An examination of pre-service teachers' capacity to create mathematical modeling problems for children

Abstract

This study examined pre-service teachers' (PSTs) capacity to create mathematical modeling problems (MMPs) for grades 1-3. PSTs created MMPs for their choice of grade level and aligned the mathematical content of their MMPs with the relevant mathematics curriculum. PSTs were given criteria adapted from Galbraith's (2007) MMP design principles, to guide their work. These criteria were then used to evaluate the resulting MMPs, leading to findings and implications relevant to two areas – mathematics teacher education and the design and evaluation of MMPs for young children. Results highlight an inclination toward creating problems for higher grade levels as well as concerns regarding both the PSTs' proficiency with the curriculum content and their capacity to create MMPs for particular content areas. Findings contribute to an important international conversation about the need for further research and development of resources aimed at supporting the integration of mathematical modeling in early childhood mathematics education.

Introduction

Problem solving has long been considered an essential element of mathematical activity and has featured in educational research and reform efforts throughout the world. Amidst discussions of the need to promote problem solving in mathematics classrooms, far less focus has been placed on measuring or developing teachers' ability to pose problems that effectively foster students' engagement in problem solving (Cai, Hwang, Jiang, & Silber, 2015; Crespo, 2003).The mathematical tasks that teachers choose to use in their classrooms strongly influence the quality of student learning and perceptions regarding the nature of mathematics (Crespo, 2015; Henningsen & Stein, 1997; National Council of Teachers of Mathematics, 1991).

Therefore, teachers need to be able to both select and create effective problems which promote learning in line with the objectives of their curriculum.

A recent increase in research focusing on problem posing has endorsed it as a valuable process for the growth and development of mathematics teachers at all levels (Cai et al., 2015; Grundmeier, 2015; Osana & Pelczer, 2015). In research evaluating the potential for problem posing to support development of primary school teachers' pedagogical content knowledge, Tichà and Hošpesovà (2010) endorse problem posing as a means to both diagnose and develop future teachers' mathematics-specific didactical competences. They also note that, as a diagnostic tool, problems posed by pre-service teachers (PSTs) can offer an opportunity to investigate their understanding of mathematics, including any misconceptions. Teacher educators can gain important insight into PSTs' mathematical knowledge by examining their choices regarding the mathematical content included in their problems.

Pedagogically, problem posing is regarded as integral to teaching and is considered a "high-leverage practice" (Ball & Forzani, 2009). While a high-leverage practice is characterized as a fundamental practice that is essential for helping students to learn, it is typically not a practice that PSTs are likely to learn on their own (Ball & Forzani, 2009). Many teachers are not active problem posers, nor do they have the required skills to pose appropriate problems (Singer & Voica, 2013). Therefore, researchers agree that it is vital for PSTs to be exposed to and supported through problem posing experiences during their initial training (Ellerton, 2015; Grundmeier, 2015; Hošpesovà & Tichà, 2015; Lavy & Shriki, 2007; Osana & Pelczer, 2015; Rosli, Capraro, Goldsby, Gonzalez y Gonzalez, Onwuegbuzie, & Capraro, 2015). While current literature lacks consensus on what a focus on problem posing should look like in teacher education, some studies have offered valuable recommendations. For instance, the research of Crespo (2003) suggests introducing and exploring non-traditional mathematical problems in teacher education classes and encourages collaborative problem posing.

One particular realm of problem posing which offers rich opportunities for analytical thinking, creativity and problem solving is mathematical modeling. Mathematical modeling is the process of mathematizing a real-world situation and employing multiple cycles of problem solving to make sense of the problem and identify a solution (Lesh & Doerr, 2003). Mathematical modeling problems (MMPs) are open-ended, real-world tasks that require the use of mathematical modeling to reach a solution. When facing an MMP, students typically move through the phases of a modeling cycle which begin with the real-life problem and include identifying relevant mathematics, organizing and representing the problem mathematically, solving the problem and interpreting the solution (OECD, 2003).

Traditionally, MMPs were only used at secondary and tertiary levels, but recent research has shown that young children can and should engage in the solving of such rich, open-ended tasks in real-world settings (English, 2006; English & Watters, 2004; Fox, 2006; Mousoulides & English, 2011; Usiskin, 2007). In addition, García, Maaß and Wake (2009) consider mathematical modeling to be a crucial part of the initial training and professional development for both primary and secondary teachers. However, both PSTs and practicing teachers find the creation of such problems very challenging (Crespo & Sinclair, 2008; Franke & Kazemi, 2001; Osana & Pelczer, 2015). While literature in this area offers support through extensive examples of MMPs and multiple frameworks for their creation and evaluation, researchers agree that more research and resources are needed to support teachers in creating and implementing MMPs in the early grades (Usiskin, 2007; Garcia & Ruiz-Higueras (2010).

Just as Tichà and Hošpesovà (2010) endorse problem posing as a diagnostic source for information regarding PSTs' knowledge and development, studying MMPs created by PSTs can offer teacher educators insight into the challenges of creating rich, real-world problems. Insight drawn from evaluating PSTs' choices regarding the mathematical content and pedagogical structure of their MMPs can also help teacher educators to address these challenges. This article presents a study aimed at gaining this insight and guided by the following questions:

1. What choices regarding mathematical content and pedagogy do PSTs make when posing MMPs for children in grades 1-3?
2. What implications do these choices have for teacher education, specifically regarding mathematical modeling?
3. How proficiently are the final-year PSTs in this study able to develop MMPs for children in grades 1-3, in accordance with criteria adapted from Galbraith's (2007) Design Principles?

Problem posing and mathematical modeling with PSTs

Problem posing can take two forms: designing new problems and reformulating existing problems (Silver, 1994). Klinshtern, Koichu, and Berman (2015) further distinguish between routine problem posing and innovative problem posing. Routine problem posing refers to the creation of "textbook-like" problems. This includes making cosmetic changes to existing problems by replacing parameters or changing the story without significantly changing the solution. They also include "in-the-moment" problems spontaneously created as examples during a lesson. Innovative problem posing involves creation of new problems by combining ideas, applying skills across different contexts or connecting different topics.

Tichà and Hošpesovà (2010) identify routine problem posing as the typical extent of PSTs' problem posing abilities in the absence of any specific coursework or intervention. They report on research showing that without specific support with problem posing, PSTs often use erroneous wording and pose problems that are, from a mathematical perspective, "uninteresting and demotivating" (p. 1948). Thus, explicit support is required to develop PSTs' ability to create the type of rich, meaningful learning experiences that come from MMPs.

Engaging students in mathematical modeling experiences offers opportunities for interpretation and argumentation that is not as common in the word problems typically used in early grade levels (English & Watters, 2004). These opportunities can enable students with a wider range of achievement levels to be highly capable, leading to increased confidence in their mathematical abilities and improved attitudes toward mathematics (English & Watters, 2004; Lesh & Doerr, 2003). Despite these benefits, mathematical modeling has only recently begun to feature in early-grade curricula. This means that those who are currently teaching or preparing to teach at the primary level would not likely have experienced this type of engagement with mathematics in the early stages of their own mathematics education. As a result, they are unlikely to be easily able to envision what these experiences might look like across all areas of their curriculum.

Frameworks for problem posing and evalation

There is a growing body of research which proposes and evaluates problem posing criteria and scoring rubrics. For example, Crespo (2015) regards problem posing as one of the highest forms of mathematical knowing and proposes a three-tiered problem posing framework for mathematics teacher education. This advocates for learning to pose problems mindfully, pose problems with students and pose personally and socially relevant problems. Rosli et al. (2015) used a problem solving and problem scoring rubric with four performance indicators in their research on middle-grade teachers' mathematical problem solving and problem posing abilities. This assessed problem structure/context; understanding of the problem; mathematical expression; and appropriateness of the problem-posing design. Grundmeier (2015) coded teacher-posed problems with a four-tiered framework assessing the plausibility and complexity of the problem and the poser's ability to provide sufficient information.

There are also a number of frameworks specific to the development of MMPs. One of the most frequently quoted frameworks, proposed by Lesh, Hoover, Hole, Kelly, and Post (2000), offers six principles for designing a modeling task. These include the reality principle, model-construction principle, self-evaluation principle, model-documentation principle, model-generalization principle, and simple-prototype principle. These principles and others were part of the coursework of participants in this study. They were thoughtfully considered by the researchers with specific regard for both the early developmental stage for which the problems would be designed and the participating PSTs' capacity to interpret and apply the framework.

For the purpose of this study, the researchers chose the frequently cited framework offered by Galbraith (2007). Its language and concepts seemed most reasonable to adapt for the context of guiding early childhood PSTs in the creation of a modeling problem. This decision was also influenced by the framework's inclusion of a Didactical Design Principle. This principle acknowledges the potential need for modeling problems to be structured into sequential questions. Particularly at the early childhood level, such sequential questions can offer scaffolding that may be essential to reaching a solution for an open-ended problem. Details of Galbraith's (2007) principles are presented in Table 1 where bold print is used to emphasize features of each principle that influenced their adaptation into the criteria used in this study.

Table 1: Principles for the design of mathematical modeling problems (MMPs)

In conjunction with his design principles, Galbraith (2007) also endorsed the value of both establishing guiding principles and considering their suitability across contexts and curricular objectives. This supports the study's focus on evaluating PSTs' capacity to create effective, curriculum-aligned modeling problems for children, when given a set of criteria adapted from Galbraith's design principles. The study also considers the suitability of such criteria for MMPs designed for grades 1-3. These criteria are presented in Table 2.

Table 2: Criteria used for creation of modeling problems with guiding principles and research

Research context and participants

Mathematical modeling has become a key component of many countries' mathematics curricula, at all levels. It features in objectives for reform initiatives and helps to define aims, objectives and desired mathematical practices in curriculum documents throughout the world (Paolucci, 2015). Thus, it has become increasingly important for teacher education programs to ensure that they are preparing their future teachers to effectively foster productive mathematical modeling in their classrooms. This is a multi-dimensional charge. First, PSTs themselves have to engage with modeling to promote metacognitive recognition of the thought processes and practices that characterize mathematical modeling. Second, PSTs must be able to design tasks and problems that effectively engage their students in the process of mathematical modeling. Finally, PSTs must be able to evaluate existing problems by recognizing key features of an effective MMP and determining the suitability of a problem for both the content of their curriculum and the developmental levels of their students.

The context of this study is a four-year Bachelor of Education (BEd) teacher preparation program with a curriculum developed to acknowledge the importance of an explicit focus on mathematical modeling in mathematics teacher education. Participants include PSTs seeking to become certified at the Foundation Phase level (grades 1-3). An integral focus of the Mathematics Education coursework in the BEd is on student thinking and reasoning, and PSTs regularly analyze and discuss students' problem solving strategies. In addition, specific units of coursework are dedicated to familiarizing PSTs with the five content areas of the Mathematics curriculum and developing the pedagogical content knowledge required to teach them. PSTs' knowledge of the content areas is further developed through task design and lesson planning. This includes three Lesson Study cycles in the final two years of the program. These cycles include planning sessions and post-lesson discussions which focus on strategies such as the strategic use of questioning to both develop and assess students' understanding.

Both problem posing and mathematical modeling are a prominent part of the curriculum throughout the BEd. In fact, mathematical modeling is a significant focus of the Mathematics Education coursework during the second, third and fourth years of the program. The PSTs' engagement with mathematical modeling in these courses aims to support development and integration of their mathematical knowledge, introduce them to the value and possibilities of mathematical modeling in grades 1-3, and prepare and empower them to implement mathematical modeling with children at these levels.

Participants in this study were at the end of their fourth and final year of the BEd program. At this stage, the program's focus on mathematical modeling includes both solving MMPs and creating, presenting and peer evaluating mathematical models designed to solve problems in real-world contexts. PSTs also revisit the theory of mathematical modeling covered in the previous year, discuss criteria for posing MMPs, analyze and categorize MMPs created in previous years, create MMPs for Foundation Phase children and improve their own MMPs through self and peer evaluation. In addition, much of the work done with mathematical modeling is done in groups, so the PSTs are accustomed to collaborative work in these areas. Purposeful and convenience sampling was employed with this cohort of 106 PSTs. All were invited to participate in the study, resulting in 93 participants.

The design of the study reflects the recommendations of researchers to provide PSTs with opportunities to explore, reflect on and evaluate examples of problems and to collaborate in posing problems (Crespo, 2003; Tichà & Hošpesovà, 2010). Prior to beginning the study, participants were given a sample MMP created by a previous cohort and asked to evaluate it using the criteria adapted from Galbraith's (2007) principles for the design of MMPs (as seen in Table 2). They discussed and evaluated additional MMPs in small groups and then shared their evaluations in a class discussion. The researchers facilitated this discussion to ensure consistent and comprehensive understanding of the criteria and proficiency with applying it.

Given schedule constraints for a group of over 100 PSTs, the researchers had a block of one hour with participants to carry out the study. The first ten minutes were spent introducing the task and answering questions. Participants then spent 50 minutes working in pairs or groups of three to create an MMP which met the given criteria. They were asked to create MMPs for children in grades 1, 2 or 3 which specifically promote application or development of mathematical concepts from the five content areas of the South African mathematics curriculum - Numbers, Operations and Relationships; Patterns and Early Algebra; Space & Shape; Measurement; and Data Handling. They were asked to identify all curricular areas involved with their MMP and to include at least one sample solution for their problems. The researchers asked for the problems to be created in English, but in cases where English was not the PSTs' first language, they were given the option of writing them in Afrikaans. These were translated prior to analysis.

While the researchers recognized the challenge of completing such a task in the allotted time, they felt that their collaboration would offer some alleviation. The PSTs did have prior experience with creating problems in a 50-minute class period. In this case, they also had an opportunity to become familiar with the criteria in advance and use it to analyze and suggest improvements to MMPs prior to the session. In cases where time constraints were a factor, the impact was seen in the sample solutions and not in the MMP itself.

Methods and analysis

This inductive study in an interpretive paradigm involved document analysis. The units of analysis were artefacts (the created MMPs) and data were mined from these artefacts through both qualitative and a quantitative analysis using a coding and scoring process. There were two aspects to this process. The first involved coding the appropriate grade level and the mathematical content involved in the MMP. The second was a scoring process in which each problem was given a score of 0, 1 or 2 for each item in a scoring rubric. The items in this rubric were aligned with the original criteria given to PSTs to guide creation of their MMPs. As previously outlined, these criteria were adapted from Galbraith (2007)'s principles for MMP design with influences from Doerr and Lesh (2011) and Kuntze (2011). While there were 6 original criterion, the scoring rubric consisted of 13 items. This was motivated by the fact that some criteria had multiple parts and needed to be broken down to accommodate for cases in which an MMP met one part of a criterion but not another. The 13 items in the scoring rubric are outlined with the findings in Table 6.

For each rubric item, a score of 2 was given if the MMP sufficiently met the criterion; a score of 1 was given if the MMP partially met the criterion; and a score of 0 was given if the MMP did not meet the criterion. Sample solutions were scored for completeness, accuracy and reasonability; however, these were not included in the overall score for the MMP. Similarities can be found between the scoring criteria used in this study and the coding criteria in the research of Crespo (2015), Grundmeier (2015) and Rosli et al. (2015) discussed above.

The 93 participants created a total of 42 MMPs. The two researchers initially coded and scored 10 out of the 42 MMPs together, with the aim of refining the scoring rubric and ensuring consistency in each researcher's approach to the coding and scoring process. The refined scoring instrument was then used by each researcher to separately score all MMPs. The individual researchers' codes and scores for each rubric item were compared for every MMP, and any discrepancies were discussed until consensus was reached. While discrepancies did arise across all criteria, they were most frequent for criterion 2a. This is consistent with concerns discussed in more detail in the following section.

Findings

The findings are presented in two categories. The first includes findings primarily related to teacher preparation. The second includes findings related to the suitability of the criteria for use in creating and evaluating MMPs for children. While the first category of findings aligns with the aims of the study and research questions, the second offers unexpected but valuable insight for considering the suitability of the criteria used in this study for the creation and evaluation of MMPs across all grade levels.

Findings – Teacher preparation

Three sets of findings with significance for teacher preparation are presented here, including targeted grade levels, curriculum alignment and ability to create MMPs which meet the given criteria. Given the option to create a problem for children in grades 1, 2 or 3, the breakdown of the PSTs' grade-level choices presented in Table 3 clearly illustrates that the majority of the problems (over 75%) were created for grade three learners. Less than 25% were created for grade two learners and less than 5% were created for grade one learners.

Table 3. Chosen grade levels for MMPs

The PSTs were also asked to align their MMP with one or more of the five content areas of South Africa's Foundation Phase Mathematics Curriculum by identifying the curricular areas from which content was required to understand and solve their problem. Analysis of their content alignment highlighted tendencies toward particular areas of the curriculum as well as issues with their ability to recognize and correctly identify concepts in particular content areas.

When the researchers compared their coding of the mathematical content involved with each MMP, discrepancies were prevalent between the researchers' codes and the content identified by the teachers. This raised concerns regarding the PSTs' ability to accurately recognize and identify the mathematical content involved in their MMPs and align them with the curriculum. To quantify these findings, each MMP was given a score from 0-3, to indicate how closely the mathematical content and curricular links identified by the PSTs aligned with those identified by the researchers. The criteria for each score and the frequency with which the PSTs accurately and completely identified the mathematical content in their MMPs is presented in Table 4.

These results show that nearly 60% of the PST teams missed or incorrectly identified at least one content area relevant to their problem.

Table 4. Content alignment of MMPs

Table 5 presents a further breakdown of these findings by indicating the frequency with which each content area was involved but not identified by the PSTs. It also presents the total frequency with which content from each curricular area was involved in an MMP. Many problems involved more than one content area, so the sum total of frequencies reported in each row will vary from the total number of problems (N = 42).

Table 5. Frequency of mathematical content in the MMPs

Table 5 also offers a comparison of the mathematical content identified by the PSTs and the researchers, by curriculum area. This highlights a few significant findings. The first is that, when given the freedom to design an MMP for any area of the curriculum, Measurement was the most commonly chosen, followed by Numbers, Operations and Relationships. The second is that, while the number of tasks which were specifically designed to incorporate Numbers, Operations and Relationships was the second highest, the researchers' codes identified content from this area of the curriculum in an additional 10 tasks not recognized by the PSTs. In fact, 24 of the 42 tasks (57.1%) involved some content categorized as Numbers, Operations and Relationships, but in almost half of these cases, the PSTs did not recognize the connection with this area of the curriculum.

The other area that was commonly involved but widely unacknowledged by the PSTs was Space and Shape. In fact, Space and Shape was the content area most commonly not identified by the PSTs when it was required by the MMP. The opposite can be said for Data Handling, which was the least prevalent area of the curriculum in the MMPs but was relatively proficiently identified in these small number of cases. It is important to mention that these findings also include consideration of the sample solutions provided by the PSTs. This is significant because it addresses the possibility of attributing unidentified content areas to varied interpretations or solutions for the MMP.

While the most significant issue was unrecognized content, in two cases, a content area was misidentified, meaning the PSTs aligned their MMP with a content area that was not relevant to their MMP. In one of these cases, the PSTs indicated that their problem involved Numbers, Operations and Relationships, but the problem only required knowledge related to Space and Shape. This was confirmed by the sample solution provided with the problem. In the other case, the PSTs indicated that the problem involved Data Handling, but the researchers could not identify any aspect of the problem or related tasks which involved the use of concepts or skills in Data Handling. This was further evidenced by the fact that no use of Data Handling appeared in the sample solution provided for the MMP.

Overall, this group of PSTs seemed most proficient in recognizing when a problem involved Measurement and Data Handling. They were less proficient in recognizing when their MMPs involved Numbers, Operations and Relationships and Patterns and Early Algebra. Space and Shape was the content which PSTs were the least proficient with identifying in a problem context. Not only was Space and Shape the area which the PSTs had the most difficulty recognizing in a task, it was also the area least purposely targeted by the PSTs in creating their problems. This was closely followed by Patterns and Early Algebra and Data Handling. This could suggest one of two things. Either the PSTs felt less comfortable with these areas than with Measurement and Numbers, Operations and Relationships or they view these areas as less conducive to real-life contexts and modeling tasks.

The third set of findings emerged from the use of the scoring rubric. As outlined above, scores of 0-2 for each item on the rubric, based on how well the associated criterion was met by the MMP. Overall, the total scores for the MMPs were notably low. Out of a total possible score of 26 (for the 13 criteria), scores ranged from 4 to 25, or 15.4% to 96.2% respectively.

The mean total score was 15.3 out of 26 (58.9%) with a standard deviation of 5.2 (20.1%). The distribution of total percentage scores for the MMPs is presented in Figure 1.

Figure 1: Distribution of total percentage scores for the MMPs

While these scores offer an indication of the overall quality of the MMPs, as determined by the rubric criteria (Table 6), they do not highlight more detailed trends of which criteria were met more successfully than others. This information is presented in Table 7 and Figure 2 below.

Table 6. Criteria used for scoring the MMPs, based on Galbraith (2007)

Table 7 presents results for each rubric item. It includes a raw score total, which indicates the sum of all 42 MMP scores for each item, and a proficiency rate, which presents this raw score as a percentage of the total possible score of 84 (2 x 42) for each item. Mean scores for each item are also presented in Table 7. The distribution of these mean scores is displayed in Figure 2.

Table 7. Overall results for each criterion

Figure 2. Overall mean scores for each MMP criterion

A number of findings arise from this distribution of scores across the criteria. The first is that, these PSTs were most proficient in constructing real-world contexts that illustrate the relevance of mathematics in their learners' lives (1a and 1b). They were similarly proficient at creating problems with at least one feasible solution that could be evaluated for accuracy and appropriateness with respect to the problem context (4, 5a and 5b) and presenting these problems at an age appropriate reading level (1c). These strengths are consistent with those described by Tichà and Hošpesovà (2010) as apparent in PSTs who had taken coursework with a specific focus on problem posing.

The MMPs earned the lowest scores on Criteria 2 and 6. This suggests that the PSTs had the most difficulty with creating tasks requiring learners to construct a mathematical representation of the context which could contribute toward finding a solution and with asking a strategic sequence of questions to help learners make sense of the problem and their solution.

These strengths and weaknesses indicate that, while the PSTs were relatively proficient in identifying and presenting relevant real-world problem contexts with feasible solutions, they had difficulty formulating problems which require students to create a mathematical representation of the context. They also struggled with asking strategically sequenced questions to help learners make sense of the mathematics involved in the task. It is important to note the use of "relatively" as a qualifier for the PSTs' proficiency with aspects of Criteria 1, 4 and 5. Mean scores for these criteria were still only 1.4 and 1.5 out of 3, which does not necessarily indicate a convincingly high level of proficiency.

Figure 3 offers an example of an MMP which helps to illustrate some of the concerns arising with regard to teacher preparation.

Figure 3. Sample MMP (1)

This MMP was created for grade 3 learners. The PSTs who created it only aligned the content with Shape and Space. The researchers identified three additional areas - Numbers, Operations and Relationships, Patterns and Early Algebra, and Measurement. This MMP also offers an example of a case in which the questions are not properly chosen or sequenced to scaffold students' efforts to solve the problem (criteria 6a, 6b and 6c). First, the jump between floor plans and consideration of the time and resources needed to build up walls makes the end goal unclear. Second, questions 4 and 5 should appear earlier in the sequence of questions if they are to effectively promote scaffolding. In addition, while question 4 does explicitly ask students to represent the context mathematically through a picture, graph, table, number sentence etc.

(criteria 2a), its placement toward the end of the question set, diminishes its role in contributing toward finding a solution to the problem (criteria 2b).

Overall, there was no clear link between the mathematical content targeted by a task and the quality of the task. There were also no perfect problems. While one was close, earning a score of 25 out of 26, all of them had some issue. This is particularly notable because these PSTs were at the end of their teacher education program. It highlights a lingering need for further preparation with both creating effective problems for their students and evaluating problems in accordance with a given criterion. This latter issue affects their ability to choose effective problems from pre-existing resources to use in their classes.

Findings – Evaluation of mathematical modeling criteria

An unexpected result from the researchers' coding process was concerns about the appropriateness of the adapted criteria for problems targeting early grade levels. While some low scores are indicative of the poor quality of the MMPs, in many cases, a low score did not necessarily mean that it was a bad problem, it just meant that it did not meet the established criteria. Conversely, the researchers were conflicted on a number of problems that earned high scores, but did not have the characteristics of a modeling problem. These problems typically were strong contextual word problems, but provided a mathematical representation or data set for students to analyze.

The two most controversial criteria were the ones with the lowest overall total mean scores – 2 and 6. While the example of building a classroom presented in Figure 3 above offers a legitimate case of an MMP with weaknesses in both of these areas, other examples seemed to fall short on these criteria because of a possible need to reconsider the thinking and mathematical reasoning typical of children in grades 1-3. Consider applying criteria 2a and 2b to the example in Figure 4. These criteria require the MMP to explicitly ask students to provide a mathematical representation of the problem that can contribute toward finding a solution.

Figure 4. Sample MMP (2)

In this example, the MMP does not explicitly require students to represent the situation mathematically. Therefore, scores of 0 for both criteria would be considered appropriate, and while this is a contextually-rich word problem, it would not seem to be an MMP. That said, to test their concerns regarding whether or not a grade 3 student could complete this problem without some form of mathematical representation, the researchers asked three separate grade 3 students from three different countries to complete this problem. In all cases, the students immediately began creating a table to model the problem – a strategy that is consistent with the sample solution provided by the PSTs for this problem (see Figure 5).

Figure 5. Sample solution provided by PSTs for Sample MMP (2)

Consequently, while requiring students to create a mathematical representation of the situation that contributes toward finding a solution is part of the criteria for an MMP, this may look different in contexts aimed at children in early grades. This makes evaluating a problem with these criteria dependent on the additional knowledge of developmentally appropriate expectations of children's mathematical reasoning in these grades. Thus, in this case, even though the problem does not explicitly ask for the context to be represented mathematically, the sample solution provided by teachers demonstrated their expectations of the implicit need for a grade 3 learner to create a model to be able to solve this problem.

Similar concerns arose regarding the suitability of criteria 6a, 6b and 6c. This requirement for scaffolding questions was motivated by Galbraith's (2007) Didactical Design Principle which says that a modeling problem

> …**may** be structured into sequential questions that…**may** be given as scaffolding hints **at the discretion of the teacher**, or be used to provide organized assistance by suggesting a line of investigation.

The words *may* and phrase *at the discretion of the teacher* appear in bold print here for the purpose of emphasizing nuances that the researchers did not incorporate into their adapted criteria. While low scores for 6a, 6b and 6c still legitimately reflect an area of PST weakness, the example presented in Figure 6 presents a case for adjusting these criteria items to allow for discretion or to allow the inclusion of scaffolding items as optional. Figure 6 offers an example of an MMP which raised these concerns. This MMP received scores of 0 for all of criteria 2 and 6 because the information was already represented mathematically in an included table and no scaffolding questions were provided. However, it is important to consider that this MMP was written for grade 1 learners.

Figure 6. Sample MMP (3)

As with the previous example the sample solution in Figure 7 acknowledges that, in most cases, learners at this level would still need to represent the information with number sentences to determine a solution.

Figure 7. Sample solution provided by PSTs for Sample MMP (3)

Upon reflection, the researchers also considered the possibility that concerns arising with regard to the suitability of the evaluation criteria could be an indication of limitations in the capacity for criteria adapted from design principles to be used for evaluation. The researchers' need to break the 6 design criteria down into 13 evaluation criteria was a factor in this concern.

Discussion and conclusions

The findings in this study highlight a need to consider a number of aspects relating to preparing teachers who can competently foster opportunities for children to meaningfully engage in mathematical modeling across the curriculum. The first aspect is the challenge of integrating mathematical modeling in early grade levels, highlighted by the PSTs' tendency toward creating problems for the highest of the grade-level options. While these findings raise this issue in the South African context, literature in this area shows that it extends far more broadly throughout the world (English & Watters, 2004; Fox, 2006).

There are a number of ways to interpret the PSTs' inclination toward creating problems for the highest of the three grade choices. One can argue that this may simply reflect the distribution of overall grade preference among the group, independent of the mathematical modeling context of this study. This is a limitation of the study and supports a recommendation that seeking insight into the PSTs' rationale for choosing a particular grade level may enhance the results of similar future research. Regardless, the uneven distribution of grade-level choices and tendency toward higher grade levels is consistent with what the researchers have found to be a notably small body of research and limited mathematical modeling resources specifically designed for teachers of early childhood grade levels (Garcia & Ruiz-Higueras, 2010).

Conclusions and implications based on these findings require consideration of multiple scenarios. On one hand, the PSTs' tendency toward higher grade levels when creating MMPs could be a result of the limited availability of examples to help conceptualize what characterizes mathematical modeling at the earliest stages in children's development of mathematical thinking. On the other hand, the fact that only two of the MMPs were created for grade 1 could also be considered a microcosm of the widespread issue that, despite researchers' recommendations that children should engage in mathematical modeling early in their mathematics education, teachers need more preparation and support in meeting this challenge. This issue has been acknowledged for over a decade (English & Watters, 2004; Usiskin, 2007), and this study confirms a persistent need for it to be better addressed both in teacher education coursework, and with more extensive and widely available resources for teaching modeling in these early grades.

In addition, while the two MMPs created for grade 1 were strong mathematical problem solving tasks, they were both among those which raised questions regarding what constitutes mathematical modeling in early grade levels. This fueled concerns about whether it is feasible to apply criteria for MMPs consistently across all grade levels. It also highlights a need for continued efforts among researchers to offer well-established conceptualizations of what mathematical modeling consists of in grades 1-3 and how this differs from the many conceptualizations of mathematical modeling prevalent in both research and curricular documents created for later stages of education when students have a greater capacity for advanced mathematical thinking. The researchers' struggle to evaluate the grade 1 MMPs by the standards set out in the criteria prompted them to re-consider the capacity for other well-established and widely-applied design and evaluation principles to be applied in early grades, such as Lesh et al. (2000). Ultimately, this further supported the conclusion highlighting a need for attention and further research aimed at increasing both the conceptual and practical mathematical modeling resources available to early grade educators.

In addition to the grade level choices in this study, the PSTs' choices regarding the content of their MMPs and their efforts to align them with the mathematical content of the curriculum suggest two important things for mathematics teacher education. The first is that, despite the unacknowledged presence of mathematical content, only two PST teams chose to create MMPs involving Space and Shape, and only three chose to create MMPs involving Patterns and Early Algebra. This suggests a need for more targeted efforts and experiences designed to increase PSTs confidence in these areas and awareness of how they can be incorporated into MMPs. This could involve exposing PSTs to a broader range of MMPs with opportunities to explicitly draw connections between these MMPs and the content of their curriculum. Not only can this serve to increase their capacity to create problems that support the learning and application of content across the curriculum, it can also serve to increase their appreciation for the value of mathematical modeling in fostering meaningful learning in all areas of mathematics.

Such experiences for PSTs can also help to address the second concern regarding their ability to align the mathematical content in their MMPs with the curriculum. While in this case, there was a genuine lack of MMPs created for Patterns and Early Algebra, the more alarming result for Space and Shape was the number of MMPs that included content in this area without the PSTs realizing it. Overall, the PSTs' recognition and alignment of the mathematics involved in their MMPs only exactly matched the researchers' coding of this content in 17 of the 42 problems. Therefore, in 60% of the cases, the PSTs missed or incorrectly identified at least one area of the curriculum that was relevant to their problem. This has two potential implications. This first is a weakness in PSTs' understanding of the content that is involved with each area of the curriculum. The second is that the uneven content distribution may suggest a limited conception of what mathematical content is conducive to being incorporated into MMPs.

Overall, the findings indicate that this group of PSTs seem most aware of the potential for creating modeling tasks that involve Measurement and are relatively confident in creating such problems (although confidence did not always indicate proficiency). The same cannot be said for Space and Shape. In 15 of the 17 MMPs (88.2%) involving Space and Shape concepts, the PSTs failed to align their MMPs with the Space and Shape area of the curriculum. Instead, they were mainly classified as Measurement and Numbers, Operations and Relationships problems. The lack of recognition of this content area combined with the fact that, in only two cases, student teachers created problems which they believed involve Space and Shape content indicates that this is a section of the curriculum that may warrant more focus to increase both student teachers' familiarity with what content is included in this strand and their confidence in creating and identifying problems involving such content.

Similar concerns arise regarding Patterns and Early Algebra. Not only did this content go unrecognized in more tasks than were correctly aligned with this area of the curriculum, only 3 of the 42 MMPs were considered by their PSTs to involve Patterns and Early Algebra. This raises questions about the PSTs' comfort level with this portion of the curriculum. Usiskin (1997) raised this issue regarding the teaching of Algebra. He reported that elements of algebra are being taught and learned in grades K-4 without the teacher realizing it.

Across the content areas, concerns about PSTs' ability to align their MMPs with the curriculum are not only relevant to their capacity to create curriculum-aligned MMPs, but also to recognize and make pedagogical choices regarding the use of existing MMPs to support learning across the curriculum. While these findings reflect the work of PSTs in a single teacher preparation program, and are specific to the context of mathematical modeling, they support an important recommendation for teacher preparation programs worldwide. Given these PSTs' experience with mathematical modeling across three years of their program, such findings demonstrate the need for explicit attention to drawing curricular connections, as well as the importance of strategic and authentic evaluation of PSTs' practical learning to ensure that coursework and experiences are best supporting desired outcomes.

These findings also suggest a need for research to consider whether this result is specific to the mathematical modeling context, or if more explicit focus is needed in general to support PSTs in creating and selecting tasks, problems and learning experiences that are appropriately aligned with their curriculum. Ball, Thames and Phelps (2008) identify knowledge of content and curriculum as a key domain of mathematical knowledge for teaching. Thus, the results of this study reveal an area of need that is of high priority in the efforts of teacher education programs to support development of future teachers' mathematical knowledge for teaching. In a worldwide education culture of curricular reform and globalization, it is important to develop PSTs' understanding of best practice, independent of specific curricular documents. However, this study also demonstrates the need for teacher education to spend time with contextually relevant curriculum documents to ensure that PSTs understand the nature of the content included in each area of the curriculum and the potential for that content to be applied and developed in the context of an MMP.

A third aspect of the findings relating to teacher preparation comes from scoring the MMPs and addresses the third research question regarding how proficiently these PSTs were able to create MMPs for Foundation Phase children, in accordance with the given criteria. Overall, the mean scores for each item on the rubric were lower than should be expected of PSTs at the end of their preparation program and establish a general need for further development in designing MMPs. While the researchers acknowledge that some things are hard to teach outside of a mathematics classroom, the PSTs' strengths turned out to be aspects that require the most experience with children. For instance, PSTs were most successful with creating appropriate and motivating contexts. This positive outcome demonstrates a valuable creativity and awareness of problem contexts that are relevant and motivating to their students and places increased value on their capacity to extend such contexts into effective MMPs.

Despite specific concerns regarding the suitability of related rubric items (criteria 2a, 2b and 6a), the results still indicate a particular need to focus on two key areas of weakness. This first is PSTs' ability to create problems which require learners to create mathematical representations of real-life contexts (criterion 2). The second is posing strategically sequenced questions which serve to both scaffold the modeling process and help the learner to make sense of the mathematics involved with the problem, in order to reach a solution (Criterion 6).

These concerns are particularly urgent given the increased emphasis being placed on mathematical modeling in curricular reform efforts throughout the world (Paolucci, 2015; Vos, 2011). In addition, while the first finding is specific to the context of mathematical modeling, the second finding is not. A teachers' ability to create strategically sequenced questions in order to promote scaffolded learning is an essential component of their capacity for effective teaching (Ball et al., 2008). Given that these PSTs were at the end of their teacher preparation program, the implications of these findings extend beyond just pre-service teacher education and support a recommendation that professional development programs for practicing teachers should also consider offering further support in these areas.

In addition to conclusions regarding the PSTs' preparation for teaching, concerns raised about the criteria used to create and evaluate the problems are also an important outcome of the study. These concerns have caused the researchers to reconsider the use of this framework in future research involving modeling with early grade levels. This was most evident in cases where the researchers considered MMPs to be of a higher quality than was reflected by their score and for MMPs which received a high score, but the researchers did not initially consider them to be modeling problems. For instance, issues around Data Handling were less about the PSTs' ability to construct a quality mathematics problem and more about their ability to construct a modeling task which involved this content area. In fact, many of the data handling problems were among what the researchers considered to be some of the best mathematical tasks, but they did not have the components of a modeling problem. They were more about interpretation of existing data than about creating and interpreting a mathematical representation of a particular context.

Ultimately, the researchers' belief that these actually did constitute MMPs for children in the earliest stages of developing mathematical thinking and reasoning fueled concerns about the suitability of widely accepted principles, criteria and frameworks for modeling problems at lower grade levels and earlier stages of development. Concluding that it may be unreasonable to expect a set of criteria for MMPs to span all levels of modeling, especially when working with young children, is consistent with Usiskin's (2007) explanation that "mathematical modeling begins in the early primary grades, even though the language and ideas of mathematical modeling are not employed" (p. 257). In fact, Usiskin (2007) classifies common arithmetic operations as mathematical models for a variety of real-world situations involving counting and measurement, noting that they "provide the basis for more sophisticated mathematical models found in algebra, geometry, analysis and statistics" (p. 257).

Final comments

The multidimensional results of this study offer both further support for existing research and contributions toward further research and program development in early childhood mathematics teacher education. While some existing literature offers valuable research and resources focused on mathematical modeling with children (English & Watters, 2004; Garcia & Ruiz-Higueras, 2011; Maaß & Gurlitt, 2011; Mousoulides & English, 2011), the PSTs' tendency to target the highest grade option in this study highlights the potential value of further research, resource development and explicit focus on mathematical modeling in early childhood education. This applies to the construction and implementation of both MMPs for early childhood mathematics education and criteria for the creation and evaluation of MMPs for these early grade levels.

Across all grade levels, researchers have noted that PSTs have difficulty engaging in problem posing (Crespo & Sinclair 2008). The problems they pose tend to lack cognitive or structural complexity (Stein, Smith, Henningsen, & Silver, 2000; Vacc, 1993) and often do not align with targeted mathematical concepts (Osana & Royea, 2011). Alignment of targeted mathematical concepts was a clear issue arising from this study. In this case, the PSTs' struggle to link their MMPs to the curriculum content areas invites further research into PSTs' capacity to recognize and understand the content involved with areas of the curriculum. It also alerts teacher educators to PSTs' potentially limited conceptions of how real-life problem contexts can incorporate particular mathematical content. As seen by Hošpesovà and Tichà (2015), analysis of MMPs posed by teachers in this study also reveal deficits in knowledge, along with "misconceptions, misunderstandings, and shortcomings" (p. 439).

Admittedly, the very nature of mathematical modeling generally allows for more than one way to solve an MMP, and in this context, it may not always be necessary to pre-determine the mathematical content that should be used to solve an MMP. However, the importance of these conclusions lies in the fact that if a teacher does not see the potential for modeling to be used to support application and development of mathematical knowledge for particular grade levels and content areas, they will likely not be motivated to incorporate mathematical modeling into their teaching of these areas or at these grade levels.

While the PSTs in this study were relatively successful with creating appropriate and motivating contexts, they were far less proficient with turning these contexts into an appropriate problem and asking strategically sequenced questions to offer scaffolding and promote meaningful learning. This has implications for both posing MMPs and problem posing in general. Similar to the disconnect noted by Crespo (2003) between PSTs' mathematical knowledge and limitations in their problem posing practices, this study highlights ways in which even programs with a well-integrated emphasis on mathematical modeling can benefit from more explicit focus on mathematical modeling across all areas of the curriculum.

An ideal next step for addressing MMP development in teacher education would be to facilitate an activity like this earlier in a program and bring these MMPs back to the PSTs to critically examine and discuss strengths, shortcomings and ways to improve the MMPs. Tichà and Hošpesovà's (2010) encourage that this be done both individually and collectively. Another recommendation would be to have students at the intended grade level attempt to solve the problems and have PSTs analyze the mathematical content that they choose to use in their efforts to find a solution. This can help to expand PSTs' thinking around using modeling problems to support all areas of the curriculum, and is consistent with Crespo's (2003) endorsement of problem posing experiences in which PSTs have the opportunity to both create problems and try them out with real students.

Overall, the challenge of using MMP design criteria to evaluate MMPs is an issue worthy of broader consideration. In particular, the results of this study establish a need to revise the associated criteria before any future use, with specific consideration given to criteria 2 and 6. The results also emphasize the need to combine existing criteria with knowledge of what is developmentally appropriate for children in lower grade levels. This encourages further consideration and identification of distinct elements of criteria appropriate for use in creating and evaluating modeling tasks specifically designed for young children.

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Tables and Figures:

Galbraith's (2007) Design Principles	
Principle 1:	Some genuine link(s) with the real world **of the students**.
Principle 2:	Opportunity to identify and specify **mathematically tractable questions** from a general problem statement.
Principle 3:	Formulation of a **solutions process** is feasible, involving the use of **mathematics accessible to students**, the making of necessary assumptions, and the assembly of necessary data.
Principle 4:	**Solution** of the mathematics for the basic problem is possible, together with **interpretation**.
Principle 5:	An **evaluation procedure** is available that enables checking for mathematical **accuracy** and the **appropriateness** of the solution with respect to the contextual setting.
Didactical Design Principle:	The problem may be structured into **sequential questions** that retain the integrity of the real situation. (These may be given as **scaffolding hints** at the discretion of the teacher, or be used to provide **organized assistance** by suggesting a line of investigation)

Table 1: Principles for the design of mathematical modeling problems (MMPs)

Criteria	Guiding Principle or Other Rationale
1. Has a genuine link to the real world with the aim of motivating students and illustrating the relevance of mathematics in their lives	Principle 1
2. Requires students to represent the context mathematically through a picture, graph, table, number sentence etc. that will help to answer related questions	Kuntze (2011)
3. Contains all information necessary to formulate a solution process which involves mathematical content, reasoning and assumptions that are age and developmentally appropriate	Principle 3
4. Has one or more feasible solution(s) and will allow students to draw conclusions about the problem context	Principle 4
5. Elicits representations and solutions which can be evaluated for accuracy and appropriateness with respect to the problem context	Principle 5
6. Offers an opportunity for scaffolded learning that preserves the reality of the context and promotes meaningful learning	Principle 2; Didactical Design Principle

Table 2: Criteria used for creation of modeling problems with guiding principles and research

Target Grade Level	**# of Problems**
Grade 1	2
Grade 2	9
Grade 3	31

Table 3. Chosen grade levels for MMPs

Content Alignment Score	**3:** Matched the researchers' coding exactly	**2:** Mostly matched the researchers' coding but one area was missing or incorrectly identified	**1:** Partially matched the researchers' coding but more than one area was missing or incorrectly identified	**0:** Did not match the researchers' coding at all
Frequency (N = 42)	17	14	7	4
Percentage	40.5%	33.3%	16.7%	9.5%

Table 4. Content alignment of MMPs

Content Area	Numbers, Operations & Relationships	Patterns & Early Algebra	Space & Shape	Measurement	Data Handling
Frequency with which content was **correctly identified**	14	3	2	18	5
Frequency with which content was required but **not identified**	10	4	15	3	1
Frequency with which content was required but **misidentified**	1	0	0	0	1
Total frequency with which content was involved in an MMP*	24	7	17	21	6
Proficiency rate with which content was correctly identified *(by curriculum area)*	58.3%	42.9%	11.8%	85.7%	83.3%

**Excludes misidentified cases*

Table 5. Frequency of mathematical content in the MMPs

Label	Criterion
1a	Has a genuine link to the real world that illustrates the relevance of mathematics in their lives
1b	Offers an age appropriate context with the aim of motivating students
1c	Presented at an age appropriate reading level
2a	Explicitly asks students to represent the context mathematically through a picture, graph, table, number sentence etc.
2b	The required representations can contribute toward finding a solution to the problem
3a	Contains all information necessary to formulate a solution process
3b	Involves mathematical content, reasoning and assumptions that are age and developmentally appropriate
4	Has one or more feasible solutions
5a	Elicits representations and solutions which can be evaluated for *accuracy* with respect to the problem context
5b	Elicits representations and solutions which can be evaluated for *appropriateness* with respect to the problem context
6a	Related questions are appropriately sequenced to promote scaffolded learning
6b	Related questions offer scaffolding which supports students in progressing toward a solution and drawing conclusions about the problem context
6c	Related questions preserve the reality of the context and promote meaningful learning

Table 6. Criteria used for scoring the MMPs, based on Galbraith (2007)

Criterion	1a	1b	1c	2a	2b	3a	3b	4	5a	5b	6a	6b	6c
Raw Score Total	65	56	60	34	28	49	52	65	59	65	36	34	40
Proficiency Rate (%)	77.4	66.7	71.4	40.5	33.3	58.3	61.9	77.4	70.2	77.4	42.9	40.5	47.6

Table 7. Overall results for each criterion

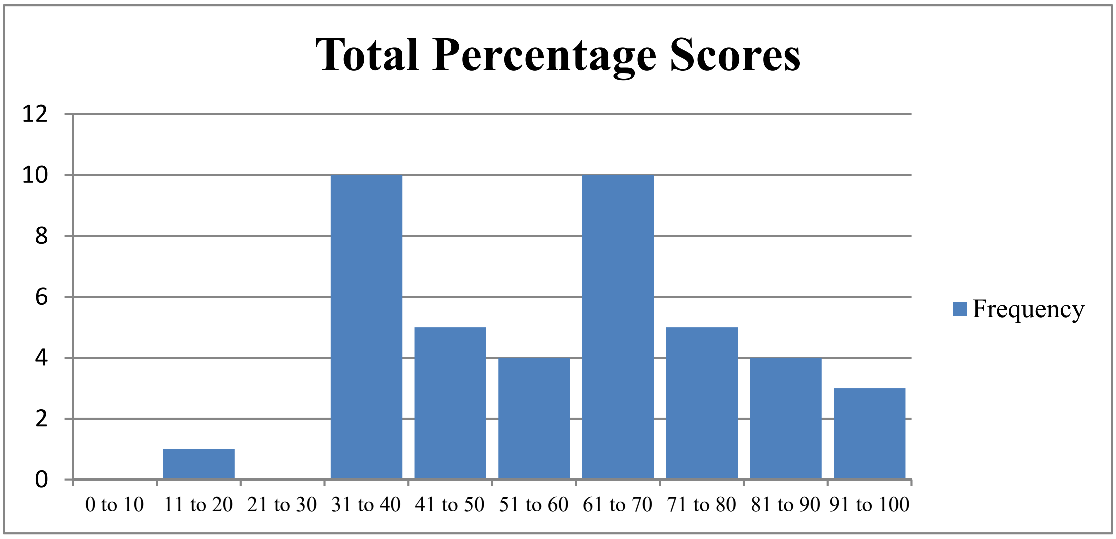


Figure 1: Distribution of total percentage scores for the MMPs

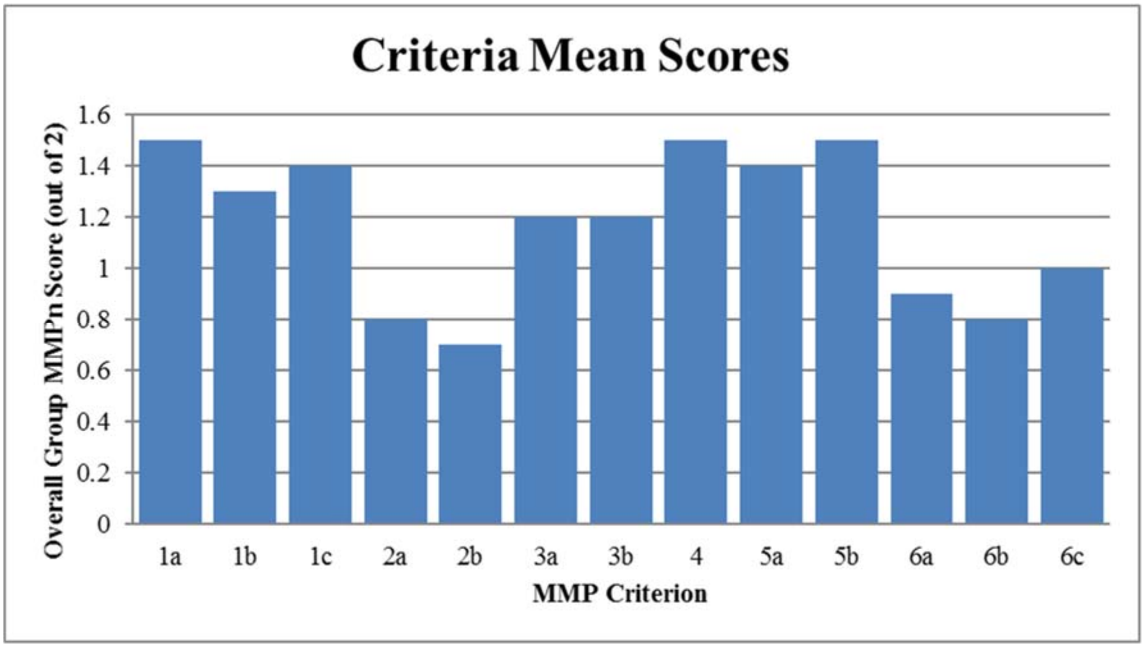


Figure 2. Overall mean scores for each MMP criterion

The school wants to build a new Grade R classroom for next year. It needs to be big enough to ~~take~~ seat 30 learners. Help the school to determine what they need.

You need to help them answer to the following questions:

1. How big should the classroom be so that the 30 children and teacher will fit in? (You have an max square meter of $72\,m^2$.)

2. Calculate how many bricks you will need to be able to build the classroom. Remember you need four walls and hight, thus length, with and hight. Take all three aspects into consideration.

3. How long will it take them to build the classroom with 2,3,4,5, 6 and 7 people? Draw a graph to explain your answer.

4. Design a classroom so that theres space for chairs, play area, tables and a carpet. (Draw your floor plan)

5. Will your classroom be large enough if they will need to fit more than 30 learners? How much more learners will be able to fit?

Figure 3. Sample MMP (1)

Early algebra: number concept development: money

Grade 3

Topic: saving pocket money

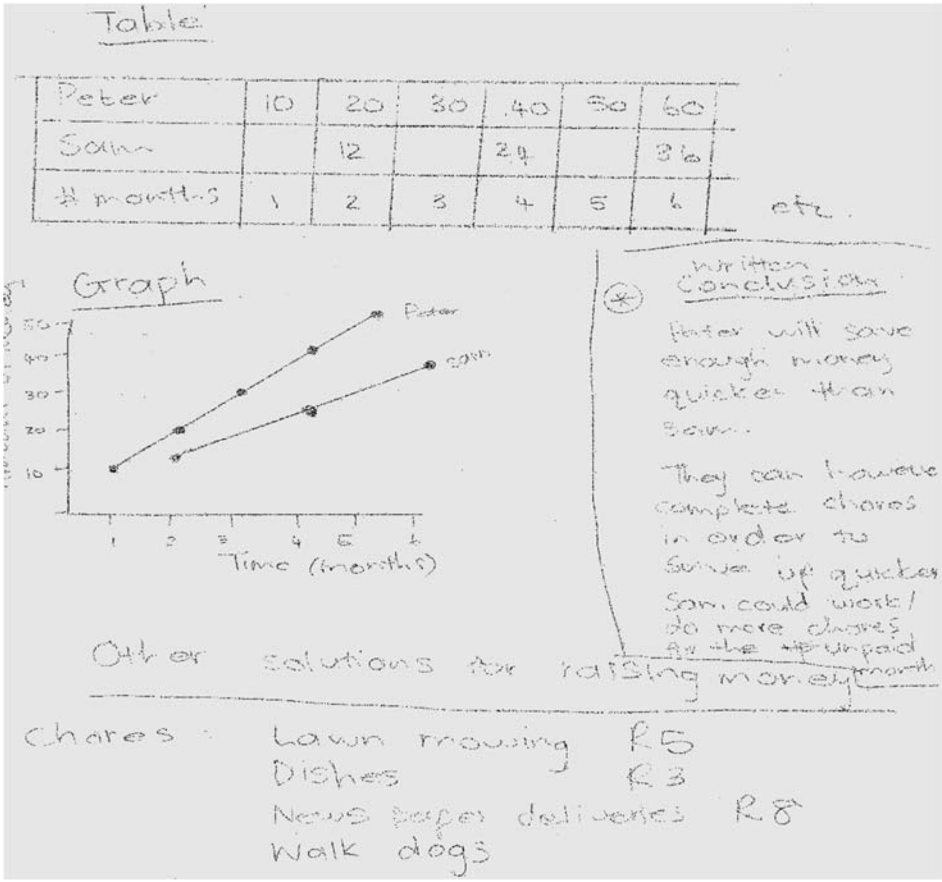
Sam and Peter want to save up for a soccer ball. A soccer ball costs R100.

Peter gets R10 per month.
Sam gets R12 every two months.

Who will be the first to save up for the soccer ball?

Think of different ways in which Peter and Sam could save enough money within 3 months.

Figure 4. Sample MMP (2)



Table

Peter	10	20	30	40	50	60
Sam		12		24		36
# months	1	2	3	4	5	6

etc.

Written Conclusion

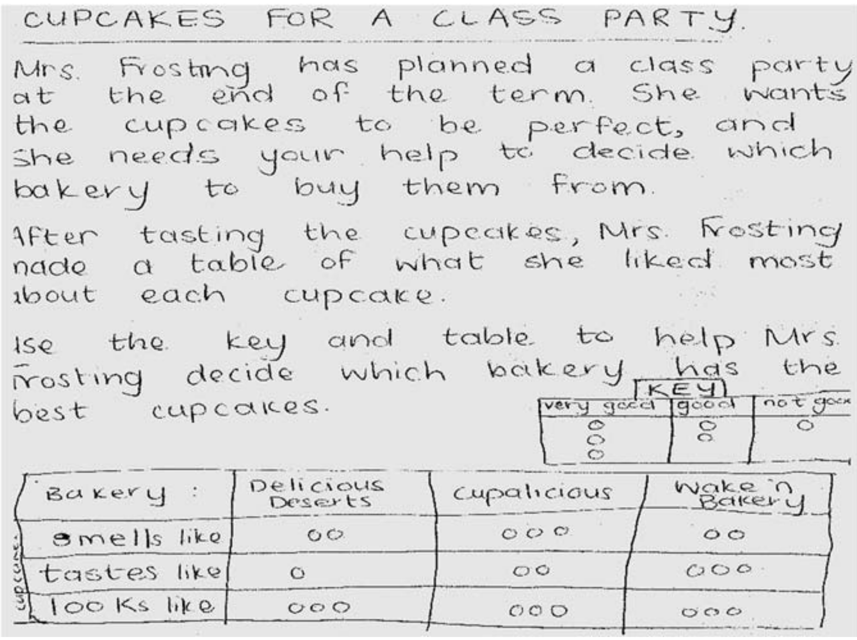
Peter will save enough money quicker than Sam.

They can however complete chores in order to save up quicker. Sam could work/ do more chores for the unpaid month

Other solutions for raising money

Chores : Lawn mowing R5
Dishes R3
News paper deliveries R8
Walk dogs

Figure 5. Sample solution provided by PSTs for Sample MMP (2)



CUPCAKES FOR A CLASS PARTY.

Mrs. Frosting has planned a class party at the end of the term. She wants the cupcakes to be perfect, and she needs your help to decide which bakery to buy them from.

After tasting the cupcakes, Mrs. Frosting made a table of what she liked most about each cupcake.

Use the key and table to help Mrs. Frosting decide which bakery has the best cupcakes.

KEY

very good	good	not good
ooo	oo	o

Bakery :	Delicious Deserts	Cupalicious	Wake 'n Bakery
smells like	oo	ooo	oo
tastes like	o	oo	ooo
looks like	ooo	ooo	ooo

Figure 6. Sample MMP (3)

- Delicious Deserts = 2 + 1 + 3 = 6 points
- Cupalicious = 3 + 2 + 3 = 8 points
- Wake 'n Bakery = 2 + 3 + 3 = 8 points

Cupalicious and Wake 'n Bakery both score high points, but we think Mrs Frosting should buy her cupcakes at Wake 'n Bakery because we think it is more important for the cupcake to taste very good than smell good.

Figure 7. Sample solution provided by PSTs for Sample MMP (3)