



A decision support system for firebase location in a nature conservation area

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Abstract

It is important that firebases are available on standby at strategic locations in a nature conservation area from where wildfire ignition points can be reached rapidly and such fires brought under control before they spread. Two facility location models are proposed in this paper which may form the basis for decision support when deciding on the locations of such firebases in a nature conservation area. Both of these models are multi-objective in nature. They are able to produce solutions that embody trade-off decisions between minimising the cost of locating firebases and maximising the coverage of key areas in a conservation area. These trade-offs may be based on a variety of coverage importance criteria, such as aiming to cover terrain portions exhibiting a steep ground slope, terrain portions that experience a high annual mean wind speed, or terrain portions in which many wildfires have ignited in the past. The coverage criteria are typically case-specific and may therefore be specified by the decision maker. Both models, as well as their approximate solution methodology, are implemented in the form of a computerised decision support system in order to render them accessible to non-mathematically inclined decision makers. The decision support system is validated by applying it to a special case study involving Table Mountain National Park, a nature conservation area in the Western Cape, South Africa.

Key words: Firebase location, nature conservation area, coverage criteria.

1 Introduction

Fynbos covers approximately 6.7 percent of the area of South Africa, and is only found in a wide coastal belt stretching from Clanwilliam on the West Coast to Port Elizabeth on the Southeast Coast. A number of conservation areas have been declared in this region to protect these unique species of flora. The region boasts sixteen national parks (spanning

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an area of 528 km²), ninety six provincial nature reserves (spanning an area of 8 924 km²), seven Department of Water Affairs and Forestry reserves (spanning an area of 118 km²), and a host of non-statutory reserves together spanning an area of 9 780 km² [49].

Fire can be both the lifeblood of fynbos and its most serious threat. This is because Fynbos is a fire-adapted family of plant species which can only survive if it experiences regular fires [13]. In the absence of fire, alien species of plants have been observed to replace fynbos gradually [23]. This poses a threat to the fynbos — not only because it is being overtaken — but also because such alien growth increases the biomass in the veld. When a fire does occur, this alien biomass increases the intensity of the fire. In some cases the fire becomes so intense that new seeds cannot germinate and, instead of fulfilling its role of rejuvenating the fynbos, the fire then kills it.

Areas inhabited by fynbos are usually subjected to prescribed and carefully controlled burning on a regular basis so as to kill alien growth and allow the fynbos to germinate. The pre-planned frequencies and locations of these prescribed fires give rise to what is called a *fire schedule*. An optimal fire schedule requires that fynbos burns approximately once every ten to fourteen years [13]. Managing fires according to this optimal frequency is, however, quite a daunting task. Apart from fires that occur due to anthropogenic causes, two other types of fires occur in fynbos conservation areas: prescribed fires (as mentioned above) and wildfires that are caused by natural elements [56]. The difficulty in fire management arises with this second type of fire.

Although models exist for predicting wildfire occurrences [2, 4, 20, 21, 46, 58], such models are usually area-specific and not very accurate, making it very hard to avoid the damaging effects of wildfires entirely. It is therefore essential that a collection of strategically placed firebases should be available in any fynbos conservation area from which natural wildfires can be brought under control quickly.

These firebases should be placed in such a manner that the largest possible number of potentially critical wildfire ignition points are accessible within a certain time frame from one of the bases. When placing these firebases in a conservation area there are many factors to take into account, such as the cost of building a firebase at a proposed location within the area, the average prevailing wind speeds and directions in various areas of the park, and the locations of firebreaks, to name but a few.

A *decision support system* (DSS) is put forward in this paper which can take *geographic information system* (GIS) data related to a nature conservation area as input and use these data to suggest an appropriate number of semi-mobile firebases for the effective management of wildfires within the area as well as suitable locations for these bases within the area. This DSS is based on two multi-objective firebase location models. The number of firebases placed may be limited either by sufficiency with respect to the (conflicting) placement criteria considered or by the conservation area's budget. A collection of high-quality trade-off firebase location solutions is recommended by the DSS from which the management of the conservation area may subjectively select a solution.

The paper is organised as follows. After briefly reviewing the literature related to firebase location in §2, two novel, multi-objective combinatorial optimisation models are derived for firebase location in §3. Since these models cannot be solved exactly for realistically sized

problem instances, the approximate solution methodology of (multi-objective) simulated annealing is described in §4, which is employed to solve the models of §3. Both models of §3 and the solution methodology of §4 are then incorporated into a computerised DSS in §5, so as to render the models accessible in various problem settings to non-mathematically inclined decision makers. The working and practical suitability of this DSS is finally demonstrated by applying it in §6 to a case study involving Table Mountain National Park, a nature conservation area in the Western Cape, South Africa, after which the paper closes in §7 with a brief summary and some ideas for possible future work.

2 Literature review

Most firebase location models in the literature are variations on, or enhancements of, five basic classes of facility location models from the realm of operations research [15, 45]. The first and simplest of these model classes is the class of *set cover models* originally proposed in the context of emergency facility location by Toregas *et al.* [59]. In a set cover model, the objective is to find a set of facility locations from among a finite set of candidate facility locations in such a manner that every demand point is covered by at least one facility while simultaneously minimising the cost associated with the entire set of facility locations. It is usually assumed that a demand point may be considered covered by a facility if the distance that separates the two does not exceed some pre-determined threshold [22]. The set cover modelling approach has two key shortcomings [15]. The number of facilities required to cover *all* demand points often exceeds the number that is practically or financially feasible. Furthermore, the demand points are treated as if they are equally important to cover — such models typically do not allow for the rating of certain demand points as more important than others in terms of coverage.

The second model class is the class of *maximum coverage models* proposed by Church and ReVelle [11]. This model class is a generalisation of the first class, addressing the above-mentioned shortcomings. A maximum coverage modelling approach allows the decision maker to specify an upper bound on the number of facilities whose placement would be practically feasible. The approach also allows for the rating of the different demand points in terms of their relative coverage importance. The objective is then to identify a set of facility locations of a cardinality not exceeding the upper bound specified, which maximises the coverage of demand weighted by the importance values of the various demand points [45]. Such *importance values* are usually based on multiple placement criteria and are specified in advance of model formulation. If the specified bound on the number of facilities to be placed is smaller than the minimum number of facilities required to cover all demand points, the model will be constrained from covering all demand points. In essence, the modelling approach therefore relaxes the condition that all demand points have to be covered.

As in the set cover modelling approach, it is required that every demand point should be covered in the class of *centre models* originally proposed by Hakimi [26], but the approach limits solutions by specifying the number of facilities that should be placed exactly [7]. Models in this class are formulated so as to minimise the maximum coverage distance between a demand point and the facility nearest to that demand point. Unlike in the

previous two model classes, centre models are therefore focussed on satisfying demand at extremal demand points, rather than at all points explicitly.

In all three model classes mentioned above, it is assumed that if a demand point is covered by a facility, all its demand is fulfilled from that facility (and it receives no demand fulfilment if it is outside the coverage radius of all facilities, if such a radius is imposed). In reality, however, this is often not an appropriate assumption — in many practical cases, demand satisfaction at a demand point is a function of the distance over which demand fulfilment occurs in the sense that the coverage benefit decreases as this distance increases [15]. In the class of *median problems*, also proposed by Hakimi [26], this assumption is relaxed by allowing any location to satisfy a fraction of any demand point's demand, with the aim of minimising the total distance over which demand is satisfied, weighted by the volume of demand satisfied, and subject to the requirement that each demand point's demand is totally satisfied by some combination of facilities.

When taking the number of facilities as a measure of the cost of locating facilities, as is the case in all four the aforementioned model classes, it is indirectly assumed that construction costs of facilities at different locations are equal. In many real-world cases, however, this is an invalid assumption. The class of *fixed charge facility location models* addresses this unrealistic assumption by allowing for the specification of the cost of building a facility at every candidate location. In this modelling paradigm, the objective is to minimise the total cost of locating facilities, which comprises a fixed cost component (*i.e.* the cost of building the facilities at the locations chosen for them) and a variable cost component (*i.e.* the cost of satisfying demand, usually modelled as in the class of median problems) [1].

One of the earliest models for the location of fire stations dates back to 1968 and is due to Hogg [29], who sought to minimise the journeying cost of fire engines from fire stations to fires in the United Kingdom city of Bristol. His model is essentially a median problem solved by a centre of gravity heuristic applied to expected fire demand data. Plane and Hendricks [47], on the other hand, adopted a set cover approach toward minimising the number of fire stations (of different companies) so as to cover expected fire call-out demand by the Denver Fire Department in the United States. Walker *et al.* [61, 62] describe a model, called the *Firehouse Site Evaluation Model*, which was developed to help fire department administrators and other local government officials to determine good locations for a city's fire houses. In this model, the user can select either a centre problem or median problem modelling approach, adopting as optimisation criterion either the expected travel time of or the distances travelled by fire engines.

Another early development was the simultaneous modelling of facility location and equipment allocation [51], which is particularly important in urban areas where high-rise buildings, for example, require very specific fire fighting equipment. Hausner *et al.* [27] conducted a cost-benefit analysis for New York City in respect of how many fire houses and fire companies the city needed, taking into account the required equipment, and they also considered heuristically where these houses and companies should be located.

All four of the aforementioned fire station location models are, however, applicable in urban settings, where the location criteria are typically dominated by city lay-outs and traffic density patterns. This is because in urban applications, fire station coverage is typically weighted by fire engine travel times and/or travel distances. Kolesar *et al.* [37],

in fact, analysed a large data set of fire engine travel times and travel distances in New York City and sought to determine an analytic relationship between these two parameters.

The body of literature on emergency response facility location in urban settings is far larger than that on firebase location in the context of wildlife management in natural areas. While Doeksen and Oehrtman [18] considered optimum locations for fire-fighting equipment in a rural fire system within a large county in Oklahoma state, their study was concerned with the protection of developed farmland and livestock instead of fauna and flora in a natural conservation area. In the context of natural conservation areas, Chow and Regan [10], proposed a location and relocation model for air tanker initial attack basing in California for regional wildland fires combatted with multiple air tankers that may be co-located at the same air base. Meyer *et al.* [43] adopted both a set cover modelling approach and a maximum coverage modelling approach in an attempt to provide decision support to Table Mountain National Park in respect of suitable firebase locations. All three the latter models were, however, based on a distance-related placement criterion.

Little attention has been afforded in the literature to the inclusion of GIS-related emergency response facility location criteria other than travel distance or response time, such as terrain elevation, terrain slope, wind data and type of terrain, which are exceedingly important criteria in nature conservation applications. A notable exception is the model of Dimopoulou and Giannikos [17], who employed a maximum coverage model, taking into account the classification of a forest by varying coverage importance in regions of different GIS classes and applied the model to the area of Parnitha, near Athens in Greece. Although Liu *et al.* [40] also employed GIS-related data in their model for emergency response facility location, this was in the urban context of covering transportation routes for hazardous materials in Singapore.

A recent development in emergency facility location within a nature conservation context has been an integrated modelling approach involving the simultaneous consideration of fuel management (the practice of maintaining forest age mosaics that result from carefully designed planting schedules and/or prescribed burning) and the traditional approaches of suppression preparedness planning of the above-mentioned models. Bettinger [6] and Minas *et al.* [44] adopted this modelling approach, but their models remain essentially single-objective. Whereas multi-objective facility location models do exist — Schilling [50] was one the early pioneers in this area — such models are few and far between. The vast majority of facility location models in general, and of firebase location models in particular, are single-objective or at most an aggregation of various placement criterion scores into one objective function when purporting to be multi-objective. Although an approximation of the Pareto front may, in principle, be traced out for a convex multi-objective model by varying the weights assigned to criteria in a single objective function, the accuracy of this approximation depends sensitively on the method used to assign weights to the criteria [54]. The most popular method of weight assignment is the *linear weighted sum method*, but many other schemes also exist [30, 31, 32, 33, 35, 48, 55]. Attempts at finding weighting methods that yield close Pareto front approximations may be tedious and are usually highly problem-specific. Other complexities that may arise when combining multiple criteria together in a single objective function are described by Das and Dennis [14].

It has, in fact, been suggested by various authors that weighting multiple objectives together in a single objective function is bad practice, because it results in poor objectivity and that the multi-objectivisation of problems that are essentially single-objective may even result in improved optimisation [38, 55, 63].

Apart from the work of Schilling [50], another example of a truly multi-objective approach to emergency response facility location is the work of Yang [65], who used a genetic algorithm to approximate solutions to a fuzzy multi-objective programming model for the optimisation of fire station locations in the Derbyshire Fire and Rescue Service in the United Kingdom. Yet another such instance is the fuzzy multi-objective programming approach of Tzeng and Chen [60] in which a genetic algorithm was employed to come up with trade-off decisions in terms of the number of fire stations required at an international airport and where these fire stations should be located. A final example is the integer goal programming approach adopted by Badri *et al.* [5] to solve their multi-objective model for urban fire station locations in which they considered seven objectives that are conflicting to some degree (including the minimisation of fixed costs, fire engine travel costs and fire engine travel times, as well as the maximisation of coverage capability). Unfortunately all of these multi-objective modelling approaches find application in an urban context, employing location criteria that are not always applicable to firebase location problems in natural conservation areas.

3 Mathematical models

In this section, two novel, multi-objective mathematical models are put forward for firebase location in conservation areas. The median problem modelling approach is followed, which is popular in the literature. Double-counting of importance values associated with covering demand points is permitted in respect of various coverage importance criteria in the first model version, but is avoided in the second version. The relative effectiveness of these two modelling approaches are compared later by applying the models to a special case study.

3.1 Double-counting model

Denote the set of firebase candidate locations in a conservation area by $\mathcal{I} = \{1, \dots, |\mathcal{I}|\}$ and the set of demand points requiring firebase coverage by $\mathcal{J} = \{1, \dots, |\mathcal{J}|\}$. Furthermore, index the set of demand point importance criteria by $\mathcal{K} = \{1, \dots, |\mathcal{K}|\}$. Examples of coverage importance criteria indexed by \mathcal{K} may be aimed toward covering locations experiencing large annual mean wind speeds (since wind accelerates the spread rate of a fire), locations at which the ground slope is very steep (since fires spread very quickly upwards against steep inclines) or locations at which large numbers of wildfires have ignited in the past. Let f_i be the fixed cost associated with locating a firebase at candidate site $i \in \mathcal{I}$ and suppose the importance value associated with covering demand point $j \in \mathcal{J}$ according to criterion $k \in \mathcal{K}$ is denoted by h_{jk} .

An $|\mathcal{I}| \times |\mathcal{J}|$ binary matrix, called the *coverage capability matrix* and denoted by \mathbf{C} , is associated with the firebase candidate locations and park demand points. The rows of this matrix represent the firebase candidate locations and its columns represent the park

demand points. The entry in row i and column j of the coverage capability matrix is

$$C_{ij} = \begin{cases} 1, & \text{if demand point } j \in \mathcal{J} \text{ is covered by a firebase at candidate location } i \in \mathcal{I}, \\ 0, & \text{otherwise.} \end{cases}$$

The binary coverage capability criterion used to populate the matrix \mathbf{C} may be based on a distance-related coverage radius or a time-constrained coverage response area around each potential firebase candidate location, as required by the decision maker(s).

Let d_{ij} represent the distance between a firebase located at $i \in \mathcal{I}$ and demand point $j \in \mathcal{J}$. Suppose further that exactly P firebases are to be placed at locations in \mathcal{I} and define the decision variables

$$x_i = \begin{cases} 1 & \text{if a firebase is placed at candidate location } i \in \mathcal{I}, \\ 0 & \text{otherwise.} \end{cases}$$

Then the model's objectives are to

$$\text{minimise } Z_0 = \frac{1}{\gamma_0} \sum_{i \in \mathcal{I}} f_i x_i \quad (1)$$

and to

$$\text{maximise } Z_k = \frac{1}{\gamma_k} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \frac{C_{ij} h_{jk}}{d_{ij}} x_i, \quad k \in \mathcal{K}, \quad (2)$$

subject to the constraints

$$\sum_{i \in \mathcal{I}} x_i = P, \quad (3)$$

$$x_i \in \{0, 1\}, \quad i \in \mathcal{I}. \quad (4)$$

The objective in (1) is to minimise the total fixed cost associated with the location of the firebases, while the objective in (2), in fact, represents $|\mathcal{K}|$ different coverage objectives according to the coverage importance criteria in \mathcal{K} . The accumulation of the coverage importance rating in (2) is scaled by dividing by the distance d_{ij} between a covering firebase at location $i \in \mathcal{I}$ and the covered demand point $j \in \mathcal{J}$. The logic behind this scaling may be elucidated by the following example. Suppose there are two firebases, each covering a specific demand point, but that one firebase is three times as far from the demand point as the other firebase. If the importance rating of the demand point is not divided by the relevant distances between the demand point and the two firebase locations as in (2), these two firebases will be considered to perform equally well in terms of covering the demand point in question according to criterion $k \in \mathcal{K}$, which would be an inappropriate reflection of reality. Constraint (3) finally ensures that exactly P firebases are located, while constraint set (4) enforces the binary nature of the decision variables.

In objective function (1), γ_0 is an upper bound on the cost of placing P firebases (*i.e.* the sum of the P largest costs in the set $\cup_{i \in \mathcal{I}} \{f_i\}$), while γ_k in (2) is an upper bound on the accumulation of importance ratings that can be achieved by P firebases according to criterion $k \in \mathcal{K}$ when viewed in isolation from the other criteria (*i.e.* the sum of the P largest values in the set $\cup_{i \in \mathcal{I}} \{\sum_{j \in \mathcal{J}} C_{ij} h_{jk} / d_{ij}\}$). The purpose of dividing by γ_0 and γ_k in

(1) and (2), respectively, is to normalise the objective function values to within the unit interval $[0, 1]$.

Although the model (1)–(4) can, in principle, accommodate any number of coverage importance criteria, it is noted that as the number of objectives increases (*i.e.* as $|\mathcal{K}|$ increases), non-dominated solutions may become rare. This is because the presence of more objectives renders domination of a candidate solution in terms *all* the objectives unlikely. In such cases, multi-objective optimisation methods may struggle to converge, as discussed by Hughes [30] and by Purshouse and Fleming [48].

Note, however, that double-counting occurs in (2), in the sense that if demand point $j \in \mathcal{J}$ is covered by $p \leq P$ firebases, then the importance value h_{jk} will be accounted for p times in (2). This process of double-counting is intentional and may, in fact, be desirable in some cases because it will encourage a measure of fail-safeness in the sense of multiple coverage of very important demand points. This might be a desirable feature in cases where firebases are unavailable to respond to fires at these important demand points (*e.g.* if multiple fires were to occur and a firebase covering an important demand point is already otherwise occupied — then there may possibly be another firebase covering the demand point if the demand point in question is considered very important).

3.2 Single-counting model

In case the practice of double-counting of coverage importance values of firebases that overlap in their coverage in the model of the previous section does not sit well with the decision maker, an alternative model, capable of avoiding such double-counting, is put forward in this section.

The practice of double-counting in the model of the previous section may be avoided by replacing Z_k in (2) by the alternative objective of

$$\text{maximising } Z'_k = \frac{1}{\gamma'_k} \sum_{j \in \mathcal{J}} \max_{i \in \mathcal{I}} \frac{C_{ij} h_{jk}}{d_{ij}} x_i, \quad k \in \mathcal{K}. \quad (5)$$

In this single-counting model version, γ'_k is again an upper bound on the accumulation of importance ratings that can be achieved by P firebases according to criterion $k \in \mathcal{K}$ when viewed in isolation from the other criteria (*i.e.* the sum of the P largest values in the set $\cup_{i \in \mathcal{I}} \{\max_{j \in \mathcal{J}} C_{ij} h_{jk} / d_{ij}\}$). Unlike in the previous model, the objective in (5) will discourage overlapping coverage of demand points by firebases.

4 Approximate solution methodology

In this section, the approximate solution methodology is described that was adopted to solve the models of §3. The solution method of choice is briefly motivated in §4.1 and described in §4.2, after which the parameter values selected for implementation are documented in §4.3. Finally, the validation of the solution methodology implementation is described in §4.4.

4.1 Choice of solution methodology

The high computational complexity associated with solving the models of §3 places a brute-force model solution (*i.e.* consideration of all $\binom{|I|}{P}$ candidate solutions in the construction of a Pareto set) out of reach of current computing technology for realistically sized problem instances. A more intelligent, almost-exact solution approach may, of course, involve a binary programming approach employing, for example, the standard branch-and-bound method [39]. In such an approach the multi-objective model nature may be accommodated by constraining all but one of the objective functions at appropriate levels and solving the single-objective problem with the remaining objective function optimised. This process may be repeated in an experimental design of “appropriate levels” of objective function limits in order to trace out a good approximation of the Pareto front in objective space. Anticipated disadvantages of this approach are two-fold: it may take excessively long to solve a single-objective iteration by the branch-and-bound method, and the number of iterations required to trace out a Pareto set approximation of suitable resolution may be very large (especially if $|\mathcal{K}|$ is large). Deb [16] also describes other disadvantages of this solution approach. These disadvantages may be alleviated to some extent by employing an advanced solution technique from the realm of combinatorial optimisation, such as Benders decomposition [12], but even such a sophisticated solution approach is expected to require long computation times for realistically sized problem instances.

An approximate solution methodology is therefore instead employed in this paper to solve the models of §3. Various (meta)heuristics (such as a variety of local search heuristics [57], the method of tabu search [24] and the method of simulated annealing [36]) were considered for this purpose. Of these, the method of *Simulated Annealing* (SA) was selected due to its flexibility, its relatively small set of parameters requiring user-specification and its ease of implementation.

4.2 The method of simulated annealing

The method of SA, originally proposed by Kirkpatrick *et al.* [36] in 1983, is an approximate solution methodology for single-objective optimisation models based on the principles of statistical mechanics traditionally employed in the annealing of solids. Various improvements to and generalisations of the original algorithm have been proposed by a variety of authors over the years [25], including extensions to multi-objective optimisation. The multi-objective extension of Smith *et al.* [53] is employed here. The algorithm requires the following sets of input data:

- an initial solution,
- an initial temperature,
- a cooling schedule,
- an epoch management protocol,
- a description of the neighbourhood structure of a candidate solution from which the next current solution may be selected, and
- termination criteria for ending the search.

The method of SA is trajectory-based, and so a single current candidate solution is main-

tained during every iteration of the algorithm. This solution is encoded in binary decision vector form as described in §3.1 (a binary vector $\mathbf{x} = [x_1, \dots, x_{|\mathcal{I}|}]$ of *weight* P — *i.e.* containing exactly P ones and $|\mathcal{I}| - P$ zeros).

The current solution of the next iteration is generated from the current solution \mathbf{x} of the present iteration by randomly selecting an alternative solution \mathbf{x}' from the neighbourhood of that solution according to a uniform distribution. If \mathbf{x}' is better than \mathbf{x} in at least one of the objective function values in (1), (2) or (5), it is accepted as the new current solution with probability 1. Otherwise, it is accepted as the new current solution with probability $\exp(-\Delta_E(\mathbf{x}, \mathbf{x}')/T_i)$, where T_i denotes the temperature during search epoch i and $\Delta_E(\mathbf{x}, \mathbf{x}')$ denotes the increase in energy associated with moving from \mathbf{x} to \mathbf{x}' in the search space. This energy increase is taken as the largest deterioration of the objective function value when moving from \mathbf{x} to \mathbf{x}' .

An epoch is either terminated by triggering a reheating cycle (*i.e.* increasing the temperature to its initial value, but retaining the current solution in a bid to promote exploration) or by lowering the temperature according to a cooling schedule for application during the next epoch (so as to encourage intensification). The well-known geometric cooling schedule

$$T_{i+1} = \alpha T_i, \quad i = 0, 1, 2, \dots, \quad (6)$$

is employed where $\alpha \in (0, 1)$ is the so-called *cooling parameter*.

The neighbourhood structure of a current solution is specified by an $|\mathcal{I}| \times |\mathcal{I}|$ binary matrix, called the *neighbourhood matrix* and denoted by \mathbf{N} . Both the rows and columns of this matrix represent firebase candidate locations. The entry in row i and column j of the neighbourhood matrix is

$$N_{ij} = \begin{cases} 1 & \text{if candidate locations } i \text{ and } j \text{ are considered to be adjacent,} \\ 0 & \text{otherwise.} \end{cases}$$

An alternative solution \mathbf{x}' is randomly selected as follows from the neighbourhood of the current solution \mathbf{x} . A unit entry of \mathbf{x} is randomly selected. Suppose this unit entry occurs in position i of \mathbf{x} (*i.e.* $x_i = 1$). Then a column, j (say), of the neighbourhood matrix \mathbf{N} is randomly selected in which there occurs a unit entry in row i , with the *proviso* that $x_j = 0$. The vector $\mathbf{x}' = [x'_1, \dots, x'_{|\mathcal{I}|}]$ is then formed by letting

$$x'_b = \begin{cases} 0 & \text{if } b = i, \\ 1 & \text{if } b = j, \\ x_b & \text{otherwise.} \end{cases}$$

During the course of execution of the algorithm, an archive of nondominated solutions to the models of §3.1 and §3.2 is maintained by inserting the current solution into the archive if it is not dominated by any candidate solution already in the archive, upon which any solutions already in the archive that are dominated by such an insertion, are removed from the archive. The algorithm is initialised with a random initial solution, which is both the first current solution and also forms the initial archive of nondominated solutions. The search is continued until the first of a number of pre-specified termination criteria is met.

4.3 Parameter tuning

Busetti [8] suggests that the initial temperature should be chosen so that approximately 80% of all non-improving moves are accepted at the start of the SA search. Such an initial temperature may be estimated by conducting a trial search during which all non-improving moves are accepted. The initial temperature for the full search may then be taken as $T_0 = \Delta^+ / \ln 0.8$, where Δ^+ denotes the mean change in objective function value as a result of accepting non-improving moves during the trial search.

The number of iterations spent by the search in epoch i is determined by a Markov chain of length L_i . As Busetti [8] states, the value of L_i should ideally depend on the optimisation problem at hand, rather than being a function of i (that is, the length L of the Markov chain should not be epoch-specific). It would, in fact, make sense to require a minimum of A_{\min} move acceptances during any epoch before lowering the temperature, where A_{\min} is a pre-specified parameter. As T_i approaches zero, however, non-improving moves are typically accepted with decreasing probability and so the number of trials expected before accepting A_{\min} moves should increase without bound as the search progresses, no matter the value of the positive integer A_{\min} . A suitable compromise is therefore to terminate an epoch once L moves have been attempted, by reheating to the initial temperature, or once A_{\min} moves have been accepted, by lowering the temperature according to the cooling schedule (whichever occurs first), for some positive integers L and A_{\min} satisfying $L > A_{\min}$. Following the rule of thumb proposed by Dreo *et al.* [19], it is acceptable to take $L = 100|\mathcal{I}|$ and $A_{\min} = 12|\mathcal{I}|$.

An initial solution of weight P was selected randomly for each run of the SA algorithm. After some numerical experimentation, it was decided to select the value $\alpha = 0.99$ for the cooling parameter in (6). This value resulted in searches uncovering high-quality trade-off solutions because of sufficient exploration taking place during the initial epochs. Finally, the SA search was terminated if any one of the following three stopping criteria was met:

- once three successive epochs had elapsed without accepting a candidate solution,
- once the temperature had decreased below a user-specified parameter $\omega \ll 1$, or
- once a user-specified CPU time Ω had elapsed since the search initialisation.

4.4 Implementation validation

In order to validate the solution methodology, it was applied to a number of small, hypothetical problem instances and the quality of its non-dominated fronts were compared to that of the true Pareto front calculated by brute force. *Hypervolume* [52] was used as the performance metric for this purpose, performing computations according to the *hypervolume by slicing objectives* algorithm of While *et al.* [64].

The results obtained by solving both models of §3 is reported for one of these problem instances, namely for a 24×24 (equidistant) discretisation grid of firebase candidate locations, requiring the placement of three firebases. This resulted in $\binom{24^2}{3} = 31\,684\,800$ possible placement combinations. Functions representing the fixed costs of placing firebases and $|\mathcal{K}| = 1$ coverage importance criterion rating were generated stochastically. These functions are shown in Figure 1.

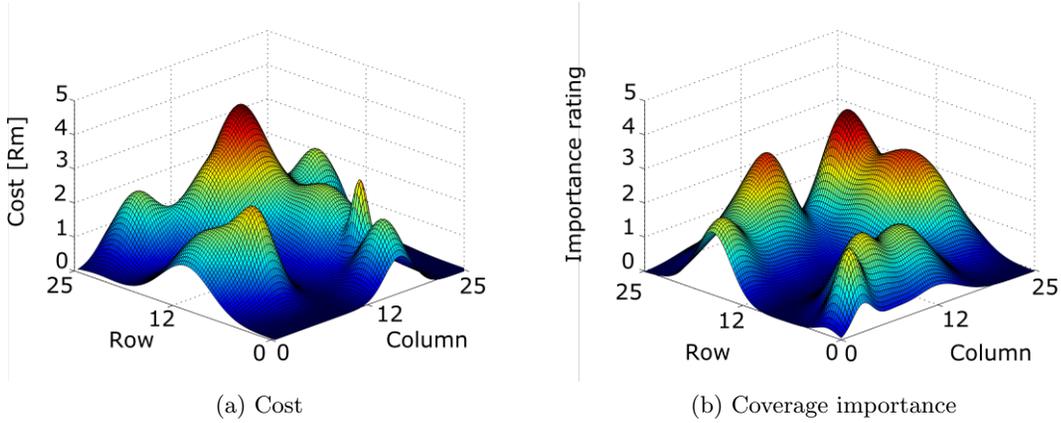


Figure 1: Placement criteria for the test problem instance.

The models of §3 were solved for the above-mentioned problem instance by the method of SA, as described above. Two types of search runs were performed in Matlab [41] — a short run lasting at most $\Omega = 1\,000$ seconds and a long run lasting at most $\Omega = 15\,000$ seconds on an Intel(R) Core(TM) i3-4030U CPU within a 64-bit Windows operating system. The hypervolumes achieved in objective space by the SA algorithm are shown in Table 1, normalised in each case according to the hypervolume of the true Pareto front (obtained by brute force, as mentioned). In all cases, the hypervolume reference point was taken as the nadir point [16].

Model	Run type	Hyper-volume	Search iterations
Double-counting	Short	93.53%	38 728
Double-counting	Long	96.44%	928 285
Single-counting	Short	98.37%	44 458
Single-counting	Long	99.65%	1 066 995

Table 1: SA hypervolume comparison for a small, hypothetical instance of the firebase location models of §3.

As may be seen in Table 1, the solution quality increases as the search is allowed to progress for longer, as expected. The long search runs are indeed capable of uncovering very high-quality nondominated fronts in objective space — in fact, finding large proportions of the true Pareto fronts. It would seem from the results of Table 1 that the double-counting model returns slightly lower-quality solutions than the single-counting model, but this is merely an artefact of the crude method of normalisation described in §3.1 (*i.e.* the choice of the normalisation constants γ_k in (2)).

5 Decision support system

The firebase location models of §3, as well as the (approximate) model solution methodology of §4, were embedded in the design of a DSS, called *Firestore Location Decision*

Support System (FLDSS). The design of this DSS is generic so that it can be applied to a wide range of application instances. In other words, FLDSS is not case-specific, but can approximate the Pareto-optimal location of a set of firebases of user-specified cardinality in various conservation area applications according to a set of user-specified importance criteria. The design of the DSS is presented in §5.1, after which a computerised concept demonstrator of FLDSS is described in §5.2.

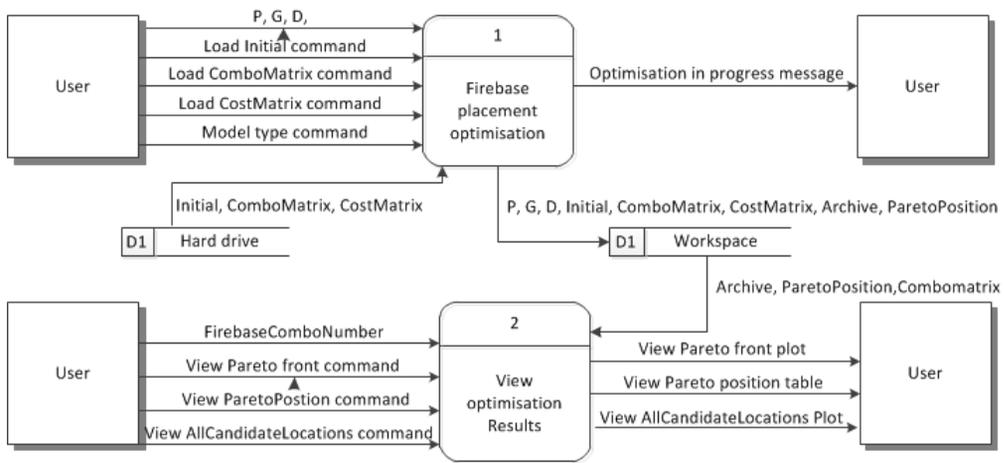
5.1 Design of the system

The primary design requirement for the FLDSS user interface was to enable the user to enter the required input data easily, and also so that the output results are easily accessible. This design is presented according to a top-down approach to diagramming data movement using *data flow diagrams* (DFDs), as described by Kendall and Kendall [34], and adopting the notation in Table 2.

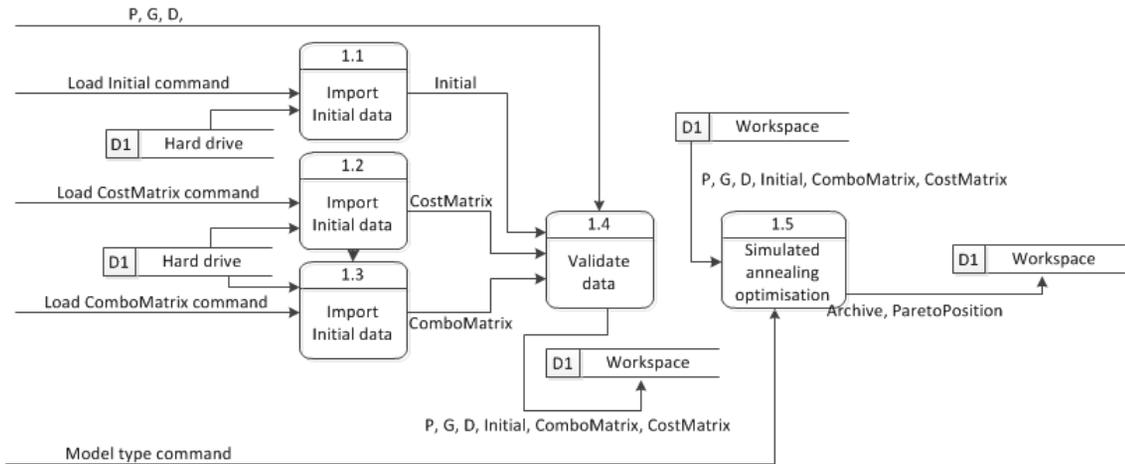
Symbol	Meaning
P	The number of firebases to be located
G	The number of times the method of SA should be re-initiated
D	The coverage capacity of a firebase, measured in units of discretisation grid cells (also called <i>matrix units</i>) as half the side length of a square with the firebase at its centre
Initial	The solution with which the SA algorithm is initialised
MatrixCombo	The matrices specifying the importance of covering each demand point with respect to the criteria in the set \mathcal{K} of §3.1 as well as the matrix that provides all the candidate firebase locations
CostMatrix	The matrix of location costs of firebases at each candidate location
Archive	The nondominated set of solutions uncovered by the SA algorithm
ParetoPosition	The positions in decision space of the firebases corresponding to solutions in the Pareto approximation set
FirestoreComboNumber	The collection of firebase location sets which form part of the Pareto approximation set, and for which the user would like to inquire specific positions within the conservation area
AllCandidateLocations	A matrix containing all the firebase candidate locations of solutions in the Pareto approximation set

Table 2: Notation used in the DFDs.

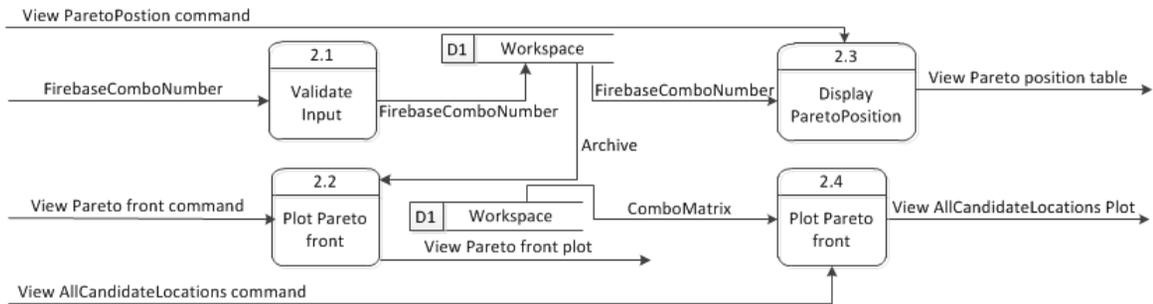
The general, high-level working of FLDSS is illustrated in DFD 0, shown in Figure 2(a), elucidating the user inputs required and the outputs displayed to the user. After receiving the number of firebases to be located, an upper bound on the number of SA search runs, the coverage capability of each firebase, a solution with which the first SA search run should be initialised, matrices of importance weightings for each of the criteria in \mathcal{K} , and a matrix of firebase location costs for all candidate sites as input from the user, the process of *Firestore placement optimisation* can be initiated according to the user's choice between the single- or double-counting model of §3. This process is labeled 1 in Figure 2(a), and the child diagram of this process is shown in Figure 2(b). Once this process has been completed, the user is informed and the output of this process (an archive of nondominated firebase location sets) is stored to the workspace. The process according to which the user may access the output of Process 1 is encapsulated in the *View optimisation results* process.



(a) Diagram 0 of the DSS showing all user inputs and outputs



(b) The child diagram of Process 1 in part (a)



(c) The child diagram of Process 2 in part (a)

Figure 2: DFD description of the design of FLDSS.

This latter process is labelled 2 in Figure 2(a), and its child diagram is shown in Figure 2(c).

5.2 DSS concept demonstrator

A screen shot of the user interface of a computerised concept demonstrator of FLDSS is shown in Figure 3. The number of firebases to be placed, the coverage capability of a firebase, and the number of times the SA algorithm should be re-initiated during its search may be entered into the text boxes labelled accordingly in the top left-hand corner of the screen. The candidate firebase locations may be specified by selecting the *Geographic matrix* button, while the cost of locating a firebase at each candidate location may be specified by selecting the *Cost Objective* button. The importance rating of covering each demand point with respect to the various objectives, may be specified by selecting the *Objective1*, *Objective2*, and *Objective3* buttons. Finally, the initial solution, may be specified by selecting the *Initial* button, upon which a file selection dialog will open. In this dialog, the user may select the appropriate Matlab type file containing the data pertaining to a location problem instance in the form of a matrix created beforehand in Matlab. Once the appropriate data have been entered and selected, the SA algorithm may be initialised by selecting either the *Single-counting model* (of §3.2) button or the *Double-counting model* (of §3.1) button.

Once the SA algorithm has completed its search in the context of the firebase location model selected, a message is displayed on the screen, guiding the user through the procedure to be followed so that the results can be accessed. When the user selects the *Plot Pareto front* button, a surface plot of the attainment front found over all the searches is plotted on the set of axes contained in the user interface. Upon selecting the *View Pareto front values* button, the search results with respect to the various objective functions are tabulated in numerical form at the bottom of the user interface. Furthermore, if the user wishes to view the actual positions in decision space corresponding to a specific set of firebase locations on the attainment front, the user is prompted to enter the firebase combination number of the location set of interest into the small text box at the bottom right of the screen and select the *Geo Location* button. Finally, if the user requires a summary of all the candidate firebase locations, along with their relative labels, her or she may select the *Position* button.

6 A case study: Table Mountain National Park

In this section, the DSS of §5 is validated by applying its concept demonstrator to a special case study involving *Table Mountain National Park* (TMNP). In §6.1, a brief introduction to the case study is given, while the input data required by the DSS in the context of TMNP are described in §6.2. Section 6.3 finally contains a summary of the results obtained by the DSS in the form of firebase locations in the park suggested by the models of §3.

6.1 The park

TMNP is situated in the Cape Peninsula, and is surrounded on all sides by either the City of Cape Town or by the Atlantic Ocean [23]. The park comprises three disconnected

natural areas, a northern section, a central section and a southern section, as shown in Figure 4. Since the City of Cape Town and TMNP are popular tourist destinations, the safety of visitors to the park and that of people living and visiting areas in close proximity to the park are of the utmost importance.

Fynbos, the dominant indigenous vegetation of the park, constitutes approximately 90% of the park's vegetation and requires a specific fire-regime in order to maintain its diversity and ecosystem processes [56]. TMNP is, however, greatly invaded by alien shrubs and trees which are not only spread by fire, but also threaten the biodiversity of the area, increasing the biomass of the veld and hence the intensities of fires in the park. This seriously influences the recovery and survival of fynbos and other natural vegetation after fires in the area [23].

It is therefore important that effective fire management takes place, and that the limited resources available for fire-fighting in the park are managed in the best manner possible. The park management is responsible for the difficult task of balancing the need to combine fire with other measures of alien plant control, maintaining features that provide safe boundaries, maintaining post-fire mosaic fynbos age patterns and providing available manpower to control and contain fires so as to reduce the risk of damage to the urban fringe of the park. Therefore, the TMNP Fire Management Plan [56] has two main objectives: to ensure the conservation and continued survival of viable populations of all indigenous biota in the area, and to minimise the potential and actual damage caused to private property by fires [23]. Major factors exacerbating the problem of providing fire protection include poor planning and unplanned wildfires as this sets back alien plant control operations and is a severe risk to the urban park fringe.

There are currently three permanent and independent firebases in the park, located as indicated in Figures 4 and 5, while a fourth semimobile firebase, situated in the north of the park, is on standby for all areas of the park and plays an integrating role between these firebases [56]. These four firebases are under extreme pressure to provide rapid response using the least amount of resources, especially during the hot, dry summer months (December to March). Fire-management teams have to keep fire-fighting costs to a minimum, and the most important factor influencing the cost of fighting a fire is the time it takes for fire-fighters to respond to a fire; response being measured as the length of time that has elapsed between when a fire is reported and when the first line of fire-fighters arrive at the scene. The longer it takes for a fire-fighting team to reach a fire, the more out-of-control the fire becomes which, in turn, increases the cost of containing and extinguishing that fire. The locations of firebases in the park significantly affect the time it takes for fire-fighters to respond to a fire, and an effective placement of these firebases can therefore greatly reduce the costs of fire management.

6.2 Input data

During a meeting with TMNP's management it was determined that fire-fighters take the following aspects into account when predicting the threat rating of a wildfire:

- the number of wildfires that have previously started in that area (*i.e.* historical data pertaining to ignition points of wildfires in the park),

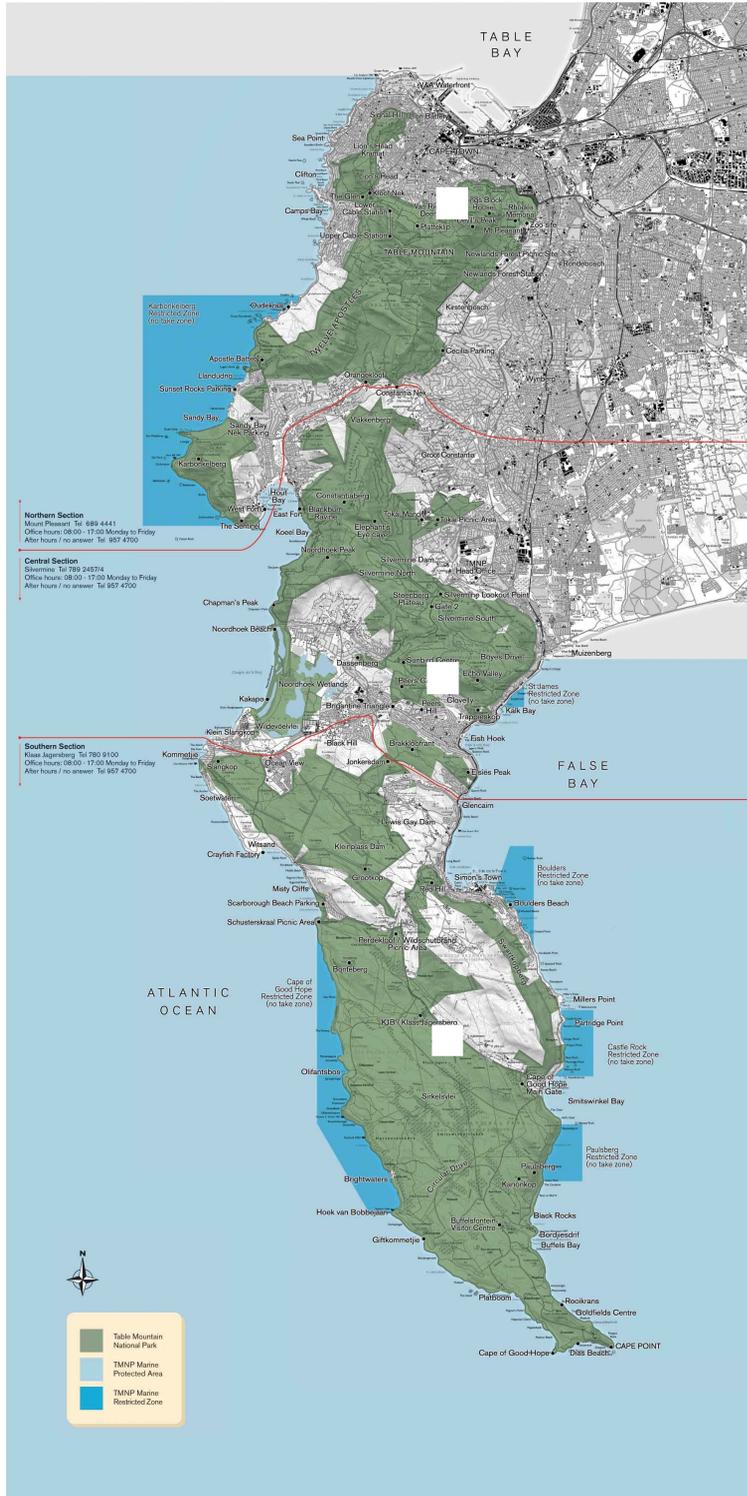


Figure 4: Map of TMNP in the Cape Peninsula.

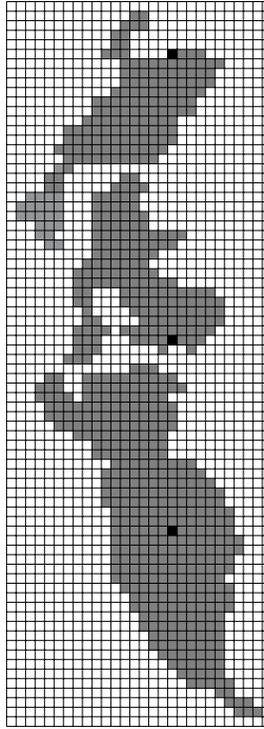


Figure 5: Discretisation of TMNP into 694 candidate firebase locations.

- the slope of the terrain in which the fire broke out (the steeper the slope, the more dangerous), and
- the wind speed in the area (the greater the wind speed, the larger the chance that the fire spreads out of control).

Despite an elaborate search by the authors, appropriate wind speed data at a resolution high enough to be meaningful could not be found for TMNP — there are only six wind speed stations in the Cape Peninsula (Signal Hill, Clifton, Muizenberg, Kommetjie, Witsands, and Cape Point) [9]. Wind speed data could therefore not be incorporated as one of the coverage criteria in the set \mathcal{K} of the coverage criteria in §3. Furthermore, in the model formulations of §3, the minimisation of the construction cost of firebases was one of the objectives. In the case of TMNP, however, the park is very well developed with respect to access roads, resulting in construction costs of firebases in various areas of the park being practically indistinguishable. Hence it was decided not to include fixed cost of firebase location explicitly in monetary terms as one of the objectives. The cost of placing firebases is rather simply taken to increase directly proportional to the number of firebases placed in the park (*i.e.* firebase location fixed costs are included implicitly by assuming them to be equal for all firebase candidate locations).

The slope of the terrain in the park was taken as the first coverage importance criterion in this case study. The greater the slope, the more important it is to cover that area by a firebase. This is because a fire spreads much quicker up a slope than along a flat plain. A contour plot of the terrain in the park may be seen in Figure 6(a). Slope data were derived from these data for TMNP using the software suite ArcGIS [3].

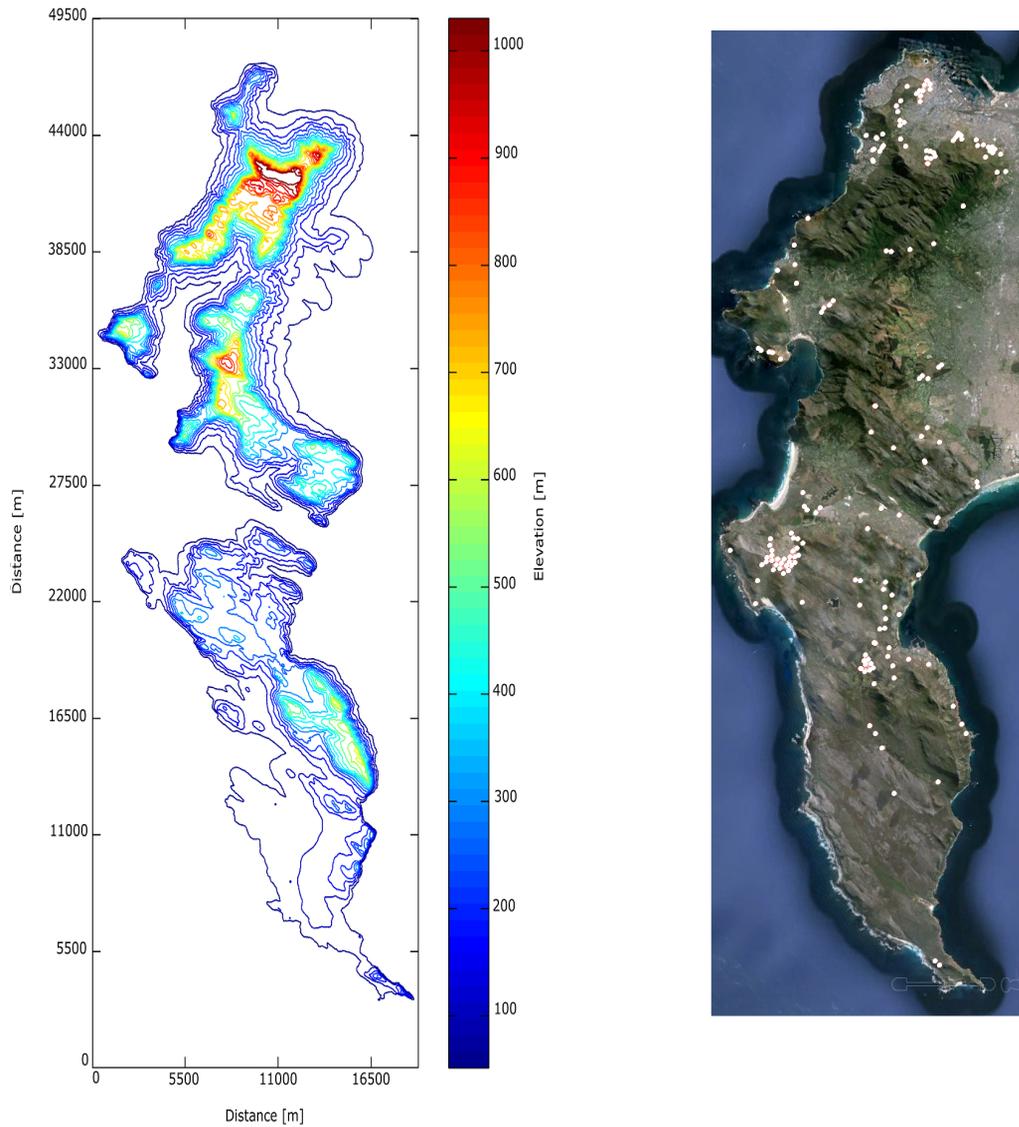


Figure 6: Firebase location criterion data for the TMNP case study.

The locations of historical wildfires in various areas within the park was taken as a second coverage importance criterion, in the sense that firebases should be placed so that they are close to areas in which many wildfires have ignited in the past. A plot of the wildfire ignition points over the past thirty years may be found in Figure 6(b).

Following Meyer *et al.* [43], the TMNP was discretised into a grid of square cells (each approximately $650 \text{ m} \times 650 \text{ m}$ in size), as shown in Figure 5. This resulted in 162, 161 and 371 candidate firebase locations in the northern, central and southern sections of the park, respectively. During a typical search for the location of three or four firebases in TMNP at this resolution, the SA algorithm would complete between 500 000 and 800 000 search iterations in total, requiring between 4 200 and 5 300 seconds of computation time in the hardware environment described in §4.4.

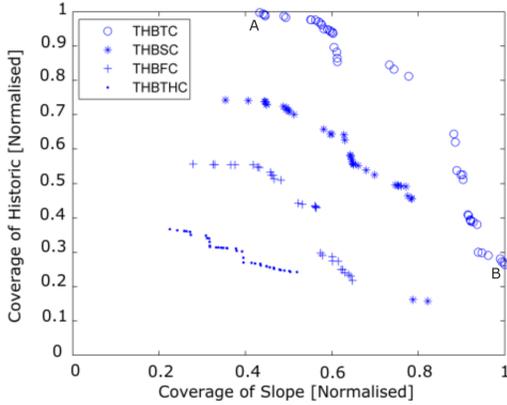
6.3 Numerical results

Using the DSS of §5, the models of §3 were solved in the context of the two TMNP covering importance criteria (of slope and proximity to historical wildfire ignition points) for four different firebase coverage capabilities, namely three matrix units, five matrix units, seven matrix units, and ten matrix units in each (horizontal and vertical) direction. It was thus confirmed that the solutions corresponding to a coverage capability of ten matrix units in each direction outperform solutions corresponding to smaller coverage capabilities. For each of the above-mentioned coverage distances, each of the models of §3 was solved in pursuit of the placement of three firebases, and then four firebases in TMNP. This gave rise to $2 \times 4 \times 2 = 16$ sets of placement results (two models, four coverage distances and two firebase set cardinalities). Each of the sixteen sets of results reported in this section is the attainment front of ten applications of the SA algorithm.

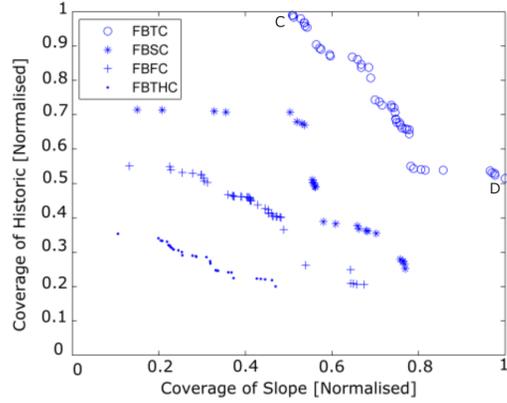
The following acronyms are used in this section for result reporting purposes in the contexts of both the single-counting and double-counting models proposed in §3: *three bases and three coverage capability* (THBTHC), *three bases and five coverage capability* (THBFC), *three bases and seven coverage capability* (THBSC), *three bases and ten coverage capability* (THBTC), *four bases and three coverage capability* (FBTHC), *four bases and five coverage capability* (FBFC), *four bases and seven coverage capability* (FBSC), and *four bases and ten coverage capability* (FBTC).

The attainment fronts uncovered by the simulated algorithm in the context of the single-counting model of §3.2 for the placement of three and four firebases are shown in Figures 7(a) and 7(b), respectively, while the attainment fronts of the double-counting model of §3.1 for the placement of three and four firebases are shown in Figures 7(c) and 7(d), respectively. The extremal solutions (*i.e.* top performing solutions with respect to each of the objectives individually) have been labelled A–H in these figures. The locations of the firebases corresponding to these extremal solutions are shown (in decision space) in Figure 8(a) for the single-counting model and in Figure 8(b) for the double-counting model.

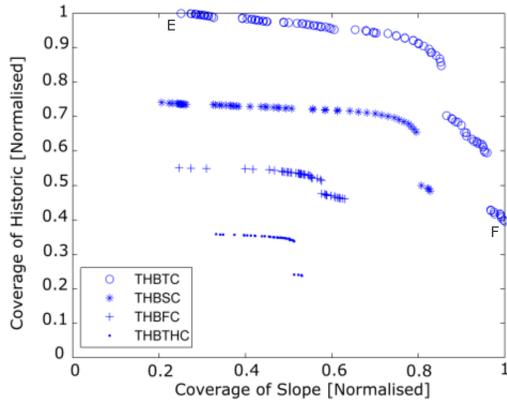
Comparing the results in Figure 8, it is clear that there is a substantial difference in the locations of solutions to the single-counting and double-counting models. As expected, the double-counting model tends to recommend placement of multiple firebases within certain



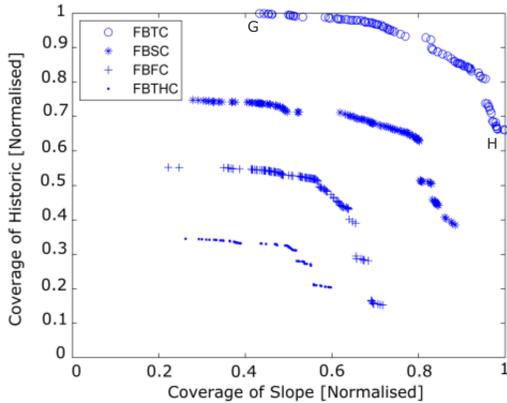
(a) Three firebases, single-counting model



(b) Four firebases, single-counting model



(c) Three firebases, double-counting model



(d) Four firebases, double-counting model

Figure 7: Attainment fronts uncovered by the SA algorithm when placing three or four firebases in TMNP.

key areas, whereas the single-counting model tends to favour placements of firebases that are spread out over various *different* important locations throughout the park. In this respect, the single-counting model seems to suggest solutions that are more practical than the double-counting model.

Comparing the results in Figure 7, it would furthermore appear that the SA algorithm consistently finds higher quality solutions for the double-counting model than for the single-counting model. This is due to the greedy way in which the performance of solutions has been normalised (as described in §3). If three firebases are, for example, to be placed, the three top-performing firebase locations are simply selected independently from one another and the performances of solutions in the attainment front are then normalised by the sum of the performance values of these three firebases. This method of normalisation therefore ignores the possibility that the coverage areas of firebases might overlap. The double-counting model also ignores the fact that firebase coverage areas may overlap, and thus in theory should be able to find better performing firebase locations

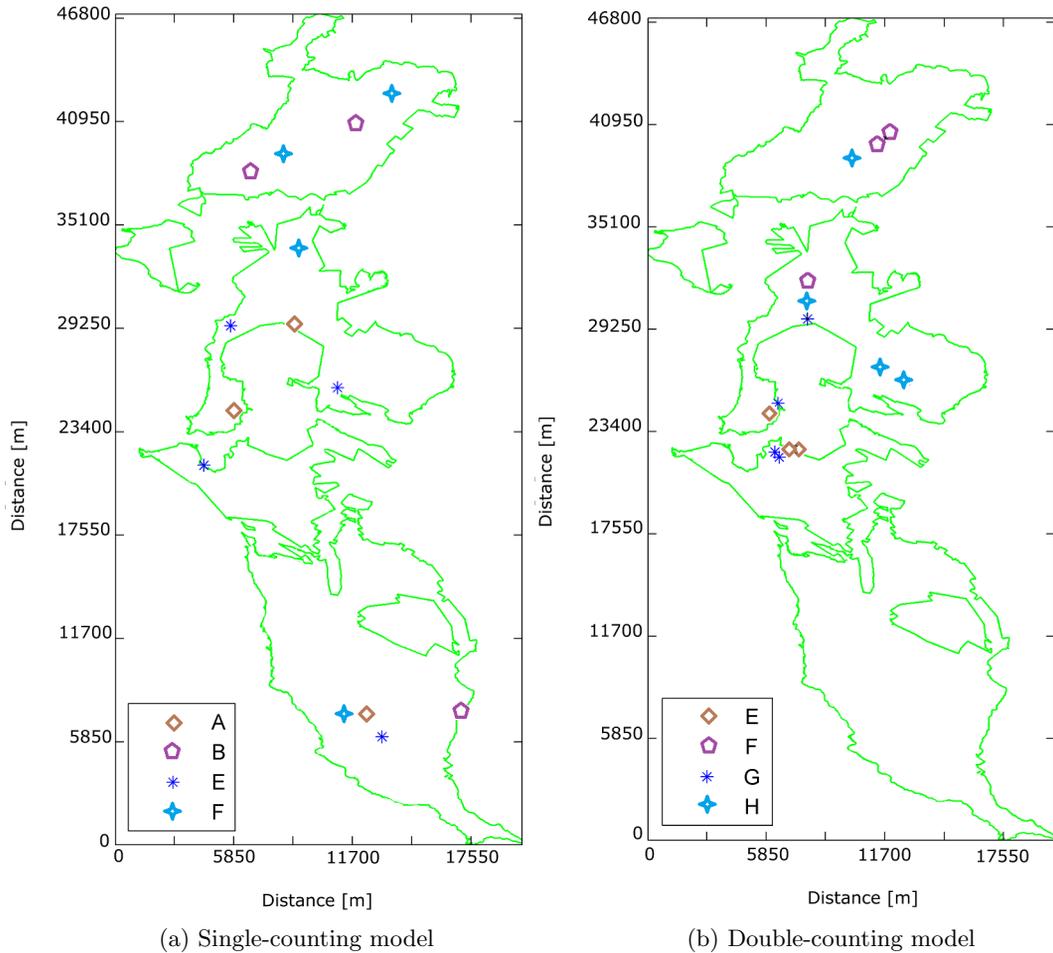


Figure 8: Solutions A–H of Figure 7 in decision space.

when adopting this method of normalisation. The single-counting model, on the other hand, takes overlapping firebase coverage areas into account and decreases the performance of the corresponding solutions accordingly. It is, however, usually the case that the top-performing firebase locations do overlap. It is therefore hard for the SA algorithm to find these top-performing firebases in the context of the single-counting model.

The performance of the existing firebases in TMNP are finally compared with those of the solutions suggested by the DSS, as reported in Figure 8. As indicated in Figures 4 and 5, there are currently three permanent, independent firebases in the park: one at Newlands, a second at Silvermine and a third at Klaasjagersberg. There is, however, also a fourth semi-mobile firebase at Kloofnek which acts as support to any area in the park.

The performance of the three permanent firebases, as well as that of all four firebases together, is compared with the performances of the DSS-suggested firebase locations in Figure 9 for a coverage capability of ten matrix units. As may be seen in the figure, the solutions suggested by the DSS in the contexts of both the single-counting and double-counting models outperform the current placement of firebases by a significant margin.

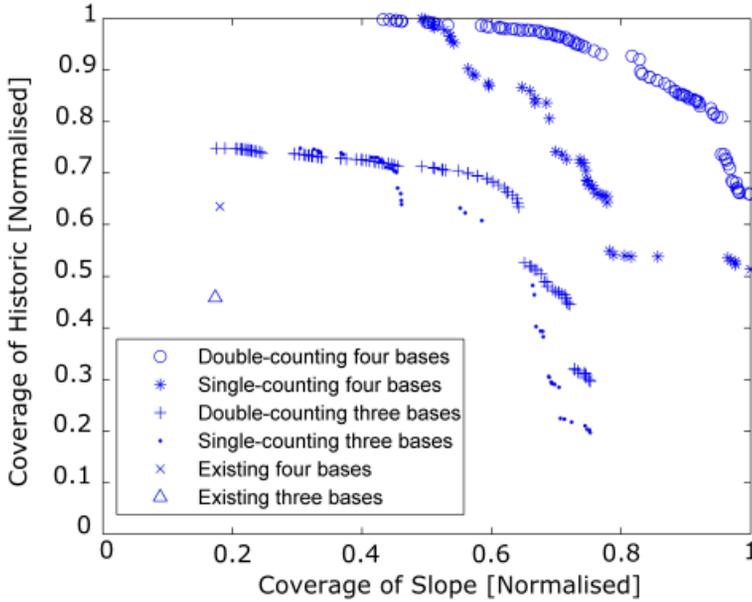


Figure 9: The performance of the current TMNP firebase locations compared to those of the DSS suggestions.

It may, however, be argued that the current firebase locations were not necessarily selected in order to maximise the coverage of key areas in the park, but that a balance between covering key areas and making it easy for personnel to reach the firebases from the city might instead have been pursued to some extent. This would explain why most of the current firebases are located near the park’s boundary.

7 Conclusion

Two novel firebase location models (a single-counting model and a double-counting model) were formulated in this paper with a view to facilitate a comparison of trade-off performances of different sets of firebase placement locations. The implementation of these models within a user-friendly computerised DSS render them capable of being applied by non-mathematically inclined decision makers to a variety of scenarios by receiving, for a specific scenario, certain geographic information (such as terrain elevation data, ground slope data and wind data), cost data, and other information. The DSS is flexible in terms of a number of user-specifiable parameters, including the coverage capability of a firebase, the number of firebases that should be placed, and the number of times that the SA algorithm should be re-initialised during its approximate solution search process. Other multi-objective firebase location models involving GIS-related location importance criteria (other than travel distance or time) in a nature conservation context, were not found in the literature. All firebase location models in the literature are capable of accommodating only a single objective function, albeit sometimes as the aggregation into one function (by linear combination, for example) of various placement criteria, or are applicable in urban

settings, as described in §2. The models in this paper are, to the best of the authors' knowledge, the first truly multi-objective mathematical treatments of the problem of nature conservation firebase location in the sense of encouraging consideration of trade-off solutions along a Pareto front. The models certainly represent a considerable improvement on the previous firebase placement models utilised within the context of TMNP [42, 43], as demonstrated in Figure 9.

The coverage capability of a firebase was modelled in this paper as a square surface area with the firebase at its centre. Modelling the coverage response area of a firebase in this manner is perhaps not realistic. Another option would be to model the coverage area of a firebase as a truncated circle or oval (when viewed from above) so as to account for certain areas that may be easier to cover as a result of terrain slope considerations, for example. Alternatively, the coverage capability of a firebase may be modelled in a time-dependent manner instead of adopting a distance-dependent modelling approach, taking into account terrain features such as contours and access roads.

Finally, the location models formulated in this paper ignore the fact that there might be existing, immovable firebases in the conservation area. The only way in which the current model can accommodate existing firebases is if the user ensures that the area represented by the current firebases' locations are rated as extremely important to cover. In this case the model will then automatically suggest placement of firebases in these locations — thereby, in essence, reproducing the existing firebases in solutions. The models put forward in this paper may, however, be extended by including a capability of both accommodating *existing* firebases in an explicit manner, and by allowing the user to limit the areas in which *new* firebases can be placed.

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