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Evaluation of the Energy Model of a Horizontally-Mounted Electric Water Heater Through Internal Temperature Measurement

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Abstract—**The resource-constraint energy sector faces an insatiable demand for energy, which necessitates improvements in efficiency. One key sector that has potential for savings is residential water heating, which makes up 32% of household energy. Previous studies have proven that with effective scheduling up to 29% savings can be achieved for a nominal consumption pattern. The model that was used to estimate the savings, calculates the energy usage for a given hot water consumption pattern and given heating schedule for a horizontally mounted water heater. This two-node model is used to aid user-informed scheduling and auto-scheduling, but was developed as a black-box model, validating the energy and not the internal temperatures, which could be misleading. This paper evaluates the accuracy of the model by performing temperature measurements inside the horizontal electric water heater. Moreover, two aspects neglected by the model are investigated: The node state transfer usage threshold, and the inter-nodal thermal resistance. The results show that the model significantly underestimates the stratification that occurs naturally. This underestimation also severely affects the modelled energy consumption and hides limitations of the model, preferring a lower threshold and higher inter-nodal resistance. The results also show that Legionella growth in the EWH could be a concern despite a high set point.**

I. INTRODUCTION

Given the increasing cost of energy and its limited resources, combined with increased awareness of climatic change, the need for lower domestic energy consumption has received increased attention. National utilities in the developing world are introducing measures to curb demand. For example, in South Africa, the national energy supplier (Eskom) has offered solar water heating rebates to users [1]. Also in South Africa, the cost of electricity has increased by 300.7% since 2007 [2]. This has led many households to look for affordable ways to lower their monthly electricity bill. On average, the electric water heater (EWH) consumes 32% of total household electricity. Given its energy consumption levels and its substantial energy-capacitive nature, many solutions for household energy savings focus on managing the energy of the EWH. These attributes have put the EWH in the spotlight of many demand side management (DSM) projects, such as for peak load shaving [3],[4]. As there are millions of EWHs across each country (South Africa has 5.4 million) they are the perfect candidates for creating small micro grids and smart networks. This requires feedback in near real-time and accurate readings from the EWHs.

Resource restrictions in a country, such as in South Africa, greatly inhibit real-time capabilities of a smart infrastructure due to the high cost and low bandwidth nature of the vast majority of internet connections. In order to utilise the available connections, optimisation strategies need to be implemented, such as keeping the amount of data transfered from each EWH to an absolute minimum. Additional information can then be obtained through simulation of each EWH using the gathered metrics to gain new information. One such model is the two-node model for horizontally mounted EWHs [5]. This model can be run on a mobile device with low computing power which thus reduces the overall system cost, making it more affordable for the masses.

A. Contribution

This paper builds on the foundation laid by the work in [5], in which a thermal energy model is developed and validated for a horizontally oriented EWH as a black box. This paper uses temperature measurements inside the EWH to avaluate the nodal temperatures modelled by Nel *et. al.*
This paper also investigates two variables neglected by the work by Nel *et. al.*, namely the inter-nodal heat transfer coefficient and the threshold used before transitioning from the one-node state to the two-node state. This paper shows that the model underestimates the natural steady-state stratification that occurs in the EWH, leading to some hidden effects (with regards to the thermal transfer coefficient and nodal state transition threshold) and underestimation of the energy that is used during consumption events (enthalpy). The lower temperatures seen at the lower part of the EWH also means that Legionella growth could be a substantial concern despite a high set point.

II. RELATED WORK

Dolan *et. al.*[6] presents a one-node model, assuming uniform temperature distribution, for an EWH. The model use a first-order differential equation to estimate the average thermal response of the water in an EWH. Fluctuations in internal temperatures due to usage events are not estimated.

Kondo *et. al.* [7] presents a two-node model, describing an upper and lower node caused by a thermocline in the EWH due to natural stratification, for a vertical EWH with two heating elements heating independently. The modeling is simplified by assuming the thermocline to be zero. The upper- and lower-nodes are simulated as individual one-nodes which consists of fixed volumes with uniform temperatures where the upper-node always has a higher temperature than the lower-node. Although the heat dissipation of each node is considered with an estimated EWH thermal resistance, the model is not validated with measured data. The consumption profiles for individual EWHs were estimated from measured average residential load profiles. The assumption of fixed node volumes and a zero thermocline within the EWH will result in erroneous energy flows for above average water usage events.

Diao *et. al.* [8] presents a one- and two-node transitioning model for vertical EWHs which consists of the one-node model from [6] and the two-node model of [7]. The model transitions to the two-node state when a withdrawal occurs and returns to a one-node state when the EWH contains only warm- or cold water. The two-node model also assumes a uniform temperature in each node as in [7] although, a varying thermocline height during withdrawal events is taken into consideration. The upper-node in the two-node model is held at constant temperature of the average withdrawal temperature, ignoring the effect of standing-losses. Since the model does not take the cross-sectional area of the EWH into consideration, it is only valid for vertical orientated EWHs. The model accuracy is not validated with measured data.

Booysen *et. al.* [9] validated Nel's EWH model with a field study in which actual consumption patterns were used with and without schedule control. It is stated that 29% savings can be achieved with informed schedule control. The savings calculations are based on estimating the internal temperatures from the EWH energy model. The case study is implemented with European Telecommunications Standards Institute (ETSI) compliant smart grid technology.

All the aforementioned contributions to modeling an EWH makes the assumption of the internal temperature being the average outlet temperature of the latest withdrawal event. This estimation can be correct at withdrawal times, although at higher resolution estimations the internal temperature will be affected due to a temperature measurement taken outside of the EWH which is exposed to the natural elements. These natural elements play a large role in measurements in the South African setup of EWHs. The Majority of the EWHs are being moved from the traditional roof attic installations to outside, for prevention of damages due to leaks. The existing EWH model is explained in the next section.

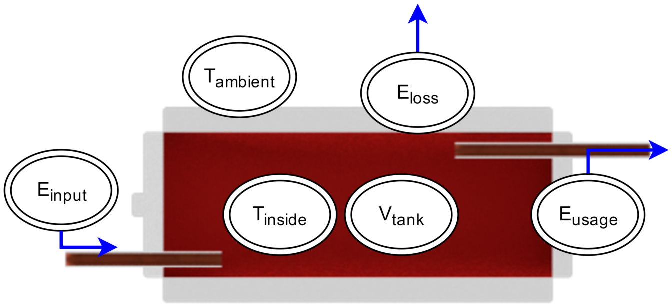

Fig. 1. One node representation

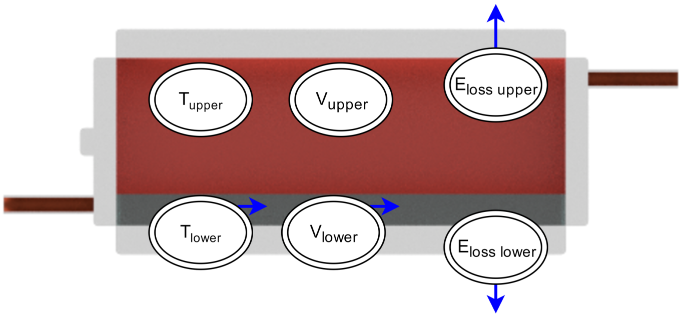

Fig. 2. Two node representation

A. *Existing EWH Model*

The existing EWH model described in detail by Nel in [5], and summarized in [9], is a verified and accurate model simulating the energy flow within the EWH. The model can be described as having two states, a one-node state and the other a two-node state. The two-node model transitions between a single node, in one-node state, to a binodal state which effectively consists of two one-node nodes known as the lower- and upper-node, with each having their separate energy input and output sources. A visual representation of the respective model nodes can be seen in figures 1 and 2.

The stratified nodes of the two-node model are separated by a thermocline, which acts as a heat transfer medium between the nodes, caused by natural stratification. The model proposed by Nel in [5] assumes no conductance through this thermocline, although Cloete [10] measured and experimented with thermocline values and implemented it into the existing EWH energy model.

The thermocline constant used in [10] is not verified with actual measurements within the EWH but with measurements taken at the EWH outlet connection.

The existing two-node model assumes a constant water withdrawal threshold. As soon as the threshold is reached, the model transitions from the one-node state 1 to the two-node state 2. This threshold is an assumption made in both [5], [10]. This threshold has not yet been physically measured or

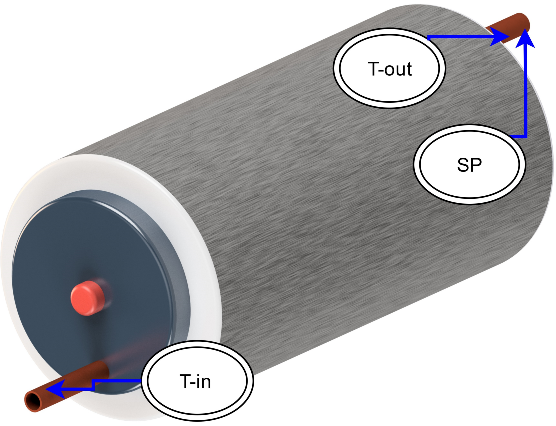

Fig. 3. Experimental setup used by Nel et. al.

determined at this time, of what the author is aware of.

The next section presents a brief overview of how Nel's EWH Model is implemented in [5].

1) One-Node Model: The one-node model, 1, treats the water mass as a uniform body, temperature change in this body happens instantaneously across the entirety of the mass. During a water withdrawal event, the average outlet temperature is used to estimate the current internal EWH temperature. The inrush water mixes instantaneously with the existing water mass, resulting in a new average internal temperature.

The energy inside the EWH (E_{inside}) consists of the input energy by the element (E_{input}), usage energy during withdrawals (E_{usage}) and energy lost to the environment (E_{loss}).

The energy balance can be depicted as follows:

$$E_{inside} = E_{input} - E_{usage} - E_{loss} \tag{1}$$

Calculating the next internal temperature can be derived from 1 with the following equation:

$$\Delta T_{inside} = \frac{E_{input}}{c\rho V_{tank}} \tag{2}$$

Where: c is the specific heat capacity of water and V_{tank} the total volume of the EWH.

The energy lost to the environment is referred to as standing losses. These losses occur when there is a temperature difference between the internal EWH temperature and the outside environment temperature.

These losses can be calculated with the following equations:

$$T_{inside}[n+1] = T_{amb}[n] + (T_{inside}[n] - T_{amb}[n])\, e^{\alpha} \tag{3}$$

$$\alpha = \frac{-t_n}{c m_{tank} R} \tag{4}$$

Where m_{tank} is the mass of water in the EWH and R is the thermal resistance of the EWH.

$$E_{loss}[n] = c m_{tank} \Delta T_{inside}[n] \tag{5}$$

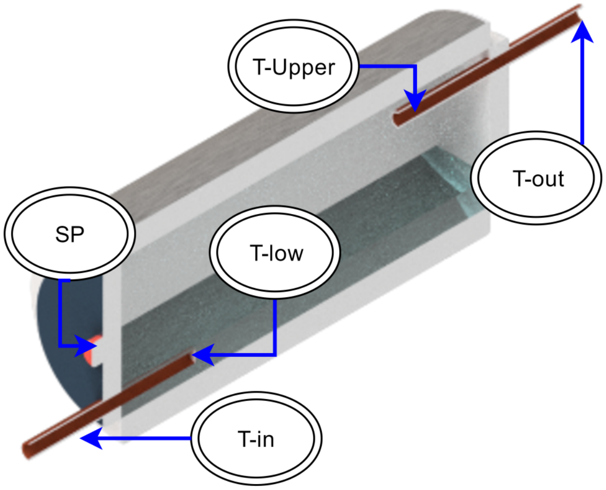

Fig. 4. Experiment of this work, showing placement of thermostat (SP) with temperature sensor.

2) Two-Node Model: The two-node model, 2, uses a volume withdrawal threshold which determines when to transition from the one-node state to the two-node state. The two-node state is when the water body splits into two sections caused by the natural stratification in the EWH when cold water enters the EWH. These sections are referred to as the lower-node (or, hypolimnion) and the upper-node (or, epilimnion). The upper-node indicates the amount of warm water left in the EWH after a withdrawal event, and the lower-node indicates the newly cold water rushed into the EWH.

With the two nodes separated into individual one-node states, each node's energy input, thermal decay and energy used can be calculated separately. The thermal resistance used to determine the thermal decay is dependent on the contact surface area of the node exposed to the environment and the opposing node. To calculate the contact surface area of each node, the Newton-Raphson method is used to first calculate the secant where the lower node contacts the edge of the cylinder, then the area of the node is calculated.

The calculation of the thermal conductance for each node enables the use of Equations 3 and 4 with the consideration of T_{inside} being the nodal internal temperature and the m_{tank} replaced with the nodal mass. Equation 5 can then be used to calculate the energy losses for the individual nodes.

The lower-node will only increase in volume when withdrawal events occur, otherwise it will continue heating until the lower- and upper-node temperatures are equal. The lower- and upper-node only then transitions back to a single one-node state.

The two-node model can also be used with a vertical EWH which reduces the complexity of the calculations since the cross-sectional area remains constant.

Despite apparently accurately describing the temperature inside the EWH, the model disregards the inter-nodal transfer of energy (inter-nodal thermal transfer resistance), and also does not consider the proposed state transfer threshold. Both

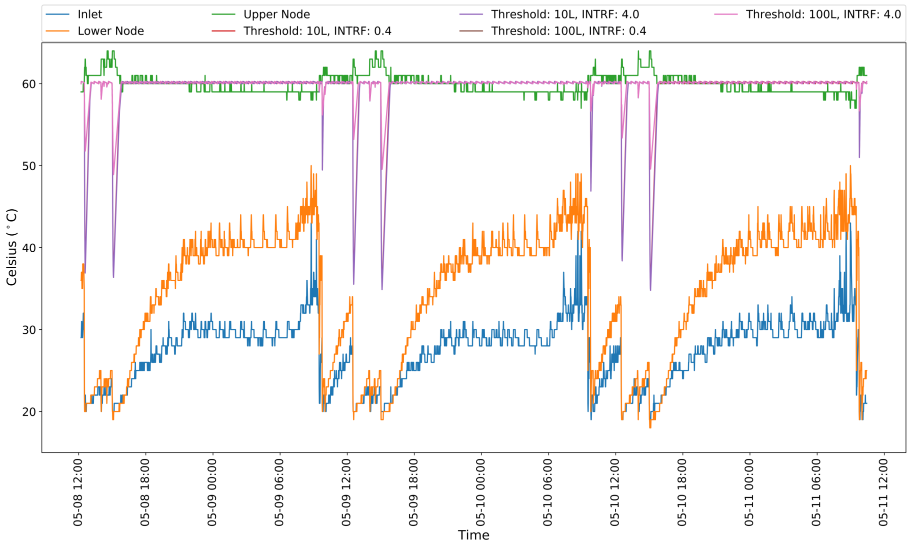

Fig. 5. Measured temperatures of lower- and upper-nodes and inlet with simulated lower node temperatures of specified threshold and Inter Nodal Thermal Resistance Factor (INTRF) values overlaid.

of which are assumed to be correct.

III. Simulation Setup

To validate the model simulations were run using the parameters as shown in Table I. Two parameters which directly influence the model, but have no concrete derivation or estimation, were varied in order to determine their impact on the simulation results. The first parameter being a threshold volume, V_{thresh}, which indicates the required volume of water to be extracted from the EWH to transition from a one-node to a two-node state. The second parameter is the Inter Nodal Thermal Resistance Factor ($INTRF$), which is a constant factor to estimate the thermocline thermal resistance based on the current thermocline area. The simulation was run as an independent simulation in that only independent external parameters were used as parameters for the simulation model in order to verify the input energy. These parameters included the measured inlet temperature, measured ambient temperature and the measured water usage.

The models discussed in [5] enabled simulation of a single EWH with given parameters. The Java implementation was able to simulate 10 days worth of data in a time of 100 milliseconds. If multiple EWH simulations were to be run, this would require sequentially simulating each individual EWH with their respective parameters. This is computationally inefficient as the time required for simulations increases N-fold for N EWHs. This model was improved on by creating a vectorised version of the model and simulator in Python. Vectorisation drastically improves the computational efficiency by utilising the parallel processing capabilities of modern 64-bit CPUs. Though the vectorised approach is slower for a single EWH simulation due to extra overhead for setting up the vector environment, however, the power really starts to show once groups of EWHs are simulated. For a single EWH simulation of 3 days the vector simulation took 733 milliseconds, where simulating 4 EWHs, each with different parameters took 751 milliseconds, an extra cost of 18 milliseconds or 2.46 % longer execution. Scaling this up further, simulating 100 EWHs, each with unique parameters, takes 830 milliseconds, an additional 97 milliseconds or 13.23 % longer execution. This clearly illustrates the power of vectorisation with single instruction multiple dimension operations, enabling multiple scenarios to be explored simultaneously.

TABLE I
Simulation Parameters

Parameter	Value
R	0.471
c	4.184 kJ/kg
Tank Volume	100 L
SP	60 C
P	2 kW
time step	1 minute
V_{thresh}	10 L, 100 L
INTRF	0.4, 4.0
T-in	Measured inlet temperature time series.
T-amb	Measured ambient temperature time series.
Schedule	Continuously on.
Hot water	Measured water used time series.

IV. Data Collection and Experimental Setup

The original experiment that was used to verify the model collected: outlet temperature, inlet temperature, ambient temperature, water usage and electricity usage. The original experiment also used the outlet for schedule control see Figure:3 as well as Table:II explaining the key inputs of the experiments.

The experiment we used to measure internal temperature, uses a thermostat that extends into the EWH. This temperature

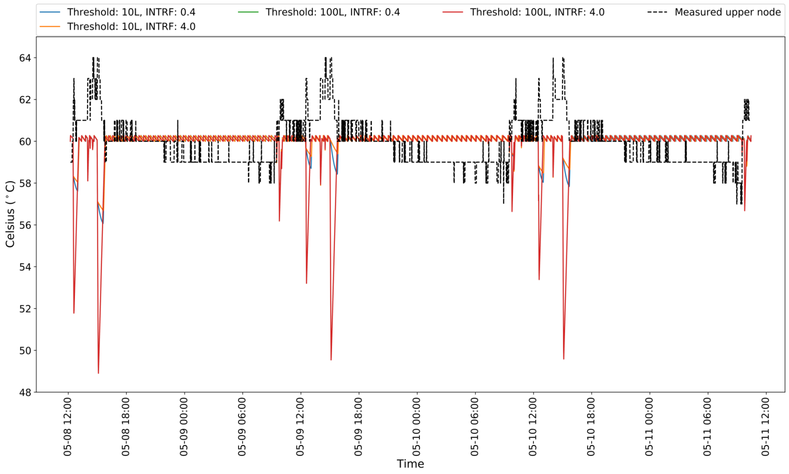

Fig. 6. Simulated temperatures of specified threshold and Inter Nodal Thermal Resistance Factor (INTRF) values for upper node with measured value as reference.

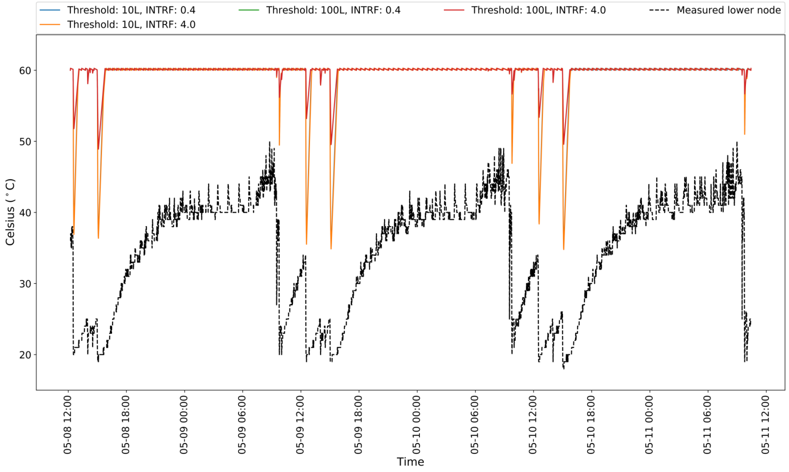

Fig. 7. Simulated temperatures of specified threshold and Inter Nodal Thermal Resistance Factor (INTRF) values for lower node with measured value as reference.

TABLE II
SYMBOL DEFINITIONS

Symbol	Definition
T-in	This is the measured temperature of the water pipe delivering cold water to the EWH.
T-out	The outlet pipe measured temperature.
SP	Temperature measurement that is used in set point control.
T-low	This temperature is being modeled in the original experiment. Now this lower node is being measured and compared to the modeled T-low.
T-Upper	This temperature is being modeled in the original experiment. Now this upper node is being measured and compared to the modeled T-Upper.

was then used for thermostat control. Figure:4 shows the main differences between the two experiments including the placement of the thermostat with an internal temperature sensor.

The internal temperature probe could measure water temperature more accurately than the outlet and be more responsive, thus lowering energy input while heating to a set display temperature.

The experiment ran with the same schedule, set temperature and water usage event timings as the original model verification experiment, [5], as to have a one to one comparison. For schedule control the EWH was set to an always on state meaning that whenever a temperature was measured that is lower than the set threshold the device would immediately turn on again. In the case of this experiment the set point temperature was set to 60 degrees. Water usage event timings were closely monitored and followed the same pattern as in the original experiment which is found in Table III.

All this data is captured by a Geasy controller [10] that has been modified to use the internal temperature for set point control. The unit then sends all the relevant data to a raspberry pi that logs and cleans the data.

TABLE III
WATER USAGE EVENT SCHEDULE.

Event time	Volume (l)
09:45	20.5
12:30	50.3
14:00	10.6
15:00	69.8

V. RESULTS

The results provide a window into the hitherto unknown internal temperature distribution inside the horizontally mounted electric water heaters. From the results illustrated in Figure 5, it is clear that the natural stratification has a significant impact on the temperature distribution inside the tank – rather than a single node state ever being reached, the lower node temperatures are always significantly less than the upper node and the modeled lower node temperature. Similarly, the upper node seems to be significantly warmer than the modeled value due to the stratification. This suggests that a gradual increase in temperature is present from the bottom to the top of the EWH, most likely due to a natural stratification that exists, even in steady state.

Although this in itself is interesting, the impact on energy is worth considering – Due to the higher temperature in the upper section of the tank, the energy that leaves the tank (per volume of water) when hot water is consumed is more than what the model predicts, meaning more energy would be required to heat the cold inlet water than would be the case if the water leaving the tank was colder. However, since the hot water is mixed with cold water to achieve a desired temperature at the point of use, it is likely that a user would use less hot water if the temperature of the hot water is higher. The nett effect on energy would therefore be less in a natural experiment, than in a controlled lab experiment with a set consumption volume.

Due to the reasons described above, it is also apparent (Figure 6 and 7) that the model performs better with a lower threshold at which the model enters into the two-node state. This is due to a lower required threshold for the nodal transition, which better emulates the evidently persistent stratification.

The results further show that the two-node model performs better with a lower inter-nodal conductance, for both the upper and lower nodal temperatures, which is, again, to the evidently persistent gradual stratification.

When comparing the measured vs. modeled energy, for different thresholds and for different INTRFs, we find that the the model underestimates the measured energy consumed. This is due to the outlet being at the highest point, which is also the highest temperatures due to the stratification. The energy is underestimated by the model, but is more accurate for the lower threshold than the higher threshold (23.7% vs. 18.6%). However, the INTRF does not affect the nett energy, which is possibly due to the comparatively small size and infrequent sequencing of the experimental events.

VI. CONCLUSION AND FUTURE WORK

In this paper we evaluated the only existing model for horizontally mounted electric water heaters, which is used in smart grid applications. The model, which uses a two-node approximation for the natural stratification is compared with actual temperature and energy measurements with a set consumption pattern under full-on thermostat control. The measurements indicate that, although the model seemingly accurately models the energy consumed by the electric water heater, it underestimates the stratification, both in the transients and in the steady-state conditions. We also find that the inter-nodal thermal resistance, which the model assumes to be infinite is overestimated, and the thresholds for state transfer also overestimated – Both of these oversights are hidden by the underestimation of the stratification. The results also show, also due to the stratification, that the model underestimates the consumption losses (enthalpy). More concerning, is the low temperatures measured in the lower node, which could lead to growth of Legionella. Future work includes thermal measurement in along the volume of the electric water heater, and evaluating the presence of bacterial growth.

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