Model Uncertainty in the Prediction of Crack Widths in Reinforced Concrete Structures and Reliability Implications

by
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DECLARATION

This research was carried out in the Civil Engineering Programme of the University of Stellenbosch, Stellenbosch under the supervision of Professor Celeste Viljoen. By submitting this dissertation electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Christina Helen McLeod　　　　　　April 2019

ABSTRACT

This research concerns the assessment of model uncertainty of serviceability limit state (SLS) crack models in reinforced concrete structures, where SLS cracking governs design. SLS are treated nominally in design standards without the extensive probabilistic calibration utilised for the ultimate limit state (ULS). Such nominal treatment is not necessarily appropriate when SLS is the governing limit state. A probabilistic approach is therefore taken in assessing SLS cracking in reinforced concrete structures such as liquid retaining structures (LRS), as a precursor to developing a safe but economical design crack width formulation for application in South Africa. In a reliability study of the current Eurocode crack model undertaken by McLeod (2013), model uncertainty displayed a significant influence on reliability. However, it was found that there is little information on the model uncertainty statistical parameters, providing motivation for this research to quantify the model uncertainty of crack models.

A probabilistic approach to SLS cracking is employed. The reliability crack model or General Probabilistic Model (GPM) may be described in this context as the best probabilistic description of expected crack widths. This research therefore included the assessment of the crack models of BS 8007 (1987), BS EN 1992-1 (2004), *fib* MC 2010 (2013) and the proposed amended EN 1992-1 model (Perez Caldentey, 2017) to establish the GPM.

Model uncertainty is defined as the ratio of the maximum experimental to predicted crack widths, w_{exp}/ $w_{predict}$. Experimental data on load-induced cracking is assembled into a database to quantify model uncertainty, including the statistical quantification of the bias and uncertainty of the prediction model under consideration. The interaction between selected model parameters and model uncertainty is investigated by means of Pearson's correlations and linear regression analyses. The crack width model for the GPM is chosen by using the model uncertainty quantification results. Reliability analyses using the First Order Reliability Method (FORM) are done to assess the importance of the respective random variables, including model uncertainty. The study confirms that model uncertainty is the dominant influence on the crack model. Factors such as the relationship between the target reliability and the reliability of the crack width formulations, and the influence of long-term shrinkage strain on reliability are also investigated. Recommendations for future research are also made.

OPSOMMING

Die navorsing ondersoek model onsekerheid van kraakwydte voorspellings in gewapende beton strukture, wat veral relevant is vir die diensbaarheid limietstaat (DLS) waar beheer van kraakwydtes die ontwerp bepaal. Nominale voorsiening vir diensbaarheid is tipies in ontwerpstandaarde, terwyl 'n heelwat meer omvattende probabilistiese aanslag gevolg word in die voorsiening vir die uiterste limietstaat (ULS). Sulke nominale voorsiening is waarskynlik onvanpas waar DLS die ontwerp bepaal. 'n Probabilistiese aanslag word hier geneem in die assessering van DLS kraakvorming in gewapende beton waterhoudende strukture, om die ontwikkeling van 'n veilig maar ekonomiese ontwerpformulering vir Suid-Afrika te ondersteun. 'n Betroubaarhiedstudie deur McLeod (2013) van die huidige Europese kraakontwerp formulering het getoon dat model onsekerheid die betroubaarheid beduidend beinvloed. Daar was egter onvoldoende informasie om die model onsekerhede statisties te kwantifiseer, 'n leemte wat deur hierdie navorsing gevul word.

'n Probabilistiese aanslag word gevolg waar die verwagte kraakwydtes deur die sogenaamde Algemene Probabilistiese Model (APM) beskryf word. Die voorgestelde gewysigde EN 1992-1 model (Perez Caldentey, 2017) bied die beste voorspellings en vorm dus die basis vir die APM. Kraakmodelle van BS 8007 (1987), BS EN 1992-1 (2004), fib MC 2010 (2013) word met behulp van die APM geassesseer.

Die model faktore word bereken as die verhouding van die maksimum gemete tot voorspelde kraakwydtes, w_{exp} / $w_{predict}$. 'n Databasis van eksperimentele kraakvorming onder las is vir die doel versamel, waaruit die model onsekerheid kwantifiseer is deur gemiddelde waardes en standaard afwykings van die model faktore te bepaal vir elk van die voorspellingsmodelle. Dit vorm die basis vir die keuse van APM. Korrelasie tussen model faktore en inset parameters kon ook bepaal word deur Pearson korrellasies en lineêre regressie analise. Die relatiewe belangrikheid van verskillende onseker inset parameters (wat model onsekerheid insluit) is bepaal deur gebruik te maak van die Eerste Orde Betroubaarheid Metode (EOBM). Die studie bevestig die belangrikheid van model onsekerheid as die primere invloed op betroubaarheid vir kraakwydte ontwerp. Die betroubaarheid van die verskillende kraak ontwerp formulerings word assesseer en vergelyk met teikenwaardes. Die invloed van langtermyn krimp onsekerheid op die betroubaarheid word ondersoek. Laastens word aanbevelings gemaak vir toekomende navorsing.

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Last but not least, a big thank you to my family and friends for their love, encouragement and support – to my husband, Johann, and my sons, Alastair and Duncan, in particular, as well as my parents and sister.

"Engineering is the art of modelling materials we do not wholly understand, into shapes we cannot precisely analyse so as to withstand forces we cannot properly assess, in such a way that the public has no reason to suspect the extent of our ignorance."
-AR Dyke at 1946 Chairman's Address to the Scottish Branch of the Institution of Structural Engineers

LIST OF TABLES

LIST OF FIGURES

TABLE OF CONTENTS

CHAPTER ONE
INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

This research concerns the assessment of model uncertainty of serviceability limit state (SLS) crack models in reinforced concrete structures, such as liquid retaining structures (LRS), where SLS cracking is the governing limit state. Durability and impermeability, critical in the function and longevity of this type of structure, are in part ensured by the control of SLS cracking. Cracking is controlled in LRS by ensuring that the predicted maximum crack width for a given configuration is less than a specified crack width limit. In addition, these crack width limits are more onerous in LRS than those of buildings. This emphasises the importance of selecting a crack model that can accurately predict crack widths that will result in a safe but economical design of LRS. To date, SLS have been treated nominally in design standards without the extensive probabilistic calibration that is utilised for the ultimate limit stats (ULS). Such nominal treatment is not necessarily appropriate when SLS is the limit state governing design. In addition, SLS considerations are becoming more important in the design of structures generally. A probabilistic approach is therefore warranted in assessing SLS cracking in reinforced concrete structures such as LRS, as a precursor to developing a safe but economical design formulation.

This probabilistic approach requires the derivation of the reliability crack model or General Probabilistic Model (GPM). The GPM may be described in this context as the best probabilistic description of expected crack widths that can be derived for a given structural configuration. Broadly, the GPM has three essential elements, namely:

(i) A best estimate prediction of expected (mean) crack width for the structural configuration considered – as described by a particular crack model. The GPM, if it is representative of the cracking mechanism, would perform consistently over a range of model parameters.

(ii) An estimate of the inherent bias of the model used to make the prediction, which is the mean value of the model factor (or model uncertainty).

(iii) An estimate of the uncertainty of the prediction which is the variance of the model factor.

In a probabilistic analysis of a structural model, the model parameters are treated as random variables (RV's) thereby taking their uncertainty into account. However, all models have uncertainty that cannot always be described by the stochastic parameters of these random variables, that is, the model uncertainty. The quantification of this model uncertainty is necessary for the GPM to properly describe the structural model over a range of parameters.

Model uncertainty may be treated as an additional random variable in the reliability model and so is quantified by its statistical parameters. In addition, when model uncertainty is significant, a more rigorous approach to quantify model uncertainty is appropriate. In a reliability study of the current Eurocode crack formulation undertaken by McLeod (2013), model uncertainty displayed a significant influence on the reliability of design formulations utilising this crack model. However, it was found that there was little information on the statistical quantification of model uncertainty, providing motivation for this research to quantify model uncertainty of crack models where SLS cracking is dominant and to employ a probabilistic approach in evaluating design formulations used for SLS crack control.

With model uncertainty specific to the model concerned, the selection of a suitable crack formulation for the probabilistic crack model or General Probabilistic Model (GPM) was required. However, the debate over the selection of this crack formulation, added to the motivation in undertaking this research. The South African civil engineering industry currently uses the withdrawn British standard BS 8007 (1987) to design water retaining structures. The current SANS 10100-1 (2004) on the design of reinforced concrete structures is presently under revision, adopting the corresponding Eurocode. At the same time, a new standard that will include the design of water retaining structures, SANS 10100-3 Draft (2015) is in the draft phase of publication. This new standard is based on the Eurocode EN 1992-3 (2004), but the scope is limited to liquid retaining structures in general, and excludes hazardous materials. An investigation of the Eurocode crack formulation is needed to ensure that it is applicable within the South African context. In addition, the Eurocode crack model has been the subject of continuing research in member countries of the European Union, to improve the current design formulation. McLeod (2013) demonstrated that the Eurocode design formulation was conservative compared to that of BS 8007, which has significant economic consequences if adopted in South Africa. In addition, whilst this design formulation results in a reasonable design in the case of buildings where cracking is less critical, it does not predict reasonably the cracking behaviour of structures such as LRS where loads are of long-term duration. Therefore, improvements to this formulation are needed. The release of the *fib* Model Code MC 2010 (Balázs *et al*, 2013), which provides an update of the fundamental crack model on which the Eurocode crack width formulation is based, has prompted proposed changes to the existing Eurocode formulation, as discussed by Pérez Caldentey (2017).

In summary, this research includes the probabilistic assessment of the design crack models of BS 8007 (1987), BS EN 1992-1 (2004), *fib* MC 2010 (2013) and the proposed amended

EN 1992-1 formulation, in addition to quantifying the model uncertainty of these crack models. The application of the crack models is within the context of the design of LRS where the control of long-term load-induced cracking, specifically that due to flexure and direct tension, is critical.

1.2 AIM AND OBJECTIVES

The main aim of this research was to establish model uncertainty of an appropriate crack width prediction model and evaluate the resulting implications on the reliability of that crack model when applied to structures where SLS governs the design. This aim may be subdivided into the following objectives as follows:

(i) Compilation of a database of experimental results for load-induced cracking of reinforced concrete from reputable sources.
(ii) Quantification of model uncertainty of select crack models using said database of experimental results.
(iii) Assessment of model uncertainty of the selected crack models with the aim to identify models that could form a suitable basis for a general probabilistic model (GPM) of crack widths. An ideal prediction model would have low bias and uncertainty, and consistent model factors over the ranges of structural configurations considered.
(iv) Reliability assessment of specific structural configurations and the design formulations of BS 8007 and MC 2010. In principle, this implies assessment of the probability of exceeding a specified crack width limit, given the structural configuration. For this purpose, a GPM derived on the basis of (iii) above is required, importantly including the quantification of model bias and uncertainty.

The calibration of partial factors of the final design formulation of the selected crack model is outside the scope of this research.

1.3 STRUCTURE OF THESIS

The structure of the thesis is as follows:

(i) Chapter 2: Literature review of current crack models and related research used in design codes, including factors influencing model uncertainty and background to standardisation for routine design.
(ii) Chapter 3: Literature review with respect to model uncertainty and reliability analysis.
(iii) Chapter 4: Quantification of model uncertainty with respect to the chosen crack models and identification of models suitable to form the basis of a GPM for crack widths.

(iv) Chapter 5: Deriving the GPM and formulating the limit state, followed by a sensitivity analysis of reliability estimates on model uncertainty and other random variables (RV's) of the GPM.
(v) Chapter 6: Reliability assessment of typical LRS wall configurations and the design formulations of MC 2010 and BS 8007.
(vi) Chapter 7: Conclusions and recommendations.
(vii) Appendices: Data sheets and graphs not presented in the main text.

The first step in this research was to undertake a literature review of research on crack models and related design standards relevant to the design of structures such as LRS, presented in Chapter 2. From this, the key issues relating to model uncertainty in crack models could be identified, which later allowed assessment of the suitability of existing models for use as a basis for a GPM. The literature review assisted in identifying the sources of uncertainty in crack models, essential in quantifying model uncertainty. In addition, the treatment of model uncertainty in probabilistic terms required investigation to develop the methodology for this research. A literature review was thus done on reliability and model uncertainty in general terms, presented in Chapter 3. This review included research on the reliability of crack models, which would then be used in the quantification of model uncertainty and the selection of the crack model for the GPM.

Reputable sources of experimental data on load-induced cracking were identified in the process of carrying out the literature review, and assembled into a database. Sources of uncertainty were identified as far as reasonably possible. Utilising the database, the ratio of the maximum experimental to predicted crack widths, $w_{exp}/ w_{predict}$, was determined which allowed statistical quantification of the bias and uncertainty of the prediction model under consideration. The database was divided into subsets according to the load type and duration, namely, flexural and tension cracking, each over the short- and long-term. Model uncertainty in each case was determined. The interaction between the selected crack model parameters and model uncertainty was investigated by means of Pearson's correlations and linear regression analyses. The crack model for the GPM was selected by using the model uncertainty quantification results to assess the performance of the selected crack models. The quantification of model uncertainty is presented in Chapter 4.

Reliability analyses using the First Order Reliability Method (FORM) were done to assess the importance of the respective random variables in the GPM, including the model uncertainty statistical parameters determined in the quantification of this variable. Typical configurations of LRS were chosen to give context to the reliability analyses. Factors such as the relationship between the target reliability and the performance of the GPM, and the influence of long-term shrinkage strain on reliability were also investigated. These reliability analyses are presented in Chapters 5 and 6.

Chapter 7 presents the final summary of results and conclusions made from this research. Recommendations for future research are also made.

A summary of the research process is given overleaf in Figure 1.1.

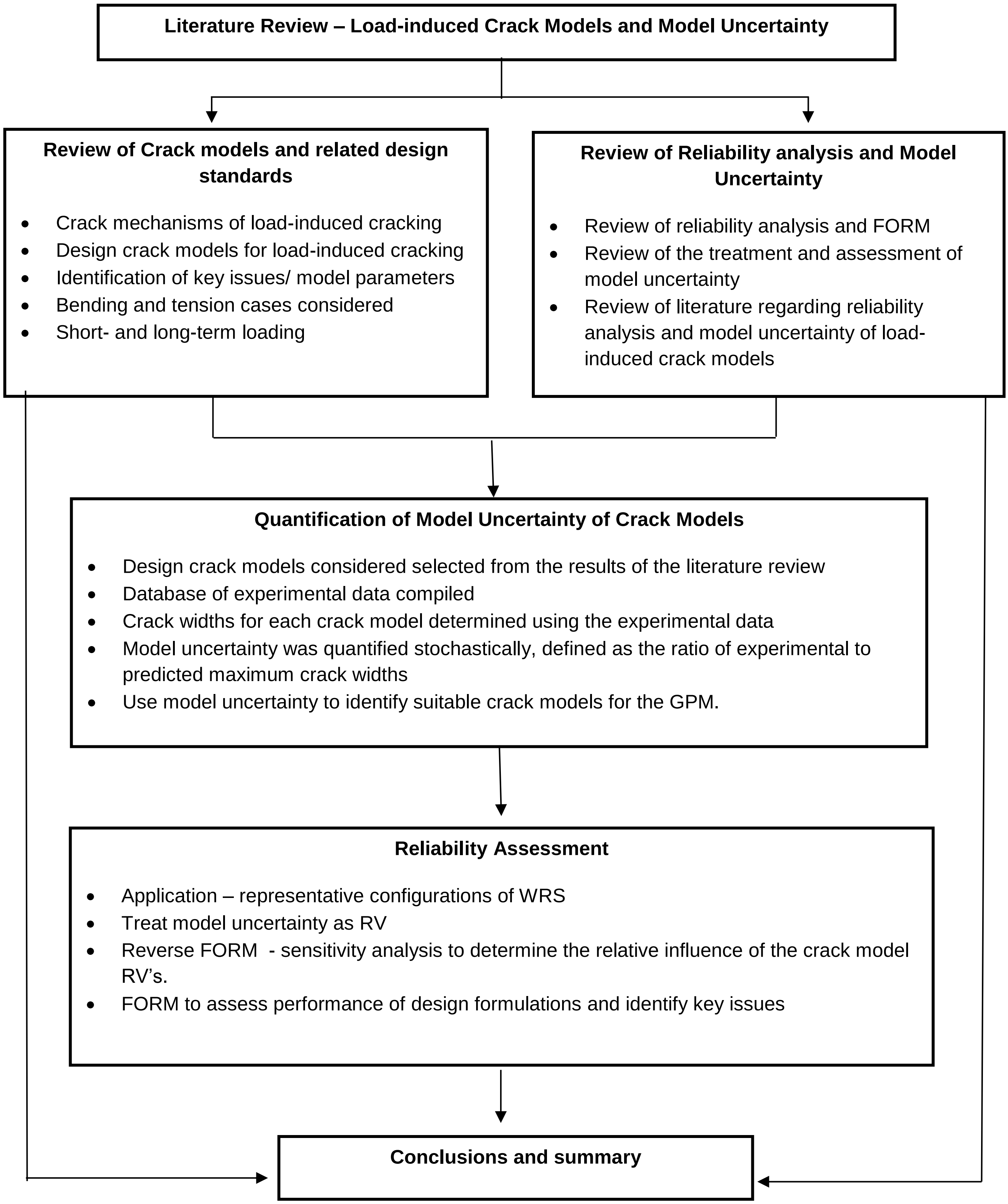


Figure 1.1: *Research process for investigation of model uncertainty of crack widths*

CHAPTER 2
REVIEW OF LOAD-INDUCED CRACK MODELS

2.1 INTRODUCTION

Cracking is considered as a serviceability limit state (SLS) by all modern design codes. To control cracking in structures such as liquid retaining structures (LRS) and so prevent leakage, maximum crack width limits are specified. The maximum predicted crack width is then calculated using the relevant design code formulation. The approaches to these formulations varies across the design standards. Cracking in reinforced concrete may be divided into two broad categories, where cracking is caused by:

- In immature concrete, cracking due to early-age restrained shrinkage.
- In mature concrete, cracking due to loading.

The scope of this research encompasses load-induced cracking, namely due to direct tensile and flexural loads, respectively. Research on flexural and direct tension cracking under short-term and long-term sustained loading was considered, as typical loading conditions in LRS. Cyclic loading, however, was outside the scope of this investigation.

In order to establish a General Probabilistic Model (GPM) applicable to load-induced cracking in South African LRS and its associated model uncertainty, a review of the most common current design code formulations for load-induced cracking was first required. This chapter constitutes a summary of research on some of these crack width formulations used in the design of LRS. The philosophy behind the derivations of these design code formulations is discussed in the following Section 2.2. The chapter presents the design crack width models first, then more detailed discussions of these models, their development and influential parameters is presented. The latter was required in order to identify sources of potential uncertainty in the quantification of model uncertainty of the GPM. A summary of current research on crack models is also given. It should be noted that, in the context of the literature review presented in this chapter, the term ‘crack model’ refers to the model describing the cracking mechanism.

2.2 REVIEW OF CURRENT DESIGN CODE MODELS

As cracking in concrete has a high degree of randomness, there is much debate over the cracking mechanism and therefore the models used to describe this type of load-induced cracking behaviour. Borosnyói and Balázs (2005) reported on over twenty crack width equations, which included design code formulations and improvements, while Lapi *et al* (2018) compared thirty formulations to experiment data, illustrating the extent of the debate. The model formulations used in design codes all assess whether serviceability limit state cracking is acceptable by comparing the predicted crack width to a limiting maximum value. The latter is decided upon depending on the use of the structure and exposure conditions. For liquid retaining structures, the general limiting crack width across most codes is 0,2 mm. Further discussion on crack width limits is given later in this chapter. Design code formulations aim to predict crack widths to at least 90 % fractile of crack widths safety. The maximum predicted crack width is taken as that at the surface of the section in tension.

Crack mechanism philosophies differ in the modelling of the development and transfer of the stresses between the steel and concrete either side of a crack in the section, and the resulting strain incompatibility. As cracking is a serviceability limit state, the stress-strain model of the section in all crack models is that of linear elastic theory. Once the stabilised crack phase has been reached, then no new cracks are formed with existing cracks widening as load increases. In the design of liquid retaining structures , the stabilised cracking phase is assumed to have been reached where all cracks have formed.

The derivation of the crack models for the design codes considered may be divided into four main approaches, namely:

(i) Analytical
(ii) Semi-analytical
(iii) Empirical relationships
(iv) Numerical modelling

Numerical crack models follow two main philosophies, namely, fracture mechanics models (discrete and smeared crack) and tension-stiffening based models. These models are generally used in finite element models. As they are not the basis for the design codes considered, the discussion is limited to the first three approaches.

2.2.1 Analytical Approach

In the analytical approach, the crack mechanism assumed is a non-linear bond-slip model, as described by Balázs (1993). The bond-slip mechanism assumes compatibility between strains in the steel reinforcement and concrete, and that stress transfer is via bond stresses at the interface between these two materials in the tensile zone in the section. Using the equilibrium of a length *dx* of steel reinforcing bar, the slip between the concrete and the reinforcing bar, s(x), may be described by the following:

$$s(x) = \int_0^x [\varepsilon_s(x) - \varepsilon_c(x)]dx$$

where $\varepsilon_s(x)$ is steel strain and $\varepsilon_c(x)$ is concrete strain in tension.

Slip between steel and concrete is assumed to occur in order to maintain the compatibility between the strains in the reinforcement and concrete in the region of cracks. Plane sections are assumed to remain plane once cracking has occurred. A crack occurs when the tensile capacity of the concrete is exceeded. At the crack, the tensile stresses are carried by the reinforcement due to the discontinuity in the concrete. Away from the crack, the tensile forces in the concrete increase as stresses are transferred from the reinforcement by bond action. The distance away from the crack at which strain compatibility between the reinforcement and concrete is regained is referred to as the transfer length. Once the tensile stresses in the concrete again reach the capacity of the concrete, another crack forms. The stabilised cracking phase is considered to have been reached when no new cracks form under increasing load, and the mean crack spacing remains constant. In this case, the mean crack spacing is twice the transfer length. The non-linear bond-slip relationship is illustrated in Figure 2.1

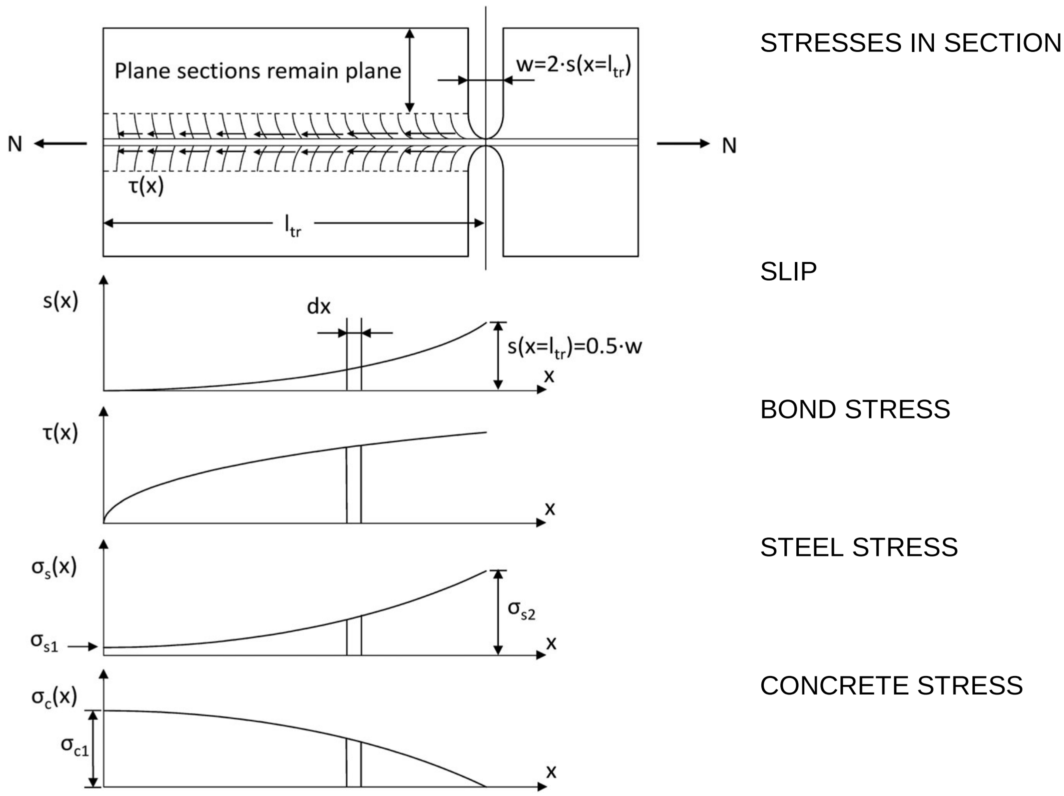


Figure 2.1: *Bond-slip relationship and associated stresses. (After Lapi et al, 2017)*

The analytical approach entails solving the differential equations of bond slip over the crack spacing. Borosnyói and Balázs (2005), on investigating flexural crack models, stated that the rigorous analytical solution of crack width (w) using the bond-slip model may be obtained by integrating the actual strains of reinforcement and concrete between cracks, that is:

$$w = \int_0^{S_r} [\varepsilon_s(x) - \varepsilon_c(x)]dx \qquad (2.1)$$

where S_r is the crack spacing of stresses between steel reinforcement and concrete between cracks. However, the solution of Equation 2.1 is not easily found.

2.2.2 Semi-Analytical Approach

As the analytical solution of equation 2.1 is not easily found, simplifications for code formulation allow for ease of use in design, resulting in semi-analytical formulations. The Eurocode and *fib* MC 2010 crack models are two such semi-analytical solutions, wherein the bond-slip relationship is simplified through the use of an average value of bond-stress, rather than the rigorous solution of bond and slip. Figure 2.2 illustrates the actual and assumed stress distributions of the bond-slip model, where the assumed bond stress is uniform across the section as a mean value.

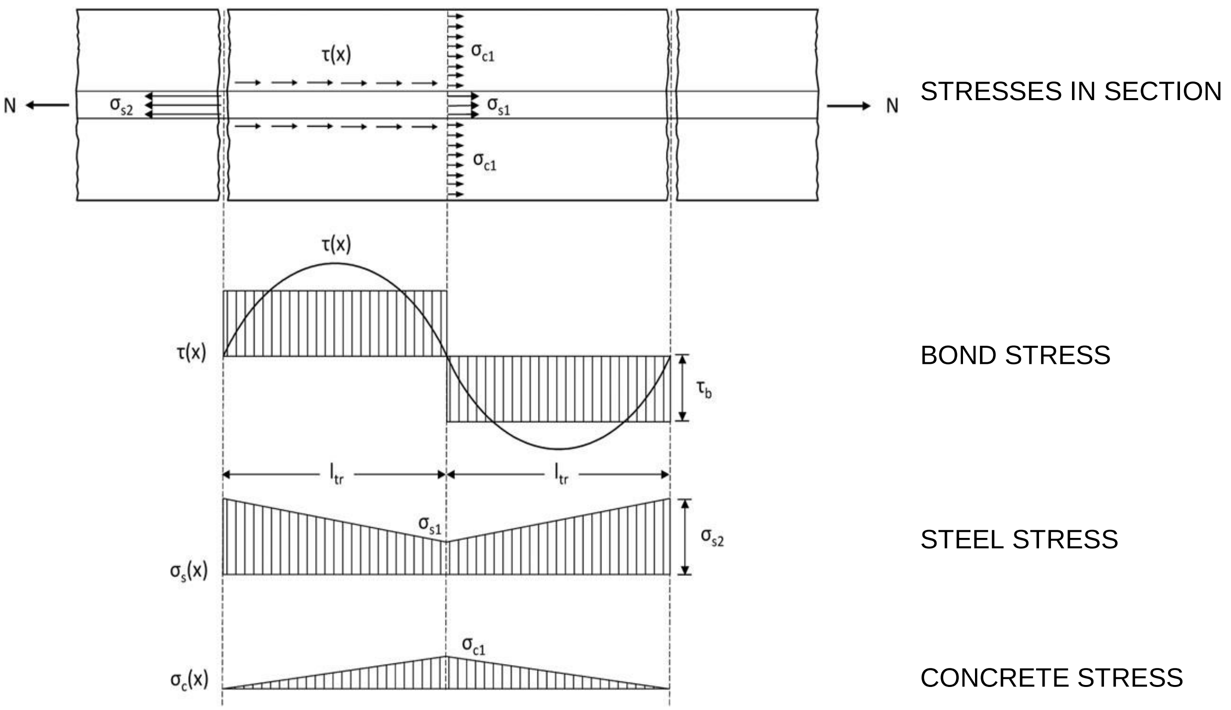


Figure 2.2: *Actual & assumed stress distributions of bond-slip model (Source: Lapi et al, 2017)*

2.2.2.1 Eurocode Crack Model

The Eurocode design code applicable to LRS is BS EN 1992 – 3 (2006) which is read in conjunction with BS EN 1992-1-1 (2004). The EN 1992 crack design equation for cracking was developed for the direct tension case, from the compatibility relationship for cracking in the stabilised crack phase, namely:

$$w_m = S_{rm}.\ \varepsilon_m \qquad (2.2)$$

where w_m is the mean crack width, S_{rm} is average crack spacing and ε_m the mean strain. The mean strain is:

$$\varepsilon_m = \varepsilon_{sm} - \varepsilon_{cm} \tag{2.3}$$

where ε_{sm} is the mean strain in the reinforcement under loading calculated using linear elastic theory. EN 1992 considers the spacing between cracks, equivalent to twice the tensile stress transfer length. The mean concrete strain, also known as tension stiffening, is calculated from:

$$\varepsilon_{cm} = k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 - \alpha_e \rho_{peff}\right) / E_s \tag{2.4}$$

where α_e is the modular ratio E_s/ E_c, $\rho_{p,eff}$ is the effective reinforcement ratio, $f_{ct,eff}$ is the mean tensile strength of the concrete at the time of cracking, E_c is the concrete modulus, and k_t is a factor dependent on the duration of load. EN 1992-1-1 recommends values for k_t of 0,6 for short-term loading and 0,4 for long-term loading. A minimum limit of 0,6 σ_s/E_s (where σ_s is the steel stress and E_s is the steel modulus) is placed on the mean strain as the model was developed for the stabilised cracking phase. At low steel stresses, the tension stiffening effect would be overestimated if the minimum stress limit was not applied, and resulting in crack widths being underpredicted.

For design purposes, the maximum crack width likely to be exceeded is required rather than the mean width. EN 1992 considers this to be that crack width having a probability of exceedance of 5%. This maximum crack width is related to the mean crack spacing by the equation:

$$w_k = (\beta_w S_{rm}).\ \varepsilon_m$$

where $(\beta_w S_m)$ is the maximum crack spacing ($S_{r,\ max}$) and the factor β_w is the ratio of $S_{r,max}$ to S_{rm}. The EN 1992 formulation uses a value of 1,7 for the factor β_w, corresponding to the 95% of crack widths. The EN 1992 design equation for the maximum expected crack width is then:

$$w_k = S_{r,max}.\varepsilon_m \tag{2.5}$$

where the EN 1992 maximum crack spacing is predicted as:

$$S_{r,max} = k_3.c + k_1 k_2 k_4 \varphi\ / \rho_{p,eff} \tag{2.6}$$

where φ is the bar diameter (mm), c is the concrete cover to the longitudinal reinforcement and k_1 is a coefficient taking into account of the bond properties of the bonded reinforcement. The coefficient k_1 has a value of 0,8 for high bond bars. The EN 1992 compatibility relationship was modified to fit flexural cracking behaviour, distinguishing between pure tensile and flexural strain distributions with the distribution of strain coefficient k_2 which is given a value of 0,5 for flexure and 1,0 for pure tension. For combined tension and flexure, intermediate values of k_2 may be calculated from the relationship $k_2 = (\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$, where ε_1 and ε_2 are the greater and lesser tensile strains, respectively, at the boundaries of the section considered assessed on the basis of a cracked section. The values of k_3 and k_4 are determined by European Union individual member countries' National Annexes. EN 1992 gives recommended values of 3,4 and 0,425 for k_3 and k_4, respectively. All k-values are empirical factors. The crack spacing is thus dependent on concrete cover (first term in Equation 2.6), and the ratio of bond stress and concrete tensile strength, and the diameter of the reinforcement (second term in Equation 2.6).

The effective reinforcement ratio, $\rho_{p,eff}$, is calculated as the ratio between the reinforcement area, A_s, and the effective area of concrete in tension, $A_{ct,eff}$. In calculating $A_{ct,eff} = b.\ h_{c,eff}$ where b is the width of the cross section, the effective depth of the tension area ($h_{c,eff}$) is taken as the lesser of h/2, 2,5(h - d) and (h – x)/3. The limiting equation depends on the type of tensile stress as well as the geometry on the section considered. The term 2,5(h – d) can be written in the form 2,5 (φ/2 + c). In other words, the effective depth in tension in this case is dependent on the diameter of the reinforcement and the cover, and independent of section thickness. This term tends to be limiting for the direct tension case, whereas (h – x)/3 tends to be limiting for the flexural case.

2.2.2.2 MC 2010 Crack Model

The MC 2010 crack model is an update on that of MC 1990 (Balázs *et al,* 2013) which was the fundamental model behind the design code crack model. The model was initially derived for the pure tension load case. The *fib* Model Codes form the basis from which the EN 1992 (2004) formulation was derived. Recent research by the *fib* Task Group 4.1 (Balázs *et al,* 2013) led to the development of the following formulation (CEB *fib* Bulletin 66, 2012). The design crack width, w_d, taken as a maximum and applied to the stabilised cracking stage, is determined from:

$$w_d = 2l_{s,max}(\varepsilon_{sm} - \varepsilon_{cm} - \eta\ \varepsilon_{sh}) \tag{2.7}$$

where $l_{s,max}$ is the length over which slip occurs, ε_{sm} is the mean steel tensile strain, ε_{cm} is the mean concrete strain in tension (or tension stiffening) and ε_{sh} is the free shrinkage strain over time. The factor η is zero for short-term cracking, and 1,0 for long-term cracking. As with the EN 1992 crack model, the design crack width corresponds to the 95 % fractile of crack widths. The ratio between the maximum and mean crack widths (β_W) is 1,7, as for EN 1992 (Pérez Caldentey, 2018).The minimum limit on mean strain of EN 1992 is also specified by MC 2010.

Figure 2.3 shows the idealised load-strain relationship for a member in pure tension. The force, N_r is that required to form the first crack. The crack formation stage is idealised here, showing a zero slope, and thus a constant cracking force, N_r.

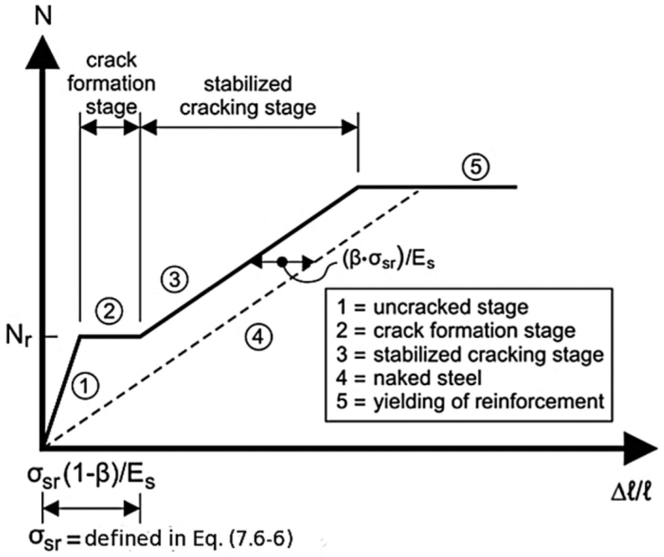


Figure 2.3: *MC 2010 simplified stress-strain relationship for cracking due to loading (Source: Balázs et al, 2013)*

Balázs *et al* (2013) stated that this idealisation was deemed acceptable by the *fib* TG 4.1 committee. Once the applied force N is greater than N_r, then the stabilised cracking phase has been reached. The influence of tension stiffening on the stresses in the section can be seen on the graph.

The mean strain to MC 2010 is determined using:

$$\varepsilon_m = \varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = \frac{\sigma_s - \beta\sigma_{sr}}{E_s} - \eta\varepsilon_{sh} \tag{2.8}$$

The first term of Equation 2.8 is the mean strain over maximum crack spacing due to loading. β is an empirical coefficient for the loading duration which influences bond and tension stiffening, and has a value of 0,6 for short-term and 0,4 for long-term loading. This is the same value taken by EN 1992 for the coefficient k_t. The empirical coefficient η describes the shrinkage contribution and has a value of 0 for short-term loading and 1,0 for long-term loading. The maximum stress, σ_{sr}, at the end of the crack formation stage for a section in pure tension is determined by:

$$\sigma_{sr} = \frac{f_{ctm}}{\rho_{p,eff}}\left(1 + \alpha_e\rho_{p,eff}\right) \tag{2.9}$$

which is the same as Equation 2.4. These equations for mean strain are stated by Balázs *et al* (2013) to be accurate for direct tensile loading but approximate for bending. However, the accuracy in the latter case is deemed to be sufficient.

MC 2010 uses 'transfer length' as opposed to 'crack spacing' where crack spacing is taken as equal to twice the transfer length. The transfer length is determined using:

$$l_{s,max} = k.c + 0.25\frac{f_{ctm}}{\tau_{cms}}.\frac{\varphi_s}{\rho_{p,eff}} \tag{2.10}$$

where c is the concrete cover, k is an empirical parameter to account for the influence of the concrete cover (k = 1,0 can be assumed), τ_{cms} is the mean bond strength between steel and concrete (considered to be evenly distributed between two cracks) and φ_s is the nominal diameter of reinforcing bars. The relationship between the concrete tensile strength and mean bond strength ($\frac{f_{ctm}}{\tau_{cms}}$) is defined as a value of 1,8 for stabilised cracking for both short and long-term loads. The first term of the MC 2010 transfer length equation allows for the influence of concrete cover on the model, which MC 1990 did not include. Research by researchers such as Pérez Caldentey *et al* (2013), showed that this was incorrect, as concrete cover has a significant influence on the transfer length and thus the crack model. The value specified for the coefficient k is smaller than the corresponding k_3 value of EN 1992, as the latter formulation is deemed to overestimate the influence of cover. The MC 2010 crack model is deemed to be valid for c ≤ 75 mm. Further discussion on the choice of values for the fixed-value coefficients is presented in Section 2.3.

A limit on concrete compressive stress of 0,6 $f_{ck}(t)$ is specified to prevent longitudinal cracking under loading which, in turn, leads to an increase in creep. If a value of 0,4 $f_{ck}(t)$ is exceeded under quasi-permanent loading, then creep has a significant effect and the effective elastic modulus for concrete, $E_{c,eff}$, is used. A limit of 0,8 f_{yk} under loading and 1,0f_{yk} under imposed deformations only is given for the reinforcement yield strength (f_{yk}).

2.2.3 Empirical Relationships

These crack models were developed from experimental research and by considering the influential parameters on cracking, such as concrete cover and concrete tensile strength, and are therefore empirical formulations. They were developed with or without explicit expressions for strain and crack spacing.

2.2.3.1 SANS 10100-1 (BS 8007) Crack Model

The South African design code for structural concrete is SANS 10100-1 (2004), based on the British Standard BS 8110 (1997). The crack model currently used by SANS 10100-1 (2004) is the same model as specified by the withdrawn British standards BS 8007 (1987) and BS 8110 (1997). South Africa currently uses BS8007 (1987) in the design of water retaining structures, although the British standards were superseded by the Eurocodes. A new South African standard SANS 10100-3: Design of liquid retaining structures (Draft) 2015 is under development and is based on the equivalent Eurocode standard, however, the BS 8007 crack model has been retained. The BS 8007 crack model was developed originally by Beeby (1979), together with extensive experimental work by various researchers. The main assumptions of this crack model, in contrast to EN 1992, is that there is no slip between concrete and reinforcement, plane sections do not remain plane, and that bond failure does not occur. The model distinguishes between flexural and direct tension loading in the determination of the maximum crack width. The maximum surface crack width, w, for flexure is calculated in BS8007 using:

$$w = \frac{3a_{cr}\varepsilon_m}{1 + 2\left(\frac{a_{cr} - c_{min}}{h - x}\right)} \tag{2.11}$$

The maximum crack width for tension is calculated from the expression:

$$w = 3\ a_{cr}\ \varepsilon_m \tag{2.12}$$

where ε_m is mean strain, c_{min} is the concrete cover, h is the section depth and x is the depth from the compression face of the section to the neutral axis. A minimum nominal cover of 40 mm is specified. Crack spacing is assumed to be a function of a_{cr}, that is, the distance from the crack considered to the nearest longitudinal reinforcing bar. According to Lapi (2017), the BS 8007 maximum crack width is taken as the value which has a probability of exceedance of 20 %.

This distance is considered as a maximum at a point mid-way between reinforcing bars in the case of a slab or wall, where a is the distance from the surface crack to the centre of the reinforcing bar, c is concrete cover, s is spacing of reinforcement and φ is bar diameter, as illustrated in Figure 2.4.

The distance a_{cr} is then:

$$a_{cr} = a - \varphi/2$$

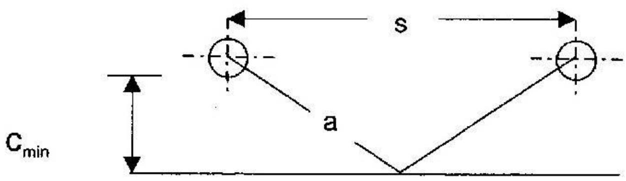


Figure 2.4: *Distance from surface crack to centre of bar for slab or wall section*

Guidance on calculating a_{cr} is given by references such as Bhatt, MacGinley and Choo (2006). The BS 8007 crack model for load-induced cracking therefore does not determine the crack spacing, which differs from the models of EN 1992 and MC 2010.

The average strain, ε_m, is calculated from:

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 \tag{2.13}$$

where ε_1 is the apparent strain at the surface and ε_2 is the tension-stiffening effect of the concrete in tension.

The apparent strain at the level of the tension reinforcement (ε_s) is calculated using elastic theory. The apparent strain at the surface (ε_1) for flexure is then determined from:

$$\varepsilon_1 = \varepsilon_s \frac{(h - x)}{(d - x)} \tag{2.14}$$

where d is the effective depth. The average strain is calculated by deducting the effect of the concrete in tension, i.e., tension stiffening from the apparent strain, ε_1. In the tension case, the apparent strain (ε_1) is equal to the steel strain (ε_s).

The equations to calculate tension stiffening strain for flexure were derived empirically and are dependent on the value chosen for the maximum crack width limit (w_{lim}) as follows:

$$\varepsilon_2 = \frac{b_t(h-x)(a'-x)}{3E_sA_s(d-x)} \qquad \text{for } w_{lim} = 0{,}2\text{ mm}$$

or,

$$\varepsilon_2 = \frac{1{,}5b_t(h\text{-}x)(a'\text{-}x)}{3E_sA_s(d-x)} \qquad \text{for } w_{lim} = 0{,}1\text{ mm} \tag{2.15}$$

where a' is defined as the distance from the compression face to the level at which the crack width is being considered, equal to the depth of section (h) in the case of a reservoir wall under bending. E_s is the steel modulus of elasticity, b_t is the width of the section in tension and A_s is the area of the tension reinforcement.

Tension stiffening strain for the tension case is

$$\varepsilon_2 = \frac{2b_t h}{3E_s A_s} \qquad \text{for } w_{lim} = 0{,}2 \text{ mm}$$

or,

$$\varepsilon_2 = \frac{b_t h}{E_s A_s} \qquad \text{for } w_{lim} = 0{,}1 \text{ mm} \tag{2.16}$$

These tension stiffening equations are specific to the type of stress due to loading and the limiting crack width. Interpolation for other crack widths therefore cannot be done which limits the application of the crack model. The BS 8007 crack width formulations are valid for a limit of 0,8 f_y/E_s for the strain in the tensile reinforcement, where f_y and E_s are the yield stress and steel modulus of the reinforcement, respectively. A limit of 0,45 f_{cu} is also placed on the concrete stress for flexural cracking, where f_{cu} is the concrete cube compressive strength at 28 days. In addition, the concrete compressive strength (cube at 28 days) should not be less than 35 MPa.

2.2.3.2 ACI Design Codes for Structural Concrete

The American concrete design codes for structural concrete treat crack control at the serviceability limit state by providing an appropriate minimum reinforcement area in the tensile zone, limiting spacing of reinforcement considerations and limiting the service tensile steel stress depending on the reinforcement provided. ACI 350-06 (ACI committee 350, 2006) concerns the design and construction of environmental engineering concrete structures including LRS and is read in conjunction with ACI 318-14 (2014) (which covers the design of concrete buildings). ACI 350 has recommendations for mix design for durability and leakage considerations. To enhance the durability and in some cases the appearance of members, ACI 318-11 (2011) prescribes rules on crack control and distribution of flexural steel. According to ACI 318, the spacing of reinforcement closest to the tension face of a flexural member must not exceed

$$s \le \begin{Bmatrix} 15\left(\dfrac{40000}{f_s}\right) - 2.5c_c \\ 12\left(\dfrac{40000}{f_s}\right) \end{Bmatrix}$$

where s is the centre-to-centre spacing of the reinforcement, f_s is the calculated service load steel stress and c_c is the least distance from the surface of the reinforcement to the extreme tension face. These limits are imposed to control cracking. The steel stress may be calculated using linear elastic theory or may be approximated as 2/3 f_y.

ACI 224.1R (2007) recommends replacing the crack width model developed by Gerley & Lutz with the empirically-derived crack model to Frosch for flexural members. The maximum allowed crack width, w_k, in SI units according to Frosch (2014), is determined using:

$$w_k = 2.\frac{\sigma_{s2}}{E_s}(1+0.0031(h-d)t_c)$$

where t_c is the effective distance between the crack considered and the nearest reinforcing bar, calculated using:

$$t_c = \sqrt{\left(\frac{a}{2}\right)^2 + (h-d)^2}$$

and a is bar spacing, h is depth of section, d is the effective depth, E_s is the steel modulus of elasticity and σ_{s2} is the stress in reinforcement after cracking. Figure 2.5 is an illustration of the crack model to Frosch.

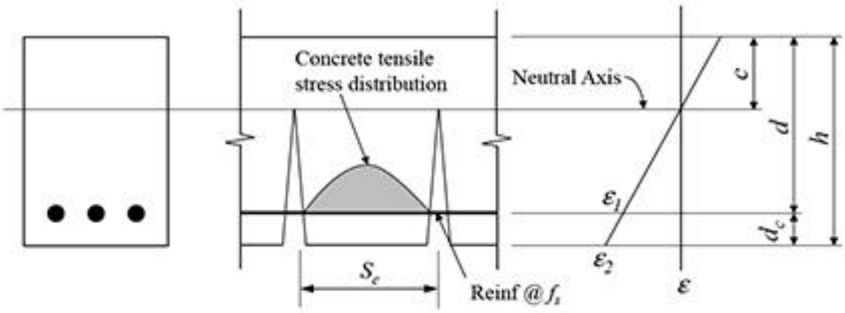


Figure 2.5. *Cracking Model to Frosch (Frosch, 2014)*

Similar to the BS 8007 crack model, the effective distance taken as equal to the distance from the centre of the reinforcement to the point on the cover furthest from the reinforcement (d_c) as shown in Figure 2.6.

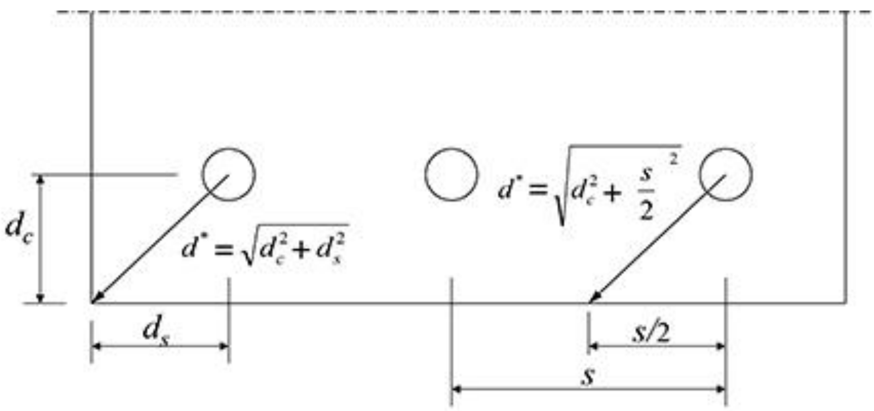


Figure 2.6: *Determination of crack spacing to Frosch crack model (Frosch, 2014)*

The ACI design standards have separate tension and flexure crack models, as BS 8007. The tension crack model formulation, in SI units, is:

$$w = 0.0605 f_s \sqrt[3]{d_c A}\ 10^{-3} \qquad \text{in SI units.}$$

where f_s is steel stress, d_c is the distance as illustrated in Figure 2.6, and A is the area of concrete symmetric with the reinforcement divided by the number of bars.

2.2.3.3 Australian Crack Model

The Australian structural concrete design codes that relate to LRS are AS 3600 (2011) and AS 3735 (2001), for general structural concrete and LRS, respectively. The Australian standards have a similar approach to crack control as the American design codes, treating crack control in terms of the member type, for example, Section 8.6 deals with crack control for tension and flexure in beams, whilst Section 9.4 deals crack control for flexure in slabs. Crack control as a serviceability limit state is deemed to be controlled by providing the appropriate minimum reinforcement area in the tensile zone, limiting the maximum spacing of reinforcement and limiting the calculated tensile steel stress depending on the reinforcement provided. Formulations for the calculation of crack width are not currently specified within the design standards. According to Gilbert (2018), the Australian concrete codes are undergoing a similar updating exercise as the equivalent South African standards with a move towards the Eurocodes proposed with modifications to the EN 1992 formulation to include shrinkage strain where applicable.

2.3 REVIEW OF RESEARCH ON CRACK MODELS

The process to establish the GPM crack model included a review of research on crack models. Influential factors within the models were identified and discussed in the following sections.

2.3.1 General Comments on Crack Models

Various researchers have compared existing code formulations and proposed crack models in an effort to improve current formulations. A wide spread of results was noted when measuring and calculating crack widths in most formulations, especially when considering sustained loading. Allam *et al* (2012) performed a crack width evaluation on flexural reinforced concrete members using the Egyptian crack model, which has a similar format to that of EN 1992, in addition to the crack models of Eurocode, and the British and the American standards. A wide spread of results was obtained from each model for the calculated crack width for short-term loading. Code formulation crack widths were compared to those obtained in short-term loading experiments done by other researchers such as Makhlouf and Malhas (Allam *et al,* 2012) on beams 400 mm deep by 600 mm wide (to address scale effect) for various steel stresses and maximum measured crack widths, over a range of reinforcement ratios and a 50 mm concrete cover. Eurocode was found to be conservative for lower steel ratios (0,62 %) and stresses up to approximately 230 MPa. For a higher reinforcement ratio of 1,4 %, EN 1992 was found to underestimate crack width by as much as a ratio (of EN 1992 to test) of 1:1,3 at higher steel stresses. For lower steel ratios, the Eurocode model overestimated crack width. It was also noted that most models underestimate crack widths at low steel stresses except Eurocode as Eurocode limits the tension stiffening effect (which is obviously more influential on the model at lower steel stresses) by means of a minimum limiting mean stress. Of importance to South African designers is that it appeared that BS 8007 underestimated crack widths at higher steel stresses. Earlier research by McLeod (2013) comparing BS 8007 and EN 1992 crack width formulations in a deterministic analysis using typical LRS wall configurations with the reinforcement ratio as the comparative parameter, determined that the EN 1992 formulation was significantly more conservative than BS 8007, especially for the tension loading case (> 38 %), for a limiting crack width of 0,2 mm and depending on the configuration. Similar conclusions were reached by Kruger (2013) using case studies of existing structures. This has economic implications for the design of South African LRS.

There is on-going debate regarding the EN 1992 crack width and crack spacing formulations, particularly on the crack spacing formulation itself and the fixed-value coefficients. The crack model has been found to be too conservative for short-term loading - all research to date points to this. In addition, EN 1992 does not model long-term cracking behaviour well, underestimating crack widths as a result. This is in part due to this model neglecting long-term effects such as long-term shrinkage strain, and is discussed further in Section 2.3.7. Some of the values of the EN 1992 Nationally Determined Parameters (NDP's) are summarised in Table 2.1 below. As a comparison, the values using the strictly analytical solution have been included. All equations have the general form, as described by Perez Caldentey *et al* (2013):

$$S_r = k_1.c + k_2 \frac{\varphi}{\rho_{eff}} \tag{2.17}$$

where k_1 is a general factor taking into account the effect of concrete cover, k_2 is a general factor for the influence of φ/ρ_{eff}, c is concrete cover, φ is bar diameter and ρ_{eff} is the effective reinforcement area. Referring to Equation 2.6, k_1 is equivalent to k_3 and k_2 is equivalent to (k_1 k_2 k_4) of EN 1992. The first term of Equation 2.17, $k_1.c$ is explained as the distance required to transmit tension forces from the bar surface to the centre of the effective cross section in tension (that is, over the depth of the concrete cover). The second term in Equation 2.17, $k_2.\varphi/\rho_{eff}$, is described as a 'direct consequence of the definition of the transfer length' and can be derived from the equilibrium of stresses between a crack and the zero slip section.

Table 2.1*: Summary of k_i values as applied to maximum crack spacing formulation*

Coefficient	Strictly Analytical (Burns, 2011)	EN 1992	EN 1992 (France) (Kruger, 2013)	EN 1992 (Germany) (Kruger, 2013	MC 2010
Bond Ribbed bars	0.5	k_1 = 0.8	0.8	1	f_{ct}/τ_{bms} = 1/1.8 = 0.556
Type of Loading, k_2	1	1 (tension) 0.5 (bending)	1 (tension) 0.5 (bending)	1	1
Concrete cover, k_3	0	3.4 (= 1.7 x 2) Includes factor $S_{r,max}/S_{rm}$ = 1.7	$c \leq 25$: 3,4 $c > 25$: 3,4 $(25/c)^{2/3}$	0	2 x k_1 2 x1.0 = 2
Factor, k_4 $S_{r,max}/S_{rm}$	2 x 0.5 (= 1.0)	0.425 (= 1.7 x 0.25)	0.278 (= 1.112 x 0.25)	0.425 (= 1.7 x 0.25)	2 x 0.25 = 0.5

The inclusion/ exclusion of the k_2 factor stems from the assumed stress distribution in the section under bending. EN 1992 considers that the stresses in the section have a triangular distribution, therefore it is necessary to transmit half the tensile force to develop a new crack in contrast to the uniform stress distribution of a section under direct tension. This means a corresponding halving of the bond term of the crack spacing equation. MC 2010 assumes that the stress variation in a section under bending within the effective concrete area (in tension) around the reinforcement is minimal, therefore the k_2 factor is not applied.

Referring to Table 2.1, the main issues under discussion are the influence of concrete cover and its inclusion in the crack model formations, the stress distribution in the element section resulting from the type of load, the influence of bond stress and the influence of φ/ρ_{eff} (the second term of Equation 2.17 for crack spacing). The extent of the influence of tension stiffening in the determination of mean strain as well as the significance of transverse reinforcement, requires investigation beyond the scope of this study, given that most crack models neglect these factors.

The following sections discuss these points in more detail.

2.3.2 Significance of Concrete Cover

Concrete cover, as already stated, influences the transfer length and crack spacing, thus crack width. The extent of the influence of concrete cover and the subsequent use of the concrete cover term in the crack spacing formulation in the semi-analytical solution has been the subject of much debate, as its inclusion is in conflict with the analytical solution. Beeby (2004) showed that concrete cover was a significant variable in determining crack spacing and width, rather than the term φ/ρ_{eff} of the crack spacing equation, on studying experimental data for specimens under flexure or pure tension. This indicates that crack models should take concrete cover into account in the crack spacing formulation. Hossin and Marzouk (2008) tested two-way spanning panels under short-term flexural loading, with a central point load on the panel, in order to assess crack spacing models. They found that an increase in concrete cover resulted in an increase in crack spacing, a doubling of the concrete cover corresponding to an approximately 40% increase in crack spacing, which is significant.

Concrete cover influences the surface crack width. The size of crack at the reinforcing bar is generally small (approx. 0,05mm), with crack width increasing away from the bar. This implies that a greater cover results in a greater crack width. So-called Goto cracking (secondary internal non-passing cracks near the bar surface especially at ribs in ribbed bars) helps to distribute slip and reduce crack opening at bar, but strain is then concentrated in the main cracks, resulting in increased crack width away from the bar and much larger surface cracks. A smaller concrete cover leads to some of the Goto cracks becoming primary cracks (that is, passing through the depth of the cover). However, a larger concrete cover implies fewer but larger surface cracks and less internal cracks with the same small crack openings at the reinforcing bar. According to Balázs *et al* (2013), despite initial proposals by some members of the *fib* TG 4.1 committee to exclude concrete cover from the transfer length/crack spacing formulation (as per the MC 1990 crack model), the final decision was to include it but to multiply by a smaller coefficient compared to the recommended EN 1992 value.

Borosnya and Snóbli (2010) investigated the crack width variation within the concrete cover of reinforced concrete members for short-term loading. They performed preliminary studies looking at the influence of concrete cover in tie elements. They noted that previous studies found that concrete cover influences the surface crack width but that investigation of the extent of that influence was still needed. Their results lead to the important conclusion that crack spacing and surface crack width are strongly dependent on concrete cover. This corroborates the findings by Hossin and Marzouk (2008) on comparing their experimental results to the MC 1990 crack spacing equation (which does not take concrete cover into account), the MC 1990 equation underpredicted the crack spacing by almost 50 %. Gilbert and Nejadi (2004) and Castel and Gilbert (2014) found that a larger cover results in wider cracks and crack spacing in separate tests on beams. As found by other researchers, they surmised that this is because some Goto cracks initiating at the bar will propagate through the cover to the surface, particularly if the cover is large. Their investigation also found that the influence of cover is less in long-term cracking than in the short-term. They concluded that this was due to long-term effects such as long-term shrinkage.

Referring to Table 2.1, research from France (Kruger, 2013) suggests that the influence of cover on crack spacing does not appear to be constant for all covers but is constant up to concrete covers of 25 mm, thereafter decreasing linearly. Balázs *et al* (2013) summarised earlier research by Beeby and Pérez Caldentey *et al,* the final conclusion of which was that concrete cover had a clear influence on crack spacing and width, therefore must be included in the crack spacing equation. Pérez Caldentey *et al* (2013) commented that cover does influence crack width and that models ignoring cover (for example, MC 1990) are 'incomplete'. It was also found that the greater the cover, the greater the crack spacing and corresponding crack width for beams under short-term loads. The mean crack spacing was described in general terms using a linear two - parameter equation, given here as Equation 2.17. Referring to Equation 2.17, as stated earlier, the coefficient K_1 here is equivalent to EN 1992 k_3, while K_2 relates to EN 1992 ($k_1.k_2.k_3$). A value between 1 and 2 times the concrete cover was recommended for K_1.

It was reported by the CEOS.fr project (2016) on thick elements that the crack spacing of the form of Equation 2.17 above is sufficient in simple cases. This would include structures such as LRS.

Considering the results of multiple researchers, it must be concluded that the concrete cover is influential in crack mechanisms therefore should be included in crack spacing formulations, for both short- and long-term cracking, although the influence of concrete cover appears to be less for long-term cracking.

2.3.3 Effective Area in Tension, $A_{c,eff}$

The effective concrete area in tension used by MC 2010 and EN 1992 is defined as shown in Figure 2.7.

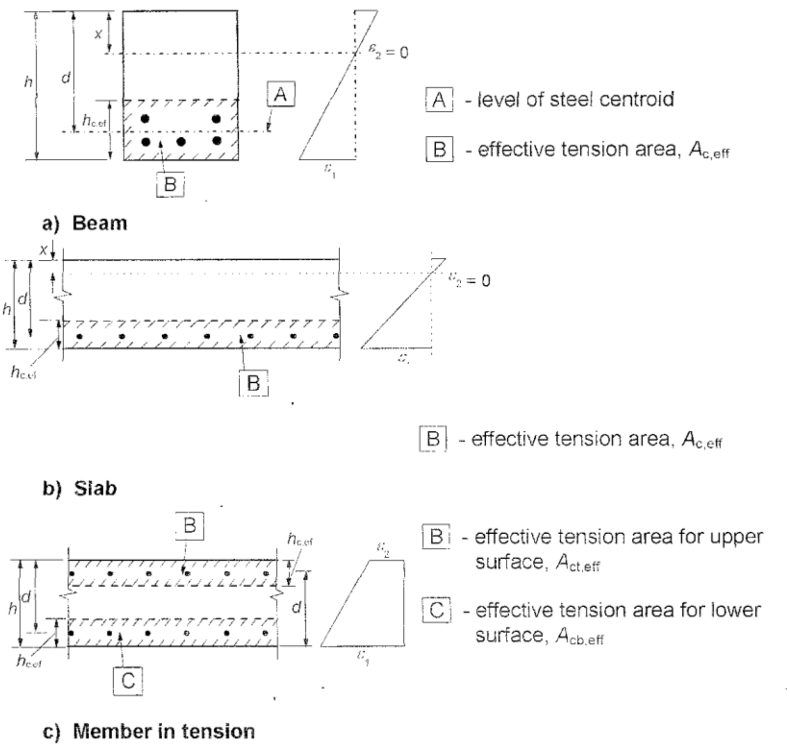


Figure 2.7: *Determination of $A_{ct,eff}$ (Source: Figure 7.1 of EN 1992-1-1)*

The rectangular areas considered are idealised with the true shape nearer ellipsoidal in shape, as shown in research by Eckfeldt (2008b) on tension members. As reported by Balázs *et al* (2013), Eckfeldt proposed a change in the MC 2010 equations to a revised model for $A_{c,eff}$ which is based on a ellipsoidal shape turned into an idealised circle with a reinforcing bar as the centre, given here as Figure 2.8.

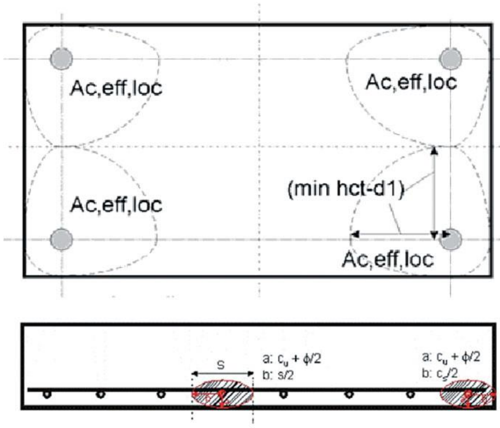


Figure 2.8: *Non-quadratic proposals for effective area of concrete in tension (Source: Balázs, 2013)*

However, it was decided by the *fib* TG 4.1 committee that as the rectangular areas give a reasonable approximation of the effective area in tension, this simplification was justified.

2.3.4 Bond Stress in Crack Spacing Formulation

Crack models such as EN 1992 and MC 2010 take bond stress and slip into account in crack spacing formulations through the use of a fixed-value coefficient, or as a ratio between concrete tensile strength and mean bond stress, respectively. Bond stresses and slip between the concrete and reinforcement cannot be measured directly, generally being determined by measuring the loss of steel stress in the reinforcing, adding some uncertainty into determining bond-slip relationships.

EN 1992 considers a value of 0,8 for the fixed bond coefficient, k_1, which is conservative and empirically-derived. MC 2010 defines the bond-slip relationship for low levels of slip under service conditions, as:

$$\tau_{bs} = \tau_{max} \cdot \left(\frac{s}{s_1} \right)^{\alpha} \quad \text{where } 0 \leq s \leq s_1$$

where τ_{bs} is bond stress, τ_{max} is bond strength depending on mean concrete compressive strength, s is slip, s_1 is the slip corresponding to the bond strength, and α is an experimental constant with value between 0 (non-linear) and 1 (linear), depending on the bond stress-slip relationship. MC 2010 assumes a value of 0,25 for α. The bond model applied in the MC 2010 crack formulation assumes a mean bond stress, where τ_m = 1/1,8 f_{ctm} for stabilised cracking. This is a conservative value for mean bond stress, considering the bond-slip relationship. Applied in the MC 2010 crack spacing equation (Equation 2.10), this equates to a value of 0,556 which is less than the Eurocode value of 0,8. In addition, bond stress decreases over time, decreasing the influence of tension stiffening, as discussed in Section 2.3.5.

2.3.5 Tension Stiffening Strain

The tension stiffening effect is found to decrease over time to approximately half its initial value, due to tensile creep and shrinkage, as well as a decrease in bond. Beeby and Scott (2006) stated that this reduction in tension stiffening occurs by about 30 days after loading. EN 1992 and MC 2010 both employ the use of the empirical k_t factor to allow for this decay, recommending values of 0,6 for short-term loading and 0,4 for long-term loading. The tension stiffening formulation of EN 1992 and MC 2010 may be derived using the equilibrium of stresses in the tensile zone, assuming elastic behaviour, as described by Beeby and Scott (2005). Creep in tension can vary considerably from compressive stress (on which the concrete creep factor and thus long-term concrete modulus), by as much as a ratio of 2, as found by Forth (2014). However, the influence of creep in the tension zone on was found to be insignificant by Beeby and Scott (2006), although it was difficult to determine exact values. The main influence on long-term tension stiffening was found to be cumulative damage over time due to the extension of cracks.

BS 8007 differentiates between bending and tension in the formulations for tension stiffening. In addition, the BS 8007 tension stiffening models, by being developed empirically, are calibrated to the specified crack width limit, namely, 0,2 or 0,1 mm. Interpolation between the tension stiffening equations may not be done for crack widths other than 0,2 and 0,1 mm. This obviously limits the use of the BS 8007 crack model if it is used in conjunction with other crack width limits specified to control leakage in LRS, especially if the crack width limit is linked to a hydraulic ratio as specified by BS EN 1992-3 (2008) for example. Crack width limits are discussed in Section 2.5.

Tension stiffening is influenced negatively by shrinkage arising prior to loading, depending on the concrete properties, section geometry and environmental conditions. The importance of mix design, curing processes and quality control on site in the construction of LRS so as to limit initial shrinkage and maintain the concrete tensile strength is thus accentuated. If earlier shrinkage strain is not taken into account, tension stiffening would be underestimated and so crack width overestimated. The factors influencing shrinkage before loading were taken into account in the EN 1992 tension stiffening model by adjusting the stress-strain curves. However, there is some evidence to suggest that tension stiffening models overestimate tension stiffening strain, so underestimate crack widths, in some instances. Researchers such as Bischoff (2001), Carreira & Chu (1986) and Kaklauskas *et al* (2015) have all studied this issue related to ties and proposed methods to improve tension stiffening models. Tension stiffening was shown to be strongly linked to the reinforcement ratio by Kaklauskas *et al* (2015), whereby tension stiffening decreases with increasing reinforcement ratios. Their analysis using test data on ties to develop a new tension stiffening model also showed that eliminating the influence of tension stiffening by adjusting the strains accordingly also improved the predictions of EN 1992, especially for lower reinforcement ratios. In the design of LRS, reinforcement ratios tend to be higher in order to meet limiting crack widths, so this effect on tension stiffening would be mitigated.

Bisch (2017) in reporting on the CEOS.fr project for large concrete structures, stated that one of the conclusions of the project was that tension stiffening is overestimated by code formulations and that the β factor of MC 2010 should be 0,36, not 0,6. This would be applicable to short-term loading. No comment was made regarding the long-term value. Debernardi and Taliano (2017) proposed a value of 0,45 for the k_t factor of the EN 1992 (and MC 2010) crack spacing equation, for short-term loading on ties, as it was concluded that the value assigned of 0,6 does not take into account the effect of secondary cracks on tension stiffening.

Beeby and Scott (2006) found that under sustained load, tension stiffening reduces rapidly and reaches its long-term value of approximately 50% of the short-term value in time of less than 30 days due to 'cumulative damage' over time. This ties in with the value for coefficient k_t in concrete strain (tension stiffening) term of the equation to calculate strain (for long-term load k_t is 0,4). They also stated that creep is a lesser factor in the decay of tension stiffening.

2.3.6 Influence of Concrete Properties

In the design of LRS, concrete needs to have a high impermeability and cracking resistance. Therefore, the mix design of concrete must be aimed at obtaining durable concrete with cracking controlled, rather than on producing high-strength concretes. The binders chosen should have a low shrinkage potential in an effort to control both early-age (immature concrete) and longer term (mature concrete) shrinkage and so, cracking. Construction practices must also be of a satisfactory standard to produce good quality concrete. Most design codes provide guidelines on durability and mix design.

BS 8007 (1987) Section 6 provides guidelines on the concrete mix design to ensure adequate strength, durability and impermeability, which includes the control of shrinkage and creep. For reinforced concrete, a minimum cement content of 325 kg/m^3 is recommended, with a maximum of 400 kg/m^3 for ordinary Portland cement (OPC) and ground granulated blastfurnace slag cements (GGBS) cements, or 450 kg/m^3 for pulverised-fuel ash (PFA) cements. A maximum of 0,55 is recommended for the water-cement ratio, reduced to 0,5 for cements containing PFA. Eurocode and MC 2010 do not have any specific guidelines regarding the concrete mix design. However, the shrinkage models themselves of these design codes take material properties such as the type of cement into account. SANS 10100 and Eurocode provide graphs to estimate shrinkage strains for design calculations. The shrinkage model of SANS 10100 does not take the cement type into account.

The draft SANS 10100-3, as given in WRC report No. 2154/1/15 (Viljoen, 2015 & Viljoen *et al,* 2015), includes recommendations on concrete mix design aimed at controlling cracking based on BS 8007 and industry experience. High strength concretes are generally not specified as they tend to have increased cracking due to higher binder contents, so a higher heat of hydration and cooling contraction, and thus a higher shrinkage potential. Characteristic concrete strengths therefore are in the range of 30 to 40 MPa (cube strength). In determining concrete compressive and tensile strengths, Viljoen *et al* (2015) determined that the Eurocode expressions for concrete strengths (Clause 3 of EN 1992-1-1, 2004) were satisfactory when applied to South African materials. Cognisance does need to be taken of the use of cylinder strengths by the Eurocodes, as opposed to cube strengths used by the South African concrete standards as the measure of the characteristic concrete compressive strength.

The structural concrete design codes tend to be prescriptive in the recommendations for mix design. Alexander and Thomas (2015) proposed a performance-based approach in terms of durability and concrete mix design (which includes crack control) that would allow for a more flexible approach in mix design whilst maintaining acceptable serviceability performance. This approach is in line with the latest developments in standards such as MC 2010. Alexander and Thomas (2015) noted that it is often difficult to verify the prescriptive specifications in practice, which means that a performance-based approach should result in a more durable concrete. Further research on this topic was recommended.

2.3.7 Influence of Long-Term Creep And Shrinkage Strain

Creep and shrinkage are two common time-dependent phenomena in concrete. Creep may be described as the increase in deformation with time due to permanent actions, and shrinkage as a decrease of volume over time as concrete ages. In terms of crack behaviour, long-term shrinkage strain decreases the mean concrete tensile strain (or tension stiffening), thus increasing the nett mean strain and crack widths, as shown by Kaklauskas *et al* (2015) where tensile stresses in concrete induced by shrinkage were found to significantly reduce cracking resistance, thus increasing cracking and crack width.

Crack models were mostly developed considering short-term shrinkage and creep, and generally do not take long-term effects into account. However, in the design of structures such as LRS, these long-term effects need to be considered. With this in mind, MC 2010, representing a more recent code development, includes long-term shrinkage in the determination of mean strain which the previous MC 1990 did not. In addition, the MC 2010 formulation when applied to long-term loading, specifies the use of the effective concrete modulus, thus taking compressive creep into account, as does the EN 1992 formulation. Thus, the effective modular ratio is used in determining stresses and strains. However, EN 1992 does not take long-term shrinkage into account. This also applies to the BS 8007 and the Frosch crack models. Both Gilbert and Nejadi (2004) and Castel and Gilbert (2014) on investigating the influence of time-dependent effects on cracking in reinforced concrete beams in separate tests, established that long-term crack widths were found to increase with time due to creep and shrinkage. Shrinkage was found to be the greater influence. Castel and Gilbert (2014) determined in their experimental research that MC 2010 gave a reasonable estimate of crack width for both long-term and short-term loading using the measured free shrinkage, ε_{sh}. However, the EN 1992 formulation underpredicted crack widths for the long-term loading case. The free shrinkage strains measured are shown in Figure 2.9.

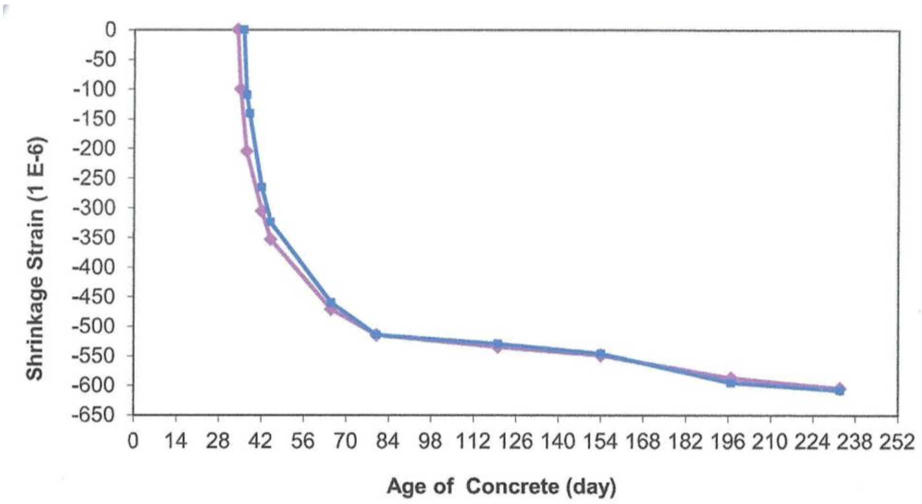


Figure 2.9: *Free shrinkage strain over time (Castel & Gilbert, 2014)*

Referring to Figure 2.9, free shrinkage strains initially increase rapidly but this rate of increase slows over the long-term.

The development of both creep and shrinkage phenomena depends strongly on the ambient humidity and environmental conditions, the dimensions of the element and the composition of the concrete. Creep is also influenced by the maturity of the concrete when the load is first applied, as well as the duration and magnitude of the loading. Creep relieves some of the shrinkage strain and reduces the concrete modulus, and in turn the modular ratio, as is well known. The age-adjusted effective concrete modulus must therefore be used for long-term loading. SANS 10100-1 and EN 1992 use the same equation to calculate the effective concrete modulus, namely:

$$E_{c,eff} = \frac{E_{c,28}}{1+\phi} \tag{2.18}$$

where $E_{c,28}$ is the concrete modulus at 28 days and ϕ is the creep coefficient. The creep coefficient depends on the age of the concrete, humidity and exposure conditions. The effective modular ratio is then used in the determination on tension stiffening strain and crack spacing. Research by Alexander *et al* (1989), (1992a) on concrete durability and the elastic modulus of concrete aggregates, showed that the elastic modulus of concrete was influenced by the type of aggregate. This needs to be considered in concrete mix design and the control of cracking.

An appropriate choice of shrinkage strain model applicable to the design of LRS was investigated as there are many models available for estimating shrinkage strain. One difficulty in measuring shrinkage and creep, and in deriving appropriate models, is the inter-dependence between the two mechanisms whereas most models treat the mechanisms separately. Theiner (2014) compared several shrinkage models to experimental work, including those of ACI, EN 1992 and MC 2010. It was found that the ACI model tended to overestimate shrinkage, whilst the MC 2010 model slightly underestimated shrinkage, with EN 1992 producing values closest to the shrinkage measured. Research on creep and shrinkage, including long-term behaviour, has also been performed by South African researchers in developing models and comparisons to other models such as those of SANS 10100, ACI and EN 1992, amongst others, using data from experiments performed using local South African materials. Mucambe (2010) investigated creep and shrinkage models applied to South African concretes. Together with work done on shrinkage models by Gaylard (2011) and Gaylard *et al* (2013), an extensive database of experimental creep and shrinkage data has been created. The SANS 10100 model was found to predict long-term (30-year) shrinkage values satisfactorily. The EN 1992 model was found to fit the South African data reasonably well, but further research was recommended. It should be noted that the creep coefficient was devised considering compressive creep.

The conclusion that can be drawn regarding the long-term effects of creep and shrinkage, is that they need to be included in crack models. A point to note is that shrinkage strains would be expected to be lower in LRS than for general structures, as all aspects of the construction and concrete mix design is such that shrinkage must be controlled. In addition, for LRS in use and thus full, the interior faces (as the more critical in terms of cracking and leakage) of the structure elements would be consistently exposed to water with a resulting reduction in long-term shrinkage. This should be taken into account when choosing the value of shrinkage strain in evaluating long-term crack widths.

2.3.8 Influence of Reinforcement Layout on Crack Patterns

Crack models treat the effect of the arrangement of reinforcement differently. Models such as those of SANS 10100 (or BS 8007) and Frosch include directly the spacing of reinforcement, as well as the area of reinforcement and diameter in the crack model. On the other hand, EN 1992 and MC 2010 do not. For a single layer of evenly spaced bars, the spacing is implied when considering diameter and area of reinforcement, but if this is not the case, for example, in beams where the arrangement of the reinforcement may be different for the same bar diameter and area of reinforcement, then the influence of the layout of the reinforcement is not considered by these crack models. In addition, much of the experimental work done on elements in tension is using simple ties with a single reinforcing bar. Whilst helpful in the development of understanding of cracking mechanisms, any influence of reinforcement configuration is then not studied. Research by Hossin and Marzouk (2008) on crack spacing for plates in flexure indicated that mean crack spacing increases with increasing bar spacing, as would be expected. Gribniak *et al* (2016) considered the effect of the arrangement of reinforcement in beams on cracking by varying the number of layers of reinforcement whilst keeping the bar diameter and area of reinforcement the same. It was found that multiple layers of reinforcement resulted in a delayed stabilised cracking phase and smaller crack widths, but that further investigation was needed to confirm these findings. Rimkus and Gribniak (2017) reported on the results of further testing on ties with differing reinforcement configurations, corroborating the earlier results reported in Gribniak et al (2016).

Most crack models do not consider the effect of transverse reinforcement, with much of the experimental work performed done on sections without transverse reinforcement. There are few studies that either include transverse secondary reinforcement, or have been done on elements such as two-way spanning slabs. Those that have been done generally have small numbers of test specimens, meaning that only qualitative conclusions may be drawn, however, this is still a useful starting point. Transverse reinforcement has been found in some studies to affect crack spacing and therefore crack widths. Rizkalla *et al* (1983) carried out short-term experiments on tension members with transverse reinforcement. Cracks tended to occur at transverse reinforcement locations, depending on the spacing of the transverse reinforcement. Specimens with transverse reinforcement at 100 mm spacing had cracks occurring at all transverse reinforcement locations. When the spacing increased to 150 and 215 mm, primary cracks occurred at the transverse reinforcement locations with subsequent cracks appearing between transverse bar locations. Only mean crack spacings were discussed, not maximum values. It would appear from these experiments that the maximum crack spacing would be influenced by transverse reinforcement if the spacing of said bars was close, but not necessarily if the spacing was further apart as cracks formed between transverse bar locations. Pérez Caldentey *et al* (2013) performed tests on twelve reinforced concrete beams for short-term loads. Under short-term loads, it was discovered that stirrups induce cracks especially when covers are smaller but it could not be said that crack spacing correlates with stirrup spacing. Cracking occurred between stirrups and did not necessarily follow stirrup spacing. It was concluded that mean crack spacing is strongly influenced by stirrups but maximum crack spacing is apparently not. The spacing of the stirrups affected the crack spacing - if the spacing was close then the crack pattern tended to follow the stirrup spacing. The final conclusion by Pérez Caldentey *et al* (2013) was that as the maximum crack spacing was the important one in crack control and was similar irrespective of whether or not the beam had stirrups, it was justified that crack models ignore stirrups.

Castel and Gilbert (2014) found that in the case of long-term loading, cracks appeared at about the stirrup locations that didn't appear in the short-term, indicating that stirrups may influence crack spacing in the long-term. It was concluded that if stirrup spacing is close, then the maximum crack spacing matches the stirrup spacing. If stirrups were wider apart (a spacing of 300 mm was given) then the maximum crack spacing was approximately half the stirrup spacing. Internal cracking also tended to occur at the stirrups which then contributed to the propagation of cracks in the long-term.

2.4 PROPOSED CHANGES TO EN 1992-1(2004) CRACK MODEL

The *fib* TG 4.1 working group on serviceability is currently, along with other researchers, looking at improving the crack model based on the latest MC 2010 crack formulation. Research has shown that the EN 1992 formulation does not consider long-term effects such as shrinkage for long-term loading situations. This would apply to liquid retaining structures where loads due to water pressure are considered as quasi-permanent loads. The EN 1992 crack model, in neglecting long-term shrinkage, thus underestimates crack widths. It was reported by the CEOS.fr project (2016) that the EN 1992 code formulation coefficients overestimates short-term crack spacing values compared to experimental results on ties but that MC 2010 produces values closer to the experimental ones.The amended EN 1992 formulation proposed by the *fib* TG 4.1 working group (Pérez Caldentey, 2017) for predicted crack width is therefore that of MC 2010 with a modification to the determination of shrinkage strain over time, where the shrinkage strain is to be multiplied by a restraint factor, R_{ax}. This allows for the restraint condition of the member to be taken into account, where if a value of 1 is used, full restraint of the member is implied. The proposed crack spacing formulation follows the format of the equation, namely:

$$S_{r,max} = 2\ c + 0.35\ k_1\ \varphi /\ \rho_{p,eff} \qquad (2.19)$$

where k_1 is a coefficient taking into account the bond properties of the bonded reinforcement. Closer inspection of this equation reveals that Equation 2.19 is the same as 2 x $l_{s,max}$ of the MC 2010 formulation, given here as Equation 2.9. Comparing Equations 2.6 and 2.19, the influence of concrete cover on crack spacing is effectively reduced from 3,4 for the factor k_3 of EN 1992 to 2 for the proposed amended EN 1992. There is still debate by the members of the *fib* TG 4.1 working group whether or not to distinguish between tension and bending as the crack spacing formulation does, or to follow the MC 2010 model, which does not. Pérez Caldentey (2017) stated that experimental evidence showed that small elements have some variation in the tensile stresses within the effective area in tension, with the stress distribution becoming uniform as the element size increases. The *fib* TG 4.1 working group recommended that further research on the MC 2010 crack model should include factors such as crack limits, the influence of the relative rib area on cracking, shrinkage and thermal effects on cracking, as well as the influence of concrete cover on crack spacing, long-term and cyclic loads, serviceability in general, and sustainability.

2.5 LIMITING CRACK WIDTH

All LRS design codes specify crack width limits for durability and water tightness, which the calculated design crack width may not exceed. The surface crack width gives an indication of the penetration of any cracks, and therefore the durability and permeability of a structure. Leakage of a LRS would obviously compromise the function of the structure, hence crack width limits are set to control leakage as well as protect the durability of the structure. Crack width limits of 0,1 mm to 0,2 mm are generally specified in the design of LRS. There is debate regarding the crack width limit for this type of structure, partly because the randomness of the cracking mechanism, and because the modelling, testing and measurement of crack widths and permeability is not straightforward. Beeby (2004), for example, gave a summary of some issues found in analysing experimental research, such as recording either the maximum or the average crack widths, but not both.

The crack width limits specified by the design codes investigated are summarized in Table 2.2. SANS 10100-1 is applicable to buildings, and therefore does not have a suitable crack width limit for LRS. As South Africa currently uses BS 8007, this code is referred to in Table 2.2.

Eurocode allows for the maximum allowable crack width, w_{k1}, to be defined in individual member countries' National Annexes. Recommendations for maximum crack width limits for liquid retaining structures are given in EN 1992-3.

Table 2.2: *Crack width limits (w_{lim}) recommended by design codes with respect to LRS*

Code	Conditions	Special condition	w_{lim} (mm)
BS 8007	Severe/ very severe exposure		0.2
	For aesthetic considerations		0.1
EN 1992-3	Tightness class 1 Limit depends on ratio hydrostatic head (h_D) to wall thickness (h). Intermediate values of h_D/h may be interpolated.	Cracks not passing through section depth of compression zone of at least x_{min} = lesser of 50 mm or 0,2h	0.3
		$h_D/h \leq 5$	0.2
		$h_D/h \geq 35$	0.05
MC 2010	Limited leakage (staining acceptable)	Compression zone minimum depth = 50 mm	0.2
	If limited leakage not acceptable		0.1
ACI	Members for use in water retaining structures		0.1

In LRS, limits are specified according to a tightness class defined by the requirements for protection against leakage, given in Table 7.105 of EN 1992-3, as follows:

- Class 0 Some degree of leakage is acceptable, or leakage of liquids is irrelevant. Class 0 structures would be those storing dry materials such as silos, so are not applicable to LRS.
- Class 1 Leakage is to be limited to a small amount. Some surface staining or damp patches is acceptable. Crack healing is expected to occur where the range for service load strain is less than 150 x 10^{-6}. There may be some cracks through the full section. Cracks are to be assumed to pass through the full section if alternate actions are applied to the section Autogenous healing of any cracks is expected to take place if the crack width limits relating to hydraulic ratio are adhered to. For intermediate values of h_D/ h, crack widths may be interpolated.
- Class 2 Leakage is to be minimal and the surface appearance is not to be impaired by staining. Cracks may not pass through the full section. To achieve this, the depth of the

compression zone is limited to a recommended value, x_{min}, the lesser of 50 mm or 0,2.h (h being the section thickness). If cracks do pass through the section, then it is expected that appropriate measures are taken, such as pre-stressing and using liners.

- Class 3 No leakage is permitted therefore special measures such as liners are to be taken.

For Class 3 structures, no specific guidance is given on the specification of liners.

The proposed SANS 10100-3 design code for LRS recommends use of the tightness classes as per EN 1992 but with less stringent crack widths limits, in keeping with the limits currently recognised by the South African industry, as the EN 1992 crack limits were deemed to be too conservative by the SANS 10100-3 Working Group (Viljoen, 2015 & Viljoen *et al*, 2015). The limits are summarised in Table 2.3.

Table 2.3: *Crack width limits to proposed SANS 10100-3 (Viljoen et al, 2015)*

Tightness Class	Requirements for leakage	Provisions to achieve leakage requirements
0	Leakage acceptable or leakage or liquids irrelevant	Surface crack width limited to 0,3 mm for acceptable durability and appearance
1	Leakage limited to small amount. Some surface staining acceptable.	Surface crack widths limited to 0,2 mm.
2	Leakage to be minimal. Appearance not to be impaired by staining.	Surface crack widths limited to 0,1 mm.
3	No leakage permitted.	Generally use special measures e.g. liners, for water tightness.

For Tightness Classes 1 and 2, a caution is given to ensure that, when calculating loads, the boundary conditions assumed are the actual ones in practice so as not in introduce any additional forces resulting in an increase in tensile forces, and therefore an increase in cracking.

LRS walls tend to have two dominant types of actions induced under loading dependent on the structure configuration, namely, flexure and direct tension for rectangular and circular reservoirs, respectively. Cracks would be expected to go through the section in the latter case, so would require a more stringent crack width limit as far as leakage is concerned. For buried LRS in flexure, as alternate actions due to soil and water pressures can occur, cracks should be assumed to pass through the full section.

As can be seen in Table 2.2, there is some variation of the limiting crack width in the design codes. There is supporting evidence for autogenous healing occurring for crack widths less than 0,2 mm when subject to low water pressures. Jones (2008) referred to a graph of the variation of crack width with hydraulic ratio illustrating autogenous healing in Tightness Class 1 structures under load-induced cracking. Figure 2.10 shows Jones's graph with the limits specified by EN 1992-3 according to Tightness Class 1 imposed. Eurocode has h_D/ h ratios higher than those by Lohmeyer and Meichner (Jones, 2008) for crack widths less than about 0,17 mm. This means that for a given water height (h_D) and section thickness (h), EN 1992-3 predicts that self-healing of cracks will occur at a higher crack width limit for crack widths less than 0,17 mm for a given hydraulic ratio

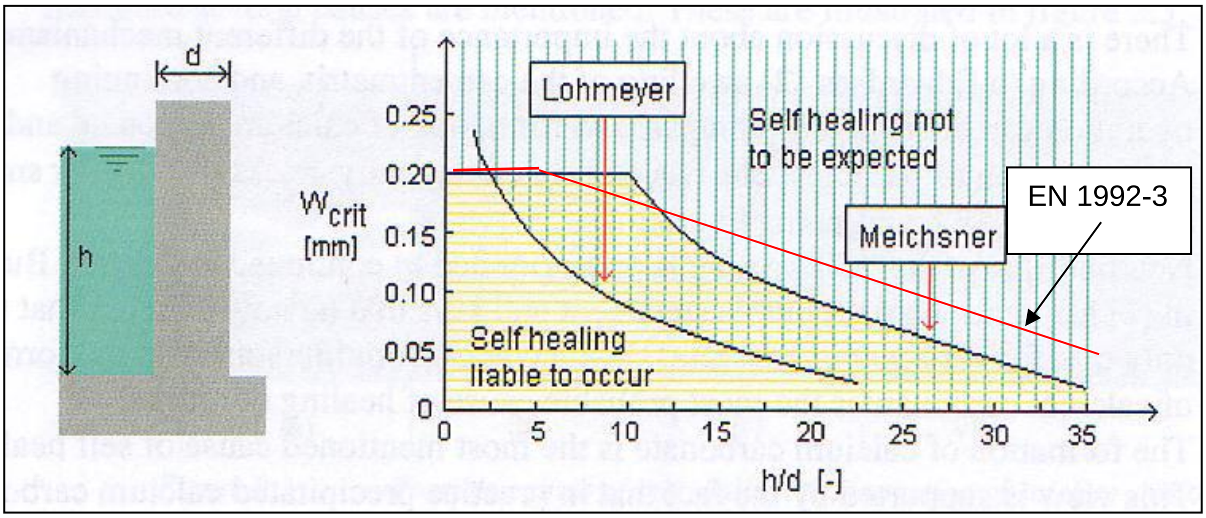


Figure 2.10:*Self-healing of cracks to Jones (2008)*

Edvardsen (1993) on performing permeability testing, proposed crack width limits based on hydraulic ratio, summarised in Table 2.4. Autogenous healing was assessed for both dormant cracks (Δ_w up to 10% cycling) and cycling loads of Δ_w up to 30% over a 24 hour period. Atkinson (2013), commenting on Edvardsen (1993), stated that the self-healing crack width limits for a maximum of 10% cycling load were more in keeping with those found in practice, rather than 30%.

Table 2.4: *Permissible crack widths for autogenous healing (Edvardsen, 1993)*

Hydraulic gradient (m/m)	w_k (mm) ($\Delta w < 10\%$)	w_k (mm) $10\% < \Delta w \leq 30\%$
40	0.10 – 0.15	≤ 0.10
25	0.15 – 0.20	0.10 – 0.15
15	0.20 - 0.25	0.15 – 0.20

From limited research and industry experience in South Africa (Viljoen, 2015 & Viljoen *et al*, 2015, McLeod, 2013), there is some evidence to suggest that cracks less than 0,2 mm in width will heal within approximately 21 days.

Several mechanisms contribute to autogenous healing, such as the precipitation of calcium carbonate, continued hydration of hardened concrete resulting in the growth of hydration products and the deposition of debris. These mechanisms and their interactions are not yet fully understood, with research ongoing. The rate of decrease of flow through a crack has been found to depend on the initial flow rate and crack width. Research has showed a rapid decrease in the initial flow through a crack within the first stages of testing by researchers such as Allen (1983). Ziari and Kianoush (2009(a) & (b)) obtained mixed results as to when flow ceases completely and at which crack widths. Ziari and Kianoush (2009a), in tests on panels under tension found that for 0,25 mm cracks under a constant head of 5 m, the flow rate decreased significantly within the first 7 hours and that self-healing did occur. They did, however, recommend in their conclusions that direct tensile cracks should be avoided in LRS. Ziari and Kianoush (2009b) in testing on panels under flexure, found that the depth of the compression zone played a key role in leakage, as well as the crack width. Seong-Tae Yi *et al* (2011) performed permeability tests on 50 mm thick sections for crack widths of 0,03 mm to 0,1 mm under varying water pressures. An allowable crack width of 0,1 mm for a hydraulic pressure of 10 kN/m^2, with a reduction in crack width to 0,05 mm for a hydraulic pressure of 25 kN/m^2, was recommended. McLeod (2103) reported on research on autogenous healing performed at the University of KwaZulu-Natal. Case studies and experimental research showed that self-healing does indeed occur, evidenced in some cracks as large as 0,4 mm. Cracks of 0,2 mm and less sealed within 72 hours in most cases.

The flow rate through a crack through the section is found to be proportional to crack width cubed, which results in a significant increase in leakage as crack width increases, as discussed by Edvardsen (1993). The effective width of the crack internally also influences the amount of leakage, as the crack profile is not uniform through the section and the effective crack width may be less than the surface crack width (taken as the limiting crack width). This may make it appear that self-healing has taken place at a larger crack width than is actually the case. Leakage rates are also influenced by crack surface roughness. Researchers therefore need to take note of this in predicting crack widths to ensure water-tightness of a structure. The research suggests that crack width limits require further investigation.

2.6 CHAPTER SUMMARY

Various crack philosophies and the resulting design crack formulations have been investigated. The literature review showed that design crack models have a high degree of uncertainty partly due to the random nature of cracking mechanism and the range of parameters that influence cracking. Models that do not allow for long-term effects for sustained loading tend to underpredict crack widths. From the review of design code formulations, the crack model of

MC 2010 (which is also the proposed updated model for EN 1992) was derived from an analytical basis, rather than an empirical one, and represents some of the most current research on load-induced cracking.

The recommended values for the fixed-value k_i coefficients of EN 1992 and MC 2010 do appear to be conservative, which would introduce some conservatism into the crack model. Concrete cover is expected to be one of the influential crack model parameters of the Eurocode-type models, as concluded by McLeod (2013). The influence of transverse reinforcement is not considered by the Eurocode-based crack models, although it appears to affect crack patterns depending on the spacing of the transverse reinforcement. From the limited studies, the maximum crack width is not significantly affected. A crack width limit of 0,2 mm is the general crack width limit specified in designing LRS.

On performing this literature review, it was found that there is a lack of experimental crack width data available for sustained loading situations, particularly for cracking due to direct tension. This would affect the quantification of model uncertainty in this case. Further discussion is presented in Chapter 3.

CHAPTER 3
REVIEW OF RELIABILITY ANALYSIS AND MODEL UNCERTAINTY

3.1 INTRODUCTION

To date, there has been little probabilistic assessment done on the serviceability limit state (SLS). In reliability-based structural design, more attention has been given to the ultimate limit state (ULS), as the critical limit state in general structures. This has resulted in serviceability limit states being treated nominally by structural design codes. Retief and Dunaiski (2009) (Section 5.9 page 51) stated that "no proper reliability assessment is generally applied to the SLS". However, in the case of liquid retaining structures (LRS), SLS cracking generally governs the design, consequently, a proper reliability assessment of crack models is required. These SLS cracking models have historically been developed mostly through experimental research under short-term loading conditions with an unspecified degree of conservatism. In developing a crack width design formulation, conservative empirical factors are chosen and applied to the crack width prediction model and related material properties, based on the judgment of the researcher and sometimes moderated by a code committee.

Holický *et al* (2009) performed a reliability study on the EN 1992-1 (2004) crack model applied to a LRS wall under pure tension, assessing loading and crack width limits. The probability of failure for varying reinforcement areas for specific crack width limits was determined and plotted. They concluded that "the accepted reliability level corresponding to a probability of 5% of exceeding the limiting crack width is not rationally justified" as the reinforcement quantities required to meet the 5% exceedance crack width limit appeared to be conservative. It was recommended that methods of probabilistic optimization considering the life cycle costs would result in an improved crack design. However, model uncertainty parameters in the reliability crack model were assumed as a first estimate and required further research. Model uncertainty treated as a random variable was found to be a significant parameter in First Order Reliability Method (FORM) analyses of the crack model by McLeod (2013). The statistical parameters of the model uncertainty random variable had to be assumed using what little information existed, confirming the need for further research.

Having discussed existing crack models in Chapter 2, this chapter thus presents a review of literature pertaining to:

- General reliability concepts and treatment in structural design standards.
- Model uncertainty assessment.
- Experimental research on the reliability of reinforced concrete crack models.

- General statistical data of crack model parameters.

3.2 RELIABILITY-BASED STRUCTURAL DESIGN AND SERVICEABILITY

Modern design standards are based on the principle of limit state design which is a semi-probabilistic procedure, derived from full probabilistic analyses. The structural performance of a structure, or its elements, is described in terms of a set of limit states that describe the fitness of the structure in performing its function. The limit states differentiate between the conditions that affect the performance of the structure. The ULS is associated with collapse of a structure or any of its elements that affect the safety of human life. The SLS is associated with loss of function under conditions of normal use. A higher reliability is typically required of the ULS than for the SLS with target reliability fundamentally dependent on the limit state under consideration. A higher reliability is generally required for more severe consequences and lower costs for increasing safety. Conversely, a lower level of reliability would be selected if the consequences of failure is low and/or the cost of increasing safety is higher.

Although modern design standards already incorporate reliability concepts, there have been improvements in recent years to limit state design. In South Africa, SANS 10160-1 (2011), which defines the basis of structural design for the other South African structural standards, was developed incorporating updated structural reliability research. It is compatible with its Eurocode counterpart, EN 1990 (2002), as well as with local conditions and practices. ISO 2394 (1998) which describes the general principles of reliability in structures, was adopted by South Africa in 2004 as SANS 2394 Ed 1 (2004). This standard was updated in 2015 and adopted in South Africa as SANS 2394 (2016), presenting and defining a framework for general structural reliability.

The design standards SANS 10160-1 (2011), EN 1990 (2002) and ISO 2394: 2015 (adopted in South Africa as SANS 2394: 2016) all differentiate between Reversible and Irreversible serviceability limits states, depending on the action or mechanism on the structure and the rate of recovery, if any, after the removal of said action. Irreversible SLS require a higher reliability level than reversible SLS. This is discussed further in Section 3.5. SANS 10160 and EN 1990 differentiate the SLS further, adding the performance level of Long-term and Appearance. SANS 2394 (2016) defines the following undesirable states that relate to reinforced concrete cracking:

- Local damage that affects appearance, efficacy or functional reliability of the structure, and

- Local damage (including cracking) that can reduce the durability of the structure or make the structure unsafe for use.

Both of these are critical in structures such as LRS as SLS cracking is the limiting criterion in the design thereof.

The *fib* Model Code, MC 2010 (2013), as reported by Bigaj-van Vliet and Vrouwenvelder (2013) has incorporated a performance-based approach to the design and assessment of concrete structures. Sustainability in general and durability as a serviceability consideration are included in assessing the performance of a structure. The latter is an important change in philosophy away from the usually more prescriptive 'deemed to satisfy' requirements given in the concrete standards, for example, regarding concrete mix design for LRS. Alexander and Thomas (2015) and Nganga, Alexander and Beushausen (2013) outlined such an approach to concrete durability with respect to corrosion and gave examples of several case studies, both local and international. 'Deemed to satisfy' requirements in design are of necessity conservative, but this can in turn, indirectly introduce undue conservatism into concrete models such as cracking. These requirements usually indirectly take any variations in materials (such as more sustainable materials) into account in their conservatism; however, this may not produce an optimum design. As durable concrete is necessary in the control of cracking, any improvement in ensuring the production of durable concrete can only benefit the performance of crack models and reduce model uncertainty.

Reliability methods provide a way of determining the performance of a structure while considering factors such as safety, sustainability and economy of design. They also provide a tool to assess new and improve existing structural models, for example, load-induced crack models in reinforced concrete liquid retaining structures . The following sections discuss the general concepts associated with reliability.

3.3 LIMIT STATE FUNCTION

The limit state is defined by the structural model written as a function, known as the limit state function or reliability performance function, defining the boundary between the safe and unsafe condition. The limit state is generically expressed as:

$$g(X_i) = R - E \tag{3.1}$$

where R is the resistance of the structural model and E the action effect, although it is not always possible to separate the function into action effects and resistance. The model parameters are treated as probabilistic basic variables of the limit state function, X_i.

Specific to the serviceability limit state (SLS), the limit state function (LSF) expressed in SANS 10160-1 (2011) has the general form:

$$g(X_i) = C - E \quad (3.2)$$

where C is the limiting value of the serviceability criterion and E is the action effects of the SLS condition. The function $g(X_i)$ will be greater than zero if the structure is safe, equal to zero at the limit state and less than zero if the structure is unsafe. Applying Equation 3.2 of the definition of the LSF to crack models, C is the crack width limit as a deterministic value and E is the probabilistic description of the predicted crack width, which may be based on a crack width prediction model with the geometric, material and loading input parameters treated as random variables. A model factor (ϑ) may be derived to probabilistically account for typical model uncertainty and bias in E, so Equation 3.1 becomes:

$$g(X_i) = C - \vartheta . E \quad (3.3)$$

The second term of the LSF of Equation 3.3 thus provides a general probabilistic model of crack widths for the structural configuration considered. Further discussion on model uncertainty is presented in Section 3.7.

3.4 LEVEL OF SAFETY MEASURED BY RELIABILITY INDEX, β

Once the limit state function is determined, the structural reliability as a measure of performance may be assessed probabilistically, that is, the probability that the limit state will be met or exceeded (probability of safety, p_s) is determined using the statistical data for the parameters (X_i) in the structural model. Conversely, the probability of failure ($g < 0$) where $p_f = 1 - p_s$ may be determined. The level of reliability of the structure is measured probabilistically by the reliability index (β) related to the probability of failure,

$$\beta = -\Phi^{-1}(p_f) \quad (3.4)$$

where Φ^{-1} is the inverse standard normal probability distribution function, as defined by SANS 2394: 2016. The relationship between the reliability index and the probability of failure is quantified in Table 3.1.

Table 3.1: *Relationship between β and p_f*

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

3.5 TARGET RELIABILITY AND DESIGN LIFE

The failure probabilities obtained from a probabilistic analysis should not exceed a specified target failure probability (p_f). Alternatively, this desired failure probability may be expressed in terms of the target reliability index (β_t), as a minimum value, specified for a given reference period, such that

$$p_f = \Phi(-\beta_t) \quad (3.5)$$

where Φ is the cumulative normal distribution function. Good quality management practices are assumed in the selection of the level of reliability such that gross errors are avoided. The probability of failure and the associated target reliability, as they are time-dependent, are related to a given reference period, defined as "a certain *a priori* specified period of time" by SANS 2394: 2016 (Cl 8.1 page 38). The standard reference period, $\beta_{t,1}$ is taken as one year. The target reliability $\beta_{t,n}$ for other reference periods, n, may be determined using that for a one-year reference period using the expression:

$$\Phi(\beta_{t,n}) = [\Phi(\beta_{t,1})]^n$$

where Φ is the distribution function of a standardized normal distribution.

SANS 2394: 2016 (Cl 8.4 page 39) describes how target reliability should be chosen by taking into account the consequence and the nature of failure, the economic losses, the social inconvenience, effects to the environment, sustainable use of natural resources, and the amount of expense and effort required to reduce the probability of failure. The choice of target reliability is also dependent on the limit state considered as this relates to the severity of the consequence of failure, and thus the related costs of failure, or conversely, of safety. The ultimate limit state is associated with structural collapse and therefore has a higher target reliability, as the costs of human safety as well as other costs such as extensive loss of function and repair costs and replacement of the structure are considered. Serviceability limit states are generally related to performance and some loss of use, inconvenience or repair costs, rather than loss of human life or injury, and therefore tend to be associated with a lower target reliability. SANS 2394 (2016) describes the choice of a suitable target reliability for SLS such that the loss of functionality is limited, and/or the occurrence of damage is within an acceptable economic level, as there is generally no risk of loss of life.

As described by sources such as Rackwitz (2000), Holicky (2009) and SANS 2394 (2016), the target reliability may be determined from a reliability cost optimisation. The aim of such a probabilistic optimisation is to determine a reliability level (the target reliability) at which the total cost of the structure is a minimum. The cost objective function, C_{tot}, describes the total cost as:

$$C_{tot} = C_o + C_1 d + \Sigma\, C_f\, p_f(d) \quad (3.6)$$

where d is the decision parameter, C_o is the initial costs independent of d, C_1 is the costs of safety dependent on d (considered as a linear relationship), C_f is failure costs and p_f (d) is the probability of failure as a function of d. The decision parameter is generally a vector of multiple decision parameters such as section geometry, material properties and reinforcement area that influence, for instance, the resistance of the structure. The costs $C_o + C_1d$ are the construction costs, while C_f p_f (d) are the expected failure costs related to each decision parameter. Considering the ULS LSF of Equation 3.1, Equation 3.5 may be rewritten as:

$$p_f(d) = P[g = R - E < 0] = \Phi(-\beta_t)$$

Thus, the target reliability is related to the decision parameter through the LSF. The minimum total cost is the derivative of Equation 3.6. The optimum d may then be determined from the condition:

$$\delta p_f(d) / \delta d = - C_1/ C_f$$

As defining all costs may be difficult, a cost optimization is performed for a range of C_1/ C_f values to find cost optimal target reliability levels. For SLS, p_f (d) may be determined using Equation 3.2. The SLS cost optimisation model derived by Van Nierop (2017) is an example of this.

Target reliabilities may be differentiated according to the type of structure and the consequences of failure. The type of structure is described in terms of its design working life, which Holický (2009) defines as “the period for which a structure or part thereof is to be used for its intended purpose with anticipated maintenance but without major repair being necessary”. Design working life as differentiated by types of structure by SANS 10160-1 (2011) is given in Table 3.2

Table 3.2: *Notional design working life to SANS 10160-1 (2011)*

Design Working Life Category	Indicative Design Working Life (Years)	Description Of Structures
1	10	Temporary structures.[a b]
2	25	Replaceable structural parts, for example bearings, agricultural structures and similar structures with low consequences of failure.
3	50	Building structures and other common structures.[c]
4	100	Building structures designated as essential facilities such as having post-disaster functions (hospitals and communication centres, fire and rescue centres), having high consequences of failure[d] or having another reason for an extended design working life.

[a] Structures or parts of structures that can be dismantled with a view to being re-used should not be considered as temporary.
[b] Refer to SANS 10160-8 for the assessment of temporary structures during execution.
[c] The design working life category applies to the reference reliability class referred to in 4.5.2.3.
[d] Consequences of structural failure could be determined in accordance with annex A.

EN1990-1, MC 2010 and SANS 2394 (2016) define similar categories for the design life of a structure. A design working life of 50 years would be chosen for liquid retaining structures (category 3), which is also the general design life for most common structures.

Consequence classes relate to the cost of failure of, for example, loss of human life, and economic, social and environmental costs. The Joint Committee on Structural Safety (JCSS, 2008) defines consequence classes by considering the risk to life cost as a ratio of the total costs of failure to construction costs. EN1990 (2002) defines reliability classes (or consequence classes) RC1 to RC3 for the ultimate limit state. RC1 has a low consequence, RC2 has a medium consequence and RC3 has a high consequence. RC2 is taken as the reference class. SANS 10160-1 (2011) defines similar reliability classes (RC1 to RC4) and gives a recommended target reliability for each consequence class. A more detailed description is given by SANS 2394: 2016 whereby five consequence classes are defined. The consequences related to each class include the expected number of fatalities. Examples of typical structures for each class are also given, but suitable target reliabilities relating to the consequence classes are not. As serviceability limit states are not usually associated with loss of human life, with the relative cost depending on performance and use of the structure, a low-order consequence class would be typically applicable. However, the consequence classes are generally derived with the ULS in mind. SANS 2394: 2016 links a quality level to the consequence classes, defining 3 levels of quality. Control organisms are recommended for each quality level. In LRS, the consequence of failure would not generally include loss of life, rather a loss of function. However, this loss of function has more serious societal consequences than usual for SLS as access to clean water may be interrupted for an extended period.

A review of the literature on target reliability confirmed that serviceability has been treated nominally by design standards. Target reliabilities for ULS have been derived through proper optimisation and calibration processes but this is not the case for SLS. The Joint Committee on Structural Safety (JCSS, 2008) made recommendations for target reliability index values (β_t) for irreversible serviceability limit states (such as concrete cracking) based on economic optimisation, given here as Table 3.3, for a one year reference period, along with the corresponding failure probabilities. The reference level of relative cost of safety is Normal.

Table 3.3: *Target Reliability Indices for Irreversible SLS – Source: JCSS (2008).*

Relative Cost of Safety Measure	SLS Target Index, β	Probability of Failure, p_f
High	1,3	10^{-1}
Normal	1,7	5.10^{-2}
Low	2,3	10^{-2}

The JCSS (2008) states that values chosen for target values may vary by about 0,3 from the β_t values of Table 3.3.

Retief and Dunaiski (2009) summarised target reliability levels recommended by ISO 2394: 1998 (SANS 2394: 2004) and EN1990: *Eurocode 1: Basis of structural design* as shown in Table 3.4, for both the ultimate and the serviceability limit states.

Table 3.4: *Target reliability levels (β) according to ISO 2394 (1998) and EN 1990 (Source: Retief and Dunaiski, 2009)*

Relative cost of safety measures	**ISO 2394 Minimum values for β**			
	Consequences of failure			
	Small	Some	Moderate	Great
High	0	1,5 (A)	2,3	3,1 (B)
Moderate	1,3	2,3	3,1(C)	3,8 (D)
Low	2,3	3,1	3,8(D)	4,3(E)

A — for serviceability limit states β = 0 for reversible and β = 1,5 for irreversible states
B — for fatigue limit states β = 2,3 to 3,1 depending on the possibility of inspection
For ultimate limit states the safety classes: C β = 3,1 D β = 3,8 E β = 4,3

Reliability Class	**EN 1990 Minimum values for β**					
	Ultimate LS		Fatigue		Serviceability LS	
Reference period	1 year	50 years	1 year	50 years	1 year	50 years
RC1	4,2	3,3				
RC2	4,7	3,8(F)		1,5 to 3,8	2,9	1,5
RC3	5,2	4,3(G)				

F — With ISO 2394 clause 4.2(b) *moderate safety costs & RC2 consequences* , but EN 1990 is more conservative; EN1990 value agrees with ISO 2394 for either *low safety cost or great consequences*
G — The EN1990 value for RC3 agrees with ISO 2394 for *low safety cost and great consequences*

ISO: 2,3 – 3,1 EN: 1,5 – 3,8 Fatigue: ISO 2394 – restricted range; EN1990 – range from *serviceability LS equivalent* to *ultimate LS*

The *fib* MC 2010 gives guidance on target reliabilities for serviceability as well as for ULS, as reported by Bigaj van Vliet and Vrouwenvelder (2013) and summarised here in Table 3.5. These SLS target reliabilities are common across the design standards of Eurocode, MC 2010 and SANS 10160.

Table 3.5: *MC 2010 SLS Target reliabilities (Source: Bigaj van Vliet & Vrouwenvelder, 2013)*

Limit State	**Target Reliability Index, β_t**	**Reference Period t_R (years)**
Serviceability, reversible	0	t_{SLS}
Serviceability, irreversible	0,7	200
	1,1	100
	1,5	50
	2,1	15
	3,0	1

SANS 2394 (2016) recommends similar target reliabilities to the earlier SANS 2394 (2004) version for the ULS. It is also stated that for ULS, the target probability of failure may be increased by a factor of 5 for a higher coefficient variation of the basic variables. Conversely, for variables with a low variability, the target probability may be reduced by a factor of 2. Bigaj van Vliet and Vrouwenvelder (2013) noted that the MC 2010 SLS target reliability indexes correspond approximately to those of ISO 2394 (SANS 2394) for small consequences of failure and moderate costs of safety measures.

Summarising, a reliability index of 1,5 for a 50-year reference period is generally recommended for irreversible serviceability states such as cracking in buildings, as highlighted in Tables 3.4 and 3.5, although the derivation of this value is unclear. It should be noted that full probabilistic analyses have not been performed to obtain this value, unlike the ULS values given in the standards. In the design of LRS, where serviceability cracking is the limiting condition and has a greater importance due to the higher consequences of a loss of function if cracking results in water leakage, a higher target reliability may be required. Retief & Dunaiski (2009) (Section 5.9, page 51) stated that target reliabilities for serviceability are an indication of appropriate levels of reliability but that 'further refinement of the scheme of target reliabilities may be feasible'. Van Nierop (2018) performed a cost optimisation to estimate an appropriate target reliability applicable for a typical LRS, the value of which was found to be greater than 2,0 (Reference period of 50 years). This is higher than the general target reliability of 1,5 for a 50 year reference period recommended by design standards such as EN 1990 and ISO 2394. The decision parameter used by Van Nierop (2018) in the optimisation of β_t the amount of reinforcement required. The resulting $\beta_{t,SLS}$ was compared to that found using the generic decision parameter used by Rackwitz (2000) for estimating ULS target reliabilities. The reinforcement area as a decision parameter was found to more cost-efficient than the generic in reducing failure probabilities, and therefore resulted in a higher assessment of the $\beta_{t,SLS}$ applicable to LRS. Some simplifications were made in this study but the results suggests that current standards are underestimating the required SLS level of reliability in at least some SLS situations.

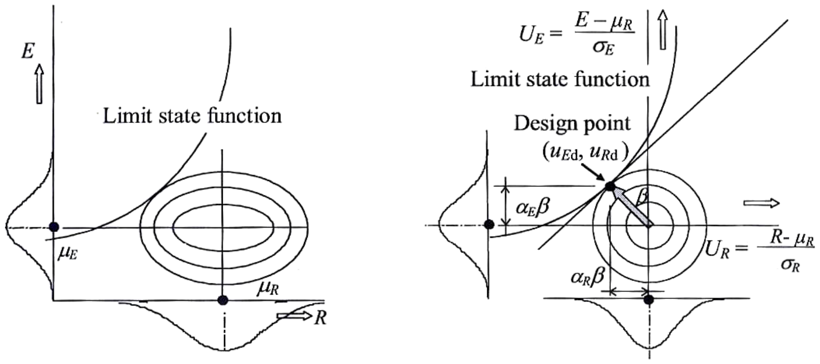
3.6 FIRST ORDER RELIABILITY ANALYSIS (FORM)

Failure probabilities can be determined using one of three methods, namely, First or Second Order Reliability Methods (FORM/SORM) depending on the available statistical data for the RV's, simulation techniques such as Monte Carlo, and numerical methods. The equation to find the probability of failure may be expressed as:

$$p_f = \int_{g(X_i)<0} f_{X_i}(X_i)\, dx_i$$

where X_i are the random variables and $f_{xi}(X_i)$ is the joint PDF of the RV's. The First Order Reliability Method provides a well-recognised method of assessing performance and is therefore the method chosen in this research. It is also well-suited to assessing non-linear performance functions such as that for crack models. The development of the FORM algorithm is well-documented, for example, by Ang and Tang (1990) therefore just a short summary is given here, illustrated in Figure 3.1 (a) and (b).

The limit state function or performance function is the failure plane on which $g(X_i) = 0$, shown in Figure 3.1 as a non-linear function.



(a) Original basic variables R and E. (b) Transformed variables U_R and U_E.

Figure 3.1: *Graphical representation of FORM (Source: Holický (2009))*

As the mathematical solution is not easily found where the basic random variables are non-normal (Figure 3.1 (a)), these are normalised, as shown in Figure 3.1 (b). In the case of a non-linear performance function, the resulting non-linear failure plane is approximated by the tangent plane that intersects the actual failure plane at the most probable failure point (the design point), as shown in Figure 3.1 (b).

Referring to Figure 3.1, the reliability index, β, is the shortest distance from the design failure point on the failure surface to the origin, and has the general equation:

$$\beta = \frac{\sum x_i'^{*}\left(\frac{\delta g}{\delta X_i'}\right)_*}{\sqrt{\sum\left(\frac{\delta g}{\delta X_i'}\right)_*^2}}$$

A first-order approximation of a Taylor series is used to find the mathematical solution of the design point and its associated reliability index. The process is an iterative one with the iterations converging at the design point, $x_i'^{*}$. The forward FORM algorithm and related formulae to determine the reliability index are summarised as follows:

1. Define limit state function, $g(X_i)$.
2. Collect statistical data for all random variables, X_i, in the limit state function (PDF, μ, σ).
3. Normalise non-Normal PDF's for the random variables such that the normalised mean μ^N and coefficient of variation σ^N obtained for each random variable at the estimated design point.
4. Initial values for the design point value for each variable (x_i) are assumed. The normalised mean is usually the initial value taken.
5. Using the limit state equation, determine the partial derivatives $\delta g/\delta X'_i$ with respect to the normalised RV's.
6. Calculate the directional cosines (α*) using the expression:

$$\alpha_i^* = (\delta g/\delta X_i')_* \,/\, \sqrt{(\Sigma(\delta g/\delta X_i')_*^2)}$$

8. Substitute the equation to calculate the new design points $x_i^* = \mu_i^N - \alpha_i^* \beta\, \sigma_i^N$ into the limit state equation $g(X_i) = 0$ and solve for the reliability index, β.
9. Repeat steps 4 to 8 until the β- value converges.

The x_i^* values obtained for the final β will be the design failure point values of the variables for the given limit state equation. The directional cosines (α_i) are also the sensitivity factors for the variables in the limit state function with values between -1 (negative influence) and 1 (positive influence) for a specific reliability index, and as such the most significant RV's can be identified. A value close to zero indicates that the random variable has little effect on reliability performance, with influence increasing as factors approach ± 1. It should be noted that $\sum \alpha^2 = 1$.

FORM analyses may be performed for time invariant or variant conditions. SLS cracking considered in this research could be considered as time invariant as any changes tend to occur slowly over time under sustained loading. Time variant analyses are more appropriate for models such as that of wind loading on structures, for example.

The reverse FORM may also be applied, whereby the design point for a given target reliability is determined. The sensitivity factors of the RV's would then be obtained for a given target reliability and may also be utilised in the process of calibrating the parameters of the model for design purposes. Theoretical partial safety factors for each basic variable, $\gamma_{x,}$ may be found for a given target reliability index from the expression:

$$\gamma_x = 1 - \alpha_x \beta w_x$$

where w_x is the CoV of the RV. Normalised distributions are assumed. Optimisation processes would then be employed to obtain the design formulation partial safety factors.

3.7 MODEL UNCERTAINTY ASSESSMENT

Uncertainty always exists in structural performance and needs to be quantified as far as practically possible to obtain a proper measure of the basis for reliability assessment. In addition to the variability of materials and loading, model uncertainty can in some cases contribute significantly. Two main types of uncertainty can be defined, namely, inherent random variability and that due to incomplete knowledge including statistical uncertainty as described in literature such as Retief (2015), Ang and Tang (1990) and the JCSS (2001, 2008).

Inherent random variability may or may not be affected by human activities, such as the uncertainties in strength values of materials or uncertainty in loads, respectively. Uncertainties are influenced by the level of production and quality control during design and construction, as discussed in Section 3.5. Uncertainty due to random variability is generally taken into account in the coefficients of variation of the random variables of the probabilistic model. The inherent random behaviour of a physical mechanism may affect the ability of a model to describe the mechanism, contributing to the overall model uncertainty which may be significant, for example, reinforced concrete cracking mechanisms which display a higher degree of randomness and therefore have a higher model uncertainty.

Mathematical simplifications of physical and probabilistic models will result in some degree of model uncertainty. The formulation of the crack model from experimental research is an example of the former, including the determination of the limiting crack width. In the stochastic model, normalizing non-normal distributions and the use of the first order approximation using FORM for the mathematical solution of the failure probability integral, would create some uncertainty in the probabilistic model. However, FORM has been developed such that this uncertainty is small. Regarding uncertainty in the material, load and geometric properties applicable to crack models, the statistical parameters thereof are relatively well researched and available in literature, discussed further in Section 3.9 and summarised in that section as Table 3.9. On developing models from experimental research, uncertainty will exist in the experiments themselves which include uncertainty in execution and test method. For well-calibrated tests that produce unbiased results, the effect of test uncertainty was found by Holický *et al* (2016) to be of minor significance when model uncertainty has CoV's greater than 0,1.

Uncertainty that is not taken into account in the statistical parameters of the model's random variables may be taken into account by treating it as a random variable commonly having either a normal or log-normal PDF as stated by JCSS (2008) applied to the model itself. Where there is not a clear indication of whether the PDF is normal or lognormal, a 2-parameter lognormal distribution may be chosen as it returns a lower reliability estimate, thus is more conservative. particularly considering the upper fractiles. This is applicable to the prediction of maximum crack widths. The CoV would be chosen to reflect the degree of uncertainty expected.

A summary of the general determination of model uncertainty is given in Figure 3.2, sourced from Holický *et al* (2016). Sources of uncertainty are divided into test and model results, and structure-specific conditions in order to identify and quantify uncertainty.

In assessing model uncertainty and resulting safety factors in code models, the relative influence of model uncertainty on the model can be divided into model uncertainty classes, described by Retief (2015) as:

- Nominal effect
- Significant effect
- Dominating effect

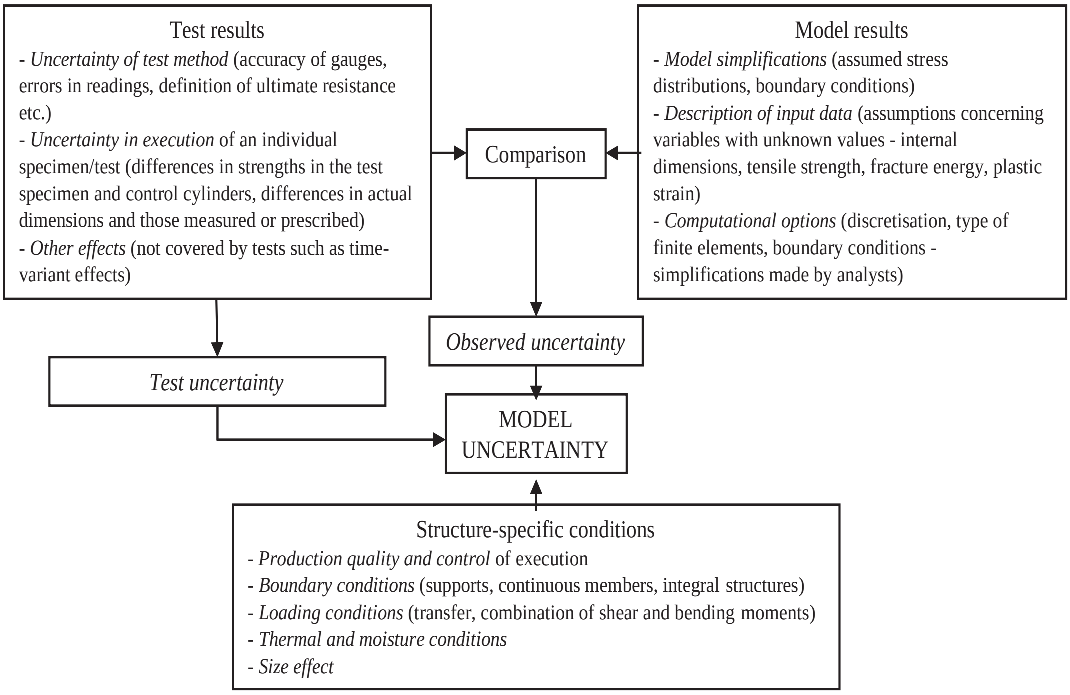


Figure 3.2: *Model uncertainty framework (Holický et al, 2016)*

When model uncertainty has a nominal effect on the model, the partial safety factors of the basic variables are adjusted to take into account model uncertainty. Model uncertainty with a sensitivity factor between 0,32 and 0,8 may be defined as having a significant effect. In this case, it is justified to include a separate model uncertainty partial safety factor in the model. Where model uncertainty is the dominating factor, usually when modelling of structural behaviour is approximate, then special attention to model uncertainty is required, including economic considerations to attain a suitable reliability. Sensitivity factors above 0,8 would imply model uncertainty has a dominating influence. Model uncertainty should then be included explicitly in the reliability analysis. The selection of the theoretical model is important in this case, as model uncertainty is specific to the chosen model. The starting point in assessing model uncertainty is the identification of the sources of model uncertainty in a particular prediction model.

Model uncertainty when treated as a random variable is indicative of the performance of a structural model. A common method of quantifying model uncertainty is to define it as the ratio between a test or actual value and the predicted value for the structural model considered. Thus, the statistical parameters of this ratio, as model uncertainty, can be determined. The mean of model uncertainty is then a measure of bias in the model, where a value of 1 indicates an unbiased model. In this context of model uncertainty, a mean greater than 1 represents underprediction in the model. Conversely, a mean less than 1 signifies an overprediction in the model. The variation of model uncertainty is the indication of uncertainty in the model – a higher variation signifying a higher uncertainty. For a model to be suitable over a reasonable range of design applications, it should capture the influence of a parameter, thereby behaving consistently for varying model parameters. The correlation of model uncertainty with model parameters gives an indication of this – strong correlations would result in drift in the mean, and therefore an inconsistent model.

3.8 RESEARCH RELATING TO RELIABILITY OF CRACK MODELS

In the scope of this thesis, a review of any research previously carried out on reliability with respect to serviceability cracking was undertaken. It was found that there has been little research done on the reliability of serviceability cracking in probabilistic terms, particularly for small crack widths. A summary of relevant research is now given.

3.8.1 Target Reliability of SLS Crack Models

In order to perform a reliability analysis, an appropriate target reliability must be chosen. A target β of 1,5 is recommended by most standards for SLS irreversible cracking in buildings. However, for structures such as LRS where SLS cracking is dominant, this value may be too low, especially as there has been little probabilistic assessment of SLS. A review of research was then done to establish a clearer idea of what value, or range of values, would be applicable to load-induced crack models.

Quan and Gengwei (2002) assessed the reliability level for cracking of reinforced concrete beams by means of an inverse FORM analysis. The crack width prediction model used was that of the Chinese code for reinforced concrete, a similar formulation to the Eurocode concrete cracking model. Therefore, some comparisons could be made in establishing values of the reliability index applicable to the crack models of MC 2010 and EN 1992. For crack widths equal to and greater than 0,2 mm, the reliability index values determined were in the range of 0 to 1,8. Markova and Holický (2001) performed a reliability analysis of cracking in a reinforced concrete slab for various design code crack formulations and found the ENV 1992-1-1 crack model (precursor of EN 1992-1-1) was sufficient for a limiting crack width of 0,3 mm (probability of exceedance of 5 %). The reliability index was determined to be above 1,5. However, crack widths less than 0,3 mm were not considered. As already discussed, Van Nierop (2018) used economic optimisation of a specific LRS and a generic model for the selection of SLS target reliability. The results of her research suggest that a β_t of at least 2,0 would be more appropriate than 1,5 for SLS cracking in the design of LRS.

The limited data on a suitable target reliability suggests that a value between 1,5 and 2,5 could be appropriate for SLS cracking with respect to structures such as LRS.

3.8.2 Model Uncertainty of Crack Models

A review of research into model uncertainty with respect to concrete cracking was done. First, sources were identified where reliability analyses were performed with at least some level of model uncertainty assessment.

An investigation into tensile load-induced cracking by Holický *et al* (2009) did a preliminary reliability analysis, using estimated statistical parameters for model uncertainty as a RV, noting that further research was required for a proper assessment of the model uncertainty. Model uncertainty was assumed to have a lognormal distribution, a mean of 1 and a standard deviation of 0,1. Comparisons were also made with general uncertainty factors in choosing the preliminary values for the crack model uncertainty. Quan and Gengwei (2002) treated model uncertainty as a random variable in the Chinese crack model, defined as the ratio of observed to predicted maximum crack widths. On analysing experimental data on short-term cracking in beams for a sample size of 116, they concluded that the model uncertainty had a log-normal distribution with a mean of 1,05 and a CoV of about 0,3. Research by McLeod (2013) showed that model uncertainty affected reliability performance of the EN 1992 crack provisions at least significantly, and possibly a dominating effect, with the effect increasing as the CoV increased particularly for the tension load case.

Othman *et al* (2014) compared experimental crack spacing data to the Dawood crack spacing formulation, using the ratio of experimental to predicted crack spacing as model uncertainty, for flexure in plates. They concluded that model uncertainty in this case had a lognormal probability distribution with a mean of 0,98 and standard division of 0,229 which equates to a CoV of about 0,23. Cervenka et al (2017) assessed the uncertainty of crack widths for the MC 2010 crack width formulation and numerical simulations, compared to the limited experiments results on 12 beams from Pérez Caldentey *et al* (2013). Their results are summarised in Table 3.6 where model uncertainty was defined as the ratio of experimental to predicted values. It was found that the numerical simulation predicted the mean crack width well but underestimated the maximum crack width, as did MC 2010. However, the results should be seen as preliminary considering the limited experimental data.

Table 3.6: *Summary of MC 2010 crack model uncertainty results from Cervenka et al (2017)*

Model Uncertainty	MC 2010		Numerical simulation	
	Mean	CoV	Mean	CoV
Mean crack width	1.34	0.42	1.09	0.35
Maximum crack width	2.15	0.38	1.53	0.36
Mean crack spacing	-	-	0.75	0.10

The above literature all indicates that although model uncertainty of crack models is dependent on the crack model itself, there is a significant randomness in the cracking mechanism considering maximum crack widths which suggests that model uncertainty of crack models is significant with a high coefficients of variation in the order of 0,2 to 0,45, depending on the crack model. The experimental data of Pérez Caldentey *et al* (2013) was reviewed and it was noted that the crack widths were estimated using measured strain and the number of cracks, as it is known to be difficult to measure crack widths accurately. All results reported here are for short-term flexural cracking. Other than the data used to quantify model uncertainty, as presented in the following chapter, no other usable sources were found with information on long-term cracking.

The review of literature showed that the information on the statistical parameters of model uncertainty assessment was limited. Further review was done to find any data on comparisons between measured and predicted crack width values, particularly for small crack widths. Such data would also aid in establishing a database of experimental results used to quantify model uncertainty. Data that could be used in the model uncertainty database are presented in Chapter 4. Results that could not be used in the database owing to insufficient information being provided, are reported here since they still add value to the discussion on model uncertainty. Most studies compared the experimental to predicted crack widths for various codes and proposed formulations directly with some linear regressions performed to assess the particular crack model's performance against the experimental data. It was noted that the datasets in many of the studies were limited.

Peiretti *et al* (Eurocode 2 commentary*, 2003*) compared test data of mean crack widths ($w_{m,exp}$) to calculated values ($w_{m,calc}$). The crack models from the CEB model code MC 1990, PrEN (ENV 1992, previous version to EN 1992) and EC2 (the then proposed EN 1992) were compared. Results were plotted, as shown in Figure 3.3. Figure 3.3 demonstrates that a wide spread in test and model crack widths was obtained. PrEN was indicated as correlating well with test crack widths, while EC2 predicted values slightly less than test values. MC90 underestimated the crack width when compared to test values. The error in the mean crack width ($w_{m,calc} - w_{m,exp}$) was then plotted against the measured crack width, shown in Figure 3.4. The mean and standard deviation for the error was determined and expressed in mm. However, the error was not related in the text to any particular crack width. The graph does indicate that there is a wide scatter in the test results compared to the calculated crack widths.

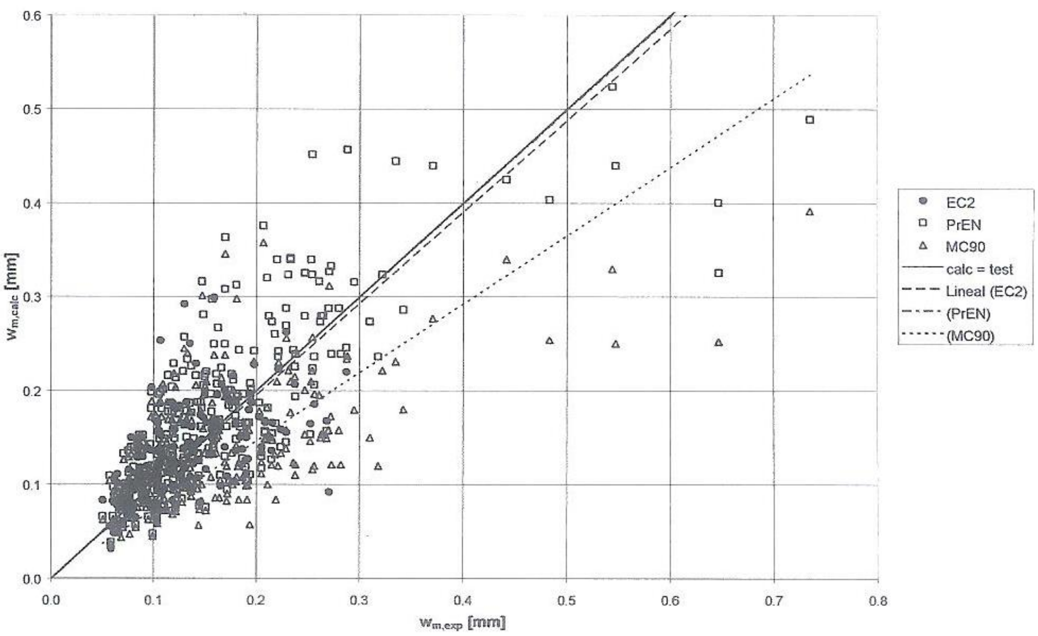


Figure 3.3: *Comparison between test and calculated mean crack widths to EC2, MC90 & PrEN. (Source: Eurocode 2 commentary (2003))*

On plotting the distribution and density functions of the error, Peiretti *et al* (Eurocode 2 commentary*, 2003*) concluded that the error had an approximately normal distribution for EN 1992.

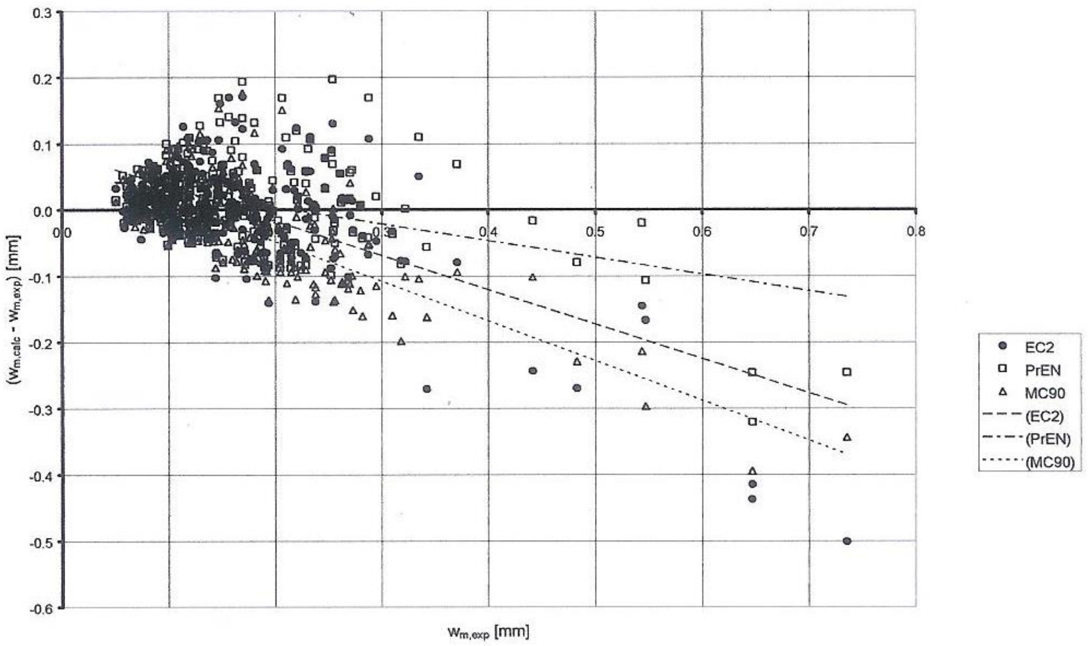


Figure 3.4: *Error in crack width (Source: Eurocode 2 commentary (2003))*

The comparison of experimental data to common design code formulations showed a large scatter of predicted crack widths, indicating that model uncertainty is significant. This provided motivation for quantifying model uncertainty of select crack width prediction models for the purpose of reliability assessment, as presented in this research

Most research on crack models, such as in the sources cited in Table 3.7, has focused on model development by means of standard statistical analysis of experimental data, whereby the design crack width is generally considered as having a 5 % probability of being exceeded. These researchers reported predicted values compared to experimental results. The ratio of experimental to predicted crack widths or spacings were then calculated in this study, along with estimates of the mean and variation in each case, using the reported experimental and predicted crack widths. Table 3.7 presents a summary of these calculations. The ratio of experimental to predicted crack widths can be taken as a measure of model uncertainty. There was insufficient data in the sources to be able to estimate the probability distributions.

Lapi *et al* (2017) reported the maximum crack width ratio as predicted values divided by the experimental values. However, to compare with the values obtained from other researchers, the results from Lapi *et al* (2017) are reported in Table 7.3 as the ratio of experimental to predicted crack widths (the inverse). The statistical analyses performed by the various researchers were aimed at fitting the experimental data to crack models, therefore did not include assessment of the probability distribution of model uncertainty.

Table 3.7: *Summary of model uncertainty factors inferred from literature, as ratios of experimental to predicted values*

Source	Load type	Duration of load	Type of element	Parameter	Measure of variation or error	n	Model uncertainty as ratio experimental to predicted values											
							EN 1992		MC 2010		MC 1990		BS 8007/ 8110		ACI Gergely & Lutz		ACI Frosch	
							μ	Variation	μ	Variation	μ	Variation	μ	Variation	μ	Variation	μ	Variation
Beeby & Scott (2005)	Tension	Short-term	Ties	w_m	CoV	244	-	-	-	-	0.81		-	-	-	-	-	-
Rimkus & Gribniak (2017)	Tension	Short-term	Ties	$S_{r,max}$	CoV	15	0.74	0.26	0.53	0.23			-	-	-	-	-	-
Eckfeldt (2009b)	Tension & flexure	Short-term	Beams & ties	w_{max}	Linear regression, y/x and R^2	176	*0.79	*0.14	-	-	1.36	*0.30	-	-	0.94	*0.49	1.15	*0.51
Van den Berg *et al* (1993)	Flexure	Short-term	Beams	w_{max}	CoV	3	-	-	-	-	-	-	1.51	0.18	1.28	0.24	-	-
Allam *et al*(2012)	Flexure	Short-term	Beams	w_{max}	CoV	3	1.06	0.15	-	-	-	-	1.56	0.19	0.90	0.19	-	-
Dawood & Marzouk (2011)	Flexure	Short-term	Plates	w_m	CoV	15	0.74	0.20	-	-	1.23	0.45	-	-	1.04	0.26	-	-
Lapi *et al* (2017)	Flexure	Short-term	Beams	w_{max}	CoV	380	1.04	0.31	1.06	0.32	1.67	0.41	-	-	0.81	0.32	1.03	0.43

*Linear regression of $w_{test}/w_{predict}$: (y/x) as mean, R^2 error not variation

Calculated in this study using source data

Referring to Table 3.7, MC 1990, which does not allow for the influence of cover in crack spacing calculation, underpredicts the maximum crack width in most cases and had the highest variation which suggests that this model does not perform well. Lapi *et al* (2017) compiled the largest database and showed that the Eurocode 2 and MC 2010 formulations generally slightly underestimate maximum crack width with means just higher than 1. Referring to the ratios calculated in this study from the results of Van den Berg (1993) and Allam *et al* (2012), the British formulation underpredicts maximum crack width for flexural cracking, with model factor mean values about 1,5. There is however a significant variation in the ratios across the data, comparing flexure and tension load cases. Ratios less than 1 suggest EN 1992 and MC 2010 overpredict maximum crack widths for tension cracking, with a significant bias. On comparing the means of model factors determined using mean crack widths to those using maximum crack widths, it is noted that the two factors are dissimilar. Sample sizes are generally small, making it difficult to compare variances.

Where test samples were of similar dimensions and reinforcement configurations, the variation was less, which would be expected. The tension load case displayed a smaller variation than for flexure. The linear regression results obtained by Eckfeldt (2009b) generally indicated the wide spread in experimental crack widths. The data sets used by Eckfeldt (2009b) were for short-term loading of beams and ties and maximum crack widths.

Lapi *et al* (2017) plotted box and whisker plots to assess the distribution of the ratio of the maximum predicted to experimental crack widths (w_{th}/ w_{exp}) for flexural cracking in beams and various crack models, presented here as Figure 3.5.

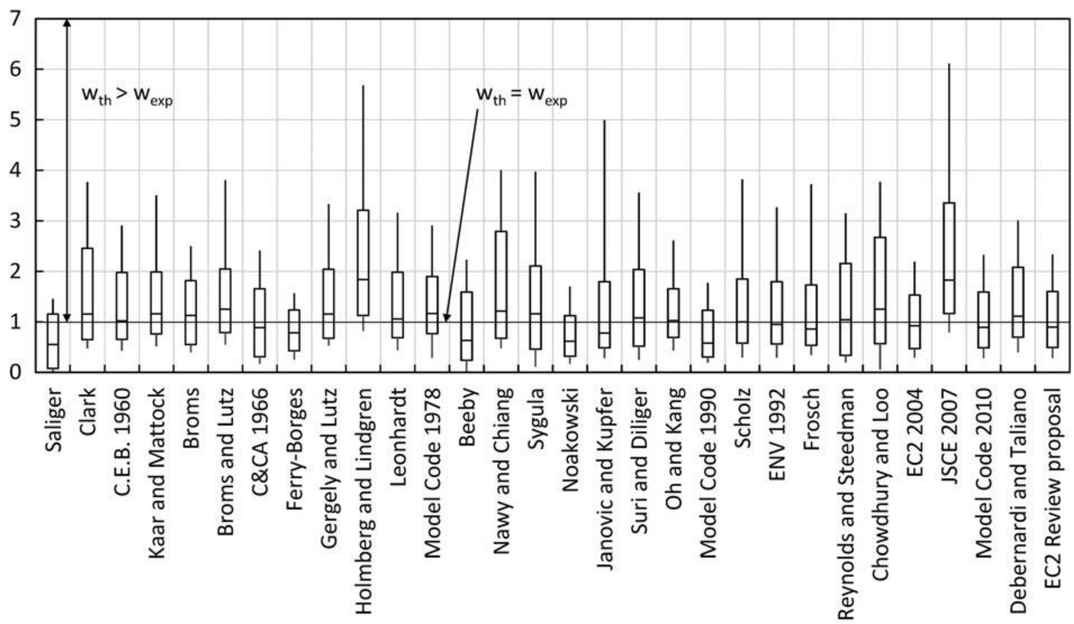


Figure 3.5: *Box and whisker plots by Lapi et al (2017) of w_{th} / w_{exp} ratios*

The data was sourced by Lapi *et al* (2017) from various researchers. The 5 % and 95 % quantiles, represented on the plot as the bottom and top of the boxes, were also determined. From these box plots, it is evident that the crack models have a wide variation, with a negative skewness. The newer code formulations have smaller tails extending beyond the 95 % quantile which suggests a better fit of these models to the experimental data. As a qualitative assessment of these results, most tend towards a skewed lognormal rather than a normal distribution.

The literature shows that due to the inherent randomness of cracking mechanisms, crack models do indeed have a high degree of uncertainty, with CoV values mostly in the range of 0,2 to 0,4, depending on the crack model. Referring to Table 3.7, there also appears to be differences in the model uncertainty values obtained for flexural and tension cracking, with the prediction of tension cracks showing more bias but less variability. There is a lack of research for long-term cracking, particularly for the direct tension case which makes the quantification of model uncertainty in this case more challenging.

3.8.3 Additional Sources of Uncertainty in Crack Models

Other than in the cracking mechanism itself and the model parameters (which are discussed in the following section), uncertainty may exist in experimental work. The size effect has been noted by a few researchers. Yasir Alam *et al* (2010) tested beams of different sizes under flexural loading. It was found that EN 1992 reasonably predicted maximum crack widths for the smaller beams, with accuracy in the predicted crack widths decreasing as the beam size increased. The conclusion was reached that this was due to the size effect which is not taken into account in the Eurocode crack model.

On studying the data on testing of reinforced concrete ties, it was noted that the lengths of these ties is often less than full scale, and reinforced only with one central bar. Whilst these experiments give a good idea of the cracking mechanism in tension load-induced cracking, basing a crack model on this data adds to model uncertainty as cracking behaviour in full size members under direct tension with multiple bars may not be fully described by the crack model. Eckfeldt (2008) showed that when determining crack spacings, which is related to crack width, the tie needs to be long enough to have sufficient crack spacings to obtain proper values for mean crack spacing, as the standard deviation of the mean crack spacing, σ_{Srm}, depends on the number of crack spacings. Eckfeldt (2008) related the expected maximum crack spacing $E(S_{r,\,max})$ to the standard deviation of mean crack widths for normally distributed variables, described by:

$$E(S_{r,max}) = S_{rm} + k_m . \sigma_{Srm}$$

where k_m is a correction factor dependent on the number of crack spacings. This concept is illustrated by Figure 3.6.

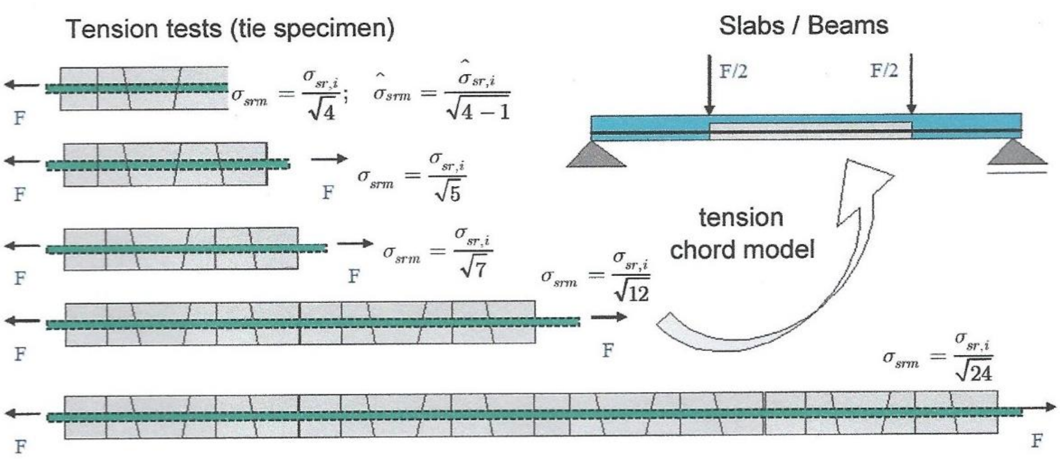


Figure 3.6: *Influence of specimen length on crack spacing* (Source: Eckfeldt, 2008)

The use of deterministic values for parameters such as bond factors would introduce some uncertainty which may be included in the model uncertainty. However, there is insufficient data in the literature to potentially treat these factors as random variables in this research. Research by Nejadi (2005) on a small sample of 12 beams concluded that, depending on the steel stress, the ratio between bond stress and concrete tensile strength was between 1,0 and 1,5 for sustained loading. This is lower than the value recommended by MC 2010 of 1,8, suggesting that MC 2010 may underestimate crack spacing and crack width for flexural cracking. Further research is needed for this parameter.

As discussed in Section 2.3.3 of Chapter 2, the EN 1992 and MC 2010 formulations specify three equations in determining the effective depth and thus the effective area of the tension zone in the cross section, the limiting equation depending on the element geometry and stress distribution. This area is idealised as rectangular although the actual shape is closer to ellipsoidal. This simplification of the effective area model would add to the overall model uncertainty, but the degree to which this would occur would be small.

3.9 GENERAL DATA FOR PROBABILISTIC PARAMETERS OF CRACK MODEL

In order to perform reliability analyses in which the influence of model uncertainty could be assessed, an investigation into the probabilistic parameters of the variables used in the calculation of crack widths was carried out, with the resulting values summarized in Table

3.8. To obtain a realistic probabilistic assessment of the crack width design formulations, the reliability analysis was applied to typical configurations of walls in LRS. Sources for the parameter values were mainly from Holický (2009), the Joint Committee on Structural Safety, JCSS (2001), Fulton's (2009) and Holický , Retief and Wium (2009), and as used by McLeod (2013). Data is reported in terms of the probability distribution type (PDF), the characteristic value, mean (μ_x) and coefficient of variation (CoV). Typical South African values correspond with the general values given in the table.

Variables which have very small coefficients of variation may be considered as deterministic values, for example, geometric properties which generally have small variations compared to actions and material properties. Geometric properties such as section thickness tend to have a normal probability distribution function (PDF).

Table 3.8 *Summary of basic variables for time-invariant reliability analysis, derived from Holický (2009), JCSS (2001), Fulton's (2009) & Holický et al (2009).*

Variable	Symbol	Units	PDF	Mean μ_x	CoV
Permanent Load	G_k	kN/m^2	N	G_k	0.03 – 0.10
Liquid Load	L_k	kN/m^2	N	L_k	0.03 – 0.10
Concrete compressive strength	f_c	MPa	LN	f_{ck}+ 2σ	0.10 – 0.18
Concrete tensile strength	$f_{c,t}$	MPa	LN	f_{ctk}+ 2σ	0.10 – 0.18
Steel modulus	E_s	GPa	Det		
Concrete modulus	E_c	GPa	Det		
Reinforcement diameter	Φ	mm	Det		
Reinforcement area	A_s, A_s'	mm^2	Det		
Concrete c/s geometry	b, h	m	N	b_k, h_k	0.005 – 0.01
Cover	c	m	BETA / Γ	c_k	0.005 – 0.015
Distance to centre of bar	a	m	BETA / Γ	c + φ/2	0.005 – 0.015
Limiting Crack width (average)	w_{lim}	mm	Det	0.05 – 0.2	
Model factor for crack width	θ_w	mm	N/ LN	1.0	0.1 – 0.3

Note: LN = log-normal, N = normal, Det = deterministic, Γ = gamma

Standard test methods are employed to determine the statistical data for material properties which tend to have normal or log-normal distributions. The characteristic value of the relevant material property is used in limit state design and corresponds to the 5% lower fractile, as defined by Cl. 5.6 of SANS 10160-1 (2011), which is taken 1,64 x standard deviation below the mean for a normal PDF.

The target design strength of concrete is related to a degree of quality control as described by EN1990-1 (2002) and SANS 2394: 2016. At least a good degree of quality control would be expected in the design and construction of special classes of structures such as water-retaining structures. The necessity of a proper mix design to ensure durable concrete should be emphasized, as was discussed in Chapter 2, so as not to introduce undue cracking or other unwanted effects in the structure that are not taken into account in the crack model.

Permanent actions or loads tend to have normal distribution which is the case for liquid loads in LRS, which are treated as quasi-permanent loading by reinforced concrete design standards. LRS undergo slow emptying and filling so as not to cause turbulence under operating conditions, meaning that liquid loads are static rather than dynamic.

Referring to Equation 3.2, the crack width limit in the limit state function of a crack reliability model was accepted as a deterministic value according to standard specification. It is recognized however that specified limits likely introduce some measure of conservatism. The specified limits were derived considering durability and self-healing of cracks, with the associated risk of leakage. However, insufficient information is available at present for the limiting crack width to be modelled as a random variable, with further research required. The reliability analysis of the EN 1992 crack model by McLeod (2013) showed that sensitivity factors and theoretical partial safety factors of the RV's were not affected significantly by the limiting crack width over a range of 0,05 to 0,2 mm for tension load-induced cracking. There was an increase of 10% in sensitivity and partial safety factors for model uncertainty as the limiting crack width decreased from 0,2 to 0,05 mm, for a model uncertainty mean of 1 and CoV of 0,3. Treating the crack width limit deterministically would thus be a reasonable assumption until probabilistic information on the likelihood of self-healing is available to improve such description. This is also in keeping with the general form of the limit state function as defined by SANS 10160 (2011), given here as Equation 3.1.

The concrete cover to reinforcement has shown to be an influential parameter in the calculation of crack widths, as discussed in Chapter 2, by various researchers in both the physical (for example, Pérez Caldentey *et al*, 2013) and reliability crack models (McLeod, 2013). Variability of the concrete cover decreases with an increasing level of quality control. Ronné (2006) reviewed local and international cover data and found that variability also decreased with increasing cover, with South African construction having a higher absolute variability for concrete cover. As a comparison, a coefficient of variation (CoV) of 0,15 % was suggested for typical British construction for a good standard of control. As LRS should have at least a good level of quality control, it can therefore be concluded that a CoV of 0,15 % would be a reasonable value for concrete cover in the probabilistic crack model. Holický *et al* (2009) concluded that a limited beta or gamma PDF for concrete cover could be reasonably approximated to a log-normal distribution.

The review on long-term shrinkage strain produced little information regarding its stochastic parameters. Numerous shrinkage strain models have been developed, as discussed in Section 2.3.7 of Chapter 2 which suggests that this mechanism is not simple to predict and therefore would have some degree of uncertainty. Alexander (Fulton's, 2008) commented that research done on the effect of aggregates on concrete shrinkage showed that shrinkage can vary by up to about 30 to 40 % depending on the type of aggregate. This could be mitigated in LRS as concretes used in this type of structure should be low-shrinkage, using the appropriate cements and aggregates, combined with good quality control during construction.

3.10 CHAPTER SUMMARY

This chapter summarises the framework for reliability analysis and the quantification of model uncertainty, related to load-induced crack width prediction models. The procedure for the First Order Reliability Method, as a suitable method of probabilistic analysis, is discussed. The limit state or performance function for a reliability assessment of this SLS is described by:

$$g(X_i) = C - \vartheta . E$$

where C is the specified crack width limit E is the predictive crack model and ϑ is the model uncertainty.

The structural performance is evaluated probabilistically and described by the reliability index, β, which is linked to a given probability of failure and reference period. The target reliability, intended to be cost optimal, has a reference level for SLS of β_t = 1,5 for a 50-year period for irreversible conditions. However, when SLS is the governing state, as in LRS, then a β of 1,5 may not be sufficient as suggested by the results of research by Van Nierop (2018). Values above 2,0 were recommended.

In general terms, sources of uncertainty in the model are identified and quantified as much as is reasonably possible. Any sources of uncertainty that cannot be quantified or have a nominal effect on the model may be included in the model uncertainty. In a reliability analysis, model certainty can be treated as a random variable and may be defined as the ratio of experimental to predicted values, for which the stochastic parameters can be determined. The extent of the treatment of model uncertainty depends on its significance in the model. In addition, model uncertainty can be used to assess the performance of structural models.

In a reliability model, the crack model parameters are treated as random variables. Their variations account for some of the uncertainty in the crack model. The fixed-value coefficients of the Eurocode and Model Code crack models may be treated deterministically. Any associated uncertainty would then be included in the model uncertainty. The limiting crack width is modelled as a deterministic value as defined by SANS 10160-1 (2011).

Long-term shrinkage strain is expected to be influential in the crack model, however, there is little information on appropriate statistical parameters if modelled as a random variable in a reliability model. The influence of shrinkage strain on crack models is therefore investigated, as presented in the following chapters. Crack models that do not include shrinkage strain for long-term cracking are expected to have a higher model uncertainty in this case as they would underpredict long-term crack widths.

With respect to load-induced crack models, it is anticipated from the literature review of the available research that model uncertainty is an influential parameter in the crack model with a CoV in the region of 0,3 to 0,4 and a lognormal distribution when considering crack models based on the Eurocode model. It was established that there have been very limited studies quantifying model uncertainty and in assessing the SLS reliability of crack models, thus justifying this research. Model uncertainty of the crack model may be defined as the ratio of experimental to predicted crack widths, and so can be quantified stochastically.

From the literature review on crack models and reliability, it was discovered that the research concentrates on short-term cracking behaviour. There is a resulting lack of information for cracking due to long-term loads particularly in the direct tension case.

CHAPTER 4
QUANTIFICATION OF MODEL UNCERTAINTY OF CRACK MODELS

4.1. INTRODUCTION

This chapter presents the quantification of model uncertainty of load-induced crack models. Model uncertainty is a significant influence on the reliability of crack width prediction models, shown in earlier research by McLeod (2013). However, as concluded from the literature review presented in Chapter 3, there is a lack of data on the stochastic values of model uncertainty, providing motivation for this research.

Model uncertainty is defined here as the ratio of the maximum measured crack width and the predicted crack width (w_{exp} / $w_{predict}$). The methodology used in quantifying model uncertainty applicable to the crack models selected is described in the following sections. The results of the model uncertainty quantification are presented and are also used to assess the crack models in developing the General Probabilistic Model (GPM). From the literature review, presented in Chapters 2 and 3, the crack models of EN 1992 and MC 2010 (which is also the proposed updated model for EN 1992) are chosen for analysis and comparison towards selecting a GPM as these models were derived from an analytical basis, rather than an empirical one, and also represent some of the most current research. BS 8007 is included for comparison as it is the model currently used by industry in South Africa. Other empirical models are not included for further study. As the GPM is to be applied to liquid retaining structures (LRS) wherein liquid loads may act as quasi-permanent loads, cracking due to both short- and long-term loading was evaluated.

The quantification of model uncertainty for a GPM crack model will allow assessment of the influence of model uncertainty on the crack model, and thus the level of further treatment of model uncertainty in reliability assessments that utilise such a GPM.

4.2 QUANTIFICATION OF MODEL UNCERTAINTY

The methodology to determine the stochastic parameters of model uncertainty as the ratio w_{exp} / $w_{predict}$ is set out in this section. A database of experimental results for load-induced cracking is first compiled. Design crack width formulations are used as a best estimate of the predicted maximum crack widths, as discussed in the following section. The experimental measured material properties, loading and sample geometry are utilised in the calculation of the predicted maximum crack widths. The w_{exp} / $w_{predict}$ ratios are then determined for each crack model and load case. A statistical analysis of the w_{exp} / $w_{predict}$ ratios for each crack model is performed, thus quantifying model uncertainty as a random variable.

4.2.1 Compilation of Experimental Database on Load-Induced Cracking

The first step in quantifying model uncertainty is to create a database using experimental data on load-induced cracking from reputable sources. The sources for each dataset are summarised in Table 4.1 and 4.2. In some cases, the data was obtained from a source that used other researchers' data, as reflected in the tables below. Full details of each test subset with the predicted crack width calculations are given in Appendix A. Only experimental datasets having all relevant details are used to ensure proper interpretation of the data and not introduce unnecessary uncertainty into the crack model.

As discussed in earlier chapters, in structures such as LRS, the critical load is due to liquid load which is considered as a quasi-permanent load over the long-term. Hence experimental data that considered long-term loading conditions was required. However, most experimental research to date has been done for the short-term loading condition. Few studies were performed for sustained loading, in particular for tension cracking. It is therefore proposed to use the short-term model uncertainty quantified in this study to assist in predicting the long-term model uncertainty, together with the limited data that does exist. Based on the literature review, presented in Chapters 2 and 3, flexure and tension load-induced cracking are treated separately.

The data is thus divided into the following load cases, according to the cause of cracking and duration of loading:

(i) Flexure, short-term loading.
(ii) Flexure, long-term loading.
(iii) Tension, short-term loading.
(iv) Tension, long-term loading.

Only experimental datasets that measured the maximum crack width were considered in this study, as this is the criterion used in the design formulations. Some researchers reported the mean crack width only, while other reported the maximum or both, depending on the objectives of their testing.

Table 4.1: *Sources of experimental data – flexural load-induced cracking*

RESEARCHER	SOURCE	ELEMENT TYPE	DURATION OF TESTS	COMMENTS	No. of samples*
Chan	Chan (2012)	Beams	Short-term	Varied reinforcement type. Links provided either over full length of beam or just outside maximum moment zone. Repeats 4 x 6 No. beams & 3 x 1 No. of different configurations.	27
Clark (1956) UPM data	Pérez Caldentey (2016) Eckfeldt (2009)	Beams and 1-way spanning slabs	Short-term	Varied cross section, reinforcement and cover	34
CUR Report No.37 UPM data	Pérez Caldentey (2016) Eckfeldt (2009)	Beams and 1-way spanning slabs	Short-term	Varied cross section, reinforcement and cover	15
Hognestad (1962) UPM data	Pérez Caldentey (2016) Eckfeldt (2009)	Beams	Short-term	Varied reinforcement and cover	30
Frosch	Frosch & Blackman (2003)	1-way spanning slabs	Short-term	Varied reinforcement spacing and area in 203 deep x 915 wide slabs. No transverse reinforcement.	4
Kenel & Marti (2002)	Burns (2011)	Beams	Short-term	3-point loading.	2
Krips (1984) UPM data	Pérez Caldentey (2016) Eckfeldt (2009)	Beam	Short-term	Single beam	1
Rimkus	Rimkus (2017)	Beams	Short-term	Varied reinforcement	6
Rusch & Rehm (1963) UPM data	Pérez Caldentey (2016) Eckfeldt (2009)	Beams and 1-way spanning slabs	Short-term	Varied cross section, reinforcement and cover	18
Attisha	Attisha (1972)	Beams	Short & long-term	Varied reinforcement + 513 samples –short-term , 5 samples long-term .	13 + 5
Illston	Illston & Stevens (1973) Stevens (1973)	Beams	Short & long-term	Varied reinforcement, cover and environment Total of 60 beams – average results reported for repeat samples.	17
Jaccoud & Favre (1982)	Burns (2011)	1-way spanning slabs	Short & long-term	Links provided. 3 beams – short-term , 6 beams - long-term	6
Nejadi (2005)	Nejadi (2005) Gilbert &Nejadi (2004)	Beams and 1-way spanning slabs	Short & long-term	Varied cover and reinforcement ratio. Repeats 2 x 4 No. beams.	12
Wu	Wu (2010) Wu & Gilbert (2013)	Beams and 1-way spanning slabs	Short & long-term	6 beams, 4 slabs Varied reinforcement in beams	8 + 2

* Final load steps considered only, EN 1992 minimum strain complied with.

Table 4.2: *Sources of experimental data – direct tension load-induced cracking*

RESEARCHER	SOURCE	ELEMENT TYPE	DURATION OF TESTS	COMMENTS	No. of samples*
Farra & Jaccoud	Farra & Jaccoud (1993)	Ties with single reinforcing bar, square cross section.	Short-term	Ties all the same dimensions Differed cements, concrete mixes and reinforcing bar diameter	71
Hartl (1977) UPM data	Pérez Caldentey (2016) Eckfeldt (2009)	Ties with single or 2 No. reinforcing bars, square cross section.	Short-term	Varied cover and reinforcement diameter. Some beam configurations repeated.	48
Hwang	Hwang (1983)	Slab elements reinforced in both directions and subject to axial tension in 1 direction	Short-term	Varied cover and reinforcement	34
Wu	Wu (2010) Wu & Gilbert (2008)	Ties with single reinforcing bar, square cross section.	Short & long-term	Ties all the same dimensions Varied reinforcement diameter. 7 short-term tests, 4 long-term	7 + 4
Eckfeldt	Eckfeldt (2009)	Ties with single or 2 No. reinforcing bars, square cross section.	Long-term	Varied width of cross section cover and reinforcement. Repeats 2 x 4 No. ties & 3 x 1 No. ties.	11

* Final load steps considered only, EN 1992 minimum strain complied with.

4.2.2 Deterministic Analysis of Predicted Crack Widths

Once the database was compiled, the best estimate of predicted crack widths could be determined using the selected design crack width prediction formulations. After reviewing the literature on crack models, the following design code formulations are considered:

(i) EN 1992 (2004).
(ii) MC 2010 (2011)
(iii) BS 8007 (1987).

With the move towards the use of Eurocodes, either in adopting or in compiling compatible South African standards, the Eurocode-based crack models, including MC 2010, are included for analysis. From the literature, these models with an analytical basis, perform reasonably compared to other crack models for short-term loading. Empirical models other than BS 8007 are not considered. The latter is included for comparison as it is in use in South Africa at present.

The design code formulations used are as described in the literature review of Chapter 2:

(i) EN 1992 - Equations 2.1 to 2.5 (Section 2.2.2.1)
(ii) MC 2010 - Equations 2.6 to 2.9 (Section 2.2.1.2)
(iii) BS 8007 - Equations 2.10 to 2.17 (Section 2.2.3.1)

The experimental measured material properties, section geometry and loading applicable to each crack width formulation were used to calculate the predicted crack widths. For any fixed-value coefficients, the recommended values were utilised. Regarding the proposed updated EN 1992 crack model, a value of 1,0 is recommended for k_2 by both EN 1992 and MC 2010 for tension cracking. For flexural cracking, values of 0,5 and 1,0 for k_2 are used by EN 1992 and MC 2010, respectively. As it is undecided which value of k_2 to use in the MC 2010 crack formulation for flexural cracking, crack widths were therefore calculated using both values for k_2 for comparison in this case.

In the long-term loading condition, the effective modulus of elasticity of concrete is calculated using the creep coefficient reported, for all crack models. Shrinkage strain in all cases is the measured value given. In most studies, standard materials and test procedures were employed to determine concrete compressive and tensile strengths, the modulus of elasticity of concrete and shrinkage strain.

A summary of the spreadsheets showing the calculation of the predicted crack widths are given in Appendix A.

4.2.3 Statistical Analysis of Model Uncertainty, ϑ_w

Model uncertainty of each crack model, ϑ_w, is defined as the ratio of the experimental to predicted crack width (w_{exp} / $w_{predict}$), as a commonly accepted way to quantify model uncertainty as a random variable and considering the form of the limit state function of Equation 3.3. Maximum crack widths are used as the design code formulations predict maximum crack width. In addition, these predicted crack widths are assessed against a crack width limit as a maximum value. It must be noted that the design predicted crack width is assumed to correspond to a 5 % probability of exceedance by design code formulations. Utilising this value, rather than an absolute maximum, introduces some bias into the mean of model uncertainty. Similarly, the crack width limit, as the crack width at which self-healing occurs, has some degree of bias. However, the extent of this bias cannot be determined, therefore is included in the model uncertainty.

The w_{exp} / $w_{predict}$ ratios for each crack model are determined using the dataset compiled of all available experimental data as listed in Tables 4.1 and 4.2. As model uncertainty is treated as a random variable, the stochastic parameters of ϑ_w are estimated for each crack model for all loading conditions considered. The model uncertainty ratio is determined for the following crack models for both flexure and tension loading:

(i) EN 1992
(ii) EN 1992 with shrinkage strain term added for long-term cracking.
(iii) MC 2010 - k_2 = 1 and 0,5 (flexure only)
(iv) BS 8007 with a crack width limit of 0,2 mm
(v) BS 8007 with a crack width limit of 0,1 mm

It must be noted that the w_{exp} / $w_{predict}$ ratios determined for the BS 8007 crack prediction formulation should be viewed with some caution. The calculation of the predicted crack widths is problematic due to this model's use of a_{cr}, defined as the distance from the crack considered at the surface to the nearest reinforcing bar in the sample cross section, rather than crack spacing, as illustrated in Figure 4.1, for single and multiple bars. The point on the cross section where the crack was measured was generally not given in the experimental records. This introduces some unknown uncertainty into the predicted crack widths due to potential mismatches between the measured and calculated positions. Therefore, where samples have multiple bars in the cross section, a_{cr} is estimated as the maximum distance either the using the spacing of the reinforcement in the sample cross section, as shown in Figure 4.1 for multiple reinforcing bars (a), or the distance from the reinforcing bar to the corner of the cross section (b).

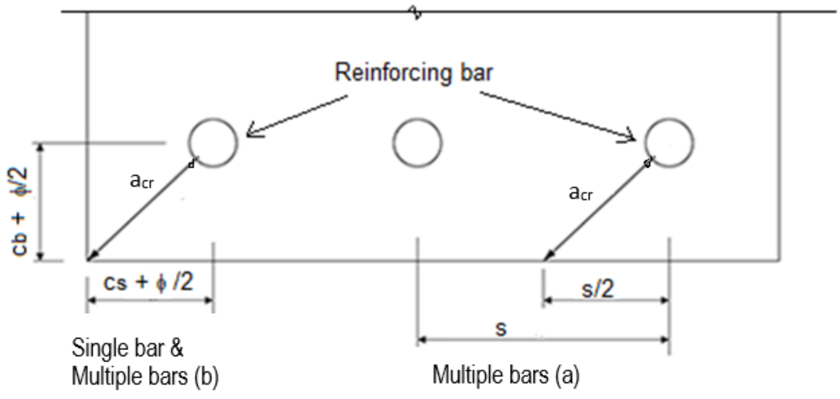


Figure 4.1: *Determination of a_{cr} to BS 8007*

In the tension case, many of the experiments consisted of ties reinforced with a single reinforcing bar. Thus, by the definition of a_{cr}, the maximum possible distance from the reinforcing bar to the crack considered is to the corner of the cross section:

$$a_{cr} = a - \frac{\varphi}{2} = \sqrt{(c_b + \frac{\varphi}{2})^2 + (c_s + \frac{\varphi}{2})^2} - \frac{\varphi}{2}$$

for a single bar, as shown in Figure 4.1.

In the MC 2010 formulation for long-term loading, the free shrinkage strain may be multiplied by the restraint coefficient, R, the value of which varies depending on the restraint condition, as per BS EN 1992-3 (2006) Figure L-1 of Annex L. A restraint factor of 1 is used in determining shrinkage strain in this case, as the measured shrinkage strain was used in the determination of mean strain which would have taken any restraint condition into account.

As the EN 1992 crack model is not expected to perform reasonably when applied to the long-term flexural loading case, the maximum crack widths are calculated first by using the design formulation as it stands, and then by modifying the existing formulation to include shrinkage strain. In this way, the EN 1992 formulation could be assessed to see if a simple modification by the inclusion of all long-term effects would be sufficient to improve its performance over time.

Some refinement of the initial database was also necessary, as follows:

- In most short-term loading tests, loading was applied in increasing increments. Only crack widths measured on the final load step were therefore considered to ensure the independence of samples and that the stabilised cracking phase was reached. The latter is in keeping with all the crack models' assumptions in predicting maximum crack widths.

- In some cases, it was found that the calculated mean strain was either very small, which results in the design crack width being underpredicted, or even negative. This is due to the tension stiffening models overpredicting the influence of the concrete in the tensile zone in the cross section where loads are small compared to the section size or reinforcement ratio, resulting in low steel stresses. Therefore, the data in these cases is disregarded. Only data where the calculated mean strain is at least the specified minimum limit of EN 1992 of 0,6 times the mean steel strain (tensile) was therefore used. This decision is justified by the fact that in liquid retaining structures , the applied loads and section geometry typically cause a mean strain well above the minimum limit. It should be noted that the BS 8007 crack model does not have a minimum limit specified for mean strain.
- Any repeat samples are averaged for the final dataset of model uncertainty values to prevent undue sample bias.

Model uncertainty, ϑ_w, is determined for each final dataset and design crack equation. A summary of the model uncertainty datasets for each load case are given in Appendix B. Standard statistical analyses of the model uncertainty data are performed to a 95 % confidence level to determine the stochastic parameters and the probability distribution thereof, relative to each crack model and load case. This confidence level is in keeping with that utilised in structural design standards. Non-parametric normality tests such as Kolmogorov-Smirnov (using Lilliefors significance correction) and Shapiro-Wilks (corrected), as appropriate, are performed to a significance p of 0,05 to establish the estimated probability distribution of the model uncertainty random variable, as required for the reliability analysis. These tests are compared to graphical methods such as probability plots and box plots to confirm the estimated probability distributions.

Through the statistical analysis of model uncertainty, a first assessment of the crack models' performances can be made. Comparisons between the crack models can be made, thus assisting in the selection of the GPM used in the reliability analysis. It must be reiterated that model uncertainty is specific to the crack model formulation, as can be seen from the statistical analyses.

As discussed in Chapter 3, a model is deemed to perform well if it shows a small bias (mean close to 1) and a low variability. With respect to crack models, a higher CoV is expected than is usually recommended for general structural model uncertainty (as Holický, 2009), due to the randomness of cracking mechanisms and the resulting difficulty in describing this behaviour in a mathematical model. In addition, consistent performance of the model over a range of parameters and conditions is desired. Pearson's correlations and scatter plots with linear regression are thus done to determine the important parameters of the crack width prediction models and their influence on the crack model considered, as well as assess consistency of performance. The Pearson's correlation coefficient, as a measure of the degree of linear association between two variables, has a value between +1 and -1. A value of 0 indicates that there is no association between the two variables. A value greater than 0 indicates a positive association between variables. Conversely, a value less than 0 indicates a negative association.

4.3 RESULTS OF MODEL UNCERTAINTY QUANTIFICATION

A summary of the results for the quantification of model uncertainty, as the ratio w_{exp} / $w_{predict}$, for each load case and crack model are presented and discussed here. The full set of results from the statistical analysis may be found in Appendix B.

4.3.1 Flexure, Short-Term Loading

After refining the database, the final sample size is 164 which is sufficient to obtain a reasonable estimate of the probability distribution and associated statistical parameters of model uncertainty. The statistical parameters of model uncertainty for flexural cracking under short-term loading are summarised in Table 4.3.

Table 4.3: *Model uncertainty statistical parameters for short-term flexural cracking*

Statistical Parameter	EN 1992	MC 2010 $k_2 = 1$	MC 2010 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Mean	1.107	1.052	1.551	1.185	1.112
Standard Error	0.033	0.031	0.05	0.035	0.040
Median	1.045	1.024	1.452	1.209	1.093
Std Deviation	0.420	0.395	0.638	0.450	0.510
COV	0.379	0.376	0.411	0.380	0.459
Sample Variance	0.176	0.156	0.407	0.203	0.260
Kurtosis	1.144	1.087	1.045	0.283	18.107
Skewness	0.813	0.65	0.863	0.136	2.642
Range	2.298	2.326	3.461	2.417	4.763
Minimum	0.350	0.295	0.286	0.154	0.149
Maximum	2.648	2.622	3.746	2.571	4.912
Count	164	164	164	164	164

Comparing the EN 1992 and MC 2010 formulations for flexure, the means obtained are 1,11 and 1,05, respectively, which is a small difference of about 5 %. MC 2010 has slightly less bias than the EN 1992 crack model, with a mean closer to 1,0. Both models have similar CoV values of about 0,38. As both crack models were derived using the same crack mechanism philosophy, this is not unexpected. On the other hand, they are substantially higher than the value of 0,1 generally recommended for model uncertainty in structural reliability models by sources such as Holicky *et al* (2009). This is indicative of the random nature of crack mechanisms and the resulting difficulties in modelling their behaviour.

The means and CoV's for the model uncertainty determined for EN 1992 and MC 2010 are comparable to those obtained by Lapi *et al* (2017) using maximum crack widths, as reported in Table 3.7 of Chapter 3. Although there is some overlap in the large dataset used by Lapi *et al* (2017) (namely, Clark (1956) and Hognestad (1962)) and that compiled for this study, the sources are largely independent. The similarity validates the short-term flexure model uncertainty values obtained in this study.

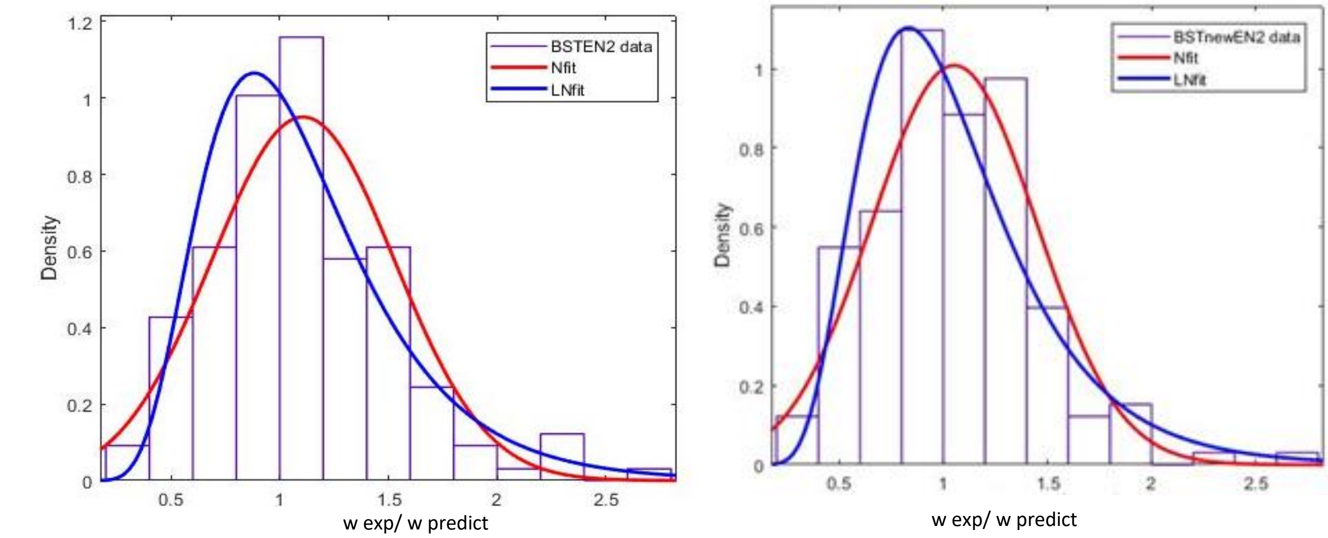
Using a value of 0,5 for the k_2 coefficient in the MC 2010 crack spacing model increases the model bias with the mean increasing substantially from 1,05 (k_2 = 1) to 1,55 (k_2 = 0,5), which is significant. However, there is just a small increase of approximately 8 % in the CoV from 0,38 to 0,41. The skewness and range also increases. Clearly, using a k_2 of 0,5 results in this crack model tending to underpredict crack widths. The results therefore suggest that it is not necessary to differentiate between flexural and tension cracking by applying the k_2 correction factor to flexural cracking.

The influence of the tension stiffening models used in the BS 8007 crack models on the statistical parameters is apparent, with the variation increasing from 0,38 to 0,46 for limiting crack widths of 0,2 mm and 0,1 mm, respectively. The range of values obtained for w_{exp} / $w_{predict}$ increases, affecting particularly the upper tail of the distribution which is important in reliability analysis. In addition, there is some uncertainty in the statistical parameters obtained, as the distance from the widest crack to the nearest reinforcing bar is assumed.

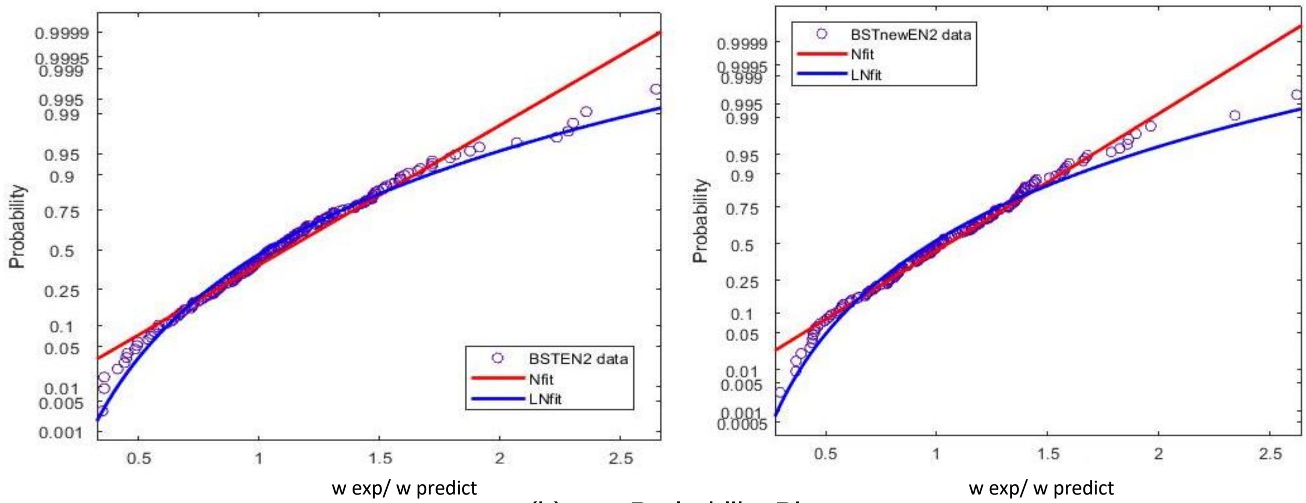
The sample size of 164 for the short-term flexural loading case is large enough to be able to reasonably estimate the main statistical moments and probability density function (pdf) of model uncertainty. Considering the skewness of each model, a similar positive skewness is observed for the MC 2010 and EN 1992 models, thus indicating that the distributions of these models tends towards the lognormal, rather than normal. The probability distributions of the Eurocode and MC 2010 crack models can be estimated from distribution plots and non-parametric tests.

Distribution fitting and non-parametric tests were employed to estimate the distributions of each of the crack models. The Kolmogorov-Smirnov test suggests that the pdf of model uncertainty of the EN 1992 crack model is not normal, but potentially normal for the MC 2010 crack model. The Shapiro-Wilks test indicates that neither crack model has a normal distribution, with significance-values less than 0,05. As the latter is the more robust test considering the sample size and high coefficient of variation, it is concluded that the pdf of both models is not normal. A summary of the histograms, probability plots and cumulative distribution plots for the EN 1992 and MC 2010 crack models for short-term flexural cracking is given in Figure 4.2. The plots for each of the Eurocode and MC 2010 crack models confirm that distributions are indeed generally not normal, tending instead towards lognormal. In addition, assuming a 2- parameter lognormal distribution for the MC 2010 crack model results in a lower prediction of reliability than the normal distribution, thus a more conservative assessment of reliability. The BS 8007 model displays a right skew, indicating that its distribution is not normal. However, the distribution changes, depending on the tension stiffening equation used, with skewness increasing substantially from 0,136 to 2,642 for limiting crack widths of 0,2 and 0,1 mm, respectively. This crack model, therefore, is not consistent in its performance.

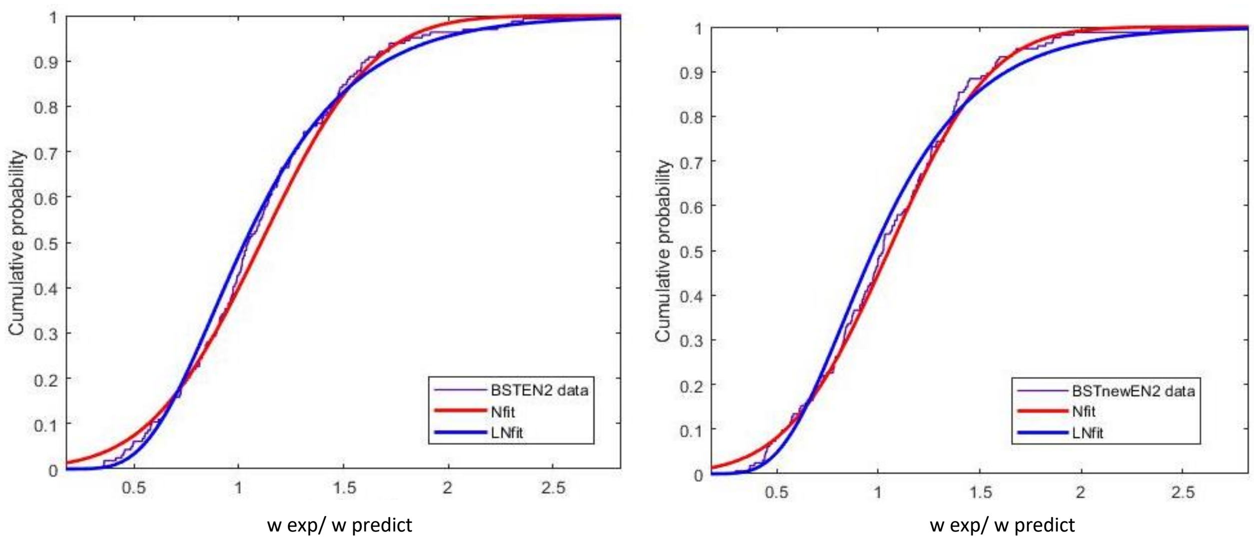
It is found that experimental bias does have some influence on the distribution of model uncertainty of the EN 1992 and MC 2010 crack models. On examining the data, the reinforcement configuration and section geometry chosen for the test samples has some influence on $w_{exp} / w_{predict}$. This was also observed by Allam *et al* (2012), as discussed in Chapter 2, and can be seen on comparing the $w_{exp} / w_{predict}$ ratios determined for the subsets of Chan (2012) and Illston and Stevens (1973). Chan (2012) tested 21 sections with h/b ratios less than 1. In this case, the means of $w_{exp} / w_{predict}$ obtained of 1,48 and 1,24 for EN 1992 and MC 2010 respectively, demonstrate a higher negative bias. Conversely, Illston and Stevens (1973) tested a total of 60 beams with a h/b ratio of 1,9. Here, the means are lower at about 0,90 and 0,86 for EN 1992 and MC 2010 respectively, showing a positive bias. There is more uniformity to the results where similar cross sections and reinforcement were used. However, where researchers varied the section geometry and reinforcement configurations, the means of $w_{exp} / w_{predict}$ are close to 1, but a greater spread of results for $w_{exp} / w_{predict}$ is obtained, that is, the model uncertainty variation increases, as found for $w_{exp} / w_{predict}$ obtained for the subset of Hognestad (Pérez Caldentey, 2016). In this case, h/b ratios (from 1 to 4), the configuration of the reinforcement and concrete cover were also varied. Some of these results may be explained by the crack models not adequately describing the crack mechanism for different configurations. In some instances, the crack models are conservative, in others, they underpredict crack widths.



(a) Histograms



(b) Probability Plots



(c) Cumulative distributions

EN 1992 crack model **MC 2010 crack model (k_2 = 1)**

Figure 4.2: *Model uncertainty* d*istribution plots for short-term flexural cracking*

The crack models considered do not allow for the effect of transverse reinforcement which could potentially decrease the maximum crack widths obtained, as discussed in Chapter 2. The crack models in this case tend to overestimate the maximum crack width, resulting in an apparent decrease in model uncertainty. However, in many of the flexural tests, transverse reinforcement was not provided which meant that this effect cannot be properly assessed.

All results from the statistical analysis may be found in Tables B.5 to B.8 of Appendix B.

4.3.2 Flexure, Long-Term Loading

After refining the data, for model uncertainty for long-term flexure, the final sample size of 30 is much smaller than for short-term loading. This sample size is sufficient to be able to reasonably estimate mean and variance, but not skewness. However, the sample size is sufficient to gain at least some insight into the behaviour of the crack models for long-term flexural loading and their influence on model uncertainty, as well as assess any relationship in model uncertainty between the short-term and long-term load cases. A summary of the statistical parameters for model uncertainty for long-term flexural cracking is given in Table 4.4.

Table 4.4: *Model uncertainty statistical parameters for long-term flexural cracking*

Statistical Parameter	EN 1992	MC 2010 $k_2 = 1$	MC 2010 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Mean	1.443	1.127	1.601	1.502	1.514
Standard Error	0.087	0.078	0.103	0.092	0.099
Median	1.591	1.099	1.582	1.564	1.576
Std Deviation	0.477	0.428	0.565	0.504	0.541
COV	0.331	0.380	0.353	0.336	0.357
Sample Variance	0.228	0.183	0.319	0.254	0.292
Kurtosis	0.753	-0.772	-0.758	0.806	0.288
Skewness	-0.835	0.356	0.313	-0.979	-0.807
Range	2.138	1.574	2.16	2.169	2.231
Minimum	0.207	0.421	0.646	0.219	0.211
Maximum	2.345	1.995	2.806	2.388	2.441
Count	30	30	30	30	30

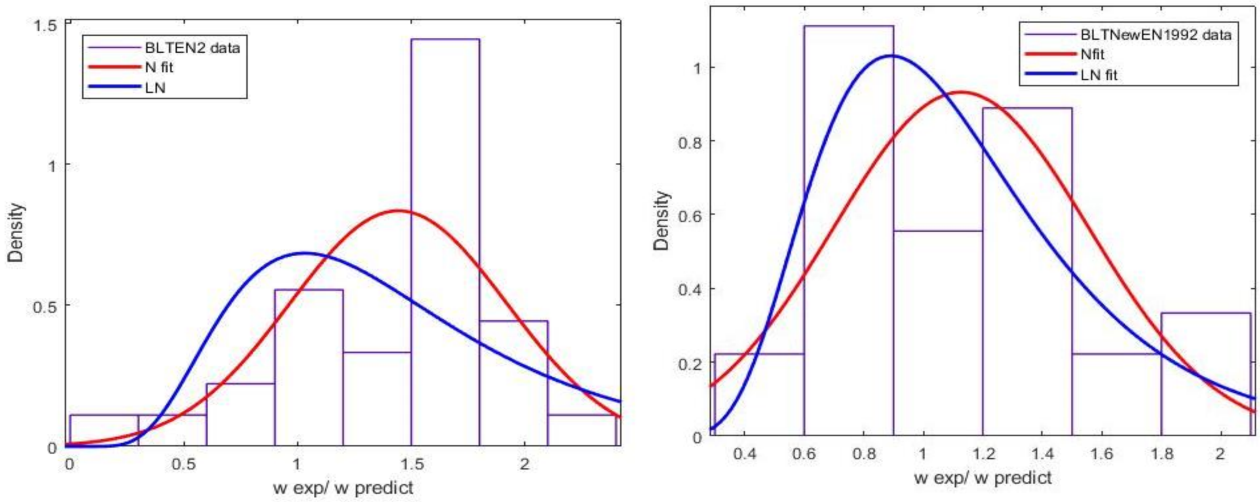
With the exception of MC 2010, the mean of the long-term model uncertainty parameter increases significantly compared to the short-term mean, showing that all other crack models underpredict crack widths over the long-term. The EN 1992 crack model uncertainty mean increases by about 30 % from 1,11 for short-term to 1,44 for long-term loading, whereas the MC 2010 mean increased by approximately 7% which is small. BS 8007 had a similar increase in the mean as EN 1992 from short to long-term loading. This demonstrates clearly that models not taking long-term shrinkage strain into account do not predict long-term behaviour adequately. Creep is considered in all crack models for long-term flexure by using the effective modulus of elasticity of concrete.

As the sample size is small, outliers significantly affect the skewness so that skewness can only be used indicatively to infer the distribution of w_{exp}/ $w_{predict}$. A larger sample size would allow true outliers to be identified. In addition, distribution plots give a broad estimate of the model uncertainty distributions of Eurocode and MC 2010, summarised in Figure 4.3 overleaf. The EN 1992 model uncertainty displays a negative skewness, whereas for short-term flexural loading, a positive skewness was observed. Again, this is due to long-term shrinkage effects not being considered by this model. On the other hand, the MC 2010 model uncertainty displays a positive skewness, although smaller than that for short-term loading, suggesting that the pdf is approximately lognormal in this case.

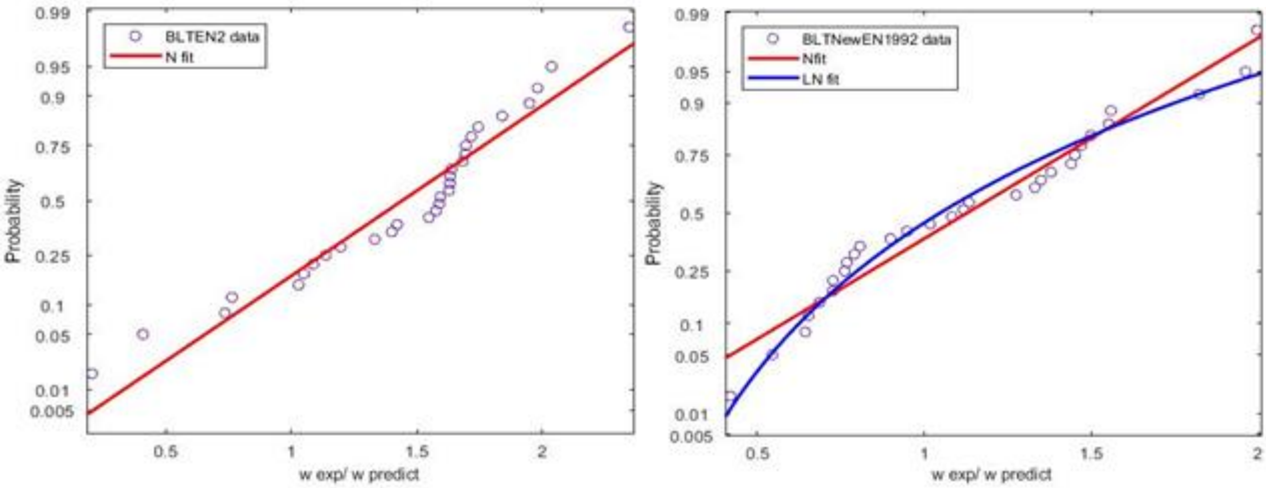
There will be some dependence of the long-term crack width data on the short-term data as independent samples were not used to obtain experimental data for long-term loading. Model uncertainty parameters for Illston (1973) are compared for short-and long-term flexural cracking, as this dataset had similar sample sizes for both load durations. Referring to Table 4.5, it can be seen that there is a substantial increase in the mean and CoV from short- to long-term loading for the crack model and BS 8007. There is an increase of about 20 % in the mean for the MC 2010 crack model but the CoV remains nearly the same. This validates that the incorporation of shrinkage strain in crack models for long-term loading is necessary.

Table 4.5: *Comparison of short & long-term flexural model uncertainty - Illston (1973)*

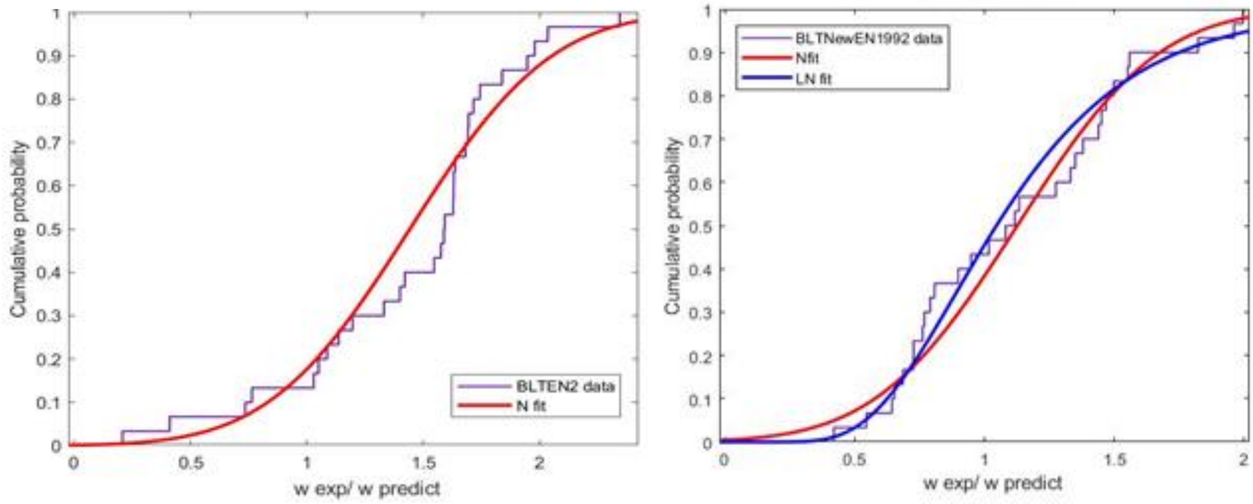
		EN 1992		MC 2010/ (k_2 = 1)		MC 2010 (k_2 = 0.5)		BS 8007 w 0,2 mm		BS 8007 w = 0.1 mm	
Load duration	n	Mean	CoV	Mean	CoV	Mean	CoV	Mean	CoV	Mean	CoV
Short-term	17	0.896	0.174	1.049	0.296	1.298	0.183	1.393	0.308	1.726	0.517
Long-term	14	1.456	0.346	1.260	0.292	1.826	0.236	1.670	0.194	1.773	0.187



(a) Histograms



(b) Probability plots



(c) Cumulative probability distributions

EN 1992 crack model **MC 2010 crack model (k_2 = 1)**

Figure 4.3: *Model uncertainty* d*istribution plots for long-term flexural cracking*

4.3.3 Tension, Short-Term Loading

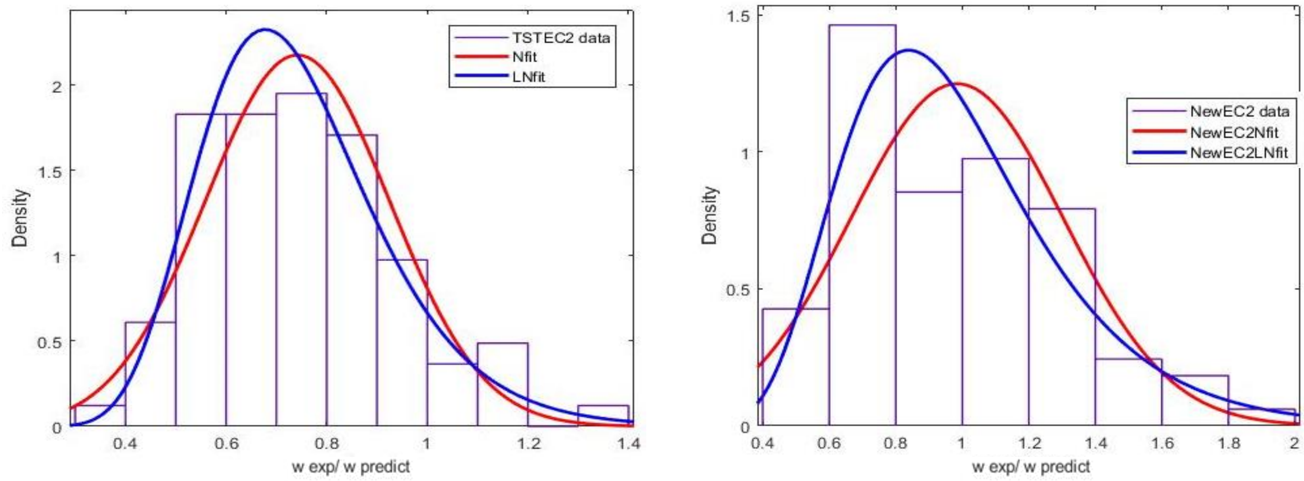
The final sample size for this load case is 82. It must be noted that as many of the elements tested were mostly ties reinforced with single reinforcing bars, the crack widths recorded may not necessarily be representative of real conditions in a structure such as a LRS where there are obviously multiple reinforcing bars in both the main and transverse directions. This requires further research to properly assess the influence of reinforcement configurations on tension cracking. The statistical parameters for short-term tension cracking to the EN 1992, MC 2010 and BS 8007 crack models are summarised in Table 4.6.

Table 4.6: *Model uncertainty statistical parameters for short-term tension cracking*

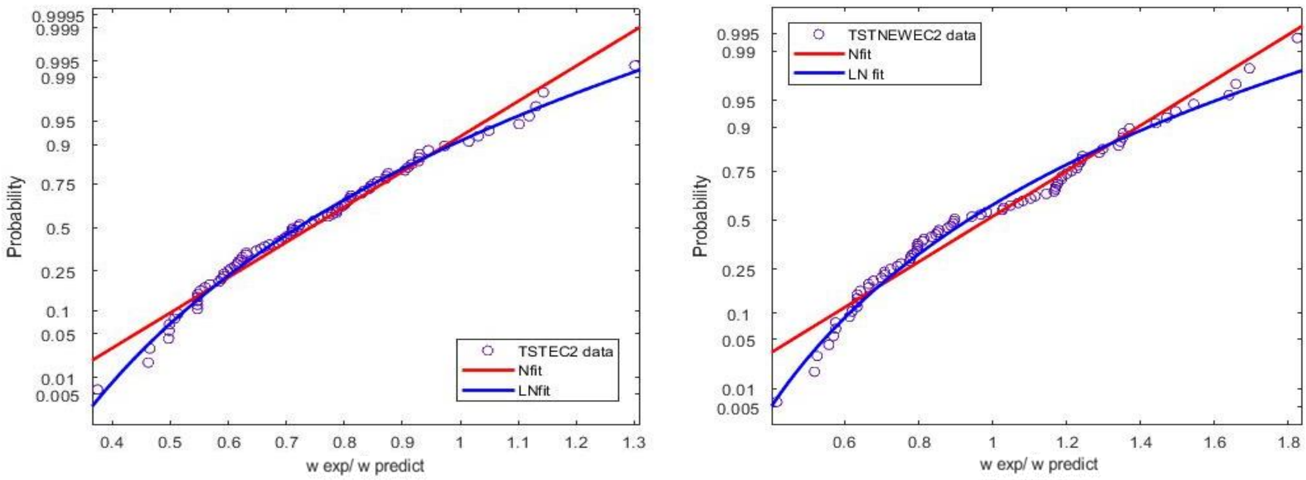
Statistical Parameter	EN 1992	MC 2010	BS 8007 w 0.2 mm	BS 8007 w 0.1 mm
Mean	0.742	0.984	1.271	1.430
Standard Error	0.020	0.035	0.032	0.041
Median	0.716	0.897	1.225	1.398
Standard Deviation	0.183	0.319	0.290	0.369
COV	0.247	0.324	0.228	0.258
Sample Variance	0.034	0.102	0.084	0.136
Kurtosis	0.174	-0.486	-0.056	1.234
Skewness	0.597	0.490	0.441	0.777
Range	0.927	1.408	1.516	2.139
Minimum	0.374	0.416	0.582	0.657
Maximum	1.301	1.824	2.097	2.796
Count	82	82	82	82

The statistical analysis showed that the EN 1992 crack model tends to be conservative, overestimating the predicted short-term tension crack widths, with a mean of 0,74. The variation in this case is smaller than that obtained for flexural cracking. The mean obtained for the MC 2010 crack model is satisfactory in terms of model performance at about 0,98, showing little bias. The MC 2010 CoV of 0,32, however, is higher than that of the EN 1992 model.

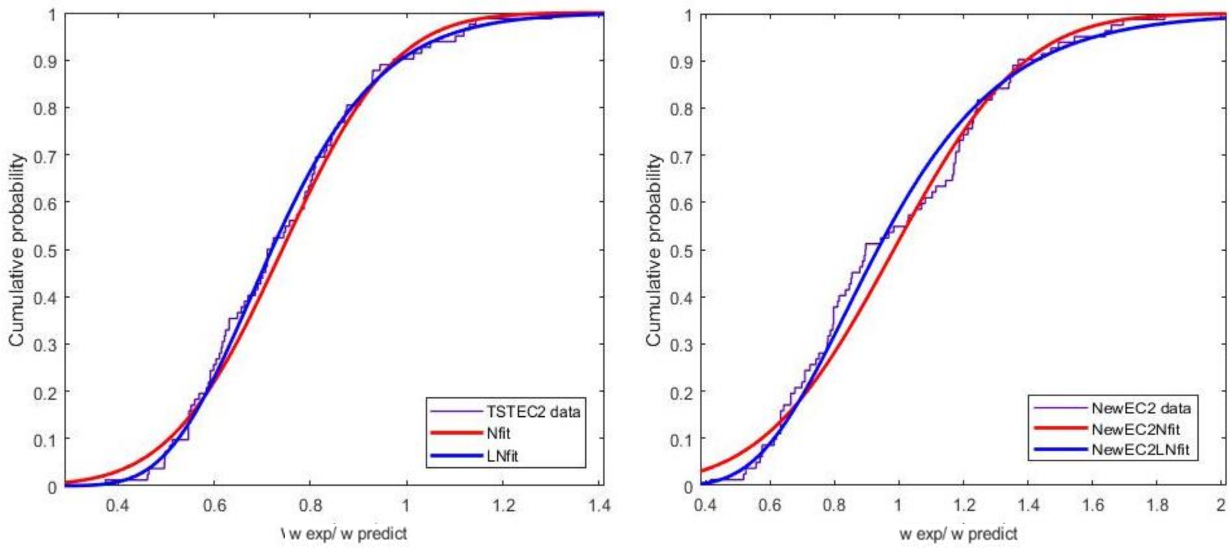
The model uncertainty distributions estimated using probability plots for the EN 1992 and MC 2010 crack models, are summarised in Figure 4.4. Referring to Figure 4.4 and Table 4.6, both crack models exhibit a positive skewness which suggests that the distribution is not normal. Considering the EN 1992 model uncertainty, the Shapiro-Wilks non-parametric test rejects the null hypothesis, suggesting that this model does not have a normal distribution. However, the Shapiro-Wilks significance factor for the MC 2010 model uncertainty is greater than 0,05, but not significantly so. Considering that the lognormal distribution produces lower reliability estimates than the normal distribution, and that the distribution of the MC 2010 crack model is not clearly normal with a positive skewness, a lognormal distribution is assumed. This would give a more conservative (lower) assessment of reliability.



(a) Histograms



(b) Probability plots



(c) Cumulative distributions

EN 1992 crack model **MC 2010 crack model**

Figure 4.4: *Model uncertainty* d*istribution plots for short-term tension cracking*

Referring to Table 4.6, the BS 8007 crack model underpredicts crack widths. It is also noted that the different tension stiffening models of BS 8007, being dependent on the limiting crack width, result in different model uncertainty statistical parameters with bias increasing significantly when tension stiffening is determined using a limiting crack width of 0,1 mm. This presents a challenge when developing a GPM, as the model is not consistent over differing crack widths limits. In addition, the BS 8007 crack width formulations for tension and flexure differ significantly, resulting in differing model uncertainties.

Comparing the short-term model uncertainty for tension to that of flexure, the means for the MC 2010 crack model are similar, although the variation for the tension case is less than that of flexure. This is probably partly due to the greater uniformity of sample configurations of the tension database compared to flexure. However, from this similarity in model uncertainty parameters, the conclusion can be made that it is not necessary to distinguish between the flexure and tension load cases for the model uncertainty parameter, if the MC 2010 crack model is considered for the GPM.

4.3.4 Tension, Long-Term Loading

The sample size in this case is very small at 15, reducing to 8 after averaging of repeat samples. The statistical analysis of the long-term, therefore, can only give an indication of the long-term model uncertainty. There is largely independence of data, as the sources of the data for short and long-term tension cracking are mostly different. The statistical parameters obtained for long-term tension cracking are summarised in Table 4.7 for the EN 1992 and MC 2010 crack models. As with the other load cases, probability plots were done, as summarised in Figure 4.5, for the EN1992 and MC 2010 crack models for long-term tension cracking.

The mean for the EN1992 crack model is about 0,90 for long-term tension cracking, compared to 0,74 for the short-term case. Considering the MC 2010 crack model, there is little difference between the means for short-and long-term tension cracking, so is more consistent than the EN 1992 crack model as the load duration increase. As with flexural cracking, this is at least in part due to the MC 2010 crack model including shrinkage strain, whereas the EN 1992 crack model does not. The MC 2010 crack model has less bias than the EN 1992 crack model. Variations are in the order of CoV 0,22 to 0,25 for both crack models, however as the sample size is small, these values are an estimate.

Table 4.7: *Model uncertainty statistical parameters for long-term tension cracking*

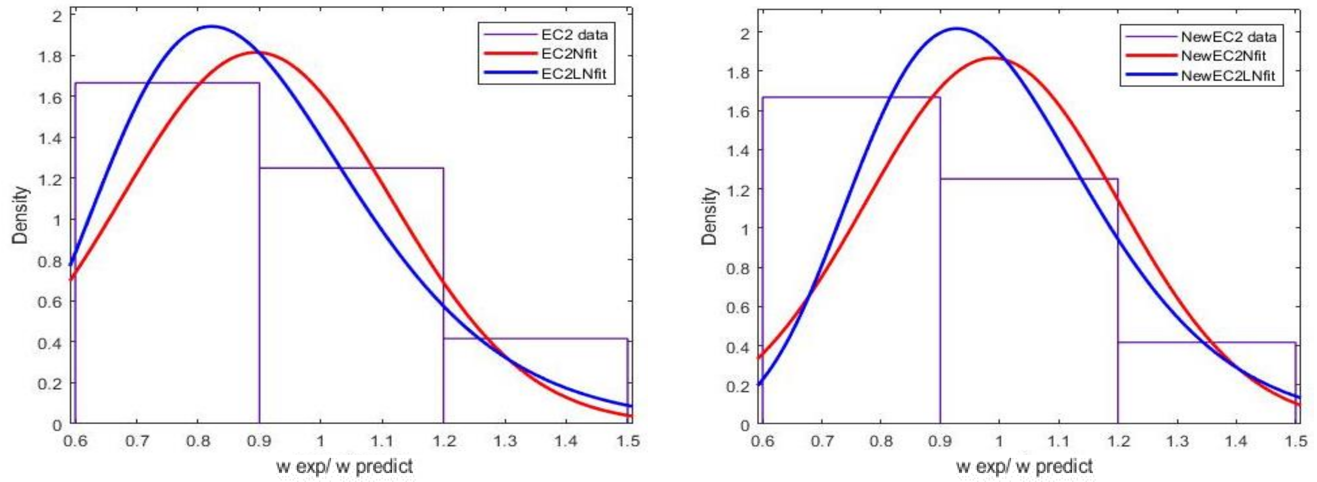
Statistical Parameter	EN 1992	MC 2010	BS 8007 w 0,2 mm	BS 8007 w 0,1 mm
Mean	0.895	0.988	1.318	1.603
Standard Error	0.078	0.076	0.211	0.280
Median	0.860	0.946	1.089	1.353
Standard Deviation	0.220	0.214	0.597	0.793
COV	0.246	0.216	0.453	0.495
Sample Variance	0.048	0.046	0.356	0.629
Kurtosis	-1.539	-0.059	3.779	2.151
Skewness	0.442	0.806	1.882	1.447
Range	0.571	0.618	1.807	2.328
Minimum	0.656	0.764	0.836	0.935
Maximum	1.227	1.382	2.643	3.262
Count	8	8	8	8

The sample size is also much too small to give a good indication of skewness, however, the values obtained for both the EN 1992 and MC 2010 crack models is positive, suggesting a lognormal rather than normal distribution. From the plots, shown in Figure 4.5, it can be concluded that the probability distributions for both crack models tend towards lognormal, even considering the limited sample size, displaying a right skew, although the sample size is too small for non-parametric tests to have any real meaning.

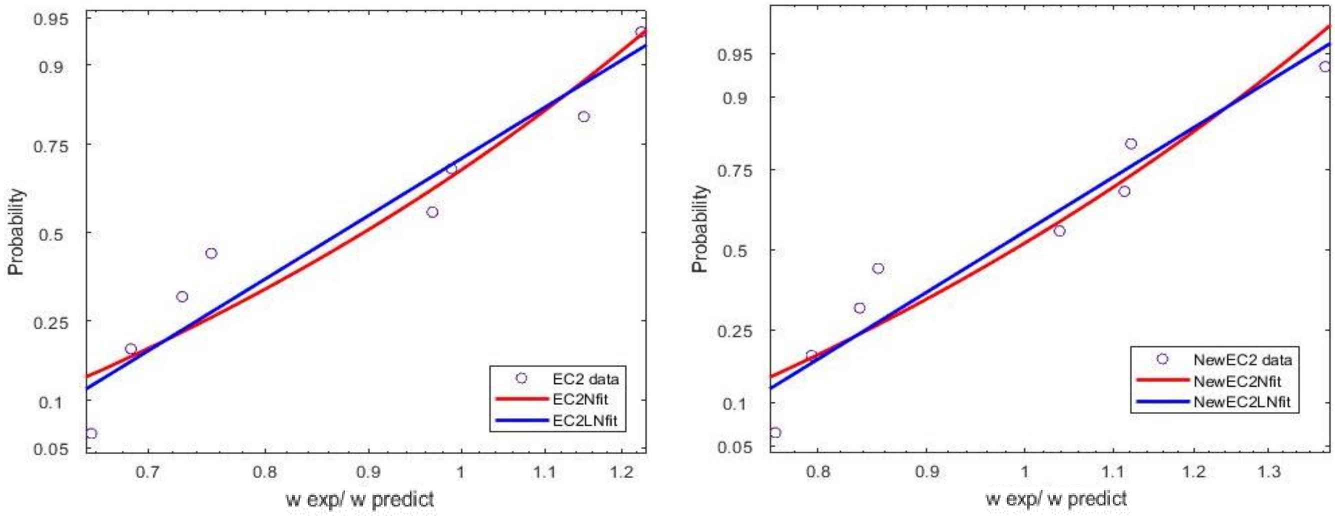
The long-term model uncertainty values are compared with the short-term case. As for flexure, the influence of shrinkage strain can be seen, with the increase in the EN 1992 model uncertainty mean from 0,74 to 0,90 for short and long-term loading, respectively. Considering the MC 2010 crack model, the means for short and long-term loading are very similar. The small sample size implies that the low CoV value obtained for long-term tension model uncertainty needs to be treated with caution.

Considering the BS 8007 crack model, as with flexure, neglecting shrinkage strain results in the model uncertainty mean and variation increasing substantially. In addition, the BS 8007 formulation for tension cracking is independent of creep and the concrete modulus. The tension stiffening formulation for a crack width limit of 0,1 mm resulted in underpredicted crack widths. There is also some uncertainty in the determination of a_{cr} in this case, due to potential mismatches between the actual and assumed distances from the bar to the crack.

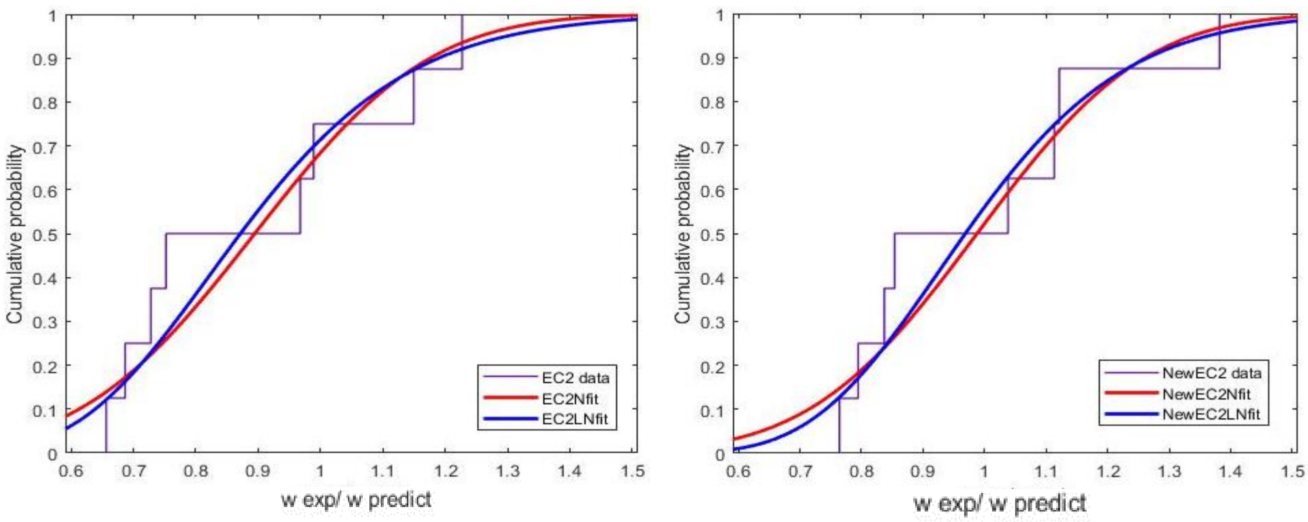
Considering that the similar variations obtained for flexural cracking for short- and long-term cracking, similar variations may be assumed for tension cracking. Further research on long-term tension loading to obtain sample sizes large enough for a good estimation of the statistical moments.



(a) Histograms



(b) Probability plots



(c) Cumulative probability distribution

EN 1992 crack model **MC 2010 crack model**

Figure 4.5: *Model uncertainty* d*istribution plots for long-term tension cracking*

4.3.5 Modification of EN 1992 For Long-Term Cracking

As discussed in Section 4.3.2, the EN 1992 crack model substantially underpredicts maximum crack widths under long-term loading conditions, with a mean of about 1,44 showing significant bias. Comparing this model to MC 2010, and from previous research as reported in the literature review in Section 2.3.7 of Chapter 2, it is concluded that this is at least in part due to the EN 1992 crack model neglecting long-term shrinkage strain. A simple modification was therefore made to the existing formulation for long term flexural cracking by including shrinkage strain in the calculation of mean strain. The model uncertainty statistical parameters, summarised in Table 4.8, were then estimated and utilised to determine if this modification improves the performance of the EN 1992 crack model. The statistical parameters for EN 1992 without long-term shrinkage strain are included in Table 4.8 for comparison.

Table 4.8: *Model uncertainty statistical parameters for modified EN 1992 – long-term flexure*

Statistical Parameter	EN 1992 No shrinkage strain	EN 1992 With shrinkage strain
Mean	1.443	0.936
Standard Error	0.087	0.076
Median	1.591	0.877
Standard Deviation	0.477	0.436
CoV	0.331	0.465
Sample Variance	0.228	0.190
Kurtosis	0.753	-1.154
Skewness	-0.835	-0.036
Range	2.138	1.531
Minimum	0.207	0.135
Maximum	2.345	1.665
Count	30	3

Referring to Table 4.8, the mean of model uncertainty improves from 1,44 to a value close to 1, when shrinkage strain is included. This implies a low bias for the modified EN 1992 crack model. However, on examining the spread of results, model uncertainty increased (with a CoV of 0,47) which demonstrates that this crack model still does not adequately predict long term cracking behaviour, both significantly under- and overpredicting crack widths.

The sample size of 33 for long-term flexure is not large enough to give a good estimate of the kurtosis and skewness of model uncertainty for the modified EN 1992 crack model but the values obtained do indicate that the distribution has nominal skewness. The box plot of model uncertainty of the modified EN 1992 crack model, given in Figure 4.6, illustrates the wide scatter of results around the mean.

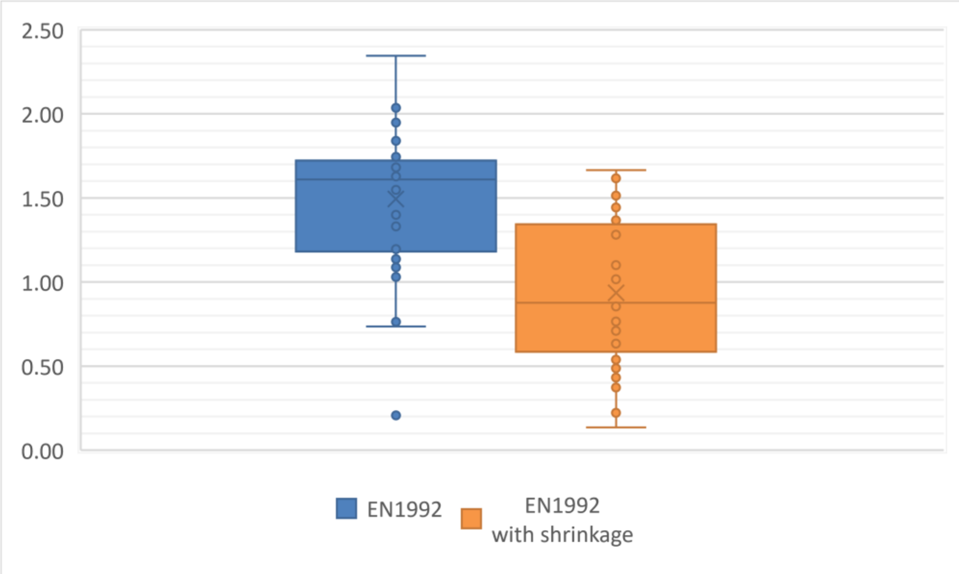


Figure 4.6: *Box plot of model uncertainty of modified EN 1992 – long-term flexure*

Comparing to EN 1992, Figure 4.6 illustrates the significant shift in the mean when shrinkage strain is not neglected in the EN 1992 crack model.

4.4 SELECTION OF CRACK MODEL FOR GENERAL PROBABILISTIC MODEL (GPM)

Through the quantification of model uncertainty, the performance of the crack models considered could be compared, thus providing a basis for the selection of a crack model of the probabilistic modelling. As discussed in Chapter 3, a model is deemed to be a suitable model if it shows a small bias (mean close to 1) and a low variability. With respect to crack models, a higher variability is expected due to the randomness of cracking mechanisms and the resulting difficulty in describing this behaviour in a crack model. In addition, the model needs to perform consistently over a range of parameters and conditions. Pearson's correlation values and scatter plots of model uncertainty factors against model parameters are therefore used to assess the performance of the crack models over the range of experimental values of the database.

4.4.1 Comparisons of Crack Models

The performances of the crack models are compared in order to select the crack model of the probabilistic model, by means of the statistical parameters obtained for each crack model, as presented in Tables 4.3 to 4.7. Box plots of model uncertainty, presented in Figure 4.7, for flexural and tension cracking, respectively, are also used for comparison.

Considering flexural cracking and referring to Table 4.3, the model uncertainty mean of 1,19 and CoV of 0,38 for BS 8007 with a limiting crack width of 0,2 mm are close to those of the EN 1992 and MC 2010 crack models. However, when a limiting crack width of 0,1 mm is assumed for BS 8007, the CoV and skewness increase. The use of an empirical tension stiffening model that is specific to a crack width limit is problematic as interpolation for other crack widths is not possible, compromising the ability of BS 8007 crack model to predict crack widths larger than 0,2 mm. The BS 8007 model uncertainty values obtained using the experimental database need to be viewed with this in mind, as most of the measured crack widths are not 0,2 or 0,1 mm. In the long-term flexural loading case, BS 8007 does not perform satisfactorily as shrinkage strain is neglected. The effect of this is demonstrated by the significant increase in the long-term model uncertainty mean for this model, reflecting a significant bias. Similar trends are noted for tension cracking.

Both the EN 1992 and MC 2010 crack models perform reasonably under short-term loading conditions for both tension and flexure. However, as EN 1992 does not consider shrinkage strain under long-term loading, the performance of this model decreases with a higher proportion of cracks widths underpredicted. Referring to Figure 4.7, the box plots confirm that if a value of 0,5 is used for k_2 for flexural cracking, the MC 2010 crack model does not perform as well as the other crack models. It is therefore recommended that a value of 1,0 be utilised for k_2.

Overall, the MC 2010 crack model predicts the maximum crack widths better than the other crack models for the experimental data, having a mean close to 1, therefore has little bias. The MC 2010 crack model has similar means and variations for model uncertainty for both the short and long-term flexural load cases, which suggests that it is not necessary to distinguish between short and long-term loading for flexure. The tension case for this model has a small positive bias and similar CoV. Given the lack of variation in configuration of most of the samples tested, it is likely that the model uncertainty mean is closer to that obtained for flexural cracking. The same model uncertainty parameter is therefore recommended for both tension and flexural cracking. This will also make it easier to derive design formulations in future. As already discussed, further research is needed for long-term tension cracking.

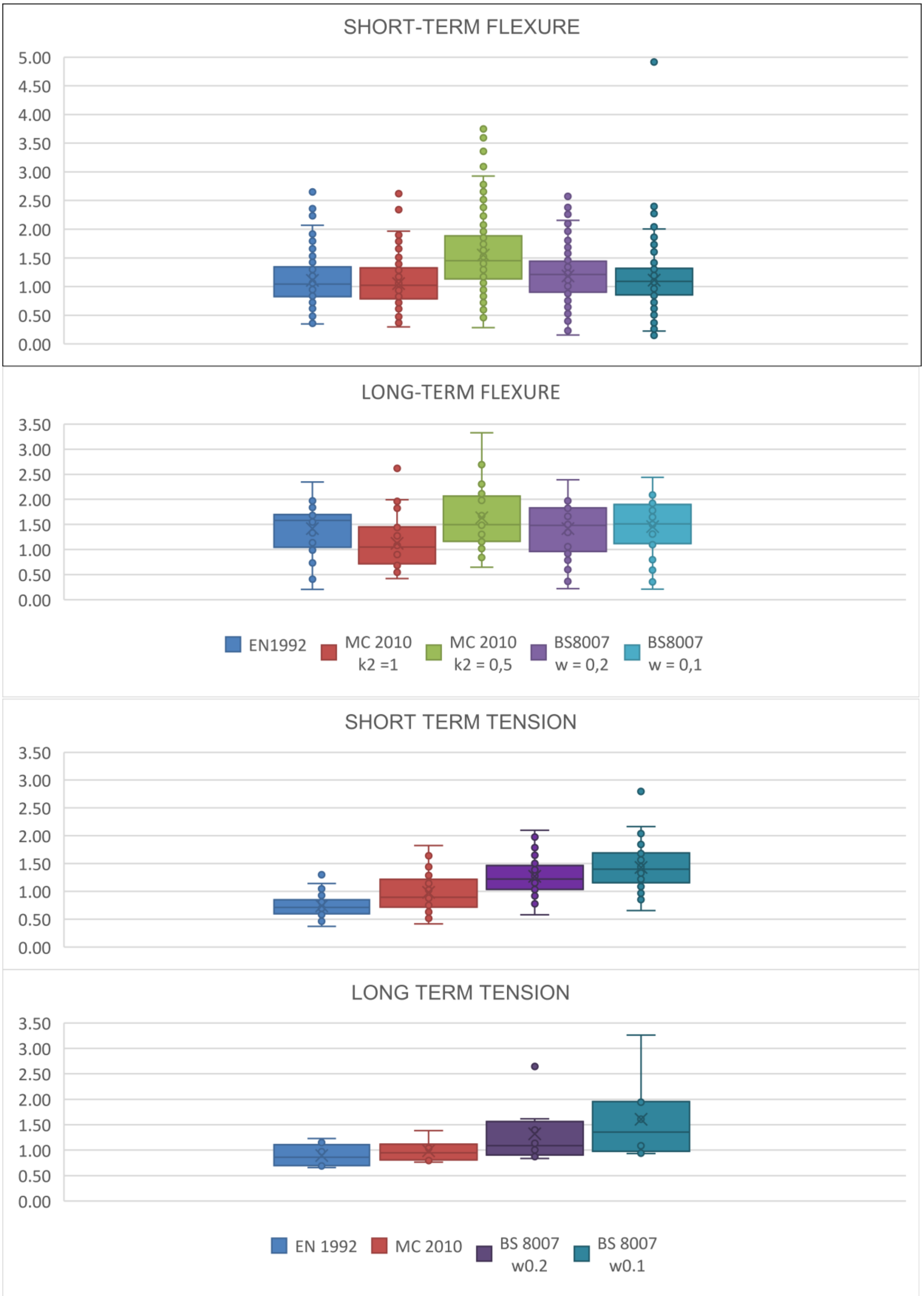


Figure 4.7: *Model uncertainty box plots comparing performance of crack models*

4.4.2 Evaluation of Crack Model Parameters and Model Uncertainty

Using the experimental database, scatter plots of the $w_{exp}/w_{predict}$ ratios for each crack model against the crack model parameters of reinforcing ratio (as % A_s), steel stress, concrete tensile strength (f_{ctm}), section thickness (h), concrete cover (c) and section width (b) were done. In addition, for long-term flexural cracking the scatter plots included comparisons with shrinkage strain. Regression analyses and Pearson's correlations between the model uncertainty and model parameters were also done. The relative effects of the parameters on model uncertainty for each model can thus be evaluated, together with further assessment of the performance of the crack models. The Pearson correlations between the model uncertainty and selected parameters is given in Table 4.9 for each load case and crack model, respectively. To illustrate the trends between model uncertainty and selected parameters, scatter plots for long-term flexure are presented in Figures 4.8 to 4.10, for the crack models of EN 1992, MC 2010 and BS 8007 (0,2 mm).

The long-term tension cracking correlations need to be viewed with caution as the sample sizes are small and there was little variation in the sample configurations and loadings. This would result in correlations regarding the geometry of the section being overestimated. For short-term tension, although the sample size is larger, most samples were ties with a single reinforcing bar. This would also influence correlations with geometric parameters. Further research using different configurations is required for tension cracking for a more accurate evaluation of this load case. It is noted that for long-term flexure, there are a few high steel stress values where very high strength reinforcement was used that would be outside the usual range of reinforcement yield strength considered for structures such as LRS. Removing the data related to these high steel stresses did not alter the correlation between steel stress and model uncertainty significantly.

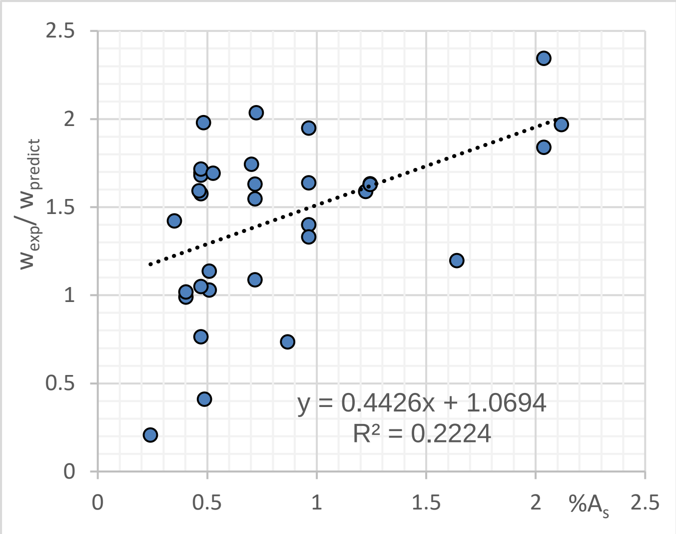
Referring to Table 4.9, model uncertainty for the EN 1992 crack model has low correlations with all parameters except concrete cover (which has a moderate correlation) for short-term flexure. However, for long-term flexural loading, there is a moderate correlation with steel stress and the reinforcement ratio, but low correlations with the remaining parameters, as also illustrated by Figure 4.8. In general, the relative correlations between model uncertainty and the selected parameters are not consistent with load duration.

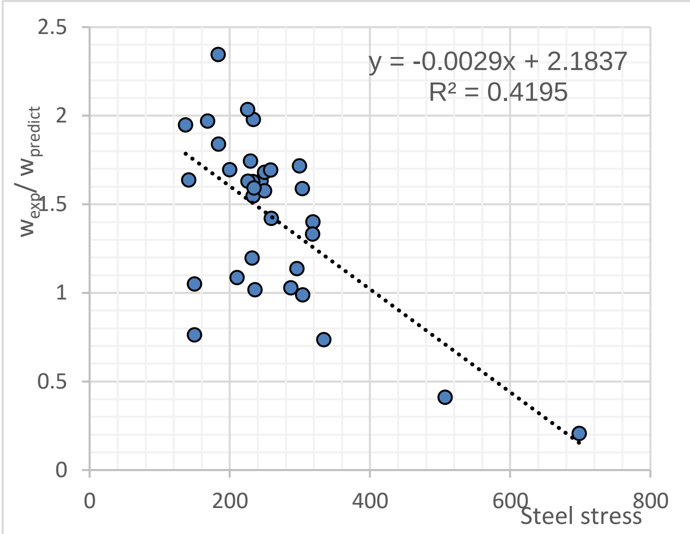
Model uncertainty for the MC 2010 crack model has little to moderate correlation with all parameters with the exception of the reinforcement ratio and long-term shrinkage strain. This crack model, although it behaves consistently over all other parameters, displays a significant drift with reinforcement ratio, as illustrated by Figure 4.9 for long-term flexural cracking. The Pearson's coefficients and the scatter plots indicate that there is a moderate to possibly strong correlation between long-term shrinkage strain and model uncertainty, plotting as the ratio of shrinkage strain to mean strain. However, given the size of the dataset in this case, further investigation is needed. Further improvements to this model are needed to obtain consistency in behaviour for variations in all parameters.

Table 4.9: *Pearson's correlation matrix between model uncertainty & model parameters*

Load Case	Parameter	EN 1992	MC 2010 k2 = 1.0	MC 2010 k2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm
Short-term flexure	Steel stress	-0.068	-0.164	-0.258	-0.165	-0.217
	h/b	-0.085	0.151	0.152	0.024	0.047
	c	-0.391	-0.189	-0.275	-0.114	-0.036
	Bar dia, φ	0.055	0.204	0.243	0.054	0.081
	f_{ctm}	0.140	0.165	-0.085	-0.133	-0.101
	% A_s	0.111	0.400	0.354	0.025	-0.086
Long-term flexure	Steel stress	-0.498	-0.371	-0.362	-0.416	-0.404
	h/b	-0.082	0.208	0.214	-0.117	0.035
	c	0.024	0.249	0.213	0.095	0.209
	Bar dia, φ	0.175	0.369	0.348	0.160	0.244
	f_{ctm}	0.015	0.207	0.197	-0.244	-0.269
	% A_s	0.330	0.608	0.575	-0.201	-0.213
	ε_{sh}	-	-0.482	-	-	-
	$\varepsilon_{sh}/\varepsilon_m$	-	-0.399	-	-	-
Short-term tension	Steel stress	0.082	0.265	-	-0.284	-0.364
	h/b	-0.504	-0.686	-	0.049	0.206
	c	0.279	0.456	-	0.332	0.230
	Bar dia, φ	0.586	0.638	-	-0.068	-0.241
	f_{ctm}	-0.341	-0.519	-	-0.030	0.080
	% A_s	0.554	0.650	-	-0.036	-0.019
Long-term tension	Steel stress	0.825	0.641	-	0.840	0.777
	h/b	0.490	0.335	-	0.602	0.479
	c	-0.825	-0.641	-	-0.840	-0.777
	Bar dia, φ	-0.414	-0.209	-	-0.544	-0.407
	f_{ctm}	-0.826	-0.724	-	-0.792	-0.773
	% A_s	-0.349	-0.156	-	-0.426	-0.334

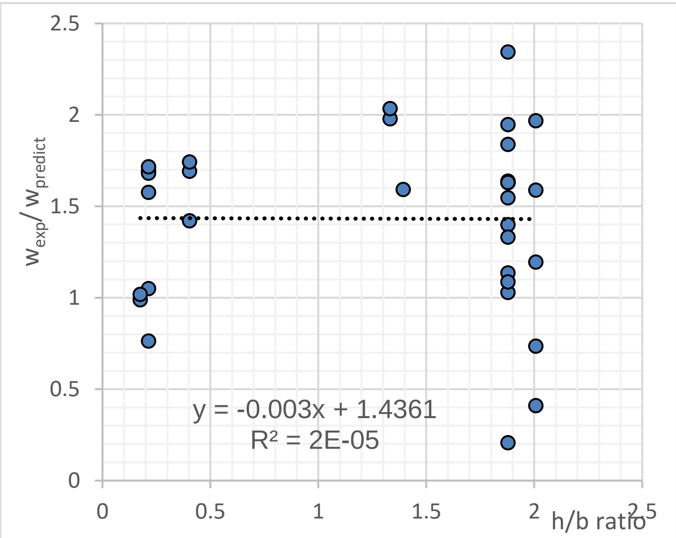
Referring to Table 4.9 and considering the BS 8007 flexural crack model, the model parameters have a low influence on model uncertainty, with Pearson's coefficients less than 0,3. This implies that this model performs consistently over a range of different parameters, for a given limiting crack width and duration of load. However, the Pearson's coefficients depend on the tension stiffening formulation and load duration. These trends are illustrated in Figure 4.10 for long-term flexural cracking to BS 8007. For short-term tension cracking to BS 8007 and a crack width limit of 0,2 mm, steel stress and concrete cover have a moderate influence. All other parameters display a very small (less than 0,1) correlation with model uncertainty. However, as with flexural cracking, the Pearson's coefficients depend on the tension stiffening formulation and load duration. The Pearson's coefficients cannot be meaningfully estimated for the long-term tension case due to the very small sample size.

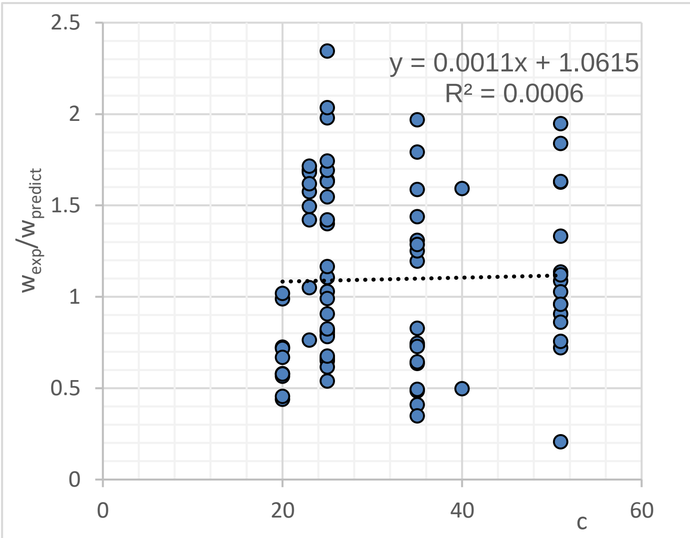




(a) Reinforcement ratio, $\%A_s$

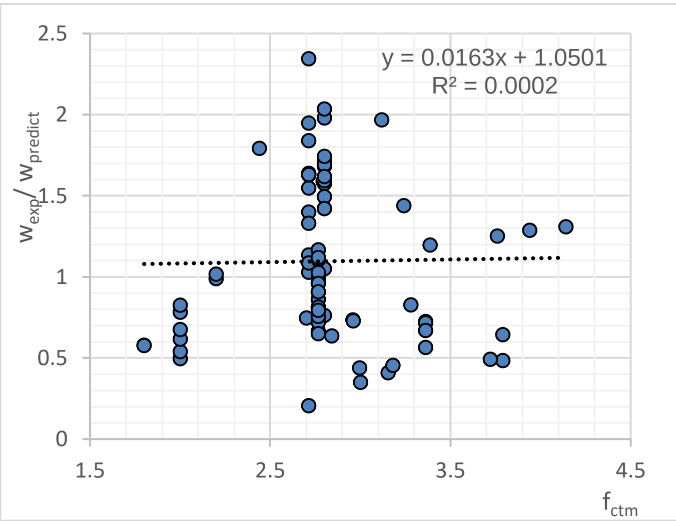
(b) Steel stress (MPa)





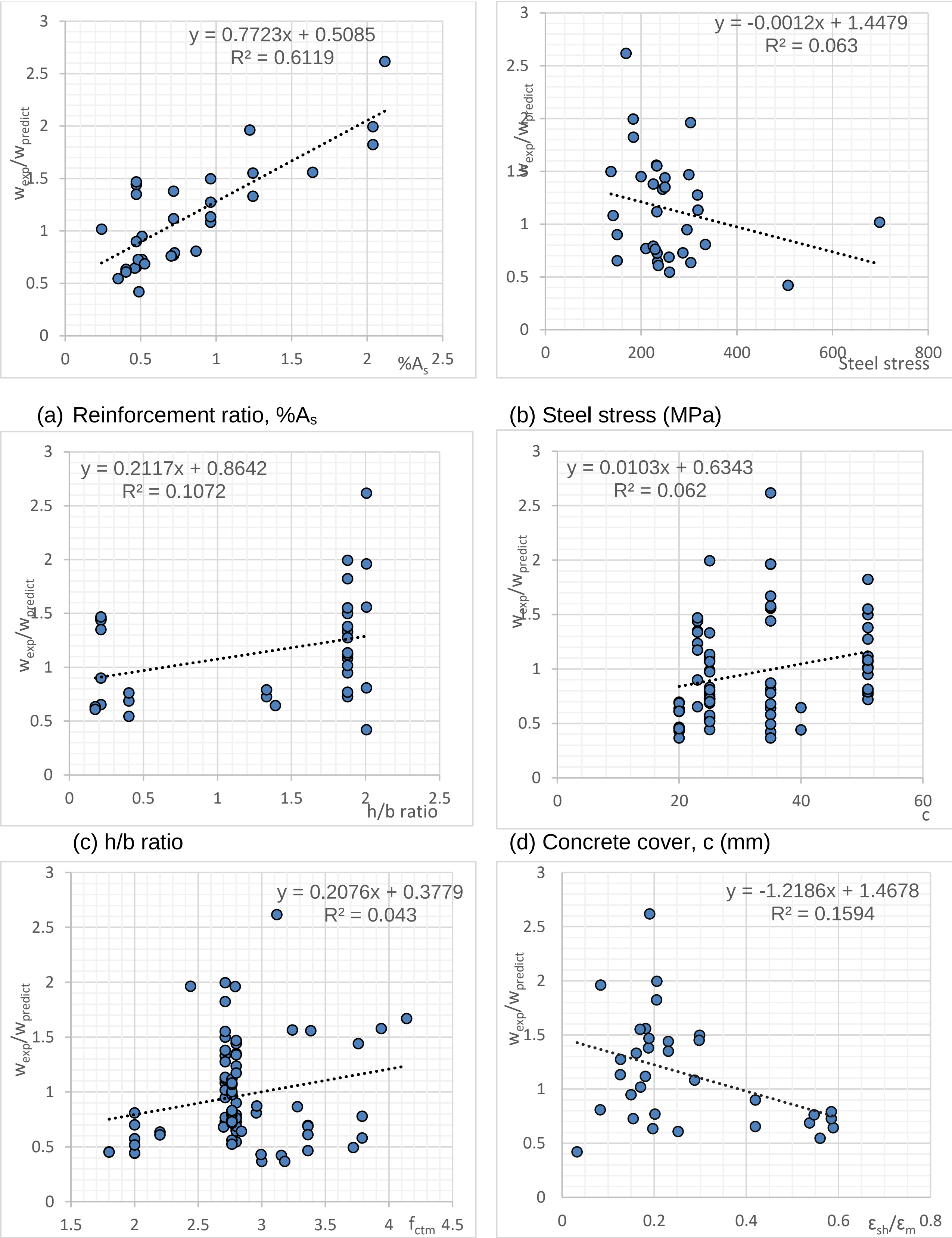
(c) h/b ratio

(d) Concrete cover, c (mm)



(e) Concrete tensile strength, f_{ctm} (MPa)

Figure 4.8: *Correlation between EN 1992 model uncertainty and select parameters Long-term flexure.*



(a) Reinforcement ratio, $\%A_s$

(b) Steel stress (MPa)

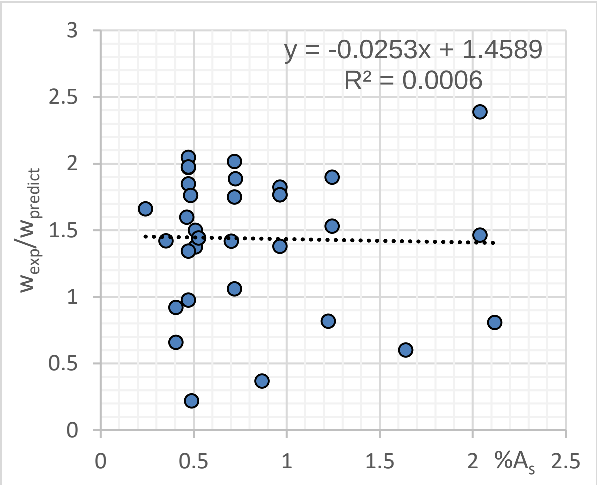
(c) h/b ratio

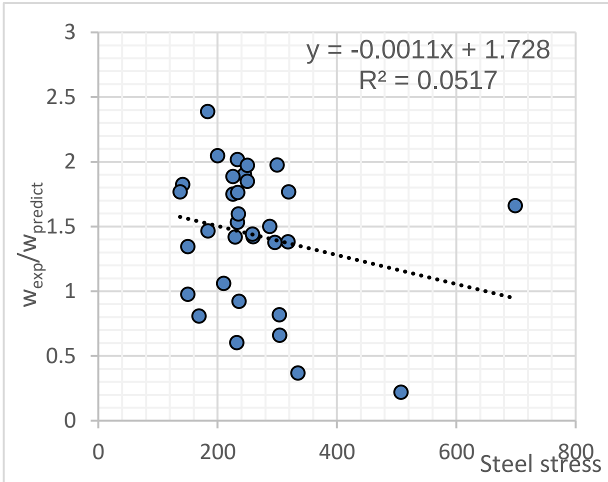
(d) Concrete cover, c (mm)

(e) Concrete tensile strength, f_{ctm} (MPa)

(f) Ratio of long-term shrinkage/ mean strain

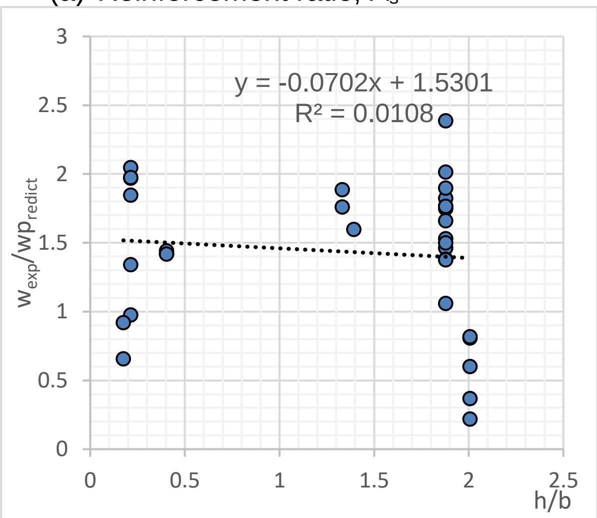
Figure 4.9: *Correlation between MC 2010 model uncertainty and select parameters – Long-term flexure.*

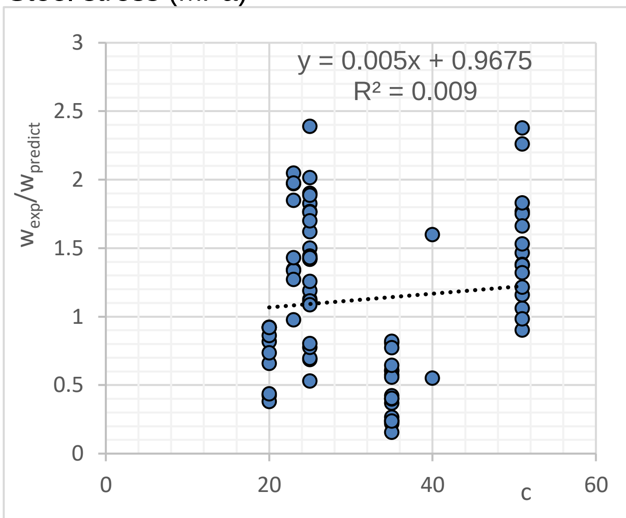




(a) Reinforcement ratio, A_s

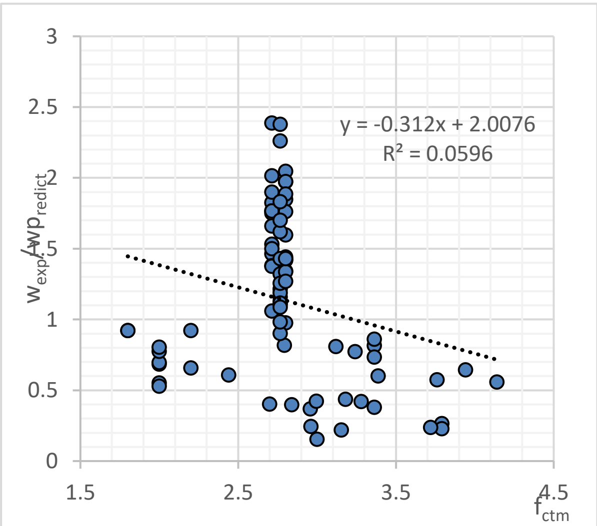
(b) Steel stress (MPa)





(c) h/b ratio

(d) Concrete cover, c (mm)



(e) Concrete tensile strength, f_{ctm} (MPa)

Figure 4.10: *Correlation between BS 8007 (w 0,2mm) model uncertainty & select parameters - Long-term flexure.*

4.5 CHAPTER SUMMARY AND CONCLUSIONS

This chapter presented the quantification of model uncertainty by means of a statistical analysis using experimental data and the crack models of EN 1992, MC 2010 and BS 8007. Both short and long-term loading were considered for tension and flexural cracking. Model uncertainty was defined as the ratio of maximum experimental to predicted cracks, $w_{exp}/ w_{predict}$ The relative performances of the crack models were also evaluated using the model uncertainty.

Crack models applied to long-term loading are found to underpredict maximum crack widths significantly if they neglect the effect of long-term shrinkage strain. This applies to the formulations of EN 1992 and BS 8007. The MC 2010 crack model, which does include long-term shrinkage strain, performs more consistently as the load duration changes. The EN 1992 crack formulation was modified to include shrinkage strain in the determination of mean strain and thus the long-term flexural maximum crack widths. It is found that the performance of this model improved in terms of the bias, with the model uncertainty mean decreasing from about 1,44 (neglecting shrinkage strain) to about 0,94 when shrinkage strain was included. However, the scatter of the model uncertainty is very high with a CoV of about 0,47. It can be concluded that the EN 1992 crack model is improved with the inclusion of shrinkage strain, however, as the variability increased, this crack model is still not adequately predicting long-term crack widths.

The BS 8007 crack model statistical parameters for short-term flexural cracking are comparable to those of EN 1992 and MC 2010. However, for tension and long-term flexure the BS 8007 crack model has a high bias compared to the MC 2010 model, with maximum crack widths underpredicted, and a high variation. The BS 8007 model uncertainty statistical parameters depend on the crack width limit chosen as this governs the tension stiffening model used in determining mean strain, which will result in bias in this model when applied to crack widths other than 0,2 mm or 0,1 mm. Considering long-term tension cracking, the tension stiffening model results in a significant underprediction of crack widths when employing a 0,1 mm crack width limit. In addition, this model, being empirically derived, does not allow for an interpolation of tension stiffening between the estimates related to crack width limits of 0,1 and 0,2 mm. Therefore, this model does not behave consistently across tension and flexural load cases and crack widths.

It was found that applying a k_2 factor of 0,5 in the crack spacing formulation of the MC 2010 crack model for flexural cracking results in the model underpredicting crack widths. This model displays a significant bias, demonstrated in the mean values of 1,55 and 1,64 obtained for its model uncertainty for short- and long-term loading, respectively. In addition, the model uncertainty has high CoV's of 0,41 and 0,38, for short- and long-term loading, respectively.

The distribution of the MC 2010 model uncertainty may be approximated as lognormal. The MC 2010 crack model exhibited a low bias with a mean of about 1,1 for both short and long-term flexural cracking. The bias decreases for tension cracking, with means just less than 1,0. The variability of model uncertainty for tension cracking is less than that of flexural cracking but it is concluded that due to the limited testing in the former, both in sample size and uniformity of test configurations, that the true model uncertainty statistical parameters for tension may well be closer to those of flexure. It is recommended that further research is performed for tension load-induced cracking over short- and long-term loading, varying the geometry and reinforcement configuration to enable a proper assessment of model uncertainty.

A significant model uncertainty variation (CoV of approximately 0,38) for both short and long-term flexural cracking is obtained for the MC 2010 crack model. The extent of the influence of model uncertainty would therefore require investigation to establish whether or not to employ a single model factor, as opposed to a scheme of partial safety factors if the MC 2010 crack model was selected for the probabilistic model.

Pearson's correlations and regression analyses were performed for each crack model uncertainty against selected model parameters in order to assess the consistency of each model. MC 2010 behaves consistently over the range of parameters, with the exception of the reinforcement ratio. Model uncertainty in this case has a significant correlation with the reinforcement ratio. Model uncertainty has a moderate correlation with long-term shrinkage strain.

Considering the results from the model uncertainty analysis for the chosen crack models, the MC 2010 crack model is selected for use in the probabilistic model, as overall it performed better than the other crack models. However, the high correlation between its model uncertainty and the reinforcement ratio, and moderate correlation with shrinkage strain, requires further investigation as this affects the ability of this crack model to behave consistently over a range of design situations.

CHAPTER 5

SENSITIVITY ANALYSIS OF MODEL UNCERTAINTY

5.1 INTRODUCTION

This chapter presents a probabilistic sensitivity analysis using a time-invariant reverse First Order Reliability Method (FORM) to assess the extent of the influence of model uncertainty on the crack width prediction model, for load-induced cracking. From the quantification of model uncertainty and subsequent assessment of selected crack models, the MC 2010 formulation was chosen for the reliability model. The model uncertainty stochastic parameters, quantified as presented in Chapter 4, as a best estimate of model uncertainty in the reliability model applicable to typical liquid retaining structures (LRS), are used in the sensitivity analysis. The sensitivity analyses are expected to verify the extent of the influence of model uncertainty on the crack width prediction model. This would have implications for any future design formulation. Given the high CoV of the model uncertainty, it is anticipated that model uncertainty would have at least a significant effect on the crack model. The sensitivity analysis consisted of:

(i) Sensitivity analyses using Reverse FORM analysis to determine the relative influences of the random variables, in particular, model uncertainty, for selected target reliability levels by means of their sensitivity factors.

(ii) As part of the reverse FORM, the theoretical partial safety factors of the RV's were determined.

The influence of model uncertainty is evaluated probabilistically for typical configurations of liquid retaining structures (LRS) as examples of structures where serviceability load-induced cracking is dominant. The effect of long-term shrinkage strain on the crack width prediction model is investigated.

5.2 RELIABILITY ANALYSIS USING FORM

The reliability model is developed following the methodology outlined in Chapter 3.

5.2.1 Reverse FORM Procedure

The reverse FORM algorithm used to perform the sensitivity analyses is as summarised in Chapter 3 Section 3.6, and given in reliability literature. To determine the theoretical partial safety factors of the random variables, γ_x, Equation 3.3 may be expressed as:

$$\gamma_x = \frac{x^*}{\mu_x}$$

where x^* is the theoretical design value at the design point, obtained from the reverse FORM analysis and μ_x is the mean value of the basic variable. The reverse FORM algorithm was performed using the software EXCEL and the add-in Solver, as summarised in Figure 5.1.

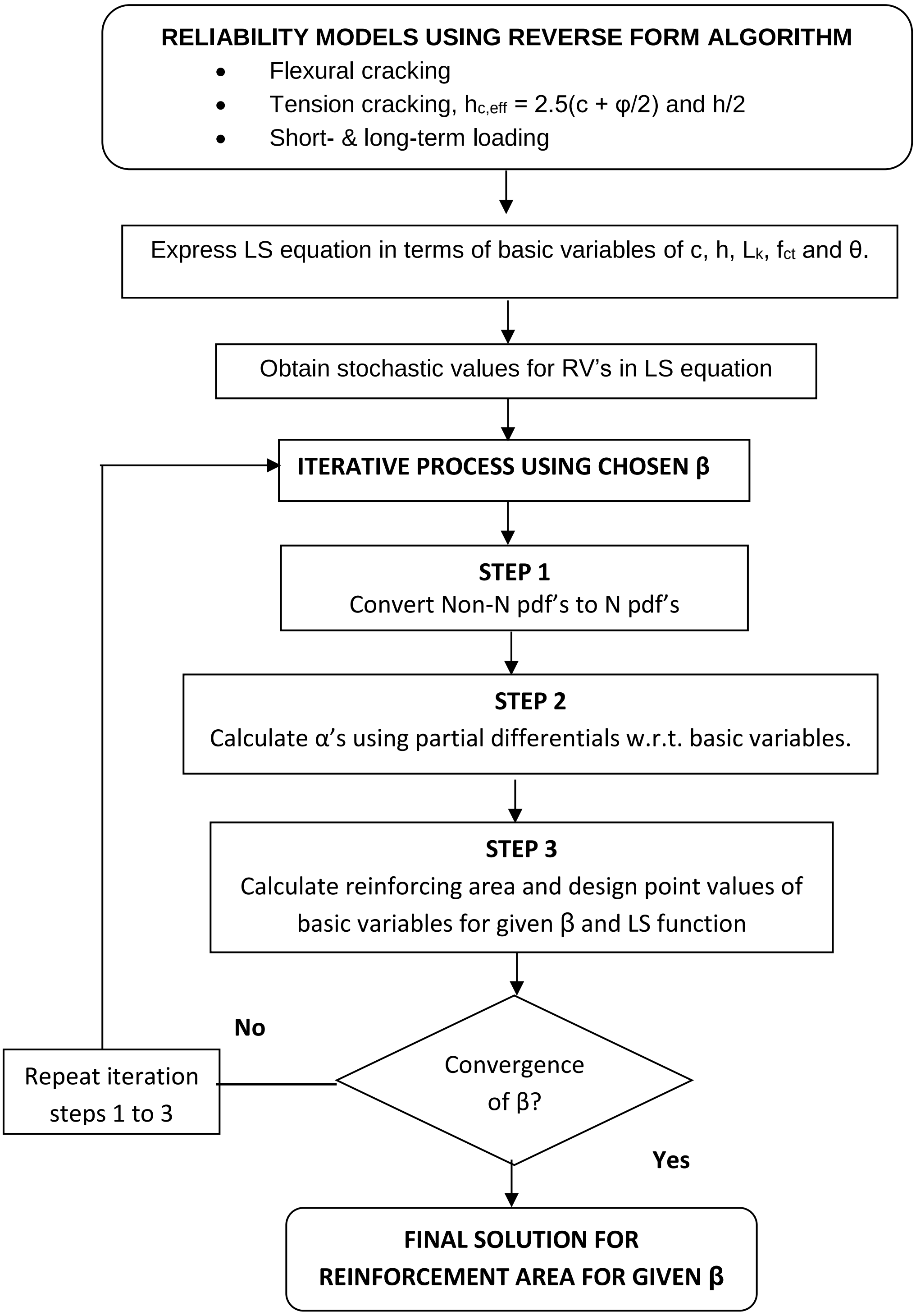


Figure 5.1: *Excel procedure for Reverse FORM*

The target reliability index, β, is first selected, then the algorithm is applied to determine the amount of reinforcement required to achieve the target reliability for a given configuration and loading. More detail on the EXCEL procedure used may be obtained in Mcleod (2013).

5.2.2 Limit State Function for Reliability Model

The FORM (forward and reverse) procedure begins with the formulation of the limit state function for the model to be analysed as discussed in Chapter 3. The general limit state function for serviceability was defined in Equation 3.3 of Chapter 3, as:

$$g(X_i) = C - \vartheta .\ E$$

Applying this to the crack model, the limit function is defined as:

$$g(X) = w_{lim} - \vartheta_w . w_{predict} \quad (5.1)$$

where w_{lim} is the specified crack width, $w_{predict}$ is the best estimate of the predicted crack width and ϑ_w is the model uncertainty treated as a random variable. Considering the form of the crack width model, it would be difficult to separate resistance (R) from actions (E) and therefore a limit state function in the form of $g(X_i) = R - E$ would not be possible.

The predicted crack width was determined using the MC 2010 formulation as the best estimate of crack width. This formulation calculates the design crack width, w_d, as a maximum as:

$$w_d = S_{r,max} \cdot \varepsilon_m = 2 l_{s,max} (\varepsilon_{sm} - \varepsilon_{cm} - \eta\ \varepsilon_{sh}) \quad (5.2)$$

where $S_{r,max}$ is the design or so-called maximum crack spacing. The design (or maximum) crack spacing and crack width correspond to the predicted 95th percentile of crack spacing and crack width, respectively. To obtain the unbiased predicted crack width, all bias is removed as much as possible from all parameters by removing all safety factors in the design formulation. In this way, the deterministic design function is converted into the reliability performance function. The steel and concrete strains calculated in Equation 5.2 are mean values. However, crack spacing and the related transfer length of Equation 5.2 are maximum values. Referring to Section 2.1.1 of Chapter 2, the ratio of maximum crack width to mean crack width in the Eurocode and MC 2010 formulations is 1,7, which is also the ratio of maximum to mean crack spacing, corresponding to the 95th percentile of crack widths. The crack spacing is thus determined by the expression:

$$S_r = (2\ c + 0.35\ k_1\ \varphi / \rho_{eff})/1{,}7$$

Or alternatively, in terms of the transfer length:

$$l_s = 2/1{,}7.\ l_{s,max} = 2/1{,}7\ (k_3.c + 0.25\ f_{ctm}/\tau_{cms}\ .\varphi_s/\rho_{eff})$$

The predicted crack width then becomes:

$$w_{predict} = 2/1{,}7.\ l_{s,max}(\varepsilon_{sm} - \varepsilon_{cm} - \eta\ \varepsilon_{sh}) \qquad (5.3)$$

where ε_{sm} is the mean strain in the reinforcement, ε_{cm} is the mean concrete strain (or tension stiffening) and ε_{sh} is mean shrinkage strain. As discussed in Chapter 2, ε_{sm} is determined using linear elastic theory and ε_{cm} is calculated using Equation 2.4.

5.2.3 Target Reliability

A SLS reliability β of 1,5 for a reference period of 50 years is used as the reference level in this reliability study in keeping with SANS 10160 (2011): Part 1. Given that an appropriate level of reliability has not been determined probabilistically where SLS cracking is the governing limit state, target reliability index values from 1,5 to 2,5 are used to assess the influence of target reliability in the crack model. These values are in keeping with the results of the study by Van Nierop (2017) on target reliability for dominant serviceability states, as is the case in the design of a LRS. A full calibration of target reliability is outside the scope of this research.

5.2.4 LRS Structural Configurations for Reliability Crack Model

The reliability crack model is assessed by application to a structure in which serviceability cracking is dominant. Representative reinforced concrete LRS configurations that result in the maximum pure tension or flexure are therefore selected from empirical evidence acquired through industry experience (for example, as given by Forth and Martin (2014)), as well as that of the researcher. A 1m length of wall in a rectangular reservoir represents the flexural loading case, whilst a 1m length of wall in a circular reservoir represents the tension loading case. Figures 5.2 and 5.3 show these configurations, where a 1 m length of wall under water pressure over its full height is considered. The top of the wall is treated as free.

In the flexural loading case, the wall behaves as a vertical cantilever and is subject to flexure about the horizontal axis of the cross section, with the maximum bending moment at the base of the wall.

The wall geometries chosen are discussed in the following section.

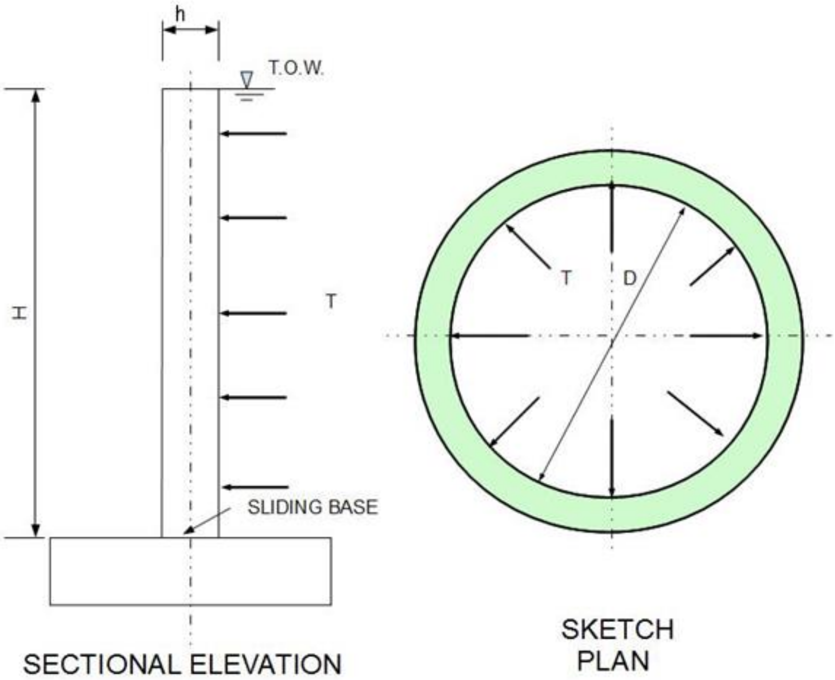


Figure 5.2: *Cross section of wall in rectangular LRS – flexural cracking case*

In the tension loading case, the fixity of the wall base affects the forces induced in the wall, namely those due to (i) tension in the horizontal plane due to hoop stresses induced by the water pressure and (ii) flexure in the vertical direction. Horizontal tension is at a maximum when the wall has a sliding base, corresponding to a negligible bending moment in the vertical plane. This is therefore the wall configuration considered for the pure tension case.

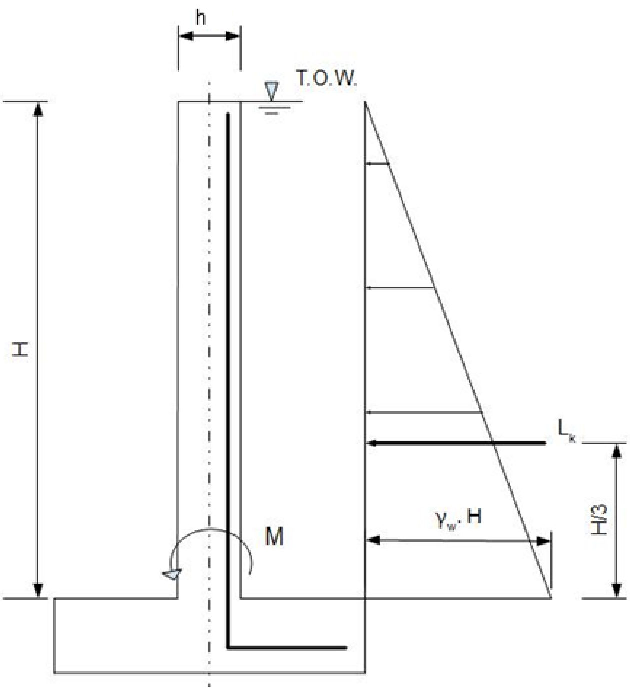


Figure 5.3: *Cross section of wall in circular LRS – tension cracking case*

In the tension load case, pinned and fixed bases result in a small moment in the vertical plane but this moment is far smaller than that of the flexural case, so as such, does not represent a critical load case for the purposes of this study.

5.2.5 Values for MC 2010 Predicted Crack Width Parameters

The parameters of the predicted crack width in Equation 5.3 are expressed as random variables. The set of appropriate random variables chosen for the sensitivity analysis is summarised in Table 5.1. The parameters of liquid load, section thickness, concrete cover and concrete tensile strength of the probabilistic model are modelled as random variables, each with its own PDF, with all other parameters taken as deterministic values.

Table 5.1: *Values of parameters for predicted crack width to MC 2010*

Variable	Symbol	Units	PDF	Characteristic Value	Mean μ_x	Std Dev. σ_x
Rectangular LRS: Height of wall	H	m	Det	5 & 7	5 & 7	-
Circular LRS: Reservoir diameter	D	m	Det	20 & 25	20 & 25	-
Concrete c/s thickness: Flexure Tension	h	mm	N	500 & 600 250	500 & 600 250	5.0 & 6.60 2.50
Concrete c/s width	b	mm	Det	1000	1.0	-
Water pressure, L_k: H 5 m H 7m	L_k	kN/m^2	N	50 70	49.05 68.67	2.45 3.43
Concrete cube strength	f_{cu}	MPa	Det	37	37	0
Concrete cylinder strength	f_{ck}	MPa	Det	30	30	0
Mean concrete tensile strength	f_{ctm}	MPa	LN	2.00	2.89	0.55
Concrete modulus, short term	E_c	GPa	Det	27.4	27.4	-
Concrete creep factor -Long term	ϕ	-	Det	1.7	1.7	-
Long term concrete shrinkage strain	ε_{sh}	-	1. Det 2. LN	-520 & -270 x10^{-6}	--400 x 10^{-6} & -200 x10^{-6}	- CoV 0.2
Shrinkage strain factor - Long term	η	-	Det	1	1	-
Reinforcement diameter	φ	mm	Det	20	0.02	-
Reinforcement area - calculated	A_s	mm^2	Det	Calculated	Calculated	-
Steel modulus	E_s	GPa	Det	200	200	-
Concrete cover	c	mm	LN	40	40	6.00
Coefficient for cover	k_1	-	Det	1	1	-
Bond coefficient (HT reinforcement)	k_2	-	Det	1/1.8 = 0.556	0.556	-
Ratio max/ mean crack width	k_m	-	Det	1.7	1.7	-
Coefficient kt : Short-term load Long-term load	k_t	-	Det	0.6 0.4	0.6 0.4	-
Crack width limit	w_{lim}	mm	Det	0.1 & 0.2	0.1 & 0.2	-
Model Uncertainty	θ_w	-	LN	1 - 1.1	1 - 1.1	CoV 0.38

Note 1: LN = log-normal PDF, N = normal PDF, Det = deterministic value.

The recommended deterministic values for the fixed-value k_i coefficients of MC 2010 are used in calculating predicted crack widths, as there is insufficient data to treat them as random variables. These coefficients were derived from experimental research in developing the crack width formulation and therefore are more empirical in nature and may add some unknown conservatism to the model. Any contribution of these coefficients to uncertainty would be taken up in the model uncertainty.

Deterministic values of 0,2 and 0,1 mm are selected for the crack width limit, corresponding to the limits set by EN 1992 (2004), and as commonly used in the South African civil engineering industry. As discussed in Chapter 3, it is acknowledged that these deterministic values are conservative estimates of the true crack width limit, but at this stage, are the best estimates thereof.

Wall thicknesses and heights are chosen using typical optimum section geometries based on empirical knowledge, as discussed, and considering economy of the structure, ease of construction and design factors such as deflection limits based the wall slenderness ratio, as follows:

(i) For flexural cracking, a wall height of 5 m with 500 (H/h of 10, m/m) and 600 mm (H/h of 8,3) wall thicknesses, respectively, is chosen to assess any influence of a change in wall thickness. A wall height of 7 m with a wall thickness of 700 mm (H/h of 10) is selected to evaluate the influence of liquid load.

(ii) For tension cracking, wall heights of 5 and 7 m were considered, with reservoir diameters of 20 and 25 m to assess the influence of liquid load. A section thickness of 250 mm is used, as discussed further in Section 5.2.6.

The reinforcement area required for each of the selected target reliabilities are calculated in the reverse FORM process for a typical bar diameter of 20 mm.

Long term shrinkage strain is selected considering the SANS 10100-1(2004) and EN 1992 shrinkage strain charts, and literature. Long-term shrinkage strain is initially modelled as a deterministic value in the reliability analyses due to a lack of information on any possible stochastic values, as an estimated mean value, not a design value. To investigate the influence of long-term shrinkage strain on the crack width prediction model, it is then treated as a random variable in the GPM for long-term flexure. Values are assumed for the stochastic parameters of long-term shrinkage strain using engineering judgement and anecdotal evidence gained from the literature review. The mean value used is the same as the deterministic value as a best first estimate. From the anecdotal evidence in the literature, as discussed in Chapter 3, a variation of 0,2 is selected. A lognormal distribution is assumed as this is the distribution commonly used for material properties, as stated by JCSS (2001).

The determination of the true stochastic parameters of long-term shrinkage strain is beyond the scope of this research.

Although the critical load case is due to a quasi-permanent liquid load, so treated as long-term loading, some analyses are also performed assuming loading of short-term duration to evaluate model uncertainty over time.

From the quantification of model uncertainty, presented in Chapter 4, for both flexure and tension cracking over the short- and long-term, the values for the model uncertainty (ϑw) stochastic parameters obtained for the MC 2010 crack model are summarised in Table 5.2.

Table 5.2: *Model uncertainty stochastic parameters for MC 2010 from analysis*

Load Case	Mean	CoV	PDF
Flexure – Short term	1.05	0.38	LN
Flexure – Long term	1.13	0.38	LN
Tension – Short term	0.98	0.32	LN
Tension – Long term	0.99	0.22	LN

In the case of tension cracking, the mean value determined from the quantification of model uncertainty is about 1,0, which is less than the mean obtained for flexure. However, as previously discussed, the experimental sample sizes were smaller and for relatively uniform test configurations, especially in the long-term tension case where only an indication of the statistical parameters could be obtained for model uncertainty. A small range of values is therefore chosen for the reliability analyses for both flexure and tension model uncertainty to ascertain if flexure and tension require separate treatment in any subsequent optimisation of a design formulation, as given in Table 5.3:

Table 5.3: *Model uncertainty stochastic parameters for FORM sensitivity analysis*

Load Case	Mean	CoV	PDF
Flexure – Short-term	1.05 & 1.1	0.38	LN
Flexure – Long-term	1.1	0.38	LN
Tension – Short- & long-term	1 and 1,1	0.30 & 0.38	LN

As discussed in Chapter 4, given that the probability distribution function (pdf) can only be estimated for tension cracking, a lognormal distribution was assumed for all load cases, as this distribution results in a more conservative (lower) estimate of target reliability than the normal distribution. In addition, previous research by sources such as Quan and Gengwei

(2002), as discussed in Chapter 3, suggests that the pdf of model uncertainty in crack models tends towards lognormal, rather than normal.

5.2.6 Load Cases for Reliability Crack Model

In the deterministic calculation of crack width, the effective section depth ($h_{c,eff}$), is taken as the lesser of (h-x)/3, h/2 or 2,5 (c + φ/2). As these expressions for $h_{c,eff}$ include different random variables, separate reliability models are required. The first expression is the limiting one in flexural cracking, thus requiring one reliability model for flexural cracking. However, for tension cracking, the limiting expression depends on the section thickness (h), the concrete cover (c) and the reinforcement diameter (φ), expressed as either h/2 or 2,5 (c + φ/2). Thus, two reliability models are required for tension cracking.

For tension cracking and so considering typical values in a circular LRS wall for a section thickness of 250 mm, both equations give the same value for $h_{c,eff}$. For a section thicknesses less than 250 mm, h/2 is the limiting equation. Conversely, for h greater than 250 mm, 2,5 (c + φ/2) is the limiting equation.

The models for the critical load cases are therefore:

(i) Model 1: Flexural load-induced cracking
(ii) Model 2 (a): Tension load-induced cracking with $h_{c,eff} = 2,5\ (c + \varphi/2)$
(iii) Model 2 (b): Tension load-induced cracking with $h_{c,eff} = h/2$

5.3 RESULTS AND DISCUSSION

The results of the reliability sensitivity analysis are presented and discussed for long-term flexural and tension cracking using the MC 2010 crack width formulation as the best estimate of predicted crack width, as described at the beginning of this chapter, namely:

(i) The relative influences of the random variables, in particular, model uncertainty, for selected target reliability levels using the sensitivity factors from the Reverse FORM analysis.
(ii) Assess potential partial safety factor schemes and the influence of model uncertainty by determining the theoretical partial safety factors for the random variables.
(iii) Indicative evaluation of the effect of long-term shrinkage strain on the reliability of the MC 2010 crack model.

5.3.1 Sensitivity Factors

The sensitivity factors of the random variables were calculated for each load case using the reverse FORM and a range of target reliability index values from 1,5 to 2,5. The influence of the random variables are then assessed for flexure and tension. Model uncertainty is of particular interest as the results from the quantification of this random variable, presented in Chapter 4, suggest that it is the dominant parameter in the reliability model. The sensitivity analysis explores further the significance of model uncertainty. In these analyses, long-term shrinkage strain is treated as a fixed value. Results are summarised into tables with selected graphs included to illustrate any trends in the sensitivity factors and thus the relative influence of the crack model variables and model uncertainty.

As the reverse FORM is performed over a range of target reliabilities, the relationship between the reliability level and the random variables can be assessed.

5.3.1.1 Model 1 - Flexural Cracking

The sensitivity factors obtained for the random variables do not vary significantly over short- and long-term flexural loading, showing that the crack model performs consistently irrespective of the load duration. The sensitivity factors of the random variables summarised in Table 5.4 for a wall height of 5m, wall thickness of 500 mm and limiting crack width of 0,2 mm demonstrate this. Increases in the mean of model uncertainty did not have any real effect on the sensitivity factors.

Table 5.4: *Sensitivity factors for flexural load-induced cracking*

Load Duration	Model Uncertainty		β	Sensitivity Factors				
	μ	CoV		α_c	α_h	α_L	α_{ft}	α_θ
Short-term	1.05	0.38	1.50	-0.138	0.048	-0.211	0.366	-0.895
			2.00	-0.148	0.046	-0.206	0.346	-0.902
			2.50	-0.159	0.045	-0.201	0.330	-0.907
	1.10	0.38	1.50	-0.140	0.048	-0.211	0.366	-0.894
			2.00	-0.150	0.046	-0.206	0.346	-0.902
			2.50	-0.161	0.045	-0.201	0.330	-0.907
Long-term	1.10	0.38	1.50	-0.159	0.032	-0.182	0.230	-0.942
			2.00	-0.171	0.032	-0.180	0.226	-0.941
			2.50	-0.184	0.032	-0.179	0.224	-0.940

The most significant parameter is found to be model uncertainty with sensitivity factors (α_ϑ) of about -0,94 across all β values for long-term cracking. This constitutes a dominant influence on the crack model and therefore justifies a rigorous treatment of model uncertainty.

Section thickness has negligible influence on the crack model, with sensitivity factors (α_h) less than 0,05. Concrete cover and liquid load has a small negative effect on the crack model, with sensitivity factors α_c and α_L, respectively, of about -0,2.

Concrete tensile strength has a small positive influence on the crack model with sensitivity factors (α_{fct}) just less than 0,3. It is noted that the sensitivity factors for all random variables do not change significantly as the reliability index increased.

The rectangular reservoir wall configurations of the flexural load case were varied to assess if the relative influences of the crack model random variables and model uncertainty remained consistent over different section geometries. Crack width limits of 0,1 and 0,2 mm were selected. It is found that the relative influences of the random variables do not change significantly, with model uncertainty remaining the dominant influence on the crack width prediction model. Figure 5.4 illustrates this dominant effect of model uncertainty on the crack width prediction model for flexural cracking, consistent over the range of parameters chosen.

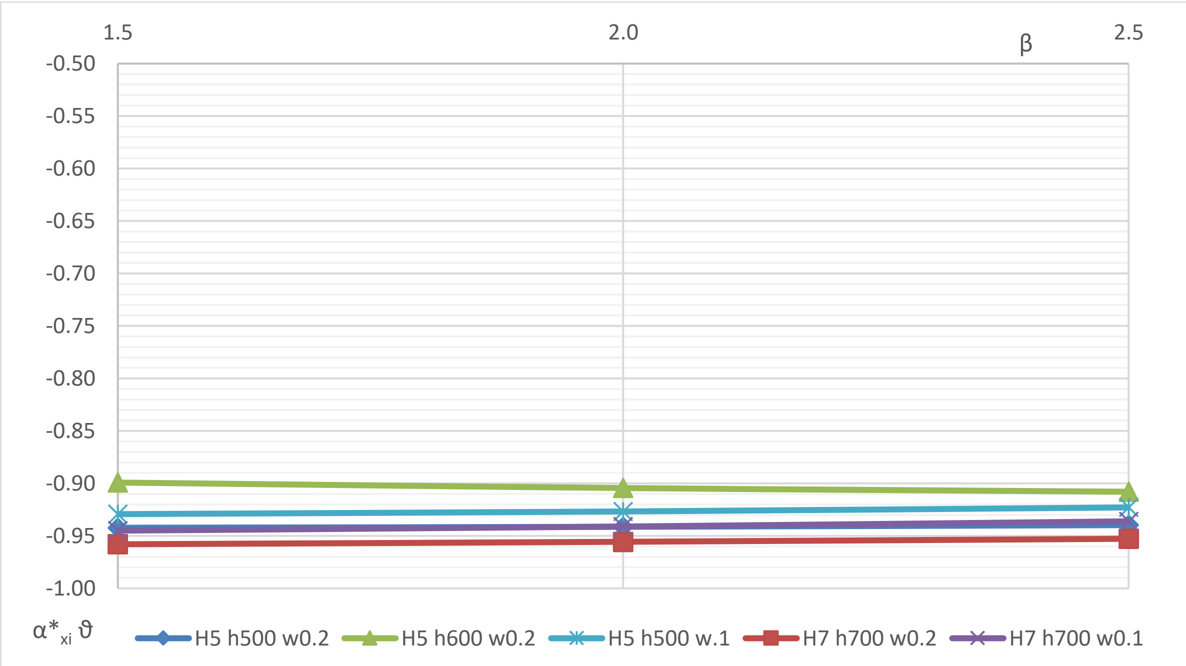


Figure 5.4: *Influence of model uncertainty - long term flexural cracking.*

5.3.1.2 Model 2 - Tension Cracking

In the case of tension cracking, two reliability models are required due to the different formulations for $h_{c,eff}$, namely Model 2(a) with $h_{c,eff} = 2,5\ (c + \varphi/2)$ and Model 2(b) with $h_{c,eff} = h/2$. The sensitivity factors obtained for each model are summarised in Tables 5.5 and 5.6, respectively, for a wall height of 5 m, wall thickness of 250 mm, reservoir diameter of 25 m and limiting crack width of 0,2 mm.

Sensitivity factors are similar for short- and long-term flexural loading for both Model 2 (a) and 2(b). Increases in the mean and variation of model uncertainty do not have any significant effect on the sensitivity factors, although the increases have a greater effect on the factors than for flexure, but this effect is still small. Similarly, the sensitivity factors remain relatively unchanged with increasing target reliability.

Table 5.5: *Sensitivity factors for tension load-induced cracking $h_{c,eff} = 2,5(c + \varphi/2)$*

Load Duration	Model Uncertainty		β	Sensitivity Factors			
	μ	CoV		α_c	α_L	α_{ft}	α_θ
Short-term	1.00	0.30	1.50	-0.164	-0.247	0.349	-0.889
			2.00	-0.162	-0.248	0.347	-0.890
			2.50	-0.161	-0.250	0.346	-0.890
	1.10	0.38	1.50	-0.134	-0.205	0.297	-0.923
			2.00	-0.133	-0.206	0.298	-0.922
			2.50	-0.131	-0.208	0.300	-0.922
Long-term	1.00	0.30	1.50	-0.251	-0.226	0.276	-0.900
			2.00	-0.249	-0.228	0.282	-0.898
			2.50	-0.247	-0.231	0.288	-0.896
	1.10	0.38	1.50	-0.205	-0.188	0.238	-0.931
			2.00	-0.203	-0.191	0.246	-0.928
			2.50	-0.201	-0.194	0.256	-0.926

Table 5.6: *Sensitivity factors for tension load-induced cracking $h_{c,eff} = h/2$*

Load Duration	Model Uncertainty		β	Sensitivity Factors				
	μ	CoV		α_c	α_h	α_L	α_{ft}	α_θ
Short-term	1.00	0.30	1.5	-0.112	-0.006	-0.233	0.326	-0.909
			2.0	-0.121	-0.006	-0.230	0.317	-0.912
			2.5	-0.130	-0.006	-0.227	0.310	-0.914
	1.10	0.38	1.5	-0.097	-0.004	-0.194	0.278	-0.936
			2.0	-0.106	-0.004	-0.192	0.273	-0.937
			2.5	-0.116	-0.003	-0.190	0.270	-0.937
Long-term	1.00	0.30	1.5	-0.119	-0.013	-0.221	0.264	-0.931
			2.0	-0.128	-0.012	-0.219	0.263	-0.931
			2.5	-0.137	-0.012	-0.218	0.262	-0.930
	1.10	0.38	1.5	-0.103	-0.010	-0.182	0.227	-0.951
			2.0	-0.111	-0.009	-0.182	0.230	-0.950
			2.5	-0.121	-0.008	-0.182	0.233	-0.948

Section thickness has a negligible effect on the crack model for an $h_{c,eff}$ of h/2 (Model 2(a)) with sensitivity factors approaching zero. It is not a parameter in the crack width when $h_{c,eff}$ is 2,5 (c + φ/2). This leads to the conclusion that section thickness could be modelled as a deterministic value rather than a random variable. As would be expected, due to the formulation of the equations for $h_{c,eff}$, the negative influence of concrete cover is greater for Model 2(a) than for Model 2(b), however, the influence remains low with sensitivity factors between -0,1 and -0,25, depending on the target reliability.

Liquid load has a slightly higher negative influence than concrete cover with sensitivity factors up to -0.25. Concrete tensile strength has a low to moderate positive influence on the tension crack model, with sensitivity factors up to about 0,35.

As can be seen from Tables 5.5 and 5.6, model uncertainty is indeed the dominant influence on the tension crack models demonstrated by sensitivity factors in the range of about -0,90 to - 0,96. This is slightly higher than those obtained for flexural cracking. As concluded for flexural cracking, a more rigorous treatment of model uncertainty is therefore justified.

The circular reservoir wall configurations and limiting crack widths were varied to evaluate whether the relative influences of the random variables for the tension load case are consistent. The sensitivity factors of the random variables were determined for the combinations of wall heights of 5 and 7 m for with reservoir diameters of 20 and 25 m, for limiting crack widths of 0,2 and 0,1 mm, respectively. As with flexure, it is found that the relative influences of the random variables do not change significantly, with model uncertainty remaining the dominant influence on the crack width prediction model. Figures 5.5 and 5.6 illustrate the dominant effect of model uncertainty for tension cracking, consistent over the range of wall geometries and crack width limits considered.

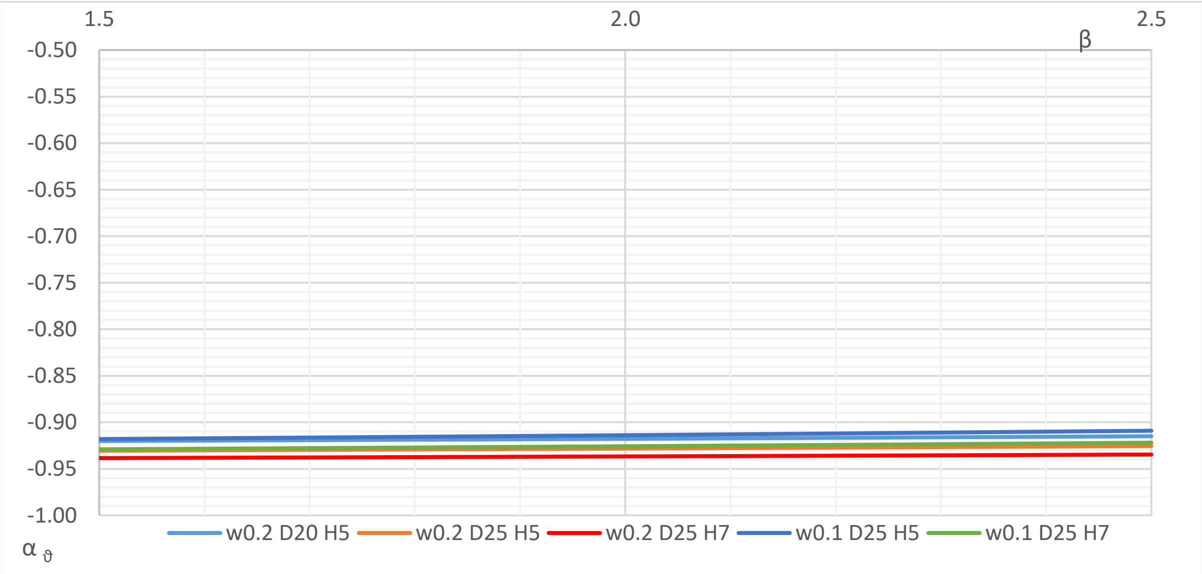


Figure 5.5: *Influence of model uncertainty – long-term tension, $h_{c,eff}$ = 2,5(c + φ/2)*

Referring to Figures 5.5 and 5.6, the influence of the random variables do not display any significant change as the target reliability increased.

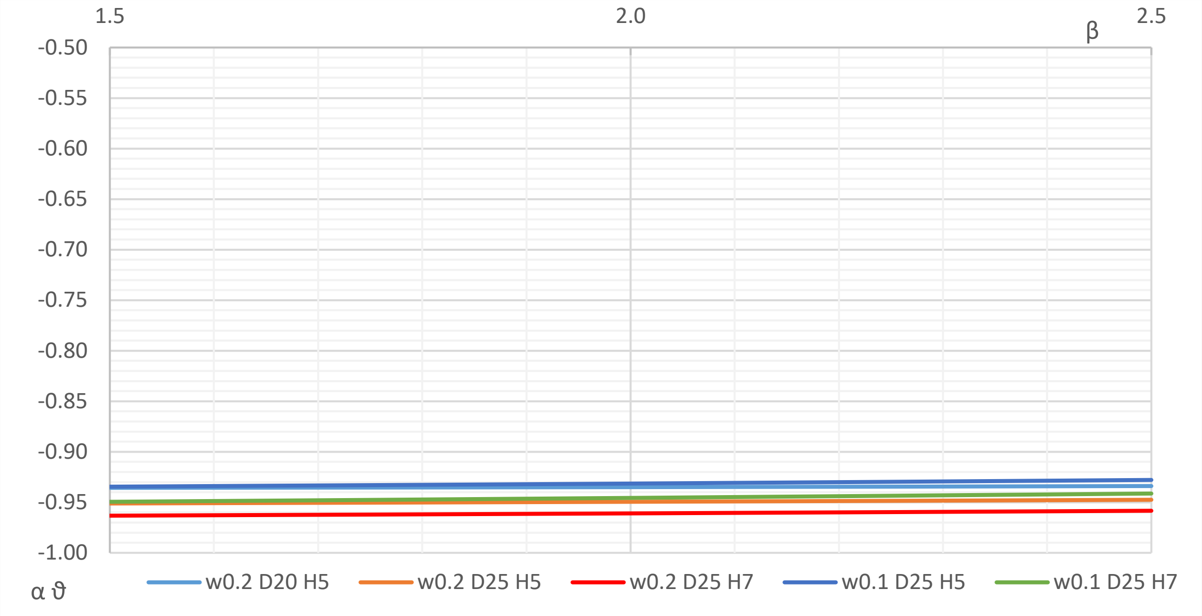


Figure 5.6: *Influence of model uncertainty - long-term tension, $h_{c,eff} = h/2$*

In summary, the sensitivity analysis shows that model uncertainty is in fact the dominant influence in the reliability model for both flexural and tension cracking.

5.3.2 Theoretical Partial Safety Factors

Using the results of the reverse FORM, the theoretical partial safety factors for each of the random variables were determined for the flexural and tension load cases. Each load case is discussed and compared in the following sections. The effect of target reliability is also assessed further. In these analyses, long-term shrinkage strain is treated as a deterministic value.

5.3.2.1 Model 1 - Flexural Cracking

The theoretical partial safety factors of the random variables for flexural loading, summarised in Table 5.7, were obtained for a wall height of 5 m, wall thickness of 500 mm and limiting crack width of 0,2 mm. It can be concluded that the partial safety factors are relatively consistent regardless of the load duration. Comparing short- and long-term loading, the partial safety factors for all random variables increase slightly as the load duration increases, but not sufficiently to differentiate between short- and long-term loading in devising a partial safety. Increasing the mean and CoV of model uncertainty also had little influence on values of the partial safety factors.

For both short- and long-term flexural cracking, the theoretical partial safety factors follow the sensitivity factors in their influence on the reliability crack model. Section thickness (γ_h)

has a partial safety factor of 1, irrespective of the level of reliability or the values of the model uncertainty. This is in keeping with sensitivity factors close to zero for section thickness and therefore negligible influence.

Table 5.7: *Theoretical partial safety factors for flexural cracking load case*

Load Duration	Model Uncertainty		β	Theoretical Safety Factors					
	μ	CoV		γ_c	γ_h	γ_L	γ_{fct}	$1/\gamma_{fct}$	γ_θ
Short-term	1.05	0.35	1.50	1.038	0.999	1.015	0.918	1.090	1.482
			2.00	1.055	0.999	1.020	0.895	1.118	1.645
			2.50	1.073	0.999	1.025	0.874	1.145	1.807
	1.1	0.38	1.50	1.036	0.999	1.014	0.923	1.084	1.487
			2.00	1.052	0.999	1.019	0.901	1.110	1.651
			2.50	1.070	0.999	1.023	0.881	1.136	1.814
Long-term	1.05	0.35	1.50	1.043	1.000	1.014	0.946	1.057	1.496
			2.00	1.046	0.999	1.018	0.928	1.077	1.660
			2.50	1.049	0.999	1.022	0.911	1.098	1.822
	1.1	0.38	1.50	1.041	1.000	1.013	0.949	1.054	1.499
			2.00	1.044	0.999	1.017	0.932	1.073	1.664
			2.50	1.047	0.999	1.021	0.915	1.093	1.828

Concrete cover (γ_c) and liquid load (γ_L) have similar low partial safety factors about 1,04 to 1,07 and about 1,02, respectively, depending on the target reliability. Concrete tensile strength (γ_{fct}), as a material resistance random variable, has a positive influence on the reliability crack model, therefore its partial safety factors are reported here as $1/\gamma_{fct}$. The partial safety factors for the concrete tensile strength are approximately 1,10 to 1,15 as target reliability increases, as indicated by its moderate sensitivity factors. The partial safety factors of concrete cover, liquid load and section thickness are not influenced to any practical degree as the reliability level increases.

Model uncertainty is clearly the dominant random variable in the MC 2010 crack width prediction model with partial safety factors from about 1,48 to 1,83, depending on the target reliability. These are significant values, particularly when compared to the remaining random variables, warranting further investigation and a proper optimisation process to determine an appropriate partial safety factor scheme for design purposes.

Referring to Figure 5.7, the effect of the reliability level on the model uncertainty partial safety factors is illustrated for different geometric wall configurations and limiting crack width, for flexural cracking. Model uncertainty is significantly influenced by the reliability level, with partial safety factors increasing by about 22 % as β is increased from 1,5 to 2,5. A proper probabilistic assessment of serviceability target reliability for SLS load-induced cracking is required particularly when this SLS governs design. The model uncertainty partial safety factors are relatively stable over the different wall configurations and limiting crack widths.

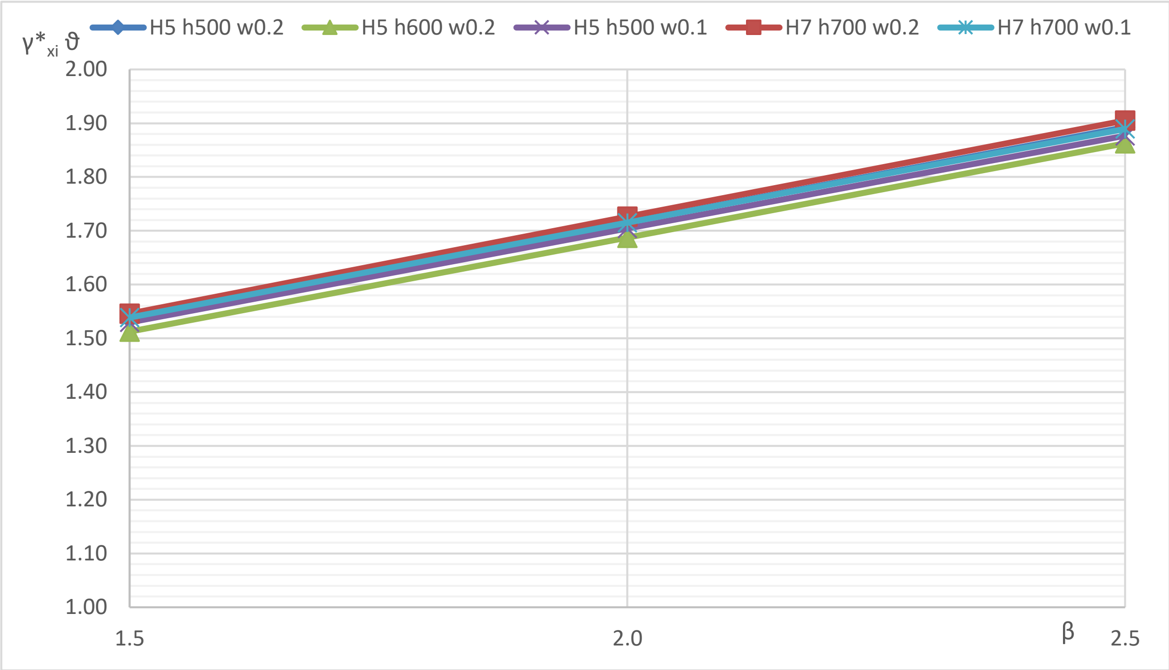


Figure 5.7: *Model uncertainty partial safety factor – long-term flexural cracking*

5.3.2.2 Model 2 - Tension Cracking

As with flexural cracking, changes in the model uncertainty stochastic parameters do not meaningfully affect the partial safety factors of the random variables. A summary of the partial safety factors for the tension load case is presented in Tables 5.8 and 5.9, reported for the two limiting equations for the effective section thickness, respectively

Table 5.8: *Theoretical partial safety factors for Tension $h_{c,eff} = 2,5(c + \varphi/2)$*

Load Duration	Model Uncertainty		β	Partial Safety Factors				
	μ	CoV		γ_c	γ_L	γ_{fct}	$1/\gamma_{fct}$	γ_θ
Short-term	1.00	0.30	1.50	1.037	1.019	0.900	1.111	1.507
			2.00	1.049	1.025	0.868	1.152	1.676
			2.50	1.060	1.031	0.836	1.196	1.846
	1.10	0.38	1.50	1.030	1.015	0.915	1.092	1.526
			2.00	1.040	1.021	0.887	1.128	1.701
			2.50	1.049	1.026	0.858	1.166	1.876
Long-term	1.00	0.30	1.50	1.057	1.017	0.921	1.085	1.513
			2.00	1.075	1.023	0.893	1.120	1.682
			2.50	1.093	1.029	0.863	1.158	1.851
	1.10	0.38	1.50	1.046	1.014	0.932	1.073	1.530
			2.00	1.061	1.019	0.906	1.103	1.705
			2.50	1.075	1.024	0.879	1.138	1.879

Referring to Tables 5.8 and 5.9, the partial safety factors obtained for long term tension loading are comparable to those of short-term loading.

Table 5.9: *Theoretical partial safety factors for Tension $h_{c,eff} = h/2$*

Load Duration	Model Uncertainty		β	Partial Safety Factors					
	μ	CoV		γ_c	γ_h	γ_L	γ_{fct}	$1/\gamma_{fct}$	γ_θ
Short-term	1.00	0.30	1.50	1.025	1.000	1.017	0.907	1.102	1.518
			2.00	1.036	1.000	1.023	0.879	1.137	1.693
			2.50	1.049	1.000	1.028	0.853	1.172	1.868
	1.10	0.38	1.50	1.022	1.000	1.015	0.921	1.086	1.533
			2.00	1.032	1.000	1.019	0.896	1.116	1.712
			2.50	1.044	1.000	1.024	0.872	1.147	1.890
Long-term	1.00	0.30	1.50	1.027	1.000	1.017	0.925	1.081	1.531
			2.00	1.038	1.000	1.022	0.900	1.111	1.707
			2.50	1.051	1.000	1.027	0.875	1.142	1.883
	1.10	0.38	1.50	1.023	1.000	1.014	0.935	1.069	1.542
			2.00	1.033	1.000	1.018	0.913	1.096	1.722
			2.50	1.045	1.000	1.023	0.889	1.125	1.900

Comparing the two models for tension cracking and referring to Tables 5.8 and 5.9, the partial safety factors for all random variables do not display any noteworthy differences, which leads to the conclusion that a single partial safety factor scheme can be selected for tension cracking, irrespective of the limiting equation of $h_{c,eff}$. Concrete cover and liquid load have similar partial safety factors of approximately 1,02 to 1,08, and about 1,03, comparable to the values obtained for flexure. Concrete tensile strength, with its moderate influence on the crack model, has a partial safety factor (as $1/\gamma f_{ct}$) of approximately 1,1 to 1,2, depending on the target reliability. Referring to Table 5.9, the partial safety factor for section thickness is 1,0 for Model 2(b), in keeping with a sensitivity factor approaching zero.

Model uncertainty has partial safety factors of about 1,5 for β of 1,5 to about 1,9 for β of 2,5, for both tension models. These are significant values, as would be expected given this variable's dominance in the reliability crack model, and comparable to those obtained for flexural cracking. Considering that, other than concrete tensile strength, the partial safety factors obtained for the remaining random variables are close to 1,0, the argument for a single model factor applied to the crack model, rather than a combination of partial safety factors, could be made. However, this does require further investigation as part of the probabilistic calibration and optimisation of the design formulation, which is outside the scope of this research.

Referring to Figures 5.8 and 5.9, the effect of the reliability level on the model uncertainty partial safety factors is illustrated for different geometric wall configurations, reservoir diameter and limiting crack width.

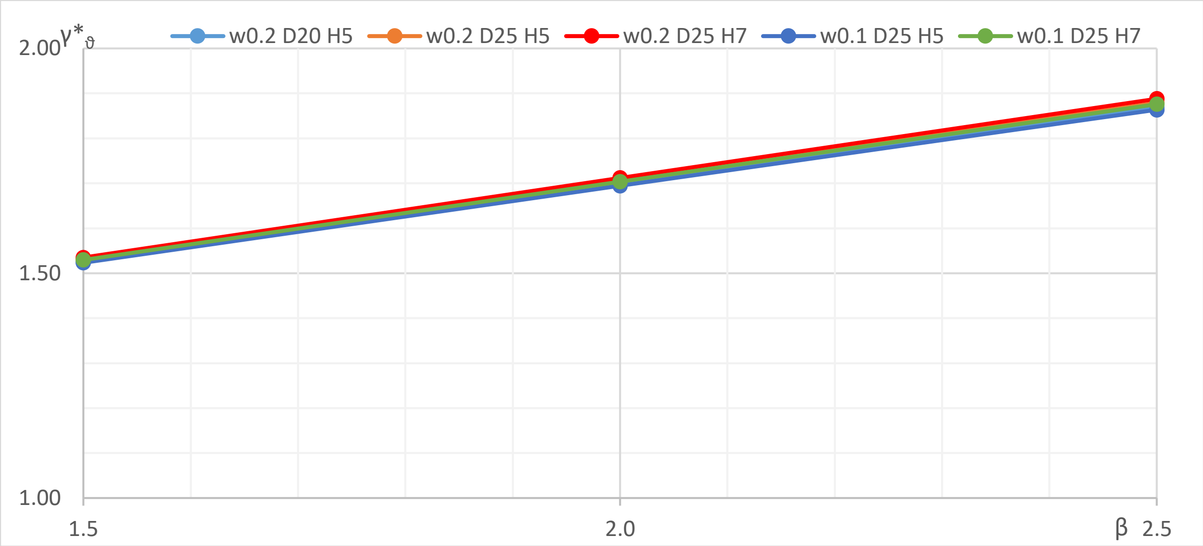


Figure 5.8: *Model uncertainty partial safety factors for long term tension, $h_{c,eff}$ = 2,5(c + φ/2)*

Model uncertainty for tension cracking is significantly influenced by the reliability level, with partial safety factors increasing by about 32 % as β is increased from 1,5 to 2,5. This would need to be considered in the optimisation of the target reliability for SLS load-induced cracking. The model uncertainty partial safety factors are relatively stable over the different wall configurations and limiting crack widths.

The partial safety factors in the tension case are similar to those obtained for flexural cracking. This suggests that the assumption made that the stochastic values for model uncertainty for flexural cracking can be utilised for tension cracking is correct, and it can be concluded that for the MC 2010 crack width prediction model, there is no need to distinguish between tension and flexural cracking in devising a partial safety factor scheme.

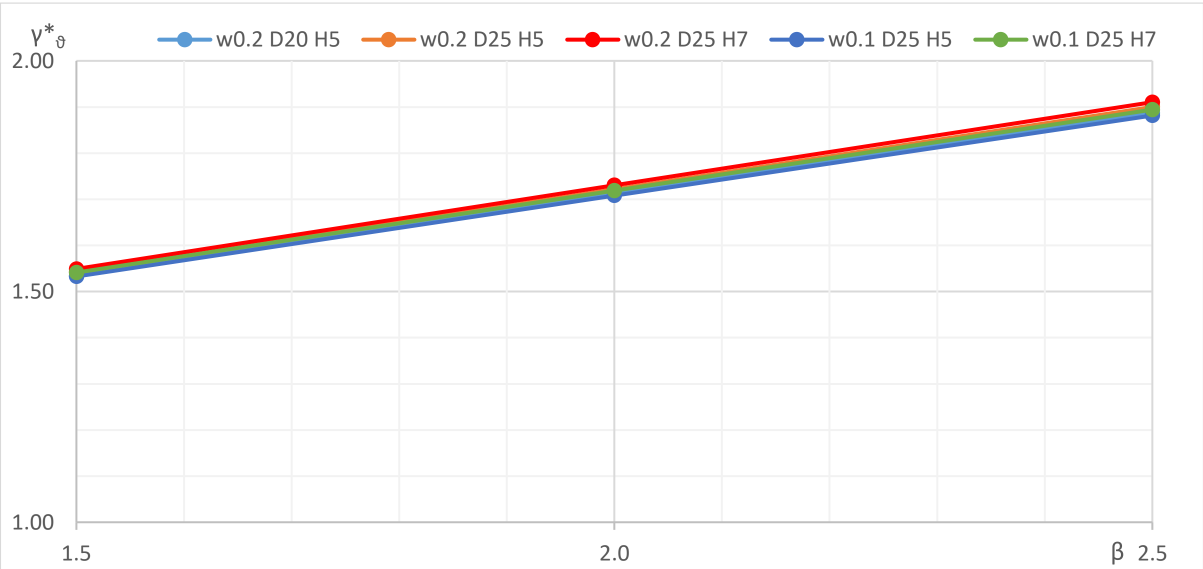


Figure 5.9: *Model Uncertainty partial safety factors for long term tension, $h_{c,eff}$ = h/2*

The sensitivity analysis confirms the relative influences of model uncertainty and the remaining random variables as determined in the correlation analyses between the model parameters of the MC 2010 crack model, reported in Chapter 4.

5.3.3 Effect of Long-Term Shrinkage Strain on MC 2010 crack with prediction model

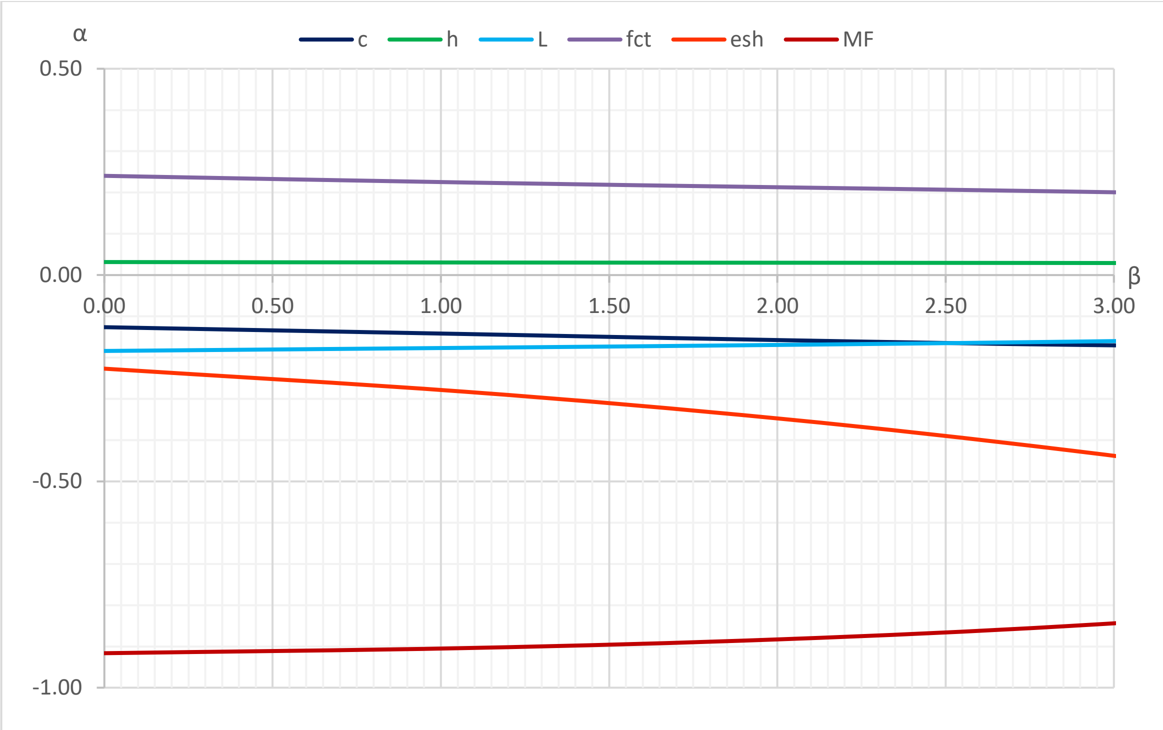
Analyses were performed with long-term shrinkage strain treated as a random variable to evaluate the effects of this parameter on the crack width prediction model, at a limiting crack width of 0,2 mm for the long-term flexure load case. Mean values of -200 x 10^{-6} (as a lower value) and -400 x 10^{-6} (as an upper value), with a CoV of 0,2, were selected for long-term shrinkage strain. Sensitivity factors ($\alpha_{\varepsilon sh}$) and theoretical safety factors were determined for long-term shrinkage strain, summarised in Tables 5.10 and 5.11, for a wall height of 5m, wall thickness of 500 mm and limiting crack width of 0,2 mm. Very similar values are obtained for a wall height of 7m and thickness of 700 mm. The sensitivity factors of the random variables remain consistent regardless of the mean value considered for shrinkage strain.

Table 5.10: *Influence of long-term shrinkage strain - sensitivity factors for long-term flexure*

Analysis	β	α_c	α_h	α_L	α_{ft}	$\alpha_{\varepsilon sh}$	α_θ
Shrinkage strain as a RV CoV 0.2	1.50	-0.150	0.030	-0.173	0.219	-0.310	-0.894
	2.00	-0.158	0.030	-0.169	0.213	-0.348	-0.883
	2.50	-0.167	0.030	-0.167	0.200	-0.387	-0.870
Shrinkage strain as deterministic value	1.50	-0.181	0.028	-0.170	0.180	-	-0.951
	2.00	-0.195	0.029	-0.169	0.179	-	-0.949
	2.50	-0.210	0.030	-0.168	0.179	-	-0.946

Referring to Table 5.10, the inclusion of shrinkage strain results in the decrease in the influence of model uncertainty. There are also small decreases in the sensitivity factors of the remaining random variables except for concrete tensile strength. For the latter random variable, there is a small increase in its influence. Sensitivity factors between 0,3 and 0,39 indicate that shrinkage strain has a moderate influence on the crack width prediction model.

Figure 5.10 illustrates the relative influences of the random variables, plotted against the reliability index. The influence of model uncertainty decreases as reliability increases with a corresponding increase in the effect of long-term shrinkage.



MF = model uncertainty, esh = long-term shrinkage

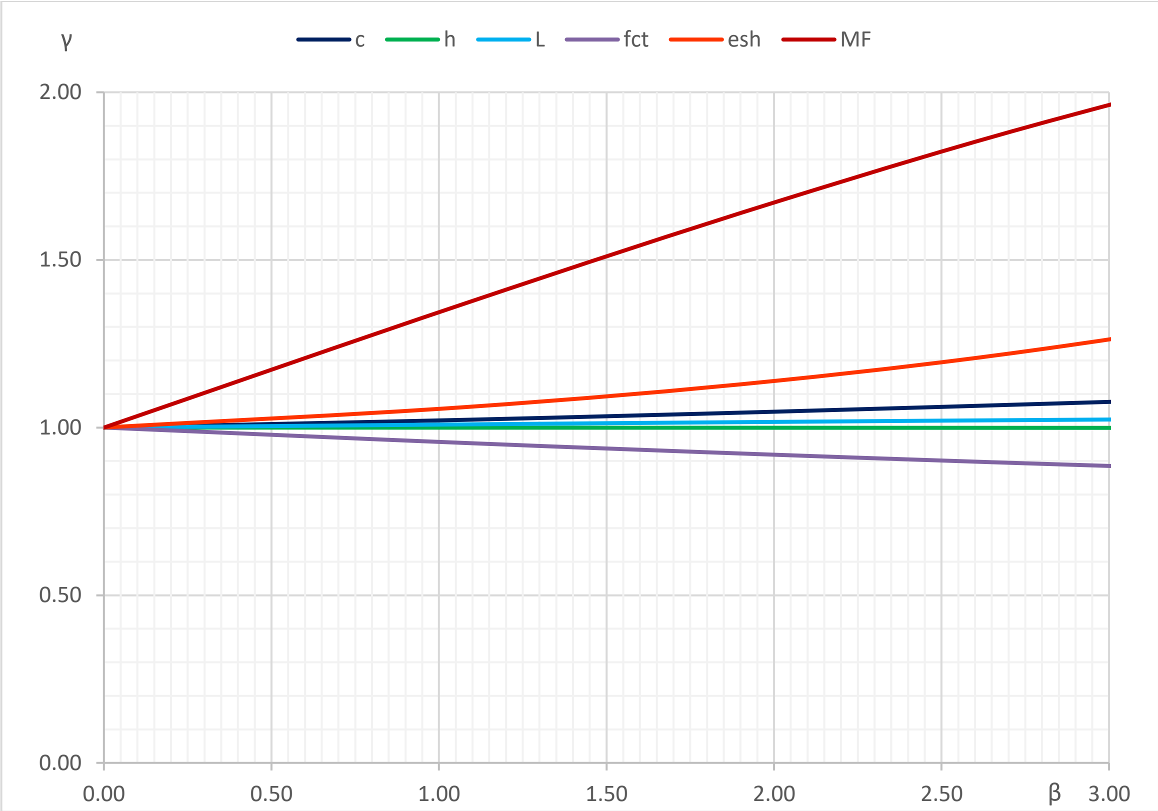
Figure 5.10: *Relative influences of the MC 2010 crack width model random variables including long-term shrinkage*

Referring to Table 5.11, the partial safety factors of concrete tensile strength, section thickness and liquid load do not vary significantly whether shrinkage strain is treated as a random variable or as a deterministic value. The partial safety factors for concrete tensile strength and model uncertainty increase slightly when shrinkage strain is modelled as a random variable.

Table 5.11: *Theoretical partial safety factors for analyses modelling long-term shrinkage strain as a RV.*

Analysis	β	Theoretical Partial Safety Factors						
		γ_c	γ_h	γ_L	γ_{fct}	$1/\gamma_{fct}$	$\gamma_{\varepsilon sh}$	γ_θ
Shrinkage strain as a random variable CoV 0.2	1.50	1.036	1.000	1.013	0.935	1.070	1.100	1.523
	2.00	1.048	0.999	1.017	0.919	1.088	1.140	1.675
	2.50	1.060	0.999	1.020	0.889	1.125	1.232	1.830
Shrinkage strain as deterministic value	1.50	1.041	1.000	1.013	0.949	1.054	-	1.499
	2.00	1.044	0.999	1.017	0.932	1.073	-	1.664
	2.50	1.047	0.999	1.021	0.915	1.093	-	1.828

Moderate partial safety factors are obtained for long-term shrinkage strain, compared to those of model uncertainty, which remains the dominant variable. Figure 5.11 illustrates this trend.



MF = model uncertainty, esh = long-term shrinkage

Figure 5.11: *Theoretical partial safety factors of random variables including long-term shrinkage strain.*

The Pearson's correlation analysis, presented in Chapter 4, indicated that there is a moderate correlation between long-term shrinkage strain and model uncertainty. However, referring to Table 5.11, treating shrinkage strain as a random variable, as opposed as deterministic, does not significantly influence the model uncertainty partial safety factors.

5.4 CHAPTER SUMMARY

The sensitivity analysis using a reverse FORM was performed to assess the relative influences of the random variables of concrete cover, section thickness, liquid load, concrete tensile strength and model uncertainty on the MC 2010 crack with prediction model. Typical configurations of walls in rectangular and circular LRS were selected, as representative of flexural and tension load-induced cracking, respectively, where SLS cracking is the governing limit state. Loading of both short- and long-term duration was considered. The latter is the critical load case in the wall of a LRS under a quasi-permanent load due to water pressure. The sensitivity analysis of the reliability model was performed for the following load cases:

(i) Model 1: Flexural load-induced cracking

(ii) Model 2 (a): Tension load-induced cracking with $h_{c,eff}$ = 2,5 (c + φ/2)

(iii) Model 2 (b): Tension load-induced cracking with $h_{c,eff}$ = h/2

From the sensitivity analysis, it is found that the relative influences of the random variables are practically the same irrespective of the section geometry, load case and load duration. Section thickness has little influence on the crack width prediction model therefore it could be modelled as a deterministic parameter rather than as a random variable. Concrete cover and liquid load have a small negative influence on the model. Concrete tensile strength has some positive influence on the model. Model uncertainty is the dominant variable with sensitivity factors exceeding 0,9. This confirms the conclusion made from the quantification of model uncertainty that it is at least a significant variable, therefore should not be treated nominally in the reliability analyses.

The theoretical partial safety factors were determined for each of the random variables. In these analyses, shrinkage strain was treated deterministically. Following the results of the sensitivity analysis, the partial safety factors for each variable are reasonably consistent irrespective of the load case and duration. The limiting formulation for the effective tension thickness used in the calculation of crack widths for tension cracking does not affect the value of the partial safety factors to any significant degree. The partial safety factor for section thickness is close to 1,0, which implies this parameter can be modelled as a deterministic value. Concrete cover and liquid load have partial safety factors of approximately 1,05, whilst concrete tensile strength has partial safety factors up to about 1,2 as $1/\gamma_{fct}$, depending on the level of reliability. The dominant influence of model uncertainty is seen in its higher partial safety factors of up to 1,9, depending on the target reliability. It is surmised that a single partial safety factor scheme may be used for the crack width prediction model, irrespective of load duration and type.

Sensitivity analyses were also performed wherein long-term shrinkage strain was treated as a random variable using assumed values for its stochastic parameters. Long-term shrinkage strain has a moderate influence on the MC 2010 crack width prediction model with sensitivity factors between 0,3 and 0,4 when included in the model as a random variable. Moderate partial safety factors of about 1,1 for β 1,5 are obtained for shrinkage strain, increasing with increasing level of reliability. Model uncertainty partial safety factors decreased slightly compared to the analyses with shrinkage strain treated as a deterministic value. Further research is recommended to obtain a proper estimate of the stochastic parameters and associated influence of long-term shrinkage strain on the model.

The level of reliability chosen has a significant influence on the crack model and model uncertainty. Given the important of SLS cracking in the design of structures such as LRS, a full probabilistic optimisation approach is therefore appropriate to obtain the design formulation for the crack width prediction model, although this is outside the scope of this research.

CHAPTER 6

RELIABILITY ASSESSMENTS OF DESIGN FORMULATIONS

6.1 INTRODUCTION

The performance of the MC 2010 and BS 8007 design formulations is investigated using the First Order Reliability Method (FORM), whereby the reliability of each formulation is determined for representative liquid retaining structures (LRS) for long-term flexural loading. The General Probabilistic Model (GPM) for crack widths is based on best estimate predictions of MC 2010 while accounting for model uncertainty as quantified by the model factor derived for MC 2010. The comparison of the reliability performance of the two design formulations MC 2010 and BS 8007 gives useful insights into both models. The South African working group (WG) on the development of SANS 10100-3 Design of liquid retaining structures would find these insights particularly valuable as the WG considers the adoption of the MC2010 formulation, while BS 8007 is currently used by the South African industry.

Parameters that affect the predicted crack width and the uncertainty of such prediction (see Chapter 4) will influence the reliability performance of design formulations. Considering serviceability (SLS) design for crack widths using the MC 2010 crack width prediction model, the sensitivity analysis in Chapter 5 shows that the model uncertainty has the highest, even dominant, influence on reliability performance, while material and geometric variability contribute less to the need for design safety margins. The sensitivity analyses modelling long-term shrinkage strain as a random variable, as presented in Chapter 5, showed that this parameter does have an influence on the crack width prediction model. In addition, the reinforcement ratio was shown to have a strong correlation with model uncertainty, as presented in Chapter 4. Further reliability analyses were therefore done to investigate the effect of shrinkage strain and the relationship between the reinforcement ratio and model uncertainty on reliability performance.

6.2 FORMULATION OF THE RELIABILITY MODEL

The reliability analyses were performed using FORM following the algorithm set out in Chapter 3, Section 3.6, and utilising software Excel, as used in the reverse FORM analyses presented in Chapter 5.

6.2.1 Limit State Function of Reliability Model

The limit state function used in the FORM analyses presented in this chapter is the same as that used in the sensitivity analysis presented in Chapter 5, that is, Equation 5.1. The crack width limit is selected as a deterministic value of 0,2 mm, as the general value used to control cracking in LRS. As previously stated, this limit, defined as that at which autogenous healing occurs, will have some unknown conservatism. Using the results from the model uncertainty quantification analyses presented in Chapter 4, and considering long-term flexural cracking, model uncertainty is treated as a random variable with a mean of 1,1, a CoV of 0,38 and lognormal distribution. The MC 2010 crack model random variables of section thickness, concrete cover, liquid load and concrete tensile strength used in the FORM analyses are as Table 5.1 of Chapter 5, for long-term flexural cracking. The material properties are representative of those used in the design of South African LRS, as given in Table 5.1. The reinforcement areas used in these FORM analyses are as described in the following Section 6.2.2.

6.2.2 Selection of LRS Configurations

A range of representative LRS wall configurations is selected for the long-term flexural cracking case. The long-term tension load case is not considered as the parameters for model uncertainty in this case can only be estimated at this stage, requiring additional research. As described in Chapter 5 and illustrated by Figure 5.2, the 1 m wall length is under a quasi-permanent water pressure. The wall configurations are selected as described in Section 5.2 of Chapter 5, and include a wall height of 6 m in addition to the 5 and 7 m wall heights considered in the sensitivity analyses presented in Chapter 5. Representative LRS wall configurations with some degree of variation in the wall slenderness are also obtained, summarised in Table 6.1. Wall thicknesses compatible with the chosen wall height and reasonable reinforcement areas are obtained from ultimate limit state (ULS) design calculations for a LRS wall under bending due to liquid load, in addition to considerations such as wall slenderness. The ULS design is to SANS 10100-1 (2004) using a permanent load partial safety factor of 1,2 applied to the liquid load, as per SANS 10160 (2011). A summary of the ULS design is given in Appendix C.

Table 6.1: LRS Wall configurations for FORM analysis

Wall Section	H (m)	h (m)	H/h (m/m)
1	5	0.50	10.0
2	6	0.60	10.0
3	7	0.70	10.0
4	5	0.60	8.3
5	6	0.75	8.0
6	7	0.85	8.2

Wall sections 1 and 3 are the configurations used in the sensitivity analyses presented in Chapter 5, repeated here for comparison and consistency.

6.2.3 Description of FORM Analyses to Explore Reliability Performance

The reliability of the MC 2010 and BS 8007 crack formulations is investigated using FORM. Using the representative LRS wall configurations of Table 6.1, the SLS design reinforcement areas are determined to meet a crack width limit of 0,2 mm for the MC 2010 and BS 8007 design formulations, respectively. A summary of the SLS design analyses is given in Appendix C. The reliabilities of the two formulations are then determined using FORM.

As part of the investigation on the effect of long-term shrinkage strain on the reliability of the MC 2010 crack width prediction model, reliability analyses using forward FORM, first treating this parameter as a deterministic value and then as a random variable (as Chapter 5), are compared. As described in Chapter 5, the mean values considered for long-term shrinkage strain are chosen considering the SANS 10100 (2004) and EN 1992-1-1 shrinkage charts, and the experimental database. The design charts are assumed to utilise design values so have some degree of conservatism. In selecting the mean value, the design value is assumed to be the 95 percentile value. As the variation is not known, CoV's of 0,15, 0,20 and 0,25 are used. Although the stochastic values of shrinkage strain are assumed, these analyses are sufficient to be able to make some judgements on the effect of this parameter on reliability. To give a broader assessment of the influence of shrinkage strain, the analyses are performed for Wall Sections 1, 2 and 3. It should be noted that as this is a preliminary assessment of shrinkage strain to gain some insight into its influence on the reliability model, FORM analyses have not been performed for an extensive range of parameters.

The Pearson's correlation analyses presented in Chapter 4 showed that there is a strong correlation between model uncertainty of the MC 2010 crack formulation and the reinforcement ratio. The influence of this correlation on reliability is investigated over a range of reinforcement areas. This range is selected using SANS 10100-1 minimum and maximum reinforcement ratios as well as practical bar spacings, considering a high tensile bar diameter of 20 mm typical in LRS. The maximum SLS reinforcement area is determined by considering a minimum practical bar spacing of 75 mm, which equates to an area of 4189 mm^2 for a bar diameter of 20 mm. A maximum reinforcement spacing of 250 mm is selected as wider spacings are beyond the limit generally considered to control cracking. In this way, the range of reinforcement areas applicable to typical LRS is devised over which the reliability of the MC 2010 prediction model can be assessed. Design values for long-term shrinkage strain were used in calculating the SLS reinforcement required.

6.3 RESULTS AND DISCUSSION

The results of FORM analyses performed for typical LRS wall configurations gave insight into the following:

(i) The reliability of the MC 2010 and BS 8007 crack formulations.
(ii) Correlation between model uncertainty and the reinforcement ratio.
(iii) Influence of long-term shrinkage strain on the MC 2010 crack width prediction model.

6.3.1 Performance of the MC 2010 crack width model

From the FORM analyses, the performance of the MC 2010 crack width prediction model is assessed for the representative LRS wall configurations over a typical range of reinforcement ratios.

Plotting the reliability index against the SLS reinforcement area as $\%A_s$, as shown in Figure 6.2, it is noted that the slenderness of the wall (as H/h) influenced the reliability level, with reliability decreasing with increasing wall slenderness and reinforcement ratio. This is in part due to the relationship between the wall height (H), liquid load (L_k) and the bending moment (M) where $M \propto H^3$ and $L \propto H^2$ thus an increase in wall height will result in increases in the liquid load and bending moment, with a corresponding decrease in reliability for the same reinforcing ratio. Figure 6.1 illustrates this trend.

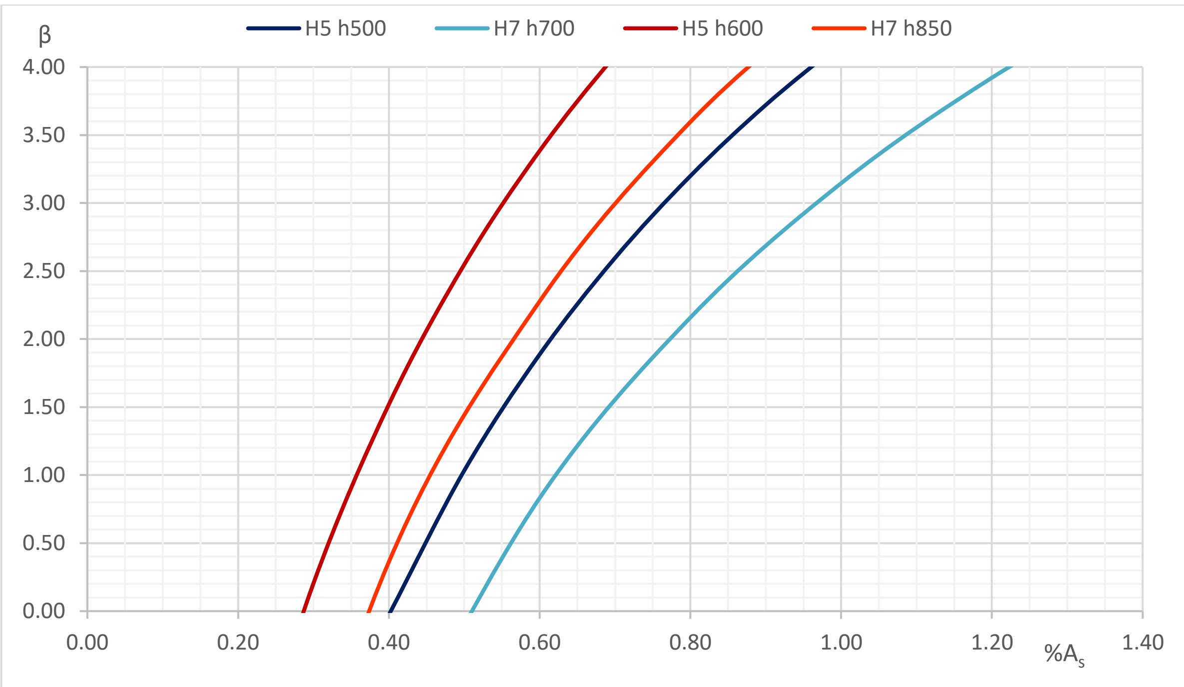


Figure 6.1: *Reliability of MC 2010 crack width prediction model*

For a given wall configuration, the reliability level increases with increasing area of reinforcement for a given configuration, as would be expected. From Figure 6.1, it is seen that the slope of the graphs decreases with increasing wall slenderness from about 8 to 10. This has economic implications for increasing the target reliability from β of 1,5 whereby the reinforcement required increases with increasing wall slenderness. In the experience of the researcher, concrete costs are higher than reinforcement costs. Therefore, an optimisation of the design LRS wall configurations is necessary to balance these costs and in obtaining a reasonable level of safety to costs of structure, to meet SLS cracking conditions.

Plotting the reliability index against the ratio of the SLS to ULS areas of reinforcement (A_{SLS}/ A_{ULS}) resulted in the graphs for each configuration falling in a relatively narrow band, as shown in Figure 6.2. This suggests that the reliability and associated A_{SLS}/ A_{ULS} ratio would be consistent for reasonable wall configurations and therefore the A_{SLS}/ A_{ULS} ratio could be used as the basis for calibration in the design of the wall. This would need to be confirmed for a wider range of configurations.

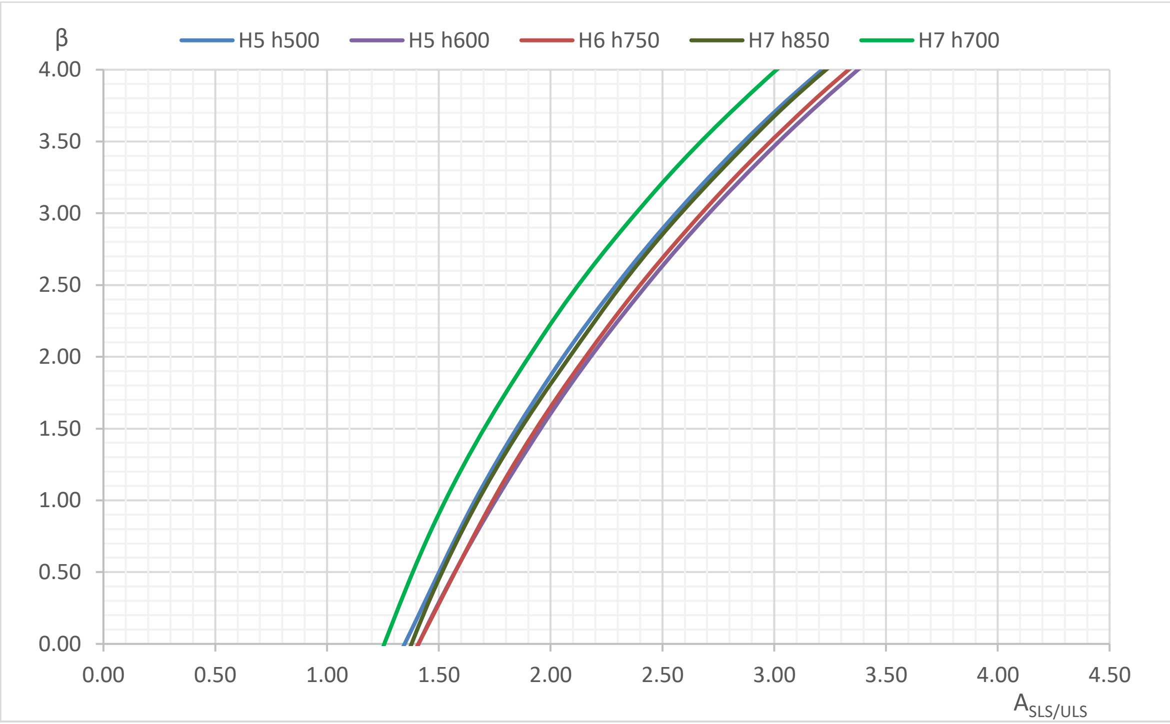


Figure 6.2: *Reliability of MC 2010 crack model using the A_{SLS}/ A_{ULS} ratio.*

6.3.2 Comparison of MC 2010 and BS 8007 design formulations

The reliability, measured by the reliability index, β, of the MC 2010 crack width prediction model is determined using FORM for each wall configuration using the design SLS reinforcement (A_{SLS}) required to meet a limiting crack width of 0,2 mm. These results are summarised in Table 6.2. The ULS reinforcement areas (A_{ULS}) and the SLS reinforcement ratios (%A_s) required to meet a target reliability β of 1,5 are included for comparison. The latter was determined in the sensitivity analyses, presented in Chapter 5, and may also be read off the graph of Figure 6.1. A deterministic value of -200 x 10^{-6} m/m was used for long-term shrinkage strain in FORM analysis, with a design value of -270 x 10^{-6} m/m used in calculating the design SLS reinforcement.

Table 6.2: *Reliability levels of MC 2010 formulation using design A_{SLS} for w = 0,2 mm*

LRS Wall Configurations				ULS Analysis	FORM using Design A_{SLS} for w = 0,2 mm				%A_s for β 1.5
Wall Section	H (m)	h (mm)	H/h	A_{ULS} (mm²)	%A_{SLS}	A_{SLS} (mm²)	A_{SLS}/ A_{ULS}	β	
1	5	500	10.0	1494	0.663	3316	2.22	2.53	0.55
2	6	600	10.0	2112	0.738	4426	2.10	2.27	0.62
3	7	700	10.0	2843	0.780	5459	1.92	2.04	0.69
4	5	600	8.3	1222	0.496	2976	2.44	2.51	0.40
5	6	750	8.0	1659	0.495	3715	2.24	2.18	0.43
6	7	850	8.2	2306	0.550	4678	2.03	1.87	0.51

Referring to Table 6.2, the reliability indexes obtained for the design SLS reinforcement to meet a 0,2 mm crack width using the MC 2010 crack formulation demonstrate that the MC 2010 design formulation is safe but conservative if aiming for a β of 1,5. The reliability indexes obtained are in the range suggested by Van Nierop (2018) of 2,0 to 2,5, for most wall configurations.

It is also noted that the reinforcement required to meet SLS cracking criteria compared to the ULS requirement is noteworthy, with A_{SLS}/A_{ULS} ratios mostly greater than 2.

For comparison between the performance of the MC 2010 and BS 8007 design formulations, FORM analyses using the GPM were performed using Wall Sections 1 to 3 (H/h of 10). The design SLS reinforcement required to meet a limiting crack width of 0,2 mm to BS 8007 was calculated for each wall section. The reliability index in each case was then determined for comparison with MC 2010. The results of these analyses are summarised in Table 6.3.

Table 6.3: *Comparison between BS 8007 & MC 2010 formulations using design A_{SLS} for w = 0,2 mm*

Wall Section				MC 2010		BS 8007	
Section	H (m)	H (mm)	H/h	%A_{SLS}	β	%A_{SLS}	β
1	5	500	10.0	0.663	2.53	0.470	0.74
2	6	600	10.0	0.738	2.27	0.496	0.41
3	7	700	10.0	0.780	2.04	0.543	0.36

BS 8007 predicts smaller reinforcement quantities than MC 2010, however, the associated reliability levels for these reinforcement areas do not meet the SLS target reliability of β 1,5. This implies that the BS 8007 design formulation is unconservative for long-term flexural cracking, which confirms the results from the quantification of model uncertainty. Referring to Table 4. 4, a model uncertainty mean of 1,5 obtained for BS 8007 indicates a significant bias in this model, which suggests that BS 8007 does underpredict crack widths for long-term flexural cracking. However, from industry experience, the BS 8007 crack width model performs satisfactorily in practice. It is surmised from the author's and industry experience that there is a conservative safety margin in the 0,2 mm crack width limit. As this parameter was modelled deterministically in the reliability model, a lower reliability would be returned for the BS 8007 crack model. This confirms the need for further research as to a suitable crack width limit, including probabilistic analysis. In assuming a lognormal distribution for model uncertainty in the GPM, the skewness of the pdf may be overestimated, thus reliability is underestimated. In addition, there may be conservative design assumptions in practice that add a safety margin that is difficult to quantify at this stage.

6.3.3 Influence of correlation between model uncertainty and reinforcement ratio

A strong correlation was found between the reinforcement ratio and model uncertainty, as discussed in Chapter 4, whereby model uncertainty bias apparently increases with increasing reinforcement ratio. This correlation is investigated here considering the range of reinforcement ratios for the typical LRS for flexural cracking, as shown in Figure 6.1.

Referring to Figure 6.1, the range of the reinforcement ratio used in the FORM analyses is approximately 0,3 to 1,0 for a reliability index range of about 1,0 to 3,0. To investigate the influence of this correlation, the scatterplot of Figure 4.9 (a) for the MC 2010 model uncertainty and reinforcement ratio over the full dataset of w_{exp}/ $w_{predict}$ ratios is replotted over the narrower range reinforcement ratios of 0,2 to 1,0, presented here as Figure 6.3, corresponding to the range used in the FORM analyses.

As is illustrated by Figure 6.3, the range of reinforcement ratios considered significantly influences the correlation between model uncertainty and the reinforcement ratio. The linear regression performed over this smaller range of reinforcement ratios indicates little correlation between the two parameters. The statistical parameters were determined for the subset of w_{exp}/ $w_{predict}$ ratios, illustrated by the box plot of Figure 6.4.

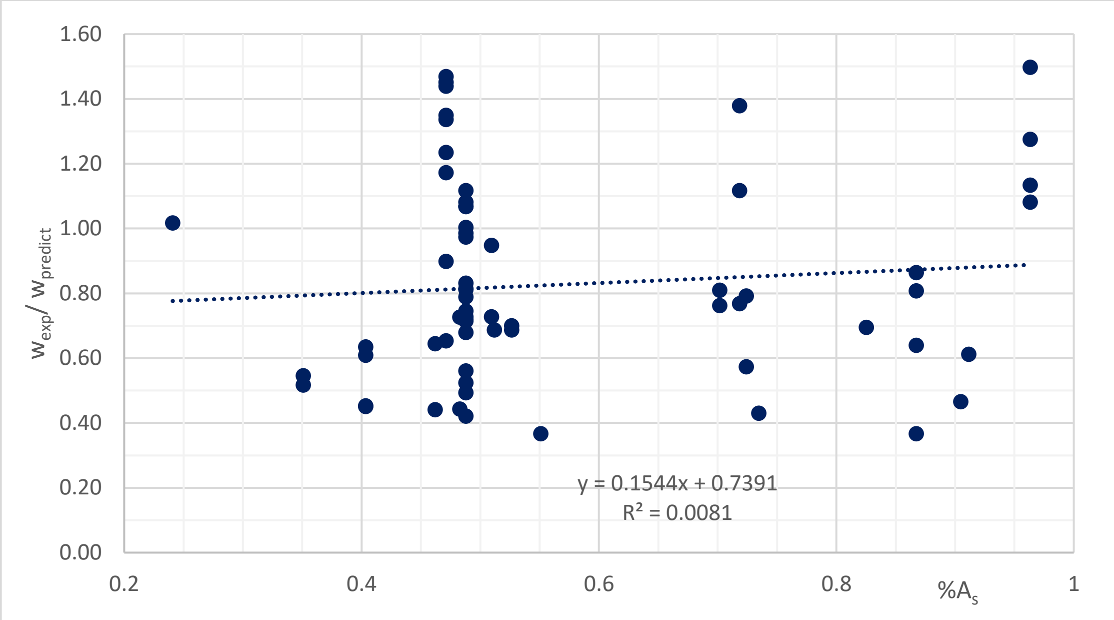


Figure 6.3: *Correlation of MC 2010 & reinforcement ratio for %A_s range of 0,25 to 1,0.*

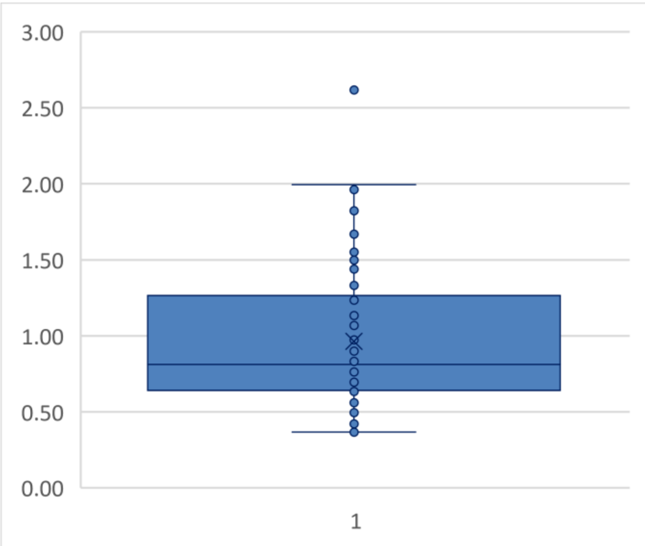


Figure 6.4: *Box plot for MC 2010 model uncertainty subset - %A_s range of 0,25 to 1,0*

The mean of model uncertainty (as w_{exp}/ $w_{predict}$) reduces from about 1,1 for the full dataset of w_{exp}/ $w_{predict}$ to about 0,96 for this subset of w_{exp}/ $w_{predict}$. In addition, and referring to Figure 6.3, it is noted that there is a large scatter in the w_{exp}/ $w_{predict}$ values over this range of reinforcement ratios, whereby the variation of model uncertainty apparently increases to 0,47 from 0,38. This illustrates the issue of the effect on reliability of a consistent model uncertainty over the range of design situations due to an interdependence with a model parameter. Reducing the model uncertainty mean results in an increase in reliability whilst increasing the variation causes a decrease in reliability. In this case, the bias, measured by the mean, shifts from negative (mean greater than 1 representing some underprediction of crack widths) to positive (mean less than 1 representing some overprediction of crack widths). To evaluate this issue further, FORM analyses were run using the model uncertainty statistical parameters obtained for the reduced reinforcement range. The results are summarised in Table 6.5. For the purposes of comparison, the FORM analyses were run assuming model uncertainty as the sole random variable. Referring to Table 6.5, a wall height of 5m with a 500 mm wall thickness and % A_s of 0,66 was used for a 0,2 mm crack width limit. To demonstrate the influence of the change in the mean and variation of model uncertainty, two analyses were run, the first with the new mean (S1), the second with both the new mean and variation (S2).

Table 6.4: *Influence of correlation between model uncertainty & reinforcement ratio on MC 2010 crack model.*

Range of %A_s	Model Uncertainty		FORM	
	Mean	CoV	β	γ_ϑ
Full database 0,2 to 2,2	1.10	0.38	2.52	2.19
Subset range 0,25 to 1,0: S1	0.96	0.38	2.88	2.36
S2	0.96	0.47	2.41	2.13

The approximately 15 % decrease in the mean results in a corresponding increase in reliability and the model uncertainty safety factor whereas the approximately 24 % increase in the variation results in a decrease of about 16 % in reliability and the model uncertainty safety factor. In this case, the nett result is a small decrease in reliability from a β of 2,52 to 2,41.

With some proper optimisation and considering real situations, it may therefore be possible to mitigate the influence of the reinforcement ratio on model uncertainty by calibrating the MC 2010 crack formulation over a range of design situations common to LRS.

6.3.4. Effect of Long-Term Shrinkage Strain on MC 2010 Crack Width Prediction Model

Long-term shrinkage strain treated as a random variable as part of the sensitivity analysis, as discussed in Chapter 5, was shown to have an influence on the performance of the MC 2010 crack width model. FORM analyses were thus performed to further assess the influence of long-term shrinkage strain on reliability. As discussed in Chapter 5, the stochastic parameters of long-term shrinkage strain were assumed as a first estimate of its effect on reliability. Long-term shrinkage strain was assumed to have a lognormal distribution and means of 200 x 10^{-6} and 400 x 10^{-6} m/m. Values of 0,15, 0,2 and 0,25 for the CoV were used to assess the extent of the effect of long-term shrinkage strain on the reliability of the

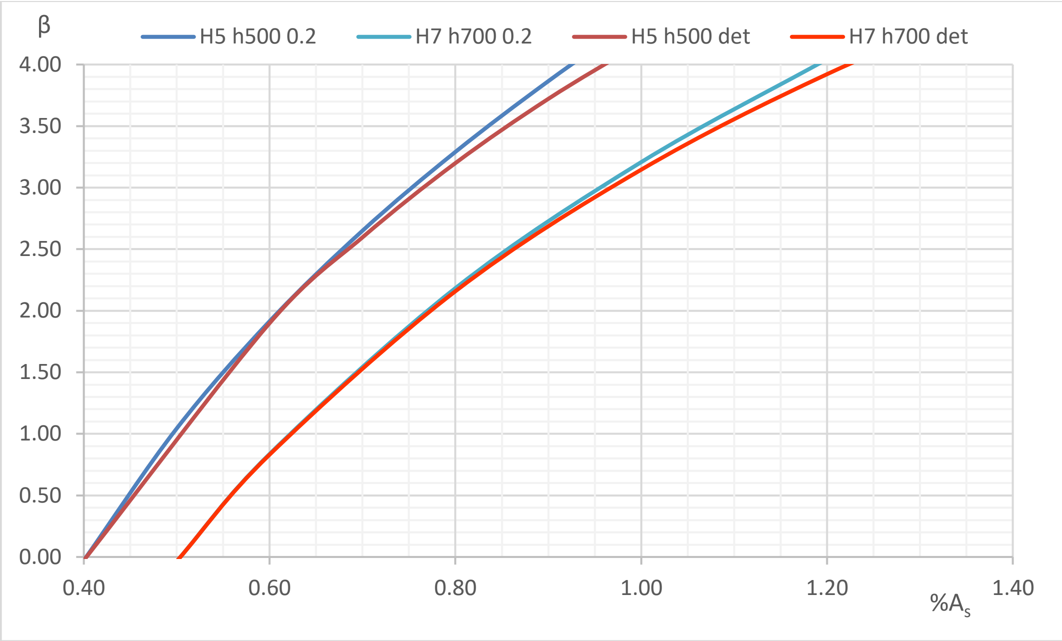
MC 2010 crack width model. Three LRS wall configurations were selected, namely, wall heights of 5, 6 and 7 m, all having a wall slenderness of 10. The reinforcement ratios determined from the SLS deterministic analysis for a crack width limit of 0.2 mm were used, as given in Table 6.2 of Section 6.3.2.

Considering a mean long-term shrinkage strain of -200×10^{-6}, the results show that the reliability of the MC 2010 crack width model is not significantly affected by long-term shrinkage strain below a β of about 2,5, as shown in Figure 6.2 (a). Above 2,5, reliability increases compared to when long-term shrinkage strain is modelled as a deterministic parameter. This increase in reliability increases with increasing β and with an increase in the mean of long-term shrinkage strain. A mean of -400×10^{-6} for shrinkage strain influences reliability above β 1,5, as shown in Figure 6.5 (b), plotted for wall heights of 5 m and 7 m with a shrinkage strain CoV of 0,2. Figure 6.5 is given overleaf.

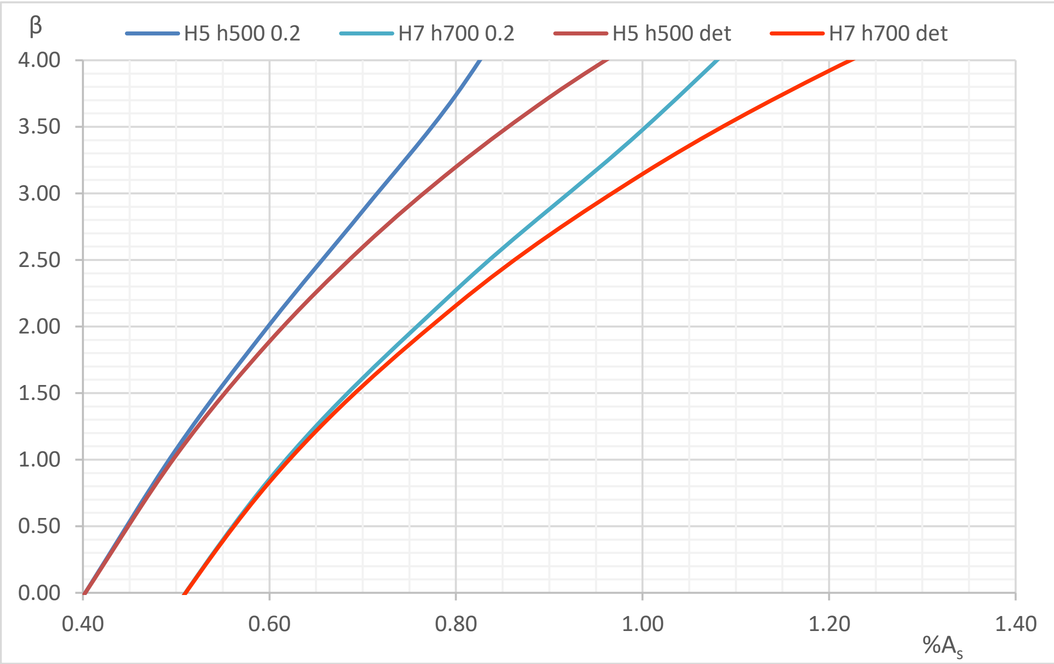
The implication of this finding is that at higher β values when shrinkage strain is treated deterministically in the reliability model, that is, for the same reliability, a larger reinforcement area is required than when shrinkage strain is modelled as a random variable. Increasing the variation of shrinkage strain increases the reliability index for a given reinforcement ratio as would be expected. However, the degree to which this increase occurs depends on the wall configuration, a higher wall resulting in a lesser increase in reliability. This in part due to the increasing influence of liquid load and the resulting steel stress, proportional to H^2 and H^3, respectively, with a corresponding decrease in the influence of shrinkage strain. Table 6.6 summarises the β values obtained for each wall configuration considering long-term shrinkage strain as a deterministic value (Det) and as a random variable with a lognormal distribution, mean of -200×10^{-6} and variations of 0,15, 0,2 and 0,25, respectively.

Table 6.5: *Influence of shrinkage strain (ε_{sh}) on reliability with changing variation.*

Wall Section					Reliability Index, β			
Section	H (m)	h (mm)	H/h	%As	ε_{sh} Det	ε_{sh} as random variable		
						CoV 0.15	CoV 0.20	CoV 0.25
1	5	500	10.0	0.663	2.35	2.35	2.39	2.41
2	6	600	10.0	0.738	2.27	2.30	2.31	2.34
3	7	700	10.0	0.780	2.04	2.06	2.07	2.09



(a) Long-term shrinkage strain -200 x 10^{-6}



(b) Long-term shrinkage strain -400 x 10-6

det – shrinkage strain modelled deterministically.
0.2 – shrinkage strain as random variable with CoV of 0,2.

Figure 6.5: *Influence of long-term shrinkage strain on reliability of MC 2010 crack model*

Whilst the true effect of long-term shrinkage strain is not known, there is enough evidence from this preliminary investigation to demonstrate that long-term shrinkage strain is influential on the reliability of the MC 2010 crack model, therefore should be modelled as a random variable. However, research is required to quantify its stochastic parameters.

6.4 CHAPTER SUMMARY AND CONCLUSIONS

The reliability analyses presented in this chapter were performed to assess model uncertainty implications on the reliability of the MC 2010 crack formulation, and to obtain insights into the reliability of the BS 8007 crack model, applied to representative LRS under long term flexural loading. The LRS wall geometry and related reinforcement quantities were chosen to first satisfy the ULS of bending, then SLS cracking for a practical range of reinforcement. The intention in this research is not to derive a design crack width formulation as this requires a full optimisation and calibration of the reliability model. However, the reliability analyses did uncover or confirm issues that would require attention in the derivation of the design model, and require further research. The conclusions that can be drawn are summarised.

The MC 2010 crack formulation appears to be safe but conservative, compared with a SLS target reliability of β 1,5, with reliability values between about 2,4 and 3,2, depending on the wall configuration. Hence, probabilistic calibration and optimisation procedures are recommended to develop a design formulation. Comparatively, the BS 8007 design formulation is shown to have a low reliability. This corresponds to the high bias obtained from the experimental database for the BS8007 crack width prediction model uncertainty, suggesting an underprediction of crack widths for long-term loading. Long-term shrinkage strain is not considered in this model. However, from industry experience, the BS 8007 crack width model performs satisfactorily in practice. It is surmised that there is a conservative safety margin in the 0,2 mm crack width limit. As this parameter was modelled deterministically in the reliability model, a lower reliability would be returned for the BS 8007 crack model than is actually the case. This confirms the need for further research as to a suitable crack width limit, including probabilistic analysis. In addition, there may be conservative design assumptions in practice that add a degree of safety that is difficult to quantify at this stage.

From the reliability analyses of the MC 2010 crack width model, it can be concluded that, providing a reasonable wall geometry and reinforcement area is chosen, the target reliability can be increased, by increasing the quantity of reinforcement without substantially increasing the costs of the structure. This has important implications given that the SLS target reliability of 1,5 is likely to be too low for situations where SLS cracking governs the design.

As concluded from the Pearson's correlation analyses between model uncertainty and crack model parameters for the experimental crack database, presented in Chapter 4, there is a strong correlation between model uncertainty and the reinforcement ratio. This correlation would cause a drift in the model bias, and therefore inconsistency in the reliability of the MC 2010 crack model. The range of reinforcement ratios of the typical design situations in LRS under bending selected for this study constitutes a subset of the range of the experimental database of Chapter 4. The correlation decreased over this smaller reinforcement ratio range. However, there was a decrease in the mean but an increase in the variation of model uncertainty. Therefore, FORM analyses were performed using these new stochastic parameters for model uncertainty, the nett result of which was a small decrease in reliability. It can be concluded that with some proper optimisation, it may be possible to mitigate the interdependence of the reinforcement ratio and model uncertainty in developing the design formulation and associated design partial safety factor scheme by considering an appropriate range of situations applicable to LRS.

Long-term shrinkage strain results in a higher reliability for the MC 2010 crack model when treated as a random variable in the reliability model at β above about 1,7, for a given reinforcement ratio and wall configuration, compared to when this parameter is modelled deterministically. This increase depends on the statistical parameters of long-term shrinkage strain and the wall configuration. Long-term shrinkage strain therefore does have at least some influence on reliability, and should be treated as random variable in the GPM. Further research is necessary to quantify the statistical parameters of long-term shrinkage strain and so determine its true effect on the reliability of the crack model.

CHAPTER 7

FINAL SUMMARY AND CONCLUSIONS

7.1 INTRODUCTION

A summary of the findings of the research is presented in this chapter. The research was conducted to investigate and quantify model uncertainty with respect to reinforced concrete crack models, together with the implications thereof on the reliability of design formulations utilising these crack models. Applications include the design of reinforced concrete liquid retaining structures (LRS) where cracking as a serviceability limit state, is the governing condition. The scope of the research covered load-induced cracking, specifically due to flexure and tension, respectively. In LRS, liquid loads are considered as quasi-permanent loads therefore the crack models were evaluated for loads of short- and long-term duration. Through the assessment of model uncertainty, the performances of the crack models were compared, leading to the selection of the MC 2010 crack model for a general probabilistic model. Probabilistic analyses were performed to assess the performance of this crack model.

7.2 LITERATURE REVIEW SUMMARY

The literature review was presented in two chapters, divided into a review of crack models and a review of reliability theory and model uncertainty related to load-induced cracking.

The literature review of research on crack models and related design standards relevant to the design of structures such as LRS was reported in Chapter 2. From this review, it was concluded that design crack formulations are likely to have a high degree of uncertainty in part due to the randomness of crack mechanisms and the range of parameters influencing cracking. Model uncertainty of the selected crack model would therefore be expected to have a high variability.

From the review of various design code formulations, those of EN 1992 and MC 2010 (which is also the proposed updated model for EN 1992) were chosen for analysis and comparison in establishing a General Probabilistic Model (GPM), as these were derived from an analytical basis, rather than an empirical one. In addition, MC 2010 represents some of the most up to date research on design crack width formulations. Research has shown that the EN 1992 crack formulation overestimates the influence of the concrete cover. The MC 2010 model reduces this by reducing the factor applied to the concrete cover. In addition, MC 2010 includes the effect of long-term shrinkage strain when determining crack widths for sustained loads, which most design standards do not. Design standards that neglect long-term shrinkage strain are found to tend to underpredict crack widths. The BS 8007 crack model was to be included for comparison as it is the model used by industry in South Africa.

The literature also showed that there is a lack of experimental research on long-term cracking. This would require further investigation in the quantification of model uncertainty.

Chapter 3 reported on the literature review of reliability and model uncertainty in general terms, including research on the reliability of crack models. From this, the key issues relating to model uncertainty in crack models could be identified, as well as confirm which models to consider for the GPM. In order to quantify model uncertainty, the sources of uncertainty in crack models were investigated and identified. The crack model parameters contributing to model uncertainty were treated as random variables in the reliability model. These are the parameters of section thickness, concrete cover, liquid load and concrete tensile strength. Their statistical parameters were established from the literature. Other parameters of the crack models were treated as deterministic values. Long-term shrinkage strain was identified as a potential source of uncertainty but little information was found on modelling it as a random variable.

The treatment of model uncertainty in probabilistic terms was investigated in developing the methodology for this research. From the literature on reliability, model uncertainty of the crack model, treated as a random variable in the reliability model, may be defined as the ratio of the experimental to predicted crack width, w_{exp}/ $w_{predict}$, and is specific to the selected model. Model uncertainty was expected to be an influential parameter in the reliability model, with a CoV in the region of 0,3 to 0,4, based on the available data on model uncertainty and the associated reliability of SLS crack models. However, this information is limited, especially with respect to long-term loading.

The literature review presented in Chapters 2 and 3 confirmed that a more rigorous treatment of model uncertainty of crack models was appropriate.

7.3 QUANTIFICATION OF MODEL UNCERTAINTY SUMMARY

Model uncertainty of the selected crack models was quantified for the crack models of EN 1992, MC 2010 (proposed amended EN 1992) and BS 8007, for both flexural and tension cracking over short- and long-term loading, as presented in Chapter 4, using the information gathered from the literature. The sources of experimental data on load-induced cracking were identified and assembled in a database, from which model uncertainty (defined as the ratio of the maximum experimental to predicted crack widths) could be determined stochastically for the selected crack models. From the literature review, It was concluded that flexural and tension cracking should be treated separately in all subsequent analyses of model uncertainty and reliability of the crack model, for both short- and long-term loads. Whilst EN 1992 and MC 2010 use one design crack width formulation, BS 8007 does not. In addition, the little information available on model uncertainty of crack models indicated that there may be differences in the statistical parameters of model uncertainty depending on the type of load-induced cracking.

Model uncertainty, defined as the ratio w_{exp}/ $w_{predict}$ using maximum crack widths, was found to have a variation greater than 0,3, irrespective of the crack model, indicating that this is a significant influence in the reliability model. The MC 2010 crack model had a low bias as indicated by the model uncertainty which had mean values between 1,0 and 1,1, a CoV of about 0,38 and approximately lognormal distribution. The statistical parameters obtained for the MC 2010 crack model uncertainty were relatively consistent for both short- and long-term loading. Applying a factor of 0,5 to the second term in the MC 2010 crack spacing equation for a flexural stress distribution resulted in a model uncertainty with a mean of about 1,6, demonstrating a high bias, together with an increased variation to about 0,41 (for short-term loading). For short-term loading, the EN 1992 and BS 8007 crack formulations have similar mean and variations to the MC 2010 model. However, as these formulations do not take shrinkage strain into account, they have a significant bias with a mean of about 1,5 for long-term loading, unlike MC 2010. The model uncertainty statistical parameters for long-term tension cracking could not be estimated to any reasonable degree due to the very small dataset in this case.

Correlation analyses using Pearson's coefficients and linear regression were done between model uncertainty and select crack model parameters, to aid in evaluating the performance of the crack models over the parameter ranges of the experimental database. The MC 2010 crack model uncertainty had low correlations with all parameters considered except the reinforcement ratio. As this correlation could affect the ability of this crack model to behave consistently over a range of structural situations, it was investigated further in the reliability analysis of the model. Long-term shrinkage strain was shown to have a moderate correlation with model uncertainty.

The performance of the EN 1992, MC 2010 and BS 8007 crack width formulations was assessed using the model uncertainty parameters determined for each crack model. Based on this, the MC 2010 (proposed EN 1992 crack model) was selected for the General Probabilistic Model as it performed reasonably well compared to the other crack models considered.

7.4 SUMMARY OF PROBABILISTIC ANALYSES

Probabilistic analyses were performed to assess the reliability performance of the MC 2010 crack model using the GPM. The First Order Reliability Method (FORM) was the reliability method used. Typical configurations of LRS walls under flexural loading due to a liquid load were selected as representative of SLS cracking where this is the governing design limit state. From the statistical analyses to determine model uncertainty, the model uncertainty of the GPM has an approximately lognormal distribution, with a mean of about 1,1 and CoV of about 0,38 for flexural cracking.

A sensitivity analysis was done in order to assess the significance of the random variables of the reliability crack model for both short- and long-term loading. The reliability of the MC 2010 crack model was also evaluated for long-term flexural cracking. The effect of the two equations to determine the limiting effective depth of the tension zone in the tension case was investigated by running sensitivity analyses for both conditions.

The South African working group on the development of SANS 10100-3 Design of liquid retaining structures is considering the adoption of the MC2010 formulation, while BS 8007 is currently used by the South African industry. Therefore, the reliability performance of the two design formulations MC 2010 and BS 8007 was investigated and compared using FORM and the GPM.

Identified in the analysis of model uncertainty as important parameters in the MC 2010 crack formulation, the influence of the reinforcement ratio and long-term shrinkage strain were investigated. As there is little information on the stochastic parameters of long-term shrinkage strain, this parameter was modelled as a deterministic value and then as a RV, assuming values for its mean, variation and probability distribution. To assess the influence of the reinforcement ratio on model uncertainty and the reliability of the MC 2010 crack model, FORM analyses were performed for the range of reinforcement ratios that would be suitable for the representative LRS wall configurations. Reinforcement area was treated deterministically as it has a small variation.

The probabilistic analyses are presented in Chapters 5 and 6.

7.5 CONCLUSIONS AND RECOMMENDATIONS

The conclusions of this research are given considering the objectives of this research. The main objective was to establish the model uncertainty of a General Probabilistic Model that would ultimately be used to develop the design crack model applicable to South African

LRS. The performance of the MC 2010 crack model was then investigated, including the significance of model uncertainty and the resulting implications on reliability.

7.5.1 Model Uncertainty of Crack Models

Model uncertainty was shown in all analyses to be a significant variable in all crack models. The sample size for long-term tension cracking was small with limited sample configurations which meant that only an estimate of the statistical parameters could be established. However, on comparing the model uncertainty statistical parameters for tension to those of flexural cracking, it was concluded that for the GPM, the model uncertainty parameters for tension would likely be in alignment with those for flexure, should a larger sample size and range of configurations become available.

Model uncertainty has a significant variation, found to be in the order of 0,38, as surmised from the literature review and given the random nature of the cracking mechanism. The sensitivity analysis of the random variables of the GPM applied to representative LRS wall configurations, presented in Chapter 5, confirmed the dominance of model uncertainty. The sensitivity factors of model uncertainty exceed 0,9, with all other variables having either negligible or small influence when shrinkage strain was modelled as a deterministic value. Liquid load and concrete cover displayed a small influence with sensitivities factors of about 0,2, while concrete tensile strength had a slightly higher influence with sensitivities factors of about 0,3. These trends are also evident in the theoretical partial safety factors where model uncertainty has partial safety factors of around 1,5 at a target reliability of 1,5, going up to about 1,8 for a target reliability of 2,5. Liquid load and concrete cover had partial safety factors of about 1,0 to 1,05, with concrete tensile strength having partial safety factors of about 1,05 to 1,1, as $1/\gamma_{fct}$. The model uncertainty partial safety factors are consistent for tension and flexural cracking, over both short- and long-term loading.

Model uncertainty of the MC 2010 crack model and the reinforcement ratio were found to have a strong interdependence in the correlation analyses, with a Pearson's correlation factor of about 0,6, as discussed in Chapter 4. The effect of this on reliability was investigated through FORM analyses using select LRS configurations under long-term flexure. The range of reinforcement ratios applicable to the representative LRS configurations chosen was more limited than that of the experimental database, as presented in Chapter 4. This resulted in an apparently lower correlation (correlation factor less than 0,1) between model uncertainty and the reinforcement ratio, suggesting that the MC 2010 crack model behaves consistently for the given design situation, namely, LRS, requiring further investigation. However, due to the correlation between model uncertainty and the reinforcement area, the model uncertainty mean drifts with an increasing reinforcement ratio over the range of the full database. Considering the narrow range of reinforcement ratios suitable for the representative LRS configurations chosen for this study, there is a decrease in the mean of model uncertainty from 1,1 to about 0,96 but an increase in the variation from 0,38 to about 0,47. On performing FORM analyses using the model uncertainty parameters obtained for the smaller reinforcement range, the decrease in the mean to below 1 resulted in an increase in reliability, but the increase in the variation decreased reliability with a resulting little change in reliability overall. Over the narrow range of reinforcement ratios considered, the results suggest that the strong correlation between this parameter and model uncertainty is mitigated. However, further investigation should be done for a greater range of LRS configurations and therefore potentially different reinforcement ratios, to establish the extent of the effect of the drift in the model uncertainty.

7.5.2 Performance of the MC 2010 and BS 8007 crack width formulations

The MC 2010 crack width formulation performed consistently for both short- and long-term load-induced cracking, which is important for ease of devising a design crack width formulation applicable to varying loading conditions. For tension cracking, there are two conditions in the determination of the effective area of the tensile zone, the limiting condition being the lesser value depending on the combination of section thickness, concrete cover and reinforcement diameter. Therefore, two reliability models were required for the tension load case. From the results of the reliability analyses, presented in Chapter 5, it can be concluded that the MC 2010 crack width formulation for tension cracking is relatively insensitive to whichever condition is limiting. This simplifies any design partial safety factor scheme as there is no need to differentiate between the two tension load formulations. The theoretical partial safety factors were found to be very similar for both flexural and tension cracking, suggesting that the same partial safety factor scheme could be used for all load types and durations in developing a design crack model.

FORM analyses were run in which long-term shrinkage strain was treated deterministically and then as a random variable with assumed stochastic parameters, for the MC 2010 crack width model. It was found that in the latter case, the reliability of the MC 2010 crack model was higher for a given wall geometry and reinforcement ratio for β greater than 1,7, depending on the value of long-term shrinkage strain. This difference increases as β increases. Shrinkage strain should therefore be modelled as a random variable as it does affect the reliability of the crack model, especially if the target reliability is to be increased from a value of β 1,5. Further research is required to establish the stochastic parameters of long-term shrinkage strain and the true effect on the reliability of the MC 2010 design crack width formulation.

The MC 2010 crack model was found to be conservative with reliability index values above β of 2,0 for the selected LRS configurations and using the design reinforcement area required to satisfy a 0,2 mm crack width limit using the MC 2010 design crack width formulation (the SLS reinforcement required was found to be at least twice that required for ULS). This suggests that there is unnecessary additional safety in the model that could be reduced on improvements to the model. Some of this additional safety could be explained in coming from sources such as, by necessity, the conservatively chosen fixed coefficients of the MC 2010 crack model, as well as the limiting crack width.

The GPM was used to assess the performance of the BS 8007 crack width formulation. The reliability of this crack model appears to be low for long-term flexural cracking, particularly when compared to MC 2010. However, from industry experience, the BS 8007 crack width model performs satisfactorily in practice. It is surmised that there is a substantial safety margin in the 0,2 mm crack width limit. There is some evidence to support this, as discussed in the literature review of Chapter 2. As the crack width limit was modelled deterministically in the reliability model, a lower reliability would be obtained than is actually the case for the BS 8007 crack model. This confirms the need for a probabilistic description of the crack width limit, as the crack width at which self-healing occurs within a reasonable time frame. In addition, there may be conservative design assumptions in practice that may add a safety margin that is difficult to quantify at this stage, that have not been taken into account in the evaluation of the BS 8007 crack width model.

The results of the analyses justify the use of probabilistic methods to assess SLS crack models and their associated model uncertainty in this research and motivate for the use of a full optimisation process to determine a design formulation. In this way, a safe but economical design scheme for SLS cracking could be devised, in particular when this is the governing limit state in design.

7.5.3 Target Reliability

From the reliability analyses, it was determined that the target reliability is an important factor in the performance of the crack model. Considering the FORM analyses, presented in Chapters 5 and 6, an increase in the target reliability results in a noteworthy increase in the reinforcement ratio for all load conditions and wall configurations. Target reliability for SLS has received nominal treatment by design standards to date, with a recommended target reliability level, β, of 1,5. However, this recommended value of 1,5 may not be sufficient for SLS cracking when this limit state is the limiting condition in the design of the structure. Given that an increase in reliability would result in increased material costs, a full probabilistic optimisation of the target reliability for SLS cracking is required, particularly when it is the governing condition in design.

7.5.4 Limitations of the Research

The reliability analysis was performed for a limited range of crack model parameters and structural configurations as the scope of the research does not extend to a detailed parametric analysis. This would form part of any future optimisation process, including all costs of safety, to establish the design crack width formulation suitable for use in LRS. However, parameters that were treated as deterministic values in the reliability analyses and may require further analysis as to whether they should be modelled as random variables were identified. These include bond stress, and the fixed coefficient, k_t, for tension stiffening strain. The results of this research should also be assessed against full scale structures.

7.5.5 Recommendations for Future Research

The study, through quantifying model uncertainty and in treating crack models probabilistically, resulted in some key issues identified for further study in improving the existing crack models that is beyond the scope of this research. A summary of these key issues is given.

Long term shrinkage strain is found to influence the reliability of the MC 2010 crack model, as previously discussed, however, research is required to determine its stochastic parameters. In quantifying model uncertainty, the long-term shrinkage strains used to determine mean strain were the experimental measured values from standard laboratory shrinkage tests, that is, the actual shrinkage strains with any experimental uncertainty incorporated in the overall model uncertainty. In practice, the shrinkage strains assumed at the design stage may vary considerably from those of the working structure which then results in an unknown increase in the model uncertainty. Structures such as reinforced concrete LRS mitigate some of this effect in their design. Concrete mixes used in LRS, for example, are such that shrinkage should be low. LRS are often buried, shielding them from low humidity, and usually filled with water which also mitigates drying shrinkage. The relationship between tensile and compressive creep, and between creep and long-term shrinkage requires further investigation in the formulation of tension stiffening models and in determining long-term crack widths.

It was determined that there is a lack of experimental data on members in direct tension for a range of configurations, especially long-term loading. Further research is therefore recommended to confirm the conclusion made in this research that for the MC 2010 crack model, model uncertainty for tension cracking has similar stochastic parameters to flexural cracking, and so refine the quantification of model uncertainty for tension cracking.

The crack models selected in this study were generally not developed considering the influence of factors such as transverse reinforcement (as discussed in Chapter 2). From the limited information available, there is some evidence to suggest transverse reinforcement does decease crack widths. This would add some conservatism to the model, the extent of which is not known. Further research is recommended particularly on members such as slabs and walls under flexure and/or or axial tension with transverse reinforcement.

The crack width limits considered in this research are the accepted values of 0,1 and 0,2 mm generally utilised in LRS. The crack width limit is modelled as a deterministic value in the probabilistic analyses. A wider range of values should be utilised in any future optimisation process. In the context of LRS, this crack width limit is determined as the crack width below which leakage will not occur. As such, the crack width limit has some unknown safety margin and is not a fixed value. A probabilistic description of the likelihood of self-healing within an acceptable time frame would aid in estimating this safety margin. The crack width limit could then be treated as a random variable in the GPM, improving the prediction of reliability of the crack width prediction. Further research is therefore needed.

Future research to improve the prediction of model uncertainty and the GPM could include finite element analysis modelling in addition to experimental work.

7.5.6 Final Comments

As a final note, models that describe complex mechanisms such as load-induced cracking in concrete have, by their nature, an inherently high model uncertainty compared to other structural models. Deterministic formulations thus, out of necessity, tend to have an unquantified conservatism to take this uncertainty into account. This research quantifies model uncertainty of select crack models and demonstrates that in treating these models probabilistically, the reliability performance of design provisions for load-induced cracking may be assessed and improved upon.

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APPENDIX A

EXPERIMENTAL DATA
USED IN
MODEL UNCERTAINTY QUANTIFICATION

Table A.1: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Attisha PhD (1972)	Attisha (1972)	Beams in flexure 4-point bending	13	A11	157	2740	305	152	261	35	2	1	19	567	210	2.44	21.80
				A12	226	2740	305	152	261	35	2	1	19	567	205	3.76	28.80
				A13	280	2740	305	152	261	35	2	1	19	567	205	3.24	30.20
				A14	310	2740	305	152	262	35	2	1	16	402	205	3.28	32.10
				A15	420	2740	305	152	262	35	2	1	16	402	207	2.84	33.40
				B11	153	2740	305	152	258	35	2	1	25	982	215	4.14	30.70
				B12	215	2740	305	152	259	35	2	1	22	760	198	3.94	27.30
				B15	480	2740	305	152	264	35	2	1	12	226	202	2.70	28.50
				B21	304	2740	305	152	278	35	2	1	25	982	215	2.96	28.38
				B22	403	2740	305	152	321	35	2	1	22	760	198	3.79	29.48
				B23	532	2740	305	152	379	35	2	1	19	567	200	3.79	27.10
				B24	659	2740	305	152	460	35	2	1	16	402	205	3.00	27.74
				B25	1080	2740	305	152	703	35	2	1	12	226	202	3.72	28.52

Table A.1 continued: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Chan (2012)	Chan (2012)	Beams all in 4 point bending Batch A - reinf Steel grade A No links in Max M zone slump 80mm	27	A7	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
				A8	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
				A9	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
				A10	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
				A11	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
				A12	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
		Batch B - reinf steel grade B No links in Max M zone slump 70mm		B7	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
				B8	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
				B9	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
				B10	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
				B11	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
				B12	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
		Batch C - reinf grade C No links in Max M zone slump 70 mm		C7	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08
				C8	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08
				C9	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08
				C10	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08
				C11	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08
				C12	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08
		Batch M - reinf grade M No links in max m zone slump 65mm		M1	524	1800	150	200	121	25	2	1	8	111	205	5.10	34.08
				M2	524	1800	150	200	121	25	2	1	8	111	205	5.10	34.08
				M3	524	1800	150	200	121	25	2	1	8	111	205	5.10	34.08
				M4	524	1800	150	200	121	25	2	1	8	111	205	5.10	34.08
				M5	524	1800	150	200	121	25	2	1	8	111	205	5.10	34.08
				M6	524	1800	150	200	121	25	2	1	8	111	205	5.10	34.08
		Beams with stirrups in M zone Averaged values for A,B and C respt, 6 beams each		A	550	1800	150	200	121	25	2	1	8	96	185	5.10	34.08
				B	523	1800	150	200	121	25	2	1	8	103	199	5.20	34.08
				C	537	1800	150	200	121	25	2	1	8	101	200	4.90	34.08

Table A.1 continued: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Clark (1956) UPM data	Caldentey (2016) Eckfeldt (2009)	Beams	34	31	310		152	304	134	12	2	1	10	157	200	2.90	30.86
				32	310		152	304	134	12	2	1	10	157	200	2.80	30.86
				33	310		152	203	134	12	2	1	10	157	200	2.50	30.86
				34	310		152	203	134	12	2	1	10	157	200	2.70	30.86
				35	310		152	190	134	12	2	1	10	157	200	2.80	30.86
				36	310		152	228	134	12	3	1	10	236	200	2.60	30.86
				37	310		152	228	134	12	3	1	10	236	200	2.40	30.86
				38	310		152	190	134	12	3	1	10	236	200	2.60	30.86
				39	276		152	190	133	12	2	1	13	265	200	2.50	30.86
				40	276		152	190	133	12	2	1	13	265	200	2.50	30.86
				41	310		152	190	133	12	2	1	13	265	200	2.70	30.86
				43	310		152	228	131	12	2	1	16	402	200	2.90	30.86
				50	310		152	190	131	12	2	1	16	402	200	2.40	30.86
				51	310		152	190	131	12	2	1	16	402	200	2.40	30.86
				C7	310		152	190	131	12	2	1	16	402	200	2.80	30.86
				C8	276		152	190	130	12	2	1	19	567	200	2.70	30.86
				C9	310		381	152	339	31	2	1	19	567	200	2.90	30.86
				C10	310		381	152	339	31	2	1	19	567	200	2.80	30.86
				C11	310		381	152	333	37	2	1	19	567	200	2.70	30.86
				C12	310		381	152	357	12	2	1	22	760	200	2.70	30.86
				M1	310		381	152	331	38	2	1	22	760	200	2.40	30.86
				M2	310		381	152	331	38	2	1	22	760	200	2.90	30.86
				M3	276		381	152	331	38	2	1	22	760	200	2.80	30.86
				M4	276		381	152	331	38	2	1	22	760	200	2.80	30.86
				M5	276		381	152	330	38	2	1	25	982	200	2.70	30.86
				M6	276		381	152	330	38	2	1	25	982	200	2.70	30.86
				A	276		381	152	344	22	2	1	29	1321	200	2.80	30.86
				B	276		381	152	328	38	2	1	29	1321	200	2.80	30.86
				C	207		381	152	328	38	2	1	29	1321	200	2.70	30.86
				31	276		381	152	328	38	2	1	29	1321	200	2.80	30.86
				32	276		381	152	328	38	2	1	29	1321	200	2.50	30.86
				33	276		584	152	531	38	2	1	29	1321	200	2.60	30.86
				34	276		584	152	531	38	2	1	29	1321	200	2.50	30.86
				35	207		584	152	528	38	2	2	35	1924	200	2.60	30.86

Table A.1 continued: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
CUR Report No.37 UPM data	Caldentey (2016) Eckfeldt (2009)	Beam	15	86	400		400	300	359	33	4	1	16	804	200	2.40	28
				87	400		400	300	349	43	4	1	16	804	200	2.40	28
				88	400		600	300	557	35	4	1	16	804	200	2.80	28
				89	250		600	300	547	45	4	1	16	804	200	2.60	28
				90	400		600	300	554	35	4	1	22	1521	200	2.70	28
				91	400		600	300	544	45	4	1	22	1521	200	2.60	28
				92	400		600	300	539	33	6	2	22	2281	200	2.20	28
				93	300		600	300	524	43	6	2	28	3695	200	2.00	28
		Slab		94	400		120	1000	101	15	10	1	8	503	200	1.80	28
				95	400		120	1000	99	15	5	1	12	565	200	2.40	28
				96	400		120	1000	89	25	5	1	12	565	200	2.40	28
				97	400		180	1000	154	18	5	1	16	1005	200	2.30	28
				98	400		180	1000	147	25	5	1	16	1005	200	2.20	28
				99	400		240	1000	214	18	10	1	16	2011	200	1.80	28
				100	400		240	1000	204	25	10	1	22	3801	200	2.90	28
Frosch	Frosch	Beams	4	B-6	379	3659	203	914	157	38	6	1	16	1206	200	3.30	34.41
				B-9	379	3659	203	914	157	38	4	1	16	804	200	3.30	34.42
				B-12	379	3659	203	914	157	38	3	1	16	603	200	3.47	35.09
				B-18	379	3659	203	914	157	38	2	1	16	402	200	3.43	34.92

Table A.1 continued: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Hognestad (1962) UPM data	Caldentey (2016) Eckfeldt (2009)	Beam	30	1	345		406	203	358	34	2	1	25	982	200	3.00	32
				2	276		406	203	360	34	4	2	19	1134	200	2.70	30
				3	345		406	203	345	34	8	2	13	1062	200	2.90	31
				4	345		406	203	332	34	15	3	10	1178	200	2.70	30
				5	345		406	203	360	34	4	2	19	1134	200	2.70	30
				6	345		406	203	358	34	2	1	25	982	200	2.50	29
				7	345		406	203	345	34	8	2	13	1062	200	2.40	28
				8	345		406	203	360	34	2	1	19	567	200	2.70	30
				9	345		406	203	345	34	4	1	13	531	200	2.70	30
				10	276		406	203	332	34	4	2	25	1963	200	2.60	29
				11	276		406	203	340	34	8	3	19	2268	200	3.00	31
				12	345		406	203	358	34	2	1	25	982	200	4.00	36
				13	345		406	203	360	34	4	2	19	1134	200	4.10	37
				14	345		406	203	345	34	8	2	13	1062	200	4.00	37
				15	345		406	203	332	34	15	3	10	1178	200	3.90	36
				16	345		406	304	345	34	12	2	13	1593	200	3.10	32
				17	345		406	203	345	34	8	2	13	1062	200	3.00	31
				18	345		406	152	345	34	6	2	13	796	200	2.90	31
				19	345		406	101	345	34	4	2	13	531	200	3.50	34
				20	345		609	203	543	34	6	2	19	1701	200	2.90	31
				21	345		406	203	360	34	4	2	19	1134	200	3.00	31
				22	345		304	203	259	34	3	1	19	851	200	2.90	31
				23	345		203	203	157	34	2	1	19	567	200	2.80	30
				24	345		406	203	386	9	2	1	22	760	200	2.80	30
				25	345		406	203	360	34	2	1	22	760	200	2.30	28
				26	345		406	203	322	73	2	1	22	760	200	2.60	29
				27	345		406	203	284	111	2	1	22	760	200	2.60	29
				28	345		406	203	360	34	2	1	22	760	200	2.90	31
				29	345		406	203	360	34	2	2	22	760	200	2.90	31
				30	345		406	203	386	9	2	1	22	760	200	2.70	30

Table A.1 continued: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Illston (1973)	Illston & Stevens 1973 Stevens (1973)	Beams A - D: Mild steel E - H: High tensile steel J - M: Very high tensile steel Ra, Rb: High tensile steel X-Z: High tensile steel Note: 2 No. each of A - Rb, except D - results already averaged	17	A	170		385	205	344	25	2	1	32	1608	200	2.77	32
				B	133		385	205	349	25	2	1	22	760	200	2.77	32
				C	171		385	205	318	51	2	1	32	1608	200	2.77	32
				D	129		385	205	323	51	2	1	22	760	200	2.77	32
				E	229		385	205	348	25	2	1	25	982	200	2.77	32
				F	220		385	205	351	25	2	1	19	567	200	2.77	32
				G	218		385	205	322	51	2	1	25	982	200	2.77	32
				H	213		385	205	325	51	2	1	19	567	200	2.77	32
				J	300		385	205	349	25	2	1	22	760	200	2.77	32
				K	274		385	205	352	25	2	1	16	402	200	2.77	32
				L	299		385	205	323	51	2	1	22	760	200	2.77	32
				M	281		385	205	326	51	2	1	16	402	200	2.77	32
				Ra	199		770	410	700	51	2	1	38	2268	200	2.77	32
				Rb	233		770	410	708	51	6	1	22	2281	200	2.77	32
				X	266		385	205	348	25	2	1	25	982	200	2.77	32
				Y	272		385	205	348	25	2	1	25	982	200	2.77	32
				Z	242		385	205	348	25	2	1	25	982	200	2.77	32
Kenel & Marti (2002)	Burns (2011)	One way spanning slabs 4-point bending test	2	B1	527	3600	200	1000	167	28	8	1	10	628	208	3.20	34
				B5	433	3600	200	1000	167	28	4	1	14	616	202	2.70	36
Krips (1984) UPM data	Caldentey (2016)	Slab	1	121	405		1000	180	965	35	4	1	16	804	200	3.30	33
Jaccoud & Favre (1982)	Burns (2011)	One way spanning slabs Transverse reinf Y08@200	3	C14	236	3100	160	750	131	23	5	1	12	565	205	2.80	30
				C24	236	3100	160	750	131	23	5	1	12	565	205	2.80	30
				C15	283	3100	160	750	131	23	5	1	12	565	205	2.80	30
Nejadi (2005)	Nejadi (2005) Gilbert & Nejadi (2004)	Beams B1 - B3 and slabs S1 - S3 4- point bending	12	B1-a	234	3500	348	250	300	40	2	1	16	402	200	2.00	23
				B1-b	160	3500	348	250	300	40	2	1	16	402	200	2.00	23
				B2-a	233	3500	333	250	300	25	2	1	16	402	200	2.00	23
				B2-b	158	3500	333	250	300	25	2	1	16	402	200	2.00	23
				B3-a	225	3500	333	250	300	25	3	1	16	603	200	2.00	23
				B3-b	135	3500	333	250	300	25	3	1	16	603	200	2.00	23
				S1-a	259	3500	161	400	130	25	2	1	12	226	200	2.00	23
				S1-b	201	3500	161	400	130	25	2	1	12	226	200	2.00	23
				S2-a	258	3500	161	400	130	25	3	1	12	339	200	2.00	23
				S2-b	178	3500	161	400	130	25	3	1	12	339	200	2.00	23
				S3-a	229	3500	161	400	130	25	4	1	12	452	200	2.00	23
				S3-b	169	3500	161	400	130	25	4	1	12	452	200	2.00	23

Table A.1 continued: *Experimental data - Short term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Rimkus (2017)	Rimkus (2017)	Beams	6	S1-1	373	3000	299	282	248	20	9	3	10	696	210	3.36	36
				S1-6	320	3000	303	271	217	20	12	3	8	603	210	3.00	34
				S2-1	380	3000	301	279	254	20	15	3	6	430	224	3.36	36
				S1-2	404	3000	300	284	273	20	5	1	14	777	211	3.36	36
				S1-4	355	3000	300	280	267	20	2	1	22	760	199	3.36	36
				S2-3	408	3000	300	282	272	20	3	1	14	466	211	3.18	35
Rusch & Reihm (1963) UPM data	Caldentey (2016) Eckfeldt (2009)	Beam	18	101	405		625	300	579	34	2	1	26	1062	200	2.70	30
				102	323		625	300	582	29	2	1	26	1062	200	2.70	30
				103	350		625	300	585	25	2	1	26	1062	200	2.70	30
				104	400		625	300	584	25	2	1	26	1062	200	2.70	30
				105	400		625	300	582	28	2	1	26	1062	200	2.70	30
				106	400		625	300	585	25	2	1	26	1062	200	2.70	30
				107	400		625	300	585	28	3	1	26	1593	200	2.70	30
				109	400		625	300	587	35	4	1	16	804	200	2.70	30
				110	450		625	300	587	35	4	1	16	804	200	2.70	30
				111	400		625	300	587	33	4	1	16	804	200	2.70	30
				112	450		625	300	587	36	4	1	16	804	200	2.70	30
				113	313		625	300	588	28	4	1	16	804	200	2.70	30
				114	450		625	300	580	29	4	1	26	2124	200	2.70	30
				115	400		1200	450	1157	29	5	1	26	2655	200	2.70	30
				117	450		625	200	584	25	2	1	32	1608	200	2.70	30
				118	300		625	200	584	25	2	1	32	1608	200	2.70	30
				119	350		625	300	587	35	4	1	16	804	200	2.70	30
				120	400		625	300	582	33	3	1	26	1593	200	2.70	30
Wu (2010)	Wu (2010)	Beams and slabs	8	BSTN2-16	318	2400	400	200	342	50	2	1	16	402	204	2.20	25
				BSTN3-16	317	2400	400	200	342	50	3	1	16	603	204	2.20	25
				BSTS2-16	371	2400	400	200	342	50	2	1	16	402	204	2.40	32
				BSTS3-16	317	2400	400	200	342	50	3	1	16	603	204	2.40	32
				SSTN4-12	422	2400	140	800	114	20	4	1	12	452	200	1.80	23
				SSTS4-12	442	2400	140	800	114	20	4	1	12	452	200	2.00	23
				SLTN4-12A	380	2400	140	800	114	20	4	1	12	452	200	1.80	23
				SLTN4-12B	295	2400	140	800	114	20	4	1	12	452	200	1.80	23

Table A.2: *Experimental data – Long term flexural cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Age at testing (days)	Span (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)	ε_{sh} x 10^{-6}
Attisha PhD (1972)	Attisha (1972)	Beams in flexure 4-point bending	5	B21	168	97	2740	305	152	258	35	2	1	25	982	215	3.12	7.75	-135
				B22	232	44	2740	305	152	259	35	2	1	22	760	198	3.39	8.54	-193
				B23	304	44	2740	305	152	261	35	2	1	19	567	200	2.79	8.24	-110
				B24	334	85	2740	305	152	262	35	2	1	16	402	205	2.96	8.43	-120
				B25	507	100	2740	305	152	264	35	2	1	12	226	202	3.15	9.05	-66
Ilston (1973)	Ilston & Stevens 1973 Stevens (1973)	Beams A - D: Mild steel E - H: High tensile steel J - M: Very high tensile steel Ra, Rb: High tensile steel X-Z: High tensile steel		A	183	365	5200	385	205	344	25	2	1	32	1608	200	2.71	10.67	-200
				B	141	365	5200	385	205	349	25	2	1	22	760	200	2.71	10.67	
				C	184	365	5200	385	205	318	51	2	1	32	1608	200	2.71	10.67	
				D	137	365	5200	385	205	323	51	2	1	22	760	200	2.71	10.67	
				E	245	730	5200	385	205	348	25	2	1	25	982	200	2.71	10.67	
				F	233	730	5200	385	205	351	25	2	1	19	567	200	2.71	10.67	
				G	233	730	5200	385	205	322	51	2	1	25	982	200	2.71	10.67	
				H	226	730	5200	385	205	325	51	2	1	19	567	200	2.71	10.67	
				J	319	730	5200	385	205	349	25	2	1	22	760	200	2.71	10.67	
				K	287	730	5200	385	205	352	25	2	1	16	402	200	2.71	10.67	
				L	318	730	5200	385	205	323	51	2	1	22	760	200	2.71	10.67	
				M	296	730	5200	385	205	326	51	2	1	16	402	200	2.71	10.67	
				Ra	210	730	5200	770	410	700	51	2	1	38	2268	200	2.71	10.67	
				Rb	699	365	5200	770	410	708	51	6	1	22	760	200	2.71	10.67	
Jaccoud & Favre (1982)	Burns (2011)	One way spanning slabs Transverse reinf Y08@200	6	C12	150	388	3100	160	750	131	23	5	1	12	565	205	2.80	9.06	-250
				C22	150	388	3100	160	750	131	23	5	1	12	565	205	2.80	9.06	
				C13	200	388	3100	160	750	131	23	5	1	12	565	205	2.80	9.06	
				C14	250	388	3100	160	750	131	23	5	1	12	565	205	2.80	9.06	
				C24	250	388	3100	160	750	131	23	5	1	12	565	205	2.80	9.06	
				C15	300	388	3100	160	750	131	23	5	1	12	565	205	2.80	9.06	
Nejadi (2005)	Nejadi (2005) Gilbert & Nejadi (2004)	Beams B1 - B3 and slabs S1 - S3 4- point bending	12	B1-a	234	400	3500	348	250	300	40	2	1	16	402	200	2.80	9.21	-825
				B1-b	160	400	3500	348	250	300	40	2	1	16	402	200	2.80	9.21	
				B2-a	233	400	3500	333	250	300	25	2	1	16	402	200	2.80	9.21	
				B2-b	158	400	3500	333	250	300	25	2	1	16	402	200	2.80	9.21	
				B3-a	225	400	3500	333	250	300	25	3	1	16	603	200	2.80	9.21	
				B3-b	135	400	3500	333	250	300	25	3	1	16	603	200	2.80	9.21	
				S1-a	259	400	3500	161	400	130	25	2	1	12	226	200	2.80	9.21	
				S1-b	201	400	3500	161	400	130	25	2	1	12	226	200	2.80	9.21	
				S2-a	258	400	3500	161	400	130	25	3	1	12	339	200	2.80	9.21	
				S2-b	178	400	3500	161	400	130	25	3	1	12	339	200	2.80	9.21	
				S3-a	229	400	3500	161	400	130	25	4	1	12	452	200	2.80	9.21	
				S3-b	169	400	3500	161	400	130	25	4	1	12	452	200	2.80	9.21	
Wu (2010)	Wu (2010)	Beams	2	SLTN4-12A	304	85	2400	140	800	114	20	4	1	12	452	200	2.20	9.24	-336
				SLTN4-12B	236	85	2400	140	800	114	20	4	1	12	452	200	2.20	9.24	

*Effective concrete modulus determined using the measured short-term modulus and creep factor
**Measured free shrinkage strain

Table A.3: *Experimental data – short term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm}	$E_{c,eff}$ (GPa)
Farra & Jaccoud (1993)	Farra & Jaccoud (1993)	Ties. 3 No. per Sample No. All reinforced with single bar	71	N10-10	441	1150	100	100	50	45	1	1	10	79	200	2.13	29.90
					427	1150	100	100	50	45	1	1	10	79	200	2.13	29.90
					413	1150	100	100	50	45	1	1	10	79	200	2.13	29.90
				N10-14	317	1150	100	100	50	43	1	1	14	154	200	2.13	29.90
					309	1150	100	100	50	43	1	1	14	154	200	2.13	29.90
				N10-20	219	1150	100	100	50	40	1	1	20	314	200	2.13	29.90
					212	1150	100	100	50	40	1	1	20	314	200	2.13	29.90
					222	1150	100	100	50	40	1	1	20	314	200	2.13	29.90
				N20-10	402	1150	100	100	50	45	1	1	10	79	200	2.47	29.10
					419	1150	100	100	50	45	1	1	10	79	200	2.47	29.10
					418	1150	100	100	50	45	1	1	10	79	200	2.47	29.10
				N20-14	314	1150	100	100	50	45	1	1	10	79	200	2.47	29.10
					299	1150	100	100	50	43	1	1	14	154	200	2.47	29.10
					312	1150	100	100	50	43	1	1	14	154	200	2.47	29.10
				N20-20	220	1150	100	100	50	43	1	1	14	154	200	2.47	29.10
					220	1150	100	100	50	40	1	1	20	314	200	2.47	29.10
					220	1150	100	100	50	40	1	1	20	314	200	2.47	29.10
				N30-10	422	1150	100	100	50	40	1	1	20	314	200	2.64	30.30
					447	1150	100	100	50	45	1	1	10	79	200	2.64	30.30
					447	1150	100	100	50	45	1	1	10	79	200	2.64	30.30
				N30-14	299	1150	100	100	50	43	1	1	14	154	200	2.64	30.30
					298	1150	100	100	50	43	1	1	14	154	200	2.64	30.30
					300	1150	100	100	50	43	1	1	14	154	200	2.64	30.30
				N30-20	221	1150	100	100	50	40	1	1	20	314	200	2.64	30.30
					220	1150	100	100	50	40	1	1	20	314	200	2.64	30.30
					220	1150	100	100	50	40	1	1	20	314	200	2.64	30.30
				N40-10	448	1150	100	100	50	45	1	1	10	79	200	2.81	30.30
					435	1150	100	100	50	45	1	1	10	79	200	2.81	30.30
					447	1150	100	100	50	45	1	1	10	79	200	2.81	30.30
				N40-14	325	1150	100	100	50	43	1	1	14	154	200	2.81	30.30
					300	1150	100	100	50	43	1	1	14	154	200	2.81	30.30
					301	1150	100	100	50	43	1	1	14	154	200	2.81	30.30
				N40-20	218	1150	100	100	50	40	1	1	20	314	200	2.81	30.30

					218	1150	100	100	50	40	1	1	20	314	200	2.81	30.30
					218	1150	100	100	50	40	1	1	20	314	200	2.81	30.30

Table A.3 continued: *Experimental data – short term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm}	$E_{c,eff}$ (GPa)
Farra & Jaccoud (1993)	Farra & Jaccoud (1993)	Ties. 3 No. per Sample No. All reinforced with single bar	71	N12-10	439	1150	100	100	50	45	1	1	10	79	200	2.6	29.00
					449	1150	100	100	50	45	1	1	10	79	200	2.6	29.00
					450	1150	100	100	50	45	1	1	10	79	200	2.6	29.00
				N12-14	292	1150	100	100	50	43	1	1	14	154	200	2.6	29.00
					326	1150	100	100	50	43	1	1	14	154	200	2.6	29.00
					326	1150	100	100	50	43	1	1	14	154	200	2.6	29.00
				N12-20	218	1150	100	100	50	40	1	1	20	314	200	2.6	29.00
					218	1150	100	100	50	40	1	1	20	314	200	2.6	29.00
					218	1150	100	100	50	40	1	1	20	314	200	2.6	29.00
				N22-10	442	1150	100	100	50	45	1	1	10	79	200	2.68	33.00
					441	1150	100	100	50	45	1	1	10	79	200	2.68	33.00
					438	1150	100	100	50	45	1	1	10	79	200	2.68	33.00
				N22-14	287	1150	100	100	50	43	1	1	14	154	200	2.68	33.00
					298	1150	100	100	50	43	1	1	14	154	200	2.68	33.00
					325	1150	100	100	50	43	1	1	14	154	200	2.68	33.00
				N22-20	220	1150	100	100	50	40	1	1	20	314	200	2.68	33.00
					220	1150	100	100	50	40	1	1	20	314	200	2.68	33.00
					221	1150	100	100	50	40	1	1	20	314	200	2.68	33.00
				N32-10	445	1150	100	100	50	45	1	1	10	79	200	2.98	31.20
					449	1150	100	100	50	45	1	1	10	79	200	2.98	31.20
					458	1150	100	100	50	45	1	1	10	79	200	2.98	31.20
				N32-14	301	1150	100	100	50	43	1	1	14	154	200	2.98	31.20
					327	1150	100	100	50	43	1	1	14	154	200	2.98	31.20
					308	1150	100	100	50	43	1	1	14	154	200	2.98	31.20
				N32-20	219	1150	100	100	50	40	1	1	20	314	200	2.98	31.20
					219	1150	100	100	50	40	1	1	20	314	200	2.98	31.20
					218	1150	100	100	50	40	1	1	20	314	200	2.98	31.20
				N42-10	453	1150	100	100	50	45	1	1	10	79	200	3.16	32.80
					478	1150	100	100	50	45	1	1	10	79	200	3.16	32.80
					448	1150	100	100	50	45	1	1	10	79	200	3.16	32.80
				N42-14	325	1150	100	100	50	43	1	1	14	154	200	3.16	32.80
					294	1150	100	100	50	43	1	1	14	154	200	3.16	32.80
					290	1150	100	100	50	43	1	1	14	154	200	3.16	32.80
				N42-20	223	1150	100	100	50	40	1	1	20	314	200	3.16	32.80

					223	1150	100	100	50	40	1	1	20	314	200	3.16	32.80
					220	1150	100	100	50	40	1	1	20	314	200	3.16	32.80

Table A.3 continued: *Experimental data – short term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm}	$E_{c,eff}$ (GPa)
Hartl (1977)	Caldentey (2016) Eckfeldt (2009)	Short ties reinforced with single bar	48	122	420		80	80	40	35	1	1	8	50	200	2.70	30.00
				123	300		80	80	40	34	1	1	12	113	200	2.70	30.00
				124	420		80	80	40	34	1	1	12	113	200	2.70	30.00
				125	300		80	80	40	34	1	1	12	113	200	2.70	30.00
				126	420		80	80	40	34	1	1	12	113	200	2.70	30.00
				127	300		80	80	40	34	1	1	12	113	200	2.70	30.00
				128	420		80	80	40	34	1	1	12	113	200	2.70	30.00
				129	300		80	80	40	30	1	1	18	254	200	2.70	30.00
				130	420		80	80	40	30	1	1	18	254	200	2.70	30.00
				131	300		80	80	40	30	1	1	18	254	200	2.70	30.00
				132	420		80	80	40	30	1	1	18	254	200	2.70	30.00
				133	300		80	80	40	30	1	1	18	254	200	2.70	30.00
				134	420		80	80	40	30	1	1	18	254	200	2.70	30.00
				135	300		80	80	40	28	1	1	24	452	200	2.70	30.00
				136	420		80	80	40	28	1	1	24	452	200	2.70	30.00
				137	300		80	80	40	28	1	1	24	452	200	2.70	30.00
				138	420		80	80	40	28	1	1	24	452	200	2.70	30.00
				139	300		80	80	40	28	1	1	24	452	200	2.70	30.00
				140	420		80	80	40	28	1	1	24	452	200	2.70	30.00
				141	300		80	80	40	34	1	1	12	113	200	3.50	33.94
				142	429		80	80	40	34	1	1	12	113	200	3.50	33.94
				143	300		80	80	40	34	1	1	12	113	200	3.50	33.94
				144	420		80	80	40	34	1	1	12	113	200	3.50	33.94
				145	300		80	80	40	30	1	1	18	254	200	3.50	33.94
				146	420		80	80	40	30	1	1	18	254	200	3.50	33.94
				147	300		80	80	40	30	1	1	18	254	200	3.50	33.94
				148	420		80	80	40	30	1	1	18	254	200	3.50	33.94
				149	300		80	80	40	30	1	1	18	254	200	3.50	33.94
				150	420		80	80	40	30	1	1	18	254	200	3.50	33.94

Table A.3 continued: *Experimental data – short term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Hartl (1977)	Caldentey (2016) Eckfeldt (2009)	Short ties reinforced with single bar		151	300		80	80	40	30	1	1	18	254	200	3.50	33.94
				152	420		80	80	40	30	1	1	18	254	200	3.50	33.94
				153	300		80	80	40	28	1	1	24	452	200	3.50	33.94
				154	420		80	80	40	28	1	1	24	452	200	3.50	33.94
				155	300		80	80	40	28	1	1	24	452	200	3.50	33.94
				156	420		80	80	40	28	1	1	24	452	200	3.50	33.94
				157	300		80	80	40	28	1	1	24	452	200	3.50	33.94
				158	420		80	80	40	28	1	1	24	452	200	3.50	33.94
				159	300		80	80	40	30	1	1	18	254	200	4.00	36.67
				160	420		80	80	40	30	1	1	18	254	200	4.00	36.67
				161	300		80	80	40	30	1	1	18	254	200	4.00	36.67
				162	420		80	80	40	30	1	1	18	254	200	4.00	36.67
				163	300		80	80	40	30	1	1	18	254	200	4.00	36.67
				164	420		80	80	40	30	1	1	18	254	200	4.00	36.67
				165	300		80	80	40	28	1	1	24	452	200	4.00	36.67
				166	420		80	80	40	28	1	1	24	452	200	4.00	36.67
				167	300		80	80	40	28	1	1	24	452	200	4.00	36.67
				168	420		80	80	40	28	1	1	24	452	200	4.00	36.67
				169	300		80	80	40	28	1	1	24	452	200	4.00	36.67

Table A.3 continued: *Experimental data – short term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Hwang (1983)	Hwang (1983)	Plates under axial tension	41	TIA	207	762	127	305	110	13	4	1	10	285	200	4.21	32.06
				TIB	207	762	127	305	102	19	4	1	13	507	200	4.21	31.23
				T2A	207	762	127	305	109	13	4	1	11	401	200	4.21	32.54
				T2B	207	762	127	305	102	19	4	1	11	401	200	4.34	33.09
				T3A	207	762	127	305	108	13	4	1	13	507	200	3.86	34.27
				T3B	207	762	127	305	102	19	4	1	13	507	200	4.21	31.99
				T4A	207	762	178	305	159	13	4	1	11	401	200	4.55	33.99
				T4B	207	762	178	305	153	19	4	1	11	401	200	4.69	33.99
				T5A	207	762	178	305	159	13	4	1	13	507	200	3.31	31.37
				T5B	207	762	178	305	152	19	4	1	13	507	200	3.79	32.27
				T6A	207	762	178	305	157	13	4	1	16	804	200	4.76	34.96
				T6B	207	762	178	305	151	19	4	1	16	804	200	4.83	34.61
				T7A	207	762	254	305	235	13	4	1	13	507	200	3.93	31.65
				T7B	207	762	254	305	229	19	4	1	13	507	200	4.96	31.72
				T8A	207	762	254	305	233	13	4	1	16	804	200	5.17	35.65
				T8B	207	762	254	305	227	19	4	1	16	804	200	4.96	34.96
				T9A	207	762	254	305	232	13	4	1	19	1140	200	4.90	34.40
				T9B	207	762	254	305	225	19	4	1	19	1140	200	4.83	35.58
				2A	207	762	178	305	153	19	4	1	11	401	200	4.57	32.30
				2B	207	762	178	305	153	19	4	1	11	401	200	4.29	34.67
				2C	207	762	178	305	153	19	4	1	11	401	200	4.53	34.77
				4A	207	762	178	305	153	19	4	1	11	401	200	4.87	32.58
				4B	207	762	178	305	153	19	4	1	11	401	200	4.47	32.56
				4C	207	762	178	305	153	19	4	1	11	401	200	4.39	31.82
				6A	207	762	178	305	153	19	4	1	11	401	200	4.51	28.91
				6B	207	762	178	305	153	19	4	1	11	401	200	4.59	33.00
				6C	207	762	178	305	153	19	4	1	11	401	200	4.08	33.85
				1	207	762	178	305	153	19	4	1	11	401	200	4.19	33.29
				2	207	762	178	305	153	19	4	1	11	401	200	4.43	35.00
				3	207	762	178	305	134	38	4	1	11	401	200	4.47	34.80
				4	207	762	178	305	134	38	4	1	11	401	200	4.33	34.14
				5	207	762	178	305	134	38	4	1	11	401	200	4.37	32.78
				6	207	762	178	305	134	38	4	1	11	401	200	4.92	35.27
				7	207	762	178	305	134	38	4	1	11	401	200	4.57	34.00

Table A.3 continued: *Experimental data – short term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Dia (mm)	A_s (mm^2)	E_s (GPa)	f_{ctm} (MPa)	$E_{c,eff}$ (GPa)
Wu	Wu (2010) Wu & Gilbert (2008)	Ties with single bar	7	STN12	442	1100	100	100	50	44	1	1	12	113	200	2.04	22.40
				STN16	522	1100	100	100	50	42	1	1	16	201	200	2.04	22.40
				STS12	442	1100	100	100	50	42	1	1	16	201	200	2.15	21.60
				STS16	448	1100	100	100	50	42	1	1	16	201	200	2.15	21.60
				LTN12A	354	1100	100	100	50	44	1	1	12	113	200	2.04	22.40
				LTN12B	177	1100	100	100	50	44	1	1	12	113	200	2.04	22.40
				LTN12C	354	1100	100	100	50	44	1	1	12	113	200	2.85	20.50

Table A.4: *Experimental data – long term tension cracking*

Researcher	Source	Element Description	No. of samples	Sample No.	Steel stress (MPa)	Age at testing (days)	Length (mm)	h (mm)	b (mm)	d (mm)	c (mm)	No. of bars	No. layers	Día (mm)	A_s (mm^2)	E_s GPa	f_{ctm} (MPa)	$E_{c,eff}$ * (GPa)	ε_{sh} ** x 10^{-6}
Eckfeldt (2008)	Eckfeldt (2008)	Ties	11	LDK1-1	350	72	3000	120	120	50	50	1	1	20	314	200	2.90	31.30	-390
				LDK1-2	350	72	3000	120	120	50	50	1	1	20	314	200	2.90	31.30	
				LDK2-1	350	62	3000	120	180	50	50	1	1	20	314	200	3.40	38.00	
				LDK2-2	350	78	3000	120	180	50	50	1	1	20	314	200	3.40	38.00	
				LDK3-1	350	50	3000	120	240	50	50	1	1	20	314	200	3.80	36.20	
				LDK3-2	350	57	3000	120	240	50	50	1	1	20	314	200	3.80	36.20	
				LDK4-1	350	98	3000	120	240	50	50	2	1	20	628	200	2.90	36.20	
				LDK4-2	350	98	3000	120	120	50	50	2	1	20	628	200	2.90	36.20	
				LDK5-1	350	117	3000	120	240	50	50	2	1	20	628	200	2.90	36.20	
				LDK5-2	350	117	3000	120	240	50	50	2	1	20	628	200	2.90	36.20	
				LDK6	350	138	3000	120	240	50	50	2	1	20	628	200	3.40	31.30	-410
Wu	Wu (2010) Wu & Gilbert (2008)	Ties with single bar	4	LTN12A	354	50	1100	100	100	50	44	1	1	12	113	200	2.04	23.60	-310
				LTN12B	177	50	1100	100	100	50	44	1	1	12	113	200	2.04	23.60	
				LTN12C	354	50	1100	100	100	50	44	1	1	12	113	200	2.04	23.60	
				LTN12D	177	50	1100	100	100	50	44	1	1	12	113	200	2.04	23.60	

*Effective concrete modulus determined using the measured short-term modulus and creep factor
**Measured free shrinkage strain

Table A.5: *Experimental & Predicted crack widths - Short term flexural cracking*

Researcher	Sample No.	No. of samples	Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm)				
				EN 1992	MC 2010 $k_2 = 1$	Proposed EN 1992 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Attisha PhD (1972)	A11	13	0.180	0.100	0.092	0.054	0.296	0.289
	A12		0.200	0.160	0.139	0.079	0.348	0.359
	A13		0.300	0.208	0.192	0.107	0.389	0.400
	A14		0.200	0.241	0.231	0.122	0.475	0.494
	A15		0.220	0.346	0.344	0.201	0.553	0.572
	B11		0.120	0.092	0.072	0.045	0.215	0.215
	B12		0.200	0.155	0.127	0.072	0.310	0.316
	B15		0.320	0.428	0.471	0.273	0.795	0.836
	B21		0.060	0.091	0.079	1.347	1.347	0.147
	B22		0.090	0.148	0.129	1.249	1.249	0.112
	B23		0.100	0.214	0.188	0.944	0.944	0.079
	B24		0.080	0.237	0.235	0.594	0.594	0.073
	B25		0.200	0.413	0.425	0.859	0.859	0.056
Chan PhD (2012)	A7	27	0.560	0.318	0.381	0.230	0.430	0.393
	A8		0.450	0.318	0.381	0.459	0.430	0.393
	A9		0.470	0.318	0.381	0.459	0.430	0.393
	A10		0.500	0.318	0.381	0.459	0.430	0.393
	A11		0.550	0.318	0.381	0.459	0.430	0.393
	A12		0.500	0.318	0.381	0.459	0.430	0.393
	B7		0.400	0.275	0.326	0.396	0.383	0.353
	B8		0.500	0.275	0.326	0.396	0.383	0.353
	B9		0.400	0.275	0.326	0.396	0.383	0.353
	B10		0.500	0.275	0.326	0.396	0.383	0.353
	B11		0.350	0.275	0.326	0.396	0.383	0.353
	B12		0.500	0.275	0.326	0.396	0.383	0.353
	C7		0.390	0.298	0.354	0.429	0.391	0.360
	C8		0.350	0.298	0.354	0.429	0.391	0.360
	C9		0.380	0.298	0.354	0.429	0.391	0.360
	C10		0.300	0.298	0.354	0.429	0.391	0.360
	C11		0.360	0.298	0.354	0.429	0.391	0.360
	C12		0.340	0.298	0.354	0.429	0.391	0.360
	M1		0.360	0.282	0.330	0.404	0.380	0.356
	M2		0.350	0.282	0.330	0.404	0.380	0.356
	M3		0.290	0.282	0.330	0.404	0.380	0.356
	M4		0.210	0.282	0.330	0.404	0.380	0.356
	M5		0.400	0.282	0.330	0.404	0.380	0.356
	M6		0.320	0.282	0.330	0.404	0.380	0.356
	A		0.471	0.318	0.381	0.459	0.430	0.393
	B		0.500	0.275	0.326	0.396	0.383	0.353
	C		0.353	0.298	0.354	0.429	0.391	0.360

Table A.5 continued: *Experimental & Predicted crack widths - Short term flexural cracking*

Researcher	Sample No.	No. of samples	Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm) EN 1992	MC 2010 $k_2 = 1$	Proposed EN 1992 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Clark (1956) UPM data	31	34	0.180	0.166	0.231	0.127	0.179	0.148
	32		0.210	0.166	0.231	0.127	0.144	0.148
	33		0.200	0.142	0.185	0.106	0.141	0.155
	34		0.180	0.138	0.180	0.103	0.141	0.155
	35		0.170	0.134	0.172	0.099	0.140	0.155
	36		0.130	0.125	0.153	0.091	0.118	0.134
	37		0.130	0.128	0.156	0.093	0.118	0.134
	38		0.180	0.116	0.136	0.083	0.110	0.126
	39		0.250	0.112	0.134	0.081	0.127	0.149
	40		0.170	0.112	0.135	0.081	0.127	0.149
	41		0.290	0.127	0.153	0.092	0.145	0.171
	43		0.210	0.127	0.151	0.091	0.156	0.190
	50		0.220	0.117	0.133	0.083	0.144	0.179
	51		0.310	0.117	0.132	0.083	0.144	0.179
	52		0.240	0.116	0.132	0.082	0.145	0.179
	56		0.180	0.094	0.101	0.065	0.125	0.160
	61		0.370	0.237	0.246	0.162	0.216	0.247
	62		0.230	0.238	0.247	0.163	0.216	0.247
	63		0.270	0.266	0.264	0.180	0.230	0.272
	64		0.190	0.120	0.136	0.085	0.173	0.187
	65		0.300	0.266	0.250	0.177	0.234	0.285
	66		0.240	0.261	0.246	0.174	0.234	0.285
	67		0.250	0.229	0.216	0.153	0.207	0.251
	68		0.340	0.229	0.215	0.152	0.207	0.251
	71		0.530	0.225	0.202	0.147	0.206	0.256
	72		0.280	0.225	0.202	0.148	0.206	0.256
	73		0.200	0.148	0.145	0.100	0.164	0.193
	74		0.290	0.217	0.186	0.141	0.204	0.261
	75		0.170	0.158	0.135	0.102	0.151	0.192
	76		0.260	0.217	0.186	0.141	0.204	0.261
	77		0.240	0.219	0.187	0.141	0.203	0.261
	81		0.320	0.245	0.236	0.164	0.213	0.243
	82		0.220	0.246	0.237	0.165	0.213	0.243
	83		0.220	0.168	0.152	0.110	0.144	0.169

Table A.5 continued: *Experimental & Predicted crack widths - Short term flexural cracking*

			Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm)				
Researcher	No. of samples	Sample No.		EN 1992	MC 2010 $k_2 = 1$	Proposed EN 1992 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
CUR Report No.37 UPM data	15	86	0.400	0.359	0.390	0.250	0.293	0.278
		87	0.350	0.417	0.425	0.284	0.285	0.332
		88	0.410	0.367	0.399	0.256	0.251	0.256
		89	0.210	0.233	0.251	0.162	0.157	0.155
		90	0.320	0.359	0.368	0.246	0.267	0.290
		91	0.460	0.444	0.448	0.302	0.307	0.342
		92	0.330	0.327	0.320	0.220	0.221	0.256
		93	0.270	0.270	0.227	0.174	0.202	0.253
		94	0.230	0.221	0.274	0.161	0.143	0.170
		95	0.230	0.256	0.336	0.191	0.184	0.227
		96	0.340	0.310	0.370	0.223	0.191	0.279
		97	0.310	0.305	0.394	0.226	0.249	0.294
		98	0.330	0.347	0.421	0.252	0.258	0.328
		99	0.300	0.249	0.292	0.178	0.205	0.235
		100	0.300	0.251	0.249	0.170	0.227	0.289
Frosch	4	B-6	0.508	0.346	0.370	0.470	0.395	0.442
		B-9	0.559	0.365	0.434	0.631	0.418	0.502
		B-12	0.660	0.417	0.528	0.794	0.424	0.551
		B-18	0.559	0.563	0.767	1.118	0.394	0.611
Hognestad (1962) UPM data	30	1	0.250	0.297	0.305	0.203	0.356	0.345
		2	0.250	0.201	0.185	0.133	0.153	0.180
		3	0.220	0.235	0.201	0.152	0.171	0.215
		4	0.250	0.220	0.173	0.139	0.167	0.225
		5	0.300	0.259	0.238	0.171	0.194	0.230
		6	0.360	0.302	0.310	0.207	0.292	0.346
		7	0.200	0.239	0.204	0.155	0.171	0.216
		8	0.220	0.322	0.359	0.226	0.288	0.326
		9	0.250	0.275	0.286	0.189	0.181	0.217
		10	0.230	0.192	0.162	0.124	0.154	0.217
		11	0.180	0.175	0.134	0.110	0.140	0.191
		12	0.280	0.285	0.295	0.196	0.294	0.345
		13	0.250	0.247	0.229	0.164	0.194	0.228
		14	0.230	0.225	0.194	0.146	0.171	0.213
		15	0.200	0.211	0.167	0.133	0.166	0.222
		16	0.270	0.233	0.200	0.151	0.171	0.215
		17	0.190	0.234	0.201	0.152	0.171	0.216
		18	0.250	0.235	0.201	0.152	0.172	0.216
		19	0.200	0.230	0.197	0.149	0.172	0.215
		20	0.270	0.258	0.237	0.170	0.178	0.209
		21	0.250	0.257	0.236	0.170	0.194	0.229
		22	0.200	0.258	0.238	0.171	0.213	0.267
		23	0.200	0.261	0.242	0.173	0.238	0.350
		24	0.190	0.125	0.155	0.092	0.242	0.251
		25	0.220	0.318	0.338	0.220	0.292	0.338
		26	0.280	0.510	0.453	0.333	0.402	0.552
		27	0.250	0.700	0.568	0.446	0.526	0.882
		28	0.180	0.308	0.330	0.214	0.304	0.350
		29	0.300	0.308	0.330	0.214	0.275	0.316
		30	0.290	0.126	0.156	0.092	0.255	0.264

Table A.5 continued: *Experimental & Predicted crack widths - Short term flexural cracking*

Researcher	Sample No.	No. of samples	Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm) EN 1992	MC 2010 $k_2 = 1$	Proposed EN 1992 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Ilston (1973)	A	17	0.100	0.991	0.986	0.069	0.090	0.074
	B		0.080	1.107	0.974	0.051	0.049	0.035
	C		0.150	0.907	1.067	0.107	0.167	0.118
	D		0.100	0.962	1.003	0.070	0.042	0.020
	E		0.180	1.167	1.068	0.108	0.126	0.108
	F		0.100	0.664	0.560	0.108	0.089	0.083
	G		0.230	1.028	1.117	0.148	0.199	0.152
	H		0.150	0.723	0.719	0.141	0.123	0.108
	J		0.180	0.812	0.715	0.157	0.151	0.135
	K		0.130	0.650	0.524	0.146	0.076	0.081
	L		0.250	0.757	0.789	0.221	0.254	0.202
	M		0.250	0.862	0.814	0.201	0.137	0.134
	Ra		0.250	0.960	0.813	0.187	0.189	0.196
	Rb		0.280	1.120	1.081	0.172	0.124	0.117
	X		0.150	0.814	0.745	0.129	0.135	0.115
	Y		0.150	0.795	0.727	0.132	0.138	0.118
	Z		0.150	0.908	0.831	0.115	0.119	0.101
Kenel & Marti (2002)	B1	2	0.600	0.408	0.574	0.297	0.356	0.424
	B5		0.450	0.427	0.162	0.326	0.392	0.586
Krips (1984) UPM data	121	1	0.350	0.157	0.501	0.201	0.221	0.202
Jaccoud & Favre (1982)	C14	3	0.200	0.134	0.162	0.097	0.150	0.193
	C24		0.190	0.134	0.202	0.097	0.150	0.193
	C15		0.270	0.167	0.178	0.121	0.189	0.254
Nejadi (2005)	B1-a	12	0.130	0.214	0.244	0.152	0.196	0.159
	B1-b		0.050	0.111	0.152	0.095	0.114	0.081
	B2-a		0.100	0.173	0.215	0.127	0.176	0.161
	B2-b		0.050	0.090	0.130	0.077	0.102	0.082
	B3-a		0.080	0.143	0.161	0.101	0.127	0.127
	B3-b		0.050	0.068	0.077	0.048	0.068	0.062
	S1-a		0.130	0.180	0.231	0.134	0.178	0.179
	S1-b		0.080	0.112	0.178	0.104	0.121	0.106
	S2-a		0.130	0.165	0.190	0.117	0.167	0.203
	S2-b		0.080	0.094	0.109	0.067	0.104	0.116
	S3-a		0.100	0.128	0.137	0.089	0.135	0.161
	S3-b		0.080	0.083	0.089	0.058	0.093	0.106
Rimkus (2017)	S1-1	6	0.124	0.171	0.178	0.147	0.152	0.166
	S1-6		0.060	0.136	0.139	0.122	0.142	0.160
	S2-1		0.102	0.142	0.149	0.141	0.118	0.137
	S1-2		0.142	0.212	0.232	0.177	0.193	0.207
	S1-4		0.140	0.247	0.301	0.221	0.369	0.400
	S2-3		0.120	0.264	0.327	0.420	0.275	0.311

Table A.5 continued: *Experimental & Predicted crack widths - Short term flexural cracking*

Researcher	Sample No.	No. of samples	Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm) EN 1992	MC 2010 $k_2 = 1$	Proposed EN 1992 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Rusch & Reihm (1963) UPM data	101	18	0.650	0.455	0.517	0.316	0.538	0.504
	102		0.350	0.318	0.365	0.220	0.363	0.372
	103		0.430	0.319	0.373	0.223	0.394	0.404
	104		0.500	0.375	0.443	0.265	0.460	0.476
	105		0.670	0.401	0.468	0.282	0.464	0.483
	106		0.450	0.371	0.436	0.261	0.460	0.475
	107		0.550	0.329	0.344	0.222	0.305	0.324
	109		0.500	0.368	0.377	0.247	0.250	0.250
	110		0.400	0.422	0.434	0.284	0.288	0.290
	111		0.510	0.356	0.371	0.240	0.243	0.243
	112		0.600	0.428	0.438	0.288	0.291	0.294
	113		0.300	0.243	0.258	0.163	0.169	0.163
	114		0.600	0.357	0.357	0.239	0.279	0.306
	115		0.300	0.331	0.341	0.223	0.262	0.255
	117		0.400	0.331	0.342	0.223	0.380	0.414
	118		0.300	0.215	0.219	0.143	0.248	0.268
	119		0.300	0.315	0.321	0.210	0.212	0.209
	120		0.600	0.369	0.378	0.248	0.319	0.342
Wu (2010)	BSTN2-16	8	0.15	0.363	0.385	0.226	0.270	0.221
	BSTN3-16		0.15	0.333	0.315	0.202	0.266	0.246
	BSTS2-16		0.15	0.420	0.508	0.326	0.320	0.269
	BSTS3-16		0.18	0.316	0.346	0.240	0.263	0.243
	SSTN4-12		0.25	0.345	0.441	0.244	0.284	0.217
	SSTS4-12		0.28	0.324	0.481	0.291	0.300	0.231
	SLTN4-12A		0.175	0.302	0.387	0.212	0.190	0.138
	SLTN4-12B		0.125	0.216	0.277	0.149	0.136	0.092

Table A.6: *Experimental & Predicted crack widths - Long term flexural cracking*

			Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm)				
Researcher	No. of samples	Sample No.		EN 1992	MC 2010 $k_2 = 1$	Proposed EN 1992 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS 8007 w = 0.1 mm
Attisha PhD (1972)	5	B21	0.060	0.106	0.083	0.066	0.272	0.274
		B22	0.090	0.178	0.139	0.107	0.366	0.371
		B23	0.100	0.233	0.191	0.142	0.465	0.473
		B24	0.080	0.265	0.244	0.173	0.543	0.559
		B25	0.200	0.455	0.447	0.295	0.867	0.902
Ilston (1973)		A	0.230	0.098	0.115	0.082	0.096	0.094
		B	0.130	0.079	0.120	0.077	0.071	0.065
		C	0.310	0.169	0.170	0.135	0.212	0.207
		D	0.230	0.118	0.154	0.111	0.130	0.119
		E	0.250	0.153	0.188	0.125	0.132	0.127
		F	0.250	0.162	0.224	0.140	0.124	0.115
		G	0.380	0.233	0.245	0.183	0.248	0.239
		H	0.380	0.233	0.276	0.192	0.217	0.201
		J	0.310	0.221	0.273	0.176	0.176	0.170
		K	0.230	0.223	0.316	0.191	0.153	0.140
		L	0.460	0.345	0.361	0.261	0.333	0.322
		M	0.380	0.334	0.401	0.268	0.276	0.252
		Ra	0.310	0.285	0.403	0.252	0.293	0.269
		Rb	0.330	1.592	0.324	0.222	0.199	0.186
Jaccoud & Favre (1982)	6	C12	0.110	0.105	0.122	0.088	0.082	0.100
		C22	0.080	0.105	0.122	0.088	0.082	0.100
		C13	0.250	0.147	0.172	0.118	0.122	0.165
		C14	0.320	0.190	0.222	0.149	0.037	0.028
		C24	0.300	0.190	0.222	0.149	0.162	0.229
		C15	0.400	0.233	0.272	0.180	0.203	0.294
Nejadi (2005)	12	B1-a	0.380	0.211	0.477	0.324	0.213	0.185
		B1-b	0.180	0.130	0.366	0.254	0.128	0.100
		B2-a	0.360	0.167	0.409	0.262	0.187	0.165
		B2-b	0.180	0.100	0.314	0.204	0.112	0.090
		B3-a	0.280	0.135	0.289	0.195	0.137	0.128
		B3-b	0.130	0.065	0.212	0.145	0.075	0.065
		S1-a	0.250	0.181	0.453	0.291	0.189	0.156
		S1-b	0.200	0.137	0.371	0.243	0.132	0.099
		S2-a	0.230	0.161	0.337	0.225	0.179	0.162
		S2-b	0.180	0.092	0.260	0.177	0.113	0.095
		S3-a	0.200	0.124	0.251	0.176	0.147	0.135
		S3-b	0.150	0.080	0.206	0.146	0.102	0.091
Wu (2010)	2	SLTN4-12A	0.300	0.303	0.473	0.230	0.190	0.138
		SLTN4-12B	0.225	0.221	0.370	0.149	0.136	0.092

Table A.7: *Experimental & Predicted crack widths - Short term tension cracking*

				Test crack width w_{exp} (mm)	Predicted Crack Width, $w_{predict}$ (mm)			
Researcher	Element Description	No. of samples	Sample No.		EN 1992	MC 2010	BS 8007 w0.2	BS 8007 w0.1
Farra & Jaccoud (1993)	Ties. 3 No. per Sample No. All reinforced with single bar	71	N10-10	0.390	0.649	0.468	0.31	0.276
				0.360	0.623	0.450	0.30	0.264
				0.390	0.597	0.431	0.29	0.251
			N10-14	0.340	0.401	0.283	0.22	0.203
				0.350	0.389	0.275	0.21	0.197
			N10-20	0.190	0.232	0.160	0.14	0.131
				0.200	0.224	0.154	0.13	0.126
				0.170	0.236	0.163	0.14	0.133
			N20-10	0.360	0.549	0.397	0.28	0.242
				0.310	0.581	0.420	0.29	0.257
				0.380	0.579	0.418	0.29	0.256
			N20-14	0.220	0.387	0.279	0.20	0.164
				0.280	0.362	0.255	0.21	0.189
				0.340	0.382	0.269	0.22	0.199
			N20-20	0.190	0.243	0.172	0.14	0.125
				0.190	0.227	0.157	0.14	0.131
				0.185	0.227	0.157	0.14	0.131
			N30-10	0.320	0.472	0.325	0.28	0.273
				0.400	0.620	0.448	0.32	0.281
				0.340	0.620	0.448	0.32	0.281
			N30-14	0.330	0.357	0.252	0.21	0.189
				0.310	0.355	0.251	0.21	0.188
				0.340	0.358	0.253	0.21	0.190
			N30-20	0.180	0.226	0.156	0.14	0.132
				0.200	0.225	0.155	0.14	0.131
				0.200	0.225	0.155	0.14	0.131
			N40-10	0.400	0.609	0.440	0.32	0.282
				0.390	0.585	0.422	0.31	0.271
				0.320	0.607	0.438	0.32	0.281
			N40-14	0.350	0.390	0.275	0.23	0.210
				0.350	0.352	0.249	0.21	0.190
				0.260	0.354	0.250	0.21	0.191
			N40-20	0.240	0.220	0.152	0.14	0.130
				0.220	0.220	0.152	0.14	0.130
				0.220	0.220	0.152	0.14	0.130
			N12-10	0.300	0.608	0.439	0.31	0.274
				0.370	0.626	0.452	0.32	0.283
				0.390	0.628	0.454	0.32	0.284
			N12-14	0.310	0.347	0.245	0.20	0.183
				0.390	0.398	0.281	0.23	0.211
				0.340	0.398	0.281	0.23	0.211
			N12-20	0.200	0.223	0.154	0.14	0.130
				0.200	0.223	0.154	0.14	0.130
				0.220	0.223	0.154	0.14	0.130
			N22-10	0.340	0.609	0.440	0.31	0.276
				0.400	0.608	0.439	0.30	0.264
				0.330	0.602	0.435	0.29	0.251
			N22-14	0.280	0.338	0.239	0.22	0.203
				0.240	0.355	0.251	0.21	0.197
				0.390	0.396	0.279	0.14	0.131
			N22-20	0.220	0.225	0.156	0.13	0.126
				0.240	0.225	0.156	0.14	0.133
				0.180	0.227	0.156	0.28	0.242

Table A.7 continued: *Experimental & Predicted crack widths - Short term tension cracking*

Researcher	Element Description	No. of samples	Sample No.	Test crack width w_{exp} (mm)	Predicted Crack Width, $w_{predict}$ (mm)			
					EN 1992	MC 2010	BS 8007 w0.2	BS 8007 w0.1
Farra & Jaccoud (1993)	Ties. 3 No. per Sample No. All reinforced with single bar		N32-10	0.300	0.591	0.427	0.31	0.277
				0.350	0.598	0.432	0.31	0.276
				0.360	0.615	0.444	0.31	0.273
			N32-14	0.290	0.348	0.246	0.20	0.179
				0.300	0.387	0.273	0.21	0.188
				0.270	0.359	0.253	0.23	0.210
			N32-20	0.210	0.219	0.151	0.14	0.131
				0.230	0.219	0.151	0.14	0.131
				0.300	0.217	0.150	0.14	0.132
			N42-10	0.340	0.592	0.428	0.32	0.279
				0.350	0.639	0.461	0.32	0.283
				0.480	0.583	0.421	0.33	0.291
			N42-14	0.320	0.379	0.267	0.21	0.191
				0.260	0.332	0.234	0.23	0.212
				0.260	0.326	0.230	0.21	0.196
			N42-20	0.220	0.221	0.153	0.14	0.131
				0.230	0.221	0.153	0.14	0.131
				0.220	0.218	0.150	0.14	0.130
Hartl (1977)	Short ties reinforced with single bar	48	122	0.270	0.447	0.324	0.264	0.231
			123	0.220	0.281	0.198	0.199	0.185
			124	0.310	0.420	0.295	0.199	0.185
			125	0.220	0.281	0.198	0.199	0.185
			126	0.310	0.420	0.295	0.290	0.276
			127	0.170	0.281	0.198	0.290	0.276
			128	0.250	0.420	0.295	0.290	0.276
			129	0.190	0.241	0.165	0.202	0.196
			130	0.280	0.348	0.239	0.202	0.196
			131	0.200	0.241	0.165	0.202	0.196
			132	0.300	0.348	0.239	0.288	0.282
			133	0.190	0.241	0.165	0.288	0.282
			134	0.290	0.348	0.239	0.288	0.282
			135	0.140	0.212	0.143	0.194	0.191
			136	0.200	0.304	0.205	0.194	0.191
			137	0.190	0.212	0.143	0.194	0.191
			138	0.270	0.304	0.205	0.274	0.271
			139	0.180	0.212	0.143	0.274	0.271
			140	0.250	0.304	0.205	0.274	0.271
			141	0.180	0.264	0.185	0.199	0.185
			142	0.260	0.413	0.290	0.199	0.185
			143	0.200	0.264	0.185	0.290	0.276
			144	0.280	0.402	0.283	0.297	0.282
			145	0.230	0.234	0.160	0.202	0.196
			146	0.330	0.341	0.234	0.202	0.196
			147	0.200	0.234	0.160	0.202	0.196
			148	0.280	0.341	0.234	0.202	0.196
			149	0.160	0.234	0.160	0.288	0.282
			150	0.240	0.341	0.234	0.288	0.282
			151	0.200	0.234	0.160	0.288	0.282
			152	0.300	0.341	0.234	0.288	0.282
			153	0.230	0.209	0.141	0.194	0.191
			154	0.340	0.300	0.203	0.194	0.191
			155	0.230	0.209	0.141	0.194	0.191
			156	0.340	0.300	0.203	0.274	0.271
			157	0.240	0.209	0.141	0.274	0.271
			158	0.350	0.300	0.203	0.274	0.271
			159	0.170	0.230	0.158	0.202	0.196

Table A.7 continued: *Experimental & Predicted crack widths - Short term tension cracking*

Researcher	Element Description	No. of samples	Sample No.	Test crack width w_{exp} (mm)	Predicted Crack Width $w_{predict}$ (mm) EN 1992	MC 2010 $k_2 = 1$	BS 8007 w0.2	BS 8007 w0.1
Hartl (1977)	Short ties reinforced with single bar		160	0.250	0.337	0.231	0.202	0.196
			161	0.210	0.230	0.158	0.202	0.196
			162	0.310	0.337	0.231	0.288	0.282
			163	0.260	0.230	0.158	0.288	0.282
			164	0.380	0.337	0.231	0.288	0.282
			165	0.200	0.206	0.139	0.194	0.191
			166	0.300	0.298	0.201	0.194	0.191
			167	0.170	0.206	0.139	0.194	0.191
			168	0.280	0.298	0.201	0.274	0.271
			169	0.190	0.206	0.139	0.274	0.271
Hwang (1983)	Plates under axial tension	41	T1A	0.120	0.205	0.187	0.128	0.105
			T1B	0.280	0.267	0.236	0.141	0.129
			T2A	0.080	0.214	0.192	0.138	0.122
			T2B	0.170	0.253	0.226	0.138	0.122
			T3A	0.110	0.221	0.198	0.141	0.129
			T3B	0.160	0.267	0.236	0.141	0.129
			T4A	0.180	0.207	0.186	0.125	0.103
			T4B	0.160	0.253	0.226	0.125	0.103
			T5A	0.160	0.230	0.205	0.131	0.114
			T5B	0.130	0.280	0.247	0.132	0.114
			T6A	0.140	0.204	0.180	0.137	0.126
			T6B	0.200	0.259	0.225	0.137	0.127
			T7A	0.120	0.219	0.196	0.117	0.092
			T7B	0.160	0.244	0.215	0.117	0.092
			T8A	0.140	0.200	0.176	0.128	0.113
			T8B	0.200	0.257	0.223	0.128	0.114
			T9A	0.140	0.198	0.173	0.130	0.120
			T9B	0.160	0.257	0.221	0.130	0.120
			2A	0.130	0.253	0.226	0.125	0.103
			2B	0.130	0.256	0.228	0.125	0.103
			2C	0.140	0.253	0.226	0.125	0.103
			4A	0.180	0.253	0.226	0.125	0.103
			4B	0.150	0.253	0.226	0.125	0.103
			4C	0.180	0.253	0.226	0.125	0.103
			6A	0.150	0.253	0.226	0.125	0.103
			6B	0.160	0.253	0.226	0.125	0.103
			6C	0.210	0.264	0.235	0.125	0.103
			1	0.120	0.260	0.231	0.125	0.103
			2	0.180	0.253	0.226	0.125	0.103
			3	0.200	0.402	0.347	0.126	0.104
			4	0.220	0.402	0.347	0.126	0.104
			5	0.220	0.402	0.347	0.126	0.104
			6	0.220	0.402	0.347	0.126	0.104
			7	0.220	0.402	0.347	0.126	0.104
Wu (2010)	Ties with single bar	7	STN12	0.310	0.622	0.444	0.37	0.343
			STN16	0.500	0.668	0.467	0.46	0.44
			STS12	0.525	0.553	0.387	0.38	0.37
			STS16	0.425	0.562	0.393	0.39	0.37
			LTN12A	0.375	0.477	0.340	0.29	0.26
			LTN12B	0.240	0.184	0.132	0.11	0.09
			LTN12C	0.475	0.431	0.308	0.29	0.26

Table A.8: *Experimental & Predicted crack widths - Long term tension cracking*

Researcher	Element Description	No. of samples	Sample No.	Test crack width wexp (mm)	Predicted Crack Width $w_{predict}$ (mm) EN 1992	MC 2010 $k_2 = 1$	BS 8007 w0.2	BS 8007 w0.1
Eckfeldt (2008)	Ties	11	LDK1-1	0.500	0.515	0.448	0.36	0.051
			LDK1-2	0.250	0.515	0.448	0.36	0.051
			LDK2-1	0.450	0.598	0.527	0.45	0.101
			LDK2-2	0.450	0.598	0.527	0.45	0.101
			LDK3-1	0.400	0.655	0.589	0.54	0.171
			LDK3-2	0.500	0.655	0.589	0.54	0.171
			LDK4-1	0.300	0.517	0.442	0.43	0.062
			LDK4-2	0.300	0.410	0.329	0.27	0.018
			LDK5-1	0.500	0.517	0.449	0.33	0.047
			LDK5-2	0.500	0.517	0.449	0.33	0.047
			LDK6	0.500	0.506	0.446	0.27	0.039
Wu (2010)	Ties with single bar	4	LTN12A	0.675	0.513	0.439	0.29	0.257
			LTN12B	0.335	0.221	0.230	0.11	0.086
			LTN12C	0.550	0.484	0.449	0.29	0.257
			LTN12D	0.150	0.192	0.241	0.11	0.086

APPENDIX B

MODEL UNCERTAINTY DATA & ANALYSIS

Table B.1: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term flexural cracking*

			w_{exp}/ $w_{predict}$ - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 $k_2 = 1$	MC 2010 $k_2 = 0.5$	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN1992	MC 2010 $k_2 = 1$	MC 2010 $k_2 = 0.5$	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm
Attisha	13	A11	1.792	1.964	3.355	0.608	0.622	No repeats				
		A12	1.252	1.440	2.525	0.574	0.557					
		A13	1.440	1.564	2.807	0.772	0.750					
		A14	0.829	0.864	1.639	0.421	0.405					
		A15	0.636	0.640	1.097	0.398	0.384					
		B11	1.309	1.668	2.692	0.558	0.557					
		B12	1.288	1.576	2.779	0.644	0.632					
		B15	0.747	0.680	1.172	0.402	0.383					
		B21	0.729	0.871	1.347	0.243	0.241					
		B22	0.645	0.777	1.249	0.264	0.260					
		B23	0.485	0.581	0.944	0.229	0.224					
		B24	0.350	0.366	0.594	0.154	0.149					
		B25	0.494	0.494	0.859	0.238	0.228					
Chan 2012	27	A7	1.759	1.471	2.440	1.302	1.426	1.587	1.327	1.303	1.174	1.286
		A8	1.414	1.182	0.980	1.046	1.146					
		A9	1.477	1.235	1.024	1.093	1.197					
		A10	1.571	1.314	1.089	1.162	1.273					
		A11	1.728	1.445	1.198	1.279	1.400					
		A12	1.571	1.314	1.089	1.162	1.273					
		B7	1.453	1.228	1.011	1.045	1.134	1.604	1.356	1.116	1.154	1.253
		B8	1.816	1.535	1.264	1.306	1.418					
		B9	1.453	1.228	1.011	1.045	1.134					
		B10	1.816	1.535	1.264	1.306	1.418					
		B11	1.271	1.074	0.885	0.914	0.993					
		B12	1.816	1.535	1.264	1.306	1.418					
		C7	1.307	1.102	0.909	0.997	1.082	1.184	0.998	0.824	0.903	0.980
		C8	1.173	0.989	0.816	0.895	0.971					
		C9	1.274	1.073	0.886	0.971	1.054					
		C10	1.006	0.847	0.699	0.767	0.832					
		C11	1.207	1.017	0.839	0.920	0.999					
		C12	1.140	0.960	0.792	0.869	0.943					
		M1	1.277	1.090	0.892	0.948	1.013	1.141	0.974	0.797	0.847	0.905
		M2	1.241	1.060	0.867	0.922	0.984					
		M3	1.029	0.878	0.719	0.764	0.816					
		M4	0.745	0.636	0.520	0.553	0.591					
		M5	1.419	1.211	0.991	1.054	1.125					
		M6	1.135	0.969	0.793	0.843	0.900					
		A	1.480	1.238	1.026	1.095	1.199					
		B	1.816	1.535	1.264	1.306	1.418					
		C	1.183	0.997	0.823	0.902	0.979					

Table B.1 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term flexural cracking*

			w_{exp}/ wpredict - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm
Clark 1956	34	31	1.087	0.779	1.421	1.005	1.220	No repeats				
		32	1.269	0.910	1.659	1.456	1.423					
		33	1.405	1.080	1.888	1.420	1.290					
		34	1.300	0.999	1.747	1.276	1.161					
		35	1.272	0.990	1.717	1.215	1.098					
		36	1.037	0.848	1.426	1.102	0.972					
		37	1.017	0.832	1.400	1.103	0.972					
		38	1.549	1.323	2.165	1.642	1.427					
		39	2.235	1.859	3.093	1.961	1.675					
		40	1.520	1.264	2.103	1.334	1.139					
		41	2.283	1.897	3.159	1.994	1.698					
		43	1.659	1.394	2.305	1.349	1.106					
		50	1.876	1.659	2.654	1.527	1.231					
		51	2.648	2.343	3.746	2.154	1.736					
		52	2.070	1.825	2.925	1.654	1.339					
		56	1.915	1.786	2.759	1.438	1.123					
		61	1.564	1.507	2.278	1.716	1.496					
		62	0.966	0.931	1.407	1.067	0.930					
		63	1.015	1.022	1.499	1.172	0.992					
		64	1.586	1.396	2.240	1.096	1.017					
		65	1.126	1.202	1.695	1.283	1.052					
		66	0.918	0.976	1.381	1.024	0.843					
		67	1.089	1.159	1.638	1.209	0.998					
		68	1.483	1.578	2.230	1.645	1.357					
		71	2.359	2.622	3.593	2.571	2.071					
		72	1.246	1.385	1.898	1.358	1.094					
		73	1.349	1.384	2.005	1.220	1.036					
		74	1.335	1.558	2.063	1.423	1.112					
		75	1.078	1.258	1.666	1.129	0.884					
		76	1.197	1.396	1.850	1.276	0.997					
		77	1.098	1.284	1.698	1.179	0.919					
		81	1.307	1.354	1.949	1.500	1.319					
		82	0.895	0.927	1.334	1.031	0.907					
		83	1.309	1.449	1.992	1.525	1.305					

Table B.1 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term flexural cracking*

			w_{exp}/ wpredict - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN1992	MC 2010 k2 = 1	MC 2010 k2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm
CUR report No.37	15	86	1.114	1.026	1.598	1.365	1.440	No repeats				
		87	0.840	0.824	1.231	1.230	1.056					
		88	1.118	1.027	1.602	1.636	1.602					
		89	0.901	0.836	1.296	1.342	1.356					
		90	0.891	0.870	1.303	1.197	1.104					
		91	1.037	1.027	1.524	1.500	1.345					
		92	1.008	1.032	1.498	1.495	1.289					
		93	1.001	1.187	1.555	1.335	1.066					
		94	1.042	0.838	1.425	1.611	1.355					
		95	0.898	0.684	1.202	1.250	1.013					
		96	1.097	0.920	1.523	1.781	1.218					
		97	1.016	0.786	1.369	1.246	1.053					
		98	0.950	0.783	1.310	1.280	1.005					
		99	1.203	1.027	1.681	1.465	1.274					
		100	1.194	1.207	1.766	1.323	1.037					
Frosch	54	B-6	1.468	1.373	1.081	1.287	1.149					
		B-9	1.530	1.288	0.886	1.337	1.113					
		B-12	1.582	1.252	0.832	1.557	1.199					
		B-18	0.992	0.729	0.500	1.417	0.914					

Table B.1 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term flexural cracking*

			w_{exp}/ wpredict - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm
Hognestad 1962	30	1	0.842	0.819	1.230	0.702	0.724	No repeats				
		2	1.242	1.354	1.881	1.634	1.388					
		3	0.936	1.092	1.446	1.287	1.023					
		4	1.136	1.441	1.797	1.500	1.111					
		5	1.157	1.262	1.753	1.545	1.307					
		6	1.190	1.163	1.742	1.234	1.042					
		7	0.835	0.979	1.293	1.170	0.925					
		8	0.684	0.612	0.972	0.763	0.676					
		9	0.910	0.875	1.324	1.382	1.153					
		10	1.197	1.422	1.859	1.492	1.058					
		11	1.028	1.342	1.640	1.284	0.943					
		12	0.982	0.950	1.432	0.953	0.812					
		13	1.011	1.093	1.528	1.287	1.098					
		14	1.022	1.186	1.577	1.347	1.080					
		15	0.949	1.196	1.499	1.202	0.901					
		16	1.158	1.351	1.789	1.576	1.254					
		17	0.811	0.947	1.254	1.108	0.882					
		18	1.063	1.241	1.643	1.457	1.159					
		19	0.871	1.014	1.345	1.165	0.931					
		20	1.048	1.141	1.587	1.514	1.293					
		21	0.974	1.059	1.474	1.288	1.091					
		22	0.774	0.840	1.171	0.941	0.748					
		23	0.767	0.828	1.158	0.841	0.572					
		24	1.515	1.225	2.075	0.785	0.757					
		25	0.692	0.650	1.000	0.754	0.652					
		26	0.550	0.618	0.840	0.696	0.507					
		27	0.357	0.440	0.561	0.475	0.283					
		28	0.584	0.546	0.842	0.593	0.514					
		29	0.973	0.909	1.403	1.093	0.948					
		30	2.304	1.863	3.156	1.139	1.099					

Table B.1 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term flexural cracking*

			w_{exp}/ wpredict - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5 (mm)	BS8007 w = 0,2 (mm)	BS8007 w = 0,1 (mm)
Illston (1973)	17	A	0.991	0.986	1.458	1.107	1.354	No repeats				
		B	1.107	0.974	1.563	1.620	2.273					
		C	0.907	1.067	1.406	0.901	1.274					
		D	0.962	1.003	1.437	2.378	4.912					
		E	1.167	1.068	1.671	1.430	1.673					
		F	0.664	0.560	0.924	1.126	1.204					
		G	1.028	1.117	1.555	1.158	1.511					
		H	0.723	0.719	1.064	1.215	1.385					
		J	0.812	0.715	1.147	1.188	1.332					
		K	0.650	0.524	0.889	1.700	1.601					
		L	0.757	0.789	1.130	0.983	1.236					
		M	0.862	0.814	1.247	1.831	1.860					
		Ra	0.960	0.813	1.337	1.320	1.279					
		Rb	1.120	1.081	1.633	2.260	2.396					
		X	0.814	0.745	1.166	1.115	1.305					
		Y	0.795	0.727	1.137	1.086	1.269					
		Z	0.908	0.831	1.300	1.257	1.483					
Krips 1984	1	121	1.148	1.260	1.743	1.585	1.734					
Kenel & Marti	2	B1	1.472	1.197	2.020	1.685	1.415					
		B5	1.053	0.784	1.381	1.149	0.768					
Jaccoud & Favre	3	C14	1.496	1.235	2.064	1.337	1.034					
		C24	1.421	1.173	1.961	1.270	0.983					
		C15	1.620	1.337	2.235	1.429	1.064					
Nejadi	12	B1-a	0.584	0.525	0.831	0.664	0.815	0.497	0.441	0.697	0.551	0.718
		B1-b	0.411	0.356	0.563	0.438	0.621					
		B2-a	0.565	0.465	0.779	0.568	0.620	0.540	0.443	0.743	0.528	0.615
		B2-b	0.515	0.422	0.707	0.489	0.610					
		B3-a	0.552	0.514	0.795	0.630	0.632	0.617	0.574	0.888	0.685	0.722
		B3-b	0.682	0.634	0.981	0.740	0.811					
		S1-a	0.692	0.556	0.946	0.731	0.727	0.677	0.517	0.880	0.696	0.739
		S1-b	0.661	0.479	0.815	0.662	0.752					
		S2-a	0.767	0.686	1.090	0.779	0.641	0.782	0.700	1.111	0.776	0.667
		S2-b	0.797	0.713	1.132	0.773	0.692					
		S3-a	0.752	0.737	1.101	0.742	0.621	0.826	0.809	1.209	0.803	0.689
		S3-b	0.899	0.882	1.317	0.864	0.757					

Table B.1 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term flexural cracking*

			w_{exp}/ wpredict - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN 1992	MC 2010 k_2 = 1	MC 2010 k_2 = 0.5 (mm)	BS8007 w = 0,2 (mm)	BS8007 w = 0,1 (mm)
Rusch & Rehm (1963)	18	101	1.468	1.258	2.054	1.208	1.290	No repeats				
		102	1.143	0.960	1.587	0.964	0.940					
		103	1.397	1.154	1.928	1.090	1.065					
		104	1.372	1.127	1.890	1.087	1.050					
		105	1.718	1.431	2.379	1.444	1.387					
		106	1.249	1.032	1.724	0.978	0.947					
		107	1.719	1.597	2.473	1.803	1.697					
		109	1.399	1.325	2.026	2.000	2.002					
		110	0.973	0.921	1.409	1.391	1.378					
		111	1.474	1.376	2.124	2.095	2.097					
		112	1.437	1.370	2.085	2.060	2.039					
		113	1.289	1.164	1.836	1.779	1.840					
		114	1.719	1.680	2.515	2.147	1.960					
		115	0.931	0.880	1.347	1.147	1.176					
		117	1.238	1.171	1.792	1.052	0.966					
		118	1.448	1.368	2.095	1.211	1.121					
		119	0.989	0.936	1.431	1.412	1.435					
		120	1.669	1.587	2.420	1.881	1.757					
Wu (2010)	8	BSTN2-16	0.413	0.390	0.664	0.556	0.678					
		BSTN3-16	0.450	0.476	0.742	0.564	0.610					
		BSTS2-16	0.357	0.295	0.460	0.468	0.557					
		BSTS3-16	0.553	0.506	0.729	0.666	0.721					
		SSTN4-12	0.726	0.566	1.026	0.880	1.151					
		SSTS4-12	0.848	0.571	0.945	0.917	1.192					
		SLTN4-12A	0.580	0.453	0.826	0.922	1.268					
		SLTN4-12B	0.578	0.451	0.841	0.920	1.353					

Table B.2: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Long term flexural cracking*

			w_{exp}/ $w_{predict}$ - All values					Mean values for repeat samples				
Researcher	n	Sample No.	EN1992	MC 2010 $k_2 = 1$	MC 2010 $k_2 = 0.5$	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm	EN1992	MC 2010 $k_2 = 1$	MC 2010 k2 = 0.5	BS8007 w = 0,2 mm	BS8007 w = 0,1 mm
Attisha	5	B21	1.96913	2.617	3.327	0.808	0.803	No repeats				
		B22	1.19671	1.558	2.064	0.601	0.593					
		B23	1.58876	1.961	2.691	0.818	0.803					
		B24	0.73541	0.808	1.157	0.368	0.358					
		B25	0.41021	0.421	0.646	0.219	0.211					
Illston and Stevens 1973	14	A	2.345	1.995	2.806	2.388	2.441					
		B	1.638	1.082	1.678	1.824	1.989					
		C	1.839	1.822	2.302	1.464	1.499					
		D	1.948	1.498	2.073	1.766	1.940					
		E	1.632	1.331	2.000	1.899	1.961					
		F	1.547	1.117	1.791	2.016	2.169					
		G	1.629	1.552	2.081	1.531	1.587					
		H	1.631	1.379	1.978	1.749	1.893					
		J	1.400	1.134	1.759	1.766	1.827					
		K	1.030	0.727	1.207	1.500	1.648					
		L	1.332	1.275	1.764	1.381	1.431					
		M	1.137	0.948	1.416	1.376	1.510					
		Ra	1.087	0.768	1.229	1.059	1.151					
		Rb	0.207	1.017	1.486	1.661	1.778					
Jaccoud & Favre (1982)	6	C12	1.051	0.899	1.253	1.343	1.096					
		C22	0.764	0.654	0.911	0.976	0.797					
		C13	1.695	1.451	2.111	2.047	1.517					
		C14	1.682	1.439	1.207	1.971	1.395					
		C24	1.577	1.349	2.012	1.848	1.308					
		C15	1.716	1.469	2.226	1.975	1.362					
Nejadi 2004	12	B1-a	1.798	0.797	1.172	1.786	2.051	1.593	0.644	0.941	1.597	1.922
		B1-b	1.387	0.492	0.709	1.408	1.794					
		B2-a	2.155	0.879	1.376	1.921	2.182	1.979	0.726	1.129	1.761	2.090
		B2-b	1.804	0.573	0.881	1.600	1.998					
		B3-a	2.067	0.969	1.435	2.037	2.191	2.036	0.791	1.165	1.886	2.092
		B3-b	2.004	0.614	0.895	1.736	1.993					
		S1-a	1.380	0.552	0.860	1.322	1.603	1.421	0.546	0.842	1.420	1.816
		S1-b	1.463	0.539	0.825	1.519	2.029					
		S2-a	1.430	0.682	1.022	1.285	1.424	1.693	0.687	1.021	1.441	1.658
		S2-b	1.955	0.691	1.019	1.598	1.892					
		S3-a	1.612	0.796	1.138	1.364	1.476	1.744	0.762	1.084	1.419	1.565
		S3-b	1.877	0.728	1.030	1.473	1.655					
Wu	2	SLTN4-12A	0.990	0.635	1.306	0.658	0.906	No repeats				
		SLTN4-12B	1.019	0.609	1.507	0.920	1.353					

Table B.3: *Model uncertainty data – w_{exp}/ wp_{redict} ratios: Short term tension cracking*

Researcher	n	Sample No.	w_{exp}/ $w_{predict}$ – All values				Mean values for repeat samples			
			EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1
Farre & Jaccoud 1994	71	N10-10	0.601	0.833	1.245	1.414	0.611	0.846	1.264	1.444
			0.578	0.800	1.196	1.366				
			0.653	0.905	1.351	1.552				
		N10-14	0.847	1.200	1.539	1.671	0.873	1.237	1.586	1.724
			0.899	1.273	1.632	1.777				
		N10-20	0.818	1.187	1.375	1.453	0.811	1.176	1.363	1.441
			0.894	1.297	1.501	1.589				
			0.721	1.045	1.212	1.280				
		N20-10	0.655	0.907	1.291	1.490	0.615	0.852	1.214	1.395
			0.534	0.739	1.055	1.208				
			0.656	0.909	1.297	1.486				
		N20-14	0.569	0.787	1.092	1.340	0.744	1.049	1.339	1.509
			0.774	1.096	1.357	1.482				
			0.891	1.263	1.568	1.705				
		N20-20	0.781	1.107	1.332	1.518	0.810	1.166	1.344	1.457
			0.836	1.212	1.368	1.446				
			0.814	1.180	1.332	1.407				
		N30-10	0.678	0.984	1.143	1.174	0.624	0.879	1.155	1.269
			0.645	0.893	1.256	1.423				
			0.548	0.759	1.067	1.209				
		N30-14	0.925	1.311	1.599	1.747	0.916	1.298	1.583	1.729
			0.873	1.237	1.508	1.648				
			0.949	1.345	1.641	1.792				
		N30-20	0.795	1.153	1.290	1.362	0.857	1.243	1.390	1.468
			0.889	1.288	1.440	1.522				
			0.889	1.288	1.440	1.522				
		N40-10	0.657	0.910	1.252	1.418	0.617	0.854	1.174	1.332
			0.667	0.923	1.266	1.441				
			0.527	0.730	1.005	1.138				
		N40-14	0.898	1.272	1.539	1.668	0.876	1.241	1.493	1.626
			0.994	1.408	1.689	1.845				
			0.735	1.042	1.250	1.365				
		N40-20	1.092	1.583	1.746	1.846	1.031	1.495	1.649	1.743
			1.001	1.451	1.601	1.692				
			1.001	1.451	1.601	1.692				
		N12-10	0.494	0.684	0.963	1.094	0.569	0.787	1.111	1.259
			0.591	0.818	1.155	1.308				
			0.621	0.860	1.214	1.374				
		N12-14	0.894	1.267	1.544	1.692	0.909	1.289	1.581	1.719
			0.980	1.389	1.709	1.851				
			0.854	1.211	1.490	1.614				
		N12-20	0.898	1.302	1.455	1.538	0.928	1.345	1.504	1.589
			0.898	1.302	1.455	1.538				
			0.988	1.432	1.601	1.692				
		N22-10	0.558	0.772	1.082	1.228	0.588	0.814	1.141	1.295
			0.658	0.912	1.277	1.450				
			0.548	0.759	1.062	1.208				

Table B.3 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term tension cracking*

			w_{exp}/ wpredict – All values				Mean values for repeat samples			
Researcher	n	Sample No.	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1
Farre & Jaccoud 1994		N22-14	0.827	1.172	1.423	1.562	0.830	1.176	1.435	1.565
			0.676	0.958	1.167	1.276				
			0.986	1.397	1.715	1.858				
		N22-20	0.976	1.415	1.584	1.674	0.945	1.370	1.534	1.621
			1.064	1.543	1.728	1.826				
			0.794	1.151	1.290	1.362				
		N32-10	0.508	0.703	0.947	1.074	0.560	0.775	1.046	1.183
			0.585	0.810	1.093	1.237				
			0.586	0.811	1.097	1.238				
		N32-14	0.833	1.180	1.394	1.522	0.787	1.115	1.323	1.439
			0.775	1.098	1.310	1.418				
			0.753	1.067	1.264	1.376				
		N32-20	0.960	1.393	1.520	1.606	1.131	1.639	1.789	1.891
			1.052	1.525	1.665	1.759				
			1.380	2.001	2.183	2.307				
		N42-10	0.574	0.795	1.050	1.187	0.648	0.898	1.188	1.341
			0.548	0.759	1.012	1.135				
			0.823	1.139	1.503	1.702				
		N42-14	0.845	1.197	1.407	1.525	0.808	1.145	1.333	1.454
			0.783	1.109	1.285	1.406				
			0.797	1.129	1.306	1.431				
		N42-20	0.994	1.441	1.561	1.647	1.015	1.471	1.592	1.681
			1.039	1.507	1.632	1.722				
			1.011	1.466	1.584	1.674				

Table B.3 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term tension cracking*

			w_{exp}/ $w_{predict}$ – All values				Mean values for repeat samples			
Researcher	n	Sample No.	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1
Hartl	48	122	0.604	0.834	1.02	1.17				
		123	0.783	1.114	1.11	1.19	0.723	1.030	1.02	1.10
		125	0.783	1.114	1.11	1.19				
		127	0.605	0.861	0.85	0.92				
		124	0.739	1.051	1.07	1.12	0.691	0.983	1.00	1.05
		126	0.739	1.051	1.07	1.12				
		128	0.596	0.848	0.86	0.91				
		129	0.790	1.150	0.94	0.97	0.804	1.171	0.96	0.99
		131	0.831	1.211	0.99	1.02				
		133	0.790	1.150	0.94	0.97				
		130	0.805	1.172	0.97	0.99	0.834	1.214	1.01	1.03
		132	0.862	1.256	1.04	1.06				
		134	0.834	1.214	1.01	1.03				
		135	0.659	0.977	0.72	0.73	0.801	1.187	0.88	0.89
		137	0.895	1.326	0.98	0.99				
		139	0.848	1.257	0.93	0.94				
		136	0.658	0.975	0.73	0.74	0.789	1.170	0.87	0.88
		138	0.888	1.316	0.98	1.00				
		140	0.822	1.219	0.91	0.92				
		141	0.683	0.972	0.90	0.97	0.721	1.026	0.96	1.03
		143	0.759	1.080	1.01	1.08				
		142	0.630	0.897	0.97	1.02	0.663	0.944	0.92	0.97
		144	0.696	0.991	0.88	0.92				
		145	0.984	1.433	1.14	1.17	0.845	1.231	1.01	1.04
		147	0.856	1.246	0.99	1.02				
		149	0.685	0.997	0.79	0.82				
		151	0.856	1.246	0.99	1.02				
		146	0.967	1.409	1.15	1.17	0.843	1.228	1.04	1.06
		148	0.821	1.195	0.97	0.99				
		150	0.704	1.025	0.83	0.85				
		152	0.879	1.281	1.04	1.06				
		153	1.103	1.635	1.18	1.20	1.119	1.658	1.20	1.22
		155	1.103	1.635	1.18	1.20				
		157	1.151	1.706	1.24	1.26				
		154	1.132	1.678	1.24	1.25	1.143	1.695	1.25	1.27
		156	1.132	1.678	1.24	1.25				
		158	1.165	1.728	1.28	1.29				
		159	0.740	1.078	0.84	0.87	0.929	1.353	1.06	1.09
		161	0.914	1.331	1.04	1.07				
		163	1.132	1.648	1.29	1.33				
		160	0.742	1.080	0.87	0.89	0.930	1.354	1.09	1.11
		162	0.920	1.339	1.08	1.10				
		164	1.127	1.642	1.32	1.35				
		165	0.969	1.436	1.03	1.05	0.904	1.341	0.96	0.98
		167	0.824	1.221	0.88	0.89				
		169	0.921	1.365	0.98	0.99				
		166	1.006	1.492	1.09	1.11	0.973	1.442	1.06	1.07
		168	0.939	1.392	1.02	1.03				

Table B.3 continued: *Model uncertainty data – w_{exp}/ $w_{predict}$ ratios: Short term tension cracking*

Researcher	n	Sample No.	w_{exp}/ $w_{predict}$ – All values				Mean values for repeat samples			
			EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1
Hwang 1983	34	TIA	0.584	0.643	0.936	1.139	No repeats			
		TIB	1.049	1.187	1.979	2.167				
		T2A	0.374	0.416	0.582	0.657				
		T2B	0.670	0.752	1.232	1.392				
		T3A	0.498	0.557	0.780	0.854				
		T3B	0.599	0.678	1.131	1.239				
		T4A	0.871	0.968	1.442	1.751				
		T4B	0.631	0.708	1.278	1.552				
		T5A	0.697	0.780	1.219	1.402				
		T5B	0.464	0.525	0.988	1.136				
		T6A	0.685	0.778	1.024	1.107				
		T6B	0.771	0.887	1.459	1.577				
		T7A	0.548	0.613	1.028	1.301				
		T7B	0.656	0.743	1.369	1.732				
		T8A	0.701	0.795	1.095	1.236				
		T8B	0.779	0.896	1.559	1.761				
		T9A	0.707	0.810	1.079	1.167				
		T9B	0.621	0.724	1.230	1.331				
		2A	0.513	0.576	1.038	1.262				
		2B	0.507	0.569	1.038	1.262				
		2C	0.552	0.620	1.118	1.358				
		4A	0.710	0.797	1.438	1.747				
		4B	0.592	0.665	1.199	1.456				
		4C	0.710	0.797	1.438	1.747				
		6A	0.592	0.665	1.199	1.456				
		6B	0.631	0.708	1.278	1.552				
		6C	0.796	0.893	1.677	2.038				
		1	0.462	0.518	0.957	1.163				
		2	0.710	0.797	1.438	1.747				
		3	0.497	0.576	1.584	1.925				
		4	0.547	0.633	1.743	2.118				
		5	0.547	0.633	1.743	2.118				
		6	0.547	0.633	1.743	2.118				
		7	0.547	0.633	1.743	2.118				
Wu 2010	7	STN12	0.498	0.699	2.359	2.62	No repeats			
		STN16	0.748	1.070	2.927	3.90				
		STS12	0.850	1.200	1.923	2.14				
		STS16	0.757	1.082	1.311	1.75				
		LTN12A	0.787	1.103	2.359	2.62				
		LTN12B	1.301	1.824	2.927	3.90				
		LTN12C	1.101	1.544	1.923	2.14				

Table B.4: *Model uncertainty data – w_{exp}/ wpredict ratios: Long term tension cracking*

			w_{exp}/ $w_{predict}$ – All values				Mean values for repeat samples			
Researcher	n	Sample No.	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1	EN 1992	MC 2010	BS8007 w = 0,2	BS8007 w = 0,1
Eckfeldt	11	LDK1-1	0.971	1.116	2.37	9.72	0.728	0.837	1.78	7.29
		LDK1-2	0.485	0.558	1.18	4.86				
		LDK2-1	0.752	0.854	1.71	4.44	0.752	0.854	1.71	4.44
		LDK2-2	0.752	0.854	1.71	4.44				
		LDK3-1	0.610	0.679	1.26	2.34	0.687	0.764	1.42	2.64
		LDK3-2	0.763	0.849	1.58	2.93				
		LDK4-1	0.581	0.678	1.18	4.85	0.656	0.795	1.54	10.62
		LDK4-2	0.732	0.911	1.91	16.39				
		LDK5-1	0.968	1.113	2.60	10.68	0.968	1.113	2.60	10.68
		LDK5-2	0.968	1.113	2.60	10.68				
		LDK6	0.989	1.121	3.17	12.99				
Wu	4	LTN12A	1.317	1.539	2.359	2.62	1.227	1.382	2.643	3.262
		LTN12C	1.138	1.225	2.927	3.90				
		LTN12B	1.517	1.453	1.923	2.14	1.150	1.038	2.643	3.262
		LTN12D	0.782	0.623	1.311	1.75				

Table B.5: *Model uncertainty stochastic parameters – Short term flexural cracking*

Researcher		EN 1992	MC 2010 $k_2 = 1.0$	MC 2010 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS8007 w = 0.1 mm
ALL	Mean	1.107	1.052	1.551	1.185	1.112
	Standard Error	0.033	0.031	0.050	0.035	0.040
	Median	1.045	1.024	1.452	1.209	1.093
	Standard Deviation	0.420	0.395	0.638	0.450	0.510
	COV	0.379	0.376	0.411	0.380	0.459
	Sample Variance	0.176	0.156	0.407	0.203	0.260
	Kurtosis	1.144	1.087	1.045	0.283	18.107
	Skewness	0.813	0.650	0.863	0.136	2.642
	Range	2.298	2.326	3.461	2.417	4.763
	Minimum	0.350	0.295	0.286	0.154	0.149
	Maximum	2.648	2.622	3.746	2.571	4.912
	Count	164	164	164	164	164
	Confidence Level (95.0%)	0.065	0.061	0.098	0.069	0.079
Attisha	Mean	0.923	1.037	1.774	0.423	0.415
	Standard Error	0.123	0.147	0.256	0.054	0.053
	Median	0.747	0.864	1.347	0.402	0.384
	Standard Deviation	0.443	0.529	0.922	0.193	0.192
	COV	0.480	0.509	0.520	0.457	0.462
	Sample Variance	0.196	0.279	0.850	0.037	0.037
	Kurtosis	-0.744	-1.341	-1.445	-1.101	-1.223
	Skewness	0.599	0.480	0.445	0.298	0.298
	Range	1.442	1.598	2.762	0.617	0.600
	Minimum	0.350	0.366	0.594	0.154	0.149
	Maximum	1.792	1.964	3.355	0.772	0.750
	Count	13	13	13	13	13
	Confidence Level (95.0%)	0.268	0.319	0.557	0.117	0.116
Chan	Mean	1.428	1.238	1.022	1.055	1.146
	Standard Error	0.099	0.089	0.081	0.065	0.073
	Median	1.480	1.282	1.026	1.095	1.199
	Standard Deviation	0.262	0.218	0.215	0.172	0.192
	COV	0.183	0.176	0.210	0.163	0.168
	Kurtosis	-1.605	-1.212	-1.997	-1.577	-1.652
	Skewness	0.200	-0.104	0.242	0.128	0.039
	Range	0.675	0.561	0.506	0.459	0.513
	Minimum	1.141	0.974	0.797	0.847	0.905
	Maximum	1.816	1.535	1.303	1.306	1.418
	Count	7	6	7	7	7
	Confidence Level (95.0%)	0.242	0.229	0.199	0.159	0.178
Clark 1956	Mean	1.443	1.358	2.076	1.413	1.204
	Standard Error	0.077	0.072	0.109	0.060	0.048
	Median	1.308	1.339	1.923	1.341	1.117
	Standard Deviation	0.451	0.421	0.635	0.351	0.281
	COV	0.312	0.310	0.306	0.249	0.234
	Sample Variance	0.203	0.177	0.403	0.123	0.079
	Kurtosis	0.529	1.524	0.728	2.553	1.516
	Skewness	1.112	1.093	1.143	1.476	1.244
	Range	1.753	1.842	2.413	1.566	1.228
	Minimum	0.895	0.779	1.334	1.005	0.843
	Maximum	2.648	2.622	3.746	2.571	2.071
	Count	34	34	34	34	34
	Confidence Level (95.0%)	0.157	0.147	0.222	0.123	0.098

Table B.5 continued: *Model uncertainty stochastic parameters – Short term flexural cracking*

Researcher		EN 1992	MC 2010 $k_2 = 1.0$	MC 2010 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS8007 w = 0.1 mm
CUR report No.37	Mean	1.021	0.938	1.437	1.404	1.214
	Standard Error	0.029	0.039	0.041	0.044	0.047
	Median	1.016	0.920	1.461	1.342	1.218
	Standard Deviation	0.111	0.152	0.152	0.172	0.183
	COV	0.109	0.162	0.106	0.123	0.151
	Sample Variance	0.012	0.023	0.023	0.030	0.033
	Kurtosis	-0.860	-0.690	-1.312	-0.071	-0.541
	Skewness	0.087	0.250	-0.072	0.850	0.576
	Range	0.363	0.523	0.479	0.584	0.598
	Minimum	0.840	0.684	1.202	1.197	1.005
	Maximum	1.203	1.207	1.681	1.781	1.602
	Count	15	15	15	15	15
	Confidence Level (95.0%)	0.062	0.084	0.088	0.095	0.101
Frosch	Mean	1.393	1.160	0.825	1.399	1.094
	Standard Error	0.136	0.146	0.121	0.059	0.062
	Median	1.499	1.270	0.859	1.377	1.131
	Standard Deviation	0.271	0.292	0.242	0.118	0.125
	COV	0.195	0.252	0.293	0.084	0.114
	Range	0.589	0.644	0.581	0.270	0.285
	Minimum	0.992	0.729	0.500	1.287	0.914
	Maximum	1.582	1.373	1.081	1.557	1.199
	Count	4	4	4	4	4
Hognestad 1962	Mean	0.985	1.049	1.476	1.140	0.931
	Standard Error	0.062	0.057	0.086	0.059	0.048
	Median	0.973	1.076	1.460	1.186	0.945
	Standard Deviation	0.341	0.310	0.471	0.322	0.265
	COV	0.346	0.296	0.319	0.282	0.284
	Sample Variance	0.117	0.096	0.222	0.104	0.070
	Kurtosis	7.138	0.427	4.723	-0.887	-0.128
	Skewness	1.866	0.178	1.268	-0.383	-0.494
	Range	1.947	1.423	2.595	1.159	1.105
	Minimum	0.357	0.440	0.561	0.475	0.283
	Maximum	2.304	1.863	3.156	1.634	1.388
	Count	30	30	30	30	30
	Confidence Level (95.0%)	0.127	0.116	0.176	0.120	0.099
Ilston	Mean	0.896	0.855	1.298	1.393	1.726
	Standard Error	0.038	0.044	0.058	0.104	0.217
	Median	0.907	0.814	1.300	1.215	1.385
	Standard Deviation	0.157	0.183	0.238	0.429	0.893
	COV	0.175	0.214	0.183	0.308	0.517
	Sample Variance	0.025	0.033	0.056	0.184	0.797
	Kurtosis	-0.874	-0.960	-0.946	0.840	11.200
	Skewness	0.132	-0.172	-0.078	1.275	3.186
	Range	0.517	0.592	0.781	1.477	3.709
	Minimum	0.650	0.524	0.889	0.901	1.204
	Maximum	1.167	1.117	1.671	2.378	4.912
	Count	17	17	17	17	17
	Confidence Level (95.0%)	0.081	0.094	0.122	0.221	0.459
Kenel & Marti	Mean	1.263	0.991	1.701	1.417	1.091
	Range	0.419	0.412	0.639	0.536	0.647
	Count	2	2	2	2	2

Table B.5 continued: *Model uncertainty stochastic parameters – Short term flexural cracking*

Researcher		EN 1992	MC 2010 k_2 = 1.0	MC 2010 k_2 = 0.5	BS 8007 w = 0.2 mm	BS8007 w = 0.1 mm
Jaccoud & Favre	Mean	1.512	1.248	2.087	1.345	1.027
	Standard Error	0.058	0.048	0.080	0.046	0.024
	Median	1.496	1.235	2.064	1.337	1.034
	Standard Deviation	0.100	0.083	0.139	0.080	0.041
	Range	0.199	0.164	0.274	0.159	0.081
	Minimum	1.421	1.173	1.961	1.270	0.983
	Maximum	1.620	1.337	2.235	1.429	1.064
	Count	3	3	3	3	3
Nejadi	Mean	0.657	0.581	0.921	0.673	0.692
	Standard Error	0.053	0.060	0.082	0.046	0.019
	Median	0.647	0.545	0.884	0.691	0.704
	Standard Deviation	0.131	0.148	0.202	0.113	0.046
	COV	0.199	0.254	0.219	0.168	0.066
	Range	0.328	0.369	0.512	0.275	0.124
	Minimum	0.497	0.441	0.697	0.528	0.615
	Maximum	0.826	0.809	1.209	0.803	0.739
	Count	6	6	6	6	6
Rimkus	Mean	0.596	0.543	0.630	0.625	0.564
	Standard Error	0.052	0.057	0.086	0.086	0.078
	Median	0.618	0.538	0.677	0.637	0.578
	Standard Deviation	0.128	0.140	0.210	0.211	0.191
	COV	0.215	0.258	0.334	0.337	0.338
	Sample Variance	0.016	0.020	0.044	0.044	0.036
	Range	0.284	0.329	0.559	0.442	0.373
	Minimum	0.441	0.367	0.286	0.410	0.369
	Maximum	0.725	0.695	0.845	0.852	0.742
	Count	6	6	6	6	6
Rusch & Reihm (1963)	Mean	1.368	1.241	1.951	1.486	1.453
	Standard Error	0.059	0.058	0.085	0.100	0.098
	Median	1.398	1.214	1.977	1.402	1.382
	Standard Deviation	0.251	0.244	0.362	0.424	0.414
	COV	0.184	0.197	0.186	0.285	0.285
	Sample Variance	0.063	0.060	0.131	0.180	0.171
	Kurtosis	-0.695	-0.916	-0.833	-1.547	-1.496
	Skewness	-0.238	0.196	-0.096	0.341	0.282
	Range	0.788	0.800	1.168	1.183	1.156
	Minimum	0.931	0.880	1.347	0.964	0.940
	Maximum	1.719	1.680	2.515	2.147	2.097
	Count	18	18	18	18	18
	Largest(1)	1.719	1.680	2.515	2.147	2.097
	Smallest(1)	0.931	0.880	1.347	0.964	0.940
	Confidence Level(95.0%)	0.125	0.121	0.180	0.211	0.206
Wu	Mean	0.563	0.464	0.779	0.737	0.941
	Standard Error	0.058	0.032	0.062	0.068	0.116
	Median	0.566	0.464	0.784	0.773	0.936
	Standard Deviation	0.163	0.091	0.175	0.193	0.329
	COV	0.289	0.197	0.224	0.262	0.350
	Sample Variance	0.027	0.008	0.031	0.037	0.108
	Range	0.491	0.276	0.566	0.453	0.795
	Minimum	0.357	0.295	0.460	0.468	0.557
	Maximum	0.848	0.571	1.026	0.922	1.353
	Count	8	8	8	8	8

Table B.6: *Model uncertainty stochastic parameters – Long term flexural cracking*

Researcher		EN 1992	MC 2010 $k_2 = 1.0$	MC 2010 $k_2 = 0.5$	BS 8007 w = 0.2 mm	BS8007 w = 0.1 mm
ALL DATA	Mean	1.460	1.175	1.657	1.480	1.491
	Standard Error	0.086	0.090	0.114	0.092	0.098
	Median	1.593	1.117	1.678	1.531	1.565
	Standard Deviation	0.479	0.499	0.636	0.511	0.547
	Sample Variance	0.229	0.249	0.404	0.261	0.299
	COV	0.157	0.212	0.244	0.177	0.200
	Kurtosis	0.746	0.809	0.112	0.394	-0.026
	Skewness	-0.864	0.857	0.643	-0.855	-0.697
	Range	2.138	2.196	2.681	2.169	2.231
	Minimum	0.207	0.421	0.646	0.219	0.211
	Maximum	2.345	2.617	3.327	2.388	2.441
	Count	31	31	31	31	31
	Confidence Level (95.0%)	0.176	0.183	0.233	0.188	0.200
Attisha	Mean	1.180	1.473	1.977	0.563	0.553
	Standard Error	0.281	0.394	0.489	0.119	0.119
	Median	1.197	1.558	2.064	0.601	0.593
	Standard Deviation	0.629	0.881	1.094	0.266	0.265
	COV	0.533	0.598	0.553	0.473	0.479
	Range	1.559	2.196	2.681	0.599	0.592
	Minimum	0.410	0.421	0.646	0.219	0.211
	Maximum	1.969	2.617	3.327	0.818	0.803
	Count	5	5	5	5	5
Ilston	Mean	1.457	1.260	1.826	1.670	1.773
	Standard Error	0.135	0.099	0.115	0.087	0.089
	Median	1.588	1.204	1.778	1.705	1.802
	Standard Deviation	0.505	0.369	0.432	0.325	0.332
	COV	0.346	0.292	0.236	0.194	0.187
	Sample Variance	0.255	0.136	0.186	0.105	0.110
	Kurtosis	2.217	-0.164	0.739	1.010	0.248
	Skewness	-0.854	0.515	0.584	0.354	0.136
	Range	2.138	1.267	1.599	1.328	1.291
	Minimum	0.207	0.727	1.207	1.059	1.151
	Maximum	2.345	1.995	2.806	2.388	2.441
	Count	14	14	14	14	14
	Confidence Level (95.0%)	0.291	0.213	0.249	0.187	0.191
Jaccoud & Favre	Mean	1.414	1.210	1.620	1.693	1.246
	Standard Error	0.166	0.142	0.229	0.177	0.106
	Median	1.629	1.394	1.633	1.910	1.335
	Standard Deviation	0.406	0.347	0.560	0.434	0.260
	COV	0.287	0.287	0.346	0.256	0.208
	Sample Variance	0.165	0.121	0.314	0.188	0.067
	Range	0.952	0.815	1.314	1.070	0.719
	Minimum	0.764	0.654	0.911	0.976	0.797
	Maximum	1.716	1.469	2.226	2.047	1.517
	Count	6	6	6	6	6
Nejadi	Mean	1.744	0.693	1.030	1.587	1.857
	Standard Error	0.095	0.036	0.050	0.081	0.089
	Median	1.718	0.706	1.052	1.519	1.869
	Standard Deviation	0.232	0.089	0.122	0.199	0.219
	COV	0.133	0.129	0.118	0.125	0.118
	Sample Variance	0.054	0.008	0.015	0.039	0.048
	Range	0.614	0.246	0.323	0.468	0.527
	Minimum	1.421	0.546	0.842	1.419	1.565
	Maximum	2.036	0.791	1.165	1.886	2.092
	Count	6	6	6	6	6
Wu	Mean	1.004	0.662	1.406	0.789	1.289
	Count	2	2	2	2	2

Table B.7: *Model uncertainty stochastic parameters – Short term tension cracking*

Researcher		EN 1992	MC 2010	BS 8007 w = 0.2 mm	BS8007 w = 0.1 mm
ALL	Mean	0.742	0.984	1.271	1.430
	Standard Error	0.020	0.035	0.032	0.041
	Median	0.716	0.897	1.225	1.398
	Standard Deviation	0.183	0.319	0.290	0.369
	COV	0.247	0.324	0.228	0.258
	Sample Variance	0.034	0.102	0.084	0.136
	Kurtosis	0.174	-0.486	-0.056	1.234
	Skewness	0.597	0.490	0.441	0.777
	Range	0.927	1.408	1.516	2.139
	Minimum	0.374	0.416	0.582	0.657
	Maximum	1.301	1.824	2.097	2.796
	Count	82	82	82	82
	Confidence Level (95.0%)	0.040	0.070	1.380	1.507
Farre & Jaccoud	Mean	0.796	1.132	0.040	0.037
	Standard Error	0.033	0.051	1.354	1.463
	Median	0.811	1.171	0.196	0.183
	Standard Deviation	0.162	0.248	0.142	0.121
	COV	0.203	0.219	0.038	0.034
	Sample Variance	0.026	0.061	-0.800	-0.679
	Kurtosis	-0.858	-0.872	0.154	0.186
	Skewness	0.118	0.123	0.744	0.708
	Range	0.571	0.865	1.046	1.183
	Minimum	0.560	0.775	1.789	1.891
	Maximum	1.131	1.639	24	24
	Count	24	24	0.083	0.077
	Confidence Level (95.0%)	0.068	0.105	1.017	1.055
Hartl	Mean	0.842	1.227	0.024	0.025
	Standard Error	0.036	0.057	1.011	1.052
	Median	0.834	1.214	0.099	0.103
	Standard Deviation	0.148	0.234	0.098	0.097
	COV	0.176	0.191	0.010	0.011
	Sample Variance	0.022	0.055	1.102	0.149
	Kurtosis	0.097	0.046	0.871	0.305
	Skewness	0.568	0.464	0.376	0.381
	Range	0.539	0.860	0.874	0.885
	Minimum	0.604	0.834	1.251	1.265
	Maximum	1.143	1.695	17	17
	Count	17	17	0.051	0.053
	Confidence Level (95.0%)	0.076	0.120	1.288	1.520
Hwang 1983	Mean	0.628	0.711	0.054	0.066
	Standard Error	0.023	0.026	1.231	1.456
	Median	0.610	0.693	1.743	2.118
	Standard Deviation	0.132	0.149	0.314	0.387
	COV	0.210	0.210	0.244	0.255
	Sample Variance	0.017	0.022	0.099	0.150
	Kurtosis	1.934	1.965	-0.250	-0.565
	Skewness	0.927	0.896	0.155	0.047
	Range	0.675	0.771	1.397	1.510
	Minimum	0.374	0.416	0.582	0.657
	Maximum	1.049	1.187	1.979	2.167
	Count	34	34	34	34
	Confidence Level (95.0%)	0.046	0.052	0.110	0.135
Wu 2010	Mean	0.877	1.240	1.432	1.639
	Standard Error	0.100	0.139	0.149	0.221
	Median	0.787	1.103	1.418	1.565
	Standard Deviation	0.264	0.368	0.395	0.586
	COV	0.301	0.297	0.276	0.357
	Range	0.803	1.125	1.231	1.839
	Minimum	0.498	0.699	0.866	0.957
	Maximum	1.301	1.824	2.097	2.796
	Count	7	7	7	7

Table B.8: *Model uncertainty stochastic parameters – Long term tension cracking*

Researcher		EN 1992	MC 2010	BS 8007 w = 0.2 mm	BS8007 w = 0.1 mm
ALL	Mean	0.895	0.988	1.318	1.603
	Standard Error	0.078	0.076	0.211	0.280
	Median	0.860	0.946	1.089	1.353
	Standard Deviation	0.220	0.214	0.597	0.793
	COV	0.246	0.216	0.453	0.495
	Range	0.571	0.618	1.807	2.328
	Minimum	0.656	0.764	0.836	0.935
	Maximum	1.227	1.382	2.643	3.262
	Count	8	8	8	8
	Confidence Level (95.0%)	0.184	0.179	0.499	0.663

APPENDIX C

RELIABILITY ANALYSES SUMMARIES

C1: Sensitivity Analysis

Table C.1: *Sensitivity analysis data – Short term flexural cracking*

θ	w (mm)	H (m)	h (mm)	%As	As	A_{SLS}/A_{ULS}	β	c*	h*	L*	ft*	θ*	d	x	hc,eff	Sr	ε (-)	wm calc	g = 0
LN(1.05; 0.38)	0.2	5	500	0.49	2468	1.65	1.5	0.041	0.500	49.8	2557	1.627	0.449	0.110	0.130	0.220	5.616E-04	1.235E-04	-8.139E-07
				0.56	2780	1.86	2.0	0.041	0.500	50.1	2488	1.939	0.448	0.116	0.128	0.199	5.185E-04	1.031E-04	3.451E-08
				0.63	3126	2.09	2.5	0.042	0.499	50.3	2427	2.313	0.447	0.122	0.126	0.181	4.776E-04	8.643E-05	6.415E-08
LN(1.10; 0.38)	0.2	5	500	0.51	2532	1.69	1.5	0.041	0.500	49.8	2557	1.704	0.449	0.112	0.129	0.215	5.480E-04	1.178E-04	-7.013E-07
				0.57	2852	1.91	2.0	0.041	0.500	50.1	2489	2.031	0.448	0.117	0.127	0.195	5.059E-04	9.846E-05	3.256E-08
				0.64	3208	2.15	2.5	0.042	0.499	50.3	2427	2.423	0.447	0.123	0.125	0.177	4.658E-04	8.253E-05	6.134E-08
			600	0.32	1935	1.44	1.5	0.040	0.599	50.0	2385	1.560	0.549	0.111	0.163	0.322	3.977E-04	1.282E-04	-3.003E-08
				0.37	2217	1.65	2.0	0.041	0.599	50.3	2289	1.847	0.548	0.118	0.160	0.284	3.808E-04	1.083E-04	-5.764E-08
				0.42	2528	1.88	2.5	0.041	0.599	50.5	2208	2.196	0.548	0.125	0.158	0.253	3.601E-04	9.109E-05	-5.188E-10
		7	700	0.69	4838	1.25	1.5	0.041	0.700	69.5	2709	1.767	0.649	0.269	0.144	0.145	7.822E-04	1.136E-04	-7.262E-07
				0.77	5408	1.40	2.0	0.042	0.700	69.8	2667	2.116	0.648	0.280	0.140	0.134	7.075E-04	9.452E-05	1.564E-08
				0.86	6043	1.57	2.5	0.043	0.700	70.1	2624	2.530	0.647	0.291	0.136	0.124	6.394E-04	7.905E-05	5.116E-09
	0.1	5	500	0.75	3732	2.50	1.5	0.041	0.500	49.8	2559	1.700	0.449	0.131	0.123	0.156	3.786E-04	5.900E-05	-3.010E-07
				0.84	4198	2.81	2.0	0.042	0.500	50.1	2489	2.022	0.448	0.138	0.121	0.143	3.492E-04	4.994E-05	-9.840E-07
				0.96	4775	3.20	2.5	0.043	0.499	50.3	2425	2.405	0.447	0.145	0.118	0.131	3.174E-04	4.156E-05	4.513E-08
		7	700	1.00	7014	1.77	1.5	0.041	0.700	69.6	2646	1.742	0.648	0.212	0.163	0.124	4.593E-04	5.709E-05	5.608E-07
				1.13	7891	1.99	2.0	0.042	0.699	69.9	2592	2.079	0.648	0.222	0.159	0.115	4.169E-04	4.809E-05	2.649E-08
				1.28	8930	2.25	2.5	0.043	0.699	70.2	2540	2.478	0.647	0.232	0.156	0.107	3.757E-04	4.034E-05	3.683E-08

Table C.2: *Sensitivity analysis data – Long term flexural cracking*

θ	w (mm)	H (m)	h (mm)	%As	As	A_{SLS}/A_{ULS}	β	c*	h*	L*	ft*	θ*	d	x	hc,eff	Sr	ε (-)	wm calc	g = 0
LN(1.05; 0.38)	0.2	5	500	0.54	2689	1.60	1.5	0.041	0.500	49.7	2659	1.672	0.449	0.171	0.109	0.181	6.62E-04	1.20E-04	-7.26E-07
				0.60	2998	1.78	2.0	0.042	0.500	49.9	2606	1.999	0.448	0.178	0.107	0.166	6.04E-04	1.00E-04	1.78E-08
				0.67	3342	1.99	2.5	0.042	0.500	50.1	2553	2.387	0.447	0.186	0.105	0.152	5.51E-04	8.38E-05	1.40E-08
LN(1.10; 0.38)	0.2	5	500	0.55	2757	1.85	1.5	0.041	0.500	49.7	2659	1.751	0.449	0.173	0.109	0.177	6.46E-04	1.15E-04	-5.92E-07
				0.61	3074	2.06	2.0	0.042	0.500	49.9	2604	2.093	0.448	0.180	0.107	0.162	5.89E-04	9.56E-05	1.53E-08
				0.69	3427	2.29	2.5	0.042	0.500	50.1	2552	2.499	0.447	0.187	0.104	0.149	5.37E-04	8.00E-05	1.00E-08
			600	0.40	2385	1.60	1.5	0.041	0.600	49.8	2561	1.709	0.549	0.185	0.138	0.237	4.95E-04	1.18E-04	-8.12E-07
				0.44	2664	1.78	2.0	0.041	0.599	50.1	2488	2.035	0.548	0.193	0.135	0.215	4.58E-04	9.83E-05	3.36E-08
				0.49	2969	1.99	2.5	0.042	0.599	50.3	2420	2.425	0.548	0.201	0.133	0.195	4.23E-04	8.25E-05	6.55E-08
	0.2	7	700	0.69	4840	1.25	1.5	0.041	0.700	69.5	2709	1.767	0.649	0.269	0.144	0.145	7.82E-04	1.14E-04	-5.94E-07
				0.77	5408	1.40	2.0	0.042	0.700	69.8	2667	2.116	0.648	0.280	0.140	0.134	7.07E-04	9.45E-05	1.29E-08
				0.86	6043	1.57	2.5	0.043	0.700	70.1	2624	2.530	0.647	0.291	0.136	0.124	6.39E-04	7.91E-05	-4.07E-10
	0.1	5	500	0.80	3997	2.68	1.5	0.041	0.500	49.7	2643	1.738	0.448	0.198	0.100	0.131	4.41E-04	5.77E-05	-2.27E-07
				0.89	4454	2.98	2.0	0.042	0.500	50.0	2582	2.070	0.448	0.206	0.098	0.121	4.01E-04	4.87E-05	-8.32E-07
				1.00	5013	3.36	2.5	0.043	0.500	50.2	2521	2.459	0.446	0.214	0.095	0.113	3.61E-04	4.07E-05	3.07E-08
	0.1	7	700	1.02	7152	1.80	1.5	0.041	0.700	69.5	2693	1.754	0.648	0.309	0.130	0.108	5.24E-04	5.67E-05	5.11E-07
				1.14	7982	2.01	2.0	0.042	0.700	69.8	2644	2.092	0.647	0.321	0.126	0.101	4.73E-04	4.80E-05	-4.71E-07
				1.29	9001	2.27	2.5	0.043	0.699	70.1	2592	2.490	0.646	0.333	0.122	0.095	4.22E-04	4.02E-05	-8.81E-08

Note: All units in kN and m unless otherwise stated.

Table C.3: *Sensitivity analysis data – Short term tension cracking, $h_{c,eff}$ = 2,5 (c + φ/2)*

θ	w (mm)	H (m)	D (m)	h (mm)	%A_s	A_s (mm^2)	A_{SLS}/ A_{ULS}	β	c*	L*	f_{ct}*	θ*	$h_{c.eff}$	S_r	ε (-)	w_m	g = 0
LN(1.00; 0.30)	0.2	5	25	250	1.12	2792	1.30	1.50	0.041	50.0	2569	1.426	0.128	0.198	7.10E-04	1.40E-04	-1.25E-09
					1.22	3060	1.43	2.00	0.042	50.3	2488	1.630	0.129	0.186	6.58E-04	1.23E-04	-4.51E-09
					1.34	3355	1.56	2.50	0.042	50.6	2409	1.864	0.130	0.176	6.09E-04	1.07E-04	2.57E-09
LN(1.10; 0.38)	0.2	5	25	250	0.75	1.43	3077	1.50	0.041	49.8	2608	1.732	0.127	0.183	6.32E-04	1.15E-04	-1.10E-09
					1.00	1.60	3443	2.00	0.041	50.1	2535	2.063	0.128	0.170	5.71E-04	9.69E-05	-7.32E-09
					1.25	1.80	3856	2.50	0.042	50.3	2462	2.456	0.129	0.158	5.15E-04	8.14E-05	1.35E-09

Table C.4: *Sensitivity analysis data – Long term tension cracking, $h_{c,eff}$ = 2,5 (c + φ/2)*

θ	w (mm)	H (m)	D (m)	h (mm)	%A_s	A_s (mm^2)	A_{SLS}/ A_{ULS}	β	c*	L*	f_{ct}*	θ*	$h_{c.eff}$	Sr	ε (-)	w_m	g = 0
LN(1.00; 0.30)	0.2	5	25	250	1.18	2948	1.37	1.50	0.042	49.9	2623	1.433	0.130	0.193	7.23E-04	1.40E-04	-3.39E-10
					1.29	3221	1.50	2.00	0.043	50.2	2550	1.638	0.132	0.184	6.65E-04	1.22E-04	-1.90E-09
					1.41	3520	1.64	2.50	0.043	50.5	2476	1.872	0.133	0.175	6.11E-04	1.07E-04	-7.83E-09
LN(1.10; 0.38)	0.2	5	25	250	1.30	3240	1.51	1.50	0.041	49.74	2652	1.739	0.129	0.178	6.45E-04	1.15E-04	-7.61E-10
					1.44	3610	1.68	2.00	0.042	49.99	2585	2.072	0.130	0.167	5.77E-04	9.65E-05	-6.75E-09
					1.61	4025	1.88	2.50	0.043	50.24	2514	2.465	0.132	0.157	5.17E-04	8.11E-05	4.05E-10

Table C.5: *Sensitivity analysis data – Short term tension cracking, $h_{c,eff}$ = h/2*

θ	w (mm)	H (m)	D (m)	h (mm)	%A_s	A_{SLS}/ A_{ULS}	A_s (mm^2)	β	c*	h*	L*	f_{ct}*	θ*	$h_{c.eff}$	S_r	ε (-)	w_m	g = 0
LN(1.00; 0.30)	0.2	5	25	250	1.18	1.37	2944	1.50	0.040	0.25	49.81	2610	1.607	0.125	0.186	6.68E-04	1.24E-04	4.97E-10
					1.31	1.52	3265	2.00	0.041	0.25	50.05	2544	1.893	0.125	0.173	6.10E-04	1.06E-04	1.00E-09
					1.45	1.69	3624	2.50	0.041	0.25	50.29	2482	2.228	0.125	0.161	5.56E-04	8.98E-05	-3.32E-09
LN(1.10; 0.38)	0.2	5	25	250	1.23	1.43	3072	1.50	0.040	0.25	49.76	2622	1.745	0.125	0.181	6.35E-04	1.15E-04	2.54E-10
					1.37	1.60	3434	2.00	0.041	0.25	49.99	2558	2.085	0.125	0.167	5.74E-04	9.59E-05	-1.62E-09
					1.54	1.79	3844	2.50	0.041	0.25	50.21	2497	2.492	0.125	0.155	5.18E-04	8.03E-05	-4.34E-11

Table C.6: *Sensitivity analysis data – Long term tension cracking, $h_{c,eff}$ = h/2*

θ	w (mm)	H (m)	D (m)	h (mm)	%A_s	A_{SLS}/ A_{ULS}	A_s (mm^2)	β	c*	h*	L*	f_{ct}*	θ*	$h_{c.eff}$	S_r	ε (-)	w_m	g = 0
LN(1.00; 0.30)	0.2	5	25	250	1.17	2931	1.37	1.50	0.041	0.250	49.9	2633	1.45	0.125	0.187	7.35E-04	1.38E-04	1.62E-10
					1.28	3193	1.49	2.00	0.041	0.250	50.1	2569	1.67	0.125	0.176	6.79E-04	1.20E-04	3.29E-10
					1.39	3481	1.62	2.50	0.042	0.250	50.4	2506	1.92	0.125	0.166	6.26E-04	1.04E-04	-8.27E-10
LN(1.10; 0.38)	0.2	5	25	250	1.29	1.50	3226	1.50	0.040	0.25	49.72	2661	1.760	0.125	0.174	6.52E-04	1.14E-04	-4.48E-10
					1.44	1.67	3588	2.00	0.041	0.25	49.94	2601	2.106	0.125	0.162	5.86E-04	9.50E-05	-5.53E-09
					1.60	1.86	3993	2.50	0.041	0.25	50.17	2540	2.517	0.125	0.151	5.26E-04	7.94E-05	-2.15E-11

Note: All units in kN and m unless otherwise stated.

Table C7: *Sensitivity factors and theoretical partial safety factors – Short-term flexure*

θ	w (mm)	H (m)	h (mm)	%As	β	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	Σ(α*xi)^2	p_f	γ c	γ h	γ L	γ ft	γ θ
LN(1.05; 0.38)	0.2	5	500	0.49	1.5	-0.138	0.048	-0.211	0.366	-0.895	1.000	6.681E-02	1.031	0.999	1.016	0.896	1.510
				0.56	2.0	-0.148	0.046	-0.206	0.346	-0.902	1.000	2.275E-02	1.044	0.999	1.021	0.868	1.686
				0.63	2.5	-0.159	0.045	-0.201	0.330	-0.907	1.000	6.210E-03	1.060	0.999	1.025	0.843	1.862
LN(1.10; 0.38)	0.2	5	500	0.51	1.5	-0.140	0.048	-0.211	0.366	-0.894	1.000	6.681E-02	1.031	0.999	1.016	0.896	1.510
				0.57	2.0	-0.150	0.046	-0.206	0.346	-0.902	1.000	2.275E-02	1.045	0.999	1.021	0.868	1.685
				0.64	2.5	-0.161	0.045	-0.201	0.330	-0.907	1.000	6.210E-03	1.060	0.999	1.025	0.843	1.862
			600	0.32	1.5	-0.100	0.072	-0.253	0.611	-0.740	1.000	6.681E-02	1.023	0.999	1.019	0.826	1.422
				0.37	2.0	-0.108	0.068	-0.245	0.566	-0.777	1.000	2.275E-02	1.032	0.999	1.025	0.785	1.590
				0.42	2.5	-0.117	0.064	-0.238	0.529	-0.804	1.000	6.210E-03	1.044	0.998	1.030	0.749	1.764
		7	700	0.69	1.5	-0.164	0.025	-0.167	0.164	-0.958	1.000	6.681E-02	1.037	1.000	1.013	0.953	1.546
				0.77	2.0	-0.178	0.025	-0.166	0.164	-0.956	1.000	2.275E-02	1.053	0.999	1.017	0.938	1.726
				0.86	2.5	-0.192	0.026	-0.165	0.165	-0.953	1.000	6.210E-03	1.072	0.999	1.021	0.922	1.905
	0.1	5	500	0.75	1.5	-0.170	0.049	-0.209	0.363	-0.891	1.000	6.681E-02	1.038	0.999	1.016	0.897	1.508
				0.84	2.0	-0.183	0.048	-0.205	0.345	-0.896	1.000	2.275E-02	1.055	0.999	1.020	0.869	1.681
				0.96	2.5	-0.198	0.047	-0.200	0.331	-0.899	1.000	6.210E-03	1.074	0.999	1.025	0.843	1.854
		7	700	1.00	1.5	-0.182	0.037	-0.185	0.246	-0.933	1.000	6.681E-02	1.041	0.999	1.014	0.930	1.532
				1.13	2.0	-0.197	0.037	-0.182	0.239	-0.933	1.000	2.275E-02	1.059	0.999	1.018	0.909	1.709
				1.28	2.5	-0.212	0.037	-0.180	0.234	-0.931	1.000	6.210E-03	1.080	0.999	1.022	0.889	1.884

Table C8: *Sensitivity factors and theoretical partial safety factors – Long-term flexure*

θ	w	H	h	%As	β	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	Σ(α*xi)^2	p_f	γ c	γ h	γ L	γ ft	γ θ
LN(1.05; 0.38)	0.2	5.0	500	0.54	1.50	-0.157	0.032	-0.182	0.229	-0.943	1.000	6.681E-02	1.035	1.000	1.014	0.935	1.537
				0.60	2.00	-0.169	0.032	-0.180	0.225	-0.942	1.000	2.275E-02	1.051	0.999	1.018	0.914	1.716
				0.67	2.50	-0.181	0.032	-0.178	0.223	-0.941	1.000	6.210E-03	1.068	0.999	1.022	0.894	1.894
LN(1.10; 0.38)	0.2	5.0	500	0.55	1.50	-0.159	0.032	-0.182	0.230	-0.942	1.000	6.681E-02	1.036	1.000	1.014	0.935	1.537
				0.61	2.00	-0.171	0.032	-0.180	0.226	-0.941	1.000	2.275E-02	1.051	0.999	1.018	0.914	1.715
				0.69	2.50	-0.184	0.032	-0.179	0.224	-0.940	1.000	6.210E-03	1.069	0.999	1.022	0.894	1.893
		5.0	600	0.40	1.50	-0.123	0.044	-0.210	0.361	-0.899	1.000	6.681E-02	1.028	0.999	1.016	0.897	1.513
				0.44	2.00	-0.132	0.042	-0.206	0.347	-0.904	1.000	2.275E-02	1.040	0.999	1.021	0.868	1.687
				0.49	2.50	-0.142	0.041	-0.203	0.336	-0.908	1.000	6.210E-03	1.053	0.999	1.025	0.841	1.863
	0.2	7.0	700	0.69	1.50	-0.164	0.025	-0.167	0.164	-0.958	1.000	6.681E-02	1.037	1.000	1.013	0.953	1.546
				0.77	2.00	-0.178	0.025	-0.166	0.164	-0.956	1.000	2.275E-02	1.053	0.999	1.017	0.938	1.726
				0.86	2.50	-0.192	0.026	-0.165	0.165	-0.953	1.000	6.210E-03	1.072	0.999	1.021	0.922	1.905
	0.1	5.0	500	0.80	1.50	-0.196	0.035	-0.185	0.250	-0.929	1.000	6.681E-02	1.044	0.999	1.014	0.929	1.530
				0.89	2.00	-0.210	0.036	-0.184	0.249	-0.927	1.000	2.275E-02	1.063	0.999	1.018	0.906	1.704
				1.00	2.50	-0.226	0.036	-0.183	0.250	-0.923	1.000	6.210E-03	1.085	0.999	1.023	0.881	1.877
	0.1	7.0	700	1.02	1.50	-0.208	0.029	-0.171	0.184	-0.945	1.000	6.681E-02	1.047	1.000	1.013	0.948	1.539
				1.14	2.00	-0.223	0.030	-0.170	0.187	-0.941	1.000	2.275E-02	1.067	0.999	1.017	0.929	1.715
				1.29	2.50	-0.239	0.031	-0.170	0.191	-0.936	1.000	6.210E-03	1.090	0.999	1.021	0.909	1.889

Note: All units in kN and m unless otherwise stated

C2: Reliability Analysis – FORM

Table C9: *Long-term flexure with deterministic ε_{sh} -200 x 10^{-6} – MC 2010 using design A_{SLS}*

H	h	%As	As	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	γ[illegible]
5	500	0.663	3316	2.348	0.042	0.500	50.081	2567.800	2.368	0.447	0.185	0.105	0.153	0.001	0.000	0.000	-0.180	0.032	-0.179	0.224	-0.940	1.
5	600	0.496	2976	2.511	0.042	0.599	50.298	2418.414	2.434	0.548	0.202	0.133	0.195	0.000	0.000	0.000	-0.142	0.041	-0.203	0.336	-0.908	1.
6	600	0.738	4426	2.265	0.042	0.600	60.003	2615.869	2.317	0.548	0.234	0.122	0.139	0.001	0.000	0.000	-0.180	0.028	-0.171	0.190	-0.949	1.
6	750	0.495	3715	2.176	0.041	0.749	60.132	2495.184	2.188	0.698	0.255	0.165	0.194	0.000	0.000	0.000	-0.131	0.038	-0.199	0.312	-0.919	1.
7	700	0.780	5459	2.044	0.042	0.700	69.836	2662.949	2.149	0.648	0.281	0.140	0.133	0.001	0.000	0.000	-0.179	0.025	-0.166	0.164	-0.955	1.
7	850	0.550	4678	1.871	0.041	0.849	69.868	2595.714	1.996	0.798	0.302	0.182	0.176	0.001	0.000	0.000	-0.135	0.032	-0.187	0.251	-0.940	1.

Table C10: *Long-term flexure with deterministic ε_{sh} -400 x 10^{-6} – MC 2010 using design A_{SLS}*

H	h	%As	As	β	c*	h*	L*	ft*	θ*	d	x	hc,eff	Sr (m)	ε (-)	wm calc	g = 0	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	γθ
5	500	0.806	4028	3.233	0.044	0.499	50.45	2476	3.232	0.446	0.198	0.100	0.133	0.000	0.000	0.000	-0.205	0.033	-0.177	0.222	-0.936	2.150
5	600	0.641	3845	3.685	0.043	0.599	50.82	2276	3.669	0.546	0.222	0.126	0.158	0.000	0.000	0.000	-0.170	0.040	-0.196	0.316	-0.912	2.277
6	600	0.882	5291	3.068	0.043	0.599	60.39	2540	3.078	0.546	0.249	0.117	0.123	0.001	0.000	0.000	-0.204	0.030	-0.170	0.190	-0.945	2.101
6	750	0.618	4631	3.199	0.043	0.749	60.68	2369	3.138	0.697	0.277	0.157	0.161	0.000	0.000	0.000	-0.154	0.037	-0.193	0.297	-0.921	2.120
7	700	0.931	6520	2.834	0.043	0.699	70.28	2595	2.848	0.646	0.299	0.134	0.118	0.001	0.000	0.000	-0.202	0.027	-0.165	0.166	-0.951	2.024
7	850	0.667	5670	2.768	0.042	0.849	70.41	2495	2.747	0.797	0.325	0.175	0.150	0.000	0.000	0.000	-0.157	0.032	-0.183	0.245	-0.939	1.987

Table C11: *Long-term flexure with ε_{sh} as random variable, mean -200 x 10^{-6} – MC 2010 using design A_{SLS}*

CoV	H	h	%As	β	c*	h*	L*	ft*	θ*	esh	d	x	hc	Sr	ε (-)	wm calc	g = 0	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	α*εsh	γθ	γεsh
0.15	5	500	0.663	2.350	0.042	0.50	50.07	2570	2.351	0.0002	0.448	0.185	0.105	0.153	0.0006	0.0001	0.00	-0.178	0.032	-0.177	0.222	-0.931	-0.138	1.822	1.057
	6	600	0.738	2.297	0.042	0.60	60.01	2615	2.329	0.0002	0.548	0.234	0.122	0.139	0.0006	0.0001	0.00	-0.179	0.028	-0.170	0.188	-0.942	-0.124	1.742	1.045
	7	700	0.780	2.056	0.042	0.70	69.84	2663	2.149	0.0002	0.648	0.281	0.140	0.133	0.0007	0.0001	0.00	-0.178	0.025	-0.165	0.163	-0.950	-0.109	1.742	1.045
0.20	5	500	0.663	2.391	0.042	0.50	50.08	2568	2.368	0.0002	0.447	0.185	0.105	0.153	0.0006	0.0001	0.00	-0.177	0.032	-0.176	0.220	-0.189	-0.923	1.839	1.090
	6	600	0.738	2.313	0.042	0.60	60.01	2615	2.329	0.0002	0.548	0.234	0.122	0.139	0.0006	0.0001	0.00	-0.178	0.028	-0.169	0.187	-0.169	-0.936	1.822	1.078
	7	700	0.780	2.067	0.042	0.70	69.84	2663	2.149	0.0002	0.648	0.281	0.140	0.133	0.0007	0.0001	0.00	-0.177	0.025	-0.164	0.162	-0.148	-0.945	1.742	1.061
0.25	5	500	0.663	2.412	0.042	0.50	50.08	2569	2.361	0.0002	0.448	0.185	0.105	0.153	0.0006	0.0001	0.00	-0.174	0.031	-0.174	0.218	-0.912	-0.243	1.747	1.079
	6	600	0.738	2.335	0.042	0.60	60.01	2615	2.329	0.0002	0.548	0.234	0.122	0.139	0.0006	0.0001	0.00	-0.177	0.028	-0.167	0.185	-0.927	-0.217	1.822	1.102
	7	700	0.780	2.094	0.042	0.70	69.84	2662	2.159	0.0002	0.648	0.281	0.140	0.133	0.0007	0.0001	0.00	-0.176	0.025	-0.163	0.161	-0.938	-0.188	1.747	1.079

Table C12: *Long-term flexure – BS 8007*

H	h	%As	As	β	c*	h*	L*	ft*	θ*	d	x	hc,eff	Sr (m)	ε (-)	wm calc (m)	g = 0	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	γθ
5	500	0.470	2350	0.74	0.040	0.500	49.39	2745	1.334	0.450	0.163	0.112	0.203	0.0007	0.0001	0.0000	-0.143	0.032	-0.185	0.237	-0.942	1.27
6	600	0.496	2976	0.41	0.040	0.600	59.07	2795	1.188	0.550	0.202	0.133	0.193	0.0009	0.0002	0.0000	-0.136	0.027	-0.177	0.196	-0.954	1.15
7	700	0.543	3801	0.36	0.040	0.700	68.88	2806	1.167	0.650	0.246	0.151	0.177	0.0010	0.0002	0.0000	-0.137	0.024	-0.170	0.166	-0.961	1.13

Note: All units in kN and m unless otherwise stated

Table C13: *Long-term flexure – MC 2010 with model uncertainty as MF*

H	h	%As	As	β	c*	h*	L*	ft*	θ*	d	x	hc,eff	Sr (m)	ε (-)	wm calc (m)	g = 0	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	γθ
5	500	0.74	3700	3.29	0.04	0.5	49.05	2890	3.289	0.450	0.193	0.102	0.137	4.428E-04	6.081E-05	0.000	0.000	0.000	0.000	0.000	-1.000	2.17
6	600	0.82	4890	3.07	0.04	0.6	58.86	2890	3.066	0.550	0.243	0.119	0.127	5.155E-04	6.524E-05	0.000	0.000	0.000	0.000	0.000	-1.000	2.10
7	700	0.86	6027	2.89	0.04	0.7	68.67	2890	2.771	0.650	0.292	0.136	0.121	5.973E-04	7.218E-05	0.000	0.000	0.000	0.000	0.000	-1.000	2.00
5	600	0.57	3432	2.62	0.04	0.6	49.05	2890	4.051	0.550	0.213	0.129	0.170	2.908E-04	4.937E-05	0.000	0.000	0.000	0.000	0.000	-1.000	2.38
6	750	0.56	4200	3.62	0.04	0.75	58.86	2890	3.290	0.700	0.268	0.161	0.172	3.531E-04	6.080E-05	0.000	0.000	0.000	0.000	0.000	-1.000	2.17
7	850	0.61	5211	3.07	0.04	0.85	68.67	2890	2.720	0.800	0.315	0.178	0.159	4.629E-04	7.352E-05	0.000	0.000	0.000	0.000	0.000	-1.000	1.98

Note: All units in kN and m unless otherwise stated

C3: Deterministic Analyses

Table C14: *ULS Bending reinforcement for Rectangular LRS Wall to SANS 10100-1 (2004)*

H (m)	h (mm)	φ = 20 mm			φ = 25 mm		
		A_{ULS}	% A_s	s (mm)	A_{ULS}	% A_s	s (mm)
5	400	1970	0.49	159	1970	0.49	249
	450	1680	0.37	187	1692	0.38	290
	500	1494	0.30	210	1502	0.30	327
	550	1344	0.24	234	1351	0.25	363
	600	1222	0.20	257	1228	0.20	400
6	500	2635	0.53	119	2652	0.53	185
	550	3588	0.65	88	2349	0.43	209
	600	4465	0.74	70	2122	0.35	231
	650	5885	0.91	53	1944	0.30	252
	700	1787	0.26	176	1794	0.26	274
	750	1659	0.22	189	1665	0.22	295
	800	1549	0.19	203	1554	0.19	316
7	550	3857	0.70	81	3881	0.71	126
	600	3441	0.57	91	3459	0.58	142
	650	3111	0.48	101	3126	0.48	157
	700	2843	0.41	110	2855	0.41	172
	750	2635	0.35	119	2644	0.35	186
	800	2459	0.31	128	2468	0.31	199
	850	2306	0.27	136	2313	0.27	212
	900	2170	0.24	145	2176	0.24	226
	950	2049	0.22	153	2055	0.22	239

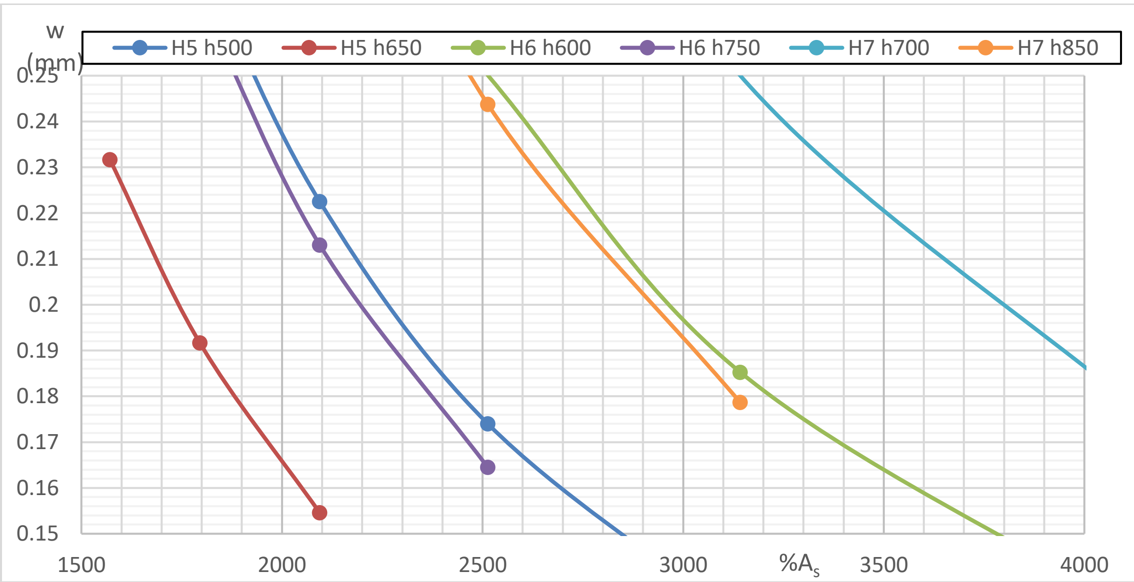

Figure C1 : *Deterministic analysis for A_{SLS} to BS 8007- flexural cracking*

Table C15: *Design SLS reinforcement using MC 2010 – long-term flexural cracking*

ε_{sh} (design)	H (m)	h (mm)	ULS As (SANS) (mm^2)	Asls/Auls	SLS %As	SLS A_s (mm^2)	$S_{r,\,max}$ (mm)	ε_{sm}	ε_{cm}	ε_m	ε final	wk (mm)	Min strain 0.6*σs/Es
-270×10^{-6}	5	500	1494	2.22	0.66	3316	256	0.00081	0.00030	0.00078	0.00078	0.200	0.00049
	5	600	1222	2.43	0.50	2976	314	0.00073	0.00036	0.00064	0.00064	0.200	0.00044
	6	600	2112	2.10	0.74	4426	233	0.00086	0.00027	0.00086	0.00086	0.200	0.00052
	6	750	1659	2.24	0.50	3715	267	0.00079	0.00031	0.00075	0.00075	0.200	0.00047
	7	700	2843	1.92	0.78	5459	207	0.00094	0.00025	0.00096	0.00096	0.200	0.00056
	7	850	2306	2.03	0.55	4678	229	0.00087	0.00027	0.00088	0.00088	0.200	0.00052
-400×10^{-6}	5	500	1494	2.77	0.83	4132	214	0.00066	0.00025	0.00093	0.00093	0.200	0.00039
	5	600	1222	3.25	0.66	3976	255	0.00055	0.00030	0.00079	0.00079	0.200	0.00033
	6	600	2112	2.56	0.90	5417	199	0.00071	0.00024	0.00100	0.00100	0.200	0.00043
	6	750	1659	2.88	0.64	4771	226	0.00062	0.00027	0.00089	0.00089	0.200	0.00037
	7	700	2843	2.35	0.95	6680	184	0.00078	0.00022	0.00109	0.00109	0.200	0.00047
	7	850	2306	2.52	0.68	5819	199	0.00071	0.00024	0.00100	0.00100	0.200	0.00043