology was suitable as it generated scientifically claimed evidence using both quantitative and qualitative data. The non-availability of specific instructional activities, synthesised hypotheses and global theoretical framework for understanding students' learning behaviours in a graphing calculator environment, required the design based study. In addition, this was an exploratory study which needed a research design for reviewing theories, hypotheses and instructional activities during the subsequent research cycles. The use of the graphing calculator in mathematics education affected the existing instructional and assessment methods, as they were not designed for technological instructions. In that regard, an appropriate design study was needed to accommodate the changing learning trajectory in the cycles.

DBR also allowed the researcher to collaborate with practitioners and peers. Peer review of instructional activities assisted in making more explicit the goals and expectations of the study. This methodology permitted the researcher to administer the mini-interviews for identifying the key items and reflecting on the results with the intent of producing feed-forwards for the subsequent cycles. In that context, the cyclic nature of the DBR assisted to generate the design principles and local instructional theory of learning quadratic inequalities in a flexible GC environment. The local instructional theory of the study was as: *The pedagogical use of the GC improves the instructional sequence of quadratic inequalities.* The main design principle of this study was: *Graphically represent quadratic inequalities in a flexible graphing calculator environment.* Other essential design principles that emerged in these three cycles were (i) the training of students to use the GC fluently to reduce chances of the limited viewing window for becoming a source of students' misconceptions and (ii) using the GC cannot address all learning styles, and must be complemented by other traditional methods.

8.4. Conclusions of the study

With the quantitative and qualitative findings of the study, the researcher concluded that the main research objective was achieved i.e., the use of the GC improved the Grade 11 students' understanding of the quadratic inequalities. The GC provided the flexible learning environment which made it possible for students to effectively interact, share mathematical ideas and create models (graphs, diagrams, number lines) which were used for developing their mathematical reasoning and problem solving skills. Students had the opportunity to make multiple representations (symbolic, graphic and numeric) of quadratic inequalities and this helped them to flexibly work within and between these representations. The use of the GC fostered the students' independent mathematical learning with the minimal guidance by the teacher-researcher. This means that the study was able to measure the true effects of the GC use on the students' understanding of mathematical concepts as observed in earlier research. With these results, the researcher was able to produce the evidenced-based heuristics (design principles) and local instructional theory for learning quadratic inequalities in a flexible GC environment. However, graphing calculator cannot orchestrate itself to articulate students' conceptual understanding of quadratic inequalities; the human instructional agent remains indispensable in appropriating it to accomplish the desired didactic goals.

8.5 Recommendations of the study

The researcher recommends the use of the findings for professional development programmes for the FET mathematics teachers so the GC could be used as an intervention tool in their classrooms. This means the findings of this study can be communicated to the teachers to encourage them to integrate the GC for improving the students' reasoning and problem solving skills of quadratic inequalities or related problems. It might be fruitful for the teachers to attend more GC workshops to enhance its efficient application with actual classroom examples of mathematics.

The researcher further recommends for the communication of the findings to the policy makers in the Department of Education about the need for permitting the use of graphing calculators both in the instruction and assessment. It might be beneficial to discuss how the GC can be used in the assessment process in educational workshops, where teachers collaboratively create both tests for the GC and non-programming calculators. The workshop should include the discussion of the nature of the questions to be assessed with the GC. Additionally, the textbook developers, including Umalusi might benefit from teachers' comments about more exploratory activities with GCs. As a result, Umalusi and curriculum developers might consider introducing calculator versions for the NSC mathematics examinations as what obtains in other countries. They might also include different types of GCs for different types of questions based on the goals of the activities.

The researcher might consider presenting different roles of the GC to subject teachers, such as computational, transformational, data collection and analysis, visualization, checking (Doerr & Zangor, 2000). This can be beneficial to them to observe different roles of the GC put into practice when learning mathematics. The findings indicated that the use of the graphing calculator improved students' performance and, their reasoning and problem-solving skills in quadratic inequalities. In addition, the use of the GC provided students with experiences of using the right strategies, connecting newly learned concepts with existing knowledge and reflecting on their solutions to check the appropriateness. The GC facilitated the use of two or more methods i.e., algebraic and graphic representations, which are instrumental for students' understanding of quadratic inequalities. Therefore, there would be a strong motivation to adopt the use of the GC as an instructional tool on a wider basis in South African secondary schools, as learners were found to be "only good at questions involving procedural knowledge in TIMSS, 2011(Reddy, et al., 2013) but poor in problem solving and higher- level cognitive abilities (Spaull, 2013)'. Such findings therefore, might become a valuable asset to inform the mathematics teachers as problem solving and reasoning skills are the cornerstones for understanding a mathematical concept. As an alternative approach, the GC can be used to achieve the main goals of the Department of Basic Education, therefore solving quadratic inequalities by integrating algebraic and graphic methods in the NSC Examination Diagnostic Reports (DBE, 2014; 2015; 2016; 2017).

The experiences gained in conducting the cyclic DBR teaching experiments of this study provided insights from which to make recommendations for future research. Because of its positive effects on students' academic achievement and its effectiveness on students' reasoning and problem-solving, the GC should be given more space in the mathematics education. Despite its undoubtedly significant role, there is a limited literature on and also not much research on graphing calculator in South Africa. This design-based study brings forth a meritorious contribution of an enquiry-based approach to the teaching and learning of mathematics in an explorative manner. There has also been little research done with the learning of quadratic inequalities in South Africa. This study then adds to that literature with reference to the teaching and learning of quadratic inequalities which is limited by the little research. Furthermore, it could be argued that there is a gap in research on teaching and learning with the GC within theoretical frameworks such as the instrumental genesis, RME and Vygotsky' socio-cultural learning theories. Further research thus is needed to fill this gap.

Additionally, future research should focus on the learners of different grades and different concepts to tap the opportunities offered by the pedagogical use of the GC in mathematics classrooms. The same opportunities should be extended to research about the teachers' and preservice teachers' perceptions in South Africa on the use of the GC in solving quadratic inequalities. The pre- and post- survey outcomes (Sections 5.6.7; 6.6.7; 7.6.7) about students' perceptions on the use of the GC to improve problem solving and reasoning could be explored in greater detail. In this case, an exploratory qualitative research could to be adapted to inquiry about how the students' reasoning and problem solving skills were supported in the GC learning environment when solving quadratic inequalities. Within that context, recommendations are made to the use of the exploratory qualitative research for providing valuable insights into how the use of the GC enhanced the students' understanding.

This study used a DBR approach to examine students' understanding of quadratic inequalities in a graphing calculator-enhanced classroom. Therefore, data in this study were collected in three cycles. This entailed that each data cycle was largely dependent on the previous for results and mostly depended on the results of the previous cycle to improve the next. It would be useful to collect similar data from one experimental school and then compare with the control school since it is one of the first of its kind in the South African mathematics curriculum.

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APPENDICES

Appendix A: Session 1: Pre-Test

This activity aims to test the prior knowledge of students in solving quadratic inequalities at the eleventh grade in South African secondary schools.

Instructions to Participants

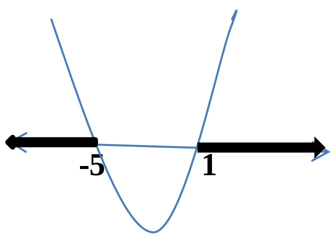
- **Please attempt to answer all questions.**

1. Given an inequality: $(x-2)(x+3)<0$, the solution is

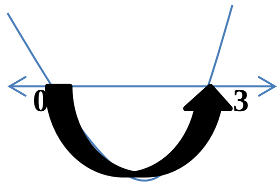
A. $-3<x<2$ **B.** $-2<x<3$ **C.** $x<-2$ **or** $x>3$ **D.** $x<-3$ **or** $x>2$ **(2)**

2. State the solution set of each diagram, indicated in bold black. (4)

2.1.



2.2.



3. Solve for x

3.1. $x^2+3x>0$ **(3)**

3.2. $(x+1)(3-x)\geq 0$ **(3)**

3.3. $(x-3)(x+2)>-4$ **(4)**

4. Given $f(x)=(x-4)^2-8$ **and** $g(x)=-2x+8$

For which values of x is $f(x)\geq g(x)$ **? (4)**

5. Determine the values of x for which $\sqrt{25-x^2}$ will be non-real (4)

6. A small manufacturer's weekly profit is given by $P(x)=-2x^2+70x$, in which x is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R600. (5)

[TOTAL MARKS 30]

APPENDIX B: Pre- intervention surveys

Indicate with an **X** the box that accurately represents your response to each statement. **SA**=Strongly Agree, **A**=Agree, **N**=Not Sure, **D**=disagree, **SD**=Strongly Disagree

ITEM	Students' perceptions about the learning of quadratic inequalities	SA	A	N	D	SD
SPQI 1	Quadratic inequalities are difficult to understand					
SPQI 2	I do not see the difference between the equation and inequality					
SPQI 3	It's difficult to determine the solution sets of quadratic inequalities after finding the critical values.					
SPQI 4	I have difficulties with determining factors of quadratic expressions (inequalities)					
SPQI 5	I don't know the difference between critical values and x-intercepts of the graphs					
SPQI 6	In order to understand the quadratic inequality topic I usually memorise it					
SPQI 7	Of all the topics I have done so far I don't like quadratic inequalities					
SPQI 8	It's difficult to use graphical sketches to determine the solution of quadratic inequality					
SPQI 9	I have never used graphical sketches when solving quadratic inequalities					
SPQI 10	Given an opportunity of not to do quadratic inequalities I was going to do so					
SPQI 11	Technology (e.g., computers) cannot help me to understand mathematics					
SPQI 12	I have never used a programmable calculator to solve quadratic inequalities					
1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree						

Appendix C: Learning Activities in Quadratic Inequalities

Session 2

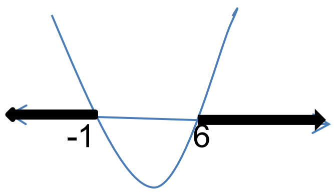
Aim: This session aims to determine the solution sets of quadratic inequalities when given in the form of graphs. Students are expected to use the zeros of the functions as the critical values when determining the inequality solution.

1.1. Given an inequality: $(x+4)(x-1)<0$, the solution is

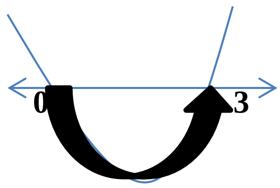
A. $-2<x<4$ **B.** $-4<x<1$ **C.** $x<-2$ or $x>4$ **D.** $x<-4$ or $x>2$

1.2. State the solution set of each diagram, indicated in bold black.

1.2.1.

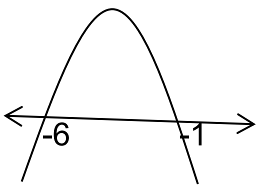


1.2.2.

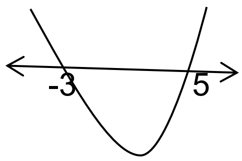


2.1. Write down the solution sets of $f(x)>0$, when $f(x)$ is represented by the drawn graphs.

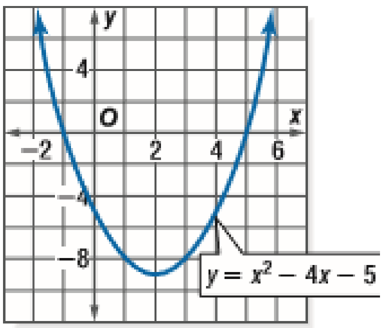
2.1.2.



2.1.2.



2.2. Examine the graph of $y=x^2-4x-5$ and then answer the following questions.



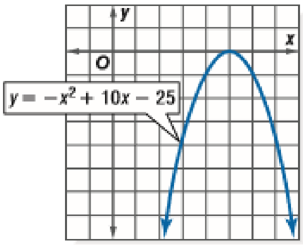
2.2.1 What are the solutions of $0=x^2-4x-5$?

2.2.2 What are the solutions of $x^2-4x-5\geq 0$?

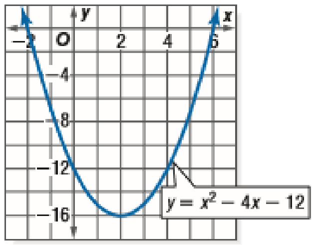
2.2.3 What are the solutions of $x^2-4x-5<0$?

2.3. Use the graph of the related function of each quadratic inequality to write its solutions

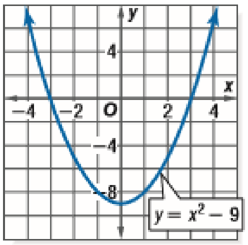
17. $-x^2 + 10x - 25 \geq 0$



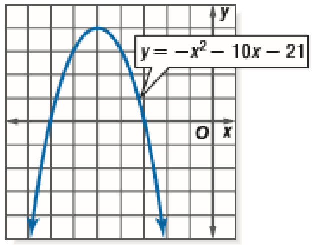
18. $x^2 - 4x - 12 \leq 0$



19. $x^2 - 9 > 0$



20. $-x^2 - 10x - 21 < 0$



Session 3

Aim: This session aims to sketch the graphs of quadratic functions using graphing calculators and then read-off the zeros of these functions. Students are expected to learn how to use the GCs through answering this session.

3. Sketch the graphs of the functions, below and then write down their zeros. You are advised to use graphing calculators when sketching these graphs.

3.1. $f(x) = x^2 - x - 12$

3.2. $g(x) = x^2 + 7x - 8$

3.3. $h(x) = 9x - 4x^2$

3.4. $f(x) = 2x^2 - 5x - 3$

3.5. $g(x) = -2x^2 + 3x + 9$

Session 4

Aim: This session aims to use graphing calculators to determine the solution sets of quadratic inequalities. Students are expected to determine the critical values that could lead to the solution set of the inequality through solving quadratic equations.

4. Use the graphing calculator to determine the solution sets of the following inequalities

4.1. $x^2 - x - 12 < 0$

4.2. $9x - 4x^2 \geq 0$

4.3. $(x-3)(x-4) < 12$

4.4. $-2x^2 + 3x + 9 \geq 0$

4.5. $9 > \ -x(x-6)$

4.6. $-x^2 - 3x + 5 \leq 0$

4.7 $x^2 - 6x + 10 \geq 2$

Session 5

Aim: This session aims to use the graphing calculators to answer the questions involving the application of quadratic inequalities. Students are expected to connect the notion of quadratic inequalities with the previous mathematical concepts in order to answer these questions.

5. Use the graphing calculator to answer these questions involving the application of quadratic inequalities.

5.1. For which values of x will Q=$\sqrt{x^2 - 8x + 12}$ be non-real?

5.2. Determine the domain of the function $f(x) = \sqrt{x^2 - 5x + 6}$

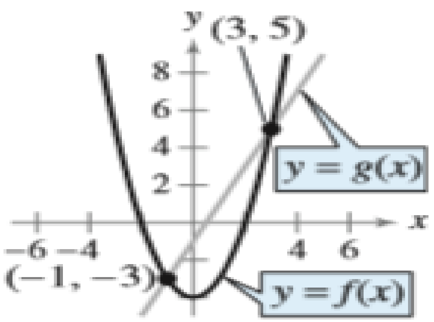
5.3. Given $T_n = 2n^2 - 3n + 2$ of the quadratic sequence. Which term of this sequence is the first term more than 407?

5.4. Sketch the graphs of $f(x) = -x^2 + 2x + 3$ and $g(x) = -2x + 3$ using the graphing calculator.

5.4.1. Determine the solution of $f(x) \geq 0$

5.4.2. Determine the solution of $f(x) \geq g(x)$

5.5. Use the graphs of the functions to solve the following:



5.5.1. $f(x) = g(x)$ 5.5.2. $f(x) \geq g(x)$ 5.5.3. $f(x) < g(x)$

Session 6

Problem-solving quadratic inequality situations

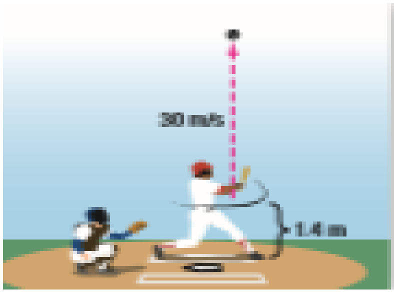
Aim: This session aims to use graphing calculators to solve the contextual problems involving the quadratic inequalities. Students are expected to develop higher-order skills and also to extend their notion of qualitative reasoning in order to solve such complex, non-routine problems.

6. Use the graphing calculators to solve the following contextual problems with the quadratic inequalities.

6.1. The height of a punted football can be modelled by the function $H(x) = -4.9x^2 + 20x + 1$, where the height $H(x)$ is given in metres and the time x is in seconds. At what time in its flight is the ball within 5 metres of the ground? Hint: the function $H(x)$ describes the height of the football. Therefore, you want to find the values of x for which $H(x) \leq 5$.

6.2. Suppose a ball's height (in meters) is given by $h(t) = -5t^2 + 20t$. When will the ball have a height of at least 15 m?

6.3. A baseball player hits a high pop-up with an initial velocity of 30 metres per second, 1.4 metres above the ground. The height $h(t)$ of the ball in metres t seconds after being hit is modelled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does a player on the opposing team have to get under the ball if he catches it 1.7 metres above the ground? Does your answer seem reasonable? Explain.



6.4. If a ball is thrown vertically upward from the ground with an initial velocity of 80 m/s, its approximate height is given by $h(t) = -16t^2 + 80t$, in which t is the time (in seconds) after the ball is released. When will the ball have a height of at least 96 m?

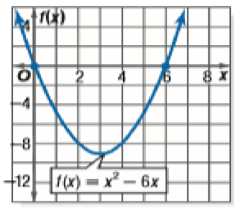
6.5*. A small manufacturer's weekly profit is given by $P(x) = -\,2x^2 + 110x$, in which x is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R1500.

Session 7

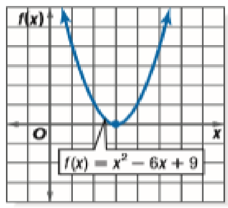
Aim: To solve quadratic inequalities without the use of graphing calculators in preparation for the post- test.

7.1. Without the use of graphing calculator, solve for questions 14, 15 and 16 when $f(x) \geq 0$ and for questions 17, 18 and 19 when $f(x) < 0$.

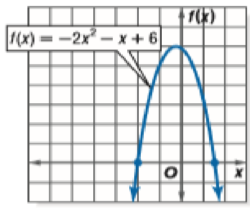
14. $x^2 - 6x = 0$



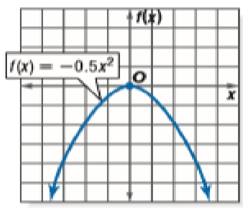
15. $x^2 - 6x + 9 = 0$



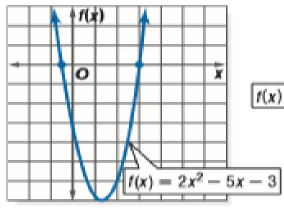
16. $-2x^2 - x + 6 = 0$



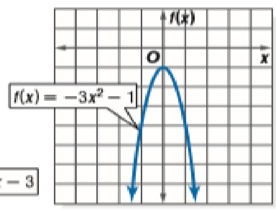
17. $-0.5x^2 = 0$



18. $2x^2 - 5x - 3 = 0$



19. $-3x^2 - 1 = 0$



7.2. Solve the quadratic inequality graphically but without the use of graphing calculator.

7.2.1. $x^2 - x - 12 \leq 0$

7.2.2 $-x^2 - 3x + 28 \leq 0$

7.2.3. $(x-3)(x+5) \leq 0$

7.3. For which values of x will Q=$\sqrt{x^2 - 8x}$ be non-real?

7.4. Sketch the graphs of $f(x) = -2x - 8$ and $g(x) = -2x^2 - 8x$ without the use of the graphing calculator.

7.4.1. Determine the solution of $g(x) < 0$
7.4.2. Determine the solution of $f(x) \geq g(x)$

7.5. If an object is thrown vertically upward from the ground with an initial velocity of 60 m/s, its approximate height is given by $h(t) = -12t^2 + 60t$, in which is the time (in seconds) after the ball is released. When will the ball have a height of at least 72 m?

7.6. A small manufacturer's weekly profit is given by $P(x) = -2x^2 + 70x$, in which x is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R600.

Session 8

Post-test (30 Marks)

Aim: To assess students' understanding of quadratic inequalities after the intervention of the graphing calculator in the 11th grade. In this session students are expected to solve quadratic inequality questions without the use of GC to demonstrate that they have developed reasoning and problem solving skills.

Instructions to Participants

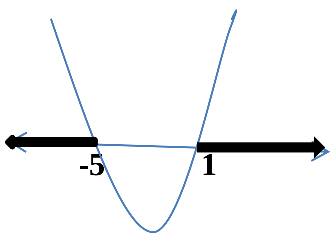
- **Please attempt to answer all questions.**

1. Given an inequality: $(x-2)(x+3) < 0$, the solution is

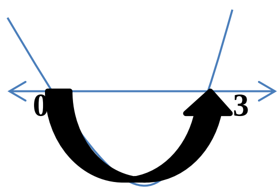
A. $-3 < x < 2$ **B.** $-2 < x < 3$ **C.** $x < -2$ **or** $x > 3$ **D.** $x < -3$ **or** $x > 2$ **(2)**

2. State the solution set of each diagram, indicated in bold black. (4)

2.1.



2.2.



3. Solve for x

3.1. $x^2 + 3x > 0$ **(3)**

3.2. $(x+1)(3-x) \geq 0$ **(3)**

3.3. $(x-3)(x+2) > -4$ **(4)**

4. Given $f(x) = (x-4)^2 - 8$ **and** $g(x) = -2x + 8$

For which values of x is $f(x) \geq g(x)$? (4)

5. Determine the values of x for which $\sqrt{25 - x^2}$ will be non-real (4)

6. A small manufacturer's weekly profit is given by $P(x) = -2x^2 + 220x$, in which x is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R6000. (5)

[TOTAL MARKS 30]

Session 9: Real-life mathematical situations: Linear inequalities

1. The maximum height of a bakkie to enter into Lakeside mall is 1.5 metres. Write down an expression representing this statement.

2. Mr Xhi earns more than R 5000 at his current job. Write down an expression representing this statement.

3. ABC spaza shop makes a profit between R2 500 and R 5 000 per week. Write down an expression representing this statement.

4. How much strength is required to throw a shot put at least 10 metres? Write down an expression representing this statement.

5. The minimum time to finish a piece of task by a learner is 3 hours. Write down an expression representing this statement.

Appendix D1: Post-intervention surveys

Indicate with an **X** the box that accurately represents your response to each statement. **SA**=Strongly Agree, **A**=Agree, **N**=Not Sure, **D**=disagree, **SD**=Strongly Disagree

ITEM	**Students' perceptions about the learning of quadratic inequalities after the graphing calculator intervention (use)**	**SA**	**A**	**N**	**D**	**SD**
SPQI 1	Quadratic inequalities are difficult to understand					
SPQI 2	I do not see the difference between the equation and inequality					
SPQI 3	It's difficult to determine the solution set of quadratic inequalities after finding the critical values.					
SPQI 4	I have difficulties with determining factors of quadratic expressions (inequalities)					
SPQI 5	I don't know the difference between critical values and x-intercepts of the graph					
SPQI 6	In order to understand the quadratic inequality topic I usually memorise it					
SPQI 7	Of all the topics I have done so far I don't like quadratic inequalities					
SPQI 8	It's difficult to use graphical sketches to determine the solution of quadratic inequality					
SPQI 9	I have never used graphical sketches when solving quadratic inequalities					
SPQI 10	Given an opportunity of not to do quadratic inequalities I was going to do so					
SPQI 11	Technology (e.g., computers) cannot help me to understand mathematics					
1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree						

Appendix D2: Post- surveys: The effects of graphing calculator on students' learning activities

Indicate with an **X** the box that accurately represents your response to each statement. **SA**=Strongly Agree, **A**=Agree, **N**=Not Sure, **D**=disagree, **SD**=Strongly Disagree

ITEM	Students' perceptions of the effects of graphing calculator on the designed sessions of learning quadratic inequalities	SA	A	N	D	SD
SPGC 1	The use of graphing calculator in learning sessions assisted me to solve symbolic (algebraic) quadratic inequalities					
SPGC 2	The use of graphing calculator in learning sessions assisted me to understand the difference between critical values and zeros of the graph					
SPGC 3	The use of graphing calculator in learning sessions assisted me to identify correctly the region of the inequality solution					
SPGC 4	The use of graphing calculator in learning sessions assisted me to transform contextual (application) problems into quadratic inequalities					
SPGC 5	The use of graphing calculator in learning sessions assisted me to use graphical sketches when solving quadratic inequalities					
SPGC 6	The use of graphing calculator in learning sessions assisted me to understand the effect of the parameter 'a' in the quadratic inequality					
SPGC 7	The use of graphing calculator in learning sessions assisted me to note that the effect of the parameter 'a' of quadratic function has the same effect on quadratic inequality					
SPGC 8	The use of graphing calculator in learning sessions assisted me to learn and understand much better quadratic inequalities					
1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree						

Appendix D3: Post-survey: The effects of graphing calculator on students' reasoning and problem solving

Please indicate with an **X** the box that accurately represents your response to each statement. **SA**=Strongly Agree, **A**=Agree, **N**=Not Sure, **D**=disagree, **SD**=Strongly Disagree

SPR	**Student's perceptions of the GC use on their reasoning in quadratic inequalities**	**SD**	**D**	**NS**	**A**	**SA**
SPR1	The graphing calculator assisted me to adequately analyse the inequality problem					
SPR2	The graphing calculator helped me use multiple approaches to solve inequality problem					
SPR3	The graphing calculator enabled me to continuously monitor my progress in solving inequality problem					
SPR4	The graphing calculator assisted me to find and use connections of previous mathematical concepts					
SPR5	The graphing calculator guided me to reflect on my solution sets of the inequality problem					
SPPS	**Student's views of the GC use on their problem-solving in quadratic inequalities**	**SD**	**D**	**NS**	**A**	**SA**
SPPS 1	The graphing calculator helped me to understand or interpret the contextual inequality problems					
SPPS 2	The graphing calculator enabled me to formulate quadratic inequalities from contextual problems					
SPPS 3	The graphing calculator assisted me to correctly solve the inequality problem					
SPPS 4	The graphing calculator assisted me to look back upon the solution found					
SPPS 5	The graphing calculator helped me to restate the inequality problem using my own words					
1 = strongly disagree, 2 = disagree, 3=not sure, 4 = agree, and 5 = strongly agree						

Appendix E1: Quadratic inequality test (QIT) rubric for pre- and post-tests

a) Overall rubric

Description	Score
• Completely blank. • Only data were written down, no attempt for solution. • Completely incorrect answer and inappropriate reasoning.	0
• Indicator of a correct strategy was written but no application. • An attempt for solution, but not completed with some unclear workings. • Correct answer but inappropriate reasoning	1
• Correct strategy was found, but the student was not able to apply it or he has not tried hard enough. • Correct answer was found, but there was no indicator as to how it was achieved, essential workings omitted.	2
• Correct strategy was identified and applied, but there was no correct answer due to some calculation errors and misconceptions. • Correct strategy was used and correct answer arrived at but some errors during the representations	3
• Complete and appropriate solution and correct answer	4

b) Rubric for quadratic inequality problem-solving test (QIPST)

Aspect rated	**Scores**			
	0	**1**	**2**	**3**
Understanding the problem	No attempt at all	Completely misinterprets the problem	Substantially interprets the problem	Correctly and completely interprets the problem
Devising a plan	No attempt at all	Completely makes a wrong inequality/model	Substantially makes a correct inequality/ model for the solution	Completely makes a correct inequality/ model and leads to a correct solution
Carrying out the plan	No attempt at all	Completely implements wrong procedures based on inappropriate model or wrong inequality	Substantially implements procedures of the correct inequality and leads to a partially correct solution	Completely and correctly implements procedures of correct inequality and leads to a correct solution
Looking back	No attempt at all	Incompletely check the solution	Substantially checks and evaluate the solution	Correctly and completely check and evaluate the solution

Appendix E2: Rubric for assessing student's reasoning skills in pre- and post-tests

	3	2	1	0
Analysing a problem	The student describes at least three relationships between mathematical concepts or terms.	The student is able to describe two relationships between mathematical concepts.	The student is able to list some mathematical concepts.	No response given
	Student is able to draw at least 2 correct conclusions about the answer to the problem.	Student is able to draw at least 1 correct conclusion about the answer to the problem.	Student draws an incorrect conclusion about the answer to the problem.	No response given
Initiating a strategy	The student lists at least 1 reasonable approach.	The student lists one unreasonable approach.	No response given	N/A
Monitoring one's progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.
Seeking and using connections	Student is able to connect this problem to previous knowledge in math before inequalities or in other subjects in at least two ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in at least one way.	No response given	N/A
Reflecting on one's solution	Student finds correct solution and decides if it is reasonable, and attempts to justify the answer.	Student finds an incorrect or partially correct solution and decides if it is reasonable, and attempts to justify it.	Student finds the correct solution but cannot justify or find out if it is reasonable.	Student finds an incorrect solution and does not decide if it is reasonable
	Student is able to make two correct inferences.	Student is able to make one correct inference.	Student attempts to make an inference, but it is incorrect.	No response given

Appendix F1: Focus group interviews with students

Thank you for doing this interview. You are requested to answer a few questions about problem solving and then solve one of the problems that were on the post-test. At all times it is important that you think out loud so that we can understand how you reason through this problem.

PROBLEM: *A small manufacturer's weekly profit is given by*$P(x) = -2x^2 + 220x$*, in which* x *is the number of items manufactured and sold. Find the number of items that must be manufactured and sold if the profit is to be greater than or equal to R6000.*

1.1 (*The teacher-researcher hands out the problem to the students)*. Please read this problem attentively and then formulate the required mathematical statement.

1.2 Without the use of the graphing calculator, solve this problem showing all the necessary workings. Do not erase any steps.

1.3 State and explain the main concept of this contextual problem.

1.4 Is there any relevant conclusion that you can make about the solution to the problem? ____

1.5 Which approaches do you think were most helpful in solving this problem-algebraic (e.g., factoring, quadratic formula) and/or graphical sketches?

Briefly explain why you selected these approaches.

1.6 Do you think the use of graphical sketch was helpful in solving this problem? Please explain your answer.

1.7 Is it possible to use both algebraic and graphical approaches in solving this problem? If yes explain your answer.

1.8 With the answers you got, do you think you have solved this problem completely and correctly? Justify your reasoning.

1.9 What conclusions can be drawn from the solution of this problem?

1.10 Is there any other relevant information that can be used to justify your solution?

1,11 Using the graphing calculator do you still get the same solution?

1.12. Please explain how helpful was the use of graphing calculator in understanding quadratic inequalities

1.13. Which prior mathematical concepts did you use to solve this problem? Please explain your answer.

Appendix F2: In-depth interviews with students

In this part of the interviews you are requested to express your feelings about how the GC use supported your learning of quadratic inequalities, for example in improving your reasoning and problem solving skills. Thank you for your time.

2.1. Does the use of a graphing calculator make the learning of the quadratic inequalities easier to understand?

____

Why or why not?

____

2.2. Does using the graphing calculator make you feel more comfortable with the quadratic inequalities?

____

Why or why not?

____

2.3. Should graphing calculators be used to solve quadratic inequalities?

____

If you feel so, how should they be used in grade 11?

____

2.4. What is your opinion on the use of the graphing calculators in learning quadratic inequalities?

____

2.5. Does using the graphing calculators improve your understanding when learning quadratic inequalities?

____

If yes, how does this happen?

____

2.6. Does the use of the graphing calculators make you write and complete your homework of quadratic inequalities?

____

Why or why not?

____

2.7. How can you explain your experiences with regard to the use of graphing calculators in the learning of quadratic inequalities?

____

2.8. Do you prefer using other methods than using the graphing calculator to solve quadratic inequalities?

____

If yes which one?

____

2.9. Does the use of a graphing calculator make you score a better mark on quadratic inequality test than any other tests? Explain you answer.

____

2.10.How would you rate your level of understanding of quadratic inequality concept after using graphing calculator, on a scale of 0 to 5, with 5 being very high?

____

APPENDIX G: OBSERVATION SCHEDULE

After each session, the researcher completed the following observation summary. These summaries were guided the researcher to plan for effective use in classrooms and, to help in interviews and with data analysis.

Observation Summary: Graphing calculator use by student

Time:_______ Date:_______

Session No.:_____________

1. Anything observed about the graphing calculator use that was salient, interesting, illuminating, or important

---

2. Anything observed about the graphing calculator use that needs to be followed up in the interviews?

----

3. Observations about the frequent use of graphs or graphing calculators

----

4. Observations about the use of graphing calculators in the reasoning process to solve quadratic inequalities.

----

5. Observations about the use of graphs or graphing calculators in the problem solving skills

---

RUBRIC FOR OBSERVATION

	1	2	3	4
Graphical approach	Does not use any approach to solve a problem	Does not use a graphical approach to solve the problem	Attempts to use a graphical approach but cannot interpret the graph to solve the problem	Uses a graphical approach to solve the problem
Graphing Calculator	Does not use the graphing calculator	Uses the graphing calculator for computation only	Uses the graphing calculator for at least one representation or feature	Uses the graphing calculator for multiple representations
Frequency	Never	Once	Sometimes	Always

APPENDIX H1: Stellenbosch University Research Ethics letter

UNIVERSITEIT
STELLENBOSCH
UNIVERSITY

NOTICE OF APPROVAL

REC Humanities New Application Form

14 February 2018

Project number: 2023

Project Title: Students' understanding of quadratic inequalities in a graphing calculator enhanced mathematics classroom

Dear Mr Levi Ndlovu

Your REC Humanities New Application Form submitted on 15 December 2017 was reviewed and approved by the REC: Humanities.

Please note the following for your approved submission:

Ethics approval period:

Protocol approval date (Humanities)	Protocol expiration date (Humanities)
14 February 2018	13 February 2021

GENERAL COMMENTS:

Please take note of the General Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

If the researcher deviates in any way from the proposal approved by the REC: Humanities, the researcher must notify the REC of these changes.

Please use your SU project number (2023) on any documents or correspondence with the REC concerning your project.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

FOR CONTINUATION OF PROJECTS AFTER REC APPROVAL PERIOD

Please note that a progress report should be submitted to the Research Ethics Committee: Humanities before the approval period has expired if a continuation of ethics approval is required. The Committee will then consider the continuation of the project for a further year (if necessary)

Included Documents:

Document Type	File Name	Date	Version
Research Protocol/Proposal	Levi Ndlovu's Revised proposal 14.12.2017 (1)	14/12/2017	
Informed Consent Form	Student Participant concent Form 14.12.17	14/12/2017	
Data collection tool	students interview guide 14.12.17	14/12/2017	
Request for permission	Levi Ndlovu's Research Request Form- signed	14/12/2017	
Parental consent form	parent consent form 15.12.17	14/12/2017	
Data collection tool	Teacher Observation guide 14.12.17	14/12/2017	
Data collection tool	pretest and post test 14.12.17	14/12/2017	
Default	pricipal consent form 15.12.17	14/12/2017	
Default	Teachers consent form 15.12.147	14/12/2017	

If you have any questions or need further help, please contact the REC office at cgraham@sun.ac.za.

Page 1 of 3

APPENDIX H2: DGE research approval letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**15 March 2018**
Validity of Research Approval:	**05 February 2018 – 28 September 2018** **2017/396**
Name of Researcher:	**Ndlovu L**
Address of Researcher:	**43 Third Avenue**
	Hillcrest
	Benoni 1501
Telephone Number:	**0 11 424 3005 074 521 1866 073 920 8056**
Email address:	**ndlovulevi@yahoo.com**
Research Topic:	**Students understanding of Quadratic inequalities in a graphing calculator enhanced mathematics classroom**
Type of Degree:	**PhD**
Number and type of schools:	**Two Secondary Schools**
District/s/HO	**Ekurhuleni North and Gauteng East**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

22/03/2018

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel. (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix K1: Request for permission to conduct research from the District Director

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

REQUEST FOR PERMISSION FROM DISTRICT DIRECTOR

TO CONDUCT A RESEARCH

STUDENTS' UNDERSTANDING OF QUADRATIC INEQUALITIES IN A GRAPHING CALCULATOR-ENHANCED MATHEMATICS CLASSROOM

Your district has been invited to participate in a research study to be conducted by **Mr Levi Ndlovu**, a PhD student in the Department of Curriculum Studies at Stellenbosch University. This study will contribute to the teaching and learning knowledge base regarding the use of graphing calculators in mathematics classrooms and their use will improve students' learning and understanding of quadratic inequalities.

1. PURPOSE OF THE STUDY

This research intends to improve students' understanding when learning quadratic inequalities. During the study the researcher will integrate graphing calculators in mathematics lessons as supporting interventional tools to enhance the development of students' reasoning, sense making and problem solving skills which are the basic aspects of conceptual understanding. The main focus is to basically minimise misconceptions and errors that may cause students not to solve quadratic inequalities successfully.

2. WHAT WILL BE ASKED OF YOU?

I would like to ask your permission to allow Hulwazi Secondary School and Crystal Park High to participate in this research after school. The following will be done during the study:

Teachers will participate in the following way:

- Teachers will conduct classroom observations for at least 6 hours, when the researcher is implementing the intervention strategy (graphing calculator) in the quadratic inequality classroom. At the end, the teachers will collectively present their findings to the researchers about the effectiveness of the graphing calculators in solving quadratic inequalities.

- Teachers will be interviewed for about 30 minutes on one-to-one basis and their experiences will be video-recorded. Interviews will be conducted during the school days at your school but once in a week.
- You and the participating schools will be given an opportunity to receive a summary of the report. I will also at the end of the research, share feedback with the mathematics teachers on the effects of integrating graphing calculators in the mathematics classroom.

Students will participate in the following way:

- Solve quadratic inequality questions in the pre- and post-tests for research purpose.
- Attend 6 one-hour lessons for solving quadratic inequalities using graphing calculators during the week.
- Be video-recorded in all their class activities that involve the use of graphing calculators to solve quadratic inequalities.
- Be interviewed for 30 minutes on one-to-one and on a focus group basis about their experiences of the use of graphing calculators to solve quadratic inequalities.

3. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts for the teachers and students to participate in the research study.

4. POSSIBLE BENEFITS TO PARTICIPANTS AND/OR TO THE SOCIETY

There are direct possible benefits for the participation of your school in this research. The use of graphing calculator in solving quadratic inequalities will most likely provide opportunities to students to better understand the concept as they will be exposed to symbolic, tabular and graphing representations. Within this graphing calculator context, students' involvement and engagement in class activities might increase due to the fact that teachers will address the content in a meaningful manner. It could be stressed that the use of different teaching methods and models will enhance students' learning and understanding. This research will also equip the participating mathematics teachers with additional pedagogical content knowledge and skills needed to teach quadratic inequalities. This study will further increase students' achievement in solving quadratic inequalities as teachers will be able to identify and select purposeful activities in the graphing calculator-enhanced classroom.

5. PAYMENT FOR PARTICIPATION

Participation of your teachers and students will be free of payment and voluntary that is there will be no remuneration. However, the researcher might provide incentives such as refreshments should funds permit.

6. PROTECTION OF YOUR INFORMATION, CONFIDENTIALITY AND IDENTITY

Any information shared by your teachers and students during this study will be protected. I will not use any name or anything else that might identify your schools, students or teachers in the written work, oral presentations, or publications. The information remains confidential at all times. All data, including field notes and video recordings, will be kept under lock and key and will the electronic versions will be digitally encrypted (password protected). Databases will be destroyed after the research has been presented and/or published which may take up to five years after the data has been collected.

7. PARTICIPATION AND WITHDRAWAL

Participating teachers and students will be free to withdraw at any time, even after they have consented to participate. They may decline to answer at any specific questions.

8. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Levi Ndlovu at 074 521 1865/ 071 920 6056, [ndlovulevi@yahoo.com] and/or the supervisors Professor MC Ndlovu [mcn@sun.ac.za] and Dr H. Wessels at 021 808 3484.

9. RIGHTS OF RESEARCH PARTICIPANTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

If you agree for this research to be conducted at your schools, please sign below. The second copy is for your records. Thank you very much for your help.

____________________________________ ____________________

Signature of District Director **Date**

APPENDIX K2: Letter of consent from principal

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REQUEST FOR PERMISSION FROM SCHOOL PRINCIPAL
TO CONDUCT A RESEARCH

STUDENTS' UNDERSTANDING OF QUADRATIC INEQUALITIES IN A GRAPHING CALCULATOR-ENHANCED MATHEMATICS CLASSROOM

Your school has been invited to participate in a research study to be conducted by Mr Levi Ndlovu, a PhD student in the Department of Curriculum Studies at Stellenbosch University. This study will contribute to the teaching and learning knowledge base regarding the use of graphing calculators in mathematics classroom and their use will improve students' learning and understanding of quadratic inequalities.

10. PURPOSE OF THE STUDY

This research intends to improve students' understanding when learning quadratic inequalities. During the study the researcher will integrate graphing calculators in mathematics lessons as supporting interventional tools to enhance the development of students' reasoning, sense making and problem solving skills which are the basic aspects of conceptual understanding. The main focus is to basically minimise misconceptions and errors that may cause students not to solve quadratic inequalities successfully.

11. WHAT WILL BE ASKED OF YOU?

I would like to ask your permission to allow three of your mathematics teachers to participate in this research. The following will be done during the study:

Teachers will participate in the following way:

- Teachers will conduct classroom observations for at least 6 hours, when the researcher is implementing the intervention strategy (graphing calculator) in the quadratic inequality classroom. At the end, the teachers will collectively present their findings to the researchers about the effectiveness of the graphing calculators in solving quadratic inequalities.
- Teachers will be interviewed for about 30 minutes on one-to-one basis and their experiences will be video-recorded. Interviews will be conducted during the school days at your school but once in a week.
- You and the participating teachers at your school will also be given an opportunity to receive a summary of the report. I will also at the end of the research, share feedback

with the mathematics teachers on the effects of integrating graphing calculators in the mathematics classroom.

Students will participate in the following way:

- Solve quadratic inequality questions in the pre- and post-tests for research purpose.
- Attend 5 one-hour lessons for solving quadratic inequalities using graphing calculators during the week.
- Be video-recorded in all their class activities that involve the use of graphing calculators to solve quadratic inequalities.
- Be interviewed for 30 minutes on one-to-one and on a focus group basis about their experiences of the use of graphing calculators to solve quadratic inequalities.

12. POSSIBLE RISKS AND DISCOMFORTS

There are no risks and discomforts for the teachers and students to participate in the research at your school.

13. POSSIBLE BENEFITS TO PARTICIPANTS AND/OR TO THE SOCIETY

There are direct possible benefits for the participation of your school in this research. The use of graphing calculator in solving quadratic inequalities will most likely provide opportunities to students to better understand the concept as they will be exposed to symbolic, tabular and graphing representations. Within this graphing calculator context, students' involvement and engagement in class activities might increase due to the fact that teachers will address the content in a meaningful manner. It could be stressed that the use of different teaching methods and models will enhance students' learning and understanding. This research will also equip the participating mathematics teachers with additional pedagogical content knowledge and skills needed to teach quadratic inequalities. This study will further increase students' achievement in solving quadratic inequalities as teachers will be able to identify and select purposeful activities in the graphing calculator-enhanced classroom.

14. PAYMENT FOR PARTICIPATION

Participation of your teachers and students will be free of payment and voluntary. That is there will be no remuneration. However, the researcher might provide incentives such as refreshments should funds permit.

15. PROTECTION OF YOUR INFORMATION, CONFIDENTIALITY AND IDENTITY

Any information shared by your teachers and students during this study will be protected. I will not use any name or anything else that might identify your school, students or teachers in the written work, oral presentations, or publications. The information remains confidential at all times. All data, including field notes and video recordings, will be kept under lock and key and will the electronic versions will be digitally encrypted (password protected). Databases will be destroyed after the research has been presented and/or published which may take up to five years after the data has been collected.

16. PARTICIPATION AND WITHDRAWAL

Participating teachers and students will be free to withdraw at any time, even after they have consented to participate. They may decline to answer at any specific questions.

17. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Levi Ndlovu at 074 521 1865/ 071 920 6056, [ndlovulevi@yahoo.com] and/or the supervisors Professor MC Ndlovu [mcn@sun.ac.za].

18. RIGHTS OF RESEARCH PARTICIPANTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

If you agree for this research to be conducted at your school, please sign below. The second copy is for your records. Thank you very much for your help.

______________________________	______________________
Signature of Principal	**Date**

Appendix K3: Student consent form

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STELLENBOSCH UNIVERSITY
STUDENT CONSENT TO PARTICIPATE IN RESEARCH

STUDENTS' UNDERSTANDING OF QUADRATIC INEQUALITIES IN A GRAPHING CALCULATOR-ENHANCED MATHEMATICS CLASSROOM

Your school has been invited to participate in a research study conducted by Mr. L. Ndlovu, from the Curriculum Studies Department at Stellenbosch University. The investigation results will contribute to the development of a PhD thesis. Your school has been selected as a possible participant in this study because this research will help you to understand better quadratic inequalities when you use graphing calculators in the classroom.

1. PURPOSE OF THE STUDY

This research intends to improve your understanding of quadratic inequalities through the use of the graphing calculators in a mathematics classroom. The aim is to minimise misconceptions and errors that usually are stumbling blocks when learners have to solve quadratic inequalities.

2. PROCEDURES

If you agree to take part in this study, you will be asked to:

- Solve quadratic inequality questions in the pre-and post-tests for research purpose.
- Attend 5 hour-lessons for learning quadratic inequalities using graphing calculators during the week.
- Be interviewed for 30 minutes on one-to-one or focus group basis about the use of graphing calculators to solve quadratic inequalities.
- Be video-recorded in all your class activities that involve the use of graphing calculators to solve quadratic inequalities.

3. POSSIBLE RISKS AND DISCOMFORTS

The only possible inconvenience might be attending lessons after school but you will only be involved for 2 weeks. There are no foreseeable risks or discomforts involved in partaking in this research.

4. POSSIBLE BENEFITS TO PARTICIPANTS AND/OR TO THE SOCIETY

There are direct benefits to you for participation in this research, which may include to increase your involvement in the mathematics classroom and to enhance your achievement in quadratic inequalities. This research will explore how your reasoning, sense making and problem solving skills may be developed to answer quadratic inequalities. This research will also equip your teachers with additional instructional skills needed to teach quadratic inequality concept. These additional skills have the potential to increase learners' participation in classroom activities as teachers will deliver the quadratic inequality concept in a realistic context.

5. PAYMENT FOR PARTICIPATION

There will be no payment for participation. Participants take part on a voluntarily basis. However, the researcher intends to provide small incentives such as soft drink with a sandwich should funds permit to encourage attendance of the voluntary sessions.

6. PROTECTION OF YOUR INFORMATION, CONFIDENTIALITY AND IDENTITY

Any information you share with me during this study and that could possibly identify you as a participant will be protected and will remain confidential and will be disclosed only with your permission or as required by law. Confidentiality will be maintained through:

- Not using your name in the final draft of the thesis, but using special coding of the data.
- Storing your personal data on a password-protected personal laptop, which include interview results and field notes, will be kept in a safe at the researcher's home.

Any information gathered from you that includes video-recordings and photographs will be made available to you on request at all times. Your personal data will be accessible to other participants only if prior consent is obtained from you. All materials gathered will be destroyed when no longer needed for the research. This information will only be used for the purpose of this research and publications that may result from the research. The results of the research study will be made available and not the identities of the learners to the school, Department of Education and other researchers on request.

7. PARTICIPATION AND WITHDRAWAL

You can choose whether to be in this study or not. If you agree to take part in this study, you may withdraw at any time without any consequence. You may also refuse to answer any questions you don't want to answer and still remain in the study. The researcher may withdraw you from this study if circumstances arise which warrant doing so to maintain the validity of the data. However, your participation in this study will improve the accuracy of the results because more responses from students better inform the study about how to continue improving students' understanding of quadratic inequalities.

8. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Levi Ndlovu at 074 521 1865/ 071 920 6056, [ndlovulevi@yahoo.com] and/or the supervisors Prof MC Ndlovu [mcn@sun.ac.za] at 021 808 3484.

9. RIGHTS OF RESEARCH PARTICIPANTS

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

DECLARATION OF CONSENT BY THE PARTICIPANT

As the participant I confirm that:

- I have read the above information and it is written in a language that I am comfortable with.
- I have had a chance to ask questions and all my questions have been answered.
- All issues related to privacy, and the confidentiality and use of the information I provide, have been explained.

By signing below, I ________________________________ agree to take part in this research study.

____ ______________________

Signature of Participant **Date**

Appendix K4: Parental consent form

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STELLENBOSCH UNIVERSITY

PARENTAL/GUARDIAN CONSENT FOR CHILD/DEPENDANT T0 PARTICIPATE IN RESEARCH

STUDENTS' UNDERSTANDING OF QUADRATIC INEQUALITIES IN A GRAPHING CALCULATOR-ENHANCED MATHEMATICS CLASSROOM

Your child/dependant has been selected to participate in a research study to be conducted by Mr Levi Ndlovu, from the Curriculum Studies Department at Stellenbosch University. The research results will be communicated in a research report to the Department of Education, aiming to improve the learning of quadratic inequality concept for your child/ dependent.

1. PURPOSE OF THE STUDY

This research intends to investigate students' understanding of quadratic inequalities in the mathematics classroom supported by the graphing calculator. The aim is to minimise misconceptions and errors that usually are stumbling blocks when learners have to solve quadratic inequalities. Students' feedback will help to improve the mathematics performance in the national public examinations.

2. PROCEDURES

If you agree that your child/dependant takes part in this study, s(he) will be asked to:

- Solve quadratic inequality questions in the pre-and post-tests for research purpose conducted at his/her regular school.
- Attend 6 hour-lessons for learning quadratic inequalities using graphing calculators once in a week.
- Be interviewed for 30 minutes on one-to-one or group basis about the use of graphing calculators to solve quadratic inequalities.
- Be video-recorded in all class activities that involve the use of graphing calculators to solve quadratic inequalities.

3. POSSIBLE RISKS AND DISCOMFORTS

The only possible inconvenience might be attending lessons after school but s(he) will only be involved for 2 weeks. There are no foreseeable risks or discomforts involved in partaking in this research.

4. POSSIBLE BENEFITS TO PARTICIPANTS AND/OR TO THE SOCIETY

There are direct benefits for participating in this research as the study will most likely increase student involvement in the mathematics classroom and then enhance his/her achievement in quadratic inequalities. This research will explore how learner reasoning, sense making and problem solving skills are developed to answer quadratic inequalities in a graphing calculator supported environment. The use of graphing calculator in solving quadratic inequalities will provide opportunities to students to better understand the concept as they will be exposed to symbolic, tabular and graphing representations.

5. PAYMENT FOR PARTICIPATION

There will be no payment for participating in the research project by learners. Participants will take part on a voluntarily basis. However, the researcher intends to provide small incentives such as soft drink with a sandwich should funds permit to encourage attendance of the voluntary sessions.

6. PROTECTION OF LEARNER INFORMATION, CONFIDENTIALITY AND IDENTITY

Any information shared by your child/dependant during this study and that could possibly identify him/her as a participant will be protected and will remain confidential and will be disclosed only with your and his/her permission or as required by law. Confidentiality will be maintained through:

- Using pseudonyms and special coding of data in the final draft of the research report.
- Storing participants' personal data on a password-protected desktop and laptops, which include questionnaire and interview results and field notes. Hard copies will be kept under lock and key both at the researcher's office and home.

Any information gathered from the learners that includes video-recordings and photographs will be made available to you on request at all times. Learners' personal data will be accessible to other participants only if prior consent is obtained from learners. All materials gathered will be destroyed when no longer needed for the research. This information will only be used for the purpose of this research and publications that may result from the research. The information collected for this research will be made available to the school, Department of Education and other researchers on request.

7. PARTICIPATION AND WITHDRAWAL

Your child/dependant can choose whether to be in this study or not. If s(he) opts to take part in this study, s(he) will still have the right to withdraw at any time without any consequence. S(he) may also refuse to answer any questions s(he) does not want to answer and still remains in the study. The researcher may withdraw any learner from this study if circumstances arise which necessitate doing so to maintain the validity of the data. However, every learner's participation in this study will improve the accuracy of the results because more responses from students will better inform the study about how to continue improving the understanding and relevance of quadratic inequalities at the eleventh grade.

8. RESEARCHERS' CONTACT INFORMATION

If you have any questions or concerns about this study, please feel free to contact Mr Levi Ndlovu at 074 521 1865/ 071 920 6056, [ndlovulevi@yahoo.com] and/or the supervisors Prof MC Ndlovu [mcn@sun.ac.za] at 021 808 3484.

9. RIGHTS OF RESEARCH PARTICIPANTS

Your child/dependant may withdraw his/her assent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your child's participation in this research study. If you have questions regarding your rights as a research participant, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

DECLARATION OF CONSENT BY THE PARTICIPANT

As the participant, I confirm that:

- I have read the above information and it is written in a language that I am comfortable with.
- I have had a chance to ask questions and all my questions have been answered.
- All issues related to privacy, and the confidentiality and use of the information I provide, have been explained.

By signing below, I ________________________________ agree that my child takes part in this research study.

______________________________________ ______________________

Signature of Parent/Guardian **Date**