The application and testing of smart beta strategies in the South African market

Jacobus Viljoen

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Supervisor : Prof WJ Conradie

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Abstract

Smart Beta portfolios have recently prompted great interest both from academic researchers and market practitioners. Investors are attracted by the performances produced by these portfolios compared to the traditional market capitalisation weighted indices. The question that this thesis attempts to answer is: Do smart beta portfolios outperform the traditional cap-weighted indices in the South African market?

According to BlackRock's smart beta guide (Ang, 2015), the smart beta strategies aim to capture stock return drivers through rules-based, transparent strategies. They are generally long only and usually implemented within an asset class, in the case of this assignment, only equity.

Smart beta is thus an investment strategy that positions itself between active and passive investing. Smart beta strategies are active in the sense that they invest in factors that drive return to improve risk-adjusted returns. In the same way, these strategies are closely related to passive strategies in that they are transparent, systematic and rules based.

In this assignent five different fundamental factor portfolios (value, quality, momentum, volatility and a combination of the four, called multi-factor) were created based on the smart beta methodology. The factors that were used are well researched in the market and have been proven to provide investors with excess return over the market.

Firstly, stock selection was done using two different techniques (time series comparison and cross-sectional comparison). The best stocks were selected based on their fundamental factor characteristics. Secondly, two different smart beta weighting strategies as well as a market-cap weighting strategy were applied to the selected stocks in order to create the various portfolios. The risk and return characteristics of the created portfolios were compared to those of the two benchmarks (JSE All Share Index and the JSE Shareholder Weighted All Share Index). The smart beta portfolios created in this thesis outperformed the benchmarks as well as the market-cap weighted portfolios.

Lastly, the estimation of the macroeconomic exposure of the smart beta portfolios using a methodology outlined in a Citi Research paper is presented (Montagu, Krause, Burgess, Jalan, Murray, Chew and Yusuf., 2015).

Opsomming

Akademiese navorsers en beleggers het in die laaste aantal jare al hoe meer in Smart Beta portefeuljes begin belang stel. Beleggers is veral beïndruk met die opbrengste wat die portefeuljes asook indekse relatief tot die bestaande markkapitalisasie geweegde indekse gelewer het. Die werkstuk probeer dus die vraag beantwoord: Gaan die smart beta portefeuljes die tradisionele markkapitalisasie geweegde indekse uitpresteer in die Suid-Afrikaanse mark?

Volgens BlackRock se smart beta gids (Ang, 2015) is die doel van die strategie om sodoende die faktore vas te vang wat die opbrengste van aandele dryf deur gebruik te maak van 'n stel vaste reëls en 'n deursigtige belegging strategie. Die smart beta portefeuljes laat oor die algemeen net die koop van aandele toe en fokus meestal op een bateklas. In die geval van hierdie werkstuk was daar gefokus op aandele.

Die smart beta belegging strategie word gesien as 'n kombinasie van aktiewe en passiewe beleggings strategieë. Dit word aktief bestuur deur in die faktore te belê wat obrengste dryf om sodoende die risiko-aangepaste opbrengs te verbeter van 'n portefeulje. Hierdie strategieë word ook passief bestuur deurdat dit deursigtig, sistematies en op 'n stel vaste reëls gebaseer is.

In die werkstuk is daar vyf smart beta portefeuljes geskep wat elkeen gefokus het op 'n spesifieke fundamentele faktor naamlik: waarde, kwaliteit, momentum, volatiliteit asook 'n portefeulje wat gefokus het op die kombinasie van die vier faktore en benoem was, die "multi-factor" model. Die faktore wat ingesluit is in die tesis is al intensief nagevors in die mark en daar bestaan betekenisvolle resultate dat die faktore die mark kan uitpresteer.

Aandele seleksie is die eerste stap in die proses en dit is toegepas deur middel van twee metodes (tydreeks vergelyking asook wat genoem was "cross-sectional" vergelyking). Die beste aandele was geselekteer gebaseer op die fundamentele faktor waarde wat gekoppel is aan 'n aandeel. Die tweede stap was om gewigte toe te ken aan die geslekteerde aandele. Dit is gedoen deur middel van smart beta strategieë asook 'n markkapitalisasie gewigstoedeling strategie. Die risiko en opbrengs eienskappe van die portefeuljes is vergelyk met die van die twee mark indekse naamlik die JSE Alle Aandele Indeks asook die JSE Aandeelhouer Geweegde Alle Aandele Indeks. Die smart beta portefueljes wat geskep is in die tesis het die twee mark indekse uitpresteer asook die markkapitalisasie geweegde portefeuljes.

Laastens was die blootstelling wat die smart beta portefeuljes gehad het aan die makro-ekonomiese faktore beraam deur gebruik te maak van 'n metodologie omskryf deur Montagu, Krause, Burgess, Jalan, Murray, Chew en Yusuf. (2015).

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1   INTRODUCTION

1.1   INTRODUCTION

Finding a strategy that beats the stock market is parallel to the search for the Holy Grail. It is a glorious and righteous quest that has gone on for centuries and will continue to do so long after we are all gone. Smart beta funds attempt to help the investor with this quest.

Smart beta funds look to improve on traditional index fund investing by strategically weighting stocks in the funds based on objective factors and weighting strategies that are described later in this thesis.

The first generation of index investing started in 1976 under the leadership of Jack Bogle at a company, Vanguard, that launched the first index mutual fund. Investors reacted positively to the idea of earning the market through low-cost investing. The most widely used strategy of indexation is the market capitalisation-weighted method. Up until 2005, this method has been regarded as the norm and has been unchallenged in financial markets.

A market cap-weighted scheme is used due to William Sharpe's (1964) capital asset pricing model (CAPM) according to which he assumed that the cap-weighted index was efficient. However, further investigation in 2006 showed that overvalued stocks were overweighted and undervalued stocks were underweighted in these indices (Hsu & Campollo, 2006). This led to a return drag.

If the two major market events, the tech bubble in 2000 and the financial crisis in 2008, are examined, it is clear that in both of these periods there were signs of overreaction in the stock market. During the tech bubble in 2000, there was some overoptimism, and during the financial crisis in 2008, some fear was shown in investors' decisions. Cisco Systems, for example, was a top-of-the-range technology company. In 2000, it was overvalued and overweighted in the market capitalisation index; it traded at a price-to-earnings (P/E) ratio of 181, more than double its P/E ratio in 1999. This company's weight rose from 1.7% in 1999 to 4.1% in 2000 in the market capitalisation index (as shown in Table 1.1). Investors knew that this share was trading far above its real market value, and the share price returned to levels that were more reasonable in the following years, leading to a significant loss for the index.

The opposite could occur when markets crash. After a market crash, stocks are usually undervalued. During the global financial crisis in 2008 the financial sector fell to severely low levels. For example, the P/E ratio of Barclays shares, whose weight in the market capitalisation index was 3% at the start of 2007, fell from USD10 to USD2.5 in 2009. Its weight decreased to 0.8% in the index (as shown in Table 1.1). At this point, Barclays was a very profitable buy due to its strong accounting fundamentals. As investors regained their trust in the market, the share price rose. The market

capitalisation index missed out on this opportunity due to the automatic rebalancing according to the share price.

Robert Arnott, Jason Hsu and Philip Moore (2005) started a new and revolutionary method for setting up an index; this method was called 'fundamental indexation'. A company's weight in this index is proportional to its accounting fundamentals, sidestepping the biggest weakness of the market capitalisation index.

Table 1.1: Performance in tech bubble (2001) and financial crisis (2008)

	Holding Data as of March 31			
Tech Bubble	**1999**	**2000**	**2001**	**2002**
Cisco Systems				
Percent in Russell 1000® Index	1.7%	4.1%	1.1%	1.3%
Percent of Economy	0.1%	0.2%	0.3%	0.4%
P/E Ratio	81.8	181.9	25.1	22.0
Global Financial Crisis	**2007**	**2008**	**2009**	**2010**
Barclays				
Percent in FTSE UK 100 Index	3.1%	2.1%	0.8%	2.7%
Percent of Economy	2.8%	3.1%	3.1%	3.5%
P/E Ratio	10.0	6.6	2.5	12.6

Source: Kalesnik (2014)

Fundamental indexation belongs to the smart beta investing family, also known as alternative investing. For an index to belong to the smart beta investing family, it should weight its constituents by measures that are unrelated to stock prices. The information obtained from financial statements and the corresponding fundamental ratios of a company are a good proxy for company size, quality and riskiness. These ratios and financial information do not contain current company prices. The fundamental indexation methodology therefore breaks the link between a company's price and its weight in an index.

Smart beta indices try to retain benefits such as preserving liquidity and capacity of market cap-weighted indices for a passive investor. Another benefit of both market cap-weighted and fundamental indices is their low volatility. Fundamental indices therefore offer similar costs and high probabilities of outperforming cap-weighted indices given that they have the same exposure to market characteristics.

1.2   PROBLEM STATEMENT

Smart beta indices concentrate on long-term investment performance. The most challenging aspect of setting up such an index is maintaining a low turnover (Arnott et al., 2005). Rebalancing these indices is regularly required. If a stock's price rises by 15%, its weight in the portfolio will also rise and therefore that stock would have to be rebalanced to its fundamental (smart beta) weight in the index.

Smart beta indices are not just a different way of weighting the stocks in a portfolio. For instance, if stocks are weighted according to their book value only, a great deal of stocks with low price-to-book ratios and substantial book values are missed out on. The end result is an index that is highly concentrated in stocks with large capitalisation and book values.

An important decision regarding smart beta investing is thus to choose which factors should be included in the investment model. The stocks are ranked according to their performance on each of these factors, and a weight could be assigned to each of them. Smart beta indices may consist of just one of these fundamental metrics, but a problem similar to the problem discussed in the previous paragraph may occur. It may be an improvement if an index consists of various fundamental factors.

A substantial amount of research regarding significant fundamentals is available. Jason Hsu and Vitali Kalesnik (2014) found that there existed a 'quant shop' that used an 81-fundamental-factor model to build their equity portfolio. These multifactor models could have been set up using data snooping, testing certain fundamentals with 10 000 back-tests, and if one is significant, it is included in a portfolio.

Campbell Harvey, Yan Liu and Heqinq Zhu (2015) retested 316 fundamental factors published in journals. They implemented a new fundamental testing framework and derived recommended statistical significance levels for current research in asset pricing. They found that many of the factors would be believed significant by chance. The most significant anomalies that they found were value, low volatility and momentum. Other risk factors such as illiquidity and the market (high-beta stocks) were also significant. This research was done with United States of America (USA) data and should thus be repeated in the South African market to see whether these anomalies would pass the test for significance.

Once significant factors are identified, an investment structure can be put into place to capture the risk premia offered by these risk factors.

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1.3   RESEARCH OBJECTIVES

The main objective of this study was to find an index, a smart beta index, that addressed the shortcomings of the cap-weighted method. This index should also retain the advantages of the cap-weighted index. These advantages include liquidity, passiveness and easy access to a wide variety of stocks.

Smart beta indices are not perfect at all; they also have some drawbacks. These include rebalancing the stocks to their original weight in the index, weighting methodologies that may be challenging and using insignificant fundamental factors. The rebalancing of this index should be well timed and effective without incurring large transaction costs.

Indices forming part of the smart beta domain may have different weighting methods, as long as a stock's weight is independent of its price or market capitalisation. In this research, various methods were tested for significant outperformance over the cap-weighted index.

1.4   LITERATUR REVIEW

In the literature review in Chapter 2, a brief description of the most common portfolio theories is presented. Smart beta investing builds on all these theories. The theories described are the efficient market hypothesis (EMH), the modern portfolio theory (MPT) and the CAPM. General aspects surrounding smart beta investing are explained. The factors on which stock selection is based, namely value, quality, volatility and momentum, are explained in detail. The impact that macroeconomic factors have on the various portfolios is explained. Macroeconomic models and factors are introduced and described briefly. Smart beta investing is introduced, and the theory underlying this strategy is explained in detail. The types of risks involved when an investor is considering this strategy are outlined and elaborated on. Various ways of compensating for and countering these risks are mentioned. Lastly, the main weighting scheme used in this study (risk parity) is described and the theory is explained in detail.

1.5   IMPORTANCE/BENEFITS OF THE STUDY

This study aimed to address the shortcomings of the market cap-weighted indices. Smart beta investing would also improve on the investment strategy called 'factor investing', which has been a focus point of quantitative active management for the past decade.

A relatively new weighting strategy, risk parity, was introduced in this study and is discussed in detail. The macroeconomic exposure of the smart beta portfolios created in this study is a new extension that has not been done in the past. When a more detailed explanation of the portfolio returns is required, some market factors may also be added to the macroeconomic model.

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1.6   CHAPTER OUTLINE

In Chapter 2, a brief literature review is presented. In Section 2.1, the classic portfolio theories namely the EMH, the MPT and the CAPM are discussed and expanded on to illustrate the fundamental viability of the smart beta strategy. In-depth research regarding smart beta investing and the contribution of the most important researchers in this field are discussed in this chapter. The problems inherent in the strategy are also addressed.

In Chapter 3, the methodology by which the data were extracted from Bloomberg using RStudio is discussed. This data were used to make informed decisions regarding the stock selection and weight allocation for the selected stocks. The methods followed for these processes are discussed in detail. The calculations of the returns for the smart beta portfolios and their benchmarks and the basic techniques used for these are presented in this chapter. Lastly, the estimation of the macroeconomic exposure of the smart beta portfolios using a methodology outlined in a Citi Research paper is presented (MontaguKrause , Burgess, Jalan, Murray, Chew and Yusuf., 2015).

The results for the various portfolios are illustrated in Chapter 4. The results are divided into sections for each smart beta portfolio. The portfolios were created through two different stock selection processes and various weighting strategies. The results for a specific portfolio were compared between the two different stock selection processes using the same weighting schemes. In the last part of this chapter, the macroeconomic exposure for each created portfolio is also illustrated and discussed.

Chapter 5 gives a broad overview of what was done in this research, and the results obtained are once again summarised, after which in Chapter 6, the macroeconomic exposures are illustrated using graphs for all the stocks. The correlations among the macroeconomic factors are discussed, and the exposures of the created portfolios to the various macroeconomic factors are summarised and explained.

2   LITERATURE REVIEW

Before the application of the theoretical views and opinions related to smart beta investing was put to the test, a clear understanding of the underlying theories, approaches and factors was needed. A great deal of international research is available, but a substantial amount of this should be retested in the South African market as most of the research was done in the USA.

2.1   FINANCIAL THEORIES

2.1.1   Efficient market hypothesis

The base for most modern financial theories has been generally referred to as the EMH, introduced by Eugene Fama (1965). An efficient market is a market in which the prices of securities 'fully reflect' available information (Fama, 1970). He divided his empirical work into three categories depending on the nature of the information subset of interest namely that:

> Strong-form test are concerned with whether individual investors or groups have monopolistic access to any information relevant for price information. Semi-strong-form tests the information subset of interest, this includes all obvious publicly available information, while in the weak form tests the information subset is just historical price or return sequences (Fama, 1970:414).

Fama found that the evidence showed a consistent positive association between day-to-day price changes and returns on stocks. He could not find evidence contradicting his weak-form and semi-strong-form tests although there was some evidence contradicting the strong-form test. He stated that a 'fair-game' model existed. This model tests whether a trader who trades only on the set of information that 'fully reflects' security prices could earn returns in excess of equilibrium expected returns.

Fama believed that securities markets were extremely efficient and that information was incorporated into share prices almost immediately. It makes sense that as soon as new information comes to light, it spreads very quickly across markets and is reflected in the prices of the securities. It was found that when this new information was unpredictable, the movements in the securities' prices were also unpredictable. Therefore, according to the EMH theory, uninformed day traders purchasing diversified portfolios should realise the same returns as the expert traders.

According to Fama's research, neither technical analysis, which studies the history of price movements, nor fundamental analysis, which is the theory of searching undervalued stocks, would give an investor the opportunity to outperform an index that holds a random selection of stocks. Burton Malkiel (1973) stated in his book *A Random Walk Down Wall Street* that a blindfolded monkey throwing darts at a selection of stocks to set up a portfolio would do as well as expert portfolio managers.

In the 21ˢᵗ century, it has become more and more evident that asset prices are predictable to some extent using fundamental and technical indicators. The new generation of economists believes that psychological and behavioural elements can also influence stock prices (Malkiel, 2013). These economists also believe that investors can earn excess risk-adjusted rates of return by using these significant fundamental and technical indicators. This is in contradiction to Fama (1970) who stated that stock prices did not have a memory; this would therefore eliminate the opportunity to predict the future stock price movements based on past performance.

Fama (1998) conducted an event study to determine whether stock prices responded efficiently to information. The events considered were announcements such as earnings surprises, stock splits, dividend actions, mergers, new exchange listings and initial public offerings. Fama found that underreaction to these events occurred as regularly as overreactions. Fama also showed that some of the return factors were only significant in certain models, depending on their weighting scheme. "For example a portfolio that gives equal weight to post-announcement returns of many stocks may produce different results from a study that weights the stocks according to their value" (Malkiel, 1973:62).

De Bondt and Thaler (1987) found that when stocks were ranked according to their 3–5-year returns, past winners turned out to be future losers and vice versa. This is due to investors' overreaction to past performance. Investors are thus subject to waves of optimism and pessimism that in return would cause stock prices to deviate from their fundamental values and would result in mean reversion of stock prices.

De Bondt and Thaler's results are consistent with the behavioural decision theory of Tversky and Kahneman (1973:81) who stated, "Investors are systematically overconfident in their ability to forecast either stock prices or future corporate earnings." This can be seen as an opportunity for investors to overweight stocks that have underperformed for long periods of time and to underweight stocks with long periods of outperformance.

In recent finance literature, there seem to be many long-term return anomalies. This does not mean that the EMH should be abandoned. "Consistent with the market efficiency hypothesis that the anomalies are chance results, apparent over-reaction of stock prices to information is about as common as under-reaction" (Fama, 1997). These long-term return anomalies are fragile. They tend to disappear when reasonable alternative approaches are used to measure them. Many predictable patterns also seem to disappear after they are published in the finance literature.

2.1.2   Modern portfolio theory

Harry Markowitz (1952), the founder of the MPT, launched an idea of efficient portfolio creation in his book *Portfolio Selection*. The Markowitz approach is a method to estimate an optimal mean-variance portfolio. The focus of this theory is that investors should maximise their expected return on a certain portfolio or minimise the amount of risk.

Markowitz stated that this portfolio selection could be divided into two stages. "The first stage starts with observation and experience and ends with beliefs about the future performance of available securities. The second stage starts with the relevant beliefs about future performance and ends with the choice of portfolio" (Markowitz, 1952:77).

Two rules are investigated in Markowitz's book. The first rule is that an investor should maximise expected returns. The second rule is that an investor should consider expected returns as desirable and variation of returns as undesirable. This rule sounds more like true investment behaviour.

If the first rule is further investigated, it is found that it fails to apply diversification to the investor's portfolio. If the investor wants to maximise the discounted value of returns, all the available capital should be placed in the security with the highest discount rate. However, any rule that prefers nondiversified portfolios to diversified portfolios should be rejected. Therefore, securities should be compared to each other in terms of their covariance. Each security has its own risk level, and by applying mathematical and statistical theory, these risks can be mitigated through diversification.

The second rule implies that the investor desires expected return and would like to minimise the variance of the returns. Markowitz found such a set of portfolios with different expected returns and variance combinations and called it the 'efficient set'. The investor could then choose a portfolio in the efficient set of combinations with a minimum variance for a given expected return. Markowitz stated that the mean-variance approach not only implied diversification; it also implied the 'right kind' of diversification for the 'right reason' (Markowitz, 1952). The efficiency of diversification is not dependent on the number of assets in a portfolio. Investing in a number of assets in the same industry will not diversify the portfolio as well as a portfolio with assets in different industries. Assets in the same industry generally perform the same over time. The same argument holds to keep the volatility of the portfolio returns as small as possible by investing in many assets. Investing in assets with low covariance among them (assets in different industries generally have lower covariance among them than assets in the same industry) lowers the variance of portfolio returns.

Investing in different geographic regions may also improve diversification. The impacts of negative events in specific industries and regions are reduced. According to Markowitz (1952), the main aim is to lower the portfolio's risk by investing in different assets across a wide range of industries and

regions. The investor should then choose the asset weights that maximise the trade-off between portfolio return and the volatility of portfolio returns. Optimal portfolio selection makes this possible.

The assumptions made by Markowitz to prove his theory are twofold. The first is that investors must desire expected returns and should be risk averse. Investors should thus desire to invest according to the framework of optimal combinations of returns and risk. The second assumption is that the set of expected returns and covariance among assets should be reasonable. "These procedures, to arrive at the reasonable values, should combine statistical techniques and the judgment of practical men" (Markowitz, 1952:91). Statistical techniques should be used to arrive at a tentative set of expected returns and standard deviations of returns. Judgement should then be used in increasing some of these expected returns on the basis of factors (economic factors, political factors and fundamental factors of the assets) not taken into account by the formal computations.

2.1.3   Capital asset pricing model

In 1964, William Sharpe introduced the CAPM. This model is used for applications such as estimating the cost of capital for firms and evaluating the performance of managed portfolios. The attraction of the CAPM is its ability to measure the relation between expected return and risk (Fama & French, 2004).

The CAPM builds on Markowitz's model according to which investors are risk averse and are only concerned about the mean and variance of their investment returns. Investors choose among a set of mean-variance-efficient portfolios. These portfolios minimise the variance of portfolio return, given expected return, and maximise expected return, given variance.

Sharpe (1964) and Lintner (1965) identified a portfolio that was mean-variance efficient by adding two assumptions to Markowitz's model. The first assumption is that investors agree on the joint distribution of asset-expected returns for a one-period forecast. The second assumption is that investors can borrow at the risk-free rate.

In Figure 2.1, the CAPM is illustrated. The horizontal axis illustrates the portfolio risk, measured as the standard deviation of portfolio returns, and the vertical axis illustrates the expected return. The minimum variance frontier, the curve labelled *abc,* illustrates the combinations of expected return and risk for portfolios of risky assets that minimise return variance at different levels of expected returns. From this figure, it can be seen that to achieve higher returns, such as a return at *a*, an investor must accept higher volatility. Thus, if no risk-free borrowing is allowed, only portfolios above *b* along the *abc* line are mean-variance efficient.

If risk-free borrowing is allowed, the efficient frontier turns into a straight line. The mean-variance-efficient frontier is obtained when the efficient frontier becomes a tangent line to the minimum-

variance frontier at point *T*, the tangency portfolio. All the efficient portfolios are thus combinations of the risk-free asset and the tangency portfolio (Tobin, 1958).

According to the CAPM, all investors see the same opportunity set. The combination of the risk-free asset and the tangency portfolio that they put in their portfolio will depend on individual investors. All investors therefore hold the tangency portfolio, and this should thus be the value-weighted market portfolio of all risky assets (Fama & French, 2004). Fama and French (2004) stated that the market portfolio should be a market cap-weighted portfolio.

Investment Opportunities
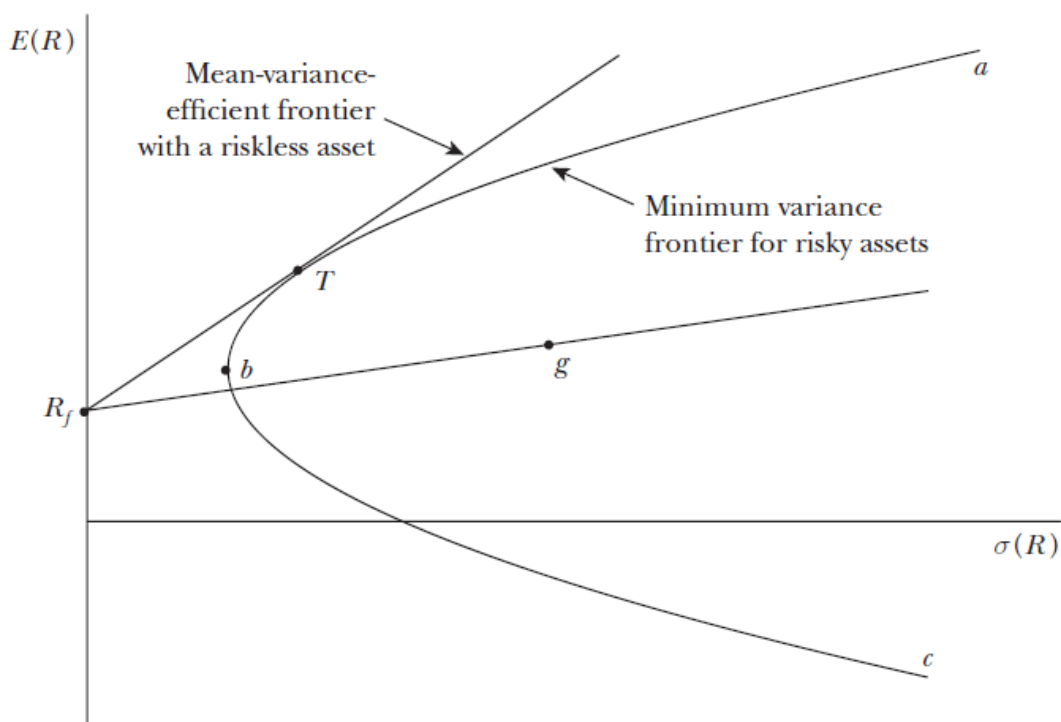
Figure 2.1: The efficient set of investment opportunities
Source: Fama and French (2004)

The Sharpe-Lintner CAPM is written as follows:

$$E(R_i) = R_f + \left[E(R_M) - R_f\right]\beta_{iM} \, , i = 1, \dots, N \tag{2.1}$$

The expected return on asset *i* is the risk-free interest rate plus a risk premium. The risk premium is equal to the asset's market beta, β_{iM}, times the premium per unit of beta risk, $E(R_M) - R_f$.

This model points out two types of risk for a portfolio: systematic and unsystematic. Systematic risk is usually called market risk; this risk cannot be diversified away and includes interest rate changes, economic changes, natural disasters and so forth. These factors all have an impact on the risk-free

rate. Unsystematic risk can be eliminated by efficient portfolio diversification. This risk is therefore linked to the individual securities within a portfolio.

2.2   FACTORS

"A factor can be thought of as any characteristic relating a group of securities that is important in explaining their returns and risk" (Bender et al., 2013:1). From the CAPM literature, it seems that the first and most important equity factor is the market. Beyond the market factor, other factors that are incorporated into portfolio construction need to be consistent over time and have strong explanatory power over a broad range of stocks.

There are three main categories of factors, namely macroeconomic, statistical and fundamental. Macroeconomic factors include measures such as interest rates, gross domestic product (GDP), inflation, and gold and oil price movements. Statistical factors are identified by using, for example, principal component analysis. The most widely used factors are probably fundamental factors, which focus on specific stock characteristics.

The most popular fundamental factors in the market today, with the most research surrounding them, are value, size, momentum, quality, volatility and dividend yield. Rosenberg and Marathe (1976) described the importance of these factors in explaining stock returns. Their discovery led to the creation of the multifactor Barra risk models. In the early 1990s, Fama and French (1992) set up a three-factor model explaining US equity market returns. The three factors were the market factor (based on the CAPM), the size factor (large- vs small-cap stocks) and the value factor (low vs high book-to-market stocks). The well-known Farma-French model is actually a four-factor model; they incorporated Carhart's (1997) momentum factor in the late 1990s.

The most popular factors are discussed below:

2.2.1   Value

The value anomaly captures the positive link between stocks that have low prices relative to their fundamental value and produces returns in excess of the cap-weighted benchmark (Bender et al., 2013). This entails that to follow the value-investing strategy, an investor needs to buy stocks that have low prices normalised by some indicator of company fundamentals such as book value, sales, earnings or dividends and to sell stocks that have high prices (also normalised). Graham and Dodd (1934) first wrote about this anomaly in their book *Security Analysis.* Basu (1997) formalised the strategy by testing the possibility that value-related variables might contradict the CAPM. He found that there was a significant positive relation between earnings yield ratios and average returns for US stocks that could not be explained by the CAPM. Rosenberg et al. (1985) and De Bondt and Thaler (1987) found a significant positive relation between price-to-book ratios and average returns.

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Various other researchers have reproduced the value effect for many different sample periods and markets.

2.2.2 Momentum

Stock prices tend to follow a trend over certain horizons; winners continue to win, and losers continue to lose. The momentum strategy therefore entails buying past winners and selling past losers. Jegadeesh and Titman (1993) showed that for the US market, the momentum strategy produced significant abnormal returns for the period from 1965 to 1989. Carhart (1997) investigated mutual fund performances and showed that the Fama-French three-factor model could be expanded into the four-factor model. The new model includes momentum as an additional explanatory variable. Rowenhorst (1998) found that an international diversified portfolio of past winners outperformed a portfolio of past losers by 1% per month. He used a sample of 2 000 European stocks in the period from 1978 to 1995. Fama and French (2012) also reevaluated their three-factor model by adding the momentum anomaly. They found strong momentum returns in North America, Europe and Asia Pacific but not in Japan in the sample period from 1989 to 2011.

Asness (1995) not only confirmed earlier findings on the momentum effect at the country level but also showed that winners and losers tended to switch around over the long term, in other words, winners underperforming and losers outperforming in 3-5 years out. In fact, empirical research suggests that the momentum effect is most prominent in the following 3-12 months, after which it will likely disappear. This implies that the momentum strategy requires relatively high turnover to work. The momentum strategy has attracted a great deal of criticism in the past by researchers searching through the same dataset and running many back-tests to find publishable results. Geczy and Samonov (2013) conducted a 212-year back-test of the momentum strategy in the US market, which showed that the momentum effect was statistically significant.

The theory underlying this anomaly is still under extensive discussion. There is no satisfactory efficient market-based theory to explain this factor. The theories that underlie this factor are mostly behavioural. Barberis, Shleifer and Vishny (1998) and Daniel, Hirshleifer and Subrahmanyam (1998) showed that this factor produced excess returns due to overreaction to news while according to Hong, Lim and Stein (2000), excess returns could be due to underreaction. These reactions drive the momentum factor.

The momentum strategy also attracts criticism regarding high turnover, crowded trading and the risk of sudden reversal (Bender et al., 2013). History shows that the probability of a short-term reversal is positively correlated with volatility and that forecasting volatility is anything but easy. In addition, like other factor strategies, momentum could go through an extended period of negative

performance. This suggests that it is perhaps better to combine momentum with other factor strategies than to use it on its own (Asness, Moskowitz & Pedersen, 2013).

2.2.3   Volatility

The volatility trading strategy requires an investor to buy stocks with lower than average volatility. This is a contradictory trading strategy due to one of the most basic principles in finance, namely that higher volatility is associated with higher returns (Blitz & Van Vliet, 2007). The CAPM showed that riskier assets should earn higher returns. The research done regarding low-volatility trading shows that the opposite is true.

In 1991, Haugen and Baker (1991) were the first to document this effect. They focused on the period from 1972 to 1989 and showed that low-volatility stocks in the US outperformed the capitalisation-weighted index. These authors were major critics of the capitalisation-weighted benchmarks. Chan, Karceski and Lakonishok (1999), Schwartz (2000), Jagammathan and Ma (2003) and Clarke, De Silva and Thorley (2013) confirmed these results using various volatility measures in the US market. Geiger and Plagge (2007), Nielsen and Subramanian (2008) and Poullaouec (2008) found this strategy to work in global markets as well.

As stated earlier, the volatility anomaly clearly contradicts the EMH and the assumptions of the CAPM. The reason for this may be behavioural. An effect called the 'lottery effect' is the most common explanation, namely that:

> people tend to take bets with a small expected loss but a large expected win, even though the probability of a loss is much higher than the win, and the weighted average of the outcome may be negative. This is similar to buying a lottery ticket whereby the customer pays a small sum for potentially winning a large amount of money albeit at a very low probability (Bender et al., 2013).

Baker, Bradley and Wurgler (2011) came up with another theory why low-volatility stocks may provide excess returns. Low-volatility stocks often have lower betas, and overweighting those leads to higher tracking errors for institutional investors. Such tracking errors need to be justified by sufficient excess returns (alpha).

2.2.4   Quality

This strategy requires an investor to seek 'high-quality' companies as they tend to outperform the market. Clearly, the problem here is to define the quality factor for a specific company. A company's quality is commonly associated with its competitiveness, efficiency, transparency, growth, financial and operating leverage, profit sustainability and return on equity.

Recently, researchers have argued that Buffett's investment strategy also include a quality component. "Buffett's strategy focusses on cheap, safe and high-quality stocks together with a great

amount of leverage to amplify the returns while sidestepping the large absolute and relative drawdowns", according to Frazzini, Kabiller and Pederson (2012).

Not a great deal of literature on why the quality factor works is available. The reason for this is that the definition of quality varies. According to the Fama-French model, all systematic risks are ultimately economic risks. In light of this argument, it is not difficult to see the connection between stock returns and quality measures such as financial leverage and earnings growth. Campbell, Polk and Vuolteenaho (2010) offered a form of this 'fundamental' explanation, arguing that the primary source of systematic risks of both growth and value stocks was the cash flow fundamentals: how a company could sufficiently produce positive cash flows from its operating activities while maintaining a good dividend pay-out ratio as opposed to market sentiments. From a corporate finance angle, it is easy to see how firm quality impacts stock prices.

> For example, a well-run company often manages its capital carefully and reduces the risk of over-leveraging or over-capitalization. The steady growth in earnings will further reduce its need for capital market financing, which will support its stock price. This will trigger a positive feedback loop making the company more competitive in the eyes of its customers and investors (Bender et al., 2013).

2.3   MACROECONOMIC FACTORS

As seen in earlier literature, a fundamental principle of finance is the trade-off between risk and return. A portfolio that is riskier in an appropriate sense may be expected to deliver excess returns over another less risky portfolio. Burmeister, Roll and Ross (2003) asked a crucial question: "What is the appropriate measure of risk?" They stated a few problems regarding using the fundamental factors explained in the previous section.

- "Most are based on accounting data, and such data are generated by rules which may differ significantly across firms."
- "Even if all firms used the same accounting rules, reporting dates differ so that it is difficult to construct time-synchronized inter-firm comparisons."
- "Most importantly, there is no rigorous theory to tell us how traditional accounting variables should be related to an appropriate measure of risk for computing the risk-return trade-off. Even if historical empirical relationships can be uncovered, without the foundations of a rigorous theory one must be concerned that any historical correlation might be spurious and subject to sudden and material change" (Burmeister et al., 2003:1).

Stephen Ross (1976) added some extra dimensions to the CAPM. Sharpe (1964) predicted that there was only one type of nondiversifiable risk influencing expected security returns, namely 'market

risk'. Ross (1976) invented the arbitrage pricing theory (APT), which is more general than the CAPM in the sense that the APT accepts a variety of different risk sources.

The APT is completely general and does not specify exactly what the risk factors are. Academic and commercial research suggests that there are several primary sources of risk that consistently impact stock returns. These risks arise from unanticipated changes in the following fundamental economic variables (Burmeister et al., 2003):

- Investor confidence
- Interest rates
- Inflation
- Real business activity
- Market index

Every portfolio of stocks has some type of sensitivity/exposure with respect to each of these risk factors. The pattern of economic betas for a stock portfolio is called its risk exposure profile. Risk exposures are rewarded with additional expected return in the market, and therefore the risk exposure profile determines the volatility and performance of a well-diversified portfolio. This profile also indicates how a portfolio may perform under different economic conditions.

In this study, the focus was on those factors that had a significant influence on stock returns in a smart beta fund. The profile mentioned in the previous paragraph may indicate how a stock or portfolio will perform under different economic conditions. Therefore, quantifying these factors (calculating the exposure that the portfolio/stock has to the factors) and accounting for their influence lead to the specific stock return. This can be very valuable in portfolio construction because it makes managing expositions more precise.

Macroeconomic factors indicate the current status of the economy or how the economy may develop in the near future in specific areas. Stocks are sensitive to macroeconomic factors such as interest rates, inflation and growth of the economy. According to Qian et al. (2007), the most commonly used macroeconomic factors are the following:

- Market return
- Change in short-term interest rate
- Change in industrial production
- Change in inflation
- Term spread
- Default spread
- Change in oil price

With the progressive globalisation of the economy and the fast development of emerging markets, it has become clear that systematic risk must be analysed at a global level. Citi Research's global risk attribution model (Montagu et al., 2015) is used to create 'global risk heat maps' to guide asset managers in best capital allocation and risk management. Global macroeconomic factors include four types of factors: interest levels, economic output, key commodities and exchange rates.

2.4   SMART BETA INVESTING

Markowitz's (1952) theory states that a well-diversified portfolio is not just a basket of randomly selected securities but an investor should also take into account the correlation among the securities that are being investigated. The CAPM states that the market portfolio is the optimal diversified portfolio, which is known as the cap-weighted index in today's terms.

Investors suffered significant losses in the period from 2000 to 2002 by investing in large-cap and growth stocks. Investors therefore increased their appetite for non-market capitalisation index-weighting strategies that broke the link between an asset's price and its weight in an index. These indices that break this link are labelled alternative indices, fundamental indices or smart beta indices (Vanguard Research, 2015).

Smart beta investing seeks to identify and capture broad, persistent drivers of return. The main idea behind smart beta investing is to search for inexpensive companies, called 'value investing', or to search for companies with high-quality balance sheets, called 'quality investing', and to allocate weights to the selected stocks to further diversify the portfolio. These investment styles are not new strategies in the world of finance. According to BlackRock's smart beta guide (Ang, 2015), the strategies aim to capture these return drivers through rules-based, transparent strategies. They are benchmark-driven versions of factor strategies, generally long only and usually implemented within an asset class, in the case of this research, only equity.

Smart beta is thus an investment strategy that positions itself between active and passive investing. Smart beta strategies are active in the sense that they invest in factors that drive return to improve risk-adjusted returns. In the same way, these strategies are closely related to passive strategies in that they are transparent, systematic and rules based. The construction of these indices is therefore based on a set of rules that requires a small amount of input from the portfolio manager. This provides investors with an index with lower fees and higher capacity than that of an active portfolio.

The EDHEC-Risk Institute (2013) stated that each smart beta solution contained risk; it grouped these risks into two categories: systematic risks and specific risks. Systematic risks are due to indices being exposed to particular risk factors depending on their construction choices. For example, an

index that favours low-volatility stocks will lead to an overexposure to some sectors (Amenc, Goltz & Martellini, 2013).

The first generation of smart beta indices was constructed around a single fundamental factor to sidestep the shortcomings of the cap-weighted indices (Amenc et al., 2013). In these indices, a distinction between the stock-picking methodology and the weighting methodology was not made. Investors were thus exposed to single risk factors that represented the source of their performances.

These risk factor exposures have been ignored in the promotion of the performance of first-generation smart beta indices (fundamental indexation). Arnott, Hsu and Moore (2005) proved that this smart beta index that outperformed cap-weighted indices did not contain any measure of the exposure of fundamentally weighted (the weights of the stocks are related to their fundamental factor; a stock with a better P/E ratio would have a higher weight) indices to different style factors (value, quality, low volatility or momentum). André Perold (2007:36) found that "fundamental indexing is a strategy of active security selection through investing in value stocks". Jun and Malkiel (2007) found that after adjusting the FTSE RAFI Index, an existing fundamentally weighted index, for its market, value and small-cap exposure, the alpha was zero.

Another aspect that can lead to concentration in a small number of stocks and reveal pronounced sector or risk factor biases is the choice of weighting scheme. For example, if the index is constructed using a volatility minimisation scheme, stocks with the least volatility are selected irrespective of their other properties.

Given this evidence, Amenc, Goltz, Lodh and Martellini (2014) examined how the stock selection stage could strongly improve performance and allow investors to tilt the risk factor according to their preference. They found that this smart beta approach not only led to better performance but also provided even more flexibility in active and passive management.

The second risk that smart beta indices are exposed to is specific risk. The weighting schemes associated with these indices rely on modelling assumptions and on parameter estimation, which can lead to lack of robustness and poor out-of-sample performance. The cap-weighted index justified its optimality with the CAPM. Any investor who moves away from this weighting scheme can be questioned on the relevance of the new model and on the robustness of its past performance, which support the investor's choice to a large degree. Smart beta is a recent research and empirical tool, so there is no possibility to verify a long-term performance. Well-informed investors will have to be clear-sighted and cannot rely on short-period past performance if they wish to achieve an out-of-sample robustness of their smart beta indices.

Specific risk is divided into two dimensions: parameter estimation risk and optimality risk. The total specific risk is equal to the sum of the parameter estimation risk and the optimality risk. The weighting schemes based on parameters such as return, volatility and correlation are subject to parameter estimation risk. The parameter estimation risk therefore increases as the number of parameters that need to be estimated increases. The maximum Sharpe ratio optimisation needs the estimation of return, volatility and correlation and thus is exposed to a possible large estimation error. The estimation error concerning the expected return parameter is the most crucial because past data hardly explain future return (Merton, 1980).

A good estimation of the parameters required for a specific scheme requires a trade-off between two components of parameter risk: sample risk and model risk. Sample risk is the risk when a parameter is estimated using sample-based information. An example is a parameter being estimated using an average of past historical data. Model risk is the risk of selecting the wrong asset-pricing model. An example of this is estimating a variable with a single-factor model while it depends on three factors.

Optimality risk is the risk of ignoring parameter estimates and replacing maximum Sharpe ratio optimisation with other optimal objectives. The new weighting scheme will be optimal under conditions that can be more or less restrictive. For example, an investor may decide to use a cap-weighted or an equally weighted portfolio; these schemes need no information regarding the risk and return characteristics of the stocks selected. Other strategies such as global minimum variance, risk parity or maximum diversification ratio strategies rely on risk parameter estimates, but by using the latter strategies, the estimation of expected returns is avoided.

To substitute one weighting scheme with another, one needs to understand how these schemes compare to one another. An investor would therefore give up on some parameter estimates rather than trying to improve these; this implies an efficiency cost related to the use of a portfolio that is suboptimal since it only overlaps with the maximum Sharpe ratio portfolio under heroic assumptions. The global minimum variance portfolio, for example, coincides with a maximum Sharpe ratio portfolio if expected returns are identical for all stocks; this is clearly not a reasonable assumption.

The second generation of smart beta investing applies a multifactor approach and also a more refined weighting strategy. The EDHEC-Risk Institute called the new generation of smart beta investing Smart Beta 2.0. In this type of investing, there is an innovative approach based on the control of both types of risks:

- The systematic risk by distinguishing the stock selection process from the application of the weighting scheme.
- The specific risk by the combination of weighting scheme to reduce estimation.

2.4.1 Systematic risk

By distinguishing between the stock selection process and application of the weighting scheme, investors can choose the risk factors to which they wish to be exposed. For example, if an investor wants to apply the smart beta scheme without incurring low-volatility risk, the scheme can be applied to the most liquid assets. Investors therefore choose their risk aversion, expected return and market beforehand. Amenc, Goltz and Lodh (2012) found that the distinction between the selection and weighting phases could add value in terms of performance and controlling the investment risks.

Table 2.1 shows how the stock selection stage reduces the exposure to some factors. Three different smart beta weighting schemes were used to show the exposure differences between the cap-weighted index and specific smart beta index. All three the indices show significant exposure to the small-cap factor when no selection is made. As soon as the focus is placed on large-cap stocks, the small-cap exposure is mitigated. Interesting to see in this table is that the market exposure is also reduced when one looks at the large-cap universe.

Table 2.1: Exposure of a smart beta-weighted portfolio versus a cap-weighted portfolio

Universe	Global Minimum Volatility (GMV)				Maximum Sharpe Ratio (MSR)				Maximum Decorrelation (MDC)			
	All stocks	Small size universe	Medium size universe	Large size universe	All stocks	Small size universe	Medium size universe	Large size universe	All stocks	Small size universe	Medium size universe	Large size universe
Market exposure of excess returns over CW	-26.20%	-25.41%	-28.03%	-23.92%	-21.92%	-23.69%	-23.94%	-20.09%	-8.60%	-7.93%	-10.57%	-6.59%
Size (Big - Small) exposure of excess returns over CW	-19.00%	-43.75%	-19.32%	1.83%	-21.13%	-46.28%	-21.40%	0.29%	-37.07%	-65.59%	-27.26%	-3.15%

Source: EDHEC-Risk Institute (2014)

The stock selection stage can be applied to avoid trading risk such as liquidity risk by selecting the most liquid assets without any impact on the portfolio performance (Amenc et al., 2013). This study was based on the top 50% of stocks in terms of liquidity in the US market. The authors found that the annualised returns and Sharpe ratios decreased slightly but not by a significant amount.

2.4.2 Specific risk

As discussed earlier, the total specific risk is equal to the sum of the estimation risk and the optimality risk. The MPT states that investors should allocate to risky assets in such a way that they achieve the highest possible Sharpe ratio. This is a complex task due to the estimation risk of the expected returns and covariance parameters. Controlling the specific risk is straightforward in the sense that an investor should look for the best trade-off between optimality risk and estimation risk.

2.5 SMART BETA WEIGHTING SCHEME

2.5.1 Risk parity (equal risk contribution)

In recent years, minimum-variance and equally weighted portfolios have grown in popularity. The reason is that their construction does not rely on expected average returns. Risk parity is the same in the sense that the risk contribution from each portfolio component is made equal; this maximises diversification of risk. The resulting portfolio is therefore similar to a minimum-variance portfolio subject to a diversification constraint on the weights of its components. Equally weighted risk contribution (risk parity) portfolios are thus more attractive than minimum-variance or equally weighted portfolios.

Markowitz (1952) formalised optimal portfolio construction as the process of efficiently allocating capital among securities in a portfolio, in a mean variance framework whereby we can assume that the rational investor seeks to maximise the expected return for a given volatility level. There are some drawbacks to this method. The first is that the allocation is excessively concentrated in a limited set of securities. The second is that the mean-variance solution is extremely dependent on the input parameters (Maillard et al., 2010). Investors prefer computationally simple and robust asset allocation methods.

Two such techniques exist, as mentioned earlier in this section: minimum-variance and equally weighted portfolios. The first one is a specific portfolio on the mean-variance efficient frontier. This portfolio is easy to compute and has a unique solution as it is independent of the expected return. However, this portfolio is often extremely concentrated in a sector or in a specific security. A simple way to sidestep this drawback is to equally weight each security in the portfolio. Benartzi and Thaler (2001), Windcliff and Boyle (2004) and DeMiguel, Garlappi and Uppal (2009) showed how this portfolio was efficient out of sample. The problem with this technique is that the diversification benefits are very limited.

An optimal choice would be to find a medium between the two abovementioned techniques. The idea would then be to equalise risk contributions from the different components in the portfolio. The risk contribution of a component is the weight of total portfolio risk attributable to that component. It

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is computed as the product of the weight in a component multiplied by its marginal risk contribution, the latter being given by the change in the total risk of the portfolio induced by an infinity small increase in the holdings of the component in question. The mathematics of this model is explained in the methodology section of this thesis. Maillard (2010) investigated the improvements that ERC portfolios made on equally weighted portfolios and found the following:

> Investigating the out-of-sample risk-reward properties of equally-weighted risk contributions (ERC) portfolios is interesting because they mimic the diversification effect of equally-weighted portfolios while taking into account single and joint risk contributions of the assets. In other words, no asset contributes more than its peers to the total risk of the portfolio (Maillard et al., 2010).

Fernholz et al. (1998) and Booth and Fama (1992) have shown that the diversification of risks can improve returns.

2.6   CONCLUSION

The purpose of this chapter was to provide an explanation of the theories on which risk parity portfolios are based; the methods for setting up these portfolios are further delved into in the next chapter. The risk parity weighting methodology is explained in mathematical terms, and the steps that are followed to set up the various portfolios are explained in detail.

3   RESEARCH METHODOLOGY

3.1   SELECTING DESIRED FACTOR EXPOSURES

In this study, the focus was on four well-known factors mentioned in earlier sections – value, quality, volatility and momentum. Factor indices fall into two major categories. The first involves selecting the stocks that are most exposed to the desired risk factor and weighting these stocks according to their price (cap-weighted index). The problem with this weighting scheme is poor diversification due to high concentration in a small number of stocks. The second method tries to maximise the exposure to a factor, either by weighting the whole of the universe on the basis of a factor or by selecting and weighting by the exposure score of the stock to that factor. This, however, does not guarantee that the indices are well diversified (EDHEC-Risk Institute, 2014).

This problem of poor diversification is countered with smart beta indices. Smart Beta 2.0 (Amenc et al., 2013) improves on the previously mentioned flaws by allowing investors to choose the risk factors that they want to be exposed to. A well-diversified weighting scheme then allows for the reduction of unrewarded or specific risks. For the purpose of this study, a risk parity weighting scheme was used and the results were compared with those of a capitalisation-weighted benchmark.

In this study, the following fundamentals, which were discussed in subsections 2.2.1 up to 2.2.4, were investigated. For the value factor, the results between the earnings yield and price-to-book ratios were compared. For the quality factor, the results between the gross profit and return-on-equity fundamentals were compared. The results obtained using a 60-day volatility and a 260-day volatility for the volatility factor were compared. Lastly, for the momentum factor, only the one-year total return was used in the stock selection process.

3.2   EXTRACTING DATA FROM BLOOMBERG AND CALCULATING Z-SCORES

The data obtained for this project were extracted from Bloomberg via RStudio. An equity screen was created in Bloomberg. The equity screen extracted all the fundamental data for each stock listed on the Johannesburg Stock Exchange (JSE) at a specific date. A cross-sectional z-score was also added to the screen. The data points were extracted at the end of each quarter beginning January 2002 and ending 30 September 2016. The cross-sectional z-score calculation can be expressed in mathematical terms as follows:

Let $x_{i,j,t}$ be the ith stock's value for the jth fundamental factor with i = 1,…,n, where n is the number of stocks listed on the JSE at a specific date, j = 1,…,m, where m is the number of fundamental

factors, and t = 1,…,T, where T is the number of quarters in the investment horizon. The mean of fundamental factor j at a specific date is calculated as follows:

$$\bar{x}_{j,t} = \frac{1}{n}\sum_{i=1}^{n} x_{i,j} \ , with \ j = 1,\dots m \ , 1 \le t \le T \tag{3.1}$$

The mean $\bar{x}_{j,t}$ is calculated for all the fundamental factors at all the rebalancing dates (quarterly). Now define $s_{j,t}$ as the standard deviation of the values for all the stocks for fundamental factor j on a specific rebalancing date:

$$s_{j,t} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_{i,j} - \bar{x}_j\right)^2} \ , 1 \le t \le T \tag{3.2}$$

The cross-sectional z-score for fundamental factor j and stock i at time t is calculated as follows:

$$z_{i,j,t} = \frac{x_{i,j} - \bar{x}_j}{s_j} \tag{3.3}$$

Daily fundamental data for all the stocks listed on the JSE from 2002 up to 2016 were downloaded from Bloomberg and stored in an RStudio file. This data were used to calculate z-scores for the stocks based on their historical fundamental factor values.

Let $x_{i,j,t}$ be the fundamental value for factor j at time t for stock i where j and i are defined as previously in this chapter. Define the average of the previous 250 values for fundamental factor j specifically for stock i as follows:

$$\bar{x}_{i,j,t} = \frac{1}{n}\sum_{k=1}^{n} x_{i,j,t-k} \ , for \ n = 250 \tag{3.4}$$

Now define $s_{i,j,t}$ as the the standard deviation of the previous 250 values for fundamental factor j specifically for stock i as follows:

$$s_{i,j,t} = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(x_{i,j,t-k} - \bar{x}_{i,j}\right)^2} \ , for \ n = 250 \tag{3.5}$$

The individual time series z-scores are then once again calculated as follows:

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$$z_{i,j,t} = \frac{x_{i,j,t} - \bar{x}_{i,j,t}}{s_{i,j,t}} \tag{3.6}$$

Daily, weekly, monthly, quarterly, semiannual and yearly price movements for all the stocks listed on the JSE and the two benchmarks were also extracted from Bloomberg. The two benchmarks were the JSE All Share Index (JALSH) and the JSE Shareholder-weighted Index (SWIX). A market capitalisation index was also created based on the same stock selection as the smart beta selection. The results were compared with the risk parity-weighted portfolios. Lastly, weekly and daily data were extracted for the macroeconomic variables. These variables were gold, oil, exchange rate, South African 10-year bond yield and GDP.

3.3   STOCK SELECTION

In this study, five different portfolios were analysed; four of the portfolios were single-factor portfolios (either value, quality, volatility or momentum), and the fifth portfolio was a multifactor portfolio (combination of all four factors).

In Section 3.2, two methods for calculating the z-scores for the stocks (cross-sectional and time series) are explained. In both cases, the top 95% of shares in terms of market capitalisation were selected before further selection was done.

Once the z-scores have been calculated, order $z_{i,j}$,$i =1,…,$n and denote the ith largest one as $z_{(i)}$.

$$\Rightarrow z_{(1),j} < z_{(2),j} < \cdots < z_{(n),j} \tag{3.7}$$

The best stocks were selected based on the calculated z-scores. For some of the fundamental factors, the best z-scores were the smallest and for others, the highest scores were the best, as outlined in Table 3.1.

Table 3.1: Z-score criteria for each fundamental factor

Fundamental factor	Good z-score	Rationale
Earnings yield	High z-score	High earnings yield anomaly
Price-to-book	Low z-score	Low price-to-book anomaly
Volatility	Low z-score	Low volatility anomaly
Gross profit	High z-score	High gross profit anomaly
Return-on-equity	High z-score	High return-on-equity anomaly
One-year return	High z-score	High momentum anomaly

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The stock selection method followed for the multifactor portfolio selected the best stocks, taking into account all four fundamental factors. This was done by adding the z-scores that were calculated for a specific stock across all the fundamental factors. Let

$z_{i,j,t}$ = z-score for *stock i* based on fundamental *factor j* at time t

m = the number of fundamental factors.

The z-score for *stock i* for the multifactor model is calculated as follows:

$$Z_{i,t} = \sum_{j=1}^{m} z_{i,j,t} \tag{3.8}$$

3.4   RISK PARITY AND MARKET CAPITALISATION WEIGHTING SCHEMES

Before the risk parity weights could be calculated, the returns of the stocks needed to be calculated for the past two years up to each rebalancing date and put into a matrix.

Let $y_{i,t}$ be the stock price of stock i at time t *for i* = 1,…,n and t = 1,…,T where n is the number of stocks listed on the JSE at a specific time and T is the number of days in the time horizon.

The one-period log return follows as:

$$r_{i,t} = \log\left(\frac{y_{i,t}}{y_{i,t-1}}\right) \tag{3.9}$$

and hence the return matrix follows as:

$$\begin{bmatrix} r_{1,1} & \cdots & r_{n,1} \\ \vdots & \ddots & \vdots \\ r_{1,T} & \cdots & r_{n,T} \end{bmatrix}$$

After all the selected stocks' returns had been calculated, another filtering process was applied, namely to delete the stocks with less than two years of data. A stock had to be listed for at least two years on the JSE to be viable for selection in one of the portfolios. One method of obtaining the risk parity weights is described in Maillard et al. (2010) and is explained below.

Consider a portfolio with weights $x' = (x_1, x_2, …, x_n)$ in n risky assets. Let σ_i^2 be the variance of asset i, σ_{ij} be the covariance between assets *i* and *j* and $\sum$ be the covariance matrix. The standard deviation of the portfolio follows as

$$\sigma(x) = \sqrt{x'\sum x} \tag{3.10}$$

$$= \sqrt{\sum_i x_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} x_i x_j \sigma_{ij}} \tag{3.11}$$

The marginal risk contribution of asset *i* is defined as the change in the volatility ($\sigma(x)$) of the portfolio for a unit change in x_i and is defined by $\delta_{x_i}\sigma(x)$.

Hence, the change in the volatility of the portfolio, if x_i changes with Δx_i, follows as

$$\Delta\sigma(x) = \delta_{x_i}\sigma(x) \cdot \Delta x_i \tag{3.12}$$

$$\Rightarrow \delta_{x_i}\sigma(x) = \frac{\Delta\sigma(x)}{\Delta x_i} \tag{3.13}$$

$$\lim_{\Delta x_i \to 0} \frac{\Delta\sigma(x)}{\Delta x_i} = \frac{\partial\sigma(x)}{\partial x_i} \tag{3.14}$$

$$= \frac{1}{2}(\sigma(x))^{-1} \cdot \left[2x_i\sigma_i^2 + 2\sum_{j \neq i} x_j\sigma_{ij} \right] \tag{3.15}$$

$$= \frac{x_i\sigma_i^2 + \sum_{j \neq i} x_j\sigma_{ij}}{\sigma(x)} \tag{3.16}$$

This implies that if the weight of the ith asset changes from 0 to x_i, the standard deviation of the portfolio changes with

$$\sigma_i(x) = \delta_{x_i}\sigma(x).x_i, \tag{3.17}$$

which is defined as the total contribution of the *i*th asset to the risk of the portfolio. Hence, the risk of the portfolio is defined as the sum of the total risk contributions:

$$\sum_i \sigma_i(x) = \frac{\sum x_i^2\sigma_i^2 + \sum_{i \neq j} x_i x_j \sigma_{ij}}{\sigma(x)} \tag{3.18}$$

$$= \frac{\sigma^2(x)}{\sigma(x)} \tag{3.19}$$

$$= \sigma(x) \tag{3.20}$$

The equal risk contribution (ERC) portfolio could now be constructed. The next step in this strategy was to find a risk-balanced portfolio so that the risk contribution, $\sigma_i(x)$, was the same for all the

securities in the portfolio. For the purpose of this study, no short-selling was allowed; therefore, the set of weights so that the risk contribution of all the assets was equal is written as follows:

$$x^* = \left\{ x \,\epsilon\, [0,1]^n : \sum x_i = 1, 0 \le x_i \le 1 \,, i = 1, \dots n, x_i.\,\delta_{x_i}\sigma(x) = x_j.\,\delta_{x_j}\sigma(x) \; for \; all \; i,j \right\} \qquad (3.21)$$

Note that

$$x_i \cdot \delta_{x_i}\sigma(x) = x_j \delta_{x_j}\sigma(x)$$

$$\Rightarrow x_i \left(\frac{x_i\sigma_i^2 + \sum_{j\neq i} x_j\sigma_{ij}}{\sigma(x)} \right) = x_j \left(\frac{x_j\sigma_j^2 + \sum_{j\neq k} x_k\sigma_{jk}}{\sigma(x)} \right)$$

$$\Rightarrow x_i \left(x_i\sigma_i^2 + \sum_{j\neq i} x_j\sigma_{ij} \right) = x_j \left(x_j\sigma_j^2 + \sum_{j\neq k} x_k\sigma_{jk} \right)$$

$$\Rightarrow x_i \left(\sum \underline{x} \right)_i = x_j \left(\sum \underline{x} \right)_j \qquad (3.22)$$

where $\left(\sum \underline{x} \right)_i$ is the ith component of the vector $\sum \underline{x}$. The set in (3.21) can thus also be written as

$$x^* = \left\{ x \in [0,1]^n : \Sigma x_i = 1 \,, x_i(\Sigma\underline{x})_i = x_j(\Sigma\underline{x})_j \,, for \; all \; i,j \right\} \qquad (3.23)$$

Consider the bivariate case (two assets in the portfolio). Let ρ be the correlation and $x' = (w, 1-w)$ the vector of weights. The vector of total risk contributions according to (3.16) follows as

$$\frac{1}{\sigma(x)} \begin{bmatrix} w^2\sigma_1^2 + w(1-w)\rho\sigma_1\sigma_2 \\ (1-w)^2\sigma_2^2 + w(1-w)\rho\sigma_1\sigma_2 \end{bmatrix}$$

To construct the ERC portfolio, the two rows were set equal:

$$w^2\sigma_1^2 + w(1-w)\rho\sigma_1\sigma_2 = (1-w)^2\sigma_2^2 + w(1-w)\rho\sigma_1\sigma_2$$

Hence,

$$w^2\sigma_1^2 = (1-w)^2\sigma_2^2 \qquad (3.24)$$

This can be rewritten as

$$w\sigma_1 = (1-w)\sigma_2$$

$$\Rightarrow w\sigma_1 = \sigma_2 - w\sigma_2$$

Thus, the optimal weights are then as follows:

$$w = \frac{\sigma_2}{\sigma_1 + \sigma_2} \tag{3.25}$$

$$1 - w = \frac{\sigma_1}{\sigma_1 + \sigma_2} \tag{3.26}$$

This solution is independent of ρ (the correlation between the assets).

The problem is solved in the same manner for the general case (n > 2; more than two assets). There are now n volatilities and $\frac{n(n-1)}{2}$ bivariate correlations. Firstly, a case where a simple analytic solution is obtained is illustrated. Assume that for every pair of variables the correlations are equal $\left(\rho_{i,j} = \rho\right)$. The total risk contribution of component j is written as

$$\sigma_j(x) = \frac{\left(x_j^2 \sigma_j^2 + \rho \sum_{j \neq i} x_i x_j \sigma_i \sigma_j\right)}{\sigma(x)}$$

$$\Rightarrow \sigma_j(x) = \frac{x_j \sigma_j \left((1 - \rho) x_j \sigma_j + \rho \sum_i x_i \sigma_i\right)}{\sigma(x)}$$

The ERC portfolio is obtained where $\sigma_i(x) = \sigma_j(x)$ for all i,j. This is equivalent to $x_i \sigma_i = x_j \sigma_j$. If constraints are added that the weights should add up to one and that no short-selling is allowed, the solution can be obtained as follows:

Let x_i be the weight of stock i and σ_i be its corresponding standard deviation. Now, if $\Sigma_i x_i = 1$ and $x_i > 0$ for all $i = 1, \dots n$ where n is the number of stocks, then

$$x_i \sigma_i = x_j \sigma_j$$

$$\Rightarrow x_i = \frac{x_j \sigma_j}{\sigma_i}$$

$$\Rightarrow \Sigma_i x_i = \Sigma_i x_j \sigma_j \sigma_i^{-1}$$

$$\Rightarrow 1 = x_j \sigma_j \Sigma_i \sigma_i^{-1}$$

$$x_j = \frac{\sigma_j^{-1}}{\sum_{i=1}^n \sigma_i^{-1}} \tag{3.27}$$

This method was therefore applied to each time series returns matrix, and a vector of weights was obtained for each rebalancing date for the selected stocks.

RStudio also has an optimisation package called 'PortfolioAnalytics' that can be used to solve the risk parity problem. With this new method, one can specify which measure of risk should be used to

estimate the risk parity weights. In this study, two methods of calculating the risk parity weights were used. The first was ERC in which the weights of the stocks were such that their contribution to the total risk of the portfolio was equal. The second method was risk parity weights. The weights were also calculated so that their contribution to the total risk of the portfolio was equal, but an objective was added to the optimisation; the standard deviation of the portfolio should be minimised.

As stated in Section 3.2, the risk parity-weighted portfolios were compared with the market capitalisation-weighted portfolios. The next step thus was to compute the market capitalisation weights for the components in the portfolio; in this case, the market cap of each stock was divided by the total market cap of all the selected stocks.

3.5   CALCULATING PORTFOLIO AND BENCHMARK RETURNS

3.5.1   Quarterly returns and cumulative returns

For the purpose of this study, the quarterly log returns of the various portfolios were compared. The SWIX and JALSH were used as benchmarks.

Let $y_{i,t}$ be the index value at time t for index I and $y_{i,t-1}$ be the index value one period back from period t for index i.

Therefore, the log return from period t-1 to period t follows as

$$r_{i,t} = \log\left(\frac{y_{i,t}}{y_{i,t-1}}\right)$$

Let $w_{1,t}, \ldots, w_{n,t}$ be the weights allocated to the n selected stocks at time t. The weighted return for the ith stock at time t is calculated as $r_{i,t} * w_{i,t-1}$. The return for the portfolio at time t is therefore written as

$$R_{p,t} = \sum_{i=1}^{m} r_{i,t} * w_{i,t-1} \qquad (3.28)$$

The cumulative returns over the entire investment horizon are calculated as follows:

$$Cumulative\ return = \left(1 + R_{p,1}\right) * \left(1 + R_{p,2}\right) * \ldots * \left(1 + R_{p,T}\right) - 1 \qquad (3.29)$$

where T is the last date in the investment horizon.

This process was followed for each portfolio constructed.

3.5.2   Relative returns

The relative return of a portfolio is the absolute return of the portfolio in excess of the absolute return of a benchmark over a period of time. Rolling quarterly relative returns for the various portfolios relative to the JSE SWIX were also calculated.

3.6   RISK METRICS

Various risk measures were estimated to compare the overall performance of the portfolios on a risky basis.

3.6.1   Value at risk

The value at risk (VaR) of a portfolio is the loss that is expected to be exceeded with a probability of x% during the next t-day holding period. Two important parameters are necessary when estimating VaR, namely the significance level and the risk horizon.

The VaR estimate will therefore be able to answer the following question: "What is the maximum percentage that I can expect to lose on my portfolio over the next day given a 95% confidence level?"

There are various ways of estimating the VaR. Three of these were implemented in this study.

3.6.1.1   Normal linear value at risk

The portfolio returns are assumed to be normally distributed, and the portfolio is linear with respect to its risk factors. The returns of the portfolio, in this section, will be denoted by X.

Let μ be the mean of the portfolio returns and σ^2 the volatility of the portfolio returns. The mean and variance of the portfolio returns are calculated at the end of each quarter throughout the investment horizon. The assumption is thus that

$$X \sim N(\mu, \sigma^2)$$

To calculate the $100\alpha\%$ normal linear VaR for the portfolio, the α quantile return such that $P(X < x_\alpha) = \alpha$ needs to be calculated. This is done as follows:

Standard normal transformation is firstly required.

$$P(X < x_\alpha) = P\left(\frac{X - \mu}{\sigma} < \frac{x_\alpha - \mu}{\sigma}\right) = P\left(Z < \frac{x_\alpha - \mu}{\sigma}\right)$$

where $Z \sim N(0,1)$. If $P(X < x_\alpha) = \alpha$, then

$$P\left(Z < \frac{x_\alpha - \mu}{\sigma}\right) = \alpha$$

$$\Rightarrow \frac{x_\alpha - \mu}{\sigma} = \Phi^{-1}(\alpha)$$

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where ϕ is the standard normal cumulative distribution function. From the symmetry of the standard normal distribution, it follows that $\Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha)$. The VaR of the portfolio with normally distributed returns is therefore equal to

$$VaR_\alpha = -x_\alpha = \Phi^{-1}(1-\alpha)\sigma - \mu$$

3.6.1.2 Historical value at risk

In this case, a large quantity of historical data were used to estimate the VaR; no assumptions were made regarding the distribution of the portfolio returns. A historical series of returns on the portfolio was constructed by holding the current portfolio weights constant and applying these to the asset returns to reconstruct a constant weighted portfolio returns series. The $100\alpha\%$ h-day historical VaR in value terms is the α quantile of the simulated portfolio returns.

The steps to estimate the VaR of a portfolio consisting of a number of assets are explained as follows: The daily historical prices of the various assets are collected over a specific horizon; in this case, it was 500 days and the historical VaR was only calculated on the last day of the investment horizon using the stocks obtained in the portfolio on that date. Five hundred scenarios can be created for what can happen between today and tomorrow. Scenario 1 is created using the percentage changes in the values of all the assets in the portfolio on the first date in the historical period and allocating these changes to the current date. Scenario 2 is created using the second day's percentage changes; 500 scenarios can thus be created. For each scenario, the percentage change in the portfolio is calculated using the weights allocated to the assets and their corresponding return for that specific scenario. A probability distribution of portfolio returns is therefore obtained, and the $100\alpha\%$ one-day historical VaR in percentage terms is the α quantile of the simulated portfolio returns. The h-day VaR is the one-day VaR estimate multiplied by $\sqrt{h}$.

When the assets underlying the portfolio are unknown, the historical returns obtained for the portfolio across the investment horizon are taken as the simulated returns and the α quantile is calculated from this probability distribution of portfolio returns.

3.6.1.3 Modified value at risk

The limitations of using the standard normal linear VaR are related to the use of a symmetrical distribution function. Zangari (1996) and Favre and Galeano (2002) provided a modified VaR calculation that took the higher moments of nonnormal distributions (skewness and kurtosis) into account through the use of a Cornish Fisher expansion. Let S be the skewness of the returns, K the

kurtosis and $z_\alpha = \Phi^{-1}(\alpha)$ the α quantile of a standard normal distribution; the Cornish-Fisher approximation $\tilde{x}_\alpha$ is as follows:

$$\tilde{x}_\alpha = z_\alpha + \frac{S}{6}(z_\alpha^2 - 1) + \frac{K}{24}z_\alpha(z_\alpha^2 - 3) - \frac{S^2}{36}z_\alpha(2z_\alpha^2 - 5)$$

Then, if μ and σ denote the mean and standard deviation of the portfolio returns, the $\alpha\%$ VaR estimate is as follows:

$$VaR_\alpha = -x_\alpha = \tilde{x}_\alpha\sigma - \mu$$

3.6.2   Maximum drawdown

A maximum drawdown is the maximum loss proportion from a peak to a trough of a portfolio before a new peak is attained. This can also be used as an indicator of downside risk over a specified time period and is written as follows:

$$\frac{Peak\ value - lowest\ value}{Peak\ value} \tag{3.30}$$

The peak value is the highest value just before the largest drop while the lowest value is the smallest (lowest) value before a new high is established. This is illustrated in Figure 3.1.
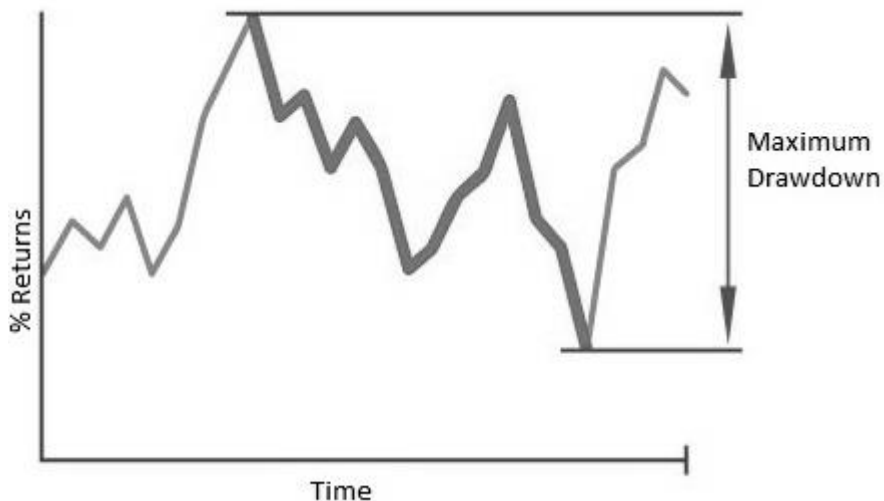
Figure 3.1: Maximum drawdown illustration

3.6.3   Drawdown duration

A measure that is used with the maximum drawdown measure is the drawdown duration. This calculates the average length of the recovery period of the drawdowns observed. This means that it will calculate the length of time that a portfolio will need to grow to its peak level after a loss has been suffered.

3.7   RISK-ADJUSTED PERFORMANCE

3.7.1   Sharpe ratio

The Sharpe ratio is a measure for calculating risk-adjusted return. This can be expressed as the average return earned in excess of the risk-free rate per unit of volatility (or total risk). The greater the Sharpe ratio, the more attractive the risk-adjusted return. Let

R_p = return of the portfolio

R_f = risk-free rate (used 6.75% annually)

S_p = volatility of the portfolio returns (using daily data for the past year).

The Sharpe ratio then follows as

$$\frac{R_p - R_f}{\sigma_p}$$

(3.31)

3.7.2   Information ratio

The information ratio in its most basic format is a ratio of portfolio returns above the returns of a benchmark divided by the volatility of the excess returns. This ratio aims to measure a portfolio manager's ability to generate excess returns relative to a benchmark but also attempts to identify the consistency of the investor. Let

R_b = return of the benchmark

S_{p-b} = tracking error (standard deviation of the differences between returns of the portfolio and the returns of the index).

The information ratio then follows as

$$IR = \frac{R_p - R_b}{S_{p-b}}$$

(3.32)

3.8   MACROECONOMIC EXPOSURE

The risk exposure to macroeconomic factors of the smart beta portfolios was calculated using Citi Research's global risk attribution model (Montagu et al., 2015). This is a macroeconomic time series risk model. The main objective of this model is to use macroeconomic factors to explain stock returns and risks.

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"The ideal factors to include in a risk model are those that best explain the movement of security prices" (Montagu et al., 2015). Some criteria that need to be specified by a viable factor are the following:

- Large total r-squared
- Significant t-statistic
- Low correlations among factors

The factors included in this study were the following: South African exchange rate, oil, gold and swap rates.

The risk factor betas, or sensitivities, were estimated for every stock by doing a regression analysis of one year of weekly stock total returns in local currency on the weekly change in the four risk factors. Let

r_{it} = the total return of the stock from period t-1 to period t

a_i = the expected total return of the stock when all the factors equal zero – the component of the stock return that is not explained by the factors in the model

β_{ij}^{ME} = the sensitivity of the stock to macroeconomic factor j

F_{jt}^{ME} = the realisation of the macroeconomic factor j during period t

e_{it} = the unsystematic component of the stock's total return that is independent of the factors during period t and distributed normally.

The macroeconomic time series factor model can be represented by the following equation for an individual stock i:

$$R_{it} = \alpha_i + \sum_j \beta_{ij}^{ME} F_{jt}^{ME} + e_{it}$$

(3.33)

The macroeconomic factors need to be adjusted for the correlations among them before the regression analysis can be implemented. The process of how this can be done is described below.

The procedure for a two-factor case is as follows:

Let

$$R_i = \alpha_i^* + \beta_{i1}^* I_1^* + \beta_{i2}^* I_2^* + c_i$$

(3.34)

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In this case, I_1^* is the first macroeconomic factor and I_2^* the second. The correlation between the two macroeconomic factors should be removed. Define I_1 as equal to I_1^*. The following regression is done to remove the impact of the first factor on the second:

$$I_2^* = \gamma_0 + \gamma_1 I_1 + d_t$$

In this case, $\gamma_0 \; and \; \gamma_1$ are regression coefficients and d_t the residuals (random term). The residuals are uncorrelated with I_1, the first factor. This is rewritten as

$$d_t = I_2^* - (\gamma_0 + \gamma_1 I_1)$$

This is therefore the new second macroeconomic factor with the impact of the first factor removed:

$$I_2 = d_t = \; I_2^* - (\gamma_0 + \gamma_1 I_1) \tag{3.35}$$

Substituting this into (3.34) leaves us with

$$R_i = \alpha_i^* + \beta_{i1}^* I_1^* + \beta_{i2}^* I_2 + \beta_{i2}^* \gamma_o + \beta_{i2}^* \gamma_1 I_1 + c_i$$

that can be rearranged into

$$R_i = (a_i^* + \beta_{i2}^* \gamma_0) + (\beta_{i1}^* + \beta_{i2}^* \gamma_1) I_1 + \beta_{i2}^* I_2 + c_i \tag{3.36}$$

The first term is a constant and can be written as α_i. The coefficient of the second term is also a constant and can be written as β_{i1}. Lastly, let $\beta_{i2} = \beta_{i2}^*$. This equation is now in its simplest form

$$R_i = \alpha_i + \beta_{i1} I_1 + \beta_{i2} I_2 + c_i \tag{3.37}$$

where I_1 is the original first macroeconomic factor and I_2 the residuals of the second factor after the first factor's impact has been removed with simple linear regression.

This procedure can be extended to a case where several factors are considered. For instance, for the third factor the impact of the first two needs to be removed in the same way:

$$I_3^* = \theta_0 + \theta_1 I_1 + \theta_2 I_2 + e_i$$

Now,

$$I_3 = e_i = I_3^* - (\theta_0 + \theta_1 I_1 + \theta_2 I_2)$$

This procedure is followed for all the factors.

For the purpose of this study, the risk factor sensitivities were estimated quarterly on a rolling basis using a year's daily data for all of the stocks; for example, on each of the rebalancing dates this

regression procedure was done for all of the listed stocks on the JSE using a year's daily log returns and estimating the sensitivities of all the stocks to the various macroeconomic factors. Let

R_{pt} = the return of the portfolio for period t

w_i = the estimated weight for stock *i*

α_i = the estimated constant term for stock *i*

β_{ij} = stock *i* sensitivity (beta) to factor *j*

I_{jt} = the value of *factor j* at time t

e_{it} = the residual term of stock *i* at time t

m = the number of stocks selected in the stock selection process at time t

k = the number of macroeconomic factors

T = the number of days in the investment horizon

after the stock selection and weight calculation procedures. The regression model for a portfolio can therefore be written as

$$R_{pt} = \sum_i w_i \alpha_i + \sum_j \left(\sum_i w_i \beta_{ij} \right) I_{jt} + \sum_i w_i e_{it} \ ; \ 1 \le i \le m \, , 1 \le j \le k, 1 \le t \le T \tag{3.38}$$

An investor may also add some market factors to the regression model; the same process needs to be followed to make sure that all the factors are uncorrelated.

3.9   CONCLUSION

In this chapter, both stock selection processes (cross-sectional and time series comparison) were explained in detail in Section 3.3. These processes were followed for each of the fundamental factors (value, quality, low volatility and momentum) as well as the multifactor model. Once the stock selection process had taken place, weights were calculated for the selected stocks based on two smart beta methodologies (risk parity and ERC) as well as the stocks' market capitalisation, as explained in Section 3.4. Stock returns as well as portfolio returns were calculated using (3.28) and (3.29). The risk and performance measures used in this study were explained in sections 3.6.1 to 3.7.2. The macroeconomic exposure was estimated only for the smart beta portfolios using the weight calculations presented in Section 3.4, and sensitivities were estimated using the process explained in Section 3.8 and equations (3.37) and (3.38).

In the next chapter, the performance of the various portfolios is illustrated through graphs and tables and further discussed. A final conclusion regarding the performance, risks and risk-adjusted returns of the various portfolios created is set out in the last part of Chapter 4.

4 FINDINGS

The results for the stock selection process, returns obtained and macroeconomic exposure are illustrated for each portfolio based on both types of selection methods in this study. The two different stock selection methods are compared to one another in each subsection based on their performance, risk and risk-adjusted returns. The graphs on the left are based on the time series comparison stock selection method, and those on the right are based on the cross-sectional method.

4.1 CUMULATIVE RETURNS

The first step in the process as stated in Section 3.1 is the stock selection method. The cumulative returns produced by using the two methods, time series comparison and cross-sectional comparison, are discussed for each fundamental factor portfolio as well as the multifactor portfolio and compared in the following subsections.

4.1.1 Value

The value portfolios were constructed by selecting the top 30% of the stocks based on their earnings yield z-scores at each rebalancing date. The z-scores were calculated using the two methods discussed in Section 3.2. Once the stocks had been selected, weights were assigned to them to calculate the portfolio returns.

The cumulative returns and relative returns are illustrated in this section for both stock selection methods based on the value factor. The legend on the right of the graph gives an indication of which line belongs to which weighting strategy. Both in Figure 4.1 and Figure 4.2 and for the rest of the figures in Section 4.1, the time series comparison stock selection method graphs appear on the left and the cross-sectional comparison graphs on the right.

Figure 4.1: Cumulative returns for both stock selection methods based on the value factor

In Figure 4.2, the legend shows the relative performance of the various portfolios compared to the SWIX. The market cap-weighted portfolio is in black, the ERC portfolio is in red and the risk parity portfolio is in green.
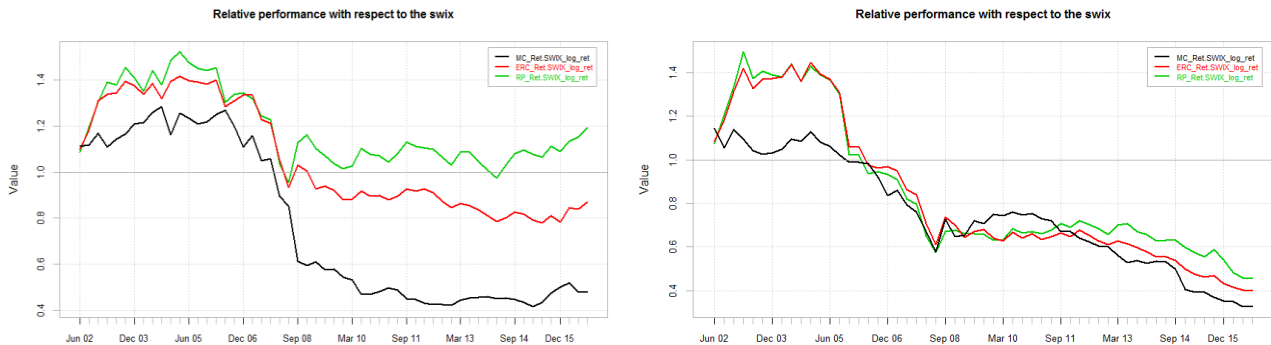
Figure 4.2: Relative performance of both stock selection methods based on the value factor

By using the value factor with time series comparison, it became clear that the risk parity weighting strategy outperformed the other strategies as well as the two benchmarks (JALSH and SWIX). When comparing the relative performance of the various weighting strategies against the SWIX, it was observed that during the financial crisis of 2008, the three created portfolios underperformed severely, especially the market cap-weighted portfolio, showed in black.

When the stock selection was done based on the cross-sectional comparison approach, it was clear that the created portfolios underperformed severely against the SWIX (these graphs are aligned on the right in Figure 4.1 and Figure 4.2). It was only up to June 2005 that the risk parity and equal risk contribution-weighted portfolios outperformed the SWIX (as seen in Figure 4.2). When dealing with the value factor, it is therefore better to base the stock selection process on the time series comparison method.

It became evident that the weighting strategy did have a significant impact on the portfolio returns.

4.1.2   Quality

The quality portfolios were constructed by selecting the top 35% of the stocks based on their gross profitability z-scores at each rebalancing date. Once again, the results for the time series comparison stock selection method are shown on the left of figures 4.3 and 4.4 and the cross-sectional comparison results on the right.

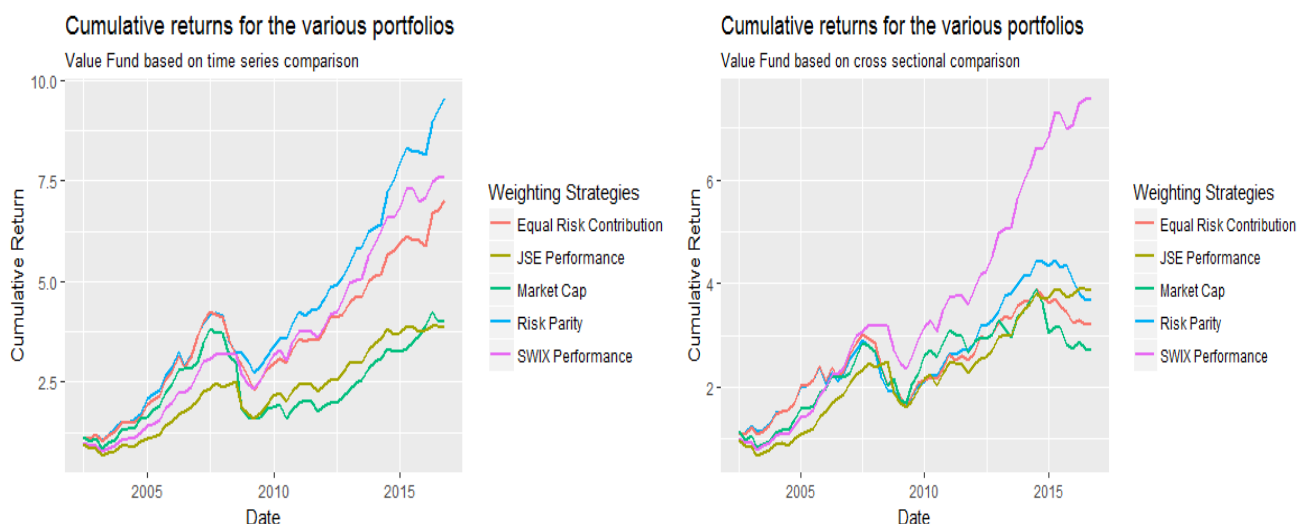**Figure 4.3: Cumulative returns for both stock selection methods based on the quality factor**
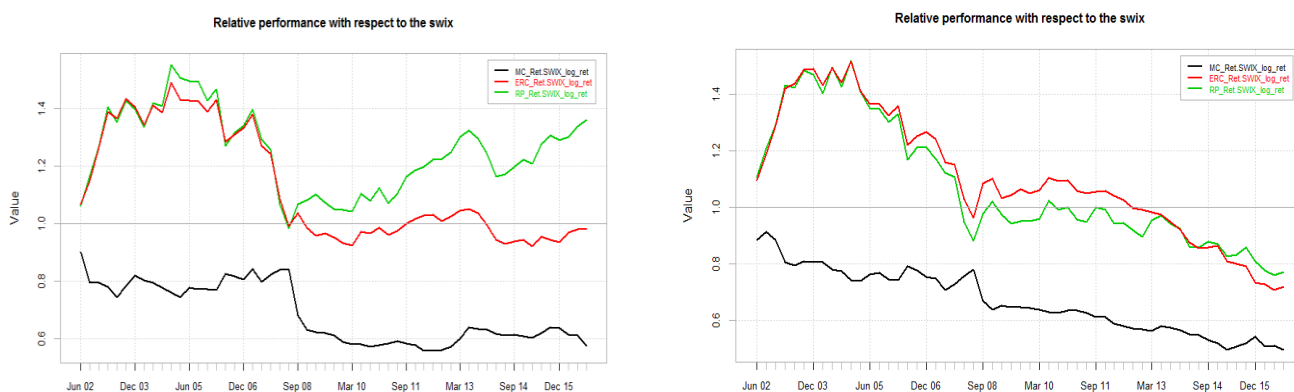
Figure 4.4: Relative performance of both stock selection methods based on the quality factor

Interpreting the results for the quality factor with the time series comparison stock selection method showed improved cumulative returns compared with the value factor. The created portfolio that performed the weakest was the market cap-weighted portfolio, outperforming the JALSH only slightly as shown in Figure 4.3 in the graph on the left. After the crisis of 2008, the risk parity-weighted portfolio, in green in Figure 4.4, quickly regained its momentum unlike with the value factor in which the risk parity-weighted portfolio struggled to regain its outperformance after the crisis.

From the start of the investment horizon, the market cap-weighted portfolio underperformed against the benchmark (SWIX) based on cross-sectional comparison stock selection. What is clear from the relative performance graph on the right is the way in which the market cap-weighted portfolio outperformed the rest of the portfolios during the financial crisis. This does not mean that this portfolio produced positive returns during this period; it just performed less badly than the rest of the portfolios. This is evident from Figure 4.3.

In this case, the portfolios for which the stock selection was based on the time series comparison method once again outperformed the portfolios based on the cross-sectional comparison method.

4.1.3 Momentum

The momentum portfolios were constructed by selecting the top 30% of the stocks based on one-year return z-scores at each rebalancing date. This was only done based on the cross-sectional comparison stock selection process.
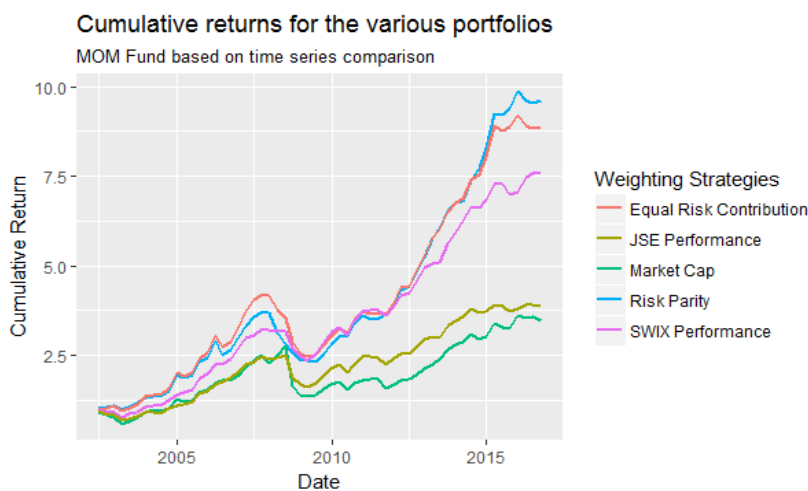
Figure 4.5: Cumulative returns based on the momentum factor
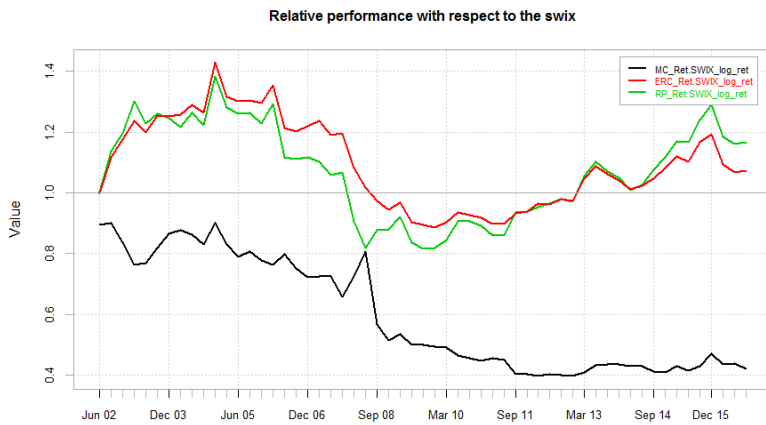
Figure 4.6: Relative performance based on the momentum factor

The risk parity portfolios once again outperformed the rest of the portfolios. The SWIX index, shown in pink in Figure 4.5, was however not far behind according to the cumulative performance graph. It can once again be observed from Figure 4.6 that the market cap-weighted portfolio outperformed the rest during the financial crisis.

4.1.4   Volatility

The volatility portfolios were constructed by selecting the top 30% of the stocks based on their 60-day volatility z-scores at each rebalancing date.
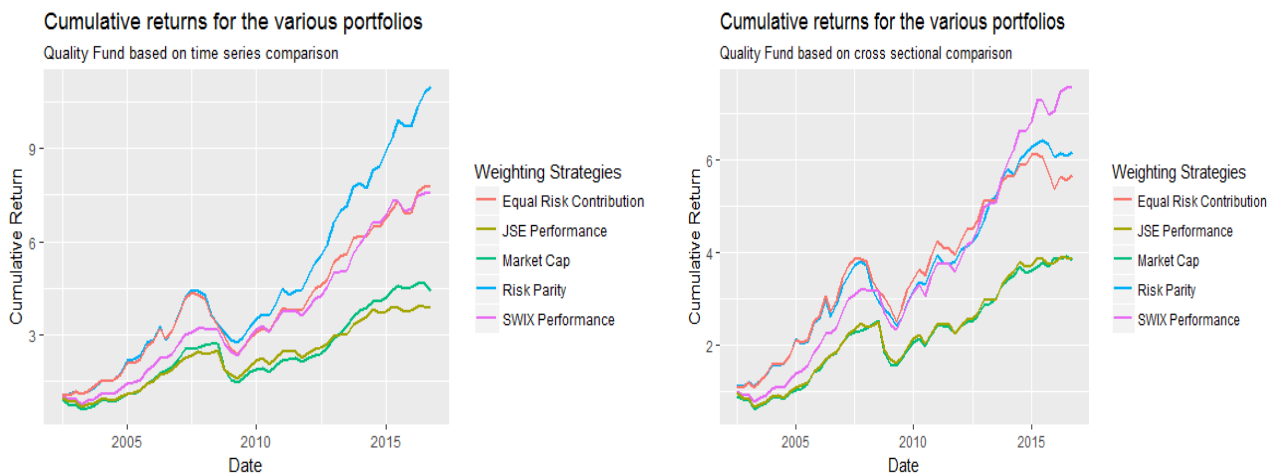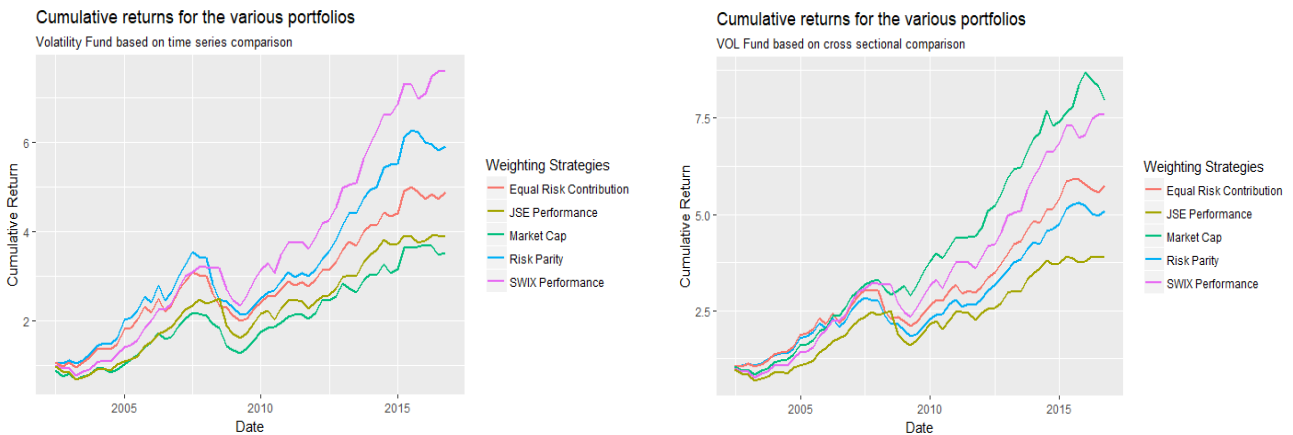

Figure 4.7: Cumulative returns for both stock selection methods based on the volatility factor
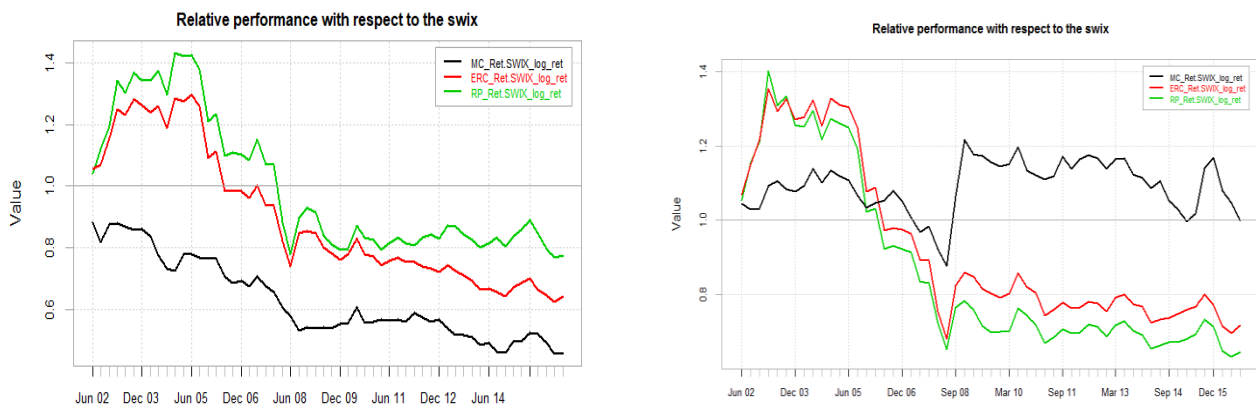
Figure 4.8: Relative performance of both stock selection methods based on the volatility factor

The best performing portfolio, based on time series comparison, according to Figure 4.7 is the SWIX index followed by the risk parity-weighted portfolio. By using this model, it was also seen that the market cap-weighted portfolio did not, like in the previous sections, outperform the rest of the portfolios during the financial crisis of 2008. The idea about which stocks would perform better during the crisis was that the low-volatility stocks combined with a risk parity weighting strategy would be optimal. According to the two graphs, on the left of figures 4.7 and 4.8, this belief was wrong. The risk parity-weighted portfolios underperformed severely during the crisis and could not regain the excess returns over the benchmark.

In the case of the cross-sectional comparison stock selection method, the market cap-weighted portfolio performed the best. As stated in the previous paragraph, it would make sense that the low-volatility factor would perform the best during the crisis of 2008. This was indeed the case according to the cumulative performance graph and relative performance graphs on the right of figures 4.7 and 4.8. The market cap-weighted portfolio showed extremely small negative returns during the time of the crisis.

4.1.5 Multifactor

The multifactor portfolios were constructed by combining the z-scores of all four previously mentioned factors, as explained with equation (3.8), and selecting the top 40% of the stocks based on their combined z-score at each rebalancing date.
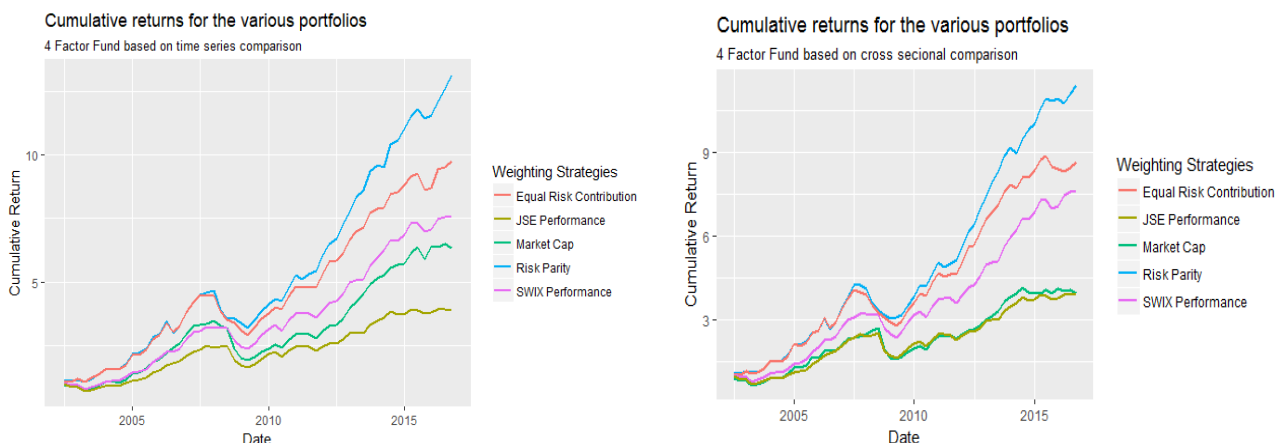
Figure 4.9: Cumulative returns for both stock selection methods based on the multifactor model
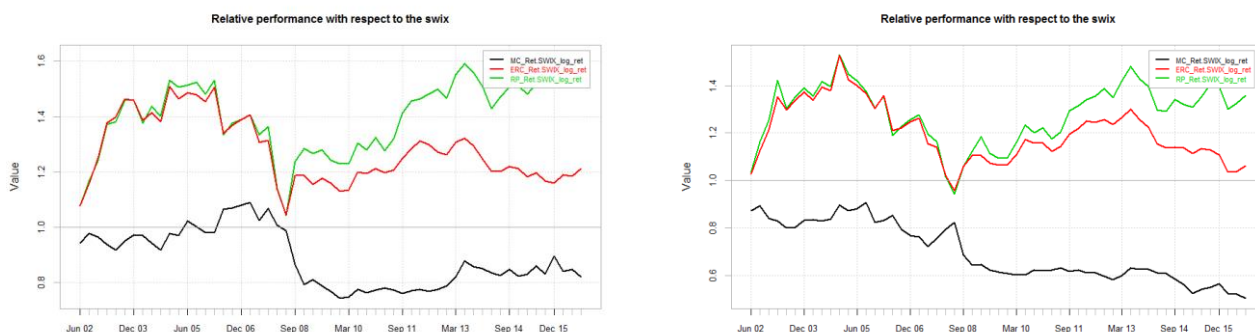
Figure 4.10: Relative performance of both stock selection methods based on the multifactor model

Combining the four factors made a significant difference in the return that the risk parity portfolios produced with both stock selection processes. The market cap-weighted portfolio underperformed against the SWIX only slightly when time series comparison was used in the stock selection process. The risk parity portfolios still underperformed against the SWIX during the crisis but quickly regained their momentum. The market cap-weighted portfolio did not show small negative returns during the crisis as in the previous subsections.

From Figure 4.9 and Figure 4.10, it can be seen that the time series comparison method produced slightly better returns than the cross-sectional method, as can be seen in all of the previous subsections of Section 4.1. The time series comparison stock selection method tends to select a portfolio that is better diversified.

4.2   DRAWDOWNS

Once the stocks had been selected and portfolio returns calculated using the various weighting methodologies explained throughout this thesis, the maximum drawdowns, drawdown durations and VaR measures could be estimated. Like in the previous section, graphs are used to illustrate the various measures for the various factor portfolios. On the left of each figure, the results are shown for the time series comparison stock selection method and on the right, the cross-sectional comparison method.

The VaR estimates were calculated at the end of the time horizon based on the portfolio returns. The methods were discussed in Section 3.6.1.

4.2.1   Value
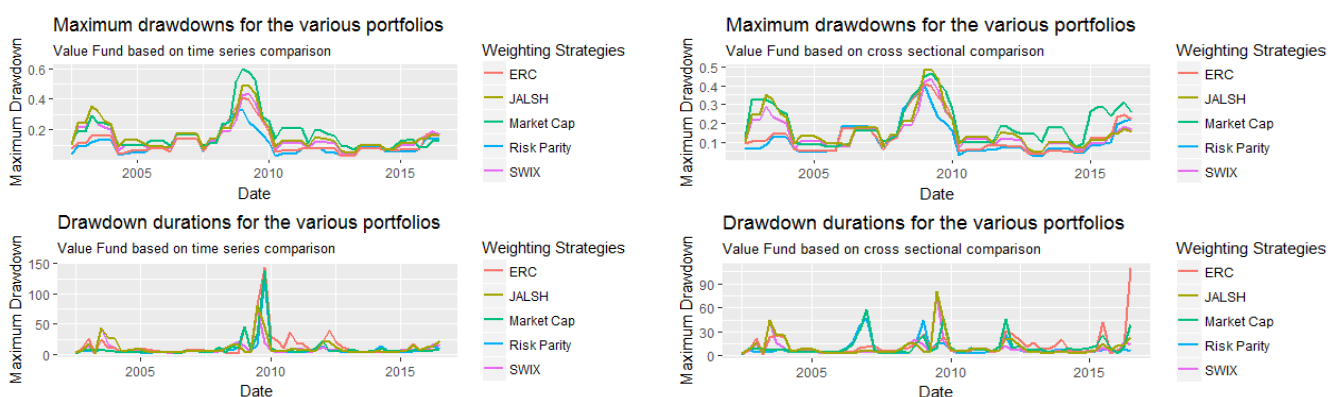
Figure 4.11: Maximum drawdowns and drawdown durations of both stock selection methods based on the value factor

As expressed in Section 4.1.1, during the financial crisis of 2008, all the created portfolios underperformed severely against the SWIX. In Figure 4.11, a sudden spike in the maximum drawdown and drawdown duration is seen during the time of the crisis. By zooming in on this time, between 2008 and 2011, it can be seen that the portfolios with the best maximum drawdown (lowest) during the peak of the crisis were the risk parity-weighted portfolios (shown in blue), reaching a maximum drawdown of 30% and 40% respectively for the time series comparison and cross-sectional comparison stock selection methods at the end of 2008.

When investigating the drawdown duration graph for the time series stock selection method, one sees that the portfolios performing the best (shortest drawdown duration) are the SWIX and JALSH. Both of these reached a drawdown duration of 80 days in the middle of 2009. This means that although the maximum drawdowns were the best for the risk parity-weighted portfolio, it still struggled to regain the momentum that it had built up before the crisis. Value stocks therefore underperformed

for a prolonged period after the peak of the financial crisis. The drawdown duration improved for the risk parity-weighted portfolios when the cross-sectional stock selection method was used. The longest drawdown duration reached for the risk parity-weighted portfolio was just under 30 days. This occurred at the end of 2008.

Table 4.1: Quarterly VaR estimates for the value portfolios and time series comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.09820	-0.11670	-0.16170	-0.10510	-0.08390
Modified VaR (95%)	-0.08090	-0.10020	-0.16840	-0.11870	-0.08900
Normal VaR (95%)	-0.07245	-0.09129	-0.13759	-0.10086	-0.079700

Table 4.2: Quarterly VaR estimates for the value portfolios and cross-sectional comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.0939	-0.1259	-0.1554	-0.1051	-0.0839
Modified VaR (95%)	-0.1050	-0.1135	-0.1438	-0.1187	-0.0890
Normal VaR (95%)	-0.09623	-0.10895	-0.13343	-0.10086	-0.07970

By investigating the daily VaR estimates for the various portfolios, it could be observed that the risk parity-weighted portfolios held the least amount of risk, after the SWIX, over the investment period if the risk of the portfolios was associated with the VaR estimates for both stock selection methods. When the stock selection was based on time series comparison, the risk parity portfolio produced higher historical VaR estimates but smaller modified and normal VaR estimates.

4.2.2 Quality

Figure 4.12: Maximum drawdowns and drawdown durations of both stock selection methods based on the quality factor

The time series comparison portfolio with the lowest maximum drawdown throughout the investment horizon was the risk parity portfolio. The drawdown duration was also the shortest throughout the

investment horizon and during the crisis, reaching a maximum of approximately 60 days in the middle of 2009.

During the financial crisis of 2008, the market cap-weighted portfolio produced the highest maximum drawdown of all the portfolios, but the drawdown duration of this portfolio (based on cross-sectional comparison) performed the best during the crisis; this can be observed in Figure 4.12 and is also evident from Figure 4.4. The drawdown duration of the risk parity-weighted portfolio based on cross-sectional comparison reached 150 days at its highest peak during the crisis. It can also be seen that since the start of 2015, the maximum drawdowns of the various portfolios started to increase and the drawdown durations were becoming longer. The longer drawdown durations can be evidence of fewer periods producing positive returns but instead delivering flat or slightly negative returns.

Table 4.3: Quarterly VaR estimates for the quality portfolios and time series comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.080400	-0.106600	-0.163100	-0.105100	-0.083900
Modified VaR (95%)	-0.084800	-0.095600	-0.144300	-0.118700	-0.089000
Normal VaR (95%)	-0.080520	-0.090403	-0.119850	-0.100863	-0.079700

Table 4.4 Quarterly VaR estimates for the quality portfolios and cross-sectional comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.081400	-0.090200	-0.116700	-0.105100	-0.083900
Modified VaR (95%)	-0.085500	-0.088400	-0.132400	-0.118700	-0.089000
Normal VaR (95%)	-0.083759	-0.090017	-0.110107	-0.100863	-0.0797004

By investigating the various VaR estimates for the different portfolios, it was also clear that the market cap-weighted portfolio produced the highest (worst) VaR estimates throughout the whole investment horizon. When the risk of a portfolio is associated with the VaR, the market cap-weighted portfolio may be the most risky. Given this riskiness, this portfolio still produced better drawdown durations than the rest of the portfolios when the cross-sectional stock selection process was followed. This shows that although the drawdowns and losses of this portfolio were significantly bigger than the losses of the rest of the portfolios, the positive returns in the periods thereafter led to the portfolio being able to regain its momentum and quickly recover its losses.

4.2.3   Momentum
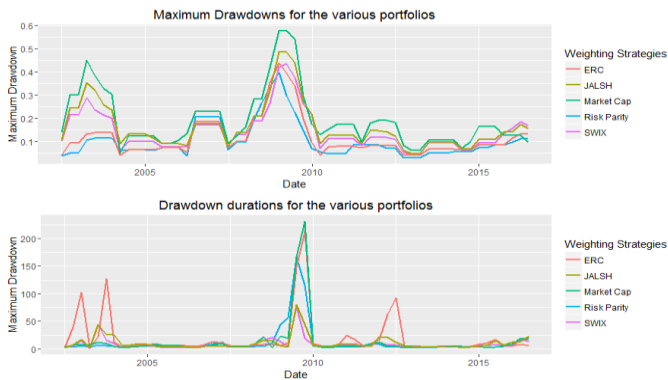
Figure 4.13: Maximum drawdowns and drawdown durations based on the momentum factor

Table 4.5 Quarterly VaR estimates for the momentum portfolios

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.088500	-0.112100	-0.142300	-0.105100	-0.083900
Modified VaR (95%)	-0.081800	-0.090400	-0.175400	-0.118700	-0.089000
Normal VaR (95%)	-0.081623	-0.090011	-0.148393	-0.100863	-0.079700

All the portfolios produced relatively high maximum drawdowns throughout the investment horizon when compared to the other factor tilts and stock selection processes. This could be expected from the momentum factor. Generally, this factor will produce higher drawdowns than the rest. However, given that a significant drawdown did occur, the expectation was that this portfolio would rebound quickly and regain its momentum quickly. It is evident from Figure 4.13 that this did not occur for the risk parity portfolios. The ERC portfolio produced a drawdown duration of 220 days during the crisis of 2008. That is almost a full business year (250 days). The market cap-weighted portfolio in this case produced the longest drawdown duration and maximum drawdowns.

However, the risk parity-weighted portfolios still produced lower (better) VaR estimates than the rest of the portfolios except for the SWIX. The market cap-weighted portfolio produced the highest VaR estimates throughout the investment horizon based on all three methods.
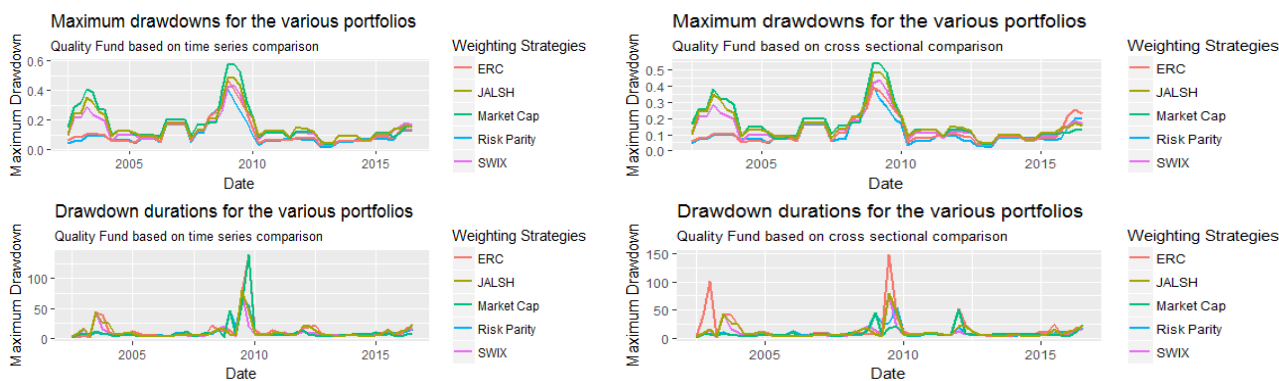
4.2.4 Volatility

Figure 4.14: Maximum drawdowns and drawdown durations of both stock selection methods based on the volatility factor

The risk parity portfolios produced the lowest maximum drawdowns compared to the other volatility factor portfolios except for the cap-weighted portfolio with cross-sectional comparison. The low performance as discussed in Section 4.1.4 could therefore have been due to lower upside returns and low negative returns that were realised over a long period of time, leading to the risk parity-weighted portfolios' performing worse than the SWIX.

Furthermore, the two portfolios with the highest maximum drawdowns were the JALSH and SWIX. The risk parity-weighted portfolio with cross-sectional comparison produced the highest drawdown duration during the crisis of 2008.

Table 4.6: Quarterly VaR estimates for the volatility portfolios and time series comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.075700	-0.101700	-0.119500	-0.105100	-0.083900
Modified VaR (95%)	-0.087300	-0.089500	-0.127300	-0.118700	-0.089000
Normal VaR (95%)	-0.085533	-0.088385	-0.114548	-0.100863	-0.079700

Table 4.7 Quarterly VaR estimates for the volatility portfolios and cross-sectional comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.079600	-0.081200	-0.068300	-0.105100	-0.083900
Modified VaR (95%)	-0.078900	-0.083800	-0.065200	-0.118700	-0.089000
Normal VaR (95%)	-0.072603	-0.075892	-0.060499	-0.100863	-0.079700

The VaR estimates for the risk parity portfolios seemed to be the lowest of all the factor tilt portfolios and stock selecting methods. The market cap-weighted portfolioproduced better VaR estimates than

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the two indices used. When stock selection was based on cross-sectional comparison, the risk parity portfolio produced the best VaR estimates.

Although the market cap-weighted portfolio performed the best based on returns and drawdowns when the cross-sectional stock selection process was used, the risk parity portfolios still produced better VaR estimates. The market cap-weighted portfolio did, however, improve significantly when compared to the previous factor tilts and stock selection methods.
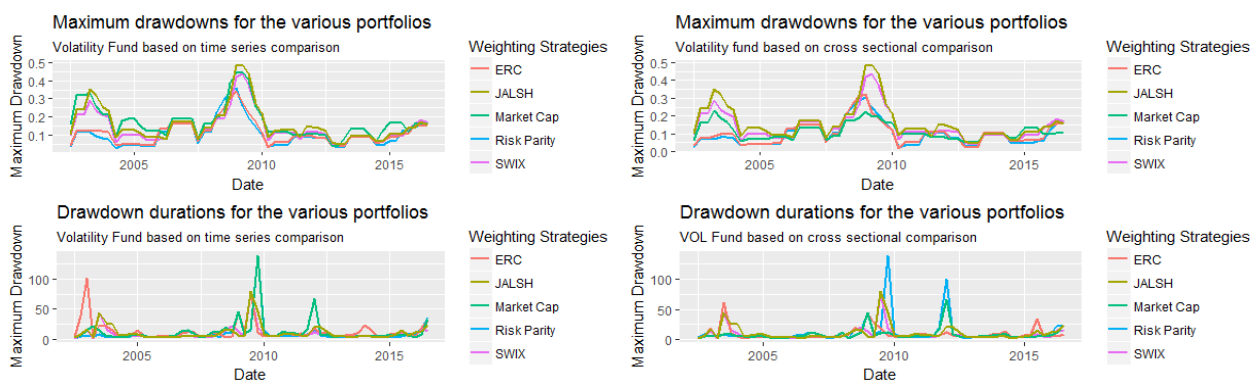
4.2.5 Multifactor
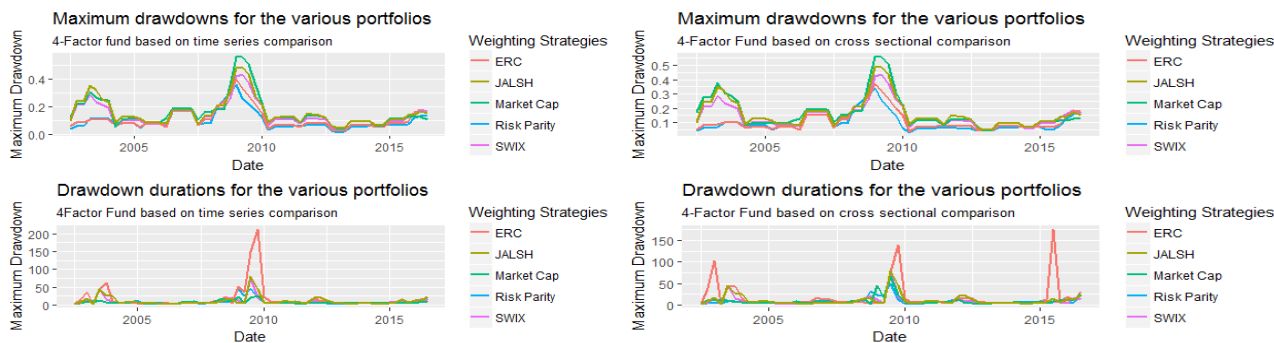
Figure 4.15 Maximum drawdowns and drawdown durations of both stock selection methods based on the multifactor portfolios

The risk parity portfolios produced the best maximum drawdowns throughout the whole investment horizon for both stock selection methods. During the crisis, the ERC portfolio produced a significantly higher drawdown duration. The portfolio with the worst maximum drawdowns was the market cap-weighted portfolio.

Table 4.8 Quarterly VaR estimates for the multifactor portfolios and time series comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.083500	-0.086900	-0.090500	-0.105100	-0.083900
Modified VaR (95%)	-0.080200	-0.085000	-0.122600	-0.118700	-0.089000
Normal VaR (95%)	-0.074073	-0.082845	-0.103849	-0.100863	-0.079700

Table 4.9 Quarterly VaR estimates for the multifactor portfolios and cross-sectional comparison

	Risk parity	ERC	Market cap	JALSH	SWIX
Historical VaR (95%)	-0.059800	-0.065700	-0.126900	-0.105100	-0.083900
Modified VaR (95%)	-0.069900	-0.073100	-0.131200	-0.118700	-0.089000
Normal VaR (95%)	-0.071984	-0.080639	-0.118495	-0.100863	-0.079700

The VaR estimates for the market cap-weighted portfolio were the largest throughout the investment horizon for both stock selection methods. The portfolios performing the best based on the VaR

estimates were the risk parity and ERC portfolios. According to Figure 4.15 however, the ERC portfolio produced the longest drawdown duration period during the financial crisis. As indicated in the previous section, the risk parity portfolio based on cross-sectional comparison produced the smallest VaR estimates. This portfolio produced the lowest VaR estimates compared to all the portfolios discussed previously.

4.3   RISK-ADJUSTED PERFORMANCE

In the previous two sections, the selection of stocks, the construction of portfolios, the calculation of returns and the calculation of drawdown measures were explained for each of the portfolios. In this section, the risk-adjusted performance measures are illustrated. As in the previous sections, graphs are used to illustrate the daily Sharpe ratios of the portfolios and tables are used to show the information ratio for each of the portfolios relative to the SWIX. On the left of each figure, the results are shown for the time series comparison stock selection method and on the right, the cross-sectional comparison method.

4.3.1   Value
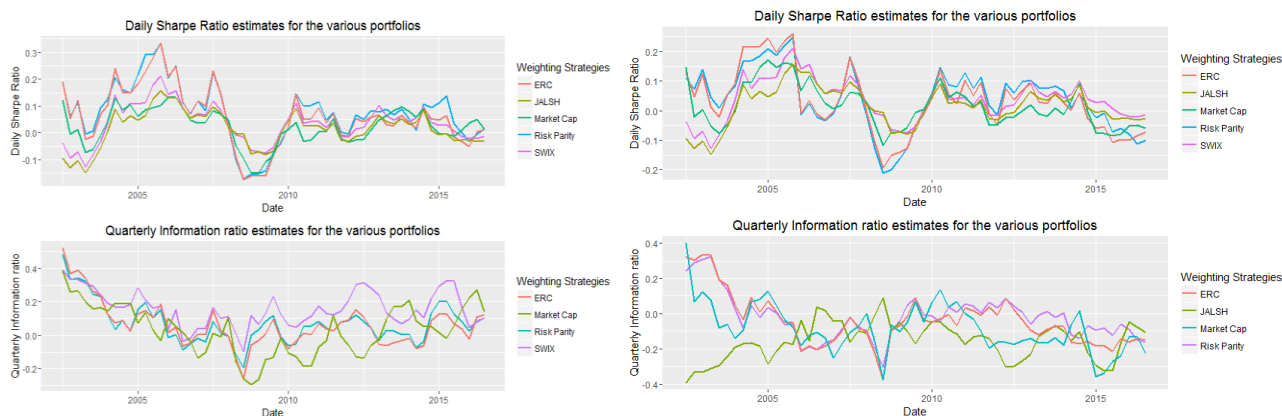
Figure 4.16: Daily Sharpe ratio estimates of both stock selection methods based on the value factor

Viewing the risk-adjusted performances of the various portfolios shows that the Sharpe ratio of the risk parity portfolios is more volatile than the rest; during the crisis of 2008, this ratio decreased from being the highest to being the lowest of all the portfolios.

Table 4.10: Information ratios based on time series stock selection for the value factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	0.1771734	-0.06733093	-0.4154537

As shown in Table 4.10, the risk parity portfolio is the only portfolio of all the value factor tilt portfolios that produced positive information ratios. This shows that the risk parity-weighted portfolio consistently outperformed the SWIX. The ERC portfolio, in second place, produced a slightly negative information ratio.

Table 4.11: Information ratios based on cross-sectional stock selection for the value factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	-0.4237772	-0.5241921	-0.6047155

As shown in Table 4.11, the annual information ratios is negative for all the created portfolios when benchmarked against the SWIX. This indicates that the SWIX consistently outperformed the three created portfolios.

4.3.2   Quality
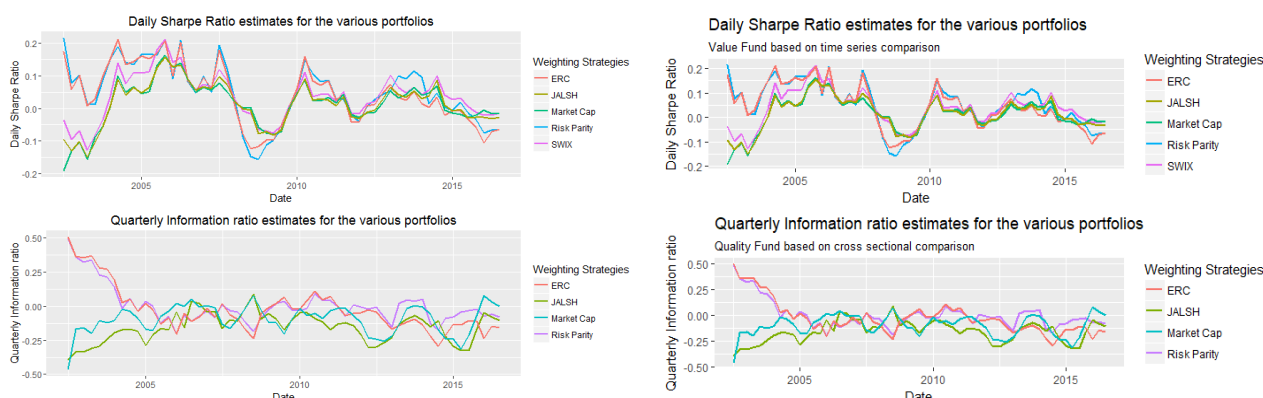
Figure 4.17: Daily Sharpe ratio estimates of both stock selection methods based on the quality factor

Table 4.12: Information ratios based on time series stock selection for the quality factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	0.2951576	0.02583083	-0.5262359

The risk parity-weighted and ERC portfolios based on time series comparison both produced positive information ratios over the investment horizon. They both therefore outperformed the SWIX on a consistent basis. This is also a significant improvement from that illustrated in the previous subsection according to which only the risk parity-weighted portfolio produced a positive information ratio.

Table 4.13: Information ratios based on cross-sectional stock selection for the quality factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	-0.157979	-0.2580767	-0.7950982

The information ratios for the three created portfolios produced negative values. Compared to the information ratios in Table 4.11, these portfolios performed slightly better but still were not able to outperform the SWIX consistently.

4.3.3 Momentum
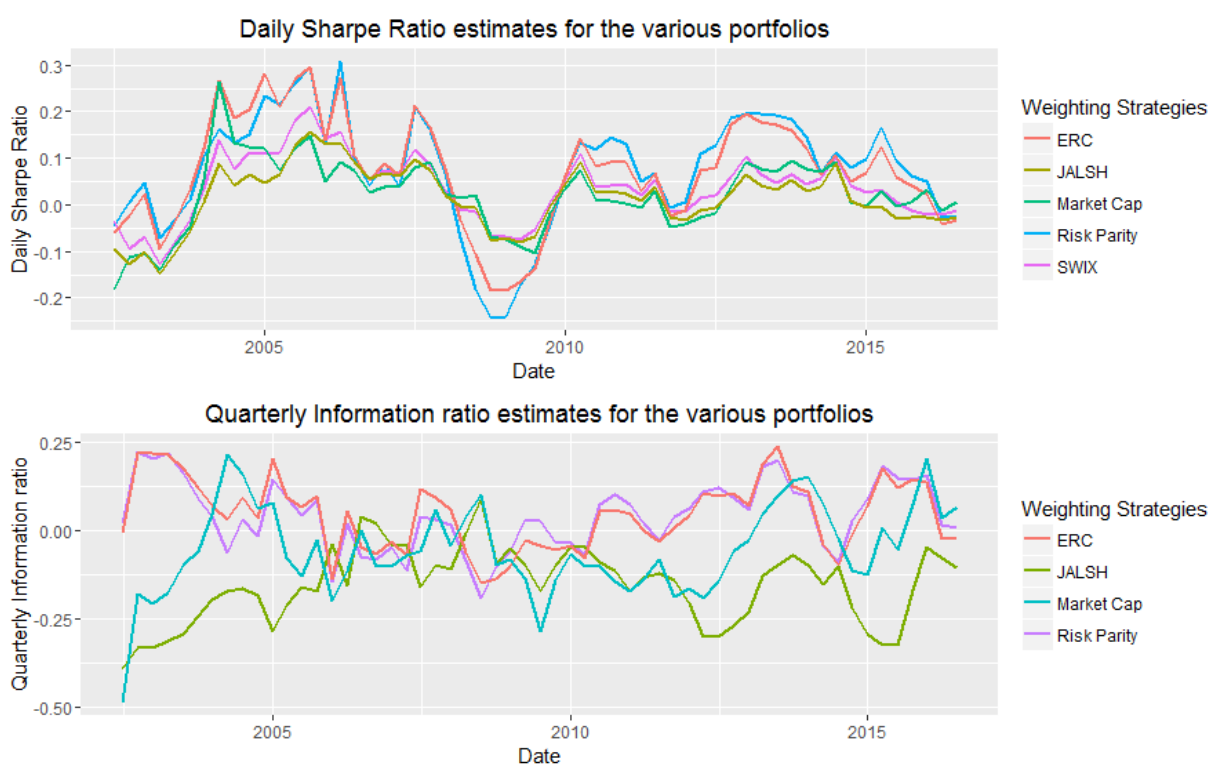
Figure 4.18: Daily Sharpe ratio estimates based on the momentum factor

In Figure 4.18, the Sharpe ratios for all the portfolios move together. If this graph is further investigated, it may be observed that when the Sharpe ratios are positive, the risk parity-weighted portfolios produce higher Sharpe ratios than the rest of the portfolios and when the Sharpe ratios are negative, the risk parity-weighted portfolios produce lower ratios than the rest of the portfolios.

Table 4.14: Information ratios for the momentum factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	0.1661329	0.1368612	-0.4942218

The information ratios for the momentum portfolios also showed positive values for the two smart beta portfolios.

4.3.4 Volatility
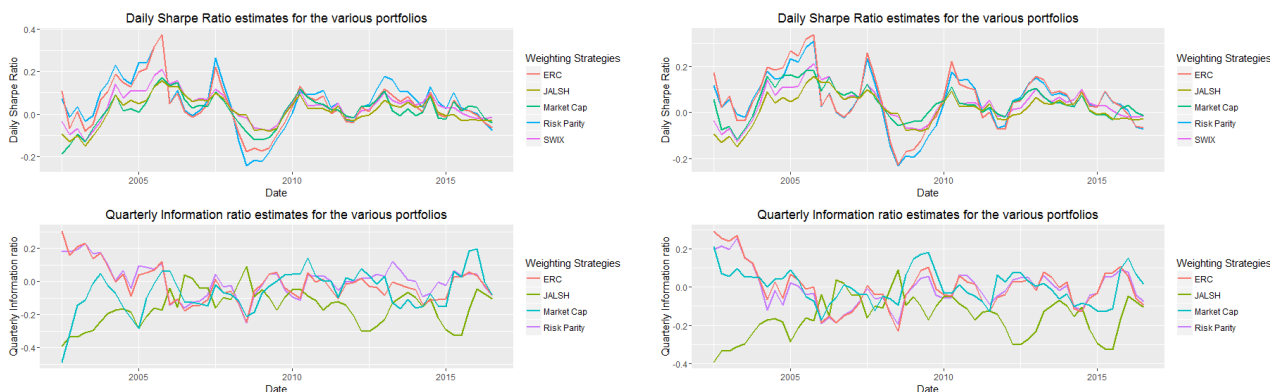
Figure 4.19: Daily Sharpe ratio estimates of both stock selection methods based on the volatility factor

In Figure 4.19, the average Sharpe ratios produced for the risk parity portfolios are slightly positive based on the performance for the investment horizon. It also seems that the risk parity portfolios produced higher Sharpe ratios than the two indices. The Sharpe ratios for the smart beta portfolio still seem to be the most volatile.

Table 4.15: Information ratios based on time series stock selection for the volatility factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	-0.1718838	-0.3468912	-0.6833739

Although the risk parity-weighted portfolio performed the best out of the three created portfolios when the information ratios were investigated, it still produced a negative ratio. The SWIX therefore outperformed the three created portfolios consistently.

Table 4.16: Information ratios based on cross-sectional stock selection for the volatility factor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	-0.2763786	-0.1947021	0.04115562

According to figures 4.7 and 4.8, the market cap-weighted portfolio outperformed the rest of the portfolios. This can also be seen from Table 4.16. The market cap-weighted portfolio produced a positive information ratio while the smart beta portfolios produced negative values. The ERC portfolio also produced better values than the risk parity-weighted portfolio.

4.3.5   Four-factor
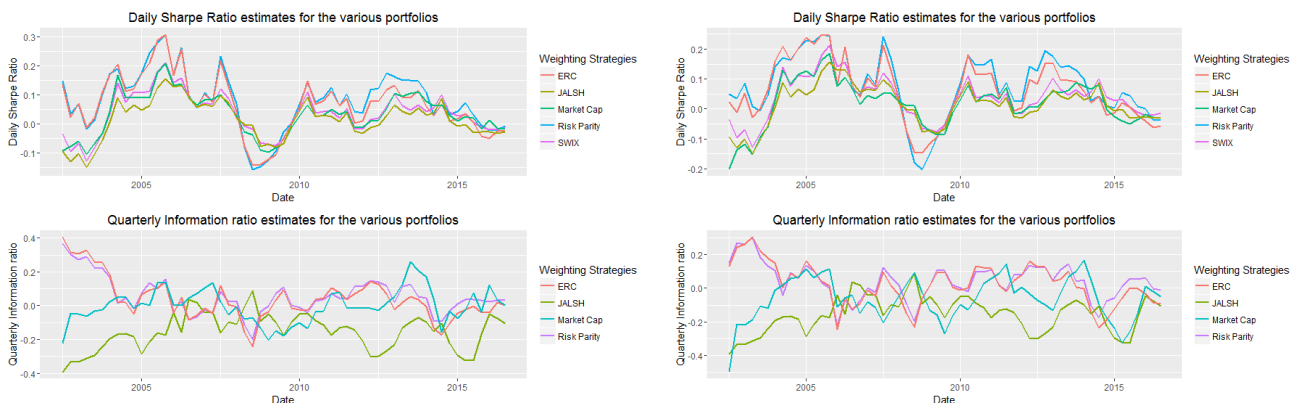
Figure 4.20: Daily Sharpe ratio estimates of both stock selection methods based on the multifactor portfolios

The Sharpe ratio for the risk parity-weighted portfolios was positive over most of part of the investment horizon except during the financial crisis of 2008 when it produced lower Sharpe ratios than the rest of the portfolios.

Table 4.17: Information ratios based on time series stock selection for the multifactor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	0.4299368	0.2228219	-0.1910707

The two smart beta portfolios produced relatively large ratios when compared to the various other factor portfolios and stock selection processes. The smart beta portfolios therefore outperformed the SWIX consistently while the market cap-weighted portfolio struggled.

Table 4.18: Information ratios based on cross-sectional stock selection for the multifactor portfolios

Information ratios	Risk parity	ERC	Market cap
SWIX	0.2964358	0.1125399	-0.6069389

The two smart beta portfolios once again outperformed the SWIX based on the information ratios. The positive ratios indicate that the two portfolios outperformed the SWIX consistently. The market cap-weighted portfolio produced a large negative ratio, as with the previous cases. Of all the created portfolios, the market cap-weighted portfolio produced a positive information ratio only once.

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4.4 GENERAL PERFORMANCE OF RISK PARITY-WEIGHTED PORTFOLIOS

Table 4.19: Performance of all the risk parity-weighted portfolios vs SWIX

	Risk Parity Weighted Portfolios										SWIX
	Value		Quality		Momentum		Volatility		Multi-Factor		
	Time Series	Cross Sectional	Time Series	Cross Sectional	Time Series	Cross Sectional	Time Series	Cross Sectional	Time Series	Cross Sectional	
Maximum Drawdown	0.3516	0.4305	0.3840	0.3647	0.3703	0.3453	0.3945	0.3515	0.3176	0.2798	0.2688
Historical VaR (95%)	-0.0982	-0.0939	-0.0804	-0.0814	-0.0986	-0.0885	-0.0757	-0.0796	-0.0835	-0.0598	-0.0839
Historical ES (95%)	-0.1272	-0.1507	-0.1242	-0.1203	-0.1302	-0.1342	-0.1372	-0.1137	-0.1286	-0.1103	-0.1367
Modified VaR (95%)	-0.0809	-0.1050	-0.0848	-0.0855	-0.0855	-0.0818	-0.0873	-0.0789	-0.0802	-0.0699	-0.0890
Modified ES (95%)	-0.1261	-0.1428	-0.1247	-0.1167	-0.1308	-0.1299	-0.1324	-0.1107	-0.1261	-0.1022	-0.1318
Annualized Return	0.1687	0.0938	0.1800	0.1339	0.1689	0.1690	0.1302	0.1188	0.1945	0.1829	0.1501
Annualized Std Dev	0.1405	0.1492	0.1540	0.1450	0.1575	0.1523	0.1462	0.1263	0.1499	0.1439	0.1445
Annualized Sharpe (Rf=6.75%)	0.6721	0.1555	0.6827	0.4231	0.6008	0.6217	0.3956	0.3722	0.7939	0.7508	0.5312

According to Table 4.19, the risk parity-weighted portfolio with the biggest maximum drawdown is the value factor portfolio of which the stock selection process was based on cross-sectional comparison. The maximum drawdown is 43.05%. This is, however, the quarterly maximum drawdown. The portfolio with the best maximum drawdown is the SWIX, with a value of 26.88%. This may be due to this portfolio's being more diversified when diversification is measured by the number of stocks held by index. Holding more stocks may smooth out the returns produced by the portfolio due to the weights allocated to the stocks being slightly less than with the created portfolios shown in Figure 4.10.

The portfolio with the highest estimated quarterly VaR was the momentum factor portfolio based on the time series stock selection process. Its quarterly VaR with p = 0.95 was estimated at 9.86%. It was expected that the momentum factor portfolio might produce the highest VaR estimates due to this factor's normally producing portfolio returns that are highly volatile. The portfolio that did best based on the VaR estimates was the four-factor portfolio based on the cross-sectional stock selection process, producing a quarterly VaR estimate of 5.98%.

Analysis of the annualised return showed that the best performing portfolio was the four-factor portfolio with time series comparison stock selection producing an annualised return of 19.45% over the investment horizon. The SWIX produced an annualised return of 15.01%, and the worst performing portfolio produced an annualised return of 9.38% that belonged to the value factor portfolios based on cross-sectional stock selection.

Lastly, investigation of the Sharpe ratios of the various portfolios showed that the four-factor portfolio, with time series stock selection, produced the highest annualised Sharpe ratio of 0.7939 (0.5312 for the SWIX) and the value factor portfolios, based on cross-sectional stock selection, produced the lowest annualised Sharpe ratio of 0.1555.

4.5   MACROECONOMIC REGRESSION MODELS

In this section, the macroeconomic exposure for each created portfolio is illustrated and further information regarding the regression model is explained.

4.5.1   Average R-squared
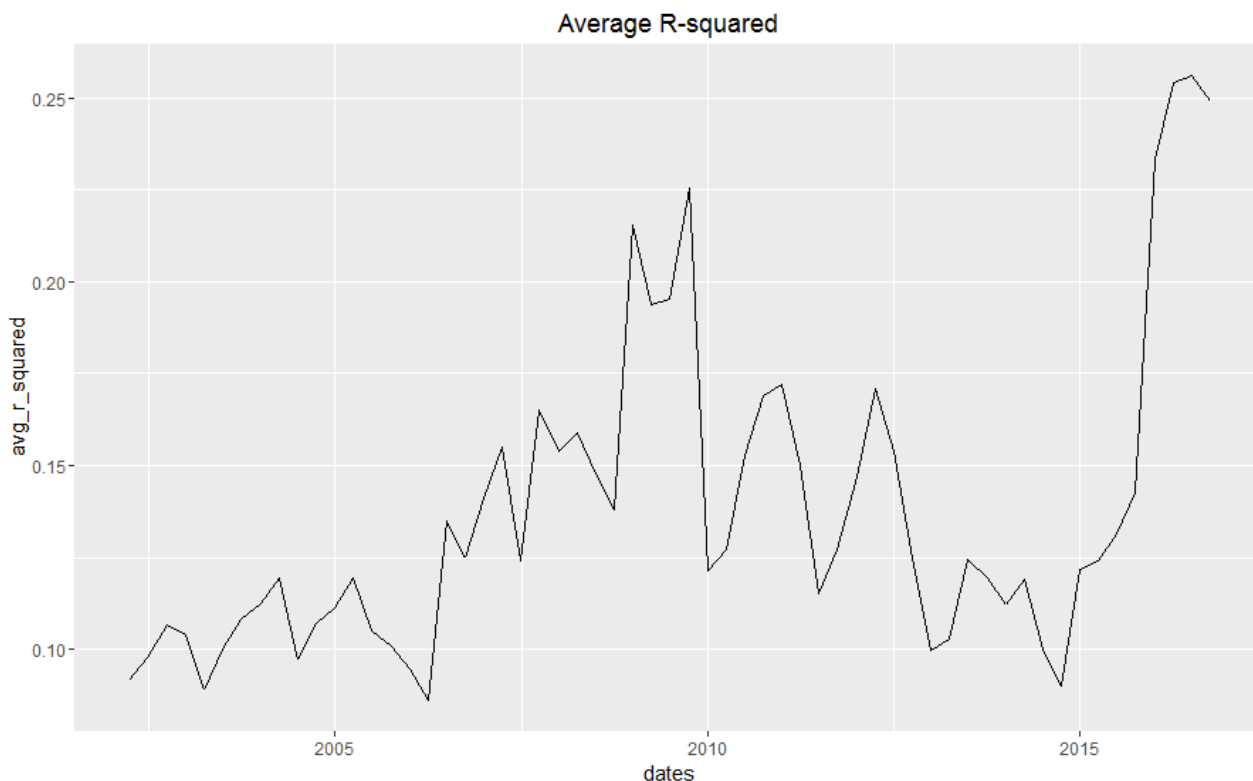
Figure 4.21: Average r-squared values for the macroeconomic regression model

As stated earlier, the macroeconomic exposure of the various portfolios had not been investigated when the smart beta portfolios were set up. To see what type of exposure the different portfolios had to macroeconomic factors, the researcher first had to look at the r-squared value that was produced when the macroeconomic model as stated in formula (3.37) was applied to each stock in the universe. The macroeconomic variables that were used were the weekly oil price, gold price, ZAR/USD currency rate and swap rates (interest rates). These were firstly transformed so that the correlation among the variables was removed using Montagu et al., (2015) as explained in Section 3.8. The r-squared value is a goodness-of-fit measure for fitting linear regression models. Its value indicates the percentage of the variance in the dependant variable that the independent variables explain collectively.

The average r-squared value, in this case, was the average of all the r-squared values obtained, at a specific date, of all the regression models applied to all the stocks in this investment universe. This means that the macroeconomic regression model was fitted to all the stocks on a specific date and

an r-squared value was obtained for each stock. The average r-squared value was then taken across all the stocks on a specific date. The results are shown in Figure 4.21.

In the years leading to the global financial crisis, the average r-squared value increased significantly to 22%. During this time, the global macroeconomic factors had a major impact on the South African market. The crisis mainly occurred in the USA, with some of its major banks being bailed out of a debt crisis, and this led to the rest of the world's also falling into a financial crisis. The average r-squared value was now even at higher levels than those leading to the great financial crisis. Although 15%, the average across the investment horizon, is a low r-squared value to have in a regression model, the reader should remember that only macroeconomic factors are included in this model and these alone do not drive financial markets. If market factors (value, quality, momentum and volatility) are also included in this regression model, a higher r-squared value may be obtained.

In Figure 4.22, the correlations among the various macroeconomic factors are illustrated in different
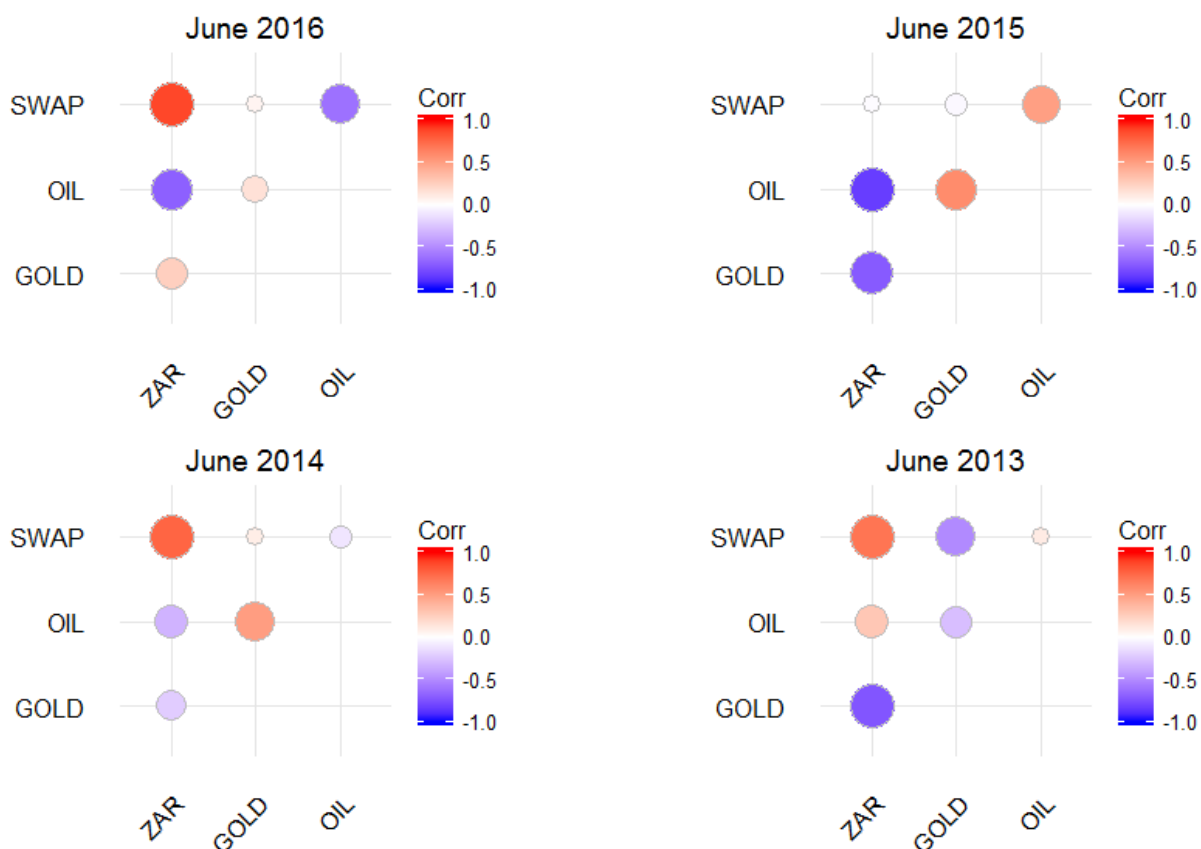
Figure 4.22: Correlation among macroeconomic variables for selected dates

time periods. The legend on the right of each figure gives an indication of the correlation between two factors. The bigger the circle, the bigger the correlation. A red circle is an indication of a positive

correlation between two factors, and a blue circle is an indication of a negative correlation between two factors.

As stated earlier in this section, before the regression model is implemented, the correlations among the variables should first be removed. This is important because as shown in Figure 6.2, there are significant correlations among some of the macroeconomic variables.

4.5.2   MacroEconomic exposure

In this section, the macroeconomic exposure of the various risk parity portfolios is illustrated for both stock selection methods. The four sets of graphs at the top of each figure give an indication of the beta between the risk parity portfolio, based on time series comparison, and each specific macroeconomic factor across the investment horizon. The four sets of graphs at the bottom of each figure illustrate the beta between the risk parity portfolio, based on cross-sectional comparison, and each specific macroeconomic factor across the investment horizon.

4.5.2.1   Value portfolio
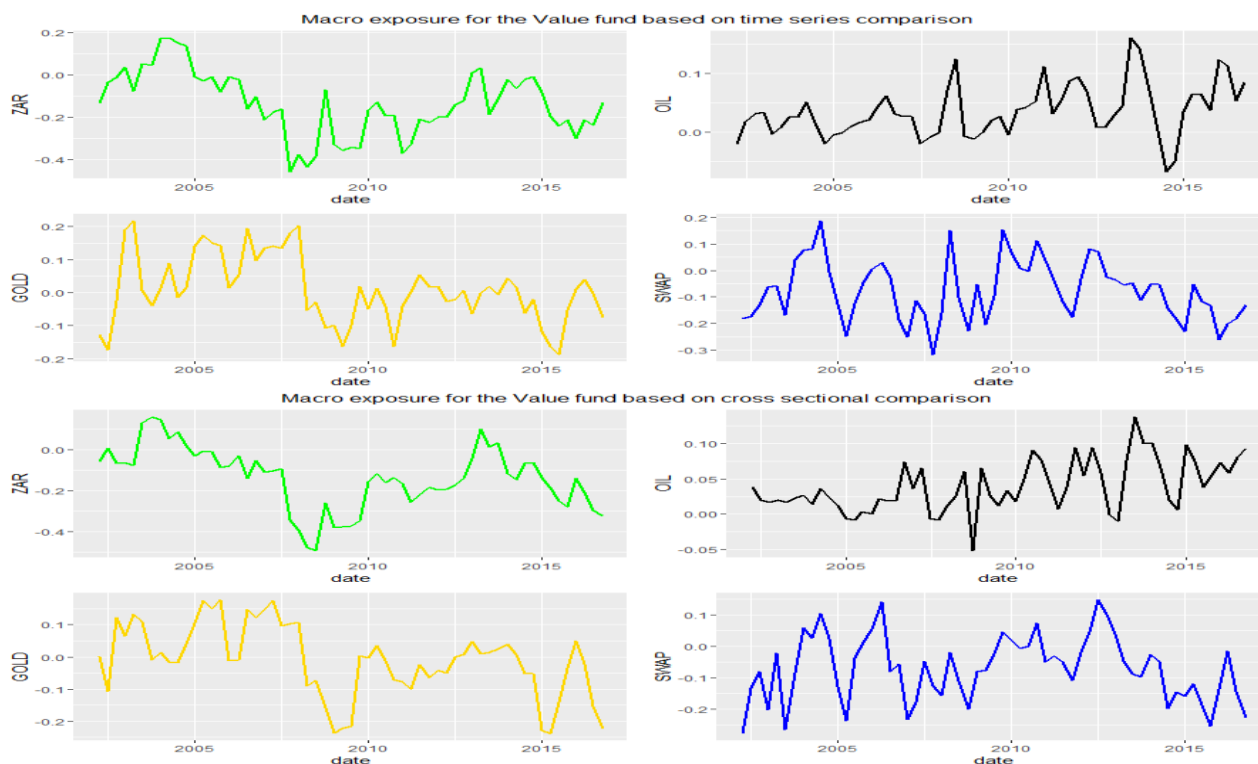
Figure 4.23: Macroeconomic exposure for the value portfolios

As seen in Figure 4.23, the exposures to the various macroeconomic factors of the value portfolios is quite volatile. There is no real trend in the data. When the USD/ZAR currency rate is regressed on the value portfolios, it looks as if the average is slightly negative. This means that when the USD/ZAR rate increases (weakening of the ZAR), the value of the portfolio decreases. This means that stocks

included in this portfolio are most probably South African incorporated stocks (most of their income is generated in South Africa). When one investigates the oil exposure, it seems that there is a positive relationship. Therefore, when oil prices increase, the value of the portfolio will increase. The swap and gold exposures are more centralised around zero; there is thus no significant relationship among the variables.

4.5.2.2 *Quality portfolio*
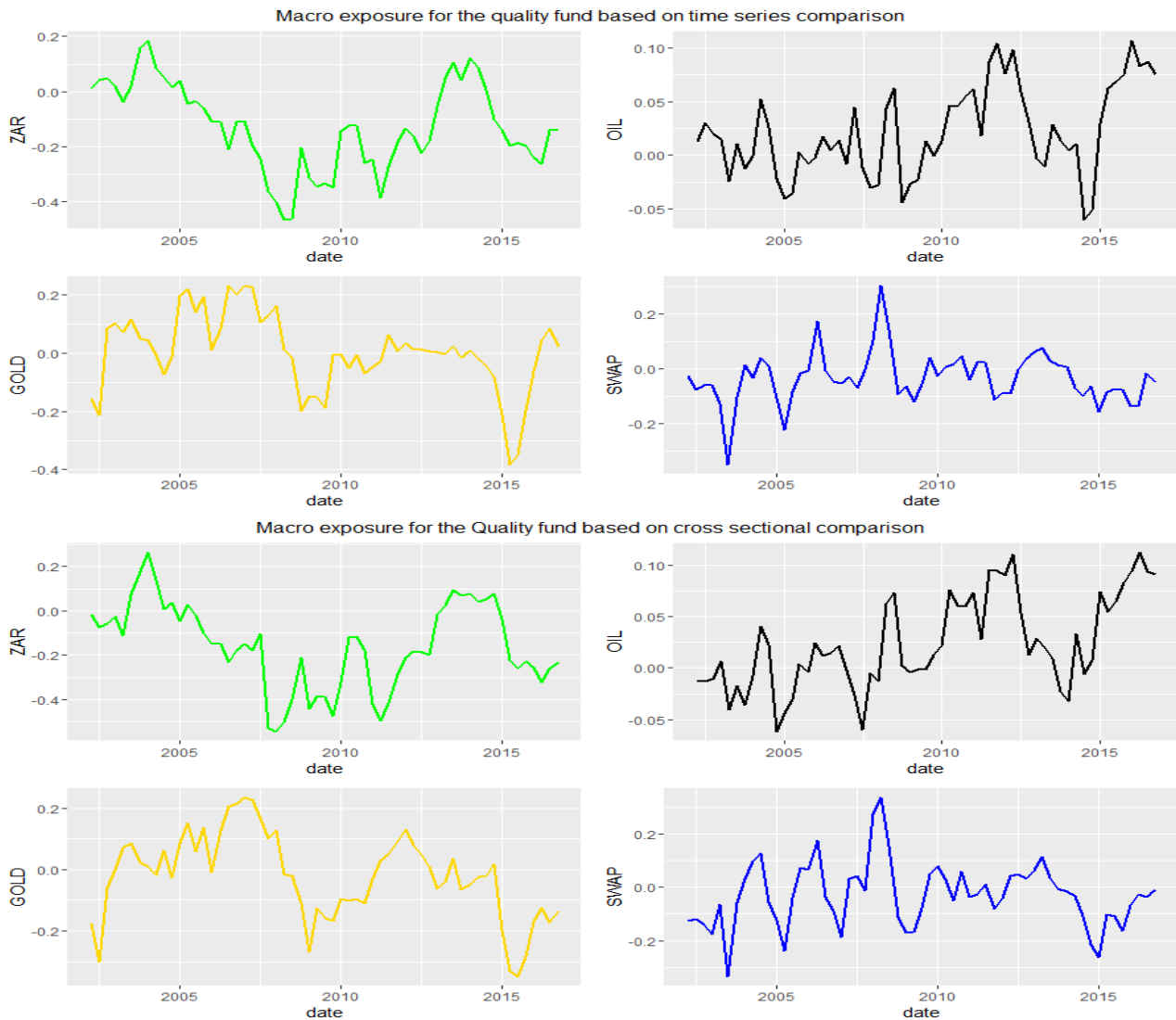
Figure 4.24: Macroeconomic exposure for the quality portfolios

The same results are shown in Figure 4.24 as in Figure 4.23, although in Figure 4.24 it seems as if there may be a more negative relationship between the change in gold prices and the change in value of the quality portfolio.

60

4.5.2.3   Momentum portfolio
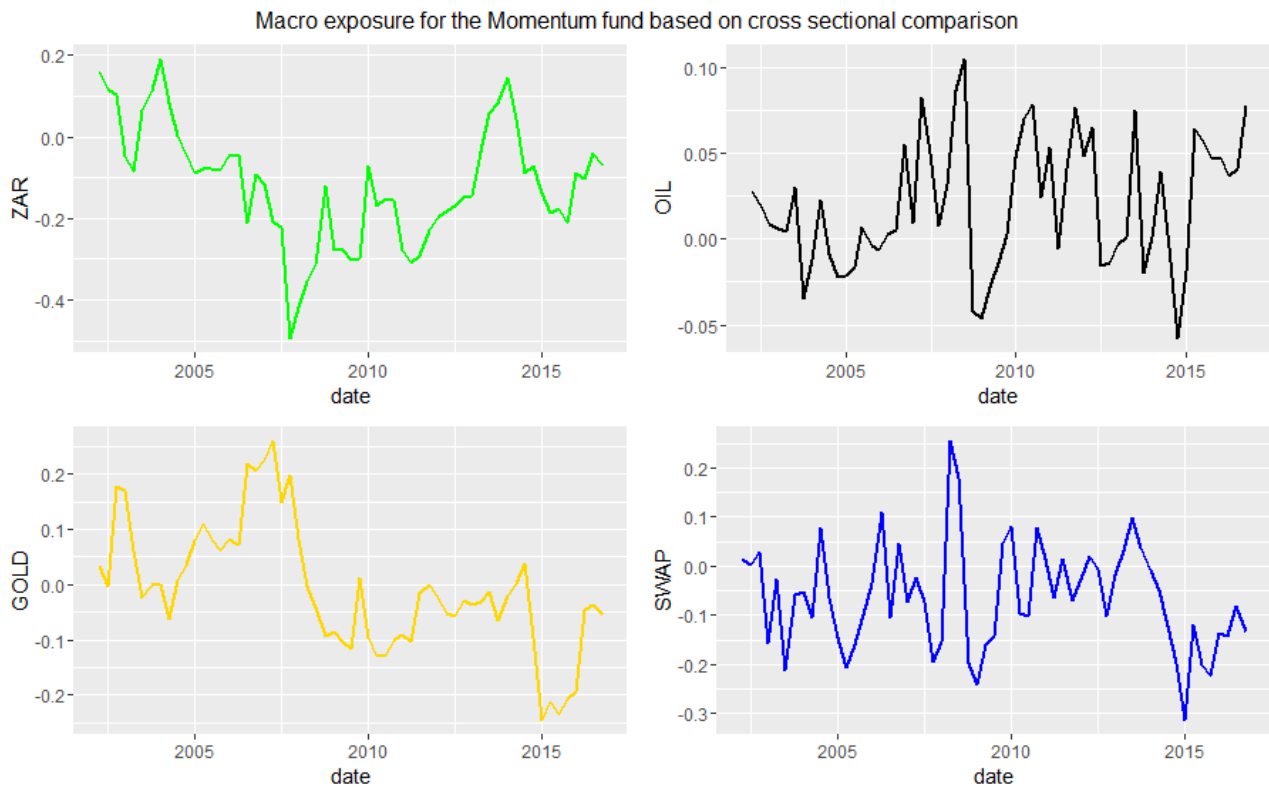
Figure 4.25: Macroeconomic exposure for the momentum portfolios

Once again, the same relationship between the USD/ZAR rate and the oil price is observed as in Figure 4.23. In this case, there is a slight negative relationship between the change in swap rates and the change in value of the momentum portfolio. This means that when swap rates increase, the value of the momentum portfolio will decrease.

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4.5.2.4 Volatility portfolio
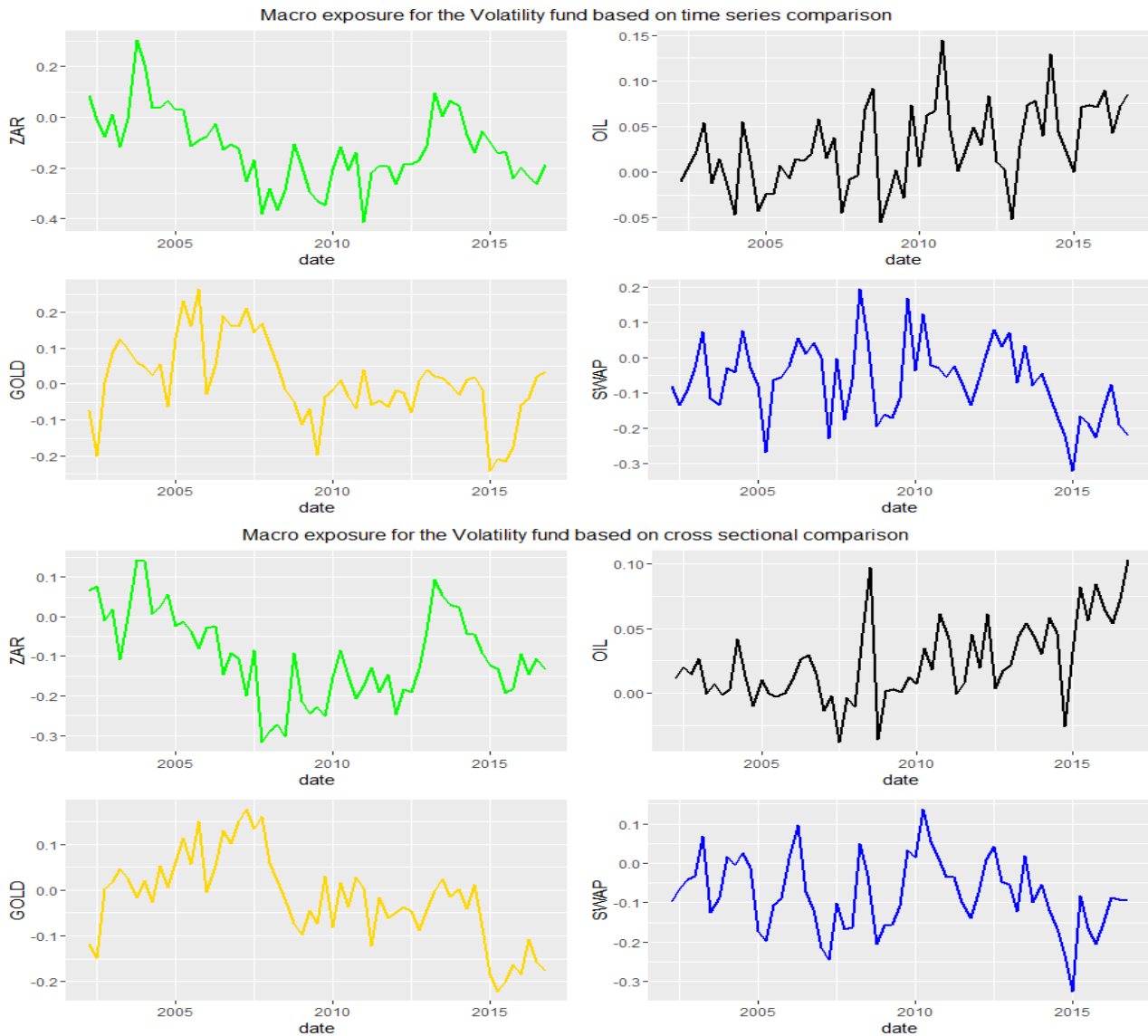
Figure 4.26: Macroeconomic exposure for the volatility portfolios

The relationship between the USD/ZAR rate and the portfolio value is once again slightly negative and slightly positive with the change in oil prices. There is also a slight negative relationship between the change in gold prices and the change in the value of the volatility portfolio.

4.5.2.5   Four-factor portfolio
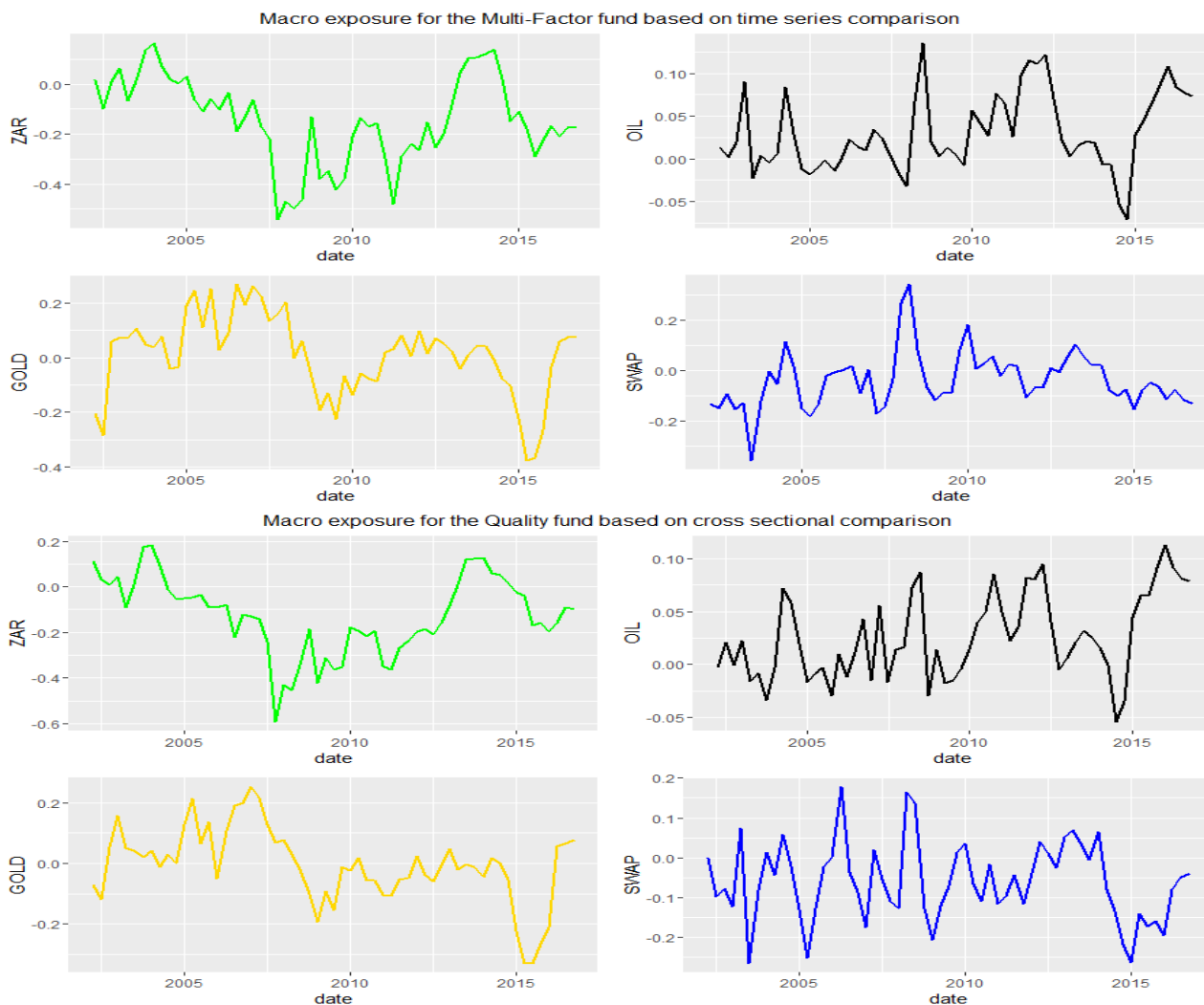
Figure 4.27: Macroeconomic exposure for the four-factor portfolios

The same results hold for the multifactor portfolios as for all the previous models.

The macroeconomic exposure part of this study was done to determine whether there was a significant exposure to macroeconomic factors. In Figure 4.21, it was seen that the average of the r-squared values across the time horizon was about 15%. This means that 15% of the variability in stock prices in the stock universe of this study is explained by the macroeconomic model illustrated in equation (3.37). Factors that can be added to this model is the value, quality, momentum and volatility factors; this may improve the average r-squared value of the current model. As seen in Figures 4.23 to 4.27, the exposure of the various portfolios to these individual factors only slightly differs from zero and no clear conclusion can be drawn with regard to this. An improvement may be to add better macroeconomic factors such as GDP values of South Africa and the USA.

5   SUMMARY, CONCLUSION AND RECOMMENDATIONS

Smart beta portfolios are still a relatively new investment strategy. This strategy is, however, slowly but surely becoming more popular in the asset management industry worldwide. In South Africa, only a handful of investment houses have started with this strategy.

Given the results presented in Chapter 4, it is clear that investors should consider a strategy like smart beta. A clear improvement can be seen between market cap-weighted portfolios and smart beta-weighted portfolios when performances are compared based on the same stock selection processes and factor tilts. The market cap-weighted portfolios outperformed the rest of the created portfolios only once and that was when the portfolios were constructed using low-volatility stocks.

As stated earlier in this thesis, a market cap-weighting strategy can create a drag on portfolio returns. This became evident when the cumulative returns were compared among the various portfolios. Not only did the performance improve when a smart beta weighting strategy was used but the risk-adjusted performance also improved. The risks of the portfolios also decreased. The risks were measured in maximum drawdowns and VaR estimates.

The portfolios that consistently produced better returns at lower risks were the four-factor smart beta portfolios (risk parity and equal risk contribution weighted). Both smart beta weighting strategies produced higher Sharpe ratios, higher information ratios and annualised returns independent of which stock selection process was used. The risks of both these portfolios were also an improvement on those of the market cap-weighted portfolio and the two benchmarks.

The stock selection process that produced the best results was the time series strategy. A reason for this improvement may be that when cross-sectional stock selection is used, a specific sector may have better fundamental factors in general and an investor may end up with a stock selection that is extremely concentrated in a specific sector. For example, the resources sector generally has lower price-to-earnings ratios than the rest of the market. A way around this problem is to standardise the fundamental value of a stock relative to itself and then compare the standardised values across the whole universe. This was explained earlier in the thesis when the calculation of the z-scores for all the stocks was presented in Section 3.2.

An improvement that can be made to the various factor tilt portfolios is that more than one fundamental factor can be used. For example, when volatility factor portfolios are set up, the investor may select the stocks based on their 60-day volatility as well as 260-day volatility. The four-factor portfolio combined the four factors used in the individual factor portfolios, and this led to a significant improvement in the returns and the risks of the portfolio.

Another improvement occurred when the ERC weighting strategy was implemented over the market cap-weighted strategy, and the risk parity-weighted portfolio even improved on the ERC method. Therefore, more thought can be put into the different smart beta-weighting strategies. The problem with this is that more optimisation would need to be done and this may become mathematically challenging. With the risk parity-weighted strategy, the objective was to minimise the standard deviation of the portfolio with certain constraints (maximum weight of 15% in a specific stock, minimum weight 0% and the sum of the weights should be 100%). A different risk measure can also be tested, for instance the expected shortfall, VaR or maximum drawdown. Another measure that an investor may implement is to set the objective to maximise the Sharpe ratio of the portfolio. As stated earlier in formula (3.31) for the Sharpe ratio, the expected return for all the selected stocks are needed to calculate this ratio. The problem with maximising this ratio is that there is a measure of estimation risk involved when the expected return is calculated.

If the Smart Beta strategy is to be implemented in the investment world, there are some factors that will compress the overall returns of the portfolio. Firstly, the trading costs will lead to a drag on the portfolio returns but as stated earlier in this thesis, the Smart Beta investment strategy is considered to be the new active strategy. In this study, rebalancing was done quarterly, just as with the major indices listed on the JSE board. Rebalancing quarterly will automatically reduce the trading costs when comparing this portfolio to an already existing actively managed portfolio.

Another factor that needs to be kept in mind is the tax implications when rebalancing portfolios. On each rebalancing date, the entire stock selection process was redone and new weights were allocated to the selected stocks. There may be times when the entire shareholding in a stock must be sold to buy into another stock.

Future research regarding this topic can also include the turnover of the portfolio to see more precisely what percentage of the portfolio is sold and bought throughout a year. If this value is too high, it will lead to significant trading costs and a great deal of portfolio returns will become diminished.

In Chapter 4.5, the macroeconomic factors and the risk parity portfolio's exposure to these factors are illustrated. It was found that the macroeconomic factors included in this study produced an average r-squared statistic of 15% across the investment time horizon. This means that relevant macroeconomic factors should be considered. Future research can also include market factors (value, quality, momentum and volatility) in the regression model. This should increase the r-squared value of the overall model.

In summary, the smart beta weighting strategy clearly made an improvement to the overall performance of the portfolios. The stock selection process can also have a significant effect on the

overall outcome of the portfolios. Further research may be done regarding stock selection techniques. It was also found that combining the risk factors or including more fundamental factors in a specific model could improve the performance of the portfolios.

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Appendix A

##Installing packages to extract data from Bloomberg and to clean the data.

```
install.packages(c("dplyr","tidyr","purrr","lubridate","Rblpapi"))
library(dplyr)
library(tidyr)
library(purrr)
library(lubridate)
library(Rblpapi)
```

##Set up a connection with Bloomberg.

```
con <- blpConnect()
```

An equity screen named AllData was done to extract all the stocks listed on the JSE All Share Index; it also looked at the relevant fundamental factors of the stocks. This was also done as a test to see whether I could do an equity screen from inside R.

```
# This is done, just to get a base of the tickers in order to extract daily, weekly
monthly etc. data and to get the corresponding rebalancing dates
eqs15 <- beqs(screenName = "AllData",date = ymd("20150101"))
TickerNames15 <- eqs15$Ticker
```

##Closing prices extraction from Bloomberg. Different periodic sample lengths were created. These were used in a tibble, and a function was mapped over all the different periodic samples, extracting periodic closing prices from Bloomberg.

```
monthly <- c("periodicitySelection"="MONTHLY",
            "periodicityAdjustment"="CALENDAR")
quarterly <- c("periodicitySelection"="QUARTERLY",
              "periodicityAdjustment"="CALENDAR")
halfyearly <- c("periodicitySelection"="SEMI_ANNUALLY",
               "periodicityAdjustment"="CALENDAR")
yearly <- c("periodicitySelection"="YEARLY",
           "periodicityAdjustment"="CALENDAR")
daily <- c("periodicitySelection"="DAILY",
          "periodicityAdjustment"="CALENDAR")
weekly <- c("periodicitySelection"="WEEKLY",
           "periodicityAdjustment"="CALENDAR")
close_prices <- tibble(selection =
list(monthly,quarterly,halfyearly,yearly,daily,weekly),
```

```
        data = map(selection,function(selection){
          bdh(securities = as.character(TickerNames15),
              fields = "PX_LAST",
              start.date=ymd("2002-01-01"),
              options=selection)
        }))
```
Importing data for cross-sectional selection

```
dates <- close_prices$data[[2]]$`SOL SJ Equity`$date
cross_sec_Fund_data <- tibble(date = dates)

eqs1 <- function(df){
  beqs(screenName = "AllData",date = ymd(df))
}

cross_sec_Fund_data <- cross_sec_Fund_data %>%
  mutate(Fundamental = map(date,eqs1))

## Need to extract all of the Tickers now, using all of the equity screen results and
run the close prices function again, the one in the previous chunk.
unlisted_data <- cross_sec_Fund_data %>% unnest(Fundamental)
TickerNames <- unlisted_data$Ticker %>% unique()

close_prices <- tibble(selection =
list(monthly,quarterly,halfyearly,yearly,daily,weekly),
        data = map(selection,function(selection){
          bdh(securities = as.character(TickerNames),
              fields = "PX_LAST",
              start.date=ymd("1999-01-01"),
              options=selection)
        }))
## Should use 3 years' data for RP weight calculation
```

##Extracting daily fundamental data for each stock. A tibble was created with a character
vector with all the fundamental factors' names. A function was written that would extract
daily data for each of these fundamentals in the tibble with the map function in R.

```
fields <- tibble(fields = c("CUR_MKT_CAP",
                            "PX_LAST",
                            "PX_TO_BOOK_RATIO",
                            "RETURN_COM_EQY",
                            "EQY_DVD_YLD_12M",
                            "GROSS_PROFIT",
                            "EARN_YLD_HIST",
                            "VOLATILITY_60D",
                            "VOLATILITY_260D",
                            "PX_VOLUME"))

extract_daily <- function(df){
  bdh(securities = as.character(TickerNames),
      fields = df,
      start.date = ymd("20000101"),
      options = daily)
}
```

```
Fundamental_Data <- fields %>%
  mutate(Fund_Data = map(fields,extract_daily))
```
Daily and weekly data of macroeconomic data

```
OIL_daily <- bdh("CL1 Comdty",
          fields = "PX_LAST",
          start.date = ymd("20000101"))
ZAR_daily <- bdh("ZAR Curncy",
          fields = "PX_LAST",
          start.date = ymd("20000101"))
GOLD_daily <- bdh("GC1 Comdty",
          fields = "PX_LAST",
          start.date = ymd("20000101"))
SWAP_daily <- bdh("SASW10 Curncy",
          fields = "PX_LAST",
          start.date = ymd("20000101"))
SA_Bond_daily <- bdh("GSAB10YR Index",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = daily)


OIL_weekly <- bdh("CL1 Comdty",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = weekly)
ZAR_weekly <- bdh("ZAR Curncy",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = weekly)
GOLD_weekly <- bdh("GC1 Comdty",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = weekly)
SWAP_weekly <- bdh("SASW10 Curncy",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = weekly)
GDP_weekly <- bdh("SACGDP Index",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = weekly)
SA_Bond_weekly <- bdh("GSAB10YR Index",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = weekly)
```
benchmark performances

```
# The JASLH and SWIX are extracted now, further attention will be given to the correct
benchmark.
JALSH <- bdh("JALSH Index",
          fields = "PX_LAST",
          start.date = ymd("20000101"),
          options = daily)
SWIX <- bdh("JSHRALTR Index",
          fields = "PX_LAST",
```

```
        start.date = ymd("20000101"),
        options = daily)
```

Necessary packages

```
library(broom)
library(modelr)
library(ggplot2)
library(stringr)
library(dplyr)
library(tidyr)
library(purrr)
library(lubridate)
library(tibble)
```

Selecting stocks for value model

```
stock_select <- function(df){
  df <- df %>% filter(!is.na(Market.Cap)) %>%
    filter(Market.Cap > quantile(Market.Cap,0.05)) %>%
    filter(!is.na(Sequential.ZScore.Current.Earnings.Yield.)) %>%
    filter(Sequential.ZScore.Current.Earnings.Yield. >
            quantile(Sequential.ZScore.Current.Earnings.Yield.,0.65)) %>%
    select(c(Ticker,GICS.Sector,Market.Cap,Earnings.Yield)) %>%
    as_tibble()
}

cross_sec_Fund_data_VALUE <- cross_sec_Fund_data %>% mutate(stocks =
map(Fundamental,stock_select))
```

Selecting stocks for volatility model

```
stock_select <- function(df){
  df <- df %>% filter(!is.na(Market.Cap)) %>%
    filter(Market.Cap > quantile(Market.Cap,0.05)) %>%
    filter(!is.na(Sequential.ZScore.60.Day.Annualized.Volatility.)) %>%
    filter(Sequential.ZScore.60.Day.Annualized.Volatility. <
            quantile(Sequential.ZScore.60.Day.Annualized.Volatility.,0.3)) %>%
    select(c(Ticker,GICS.Sector,Market.Cap,Volat.D.60)) %>%
    as_tibble()
}

cross_sec_Fund_data_VOL <- cross_sec_Fund_data %>% mutate(stocks =
map(Fundamental,stock_select))
```

Selecting stocks for quality model

```
stock_select <- function(df){
  df <- df %>% filter(!is.na(Market.Cap)) %>%
    filter(Market.Cap > quantile(Market.Cap,0.05)) %>%
    filter(!is.na(Sequential.ZScore.LF.Gross.Profit.)) %>%
    filter(Sequential.ZScore.LF.Gross.Profit. >
            quantile(Sequential.ZScore.LF.Gross.Profit.,0.35)) %>%
    select(c(Ticker,GICS.Sector,Market.Cap,Gross.Profit.LF)) %>%
    as_tibble()
}
```

```
cross_sec_Fund_data_Quality <- cross_sec_Fund_data %>% mutate(stocks =
map(Fundamental,stock_select))
```

Selecting stocks for momentum model

```
stock_select <- function(df){
  df <- df %>% filter(!is.na(Market.Cap)) %>%
    filter(Market.Cap > quantile(Market.Cap,0.05)) %>%
    filter(!is.na(Sequential.ZScore.1.Year.Total.Return.)) %>%
    filter(Sequential.ZScore.1.Year.Total.Return. >
            quantile(Sequential.ZScore.1.Year.Total.Return.,0.7)) %>%
    select(c(Ticker,GICS.Sector,Market.Cap,Total.Return.Y.1)) %>%
    as_tibble()
}

cross_sec_Fund_data_MOM <- cross_sec_Fund_data %>% mutate(stocks =
map(Fundamental,stock_select))
```

Selecting stocks for multifactor model

```
stock_select <- function(df){
  df <- df %>% filter(!is.na(Market.Cap)) %>%
    filter(Market.Cap > quantile(Market.Cap,0.05)) %>%
    filter(!is.na(Sequential.ZScore.Current.Earnings.Yield.) &
            !is.na(Sequential.ZScore.1.Year.Total.Return.) &
            !is.na(Sequential.ZScore.LF.Gross.Profit.) &
            !is.na(Sequential.ZScore.60.Day.Annualized.Volatility.)) %>%
    mutate(Z_ALL = Sequential.ZScore.Current.Earnings.Yield. +
Sequential.ZScore.1.Year.Total.Return. +
            Sequential.ZScore.LF.Gross.Profit. -
Sequential.ZScore.60.Day.Annualized.Volatility.) %>%
    filter(Z_ALL >
            quantile(Z_ALL,0.4)) %>%

select(c(Ticker,Short.Name,GICS.Sector,Market.Cap,Earnings.Yield,Total.Return.Y.1,Gross
.Profit.LF,Volat.D.60)) %>%
    as_tibble()
}

cross_sec_Fund_data_4Factor <- cross_sec_Fund_data %>% mutate(stocks =
map(Fundamental,stock_select))
```

Necessary packages

```
library(quantmod)
library(broom)
library(modelr)
library(ggplot2)
library(stringr)
library(dplyr)
library(tidyr)
library(purrr)
library(lubridate)
```

Calculating weekly log returns for all the shares and macro factors

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This was done for all the shares so that with the stock selection process, I could just filter the necessary stocks and their exposure for a given rebalancing date.

```
## exclude NA's due to OIL and GOLD being american data, some dates may be missing
options(na.action = na.exclude)
## daily per share date for fitting macro model for each share
per_share <- close_prices$data[[6]] %>%
  bind_rows(.id="Ticker") %>%
  group_by(Ticker) %>%
  nest()

## Calculates Daily log Return and put into Tibble
log_returns <- function(df){
  close <- df$"PX_LAST"
  date <- filter(df,date>date[1]) %>% select(date)
  ret <- tibble(date = date$date, log_ret = diff(log(close)))
}
## Calculate macroeconomic changes
ZAR_change <- log_returns(ZAR_weekly) %>% dplyr::rename(ZAR_Change = log_ret)
Oil_change <- log_returns(OIL_weekly) %>% dplyr::rename(OIL_Change = log_ret)
GOLD_change <- log_returns(GOLD_weekly) %>% dplyr::rename(GOLD_Change = log_ret)
SWAP_change <- log_returns(SWAP_weekly) %>% dplyr::rename(SWAP_Change = log_ret)
############## Need to get data for 10y bond and also CPI

### This function is created twice due to the macro factors added in the second one.
log_returns <- function(df){
  close <- df$"PX_LAST"
  date <- filter(df,date>date[1]) %>% select(date)
  ret <- tibble(date = date$date, log_ret = diff(log(close))) %>%
    merge(ZAR_change) %>%
    merge(Oil_change) %>%
    merge(GOLD_change) %>%
    merge(SWAP_change)
}

## add weekly log returns to the weekly per_share data and also the weekly the change
in macro factors
## The macro factors are added for each share, it is easier to do a regression that
way.
## i know that the dates match
per_share <- per_share %>% mutate(ret = map(data,log_returns))
```

Grouping the data according to the rebalancing dates

A tibble with the rebalancing dates was created at first. Yearly data for each stock, up to a rebalancing date, were put into the next column as a list of tibbles.

```
dates <- close_prices$data[[2]] %>%
  bind_rows(.id="Ticker") %>%
  group_by(date) %>%
  nest() %>%
  filter(date >= date[13]) %>%
```

```
    select(date)

# Creating a tibble with the rebalancing dates
macro_per_share <- tibble(dates = dates$date)

# Getting the data for each stock for the past year for the corresponding date
collect_data <- function(df){
  per_share %>%
    unnest(ret) %>%
    filter(date <= df,date >= df - years(1)) %>%
    group_by(Ticker) %>%
    nest()
}

macro_per_share <- macro_per_share %>%
  mutate(data = map(dates,collect_data))
```

Calculating the macro exposure of each stock using Gruber

This was based on a time series regression over the past 52 weeks. Four linear regression
functions were written to make the macroeconomic factors uncorrelated with each other.

```
macro_exposure <- function(df){
  macro_model <- function(df){
    lm(OIL_Change ~ ZAR_Change, data = df)
  }

  macro1 <- df %>% mutate(model = map(data,macro_model))
  macro1 <- macro1 %>% mutate(resids = map2(data,model,add_residuals))

  macro1 <- macro1 %>% unnest(resids) %>% dplyr::rename(OIL = resid) %>%
group_by(Ticker) %>% nest()


  macro_model <- function(df){
    lm(GOLD_Change ~ ZAR_Change + OIL, data = df)
  }
  macro2 <- macro1 %>% mutate(model = map(data,macro_model))
  macro2 <- macro2 %>% mutate(resids = map2(data,model,add_residuals))

  macro2 <- macro2 %>% unnest(resids) %>% dplyr::rename(GOLD = resid) %>%
group_by(Ticker) %>% nest()

  macro_model <- function(df){
    lm(SWAP_Change ~ ZAR_Change + OIL + GOLD, data = df)
  }
  macro3 <- macro2 %>% mutate(model = map(data,macro_model))
  macro3 <- macro3 %>% mutate(resids = map2(data,model,add_residuals))

  macro3 <- macro3 %>% unnest(resids) %>% dplyr::rename(SWAP = resid) %>%
group_by(Ticker) %>% nest()


  macro_model <- function(df){
```

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```
    lm(log_ret ~ ZAR_Change + OIL + GOLD + SWAP , data = df)
  }

  macro_final <- macro3 %>% mutate(model = map(data,macro_model))
  macro_final <- macro_final %>% mutate(resids = map2(data,model,add_residuals))


}
# This function of linear regression functions was mapped over all the rebalancing
dates and over all the stocks, given their yearly data up to that specific date
exposure_per_reb_date_gruber <- macro_per_share %>% mutate (exposure =
map(data,macro_exposure))
```

Necessary packages

```
library(broom)
library(modelr)
library(ggplot2)
library(stringr)
library(dplyr)
library(tidyr)
library(purrr)
library(lubridate)
library(tibble)
library(plyr)
#library(FRAPO)

p1 <- cross_sec_Fund_data_4Factor$stocks[[59]] %>% ggplot() +
geom_bar(aes(GICS.Sector,fill = GICS.Sector),show.legend = F) + theme(aspect.ratio=1) +
coord_polar()
p2 <-cross_sec_Fund_data_4Factor$stocks[[59]] %>% ggplot() +
geom_bar(aes(GICS.Sector,fill = GICS.Sector),show.legend = F) + theme(aspect.ratio=1) +
coord_flip()
grid.arrange(p1,p2,ncol=2)
```

In this file, I wanted to calculate the weights assigned to each stock that had been selected by the corresponding fundamental factor. I used the risk parity approach. At the end of the stock selection process, I was left with a tibble with the corresponding quarterly rebalancing

dates in a vector and then a list of tibbles with the selected stocks corresponding to the dates.

Merging the selected stocks with their daily log returns

```
# close_price row number five consist out of the daily closing prices
# log returns are calculated using this function
log_returns <- function(df){
close <- df$"PX_LAST"
date <- dplyr::filter(df,date>date[1]) %>% select(date)
ret <- tibble(date = date$date, log_ret = diff(log(close)))
}

# nesting the daily prices for each stock.
logreturns <- close_prices$data[[5]] %>%
  bind_rows(.id = "Ticker") %>%
  group_by(Ticker) %>%
  nest() %>%
  mutate(logret = map(data,log_returns))  # Mapping the log returns function over each
stock's daily close prices


## This function takes the stocks list and dates from a specific model into account in
collecting the daily return data for each selected stock for the past year.
# The reason for doing this, is to calculate the risk parity weights, calculating
variance/covariance matrix.
Returns <- function(df1,df2){
  Tickers <- df1 %>% select(Ticker)
  logreturns %>%
    unnest(logret) %>%
    group_by(Ticker) %>%
    filter(Ticker %in% Tickers$Ticker) %>%
    filter(date <= df2,date >= df2 - years(2)) %>%
    nest() %>%
    left_join(df1,by = "Ticker")
}
## Example for the Momentum model
# This will be done for each model
Closing <- cross_sec_Fund_data_4Factor %>%
  mutate(returns = map2(stocks,date,Returns))

# Remove the stocks with less than 500 days of data points
# add a column to each returns tibble, which will show the number of rows for a
specific share
complete_data <- function(df){
  extension <- df %>% mutate(size = map_dbl(df$data,NROW))
  filter(extension,size >= max(size)-5) %>%
    select(Ticker,data,Market.Cap)
}

Closing <- Closing %>% mutate(returns = map(returns,complete_data))

## Creating a matrix with all the returns for the selected stocks.
Return_Matrix <- function(df){
  joined_data <- df$data %>%
```

```
  join_all(by = "date") # Comes from the plyr package
  colnames(joined_data) <- c("date",df$Ticker)
  joined_data
}

Closing <- Closing %>%
  mutate(ret_matrix = map(returns,Return_Matrix))

## Return matrix for the use of the equal risk contribution methodology.
ERC_WEIGHTS <- function(df){
  cov_matrix <- df[,2:length(colnames(df))] %>%
    cov(use = "complete.obs")
  ERC_weights <- FRAPO::PERC(Sigma = cov_matrix,percentage = FALSE)
  ERC_weights <- as.data.frame(FRAPO::Weights(object = ERC_weights))
  ERC_weights <- tibble(Ticker = c(rownames(ERC_weights)),ERC_Weight =
ERC_weights$`FRAPO::Weights(object = ERC_weights)`)
}
Portf_Weights <- Closing %>%
  mutate(ERCWeights = map(ret_matrix,ERC_WEIGHTS)) %>%
  select(date,ERCWeights)
```

Market cap weights for the volatility model

```
MC_weights <- Closing %>% mutate(MC_Weight = map(returns,function(df){
  df %>%
    mutate(MC_Weight = Market.Cap/sum(Market.Cap)) %>%
    select(c(Ticker,MC_Weight))
}))

MC_weights <- MC_weights %>% select(date,MC_Weight)
```

Risk parity weights

```
# The Portfolio Analytics package is used in order to do the optimisation to estimate
the risk parity weights. The risk proxy used is the expected shortfall
# Further investigation will be put into the risk proxy
#install.packages("DEoptim")
#install.packages("ROI")
#install.packages("timeSeries")
#install_github("R-Finance/PortfolioAnalytics")
#install_github("cran/ROI.plugin.glpk")
#install_github("cran/ROI.plugin.quadprog")
library(devtools)
library(DEoptim)
library(ROI)
library(PortfolioAnalytics)
require(ROI.plugin.glpk)
require(ROI.plugin.quadprog)

# This function runs the optimization in order to estimate the risp parity weights,
given an xts object.
RP_Weights <- function(df){
returns <- df
rownames(returns) <- returns[[1]]
returns <- as.xts(returns[,2:length(colnames(returns))])
```

```r
stock_names <- colnames(returns)
pspec <- portfolio.spec(assets = stock_names)
pspec <- add.constraint(portfolio = pspec,
                        type = "weight_sum",
                        min_sum = 1,
                        max_sum =1)
pspec <- add.constraint(portfolio = pspec,
                        type = "box",
                        min = 0.001,
                        max = 0.15)
pspec <- add.objective(portfolio =  pspec ,
                       type= "risk_budget",
                       name = "StdDev",
                       min_concentration = TRUE)
opt <- optimize.portfolio(returns,
                          pspec,
                          optimize_method = "ROI")
RP_weights <- opt$weights
}
# Mapping the optimization function over all the periods
Closing <- Closing %>% mutate(RP_weights = map(ret_matrix,RP_Weights))

# The weights need to be merged with the returns list of tibbles.
# This is done for the risk parity weights
stock_weights <- function(df1,df2){
  df2 %>%
    left_join(df1 %>%
              tibble(Ticker = c(Ticker = df2$Ticker),RP_Weight = .),by = "Ticker")
}

## adding the RP, MC and ERC weights
Closing <- Closing %>%
  mutate(returns = map2(RP_weights,returns,stock_weights)) %>%
  mutate(returns = map2(returns,MC_weights$MC_Weight,left_join)) %>%
  mutate(returns = map2(returns,Portf_Weights$ERCWeights,left_join))
```

Calculating the benchmark's performance

```r
dates <- Closing$date
JALSH_returns <- JALSH %>%
  filter(date %in% dates) %>%
  mutate(JSE_log_ret = c(0,diff(log(PX_LAST)))) %>%
  mutate(JSE_cum_ret = cumprod(1+JSE_log_ret))
SWIX_returns <- SWIX %>%
  filter(date %in% dates) %>%
  mutate(SWIX_log_ret = c(0,diff(log(PX_LAST)))) %>%
  mutate(SWIX_cum_ret = cumprod(1+SWIX_log_ret)) %>%
  select(-PX_LAST)
# There is actually one quarterly date missing in the SWIX data. Won't make difference
to cumulative returns at the end.
```

The total return for each period

```r
# The log returns for each asset on the JALSH for each 3-month period
Quarter_logreturns <- close_prices$data[[2]] %>%
```

```
  bind_rows(.id = "Ticker") %>%
  group_by(Ticker) %>%
  nest()

log_returns2 <- function(df){
close <- df$"PX_LAST"
date <- dplyr::filter(df,date >= date[1]) %>% select(date)
ret <- tibble(date = date$date, log_ret = c(0,diff(log(close))))
}

Quarter_logreturns <- Quarter_logreturns %>% mutate(logret = map(data,log_returns2))

# Leading the dates one period forward in order to get the return achieved over a 3-
month period when mapping the Tot_return function
dates <- Closing$date %>% lead()

# This function mutates the corresponding return for an asset over the next 3 months
with all the assets selected in the model.
Tot_return <- function(df1,df2){
  Tickers <- df1$Ticker
  b <- Quarter_logreturns %>%
    unnest(logret) %>%
    filter(date == df2,Ticker %in% Tickers) %>%
    right_join(df1,by="Ticker") %>%
    mutate(RP_tot_ret = RP_Weight*log_ret,
           MC_tot_ret = MC_Weight*log_ret,
           ERC_tot_ret = ERC_Weight*log_ret)
}

# Mapping the function and adding a new list : tot_ret to the closing tibble
# The output, a tibble with all the selected stocks with their corresponding weights
and 3 month log returns and total returns (weight*return)
Closing <- Closing %>%
  mutate(tot_ret = map2(returns,dates,Tot_return))



# get the total return for a 3-month period.
#Mapping a sum_returns function over the total return list in the closing tibble
RP_Ret <- Closing %>%
  mutate(RP_Ret = map_dbl(tot_ret,function(df) sum(df$RP_tot_ret)),
         MC_Ret = map_dbl(tot_ret,function(df) sum(df$MC_tot_ret)),
         ERC_Ret = map_dbl(tot_ret,function(df) sum(df$ERC_tot_ret))) %>%
  select(date,RP_Ret,MC_Ret,ERC_Ret)

# adding the cumulative return tibble to the RP_Ret_MOM model
RP_Ret_4FACTOR <- RP_Ret %>%
  mutate(RP_cum_ret = cumprod(RP_Ret+1) ,
         MC_cum_ret = cumprod(MC_Ret+1),
         ERC_cum_ret = cumprod(ERC_Ret+1)) %>%
  dplyr::rename(Rebalancing_Date = date) %>%
  mutate(date = lead(Rebalancing_Date))

# This tibble gives a summary of the performance up to a specific date.

# Adding the benchmark's performance.
```

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```
# In this case only the JSE All Share index is used, since it had data from before
2001.
# If another benchmark need to be added, the starting date needs to be changed since I
compare RP_Ret_VALUE returns
RP_Ret_4FACTOR <- RP_Ret_4FACTOR %>%
  left_join(JALSH_returns,by = "date") %>%
  left_join(SWIX_returns,by = "date") %>%
  select(-PX_LAST) %>%
  dplyr::rename(End_Date = date) %>%
  filter(Rebalancing_Date < ymd("20160930"))

  # Omitting the last row, which has no data due to the dates moving one period forward

ggplot(RP_Ret_4FACTOR) +
  geom_line(aes(x=End_Date,y=RP_cum_ret,colour = "Risk Parity"),size =1) +
  geom_line(aes(x=End_Date,y=ERC_cum_ret,colour = "Equal Risk Contribution"),size =1) +
  geom_line(aes(x=End_Date,y=MC_cum_ret,colour = "Market Cap"),size =1) +
  geom_line(aes(x=End_Date,y=SWIX_cum_ret,colour = "SWIX Performance"),size =1) +
  geom_line(aes(x=End_Date,y=JSE_cum_ret,colour = "JSE Performance"),size =1) +
  labs(x="Date",y= "Cumulative Return", title = "Cumulative returns for the various
portfolios",subtitle = "4 Factor Fund based on cross secional comparison") +
  scale_color_discrete(name = "Weighting Strategies")

#relative performance
portf <- RP_Ret_4FACTOR %>% na.omit() %>% select(End_Date,MC_Ret,ERC_Ret,RP_Ret)
bench <- RP_Ret_4FACTOR %>% na.omit() %>% select(End_Date,SWIX_log_ret)
rownames(portf) <- portf$End_Date
rownames(bench) <- bench$End_Date
portf <- portf %>% select(MC_Ret,ERC_Ret,RP_Ret) %>% as.xts()
bench <- bench %>% select(SWIX_log_ret) %>% as.xts()
chart.RelativePerformance(portf,bench,main = "Relative performance with respect to the
swix",legend.loc = "topright")
```
Performance table

```
new_portf <- RP_Ret_4FACTOR %>%
  filter(End_Date <= ymd(20161030)) %>%
  select(End_Date,RP_Ret,ERC_Ret,MC_Ret,JSE_log_ret,SWIX_log_ret)

rownames(new_portf) <- new_portf$End_Date
new_portf <- new_portf %>% select(RP_Ret:SWIX_log_ret)

new_portf <- new_portf %>% as.xts()

VaR(new_portf,p = 0.95,method = "gaussian")

table.AnnualizedReturns(new_portf,Rf = 0.0675/4,scale = 4)
table.DownsideRisk(new_portf,ci = 0.95,scale = 4,Rf = 0.0675/4,p = 0.95)

InformationRatio(new_portf[,1:3],new_portf[,5],scale = 4)
```

Combining the macro exposure of the selected stocks with the tibble containing stock weights

Made sure that the macroeconomic file was executed before the following chunks.

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```
# A loop is run over all the rebalancing dates in the outer_combine function
# In the inner map function, the macro exposures are combined with their corresponding
shares for the specific date.
outer_combine <- function(df,df1){
  exposure <- exposure_per_reb_date_gruber %>%
    filter(dates == df) %>%
    select(exposure)
  df1 %>%
    mutate(macro_exp = map(Ticker,function(df1){
    ticker <- df1
    model <- exposure$exposure[[1]] %>% filter(Ticker == ticker[[1]])
    macroexp <- tibble(Alpha = model$model[[1]]$coefficients[["(Intercept)"]],
                       ZAR = model$model[[1]]$coefficients[["ZAR_Change"]],
                       OIL = model$model[[1]]$coefficients[["OIL"]],
                       GOLD = model$model[[1]]$coefficients[["GOLD"]],
                       SWAP = model$model[[1]]$coefficients[["SWAP"]])
  }))
}
Macro <- Closing %>%
  mutate(macro_exp = map2(date,returns,outer_combine)) %>%
  select(date,macro_exp)

#The output can be compared to the specific model's output, to check if all the stocks
are included and all their macro exposures are linked up
Macro <- Macro %>%
  mutate(macro_exp = map(macro_exp,function(df) df %>% unnest(macro_exp)))
```

Calculating the total macro exposure of the portfolio for a specific month

```
Tot_macro <- function(df){
  exp <- tibble(Alpha = df$RP_Weight*df$Alpha,
                ZAR = df$RP_Weight*df$ZAR,
                OIL = df$RP_Weight*df$OIL,
                GOLD = df$RP_Weight*df$GOLD,
                SWAP = df$RP_Weight*df$SWAP)
  tot_exposure <- tibble(Alpha = sum(exp$Alpha),
                         ZAR = sum(exp$ZAR),
                         OIL = sum(exp$OIL),
                         GOLD = sum(exp$GOLD),
                         SWAP = sum(exp$SWAP))
}
Total_exposure <- Macro %>% mutate(tot_exp = map(macro_exp,Tot_macro))
Total_exposure_4FACTOR_cross <- Total_exposure %>% unnest(tot_exp)

p1 <- ggplot(Total_exposure_4FACTOR_cross) +
  geom_line(aes(date,ZAR),colour = "green",size = 1)
p2 <- ggplot(Total_exposure_4FACTOR_cross) +
  geom_line(aes(date,OIL),size = 1)
p3 <- ggplot(Total_exposure_4FACTOR_cross) +
  geom_line(aes(date,GOLD),colour = "gold",size=1)
p4 <- ggplot(Total_exposure_4FACTOR_cross) +
  geom_line(aes(date,SWAP),colour = "blue",size=1)
grid.arrange(p1,p2,p3,p4,top = textGrob("Macro exposure for the Quality fund based on
cross sectional comparison"))
```

Calculating daily performance of the portfolio

```r
# The daily performance of the portfolio is calculated in order to compute VaR and
other risk measures at portfolio level
Closing <- Closing %>% filter(date < Closing$date[[59]])

dates <- Closing$date
combine_daily_returns <- function(df1,df2){
  Tickers <- df1$Ticker
  logreturns %>%
    unnest(logret) %>%
    filter(date <= df1$date[[1]],date > df2) %>%
    filter(Ticker %in% Tickers) %>%
    group_by(Ticker) %>%
    nest()
}

daily_performance <- Closing %>%
  mutate(d_performance = map2(tot_ret,dates,combine_daily_returns))

Closing <- Closing %>%
  mutate(d_performance =
map2(Closing$tot_ret,daily_performance$d_performance,function(df1,df2){
  left_join(df1,df2,by="Ticker")
}))



daily_performance <- Closing %>% mutate(D_performance = map(d_performance,
function(df){
  df %>%
    select(Ticker,Market.Cap,RP_Weight,MC_Weight,ERC_Weight,data.y) %>%
    unnest(data.y) %>%
    mutate(RP_daily_ret = RP_Weight*log_ret, MC_daily_ret = MC_Weight*log_ret,
ERC_daily_ret = ERC_Weight*log_ret) %>%
    group_by(date) %>%
    nest() %>%
    dplyr::rename(end_date=date)
}))


tot_daily_performance <- function(df2){
  df2 %>% mutate(daily_portfolio_return = map(data,function(df1){
  tibble(RP_tot_daily_ret = sum(df1$RP_daily_ret,na.rm=T),
        MC_tot_daily_ret = sum(df1$MC_daily_ret,na.rm=T),
        ERC_tot_daily_ret = sum(df1$ERC_daily_ret,na.rm=T))
  }
  ))
}

daily_performance <- daily_performance %>%
  mutate(daily_portfolio = map(D_performance,tot_daily_performance)) %>%
  select(date,daily_portfolio)

daily_performance <- daily_performance %>%
  mutate(daily_performance=map(daily_portfolio,function(df){
  df %>% unnest(daily_portfolio_return)
```

```
}))

daily_performance_4FACTOR_Cross <- daily_performance %>%
  unnest(daily_performance)

bench_returns <- function(df){
  date <- dplyr::filter(df,date>date[1]) %>% select(date)
  close <- df$PX_LAST
  ret <- tibble(end_date = date$date, swix_daily_ret = diff(log(close)))
}

SWIX1 <- SWIX %>% bench_returns()
JALSH1 <- JALSH %>%
  bench_returns() %>%
  dplyr::rename(JALSH_daily_ret = swix_daily_ret)

daily_performance_4FACTOR_Cross <- daily_performance_4FACTOR_Cross %>%
  left_join(SWIX1) %>%
  left_join(JALSH1)
```

Comparing portfolio performances

```
dates <- daily_performance_4FACTOR_Cross %>% filter(end_date %in% date) %>%
select(end_date)

drawdowns <- function(df1){
  new <- daily_performance_4FACTOR_Cross %>%
    filter(end_date <= df1,end_date >= df1 - years(1)) %>%

select(c(RP_tot_daily_ret,MC_tot_daily_ret,ERC_tot_daily_ret,swix_daily_ret,JALSH_daily
_ret))
  tibble(RP_Drawdown = new$RP_tot_daily_ret %>% maxDrawdown(na.rm=T),
         ERC_Drawdown = new$ERC_tot_daily_ret %>% maxDrawdown(na.rm=T),
         MC_Drawdown = new$MC_tot_daily_ret %>% maxDrawdown(na.rm=T),
         SWIX_Drawdown = new$swix_daily_ret %>% maxDrawdown(na.rm=T),
         JALSH_Drawdown = new$JALSH_daily_ret %>% maxDrawdown(na.rm=T))
}

DrawdownRecovery <- function(df){
  new <- daily_performance_4FACTOR_Cross %>%
    filter(end_date <= df, end_date >= df - years(1)) %>%
    select(c(end_date,RP_tot_daily_ret:JALSH_daily_ret))
  rownames(new) = new$end_date
  new <- new %>% select(RP_tot_daily_ret:JALSH_daily_ret) %>% as.xts() %>%
AverageRecovery()
  tibble(RP_DrawdownDuration = new[[1]],
         ERC_DrawdownDuration = new[[2]],
         MC_DrawdownDuration = new[[3]],
         SWIX_DrawdownDuration = new[[4]],
         JALSH_DrawdownDuration = new[[5]])
}

drawdowns_4FACTOR_cross <- dates %>%
  mutate(DD = map(end_date,drawdowns)) %>%
  unnest(DD)
```

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```
drawdownDurations_4FACTOR_cross <- dates %>%
  mutate(DD = map(end_date,DrawdownRecovery)) %>%
  unnest(DD)

p1 <- ggplot(drawdowns_4FACTOR_cross) +
  geom_line(aes(x=end_date,y=RP_Drawdown, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=SWIX_Drawdown,colour="SWIX"),size=1) +
  geom_line(aes(x=end_date,y=ERC_Drawdown,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_Drawdown,colour="Market Cap"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_Drawdown,colour="JALSH"),size=1) +
  labs(x="Date",y= "Maximum Drawdown", title = "Maximum Drawdowns for the various
portfolios",subtitle = "4FACTOR Fund based on time series comparison") +
  scale_color_discrete(name = "Weighting Strategies")

p2 <- ggplot(drawdownDurations_4FACTOR_cross) +
  geom_line(aes(x=end_date,y=RP_DrawdownDuration, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=SWIX_DrawdownDuration,colour="SWIX"),size=1) +
  geom_line(aes(x=end_date,y=ERC_DrawdownDuration,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_DrawdownDuration,colour="Market Cap"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_DrawdownDuration,colour="JALSH"),size=1) +
  labs(x="Date",y= "Maximum Drawdown", title = "Drawdown durations for the various
portfolios",subtitle = "4FACTOR Fund based on cross sectional comparison") +
  scale_color_discrete(name = "Weighting Strategies")

grid.arrange(p1,p2,nrow =2)

varestimates <- function(df1,df2){
  new <- daily_performance_4FACTOR_Cross %>%
    filter(end_date <= df1,end_date >= df1 - years(1)) %>%

select(c(RP_tot_daily_ret,MC_tot_daily_ret,ERC_tot_daily_ret,swix_daily_ret,JALSH_daily
_ret))
  tibble(RP_VaR = new$RP_tot_daily_ret %>% VaR(p=.95,method = df2) %>% as.double(),
         ERC_VaR = new$ERC_tot_daily_ret %>% VaR(p=.95,method = df2)%>% as.double(),
         MC_VaR = new$MC_tot_daily_ret %>% VaR(p=.95,method = df2) %>% as.double(),
         SWIX_VaR = new$swix_daily_ret %>% VaR(p=.95,method = df2) %>% as.double(),
         JALSH_VaR = new$JALSH_daily_ret %>% VaR(p=.95,method = df2) %>% as.double())
}
VaR_4FACTOR_cross_hist <- dates %>%
  mutate(daily_VaR = map2(end_date,"historical",varestimates)) %>%
  unnest(daily_VaR)

VaR_4FACTOR_cross_gaus <- dates %>%
  mutate(daily_VaR = map2(end_date,"gaussian",varestimates)) %>%
  unnest(daily_VaR)

VaR_4FACTOR_cross_mod <- dates %>%
  mutate(daily_VaR = map2(end_date,"modified",varestimates)) %>%
  unnest(daily_VaR)

p1 <- ggplot(VaR_4FACTOR_cross_hist) +
  geom_line(aes(x=end_date,y=RP_VaR, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=SWIX_VaR,colour="SWIX"),size=1) +
  geom_line(aes(x=end_date,y=ERC_VaR,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_VaR,colour="Market Cap"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_VaR,colour="JALSH"),size=1) +
```

```
    labs(x="Date",y= "Daily VaR", title = "Daily historical VaR estimates for the various
portfolios",subtitle = "Value Fund based on cross sectional comparison") +
  scale_color_discrete(name = "Weighting Strategies")

p2 <- ggplot(VaR_4FACTOR_cross_gaus) +
  geom_line(aes(x=end_date,y=RP_VaR, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=SWIX_VaR,colour="SWIX"),size=1) +
  geom_line(aes(x=end_date,y=ERC_VaR,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_VaR,colour="Market Cap"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_VaR,colour="JALSH"),size=1) +
  labs(x="Date",y= "Daily VaR", title = "Daily Normal linear VaR estimates for the
various portfolios",subtitle = "Value Fund based on cross sectional comparison") +
  scale_color_discrete(name = "Weighting Strategies")

p3 <- ggplot(VaR_4FACTOR_cross_mod) +
  geom_line(aes(x=end_date,y=RP_VaR, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=SWIX_VaR,colour="SWIX"),size=1) +
  geom_line(aes(x=end_date,y=ERC_VaR,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_VaR,colour="Market Cap"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_VaR,colour="JALSH"),size=1) +
  labs(x="Date",y= "Daily VaR", title = "Daily Monte Carlo VaR estimates for the
various portfolios",subtitle = "Value Fund based on cross sectional comparison") +
  scale_color_discrete(name = "Weighting Strategies")

grid.arrange(p1,p2,p3)

SharpeRatioEstimates <- function(df){
  new <- daily_performance_4FACTOR_Cross %>%
    filter(end_date <= df,end_date >= df - years(1)) %>%

select(c(RP_tot_daily_ret,MC_tot_daily_ret,ERC_tot_daily_ret,swix_daily_ret,JALSH_daily
_ret))
  tibble(RP_SR = (mean(new$RP_tot_daily_ret,na.rm = T) -
0.09/252)/sd(new$RP_tot_daily_ret,na.rm = T),
        ERC_SR = (mean(new$ERC_tot_daily_ret,na.rm = T) -
0.09/252)/sd(new$ERC_tot_daily_ret,na.rm = T),
        MC_SR = (mean(new$MC_tot_daily_ret,na.rm = T) -
0.09/252)/sd(new$MC_tot_daily_ret,na.rm = T),
        SWIX_SR = (mean(new$swix_daily_ret,na.rm = T) -
0.09/252)/sd(new$swix_daily_ret,na.rm = T),
        JALSH_SR = (mean(new$JALSH_daily_ret,na.rm = T) -
0.09/252)/sd(new$JALSH_daily_ret,na.rm = T))
}

IR <- function(df){
  new <- daily_performance_4FACTOR_Cross %>%
    filter(end_date <= df, end_date >= df - years(1)) %>%
    select(end_date,RP_tot_daily_ret:ERC_tot_daily_ret,JALSH_daily_ret)
  new2 <- daily_performance_4FACTOR_Cross %>%
    filter(end_date <= df, end_date >= df - years(1)) %>%
    select(end_date,swix_daily_ret)
  rownames(new) <- new$end_date
  rownames(new2) <- new2$end_date
  new <- new %>% select(RP_tot_daily_ret:ERC_tot_daily_ret,JALSH_daily_ret) %>%
as.xts()
  new2 <- new2 %>% select(swix_daily_ret) %>% as.xts()
  IRs <- InformationRatio(new,new2,scale = 4)
```

```
  tibble(RP_IR = IRs[[1]],
         MC_IR = IRs[[2]],
         ERC_IR = IRs[[3]],
         JALSH_IR = IRs[[4]])
}

IR_4FACTOR_cross <- dates %>%
  mutate(Q_IR = map(end_date,IR)) %>%
  unnest(Q_IR)

SharpeRatio_4FACTOR_cross <- dates %>%
  mutate(daily_SR = map(end_date,SharpeRatioEstimates)) %>%
  unnest(daily_SR)

p1 <- ggplot(SharpeRatio_4FACTOR_cross) +
  geom_line(aes(x=end_date,y=RP_SR, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=SWIX_SR,colour="SWIX"),size=1) +
  geom_line(aes(x=end_date,y=ERC_SR,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_SR,colour="Market Cap"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_SR,colour="JALSH"),size=1) +
  labs(x="Date",y= "Daily Sharpe Ratio", title = "Daily Sharpe Ratio estimates for the
various portfolios",subtitle = "Value Fund based on time series comparison") +
  scale_color_discrete(name = "Weighting Strategies")

p2 <- ggplot(IR_4FACTOR_cross) +
  geom_line(aes(x=end_date,y=RP_IR, colour = "Risk Parity"),size=1) +
  geom_line(aes(x=end_date,y=JALSH_IR,colour="JALSH"),size=1) +
  geom_line(aes(x=end_date,y=ERC_IR,colour="ERC"),size=1) +
  geom_line(aes(x=end_date,y=MC_IR,colour="Market Cap"),size=1) +
  labs(x="Date",y= "Quarterly Information ratio", title = "Quarterly Information ratio
estimates for the various portfolios",subtitle = "4FACTOR Fund based on cross sectional
comparison") +
  scale_color_discrete(name = "Weighting Strategies")

grid.arrange(p1,p2)
```