THE CONCURRENT DEVELOPMENT OF MATHEMATICAL MODELLING AND ENGINEERING TECHNICIAN COMPETENCIES OF FIRST-YEAR ENGINEERING TECHNICIAN STUDENTS

BY

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Abstract

Mathematics contributes significantly towards engineering education, denoting the prominance of possessing mathematical competence. Motivation for the study originated from observing students' modest levels of mathematical reasoning and understanding, problem-solving and meta-cognitive abilities. A gap in literature was exposed for enhancing engineering technician students' competencies to proceed towards successful mathematical thinkers and doers. This study serves to fill this gap, by answering the research question regarding the extent to which engineering technician and mathematical modelling competencies can co-develop, to produce a deeper understanding of mathematics within the context of a mathematical modelling course for first-year engineering technician students who are not strong in mathematics.

The study aimed to develop a qualitative and quantitative profile that characterises the design in practice, commanding Design-Based Research methodology. Twelve first-year engineering technician students, volunteered to partake in a mathematical modelling course of one semester. They worked in small groups on model-eliciting activities that required the construction of models to describe, analyse and solve real-world problems. Qualitative data sources included video and audio recordings, observation instruments, informal discussions, students' written work, and field notes. Analysis was done throughout the experiment.

The students revealed improvements in all the competency categories, with the most prominent development occurring in generalising (cognitive) and management (meta-cognitive) competencies. Mathematical ideas and higher-order thinking develop interactively, and the characteristics of being deeply involved in solving model-eliciting activities allowed for the stimulation of reflective activities.

Explanations on how the competencies advanced, exposed an intricate web of teacher beliefs, classrooms norms that foster socio-constructivist forms of learning and teaching, and model-eliciting activities designed to develop higher-order understanding. Combined with formative assessment methods to describe the nature of the students' constructs, a local instructional theory was constructed that explains how mathematical modelling and engineering technician competencies can co-develop through mathematical modelling, and how to support competence development for improved mathematical reasoning and understanding.

Keywords

mathematical modelling, model-eliciting activities, competencies, mathematical modelling competencies, engineering technician competencies, Design-Based Research, mathematical reasoning and understanding.

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Abstrak

Wiskunde dra beduidend by tot ingenieursopleiding, wat die belangrikheid van wiskundige bevoegdhede impliseer. Motivering vir die studie spruit uit die waarneming van studente se beskeie vaardighede van wiskundige beredenering en begrip, probleemoplossing en meta-kognitiewe vermoëns. 'n Gaping is blootgelê in die literatuur om suksesvol ingenieurstegnici se vaardighede te ontwikkel tot suksesvolle wiskundige denkers en gebruikers. Die studie dien om hierdie leemte te vul deur die navorsingsvraag te beantwoord oor die mate waarin ingenieurstegniese en wiskundige modelleringsbevoegdhede gelyktydig kan ontwikkel om 'n dieper begrip van wiskunde te lewer, binne die konteks van 'n wiskundige modelleringskursus vir eerstejaar ingenieurstegnikus studente wat nie vaardig in wiskunde is nie.

Die studie poog om 'n kwalitatiewe en kwantitatiewe profiel te ontwikkel wat die ontwerp in die praktyk op die voorgrond plaas en dus ontwerp-gebaseerde navorsing onderskryf. Twaalf eerstejaar ingenieurstegniese studente wat 'n oorbruggingsprogram gevolg het, het vrywillig aan 'n wiskundige modelleringskursus van een semester deelgeneem. Hulle het in groepies aan modelleringsaktiwiteite gewerk wat model-konstruksie vereis het om werklike probleme te beskryf, ontleed en op te los. Kwalitatiewe databronne sluit in video- en klankopnames, waarneming- en refleksie-instrumente, informele besprekings, studente se geskrewe werk en veldnotas. Voortdurende analise is verder ontwikkel deur Gevallestudie Navorsing.

Die studente het verbeterings in al die bevoegdheidskategorieë gewys, met die prominentste ontwikkeling in veralgemenings- (kognitief) en bestuurs- (meta-kognitief) bevoegdhede. Wiskundige idees en hoër-orde denke is deur dinamiese interaktiwiteit ontwikkel, en die eienskappe van modelleringsontlokkende aktiwiteite (MOAe) stimuleer reflektiewe aksies.

Besprekings oor hoe die bevoegdhede ontwikkel het, het die komplekse web van onderwyser-oortuigings, klaskamer-norme wat sosio-konstruktivisme ondersteun, en MOAe wat ontwerp is om hoër-orde begrippe te ontwikkel, blootgelê. Gekombineer met formatiewe assesserings-metodes, is 'n Lokale Onderrigteorie (LOT) opgestel wat verduidelik hoe ingenieurstegniese en wiskundige modelleringsbevoegdhede kan ontwikkel via wiskundige modellering, en ook hoe om ondersteuning te bied vir beter wiskundige beredenering en begrip.

Sleutelwoorde

wiskundige modellering, bevoegdhede, wiskundige modelleringsbevoegdhede, ingenieurstegniese bevoegdhede, ontwerp-gebaseerde navorsing, wiskundige beredenering en begrip, lokale onderrigteorie

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CHAPTER 1
OVERVIEW, BACKGROUND AND MOTIVATION

The man ignorant of mathematics will be increasingly limited in his grasp of the main forces of civilisation ~ John Kemeny (1926 – 1992)

1.1 OVERVIEW

This chapter provides an overview of the purpose and focus of this study. It serves to direct the reader to the prominence of developing engineering technician students' mathematical modelling as well as their engineering competencies through mathematical modelling. The importance for engineering students to develop an understanding of mathematics as well as the current gap in mathematics and engineering education will be discussed. An explanation as well as the motivation of the Realistic Mathematics Education (RME) theory that supports mathematical modelling will be provided. The purpose, aims and methodology will also be outlined, as well as the various methods of data collection and analysis that will be used.

1.2 BACKGROUND

Wealth creation is of critical importance for South Africa locally as well as globally. The New Partnership for Africa's Development (NEPAD), which is the technical arm of the African Union, define their vision as eradicating poverty and placing their countries, individually and collectively, on a path of sustainable growth and development whilst actively participating in the world economy (NEPAD, 2016). South Africa's role in NEPAD is one of leadership in the continent's recovery, but to play this role effectively, the country needs a sound and growing economy combined with a first world economic infrastructure to support this growth. Both social as well as economic infrastructure development rely heavily on the engineering profession, and the competence of South Africa's engineering professionals must therefore be ensured. Lawless' (2005:8) statement that mathematics contributes significantly towards engineering education, complements the introductory quote above by John Kemeny. Kemeny also emphasised the importance not to view mathematics as a distinctive subject matter, but as an indispensable tool to improve our understanding of the world around us (Kemeny, 1959:577; Knudsen, 1960:17). The significance of mathematics granted by the engineering profession, thus compelled Lawless to accentuate the prominence of possessing *mathematical competence* for engineering professionals [and technicians – LdV] (Lawless, 2005:8). Professional engineers, technologists and technicians constantly need to evaluate, analyse, interpret and solve real-world problems (Biembengut & Hein, 2007:422).

One of the goals of mathematics education is the promotion of *efficient mathematical thinkers* (Hoyles & Noss, 2007:79). Resnick in Schoenfeld (1992:360) describes a good mathematical thinker as one that not only acquires a specific set of skills, strategies or knowledge, but that also obtains the habits of interpretation and sense-making. Furthermore, mathematical thinkers should also value the importance of representational fluency, as it is "at the heart of what it means to understand most mathematical constructs" (Lesh, 2000:180). The attainment of such habits is acquired through a socialisation process rather than an instructional process, and mathematics teaching and learning should therefore take place in a social context. Being a member of a community, collaborating and communicating with others, as well as knowing how to use resources, is part of what constitutes mathematical thinking and knowing (Schoenfeld, 1992:341-4).

The synergy between the goals of mathematics and engineering education in terms of the importance granted to problem-solving and mathematical understanding competencies, led the researcher to investigate the current situation in both disciplines. This investigation revealed crucial mismatches and prompted the motivation for this research study.

1.3 MOTIVATION FOR THE STUDY

On investigating the current situation in both mathematics and engineering education, the following mismatches emerged:

1.3.1 Poor understanding of Mathematics

Mathematics education does not yield the mathematical thinkers as is required. Lawless (2005:82), of the South African Institute of Civil Engineers (SAICE): Professional Development and Projects, noted that many undergraduate students reveal a poor understanding of mathematics in the classroom and they have not grasped the basic mathematical principles to continue with mathematical studies beyond school level. In the mathematics education literature, mathematical knowledge often focuses on the content aspects (Chick & Stacey, 2013:122). To come to terms with the content, students tend to revert to rote knowledge and less time is devoted to procedural and conceptual knowledge (Eisenhart et al., 1993:10). This narrow orientation is what Skemp (1976:22) describes as *instrumental understanding*: the majority of students have learnt how to do numerical computations at the expense of relational understanding. However, Mathematics is regarded as the most abstract and powerful of all theoretical systems, and cannot be understood by utilising only our short-term memory. Short-term memory can only store a limited number of words or symbols. Students have been taught to manipulate symbols with little meaning attached according to many rote-memorised rules. These unconnected rules are meaningless and far more difficult to remember than an integrated conceptual structure (Skemp, 1987:17-18). *Relational understanding*, in contrast to instrumental understanding, focuses on a greater cognitive connectivity of the mathematical knowledge – it involves knowing the 'what' as well as the 'why'. Sierpinska (1990:35) distinguishes between these two types of understanding in terms of '*depth of understanding'*. The depth of understanding is directly related to an increase in complexity and richness of knowledge, asking for a more holistic picture of mathematics education and it develops over a period of time. However, Anderson in Crouch and Haines (2004:198) found that even final year mathematics undergraduates display a lack in relational understanding and tend to revert to memorising when solving test questions, rather than retaining and building upon a strong and coherent structure in mathematics. These students experience a gap in terms of knowledge and abilities to construct viable mathematical solutions from real-world problems. Singh and White (2006:51) emphasise this mismatch between the mathematical learning in high schools and the competencies needed at university as well as in the professional workplace.

Lawless (2005)) believes that the standard of education, drop-out rates at tertiary institutions and 'fast tracking' are major influences why students do not develop the necessary competencies to foster a deeper understanding of mathematics. Publicised statistics of the South African Institute of Race Relations (SAIRR, 2016) show that in 2014 only 263 903 (41%) of the 644 536 NSC candidates were enrolled for Mathematics in 2015, with only 7% of them passing Mathematics with a mark of 70% of more.

1.3.2 Inadequate problem-solving abilities

Clarke in Lawless (2005:82) noted that this poor knowledge base restricts the students from being able to solve real-world mathematical problems, which is a primary task of any engineer. Being poor problem-solvers, they lack critical and reflective thinking abilities to construct practical mathematical solutions from real-world problems, and this gap continues to widen. To further accentuate this view, Singh and White's (2006:48-49) quantitative and qualitative study regarding engineering students' thinking and reasoning capabilities in solving non-routine problems with mathematics, revealed the following mismatches:

- Students' inabilities to unpack subject knowledge in mathematics contribute to high drop-out rates of university students.
- Students do numerical computations as a procedure and they have not been taught to think and solve problems. The goal for most of the students are to find an algorithm to produce an instant answer. They can carry out a procedure when presented in symbolic form, but struggle with solving problems presented in words.
- Students' main difficulty lies in understanding the problem rather than executing the procedures. Developing instrumental understanding is prioritised above relational understanding.
- After doing meaningless computations, students often do not know what is represented by the numbers they obtained.

1.3.3 Unsatisfactory meta-cognitive abilities

Furthermore, Woollacott (2003) stresses a concern for the South African engineering students that are under-prepared and not successful in achieving their qualifications. Apart from an inadequate knowledge base, gaps also emerged in terms of meta-cognitive competencies, such as team-working, decision-making and effective communication, even though such skills are regarded as essential building blocks to become successful students, as well as professional engineering technicians (Marra, Steege, Tsai, & Tang, 2016).

1.4 DEFINING THE GAP

The motivation for the study as detailed in Section 1.3, explicated a gap in literature to fruitfully enhance engineering technician students' competencies to proceed towards successful mathematical problem-solvers, mathematical thinkers and mathematical doers. Literature (Blomhøj & Jensen, 2007; ECSA, 2014; Hoyles & Noss, 2007; Kaiser, 2007; Kilpatrick, Swafford, & Findell, 2001; MaaB, 2007; Niss & Højgaard, 2011; Passow, 2012; Rugarcia, Felder, Woods, & Stice, 2000; Rychen & Salganik, 2003; Woollacott, 2003, 2007) addresses many important engineering technician and mathematical competencies that can assist students to develop mathematical understanding and problem-solving abilities (Sections 3.5 and 3.6). However, no in-depth studies were found that identified and investigated the engineering technician competencies that can co-develop with mathematical modelling competencies through modelling-based mathematics teaching and learning, and this study will thus attempt to fill this gap in knowledge in the field of mathematics and engineering education. By *co-development*, the researcher refers to the *simultaneous development* of engineering technician and mathematical modelling competencies, while the focus remains on mathematics teaching and learning. Therefore, crucial engineering technician competencies will be investigated and identified and the development of these competencies will be followed at the same time as the mathematical modelling competencies.

1.5 ADDRESSING THE GAP

Galbraith (2007:60) believes that mathematical modelling has the potential to address the gap between applying mathematics in the real-world and addressing mathematical concerns in the classroom, without preparing the students narrowly for an agenda dictated by the workplace. Crouch and Haines' (2007:91) study indicates that mathematical modelling allows for opportunities to link knowledge acquired from one domain to another due to students' development towards stronger engagements and motivation. Mathematical modelling is a tool to facilitate conditions for learning how to formulate, solve and make decisions regarding engineering problems (Biembengut & Hein, 2007:422). Students who are engaged in mathematical modelling tasks, learn to make connections between real-world problems and mathematics. During this process, students learn to develop 'mathematical thought' competencies to abstract critical information, to mathematise, interpret, verify and communicate solutions to others. Students learn to shape the messy problems into tractable ones, figure out what data they need and define problems in their own way which can ultimately lead to new and creative models, unlike a traditional course where students work through variations of the same problems solved by others. This push towards self-sufficiency may cause students to feel initially hopeless and scared, but as students grapple with the problems and gain momentum, the sense of achievement outshines their efforts, and students become grateful for the experience (Garfunkel & Montgomery, 2016:80). These competencies are all critical for the Engineering profession as well. The tools they learn now can be applied to the many serious problems that they will face in the real-world (Parmjit & White, 2006:36). Mathematical modelling therefore can eliminate the problem of rote learning, and relational understanding becomes the focus in mathematical teaching and learning. Kaiser identifies one of the goals of mathematics education as "the development of students' capacities to use mathematics in their present life as well as in their future lives, which calls for the importance of stimulating modelling competencies" (Kaiser, 2007:110). A variety of competencies are needed to master mathematics, of which *mathematical competencies* and *mathematical modelling competencies* are central. Engineering students need to use mathematics: through mathematising, they get the opportunity to experience the interconnections of university mathematics with other relevant areas of mathematical application (Parmjit & White, 2006:34). These experiences accentuate Kaiser's urge to include real-world examples to solve real-world problems in mathematics education (Kaiser, 2013:1). When engineering students engage in mathematical modelling, they learn to understand and interpret various kinds of abstract structures (Hoyles & Noss, 2007:79), and ultimately progress towards *efficient mathematical thinkers*. The Science, Technology, Engineering and Mathematics (STEM) curriculum in the US follow a similar approach by blending the learning environment to show the students the application of scientific methods to everyday life, as it focuses on the real-world applications of problem-solving (White, 2014).

Wessels (2014:1-3) remarked that model-eliciting activities (MEAs) do not only offer students the opportunities to develop competencies and creativity, but being closely connected to real-world contexts, students learn to construct meaningful mathematics, rather than just being involved in the regurgitation of mathematical knowledge. Aligning with the Neo-Vygotskian approach (Zbiek

& Conner, 2006:90) to current mathematics teaching and learning, students bring their own unique sets of knowledge and experiences regarding mathematics and the real-world into the classroom. While being engaged in MEAs, students actively learn mathematics by continuously connecting and altering old and new pieces of knowledge, which leads to improved understanding. These thought-revealing activities necessitate the construction of models to describe, analyse and solve real-world problems, and multiple approaches are used to investigate, explain, solve and justify their solutions. Real-world problems are solved in complex settings that are submerged in human preferences and social dynamics. By participating in solving MEAs, students can become better problem-solvers, while teachers acquire sensitivity to design situations that engage learners in productive mathematical thinking – one of the goals of mathematics education (Yildirim, Shuman, Besterfield-Sacre, & Yildirim, 2010:831).

Diefes-Dux, Moore, Zawojewski, Imbrie, and Follman (2004:F1A-3) advised that, from an engineering perspective, MEAs can assist undergraduate engineering students to develop higher-order understandings of problems that can lead to solutions where the emphasis is not only placed on the product as seen in traditional engineering education, but also on the process. This shift towards the problem-solving process indicates the main difference between practising engineering and educating future engineers. This holistic approach to teaching and learning mathematics asks for the consideration of a multi-disciplinary view to mathematics education, while still retaining fundamental rigor and discipline to provide as many opportunities as possible for the students to develop the necessary competencies.

This study thus hope to make a meaningful contribution to address the gap as defined in Section 1.4 by introducing a mathematical modelling course to the first-year engineering technician students. During this course, the focus will remain on improving the students' reasoning and understanding of mathematics, by developing specific competencies that are required from both mathematics and engineering technician professions. The data used in this document was generated during this modelling course. The process of investigating and selecting the competencies will be discussed in the following section.

1.6 INVESTIGATING COMPETENCIES

To align with the requests of the workplace, it is necessary to examine the competencies required from professional engineers to support and assist the students with successful completion of their studies. The Engineering Council of South Africa (ECSA) is a statutory body established in terms of the Engineering Profession Act (EPA), 46 of 2000 (ECSA, 2015). They are the only body in South Africa that is authorised to register engineering professionals and bestow engineering titles on persons who have met the mandatory professional registration criteria. ECSA's mission statement focuses on establishing a South African Engineering profession that can successfully fulfil the necessary roles for establishing socio-economic growth in the country. These obligations can only be met if the competencies of individuals are ensured. ECSA has identified crucial cognitive as well as meta-cognitive competencies and proficiencies required from professional engineering technicians which agree with many proficiencies required for mathematical modelling. By mapping the mathematical modelling competencies as identified in literature to the engineering technician competencies as suggested by national and international professional accrediting engineering bodies, competencies relevant to this study will be identified and investigated (Chapter 3).

1.7 THEORETICAL SUPPORT

The theoretical support in this study for mathematical modelling is rooted in socio-constructivism and Realistic Mathematics Education (RME). The socio-constructivist approach emphasises the need for understanding, while RME is designed to answer for the quest for educational change. The combination of these two approaches answers to the underlying philosophy of design-based research: "you have to understand the innovative forms of education that you might want to bring about to be able to produce them, or rather, if you want to change something, you have to understand it, and if you want to understand something, you have to change it" (Gravemeijer & Cobb, 2006:17).

1.7.1 Constructivism and socio-constructivism

Learning is culturally shaped and defined, hence people develop their understandings from participating in 'communities of practice' (Lave, 1993:201). Students should see the world through the lens of a mathematician, just like apprentices living amongst masters and picking up their values and perspectives and learning their skills. Even though these values are not part of the formal curriculum, they are central defining features of the environment. Classroom environments must therefore be designed to allow students to experience mathematics in similar ways that practitioners do. From the constructivist's perspective, learning is a process whereby learners actively construct their own understanding, and not passively copy the understanding of others (Parmjit & White, 2006:34). Constructivism therefore requires a learner-centred, problem-centred and collaborative learning and teaching approach (Knott, 2014:4). Aligning with the Neo-Vygotskian approach (Zbiek & Conner, 2006:90) to current mathematics teaching and learning, we find that each student brings his or her own unique set of knowledge and experiences about mathematics and the real-world into the classroom, which in turn influences his or her interpretation of the situation. A person who actively learns mathematics, makes continuous connections between old and new pieces of knowledge. Previous knowledge gets altered and leads to improved understanding.

This learner-centred, problem-centred and collaborative learning and teaching approach denotes mathematics education as a social activity, as students construct their knowledge more effectively when it is embedded in a social process (Zulkardi, 1999:9). Socio-constructivism is derived from social-constructivism, but only relates to mathematics education with similar characteristics as RME. However, the main difference between socio-constructivism and RME is that the teacher does not use heuristics in the socio-constructivist approach to solve problems or to investige ways to find solutions. The subjective meanings that students develop are formed through interaction with others, depicting an inductive process of developing a theory or pattern of meaning (Creswell & Poth, 2017:25). This inductive process contrasts deductive reasoning, as the latter refers to making conclusions based on previous known facts. By employing inductive reasoning, a conclusion is obtained based on a set of observations. To enhance interaction amongst the students, the researcher needs to design open-ended questions and to focus on the processess of interaction within specific contexts, to interpret the meanings others have about the world. A comprehensive explanation on socio-constructivism will be provided in Chapter 2.

1.7.2 Realistic Mathematics Education (RME) as theoretical support for mathematical modelling

The RME theory offers three design heuristics, namely *guided reinvention, didactical phenomenology* and *emergent modelling*. This domain-specific instructional theory for mathematics education is based on the ideas of Freudenthal (1981:7-8) that mathematics needs to be *connected to reality* and must be of *human value*. The use of realistic contexts that make sense to the students, became one of the determining characteristics of RME. Mathematics must be close to children and relevant to their everyday lives to be of human value (Zulkardi, 1999:3). The term 'realistic' refers more to the intention that students should be offered problem situations which they can imagine, rather than to the 'realness' or 'concreteness' of the problems; thus 'real' as in the students' minds (Van den Heuvel-Panhuizen, 2003:10). RME offers the platform for student understanding to be secure, while it continuously expand in their learning processes (Freudenthal, 2006), as it is rooted in contexts and mental images which allow students to take ownership of the mathematics (Dickinson & Hough, 2012:1). They regard their acquired knowledge as their own private knowledge for which they themselves are responsible, while being active participants in the teaching-learning process that takes place within the social context of the classroom (Larsen, 2013:2; Van den Heuvel-Panhuizen, 2003:11).

1.7.2.1 Guided Reinvention

The importance of mathematics having human value, stipulates a connection between mathematics and human activities, which can be explained by RME's heuristic of *guided reinvention*, the key principle of RME (Gravemeijer, 2004:114). Through the process of guided reinvention, students' current ways of reasoning can be developed into more sophisticated ways of mathematical reasoning (Gravemeijer, 2004:105). Freudenthal (2006:60) commented that the knowledge that students obtain through informal activities is better retained and more readily available than when it is imposed by others. These informal strategies can emerge in formal knowledge through guided reinvention**,** as students experience a similar process compared to the process by which mathematics was invented. Attainment of formal knowledge occurs while they actively take part in abstracting, schematising, formalising, algorithmatising, verbalising, etc. (Freudenthal, 2006:49,100). This study will therefore focus on applying carefully chosen sequences of examples which have the potential to elicit this growth in understanding as well as appropriate teacher interventions, depicting the students as active participants in the teacher-learning process (Freudenthal, 2006:85). The entire learning environment (classroom norms and culture) must allow for the students to regain higher levels of comprehension, while the guide should constantly provoke reflective thinking (Freudenthal, 2006:100). The diagram in Figure 1.1 illustrates a model of guided reinvention:

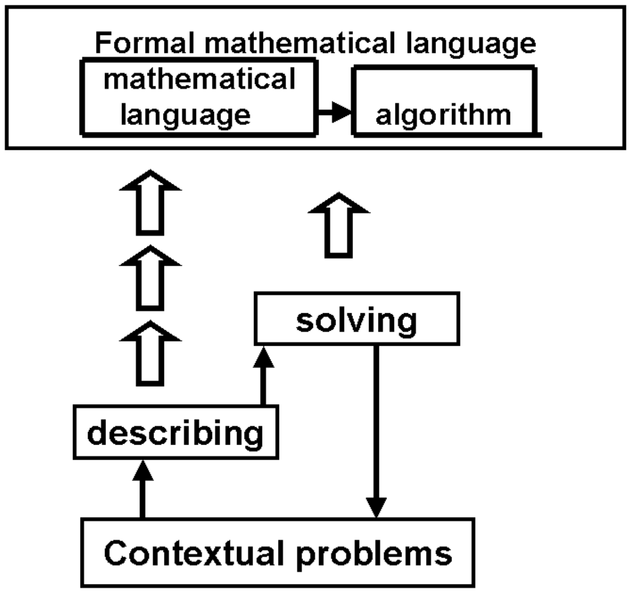


Figure 1.1- Guided Reinvention Model (Gravemeijer, 1994)

1.7.2.2 Mathematising

Closely connected to RME's guided reinvention heuristic, is mathematising. Freudenthal uses the term 'mathematising' to explain mathematics as an "activity of solving problems and looking for problems, and more generally, the activity of organising matter from reality or mathematical matter" (Van den Heuvel-Panhuizen, 2003:11). He proposes that mathematising should incorporate the "entire organising activity of the mathematician". Such an activity can comprise of mathematical content, mathematical expressions, even lived experiences expressed in everyday language (Freudenthal, 2006:31). Mathematising also includes the act of reflecting on one's own mathematical activities which may prompt a change of perspective. The changed perspectives can result in two actions: either to rethink or redo the process, or it may lead to axiomatising (Freudenthal, 2006:36). De Villiers (1986:8,15) explains axiomatising as the creation of new knowledge, as well as the reorganisation of existing mathematics and he emphasises the importance of mathematics teaching and learning to begin with questions and only end in axioms. As mathematising refers to an activity and not a body of mathematical knowledge, Freudenthal (2006) stresses that mathematics can best be learned by doing. He describes such an activity as being involved in meaningful training, which allows for the opportunity of prospective and retrospective learning – past and future learning processes must be integrated. A tight intertwinement of learning strands assists with the integration of the whole learning process. Through *reflection*, *group cooperation* and *interactive communication* with the guide and between the students, students can experience the various levels of mathematising while the entire process is situated within a rich context (Freudenthal, 2006:121). As such, mathematising is regarded as the core goal of mathematics education (Van den Heuvel-Panhuizen, 2003:11). From a pedagogical point of view, mathematising of real-world situations should not be demonstrated by the teacher, but it should rather be reinvented by the student while switching back and forth between realities – natural, social, and mathematical (Freudenthal, 2006:85).

- **Horizontal and vertical mathematising**

  In 1978, Treffers categorised mathematising in horizontal and vertical mathematising (Menon, 2013:3). *Horizontal mathematisation* refers to the movement between the real-world situation and the world of symbols, or rather, as Freudenthal (2006) explains, "going from the world of life into the world of symbols". As the learning process starts with contextual problems, students apply horizontal mathematisation to gain an informal or formal mathematical model (Zulkardi, 1999:4). Mathematical tools are selected and used to solve a problem situated in the real-world (Van den Heuvel-Panhuizen, 2003:12). The rework of a problem to evolve in a problem statement that can be solved with mathematics, involve horizontal mathematisation. When problems are introduced to students that are projected at a level too abstract to allow them to construct a meaning of the problem, it first has to be transformed through inductive reasoning before it can be solved (Menon, 2013:3).

  Once the student has achieved this form of representation, the representation can be used as a tool to work with new situations through activities such as generalisation and symbolisation. *Vertical mathematisation* refers to all kinds of reorganisations and operations which students do within the mathematical system itself – activities such as solving, comparing and discussing

(Van den Heuvel-Panhuizen, 2003:12). Freudenthal expressed vertical mathematising as "moving within the world of symbols", and he suggests that vertical mathematising is "most likely the part of the learning process where the bonds with reality can be loosened and eventually cut" (2006:68). Although the differences between horizontal and vertical mathematisation are not clear-cut, the worlds are not separate either (Van den Heuvel-Panhuizen, 2003:12). This fusion can be explained where the student selects and uses symbols and mathematical language to describe phenomena, and then engages in mathematical language, reasoning, and representations (Section 3.6.3). Once the student can interpret the solution and apply the model to another situation, mathematical understanding is gained.

Treffers (1987) in Freudenthal (2006:135-136) classified mathematics education into four categories with regards to horizontal and vertical mathematising, differentiating RME from other approaches to mathematics education:

- **Mechanistic (Traditional) approach:**

  Memorising of patterns or algorithms and drill-practice characterise this approach. Mathematical understanding does not take the central stand, and neither horizontal nor vertical mathematics is used.

- **Structuralist (New Math) approach:**

  This approach regards students as empty vessels (tabula rosa) where they reiterate the teachings of well-structured subject matter. Being able to replicate processes and procedures correctly determine their mathematical success, regardless of whether they have insight in the situation or not. Furthermore, as it starts in an 'ad hoc' created world with no connection to the students' world of life, it also obstructs the use of geniune mathematising.

- **Empiristic approach:**

  The approach focuses on real-world problems and the students are introduced to experiences which are useful to them. However, they do not get the opportunities to systemise and rationalise these experiences. Their familiar ways of doing are not challenged to expand their reality comfort zone – students are not required to come forward with a formula or a model and vertical mathematising is not exercised.

- **Realistic approach:**

  The starting point of learning mathematics is a real-world problem situated within a specific context. By using horizontal mathematising, the real-world situation is explored and understood. Students organise the problem and try to find the mathematical aspects to explain and solve the problem. By applying vertical mathematising, mathematical concepts are developed. Thus, the student is stimulated to 'reinvent' mathematics in a meaningful way.

1.7.2.3 Didactical and historical phenomenology

The second RME design heuristic, *didactical phenomenology*, is closely related to the guided reinvention principle, as it informs the educator about possible reinvention routes. Didactical phenomenology focuses on the relation between the mathematical '*thought thing'* and the '*phenomenon'* that it describes and explains (Gravemeijer, 2004:115). Freudenthal (1986:10) describes didactical phenomenology as a cognitive process that deals with a learning and teaching matter. Historically mathematics has evolved through practical problem-solving, which today drives the process of finding a variety of problem situations where, through generalising and formalising specific situated problems, formal mathematics (vertical mathematising) can come into being (Gravemeijer & Terwel, 2000). To be informed about possible reinvention routes, both historical as well as didactical phenomenology should be considered. *Historical phenomenology* strengthens didatical phenomenology, as students often encounter similar obstacles with which people have grappled in the past (Bakker, 2004:51).

As far back as 1971, Scandura (1971:23-24) commented about the significance of historical phenomonology:

> The major advantage man has over other animals is his ability to learn and communicate by verbal means. Man's knowledge has reached the fantastic point it has today for precisely that reason: The next generation does not need to discover for itself everything known to the previous generation.

The literature discussions on mathematical modelling and mathematical modelling competencies in subsequent chapters, as well as the knowledge gained from the past about students' difficulties in understanding problems, constructing models, and generalising results, together with the students' prior knowledge, will form the basis of the didactical phenomenology, which in turn forms a basis for the hypothetic learning trajectory (HLT) (Section 4.3.1).

1.7.2.4 Emergent modelling – the level raising power of models

The purpose of using models is to solve authentic and contextual problems (Zulkardi, 1999:7). The reinvention process characteristic of RME also gives rise to emergent models: through the process of *mathematisation*, the students' informal and intuitive *model of* the situation can later evolve into a *model for* more formal activity, allowing them to acquire a more sophisticated and formal way of working and reaching higher levels of comprehension (Dickinson & Hough, 2012:1). A concept revolves into a *model for* when students can apply the concept and use their knowledge in a new situation, thus the transition from *model of* to *model fo*r represents the ability to progress towards more generalised mathematical activity (Larsen, 2013:2), with reality being trimmed according to the mathematician's needs and preferences. Freudenthal (1975) explained *models of* something as after-images of a piece of given reality, while *models for* something refer to the pre-images for a piece of reality to be created. The formal model becomes an entity of its own and allow for the opportunity to engage in mathematical reasoning. Vertical mathematisation is thus closely related to trajectories of learning (Menon, 2013:3).

Van den Heuvel-Panhuizen (2003:13) describes this progress as a process where students progress from a stage where they devise informal context-connected solutions to finally gaining the insight into the principles of the problem and they are able to understand the 'big picture'. Level raising involves discovering that one's mathematical knowledge is too simple to construct a specific task, which can then resolve in accessing new mathematical understanding (Dekker & Elshout-Mohr, 1998:305). As a result of reflection (through interaction and by their 'own productions'), students' mathematising become more formal. Such level raising does not necessarily occur in a step-by-step format, but rather in episodes of jumps or discontinuities (Freudenthal, 2006:96). Both vertical and horizontal mathematising are of equal value, and both of these activities can take place on all levels of mathematical activity (Van den Heuvel-Panhuizen, 2003:12). Therefore, both horizontal and vertical mathematising can cause a jump from reality to new mathematical understanding and concept development (Freudenthal, 2006:101).

What makes level theory so important in RME, is the fact that teaching and learning should start at the first level that deals with contextual situations that is familiar to the students. Again, through the process of *guided reinvention* and subsequent *progressive mathematising*, the students progress from one level of thinking to the next (Zulkardi, 1999:6). Students play the most important role in RME as they pass through these various levels of mathematisation while reinventing their own mathematics (Van den Heuvel-Panhuizen, 1996:14). From a pedagogical point of view, a delicate, though crucial balance needs to be established throughout the mathematising process between the force of teaching and the freedom of learning (Freudenthal, 2006:55). Teachers need to introduce activities that elicit this growth in understanding, which again emphasises the importance of the intertwinement with didactical phenomenology.

Gravemeijer (1999:163) suggests four levels of models in designing RME lessons (Figure 1.2):

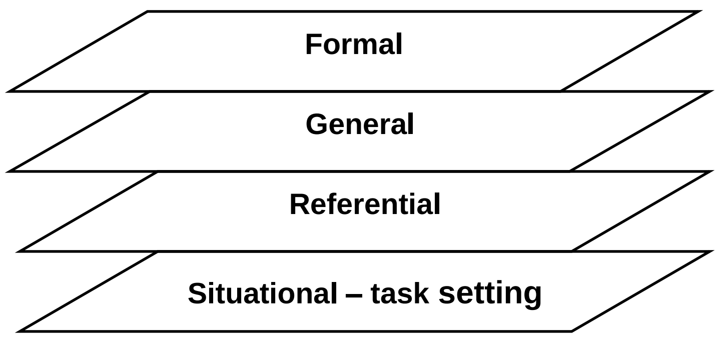


Figure 1.2 - Level Raising of Mathematical Activities (Gravemeijer, 1999)

- The first level, the *situational level*, deals with the interpretations and solution strategies that depend on understanding the domain-particular, situational problem.
- The *referential level* becomes the *model of* the situation, where the model explains and describes the problem.
- The *general level* refers to the *model for* more formal mathematical activities which now dominates over the situation-specific imagery. The acquired mathematical concepts can now be applied to a new situation.
- The fourth level, the *level of formal mathematics*, works with conventional procedures and notations and allows for opportunities to reach higher levels of comprehension, and is no longer dependent on the support of the models (Gravemeijer, 1999:163; Zulkardi, 1999:7).

Models therefore allow for flexible movements to higher levels of mathematical activities while movement from the world of mathematics to the reality situation stays put.

1.7.3 Tenets of guided reinvention

By combining Freudenthal's didactical phenomenology and Treffer's mathematisation classifications, Treffers in Freudenthal (2006:118) introduced five tenets about guided reinvention which will also apply to the evolving classroom learning environment during this design-based research study:

- Carefully selected learning situations must be designed that align with the students' current realities (contextual), appropriate for horizontal mathematising.
- Means and tools must be available for vertical mathematising. As this setting offers unlimited opportunity for improvisation by the student, the researcher needs carefully designed instructional plans that allow her to take advantage of the class situation as it presents itself at any given moment.
- Guided reinvention is based on the principle of interactive instruction – mutual relations must be established between the student and the researcher as well as among students. The researcher remains in the background to allow students the opportunity for efficient reinventing of mathematics.
- While the researcher remains in the background, the students are motivated to produce their own work, which results in the reinvention of solutions as well as problems.
- Learning strands must be intertwined as they aim to integrate past and future learning processes. Freudenthal proposes that learning should be "organised in strands which are mutually intertwined as early and as long as possible" (2006:118).

RME offers the framework where students are allowed learning opportunities to invent powerful mathematics through the process of guided reinvention. This research study investigates engineering technician students' mathematical modelling and engineering competencies within such a framework. Careful consideration will be given to the design of instructional methods and materials while respecting RME principles of establishing a classroom environment conducive for learning and teaching mathematics, the roles of the students and the teacher, as well as the other relevant factors that relates to RME.

1.7.4 Rationale of RME

The justification or reasoning of RME theory is to try to construe learning paths along which students can reinvent mathematics (Gravemeijer, 2004:107). Such learning paths constitute of instructional activities that can be used to elicit the reinvention process. However, the aim is not to design an instructional activity to be used intact, but to provide support to educators in the form of support materials and methods which they can adapt to use in their specific environments (Gravemeijer, 2004:107). This ‘support’ can be referred to as a ‘*Local Instructional Theory*’ (LIT) and functions as a stepping stone for the educator towards reinventing mathematics. Gravemeijer (2004:107) uses the term ‘*Hypothetical Learning Trajectory*’ (HLT) to refer to the educator’s intended path of work development towards a specific goal within a specific environment (classroom) on a day-to-day basis. The current knowledge level of the student needs to be considered to ensure progressive movement towards the envisaged learning goal. The HLT will continually change and adapt to support the learning and teaching towards a specific goal. By observing the enactment of the HLT, a LIT can be developed that describes the envisioned learning route relating to a set of instructional activities.

Therefore, the insight of a LIT is used by the educators to choose instructional activities and to design HLT’s for their own students. One of the aims of the study will be to provide an understanding of how and why specific classroom interventions can or cannot elicit the development of the specific competencies through the designing of a HLT that will form a learning trajectory to establish a framework for a LIT. The researcher aims to provide a modest, local theory on how engineering technician students can co-develop mathematical modelling as well as engineering technician competencies through the use of mathematical modelling for similar situations.

1.8 STATEMENT OF THE PROBLEM

The importance of competencies required by today’s engineering technicians cannot be overemphasised. Many of these engineering competencies align with the competencies needed in mathematics, as the possession of mathematical abilities is one of the most important qualities of successful professional engineers. Manipulating and solving of routine mathematical problems can only support the development of such competencies to a very limited degree. Mathematical modelling gives a student the opportunity to experience mathematics not as a subject dictating the reproduction of learnt procedures and algorithms, but allows him or her to be engaged in sense-making, simplifying, model-building, horizontal as well as vertical mathematising, comparing, organising mathematical matter, communicating and justifying while working with real life problems. Within the modelling activity, various competencies can be identified during each mathematisation process.

As explained in Section 2.3.6, carefully designed contextual problems allow students to develop and share their ways of thinking with one another. While students explain, justify and defend their solution processes, they learn to think and reason mathematically, and ultimately they increase their own levels of mathematical understanding. This denotes the close relation between mathematical reasoning and understanding, as mathematical understanding can be gained from effective mathematical reasoning. Throughout this study, these two intertwined concepts will be exploited collectively.

This study will seek to provide insight into the co-development of engineering and mathematical modelling competencies of first-year engineering technician students who are not strong in mathematics, with the aim to develop more sophisticated ways of mathematical reasoning. Relevant mathematical modelling and engineering technician competencies will be examined and mapped to establish the competencies essential to both disciplines. In addition, this study will attempt to determine how such competencies can be developed and measured in the students' work. The study will be placed within a socio-constructivist framework of teaching and learning mathematics where real-world contextual modelling problems will be solved by the students in the classroom.

1.8.1 Main research question

To what extent can engineering and mathematical modelling competencies co-develop to produce a deeper understanding of mathematics within the context of a mathematical modelling course for first-year engineering technician students who are not strong in mathematics?

1.8.2 Sub-research questions

Relating to the main research question, the following sub-questions emerged:

Sub-question 1:	How/where does mathematical modelling fit into the context of mathematical teaching approaches to develop mathematical reasoning and understanding?
Sub-question 2:	Which engineering technician and mathematical modelling competencies can co-develop through mathematical modelling?
Sub-question 3:	How do engineering and mathematical modelling competencies co-develop to nurture reasoning and deeper understanding of mathematics?
Sub-question 4:	How can competence development and mathematical reasoning be measured in the students' work?

1.8.3 Aims of the study

This study aims to investigate the development of engineering and mathematical modelling competencies through mathematical modelling. To allow for the development of such competencies through mathematical modelling, the study will adapt the RME theory which also complements the socio-constructivist approach to teaching and learning of mathematics. Through the processes of guided reinvention, didactical and historical phenomenology as well as emergent modelling, higher levels of comprehension and more effective learning take place. The following chapters will focus on the various perspectives of mathematical modelling teaching and learning, to place modelling in the context of mathematical teaching approaches. The competencies of engineering technicians will be identified and analysed in terms of mathematical competencies. The different kinds of mathematical modelling competencies will be characterised from existing literature and mapped to the required engineering competencies. Once the modelling process and mathematical modelling competences have been explored, a hypothetical learning trajectory (HLT) will be defined and used as a starting point for the experiment. The HLT will be based on phenomenological analyses and current literature, where after the researcher will select activities based on her anticipation of how the students' modelling competencies may develop. Through a thorough documentation process by means of video recordings, observations, field notes, informal interviews, group presentations, as well as students' written work, the possible competence development will be analysed and compared to the intended learning path. The development of mathematical modelling competencies in groups will be explored by a collective analysis of qualitative data. Through a process of reflection, the various competence developments will be analysed, resulting in the establishment of a local instructional theory (LIT) for developing mathematical modelling competencies.

It is wished that the participating students will acquire the necessary competencies to carry out modelling activities independently, to extract relevant mathematical questions and to independently develop solutions to real-world problems. Also, like Kaiser (2007:112), it is also hoped that students will learn to work purposefully and on their own with real-world problems, that they will experience the feelings of uncertainty and insecurity which are characteristics of real applications of mathematics in everyday life. Furthermore, a primary goal is that students' mathematical world views and beliefs are broadened, that they will be able to carry out a whole-scale mathematical modelling process, covering all the required mathematical modelling competencies.

Specific issues relating to the sub-questions will be addressed. There is a strong interconnection between the sub-questions and the way in which those questions will be answered. The answers will be provided by following a number of related aims for each of the four sub-questions:

Sub-question 1: **How/where does mathematical modelling fit into the context of mathematical teaching approaches to develop mathematical reasoning and understanding?**

Aim 1 Investigate the various perspectives of mathematical modelling teaching and learning and place mathematical modelling in the context of mathematical teaching approaches (Section 2.3).

Aim 2 Explore the theoretical underpinnings of mathematical modelling and model-eliciting activities to enhance reasoning and understanding of mathematics (Sections 2.4 and 2.5).

Aim 3 Explain the potential benefits of teaching and learning mathematics through mathematical modelling (Section 2.6).

Sub-question 2: **What engineering technician and mathematical modelling competencies can co-develop through mathematical modelling?**

Aim 4 Explore the most essential engineering technician competencies that are required from the engineering discipline (Section 3.5).

Aim 5 Identify the mathematical modelling competencies that can be developed through mathematical modelling as suggested by literature (Section 3.6).

Aim 6 Establish the specific competencies that form the focus of this study to address improved reasoning and understanding of mathematics (Section 3.7).

Sub-question 3: How do engineering and mathematical modelling competencies co-develop to nurture reasoning and deeper understanding of mathematics?

Aim 7 Explain how design-based research (DBR) methodology combined with case study research can be used as a vehicle to investigate the co-development of mathematical modelling and engineering technician competencies for the fostering of reasoning and understanding of mathematics (Section 4.3 and 4.4).

Aim 8 Explore the design and use of instructional activities (MEAs) that can elicit opportunities for such competence development (Section 2.5, Section 4.3.1.5, and Section 5.2).

Sub-question 4: How can competence development and mathematical reasoning be measured in the students' work?

Aim 9 Identify assessment instruments and data collection methods that will assist in obtaining unbiased and reliable results (Section 4.3.1.6 and 4.3.2.2).

Aim 10 Explain the data analysis processes that will apply when investigating possible competence development. (Sections 4.3.2, 4.3.3 and 5.2)

Aim 11 Explore competence development in individuals, groups, as well as in the whole class, through an analysis of qualitative data derived from the students' modelling activities (Section 5.2).

Aim 12 Define a hypothetical learning trajectory (HLT) from the results of a pilot study and the pre-intervention interviews, to be used as a starting point for the design experiment (Section 5.2).

Aim 13 Establish a learning trajectory that not only addresses classroom norms and discourse, but also explains how the possible shifts in students' reasoning abilities occur (Section 5.4).

1.9 RESEARCH METHODOLOGY

In striving to find a way to help the students to develop the required competencies and to improve their own current ways of thinking into more sophisticated ways of mathematical reasoning within a naturalistic setting, this study requires a detailed investigation of competence development with an interactive, iterative and interventionist approach within a socio-constructivist perspective. For this reason, a combination of design-based research (DBR) and case study research is appropriate. These two research methods go hand in hand, as outputs of DBR can be further developed through case studies, which provide us with rich data. Barab and Squire (2004:2) defined the purpose of DBR as "the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching". This study takes place in the messy and complex classroom situations that characterise real-life learning within a specific context relevant to engineering technician students.

As Eisenhardt (1989:534-540) explains, case study research aims to provide descriptions and understandings by combining data collection methods such as interviews, questionnaires, observations and field notes. Rather than developing theories and instructional materials, case study research studies a single case, or a few cases, to understand a larger population of similar cases (Gerring, 2006). In both DBR and case study research, triangulation is obtained through multiple data collection methods, and it provides strong substantiation of constructs that aim to further support localised theory building. Triangulation allows for overlaps in data analysis, which offers the researcher the flexibility to make necessary adjustments during the experimental cycles when specific conjectures are generated and tested. While DBR allows for the examination of small groups of four students each, a further case study will be conducted on the "weakest" and "strongest" case in as much depth as is feasible, to allow for an improved theory. Case study research therefore complements DBR and the researcher will attempt to provide explanations on competence development of the whole class, groups, as well as individual students.

1.9.1 DBR methodology

DBR consists of three distinct phases: a) the design and preparation phase, b) the implementation of problems within a socio-constructivist perspective, and c) a reflection phase, consisting of a thorough data collection and analysis process to assist towards further refining and revising of instruments, which again feeds a new design phase (Godino et al., 2013:3; Gravemeijer & Cobb, 2006:19). The retrospective analysis of the pilot study that took place prior to the main study, formed the basis of this design research. Two main arguments resulted from the findings of the pilot study: procedural understanding and memorising seems to be of primary importance to these students when they learn mathematics, and the data indicated that students tend to do what they are told, which is a far cry from the educational goal of mathematical modelling. The results of the pilot study serve as a link between the literature study and the teaching experiment. To explain the methodology and the development of a hypothetical learning trajectory in more detail, a discussion on the three phases of DBR follows:

- Design and preparation phase (Phase 1): To answer the research question, a hypothetical learning trajectory (HLT) will be constructed that serves as a guide to select and develop the instructional materials and to select and design specific activities. The HLT's function evolves throughout the research experiment, depending on the phases of DBR (Bakker, 2004). Simon (1995) explains the HLT as threefold: it consists of "the learning goal that defines the direction, the learning activities, and the hypothetical learning process – a prediction of how the students' thinking and understanding will evolve in the context of the learning activities". The HLT does therefore not only include information about the learning tasks, but also provides robust explanations relating to the variables that can affect the outcomes. Such variables include teachers, students, classroom ethos, contextual activities and reflection tools, as these are all inputs that are part of the working whole (Reeves, 2006). Six activities from literature that had proven successful in competence development will be adjusted by considering both the students' current levels of understanding, as well as the results of the pilot study. Expectations about the students' learning and reasoning are based on past literature, as well as the findings of the pilot study. While planning and designing the activity, anticipated questions are prepared to assist students with possible difficulties and to guide them towards possible solution paths (Wake, Swan, & Foster, 2016).
- The second phase of DBR, the implementation phase, involves empirical data generation while the students are engaged in the activities. These activities and instruction methods must be appropriate according to the HLT, which serves as a guideline to the researcher to ensure that all scaffolding and observations are focused on developing the students' competencies (Bakker, 2004:42). The anticipated questions that were prepared during the planning phase,

assist the students with possible difficulties during the activity. Bakker (2004:43) regards the answers to these questions as "an important source of information for the evolving HLT". Changes to the HLT are made where anticipated behaviours do not correspond with the actual incidents as they play out. The cyclic nature of DBR allows for continuous revising and refining of instruments (and accompanied changes to the HLT), based on theoretical arguments from the literature, as well as observations during the lessons. The focus is to create instructional materials as well as an instruction theory that can be used in other local settings. To keep track of the possible changes in the HLT, various data collection methods will be used such as interviews, walk-throughs, students' written work, reflection instruments, audio and video recordings and field notes.

- During the third phase of DBR, retrospective analysis of the data (that is generated while the students are actively engaged in the activities) are conducted. Analysing the HLT against the students' actual learning, enables the researcher to answer to the research questions and to contribute to an instructional theory (Bakker, 2004:45). All episodes relating to competence development will be transcribed, coded, analysed and tested against the various data collection instruments (audio/video recordings, students' written work, informal interviews, walk-throughs, field notes, reflection instruments) and will be compared to current literature to search for confirmation and counter-examples. The HLT and the research questions will continuously serve as a guide for making decisions on students' competence development. A thorough analysis of data is crucial in both DBR and case study research, where the data needs to be cross-examined repeatedly to allow for the theory to relate closely to the data collection and analysis processes. The researcher must continually grapple with questions such as: *What is happening here?* and *What can be truthfully derived from this data?* By iteratively moving forwards and backwards between collecting and analysing data, by constantly comparing data with prior data and against literature, checking for ideas, refining emerging ideas, and by constructing abstract categories from data analysis, the emergent levels of analyses are raised while new ideas, questions and deeper refinements of earlier concepts can also emerge. The decision on what data to collect and how to collect it next, is based on this analysis and this uncertain nature of the learning process requires constant adjustments of every aspect of the HLT (Simon & Tzur, 2004:93). The results of this analysis will thus further develop the HLT and answer the research questions.

1.9.2 Population and sampling

Purposive homogenous sampling applies to this study, as all the students are first-year civil engineering students studying at a University of Technology. The students that participate in the research experiment will not be the same students that enrol for the pilot study. These students have had no prior experience with mathematical modelling activities. They also did not meet the entrance requirements for studying engineering. Since 2006 institution offers a model of extended education, known as the *Access course*, to these struggling students, which means an additional 6 months of university preparation. Through more support and intensive tutoring, students enhance their chances of succeeding in the university's mainstream programmes.

All the students in the class will be invited to take part in this study. It is anticipated that the class size will be between 10 and 20. The students will be allowed to form their own groups of three or four students each. The current literature supports the importance of developing modelling competencies – especially meta-cognitive competencies – in small groups, as it enables observers to hear the thoughts of students without interfering in the process (Artz & Armour-Thomas, 1992:168). A pilot study of one semester will commence in July 2016, where-after the main research study will follow for a further semester. During the first week of the research experiment, the students will work on modelling activities for two days, five hours per day. Thereafter the students will meet once a week for two hours. Qualitative data will be generated from the students by using reflection tools, informal discussions, observations, field notes, written work and video/audio recordings. The study takes place in the classroom to keep it as close as possible to a naturalistic setting.

1.9.3 Data collection and assessment instruments

Appropriate data collection and assessment instruments will be designed to assist in documenting possible ways to enhance the development of students' mathematical modelling and engineering technician competencies during modelling activities. The activities used will be sourced from existing literature, to further support validity and reliability of the study and they will be subjected to a pilot study to assess its usefulness. Instruments and tasks will be adjusted to align with the findings of the pilot study. The development of a baseline assessment will assess the learners' prior knowledge of mathematics to establish what the students can learn through support. The results will be used as starting points towards developing an initial HLT. Engineering and mathematical modelling competencies will be documented throughout the entire experiment. To keep track of the possible changes in the HLT, various data collection methods will be used such as interviews, walk-throughs, students' written work, reflection instruments, audio and video recordings and field notes to ensure triangulation.

In this study, the researcher is the designer as well as the evaluator of the program. This results in the researcher playing conflicting roles of advocate and critic (McKenney, Nieveen, & van den Akker, 2006:83), which can result in a threat to validity. To overcome these threats, the following strategies of McMillan and Schumacher (2006:327) will be implemented:

- The researcher must have personal awareness.
- The researcher must facilitate the participants and allow them to voice their own opinions and ideas.
- The researcher's focus must be to collect a 'true version' of the phenomena.
- The researcher must attempt to deliver work of superior standard and present the report accurately.

Apart from the teacher's expectations, many more issues matter to students in the classroom. The conclusions that are drawn from data collection instruments can supply information about goals and needs, but as Lerman (2001:106) puts it:

> … we are forced to admit that we cannot arrive at what is going on for students in our studies, only what they might choose to tell us or what we might conjecture from studying voice inflections, gestures and so on… [Whether their behaviours (LdV)] were for the benefit of their standing in conversation with the researcher or an appropriate reflection of their interactions across time we cannot know and can only surmise."

However, the multitude of assessment and data collection instruments that will apply to this study, will enable the researcher to obtain at least six competence ratings per task, per competency and per student. Scaffolding will be provided throughout the six activities only as and when required, without any direct instruction. Working with a small number of students over one semester within a socio-constructivist classroom environment, can encourage the students to progressively act more spontaneous and to voice their opinions and thoughts about possible problems and solution paths whilst respecting the opinions of others. By considering all the above aspects, the researcher will strive to portray all the events as accurately as possible.

1.9.4 Rigour, relevance and collaboration

Tenets that shape design research are rigour, relevance and collaboration. The study aims to adhere to these principles as far as possible. The tasks will be carefully selected from existing literature and the entire experiment will be documented in a variety of formats and over a period of time to adhere to these standards. The study also aims to produce relevant material for the context and culture in which engineering students studying mathematics at a University of Technology will implement it. The entire process will be conducted in collaboration with the students. The data collection procedures (the modelling exercises) will be mutually beneficial, as it will aim to address the development of engineering and mathematical modelling competencies, while simultaneously offering meaningful experiences for the students.

1.9.5 Ethical considerations

Permission was granted by the Ethics Review Committee of Stellenbosch University (Appendix T) as well as the Durban University of Technology (Appendices O and Q) to conduct the research. All measures were taken to minimise risks and maximise benefits during the study. Informed consent was given by the head of department as well as the participants. The researcher used a coding system to protect the privacy of the participants (Chapter 4).

1.10 DELINEATION AND LIMITATIONS

The study intends limiting its scope to 10 to 20 first-year engineering technician students studying at the Civil Engineering Department of a University of Technology in South Africa. This means that the results cannot necessarily be generalised over a wide spectrum. The study investigates competence development with the aim to support a deeper understanding of mathematics and problem-solving over six months, which limits the analysis of determining the impact of the mathematical modelling course over the duration of their study careers. A further limitation of this study relates to researcher and facilitator resources. Having participating facilitators or volunteers may prove beneficial to the study, as the researcher does not have the benefit of discussing the students' progress with others. However, triangulation allows the researcher to identify the necessary and crucial aspects of competence development.

1.11 CHAPTER DIVISIONS

Chapter 1 aims to provide the background as well as motivation for the study. The importance of developing engineering technician students' mathematical modelling and engineering competencies, as well as the RME theory which supports mathematical modelling – the instructional theory for teaching and learning mathematics to investigate modelling competencies – are discussed. The study's purpose, aims and objectives are established, where-after a short description of the research design stipulates the direction for the study.

Chapter 2 focuses on the various perspectives on mathematics education and mathematical modelling in specific by reviewing past and current literature. This chapter will serve to provide a framework for teaching and learning mathematics through mathematical modelling. The process of mathematical modelling and the importance of designing and selecting model-eliciting activities will be clarified through literature to serve as a basis of the study.

Chapter 3 offers a theoretical understanding of engineering, mathematics and mathematical modelling competencies, particularally the competencies that allow for the successful accomplishment of mathematical modelling tasks and the complementing competencies that are needed for engineering technicians to successfully fulfil their professional roles in their current and future lives. An investigation towards competencies relevant to this study will be explained and motivated to ensure the development of relevant competencies for engineering technicians through mathematical modelling.

Chapter 4 concentrates on the methods and procedures that apply in generating qualitative data for the study. The motivation and understanding of design-based research (DBR) methodology will be clarified by offering explanations on how the hypothetical learning trajectory develops during the various phases of DBR. Motivation for supplementing the methodology with case study research will also be supported.

Chapter 5 is concerned with the retrospective analysis and results of the study. All the MEAs will be analysed and explained in terms of their role to enhance competence development for developing deeper mathematical understanding and reasoning. The construction of a learning instructional theory (LIT) through the cyclic and iterative use of real-world problems will be explained, as well as how the researcher/facilitator constantly compared expected behaviours with actual incidents as they played out, to determine whether the students' competencies have developed to such an extent to allow a deeper undertanding of mathematics.

Chapter 6 is the concluding chapter, where the main findings of this study will be summarised. Its contribution to knowledge, limitations and recommendations for further research will also be provided.

CHAPTER 2

MODELLING IN MATHEMATICS EDUCATION

Tell me, and I will forget. Show me, and I may remember. Involve me, and I will understand ~ Confucius (551 – 479BC)

2.1 OVERVIEW

Industry advisors to university programmes consistently emphasise the need for workers who are proficient in sense-making of complex real-world systems, team-workers, flexible to apply their knowledge to various domains, are able to plan and work collaboratively with different levels and types of participants, and share and re-use conceptual tools (Lesh & English, 2005:487). This chapter will explain the theoretical perspectives of modelling in mathematics education to allow for the realisation of such requests.

Chapter 1 exposed the current gap in mathematics and engineering technician education, and explained the significance of developing students' mathematical modelling competencies within the framework of Realistic Mathematics Education (RME). RME adopts the view of mathematics being experientially real for the student and it approaches learning through a process of guided reinvention, whereby the student reinvents mathematics through the guidance and constant provoking of reflective thinking from the teacher. The process of reinvention assists the students towards increased understanding, and thereby also complements the above quote of the Chinese teacher, editor, politician, and philosopher. This guidance requires a more contemporary approach towards teaching and learning, as the traditional approach of 'chalk and talk' does not allow students to reinvent their own mathematics.

Chapter 2 will commence by discussing the constructivist approach towards teaching and learning, which will be used throughout this study. The current perspectives that support the movement towards mathematical modelling, will be explained to better understand the implications of a mathematical modelling perspective towards the teaching and learning of mathematics. This discussion will be followed by a detailed investigation into the theory of models and mathematical modelling. As the instructional activities must be designed to allow the researcher to investigate students' thinking and understanding about mathematics while constructing models, these model-eliciting activities (MEAs) need to adhere to specific design principles. A in-depth explanation of MEAs will be provided, followed by closing arguments for including mathematical modelling in mathematics education. By completion of this chapter, the first sub-question and accompanied aims of the research question will be answered as formulated in Section 1.8.3:

Sub-question 1: **How/where does mathematical modelling fit into the context of mathematical teaching approaches to develop mathematical reasoning and understanding?**

Aim 1 Investigate the various perspectives of mathematical modelling teaching and learning and place mathematical modelling in the context of mathematical teaching approaches (Section 2.3).

Aim 2 Explore the theoretical underpinnings of mathematical modelling and model-eliciting activities to enhance reasoning and understanding of mathematics (Sections 2.4 and 2.5).

Aim 3 Explain the potential benefits of teaching and learning mathematics through mathematical modelling (Section 2.6).

2.2 CONSTRUCTIVISM AND SOCIO-CONSTRUCTIVISM AS THEORIES FOR MATHEMATICS EDUCATION AND MODELLING

Pólya (1945), as mathematician with a strong philosophical slant about the teaching and learning of mathematics through problem-solving, with other mathematical philosophers, have been instrumental in changing the traditional view of mathematics. Pólya promoted a problem-solving approach to mathematics teaching and learning that involves four steps in the problem-solving process: understanding the problem; developing a plan based on connections made; carrying out the plan; and evaluating that solution in retrospect. Today psychologists recognise that culture shapes the cognitive development of a child by determining what and how a child will learn, thereby employing constructivism as a theory of learning, which stems from both philosophy and psychology (Doolittle & Camp, 1999). Historically, two main perspectives on constructivism exists: cognitive constructivism (Piaget) and social-constructivism (Vygotsky), or rather socio-constructivism in terms of mathematics teaching and learning (Kanselaar, 2002:1). Piaget (1896-1980) promoted the idea that human intellect develops through adaptation and organisation. Adaptation refers to assimilation of external events into thoughts, as well as the accommodation of new mental structures into the mental environment (Piaget, 1964).

The Russian psychologist, Lev Vygotsky (1896-1934), paved the way towards a socio-cultural theory, which emphasises the role in the development of cooperative dialogues between children and more knowledgeable members of society (Vygotsky, 1967). Vygotsky's socio-constructivist perspective on learning is that the learner is a member of a socio-cultural group from which resources are drawn. He introduced the *Zone of Proximal Development* (ZPD) as the difference between what a learner can do by using his prior knowledge without help and what the learner can do with the help from a teacher or more knowledgeable peers (Lesh & Lehrer, 2003:121). His theory proposed that a group who is exposed to an environment to function just beyond their levels of competence, referring to more capable peers, can contribute more to one another's understanding than when learning individually. A learner's ability to learn from experience depends on the quality of the prior knowledge that the learner has. Learning occurs when the learner's prior knowledge is increased to a higher level (ZPD) (Woolfolk, Winne, & Perry, 2015:42-48). Vygotsky believed that every function of a child's development appears first on a social level and later on the individual level, when discussions with peers become internalised as thought, and ultimately socially constructed learning (Ferguson, 2005:6; Vygotsky, 1967). Creswell (2009:25-26) explains socio-constructivism as seeking to understand the world in which people live and work, which leads to the development of subjective meanings of their own experiences that are often socially and historically negotiated.

In mathematics education particularly, the socio-constructivist perspective shifts the emphasis towards the experiential side of mathematics which offers us an image of mathematics as a *human-friendly, socially constructed product* of human activity. The teacher has less control over the answers, the methods applied by the students, and the content choice of the lesson. Students gain control over the methods they apply to solve mathematical problems, and then finally over the content itself. Mathematical knowledge is not passively received, but is actively constructed by the cognising subject. Also, certainty and truth can hardly be found, but students can construct viable explanations of their experiences. This emphasises the fact that knowledge is situated and therefore the context must always be considered by both teacher and student.

Guided learning, or scaffolding, refers to this process where a teacher gives aid to the student in his or her ZPD. During scaffolding, a teacher can guide students by asking focused questions that can enable them to fill the knowledge gaps that they may have. Teachers should therefore always be sensitive to students' prior knowledge and the processes by which they make sense of phenomena, to enable effective teaching and learning (Woolfolk et al., 2015:50). Open-ended questions allow students to construct meanings of the contexts through discussions and interactions with others, which again enables the teacher to get an idea of the students' current understandings as well as misunderstandings (Creswell, 2009:26). Therefore, from the constructivist's perspective, the teacher has to equip students with conceptual understanding of the process skills, that allow students to individually or collectively develop a repertoire for constructing powerful mathematics that concur with viable mathematical knowledge. Continual assessment of instructional preferences is important, as it promotes student understanding and the constructing of new pedagogical knowledge from classroom practices. The assessment instruments and methods resulting in new pedagogical knowledge, are discussed in detail in Chapters 4 and 5.

Socio-constructivists have distinct epistemological approaches to teaching and learning mathematics. Firstly, the teacher must give the *student autonomy* to find the processes that can lead to a solution and the learning *environment must be designed to support and challenge student thinking* – the design of classroom environments are more emphasised than instructional sequence. The classroom environment which engages students in explorations, sense-making and discussing ideas is not dominated by the teachers, but the responsibility of learning is located in the students themselves and does not primarily rest with the teacher. The teacher's role as facilitator ensures that all students are given equal opportunities to experiment with ideas of their current understanding. Furthermore, the *testing of ideas* against alternative viewpoints should be encouraged at all times. Lastly, opportunities must be provided for *support and reflection* on both the content learnt and the learning process (Ferguson, 2005:19). This viewpoint is very closely related to RME as explained in Chapter 1, as *reflection*, *group cooperation* and *interactive communication* with the guide and between the students, allow students to experience the various levels of mathematising while the entire process is situated within a rich context (Freudenthal, 2006:121). Through teacher facilitation the students are expected to engage in challenging dialogue to explore the relevance of their ideas when solving mathematical problems, and to reflect on the solution paths that they design. This approach assists students to actively create, interpret and reorganise knowledge in individual ways (Ferguson, 2005:19).

By fostering such learning environments, students learn to value and appreciate the input from their peers, which results in successful collaborative work. A community of learners is established where students learn from one another, rather than the traditional approach where all students learn the same thing at the same time (Collins, Joseph, & Bielaczyc, 2004:24). Mathematic modelling education raises the need for a socio-constructivist view on teaching and learning. Section 2.5.4 further explains the role of the teacher during modelling tasks to support the development of students' mathematics understanding. The various perspectives on modelling in mathematics education will be explained to place this study in a solid theoretical framework, whereafter a detailed discussion of models, modelling and model-eliciting acivities will commence.

2.3 PERSPECTIVES ON MODELLING IN MATHEMATICS EDUCATION

The earlier interpretations of modelling in mathematics education were mainly based on two main perspectives: It had to be useful to solve practical problems (pragmatic perspective), and it focused on the students' abilities to create relations between mathematics and the real-world (scientific-humanistic). Over time, further differentiations in these interpretations came forward. The first differentiation focused on the epistemological goals where models are constructed from real-world situations to develop a mathematical theory. Secondly, an emancipatory perspective developed into socio-critical ideas of mathematics teaching and learning, while the third differentiation focused on the interdisciplinary nature of mathematical modelling to serve scientific, mathematical and pragmatic purposes in a complementing relation to one another (Kaiser & Sriraman, 2006:302).

Over time, new perspectives were identified and, to gain a better understanding of these interrelated approaches adopted by researchers and practitioners, Kaiser and Sririman's (2006) seminal classification can assist researchers to frame their work within and across specific perspectives. These five perspectives are: realistic or applied modelling, contextual modelling, educational modelling, socio-critical modelling and epistemological or theoretical modelling. The various perspectives that relate to this study will be explained in subsequent sections, to place this study within a specific theoretical framework. These perspectives are driven by their aims concerning application and modelling, for example whether pedagogical, psychological, subject-related, or science-related goals are prioritised (Kaiser & Sriraman, 2006:304-6). An explanation on the perspective follows:

2.3.1 Realistic or applied modelling

The realistic perspective focuses on *authentic examples from industry and science*. Here modelling is viewed as an activity to solve authentic problems, and it is not for the purpose of theory development. (For the purposes of this study, the term 'authentic' refers to aspects of a task that can be considered as *simulations of reality* situated within a context that is real to the students' experiences in their daily lives (Vos, 2011).) As the main criteria for the students' learning is based on the solving of *real-life problems*, this perspective takes the subject area of the application of mathematics very seriously and regards modelling as an interdisciplinary problem-solving activity (Blomhøj, 2009:2). This view on realistic problems aligns with RME's domain-specific instructional theory for mathematics education, that mathematics needs to be connected to reality and must be of human value (Section 1.7). Pollak (1969) emphasises that problems must always remain useful, meaningful and as close as possible to real-world contexts. Modelling is born in the 'unedited' world. Explicit attention is given during the beginning of the modelling process to translate the real-world problem into a mathematical formulation. Once solved, the modeller explicitely needs to reconcile the mathematical result again with the original context. This process allows for multiple solution paths, and the result needs to be both mathematically correct as well as reasonable in the real-world contexts (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016:9). The main difference between mathematical modelling and applied mathematics is the emphasis that mathematical modelling puts on the transition from a real phenomenon towards a mathematical problem. In applied mathematics, the distinction between the real-world model and the mathematical model is not always clear-cut, while this process is regarded as the core of modelling (Kaiser & Schwarz, 2006:197).

The importance for engineering technician students to understand the relevance of mathematics in their everyday lives, in their studies, as well as in their future careers cannot be overstated. In Sections 1.3 and 1.4, the main concerns in the engineering workplace came down to the inadequate development of students' meta-cognitive, problem-solving, reasoning, and mathematical understanding abilities. Teaching mathematical modelling by considering the realistic perspective, allows students to gain an understanding of the importance of mathematics in their current and future lives, and thereby they can also acquire the competencies to enable them to successfully solve real-world problems.

2.3.2 Contextual modelling

The solving of word problems *within context* takes priority in this perspective. Problems start from meaningful situations, where student-groups can engage with meaningful problems which again emphasise the importance of conceptual systems as fundamentally social in nature. Conceptual systems are regarded as human constructs and are represented through spoken language, written language, diagrams and graphs, as well as through concrete models and experience-based metaphors. Knowledge is organised around experiences as much as around abstractions, and because humans need to be able to explain and understand it, such knowledge continually changes as the needs of humans change (Kaiser & Sriraman, 2006:306). This research perspective focuses foremost on the development and testing of model-eliciting activities (MEA's), guided by six principles (Section 2.5): the reality principle, the model construction principle, the self-evalulation principle, the construct documentation principle, the construct generalisation principle, and the simplicity principle (Lesh & Doerr, 2003). By entertaining these principles, the modelling approach to teaching and learning is not constrained to finding a solution for a specific problem, but it can be developed to create a system of generalisable relationships. MEAs have the potential to reveal students' thought processes explicitly through their descriptions, explanations, justifications and representations while being engaged in the tasks, and it also fosters self-evaluation skills (Doerr, 2006:255-6). Iversen and Larson (2006:281) believe that, by being engaged in MEAs, students should ideally be able to use real-world mathematics outside the classrooms - a core function of professional engineering technicians. This contextual modelling perspective differs from the realistic perspective in that *didactical design* of modelling eliciting activities focuses on carefully structured activities to *support the students' learning*, and thereby regarding mathematical modelling as a special type of problem-solving (Blomhøj, 2009:4). While the students grapple with unfamiliar and non-routine realistic problems, they develop both cognitive as well as meta-cognitive competencies, while the context problems support a reinvention process that enable students to develop a deeper understanding of formal mathematics.

Therefore, modelling problems as reality-based contextual examples should be an important component in mathematics education.

2.3.3 Educational modelling

The important role of mathematics as a gatekeeper to engineering studies requires transparency of the transition to mathematics beyond school education to demonstrate the relevance of mathematics to engineering technician students (Michelsen, 2006:269). The integration of real-world problems with mathematics and the interplay between mathematics and engineering, drives the structuring of teaching and learning mathematics, to attempt to eliminate the problem of isolation, which can be to the disadvantage of both mathematics and engineering education. This interdisciplinary approach allows for the intertwinement of aspects of mathematics and engineering education.

Modelling from an educational perspective concerns the integration of models and modelling in the teaching of mathematics and has two main aims: Firstly, modelling is used as a means for teaching and learning mathematics through the structuring of learning processes. Galbraith (2007:47) refers to this approach as *modelling-as-vehicle*. Here mathematical modelling is treated as submissive to other curriculum purposes. The goal is to provide an alternative setting where the students learn mathematics without the primary goal of becoming proficient modellers. The mathematics that they learn does not include mathematical modelling as an explicit area of study. This approach regards mathematical modelling only worthwhile if it provides the students with significant opportunities to develop a deeper understanding of curricular mathematics (Zbiek & Conner, 2006:90). The purpose of modelling is to develop, motivate and illustrate the relevance of a specific mathematical content. These learning processes are aimed to develop students' understanding of concepts. Secondly, modelling also fosters the advancing of mathematical modelling competencies (Blomhøj, 2009:5; Kaiser & Sriraman, 2006:305). This approach, denoted as *modelling-as-content* by Galbraith (2007:47), refers to mathematical modelling as the instrument to provide students with the abilities that are relevant to their mathematical learning, as well as to enable them to learn and apply problem-solving competencies needed for model real-world situations. Galbraith argues that it is practical to apply a *combination of these approaches* by appreciating the curricular pressures, but also to allow for space to include modelling problems (Galbraith, 2007:47).

This study will primarily focus on the second aim, as the development of competencies to acquire a deeper understanding of mathematics and problem-solving drives this study. Didactical questions relating to educational goals, motivations for teaching and learning mathematical modelling, the organising of modelling tasks in the curricula, as well as the implementation and assessment of modelling activities, fall under this research perspective (Blomhøj, 2009). Niss (1989) and Blum and Niss (1991) contributed significantly to this research perspective in their discussions about the basic notions in the field, such as models, modelling, the modelling cycle or modelling cycles, modelling applications and competencies (Blomhøj, 2009:5-6). In Sections 2.4 and 2.5, as well as in Chapter 3, the relation of these notions to mathematics teaching and learning, exposes the importance of an educational perspective on modelling.

2.3.4 Cognitive modelling

The traditional approach to teaching and learning mathematics neglects deeper and higher-order thinking. Students easily forget disorganised lists of facts and skills, and, when such facts and skills are mastered one-at-a-time and in isolation, they do not necessarily know when to choose which one to use in specific situations. Many of the most important constructs that students need to learn, relate to models and complex conceptual systems that are used to facilitate explanations or constructions of other real-world complex systems, and can thus not be truncated to facts and skills. These *ways of thinking* of the students tend to persist, as modelling allows their understandings to be rooted in contexts to which they can relate (Lesh & Clarke, 2000:120-122).
Piaget (1964:176), one of the earliest researchers to reveal the holistic nature of important conceptual systems that students develop to make sense of their mathematical experiences, emphasised that the development of conceptual systems involve more than the ‘sum of the parts’. The students’ experiences that they try to describe, explain or predict, lead to conceptual reorganisations and such development takes place in the following dimensions: concrete-abstract, simple-complex, intuitive-formal, situated-decontextualised, or particular-general (Lesh & Lehrer, 2003:120). The main focus of cognitive objectives is on students’ *interpretations of situations* rather than on their actions in these situations. As mathematical modelling is concerned with constructing models from messy real-world situations, students learn to generate mathematical constructs through developing ways of thinking that may cause previously existing

conceptual systems to be integrated, differentiated, extended, or refined in significant ways. The symbolic descriptions that students generate are aimed to be shareable, reusable and easily modifiable, resulting in the development of significant forms of generalisations and higher-order thinking (Lesh & Clarke, 2000:142). In Section 2.4.2, the cognitive activities relating to the various phases of the modelling process, as well as the transition from one phase to another, are explained.

This perspective towards teaching and learning of mathematics focuses predominantly on the process of *sense-making*. Students learn to develop a deeper understanding of mathematics through mathematical modelling as they grapple with context-bound activities. Students do not only develop and construct mathematical knowledge, but they also acquire mathematical modelling competencies, as will be discussed in Chapter 3.

2.3.5 Socio-critical modelling

The important role of mathematics in society, the support of critical thinking, as well as the role, nature and functions of mathematical models are emphasised in this perspective. AlrØ and Skovmose in Barbosa (2006:294) regard critical epistimology as "a theory of developing or constructing knowledge, where critique of what is learned is seen as part of the learning process". Because critical thinking is regarded as one of the central goals of mathematics teaching and learning, reflexive discussions that surface during the modelling process are seen as an indispensable part of the modelling process (Kaiser & Sriraman, 2006). The modelling process allows students to develop tools to discuss and critique their models, which reveal their current understanding and which can in turn lead to refinements of their solution methods. This perspective also emphasises the fact that the modelling activity has to be a problem – and not an exercise – for the students, and it needs to be extracted from everyday life or from other sciences that are not pure mathematics to address the role of mathematical models in society (Barbosa in Barbosa (2006:294)). Socio-critical modelling mainly focuses on discourse to understand aspects of students' cognition in modelling, as it reveals some of their internal processes (Blomhøj, 2009:9). The *development of a reflexive discourse* related to the modelling process, refers to all types of language including signs, gestures, artefacts and mimics (Barbosa, 2006). Student discourse denotes mathematical models that are not neutral descriptions of reality, but elicit the learning of mathematical concepts and the development of modelling competencies during the modelling processes that provide students the support for understanding a current social situation and thereby they could become critical, engaged citizens (Barbosa, 2006:295).

Teaching mathematical modelling under the social-critical perspective can thus empower students to *reflect critically* on societal issues through criticising their own modelling processes in real-world situations, which is one of the primary goals of this perspective (Blomhøj, 2009:9).

2.3.6 Epistemological or theoretical modelling

Discussions on modelling also started from a theory-related background. The epistemological perspectives refer to approaches such as Chevallard's approach of mathematical praxeologies, the anthropological theory of didactics (ATD), Brousseau's theory on *contract didactique,* and the theory of RME. In short, this perspective focuses on the development of theories of learning and teaching.

Chevallard in Garcia et al (2006:226) promoted the idea that mathematical work should not be limited to a specific confinement or theme, without reflecting on its relation to other levels of praxeologies (Garcia, Pérez, Higueras, & Casabó, 2006:226). Chevallard's approach of mathematical praxeologies denotes the intertwinement of mathematics and modelling activities (Kaiser & Sriraman, 2006:306). It proposes modelling through the notion of mathematical and didactical praxeologies and introduces the above scale of levels of mathematics and didactic determination to overcome the compartmental problem of mathematics education.

The Anthropological Theory of Didactics (ATD) is a theory or perspective that builds on Chevallard's approach of mathematical praxeologies and allows for the diffusion of tasks, techniques and discourse to explain and justify the mathematical activity performed, making the process and the product of mathematical modelling two sides of the same coin (Garcia et al., 2006:226-7). ATD's view on mathematical modelling is also that it cannot be considered independently from mathematics, whether one views the modelling process as an object to be taught, or as a tool to learn and teach mathematics. When teaching mathematics through mathematical modelling with the main goal of constructing new knowledge, the student's model becomes part of his or her mathematical heritage. In this process, modelling competencies and the accompanied development of problem-solving competencies are also promoted. ATD refers to an integrated curriculum where modelling questions are integrated with mathematical themes (Artigue & Blomhøj, 2013:807). When viewing the *real world as an institution*, the real-world problems need to be solved by utilising various techniques, justifications and validations from the real world. The *mathematical world as an institution*, produce mathematical knowledge by utilising various techniques, justifications and validations from the mathematical world. This perspective differs slightly from the realistic perspective, as the realness of the problems is not taken as seriously in ATD.

The research domain of RME focuses on mathematics as a human activity by construing learning trajectories to explain the acquisition of mathematical knowledge from a real-world situation through the process of mathematisation. The emergence of the RME movement, inspired by Freudenthal's idea of mathematics being a human activity, provides a drive for contexts to support the understanding of mathematical problems. Freudenthal (2006:135) expanded the structuralistic view on mathematising and included not only thinking and reasoning that focus on set theory and abstract deductive structures of mathematics, but emphasised that mathematics learning must be presented as the inversion of the "antididactic inversion" where a mathematical problem arises from a real situation. He motivated his point of view by claiming that "raising a problem is mathematics too" (Freudenthal, 1991:135). Therefore, in RME theory, real-world situations are regarded as the starting point from where students explore and reinvent mathematics that is experientially real to them, as opposed to the deductive approach of starting with the product of mathematisation. The teacher's role diverts from the traditional perspective of *talk and chalk*, towards a *facilitator of learning* by utilising rich contextual problems and asking open-ended questions, while focusing on the cultivation of a learning environment where the students' ideas and solution methods serve as a basis for classroom discourse (Widjaja, Dolk, & Fauzan, 2010:169). The design of these contextual problems is also a priority, as it aims to support the students to develop and share their ways of thinking with one another. This perspective views mathematical modelling as a tool where students learn to explain, justify and defend their reasoning, rather than just presenting the teachers with given answers to problems posed. They learn to investigate the problem, to think and reason mathematically, to listen to other viewpoints and ultimately they increase their own levels of mathematical understanding. By employing the methodology of design-based research in this study, the researcher aims to develop a domain specific instructional theory (LIT) that explains students' competence development to nurture a deeper understanding of mathematics (Section 5.4).

The work done by Treffers in 1978 distinquishes two forms of mathematisation: horizontal and vertical mathematisation (Freudenthal, 2006). As explained in Section 1.7.2.2 of Chapter 1, horizontal mathematisation translates the real-world problem into a mathematical formulation, while vertical mathematisation is more concerned with reflecting and working within the world of mathematics where a new mathematical reality is constructed. Gravemeijer's *model-of* and *model-for* explains the roles of models in horizontal and vertical mathematisation as models evolve from context-specific to more generic models that are detached from the original situation (Artigue & Blomhøj, 2013:804). As mentioned in Section 1.7.2.4, the four different levels of activities that are involved in this transition between context-specific and formal mathematics, can be illustrated as follows:

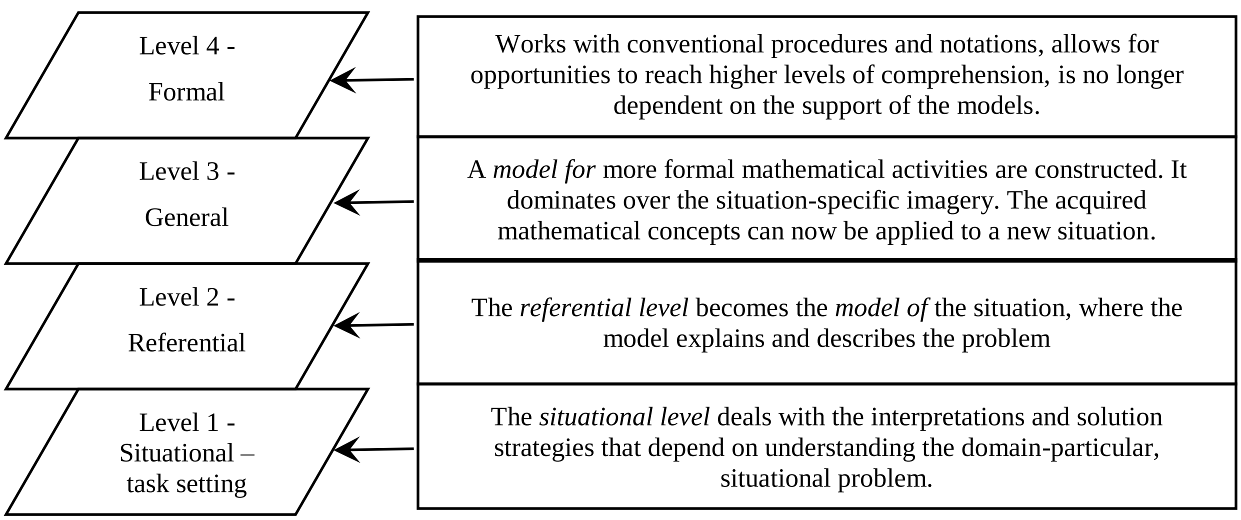


Figure 2.1 - Level Raising of Mathematical Activities (Gravemeijer, 1999)

Guided reinvention is a key principle of RME, which results in the designing of a learning trajectory to allow the students to reinvent their own mathematics and progressively develop more formal and meaningful knowledge and strategies from their initial disorganised and informal ways of thinking. Again, the guidance of the teacher is essential to successfully implement such trajectories. In RME, conceptual development is not emphasised as much as the role of the teacher and the didactical design of the activities. The core motives for learning and teaching mathematics in this theoretical perspective, are mathematising and modelling. Kilpatrick, Swafford and Findell

(2001) exposed five interwoven and interdependent components of mathematical proficiency as conceptual understanding, procedural fluency, strategic fluency, adaptive reasoning and productive disposition (Section 3.6.1). However, Artigue and Blomhøj (2013:805) denote the essential role that modelling and mathematisation play in developing conceptual knowledge. By engaging in real-world situations, modelling serves as a bridge between mathematical concepts and real-world situations, and students get the opportunity to develop modelling competencies and simultaneously foster a deeper understanding of mathematics.

The various perspectives on mathematical modelling imply different, but not mutually exclusive, implications towards the teaching and learning of mathematics. All these perspectives support the movement towards mathematical modelling, and the strong links between the various perspectives are emphasised. For the purposes of this research study, modelling will mainly relate to the epistemological perspective, as a learning trajectory will be construed that explains the students' development paths in mathematical modelling competencies that can lead them to acquire deeper mathematical understanding. However, complementing overlaps in other perspectives will also apply, such as the realistic perspective which focuses on the transition of real-world extra-mathematical situations to its mathematical formulations, and again on the transition back to the real-world situation that again supports improved reasoning and understanding of mathematics. The tasks will adhere to the design principles of model-elicing activities, indicating the importance of the contextual perspective. As the study focuses on competence development, and not explicitly on concept development, the educational perspective will focus on *modeling-as-vehicle*. Regarding the cognitive perspective, the development of a deeper understanding of mathematics by focusing on sense-making processes will also apply. Reflective discussions allow students to refine their existing mathematical understanding to progress to a more formal knowledge of the subject matter. This socio-critical modelling perspective will motivate reflexive discourse, which can again promote the development of modelling competencies as students learn to understand the complex problem-solving situation and can develop into critical, engaged citizens. This study will reflect the view that mathematics and engineering education are closely connected fields, and that mathematics also has a strong experimental component, similarly to engineering sciences. In the attempt to connect mathematics education to engineering education, an explicit role will be given to solving real-world problems through mathematical modelling. The theory of realistic mathematics education (RME) allows for the connection of mathematics with daily life, paying specific attention to the delicate role of the teacher to support and guide the students' development processes, and to interact with the students to contribute to the negotiation of meaning. To become successful modellers, students need to be able to complete non-routine and complex problems with mathematics. Learning these competencies is thus not only integrated with the teaching and learning of a specific concept, but attention is also given to meta-cognition and heuristics. Such competencies will be explored in detail in Chapter 3.

By stipulating the study in a clear theoretical perspective, the researcher has attended to Aim 1 of the research question, namely to *investigate the various perspectives of mathematical modelling teaching and learning, and to place mathematical modelling in the context of mathematical teaching approaches* (Section 1.8.3). In addressing the second and third aims of the research question, the following sections provide an in-depth explanation of models, modelling and MEAs, followed by the learning and teaching advantages that can be realised through mathematical modelling.

2.4 MATHEMATICAL MODELLING

Mathematical modelling is the dynamic process whereby conceptual, real-world and mathematical models are created and manipulated during problem-solving, with the emphasis on the structuring of ideas, the connecting of knowledge and the adaptation of big ideas to new contexts (Hamilton, Lesh, Lester, & Brilleslyper, 2008:8). In literature, many different definitions and opinions on mathematical modelling are to be found (Ang, 2009:161; Kaiser & Sriraman, 2006:303; Lingefjärd, 2006). Dossey (2002:85,114) defines mathematical modelling as "the process of representing real-world situations through mathematics", while Ogborn in Bahmaei (2013:35) describes mathematical modelling in general terms as "thinking about one thing in terms of simpler artificial things". These "simpler artificial things" refer to mathematical vocabulary and syntax. Lingefjärd in Bahmaei (2013:35) complemented this view by defining mathematical modelling as a "mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyses, obtaining mathematical results, and reinterpreting the model". The activities of observing, speculating, analysing, solving and interpreting within a socio-constructivist framework, require the students to be continually engaged in discourse to explain and justify their solution processes to one another, which in turn nurtures improved reasoning skills that again allows a platform for improved mathematical understanding.

In line with Freudenthal's RME theory as discussed in Chapter 1, this study views mathematical modelling from the stance that real-world situations are regarded as the starting point from where students explore and reinvent mathematics that is 'experientially real' to them, linking mathematical modelling explicitly to real-world contexts. To model a phenomenon, thus refers to the connecting of mathematical concepts and operations with reality and to symbolically describe a specific situation using mathematical concepts such as functions and equations. By creating a model, the modeller moves from the real-world into the abstract world of mathematical concepts, where the mathematical model is built. The resulting models are applicable to a given reality and can be generalised to interpret the solutions derived from them – the model is 'solved' by using mathematical or statistical techniques. (Albarracín & Gorgorió, 2012:22; Edwards & Hamson, 1989:2). Finally, we re-enter the real-world and translate the mathematical solution into a useful solution to the real problem. In checking the model against reality, reflecting and refining the model, the modeller iteratively moves backwards and forwards between the real-world and the mathematical world, to ensure that the mathematical solution is correct and reasonable in the real-world context. This process thus iteratively maps the students' understanding of the real-world with their mathematisations of that world (Eames, Brady, & Lesh, 2016:229). The following diagram by (Cirillo et al., 2016:6) depicts the explicit translation to and from the real-world and mathematical world:

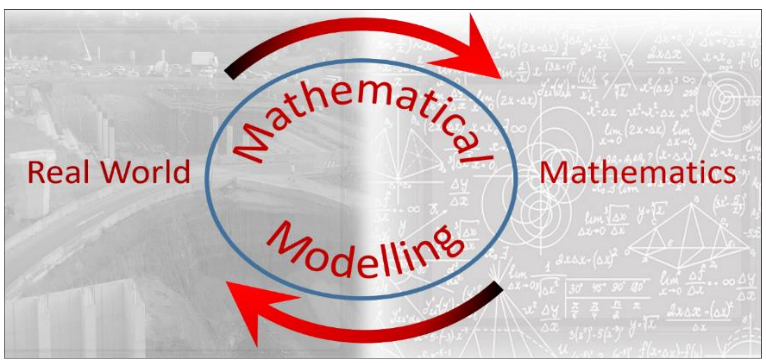


Figure 2.2 - The cycle of connecting the real world and mathematics (Cirillo et al., 2016:6)

The explicit attention of extracting the problem outside of mathematics into a mathematical formula and reconciling the mathematical solution and the real-world situation distinguishes mathematical modelling from other applications of mathematics, for instance, problem-solving. The latter "either does not refer to the real-world at all, or, if it does, it usually begins with the idealised real-world situation in mathematical terms and ends with a mathematical result" (Cirillo et al., 2016:10). Once the mathematics have been applied to solve a specific real-world problem in mathematics application, the context is no longer required. In contrast, in mathematical modelling the focus remains on the investigation of a specific phenomenon. The mathematics to be used is simply a means to understand how to solve the problem (Galbraith & Stillman, 2001). With mathematical modelling, the task setter starts with reality and then looks at mathematics before finally returning to reality to evaluate the usefulness of the mathematical model for describing or analysing a real situation. This continual engagement with the real-world model emphasises the importance of focusing on the nature of the tasks to allow for accurate interpretations of the given information as well as the desired outcomes. To develop sufficiently useful models thus requires a series of iterative modelling cycles, involving quantifying, organising, systematising, dimensionalising, coordinatising, and all the facets of mathematising.

The essense of modelling is translating the real-world problem into a mathematical form (Bahmaei, 2013:46). This process involves discussions to clarify the problem, identify the necessary variables, estimate, approximate and make decisions regarding possible courses of action to take, while considering aspects such as time, money, and other logistical matters. As mathematical learning is regarded as a socialisation process, mathematical modelling provide opportunities for group-work and critical discourse is exercised when the students seek to make connections between the real-world and mathematics, as well as connections within mathematics. The possible solution method as well as results are communicated to others to share thoughts, rather than to introduce new information. Interacting among the students occurs throughout the modelling process (Zbiek & Conner, 2006:102-105). This interactive environment provides a platform for developing more sophisticated ways of mathematical reasoning. These processes are cyclic and ongoing, and the modeller's path is non-linear: each modelling cycle tends to involve somewhat different interpretations of the given, goals and possible solution steps (Lesh & English, 2005:489).

Cirillo et al. (2016:8) identified the following five features as common characteristics of mathematical modelling:

1. An *authentic connection* to the real-world commences with ill-defined, often messy problems for which no exclusive correct answer exists. Vague problems create an opportunity for creative interpretation by doing some research or brainstorming with the initial goal to clarify what the model will predict or explain about the real-world (Bliss, Fowler, & Galluzo, 2014:10).
2. Real-world phenomena are *examined and explained* through mathematical modelling, and predictions about future behaviours of a system of the real-world can be made. The real-world context must not be under-estimated, as it is in fact this real-world situation that initiates the motivation to engage in modelling. By examining and explaining the phenomena through modelling, the modeller learns to understand or predict something about the real-world.
3. The modeller is required to display *creativity in making choices, assumptions and decisions*. Modellers need to determine the important aspects of the situation and be able to discount irrelevant data. In working with the data, they need to apply creativity to piece the relevant aspects of the situation together in a meaningful way (Bliss et al., 2014:19). These decisions are based on their prior knowledge about the situation and their mathematical world.
4. Mathematical modelling has an *iterative* nature. The modeller iteratively moves forwards and backwards between the real-world and the mathematical world as new insights emerge while the modeller grapples with the problem. The predictions of the model and the model's actual behaviour can also cause an imbalance which requires the modeller to iteratively engage in the process until a level of equilibrium has been reached.
5. The fact that *no one clear, unique approach* or answer exists, allows for multiple paths to be explored by the mathematical modeller.

Critical elements of the modelling approach are models, symbol systems and representational media. The symbols used are given a systematic semantic interpretation and they can be consistently and coherently used to mean something. The heart of modelling is to have an image or visualisation of the problem situation. Representations of real life or actual situations are the actual models and such representations can be pictures, drawings or graphs, symbols, tables or numbers and verbal representations. Learners need to be encouraged to use a variety of representations to model the actual situation and to deepen their understanding.

Various modelling cycles exist in literature (Blomhøj & Jensen, 2003:125; Blum & Leiβ, 2007:225). These cycles denote the general processes involved in modelling and are illustrated in Figure 2.3 below. A typical cycle comprises of the following processes:

a) *Understanding and formulating the task* that will guide you to identify the characteristics of the perceived reality that needs to be modelled;

b) *Simplifying* the real problem into a real model by abstracting critical elements to make a mathematical representation possible (systematisation);

c) *Mathematising* the real model into a mathematical model – the model is translated into mathematics that leads to a mathematical model of the real-world situation;

d) Searching for a solution from the mathematical model – *mathematical analysis;*

e) *Interpreting* the solution of the mathematical model; and

f) *Validating* the solution within the context of the real-life problem. This process is repeated if the mathematical solution is unsatisfactory. The start as well as the end of this process is in the *real-world*.

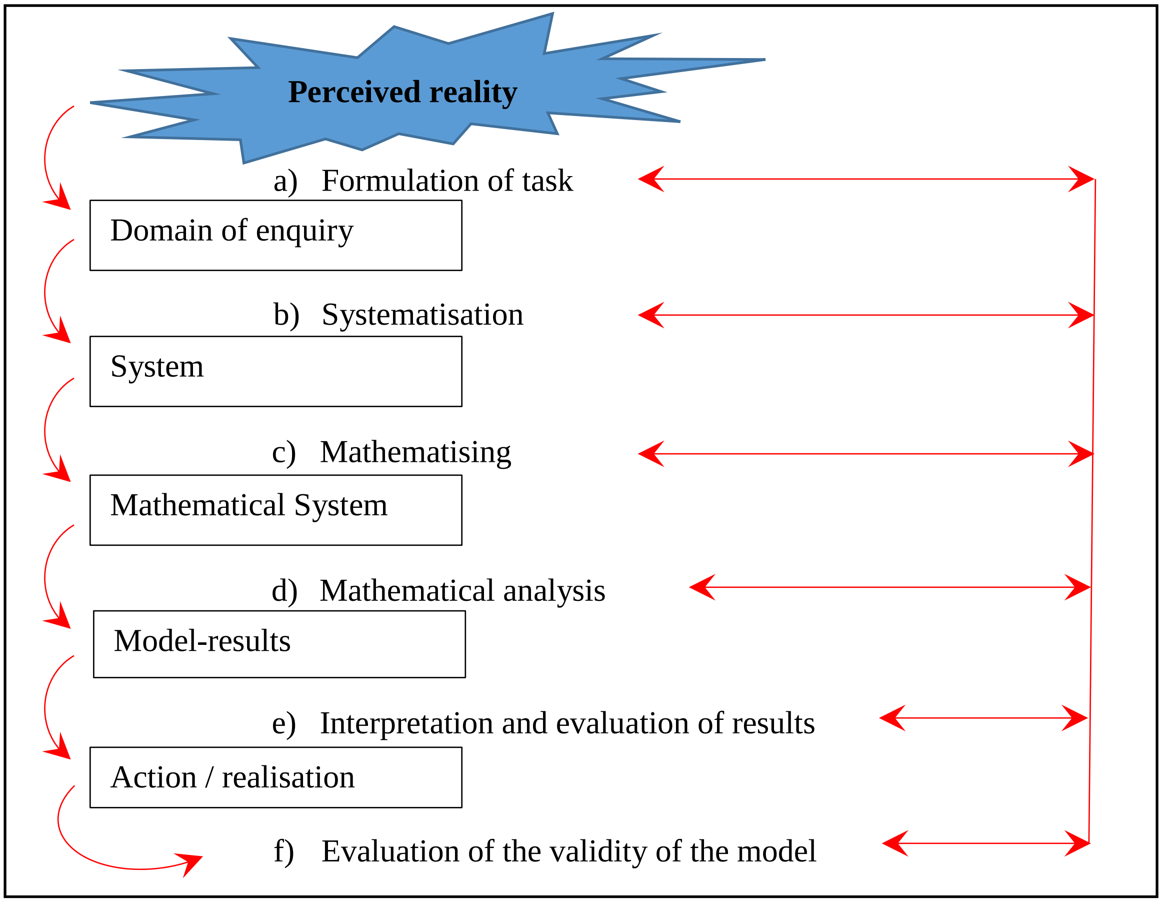


Figure 2.3 - A graphic model of a mathematical modelling process (Blomhøj & Jensen, 2003)

The manner in which models develop, can be explained as follows: When students react to modelling problems, they learn to distinguish the irrelevant features that are inevitably embedded in specific models, and they start to recognise the general, abstract concepts that the various models intend to convey. These opportunities allow students to learn to compare and contrast models, to think about the similarities and differences among them, and to investigate the relationships among alternative models. Their interpretations progress through a series of modelling cycles in which the results of each cycle produce new kinds of information which the students have to consider, and this might lead to further refinements of their underlying interpretations (emergent modelling). These thought processes allow students to develop new ways of thinking that involve the unravelling of unstable conceptual systems as well as the accumulating of stable systems. As their interpretations become more sophisticated, students often progress from going beyond thinking *with* a given construct to thinking *about* the construct. This process leads students to generalise specific mathematical knowledge from the initial situation to an unfamiliar, though similar situation. The focus of mathematical modelling is therefore not in finding solutions, but model development nearly always moves from situated cognition to generalisable and sharable knowledge that can be used and reused (Dienes, 1968; Lesh & Clarke, 2000:132; Lesh & Zawojewski, 2007). Kaiser (2007:111) explains that, when students learn to model, they also develop needed competencies to use their mathematics for the solution of problems in their daily life and from sciences. Expertise in the techniques of mathematics, statistics and computing is no longer sufficient skills, but general qualities such as enthusiasm for the task, being able to use intuition and experience, are some of the many other worthy attributes that also need to be acquired.

Therefore, improvements in students' understanding tend to be multi-dimensional and unbounded as it occurs along various independent dimensions such as concrete-abstract, intuitive-formal, or situated-decontextualised, similar to the rules that apply to other complex and continually adapting systems (Lesh & Lehrer, 2003:120).

2.4.1 Defining models of mathematical modelling

One definition for a 'model' to be found in The Oxford American Dictionary and Language Guide (Abate, 1999) includes "a simplified (often mathematical) description of a system, etc., to assist calculations and predictions". Within the mathematical modelling arena, different meanings are attached to the word *model*.

Lesh and Harel (2003:159) explain models as "conceptual systems that are used to construct, describe or explain other systems". These *conceptual models* or systems are non-physical objects, such as ideas or concepts, which are also regarded as human constructs and are fundamentally social in nature. Borromeo Ferri (2006:92) refers to conceptual models as mental representations of a situation (MRS), where the students create a mental reconstruction of the situation, which again is subjective and depends on the students' own mathematical thinking style – some students focus on the numbers or facts given in a problem, while others create visual images that have strong connections to their own experiences. The mental representations are embedded in naïve simplifications and individual preferences. The focus on conceptual systems thus imply a further dimension to models: models are created by modellers who are members of a socio-cultural group from where resources are drawn. In mathematics education particularly, models offer us an image of mathematics as a human-friendly, socially constructed product of human activity. Models are thus actively constructed by students to provide viable explanations for their experiences.

Conceptual models give rise to *real-world models*, expressing the students' mental understandings through various representational media. The representational media can include written symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experience-based metaphors (Lesh & Harel, 2003:159). Freudenthal's theory on RME promotes the use of a real-world model to describe a specific situation within a specific context with the aim to connect mathematics to reality and to emphasise the human aspect of mathematics. Students' world-views are interpreted differently, and such a real-world model will express their own subjective understandings of the situation in hand, emphasising the situated character of knowledge.

Through the process of horizontal mathematising as explained in Section 1.7.2, the students translate the real-world problem to a mathematical formulation. By reflecting and working within the world of mathematics (vertical mathematising), a new mathematical reality is constructed, denoting the *mathematical model*. This model represents a simplified mathematical representation or analogy of some aspect of realitiy, for the purposes of description and/or calculation. To summarise, a mathematical model can be regarded as some kind of representation, for the purpose of explaining something (i.e. a conceptual system, knowledge, or a reality) by means of mathematics, situated in the mind of the student or in some form of representational media.

Models mainly contribute to understand specific phenomena and they need to be purposeful, as they can only be examined in relation to their purpose (Edwards & Hamson, 1989:3; Starfield, Smith, & Bleloch, 1993). To enhance the sense-making role of models, Schorr and Koellner-Clark (2003) describe a model as

> … a way to describe, explain, construct or manipulate an experience, or a complex series of experiences. Models are organised around a situation or an experience. A person interprets a situation by mapping it into his or her own internal model, which helps him or her make sense of the situation. Once the situation has been mapped into the internal model, transformations, modifications, extensions, or revisions within the model can occur, which

in turn provide the means by which the person can make predictions, descriptions, or explanations for use in the problem situation.

2.4.2 The modelling cycle

The following diagram depicts the very familiar representation of the modelling cycle adapted from Blum and Leiβ (2005:1626):

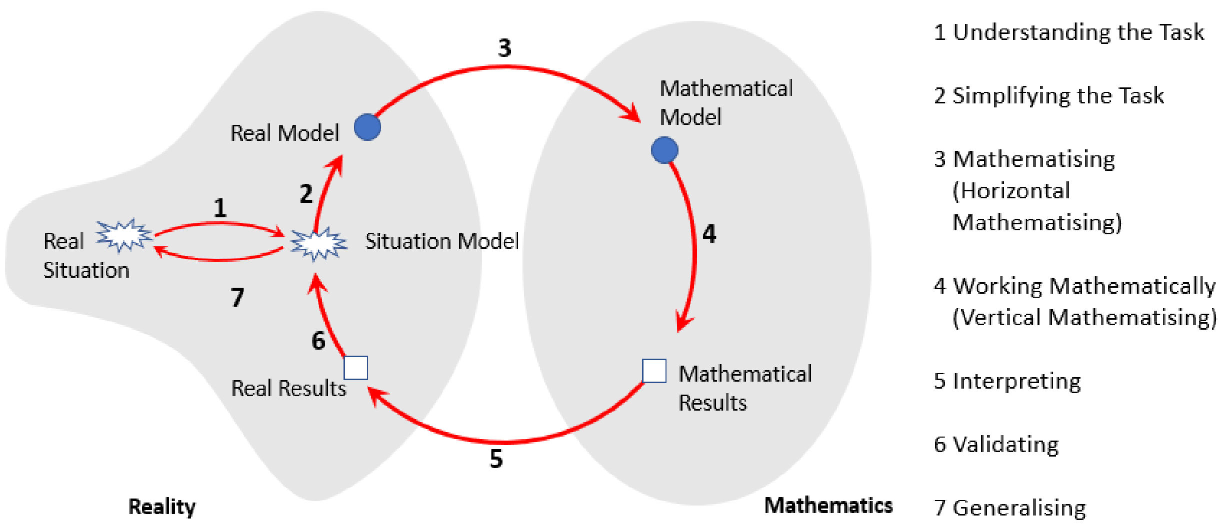


Figure 2.4 - The modelling process as adapted from Blum and Leiβ (2005:1626)

The above modelling processes denote multiple cycles of interpretations, descriptions, conjectures, explanations and justifications. Students iteratively refine and reconstruct their understandings as they interact and collaborate with others. These interactions promote modelling competence, as they learn to reason and to construct their own understanding of mathematics through abstracting of critical features, solving problems, presenting solutions and learning from one another (Doerr & Lesh, 2003:12). The process of mathematical modelling is instrumental in the development of students' abilities to understand, predict and control real-world situations while developing meaningful mathematics. Most important are the views of Dubinsky and Tall (1991:243) that learning modelling needs to be experienced; it is not a spectator sport or something that one can learn from a textbook. The various phases of the modelling cycle as denoted in Figure 2.2 can be explained as follows:

2.4.2.1 Understanding the task

This process gives rise to a situation model. Students deal with unstructured problem situations in the real-world where neither the purpose nor the mathematical entity is suggested explicitly. (Zbiek & Conner, 2006:92). Their initial goal is to make mental representations of the situation, which is given in the problem (Borromeo Ferri, 2006:92).

The modelling process starts in the real-world, when the student recognises the existence of a problem and the need to solve it (Albarracín & Gorgorió, 2012:21). Students start with their own thinking and current knowledge and use previous experiences to make sense of the new problem (Chick & Stacey, 2013:134). While the students actively *explore* the real-world situation by means of questioning, researching, brainstorming, clarifying, or attending carefully to certain information about the problem, they learn new information about the situation that was not originally apparent to them. They simplify (often intuitively and unintentional) the real-world situation by connecting the essential concepts regarding the problem (Borromeo Ferri, 2006:92). In sharing their knowledge with others while working on it to understand different aspects of a domain, they have to formulate the task in their own language to guide them to identify the characteristics of the perceived reality that needs to be modelled (Blomhøj & Jensen, 2003:125; Blum & Leiβ, 2007:225). This formulation of the task can be regarded as the first goal of the modelling process. When collecting relevant information, students choose what aspects of the situation they deem as relevant and ignore aspects that they assume irrelevant. These choices are based on the students' knowledge about both the real-world situation and about mathematics. The choices the students make to decide what information to use, will affect all subsequent processes and students will continuously revisit and evaluate earlier decisions. This phase represents Gravemeijer's (1999:163) *situational level*, as it deals with the interpretations and solution strategies that depend on understanding the domain-particular, situational problem.

2.4.2.2 Simplifying the task

This phase denotes the transition process from the students' mental representations of the situation to a real model, which is an idealised version of the real situation. This activity relates to making appropriate and efficient assumptions to further simplify and understand the real-world problem.

Students recognise certain conditions and constraints that may or may not work in the specific problem situation, as well as quantities that can influence the situation. Zbiek and Conner (2006:93) emphasise the importance of assumptions, as each modeller relies on his or her unique set of knowledge, intuitions and conceptions about the mathematics and the real-world, which in turn influences his or her interpretation of the situation, as well as the use of mathematical ideas. This activity is still based on the existing framework of the students, they need to identify various patterns, relationships and regularities. The experience becomes meaningful to the students when they relate the situation with similar ideas and constructs that they have dealt with in the past. Depending on the specific problem, a demand for extra-mathematical knowledge may surface (Borromeo Ferri, 2006:92). Their external representations will demonstrate whether they can identify patterns and relationships within the problem situation. This phase correlates with Gravemeijer's (1991:163) *referential level* as it becomes the '*model of*' the situation, where the model explains and describes the problem.

2.4.2.3 Mathematising (horizontal mathematising)

As discussed in Section 1.7.2.2, mathematising relates to the entire mathematical environment that includes both horizontal as well as vertical mathematising (Freudenthal, 2006:31). However, in the context of this modelling cycle diagram (Figure. 2.2), this phase focuses mainly on horizontal mathematisation. During this activity, students' verbal statements progress from a reality level to a mathematical level. They choose appropriate mathematical symbols and use those symbols to set up the mathematical model. Students gradually trim away the reality through processes such as making assumptions, generalising and formalising, which promote the mathematical features of the situation (De Lange, 1987:84). Students are typically concerned with the identification and describing of specific mathematics in a general context, schematising, formulating and visualising the problem in different ways, discovering relations, discovering regularities, recognising isomorphic aspects in different problems and transferring the real-world problem to a mathematical problem (Üzel & Mert Uyangör, 2006:1953). They must choose what aspect of the situation to focus on, ignore the aspects that they assume irrelevant, and decide how to formulate the real-world-situation mathematically. Students use and switch between their different representations, by using symbolic, formal and technical language and operations (De Lange, 1987:85).

Students construct a mathematical *model of* the real-world model, which, through working on it, emerges as a *model for* reasoning mathematically. Therefore, it represents a movement from horizontal mathematising to working with the mathematical symbols (vertical mathematising), a process called *objectification*. In objectification, symbols are used to objectify the mathematical object. However, the means of objectification does not only include symbol systems, but also refers to artefacts (e.g. rulers, calculators and computers) and linguistic devices (e.g. metaphors) to represent mathematical objects (Radford, 2002:14). During objectification, both *horizontal and vertical mathematising* (Figure 2.2) are prevalent as it signifies the transition between the two. The differences between horizontal and vertical mathematising are not always clear-cut, nor are the various stages of the modelling cycle. Although certain actions can explicitly be linked to specific phases of the modelling cycle, they are *not mutually exclusive* and can also occur in other phases of the cycle (Zbiek & Conner, 2006:102). To gain a better understanding of the interrelated activities within the modelling cycle, it is necessary to pay attention to each individual activity within the modelling cycle.

2.4.2.4 Working mathematically (vertical mathematising)

This process denotes mathematical analysis, as the students search for a solution from the mathematical model and they test their solutions. Students iteratively rephrase the problem and refine and test their symbolisations. The symbolisations still relate to the real situation, but become more general through the activity of formalising. The reasonableness of the model gets repeatedly tested. A model which is too complex, cannot be solved and a model which is too simple does not produce accepted solutions. A model can be simplified by introducing restrictions, overlooking variables and assume relationships. One the other hand, introducing more variables, considering more intrinsic relationships and so forth, can refine a model to the desired degree. Another option might be to reject the model altogether and to start all over again, by redefining the original problem. Learning of mathematics occurs as students generate and validate mathematical models. It can be seen as a process of making new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. In the mathematical modelling environment, understanding can arise from the corrected connections, as well as from new connections and knowledge (Zbiek & Conner, 2006:89).

The movement from the real-world to the world of mathematics, as explained in the previous phase, represents horizontal mathematising. Vertical mathematisation, on the other hand, refers to the reorganisation within the mathematical system itself. Activities relating to the latter are: representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, analysing, formulating a mathematical model, interpreting, examining and generalising (Üzel & Mert Uyangör, 2006; Zbiek & Conner, 2006:99).

2.4.2.5 Interpreting

Interpreting the mathematical results, reflecting on mathematical arguments, explaining, justifying, communicating and critiquing the model and its limits are typical activities required of this phase. Reorganising takes place within the mathematical system. By analysing (manipulating and interpreting) the mathematical entity, new parameters and properties are established. The appropriateness of solutions are examined when the students *evaluate* and *reflect* on their solutions and reconcile it to the original situations. Mathematical thinking now involves abstraction and deductive reasoning and skills, acquired within this process of *vertical mathematisation* (Zbiek & Conner, 2006).

2.4.2.6 Validating

The model may be revised and validated according to the context if it seems inappropriate, denoting once again the movement back to the real-world situation (Blomhøj & Jensen, 2003:125; Blum & Leiβ, 2007:225). An inappropriate model refers to a model that is not useful for the prediction or action in the real-world context (Lesh & Zawojewski, 2007:784). When the student encounters a problem during the verification phase, the cyclic process of expressing, testing and revising the trial solution is repeated. The start as well as the end of this process is in the *real-world.* In completing this phase, Gravemeijer's (1991:163) *general level* of mathematical activity has been reached, and a *model for* more formal mathematical activities has been established and dominates over the situation-specific imagery (Figure 2.1). The acquired mathematical concepts can now be applied to a new situation.

2.4.2.7 Generalising

When a modeller creates a mathematical model to meet a specific need for a specific purpose, he or she uses the constructed model to make sense of the problem situation and to verify the solutions that were obtained. By solving different real-world problems, it often happens that certain similarities or pattern regularities amongst the different situations emerge. A tool that was developed for one problem, can be adapted to be used in other similar situations and can be shared by other people. This implies that professional modellers compare and contrast their conventional models and start to think *about* the mathematical models, rather than *with* the mathematical models. By thinking *about* the models, modellers get the opportunities to adapt their models to other situations to meet new challenges, and thereby allowing the models to be shareable and generalisable (Lesh & Zawojewski, 2007:788). For example, in engineering fields, problem solvers continually grapple with new kinds of systems and new kinds of problems in their work environments. The ability to generalise knowledge plays and important part in the learning and teaching of mathematical modelling, which explains why the researcher adapted the original representation of the modelling cycle by Blum and Leiβ (2005:1626), to incorporate the activity of *generalising* as well.

During the last phase of the modelling cycle, students are offered opportunities to adapt their own model, or another model recently explored, to a new situation (Lesh & Zawojewski, 2007:788). This last stage is regarded as one of the most important activities in the emergent modelling process. The mathematical model is now completely detached from the original context and it is developed into a reusable and sharable system to interpret other contextual problems (Üzel & Mert Uyangör, 2006). Students acquired the *formal knowledge* when this level is reached and new concepts can be discovered (Gravemeijer, 1999:163). Independent reasoning and acting takes place, and the students can adapt the rule to use in another situation and to make predictions. As there is not just one right and proper model for a specific problem, the success of a model generally depends on how easy it is to use, and how accurate its predictions are. Models have limitations, which can be a cost or resources, or even a mathematical restriction (Edwards & Hamson, 1989:3).

A successful or convincing model also depends on the modeller's beliefs and opinions. One of the important functions of a mathematical model is to provide a truthful mathematical connection between one's presuppositions and one's observations. As one's beliefs can serve as an indicator to what one considers to be real and what one dismisses as only apparent, it can serve as a strong predictor of the type of model that one builds (Byl, 2003:5).

All these complex and intertwined processes that are involved in the various modelling phases as discussed, require tasks that are carefully designed as well as implemented to allow the students to experience all the facets of the mathematical modelling enterprise. This study will adhere to the principles of contextual modelling (Section 2.3.2) and careful attention will be given to the didactical design of modelling eliciting activities to structure such activities that support the students' learning. The next section explains what model-elicing activies (MEAs) are, its' accompanied design principles, the possible factors that can influence successful implementation, as well as the crucial role of the facilitator during modelling activities that take place in the classroom.

2.5 MODEL-ELICITING ACTIVITIES (MEAs)

MEAs are types of problems that simulate real-world, client-driven instructions, which students solve in small groups over one or two class periods. Such tasks require the students to interpret complex real-world situations mathematically, and also necessitate the construction of mathematical descriptions and explanations to make an informed decision for a realistic client. Students need to produce descriptions, procedures and solution methods (instead of a one-word or one-number answer), which explicitly reveal how they approach and solve a given real-world situation. MEAs allow for multiple methods to solve problems through processes involving mathematising in all its facets: quantifying, dimensionalising, coordinating, categorising, algebratising, systematising the relevant objects, relationships, actions, patterns and regularities (Lesh & Doerr, 2003:5).

A model-eliciting activity consists of four sections. The first section is a reading passage to generate students' interests and discussions about the context of the situation. This is followed by a readiness questioning section, where students ask questions about the preceding reading passage.

These questions can be simple comprehension questions or questions regarding the *interpretation* of the data. The second section's purpose is to ensure that the students have the *foundational knowledge to solve* the problem and *to understand the context* of the problem situation. The next section deals with data, where students can use diagrams, charts, maps, tables and so forth and is linked to the previous questioning section. The fourth section relates to the problem-solving task where the students solve a mathematical complex problem for a hypothetical client. This model is then *generalised* to subsequent situations. The last two sections comprise the bulk of the mathematics of the model-eliciting activities (Chamberlin & Moon, 2005:39).

The cyclic nature of MEAs allows the students to repeatedly reveal, test and refine their ways of thinking. Students go through sequences of interpretation-development cycles where they think about the problem situation and processes in different ways. Their interpretations of the problem situations mature while progressing through the various sequences. The transition from initial to final interpretations (local conceptual development) can be described as follows (Lesh, Hoover, Hole, Kelly, & Post, 2000:597): Students normally initiate the activities from disorganised and inconsistent ways of thinking about their goals and possible solution steps. They generally recognise the need to develop a model, but ignore difficulties relating to surface-level details or gaps in the data. As they progress through the iterative sequences of interpretation-development cycles, they start to apply more sophisticated ways of thinking and focus on relationships, patterns or trends in the data. As initial interpretations organise and simplify the situation, new information gets noticed that create a need for further refinement or elaboration of descriptions or interpretations. The new interpretations that emerge can create the need for another round of noticing additional information. These intermediate interpretations can still be unstable and evolving and the students repeat these 'express-test-revise' cycles until they have produced their desired results without further adjustments (final interpretations) (Lesh et al., 2000:597-600). Iterative cycles can yield new cognitive structures and understandings more effectively than single iteration application of textbook formulas. These series can be documented to reflect Piagetian-like stages of concept development where continual tension between accommodation (a state when one modifies one's viewpoint) and assimilation (integrate new ideas toward the solution of the problem) characterises the process of learning (Harel, 2008b:897) (See Section 2.2).

Whereas the traditional problem-solving goal is to process information with a given procedure, model-eliciting activities emphasise the active sense-making of meaningful situations through invention, extending and iterative refining of the students' own mathematical constructs (Bahmaei, 2013:35). The emphasis moves away from imparting strategies and skills, towards allowing the *elicitation of a model* that the group of students use and *iteratively revise*, to interpret or make sense of a problem. This eliciting and multi-cycle revision of models is the foundational strategy of MEA design (Hamilton et al., 2008:10). Students focus on the structural and systemic characteristics of what have been modelled or constructed. The final solution portrays what the students value as important aspects of their mathematical thinking (Lesh & Zawojewski, 2007:784). This process of solving the problems is thus emphasised far more than the solution itself, which is in contrast with traditional problem-solving activities (Diefes-Dux et al., 2004:F1A-3).

The progression from students' initial immature, primitive, or unstable interpretations into more sophisticated solutions, are expressed in a variety of *representational* media – spoken language, written symbols, graphs, graphics and experience-based metaphors. When the students work with the MEAs, they have to describe, explain, create and mathematise constructions which directly reveal their interpretations of the mathematical situations that they are confronted with. The different representations accentuate different aspects of the problem situation. By interpreting the problem-solving situations mathematically, their interpretations can go beyond *mathematics and also include feelings, dispositions, values and beliefs* (Lesh & Zawojewski, 2007:784-785), which again paves the way towards developing a holistic set of competencies to be used in their future lives and careers. These activities can generate trails of documentation that go far beyond providing information about the final result, and also reveal information regarding the students' ways of thinking, to support the productivity of ongoing learning or problem-solving experiences (Lesh et al., 2000:594). Instead of having to produce brief answers to questions formulated by others, model-eliciting activities require students to be deeply engaged in mathematising to develop, explain and interpret specific situations by themselves.

The desired mathematical understanding and abilities that students need to develop are only meaningful *within social contexts*. Because communication, justification and argumentation is a crucial part of mathematics, student-to-student and student-to-teacher interactions are just as relevant as student-to-perform interactions. An important dimension of conceptual development in the Vygotskian perspective on learning (Vygotsky, 1967) involves the continuing internalisation of external processes and functions. During group-work, students are more willing to externalise their ways of thinking that might remain internal otherwise. Also, through developing of shared knowledge, rather than personal knowledge, the goals of learning can be realised easier (Davies, 2009; Lesh et al., 2000:608). Due to the nature of model-eliciting activities, groups of students work together on the problems which need to be solved for the specific clients. Groups are also important for the following two reasons: firstly, there is a time constraint on solving the problem. Group-work offers the luxury of multiple perspectives and they can therefore come to a conclusion in less time. Secondly, professional engineers are required to work in teams and these exercises assist students towards effective collaborative skills (Moore & Diefes-Dux, 2004:10). The group members are actively involved in justifying and explaining their actions, predicting the outcomes, monitoring and assessing their progress, as well as integrating and communicating their results to others in a useful and meaningful way. These actions once again put a high demand on meta-cognitive competencies – planning, monitoring, controlling – as well as on competencies such as communication and representational fluency. Group-work thus assist students to externalise and reveal their thinking throughout the activities, as well as to reflect upon their current thinking strategies to make appropriate revisions to their solutions.

For MEAs to be implemented effectively to realise the possible benefits as discussed here, they need to adhere to strict design principles, which will be explained in the following section.

2.5.1 Design principles of MEAs

To ensure that a model-eliciting activity will meet the required and intended learning characteristics, designers rely on six design principles. These six principles are crucial in guiding the development of a model-eliciting activity and they serve as a benchmark. By returning to these principles, designers of model-eliciting activities can verify that the students' *growth of ideas* were considered. The growth of ideas refer to the growth in the students' own knowledge and ideas when they bring it to the classroom and the subsequent transformation of these ideas and knowledge to a more advanced state (Moore & Diefes-Dux, 2004:9). Chapter 5 discusses how the participants in this research study developed their ways of thinking, ways of understanding, and other complementing competencies. These principles are as follows (Chamberlin, 2004:53-60):

2.5.1.1 Reality principle (Personal meaningfulness)

The scenario that is presented has to be realistic to the student. This principle ensures increased student interest and it simulates activities of applied mathematicians in real-world problem situations. Students learn to interpret the situation meaningfullly, based on their individual levels of mathematical ability and general knowledge (Chamberlin, 2004:53). The more realistic the model-eliciting activity, the more likely it is that the problem will have more than one reasonable solution. Situated cognition refers to learning and problem-solving in context, which allows for the emergence of significant types of mathematical thinking. Students learn to make sense of real-world experiences from different topic areas, while they organise their mathematical ways of thinking around problem contexts (Lesh & Zawojewski, 2007:798).

2.5.1.2 Model construction principle

MEAs emphasise the *elicitation of conceptual models* which in turn generates the need for iterative revisions and refinements to elicitate students' underlying strategies about structuring and understanding real-world problems. Therefore, MEAs aim to leverage the models and conceptual systems that the students already possess. When constructing models, life-long learning skills required for future professionals can be nurtured, while students develop abilities to verbally articulate models and clarify connections between ideas (albeit primitive), or to elaborate on why a model may or may not work. Students learn to succeed in taking responsibility for the task and ultimately for their own learning, rather than expecting the teacher to teach everything that must be learned (Hamilton et al., 2008:7,9). A successful response to the problem should demand the *creation of a model for a mathematically significant situation*. By generating descriptions, explanations or procedures, students' interpretations of the situation as well as the types of mathematical quantities, relationships, operations and patterns that they consider are externalised.

2.5.1.3 Self-assessment principle

The assessment of a mathematical model is an embedded feature of the activity of modelling and it enables the student to make judgements about a model in testing whether the model serves the purpose for which it was designed. Assessment of models informs the modeller about possible decisions to take, assumptions to be made or altered, as well as possible next steps to be taken during the iterative process of modelling. Formative self-assessment tools such as status updates, reflection tools, or letters to their 'clients', can be used to assist students to judge the viability of their models for themselves, when they iteratively map their initial or intermediate models back to the real-world (Eames et al., 2016:230). Status updates are used to map students' early interpretations and models against the needs of the client. By using a variety of reflection tools, students can develop their own personal models of modelling. Examples of such reflection tools include the changing roles of the individuals during the modelling activity, the values, attitudes or feelings that can contribute to higher levels of engagement, and the problem-solving strategies that are productive during the different stages of modelling. These tools can form a potential direction for developing student-based self-assessments (Hamilton et al., 2008:16). Reflection tools assist students in creating auditable trails of their thinking strategies throughout the activity. Finally, when writing a 'letter to the client', students learn to describe and defend their ways of judging their solution processes (Eames et al., 2016:234). Reflection tools relevant to this study are attached in Annexures A-D.

The self-assessment principle thereby allows the students to assess the appropriateness and usefulness of their own solutions without the input from a teacher (Chamberlin & Moon, 2005:40). It also allows for established criteria that the students can identify themselves, and use it to test or revise their current ways of thinking (Chamberlin, 2004:54), relating to both cognitive and meta-cognitive understandings that students develop throughout the modelling processes.

2.5.1.4 Model documentation principle

Instead of having to produce brief answers to questions formulated by others, model-eliciting activities require students to be deeply engaged in mathematising to develop, explain and interpret specific situations by themselves. The modelling processes are documented, which *generate trails of documentation* that go far beyond providing information about the final result, but also reveal information regarding the students' ways of thinking, and as such also support the productivity of ongoing learning or problem-solving experiences and promote development of mathematical competence (Lesh et al., 2000:594). Also, the students learn to visualise and reflect on their thinking when explaining and describing their work in their groups (Chamberlin, 2004:54).

2.5.1.5 Construct share-ability and re-usability principle

The response to a model is claimed to be successful when it can be generalised to other situations using a similar model. Thus the students need to produce solutions which are share-able and re-usable. This design characteristic allows students to go beyond their personal ways of thinking to develop a more general way of thinking. By generalising their ways of thinking, students are enabled to produce powerful mathematics (Chamberlin, 2004:54), resulting in significant forms of generalisations and higher-order thinking (Lesh & Clarke, 2000:142). See section 2.4.2.7 for more information about the value of generalisation skills.

2.5.1.6 Effective prototype principle

The student's model should be easily interpreted by others (Chamberlin & Moon, 2005:40). This principle differs from the construct shareability and reusability principle, in that students may use this prototype in similar, but not parallel situations. This principle focuses on simplifying the model without losing mathematical significance. The crux remains that the model produced must be as simple as possible, yet still mathematically significant for the purpose for that it is constructed for (Moore & Diefes-Dux, 2004:10).

The importance of carefully designed model-eliciting activities are emphasised in Lesh et al. (2000:633-634)'s warning that students would not be expected to invent powerful constructs unless all the above principles apply:

- By applying and extending their own personal knowledge and experience, students try to make sense of a situation (*the reality principle*).
- Students recognise and value the need to construct a specific model for a specific purpose (*the model construction principle*).
- Students consider various ways of thinking about the problem situation (*the self-assessment principle*). Critiquing and judging the quality of alternative ideas and results, which often assumes multiple iterations of the modelling cycle must be apparent.
- They externalise their internal thoughts about the problem situation, which allow them to revise and refine their understandings about it (*the construct documentation principle*).

- They acknowledge the necessity to obtain a general solution for a group of specific situations, to be shareable and reusable in other similar cases (*the construct share-ability and reusability principle*).
- The students use their resulting models as a prototype for thinking about other structurally similar situations (*the effective prototype principle*). By doing so, students learn to focus on the important and useful ideas, while avoiding unnecessary complex computations.

This careful attention to the design of model-eliciting activities, is one of the reasons why model-eliciting activities are such powerful tools for investigating students' development of necessary mathematical competencies. However, the proper implementation of MEAs is just as crucial as the designing of the activities.

2.5.2 Benefits of MEAs

Model-eliciting activities (MEAs) were initially created by mathematics researchers during the mid-1970s with two main objectives: Firstly, to *encourage students* to develop models to solve complex mathematical problems similar to what mathematicians would do, and secondly, to *provide researchers with rich information* about the development of students' problem-solving competencies and the accompanied growth of mathematical cognition (Chamberlin & Moon, 2005:37; Hamilton et al., 2008:4). However, research in MEAs in classroom settings over the years prove not only to benefit researchers, but also observable changes among students and teachers were experienced (Chamberlin & Moon, 2005:44).

As model-eliciting activities are designed to stimulate certain types of enquiry and development without any direct instruction from the teacher, it is possible not only for high-ability students, but also for average-ability students to invent constructs that are more powerful than constructs that teachers have taught them by using traditional methods. Under-achieving students often seem to disconnect mathematics in the real-world from school mathematics, but model-eliciting activities have the potential to close the gap between applying mathematics in the real-world and experiencing mathematics in the classroom. By being engaged in MEAs, students learn to develop models, metaphors and other descriptive systems for making sense of familiar experiences, without having to use clever language and notation systems (Hamilton et al., 2008; Lesh & Doerr, 2003:5). They learn to use the resources they have at their disposal to express the 'new' ideas that they are expected to learn, while engaging in interdisciplinary, non-routine problem-solving activities (Lesh et al., 2000:632). The students that participate in this research study, are traditionally ‘weak’ mathematics students. They have not met the entrance requirements for studying engineering and therefore they enrolled on a six-month bridging course for additional support. It is thus hoped that these ‘weak’ students will also experience the meaningful facets of mathematical modelling.

While students invent, extend, revise or refine powerful mathematical constructs, it is possible to recognise and reward a wider range of mathematical abilities, as well as to recognise and reward a wider range of students who have these abilities (Chamberlin & Moon, 2005:44; Lesh & Doerr, 2003:24). Benefits for students are many and varied, which include the capacity to work in complex task-settings that involve human preferences, values and social dynamics, with competing constraints that are often unrelated to the underlying science or technology (Hamilton et al., 2008:2). Also, students can develop conceptual foundations to gain increased understanding in areas such as mathematics, engineering, technology and business, while they improve problem-solving and team-work skills (Yildirim et al., 2010:831).

Authentic mathematics learning, through the active engagement with model-eliciting activities (MEAs) also benefits educators. MEAs allow educators to investigate students’ thinking via three pathways. Firstly, they can become more familiar with the students’ thinking by acting as a meta-cognitive coach, while the students engage in model-eliciting activities through posing open-ended questions. Secondly, as students document their models in written form, it enables the educators to engage in a detailed analysis of the students’ thinking at a later stage. Thirdly, while the students present their models to the class and they answer to the questions probed, their thinking can also be investigated (Chamberlin & Moon, 2005:44).

Resnick in Kanselaar (2002:2) noted important differences between learning in school versus in the workplace. Traditional school learning implies individual cognition, while collaborative learning is often regarded as the preferred learning method in the workplace, as thinking is distributed across members of a team. Secondly, rather than entertaining pure mentation, the workplace is more focused on handling concrete, situated problems. Well-designed MEAs assist in blurring the boundaries between school and workplace, as the students are engaged in situated learning. They can develop team-working skills while they contribute meaningfully to the tasks, and simultaneously enhance their cognitive as well as meta-cognitive abilities.

2.5.3 Factors for implementing MEAs successfully

Although the literature promotes three distinct instructional benefits related to MEAs named improved conceptual understanding, problem-solving, and teamwork skills, such benefits can only realise if the MEAs are implemented successfully. Successful implementation depends on many aspects, such as the purpose of the MEA, the time allowed to complete such exercises, the quality and scope of guidance provided, the feedback offered during and at the end of the activities, as well as the educator's competence in using MEAs. Yildirim et al. (2010:837) suggest that careful attention should be given to the various aspects to ensure successful implementation of the MEAs. An explanation regarding these factors and how it will apply to this research study, follows.

2.5.3.1 The nature of embedded concepts

Yildirim et al. (2010) propose that embedded concepts are key to implementing MEAs successfully. For this reason, the MEAs in this study have targeted proportions and ratios, Pythagorean and Euclidean geometry, quality control, and multi-criteria decision making. Students will need appropriate guidance throughout the implementation process to ensure that they recognise and construct a proper understanding of the embedded concepts (Section 5.2).

2.5.3.2 MEAs' purpose in conceptual understanding

The purpose for developing MEAs can be either to stimulate the integration of concepts from earlier courses, to reinforce concepts that were recently introduced, to discover new concepts through guided discovery from the instructor, or a combination of these three roles (Yildirim et al., 2010:838).

Even though concept development is regarded as an important purpose of MEAs, this design study's focus is on *competence development.* It was therefore decided that the MEAs role in conceptual understanding will primarily be to reinforce previously introduced concepts and allow for integration of concepts from earlier courses. The MEAs in this study target various mathematical concepts, all of which the students have already experienced in high school. The mathematical concepts that arise from the activities are concepts that the students were supposed to have encountered before. It is also important to note again that the students attending this research study were all part of a bridging course – their matric marks were not sufficient for their acceptance in the mainstream engineering program. The success of the MEAs in this study will thus depend on the difficulty to understand and recognise the mathematics to be used.

2.5.3.3 Team sizes

Ahn and Leavitt in Yildirim et al.(2010:838) suggest that the team sizes should preferably be between three to four persons per team. They also noted that too many teams can lead to decreased instructor guidance and have a negative impact on the outcome of the MEAs. This research study will divide the students in groups of four students each to allow for adequate instructor guidance.

2.5.3.4 Instructor's experience with MEAs

Yildirim et al. (2010:838) noted that instructors' familiarity with prior MEA research, their experience in constructing and preparing MEAs for implementation, as well as their appreciation for the benefits relating to solving MEAs, are strong predictors for successful implementation. For this study, the researcher familiarised herself with MEAs through thorough research, and MEAs were constructed and implemented in the pilot study (adhering to the required design principles) to enhance chances of success during the final experiment (Section 4.3.1).

2.5.3.5 Instructor guidance

Sufficient guidance must be given to students during the execution and feedback phases. The instructor needs to identify students' mistakes and assist them to reflect on their work and redirect them. However, it is important that the instructor does not offer too much assistance, but rather probe students to reflect on their work and clear up misunderstandings. By reflecting on their work and justifying their ways of thinking, the students acquire confidence to continue with the problem, which in turn allows them to develop and construct their own strategies. Corrective guidance assists students to remain focused and to recognise and use the mathematics required to solve the problem. Appropriate teacher guidance as required for successful teaching and learning of mathematics through mathematical modelling, as well as the additional roles of instructors to support the successful implementation of MEAs, are described in Sections 2.2, 2.3.6 and 2.5.4, and will also apply to this research study. Section 5.2 explains how guided instruction was employed in this study.

2.5.3.6 Time allowed for determining a solution

When given enough time, students tend to display more depth in their solution processes and contributes positively to deeper conceptual understanding. However, when students are allowed to work outside the classroom, they do not receive the guidance that they may require for successful completing the modelling task and often continue working in the wrong direction. To allow for effective guidance and scaffolding, all the tasks in this study will be done in the classroom, and each task will last between one to three classroom periods.

2.5.3.7 Feedback

Feedback is considered as one of the most critical influences on student learning. Hattie and Timperley (2007:81) describe feedback as information that is provided by a teacher, peer, book, experience, etc., that explains aspects about one's performance or understanding of a task. Hattie and Timperley distinguish between three important and interrelated categories of feedback questions: firstly, feedback relating to how well a task is understood; secondly, feedback relating to the processes required to understand or perform the task; and thirdly, feedback relating to self-regulation. By attending to these three interrelated categories, the gap between the goal of learning and the actual attainment of this goal can be minimised.

Feedback on the *task* is more effective when the teacher applies strategies to address students' misinterpretations, rather than their lack of understanding. By providing the students with cues about erroneous ideas and interpretations, students can learn to develop more effective strategies to understand the material. On the *process* level, the goal is to assist the students towards directions for searching and strategising. By sensitising the students to strategise their work processes, they develop self-regulation competencies to manage and monitor their own work. In addressing the necessary *regulatory processes* required for the task, the students' beliefs about mathematics are shaped and they learn to perceive mathematics as useful and worthy of engaging in. To provide feedback that adequately addresses these concerns, requires a classroom climate where rich learning opportunities are provided for all students, where students learn to respect one another, where they can collaboratively share their ideas and opinions, learn from one another, are prepared to question or reflect on what they know and understand, be willing to engage in self-regulation, in critical thinking and reflection to develop the needed competencies and to achieve a deeper understanding of mathematics. Feedback should not only be provided by the teacher, but the students should also engage in self-assessment and evaluation strategies. When feedback is provided and directed at the right level, students can develop the necessary strategies needed for effective and efficient learning (Hattie & Timperley, 2007:91).

The information and interpretations provided by assessments should comprise of enough dimensions that do not only provide the students with feedback on learning issues, but to also inform the teacher about possible areas in teaching that can be addressed to further support the students towards increased comprehension of the tasks (Hattie & Timperley, 2007:104). Formative and summative assessment instruments were designed for this research study to provide ample information on competence development. A discussion on the relevance of formative and summative assessment is provided in Section 5.3. Not only are these instruments to be used by the educator, but self-assessment instruments (Appendices A–D) were also designed to support the students towards competence development and improved understanding of mathematics and problem-solving.

These seven factors. as identified by Yildirim et al. (2010), should not be regarded as independent of one another, but careful consideration should be given to every aspect to allow for the successful implementation of MEAs to foster competence development and a deeper understanding of mathematics and problem-solving.

2.5.4 Further roles of the teacher during MEA implementation

The preceding discussions on modelling and mathematics education emphasised the critical role of the teacher as guide and scaffolder to allow students the opportunities to reinvent mathematics and to develop critical thinking abilities. Doerr (2006:256) further accentuates three very important dimensions of teachers' knowledge within the context of a modelling task: to understand the multiple ways that students' thinking may develop, to acquire ways of listening to (or recognising) that development, and to learn to respond with pedagogical strategies that can support that development. Research on teachers' professional development indicate the importance of teachers to attend to and understand the ways that students think and approach mathematical tasks, as it initiates meaningful interactions with students to promote effective learning (Doerr, 2006:255; Franke & Kazemi, 2001:105; Simon & Schifter, 1991:329). Expertise in the complex learning and teaching environment proposed by MEAs, requires teachers to not merely follow fixed processes and procedures, but to adapt their ways of teaching as the students' knowledge develop along multiple dimensions within particular contexts and purposes.

During model-eliciting tasks, the teacher has to develop techniques to *understand the students' multiple ways of interpreting and approaching a problem*. The teacher needs to be aware of the multiple paths that the students can take to revise and refine their initial ideas. Doerr (2006:265)'s case study emphasises the effectiveness of teachers that communicate the importance of engaging deeply with mathematics - even when they are confronted with difficulties - to the students. The preparation of an anticipatory trajectory about students' understanding can further assist the teacher to anticipate possible misunderstandings and problems that the students may encounter. Sufficient planning and provision for such pitfalls enable the teacher to motivate the students to express, test and revise their own models and to motivate the students to explain, justify and reflect on their own ideas and constructs.

Doerr's second dimension (recognising students' development) can be interpreted as *listening to the students' voices*. Davis in Doerr (2006:256) distinguishes between three ways of teachers' listening to students' ways of thinking: In listening the *evaluative* way, the teacher focuses on the end-results to determine the correctness of the students' answers. With the *interpretive* ways of listening, the teachers require the students to respond to their answers in an elaborated way. The third way of listening, the *hermeneutic* way, is promoted with learning and teaching mathematics through mathematical modelling. Here the goal is to engage with the students in meaningful ways and to participate with the student in revising and refining both the student's as well as the teacher's existing knowledge and understanding, without having prior expectations about what the students' responses may be. Again, a thorough planning of the anticipated process can enable the teacher to be prepared for multiple interpretations and responses, which can be added to the schema for future use in other relevant tasks. The teacher experiences a shift from teaching what needs to be taught, to 'see' and interpret the knowledge that the students already have in their repertoire (Doerr, 2006:266).

Doerr and Lesh (2003) and Lesh and Doerr (2000) in Doerr (2006:256) emphasise the importance of a teacher to *respond in ways that will support students' conceptual development* towards more refined, generalised, flexible, and integrated ways of thinking. Teachers are confronted with pedagogical challenges when they understand and see students' ways of thinking during mathematical activities that support student development. The ways that they respond to these challenges depends on their own perceptions and interpretations of the classroom situation. By asking students to explain the strategies that they used to arrive to meaningful solutions, the teacher creates a situation in the classroom where students have the opportunities to share and justify their ways of thinking with one another. Through meaningful discourse, the students can continually revise and refine their ways of thinking. Doerr (2006:267) suggests that, by preparing and continually updating and refining an anticipated schema about the students' ways of thinking, allows teachers to develop new and improved ways of responding to the students' problems to further develop their emerging models.

In this research study, the researcher will aim to select and adapt MEAs from existing literature that adhere to the design principles as stipulated in Section 2.5.1. Attention will be given to the various factors that can influence the successful implementation of MEAs, as well as the crucial and complex role of the teacher to support and guide the students towards a deeper understanding of mathematics and problem-solving.

A description has been provided to situate this study within the perspective of teaching and learning mathematics in the framework of RME and socio-constructivism. Furthermore, mathematical modelling, the modelling cycle, as well as the design and implementation of model-eliciting activities as explained, provide the required background for engaging in mathematics teaching and learning through mathematical modelling. These explanations answered to the second aim of the research question, namely to explain the theoretical underpinnings of mathematical modelling and model-eliciting activities that can promote reasoning and understanding of mathematics (Section 1.8.3). However, apart from understanding what models and modelling entail, it is even more important to understand *why it is worth doing* modelling. The following section will discuss the motivation for learning and teaching mathematics through mathematical modelling.

2.6 MOTIVATION FOR TEACHING AND LEARNING MATHEMATICS THROUGH MATHEMATICAL MODELLING

One of the main objectives of mathematical modelling is to provide an approach to *formulate, explain and predict real-world problems* in terms of mathematics. The activity of mathematical modelling *promotes the development of many competencies* which are indispensable in the professional workplace. When engineering students enter employment, they have to formulate and tackle real-world problems in terms of mathematics (Edwards & Hamson, 1989:iii). Mathematical modelling thus offers the possibility to assist engineering students to become more interested in their future careers whilst engaged in mathematics in an engineering context (Moore & Diefes-Dux, 2004:9). The students learn to apply mathematics in their daily lives, and thereby gain a *better understanding of the world they live in, as well as the utility of mathematics* (Mischo & Maaβ, 2012:3). Models are useful in representing aspects of reality that are hard to visualise, and they can thus function as metaphors and analogies that can lead to *important physical discoveries*. Hunt stresses the importance of mathematical modelling to *understand and deal with society's major challenges*. Policy-makers and industrialists – to name a few – base many decisions on the results of mathematical calculations (Hunt, 2007:2).

People learn to understand, interpret, and forecast future trends, based on their engagement with scientific data. From an instructional point of view, the focus is shifted towards engaging the students in *revising, refining and extending their own ways of thinking* about a problem, or rather, to engage the students in addressing their habits of mind. Cuoco, Goldenberg and Mark (1996:378) describe habits of mind as "mental habits that allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations". Traditional teaching and learning of mathematics often only prepare students for life after school by giving them a bag of facts. Instead of learning such already established results, Cuoco et al. (1996) appeal that students must be given the opportunity to acquire the habits of mind used by the people who created those results. The goal should thus be to assist students to learn and to adopt some of the ways in which mathematicians think about problems. This view also complements the principle that students' ways of understanding are impacted by the ways of thinking they possess (Harel, 2008b). To elaborate, Harel (2008a:487) interprets mathematics as a combination of two sets: The first set refers to all the *ways of understanding* as regarded by the mathematics community as correct and useful to solve mathematical problems throughout history. The second set incorporates all the *ways of thinking* that are characteristics of the mental acts whose products comprise of the first set. To summarise, ‘ways of understanding’ refer to a cognitive product of a person’s mental actions, while ‘ways of thinking’ relates to the person’s cognitive characteristics and are highly intertwined. Operational thought characteristics manifest itself through mathematical modelling as students determine their own strategies and solution paths (ways of thinking) to anticipate the possible outcomes (ways of understanding) of their modelling activities (Harel, 2008b:899).

Mathematical modelling has the potential to address the co-development of students’ ways of thinking and ways of understanding. During modelling, students learn to recognise important ideas from ill-posed problems, they elicit meaning to the problems through defining, systematising, abstracting and connecting logical concepts, while continuously searching for new ways of describing situations. This gives rise to students being able to access various tools to use in developing their ways of thinking and understanding mathematics while reinventing their own mathematics. The focus moves from what mathematicians *say*, to what mathematicians *do*: students now join in the process of creating, inventing, conjecturing and experimenting. Modelling assists students towards making logical and heuristic connections between new and old ideas while encountering a true research experience.

Model-eliciting activities are activities that elicitate the need to develop mathematical models towards *finding successful solutions of problems*. The creation of these models are regarded as the most powerful mathematical activities in which a student might engage. By engaging in mathematical modelling, the *interconnectedness of mathematics* can be experienced. Glas in Chamberlin and Moon (2005:42) listed four educational outcomes that can be achieved by modelling in the mathematics classroom. Models and modelling can help students to:

- recognise the interconnectedness of mathematics and other disciplines;
- recognise the various perspectives on a domain of knowledge;
- develop their creativity in mathematical thought; and
- be able to consider mathematics as practical and applicable in many ways.

Models also bridge the gap between appearance and reality, or rather, they provide useful bridges that *connect our worldview with our experiences*. A person’s view of reality and how knowledge is required, depends on the person’s worldview, which again is based on his or her most basic beliefs. Thus, his or her beliefs or worldview presuppositions will have an effect on the choice of models to be used (Byl, 2003:1,15). These benefits emphasise the important use of such a tool in the learning and teaching of mathematics. Niss in Bahmaei (2013:46) proposed the following five arguments for the inclusion of mathematical modelling in mathematics education:

- The *formative argument* focuses on the students' personal mathematical development regarding general capabilities and attitudes, as well as open-mindedness and self-confidence;
- The *critical competence argument* emphasises the importance to make students aware of the use and possible misuse of mathematics in society;
- The *utility argument* stresses the usefulness of mathematics in all domains – mathematical as well as extra-mathematical situations;
- The *picture of mathematics argument* aims to provide the students with a rich faceted picture of mathematics as a science and an integral part of society and culture;
- The *promoting mathematics learning* argument focuses on the instrumental aspects of modelling in the students learning of mathematical knowledge.

Other advantages relating to the learning and teaching of mathematics through mathematical modelling are as follows:

2.6.1 Conceptual and procedural understanding

Zbiek and Conner (2006:105-110) accentuate changes in both conceptual and procedural understanding that takes place during the modelling process. Cognitive learning styles can be characterised in two contrasting categories: surface or shallow processing styles, or a deep-processing approach. The former approach relates to the rehearsing and memorising of study material, characteristics of the traditional approach to mathematicas learning and teaching where students prefer a linear task and pay attention to operational detail and procedural information. This learning style relates to instrumental understanding where students merely apply previous learned procedures and algorithms without necessarily gaining a relational understanding of the task (Skemp, 1976). Refer to Section 1.3.1 for further discussions on instrumental and relational understanding. However, mathematical modelling allows students to develop holistic strategies that exhibit a global approach towards learning as they learn to focus on main ideas and construct overall conceptions. In this deep-processing approach, students are actively engaged in understanding the material, by relating the ideas and arguments of others to their own experiences, as well as to the evidence provided (Boekaerts, 1999:447). This aspect of learning is in line with the constructivist approach as used in the RME theory of learning (Section 1.7). As students are actively involved in their own learning processes, learning can occur while students observe mathematically if new connections are made.

Mathematical insight can develop through real-world insight when students value the importance of mathematics in the real-world contexts. Insightful learning is an active process, whereby the student links new knowledge to old knowledge, with the new knowledge having some meaning for the student. Insightful learning is situated and thus takes place in a certain context and is regulated by meta-cognition and motivation (Maaß & Mischo, 2011:110) Then, by combining a set of properties and parameters into an entity, conceptual understanding of mathematical entities can develop. Both conceptual and procedural understanding can increase through analysing situations, evaluating mathematical ideas, and comparing and reflecting on their own mathematical thoughts in collaboration with others. The most meaningful conceptual developments are likely to occur while students are challenged to repeatedly express, test, and revise their own current ways of thinking, rather than being led by their teachers' ways of thinking (Harel, 2008a:489; Lesh & English, 2005:489). As a result of describing their own problem-solving processes, students gradually develop dynamically changing and manipulatable problem-solving personalities, comprising of meta-cognitive functions, problem-solving strategies, beliefs, attitudes, feelings and values (Lesh & Zawojewski, 2007:770-8), that are to be discussed next.

2.6.2 Higher-order thinking – meta-cognition

Resnick (1987:44) noted that "higher order thinking has always been a major goal of elite educational institutions. The current challenge is to find ways to teach higher order thinking within institutions committed to educating the entire population". The higher-order thinking characteristics that he refers to, include an awareness of one's strengths and weaknesses relating to solving problems, the development of self-regulatory abilities to monitor and regulate efforts, as well as the motivation and perseverance to grapple with a task to obtain a well-designed solution to a specific problem (Lesh & Zawojewski, 2007:770-8). These characteristics all belong to meta-cognition. Where cognition is involved in doing, meta-cognition relates to choosing and planning what to do, and then monitoring what has been done (Garofalo & Lester Jr, 1985:164). Resnick

(1987), Schoenfeld (1992:334) and Silver (1982) were influential in drawing attention to the importance of meta-cognition. They regard meta-cognitive actions as the driving forces in solving real-world problems. Students who lack meta-cognitive competencies, are often not able to direct their own learning (Maaß, 2006:118; Wolters, Yu, & Pintrich, 1996:215).

Research indicates that meta-cognition is a thinking process common to all branches of mathematics (Goos & Galbraith, 1996:229). A person who thinks mathematically has a specific way of seeing, representing and analysing real-world situations. Such persons have acquired the habits of interpretation and sense-making as discussed in the introduction of this section. Studies indicated that students who were taught in the traditional framework, believe that answers and methods to problems will be provided to them. As they did not learn to figure out methods by themselves, they play a passive role in learning mathematics and think of mathematics as 'handed down' by experts for them to memorise (Parmjit & White, 2006:343). This approach to mathematics teaching and learning has serious ramifications for the development of meta-cognitive skills.

Mathematical modelling can foster the development of such meta-cognitive skills. Middleton, Lesh and Heger's (2003:425) study revealed that, during problem-solving episodes, students' abilities to take part in meaningful discourse improve while they externalise their mathematical thinking to group members when proposing their varying perspectives. By coordinating, negotiating, and sometimes rejecting their ideas as a result, students overtly assess and revise their mathematical ways of thinking. The role of meta-cognition in mathematical problem-solving is concerned with two related components: the *knowledge of one's own thought processes* (meta-cognitive knowledge), as well as the *regulation and monitoring of one's activities* (self-regulation) during the problem-solving task (Flavell, 1976; Lester & Kehle, 2003:508).

2.6.2.1 Meta-cognitive knowledge

Meta-cognition thus includes reflecting on cognitive activities, as well as making decisions to alter the activities when needed (Artz & Armour-Thomas, 1992:139). This higher-order cognition can be described as the *thinking and management about one's own thinking*, or rather knowledge about knowing and learning. Knowledge about cognition can be categorised (although not always clear-cut) according to whether it bears upon the influence of *person*, *task or strategy* factors as follows:

- Meta-cognitive knowledge included in the *person* category (declarative meta-cognition):

This category refers to the ways we think about our own thinking, about tasks, and the strategic knowledge relating to cognitive strategies that students use to learn, remember and understand the material that they study. The student knows *what* different types of strategies are available for memory, thinking, problem-solving, etc.

- Knowledge falling in the *task* category (procedural meta-cognition):
  Procedural meta-cognition relates to the planning and monitoring of one's own actions. When students acquire this, they know *how* to use and enact different cognitive strategies.
- Meta-cognitive knowledge about *strategies* (motivational metacognition):
  When empowered with motivational meta-cognition, students' beliefs, affects and motivation guide them to know *when* and *why* to use different cognitive strategies (Garofalo & Lester Jr, 1985; Maaß, 2007:65; Pintrich, Wolters, & Baxter, 2000).

2.6.2.2 Self-regulated learning

Self-regulated learning incorporates meta-cognitive knowledge, as such knowledge provides direction for making decisions to employ a specific strategy, and to monitor one's understanding of a task (Garofalo & Lester Jr, 1985:165). Self-regulation refers to the ability to develop knowledge, skills and attitudes to be transferred from one learning context to another. To regulate one's own thinking and learning processes, three skills need to be applied: *planning*, *monitoring* and *evaluating* (Schraw, 1998:26). Planning involves the decisions to be made regarding time management, what strategies to use, how to approach the problem, etc. The core components of self-regulation is the real-time monitoring and assessing of one's progress, and the subsequent reacting in response to these assessments (Schoenfeld, 1992:355). Monitoring refers to one's awareness of comprehension and task performance. Evaluating deals with reflection and making judgements about the processes and outcomes, as well as acting on them (Pintrich et al., 2000:45). These management strategies enable the students to *manage and control* the material and resources that they have at their disposal to reach their goals, as well as to *persevere and maintain their intellectual engagement* when grappling with difficult tasks. In a study of Pintrich and De Groot (1990:37-40), self-regulation proved to be a strong predictor of academic performance, suggesting that the use of self-regulating strategies such as comprehension, orienting, monitoring, goal setting, planning, effort management, persistence, evaluating and correcting are essential for academic performance in various types of classroom tasks. Students develop the abilities to set their own goals, to apply appropriate methods and techniques related to the problem content and goal, as well as to judge their own process (Boekaerts, 1999:449). This managerial or strategic aspect of self-regulated learning is regarded by Schoenfeld (1983:20) of utmost importance in problem-solving, as it allows the students to select a framework for the problems that they need to solve.

The application of self-regulation can be interpreted as follows: When a framework is established, certain branch points come forward where the student needs to decide which direction a solution should take. During this process, new information continuously emerges which requires rethinking and new decisions are made to decide whether the existing solution path should be abandoned, or what (if anything) should be retrieved from previous abandoned paths or from paths that are not taken. The implementation is continually compared against the student's expectations to determine whether more interventions are needed. This interpretation suggests that students must be able to understand not only the *what* of cognitive strategies, but also *how* and *when* to use strategies appropriately.

The process of mathematical modelling offers a platform to improve self-regulated learning. While students are engaged in a modelling task, they learn to select, combine and coordinate cognitive strategies in effective ways, and thereby improve their abilities to direct their own learning and to acknowledge and use meta-knowledge (Boekaerts, 1999:454; Pintrich & De Groot, 1990:33). Mathematical modelling also allows for student reflection while students repeatedly express, test and revise their own ways of thinking about the solution. By using a variety of reflection tools, students can develop their own personal models of modelling. Examples of such reflection tools include the changing roles of the individuals during the modelling activity, the values, attitudes or feelings that can contribute to higher levels of engagement, and the problem-solving strategies that are productive during the different stages of modelling. These tools can form a potential direction for developing student-based self-assessments (Hamilton et al., 2008:16). Mathematical ideas and higher-order thinking develop interactively, and model-eliciting activities have the characteristics to elicitate such reflective activities (Lesh & Zawojewski, 2007:770-778).

2.6.2.3 Motivation

Closely related to self-regulation, is motivation. Developing the above strategies in isolation do not promote student achievement, but students must also be *motivated to use* the strategies as well as to *regulate* their cognition and effort (Pintrich & De Groot, 1990:33).

Zbiek and Conner (2006: 105-6) also recognised three different types of motivation while students are engaged in mathematical modelling activites. The first type is *confirmation that real-world situations appeal to some learners*. Students may be excited by the appeal of the context or they may see a connection between some mathematics and some real-world issues which allows them to believe that mathematics may be useful. However, this type of motivation does not necessarily develop further as the student is involved with the modelling tasks or in studying mathematics. The second type of motivation is simply motivation *to continue studying mathematics* in general as students believe that mathematics may be a tool to unravel complex real-world phenomena. These students understand that mathematics is applicable to the real-world. In the third type of motivation *to learn new mathematics* as the modeller actively seeks understanding of the needed mathematics by altering existing knowledge or adds new connections to known pieces of knowledge (Zbiek & Conner, 2006:90). This need for a deeper understanding can be experienced within several subprocesses: mathematising, combining, analysing, examining and communication.

Fielding-Wells, O'Brien, and Makar (2017:238) emphasise the strong connection that exists between motivation and engagement. Motivated students are more willing to focus their attention on a particular goal, which intensifies their persistance to continue to carry out specific activities. Engagement, that can be observed through the students' interactions with learning, can thus be seen as a consequence of motivation, which is more challenging to observe. Students' engagement with a task can be directed to their beliefs and attitudes, their general conduct and commitment towards learning, or their desire to invest in learning. Again, their desire to invest in learning can also originate from different sources. Firstly, students' academic performance can be a motivation for learning. However, Wigfield, Eccles, Roeser, and Schiefele (2008:69) note that individual beliefs, values and goals are key sources of motivation. The acknowledgement of the psychological, social and cultural influence on students' motivation and accompanied learning, as well as the importance of connecting learning to real-world problems and experienes, makes mathematical modelling the ideal tool for promoting modelling competencies and developing a deeper understanding of mathematics and problem-solving.

2.6.2.4 Productive disposition – beliefs

When students are immersed in sense-making situations which they can relate to their own direct experiences, their understanding of the scope and usefulness of mathematics is deepened and broadened (Bahmaei, 2013:46). Kilpatrick, Swafford and Findell (2001) recognised five interwoven and interdependent components to constitute one's mathematical proficiency: conceptual understanding, procedural fluency, strategic fluency, adaptive reasoning and productive disposition. The first four strands relate more to cognitive processes, whereas productive disposition encompasses issues such as a person's affect, beliefs and identity – aspects that are necessary to develop the other strands effectively, and vice versa. Productive disposition is defined as the "tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001:131).

The tendency to see sense in mathematics is in essence one's beliefs about the nature of mathematics (Siegfried, 2012:25). Beliefs can be interpreted as an individual's understandings and feelings that shape the ways that the individual conceptualises and engages in mathematical behaviour (Schoenfeld, 1992:359). It differs from knowledge in the ways that beliefs can be held at various levels of conviction, are not agreed upon by all, are not absolute, but can change over time, and are not subjected to vigorous testing. People possessing productive disposition, are confident in their mathematical knowledge and abilities, and are willing to engage in complex problem-solving activities to increase their knowledge and abilities – they are doers of mathematics, not passive 'learners' of mathematics (Siegfried, 2012:3). Resnick (1987:41) affiliates 'disposition' with a 'habit of thought', which can be *learned and taught*, implying that humans are not born predisposed to a specific affection towards mathematics, but it can be altered in a positive direction through effective learning and teaching. This statement implies that not only the beliefs of the students, but also those of the teachers need to be altered.

- **Beliefs of students**

Students have cultural assumptions about mathematics (beliefs, attitudes, feelings or dispositions) that are shaped by their school experiences, which can have powerful (and often negative) consequences. For example, traditionally, students did the mathematics as requested by the teacher. They learned to know the mathematics through remembering and applying correct rules and algorithms, and the mathematical truth was determined when the answer was endorsed by the teacher. Getting to the correct answer quickly, was a determining factor for students to be successful in mathematics, which ultimately caused students to give up on a problem after unsuccessful attempts, without any perseverance. Also, as traditional word-problems tend to be whimsey, students ignore the context of the problems and only focus on performing the algorithm and writing down the answer (Schoenfeld, 1992:359).

Some of the common beliefs that can lead to strong and often negative influences on students' mathematical thinking, is the belief that mathematics problems have only one correct answer, which is produced by only one correct way of solving it. Further negative beliefs are that the understanding of mathematics is reserved for the priviledged gifted students and should be memorised by the rest, that mathematics is a solitary activity done in isolation, and that mathematical problems should be solved in a short period of time, and has no relevance to the real-world. Lastly, students often believe that formal proof is irrelevant and unnecessary to the processes of discovery or invention (Lesh & Zawojewski, 2007:776).

Mathematical modelling allows for opportunities for students to alter these beliefs as students learn to grapple with meaningful situations and develop new knowledge by building and expanding their existing knowledge base. Meaningful situations motivate students to persevere in their tasks, and it also allows for opportunities to view mathematics as useful when students learn to apply the mathematics in various real-world situations. Research has proven that when average-ability students can recognise the need for a specific type of conceptual tool, they often are able to develop concepts that are both powerful and mathematically important (Lesh & Doerr, 2003:11,24).

- **Beliefs of teachers**

The mathematical beliefs that teachers hold also contribute to mathematics teaching efficacy and directly affect the beliefs that their students hold. The set of teachers' predetermined beliefs about mathematics and the teaching and learning of mathematics also include cultural assumptions. Such assumptions are often based on their prior experiences as students, which often relates to teachers holding a traditional view of mathematics, where memorisation rather than understanding plays a major role (Briley, 2012:3). Teachers' knowledge, beliefs, decisions and actions affect what is ultimately learned, though students' expectations, knowledge, interest and responses also play a crucial role. Students' negative beliefs and dispositions towards mathematics teaching can be limited through changes in instruction, teachers' practices, the curriculum, or the culture of the classroom (Lesh & Zawojewski, 2007:776).

The view and attitudes of mathematics teachers who teach through mathematical modelling, requires various changes to their traditional outlook to conform to a constructivist perspective on teaching and learning mathematics as discussed in Section 2.2. Bahmaei (2013:46) suggests that educators need to try to:

- modify their attitudes towards mathematics if their attitudes are negatively influenced by their own mathematics learning experiences;
- revise their beliefs about the usefulness of mathematical knowledge in real-world problem-solving;
- amend their current classroom practices to allow real-world situations to become the starting points for mathematical activities;
- listen to the ways of thinking and mathematical ideas of students with diverse cultural, ethnic, linguistic communities.

These changes in the teachers' beliefs and behaviours can lead to a classroom culture that supports learning with understanding, thus motivating the students to learn through mathematical modelling. Galbraith and Stillman (2001) in Bahmaei (2013:46) commented that a teacher has to be *willing and ready* to create and manage open situations. Such situations are continuously transforming and the teacher cannot foresee the final result. One of the main differences between teaching and learning modelling and other teaching strategies, is the use of students' own initial ideas that are relevant, albeit primitive, egocentric and biased, rather than relying exclusively on the descriptions of experts' behaviours and experiences. The goal is now to improve their abilities to use, extend, refine and develop their own ideas regarding the problems they are solving (Lesh & Zawojewski, 2007:794).

2.6.3 Representational fluency

Mathematical thinkers always value the importance of representational fluency, as it is "at the heart of what it means to understand most mathematical constructs" (Lesh & Doerr, 2003:16). To enhance the communication capability and conceptual flexibility needed to solve real-life problem situations, such fluency is critical. Professional problem-solvers often work in teams with diverse specialities and they need to represent important concepts to team members in various forms. Representations of real-life or actual situations are conceptual systems (human constructs) that are represented through spoken language, written language, diagrams and graphs, as well as through concrete models and experience-based metaphors. The conceptual models that students create have both internal as well as external components. The internal components refer to the mental constructs or conceptual systems, while the external components refer to the artefacts of representations (Lesh & Carmona, 2003:71). We can thus regard representations as the projection of the internal conceptual systems onto the external world.

Freudenthal's theory on RME promotes the construction of a real-world model to describe a specific situation within a specific context with the aim to connect mathematics to reality and to emphasise the human aspect of mathematics. Students' world-views are interpreted differently, and their real-world models express their own subjective understandings of the situation on hand, emphasising the situated character of knowledge. Representational systems throughout the modelling process should aim to assist the individual student as well as the group to develop a deeper understanding of the problem at hand.

When the students work with the MEAs where they have to describe, explain, create and mathematise constructions which directly reveal their interpretations of the mathematical situations that they are confronted with. The various representations accentuate different aspects of the problem situation and students can learn to move flexibly from one representation to another while examining and critiquing their understandings of the situation. By interpreting the problem-solving situations mathematically, their interpretations can "go beyond mathematics and also include feelings, dispositions, values and beliefs" (Lesh & Zawojewski, 2007:784-785), which again paves the way towards developing a holistic set of competencies to be used in their future lives and careers. These activities can generate *trails of documentation* that do not only provide information about the final result, but also reveal information relating to the students *ways of* *thinking* and as such also support the productivity of ongoing learning and problem-solving development (Lesh et al., 2000:594).

2.6.4 Cooperative classroom environments

As the modelling approach in this study focuses on students working in small groups of four, it is essential to investigate the possible advantages of a cooperative learning environment while being engaged in mathematical modelling.

Literature indicates that meta-cognitive competencies can be developed when classes are designed appropriately, as it develops simultaneously with subject knowledge (Maaß, 2006:118). Maaβ points out that meta-cognition is developed where the classroom environment is branded by discourse, individual perceptions, and discussions of different arguments, together with a search for understanding, comprehension, systematisation, questioning, inquiry, and reflection. Self-monitoring can be supported by the educator by asking open-ended questions relating to the *why* and *what* the students are doing. Successful classroom performance presupposes the existence of both the *will* and the *skill*, which allows a dynamic interaction between motivation, cognition, and meta-cognition throughout the learning process and over time (Metallidou & Vlachou, 2007:4). Students' learning styles influence how they perceive, interact with, and respond to a learning environment. Hall in Ma and Ma (2014:2) defined a learning style as "the way in which a person begins to concentrate on, process, internalise and remember new and difficult academic information". Learning styles can pertain to either students' individual processing of information, or to students' individual relationships with other. Generally, when dealing with higher-order tasks such as problem-solving, cooperation results in more effective learning than individual learning. A student's learning style can have a profound impact on his or her mathematics performance. People are not born with a predetermined learning approach, but as explained in Section 2.2, they learn to conduct their learning through socialisation, which is unique to each culture (Ma & Ma, 2014:3).

Highly successful students are skilful users of advanced learning strategies in their learning and they know how to utilise meta-cognitive skills in the process of their learning. (Ma, Jong, Yuan, Meyer, & Benavot, 2013:225). Students' experiences in the classrooms influence largely their beliefs about formal mathematics, which again shape their behaviour in ways that have extraordinarily powerful (and often negative) consequences. The mathematical practice in the classroom shapes and is shaped by beliefs of students (and teachers, and society). These implications are far-reaching, as the classroom culture determines in part students' choices of future courses, intended majors, and subsequent approaches to mathematics instruction taken by teachers (Lesh & Zawojewski, 2007:778).

This section provided an explanation about the potential benefits of teaching and learning mathematics through mathematical modelling to students, teachers and researchers and thereby addressed the third aim of the research question (Section 1.8.3). As argued, these benefits are far-reaching and allow students to experience the world of mathematising in all its facets. The first sub-question of the research question has also been attended to, as the discussions in this chapter positioned mathematical modelling as a central tool to develop mathematical reasoning and understanding.

2.7 CONCLUSION

Schoenfeld (1992:335) summarises the mathematics enterprise as one comprising of a fundamentally social activity, where the communities of trained practitioners engage through observations, studies and experimentation in the science of patterns. The *mathematical tools* needed for learning to think mathematically are abstractions, symbolic representations and symbolic manipulations. However, the *ability* to think mathematically is far more than being trained to use such tools. One also needs to develop a *mathematical point of view* – being able to assess the mathematisation processes and know when and how to apply them – as well as *competence* with the tools of the trade towards the goal of understanding structure, or *mathematical sense-making* (Schoenfeld, 1992:335). It is therefore not surprising that modelling is a central theme in mathematics education. As learning and understanding mathematics empower us to understand the world around us, to cope with everyday problems and to prepare us for future professions, it is a discipline which every human being ought to learn. Nearly all questions concerning human learning and teaching of mathematics, affect and are affected by relations between mathematics and the real-world (Blum, 2002:151).

Throughout this chapter literature has proven that *mathematical modelling* allows students to develop a mathematical point of view and mathematical sense-making. MEAs put students in situations where they are confronted with the need to construct a mathematical model or models, and while producing their products or solutions, trials of documentation are generated that reveal explicitly important characteristics of their underlying ways of thinking and ways of understanding. Such information benefits the researchers, the teachers as well as the students. Opportunies for students to develop their mathematisation processes which extend beyond their regular curriculum are allowed through the use of MEAs (English, 2007:8). Interdisciplinary learning is fostered while students work on real-world situations and learn to apply their mathematical knowledge and skills in their daily lives. Mathematical modelling does not only nuture the development of cognitive competencies, but meta-cognitive competencies such as self-regulation, motivation, ways of thinking, and beliefs are also developed. This holistic view on teaching and learning supports students to successfully bridge the gap between mathematics learned in school and mathematics needed for their future careers.

This chapter aimed to answer to the first sub-question of the research question (Section 1.8.3): How/where does mathematical modelling fit into the context of mathematical teaching approaches to develop mathematical reasoning and understanding? To answer this question, the researcher investigated and explored three dimensions: Firstly, and investigation of the various perspectives of mathematical modelling teaching and learning was conducted in Section 2.3, to place mathematical modelling in the context of mathematical teaching approaches. Secondly, the theoretical underpinnings of mathematical modelling and model-eliciting activities were explained in Sections 2.4 and 2.5, while the benefits of teaching and learning mathematics through mathematical modelling were explored in Section 2.6.

Chapter 3 builds on this chapter and investigates relevant competencies relating particularly to engineering technician students' future careers. These competencies will then be mapped to mathematical modelling competencies, to determine the relevant competencies to be investigated in this study. A taxonomy to guide the development of such competencies will also be designed, as clear guidelines and assessment methods are required to allow for determining the level of proficiency in any one of these skills.

CHAPTER 3
ENGINEERING AND MATHEMATICAL MODELLING COMPETENCIES

… sweep streets like Michelangelo painted pictures. Sweep streets like Beethoven composed music … Sweep streets like Shakespeare wrote poetry. Sweep streets so well that all the hosts of Heaven and Earth will have to pause and say, "Here lived a great street sweeper who swept his job well." ~ *Martin Luther King Jr (1929 – 1968)*

3.1 OVERVIEW

Chapter 1 explained the motivation for the study and the consequent gap in mathematics and engineering technician education. Within the South African context, Woollacott (2003:2) stresses the concern for students that are under-prepared to successfully complete their qualifications, or to succeed in their professional lives. Lawless (2005:3-4) ascribes the cause of the widening gap to a variety of reasons, of which the standard of education, drop-out rates at tertiary institutions and 'fast tracking' are considered to be major role players. She also comments that South African school learners have not grasped the basic mathematical principles, and she believes that changes in school mathematics do not adequately support the students who wish to continue with mathematical studies beyond school level (Lawless, 2005:82). Internationally, many deficiencies in engineering education have been identified. Anderson in Crouch and Haines (2004:198) focused on the students' inabilities to construct viable mathematical solutions from real-world problems and noted that rote learning still seems to be a preferred method to relational understanding. Newly graduated engineers often experience difficulties to solve problems in real-world contexts, as they have to consider many and different factors, a far cry from the routine problem-solving exercises they were doing at school or university (Spinks, Silburn, & Birchall, 2006). Team-working, decision-making, reasoning and communication are also considered as major deficiencies in establishing competitiveness and performances of engineering businesses (Bennett, 2002:470; Meier, Williams, & Humphreys, 2000:383).

The discussion emphasises the current gap in education to enhance engineering technician students' abilities to solve real-world problems, which is a primary task of any engineer (Parmjit & White, 2006:51). Section 1.4 noted that no in-depth studies were found that identified and investigated the engineering technician competencies that can co-develop with mathematical modelling competencies through modelling-based mathematics teaching and learning, and this study will thus attempt to fill this gap in knowledge in the field of mathematics and engineering education.

The literature study in Chapters 1 and 2 suggests that, within a socio-constructivist framework, mathematical modelling is a tool which can be used to develop students' problem-solving abilities, and allow them to reach deeper levels of understanding. This chapter will build on the previous chapter's explanation of modelling and model-eliciting activities, and aim to identify and examine relevant competencies for successful mathematical modelling and engineering education. Firstly, an understanding of the engineering profession and related requirements for successful engineering technicians will be discussed, followed by an investigation and understanding of engineering competencies. A theoretical understanding of the mathematical and mathematical modelling competencies required from competent modellers will then be provided through a thorough investigation of present and past literature. Once the mathematical modelling competencies have been identified, they will be compared to engineering technician competencies. These comparisons will serve as confirmation that the competencies under investigation have the potential to benefit the engineering students in their future professional lives, in becoming engineers who have done their jobs well, as expressed in the above quotation of Martin Luther King Jr (West, 2015:67). The rationale is to strengthen the support and motivation to investigate those specific competencies in this study. It also serves to guide the focus towards engineering technician students specifically, by developing a hypothetical learning trajectory (HLT) to investigate the development of the relevant competencies. As this study takes place in the civil engineering department of a university of technology, mathematical modelling competencies need to be identified specifically for engineering technician students' purposes. This chapter will thus address Aims 4, 5 and 6 of the research question to determine the required engineering technician and mathematical modelling competencies that can co-develop through mathematical modelling, leading to a deeper understanding of mathematics (Section 1.8.3):

Sub-question 2: What engineering technician and mathematical modelling competencies can co-develop through mathematical modelling?

Aim 4 Explore the most essential engineering technician competencies that are required from the engineering discipline (Section 3.5).

Aim 5 Identify the mathematical modelling competencies that can be developed through mathematical modelling as suggested by literature (Section 3.6).

Aim 6 Establish the specific competencies that form the focus of this study to address improved reasoning and understanding of mathematics (Section 3.7).

3.2 THE CHANGING LANDSCAPE OF ENGINEERING EDUCATION

Engineering is concerned with the application of science, mathematics and technology to adhere to the requirements of society, to assist towards economic development, and to provide various services to society. Thus, engineering is an activity of solving problems in the real-world (IEA, 2013). We live in a world where ever-increasing change is a constant factor; what once took years to become obsolete can now become outdated within months or even weeks. The flood of information that is instantly available through internet and virtual environments, continue to grow every day as our society becomes increasingly globalised. This global society requires globalised markets, as domestic markets are unlikely to compete in the future if industries cannot compete internationally. The well-defined body of knowledge that was needed for an engineer became more complex over time as they work in multi-disciplinary environments, where collaboration with various other disciplines are fundamental aspects for improved technological development. Sustainability of our environment has received enormous attention over the past years, to such an extent that people's health and safety aspects became a critical consideration, as well as the possible exhaustion of non-renewable resources. Thus, apart from profit and mass production, a growing awareness towards a social responsibility for society and our ways of life is maturing. No longer can decisions be entertained without consideration for all possible consequences, as they can directly influence the well-being of society if appropriate actions are not taken when required. Globalisation implies being part of a society, which means that decision-making processes require the participation of all possible role players to gain a better understanding of the consequences of those decisions (Duderstadt, 2008:2; Male, 2010:26; Rugarcia et al., 2000:3-5).

It is within this complex environment that we need to critically scrutinise the future of engineering education, as competency gaps can occur when engineering education cannot conform to the requests of the ever-changing workplace (Male, 2010:27). A study of Spinks et al. (2006:19) indicates that the roles of engineering graduates have changed significantly during the past years. Today, engineering graduates are not only expected to design and produce new products and to solve customers' problems, but many of them are also responsible for the sales and marketing of such products and services. This situation causes deviation in their roles as traditional engineers and consequently has far-reaching implications for engineering education as well. Not only does the educational institution have to deal with academic excellence, but it also needs to address the gaps between the competencies that are developed in mathematics and engineering education, and the competencies that are demanded from the ever-changing workplace (Brunhaver, Korte, Barley, & Sheppard, 2017:1). As globalisation caused the need for rapid developments and changes in organisations during the past few years, business competitiveness became progressively important and organisations that possess the abilities to adapt and compete in such ever-changing environments on the basis of their core competencies and skills, rather than focusing on specific job functions, have shown to be successful in sustaining their competitive edge (Lawler, 1994:3). Lawler further suggests that more attention should be given to the development of individuals and their needed competencies, rather than treating engineering jobs as the building blocks of complex organisations. To address such development of engineering technician competencies, the following section aims to focus on consistent themes that appear in the literature.

3.3 REQUIREMENTS OF A SUCCESSFUL ENGINEERING TECHNICIAN

A multitude of competencies are needed for engineering technician students to be productive and effective in their careers, as they consistently have to adapt to the challenges and changing demands from the workplace (Rugarcia et al., 2000:5). Research indicated that problem-solving, team-work and effective communication are some of the most important aspects that need to be addressed. Meier et al. (2000:377,378) noted that technical competent employees who embrace skills such as problem-solving and effective team-work, learn to understand how their teams fit into the bigger picture of their organisations and can add value to their workplace and thereby promote agile competitiveness. The Accreditation Board for Engineering and Technology (ABET) also emphasised the importance of being able to function as a team member within a multi-disciplinary environment, to understand the importance of responsibilities, and to be able to engage in effective communication (Meier et al., 2000:384). Plonka, Hillman, Clarke, and Taraman (1994:693) investigated competency requirements for the manufacturing engineer of the 21st century and central to the core of their evolutionary model, is the “need to stretch one’s self within a team environment”, denoting the need to develop intellectual capacity, effective communication and team-work. Furthermore, professional competencies relating to aspects such as organising, motivation, effective communication, initiative, creativity, problem-solving and leadership are required, as these skills can be applied across professions and they allow people to participate in a flexible and adaptable workforce (Bennett, 2002:457). Rychen and Salganik (2002:11) describe a competent individual as one that can successfully participate in the world of work, in his or her community, and within society. To summarise, the requirements as stipulated by the engineering profession do not only focus on theoretical and technical knowledge, but also on professional competencies, denoting a change in focus from job-based to competency-based organisations (Brunhaver et al., 2017:1). To gain an understanding of the goal of engineering technician education, the profile of a successful professional engineering technician will be investigated with the purpose to address the fourth aim of the research question, namely to explore the most essential engineering technician competencies that are required from the engineering discipline.

3.4 GOALS OF ENGINEERING TECHNICIAN EDUCATION

During the Industrial Revolution, the engineering profession was essentially a practical profession. Not only did engineers envisage and design machinery, they were also responsible for the construction of their designs. In the early twentieth century, a shift towards science-based engineering occurred. This caused a divide between the science-based professional engineer and the engineer skilled in applying the technology, which resulted in the birth of the engineering technologist (IEA, 2015). Over time, further distinctions were made, and today specific roles exist for a professional engineer, an engineering technologist, as well as an engineering technician.

To provide quality engineers that are much needed today, the International Engineering Alliance (IEA) was formed in 1989, with the main goal of improving engineering education globally.

Through agreements with 18 countries worldwide, this Alliance aims to improve competence in engineering education globally, to allow for the recognition of engineering qualifications amongst the members of the IEA (2015). The Engineering Council of South Africa (ECSA) became a member of this Agreement in 1999. The IEA represents three constituents that deal with the recognition of educational programs: the Washington Accord for professional engineers, the Sydney Accord for engineering technologists, and the Dublin Accord for engineering technicians.

Furthermore, four more constituents are involved with the competence standards as well as the mutual recognition of experienced engineering professionals: the International Professional Engineers Agreement (IPEA), the APEC Engineer Agreement with links to the Asia Pacific Economic Cooperation, and the International Engineering Technologist Agreement (IETA). The fourth Agreement, the Agreement for International Engineering Technicians (AIET), was established in 2015. The AIET agreed to an international standard of competence of practicing technicians that needs to be assessed to obtain registration in other member countries (IEA, 2015). Through these agreements, the IEA is regarded as an authoritative body on engineering education and professional standards. The IEA defines the fundamental purpose of engineering education as

> … building a knowledge base and attributes to enable the graduate to continue learning and to proceed to formative development that will develop the competencies required for independent practice (IEA, 2013).

During the inception years of the IEA, attention was focused on the inputs and processes of education. Curriculum structure, content and technical depth were the main concerns. The mid-1990s introduced a renewed interest in the relationship between the characteristics of modern society and the required attributes of engineers to be able to function effectively, resulting in the formulation of the *Graduate Attribute* document (IEA, 2015). The IEA laid down a taxonomy of graduate attributes, suggesting the crucial attributes which are expected from graduates. They compiled classifications for professional engineers, engineering technologists and engineering technicians. This study focuses specifically on the latter classification, as it is situated within a civil engineering department of a university of technology. The purpose of this classification was to guide the institutions towards a common reference in describing the outcomes of their qualifications, which serves as the first step towards global consensus on educational outcomes and professional competencies (IEA, 2013).

The graduate attribute profiles as stipulated by the Dublin Accord for engineering technicians, are as follows (IEA, 2013):

Table 3.1 - Graduate attribute profiles for engineering technicians

Differentiating characteristic	**Dublin accord - graduate attributes for engineering technicians**
Engineering knowledge	Apply knowledge of mathematics, natural science, engineering fundamentals and an engineering specialisation to wide practical procedures and practices, as specified in 1 to 4 respectively: 1. A descriptive, formula-based understanding of the natural sciences applicable in the sub-discipline. 2. Procedural mathematics, numerical analysis, statistics applicable in a sub-discipline. 3. A coherent procedural formulation of engineering fundamentals required in an accepted sub-discipline. 4. Engineering specialist knowledge that provides the body of knowledge for an accepted sub-discipline.
Problem analysis – complexity of analysis	Identify and analyse well-defined engineering problems reaching substantiated conclusions using codified methods of analysis specific to their field of activity (Also related to 1-4): 1. A descriptive, formula-based understanding of the natural sciences applicable in the sub-discipline. 2. Procedural mathematics, numerical analysis, statistics applicable in a sub-discipline. 3. A coherent procedural formulation of engineering fundamentals required in an accepted sub-discipline. 4. Engineering specialist knowledge that provides the body of knowledge for an accepted sub-discipline.
Design / development of solutions: breadth and uniqueness of engineering problems i.e. the extent to which problems are original and for which solutions have previously been identified or codified	Design solutions for well-defined technical problems and assist with the design of systems, components or processes to meet specified needs with appropriate consideration for public health and safety, cultural, societal, and environmental considerations. Apply knowledge that supports engineering design based on the techniques and procedures of a practice are required.
Investigation: breadth and depth of investigation and experimentation	Conduct investigations of well-defined problems, locate and search relevant codes and catalogues, conduct standard tests and measurements.
Modern tool usage: level of understanding of the appropriateness of the tool	Apply appropriate techniques, resources, and modern engineering and IT tools to well-defined engineering problems, with and awareness of the limitations. Codified practical engineering knowledge in recognised practice area, is required.

Differentiating characteristic	Dublin accord - graduate attributes for engineering technicians
The engineer and society: level of knowledge and responsibility	Demonstrate knowledge of the societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to engineering technician practice and solutions to well-defined engineering problems. Knowledge of issues and approaches in engineering technical practice is required, relating to ethics, financial, cultural, environmental and sustainability impacts.
Environment and sustainability: types of solutions	Understand and evaluate the sustainability and impact of engineering technician work in the solution of well-defined engineering problems in societal and environmental contexts. Knowledge of issues and approaches in engineering technical practice is required, relating to ethics, financial, cultural, environmental and sustainability impacts.
Ethics: understanding and level of practice	Understand and commit to professional ethics and responsibilities and norms of technician practice. Knowledge of issues and approaches in engineering technical practice is required, relating to ethics, financial, cultural, environmental and sustainability impacts.
Individual and team-work: role in, and diversity of team	Function effectively as an individual, and as a member in diverse technical teams.
Communication: level of communication according to the type of activities performed	Communicate effectively on well-defined engineering activities with the engineering community and with society at large, by being able to comprehend the work of others, document their own work, and give and receive clear instructions.
Project management and finance: level of management required for differing types of activity	Demonstrate knowledge and understanding of engineering management principles and apply these to one's own work, as a member or leader in a technical team and to manage projects in multidisciplinary environments.
Lifelong learning: preparation for and depth of continuing learning	Recognise the need for, and can engage in independent updating in the context of specialised technical knowledge.

As indicated in Table 3.1, the profile of an engineering technician stretches much further than the mere attainment of engineering knowledge. Engineers deal with real-world problems, and they need to adhere to specific standards regarding risk-management, responsibility, ethical behaviour, resource management, health and safety issues, as well as adequate consideration for the environment. This holistic view of the profession requires students to develop a multitude of competencies to complement the academic knowledge they acquired during their studies.

Competence can be defined as

> … the ability to meet complex demands successfully through the mobilisation of mental prerequisites. Each competence is structured around a demand and corresponds to a

> combination of interrelated cognitive and practical skills, knowledge, motivation, values and ethics, attitudes, emotions and other social and behavioural components that together can be mobilised for effective action in a specific context (Rychen & Salganik, 2003:4).

Rychen and Salganik (2002) identified three common competency characteristics:

- Firstly, competencies need to be *put in action* within a specific *context*, as this is the only way that they are observable. Apart from considering a specific context, competencies also depend on external demands and personal dispositions.
- Competencies can be *developed throughout one's life* and in a variety of settings by being actively engaged in various actions and interactions in both formal and informal educational contexts. *Learning environments* that are conducive to competence development therefore need to be created.
- Competencies encompass a *value aspect* and are regarded in various levels of importance, depending on the society's values. The competencies that will be investigated in this study, will have to complement the visions of both engineering and mathematics education (Rychen & Salganik, 2002:9-11).

The above characteristics denote cognitive as well as meta-cognitive aspects of competencies and cannot be acquired through rote learning. They require engagement with real-world problems and situations to promote the development of such competencies (Harris & Philander, 2015:26). To identify the development of specific engineering technician competencies relevant to this study, an investigation of past and present literature will be done in the following section.

3.5 INVESTIGATING ENGINEERING COMPETENCIES

This section investigates competence taxonomies from past and present literature, to allow the researcher to select specific competencies that serve the dual roles of developing mathematical reasoning and understanding through mathematical modelling, as well as attending to the gaps and demands of the engineering workplace. The responsibility to ensure that graduates are prepared for future engineering work and careers, rests heavily on the shoulders of educators (Male, 2010:25). Traditionally, engineers classified their work into technical work and non-technical work. Faulker (2007:349-350) believes this tendency is both flawed and harmful to the profession, as a focus on generic attributes in isolation can contribute to engineers' perceptions that generic attributes indicate low technical, theoretical, and practical skills. Meier et al. (2000:378) recommend a cross-disciplinary curriculum for engineering students to address the knowledge, skills and behavioural requirements for the 21st century. They propose a curriculum where technicians can understand problem-solving, where they can effectively fulfil their roles in various teams and where they gain an understanding of how their teams fit into the entire system, rather than to be isolated from the broader perspective of the firm.

To establish improved engineering education, Male (2010) profiled graduate competencies across various engineering disciplines in Australia, Europe, New Zealand and the USA. She distinguished between generic competencies, engineering-specific competencies, and generic engineering competencies. While engineering-specific competencies address the engineers' technical and theoretical skills, and generic competencies relate to non-technical competencies such as communication and management skills, *generic engineering competencies encompass both technical and non-technical competencies* required from graduates across all engineering disciplines (Male, 2010:26). Generic engineering competencies enable engineering individuals to successfully contribute to a well-functioning society (Male, 2010:41). Based on responses to demands from the workplace, Male, Bush, and Chapman (2009:5) identified eight crucial generic engineering competencies: communication, team-work, professional attitudes, engineering business skills, problem-solving, critical thinking, creativity, and practical engineering skills. Male denotes the key features of these competencies as knowledge, skills, attitudes and dispositions. All of these competencies are interrelated and vary in importance, depending on the context of specific situations (Male, 2010:42). The selection of competencies always depends on the stakeholders and their purpose, and therefore, the competencies relating to this study, will be selected to advance engineering technician students' mathematical understanding and problem-solving abilities.

The Accreditation Board for Engineering and Technology (ABET, 2017) established an accredited engineering program that assists students in attaining specific competencies to complete their studies. Passow (2012:95) investigated the ABET competencies to determine which of those competencies can be regarded as most important by engineering graduates. The competencies established as the most crucial in his study were team-work, communication, data analysis and problem-solving, denoting a balanced inclusion of both technical and non-technical competencies.

Apart from investigating the competencies that are required to work successfully as an engineer, it is also vital to recognise future needs. The rapid pace of change in both technological as well as non-technological aspects of work, compels industries towards increased focus on problem-solving, on creating systems solutions for solving clients' problems, and on increased complex management tasks to sustain value-added services (Spinks et al., 2006:4). Thus, technical expertise, being able to work across boundaries, and having the ability to work creatively and innovatively while employing relevant leadership and communication skills, will be needed to guide industries towards success in the future (Spinks et al., 2006:5). Further taxonomies relevant to the development of engineering technician students' competencies, are as follows:

3.5.1 The DeSeCo's taxonomy

The Definition and Selection of Competencies Steering Group (DeSeCo) established a taxonomy to determine the key competencies to think critically and reflectively, and to be able to apply a holistic approach to life. They focused on the economic as well as the social arena to identify relevant competencies (OECD, 2005). As far as the economic arena is concerned, DeSeCo examined competencies that can have an impact on the achievement of economic growth, and investigated competencies that can possibly influence participation in the labour markets (Rychen & Salganik, 2003:7). Competencies were then classified in three main categories: interaction in groups, autonomous work, and the ability to use tools interactively. Each of these categories was further described in terms of key competencies. These key competencies are indicated in Table 3.2:

Table 3.2 - DeSeCo taxonomy of key competencies (Ryche and Salganik, 2003:5)

Key competencies as identified by DeSeCo	
Category	**Key competency**
Interacting in socially heterogeneous groups.	• Relating well to others. • Cooperating. • Managing and resolving conflict.
Acting autonomously.	• Acting within the big picture or the larger context. • Forming and conducting life plans and personal projects. • Defending and asserting one's rights, interest, limits and needs.
Using tools interactively.	• Using language, symbols and text interactively. • Using knowledge and information interactively. • Using technology interactively.

Meta-cognitive competence development is again emphasised in the above classification. Professionals need to acquire the abilities to plan, work and interact in meaningful and effective ways, that can result in enhanced performance of organisations to acquire competitive advantages (Lawler, 1994:2). Furthermore, this classification also relates to the graduate attribute profiles for engineering technicians, as stipulated by the Dublin accord and explained in Table 3.1. Specific characteristics that are of interest in both taxonomies, address crucial attributes such as tool usage, individual and team-work, communication, management, and lifelong learning.

3.5.2 Competence classifications of Rugarcia, Felder, Woods and Stice

Complementing the classification of DeSeCo above, Robinson, Sparrow, Clegg, and Birdi (2005:124) identified the following competencies as crucial towards the development of an organisation's performance: technical competencies, motivation, problem-solving, decision-making, team-work, management and leadership, communication, planning, innovation and strategic awareness of the "wider business and customer context". Management and leadership refer to the ability to manage tasks, risks, and people. To contribute to an organisation's competitive advantage, engineers need to manage and apply technical knowledge and resources, they need to bridge cultural gaps between knowledge workers and managers, and they need to draw on personal attributes such as integrity, creativity, willingness to lead and the need for accomplishment (Rifkin, Fineman, & Ruhnke, 1999:54). Rugarcia et al. (2000:5) categorised the profiles of effective engineers in three components: knowledge, skills, and attitudes. By adapting this classification, competencies can be categorised as follows:

3.5.2.1 Knowledge

Knowledge relates to what the student knows and understands. As the collection of information that engineers are required to possess, increases at a faster rate than what the curriculum can cover, it will be impossible to teach engineering students everything they need to know to operate as competent workers. It is therefore important that students learn to integrate knowledge across disciplines, and to develop the critical skills to appropriately use such knowledge (Rugarcia et al., 2000:5).

3.5.2.2 Skills

Crucial cognitive and meta-cognitive skills required by engineering technicians to manage and apply their knowledge, can be divided in seven categories (Rugarcia et al., 2000:7):

- **Independent, interdependent and lifelong learning skills**

The Perry schema of intellectual and ethical development explains the progression of increasing cognitive development of students (Moore, 1994:46). Students undergo a shift in thinking, from being a *dependent learner* and view the teacher's knowledge as absolute and true, to becoming more independent in their learning and regarding knowledge from a variety of sources which may not always be absolute and true. They learn to critically evaluate the knowledge by looking at it from various perspectives, to determine what is known and what is unknown, and to acquire the necessary new knowledge and apply it in their specific situations. When they can elaborate on their knowledge, they can adjust and apply it in future applications (generalisation) and thereby reach the level of *independent learning*. However, educators should aim to further motivate the students to progress towards *interdependent learners* where they can perceive knowledge in context and experience the powerful resources of learning from peers. Being interdependent learners, allow the students to strengthen their abilities in assessing their own work as they communicate newly-acquired information to others, and they learn to use the strength of a group to compensate for their own shortcomings (Hofer & Pintrich, 1997; Rugarcia et al., 2000:8).

- **Problem-solving, critical thinking and creativity**

As explained in Chapter 2, problem-solving encompasses all aspects of understanding a problem in context, formulating a plan towards a possible solution, identify relevant and irrelevant aspects of the situation, stipulating assumptions, working through the problem, evaluating the problem, making adjustments and testing, finding alternative solutions, critiquing the solutions and their ways of thinking, judging and arguing ideas and concepts for further validation and plausibility of the solution paths and outcomes, and generalising solutions to be able to apply in similar scenarios in the future. These attributes assist students towards developing the competence to recognise the need for *critical and reflective thinking*, to address the complex demands and challenges of modern life (Rychen & Salganik, 2002:9-11).

- **Team skills**

Engineers are ordained to work cooperatively, and therefore they need to acquire skills such as listening, understanding, and to have consideration for others' perspectives and needs. They also need to develop leadership skills, management skills, the ability to take responsibility for the decisions they make and the know-how to handle possible interpersonal conflict, to name a few. Such skills not only assist towards professional and ethical behaviours as stipulated in the IEA Agreements (IEA, 2013), but it will also be of crucial importance in confronting the future technological and social challenges (Bennett, 2002:457; Meier et al., 2000:377,378; Plonka et al., 1994:693; Rugarcia et al., 2000:8). Complementing, but dissimilar to team skills, are collaborative skills. Collaborative skills are needed in the engineering industry to allow engineers to function in multidisciplinary teams and to develop effective communication (Marra et al., 2016). Whereas teams work towards shared goals, the shared goal is often just a small part of a collaborator's responsibility. Inputs from various collaborators are often required to solve a specific problem, and effective interaction is commanded in such cases (Robinson et al., 2005:125).

- **Communication skills**

Engineering work occurs in both the object-world as well as the social-world (Bucciarelli & Kuhn, 1997:220). While the former deals with traditional problem-solving activities, the latter focuses on interactions that take place during such activities. Team-work is a fundamental skill to be acquired by engineers, as they need to communicate ideas clearly and effectively across cultures, disciplines and languages with a multiple of stakeholders. The stakeholders can be other engineers, client companies, accountants, managers and the general public, to name a few (Male, 2010:27; Rugarcia et al., 2000:9). Communication skills include verbal, written, and presentation skills, and are interrelated with other people-based skills, such as management, leadership, team-work and argumentative skills. These skills are important throughout an engineer's career, and should already be emphasised during the students' formal study careers (Spinks et al., 2006:35).

- **Assessment skills**

Professional engineering technicians should also be able to assess accurately their own, as well as others' knowledge and skills. Self-assessment empowers oneself and enhances effective and confident learning (Rugarcia et al., 2000:9).

- **Integration of disciplinary knowledge**

Integrating knowledge obtained in the various subject areas can assist towards increased problem-solving skills. Engineering technicians need to learn the value of applying integrated knowledge to become competent problem-solvers, as real-world problems seldom require problem-solvers to view a challenge from a narrow context of an individual course (Rugarcia et al., 2000:9).

- **Managing change**

The changing nature of the world around us, technologically as well as social, prompts engineers to continually adapt and change to survive. They will have to acquire skills to use and adapt their current knowledge in new and different situations (Rugarcia et al., 2000:10).

3.5.2.3 Attitudes and values

Engineering decisions must always incorporate social, ethical and moral consequences (Rugarcia et al., 2000:10). Vesilind (1988:290) defines ethics as "the study of systematic methodologies which, when guided by individual moral values, can be useful in making value-laden decisions". Morrill in Vesilind (1988:292) explains moral values as a configuration of choices that results in satisfaction, fulfilment or meaning. Engineering technicians need to learn to make decisions about such qualities – being a good example to these engineering technician students is probably the most effective way of transmitting such skills (Vesilind, 1988:293).

3.5.3 Woollacott's taxonomy of engineering competency

A taxonomy developed by Campbell, McCloy, Oppler, and Sager (1993), focuses on disciplines from the perspective of human resource management, industrial psychology, and business management. They created the following categories of performance components to determine how job effectiveness can be accomplished through various aspects of an individual's work: job-specific task proficiency, non-job-specific task proficiency, communication, productive personal behaviours relating to demonstrating effort and maintaining personal discipline, and productive interactions with people. Such interactions relate to the facilitating of peer and team performance, supervision and leadership, as well as management and administration. Williams in Woollacott (2003) went a step further and proposed the importance of adaptive performance. He believes that job effectiveness also depends on one's ability to adapt to the ever-changing nature of modern work environments.

To address the concerns of high attrition rates and the diverse character of engineering students in South Africa, Woollacott (2003:2; 2007:7) investigated the development of important academic and professional engineering competencies. In combining many of the above aspects of engineering competence, including the documentation produced by professional and national bodies for accrediting engineering educational programs such as ECSA and ABET, as well as perceptions of employers and experienced practitioners from investigations done in over 20 countries over a span of 20 years, Woollacott (2003:7) designed a taxonomy for understanding the goal of engineering education by focusing on engineering competencies – what it means to be a competent engineer. He categorised engineering work in *core work functions* and *support work functions* (Woollacott, 2003:3). Core work functions relate to work initiating, planning, acquiring resources, performing sub-tasks and integrating sub-tasks. Support functions refer to those functions that support the core functions, i.e. managing one's work, evaluating effectiveness, interacting, communicating and resource management. Through this classification, he raised the importance of interrelated competencies: the different types of work require a different combination of competencies. Woollacott emphasised the fact that competencies are all intertwined and depending on the type of work, the extent and complexity of the combinations of competencies vary (Woollacott, 2007:6).

A definition of engineering competency is expressed in the form of Woollacott's 'Taxonomy of engineering competency' (Woollacott, 2003:12-13):

Table 3.3 - Woollacott's taxonomy of engineering competencies (Woollacott, 2003:12-13)

Major areas of proficiency	**Sub-categories**	**Competency: An ability to…**
1. Engineering-specific work	General engineering work	• Perform the different aspects of any engineering work or task namely **initiating and planning** the work/task, acquiring the **resources** needed, performing sub-tasks and **evaluating and synthesizing** results. • **Use** appropriate engineering and computer **methods, skills and tools** and properly assess, **analyse and interpret** the results they yield. • **Evaluate** effectiveness, productivity, profitability, quality, service, impact or implications of any aspect of work done or planned and a disposition to do so. • **Arrange, sort, retrieve and properly assess** data, knowledge and ideas.

Major areas of proficiency	Sub-categories	Competency: An ability to…
	Specialist engineering work	• Perform analytical work to solve existing and anticipated engineering problems and model relevant systems by **applying knowledge** of mathematics and the natural, engineering and computational sciences, as well as by **identifying, assessing, formulating and solving** divergent (relating to generative design questions) and convergent (relating to deep reasoning questions) engineering problems in a creative and innovative way. • Perform **design** work by converting concepts and information into detailed plans and particularities for the development, manufacture or operation of systems, processes, products or components that meet desired needs. • **Plan and perform investigations** to **test** that a design or product meets particularities, to develop products, components, systems or processes, or **search for new knowledge** that can be applied for the advancement of engineering practice.
	Engineering mixed with other work	• **Integrate** specialist engineering work appropriately with work relating to core functions, management, administration, supervision, projects, sales, consulting, entrepreneurship or teaching to achieve the broader aims of the business enterprise or stated objectives.
2. Non-engineering-specific work	General	• **Perform tasks and execute behaviours** not specific to one's specific job. • **Manage one's personal work** effectively to ensure that all aspects are properly coordinated, are progressing in a satisfactory manner and that problems that arise are dealt with appropriately. • **Support and help peers** and facilitate group functioning by being a good model, keeping the group directed and reinforcing participation by other group members. • Ensure that the **resources and capacity** to do good work are maintained, sustained, and where necessary, developed further.
	Supervision, leadership	• Influence the performance of subordinates through interpersonal **interaction** and influence, modelling, goal-setting, coaching and providing reinforcement. • Function as a **supervisor** in the 'line production' activities of the enterprise at the appropriate designated position in the supervision hierarchy.
	Management, administration	• Articulate **goals** for a unit or enterprise, **organise** people or resources to achieve these, **monitor** progress, help **to solve problems** or overcome crises that stand in the way, **control** expenditures, and **represent** the unit in dealing with other units or clients. • **Manage** a project and ensure that it is completed successfully, on time and within budget.

Major areas of proficiency	Sub-categories	Competency: An ability to…
3. Communication	General	• **Effectively exchange**, transmit and express – verbally, graphically and in writing – **knowledge and ideas** to achieve set objectives when communicating with colleagues, peers, clients, superiors, subordinates, engineering audiences and the larger community.
4. Inter-personal interactions	General	• **Interact effectively** and positively with colleagues, clients, superiors, subordinates, engineering audiences and the larger community. • Function effectively on **multi-disciplinary** teams through personal contributions and interactions with others that enhance their contributions.
5. Dispositions	See continuation of this table below	

Continuation of the above table, to further detail the dispositions:

Table 3.4 - Continuation of Woollacott's taxonomy of engineering competencies (Woollacott, 2003:12-13)

Dispositions		
Major areas of proficiency	**Sub-categories**	**Competency: An ability to…**
5.1 Personal dispositions	General	• Agreeable personal style, characteristics and self-management including maturity, initiative, enthusiasm, poise, appearance, values, goals, outlook and motivation. • Disposed to consistent commitment to all job tasks, to working at a high level of intensity and the willingness to keep working under adverse circumstances and to expend extra effort when required. • Disposed to taking responsibility within own limits of competence. • Interest and knowledge in contemporary issues.
	Discipline	• Disposed to maintaining personal disciplines and avoiding negative behaviours. • Being critically aware of the need to act professionally and ethically. • Being critically aware of the impact of engineering activity in a global/social setting.
5.2 Adaptive dispositions	Self-development	• Disposed to improving personal competencies in general. • Understands nature and importance of effective learning skills and can apply them. • Able to assess one's own performance effectively and accurately. • Disposed to improving critical knowledge, skills and dispositions to sustain or improve one's reputation and advancement prospects.

Dispositions		
Major areas of proficiency	**Sub-categories**	**Competency: An ability to…**
	Life-long learning	• Understands the requirement to maintain continued competence. • Able to and disposed to engage in independent and interdependent life-long learning through well-developed learning skills.
	Change management	• Able to manage the impact of change effectively and flexible, and to engage in new learning in coping with change.
5.3 Particular productive dispositions	Achievement orientation	• Works to meet required standards but also creates own measures of excellence. • Disposed to improve performance or improve morale, revenues or customer satisfaction by making specific changes in the system or in own work methods. • Sets and acts to reach challenging goals for self or others. • Innovates.
	Impact and influence	• Gives presentations tailored to audience, calculates the impact or own actions/words and adapts presentations or discussion to appeal to the interest and level of others. • Shows concern with professional reputation.
	Conceptual thinking	• Recognises key actions and underlying problems by observing discrepancies, trends and inter-relationships, crucial differences, past discrepancies. • Able to condense large amounts of information in a useful manner. • Makes connections and patterns by pulling together ideas, issues and observations into a single concept and identifies key issues in complex situations.
	Analytical thinking	• Anticipates obstacles, breaks problem apart systematically, makes logical conclusions, and sees consequences and implications.
	Initiative	• Persists in problem-solving when things do not go smoothly. • Exceeds job description. • Addresses problems before asked to. • Creates opportunities.
	Self-confidence	• Expresses confidence in own judgement. Sees self as a causal agent, prime mover. • Seeks challenges and independence, welcomes challenging assignments, seeks additional responsibility, states own position clearly and confidently.
	Interpersonal understanding	• Understands attitudes, interests, and needs of others and is good at discerning unspoken thoughts, concerns or feelings.
	Concern for order	• Seeks clarity of roles and information, checks quality of work/information, keeps records and an organised workplace, monitors data, projects and the work of others.

Dispositions		
Major areas of proficiency	**Sub-categories**	**Competency: An ability to…**
	Team-work and cooperation	• Genuinely values others' input and expertise and is willing to learn from others. • Empowers others, encourages those who perform well and gives them credit.
	Expertise	• Applies technical knowledge to achieve additional impact, goes beyond simplify answering a question and helps resolve others' technical problems. • Exhibits active curiosity to discover new things, makes major efforts to acquire new skills and knowledge, and to maintain an extensive network of relevant contacts.
	Customer service orientation	• Seeks information about the real, underlying needs of the client, beyond those expressed initially, and matches these to available (or customized) products or services.

Apart from the fact that Woollacott's taxonomy is framed within solid literature research, it addresses a very specific character of this study: first-year students in South Africa are not well prepared to enter engineering programs. As explained in Chapter 1, this study is concerned with first-year students that did not meet the entrance requirements for studying engineering at a University of Technology. The students are enrolled on a bridging course, which involves an additional six months of university preparation. Through more support and intensive tutoring, students enhance their chances of succeeding in the university's mainstream programs. The perspectives that Woollacott included in his study are comprehensive, as such that they include the categorisation of the major components as indicated in the DeSeCo taxonomy in Table 3.2, the classifications by Rugarcia et al. in section 3.5.2, the main components as explained by Campbell and Williams, perspectives from professional accreditation engineering bodies (ECSA and ABET), as well as perspectives from human resource management, educators and practitioners (Woollacott, 2003:10).

3.5.4 ECSA's taxonomy for competence development

The Higher Education Qualifications Sub-Framework (HEQSF) describes the qualification of engineering technicians as follows:

> This qualification primarily has a vocational orientation, which includes professional, vocational, or industry specific knowledge that provides a sound understanding of general theoretical principles as well as a combination of general and specific procedures and

> their application. The purpose of the Diploma is to develop graduates who can demonstrate focused knowledge and skills in a particular field. Typically, they will have gained experience in applying such knowledge and skills in a workplace context. A depth and specialisation of knowledge, together with practical skills and experience in the workplace, enables successful learners to enter a number of career paths and to apply their learning to particular employment contexts from the outset. Diploma programmes typically include an appropriate work-integrated learning (WIL) component (SAQA, 2013).

The South African Qualifications Authority's (SAQA) mission is to ensure the development and implementation of a National Qualifications Framework (NQF). The objectives of the NQF is outlined in the NQF Act No 67 of 2008 as follows:

- To create a single integrated national framework for learning achievements;
- Facilitate access to, and mobility and progression within, education, training and career paths;
- Enhance the quality of education and training; and
- Accelerate the redress of past unfair discrimination in education, training and employment opportunities (SAQA, 2008).

To allow for the implementation of the NQF, SAQA adopted Critical Cross-Field Outcomes (CCFO), that refer to those generic outcomes that inform all teaching and learning. For example, CCFOs include outcomes such as working effectively with others as a member of a team, and/or collecting, analysing, organising and critical evaluating of information. The Engineering Council of South Africa (ECSA) is a member of IEA and strives to comply with the IEA agreement to be committed towards developing and recognising competence for independent practice in engineering, whilst answering to SAQA's stipulations about engineering technician education. ECSA describes the purpose of an engineering technician qualification as follows:

> The primary purpose of this vocationally-oriented diploma is to develop focused knowledge and skills as well as experience in a work-related context. The Diploma equips graduates with the knowledge base, theory, skills and methodology of one or more engineering disciplines as a foundation for further training and experience towards becoming a competent engineering technician. This foundation is achieved through a thorough grounding in mathematics and natural sciences specific to the field, engineering sciences, engineering design and the ability to apply established methods. Engineering

> knowledge is complemented by methods for understanding of the impacts of engineering solutions on people and the environment. This standard is designed to meet the educational requirements towards registration as a Candidate or Professional Engineering Technician with the Engineering Council of South Africa and acceptance as a candidate to write the examinations for Certificated Engineers (ECSA, 2015).

The members of the IEA play a central role with respect to a unified understanding of competence at all levels of engineering, as the necessity of engineers to be able to work beyond countries, has increased significantly over the past decades (Lucena, Downey, Jesiek, & Elber, 2008). In complying with the IEA agreement, as well as SAQA's stipulations, ECSA established the competencies that are required to be registered as a professional engineering technician. The essential competencies as identified by ECSA are compiled in Table 3.5 (see below), and are grouped in cognitive and meta-cognitive competencies. To establish assessment guidelines (Section 3.8) that can follow the development of all the relevant competencies through mathematical modelling, the researcher will undertake a mapping process to relate the engineering competencies to the mathematical modelling competencies and vice versa. This mapping process will be explained in Section 3.7. To facilitate the mapping process, engineering technician competencies and sub-competencies are coded as follows:

Engineering technician competencies : ETC-X

Engineering technician sub-competencies : ETSC-X

Cognitive competencies for engineering technicians as identified by ECSA (2014):

Table 3.5 – Engineering Technician Competencies (cognitive) as identified by ECSA

ETC-X	**Engineering Technician Competencies**	**What the engineering technician does to display the competency (Engineering technician sub-competencies)**	**ETSC-X**
ETC-01	Define, investigate and analyse engineering problems	• Interprets the client's requirements, leading to an agreed statement of requirements. • Clarifies requirements, drawing issues and impacts to the client's attention. • Identifies design aspects standards, codes and procedures to be followed.	• ETSC-01 • ETSC-02 • ETSC-03
ETC-02	Design or develop solutions to engineering problems	• Gathers information required for problem analysis. • Identifies acceptance criteria for work product. • Verifies that the design problem is amenable to solution by candidate's techniques. • Documents functional solution requirements and gains client acceptance.	• ETSC-04 • ETSC-05 • ETSC-06 • ETSC-07

ETC-X	Engineering Technician Competencies	What the engineering technician does to display the competency (Engineering technician sub-competencies)	ETSC-X
ETC-03	Comprehend and apply knowledge embodied in engineering practices	• Displays mastery of established methods, procedures and techniques in the practice area. • Applies knowledge underpinning methods, procedures and techniques to support technician activities. • Displays working knowledge of areas that interact with the practice area.	• ETSC-08 • ETSC-09 • ETSC-10
ETC-04	Recognise and address the reasonably foreseeable social, cultural and environmental effects of engineering activities	• Identify interested and affected parties and their expectations. • Identify environmental impacts of the engineering activity. • Identify sustainability issues. • Propose measures to mitigate negative effects of engineering activity. • Communicate with stakeholders.	• ETSC-11 • ETSC-12 • ETSC-13 • ETSC-14 • ETSC-15
ETC-05	Meet all legal and regulatory requirements related to health and safety requirements in engineering activities	• Identify applicable legal, regulatory and health and safety requirements for the engineering activity. • Select safe and sustainable materials, components, processes and systems, seeking advice when necessary. • Apply defined, widely accepted methods to identify and manage risk.	• ETSC-16 • ETSC-17 • ETSC-18
ETC-06	Conduct engineering activities ethically	• Identify the central ethical problem. • Identify affected parties and their interests. • Search for possible solutions for the dilemma. • Select and justify solution that best resolves the dilemma.	• ETSC-19 • ETSC-20 • ETSC-21 • ETSC-22
ETC-07	Exercise sound judgement during engineering activities	• Considers the interdependence, interactions, and relative importance of factors. • Foresees consequences of actions. • Evaluates a situation in the absence of full evidence. • Draw on experience and knowledge.	• ETSC-23 • ETSC-24 • ETSC-25 • ETSC-26

Meta-cognitive Competencies that are of specific importance to engineering technicians:

Table 3.6 – Engineering Technician Competencies (meta-cognitive) as identified by ECSA

ETC-X	Engineering Technician Competencies	What the engineering technician does to display the competency (Engineering technician sub-competencies)	ETSC-X
ETC-08	**Management** (Personal and work process management abilities)	• Work effectively in a team environment. • Manage people, work priorities, work processes and resources. • Maintain professional and business relationships.	• ETSC-27 • ETSC-28 • ETSC-29
ETC-09	**Communication**	• Write clear, concise, effective, technically, legally and editorially correct reports. • Read and evaluate technical and legal matter. • Receive instructions, ensuring correct interpretation. • Issue clear instructions to subordinates using appropriate language and communication aids. • Make oral presentations using structure, style, language, visual aids and supporting documents appropriate to the audience and purpose.	• ETSC-30 • ETSC-31 • ETSC-32 • ETSC-33 • ETSC-34
ETC-10	**Responsibility**	• Demonstrates a professional approach always. • Has due regard to technical, social, environmental and sustainable development considerations. • Takes advice from a responsible authority on any matter considered to be outside area of competence. • Evaluates work output, revises as required and takes responsibility for work output.	• ETSC-35 • ETSC-36 • ETSC-37 • ETSC-38

The taxonomies of engineering competencies as discussed here, serve to provide an understanding of the goal of engineering and what it means to be a competent engineer. This explanation of the most essential engineering technician competencies that are required from the engineering discipline, addresses the fourth aim of the research question (Section 1.8.3). The remainder of this chapter will provide an in-depth explanation of mathematical competencies and mathematical modelling competencies to enhance mathematical reasoning and understanding. The engineering competencies will thereafter be mapped against mathematical modelling competencies to establish the relevant competencies to investigate and assess in this study. The mapping process will allow the researcher to establish the relevant competencies that benefit engineering technician students in both mathematics as well as engineering education.

3.6 MATHEMATICAL AND MATHEMATICAL MODELLING COMPETENCIES

A theoretical explanation on mathematical modelling and model-eliciting activities were provided in Chapter 2. When students engage in model-eliciting activities, they are required to construct a model of the real-world problem, and then a mathematical model to solve the problem. Students thus need to reinvent mathematics to suit the specific problem situation. To be able to successfully complete such activities, students need to engage in all the phases of mathematical modelling, which in turn require a multitude of competencies. These competencies include mathematical modelling competencies as well as mathematical competencies.

This section will attempt to examine the required mathematical and mathematical modelling competencies for successful learning and teaching of mathematics, to gain a better understanding about the importance for students to develop these competencies. Once the required competencies are established, a mapping process will follow to determine the specific competencies that will be investigated and assessed in this study.

3.6.1 Mathematical competencies

To be competent in a field, requires mastery of all the essential aspects of that specific field. In mathematical terms, a competent mathematician should possess the knowledge of mathematics, accompanied with the understanding, doing and using of mathematics in different contexts where mathematics can play a role. Furthermore, mathematical competence denotes having an opinion to reason and judge mathematical activities as they play out in the relevant contexts. Jørgensen in Blomhøj and Jensen (2007:47) explains the meaning of mathematical competence as “someone’s insightful readiness to act in response to a certain kind of mathematical challenge of a given situation”.

Jensen (2007:142) highlights that being immensely insightful, does not lead to any competency, unless it is put into *action*, which indicates that all competencies have a sphere of exertion. The activities embedded in competence development therefore suggest that competencies are framed by the historical, social, and psychological circumstances of the given situation within the modelling problem. Niss’s (2003:4) search for what it means to possess the competencies to master mathematics, led to the initiation of the Danish KOM project, an abbreviation for “Competencies and the Learning of Mathematics”. He defined mathematical competence as the ability to *understand*, *judge*, *do* and *use* mathematics in a variety of contexts and situations. Niss and his colleagues identified *eight interrelated mathematical competencies*, which they divided in two groups: the ability to ask and answer questions, and the ability to deal with mathematical language and tools (Niss, 2003:7-9). The competencies concerning the first group relate to thinking mathematically, posing and solving mathematical problems, modelling mathematically, and reasoning mathematically. The second group relates to the competencies of representing mathematical entities, handling mathematical symbols and formalisms, communicating, and tool handling. Even though the competencies are independent and relatively distinct, all the competencies are related, and they are not acquired in isolation from one another. Figure 3.1 denotes a visual representation (known as the KOM flower) to support the understanding of these interrelated competencies:

Danish KOM flower (Competencies and the learning of mathematics)

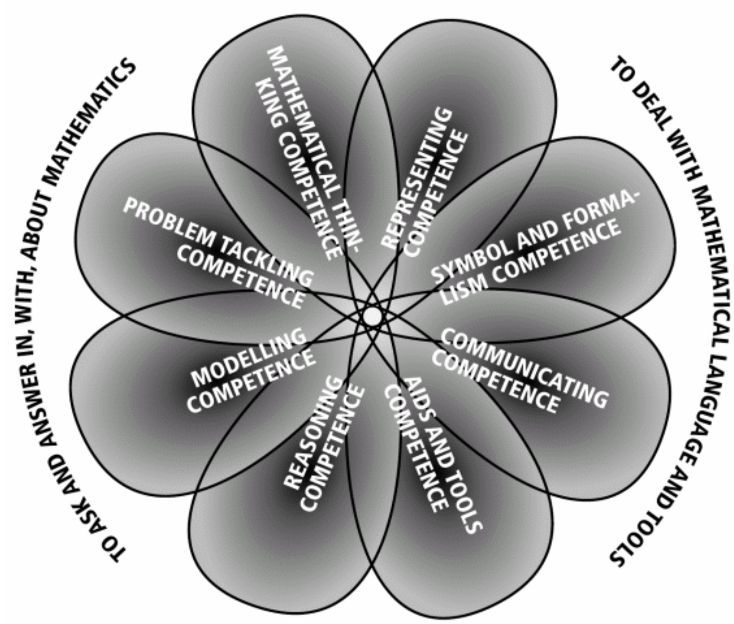


Figure 3.1 – A visual representation of interrelated mathematical competencies (Niss and Højgaard, 2011:51)

To understand the interrelated, but distinct nature of these competencies, it is necessary to explain what each competency means, and how they relate to one another. The first group of competencies, to *ask and answer questions relating to mathematics*, focuses on the nature of mathematical questions and answers, and not on the content of the questions or answers. One of the goals of mathematics education is the promotion of efficient mathematical thinkers (Hoyles & Noss, 2007:79). Resnick in Schoenfeld (1992:360) describes a good mathematical thinker as one that not only acquires a specific set of skills, strategies or knowledge, but that also obtains the habits of interpretation and sense-making. Furthermore, mathematical thinkers should also value the importance of representational fluency, as it is “at the heart of what it means to understand most mathematical constructs” (Lesh, 2000:180). The attainment of such habits is acquired through a socialisation process rather than an instructional process, and mathematics teaching and learning should therefore take place in a social context. Being a member of a community, collaborating and communicating with others, as well as knowing how to use resources, is thus part of what constitutes mathematical thinking and knowing (Schoenfeld, 1992:341-4). The *mathematical thinking competence* emphasises one’s ability to pose mathematical questions and to have an insight into the types of answers that can be expected. To abstract the properties of mathematical concepts and to understand and generalise the results, together with the ability to distinguish between different types of mathematical statements, theorems, definitions, and conjectures, are all part of this competency (Niss & Højgaard, 2011:52-53). The *problem-tackling competence* involves being able to formulate problems and solve the problems in their mathematical formulated form. Solving of these mathematical problems do not refer to the activating of routine procedures, but rather to engage in a mathematical investigation. As the same task cannot be considered as routine to everyone, the notion of a mathematical problem is thus not absolute. Also, questions often pose new problems, which again expose the relation to the mathematical thinking competency. However, the mathematical thinking competency is not concerned with the solving of a mathematical problem, but rather with distinguishing between definitions and theorems (Niss & Højgaard, 2011:55-56). *The modelling competence* encompasses a range of different elements, such as the structuring of a real situation, mathematising the situation, working with the resulting mathematical model, analysing, validating, managing and communicating aspects about the modelling process with others. All the activities surrounding mathematical modelling are explained in detail in Chapter 2. The modelling competency part relating to solving the mathematical formulation, is closely connected to the problem-tackling competency. Achieving *reasoning competence*, allows one to put an argument forward in supporting a mathematical claim. It is therefore important to include in this competency the necessity of understanding mathematical proofs, to justify the correctness of answers to problems. Justifying solutions and processes indicate a close connection to the problem-tackling and modelling competencies (Niss & Højgaard, 2011:60-61).

The second group of competencies, that deals with the handling of mathematical language and tools, covers the remaining four competencies. The *representing competence* enables one to understand and utilise different kinds of representations, to understand the role of the various representations, and to be aware of the connections between them (Niss & Højgaard, 2011:65). The handling of mathematical symbols and formal language, the translation between the symbolic language and the natural language, as well as the utilisation of symbolic statements and expressions, relate to the *symbol and formalism competence*. It is closely connected to the representing competence, but differs in the sense that it also focuses on the mathematical symbols' meaning, character, status and ways of using the symbolic statements and expressions (Niss & Højgaard, 2011:67). *Communicating competence* relates to the interpretation of others' mathematical expressions, whether it may be in written, oral or visual format. It also includes the ability to express oneself with theoretical or practical precision about specific mathematical matters. In communicating a message, various forms of representations can be used, emphasising a close link to the representing competence. However, communication skills also relate to a specific social context, as messages about mathematics also incorporate the sender and/or the receiver's background and perspectives (Niss & Højgaard, 2011:67). Knowing the various tools to use and having an insight to their possibilities and limitations in specific mathematical applications, relate to the *aids and tools competence*. Such tools, which include spreadsheets, calculators, arithmitic and graphic programmes, are closely connected to the representing competency, and are also linked to the symbol and formalism competency (Niss & Højgaard, 2011:68-69).

The KOM flower in figure 3.1, together with the above descriptions of the eight competencies, highlight their interrelated characteristics, but also denote their explicit purposes and offer a holistic view of mathematics teaching and learning. This study's focus will be primarily on mathematical modelling competencies, even though aspects of the other mathematical competencies will also be incorporated. It is noteworthy that the development of mathematical modelling competencies and concepts are constructed socially and situated in context. This results in viewing mathematical problem-solving as a complex mathematical activity (Lester & Kehle, 2003:508). Even though competence development, and not concept development, is the focus of this study, concept development relies heavily on the development of competencies such as beliefs, feelings, dispositions, values as well as other components of a complete mathematical modelling persona. Mathematical modelling is such a tool that provides the platform for developing these necessary cognitive and meta-cognitive competencies. The competencies as discussed have an analytical (investigative) and a productive aspect. The analytical side focuses on understanding, interpreting, examining and assessing of mathematical phenomena and processes, while the productive side focuses on the activity of carrying out the various processes (Niss, 2003:9).

To further address the interdependent nature of mathematical competencies, the research done by Kilpatrick, Swafford and Findell (2001) was also examined. They use the term *mathematical proficiency*. To gain an understanding of mathematical proficiency, they explain it in terms of five interwoven and interdependent components: conceptual understanding, procedural fluency, strategic fluency, adaptive reasoning and productive disposition (Kilpatrick et al., 2001:5). It is believed that each strand of mathematics proficiency should be developed in synchrony with the others, denoting that mathematical proficiency requires the presence of all five of these. *Conceptual and procedural fluency* continually interact. Conceptual understanding refers to the ability to understand mathematical ideas and to use one's mathematical knowledge in new situations and contexts. As one gains conceptual understanding, procedures are easier to remember, and they become more automatic. This enables a student to think about other aspects of a problem, which in turn can lead to improved understanding. Reflecting on why a procedure works, strengthens existing conceptual understanding. Procedural fluency refers to the ability of performing procedures with flexibility, accuracy and efficiency. It also refers to knowing *how* and *when* to use those procedures. Proficiency is acquired over time, as students become more proficient when they spend sustained periods of time doing mathematics and building connections between new and old knowledge. The capability to think logically about the relationships between concepts and to apply adaptive reasoning, also applies to every domain of mathematics (*adaptive reasoning* and *strategic competence*), such as the tendency to see mathematics as useful and worthwhile to persist in solving mathematical problems (*productive disposition*) (Kilpatrick et al., 2001:134-141). This perspective on mathematics is a product of interactions among the educator, the students and the mathematics that take place within a socio-constructivist framework where the RME theory of guided reinvention, didactical phenomenology and emergent modelling plays an important role. The following diagram accentuates the interactive character of teaching and learning for mathematical competency:

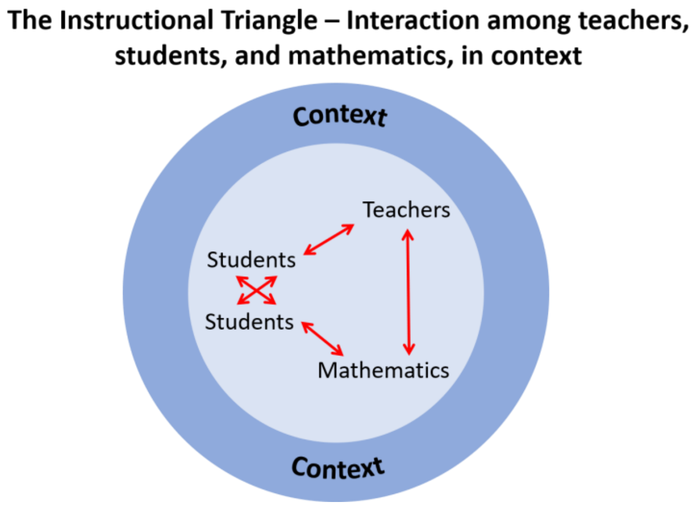


Figure 3.2 - Representation of the interactive nature between teachers, students and mathematics, adapted from Cohen and Ball (1999)

As aforementioned (Section 3.5), the development of mathematical proficiency rests on the shoulders of educators to ensure effective teaching of mathematics. Kilpatrick et al. (2001:313-4) regard *effective teaching* as teaching that fosters the development of mathematical proficiencies *over time* when students develop the crucial mathematical knowledge, skills and abilities. Complementing the roles of the teacher as discussed in Section 2.5.4, Hiebert, Morris, and Glass (2003:202-204) suggested the following two goals for educators to assist students towards mathematical competence:

- **Teachers need to become mathematically proficient**

Again, the intertwinement of Kilpatrick et al.'s five strands are emphasised:

  - Conceptual (theoretical, abstract) understanding – knowledge of mathematical concepts, operations and relations;
  - Procedural (practical, routine) fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
  - Strategic competence – ability to formulate, represent, and solve mathematical problems; and

- Adaptive reasoning – capacity for logical thought, reflection, explanation and justification
- Productive disposition – Kilpatrick et al. (2001:131) explain productive dispositions as the "… tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics". This strand relates to the teachers' mathematical knowledge, their attitudes and beliefs that they possess about mathematics, and how they can relate to being a learner and a doer of mathematics (Siegfried, 2012:3).

- **Teachers need to learn to teach**

Over time, teachers need to develop the knowledge, competencies and dispositions to learn to teach, to support the students in mathematical proficiency. Learning to teach is a longitudinal process, whereby teachers learn to take advantage of new knowledge generated by themselves and by others (Hiebert et al., 2003:205). Such teachers adapt the disposition to continually *reflect* on the outcomes of their teaching practices to maximise the benefits for both students and teachers. Also, *collaboration* with other teachers who share the same learning goals for students continuously shapes the teaching experience for increased excellence (Hiebert et al., 2003:211-217).

Fung and Siu (2015) further emphasise that students need to be able to experience the teacher's intellectual life; they need to experience the nature of mathematical activities which the teacher engages in, combined with the mental processes which it entails, as well as the mathematical knowledge that is required to successfully carry out problem-solving activities. These experiences are all necessary aspects to guide students towards mathematical proficiency.

3.6.2 Mathematical modelling competencies

As already noted, mathematical modelling plays a central role in mathematics education, due to its nature to develop mathematical competence. To build on the definition of mathematical competencies in the previous section, mathematical modelling competence is defined as "being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context" (Blomhøj & Jensen, 2007:48). Such abilities do not only relate to the *readiness to perform* a certain activity within a specific context, but also the *willingness to put these skills into action* (Maaß, 2006:117). The RME principle of guided reinvention probes the educator to help students to acquire skill, meta-knowledge and will which together can promote expertise. Competencies have a subjective and a socio-cultural side, as they are tied to an individual, and they are always relative to the surroundings (situated) (Blomhøj & Jensen, 2003:126-7). Competencies are also acquired by intuition, experience and common sense (Edwards & Hamson, 1989:89). Mathematical modelling competencies are not only necessary for successful mathematical modellers, but they also assist students generally in preparing for future jobs and successful university education (Aliprantis & Carmona, 2003:257). It encourages them to learn to recognise the importance of being able to develop a mathematical model from a real-life situation and to effectively explain such a model to someone else, which, in turn, answers the problem of the mathematics and engineering workplace that students have difficulties in solving real-world problems (Section 1.3).

The characteristics of mathematical modelling, the typical phases, as well as the rationale of teaching and learning mathematics through mathematical modelling, were explained in detail in Chapter 2. Kaiser and Schwarz (2006:196) summarise an ideal-typical description of the modelling processes as follows: The process starts with a real-world problem, which needs to be simplified or structured to create a real-world model. Through mathematising, the real-world model is translated to a mathematical model that describes the real situation in mathematical symbols. Through further mathematisation, mathematical results are produced, which again need to be reconciled with the real-world situation. The results are validated to ensure mathematical accuracy and relevance to the original problem, and an iteration of the process follows where the problem solutions are unsatisfactory.

While modelling, the modellers rely on their unique sets of knowledge, intuitions and conceptions about the mathematics and the real-world, and to make certain assumptions about the problems they face. These assumptions, in turn, influence their interpretations of the situation, as well as the use of mathematical ideas. Multiple journeys between conditions, assumptions, properties and parameters make the modelling path a non-linear path, and students continually move forwards and backwards between the original situation and the specific real-world problem that needs to be solved (Zbiek & Conner, 2006:102).

To carry out the modelling processes successfully and autonomously, Kaiser (2007:111) suggests the following competencies as fundamental:

- Competency to solve at least part of a real-world problem by making use of a *mathematical description developed by oneself*;
- Competency to *reflect* on one's own work by activating meta-knowledge about the modelling process – *monitoring and controlling* the entire modelling process;
- Being *aware of the connections* between mathematics and reality;
- Understand the view that mathematics is a *process and not only a product*;
- Understand how mathematical modelling is *dependent on the aims* and the available mathematical *tools*, as well as the students' mathematical *competencies*;
- Social competencies, such as being able to *communicate* mathematical ideas and to work in *groups* are also regarded as essential. Galbraith (2007:56) agrees with Kaiser that students should develop the ability to report and communicate the outcomes, both verbally and in writing. Social competencies relate to being comfortable with team activities, and therefore the students should strive to obtain *well-developed interpersonal skills*. Rather than relying on the educator or on textbooks, students can learn to *defend and critique* their ideas by proposing justifications, by explaining their approaches, and by suggesting alternatives.

Competent mathematicians thus do not only possess more extensive and better organised knowledge than novices, but they also exercise *better control when solving problems*. This dimension goes beyond cognition to metacognition (Goos & Galbraith, 1996:203). Considering the importance of meta-knowledge, Maaβ (2007:76) identified the following competencies as being part of modelling competencies:

- Partial competencies for conducting single phases of a modelling process
    - Understanding the problem;
    - Simplifying the problem;
    - Horizontal mathematising;
    - Vertical mathematising (working mathematically);
    - Interpreting; and
    - Validating the results.
- Meta-cognitive modelling competencies

- Competencies to structure real-world problems and to work with a *sense of direction*;
- *Argumentation* competencies – being able to argue in relation to the modelling process and to write down the argumentation; and
- Competencies to understand the possibilities that mathematics offer to solve real-world problems and to regard it as positive, complementing Kilpatrick et al.'s views on the importance of *productive disposition* (Section 3.6.1).

Based on these necessary conditions for successful learning and teaching of mathematics through mathematical modelling, the following section aims to establish a mathematical modelling competency taxonomy for increased mathematical reasoning and understanding that addresses the fifth aim of this study (Section 1.8.3).

3.6.3 Establishing a mathematical modelling competency taxonomy

Many and varied mathematical modelling competencies exist, but as Kaiser (2007:111) noted, the competencies as discussed in the previous section are crucial competencies for successful mathematical modelling. The competencies can be categorised as modelling competencies and modelling sub-competencies. When the students carry out the single steps of the modelling process, the sub-competencies refer to the skills and processes that need to be carried out to master the competencies. In listing the activities relating to the seven steps of the modelling cycle (Figure 2.4) as discussed in Section 2.4.2, a differentiation between competencies and sub-competencies are summarised in Table 3.7:

Table 3.7 - Differentiating between competencies and their related sub-competencies

Competencies and related sub-competencies	
Competencies	**Sub-competencies**
Understanding	Recognising the existence of and the need to solve a problem, referring to previous experiences to make sense of the problem, questioning, researching, brainstorming, clarifying, attending carefully to certain information about the problem, simplifying the real-world situation by connecting the essential concepts, formulate the task in own language, distinguish between relevant and irrelevant information.
Simplifying	Make appropriate and efficient assumptions to further simplify and understand the real-world problem, recognise conditions and constraints that may or may not work in the specific problem situation, as well as quantities that can influence the situation, identify patterns, relationships and regularities, relate situation to similar ideas and constructs previously experienced, representing ideas externally.
Horizontal mathematising	Progressing from a reality level to a mathematical level, choosing appropriate mathematical symbols, using those symbols to set up the mathematical model, trimming

Competencies and related sub-competencies	
Competencies	**Sub-competencies**
	away the reality through processes such as identifying and describing specific mathematics in a general context, schematising, formulating and visualising the problem in different ways, discovering relations, discovering regularities, recognising isomorphic aspects in different problems and transferring the real-world problem to a mathematical problem, choosing aspects to focus on, ignoring irrelevant information, switching between different representations by using symbolic, formal and technical language and operations.
Vertical mathematising	Rephrasing the problem, refining and testing the symbolisations, adding or eliminating restrictions, variables and assumptions, making new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge, representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, analysing, formulating, interpreting, examining and generalising models.
Interpreting	Interpreting the mathematical results, reflecting on mathematical arguments, explaining, justifying, communicating and critiquing the model and its limits, examining the appropriateness of solutions, evaluating, reflecting and reconciling the solutions to the orginal situation.
Validating	Critically checking and reflecting on solutions, reviewing parts of the process, reflecting on other ways to solve the problem, generally questioning the model.
Generalising	Independent reasoning and acting, adapting a rule to use in another situation, and making predictions.

In Section 2.6.2, one of the benefits of teaching and learning mathematics through mathematical modelling is increased meta-cognitive competency development throughout all the phases of the modelling process. Such benefits relate specifically to the competencies of planning, monitoring, managing, controlling, evaluating, motivation and productive disposition. These competencies complement the meta-cognitive competencies of working with a sense of direction, being able to engage in argumentation, and to see mathematics as valuable to solve real-world problems, as suggested by Maaβ (2007:76) in Section 3.6.2. The development of meta-cognitive competencies is not restricted to specific phases, but are intertwined and can develop throughout the modelling cycle.

The importance of *mathematisation* is attended to in Section 1.7.2.2, specifically the importance of horizontal and vertical mathematising. Horizontal mathematising denotes the process where the student moves from the real-world to the world of mathematics. While attempting to understand a realistic problem, students construct a mathematical *model of* the real-world model, which is the first step in emergent modelling. The focus is on acquiring mathematical tools to be used to organise and construct their pre-informal knowledge to solve a real-world problem. Through further interpretations of the real-world model, students make use of their intra- and extra-mathematical experiences which, through working on the problem, emerge as a *model for* reasoning mathematically (Section 1.7.2.4). This represents a movement from horizontal mathematising to working with the mathematical symbols (vertical mathematising), a process called *objectification*. In objectification, symbols are used to objectify the mathematical object. However, the means of objectification do not only include symbol systems, but also refer to artefacts (e.g. rulers, calculators and computers) and linguistic devices (e.g. metaphors) to represent mathematical objects (Radford, 2002:14). During objectification, both *horizontal and vertical mathematising* (Figure 2.2) are prevalent as they signify the transition between the two. Vertical mathematisation thus relates specifically to all kinds of re-organisations and operations that the students perform within the mathematical world (Van den Heuvel-Panhuizen, 2003:12). The competency to mathematise (both horizontally and vertically), is viewed by De Lange as the most important aspect of mathematical modelling (De Lange, 1987:84-85). Even though horizontal and vertical mathematisation denote two separate definable processes, they cannot be separated, and are complementary in nature and purpose. Furthermore, even though certain actions can explicitly be linked to specific phases of the modelling cycle, they are *not mutually exclusive* and can also occur in other phases of the cycle.

This fusion between horizontal and vertical mathematising was recognised by Knott (2014:64), and she proposed a taxonomy for competencies and their accompanied sub-competencies that shows the well-established phases of the modelling process and accompanied competencies, as well as the progression of horizontal to vertical mathematisation. The interrelated nature between the two can be seen during the symbolising phase, as it relates to the selecting of appropriate tools and symbols, as well as the utilising of such tools to formulate a mathematical problem. The symbolising phase's focus is to create a mathematical *model of* the real-world model. Knott's taxonomy is represented in the following table:

Table 3.8 - Number pattern competencies for mathematising (Knott, 2014:64)

<table>
<tr><th colspan="3">Knott's number pattern competencies for mathematising</th></tr>
<tr><th></th><th>Competencies</th><th>Sub-competencies</th></tr>
<tr><td rowspan="4">Horizontal</td><td>Internalising</td><td>• Understanding the problem.
• Distinguishing between relevant and irrelevant information.
• Simplifying the situation.</td></tr>
<tr><td>Interpreting</td><td>• Making assumptions.
• Identifying conditions.
• Identifying constraints.
• Recognising quantities that influence situation.</td></tr>
<tr><td>Structuring</td><td>• Setting up a real model.
• Naming quantities.
• Identifying key variables.
• Recognising patterns.
• Recognising relationships.</td></tr>
<tr><td rowspan="2">Symbolising</td><td>• Choosing appropriate mathematical symbols.
• Using symbols.</td></tr>
<tr><td rowspan="4">Vertical</td><td>• Setting up a mathematical model.
• Switching between symbolisations.</td></tr>
<tr><td>Adjusting</td><td>• Rephrasing the problem.
• Refining.
• Using and switching between operations.</td></tr>
<tr><td>Organising</td><td>• Viewing the problem in a different form.
• Use mathematical knowledge to solve problem.
• Using heuristics.
• Combining.
• Integrating.</td></tr>
<tr><td>Generalising</td><td>• Establishing similar relationships in different problems.
• Independent reasoning and acting.</td></tr>
</table>

The above taxonomy categorises the competencies of *internalising*, *interpreting* and *structuring* as horizontal mathematising, since they denote moving from the real-world problem to the world of mathematics. Freudenthal in Van den Heuvel-Panhuizen (2003:12) also warned against a clear-cut approach between horizontal and vertical mathematising, as the differences between these two worlds are not separate. The modelling competency of *symbolising* denotes this fusion between the two ways of mathematising: During *symbolising*, the student uses symbols and mathematical language to describe phenomena which typically represent horizontal mathematisation. However, symbolising also refers to being engaged in mathematical language and mathematical representations, denoting vertical mathematising. *Adjusting*, *organising* and *generalising* characterise vertical mathematisation processes, as the modeller moves within the mathematical world, and from the world of mathematics back to the real-world situation.

By combining the taxonomies in Tables 3.7 and 3.8, and the relevant competencies from literature as discussed in this chapter, the following taxonomy in Table 3.9 (below) is designed to use as a starting point for identifying mathematical modelling competencies for engineering technician students. To facilitate the mapping process in Section 3.7, the mathematical modelling competencies and sub-competencies must also be coded, to correspond with the engineering technician competencies and sub-competencies:

Mathematical modelling competencies : MMC-X

Mathematical modelling sub-competencies : MMSC-X

Mathematical modelling competencies and related sub-competencies:

Table 3.9 - Mathematical modelling competencies and sub-competencies

A model for mathematical modelling competencies			
	Competencies	**Sub-competencies**	**MMSC-X**
Horizontal	Internalising (MMC-01)	• Recognising the existence of and the need to solve a problem. • Referring to previous experiences to make sense of the problem. • Questioning, researching, brainstorming, clarifying, attending carefully to certain information about the problem. • Simplifying the real-world situation by connecting the essential concepts. • Formulating the task in own language. • Distinguishing between relevant and irrelevant information.	• MMSC-01 • MMSC-02 • MMSC-03 • MMSC-04 • MMSC-05 • MMSC-06
	Interpreting (MMC-02)	• Making appropriate and efficient assumptions to further simplify and understand the real-world problem. • Recognising conditions and constraints that may or may not work in the specific problem situation. • Recognising quantities that can influence the situation. • Representing ideas externally.	• MMSC-07 • MMSC-08 • MMSC-09 • MMSC-10
	Structuring (MMC-03)	• Setting up a real model. • Identifying patterns, relationships and regularities. • Relating the situation to similar ideas and constructs previously experienced. • Representing ideas externally.	• MMSC-11 • MMSC-12 • MMSC-13 • MMSC-14
	Symbolising (MMC-04)	• Progressing from a reality level to a mathematical level. • Choosing appropriate mathematical symbols. • Using those symbols to set up the mathematical model.	• MMSC-15 • MMSC-16 • MMSC-17

A model for mathematical modelling competencies			
	Competencies	**Sub-competencies**	**MMSC-X**
		• Trimming away the reality through processes such as identifying and describing specific mathematics in a general context.	• MMSC-18
		• Schematising, formulating and visualising the problem in different ways.	• MMSC-19
		• Discovering relations and regularities.	• MMSC-20
		• Recognising isomorphic aspects in different problems.	• MMSC-21
		• Transferring the real-world problem to a mathematical problem.	• MMSC-22
		• Choosing aspects to focus on, ignoring irrelevant information.	• MMSC-23
		• Switching between different representations by using symbolic, formal and technical language and operations.	• MMSC-24
Vertical		• Rephrasing the problem.	• MMSC-25
		• Refining and testing the symbolisations.	• MMSC-26
		• Switching between symbolisations.	• MMSC-27
		• Adding or eliminating restrictions, variables and assumptions.	• MMSC-28
		• Making new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge.	• MMSC-29
		• Representing a relation in a formula.	• MMSC-30
		• Proving regularities.	• MMSC-31
	Adjusting (MMC-05)	• Interpreting the mathematical results.	• MMSC-32
		• Reflecting on mathematical arguments, explaining, justifying, communicating and critiquing the model and its limits.	• MMSC-33
		• Examining the appropriateness of solutions, evaluating, reflecting and reconciling the solutions to the original situation.	• MMSC-34
		• Refining and adjusting models, using different models, combining and integrating models.	• MMSC-35
	Organising (MMC-06)	• Critically checking and reflecting on solutions, reviewing parts of the process, reflecting on other ways to solve the problem.	• MMSC-36
		• Generally questioning the model.	• MMSC-37
		• Analysing, formulating, interpreting, and examining models.	• MMSC-38
	Generalising (MMC-07)	• Independent reasoning and acting.	• MMSC-39
		• Adapting a rule to use in another situation.	• MMSC-40
		• Making predictions.	• MMSC-41

In the above table, the sub-competencies relate to the partial competencies for conducting single phases of a modelling cycle as identified by Maaβ (2007:76). The meta-cognitive competencies as discussed in this chapter, specifically competencies such as planning, monitoring, managing, reflecting, evaluating, communicating, argumentation, working with a sense of direction, and viewing mathematics as useful and worthwhile to persist in solving mathematical problems, are implicitly linked to all the phases of the modelling cycle. The relevant cognitive and meta-cognitive mathematical modelling competencies that can be developed through mathematical modelling have been established from past and present literature, and this serves to answer the fifth aim of the research question (Section 1.8.3).

The following section focuses on the mapping process of the engineering technician competencies to the above mathematical modelling competencies, to allow for the concurrent development of mathematical modelling and engineering technician competencies that compliments both mathematics and engineering education.

3.7 MAPPING ENGINEERING COMPETENCIES TO MATHEMATICAL MODELLING COMPETENCIES

The synergy between the goals of mathematics and engineering education exposed the current competency gap relating to students' poor understanding of mathematics, their inadequate abilities to engage in solving real-world problems, as well as poor meta-cognitive skills, as described in Section 1.3. These competencies are all critical for the Engineering profession as well. The tools they learn now can be applied to the many serious problems that they will face in the real-world (Parmjit & White, 2006:36). An investigation on mathematical modelling in Chapter 2 revealed that mathematical modelling has the potential to close this gap, and to develop the students' competencies to reason and understand the mathematics to be used in their present and future lives (Kaiser, 2007:110). Engineering students need to use mathematics: through mathematising, they get the opportunity to experience the interconnections of university mathematics with other relevant areas of mathematical application and ultimately progress towards efficient mathematical thinkers (Parmjit & White, 2006:34). To align with the requests of the workplace, the competencies required from professional engineering technicians were examined (Section 3.5) to support and assist the students with successful completion of their studies. By mapping the mathematical modelling competencies as identified in literature to the engineering technician competencies as suggested by national and international professional accrediting engineering bodies, competencies that support increased reasoning and understanding of mathematics can be identified and investigated. By identifying such competencies, the sixth aim of the research question will be addressed (Section 1.8.3). This mapping process needs to adhere to strict guidelines to ensure the investigation and assessment of reliable and valid competencies. Woollacott (2003:2) proposed the following criteria for selecting specific competencies:

- **Relevance**

All the descriptions that are useful to inform the engineering as well as the mathematics curriculum are regarded as relevant. Not only will the direct issues that relate to performance in engineering and mathematics be considered, but also the indirect aspects. For example: communication, management and responsible behaviour abilities can also affect general performance in both engineering and mathematical tasks.

- **Significance**

The competency needs to be significant in terms of research evidence, or in terms of its importance granted by professional bodies.

- **Personal judgement**

Competencies related to the mismatches as discussed in this study will be considered.

- **Detail**

The scope of detail must be manageable and useful for categorising competencies. Categories should be arranged in such a way that they make intrinsic and relational sense.

Considering both the engineering competencies as required by industry, as well as the mathematical modelling competencies required to successfully carry out all the processes of a modelling activity, the competence taxonomy as presented in Table 3.9 will be adapted to document possible development of students' mathematical modelling competencies. Engineering technician competencies were investigated by detailing the technicians' tasks relating to the specific competencies, as summarised in Table 3.6. The relation between engineering technician competencies and mathematical modelling competencies is expressed in Table 3.10 below:

Table 3.10 - Relation between engineering technician and mathematical modelling competencies

Relation between engineering technicians' tasks and modelling tasks in the mathematics classroom		
Engineering technician competencies	**What the engineering technician does (ETSC)**	**What the mathematics modelling student does (MMSC)**
Define, investigate and analyse engineering problems (ETC-01)	• Interprets the client's requirements, leading to an agreed statement of requirements (ETSC-01). • Clarifies requirements, drawing issues and impacts to the client's attention (ETSC-02). • Identifies design aspects standards, codes and procedures to be followed (ETSC-03).	• The student recognises the existence of and the need to solve a problem. (Internalising – MMSC-01) • The student refers to previous experiences to make sense of the problem. (Internalising – MMSC-02) • The student explains/notes important information to simplify the problem. (Internalising – MMSC-03) • The student simplifies the problem by connecting the essential concepts. (Internalising – MMSC-04) • The student formulates the problem in his/her own words. (Internalising - MMSC-05) • The student makes relevant assumptions for the problem to further simplifies the situation. • Assumptions are stipulated clearly and coherently whilst consideration for the consequences of the assumptions have been made. (Interpreting - MMSC-07)
Design or develop solutions to engineering problems (ETC-02)	• Gathers information required for problem analysis (ETSC-04). • Identifies acceptance criteria for work product (ETSC-05). • Verifies that the design problem is amenable to solution by candidate's techniques (ETSC-06). • Documents functional solution requirements and gains client acceptance (ETSC-07).	• The student formulates the problem in his/her own words. (Internalising - MMSC-05) • The student recognises the information relevant to the situation and discards irrelevant information (Internalising – MMSC-06) • The student notes conditions and constraints that will/will not work for the problem situation (Interpreting – MMSC-08) • The student recognises quantities that can influence the situation. (Interpreting – MMSC-09) • The student represents ideas externally. (Interpreting – MMSC-10) • The student creates a realistic representation of the original situation, which becomes a 'model of' the original real-world problem situation. (Structuring – MMSC-11) • The student identifies patterns, relationships and regularities. (Structuring – MMSC-12) • The student relates the situation (real-model) to similar ideas and constructs previously experienced. (Structuring – MMSC-13) • The student represents ideas relating to the real model externally. (Structuring – MMSC-14)

Relation between engineering technicians' tasks and modelling tasks in the mathematics classroom		
Engineering technician competencies	**What the engineering technician does (ETSC)**	**What the mathematics modelling student does (MMSC)**
Comprehend and apply knowledge embodied in engineering practices (ETC-03)	• Displays mastery of established methods, procedures and techniques in the practice area (ETSC-08). • Applies knowledge underpinning methods, procedures and techniques to support technician activities (ETSC-09). • Displays working knowledge of areas that interact with the practice area (ETSC-10).	• Progressing from a reality level to a mathematical level. (Symbolising – MMSC-15) • Choosing appropriate mathematical symbols. (Symbolising – MMSC-16) • Using those symbols to set up the mathematical model. (Symbolising – MMSC-17) • Trimming away the reality through processes such as identifying and describing specific mathematics in a general context. (Symbolising – MMSC-18) • Schematising, formulating and visualising the problem in different ways. (Symbolising – MMSC-19) • Discovering relations and regularities. (Symbolising – MMSC-20) • Recognising isomorphic aspects in different problems. (Symbolising – MMSC-21) • Transferring the real-world problem to a mathematical problem. (Symbolising – MMSC-22) • Choosing aspects to focus on, ignoring irrelevant information. (Symbolising – MMSC-23) • Switching between different representations by using symbolic, formal and technical language and operations. (Symbolising – MMSC-24) • Rephrasing the problem. (Structuring – MMSC-25) • Refining and testing the symbolisations. (Structuring – MMSC-26) • Switching between symbolisations. (Structuring – MMSC-27) • Adding or eliminating restrictions, variables and assumptions. (Structuring – MMSC-28) • Making new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. (Structuring – MMSC-29) • Representing a relation in a formula. (Structuring – MMSC-30) • Proving regularities. (Structuring – MMSC-31)
Recognise and address the reasonably foreseeable social, cultural and environmental effects of	• Identify interested and affected parties and their expectations (ETSC-11). • Identify environmental impacts of the engineering activity (ETSC-12).	• Recognise conditions and constraints relevant to the problem. (Interpreting – MMSC-08) • Interpret the mathematical results. (Adjusting – MMSC-32)

Relation between engineering technicians' tasks and modelling tasks in the mathematics classroom		
Engineering technician competencies	**What the engineering technician does (ETSC)**	**What the mathematics modelling student does (MMSC)**
engineering activities (ETC-04)	• Identify sustainability issues (ETSC-13). • Propose measures to mitigate negative effects of engineering activity (ETSC-14). • Communicate with stakeholders (ETSC-15).	• Rephrase the problem and question own model. (Adjusting – MMSC-33) • Review or refine parts of the model or go through the entire modelling process if the solution does not fit the situation. (Adjusting – MMSC-34) • Adapt the model to make sense in a specific situation. (Adjusting – MMSC-35) • Critically checking and reflecting on solutions, reviewing parts of the process, reflecting on other ways to solve the problem. (Organising – MMSC-36) • Generally questioning the model. (Organising – MMSC-37) • Analysing, formulating, interpreting, and examining models. (Organising – MMSC-38)
Meet all legal and regulatory requirements related to health and safety requirements in engineering activities (ETC-05)	• Identify applicable legal, regulatory and health and safety requirements for the engineering activity (ETSC-16). • Select safe and sustainable materials, components, processes and systems, seeking advice when necessary (ETSC-17). • Apply defined, widely accepted methods to identify and manage risk (ETSC-18).	
Conduct engineering activities ethically (ETC-06)	• Identify the central ethical problem (ETSC-19). • Identify affected parties and their interests (ETSC-20). • Search for possible solutions for the dilemma (ETSC-21). • Select and justify solution that best resolves the dilemma (ETSC-22).	
Exercise sound judgement during engineering activities (ETC-07)	• Considers the interdependence, interactions, and relative importance of factors (ETSC-23). • Foresees consequences of actions (ETSC-24). • Evaluates a situation in the absence of full evidence (ETSC-25). • Draw on experience and knowledge (ETSC-26).	• Critically checking and reflecting on solutions, reviewing parts of the process, reflecting on other ways to solve the problem. (Organising – MMSC-36) • Generally questioning the model. (Organising – MMSC-37) • Analysing, formulating, interpreting, and examining models. (Organising – MMSC-38)

Relation between engineering technicians' tasks and modelling tasks in the mathematics classroom		
Engineering technician competencies	**What the engineering technician does (ETSC)**	**What the mathematics modelling student does (MMSC)**
		• General or independent reasoning and acting – applying of deductive reasoning to prove the solutions. (Generalising – MMSC-39) • Establish similar relationship in different situations by adapting some of the rules. (Generalising – MMSC-40) • The successful model is easy to use, and the student can predict and generalise to explore further applications. (Generalising – MMSC-41)

From the above competence classifications, the importance of mathematics and problem-solving stand out. Two of the competence categories as required by ECSA (regulatory requirements related to health and safety, and ethical behaviour) are not directly related to mathematics education in the context of this study, and will therefore not form part of this study. However, Table 3.10 exposes the relevance of *all* the selected mathematical modelling competencies to engineering technician students. By coding all the relevant competencies as explained in Sections 3.5.4 and 3.6.3, a mapping process can commence to allow forward and backward movement between the related engineering and mathematical modelling competencies (Table 3.11), and between competencies and related sub-competencies (Table 3.12). Engineering technician and mathematical modelling competencies and sub-competencies were mapped as follows:

Table 3.11 - Mapping engineering technician and mathematical modelling competencies

Mapping engineering technician competencies to mathematical modelling competencies	
Engineering Technician Competencies	**Mathematical Modelling Competencies**
ETC-01	MMC-01, MMC-02
ETC-02	MMC-01, MMC-02, MMC-03
ETC-03	MMC-04
ETC-04	MMC-02, MMC-05
ETC-07	MMC-06, MMC-07

Mapping mathematical modelling competencies to engineering technician competencies	
Mathematical Modelling Competencies	**Engineering Technician Competencies**
MMC-01	ETC-01, ETC-02
MMC-02	ETC-01, ETC-02, ETC-04
MMC-03	ETC-02
MMC-04	ETC-03
MMC-05	ETC-04
MMC-06	ETC-07
MMC-07	ETC-07

Table 3.12 - Mapping competencies to sub-competencies

Mapping mathematical modelling competencies to mathematical modelling sub-competencies	
Mathematical modelling competencies	**Mathematical modelling sub-competencies**
MMC-01	MMSC-01 to MMSC-06
MMC-02	MMSC-07 to MMSC-10
MMC-03	MMSC-11 to MMSC-14
MMC-04	MMSC-15 to MMSC-31
MMC-05	MMSC-32 to MMSC-35
MMC-06	MMSC-36 to MMSC-38
MMC-07	MMSC-39 to MMSC-41

Mapping engineering technician competencies to engineering technician sub-competencies	
Engineering technician competencies	**Engineering technician sub-competencies**
ETC-01	ETSC-01 to ETSC-03
ETC-02	ETSC-04 to ETSC-07
ETC-03	ETSC-08 to ETSC-10
ETC-04	ETSC-11 to ETSC-15
ETC-05	ETSC-16 to ETSC-18
ETC-06	ETSC-19 to ETSC-22
ETC-07	ETSC-23 to ETSC-26

To identify the cognitive modelling competencies relevant to this study, this mapping process must be expanded to indicate the engineering and mathematical modelling sub-competencies that can be developed within the model as established in Table 3.9. Table 3.13 (below) further distinguishes between the competencies and sub-competencies in terms of horizontal and vertical mathematising:

Table 3.13 - Mapping process to identify competencies to follow and assess

Related engineering technician and mathematical modelling sub-competencies			
	Mathematical modelling competency	**Related engineering technician sub-competency**	**Related mathematical modelling sub-competency**
Horizontal Mathematising	MMC-01	ETSC-01, ETSC-02, ETSC-03, ETSC-04, ETSC-05, ETSC-06, ETSC-07	MMSC-01, MMSC-02, MMSC-03, MMSC-04, MMSC-05, MMSC-06
	MMC-02	ETSC-01, ETSC-02, ETSC-03, ETSC-04, ETSC-05, ETSC-06, ETSC-07, ETSC-11, ETSC-12, ETSC-13, ETSC-14, ETSC-15	MMSC-07, MMSC-08, MMSC-09, MMSC-10
	MMC-03	ETSC-04, ETSC-05, ETSC-06, ETSC-07	MMSC-11, MMSC-12, MMSC-13, MMSC-14
	MMC-04	ETSC-04, ETSC-05, ETSC-06, ETSC-07, ETSC-08, ETSC-09, ETSC-10	MMSC-15, MMSC-16, MMSC-17, MMSC-18, MMSC-19, MMSC-20, MMSC-21, MMSC-22, MMSC-23, MMSC-24
Vertical Mathematising	MMC-04		MMSC-25, MMSC-26, MMSC-27, MMSC-28, MMSC-29, MMSC-30, MMSC-31
	MMC-05	ETSC-11, ETSC-12, ETSC-13, ETSC-14, ETSC-15	MMSC-32, MMSC-33, MMSC-34, MMSC-35
	MMC-06	ETSC-23, ETSC-24, ETSC-25, ETSC-26	MMSC-36, MMSC-37, MMSC-38
	MMC-07	ETSC-23, ETSC-24, ETSC-25, ETSC-26	MMSC-39, MMSC-40, MMSC-41

This mapping process serves to assist towards the goal of understanding what the competencies mean and to determine a way to recognise and identify these competencies in the students' work. This will also serve to assist in constructing a hypothetical learning trajectory (Section 5.2). Table 3.14 exposes the relation between engineering competencies and mathematical modelling competencies, and serves as a model for engineering and mathematical modelling competence development to answer to the search for *cognitive competencies* to be investigated and assessed:

Classification of mathematical modelling competencies to investigate and assess:

Table 3.14 – Classification of mathematical modelling competencies to investigate and assess (cognitive)

	Modelling Competencies	Related engineering technician sub-competencies – Cognitive	What the mathematics student does
Horizontal	Internalising	• Identifies design aspects standards, codes and procedures to be followed. • Gathers information required for problem analysis. • Identifies acceptance criteria for work product. • Verifies that the design problem is amenable to solution by candidate's techniques. • Documents functional solution requirements and gains client acceptance.	• Recognising the existence of and the need to solve a problem. • Referring to previous experiences to make sense of the problem. • Questioning, researching, brainstorming, clarifying, attending carefully to certain information about the problem. • Simplifying the real-world situation by connecting the essential concepts. • Formulating the task in own language. • Distinguishing between relevant and irrelevant information.
	Interpreting	• Interprets the client's requirements, leading to an agreed statement of requirements. • Clarifies requirements, drawing issues and impacts to the client's attention. • Make assumptions. • Identifies accepted criteria for work product. • Consider practical, economic, social, environmental, quality assurance, safety and statutory factors that can influence the situation. • Identify conditions and constraints, also in terms of the efficient utilisation and interaction of people, materials, machines, equipment, means and funding.	• The student makes relevant assumptions regarding the problem and further simplifies the situation. Assumptions are stipulated clearly and coherently whilst consideration for the consequences of the assumptions have been made. • The student can recognise quantities and variables that can influence the problem situation and how they relate to the problem. • The student notes conditions and constraints that will/will not work for the problem situation.
	Structuring	• Innovative planning and design (setting up a situation model). • Construct Relations – maintain a good balance between the effectiveness of the solution process and the time/cost involved. Consider the impact of decisions on social, safety and environmental aspects, considering all relevant legislation. • Verifies that the design problem is amenable to solution by candidate's techniques.	• The student creates a realistic representation of the original situation, which becomes a 'model of' the original real-world problem situation. • The student can identify and construct relations between key variables. • Relating the situation to similar ideas and constructs previously experienced. • Representing ideas externally.

	Modelling Competencies	**Related engineering technician sub-competencies – Cognitive**	**What the mathematics student does**
	Symbolising	• Insight - apply an acceptable level of understanding and technological knowledge to execute engineering decisions. • Take effective decisions where the technical tools at their disposal are insufficient to provide solutions. • Approach problems methodically – comprehend and apply knowledge – principles, specialist knowledge, jurisdictional and local knowledge. • Displays mastery of established methods, procedures and techniques in the practice area. • Applies knowledge underpinning methods, procedures and techniques to support technician activities. • Displays working knowledge of areas that interact with the practice area.	• Transferring the real-world problem to a mathematical problem. • Choosing appropriate mathematical symbols: properties and parameters that correspond to the situational conditions and assumptions that are specified by the modeller. • Using those symbols to set up the mathematical model. • Schematising, formulating and visualising the problem in different ways. • Discovering relations and regularities. • Recognising isomorphic aspects in different problems. • Choosing aspects to focus on, ignoring irrelevant information.
Vertical			• Trimming away the reality through processes such as identifying and describing specific mathematics in a general context. • Switching between different representations by using symbolic, formal and technical language and operations. • Mathematical reasoning – students make use of heuristic strategies. While students mathematise the problem, they translate and communicate the structure of the situation into mathematical language. • Setting up a mathematical model – the student creates a 'model of' by translating the structure of the situation into mathematical language to solve the problem. • Rephrasing the problem. • Refining and testing the symbolisations. • Switching between symbolisations. • Adding or eliminating restrictions, variables and assumptions. • Making new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge.

	Modelling Competencies	Related engineering technician sub-competencies – Cognitive	What the mathematics student does
	Adjusting	• Refining of the engineering design. • Testing. • Engineers must keep themselves informed of new technological developments in their various fields. • Identify interested and affected parties and their expectations. • Identify environmental impacts of the engineering activity. • Identify sustainability issues. • Propose measures to mitigate negative effects of engineering activity. • Communicate with stakeholders.	• The student adapts the model so that it makes sense in the specific situation. • The student rephrases the problem and question his/her own model. • The student reviews or refines parts of the model or go through the entire modelling process if the solutions do not fit the situation. • The student creates a 'model for'. • The student is capable to derive an elegant solution for the problem.
	Organising	• Evaluating and engineering judgement – the work must be aimed at the full development of the suggested solution to the problem through a process of synthesis, with the application of all information acquired during the problem investigation, also using the design, development and communication. • Consider all relevant engineering principles that can influence the solution – recognise and address the reasonably foreseeable social, cultural and environmental effects, and meet all legal and regulatory requirements. • Considers the interdependence, interactions, and relative importance of factors. • Foresees consequences of actions. • Evaluates a situation in the absence of full evidence. • Draw on experience and knowledge.	• Viewing the problem in a different form. • Reflects on the real problem and use mathematical knowledge to solve the problem. • Create a 'model for'. • Validate the solution. • Critically checking and reflecting on solutions, reviewing parts of the process, reflecting on other ways to solve the problem. • Generally questioning the model. • Analysing, formulating, interpreting, and examining the model.
	Generalising	• Holistic approach to engineering activities and reasoning. • Considers the interdependence, interactions, and relative importance of factors.	• Establish similar relationship in different situations by adapting some of the rules. • General or independent reasoning and acting – applying of deductive reasoning to prove the solutions. • Your successful model is easy to use and you predict and make generalisations to explore further applications.

Although the above table implicitly links meta-cognitive competencies to all the modelling tasks, it is necessary to explicitly describe the three most crucial meta-cognitive competencies - management, responsibility and communication - required from engineering technicians (Table 3.6). The following table serves to classify the meta-cognitive mathematical modelling competencies to investigate and assess.

Table 3.15 - Classification of mathematical modelling competencies to investigate and assess (meta-cognitive)

Engineering technician – meta-cognitive competencies	**Engineering technician sub-competencies - meta-cognitive**	**Mathematical modelling sub-competencies – meta-cognitive**
Management (Personal and work process management abilities) Engineers must increasingly develop the ability to use their theoretical and practical knowledge to an advanced level without constant supervision	• Work effectively in a team environment. • Manage people, work priorities, work processes and resources. • Maintain professional and business relationships.	• Self-directed learning Students actively plan, monitor, evaluate, reflect, direct and regulate their own learning processes. Students display the competency to reflect on their work by activating meta-knowledge about the modelling process – monitoring and controlling the entire modelling process. They evaluate the solution paths that they designed through reflection and by making judgements about the processes and outcomes. Reflective activities are activities such as talking and writing about the processes they have gone through, making posters and reporting to the class, drawing up concept maps of a topic, or sharing attainment targets. • Productive disposition Students recognise the possibilities that mathematics offers for the solution of real-world problems and regard these possibilities as positive. • Group work Students work effectively in a team environment toward group goals. They respect one another's ideas and take turns to assume leadership, displaying teamwork, leadership, project management and communication skills.
Communication	• Write clear, concise, effective, as well as technically, legally and editorially correct reports. • Read and evaluate technical and legal matter. • Receive instructions, ensuring correct interpretation. • Issue clear instructions to subordinates using appropriate language and communication aids. • Make oral presentations using structure, style, language, visual aids and	• Share Students share their ideas with one another within the group. The focus of communication is to share thoughts, rather than to introduce new information. Ideas, information or other details about the process can occur verbally, through motions, through written or pictorial work. Students reflect on their work while communicating their solutions to one another. This may arise in the need to reconcile, modify or justify details of the mathematical processes and products. • Reading Competence Student have culture-specific knowledge about facts and semantics to understand words and expressions mentioned in the question or situation. • Group work

	supporting documents appropriate to the audience and purpose.	Group-based discourse sets up optimal opportunities for individuals' ideas to be challenged within their zone of proximal development, leading to the further development of those ideas. Group members listen carefully to others' ideas and offer constructive feedback when appropriate. • Oral Presentations Communicate clearly and effectively.
Responsibility	• Demonstrates a professional approach always. • Has due regard for technical, social, environmental and sustainable development considerations. • Takes advice from a responsible authority on any matter considered to be outside his/her area of competence. • Evaluates work output, revises as required and takes responsibility for work output.	• Sense of Direction The student considers all necessary factors that can have an influence on the problem solution, collaborates with others where necessary, and validates and refines the work continuously. The student gathers an overall view of the real-world problem which helps him/her to work more purposefully. Through reflection and communication, the student can justify his/her thought processes to connect the real-world problem with the solutions.

Through this thorough review of present and past literature, the researcher identified the ten modelling competencies that will form the focus of this study. This collection of cognitive and meta-cognitive competencies to be investigated focuses collaboratively on the advancing of mathematical reasoning and understanding, and also serves to answer aim six of the research question (Section 1.8.3). To measure the students' proficiency in any one of these competencies, clear guidelines and assessment methods should be established. A discussion on how such competencies will be assessed, is presented in the following section.

3.8 ASSESSING COMPETENCIES

The first goal of the study has now been established, as the ten mathematical modelling competencies have been identified, together with a description of how each competency can present itself in the mathematics classroom (Tables 3.14 and 3.15). To allow for valid assessments of activities, the assesssment instruments must be designed appropriately to fulfill the educational goal of competence development that supports a deeper understanding of mathematics. The assessment instruments need to identify the existence and range of the students' competencies in relation to the mathematical activities that they are involved in, to reflect the level of their modelling competencies (Niss, 2003:10). Jensen (2007:141) proposes the need of a multi-dimensional approach to make valid competence assessments, where students' work can be scored along more than one dimension and not only indicate the presence or absence of such a competency. His assessment proposal concentrates on the degree of coverage, the radius of action, as well as the students' technical levels. In mathematical modelling, *degree of coverage* addresses which part of the modelling process the student can work with and reflect on. The *radius of action* relates to the variety of situations in which students can perform mathematical modelling activities, while the *technical level* refers to the kind of mathematics that students use and how flexible they are using their mathematical 'toolbox' (Jensen, 2007:144). Jensen emphasises the importance of all three dimensions when competencies are assessed, to make a valid and reliable judgement regarding the students' progression (or absence of it) in mathematical modelling competency development.

Assessment criteria are descriptive statements to allow judgements to be made about the marks awarded, and are communicated to the students before the tasks are set. The tasks that relate to this study, adhere to the design principles as laid out in Section 2.5.1. Modelling activities that are designed according to these principles, promote the development of the ten competencies which will be assessed in this study. Marking guidelines are required to provide a link between the assessment task and the competencies that must be assessed, and allows for consistency in assessing the students' work over time. Furthermore, effective assessment allows for valuable information about the students' performances, and to enable effective decision-making processes for improved student learning and teaching.

To measure the progression of competencies and not merely the presence or absence thereof, the researcher adapted Jensen's (2007) multi-dimensional model, and designed a Group Modelling Competency Observation Sheet to investigate the degree of mastery within each competency (Table 3.16 below):

Table 3.16 - Group Modelling Competency Observation Guide

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory	Emergent / Developing	Proficient	Exemplary
			0	1	2	3
Horizontal	**Internalising**	Understand the problem. (Technical Level)	You failed to identify, summarise or explain the main problem or question in your own words.	You identified main issues but did not summarise or explain them clearly or sufficiently.	You successfully identified and summarised the main issues, but did not explain why/how they are problems or create questions.	You clearly identified and summarised main issues and successfully explained why/how they are problems or questions.
		Collect relevant information. (Degree of Coverage)	You gathered information that lacks relevance, quality and balance.	Your response was not completely related to the problem.	You used all relevant information from the problem for working towards a solution.	You uncovered hidden or implied information not readily apparent.
		Simplify the situation. (Radius of Action)	You were unable to recognise and connect essential concepts about the problem.	Your situational model was essentially correct, but not all concepts were accurately represented.	Your situational model was complete and accurate.	You used multiple representations for explaining and simplifying the problem.
	Interpreting	Assumptions (Degree of Coverage)	Your assumptions were not appropriate for the problem, you did not simplify the problem.	You used an oversimplified approach and assumptions to the problem, you did not explain all the important information to simplify the problem.	You chose appropriate, efficient assumptions for simplifying and solving the problem.	You chose innovative and insightful assumptions and showed consideration for the consequences of the assumptions clearly and coherently.
		Determine particularities – recognise factors that can influence the situation. (Technical Level)	You did not recognise the information relevant to the situation and discarded irrelevant information that have an influence on the problem.	You recognised some quantities and variables and discarded some irrelevant information that could influence the problem.	You recognised important quantities and variables in the problem and you were able to discard irrelevant information that could influence the problem.	You created a general rule or formula for solving related problems.
		Establish conditions and constraints. (Radius of Action)	You were unable to recognise conditions that will/will not work for the problem.	You established vague conditions under which the problem will/will not work.	You established clear conditions and constraints for a successful solution to the problem.	You established clear conditions and constraints, as well as explanations for such conditions and constraints.
	Structuring	Innovative planning and design (setting	You were unable to recognise and connect	Your situational model was essentially correct,	Your situational model was complete and accurate (‘model of’).	You used multiple representations for

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory	Emergent / Developing	Proficient	Exemplary
			0	1	2	3
		up a situational model). (Radius of Action / Degree of Coverage)	essential concepts about the problem.	but not all concepts were accurately represented.		explaining the problem ('model of').
		Construct relations – Consider the interdependence, interactions, and relative importance of various factors. (Radius of Action / Technical Level)	You were unable to recognise relationships between variables.	You recognised some patterns and/or relationships.	You recognised important relationships between the variables in your problem.	You created a general rule or formula for solving related problems.
	Symbolising	Choose appropriate symbols. (Technical Level)	The mathematical tools you chose would not lead to a correct solution.	The mathematical tools you chose would lead to a partially correct solution.	The mathematical tools you chose would lead to a correct solution.	You chose mathematical tools that would lead to an elegant solution.
Vertical		Using the symbols. (Radius of Action)	Your use of mathematical symbols will not explain the problem or lead to a satisfactory solution.	Your use of mathematical symbols was partially correct.	You used mathematical symbols effectively - your model can lead to a correct solution.	You explained and described the symbols used in your model, as well as possible alternative methods for working with the problem.
		Approach problems methodically. (Degree of Coverage)	Errors in reasoning were serious enough to flaw your solution. You were unable to translate the structure of the situation into mathematical language.	You made minor errors in your attempt to communicate the structure of the situation into mathematical language.	Your mathematical reasoning was essentially accurate.	All aspects of your mathematical reasoning were completely accurate.
		(Technical Level)	Your mathematical model will not explain the problem or lead to a satisfactory solution.	Your mathematical model will lead to a partially correct solution.	Your mathematical model can lead to a correct solution.	You translated the structure of the situation into mathematical language and solved the problem successfully.

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory **0**	Emergent / Developing 1	Proficient 2	Exemplary 3
	Adjusting	Refining and Testing. (Radius of Action)	You found a solution and then stopped.	You found multiple solutions, but not all of them were correct.	You found multiple solutions using different interpretations of the problem, you reviewed or refined parts of the model or went through the entire modelling process when the solutions did not fit the situation (‘*model for*’).	You related the underlying structure of the problem to other similar problems (‘*model for*’).
		Explaining (Degree of Coverage)	You gave no explanation for your work.	Your explanation was redundant at places.	Your solution flowed logically from one step to the next.	You gave an in-depth explanation of your reasoning.
		Capable to derive to an elegant solution of the problem. (Technical Level)	Your methods were clumsy and inappropriate.	The methods you used led to a partially correct solution.	The methods you used led to a correct solution.	You applied methods elegantly, which led to the correct solutions.
	Organising	Evaluating and judgement. (Degree of Coverage / Technical Level)	You did not evaluate your work, and little or no connections were made between the mathematical model and the real-world problem.	You made attempts to analyse, evaluate or judge your work, but the connections between your work and the real-world problem were limited.	You offered substantial information, evidence of analysis, synthesis and evaluation; general connections are made, but are sometimes too obvious or not clear.	Rich in content, insightful analysis, synthesis and evaluation, clear connections made to real-life situations or to previous content.
		Reflection – consider relevant principles that can influence the solution. (Reflecting on own thought processes) (Degree of Coverage / Radius of Action)	You did not reflect on your own thinking (viewing problem in different form).	You identified some perspectives about the problem, but did not consider alternate points of view.	You identified strengths and weaknesses in your own thinking, you recognized alternative perspectives about the problem when comparing to others.	You identified strengths and weaknesses in your own thinking, you recognized alternative perspectives about the problem when comparing to others, and evaluated them in the context of alternate points of view.

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent / Developing 1	Proficient 2	Exemplary 3
	Generalising	Establish similar relationship in different situations by adapting some of the rules. (Technical Level)	You found no connections to other disciplines or mathematical concepts.	Your solution hinted at a connection to an application or another area of mathematics.	You connected your solution process to other problems, areas of mathematics, or applications. Predictions can be made from the model.	Your connection to a real-life application was accurate and realistic, the model is easy to use, and the predictions are accurate.
		General or independent reasoning – applying deductive reasoning to prove the solutions. (Degree of Coverage)	You exhibit an inability to identify a generalisation when presented with a specific situation.	With assistance, you identified a partially correct generalisation when presented with a specific situation.	You exhibit the ability to identify a generalisation when presented with a specific situation, but require assistance.	You exhibit the ability to identify a generalisation easily when presented with a specific situation.
		The successful model is easy to use and allows for predictions. (Radius of Action)	The complicated model cannot be detached from the current context.	With minor adjustments, the model can be used in other related situations.	The model can be transferred to other similar situations, but needs minor simplifications.	The model can easily be adapted in another related situation.
Meta-cognitive	**Management**	Self-directed learning. (Degree of Coverage)	You were not able to direct your own learning, and tried to find someone to direct your activities.	You complete your tasks through guided learning and searched for confirmation throughout your work.	You set goals and managed your own learning. You designed the mathematical model independently and considered feedback from others to find the solutions.	You set goals and managed your own learning. You designed the mathematical model independently and reflect and evaluate your work critically to improve your learning.
		Group Self-efficacy. (Radius of Action)	Your approach to the task and to the team was hostile and uninterested.	You attempted to complete the task, but you seemed unsure about your role and your abilities during the team activity.	You approached the task with positive expectations about finding solution strategies.	You approached the task with positive expectations about finding solution strategies and you communicated your ideas to other team members in a positive and productive manner.

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory	Emergent / Developing	Proficient	Exemplary
			0	1	2	3
		Productive disposition. (Technical Level)	You showed no evidence of engaging with the task, mathematical or otherwise. The lack of effort can be attributed to either disinterest or a lack of capability.	You engaged with the task and with the mathematics, but you made little progress towards understanding the mathematics. However, you were willing to engage with the mathematics of the task.	You showed strong evidence of engaging with the task and the mathematics, but the quality of engagement was somewhat shallow or only related to a small aspect of the work.	You showed very strong evidence of engaging with both the task and the mathematics. You tried to help fellow group members through explaining the work and approached it from various perspectives. You were persistent in continuing with the work until you reached an acceptable solution.
	Communication	Sharing Ideas. (Technical Level)	You can communicate your ideas, but you make mistakes in content and reasoning or misread your audience.	You can communicate ideas clearly and accurately and with some awareness of the context.	You communicate ideas clearly, accurately and appropriately, and give arguments and reasons for your beliefs.	You can communicate convincing arguments clearly and accurately with a level of comprehensiveness and conciseness appropriate to the audience.
		Reading Competence. (Degree of Coverage)	The response shows an inability to construct a literal meaning of the selection and may focus only on the reader's own frustration or indicate that the reader gave up.	The response correctly identifies some main ideas, focuses on isolated details or misunderstands or omits some significant supporting details.	Indicates an understanding of the main ideas and relevant and specific supporting details. Uses information from textual resources to clarify the meaning and form conclusions.	Indicates a thorough and accurate understanding of main ideas and all significant supporting details, including clarification of the complexities. Uses relevant and specific information from textual resources to clarify meaning and form conclusions.

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory **0**	Emergent / Developing 1	Proficient 2	Exemplary 3
		Group Work (Radius of Action)	You did not collaborate with your team members. You either showed no interest, made fun of the work, had a negative attitude, contributed very little to group effort, or did not perform the duties of the assigned team role.	You occasionally helped to complete group goals, you finished your individual task, but did not assist the team members. You performed some of the duties of the assigned team role.	You usually help to complete group goals with a positive attitude about the tasks and work of others. You assisted team members in the finished project and performed nearly all the duties of the assigned team role.	You work to complete all the group goals while maintaining a positive attitude about the tasks and work of others. All team members contributed equally, and you performed all the duties of the assigned team role.
		Interaction (Technical Level)	No participation.	The speaker makes many grammatical mistakes, using very simplistic, bland language.	The speaker uses language which is appropriate for the task.	The speaker uses language in highly effective ways to emphasise the meaning of the message.
	Responsible behaviour	Sense of Direction (Degree of Coverage / Radius of Action)	You do not seek ways to improve personal or group performance and seem to be lost regarding what must be done.	You sometimes seek ways to improve personal or group performance. You seem occasionally lost to what must be done.	You seek ways to improve personal or group performance and work towards a goal.	You always seek ways to improve personal or group performance and continually connect your processes to your intended outcome.
		Independent Work (Technical Level)	You need constant supervision.	You need regular supervision and direction.	You need some supervision and reassurance.	Minimal supervision and reassurance are needed.

The above observation sheet addresses all competencies to be investigated as laid out in this chapter. Also, the three dimensions of Jensen's assessment model are indicated in the table. The purpose of this observation sheet is to record observations of students while they are engaged in thought-provoking problem-solving activities, to gain an understanding of their current levels of mathematical modelling competencies. The information gained by observing the students, will assist the researcher in motivating subsequent activities to allow for competence development in mathematical modelling. The observation sheet indicates how the students' competencies will be measured. However, various data collection methods will be used during the activities, such as interviews, walk-throughs, students' written work, reflection instruments, audio and video recordings and field notes to ensure triangulation. A detailed explanation on the assessment instruments will be provided in Chapters 4 and 5.

3.9 CONCLUSION

This chapter answers to the dilemma of identifying relevant mathematical modelling competencies to develop more sophisticated ways of mathematical reasoning while preparing engineering students towards the successful completion of their qualifications, and thereby attended to the second sub-question and accompanied aims of the research question (Section 1.8.3). Through an in-depth study of past and current literature, the researcher was able to map engineering technician competencies to mathematical modelling competencies, and various competence taxonomies from literature were combined to explain how the mathematical modelling competencies can assist in promoting engineering proficiency. Competencies relevant to this study include seven cognitive competencies directly related to the modelling process (internalising, interpreting, structuring, symbolising, adjusting, organising, generalising), as well as three meta-cognitive competencies (management, responsibility and communication). Research indicated that the consideration of both 'soft-skills' and technical expertise are necessary to develop a deeper understanding of mathematics (Bennett, 2002:457; Bucciarelli & Kuhn, 1997:220; ECSA, 2015; IEA, 2015; Male, 2010:26; Male et al., 2009:5; Meier et al., 2000:277; Plonka et al., 1994:693; Robinson et al., 2005:124; Rugarcia et al., 2000:7; Rychen & Salganik, 2003). These competencies can assist the engineering technicians to be productive and effective in their future careers, and to be able to adapt to the challenges of an ever-changing society. As discussed throughout this chapter, the competencies are many and varied, and, depending on the task, various combinations of the competencies are required. Thus, the competencies to investigate are not mutually exclusive, but interrelated.

Chapter 4 builds on the theory as laid out in these first three chapters, and aims to provide a theoretical explanation of the methodology (design-based research and case study research) to be used in this study. Case study research will serve to complement DBR, as a few single cases will be studied further to gain a deeper understanding of the larger population. It is expected that the case study research will offer further explanations and understandings of the resulting data.

The various phases of DBR will also be explained, where after the formulating of a hypothetical learning trajectory (HLT) will follow. DBR assists in developmental research to test and refine educational designs, based on the theoretical principles as derived from literature. Students' zones of proximal development (ZPD) need to be determined, to establish what the students are ready to learn through guidance, and activities will be designed and motivated to further the students' understanding of mathematics. DBR's nature of design, enactment and reflection is an iterative process and will be explained and motivated further. Due to the nature of a qualitative study, methodological concerns are real, and will be addressed appropriately.

CHAPTER 4
RESEARCH DESIGN AND METHODOLOGY

Mathematical discoveries, small or great, are never born of spontaneous generation. They always presuppose a soil seeded with preliminary knowledge and well prepared by labour, both conscious and subconscious ~ Jules Henri Poincaré (1854 – 1912)

4.1 OVERVIEW

During the previous chapters, an in-depth theoretical explanation about mathematical modelling, mathematical modelling activities, engineering education, engineering technician and mathematical modelling competencies paved the way to focus on the main research problem as posed in Chapter 1: How could first-year engineering technician students co-develop the mathematical modelling and engineering technician competencies to assist them towards improved mathematical reasoning and understanding that are necessary for successful completion of their studies? What are those competencies that need to be developed and why are they regarded as relevant? Does exposure to mathematical modelling and the development of mathematical modelling competencies have a significant impact on the students' mathematical reasoning and understanding?

A theoretical explanation of MEAs, the roles of the teacher in implementing MEAs, together with the possible benefits that can be realised in teaching and learning mathematics through model-eliciting activities, are presented in Section 2.5. This chapter will build on this understanding, and aim to partially answer the third and fourth sub-questions of the research question:

Sub-question 3: How do engineering and mathematical modelling competencies co-develop to nurture reasoning and deeper understanding of mathematics?

Aim 7 Explain how design-based research (DBR) methodology combined with case study research can be used as a vehicle to investigate the co-development of mathematical modelling and engineering technician competencies for the fostering of reasoning and understanding of mathematics (Section 4.3 and 4.4).

Aim 8 Explore the design and use of instructional activities (MEAs) that can elicit opportunities for such competence development (Section 2.5, Section 4.3.1.5, and Section 5.2).

Sub-question 4: How can competence development and mathematical reasoning be measured in the students' work?

Aim 9 Identify assessment instruments and data collection methods that will assist in obtaining unbiased and reliable results (Sections 4.3.1.6 and 4.3.2.2).

Aim 10 Explain the data analysis processes that will apply when investigating possible competence development. (Sections 4.3.2, 4.3.3 and 5.2)

This study examines engineering technician students' mathematical modelling and engineering competence development in groups, while students are engaged in mathematical modelling activities. During this teaching experiment, all relevant data generation processes that assist in competency development, will be documented. As the study tries to create an understanding of a real teaching and learning problem within a classroom setting, the methodology of design-research is apt. Furthermore, the teaching experiment will be guided by the RME instructional theory's three heuristics named guided reinvention, didactical phenomenology, and emergent modelling (Section 1.7.2). To establish what students can learn with support, their prior knowledge and levels of understanding were determined during a pilot study. A comprehensive study on past and present literature (Chapters 2 and 3) together with the data gathered from the pilot study, will assist the researcher towards developing an instructional strategy for the hypothetical learning trajectory (HLT). This comprehensive and systematic process of constructing the HLT emphasises Poincaré's sentiment of meticulous work ethics when attempting to generate new mathematical ideas (Delury, 1912:316).

Design-based research (DBR) follows a specific methodological approach whereby learning in context is studied through the design and analysis of instructional strategies and tools (Godino et al., 2013:2). This chapter will commence by clarifying the motivation and understanding of this specific research methodology. Barab and Squire (2004:2) define the purpose of DBR as "the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching". This study takes place in the messy and complex classroom situations that characterise real-life learning within a specific context that is relevant to engineering technician students.

In addition, the outputs of DBR can further be developed through Case Study Research, which can provide us with rich data (Section 4.4). As Eisenhardt (1989:534-540) explains, case study research aims to provide descriptions and understandings by combining data collection methods such as interviews, questionnaires, observations and field notes. Rather than developing theories and instructional materials, case study research studies a single case, or a few cases, to understand a larger population of similar cases (Gerring, 2006). In both DBR and case study research, triangulation is obtained through multiple data collection methods, and they provide strong substantiation of constructs that further aim to support localised theory building.

During the design experiment, student volunteers will work in small groups on various modelling tasks (Appendices K-P). The first week will involve six sessions of two hours each, where after weekly sessions of two hours each will take place for the duration of the study for one semester. All findings of the research study will be detailed in Chapter 5.

Sections 4.3 and 4.4 will concentrate on the methods and procedures that are applicable in generating qualitative data for the study with the goal to answer Aims 7 and 8 of the of sub-questions. As to how the MEAs are used during the teaching and learning experiment, will be explained in Section 5.2 to fully address Aim 8. The fourth sub-question will also be unravelled by attending to Aims 9 and 10 of the research question, to identify the various assessment instruments and data collection methods to assist in obtaining unbiased and reliable results (Sections 4.3.1.6 and 4.3.2.2). An in-depth discussion of the planning, implementing and analysis phases of DBR serves to partially answer Aim 10.

4.2 MOTIVATION FOR DESIGN-BASED RESEARCH (DBR)

As the foundations of underlying learning and teaching theories changed over the years from behaviourism (traditional perspectives) to cognitivism and later to social constructivism (contemporary perspectives), improved education was anticipated in all contexts (Reeves, 2006:52-53) (Chapter 2). However, studies (Brown, 1992:143; Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006:4; Walker, 2006:8) indicate that educational research and development as a whole has been a failed enterprise. Conventional approaches to educational research have not yet experienced the callibre of intellectual breakthroughs when comparing to research in other fields such as medicine, engineering or the sciences (Hamilton et al., 2008:2). Also, no measureable large scale improvements in teaching or learning practices have been noted. In fact, the Institute of Race Relations' (SAIRR, 2016) publicised statistics show that the average numeracy scores (average mathematics scores) of 2014'2 Grades 4 and 5 learners were 37%, while the average mathematics score of 2014's Grade 9 learners amounted to 11%. Also, only 263 903 (41%) of the 644 536 National Senior Certificate candidates were enrolled for Mathematics in 2015 with only 7% of them passing Mathematics with a mark of 70% of more. Reeves (2006:53) and Reimann (2011:37) blames part of these poor performances on research, and believe that educational research focuses too much on scientific proofs and hypotheses, in stead of making learning research more relevant for classroom practices. Cronbach in Reeves (2006:55) warned against the dangers of discounting local conditions, as the applications of generalised results are working hypotheses, not conclusions. Traditional educational research tend to conduct once-off quasi-experimental studies, which are not necessasirily linked to a robust research agenda and normally entertain weak links with practice (McKenney et al., 2006:72).

The movement towards socio-constructivism has raised the need for different research designs, as socio-constructivism promotes the idea that mathematics education should benefit from students' own ideas and inventions within an interactive learning environment (Gravemeijer, 2004:106). An instructional design was needed to help the students to develop their own current ways of reasoning into more sophisticated ways of mathematical reasoning, hence the renewed interest in design-based research. The purpose of design research is to "develop theories of the processes of learning as well as the means designed to support that learning in naturalistic settings" (Gravemeijer & Cobb, 2006:18). This study will attempt to *develop an understanding of the co-development of mathematical modelling and engineering technician competencies, together with instructional activities and other means, to support competence development for improved mathematical reasoning and understanding in the classroom*. Research questions must aspire to result in knowledge that is both exploitable and open to validation. Therefore, the ultimate aim is not to directly apply a theory to existing problems and evaluate whether it works or not, but rather to strive to find a practical and effective solution to a real teaching and learning problem by incorporating an interactive learning environment within a specific context. This research study occurs within such a classroom ethos with the facilitator as researcher.

More effective educational interventions can be developed and learning opportunities can occur during the design research process (McKenney et al., 2006:72). Gravemeijer (2004:104) argues that design research provides "an empirically grounded theory on how the intervention works". By carefully reporting and documenting on all the design processes, relevant data can be collected and new theories can be developed by using this data. The above complements McKenney et al.'s (2006:72) view that design research is not only judged on the merits of disciplined quality, but also on its impact on practice. Van den Akker et al. (2006:5) characterises design research as follows:

- The research aims to design an *intervention* in the real-world – the focus is on complex problems in real contexts in collaboration with practitioners;
- The research makes use of *multiple cycles* of analysis, design and evaluation;
- The focus of the research is to understand and improve interventions, thus *process oriented*;
- The design can be measured by its practicality for users in real contexts – the research is *useable*;
- The design is partly based on theoretical propositions and contributes to theory building. However, design research is not done to test theories, but rather to discover ways based on theories to determine the *effectiveness of the theories in practice*.

The theoretical support for design-based research is rooted in both socio-constructivism (Section 2.2) as well as Realistic Mathematics Education (RME) (Section 1.7.2).

4.2.1 Theoretical Explanations of DBR

Design-Based Research (DBR) is a methodological approach whereby *learning in context* is studied by designing and analysing instructional strategies and tools. As mentioned before, DBR approaches educational research as a fusion between empirical educational research and the theory-driven design of learning environments, and therefor no priori hypotheses exist (Godino et al., 2013:2,6). The methodology of DBR explains how, when, and why educational innovations may or may not work in practice (Design-Based Research Collective, 2003:5). As the aim remains to develop a product – curriculum, sequence of lessons, educational software, and so forth – from research, Hjalmarson and Lesh in Godino et al. (2013:7) suggest that DBR can be considered as a form of engineering inquiry.

The new theories that can be produced through DBR as per Barab and Squire (Section 4.1), embody:

- how mathematics learning can be improved in realistic contexts based on research results and what instructional resources to be used (paradigmatic issues);
- a design based on various interpretive frameworks and which emerges from the data (theoretical assumptions);
- a qualitative and quantitative character consisting of a planning, experimentation and retrospective phase (methodology); and
- instructional resources and local emerging theories that emerge from the above activities (methodology) (Godino et al., 2013:6).

The Design-Based Research Collective (2003:5) attributes five characteristics to the research method: iterative, intertwinement, shareable, contextual, and the connection of processes to outcomes. These five characteristics can be explained as follows:

4.2.1.1 Iterative

The iterative nature of the research and development process takes the form of *continuous cycles* of design, enactment, analysis and redesign (Design-Based Research Collective, 2003:5). A design needs to be regarded as a unified system that *changes continually* as a result of the continual evaluation and which again results in the adaption of the design (Brown & Campione, 1996). To progressively refine a design, the practitioner implements the first version of the design – that was designed with carefully planned procedures and materials – in the natural classroom setting. By analysing and reflecting on the data that are produced, this flexible design gets repeatedly revised until “all the bugs are worked out”, or rather, a local instructional theory (LIT) has been established (Collins et al., 2004:18,20).

4.2.1.2 Intertwinement

This principle is based on the idea that the designing of learning environments are intertwined with the development of theories (Design-Based Research Collective, 2003:5). However, previous theories are not avoided or ignored in DBR, but rather serves to encourage theory building as unobservable elements can also be incorporated. As explained by Cobb, Confrey, Lehrer and Schauble (2003:9), “design experiments are conducted to develop theories – though humble in that they target domain-specific learning processes – not merely to empirically tune ‘what works’”, and should thus aim towards the dual goals of developing theory as well as making a contribution to practice.

DBR goes beyond designing and implementing a specific intervention. These interventions represent specific theoretical underpinnings regarding teaching and learning, and it is bounded by interrelations with theory, designed products and practice (Design-Based Research Collective, 2003:6). DBR does not adopt specific theoretical frameworks, but rather focuses on the design, implementation and evaluation of educational interventions in naturalistic contexts without explicit interest in epistemological questions (Godino et al., 2013:5). Although the approach towards DBR is mainly qualitative or mixed, DBR tends to a qualitative posture, assuming that theories emerge from the data and vice versa.

Even though the focus of this research study is to investigate and support students’ efforts to develop mathematical modelling and engineering technician competencies with the goal of cultivating a deeper understanding of mathematics, DBR allows for the opportunity to also improve curriculum design, and simultaneously yield findings concerning certain aspects of students’ understanding of mathematical modelling, their abilities to delve in argumentation and reflection, as well as the role that mathematical modelling activities and the social interactions around them play in such processes. These findings can also provide insights over a period of time into the complex process of developing modelling competencies and helping to understand and improve the role of the teacher, as well as the role of the learning materials. The role of emergent features should be considered throughout the experiment, since emergent behaviours of students in response to activities can also drive the refinement and further development of both the intervention, as well as the development of the theory.

4.2.1.3 Shareable

“Sustainable innovation requires understanding of how and why an innovation works within a setting over time and across settings and generate heuristics for those interested in enacting innovations in their own local contexts” (Brown & Campione, 1996).

Because both the possible contribution to a theory of learning and the possible contribution to practice are regarded as important, it is vital that the researcher aims to design an intervention that has the possibilities to migrate to other classrooms as well (Brown, 1992:143). Successful interventions in educational research can easily be regarded as 'fond illusions' as it might not necessarily be shareable to settings outside the innovator's control. By unpuzzling variables, theoretical clarity can be enhanced and the necessary and sufficient aspects of the intervention can be disseminated. However, this is no easy task due to the complex and 'messy' environment in which teaching and learning takes place. Essential features need to be determined and must be in place to allow for the required change to take place. Adequate documentation procedures of the entire design process, together with the use of technology such as video recordings, are valuable resources in documenting conceptual changes, and can be made available to continue exploring data as new and more powerful theories emerge (Brown, 1992:171-174).

To answer to the shareability dilemma, DBR requires methods such as descriptive databases, systematic analyses of data with carefully defined measures, as well as periods of reflection during which data are intepreted from a variety of perspectives.

4.2.1.4 Contextual

Research must explain how the design works in an authentic setting. Apart from assessing the research project as a success or a failure, attention must be given to the interactions that can refine our understanding of the learning issues involved. By attending to context it not only produces a better understanding of the intervention, but it could also improve the theory on teaching and learning (Design-Based Research Collective, 2003:7). This description will further enhance usability prospects.

The unpredictable and complex nature of a classroom requires that contextual variables also include local factors (e.g. describe the setting for which the design is created), as well as system factors (e.g. large-scale assessments) (McKenney et al., 2006:76). The method that is applied in DBR assumes that phenomena are dependent on the context and changes in students' behaviour that can result from interactions of the various factors (Design-Based Research Collective, 2003:6). In identifying the relevant contextual factors, our understanding of the nature of the intervention itself can be enhanced. Detachment of interventions from practice can cause a researcher's intended design to differ substantially from the enacted design (Collins et al., 2004:17). Robinson (1998) comments that educational research detached from practice also risks the incompleteness of knowledge about which factors are relevant for prediction. Rather, educational interventions must be viewed holistically, and therefore interventions must be endorsed through the interactions between materials, the facilitator, and students, since many factors can determine the outcome of a specific intervention (Design-Based Research Collective, 2003:5). The research findings that result from close proximity to real schools and close cooperation with teachers and students in an authentic setting have the potential to be implemented more easily and rapidly in classrooms in general (Reimann, 2011:38).

Some of the factors that need careful consideration are:

- *Ideal classroom environments* have to be established that allow for students to spontaneously propose new ideas; they must be able to share their learning experiences and produce products that they can demonstrate to others. The establishment of such classroom environments can have a major influence on obtaining the intended goals of the research study and the functionality of a classroom, as a learning environment is thus central to design experiments (Brown, 1992).
- Students must be encouraged to engage in *self-reflective learning and critical thinking* (Brown, 1992:150).
- Teachers serve as *active role models* and *guided teaching and learning* take on an important role. Through continually diagnosing students' understanding, their zones of proximal development can be determined. The teachers need to observe their students' thinking to identify their conceptual strengths and weaknesses. Through guided reinvention, they must then try to help students to strengthen their relevant concrete, intuitive, and informal conceptual foundations, to proceed to levels that they would not have reached without expert guidance (Lesh et al., 2000:631).
- *Technology* encourages intentional learning and promotes reflection and communication (Brown, 1992:150).
- *Assessment methods* must focus on students' abilities to discover and use knowledge, and not merely to test how much of the knowledge they are able to retain (Brown, 1992:150).

- Brown regards human memory's most interesting aspect as the fact that people have knowledge and beliefs about it. Human memory includes *meta-cognition* – competencies that will also be investigated in this study – which can only be developed when students grapple with meaningful, contextual material. Consideration therefore, must also be given to the social context of learning and collaborative cognition which takes place in everyday life (Brown, 1992:146-147).

The above aspects denote the importance of continuous interplay between teacher training, curriculum selection, and testing, as changes in the one affects outcomes in the other. Figure 4.1 indicates the importance of considering the role of the teacher, the student, the type of curriculum as well as the place of technology, as inputs into the work as a whole (Brown, 1992:141):

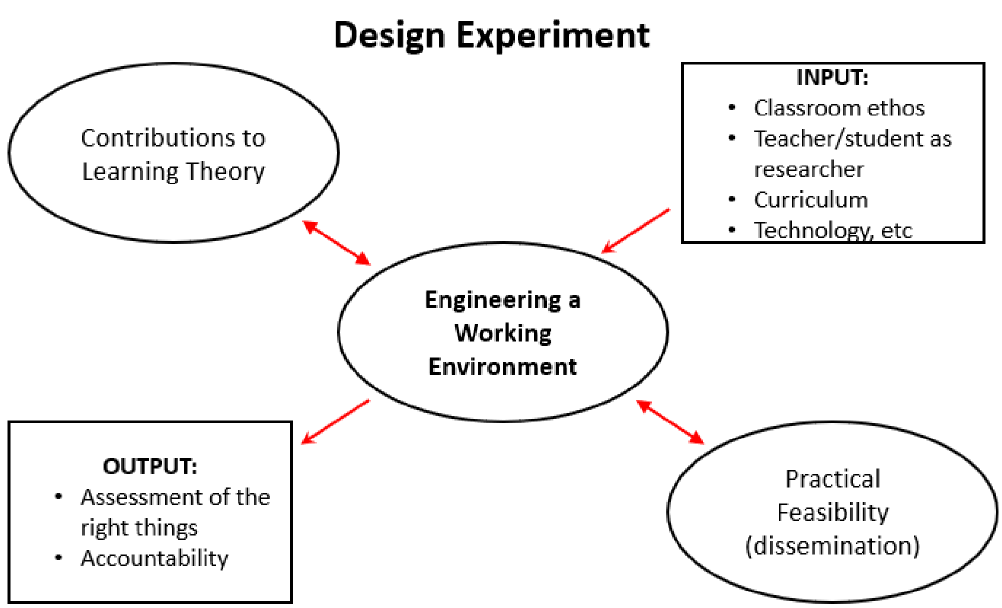


Figure 4.1– The complex features of design experiments (Brown, 1992:142)

Even though contextualisation is important, attention must also be given to proceed toward *generalising* from those settings, to guide the design process and to move forward to more formal ways of thinking (Collins et al., 2004:21). Chapter 5 details how the students in this study progressed from their initial context-bound descriptions to more formal ways of mathematical reasoning and thinking.

4.2.1.5 Connecting Processes to Outcomes

Knowledge that directly relates to educational practices is generated through the trails of careful documentation throughout the research study that links the processes of enactment to the research outcomes. These trails of documentation serve as critical evidence in explaining why specific outcomes occur. Triangulation (Section 4.5.2) is such a method which allows for connecting intended and unintended outcomes to the processes of enactment (Design-Based Research Collective, 2003:7).

4.2.2 DBR outputs

Design research results in three levels of outputs: primary output (design principles), secondary output (societal contribution) and tertiary output (the professional development of participants) (McKenney et al., 2006:72). These outputs are further shaped by a set of tenets in a particular domain. The tenets are rigor, relevance and collaboration and the connection between the tenets and the outputs can be explained as follows:

4.2.2.1 Primary output (design principles) shaped by rigor

Primary output relates to the resulting knowledge that is generated. It orients the researcher to gain an understanding of what is learnt from the experiment. These theoretical insights that are developed through the process of design research are referred to as *design principles* (McKenney & Reeves, 2013:34). Design principles need to be interpreted and must be viewed as a guide towards understanding and replicating other similar local experiences in practice. Interpretation of design principles occurs inductively from prior examples of success and are continually refined during the experiment (Bell, Hoadley, & Linn, 2004:83-84). The tight link between empirical research and theory, triggers strong principles that can answer the research questions that need to be addressed.

The generated knowledge (design principles) must adhere to *rigorous* standards. Issues to be addressed will be internal validity (the extent to which causal relationships can be based on findings), external validity (generalisability), reliability (the extent to which the operations of the study can be repeated with the same resuls), trustworthiness, and utilisation (whether the results of the study is useful or not) (Section 4.5).

4.2.2.2 Secondary output (societal contribution) shaped by relevance

The secondary output refers to the *societal contribution*. This output often contains products or programs that are of value to the community, for example materials that can be used in the classroom (curricular products). The resulting material must be *relevant* to the context and culture in which engineering students studying mathematics at their specific institution will implement it.

4.2.2.3 Tertiary output (participants' professional development) shaped by collaboration

Thirdly, the tertiary output denotes the contribution that the design research activities make in terms of the *participants' professional development*. Apart from actively taking part in the activities, students' own professional development can also be enhanced while the researcher or practitioner applies data collection methods such as interviews, walk throughs, discussions, observations and questionnaires which can stimulate dialogue, reflection and engagements among the participants (McKenney et al., 2006:72).

The design and development must be conducted in *collaboration* with all the stakeholders (students, lecturers, researcher and the University of Technology where the study takes place). While addressing research needs, meaningful experiences must be offered to the students and the data collection procedures must also aim to be mutually beneficial.

This discussion on DBR exposes the consistent and logical methodology that is used to link theoretical research and educational practice. The investigation of both the design of the intervention, as well as its effect on learning in a specific context, can produce explanations and principles of innovative learning to be localised and used in new settings. To provide robust explanations, this research study will constantly concentrate on the variables that can effect the outcome such as teachers, students, classroom ethos, activities and reflection tools, as they are all inputs into the work as a whole (Reeves, 2006:58). To summarise, the Design-Based Research Collective (2003:7) reflects on DBR as follows:

> Design-based research goes beyond perfecting a specific product. The intention of design-based research in education is to inquire more broadly into the nature of learning in a complex system and to refine generative or predictive theories of learning. Models of successful innovation can be generated through such work – models, rather than specific artifacts or programs, are the goal.

The following section will examine the various phases of the DBR process to inform the reader how the DBR methodology is implemented in this study to investigate the co-development of mathematical modelling and engineering technician competenencies, and thereby also serves to address Aim 7 of the research question (Section 1.8.3).

4.3 DBR METHODOLOGY

Confrey in Reimann (2011:38) describes DBR as a study that

> … seeks to document what resources and prior knowledge the students bring to the task, how students and teachers interact, how records and inscriptions are created, how conceptions emerge and change, what resources are used, and how teaching is accomplished over the course of instruction, by studying student work, video records, and classroom assessments.

The extensive scope of processes and context that is considered as relevant, forms an essential aspect of a design study. DBR comprises of three phases of a macro-cycle of design research together with the related methodological issues (Godino et al., 2013:3; Gravemeijer & Cobb, 2006:19):

- Phase 1: Planning and preparation for the experiment, which entails an in-depth literature study, a pilot study, as well as planning for the teaching experiment;
- Phase 2: Experimentation to support learning in the classroom, involving the empirical data generation; and
- Phase 3: Conducting retrospective analysis of the data generated alongside the experiment, consisting of transcriptions, coding and analyses of the various experiments.

4.3.1 Phase 1 of DBR – Planning

The main goal of this first design phase is to formulate the ‘first draft’ of the local instructional theory (Section 5.4), which will be elaborated and refined during the experiment, as well as to clarify the study’s theoretical intent (Gravemeijer & Cobb, 2006:19). This phase focuses on the establishment of learning goals, the collection and possible adjustment of activities relating to the learning goals, as well as discussions about the formulation of a hypothetical learning trajectory

(HLT) that needs to be developed. The HLT comprises of suggested (anticipated) learning goals, a suggested learning activity, the anticipated thinking and learning of the students, as well as the researcher's expectations on how the development of mathematical modelling competencies can be explained (Simon, 1995). This first draft of the local instructional theory – the HLT – thus forms the heart of Phase 1's first iteration.

Section 2.3 dealt with the various perspectives on modelling in mathematics education, and the theoretical perspective applies primarily to this study (Section 2.3.6), even though aspects of all the perspectives are intertwined. Relating to the educational perspective (Section 2.3.3), mathematical modelling fosters the advancing of mathematical modelling competencies (Blomhøj, 2009:5; Kaiser & Sriraman, 2006:305). This approach, denoted as *modelling-as-content* by Galbraith (2007:47), refers to mathematical modelling as the instrument to provide students with the abilities that are relevant to their mathematical learning, as well as to enable them to learn and apply problem-solving competencies needed for modelling real-world situations. This study focuses on modelling-as-content, as the development of competencies to acquire a deeper understanding of mathematics and problem-solving drives this study. The introduction session explains the modelling process, and a diagram of the modelling process will be handed out to all the students for reference purposes throughout the study. By understanding the processes of mathematical modelling, students can learn ways of approaching real-world problem solving tasks. Julie and Mudaly (2007:504) noted that the modelling-as-content perspective focuses primarily on the construction of models, rather than the successful execution of specific mathematical procedures. The students are free to use their own unique mathematical repertoire for explaining and solving meaningful real-world problems.

Once the study's theoretical intent is clarified (Section 4.3.1.1), the students' prior knowledge will be assessed by means of a pilot study (Section 4.3.1.2), where after MEAs must be designed that adhere to specific design principles (Section 4.3.1.5). By entertaining these fundamental principles, students are allowed to reinvent their own mathematics, they can progress their mathematical thinking and they can construct new knowledge through the building of models, while actively participating in their own learning.

The purpose of this research inquiry is focused on how to improve (not prove) competence development. Herrington, McKenney, Reeves, and Oliver (2007:6) emphasise the intention of design research to use the context "to gain an understanding which will have meaning beyond the immediate setting”, therefore pointing to two or more design cycles. The first iteration of the DBR design cycle differs from the second and further iterations. Second and further iterations are based on the analyses and evaluation of the previous cycles to determine what decisions to make for the so-called ‘next steps’. This cyclic process gets repeatedly revised until the experiments are carried out successfully.

4.3.1.1 Stipulation of the study’s theoretical intent (preparing for the first iteration of Phase 1)

To clarify this experimental study’s theoretical intent, a comprehensive study of past and present literature on mathematical modelling, modelling problems, engineering and mathematical modelling competencies was carried out (Chapters 2 and 3). After providing a clear understanding of all the necessary theoretical concepts, the mathematical modelling competencies were mapped to the competencies required for engineering technician students’ professional lives, to promote students’ mathematical reasoning and understanding. This thorough literature study contributed to orientate the researcher and to select well-judged tasks and instruments (Sections 4.3.1.5 and 4.3.1.6). A pilot study took place to assist with the selection of the tasks and to design and adapt the required instruments to enable the identification of first-year engineering technician students’ current levels of learning in terms of their prior instructional experiences.

4.3.1.2 Instructional starting points – strategies and tasks (First iteration)

In determining instructional starting points, the researcher had to consider the most effective way of answering to the research question. The motivation for this research study came from the researcher’s observation of students’ inabilities to use relational understanding effectively. The researcher anticipated students to use instrumental understanding and to go through meaningless steps of applying previously learned procedures and algorithms without much relational understanding (Skemp, 1976). This inability of students to gain a holistic view of mathematics, limit them to acquire competencies such as deeper understanding, being able to simplify complex problems, to analyse, argue, organise, evaluate and reflect on processes and procedures (Chapter 2). Students’ leniency towards instrumental understanding rather than relational understanding also limit their meta-cognitive development. By not being actively engaged in problem-solving activities, they struggle to develop competencies such as managing their thinking processes, communicating mathematical ideas clearly and effectively, and taking responsibility for their own decisions (Skemp, 1976).

Activities were selected based on the results of existing literature, to allow for engineering technician and mathematical modelling competency development, and specifically the ten competencies identified in this study. The literature study in Chapter 2 indicated that mathematical modelling can promote the development of such conceptual structures over time, which in turn raised the need for using MEAs. MEAs are typically solved in small groups during one or two class periods, while students are iteratively engaged in expressing, testing and revising models to solve contextual problems (Hamilton et al., 2008:2,4). While trying to *understand and making sense* of the problem, students learn to take *responsibility* for their own learning. Through *reflection* and *communication*, they learn to justify their thought processes and connect the real-world problem to their solutions. These activities can enable students to perceive mathematics as useful and worthwhile, and they progressively learn to *manage* their own learning and develop into effective learners and doers of mathematics. These meta-cognitive competencies that emerge by means of MEAs – responsibility, communication and management – have also been identified by ECSA as important meta-cognitive competencies for engineering technicians (Chapter 3). Adhering to the six design principles of MEAs, trails of documentation will be produced, which is also one of the primary characteristics of DBR.

Working in *small problem-solving groups* also provide natural settings for interpersonal monitoring and regulating of members' goal-directed behaviours (Artz & Armour-Thomas, 1992:138). Biccard and Wessels (2011:382) also suggest that by exposing students to a broad range of peers, can benefit interaction, communication and reflection between the group members. The theoretical understanding of group work (Chapter 2) explained the possibility of *studying meta-cognitive behaviour* while students talk about the problem, *as well as cognitive behaviour* when the students are actively involved in constructing solutions for the problems. Continual examination of students' understanding and reasoning can gain an insight into why they used specific approaches and thereby the instructor can gain an understanding of the students' prior instruction (Larsen & Lockwood, 2013:2).

The pilot study served as a link between the literature study and the teaching experiment, and assisted towards ascertaining data integrity, as data integrity is dependent on the tasks and instruments used. During the pilot study data collection methods were tested for two purposes: to test for rich and meaningful data, and to use the data to begin the analysis process to inform future phases of the study. Also, students' current levels of understanding could be determined. Reflection tools, written work, informal discussions, group assessments, observations, video and audio recordings, and field notes were used to gather relevant data. These assessments instruments have, if designed and used correctly, the potential to reveal critical information about the students' prior instruction. Modelling tasks were selected and sequenced to reveal the usefulness and importance of developing mathematical modelling competencies to the students. Each instructional activity required planning and anticipation of possible problems that might arise from the students' side during the experiment. As the research literature only provided limited guidance, reports of the students' learning processes in the specific domain, as well as the tasks and tools that may enable or support competence development, were relevant for constructing a hypothetical learning trajectory (HLT). Consideration for classroom norms and classroom discourse was also included in the design, as well as the proactive role of the teacher.

- The Pilot Study

The pilot study took place between August and November 2016 with first-year engineering technician students. The students that took part in this pilot study did not meet the necessary entrance requirements for studying engineering at the University of Technology, and they were enrolled in a bridging course. The pilot class consisted of 12 students, grouped in three groups of four students each. The groups met on a weekly basis for two hours, and they worked within their groups on various modelling tasks. However, due to student unrests, the researcher was not able to conduct as many modelling tasks as she planned. Data collection strategies such as modelling activities, data recordings, interviews, questionnaires, informal discussions, observations, and field notes were used throughout the experiment.

The following tasks that were chosen for the pilot study were all adapted from existing literature, to strengthen the validity and reliability of the research experiment further.

1. Lawnmower Task
2. Paper Airplanes
3. Tidal Power

4. Product Coding
5. Turning Tyres
6. Filling Dams
7. Measuring Study Effectiveness
8. Efficient Storage

The pilot study yielded valuable insights, especially in terms of the practicality of the study. The time spent on the activities were far more than planned, problems were experienced related to the quality of the video and audio recordings, and time was lost due to student unrest. The instruments were continuously adapted, shaped and changed to acquire all possible data relevant to the purpose of the study. The researcher did not explain the modelling cycle and modelling competencies to the students prior to attempting the first activity, which caused very slow progress and led to great confusion among the students in terms of what was expected from them. The researcher then had to plan for an introductory discussion on what mathematical modelling is, what mathematical modelling competencies entail, as well as why it is important for the engineering students to develop such competencies.

Furthermore, the students' work relating to their problem-solving abilities revealed their inclination to follow step-by-step procedures to solve problems – they did not understand how they could benefit from an activity without clear steps to follow towards solving the problem. The students tend to be unreceptive in the first few activities, as they constantly waited on the facilitator to guide them towards possible next steps. It was only during the third activity that the students were actively engaged in trying to make sense of the problem, since by that time they have come to realise that the facilitator would not employ direct instruction. Memorisation was the preferred method of learning mathematics, as students constantly tried to apply learned procedures without relevance to the problems posed.

Some of the main benefits that arose from this pilot study was the lessons learnt that related with how to deal with practical issues (i.e. time constraints, equipment, etc.) and to ensure that the collected data are relevant for the purpose of the study. The researcher gained some experience with facilitating modelling activities, especially as to when, and how much to guide a class without exercising direct instruction.

4.3.1.3 Second and further iterations

To ensure alignment with the results of the pilot study and to verify that decisions are influenced by one of the goals of meeting the triangulation criteria, namely the utilisation of multiple methods in various formats, the researcher adapted a pre-intervention questionnaire designed by Kloosterman (2006) to be used with the student groups that participated in the main study. The assessment and reflection on these interviews are covered in depth in Chapter 5. Two main arguments that resulted from the questionnaire and which complemented the findings of the pilot study, were:

- Procedural understanding and memorising seems to be of primary importance to the students when they learn mathematics; and
- The data indicates that students tend to do what they are told, a far cry from the educational goal of mathematical modelling (Chapter 5).

The complementing results of the pilot study and the pre-intervention questionnaire (Appendix J), together with the findings of current literature (Chapters 2 and 3), further substantiated the researcher's beliefs about students' innate incapacity to develop deeper mathematical reasoning and understandings to successfully solve real-world problems.

Based on these findings, it was decided to use the following activities, which involved 3 full days (2 sessions of 2.5 hours per day), as well as an additional 11 weeks (1 session per week of 2 hours):

1. Pre-Assessment Questionnaire (to confirm alignment with the pilot study)
2. Lawnmower Task
3. Paper Airplanes
4. Tidal Power
5. Product Coding
6. Turning Tyres
7. Finding a lost cell phone

4.3.1.4 Selection of participants (second and subsequent iterations)

Purposive homogenous sampling applied to this experiment, as all the students were first-year engineering students studying at a University of Technology in South Africa. The group of students that took part in the main research study was not the same students that enrolled for the pilot study. These students have had no prior experience with mathematical modelling activities. They also did not meet the entrance requirements to be allowed access to study engineering. Since 2006 the Institution offers a model of extended education, known as the *Access course* for these struggling students, which means an additional six months of university preparation. Through more support and intensive tutoring, students enhance their chances of succeeding in the university's mainstream programmes. Previous research studies revealed that the Access program is not only an effective tool to allow struggling students a chance to study Civil Engineering, but that the difference in course duration of Access and mainstream students completing a National Diploma in Civil Engineering at the particular University of Technology is also insignificantly small (De Villiers, 2013). Furthermore, studies indicate that low-achieving students have the potential to excel in mathematical modelling (Aliprantis & Carmona, 2003:261; Knott, 2014:149; Lesh & Clarke, 2000:633-634; Maaß, 2005:71).

All the students in the class were invited to take part in this study, but only 12 of the 150 students were interested to participate in the research experiment. They were grouped in 3 groups of 4 students each. The first week of the research experiment the students worked on modelling activities for three days, six sessions of two hours each. From there on they met once a week for two hours. The current literature supports the importance of developing modelling competencies – especially meta-cognitive competencies – in small groups, as they enable observers to hear the thoughts of students without interfering in the process (Artz & Armour-Thomas, 1992:168). Qualitative data were generated from the students by making use of reflection tools, informal discussions, observations, field notes, written work, and video/audio recordings. The study took place in the classroom to keep it as close as possible to the natural classroom environment. The duration of the experiment was one semester.

Flyers were handed out to the first-year engineering students during registration week, inviting them to take part in the research experiment. The students received additional letters including student information sheets and consent forms on the first day of the experiment. The letters were discussed in a class setting and signed by the students once they were satisfied with the subject matter. Permission was also granted from the University of Technology (Appendix R).

4.3.1.5 Designing and selecting instructional activities

The core of design-based research is based on classroom teaching experiments to develop instructional sequences (Godino et al., 2013:5). These instructional sequences are underpinned by local instructional theories. In working toward given end goals**,** teachers can proactively support students' mathematical development. However, it demands that teachers have to anticipate students' thinking and learning experiences to ensure that they are aligned with the end goals. Simon (1995) in Gravemeijer (2004:107) introduced *hypothetical learning trajectories (HLT)*: in designing a teaching experiment, the researcher needs to develop sequences of instructional activities representing assumptions about the students' learning paths. An anticipatory thought experiment will lead the researcher to imagine how the planned teaching and learning process can be carried out within the classroom context, and also what thought activities the students may engage in while carrying out the activities. Through a thorough analysis of the actual processes as they unfold, the researcher gathers valuable information that is used to revise or refine further instructional activities that corresponds with the revised learning trajectory. This results in the formulation of a conjectured local instruction theory (Gravemeijer, 2004:108). Gravemeijer distinguishes between hypothetical learning trajectories and local instruction theories in that the former refers to the planning of instructional activities in a specific classroom on a daily basis, whereas local instruction theories explain the envisioned learning route, relating to an entire set of instructional activities (Gravemeijer, 2004:107).

An instructional theory comprises of three elements: learning goals, planned instructional activities, and an envisioned learning process. The envisioned learning process anticipates how students' thinking and understanding might evolve when the instructional activities are employed in the classroom, as well as possible means of supporting that learning processes (Gravemeijer & Cobb, 2006:19). The support refers to the instructional activities, an envisioned classroom culture, as well as the proactive role of the teacher/facilitator (Chapter 2).

Three important aims of the design experiment are:

- the constitution of a local instructional theory,
- to place classroom events in broader context by considering the various roles of the teachers, the students, and the classroom norms, and

- the development of new categories that are invented and embedded in a supportive theoretical framework that can assist in generating, selecting, and assessing design alternatives (Gravemeijer & Cobb, 2006:22-23).

The iterative character of DBR is revealed when student inputs and assessments of actual understanding can repeatedly lead to refining and adapting of the conjectured local instructional theory. These continual adaptations are guided by a “possibly still emergent domain-specific instruction theory” (Gravemeijer & Cobb, 2006:22).

The approach that is followed when using and adapting existing materials in DBR is also guided by RME theory (Chapter 1). RME’s three design heuristics – guided reinvention, didactical phenomenology and emergent modelling – drive the design towards a possible learning path with proposed instructional activities to complement the learning path. As Gravemeijer (2004:109-110) explains, this basically means that the researchers need to consider all the possible mental activities that students can experience while they are engaged in the instructional activities. They then need to envision how those mental activities can assist the students to further develop their mathematical understanding. The researcher’s anticipated expectations on how the students’ thinking and understanding evolves give way to a *proposed activity*, which gets revised as new information emerges. A continuous reconciliation transpires between planning, adapting, adjusting and refining, conforming again to the iterative nature of DBR.

As the investigation of mathematical modelling and engineering technician competencies remained the focus of this study, activities had to be selected that could elicit growth in modelling competencies. Caron and Bélair (2007:127-128) emphasise the fact that solid modelling competencies cannot be developed in isolation in a single course, but need to be disseminated throughout the entire mathematical course (algebra, analysis, etc.). This holistic view on mathematical modelling teaching and learning, guided the researcher to focus on a wide range of modelling activities and not only on one specific content area of mathematics.

To ensure that the model-eliciting activities will meet the required and intended learning characteristics, the researcher devised a checklist to warrant conformation to all necessary aspects when designing and selecting instructional sequences of activities. Various factors need to be considered when designing a MEA. A theoretical understanding of MEAs is provided in Section 2.5. The researcher summarised the various design principles and required factors as discussed in that section to guide her towards the successful design and implementation of MEAs by constructing the following diagrams indicating the design principles and the curricular characteristics that must flow from those design principles (Figures 4.2 and 4.3). These diagrams functioned as checklists when designing or selecting an MEA to allow for a thorough investigation of students' mathematical modelling competencies:

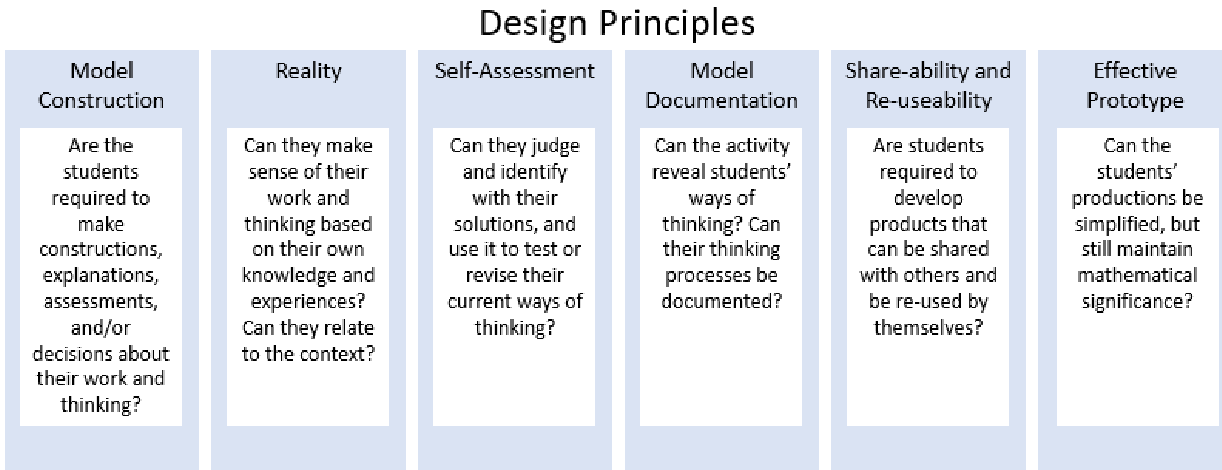
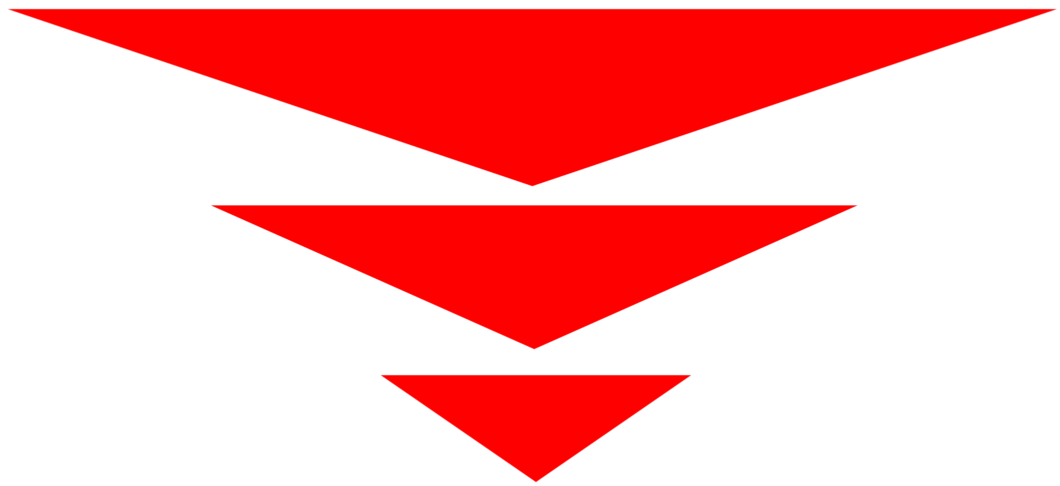


Figure 4. 2 - Design principles for model eliciting activities (Adapted from Lesh et al. (2000))



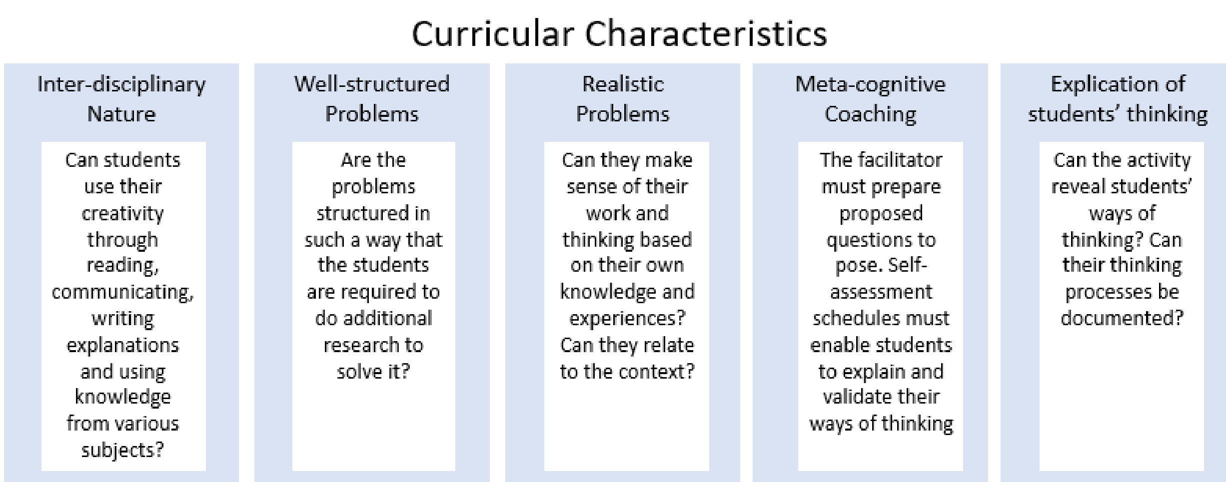


Figure 4. 3 - Curricular characteristics that must flow from the design principles (Adapted from Chamberlin and Moon (2005))

An explicit commitment of instructional designers to adhere to the above principles, offers students the opportunities to experience mathematical modelling in all its facets and allows for the possible development of the ten modelling competencies as suggested in this study. The ten competencies that will be investigated to promote students' mathematical reasoning and understanding, are the seven cognitive competencies of internalising, interpreting, structuring, symbolising, adjusting, organising and generalising, and the three meta-cognitive competencies focusing on management, communication and responsibility (Section 3.7). This discussion about the design and use of instructional activities to elicit opportunities for competence development in this section, emphasises the importance of the careful design of model-eliciting activities, as it does not only need to answer to the design principles as indicated in Figure 4.2, but it also needs to address particular curricular characteristics that can develop students' current ways of thinking to more sophisticated mathematical reasoning by focusing on inter-disciplinary, well-structured and realistic problems, while the facilitator also remains aware of meta-cognitive coaching (Figure 4.3). This section thus also answers to Aim 8 of the research question, namely to explore the design and use of instructional activities that can elicit opportunities for competence development (Section 1.8.3).

The planning, design, selection and motivation of each of the selected activities are discussed in detail in Section 5.2. The remainder of this first phase of the design process, focuses on the design and selection of relevant and meaningful research instruments to assist in obtaining unbiased and reliable results, and consequently also partially attend to Aim 9 of the research question (Section 1.8.3).

4.3.1.6 Designing and selecting research instruments

Apart from video and audio tapes as well as researcher field notes, various instruments have been designed to assist in the documenting of possible modelling competence development. Data was also generated through observations, discussions, oral and written work. This section will explain how the designed instruments contribute to the truthful and reliable documenting of competency development.

- **Instrument 1 – Pre-Intervention Interview**

Pre-intervention interviews were conducted with all students within their groups, to gain an understanding about their expectations of the research experiment. Students were asked questions regarding their beliefs about mathematics and how they perceive themselves as students (Appendix J).

- **Instrument 2 – Group Modelling Competency Observation Guide**

Based on the combined work of Arter and McTighe (2001), Jensen (2007) and Knott (2014), the researcher designed a rubric to allocate scores that relate to the degree to which the students display the relevant competencies while carrying out modelling tasks. This rubric was designed to allocate levels of mastery of all the competencies under investigation, and not merely the presence or absence thereof. These competencies compliment the competencies as requested by international engineering authorities (Chapter 3). By making use of coding, the researcher used this rubric and indicated the level of mastery of the various groups throughout the activity (Appendix D).

- **Instrument 3 – Status Update**

Reflecting on and assessing one's model can guide the modeller about possible decisions to take, assumptions to be made or altered, as well as possible next steps to be taken during the iterative process of modelling (Yildirim et al., 2010:835). Formative self-assessment tools such as status updates were used to assist students to judge the viability of their models for themselves when they iteratively map their initial or intermediate models back to the real-world (Eames et al., 2016:230). Status updates are used to map students' early interpretations and models against the needs of the client. This reflection tool assists students in creating auditable trails of their thinking strategies throughout the activity. Finally, when writing a 'letter to the client', students learn to describe and defend their ways of judging their solution processes (Eames et al., 2016:234). The status update allows for competencies such as internalising, interpreting, structuring, and symbolising to surface (Appendix A).

- **Instrument 4 – Quality Assurance Assessment – Comprehensive solutions**

To rate the comprehensiveness of students' solutions, Lesh and Clarke (2000:145) designed a Quality Assurance Rubric to help educators and students to evaluate the products that are developed in response to model eliciting activities, with the following characteristics: the goal is to develop a conceptual tool; the client who needs the tool is clearly identified; the client's purposes are known; and the tool must be sharable with other people and must it be useful in situations where the data are different from those specified in the problem. Trustworthiness issues are strengthened, as this rubric is a familiar rubric in the mathematical modelling arena. Specific competencies that could be identified from this rubric were internalising, symbolising, adjusting, organising and generalising (Appendix C).

- **Instrument 5 – Student Reflection Guide**

The student reflection guide served as a guide to the researcher to determine possible 'next steps' towards developing a deeper understanding of mathematics. The students have to reflect on the activity and it allows for strengths and weaknesses to surface. Explanations about their strengths and weaknesses allow the researcher to adapt the subsequent activities where necessary, and it also offers guidance as to whether the scaffolding was adequate or not – (Appendix E).

- **Instrument 6 – Group Reporting Sheet**

This reflection tool was adapted from Biccard and Wessels (2011:233). The group reporting sheet assists the students to report on their work in such a way that the researcher can identify the possible presence of competencies. This tool supports students in working with a sense of direction, as it keeps them focused on their goal. Specific competencies emphasised in this instrument are competencies relating to internalising, interpreting, symbolising, adjusting, organising and management – (Appendix F).

- **Instrument 7 – Group Functioning Sheet**

This instrument is adapted from Hamilton, Lesh, Lester, and Yoon (2007:364). Students reflect on their activities during the initial, middle and final stages of the modelling task. They also need to allocate the time that was spent on brainstorming, working, considering alternative approaches, communicating ideas and finding a working strategy. Identifying the presence and sphere of meta-cognitive competencies such as management, communication and responsibility, was the focus of this instrument (Appendix G).

- **Instrument 8 – Written and Oral Work Guide – Poster Presentation.**

The purpose of a poster is to present the main ideas of the project verbally and visually, and thereby needs to be designed with a logical layout without too many words. It is important to convey the required information and it should also include headings, pictures and/or diagrams. The outline of the problem as well as the reasons for the outcome should be conveyed clearly in the students' own work. The mark scheme for the posters is adapted from Berry and Nyman (1998:112).. – (Appendix H).

- **Instrument 9 – Student Reports**

Another means of evaluating the success of MEA implementation is to use student reports. Student reports refer to the typical 'letter to the client' where the students need to explain the model that they have developed to resolve the problem. Assumptions should be clearly outlined as it can have a substantial effect on the solutions provided (Yildirim et al., 2010:835). Information that can be extracted from these reports include the students' level of understanding of the problem, their abilities to extract relevant information, whether they received adequate guidance, if enough time was allowed to complete the activity, as well as whether they were able to effectively communicate their solution methods.

- **Instrument 10 – Post Intervention Questionnaire (meta-cognitive competencies)**

A post intervention questionnaire will also be handed out at the end of the modelling course to gain more insight into the students' opinions about the mathematical modelling course. The researcher anticipates finding relevant information relating to meta-cognitive competencies based on the students' answers about their feelings and beliefs. These questions were sourced from (Berry & Nyman, 1998:108-110) – (Appendix I).

Due to the variety of factors that need to be considered when planning this design experiment, the researcher created a further checklist (Figure 4.4 below) to ensure that all the activities and assessment instruments adhere to the requirements of the three design heuristics of the RME theory, as discussed in Section 1.7:

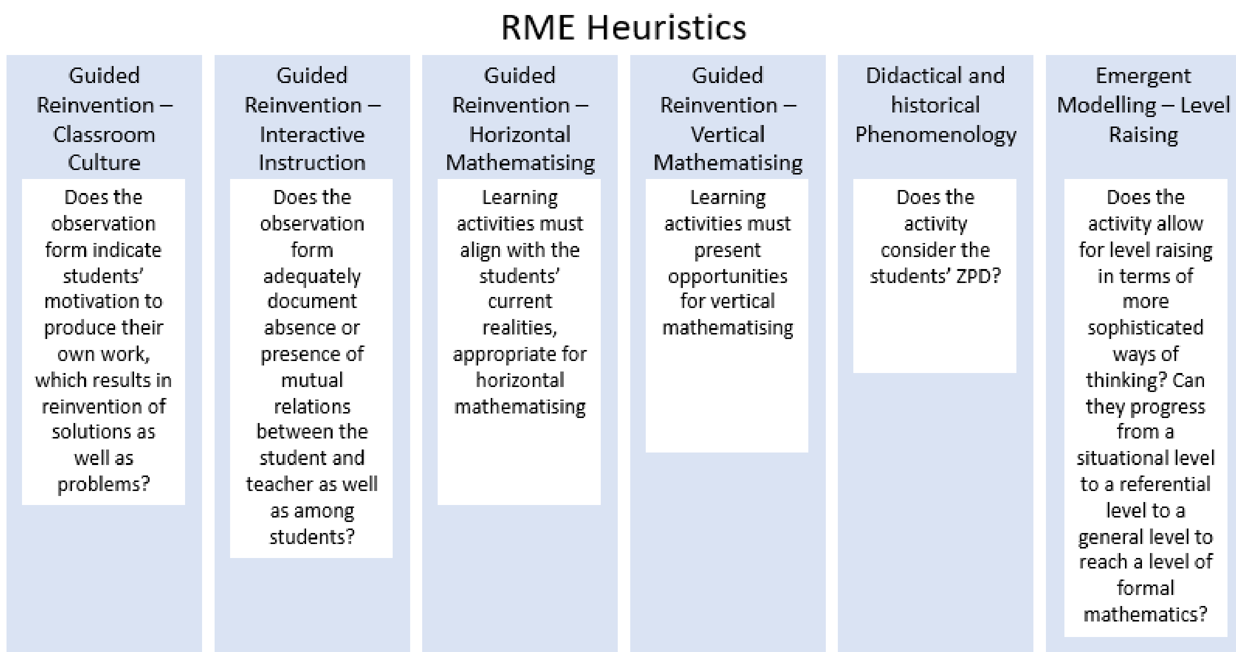


Figure 4. 4 - RME Principles and characteristics to consider when designing MEAs and Assessment Instruments

The assessment instruments as identified in this section, partially answers to Aim 9 of the research question (Section 1.8.3). These instruments need to recognise the existence and range of the students' competencies in relation to the mathematical activities that they are involved in, to reflect the level of their modelling competencies. It thus does not merely serve to indicate whether the competencies exist or not, but asks for a holistic and truthful interpretation of the students' work and ways of thinking. Of primary importance are the questions that are asked in the reflection tools and questionnaires, as the researcher has to continually inspect the questions to ascertain the quality and relevance of the students' answers to allow for a wide variety and adequate quality of relevant data to be collected. By employing a vast range of assessment instruments, triangulation assists towards obtaining a more holistic and contextual portrayal of the phenomenon under investigation, as variances can be uncovered which otherwise would not have surfaced (Jick, 1979:603).

Once all the planning and designing of the activities, instruments, classroom culture and anticipated learning and teaching has been done, together with consideration of the students' anticipated ways of thinking by analysing the results of the pilot study, the second phase of the design study can commence.

4.3.2 Phase 2 of DBR – The teaching experiment

This second phase of the design experiment represents the implementation of the modelling activities for testing, improving and developing an understanding of how the conjectured local instruction theory functions. Gravemeijer suggests that the design must consist of a cyclic process of (re)designing and testing instructional activities and other aspects of the design (Gravemeijer & Cobb, 2006:24).

During the experiment and retrospectively, the actual processes of students participation and learning are analysed; thus cyclic processes of thought experiments and instructional experiments make up the design experiment. The researcher starts by anticipating the mental activities which the students would engage in. During the activity the researcher has to determine to what extent the students' actual activities correspond to and differ from the anticipated ones, to support decisions about revised follow-up activities, this entails the term which Simon in Gravemeijer calls '*hypothetical learning trajectories*' (Gravemeijer & Cobb, 2006:28).

These modelling activities were conducted during the first semester of 2017. Six sessions of two hours each occurred within the first week to allow for possible student unrest later in the semester. In the remainder of the sessions, the twelve students met once per week for two hours on Wednesday mornings in a mathematics tutorial classroom. The first session consisted of an orientation session. Students were handed out a pre-intervention questionnaire, to assist the researcher to gain insights in their current beliefs about mathematics and mathematics education. The pre-intervention questionnaire also indicated that the students have had no prior exposure to mathematical modelling. Also, the pilot study indicated that students needed more information on the modelling process as well as on modelling competencies. Whole class discussions prevailed while the entire modelling process was discussed. Due to the students' inexperience with mathematical modelling, they were very curious about how to find mathematics in real-world situations. The students also favoured the idea that they could work in groups and use the ideas of their fellow students to gain a deeper understanding of the problem posed. The difference between traditional word-problems and modelling was explained extensively. The importance of being an effective and productive engineering technician was discussed, accompanied with the requested engineering competencies of international engineering bodies. The researcher explained to the class how mathematical modelling competencies compliment engineering competencies and why it is important for engineering students to develop the ten modelling competencies as proposed in Chapter 3. The students then studied the information as well as the consent forms. The content on the forms was discussed and the researcher ensured that all students understood their responsibilities and rights regarding their participation in the study.

The second session commenced with the students dividing themselves in three groups of four each. All the tasks that the students had to work on adhered to the design principles of MEAs, and they had to develop a product or create a tool for a specific client. After completing the first task, the students had to present their solution methods. Throughout the activity the researcher's role as facilitator and guide was carefully controlled by the researcher, as interventions had to be limited not to allow direct instruction, but rather to create opportunities for student reflection. The researcher continuously probed the students to explain their work to assist them to develop a sense of direction and to self-assess their progress. Reflection and assessment instruments were also handed out to the students for monitoring competence development.

During the teaching experiment, a total of six modelling tasks were completed. Group discussions were either audio or video recorded and all written documentation was preserved for future use. After each session, the researcher's notes, the audio and video transcripts, as well as all the various research instruments were analysed and unified to ascertain an index for each of the ten competencies. The development of these competencies were than graphed for representational purposes (Chapter 5).

4.3.2.1 Progress of the design experiment:

The reflection instruments were used throughout the experiments, but not all instruments were used with each activity. The use of the reflection instruments allowed for adequate data generation for exploring modelling competency development. Table 4.17 (below) indicates how the study progressed, as well as how and when the various instruments were used to allow for duplication of this research study. Motivation for the use of particular instruments during the various activities are described in Chapter 5.

Table 4.1 – Progress plan and accompanied assessment instruments of the study

The course of the design experiment:		
Sessions	**Activities**	**Instruments Used**
WEEK 1 Session 1&2: Morning and afternoon sessions of 2 hours each	**Orientation Session** Students had one-on-one interviews with the researcher, answering the questions of the Pre-Intervention Interview (Appendix J), whereafter the researcher/facilitator explained the modelling process and modelling competencies to them. Students divided themselves in three groups of four each. Time was spent to get to know fellow group members. Students studied and completed the ascent and consent forms and the researcher answered their questions related to the purpose of this study.	• Pre Intervention Questionnaire (Appendix J)
WEEK 1 Session 3&4: Morning session of 2 hours, afternoon session of 1 hour	Activity 1 – Lawnmowing Problem	• Status Update Report (Appendix A) • Quality Assurance Guide (Appendix C) • Group Modelling Competency Sheet (Appendix D) • Group Reporting Sheet (Appendix F) • Poster Presentation (Appendix H)
WEEK 1 Session 5&6: Morning and afternoon sessions of 2 hours each	Activity 2 – Paper Airplanes	• Status Update (Appendix A) • Researcher Observation Guide (Appendix B) • Quality Assurance Guide (Appendix C) • Group Modelling Competency Sheet (Appendix D) • Student Reflection Guide (Appendix E) • Student Reports

The course of the design experiment:		
Sessions	**Activities**	**Instruments Used**
WEEK 2-3 Session 7&8: Weekly sessions of 2 hours each	Activity 3 – Tidal Power Plants	• Status Update (Appendix A) • Researcher Observation Guide (Appendix B) • Quality Assurance Guide (Appendix C) • Group Modelling Competency Sheet (Appendix D) • Group Functioning Sheet (Appendix G) • Poster Presentation (Appendix H)
WEEK 4-5 Session 9&10: Weekly sessions of 2 hours each	Activity 4 – Product Coding	• Status Update (Appendix A) • Group Modelling Competency Sheet (Appendix D) • Researcher Observation Guide (Appendix B) • Quality Assurance Guide (Appendix C) • Student Reflection Guide (Appendix E) • Group Reporting Sheet (Appendix F)
WEEK 6-8 Session 11,12&13: Weekly sessions of 2 hours each	Activity 5 – Turning Tyres	• Status Update (Appendix A) • Researcher Observation Guide (Appendix B) • Group Modelling Competency Sheet (Appendix D) • Quality Assurance Guide (Appendix C) • Group Functioning Sheet (Appendix G)
WEEK 9-11 Session 14,15&16: Weekly sessions of 2 hours each	Activity 6 – Finding the Cell Phone	• Group Modelling Competency Sheet (Appendix D) • Researcher Observation Guide (Appendix B) • Status Update (Appendix A) • Student Reflection Guide (Appendix E) • Group Functioning Sheet (Appendix G) • Poster Presentation (Appendix H)

Status Update Reports were handed out to students while they were participating in the activities and they had to complete the report during the horizontal/vertical bridging phases of their modelling process. These reflection tools allowed the students to reflect upon their situational model and to ensure that they develop a sense of direction by reflecting upon the initial phases and deciding upon ‘next steps’. The Competency Rubric and the Researcher Observation Guide were used by the researcher/instructor and the presence or absence of meta-cognitive development was noted on the Researcher Observation Guide in more detail. The Competency Rubric also assisted the researcher as to the extent of competence development that took place within each group activity. The Quality Asssurance Guide was also used throughout the activities, as it also allowed the groups to reflect on their progress and to discuss their solution methods. It also assisted the researcher in ensuring that all MEAs adhere to the set design principles (Chapter 3). Group Reporting Sheets also aided the students in developing a sense of direction and learn to reflect on their work. During the pilot study students preferred to use the Group Reporting Sheet rather than the Quality Assurance Guide, as it was easier for them to document their progress and as such more data was revealed to the researcher. Student Reflection Guides and Presentation Forms were completed at the end of the activities.

4.3.2.2 Data collection procedures

This section will address the second part of Aim 9 of the research question, namely to identify data collection methods that will assist in obtaining unbiased and reliable results (Section 1.8.3). DBR continually tests and revises speculations about potential learning processes and the means that support such learning processes. Data collection procedures thus need to be planned carefully to produce data that can answer to such speculations and can undergo rigorous analysis during the retrospective phases (Reimann, 2011:41). Such data should not only include information about student learning and classroom practices, but the learning process of the researcher should also be an integral part of the data.

Information was collected through multiple methods. Reflection instruments were handed out to the students and all sessions were observed using observation guides, field notes, video and/or audio recordings. All such reflection tools were clearly defined regarding their purpose. These tools assisted the students as well as the researcher in gaining a better understanding about their ways of thinking and acting about mathematical problems. The questionnaires related to either mathematical questions that needed to be answered, or it addressed issues regarding students' beliefs about the mathematics and how they experienced it.

Informal discussions took place during and on completion of the activities, which mainly revolved around the participants explaining their methods applied and strategies used during the activities, as well as their experiences during the problem-solving activity. These discussions took place in the classroom where participants were free to share their experiences and beliefs about mathematics education and mathematical modelling, what they envisage about the content and context of the course, and what they have learned/not learned. During these informal discussions, the participants used the opportunity to share, compare, explain and justify their mathematical modelling activities' outcomes with the rest of the class and with the researcher, who also acted as the facilitator.

Due to practical problems related to memory capacity and battery life as well as the quality of the video/audio equipment, it was not possible to record entire lessons. However, various parts of each activity were recorded, while field notes and researcher observation sheets further assisted the researcher in documenting her observations. Details about each activity's data collection processes will be discussed in detail in Section 5.2.

4.3.3 Phase 3 of DBR – Retrospective analysis

The goal of this phase is to find ways to understand the co-development of engineering technician and mathematical modelling competencies through mathematical modelling to promote mathematical reasoning and understanding, based on observations and inferences made during the experiment. Data that are used include video/audio tapes, observations, reflection tools of students and of the researcher, as well as field notes. To ascertain credibility, the entire data set must be scrutinised and analysed and all phases of the analysis process are documented. Possible changes to the conjectured local instructional theory must also be documented and implemented in the follow-up preliminary phase.

In adhering to specific processes and procedures, the results of this study can be justified as empirically grounded, while simultanously addressing the outputs of DBR in terms of knowledge as well as products. The scientific outputs in the form of design principles, contain substantive and procedural knowledge that provides an accurate portrayal of the procedures, results, as well as the context. Such descriptions allow others to establish which insights might be relevant to their own specific situations. Each intervention will be carefully described as well as analysed to provide such information (Chapter 5). Practical outputs (designed artefacts) in this study will relate to the materials to be used in the classroom, as discussed in Sections 4.3.1.5 and 4.3.1.6. Instructional activities and measuring instruments will be continually adapted and refined where and when necessary, and these adjustments as well as the results relating to professional development will also be exposed in Chapter 5.

A thorough *analysis of data* is crucial in design-based research studies where the data need to be cross-examined repeatedly to allow for the theory to relate closely to the data collection and analysis processes, or, as Charmaz (2008:160) noted, "the method does not stand outside the research process; it resides within it". By iteratively moving forwards and backwards between collecting and analysing the data against theory, by constantly comparing data, checking for ideas, refining emerging ideas and constructing abstract categories from data analysis, the emergent levels of analyses are raised while new ideas, questions and deeper refinements of earlier concepts can also emerge.

Emergent logic typically follows a process of systematical data collection and analysis of the data which *occurs immediately after each data collection process* and not after all the data have been collected. The decision on what data to collect and on how to collect the data next is based on this analysis (Bruce, 2007:57). Emergence assumes epistemological understandings and a theory of time. It deals with movement, process and change: moving chronologically from a past to the immediacy of the present and implies a future cannot necessarily be anticipated (Charmaz, 2008:157). Emergence thus allows for the possibility of the researcher to gain new insights which must be strengthened by careful documenting the researcher's observations, and such insights will also be subjected to testings for substantiation, as well as for negative cases (Harry, Sturges, & Klingner, 2005:4). This study will aim to continually search for theoretical understanding between the data collected and the emergent interpretation of the data.

Charmaz (2008:155) mentions the following important principles that need to be adhered to during this phase:

- Researchers need to restrain their own preconceived ideas about the research problem and the data, and allow the participant's voice to be heard.
- Data collection and analysis should be used simultaneously to inform each other.

- The researcher has an obligation toward sensitivity regarding possible explanations and understanding of the data. Strauss and Corbin (1990:87) suggest that sensitivity can be accomplished through activities such as questioning, the flip-flop technique, and far-out comparisons to stimulate reflective behaviour about the data collected.

The incorporation of RME theory throughout the experiment, guides the design and it also offers a framework to interpret the students' learning processes (Gravemeijer & Cobb, 2006:23). The interactive character of a method that continually emerges from the researcher's questions, choices and strategies, and in turn progressively shapes and grows the researcher's perspectives, denotes an emergent structure of enquiry, which is well in line with DBR principles. Within the constructivist framework, DBR is concerned with a thorough analysis of theory, as well as a critical analyses of the researcher's own epistemological ideas, research principles and practices. Procedures used to interpret and organise data, typically consist of coding, theoretical saturation, memo writing, and diagramming (Strauss & Corbin, 1990:12).

4.3.3.1 Coding

DBR methods require simultaneous data collection and data analyses through processes of comparing and analysing, which can be accomplished by means of coding. By continually moving forwards and backwards among the data, the researcher codes events and actions in the data and compare them with one another to create conceptual categories. Two types of comparisons pertain to this study: constant comparisons of incident to incident, and theoretical comparisons that compare categories to similar or different concepts (Strauss & Corbin, 1990:94). Theoretical comparisons occur by means of flip-flop techniques as well as systematic comparisons. The former focuses on examining opposites or extremes to reveal significant properties. Flip-flop techniques assist the researcher in gaining a better understanding and they elicit more questions to be asked to the participants, which again leads to further sampling along conceptual lines during data collection. By employing systematic comparisons, an incident in the data gets compared to literature or to a previous experience, and it stimulates researcher sensitivity towards properties and dimensions in the data that was not noticed previously (Strauss & Corbin, 1990:96). This open coding process therefore allows the researcher to gain greater insight in the data, and thereby understanding the meaning of the data collected and it also serves to direct the researcher to consider various alternatives (Charmaz, 2008:164).

During the coding activity, the researcher starts to interpret and abstract the meaning of the data. Focused codes assist in determining the best interpretations of the empirical phenomenon (Charmaz, 2008:164). More levels of coding can occur as the researcher continues to compare and integrate the various code clusters in relation to one another. Further comparisons between the categories assist in investigating interrelationships among them. This action can lead to progressive refinements of the categories, which can ultimately lead to theory development (Harry et al., 2005:5).

4.3.3.2 Theoretical saturation

The ultimate measure to determine whether or not to conclude the data gathering process is theoretical saturation (Strauss & Corbin, 1990:158). Strauss and Corbin (1990:212) warned about the importance of theoretical saturation to allow the theory to develop evenly, concise and with precision. Theoretical saturation occurs when a) continued data collection does not reveal any new or useful data regarding a category, b) the categories are well developed in terms of properties and dimensions, and c) the relationship between the categories are well established and validated. The careful selection and use of multiple sources of data collection strategies (triangulation) further assist in theoretical saturation of data (Harry et al., 2005:4). Saturation of data does not necessarily mean that no new data is likely to emmerse, it merely indicates the point in research where further data collection would not be productive, or, as Strauss and Corbin (1990:136) remarked, “the new that is uncovered does not add that much more to the explanation at this time”.

4.3.3.3 Memo writing

Glaser and Holton (2004:18) express the goal of memoing to be the development of ideas on categories as descriptions are raised to conceptual understandings. Through writing memos once the data is collected, the researcher ‘submerges’ in the data and gets the opportunity to interrogate the data rather than just summarise it. The researcher’s thinking is extended and new ideas can emerge as discoveries unfold. Memos thus allow for the opportunity to explore certain concepts or ideas in more detail. Memos record the progress, thoughts, feelings and directions of the research and researcher. This recording process slows the researcher’s pace and force reflection on the reasoning of how categories integrate and fit into the bigger picture. When using sporadically, the final product can lack density and integration (Strauss & Corbin, 1990:218). Apart from the checklists as described in Section 4.3.1 during the first phase of the design cycle, the researcher created a further checklist (Figure 4.5 below) based on the suggestions of Van den Akker et al. (2006:5), to ensure that the principles and characteristics of DBR as explained in this chapter, are adhered to.

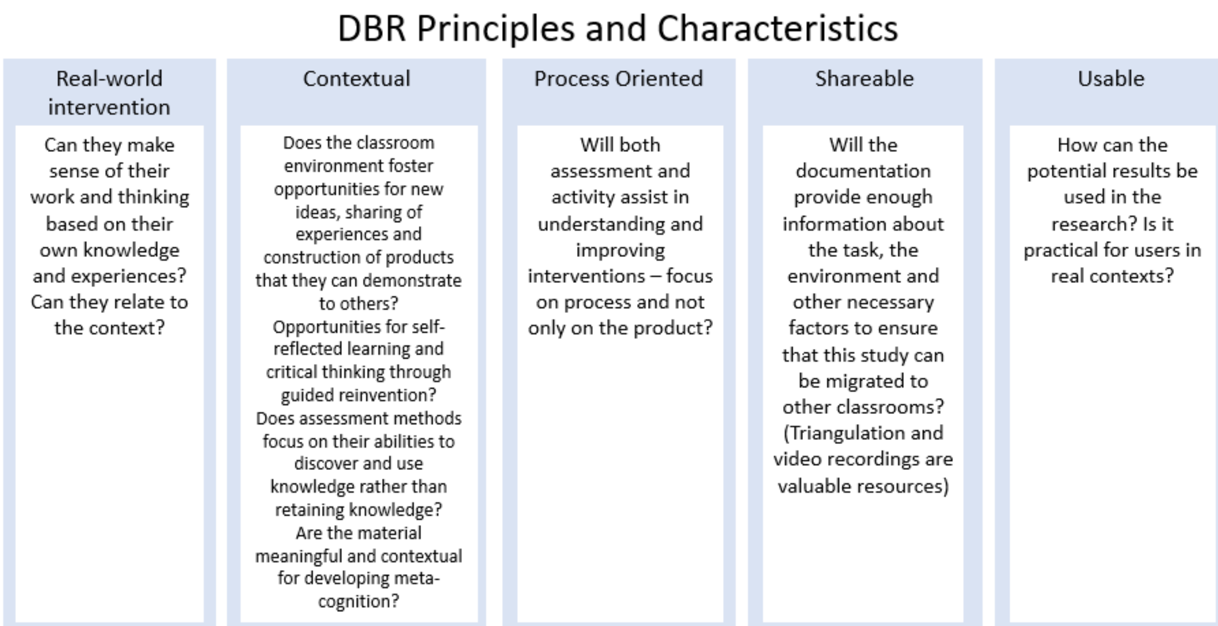


Figure 4. 5 - Principles and characteristics of DBR (Adapted from Van den Akker et al. (2006:5))

This section stipulated how DBR methodology can be used as a vehicle to investigate the co-development of mathematical modelling and engineering technician competencies for the fostering of reasoning and understanding of mathematics, and thus answered to Aim 7 of the research question (Section 1.8.3). The following diagram (Figure 4.6) is a graphical representation summarising the DBR process as explained (researcher's own construction), to further guide the researcher in considering all necessary aspects of design, enactment and analysis.

Design-based research in educational research

It is an inter-disciplinary mixed-method research approach where not only students' experience changes concerning their understandings, but researchers' conceptions regarding teaching and learning also evolve.

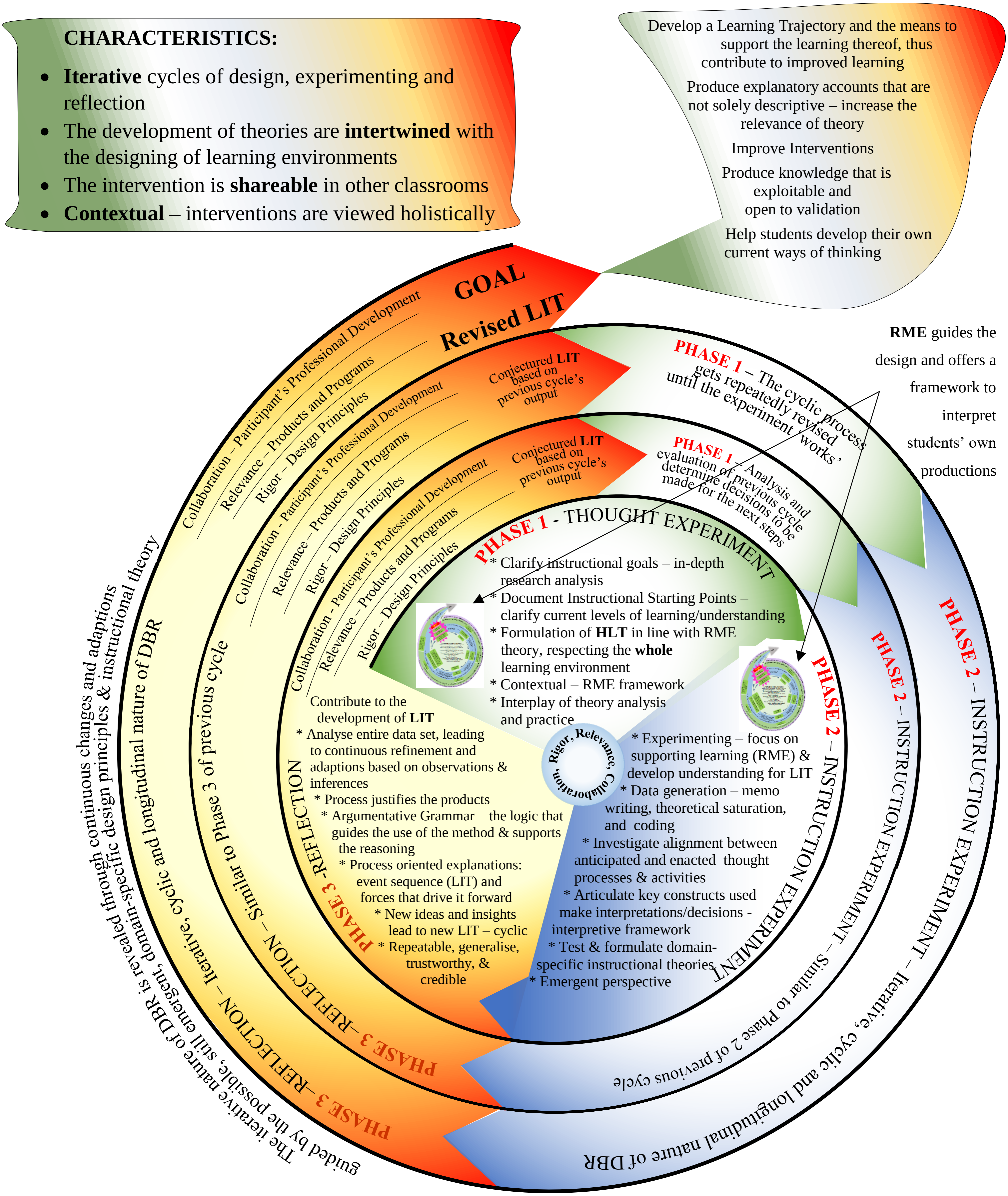


Figure 4. 6 - DBR process to serve as final checklist for planning, enacting and analysing the design experiment

4.4 CASE STUDY RESEARCH AS A COMPLEMENTARY EXTENTION OF DBR

While DBR allows for the examination of small groups of four students each, a further case study will be conducted on the 'weakest' and 'strongest' case in as much depth as is feasible, to allow for an improved theory. Case study research therefore complements DBR and the researcher will attempt to provide explanations on variations of competence development among individual students as well. Among the variety of definitions of case study research, the researcher uses the definition as proposed by VanWynsberghe and Khan (2007:80):

> Case study is a transparadigmatic and transdisciplinary heuristic that involves the careful delineation of the phenomena for which evidence is being collected (event, concept, program, process, etc).

Van Wynsberghe and Khan (2007) emphasise that case study cannot be defined as a method, a research design, or a methodology. They argue that data are not prescriptively collected in case studies, that there does not exist clear guidance on the research process in terms of collecting, analysing and interpreting the data, and that case study does not provide a theory of how research should proceed, what methods should be used and how the data are mapped onto the resulting theory. To further support their definition, they suggested seven features in a prototypical case study (VanWynsberghe & Khan, 2007:83-84):

- Small N

The case study focuses in-depth on a specific unit of analysis. The extension of DBR to case study research will focus in-depth on the difference in competence development between the strongest and weakest case that resulted from the design-based research experiment.

- Contextual detail

The study needs to be stipulated within a particular context, to provide the reader with the possibility to be immersed in the situation, by paying careful attention to detailed and contextualised analysis of all relevant activities.

- Natural settings

Similar to DBR, case study research implies little control over variables, as the experiment unfolds in natural complex settings.

- Boundedness

Explicit attention to the place and time of the study allows for boundaries that enable the researchers to develop focused hypotheses by demarcating what is studied, and what falls outside the case. This study clearly stipulated such boundaries, and the variation of competence development between the best and weakest case that resulted from this DBR study, will be investigated.

- Working hypotheses and lessons learned

Hopefully new lessons would be learned regarding the differences in competence development as new information becomes available during further data collection and analysis in the case study.

- Multiple data sources

Post-questionnaires (Appendix I), observations and interviews will allow for the development of converging lines of inquiry, which again answers to the triangulation dilemma (Section 4.5.2).

- Extendibility

Hopefully this case study will provide opportunities to further enrich the researcher's understanding about the variance in competence development between the strongest and weakest case.

Yin (1981:59) categorised case study research's distinguished characteristic as an attempt to examine "a contemporary phenomenon in its real-life context, especially when the boundaries between phenomenon and context are not clearly evident". This design experiment occurred in messy, meaningful, real-life situations, where numerous factors influenced other depending variables of interest. These factors included teachers, students, classroom ethos, contextual activities, and reflection tools, as they were all inputs of the working whole (Reeves, 2006). Robust explanations relating to these variables will be provided during the construction and refinement of the HLT (Section 5.2).

As already indicated, the cases to be studied will be limited to the two students that resulted as the best and worst cases concerning competence development during the DBR experiment. Evidence will be collected relating to the ten mathematical modelling competencies as investigated in Chapter 3, and these competencies will thus represent the unit of analysis (the phenomenon for which evidence will be collected).

Rule and John (2015:2) emphasise the importance of a dialogic engagement between theory and case study, as theory can be generated from practice and vice versa, as the practitioner can contribute to theory building from the perspective of practice. Their dialogical model implies an interactive dialogue between theory and practice throughout the research process. Throughout the

DBR experiment, continual engagement between theory and practice allowed the researcher to come to insightful understandings of how competence development with the aim to develop mathematical reasoning and understanding, can be achieved. During the subsequent case study, continual interplay between theory and practice will occur to develop a deeper understanding of how and why students' competence development vary.

4.5 METHODOLOGICAL CONCERNS

While designing and implementing the research study, the researcher needs to be continually engaged in critical questions such as:

- How did I find out what I found out?
- Do I assert findings on the basis of values and hopes?
- What are my sources of knowledge?
- How do I deal with my own biases?
- How do I try to acknowledge bias?
- How do I control bias?
- Do I apply enough methods not only to get answers to the questions asked?
- How can I obtain evidence of what is going on inside students' heads?
- When observing, does it only include the students' work, or do I see anything else - their feelings?

Critical thinking about all possible aspects relevant to this design research study elicited certain methodological concerns. Those of specific relevance are discussed and explained in this section.

4.5.1 Reliability and validity in qualitative research studies

Joppe in Golafshani (2003:589-599) defines reliability and validity as follows:

> The extent to which results are consistent over time and an accurate representation of the total population under study is referred to as **reliability** [highlighting - LdV] and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable.
>
> **Validity** [highlighting - LdV] determines whether the research truly measures that which it was intended to measure or how truthful the research results are.

Relating to quantitative research, the results thus need to be replicable to be reliable and the measurement tools should measure what they are designed for to measure.

However, qualitative research produces findings that arrived from messy, real-world situations, with the aim to enhance insight and understanding of the problem to use in other similar situations (Golafshani, 2003:600). Qualitative researchers are also involved in real-world situations that are subject to change, and thus need to truthfully record the events prior and after the changes occurred. The importance of meticulous recording of events signifies a shift towards focusing on the credibility of the results. Research validity and reliability are thus intertwined with credibility of the research, which depends on the ability and effort of the researcher. Eisner (2017:58) characterises a good qualitative study as one "that help us understand a situation that would otherwise be enigmatic or confusing". This characteristic accentuates the main difference in reliability between quantitative and qualitative research. In quantitative research, reliability is concerned with the "purpose of explaining", while reliability focuses on the "purpose of understanding" in qualitative research (Golafshani, 2003).

Relating to the generalisation of results, one should also distinguish between quantitative and qualitative research approaches. The ability to generalise results is a common test of validity in quantitative studies, but Paton in Golofshami (2003:603) noted that, in qualitative studies, generalisability depends on the case selected and studied. This aspect of dissemination will be discussed further in Section 4.5.4. Various methods are employed in this study to ascertain trustworthy, reliable and valid results, which will be explained in subsequent sections.

4.5.2 Triangulation

Denzin in Jick (1979:602) broadly defined triangulation as "the combination of methodologies in the study of the same phenomenon", originating from the belief that, when two or more methods produce complementary findings, the validity of such findings is enhanced. Relating to this study, the development of engineering technician and mathematical modelling competencies are studied with the help of several instruments, such as informal discussions with the students, observing their behaviours and actions while grappling with problem-solving activities, analysing reflection questionnaires, evaluating their final products, memo writing, and so forth. All these methods are employed to examine the same dimension – engineering technician and mathematical modelling competence – of the research problem. Of primary importance are the questions that are asked in the reflection tools and questionnaires, as the researcher has to continually inspect the questions to ascertain the quality and relevance of the students' answers to allow for a wide variety and adequate quality of relevant data to be collected.

Apart from enhancing reliability and validity issues, triangulation also assists towards obtaining a more holistic and contextual portrayal of the phenomenon under investigation, as variances can be uncovered which otherwise would not have surfaced (Jick, 1979:603). Thus triangulation does not only examine one phenomenon from a variety of perspectives, but it also enhances understanding of the emergence of new dimensions. Analyses of the multiple methods of data collection allow compensation for the weaknesses of a single method by the counter-balancing strength of another (Jick, 1979:604).

The in-depth literature study as presented in Chapters 2 and 3, indicated that the use of MEAs can be a productive tool to investigate students' engineering technician and mathematical modelling competence. One focus of the research is to document and examine the symptoms of mathematical competence. How would the researcher know whether and to what extent the students' competencies have developed? Again, falling back on past and present literature, several techniques apply:

- Pre-intervention questionnaires allow the researcher to gain an insight in their beliefs about mathematics;
- Observe the students' competence development while working on problem-solving activities to identify whether they use such competencies and to what extend (observations by the researcher aided by audio and video recordings);
- Complete the various reflection tools to elicit their understanding and development;
- Student worksheets allow the researcher to clearly recognise the presence or absence of specific competencies;
- Presentations and reports also elicit both cognitive as well as meta-cognitive understanding; and
- Informal discussions and questions are continually employed to answer to queries that can arise while data collection takes place.

All these data collection instruments are analysed and coded immediately after the data collection process, and the extent to which competencies surface in each instrument is rated on a scale of 0-

3. These various techniques and instruments generate a comprehensive picture of competence development during the course of the study. The analysis of the above methods and an explanation on variances in the findings will be discussed in Chapter 5.

4.5.3 Objectivity and sensitivity

The method implied in this study comprises of continual data analysis from the very first interview and observation, since analysis drives the data collection. This interplay between analysis, data and theory, requires the researcher to immerse in the data which results in the researcher being moulded by the data and vice versa. This mutual moulding process asks for great care to always maintain a balance between objectivity and sensitivity (Strauss & Corbin, 1990:42). Objectivity allows for the generation of unbiased and accurate interpretations of events, while sensitivity enables the researcher to acknowledge delicate tones and connotations in the data that can ultimately lead to making connections between concepts.

Being the designer and evaluater of the program, can cause conflicting roles of advocate and critic. Even though a state of complete objectivity is impossible, appropriate measures can be taken to minimise subjective analysis. To address such concerns, measures can be taken such as listening to the participants' voices, cautiously observing what they do, and documenting it as truthful as possible. Methodologically, by applying the method of constant comparisons, and by referring to literature and past experiences about similar phenomena to stimulate thinking about properties and dimensions to be used, are two ways to assist the researcher in gaining some perspective while examining aspects of data (Strauss & Corbin, 1990:44). Strauss and Corbin suggested additional techniques to assist in minimizing these risks: a) applying a wider range of data-gathering techniques and approaches, referred to as triangulation (Section 4.5.2), or b) to obtain multiple opinions of an event, or c) to occasionally revisit your assumptions about the participants against incoming data, or d) to step back and reflect about what is happening, and whether your ideas about the data fit in with the reality of the data.

Furthermore, the relationship between the researcher and the participants can also affect the quality of the outcome. Linder (2011:60) recommends that the following factors need to be considered to guide the researcher towards becoming an influential facilitator:

- Credibility – the facilitator must display content and pedagogical knowledge, effectiveness and professionalism;

- Assistance – facilitate and guide the participants when necessary;
- Motivation – knowledge of content, pedagogy and the facilitator's own experiences;
- Management – be responsible for meaningful context, content and materials; and
- Personality – positive presentation of the facilitator.

During the entire experiment, the researcher will take all necessary precautions to ensure that she approaches the study in a state of neutrality, that the participants' voices are respected above that of her own. In design research, the researcher's rigorous analysis of a specific problem can lead to decisions for further interventions. Field notes including the dates and times of activities, field journals that keep track of decisions made, written accounts of ethical considerations, coding and categorising for data collection, and analysis and interviews can all assist towards substantial findings. As the data are gathered from a number of different sources, problems and interventions can be identified to assist in the possible development of students' engineering technician and mathematical modelling competencies, and thereby further support them towards increased mathematical reasoning and understanding.

4.5.4 Argumentative grammar

Argumentative grammar guides the researcher to ensure that the learning process justifies the products of the research project, or rather, as Kelly (2004:118) noted, that it "is the logic that guides the use of a method and that supports reasoning about its data. It supplies the logos in the methodology and is the basis for the warrant for the claims that arise". Engeström (2011:607) regards argumentative grammar as the golden thread that connects the theory, methods and empirical research in a research approach.

One of DBR's outputs is to establish a learning trajectory that is framed within a specific interpretive framework (Chapter 2) and consists of learning activities and shifts in students' reasoning (Reimann, 2011:44). The researcher needs to determine that such shifts in reasoning and the accompanied development of a specific competence resulted from *the support provided by the instructional design*, and that it did not occur due to a mere sequence of events. The motivation of this study came from the researcher's past experiences with further confirmation through past and current literature, that the typical mathematics instruction did not adequately develop the mathematical reasoning and understanding as required from engineering technician students

(Chapter 3). An interpretive framework derived from current literature allows the description and explanation of changes in students' reasoning and competencies, in terms of abstract conceptions of learning. The framework not only addresses issues of learning, but also classroom norms and discourse. As the experiment takes place over of a period of time, this longitudinal study is covered in copious descriptions of how their reasoning and competence development evolve as a reorganisation of prior forms of reasoning (Reimann, 2011:44).

To come back to generalisation of results as mentioned in Section 4.5.1, the above explanation paves the way to allow for *replicability*, or rather *dissemination*, which is one of the goals of DBR. It will undeniable be a meaningless task to investigate students' competency development without being able to replicate the study to a certain extent. However, *the intent of the experiment is not one of exact duplication, but rather to inform other practitioners or researchers to differentiate between the necessary and contingent aspects of the design, while they customise the design in their own settings* (Cobb, Jackson, & Dunlap, 2016:22). As design research is concerned with explanations of the processes and mechanisms that caused the changes in learning (and not comparing generalised results to quantitative studies emerged in very large sample sizes), the researcher is interested in the "mechanisms through which and the conditions under which the causal relationship holds", to explain the developmental process of a single case (Cobb et al., 2016:4). The retrospective phases of this study is primarily concerned with the analysis and documenting to explain – based on the data and on theory – how successive forms of reasoning emerge as a restructuring of prior forms of cognition, and to identify the critical and necessary aspects of the entire learning environment that can support students' development of engineering technician and mathematical modelling competencies to enhance their mathematical understanding and reasoning. This emerged domain-specific local instructional theory not only explains the possible development of such competencies, but the relevant aspects of the classroom learning environment and other supports needed for learning are also addressed.

Credibility, trustworthiness and validity of the findings are illustrated by the rigorous data collection and analysis procedures which are thoroughly documented. The analysis of this longitudinal data set is also both systematic and thorough, and thereby it further enhances the credibility of the findings of the experiment.

4.5.4.1 Bartlett Effect

Brown (1992:162) warned against the risk of being accused of the *Bartlett Effect*. This methodological issue relates to running the risk of data being misrepresented, especially when conducting studies using small samples. She suggested that meticulous observation and recording of events as they unfold can enable the researcher to access and analyse the events of interest. Schoenfeld in Brown (1992) suggested his solution to this problem in keeping all data records and scoring materials transparent and available to the field. This research study will aim to carefully observe all relevant data, instruments used, students work, and researcher coding and analyses notes, as far as possible.

4.5.4.2 Hawthorne Effect

A further concern relating to interventionist design studies is the *Hawthorne Effect* (Brown, 1992:163). The Hawthorne Effect is concerned with the fact that the intervention is likely to have a positive effect, simply because of the consideration of the researchers to the subject's welfare. An in-depth study of past and present literature (Chapter 2 and 3) indicates that the intervention of model-eliciting activities has the possibility of improving students' mathematical modelling competencies. A close connection thus exists between the cognitive activities practiced and the type of improvements anticipated. The researcher particularly makes use of activities that comply with the design heuristics as defined in the literature, while instilling a classroom culture conducive for cooperative learning with an ethos where individual responsibility and group collaboration are the norm. This complex social environment makes it impossible to isolate one component. Like Aristotle who said, "the whole really is more than the sum of its parts", Brown (1992:166) also emphasises the interdependance of the learning effects as outcomes of the cognitive interventions. To conclude, the researcher of this study shares the sentiments of Brown (1992:167) that the consequences of the Hawthorne Effect, named improved mathematical competencies of students, are exactly what the researcher is hoping for.

4.5.5 Strategies to enhance rigor

To attain reliability and validity of qualitative research, Morse, Barrett, Mayan, Olson, and Spiers (2002:17) proposed verification strategies to be built in the research process for ensuring rigor (also one of the outputs of DBR). These verification strategies assist the researcher to remain focused on the direction of the analysis, as well as the development of the study. The iterative processes offered by DBR, require the researcher to adhere to ongoing analysis and reflection in the formulation of conjectures and questions. Responsiveness, sensitivity, creativity, insight and willingness to abandon poorly supported ideas and preconceptions, are all crucial to the attainment of reliability and validity. Complementing verification strategies that apply to this study, include:

4.5.5.1 Methodological coherence

Methodological coherence ensures congruence between the research question and the componenets of the methods (Morse et al., 2002:18). The intertwinement between the research method and question is achievable through the process of argumentative grammar as explained in Section 4.5.4. Possible adjustments to the activities, reflection and assessment instruments need to be refined continually by adjusting the scaffolding and modifying methods. The design of the data collection and assessment instruments took place during the first phase of DBR (Section 4.3.2), and possible adjustments to these instruments are explained in Section 5.2 where the activities play out.

4.5.5.2 Appropriate sampling

Choosing the participants who best represent the research topic is vital. As this study is concerned with the concurrent development of mathematical modelling and engineering technician competencies for enhancing mathematical reasoning and understanding, 12 first-year engineering technician students volunteered to partake in this study over one semester. This process took place during the first phase of DBR, and is explained in Section 4.3.1.5.

4.5.5.3 Concurrent collection and analysis of data

By collecting and analysing the data concurrently, a mutual interaction between what is known and what one needs to know, can be established. During the second phase of DBR, data collection occur while the students are engaged in the activities (Section 4.3.2), and further data collection and analyses follow immediately after the students completed the activities during the reflexion phase of DBR (Section 4.3.3). The data are continually compared with the theoretical understandings about competence development and mathematical modelling as stipulated in Chapters 2 and 3.

4.5.5.4 Theoretical Thinking

Again, throughout all the phases of DBR, continuous interplay between data and theory is required to allow the researcher to gain an understanding about the real teaching and learning problem within the socio-constructivist classroom environment (Sections 4.3.1, 4.3.2 and 4.3.3).

4.5.5.5 Theory Development

The development of new theories is one of the outputs of DBR, and the construction of a LIT provide information about possible ways to enhance the students' competencies for improved mathematical understanding and reasoning (Section 5.4).

Together, all these strategies as required by DBR contribute to reliability and validity, and thus ultimately enhance the rigor of this qualitative inquiry. By adhering to rigorous standards in qualitative research, trustworthiness can be obtained. Complementing these rigorous strategies, further strategies and accompanied criteria that apply to this study for establishing trustworthiness are explained in the following section.

4.5.6 Trustworthiness

Krefting (1991) proposed strategies and accompanied criteria for the establishing of trustworthiness. The following table is adapted from Krefting (1991:217), indicating how the particular criteria will be met in this study:

Table 4.2 - Summary of strategies with which to establish trustworthiness (adapted from Krefting (1991:217))

Strategy	**Criteria**
Credibility	Prolonged and varied field experience The duration of this study is one semester, and DBR requires that the study is covered in copious descriptions of how the students' reasoning and competence development evolve (or not) as a reorganisation of prior forms of reasoning. The extended time period also allow increased rapport between student and researcher, and participants may expose valuable information that would not have been exposed during the beginning of the project Reflexivity (field journal) A qualitative approach to research asks the researcher to be reflexive as the researcher becomes a participant and cannot separate herself from the study The role of the researcher is thus crucial, and is explained in detail in section 2.5.4. Krefting (1991:218) noted that using a field journal can assist the researcher to reflect on her thoughts, feelings, ideas, and hypotheses generated while in contact with the participants, as reflecting on field journals can elicit biases and preconceved assumptions. Triangulation See Section 4.5.2.

Strategy	Criteria
	Member checking and peer examination As the researcher will be the sole designer and evaluator of the program, can cause conflicting roles of advocate and critic. To overcome these hurdles, the following apply (Section 6.4): • The researcher had personal awareness. • The researcher facilitated the participants and allowed them to voice their own opinions and ideas. • The researcher's focus remained to collect a 'true version' of the phenomena. • The researcher attempted at all stages to deliver work of superior standard and to present an accurate report of her findings. Interview technique May in Krefting (1991:220) noted that credibility can be enhanced by reframing questions, and repetition or expantions of questions. The vast range of reflection and assessment instruments as discussed in Sections 4.3.1.5 and 4.3.1.6 were designed to be used throughout all the activities in this study. Structural coherence Ensure that all inconsistancies between the data and the interpretation of the data are explained (Section 5.2).
Transferability	This study involves only 12 first-year engineering technician students, signifying that the results cannot necessarily be generalised over a wide spectrum. As explained in Section 4.5.4, design research is concerned with explanations of the processes and mechanisms that caused the changes in learning, to explain the developmental process of a single case (Cobb et al., 2016:4). The focus is thus not on the comparisons of generalised results to quantitative studies that emerged in large sample sizes or to test hypotheses, but to develop a qualitative and quantitative profile that characterises the design in practice. DBR's consistent and logical methodology is used to link theoretical research and educational practice. The design (Chapter 4), the interventions (Section 5.2), as well as its effect on learning in a specific context (Sections 5.2 and 5.3), provide explanations and principles of innovative learning to be localised in new settings (Section 4.2.2).
Dependability	Dependability audit An auditable description of how the activities play out is required to enable the researcher to follow the decision trails used when reflecting and refining decisions based on the data. Dense description of research methods The research method needs to be explained in detail, as understanding of every aspect of the methodology is vital for allowing informed opinions (Section 4.3). Triangulation See Section 4.5.2. Peer examination Again, the researcher is the designer and evaluater of the program. Refer to "Member checking and peer examination" earlier in this table. Code-recode procedure Repeat coding after a few days to confirm/disconfirm previous coding results.
Confirmability	Confirmability audit Again, the researcher is the designer and evaluater of the program. Refer to "Member checking and peer examination" earlier in this table. Triangulation See section 4.5.2 Reflexivity

The above table serves to provide a further scaffold for the researcher to adhere to the demands of trustworthiness.

4.6 CONCLUSION

This chapter aimed to provide an explanation of the research methodology employed in the research experiment and thereby addressed Aim 7 of the research question (Section 1.8.3). Design experiments were developed to carry out developmental research to test and refine educational designs based on the theoretical principles derived from literature. Case study research was combined with DBR (Section 4.4) to allow for an improved theory, and to attempt to provide explanations on variations of competence development among individual students as well. Throughout the study, an explicit interpretive framework guided the entire process.

The processes of planning for a conjectured learning trajectory, implementing it and reflecting upon it, were explained in Sections 4.3.2 and 4.3.3, and partially addressed Aim 10 as specified in Section 4.1. These processes were iteratively repeated to allow the researcher to build on the knowledge and insights gained from coding, analyses and scrutinising of the data, while comparing it to present and past literature. With each iteration, the learning trajectory was examined and refined to eventually allow for an improved learning instructional theory (LIT) to emerge from the data. This emerged theory of teaching and learning will also be tested against the outputs of DBR, named design principles, the societal contribution, and the participants' professional development (Section 5.2). The LIT comprises of learning goals, planned instructional activities, and an envisioned learning process.

Each MEA was designed according to the design principles as stipulated in Section 2.5.1 to allow for the development of mathematical modelling and engineering technician competencies. The activities were designed to create situations in which students were required to produce their own cognitive constructs in observable forms (Section 4.3.1.5). The task descriptor elicited the need to construct a mathematical model that did not reside in the problem statement. This discussion served to attend to Aim 8 as stipulated in the introduction paragraphs of this chapter.

The assessment instruments applicable to this study were designed to recognise and describe the nature of the students' constructs, as well as to offer guidelines to both facilitator/researcher and students, while comparing the usefulness of their (alternative) ways of thinking (Lesh & Clarke, 2000:127). This detailed discussion of the assessment instruments and data collection methods that apply to this study, also served to address Aim 9 of the research question (Section 1.8.3).

Even though this study anticipates development of students' mathematical modelling competencies, none of the competencies under investigation was defined in terms of what it would mean to possess such a competence to perfection. It is important not to view a LIT as the end-all to student learning challenges. Competence development will always remain a growing process, directing to the opinion that all students, regardless at what level they are, should always strive to continue to develop themselves further, as no final stage of expertise exists (Schorr & Lesh, 2003:145).

The enormous and overwhelming amount of data that have been collected by means of the methodology as described in this chapter, allowed for the unfolding of a comprehensive picture of students' competence development. Furthermore, the quality of the data should be of acceptable standard to answer to the research questions as discussed in Section 4.1, to contribute positively and meaningfully to mathematical education research. The following chapter describes the most important and significant results of the study.

CHAPTER 5

RETROSPECTIVE ANALYSIS AND RESULTS OF THE STUDY

Teaching is really about helping those I work with to become open-minded and inquisitive thinkers who are willing to – even hungry to – ask questions, gather evidence, and sort through ideas in a reasoned way ~ Alan H. Schoenfeld (2009:1)

5.1 OVERVIEW

Herrington et al. (2007:1), as well as McKenney and Reeves (2014:132) explain design-based research as a process that aims to generate new and usable knowledge (design principles) and to develop interventions in practice, while making use of existing knowledge in finding solutions to realistic classroom problems. Expressing such design principles requires rigorous and reflective inquiry to test and progressively refine innovative learning environments (Brown, 1992).

Not only will this chapter aim to provide credible evidence for local gains as a result of a specific design, but it will also strive simultaneously to discover new knowledge that can inform the work of others that face similar problems (McKenney & Reeves, 2014:131). Barab and Squire (2004:6) explain the requirements of Design-based Research (DBR) as

> … more than simply showing a specific design works but demands that the researcher (move beyond a specific design exemplar to) generate evidence-based claims about learning that address contemporary theoretical issues and further the theoretical knowledge of the field.

Design interventions will constantly be connected with existing theory, while being sensitive to the possibilities of emerging new theories. A thorough description of the context in which DBR is carried out was explained in Chapter 4, with the intention to adequately investigate the variables to answer the research questions. Results of all activities will be documented carefully, and visual summaries will be provided to support the relevance of the trends and patterns in the data relating to the research questions. This chapter will attempt to address Aims 11 to 13 of the research question, with the purpose of concluding the discussion on the last sub-research question (Section 1.8.2):

Aim 11 Explore competence development in individuals, groups, as well as in the whole class, through an analysis of qualitative data derived from the students' modelling activities (Section 5.2).

Aim 12 Define a hypothetical learning trajectory (HLT) from the results of a pilot study and the pre-intervention interviews, to be used as a starting point for the design experiment (Section 5.2).

Aim 13 Establish a learning trajectory that not only addresses classroom norms and discourse, but also explains how the possible shifts in students' reasoning abilities occur (Section 5.4)

Regarding Aim 11, competence development will be explored in detail during the implementation and reflection stages of the six model-eliciting activities (MEAs) (Section 5.2). All episodes relating to competence development will be transcribed, coded, analysed and tested against the various data collection instruments as discussed in each activity, and they will be compared to current literature to search for confirmation and counter-examples. By moving forwards and backwards between collecting and analysing data, by constantly comparing data with prior data and against literature, checking for ideas, refining emerging ideas, and by constructing abstract categories from data analysis, the emergent levels of analyses are raised while new ideas, questions and deeper refinements of earlier concepts can also emerge (Simon & Tzur, 2004).

As discussed in Chapter 1, this study adapted the RME theory whereby learning in context and guided reinvention are motivated to allow students to take ownership of the mathematics (Dickinson & Hough, 2012:1). Students regard their acquired knowledge as their own private knowledge for which they themselves are responsible, while being active participants in the teaching-learning process that takes place within the social context of the classroom (Larsen, 2013:2; Van den Heuvel-Panhuizen, 2003:11). For further support of successful learning, Gravemeijer (2004:107) introduced the construction of a '*Hypothetical Learning Trajectory*' (HLT). This HLT consists of learning goals, planned instructional activities, and an envisioned learning process. The envisioned learning process anticipates how students' thinking and understanding might evolve when the instructional activities are employed in the classroom, as well as possible means of supporting that learning processes (Gravemeijer & Cobb, 2006:19). The support also refers to an envisioned classroom culture, the use of technology, and the proactive role of the teacher/facilitator (Chapter 2). To give emphasis to Schoenfeld's (2009:1) quote above, this support remains focused on the learning goal of nurturing inquisitive thinkers to develop mathematical reasoning and understanding.

The HLT thus serves to guide the educator's intended path of work development aiming for a specific goal within a specific environment (classroom) on a day-to-day basis. The current knowledge level of the student needs to be considered, to ensure progressive movement towards the envisaged learning goal. The HLT will continually change and adapt to support the learning and teaching towards a specific goal. To keep track of the possible changes in the HLT, various data collection methods will be used, e.g. interviews, walk-throughs, students' written work, reflection instruments, audio and video recordings, and field notes (Section 4.3.1.6). This variation in data collection methods will further assist in enhancing triangulation (Section 4.5.2). Section 5.2 addresses the changes and refinements of the HLT throughout the activities. By observing the enactment of the HLT, a *Local Instructional Theory* (LIT) will be developed, that describes the envisioned learning route relating to the set of instructional activities. The construction of the HLT (Section 5.2) and LIT (Section 5.3) will thus serve to answer to Aims 12 and 13 of the research question.

5.2 ASSESSING AND REFLECTING ON ACTIVITIES

To adhere to the reliability and credibility of a design research study, it is vital to describe as truthful as possible how the activities played out during the experiment, to acquire an understanding of how the competencies developed through engagement with model-eliciting activities (MEAs). All the activities will be described in terms of how they materialised in the Design-Based Research (DBR) methodology of planning, implementing and reflecting (Section 4.3). When reporting on DBR experiments, a thorough explanation needs to be provided of how the design was implemented in the settings (Collins et al., 2004:38). This section will aim to provide such information.

Auditable trials of documentation that will be collected during each MEA, will be based on representative samples of students' work that consist of project-sized activities. The goal of such activities is to emphasise deeper and higher-order understanding, where students have to express the abilities to make and justify their own decisions (Lesh & Clarke, 2000:116). Dynamic feedback will be provided during the various stages of the solution process, as well as after completion of the MEAs, to reinforce student understanding and to correct applicable misconceptions (Yildirim et al., 2010:842). Lesh and Clarke (2000:116) note that such dynamic feedback have the potential to encourage students towards 'increasingly better' productions and understandings.

Assessment instruments were designed to replace traditional checklists, as not all forms of learning consist of pre-defined rules (Appendices A– I). Complex conceptual systems usually involve more than the sums of their parts, and to define observable and assessable goals of instruction, students' interpretations of problem situations are just as important as the ways that they act upon such problems. MEAs are designed (Sections 4.3.1.5 and 5.2) to create situations in which students are required to produce their own cognitive constructs in observable forms. The task descriptor of an MEA elicits the need to construct a mathematical model that does not reside in the problem statement. The assessment instruments applicable to this study are designed to recognise and describe the nature of the students' constructs, and to offer guidelines to both facilitator/researcher and students, while comparing the usefulness of their (alternative) ways of thinking (Lesh & Clarke, 2000:127). Each assessment instrument will have an explicit goal to contribute towards fostering increased mathematical reasoning and understanding.

The Status Update Report (Appendix A) requires students to reveal their ways of thinking, as they cannot only produce solutions, but descriptions, constructions, explanations and justifications are needed as well. Their mathematical interpretations explicate the mathematical knowledge that the students consider – the processes as well as the products interact continually. The students need to also think *about* their constructs to be able to share, modify, transport and reuse them, which in turn creates possibilities for generalisation (Lesh & Clarke, 2000:128). Complementing the Status Update Report is the Student Reflection Guide (Appendix E). It serves as a guide to the researcher to determine possible 'next steps' towards developing a deeper understanding of mathematics. The students have to reflect on the activity and allows for strengths and weaknesses to surface. Explanations on their strengths and weaknesses allow the researcher to adapt for subsequent activities where necessary and it also offers guidance as to whether the scaffolding is adequate or not. The Group Reporting Sheet assists the students to report on their work in such a way that the researcher can identify the possible presence of competencies (Appendix F). This instrument supports students in working with a sense of direction, as it keeps them focused on their goal. The goal of the

Group Functioning Sheet is to allow students to reflect on their activities during the initial, middle, and final stages of the modelling task (Appendix G). They also need to allocate the time that was spent on brainstorming, working, considering alternative approaches, communicating ideas and finding a working strategy. Identifying the presence and sphere of meta-cognitive competencies such as management, communication and responsibility are the focus of this instrument. The Research Observation Guide (Appendix B) serves to assist in recognising important characteristics of the relevant systems that occur, while the Quality Assurance Guide (Appendix C) is used as a tool to assess the relative strengths and weaknesses of the students' systems that are elicited. Appendix H represents an assessment guide for the students' poster presentation. Students have to convey the required information with a clear and logical layout, without too many words. A post intervention questionnaire will also be handed out at the end of the course, to gain more insight into the students' opinions about the mathematical modelling course (Appendix I). The researcher anticipates the finding of relevant information relating to meta-cognitive competencies based on the students' answers about their feelings and beliefs. As already explained in Chapter 4, the Group Competency Observation Sheet (Appendix D) allocates marks for each competency between 0 and 3. Even though this study anticipates development of students' mathematical modelling competencies, none of the competencies under investigation was defined in terms of what it would mean to possess such a competence to perfection. It is important not to view a LIT as the end-all to student learning challenges. Competence development will always remain a growing process, directing to the opinion that all students, regardless at what level they are, should always strive to continue to develop themselves further, as no final stage of expertise exists (Schorr & Lesh, 2003:145).

The mathematical modelling competencies that need to be assessed involve internalising, interpreting, structuring, symbolising, adjusting, organising, generalising, managing of the task, communication and responsibility, while the accompanied engineering technician competencies relate to defining, investigating and analysing of a problem, designing of solutions, comprehend and apply applicable knowledge to solve a problem, recognition of factors that can affect the activities, and sound judgement. As stated earlier, such complex systems cannot be reduced to a checklist of condition-action rules. Therefore, assessing implies a continuous adapting of programs, of teachers' behaviours and beliefs, and of students' development describing and explaining complex systems. Interpretations tend to develop through a series of modelling cycles, in which the result of one cycle informs decisions and actions to be made for following cycles to allow for additional refinements in the underlying interpretations (Lesh & Clarke, 2000:133).

5.2.1 Assessment of pre-intervention interviews (Beliefs about mathematics)

A student's belief is something that he or she knows or feels that involves effort (Kloosterman, 2006:248). In this case, effort relates to learning mathematics and developing mathematical modelling and engineering technician competencies. Kloosterman (2006:248) explains that the efforts that individuals put forth, depend on their perceptions of whether such efforts might enhance their chances to fulfil specific goals, particularly how the effort affects mathematics learning. Motivation to learn, results from students' beliefs about mathematics as a discipline, how they perceive themselves as mathematics students, the role of the educator, as well as other beliefs about mathematics learning (Section 2.5).

The 12 first-year engineering students who enrolled in the research experiment, had no prior experience of mathematical modelling. To gain a better understanding of their beliefs about mathematics and how they perceived themselves as students, the researcher adapted the student interview instrument as suggested by Kloosterman (2006:266). This instrument was developed to assess the key motivational factors. Interviews were held within groups where each student was asked to verbally respond to every question. As Kloosterman (2006:249) clarifies, one of the major benefits of an interview as opposed to a questionnaire is, that interviews allow the researcher to ask follow-up questions or prompts where students fail to produce detailed answers. This assessment instrument allowed the researcher to gain an understanding of how students' beliefs can possibly influence their actions (Appendix J). All interviews were video and/or audio recorded.

5.2.1.1 Reflecting on the interviews

Through careful documentation, analysis, coding and categorising of data, the following emerged from the interviews:

Question 1 (feelings about mathematics):

	Scale	Response
On a scale of 1 to 10, with 10 being the highest, how much do you like mathematics? (Look for topics the students like, such as likes fractions but dislikes algebra. Also look for level of challenge student prefers – are textbook exercises boring? Are story problems too hard?)		
	1	0
	2	0
	3	0
	4	2
	5	2
	6	4
	7	2
	8	2
	9	0
	10	0
Are there some parts of mathematics you like and some you don't? Please explain. (Look for topics the students like, such as likes fractions but dislikes algebra. Also look for level of challenge a student prefers - are textbook exercises boring? Are story problems too hard?)		

Figure 5.1 - Question 1 – Pre-intervention interview

When asked about their perspectives on mathematics learning, all the students believed that memorising is crucial to understanding mathematics. Ten of the students preferred to learn mathematics by following procedures, rather than to try to understand mathematics. However, two students agreed that, to understand mathematics is just as important as to memorise the rules. Students predominantly referred to mathematics as rule-based activities (8), but conceptual understanding was mentioned as well (2). One student's response was very different to the other students, as she referred to mathematics as 'building on previous knowledge', and thereby displayed the ability to connect new knowledge with previously learned knowledge, to expand her knowledge base. Content-wise, only 5 students (who all agreed they liked mathematics) indicated a specific preference. One of the students' mentioned that he preferred algebra to geometry, while the other four students were mainly opposed to content areas that involve in-depth explanations, e.g. word problems. Again, only one student indicated that she liked to find various approaches to answer mathematical problems.

Question 2 (effort):

Do you work hard in the mathematics class?	Yes	No
	8	4
Do you always do everything the teacher assigns?	4	8
In general, what influences you to work hard in mathematics? (Although this issue comes up again later, if there is any evidence of task orientation, ability orientation, or any type of social orientation, make sure it is noted)		
How do you like mathematics in comparison with other subjects? Are the factors that make you work hard in other subjects different from the ones that make you work hard in mathematics?		

Figure 5.2 - Question 2 – Pre-intervention interview

Question 3 (goal orientation and effort):

How often do you work hard in mathematics just to learn the material? (Look for evidence of task-orientation – motivation just to learn the material or accomplish the task)

Figure 5.3 - Question 3 – Pre-intervention interview

How hard students work, meant different things to different students. Two students held strong beliefs that hard work over time, results in success in mathematics and that this success carries a responsibility, while seven students considered mathematics as less important and believed that a pass in mathematics is more than sufficient to complete their engineering technician program successfully. One student struggled to see mathematics as something worthy of understanding and doing. Only 4 of the students attended to their homework on a regular basis. Homework was done with little evidence of it reflecting on their work. Solutions for the homework were normally given to the students at a later stage. Five students remarked that they would attend to their homework more regularly, if they received more direct instructions from teachers to 'know what steps to follow'. One student commented that, if the work is enjoyable, he would attend to it, while another student does her homework just to please her teacher. This complements Kloosterman's (2006:248) view that,

> … in reform-oriented classrooms where math instruction includes focus on reasoning, problem-solving and concept understanding, students may quit trying to learn when the instructor stresses comprehension of concepts over memorisation of steps to get to an answer.

All but one student were mainly motivated by their grades, parents and teachers. She valued the satisfaction of a deeper understanding as worthy to engage deeply in mathematics. The data also indicated that the students distinguished between mathematics and other subjects in terms of understanding and thinking skills. Surprisingly, nine students agreed that they prefer mathematics, although their reasons were more related to memorisation and procedural rules than experiencing the benefit of acquiring a deeper understanding of the subject matter. Only two students preferred mathematics to other subjects, because of its' nature to develop critical thinking abilities. Another aspect that emerged during the interviews, was the students' concern, or rather, dissatisfaction, with the teacher if he or she did not offer clear guidelines and instructions as to how to proceed with questions. Schoenfeld (1983:3;166;168) warns that students' limited beliefs about mathematics and their accompanied deficiencies to regulate and monitor their work, halters efficiencies when engaging in higher-order thinking processes, such as mathematical problem-solving.

Question 4 (self-confidence):

How good are you at mathematics? How do you know it? (Look for how judgements are made and what it means to a student to be good in mathematics – if needed, probe to see if the student compares him/herself to other students. Also, are judgements made on the basis of grades? Teacher comments? Is there any evidence of internal factors such as 'I know I am good because I understand mathematics'?)
Are you better at some kinds of mathematics than others? For example, are you better at long division than you are at fractions, or are you better at computations than problems that require a lot of thinking?

Figure 4.4 - Question 4 – Pre-intervention interview

The majority of the students (9) rated their accomplishments in mathematics as average, and determined their mathematical successes in terms of their class positions, not in terms of acquiring an understanding of the mathematical content. Students were mainly goal orientated – marks guided them regarding how much and hard they needed to work. Eleven students preferred procedural mathematics and did not necessarily appreciate the reward of accomplishment after grappling with a problem; they tend to lose interest before the problem was solved. It appeared that students regarded mathematics learning and understanding to be the same as to know how a procedure works. However, one student commented that, even though she regarded herself as competent in memorising rules, she enjoys the challenge of 'harder mathematics' like word problems.

Question 5 (natural ability in mathematics):

Do you think it takes a special talent to do well in mathematics? Do you have such talent? Can people do OK in mathematics even without special talent?
When someone makes mistakes in mathematics, does it mean that person is dumb in mathematics? (Probe for explanation – specifically if a student feels that making mistakes is part of the learning process)
How important is memorising in mathematics? Are you good at memorising? Can someone who is not very good at memorising be good in mathematics? (or even OK in mathematics?)

Figure 5.5 - Question 5 – Pre-intervention interview

Students also expressed various opinions about whether one should have talent or not, to excel in mathematics. Two students admitted that engagement with interesting concepts could cultivate determination, as one works harder to understand the task at hand, which can eventually lead to success. All the students agreed that making mistakes in mathematics allows one to learn and develop an understanding of a specific concept. However, ten students indicated that they normally struggled to complete tasks which required more than procedural understanding. Eight students remarked that memorising skills determine whether you are good or not at mathematics, as “word sums only counts about 10-15% of the total mark”.

Question 6 (mathematics content):

Suppose an alien from outer space landed in your back yard and started asking you what mathematics was like in South Africa. What would you tell him? What words best describe mathematics? (Try to get a sense of how much the student sees mathematics as 'rule-based' and believes that it involves complex problems as opposed to textbook exercises.)

Figure 5.6 - Question 6 – Pre-intervention interview

Ten students predominantly referred to mathematics as a rule-based subject. However, one student commented that mathematics allowed him to understand things from everyday life which he never understood before. Another student explained that mathematics helped her to understand other subjects as well. The student responses were related more to what they did in the mathematics classroom, than what they thought mathematics meant.

Question 7 (communication):

How important do you regard mathematics learning as a collaborative activity?

Figure 5.7 - Question 7 – Pre-intervention interview

None of the students were used to group work, they all regarded mathematics as a solitude activity. This behaviour is indicative of students who lack exposure to mathematical modelling problems.

5.2.1.2 Lessons learned from the pre-intervention interviews

To summarise, the findings of the interview questions were considered as valuable lessons to be learned for the researcher as it suggested that

- procedural understanding and memorising seem to be of primary importance to the students when they learn mathematics;
- the data indicates that students tend to do what they are told, far-removed from the educational goal of mathematical modelling;
- as the students are not familiar with group work, classroom norms need to be established clearly to allow the students to engage in argumenting, reflecting and justifying of their work; and
- the students are not familiar with learning through guided reinvention, therefore the facilitator will have to ensure that constructivism norms are well established for the successful implementation of the mathematical modelling problems (Sections 2.2 and 2.6.4).

5.2.2 Design cycle 1 – Lawn Mowing Task

The *Lawn Mowing Task* was adapted from modelling tasks designed by Singh and White (2006:44). Their study investigated students' preferred use of relational understanding versus instrumental understanding. This was the first of six modelling tasks intended to elicit the development of students' conceptual systems, and this task focused particularly on ratios. The task posed to the students was to create a model for a garden service company, where they had to determine the number of hours required to mow the lawn of a school, depending on the available staff complement. Students had to apply knowledge of ratios to determine a successsful solution to the problem. Due to the students' lack of exposure to mathematical modelling, group work and problem-solving, the researcher decided to use this MEA as a 'warm-up' activity. Students had already spent a morning session introducing themselves, expressing their understanding of the research experiment and what they were hoping to gain from the experiment, completing the pre-intervention questionnaire (Section 5.2.1.1). The researcher was hoping that, by starting with a relative undemanding task, the students would divert from instrumental understanding and solitude working, to openly discuss ideas about solving the given problem. It was also anticipated that students would establish a situational model without guidance from the facilitator, as model eliciting is one of the main tasks of engineering design. The Lawn Mowing Task of Singh and White (2006:44) was adapted as follows (Figure 5.8):

Mr. Green, the small business owner of *'Keep it Clean and Green'* garden services, has asked your consultancy firm to provide him with a model to improve the planning of his daily workload. One of the weekly responsibilities of *Keep it Clean and Green* garden services, includes the upkeep of the local school's lawn. The lawn gets mowed every week and Mr. Green needs all of his workers to complete the task in 5 hours. His total staff compliment is made up of 9 workers.
However, an epidemic has broken out in the area, which caused Mr. Green to adjust his weekly timetable. This week, 3 of his workers are absent. Mr Green needs to plan for future absenteeism, as he is currently unable to commit to all his clients due to a shortage of staff and needs to re-schedule all his appointments. As the job at the school requires more manpower and time than any of his other clients, he needs to prioritise this job to remain in business. Your task is to develop a model for Mr. Green to project how long it will take to mow the school's lawn given that some of his workers may be absent. The solution must be in the form of a poster presentation.

Figure 5.8 -The Lawn Mowing Task as adapted from Singh and White (2006:44)

5.2.2.1 Planning for the Lawn Mowing Task

As discussed in the introductory section of this chapter, the planning and implementation part of each activity consists of the construction, adaption and refinement of the hypothetical learning trajectory (HLT), which comprises of the goal for the students' learning, the mathematical tasks that will be used to promote students' learning (Figure 5.8), and hypotheses about the processes of the students' learning within a constructivist framework (Simon & Tzur, 2004:93). The HLT is based on phenomenological analyses and current literature, and this activity was selected based on the researcher's anticipation of how the students' modelling competencies may develop. The motivation for this activity came from the researcher's observation of students' inabilities to use relational understanding effectively, and this observation was confirmed by literature (Skemp,

1976) during the pilot study (Section 4.3.1.2), and again in feedback received from the pre-intervention interviews (Section 5.2.1).

Harel and Sowder (2007:4) accentuate the importance for teachers to identify students' current knowledge, regardless of its quality, to help them to gradually refine it. Their view is based on Vygotsky's (1967) recognition that the construction of new knowledge depends on students' prior knowledge. Thus the mapping of the Zone of Proximal Development (the region of competence that a student can traverse with and without aid) needs to take place before attempting to engage in new knowledge (Brown, 1992:157). Students' competencies are never expected to be 'completely undeveloped', or 'completely mastered', but rather at some intermediate stage of development (Lesh & Harel, 2003:176). In general, the challenge for students is to extend, revise, reorganise, refine, modify and adapt the constructs that they do have at their disposal. Students' emerged knowledge should therefore be viewed as "*developing* rather than as being in a state of learned versus not learned, or solved versus not solved" (Lesh & Zawojewski, 2007:790). It should be the student's goal to play an active role in meaningfully assimilating knowledge for his or her existing mental framework, to enhance mathematical modelling and engineering technician competence development.

The researcher anticipated students to use instrumental understanding and to go through meaningless steps of applying previously learned procedures and algorithms without much relational understanding. This MEA was based on the studies of Singh and White (2006:44), who investigated first-year engineering students' abilities to apply and use previously learned mathematics, their understanding of mathematics, abilities to reconstruct understandings and to connect mathematical concepts to allow for a deeper understanding. Their study aimed to provide an understanding as to what extent students perform meaningless algorithmic procedures, rather than having good reasoning abilities.

As the HLT also includes hypotheses about the processes of learning (Section 4.3.1.5), the researcher followed Wake et al.'s (2016:252) suggestion to predict areas of difficulty and to prepare possible feedback questions to use when the students grapple with the problems. The anticipated difficulties and accompanied questions are indicated in the Table 5.1 below:

Table 5.1 - Anticipated difficulties – MEA-1, adapted from Wake et al. (2016:252)

Anticipated Difficulties	**Suggested Questions and Prompts**
Students start detailed calculations before planning an approach. For example, they start with the first numbers, then do meaningless multiplication and division with the following.	Describe in words a plan for tackling this problem. What are the key decisions you have to make? Which information are you going to focus on at the start; which will you ignore (if any)?
Students ignore one or more constraints.	Does your solution make any sense? Does it take longer or shorter when there are more or less workers?
Students do not justify decisions made. For example, they state a solution with no explanation	Why have you chosen to apply this formula? How can you be sure this is the best solution?
Students leap to conclusions.	Have you taken all the issues into account? Does your solution make sense in the case where only 4 workers pitch?
Students do not understand the concept of the problem.	What is your main objective when trying to solve the problem?
Students do not grasp the meaning of their calculations. For example, students might perform a sensible calculation but not understand what their answer represents	What does this figure represent? Does it represent a number of people or the hours worked?
Students only write numbers with no justifications.	Where have these figures come from? Do you know what they represent? Are you able to justify why you have used these numbers?

The HLT has now been constructed, which will continually be adapted and refined as new information and observations emerge during the course of the six modelling tasks. Defining the HLT from the results of the pilot study and the pre-intervention interviews, also serves to address Aim 12 of the research question (Section 1.8.3)

5.2.2.2 Implementing the Lawn Mowing Task

During the introductory sessions of the experiment, the researcher explained to the class what mathematical modelling entails, as well as the importance of striving towards becoming a successful mathematical modeller (Section 4.3.2). The mathematical modelling and accompanied engineering technician competencies that formed the focus of this study were then discussed, and the researcher explained the importance for engineering technicians to develop such competencies. Due to limited exposure to constructivist learning environments, students were reluctant to ask any questions and displayed behaviour indicative of instrumental understanding. They wanted to do what the teacher asked without necessarily developing an understanding of the tasks, which denotes typical behaviour of the traditional teaching paradigm. Briley (2012:3) noted that, within the traditional perspective of mathematics, doing what is told by the instructor rather than understanding a task, plays a big role. Traditional teaching and learning of mathematics revolves around demonstrating facts, rules, skills and processes, after which student activities are monitored by the teacher while they practice the preceding items. The teacher corrects errors as they occur. However, teaching within the models-and-modelling perspective's framework, the focus is on carefully structured activities. Within such experiences, the students confront the need for significant mathematical constructs where they repeatedly express, test, refine and revise their current ways of thinking (Lesh & Doerr, 2003:31-32).

Implementation of the MEA followed the following format:

- Informational texts were handed out to the students for reading, where after the facilitator led the class discussion.
- Students worked in three groups of four students each throughout the modelling activity.
- Readiness questions were prepared (Section 5.2.2.1), based on the students' anticipated understandings and misconceptions. Classroom discussions (either whole class or group discussions) ensured understanding of the task.
- Brainstorming among the groups were continually promoted, especially during the initial stages where the students need to find ways of solving the client's problem.
- As the teams selected their own ideas to be developed further, they were motivated to create procedures to solve the client's problem.

- Guiding questions were asked by the researcher/facilitator, while the students created their procedures to address relevant issues that arose.
- The teams tested, evaluated and revised their procedures as necessary and presented it to the class, taking turns to lead the presentations.
- Peer critique and discussions followed the presentations.
- Students' modelling competencies were investigated and documented throughout the experiment by means of various data collection strategies: a Status Update Report (Appendix A), a Quality Assurance Guide (Appendix C), a Group Modelling Competency Observation Guide (Appendix D), a Group Reporting Sheet (Appendix F), a Poster Presentation (Appendix H), video and audio recordings, students worksheets, observation from the researcher by doing walk-throughs, as well as informal interviews. By utilising multiple data gathering instruments, the validity of the findings were enhanced (Section 4.5). Furthermore, field notes, informal discussions, and video tapes were used to facilitate and monitor the students' possible competence development.

5.2.2.3 Reflecting on the Lawn Mowing Task

An unexpected dilemma surfaced during the initial stages of this task, which the researcher did not anticipate. The students' (Group A in particular) revealed an ill-equipped knowledge of the English vocabulary. English is the second language of all the students that took part in the study. Seven of the students did not understand the meaning of '*mowing lawns*'. When asking them what they found difficult to understand, they pointed to both words. This language barrier that was exposed, provided a rich opportunity for group discussions, and led to reconstructing of the sentence to *'the garden services had to cut the grass'*. The students were able to apply code-switching, which allowed them to proceed with the task. Lehrer and Schauble (2000:51) commented that students can develop representational fluency and increased conceptual understanding by using their own, albeit primitive, languages, diagrams, and other media to express their thinking processes.

The purpose of selecting this specific task, was to get the students to construct a model and to apply relational understanding, while they were exposed to work in small groups and had to communicate clearly and effectively to one another what they envisaged to do. Their inexperience with learning and teaching within a constructivist environment emerged early during the activity, as all groups halted their investigations to wait for the researcher for direct instruction, whenever they were confronted with a problem. Group C seemed specifically confused when realising that the researcher would not answer their questions directly. Even though the researcher explained her role to them prior to starting the modelling activity, they were still waiting and hoping for clear guidelines and direct instructions to follow. This inability of the students to carry on, exposed their low degrees of employing management and responsible behaviour competencies. Such competencies are often exposed *between* episodes of action, where the quality of their decisions has the possibility of 'making or breaking' a problem-solving attempt (Schoenfeld, 1983:2). Students reverted to brainstorming and group discussions only after they realised that they would not get more help from the facilitator/researcher.

The initial disorganised and inconsistent interpretations and ways of thinking regarding what was given, what their goals were, and what possible steps to be taken towards producing a solution, were characteristics of the all the students' early interpretations. Inconsistency in their thought processes, led them to switch from one way of thinking to another without noticing the changes. During those early stages, they generally recognised the need to develop a simplified model, but ignored the difficulties relating to surface-level details or gaps in the data (Lesh et al., 2000:597). An initial interpretation of Group C's understanding as presented in the following image (Figure 5.9), indicates their lack of understanding of the content:

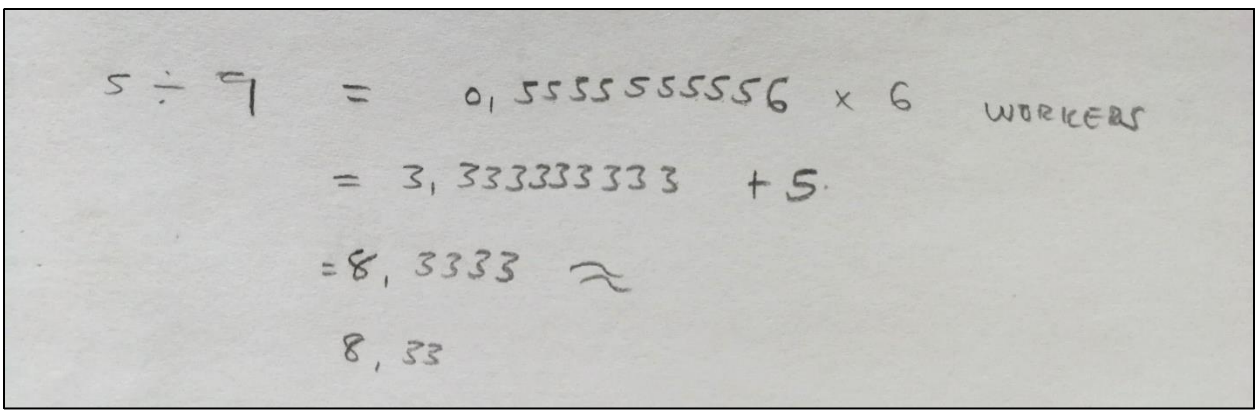


Figure 5.9 - Group C's first attempt to solve MEA-1

Groups A and C wanted to engage in detailed calculations, before trying to gain an understanding about the content. Both groups' understandings were flawed, in that they thought that, if it takes nine workers to finish the job in five hours, it should take one worker 5/9 hours to finish the same job (Figure 5.2). Again, their behaviour pointed to instrumental understanding, as they proceeded to manipulate the numbers without considering the meaning of the problem or of their mathematical descriptions. Their immediate interpretations led these two groups to believe that each worker only works for 0.5556 hours, instead of all workers working for the entire period. These interpretations of Groups A and C resulted in meaningless cross-multiplication strategies, which gave rise to answers that did not make sense (Figure 5.3). The desire to use instrumental rather than relational understanding, was prevalent throughout these stages. Group C's initial aim was to solve the problem by using as much data as possible. Due to the inability to understand the context, the group was distracted and did not have a clear goal in mind. A student in Group A reminded the group about 'inverse relations' and tried to implement it, but was unable to explain it when probed by the researcher/facilitator. An example of this attempt by Group A is represented in Figure 5.10:

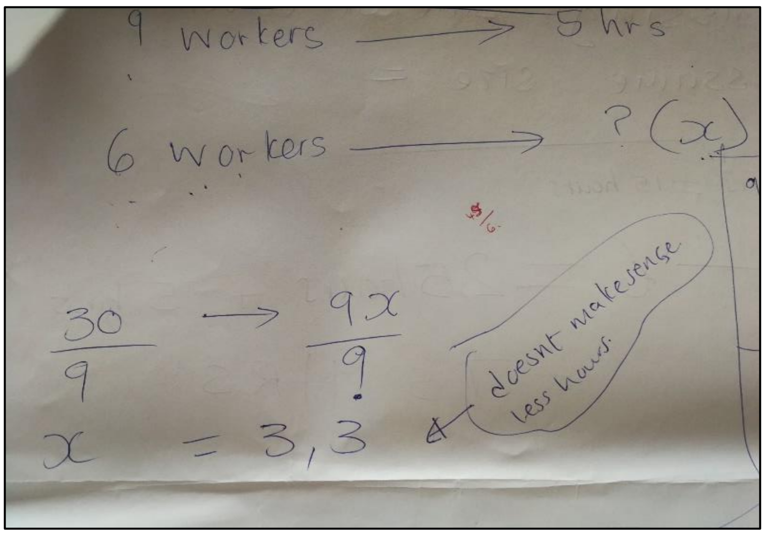


Figure 5.10 - Cross-multiplication attempts by Group A - MEA-1

Group B illustrated their understanding by drawing a picture of a sports field, and dividing it in nine sections. This illustration allowed the group to create a primitive situational model, from where they were able to apply relational understanding and explained that the job would take longer when employing less workers. Relational understanding was not exposed until their situational models were clearly established, which in turn assisted them to gain a deeper understanding about context of the problem. Even though their understanding remained primitive, the modelling problem obliged them to discuss and represent their situational models.

Upon reflection, Group A realised that their solution methods did not make any sense, and eventually decided to start all over by creating pictorial representations and to continually evaluate their representations throughout the activity. Group C also reverted to a pictorial representation of the situation, which ultimately allowed all three groups to gain a sense of direction whilst continually considering their situational models. The initial conceptual models that the students created, had flaws in both internal as well as external components. Internal components refer to constructs or conceptual systems, while external components refer to their representations (Lesh & Carmona, 2003:71). The students' initial flawed representations forced them to reconsider alternatives, which assisted them towards developing a better understanding of the problem, resulting in improved mathematical explanations, which in turn pointed to increased understanding of the conceptual system. The three groups' solution methods are represented in the following illustrations (Figure 5.11):

Solution methods of Groups A, B and C for MEA-1 – Lawn Mowing Task

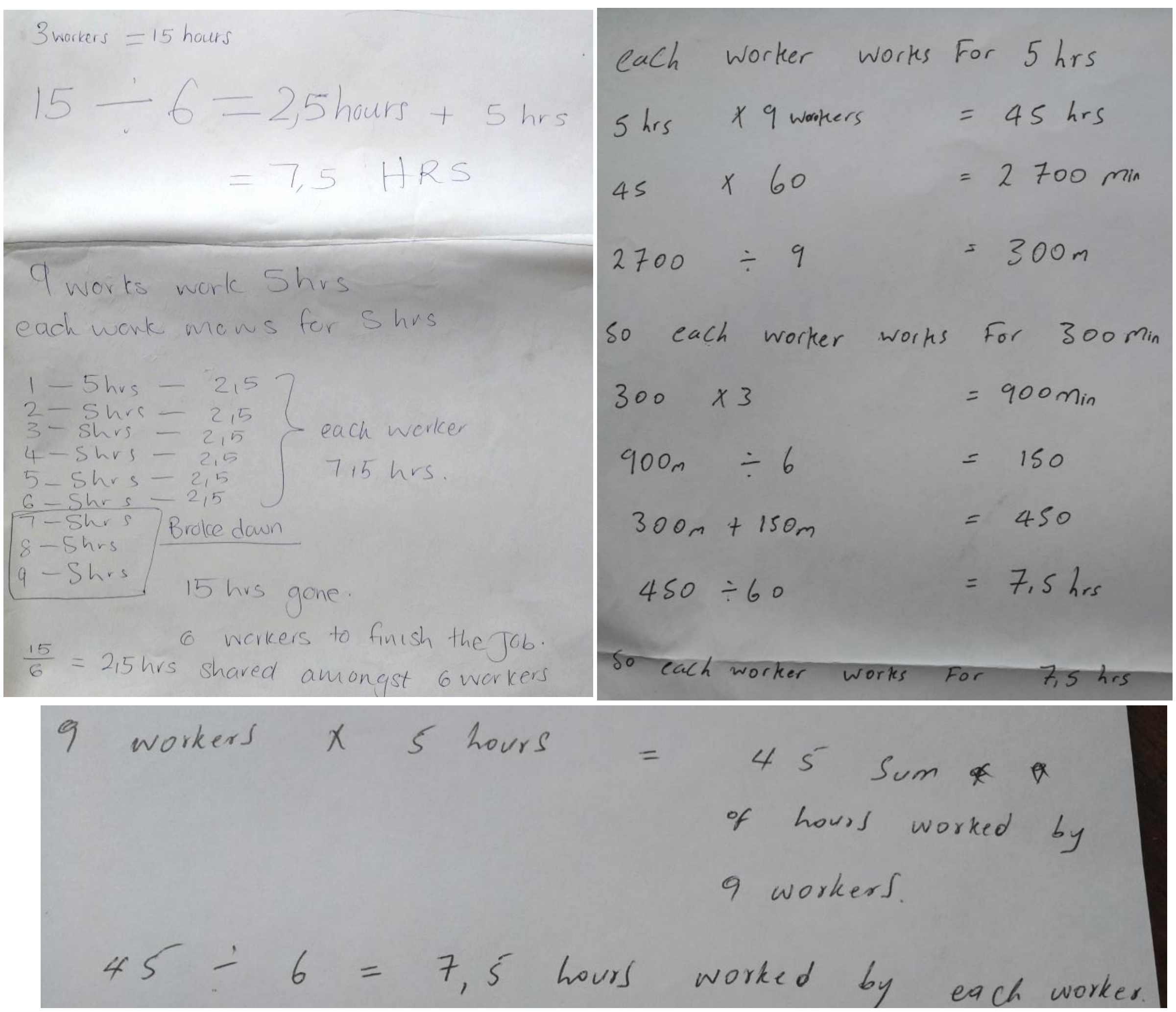


Figure 5.11 - Solution methods presented by Group A (top left), Group B (top right), and Group C (bottom)

The students were surprised when they presented their solution methods to the class, because even though all the groups managed to determine the correct solution, their solution methods all varied. One of the characteristics of MEAs is exactly this aspect: there are many different ways of reasoning about a problem. This differentiation of alternative ways of thinking emphasised Lesh and Harel's (2003:187) observation that development in mathematical modelling competencies do not simply progress along ladder-like sequences. The overall task was completed in a very primitive way, and the students' inexperience with mathematical modelling problems surfaced during their unsuccessful attempts to try and manipulate numbers in meaningless ways. During the group discussion at the end of this session, all students agreed that they needed to rely more on group work, group discussions and representations to assist them towards understanding and ultimately solving MEAs. Meta-cognitive competencies, such as management skills, communication, responsibility and motivation were initially almost non-existing as they were waiting on guidance from the facilitator/researcher, but they managed to expose such competencies (however primitive) during the latter part of the activity. Students slowly took charge of their processes and ideas (again very primitive) and, by communicating their ideas to one another, they gradually experienced the value of mathematics. One of the student's comments at the end of the session was:

B1: "*Ma'am, we can use the maths that we have learned long ago to solve this problem!!*"

To experience the utility of mathematics can serve as a motivation for the students to carry on and complete the tasks. Data were collected, analysed, coded and categorised by means of the various assessment instruments. The results of the analyses, together with memos which were compiled after collecting the data, revealed particular competence development of the whole class and per group, in terms of mathematical modelling competencies (MMC) and engineering technician competencies (ETC). The data table (Table 5.2) serves to complement the subsequent graphical representations (Figures 5.12 and 5.13):

Table 5.2 - Results of competence assessment - MEA1 - Lawn Mowing Task

Competence assessment of the Lawn Mowing Task:

Mathematical modelling competencies	
Internalising	0.93
Interpreting	0.98
Structuring	1.01
Symbolising	1.21
Adjusting	1.17
Organising	1.02
Generalising	0.56
Management	0.91
Communication	1.14
Responsibility	1.01

Engineering technician competencies	
Define, Investigate & Analyse Problems	0.95
Design/Develop Solutions	0.97
Comprehend and Apply Knowledge	1.13
Recognise and Address Factors	1.07
Sound Judgement	0.99
Management	0.91
Communication	1.14
Responsibility	1.01

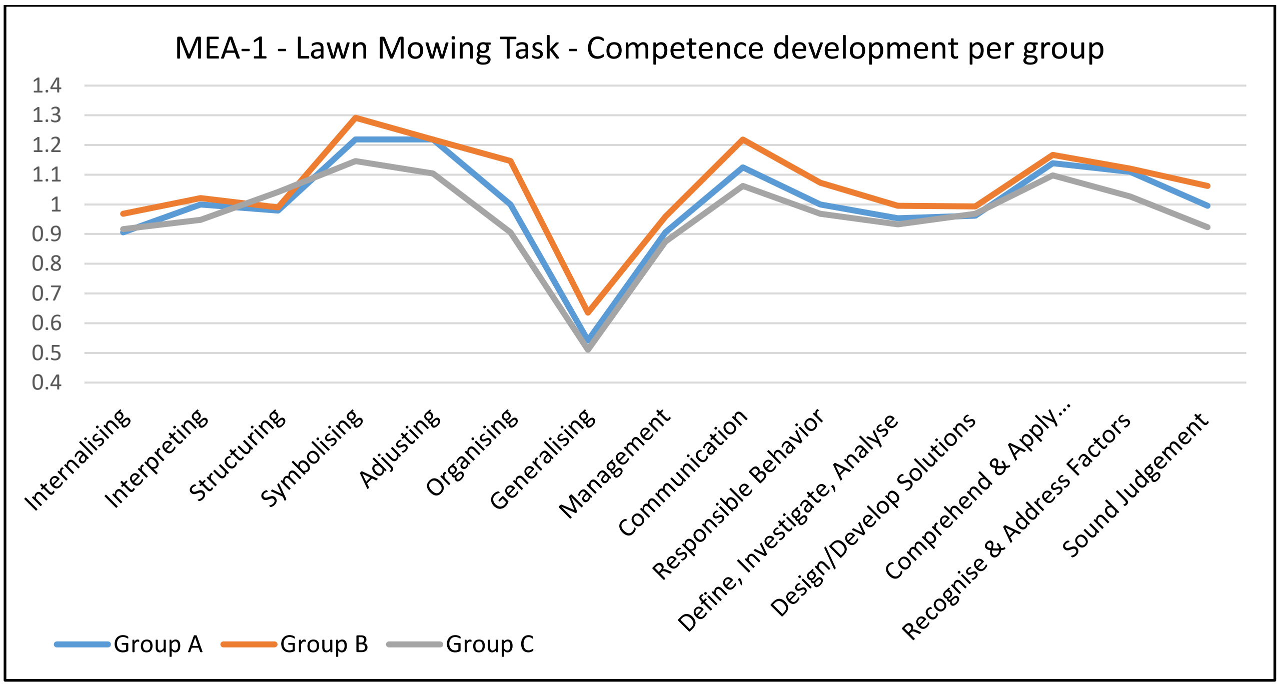


Figure 5.12 - Competence development per group - MEA-1

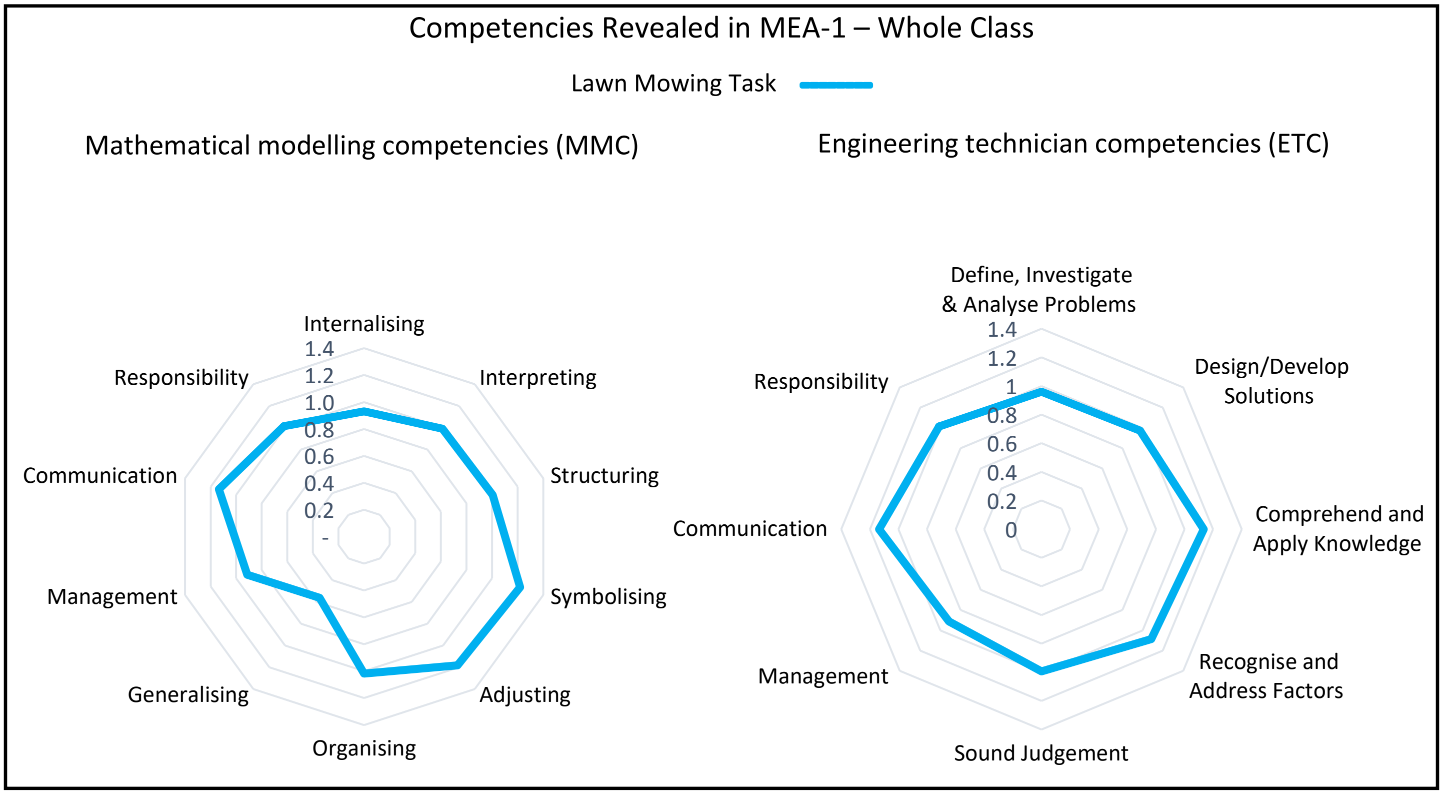


Figure 5.13 - Graphical representation of competence development – MEA-1 – Lawn Mowing Task

As remarked earlier in this section, the Group Modelling Competency Observation Guide (Appendix D) allocates marks for each competency between 0 and 3. The above graphs emphasise the difficulties that students experienced in all competency domains. Of the mathematical modelling competencies relevant to this study, internalising, interpreting and generalising seemed to be the most problematic cognitive competencies. Primitive development of these problem-solving competencies explains the poor outcomes of the engineering technician competencies relating to define, investigate, analyse and design solutions and practising sound judgement. The researcher's anticipated beliefs about students' inabilities to understand and interpret problem situations, as well as to generalise their mathematical models, were confirmed. Hamilton et al. (2008:7) indicated that, in their research study, only a few undergraduates managed to reach a satisfactory level of generalisation, and only managed to generalise by relying on trial-and-error methods.

This longitudinal study will aim to assist the students in progressing through the various sequences of interpretation-development cycles, which have the potential to develop more sophisticated ways of thinking. Instead of merely organising and processing bits and pieces of data, students can learn to focus on relationships, patterns or trends in the data (Lesh et al., 2000:599). They start the process by interpreting and simplifying the situation, where after new information are noticed, which can create the need for further refinement or elaboration of descriptions or interpretations. The new interpretations that emerge can again create the need for another round of observing additional information. These interpretations can be unstable and evolving and the students repeat these interpretation-development cycles until they have produced their desired results without further adjustments (Lesh et al., 2000:600). Further instructional tasks were designed to match their thinking levels to allow for the progressive development of engineering technician and mathematical modelling competencies, the goal of the learning trajectory.

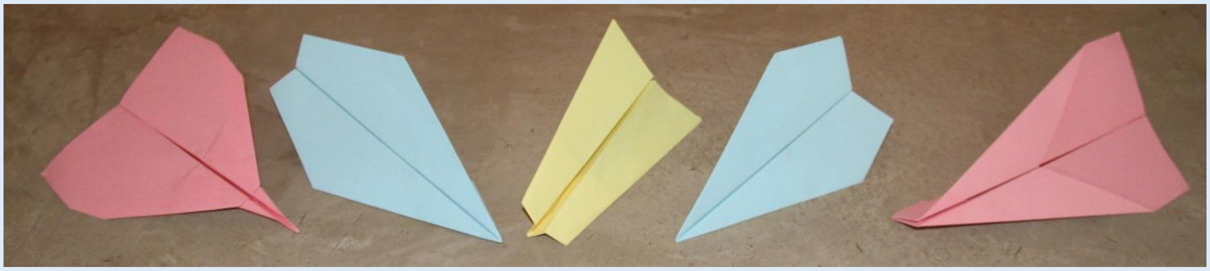
The competencies of communication (1.3) and symbolising (1.2) ranked the highest of all, but were at unsatisfactory low levels. Communication did not provide evidence of appropriate reasoning or initial understanding of the main ideas, but the groups displayed a positive attitude about the tasks and work of others, and all the team members assisted to finish the project satisfactory. All groups were able to reach a satisfactory solution and used various ways of mathematising the problem scenario (Figure 5.11). Although their symbolisation competencies were very basic, they were able to select appropriate symbols to reach correct solutions.

5.2.3 Design cycle 2 – Paper Airplane Task

The results of the Lawn Mowing Task revealed the students' inexperience with mathematical activities, where the problem statement does not explicitly refer to the mathematical concept that needs to be used (e.g. ratio, areas, volumes). It was observed that they found it difficult to interpret, formulate, and define the problem, before attempting to develop a solution method. This second MEA was selected for two reasons. Firstly, the researcher reckoned that the scenario would be realistic to the students to ensure increased student interest, and would hopefully stimulate students to interpret the situation meaningfully and thereby overcome some of the difficulties experienced in the first MEA. Secondly, this MEA was selected due to its strong link to a second semester program, named *Survey for Civil Engineers*. The researcher searched for an activity that did not only develop sound problem-solving competencies, but also had the potential to assist in blurring the boundaries between mathematics education and engineering education. *Survey for Civil Engineers* is a program based on sound geometric and trigonometric principles, which further influenced the researcher's decision to include this MEA in the design experiment.

The MEA required the students to assist judges from a local high school to create a procedure to determine the most accurate paper airplane in a competition (Appendix L). This activity provided the students with opportunities to work in teams, and to create a system to judge a paper airplane contest. Mathematical ideas can be acquired from real-life situations while making mathematical connections through problem-solving. The students had to consider the relevance and importance of certain given information to create meaningful solutions. The latter part of the modelling activity was particularly focused on the competency of generalising. The modelling activity was based on the paper airplane activity designed by Eames et al. (2016:230):

Letter from the client:



Pietermaritzburg High School
165 School Road
Pietermaritzburg
3201

Dear engineering team

A paper airplane contest is again planned to be held at our school next year. Prizes will be awarded for characteristics for example, the most accurate, best floater, fanciest flyer, and most creative airplane. However, there exists a lot of controversy about which planes really should win several of the contests. Arguments arose for two main reasons:

a. Differences may not be large between planes or pilots who are ranked 1st, 2nd, and 3rd; and
b. Planes often fly quite differently when different pilots toss them.

The judges want to have better and more quantitative rules for judging planes for each award, and as much as possible, they want their judgments to depend on clear rules or formulas. Three judges are going to continue their policy of having at least three different pilots fly each airplane, and an award will be given to the best paper airplane. They therefore need a procedure that can somehow factor out the pilot factor when judging the planes.

Please help the judges to plan for the paper airplane contest and provide us with a report that explain how they can use information of the kind shown in the diagram and data table provided in to give awards for the plane that is the most accurate.

Thank you,

Mr Bird
Head of Science Department

Figure 5.14 - The Paper Airplane Task as adapted from Eames et al. (2016:230)

The following data sheet (Table 5.3), as well as the graphical representation of previous contests' landing positions (Figure 5.15), accompanied the letter from the client:

Table 5.3 - Data sheet of previous contests' landing positions (MEA-2)

Landing positions of paper airplanes											
No	**Plane**	**Pilot**	**Flight Distance**	**Angle Error**	**Flight Time**	**No**	**Plane**	**Pilot**	**Flight Distance**	**Angle Error**	**Flight Time**
1	W	A	39.40	(33.00)	4.70	**19**	Y	A	30.00	(23.60)	6.00
2	W	B	34.70	(1.80)	2.40	**20**	Y	B	38.40	10.00	5.80
3	W	C	31.10	4.10	1.90	**21**	Y	C	32.60	27.70	4.60
4	X	A	28.60	(31.80)	2.30	**22**	Z	A	33.50	(29.00)	6.40
5	X	B	26.40	12.10	2.10	**23**	Z	B	34.40	(14.90)	3.40
6	X	C	19.90	43.20	2.20	**24**	Z	C	31.80	(2.00)	6.00
7	Y	A	31.90	(20.10)	3.20	**25**	W	A	30.10	(37.00)	2.40
8	Y	B	40.60	11.00	5.70	**26**	W	B	33.10	(31.40)	2.00
9	Y	C	39.20	11.40	7.80	**27**	W	C	26.90	0.00	1.30
10	Z	A	38.30	(18.40)	5.00	**28**	X	A	33.10	25.30	3.00
11	Z	B	46.10	(6.90)	7.80	**29**	X	B	25.60	10.00	3.80
12	Z	C	35.20	16.30	6.00	**30**	X	C	32.90	40.00	3.30
13	W	A	30.40	(27.60)	1.80	**31**	Y	A	25.00	12.80	3.80
14	W	B	43.00	(14.90)	3.90	**32**	Y	B	31.10	(4.10)	4.70
15	W	C	39.70	17.80	2.40	**33**	Y	C	34.80	9.20	6.60
16	X	A	23.30	22.00	1.60	**34**	Z	A	32.00	(10.00)	6.40
17	X	B	31.80	2.00	3.50	**35**	Z	B	31.90	6.00	4.10
18	X	C	20.60	(9.30)	1.00	**36**	Z	C	48.20	5.30	7.20

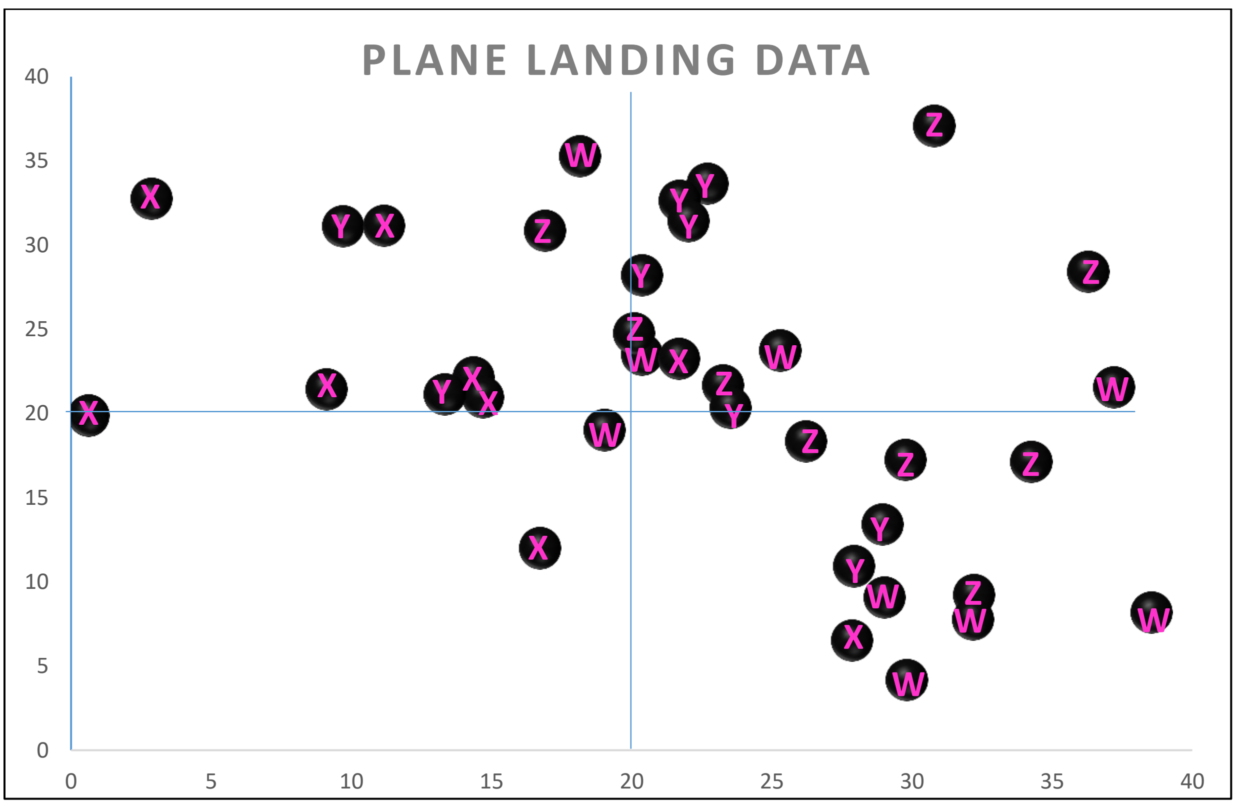


Figure 5.15 - Graphical display of previous contests' landing positions – (MEA-2)

5.2.3.1 Planning for the Paper Airplane Task

The goal for the students is to develop a productive procedure in a competition, based on specific landing data that are provided. In reaching this goal, the students will have to use numeric data to create a productive procedure to determine the most accurate paper plane. They need to consider how to use and exclude particular data, where after they need to make decisions about whether or not their solutions meet the needs of a client. Once solved, the solutions must be communicated in a clear and concise report to the client. In this activity, the facilitator will monitor the students' abilities to make mathematical connections, and to engage in problem-solving.

As the class was still not familiar with modelling problems, the researcher had to refine the HLT further, and followed suggestions based on The Summer Job MEA (Chamberlin, 2005) by communicating the following information to the students prior to start working on the activity:

- MEAs are longer problems without immediate or observable solutions.
- MEAs are often defined by problems having more than one solution and different ways of reasoning about the problem.
- The students need to present their solution methods to the class.
- The facilitator will not answer any direct questions such as 'Is this correct?' or, 'Am I done?' However, clarification questions may be answered, but direct instruction will not be allowed.
- Students will be reminded to constantly concentrate on the problem to confirm whether they have answered the question satisfactorily.

Readiness questions were prepared based on suggestions by Chamberlin and Moon (2005:39), to ensure understanding of the text, as it included basic comprehension questions, inference questions as well as questions related to data interpretation:

- What is the problem?
- Who is the client in this problem?
- What does the client need? What is the product that you need to produce? (A fair system needs to be developed whereby prizes can be rewarded for paper airplanes based on previous data records)
- How do you decide which airplane is the most accurate?
- What do you need to include in your letter? (procedure for selecting the best paper airplane)

In predicting areas of difficulty, assist the researcher to prepare possible feedback questions, while the students grapple with the problems. The anticipated difficulties and accompanied questions are indicated in Table 5.4 below, and were adapted from Wake et al. (2016:252):

Table 5.4 - Anticipated difficulties – MEA-2, adapted from Wake et al. (2016:252)

Anticipated Difficulties	**Suggested Questions and Prompts**
The students will make mistakes in measuring the difference between the target and the landing positions, due to the extent of information provided on the data sheets. For example, students might not understand the meaning of angle errors.	The instructor needs to continually prompt for explanations and ask them whether their solutions are realistic or not. Let them explain it visually to assist them in understanding how the solution will differ depending on where the planes land.
Students might get confused with detail provided such as flight times, or may not be able to grasp the meaning of their calculations.	They need to be able to explain the answers of their calculations meaningfully. They need to be clear on what their solutions represent.
Students start detailed calculations before planning an approach. For example, they may start using all information, such as flight times, without understanding the goal of determining wat an ‘accurate’ paper plane means to them.	Describe in words a plan for tackling this problem. What are the key decisions you have to make? Which information are you going to focus on at the start? Which will you ignore (if any)?
Students do not justify decisions made. For example, they state a solution with no explanation.	Why have you chosen to apply this formula? How can you be sure this is the best solution?
Students leap to conclusions.	Have you taken all the issues into account? Does your solution make sense in if you add more landing positions for the various planes?
Students do not understand the concept of the problem.	What is your main objective when trying to solve the problem?
Students do not grasp the meaning of their calculations. For example, students might perform a sensible calculation but not understand what their answer represents.	What does your answer represent? The distance of the flight? The distance from the target?

Anticipated Difficulties	**Suggested Questions and Prompts**
Students only write numbers with no justifications.	Where have these figures come from? Do you know what they represent? Are you able to justify why you have used these results?

5.2.3.2 Implementing the Paper Airplane Task

Implementation of the MEA followed the following format:

- This model-eliciting activity followed the instructional model of Chamberlin and Moon (2005:39). The first section was the reading passage (individual reading) to generate students' interests and discussions about the context of the situation. The students worked in small groups, and the task description, together with the images and data table (Appendix L) were handed out.
- Self-assessment instruments were also handed out, including the Status Update Report (Appendix A), the Quality Assurance Guide (Appendix C).
- The researcher facilitated the meaning of the information in the data tables and the graph. Before the students started working on the problem, they threw a few paper airplanes and the flight variations were pointed out by the researcher. Students were made attentive to the fact that, depending on who the pilot was, the flight path of the plane differed as well. The researcher continuously drew the students' attention to the Status Update Report to assess their current progress and to ensure that the students acquired a foundational knowledge to solve the problem and to understand the context of the problem situation.
- The readiness questions were prepared (see above), based on the students' anticipated understandings and misconceptions. Class discussions (either the whole class or group discussions) ensured understanding of the task.
- Brainstorming among the groups were continually promoted, especially during the initial stages, while the students needed to find ways of solving the client's problem. A Student Reflection Guide (Appendix E) was also handed out to the students to assist in reflecting on the activity, as it allowed for strengths and weaknesses to surface.
- As the teams selected their own ideas to be developed further, they were motivated to create procedures to solve the client's problem.

- The researcher/facilitator made use of a Researcher Observation Guide (Appendix B) and a Group Modelling Competency Observation Guide (Appendix D) to assist in recognising students' competence development, and to inform her as to when and where scaffolding might be required.
- Guided questions were asked by the facilitator/researcher, while the students created their procedures to address relevant issues that arose.
- The teams tested, evaluated and revised their procedures as necessary and presented it to the class, taking turns to lead the presentations.
- Peer critique and discussions followed the presentations.
- The mathematical approach and effectiveness of the solutions were also discussed.

The reflection reports and guides that were handed out to the students assisted them in formative self-assessment, where they had the opportunity to verify their early or intermediate models against the problem stated (Eames et al., 2016:233). Presentations, student reports and subsequent discussions at the end of the session, further motivated the students to assess the validity of their models against the needs of the client.

5.2.3.3 Reflecting on the Paper Airplane Task

The majority of the students understood that they had to determine the most accurate paper airplane. However, inconsistencies amongst the groups appeared when they had to explain their understandings about the meaning of an accurate plane. Both groups A and B initially considered measuring the distances of each flight, while Group C decided to also investigate the properties of the best floater, fanciest flyer, and most creative airplane. The multitude of strategies that Group C put forward, confused them and prevented them from noticing useful similarities and patterns that could guide them towards a possible next step. Lesh and Fennewald (2013:26) commented that, the understanding of when and why to use strategies, e.g. looking for similar problems, can lead to both negative as well as positive outcomes. The discernment ability to judge when and why to use such strategies are at the "heart of what it means to understanding them" (2013:26). After the group discussion, Group C decided to ignore the other aspects of the competition and to rather focus on the plane that lands closest to the target.

Another problem that emerged, was the fact that all the groups initially treated the data as 'future data', and did not initially recognise the need to provide the client with a *procedure* to determine the best plane. The students focused on using the data of the previous competition to determine which plane was the most accurate, rather than providing an effective procedure. Furthermore, most of the students wanted to incorporate all the data in their initial interpretations. Only four students, Students A1, A3, B1, and C3, were able to recognise irrelevant data and make assumptions on what they regarded as important. Students A2, B2 and C2 considered measuring the distances to the target by using rulers, indicating the use of oversimplifications, typical of novice modellers (Eames et al., 2016:230).

After lengthy group discussions and analysis of the data provided, the groups explained their first modelling attempts (consisting of primitive and oversimplified models) to one another, where after they all agreed upon defining the most accurate plane to be the plane with the shortest average distance to the target. They then regrouped and attempted to determine the distances between the landing positions and the target. Even though no student could explain the meaning of *angle error*, they realised that the angle error must be used to calculate the coordinates of the landing positions. Due to time constraints, the researcher called for a whole class discussion to guide them towards understanding of the term *angle error*:

R: *"Please tell me what you understand by 'angle error'?"*

B2: *"It must be the error of some or other angle? I think?"*

B1: *"The error of an angle – this must mean that you aimed in the wrong direction?"*

R: *"B1, can you perhaps try to draw a picture on the whiteboard to explain to us what you mean?"*

B1 drew the following picture:

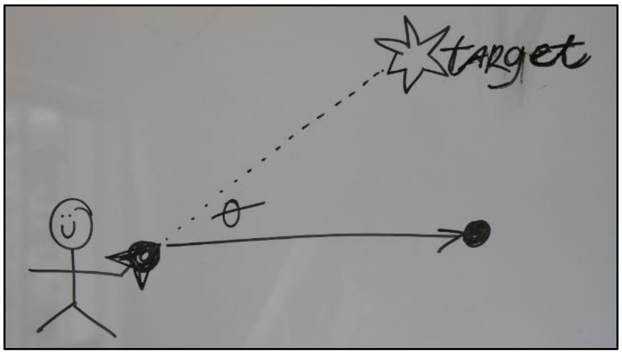


Figure 5.16 – Student B1's explanation of angle error

B1: *"You see, Ma'am (pointing to the θ), it is the error in degrees between your target and where you actually landed."*

B2: *"But ... Here are negative numbers – for instance a MINUS 33 degrees?"*

C3: *"Oh, I remember! It indicates the direction of the angle, clockwise is negative, anti-clockwise is positive."*

R: *"So what does the minus 33 degrees mean to you?"*

C3: *"The plane did not fly in the right direction, it was 33 degrees away from the target ... clockwise."*

C3: *"But all these landing positions are in the first quadrant, negative moves to the fourth quadrant…?"*

R: *"What is the angle of the target?"*

A4: *"The target is at 45 degrees, because the target is at position (20,20)."*

A3: *"???Mmmmm so ... if it is -33 degrees, it means that my actual angle is ... 45-33=12 degrees?"*

A3: *"Yeah, it must be right, this one looks more or less like 12 degrees. Let me see if I can judge the correctness of the distances."*

The slight scaffolding provided by the researcher allowed the students to engage in dialogue, and it was not necessary to entertain any direct instruction regarding the meaning of angle error. Constructive discourse within the groups assisted the students to build on one another's current understanding and their current ways of reasoning were developed into slightly more sophisticated ways of mathematical reasoning (Gravemeijer, 2004:105). All the students managed to gain a better understanding of the meaning of *angle error*, and continued working in their groups to find a possible solution to the problem. Through the process of *guided reinvention* and subsequent *progressive mathematising*, the students progressed from one level of thinking (measuring the distances with a ruler) to the next (using trigonometric and geometric principles to determine the distances algebraically) (Zulkardi, 1999:6).

Both Groups A and C continued to calculate the distances between the 36 landing positions and the targets, where after they grouped the planes together to determine the most accurate plane based on the lowest average. Only when Groups A and C were writing their letter to the school, the two groups realised that they had to explain a *procedure* and did not have to calculate the most accurate plane based on the previous competition's results (sudden insight). Group B's continual reference to the Status Update sheets, allowed them to realise this mistake earlier, and they did not spend as much time to calculate every plane's distance to the target. Self-assessment of the task and keeping a sense of direction while working, allowed this group to understand the impact of their strategies and they were in a better position to alter the course of the activity towards improving their models (Schoenfeld, 1983:24). By managing the process, they exposed more effective organising, management as well as responsible behaviour competencies, and they were able to monitor and implement their strategy to a satisfactory extent.

All groups represented their solution procedures orally and in the form of a report at the end of the session. Upon reflection, they all agreed that they would make more use of brainstorming and group work to gain an understanding of the problem before trying to work on meaningless calculations. The concept of collaborative work came out strongly in the reflection stage of this activity, as students realised the value of learning from one another.

The following table (Table 5.5) and graphs (Figures 5.17 and 5.18) indicate the changes in mathematical modelling competencies and accompanied engineering technician competencies by allocating competency scores using the Group Modelling Competency Observation Guide (Annexure D):

Table 5.5 - Results of competence assessment - MEA-2 - Paper Airplane Task

Competence assesment of the Paper Airplane Task

Mathematical modelling competencies	
Internalising	1.16
Interpreting	1.12
Structuring	1.19
Symbolising	1.46
Adjusting	1.28
Organising	1.13
Generalising	0.82
Management	1.05
Communication	1.24
Responsibility	1.17

Engineering technician competencies	
Define, Investigate & Analyse Problems	1.14
Design/Develop Solutions	1.16
Comprehend and Apply Knowledge	1.31
Recognise and Address Factors	1.20
Sound Judgement	1.16
Management	1.05
Communication	1.24
Responsibility	1.17

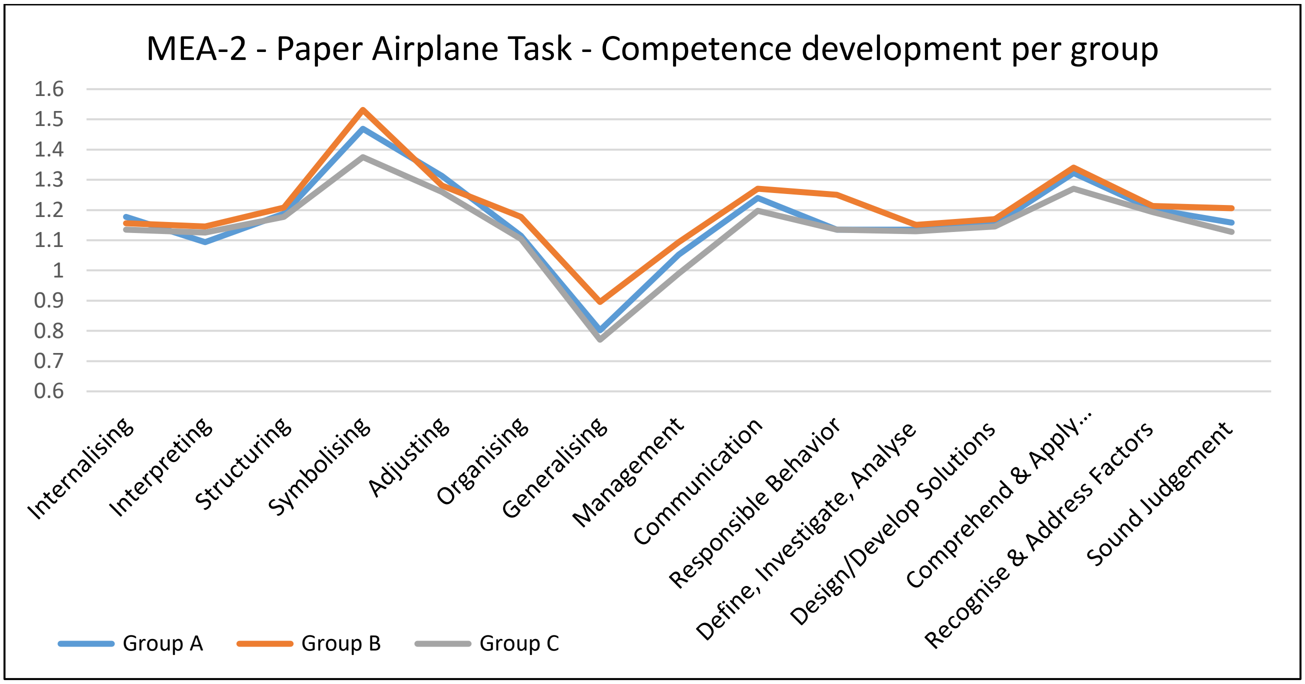


Figure 5.17 - Competence development per group - MEA-2

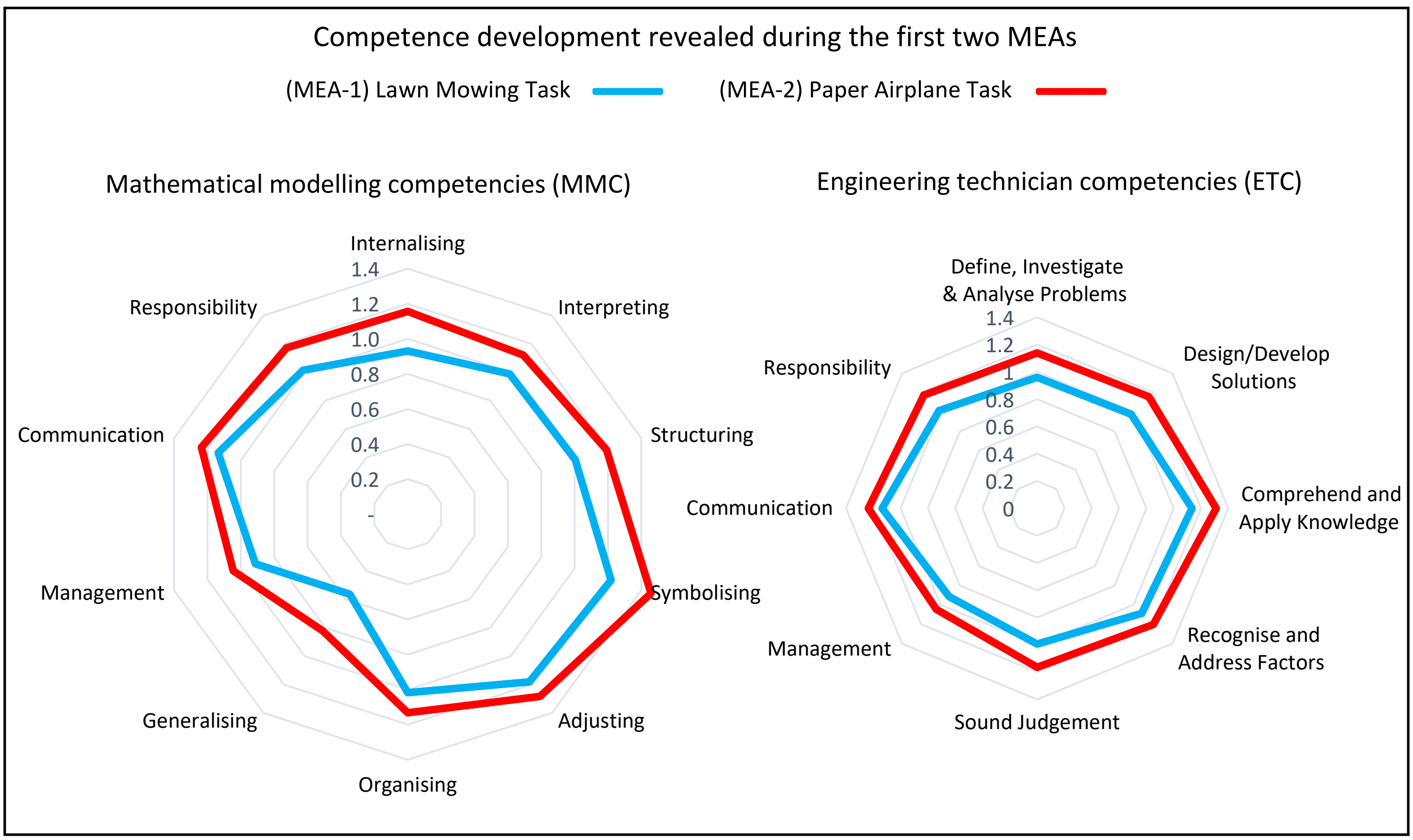


Figure 5.18 - Whole class competence development – MEA-1 and MEA-2

Marks for each competency were again allocated between 0 and 3. Figure 5.12 indicates a slight progress in the level of mastery in all competency categories when compared to the first MEA. The graphs indicate the students' immature understanding and interpretation of the real-world problem, as well as their current inabilities to evaluate their mathematical models against the real-world situation. Due to their misunderstanding of the client's problem to create a procedure rather than a solution, their initial models ignored this aspect and their generalisation competencies remained weaker than the other competencies.

As far as meta-cognitive development was concerned, the students had a slightly better understanding of what was expected from them, as they realised that they had to engage in effective group work and brainstorming to develop a deeper understanding of the MEA. Clear and meaningful communication was a little bit more prevalent during this second activity. However, Groups A and C struggled to manage the process consistently and only realised that they performed unnecessary calculations when they presented their model. Again, once the students managed to obtain an understanding of the task, they were able to select basic mathematical tools to lead to partially correct solutions.

Once again, this problem focused on competency development, and not concept development, as the study investigates the co-development of mathematical modelling and engineering technician competencies with the aim to develop a deeper understanding of mathematics. The elicitation of models are crucial in this experiment to assist the students in learning to solve problems that do not have clear solution paths to be followed. The students' work still indicated low levels of mathematics understanding, which is expected from students who are underprepared in mathematics when entering an engineering program.

5.2.4 Design cycle 3 – Tidal Power Task

This MEA, adapted from Hamilton et al. (2008:6), required students to act as engineering consultants, and to provide the City Council for Sea Shell Island with one or more designs to increase the Islands' generated power (Appendix M). Students had to build and test models to describe and justify various excavation options to increase the energy in the dam. Big ideas that students encountered was the proportionality between work and energy, the fact that gravity plays a role in determining the output energy, as well as consideration in terms of the average height of the excavations. This activity has strong links with Physics, another first-year engineering subject, because of the knowledge required regarding potential energy, gravitation and output energy. Also, this activity was considered to be extremely relevant to the engineering students, due to the importance of environmental considerations in today's society. Students had to apply knowledge of Geometry to determine a successsful solution of how to increase the output energy by 15%.

Background

The City Council of See Shell Island has asked your engineering firm to provide an analysis of their tidal power plant. Due to population and business expansion on Sea Shell Island, there is a need to obtain more energy from the power plant. In particular, the City Council is looking to increase energy production at the plant with around 15%.

Tidal power plants generate electricity by trapping water from the rising tide behind a dam, and then letting it out so that it turns one or more turbines. Currently, Sea Shell Island has a tidal power plant whose basin is 200 meters across and goes 400 meters inland. Openings in the dam allow water to enter and leave the basin as the tide rises and falls. This passing of water through the dam generates energy that can be stored, processed and distributed. The amount of energy generated, is directly proportional to the amount of work required to fill the dam. In the case of the Sea Shell Island plant, the energy produced each time the dam empties, is 70% of the work required to fill it. The depth of the basin is 10 meters at the dam wall and gradually decreases to ground level at 400 meters inland. The bottom of the basin follows the shape of a trapezoid: (See accompanied diagram)

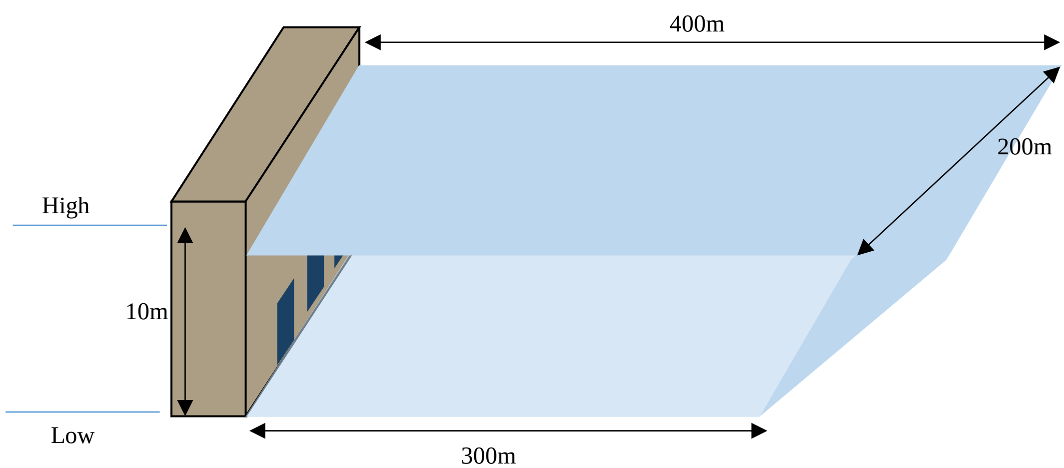


Figure 5.19 – Background on MEA-2 (Tidal Power Task) as adapted from Hamilton et al. (2008:6)

Task

Write a report for the City Council that addresses the current particularities of the power plant and provides at least two alternative designs for achieving a 15% net gain in power output. Note that the City Engineer will read the study and represent your findings to the Council. It is appropriate to provide detailed calculations along with relevant explanations for any solutions that you propose. Any charts and graphs you use can be incorporated into the report. Carefully consider the council's desire to increase the energy production by 15%. Discuss different construction options on the basin to achieve this result. Note that there is open space for another 80 meters inland, but beyond that there are buildings and roads. The community has expressed a preference for retaining as much open space as possible. You should consider options that require excavating the minimal amount of earth since the cost of the project will be directly proportional to the volume of earth that needs to be excavated.

Figure 5.20 – MEA-2 (Tidal Power Task) as adapted from Hamilton et al. (2008:6)

Current particularities

You need to specify the current volume of the dam, as well as the proposed new volume of the dam	
Density of water (ρ)	1000 kg/m^3
Gravity (g):	9,8 m/s^2
The gravitational potential energy of the water mass:	$PE_g = mgh = \frac{kg.m}{s^2}.m = Work \times m = Newton \times m = Joules$ where m represents the mass of the water in kg, g represents gravity (m/s^2), and h represents the **average height** of the dam (m).
Power produced as water flows through the turbine	$P = \frac{PE_g}{t}$

Figure 5.21 - Further specifications relating to MEA-2 (Tidal Power Task)

Assumptions

Changing the width of the dam is not practical since it already exists. Therefore all construction should be focused on changing the basin. The dam can be closed (thus not allowing any water to enter) so that construction can be accomplished. Currently the vertical cross section of the dam is in the form of a trapezoid, but this is not necessarily required to be the case after excavation work is done.

Figure 5.22 - Assumptions regarding MEA-2 (Tidal Power Task)

5.2.4.1 Planning for the Tidal Power Task

Again, to further elaborate on the HLT, readiness questions (adapted from Chamberlin and Moon (2005:39)) were prepared in anticipation that the students would struggle with the understanding of the text. Suggested questions and prompts were:

- Who is your client?
- Describe in your own words what the problem is.
- Explain in words how you want to attempt this problem.

- What are the key decisions that you have to make?
- Which information is important to start with, and which will you ignore?
- Do you have enough resources to be able to work towards a solution?
- Explain your solution to me - justifying the decisions you made.

It was also anticipated that not all the students would automatically grasp what was expected from them, hence the researcher prepared questions to guide them towards gaining an understanding of the problem posed. These questions also served to further finalise the HLT, and they were adapted from the suggested questions by Wake et al. (2016:252):

Table 5.6 – Anticipated difficulties – MEA-3, adapted from Wake et al. (2016:252)

Anticipated Difficulties	**Suggested Questions and Prompts**
Students start detailed calculations before planning an approach. For example, they start with the first numbers, then do meaningless multiplication and division with the following.	Describe in words a plan for tackling this problem. What are the key decisions you have to make? Which information are you going to focus on at the start; which will you ignore (if any)?
Students do not understand the concept of the problem.	What is your main objective when trying to solve the problem?
Students ignore one or more constraints.	What are your assumptions, if any? What do you plan to do to simplify your problem?
Students do not justify decisions made. For example, they state a solution with no explanation.	Why have you chosen to apply this formula? How can you be sure this is the best solution?
Students leap to conclusions.	Have you taken all the issues into account? Does your solution make sense if the size or shape of the excavation differs? What about variation in the height of the dam wall?
Students do not grasp the meaning of their calculations. For example, students might perform a sensible calculation but not understand what their answer represents.	What does this answer represent? Did you answer the clients' question? Why did you decide on this specific excavation design?
Students only write numbers with no justifications.	Where have these figures come from? Do you know what they represent? Are you able to justify why you have used these numbers?

Anticipated Difficulties	Suggested Questions and Prompts
The students do not adjust the average height of their dam according to the excavations that they propose.	Explain how the height of the dam influence the energy generated. Let them explain it visually to assist them in understanding how the solution will differ depending on how the excavation is planned
Students might get confused with units of measurements, or may not be able to grasp the meaning of their calculations.	They need to be able to explain the answers of their calculations meaningfully, they need to be clear on what their graphs/diagrams/solutions represent.
Students may not understand the fact that only 70% of the power generated can be used.	Probe them for explicating their thinking, by asking questions such as: 'Can you explain why they plan for 70% usage, and not 100%?'

5.2.4.2 Implementing the Tidal Power Task

To build from the students' zone of proximal development, they were guided by the instructor to reinvent their own constructions. To ensure the students acquired a foundational understanding of how tidal energy works, the following YouTube video clips were shown to the class:

Table 5.7 - Tidal power video clips

YouTube video clips explaining tidal power			
Tidal Power 101	**Tidal barrage generation systems**	**Hydro Power 101**	**Tidal Power**
			
(Student_Energy, 2015b)	(Allaboutrenewables, 2013)	(Student_Energy, 2015a)	(SarahGotsMadSkillz, 2012)

At the end of the video clips, a whole class discussion followed to verify the level of their current understanding of tidal power plants. The facilitator probed the students to think about various factors that could contribute to the power and energy in the dam, e.g. height, gravity, the weight of the water, the amount of water. Students had to express their understanding of tidal energy verbally, to ensure adequate comprehension of the content.

Implementation of the MEA followed the following format:

- Informational texts were handed out to the students for reading, where after the facilitator led the class discussion.
- Students worked in three groups of four students each throughout the modelling activity.
- Readiness questions were prepared (see above), based on the students' anticipated understandings and misconceptions. Class discussions (either whole class or group discussions) ensured understanding of the task.
- Brainstorming among the groups were continually promoted, especially during the initial stages where the students needed to find ways of solving the client's problem.
- As the teams selected their own ideas to be developed further, they were motivated to create procedures to solve the client's problem.
- Guiding questions were asked by the facilitator while the students created their procedures to address relevant issues that arose.
- The teams tested, evaluated and revised their procedures as necessary and presented it to the class, taking turns to lead the presentations.
- Peer critique and discussions followed the presentations.
- The following research instruments were used in this activity: a Status Update Report (Appendix A), Researcher Observation Guide (Appendix B), Quality Assurance Guide (Appendix C), Group Modelling Competency Observation Guide (Appendix D), Group Functioning Sheet (Appendix G), and a Poster Presentation Guide (Appendix H) at the end of the session.

5.2.4.3 Reflecting on the Tidal Power Task

There was a clear indication that the students have learned certain lessons from the previous activities, as increased *communication skills* and *collaborate work* were far more observable during this activity. Even though it seemed as if Group A deviated from the topic, they discussed the effects of inadequate power supply in their day-to-day lives and in so doing, they expressed an interest in the problem situation. Various alternatives were suggested, also indicating their understanding for the potential of multiple solution paths. Groups A and B expressed appreciation for teamwork to build on one another's understanding, by taking turns to explain sections of the problem to one another. Students frequently reverted back to the YouTube video clips to gain a better understanding of hydroelectricity, as this concept was new to them. With classroom designs branded by discourse, individual perceptions and discussions of different arguments, together with a search for understanding, comprehension, systematisation, questioning, inquiry and reflection, meta-cognition developed simultaneously with their subject knowledge (Maaß, 2006:118). Lesh, Lester, and Hjalmarson (2003:384) explain that both meta-cognitive thinking and cognitive activities interact, and development in one domain can lead to further development in the other domain. Self-monitoring was supported by the facilitator/researcher in asking questions to 'why' and 'what' the students were doing. Students expressed the will to make sense and interpret the problem situation, as they continually interacted with one another and with the video clips, which in turned fostered their motivation, cognition, and meta-cognition during the learning process and over time (Metallidou & Vlachou, 2007:4).

While trying to explain what they wanted to achieve in the end, the groups demonstrated their first attempts to create a *model of* the situation. Group A drew a picture of a dam, representing the current situation, and then added an expansion, representing the proposed excavations. However, their model assumed (incorrectly though) that the dam's volume needed to be increased by 15%. Groups B and C reverted to the physics calculations of potential energy as supplied in the data to determine the required increase in potential energy to assist with decisions regarding excavations. Through the process of mathematisation, the students' informal and intuitive *model of* the situation gradually revolved into a *model for* explaining tidal pool excavations, which represents their (even though basic) abilities to progress toward more generalised mathematical activities (Larsen, 2013:2).

Both Groups A and B established a reasonable understanding about what information they needed to extract to continue with the problem, but they were uncertain about the meanings of their calculations. Group B again displayed increased management and monitoring skills as they decided to list all outstanding information, where after they proceeded to attend to those gaps. There were two issues in which all the groups exposed misunderstandings: the result of the 15% increase in energy, and how to calculate the volume of the dam. The following discussion took place in Group A:

A4: *"Ma'am, please explain the 70% and 15% stuff?"*

R: *"What about the 70% or 15% do you not understand?"*

(Students experienced difficulty to understand the fact that only 70% of the energy used to fill the dam could be used to generate energy. They insisted that energy 'cannot be lost').

A4: *"How can I increase the output energy if it only uses 70%? Can we not let it use all 100%?"*

R: *"Any ideas on this?"*

A1: *"If energy cannot disappear, it needs to be somewhere ..."*

R: *"So ... where is it?"*

A1: *"I don't know, but ... it has to be transformed in another kind of energy..."*

A1: *"What about the kinetic energy??"*

At this stage the researcher suggested that they consider all alternatives again. Student A2 suggested they look at a YouTube video clip again to search for possible clues.

A1: *"See, the water moves in the dam ... maybe this is the kinetic energy? Can it be 30%? Let us assume so ..."* (First assumption noted explicitly in this activity)

A2: *"OK then ... we need to find the PE in the dam, then take 70% of it, this is the energy we have at the moment. Now we have to find 15% of this 70% and add it to the 70% . This is the power they want to generate, I think..."*

A2: *"So ... we need 70 + 15 = 85% of the current energy in the dam?"*

Group A still struggled with the percentage increase: (They did not realise that 15% increase does not equal 85%)

R: *"Can you explain this 85% to me?"*

A2: *"Yes Ma'am, if we increase the 70% with 15%, we add 15% to the 70%"*

A4: *"No, I don't know, but it seems wrong. If you increase 70 by the number 15, you get 85. But, it you increase 70 by 15%, you get 70 * 1.15 = 80.5"*

(Student used his calculator)"

R: *"Can you explain this to me? Maybe by an example?"*

A4: *"Not really, I remember the rule!"*

A3: *"I think I would like to explain it – you only increase the current number by a **proportion** – in this case 15%, which is 15/100 of the current energy."*

R: *"I want you all to consider the following: If the monthly rent for my flat is R3000, but the landlord want to increase it by 10% as from next month – How much rent will I have to pay next month?"*

Again, students' dependence on memorising facts came to the forth as A4 struggled to explain the 15% increase, but knew the procedure to increase 70 by 15%. The facilitator deemed it necessary to ensure that the students' foundational knowledge about percentage increases were sound, hence the discussion about percentage increases on monthly rent. After the discussion, Group A managed to engage further in this activity, and transferred the principles from the rental increase to this situation. All the students in the group worked on the problem and were able to explain their results correctly. The group then proceeded to determine the volume of the dam. When the three groups engaged in the dam's volume, none of them could remember the formula for determining the volume of a trapezoidal shape. Some students referred to the Internet and found the formula for calculating the volume, others were still unsure of how to proceed.

Group B's discussion on how to calculate the dam's volume was as follows:

B4: *"The PE is given by the formula mgh, and we do not know what m is."*

B3: *"Try using the formula for Volume."*

B4: *"OK, then Volume = Mass x 1000 – this does not really help me a lot."*

B1: *"No man, we have to calculate the volume of the dam before we can find the mass."*

R: *"Is there a way that you can try to simplify the shape of the dam as indicated in the text?"*

B1 suggested they split the shape in a rectangular block and a prism, then calculate the volume of each shape and add it together. After numerous attempts, all 3 groups obtained a solution method (mathematical model) for determining the volume of the dam. All groups carried on and calculated the mass of the water by applying the given formula. When calculating the existing PE, all groups used '*h*' as the actual height of the dam, no one considered the meaning of '*h*' in terms of a tidal dam. The researcher decided to show them video clips on how hydroelectricity gets generated by a waterfall as well as tidal power and asked them to explain the differences and similarities in how to calculate PE within their own groups.

R: *"Are there any similarities or differences between the calculation of the height to find the PE of a waterfall and of a tidal dam?"*

The researcher did walk-throughs and observed their thinking:

Group B: *"The formula for PE is mgh."*

B3: *"With a waterfall, all the water runs from the top to the bottom, while the water that sits in the middle or bottom of the dam does not travel the same height as the water on top."*

R: *"So ... What about h?"*

B2: *"I guess we have to calculate the PE of each drop and add it together? Oh no, this will take forever!!!"*

B1: *"Well, if we have to add all the drops' PE together, maybe it is the same as averaging it out? Then we take the drop in the middle of the dam and use that as h ... Which means we should use average height to find the PE."*

B3: *"This then means that half of the volume must be above the average height and the other half below the average height ..."*

B1: *"So ... we cannot use 5 as avg height, we must first get the volume of the dam, then split it in half, then use the volume formula to determine the average height?"*

Group A's discussion:

A1: *"PE = mgh. On the video clip all the water came from the same height, which is different from the tidal dam. Here some water drops travel a bigger vertical distance than other drops. We then need to consider the average height of the dam."*

A3: *"In the case of the waterfall, all the drops travel the same vertical distance – not with the tidal dam though ... You are right, we need to find the average!!! There must be a specific height where half of the water is above it and half of the water underneath it? Let's try ..."*

Group A realised that they had to consider the water's average height, but were not sure as to how to progress further and decided to assume the dam's average height to be 7 meters, rather than to calculate it. What surfaced from these activities, was the students' willingness to create situational models and to *make assumptions* to simplify the process; this was an activity that they have not considered before. Students worked on various solution paths; Group C decided just to halve the height of the dam, while Group B realised that they had to consider the volume of the dam to determine the average height of the dam. Zbiek and Conner (2006:93) emphasise the importance of assumptions, as each modeller relies on his or her unique set of knowledge, intuitions and conceptions about the mathematics and the real-world, which in turn influences his or her interpretation of the situation, as well as the use of mathematical ideas. The non-linear modelling path is prevalent during mathematising, due to the multiple journeys between conditions and assumptions and properties and parameters (Zbiek & Conner, 2006:102). The students continually moved forwards and backwards between the original situation and the specific real-world problem that needed to be solved.

After calculating the average height of the dam, the researcher asked each group to present their solution methods for determining the average height to the class. Group C then realised that they must either simplify the problem to only allow for a rectangular shaped dam, or to adjust their formulas. They decided to simplify the problem by changing the shape of the dam, not realising that it could potentially make a huge difference to the final outcome. Even though the students started this activity by displaying a distinct growth in abilities to internalise and structure real-world problems, their attempts to symbolise and adjust the situational models progressed slowly and they frequently applied methods that would only lead to partially correct solutions. Their limited mathematical knowledge was again exposed whilst trying to adjust and solve their mathematical models. Explanations of their mathematical models also seemed to be redundant at stages, some students struggled to explain how the potential energy would be influenced by the shape of the dam, the location, and the shape of the anticipated excavation.
The activity ran over two sessions (two and a half hours per session). The last session focused on the generalisation of their solutions, which proceeded far smoother than anticipated. All the groups were able to explain their models, and could also describe how changes in the mathematical model

would affect changes in the real-world solution. This unexpected development could be contributed to the fact that the second session focused in-depth on the vertical mathematisation processes. Though students only exhibited little progress regarding development in symbolising and adjusting competencies, the time spent on these activities allowed for the students to gain a clear understanding of their own mathematical models as they were able to more comfortably address generalisation issues. Also, this activity was the students' third MEA and the importance of creating a solution procedure rather than a specific solution for the client, was discussed in all the previous modelling sessions. It seemed as if the students slowly developed an understanding of the importance to create a model for a generalised real-world situation and not only a specific solution. Students' evidence of engagement in the modelling tasks increased substantially from the first activity, and they commented frequently about their admiration for the 'practical mathematics' which they have experienced in the mathematics modelling classroom.

The following tables (Table 5.8) graphs (Figures 5.23 and 5.24) represent the students' changes in competence development as discussed above:

Table 5.8 - Results of competence assessment - MEA-3 - Tidal Power Task

Competence assesment of the Tidal Power Task:

Mathematical modelling competencies	
Internalising	1.34
Interpreting	1.28
Structuring	1.30
Symbolising	1.50
Adjusting	1.38
Organising	1.19
Generalising	1.15
Management	1.30
Communication	1.33
Responsibility	1.24

Engineering technician competencies	
Define, Investigate & Analyse Problems	1.31
Design/Develop Solutions	1.31
Comprehend and Apply Knowledge	1.39
Recognise and Address Factors	1.33
Sound Judgement	1.30
Management	1.30
Communication	1.33
Responsibility	1.24

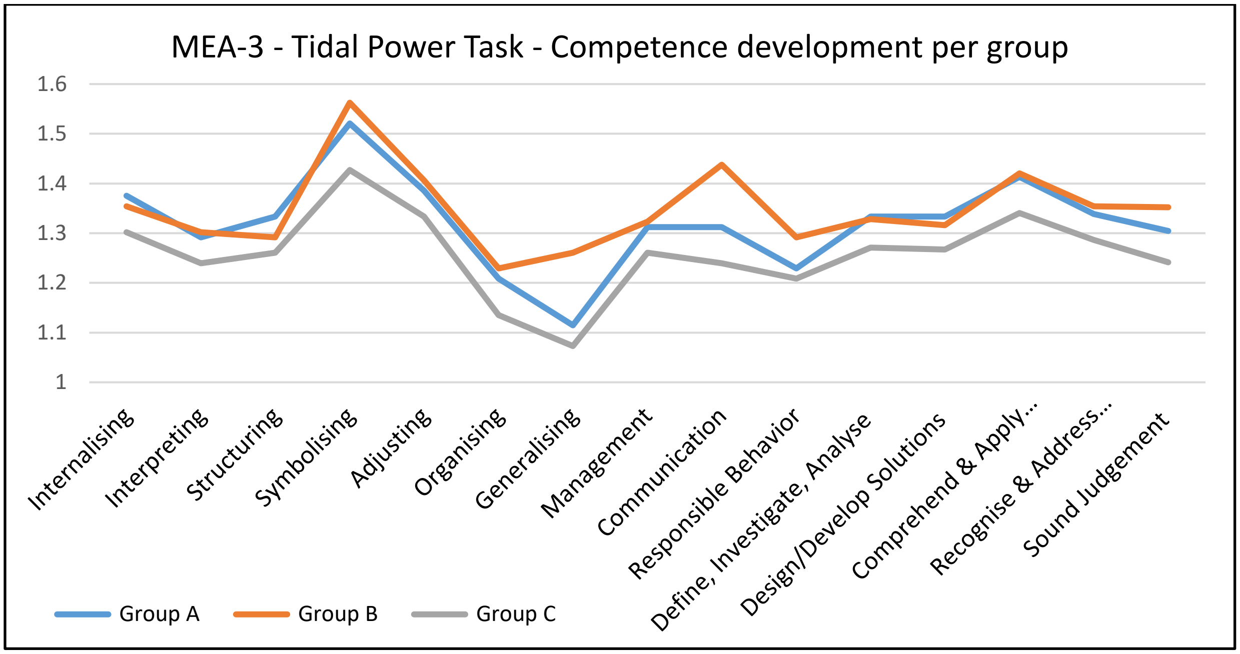


Figure 5.23 - Competence development per group - MEA-3

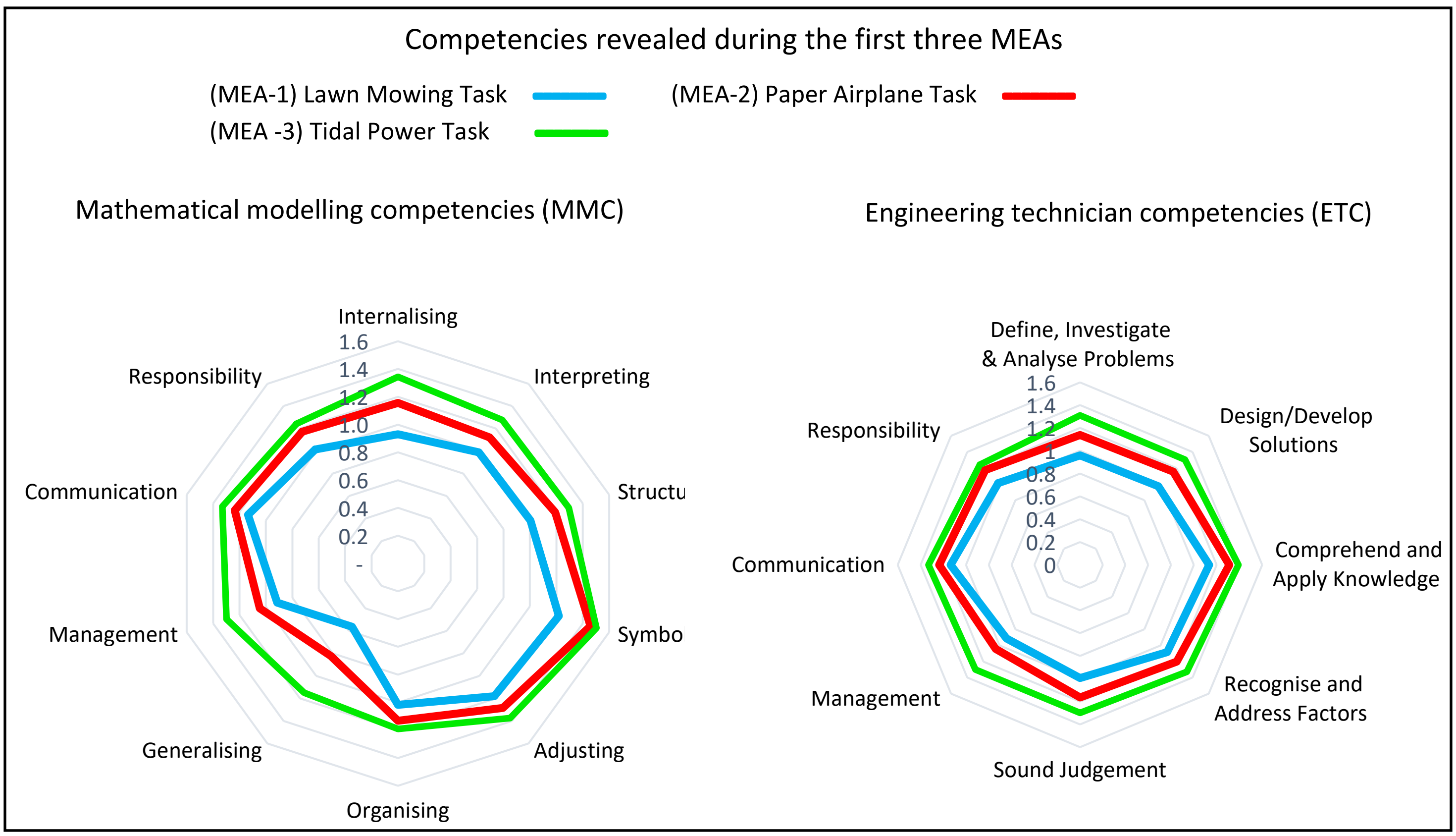


Figure 5.24 - Whole class competence development – MEA-1 to MEA-3

This third MEA allowed the groups to engage actively in mathematical analysis. Resnick (1988:7) describes mathematical problem-solving as a process of building a mathematical interpretation of a situation, and thereafter mathematising the interpretations. The students were required to do online research to enhance their understanding of how tidal pools work. The research helped them to understand and interpret the context, to find ways for calculating the dam's volume, and to discover the mathematics to be used. None of the students were familiar with tidal pools, which explains the additional time that was spent on context familiarising. Furthermore, they had to actively analyse important equations that they previously only applied to a very limited extent in Physics lectures. The student groups spent vast amounts of time on researching, and persistently searched for solutions to determine the optimum excavations for the dam. There strategies were adapted frequently when they were not satisfied with their results after reflecting on the meaning of their solutions. During this MEA, the students continued to display consistent improvements in their abilities to solve real-world problems, and all groups remained focused on their goal of finding an effective design to expand the dam and thereby increasing the potential energy output.

Piaget (1896-1980) promoted the idea that human intellect develops through adaptation and organisation. Adaptation refers to assimilation of external events into thoughts, as well as the accommodation of new mental structures into the mental environment (Piaget, 1964:176) (Section 2.2). The students' experiences while interpreting the situation, led to conceptual reorganisations. As mathematical modelling is concerned with constructing models from messy real-world situations, students learned to generate mathematical constructs through developing ways of thinking that caused their previously existing conceptual systems to be integrated, differentiated, extended, or refined in significant ways. (Lesh & Clarke, 2000:142). Section 2.5 explains the cyclic nature of MEAs. Students repeatedly revealed, tested and refined their ways of thinking. As they progressed through the iterative sequences of interpretation-development cycles, they started to apply more sophisticated ways of thinking and focused on relationships, patterns or trends in the data. These series can be documented to reflect Piagetian-like stages of concept development, where continual tension between accommodation (a state when one modifies one's viewpoint) and assimilation (integrate new ideas toward the solution of the problem) characterises the process of learning (Harel, 2008b:897).

The above graphs (Figures 5.23 and 5.24) signify slow, but consistent improvements in all the competencies as identified in Section 3.7. Particular progress was noted relating to the competencies of generalising (1.15) and management (1.3). The YouTube videos and additional time spent on researching tidal pools, could be two of the contributing factors that supported growth of these competencies, as the students were able to gain a reasonable understanding of the problem content. Generalising is concerned with vertical mathematising and denotes activities of more advanced mathematical thinking (Section 5.3). Tall (1991:20) explains that mathematical thinking progresses logically "from describing to defining, from convincing to proving". The student groups can thus not justify their solution methods prior to describe and mathematically define the problem situation. Apart from generalising competencies, the continual exposure to the context increased the groups' willingness to approach the task with positive expectations about finding solutions strategies, and thereby improved their management competencies.

5.2.5 Design cycle 4 – Product Coding Task

This activity, adapted from Galbraith (2009:15), required the students to verify and correct barcodes for a large supermarket by building and testing mathematical models, to explain and judge the validity of different product codes in terms of both EAN and ISBN systems (Appendix N). Even though this activity contained more structured questions than the previous activities, it still adhered to all the design principles of MEAs: the model construction, reality, self-assessment, model documentation, sharable and reusable and effective prototype principles (Sections 2.5.1 and 4.3.1.5, Figures 4.2 and 4.3).

Background

The idea of placing a barcode on a product originated in the 1930's, but the first barcode reader was not built until 1952. In 1974, the first retail product (a packet of chewing gum) was sold using a barcode reader at a supermarket in Ohio. Barcodes allow for instantaneous processing of information by computers and is used almost all over the world. South Africa uses the European Article Numbering Code containing 13 digits (EAN-13), which is one of the most commonly used systems worldwide. The following figure represents and ISBN-EAN barcode.

A sample barcode:

0 36000 29145 2

The following codes were respectively taken from an iodised salt product and an instant coffee product, sold in a Spar Supermarket.

600 102102102 3 (Wellington's tomato sauce)

600 100715730 2 (Cerebos iodised table salt)

The left-most digit (called the 0th digit) together with the next 1 or 2 digits (called the 1st and 2nd digits) indicates the country of manufacture (for example, 600 and 601 represents South Africa, 76 represents Switzerland and 94 represents New Zealand).

The next 9 or 10 digits (depending on how many digits are used for the country code) identify the manufacturer, as well as the specific product. The final number is a *check digit*. All products that are manufactured by a specific company, will use the same manufacturer code.

When the label is scanned, the barcode identifies the item of which the price is stored in the retailer's database. When a barcode is read, the computer will verify that the check digit is correct, before processing the number. If an error is detected, the computer will indicate an error.

This can happen for example, if a paper label on a can is distorted, as it can cause a digit to be misread. A checkout attendant will then have to manually enter the barcode. This procedure is also subject to error.

The check digit works as follows: Using the first 12 digits in the code, the check digit satisfies the condition that:

3 × (sum of even digits) + 1 × (sum of uneven digits) + check digit is divisible by 10. (Here 3 and 1 are referred to as *weights*).

Figure 5.25 – Background information on EAN product codes, as adapted from Galbraith (2009:15)

Background to ISBNs (International Standard Book Number):

Every new *book* that gets published, gets allocated with an ISBN, which is a unique identifier. Each ISBN has four parts, which are separated by blanks or hyphens.

1. A group identifier (this identifies the particular country participating in the ISBN system)
2. A publisher's identification number (variable length)
3. A title number (variable length)
4. A single check digit (0, 1, 2, … 9, X)

For example, paperback editions of the trilogy *Lord of the Rings* by JRR Tolkien, published by HarperCollins Publishers, have the following ISBNs:

Fellowship of the Ring	0 26110 357 1
The Two Towers	0 26110 358 X
The Return of the King	0 26110 359 8

Because every book is uniquely identified by its ISBN, it is important to guard against errors (for example, transcription errors) that could have serious consequences for ordering and charging, specifically with automatic coding procedures. The check digit is assigned to take care of this and is calculated as follows (using *Fellowship of the Ring* as an example):

$10\times0 + 9\times2 + 8\times6 + 7\times1 + 6\times1 + 5\times0 + 4\times3 + 3\times5 + 2\times7 = 120$ + check digit must be a multiple of 11. Thus the check digit in this case equals 1.

Note that, because of the division by 11, the check digit can sometimes turn out to be 10. Because 10 cannot be represented by a single digit, the Roman number for 10 (**X**), is used to denote the check digit. An example of a Roman check digit appears in the barcode for *The Two Towers* above.

Bookland and EAN-ISBN codes:

EAN and ISBN codes come together in the publishing industry. Since book publishers commonly publish in a variety of countries, the book industry has designated and imaginary 'country' called Bookland, as the wonderful place where all books are produced, together with its own special prefix '978'. An EAN-ISBN code for a book starts with 978 and follows with the first nine digits of the ISBN, concluding with a check digit calculated according to the EAN rule. Thus only rarely will this check digit be the same as the one in the ISBN. Commonly, a second shorter code is printed as well, which gives the price in whatever currency is appropriate.

Figure 5.26 – Background information on EAN-ISBN product codes, as adapted from Galbraith (2009:15)

Task:

A large supermarket, ABC Bargains, experienced huge problems with the printing of their barcodes. They called the help of your development team to assist with some specific issues that they picked up. The manager asked you to assist and correct (where necessary) the following, and to provide a thorough explanation on each of these matters:

1. Verify the check digit for the other two titles of the *Lord of the Rings* trilogy.
2. The manager wants to change the ISBN code for the *Lord of the Rings* trilogy to EAN-ISBN codes. Find the check digits for the EAN-ISBN codes that would be allocated to the *Lord of the Rings* trilogy.
3. An ISBN was incorrectly recorded as 540 12156 5 by omitting the group (country) identifier. Correct the code by adding the missing digit.
4. A printing flaw caused a digit in an ISBN to be illegible. The number appeared as 0 853□2 456 6. Find the missing digit to correct the code.
5. In copying an ISBN, two of the adjacent digits were accidentally transposed, and the code was printed as 0 340 39155 X. Find and correct the error.
6. The manager of the store is concerned that the printing company printed the first three digits of one of their products in the wrong sequence. Prove that, if *a*, *b*, and *c* are digits such that 1 867751*ab c* is a correct ISBN, then 1 687751*ab c* cannot be an ISBN.

A report needs to be submitted to the manager and all of the above issues need to be explained. The report needs to include all the necessary calculations and explanations to assist them in dealing with future printing problems.

Figure 5.27 – Product Coding Task, as adapted from Galbraith (2009:15)

5.2.5.1 Planning for the Product Coding Task

Although relative straightforward and routine arithmetic procedures were required, the error-correction activities were anticipated to challenge the students to delve deeper into problem-solving. The initial routine work was hoped to boost student confidence and motivate them to carry on with the more challenging aspects of this activity. The students were also required to devise numerical proofs which could uncover important information about their abilities to strive towards generalising of mathematical solutions.

Readiness questions were prepared to guide the students towards understanding the text, as suggested by Chamberlin and Moon (2005:39):

- Who is your client?
- Describe in your own words what the problem is.

- Explain in words how you want to attempt this problem.
- What are the key decisions that you have to make?
- Which information is important to start with, and which will you ignore?
- Do you have enough resources to be able to work towards a solution?
- Explain your solution to me and justify your decisions.

It was anticipated that not all the students would automatically grasp all required aspects, therefor the researcher prepared questions to guide them towards gaining further understanding of the problem posed, based on the work of Wake et al. (2016:252):

Table 5.9 – Anticipated Difficulties – Task 4 (Product Coding)

Anticipated Difficulties	**Suggested Questions and Prompts**
Students start detailed calculations before planning an approach. For example, they start with the first numbers, then do meaningless multiplication and division with the following.	Describe in words a plan for tackling this problem. What are the key decisions you have to make? Which information are you going to focus on at the start; which will you ignore (if any)?
Students do not understand the concept of the problem.	What is your main objective when trying to solve the problem?
Students ignore one or more constraints.	What are your assumptions, if any? What do you plan to do to simplify your problem?
Students do not justify decisions made. For example, they state a solution with no explanation.	Why have you chosen to apply this formula? How can you be sure this is the best solution?
Students leap to conclusions.	Have you taken all the issues into account? Does your solution make sense if we consider as different, but similar kind of problem, e.g. the value of one of the digits differs?
Students do not grasp the meaning of their calculations. For example, students might perform a sensible calculation but not understand what their answer represents.	What does this answer represent? Did you answer the clients' question?
Students only write numbers with no justifications.	Where have these figures come from? Do you know what they represent? Are you able to justify why you have used these numbers?

Anticipated Difficulties	**Suggested Questions and Prompts**
Students might get confused when determining the validity of the check digit with EAN codes and ISBN codes.	They need to be able to explain the answers of their calculations meaningfully, they need to be clear on what their solutions represent.

5.2.5.2 Implementing the Product Coding Task

To build from the students' zones of proximal development, the facilitator/researcher had to guide them once again to reinvent their own constructions. Direct instruction was prevented at all times. The facilitator/researcher showed the students barcodes of a few products, and asked them to describe their understanding of how barcodes are designed and what the numbers represent. It was anticipated that the majority of students would understand that barcodes primary convey information about the product and not the price.

Implementation of the MEA followed the following format:

- Informational texts describing the background context were handed out to the students for reading, where-after the facilitator led the class discussion.
- Students worked in three groups of four students each throughout the modelling activity.
- The readiness questions were prepared (see above), based on the students' anticipated understandings and misconceptions. Class discussions (either whole class or group discussions) ensured understanding of the task.
- Further informational texts, including the letter from the client, were handed out to the students.
- Brainstorming among the groups were continually promoted, especially during the initial stages where the students needed to find ways of solving the client's problem.
- As the teams selected their own ideas to be developed further, they were motivated to create procedures to solve the client's problem.
- Guiding questions were asked by the facilitator while the students created their procedures to address relevant issues that arose.
- The teams tested, evaluated and revised their procedures as necessary, and presented it to the class, taking turns to lead the presentations.
- Peer critique and discussions followed the presentations.

- Students were handed a Status Update Report (Appendix A), a Quality Assurance Guide (Appendix C), a Students Reflection Guide(Appendix E), and a Group Reporting Sheet (Appendix F) to monitor their progress, while the facilitator used the Researcher Observation Guide (Appendix B) and the Group Modelling Competency Observation Guide (Appendix D) in conjunction with audio and video recordings, walk-throughs, informal discussions, field notes and memos, to ascertain the students'competence development.

The modelling activity was split over two weekly sessions of two hours each. This MEA differed from the previous MEAs, as they had to answer six distinct questions (see previous section). The students' previous modelling attempts, as well as existing literature (Haines & Crouch, 2013:4), indicated that students experience fundamental difficulties during the initial phases of the modelling process - understanding and interpreting the real-world situation. This aspect motivated the researcher to further investigate the students' abilities to make sense of the real-world problem, and only handed the background text about the workings of product codes to the students, dissimilar to previous MEAs where the students received the background information and the tasks on commencing with the MEA.

The actual task as requested by the client, was only handed out during the second session (the following week), which meant that the students had two hours to familiarise themselves with how EAN and IBSN barcodes work. While the students tried to investigate, understand, interpret and communicate the context to one another in their groups, the researcher gathered data by means of audio/video recordings, walk-throughs, informal discussions, field notes and observations. The aim of the first session was to guide the students through the processes and workings of barcoding as to allow all students to gain an adequate understanding of the context prior to answering any questions from the client. All three groups were engaged in building situational models to ascertain their understanding of barcodes. Group discussions explicated the students' understanding.

Due to the many different questions that needed to be answered by the client, the researcher did not use the Status Update Report halfway through the MEA, but requested it from the various groups at different times while they were engaged with different aspects of the task. The researcher/facilitator was also constantly involved in informal discussions with the groups.

5.2.5.3 Reflecting on the Product Coding Task

Students' poor mathematical abilities were exposed when all the groups misinterpreted the meaning of *even* and *uneven* digits in the text. Within their groups it became clear that the students regarded even digits as digits representing even values, rather than the positions of the digits. The researcher did not anticipate this misunderstanding. Figure 5.28 below denotes Group A's (mis)understanding of even and uneven digits:

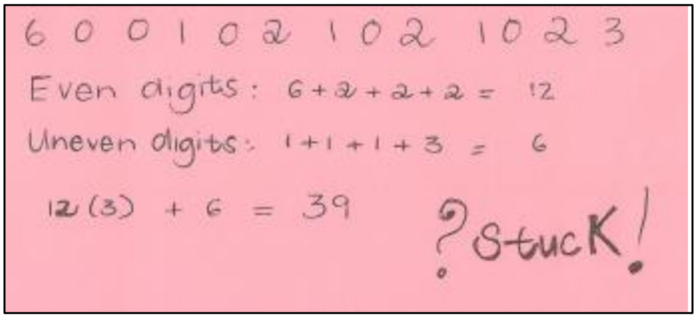


Figure 5.28 – Group A's (mis)understanding of even and uneven digits

On reflection, the researcher should perhaps consider adapting the text to allow for easier reference to the positions rather than the values of the digits. After explaining (in a whole class discussion, as this problem seemed to be a difficulty for all three groups) that even and uneven digits represent ordinal numbers (the position of the digits) rather than cardinal numbers (the value of the digits), the groups engaged in further exploration of the text. After this discussion Group A adjusted their situational model, as denoted in Figure 5.29 below:

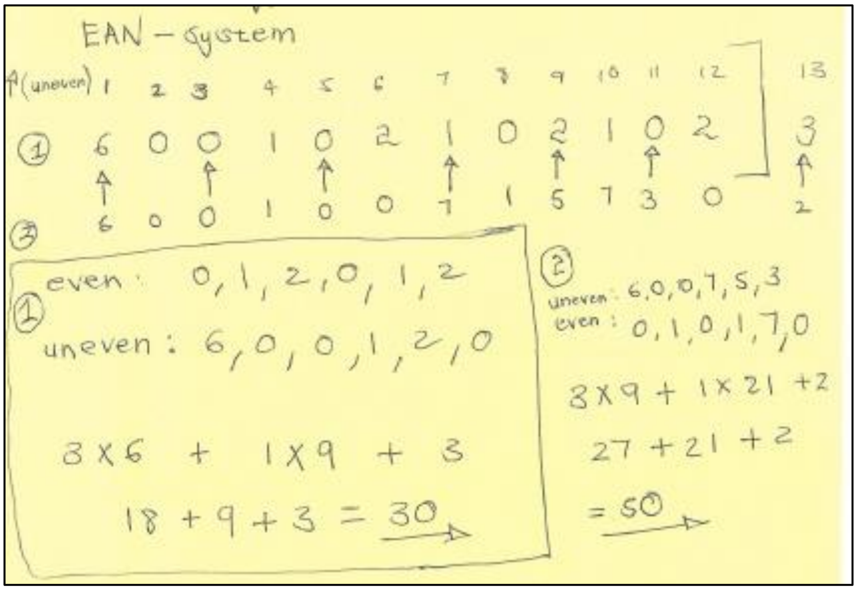


Figure 5.29 - Group A's adjusted model to explain even and uneven digits of bar codes

Group B proceeded to calculate check digits, and presented their initial model as follows (Figure 5.30):

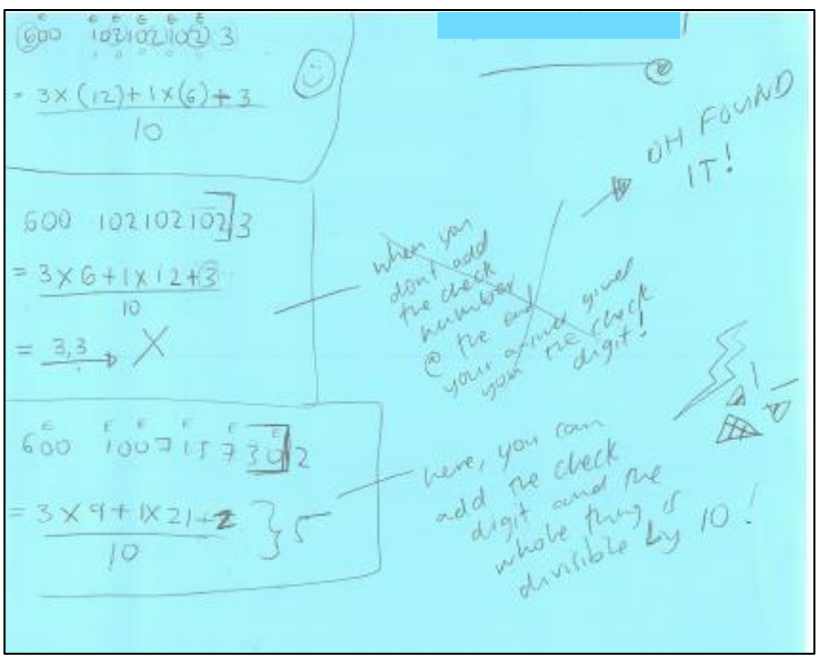


Figure 5.30 - Group B's representation of product codes

Once they were satisfied with their situational model, Group B decided to pose problems to one another, to further enhance their understanding of the content. (At this stage, they still did not receive any problems to be solved by the client):

B2: *"I wonder what will happen if we say, anyone of the digits were scratched out – will our model allow us to find the correct digit?"*

This was the first time that students spontaneously generated possible problem questions relating to the text. In discussing this question, the students engaged in the mathematical analyses and a productive discourse took place in this group as the possible solution methods were discussed and justified among the group. Adjusting, organising and management competencies improved substantially since the first MEA, as they were continually engaged in adjusting and reflecting on their own models.

Group C also decided to test their understanding, and searched for objects in the classroom to analyse the barcodes.

C1: *"The barcodes for the A4 Mondi Rotatrim paper is 6003977000602. This number is very different from the black whiteboard marker (Artline 500A) which is 4974052809743. But, if I compare the black whiteboard marker with a blue Artiline 517 marker, only the last five digits differ."*

C3: *"Hmmm ... we know it is manufactured in the same country, both are pens, type and colour of the pens differ. Let's see if we compare it with pens of the same type, where only the colour differs."*

The students copied the codes of Artline 500A and Artline517 in a table, and used different colours to further develop an understanding of similar and different codes.

Group C represented the situation as follows (Figure 5.31):

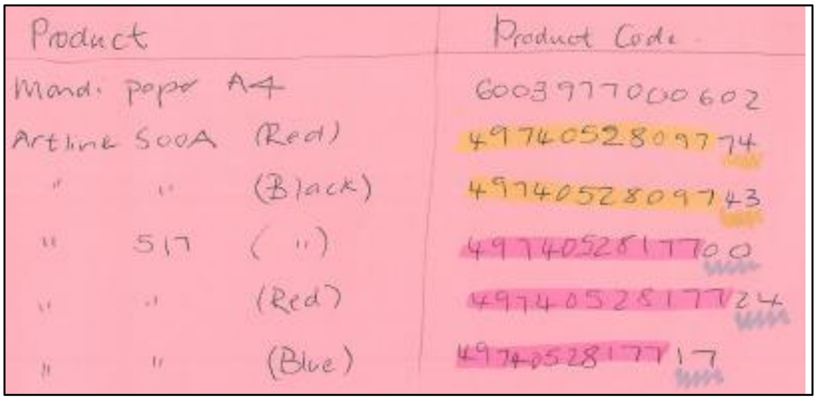


Figure 5.31 – Group C compared barcodes of various products

C2: *"Look here .... when there are huge differences, like paper and pens, the codes differ a whole lot. When the only difference is the colour of pen, it looks like it is only the last two digits that differ."*

R: *"So what does this tell you?"*

C1: *"I think it is because the barcode only represents the country and product, not the price. Prices vary from shop to shop. Similar products will then have kind of similar barcodes. Here we can see that, if only the colour changes from one pen to another, the last two digits differ, even though they would normally cost the same in a shop".*

Understanding and internalising competence development were noted here in all the groups, as they were able to explain their ways of thinking about the content and simplify the situation. Variables of interest were also recognised while they tried to gain deeper understanding of the content. The students managed to test and revise their own models while developing internalising, interpreting, structuring and adjusting competencies, which are important competencies to allow progression from horizontal to vertical mathematising. An interesting observation was the groups' spontaneous movement between horizontal and vertical mathematising. The groups were able to create situational models, and by suggesting their own problems to their teams, they rearranged and adjusted their models accordingly (vertical mathematising), leading to improved generalising competencies.

By inventing their own problem questions, Group B signified a satisfactory understanding of how the EAN digit system functions. The students were engaged in meaningful discourse in their respective groups. Their understandings about the meaning and purpose of product codes, forced them to use their common-sense knowledge and experience about the real-world as ‘helping agents’ to solve the real-world problem mathematically (Bonotto, 2013:400). This intertwinement between everyday life reality and classroom mathematics allowed for active engagements, as the students instinctively took part in communicating their understanding of the problem situations to one another.

The success that students experienced in teamwork during the previous weeks' activities, served as a confirmation to all three groups to regard brainstorming and sharing of ideas as vital. All three groups decided to read the passage and discuss it afterwards within their groups. While reading about the ISBN barcoding systems, a confusion in all three groups relating to the **Roman X** emerged. Even though they reread the paragraph explaining the concept a few times, Group B wanted to treat **X** as an unknown variable, rather than a number with a value of ten. Their presentations also indicated this misconception, as they tried to move **X** to be the subject of the equation:

$X = 10(0) + 9(2) + 8(6) + 7(1) + 6(1) + 5(0) + 4(3) + 3(5) + 2(8) + 1(????)$

$X = 122 + Y$

The researcher realised (unexpectedly) that the students struggled to think of X in mathematical terms anything other than an unknown variable which needs to be solved. The following group discussion took place:

R: *“What is the IsiZulu word for the number ‘ten’?”*

B2: *“Kugishumi”*

R: *“Please write the two words on the white board?”*

(B2 wrote TEN and KUGISHUMI on the white board.)

R: *“Is there any difference in the meaning between TEN and KUGISHUMI?*

B2: *“No Ma’am, same thing.”*

R: *“So, even though the words look different when you write it on the board, they mean the same thing. How can you use this analogy to explain the X in the barcode?”*

B3: *"I can see where this is going – Roman ten does not look like decimal ten, but means the same thing. But ... if we replace X with 10, we have added another digit. This will then be wrong ... I think.."*

B1: *"'So, why do we not scratch the X out and replace it with a 10 and then PRETEND that 10 is only one digit? We can put the 10 in brackets and pretend it is only one character, not two."*

This representation allowed the group to excel their understanding, and they were also able to carry on and – by trial and error – convert ISBN barcodes to EAN barcodes successfully. The researcher did no further questioning, as their verbal and written work clearly indicated that they progressed towards a deeper understanding of the context, as denoted in the following illustrations (Figure 5.32). Note the bracket around the **10**, and around the **X**, to illustrate one character:

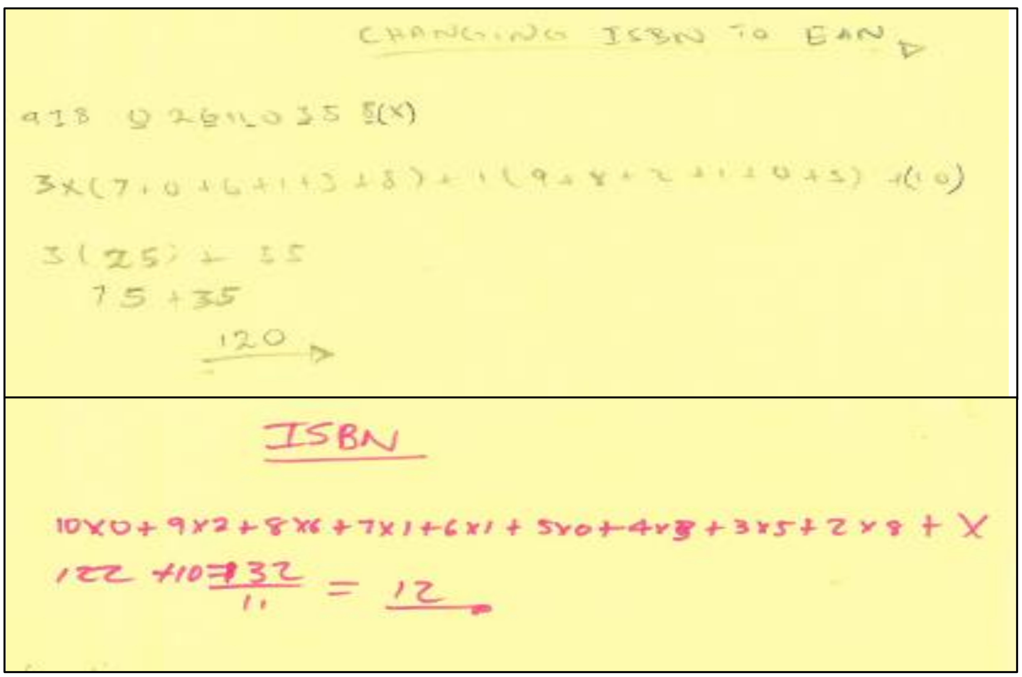


Figure 5.32 - Group B represented the Roman X

The client's letter was handed out in the beginning of the second session. After evaluating their work of the previous week, the students were enthusiastic to deal with the client's letter. With all the students actively engaged in the tasks, the groups were able to complete the task within 80 minutes. The last half an hour were used for group presentations. All the groups seemed to enjoy the presentations, and they were able to explain the various types of product codes without hesitating about what they were doing. They definitely experienced a feeling of *working with a sense of direction*, and they were motivated to present their solutions in a clear and concise manner. Competencies that were revealed during this activity, are indicated in the following table (Table 5.10) and accompanied graphs (Figures 5.33 and 5.34):

Table 5.10 - Results of competence assessment - MEA-4 - Product Coding Task

Competence assessment of the Product Coding Task:

Mathematical modelling competencies	
Internalising	1.58
Interpreting	1.43
Structuring	1.49
Symbolising	1.60
Adjusting	1.63
Organising	1.46
Generalising	1.28
Management	1.67
Communication	1.47
Responsibility	1.53

Engineering technician competencies	
Define, Investigate & Analyse Problems	1.50
Design/Develop Solutions	1.50
Comprehend and Apply Knowledge	1.58
Recognise and Address Factors	1.53
Sound Judgement	1.48
Management	1.67
Communication	1.47
Responsibility	1.53

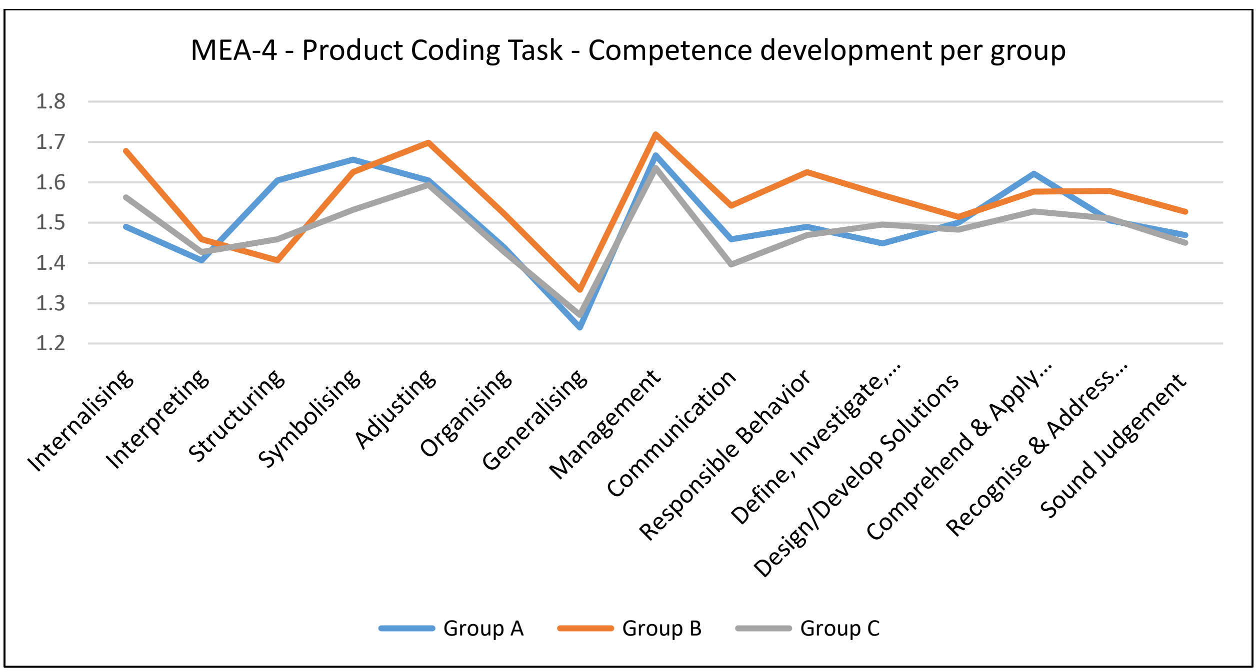


Figure 5.33 - Competence development per group - MEA-4

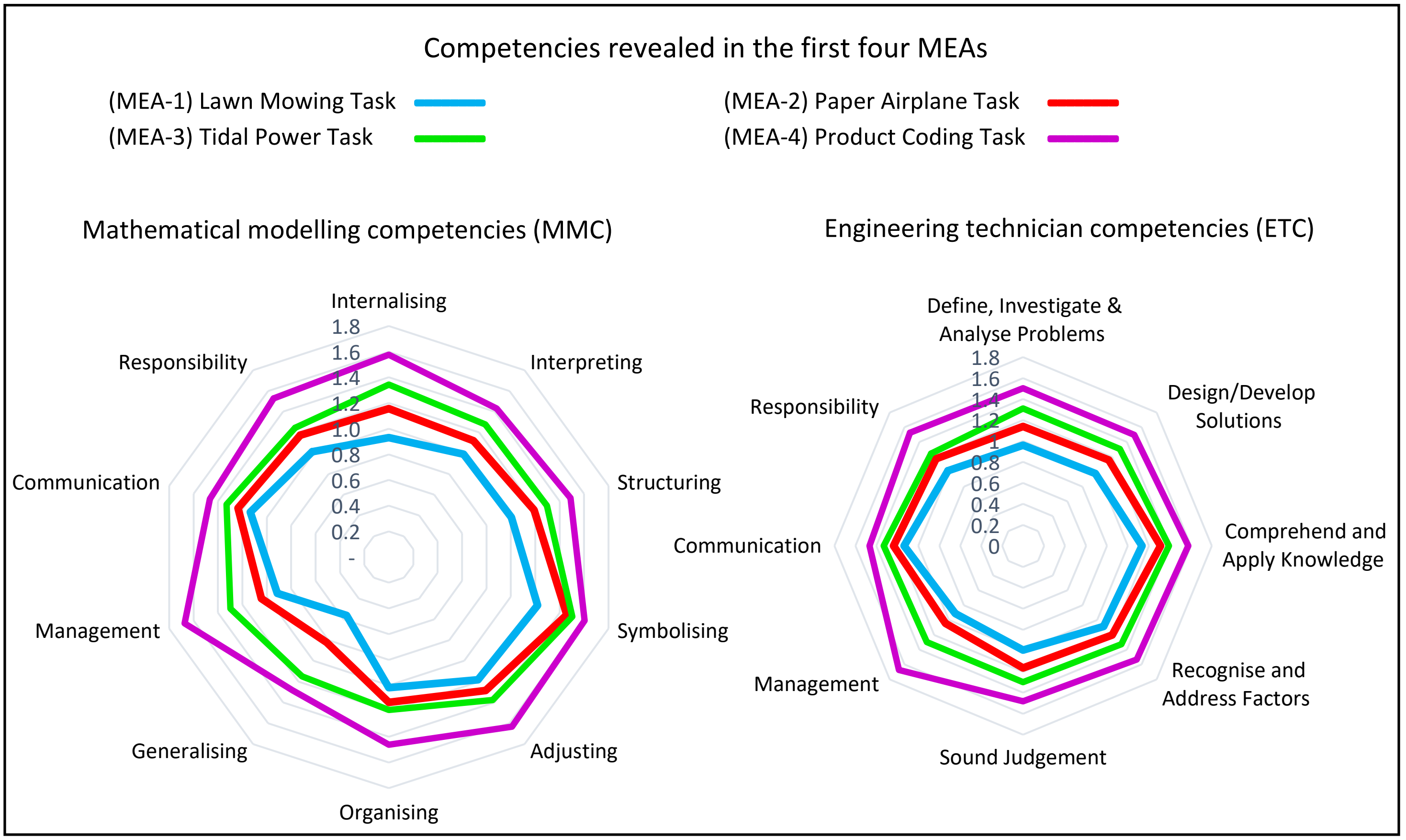


Figure 5.34 - Whole class competence development – MEA-1 to MEA-4

From the above graph (Figure 5.34) denoting the students' mathematical modelling technician competence development during the past four MEAs, the following mathematical modelling competence categories displayed the biggest improvements: internalising, adjusting, organising, management, and responsible behaviour.

The student volunteers that took part in this study were homogenous, as none of the students met the requirements for studying civil engineering. Their matric marks for mathematics ranged between 40% and 49%. By the time of engaging in this fourth MEA, the students were already familiar with constructive classroom norms that promoted effective discourse, which resulted in more spontaneous and constructive classroom discussions. There were no major discrepancies in the competence development between the three groups. The researcher contributed this aspect to two reasons: Firstly, all the students came from a background with mediocre academic achievements (which can be contributed to many various aspects), and secondly, due to their continual engagement with one another and the establishment of a constructivist classroom culture where effective discourse prevailed, they were all willing to share their thoughts openly and honestly (increased communication competence). They were all willing to learn from one another, which resulted in increased management and responsible behaviour competencies. Growth in these meta-cognitive competencies may also be contributed to the fact that the students were under no pressure to answer particular questions from the client during the first session, but they were only required to familiarise themselves with the content.

Continual exposure to mathematical modelling activities over a period of time, allowed the students to gradually develop the abilities to organise themselves within their groups, and to internalise, adjust and organise their understanding of the content through building models. The above graphs show that this development grew stronger during each subsequent activity. Hamilton et al. (2008:5) noted that MEAs assist students to become better problem-solvers, especially the students that do not perform well in traditional mathematics curriculum settings, which is characteristic of the participants in this study. This was the first activity where the students were allowed to use one full session to just gain an understanding of the content. This additional time could also contribute to the reason why the students displayed a slightly deeper understanding of the problems posed than before. Group discussions were also more productive, because each student had a better understanding of his/her roles in the groups and shared responsibility to solve the real-world problem. As suggested by Yildirim et al. (2010:842), feedback was provided at various points during the solution process to assist the students in identifying and correcting possible misconceptions, and to guide them towards achieving the desired solution.

5.2.6 Design cycle 5 – Turning Tyres Task

The goal of this fifth MEA was to further enhance the development of engineering technician and mathematical competencies relevant to this study. The activity offered students an engineering problem where they had to work in teams to design a procedure for a client to select the best tyre material for a specific situation (Appendix O). The activity was based on the work of CPALMS, the State of Florida's official source for standards information and course descriptions. CPALMS was created by the Florida Center for Research in Science, Technology, Engineering and Mathematics (FRC-STEM) at the Florida State University (CPALMS, 2012).

Letter from the client:

BESTYRES
825 Long Drive Ave
Mkondeni,
Pietermaritzburg, 3200

Dear engineering team

Our company, “BesTyres” is responsible for supplying custom-made vehicles that are suitable and economical to use in various landscapes. We have recently received a request from Lesotho’s government to produce a tyre that is appropriate to be used on both off-road as well as on-road terrains. We need your team to develop a procedure to select the optimal tyre materials to suit their needs.

Please furnish us with a report that ranks your choices of material from best to worst and motivate your decisions in detail by providing procedures to us to be able to use in the future. The final cost of material for each tyre needs to be included. We are only concerned with the following sizes and aspect ratios:

Sizes: 200, 265 and 330

Aspect Ratios: 45% and 88%

Thank you,

John Car
“BESTYRES” President

Figure 5.35 - Letter from the client, Turning Tyres Task as adapted from CPALMS (2012).

Accompanied data sets:

Tyre material types data set:

Tyre Material Type	Durability (1-10)	Defect Rate* (%)	Performance (1-10)		Cost per Sq. Inch
			Off-road	On-road	
Material A	4	2.41%	6	6	ZAR 0.54
Material B	7	0.28%	6	4	ZAR 2.40
Material C	7	0.41%	3	7	ZAR 1.60
Material D	6	0.23%	8	3	ZAR 0.90
Material E	8	0.52%	5	6	ZAR 1.20

* The defect rate refers to the probability that a tyre will be defective. Upper limits come from the Firestone tyre recall in 2000 and other values were based on a report on defect rates.

Performance rates of change data set:

Measurement	Change	Durability	Performance	
			On-road	Off-road
Section Width	+ 10 mm	+ 0.1	+ 0.3	- 0.2
Aspect Ratio	+ 10%	- 0.2	- 0.2	+ 0.5

Performance: For every 10% increase in Aspect Ratio, the off-road performance goes up by 0.5 unit, and on-road performance decreases by 0.2. For every 10mm increase in section width, on-road performance increases by 0.3, durability increases by 0.1, and off-road performance decreases by 0.2.

Figure 5.36 - Datasets for Turning Tyres Task as adapted from CPALMS (2012)

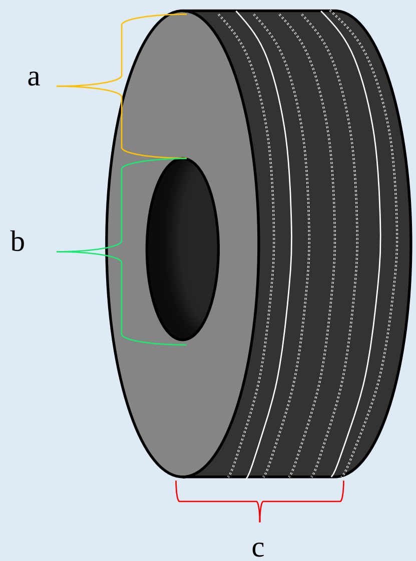
Further background:

Baseline tyre size:

All tyre material types were rated with a P225/60R16.5 tyre. This means the Section Width is 225mm, Aspect Ratio is 60% of that, and the Rim Size is 16.5" (Surface area of tyre = 1483.5 sq.in.).

- In the information for the measurements of the tyre, "P" denotes that it is a tyre for a "Passenger" vehicle. This will not necessarily be the case for your tyre and can be ignored.
- The number following it denotes the Section Width in mm (225). This means that the width of the tyre from inner sidewall to outer sidewall is 225mm.
- The next number (60) is the Aspect Ratio, which is given by the ratio of the Sidewall Height to the Section Width. It means that the height of the Sidewall is 60% that of the Section Width.
- Finally, the last letter/number combination (R16.5) gives the Rim Diameter in inches. This is the diameter of the hollow section in the centre of the tyre.

A diagram of a tyre and its parts:



a = Sidewall height
b = Rim diameter
c = Section width
Aspect ratio = $\frac{Sidewall\ height}{Section\ width}$ x 100

Helpful equations for new qualities of durability and performance, denoted by "Quality" (Q):

$$Q_n = Q_o + \frac{\alpha}{10}(SW - 225) + \frac{\beta}{10}(AR - 60)$$

Q_n = new value of quality (e.g. durability, performance)
Q_o = original value of quality
α = increment of change with regard to Section Width (e.g. durability changes by 0.1)
SW = New value of Section Width (200, 265, or 330)
β = increment of change with regard to Aspect Ratio (e.g. durability changes by -0.2)
AR = New value of Aspect Ratio (45 or 85)

Note: The conversion of inches to millimetres is 1 inch = 25.4 millimetres

Figure 5.37 - Further informational texts on Turning Tyres Task as adapted from CPALMS (2012)

5.2.6.1 Planning for the Turning Tyres Task

Big ideas that was anticipated for students to encounter, concerned the application of geometric concepts through modelling, the designing of a functional spreadsheet for calculating various scenarios, and to represent their solutions graphically in MicroSoft Excel (2003). Technology, mathematics as well as engineering concepts were integrated throughout this activity. It was anticipated that the students would struggle with the large amount of data, and the researcher planned an introductory session on MicroSoft Excel (2003) prior to embarking on the MEA.

The readiness questions that were prepared, were based on suggestions by CPALMS to ascertain the students' understanding of the problem context. These questions were asked after they read the passage and studied the data sets within their small groups. The readiness questions assisted the researcher/facilitator to ensure that they understood the problem before they started brainstorming and working with the data:

Readiness Questions:

1.1 What is the problem?

1.2 Who is the client in this problem?

1.3 Who are the customers that the letter refers to?

1.4 What does the client need?

1.5 What does the customers need?

1.6 What do you need to include in your letter?

Anticipated difficulties and accompanied questions were again predicted and are summarised as follows (adapted from Wake et al. (2016:252)):

Table 5.11 – Anticipated Difficulties – Task 5 (Turning Tyres)

Anticipated issues	Suggested questions and prompts
Students do not understand the real problem, thinking that they need to select the best tyre, instead of selecting the best tyre material for a specific tyre size and aspect ratio.	Describe in words why the BesTyres company need your help. What information do they want from your procedure?

Anticipated issues	**Suggested questions and prompts**
Students do not understand the concept of the problem.	What is your main objective when trying to solve the problem?
Students want to start detailed calculations before understanding the data provided. For example, they want to apply the equation provided without understanding the meaning of the data, especially what aspect ratio and section width mean.	Describe in words a plan for tackling this problem. What are the key decisions you have to make? Which information are you going to focus on at the start, and which will you ignore (if any)?
Students struggle to create a working model for determining the cost of the tyres.	What information do you need to calculate the cost of a tyre? Upon realising that they need to find the area, then: How can you use the cylinder formula in this problem?
Students ignore one or more constraint. For example, they calculate areas and ignore the fact that they work with mm and inches.	Does your solution make any sense? If you need to show me an area with this size – how big/small will it be? Is this a reasonable solution?
Students do not justify decisions made. For example, they state a solution with no explanation.	Why have you chosen to apply this formula? How can you be sure this is the best solution?
Students leap to conclusions.	Have you taken all the issues into account?
Students do not grasp the meaning of their calculations. For example, students might perform a sensible calculation but not understand what their answer represent.	What do these figures represent? How will you determine an optimal solution based on this data?
Students only write numbers with no justification.	Where have these figures come from? Do you know what they represent? Are you able to justify why you have used these numbers?
Students represent their solutions in graphs, but the graphs do not represent the real-world solution.	What are you trying to explain with your graphical representations? Do your graphs show the information that you want it to show? Can your client use your datasheet to predict optimal solutions in the future?

5.2.6.2 Implementing the Turning Tyres Task

Due to the vast number of calculations and demand to represent their solutions graphically, the researcher first introduced some of the basic features of MicroSoft Excel (2003) to assist the students in managing their calculation workload. They have been exposed to Excel (2003) before, but on a very limited scale. The classroom was therefore moved to one of the computer laboratories to allow all students access to computers. As anticipated, the students displayed a very basic understanding of MicroSoft Excel (2003). They were only able to do elementary computations, but with guidance, they gradually managed to create graphs that represented the data from tables that they created. The researcher taught them how to design spreadsheets when working with large volumes. Chao, Empson and Shechtman (2013:557) comment that students' understanding of mathematical constructs increase when they engage with technology to expand the variety of their representations.

Guided questions were asked throughout the MEA, and feedback was provided to all groups to ensure that the students grasped the necessary concepts and to address possible misconceptions. Upon completion of the activity, the students had to present a calculation spreadsheet with accompanied graphs, and the groups had to explain their solutions orally.

Implementation of the MEA followed the following format:

- An introductory session to MicroSoft Excel (2003) familiarised the students to design tables, create and copy formulas, and produce accompanied graphs for representational purposes.
- Informational texts were handed out to the students for reading, where after the researcher/facilitator led the class discussion.
- Students worked in three groups of four students each throughout the modelling activity.
- The readiness questions were prepared (see above), based on the students' anticipated understandings and misconceptions. Class discussions (either whole class or group discussions) enhanced understanding of the task.
- Brainstorming among the groups were continually promoted, especially during the initial stages where the students need to find ways of solving the client's problem.
- As the teams selected their own ideas to be developed further, they were motivated to create procedures to solve the client's problem.

- Guiding questions were asked by the facilitator while the students created their procedures to address relevant issues that arose.
- The teams tested, evaluated and revised their procedures as necessary and presented it to the class, taking turns to lead the presentations.
- Peer critique and discussions followed the presentations.
- Students were handed a Status Update Report (Appendix A), a Quality Assurance Guide (Appendix C), a Student Reflection Guide (Appendix E), and a Group Functioning Sheet (Appendix G) to monitor their progress. The facilitator used the Researcher Observation Guide (Appendix B) and the Group Modelling Competency Observation Guide (Appendix D) to assess the students' competence development.

5.2.6.3 Reflecting on the Turning Tyres Task

Unlike the last MEA, the researcher handed out the informational data at once, hoping that the students would have learnt from the previous activity the importance of understanding the problem content, prior to working with the data. Both Groups A and C initially misinterpreted the problem statement, thinking that they had to determine the best tyre, rather than the most effective material types for specific tyre sizes and aspect ratios. The facilitator/researcher called a whole class discussion. Students explained their different understandings to the class, and after consolidating their views, they all agreed with a student of Group B:

B1: *"We need to find a way to teach them how to select the best materials depending on where they want to drive with their vehicles."*

The reason for engaging in a whole class discussion again followed from Vygotsky's (1978) recognition, that the construction of new knowledge depends on students' prior knowledge. The value of continuous exposure to mathematical modelling emerged, as some of the students already adapted the approaches to MEAs in recognising that not only solutions, but also mathematical models, had to be part of their products as well. Group A's initial understanding of the problem was modelled as follows (Figure 5.38):

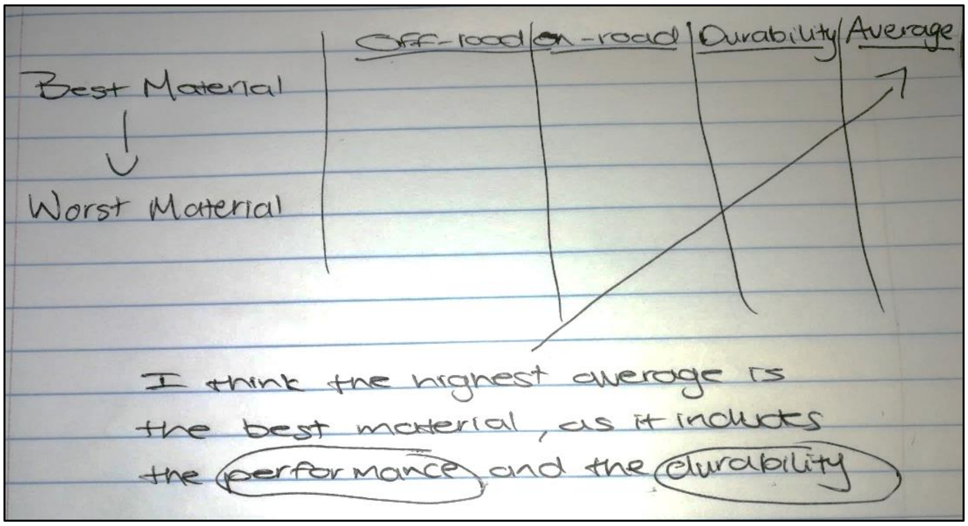


Figure 5.38 - Group A's action plan for MEA-5

The students' abilities to construct conceptual models of the problem situation have increased substantially. They progressively constructed more enriched situational models to improve their understanding of calculating the surface areas of cylinders, to determine the cost of the tyres. An example of such a model from Group B follows:

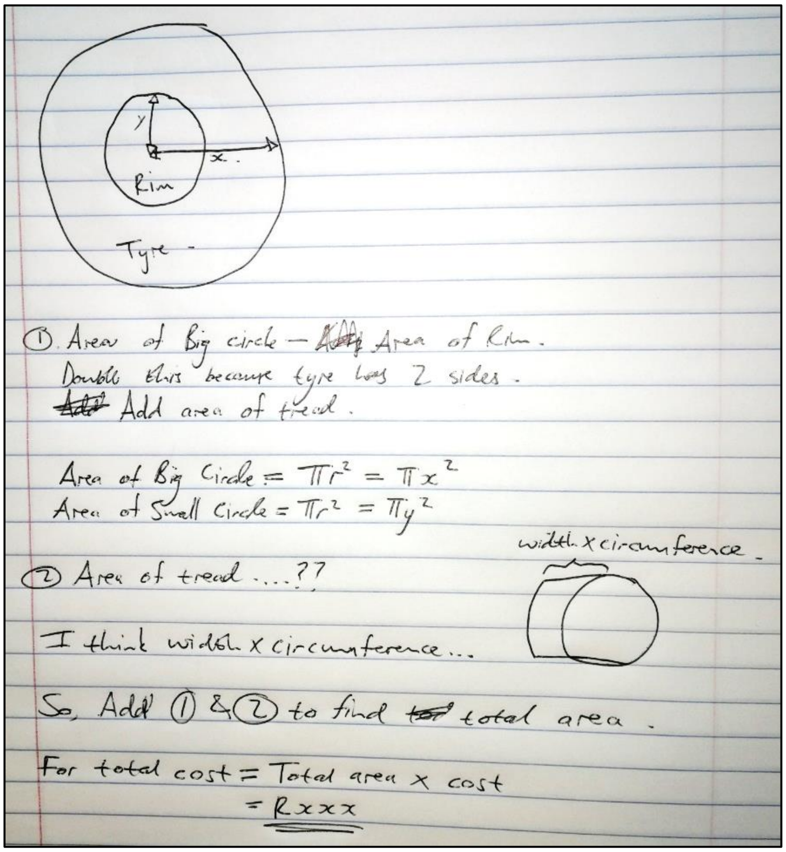


Figure 5.39 - Group B's illustration to determine the cost of the tyres

The informational text provided the surface area of the baseline tyre. Group B suggested that they calculate the surface area of the tyre in the example, to confirm the accuracy of their calculations. As anticipated, two problems surfaced: students did not understand what aspect rations meant, and they struggled to convert their calculations from inches to millimetres, and vice versa. After suggesting to them to re-read the data sets, all three groups slowly advanced towards more sophisticated understandings of aspect ratios. An aspect that unfolded here, was the students' increased abilities to intentionally manage their problem-solving approaches, by organising episodes of silent reading and brainstorming within their groups. This ability denotes a development in meta-cognitive competencies. By using Group B's suggestions, the other groups also used the example of the baseline tyre size to test their understandings.

Whole class discussions followed, where the students explained their current (mis)understandings and tried to gain further comprehension of the task. Students continually reverted to drawing models to improve their understanding of aspect ratios, where after they proceeded with calculating the areas of the tyres. During all the phases of the modelling cycle, students exposed various ways of thinking about the problem, and continued to switch back and forth among the different ways of thinking and reasoning. Here follows a transcript showing how Group B's ways of thinking about aspect ratios developed differently:

B2: *"So… what is aspect ratios? Same as tyre sizes?"*

B3: *"Nope, it reads that, whatever it is, it increases with a percentage, not an amount – it probably means something like performance?"*

B1: *"Wait, here is a formula of aspect ratio on the last page, but the equations that they gave, looks too difficult to understand …"*

B4: *"You have lost me now. Maybe we should just go back and try to make a drawing of what we understand."*

The group actively engaged in trying to understand this new concept. However, they kept going back to the equation of performance and durability qualities, and ignored the explanations provided on aspect ratios in both informational texts (Figures 5.36 and 5.37). The two students struggled to distinguish between using aspect ratios to calculate the cost, and using aspect ratios to calculate the difference in performance and durability.

B4: *"Let us rather do some quiet reading for five minutes, then see what we can come up with."*

After spending time on quiet reading, B4 continued:

B4: *"You know, if you look at the explanation of the codes for a baseline tyre, it makes sense. It says that the aspect ratio is 60% of the section width, which means that, comparing it to the formula they gave us, then* $60\% = \frac{sidewall\ height}{section\ width}\ x\ 100$.*"*

Note B4's error in using the percentage on the left-hand side of the equation, but multiplying 100 on the right-hand side. B4 managed to focus on the correct data in order to calculate the cost of the tyres, but his application of the formula led to a meaningless solution. He then asked his group to make further suggestions, and revisited the text again. By this time, B1 had finished reading the material and contributed to the discussion as follows:

B1: *"Well, I was reading through the material again, not having a clue what was meant by this. But, once I drew a model that looks very similar to the one in the text, I just substituted the numbers of the baseline tyre size with a, b and c, and realised that we can calculate the* *cost of the tyre by using the aspect ratio. To calculate the cost, we need to multiply the total area of the tyre with the cost. But… the total area must not include the rim as well, so the sidewall height will help us to find the radii for both circles - the big one and the small one for the rim. Look here:"*

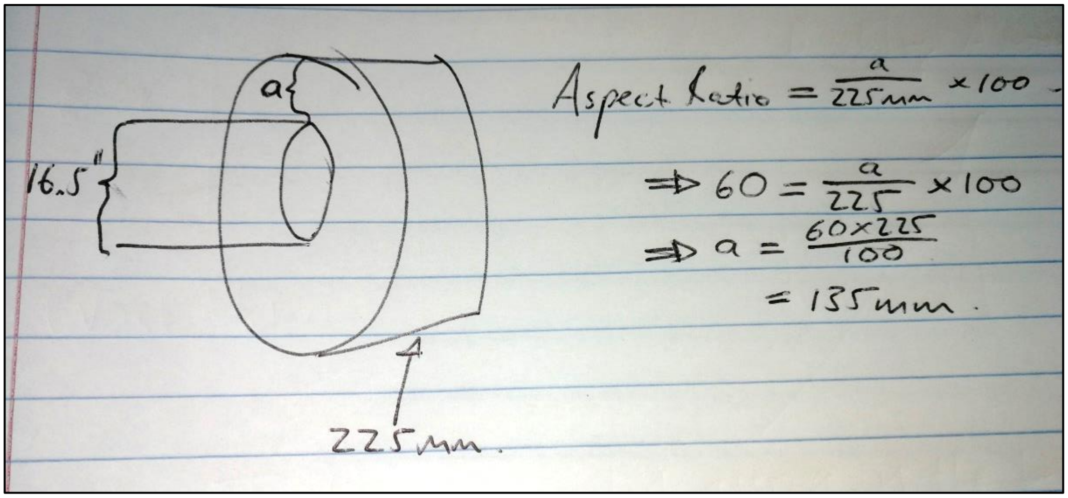


Figure 5.40 - Student B1's understanding of aspect ratio

B1: *"The dataset for performance rates now also makes sense to me, because a racing car's sidewall heights are very small to allow for good performance. Think about racing cars' performances and tractors' performances: there is a huge difference in these vehicles' sidewall heights. So – this is the reason why aspect ratio can affect on-road and off-road performances."*

As discussed in Section 2.5, by adhering to the reality principle when designing MEAs, the task designer ensures that the presented situation is realistic to the student. This principle allows for increased student interest, and it simulates activities of applied mathematicians in real-world problem situations. Students learn to interpret the situation meaningfullly, based on their individual levels of mathematical ability and general knowledge (Chamberlin, 2004:53). Students learn to make sense of real-world experiences from different topic areas, while they organise their mathematical ways of thinking around problem contexts. (Lesh & Zawojewski, 2007:798). Under-achieving students often seem to disconnect mathematics in the real-world from school mathematics, but as noted in these comments by student B1, MEAs have the potential to close the gap between applying mathematics in the real-world and experiencing mathematics in the classroom. By being engaged in MEAs, students learn to develop models, metaphors and other descriptive systems for making sense of familiar experiences, without having to use clever language and notation systems (Hamilton et al., 2008; Lesh & Doerr, 2003:5). They learn to use the resources they have at their disposal to express the 'new' ideas that they are expected to learn, while engaging in interdisciplinary, non-routine problem-solving activities (Lesh et al., 2000:632). Whilst B2 and B3 were still attempting to construct situational models, B4 already disregarded unnecessary information for calculating the surface area, although his equation were erroneous. However, B1 jumped from constructing a situational model to achieving an understanding of how to use the aspect ratio calculation for determining the sidewall heights, as well as to distinguish between the two different uses of aspect ratios that were both required in this MEA. The above transcript indicates that differentiation in their ways of thinking did not only occur between groups, but also among individuals within the groups. This gradual differentiation and integration of alternative ways of thinking emphasised Lesh and Harel's (2003:187) observation that development in mathematical modelling competencies do not simply progress along ladder-like sequences. The students' continual forward and backwards movement between the real-world context and their models and assumptions, indicates that the transition from one stage to another is not necessarily dependent upon the successful completion of the previous modelling stage, which also supports Voskoglou's (2007:156) stochastic model of the modelling process. The students' behaviour corresponded with Doer's (2007:69) suggestion that students exploit a diversity of approaches when engaging with modelling tasks.

Relating to errors in the conversion between inches and millimetres: Group C ignored the fact that the rim size was stated in inches, while the section width was given in millimetres. These computation errors resulted in students not being able to make sense of their solutions when they compared their values to the real-world context. Only after the facilitator probed the students to explain their results in terms of actual lengths, were they able to recognise and correct their mistakes. This misinterpretation of the data can be contributed to the students' deficient mathematical knowledge.

The groups proceeded to capture the data in MicroSoft Excel (2003). Three students (A2, C2 and C3) needed assistance in creating absolute references to cells for copying formulae to other cells.

Their situational models allowed the groups to calculate the correct surface areas, but seven students (across the three groups) converted their results to square inches erroneously. The facilitator/researcher called for a whole class discussion, when student B1 offered to explain why she divided the result by 25.4 squared:

B1: *"It is like ... you know, when you have an area, you always square your units – so ... you need to divide by 25.4 squared and not only by 25.4".*

The facilitator/researcher asked B1 to draw a square on the white board to represent one squared inch. B1 was then asked to use that illustration to explain her understanding of how many squared millimetres are in one squared inch. The student represented each side of the square as 1inch= 25.4mm. From there she calculated the surface are as 25.4mm x 25.4mm = 645.16mm^2. This model enabled the struggling students to understand the conversion process, and they carried on calculating the cost of the various tyre size/aspect ratio combinations. These basic mistakes were once again symptomatic of the students' weak knowledge base of mathematics.

After calculating the costs of the tyres, the groups attempted the durability and performance calculations. Again, students applied various methods to proceed. Group A only focused on the equation provided at the end of the text that explains new durability and performance calculations, while Group B focused on the performance rates data set to make sense of the changes in durability and performances, and Group C decided to use both methods. Application of the given formulae for determining durability and performance qualities did not present a problem in Groups A or C. However, understanding the equation (mathematical sense-making) seemed to be a concern for both groups. They all struggled to understand the incremental changes with regard to section width and aspect ratios. The facilitator asked B1 to repeat his explanation of how aspect ratios can influence the performances of vehicles to the whole class, where after the students reflected on their methods and solutions to confirm that it made sense. They then reverted to their Excel (2003) worksheets and captured the data and formulae without much effort. Haines and Crouch (2013:4) observed that these initial phases of the modelling processes (understanding of the real-world context and problem) are of specific difficulty to the students in mathematics education. However, these are the exact parts that assume critical importance outside education.

The next step was to create a graphical representation of their results. Some difficulties were experienced in all groups. Group A did not consider the cost of tyres as a determinant factor to find the best material, and they decided to use the average value of all their on- and off-road performance results and the durability results to determine the best material for each tyre size and aspect ratio: (Figure 5.41)

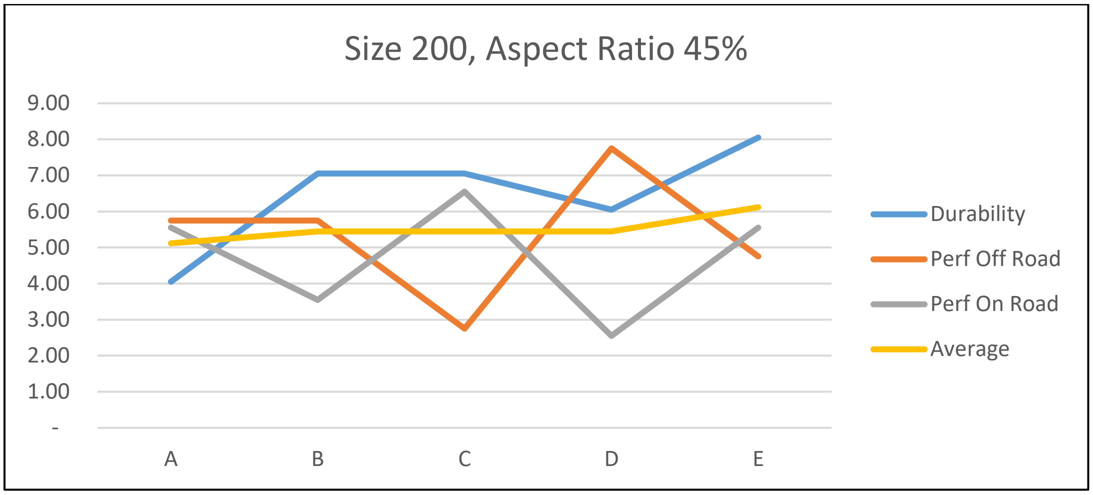


Figure 5.41 – Group A's graphical representation of the various tyre materials

By analysing the graph, they concluded that, for section width 200mm and aspect ratio of 45%, they would consider the type E material, as its average value was the highest. When asking them for the solution process, they answered that they would give the client a spreadsheet and they would highlight the cells where the client can change variables such as aspect ratio and section widths. Their formulae were linked to those cells, and would therefore display the updated performance figures. Group A also assumed that the cost of the tyres was not relevant to the task, and decided to ignore that data. This assumption can be associated with strategic choices in their solution process, as it could have an effect on their solution path. As Galbraith and Stillman (2001:305) explain, such assumptions are typically made when interim results have been obtained. The students did not foresee the difficulty in considering the best materials based on both design as well as cost at the outset.

Group C presented their results as follows (Figure 5.42):

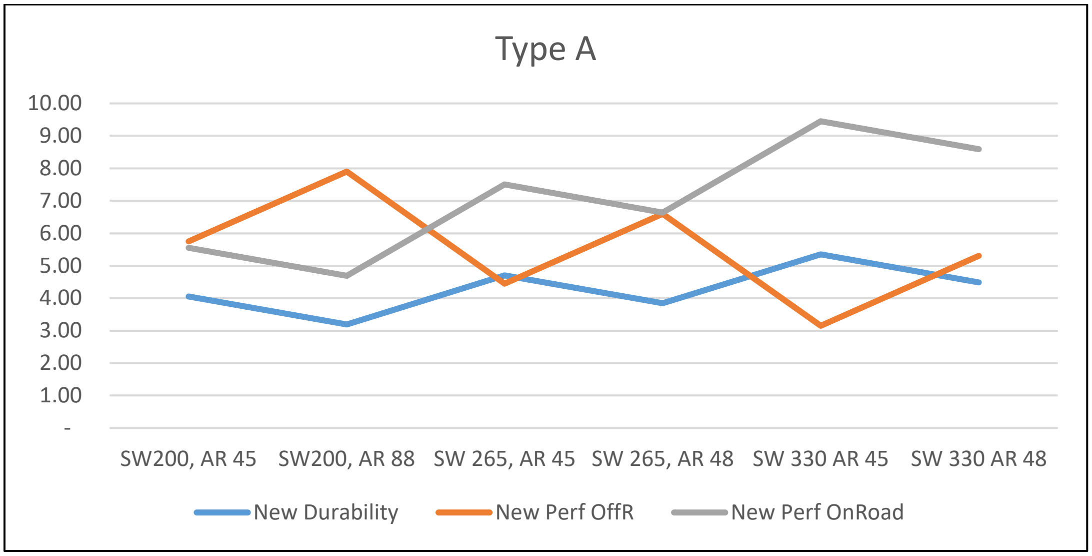


Figure 5.42 – Group C's representation of the best tyre material

Group C's illustrations did not answer the problem clearly, as their graphs compared the durability and performances of various tyre sizes manufactured with each type of material, rather than comparing the various material types as requested by the client. This representation indicates that the group did not revisit the client's letter before representing their findings, but they assumed that they answered to the letter once their calculations had been done. They struggled to maintain their sense of direction, causing weaker responsible behaviour competence ratings than the other groups. Group C presented their five graphs (each per tyre type), and when explaining the solution, their interpretations were flawed and they only realised their representational errors when they tried to explain the procedure to find the best material type. Lesh and Doerr (2003:16) emphasise the importance of representational fluency, as it is "at the heart of what it means to understand most mathematical constructs". This group's lack of representing their data fluently, also indicated low levels of the competencies organising and generalising, as they were unable to connect their results (even though correct) to the real-world situation.

Group B used a similar tactic as Group A, but tried to incorporate the cost as well. They represented the cost on the same axis:

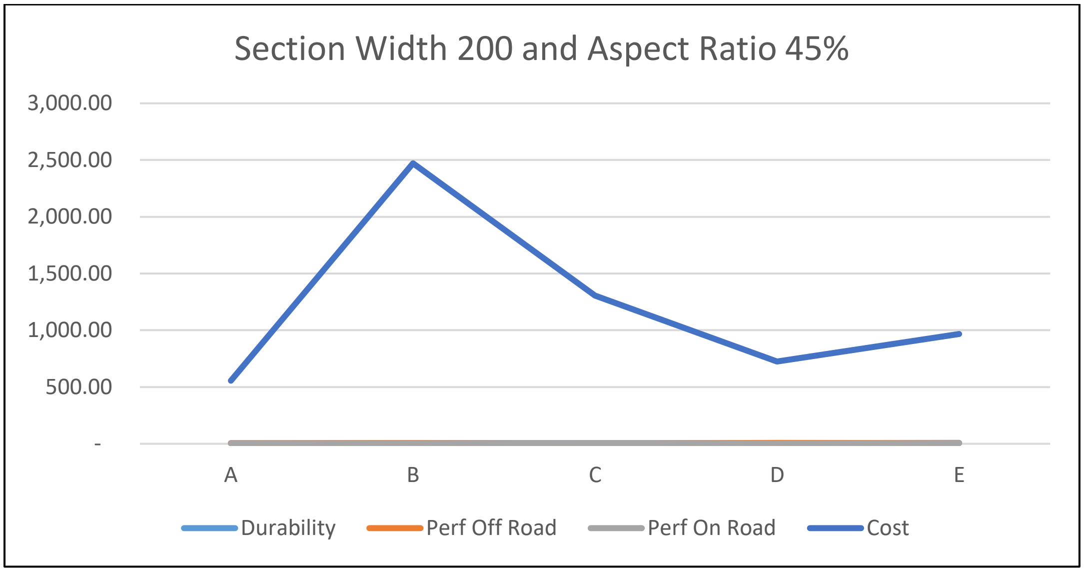


Figure 5.43 – Group B's model without secondary axis

Even though this group's representations were also flawed, they understood that there were huge differences in values between the cost of the tyres, and the durability and performance ratings, and they decided not to ignore the cost aspect. Based on their graphical solution only, they were unable to make a sensible conclusion relating to durability and performances. At the end of the presentations, the facilitator called for a whole class discussion again, and showed the students how to use secondary axes in Excel (2003). Group B then adjusted their graph as follows:

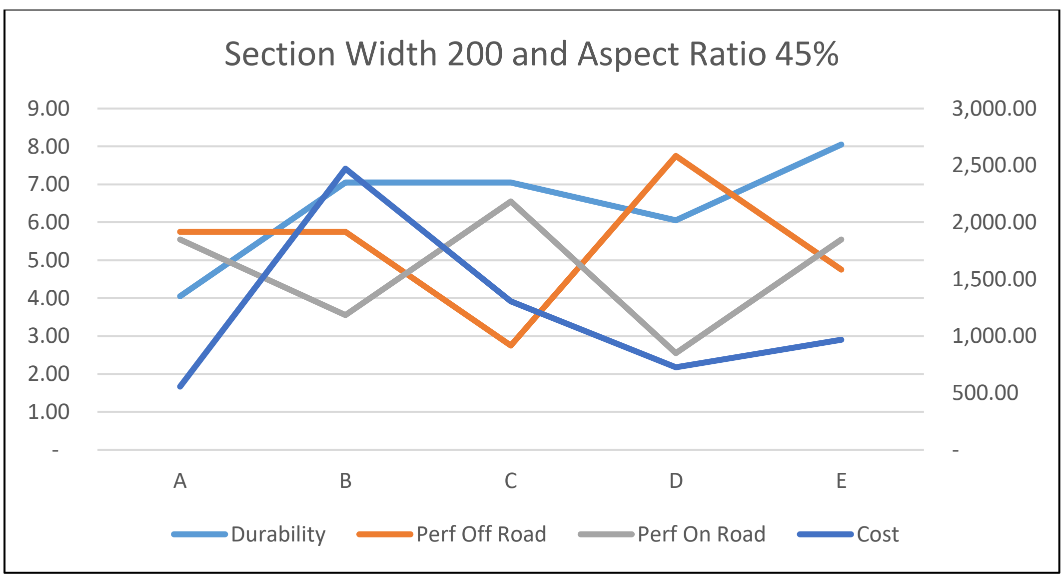


Figure 5.44 – Group B's adjusted graph – secondary axis included

The entire process was prolonged and lasted for three weeks. Being the first time that students used MicroSoft Excel (2003) while engaging with MEAs, the researcher was surprised to see that students planned their Excel (2003) tables thinking in the same ways that they did while planning their situational models during the previous MEAs. Again, the continued exposure to mathematical modelling enabled the students to utilise the competencies they developed by engaging in such activities. The development of competencies as revealed during the past five MEAs, are indicated in the following table (Table 5.12) and graphs (Figures 5.45 and 5.46):

Table 5.12 – Results of competence assessment – MEA-5 – Turning Tyres Task

Competencies assessment of the Turning Tyres Task:

Mathematical modelling competencies	
Internalising	1.69
Interpreting	1.54
Structuring	1.65
Symbolising	1.72
Adjusting	1.77
Organising	1.70
Generalising	1.50
Management	1.85
Communication	1.60
Responsibility	1.77

Engineering technician competencies	
Define, Investigate & Analyse Problems	1.62
Design/Develop Solutions	1.63
Comprehend and Apply Knowledge	1.71
Recognise and Address Factors	1.65
Sound Judgement	1.64
Management	1.85
Communication	1.60
Responsibility	1.77

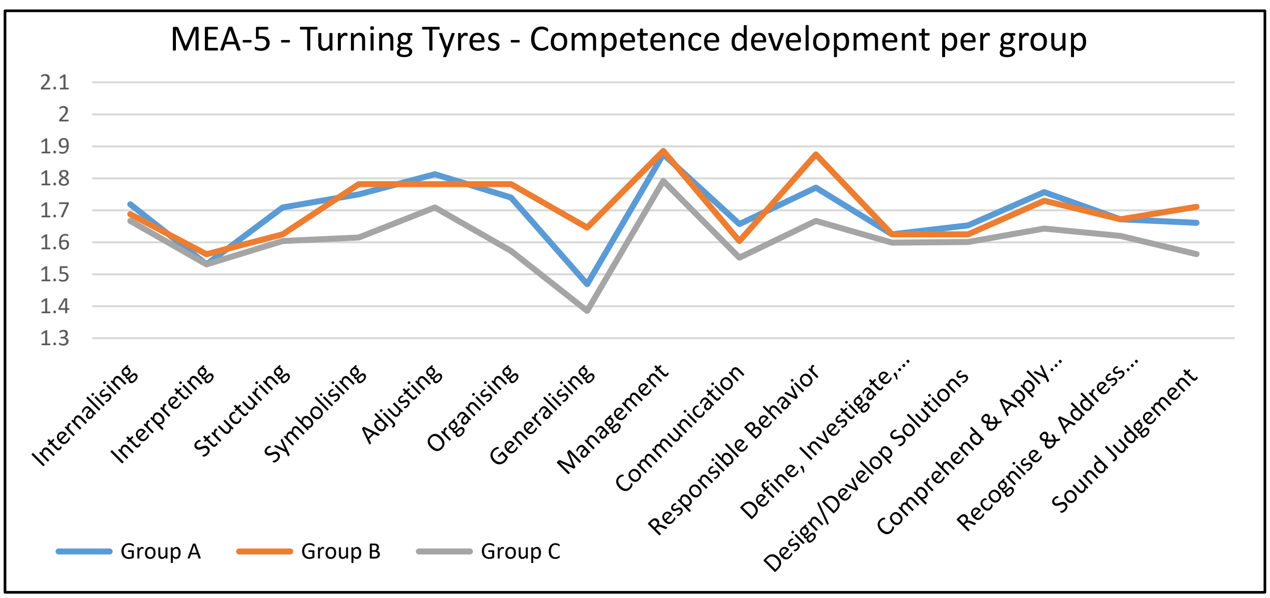


Figure 5.45 - Competence development per group - MEA-5

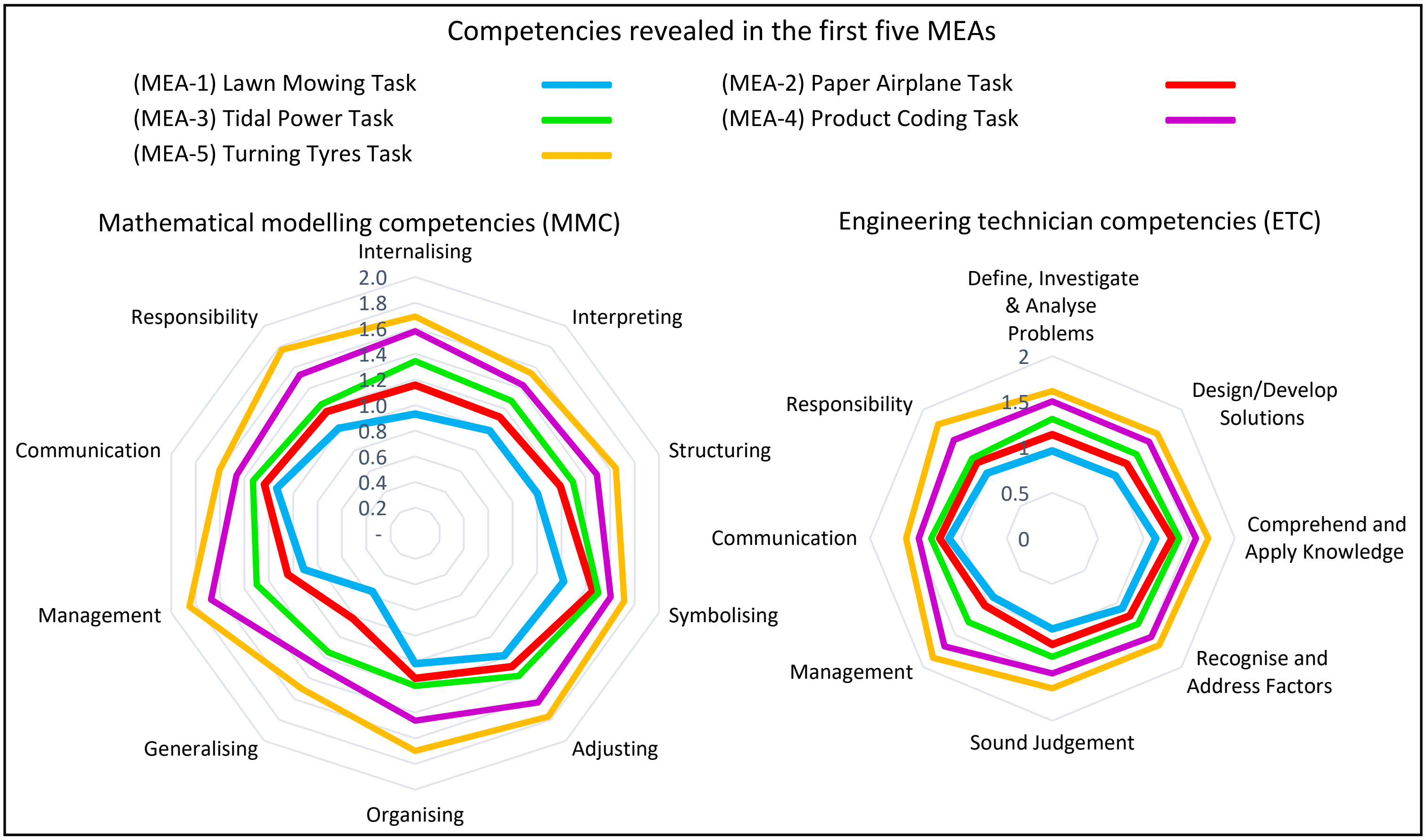


Figure 5.46 - Whole class competence development – MEA-1 to MEA-5

Adjusting, organising and generalising models play an important part in the learning and teaching of mathematical modelling. This model-adaptation activity provided opportunities for students to adapt and explore their models to new situations (Lesh & Zawojewski, 2007:788). Gravemeijer and Terwel (2000) emphasised the importance of selecting mathematical tasks that have the potential to be generalised from a situation-specific setting to be used as a *model for* more formal mathematical activities through the process of guided reinvention. Group B's graphical representations indicated a significant improvement in the competency of generalising, which again highlights an important aspect of longitudinal exposure to mathematical modelling; students' informal and intuitive *model of* the original situation evolved into a *model for* more formal activity as they acquired higher levels of comprehension over time (Dickinson & Hough, 2012:1). See Section 5.3, Figure 5.59 for more detail on Group B's progress relating to these competence developments. The model-construction processes of mathematical modelling allowed the students to integrate, differentiate, extend and refine their previous existing conceptual systems to develop new and improved ways of thinking to solve real-world problems (Lesh & Clarke, 2000).

5.2.7 Design cycle 6 – Find the Cell Phone Task

The results of the previous MEAs still revealed relative low levels of most of the competence domains under investigation. It was observed that the students still found it difficult to interpret, formulate, and define the problem, before attempting to develop a solution method. Again, the scenario in this activity was selected hoping that it would be realistic to the students to ensure increased student interest, and that it would hopefully stimulate students to interpret the situation meaningfully to overcome some of the difficulties experienced so far. This MEA was, similar to the Paper Airplane Task, selected due to its' strong link to a second semester program, '*Survey for Civil Engineers*'. This activity should not only develop sound problem-solving competencies, but it also has the potential to assist towards blurring the boundaries between mathematics education and engineering education. Students were required to locate a missing cell phone, by applying strong geometric and trigonometric principles. Big ideas in this activity involved the application of geometric concepts through modelling, the design of a functional spreadsheet for calculating various scenarios, as well as the graphical representation of students' solutions by using GeoGebra software. To further contextualise the activity, the researcher incorporated a topographic map of the region in the vicinity of the students' university.

The MEA, which was based on the Lost Cell Phone Problem by Anhalt and Cortez (2015:449) (Appendix P), provided students with the opportunity to develop mathematical ideas from real-life situations, by making mathematical connections through problem-solving. Students had to consider the relevance and importance of certain given information to create a meaningful solution.

Background:

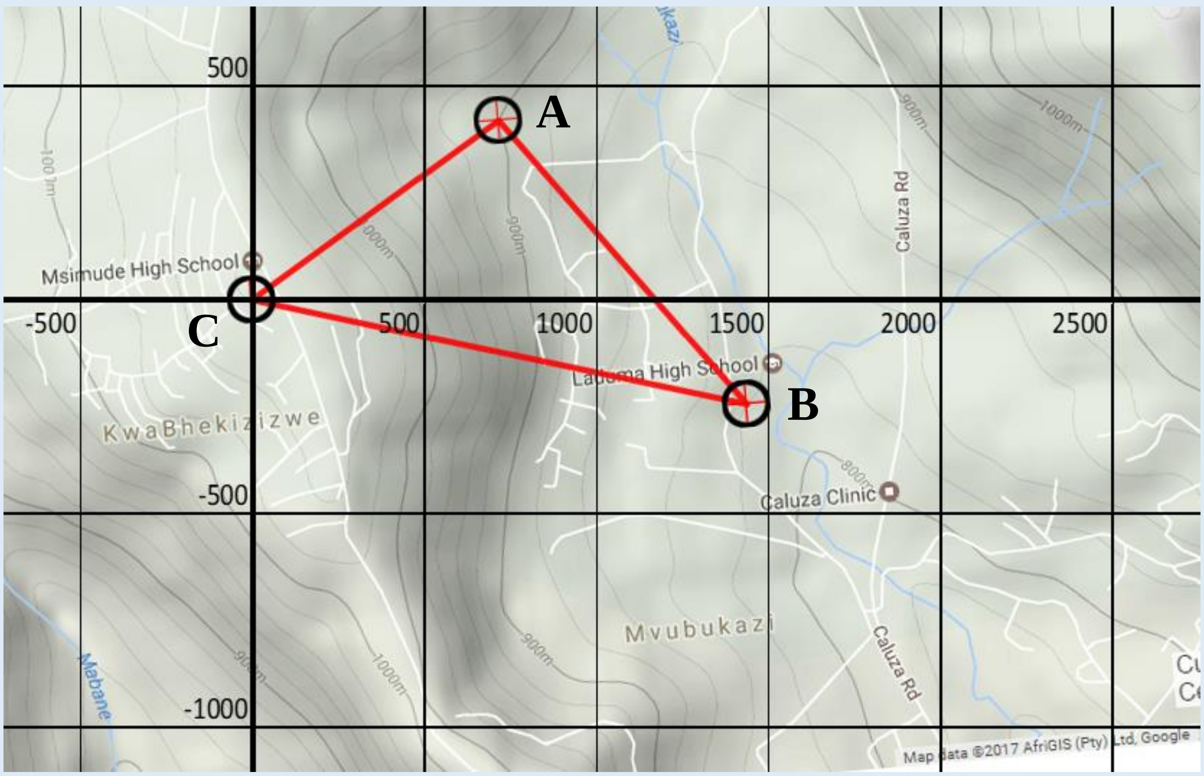
Electromagnetic radio waves, or radio frequency (RF energy) is emitted when you make a call on your cell phone. The cell phone tower's antenna that is the closest to your phone will receive these radio waves. Cell phone towers have antennas at the top of the towers that can both receive, as well as transmit, signals from your phone. Once the tower has received a signal from your phone, the signal is transmitted to a "switching center" – a telephone exchange for mobile phones. This connects your call to another phone or to another telephone network. The geographical area in which a cell phone tower is located, is known as a "cell" (from there the name "cell phones"). Some cell phone towers have larger cells than others, depending on the traffic that is required during peak times. Due to this reason, the cells of the towers in city centres are normally smaller than cells in less populated areas. "Hand-overs" or "hand-ons" occur when you cross the border between two cells. The new cell will automatically take over and this process is controlled by a computer in the switching centre.

Figure 5.47 - Background on cell phone towers, as adapted from Anhalt and Cortez (2015:449))

Task:

A detective company has called for your services to assist them in solving a murder case. A young man was found murdered only a short distance from the local university. After consulting with his family and friends, new information surfaced indicating that he could have been a victim of cell phone abuse. No cell phone was recovered at the murder scene, but three cell phone towers in the vicinity were able to detect a signal. A coordinate system used by the city, indicates that the cell towers are located at $(x; y)$ coordinates, measured in meters from one of the cell towers (see topographic map). You have been asked to create an approach for finding the location of the lost cell phone, and to explain your reasoning as to assist them in finding lost phones based on this information in the future.

A topographic map of the three towers provides the following information:



(Google Maps, 2017)

Tower A is at position $(787; 455)$, cell tower B is at position $(1478; -194)$, and cell tower C is at position $(0; 0)$. Tower A detects the signal at a distance of 603.5 meters. Tower B detects the signal at a distance of 804 meters, and tower C detects the signal at a distance of 760.6 meters.

Figure 5.48 - Cell Phone Tower Task, as adapted from Anhalt and Cortez (2015:449)

5.2.7.1 Planning for Find the Cell Phone Task

As mentioned in the introduction session, big ideas in this activity involved the application of geometric concepts through modelling, the design of a functional spreadsheet for calculating various scenarios, exposure to topographic maps, as well as the graphical representation of students' solutions by using GeoGebra software.

The readiness questions that were prepared were based on suggestions by Chamberlin and Moon (2005:39), to ascertain the students' understanding of the problem and of its content. These questions were asked after reading the passage and studying the data sets within small groups. The readiness questions assisted the facilitator to ensure that they understood the problem, before they started brainstorming and working with the data:

Readiness Questions:

1.1 What is the problem?

1.2 Who is the client in this problem?

1.3 Who are the customers that the letter refers to?

1.4 What does the client need?

1.5 What does the customers need?

1.6 What do you need to include in your letter?

For this activity, the anticipated questions and prompts were not presented in table format, due to the algebraic and geometric explanation of the planned activity. A thorough discussion of such anticipated problems as well as possible solutions will now follow:

The researcher anticipated that the students would assume that the distances from the top of the towers to the cell phone are horizontal distances, and thereby ignoring the height of the antennas and the topographic map. On the basis of these assumptions, it was expected that they would draw circles around the towers with radii equal to the distances to the cell phone, recorded by the antennas. This geometric approach can be represented algebraically by solving the following system of equations, which constitutes the following model:

$$(x-787)^2+(y-455)^2=603.5^2$$

$$(x-1478)^2+(y+194)^2=804^2$$

$$x^2 + y^2 = 760.6^2$$

By solving this system of equations, the students should find that the three circles do not intersect at a single point, even though these points appeared to be close to one another. A geometric solution of the groups' first anticipated approach is illustrated in Figure 5.49:

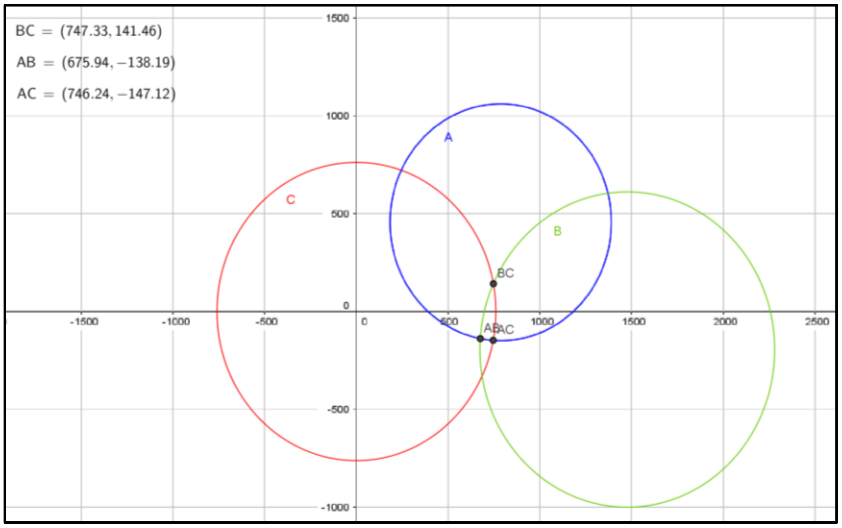


Figure 5.49 – Anticipated result of the students' first attempt in the modelling cycle

After solving their initial interpretations of the problem, the facilitator/researcher will map the students' graphs on a topographic map to allow them access to more information for further assumptions, analyses and testing. Figure 5.50 below illustrates an example of the mapped results.

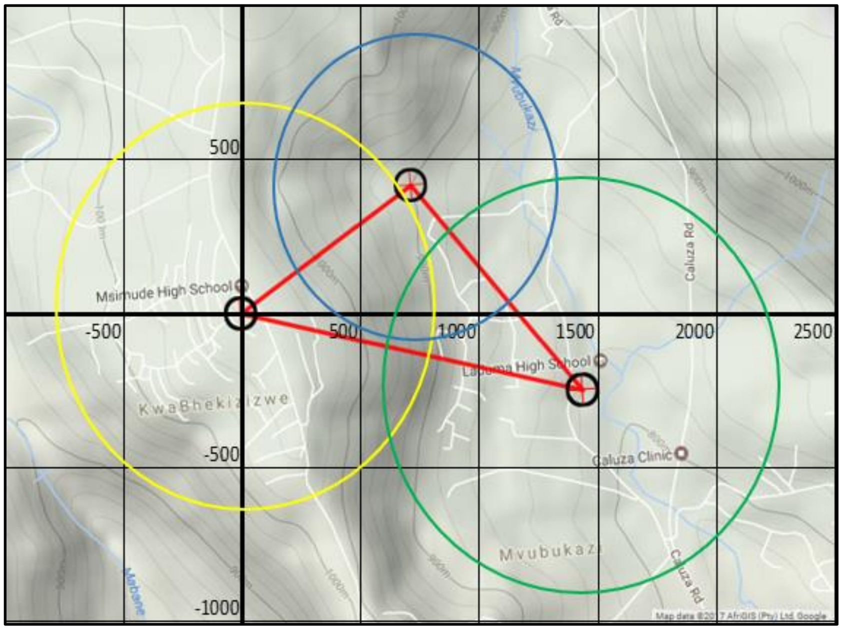


Figure 5.50 – Anticipated initial interpretations mapped on topographic map (Google Maps, 2017)

Results from previous MEAs and literature (Chao et al., 2013:557), indicated that students' understanding of mathematical constructs increase when they engage with technology to expand the variety of their representations. This served to support the researcher's decision to introduce the class to the software program GeoGebra to further assist the students to represent their results graphically. Once they have established a foundational knowledge of GeoGebra, the researcher/facilitator will plot their own solutions on a topographic map. On the basis of these findings, the students may interpret the location of the cell phone as one of the three points, or somewhere in the region that contain all three points. She will ask the class to determine the area of the triangular region (9 930 m^2). This area may still be too large to find a lost cell phone. Possible questions to the class will include:

- How can you improve our model to reduce the area under investigation?
- How can you modify your model to give a more specific answer by revisiting the given data, or by establishing new assumptions?
- What else do you know about cell phone towers that we have not considered? (Signal receivers are at the top of the towers, not at the bottom. When the students discuss these properties, the facilitator will suggest to them to use a height of 200 meters for all three cell phone towers)

- What about the data provided by the topographic map? How does that influence your distance from the tower to the cell phone? (The instructor will now show them the mapping of the GeoGebra graph on the topographic map):

On investigating the topographic map, and by taking the height of the towers in consideration, the instructor will guide the class towards re-thinking the meaning of the distances between the cell phone towers and the phone. These discussions have the potential to develop and formulate a modified model. Students may be looking at the various contour lines within the region which they calculated, and attempt to create new or modified versions of their model. In taking the topographical map into consideration and assuming that the antennas are situated on top of the 200 meter high towers, various models can be produced. The following diagram displays the heights of the antennas above sea level. The height of the cell phone above sea level is assumed to be 940m, based on the information from the topographic map.

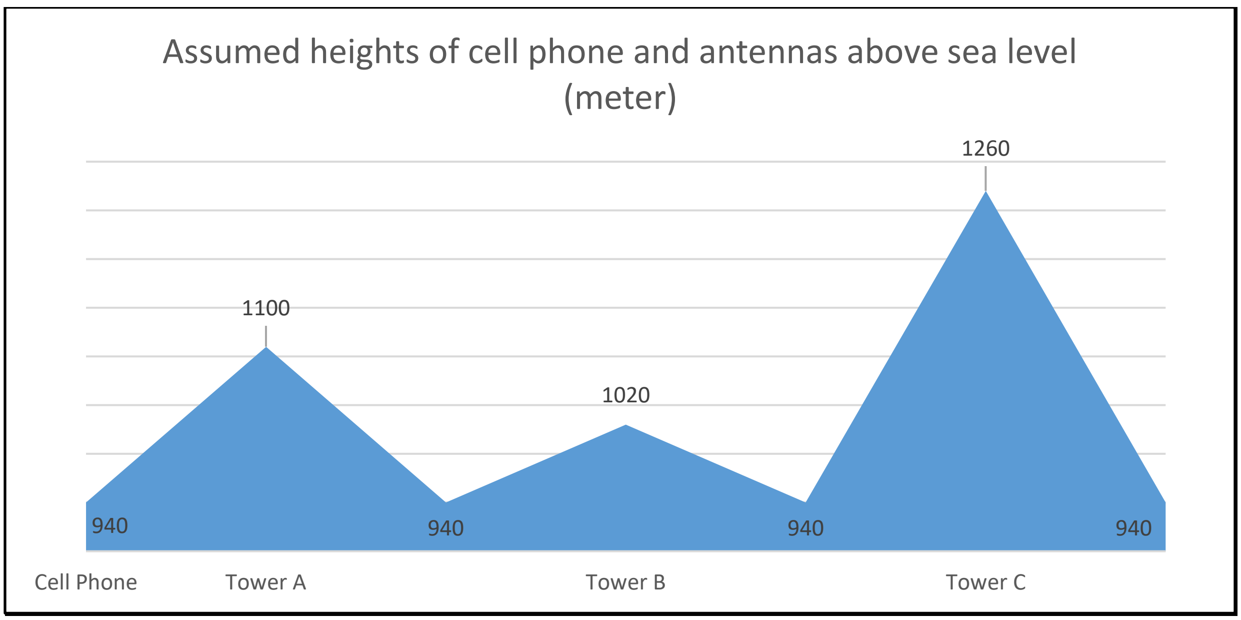


Figure 5.51 – Model of cell phone and towers above sea level

When considering the heights and radii, the students should calculate new regions with far more accurate solutions than their original model. At the elevation of 940 meters and with the assumption that all towers' antennas are 200 meters above ground level, the modified models can be represented as follows:

$$(x - 787)^2 + (y - 455)^2 = 581.9^2$$
$$(x - 1478)^2 + (y + 194)^2 = 800^2$$
$$x^2 + y^2 = 690^2$$

This algebraic model results in the following:

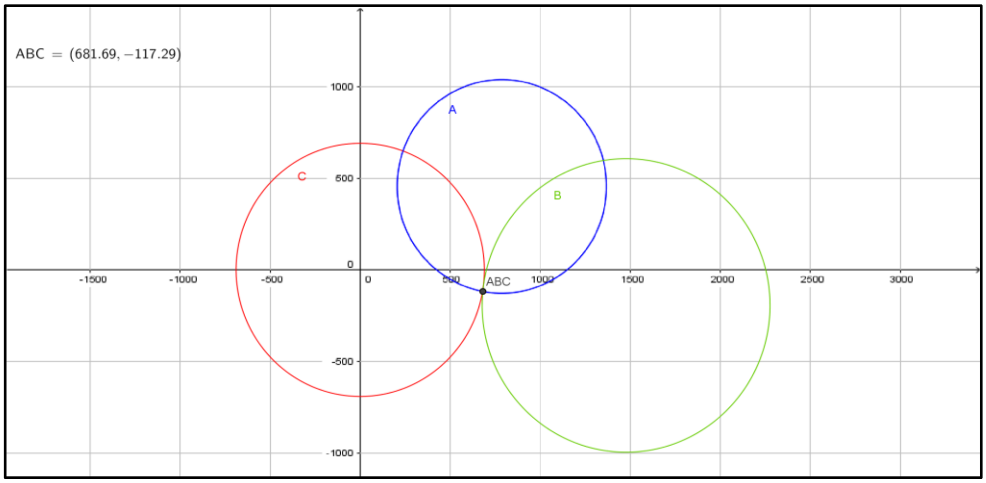


Figure 5.52 – Anticipated model explaining the location of the cell phone

After projecting the above graph on a topographic map, their final location can be illustrated as follows (Figure 5.53):

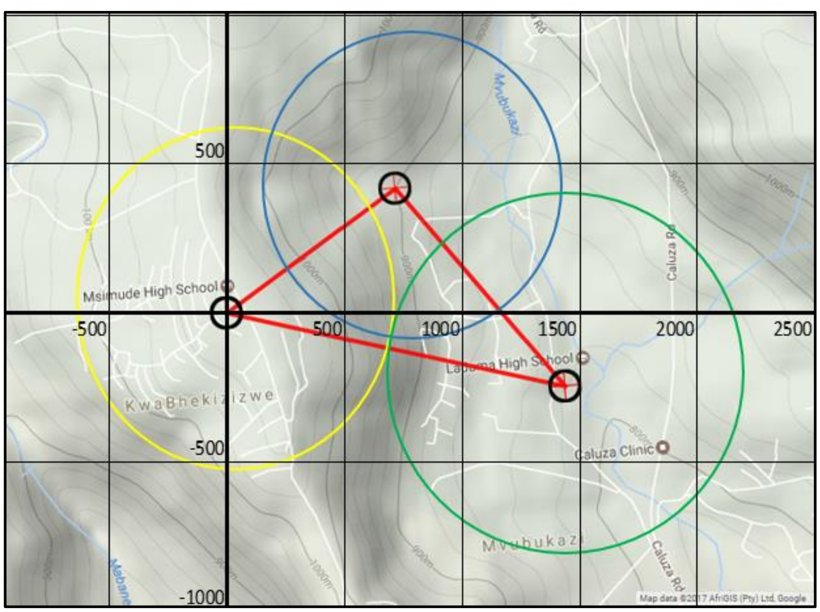


Figure 5.53 – Location of cell phone indicated on topographic map (Google Maps, 2017)

5.2.7.2 Implementing Find the Cell Phone Task

Implementation of the MEA followed the following format:

- Informational texts were handed out to the students for reading, where after the facilitator led the class discussion.
- Students worked in three groups of four students each throughout the modelling activity.
- The readiness questions were prepared (see above), based on the students' anticipated understandings and misconceptions. Class discussions (either whole class or group discussions) ensured understanding of the task.
- Brainstorming among the groups were continually promoted, especially during the initial stages where the students need to find ways of solving the client's problem.
- As the teams selected their own ideas to be developed further, they were motivated to create procedures to solve the client's problem.
- Guiding questions were asked by the facilitator while the students created their procedures to address relevant issues that arose.
- The teams tested, evaluated and revised their procedures as needed and presented them to the class, taking turns to lead the presentations.
- Peer critique and discussions followed the presentations.
- The mathematical approach and effectiveness of the solutions were also discussed.

The Status Update Report (Appendix A), the Student Reflection Guide (Appendix E), the Group Functioning Sheet (Appendix G), and the Poster Presentation Guide (Appendix H) assisted the students in formative self-assessment where they had the opportunity to verify their early or intermediate models against the problem stated (Eames et al., 2016:233). Presentations and subsequent discussions at the end of the session further motivated the students to assess the validity of their models against the needs of the client. Similar to all the previous MEAs, the facilitator/researcher utilised a variety of assessment instruments (Appendix B and D), video and audio recordings, walk-throughs, informal discussions, field notes and memos, to enhance honest and objective reporting on the students' competence development. The instruments were also used to allow the researcher to see when scaffolding and the necessary guidance were required.

5.2.7.3 Reflecting on Find the Cell Phone Task

Once the students have read the instructions, Group C's initial response was to find the midpoint between the three antennas by using the *midpoint theorem of triangles*. After a lengthy discussion within the group, they realised that they had to consider the distances between the antennas and the cell phones, and their primitive model did not allow for this addition. They then decided to start from the beginning again, and similar to Groups A and B, they reverted to drawing circles around the antennas depicting possible locations of the cell phones. The facilitator/researcher did not offer any scaffolding at this stage, and as anticipated, all groups used the circle equations and attempted to solve the simultaneous equations algebraically. However, they soon realised that there was no point of intersection between the three circles. The students exposed obstacles when trying to draw graphs in Excel (2003), where after the facilitator/researcher decided to introduce the class to GeoGebra. GeoGebra is a geometry software package that can provide for both graphical and algebraic input. As this was new to the students, almost 45 minutes were spent to explain the software package and to allow the students to test their understanding by entering various algebraic equations. Once they had a basic understanding of the program, the researcher asked them to continue with the MEA. The students were very excited to enter the data in GeoGebra, which confirmed their algebraic solutions that no one point of intersection existed between the three circles. The students were by now used to mathematical modelling examples, and were searching for information that they could possibly have missed during their previous readings of the text. Group A decided to first determine the size of the area where the cell phone could be found.

Due to limited mathematical content knowledge, the students were confused about how to determine the area between the three points of intersection. The groups searched for more guidance on the Internet, which lasted about twenty minutes, but they were eventually able to determine the size of the land and realised that the solution that they offered, could be improved. However, all three groups were unsure about their next steps. Due to time constraints, the researcher reverted to her proposed questions within a whole class discussion:

R: *"How can you improve your model?"*

No student were able to suggest any alternatives, and the researcher decided to ask them to explain how a tower can signify the distance to a cell phone.

A4: "*Ma'am, it is the antennas on the tower that get the signal, not the tower itself ...*"

B1: "*Yes – the antennas are ... I think ... at the top of the tower, and from there they can pick up the distance*."

A3: "*Wait – this means the distance is not horizontal, but diagonal? Am I correct?*"

C1: "*I think so, but the problem is that we do not know what the height of the towers are ...*"

The researcher then took a pointer and attached a string to the tip of the pointer.

R: "*Please show me what you meant with diagonal distance by using the pointer and the string, B1?*"

B1 used the model and explained why they have used the 'incorrect' radii to find the position of the cell phone.

C4: "*So ... this is why all three circles overlap ... Our radii are by far too big!*"

At this stage, all the students realised their mistake and the researcher directed them back to their groups. Within their groups, they reread the passage to see whether the height of the tower was included in the data.

C2: "*Ma'am, they never stipulated the height of the tower – do you think they forgot to include it, or what should we do now*?"

C1: "*Why do we not assume a height of the towers, Mrs de Villiers has recommended us to make lists of assumptions many times from before?*"

R: "*Ok, you can all assume the height of the towers to be 200 meters. How will this impact your solution procedure?*"

A1: "*Well, it will most definitely limit the size of the land where they need to search for the missing cell phone!*"

The students started to work individually to calculate the new radii of the towers by applying Pythagoras Theorem, and taking in consideration the height of the antennas. However, there still remained an overlap in their graphical representations, even though it was smaller than their original models. The following graph (Figure 5.54) illustrates Group B's interpretation (the other groups' graphical representations were very similar):

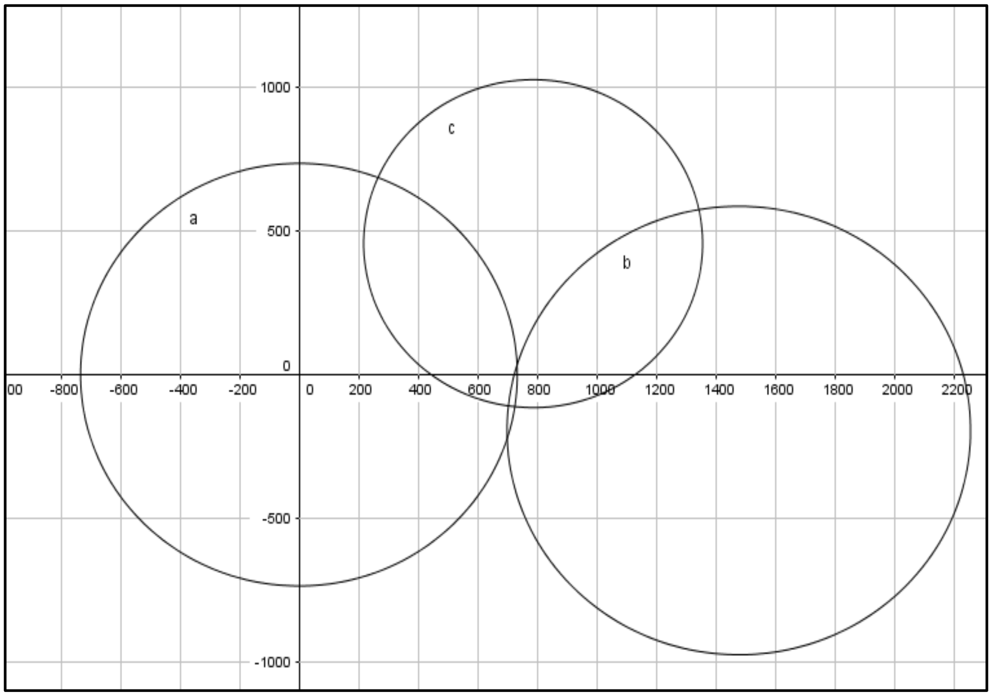


Figure 5.54 – First attempt of Group B to further refine and revise their model

The groups' representations were all very similar. All groups were very pleased with their results, as the proposed search area was substantially narrowed. They only represented their results graphically and not algebraically. The researcher was at this stage sure that all students had the know-how of representing the results algebraically as well, and were satisfied with their representations.

The fact that the three circles still did not intersect at one point, was a concern to student A2.

A2: "*Ma'am, there should be something else that we need to consider to get a perfect intersection – does it have anything to do with the map?*"

R: "*What does this map mean?*"

Some of the students had a vague understanding of topographic maps. This activity was designed in conjunction with the Survey lecturer. *Survey for Civil Engineers* is one of the subjects that the students will be enrolled for during the following semester, and the researcher asked the Survey lecturer to explain how topographic maps work and to give them some background as to how they will use such maps in their future studies. The discussion with the lecturer further motivated the students, as they obtained a richer understanding as to what would be expected from them in the future and it also instilled the value of mathematical education in them. One of the students commented:

C3: "*Thank you, Ma'am, it was great to see that we learn all this work to be used again in the future. I am far more committed to try to understand mathematics, as I can see that it will help me in my future studies!*"

R: "*How are you going to use the data provided by the topographic map? Will it influence your solution procedure? If yes, how?*"

B1: "*I guess we have to take the height between the antennas and the cell phone in consideration, so the vertical distances can increase even more, and we can perhaps find a point of intersection?*"

The researcher mapped all three groups' GeoGebra representations on a topographical map, and in all cases the possible location of the cell phone was depicted as 940 meters above sea level. Further brainstorming took place within the groups and all groups eventually managed to determine a point of intersection after adjusting the radii of the towers to allow for the effect of the landscape. The time spent on GeoGebra training, plus the additional discussions about topographic maps, caused delays in this activity, and it eventually took three weeks to complete.

Finally, tabular and graphical representation of the development of both mathematical modelling competencies and engineering technician competencies across the experiment that consisted of six MEAs, follows in Table 5.13 and Figures 5.55 and 5.56 below:

Table 5.13 - Results of competence assessment - MEA-6 - Find the Cell Phone Task

Competence assessment of the Find the Cell Phone Task:

Mathematical modelling competencies	
Internalising	1.83
Interpreting	1.77
Structuring	1.82
Symbolising	1.90
Adjusting	1.97
Organising	1.88
Generalising	1.61
Management	2.03
Communication	1.80
Responsibility	1.94

Engineering technician competencies	
Define, Investigate & Analyse Problems	1.80
Design/Develop Solutions	1.81
Comprehend and Apply Knowledge	1.90
Recognise and Address Factors	1.87
Sound Judgement	1.83
Management	2.03
Communication	1.80
Responsibility	1.94

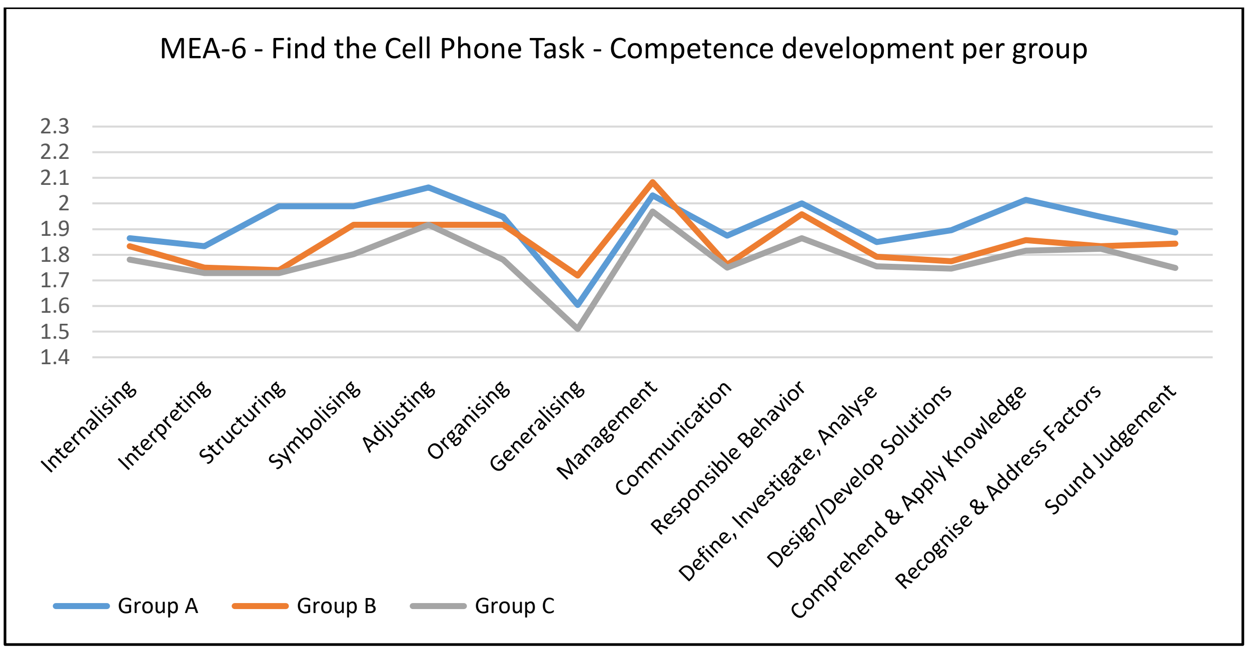


Figure 5.55 - Competence development per group - MEA-6

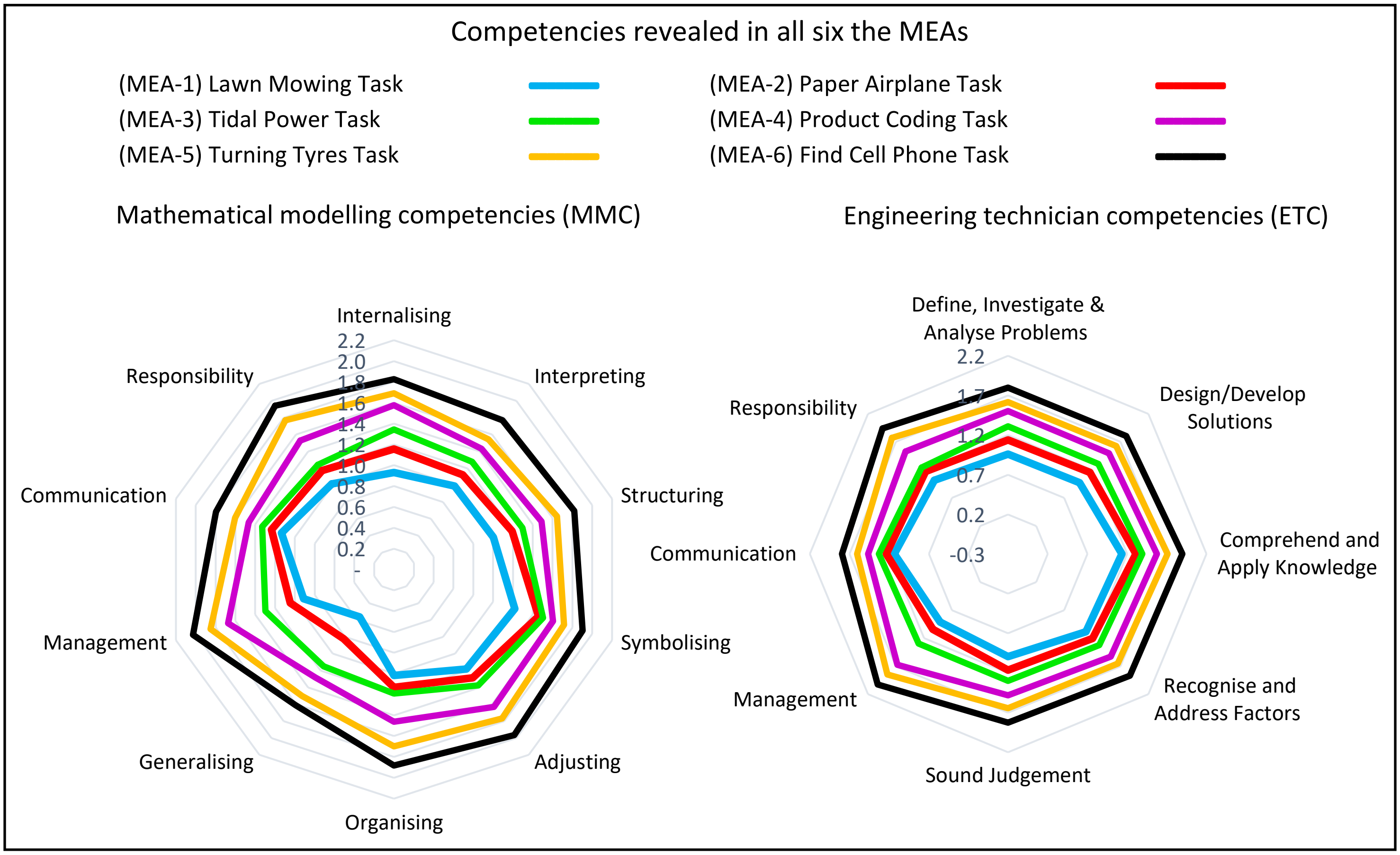


Figure 5.56 - Whole class competence development – MEA-1 to MEA-6

Figure 5.56 above depicts the changes in competencies for the whole class for the entire design experiment. Another illustration (for clarity purposes) of the same data can be seen in Figure 5.57 below:

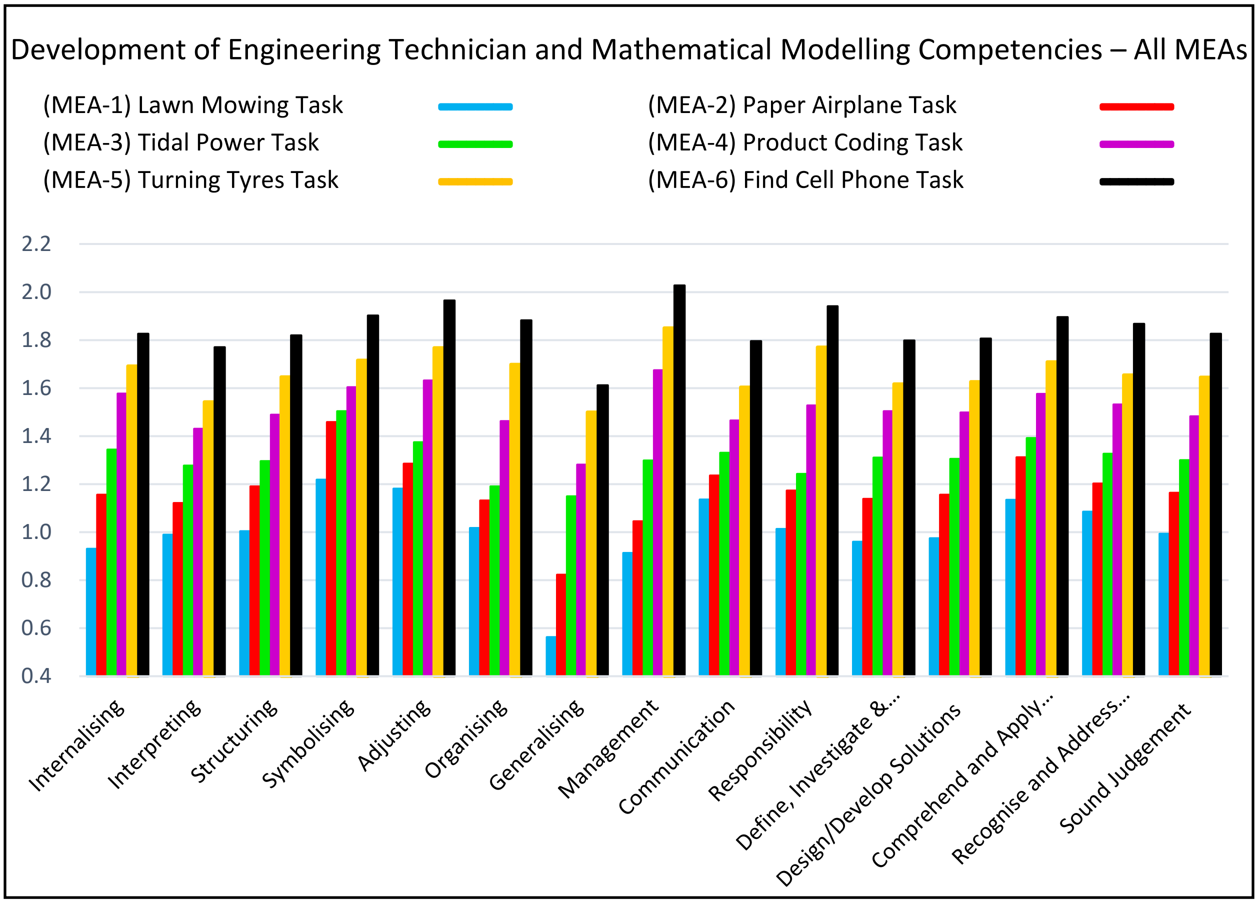


Figure 5.57- Whole class competence development in all activities

The above graphs indicate very slow, but consistent progress in all the competency domains. This gradual increase in engineering technician as well as mathematical modelling competencies complements Lesh and Harel's (2003:176) views that students' levels of development do not only change during the course of one modelling activity, but that students' progress from one activity to another which is structurally similar.

By the time the students were given this last MEA, they possessed a more distinct understanding of the modelling process, including activities such as *group work* and brainstorming. Students took turns to solve the relevant algebraic equations, to compose graphs and to explain their various solutions paths to one another. Even though the representations seemed to be problematic for some students, they still recognised the need for creating (*structuring*), revising and refining their models, albeit primitive at times and containing errors that would not lead to adequate solution paths. Cox and Brna (2016:11) noted in their research, that novice problem solvers do not spontaneously connect information to external or internal representations, but they seem to require encouragement to do so. When information is given to students in the form of diagrams, they often regard such diagrams as intructors' teaching aids, and not necessarily as tools that they can use to advance their problem-solving capablities. To answer to this dilemma, the researcher followed advice from Fry (1981), Cox (1999:359) and Schwonke, Berthold, and Renkl (2009:1227), by teaching graphical literacy skills (GeoGebra) explicitly to the students.

The long-term exposure to mathematical modelling instilled further qualities in the students, and growth in their abilities to *internalise* the problem situation can be noted in Figure 5.57. While the students leaned towards finding numbers and doing meaningless calculations in the initial activities, they eventually proceeded towards searching intentionally for a deeper understanding of the problem.

The competency of *interpreting* only seemed to improve during the groups' second and further iterations of the modelling process. During their first iterations, all groups created assumptions unintentionally (e.g. antennas as bottom of towers, surface area is flat) and only considered and recognised factors that could influence the situation during subsequent iterations.

Mathematical modelling's nature of continuous revising and refining students' complex conceptual systems, has also assisted them to develop their ability to explore alternative options and not to view their initial solutions as a final product. The groups developed the realisation to incorporate assumptions that could lead to more elegant solutions, even though they were not always sure what assumptions to use. This ability pointed towards a gradual growth in the mathematical modelling competency of *adjusting.*

The development of students' capacities to evaluate and judge their solution processes during this last MEA, came as a surprise to the researcher. All the groups immediately queried their initial results as being too vague and they realised that they had to reflect on their work to produce more elegant solutions. The three groups' attempt to calculate the possible area under investigation and realisation upon reflection, that the search area was still too large, indicated that they did not only attempt to produce solutions anymore, but that they advanced to a level where they also considered improved accuracy and elegant solution processes. This manifestation is indicative of improvement in their *organising* competencies.

Generalising of their solution procedures was displayed during their presentations, as the students did not only represent their solutions graphically, but they also explained their solution procedures (mathematical models) independent of specific coordinates, heights of antennas, or locations of cell phones above sea level.

The students were all actively engaged in the activity and revealed characteristics of *responsible behaviour* as they all worked enthusiastically within their groups to obtain elegant solutions. The introduction to GeoGebra was experienced in a very positive manner, and the students were excited to work with this powerful software program. Furthermore, they realised the value of mathematical modelling even more when they were exposed to other engineering disciplines such as Engineering Survey and Design. Hopefully their positive attitude towards other engineering disciplines will assist them to be more motivated in both subject areas.

All the MEAs as described in the above six design cycles, served to further revise and refine the HLT as defined from the results of the pilot study and the pre-intervention interviews (Sections 5.2.1 and 5.2.2.1). The HLT comprises of learning goals, planned instructional activities, and envisioned learning processes. An envisioned learning process anticipates how students' thinking and understanding might evolve when the instructional activities are employed in the classroom, as well as the possible means of supporting that learning processes (Gravemeijer & Cobb, 2006:19). The support also refers to an envisioned classroom culture, and the proactive role of the teacher/facilitator which was described throughout this section. The HLT was continually revised and refined during the various planning, implementing and reflecting phases of the subsequent design cycles. All the aspects of the HLT, together with possible changes to them, were constantly discussed and refinements were noted as new information emerged during the experiment. This section thus aimed to answer to Aim 12 of the research question, which asked for the establishment of an HLT. By observing the enactment of the HLT, a local instructional theory (LIT) will be developed that describes the envisioned learning route relating to a set of instructional activities. Section 5.3 investigates the results as presented here in further detail, to assist in the formation of the LIT (Section 5.4). The LIT's task to explain how the possible shifts in students' reasoning abilities occurred, will also serve to satisfy the last aim (Aim 13) of the research question (Section 1.8.3).

5.3 FURTHER REFLECTIONS ON THE RESULTS OF THE STUDY

This section aims to provide further explanations on how students' mathematical modelling and engineering technician competencies co-developed by engaging in MEAs, for the purpose of developing increased mathematical reasoning and understanding. To allow for uncluttered representations of the data, the researcher applied the codes as explained in Section 3.5.4, for each mathematical modelling and engineering technician competency. A summary of the codes follows again in Table 5.14 below:

Table 5.14 - Mathematical modelling and engineering technician competence codes

Code	Competency	Code	Competency
MMC-1	Internalising	MMC-9	Communication
MMC-2	Interpreting	MMC-10	Responsibility
MMC-3	Structuring	ETC-1	Define, Investigate & Analyse
MMC-4	Symbolising	ETC-2	Design/Develop Solutions
MMC-5	Adjusting	ETC-3	Comprehend & Apply Knowledge
MMC-6	Organising	ETC-4	Recognise & Address Factors
MMC-7	Generalising	ETC-7	Sound Judgement
MMC-8	Management		

Construction of the LIT involves explanations of how the students' reasoning abilities occur. Prior to this discussion, it is vital to understand the magnitude of competence development that transpired during the design experiment. Section 5.2 detailed the construction of the HLT during each design cycle's planning, implementing and reflecting phases. The changes as discussed relating to mathematical modelling and engineering technician competencies, referred to the whole class. However, design-based research (DBR) allows for the investigation of small groups, and the results were further analysed to also reflect the development of the relevant competencies between the three groups. The similarities and differences in competence development among Groups A, B and C, are illustrated in Figures 5.58 to 5.60:

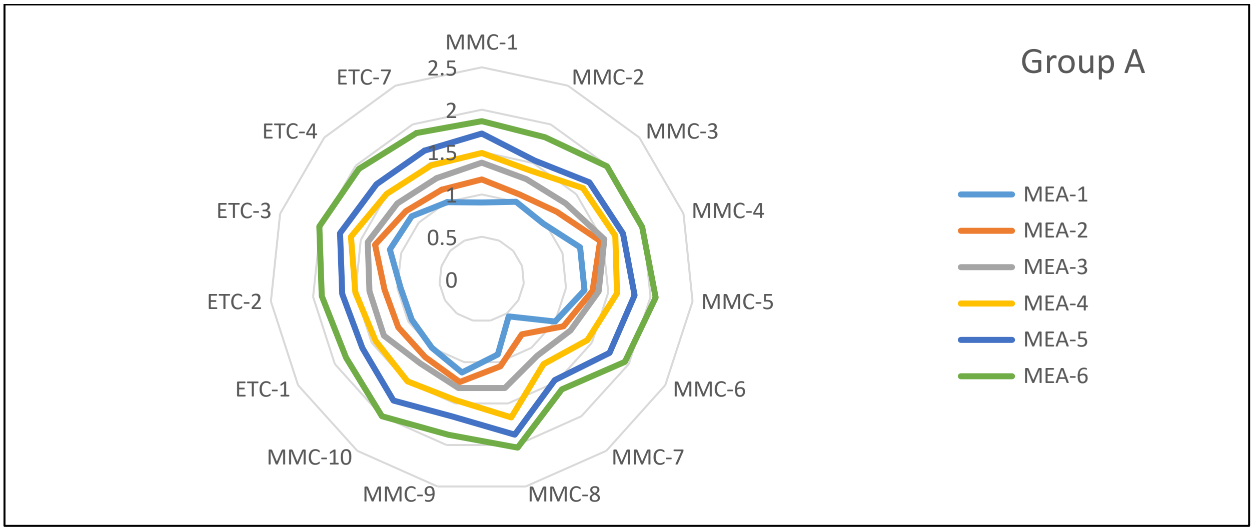


Figure 5.58 - Group A - Development of all competencies

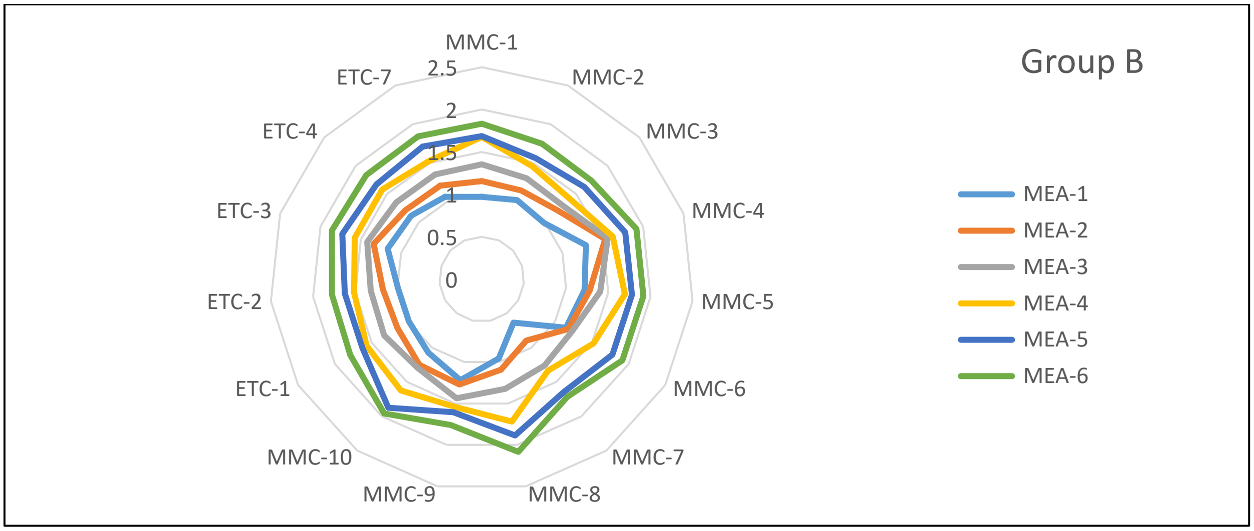


Figure 5.59 - Group B - Development of all competencies

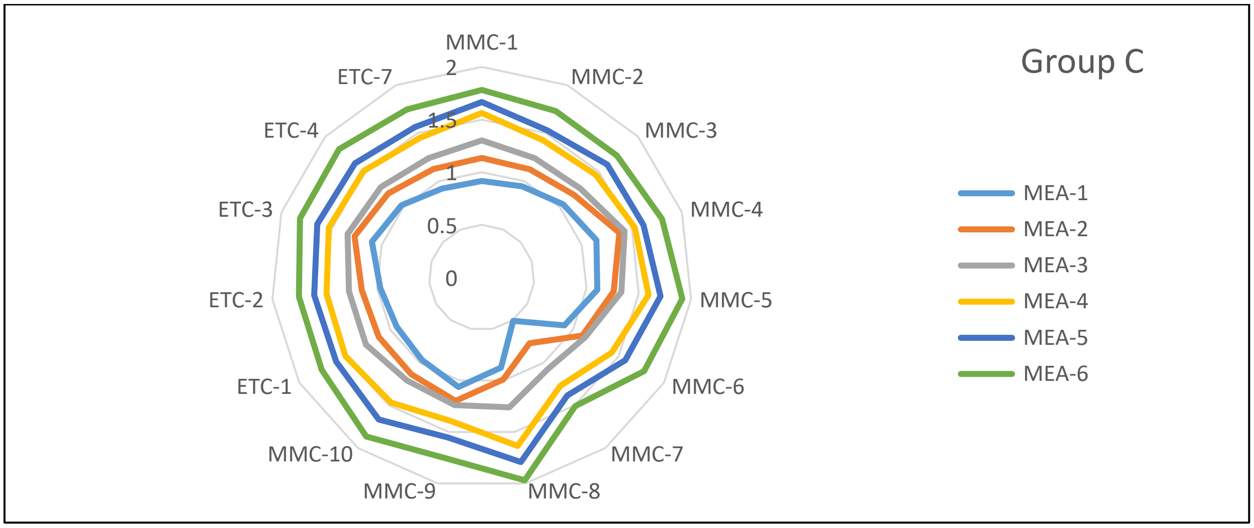


Figure 5.60 - Group C - Development of all competencies

As mentioned in Section 1.9.2, and earlier in this Chapter, this homogenous group of students were first-year civil engineering students, studying at a university of technology. They did not meet the entrance requirements for studying engineering, hence they were enrolled on a bridging course. No significant differences were exposed when analysing the competence development among the three groups. By the end of the design experiment, all the groups displayed a slightly stronger inclination towards the cognitive competency of adjusting (MMC-5), and the meta-cognitive competency of management (MMC-8).

Relating to the competency of adjusting (MMC-5), development occurred more visibly during the last three MEAs. This developmental jump was expected during the later stages of the experiment, as the students were by then familiarised with mathematical teaching and learning in a socio-constructivist environment through engaging in MEAs. Mathematical modelling's nature of continuous revising and refining students' complex conceptual systems, compelled the students to explore alternative options, and not to view their initial solutions as a final product (Section 5.2.7). The adjusting competency as explained in Sections 3.7 and 3.8, refers to the ability

- to interpret mathematical results;
- to rephrase problems;
- to question one's own model;
- to refine parts of the model or to go through the entire modelling process if the solution does not fit the situation; and
- to adapt the model to make sense of a specific situation (Section 3.7)

Management strategies (MMC-8) enabled the students to manage and control the material and resources that they had at their disposal to reach their goals, as well as to persevere and maintain their intellectual engagement when grappling with difficult tasks. MEAs allowed the students to develop the abilities to set their own goals, to apply appropriate methods and techniques related to the problem content and goal, as well as to judge their own processes (Boekaerts, 1999:449). This managerial or strategic aspect of self-regulated learning is regarded by Schoenfeld (1983:20) of utmost importance in problem-solving, as it allows the students to select a framework for the problems that they need to solve (Section 2.6.2.1).

Section 3.7 defines management competencies relating to this study, as

- self-directed learning, where students actively plan, monitor, evaluate, reflect, direct and regulate their own learning processes. These students display the competency to reflect on their work by activating meta-knowledge about the modelling process – monitoring and controlling the entire modelling process (Schraw, 1998:26). They evaluate the solution paths that they designed through reflection and by making judgements about the processes and outcomes (Pintrich et al., 2000:45). Reflective activities are activities such as talking and writing about the processes they have gone through, making posters and reporting to the class, drawing up concept maps of a topic, or sharing attainment targets;
- productive disposition, referring to the ability of students to recognise the possibilities that mathematics offers for the solution of real-world problems and to regard these possibilities as positive; and
- group work, denoting students that can effectively work in a team environment toward group goals. They respect one another's ideas and take turns to assume leadership, displaying teamwork, leadership, project management and communication skills.

All three groups denoted low levels of management competencies during the beginning phases of the experiment. This was anticipated, as they were not familiar with socio-constructivist classroom norms where the teacher acted as facilitator and not as a direct instructor. During each subsequent MEA, the students progressively acquired the ability to work collaboratively within their groups while they planned, executed and reflected on their various solution paths. The assessment instruments further created a platform for students to develop reflecting skills, as they had to continually engage in justifying and improving their initial models. Already after the first MEA, the students were positively surprised about the utility of mathematics to solve real-world problems (Section 5.2.2.3). They also recognised the value of engaging in collaborative work, but it was only during the last three activities that they exposed significant improvements in their abilities to monitor and regulate their learning processes. In a study of Pintrich and De Groot (1990:37-40), self-regulation proved to be a strong predictor of academic performance, and suggested that the use of self-regulating strategies such as comprehension, orienting, monitoring, goal setting, planning, effort management, persistence, evaluating and correcting are essential for academic performance in various types of classroom tasks. The results of this design study indicate not only improved management competence, but also improved cognitive competence in all groups (Section 5.2).

When investigating the competence ratings for the groups at the end of the sixth MEA, all three groups still displayed low levels of generalising competencies (Figures 5.58 to 5.60). Freudenthal (1968) emphasised the importance of learning mathematics (mathematising) by doing (Section 2.4.2.3), and he distinguished between horizontal mathematising (translating the problem from the real-world situation to mathematics) and vertical mathematising (manipulating and moving within the mathematical world itself) (Van den Heuvel-Panhuizen, 2003). Horizontal mathematising thus enables the user to engage in further mathematical analysis (vertical mathematising). One of the aims of modelling, is to generalise the solution to use in other similar situations. However, Zazkis and Applebaum (2007:2396) noted that reconstructive generalising (where an existing schema is reconstructed to be applied elsewhere), is an identifier of advanced mathematical thinking, and should not be expected from novice modellers.

The literature discussions on mathematical modelling and mathematical modelling competencies throughout all the previous chapters, as well as the knowledge gained from the pilot study and the pre-intervention interviews (Section 5.2.1), emphasised the students' inabilities to generalise their mathematical results. The HLT that was constructed in Section 5.2 was based on this concern, and activities were designed and adapted to support growth in these particular skills. Gravemeijer and Terwel (2000) noted that formal mathematics come into being through generalising specific situated problems (Section 1.7.2). Students thus need to learn to generalise problem situations to other similar scenarios, which promotes level raising theory. Level raising theory denotes the process whereby students' learning trajectories evolve from constructing a *model of* a particular situation, to constructing a *model for* mathematical reasoning and understanding (Menon, 2013:3).

The MEAs that were designed for this study, allowed the students to move from situated cognition to generalisable and sharable knowledge that could be used and reused. As each subsequent MEA played out, students slowly increased their independent reasoning and acting abilities.

All the MEAs in the experiment adhered to the realistic design principle (Section 2.5.1.1). This principle enhances student interest, and careful attention was given to the didactical design to structure the activities to support students' learning. Contextualised modelling allowed for the emergence of significant types of mathematical thinking. Students learned to make sense of real-world experiences from different topic areas, while they organised their mathematical ways of thinking around problem contexts. (Lesh & Zawojewski, 2007:798). However, Section 2.4 explains that the focus of mathematical modelling is not in finding solutions, but model development nearly always moves from situated cognition to generalisable and sharable knowledge that can be used and reused. (Dienes, 1968; Lesh & Clarke, 2000:132; Lesh & Zawojewski, 2007). Kaiser (2007:111) explains that when students learn to model, they also develop needed competencies to use their mathematics for the solution of problems in their daily life and from sciences. Modellers thus need to develop both cognitive and meta-cognitive competencies in sync.

Section 5.2 details how the students in this study progressed from their initial context-bound descriptions to more formal ways of mathematical reasoning and thinking. Even though the generalising competency remained weaker than the other competencies in all three groups, care must be taken not to focus on the specific competence values that were generated in the assessments, but to view this development holistically. When comparing the three groups' competence development in terms of growth, a very different scenario is presented in Figure 5.61 and Table 5.15:

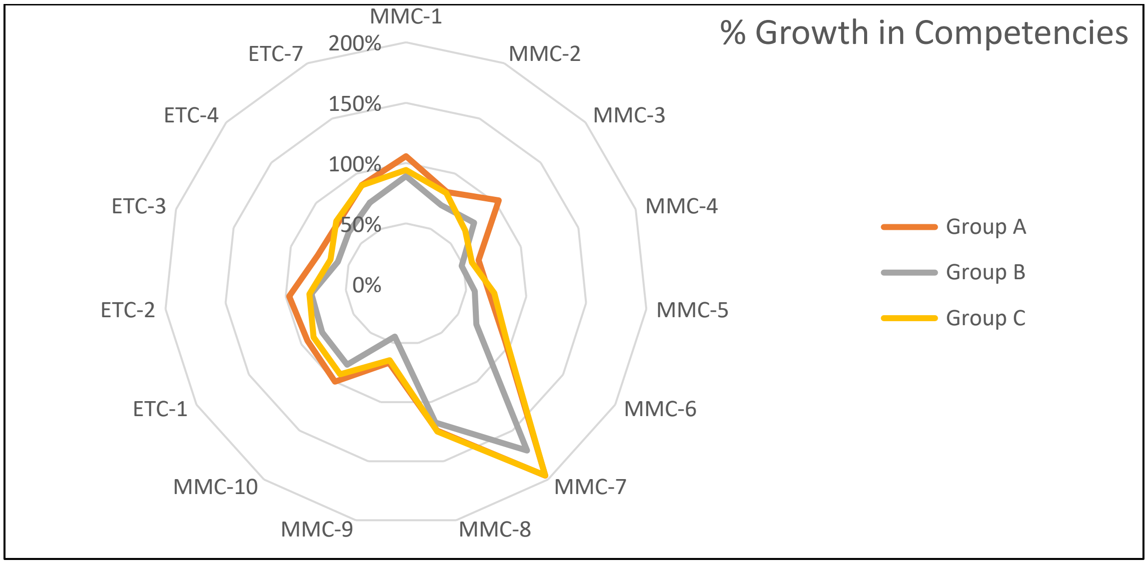


Figure 5.61 - Percentage growth in competence development per group

Table 5.15 - Data set indicating percentage growth in competence development

Competence growth in all competencies per group and per class (%)				
Competencies	**Group A**	**Group B**	**Group C**	**Whole Class**
MMC-1	106%	89%	94%	96%
MMC-2	83%	71%	82%	79%
MMC-3	103%	76%	66%	81%
MMC-4	63%	48%	57%	56%
MMC-5	69%	57%	74%	66%
MMC-6	95%	67%	97%	85%
MMC-7	196%	170%	196%	186%
MMC-8	124%	117%	125%	122%
MMC-9	67%	44%	65%	58%
MMC-10	100%	83%	92%	91%
ETC-1	94%	80%	88%	87%
ETC-2	97%	79%	80%	85%
ETC-3	77%	59%	66%	67%
ETC-4	76%	64%	78%	72%
ETC-7	90%	74%	89%	84%

The above graph and accompanied table denote exceptional improvements in all competency categories, in particular generalising competencies (MMC-7) and management competencies (MMC-8). Even though the student groups' generalising competencies still remained weaker in comparison with other competencies, their growth in generalising competencies increased substantially.

The results of the competence development comparisons between the three groups, indicated a very similar growth pattern for all three groups. In search for variances in the data, the researcher complemented the design study with case study research. The robust data collection and analysis procedures as discussed in detail throughout Chapter 4 and in Section 5.2, enabled her to also investigate competence development among individual students. While DBR allows for the examination of small groups of four students each, case study research studies a single case, or a few cases, to understand a larger population of similar cases (Gerring, 2006). The weakest and strongest cases to be studied, were determined by examining the changes in their competence development between the first and last MEAs. Evidence was collected relating to the ten mathematical modelling competencies as investigated in Chapter 3, and these competencies collectively represented the unit of analysis (the phenomenon for which evidence will be collected). Apart from the data that were collected and analysed during and after the design experiment, the researcher also studied the results of the post intervention questionnaire (Appendix I), which was handed out at the end of the course. This questionnaire addressed issues relating to the students' opinions about the mathematical modelling course (Appendix I). Rule and John (2015:2) emphasise the importance of a dialogic engagement between theory and case study, as theory can be generated from practice and vice versa. The researcher hoped that case study research's continual engagement between theory and practice, would assist the researcher to come to insightful understandings of how and why students' competence development vary. The following graphs (Figures 5.62 to 5.73) indicate each individual student's competence development as they progressed during the six MEAs:

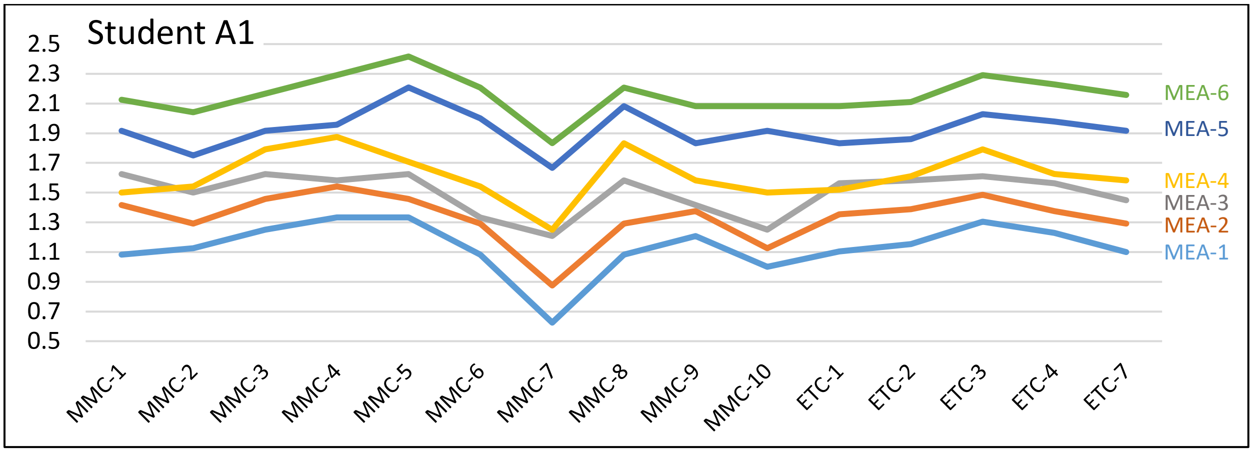


Figure 5.62 - Student A1's competence development during the six MEAs

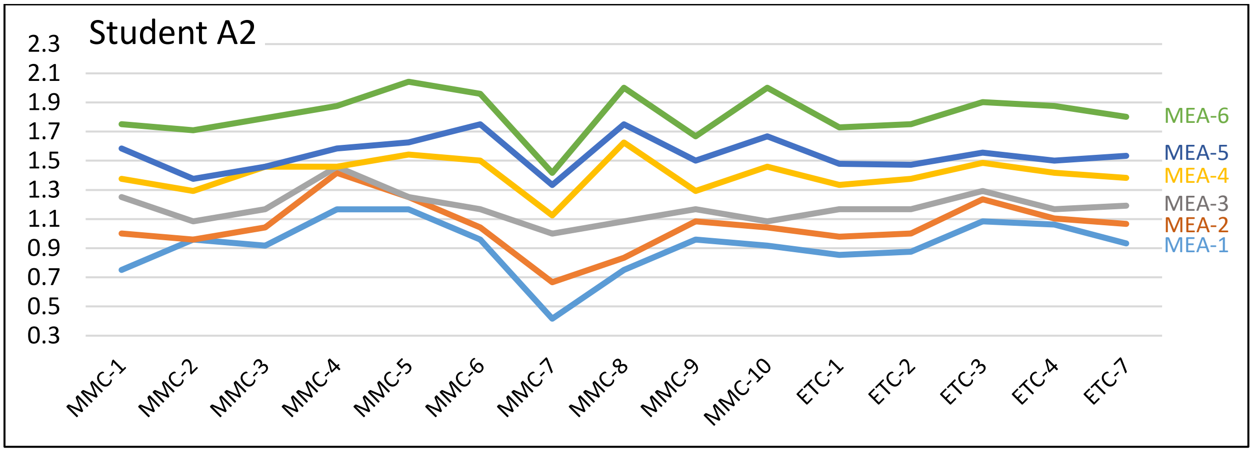


Figure 5.63 - Student A2's competence development during the six MEAs

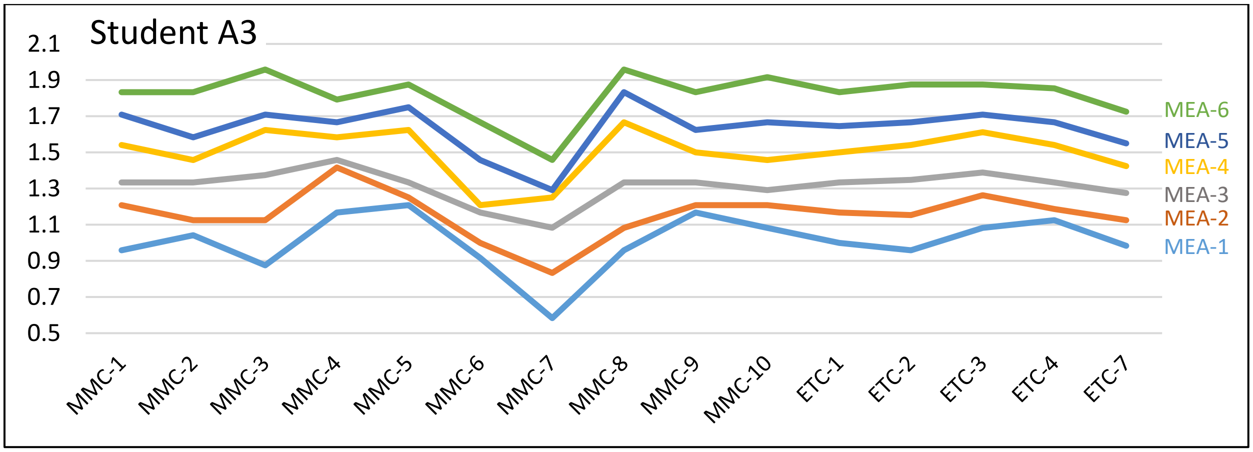


Figure 5.64 - Student A3's competence development during the six MEAs

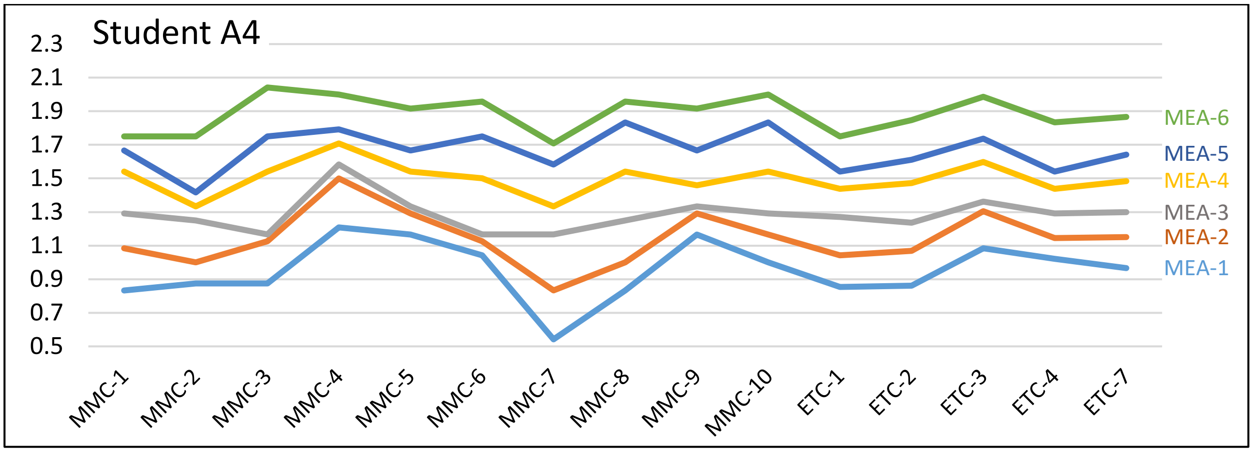


Figure 5.65 - Student A4's competence development during the six MEAs

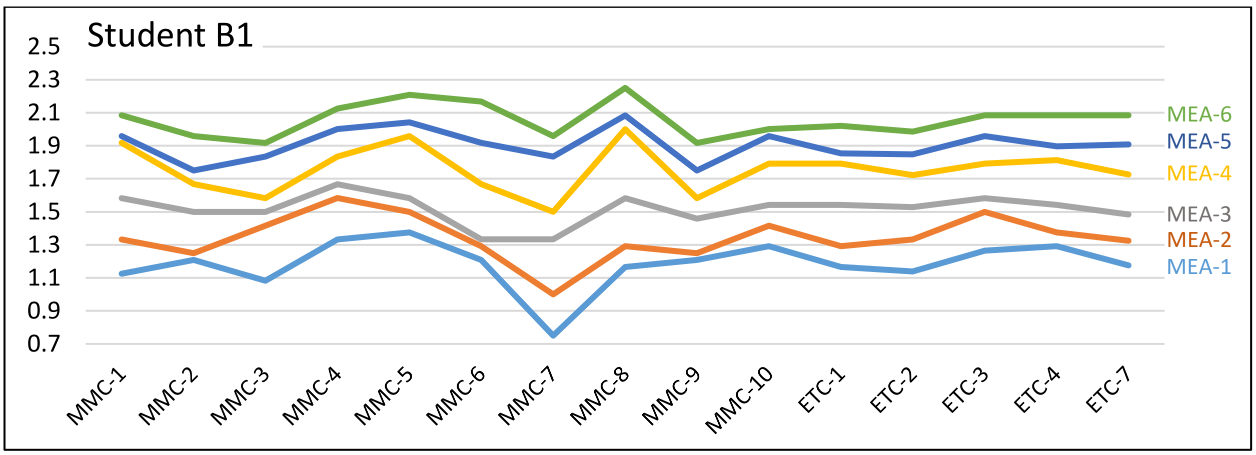


Figure 5.66 - Student B1's competence development during the six MEAs

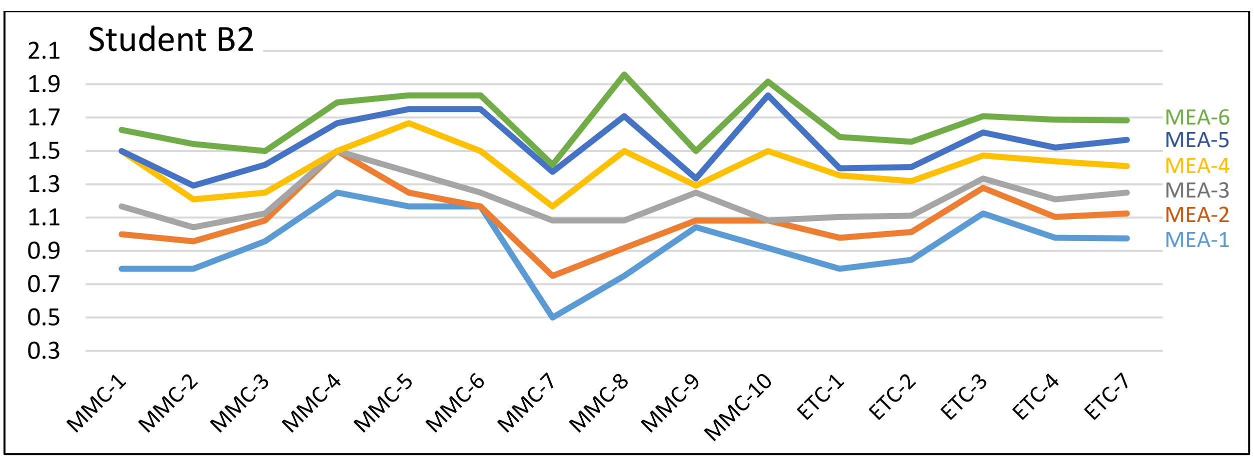


Figure 5.67 - Student B2's competence development during the six MEAs

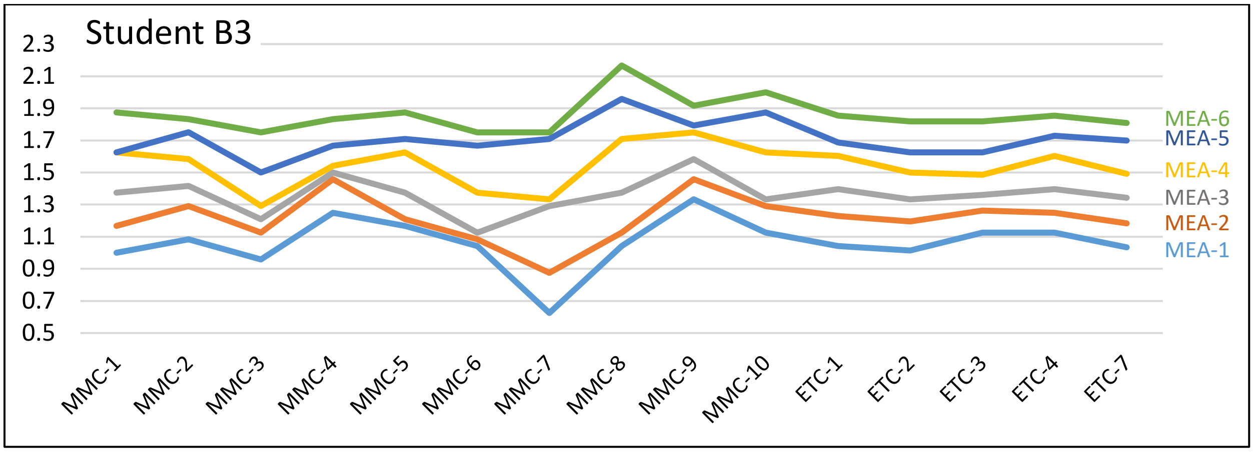


Figure 5.68 - Student B3's competence development during the six MEAs

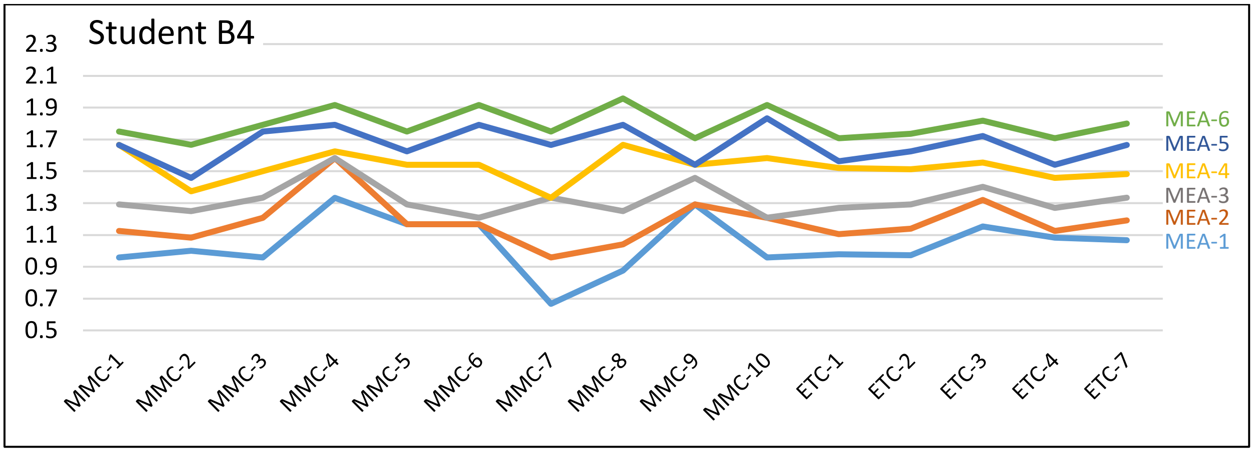


Figure 5.69 - Student B4's competence development during the six MEAs

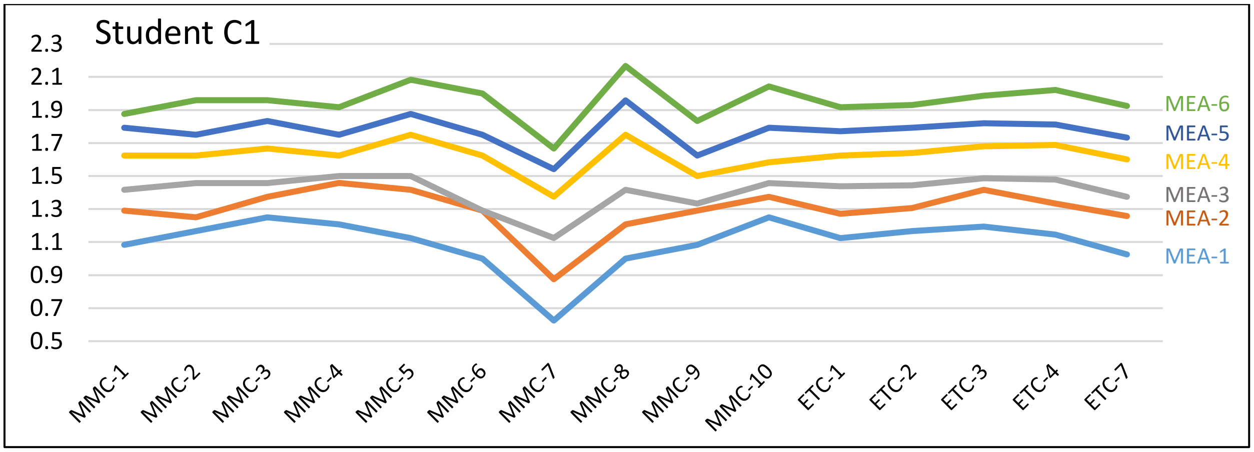


Figure 5.70 - Student C1's competence development during the six MEAs

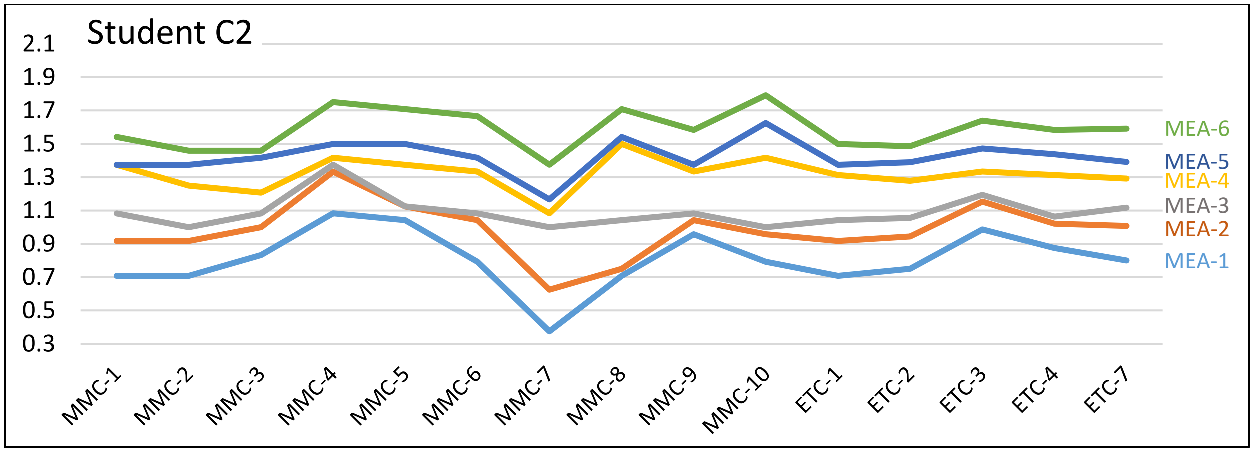


Figure 5.71 - Student C2's competence development during the six MEAs

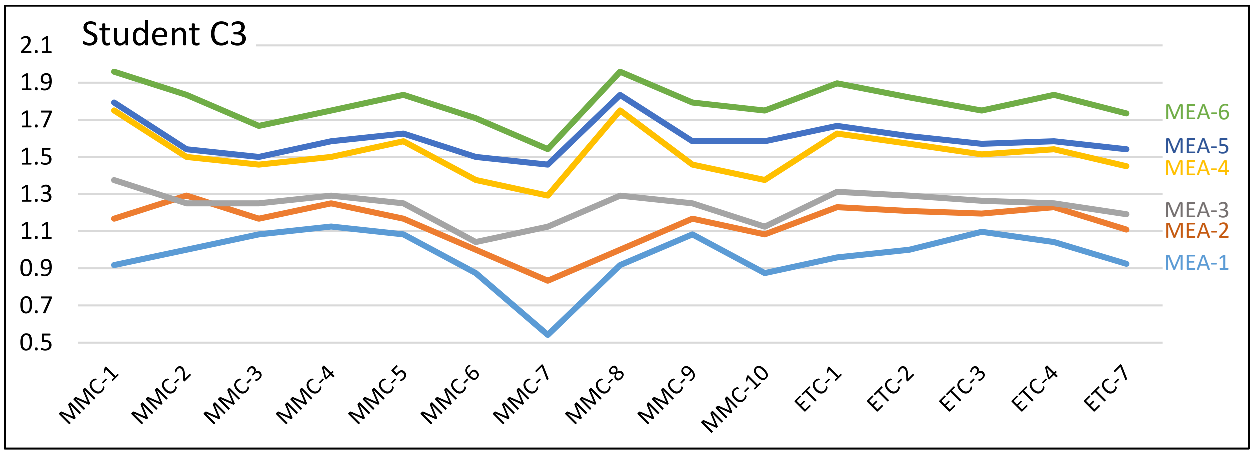


Figure 5.72 - Student C3's competence development during the six MEAs

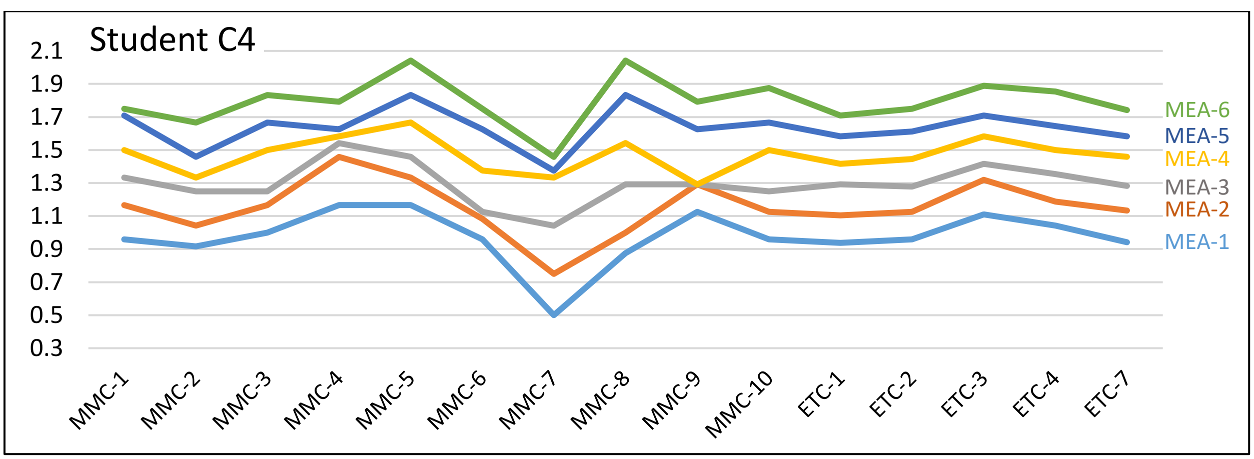


Figure 73 - Student C4's competence development during the six MEAs

The above graphs again denote slow, but consistent competence development patterns in all competence domains for all the students. These similarities probed the researcher to further investigate the students' individual competence development. The specific cognitive and meta-cognitive competencies that relates to this design experiment, were selected based on their importance in mathematics and engineering education (Section 3.7). As all competencies are regarded with equal importance to develop mathematical understanding and reasoning abilities in both mathematics and engineering education, it was decided to use the average values of the resulting competence ratings to compare the students' competence development paths. Figure 5.74 below represents the students' average development in all the cognitive and meta-cognitive competencies while they were engaged in the six activities:

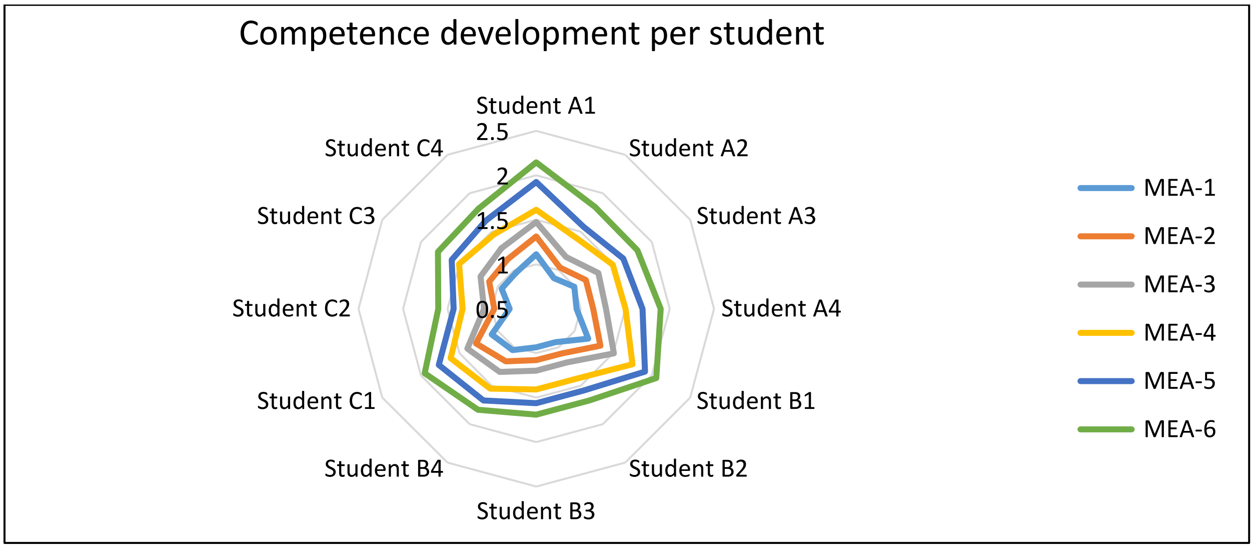


Figure 5.74 - Average progress in competence development per student

The researcher wanted to identify the best and worst cases in terms of competence development, with the intention to reach a better understanding of how and why students' competence development vary. This study was concerned with the development of competencies, regardless of the students' cognitive and meta-cognitive abilities prior to the course. For this reason, it was decided to investigate the growth patterns in competence development per student. Comparative analyses yielded the following results (Figure 5.75):

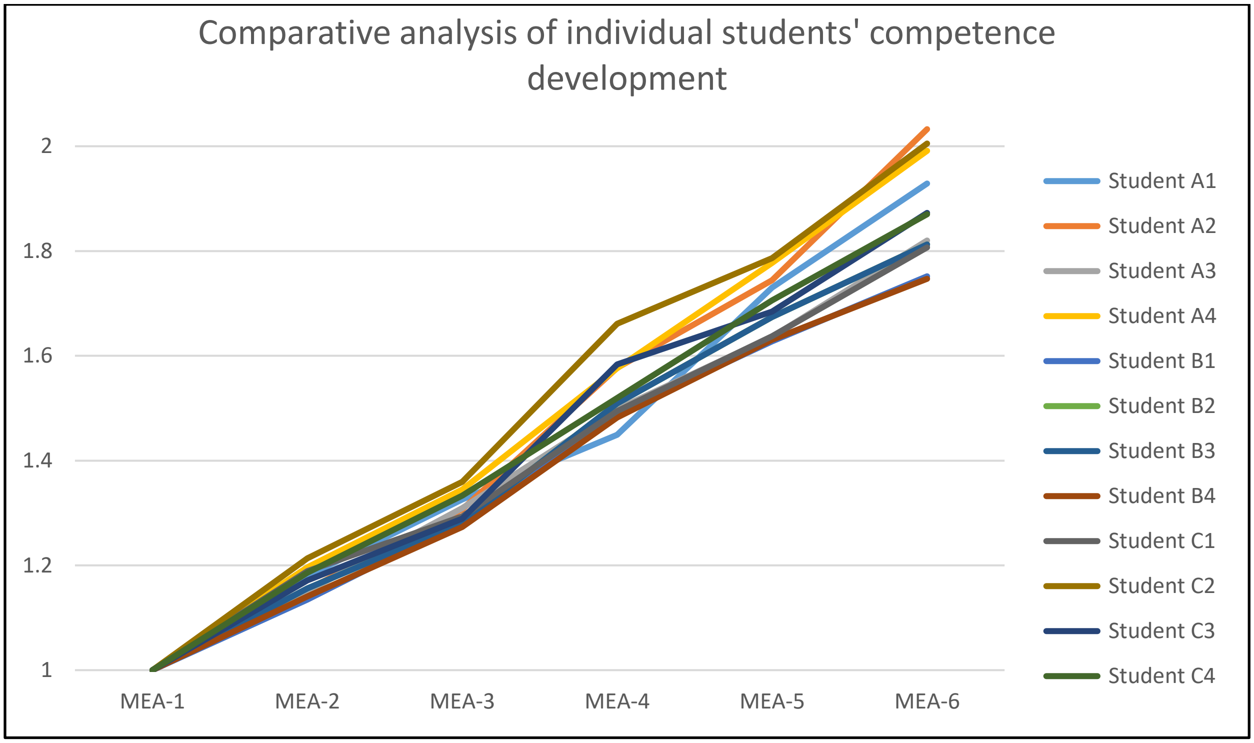


Figure 5.75 - Comparisons of individual students' competence development

The resulting growth rates in the students' average competence development over the six MEAs, are denoted in the following table (Table 5.16):

Table 5.16 – Individual students' average competence development

Average competence development per student (%)			
Student A1	92.88%	Student B3	81.25%
Student A2	103.26%	Student B4	74.70%
Student A3	82.01%	Student C1	80.69%
Student A4	99.13%	Student C2	100.52%
Student B1	75.18%	Student C3	87.28%
Student B2	81.25%	Student C4	87.01%
Class average (% growth)			**87.10%**

Significant growth in competence development was experienced across all students. Students A2 and B4 were selected as two cases for further investigation, since these two students' competence growth embodied the best case (Student A2) and the worst case (Student B4).

The results of the competence ratings of Student A2 and Student B4 at the end of each MEA, are illustrated below in Table 5.17:

Table 5.17 - Competence development of Students A2 and B4

Development of combined competencies – Students A2 andB4							
	MEA-1	**MEA-2**	**MEA-3**	**MEA-4**	**MEA-5**	**MEA-6**	**Percentage growth in competence development**
Student A2	0.9	1.0	1.2	1.4	1.6	1.8	103.26%
Student B4	1.0	1.2	1.3	1.5	1.7	1.8	74.70%

The above table results from the analyses during the design experiment. Both students' again displayed similar progress. Student A2's competence rating at the end of the first MEA was slightly weaker than Student B4's, hence the huge difference in overall competence growth. It was then decided to use the post intervention questionnaire (Appendix I), which was handed out at the end of the course. The purpose of this questionnaire was to gain more insight into the students' opinions about the mathematical modelling course. The questionnaire comprised of five questions, which were taken from Berry and Nyman (1998:108-111). Here follows the two students' responses to the questions:

Question 1: ***"A course should challenge students to stretch themselves intellectually. Did this course challenge you in ways which strengthened your ability to think and learn? And if so, how?"***

A2: *"It made me think about how the mathematics we learned, can be applied in the real-world. I can now understand errors in calculations that never made sense to me. I remember the 15% which I had to add to the 70% in the Tidal Power Task. This is the first time that these computational errors make sense to me."*

B4: *"I learned to approach problems very differently. I will now first make sure that I understand what the problem is, before trying to manipulate all the numbers I see in the question."*

Question 2: ***"Regardless of your own level of enjoyment or success in this course, do you consider the course content to have been worthwhile for your education? Why or why not? How serious was your own effort to understand and master the material covered in this course?"***

A2: *"The course helped me to think differently about mathematics, I will first try to understand the content before trying to just memorising the work."*

B4: *"Yes, it helped me to think more about the mathematics, and not just doing it. Also, I enjoyed working in groups, as we had different views of a problem which helped me to understand it better."*

Question 3: ***"In a liberal arts setting, courses should increase students' awareness of connections between related areas within their own major discipline, as well as those between their own and other disciplines. Comment on those connections you became aware of during the course."***

A2: *"I enjoyed the Cell Phone Task, as we learned about topographic maps and mathematics simultaneously. I can now see why maths is important for my career."*

B4: *"I liked the fact that you have to consider all the maths that you know, as the course does not tell you what maths to use, it just gives you a real problem which you can come across in the real world, and you have to find the maths."*

Question 4: ***"What advice would you give to other students who were planning to take this course? If you had known earlier what you know now, would you approach your own work in this course differently?"***

A2: *"This is no ordinary class. To really progress, you need to be able to work with other students, and not be shy to voice your opinions. It does not matter if your opinion is wrong (mine was wrong many times!), it helps to discuss the problem and you learn a lot from speaking to your friends about the mathematics. If I could start over, I would not have been shy about my maths knowledge. I thought I knew nothing about maths, but this course helped me to remember the maths that I have forgotten long ago!"*

B4: *"To focus on modelling helps one to think about everything very differently. I think I will advise everyone to take this course, as you approach all problems differently. To draw a simplified model of what I understand, help me to remain focused to find an answer to the problem."*

Question 5:	***"Did you learn anything in this course that surprised you? Did you ever surprise yourself?"***
A2:	*"I was surprised that I actually enjoyed a maths class!"*
B4:	*"I was surprised that, by building a model and finding the maths and going back to the model, helps a lot to understand a problem. I will now try to use this method in problems that I may be confronted with in the future."*

The above questions that were asked in the post intervention questionnaire, related to meta-cognitive competencies. The two students' (A2 and B4) beliefs about mathematics, and how they perceived themselves as users of mathematics, were exposed. In Section 2.6.2.4, Kilpatrick, Swafford and Findell (2001) described productive disposition as an interwoven component of one's mathematical proficiency. Productive disposition is defined as the "tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001:131), thus it encompasses issues like a person's affect, beliefs and identity. The tendency to see sense in mathematics is in essence one's beliefs about the nature of mathematics, and can change over time (Siegfried, 2012:25). Resnick (1987:41) affiliates 'disposition' with a 'habit of thought', which can be *learned and taught*, implying that humans are not born predisposed to a specific affection towards mathematics, but it can be altered in a positive direction through effective learning and teaching. This statement implies that not only the beliefs of the students, but also those of the teachers, need to be altered.

Some of the common beliefs that can lead to strong and often negative influences on students' mathematical thinking, is the belief that mathematics problems have only one correct answer, which is produced by only one correct way of solving it. Other negative beliefs are that the understanding of mathematics is reserved for the priviledged gifted students and should be memorised by the rest, that mathematics is a solitary activity done in isolation, and that mathematical problems should be solved in a short period of time, and has no relevance to the real-world. Lastly, students often believe that formal proof is irrelevant and unnecessary to the processes of discovery or invention (Lesh & Zawojewski, 2007:776). These negative beliefs were exposed during the pre-intervention interviews. However, during this study, mathematical modelling allowed the students to alter these beliefs as they learned to grapple with meaningful situations and developed new knowledge by building and expanding their existing knowledge base. Meaningful situations motivated the students to persevere in their tasks, and it also allowed for opportunities to view mathematics as useful when they applied the mathematics in real-world situations (Section 5.2).

Lesh and Zawojewski (2007:776) also emphasised that students' negative beliefs and dispositions towards mathematics teaching can be limited through changes in instruction, teacher's practices, the curriculum, or the culture of the classroom. The researcher as teacher followed the advice from Galbraith and Stillman (2001) to be *willing and ready* to create and manage open situations, which continuously transformed as the students'own initial ideas were regarded as relevant, rather than relying exclusively on the decriptions of experts' behaviours and experiences.

Both students displayed positive feedback, especially in terms of group work, the utility of mathematics, the systematic way of thinking about problems offered by mathematical modelling, and their enjoyment of mathematics. Even though these two students' competence development in terms of percentages indicate a large discrepancy (74.70% vs. 103.26%), the real results were very similar (Table 5.17). Throughout the six MEAs, the students went through sequences of interpretation-development cycles where they thought about the solution processes in different ways (Section 5.2). Chamberlin (2004:53) explains that the variances in the students' interpretations of the problem situations are also based on their individual levels of mathematical ability and general knowledge.

The students' explicit references to reasoning and mathematical understanding during the post intervention interviews were interesting. The goal of this study was to co-develop specific mathematical modelling and engineering technician competencies to foster more productive reasoning and understanding of mathematics. *The case study thus served as confirmation to the researcher that the competencies which were investigated, were indeed the competencies that promote students' reasoning and mathematical understanding abilities*. Chapter 3 explained the detailed investigation of the competencies that can significantly influence mathematical reasoning and understanding, as well as technician engineering education. These competencies were all defined, and by mapping the engineering technician and mathematical modelling competencies, the strong relation between mathematical modelling competencies and engineering technician competencies was exposed. The students' continual exposure to, and active engagement in mathematical modelling, allowed the researcher to investigate the development of these competencies for the whole class, per group and per individual student. The subsequent case study research again emphasised that, even though students progressed differently at different stages of a modelling cycle, all students' are able to progress and develop their mathematical modelling and engineering technician competencies through engagement with mathematical modelling, regardless of the abilities and knowledge that they bring to the class.

5.4 A LOCAL INSTRUCTIONAL THEORY (LIT) FOR THE CO-DEVELOPMENT OF ENGINEERING TECHNICIAN AND MATHEMATICAL MODELLING COMPETENCIES

This study attempted to develop an understanding of the co-development of mathematical modelling and engineering technician competencies, together with instructional activities and other means, to support competence development for improved mathematical reasoning and understanding. The explanations provided in Sections 5.2 and 5.3 on how the competencies advanced, must never be viewed in isolation. The intricate web of teacher beliefs, classrooms designed to foster socio-constructivist forms of learning and guided teaching, meaningful contextual activities designed to elicit the need for model-construction to develop deeper and higher-order understanding, requires from students to engage in critical thinking to explain and justify their solution processes (Brown, 1992:141; Lesh & Clarke, 2000:127). All the aspects of this complex web, together with the assessment methods designed to recognise and describe the nature of the students' constructs, while simultaneously offering guidelines to both teachers and students, were inputs for revising and refining the HLT to progress towards the construction of a local instructional theory (LIT), and thereby undertook to answer Aim 13 of the research question (Section 1.8.3).

This study was concerned with the amalgamation of instructional tasks, students' competence development, the role of the researcher/facilitator/teacher, classroom norms, and formative assessment instruments. Meaningful and contextual activities were designed to allow for the elicitation of conceptual constructs, and the experiment unfolded in a learning community that fostered socio-mathematical norms, to optimise learning through group discourse. The researcher/facilitator created and managed open situations which continually transformed, as students' own constructs were used to develop, extend, and refine their mathematical understanding and problem-solving abilities. The ultimate goal of this study remained the co-development of engineering technician and mathematical modelling competencies to enhance students' mathematical reasoning and understanding. Lesh and Clarke (2000:132-133) noted that the development of students, teachers and programs, is interdependent and inseparable. Care must be taken to not contribute the development of specific competencies solely to the result of passive acceptance of information, but also to regard teachers and facilitators as integral role players in the development of students' problem-solving competencies.

Doerr and Lesh (2003) and Lesh and Doerr (2000) in Doerr (2006:256) emphasise the importance of a teacher to *respond in ways that will support students' conceptual development* towards more refined, generalised, flexible, and integrated ways of thinking. By asking students to explain the strategies that they have used to arrive to meaningful solutions, the researcher/facilitator created situations in the classroom where students had the opportunities to share and justify their ways of thinking with one another. Through meaningful discourse, the students continually revised and refined their ways of thinking. Anticipated schemas about the students' ways of thinking were prepared prior to each MEA and updated and refined as the experiment unfolded. This schema further assisted the researcher/facilitator to develop new and improved ways of responding to the students' problems, to promote mathematical reasoning and understanding.

Collins et al. (2004:20-21) noted three aspects of design experiments that influence both the methodology and the assessment methods. These three aspects relate to the real-life context of the problem situations, the aim of the design study, and the consideration for all the participants in the experiment. This design experiment occurred in messy, meaningful, real-life situations, where numerous factors influenced other depending variables of interest. These factors included teachers, students, classroom ethos, contextual activities, and reflection tools, as they were all inputs of the working whole (Section 1.9.1). Robust explanations relating to these variables were provided during the construction and refinement of the HLT (Section 5.2). As discussed in Section 4.2.1, the thorough investigation of the variables, combined with adequate documentation procedures of the entire design process, enhanced the theoretical clarity of the study, to further aspire to increase the possibility for this study to migrate to other classrooms as well. However, this study only involved 12 first-year engineering technician students, who studied at a civil engineering department of an University of Technology in South Africa. This means that the results cannot necessarily be generalised over a wide spectrum. As explained in Section 4.5.4, design research is concerned with explanations of the processes and mechanisms that caused the changes in learning, to explain the developmental process of a single case (Cobb et al., 2016:4). The focus is thus not on the comparisons of generalised results to quantitative studies that emerged in large sample sizes. The researcher shares the sentiment of Olson in Engeström (2011:598) that

> … the reputation of educational research is tarnished less by the lack of replicable results than by the lack of any deeper theory that would explain why the thousands of experiments that make up the literature of the field appear to have yielded so little.

The aim of this study was not to test hypotheses, but to develop a qualitative and quantitative profile that characterises the design in practice. DBR's consistent and logical methodology was used to link theoretical research and educational practice. The design (Chapter 4), the interventions (Section 5.2), as well as its effect on learning in a specific context (Sections 5.2 and 5.3), were investigated to provide explanations and principles of innovative learning to be localised in new settings (Section 4.2.2).

The six MEAs that were desiged for this study (Section 4.3.1.5 and Appendices K–P), allowed students to express their ways of thinking in ways that they could test, revise and refine it themselves, and simultaneously offered opportunities for the researcher to examine specific aspects of the students' development within mathematically enriched environments. Lesh and Clarke (2000:137) and Lesh and Harel (2003:176) commented that these opportunities permit researchers to make first-hand observations about their students' ways of thinking. The MEAs were designed for the students to create initial (even 'wrong') models and to improve such models through the processes of testing, refining, adjusting and revising. The focus was therefore on improving models, rather than regarding the students' initial primitive models as 'failing solution paths' (Zawojewski, 2013:239). As a meta-cognitive coach, the researcher interacted with the students by probing, instead of asking questions. Self-assessment principles also applied, whereby the students learned to direct, explain and validate their own ways of thinking. The trails of documentation that students left behind, provided invaluable information to the researcher about students' thought processes. Students decided within their groups how to solve a specific problem, and they then needed to be creative to identify the necessary mathematical processes to lead them towards becoming consumers of mathematics (Bahmaei, 2013:46; Chamberlin & Moon, 2005:41; Galbraith & Stillman, 2006:143; Lesh & Zawojewski, 2007:794; Maaß, 2006:115).

In reponse to the MEAs, assessment instruments were designed to identify and analyse the steps that the students took when they solved real-world problems. These assessment instruments were developed to provide insights into the level of the students' conceptual learning, and also assisted the researcher to assess meta-cognitive processes, in particular teamwork (communication skills), management, and responsible behaviour (Yildirim et al., 2010:234). This study used various methods for collecting student responses for MEAs, e.g. Status Update Reports, Researcher Observation Guides, Quality Assurance Guides, Group Modelling Competency Observation Guides, Poster Presentation Guides, Student Reflection Guides, Group Functioning Sheets, informal interviews, walk-throughs, video and audio recordings (Section 4.3.1.6 and Appendices A-I).

Assessment methods applicable to this study, were predominantly formative rather than summative. Whereas summative assessment methods test students' work towards the end of instruction when it is often too late to repair misconceptions, Schoenfeld (2013:19) explains the purpose of formative assessment "to find out what the student knows while you can still do something about it". Formative assessment is concerned with putting students in situations where they need to grapple with the problem content to construct meaningful solutions. Furthermore, Schoenfeld (2013) considers quality feedback as crucial to develop and assist the students toward improving the quality of their work, while summative assessments often only assign scores to tests with minimal feedback to students. The feedback that was provided during formative assessments, made it possible for the researcher and the students to identify strengths and weaknesses, to suggest strategies for improvement, and to reflect on teacher-student relationships, specifically when the researcher entered into a discussion with the students about their thought processes (Yildirim et al., 2010:839).

Teaching programs can also develop in directions that are continually 'better' without basing the next steps on preconceived notions of 'best'. Self-documenting activities served to encourage the students and the researcher, while it simultaneously produced trails of documentation that exposed significant features about the nature of what was being learned (Lesh & Clarke, 2000:136).

As mentioned before in this chapter, growth in one competency does not occur in isolation, but many interrelated variables should be considered. The classroom' functionality as a learning environment, and the continuous interplay between the teacher, the student and the tasks, are all considered inputs into the working whole. Changes in one aspect can affect the outcomes in the other (Brown, 1992:141).

The students' responses to the six MEAs triggered the development of their internalising and interpreting competencies through a series of modelling cycles and tasks, where they continually considered new and different kinds of information. This new information thus created the need for further refinements of their initial interpretations. Unstable conceptual systems evolved into more sophisticated interpretations and the students slowly progressed in learning to think – in the words of Lesh and Clarke (2000:132) – "'*about'* a given construct, and not only '*with'* a given construct".

Similar to cognitive competencies, the study also aimed to investigate complementing engineering technician and mathematical modelling *meta-cognitive* competencies. Garofalo and Lester Jr (1985:171) noted that awareness and control of knowledge are crucial ingredients of successful cognitive performances. Brown and Smiley (1977:7) researched learners' abilities to identify important information in texts and concluded that such strategic behaviours as monitoring and assessing one's work, develop over long periods of time and novices still need to develop such skills to use them effectively. The limited knowledge that students have about their own capabilities and limitations with respect to mathematics (beliefs), about the nature of mathematical tasks, as well as about strategies to aid in problem-solving situations, are all crucial in complex problem-solving activities and these all need to be coordinated. Such limitations caused difficulties for the students to analyse data, to monitor progress and to evaluate solutions, which are skills that often lack in undergraduate students (Schoenfeld, 1983).

The mapping of the competencies related to the two disciplines (mathematics and engineering education), allowed the researcher to limit the huge number of different meta-cognitive competencies. The mapping process resulted in focusing on the meta-competencies of communication, management and responsible behaviour. Schoenfeld explains managerial decisions as those decisions that impact the solution process and how one's resources are allocated during a problem-solving task (1983:2). He further suggests that heuristic fluency "is of limited value if the heuristics are not 'managed' properly". Managerial decisions are made between episodes of action, which has the possibility of 'making or breaking' a problem-solving attempt based on the quality of such decisions (Schoenfeld, 1983:2). Problem-solving managing does not only involve making decisions regarding what path to choose, but also incorporates assessments of the students' progress. This study distinguished between management and responsible behaviour competencies, in that the latter competency is focused on working with a sense of direction (clear end goal at all times), as well as being able to work independently within groups without the constant supervision of a teacher or facilitator (Section 3.6).

This chapter built on steadily towards describing a learning instructional theory (LIT) that can support the co-development of engineering technician and mathematical modelling competencies for enhancing mathematical reasoning and understanding. All aspects of learning and teaching mathematics through mathematical modelling, by utilising MEAs within a socio-constructivist learning-teaching environment that adheres to the principles of the RME theory, were explained, which served to address the last aim of the research question. This last aim, Aim 13, required the construction of a learning trajectory that not only addresses classroom norms and discourse, but also explains how the possible shifts in students' reasoning abilities occur (Section 1.8.3).

5.5 CONCLUSION

This chapter explains the complex processes of assessing the possible co-development of engineering technician and mathematical modellers' cognitive as well as meta-cognitive competencies. Throughout the teaching experiment, adequate data were collected and produced for evaluation of such competencies. Argumentative grammar (an integral part of DBR) allowed for connecting the theory, methods and empirical research, thus guided the researcher to ensure that all learning processes justified the products of this study. Possible methodological dilemmas and short-comings to this study will be discussed in Chapter 6.

Both cognitive and meta-cognitive competencies were investigated. The results of this study shows that mathematical modelling competencies do not develop in isolation but, as Brown (1992:146-147), Garofalo and Lester Jr (1985:163) and Schoenfeld (1983:3) noticed, the knowledge and beliefs that people have about their own mathematical learning (person, task and strategy), have a profound impact on how students develop cognitive intelligence relating to understanding problems, interpreting information, building and analysing conceptual models, and reorganising and adjusting their ways of thinking to proceed towards obtaining solutions of real-world scenarios. This intertwining character of competence development probed the researcher to employ a study which was longitudinal in nature, as the cognitive changes could be determined appropriately over a period of time and not merely over one or two days. Also, teaching did not consist of straight didactic teaching methods. Interventions were not one-on-one, but followed Brown's (1992:147) suggestion to incorporate the social context of learning, typical of everyday learning and teaching in the classroom. The designed activities allowed students to externalise their thought processes and it also provided a structure to scaffold student discourse towards taking ownership of their knowledge. The content of the tasks were focused on themes that allowed the students to engage in deeper levels of understanding. As the activities unfolded, the researcher regarded it necessary to introduce a technological environment to further encourage intentional learning, reflection and communication. Such opportunities enabled the students to focus on their abilities to discover and use knowledge, rather than merely retaining it (Brown, 1992:150).

The synergy between the goals of mathematics education and engineering education was exposed in Chapter 3, which allowed for the selection of specific competencies to be investigated and assessed in this study through mathematical modelling. As discussed, results of this study indicated very slow but consistent progress in all competencies. Even though competence development seems to be a slow and cumbersome process when reflecting on the progress as assessed during this study one semester, the researcher wants to draw the reader's attention to Bereiter's (2002:323) correlation between the development of the automobile and the possible road of development in education. He commented that the automobile development process was driven by an awareness of potential that was not fully grasped at the outset. However, during a process that unfolded over a period of almost eight decades, potential continued to unfold as the development proceeded. Bereiter (2002:327) thus pleaded for educational visionaries, rather than early adopters. In his words:

> Design research is not defined by its methods but by the goals of those who pursue it. Design research is constituted within communities of practice that have certain characteristics of innovativeness, responsiveness to evidence, connectivity to basic science, and dedication to continual improvement (Bereiter, 2002:321).

As a design researcher, boundary-crossing between observer and actor allowed for close attention to negative results, misconceptions and incorrect interpretations, which could be addressed as they emerged. In the course of the design cycles and accompanied research, emergent goals continually arose and evolved, but the main focus remained on competence development that promote increased reasoning and mathematical understanding. Further development revealed further potential, and DBR created the platform to hold onto a visionary quality that was driven by the potential of developing students' mathematical reasoning and understanding. This visionary quality allowed the researcher to see ways that mathematical modelling can help in achieving these long-term goals. As the research continually produced findings that were fed back into further design cycles, new potential was noticed, complementing Whitehead in Bereiter's (2002:325) view that "the greatest invention of the nineteenth century was the invention of the method of invention" – alias Design-Based Research.

CHAPTER 6

SUMMARY, CONTRIBUTIONS AND RECOMMENDATIONS

An investment in knowledge pays the best interest ~ Benjamin Franklin (1706 – 1790)

6.1 SUMMARY

This study aimed to address the current gap in mathematics and engineering education. Mathematics contributes significantly towards engineering education, as engineers constantly need to evaluate, analyse, interpret and solve real-world problems, denoting the prominence of developing mathematical competence for engineering professionals. One of the goals of mathematics education is the promotion of efficient mathematical thinkers, where students obtain the habits of interpretation and sense-making (Schoenfeld, 1992). The goal of the study was to provide an explanation of how engineering technician and mathematical modelling competencies can co-develop through mathematics education to enrich mathematical reasoning and understanding, which led to the construction of the research question:

> *To what extent can engineering and mathematical modelling competencies co-develop to produce a deeper understanding of mathematics within the context of a mathematical modelling course for first-year engineering technician students who are not strong in mathematics*?

This research question provoked the investigation of further sub-questions:

Sub-question 1: How/where does mathematical modelling fit into the context of mathematical teaching approaches to develop mathematical reasoning and understanding?

Sub-question 2: Which engineering technician and mathematical modelling competencies can co-develop through mathematical modelling?

Sub-question 3: How do engineering and mathematical modelling competencies co-develop to nurture reasoning and deeper understanding of mathematics?

Sub-question 4: How can competence development and mathematical reasoning be measured in the students' work?

By thinking methodologically, this study's focus was driven by the main purpose of finding ways to develop the students' reasoning and mathematical understanding. Literature (Chick & Stacey, 2013; Eisenhart et al., 1993; Lawless, 2005; Singh & White, 2006; Woollacott, 2003), exposure to mathematics and engineering education, and to the engineering profession, all pointed to poor understanding and reasoning abilities as crucial set-backs of successful students and professionals. These current pitfalls motivated the researcher to engage in a study to address the gap in knowledge of how to develop the competencies to allow students to enrich their reasoning and understanding of mathematics by employing mathematical modelling (Sections 1.4 and 1.5). In answering the research question, gaps could be identified and subsequent alternatives could be employed, such as modelling approaches to teaching and learning mathematics, which were discussed in Chapter 2.

A research procedure was required to meet the needs of the research question and accompanied sub-questions and aims. Veresov (2014:215-228) pointed out in his writings on method, methodology and methodological thinking, that the research question should always drive the research methods, and not vice versa. A real answer to a real teaching and learning problem was sought, and design-based research (DBR) created a platform to hold onto a visionary quality that was driven by the potential of developing students' mathematical reasoning and understanding. The outputs of DBR were further developed through case study research, as case study research aims to provide descriptions and understandings by combining various data collection methods (Chapters 4 and 5).

The results of this study indicated very slow, but consistent progress in all competencies (Chapter 5). It also revealed that, with continual exposure to mathematical modelling, even under-privileged students are able to progress towards increased mathematical understanding and reasoning. The strong relation that was exposed in Chapter 3 between the goals of mathematics and engineering education, paved the way to investigate the co-development of both engineering technician and mathematical modelling competencies through mathematical modelling. The competencies that were selected to follow and assess, did not develop in isolation. The knowledge and beliefs that people (both students and educators) have about their own mathematical learning, have a profound impact on how students develop cognitive intelligence relating to understanding problems, interpreting information, building and analysing conceptual models, and reorganising and adjusting their ways of thinking to proceed towards obtaining solutions of real-world scenarios

(Schoenfeld, 1983:3). Section 5.4 exposed a learning instructional trajectory (LIT) that is required for co-developing mathematical modelling and engineering technician competencies through mathematical modelling, to nurture reasoning and deeper mathematical understanding. This LIT as presented in Chapter 5 comprised of learning goals, planned instructional activities, and an envisioned learning process.

This chapter will conclude the study by discussing the study's modest contributions to knowledge, critique of the research, the limitations of the study, the methodological dilemmas experienced, as well as suggestions for further research.

6.2 CONTRIBUTION TO KNOWLEDGE

To explain the contribution to knowledge, this section will be dealt with in four parts. Firstly, the researcher will summarise the gap in mathematics and engineering technician education, followed by a rephrasing of the current gap in mathematics and engineering technician education that she observed to elicit her contribution to knowledge. Thirdly, an explanation will be presented to provide evidence for allowing the claim to a modest contribution that is plausible and can be defended through the rigor of this research approach and methodology. Lastly, justification for the researcher's claim of contribution to knowledge will be provided.

6.2.1 Gap in knowledge

By undertaking an investigation of both mathematics and engineering education, the goals of mathematics and engineering education both pointed towards the importance of mathematical understanding, real-world problem-solving, and adequate meta-cognitive competencies (Section 1.3). Woollacott (2003) stresses a concern for the South African engineering students, as the current engineering technician education does not adequately prepare them to successfully achieve their qualifications. Apart from addressing knowledge base concerns, educational gaps occurred in terms of developing meta-cognitive competencies, such as team-working, decision-making and effective communication, even though such skills are regarded as essential building blocks to become successful students, as well as professional engineering technicians (Marra et al., 2016). These concerns motivated the researcher to engage in a study to fruitfully enhance engineering technician students' competencies to proceed towards successful mathematical problem-solvers, mathematical thinkers and mathematical doers. Section 1.4 noted that no in-depth studies were found that identified and investigated the engineering technician competencies that can co-develop with mathematical modelling competencies through modelling-based mathematics teaching and learning, and this study served to fill this gap in knowledge in the field of mathematics and engineering education.

6.2.2 Rephrasing the gap as a modest contribution to knowledge

To contribute to knowledge, the gap as explained above was addressed as follows :

- An in-depth literature review (Chapter 2) indicated that mathematical modelling has the potential to provide students with opportunities to engage in real-world problem-solving where they learn to understand, interpret, solve, reflect and justify their solution methods. From an instructional point of view, the focus is shifted towards engaging the students in revising, refining and extending their own ways of thinking about a problem, to engage the students in addressing their habits of mind. Cuoco, Goldenberg and Mark (1996:378) describe habits of mind as "mental habits that allow students to develop a repertoire of general heuristics and approaches that can be applied in many different situations".

  Model-eliting activities (MEAs) were sourced from literature (Section 4.3.1.5), and the researcher adapted some of the tasks to suit local conditions. MEAs allow for multiple methods to solve problems through processes involving mathematising in all its facets: quantifying, dimensionalising, coordinating, categorising, algebratising, systematising the relevant objects, relationships, actions, patterns and regularities (Lesh & Doerr, 2003:5). The cyclic nature of MEAs allow the students to repeatedly reveal, test and refine their ways of thinking. Their interpretations of the problem situations mature while progressing through the various sequences. MEAs emphasise the active sense-making of meaningful situations through invention, extending and iterative refining of the students' own mathematical constructs (Bahmaei, 2013:35). The emphasis moves away from imparting strategies and skills, towards allowing the elicitation of a model that the group of students use to interpret or make sense of a problem (Hamilton et al., 2008:10). The students' final solutions portray what they value as important aspects of their mathematical thinking (Lesh & Zawojewski, 2007:784). This process of solving the problems is thus emphasised far more than the solution itself, which is in contrast with traditional problem-solving activities (Diefes-Dux et al., 2004:F1A-3). By interpreting the problem-solving situations mathematically, their interpretations can go beyond mathematics and also include feelings, dispositions, values and beliefs (Lesh & Zawojewski, 2007:784-785), which again paves the way towards developing a holistic set of competencies to be used in their future lives and careers. These activities can generate trails of documentation that also reveal information about the students' ways of thinking, to support the productivity of ongoing learning or problem-solving experiences (Lesh et al., 2000:594). Instead of having to produce brief answers to questions formulated by others, model-eliciting activities require students to be deeply engaged in mathematising to develop, explain and interpret specific situations by themselves.

- Chapter 3 identified particular competencies that could develop the students' problem-solving and meta-cognitive abilities, to ultimately enhance their mathematical reasoning and understanding.
- A mapping process between mathematical modelling and engineering technician competencies allowed the researcher to simultaneously follow and assess the most essential competencies in both disciplines.
- Furthermore, competence development had to be followed and assessed. In combining assessment rubrics from literature (Arter & McTighe, 2001), a new Group Modelling Competency Observation Guide was designed to reflect the students' competence progress (Table 3.16). This observation guide was used in conjunction with various other assessment and reflection instruments, as well as informal interviews, walk-throughs, field notes, memos, video and audio recordings (Section 4.3.1.6).
- Chapter 5 provided an explanation of how students can co-develop both mathematical modelling and engineering technician competencies to allow a deeper understanding of mathematics, to be able to engage successfully in all the facets of mathematising, and to develop the meta-cognitive competencies required by the engineering technicians of today.

6.2.3 Evidence of a modest contribution to knowledge

Evidence of the contribution to educational knowledge was exposed through the methodology of design research. The purpose of design research is to “develop theories of the processes of learning as well as the means designed to support that learning in naturalistic settings” (Gravemeijer & Cobb, 2006:18). The ultimate aim was not to directly apply a theory to existing problems and evaluate whether it works or not, but rather to strive to find a practical and effective solution to a real teaching and learning problem, by incorporating an interactive learning environment within a specific context.

Gravemeijer (2004:104) argues that design research provides “an empirically grounded theory on how the intervention works”. By carefully reporting and documenting on all the design processes, relevant data were collected and new theories were developed by using this data (Section 5.4). The processes of planning for a conjectured learning trajectory, implementing it and reflecting upon it, were iteratively repeated to allow the researcher to build on the knowledge and insights gained from coding, analyses and scrutinising of the data, while comparing it to present and past literature (Section 5.2). With each iteration, the learning trajectory was examined and refined to allow for an improved learning instructional theory (LIT) to emerge from the data (Section 5.4). As already stated in Section 6.1, the LIT comprises of learning goals, planned instructional activities, and an envisioned learning process. This emerged theory of teaching and learning was tested against the outputs of DBR: primary output (design principles), secondary output (societal contribution) and tertiary output (the professional development of participants) (McKenney et al., 2006:72). These outputs were further shaped by the principles of rigor, relevance and collaboration.

The scientific outputs in the form of *design principles*, contain substantive and procedural knowledge that provides an accurate portrayal of the procedures, results, as well as the context. The descriptions on how the students’ competencies co-developed to increase their reasoning and mathematical understanding are detailed in Section 5.4, which serve as a guide to inform others about relevant insights to their own specific situations. Each intervention was carefully described and analysed to provide such information. The tight link between empirical research and theory, triggered strong principles that can answer to the above research question. Meticulous data collection and analysis served to ensure that the study adhered to rigorous standards. The way in which the researcher handled methodological concerns is laid out in Section 6.5.

The second output, the *societal contribution*, relates to the materials that were used in the classroom, as discussed in Sections 4.3.1.5 and 4.3.1.6. Instructional activities and measuring instruments were continually adapted and refined, and these adjustments were discussed throughout Chapter 5. The activities and measuring instruments are valuable products to society, and are relevant to the context and culture in which engineering technician students studying mathematics at their specific institution will implement them.

Thirdly, the tertiary output denotes the contribution that the design research activities make in terms of the *participants' professional development*. Apart from actively taking part in the activities, students' own professional development can also be enhanced while the researcher or practitioner applies data collection methods such as interviews, walk throughs, discussions, observations and questionnaires which can stimulate dialogue, reflection and engagements among the participants (McKenney et al., 2006:72). The model-eliciting activities, together with the data collection procudures, were mutually beneficial to both the researcher and the students, as the researcher addressed research needs, while the students were enriched through exposure to modelling tasks. Students' positive stance at the end of the course, combined with their increased abilities to engage in mathematising, enhanced their own professional development. Furthermore, the design and development were conducted in *collaboration* with all the stakeholders – students, lecturers, researcher, and the University of Technology where the study took place.

These three outputs of design based research thus serve as a contribution to address educational ways of nurturing students' mathematical modelling and engineering technician competencies through mathematical modelling.

6.2.4 Justifying the claim

The researcher's claim for a modest contribution to knowledge can be justified based on the new understandings that emerged from existing issues. Parmjit and White (2006:36) emphasised the importance of developing critical competencies for professional engineers to confront serious problems in the real world. Kaiser (2007:110) identified one of the goals of mathematics education as "the development of students' capacities to use mathematics in their present life as well as in their future lives, which calls for the importance of stimulating modelling competencies" (Kaiser, 2007:110). Blomhøj (2009:4) regarded mathematical modelling as a special type of problem-solving that promotes students' learning and understanding, as contextual problems support a reinvention process that enables students to develop the required competencies that allow a deeper understanding of formal mathematics.

The focus of this research was to find ways to develop the students' reasoning and mathematical understanding to address the educational concerns as discussed in Chapter 1. The researcher aimed to provide an understanding of how both mathematical modelling competencies and engineering competencies co-develop in students, by introducing mathematical modelling to the students. By mapping the essential mathematical modelling and engineering technician competencies, and by designing a Group Modelling Competence Observation Guide (Appendix D), it became possible to follow and assess the students' co-development of both disciplines' competencies. The research resulted in a learning instructional theory (LIT), that explained how the shifts in the students' reasoning and mathematical understanding occurred, while they grappled with modelling activities (Section 5.4).

6.3 CRITIQUE OF THE RESEARCH

Upon reflecting on this research study, the thought of approaching my study differently provoked a few concerns: What if I chose another tool than mathematical modelling to nurture the students' mathematical understanding? What about a deductive approach to eliminate the problem of validity? What would have happened if I chose to do a laboratory setting rather than the classroom setting? How would these approaches have altered the design, fieldwork, findings, conclusions and outcomes of my study?

Chapters 2 and 3 exposed the consequences of learning and teaching mathematics through direct instruction as opposed to mathematical modelling. Whereas the traditional framework of mathematics education predominantly focuses on mastering skills and facts, students tend to revert to rote learning, which results in the neglect of deeper and higher-order thinking (Lesh & Clarke, 2000:120-122). They learn to master facts and skills one step at a time and in isolation, and they do not necessarily know when to choose which fact or skill to use in a specific situation (Section 2.3.4). Students believe that the answers and methods to problems will be provided for them, thus they tend to play a passive role in learning mathematics, and they think of mathematics as 'handed down' to them by experts to memorise (Parmjit & White, 2006:343). This passive role can further affect their beliefs about mathematics, such as that mathematics is reserved for the priviledged gifted students and should be memorised by the rest, that mathematics is a solitary activity done in isolation, and that mathematical problems should be solved in a short period of time, and has no relevance to the real-world. Lastly, students often believe that formal proof is irrelevant and unnecessary to the processes of discovery or invention (Lesh & Zawojewski, 2007:776).

Mathematical modelling regards real-world situations as the starting point from where students explore and reinvent mathematics that is experientially real to them, as opposed to the deductive approach of starting with the product of mathematisation. MEAs emphasise the active sense-making of meaningful situations through invention, extending, and iterative refining of the students' own mathematical constructs (Bahmaei, 2013:35). This *process* of solving the problems is emphasised far more than the solution itself, which is in contrast with traditional problem-solving activities (Diefes-Dux et al., 2004:F1A-3). During this process, students learn to develop 'mathematical thought' competencies to abstract critical information, to mathematise, interpret, verify, and communicate solutions to others. Students learn to generate mathematical constructs through developing ways of thinking that may cause previously existing conceptual systems to be integrated, differentiated, extended, or refined in significant ways. Furthermore, mathematical modelling allows students the opportunities to alter their beliefs about mathematics as they grapple with meaningful situations and develop new knowledge by building and expanding their existing knowledge base. Meaningful situations motivate them to persevere in their tasks, and students can view mathematics as useful when they learn to apply the mathematics in various real-world situations (Lesh & Doerr, 2003:11,24).

Section 2.5.2 noted another important aspect of mathematical modelling which is very significant to this study: As the activities are designed to stimulate certain types of enquiry and development without any direct instruction from the teacher, it is possible not only for high-ability students, but also for average-ability students to invent constructs that are more powerful than constructs that teachers have taught them by using traditional methods. Under-achieving students often seem to disconnect mathematics in the real-world from school mathematics, but model-eliciting activities have the potential to close the gap between applying mathematics in the real-world and experiencing mathematics in the classroom. By engaging in MEAs, students learn to develop models, metaphors, and other descriptive systems to making sense of familiar experiences, without having to use clever language and notation systems (Hamilton et al., 2008; Lesh & Doerr, 2003:5). The researcher is of the opinion that the above explanation places mathematical modelling as the most preferred and solid tool for investigating competencies to acquire a deeper understanding of mathematics, as the outcomes of this study would not have been possible if the students were not allowed to experience mathematics in all its facets.

Approaching this study from a deductive perspective, would have allowed the researcher to make claims about the generalisability of the results, as deductive analysis refers to "data analyses that set out to test whether data are consistent with prior assumptions, theories, or hypotheses identified or constructed by an investigator" (Thomas, 2006:238). Thomas (2006:238) defines inductive analysis as "approaches that primarily use detailed readings of raw data to derive concepts, themes, or a model through interpretations made from the raw data by an evaluator or researcher". While deductive analyses have the inherent danger of ignoring key themes due to preconceptions about data collection and analysis procedures, the primary purpose of the inductive approach is to meticulously collect, analyse, and code the raw data to allow the findings to emerge from the data. This study tried to create an understanding of a real teaching and learning problem within a classroom setting, thus the need for an inductive approach, which was complemented by the methodology of DBR. As noted in Section 4.2, the ultimate aim of this study was not to directly apply a theory to existing problems and evaluate whether it works or not, but rather to strive to find a practical and effective solution to a real teaching and learning problem, by incorporating an interactive learning environment within a specific context. The retrospective phases of this study (Section 4.3.3) were primarily concerned with the analysing and documenting to explain – based on the *data and on theory* – how successive forms of reasoning emerged as a restructuring of prior forms of cognition, and to identify the critical and necessary aspects of the entire learning environment that can support students' development of engineering technician and mathematical modelling competencies to enhance their mathematical understanding and reasoning. This emerged domain-specific local instructional theory (Section 5.4) did not only explain the possible development of such competencies, but also addressed the relevant aspects of the classroom learning environment and other supports needed for learning. As stated in Chapter 5, the research continually produced findings that were fed back into further design cycles which led to the noticing of new potential, complementing Whitehead in Bereiter's (2002:325) view that "the greatest invention of the nineteenth century was the invention of the method of invention" – alias Design-Based Research.

In Brown's (1992:152-153) studies she noted, that by switching back and forth between laboratory and classroom settings during research experiments, "enriches my understanding of a particular phenomenon", as her laboratory work informed her classroom observations, and vice versa. It could thus be argued that this study's results which were obtained in the complex classroom environments, should have been tested in a laboratory under more controlled conditions as well. Such controlled conditions could in turn expose other facets of competence development, that could again be further advanced in the classroom settings. However, due to many constraints such as time and resources, it was impossible for the researcher to combine laboratory work with classroom work. Having to choose between the two, there was no option but to choose the classroom, as learning takes place in a social context, of which one such setting is the classroom (Vygotsky, 1978). The classroom also offered opportunities to adhere to all the principles of successful implementation of MEAs, in particular the design of a classroom culture that was branded by discourse, individual perceptions, discussions of different arguments, combined with a search for understanding, comprehension, systematisation, questioning, inquiry, and reflection, as discussed in Section 2.6.4. By entertaining such a classroom ethos over a period of time, the students' confident levels increased, which made them feel comfortable to voice their opinions, and to communicate and justify their solution methods. The thorough documentation processes revealed valuable information regarding the students' ways of thinking, and also supported the productivity of ongoing learning or problem-solving experiences and promoted development of mathematical competence (Lesh et al., 2000:594). The students also learned to visualise and reflect on their thinking when they explained and described their work in their groups (Chamberlin, 2004:54).

6.4 DELINEATION AND LIMITATIONS

This research study involved 12 first-year engineering technician students studying at the Civil Engineering Department of Durban University of Technology, South Africa. As explained in Sections 1.8.2 and 4.3.1.3, these students have had no prior experience with mathematical modelling activities. They also did not meet the entrance requirements for studying engineering and were enrolled for a bridging program. The small number of participants imply that the results could not necessarily be generalised over a wide spectrum. However, Section 6.5 explains the purpose of investigating a small number of students to understand how competence development – with the aim to support a deeper understanding of mathematics and problem-solving – occur, and how the results can be disseminated.

The duration of this study was one semester, which limited the analysis of determining the impact of the mathematical modelling course over the duration of their study careers.

Another limitation of this study relates to researcher and facilitator resources. Having participating facilitators or volunteers may prove beneficial to the study, as the researcher did not have the benefit of discussing the students' progress with others. However, triangulation allowed the researcher to identify the necessary and crucial aspects of competence development (Sections 4.5.2 and 6.5). In this study, the researcher was the designer as well as the evaluator of the program. The researcher is also the mathematics lecturer of the students. This results in playing conflicting roles of advocate and critic (McKenney et al., 2006:83), which could result in a threat to validity. To overcome these hurdles, the researcher implemented the following strategies suggested by McMillan and Schumacher (2006:327):

- The researcher had personal awareness.
- The researcher facilitated the participants and allowed them to voice their own opinions and ideas.
- The researcher's focus remained to collect a 'true version' of the phenomena.
- The researcher attempted at all stages to deliver work of superior standard and to present an accurate report of her findings.

An added limitation could relate to the homogenous character of the group of participants. Apart from being in a bridging program, all the students were Zulu-speaking, and English was their second language. In the diverse and multi-cultural society that we find ourselves in South Africa, the researcher is of the opinion that this course would also have had a positive effect on a more diverse group of participants. While engaged in the mathematical modelling activities, the students in this study learned to work collaboratively and progressed to more mature levels of communicating ideas openly and effectively to one another. The students learned to listen to one another, and to regard one another's different opinions and ideas with respect. This open environment where students freely voiced their opinions, could perhaps have assisted in fostering empathy and understanding for different races and cultures as well, complementing Kaiser's (2007:111) view that mathematical modelling has the possibilities to develop many different competencies to successfully engage in everyday life.

6.5 HANDLING OF METHODOLOGICAL CONCERNS

This research study followed an inductive approach, and 12 participants volunteered to take part in the study that lasted for one semester. Brown (1992:154) commented that the choice between nomothetic and idiographic approaches to research, is one of the major decisions to be made when orchestrating social science research. Rather than approaching the design from a nomothetic stance, the idiographic approach applied to this study, as it attempted to provide a more complete understanding of a small group of students. This might lead to limited generalisability and predictability of results, and thereby could have a negative effect on the reliability of the study. However, Section 4.2 discussed the dangers of discounting local conditions, as the applications of generalised results are working hypotheses and not conclusions. Traditional educational research tend to conduct once-off quasi-experimental studies, which are not necessarily linked to a robust research agenda, and normally entertain weak links with practice (McKenney et al., 2006:72; Reeves, 2006). Instead of comparing generalised results to quantitative studies that emerge in very large sample sizes, design research is concerned with explanations of the processes and mechanisms that caused the changes in learning. The researcher was interested in the "mechanisms through which and the conditions under which the causal relationship holds" (Cobb et al., 2016:4), to explain the developmental process of three small groups. As noted in Section 4.5, it would not be a meaningful task to investigate students' competency development without being able to replicate the study to a certain extent. However, the intent of the experiment is not one of exact duplication, but rather to inform other practitioners or researchers to differentiate between the necessary and contingent aspects of the design, while they customise the design in their own settings (Cobb et al., 2016:22).

Triangulation (Section 4.5.2) served to ascertain trustworthy, reliable and valid results for this study. The development of modelling competencies were studied with the help of various instruments, such as informal discussions with the students, observing their behaviours and actions while grappling with problem-solving activities, analysing reflection questionnaires, and evaluating their final products. All these methods were employed to examine the same dimension of the research problem – engineering technician and mathematical modelling competence to enhance reasoning and mathematical understanding. Credibility, trustworthiness, and validity of the findings were illustrated by the rigorous data collection and analysis procedures which were thoroughly documented in Chapter 5. The analysis of this longitudinal data set was both systematic and thorough, and thereby it further enhanced the credibility of the findings of the experiment.

During the entire experiment, the researcher took all necessary precautions to ensure that she approaches the study in a state of neutrality – the participants' voices were respected over that of the researcher. In design research, the researcher's rigorous analysis of a specific problem can lead to decisions for further interventions. Field notes including the dates and times of activities, field journals that keep track of decisions made, written accounts of ethical considerations, coding and categorising for data collection and analysis, and interviews all assisted towards substantial findings. This attention to detail assisted towards a truthful representation of how the study played out, and thereby dealt with the Bartlett Effect (Section 4.5.4.1).

Argumentative grammar (Section 4.5.4) guided the researcher's study to ensure that the learning process justified the products of the research project, or rather, as Kelly (2004:118) noted, that it "is the logic that guides the use of a method and that supports reasoning about its data. It supplies the logos in the methodology and is the basis for the warrant for the claims that arise". Engeström (2011:607) regards argumentative grammar as the golden thread that connects the theory, methods, and empirical research in a research approach. The local instructional theory (LIT) that was explained in Section 5.4, was framed within a specific interpretive framework (Chapter 2), and consisted of learning activities and explanations about the students' shifts in reasoning. An interpretive framework did not only address issues of learning, but also classroom norms and discourse. As the experiment took place over one semester, this longitudinal study was covered in copious descriptions of how their reasoning and competence development evolved as a reorganisation of prior forms of reasoning.

Furthermore, credibility, trustworthiness, and validity of the findings were illustrated by the rigorous data collection and analysis procedures as provided by DBR's strong methodology which are thoroughly documented. The analysis of this longitudinal data set was both systematic and thorough, and thereby further served to enhance the credibility of the findings of the experiment.

6.6 AN AGENDA FOR FURTHER RESEARCH

The main purpose of this study was to determine the extent to which engineering and mathematical modelling competencies can co-develop to produce a deeper understanding of mathematics, within the context of a mathematical modelling course for first-year engineering technician students who are not strong in mathematics. To answer to this purpose, it was necessary to separate it in four goals:

- The first goal was to determine how/where mathematical modelling fits into the context of mathematical teaching approaches to develop mathematical reasoning and understanding (Chapter 2),
- Secondly, the particular engineering technician and mathematical modelling competencies had to be identified that could co-develop through mathematical modelling (Chapter 3).
- The third goal was to determine how engineering and mathematical modelling competencies co-develop to nurture reasoning and deeper understanding of mathematics (Chapters 4 and 5).
- Lastly, the process of measuring competence development and mathematical reasoning in the students' work had to be established (Chapters 4 and 5).

During the course of this research study, all four goals were achieved. However, as indicated in the following paragraphs, further research in this domain of education is desired:

- For practical reasons, the duration of his study was limited to one semester, which made it impossible to determine the effect of the mathematical modelling course over the duration of the participants' study careers. Even though the students displayed increased competence development and mathematical understanding as the experiment progressed, it would have been a valuable contribution to determine the long-term impact of this course on the students' future studies and their professional lives. This limitation would thus suggest a further study.

- The ultimate aim of this study was to strive to find a practical and effective solution to a real teaching and learning problem, and the study therefore incorporated an interactive learning environment within a specific context. Reimann (2011:38) argues that research findings that result from close proximity to real schools and close cooperation with teachers and students in an authentic setting, have the potential to be implemented more easily and rapidly in classrooms in general (Section 4.2.1.4). However, in Section 6.3, Brown (1992:152.153) commented that, by combining classroom work with laboratory work, enriched her understanding of the phenomena that she investigated, and that her laboratory work further informed her classroom observations and vice versa. By utilising both settings, other facets of competence development could have been exposed that were perhaps ignored in this study. Further studies on how to co-develop mathematical modelling and engineering technician competencies in both settings will thus be valuable.
- The eight mathematical competencies as identified by Danish KOM project (Figure 3.1), as well as the explanation of mathematical proficiency by Kilpatrick, Swafford and Findell (2001) (Section 3.6.1), expose the interwoven and interdependent character of the components that are required to possess the competencies to master mathematics. These intertwined relations further expose the complexity of mathematical modelling. Binder and Desai (2011) and Irish and Piguet (2013) in Irish (2016:6144) explain the semantic network in the brain as

> … an individual's store of general conceptual knowledge accumulated and abstracted from previous experiences, without a specific spatial or temporal context. The flexible nature of semantic representations is posited to underlie a host of sophisticated cognitive endeavors, such as language, social cognition, and the capacity to mentally project oneself backward and forward in subjective time.

A person's brain creates a web of interconnected memories, of which each one is tied to many other related memories. A study by Zhou et al. (2018:360-370) used functional magnetic resonance imaging (fMRI) to examine neural basis of mathematical problem solving. They found that mathematical problem-solving involves detailed semantic processing, due to the involvement of conceptual knowledge during problem-solving activities. The result of their study shows that the semantic network in the brain promotes problem-solving. To enrich this study towards finding improved ways of supporting mathematical reasoning and understanding, a further study is recommended to investigate approaches to optimise the semantic processing of the brain with specific relation to mathematical modelling and engineering technician competencies.

REFERENCES

Abate, F. R. (1999). *The Oxford American dictionary and language guide*: Oxford University Press, USA.

ABET. (2017). Accreditation board for engineering and technology. Accessed on 28 November 2017. Retrieved from http://www.abet.org/accreditation/accreditation-criteria/accreditation-policy-and-procedure-manual-appm-2017-2018/#fourb

Albarracín, L., & Gorgorió, N. (2012). Inconceivable magnitude estimation problems: an opportunity to introduce modelling in Secondary School. *Journal of Mathematical Modelling and Application, 1*(7), 20-33.

Aliprantis, C., & Carmona, G. (2003). Introduction to an economic problem: A models and modeling perspective. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 255-264). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Allaboutrenewables. (2013). "Tidal barrage generation systems." Online video clip. *YouTube.* YouTube, 22 Aug 2013. Accessed on 25 July 2016. Retrieved from https://www.youtube.com/watch?v=LfCYXnlMtm8

Ang, K. (2009). Mathematical modelling and real life problem solving. In B. Kaur, B. Yeap, & M. Kapur (Eds.), *Mathematical Problem Solving: Association of Mathematics Educators Yearbook 2009* (pp. 159-182). Singapore: World Scientific.

Anhalt, C. O., & Cortez, R. (2015). Mathematical modeling: A structured process. *Mathematics Teacher, 108*(6), 446-452.

Arter, J., & McTighe, J. (2001). *Scoring rubrics in the classroom: Using performance criteria for assessing and improving student performance*. Thousand Oaks, California: Corwin Press.

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 45*(6), 797-810.

Artz, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and instruction, 9*(2), 137-175.

Bahmaei, F. (2013). Mathematical modelling in university, advantages and challenges. *Journal of Mathematical Modelling and Application, 1*(7), 34-49.

Bakker, A. (2004). *Design research in statistics education: On symbolizing and computer tools.* (Doctor of Philosophy thesis), Utrecht University,

Barab, S., & Squire, K. (2004). Design-based research: Putting a stake in the ground. *The journal of the learning sciences, 13*(1), 1-14.

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(3), 293-301.

Bell, P., Hoadley, C. M., & Linn, M. C. (2004). Design-based research in education. In M. Linn, E. Davis, & P. Bell (Eds.), *Internet environments for science education* (pp. 73-85). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Bennett, R. (2002). Employers' demands for personal transferable skills in graduates: a content analysis of 1000 job advertisements and an associated empirical study. *Journal of Vocational Education and training, 54*(4), 457-476.

Bereiter, C. (2002). Design research for sustained innovation. *Cognitive Studies, 9*(3), 321-327.

Berry, J., & Nyman, M. A. (1998). Introducing mathematical modelling skills to students and the use of posters in assessment. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 8*(2), 103-115.

Biccard, P. (2010). *An investigation into the development of mathematical modelling competencies of grade 7 learners.* (Masters of Education), Faculty of Education, Stellenbosch University, (in press).

Biccard, P., & Wessels, D. C. (2011). Documenting the development of modelling competencies of Grade 7 mathematics students. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling* (pp. 375-383). New York: Springer.

Biembengut, M. S., & Hein, N. (2007). Modeling in engineering: Advantages and difficulties. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling ICTMA 12: Education, Engineering and Economics* (pp. 415-423). Chichester: Horwood Publishing.

Bliss, K. M., Fowler, K. R., & Galluzo, B. J. (2014). *Math Modeling: Getting Started & Getting Solutions*. Philadephia, PA: Society for Industrial and Applied Mathematics.

Blomhøj, M. (2009). *Different perspectives in research on the teaching and learning mathematical modelling: Categorising the TSG21 papers*. Paper presented at the Mathematical applications and modelling in the teaching andlearning of mathematics. 11th International Congress on Mathematical Education, Monterrey, Mexico, July 6-13, 2008.

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching mathematics and its applications, 22*(3), 123-139.

Blomhøj, M., & Jensen, T. H. (2007). What's all the Fuss about Competencies? In P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 45-56): Springer Science & Business Media.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education – Discussion document. *Educational Studies in mathematics, 51*(1-2), 149-171.

Blum, W., & Leiβ, D. (2005). *"Filling Up "-the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks*. Paper presented at the Cerme 4–Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education, Accessed on 20 May 2016. Retrieved from https://www.researchgate.net/publication/46672512

Blum, W., & Leiβ, D. (2007). How do students and teachers deal with modelling problems? In C. R. Haines, B. Galluzzo, W. Blum, & S. Khan (Eds.), *Mathematical Modelling: Education, Engineering and Economics - ICTMA 12* (pp. 222-231). Sawston, Cambridge: Woodhead Publishing Limited.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects—State, trends and issues in mathematics instruction. *Educational Studies in mathematics, 22*(1), 37-68.

Boekaerts, M. (1999). Self-regulated learning: Where we are today. *International journal of educational research, 31*(6), 445-457.

Bonotto, C. (2013). Realistic mathematical modeling and problem posing. In R. A. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies - ICTMA 13* (pp. 399-408). Dordrecht: Springer Science & Business Media.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(2), 86-95.

Briley, J. S. (2012). The Relationships among Mathematics Teaching Efficacy, Mathematics Self-Efficacy, and Mathematical Beliefs for Elementary Pre-Service Teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal, 5*, 1-13.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The journal of the learning sciences, 2*(2), 141-178.

Brown, A. L., & Campione, J. C. (1996). Psychological theory and the design of innovative learning environments: On procedures, principles, and systems. In L. Schauble & R. Glaser (Eds.), *Innovations in learning: New environments for education* (pp. 289-325). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Brown, A. L., & Smiley, S. S. (1977). Rating the importance of structural units of prose passages: A problem of metacognitive development. *Child development, 48*(1), 1-8.

Bruce, C. D. (2007). Questions arising about emergence, data collection, and its interaction with analysis in a grounded theory study. *International Journal of Qualitative Methods, 6*(1), 51-68.

Brunhaver, S. R., Korte, R. F., Barley, S. R., & Sheppard, S. D. (2017). Bridging the gaps between engineering education and practice. In R. Freeman & H. Salzman (Eds.), *U.S. Engineering in the Global Economy*: University of Chicago Press.

Bucciarelli, L. L., & Kuhn, S. (1997). Engineering education and engineering practice: Improving the fit. In S. R. Barley & J. Orr (Eds.), *Between Craft and Science: Technical Work in U.S. Settings* (pp. 210-229): Cornell University Press.

Byl, J. (2003). *Mathematical models and reality.* Paper presented at the Proceedings of the 2003 Conference of the Association for Christians in the Mathematical Sciences.

Campbell, J. P., McCloy, R. A., Oppler, S. H., & Sager, C. E. (1993). A theory of performance. In N. Schmitt & W. Borman (Eds.), *Personnel selection in organizations* (pp. 35-70). San Francisco: Jossey Bass.

Caron, F., & Bélair, J. (2007). Exploring university students' competencies in modelling. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling: Education, Engineering and Economics - ICTMA 12* (pp. 120-129). Sawston, Cambridge: Woodhead Publishing Limited.

Chamberlin, M. (2004). Design Principles for Teacher Investigations of Student Work. *Mathematics Teacher Education and Development, 6*, 52-65.

Chamberlin, M. T. (2005). Teachers' Discussions Of Students' Thinking: Meeting The Challenge Of Attending To Students' Thinking. *Journal of Mathematics Teacher Education, 8*(2), 141-170.

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Prufrock Journal, 17*(1), 37-47.

Chao, T., Empson, S. B., & Shechtman, N. (2013). A principal components model of Simcalc mathworlds. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies - ICTMA 13* (pp. 555-560). New York: Springer Science & Business Media.

Charmaz, K. (2008). Grounded theory as an emergent method. In S. Hesse-Biber & P. Leavy (Eds.), *Handbook of emergent methods* (pp. 155-170). New York: The Guilford Press.

Chick, H., & Stacey, K. (2013). Teachers of mathematics as problem-solving applied mathematicians. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 121-136.

Cirillo, M., Pelesko, J., Felton-Koestler, M., & Rubel, L. (2016). Perspectives on modeling in school mathematics. In C. Hirsch & A. McDuffie (Eds.), *Mathematical Modeling and Modeling Mathematics* (pp. 3-16): Reston, VA: The National Council of Teachers of Mathematics.

Cobb, P., Confrey, J., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher, 32*(1), 9-13.

Cobb, P., Jackson, K., & Dunlap, C. (2016). Design research: An analysis and critique. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 481-503). New York: Routledge.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *The journal of the learning sciences, 13*(1), 15-42.

Cox, R. (1999). Representation construction, externalised cognition and individual differences. *Learning and instruction, 9*(4), 343-363.

Cox, R., & Brna, P. (2016). Twenty Years on: Reflections on "Supporting the Use of External Representations in Problem Solving". *International Journal of Artificial Intelligence in Education, 26*(1), 193-204.

CPALMS. (2012). Turning Tires Model Eliciting Activity. Accessed on 18 March 2017. Retrieved from http://www.cpalms.org/Public/PreviewResourceLesson/Preview/28057

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: Sage Publications.

Creswell, J. W., & Poth, C. N. (2017). *Qualitative inquiry and research design: Choosing among five approaches*. Thousand Oaks, CA: Sage publications.

Crouch, R., & Haines, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology, 35*(2), 197-206.

Crouch, R., & Haines, C. (2007). Exemplar models: Expert-novice student behaviors. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling, education, engineering and economics: The ICTMA 12 Study* (pp. 90-100). Chichester: Horwood Publishing.

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *The Journal of Mathematical Behavior*, *15*(4), 375-402.

Davies, W. M. (2009). Groupwork as a form of assessment: common problems and recommended solutions. *The International Journal of Higher Education Research, 58*(4), 563-584.

De Lange, J. (1987). *Mathematics, insight and meaning: Teaching, learning and testing of mathematics for the life and social sciences.* (Ph.D. Doctoral Dissertation), OW&OC, Utrecht University,

De Villiers, L. (2013). *The Effect of the Access program on students' academic performances at the Department of Civil Engineering, Durban University of Technology, Pietermaritzburg campus.* Paper presented at the Department of Civil Engineering Pietermaritzburg, Scholarly Colloquium 2013.

De Villiers, M. D. (1986). *The role of axiomatization in mathematics and mathematics teaching*: RUMEUS, Stellenbosch University.

Dekker, R., & Elshout-Mohr, M. (1998). A process model for interaction and mathematical level raising. *Educational Studies in mathematics, 35*(3), 303-314.

Delury, A. T. (1912). Jules Henri Poincaré. *Journal of the Royal Astronomical Society of Canada, 6*, 309.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational researcher, 32*(1), 5-8.

Dickinson, P., & Hough, S. (2012). Using realistic mathematics education in UK classrooms. *Centre for Mathematics Education, Manchester Metropolitan University, Manchester, UK*, Accessed on 21 April 2016.Retrieved from http://www.mei.org.uk/files/pdf/rme_impact_booklet.pdf

Diefes-Dux, H. A., Moore, T., Zawojewski, J., Imbrie, P., & Follman, D. (2004). *A framework for posing open-ended engineering problems: Model-eliciting activities.* Paper presented at the 34th Annual Frontiers in Education, 2004.

Dienes, Z. P. (1968). Some basic processes involved in mathematics learning. In J. Scandura (Ed.), *Research in mathematics education* (pp. 21-34). Washington DC: National Council of Teachers of Mathematics (NCTM).

Doerr, H. M. (2006). Teachers' ways of listening and responding to students' emerging mathematical models. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(3), 255-268.

Doerr, H. M. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 69-78). New York: Springer Science & Business Media.

Doolittle, P. E., & Camp, W. G. (1999). Constructivism: The career and technical education perspective. *Journal of Career and Technical Education, 16*(1), Accessed on 24 January 2016.Retrieved from http://dx.doi.org/10.21061/jcte.v16i1.706

Dossey, J. A., McCrone, S., Giordano, F. R., & Weir, M. D. (2002). *Mathematics methods and modeling for today's mathematics classroom: A contemporary approach to teaching grades 7-12*. Pacific Grove: Brooks/Cole.

Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 231-248). Dordrecht: Kluwer.

Duderstadt, J. J. (2008). *Engineering for a changing world: A roadmap to the future of engineering practice, research, and education*. Retrieved from http://milproj.dc.umich.edu/

Eames, C. L., Brady, C., & Lesh, R. (2016). Formative Self-Assessment: A Critical Component of Mathematical Modeling. In C. Hirsch & A. McDuffie (Eds.), *Annual Perspectives in Mathematics Education 2016: Mathematical Modeling and Modeling Mathematics* (pp. 229-238): ERIC.

ECSA. (2014). Engineering Council of South Africa. *Qualification Standard for Diploma in Engineering: NQF Level 6.* Access date 3 December 2016. Retrieved from https://www.ecsa.co.za/register/Professional%20Engineers/R-08-PN.pdf

ECSA. (2015). Engineering Council of South Africa. *Qualification Standard for Diploma in Engineering: NQF Level 6.* Accessed on 29 April 2017. Retrieved from https://www.ecsa.co.za/education/EducationDocs/E-02-PN.pdf

Edwards, D., & Hamson, M. (1989). Modeling skills. In *Guide to Mathematical Modeling. Mathematical Guides.* (pp. 67-99). London: Palgrave.

Eisenhardt, K. M. (1989). Building theories from case study research. *Academy of management review, 14*(4), 532-550.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education, 24*(1), 8-40.

Eisner, E. W. (2017). *The enlightened eye: Qualitative inquiry and the enhancement of educational practice*: Teachers College Press.

Engeström, Y. (2011). From design experiments to formative interventions. *Theory & Psychology, 21*(5), 598-628.

English, L. D. (2007). *Interdisciplinary modelling in the primary mathematics curriculum.* Paper presented at the 30th Mathematics Education Research Group of Australasia Annual Conference, Adelaide.

Faulkner, W. (2007). Nuts and Bolts and People' Gender-Troubled Engineering Identities. *Social studies of science, 37*(3), 331-356.

Ferguson, J. W. (2005). Two Cases of College Instructors' Application of Constructivist Principles. *College Quarterly, Seneca College of Applied Arts and Technology, Toronto, 8*(3), 1-21.

Fielding-Wells, J., O'Brien, M., & Makar, K. (2017). Using expectancy-value theory to explore aspects of motivation and engagement in inquiry-based learning in primary mathematics. *Mathematics Education Research Journal, 29*(2), 237-254.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. *The nature of intelligence, 12*, 231-235.

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into practice, 40*(2), 102-109.

Freudenthal, H. (1981). Major problems of mathematics education. *Educational Studies in mathematics, 12*(2), 133-150.

Freudenthal, H. (1986). *Didactical phenomenology of mathematical structures* (Vol. 1). Dordrecht: D Reidel Publishing Company.

Freudenthal, H. (2006). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer Academic Publishers.

Fry, E. (1981). Graphical literacy. *Journal of Reading, 24*(5), 383-389.

Fung, C., & Siu, M. (2015). *Mathematics in teaching and teaching of mathematics.* Paper presented at the 22nd Advanced Metallization Conference, Colorado Springs.

Galbraith, P. (2007). Dreaming a 'possible dream': More windmills to conquer. *Mathematical modeling (ICTMA 12): Education, engineering, and economics*, 47, 44-62.

Galbraith, P. (2009). *Macmillan Senior Mathematical Modelling and Applications* (D. Fry Ed. 1st ed. ed.). South Yarra: Ingrid Kemp.

Galbraith, P., & Stillman, G. (2001). Assumptions and context: Pursuing their role in modelling activity. *Modelling and mathematics education, ICTMA, 9*, 300-310.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(2), 143-162.

Garcia, F. J., Pérez, J. G., Higueras, L. R., & Casabó, M. B. (2006). Mathematical modelling as a tool for the connection of school mathematics. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(3), 226-246.

Garfunkel, S., & Montgomery, M. (2016). Guidelines for assessment and instruction in mathematical modeling education (GAIMME) report. *Boston/Philadelphia: Consortium for Mathematics and Its Applications (COMAP)/Society for Industrial and Applied Mathematics (SIAM).*

Garofalo, J., & Lester Jr, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education (NCTM), 16*(3), 163-176.

Gerring, J. (2006). *Case study research: Principles and practices* (J. Gerring Ed.). New York: Cambridge University Press.

Glaser, B. G., & Holton, J. (2004). Remodeling Grounded Theory. *Forum: Qualitative Social Research - Remodelling Grounded Theory, 5*(2), Accessed on 30 March 2016.Retrieved from http://www.qualitative-research.net/index.php/fqs/article/view/607/1315Volume.

Godino, J. D., Batanero, C., Contreras, A., Estepa, A., Lacasta, E., & Wilhelmi, M. R. (2013). *Didactic engineering as design-based research in mathematics education.* Paper presented at the Proceedings of the Eight Congress of the European Society for Research in Mathematics Education.

Golafshani, N. (2003). Understanding reliability and validity in qualitative research. *The qualitative report, 8*(4), 597-606.

Google Maps. (2017). KwaBhekizizwe Map. Accessed on 5 April 2017. Retrieved from https://mappingsupport.com/p/gmap4.php,

Goos, M., & Galbraith, P. (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in mathematics, 30*(3), 229-260.

Gravemeijer, K. (1999). How Emergent Models May Foster the Constitution of Formal Mathematics. *Mathematical thinking and learning, 1*(2), 155-177. doi:10.1207/s15327833mtl0102_4

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical thinking and learning, 6*(2), 105-128.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. Van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17-51). London and New York: Routledge.

Gravemeijer, K., & Terwel, J. (2000). Hans Freudenthal: a mathematician on didactics and curriculum theory. *Journal of curriculum studies, 32*(6), 777-796.

Haines, C. R., & Crouch, R. (2013). Remarks on a modeling cycle and interpreting behaviours. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies - ICTMA 13* (pp. 145-154). Dordrecht: Springer Science & Business Media.

Hamilton, E., Lesh, R., Lester, F., & Brilleslyper, M. (2008). Model-eliciting activities (MEAs) as a bridge between engineering education research and mathematics education research. *Advances in Engineering Education, 1*(2), 1-25.

Hamilton, E., Lesh, R., Lester, F., & Yoon, C. (2007). The use of reflection tools to build personal models of problem-solving. In R. Lesh, E. Hamilton, & J. Kaput (Eds.), *Models & Modeling as Foundations for the Future in Mathematics Education*: Lawrence Earlbaum Associates, NJ, Inc.

Harel, G. (2008a). DNR perspective on mathematics curriculum and instruction, Part I: focus on proving. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 40*(3), 487-500.

Harel, G. (2008b). A DNR perspective on mathematics curriculum and instruction. Part II: with reference to teacher's knowledge base. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 40*(5), 893-907.

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester Jr (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 805-842): National Council of Teachers of Mathematics.

Harris, F., & Philander, O. (2015). *Developing engineers fit for the 21st century: Integrated learning in the CPUT Motorsport Programme.* Paper presented at the The Third Biennial Conference of the South African Society for Engineering Education, University of KwaZulu Natal, Durban.

Harry, B., Sturges, K. M., & Klingner, J. K. (2005). Mapping the process: An exemplar of process and challenge in grounded theory analysis. *Educational researcher, 34*(2), 3-13.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of educational research, 77*(1), 81-112.

Herrington, J., McKenney, S., Reeves, T., & Oliver, R. (2007). Design-based research and doctoral students: Guidelines for preparing a dissertation proposal.Retrieved from www.editlib.org/d/25967/ proceeding_25967.pdf.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An``experiment"model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6*(3), 201-222.

Hofer, B. K., & Pintrich, P. R. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of educational research, 67*(1), 88-140.

Hoyles, C., & Noss, R. (2007). Learning By Constructing and Sharing Models. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA12): Education, Engineering and economics* (pp. 79-88). Chichester, UK: Horwood Publishing.

Hunt, J. F. (2007). Communicating Big Themes in Applied Mathematics. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA12): Education, Engineering and economics* (pp. 1). Chichester, UK: Horwood Publishing.

IEA. (2013). Graduate attributes and professional competencies. Accessed on 2 November 2016. Retrieved from http://www.ieagreements.org

IEA. (2015). A history of the international engineering alliance and its constituent agreements. Accessed on 15 September 2016. Retrieved from http://www.ieagreements.org

Irish, M. (2016). Elucidating a Core Semantic Network in the Brain—Implications for Disorders of Semantic Cognition. *The Journal of Neuroscience*, *36*(23), 6144-6146. doi:https://doi.org/10.1523/JNEUROSCI.0917-16.2016

Iversen, S. M., & Larson, C. J. (2006). Simple thinking using complex math vs. complex thinking using simple math—A study using model eliciting activities to compare students' abilities in standardized tests to their modelling abilities. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education*, *38*(3), 281-292.

Jensen, T. H. (2007). Assessing mathematical modelling competency. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modeling (ICTMA 12): Education, Engineering and Economics* (pp. 141-148). Chichester, UK: Horwood Publishing.

Jick, T. D. (1979). Mixing qualitative and quantitative methods: Triangulation in action. *Administrative science quarterly*, *24*(4), 602-611.

Julie, C., & Mudaly, V. (2007). Mathematical modelling of social issues in school mathematics in South Africa. In *Modelling and applications in mathematics education* (pp. 503-510): Springer.

Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 110-119). Chichester, UK: Horwood Publishing.

Kaiser, G. (2013). Introduction: ICTMA and the teaching of modeling and applications. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies. International Perspectives on the Teaching and Learning of Mathematical Modelling.* (pp. 1-2). Dordrecht: Springer.

Kaiser, G., & Schwarz, B. (2006). Mathematical modelling as bridge between school and university. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(2), 196-208.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(3), 302-310.

Kanselaar, G. (2002). Constructivism and socio-constructivism. *Constructivism and socio-constructivism*, 1-7, Accessed on 19 November 2015 Retrieved from http://dspace.library.uu.nl/handle/1874/6684.

Kelly, A. (2004). Design Research in Education: Yes, but is it Methodological? *Journal of the Learning Sciences, 13*(1), 115-128.

Kemeny, J. G. (1959). Mathematics without numbers. *Daedalus, 88*(4), 577-591.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping Children learn Mathematics*. Washington DC: National Academy Press.

Kloosterman, P. (2006). Beliefs about Mathematics and Mathematics Learning in the Secondary School: Measurement and Implications for Motivation. In G. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (Vol. 31, pp. 247-269). Norwell, MA: Kluwer.

Knott, A. (2014). *The Process of Mathematisation in Mathematical Modelling of Number Patterns in Secondary School Mathematics.* (Master of Education), Faculty of Education, Stellenbosch University, (in press).

Knudsen, R. D. (1960). Philosophy. *Journal of the American Scientific Affiliation, 12*, 17-18.

Krefting, L. (1991). Rigor in qualitative research: The assessment of trustworthiness. *American journal of occupational therapy, 45*(3), 214-222.

Larsen, S. (2013). A local instructional theory for the guided reinvention of the group and isomorphism concepts. *The Journal of Mathematical Behavior, 32*(4), 712-725.

Larsen, S., & Lockwood, E. (2013). A local instructional theory for the guided reinvention of the quotient group concept. *The Journal of Mathematical Behavior, 32*(4), 726-742.

Lave, J. (1993). The practice of learning. In S. Chaiklin & J. Lave (Eds.), *Understanding Practice* (pp. 200-208): Cambridge University Press.

Lawler, E. E. (1994). From job-based to competency-based organizations. *Journal of organizational behavior, 15*(1), 3-15.

Lawless, A. (2005). *Numbers & Needs: Addressing Imbalances in the Civil Engineering Profession*. Halfway House: South African Institution of Civil Engineering (SAICE).

Lehrer, R., & Schauble, L. (2000). Inventing data structures for representational purposes: Elementary grade students' classification models. *Mathematical thinking and learning, 2*(1-2), 51-74.

Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in mathematics, 46*(1-3), 87-113.

Lesh, R. (2000). Beyond constructivism: Identifying mathematical abilities that are most needed for success beyond school in an age of information. *Mathematics Education Research Journal, 12*(3), 177-195.

Lesh, R., & Carmona, G. (2003). Piagetian conceptual systems and models for mathematizing everyday experiences. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 71-96). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc. .

Lesh, R., & Clarke, D. (2000). Formulating Operational Definitions of Desired Outcomes of Instruction in Mathematics and Science Education. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 113-149). Magwah, New Jersey: Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. (2003). Foundations of a model and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-34). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Lesh, R., & English, L. D. (2005). Trends in the evolution of models & modeling perspectives on mathematical learning and problem solving. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 37*(6), 487-489.

Lesh, R., & Fennewald, T. (2013). Introduction to Part I Modeling: What Is It? Why Do It? In R. A. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies* (pp. 5-10). New York: Springer.

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical thinking and learning, 5*(2-3), 157-189.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities for students and teachers. In A. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-645). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical thinking and learning, 5*(2-3), 109-129.

Lesh, R., Lester, F. K., & Hjalmarson, M. (2003). A models and modeling perspective on metacognitive functioning in everyday situations where problem solvers develop mathematical constructs. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 383-403). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester Jr (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematic* (Vol. 2, pp. 763-804). Charlotte: Information Age.

Lester, F., & Kehle, P. (2003). From problem solving to modeling: The evolution of thinking about research on complex mathematical activity. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 501-518). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Linder, S. M. (2011). The Facilitator's Role in Elementary Mathematics Professional Development. *Mathematics Teacher Education and Development, 13*(2), 44-66.

Lingefjärd, T. (2006). Faces of mathematical modeling. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(2), 96-112.

Lucena, J., Downey, G., Jesiek, B., & Elber, S. (2008). Competencies Beyond Countries: The Re-Organization of Engineering Education in the United States, Europe, and Latin America. *Journal of Engineering Education, 97*(4), 433-447.

Ma, V. J., & Ma, X. (2014). A comparative analysis of the relationship between learning styles and mathematics performance. *International Journal of STEM Education, 1*(1), 1-13.

Ma, X., Jong, C., Yuan, J., Meyer, H., & Benavot, A. (2013). Exploring reasons for the East Asian success in PISA. In H. Meyer & A. Benavot (Eds.), *PISA, power, and policy: The* *emergence of global educational governance* (pp. 225-246). Oxford, United Kingdom: Symposium Books.

MaaB, K. (2007). Modelling in class: What do we want students to learn. In C. R. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modeling (ICTMA 12): Education, Engineering and Economics* (pp. 63-78). Chichester, UK: Horwood Publishing.

Maaß, K. (2005). Barriers and opportunities for the integration of modelling in mathematics classes: results of an empirical study. *Teaching mathematics and its applications, 24*(2-3), 61-74.

Maaß, K. (2006). What are modelling competencies? *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(2), 113-142.

Maaß, K. (2007). *Modelling in class: what do we want the students to learn?* Paper presented at the Mathematical Modelling (ICTMA 12): Education, Engineering and Economics: Proceedings from the Twelfth International Conference on the Teaching of Mathematical Modelling and Applications.

Maaß, K., & Mischo, C. (2011). Implementing Modelling into Day-to-Day Teaching Practice – The Project STRATUM and its Framework. *Journal für Mathematik-Didaktik, 32*(1), 103-131.

Male, S. A. (2010). Generic engineering competencies: A review and modelling approach. *Education Research and Perspectives, 37*(1), 25.

Male, S. A., Bush, M. B., & Chapman, E. S. (2009). *Identification of competencies required by engineers graduating in Australia.* Paper presented at the 20th Annual Conference for the Australasian Association for Engineering Education, 6-9 December 2009: Engineering the Curriculum.

Marra, R. M., Steege, L., Tsai, C.-L., & Tang, N.-E. (2016). Beyond "group work": an integrated approach to support collaboration in engineering education. *International Journal of STEM Education, 3*(1), 17.

McKenney, S., Nieveen, N., & van den Akker, J. (2006). Design research from a curriculum perspective. In J. Van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 67-90). London: Routledge.

McKenney, S., & Reeves, T. C. (2013). *Conducting educational design research*. London: Routledge.

McKenney, S., & Reeves, T. C. (2014). Educational design research. In J. Spector, M. Merril, J. Elen, & M. Bishop (Eds.), *Handbook of Research on Educational Communications and Technology* (4 ed., pp. 131-140). New York: Springer.

McMillan, J., & Schumacher, S. (2006). *Research in Education: Evidence–Based Inquiry*. Boston: Pearson.

Meier, R. L., Williams, M. R., & Humphreys, M. A. (2000). Refocusing our efforts: assessing non-technical competency gaps. *Journal of Engineering Education, 89*(3), 377.

Menon, U. (2013). *Mathematisation-vertical and horizontal.* Paper presented at the Fifth International Conference to Review Research on Science, Technology and Mathematics Education (epiSTEME 5) Delhi.

Metallidou, P., & Vlachou, A. (2007). Motivational beliefs, cognitive engagement, and achievement in language and mathematics in elementary school children. *International Journal of Psychology, 42*(1), 2-15.

Michelsen, C. (2006). Functions: a modelling tool in mathematics and science. *Zentralblatt Fur Didaktik Der Matematik (ZDM): The International Journal on Mathematics Education, 38*(3), 269-280.

Middleton, J., Lesh, R., & Heger, M. (2003). Interest, identity, and social functioning: Central features of modeling activity. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 405-431). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Mischo, C., & Maaβ, K. (2012). Which personal factors affect mathematical modelling? The effect of abilities, domain specific and cross domain-competences and beliefs on performance in mathematical modelling. *Journal of Mathematical Modelling and Application, 1*(7), 2-19.

Moore, T., & Diefes-Dux, H. (2004). *Developing model-eliciting activities for undergraduate students based on advanced engineering content.* Paper presented at the 34th Annual Frontiers in Education, 2004.

Moore, W. S. (1994). Student and faculty epistemology in the college classroom: The Perry schema of intellectual and ethical development. In K. Prichard & R. Sawyer (Eds.), *Handbook of college teaching: Theory and applications* (pp. 45-67). Westport, CT: Greenwood Press.

Morse, J. M., Barrett, M., Mayan, M., Olson, K., & Spiers, J. (2002). Verification strategies for establishing reliability and validity in qualitative research. *International Journal of Qualitative Methods, 1*(2), 13-22.

NEPAD. (2016). The mission of NEPAD? To be a driving force for Africa. Accessed on 16 November 2016. Retrieved from http://www.nepad.org/content/mission-nepad-be-driving-force-africa

Niss, M. (1989). Aims and scope of applications and modelling in mathematics curricula. In W. Blum (Ed.), *Applications and modelling in learning and teaching mathematics* (pp. 22-31). Chichester, UK: Horwood.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagtsis & Papastavrides (Eds.), *3rd Mediterranean Conference on Mathematical Education, 3-5 January 2003* (pp. 115-124). Athens: The Hellenic mathematical society, 2003.

Niss, M., & Højgaard, T. (2011). *Competencies and Mathematical Learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilde University: IMFUFA. .

OECD. (2005). Organisation for Economic Co-operation and Development. *The Definition and Selection of Key Competencies: Executive Summary*. Accessed on 29 March 2017. Retrieved from http://www.oecd.org/dataoecd/47/61/35070367.pdf

Parmjit, S., & White, A. (2006). Unpacking first year university students' mathematical content knowledge through problem solving. *Asian Journal of University Education, 1*(1), 33-56.

Passow, H. J. (2012). Which ABET competencies do engineering graduates find most important in their work? *Journal of Engineering Education, 101*(1), 95.

Piaget, J. (1964). Part I: Cognitive development in children: Piaget development and learning. *Journal of research in science teaching, 2*(3), 176-186.

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of educational psychology, 82*(1), 33.

Pintrich, P. R., Wolters, C. A., & Baxter, G. P. (2000). Assessing Metacognition and Self-Regulated Learning. In G. Schraw & J. Impara (Eds.), *Issues in the Measurement of Metacognition*. Lincoln, NE: Buros Institute of Mental Measurements.

Plonka, F., Hillman, J., Clarke, M., & Taraman, K. (1994). *Competency requirements in the Greenfield paradigm: the manufacturing engineer of the 21st century*. Paper presented at the Twenty-Fourth Annual Conference Proceedings of Frontiers in Education Conference.

Pollak, H. O. (1969). How can we teach applications of mathematics? *Educational Studies in mathematics, 2*(2), 393-404.

Pólya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method,* . Princeton, New Jersey: Princeton University.

Radford, L. (2002). The seen, the spoken and the written: A semiotic approach to the problem of objectification of mathematical knowledge. *For the learning of mathematics, 22*(2), 14-23.

Reeves, T. C. (2006). Design research from a technology perspective. In J. Van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (Vol. 1, pp. 52-66). London, UK: Routledge.

Reimann, P. (2011). Design-based research. In L. Markauskaite, P. Freebody, & J. Irwin (Eds.), *Methodological choice and design: Scholarship, policy and practice in social and educational research* (pp. 37-50). New York: Springer.

Resnick, L. B. (1987). *Education and learning to think*. Washington DC: National Academy Press.

Resnick, L. B. (1988). Treating mathematics as an ill-structured discipline. In R. Charles & E. A. Silver (Eds.), *Research Agenda for Mathematics Education: Teaching and Assessing Mathematical Problem Solving*. Reston, VA: National Council of Teachers of Mathematics.

Rifkin, K. I., Fineman, M., & Ruhnke, C. H. (1999). The Human Side: Developing Technical Managers—First You Need a Competency Model. *Research-Technology Management, 42*(2), 53-57.

Robinson, M. A., Sparrow, P. R., Clegg, C., & Birdi, K. (2005). Design engineering competencies: future requirements and predicted changes in the forthcoming decade. *Design Studies, 26*(2), 123-153.

Rugarcia, A., Felder, R. M., Woods, D. R., & Stice, J. E. (2000). The future of engineering education I. A vision for a new century. *Chemical Engineering Education, 34*(1), 16-25.

Rule, P., & John, V. M. (2015). A necessary dialogue: Theory in case study research. *International Journal of Qualitative Methods, 14*(4), 1609406915611575.

Rychen, D., & Salganik, L. (2002). Strategy Paper: An Overarching Frame of Reference for a Coherent Assessment and Research Program on Key Competencies. *Definition and selection of competencies (DeSeCo): theoretical and conceptual foundations*.

Rychen, D., & Salganik, L. (2003). *Highlights from the OECD Project Definition and Selection Competencies: Theoretical and Conceptual Foundations (DeSeCo).* Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.

SAIRR. (2016). Schools drag South Africa down. *Accessed on 4 April 2016*, *2*(294).Retrieved from http://irr.org.za/reports-and-publications/fast-facts/fast-facts-2016/schools-drag-south-africa-down-2013-february-2016.

SAQA. (2008). South African Qualifications Authority. *Accessed on 16 October 2016.* Retrieved from http://www.saqa.org.za/docs/pol/2000/curriculum_dev.pdf

SAQA. (2013). *National Qualifications Framework Act 67 of 2008: Publication of General and Further Education and Training Qualifications Sub-framework and Higher Education Qualification Sub-framework of the National Qualifications Framework.* South African Government Gazette, No 36797. Pretoria: Government Printer.

SarahGotsMadSkillz. (2012). "Tidal Power." Online video clip. *YouTube.* YouTube, 17 June 2012. Accessed on 24 July 2016. Retrieved from https://www.youtube.com/watch?v=GS9V1Jt5RfI

Scandura, J. M. (1971). *Mathematics: Concrete behavioral foundations*. New York: Harper & Row.

Schoenfeld, A. H. (1983). Episodes and Executive Decisions in Mathematical Problem Solving. In R. A. Lesh & M. Landau (Eds.), *Acquisition of Mathematical Concepts and Processes*. New York: Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: Macmillan.

Schoenfeld, A. H. (2009). Why do we teach? *Kappa Delta Pi Record, 46*(1), 26-29.

Schoenfeld, A. H. (2013). Mathematical modeling, sense making, and the common core state standards. *Journal of Mathematics Education at Teachers College*(4), 6-17.

Schorr, R., & Lesh, R. (2003). A modeling approach for providing teacher development. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 141-158). Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

Schorr, R. Y., & Koellner-Clark, K. (2003). Using a modeling approach to analyze the ways in which teachers consider new ways to teach mathematics. *Mathematical thinking and learning, 5*(2-3), 191-210.

Schraw, G. (1998). Promoting general metacognitive awareness. *Instructional science, 26*(1-2), 113-125.

Schwonke, R., Berthold, K., & Renkl, A. (2009). How multiple external representations are used and how they can be made more useful. *Applied Cognitive Psychology, 23*(9), 1227-1243.

Siegfried, J. M. (2012). *The hidden strand of mathematical proficiency: defining and assessing for productive disposition in elementary school teachers' mathematical content knowledge*. San Diego, CA: University of California/San Diego State University.

Sierpinska, A. (1990). Some remarks on understanding in mathematics. *For the learning of mathematics*, *10*(3), 24-26.

Silver, E. A. (1982). *Thinking about problem solving: Toward an understanding of metacognitive aspects of mathematical problem solving.* Paper presented at the Conference on Thinking, Suva, Fiji.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114-145.

Simon, M. A., & Schifter, D. (1991). Towards a Constructivist Perspective: An Intervention Study of Mathematics Teacher Development. *Educational Studies in mathematics, 22*(4), 309-331.

Simon, M. A., & Tzur, R. (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical thinking and learning*, *6*(2), 91-104.

Singh, P., & White, A. (2006). Unpacking First Year University Students' Mathematical Content Knowledge Through Problem Solving. *Asian Journal of University Education*, *2*(1), 33-56.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching*, *77*, 20-26.

Skemp, R. R. (1987). *The psychology of learning mathematics*. London: Penguin.

Spinks, N., Silburn, N., & Birchall, D. (2006). Educating engineers for the 21st century: The industry view. *London: The Royal Academy of Engineering*.

Starfield, A. M., Smith, K., & Bleloch, A. L. (1993). *How to model it: Problem solving for the computer age*: McGraw-Hill, Inc.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, CA: Sage.

Student_Energy. (2015a). "Hydro Power 101." Online video clip. *YouTube*. YouTube, 17 May 2015. Accessed on 25 July 2016. Retrieved from https://www.youtube.com/watch?v=q8HmRLCgDAI

Student_Energy. (2015b). "Tidal Power 101." Online video clip. *YouTube*. YouTube, 17 May 2015. Accessed on 25 July 2016. Retrieved from https://www.youtube.com/watch?v=VkTRcTyDSyk

Tall, D. (1991). *Advanced mathematical thinking*. Dordrecht, The Netherlands: Kluwer.

Thomas, D. R. (2006). A general inductive approach for analyzing qualitative evaluation data. *American journal of evaluation, 27*(2), 237-246.

Treffers, A., & Vonk, H. (1987). *Three dimensions: A model of goal and theory description in mathematics instruction - The Wiskobas Project*. Dordrecht: D Reidel Publishing.

Üzel, D., & Mert Uyangör, S. (2006). Attitudes of 7th class students toward mathematics in realistic mathematics education. *International Mathematical Forum, 1*(39), 1951-1959.

Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). Introducing educational design research. In J. Van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 3-7). London: Routledge.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and Realistic Mathematics Education*. Utrecht: Freudenthal Institute.

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in mathematics, 54*(1), 9-35.

VanWynsberghe, R., & Khan, S. (2007). Redefining case study. *International Journal of Qualitative Methods, 6*(2), 80-94.

Veresov, N. (2014). Method, methodology and methodological thinking. In *Visual Methodologies and Digital Tools for Researching with Young Children* (pp. 215-228): Springer.

Vesilind, P. A. (1988). Rules, ethics and morals in engineering education. *Engineering Education, 78*(5), 289-293.

Vos, P. (2011). What Is 'Authentic'in the Teaching and Learning of Mathematical Modelling? In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in the Teaching and Learning of Mathematical Modelling* (pp. 713-722). New York: Springer.

Voskoglou, M. G. (2007). A stochastic model for the modelling process. *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics, Chichester, Horwood Publishing*, 149-157.

Vygotsky, L. S. (1967). Play and its role in the mental development of the child. *Soviet psychology, 5*(3), 6-18.

Vygotsky, L. S. (1978). Interaction between learning and development. In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *Mind in Society. The development of higher psychological processes* (pp. 79-91). Cambridge, MA: Harvard University Press.

Wake, G., Swan, M., & Foster, C. (2016). Professional learning through the collaborative design of problem-solving lessons. *Journal of Mathematics Teacher Education, 19*(2-3), 243-260.

Walker, D. (2006). Toward productive design studies. *Educational design research*, 8-13.

Wessels, H. M. (2014). Levels of mathematical creativity in model-eliciting activities. *Journal of Mathematical Modelling and Application, 1*(9), 22-40.

West, C. (2015). *The Radical King* (Vol. 11). Boston, Massachusetts: Beacon Press.

White, D. W. (2014). What Is STEM Education and Why Is It Important? *Florida Association of Teacher Educators Journal, 1*, 1-9.

Widjaja, W., Dolk, M., & Fauzan, A. (2010). The role of contexts and teacher's questioning to enhance students' thinking. *Journal of Science and Mathematics Education in Southeast Asia, 33*(2), 168-186.

Wigfield, A., Eccles, J. S., Roeser, R. W., & Schiefele, U. (2008). Development of achievement motivation. *Child and adolescent development: An advanced course, 1*, 406-434.

Wolters, C. A., Yu, S. L., & Pintrich, P. R. (1996). The relation between goal orientation and students' motivational beliefs and self-regulated learning. *Learning and Individual Differences, 8*(3), 211-238. doi:http://dx.doi.org/10.1016/S1041-6080(96)90015-1

Woolfolk, A., Winne, P. H., & Perry, N. E. (2015). *Educational psychology*: Pearson Canada.

Woollacott, L. C. (2003). *Dealing with under-preparedness in engineering education. Part 1: Defining the goal. A taxonomy of engineering competency.* Paper presented at the Proceedings of the 2003 American Society for Engineering Education E-Conference, University of Witwatersrand.

Woollacott, L. C. (2007). *The goals of engineering education: A rationale for a universal document based on the CDIO syllabus and the taxonomy of engineering competencies.* Paper presented at the 3rd International CDIO Conference, Cambridge, Massachusets, USA.

Yildirim, T. P., Shuman, L., Besterfield-Sacre, M., & Yildirim, T. (2010). Model eliciting activities: assessing engineering student problem solving and skill integration processes. *International Journal of Engineering Education, 26*(4), 831-845.

Yin, R. K. (1981). The case study crisis: Some answers. *Administrative science quarterly, 26*(1), 58-65.

Zawojewski, J. (2013). Problem solving versus modeling. In R. A. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies - ICTMA 13* (pp. 237-243). New York: Springer.

Zazkis, R., & Applebaum, M. (2007). Advancing mathematical thinking: Looking back at one problem. *Congress of the European Society for Research in Mathematics Education (CERME)-5 Working group 14.* Accessed on 18 Sept 2017. Retrieved from http://ermeweb.free.fr/CERME5b/WG14.pdf

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in mathematics, 63*(1), 89-112.

Zhou, X., Li, M., Li, L., Zhang, Y., Cui, J., Liu, J., & Chen, C. (2018). The semantic system is involved in mathematical problem solving. *NeuroImage, 166*, 360-370.

Zulkardi, Z. (1999). How to Design Mathematics Lessons based on the Realistic Approach. *Accessed on 16 December 2015*.Retrieved from http://eprints.unsri.ac.id/692/1/rme.html.

APPENDICES

ASSESSMENT INSTRUMENTS

APPENDIX A – STATUS UPDATE REPORT

Task:	Date:
	Name:
	Group:

	Question	**Answer**
1	What is the problem about?	
2	What do we want to achieve at the end?	
3	What information do we need to answer this problem? What assumptions can we make to simplify the problem?	
4	What have you done to work out a solution?	
5	What do you need to do now?	
6	What did you need help with and why?	
7	What will you do different or the same next time and why?	

(Adapted from Biccard (2010))

APPENDIX B – RESEARCHER OBSERVATION GUIDE

DATE		**GROUP**		**TASK**
Competency				
Management Group: Self-directed learning – reflective activities	Talking	Writing / Reporting	Judging their progress	Leadership roles
Group: Self-efficacy	Their beliefs to execute the task successfully			
Communication	Sharing ideas	Reading Competence	Group Discussions	Oral Presentations
Responsibility	Sense of Direction – working towards a goal, justifying their thought processes to connect the real-world problem with their solutions.			

APPENDIX C – QUALITY ASSURANCE GUIDE

<table>
<tr><th>Performance Level</th><th>How useful is this product?</th><th>What might the client say?</th><th>What questions should be asked?</th></tr>
<tr><td>Requires Redirection</td><td>The product is on the wrong track. Working longer or harder won't work. The students may require some additional feedback from the teacher.</td><td>"Start over. This won't work. Think about it differently. Use different ideas or procedures."</td><td rowspan="4">To assess work, put yourself in the role of the client. To do this, it is necessary to be clear about answers to the following questions:
1. Who is the client?
2. What conceptual tool does the client need?
3. What does the client need to be able to do with the tool?

Then, the quality of students' work can be determined by focussing on the questions – How useful is the tool for the purposes of the client?
To assess usefulness, and to identify strengths and weaknesses of different results that students produce, it would be helpful to consider the following question:
1. What information, relationships and patterns does the tool take into account?
2. Were appropriate ideas and procedures chosen for dealing with this information?
3. Were any technical errors made in using the preceding ideas and procedures?

But. The central question is – Does the product meet the client's needs?</td></tr>
<tr><td>Requires Major Extensions or Refinements</td><td>The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.</td><td>"You are on the right track, but this still needs a lot more work before it will be in a form that is useful."</td></tr>
<tr><td>Requires Only Minor Editing</td><td>The product is nearly ready to be used. It still needs a few small modifications, additions or refinements.</td><td>"Hmmmm, this is close to what I need. You just need to add or change a few small things."</td></tr>
<tr><td>Useful for this Specific Data Given</td><td>No changes will be needed to meet the immediate needs of the client.</td><td>"Ahhh, this will work well as it is. I won't even need to do any editing."</td></tr>
<tr><td>Sharable or Reusable</td><td>The tool not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.</td><td>"Excellent, this tool will be easy for me to modify or use in other similar situations – when the data are slightly different."</td><td>The product should make it clear that:
1. The students went beyond producing a tool that they themselves could use to also produce a tool that others could use – by including needed explanations, and by making it as simple, clear, and well organised as possible;
2. The students went beyond thinking with the tool to also think about it – by identifying underlying assumptions (so that others know when the tool must be modified for use in similar situations);
3. The students went beyond blind thinking to also thinking about their thinking – by recognising strengths and weaknesses of their approach compared with other possible alternatives.</td></tr>
</table>

(Lesh & Clarke, 2000:145)

APPENDIX D – GROUP MODELLING COMPETENCY OBSERVATION GUIDE

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent / Developing 1	Proficient 2	Exemplary 3
Horizontal	**Internalising**	Understand the problem	You failed to identify, summarise or explain the main problem or question in your own words.	You identified main issues but did not summarise or explain them clearly or sufficiently	You successfully identified and summarised the main issues, but did not explain why/how they are problems or create questions	You clearly identified and summarised main issues and successfully explained why/how they are problems or questions
		Collect relevant information	You gathered information that lacks relevance, quality and balance	Your response was not completely related to the problem	You used all relevant information from the problem for working towards a solution	You uncovered hidden or implied information not readily apparent
		Simplify the situation	You were unable to recognise and connect essential concepts about the problem	Your situational model were essentially correct, but not all concepts were accurately represented	Your situational model was complete and accurate	You used multiple representations for explaining and simplifying the problem
	Interpreting	Assumptions	Your assumptions were not appropriate for the problem, you did not simplify the problem	You used an oversimplified approach and assumptions to the problem, you did not explain all the important information to simplify the problem	You chose appropriate, efficient assumptions for simplifying and solving the problem	You chose innovative and insightful assumptions and showed consideration for the consequences of the assumptions clearly and coherently
		Determine particularities – Recognise factors that can influence the situation	You did not recognise the information relevant to the situation and discarded irrelevant information that have an influence on the problem	You recognised some quantities and variables and discarded some irrelevant information that could influence the problem	You recognised important quantities and variables in the problem and you were able to discard irrelevant information that could influence the problem	You created a general rule or formula for solving related problems
		Establish Conditions and Constraints	You were unable to recognise conditions that will work/not work for the problem	You established vague conditions under which the problem will work/not work	You established clear conditions and constraints for a successful solution to the problem	You established clear conditions and constraints, as well as explanations for such conditions and constraints
	Structuring	Innovative planning and design (setting up a situational model)	You were unable to recognise and connect essential concepts about the problem	Your situational model were essentially correct, but not all concepts were accurately represented	Your situational model was complete and accurate (‘model of’)	You used multiple representations for explaining the problem (‘model of’)

	Mathematical Modelling Competency	**Sub-modelling competencies that support the modelling competency**	**Unsatisfactory** **0**	**Emergent / Developing** 1	**Proficient** 2	**Exemplary** 3
		Construct relations – Consider the interdependence, interactions, and relative importance of various factors	You were unable to recognise relationships between variables	You recognised some patterns and/or relationships	You recognised important relationships between the variables in your problem	You created a general rule or formula for solving related problems
	Symbolising	Choose appropriate symbols	The mathematical tools you chose would not lead to a correct solution	The mathematical tools you chose would lead to a partially correct solution	The mathematical tools you chose would lead to a correct solution	You chose mathematical tools that would lead to an elegant solution
Vertical		Using the symbols	Your use of mathematical symbols will not explain the problem or lead to a satisfactory solution	Your use of mathematical symbols were partially correct	You used mathematical symbols effectively, your model can lead to a correct solution	You explained and described the symbols used in your model, as well as possible alternative methods for working with the problem.
		Approach problems methodically	Errors in reasoning were serious enough to flaw your solution. You were unable to translate the structure of the situation into mathematical language	You made minor errors in your attempt to communicate the structure of the situation into mathematical language	Your mathematical reasoning were essentially accurate.	All aspects of your mathematical reasoning were completely accurate
			Your mathematical model will not explain the problem or lead to a satisfactory solution	Your mathematical model will lead to a partially correct solution	Your mathematical model can lead to a correct solution	You translated the structure of the situation into mathematical language and solved the problem successfully.
	Adjusting	Refining and Testing	You found a solution and then stopped	You found multiple solutions, but not all were correct	You found multiple solutions using different interpretations of the problem, you reviewed or refined parts of the model or went through the entire modelling process when the solutions did not fit the situation (‘*model for*’)	You related the underlying structure of the problem to other similar problems (‘*model for*’)

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent / Developing 1	Proficient 2	Exemplary 3
		Explaining	You gave no explanation for your work	Your explanation was redundant at places	Your solution flowed logically from one step to the next	You gave an in-depth explanation of your reasoning
		Capable to derive to an elegant solution of the problem	Your methods were clumsy and inappropriate	The methods you used led to a partially correct solution	The methods you used led to a correct solution	You applied methods elegantly that led to the correct solutions
	Organising	Evaluating and judgement	You did not evaluate your work, and little or no connections were made between the mathematical model and the real-world problem	You made attempts to analyse, evaluate or judge your work, but the connections between your work and the real-world problem were limited	You offered substantial information, evidence of analysis, synthesis and evaluation; general connections are made, but are sometimes too obvious or not clear	Rich in content, insightful analysis, synthesis and evaluation, clear connections made to real-life situations or to previous content
		Reflection – Consider relevant principles that can influence the solution (Reflecting on own thought processes)	You did not reflect on your own thinking (viewing problem in different form)	You identified some perspectives about the problem, but did not consider alternate points of view.	You identified strengths and weaknesses in your own thinking, you recognized alternative perspectives about the problem when comparing to others.	You identified strengths and weaknesses in your own thinking, you recognized alternative perspectives about the problem when comparing to others, and evaluated them in the context of alternate points of view.
	Generalising	Establish similar relationship in different situations by adapting some of the rules	You found no connections to other disciplines or mathematical concepts	Your solution hinted at a connection to an application or another area of mathematics	You connected your solution process to other problems, areas of mathematics, or applications. Predictions can be made from the model.	Your connection to a real-life application was accurate and realistic, the model is easy to use and the predictions are accurate
		General or independent reasoning – applying deductive reasoning to prove the solutions	You exhibit an inability to identify a generalisation when presented with a specific situation	With assistance, you identified a partially correct generalisation when presented with a specific situation	You exhibit the ability to identify a generalisation when presented with a specific situation, but require assistance.	You exhibit the ability to identify a generalisation easily when presented with a specific situation

	Mathematical Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent / Developing 1	Proficient 2	Exemplary 3
		The successful model is easy to use and allows for predictions	The complicated model cannot be detached from the current context	With minor adjustments, the model can be used in other related situations	The model can be transferred to other similar situations, but needs minor simplifications.	The model can easily be adapted in a another related situation
Meta-cognitive	**Management**	Self-directed learning	You were not able to direct your own learning, and tried to find someone to direct your activities	You complete your tasks through guided learning and searched for confirmation throughout your work	You set goals and managed your own learning. You designed the mathematical model independently and considered feedback from others to find the solutions	You set goals and managed your own learning. You designed the mathematical model independently and reflect and evaluate your work critically to improve your learning
		Group Self-efficacy	Your approach to the task and to the team was hostile and uninterested	You attempted to complete the task, but you seemed unsure about your role and your abilities during the team activity	You approached the task with positive expectations about finding solution strategies	You approached the task with positive expectations about finding solution strategies and communicated your ideas to other team members in a positive and productive manner
		Productive disposition	You showed no evidence of engaging with the task, mathematical or otherwise. The lack of effort can be attributed to either disinterest or a lack of capability	You engaged with the task and with the mathematics, but you made little progress towards understanding the mathematics. However, you were willing to engage with the mathematics of the task	You showed strong evidence of engaging with the task and the mathematics, but the quality of engagement was somewhat shallow or only related to a small aspect of the work	You showed very strong evidence of engaging with both the task and the mathematics. You tried to help fellow group members through explaining the work and approached it from various perspectives. You were persistent in continuing with the work until you reached an acceptable solution.
	Communication	Sharing Ideas	You can communicate your ideas, but you make mistakes in content and reasoning or misread your audience	You can communicate ideas clearly and accurately and with some awareness of the context	You communicate ideas clearly, accurately and appropriately, and give arguments and reasons for your beliefs	You can communicate convincing arguments clearly and accurately with a level of comprehensiveness and

	Mathematical Modelling Competency	**Sub-modelling competencies that support the modelling competency**	**Unsatisfactory** **0**	**Emergent / Developing** 1	**Proficient** 2	**Exemplary** 3
						conciseness appropriate to the audience
		Reading Competence	The response sows an inability to construct a literal meaning of the selection, may focus only on the reader's own frustration or indicate that the reader gave up	The response correctly identifies some main ideas, focuses on isolated details or misunderstands or omits some significant supporting details.	Indicates an understanding of the main ideas and relevant and specific supporting details. Uses information from textual resources to clarify the meaning and form conclusions	Indicates a thorough and accurate understanding of main ideas and all significant supporting details, including clarification of the complexities. Uses relevant and specific information from textual resources to clarify meaning and form conclusions
		Group Work	You did not collaborate with your team members. You either showed no interest, made fun of the work, had a negative attitude, contributed very little to group effort, or did not perform the duties of the assigned team role.	You occasionally helped to complete group goals, You finished your individual task, but did not assist the team members. You performed some of the duties of the assigned team role	You usually help to complete group goals with a positive attitude about the tasks and work of others. You assisted team members in the finished project and performed nearly all the duties of the assigned team role.	You work to complete all the group goals while maintaining a positive attitude about the tasks and work of others. All team members contributed equally, and you performed all the duties of the assigned team role
		Interaction	No participation	The speaker makes many grammatical mistakes, using very simplistic, bland language.	The speaker uses language which is appropriate for the task	The speaker uses language in highly effective ways to emphasise the meaning of the message
	Responsible behaviour	Sense of Direction	You do not seek ways to improve personal or group performance and seem to be lost regarding what must be done.	You sometimes seek ways to improve personal or group performance, seem occasionally lost to what must be done.	You seek ways to improve personal or group performance and work towards a goal.	You always seek ways to improve personal or group performance and continually connect your processes to your intended outcome.
		Independent Work	You need constant supervision	You need regular supervision and direction	You need some supervision and reassurance	Minimal supervision and reassurance is needed.

APPENDIX E – STUDENT REFLECTION GUIDE

NAME OF TASK:____

Please mention the mathematical 'big ideas' and skills that you used to solve this activity (e.g. ratios, proportions, forces, etc)	
Symbolising	
After solving this activity, circle the score that best describes how well you understand the mathematical ideas you used: Not at all - 0 A little bit - 0.75 Some - 1.5 Most of it - 2.25 All of it - 3	
Symbolising	
How difficult do you think this activity was? Circle your choice: Easy - 3 Little challenging - 2.25 Somewhat challenging - 1.5 Challenging - 0.75 Very difficult - 0	
Meta-cognition	
Explain why you feel that way:	
Meta-cognition	
After seeing all of you classmates' presentations, what do you think you would do differently with your presentation?	

APPENDIX F – GROUP REPORTING SHEET

DATE	**GROUP**	**TASK**
This problem is about	What do we want to achieve at the end?	What information do we need to answer the problem?

What assumptions can we make to simplify this problem?	What are the potential consequences of our assumptions?	What have we done to work out a solutions	Why did we follow this thought process?

<table>
<tr><td>What do we need to do now?</td><td>Why do we want to do this?</td><td>What do we need help with?</td><td>Why do we need help with it?</td></tr>
<tr><td colspan="2">What will we do differently next time?</td><td colspan="2">Why will we do it differently?</td></tr>
</table>

Adapted from Biccard and Wessels (2011:233)

APPENDIX G – GROUP FUNCTIONING SHEET

Brief description:
The thinking that takes place in your group involves several processes: diversity, selection, communication, and preservation of ideas.
Report using a scale of A to E, how you think your group participated in each of the four processes as your group worked on this activity.

DIVERSITY	EARLY	MIDDLE	LATE
A. Spent all of our time brainstorming ideas			
B. Spent most of our time brainstorming ideas			
C. Divided our time equally between brainstorming and working with current ideas			
D. Spent most of our time working with current ideas, did a little bit of brainstorming			
E. Spent no time brainstorming			

SELECTION	EARLY	MIDDLE	LATE
A. Our attention was focused on one approach			
B. Our attention was focused mainly on one approach, with a backup idea			
C. Our attention was focused on two approaches equally			
D. Our attention was focused on several approaches			
E. Our approaches were all being considered equally			

COMMUNICATION	EARLY	MIDDLE	LATE
A. There was a lot of communication – both written and verbal			
B. There was a lot of communication – either written or verbal			
C. There was some communication – written or verbal			
D. There was a little communication – written or verbal			
E. There was no communication			

PRESERVATION	EARLY	MIDDLE	LATE
A. Our strategy was reusable			
B. Our strategy needed modification			
C. Our strategy was only applicable to this problem			
D. Our strategy was weak			
E. We did not have a working strategy			

(Adapted from Hamilton et al. (2007))

APPENDIX H – POSTER PRESENTATION – WRITTEN AND ORAL WORK GUIDE

DATE	**GROUP**	**TASK**

	High (5)	**(4)**	**(3)**	**(2)**	**Low (1)**
Designs the poster **logically**: Layout is logical and easy to follow					
Uses Illustrations **effectively**: Illustrations are necessary and sufficient to aid understanding of the text					
Text is **concise**: Overall presentation is of agreed size. Uses English correctly					
Understanding: Demonstrates understanding of project through discussion					

(Adapted from Berry and Nyman (1998:112))

APPENDIX I – POST INTERVENTION QUESTIONNAIRE

Questions asked in the post intervention questionnaire to ascertain the students feelings and believes about the mathematical modelling course were taken from Berry and Nyman (1998:108-111):

1. A course should challenge students to stretch themselves intellectually. Did this course challenge you in ways which strengthened your ability to think and learn? And if so, how?

2. Regardless of your own level of enjoyment or success in this course, do you consider the course content to have been worthwhile for your education? Why or why not? How serious was your own effort to understand and master the material covered in this course?

3. In a liberal arts setting, courses should increase students' awareness of connections between related areas within their own major discipline as well as those between their own and other disciplines. Comment on those connections you became aware of during the course.

4. What advice would you give to other students who were planning to take this course? If you had known earlier what you know now, would you approach your own work in this course differently?

5. Did you learn anything in this course that surprised you? Did you ever surprise yourself?

MODEL-ELICITING ACTIVITIES

APPENDIX J – ACTIVITY 0 – PRE-INTERVENTION INTERVIEW

1. Feelings about Mathematics	On a scale of 1-10, with 10 being the highest, how much do you like mathematics? Are there some parts of mathematics you like and some you don't? Please explain.
2. Effort in Mathematics	How hard do you work in the mathematics class? Do you always do everything the teacher assigns?
	In general, what influences you to work hard in mathematics? Is there anything that causes you to work very hard?
	How do you like mathematics in comparison to other subjects? Are the factors that make you work hard in other subjects different from the ones that make you work hard in mathematics?
3. Goal Orientation and Effort	How often do you work hard in mathematics just to learn the material?
4. Self-confidence in Mathematics	How good are you at mathematics? How do you know it?
	Are you better at some kinds of mathematics than others? For example, are you better at long division than you are at fractions, or are you better at computations than problems that require a lot of thinking?
5. Natural Ability in Math	Do you think it takes a special talent to do well in mathematics? Do you have such talent? Can people do OK in mathematics even without special talent?
	When someone makes mistakes in mathematics, does it mean that person is dumb in mathematics?
	How important is memorising in mathematics? Are you good at memorising? Can someone who is not very good at memorising be good in mathematics? (or even OK in mathematics?)
6. Mathematics Content	Suppose an alien from outer space landed in you back yard and started asking you what mathematics was like in South Africa. What would you tell him? What words best describe mathematics?
7. Communication	How important do you regard mathematics learning as a collaborative activity?

(Adapted from Kloosterman (2006:266))

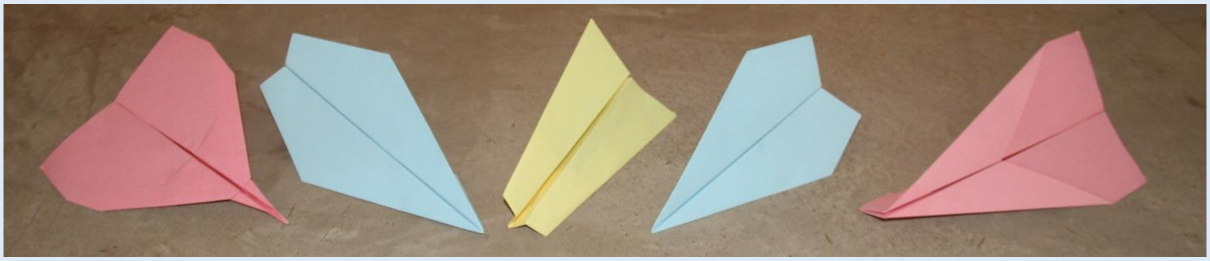
APPENDIX K – ACTIVITY 1 – LAWN MOWING TASK

The Lawn Mowing Task was adapted from modelling tasks designed by Singh and White (2006:44).

Mr. Green, the small business owner of *'Keep it Clean and Green'* garden services, has asked your consultancy firm to provide him with a model to improve the planning of his daily workload. One of the weekly responsibilities of *Keep it Clean and Green* garden services, includes the upkeep of the local school's lawn. The lawn gets mowed every week and Mr. Green needs all of his workers to complete the task in 5 hours. His total staff compliment is made up of 9 workers. However, an epidemic has broken out in the area, which caused Mr. Green to adjust his weekly timetable. This week, 3 of his workers are absent. Mr Green needs to plan for future absenteeism, as he is currently unable to commit to all his clients due to a shortage of staff and needs to re-schedule all his appointments. As the job at the school requires more manpower and time than any of his other clients, he needs to prioritise this job to remain in business. Your task is to develop a model for Mr. Green to project how long it will take to mow the school's lawn given that some of his workers may be absent. The solution must be in the form of a poster presentation.

APPENDIX L – ACTIVITY 2 – PAPER AIRPLANES

The Paper Airplanes task was adapted from Eames, Brady and Lesh (2016:230), and requires the students to assist judges from a local high school to create a procedure to determine the most accurate paper airplane in a competition. This activity provides the students with opportunities to work in teams, and to create a system to judge a paper airplane contest. Mathematical ideas can be acquired from real-life situations while making mathematical connections through problem-solving. The students have to consider the relevance and importance of certain given information to create meaningful solutions.



Pietermaritzburg High School

165 School Road
Pietermaritzburg
3201

Dear engineering team

A paper airplane contest is again planned to be held at our school next year. Prizes will be awarded for characteristics for example, the most accurate, best floater, fanciest flyer, and most creative airplane. However, there exists a lot of controversy about which planes really should win several of the contests. Arguments arose for two main reasons:

c. Differences may not be large between planes or pilots who are ranked 1st, 2nd, and 3rd; and
d. Planes often fly quite differently when different pilots toss them.

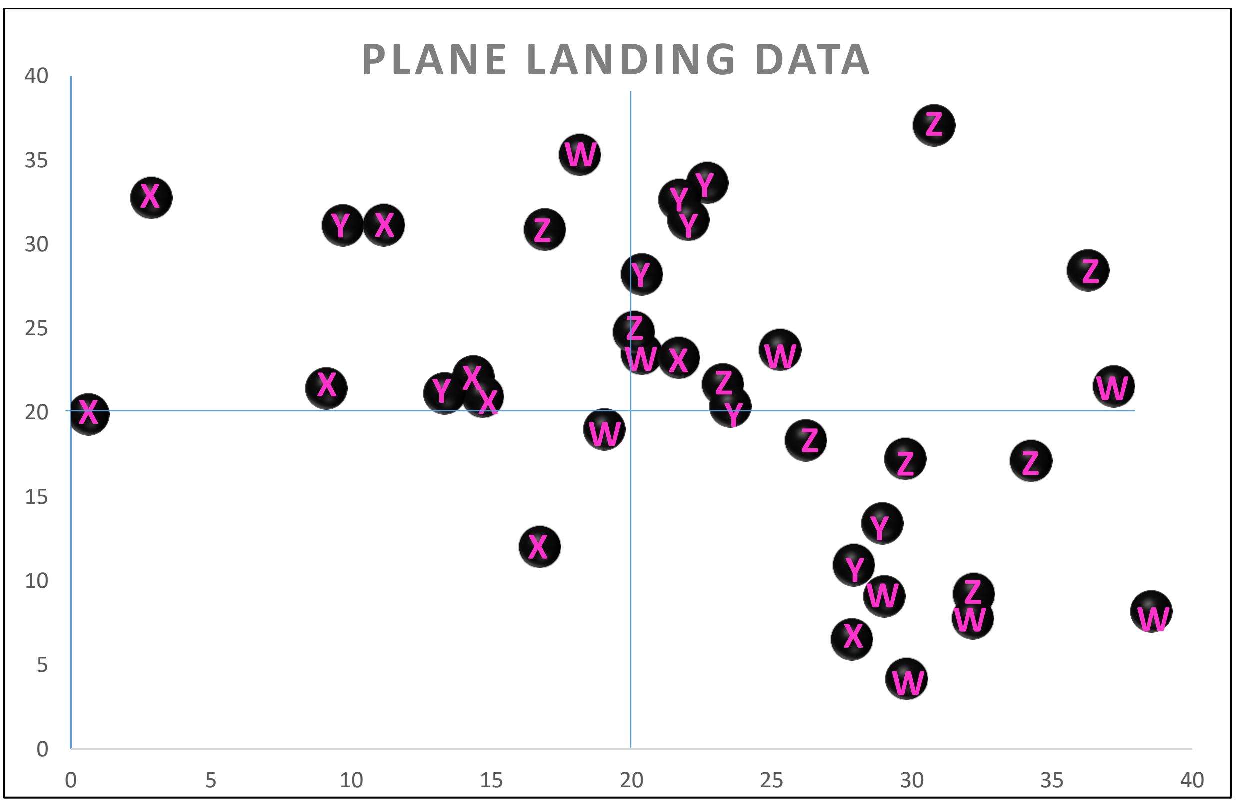
The judges want to have better and more quantitative rules for judging planes for each award, and as much as possible, they want their judgments to depend on clear rules or formulas. Three judges are going to continue their policy of having at least three different pilots fly each airplane, and an award will be given to the best paper airplane. They therefore need a procedure that can somehow factor out the pilot factor when judging the planes.

Please help the judges to plan for the paper airplane contest and provide us with a report that explain how they can use information of the kind shown in the diagram and data table provided in to give awards for the plane that is the most accurate.

Thank you,

Mr Bird
Head of Science Department

Landing positions of paper airplanes											
No	**Plane**	**Pilot**	**Flight Distance**	**Angle Error**	**Flight Time**	**No**	**Plane**	**Pilot**	**Flight Distance**	**Angle Error**	**Flight Time**
1	W	A	39.40	(33.00)	4.70	**19**	Y	A	30.00	(23.60)	6.00
2	W	B	34.70	(1.80)	2.40	**20**	Y	B	38.40	10.00	5.80
3	W	C	31.10	4.10	1.90	**21**	Y	C	32.60	27.70	4.60
4	X	A	28.60	(31.80)	2.30	**22**	Z	A	33.50	(29.00)	6.40
5	X	B	26.40	12.10	2.10	**23**	Z	B	34.40	(14.90)	3.40
6	X	C	19.90	43.20	2.20	**24**	Z	C	31.80	(2.00)	6.00
7	Y	A	31.90	(20.10)	3.20	**25**	W	A	30.10	(37.00)	2.40
8	Y	B	40.60	11.00	5.70	**26**	W	B	33.10	(31.40)	2.00
9	Y	C	39.20	11.40	7.80	**27**	W	C	26.90	0.00	1.30
10	Z	A	38.30	(18.40)	5.00	**28**	X	A	33.10	25.30	3.00
11	Z	B	46.10	(6.90)	7.80	**29**	X	B	25.60	10.00	3.80
12	Z	C	35.20	16.30	6.00	**30**	X	C	32.90	40.00	3.30
13	W	A	30.40	(27.60)	1.80	**31**	Y	A	25.00	12.80	3.80
14	W	B	43.00	(14.90)	3.90	**32**	Y	B	31.10	(4.10)	4.70
15	W	C	39.70	17.80	2.40	**33**	Y	C	34.80	9.20	6.60
16	X	A	23.30	22.00	1.60	**34**	Z	A	32.00	(10.00)	6.40
17	X	B	31.80	2.00	3.50	**35**	Z	B	31.90	6.00	4.10
18	X	C	20.60	(9.30)	1.00	**36**	Z	C	48.20	5.30	7.20



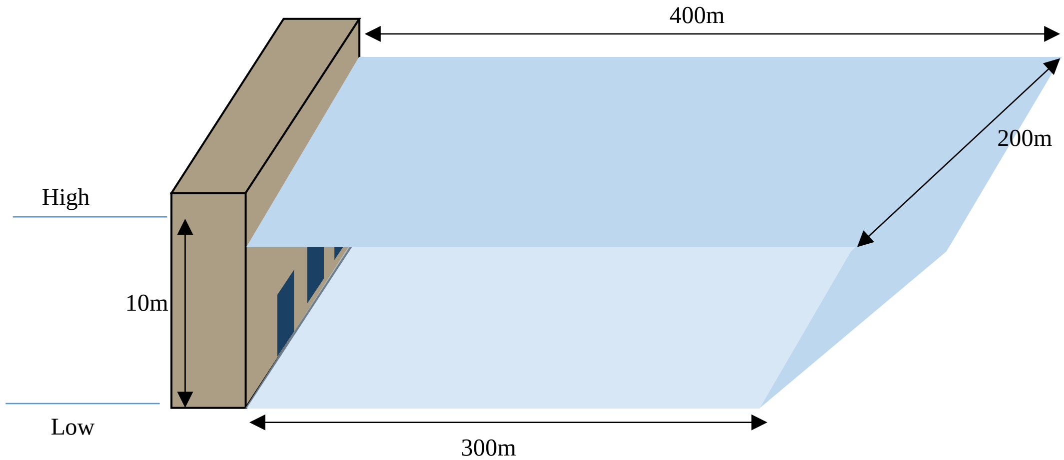
APPENDIX M – ACTIVITY 3 – TIDAL POWER TASK

The development of mathematical ideas from real-life situations – Geometry (adapted from Hamilton et al. (2008:6))

Background

The City Council of See Shell Island has asked your engineering firm to provide an analysis of their tidal power plant. Due to population and business expansion on Sea Shell Island, there is a need to obtain more energy from the power plant. In particular, the City Council is looking to increase energy production at the plant with around 15%.

Tidal power plants generate electricity by trapping water from the rising tide behind a dam, and then letting it out so that it turns one or more turbines. Currently, Sea Shell Island has a tidal power plant whose basin is 200 meters across and goes 400 meters inland. Openings in the dam allow water to enter and leave the basin as the tide rises and falls. This passing of water through the dam generates energy that can be stored, processed and distributed. The amount of energy generated, is directly proportional to the amount of work required to fill the dam. In the case of the Sea Shell Island plant, the energy produced each time the dam empties, is 70% of the work required to fill it. The depth of the basin is 10 meters at the dam wall and gradually decreases to ground level at 400 meters inland. The bottom of the basin follows the shape of a trapezoid: (See accompanied diagram)



Task

Write a report for the City Council that addresses the current particularities of the power plant and provides at least two alternative designs for achieving a 15% net gain in power output. Note that the City Engineer will read the study and represent your findings to the Council. It is appropriate to provide detailed calculations along with relevant explanations for any solutions that you propose. Any charts and graphs you use can be incorporated into the report.

Carefully consider the council's desire to increase the energy production by 15%. Discuss different construction options on the basin to achieve this result. Note that there is open space for another 80 meters inland, but beyond that there are buildings and roads. The community has expressed a preference for retaining as much open space as possible. You should consider options that require excavating the minimal amount of earth since the cost of the project will be directly proportional to the volume of earth that needs to be excavated.

Current particularities

You need to specify the current volume of the dam, as well as the proposed new volume of the dam	
Density of water (ρ)	1000 kg/m^3
Gravity (g):	9,8 m/s^2
The gravitational potential energy of the water mass:	$PE_g = mgh = \frac{kg.m}{s^2}.m = Work \times m = Newton \times m = Joules$ where m represents the mass of the water in kg, g represents gravity (m/s^2), and h represents the **average height** of the dam (m).
Power produced as water flows through the turbine	$P = \frac{PE_g}{t}$

Assumptions

Changing the width of the dam is not practical since it already exists. Therefore all construction should be focused on changing the basin. The dam can be closed (thus not allowing any water to enter) so that construction can be accomplished. Currently the vertical cross section of the dam is in the form of a trapezoid, but this is not necessarily required to be the case after excavation work is done.

APPENDIX N – ACTIVITY 4 – PRODUCT CODING TASK

This activity, adapted from Galbraith (2009:15), required the students to verify and correct barcodes for a large supermarket by building and testing mathematical models, to explain and judge the validity of different product codes in terms of both EAN and ISBN systems

Background

The idea of placing a barcode on a product originated in the 1930's, but the first barcode reader was not built until 1952. In 1974, the first retail product (a packet of chewing gum) was sold using a barcode reader at a supermarket in Ohio. Barcodes allow for instantaneous processing of information by computers and is used almost all over the world. South Africa uses the European Article Numbering Code containing 13 digits (EAN-13), which is one of the most commonly used systems worldwide. The following figure represents and ISBN-EAN barcode.

A sample barcode:

0 36000 29145 2

The following codes were respectively taken from an iodised salt product and an instant coffee product, sold in a Spar Supermarket.

600 102102102 3 (Wellington's tomato sauce)

600 100715730 2 (Cerebos iodised table salt)

The left-most digit (called the 0^{th} digit) together with the next 1 or 2 digits (called the 1^{st} and 2^{nd} digits) indicates the country of manufacture (for example, 600 and 601 represents South Africa, 76 represents Switzerland and 94 represents New Zealand).

The next 9 or 10 digits (depending on how many digits are used for the country code) identify the manufacturer, as well as the specific product. The final number is a *check digit*. All products that are manufactured by a specific company, will use the same manufacturer code.

When the label is scanned, the barcode identifies the item of which the price is stored in the retailer's database. When a barcode is read, the computer will verify that the check digit is correct, before processing the number. If an error is detected, the computer will indicate an error.

This can happen for example, if a paper label on a can is distorted, as it can cause a digit to be misread. A checkout attendant will then have to manually enter the barcode. This procedure is also subject to error.

The check digit works as follows: Using the first 12 digits in the code, the check digit satisfies the condition that:

3 × (sum of even digits) + 1 × (sum of uneven digits) + check digit is divisible by 10. (Here 3 and 1 are referred to as *weights*).

Background to ISBNs (International Standard Book Number):

Every new *book* that gets published, gets allocated with an ISBN, which is a unique identifier. Each ISBN has four parts, which are separated by blanks or hyphens.

5. A group identifier (this identifies the particular country participating in the ISBN system)
6. A publisher's identification number (variable length)
7. A title number (variable length)
8. A single check digit (0, 1, 2, … 9, X)

For example, paperback editions of the trilogy *Lord of the Rings* by JRR Tolkien, published by HarperCollins Publishers, have the following ISBNs:

Fellowship of the Ring	0 26110 357 1
The Two Towers	0 26110 358 X
The Return of the King	0 26110 359 8

Because every book is uniquely identified by its ISBN, it is important to guard against errors (for example, transcription errors) that could have serious consequences for ordering and charging, specifically with automatic coding procedures. The check digit is assigned to take care of this and is calculated as follows (using *Fellowship of the Ring* as an example):

$10\times0 + 9\times2 + 8\times6 + 7\times1 + 6\times1 + 5\times0 + 4\times3 + 3\times5 + 2\times7 = 120$ + check digit must be a multiple of 11. Thus the check digit in this case equals 1.

Note that, because of the division by 11, the check digit can sometimes turn out to be 10. Because 10 cannot be represented by a single digit, the Roman number for 10 (**X**), is used to denote the check digit. An example of a Roman check digit appears in the barcode for *The Two Towers* above.

Bookland and EAN-ISBN codes:

EAN and ISBN codes come together in the publishing industry. Since book publishers commonly publish in a variety of countries, the book industry has designated and imaginary 'country' called Bookland, as the wonderful place where all books are produced, together with its own special prefix '978'. An EAN-ISBN code for a book starts with 978 and follows with the first nine digits of the ISBN, concluding with a check digit calculated according to the EAN rule. Thus only rarely will this check digit be the same as the one in the ISBN. Commonly, a second shorter code is printed as well, which gives the price in whatever currency is appropriate.

Task:

A large supermarket, ABC Bargains, experienced huge problems with the printing of their barcodes. They called the help of your development team to assist with some specific issues that they picked up. The manager asked you to assist and correct (where necessary) the following, and to provide a thorough explanation on each of these matters:

7. Verify the check digit for the other two titles of the *Lord of the Rings* trilogy.
8. The manager wants to change the ISBN code for the *Lord of the Rings* trilogy to EAN-ISBN codes. Find the check digits for the EAN-ISBN codes that would be allocated to the *Lord of the Rings* trilogy.
9. An ISBN was incorrectly recorded as 540 12156 5 by omitting the group (country) identifier. Correct the code by adding the missing digit.
10. A printing flaw caused a digit in an ISBN to be illegible. The number appeared as 0 853□2 456 6. Find the missing digit to correct the code.
11. In copying an ISBN, two of the adjacent digits were accidentally transposed, and the code was printed as 0 340 39155 X. Find and correct the error.
12. The manager of the store is concerned that the printing company printed the first three digits of one of their products in the wrong sequence. Prove that, if *a*, *b*, and *c* are digits such that 1 867751*ab c* is a correct ISBN, then 1 687751*ab c* cannot be an ISBN.

A report needs to be submitted to the manager and all of the above issues need to be explained. The report needs to include all the necessary calculations and explanations to assist them in dealing with future printing problems.

APPENDIX O – ACTIVITY 5 – TURNING TYRES TASK

The activity was based on the work of CPALMS, the State of Florida's official source for standards information and course descriptions. CPALMS was created by the Florida Center for Research in Science, Technology, Engineering and Mathematics (FRC-STEM) at the Florida State University (CPALMS, 2012).

Letter from the client:

BESTYRES
825 Long Drive Ave
Mkondeni,
Pietermaritzburg, 3200

Dear engineering team

Our company, "BesTyres" is responsible for supplying custom-made vehicles that are suitable and economical to use in various landscapes. We have recently received a request from Lesotho's government to produce a tyre that is appropriate to be used on both off-road as well as on-road terrains. We need your team to develop a procedure to select the optimal tyre materials to suit their needs.

Please furnish us with a report that ranks your choices of material from best to worst and motivate your decisions in detail by providing procedures to us to be able to use in the future. The final cost of material for each tyre needs to be included. We are only concerned with the following sizes and aspect ratios:

Sizes: 200, 265 and 330

Aspect Ratios: 45% and 88%

Thank you,

John Car

"BESTYRES" President

Accompanied data sets:

Tyre material types data set:

Tyre Material Type	Durability (1-10)	Defect Rate* (%)	Performance (1-10)		Cost per Sq. Inch
			Off-road	On-road	
Material A	4	2.41%	6	6	ZAR 0.54
Material B	7	0.28%	6	4	ZAR 2.40
Material C	7	0.41%	3	7	ZAR 1.60
Material D	6	0.23%	8	3	ZAR 0.90
Material E	8	0.52%	5	6	ZAR 1.20

* The defect rate refers to the probability that a tyre will be defective. Upper limits come from the Firestone tyre recall in 2000 and other values were based on a report on defect rates.

Performance rates of change data set:

Measurement	Change	Durability	Performance	
			On-road	Off-road
Section Width	+ 10 mm	+ 0.1	+ 0.3	- 0.2
Aspect Ratio	+ 10%	- 0.2	- 0.2	+ 0.5

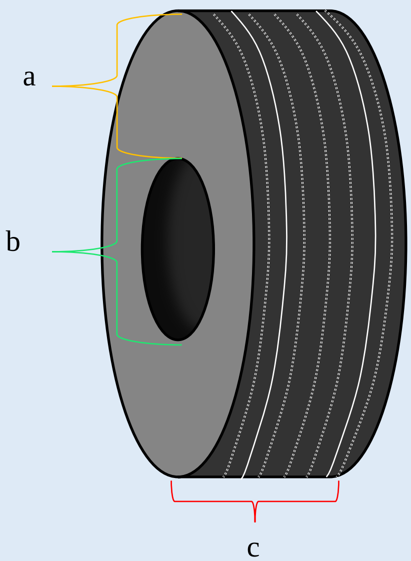
Performance: For every 10% increase in Aspect Ratio, the off-road performance goes up by 0.5 unit, and on-road performance decreases by 0.2. For every 10mm increase in section width, on-road performance increases by 0.3, durability increases by 0.1, and off-road performance decreases by 0.2.

Further background:

Baseline tyre size:
All tyre material types were rated with a P225/60R16.5 tyre. This means the Section Width is 225mm, Aspect Ratio is 60% of that, and the Rim Size is 16.5” (Surface area of tyre = 1483.5 sq.in.).

- In the information for the measurements of the tyre, “P” denotes that it is a tyre for a “Passenger” vehicle. This will not necessarily be the case for your tyre and can be ignored.
- The number following it denotes the Section Width in mm (225). This means that the width of the tyre from inner sidewall to outer sidewall is 225mm.
- The next number (60) is the Aspect Ratio, which is given by the ratio of the Sidewall Height to the Section Width. It means that the height of the Sidewall is 60% that of the Section Width.
- Finally, the last letter/number combination (R16.5) gives the Rim Diameter in inches. This is the diameter of the hollow section in the center of the tyre.

A diagram of a tyre and its parts:



a = Sidewall height
b = Rim diameter
c = Section width
Aspect ratio = $\frac{Sidewall\ height}{Section\ width}$ x 100

Helpful equations for new qualities of durability and performance, denoted by “Quality” (Q):

$$Q_n = Q_o + \frac{\alpha}{10}(SW - 225) + \frac{\beta}{10}(AR - 60)$$

Q_n = new value of quality (e.g. durability, performance)
Q_o = original value of quality
α = increment of change with regard to Section Width (e.g. durability changes by 0.1)
SW = New value of Section Width (200, 265, or 330)
β = increment of change with regard to Aspect Ratio (e.g. durability changes by -0.2)
AR = New value of Aspect Ratio (45 or 85)

Note: The conversion of inches to millimetres is 1 inch = 25.4 millimetres

APPENDIX P – ACTIVITY 6 – FINDING THE CELL PHONE TASK

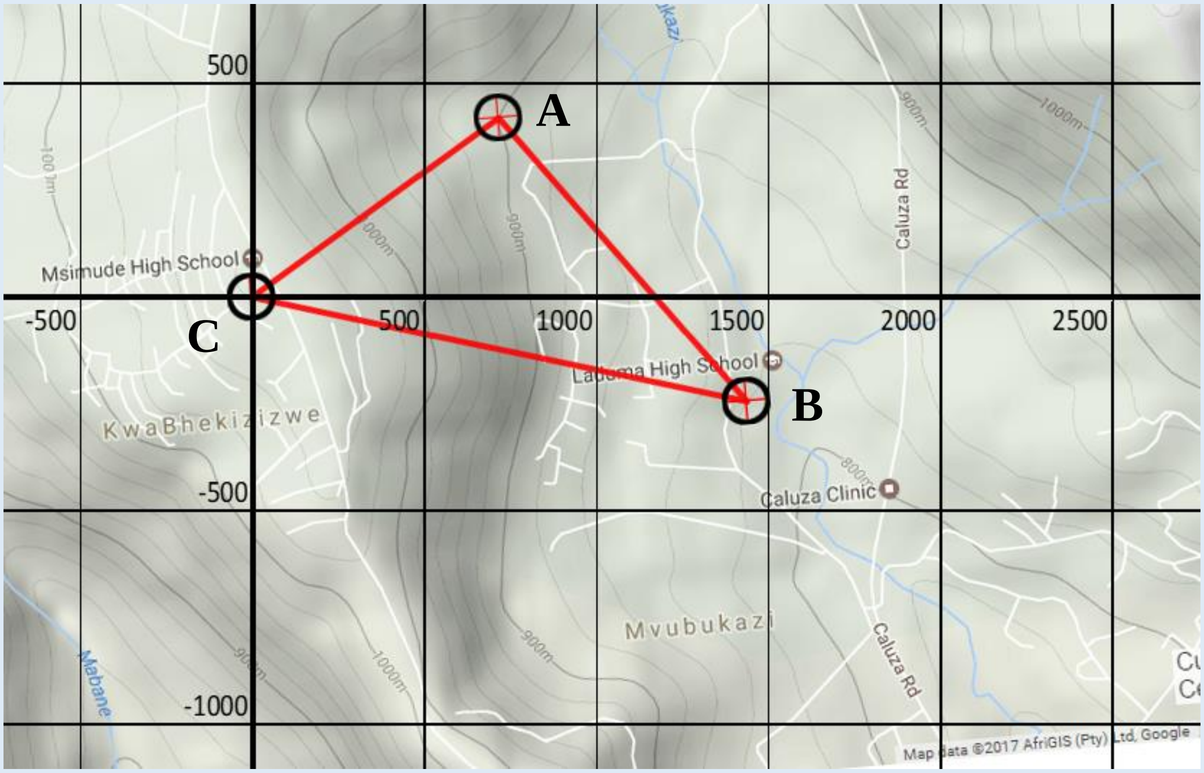
This MEA, which was based on the Lost Cell Phone Problem by Anhalt and Cortez (2015:449), provided students with the opportunity to develop mathematical ideas from real-life situations, by making mathematical connections through problem-solving. Students had to consider the relevance and importance of certain given information to create a meaningful solution.

Background:

Electromagnetic radio waves, or radio frequency (RF energy) is emitted when you make a call on your cell phone. The cell phone tower's antenna that is the closest to your phone will receive these radio waves. Cell phone towers have antennas at the top of the towers that can both receive, as well as transmit, signals from your phone. Once the tower has received a signal from your phone, the signal is transmitted to a "switching center" – a telephone exchange for mobile phones. This connects your call to another phone or to another telephone network. The geographical area in which a cell phone tower is located, is known as a "cell" (from there the name "cell phones"). Some cell phone towers have larger cells than others, depending on the traffic that is required during peak times. Due to this reason, the cells of the towers in city centres are normally smaller than cells in less populated areas. "Hand-overs" or "hand-ons" occur when you cross the border between two cells. The new cell will automatically take over and this process is controlled by a computer in the switching centre.

Task:

A detective company has called for your services to assist them in solving a murder case. A young man was found murdered only a short distance from the local university. After consulting with his family and friends, new information surfaced indicating that he could have been a victim of cell phone abuse. No cell phone was recovered at the murder scene, but three cell phone towers in the vicinity were able to detect a signal. A coordinate system used by the city, indicates that the cell towers are located at $(x; y)$ coordinates, measured in meters from one of the cell towers (see topographic map). You have been asked to create an approach for finding the location of the lost cell phone, and to explain your reasoning as to assist them in finding lost phones based on this information in the future. A topographic map of the three towers provides the following information:



(Google Maps, 2017)

Tower A is at position $(787; 455)$, cell tower B is at position $(1478; -194)$, and cell tower C is at position $(0; 0)$. Tower A detects the signal at a distance of 603.5 meters. Tower B detects the signal at a distance of 804 meters, and tower C detects the signal at a distance of 760.6 meters.

COMPETENCE MAPPING

APPENDIX Q – MAPPING ENGINEERING TECHNICIAN COMPETENCIES AND MATHEMATICAL MODELLING COMPETENCIES

	Modelling Competencies	What the engineering technician does
Horizontal	Internalising	Interprets the client's requirements, leading to an agreed statement of requirements.
		Gathers information required for problem analysis.
		Clarifies requirements with the client
		Interprets the client's requirements, leading to an agreed statement of requirements. Identify legal, regulatory, health and safety requirements Identify ethical problems, affected parties and their interests
	Interpreting	Identifies design aspects standards, codes and procedures Identifies acceptance criteria for work product Identify applicable legal, regulatory and health and safety requirements for the engineering activity.
		Identifies design aspects standards, codes and procedures Recognise the legal issues that can influence the situation Recognise the effect of ethical problems
		Identifies design aspects standards, codes and procedures Verifies that the design problem is amenable to solution by candidate's techniques. Identify environmental impacts of the engineering activity. Identify sustainability issues.
	Structuring	Documents functional solution requirements and gains client acceptance.
		Displays mastery of established methods, procedures and techniques in the practice area. Displays working knowledge of areas that interact with the practice area Applies codified knowledge in related areas: financial, statutory, safety, management Considers the interdependence, interactions and relative importance of factors
	Symbolising	Displays mastery of established methods, procedures and techniques in the practice area
		Applies knowledge underpinning methods, procedrues and techniques to support technician activities
Vertical		Applies codified knowledge in related areas: financial, statutory, safety, management Displays working knowledge of areas that interact with the practice area
	Adjusting	Uses information technology effectively as required by the practice area. Evaluate each solution using the interests of those involved
		Search for possible solutions for the dilemma Displays working knowledge of areas that interact with the practice area
		Select and justify solution that is best resolves the dilemma
	Organising	Propose measures to limit negative effects Considers a limited number of factors, some of which may not be well defined Draw on experience and knowledge. Foresees consequences of actions Evaluate each solution using the interests of those involved Displays knowledge of areas that interact with the practice area
	Generalising	Evaluates a situation in the absence of full evidence

Engineering Technician Competencies as described by ECSA
Define, investigate and analyse complex engineering problems
Design or develop solutions to complex engineering problems
Comprehend and apply advanced knowledge: principles, specialist knowledge, jurisdictional and local knowledge
Recognise and address the reasonable foreseeable socia, cultural and environmental effects of complex engineering activities
Meet all legal and regulatory requirements and protect the health and safety of persons in the course of his or her complex engineering activities
Conduct engineering activities ethically
Exercise sound judgement in the course of complex enineering activities

PERMISSION AND ETHICAL CLEARANCE

APPENDIX R – PERMISSION FROM DURBAN UNIVERSITY OF TECHNOLOGY

DUT DURBAN UNIVERSITY OF TECHNOLOGY

FACULTY OF ENGINEERING & THE BUILT ENVIRONMENT

DEPARTMENT OF CIVIL ENGINEERING

Department of Civil Engineering
Faculty of Engineering and the Built Environment
Durban University of Technology
Indumiso Campus, F, Sithole Road
Pietermaritzburg, 3201

P O Box [illegible], Scottsville,
Pietermaritzburg, 3209
South Africa
Tel. 033-845 9000
Fax: 033-845 8941
Email: [illegible]@dut.ac.za

www.dut.ac.za

Tuesday, 24 May 2016

Mrs Lidamari de Villiers

41 Windermere Road

Wembley

Pietermaritzburg

3201

Dear Mrs. Lidamari de Villiers

PERMISSION TO CONDUCT RESEARCH: PROJECT

The Department of Civil Engineering, Pietermaritzburg, of Durban University of Technology hereby grants you permission to conduct research in its institution as per application. We have noted the following details pertaining to this research;

<u>Topic of Research:</u> Investigating the development of mathematical modelling competencies of first year engineering students

<u>Nature of qualification</u>: M.Ed (Mathematics Education)

<u>Name of Institution</u>: University of Stellenbosch

Upon completion of the research project the researcher is obliged to furnish the Department of Civil Engineering with a copy of the research report, ideally in bound format.

The Department wishes you success in your academic pursuit.

Yours Sincerely

Tom McKune Pr.Tech.Eng

Head: Department of Civil Engineering

DUT DURBAN UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING
PIETERMARITZBURG
PO Box 10112
Scottsville, 3209, South Africa

APPENDIX S – ETHICAL CLEARANCE FROM DURBAN UNIVERSITY OF TECHNOLOGY

Protecting Human Subject Research Participants

Certificate of Completion

The National Institutes of Health (NIH) Office of Extramural Research certifies that **Lidamari De Villiers** successfully completed the NIH Web-based training course "Protecting Human Research Participants".

Date of completion: 08/16/2012

Certification Number: 962809

APPENDIX T – ETHICAL CLEARANCE FROM STELLENBOSCH UNIVERSITY

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Approval Notice
New Application

18-Aug-2016
De Villiers, Lidamari L

Proposal #: SU-HSD-002606
Title: Investigating students' mathematical modelling competencies

Dear Mrs Lidamari De Villiers,

Your **New Application** received on **20-Jul-2016**, was reviewed
Please note the following information about your approved research proposal:

Proposal Approval Period: **15-Aug-2016 -14-Aug-2019**

Please take note of the general Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

Please remember to use your **proposal number** (**SU-HSD-002606**) on any documents or correspondence with the REC concerning your research proposal.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Also note that a progress report should be submitted to the Committee before the approval period has expired if a continuation is required. The Committee will then consider the continuation of the project for a further year (if necessary).

This committee abides by the ethical norms and principles for research, established by the Declaration of Helsinki and the Guidelines for Ethical Research: Principles Structures and Processes 2004 (Department of Health). Annually a number of projects may be selected randomly for an external audit.

National Health Research Ethics Committee (NHREC) registration number REC-050411-032.

We wish you the best as you conduct your research.

If you have any questions or need further help, please contact the REC office at 218089183.

Included Documents: