

INVESTIGATING THE BELIEFS ABOUT PROBLEM-SOLVING OF
MATHEMATICS TEACHERS AT INDEPENDENT SECONDARY
SCHOOLS IN SOUTH AFRICA

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DECLARATION

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ABSTRACT**INVESTIGATING THE BELIEFS ABOUT PROBLEM-SOLVING OF MATHEMATICS
TEACHERS AT INDEPENDENT SECONDARY SCHOOLS IN SOUTH AFRICA**

The South African National Curriculum Statement (NCS) for Mathematics calls for teaching that facilitates and promotes problem-solving as a routine activity and which promotes learning and teaching from a constructivist paradigm. This is in line with international curricula. However, teachers have found it difficult to incorporate problem-solving into their teaching and to include problem-solving questions in their school-based assessments, even in privileged school environments.

Researchers argue that teachers' beliefs about the nature of Mathematics and the teaching and learning of Mathematics significantly influence their teaching practices. Teachers who hold a constructivist belief about the teaching and learning of Mathematics have been shown to incorporate problem-solving in their teaching more readily. However, those with traditionalist beliefs find it difficult to include the reformed practices called for by the reformed curricula. This research therefore investigated the beliefs of practising secondary Mathematics teachers about problem-solving, the nature of Mathematics, and the teaching and learning of Mathematics, and how their beliefs related to their implementation of problem-solving activities as required by the NCS.

This research is a quantitative case study, augmented by open-ended questions and semi-structured interviews. The participants were Mathematics teachers working at predominantly privileged secondary schools in South Africa. As these schools offer a positive teaching and learning environment with qualified and experienced teaching staff, a reasonable expectation would be that the aims and objectives of the reformed curriculum would be met. Data was collected through the use of questionnaires with further data collection taking place in the form of semi-structured interviews. The data from 95 completed questionnaires were analysed, followed by the semi-structured interviews that were used only for the purposes of informing future research, as only two interviews could be completed.

The theoretical framework used for the study is based on the three belief dimensions of Mathematics teachers:

- (i) the nature of the discipline of Mathematics;
- (ii) the teaching of school Mathematics;
- (iii) the learning of school Mathematics.

Teachers' beliefs were further categorised on a continuum moving from a traditional belief to a constructivist belief within these three dimensions. In addition, teachers' beliefs were investigated in relation to their teacher-training qualifications and their academic qualifications in Mathematics. Both these aspects have been shown in prior research to have an influence on a teacher's beliefs.

The research indicated that:

- (i) the participants hold traditionalist beliefs in general about the nature of Mathematics, while they hold constructivist beliefs about the teaching and learning of Mathematics;
- (ii) the participants with post-reform qualifications are more likely to hold constructivist beliefs in general than their colleagues with pre-reform qualifications;
- (iii) the participants with only a Mathematics 1 qualification, and who have both pre- and post-reform professional qualifications, are more likely to have traditionalist beliefs;
- (iv) the participants (most of whom are qualified and experienced educators) struggle to describe adequately aspects of teaching and learning that involve problem-solving.

The study therefore informs schools and teacher-training institutions regarding aspects of teachers' professional development – that the focus should be on the development of constructivist beliefs which will encourage problem-solving as a routine activity in classroom practice.

OPSOMMING

'n ONDERSOEK NA DENKWYSES EN HOUDINGS OOR WISKUNDEPROBLEEMOPLOSSING VAN ONDERWYSERS BY ONAFHANKLIKE SEKONDÊRE SKOLE IN SUID-AFRIKA.

Die nasionale Suid-Afrikaanse Wiskundekurrikulum, soos ander internasionale kurrikulums, vereis dat die onderrig van probleemoplossing in Wiskunde as 'n roetine aktiwiteit gefasiliteer word en dat 'n konstruktiewe onderrigparadigma bevorder word. Onderwysers vind dit egter moeilik om probleemoplossing by hul onderrig te inkorporeer en ook om probleemoplossingstipe vroeë by hul skoolgebaseerde assesserings in te sluit - selfs by skole in wat in gunstige omstandighede funksioneer.

Navorsers argumenteer dat onderwysers se denkwyses rakende die aard van Wiskunde en die leer en onderrig van Wiskunde, 'n beduidende invloed op die onderwyser se onderrigpraktyk het. Onderwysers wat 'n konstruktiewe denkwyse oor die leer en onderrig van Wiskunde het, toon 'n groter gewilligheid om probleemoplossing by hul onderrig in te sluit. Terwyl dié met tradisionele denkwyses dit moeilik vind om die hervormde praktyke in hul onderrig, soos vereis word deur die hervormde kurrikulums, te inkorporeer. Die studie ondersoek dus sekondêre Wiskunde onderwysers se denkwyses oor probleemoplossing, die aard van Wiskunde en die leer en onderrig van Wiskunde en hoe dit verband hou met die implementasie van probleemoplossingsaktiwiteite soos deur die Nasionale Wiskundekurrikulum vereis word.

Die navorsing is 'n kwantitatiewe gevallestudie wat deur kwalitatiewe ope vroeë met semi-gestruktureerde onderhoude aangevul word. Die deelnemers aan die studies was Wiskunde onderwysers vanaf oorwegend goeie sekondêre skole in Suid-Afrika. Aangesien die skole oor positiewe onderrig- en leeromgewings met goed opgeleide en ervare onderwysers beskik, is daar 'n redelike verwagting dat die doelstellings en doelwitte van die hervormde kurrikulum bereik sal word. Data insameling het deur die voltooiing van vraelyste, gevolg deur semi-gestruktureerde onderhoude plaasgevind. Data insameling en analise is deur die gebruik van 95 voltooië vraelyste voltooi, gevolg deur die semi-gestruktureerde onderhoude.

Die data van die onderhoude is slegs gebruik om die studie aan te vul en om 'n basis vir toekomstige studies te bied.

Die teoretiese raamwerk wat in die studie gebruik is, is op die drie aspekte van Wiskunde onderwysers se denkwyses gebaseer:

- (i) Die aard van Wiskunde as 'n dissipline.
- (ii) Die onderrig van Wiskunde op skoolvlak.
- (iii) Die leer van Wiskunde op skoolvlak.

Onderwysers se denkwyses word verder op 'n kontinuum, wat vanaf 'n tradisionele denkwyse tot 'n konstruktiewe denkwyse beweeg, gekategoriseer. Onderwysers se denkwyses word verder, in verband met hul onderwyskwalifikasie en akademiese kwalifikasie in Wiskunde, ondersoek. Vorige navorsing het getoon dat beide dié aspekte 'n onderwyser se denkwyse beïnvloed.

Die navorsingstudie het die volgende getoon:

- (i) Oor die algemeen behou die deelnemers tradisionele denkwyses oor die aard van Wiskunde, maar in teenstelling behou hul konstruktiewe denkwyses oor die leer en onderrig van Wiskunde.
- (ii) Deelnemers met post-hervorm kwalifikasies is meer geneig om konstruktiewe denkwyses te behou as die pre-hervorm gekwalifiseerde deelnemers.
- (iii) Deelnemers met Wiskunde 1 as kwalifikasie is meer geneig om tradisionele denkwyses oor beide pre- en post-hervormde professionele gekwalifiseerde deelnemers te behou.
- (iv) Die deelnemers, wat meestal goedopgeleide en ervare onderwysers uitmaak, vind dit moeilik om aspekte van probleemoplossing binne die leer en onderrig daarvan voldoende te bespreek.

Daarom beveel die studie aan dat skole en ander onderwys- en opleidingsinstansies op die aspekte van professionele opleiding fokus wat die ontwikkeling van konstruktiewe denkwyses bevorder en probleemoplossing as 'n roetine aktiwiteit promoveer.

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LIST OF ABBREVIATIONS

CAPS	Curriculum Assessment Policy Statement
DBE	Department of Basic Education
FET	Further Education and Training
HDE	Higher Diploma in Education
HOD	Head of Department
IEB	Independent Examination Board
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
NQF	National Qualifications Framework
OBE	Outcome-based Education
PGCE	Postgraduate Certificate in Education
SAGs	Subject Assessment Guidelines
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1

INTRODUCTION AND OVERVIEW

“[...] how a teacher conceptualizes mathematics has direct impact on [his/her] teaching and so if there is to be any change in his/her instructional practices, beliefs must first be addressed.” (Cross 2009)

1.1. Background to the study

As a Mathematics teacher and Head of Department (HOD) of Mathematics at an independent secondary school in South Africa registered with the Independent Education Board (IEB), and as National Moderator for School-based Assessment (SBA) in Mathematics for all IEB registered schools, I have had the opportunity to interact with Mathematics teachers throughout the IEB and to moderate numerous assessment tasks from most schools registered with the IEB.

I have found that although mathematical problem-solving is a curriculum-specific aim in the South African national Mathematics curriculum (Department of Basic Education 2011a: 8) and is required to form part of all SBA tasks (Department of Basic Education 2011a: 53; SAGs IEB 2018), many teachers struggle to include mathematical problem-solving in their teaching as a routine activity and in their assessment tasks. Teachers either omit problem-solving from their assessments or wrongly categorise problems as “mathematical problem-solving”. The latter shows a lack of understanding of what constitutes mathematical problem-solving.

Independent schools registered with the IEB predominantly consist of secondary schools proposing to offer quality education with high academic standards, qualified teaching staff, small classes, structured, safe and nurturing learning environments, accountable school governance and adequate learning resources (AdvTech 2018; Curro 2018; IEB 2018a; ISASA 2015a; ISASA 2015b; Reddam House 2018). As these schools offer a positive teaching and learning environment with qualified and experienced teaching staff, a reasonable expectation would be that the aims and objectives of the Curriculum Assessment Policy Statement (CAPS) for Mathematics and the IEB’s Subject Assessment Guidelines (SAGs) for Mathematics should be enacted. I have, however, found that teachers struggle with the

concept of mathematical problem-solving and how to make it part of their routine classroom practice and assessments.

As teachers' beliefs significantly influence individuals' perceptions and interpretation of a situation as well as the practice they engage in (Skott 2015: 6), the apparent difficulty in including problem-solving as a routine activity may partially be due to the individual teacher's beliefs about (i) problem-solving, (ii) the nature of Mathematics and (iii) the teaching and learning of Mathematics. These observations prompted the desire to investigate the beliefs of secondary school Mathematics teachers within independent schools registered with the IEB and how their beliefs might be affecting classroom practice.

The following sections of this chapter summarise the rationale to the study (Section 1.2), the problem statement (Section 1.3), the research question (Section 1.4), the purpose and significance of the study (Section 1.5), research design and methodology (Section 1.6) and the chapter overview of the thesis (Section 1.7).

1.2. Rationale to the study

1.2.1. Importance of beliefs

Developing 21st century skills requires learners to engage with authentic real-world problems (Bransford, Brown and Cocking 2004; Trilling & Fadel 2009) so that they can: “think and read critically, [...] express themselves clearly and persuasively, [and can] solve complex problems” (Bransford *et al.* 2004: 4). To this end, reforms in both national and international Mathematics curricula have included problem-solving either as an outcome or an instructional approach (Pellegrino & Hilton 2012; Ministry of Education Singapore 2012; CCSS 2018).

Teachers both nationally and internationally are finding it difficult to introduce problem-solving into their classrooms as a routine activity (Mayer 1998; Jonassen 2000; Brodie & Pournara 2005; de Freitas & Zolkower 2011; Stols 2013). Various studies have identified a number of factors that influence teachers’ classroom practices and the implementation of reforms in the classroom. These factors include: pedagogical knowledge, content knowledge, pedagogical content knowledge, resources, curriculum, standards, goals, efficacy, culture, socio-economic factors and time constraints (Howie 2003; Moses & Mji 2006; Paolucci 2008; Speer 2008; Cross Francis 2014). But teachers’ beliefs regarding Mathematics and Mathematics education have been identified as the most significant influence on their classroom practices (Ernest 1989; Hiebert *et al.* 2003; Webb & Webb 2004; Li & Yu 2010; Cross Francis 2014; Xenofontos & Andrews 2014). Educational research into the beliefs of teachers is therefore fundamental for a clearer understanding of teachers’ decisions regarding the inclusion of problem-solving as a routine activity. The research is also necessary for facilitating the implementation of mathematical problem-solving as a curriculum aim and objective (Hart 1987; Shahvarani & Savizi 2007; Paolucci 2008; Beswick 2012; Xenofontos & Andrews 2014).

1.2.2. Teachers' beliefs

A main challenge has been to define the construct of beliefs (Cross Francis 2014; Skott 2015). Research interest into teachers' beliefs grew out of the desire to understand better the motivation behind teachers' practices. The construct of beliefs can be defined as:

“[the] embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it, developed through membership in various social groups; these beliefs are considered by the individual to be true” (Cross 2009).

Beliefs are cognitive constructs that are strong predictors of behaviour (Ernest 1989; Speer 2008; Cross Francis 2014). An individuals' perceptions and interpretations of a situation as well as the practices they engage in is significantly influenced by their beliefs (Skott 2015). The view that teachers' beliefs significantly influence their individual instructional practices is the main driving force behind research efforts into this construct.

It is therefore evident that for the purpose of this study, the beliefs held by Mathematics teachers are of great significance in understanding teaching methodologies and the implementation (or the lack thereof) of problem-solving in instruction. The construct of beliefs is further explored in Chapter 2.

1.2.3. Mathematics teachers' beliefs

From literature three dimensions can be identified when investigating the beliefs of Mathematics teachers. These include beliefs about:

- (i) the nature of Mathematics (the discipline);
- (ii) the teaching of Mathematics (school);
- (iii) the learning of Mathematics (Ernest 1989; Beswick 2012; Paolucci 2015).

In addition, Ernest (1989) theoretically places teachers' beliefs about the nature of Mathematics into three categories:

- (i) Mathematics as a set of effective rules and facts (an instrumentalist or traditionalist belief);
- (ii) Mathematics as a unified body of knowledge that changes over time only by new discoveries (a Platonist or formalist belief);
- (iii) and Mathematics as a dynamic, continually expanding field of human creation and invention (a constructivist belief).

Further, the role of the teacher can be linked to these three categories. The teacher can be regarded as:

- (i) an instructor in the instrumentalist view;
- (ii) an explainer in the Platonist view;
- (iii) a facilitator in the constructivist view.

Teachers' views about the nature of Mathematics are in alignment with their beliefs about the teaching of Mathematics as shown in Table 1.1. The beliefs in the same row are seen to be theoretically consistent with one another and those in the same column are seen as a continuum (Beswick 2012). The current study investigates the beliefs of Mathematics teachers through this theoretical framework. Beliefs about the nature, teaching and learning of Mathematics; problem-solving within a constructivist learning environment; and the theoretical framework are further reviewed and presented in more detail in Chapter 2.

Table 1.1 A continuum of Mathematics Teachers' Beliefs

Beliefs about the nature of Mathematics	Beliefs about the teaching of Mathematics	Beliefs about the learning of Mathematics
Instrumentalist Set of unrelated but effective rules and facts.	Traditionalist perspective The teacher as instructor towards mastery in application. Content-focused with an emphasis on performance. Knowledge is transmitted. The focus is on the teaching the formulas and processes.	Traditionalist perspective Skill mastery, passive reception of knowledge. Passive receiver of knowledge. Learning is an independent and isolated event.
Platonist Static and unified body of knowledge that is discovered, not created.	Formalist perspective The teacher as explainer of existing knowledge. Content-focused with an emphasis on understanding.	Formalist perspective Receiver of knowledge but with the emphasis on the learner actively constructing knowledge and understanding.
Problem-solving A dynamic, continually expanding field of human creation and invention. Engaging in Mathematics is a process rather than a product.	Constructivist perspective The teacher as facilitator of the learning process. Learner-focused. Activities are interactive and learner-centred. Instruction emphasis is on solving problem, generative learning processes and explorative learning.	Constructivist perspective Autonomous exploration through problem posing and problem-solving. Learner takes responsibility for their own learning. Learner socially constructs mathematical knowledge.

It is important to note that Table 1.1 describes a theoretical construct regarding the beliefs of Mathematics teachers and does not propose a direct link between these beliefs and individual teacher practices. The continuum between an instrumentalist belief and that of a problem-solving/constructivist belief system was used to categorise and investigate the problem-solving beliefs of Mathematics teachers at IEB-registered secondary schools.

1.2.4. Mathematical problem-solving in reformed curricula

Since the early 1980s, reformed curricula in Mathematics education have included mathematical problem-solving as an outcome and/or an instructional approach (NCTM 1980; Cockcroft 1982; HMI 1985; Ernest 1989). Firstly, problem-solving is explicitly formulated as a curriculum outcome to be achieved. Examples of this are found in the process standards of the National Council of Teachers of Mathematics (NCTM) (Mathematics 2014) and the South African CAPS document (Department of Basic Education 2011a). Secondly, problem-solving is seen as an instructional approach in itself. The NCTM (Mathematics 2014) and Singapore Mathematics Curriculum (Ministry of Education Singapore 2012) emphasise this. As such, Mathematics is taught through the process of engaging learners in problem-solving. The content and problem activities are designed in such a way as to introduce the mathematical content.

1.2.5. Problem-solving in the South African Mathematics curriculum

The development of problem-solving skills is stated as a specific aim in the CAPS Further Education and Training (FET) phase for Mathematics. This curriculum covers the final phase of schooling in South Africa and prepares learners for the National Senior Certificate (NSC) examinations. The FET phase is taught by secondary school teachers in South Africa. Mathematical problem-solving is described in the CAPS as a means of understanding the world around us and of teaching us to think critically. Learners are expected to develop problem-solving skills and to use these mathematical process skills to identify, investigate and solve problems creatively and critically (Department of Basic Education 2011a: 8). Problem-solving activities are required within each topic in the curriculum. Furthermore, a minimum of 15% of questions in each SBA task should be concerned with problem-solving. Questions related to problem-solving include:

- (i) non-routine problems (which are not necessarily difficult);
- (ii) problems that require higher order reasoning and processing skills;
- (iii) problems that might require the ability to break the problem down into its constituent parts.

Problem-solving is therefore required throughout the Mathematics curriculum in secondary schools and questions related to problem-solving are required in all assessments, thus making it an essential part of the teaching practice of this phase. Furthermore, as a teacher's beliefs are a significant factor in teaching practice, investigating their beliefs about problem-solving becomes crucial to the enactment of the aims of the curriculum.

1.2.6. Problem-solving as routine activity in South African schools

Local studies have investigated teachers taking up problem-solving as a routine activity and have concluded that teachers find it difficult to introduce mathematical problem-solving into their Mathematics lessons (Brodie & Pournara 2005; Webb & Webb 2008; Stols 2013). A local study of 18 DBE schools in Gauteng, South Africa, concluded that teachers were found to spend more time on topics that are procedural and avoided topics that required conceptual understanding and problem-solving (Stols 2013). A further South African study by Brodie and Pournara (2005) reported that some teachers believe that perceived lower-ability learners cannot solve problems on their own. In addition, they believe that incorporating problem-solving questions into lessons involving lower-ability learners will be too time consuming, and this is therefore not included in their teaching. International studies by Silver, Ghousseini, Gosen, Charalambous and Strawhun (2005) and de Freitas and Zolkower (2011) support these findings, but go further to show that teachers are not comfortable with solving non-routine questions themselves, and that teachers find it challenging to deal with the variety of solution strategies that are characteristic of solving non-routine problems. At most, learners are given well-structured problems (routine problems) and are rarely required to solve meaningful problems as part of the curriculum (Jonassen 2000).

In spite of the demand for Mathematical problem-solving in the national curriculum, official reports regarding the NSC examinations consistently indicate the absence of such learning in South African schools. The diagnostic reports on the NSC examinations in 2014, 2015 and 2016 state: "In many cases, participants appeared to cope with lower order questions that require the application of routine procedures that were taught in the classroom. However, where the questions required independent or creative thought, learners were unable to cope. This relates to analytical, evaluative or problem-solving questions." This leads to the key recommendation for teachers to create a learning environment in which learners have the

opportunities to reflect, analyse and evaluate subject content, in order to construct their holistic understanding and to apply the knowledge they have acquired in unseen problems (Department of Education 2014; 2015; 2016; 2017).

These reports and studies conclude that mathematical problem-solving is not a routine activity in Mathematics classrooms in South Africa and that teachers are not creating learning/instructional environments that allow learners to reflect, analyse, evaluate and construct the knowledge needed to engage in Mathematical problem-solving.

However, no study on problem-solving as a routine activity has been conducted in independent schools registered with the IEB and neither was mention made of this in the DBE report on the NSC IEB examination. This makes the current study unique and significant in that it will add to the research on problem-solving within the South African schooling environment.

1.3. Problem statement

The rationale for the study identifies the problem statement for this study:

Mathematical problem-solving is not a routine activity in the learning environment as required by the national curriculum for Mathematics, and as can be expected in privileged schools.

Given that teacher beliefs strongly influence teaching practices the following research question will guide the study.

1.4. Research question

Pertaining to secondary Mathematics teachers in independent schools registered with the IEB in South Africa,

- (i) What are the beliefs of practising secondary Mathematics teachers about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics?
- (ii) How do their beliefs relate to their implementation of problem-solving activities as required by the National Curriculum Statement?

1.5. Purpose and significance of the study

The purpose of the study is to investigate the beliefs of Mathematics teachers about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics. The investigation will not only add to current research into the beliefs of Mathematics teachers, but will also aim to identify the causes of the apparent lack of implementation of problem-solving as a routine activity at secondary schools registered with the IEB. As these secondary schools function within the wider secondary schooling community in South Africa, this research study will also add to the research on teachers' beliefs and the lack of problem-solving as a routine activity within secondary classrooms in South African schools. The research will be done on the basis that teaching practice is significantly influenced by belief and that fundamental beliefs are an obstacle to the implementation of curriculum aims (Handel 2003).

This research study will offer information on classroom practices by highlighting areas of concern and good practice. The research will add to the international body of research conducted on the beliefs about the nature of Mathematics and Mathematics education, as the context in which the investigation takes place is unique. As mentioned in the previous section (1.2.6) this type of study has not been conducted within independent schools registered with the IEB nor have the beliefs of secondary school Mathematics teachers (to the knowledge of the researcher) been investigated in South Africa. In addition, few studies have specifically examined teachers' beliefs in relation to mathematical problem-solving (Xenofontos & Andrews 2014).

The study will also hold significance for several different parties. These include:

- Teacher-training institutions in South Africa: Information collected from the research will offer a means to improve and develop teacher-training workshops which would support the implementation of a Mathematics curriculum oriented to problem-solving;
- Researchers: describing the link between teachers' practices and beliefs will allow educational researchers to understand the teaching process better;
- Schools and teacher employment agencies: The research questionnaire and interviews can be used to evaluate the beliefs and classroom practices of current and potential Mathematics teaching staff.

1.6. Research design and methodology

The study employed two data-collection instruments: a questionnaire which was then followed by one-to-one semi-structured interviews. The questionnaire was designed to collect information on the beliefs of secondary school Mathematics teachers about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics.

The research participants were secondary-school Mathematics teachers from 213 registered IEB schools. The questionnaire was distributed to the target population on various occasions. Initially emails were sent to all schools registered with the IEB requesting teachers to complete an online version of the questionnaire. To raise the response rate, a paper-based version of the questionnaire was distributed (and responses collected) at various IEB conferences, workshops and marking sessions. A total of 124 participants took part in the survey with 95 fully completed questionnaires collected. As the schools are independent, there is no data available on the numbers of Mathematics teachers working within this sector. The questionnaire did not require the teacher to identify the school he/she worked for and could be completed anonymously. However, according to the email addresses provided, teachers from at least 52 different schools (24% of IEB-registered schools) took part in the study. This provides at least a representative sample of schools within the targeted educational sector.

The initial data analysis was used to develop the questions for the semi-structured interviews. These interviews included discussions around classroom practices which probed the participants' espoused beliefs further. In addition, instructional approaches to mathematical problem-solving were also investigated during the interviews. Interviewing all participants of the study was not feasible during the time of this study. Two participants were interviewed in sessions of ± 30 minutes with an additional pilot study done before the two formal interviews. The two participants were chosen on the basis of their belief scores from the quantitative section of the questionnaire, their academic qualifications and their professional qualifications.

A more detailed discussion of the research design and methodology used for creating the questionnaire and analysing the results is included in the research design and methodology chapter of this study. Ethical considerations are also addressed in a later section of this study.

1.7. Chapter overview

This introductory chapter has provided the researcher's background, context of the study and rationale to the study. The rationale of the study included sub-sections that introduced relevant concepts which are further explored in the literature review chapter. Further, this chapter offered sections on the purpose and significance of the study and a summarised section on the research design and methodology of the study. Chapter Two presents a review of literature relevant to the study. Chapter Three details the methodology used during the study, describes the research design, data-collection tool and data-analysis approach. Chapter Four discusses the background data collected from the participants. Chapter Five discusses the quantitative data collected in the questionnaire from the participants while Chapter Six discusses the qualitative data collected in the questionnaire. Chapter Seven examines the qualitative data collected from the semi-structured interviews and Chapter Eight summarises and concludes the study. Chapter Eight also discusses limitations of the findings and identifies areas of future research.

CHAPTER 2

LITERATURE REVIEW

2.1. Introduction

This chapter reviews research conducted on the beliefs of Mathematics teachers and the relationship between beliefs and practice. A short review of reforms in Mathematics education, teacher education and teachers' mathematical knowledge is given in support of this study.

2.2. International and national reforms in Mathematics education

Since the early 1980s reformed curricula in Mathematics education have encouraged the focus of Mathematics teaching to shift towards problem-solving and learning so that the learners are empowered to become creative and confident problem-solvers. In addition, Mathematics teaching should include problem-solving at all levels in schools (NCTM 1980; Cockcroft 1982; HMI 1985; Ernest 1989). This is a move away from an instrumentalist or Platonist view of the nature of Mathematics which emphasises numeracy, facts, rules and procedures in the teaching of the subject, while ignoring links within different areas within the subject (Ernest 1989). The reformed view of Mathematics moves towards a constructivist theory of learning and teaching that encourages problem-solving and the application of higher-order thinking skills, linking concepts between various subjects. This shift away from an instrumentalist view in the 1960s to a constructivist, problem-solving view regarding Mathematics and the teaching and learning of Mathematics has culminated in the 21st century skills framework that requires learners to engage with solving authentic real-world problems (Bransford et al. 2004: 4; Trilling & Fadel 2009). This is an educational framework that aims to create learners who can think and read critically and solve complex problems (Bransford et al. 2004). To enact these reforms, national and international educational governing bodies have included problem-solving in their assessment objectives, syllabus outcomes and instructional approaches in Mathematics education (Department of Basic Education 2011b; Pellegrino & Hilton 2012; Ministry of Education Singapore 2012; Cambridge International Examinations 2016; CCSS 2018; IGCSE 2019).

In South Africa, various educational reforms were initiated following the first democratic general elections in 1994. Major changes to the educational system made in 1996 saw the introduction of a new curriculum for schools. “Curriculum 2005” was an outcomes-based education curriculum (De Waal 2004; The Council on Higher Education 2010), which involved a constructivist view of the teaching and learning of Mathematics (Molefe & Brodie 2010). Further curriculum reforms were introduced in the following decades that were aligned with a constructivist, problem-solving view of Mathematics and the teaching and learning of Mathematics.

The current national curriculum in South Africa aims to produce learners who are able to:

- (i) identify and solve problems and make decisions using critical and creative thinking;
- (ii) work effectively as individuals and with others as members of a team;
- (iii) organise and manage themselves and their activities responsibly and effectively;
- (iv) collect, analyse, organise and critically evaluate information (rather than rote and uncritical learning of given truths);
- (v) demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation
(Department of Basic Education 2011a: 5).

In addition, the national Mathematics curriculum specifies problem-solving as a specific aim: “To develop problem-solving and cognitive skills” (Department of Basic Education 2011a, p.8).

These aims all fit within the broader skills set as required by the framework for 21st century skills (Trilling & Fadel 2009; Partnership for 21st Century Learning 2015). The South African current national curriculum for Mathematics therefore requires the teacher to hold a constructivist view of the teaching and learning of Mathematics with problem-solving as a specific aim, similar to that which is required by international Mathematics curricula.

2.3. The concept of beliefs in education

This section discusses the conceptualisation of beliefs and introduces its importance to educational research.

A main challenge in the research of teachers' beliefs has been to define the construct of beliefs (Cross Francis 2014; Skott 2015). Cross (2009) defines beliefs as the “embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it, developed through membership in various social groups; these beliefs are considered by the individual to be true”. Beliefs are implicitly or explicitly held subjective conceptions that the individual holds to be true even if others do not agree with these (Op't Eynde, De Corte & Verschaffel 2002). This truth then forms the basis and gives relevance to beliefs regarding teaching practices. Beliefs are cognitive constructs that are strong predictors of behaviour. Beliefs therefore significantly influence individuals' perceptions and interpretations of a situation as well as the practices they engage in (Skott 2015). Beliefs are stable and often beyond the immediate control of the person. Beliefs are highly resistant to change, develop over considerable time and often resist change even with training and experience (Cross 2009; Cross Francis 2014).

The nature of beliefs is contextual and when a teacher fails to enact an espoused belief, it could be explained as another belief taking precedence in a certain context (Beswick 2007). This is supported by the view that the organisation of a person's beliefs can be considered to be quasi-logical as described by Green (1971), cited by Cross Francis (2014). According to Cross (2009) this quasi-logical organisation implies that beliefs are not organised based on the *content* of the belief but on *how* they are held. Beliefs are held in different belief dimensions; this means that individuals can hold two incompatible, inconsistent beliefs without internal conflict. These conflicting beliefs are often upheld by a third. An example given by Cross (2009) is that a teacher can believe that “schools should be an environment where students are provided with all opportunities to excel” but then also hold the belief that “students who are not in the gifted classes should not be recommended for advanced math courses”. These conflicting beliefs are acceptable to the teacher because other beliefs support this, such as the belief that “ability is fixed” (Green 1971; Cross 2009; Beswick 2007). The beliefs on ability can be associated with a traditional view of mathematical ability in which a learner's ability is seen as fixed. This is in contrast to a constructivist belief regarding

mathematical ability in which ability develops through the construction of knowledge (Stipek & Givvin & Salmon & MacGyvers 2001).

The view that teachers' beliefs significantly influence their instructional practices is the main driving force behind research efforts into teacher's beliefs. The effect of teachers' beliefs on the support of educational reforms in Mathematics classrooms has been an important focus of educational research (Ernest 1989; Speer 2008; Cross Francis 2014).

2.4. Research into the beliefs of Mathematics teachers

A teacher's instructional practices are influenced by a number of factors both within and beyond the teacher's control. These factors include:

- (i) Knowledge factors: content knowledge, pedagogical knowledge, pedagogical content knowledge (Ball 1990; Moses & Mji 2006; Hill & Sleep & Lewis 2007; Speer 2008);
- (ii) School factors: curricular context, curriculum material, management, classroom management, time constraints, expectations of parents, learners and schools (Herbel-Eisenmann, Lubienski & Id-Deen 2006; Paolucci 2008);
- (iii) Psychological factors: goals and efficacy (Speer 2008);
- (iv) Socio-historic and socio-economic factors (Howie 2003; Sztajn 2003);
- (v) Contextual factors and student needs (Sztajn 2003; Herbel-Eisenmann, Lubienski & Id-Deen, 2006; Speer 2008).

While these factors all contribute to the teaching environment and influence a teacher's instructional practices to some degree, beliefs are one of the most influential factors.

Research has shown that a teacher's beliefs about Mathematics and the teaching and learning of Mathematics are significant factors in predicting classroom practices (Shirk 1972; Bawden & Rurke & Duffy 1979; Thompson 1984; Ernest 1989; Hiebert et al. 2003; Li & Yu 2010; Webb & Webb 2004; Cross Francis 2014; Xenofontos & Andrews 2014).

Thompson (1984) notes that prior to 1984, educational research into instructional practices focused on teachers' content knowledge of Mathematics and the impact this had on their instructional practices. The research did not focus on the impact that teachers' conceptions

(beliefs, views and preferences) of Mathematics and the teaching and learning of Mathematics had on a teacher's instructional practices (Thompson 1984). There were two reasons for this:

- (i) teacher behaviour and practices in class can be observed, while cognition has to be inferred by the researcher;
- (ii) the view that the factors causing a teacher to "act" in a certain way are external to the person (Fenstermacher 1978; Thompson 1984).

The few studies that did focus on teacher cognitive conceptions suggest that teachers' decision-making during lessons was based on instinct and intuition and not on rational and reflective thought. However, it is argued that even if a teacher were acting unconsciously, there would still be the potential that these cognitive *conceptions* (emphasis mine) would lead to habitual practices in the teacher's lessons. Therefore, any attempt to improve or reform teacher practices should include the understanding of the cognitive beliefs, views and preferences of the teacher (Thompson 1984; Ernest 1989). In addition, the need for research into teachers' thought structures arose out of educational reforms in Mathematics proposed and recommended by international government bodies and research publications in the early 1980s (NCTM 1980; Cockcroft 1982; HMI 1985). These reforms shifted the focus of Mathematics teaching towards problem-solving and the application of higher-order cognitive skills and away from procedural knowledge and the use of this knowledge in routine activities. A fundamental change was therefore required in teaching practices, where the role of the teacher would shift from instructor to facilitator. The teacher would have to adapt to the new role and acquire the necessary pedagogical knowledge. Further, Ernest (1989) emphasises the point that to implement these reforms would require a fundamental understanding of a teacher's beliefs and the influence of beliefs on instructional practices.

Research into teacher cognition was termed the "missing" programme by Shulman (1986) as educational research up to the early 1980s overlooked the psychological foundations of teacher practices. Investigations into aspects of a teacher's psychology of teaching identified two distinct facets: (i) A teacher's **thought processes** which include planning, reflection and interactive decision-making (ii) and a teacher's **thought structures** which include the beliefs and knowledge that are stored as schemas in the mind of the teacher. Research into the psychology of teaching tended to focus on the thought processes of the teacher and how these influenced instructional practices and not on the thought structures of the teacher.

Ernest (1989) notes that previous research has pointed to the complexity of the relationship between teachers' beliefs and their instructional practices. To give direction and foundation to research into teacher beliefs, views and perspectives about Mathematics and the teaching and learning of Mathematics, Ernest proposes an analytical model of the different types of Mathematics teachers' beliefs as given by their relationship to instructional practices. The model for investigating a teacher's cognitive thought structures (specifically beliefs) comprises three components: the teacher's beliefs about (i) the nature of Mathematics, (ii) the nature of Mathematics teaching and (iii) the process of learning Mathematics. The three components are discussed separately in the following sections.

2.5. Beliefs about the nature of Mathematics

A teacher's belief or conception regarding the nature of Mathematics can be categorised under three philosophies of Mathematics as described by Thompson (1984). These philosophies are important as they underpin teachers' beliefs and ultimately influence their instructional approach (Thompson 1984; Ernest 1988). They include:

- (i) The instrumentalist view: Mathematics is viewed as a useful set of rules, procedures, skills and facts;
- (ii) The Platonist view: Mathematics is viewed as a static body of knowledge. Mathematics is a static, unchallengeable product of discovery and not a creative act of humanity;
- (iii) The problem-solving view: Mathematics is viewed as a continuously expanding and creative field of human knowledge. Mathematics results are not set and are open to revision and human inquiry (Thompson 1984; Ernest 1988; Ernest 1989; Cross 2009).

The three psychological systems of belief are related to practical classroom outcomes as it could be expected that a teacher with a problem-solving view of Mathematics would accept alternative methods and approaches to problems in class, while a teacher with a Platonist or instrumentalist view would look for a single method for solving the problem (Ernest 1988). The following section discusses teacher beliefs about the teaching of Mathematics.

2.6. Beliefs about the teaching of Mathematics

Personal views about the nature of Mathematics teaching have an effect on how individual teachers see themselves while teaching the subject and the various activities they would engage in. The individual roles assumed by the teacher's guide their choice of classroom activities. Three roles are identified with associated intended outcomes:

- (i) Instructor: skills mastery with correct performance;
- (ii) Explainer: conceptual understanding with unified knowledge;
- (iii) Facilitator: confident problem-posing and -solving (Ernest 1988).

Further, research differentiates a teacher's beliefs about the teaching of Mathematics into three different perspectives: the traditionalist, formalist and constructivist views (Ponomareva & A. Kardanova & E. Hannula & M. Pipere & A. Lepik, M. 2015). These beliefs can be associated with the previously identified roles in the following ways:

- (a) Traditionalist belief about the teaching of Mathematics: In this belief teachers view themselves as instructors and Mathematics teaching is seen as the mastering of procedural skills and the using of rules and formulas by the learner. The teachers believe their main role is to present the material clearly, correctly, precisely and to give correct procedures and methods to problems. Teaching is content-focused with the emphasis on performance.
- (b) Formalist belief about the teaching of Mathematics: In this belief the teachers view themselves as explainers and Mathematics teaching is about the correct use of mathematical language, rigorous proof, logic and exact definitions. Teaching is content-focused with the emphasis on conceptual understanding.
- (c) Constructivist belief about the teaching of Mathematics: In this belief the teacher's view is that knowledge is actively constructed by the learner and is built on existing knowledge in the context of prior experience (Beswick 2005). The teachers view themselves as facilitators in the construction of knowledge. Mathematics teaching is about facilitating the construction of mathematical knowledge with a focus on developing thinking processes

and creative steps during problem-solving activities. Teaching is learner-focused with emphasis on social interactions (Ernest 1988; Van Zoest, Jones & Thornton 1994; Adam 2012; Ponomareva et al. 2015).

A teacher's beliefs about the learning of Mathematics are closely linked to the teaching of Mathematics as described in the following section 2.7.

2.7. Beliefs about the learning of Mathematics

The third component of a Mathematics teacher's beliefs is how a teacher views the learning of Mathematics. This includes the teacher's view of the processes involved in learning Mathematics, the learner's role during the learning process and the appropriate tasks in which they should engage. A teacher's beliefs of the learning of Mathematics is vital to the way the learner experiences the learning of Mathematics (Ernest 1989). Ernest (1988) identifies four simplified models for the learning of Mathematics:

- (i) compliant behaviour and mastery of skills;
- (ii) receptor of knowledge;
- (iii) active construction of understanding;
- (iv) exploration and autonomous pursuit of own interest.

These models can similarly be grouped within the three approaches listed in section 2.6.

- (i) Traditionalist belief of the learning of Mathematics: Mathematics learning is about skill mastery and the passive reception of knowledge.
- (ii) Formalist belief about the learning of Mathematics: Mathematics learning is about the reception of knowledge but with an emphasis on the active construction of understanding.
- (iii) Constructivist belief about the learning of Mathematics: Learning is constructed on existing knowledge in the context of prior experience (Beswick 2005). Mathematics learning take place through autonomous problem-posing and -solving and through the active construction of knowledge by the learner (Ernest 1989; Ponomareva et al. 2015).

The following section gives a theoretical framework within which teachers' beliefs about the nature of Mathematics, the teaching and learning of Mathematics, and problem-solving can be investigated.

2.8. A framework for investigating Mathematics teachers' beliefs

The theoretical framework in Table 2.1 is adapted from the framework given by Beswick (2005). The framework in Table 2.1 combines findings from the literature reviewed in the previous three sections as a means of providing a theoretical framework within which to investigate Mathematics teachers' beliefs. Table 2.1 also displays a summary of the three components of a Mathematics teacher's beliefs across the three philosophical views. As with the framework given by Beswick (2005), beliefs in the same row are seen to be theoretically consistent with one another and those in the same column are viewed as a continuum..

Table 2.1 A Continuum of Mathematics Teachers' Beliefs

Beliefs about the nature of Mathematics	Beliefs about the teaching of Mathematics	Beliefs about the learning of Mathematics
Instrumentalist Set of unrelated, but effective, rules and facts.	Traditionalist perspective The teacher as instructor towards mastery in application. Content-focused with an emphasis on performance. Knowledge is transmitted. The focus is on the teaching formulas and processes.	Traditionalist perspective Skill mastery, passive reception of knowledge. Passive receiver of knowledge. Learning is an independent and isolated event.
Platonist Static and unified body of knowledge that is discovered, not created.	Formalist perspective The teacher as explainer of existing knowledge. Content-focused with an emphasis on understanding.	Formalist perspective Receiver of knowledge but with the emphasis on the learner actively constructing knowledge and understanding.
Problem-solving A dynamic, continually expanding field of human creation and invention. Engaging in Mathematics is a process rather than a product.	Constructivist perspective The teacher as facilitator of the learning process. Learner-focused. Activities are interactive and learner-centred. Instruction emphasises problem solving, generative learning processes and explorative learning.	Constructivist perspective Autonomous exploration through problem posing and problem solving. Learner takes responsibility for own learning. Learner socially constructs mathematical knowledge.

It is evident from the literature reviewed in the previous sections and summarised in Table 2.1 that the problem-solving view of the nature of Mathematics and its associated characteristics, which have to do with teaching and learning, are consistent with a constructivist view of learning.

The next section (2.9) further reviews the literature on Mathematics teachers' beliefs but the focus is on the relationship between their espoused beliefs and their instructional practices.

2.9. Research into the relationship between Mathematics teachers' beliefs and their classroom practices

Research questions on the relationship between beliefs and instructional practices fall most often within one or more of the following specific questions (Ernest 1989):

- (i) Are there differences between a teacher's professed beliefs and his/her instructional practices?
- (ii) How can these differences be explained?
- (iii) Are different instructional practices related to different beliefs?

This study relates to (i) and (iii) as it investigates the teachers' espoused beliefs and their alignment with the aims and objectives of the national curriculum.

The research questions in the reviewed studies, and in this study, relate to (i) and (iii) above. The participants in the reviewed studies are predominantly in-service and/or secondary school Mathematics teachers. This study involved secondary school Mathematics teachers as participants.

Research by Cross (2009) on five in-service teachers revealed that in general, beliefs were very influential in their day-to-day teaching strategies and that their beliefs about the nature of Mathematics were the primary source for beliefs about the teaching and learning of Mathematics. The study went further and investigated how teachers' beliefs hindered or supported the implementation of reformed-orientated teaching practices. The study concluded that the participating teachers all held strong beliefs that stemmed from their own schooling experience. In addition, a number of the participants held beliefs that did not align with the aims and objectives of their requisite curriculum, that is, the aims of the National Council of

Teachers of Mathematics (NCTM). These teachers emphasised the use of practice and not the use of reasoning and problem-solving in their teaching as required by their curriculum. Although the study concluded that the beliefs held by the teachers shaped their instructional practices, it is noted that research has been inconclusive regarding the degree of influence that beliefs have on instructional practices. There is no linear relationship between beliefs and practices and other factors do influence classroom practices. Additionally, contradictions between beliefs and practices are often noted where teachers espouse reformed-orientated beliefs which are not evident in their teaching practices (Thompson 1984; Stipek et al. 2001; Beswick 2007; Cross 2009). Cross (2009) cautions that studies into the relationship between beliefs and instructional practices should not only consider the verbalised beliefs of the teacher but should also seek evidence in classroom practices. Further, Cross (2009) notes in her study that beliefs about the teaching and learning of Mathematics differ according to the ability of the group being taught and also differ according to the section of the syllabus being covered. These contradictory views are evident in other studies (Fuchs & Fuchs & Hamlett & Karns 1998; Torff & Warburton 2005). Such conflicting beliefs are often upheld by a third belief, as discussed in section 2.3. Similar contradictory beliefs were evident in this study and are discussed further in Chapter 5.

Cross (2009) concluded that for learners to become active, creative, think critically and able to solve problems, it is necessary for teachers to possess beliefs that support a learning environment which is learner-centred and would develop problem-solving skills. Cross concludes that teachers who do not hold these beliefs should be enrolled in programmes that can develop these beliefs, more specifically their beliefs about the nature of Mathematics. Belief change should be an ongoing process of reflection, confrontation and awareness of one's beliefs. Pre-service and in-service training can begin the process of changing beliefs but teachers must be confronted continuously with experiences that challenge their beliefs as only then is belief change likely to become permanent. Further, targeting pre-service and in-service teachers' Mathematics content knowledge rather than teaching methodologies is more likely to enact the desired belief change, as courses focusing on Mathematics methodology have not had the desired effect (Cross 2009).

An investigation of secondary Mathematics teachers who held beliefs that were consistent with the tenets of constructivism was conducted by Beswick (2007). Again, the study investigated the nature and relationship between beliefs and whether these beliefs were a predictor of classroom practice. Beswick acknowledges the complex nature and relationship

between beliefs and instructional practices. In addition, even the direction of influence is disputed by a number of studies, as some studies suggest that teaching practice influences the beliefs of the teacher and vice versa (Handel 2003; Beswick 2005; Beswick 2007).

Beswick notes that very few studies into teachers' beliefs and the relationship between beliefs and practice have focused on improving Mathematics education. Her study identifies nine beliefs which are broadly held by teachers who create constructivist learning environments promoting problem solving. The study hypothesises that enacting these nine beliefs would directly improve the learning environment in Mathematics classrooms, and in so doing bring about the required reforms. The nine beliefs were grouped by Beswick (2007: 114) within the three belief dimensions as described previously in this chapter. A summary of this grouping is given below.

(i) Beliefs about the nature of Mathematics:

1. Mathematics is about connecting ideas and sense making: a problem-solving view of Mathematics;
2. Mathematics is fun. This leads to a confident and genuine interest in Mathematics.

(ii) Beliefs about the teaching of Mathematics:

3. It is the teacher's responsibility to control the classroom discourse. In other words, it is the teacher's responsibility to make sure that the construction of knowledge takes place in the classroom;
4. It is the teacher's responsibility to facilitate the construction of mathematical knowledge;
5. It is the teacher's responsibility to induct learners into the accepted ways of thinking and communicating in Mathematics;
6. The teacher is the authority with respect to acceptable behaviour that is expected of a learner (social norms) in the classroom;
7. It is the teacher's responsibility to engage in continued professional development.

(iii) Beliefs about the learning of Mathematics:

8. Students' learning is unpredictable. This belief is required to create an environment in which knowledge is individually and socially constructed;
9. All learners can learn Mathematics.

Beswick argues that teachers holding these nine beliefs are more likely to create a constructivist-learning environment. The nine beliefs can be categorised under Ernest's (1989) problem-solving philosophical beliefs about the nature of Mathematics as cited by Beswick (2005; 2012). This again links the constructivist learning theory with the problem-solving view about the nature of Mathematics.

Research of teachers' beliefs most often investigates the beliefs held on the continuum between a traditional beliefs and a reformed problem-solving beliefs about instructional practices. In theory, traditional instruction is associated with a behaviourist learning theory, while a progressive/reformed instruction is associated with a socio-constructivist theory of learning. In traditional instructional practices, rote learning of formula, rules and procedures is emphasised. Learning is an independent and isolated event with knowledge transmitted from teacher to learner. In contrast, socio-constructivist instructional practices emphasise problem-solving, metacognition and discovery. Learning happens collaboratively (socially) with the learners actively involved in their learning with teachers facilitating the discussions and guiding the construction of knowledge (Handel 2003).

Handel (2003) argues in his study that teachers' beliefs originate from their own traditional schooling and that the beliefs reproduced in the classroom environment are due to the conservative or traditional nature of schools which in turn reinforces the traditional beliefs. Teachers' beliefs affect their teaching. Therefore, a teacher's beliefs will shape the way a teacher thinks and feels about Mathematics and the teaching and learning of Mathematics (Stipek et al. 2001; Oksanen & Hannula 2013). If a teacher's beliefs do not support the aims and beliefs of curriculum reforms, then this might hinder the implementation of these reforms. Handel (2003), however, adds that even if teachers' beliefs match those of the reform curricula, the traditional nature of the schooling system will make it difficult for the teachers to enact their espoused beliefs. The relationship between beliefs and practice is complex and a number of external factors also influence practices. These include pressure from school and parents, lack of preparatory time to cover content and the challenges posed by different learner abilities (Handel 2003).

A number of studies involving pre-service teachers found that they largely hold beliefs which are traditional in nature. For example:

- Mathematics is either right or wrong (Benbow 1993; Nisbet & Warren 2000);
- Mathematics requires neatness and speed (Civil 1990);
- Mathematics requires logic and not intuition (Frank 1990);
- The learning of Mathematics is based on the memorisation of facts and rules (Lappan & Evan 1989; Wood & Floden 1990; Southwell & Khamis 1989; Benbow 1993; Foss & Kleinsasser 1996);
- Mathematical ability is innate (Frank 1990; Foss & Kleinsasser 1996).

Further, a study by Howard, Perry and Lindsay (1997) with 249 secondary in-service teachers in Sydney, Australia, showed two distinct belief groupings of teachers. The larger group could be associated with an instrumentalist/traditionalist view of the teaching and learning of Mathematics while the second smaller group held more constructivist views associated with the problem-solving view of the teaching and learning of Mathematics. A similar grouping in many of the beliefs held by the participants in this study was also noted. In addition, other studies involving in-service teachers also concluded that the participants largely hold traditionalist beliefs about the teaching and learning of Mathematics (Handel & Herrington 2003; Handal & Bobis 2004).

In addition to these influencing factors, researchers have suggested that professional development programmes designed to reform teachers' beliefs have largely been ineffective, as teachers filter what they learn through their existing beliefs (Cohen 1990; Stipek et al. 2001).

In conclusion, the reviewed studies on the relationship between beliefs and classroom practices found that the relationship is complex in nature, the participating teachers largely hold traditional beliefs, and traditionalist views could be associated with traditional practices. However, all studies concluded that teachers' espoused beliefs were not always evident in observations of their classroom practices because of other prioritised beliefs and/or external factors.

2.10. Mathematical knowledge of teachers

A teacher's mathematical knowledge provides the foundation for a teacher's pedagogical knowledge and teaching practices (Shulman 1986; Ernest 1989; Ball, Thames & Phelps 2008). A teacher's mathematical knowledge includes not only basic factual knowledge of Mathematics but also the conceptual knowledge of the underlying structures and principles of the discipline. This means that a teacher needs to understand, not only the *how* but also the *why* of any mathematical content (Shulman 1986; Ball et al. 2008). A teacher's understanding of learners' methods, learners' misconceptions, and the teacher's explanations and demonstrations are all dependent on the teacher's mathematical knowledge (Ernest 1989). Mathematical knowledge is therefore one of the most important factors regarding the quality of Mathematics teaching in the classroom (Blömeke & Delaney 2012). In addition, a teacher's mathematical knowledge is a significant influencer of learner achievement even in lower grades (Ball & Hill & Bass. 2005; Hill & Rowan & Ball & 2017; Baumert & Kunter & Blum & Brunner & Jordan & Klusmann & Krauss & Neubrand & Tsai 2017).

Further, Manouchehri and Goodman (2000) noted a clear difference in the beliefs of teachers with different levels of mathematical knowledge and concluded that mathematical knowledge was the greatest influence in evaluating and implementing the reforms required by a problem-solving approach to the teaching and learning of Mathematics. Moreover, pre-service teachers with the desired progressive beliefs have attributed the development of these preferred beliefs to tertiary Mathematics courses and not school-level Mathematics (Paolucci 2008; Paolucci 2015). Advanced courses in Mathematics seem to have the potential to influence a teacher's beliefs and the implementation of reforms within curricula. It is therefore understandable that teachers and mathematicians have indicated that the limited understanding of what Mathematics is contributes to the inability of teachers to implement reform practices in the classrooms (Blömeke & Delaney 2012).

Considering a teacher's mathematical content knowledge when investigating teachers' espoused beliefs and the implementation of problem-solving as routine activity in the classroom is therefore important in this study.

2.11. Teacher professional qualifications and development

Following the first general election of 1994 and the removal of the apartheid education system, various reviews of South Africa's educational system were initiated. Changes to the educational system were introduced in 1996. Major changes came in the form of the National Qualifications Framework (NQF) and a new schooling curriculum "Curriculum 2005"; both were based on the outcomes-based education (OBE) theory (De Waal 2004; The Council on Higher Education 2010). These reforms included changes to educator qualifications that by April 2002 would reflect the Norms and Standards for Educators (South Africa 2000). The reforms included a constructivist and social view of the teaching and learning of Mathematics (Molefe & Brodie 2010). The Postgraduate Certificate in Education (PGCE) was introduced as a post bachelor's degree qualification offered at tertiary level in South Africa. The PGCE is a general teacher qualification which prepares secondary school educators for teaching the aims and objectives of the national curriculum (South Africa 2000; The Council on Higher Education 2010). This teacher qualification replaced the Higher Diploma in Education (HDE), with the first graduates at universities in 2002 (University of Stellenbosch 2001; Molefe & Brodie 2010).

To be allowed to practise as a Mathematics teacher at secondary school level, a suitable degree or post-degree certificate is required in South Africa. Currently, pure Mathematics at first-year level is required as a prerequisite to enrol for the subject Didactic Mathematics (Grades 7-9) in a PGCE programme (University of South Africa 2018), while Mathematics at second-year level is required as a prerequisite to enrol for the subject Didactic Mathematics (Grades 7-12) in a PGCE programme. This qualification prepares teachers for offering Mathematics in secondary schools (University of Stellenbosch 2018; University of Cape Town 2018; University of South Africa 2018).

Often teachers are not seen as learners during in-service training. Instead, the teacher is simply seen as lacking the required skills or knowledge and their prior knowledge, experiences and beliefs are not taken into account. This is ironic as teachers are taught to view learning as a constructive process (Ball 1988). Understanding teachers' beliefs and views on Mathematics and the teaching and learning of Mathematics is therefore of great importance. It will enhance teacher training and help implement the required reforms in teaching.

If Mathematics education is expected to include a significant amount of practical work, investigations, communication, collaboration, critical thinking and problem-solving, then so too must teacher education (Cockcroft 1982).

Teacher education and in particular teacher education in South Africa is of significance to this study as it can be argued that teacher training also affects the beliefs of Mathematics teachers. Investigating the beliefs held by teachers trained under pre-reformed and post-reformed curricula will not only enrich this study but could also inform the teacher-training programmes in South Africa.

2.12. Summary

The main areas discussed in this literature review have included the three components of Mathematics teachers' beliefs: (i) beliefs about the nature of Mathematics, (ii) beliefs about the teaching of Mathematics, and (iii) beliefs about the learning of Mathematics. A framework within which these three components can be investigated was presented. The chapter has examined the concept of belief structures and presented research on the relationship between teachers' beliefs and their instructional practices. The role of problem-solving within a constructivist learning environment has also been highlighted. Finally, the significance of mathematical knowledge and the impact of teacher training on teachers' beliefs have been presented. The following chapter describes the research design and methodology used in this study.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

The study was designed to investigate and explore the beliefs of participating teachers about the nature of Mathematics, the teaching and learning of Mathematics and mathematical problem-solving. This chapter details the research design and methodology employed to do so.

3.1. Research methodology

A multi-method approach was utilised in this research study. Methodological triangulation was used as described by Cohen, Lawrence and Morrison (2007: 143). This methodological approach utilises both quantitative and qualitative research methods. A questionnaire containing both quantitative and qualitative items was employed to collect data on the research objectives. Using both methods within the same questionnaire increases the validity and reliability of the results (Cohen et al. 2007:143). Results and findings from the analysis of the questionnaire data were further explored through one-on-one semi-structured interviews. This qualitative research method further increased the validity and reliability of the research study.

3.2. Subject population

The research question identifies the subject population as secondary school Mathematics teachers. The subject population of secondary school Mathematics teachers from independent schools registered with the Independent Examinations Board (IEB) in South Africa was chosen for the following two reasons:

- (i) the researcher's involvement within the independent school sector and schools registered with IEB would facilitate the collection of data;
- (ii) the independent schools registered with the IEB predominantly consist of schools that claim they offer quality education with high academic standards, small classes, structured, safe and nurturing learning environments, accountable school

governance and adequate learning resources (AdvTech 2018; Curro 2018; IEB 2018a; ISASA 2015a; ISASA 2015b; Reddam House 2018).

At the end of 2017, the IEB had 213 registered schools (with full-time candidates) with 11 322 learners writing the National Senior Certificate (NSC) as their school-leaving examination of which 98,76% achieved a pass result (IEB 2017a). This pass rate is much higher than the 75,1% pass rate of the Department of Basic Education (DBE). In particular, for Mathematics the pass rate for the DBE was 51,9% (Department of Basic Education 2017) and for the IEB, it was 95,1% of which 26,5% achieved a pass higher than 80% compared to the 0,9% in the state exams (IEB 2017b). The IEB schools therefore produce notable higher results compared to the DBE in the NSC examinations. This can be seen as evidence of the quality, in general, of the teaching environment offered at these schools. With the good quality of school and teaching environments, Mathematics teachers from IEB schools in general would have fewer outside factors that influence their teaching practices. Beliefs would therefore be a remaining factor to investigate when looking at the apparent lack of problem-solving as a routine activity in their classrooms.

The national Mathematics curriculum specifies problem-solving as a specific aim: “To develop problem-solving and cognitive skills”, (Department of Basic Education 2011a: 8). In addition, both the Curriculum Assessment Policy Statements (CAPS) and the Subject Assessment Guidelines (SAGs) for Mathematics require problem-solving to form part (15%) of all School-based Assessments (SBA) tasks. As given in the second reason for the choice of subject population, the schools registered with the IEB claim to offer a positive teaching and learning environment, and together with a qualified and experienced teaching staff (see Chapter 4), a reasonable expectation would be that the objectives and requirements of the national curriculum should be enacted, in particular with regard to the need for problem-solving. Further, the good quality teaching environment together with teachers’ skill levels minimises the outside factors that influence teacher practices. Various studies have identified a number of factors that influence teachers’ classroom practices and the implementation of reforms in the classroom. The factors include: pedagogical knowledge, content knowledge, pedagogical content knowledge, resources, curriculum, standards, goals, efficacy, culture, socio economic factors and time constraints (Howie 2003; Moses & Mji 2006; Paolucci 2008; Speer 2008; Cross Francis 2014). But teachers’ beliefs about Mathematics and Mathematics education have been identified as the most significant influence on their classroom practices (Ernest 1989; Hiebert et al. 2003; Webb & Webb 2004; Li & Yu 2010;

Cross Francis 2014; Xenofontos & Andrews 2014). It was therefore deduced by the researcher that if specific aims, objectives and requirements of the curriculum pertaining to problem-solving were not pursued by the teachers, that beliefs (for this subject population) would be a significant contributor to the lack of problem-solving as routine activity in their classroom practices. The research collection instruments were therefore been designed to collect data from IEB schools on secondary Mathematics teachers' beliefs about the nature of Mathematics, the teaching and learning of Mathematics and about problem-solving.

The research participants are secondary school Mathematics teachers from 213 (as end 2017) registered IEB schools. As the schools are independent there is no data available on the number of Mathematics teachers working within this sector. The questionnaire did not require the teacher to identify the school he/she worked at. In hindsight this should have been included. This would have given a clearer idea of how many schools were represented in the study. However, according to the email addresses provided by the participants, at least 52 different schools could be identified. This represents 24% of IEB registered schools. According to literature at least 30 cases should be considered if some sort of statistical analysis is to be used on the data sample (Cohen et al. 2007: 101). This requirement has been met and a representative sample of the schools (from targeted educational sector) was therefore present in the sample.

3.3. The questionnaire

A questionnaire as the data collection instrument for this study had the following advantages:

- (i) the whole subject population could be targeted quickly, easily and inexpensively by using online survey software and distributing the survey request by email to the subject population (Cassim 2017). A larger representative sample would potentially increase the credibility and accuracy of the findings;
- (ii) anonymity of the participating teachers could be guaranteed as the survey did not require teachers to identify themselves or their teaching institution;
- (iii) the quantitative sections of the instrument provided structured and numerical data that could easily be administered and analysed by the researcher (Cohen et al. 2007: 317);

- (iv) the qualitative section of the instrument provided open-ended responses within a traditional quantitative data collection instrument. In addition, this allowed the respondents flexibility in their responses. The respondents could therefore give honest and personal views rather than the ticking of boxes as in the quantitative section.

The disadvantages of using a questionnaire are summarised below (Cohen et al. 2007; Cassim 2017):

- (i) the traditional low response rate to a questionnaire.
To counter this, a number of subsequent occasions were identified, after the initial requests via email to complete the survey, at which time a paper-based survey was personally distributed and collected by the researcher to increase the response rate;
- (ii) the quantitative questions do not fully capture the reasoning behind certain findings (Cassim 2017).

The data questionnaire therefore, in addition to the quantitative questions, contains open-ended questions to elicit the participants' own interpretation and responses to certain topics. These were used as a comparison to the findings from data collected from the quantitative section. In addition, the findings were further investigated in the interview phase of the study;

- (iii) the impersonal nature of collecting data via a questionnaire.

To counter this, the researcher addressed the participants in a personal letter, asking for assistance in the study so as to elicit more questionnaire responses. In addition, on various occasions the researcher personally addressed a body of teachers on the rationale for the study, asking them for assistance in the study;

- (iv) the danger of bias if questions are not worded carefully.

To limit the possibility of bias, the questionnaire was reviewed by the study leader, the ethics committee and in a pilot study. The pilot questionnaire was completed by two qualified and experienced secondary Mathematics teachers from an independent school in South Africa. The questionnaire was adapted after the input from these sources.

The following sections detail the questionnaire design, data collection and analysis. The complete questionnaire is given in Appendix E.

3.3.1. Questionnaire design

The questionnaire was designed with the following research objectives:

- (i) to collect background information on the participants, including their academic and professional qualifications;
- (ii) to investigate participants' beliefs about problem-solving;
- (iii) to investigate participants' beliefs about the nature of Mathematics;
- (iv) to investigate participants' beliefs about the teaching and learning of Mathematics.

The final version of the questionnaire was developed after consideration of several existing questionnaires created for and used in other published research studies (explained further in 3.3.1). During the developmental phase, various academic professionals were consulted. Firstly, the study leader was consulted throughout the drafting of the questionnaire items; secondly a statistician was consulted (from the Centre for Statistical Consultation at the University of Stellenbosch) to assist with the design of the quantitative items; and finally, input was given by the Research Ethics Committee (Humanities) from the University of Stellenbosch.

In addition, three pilot questionnaires were completed by secondary Mathematics teachers from an IEB school. Piloting the questionnaire increases the reliability, validity and practicability of the questionnaire (Cohen et al. 2007: 341). These results were excluded from the final collected data to decrease the potential of bias occurring (Cassim 2017: chapter 3). A short interview to address and discuss concerns about the pilot questionnaire was held with each of the three participants. This not only assisted with the design of the questionnaire but also with the intended future interviews regarding the research. Each participant was assured of confidentiality before their participation in the pilot study.

The final version of the questionnaire was converted to an on-line version and uploaded to the University of Stellenbosch's online survey software, Checkbox. Using an online survey assisted in reaching the entire target population via email. The data could also at any time

during the collection phase be downloaded and observed by the researcher and study leader. In addition, the online version also makes the analysis of data easier as the data is already available in electronic form. The following section details the particular items of the questionnaire.

Questionnaire Section 1: Background information

This section includes questions on participants' teaching experience, age, academic and professional qualifications. This information informed the investigation of academic and professional qualifications of the participants and how this correlated with their beliefs.

Questionnaire Section 2: Qualitative Section

This section contains open-ended questions to elicit qualitative responses. It has items adapted from a questionnaire used by Paolucci (2008) and from an interview protocol by Xenofontos and Andrews (2014). Paolucci (2008) investigated the beliefs of prospective Mathematics teachers at Irish universities, with the aim of exploring how their tertiary studies in Mathematics have aligned their mathematical beliefs with reforms in the Mathematics curriculum (Paolucci 2015). The study by Xenofontos and Andrews (2014) compares prospective elementary teachers' beliefs (at the end of their studies) about Mathematics according to the three dimensions of teacher beliefs. Questions used in the interview protocol of the study by Xenofontos and Andrews (2014) were adapted and incorporated in this section of this study.

Questions in this section were grouped to reflect the belief dimensions:

- R1-R3: Investigates the participant's beliefs about the nature of problems and problem-solving within the **discipline** of Mathematics.
- R4-R8 Investigates the participant's beliefs about the nature of problems and problem-solving within the **teaching** of Mathematics.
- R9-R11 Investigates the participant's beliefs about the nature of problems and problem-solving within the **learning** of Mathematics.

- R12-R14 Investigates the participant's own beliefs about **being** a problem-solver.

Questionnaire Section 3: Quantitative section

The quantitative data questionnaire is divided into five sections. The first two sections include questions that investigate the theoretical dimensions of the beliefs of Mathematics teachers. The following three sections include questions that further investigate the teachers' beliefs and practices on (i) Grade 12 assessments, (ii) instructional approaches relating to problem-solving and (iii) time spent in class on problem-solving. The five sections are:

1. Beliefs about the nature of Mathematics (NM1-NM5);
2. Beliefs about the teaching and learning of Mathematics (BM1-BM10);
3. Views on Grade 12 Assessment (BM11: P1-P4);
4. Instructional approach to problem-solving (BM12: PS1-PS3);
5. Class-time per week on non-routine problem-solving (BM13).

The quantitative items were items adapted from questions used in the following studies:

- (i) A study by Zakaria and Murisan (2010) investigated the beliefs of trainee Mathematics teachers (n=100). The questionnaire items used in the quantitative section of this study were adapted from the Mathematics Beliefs Instrument used in earlier studies on teachers' beliefs by Hart (1987). The Cronbach alpha reliability index for beliefs about the nature of Mathematics was reported at 0,74; and beliefs about the teaching of Mathematics at 0,84; and learning of Mathematics at 0,70 in the study by Hart. These values show a strong internal consistency (Cohen et al. 2007: 506) which is in contrast to the values reported in this study. The reliability analysis for this study showed a very low internal consistency between the items in the two belief dimension categories: for the nature of the discipline of Mathematics items at 0,42 and for beliefs about the teaching and learning of Mathematics items at 0,55. The reliability issue with this data instrument was addressed in the data analysis phase of the study by analysing question items individually.
- (ii) A study investigating beliefs by Webb and Webb (2008) at the Nelson Mandela Metropolitan University in Port Elizabeth used the Standards

Belief Instrument by Zollman and Mason (1992), and question items developed and tested by Pehkonen and Törner (2004). Their study also investigated beliefs of pre-service teachers. Items from this instrument were also adapted and included in the quantitative section of this study to investigate beliefs of Mathematics teachers. Reliability scores on internal consistency for these studies were not given.

A four-point Likert scale ranging from 1 (strongly disagree) to 4 (strongly agree) is used to investigate teachers' beliefs on the discipline of Mathematics (NM1-NM5) and their beliefs on the teaching and learning of Mathematics (BM1-BM10). A rating of 1 indicated a strongly instrumentalist view and a 4 showed a strongly constructivist/problem-solving view by the respondent. Reversed items are indicated by (-) as explained below. During the data analysis the scores were mapped on a scale from 0 to 4 marks and a percentage was assigned.

Negative items scores were reversed. For example, strongly agreeing with this statement, *mathematical problems can be done correctly in only one way*, shows a strongly traditionalist view of mathematical problems. This question would be a reversed item and a response that strongly agreed with this item would be given a 0. A mean score equal or higher than 2 indicates a strong constructivist view by the participants on the particular question. The belief dimensions on teaching and learning are grouped together in this section, in accordance with the belief investigation instruments on which this section is based. An analysis of these belief items is given further attention in Chapter 5.

The curriculum requires that all school-based assessment tasks include problem-solving items (15%) and, as assessment often dictates classroom practices (Shepard 2000), teachers' views on what is important to excel in summative assessments were investigated in Question BM11. Thus, Question BM11 investigated the teachers' views on what skills or knowledge are important for learners to achieve a high result in Mathematics. Again, a four-point Likert scale was used with 1 (not important) and 4 (very important). Results from this question were inconclusive and not included in the data analysis in the study. The question should have included queries that elicited the participants' beliefs about problem-solving in assessment. However, the question was revisited in the interviews.

Question BM12 investigated the time allocated to problem-solving questions by participants. This question was included to measure the importance placed on problem-solving activities by the participants. It was also included to add to information and triangulate information

collected in Section 2 (open-ended questions), particularly about constraints on including problem-solving as a routine activity in participants' teaching practices.

Question BM13 consists of three sub-questions that investigated problem-solving instructional approaches in the classroom. For this question a four-point Likert scale was again used with 1 (never) and 4 (always). This question was designed in the hope to elicit the participant's beliefs about problem-solving instruction. The results were, however, inconclusive and not included in the data analysis phase. Nevertheless, the question was included in the semi-structured interviews in order to gain insight about the participant's view on problem-solving instructional practices.

3.3.2. Questionnaire data collection

The data collection via the questionnaire was conducted on a number of occasions. The collection process is given in more detail below. For each occasion, permission to conduct and complete the questionnaire was requested from the participant, school and the IEB. A number of collections were done to increase the response rate after the initial request by email to all IEB schools.

Step 1: A letter was emailed to the IEB to ask for permission and assistance in conducting the research (see Appendix D).

Step 2: An email was sent to all IEB schools to ask for permission and assistance in conducting the research. The email contained a letter to the head of the school (see Appendix B) and a letter to the teachers (see Appendix A). The letters contained the required information as requested by the University of Stellenbosch's research and ethics policy. The email also contained the link to the online questionnaire.

Step 3: The initial emails to schools were followed by additional emails requesting participation in the study. Email groups of the target population were also informed and requested to participate in the study.

Step 4: To increase the response rate, printed versions of the questionnaire were distributed at various IEB events. These included national marking sessions, regional cluster meetings and national conferences.

Step 5: All completed paper-based questionnaire data was captured electronically by the researcher. Again, the online survey software was used for this. The data was securely stored and it could then be electronically analysed with data analysis software. Microsoft Excel was mainly used for the data analysis. In addition, initial data analysis was conducted by the Statistical Consultation Services at the University of Stellenbosch.

The following ethical issues were considered and addressed in the questionnaire data collection phase:

- (i) ethical consent (see Appendix C for the letter of consent). A similar consent form was used on the opening page of the electronic survey;
- (ii) rights to withdraw or not to complete (see Appendix A and C). Participants could at any time opt out of the online survey;
- (iii) issue of beneficence (see Appendix A). The letter detailed the reason and need for the study including the benefit of the research to the participants' profession;
- (iv) issue of non-maleficence (see Appendix A and C);
- (v) issues of confidentiality and anonymity guaranteed (see Appendix A and C). All data collected in the study was stored on a secure, password-protected cloud drive with access by the researcher and study leader only.

The full questionnaire was submitted to the University of Stellenbosch's ethical committee who approved the instruments for the study with stipulations given (Proposal #: SU-HSD-002592).

3.3.3. Questionnaire data analysis

The analysis of the questionnaire data was converted from the collected paper-based questionnaires and Checkbox software into an excel format. This allowed the researcher the use of the data analysis tools of the software. The data was then analysed in conjunction with a statistician from the Centre for Statistical Consultation at the University of Stellenbosch. The same statistician was consulted during the design of the quantitative section of the questionnaire. Further analysis is given in Chapters 4–6.

3.4. The interview

Semi-structured interviews of 30 minutes were used to further explore and investigate certain topics and findings from the analysis phase of the questionnaire data. The interview consisted of a number of open-ended questions. The guide to the semi-structured interview is given in Appendix F.

The purpose of the interview for this study was to:

- (i) collect data to investigate the research questions and objectives;
- (ii) allow for methodological triangulation;
- (iii) validate data collected from the questionnaires;
- (iv) further investigate findings and topics from the questionnaire analysis.

Some advantages to using an interview are that it (Laxton 2004):

- (i) allows for exploring responses, hypotheses and findings from other data collection methods in further detail;
- (ii) allows for measured responses by the interviewee. Participants' time might have been restricted and they were rushed to complete the questionnaire.

Some disadvantages to using an interview are that (Laxton 2004):

- (i) the number of respondents who can be reached is limited;

- (ii) the interview is prone to bias and subjectivity on the part of the interviewer, as there is the potential for the interviewer to lead the interviewee to certain responses.

To decrease the bias and subjectivity and to enhance the reliability of the interview, Cohen et al. (2007) suggest (i) pilot interviews, (ii) structured interviews with the same questions to all participants (iii) being careful to avoid leading questions. These methods were employed for the interviews in this study.

A semi-structured type of interview technique was chosen as the researcher wished to explore certain findings and further investigate topics from the questionnaire data. The semi-structured interview allows for the researcher to compare responses to set questions, where an open conversational interview would not have allowed this. This further strengthens the internal validity of the study. The semi-structured interview also allows the researcher to re-order and reword questions, which would not be possible with a structured or closed interview. However, this could lead to the omission of important topics from the interview, if the interviewer is not careful (Cohen et al. 2007; Cassim 2017).

The semi-structured interview guide (see Appendix F) was designed to further the research objectives as given in section 3.3.1 but also to further investigate the findings and some topics arising from the questionnaire data. The allocated time of the interview was carefully managed to be 30 minutes so as to prevent interviewee fatigue.

All transcripts and recordings of the pilot interview and interviews are stored on a password-protected and secure cloud-drive. Access to the folder is restricted to the researcher and study leader.

3.4.1. The interview design

The semi-structured interview guide (see Appendix F) was designed to implement the research objectives but also to investigate further the findings and some topics arising from the questionnaire data.

During the analysis of the open-ended questions and the quantitative items, it was felt that certain questions should be further investigated in the interviews. The aim of the study was to investigate the Mathematics teachers' beliefs and how these relate to their implementing problem-solving as a routine activity in their classroom practices. The following questions from the questionnaire were revisited in the interviews to obtain further clarity:

1. What is a mathematical problem to you? (Questionnaire R1-R2)
 - Can you give me an example of a mathematical problem in class?
2. Could you describe a mathematical problem-solving activity you use in class to me? (Questionnaire R3-R4)
 - What is your role during the task/activity? (Questionnaire R4 and R8)
 - Could you describe your role and what the learners would be doing? (Questionnaire R4 and R11)
 - Is a word problem always problem-solving? (Questionnaire R2)
 - Would you engage all learners of all abilities in problem-solving activities? (Questionnaire NM5 and BM7)
3. Could you, *in your own words*, describe a problem-solving orientated teaching methodology? (Questionnaire R5)
 - What implication would this sort of approach have on your teaching? (Questionnaire R6)
 - What constraints are there to this sort of teaching? (Questionnaire R6)
4. Do you believe that using problem-solving type and non-routine type problems is beneficial to learners' results and progress in Mathematics? (Questionnaire R9 and BM11)
5. Do you enjoy solving mathematical problems? What types do you like? (Questionnaire R12-R14)
6. Do you feel that Mathematics as delivered in our classrooms promotes creativity and originality in all learners? (Questionnaire BM8)

The use of the particular questions in the interview phase is further explained in the results and discussion chapters (Chapters 4–7).

3.4.2. The interview data collection

Potential interviewees were identified according to their belief scores, and their academic and professional qualifications. A letter requesting an interview was emailed to the potential interviewees (see Appendix I). A time and place was then agreed with the interviewees and consent for the interview given by email.

Because of time limitations only two teachers who completed the questionnaire were interviewed. The two interviews took place at the end-of-year marking session held by the IEB. Both interviewees were again informed about the research and asked to give consent to the interview and the recording. They were also assured that they could stop the interview at any time and ask not to continue and that their identities would be kept confidential in the reporting phase of the study. The interviews were conducted according to the guidelines as given by Cohen et al. (2007: 366). The allocated time of the interview was carefully managed to be 30 minutes so as to prevent interviewee fatigue. The recordings of the interviews were stored on a password-protected and secure cloud-drive. Only the researcher and study leader has access to the interview recordings.

3.4.3. The interview data analysis

The interviews were transcribed (see Appendix G and H) and analysed (see Chapter 7), again using the guidelines as outlined by Cohen et al. (2007: 366). Each participant was given a pseudonym to protect their identity in the reporting of this study. Because of time constraints only two interviews were conducted and the interview data is therefore an insufficient data source for this study. However, the interview data and analysis will be used to inform future studies.

3.5. Further ethical considerations

All research was conducted according to the Stellenbosch University's (SU) Research Ethics Policy (University Stellenbosch 2016).

The research complies with the principles of the SU Research Ethics Policy on Educational Research in the following ways:

- (i) The relevance of the study to the broader community is detailed in section 1.5 of this study;
- (ii) The research is based on quantitative and qualitative research methods in education;
- (iii) Approval to conduct the research is obtained from the relevant authorities; this includes the IEB and the individual participants;
- (iv) Participants are asked to complete a consent form, (see appendices);
- (v) All participants are informed of the purpose and how the results will be disseminated in the cover letter of the questionnaire;
- (vi) Participants, the IEB and their schools are assured of their privacy in the cover letter and guaranteed that no personal and/or school information will be used or divulged for the purpose of the study. Anonymity of the participants and their schools will be ensured by a coding system in the data analysis and reporting phase of the study;
- (vii) Participants in the questionnaire are spread across the whole population of teachers within the IEB school, to ensure there is no bias involved;
- (viii) Participants in the interviews were chosen according to the data collection procedure detailed in this proposal. Again, no bias is involved;
- (ix) Participants are allowed to complete an online or paper-based questionnaire in their own time, and interviews will be scheduled within a time period that suits the participant.

The research complies with the Data Acquisition and Management section of the SU Research Ethics Policy on Educational Research in the following ways:

- (i) Data was collected via an online and paper-based questionnaire;
- (ii) Data was collected from structured interviews, using a voice recording device;
- (iii) Consent to record the interviews was obtained before the interview was scheduled;
- (iv) Data was stored electronically and saved in a cloud-drive folder. Both the researcher and study leader have access to this cloud-drive folder.

3.6. Summary

In Chapter 3, the research design and methodologies for this study are outlined. The collected data is presented, analysed and discussed in the following four chapters. Chapter 4 gives a summary of the background information of the participants. Chapter 5 reports and discusses the open-ended question data, Chapter 6 presents the quantitative data collected, Chapter 7 presents and discusses the interviews. Finally, Chapter 8 summarises the study and presents limitations, recommendations and final conclusions of this study.

CHAPTER 4

RESULTS AND DISCUSSION: INTRODUCTION

The results from the data collection are reported and discussed in the following chapters. The participants' background information is given in this introductory chapter which is followed by three chapters on the qualitative, quantitative and interview data. Chapter 8 gives a summary of the results and final conclusions to this study. Chapter 8 also presents the limitations of this study, followed by recommendations for future studies.

Background information on the participants

The participants are secondary school teachers of Mathematics who are employed by schools registered with the IEB in South Africa. A total of 124 questionnaires were received from which 95 completed questionnaires were collected.

The gender distribution of the population is included in the background data. The sample group has 32% male and 68% female participants. This is a slightly higher male representation than the South African national teacher ratio of 26,5% male teachers to 73,5% female (PMG 2017). The only relevance of this insert to the analysis of the data is that it gives an indication of the gender split of the sample versus gender split of teachers nationally in South Africa.

Information about teaching experience was collected based on the number of years' participants had been teaching. Teacher training information was gathered based on professional qualifications (see Table 4.1). Of the participants, 65% had more than ten years of teaching experience, indicating an experienced Mathematics staff as claimed by the schools registered with the IEB. Considering teachers' professional training qualifications, 41% had an HDE, a pre-reform qualification as discussed in Chapter 2 (see Section 2.11). Further, 37% had a PGCE and the remaining 22% indicated "other" as qualifications. Of the "other" professional qualifications, 14% indicated they had different teacher qualifications to the HDE and PGCE, for example, a bachelor's degree in education (B Ed). The remaining 8% did not submit any relevant teachers' training qualification. Therefore, of the participants, 92% indicated relevant professional qualifications. Again, this supports the claim by the schools registered by the IEB that they offer schooling by qualified professional teachers. As

discussed in Chapter 2, teachers' experience and training are relevant to their beliefs about the nature of Mathematics, the teaching and learning of Mathematics and about problem-solving. The participants' experience and professional qualifications in relation to their belief dimensions are reported on and discussed in Chapters 5 and 6. Unfortunately, there is no reported data available to compare the experience and qualifications of the sample to the secondary Mathematics teacher population in South Africa. Information was requested from the Western Cape Education Department and the Department of Basic Education (DBE) without any reply within the study period.

The participants' level of mathematical knowledge is relevant to this study, as discussed in Chapter 2 (see section 2.10). The results on Mathematics teachers' beliefs in relation to their level of mathematical knowledge will be reported on and discussed further in Chapters 5 and 6. Only 4% of participants in this study had no tertiary level of Mathematics knowledge. The 6% who indicated "other" as Mathematics knowledge did not give any indication as to what this might be. Furthermore, it was not possible to establish this "other" knowledge from their highest academic qualification.

The following table (Table 4.1) summarises the relevant background information of the participants in this study. The information is further used in the results and discussion in chapters 5 and 6.

Table 4.1 Background information on participating teachers

	n=95	%
Gender		
Male	30	32%
Female	65	68%
Number of years teaching Secondary Mathematics		
[0;10]	33	35%
(10;20]	29	31%
(20;30]	23	24%
(30;40]	9	9%
(40;50]	1	1%
Professional Qualifications		
PGCE	35	37%
HDE	39	41%
Other	21	22%
Pre-reform	47	49%
Post-reform	48	51%
Academic Qualifications		
Mathematics 1	16	17%
Mathematics 2	27	29%
Mathematics 3	31	33%
Honours in Mathematics	7	7%
Master's in Mathematics	4	4%
None (no tertiary Mathematics)	4	4%
Other (not specified)	6	6%

The following four chapters present further data collected together with discussions on the data reported on in this chapter. The quantitative analysis is given first as this was the initial section to be analysed. In addition, the main findings of the study emanate from the quantitative data analysis.

CHAPTER 5

RESULTS AND DISCUSSION: QUANTITATIVE DATA

This chapter presents and discusses the quantitative data collected via the questionnaire. The relevant quantitative results are presented and discussed under three headings: teachers' beliefs across belief dimensions (5.2), teachers' mathematical knowledge and their beliefs (5.3), and teachers' professional qualifications and their beliefs (5.4). The three headings discuss findings that became apparent during the analysis phase of the data from the questionnaire. The introduction to the chapter gives a short summary of the design of the relevant quantitative data sections in the questionnaire. This is followed by detailed discussions on the quantitative data.

5.1. Introduction

The quantitative data section of the questionnaire investigated the theoretical dimensions of the beliefs of Mathematics teachers in independent schools registered with the IEB in South Africa with regard to:

- (i) beliefs about the nature of Mathematics (NM1-NM5);
- (ii) beliefs about the teaching and learning of Mathematics (BM1-BM10).

The data collected in this section will in part contribute to investigating the research questions to this study, namely:

Pertaining to secondary Mathematics teachers in independent schools registered with the Independent Examination Board (IEB) in South Africa,

- (i) *What are practising, secondary-school Mathematics teachers' beliefs about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics?*
- (ii) *How do their beliefs relate to their implementation of problem-solving activities as required by the National Curriculum Statement?*

The quantitative data for the questionnaire is based on a four-point Likert scale: Strongly Disagree (SD), Disagree (D), Agree (A) or Strongly Agree (SA) with the statement; a response of one (SD) is associated with a strongly traditionalist view, and a four (SA) with a strongly constructivist belief held by the respondent. Some of items are reversed items and are indicated by (-). During the data analysis the scores were mapped on a scale from 0 to 3 marks and a percentage was assigned to the response. Negative item scores were reversed. For example, strongly agreeing with the statement, “mathematical problems can be done correctly in only one way” shows a strongly traditionalist belief of mathematical problems. This question is a reversed item and a response that strongly agreed with this item would be given a 0 mark. If the average score for this questionnaire item is higher than or equal to 2, it will indicate a strongly constructivist view by the participants (on average) about the particular belief item. Of the 124 received questionnaires, 95 were completed and the data of the completed questionnaires is reported on and discussed in this chapter.

5.2. Teachers’ beliefs across belief dimensions

***Finding 1:** There is a difference between teachers’ belief dimensions about the nature of Mathematics and the teaching and learning of Mathematics.*

The belief scores of the participants showed a lower average belief score across items that investigated the teachers’ beliefs about the nature of Mathematics (NM) versus beliefs about the teaching and learning of Mathematics (BM). This might indicate that teachers hold different views across belief dimensions. In other words, a teacher could have constructivist views about the teaching and learning of Mathematics but have a traditionalist view about the nature of Mathematics. This chapter presents the data collected per belief dimension. Firstly, in 5.2.1, there is a report and discussion of the data on the beliefs of Mathematics teachers about the nature of Mathematics and secondly, in 5.2.2, there is a report and discussion of the data on the beliefs of Mathematics teachers about the teaching and learning of Mathematics.

5.2.1. Belief Dimension: Beliefs about the nature of the discipline of Mathematics

Table 5.1 displays the collected data from this section of the questionnaire. The percentage response is given per question item. Participants were asked to indicate if they Strongly Disagreed (SD), Disagreed (D), Agreed (A) or Strongly Agreed (SA) with the statement. Reversed items are indicated by a (-). The shaded blocks indicate the highest percentage response per question item.

Table 5.1 Participants' responses to questions on their beliefs about the nature of Mathematics

Code	Item	SD (%)	D (%)	A (%)	SA (%)
NM1(-)	Mathematical problems can be done correctly in only one way.	88	10	0	2
NM2(-)	Some people have a natural talent for Mathematics.	5	14	34	47
NM3	Mathematics is primarily a formal way of representing the real world.	5	19	48	28
NM4(-)	In Mathematics something is either right or wrong.	20	38	30	12
NM5(-)	Some people are good at doing Mathematics and some are not.	10	30	39	21

(-) Negative items. SD: Strongly Disagree; D: Disagree; A: Agree; SA: Strongly Agree

Item NM1: Responses show a strongly constructivist belief with 98% of the responses strongly disagreeing or disagreeing with the statement that Mathematical problems can only be done in one way. This is a negative question item, meaning agreeing or strongly agreeing with the statement indicates a traditionalist belief and not a constructivist belief. The results correspond with the results from the study by Zakaria and Musiran (2010). The study by Zakaria and Musiran investigated the beliefs about the nature of Mathematics and the teaching and learning of Mathematics of 100 Mathematics teacher trainees in higher institutions in Malaysia.

Item NM2: Responses show a strongly traditionalist belief with 80% of respondents agreeing or strongly agreeing with the statement that Mathematics ability is an innate talent. This is contrary to constructivist theory that allows for knowledge to be constructed independently of the talent of the person (Ultanir 2012; Sidney 2015).

Item NM3: Responses show a strongly constructivist view with 76% of the respondents agreeing or strongly agreeing with the statement that Mathematics is primarily a formal way of representing the real world. The socio-constructivist theory of Mathematics holds in part that Mathematics is useful in real life and that Mathematics empowers us to understand better the world we live in (Op 't Eynde et al. 2006).

Item NM4: Some 58% of the respondents disagreed or strongly disagreed with the statement that in Mathematics, something is either right or wrong. This indicates a constructivist belief regarding the statement. However, a large percentage (42%) agreed or strongly agreed with the statement. This indicates that there are nevertheless a large number of the participants who hold a traditionalist belief regarding the statement.

Item NM5: Some 60% of the respondents agreed or strongly agreed with the statement that some people are good at doing Mathematics and some are not. While this result supports the result of NM2 (some people have a natural talent for mathematics), there is now a lower percentage of the participants holding a traditionalist view of the statement. This indicates that individual participants hold contradictory beliefs with regard to similar statements within this belief dimension.

The following graph (Figure 5.1) displays the data from Table 5.1 with percentages per bar given below the graph. The responses that agree and strongly agree (A/SA) with a question item are grouped together. In addition, those who disagree and strongly disagree (D/DS) with a question item are grouped together. Reversed items are indicated by (-). These groupings give a combined and simplified view of how the participants responded to each question item. This further adds to the analysis.

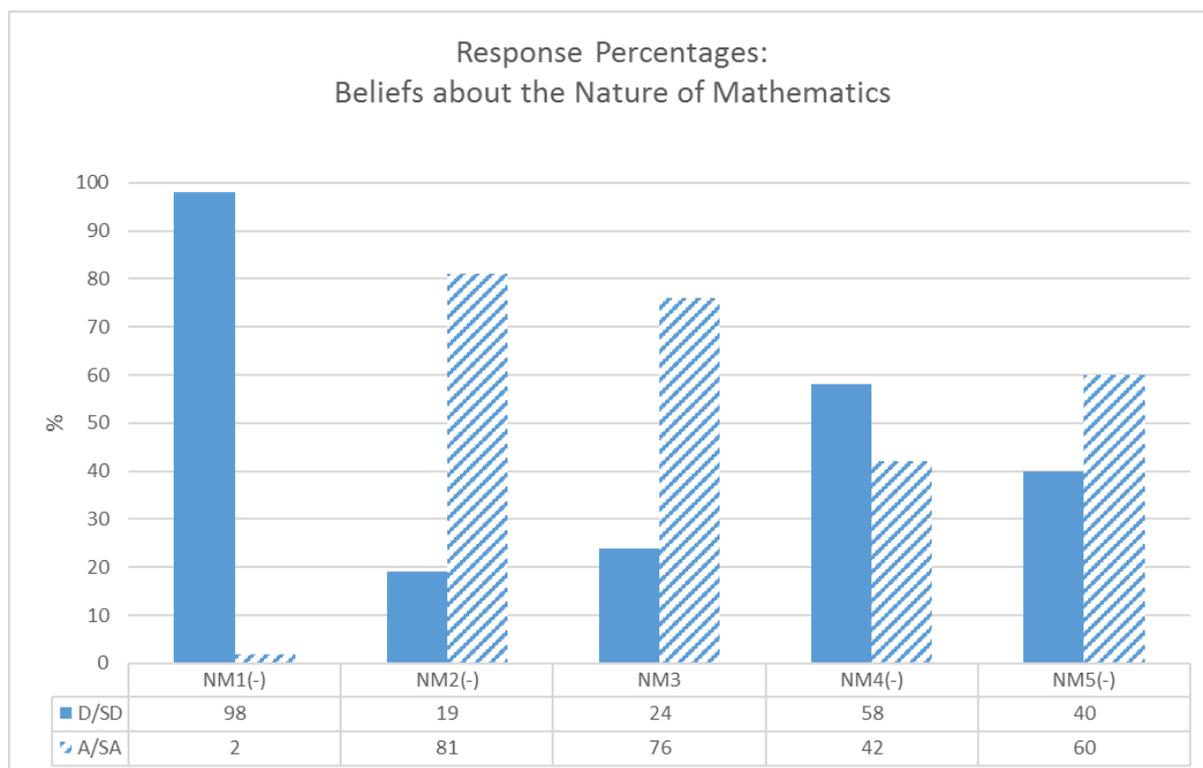


Figure 5.1 Response percentages: Questions on beliefs about the nature of Mathematics.

Note that NM1 and NM4 are reverse scored and that disagreeing with the statements would indicate a constructivist belief regarding the question item and not a traditionalist belief. Question items NM1 (mathematical problems can be done correctly in only one way), NM3 (Mathematics is primarily a formal way of representing the real world) and NM4 (in Mathematics something is either right or wrong) therefore these three items that elicited more responses that aligned with a constructivist belief. NM2 (some people have a natural talent for Mathematics) and NM5 (some people are good at doing Mathematics and some are not) show more participants agreeing with a traditionalist belief about the item.

For three out of the five questions on beliefs about the nature of Mathematics, there were more constructivist beliefs than traditionalist beliefs indicated by the participants. However, a closer look at the mean scores per question item (Figure 5.2) indicates that only for NM1 do the participants on average have constructivist beliefs about the question item. This is discussed in more detail in the following paragraphs.

Figure 5.2 (below) displays the mean score per question item. Responses from the Likert-scale questions were mapped on a scale from zero to three marks. Negative item scores were

reversed. A mean score per question equal to or higher than 2 indicates a strongly constructivist belief by the participants on the question.

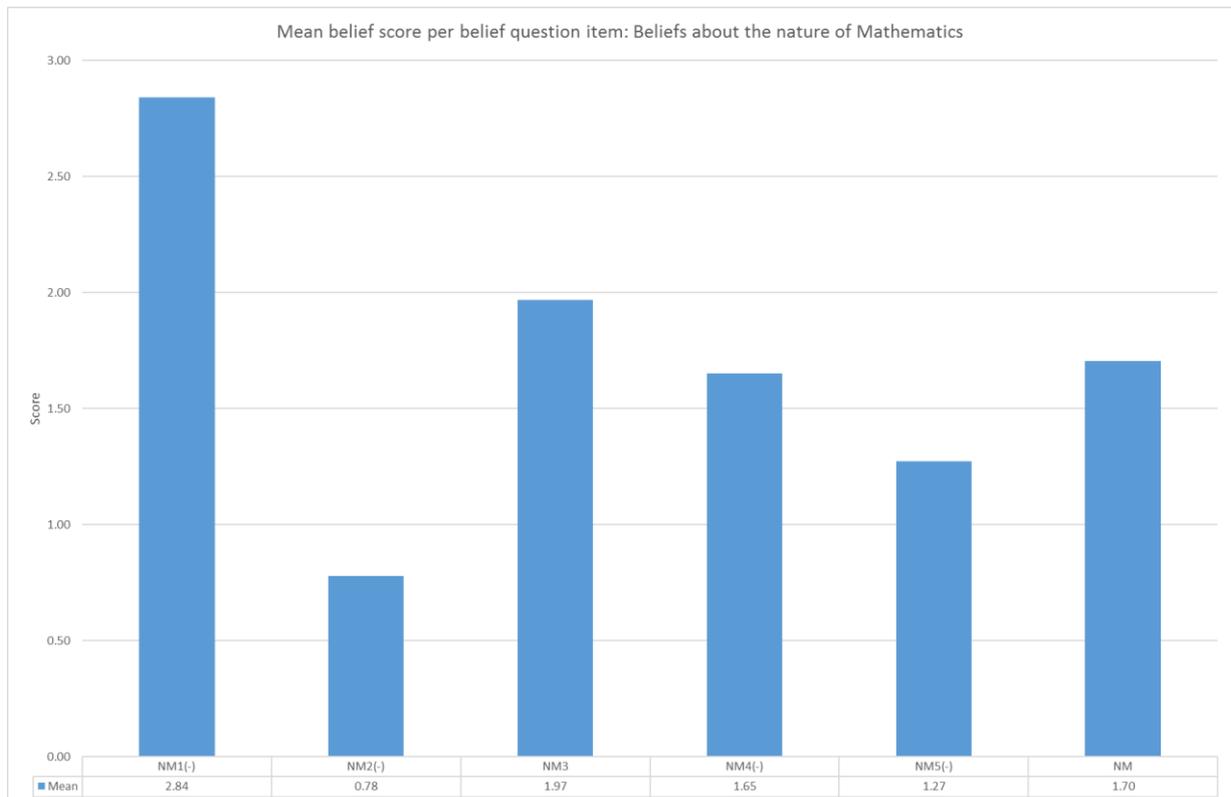


Figure 5.2 Mean belief score per belief question: Beliefs about the nature of Mathematics

Question item NM1 is an exception with four out of the five items showing a mean score below two. This indicates beliefs that lean towards traditionalist beliefs by the participants. In addition, the overall mean score for all the responses in this belief dimension is below two (NM = 1,70). From the mean scores, we can infer that the participants on average hold beliefs about the nature of the discipline of Mathematics that lean more towards a traditionalist belief than a constructivist view.

In conclusion, while three out of the five question items have more participants selecting responses that can be associated with a constructivist belief of the item, a closer look at the average response score per item suggests that on average the participants hold traditionalist beliefs on four out of the five question items.

The dominant traditionalist view about the nature of Mathematics can influence the integration of constructivist instructional practices in the classroom. The next section will investigate data collected for the belief dimension on the teaching and learning of Mathematics.

5.2.2. Belief dimension: Beliefs about the teaching and learning of Mathematics

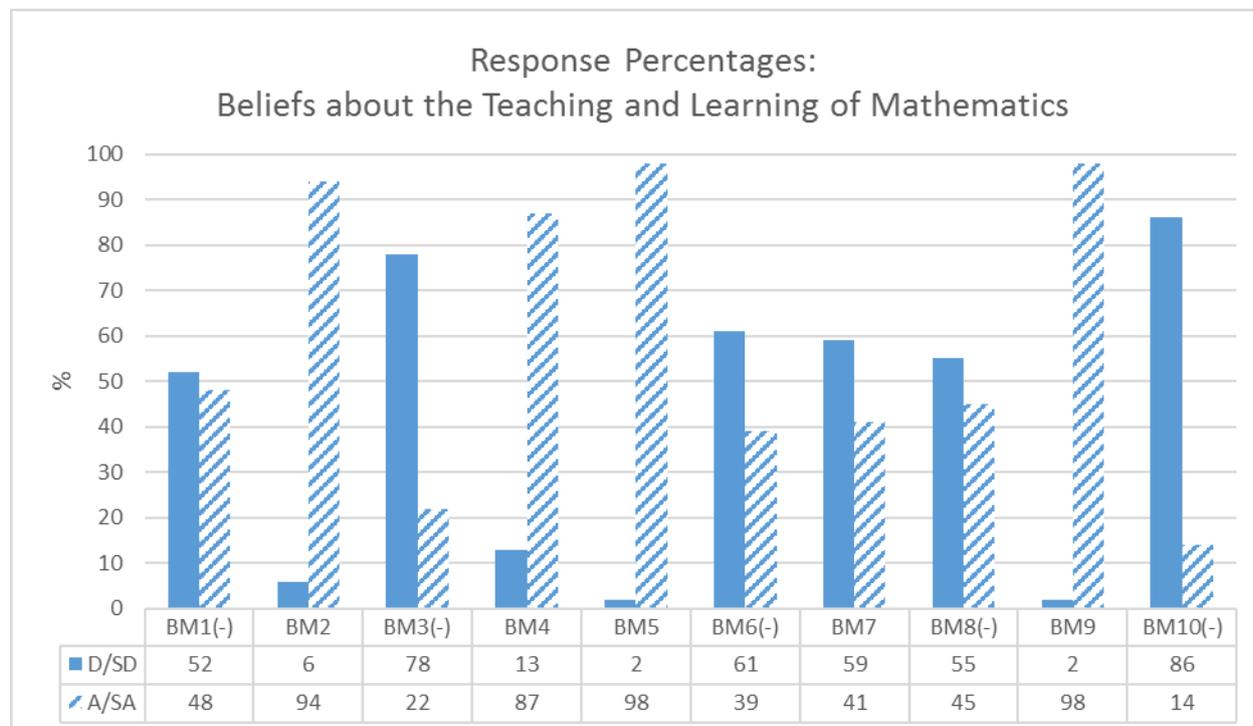
This section provides a report and discussion regarding the data collected from question items BM1 to BM10 which investigated the participants' beliefs about the teaching and learning of Mathematics. The following table (Table 5.2) displays the percentage responses to the Likert-scale question items. The participants were asked to indicate whether they Strongly Disagree (SD), Disagree (D), Agree (A) or Strongly Agree (SA) with the given statement. The shaded blocks in the Table indicate the highest percentage response to the question item. A total of 95 completed questionnaires were analysed for this data.

Table 5.2 Participants' responses to questions on their beliefs about the teaching and learning of Mathematics

Code	Item	SD (%)	D (%)	A (%)	SA (%)
BM1(-)	Mathematics should be taught as a collection of procedures (skills) and algorithms.	15	37	34	14
BM2	More than one representation should be used when teaching a maths concept.	0	6	45	48
BM3(-)	Good Mathematics teachers show you exactly how to get to the answer.	33	45	17	4
BM4	Good reasoning should be regarded more important than getting to the correct answer.	1	12	37	51
BM5	Learning Mathematics is an active process with learners actively involved in their learning.	1	1	17	81
BM6(-)	To solve a Mathematical problem you need to teach the correct procedure.	12	49	31	8
BM7	Problem-solving activities form part of the general teaching of lower ability groups.	18	41	30	11
BM8(-)	In teaching Mathematics logic is promoted, whereas creativity and originality are not stressed.	22	33	37	9
BM9	Teaching Mathematics provides an excellent opportunity to promote the development of the learners' thinking.	1	1	18	80
BM10(-)	Mathematics teaching is especially meant for mathematically talented learners.	41	45	14	0

(-) Negative items. SD: Strongly Disagree; D: Disagree; A: Agree; SA: Strongly Agree

In the following graph (Figure 5.3) the responses that agree and strongly agree (A/SA) with a question item are grouped together. So too, those that disagree and strongly disagree (D/SD) with a question item are grouped together. The percentages per grouping are displayed in the bar graph (Figure 5.3). These groupings give a combined and simplified response of how the participants responded to each question item.



(-) Negative items. (D/SD): Disagree/Strongly Disagree (A/SA): Agree/Strongly Agree

Figure 5.3 Response percentages: Beliefs about the teaching and learning of Mathematics.

As indicated in Figure 5.3, only question item BM7 (problem-solving activities form part of the general teaching of lower-ability groups) has more participants selecting to disagree or strongly disagree with a positive item. This indicates that participants in general hold a traditionalist belief of this question item. Responses to all other question items indicate a higher frequency of constructivist belief held by the participants. Selected question items regarding participants' beliefs about the teaching and learning of Mathematics are further investigated later in this section.

The following graph (Figure 5.4) displays the mean score per question item on participants' beliefs. Responses from the Likert-scale questions for this section were again mapped onto a zero to three-mark scale and a percentage was assigned to each response. The scores of negative items were reversed. For example, strongly agreeing with the statement, "good

Mathematics teachers show you exactly how to get to the answer” shows a strongly traditionalist belief of the teaching of mathematical problems. This question would be a reversed item and a response that strongly agreed with this item would be given a zero mark. A mean score per question item equal to or higher than two will indicate a strong constructivist belief, on average, by the participants on the particular question item.

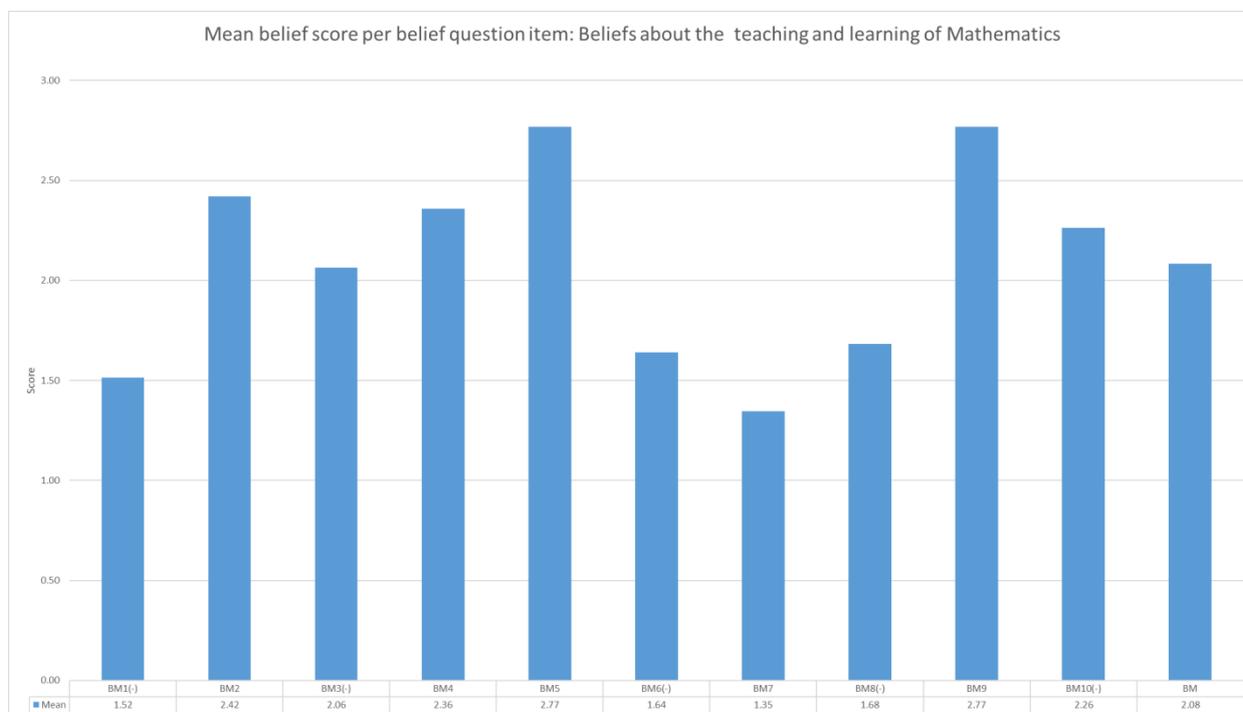


Figure 5.4 Mean belief score per belief question: Beliefs about the teaching and learning of Mathematics

In the data displayed in Figure 5.4 (above), six out of the ten items have mean scores above two which indicates, on average, participants hold strongly constructivist views on the question items. Four of ten question items have mean scores below two; however, three of the items, BM1 (Mathematics should be taught as a collection of procedures (skills) and algorithms), BM6 (to solve a mathematical problem you need to teach the correct procedure) and BM8 (in teaching Mathematics, logic is promoted, whereas creativity and originality are not stressed) have more constructivist belief responses than not (see Figure 5.3). This seems to indicate that in general, the participants lean towards a constructivist belief regarding these items.

The three question items (BM1, BM6 and BM8) are discussed in greater detail now as the data seem to indicate a dichotomy in views by the participants. This is followed by further discussions of selected question item results.

- Item BM1: (Mathematics should be taught as a collection of procedures (skills) and algorithms): some 52% of the teachers disagreed or strongly disagreed with the statement. Even though a higher percentage of participants disagreed with this statement, a large percentage (48%) still agreed with the statement that Mathematics should be taught as a collection of procedures. There is only a 4% difference between teachers who believe that Mathematics should be taught as a set of procedures (48%) and those who disagree (52%). This indicates a traditionalist belief by many of the participants on the aspect of the teaching and learning of Mathematics.
- Item BM6 (to solve a mathematical problem you need to teach the correct procedure): A majority of participants (61%) disagreed or strongly disagreed with the statement, with 39% of the participants indicating that a correct procedure should be taught to solve a mathematical problem. While there were fewer participants (only 39%) holding a traditionalist belief of this question item in comparison to those participants (48%) who held traditionalist beliefs in response to BM1, there are still a large number of teachers holding traditionalist beliefs about problem-solving. The lower percentage (39% versus 48%) may indicate that some teachers believe that problem-solving can be taught as a set of procedures and algorithms.
- Item BM8 (In teaching Mathematics logic is promoted, whereas creativity and originality are not stressed). Of the participants, 55% disagreed with this statement, while 45% felt that creativity and originality are not stressed. This might be an indication of how they interpret the curriculum and might not indicate their beliefs about the teaching of Mathematics. The question item is therefore investigated further in the interviews.
- Item BM7 (problem-solving activities form part of the general teaching of lower ability groups). The only question item with a mean belief score below two and thus indicating more participants agreeing with a traditionalist view of the statement, is BM7. This indicates that problem-solving is not a routine activity in many of the Mathematics classes where the learners are deemed to be of “lower” ability.

As indicated in the literature review, different beliefs are held in different belief dimensions. This means that individuals can hold two incompatible, inconsistent beliefs without internal conflict. These conflicting beliefs are often upheld by a third belief. An example given by Cross (2009) is that a teacher can believe that “schools should be an environment where students are provided with all opportunities to excel” but can then also hold the belief that

“students who are not in the gifted classes should not be recommended for advanced math courses” (Cross 2009). These conflicting beliefs are acceptable to the teacher because another belief supports this; the belief that “ability is fixed” (Green 1971; Cross 2009; Beswick 2007). Similar results are evident in the data collected in this study, as detailed in the following paragraphs.

Conflicting beliefs are held between BM7 (problem-solving activities form part of the general teaching of lower ability groups) and BM10 (Mathematics teaching is especially meant for mathematically talented learners). Most of the participants (59%) disagreed with BM7, indicating that the majority of the participating teachers feel that problem-solving is not for lower-ability learners. The conflicting belief is evident in BM10 with only 14% of the participants agreeing with the statement. The majority of the participants therefore feel that Mathematics teaching is for all ability groups. NM2 (some people have a natural talent for Mathematics) and NM5 (some people are good at doing Mathematics and some are not) have 80% and 60% respectively of the participating teachers agreeing with the statements. This seems to indicate that most of the participating teachers feel that mathematical ability is fixed. This could explain the existence of the conflicting beliefs held between BM7 and BM10 in this study. However, this might only be a reflection of the teachers’ classroom reality; many might be teaching in mixed-ability classes. They are therefore teaching Mathematics to all ability groups and not just to the talented few. In addition, problem-solving is described as a higher-order skill in the curriculum document (Department of Basic Education 2011a, p.53), and this could be interpreted to mean that only high ability and the talented learners can engage in problem-solving. This might be the case as often the *most difficult* and *problem-solving* question is included as the last question in the school-based assessment tasks. This question item is therefore investigated further in the interviews.

The mean score for all the question items regarding the participating teachers’ beliefs about the teaching and learning of Mathematics indicates a strongly constructivist belief about this belief dimension (BM=2,08). In addition, the participants indicated more traditionalist than constructivist beliefs in response to only one (BM7) of the ten question items. Further, six out of the ten items have strongly constructivist belief mean scores. There seems to be a strongly constructivist response by the participants to items in this belief dimension, in general, with the mean scores (see Figure 5.4) supporting this.

5.2.3. Conclusion: Beliefs about the nature of Mathematics, and beliefs about the teaching and learning of Mathematics

Firstly, the quantitative data indicate a contradiction between the participants' beliefs about the nature of Mathematics and the teaching and learning of Mathematics. The participants lean towards a traditionalist belief regarding the nature of Mathematics while in general have a strongly constructivist belief about the teaching and learning of Mathematics. The correlation coefficient ($\rho = 0,24$) between the participants' average scores on the nature of Mathematics and the belief dimension on the teaching and learning of Mathematics, shows only a slight relationship (Cohen et al. 2007). Overall it seems that a constructivist belief in the nature of Mathematics does not imply a constructivist belief about the teaching and learning of Mathematics.

Secondly, the analysis of the data indicates that a large group of the participants believes problem-solving should not form part of the teaching and learning of Mathematics for lower-ability learners; that doing Mathematics should be taught as a set of procedures; and that the teaching of Mathematics should emphasise logic rather than creativity.

The aims and objectives of the Mathematics curriculum promote problem-solving, creativity and the conceptual teaching of Mathematics for all learners regardless of ability (Department of Basic Education 2011a: 8). It stands to reason that problem-solving will not be a routine activity even in privileged schools if as many as 48% of the participants hold beliefs that contradict the curriculum aims.

The following section discusses the second finding related to teacher mathematical knowledge.

5.3. Teachers' mathematical knowledge and their beliefs

Finding 2: Teachers' level of mathematical knowledge in relation to their beliefs about (a) the nature of mathematics, (b) the teaching and learning of mathematics.

Teachers' mathematical knowledge has a significant impact on learners' progress and achievements in Mathematics (Hill et al. 2017; Baumert et al. 2017). Further, Manouchehri and Goodman (2000) note a clear difference in the beliefs of teachers with different mathematical knowledge and conclude that mathematical knowledge is the greatest influence in evaluating and implementing the reforms required of a constructivist approach, which includes problem-solving, in the teaching and learning of Mathematics. In addition, pre-service teachers with the desired beliefs (constructivist beliefs) have attributed the development of these beliefs to tertiary Mathematics courses and not to school-level Mathematics (Paolucci 2008; Paolucci 2015). Therefore, tertiary-level Mathematics has the potential to influence prospective beliefs of teachers and the implementation of reforms within curricula.

The studies by Manouchehri and Goodman (2000) and Paolucci (2008; 2015) conclude that Mathematical knowledge has an influence on teachers' beliefs, but neither investigate the beliefs of Mathematics teachers at the level of tertiary Mathematics knowledge.

5.3.1. Participants' background information in relation to mathematical knowledge

Participants were asked to indicate their tertiary-level academic qualifications in Mathematics in the questionnaire. Figure 5.5 and Figure 5.6 which follow display the participants' mathematical knowledge as percentages and numbers of participants respectively.

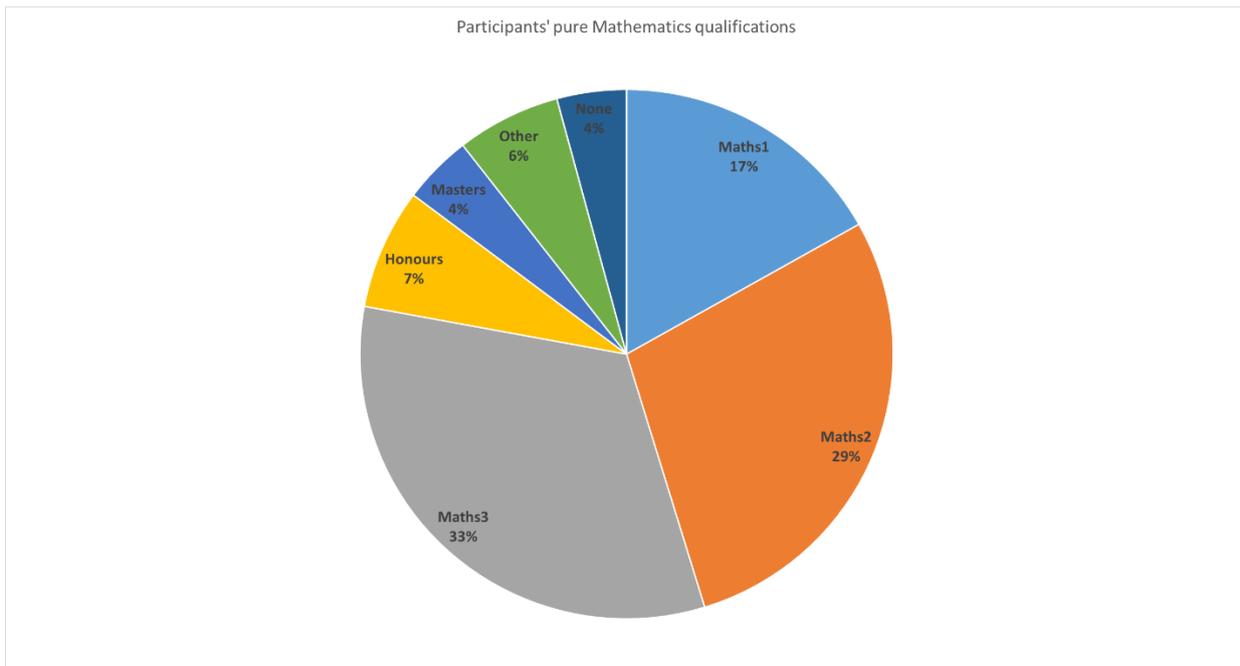


Figure 5.5 Participants' pure Mathematics qualifications

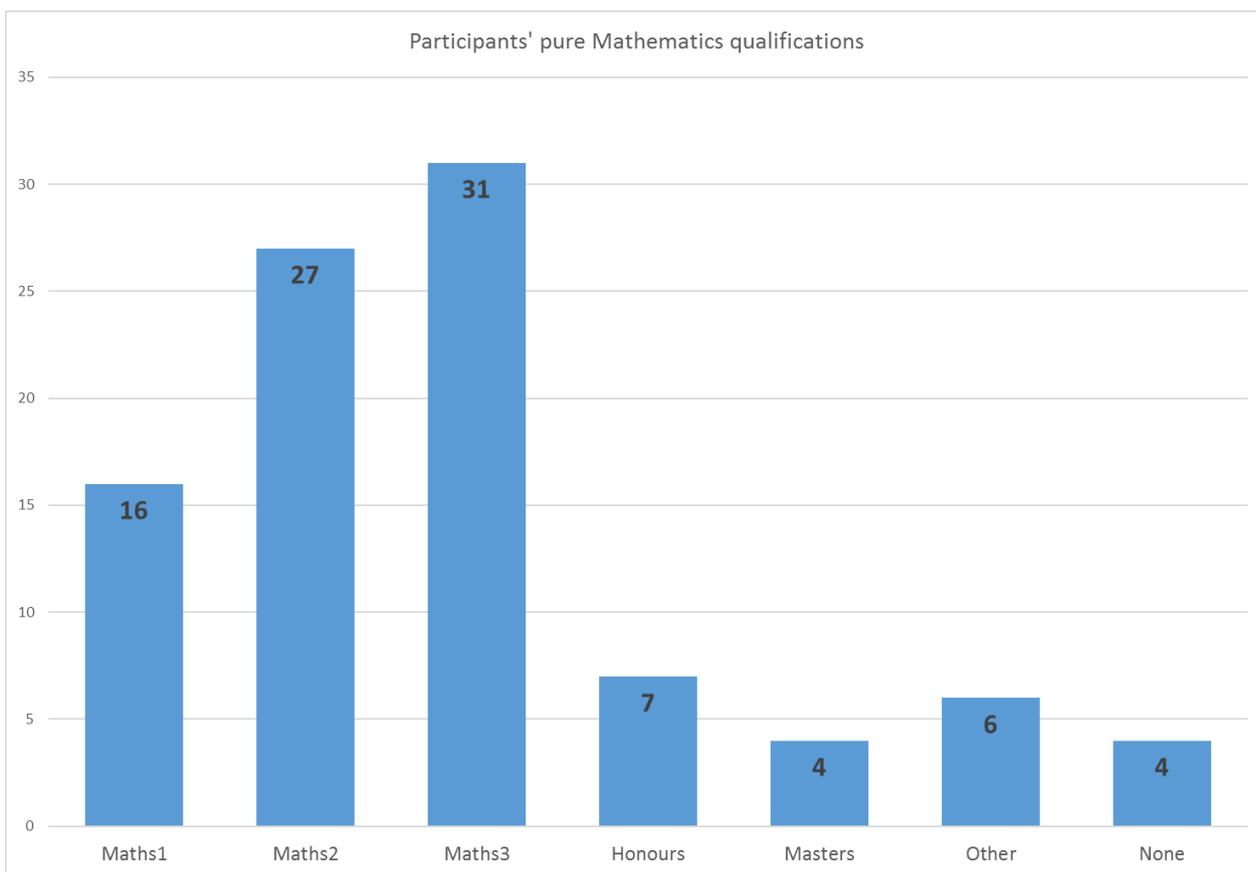


Figure 5.6 Participants' pure Mathematics qualifications

In this study, 85 of the 95 participants were teachers with tertiary qualifications that included pure Mathematics as a subject; six indicated “other” tertiary qualifications that included Mathematics; and only four indicated that they had no Mathematics at tertiary level. The four who had no background in pure Mathematics indicated that they possessed the following tertiary qualifications: Bachelor of Arts degree, Quantity Surveying qualification, Master’s in Science and a Higher Diploma in Education. Of the ten without a qualification in pure Mathematics, nine had a professional teaching qualification, allowing them to teach in South African schools. The candidate without a teaching qualification indicated that he had a tertiary qualification with pure Mathematics as subject, but he did not indicate which level this was. Currently, pure Mathematics at second-year level is a prerequisite for the study of the subject Didactic Mathematics in the PGCE qualification. This qualification prepares teachers for offering Mathematics to Grades 7 to 12 learners (University of Stellenbosch 2018; University of South Africa 2018; University of Cape Town 2018). There are other teacher qualifications at South African tertiary institutions which allow teachers to teach Mathematics in secondary schools, but which require only pure Mathematics 1 (University of South Africa 2018) or do not require pure Mathematics as a prerequisite (University of Stellenbosch 2018; University of South Africa 2018; University of Cape Town 2018). However, these only prepare teachers for teaching Mathematics to learners in Grades 7 to 9.

Of the participants, 16 (19%) indicated that they have a qualification in Mathematics 1 only. Although this level of Mathematics does not qualify a teacher to teach Mathematics at the Further Education and Training (FET) Phase (Grades 10 to 12) of schooling in South Africa, it does, however, allow a candidate to study to teach Mathematics at the Senior Phase (Grades 7 to 9). Secondary schooling in South Africa covers Grades 8 to 12 Mathematics (DBE SA 2015, p.4). The 16 participating teachers with Mathematics 1 as qualification all had suitable qualifications to teach up to Grade 9 level: five had a PGCE, six had an HDE and five indicated that they had a B Ed. The five with a B Ed did not indicate their major subject in the degree.

The background information shows that the majority of the teachers taking part in the study are qualified to teach Mathematics to secondary school learners in South African schools, with 44% holding mathematical subject knowledge that is higher than required (Mathematics 3 and postgraduate qualifications) to teach Mathematics in secondary schools in South Africa. This supports the claims of the schools registered with the IEB that their staff is qualified.

The following section reports on and discusses the teachers' belief scores per belief dimension in relation to Mathematics qualifications. The purpose is to examine the possible differences in beliefs against qualifications in Mathematics.

5.3.2. Beliefs about the nature of Mathematics in relation to participants' mathematical knowledge

To investigate the teachers' beliefs about the nature of Mathematics against their Mathematics knowledge, an analysis was made according to each participant's belief score. As with the previous analysis, a mean belief score per participant of two and higher indicates a strongly constructivist view about the belief dimension, while a score below two would indicate a view leaning towards a traditionalist view about the belief dimension. The bar graph (Figure 5.7 below) displays the frequency of participants with a mean belief score above two versus those below two, against their mathematical knowledge.

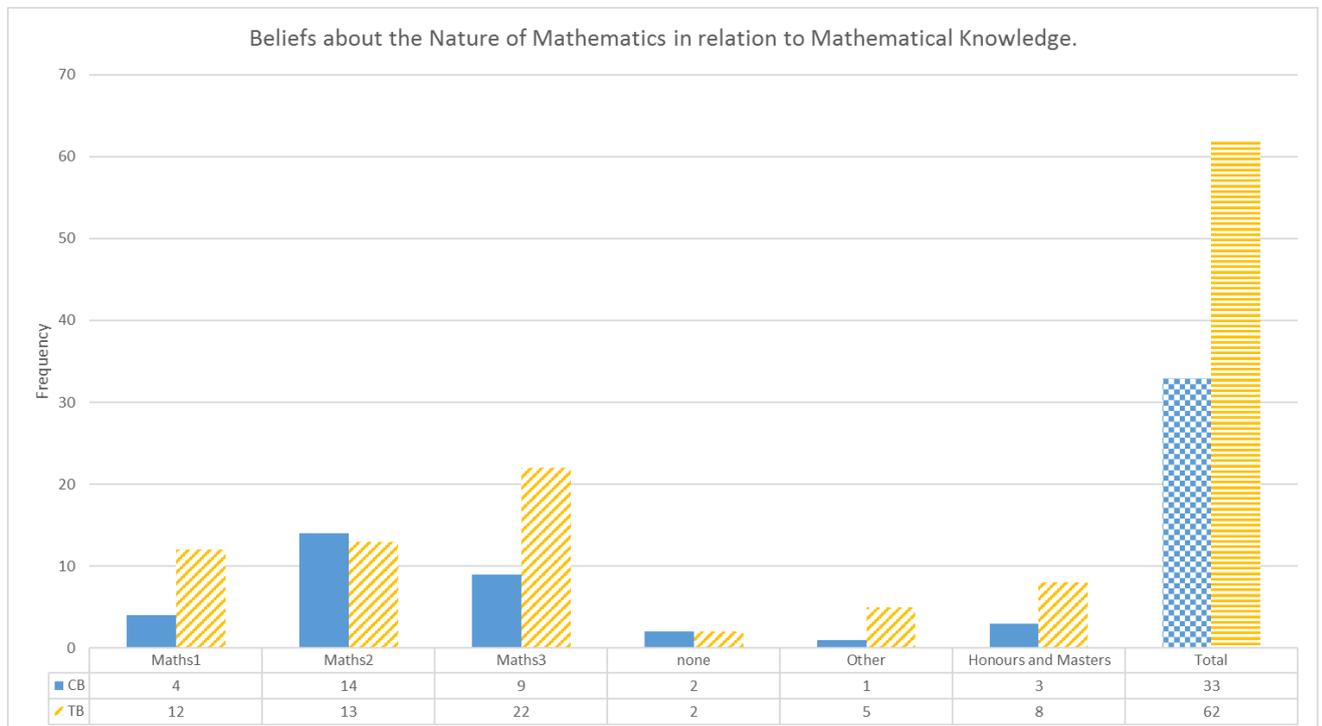


Figure 5.7 Beliefs about the nature of Mathematics in relation to mathematical knowledge. Constructivist beliefs – (CB); traditionalist beliefs – (TB)

The following graph (Figure 5.8) displays the distribution of mathematical qualifications of participants with a mean belief score of two and higher, that is, those who have strongly constructivist beliefs within the belief dimension, the nature of Mathematics. Figure 5.9 displays the distribution of Mathematics qualifications of participants with a mean belief score below two, that is, those who lean towards more traditionalist beliefs within the belief dimension.

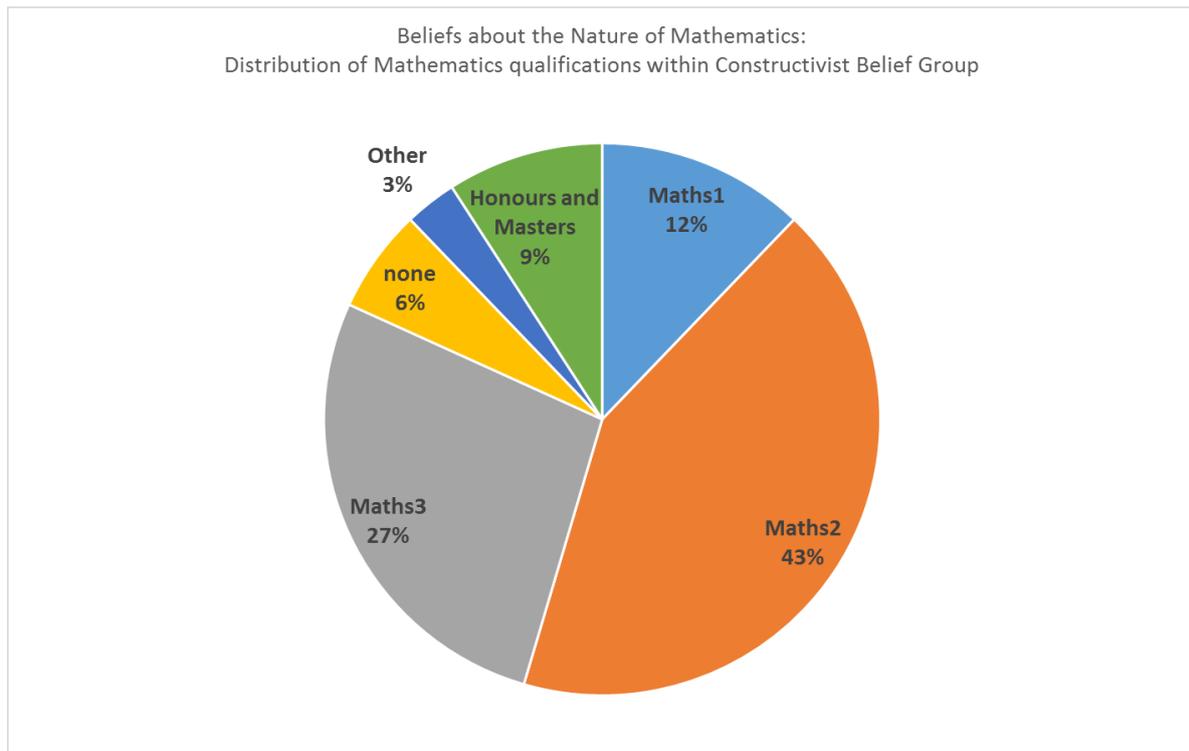


Figure 5.8 Beliefs about the nature of Mathematics: Distribution of Maths qualifications within Constructivist Belief Group

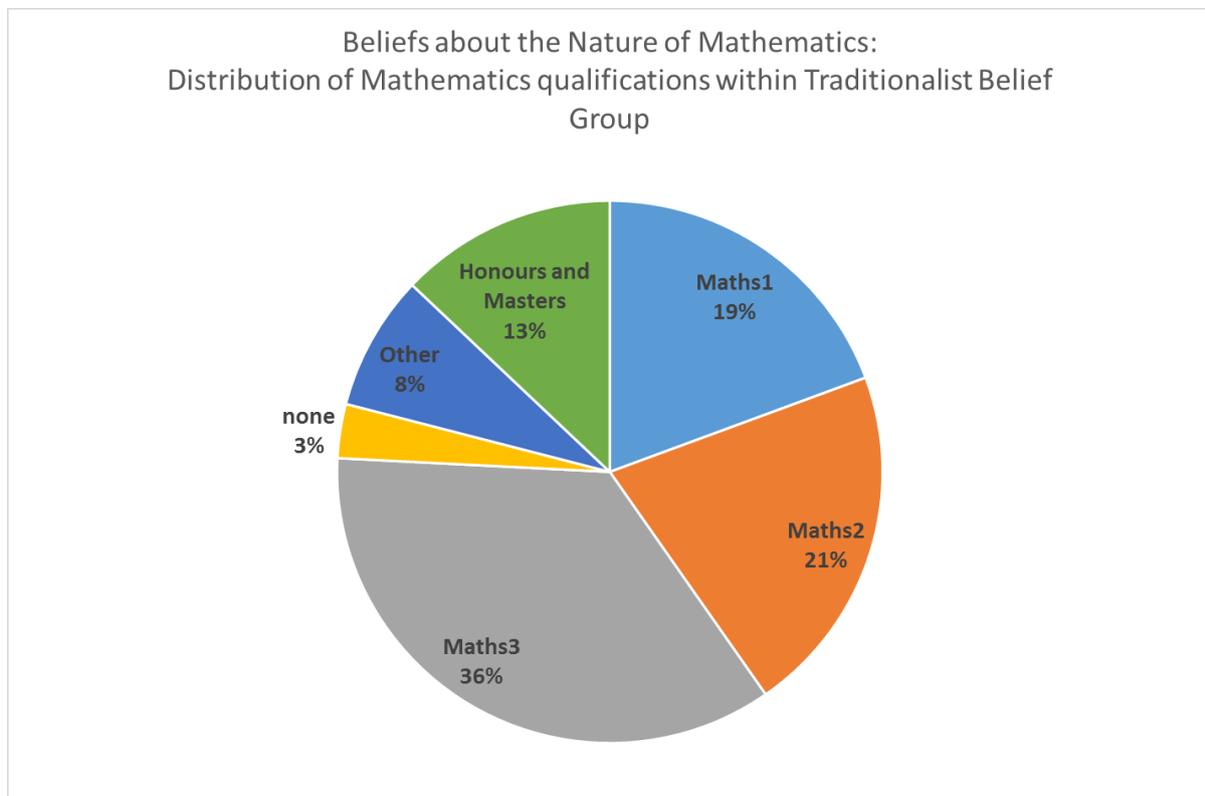


Figure 5.9 Beliefs about the nature of Mathematics: Distribution of Mathematics qualifications within Traditionalist Belief Group

In the bar graph (Figure 5.7), only participants with a Mathematics 2 qualification showed a higher percentage of respondents falling within the constructivist belief group. In addition, in the pie chart (Figure 5.8), the Mathematics 2 group has the highest percentage (43%) of participants with a mean score of two and higher. This seems to suggest that the participants with a Mathematics 2 qualification hold a strongly constructivist view of beliefs about the nature of Mathematics. However, the other qualification groupings all lean towards traditionalist views about beliefs regarding the nature of Mathematics. From the data, one can argue that as a group, the participating teachers predominantly align their views with a traditionalist view of the nature of Mathematics, as supported by the first finding in this chapter. There is no clear indication that a lower, higher or no Mathematics qualification would lead to a particular view regarding the nature of Mathematics. The following section looks at Mathematics teachers' beliefs about the teaching and learning of Mathematics in relation to their mathematical qualifications.

5.3.3. Beliefs about the teaching and learning of Mathematics in relation to participants' mathematical knowledge

The bar graph (Figure 5.10) below displays the number of participants with a mean score of two or higher against the number of participants with a belief score lower than two. A belief score of two or higher indicates strongly constructivist beliefs (CB) within this belief dimension, the teaching and learning of Mathematics, while a lower score indicates beliefs that lean towards traditionalist beliefs (TB) within this belief dimension. As stated in section 5.3.1, ten of the participants did not select one of the pure Mathematics options in the questionnaire, six indicated "other" tertiary qualifications that included Mathematics and only four indicated that they had no Mathematics at tertiary level.

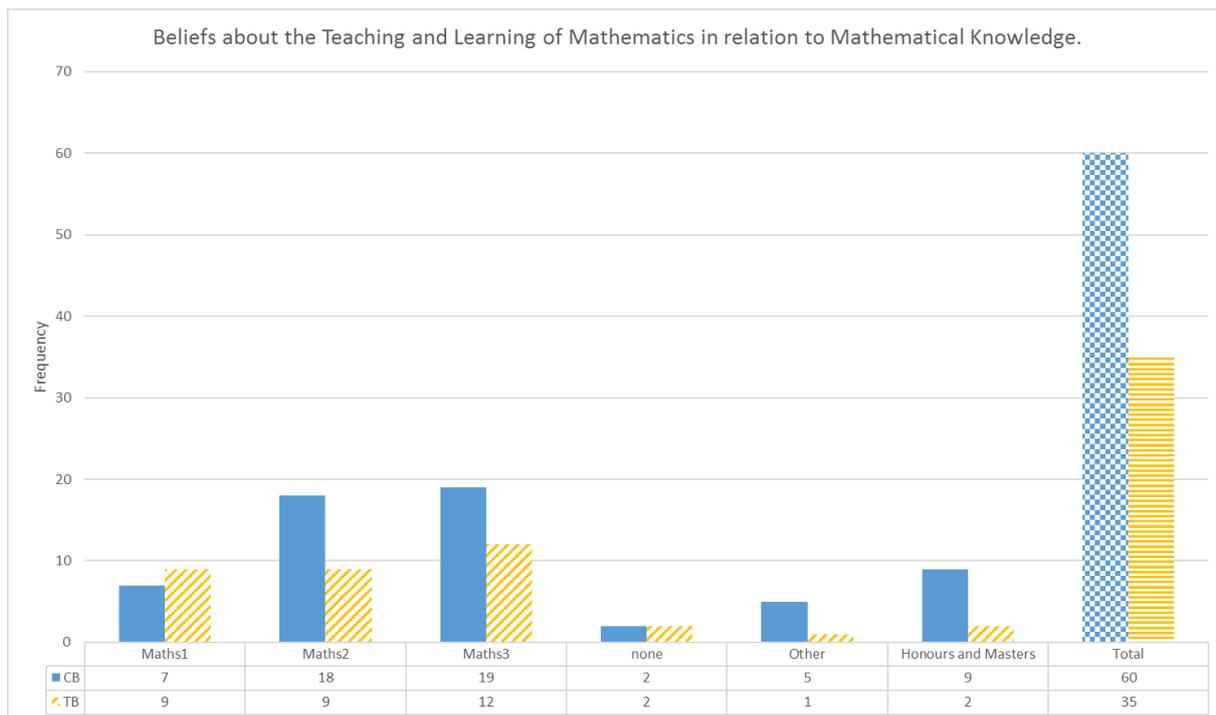


Figure 5.10 Beliefs about the teaching and learning of Mathematics in relation to mathematical knowledge. Constructivist beliefs – (CB); traditionalist beliefs – (TB)

The following Figure 5.11 displays the distribution of participants' qualifications in mathematical knowledge with a mean belief score of two and higher; for example, those who have strongly constructivist beliefs within this belief dimension, the teaching and learning of Mathematics. Figure 5.12 displays the distribution of qualifications in mathematical knowledge of participants with a mean belief score below two, that is, those who lean towards more traditionalist beliefs within this belief dimension.

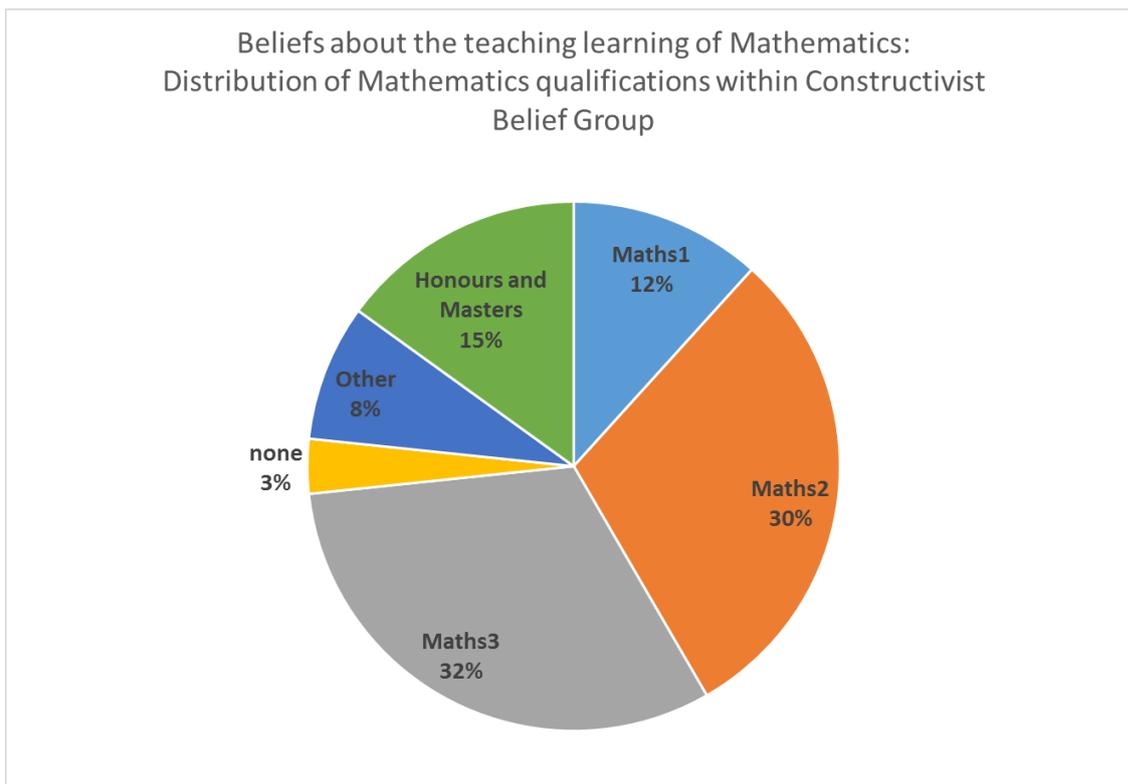


Figure 5.11 Beliefs about the teaching and learning of Mathematics: Distribution of Mathematics qualifications within Constructivist Belief Group

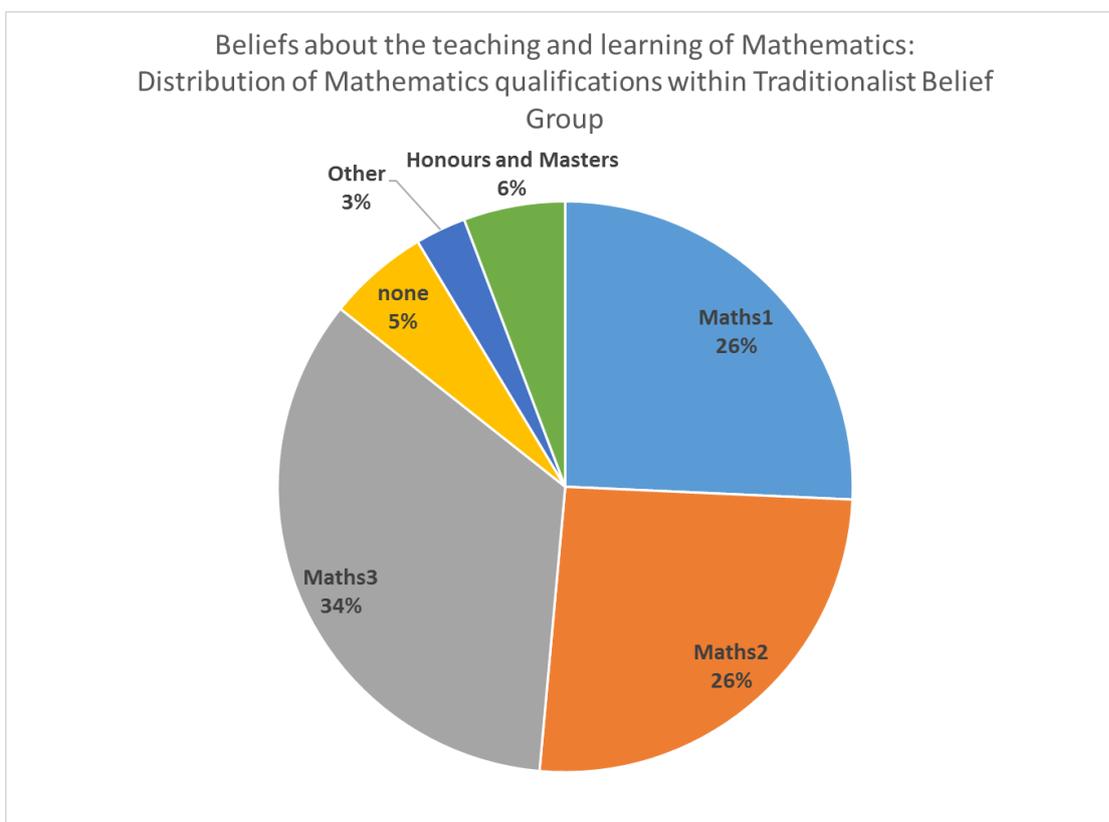


Figure 5.12 Beliefs about the teaching and learning of Mathematics: Distribution of Mathematics qualifications within Traditionalist Belief Group

The first finding indicates that in general, the participants' beliefs are aligned with a constructivist belief within the belief dimension on the teaching and learning of Mathematics. This is again evident when looking at the data displayed in the bar graph (Figure 5.10). This graph indicates that there are more participants with belief scores higher than two in all the mathematical knowledge groups, except for the Mathematics 1 grouping within the belief dimension about the teaching and learning of Mathematics. In addition, the Mathematics 1 group of participants predominantly (75%) hold traditionalist beliefs about the nature of Mathematics (see Figure 5.7). Having only Mathematics 1 as mathematical knowledge is therefore a strong indicator that a teacher will hold stronger traditionalist beliefs in both belief dimensions. With regard to beliefs about the teaching and learning of Mathematics, 60 of the 95 (63%) have belief scores equal or higher than two, indicating a strongly constructivist view by these participants. However, there is the concern that 37% of the participants hold beliefs that tend towards a traditionalist view in this belief dimension. A large percentage of the teaching body entrusted with teaching according to the aims and the objectives of the curriculum still hold many traditionalist beliefs on the teaching and learning of Mathematics distributed across all the mathematical qualifications. This, together with the high percentage (75%) of teachers' beliefs that tend towards traditionalist beliefs about the nature of Mathematics (see Figure 5.7), is problematic for the continued implementation of reforms in the teaching and learning of Mathematics.

5.3.4. Conclusion: Teachers' mathematical knowledge and their beliefs

In conclusion, in the collected data for this study, there seems to be no relationship between higher mathematical knowledge qualifications and teachers holding constructivist beliefs. The data does, however, indicate that teachers with Mathematics 1 as the highest Mathematics qualification are more likely to have traditionalist beliefs about the nature of Mathematics and about the teaching and learning of Mathematics than teachers with any other levels of Mathematics qualifications.

5.4. Teachers' professional qualifications and their beliefs

Finding 3: Teachers' teaching qualifications influence their beliefs about (a) the nature of Mathematics, and (b) the teaching and learning of Mathematics.

Literature indicates that a teacher's mathematical belief system is formed largely during the student-apprenticeship years while at school (Handel 2003) and that beliefs are either reinforced at tertiary level or challenged and changed, as concluded by Paolucci (2015). The quantitative analysis on teacher mathematical knowledge from the previous section shows little evidence that the participants with higher than required mathematical knowledge have higher constructivist belief scores. The data does, however, indicate that the participants hold some opposing beliefs between the two belief dimensions and that participants with Mathematics 1 show traditionalist beliefs across the belief dimensions.

This section further explores the research question by looking at the participants' professional teaching qualifications. In this study, the teachers' beliefs were investigated according to the period in which they studied for their professional qualification, as a significant change in teacher training and the qualification curriculum took place in the late 1990's in South Africa. These curricula reforms included changes to educator qualifications that reflected the Norms and Standards for Educators (2000) by April 2002 (South Africa 2000). The reforms incorporated a constructivist and social belief of the teaching and learning of Mathematics (Molefe & Brodie 2010). In secondary teacher training, the change was made from an HDE qualification to the PGCE, with first graduates at universities in 2002 (University of Stellenbosch 2001; Molefe & Brodie 2010).

The PGCE is a post-degree qualification offered at tertiary level in South Africa. PGCE is a general teacher qualification which prepares secondary school educators for teaching the aims and objectives of the national curriculum (South Africa 2000; The Council on Higher Education 2010). A PGCE with a Didactic Mathematics as a subject prepares teachers for offering Mathematics as a subject in secondary schools.

In light of the changes to curricula in 1996, teacher qualification data were analysed according to those with a PGCE and those with an HDE. Of the 95 participating teachers, 21 indicated "other" as a professional qualification. Of the 21 with other professional

qualifications, 13 had tertiary qualifications in education: Bachelor of Education degrees (10), Advanced Certificate in Education (1), Master of Education degree (1) and Doctor of Education degree (1). The remaining eight indicated no formal tertiary teacher training. However, seven out of the eight had Mathematics 2 or 3 as tertiary Mathematics qualification. This supports the claim by schools registered with the IEB that they employ qualified staff. The one remaining participant indicated a bachelor's degree without giving any indication of the level of Mathematics knowledge.

In addition, data were analysed according to when the teachers obtained their teaching qualifications. Teachers qualifying before 2002 were placed in a pre-reform group and those with later qualifications in a post-reform group. This was done as curriculum reform across all educational qualifications was enacted by 2002 (Molefe & Brodie 2010) with the first graduates in 2002 (University of Stellenbosch 2001; Molefe & Brodie 2010). The first table (Table 5.3) gives the teacher numbers per professional qualification grouping. The graphs (Figure 5.13 and Figure 5.14) display the distribution of the qualification in the sample group.

Table 5.3 Teacher Qualification Groupings

Qualification Grouping	Frequency
PGCE	35
HDE	39
Other Education Qualification	13
Other	8
Total	95
Pre-reform	47
Post-reform	48
Total	95

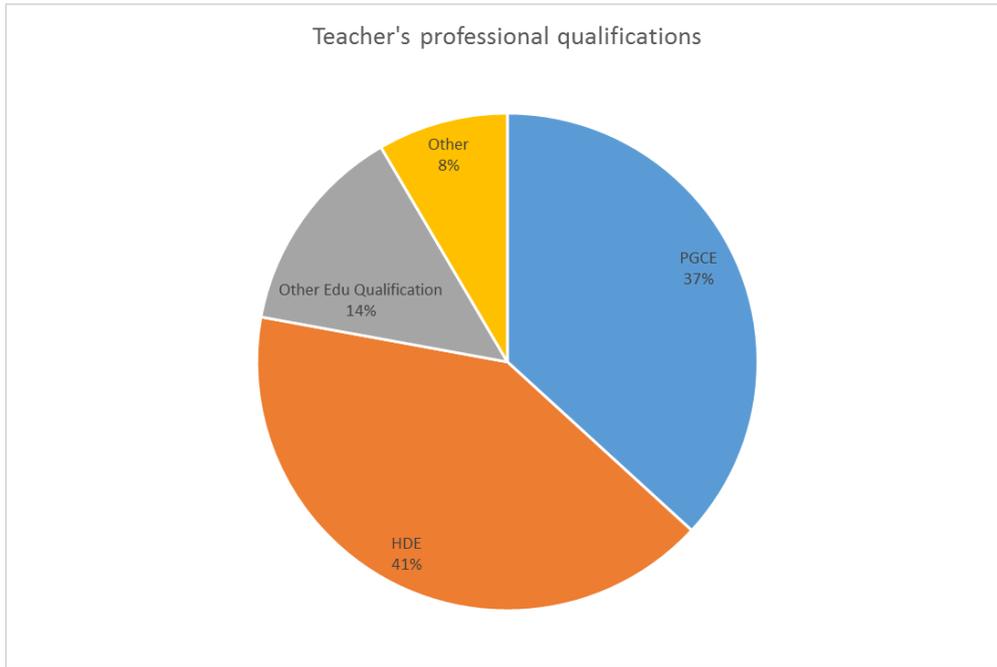


Figure 5.13 Teachers' professional qualifications



Figure 5.14 Teachers' professional qualifications according to time period

The following sections (5.4.1 and 5.4.2) report on and discuss the participating teachers' beliefs about (a) the nature of Mathematics and (b) the teaching and learning of Mathematics against their teacher-training qualifications.

5.4.1. Teachers' beliefs regarding the nature of Mathematics in relation to professional teacher qualifications

To investigate the teachers' beliefs about the nature of Mathematics in relation to their professional teacher qualifications, an analysis was made according to each participant's belief score. As with the previous analysis, a mean belief score per participant of two and higher indicates a strongly constructivist belief about the belief dimension, while a score below two indicates a view leaning towards a traditionalist belief about the belief dimension. Figure 5.15 (below) displays the frequency of participants with a mean belief score above two (CB) versus those below two (TB), against their professional qualifications.

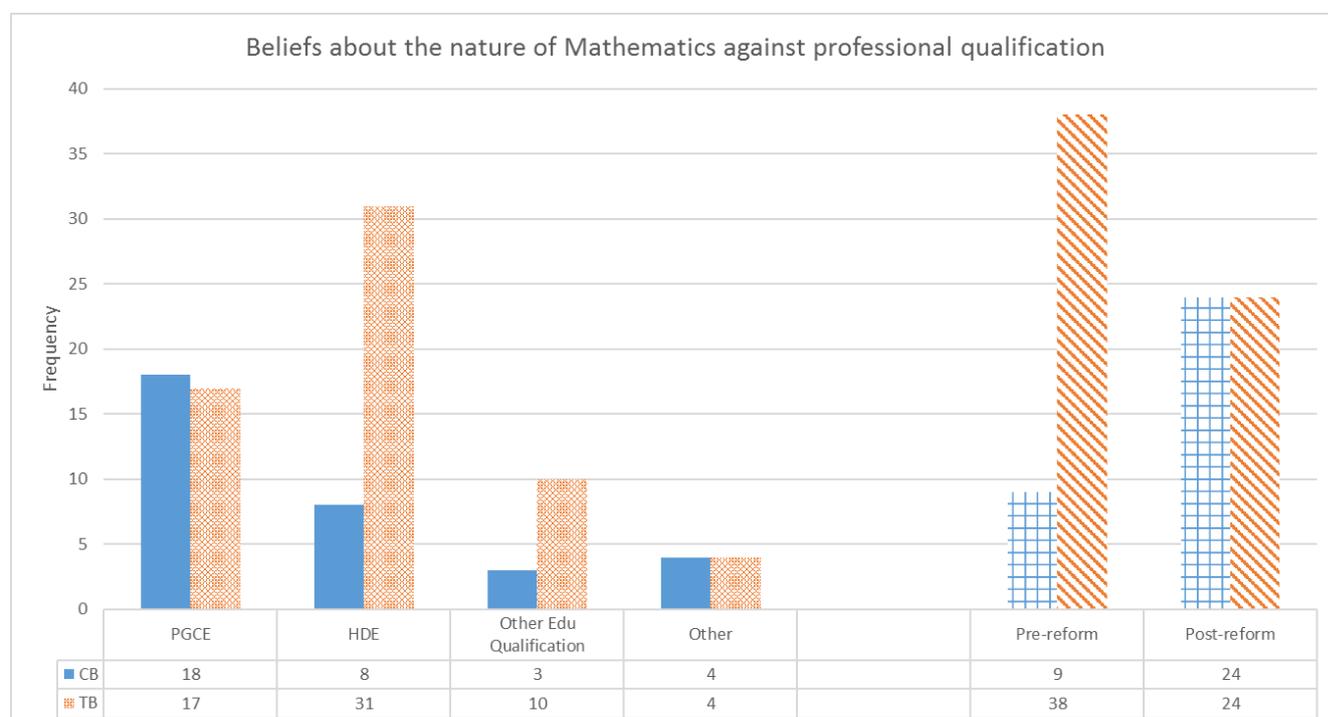


Figure 5.15 Beliefs about the nature of Mathematics against professional teaching qualification. Constructivist belief – (CB); traditionalist belief – (TB)

The data collected shows that 31 out of the 39 participants (82%) with an HDE qualification have belief scores below two, while only 17 out of the 35 (49%) with a PGCE do. The data indicates that those participants with an HDE (pre-reform qualification) are more likely (82%) to hold beliefs that tend towards a traditionalist belief about the nature of Mathematics than those with a PGCE (post-reform qualification) where 49% lean towards a traditionalist

belief about the nature of Mathematics. The pre- and post-reform groupings therefore show similar results with 38 out of the 47 (81%) participants with pre-reform qualifications scoring below two. This indicates that, as a group, their views lean towards traditionalist beliefs about the nature of Mathematics. In contrast, only 50% of the post-reform participants have scores below two. The following graphs (Figure 5.16 and Figure 5.17) display the distribution of professional qualifications of those participants with a mean score of two or more. These participants all have a strongly constructivist belief (average belief scores equal or higher than two) about the nature of Mathematics.

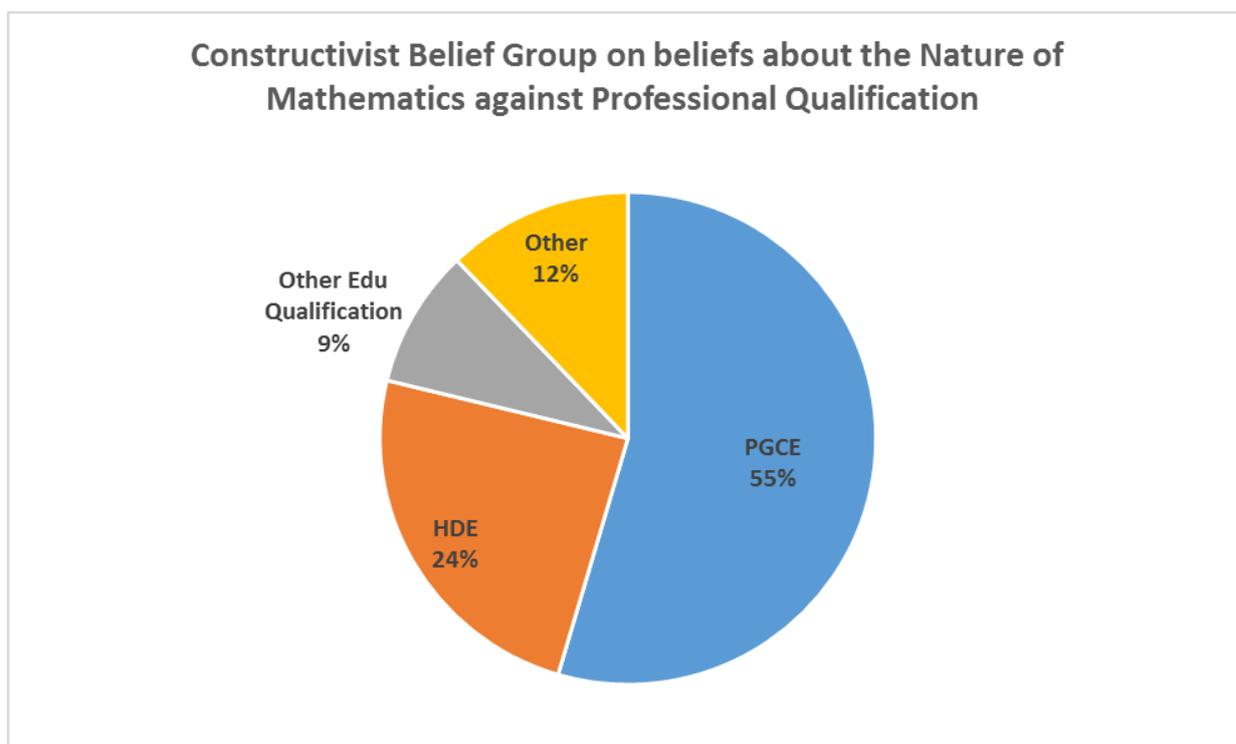


Figure 5.16 Constructivist belief group on beliefs about the nature of Mathematics against professional qualification

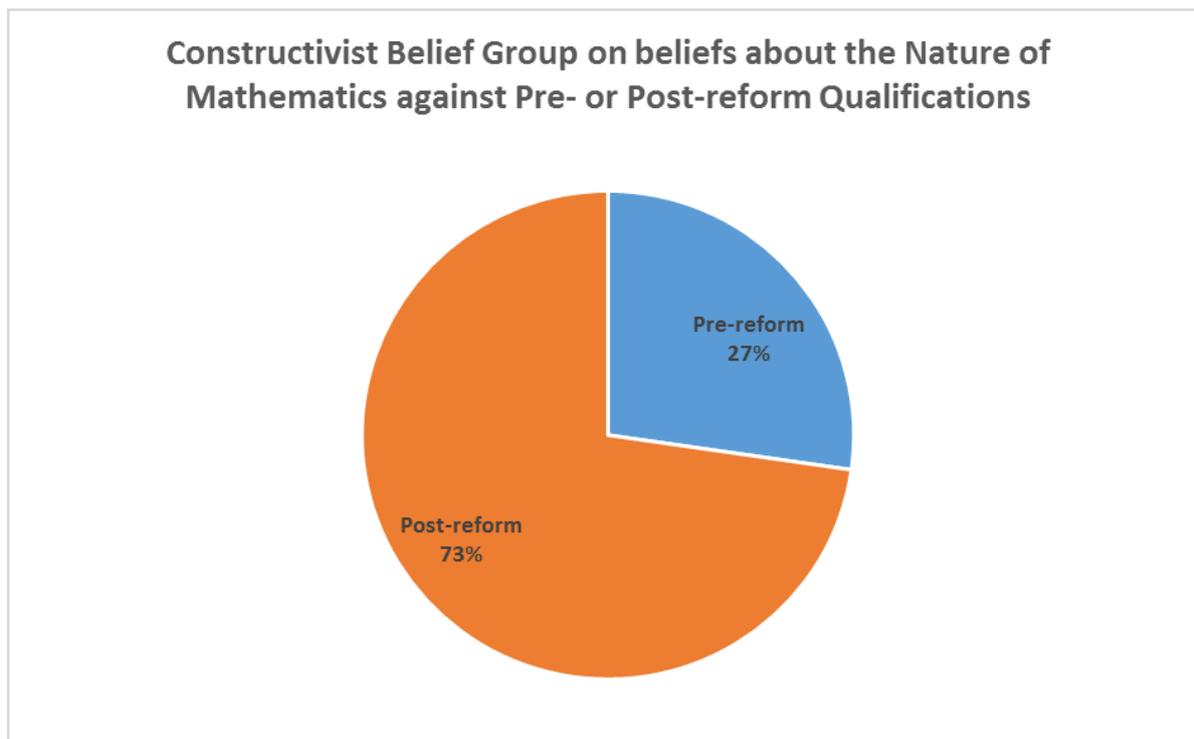


Figure 5.17 Constructivist belief group on beliefs about the nature of Mathematics against pre- or post-reform qualifications

Of the participants with a strongly constructivist view about the nature of Mathematics, 55% hold PGCE qualifications; in addition, 73% have post-reform qualifications.

The data seems to indicate that participants with a PGCE or post-reform professional qualification are more likely to hold strongly constructivist beliefs about the nature of Mathematics, while those with an HDE or pre-reform professional qualification are more likely to have views that lean towards traditionalist beliefs about the nature of Mathematics. As beliefs influence teacher classroom practices (Ernest 1989; Speer 2008; Cross 2009; Cross Francis 2014), this could indicate that teachers with a post-reform qualification are more likely to act according to their constructivist beliefs when teaching.

The following section investigates teachers' beliefs about the teaching and learning of Mathematics against their professional qualifications.

5.4.2. Beliefs about the teaching and learning of Mathematics in relation to professional teacher qualifications

To investigate the teachers' beliefs about the teaching and learning of Mathematics in relation to their professional teacher qualifications, an analysis was made according to each participant's belief score. As with the previous analysis, a mean belief score per participant of two and higher indicates a strong constructivist belief within this belief dimension, while a score below two indicates a view leaning towards traditionalist beliefs about this belief dimension. The bar graph (Figure 5.18) below displays the frequency of participants with a mean belief score above two versus those below two, against their professional qualifications.

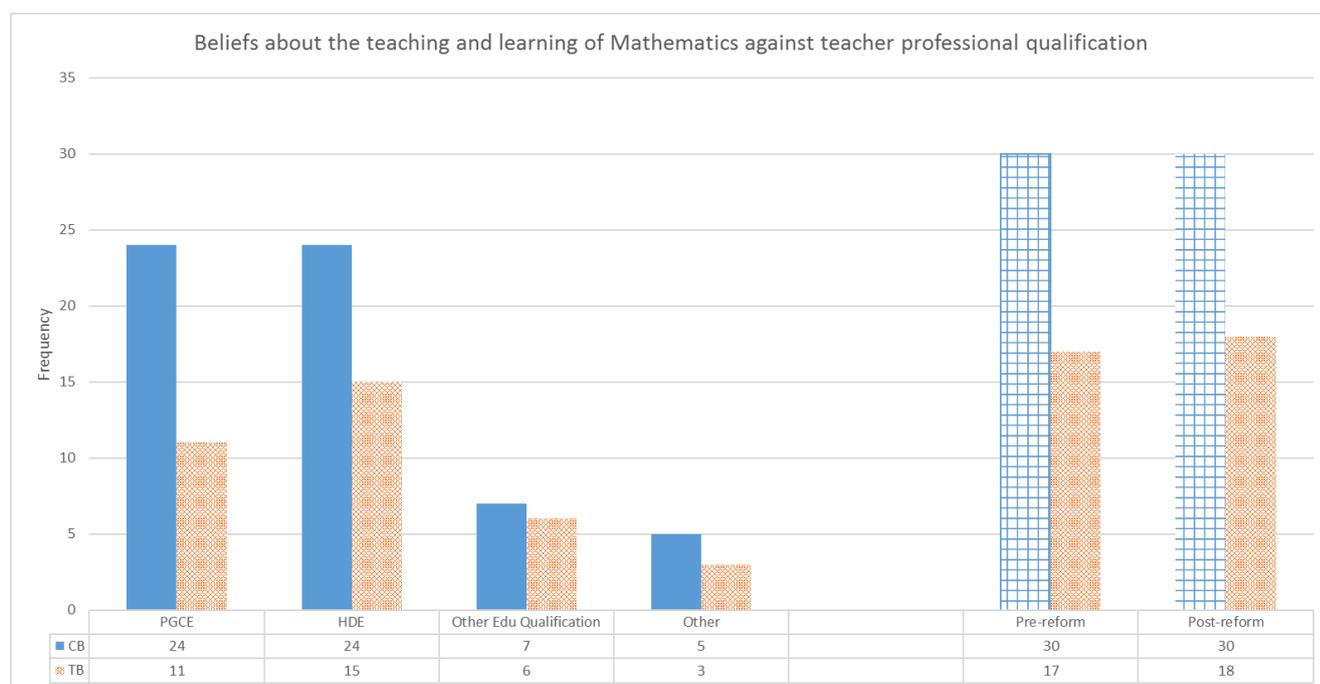


Figure 5.18 Beliefs about the teaching and learning of Mathematics against professional teaching qualification. Constructivist belief – (CB); traditionalist belief – (TB)

A higher number of participants have strongly constructivist views across all the qualification groupings, as shown in the bar graph (Figure 5.18). In particular, the PGCE grouping has the highest percentage (69%) of all qualification groupings, with strongly constructivist views about the teaching and learning of Mathematics.

The following graphs (Figure 5.19 and 5.20) display the professional qualifications distribution of those participants with a mean score of two or more. These participants have strongly constructivist beliefs about the nature of Mathematics.

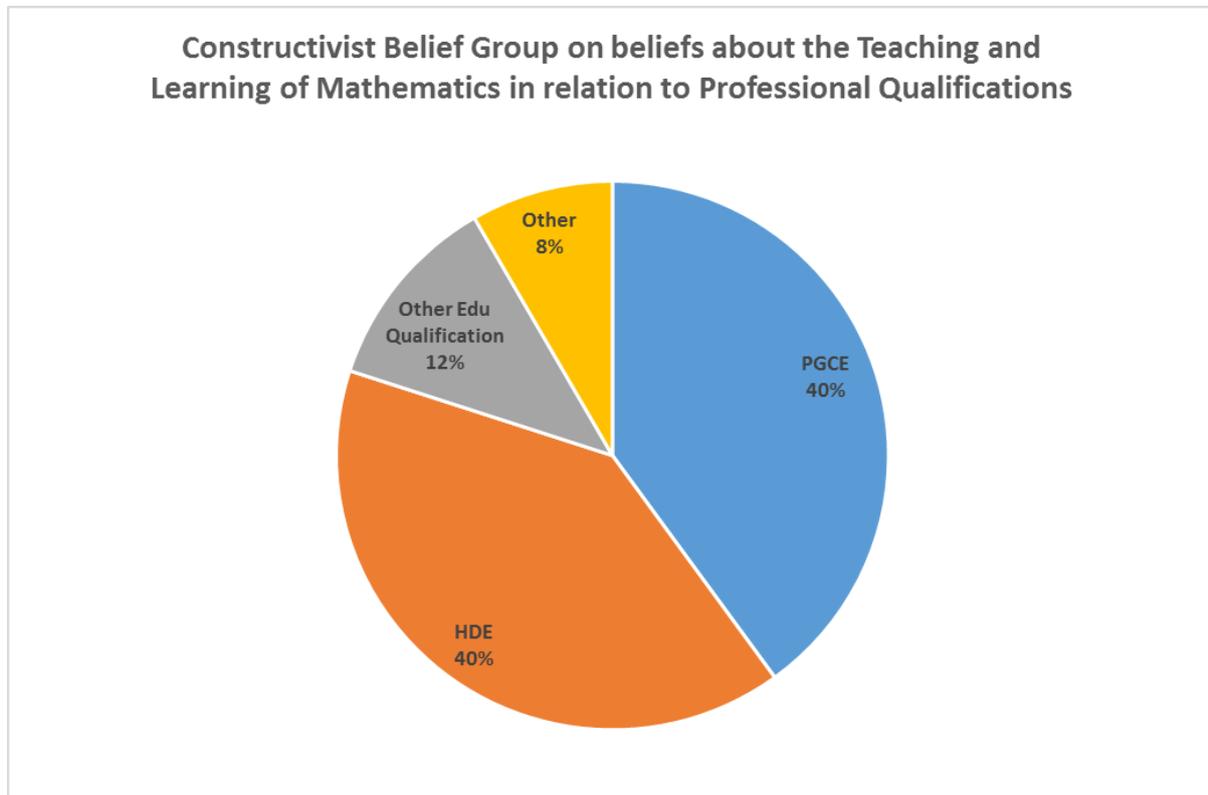


Figure 5.19 Constructivist belief group on beliefs about the teaching and learning of Mathematics in relation to professional qualifications



Figure 5.20 Constructivist belief group on beliefs about the teaching and learning of Mathematics in relation to pre- or post-reform qualifications

With reference to this belief dimension (the teaching and learning of Mathematics), there is no dominant qualification in the constructivist view grouping between pre- and post-reform qualifications. However, when considering the traditionalist view grouping (Figure 5.21 below), it is evident that the professional qualification held by most is the HDE qualification.

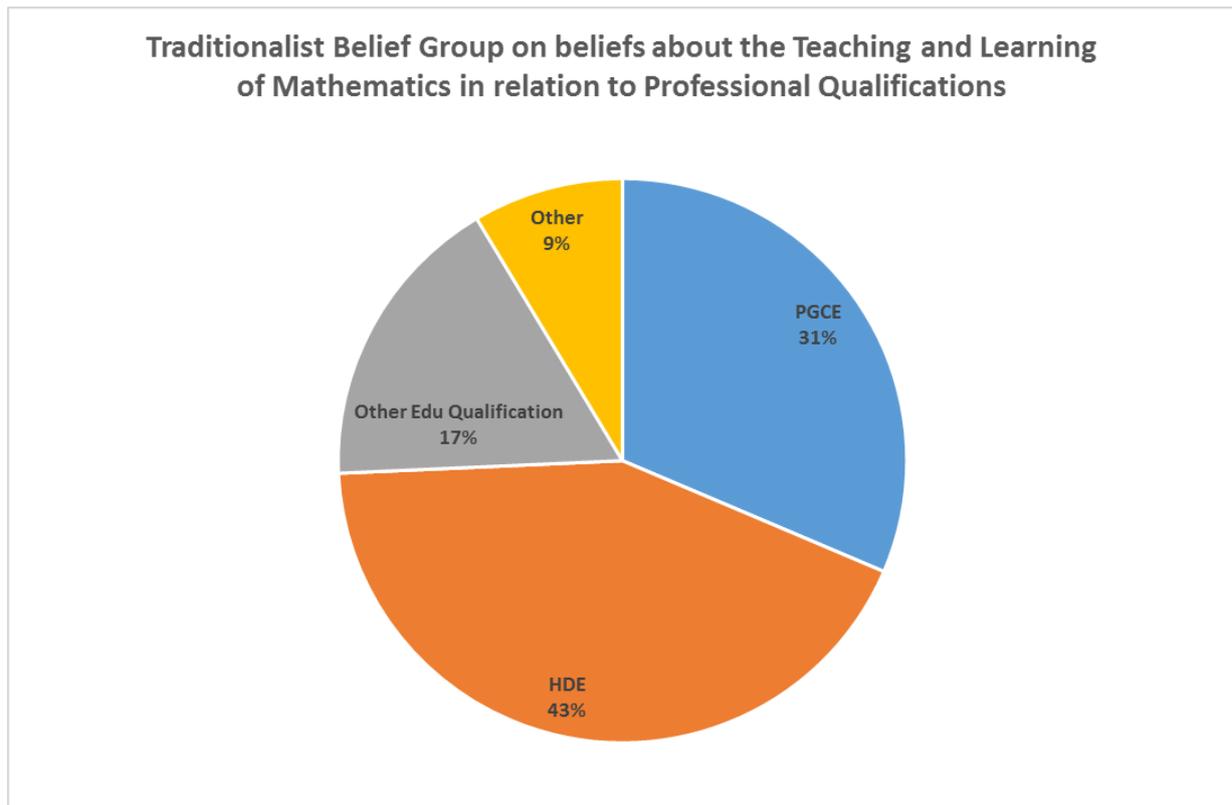


Figure 5.21 Traditionalist belief group on beliefs about the teaching and learning of Mathematics in relation to professional qualifications

The data indicates that participants with a PGCE qualification are more likely to hold a constructivist belief about the teaching and learning of Mathematics than those participants with an HDE qualification. However, there is no indication from the data that participants with a pre-reform qualification are more likely to hold constructivist beliefs about the teaching and learning of Mathematics than participants with a post-reform qualification.

The following section describes participants' responses per question item against professional qualification grouping.

5.4.3. Participants' responses per question against professional qualification groupings

The following two tables display the responses received (%) per belief question investigating the beliefs held by the teachers about the nature of Mathematics (Table 5.4) and the teaching and learning of Mathematics (Table 5.5), in relation to professional qualifications. Responses agree and strongly agree (A/SA) with a question item were grouped together and strongly disagree and disagree (SD/D) were grouped together. Strongly agreeing or agreeing with a statement indicates a constructivist belief of the statement in the questionnaire, unless the question was reversed, in which case it indicates a traditionalist belief of the question item by the participants in the qualification grouping. Strongly disagreeing, or disagreeing, with a statement indicates views that lean towards a traditionalist belief of the statement in the questionnaire, unless the question was reversed, in which case it indicates a constructivist belief of the question item by the participants in the qualification grouping. Reversed statements are indicated with a (-) in the table. Data for the following question items about the nature of Mathematics is displayed in Table 5.4.

- NM1(-) Mathematical problems can be done correctly in only one way.
- NM2(-) Some people have a natural talent for mathematics.
- NM3 Mathematics is primarily a formal way of representing the real world.
- NM4(-) In Mathematics something is either right or wrong.
- NM5(-) Some people are good at doing Mathematics and some are not.

Table 5.4 Responses per belief statement about the nature of Mathematics against professional qualification

Responses per qualification group (given as %) in the Nature of Mathematics belief category					
HDE (39 Participants)					
	NM1(-)	NM2(-)	NM3	NM4(-)	NM5(-)
SD/D	97	18	26	59	23
A/SA	3	82	74	41	77
PGCE (35 Participants)					
	NM1(-)	NM2(-)	NM3	NM4(-)	NM5(-)
SD/D	97	23	20	54	60
A/SA	3	77	80	46	40
Other Education (13 Participants)					
	NM1(-)	NM2(-)	NM3	NM4(-)	NM5(-)
SD/D	100	8	31	62	23
A/SA	0	92	69	38	77
Other (8 Participants)					
	NM1(-)	NM2(-)	NM3	NM4(-)	NM5(-)
SD/D	100	25	38	50	50
A/SA	0	75	62	50	50
Pre-reform (47 Participants)					
	NM1(-)	NM2(-)	NM3	NM4(-)	NM5(-)
SD/D	98	15	26	60	19
A/SA	2	85	74	40	81
Post-reform (48 Participants)					
	NM1(-)	NM2(-)	NM3	NM4(-)	NM5(-)
SD/D	98	23	25	54	58
A/SA	2	77	75	46	42

Data for the following question items about the teaching and learning of Mathematics is displayed in Table 5.5.

- BM1(-) Mathematics should be taught as a collection of procedures (skills) and algorithms.
- BM2 More than one representation should be used when teaching a mathematical concept.
- BM3(-) Good Mathematics teachers show you exactly how to get to the answer.
- BM4 Good reasoning should be regarded more important than getting to the correct answer.

- BM5 Learning Mathematics is an active process with learners actively involved in their learning.
- BM6(-) To solve a mathematical problem you need to teach the correct procedure.
- BM7 Problem-solving activities form part of the general teaching of lower ability groups.
- BM8(-) In teaching Mathematics logic is promoted, whereas creativity and originality are not stressed.
- BM9 Teaching Mathematics provides an excellent opportunity to promote the development of the learners' thinking.
- BM10(-) Mathematics teaching is especially meant for mathematically talented learners.

Table 5.5 Responses per belief statement about the teaching and learning of Mathematics against professional qualification

Responses per qualification group (given as %) in the teaching and learning of Mathematics belief category										
HDE (39 Participants)										
	BM1(-)	BM2	BM3(-)	BM4	BM5	BM6(-)	BM7	BM8(-)	BM9	BM10(-)
SD/D	44	8	77	10	0	64	59	46	0	90
A/SA	56	92	23	90	100	36	41	54	100	10
PGCE (35 Participants)										
	BM1(-)	BM2	BM3(-)	BM4	BM5	BM6(-)	BM7	BM8(-)	BM9	BM10(-)
SD/D	51	6	74	11	3	60	57	66	3	91
A/SA	49	94	26	89	97	40	43	34	97	9
Other Education (13 Participants)										
	BM1(-)	BM2	BM3(-)	BM4	BM5	BM6(-)	BM7	BM8(-)	BM9	BM10(-)
SD/D	54	0	77	8	0	46	54	54	0	69
A/SA	46	100	23	92	100	54	46	46	100	31
Other (8 Participants)										
	BM1(-)	BM2	BM3(-)	BM4	BM5	BM6(-)	BM7	BM8(-)	BM9	BM10(-)
SD/D	75	12	100	37	12	62	75	62	12	75
A/SA	25	88	0	63	88	38	25	38	88	25
Pre-reform (47 Participants)										
	BM1(-)	BM2	BM3(-)	BM4	BM5	BM6(-)	BM7	BM8(-)	BM9	BM10(-)
SD/D	49	6	81	11	0	62	57	49	0	85
A/SA	51	94	19	89	100	38	43	51	100	15
Post-reform (48 Participants)										
	BM1(-)	BM2	BM3(-)	BM4	BM5	BM6(-)	BM7	BM8(-)	BM9	BM10(-)
SD/D	52	6	75	15	4	58	60	62	4	88
A/SA	48	94	25	85	96	42	40	38	96	12

The responses per belief question item against professional qualification grouping in Table 5.5 show inconsistencies between the qualification groupings for the following question items:

Item NM5 (some people are good at doing Mathematics and some are not) has more participants per qualification grouping agreeing with the statement, except for those with a PGCE qualification as 60% of the participants with a PGCE qualification disagreed with the statement. This indicates that the PGCE qualification has more participants with constructivist beliefs regarding this statement. This is in contrast to the other qualification groupings that have more participants with a traditionalist belief about this question item.

Item BM1 (Mathematics should be taught as a collection of procedures (skills) and algorithms) has more participants with an HDE qualification agreeing (56%) with this statement than not. Therefore, the HDE qualification has more participants with a traditionalist belief regarding this statement. This is inconsistent with the remaining qualification groups that have more participants with a constructivist belief regarding this statement. Similarly, the pre-reform group has more participants (51%) agreeing with this statement, in contrast with the post-reform group with only 48% agreeing with this statement. This difference is also evident from the pre-reform and post-reform groupings.

In response to item BM8 (in teaching Mathematics logic is promoted, whereas creativity and originality are not stressed), the HDE qualification group is the only group with most (54%) agreeing with this statement. Therefore, the HDE group has more participants with a traditionalist belief of this statement than not. This is inconsistent with the remaining qualification groups that have more participants with a constructivist belief of this statement. The pre-reform grouping has 51% agreeing with this statement while the post-reform grouping have only 38% of the group agreeing with it. Again, this indicates that the percentage of participants in the post-reform group who hold constructivist views about this item is larger than that in the pre-reform group.

Item BM6 (to solve a mathematical problem you need to teach the correct procedure) has participants across the professional qualifications who mostly disagree with this statement except for participants with “other” educational qualifications, who mostly agree (54%) with this statement, indicating views that align more with a traditionalist view of the statement.

5.4.4. Conclusion: Teachers' professional qualifications and their beliefs

In conclusion, the data seems to indicate that participants with a PGCE or post-reform professional qualification are more likely to hold strongly constructivist beliefs about the nature of Mathematics. In addition, participants with a PGCE qualification are more likely to hold constructivist beliefs about the teaching and learning of Mathematics than those participants with an HDE qualification. Further, participants with a PGCE qualification are the only ones who predominantly disagree with the statement that some people are good at Mathematics and others are not. This implies that more participants with a PGCE qualification have constructivist beliefs regarding this statement.

In contrast, participants with an HDE or pre-reform professional qualification are more likely to hold beliefs that lean towards traditionalist beliefs about the nature of Mathematics. In addition, participants with an HDE also make up the largest number of participants with traditionalist beliefs of the teaching and learning of Mathematics. Further, more participants with an HDE qualification agree with the statements that (i) Mathematics should be taught as a collection of procedures (skills) and algorithms, and (ii) in teaching Mathematics, logic is promoted, whereas creativity and originality are not stressed. This implies that traditionalist views regarding these two statements are dominant among the participants with an HDE. This is inconsistent with the dominant constructivist views regarding these two statements held by participants with the other professional qualifications. Finally, this seems to imply that participating teachers with PGCE or post-reform qualifications are more likely to align with constructivist beliefs than those with other qualifications.

The following chapter reports on and discusses the qualitative data collected via open-ended questions from the questionnaire. The open-ended questions further investigate the participants' beliefs about the nature of Mathematics, the teaching and learning of Mathematics and, more specifically, their beliefs about mathematical problem-solving. The open-ended questions are there in part to triangulate the data collected in the quantitative section, but importantly they give the participants the freedom to answer a question without the constraints of a quantitative question item. The hope is that this would lead to richer data on the participants' espoused beliefs.

CHAPTER 6

RESULTS AND DISCUSSION: QUALITATIVE DATA

This chapter presents and discusses the qualitative data collected via the questionnaire. The open-questions in the questionnaire explore the participants' beliefs about

- (i) the nature of problems and problem-solving within the discipline of Mathematics;
- (ii) the nature of problems and problem-solving within the teaching of Mathematics;
- (iii) the nature of problems and problem-solving within the learning of Mathematics;
- (iv) their own beliefs about being a problem-solver.

The participants' responses per belief dimension are also discussed in relation to their academic and professional qualifications, thus adding to the findings made in the quantitative data analysis. The introduction to the chapter is followed by a detailed discussion of the responses to each open question, collected via the questionnaire.

6.1. Introduction

The results are discussed in terms of to the theoretical framework given in the following table (Table 6.1) and as discussed in Section 2.8.

Table 6.1. A continuum of Mathematics teachers' beliefs.

Beliefs about the nature of Mathematics	Beliefs about the teaching of Mathematics	Beliefs about the learning of Mathematics
<p>Instrumentalist Set of unrelated, but effective, rules and facts.</p>	<p>Traditionalist perspective The teacher as instructor, aiming at mastering the application. Content-focused with an emphasis on performance. Knowledge is transmitted. The focus is on teaching formulas and processes.</p>	<p>Traditionalist perspective Skill mastery, passive reception of knowledge. Passive receiver of knowledge. Learning is an independent and isolated event.</p>
<p>Platonist Static and unified body of knowledge that is discovered, not created.</p>	<p>Formalist perspective The teacher as explainer of existing knowledge. Content-focused with an emphasis on understanding.</p>	<p>Formalist perspective Receiver of knowledge but with the emphasis on the learner actively constructing knowledge and understanding.</p>
<p>Problem-solving A dynamic, continually expanding field of human creation and invention. Engaging in Mathematics is a process rather than a product.</p>	<p>Constructivist perspective The teacher as facilitator of the learning process. Learner-focused. Activities are interactive and learner-centred. Instruction emphasises problem solving, generative learning processes and explorative learning.</p>	<p>Constructivist perspective Autonomous exploration through problem posing and problem-solving. Learner takes responsibility for own learning. Learner socially constructs mathematical knowledge.</p>

The qualitative section of the questionnaire consisted of 14 open-ended questions. These questions were grouped in the following categories and set out to investigate:

- (i) The participant's beliefs about the nature of problems and problem-solving within the discipline of Mathematics. (R1-R3)
- (ii) The participant's beliefs about the nature of problems and problem-solving within the teaching of Mathematics. (R4-R8)
- (iii) The participant's beliefs about the nature of problems and problem-solving within the learning of Mathematics. (R9-R11)
- (iv) The participant's own beliefs about being a problem-solver. (R12-R14)

These categories correspond with the belief dimensions about (i) the nature of Mathematics and (ii) the teaching and learning of Mathematics under which the quantitative data was discussed. However, the focus of the qualitative enquiry was to elicit responses from the participants about beliefs concerning mathematical problems and problem-solving under these belief dimensions. These open-ended questions also further investigated the research question to this study:

Pertaining to secondary Mathematics teachers in independent schools registered with the Independent Examination Board (IEB) in South Africa,

- (i) *What are practising secondary Mathematics teachers' beliefs about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics?*
- (ii) *How do their beliefs relate to their implementation of problem-solving activities as required by the National Curriculum Statement?*

Each open-ended question is also analysed according to the individual participant's academic and professional qualifications and this builds on Chapter 5's findings.

6.2. Participants' beliefs about the nature of problems and problem-solving within the discipline of Mathematics

R1. What is a mathematical problem to you?

A mathematical problem in Mathematics education can be defined as an exercise given to learners to consolidate new procedures and concepts learned in class (Schoenfeld 1992; Xenofontos & Andrews 2014) or as a task that requires learners to solve unseen/non-routine problems (Schoenfeld 2013). Further, a problem task can be either a purely abstract problem or an applied problem within a real situation (Xenofontos and Andrews 2014). When a mathematical problem is unseen or non-routine it is defined as problem-solving in the national curriculum (Department of Basic Education 2011a: 53). A consequence of this view of mathematical problem-solving is that the cognitive demand of a mathematical problem is dependent on the solver (Xenofontos & Andrews 2014).

The Curriculum and Assessment Policy Statement (CAPS) and Subject Assessment Guidelines (SAGs) define mathematical problems according to four cognitive levels (Department of Basic Education 2011a; IEB 2018b). The cognitive levels are based on those defined in the TIMSS (Trends in International Mathematics and Science Study) of 1999 (Department of Basic Education, 2011a: 53). This cognitive taxonomy classifies a mathematical problem according to the complexity of the problem for the solver (Polya 1946; Schoenfeld 2013; Xenofontos and Andrews 2014). The classification of mathematical problems according cognitive demand is given in Table 6.2 with descriptions (Department of Basic Education, 2011a: 53).

Table 6.2. Classification of mathematical problems according to cognitive levels, with descriptions

Problem classification	Cognitive-level Descriptor	Descriptions of skills to be applied in the problems
Knowledge/routine procedure	Knowledge	<ul style="list-style-type: none"> • Straight recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary
	Routine procedure	<ul style="list-style-type: none"> • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formulae on the information sheet (no changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve a few steps • Identification and use (after changing the subject) of correct formulae • Generally similar to those encountered in class
Problem-solving/complex	Complex procedure	<ul style="list-style-type: none"> • Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real-world context • Could involve making significant connections between different representations • Requires conceptual understanding
	Problem-solving	<ul style="list-style-type: none"> • Non-routine problems (which are not necessarily difficult) • Higher-order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts

(Department of Basic Education 2011a: 53)

This open-question, “what is a mathematical problem to you?” investigates the participants’ beliefs about what constitutes a mathematical problem. This is relevant, as in part, individual teachers’ goals for their Mathematics instruction will depend on their perspective of what defines a mathematical problem (Schoenfeld 1992). This implies that if teachers hold the view that mathematical problems are generally problem-solving questions, then their teaching would more likely routinely involve problem-solving questions which would then promote the aims of the curriculum.

Table 6.3 below displays the percentage of responses according to the classifications of mathematical problems, as given in Table 6.2. In addition, the classifications are displayed in relation to the participants’ academic and professional qualifications. Examples of responses regarding the classification of mathematical problems are:

- An “other” response: *“Any question that challenges a learner in the respect of any maths.”*
- Routine response: *“A scenario that has a definite/clear equation or set of equations directly associated with it.”*
- Problem-solving response: *“A question/task that requires application of skills and knowledge rather than routine substitution or repetition. May involve context.”*

Table 6.3. Classification of responses to R1: What is a mathematical problem to you?

Q11/R1 What is a mathematical problem to you?					
		n	%		
Knowledge/Routine Procedure		10	11		
Problem-Solving/Complex		60	63		
Other		25	26		
		95	100		
Real life/Word problem response		17	18		
Academic Qualifications	n	Knowledge/Routine Procedure (%)	Problem-Solving/Complex (%)	Other (%)	Real life/Word problem response (%)
Maths1	16	0	56	44	0
Maths2	27	19	59	22	19
Maths3	31	3	74	23	26
Honours	7	14	57	29	29
Masters	4	25	25	50	0
None	4	25	75	0	25
Other	6	17	67	17	17
	95				
Professional Qualifications	n	Knowledge/Routine Procedure (%)	Problem-Solving/Complex (%)	Other (%)	Real life/Word problem response (%)
PGCE	35	14	60	14	0
HDE	39	8	64	8	13
Other	21	10	67	10	38
Pre-Reform	47	9	64	28	19
Post-Reform	48	13	63	25	17

The responses indicate that 63% of the respondents viewed a mathematical problem as a problem-solving or complex problem type of activity. Of those who indicated that mathematical problems are of a problem-solving/complex problem type, the largest group (28%) indicated that in their opinion, a mathematical problem could be described as a worded situation that is often relevant to real life or a specific context. As this is the descriptor given by most of the participants, this view is further investigated in the interviews. In addition, only 11% of the responses described a mathematical problem as a procedural and/or knowledge type of activity. It is concerning that 26% of the responses could not adequately describe a mathematical problem in such a way so that it could be classified. Given that teachers' instructions are, in part, dependent on their beliefs about what constitutes a mathematical problem, it is of concern that a large section (37%) of this sample group could not adequately describe what a mathematical problem is, or they classified it as knowledge or a routine procedure.

When responses to this question (R1) are considered in relation to academic qualifications, 74% of the participants with a Mathematics 3 qualification describe a mathematical problem as something that involves problem-solving or as a complex problem. Of all the academic qualification groups, more participants with a Mathematics 3 qualification view mathematical problems as a problem-solving/complex problem. When responses to this question (R1) are considered in relation to the professional qualification groupings, the dominant belief is that the term "mathematical problem" indicates problem-solving or a type of complex problem.

R2. What characteristics should a good mathematical problem have?

The responses to this question reveal teachers' views about the characteristics of a mathematical problem. Table 6.2 was again used to code responses. The responses in the problem-solving/complex category contained the following keywords: "challenging, creativity, out of the box, out of comfort zone, multiple steps, unseen, application of more than one concept, relevant, room for 'messy mathematics', analyse the mathematics, interesting, inspire curiosity". Those responses that saw a good mathematical problem as involving both knowledge/routine procedures and problem-solving/complex characteristics were placed under the problem-solving/complex category. Examples used in the coding were:

- An “other” response: *“It has to make them think really hard and maybe force them to work in groups or to talk to a classmate in that regard.”*
- Knowledge/routine procedure response: *“It should require pupils to think and to have to apply those skills and concepts that have been taught.”*
- Problem-solving/complex procedure response: *“[...], a mathematical problem should focus on a real-life situation where the learner can ‘imagine’ the scenario/context. In other words, the problem (when solved) should have meaning to the learner and not just be another problem to solve/a regurgitation of facts/theory learned. The mathematical problem should ideally incorporate a variety of mathematical concepts that the learner needs to take into consideration or needs to apply in order to reach a solution to the problem. Basic skills should also be incorporated into the problem.”*

Table 6.4 below displays (as a percentage) the number of responses by the participants per problem type. In addition, the number of responses per problem type, in relation to academic and professional qualifications, are also shown as a percentage.

Table 6.4. Classification of responses to R2: What characteristics should a good mathematical problem have?

Q12/R2 What characteristics should a good mathematical problem have?				
		n	%	
Knowledge/Routine Procedure		21	22	
Problem-Solving/Complex		61	64	
Other		13	14	
		95	100	
Academic Qualifications	n	Knowledge/Routine Procedure (%)	Problem-Solving/Complex (%)	Other (%)
Maths1	16	19	69	13
Maths2	27	15	70	15
Maths3	31	29	52	19
Honours	7	43	57	0
Masters	4	0	100	0
None	4	0	100	0
Other	6	33	50	17
	95			
Professional Qualifications	n	Knowledge/Routine Procedure (%)	Problem-Solving/Complex (%)	Other (%)
PGCE	35	14	77	9
HDE	39	28	59	13
Other	21	24	52	24
Pre-Reform	47	32	55	13
Post-Reform	48	13	63	25

As shown in Table 6.4, some 64% of the participants' responses regarding the characteristics of a good mathematical problem were that such problems should be of the problem-solving/complex type. This indicates that the majority of the participants would most likely promote problem-solving as a routine activity in their teaching. However, 14% could not adequately describe the characteristics of a mathematical problem and 22% described a good mathematical problem as a routine procedure. Further, a high percentage (36%) – which represents more than one out of three – did not categorise a good mathematical problem as complex or of the problem-solving type. This is of concern as an individual teacher's instruction depends on his or her personal perspective of what defines a good mathematical problem (Schoenfeld 1992).

When investigating responses in relation to the academic qualifications of the participants, 69% of participants with Mathematics 1 and 70% of those with Mathematics 2 believed that the characteristics of a good mathematical problem should be of the problem-solving/complex type. This is a significantly higher percentage than that of participants with Mathematics 3 (52%). While the groups with honours, master's and other academic qualifications all have more participants holding a problem-solving belief about this question, the number of participants per qualification group is statistically very small.

Responses in relation to professional qualifications showed that a higher percentage of participants with a PGCE, or a post-reform qualification in education, characterised a good mathematical problem as being of the problem-solving/complex type. Some 77% of the PGCE group and 63%, and of the post-reform group held this view. Although more than half of participants in the HDE group (59%) and pre-reform group (55%) were classified as problem-solving/complex, these percentages were significantly smaller than those of the PGCE and post-reform groups.

The results, when considering the academic qualification groupings, are not consistent with the results for the previous question (R1). The first open-ended question (R1) investigated the participant's view regarding what constitutes a mathematical problem, while this question (R2) investigated the participant's view about the characteristics of a good mathematical problem. One would expect that participants who have problem-solving/complex views of R1 would hold similar views about R2. The data, however, shows that this is not so. Of the participants with a Mathematics 3 qualification, 74% described a mathematical problem as problem-solving/complex, while only 52% described the characteristics of a good

mathematical problem as problem-solving/complex. This same inconsistency is evident in the responses of those with Mathematics 1 and Mathematics 2. Here, however, the percentage responses are higher. The contingency table (Table 6.5) clearly shows this inconsistency with only 37 out of the 95 (39%) participants giving responses that can be classified under the problem-solving/complex category for both questions.

These conflicting views regarding what constitutes a mathematical problem, and what the characteristics are of a good mathematical problem, could be related to the problem-solver. The first question might have been viewed as relating to the teacher (i.e., what is a mathematical problem to the teacher) and the second as relating to the learner (i.e., what would be the characteristics of a good mathematical problem to be given to a learner). Further, if only 39% (37 out of 95 participants, see Table 6.5) of the participants hold problem-solving/complex views in response to both R1 (what is a mathematical problem?) and R2 (what characteristics should a good mathematical problem have?) then the majority of the participating teachers hold conflicting views of this important concept that informs instructional practice.

Table 6.5. Contingency table: R1 vs R2 responses

What is a mathematical problem to you?	What characteristics should a good mathematical problem have?			
	Routine/Procedure	Non-routine/Complex	Other	
Routine/Procedure	3	7	0	10
Non-routine/Complex	13	37	10	60
Other	5	17	3	25
	21	61	13	95

R3. What does mathematical problem-solving mean to you?

This question specifically refers to mathematical problem-solving and it aims to investigate how closely the teachers' views of mathematical problem-solving correspond to the definition given in the CAPS document. The CAPS documentation gives the following description of problem-solving (Department of Basic Education 2011a):

- Mathematical problem-solving allows us to make sense of the universe around us and most of all to teach us to think creatively;
- Mathematical problem-solving includes the “when” and “why” problem types and not just the “how”;
- It involves non-routine/unseen problems;
- It allows for the development and use of higher order reasoning processes;
- It also involves the ability to break a problem down, and to solve it in this way.

This description was used in the categorisation of responses to R3. Responses that did not fit the above description or were not clear in their meaning were categorised as “Align with CAPS-No”. Those that did fit with the description were called “Align with CAPS-Yes”.

Examples relating to the coding are:

Align with CAPS-No: *“Finding patterns to a solution” or “Seeking pieces of a puzzle and putting it together.”*

Align with CAPS-Yes: *“The application of knowledge to break down and solve more diverse or complex mathematical questions/applications. Breaking down of larger tasks into smaller bits that are solvable using concepts that have been learnt.”* A further example is: *“When the learner can apply what they've learnt in a unfamiliar context.”*

Table 6.6 (below) displays the responses that were aligned with the description and those that did not.

Table 6.6. Classification of responses to R3: What does mathematical problem-solving mean to you?

Q13/R3 What does mathematical problem-solving mean to you?			
		n	%
Align with CAPS-Yes		49	51
Align with CAPS-No		46	49
		95	100
Academic Qualifications	n	Align with CAPS-Yes (%)	Align with CAPS-No (%)
Maths1	16	63	37
Maths2	27	37	63
Maths3	31	58	42
Honours	7	57	43
Masters	4	25	75
None	4	50	50
Other	6	67	33
Professional Qualifications	n	Align with CAPS-Yes (%)	Align with CAPS-No (%)
PGCE	35	49	51
HDE	39	54	46
Other	21	52	48
Pre-Reform	47	51	49
Post-Reform	48	52	48

The participants were divided in their responses, with 51% giving responses that were aligned with the description of mathematical problem-solving and 49% that were not aligned. This dichotomy in responses is also evident when looking at the results relating to pre- and post-reform professional qualifications. However, 63% of participants with a Mathematics 1 qualification gave a response that was aligned with that of the National Curriculum Statement (NCS), while Mathematics 2 participants had the largest number of participants (63%) who did not have views of mathematical problem-solving in line with the description given in the NCS. Too few participants with other academic qualifications took part in the survey to make any statistical comment.

In conclusion, when considering the responses to the first three open-questions (R1: “What is a mathematical problem to you?”; R2: “What characteristics should a good mathematical problem have?”; and R3: “What does mathematical problem-solving mean to you?”), only 21 out of the 95 (22%) participants gave responses that could be classified as problem-solving/complex and gave a description of problem-solving that was aligned with that given in the national curriculum. This indicates that among the majority of participants there are

conflicting views on mathematical problems and mathematical problem-solving. In addition, this seems to indicate that a significant number of participating teachers

- (i) do not have the basic knowledge about mathematical problems and problem-solving required to implement the aims of the national curriculum, regardless of academic or professional qualifications;
- (ii) hold traditionalist views about mathematical problems and problem-solving within the belief dimension relating to the nature of Mathematics.

The real concern is that the data seem to imply that a large percentage of participants hold beliefs about mathematical problem-solving that are in conflict with what is set out in the curriculum.

6.3. Participants' beliefs about the nature of problems and problem-solving within the teaching of Mathematics

R4. What is the teacher's role during problem-solving activities in class?

This question investigates the views of participating teachers on the role of the teacher in a problem-solving activity in class.

Responses were grouped according to the descriptions of the role of the teacher in the problem-solving/constructivist category. These descriptions are given below in Table 6.7. (For a full table showing descriptors for all belief categories, see Section 2.8.)

Table 6.7 Descriptors for belief category: problem-solving/constructivist

Problem-solving:	Constructivist teaching perspective:	Constructivist learning perspective:
A dynamic, continually expanding field of human creation and invention. Engaging in Mathematics is a process rather than a product.	The teacher as facilitator of the learning process. Learner-focused. Activities are interactive and learner-centred. Instruction emphasis is on solving problems, generative learning processes and explorative learning.	Autonomous exploration through problem-posing and problem-solving. Learner takes responsibility for their own learning. Learner socially constructs mathematical knowledge.

Responses were categorised according to whether the participants adequately or inadequately described the role of the teacher. Examples of this are:

- Inadequate response: *“Assisting and leading.”*
- Adequate response: *“[...] facilitate the process and not the answer.”*

The results from the data collection are displayed in Table 6.8. The table gives the number of responses (as a percentage) categorised according to the given criteria. In addition, the responses are categorised according to academic and professional qualifications.

Table 6.8. Classification of responses to R4: What is the teacher's role during problem-solving activities in class?

Q14/R4 What is the teacher's role during problem-solving activities in class?			
		n	%
Adequate Response		62	65
Inadequate Response		33	35
		95	100
Academic Qualifications	n	Adequate Response (%)	Inadequate Response (%)
Maths1	16	81	19
Maths2	27	48	52
Maths3	31	74	26
Honours	7	86	14
Masters	4	25	75
None	4	75	25
Other	6	50	50
Professional Qualifications	n	Adequate Response (%)	Inadequate Response (%)
PGCE	35	66	34
HDE	39	59	41
Other	21	76	24
Pre-Reform	47	62	38
Post-Reform	48	69	31

Of all the participants, 65% described the teacher's role as one that corresponded with the description of the problem-solving/constructivist category. However, more than a third of the participants (35%) did not describe the teacher's role as such. This is concerning as the teacher's role is essential in creating the learning environment that will promote and encourage the aims of the curriculum.

Regarding qualification groupings, the majority of participants gave responses that corresponded to a constructivist role of the teacher during a problem-solving activity in each of the academic qualification groupings. However, this did not hold true for the Mathematics 2 and master's groupings, where 48% and 25% respectively gave constructivist responses. The master's group of four participants is too small to make any statistical inference regarding the group.

The results from the professional-qualification groupings showed that a larger percentage in the PGCE (66%) and "other" (76%) qualification groups gave responses that corresponded with the role of a teacher in the problem-solving/constructivist belief category, as opposed to those in the HDE qualification group (59%). This is also seen in the post-reform qualification

group, where 69% of the group's participants gave responses that aligned with the role description of a teacher in the problem-solving/constructivist belief category. Moreover, the majority of the participants across all the professional qualifications gave responses that were aligned with a constructivist view relating to the role of the teacher in a problem-solving activity.

In conclusion, a higher percentage (69%) of participants with post-reform teaching qualifications described the role of the teacher as falling within the problem-solving category. The percentage is higher than participants within the pre-reform qualification group (62%). This seems to suggest that teachers with post-reform teaching qualifications have a higher likelihood of having a problem-solving belief relating to the role of the teacher when dealing with problem-solving activities, than those with pre-reform teaching qualifications. This is a positive conclusion, but still 31% of teachers with a post-reform qualification did not hold beliefs relating to the role of the teacher that fall within the problem-solving belief category; nor did they adequately describe the role of the teacher.

R5. Do you think that the Mathematics curriculum in its current state can be taught by using a problem-solving approach? If not, please elaborate.

According to national and international studies, teachers find it difficult to introduce problem-solving as a routine activity in their lessons (Mayer 1998; Jonassen, 2000; Brodie & Pournara 2005; Webb & Webb, 2008; de Freitas & Zolkower, 2011; Stols, 2013). This question aims to investigate whether, in the opinion of the participating teacher, the current national curriculum can be taught by using a problem-solving approach. This question also links to the second research question in that it investigates teachers' views on implementing problem-solving as a routine activity.

The following table (Table 6.9) displays the collected responses. In addition, the responses are grouped according to academic and professional qualifications.

Table 6.9 . Classification of responses to R5: Do you think that the Mathematics curriculum in its current state can be taught by using a problem-solving approach? If not, please elaborate.

Q15/R5 Do you think that the Mathematics curriculum in its current state can be taught by using a problem-solving approach? If not, please elaborate.				
		n	%	
Yes		64	67	
No		31	33	
Total		95	100	
<i>Time as reason</i>		33	35	
Academic Qualifications	n	Yes (%)	No (%)	Time (%)
Maths1	16	63	38	13
Maths2	27	78	22	37
Maths3	31	61	39	42
Honours	7	86	14	43
Masters	4	75	25	25
None	4	75	25	50
Other	6	33	67	33
Professional Qualifications	n	Yes (%)	No (%)	Time (%)
PGCE	35	69	31	31
HDE	39	67	33	38
Other	21	67	33	33
Pre-Reform	47	68	32	38
Post-Reform	48	67	33	31

Of all the participants, 67% agreed that the Mathematics curriculum could be taught by using a problem-solving approach. This should, however, be viewed together with the results from the first three open questions where very few of the participants could adequately describe a mathematical problem or problem-solving. In general, 67% teachers agreed that the Mathematics curriculum could be taught by using a problem-solving approach, but they expressed concerns and stated that there were constraints to this approach. Time constraints, including curriculum-content overload, was mentioned most often (35%) as a restriction in the implementation of a problem-solving approach to the teaching and learning of Mathematics. Other frequent responses included the following constraints to the implementation of a problem-solving approach:

- Curriculum is focused on procedures and not application of skills;
- Problem-solving is extension work only and a supplement to the work done in class;
- The availability of teaching resources is an issue;
- Only certain topics are covered;
- Curriculum focus and delivery in higher grades is on summative assessment;
- There are concerns about practical implementation.

Responses categorised according to academic qualifications reflect the general distribution of data within all academic qualification groupings. Only the majority of participants with “other” academic qualifications disagreed with a problem-solving approach. Some 78% of participants with Mathematics 2 and 86% of those with an honours degree agreed with the problem-solving approach. These groups showed the highest approval for the problem-solving approach.

Responses categorised according to professional qualifications showed similar percentage distributions across all qualification groupings. This seems to indicate that post-reform training has not altered the beliefs of teachers drastically as similar numbers of participants with pre- and post-reform qualifications believe that the curriculum can be taught by using a problem-solving approach. However, the participants were reflecting on their views regarding problem-solving and this result should be interpreted with this in mind and their interpretation of the curriculum.

From the quantitative data (BM13) collected, only 11 (12%) of the participants indicated that they spent more than two hours per week on average working on problem-solving questions. Eight of the 11 participants indicated that the curriculum in its current state could be taught by using a problem-solving approach. This seems to imply that while a majority of the participants felt that the curriculum could be taught using a problem-solving approach, very few indicated that they spent substantial class time on incorporating problem-solving questions in their lessons.

The following question (R6) further investigated the constraints to a problem-solving teaching approach.

R6. What are the external constraints in your teaching experience, which limit you from incorporating a teaching methodology oriented towards problem-solving?

In response to this question, time was identified most often (67%) as a constraint when incorporating problem-solving as a routine activity in the lesson. Time to prepare, length of lessons and extra-curricular demands were all mentioned as factors that affected time constraints. The size of the syllabus was mentioned in 25% of the responses as a time constraint. This time constraint was most often mentioned by the participants voicing serious

concern about the amount of work and the relative shallowness of the content that is covered in secondary Mathematics.

A resistance to change, by learners and parents, was mentioned in 16% of the responses, with 7% clearly pointing at the “marks-driven” mentality of the school communities in which they work. While general teaching resources are never mentioned, which is to be expected from the sample group of schools, a lack of textbook material and other teaching resources specifically focused on encouraging and promoting problem-solving was given as a constraint in 12% of the responses. In addition, teachers’ pedagogical knowledge and training in a problem-solving methodology was given by 5% of the participants as a constraint.

The lack of learner experience in problem-solving and problem-solving methods as they move into the high school was given by 6% of the participants as a constraint. In addition, low mathematical ability in learners was seen as a constraint by 8% of the participants.

In conclusion, while many felt that time is the major issue that limits the incorporation of problem-solving as a routine activity in their lessons, only 3% of the participants gave teacher anxiety/resistance as a reason for the lack of problem-solving activities in their lessons. This is a positive statement made by the group, as very few listed their own anxiety/resistance as a constraint. In general, one could conclude that the teachers are willing to incorporate problem-solving as a routine activity given certain conditions, such as if more time was made available to prepare, the syllabus content was adjusted, problem-solving orientated teaching resources were made available, school communities were supportive and teacher training was offered.

The following are some responses given by the participants. These give a general view of the participants’ responses to this question.

“Time. There is only so much time in a day and we are realistically competing with several other educationally relevant subjects and experiences. Thus we focus on drilling the skills that will give the learner the highest mark in the final matric exam. Problem-solving will simply have to wait until after school (sadly).”

“The method I was trained, the lack of available and suitable material to use as resources. Knowing how to fit it in amongst the basic content that should be done. Basically: training in how to teach in this way. The ability of learners in the classroom differs and the number of learners in the classroom, working without classroom assistants. Facilitating of these classes

take a lot of time to prepare, but once done it is possible and wonderful albeit time consuming in the classroom.”

“Mostly time, but also, after years of teaching, it is difficult for me to think up problem-solving questions and activities – mostly due to demands on my personal time, but obviously also an issue as far as time in the syllabus is concerned. The emphasis on calculations in the matric paper make it difficult to align real life problem-solving with maths class problem-solving, is doing a presentation to the class seen as a 'maths' activity?”

“As mentioned above, there is a time limit on finishing the curriculum. In addition to this our textbooks have been aligned to the curriculum, so they also do not encourage heuristic strategies. Which I agree with, as learners are very scared when faced with a ‘blob’ of writing they are supposed to interpret mathematically.”

“Time and weak students. Students of differing abilities in the same class, meaning that the pace might be a bit slower.”

R7. In which ways and for what purpose can problem-solving activities be used in Mathematics lessons?

This question further explores the participants' views on problem-solving and problem-solving activities. The aim of the question is to investigate for what purpose and in which way problem-solving activities are used by the participating teachers in their teaching. Firstly, the responses were evaluated according to the purpose of problem-solving activities and secondly, according to the ways in which problem-solving activities are to be used in lessons.

The purpose/aims of problem-solving activities in the Mathematics lesson

According to the NCS (Department of Basic Education 2011a), the aim and purpose of mathematical problem-solving activities are to:

- allow us to make sense of the universe around us and most of all to teach us to think creatively;
- evaluate the “when” and “why” problem types and not just the “how”;
- solve non-routine/unseen problems;
- develop and use higher order reasoning and processes;
- develop the ability to break a problem down and in this way solve it.

Using the description given by the curriculum statement and the responses by the teachers, the following categories were identified and used to group the data:

- makes sense of the universe around us (mathematical modelling) (MS);
- develops conceptual knowledge of the content, critical thinking, creative thinking and other higher order thinking skills (DC);
- develops skills to solve non-routine/unseen problems (DS);
- develops the skill to break a problem down and in this way solve it (DSB);
- none/other (N/O).

The data was further grouped according to academic and professional qualifications. Table 6.10 (below) displays the data collected from the participants.

Table 6.10. Classification of responses to R7: the purpose/aims of problem-solving.

Q17/R7 In which ways and for what purpose can problem-solving activities be used in Mathematics lessons?						
Purpose/Aims of Problem-solving		n	%			
Makes sense of the universe around us (mathematical modelling) (MS)		13	14			
Develops conceptual knowledge of the content, critical thinking, creative thinking and other higher order thinking skills (DC);		36	38			
Develops skills to solve non-routine/unseen problems (DS)		4	4			
Develops the skill to break a problem down and in this way solve (DSB)		2	2			
None/other (N/O)		40	42			
		95	100			
Academic Qualifications	n	MS (%)	DC (%)	DS (%)	DS (%)	N/O (%)
Maths1	16	13	31	6	0	50
Maths2	27	19	30	0	0	52
Maths3	31	13	55	3	3	26
Honours	7	14	43	0	14	29
Masters	4	25	25	50	0	0
None	4	0	25	0	0	75
Other	6	0	17	0	0	83
Professional Qualifications	n	MS (%)	DC (%)	DS (%)	DS (%)	N/O (%)
PGCE	35	9	31	3	6	51
HDE	39	23	38	5	0	33
Other	21	5	48	5	0	43
Pre-Reform	47	19	38	6	0	36
Post-Reform	48	8	38	2	4	48

A substantial number (42%) of participants did not include a purpose statement or they included a statement that was unrelated to the question, for example, “*Discovery items and group problem solving*”. This is of concern as this indicates that for many teachers (at least two out of five), the purpose of problem-solving was not aligned with that of the curriculum, nor could they adequately respond to this question. This is supported by the data collected from question Q13/R3 (what does mathematical problem-solving mean to you?), where 49% of the participants gave responses that were not aligned with the curriculum statement. Further, more than a quarter (26%) of the participants gave responses that were not aligned with the description of problem-solving and the purpose/aims of problem-solving instruction. It is evident that a large section of the sample group could not adequately describe problem-solving and/or the need for problem-solving activities in mathematical education. Consequently, this would have an impact on the incorporation of problem-solving as a routine activity in their lessons.

Of the participants who supported the curriculum statement’s description of mathematical problem-solving, the largest percentage (38%) described the purpose of problem-solving as a means for developing conceptual knowledge of the content and/or higher order thinking skills. Only 14% (13 participants) indicated that the purpose of problem-solving was to

engage in mathematical modelling. Of the 13 participants, nine had pre-reform professional qualifications and ten had Mathematics 2 or higher academic qualifications.

Further, when investigating the responses according to academic and professional qualifications, participants with Mathematics 1 and Mathematics 2 respectively had 50% and 51% of their responses not aligning with the curriculum statement. For example,

“To enhance the learning experience. To create a love for Mathematics” or “To introduce a section of work or just as an alternate teaching method”.

Only 26% of participants with Mathematics 3 as an academic qualification did not support the purpose as given by the curriculum statement. It is also evident from the data collected that, although these were very small sample groups, participants with “none” or “other” academic qualifications had very few participants who could adequately describe the purpose of problem-solving: 25% and 17% respectively.

Some 48% of participants with a post-reform professional qualification gave responses that did not support the purpose as given by the curriculum statement. Moreover, 51% of the PCGE group responded with statements that did not support the purpose as given by the curriculum statement or they gave responses that could not be categorised. In the pre-reform professional qualification groups, 36% of the participants did not give a purpose in their statements or they gave a purpose that did not support the curriculum statement. Further, with similar results, 48% of the post-reform qualification group could not adequately describe the purpose of problem-solving activities while only 36% of the pre-reform qualification group could not. This seems to imply that participants with post-reform teacher-training qualifications are less likely to hold beliefs that are aligned with the curriculum about the purpose of problem-solving than the pre-reform group. This could be due to the training the post-reformed group received.

In conclusion, the data indicates that participants with Mathematics 3 and/or pre-reform professional qualifications have a higher probability of having views that are aligned with the purpose of problem-solving as given by the curriculum statement, versus participants with lower Mathematics qualifications and/or post-reform professional qualifications. While it could be expected that participants with a higher academic knowledge of Mathematics should be more experienced in the purpose of problem-solving, it could be expected that teachers trained in the post-reform period should have a clearer understanding of the purpose of problem-solving. This is not the case and this could be due to the teaching experience in

years of the pre-reform group. Further, there are also 19 participants holding pre-reform and Mathematics 3 qualifications versus 12 participants holding post-reform and Mathematics 3 qualifications. This slightly skews the results in favour of the pre-reform group.

In which ways are problem-solving activities used in the Mathematics lesson?

The responses to this were analysed and grouped within four categories: Problem-solving activities were used as:

- fun/interesting activities for learners (FA);
- a means of introducing new concepts by exploring/discovering/investigating (NC);
- extension and/or enrichment (EE);
- none/other (N/O).

The responses placed in the none/other category indicated there had been no response or a response had been given that had not answered the question. The data was further grouped according to academic and professional qualifications. Table 6.11 (below) displays the data collected from the participants.

Table 6.11. Classification of responses to: Which ways can problem-solving activities be used?

Q17/R7 In which ways and for what purpose can problem-solving activities be used in Mathematics lessons?					
Which ways can problem-solving activities be used?					
	n	%			
Fun/interesting activities for learners (FA)	13	13	14		
A means of introducing new concepts by exploring/discovering/investigating (NC)	48	48	51		
Extension and/or enrichment (EE)	11	11	12		
None/other (N/O)	23	23	24		
	95	95	100		
Academic Qualifications					
	n	FA (%)	NC (%)	EE (%)	N/O (%)
Maths1	16	13	31	0	50
Maths2	27	19	30	0	52
Maths3	31	13	55	3	26
Honours	7	14	43	14	29
Masters	4	25	25	0	0
None	4	0	25	0	75
Other	6	0	17	0	83
Professional Qualifications					
	n	FA (%)	NC (%)	EE (%)	N/O (%)
PGCE	35	9	31	6	51
HDE	39	23	38	0	33
Other	21	5	48	0	43
Pre-Reform	47	19	38	0	36
Post-Reform	48	8	38	4	48

Almost a quarter (24%) of the participants did not give a response or they gave a response that did not answer this question. Of the adequate responses received, the majority (51%) described the use of problem-solving as a means of introducing a new concept. Very few

(12%) indicated that they would use problem-solving as extension and/or enrichment exercises and 14% used problem-solving as an interesting/fun activity.

A similar distribution is evident when investigating the responses according to academic and professional qualifications. Those participants with Mathematics 2 and Mathematics 3 qualifications had the lowest number of responses falling within the none/other category – 15% and 19% respectively. In the professional qualifications grouping, participants with a PGCE gave the lowest number of responses falling within the none/other category: 20%.

Finally, of the 95 participants, only ten (11%) could not adequately describe the purpose and could also not provide an adequate way in which to use problem-solving activities in a lesson. The majority of the participants therefore gave an adequate purpose or an adequate way in which problem-solving activities could be used during a lesson, which was aligned with the aims of the curriculum.

R8. Suppose you were teaching a class of learners. How would you describe a mathematical problem-solving activity on a topic of your choice? What would you, as the teacher, be doing? What would the learners be doing?

The responses were categorised according to the role of the teacher and role of the learner during a problem-solving activity. Very few responses gave details of the activity itself. If there was no response, it was not included in the tally. Again, the beliefs framework (Table 6.12) was used to code the responses according to the role of the teacher and the role of the learner during a problem-solving activity.

Table 6.12. Descriptors for belief category: Problem-solving/constructivist

Beliefs about the nature of Mathematics	Beliefs about the teaching of Mathematics	Beliefs about the learning of Mathematics
<p>Problem-solving: A dynamic, continually expanding field of human creation and invention. Engaging in Mathematics is a process rather than a product.</p>	<p>Constructivist perspective: The teacher as facilitator of the learning process. Learner-focused. Activities are interactive and learner-centred. Instruction emphasis is on solving problems, generative learning processes and explorative learning.</p>	<p>Constructivist perspective: Autonomous exploration through problem posing and problem-solving. Learner takes responsibility for own learning. Learner socially constructs mathematical knowledge.</p>

The responses were coded to either correspond with the constructivist statements given in Table 6.12 or not. As the constructivist learning theory corresponds with a problem-solving belief about the nature of Mathematics, this dimension is the one investigated in this question. Responses were tabled according to the role of the teacher (Table 6.13) and the role of the learner (Table 6.14). In addition, each table includes the responses in relation to academic and professional qualification groupings.

Responses about the role of the teacher

Table 6.13 . Classification of responses to R8: Suppose you were teaching a class of learners. How would you describe a mathematical problem-solving activity on a topic of your choice? What would you, as the teacher, be doing?

Q18/R8 Suppose you were teaching a class of learners. How would you describe a mathematical problem-solving activity on a topic of your choice? What would you, as the teacher, be doing? What would the learners be doing?			
Role of the teacher		n	%
Constructivist belief		86	92
Other		9	8
Total		95	100
Academic Qualifications	n	Constructivist belief (%)	Other (%)
Maths1	16	94	6
Maths2	27	93	7
Maths3	31	84	16
Honours	7	100	0
Masters	4	100	0
None	4	75	25
Other	6	100	0
	95	92	8
Professional Qualifications	n	Constructivist belief (%)	Other (%)
PGCE	35	89	11
HDE	39	95	5
Other	21	86	14
Pre-Reform	47	91	9
Post-Reform	48	90	10

Example of responses:

Other response: *“How you would teach Fractions using problem-based approach. Maybe bring Pizza to class and learners must divide equally and start you lesson from there”*; or *“sequences and number”*.

Constructivist belief response: *“I would give them a problem and allow them to work on it in groups. I would move between the groups observing the conversations and giving advice”*; or *“The idea would be for them to do most of the work. So I would introduce a problem and then give them the tools to complete it and allow them free reign to investigate and attempt to solve themselves in groups.”*

Considering the responses, 92% correspond with a constructivist belief regarding the role of the teacher during a problem-solving activity. There is a very high percentage (84%–100%) of responses across the academic and professional qualifications groupings, which correspond to the description of the role of the teacher in a problem-solving/constructivist belief. This is a much higher percentage than the 65% that adequately described the role of the teacher during a problem-solving activity in question R4.

Responses about the role of the learner

Table 6.14. Classification of responses to R8: Suppose you were teaching a class of learners. How would you describe a mathematical problem-solving activity on a topic of your choice? What would the learners be doing?

Q18/R8 Suppose you were teaching a class of learners. How would you describe a mathematical problem-solving activity on a topic of your choice? What would you, as the teacher, be doing? What would the learners be doing?			
Role of the learner			%
Constructivist belief		84	88
Other		11	12
		95	100
Academic Qualifications	n	Constructivist belief (%)	Other (%)
Maths1	16	81	19
Maths2	27	89	11
Maths3	31	87	13
Honours	7	86	14
Masters	4	100	0
None	4	100	0
Other	6	100	0
	95	92	8
Professional Qualifications	n	Constructivist belief (%)	Other (%)
PGCE	35	89	11
HDE	39	90	10
Other	21	86	14
Pre-Reform	47	89	11
Post-Reform	48	88	13

Again, the majority of the participants held constructivist beliefs regarding the role of a learner during a problem-solving activity. Very few participants (12%) did not describe a constructivist view of the role of the learner, irrespective of their academic or professional qualifications. There was no indication from the data that different academic and/or professional qualifications influenced the view of the teachers with regard to this question.

It is clear from the data that the participants in the study could describe the role of the teacher and the learner during a problem-solving activity that was in line with the constructivist learning theory and the problem-solving belief category.

6.4. Participants' beliefs about the nature of problems and problem-solving within the learning of Mathematics

R9. Do you believe that consistent exposure of learners to mathematical problem-solving situations and solving non-routine problems is necessary for achieving a good Mathematics result in the final Grade 12 examinations? Please elaborate on your answer.

This question aimed to explore the participants' views regarding the use of mathematical problem-solving and the solving of non-routine problems in succeeding and achieving high results in the final Grade 12 Mathematics examinations.

The following table (Table 6.15) displays the responses collected for this question. In addition, Table 6.15 displays the teachers' responses according to academic and professional qualifications.

Table 6.15. Classification of responses to R9: Do you believe that consistent exposure of learners to mathematical problem-solving situations and solving non-routine problems is necessary for achieving a good Mathematics result in the final Grade 12 examinations? Please elaborate on your answer.

Q19/R9 Do you believe consistent exposure of learners to mathematical problem-solving situations and solving non-routine problems is necessary for achieving a good Mathematics result in the final Grade 12 examinations? Please elaborate on your answer.				
				%
Yes		73	77	
No		19	20	
Other		3	3	
		95	100	
Academic Qualifications				
	n	Yes (%)	No (%)	Other (%)
Maths1	16	88	13	0
Maths2	27	70	30	0
Maths3	31	81	16	3
Honours	7	71	14	14
Masters	4	75	25	0
None	4	75	25	0
Other	6	67	17	17
	95	77	20	3
Professional Qualifications				
	n	Yes (%)	No (%)	
PGCE	35	77	20	3
HDE	39	67	28	5
Other	21	95	5	0
Pre-Reform	47	72	23	4
Post-Reform	48	81	17	2

The data collected shows that the majority of participants (77%) regarded the use of mathematical problem-solving as necessary for success in school-level Mathematics. Consistent results were recorded across academic and professional qualifications. However, the post-reform group had 81% of the participants indicating that exposing learners to problem-solving was necessary while only 72% of the pre-reform group indicated this.

Various reasons were given by participants who indicated “yes” to this question, with *development of higher-order thinking skills* (28 responses) and the *development of confidence* (18 responses) most often given as responses. Out of the 19 “no” responses, 11 (58%) indicated that rote learning of procedures and the curriculum’s summative assessment requirements expected very little problem-solving from the learners. It was therefore not necessary, according to the participants, to expose learners to problem-solving. The curriculum’s summative assessment requirements prescribe that a maximum of 15% of assessments should involve problem-solving questions (Department of Basic Education 2011a; SAGs IEB 2018). Learners are largely expected to engage with procedural knowledge

and skills in the assessments as required by the curriculum statement (Department of Basic Education 2011a, p.53).

Whereas 77% of the participants viewed problem-solving as necessary for achieving high results in Mathematics, only 54% noted that they spent more than one hour per week (BM13 of questionnaire) on problem-solving activities. There seems to be a professed necessity for problem-solving activities expressed by the participants, but in practice this is not realised.

R10. What should someone do, in your opinion, in order to improve his/her problem-solving skills?

The practising of mathematical problems, of varying cognitive demands, is given by 66% of the participants as a way of improving problem-solving skills. Some 12% of the participants said that practising questions from Olympiad and Mathematics competitions was the best way to practise and improve problem-solving skills.

Of all the participants, 5% mentioned a specific heuristic method and another 6% mentioned the need to learn specific skills and/or heuristic methods to improve problem-solving skills.

A solid understanding of key concepts and procedural knowledge was given by 11% of the teachers as being critical for improving problem-solving skills. In addition, working in groups or with a mentor was a learning method given by 8% of the respondents.

In conclusion, most of the teachers stated that the practising and drilling of problem-solving questions was a way of improving mathematical problem-solving skills. Apart from Olympiad-type of questions, there was no description given of what sort of question should be practised. The concern is that on its own, the drilling of procedures will not develop the broad set of mathematical problem-solving skills that mathematicians have in mind (Schoenfeld 1999). The vague and generic responses given to this question indicate that teachers are uncertain about how to improve problem-solving skills. The data seems to imply that the participants feel the acquiring of specific problem-solving skills and the learning of heuristic methods are not required and the hope is that through practice alone a learner will acquire these skills.

R11. Suppose some learners face difficulties during the problem-solving activity. What would you do in order to help them?

According to Schoenfeld (1992) the teacher should take the role as facilitator during activities when learners are working on questions. This view is in line with the role of a teacher in the constructivist classroom where the teacher creates the learning environment in which the learner can construct the required knowledge through active participation and self-direction (Ultanir 2012).

This question aims to investigate the teacher's teaching methodologies on how to assist learners with problem-solving activities. Key words used by respondents included: "*Guide, give hints, leading question to encourage learners, scaffold problem, and give simple examples, group work.*" The following table (Table 6.16) displays the coded responses. The responses were coded into four main categories: guided/facilitated learning, scaffolding/deconstructing the problem, group work/collaborative learning, and other.

Table 6.16. Classification of responses to R11: Suppose some learners face difficulties during the problem-solving activity. What would you do in order to help them?

Q21/R11 Suppose some learners face difficulties during the problem-solving activity. What would you do in order to help them?					
			%		
Guide		57	61		
Scaffold		27	28		
Group work		8	9		
Other		3	2		
		95	100		
Academic Qualifications	n	Guide (%)	Scaffold (%)	Group Work (%)	Other (%)
Maths1	16	75	19	6	0
Maths2	27	59	30	4	7
Maths3	31	52	32	13	3
Honours	7	71	29	0	0
Masters	4	50	25	25	0
None	4	75	25	0	0
Other	6	50	33	17	0
	95	61	28	9	2
Professional Qualifications	n	Guide (%)	Scaffold (%)	Group Work (%)	Other (%)
PGCE	35	63	26	9	6
HDE	39	56	33	8	3
Other	21	62	24	10	5
Pre-Reform	47	57	30	9	4
Post-Reform	48	63	27	8	2

The responses show a strongly constructivist view of the learning environment by the participants, with guided/facilitated learning given as an example by 61% of the participants. However, it is noticeable that collaborative learning was given by only 9% of the participants as a way of assisting learners with a problem-solving activity. In addition, none of the responses included the developing of problem-solving skills and methods to aid the learners, for example the use of Polya's four-step approach to problem-solving (Polya, 1945; Schoenfeld, 1987).

Looking at the academic-qualifications groupings, teachers with Mathematics 1 had the highest percentage of responses that included guided/facilitated learning as a way of assisting learners in a problem-solving activity. For all other qualifications, the dominant response was also guided/facilitated learning.

Some 63% of participants with a post-reform teaching qualification gave guided/facilitated learning as a way of assisting learners, compared to those with a pre-reform qualification (57%). Scaffolding and group work had a higher percentage representation in the pre-reform group than in the post-reform group.

In conclusion, guided/facilitated learning is given most often as a means of assisting a learner during a problem-solving activity. This is in agreement with the role of the teacher in a problem-solving activity (see Table 2.1). The responses are not clear on how the guided/facilitated learning is to take place and if scaffolding, peer collaboration and other approaches are part of the guiding process. The incorporation of general problem-solving skills and heuristics in the teaching is not given as a means of assisting the learner in solving problems by any of the participants.

6.5. Participants' own beliefs on being a problem-solver

R12. With regard to Mathematics, would you describe yourself as a problem-solver?

This question investigates the individual participants' own views about themselves as problem-solvers.

The following table (Table 6.17) displays the participants' responses to this question according to academic and professional qualifications.

Table 6.17. Classification of responses to R12: With regard to Mathematics, would you describe yourself as a problem-solver?

Q22/R12 With regard to Mathematics, would you describe yourself as a problem-solver?			
		n	%
Yes		71	76
No		24	24
		95	100
Academic Qualifications	n	Yes (%)	No (%)
Maths1	16	69	31
Maths2	27	70	30
Maths3	31	81	19
Honours	7	86	14
Masters	4	100	0
None	4	75	25
Other	6	50	50
	95	76	24
Professional Qualifications	n	Yes (%)	No (%)
PGCE	35	71	29
HDE	39	79	21
Other	21	71	29
Pre-Reform	47	81	19
Post-Reform	48	69	31

A large percentage of the participants (76%) viewed themselves as problem-solvers. This percentage increases with higher qualifications in Mathematics. The pre-reform qualification group also holds a considerably higher percentage (81%) of participants viewing themselves as problem-solvers, while only 69% of the post-reform group held this view. This might be because the pre-reform group have substantially more teaching experience.

In conclusion, while a high percentage of the participants viewed themselves as problem-solvers, almost a quarter (24%) of them did not see themselves as problem-solvers. This belief might influence how and if problem-solving activities are incorporated in their lessons. The following questions (R13 and R14) further explored the issue of how the participants view themselves as problem-solvers.

R13. How do you feel when faced with solving a non-routine and unseen mathematical problem?

This question continues the investigation of how the participants view and interact with non-routine and unseen mathematical problems. Involvement with these types of questions, which are fundamentally problem-solving in nature, is required by the national curriculum. Once again, this question investigated the beliefs of teachers regarding problem-solving, as their beliefs are crucial to the incorporation of problem-solving as a routine activity in their teaching.

The participants' responses were broadly grouped into positive views and negative views. Responses containing key-words such as *excited, eager, enjoy the challenge, love it, good feeling*, etc., were all grouped under positive views. Responses containing key-words such as *nervous, challenging, anxious, stressed, intimidated, panic, frustration, insecure, out of comfort zone*, etc., were grouped under negative views.

Table 6.18 (below) displays the data collected for this question. The results are also displayed according to academic and professional qualifications.

Table 6.18. Classification of responses to R13: How do you feel when faced with solving a non-routine and unseen mathematical problem?

Q23/R13 How do you feel when faced with solving a non-routine and unseen mathematical problem ?			
		n	%
Positive		67	71
Negative		28	29
		95	100
Academic Qualifications	n	Positive (%)	Negative (%)
Maths1	16	56	44
Maths2	27	74	26
Maths3	31	65	35
Honours	7	86	14
Masters	4	100	0
None	4	75	25
Other	6	83	17
	95	77	23
Professional Qualifications	n	Positive (%)	Negative (%)
PGCE	35	89	11
HDE	39	62	38
Other	21	57	43
Pre-Reform	47	60	40
Post-Reform	48	81	19

The majority of the participants (71%) have positive views or feelings when dealing with non-routine type questions. Participants with Mathematics 1 qualifications have the highest percentage (44%), per qualification grouping, of negative responses, whereas participants with Mathematics 2 qualifications show the highest percentage (74%) of positive views or feelings towards non-routine type questions. All other academic-qualification groupings show a higher percentage of positive views as opposed to negative views.

Participants with a post-reform qualification hold a significantly higher percentage of respondents with positive views (81%) versus pre-reform participants, with only 60% of this group voicing positive feelings towards non-routine type questions. Solving non-

routine/unseen type questions is fundamentally mathematical problem-solving. There is therefore an apparent contrast to the data collected in previous questions (R12), where 81% of the pre-reform group viewed themselves as problem-solvers, while only 69% of the post-reform group viewed themselves as problem-solvers. The following two tables (Tables 6.19 and 6.20) further analyse this seeming inconsistency in beliefs regarding problem-solving.

Table 6.19 Contingency table: Views of problem-solvers vs views of “not problem-solvers”

	Positive (%)	Negative(%)
Problem Solver (%)	59	16
Not Problem Solver (%)	11	14

Of the participants, the majority (59%) who view themselves as problem-solvers express positive feelings towards solving non-routine/unseen type questions. This indicates a trend that those who view themselves as problem-solvers are more likely to have positive feelings towards problem-solving. The remaining 41% of the participants expressed either negative views towards non-routine/unseen type problems or did not view themselves as problem-solvers.

From the information given in Table 6.20 the pre-reform group has a high percentage of participants who viewed themselves as problem-solvers (26%) but with negative feelings when faced with these problems. This in part explains the conflicting data, but the view of the participant as to what a problem is and what problem-solving requires of the learner could also explain the responses to the two questions.

Table 6.20 Contingency table: View as problem-solver vs views about solving non-routine/unseen. Pre- and Post-reform groupings. Given as percentages.

Post-reform educational qualification		
	Positive View	Negative View
Problem Solver	63	6
Not Problem Solver	18	13
Pre-reform educational qualification		
	Positive View	Negative View
Problem Solver	55	26
Not Problem Solver	4	15

The results are consistent across the qualification groupings. Participants who viewed themselves as problem-solvers also held positive feelings towards solving non-routine/unseen questions, but this was to be expected with 59% of the participants overall falling within this

category. However, a large percentage (26%) of the participants with a pre-reform qualification viewed themselves as problem-solvers but expressed negative feelings towards solving non-routine/unseen questions.

R14. What do you do when faced with solving a difficult problem in Mathematics? How do you go about solving the problem in general?

The question seeks to investigate the various methods and thinking strategies in which the participants engage when attempting to solve a difficult problem. Responses were coded and when specific methods were given, these were categorised according to the heuristic methods as identified by Pólya (1946). Table 6.21 (below) displays the data collected from responses to this question. A response from a participant was coded as “none” if the response did not contain a specific and/or clear method. Once more, the data was grouped according to academic and professional qualifications.

Table 6.21. Classification of responses to R14: What do you do when faced with solving a difficult problem in Mathematics? How do you go about solving the problem in general?

Q24/R14 What do you do when faced with solving a difficult problem in Mathematics? How do you go about solving the problem in general ?								
Heuristic Methods			%					
Solve a simpler problem (SS)		7	7					
Visual Representation (VR)		14	15					
Trial and Error (EE)		6	6					
Look for a Pattern (LP)		4	4					
Decomposing and Recombining (DR)		7	7					
Do you know a related problem? (RP)		17	18					
None		40	42					
		95	100					
Academic Qualifications	n	SS (%)	VR (%)	EE (%)	LP (%)	DR (%)	RP (%)	None (%)
Maths1	16	0	19	13	6	0	6	56
Maths2	27	4	19	7	7	0	22	41
Maths3	31	10	6	3	3	16	26	35
Honours	7	29	14	0	0	14	0	43
Masters	4	0	25	25	0	0	0	50
None	4	25	25	0	0	0	0	50
Other	6	0	17	0	0	17	33	33
Professional Qualifications	n	SS (%)	VR (%)	EE (%)	LP (%)	DR (%)	RP (%)	None (%)
PGCE	35	14	6	14	3	6	11	46
HDE	39	3	21	0	5	10	26	36
Other	21	5	19	5	5	5	14	48
Pre-Reform	47	4	19	0	6	11	23	36
Post-Reform	48	10	10	13	2	4	13	48

The largest percentage (42%) of the responses did not contain a clear method or “rule of thumb” for use when engaging with solving a difficult mathematical problem. Often “use various methods” and “sleep on it” were used as ways of engaging with the difficult problem. Of the identifiable heuristic methods, using a related problem (18%) and using a visual representation of the problem (17%) were most often stated by the participants. Participants with a Mathematics 1 qualification (56%) contained the highest percentage of “none” statements. It is also notable that with the exception of the participants with master’s degrees

(but here we have a sample of only four), the higher the academic qualification in Mathematics the more the statements contained specific methods to solve problems. Post-reform participants gave the highest percentage (48%) of “none” statements in the professional-qualifications groupings. The pre-reform group has markedly fewer participants in the “none” statement category (36% vs 48%). This seems to indicate that participants with pre-reform educational qualifications have specific methods (in general) to solve difficult problems, while post-reform participants do not. However, pre-reform participants have substantially more teaching experience.

In conclusion, it is concerning that only 58% of the participants could give an identifiable method on how to approach difficult problems when solving mathematical problems. Of the participants, 32% at least stated that they would try and understand the problem first, which is the first step in Pólya’s four-step process. Further, one participant mentioned the use of heuristic methods while another participant described the problem-solving process as given by Pólya. It is clear that a substantial number of participants of this experienced sample-group of secondary Mathematics teachers do not have a clear methodology to fall back on when dealing with difficult-to-solve Mathematics problems (problem-solving). At least two out of five teachers could not give an identifiable method to start with. This could only make it difficult to include problem-solving as a routine activity in the schools.

6.6. Conclusion

This chapter reported on and discussed the participants’ responses to the open-ended questions in the questionnaire. The research questions

- (i) *What are practicing secondary Mathematics teachers’ beliefs about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics?*
- (ii) *How do their beliefs relate to their implementation of problem-solving activities as required by the National Curriculum Statement?*

were both investigated within this chapter under the following four categories. A short summary and conclusion are given in relation to each category.

- (i) Questions R1 to R3 investigated the participants' beliefs about the nature of problems and problem-solving within the discipline of Mathematics. The majority (63%) of the participants' responses showed that they viewed a mathematical problem as a problem-solving/complex activity. Some 64% of the participants gave characteristics of a good mathematical problem that corresponded with problem-solving/complex questions. Only 37% of the participants gave responses to both these concepts that could be classified as being of the problem-solving/complex type. A large percentage of the participants therefore held differing views about an important concept that informs instructional practice. Further, only 22% the participants gave responses that could be classified as problem-solving/complex and gave a description of problem-solving aligned with the national curriculum. This implies conflicting beliefs by the majority of participants about the nature of problems and problem-solving.
- (ii) Questions R4-R8 investigated the participants' beliefs about the nature of problems and problem-solving within the teaching of Mathematics. The majority of responses by the participants on the role of the teacher and the role of the learner during a problem-solving activity corresponded with a constructivist view about the teaching of Mathematics. In addition, only 11% of the participants could not adequately describe the purpose nor could they give an adequate way in which problem-solving activities could be used during a lesson, in accordance to the aims of the curriculum. However, while the majority of participants were willing to incorporate problem-solving as a routine activity in their lessons, many cited constraints such as inadequate time, the syllabus content, lack of teaching resources oriented towards problem-solving, lack of school-community support and lack of teacher training.
- (iii) Questions R9-R11 investigated the participants' beliefs about the nature of problems and problem-solving within the learning of Mathematics. The view of the majority (77%) of participants was that engaging learners in problem-solving activities was necessary within the learning of Mathematics. Guiding the learner during the problem-solving activity was given most often (61%) as a means to assist a learner during the activity. Further, the majority of participants (66%) said learners could improve

their problem-solving skills by practising as a way of improving problem-solving skills. However, the responses were vague and very few suggested the learning of specific problem-solving skills (6%) and/or heuristic methods (5%). The responses to this category of questions seem to imply that while the teachers believe in the importance of problem-solving activities in the learning of mathematics, there is uncertainty and a lack of knowledge (of methodologies, heuristics and problem-solving skills) regarding how to include problem-solving as a routine activity in the class.

- (iv) Questions R12-R14 investigated the participants' own beliefs about being problem-solvers.

The majority (59%) of the participants viewed themselves as problem-solvers and had positive feelings towards problem-solving type activities. However, a large percentage (41%), at least two out of five participants, either had negative feelings towards problem-solving activities or did not see themselves as problem-solvers. This could have a negative effect on including problem-solving as a routine activity in the classroom. Further, a large percentage (42%) of participants could not give an identifiable method of approaching a problem-solving question. This might be the reason for many feeling that they are not problem-solvers and/or the negative feelings they experience when dealing with problem-solving questions.

The following chapter reports on and discusses the data collected in the interviews. The chapter details interviews conducted with two secondary Mathematics teachers from schools registered with the IEB.

CHAPTER 7

RESULTS AND DISCUSSION: INTERVIEWS

7.1. Introduction

Participants in the interview were contacted in advance via email. In the email the participant was asked to take part in a short, 30-minute semi-structured interview at a time and place of their choosing. The participants were assured that the interview was voluntary and that no personal information would be shared or communicated in the reporting phase of this study.

The participants were asked a number of questions similar to those in the questionnaire. These questions were constructed throughout the analysis of the questionnaires. The interviews further investigated the teachers' beliefs about problem-solving, the nature of Mathematics, and the teaching and learning of Mathematics. Importantly, the interviews were conducted to triangulate the collected data and to add further information to the study. The following questions guided the semi-structured interviews (see also Appendix F). The questions from the questionnaire to which the interview questions relate are given in brackets.

1. What is a mathematical Problem to you? (R1-R2)
 - Can you give me an example of a mathematical problem in class?
2. Could you describe a mathematical problem-solving activity you use in class to me? (R3-R4)
 - What is your role during the task/activity? (R4 and R8)
 - Could you describe your role and what would the learners be doing? (R4 and R11)
 - Is a word problem always problem-solving? (R2)
 - Would you engage all learners' abilities in problem-solving activities? (NM5 and BM7)
3. Could you "in your own words" describe a problem-solving orientated teaching methodology? (R5)
 - What implication would this sort of approach have on your teaching? (R6)
 - What constraints are there to this sort of teaching? (R6)

4. Do you believe that using problem-solving and non-routine problems are beneficial to learners' results and progress in Mathematics? (R9 and BM11)
5. Do you enjoy solving mathematical problems? What types do you like? (R12-R14)
6. Do you feel that Mathematics as delivered in our classrooms promotes creativity and originality in all learners? (BM8)

A pilot interview was conducted before data collection started. This informed both the questionnaire design and the following structured interviews. For this study only two participants were interviewed. They were chosen on the basis of their belief scores, academic and professional qualifications.

The first interviewed teacher was male with 10 years of teaching experience, a B.Ed. degree and Mathematics 3 as an academic qualification. His professional teaching degree was achieved post-reform. John's average rating derived from the Likert-scale was 2,0 for the section on beliefs about the nature of Mathematics and 1,8 for the section on beliefs about the teaching and learning of Mathematics. This indicates a stronger constructivist view about beliefs related to the nature of Mathematics than about beliefs related to the teaching and learning of Mathematics. John's interview responses corresponded with his open responses in his questionnaire feedback. John is a deputy of academics and head of the Mathematics department at his school. He indicated on his questionnaire that he spends more than three hours a week on problem-solving, preferring to start a new section of work with a problem to solve. (Full transcription of this interview is given in Appendix G.)

The second interviewee was a female teacher with 27 years of teaching Mathematics in a secondary school. She has a Higher Diploma in Education (HDE) and Mathematics 2 as an academic qualification. The HDE is a qualification attained pre-reform. She is a senior Mathematics teacher and head of her Mathematics department at her school. Liz scored 1,4 for the section on beliefs about the nature of Mathematics and 1,6 for the section on beliefs about the teaching and learning of Mathematics. Her responses to the questionnaire items and belief scores indicated a generally traditionalist view regarding the nature of Mathematics and about the teaching and learning of Mathematics. She indicated on her questionnaire that she

spends one to two hours of class-time per week on non-routine/problem-solving type questions. (Full transcription of this interview is given in Appendix H.)

7.2. Interview 1 discussion

John had a strong constructivist/problem-solving view on what constitutes

- (i) a mathematical problem: *“I think [...] when someone has to solve a problem that is posed. A problem that can be open or closed. Using prior knowledge and the application thereof.”*
- (ii) a mathematical problem-solving activity: *“[...] I think problem-solving is when the learner does not immediately know what to do. It must not be routine. It must not be something done previously. They should use content knowledge... there should be links. A good problem-solving activity should include more than one content area.”*
- (iii) the role of the teacher and learner during a problem-solving activity: *“I am the facilitator. I answer them by asking more questions. The kids are busy investigating [...] They interrogate. They try to make connections and understand the connections between the different concepts.”*

John felt that lower-ability learners should be exposed to problem-solving activities and that they would benefit from this, but that different teaching strategies were required when working with lower-ability learners: *“You need to change how you answer their questions to how you would answer a strong kid. You need to be aware of their learning difficulties. It takes much more patience and different strategies. It is not always more difficult but takes more endurance.”* All learners should be exposed to problem-solving activities and these encouraged learners to engage with problems they might not have previously engaged with: *“Our kids will now not just leave a final paper question blank. They will play around with the questions. The biggest benefit is that they will not just leave a question blank as they did in the past. None of our kids in our June exams and prelims left any question unanswered.”*

His responses indicated that he believes teachers teach the way they were taught at school and that teachers who believe in a reformed Mathematics curriculum are needed for its implementation: *“There was a lot of resistance from staff. It was not easy. Naturally you do*

what is comfortable. There is a fear. You teach the way you were taught [taps on desk]. To change this, you try and do what is in the best interest of the learners [which is not always] the easiest way to teach.”

John personally enjoys the challenges posed by mathematical problem-solving. He is very comfortable about his mathematical abilities, but he is willing to admit to the learners if he does not immediately know the answer or how to get to the answer. He feels strongly that this is crucial in demonstrating the problem-solving process: *“I am not one [a teacher] that pretends I know it all. I like to share with the learners how I approach and solve problems [...] admitting that you don’t know to the kids when they ask. I don’t feel threatened. It is important to me for Mathematics that the kids know that you’re also a learner. That your part of the process.”*

John’s responses indicated that he did not believe our classrooms promote creativity or originality: *“No, I think we isolate ourselves.”* He feels our approach to teaching in general is a *“one size fits all”* approach and that to solve this we need *“a good problem-solving problem”* and to go outside the classroom.

John indicated that problem-solving should be a routine activity in our classrooms: *“I have a problem with problem-solving only being part of an examination. We try to do a lot of problem-solving during class time. For me problem-solving is part of learning and not only part of assessment, but part of assessment for learning. The kids should learn from a problem not just for marks. Naturally our exams have to be weighted in a certain prescribed way with a certain percentage for problem-solving. It is easier for the kids if they do it [problem-solving] on a regular basis. You don’t have to know immediately what to do- play around, poke around in the dark and build a conjecture.”*

It was clear from his responses that John had researched, managed and implemented a problem-solving approach to teaching in his Mathematics classroom. His views, as voiced in the interview, show beliefs about problem-solving that are aligned with the requirements of the national curriculum. He did, however, voice concerns regarding time, resources and teacher beliefs as constraints to implementing this approach.

7.3. Interview 2 discussion

Similar to the responses from John, Liz had constructivist/problem-solving views about what constitutes

- (i) a mathematical problem: *“Depends on the context that you are in at the time and the pupils you’re working with at the time. Depends on what you want the answer out of it to be. If you want [...] pupils to think, you are going to ask the question in a certain way as a problem. If just want a set answer, you will ask it in a different way. I think it depends on what you want out of it as to what you decide to define as a problem.”*
- (ii) mathematical problem-solving: *“I think the idea of problem-solving is to get pupils to think. I think you have to take away [the] classroom situation if that is where you are. Get away from the idea that there is one right answer and I want that answer. You want to see what pupils are thinking. You want to see how pupils are thinking. And I think that is what your aim is, to get them to think whether they end up at the right place or not. You want to first get them thinking.”*
- (iii) a problem-solving activity: *“I think it has to be open-ended. I think you got to be [...] I am going from the idea that I don’t want 10 to be the right answer. I want them to think about various solutions. Can you [learner] give me an idea of ways to get solutions [...] to a particular question? We want to put JoJo tanks outside. Can you [learners] give me an idea of what size JoJo tanks we would need? Where would we place them? Why we would need them there? Would it be better having this kind or that kind and why? Explaining, thinking, getting them thinking and then bringing the Maths in, sometimes incidentally or accidentally but on purpose.”*
- (iv) the role of the teacher and learner during a problem-solving activity: *“The kids need to be the ones coming up with the ideas. They need to be the ones thinking about what? Why? What could be? and problems with their answer. Discussing where the problems will come up with their particular answer or solution. I think the role of the teacher is to suggest options if they are stuck. If they heading off on major tangents that are not going to*

head in the direction you need them at that particular point. To redirect them back to your aim and intention, but not to close doors and say you're doing the wrong thing. Do it this way."

Liz indicated that lower-ability learners are often better at problem-solving as they *"have real life solutions to something that makes it more practical"* and sometimes have *"interesting ways"* and *"ingenious ways"* of solving problems, but that lower-ability learners will need a different teaching strategy when solving problems.

While Liz gave responses that can be categorised as her holding a constructivist/problem-solving view of the teaching and learning of Mathematics, in practice she does not often involve her classes in problem-solving activities. She indicated that mathematical problem-solving was *"not what I ever do in class. Because in class we just never ever have the time to do it. Something I know I definitely gloss over. [...] Unfortunately, that is where our learners lose out. The stronger learners lose out; The stronger the class, you often have less time to do the problem-solving. You're pushing them to get higher marks. Your middle groups you're pushing to get better marks than than they are doing. And you just don't have the time to do that."* In addition, she responded that pupils could achieve very good results in Mathematics (*"I think they can work through past papers and get an A"*) without being a good problem-solver or engaging in problem-solving activities. She did, however, feel that the learners would benefit from problem-solving activities, although she said, *"I think with the amount of time, we are very limited to cover the basics of the syllabus."*

Liz did not see word problems as always being a problem-solving question and that *"it [a word problem] often ends up being a routine question just disguised as different words. [...] I don't think this is problem-solving."* In addition, learners saw word problems as being difficult because *"there are words in the sum"*.

Liz commented on the benefit of teaching learner's heuristics to aid problem-solving. Junior learners took part in a workshop on problem-solving that involved identifying and using heuristics to solve problems. This benefited them in their Olympiad results and made them more *"confident at trying the problem-solving type questions that are different"*. Again, while Liz commented on the benefits of introducing problem-solving in her teaching, this was done on an ad hoc basis when there was time or it was not done at all. *"Unfortunately, we could not do it [problem-solving workshop] this year because we ran out of time. Maybe next*

year when we have an additional lesson. So we're hoping to do more of that non-routine type stuff in class."

Liz's responses indicated that in general she feels anxious when solving mathematical problems. She responded by saying this was something *"I had to get my head around"*. Her response to mathematical problem-solving was negative: *"I have always been taught that there is a right and a wrong answer and Maths is the aim to get the right answer. I was never very strong in Maths myself. The whole idea of playing around and trying answers is very foreign to me. It is something that I have [...] really tried to work on. For me it is very uncomfortable not knowing where it is going."* She feels comfortable if there is a taught and given process to get to an answer. Her responses clearly indicate traditionalist beliefs about the teaching and learning of Mathematics: *"I want a method to get there. And that is the way I have always been trained and taught and practised to do it. [...] If I know where it is going, I am more comfortable with it. If I get a book of problem-solving, I want a set of answers available that if I can't find it, I know where to find it. I am not comfortable going into class without knowing that I don't know the answer or how to get there [...] And it is not always easy to say I don't know and ask for one of them to explain. 'Who can help me?' And that has taken a long time myself getting my head around saying 'I don't know. Someone get up and explain. Let's figure it out.'"* Her responses, however, indicate a willingness to engage in problem-solving and to make it part of her teaching. *"It has only been the last 5 years or 6 years where I had to get my head around doing it differently. I think it has only been the last 5 or 6 years that we have been doing this non-routine stuff [...] It has taken a long time myself getting my head around saying I don't know [and to say to] someone [learner] 'Get up. Explain. Let's figure it out.'"*

Liz indicated that Mathematics was not her preferred subject at university: *"I hated it. I loved Maths at school. It was a comfortable environment. Varsity, I was always out of my depth. My lecturers would always say, 'It is up to you to figure it out. We have no time to help the students.' So I battled to get those 50%. Maths was a scrape through by the skin of my teeth."*

Liz is a senior and experienced Mathematics teacher. She received her teacher training pre-reform. Her views and responses to questions on problem-solving are aligned with that of the national curriculum. However, it is clear from responses to questions on classroom practice that traditional methods are practised and preferred. Time is given as the major reason for not including problem-solving as a routine activity in her teaching, but she often voices her

anxiety in dealing with non-routine type questions and “uncomfortable” feelings when dealing with problems that do not have clear methods and to which she does not know the answers. Her interview responses reflect the traditionalist scores on both the nature of Mathematics and the teaching and learning of Mathematics as given by the questionnaire data.

7.4. Conclusions

In conclusion, the interviews give a snapshot of the beliefs held by two participating teachers. Both are senior teachers with experience but one has pre-reform training and the other has post-reform training. In addition, the post-reform teacher holds a higher academic qualification in Mathematics than the pre-reform teacher. While both teachers have constructivist beliefs regarding mathematical problem-solving and the teaching and learning of problem-solving, in their teaching practice there appears to be a real difference. The post-reform teacher with a preference for problem-solving engages in problem-solving as a routine activity in his teaching in contrast to the pre-reform teacher. The pre-reform participant, while showing a willingness to engage learners in problem-solving activities, and understanding the benefit of it, does not include problem-solving as a routine activity in her teaching. It is also evident from the interviews that the teacher with a higher Mathematics qualification expresses positive feelings towards engaging in problem-solving activities, while the teacher with a lower Mathematics qualification expresses anxiety when engaging herself and learners in problem-solving activities.

CHAPTER 8

SUMMARY, CONCLUSIONS, LIMITATIONS AND RECOMMENDATIONS

The purpose of this study was to examine the beliefs of secondary Mathematics teachers at independent schools in South Africa that are registered with the IEB. In particular, their beliefs about the nature of Mathematics, the teaching and learning of Mathematics and problem-solving were examined. Consequently, the study was conducted to address the following research questions:

- (i) What are practising secondary Mathematics teachers' beliefs about problem-solving, the nature of Mathematics and the teaching and learning of Mathematics?
- (ii) How do their beliefs relate to their implementation of problem-solving activities as required by the National Curriculum Statement?

The choice of subject population was significant for two reasons. The first was the researcher's involvement at IEB schools as a national moderator of school-based assessment. The researcher's own experience in observing, moderating and working with schools and teachers connected to the IEB would facilitate and inform the research study. Secondly, the schools registered with the IEB offer quality education with experienced and qualified teachers. It would be reasonable to expect that teachers from these schools would in general experience fewer outside factors that negatively influence their teaching practices or prevent the implementation of the aims and objectives of a reformed curricula. As beliefs are a significant factor in classroom practices and the implementation of reforms (Handel 2003), investigating these teachers' beliefs would therefore be the main focus of this research study. Further, to this end, the problem statement: "*Mathematical problem-solving is not a routine activity in the learning environment as required by the national curriculum*" would be addressed.

The significance of the research study is not only that it adds to the international body on research into the beliefs of Mathematics teachers, but that it also explores their problem-solving beliefs, as few studies have specifically examined these (Xenofontos & Andrews 2014). Further, as the IEB-registered schools function within the wider secondary schooling community in South Africa, this research study will also add to the research on teachers'

beliefs and the lack of problem-solving as a routine activity within secondary classrooms in South African schools.

The study also examined teachers' beliefs in relation to their mathematical knowledge and professional teaching qualifications. Both mathematical knowledge and professional qualifications are discussed in detail in the literature review as possible influences of participants' beliefs. The data throughout the study was therefore analysed in relation to these two aspects of teacher training. This added to the significance and uniqueness of this study in the following two ways:

- (i) While research has been done on the beliefs of Mathematics teachers and the relationship of these beliefs to their mathematical knowledge, research has not been conducted on teachers' specific, tertiary Mathematics qualifications in relation to their beliefs. No such research has been done in the South African context. The investigation will therefore be able to inform teacher workshops and professional development initiatives that focus on teachers' mathematical knowledge.
- (ii) In the South African context, research has not been done on the relationship between a teacher's professional teaching qualification and the teacher's beliefs. Again, this will inform pre-service and in-service professional development initiatives for teachers in the future.

To address the research questions, data was collected through the administration of a questionnaire that included both quantitative items and open-ended questions. This research is a quantitative case study augmented by open-ended questions and semi-structured interviews. After an initial analysis of the data collected from the questionnaires, semi-structured interviews were conducted. The semi-structured interviews further investigated topics and findings from the initial analysis of the questionnaire data.

The following sections in this chapter provide conclusions (Section 8.1); summarise the main findings (Section 8.2); outline the limitations (Section 8.3); and finally give recommendations (Sections 8.4) for further studies.

8.1. Conclusions

As with findings from other studies (Thompson 1984; Cross 2009) the teachers in this study had beliefs that reflected a combination of traditional beliefs and constructivist beliefs about the nature of Mathematics and the teaching and learning of Mathematics. The participants had beliefs about the nature of Mathematics that leaned towards traditionalist beliefs, while in general their beliefs about the teaching and learning of Mathematics showed strongly constructivist beliefs.

I set out with the following expectations that:

- (i) Participants with a higher level of tertiary academic Mathematics qualifications would show beliefs that aligned with a constructivist view regarding the nature of Mathematics, and the teaching and learning of Mathematics. Their beliefs on problem-solving would also support the aims and objectives of a reformed curriculum.
- (ii) Participants with post-reformed teaching qualifications (those teachers with a PGCE, B.Ed. or other qualification attained after 2002) would show beliefs aligned with a constructivist perspective of the nature of Mathematics, and the teaching and learning of Mathematics. In addition, teachers with post-reformed qualifications would have beliefs that would contribute to incorporating problem-solving as a routine activity in classroom practices.

Both expectations seem to be justified in part regarding certain beliefs held by the participants but no general conclusion could be made. My recommendation would be that the relationship between teachers' academic and professional qualifications and their beliefs should be researched in further detail in future studies.

Considering the participants' mathematical knowledge in relation to their beliefs, there seems to be no clear indication from the collected data that there is a positive correlation between higher mathematical qualifications and teachers holding constructivist beliefs. However, the data does indicate that teachers with Mathematics 1 as the highest Mathematics qualification, are more likely to have traditionalist beliefs about the nature of Mathematics and about the teaching and learning of Mathematics than teachers with any other levels of Mathematics qualification. Of the sixteen participants with Mathematics 1 academic qualifications, six had an HDE, four had a PGCE and six were in possession of a B.Ed. Of the sixteen participants,

nine belonged to the post-reform group and seven to the pre-reform group. Teachers with Mathematics 1 are fairly evenly distributed across the professional qualification groupings.

Considering the participating teachers' professional qualifications, the research indicated that those with post-reform teacher training were more likely to hold strongly constructivist beliefs about the nature of Mathematics. In addition, those post-reform trained teachers, in particular those with a PGCE, were more likely to hold constructivist beliefs about the teaching and learning of Mathematics than those with a B.Ed. or HDE. Those teachers with pre-reform qualifications and in particular HDE qualifications made up the largest number of participants with a traditionalist belief regarding the teaching and learning of Mathematics. While the research seems to indicate that teachers with post-reform qualifications (these include PGCE-qualified teachers) were more likely to align with having constructivist beliefs and thereby with the aims and objectives of the reformed Mathematical curricula, the concern (for training institutions) should be that a large percentage of both post- and pre-reformed trained participants in this study still held various traditionalist beliefs about the teaching and learning of Mathematics.

The teachers' beliefs and espoused classroom practices in relation to problem-solving were investigated in the open-ended questions. Here it was found that:

- (i) The majority of the participants (at least six out of ten) could give a description of the role of the teacher and the role of the learner during a mathematical problem-solving activity that aligned with a constructivist perspective during a mathematical problem-solving activity;
- (ii) The majority (89%) could describe the purpose of a problem-solving activity and/or provide a way in which to use problem-solving activities in their lessons that aligned with a constructivist perspective of the teaching and learning of Mathematics;
- (iii) The majority (67%) believed that the curriculum in its current state could be taught by using a problem-solving approach which supported the aims and objectives of reformed curricula;
- (iv) The majority (77%) of the participants held beliefs that mathematical problem-solving was necessary for the teaching and learning of Mathematics, which supports the aims and objectives of a reformed curricula;

- (v) The majority viewed themselves as problem-solvers with positive feelings (59%) when engaging with problem-solving. It can be expected that teachers who have a positive view of problem-solving and positive feelings about problem-solving would more readily engage with problem-solving activities in their lessons. This will again support the aims and objectives a reformed curriculum.

However,

- (i) The majority of the participants cited various constraints to introducing problem-solving as a routine activity in their teaching;
- (ii) Some 42% of the participants could not give an identifiable method on how to engage with a difficult (unseen) problem to solve. Participants with a Mathematics 1 qualification were among the highest percentage (56%) of participants not able to give an identifiable method. Again, participants with only Mathematics 1 as an academic qualification did not show the expected beliefs. This is of great concern as at least two out of five participants could not give an identifiable method. This would surely influence classroom delivery;
- (iii) A large percentage held differing beliefs regarding what constituted a mathematical problem and the characteristics of a good mathematical problem;
- (iv) Many (at least two out of every five teachers) feared problem-solving and/or did not see themselves as problem-solvers.

The responses seem to indicate that while there is a belief that problem-solving is necessary and that the role of the teacher and learner is clear to the teacher, there is a lack of knowledge about problem-solving and problem-solving teaching strategies among a large percentage of the participating teachers, irrespective of their professional qualifications. The implications for professional development are significant as teachers' beliefs have been shown to significantly influence classroom practices. The study therefore would suggest that professional development programmes should focus on the following, so as to promote a constructivist perspective regarding the nature of Mathematics and the teaching and learning of Mathematics, and to encourage problem-solving as a routine activity in teaching practices:

- (i) Engaging teachers in problem-solving activities, which would be more than “how to do” problem-solving, as part of professional development, and in this way alleviating fear, anxiety and building confidence;
- (ii) Aligning teacher’s beliefs with the constructivist perspective regarding the nature of Mathematics and the teaching and learning of Mathematics, and thereby promoting the aims and objectives of a reformed curriculum;
- (iii) Clearly identifying methods (heuristics) and other problem-solving techniques for teachers to use during problem-solving activities in class;
- (iv) Supplying teachers with adequate resources to support and implement problem-solving as a routine activity in their teaching practice;
- (v) Arranging targeted and specific workshops that address traditionalist views regarding the nature of Mathematics and the teaching and learning of Mathematics;
- (vi) Upskilling those teachers with a Mathematics 1 qualification to a higher tertiary Mathematics qualification. Seven out of the sixteen participants with a Mathematics 1 qualification said that they were fearful of engaging with problem-solving activities. It can be expected that with a higher level of Mathematics competency, the confidence of the teachers would also improve.

While only two interviews were conducted, the interviews did give a snapshot of the beliefs held by the two participating teachers. Both teachers have constructivist views regarding mathematical problem-solving and the teaching and learning of problem-solving, but in their teaching practices, there appear to be real differences. The post-reform teacher with a preference for problem-solving engages in problem-solving as a routine activity in his teaching in contrast to the pre-reform teacher. The pre-reform participant, while showing a willingness to engage learners in problem-solving activities, and expressing a belief in the benefit of it, does not include problem-solving as a routine activity in her teaching. It is also evident from the interviews that the teacher with a higher Mathematics qualification expresses positive feelings towards engaging in problem-solving activities, while the teacher with a lower Mathematics qualification experiences anxiety when engaging learners or herself in problem-solving activities. The responses to the interview questions supported the data collected from the questionnaire and the findings made in this section. In addition, in both interviews, the participants found it difficult to articulate definitions, teaching methodologies and instructional practices that involved problem-solving.

8.2. Limitations to the study

At every stage of the study care was taken to ensure the validity and the trustworthiness of the findings and to increase the reliability of the study. However, as with any other study, there were limitations.

The population group consisted only of teachers from a small sector within the secondary schooling environment in South Africa. The sample group was however, large enough to provide a statistically representative set of data that was still small enough to be manageable. Nevertheless, the sample size was too small to analyse statistically some of the underlying variables. For example, participants with master's degrees and honours degrees in Mathematics were represented by only four and six participants respectively. This made any analysis of beliefs in relation to these qualification groupings difficult.

The written responses were limited in how much information they could provide and more interviews would have offered richer information regarding the participants' beliefs and classroom practices. The limited time for this study prevented the researcher from conducting sufficient interviews to enhance the overall study. In addition, classroom visits and detailed observations with pre-interviews and post-interviews would have provided further data regarding the relationship between espoused beliefs and actual teacher practice. Again, the limited time and resources available for this study did not allow for this.

8.3. Recommendations

A number of recommendations for future studies can be made:

- The study only focused on a small privileged sector within the South African schooling section. A larger population group would further the research conducted in this study.
- Beliefs in relation to mathematical knowledge can be further explored. The data was inconclusive with the current sample group. I would suggest selecting a representative sample group from each academic qualification grouping to see if there is a statistical difference in their beliefs and classroom practices.
- Beliefs in relation to teachers with pre-reform and post-reform qualifications in South Africa can be further explored. A representative sample group could be selected from each teacher training qualification category to see if there is a difference in beliefs and classroom practices.
- Teachers' espoused beliefs should be compared with classroom practices. Further studies should involve classroom visits and teaching observations.

8.4. Summary

Firstly, this study has shown that teachers with post-reform teaching qualifications do not necessarily hold constructivist beliefs regarding the nature of and/or the teaching and learning of Mathematics. However, teachers with post-reform qualifications are more likely to hold constructivist beliefs in general than their colleagues with pre-reform qualifications.

Secondly, holding a higher academic qualification in Mathematics also does not ensure constructivist beliefs. However, participants with only a Mathematics 1 qualification are more likely to have traditionalist beliefs across participants with both pre- and post-reform qualifications.

Thirdly, the study has shown that within the privileged teaching environment many teachers still hold beliefs that are traditionalist in nature.

Finally, many educators, even though qualified and experienced, struggled to describe adequately aspects of teaching and learning that involve problem-solving. Very few were them is able describe “how to do” problem-solving.

In conclusion, I strongly recommend that teacher professional development (both pre-service and in-service) focuses on enhancing mathematical knowledge to at least the level of Mathematics 2 and to include the training that provides teachers with pedagogical knowledge of mathematical problem-solving. This would enable them to include problem-solving as a routine activity in their daily lessons.

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APPENDICES

Appendix A: Letter to participants

Investigating the beliefs about problem-solving of mathematics teachers at independent secondary schools in South Africa.

Dear Colleague

I am conducting research for my M.Ed. degree at Stellenbosch University and I would appreciate your contributions as qualified and experienced teachers. The study aims to describe and analyse mathematics teachers' views and beliefs about problem-solving in independent schools. This will be the first study of its kind among independent schools in South Africa. The hope is that the study will shed light on why we find it difficult to include problem-solving as a routine activity in our teaching. Also information collected from the research will offer a means to develop better teacher training workshops to support the implementation of a problem-solving orientated mathematics curriculum.

I would appreciate your participation in this study by completing an online questionnaire and possibly participating in an interview with me. There is no payment if you take part in the study. The questionnaire will be opened online in June 2016, and the interview will be arranged at a time and place convenient to you, sometime between June and August 2016. The questionnaire should take you no longer than 30 minutes and the interview will be restricted to 60 minutes.

The questionnaire asks for your name and the name of the school/institution where you are currently employed. This is purely for administrative purposes and for possible interviews. Your personal information and that of your school will not be used in any way during the analysis and/or the publication of my results. A coding system will be used to assure anonymity.

If you consent and you are approached for an interview, the interview will be video-recorded in a way that shows only your hands, and captures your voice. Once again, care will be taken not to use identifiable names, and your identity will not be revealed in the analysis and report of the data. Again a coding system will be used when the research is analysed. You as participant can at any stage request a copy of the recorded interview. Only the researcher and supervisor will have access to the recordings and the data collected.

There is no obligation to take part in the questionnaire or in the interview. The results will not be used in any way to influence your position as a teacher at your school. All research will be conducted in accordance with Stellenbosch University's Research and Ethics Policy. If you wish to discuss any aspect of this research, please feel free to contact me at stewil@bridgehouse.org.za or my Supervisor Dr. Erna Lampen ernalampen@sun.ac.za .

You may withdraw your consent at any time and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research subject, contact Ms Maléne Fouché [mfouche@sun.ac.za; 021 808 4622] at the Division for Research Development.

Thank you very much for taking the time to support me in this study and to improve the quality of teaching in our schools.

Yours sincerely,

Stephan Willers

A handwritten signature in black ink, appearing to read 'Stephan Willers', with several horizontal and diagonal lines drawn over it.

Appendix B: Letter to head of school

Dear Head of School

Request to conduct interviews with Mathematics teachers.

My name is Stephan Willers and I am busy with collecting data for my Master's degree at the University of Stellenbosch. I have been a teacher at Bridge House School for the last ten years and currently serve as the National SBA Moderator for Mathematics.

The focus of my study is on Mathematics teachers' beliefs about Mathematics and the teaching and learning of Mathematics, in particular problem-solving. My problem statement is that although problem-solving is a critical outcome of the CAPS document (the SAGs for Mathematics require problem-solving to be included in all assessments) there is still a lack of problem-solving happening in our classes. I have chosen IEB schools not only because I am familiar with the teachers and the schools, but because for the most part schools registered with the IEB are well-resourced and well-managed. Mathematics teachers at these schools are therefore faced with minimal external factors that influence their teaching and they should find it easier to deliver the outcomes of the curriculum.

My aim with this study is to firstly shed light on IEB teachers' views and beliefs of problem-solving within Mathematics education, and secondly to inform teacher training programmes. This will assist in improving the teaching of problem-solving in our schools.

I would greatly appreciate it if I could ask your Mathematics teachers to participate in an online survey and a possible future interview. With your approval I will wish to contact them in regards to this research. Their participation is voluntary and all personal and school related information will be kept confidential. All research will be conducted in accordance with Stellenbosch University's Research and Ethics Policy. I will be happy to share the conclusions of my study with you after publication.

Thank you in advance for considering my request.



Stephan Willers

stewil@bridgehouse.org.za

Appendix C: Consent form teacher participant

Investigating the beliefs about problem-solving of mathematics teachers at independent secondary schools in South Africa.

Consent form: Teacher participant

Dear Teacher

Please read and sign, and return to me:

I, _____ have been informed of the nature of the research about teachers' beliefs about problem-solving in independent schools in South Africa. I understand that I am not under any obligation to take part in the study. I understand, that should I choose to participate, my identity will not be revealed at any stage by using my name, or by indicating at which school I am teaching, or in any other way. I understand that the information I supply will not be used in any way to influence my position as teacher at an independent school. All research will be conducted in accordance with Stellenbosch University's Research and Ethics Policy.

I hereby consent voluntary to (tick each item appropriately)

- | | | |
|--|-----|----|
| 1. Complete the online questionnaire | YES | NO |
| 2. Participate in an interview with the researcher, should I be selected | YES | NO |

I give permission (tick each item appropriately) for

- | | | |
|--|-----|----|
| 1. The interview to be video recorded, focussing on my hands | YES | NO |
| 2. The data from the interview and questionnaire to be used in research reports, provided that my identity is not revealed in any way. | YES | NO |

Name: _____

Date: _____

Signature: _____

Yours sincerely,

Stephan Willers _____



Appendix D: Letter to Independent Examinations Board

Dear Me Oberholzer (*CEO of the IEB*)

Request to administer research questionnaire to independent schools registered with the IEB.

I have been a teacher at Bridge House School for the last ten years and currently serve as the National SBA Moderator for Mathematics. My observations as SBA Moderator stimulated my interest in the beliefs about problem-solving of teachers at independent schools. With this question I have enrolled for a Master's degree at Stellenbosch University.

The focus of my study is on Mathematics teachers' beliefs about Mathematics and the teaching and learning of Mathematics, in particular problem-solving. My problem statement is that although problem-solving is a critical outcome of the CAPS document (the SAGs for Mathematics require problem-solving to be included in all assessments) there is still too little problem-solving happening in our classes. I have chosen IEB schools not only because I am familiar with the teachers and the schools, but because for the most part schools registered with the IEB are well-resourced and well-managed. Mathematics teachers at these schools are therefore faced with minimal external factors that influence their teaching and they should find it easier to deliver the outcomes of the curriculum.

My aim with this study is to firstly shed light on the teachers' views and beliefs about problem-solving within Mathematics education, and secondly to inform teacher education programmes. I trust that my study will contribute to improving the teaching of problem-solving.

I believe the results from this study will provide valuable information to the IEB in the quest for quality schooling. Therefore, I ask permission to administer the attached questionnaire via online software to schools registered with the IEB. Names of participants and schools will be protected and only used for my own administrative purposes. Permission will also be requested from the Heads of the schools and the teachers. All research will be conducted in accordance with Stellenbosch University's Research and Ethics Policy.

In particular, I request permission to

- 1) Administer a questionnaire to mathematics teachers at independent schools registered with the IEB. The teachers will be informed about the nature of the study, and ensured that participation is voluntary and has no effect on their practices or careers.
- 2) To further interview a sample of teachers, again under conditions of protection of their identities, and if they voluntarily choose to participate. These interviews will not interfere with their teaching duties.

I trust that you will give your consent and I await your response with anticipation. Please do not hesitate to contact me if you need more information or want to discuss any aspect of the study with me.

Yours sincerely,



Stephan Willers

stewil@bridgehouse.org.za

Appendix E: Questionnaire

An online version of this questionnaire was used in addition to the paper based questionnaire. The online questionnaire included an introductory page with electronic consent. The University of Stellenbosch's Checkbox survey software was used for this purpose. The URL <https://sunsurveys.sun.ac.za/problemsolvingbeliefs.aspx> was used.

Background Information (Section 1)

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QUESTIONNAIRE

Investigating the beliefs of Mathematics teachers within independent secondary schools in South Africa about problem-solving within Mathematics education.

Please take the time to complete this questionnaire fully and to answer honestly. Space is provided at the end of this survey for you to add any further comments. If you wish to discuss any aspect of this research, please feel free to contact me at stewil@bridgehouse.org.za. All research will be conducted in accordance with Stellenbosch University's Research and Ethics Policy. Thank you, in advance, for your contribution.

1. Name and Surname: _____

2. E-Mail: _____

3. Gender:

M	F
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4. Number of years of teaching
Mathematics: _____

5. Number of years teaching
Mathematics at IEB School:

6. Educational qualification:

PGCE	HDE	Other:	
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7. Highest academic qualification:

Degree	

8. If Mathematics was part of your tertiary studies, please indicate your highest level of Mathematics qualification.

Mathematics I	Mathematics II	Mathematics III	Honours level Mathematics
Masters' level Mathematics	Other:		

Reflections (Section 2)

Reflections (Section 2)

This section carries the most weight in my research. Please take some time to reflect on your own experiences as a teacher and your classroom practices when answering the questions. It is advised to read all the questions through first before answering.

R.1. What is a mathematical problem to you?

R.2. What characteristics should a good mathematical problem have?

R.3. What does mathematical problem-solving mean to you?

R.6. What are the external constraints in your teaching experience, which limits you from incorporating a problem-solving orientated teaching methodology?

R.7. In which ways and to what purpose can problem-solving activities be used in Mathematics lessons?

R.8. Suppose you were teaching a class of learners. How would you describe a Mathematical problem-solving activity on a topic of your choice? What would you, as the teacher, be doing? What would the learners be doing?

R.11. Suppose some learners face difficulties during the problem-solving activity.
What would you do in order to help them?

R.12. Would you describe yourself as a problem solver? With regards to
Mathematics.

R.13. How do **you** feel when faced with a non-routine and unseen Mathematical
problem to solve?

Quantitative data (Section 3)

Section 3

Nature of the discipline of Mathematics:

Please circle your option to each question in the table below.

(1: strongly disagree; 2: disagree; 3: agree; 4: strongly agree)

NM1 (-)	Mathematical problems can be done correctly in only one way.	1	2	3	4
NM2 (-)	Some people have a natural talent for mathematics.	1	2	3	4
NM3	Mathematics is primarily a formal way of representing the real world.	1	2	3	4
NM4 (-)	In Mathematics something is either right or wrong.	1	2	3	4
NM5 (-)	Some people are good at doing Mathematics and some are not.	1	2	3	4

Beliefs about the teaching of Mathematics:

Please circle your option to each question in the table below.

(1: strongly disagree; 2: disagree; 3: agree; 4: strongly agree)

BM1 (-)	Mathematics should be taught as a collection of procedures (skills) and algorithms.	1	2	3	4
BM2	More than one representation should be used when teaching a maths concept.	1	2	3	4
BM3 (-)	Good Mathematics teachers show you exactly how to get to the answer.	1	2	3	4
BM4	Good reasoning should be regarded as more important than getting to the correct answer.	1	2	3	4
BM5	Learning Mathematics is an active process with learners actively involved in their learning.	1	2	3	4

BM6 (-)	To solve a Mathematical problem you need to teach the correct procedure.	1	2	3	4
BM7	Problem-solving activities form part of the general teaching of lower ability groups.	1	2	3	4
BM8 (-)	In teaching Mathematics logic is promoted, whereas creativity and originality are not stressed.	1	2	3	4
BM9	Teaching Mathematics provides an excellent opportunity to promote the development of the learners' thinking.	1	2	3	4
BM10 (-)	Mathematics teaching is especially meant for mathematically talented learners.	1	2	3	4

BM11: To achieve a high result at the end of grade 12 in Mathematics, how important do you think it is for students to?

(1: not important to 4: very important)

P1 (-)	Remember procedures.	1	2	3	4
P2 (-)	Think in a structured and sequential manner.	1	2	3	4
P3	Be able to supply logical reasons to support solutions.	1	2	3	4
P4 (-)	Prepare by doing old examinations papers.	1	2	3	4

BM12: Problem-solving as an instructional approach:

Please circle your option to each question in the table below.

(1: never; 2: occasionally; 3: often; 4: always)

PS1	When problem-solving in class, I teach learners to identify key words and then use taught methods (heuristic methods) and procedures to solve the problem.	1	2	3	4
PS2	I incorporate problem-solving only at the end of each section of work after learners have been taught the necessary procedure and techniques to solve the problems.	1	2	3	4
PS3	I start a new concept first with a problem to solve and then through exploring the problem we engage in procedures and techniques to assist us in solving the problem.	1	2	3	4

BM13	How much time (in hours) on average do you spend on non-routine and unseen type Mathematics questions per week in your class? Please circle your answer.	1 (less than 1)	2 (1 to 2)	3 (2 to 3)	4 (more than 3)
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Thank you for completing this questionnaire. Your opinions and contributions are valued and appreciated.

Any further opinions and suggestions can be emailed to me: stewil@bridgehouse.org.za or handed in with this survey.

Appendix F: Semi-structured interview guide

1. What is a Mathematical Problem to you?
 - Can you give me an example of a Mathematical problem in class?
2. Could you describe a mathematical problem-solving activity you use in class to me?
 - What is your role during the task/activity?
 - Could you describe your role and what would the learners be doing?
 - Is a word problem always problem-solving?
 - Would you engage all learners of all abilities in problem-solving activities?
3. Could you “in your own words” describe a problem-solving orientated teaching methodology?
 - What implication would this sort of approach have on your teaching?
 - What constraints are there to this sort of teaching?
4. Do you believe that using problem-solving type and non-routine type problems are beneficial to learner’s results and progress in Mathematics?
5. Do you enjoy solving Mathematical problems? What types do you like?
6. Do you feel that Mathematics as delivered in our classrooms promote creativity and originality in all learners?

Appendix G: Semi-structured Interview Transcription Interview 1

B. Ed (Hons)

Maths 3

10 years teaching

NM 2

BM 1.8

Date and Time: 11 Dec 2017 at 10:05

Duration: 28:29

Interviewer:

What is a mathematical problem to you?

John:

I think [...] when someone has to solve a problem that is posed. A problem that can be open or closed. Using prior knowledge and the application thereof. [long pause] This is a difficult question.

Interviewer:

Maybe give an example?

John:

Is this focused on problem-solving?

Interviewer:

No, this is an open question.

John:

A problem that uses prior mathematical knowledge to solve. It can be multi-faceted, open or closed. The child should make links between different content areas. Maybe I should come back to this question later?

Interviewer:

Fine, it is an open question. There is no right or wrong answer to this.

John:

So if you give me a question: "Is this a mathematical problem or not?" So it must be more. I think the problem with a mathematical problem is that it is called a mathematical sum. This refers to it only being an arithmetic sum like $2+4$ or a multiplication [...] but I mean a mathematical problem is more than just a sum. It can be so multi-faceted. It can be integrated, can be composited and consist of lots of small branches.

Interviewer:

So what is $x + x$ to you?

John:

It is a routine procedure. To solve a mathematical problem I might use the skill of $x + x$ to solve the problem.

Interviewer:

How would you describe a mathematical problem activity? How would you describe this in your classroom? How would you describe your role and that of the learner during these activities?

John:

For the first couple of years of my teaching I always thought that you start with the easy problems and then build up to the difficult problem. We at our school now, for the last couple of years, try to start with a problem as an introduction. We don't do the problem at the end. You pose the problem to the learners and then you ask them, "What do we need to solve this problem?" From there we move to cover the content needed.

Interviewer:

Could you think of an example?

John:

Grade 10 and 11 Analytical geometry lends itself nicely to this. We could draw a tangent to a circle. Then ask the question, "How would we find the tangent to the circle? What skills do we need? We need to know about the gradient of the radius of the circle." At this stage the kids would have knowledge from Euclidean geometry.

Interviewer:

How would you describe this activity in class?

John:

We would start with a chapter. We would start with a problem that the learners would play around with for 5 to 10 min. We would tell them that even though they do not have all the knowledge to solve the problem, they should try and also identify the missing links.

Interviewer:

What are the learners busy doing during this activity?

John:

I am the facilitator. I answer them by asking more questions. The kids are busy investigating [...] they interrogate, they try to make connections and understand the connections between the different concepts. [...] but it is not always the case. In other cases like Grade 12 you would not always start with problem-solving. Then problem-solving is something where you take together many different sections and see if they [learners] can with little information describe a whole concept and understand what you are busy with.

Interviewer:

Is this problem-solving you're describing part of an assessment?

John:

I have a problem with problem-solving only being part of an examination. We try to do a lot of problem-solving during class time. For me problem-solving is part of learning and not only part of assessment, but part of assessment for learning. The kids should learn that solving problems are not just for marks. Naturally our exams have to be weighted in a certain prescribed way with a certain percentage for problem-solving. It is easier for the kids if they do it [problem-solving] on a regular basis. You don't have to know immediately what to do, play around poke around in the dark and build a conjecture.

Interviewer:

What is problem-solving for you? What would you classify as a problem-solving activity?

John:

Well once again, I think problem-solving is when the learner does not immediately know what do to. It must not be routine. It must not be something done previously. They should use content knowledge, there should be links. A good problem-solving activity should include more than one content area.

Interviewer:

Is this what you see as links made?

John:

Yes, Euclidean and Analytical geometry.

Interviewer:

What do you understand as a word-sum?

John:

For me a word sum is a normal problem translated into a language problem. So the better your comprehension skills the easier you would be able to translate it into a mathematical language you [learner] can understand. The stronger the kid's first language is, the easier they can translate.

Interviewer:

I would like to know if a word problem is always classified as problem-solving

John:

No, I don't think so. You can easily conceptualise a problem. Like the trig problems. It could be a routine activity.

Interviewer:

So is a problem-solving question always cognitively more demanding?

John:

I think there are different classes of problem-solving questions. It depends on if the learner could easily translate into algebra if a word problem. Then it is easier. If you look at the last questions in the Grade 12 paper [those classified as problem-solving], that type of problem-solving. We get kids that do much better in Section B [problem-solving and complex procedures] than Section A [mostly routine procedure]. I think there is an interesting link;

you need to know Mathematics and the concepts. It does not mean that if you're very good in problem-solving then you know your routine knowledge, but it does not mean if you get very, very good at routine procedural and content concepts that you would become an excellent problem solver. You should be exposed to it [problem-solving]. You need to be able to play with it [the problem] if you don't know what to do.

Interviewer:

So how do the learners know what to do then?

John:

I think it is a teaching strategy. They need to get comfortable to deal with problems they don't know what to do with.

Interviewer:

Is there a specific methodology you follow to teach them this?

John:

We do many heuristics with the learners in Grade 8 and 9. We spend about 2 months focusing on heuristics. We divide it into the 6 heuristics of Pólya. They need to be taught what to do if they don't know what to do.

Interviewer:

Could you in own words describe a problem-solving teaching methodology? What effect does this sort of approach have on you teaching?

John:

When we first started it four years ago I tried to change my whole teaching. I have always thought that problem-solving comes at the end of a section/chapter and is not part of the process. The difficult problems are not part of the process. I did not find it to go smoothly. I found it difficult, the kids found it difficult as you could not just give them the answer. You answered them with more questions. You have to hold their hands but not pull them through. The kids found it uncomfortable, in the beginning.

Interviewer:

And the teacher?

John:

There was a lot of resistance from staff It was not easy. Naturally you do what is comfortable. There is a fear. You teach the way you were taught [taps on desk]. To change this, you try and do what is in the best interest of the learners, not the easiest way to teach.

Interviewer:

Which other factors influenced the implementation of a problem-solving approach?

John:

At the beginning I was worried about time. But if you spend so much time on problem-solving in the beginning you actually go through the syllabus much quicker.

Interviewer:

Other factors?

John:

You need people to believe in it, you need buy in. Developing resources is an issue. So is planning as you need to prepare very well. My thinking with problem-solving is to enrich the content for the kids.

Interviewer:

Do you think that this problem-solving and non-routine type problem-solving benefit their results at the end of Grade 12 Mathematics?

John:

Yes, I think so. Our kids will now not just leave a final paper question blank. They will play around with the questions. The biggest benefit is that they will not just leave a question blank as they did in the past. None of our kids in our June exams and prelims left any question unanswered.

Interviewer:

Do you like solving mathematical problems and which type of problems?

John:

O yes! I would do it all the time. To go back to what is a mathematics problem, I don't always think a mathematical problem is always like an Olympiad type problem, it is a type of a problem. There is more and we should be careful of telling the kids that those are the only types of mathematics problems. You can contextualise a normal problem. It can be something that they don't know [how to do] or you don't. I made the mistake with introducing Matrices and I wasn't sure why my determinant kept on being equal to zero. What now? It turned into a wonderful problem-solving lesson.

Interviewer:

How did you feel in this situation?

John:

I am not one [teacher] that pretends I know it all. I like to share with the learners how I approach and solve problems. There is 20 seconds of not knowing what is going on, but then you work with the kids. Admitting that you don't know to the kids when they ask. I don't feel threatened. It is important to me for Mathematics that the kids know that you're also a learner. That you're part of the process.

Interviewer:

Last question. Do you feel that Mathematics as delivered in our classrooms promotes originality and creativity in all our learners?

John:

For me it's a big part in all the classrooms. I have a big problem with how Mathematics is presented in our classrooms because it is this 'one size fits all'. You have kids in your class that want to study engineering and then you have those that just want 50%, those that struggled with Maths that took up the challenge. Our normal teaching does not promote this, if you just use a textbook you would not reach this.

Interviewer:

And creativity? Do we promote this?

John:

No, I think we isolate ourselves. We [in his school] are going to next year start a project with the arts department with structures and trigonometry. I think there is so much content in the CAPS. I don't have issues with leaving out content and allowing the kids to play.

Interviewer:

When you speak to people they can't attach originality and creativity to Mathematics. Can you comment on this?

John:

I believe kids have to go outside the classroom and do Maths. Looking at a wine barrel and measurement. How do we solve this? A good problem-solving problem in my view. What was the best strategy?

Interviewer:

How do lower ability learners fit into your learning and teaching environment? What is your feeling?

John:

Everyone benefits from problem-solving. The weaker learners gain confidence when they get something right. It is more difficult to work with the weaker learners; you need to change your teaching strategies. You need to change how you answer their questions to how you would answer a strong kid. You need to be aware of their learning difficulties. It takes much more patience and different strategies. It is not always more difficult but takes more endurance.

Interviewer:

Thank you, anything else you would like to add?

John:

Yes, the question about: "What is a mathematical problem?" It is so wide. A problem is supposed to be a problem; you need to be able to make sense out of it. I don't think you make sense out of a routine procedure. You need a strategy to solve it. I think that is part of the problem, our kids think it is supposed to be easy. You are supposed to struggle with a mathematical problem.

Appendix H: Semi-structured Interview Transcription Interview 2

HDE

Maths 2

27 years teaching

NM 1.4

BM 1.6

Date and Time: 11 Dec 2017 at 15:10

Duration: 23:13

Interviewer:

What is a mathematical problem to you?

Liz:

It is a very wide question. Depends on the context that you're in at the time and the pupils you're working with at the time. Depends on what you ... what you want the answer out of it to be. If you want [...] pupils to think, you are going to ask the question in a certain way as a problem. If you just want a set answer, you will ask it in a different way. I think it depends on what you want out of it as to what you decide to define as a problem.

Interviewer:

Linking to this, could you describe a mathematical problem-solving activity?

Liz:

Depends on if you want a right answer or a wrong answer. I think the idea of problem-solving is to get pupils to think. I think you have to take away from a classroom situation if that is where you are. Get away from the idea that there is one right answer and I want that answer. You want to see what pupils are thinking. You want to see how pupils are thinking. And I think that is what your aim is, to try and get them to think whether they end up at the right place or not. You want to first get them thinking. I don't think that is what we do when we are testing.

Interviewer:

How would you describe a mathematical problem-solving activity?

Liz:

I think it has to be open-ended. I think you've got to be [...] I am going from the idea that I don't want 10 to be the right answer. I want them to think about various solutions. Can you give me an idea of ways to get solutions to a particular question? We want to put JoJo tanks outside. Can you give me an idea of what size JoJo tanks we would need? Where would we place them? Why we would need them there? Would it be better having this kind or that kind and why? Explaining, thinking, getting them thinking, and then bringing the Maths in, sometimes incidentally or accidentally but on purpose.

Interviewer:

What would your role be during these sort of activities? And could you describe the kids' role during these activities?

Liz:

The kids need to be the ones coming up with the ideas. They need to be the ones thinking about what, why, what could be and problem with their answer. Discussing where the problems will come up with their particular answer or solution. I think the role of the teacher is to suggest options if they are stuck. If they're heading off on major tangents that are not going head in the direction you need them at that particular point. To redirect them back to your aim and intention, but not to close doors and say: "You're doing the wrong thing. Do it this way." You want to tray and I think it is very difficult especially for teachers teaching for a long time with a right answer wrong answer thing. It is more of a case of there are lots of ways of getting there and try and help pupils to take their route back in their direction rather than in your direction.

Interviewer:

Would you involve learners from all ability groups in this sort of activity?

Liz:

I think often weaker ability pupils are better at problem-solving. They are not necessarily better at getting the right answer to the mathematical problems we give kids in class. I think they often come up with better solutions and better ways of doing it, whereas a stronger mathematics student will come up with a mathematical formula that will get there and they will come up with it quickly with that. [...] I think often mixed ability. But depending on where you're going, maybe a different interaction at different stages of the problem. Let groups interact with each other once they have time to interact with the problem.

Interviewer:

But you won't exclusively be doing problem-solving with high ability kids?

Liz:

No. I have had some very interesting ways of solving problems from my Math lit kids. They have come up with some very ingenious ways of solving problems. While the Maths kids will come up with a formula or try and think of something. Whereas a Math Lit pupil would come from real life, and have a real life solution to something that often makes it more practical.

Interviewer:

Could you describe what you would see as mathematical problem-solving?

Liz:

Ideally it is not whatever I ever do in class. Because in class we just never ever have the time to do it. Something I know I definitely gloss over. Give it to pupils as a question, take this one home and go and think about it. Ideally it is a week-long kind of problem. Here is a task. Go and find a solution. But practically we just don't have the time. Unfortunately, that is where our learners lose out. The stronger learners lose out, the stronger the class you often have less time to do the problem-solving. You're pushing them to get higher marks. Your middle groups you're pushing to get better marks than they're doing. And you just don't have the time to do that.

Interviewer:

Do you then feel that engaging kids in problem-solving, non-routine type questions is not beneficial to their results in Grade 12?

Liz:

No, I think it is hugely beneficial. Not necessarily to their results in Grade 12 but to their results thereafter. I think with the amount of time we are very limited to cover the basics of the syllabus.

Interviewer:

So do you think a kid could do very well without doing any problem-solving type activity in their schooling?

Liz:

Currently yes. I think they can work through past papers and get an A, without being a good problem solver. I think your good problem solver [...] will often not get the highest they could have got. They often look at problem-solving ways of doing the questions. [...] They then do not finish the paper. Ideally it would be lovely to have a week to do completely random unseen problem-solving that has nothing to do with syllabus, with what is prescribed. We just don't have the time.

Interviewer:

Do you think that every and any word type problem is problem-solving?

Liz:

Definitely not. I think a lot of it is words that are disguising [...] normal routine questions. And they just see it as there are words in the sum so it is going to be difficult. Maybe I'll just leave it out. And it often ends up being a routine question just disguised as different words. [...] I don't think this is problem-solving.

Interviewer:

Would you ever teach the kids heuristics for problem-solving?

Liz:

We ran a very successful workshop with our Grade 8s and with our Grade 9s last year. We did very similar things [to a problem-solving development workshop for teachers]. We had the whole day. We broke them up in groups. I think we started with 5 different group activities. Things like goal posts with ropes, get through the quickest way. Each person can only touch three ropes. Here are some matchboxes you can only touch three of them, which one is the heaviest? Those kinds of things. Spaghetti and marshmallows make the highest tower. And then from there, after we have done some chatting to them. From there what did they do and how did they go about it? We then did some problem-solving from the workshop [workshop on problem-solving] to a similar way we did [the teachers]. We then gave them a booklet afterwards of similar questions that they could take away and then in class after they have done their equations or finished work they could take out their problem-solving booklet. They could work at it and challenge each other and work at it. This seems to have benefited looking at Maths Olympiad results this year. In those particular two grades we had some very nice [results]. They are a lot more confident at trying ... in trying the problem-solving type

questions that are different. As opposed to the seniors that say they have not been taught this. The babies are much more keen to go for it. May be it is trial and error. May be it is this. It will be interesting following that group to see if it has had more benefit. Unfortunately, we could not do it this year because we ran out of time ... maybe next year when we have an additional lesson. So we're hoping to do more of that non-routine type stuff in class.

Interviewer:

What do you understand when I describe a problem-solving orientated teaching methodology?

Liz:

I think it is going more from saying "Today we're doing equations" to "Here are some questions and let's see which ways or how we could go about doing it". Almost flipping the classroom type terminology. We are going to be doing quite a lot of that year after next and we're building up to that. We're building a middle school. We're doing problem based learning with our 7,8 and 9s. We are going to bring the 7s across to the high school. And we're going to do a lot of problem based learning and project based learning. I think those are closely tied together - problem based and project based. I am not sure if there is a difference between them, but that is the route we are going. Some of the admin staff have started investigations into it. So I know we're changing the route we're going with Maths for 7, 8 and 9. So we bring that in the year after next.

Interviewer:

So what do you think the constraints would be?

Liz:

I think it - Maths - would be one of the subjects and physical science would still be one of the subjects. I think it is going to change the order in which we teach things. I think we would start with the useful kind of probability, measurement kind of topics and work backwards to develop the algebra. Putting specific teachers into it. I will tell you more next year.

Interviewer:

Do you enjoy solving mathematical problems? How does it make you feel? Does it make you feel anxious working with problem-solving activities?

Liz:

It is something I had to get my head around. Because generally [it makes me] feel very anxious. Changing my [...] way of thinking. I have always been taught that there is a right and a wrong answer and Maths is the aim to get the right answer. I was never very strong in Maths myself. The idea whole idea of playing around and trying an answer is very foreign to me. It is something that I have [...] really tried to work on. For me it is very uncomfortable not to know where it is going. It is my own personality.

Interviewer:

When you said there is a right or wrong answer. Were you saying a right or wrong process to get to an answer or a right answer?

Liz:

Pretty much both. Because the aim was that if this is the answer then there is only one way to get there.

Interviewer:

And your feeling about that now is?

Liz:

I want a method to get there. And that is the way I have always been trained and taught and practised to do it. It has only been the last 5 years or 6 years where I had to get my head around doing it differently. I think it has only been the last 5 or 6 years that we have been doing this non-routine stuff. If I know where it is going, I am more comfortable with it. If I get a book of problem-solving, I want a set of answers available that if I can't find it, I know where to find it. I am not comfortable going into class without knowing [that I don't know] the answer or how to get the answer to question 7. Because I know the first kid that walks in will say, "How do you do question 7?" And it is not always easy to say "I don't know" and ask for one of them who can help me to explain. And that has taken a long time myself getting my head around saying: "I don't know. Someone get up and explain. Let's figure it out."

Interviewer:

What is your background at university?

How did you experience Mathematics at varsity?

Liz:

I hated it. I loved Maths at school. It was a comfortable environment. Varsity I was always out of my depth. My lecturers would always say: "It is up to you to figure it out. We have no time to help the students." So I battled to get those 50%. Maths was a scrape through by the skin of my teeth. Now I actually like teaching Maths. I like working with the weak students. I like watching the development of the others. But when the strongest ones come and ask "How do you do this?" And I have to say "I don't know". I find it very uncomfortable.

Interviewer:

Do you think the way that Mathematics is offered in our classes promotes originality and creativity from our learners in their work?

Liz:

Depends very much on the teacher. It depends on the section and very much on the class. I find girls very much wanting process-based answers in an all girls' school. In an all boys' school they are very happy with something non routine. Because they're never sure if it's something [...] they can figure out anyway. The more confident pupils are very happy to do whatever they feel like. They are prepared to investigate and keep going with the interest. Pupils that are not interested want a much more 'this is the way you do it and this is the method'. Okay I can achieve something. As long as they can achieve something they feel more comfortable to explore a bit more.

Interviewer:

Do you think the syllabus or the way it is presented, promotes originality and creativity?

Liz: No.

Appendix I: Example of interview request email

Dear ...

I trust you are doing well. I am sure you are looking forward to the end of the term and the year. I have a favour to ask. As part of my studies I need to do a few short interviews. I was hoping you would have some time to have a short chat to me about your views of problem-solving and the teaching of Mathematics. I was thinking maybe at marking? The interview data is treated as confidential and no school or personal details are represented in the study.

If you're not available, I completely understand. It is purely voluntary.

Kind regards,