Credit and Debit Value Adjustment Estimations in the Data Sparse South African Market

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Abstract

During 2014, the International Accounting Standards Board (IASB) implemented a new standard for measuring the fair value of assets through the International Financial Reporting Standards (IFRS) 13 guidance. The newly introduced guidelines have probed market participants to adjust their valuation of financial positions for material counterparty credit risk (CCR) in the over-the-counter (OTC) market

Five different models are implemented in this research for the purpose of calculating the credit value adjustment (CVA) and debit value adjustment (DVA) of an interest rate swap portfolio between a South African corporate treasurer, Eskom, and a generic South African tier 1 bank. The models differ from simple to complex. The Monte Carlo (MC) simulation model is assumed to be the most accurate, since it involves the simulation of expected exposure and the modelling of the short-rate.

Corporate treasurers do not always have the necessary resources to calculate CVA by means of a sophisticated approach. Due to input data and resource challenges, corporate treasurers need to consider creative alternative methods to include CCR in their fair value adjustments. Therefore, semi-analytic methods and input approximation methods were considered in this research. It was found that simpler semi-analytic approximation methods do not possess the complexity needed to deal with the complexity of netting and collateral agreements. They serve as good approximations to quickly estimate a ball-park CVA, but lack the accuracy of the MC based approach.

Opsomming

Die *International Accounting Standards Board* (IASB) het gedurende 2014 ‘n nuwe standaard geïmplementeer ten opsigte van die meting van die billike mark-waarde van bates onder die nuwe *International Financial Reporting Standard* (IFRS) 13 leiding. Hierdie nuwe leiding het mark belanghebbers gepeil om aanpassings te maak tot hul finansiële posisies ten opsigte van teenparty kredietrisko in die oor-die-toonbank mark.

Vyf verskillende modelle word in hierdie studie geïmplementeer vir die berekening van kredietwaardeaanpassing en debietwaardeaanpassing, van ‘n portefeulje bestaande uit rentekoers uitruilkontrakte tussen die Suid-Afrikaanse korporatiewe tesourier Eskom en ‘n generiese Suid-Afrikaanse vlak 1 bank. Die modelle wissel van eenvoudig tot kompleks. Die Monte Carlo model word aanvaar as die mees akkuraatste, vanweë sy komplekse onderliggende modellering van die kort-rentekoerse, asook sy onderliggende verwagte krediet blootstelling simulasie.

Korporatiewe tesouriers beskik dikwels nie oor die nodige hulpbronne om kredietwaardeaanpassing s te bereken met ‘n gesofistikeerde model nie. As gevolg van data en ander hulpbron uitdagings, berus dit op die korporatiewe tesouriers om met kreatiewe alternatiewe voorendag te kom vir die hantering van kredietwaardeaanpassings tot hul finanisiële posisies. Dus moet semi-analitiese metodes en data beramings ondersoek word. In die studie word gevind dat hierdie eenvoudiger semi-analitiese metodes nie oor die nodige kompleksiteit beskik om komplekse netting en kollateraal kontrakte, wat met baie afgeleide instrumente gepaard gaan, te hanteer nie. Hulle dien egter as goeie metodes om vining ‘n beraming van kredietwaardeaanpassing te bereken, alhoewel hulle nie so akkuraat is soos die meer kompleke Monte Carlo en Swaption modelle nie.

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DEFINITIONS

THEOREMS

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List of Abbreviations

AIC	Akaike Information Criterion
ASW	Asset Swap Spread
BIS	Bank for International Settlements
BPS	Basis-Points
CCR	Counterparty Credit Risk
CDS	Credit Default Swap
CLN	Credit Linked Note
CRA	Credit Rating Agency
CSA	Credit Support Annex
CVA	Credit Value Adjustments
DCF	Discounted Cash Flow
DVA	Debit Value Adjustments
EE	Expected Exposure
EPE	Expected Positive Exposure
ENE	Expected Negative Exposure
FVA	Funding Value Adjustments
IASB	International Accounting Standards Board
IFRS	International Financial Reporting Standards
ISDA	International Swaps and Derivatives Association
LGD	Loss Given Default
LLN	Law of Large Numbers
MC	Monte Carlo
NA	Netting Agreement
OTC	Over-the-counter
PFE	Potential Future Exposure
SOE	Sovereign Owned Entities
VaR	Value at Risk

CHAPTER 1
INTRODUCTION

The credit market has become an imperative facet in the dynamics of financial markets following the 2008 financial crisis. The financial world realised that no investment can be assumed to be risk-free and that counterparty credit risk (CCR) should be taken into account when embarking on business ventures or when entering into financial contracts. Aroro et al. (2012) notes, that, although CCR has been around for a decade before the crisis, it became apparent that very few people fully understood CCR or indeed applied it correctly.

During 2014, the International Accounting Standards Board (IASB) implemented a new standard for measuring the fair value of assets through the International Financial Reporting Standards (IFRS) 13 guidance. The latest version of the guidance has been effective for any annual financial reporting period beginning at or after 1 July 2014. IFRS 13, Fair Value Measurement, defines the fair value of financial instruments as the price that would be received from selling an asset or paid to transfer a liability in an orderly transaction between two market participants at the measurement date under current market conditions (IFRS 13: B2).

IFRS 13 states that financial institutions must account for counterparty default risk and own non-performance risk into its fair value measurements through fair value adjustments. These fair value adjustments form part of a set of adjustments often referred to as xVA, which includes (but not limited to) credit value adjustments (CVA), debit value adjustments (DVA) and funding value adjustments (FVA). CVA refers to the adjustment made to include the counterparty's credit risk into the fair value measurement of an asset, whilst DVA assesses the impact of its own. Only CVA and DVA will be considered in this research.

Four of the leading professional services firms in the financial world, namely EY (2014), Deloitte (2013), PWC (2012) and KPMG (2012) have each published a fair value measurement paper wherein the firms discuss different methodologies to estimate CVA and DVA. The problem is that the introduced methodologies and models are explained in very broad terms and have little academic research or rigorous analysis accompanying them. Some of the models introduced are extremely complex, while others are utterly basic. Deloitte (2013:3) states that judgement will be required to determine the most appropriate approach under particular circumstances. This notion is supported by Pykhtin (2010) who proposed a semi-analytical method for calculating collateralised expected exposure (EE), which is proven to outperform some rigorous Monte Carlo driven alternatives.

The choice of method could proof difficult for corporate treasurers where underlying modelling data is sparse, technology resources are insufficient, and knowledge in complex derivatives are limited. Due to the relatively small size of corporate treasurers' financial instrument portfolios, it is in most cases more efficient to use a simpler semi-analytical approach which requires less programming power. When considering treasurers based and operating in developing countries, such as South Africa, this typically proves to be more valid. South Africa has a shortage in quantitative skills and most of the available skills are lured to big banks. According to Kumarasiri and Fischer (2011) developing countries also experience more difficulty due to inactive markets causing unreliable fair value estimates with large bid-offer spreads. These markets are also more easily influenced by a few market participants and transactions. This research sets out to answer the following question:

Which Credit Value Adjustment and Debit Value Adjustment estimation methodologies are more appropriate to use for a corporate treasurer in the South African market under particular circumstances?

The above stated question can be divided into three sub questions within the context of this research:

- What is an appropriate probability of default estimation methodology to use for a corporate treasurer in the South African market under particular circumstances for the purpose of calculating a trade's debit value adjustment?
- What is an appropriate probability of default estimation methodology to use for a tier one bank in the South African market under particular circumstances for the purpose of calculating a trade's credit value adjustment?
- Which expected exposure estimation methodologies are more appropriate to use for a corporate treasurer in the South African market under particular circumstances for the purpose of calculating a trade's credit and debit value adjustments?

The research will be commenced by a detailed literature review focussing on different researched application for a CCR world. The literature provides the theory and background needed to calculate xVA. The research will be conducted from the point of view of a South African corporate treasurer that needs to include non-performance adjustments in their fair value measurements when reporting financial positions in their financial statements. Due to the lack of most corporate treasurers' technical ability, sparse data and lack of knowledge in complex derivatives, more simple techniques for estimating value adjustments are considered. A fictitious portfolio consisting of interest rate swaps is created and evaluated. This is the instrument predominantly traded by corporate treasurers to manage interest risk arising from day-to-day business operations. The theory and application behind interest rate swap valuation is valid for other more basic derivatives such as forwards or contracts with straightforward known market variables underlying the derivatives.

Under IFRS 13, a 'fair value hierarchy' is defined which specifies which model inputs should ideally be used. Within the guidance it stipulates that observable inputs are preferred above approximated or unobservable ones. It also stipulates that the inputs used, should be a realistic presumption of what other market participants would use in their calculations. The research will adhere to these guidelines as far as the developing South African market allows it. All observable data will be sourced from Bloomberg, for which the University of Stellenbosch has a licence.

The methodology for estimating credit and debit adjustments is threefold, since the estimation consists of three components: loss given default, expected exposure and probability of default. That is,

$$CVA = LGD \times Exposure_i \times Probability\ of\ Default_i \times Discount\ Factor_i$$

Where i represents the time point i, valuation time point $\leq i \leq$ contract maturity

Loss Given Default: This component will be assumed by considering different estimates regularly applied by active market participants.

Expected Exposure: Future exposure underlying derivative contracts will be estimated by making use of five different methodologies based on models presented by Ernst and Young (2014). It is important to keep in mind that DVA is calculated on a similar way to CVA, the only difference being a change in counterparty perspective.

Probability of Default: Exposures are weighted by the probability of default at time points throughout the lifetime of financial contracts. It will be estimated by initially estimating credit spread curves for tier one South African banks through a multiple regression analysis and credit spread curves for local corporate treasurers by calibrating to traded bond zero coupon spreads or prices. Flannery et al. (2010) found that using credit spreads as a probability of default measure is a better alternative than using credit ratings, mainly due to the dynamic nature of credit spreads. This notion fits suitably into the IFRS guidelines regarding model input selection.

Hazard rate curves will be stripped from the credit curves in order to estimate probability of default and survival over any definable time interval within the lifetime of a contract. The goodness of these probability estimates will be measured by comparing them to the markings of active South African banks.

After all of the components are estimated, the research will conclude by combining the three components to arrive at credit and debit value adjustments. The results will be compared to rigorous Monte Carlo estimates.

The outline of this research paper is presented below:

- In Chapter 2 in a literature review is presented which introduces the theory and background needed to calculate CVA and DVA.

- In Chapter 3 the methodology and results are presented for the estimation of *own* probability of default measures from the perspective of a corporate treasurer in the South African market. The estimation approach is based on credit default spread approximation by using observed zero-coupon bond spreads and active trading bond prices.

- In Chapter 4 the methodology and results are presented for the estimation of *counterparty* probability of default from the perspective of a corporate treasurer in the South African market. The estimation approach is based on credit default spread approximation through multi-linear regression.

- In Chapter 5 the different estimation methods of CVA and DVA, as introduced in Chapter 2, are evaluated by making use of a hypothetical portfolio of interest rate swaps and inputs estimated in Chapters 3 and 4.

- In Chapter 6 the results of the CVA calculations of Chapter 5 are discussed and compared.

CHAPTER 2
LITERATURE REVIEW

In this literature review, theory and background needed to calculate CVA and DVA are provided. As stated in the previous chapter, credit type value adjustments have become a prominent feature of derivative valuation after newly introduced guidance by the IASB.

Firstly, a general introduction is presented covering the background of the over-the-counter financial trading market and the inclusion of xVA within a credit risk exposed world. The conceptual background and mathematical theory applicable to CRR related metrics and definitions form the topic of the second section. This section comprises of derivative pricing fundamentals and the essential building blocks of xVA, namely exposure, probability of default and LGD. Exposure mitigating strategies are also presented as a subsection. In the third section pricing methodologies of interest rate swaps and swaption derivative products are presented. This includes the introduction of interest rate modelling and Monte Carlo (MC) simulation methods, which form part of the pricing mechanisms of these types of instruments.

The pricing methodologies will help estimate the exposure of the derivatives for the research's hypothetical portfolio, which in turn is used for CVA/DVA is calculations in Chapter 5. In the last section of this chapter the pricing principals of CCR are formally presented which includes the aforementioned LGD, exposure and probability of default components.

2.1 INTRODUCTION

2.1.1 THE FINANCIAL MARKETS

Financial derivatives are financial products that are traded by investment banks, private investors, corporate treasurers and fund managers. Their values are determined by more basic underlying variables which typically follow dynamic processes over time. These variables often take on the form of the prices of traded financial instruments or other observable or implied market variables. Financial derivatives can however be constructed using any measurable underlying, even non-financial, such as the prices of corn or the amount of rainfall in a specified period of time. In order to fully understand credit type adjustments required by IFRS 13, what is being adjusted should be understood. Bingham (2013:2-3) provides the following formal definition,

Definition 1.1 (Financial Derivatives) *A derivative security, or contingent claim, is a financial contract whose value at expiration date T is determined exactly by the price (or prices within* *a pre-specified time interval) of the underlying financial assets (or instruments) at time T within the time interval [0,T].*

There are ample reasons for an entity to trade financial derivatives. This research evolves around the valuation of derivatives and subsequent compulsory credit adjustments from the perspective of a South African corporate treasurer. Hull (2009:9-14) states the reason why corporate treasurers enter into derivative transactions can be classified either under hedging, speculation, or arbitrage. Hedging refers to the strategy of protecting a current financial position against possible price movements and therefore this strategy's main purpose is to mitigate risk. Speculation occurs when a market participant believes that a certain underlying market variable will move in a particular direction or through a specific path over time. With this strategy, entities do not want to decrease their exposure, but they rather want to take a position in the market. Speculators enter into derivative contracts to make profit on their conceived beliefs regarding market movements. Derivatives are particularly popular for this purpose, since it is possible to effectively leverage a position or obtain exposure to financial assets without legal ownership. Leveraging enables traders to gain large amounts of exposure by placing relatively small amounts of investment capital.

Corporate treasurers are rarely speculators in the financial derivatives market as they tend to rely on their day-to-day operations to reach growth margins. Through market observation and empirical findings by Tufano (1996) and Brown (2001) it can be concluded that corporate treasurers most often act as hedgers to protect against operational resource price volatility. Bartram et al. (2003:13-14) conducted a study on US corporate treasurers and found that interest rate derivatives were the type of derivative actively traded by most firms (40.3% of firms). In a rest of the worldwide context, one can expect forex trading to carry a larger weight than interest rate derivatives due to conversions to and from the universally traded USD currency. In this research, however the focus will be on interest rate derivatives, due to their more complex nature. Note here, however, that the underlying principals discussed in this research are easily transferrable to other linear derivative classes.

The derivatives market has grown considerably over the past fifteen years and are traded either in the form of standardised structures in regulated exchanges or in the over-the-counter market. The over-the-counter market is a customised financial market where the characteristics of derivative products are constructed and decided upon by the entities bound under the derivative's transaction contract.

From these early years of derivatives trading the industry has changed considerably. The open outcry system has been replaced by electronic trading where counterparties are easily paired up via electronic platforms. New derivative structures are created yearly and introduced into the market as the market participants' need change due to market trends or newly introduced guidelines. In the modern financial era of derivatives trading derivative contracts are often designed to reference the change of underlying financial variables and the delivery of physical assets are in most cases avoided through inherent product design. Netted cash payments are generally exchanged between counterparties as a monetised amount.

The over-the-counter market has surpassed standardised exchanges in terms of trade volume in recent years, since it has become easier to find counterparties willing to enter into customised contracts which suit both parties' needs. Liquidity is ever improving with all the different types of traders flooding the market. In the Bank for International Settlements' (BIS) December 2016 study of the OTC market, it is noted that swaps are the most traded interest rate derivative, with a $311bn notional outstanding against $418bn in total. Interest rate swaps was also found to be the largest financial derivative across all types.

The development of the OTC however brings with it an inherent risk. Credit risk arises in these semi-unregulated OTC markets. This is the risk that the counterparty in a transaction is not able to meet all of its financial obligations. This risk is referred to as CRR and will be formally introduced in the following subsection. According to new guidelines market participants need to adjust their fair-value valuations to include this risk in their financial pricing and reporting.

2.1.2 COUNTERPARTY CREDIT RISK (CCR)

In this subsection a brief introduction on credit risk, and the impact thereof, will be provided. As mentioned in the previous subsection, market participants need to adjust their fair-value measurements according to embedded credit risk within OTC derivative transactions.

It is possible to measure this embedded credit risk by considering the credit market. The credit market is simplistically the market where credit exposure is exchanged through credit derivatives. Participants in the market issue new debt in order to fund expansion projects or to pay for current operational costs. This debt is then traded in the secondary market on a regular basis depending on the liquidity thereof. Credit risk associated with the issued debt can then be hedged by credit derivatives which are also sold onto the secondary market.

Credit risk is embedded in most marked-to-market OTC type derivatives. Even if a market participant is well in-the-money, there is still the risk that the counterparty will be unable to make payments.

With the introduction of IFRS13 a few new guidelines have come to pass which stipulates how market participants must account for the possible event of non-performance either by the counterparty or by the entity itself in their fair value estimations. Non-performance in this regard can be viewed as the event where either of the parties in a financial transaction is unable or willing to meet financial obligations stipulated in the contract. According to IFRS 13 credit adjustments should be market-based and measured similarly as the average market participant would. These adjustments take on the form of CVA and DVA that need to be made on all relevant derivatives of an entity.

The main building blocks that define credit risk are the amount of exposure to a legal counterparty, the creditworthiness of that counterparty and the amount of capital that can be retained in the event of default. These factors will be discussed in the following section under probability of default, exposure and loss given default. As an approximation CVA can be calculated as the following:

$$CVA = LGD \times Exposure_i \times Probability\ of\ Default_i \times Discount\ Factor_i$$

Where i represents the time point i, valuation time point $\leq i \leq$ contract maturity

2.2 CONCEPTUAL BACKGROUND

In this section, background and mathematical theory applicable to CRR related metrics and definitions are presented. It comprises of derivative pricing fundamentals and the essential building blocks of xVA, namely exposure, probability of default and LGD. Exposure mitigating strategies and credit quality measures are also presented as subsections of exposure and probability of default respectively.

2.2.1 PRESENT VALUE

According to IFRS, all financial derivatives (assets and liabilities) should be measured at fair value. KMPG (2012:1) interprets the newly stated IFRS guidelines as follow. If a financial derivative has a quoted price in an active market, then that price should be taken as the derivative's fair value. It is assumed that all possible fair-value adjustments like credit risk, liquidity, funding and administration costs are already included in this quoted price. However, the majority of the derivatives market, unlike say the futures market, is not made up out of standardised termed transactions with observable prices in the market. Most derivatives are tailored for specific structured deals with counterparties to fit the unique needs of each entity. Therefore, the value of these derivatives that trade in the OTC market must be agreed upon by all parties involved in the financial transaction. This pricing is normally done according to academic and accounting literature, through the risk-neutral discounting of future payments.

These OTC derivatives are then initially priced to potential clients or counterparties as the fair-value of the instrument excluding credit value adjustments. Once a willing counterparty has been found in the market place, estimates around credit value adjustments are made and agreed upon between both parties.

In order to understand CCR adjustments fully, a proper understanding of financial derivative pricing is needed. After derivatives are priced or potential future exposures calculated, it becomes possible to adjust these measurements according to their embedded credit risks.

The notion of time value of money is an important concept within financial pricing. The same amount of money is worth less at present value relative to future time points. This is due to the interest earning potential of the capital. One would rather take a certain amount of money today versus the same amount in the future; since it can be invested over the year period to generate a return.

In this research this return will to be taken to be a risk-free rate, since the risk-neutral expectation is assumed to be true. The impact of time value of money is incorporated in the calculation of the price of financial instruments through the discounted cash-flow valuation (DCF) model. In this model all future payments are converted back to their equivalent value as at the valuation date to arrive at the fair value of a financial instrument. Franzen (2014:24-25) and Ahlberg (2013:8) have defined the present value of an asset along similar lines as presented below. The present value $V(t,T)$ of a financial derivative for any time $0 < t < T$ with maturity T, under a risk neutral expectation measure $\mathbb{E}^Q$ can be given as,

$$V(t,T) = \mathbb{E}^Q\left[\sum_{u\in[t,T]} C(u)\,P(t,u)|\mathrm{F}_t\right]$$

Where,

u	$\mathit{Discrete\ time\ point\ between\ time\ interval}\ (t,T]$
$C(u)$	$\mathit{The\ payoff\ at\ time\ point\ u}$
$P(t,T)$	$\mathit{Discount\ factor\ between\ time\ t\ and\ u}$
F_t	$\mathit{Market\ filtration}$

Brigo (2006:2) and Jones (2010:11) provide summaries of the money market account. The following explanation is adapted from their work. The money market account provides a thorough discounting framework through the introduction of the instantaneous rate. Let A_t be the value of 1 unit of money at time t, after it was invested in the money market account at time 0. This account assumes that invested capital earns the instantaneous riskless interest rate r_t throughout the life of the contract. In this research this rate will be referred to as the short rate. It can now be deducted that the stochastic process of A_t can be expressed by the following:

$$dA_t = r_t A_t dt$$

Where, the notation dt represents an instantaneous short period of time. Note that a detailed introduction to stochastic processes is provided in Appendix A. By considering the above equation it can be derived that in the money market account the drift is only dependent on the current short rate and the value level of A_t. From this the value of the account at time t can be derived as:

$$B_t = A_t \exp\left(\int_0^t r_s ds\right)$$

Zero and Forward Interest Rates

In order to value all derivative sufficiently two important interest rates need to be introduced in addition to the short rate. These are the zero-coupon and forward interest rates. A complete discussion on interest rate conventions and dynamics is presented in Appendix A. Hull (2009:78-85) and Sorenson (1994:4-5) provides an intuitive explanation of zero-coupon and forward interests, summarised here.

The T year zero coupon (zero for short) rate is the interest rate earned over a T year investment period where the notional value and all accrued interest is exchanged at the end of the T year investment period. Define the continuously compounded zero rate as $R(t,T)$ and $P(0,T)$ as the investment amount that needs to be invested as time 0 to achieve a 1 unit return at the end of time T. Thus,

$$P(t,T) = 1 \times e^{-R(0,T)(T-t)}$$

$$\rightarrow R(t,T) = -\frac{1}{T-t}\ln\big(P(t,T)\big)$$

The above is illustrated graphically in Figure 2.1 through the depicted time line, where $R(0,t_1), R(0,t_2)$ and $R(0,t_3)$ denote the respective zero rates for time points t_1, t_2 and t_3:

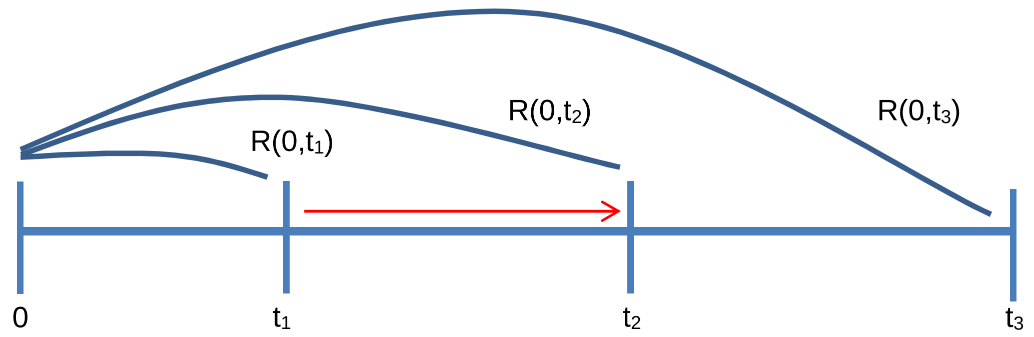

Figure 2.1: Zero Rate Diagram

It is possible to determine the rate implied by these zero rates between any of the points t_1, t_2, and t_3, while currently standing at time point 0. This rate is referred to as the forward rate between say time t_1 and t_2, as calculated from time 0 and denoted by $F_0(t_1, t_2)$. This is represented in Figure 2.1 by the red line. More formally the forward rate for any arbitrary time period between $T_{i-\delta}$ to T_i with time period length δ can be written as:

$$F(T_{i-\delta}, T_i) = \frac{R(0, T_i)T_i - R(0, T_{i-\delta})T_{i-\delta}}{T_i - T_{i-\delta}}$$

2.2.2 EXPOSURE

In order to determine CCR fair value adjustments, institutions need to quantify and measure the exposure that could be lost if the counterparty defaults. Thus, the expected exposure (EE) needs to be estimated. In broad terms, expected exposure is the exposure to the counterparty at the point of default – this could be either positive or negative (Gregory, 2012:127).

The exposure on an underlying financial instrument is directly linked to CCR. If an institution has a positive marked-to-market value in an underlying financial transaction with some counterpart, the said institution is exposed to CCR. This is a far greater risk than say profit at risk, since these exposures often include large notional amounts.

CCR is mostly observed in the OTC market, where there are usually no third party counterparty credit houses or fully collateralised agreements involved to mitigate credit risk. In exchange traded markets credit risk is already embedded in the quoted market price. It should be kept in mind that exposure is only relevant in the event of default and is therefore conditional on the event of default taking place. Default events typically become very technical and notice of default is usually only given a month or two after default occurs.

Pykhtin (2007:17) has formally expressed exposure as the following:

$$Exposure(t) = \max(V(t,T),0)$$

Where $V(t,T)$ is the fair value of a contract at time t with maturity T. It is noted that only positive fair value estimates are taken. For example, if the fair value of the contract is negative, it means that the institution owes the counterparty and thus there is no associated credit risk to take into consideration. Since fair values of financial instruments can are calculated with a risk-neutral expectation, it follows that exposure is also assumed to be a risk-neutral expectation (Ahlberg, 2013:8).

Although CRR has been around since the start of financial contracts, its impact only became clearly evident during the 2008 financial crisis. Klacar (2013) notes the crisis as the reason why forecasting exposures have become crucial and why guidelines around credit risk have tightened in recent years. For some financial transactions, such as normal deposits or bonds, calculating the exposure to a particular counterparty is straightforward at all future time points. This is because all cash flows are known in advance. However, for derivatives with embedded floating legs it becomes increasingly difficult, because all cash flows are not known from any future valuation time point. This makes sensitivity modelling of derivative underlying interest rates, forex and others of utmost importance to estimate future exposure and the associated risk.

Exposure estimation also becomes tricky if it is unsure whether a fair value amount will be positive or negative at a certain point in time. An example of such a contract is a Forward Rate Agreement (FRA), where a party pays a fixed rate on a predefined notional and the other pays a floating rate on the same notional at maturity. It is then uncertain whether the net payment to be made at the end of the contract is in favour of the fixed payer or receiver. Contracts like these are commonly referred to as bilateral derivative contracts, where it is not certain which party will be in- or out-the-money at defined payment time points. Unilateral contracts do not have this issue, as one party will always be exposed to the credit risk.

The two most popular estimation methods of exposure are expected exposure and potential future exposure. These will be introduced in the subsequent sections 2.2.2.1 and 2.2.2.2.

2.2.2.1 Expected Exposure

The expected exposure of a financial contract is defined as the average exposure over the life of the financial contract until maturity. It can be written as the following,

$$EE_t = \frac{1}{N}\sum_{i=1}^{T} \max(V(t_i,T),0)\,,$$

Where,

$V(t_i, T)$	$Fair\ value\ amount\ of\ the\ contract\ at\ time\ t_i\ with\ maturity\ T$
N	$Number\ of\ valuation\ time\ points$

It is noted through the law of large numbers, that the larger the value of N, the more accurate the estimate of EE_t.

Theorem 2.1 (The Law of Large Numbers) *Let $X_1, \ldots, X_n$ be independently identically distributed (i.i.d) random variables with mean θ and variance σ^2. Then for any given $\delta > 0$, $P(|\hat{\theta}(n) - \theta| > \delta) \to 0$ as $n \to \infty$. (Rice, 2007:175).*

2.2.2.2 Potential future exposure

Potential future exposure (PFE) follows the same underlying principals as a Value at Risk estimate. It is a more conservative estimate than the previously discussed expected exposure. Instead of considering the average of expected exposures at chosen time points it tries to gauge what the x% biggest exposure is likely to be over the life of the financial contract. Exposure varies over time due to defined cash flows occurring at different times over the term structure. This method considers the vector of say N expected exposures at N different time points and then takes a defined quantile as the PFE estimate.

Again, only positive fair value estimates are considered. Popular quantile estimates to consider are 95% and 99% (similar to VaR). For example, the 99% PFE estimate represents the exposure level that is expected to be exceeded with a probability of 1% over the life of the contract. In the graph below an illustration is given of an expected exposure estimate against a 99% PFE estimate. The grey area represents all positive fair value estimates used in the estimation.

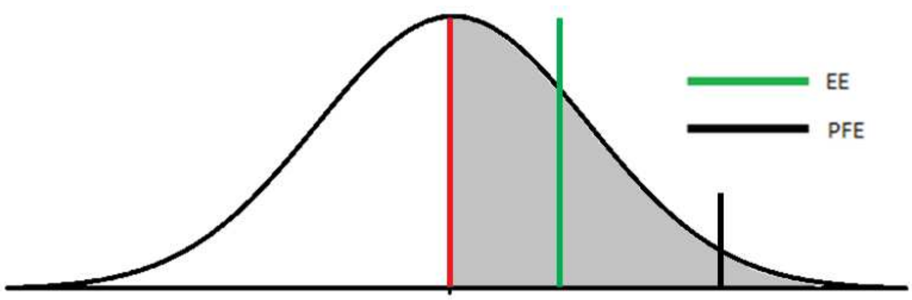

Figure 2.2: EE vs PFE

2.2.3 MITIGATING EXPOSURE

As was seen in the previous section, credit risk poses a real risk to entities with exposure at some point in a financial contract. There are a couple of methods which can be implemented to help mitigate CCR, which will be discussed in the subsections below.

2.2.3.1 Netting Agreements

IFRS 13 allows entities to reduce their CCR through netting agreements, given that these agreements are well documented and legally enforced through a master netting agreement upholding the new IFRS 7 amendment on *Disclosures – Offsetting Financial Assets and Financial Liabilities.* The instructions and requirements of a netting agreement are normally documented in a Credit Support Annex (CSA) agreed on by both counterparties. Counterparties should be well familiar with this annex.

These agreements, with countless variations, make it possible to net exposures across different contracts between the same two counterparties. Such netting agreements can pertain to variations of periodic payment netting as well as settlement netting in the event that one of the counterparties defaults. Ernst & Young (2014:9) makes the observation that netting reduces the gross payments that need to occur between counterparties having the added benefit of strengthening both parties' cash flows. Before entering into such a netting agreement parties must ensure they have appropriate systems in place able to handle all the complexities in the netting contract.

2.2.3.2 Collateral

Most of the introductory material presented here is adapted from Gregory (2012:67-80). Another way of mitigating some of one's credit exposure is by entering into collateral agreements with counterparties. Such agreements limit the credit exposure throughout the life of the contract by requiring the out-of-the-money party to post a certain level of collateral to the in-the-money party. This decreases the potential credit exposure of both parties, since when one of the parties enters default, the other party can hold on to the posted collateral to reduce losses.

Collateral can take on the form of cash or some predefined security. This security is almost always defined as some liquid security which is ideally independent of the performance of the counterparty. If the performance of the security and the counterparty is dependent, it leads to a concept called wrong-way risk. Further adjustments need to be made to account for the impact of wrong-way risk embedded in derivative transactions. This is normally done via a monthly dynamic provision. Wrong-way risk is formally presented by Hull (2011:5) as the following,

Definition 2.2 (Wrong-way Risk) *A situation where there is a positive dependence between a counterparty's probability of default and a dealer's exposure to said counterparty, so that the probability of default by the counterparty tends to be high (low) when a dealer's exposure to the counterparty is high (low).*

For example, say that company A has a derivative contract in a unilateral repo transaction with some African sovereign. If a collateral agreement is put in place it would be very unwise for company A to agree to the sovereign posting collateral on its own government bonds. Should the sovereign default, the value of its bonds are also sure to depreciate making the value of the collateral less at the point in time where it is needed the most. Gap risk is then also reduced. Gap risk is associated with the risk that the value of collateral can decrease from the point of default to collateral delivery.

In most cases collateral does not eliminate credit risk completely. Often an exposure threshold is defined from which point collateral has to be posted. If market movements adversely affect the out-of-the-money party in a bilateral or unilateral derivative transaction to the point where exposure increases past the defined collateral threshold level (haircut), the in-the-money party has the option to call for collateral.

Collateral has only to be topped-up to the defined threshold; however, timing of payment and collateral levels tend to be a negotiation between the two counterparties based on the grounds of good business relations. If collateral has been posted previously and the exposure drops back to below the predefined threshold due to market movements, collateral is given back. In the CSA full details regarding the underlying collateral is given. This specifies when collateral ownership transfer occurs and how interest and other gains on collateral should be treated.

Sometimes counterparties enter a fully collateralised agreement with a particular counterparty. Collateral is then transferred, usually on a daily basis, into a marginal account managed by a third party called a Credit Clearing Party or House (CCP). A CCP reduces the operational intensity of such an agreement. According to KPMG (2012:15) accounting literature, if a contract is daily collateralised, it can be assumed that inter-day movements are insignificant and the contract is seen as credit risk free. Therefore, no CVA adjustments need to be made on the fair-value measurement of the underlying asset.

Similarly, to netting agreements, collateral agreements are also governed through general terms published by the International Swaps and Derivatives Association (ISDA) which applies to all OTC derivative transactions. All tailor-made specifications are then specified within a CSA which regulates the specific contract.

From a pricing perspective collateralised derivative contracts are worth more than similar uncollateralised contracts. The difference in fair value measurement tends to diverge even further during market stress conditions when everybody wished they have taken out some protection.

When pricing collateralised derivatives many market participants in developed markets discount estimated future cash flows by the rate associated to cash collateral posted. This is typically taken as relevant currency overnight index benchmark rates such as the SONIA (Sterling Overnight Index Average) in Britain and ENONIA (Euro Overnight Index Average) in Europe.

South Africa does not have an actively liquidly trading overnight index. Therefore, in this research alternative discount rates are ignored. As will be seen in Chapter 5, exposures are adjusted accordingly in the calculation of CVA for collateralised deals.

2.2.4 LOSS GIVEN DEFAULT

Once a company has entered default, the company still has available cash to pay some of its obligations. The portion of exposure that is expected to be paid in the event of default is called the recovery rate. Loss given default is then defined as,

$$LGD = (1 - Recovery\ Rate)$$

This amount can then be multiplied by the exposure at the point of default to arrive at the true loss originating from the default event. Since LGD is unknown before a default event, LGD needs to be modelled or an assumed LGD estimate should be taken according to that taken by the average market participants, as stipulated by IFRS 13. In this research the recovery rate (RR) is assumed to be deterministic over time. Assumed RR's will be presented in Chapter 3 and 4 for the corporate treasurer and banks.

2.2.5 PROBABILITY OF DEFAULT

Probability of default is exactly what the name suggests. It is a measure of the likelihood of an entity entering financial default during a specific time period. More broadly it is the likelihood that a borrower or counterparty is unable to meet all of its financial obligations.

Default is defined by the ISDA as a credit event defined along the following lines:

- *Bankruptcy:* The reference entity has filled for relief under bankruptcy law.
- *Obligation acceleration:* Obligation becomes due and payable before its normal expiration date.
- *Failure to pay:* Failure of the reference entity to make due payments.

- *Repudiation/Moratorium:* Repudiation occurs when a borrower refuses to honour the terms of a contract and therefore stops making payments.
- *Restructuring:* Reduction and renegotiation of delinquent debts in order to improve or restore liquidity.

Amendments to the default definition can be made and documented in the CSA for specific derivative transactions according to tailored exceptions or additions agreed upon between the two parties bound under the contract.

After the financial crisis probability of default has enjoyed much needed attention. It has begun to form a central part of risk frameworks in both the market and credit risk spaces. If a bank for example wants to make use of internal model based approached, probability of default modelling should be done at an extensive level to provide comfort to regulators. According to (Iqbal 2012:3), probability of default is of such significance, because it forms a core part of capital allocation, assists greatly to the pricing of certain financial products, makes sure client judgement is carefully evaluated, serves as part of regulatory compliance and gives an overall indication of embedded issuer risk.

2.2.6 CCR MEASURES

In the following few subsections different types of inputs are introduced that can be used to estimate credit risk over time. Some of these are of historical nature, live dynamic measures of credit risk and some are financial inputs that try to paint a qualitative picture of the underlying health of a company through fundamental accounting based measures. It should be kept in mind that according to the new IFRS definition of fair value, inputs that are observable in the market and that would most likely be used in the estimation by an average market participant should be ranked as more appropriate.

2.2.6.1 Credit Default Swap Spread

In order to estimate the probability of default of counterparties or assess their general creditworthiness, one can look to the credit market for assistance. In the credit market quotes are available that represents the price of a particular entities credit risk. Credit default swap (CDS) quotes measure the associated credit risk of a particular counterparty at a specific maturity. CDS quotes (also referred to as credit default spreads) are the premiums paid to accept the credit risk in a credit derivative instrument called a credit default swap. Credit default swaps are financial instruments designed to transfer some underlying credit risk between parties.

The seller in such a contract is obligated to make a payment to the buyer in the event of a credit event in an underlying reference asset. The different conditions which constitute a credit event are specified in the legally binding contract inception. In return for this protection or insurance given to the buyer of the contract, the buyer makes an upfront payment or periodic payments to the seller. This premium paid by the buyer is defined as the credit default spread of the derivative. It is the price paid for the credit protection bought by the buyer of the credit default swap. Similarly, to interest rates these spreads are denoted in basis points (bps).

There are different reasons a company would enter into such a contract. The buyer might own the underlying asset (normally a bond or loan) and he is concerned that the bond issuer will default and will not be able to pay the coupons due over time. When entering into a long credit default swap agreement the buyer receives protection against this event coming to fruition.

The seller receives the credit default spread (premium) without posting any upfront investment capital. It takes on the risk of the reference asset defaulting or triggering a credit event. In effect the seller is speculating that the underlying asset will not default. The payment profile of a credit default swap is presented in Figure 2.3.

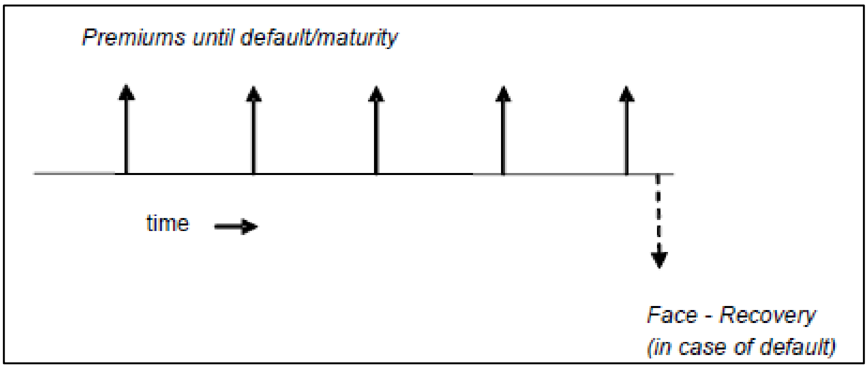

Figure 2.3: CDS Diagram

As the contract progresses through time towards maturity, the price of the contract changes according to changes in the credit quality of the reference asset. If the credit quality of the reference asset deteriorates the underlying credit spread of the CDS increases. This is because if the reference asset becomes riskier, it becomes more expensive to buy protection on it.

Similarly, to a company's share price the credit spreads of company has dynamic credit default spread quotes which represents the price to buy protection at on the specific company in the market across a defined term structure.

Like interest rates, credit spreads differ in value across different maturities. Therefore, credit spreads are quoted across a term structure curve. One can think of a particular credit spread time point as the price to buy protection for an asset for that particular maturity. Credit curves are usually monotone increasing, with the price of protection increasing for longer maturities. Credit spreads are typically interpreted as the spread in bps above the 'basic risk' curve level or a level perceived to be riskless. In this research the par quoted ZAR swap curve will be taken as the basic risk curve.

Most big banks and large corporates do have liquid Credit spreads available through a data provider such as Bloomberg or MarkIT. These are not live quotes that can be used directly to enter into credit derivative transactions, but rather gives an indication where the market perceives the credit risk to be priced.

Since these credit spread curves represent the credit risk on a particular entity, probability of default estimates can be stripped from them for any specified quoted time period. The framework for probability of default stripping is illustrated in Figure 2.4.

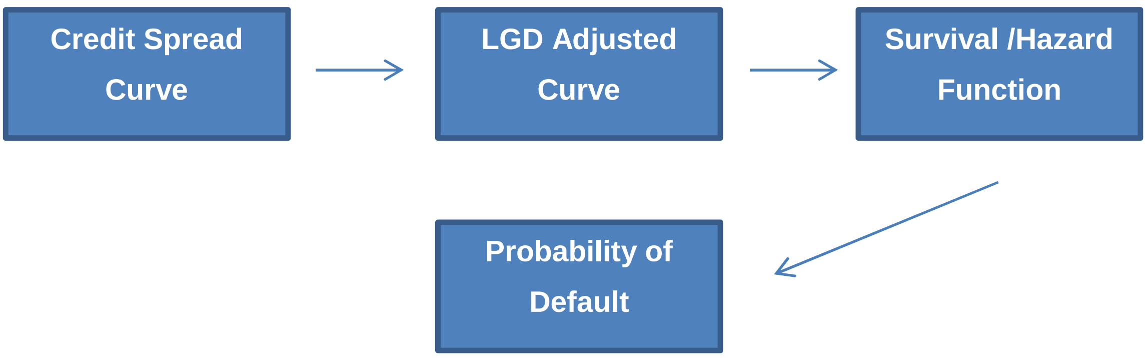

Figure 2.4: Probability of Default Stripping

The process of probability default stripping and survival theory is discussed in Duffie (1999:78-83) and Cerny (2014:3-4). The following explanation of the process is adapted from their work.

Suppose F is the cumulative distribution of the time of default denoted by τ. Then the following probability of default function can be written over an infinitesimally small time interval dt,

$$P(t < \tau < t + dt | \tau \geq t) = \frac{P(t < \tau < t + dt, \tau \geq t)}{P(\tau \geq t)}$$

$$= \frac{F(t + dt) - F(t)}{S(t)}$$

$$= \frac{F(t + dt) - F(t)}{S(t)dt}$$

Since $dt \to 0$, the above can be written as,

$$= f(t)dt\frac{1}{S(t)}$$

$$= -S'(t)dt\frac{1}{S(t)} = \lambda(t)dt$$

Where,

$\lambda(t)$ Hazard rate at time t

$S(t)$ Probability of survival at time t, denoted by $P(\tau \geq t)$

$f(t)$ Density function of τ

In this research the f will be assumed to be an exponential density function due to its memoryless property. Hazard rate and survival curves can only be estimated appropriately from the credit spread curves, after incorporating the assumed recovery. The credit spread curve is divided by the assumed LGD constant to arrive at an adjusted credit spread curve denoted by say $C(t)$. After $C(t)$ has been obtained the survival function curve can be calculated by $S(t) = \mathrm{e}^{-\mathrm{C(t)}t}$.

A hypothetical example of a cumulative survival rate function is depicted in Figure 2.5 below. It can be plotted as a decreasing cumulative function over time, with values between 1 and 0.

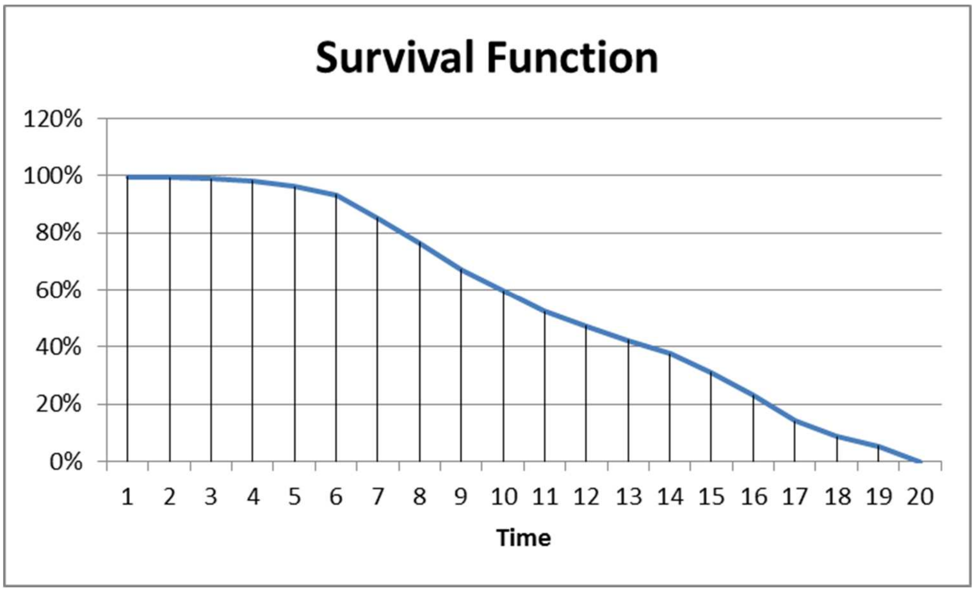

Figure 2.5: Survival Function

It makes intuitively sense that the function depicted in the graph above is monotone decreasing since the probability that a company finds itself in default increases over time. Once a company is in default, it remains in this defaulted state while all transactions are unwound or restructured. Once a company defaults, net exposure is calculated at the point of default.

It is now straightforward to calculate the probability of default between say time points t_3 and t_1, by simply taken the difference $S(t_3) - S(t_1)$. Estimated exposures at different time point can then be weighted by their probability of default in the CVA calculation. This will be covered in detail in Chapter 5.

2.2.6.2 Asset Swap Spreads versus Zero-volatility-spreads

In this section probability of default estimation is considered for the case when a specific entity does not have a quoted liquid credit spread, but bond information on issued debt. It will be seen that zero-volatility bond spreads can be taken as to help imply an entity wide credit spread curve.

When an analyst has two yield curves, the difference between the two is defined as a particular spread expressed in basis points. There are mainly two types of spreads that traders consider to assess the relative value of a bond. These are the asset swap spread (AWS) and the zero volatility spread (z-spread)

The following introduction to these spreads are based on summarised material presented by Choudry (2005:2-5). The asset swap spread and z-spreads of a bond are variables that give an indication of the relative value of the specific bond. Both of these measures try to quantify the embedded credit risk in a bond. However, they are calculated in slightly different way.

The AWS is defined as the difference between the yield-to-maturity of the bond and the interpolated interest rate obtained by linear interpolation of the par swap curve or treasury curve. The biggest difference between this spread and the zero-volatility spread is that ASW is an actively traded spread, while the z-spread is not. The ASW is used in valuing an asset swap instrument. An asset swap is an instrument that combines an interest rate swap with a cash bond. It is an instrument where the bond holder receives a floating coupon, measured as the spread above LIBOR, and the bond holder pays a fixed coupon. The determined floating spread is the asset-swap spread. Asset swaps are usually transacted at the bond's par price.

Therefore, the asset-swap spread incorporates the difference between the par price and the market price, as well as the difference between the fixed swap rate and the bond's coupon rate. This is the reason why traders prefer to consider the asset swap spreads between bonds to gauge relative value, since it provides additional information regarding the inherent credit risk of the company. For example, if the ASW of a bond is quoted as 121bps on say Bloomberg, it means that this is the spread above LIBOR that an investor would receive in an asset swap instrument with the bond as underlying.

A bond's z-spread is calculated slightly differently. It is defined as the constant spread that must be added to a swap or treasury ('basic risk') curves such that the present value of the bond's cash flows equals the price of the bond in the market.

Since these spreads give an indication of embedded asset credit risk, they can therefore be used as approximations for credit spreads with the tenor relating to the time left on the referenced asset until maturity. If an entity has enough liquid debt trading in the secondary market, the z-spreads or ASW, of the bonds can be used to calibrate a spread curve for the entity as a whole over an entire term structure. The choice of assets to include in such a calibration is of utmost importance, as they need to be a good representation of the entity's overall credit risk.

2.2.6.3 Credit Ratings

In the financial world there are bodies called credit rating agencies (CRA) that assess the creditworthiness of corporates and sovereigns and then assign a specific rating to these entities. The three most acclaimed and market influencing agencies are Standard & Poors, Moody's, and Fitch. They decide on a specific rating by considering underlying factors of an entity's credit risk through credit information conveyed to them by financial statements, market participants or general public information.

In order for an entity to be rated it needs to pay an annual fee to these agencies and share some disclosed financial fundamentals. It is beneficial for an entity to be rated since it provides market confidence to investors regarding the financial well-being of the company. Investors are then in a more equipped position to make financially sound investment decisions to suit their risk and return appetites.

The rating of a well respective agency can shed light on the creditworthiness of an entity. Usually CRAs also include an outlook in their assessment which helps investors to make forecasts on the future financial well-being for companies or sovereigns. A scaling table is provided in Chapter 4 which can be used to compare the given rating of the three largest credit rating agencies. A 'A' type rating is assigned to entities perceived to have the most attractive credit risk attributes. A 'BBB-' rating by Fitch and Standard & Poors and the equivalent Baa3 is seen as the tipping point between investment grade and sub-investment grade. Chaplin (2005:9) notes that investment grade entities are rated up to BBB- and that their credit spreads are typically trading up to 300bps above LIBOR.

All entities rated at or above this level are judged to be likely to be able to meet their financial obligations, while the rest are not. As it is in the investment landscape, products with a higher embedded risk also carry a higher expected return profile.

Conclusions can be drawn from these credit ratings in respect to their underlying probability of default. Since agencies assign different ratings to different entities, these ratings can be used to group entities with a certain credit risk profile together. Subsequently these grouped entities can then be assigned a particular probability of default term structure.

This aspect of assessing credit risk in a standardised framework, really appeals to IFRS requirements regarding CVA calculation inputs. Having a consistent method across the market to reference probability of defaults is favourable according to IFRS 13, which states that credit adjustments should be market based and measured from the perspective of the average market participant. If the entire market has access to the standard credit ratings assigned to all companies, there will be consistency in the view how the market measures creditworthiness.

However, the problem with this method is that CCR change over time and that these non-dynamic ratings would then become stale and inaccurate. Rating agencies only update sovereign and company ratings annually or at most biannually. When some external credit event impacts a company, the company's credit quality tends to deteriorate quickly. Once a company is in financial trouble the lagged credit ratings will not reflect the company's poor credit quality and distress accurately.

2.2.6.4 Historical Defaults

When statistically modelling any dataset, the more data one has to work with, the more accurate estimates. For example, the modelling of the probability of default of clients in the personal banking space is more accurate than corporate banking due to the abundance of historical default data available. However, a stumbling block comes into play when considering low default portfolios. Large structured derivatives transactions are normally executed with large corporates or banks, where there are fewer market participants and fewer resulting defaults. Although there are fewer defaults, these defaults should still at least be considered when estimating probability of default.

The biggest problem with historical estimates is that they often tend to take on the form of long-term estimate that does not move dynamically over time as the creditworthiness of entities change. This also introduces the problem that probability of default will not be consistently measured by market participants.

2.2.6.5 Comparison of CCR data inputs

Advantages and disadvantages are listed below for each of the inputs for the calculation of credit adjustments. Ernst and Young (2014) has ranked the inputs from most to least appropriate as can be seen below in Table 2.1.

According to IFRS one should make use of data inputs that are must likelihood to be used by the average market participant and that all observable relevant data should not be discarded in the estimation of credit value adjustments.

So does one want to capture with this probability estimate? It is about dynamically estimating the probability of default over different maturities looking forward in time.

Since credit curves are the already in this sought after form it will be the most logical starting block. When considering Table 2.1 it can be seen that credit curve information sourced through data service providers also rank high when compared to the requirements of accounting literature for the estimation of credit value adjustments. Service providers usually provide credit spread data for standard discrete time points. Estimates between these observed time points are then normally calculated by means of a simple linear interpolation function. A full credit spread curve is then obtained across all time points from which probability of default estimates can be calculated.

	Market observable?	Easy to source?	Data is current?	Not too many assumptions
Direct CDS quotes				
Data provider CDS				
Debt issuance				
Sector CDS				
Credit ratings				
Historical defaults				

Table 2.1: CRR Measures (Source: EY 2014)

In section 2.2, background and mathematical theory applicable to CRR related metrics and definitions were presented. The different building blocks of xVA were introduced and the section was concluded with a comparison between different data inputs for the measurement of CCR. In the next section interest rate swaps and swaptions derivatives are introduced which will be used in the calculation of CVA and DVA. The modelling and pricing of these derivatives are presented.

2.3 PRICING INTEREST RATE DERIVATIVES

The pricing of financial derivatives in this research will follow a risk-free valuation. The section starts off with an introduction to interest rate swaps. This is important since the hypothetical portfolio introduced in Chapter 5 to calculate xVA, consists of interest rate swaps with differing contract specific characteristics.

Pricing derivatives with an embedded floating leg or option functionality can prove to be trickier than most other vanilla derivatives. For example, the floating leg of interest rate derivatives payoffs dependent on the level of some reference interest rate at particular time points through the lifetime of a financial contract. To value non-linear interest rate derivatives, the short rate needs to be modelled over time. The modelling of the short rate r will form the topic of the second subsection. Monte Carlo simulation is then summarised, which will be the simulation method to model r accurately.

The section is concluded by the introduction of an interest rate swap option also known as swaptions. Underlying concepts of these instruments are needed for one of the CVA estimation methods presented in Chapter 5.

2.3.1 INTEREST RATE SWAP INTRODUCTION

Swaps are high volume traded financial derivative instruments traded predominantly in the over-the-counter market. In a swap two counterparties enter the contract where payments are exchanged between the two on predetermined payments dates. The design of the payments is agreed upon between the counterparties themselves before entering into the swap agreement. Underlying payment exchanges are dependent and derived from the future value of some underlying market variable. Popular market variables used in swaps are forex and interest rates.

In an interest rate swap a series of cash flows are exchanged between counterparties at specified periodic time intervals, referred to as cash flow or payment dates, based on the interest rate of a predefined principal notional. In a plain vanilla swap the one party pays a fixed interest rate throughout the lifetime of the contract while the other pays an interest rate linked to some floating referenced one. Unlike a bond or other fixed income products, the notional is not exchanged at maturity. Payments are normally netted and the party with a negative marked-to-market pays the owed amount to the other party.

The fix payer therefore believes that the referenced floating interest rates are going to increase over the lifetime of the swap. If this happens he will be paying a smaller interest rate and receiving a higher interest rate based on the same notional amount. The floating payer believes that the opposite will occur in the interest rate market over time.

Another major reason for entities to enter into such swap agreements comes down to comparative advantage. Comparative advantages come into play when differently rated companies enter the debt market. Badly rated entities are usually offered better floating rates than fixed rates and vice versa for companies with a high rating. In most cases interest rate swaps are used to switch future floating obligations to fixed or vice versa. It is therefore a popular hedging tool to mitigate the risk of other exposures already on book, especially for corporate treasurers as discussed in section 2.1.1. The above described plain vanilla interest rate swap and is illustrated in Figure 2.6 below:

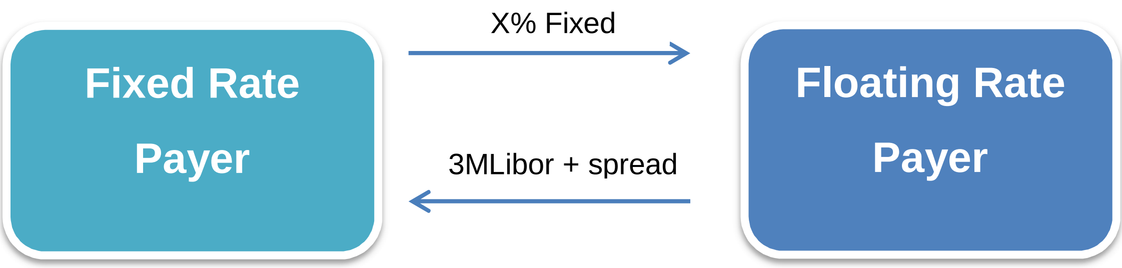

Figure 2.6: Interest Rate Swap Diagram

The floating rate is often taken as the interbank borrowing rate (e.g. LIBOR) for a specified tenor or some other quoted basic risk rate. In some swaps a fixed spread is added to the floating rate in order for the swap to be valued at par. The time interval between swap payments is generally made the same length as the referenced interest rate tenor. Suppose that swap payments are exchanged quarterly. Then it is the norm that the floating interest rate references the 3-month quoted rate of some reference interest rate.

If the 3-month interest rate of some underlying published reference interest rate is used, then payments between the two parties can be fixed 3-months prior to a payment date. This is called the fixing date and allows both counterparties to know the exact exchange amount at the next payment date. Payments past the 3-month point are still unknown and therefore need to be modelled to gauge the possible future payments of the floating rate leg of the swap.

2.3.2 INTEREST RATE MODELLING

This section introduces the fundamental theory of the instantaneous short rate and the modelling thereof as presented by Glasserman (2004:108-112). It is important to model the short-rate for non-linear interest rate derivatives. These derivatives are in turn important for the calculation of CVA and DVA. For linear derivatives the forward rate can be used instead of the short-rate.

Firstly, consider an investment in a money market account earning the defined short rate $r(s)$ at time time s which grows from a value of 1 at time zero to the following value at time t as presented in section 2.2.1,

$$B(t) = \exp\left(\int_0^t r(s)\,ds\right)$$

The above result is the numeraire for risk neutral pricing. The underlying theory of numaraires is presented in Appendix A. This implies that the price of a security at time 0 with a payoff of X at time T is an expectation of $\frac{X}{B(T)}$. For a zero-coupon bond paying 1 unit of money at maturity T, the price of the bond is therefore given by,

$$B(0,T) = \mathbb{E}^Q\left[\exp\left(-\int_0^T r(s)\ ds\right)\right]$$

The modelling of the short rate r is therefore of utmost importance for the discounting of future expected payoff for the pricing of derivatives in a risk-neutral world. Modelling of the instantaneous short rate assumes that investors are operating in the defined risk neutral world where they are indifferent towards risk. Expected payoffs f_T can be calculated for future time points and then discounted to the valuation date by using the average of the short rate r to arrive at the appropriate fair value.

$$E\left(\exp\left(-\bar{r}(T-t)\right) f_T\right)$$

Therefore, the short rate should be simulated between time t and T and then used as the discounting rate in a risk neutral world. Zero coupon bonds will be used in this risk neutral world defined by:

$$P(t,T) = \mathbb{E}^Q\left(\mathrm{e}^{-\bar{r}(T-t)}\right)$$

Denote $R(t,T)$ as the continuously compounded riskless zero interest rate over a time period $(T-t)$ at time t. Then it can be written that,

$$P(t,T) = e^{-R(t,T)(T-t)}$$

$$\rightarrow R(t,T) = -\left(\frac{1}{T-t}\right)\ln(P(t,T))$$

$$\rightarrow\ R(t,T) = -\left(\frac{1}{T-t}\right)\ln E\left(\mathrm{e}^{-\bar{r}(T-t)}\right)$$

Note that, the denoted term $\mathbb{E}^Q$ in the above equation refers to the expectation in risk neutral world. The equations above enable the entire term structure of interest rates to be obtained from the simulated value r. The initial zero curve and its change through time is completely captured. The only thing needed is to define the stochastic process of the instantaneous short rate.

In valuing instruments, literature widely assumes that interest rates are lognormally distributed. This assumption makes interest rate modelling simpler since modelling is based on a predefined distribution; however, this assumption exhibits the shortcoming that it does not provide any description of the time stochastic process that interest rates follow over time. Under the log-normal assumption the zero rate from time t to T is given by the following:

$$Z(t,T) = Z_0 \exp\left(-\frac{1}{2}\sigma^2(T-t) + \sigma\sqrt{T-t}\epsilon\right)$$

Where,

ϵ	*The standard normal variable*
σ	*Constant assumed rate volatility*
Z_0	*The forward rate applying from time t to time T as observed from time* 0 $(Z_0 \equiv F(0,t,T))$

This is chosen to ensure that the mean of the distribution of zero rates is centred on the forward rate. The stochastic discount factor from time t to time T is then:

$$P(t,T) = e^{-Z(t,T)(T-t)}$$

The same notation $P(t,T)$ is given for the discount factor as was presented in section 2.2.1 where discount factors are equivalent to zero coupon bonds. However, this simulates rates under the t-forward measure and according to (Fabozzi, 2010:56-64) if the expectation of forward discount factors is taken under this measure the no-arbitrage condition does not hold i.e.

$$\frac{P(0,T)}{P(0,t)} \neq \mathbb{E}_t\left[e^{-Z(t,T)(T-t)}\right]$$

Therefore, an alternative is needed to overcome the above mentioned problems. One alternative is to construct a term structure model by modelling the behaviour of a short term interest rate r by using its assumed stochastic process. The short term interest r is defined at time t as the interest rate applied to an infinitesimally short time period of time.

Stochastic processes for short rates can be categorised into two categories namely Equilibrium Models or No-Arbitrage Models. The theory outlined below is adapted from work presented by Hull (2009:673-683), Svoboda (2002:6) and Vasicek, (1977:181). In one-factor equilibrium models there is only one source of uncertainty. The drift rate and the standard deviation are assumed to be functions of the short rate. Stochastic processes are introduced in Appendix A in detail.

$$dr = m(r)dt + \sigma(r)dt$$

The most popular one-factor equilibrium model is the Vasicek model, which incorporates mean reversion into the drift rate. The short rate is assumed to pull back to its long run level b at a rate of a.

$$dr = a(b - r)dt + \sigma dz$$

Mean reversion is a well-known statistical phenomenon. It implies that when the short rate is above value level b it causes a negative drift. When the short rate is below value level b it causes a positive drift. It is regularly used as motivation for outright or relative value trading across most market variables. Within the interest rate space there are economic fundamentals that support the mean reversion argument. When rates are high, economic growth declines, because fewer borrowers are willing to enter the market at these high interest rate levels. When rates are low, borrowers perceive the cheaper price of lending and demand of funding increase which causes interest rates to rise.

The Cox Ingersoll and Ross model adds to the Vasicek model by forcing interest rates to be non-negative.

$$dr = a(b - r)dt + \sigma\sqrt{r}\, dz$$

There are arguments that this assumption is practically flawed, since negative rates are for example observed in Japan in the second decade of the twenty first century. This model also implies that the standard deviation decreases as the short-rate increases.

Many more adjustments can be found in the literature. Brennan and Schwartz (1982) proposed a two-factor model where the long term level b follows a stochastic process in itself. They also suggest that the long term level be chosen as the yield on a perpetual bond that pays 1 unit of currency annually.

The biggest disadvantage of equilibrium models is that they do not automatically fit the current interest rate term structure. Through adjustments the term structure of some equilibrium models are dependent on time, but since the term structure is an output it is not an exact fit. No-arbitrage models overcome this problem, whereas in these models the interest rate term structure is an input which in turn provides an exact fit of the current term structure shape. This is done by defining the drift rate of the short rate as a function of r as well as time. If the zero curve is upward sloping between certain time points, the drift will be positive for this time period. If the zero curve is downward sloping between certain time points, the drift will be negative for this time period.

Svoboda (2002:208) notes that the choice of interest rate modelling model in South Africa, is heavily reliant on available modelling data. The Hull-White (one-factor) model is a no-arbitrage model which extends the Vasicek model for which all the needed data is available in a South African context. It is characterised as a Vasicek model, but the mean reversion rate is dependent of time. This can be formulated as follow,

$$dr = [\theta(t) - ar]dt + \sigma dz$$

Where, a and σ are constants. The reversion rate within this model takes the form of $\frac{\theta(t)}{a}$, where r reverts back to the level at a rate a. The value of θ can be calculated by using the initial term structure of an assumed risk-free interest rate curve (for example JIBAR).

$$\theta(t) = F_t(0,t) + a\,F(0,t) + \frac{\sigma^2}{2a}\left(1 - \exp(-2at)\right)$$

Where, $F(0,t)$ is defined as the instantaneous forward rate for a maturity t as seen at time zero. Since the present time is taken as zero, values for $F(0,t)$ is known for all values of t up to maturity say T. $F_t(0,t)$ is defined as the first partial derivative of the particular forward rate with respect to time t.The last term is significantly small and can be ignored in the calculation of θ. The equation for θ therefore, implies that the average direction that the short rate will be moving in the future is approximately equal to the slope of the known instantaneous forward curve. This is an important characteristic of the model that explains how r changes through time. When r therefore deviates from the initial forward curve it reverts back to it at a rate a. The proof of $\theta(t)$ is outlined at the end of this section.

Bond prices, within a risk-neutral framework, at time t is given by

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

Where,

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$

and

$$lnA(t,T) = ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aT} - e^{-at})^2(e^{2at} - 1)$$

Proof: Relationship between the future rate and the forward rate which implies $\theta(t)$

Through Ito's Lemma the result of $P(t,T)$ above and by noting that the drift of $F(0,T)$ is zero in world that is risk neutral with respect to $P(t,T)$ the following stochastic processes can be written as

$dP(t,T) = \cdots - \sigma B(t,T)P(t,T)dz$

$dF(0,T) = \cdots + \sigma e^{-a(T-t)}dz$

A world forward risk neutral with respect to $P(t,T)$ implies a market price of risk of $-\sigma B(t,T)$. When a numeraire change takes place where the market price of risk is changed to zero, the drift of $F(0,T)$ increases to $\sigma^2 e^{-a(T-t)}B(t,T)$. Integrating over the life of the contract (between 0 and T) the total growth of the forward rate is $\frac{\sigma^2}{2a^2}(1-e^{-aT})^2$. Since the futures price has zero drift in this world, the forward price must equal the futures price at maturity. Also at time zero, the futures price must exceed the forward price by $\frac{\sigma^2}{2a^2}(1-e^{-aT})^2$.

Between interval $t_1$and t_2 where $t_1 < t_2$, the drift of the forward rate can then be calculated as

$$\frac{\sigma^2 B(t,t_2)^2 - \sigma^2 B(t,t_1)^2}{2(t_2-t_1)} = \frac{\sigma^2}{2a^2(t_2-t_1)}[e^{at}(-2e^{-at_2}+2e^{-at_1}) + e^{2at}[e^{-2at_2} - e^{-2at1}]$$

Integrating between time 0 and time t_1 it is seen that the amount by which the futures rate exceeds the forward rate at time zero is given by,

$$\frac{\sigma^2}{2a^2(t_2-t_1)}[((e^{at_1}-1)(-2e^{-at_2}+2e^{-at_1}))/a \;+ (e^{2at_1}-1)(e^{-2at_2}-e^{-2at_1})/2a]$$

$$= \frac{\sigma^2 B(t_1,t_2)}{4a^2(t_2-t_1)}\left[4(1-e^{-at_1}) - (1-e^{-2at_1})\left(1+e^{-a(t_2-t_1)}\right)\right]$$

$$= \frac{B(t_1,t_2)}{t_2-t_1}[B(t_1,t_2)(1-e^{-2at_1}) + 2aB(0,t_1)^2]\frac{\sigma^2}{4a}$$

Define $G(0,t)$ as the instantaneous futures rate for maturity t so that

$$G(0,t) - F(0,t) = \frac{\sigma^2}{2a^2}(1-e^{-at})^2$$

And,

$$G_t(0,t) - F_t(0,t) = \frac{\sigma^2}{a}\,(1-e^{-at})e^{-at}$$

In a risk neutral world $r = G(0,t)$. Therefore,

$$\theta(t) - ar = G_t(0,t) - a[r - G(0,t)]$$

$$\rightarrow \theta(t) = G_t(0,t) + aG(0,t)$$

$$= F_t(0,t) + aF(0,t) + \frac{\sigma^2}{a}(1 - e^{-at})e^{-at} + \frac{\sigma^2}{2a}(1 - e^{-at})^2$$
$$= F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})$$

The basic principles underlying interest rate modelling were introduced in this section. This included the fundamentals underlying the short-rate, stochastic processes and interest rate models. The Vasicek model was presented in more detail since this is the interest rate model that will be applied in the modelling of the short-rate in Chapter 5.

2.3.3 MONTE CARLO SIMULATION

In this section, a general theoretical background on Monte Carlo simulation is presented. It is important to understand the fundamentals since Monte Carlo (MC) will be used in Chapter 5 to estimate the instantaneous short rate, used in the valuation of interest rate swaps and their CVA. The summary of MC simulation presented here is based on work by Van der Merwe (2010:6-7) and others as cited in the literature throughout this section.

Mackay (2011) described the purpose of MC simulation as the solution for the following two statistical problems:

- To generate samples $\{x^{(n)}\}_{n=1}^{N}$ given an underlying probability distribution of P(x).
- To estimate expectations of functions under this given probability distribution P(x).

The second problem can be solved after the sampling in the first problem has been completed. For example, consider the function $f(x)$ over an integrable unit interval [0,1], $\theta = \int_0^1 f(x)dx$. The expectation $\theta = E[f(U)]$, with $U \sim Uniform(0,1)$, can then be estimated through the average of randomly sampled n variables $U_1, U_2, \ldots U_n$. The function f can be calculated at these n points, summed up and averaged by dividing by the amount of sample points to arrive at an estimation for θ,

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} f(U_i)$$

Glassserman (2004) considers MC simulation to be based on the analogy between probability and volume or the amount of sample points. Under the Law of large numbers, the estimation $\hat{\theta}$ of the true value of θ is improved as $n \to \infty$.

Hull (2208:778) describes the application of MC for the pricing of derivatives as the procedure to randomly sample changes in underlying market variable inputs in pricing models. MC makes use of the risk-neutral valuation result as an expectation assumption in this research. In other words, expected payoffs in a derivatives transaction are calculated in risk-neutral world and then discounted using a risk-free interest rate or approximation thereof. The standard MC algorithm for financial derivative pricing is outlined below, when considering a single underlying market variable X that provides a payoff at time T under a constant interest rate assumption.

Algorithm 2.1 (General MC Simulation)

1. *Sample random paths for X in a risk-neutral world.*
2. *Calculate the payoff from this derivative at time T.*
3. *Repeat steps1 and 2 to get many sample values of the payoff form the derivative in a risk-neutral world.*
4. *Calculate the mean of the sample payoffs to get an estimate of the expected payoff value at time T.*
5. *Discount this expected payoff at the risk-free rate to get an estimate of the present value of the financial derivative.*

2.3.4 INTEREST RATE SWAP VALUATION

Generally, interest rate swaps are constructed in such a way that their value is zero from both parties' perspective. This is referred to as the pricing at par, where no one of the parties has an advantage due to competitive pricing or favourable client relation. The value can then change during the life of the contract to either negative or positive depending on which side of the contract you are invested. Work presented in this section is based on previous findings and interpretations by Hull (2009:159-163), Lesniewsky (2008:6-8) and Ahlberg (2013:42).

Swaps can be valued by making use of basic bond pricing arguments. In a swap the underlying swap principal is not exchanged between the two parties. However, it is noted that when it is assumed that principal payments are both received and paid at swap maturity it does not change the value of the swap, since the same payment is paid and received and discounted using the same discount factor. By synthetically adding these principal payments a swap, from the point of view of the floating-rate payer, it can be seen as a portfolio consisting of a long position in a normal fixed rate bond and a short position in a floating-rate bond. Therefore, by looking at the cash flow payments in isolation the following is deducted:

$$Swap_{value} = Fixed\ Rate\ Bond\ Price\ - Floating\ Rate\ Bond\ Price$$

Following the same arguments, the value of swap from the point of view of a fixed-rate swap payer is:

$$Swap_{value} = Floating\ Rate\ Bond\ Price - Fixed\ Rate\ Bond\ Price$$

The value of a fixed rate standard bond can easily be calculated by discounting future cash flows. The problem is that the values of future floating payments are unknown. However, this problem can be overcome by noting that immediately after each swap payment, the value of the bond is at par (the value of the principal). Let us denote the principal as L and the known next payment at time $t*$ as $k*$. The payment is known since the floating rate is fixed at the previous payment time point. It is assumed that that the time length between payment dates are $t*$ and it is assumed that for this illustration that the valuation date is at time point zero. Immediately after the next payment $k*$ the value of the floating rate bond will simply be L and just before the payment it will be $L + k*$. This is true because after each payment exchange the swap is valued at par. If we assume the continuous discounting rate at time $t*$ is $r*$, the value of a floating rate bond today can then be taken as:

$$(L + k^*)e^{-r^*t^*}$$

The above argument can be used in the pricing of plain vanilla swaps as will be formally outlined below. The argument is only valid if it is assumed that the spread is equal to the discounting rate.

Let $t_1 < \cdots < t_N$ denote the N interest payment dates of the swap and suppose the valuation time is taken as $t_o = 0$. The fixed leg of the swap can then be valued at time t by calculating the sum of all future cash flows discounted to valuation date time t. Note that in the equation below that $0 \leq t \leq N$, where N is the maturity date.

$$V_{fix_t} = \sum_{i=1}^{N} (t_i - t)s_k P(0, t_i)$$

Where V_{fix_t} is the fair-value of the fixed leg of the swap at time t, s_k is the fixed coupon rate (swap rate), $P(0, T_i)$ is price at time zero of a zero coupon bond yielding one unit of money at time T_i representing the discount factor for the ith payment date and $(t_i - t)$ is the day-count fraction applied to each period. Along the same notation the value of the floating payment leg can be written as,

$$V_{float_t} = \sum_{i=1}^{N} (t_i - t_{i-1})L_i P(t, t_i)$$

Where,

$$L_i = F(t_{i-1}, t_i)$$

$$\rightarrow \ = \frac{R(t, t_i)t_i - R(t, t_{i-1})t_{i-1}}{t_i - t_{i-1}}$$

$$\rightarrow \ = \frac{1}{(t_i - t_{i-1})}\left(\frac{1}{P(t_{i-1}, t_i)} - 1\right)$$

is the forward rate for settlement at t_{i-1}.

It can be shown that $V_{float_t} = 1 - P(t, T_N)$ by replacing L_i in the equation above and by using the argument described earlier in the section that a spot settled floating rate bond, paying LIBOR and repaying the maturity, is valued at par if the discounting spread is equal to the spread above floating.

The net value of the swap can be calculated from as the difference between the value of the fixed leg and the floating leg. The sign of the marked-to-market dependents on which side of the swap an investor is invested. For a fixed payer the value of the swap can be calculated by,

$$V_{swap} = V_{fix} - V_{float}$$

The problem with the above however remains that it is unknown what the floating interest rate LIBOR will be at future payment time points in order to calculate $P(t, T_N)$. Interest rates therefore need to be modelled through some simulated stochastic process as was discussed in section 2.3.2.

2.3.5 SWAPTIONS

In section the valuation of a European swaption interest rate derivative is introduced. This is needed since one of the CVA and DVA estimation methods presented in Chapter 5 uses the underlying principles of swaptions. The following is adapted from work by Cerny (2014) and Hull (1009:650-654) The option component embedded in swaptions is linked to the event of a counterparty entering default or not.

In short the holder of a swaption contract has the right to pay a swap rate C_k in an interest rate swap starting in T years time that lasts n years on a notional L. By using Black's model presented in Appendix A, the value of the swaption is given by,

$$V_{pay}(0, T, T_n) = \sum_{i-1}^{n} P(0, T_i)\, L\, (s_0\Phi(d_1) - s_k\, \Phi(d_2))$$

Where,

$P(0, T_i)$	$Discount\ factor\ between\ time\ t\ and\ u$
Φ	$Cumulative\ probability\ function\ for\ a\ standardised\ normal\ distribution$
s_0	$Forward\ swap\ rate$
s_k	$Swaption\ strike\ rate$

$$d_1 = \frac{\ln\left(\frac{s_0}{s_k}\right) + \frac{\sigma^2 T_n}{2}}{\sigma\sqrt{T_n}}$$

$$d_2 = d_1 - \sigma\sqrt{T_n}$$

Swaptions will be used in one of the CVA/DVA calculation method proposed by Ernst & Young (2014). The option component embedded in swaptions is used to model the binary event of a counterparty defaults or not. The swaption approach will be discussed in detail in Chapter 5.

2.4 THEORETICAL PRICING OF CVA AND DVA

Post 2008 most large financial institutions have started to include a Credit Value Adjustment in their fair value assessments after the fallout of the financial crisis. It was clear to see the devastating impact that non-performance can have on an entities' asset books and the financial world realised that this impact needs to be accounted for on balance and income sheets to arrive at a true representation of financial position. Since no definite method is prescribed by the guidelines, many derivatives dealers, financial consultants and end-users have come up with various methods to account for the effect of credit risk in their fair value adjustments of their OTC derivatives. (Ernst and Young, 2014:2). These types of adjustments will be covered in detail later in the Chapter 6.

The method chosen by entities depends on an array of factors. For example, an entity might not possess of required technical human resources, financial resources or IT capability to implement highly complex and sophisticated methodologies. It also depends on the size of the entity's derivative book and the percentage impact such adjustments have an overall size of assets and liabilities. The presence of credit risk mitigating factors also plays a large role. Risk mitigating tools like collateral or netting agreements tend to complicate the calculation of value adjustments as was discussed in section 2.2. It will be seen in Chapter 6 that different methods work better for certain types of derivatives and the extent of their moneyness.

When a trading desk or any other participant in the financial market calculates the price of a certain derivative or financial product, the participant normally uses risk-neutral pricing principals to arrive at a fair market price. A spread is then applied or subtracted from this price depending on trade strategy. Therefore, no adjustment has been made for CCR (Ahlberg, 2013:7; Gregory, 2012:241). CVA aims to capture this CCR and adjust the fair value amount of a particular trade or group of trades facing a particular counterparty.

Since DVA and CVA are estimated in the same manner, the main focus in this chapter will lie with CVA. When calculating the value adjustment needed for credit exposure to a counterpart the adjustment is referred to as CVA, when considering the adjustment needed due to your own credit obligations the adjustment is called DVA.

CVA is mathematically outlined below:

Since CVA is an adjustment made to the risk neutral valuation of a financial derivative, it can be defined as the difference between the valuation where CCR is included ($\tilde{\mathrm{V}}(\mathrm{t},\mathrm{T})$) and the case where it is not, $\mathrm{V}(\mathrm{t},\mathrm{T})$. Although exposure is linear, due to netting sets and collateral across different transaction with a specific counterparty CVA is not linear and additive with respect to individual transactions (Gregory, 2012:243).

Unilateral CVA Valuation

Below the CVA amount is defined for a derivative contract at time t with maturity T.

$$CVA(t,T) = \tilde{V}(t,T) - V(t,T)$$

Where, $CVA(t,T) \approx LGD \sum_{i=1}^{N} DF_{t_i} EE_{t_i} PD(t_{i-1},t_i)$

Proof. Consider how the event of default will influence the fair value, mainly due to the early termination of the contract. Denote τ as the time point of default, Q as the risk neutral measure and θ as the constant recovery in the event of default. Let $V(t,T)^+ = \max(V(t,T),0)$, $V(t,T)^- = -\max(-V(t,T),0)$.

$$\tilde{V}(t,T) = \mathbb{E}^Q\left[\, \mathbb{1}_{[\tau>T]}V(t,T) + \mathbb{1}_{[\tau\le T]}V(t,\tau) + [\, \mathbb{1}_{[\tau\le T]}(\theta\, V(t,T)^+ + V(\tau,T)^- \,]\right]$$

$$= \mathbb{E}^Q\left[\, \mathbb{1}_{[\tau>T]}V(t,T) + \mathbb{1}_{[\tau\le T]}V(t,\tau) + \mathbb{1}_{[\tau\le T]}\big((\theta-1)V(\tau,T)^+ + V(\tau,T)\big)\right]$$

$$= V(t,T) - \mathbb{E}^Q\left[(1-\theta)\mathbb{1}_{[\tau\le T]}V(\tau,T)^+\right]$$

Since $1-\theta = LGD$, the CVA in the second term in equation above, can be written as the expectation of the irrecoverable exposure in a risk-neutral world as,

$$CVA(t,T) = \mathbb{E}^Q\left[(1-\theta)\mathbb{1}_{[\tau\le T]}V(\tau,T)^+\right] = -\mathbb{E}^Q \int_t^T LGD\, V(u,T)^+ dS(u)$$

Where $S(t) = P(\tau > t) = 1 - P(\tau \le t)$ is defined as the survival function of the considered counterparty. In the case of DVA this will refer to own survival. In the above equation it is assumed that risk-neutral discounting is already included in the estimate. If this was not the case CVA can be represented by the following equation,

$$CVA(t,T) = -\mathbb{E}^{Q}\int_{t}^{T} LGD\; P(t,u)V(u,T)^{+}\,dS(u)$$

Where $P(t,u)$ is defined as the risk-neutral discount factor between time t and T. The hazard rate, as was presented in section 2.2.6.1, can also be incorporated in the equation above. The hazard rate was presented previously as,

$$\lambda(t)dt = -S'(t)dt\,\frac{1}{S(t)}$$

$$\rightarrow \quad \lambda(t) = -\frac{S'(t)}{S(t)}$$

By substituting the hazard rate into the CVA equation the following is obtained

$$CVA(t,T) = (LGD)\;\mathbb{E}^{Q}\int_{t}^{T} P(t,u)\;V(u,T)^{+}\lambda(u)S(u)du$$

The LGD can be taken out from the integral when it is assumed that the LGD is constant or deterministic. It is noted by Pykhtin (2007:38) that when working in a risk-neutral world the following is true,

$$E_{d}(t,T) = \mathbb{E}^{Q}\,[P(t,u)V(u,T)^{+}]$$

This observation can be substituted into the CVA equation,

$$CVA(t,T) = (LGD)\int_{t}^{T} EE_{d}(u,T)\,\lambda(t)S(u)du$$

Lastly through Monte Carlo integration, as introduced in section 2.3.3, can be used to move from integration to a discrete framework approximation. The final form of unilateral CVA is therefore given as

$$CVA(t,T) = LGD\sum_{i=1}^{N} DF_{t_i}\,EE_{t_i}PD(t_{i-1},t_i)$$

Where DF_{t_i} is denoted as the risk-free discount rate at time t_i to zero, EE_{t_i} is denoted as the expected exposure of the financial derivative at time t_i and $PD(t_{i-1},t_i)$ is the weighted exposure at t_i's probability of default defined between time points t_{i-1} and t_i.

CVA including Wrong-way Risk

When including wrong-way risk into the CVA calculation one needs to incorporate the linear dependence implied by WWR. This dependence is the inherent dependence between the underlying asset of the derivative and the default time. The dependence is measured by means of an estimated correlation coefficient. Cerny (2014:4-9) notes that by introducing WWR, an addition charge is assumed above the unilateral estimated CVA. This section and the introduction of swaptions in the next section are adapted from Cerny's work. CVA including WWR is denoted by CVA_W and can be presented as,

$$CVA_W(t,T) = LGD\left[cov\left(\mathbb{1}_{[\tau\leq T]}, V(\tau,T)^+\right) + \mathbb{E}^Q\, \mathbb{1}_{[\tau\leq T]}\, \mathbb{E}^Q[V(\tau,T)^+]\right]$$

$$CVA_W(t,T) = LGD\; cov\left(\mathbb{1}_{[\tau\leq T]}, V(\tau,T)^+\right) + CVA(t,T)$$

The covariance term of the exposure and the default time can be given for correlation coefficient $\rho \in [-1,1]$ as,

$$cov\left(\mathbb{1}_{[\tau\leq T]}, V(\tau,T)^+\right) = \rho\sqrt{var\left(\mathbb{1}_{[\tau\leq T]}\right) var(V(\tau,T)^+)}$$

The indicator of the default time can either be assumed to be Bernoulli distributed depending on the time variant hazard rate λ,

$$var\left(\mathbb{1}_{[\tau\leq T]}\right) = \left[1 - p\left(\lambda(T)\right)\right] p\left(\lambda(T)\right)$$

Or it is often be assumed that the default time is exponentially distributed with constant parameter λ

$$var\left(\mathbb{1}_{[\tau\leq T]}\right) = e^{-\lambda T}(1 - e^{-\lambda T})$$

CVA of interest rate swaps using swaptions

In this subsection the CVA estimation of a plain vanilla interest rate swap is considered by making use of a swaption valuation approach. This method serves as an alternative for the normal unilateral approach. In this method the optionality embedded in swaptions are represented by a counterparty entering default.

Consider a contract that gives the holder the right, but not the obligation, to enter a vanilla interest rate swap at time t. This is the same as receiving a payoff of $\max(Swapvalue_t, 0)$ at time t. For CVA calculation purposes the probability of the holder entering the swap can be replaced by the probability of default before time t. Sorensen (1994:24) was the first to present the swaption approach for valuing the CVA of an interest rate swap formally. He noted that the CVA can be represented by the summation of a series of swaption valuations through the life of the interest rate swap as seen from the current point in time (weighted by the probability of the default). Each swaption is then interpreted as the cost of replacing the remainder of the swap that was lost due to counterparty default.

The CVA of an interest rate swap will be denoted by $\mathrm{CVA}_{\mathrm{IRS}}$. Cerny (2014) summarised Sorenson's work and showed that the CVA_{IRS} can be estimated by weighting swaption prices at each payment time point by the probability of default. The equation is presented as,

$$CVA_{IRS} \approx LGD \sum_{i=1}^{n-1} \left(S(T_i) - S(T_{i+1})\right) V(t, T_{i+1}, T)$$

Where n is the number of swap payments with maturity T and $V(t, T_{i+1}, T)$ denotes the present value of a swaption at time t with option expiry at time T_{i+1}.

2.5 CVA AND DVA ESTIMATION METHODS

In this section different approximation method of CVA/DVA is introduced as presented of Ernst and Young (2014).

Expected Future Exposure Approach

$$CVA = LGD \sum_{t=1}^{T} DF_t EPE_t PD_{(t,t+\delta)}$$

Where,

LGD $\quad$ $Deterministic\ assumed\ LGD$

EPE_t $\quad$ $Expected\ positive\ exposure\ at\ discrete\ time\ point\ t$

PD_t $\quad$ $Probability\ of\ defualt\ esitmate\ between\ discrete\ time\ points\ t\ and\ t + \delta$

DF_t $\quad$ $Applicable\ discount\ factor\ at\ time\ point\ t$

This method simulates expected positive exposure from the inception of the contract up to maturity T for discrete time points $t < T$. The more discrete time point used in the simulation, the more accurate is the CVA estimated according the Law of Large Numbers (LLN). CVA is then estimated by weighting expected exposures by the associated probability of default and LGD.

Swaption Approach

$$CVA = LGD \sum_{t=1}^{T} PD_{(t-\delta,t)} Swaption_t$$

Where,

$Swaption_t$ $\quad$ $The\ value\ of\ a\ swaption\ contract\ with\ expiry\ at\ time\ point\ t$

This method models EPE as a series of swaptions. The optionality part of the valuations refers to either the scenario where the counterparty defaults or not at each time point.

Current Exposure Linear Model

$$CVA = LGD \sum_{t=1}^{T} EPE_t \, PD_{t-\delta,t}$$

In this method the current exposure of the hypothetical portfolio is calculated. Thereafter an appropriate diminishing exposure model is applied to the current exposure over the lifetime of the portfolio.

Constant Exposure

This method is similar to the previous one; the only difference is that the estimated exposure as measured at time zero is kept constant throughout the life of the contract.

Duration Approach

$$CVA = MtM\ x\ Credit\ Spread\ x\ Duration$$

This method makes use of duration to measure how much the fair value changes when credit spreads are introduced to the discounting rate. The duration serves as an estimate of the average lifetime of the exposure

2.6 SUMMARY

In this literature review, theory and background needed to calculate CVA and DVA was provided. A general introduction was presented covering the background of the over-the-counter financial trading market and the inclusion of xVA within a credit risk exposed world. The conceptual background and mathematical theory applicable to CRR related metrics and definitions were also addressed.

The pricing methodologies presented in section 2.3 will help estimate the exposure of the derivatives for the research's hypothetical portfolio. In the last section of this chapter the pricing principals of CCR are formally presented which includes the aforementioned LGD, exposure and probability of default components.

In Chapter 3 and 4 the probability of default and LGD estimates for corporate treasurers and South African banks will be considered. The application of each of the CVA methods introduced in section 2.5 will be considered in Chapter 5, through the means of a numerical example based on a fictitious portfolio of interest rate swaps between a South African corporate treasurer (Eskom) and a large tier one South African bank.

CHAPTER 3

Own Probability of Default

3.1 INTRODUCTION

In this chapter the methodology and results are presented for the estimation of own probability of default from the perspective of a corporate treasurer in the South African market. More specifically the biggest corporate in South Africa, namely Eskom, will be used in the research as an example. According to the Department of Public Enterprises of South Africa, Eskom is wholly government owned and is the South African electricity utility body. It is the world's eleventh-largest power utility in terms of generating capacity. Eskom was chosen for the research since it is one of the largest corporate treasurers in South Africa, with subsequently the most liquid available credit and debt data.

Corporate treasurers want to hedge their market risk by entering into a variety of derivative transactions. The only market-makers within the South African market to facilitate such transactions are the five biggest local investment banks: Standard Bank, Investec, ABSA Capital, Nedbank, and Rand Merchant Bank.

For corporate treasurers to value these derivative transactions according to IFRS guidelines, they need to account for CCR through CVA and DVA. In this chapter one of the building blocks of DVA will be considered; the probability of the corporate treasurer itself, entering default.

In the beginning of the chapter different probability of default measures are introduced and compared. In the second section directly sourced credit spread curves from Bloomberg is considered. Since the available data is rather illiquid some credit spread approximation is needed. The different approximation possibilities are then considered in the subsequent section. This includes a summary of how probability of default estimates are stripped from credit spread curves. A comparison discussion and is the topic of the fourth section and it is found that using publicly traded debt is the best approximation for the Eskom credit spread curve. The chapter is concluded by a deliberation of the chosen LGD assumption.

3.2 CHOICE OF PROBABILITY OF DEFAULT METRIC

As discussed in Chapter 2, determining the probability of default of specific entities is not straightforward. The creditworthiness and the underlying financial health of an entity depend on a wide range of factors that change over time. According to Standard & Poors (2011) it can depend on the credit quality of securitised assets, legal and regulatory risks, the payment structure, cash flow mechanics, operational and counterparty risks. Rating agencies use these types of measures to arrive at a specific rating of a particular entity.

Using the probability of defaults implied by a credit agency is one option which would create a consistent standard for measuring credit risk by market participants for all entities in the market place. In this method the credit rating or aggregate rating of a few rating agencies are linked to different probability of defaults. For example, AAA rated entities might be assigned a fixed probability of 2% over a 1-year time horizon.

Having a consistent method across the market to reference probability of defaults is favourable according to IFRS 13 which states that credit adjustments should be market based and measured from the perspective of the average market participant. If the entire market has access to the standard credit ratings assigned to all companies, there will be consistency in the view how the market measures creditworthiness. A comparative table of the three largest rating agency's measurement metrics are presented in Table 3.1.

Fitch	S&P	Moody's
AAA	AAA	Aaa
AA+	AA+	Aa1
AA	AA	Aa2
AA-	AA-	Aa3
A+	A+	A1
A	A	A2
A-	A-	A3
BBB+	BBB+	Baa1
BBB	BBB	Baa2
BBB-	BBB-	Baa3
BB+	BB+	Ba1
BB	BB	Ba2
BB-	BB-	Ba3
B+	B+	B1
B	B	B2
B-	B-	B3
CCC	CCC	Caa
CC	CC	Ca
C	C	C
D		D

Table 3.1: Credit Ratings

The problem, however, with this method is that CCR change over time and that these non-dynamic ratings would then become stale and inaccurate. Rating agencies only update sovereign and company ratings annually or at most biannually. When some external credit event impacts a company, the company's credit quality tends to deteriorate quickly. Once a company is in financial trouble the lagged credit ratings will not reflect the company's poor credit quality and distress accurately.

Internally modelled probability of default exhibits the same flaw. Since historical estimates are calculated by using historical defaults, these estimates are also not dynamic over time. Moreover, most companies do not have access to the already sparse historical default data. In the South African context only Saambou and African Bank have defaulted in the banking sector which categorised as tier 1, which does not provide enough default data to model off. Due to data differences and inherent complexities within these models, market participants will not have a consistent view of companies' creditworthiness as required by IFRS 13.

One needs to look at some other dynamic measure. Since the probability of default is linked to the risk of a counterparty being unable to meet all of its financial obligations, the probability of default is linked to its credit risk. Since the credit market has grown considerably in the last couple of decades, liquid credit pricing has increased significantly. Credit risk can now be measured in the market through credit spreads added to structured products to account for the CCR embedded in transactions.

Since no South African corporate treasurer has reliable live liquid CDS pricing available, an alternative is to consider quotes from data providers. Data providers like Bloomberg provide dynamic credit spread quotes for companies across different time maturities. Even though this is the case, CDS pricing for Eskom is only available in USD and not in local currency ZAR. This is mainly due to foreign funded investments dominating the market. This will change and the market is expected to grow with big strides over the next five- to ten years.

Probability of defaults can then be stripped from these curves as was presented in section 2.2.6.1.

3.3 SOUTH AFRICAN CORPORATE TREAUSERERS' CREDIT SPREADS

South Africa's credit market is not as liquid as those of the developed countries. The major banks in South Africa have only recently established designated credit trading desks which provide credit pricing for the broader market.

Even though these desks have been operating for a couple of years, the credit market is still in its infancy. Proper liquid pricing from data providers is only available on South Africa's government itself and on its biggest corporate treasurer, namely Eskom. Even the credit spread data available on Eskom is quite illiquid in the short-end of the curve.

The problem with using credit spreads for CVA or DVA calculations purposes is that in South Africa's illiquid credit market there are very little liquid credit spreads available on local corporates. Below follows a depiction of Eskom and South African Sovereign' credit spreads, as at 29 July 2016, as sourced from Bloomberg.

In this research Bloomberg will be used to extract credit data. Bloomberg is chosen since it is widely used in South African banks and it is not too expensive for corporate treasurers to acquire one licence for their entire organisation.

Credit spreads are published for the 3M, 6M, 9M, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y and 10Y points and are linearly interpolated for all time points in between.

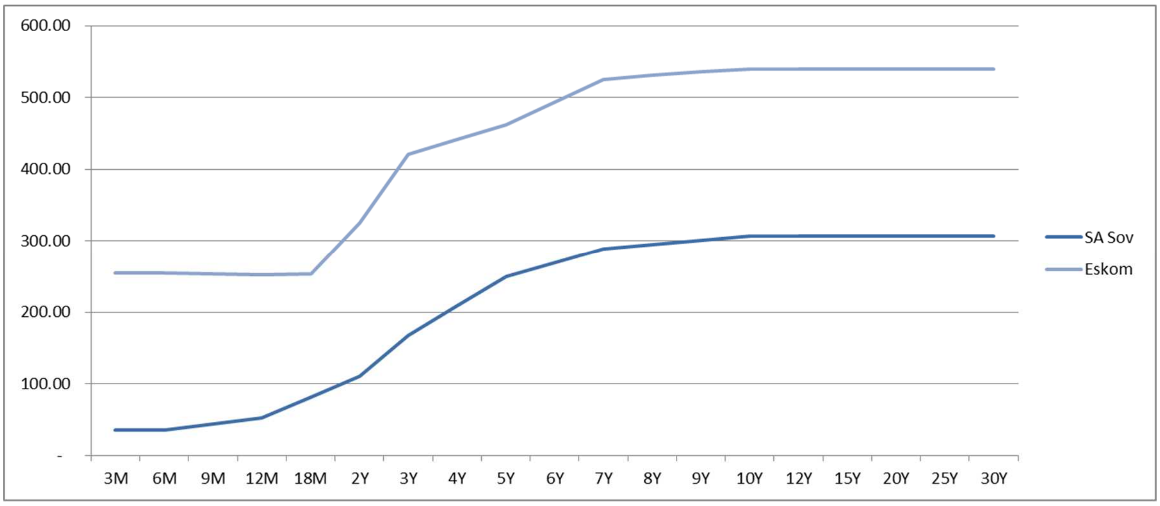

Figure 3.1: Observed Bloomberg CDS Curve

As can be seen in Figure 3.1, the Eskom credit spread is trading higher than the Sovereign one. Intuitively this makes sense. It is expected that to buy credit default protection on Eskom, or any other corporate, would cost more than buying protection on the Sovereign the corporate is operating in.

The uneven irregular shape of the Eskom curve suggests stale or illiquid underlying data. Especially in the short end of the curve, data seems to be sparser than at the longer maturities. The curve flattens out for tenors shorter than 18-months, since no data is available in the market for these points. It is very seldom that a treasurer or bank would buy protection for such a short maturity. CDS protection buying and selling normally occurs beyond a three-year maturity point. The five-year point is the most liquid across all credit markets. The five-year point is seen as the standard tenor for quoting credit spreads and it can be assumed that the 5-year spread in the graph below is the most accurate. The choice of tenor point is also supported by BIS (2016:62), which requires that all default measures be calibrated off the 5-year point in the case of single point calibration.

3.4 ALTERNATIVE CREDIT SPREAD CURVE PROXIES

It was seen in the previous section that the Bloomberg sourced Eskom credit spread curve is rather illiquid. It does not appear to be a liquid reliable measure to represent the credit quality of Eskom across all maturities.

However according to IFRS 13, all observable credit risk inputs must be included in the calculation of DVA and CVA. In this section different approximations are considered that can better represent Eskom's true credit spread curve, by making use of observable debt related inputs.

3.4.1 OBSERVABLE CREDIT SPREAD CALIBRATION

Since the Bloomberg Eskom curve does not appear liquid across all maturities, the ones which are liquid can be used as calibration points for the other time points. It was already mentioned in section 3.3 that the five-year point is the most liquid and therefore the most reliable point to use in a calibration. Through credit market demand and supply factors this is the tenor that is mostly quoted in the market.

Eskom is government owned and its credit spread tracks movements in sovereign quite closely as can be seen in the 5-year historical log returns graph below. It is therefore possible to mark the Eskom credit spread as a fixed factor of the South African credit spread, as observed in the relative relationship between the two in the 5-year tenor.

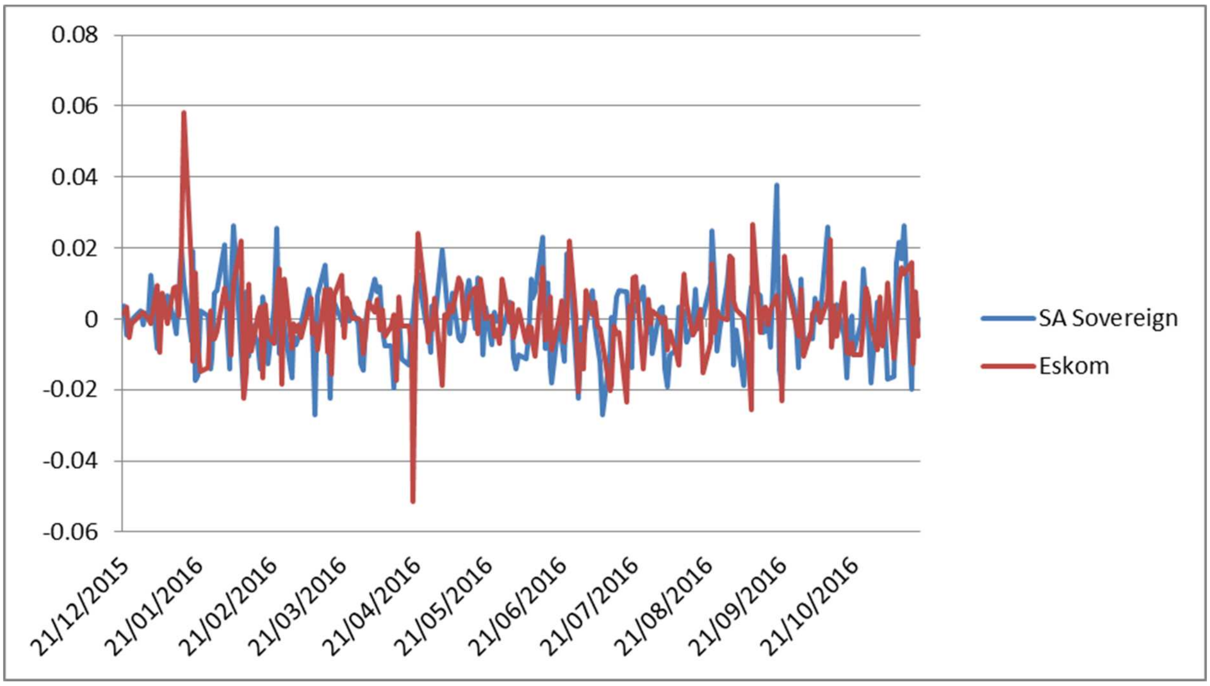

Figure 3.2: Eskom vs SA Sovereign Log Returns

There is such a close relationship since Eskom is completely government owned and if South Africa were to fail, it is likely that all government owned corporate treasurers will be in severe financial difficulty. It is therefore assumed that that if a corporate treasurer were to fail, it is possibly due to some non-idiosyncratic sovereign linked event that would affect all the SOEs (Sovereign Owned Entities). Therefore, the assumption can be made that movements in the credit spread of government owned corporate treasurers are closely related to the moves in Sovereign.

Since the 5-year Eskom and South African sovereign credit spreads are always liquidly available from Bloomberg, the relationship between the two can be used to infer the other time point credit spreads of Eskom.

Idiosyncratic movements can therefore be included as the changes in the relative relationship between the 5-year SA Sovereign and Eskom credit spread points. The bulk of Eskom's credit quality will however be driven by movements in the underlying sovereign credit spread level. When calibrating on the 29th of July 2016, a factor of 1.85 is calculated for the 5-year point which translate to the following implied Eskom credit spread curve, depicted in Figure 3.3 below. Note that the y-axis is measured in bps.

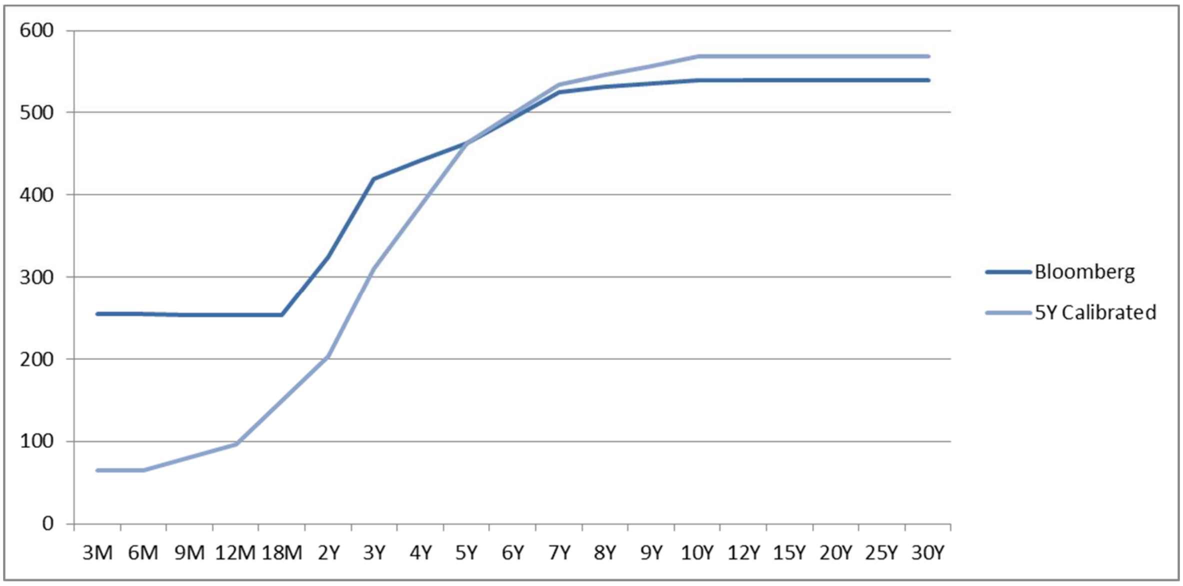

Figure 3.3: 5Y Calibrated Eskom CDS Curve

The calibrated graph exhibits a much smoother and theoretically expected shape. The short end is calibrated to have a smooth downward sloping shape instead of the observed stale illiquid one. It provides a much better expected fit. The biggest difference between the illiquid observed and calibrated graphs lies in this short end of the curve.

In terms of measured SPV01 exposure the differences in the calibrated model versus the Bloomberg observed one does not raise concern. SPV01s are equivalent to PV01s with the only difference being that SPV01s are measured off a credit spread curve whilst PV01s are measured from an interest rate curve. SPV01 represents the profit gain or loss that a one basis-point move in the credit spread curve infers. This is because the closer an exposure rolls to maturity; the smaller credit risk becomes due to the diminishing probability of default weighting.

Since SPV01s decrease over time, the value of SPV01s over short time duration becomes diminishing small. An enormous notional amount is needed on a CDS or some other debt to generate a large amount of sensitivity exposure in the short term. Therefore, the exposure impact will be less on the short end of the curve than the long. This provides comfort for the relatively large differences in the short term, although the calibrated curve is perceived to be more correct.

The reason why a fixed factor is taken is to ensure that the resulting Eskom credit curve has a smooth shape similar to that of the liquid sovereign. For example, if different factors were observed for the short and long end, it would imply an Eskom credit curve with a severe kink between the short and long end time point divider.

3.4.2 DEBT FUNDED APPROXIMATIONS

An alternative to using a company's sourced or observed credit spread is to use some other measure of a company's debt profile which can be translated to a credit spread. The characteristics of issued debt can be used to assess the creditworthiness of an entity.

This can be done by considering the implied bond prices and associated credit spreads above a so called basic risk curve. Observable asset swap spreads and zero-coupon spreads can be used as approximations for an entity's credit default swap spreads (i.e. credit spreads).

The asset swap spread and z-spreads of a bond are variables that give an indication of the relative value of the specific bond. Both of these measures try to quantify the embedded credit risk in a bond. However, they are calculated in slightly different ways. The following introduction to these spreads are based on summarised material presented by Lehman Brothers (2004:8-12) and Choudry (2005:2-5).

According to Choudry (2005), the AWS is defined as the difference between the yield-to-maturity of the bond and the interpolated interest rate obtained by linear interpolation of the swap curve or treasury curve. It is thus the spread used in valuing an asset swap. An asset swap is an instrument that combines an interest rate swap with a cash bond. Say that the bond holder receives a floating coupon, measured as the spread above LIBOR, and the bond holder pays a fixed coupon. The determined floating spread is the asset-swap spread. Asset swaps are usually transacted at the bond's par price.

The implied zero-coupon curve, implied from the underlying swap curve, is used in the asset swap valuation. The asset swap spread is then defined as the floating spread that the bond holder receives in the asset swap so that the bond price, calculated with the implied zero curve, equates to the bond's market price. Therefore, the asset-swap spread incorporates the difference between the par price and the market price, as well as the difference between the fixed swap rate and the bond's coupon rate. This is the reason why traders like to consider the asset swap spreads between bonds to gauge relative value, since it provides additional information regarding the inherent credit risk of the company.

A bond's z-spread is calculated slightly differently. It is defined as the constant spread that must be added to a swap or treasury ('basic risk') curves such that the present value of the bond's cash flows equals the price of the bond in the market.

If the z-spread and the asset swap spread of a particular asset follow each other closely and are calculated at roughly the same bps, it intuitively means from their definitions that the asset's theoretical price is very close to its market implied price. This in turn shows that the asset is liquid, since there are enough demand and supply players in the market to drive the asset's price back to its theoretical value. If this is the case, either of the two measures can be used as a proxy for the credit spread.

3.4.3 CHOICE OF DEBT

Since there are not always observable CDS pricing available in the market or even reliable credit spread data from financial data providers, one needs to start looking at alternatives. One of the alternatives is to use liquid bonds issued by a company in the market and use these bonds' spreads to approximate credit spread curves. Aside from liquidity, the main reason why two similar bonds would trade at differing prices is differing probabilities of default. In order to use bonds the liquidity of these bonds needs to be evaluated and found to be adequate.

As was discussed in the previous section, when the z-spread and the ASW spread are nearly identical it is an indication that the specific asset is liquid enough to give an accurate reflection of credit risk. In the case of Eskom, its different issued debts can be analysed to test whether a particular asset is liquid enough to obtain a reasonable proxy spread off.

Issued bonds that are currently trading in the secondary market are considered, since these unlike simple loans or other debt instruments, present more accurate current credit risk estimates, because they trade actively in the secondary market. Below a list is given of all Eskom's current issued bonds up to 2027, together with a percentage difference between their ASW and z-spreads.

Bond Name	Currency	Class	Rating	Issued	% Spread Difference
ESK 27/12/2018	ZAR	SR Unsecured	NA	R 5,000,000,000	19%
ESK 14/03/2020	ZAR	SR Unsecured	NA	R 5,000,000,000	3%
ESK 01/08/2020	ZAR	1st Lien	NA	R 11,040,681,000	7%
ESK 26/01/2021	USD	SR Unsecured	BB	$ 1,750,000,000	2%
ESK 14/03/2022	ZAR	SR Unsecured	NA	R 5,000,000,000	4%
ESK 06/08/2023	USD	SR Unsecured	BB	$ 1,000,000,000	2%
ESK 14/03/2024	ZAR	SR Unsecured	NA	R 5,000,000,000	5%

Table 3.2: Eskom Bonds

A good indication of the embedded credit risk can be seen from the type of debt class. Different classes of debt refer to the collateral hierarchy in the case of the issuer defaulting. Senior secured (SR) debt holders are paid back first when the company begins liquidating its assets to pay its debts. The highest rank in the secured tier is the referred to as the '1st lien' class. A lien refers to the legal right of a creditor to seize the property from a borrower that has failed to meet its financial obligations. These debt holders are paid before any other secured debt holders. After the secured debt holders, unsecured debt holders are paid.

For the purposes of gauging the credit risk of an entity through issued debt, it would be best to look at unsecured debt. Within this debt class collateral ranking is not weighted into the bond prices as much and it gives a truer reflection of the risk of the issuer defaulting.

If debt is rated by a rating agency it provides liquidity comfort in the sense that it is traded frequently enough in the secondary market for rating agencies to take time and assess the debt. The size of issuance and uptake also gives an indication of liquidity. The more debt issued, the more debt is available to be exchanged in the secondary market. Liquidity is important since these assets then provide a dynamic real time reflection about the market's perception regarding the embedded credit risk of the asset.

In the last column of Table 3.2, a percentage difference between each asset's zero-coupon spread and asset swap spread is given. As was discussed in the previous section, the smaller this difference the closer is the implied price of the bond to its theoretical value. This also gives an indication of liquidity. If traders know assets to be liquid and there are substantial differences between these two spread metrics, the market perceives the asset to be mispriced and in turn provides profitable trading opportunities.

From the above table the USD denoted Eurobonds seems to come out on top in all of the listed criteria for liquidity. Eurobonds are bonds which are traded in one sovereign region with a foreign denoted currency.

For the proxy methodology the three Eurobonds 2021, 2023 and 2025 was identified in the assessment of credit risk and the proxy of a credit spread. The three bonds are sufficiently spaced across a term structure stretching nine years. This will allow for sensible interpolation of missing tenors on the approximated credit spread curve.

3.4.4 ZERO COUPON SPREAD CALIBRATION

The zero-coupon spreads of the selected issued debt in the previous section can be used to approximate a credit spread curve across different maturities in relation to the South African credit spread curve. As was discussed in section 3.3, South Africa sovereign is a major driver of Eskom and therefore it makes sense to use it as a calibration proxy.

Z-spreads and ASW reflect the credit risk of the respective assets and can be used as the bps approximation of the associated credit spread for the respective asset's maturity. Since the z-spreads and ASW of all three selected bonds are almost equal, it does not matter which measure is used in the approximation. In this research the z-spread will be used, since it is the spread most widely used to assess the relative value of bonds in practice.

This is done by finding a single optimising factor of the South African sovereign credit spread curve to represent the approximated Eskom credit spread curve. The factor is chosen by calculating the minimum mean squared error between the observed z-spreads of the bonds and their associated implied credit spreads. As noted in section 3.3 a single factor is chosen across all tenors in order to keep the shape of the implied Eskom credit spread curve regular.

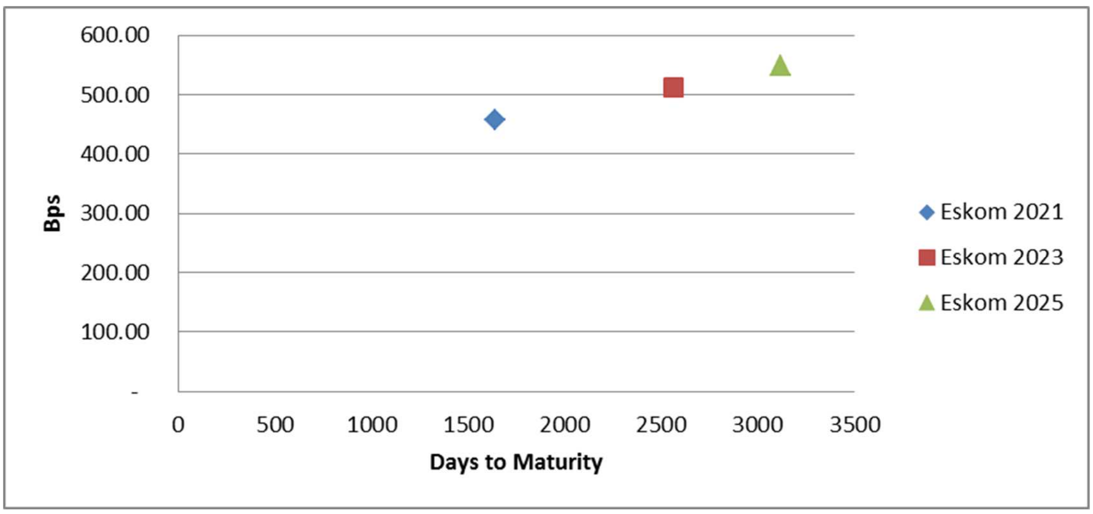

Figure 3.4: Eskom Eurobonds Z-Spreads

The three bond's z-spreads are presented above as observed on the 29th of July 2016. Intuitively it makes sense that the longer dated the bond, the larger the z-spread in bps. This is because the z-spread represents an approximation of the associated credit spread. The further out the maturity, the higher is the associated default risk and therefore it is more expensive to buy credit protection on these longer dated maturities. There is more time for Eskom to default on its coupon obligations in the longer termed assets.

The minimum mean squared error factor is calculated as 1.851573. This factor is then multiplied by the observed South African credit spread curve. Unobservable maturities are obtained through linear interpolation.

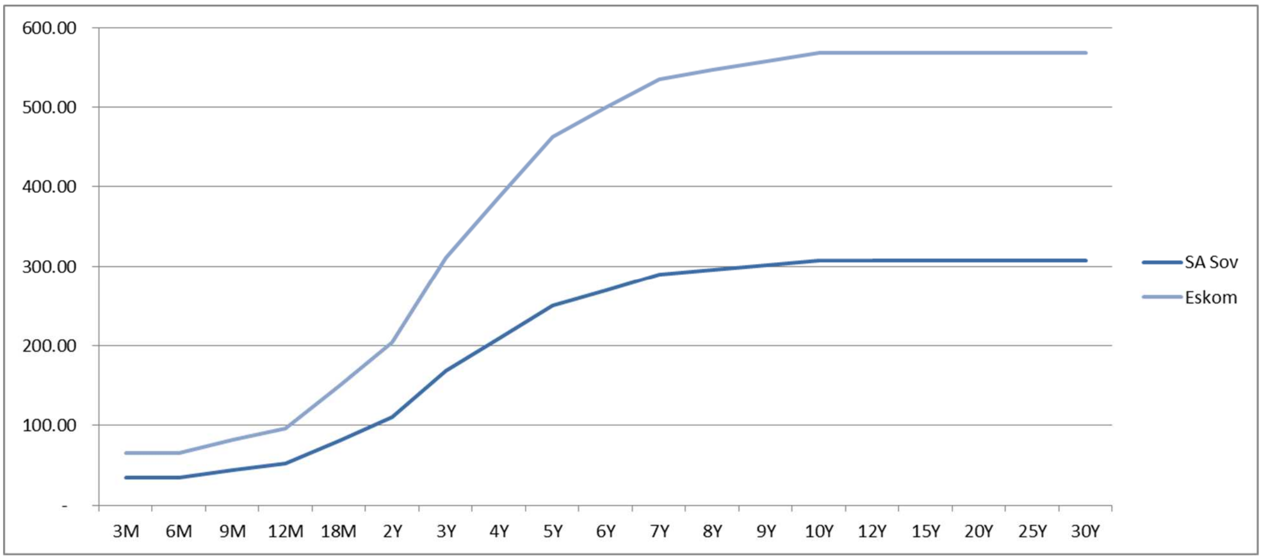

Figure 3.5: Z-spread Calibrated Eskom CDS Curve

3.4.5 FULL ASSET REVALUATION CALIBRATION

In this section a more complex calibration is presented which uses the market prices of the three Eurobonds instead of their observed z-spreads. A single factor multiplied across the South African sovereign curve is once again calibrated such that the observed bond prices in the market equal the implied theoretical bond prices. Before this can be discussed in full the mechanics of stripping probability of default from a credit spread must be presented.

Hazard rate and survival curves can only be estimated appropriately from the credit spread curves after incorporating the assumed recovery. The credit spread curve is divided by the assumed LGD constant to arrive at an adjusted credit spread curve denoted by say $C(t)$. After $C(t)$ has been obtained the survival function curve can be calculated by $S(t) = \mathrm{e}^{-\mathrm{C(t)}t}$.

It is now straightforward to calculate the probability of default between say time points t_3 and t_1, by simply taken the difference $S(t_3) - S(t_1)$.

To calculate the theoretical value of a bond by incorporating the embedded credit risk is outlined below. The crux of the pricing methodology is the weighting of future coupons and the notional amount by the probability of being realised by using the survival function.

$$Bond\ Price = PV\ of\ Notional\ Leg + PV\ of\ Coupon\ Leg + PV\ of\ Default\ Leg$$

Where,

$$PV\ Notional\ Leg = \sum (N)(DF)(SF)$$

$$PV\ Coupon\ Leg = \sum (N)(c)(DF)(SF)$$

$$PV\ Default\ Leg = \sum (N)(RR)(DF_i)(DP_i)$$

Where N is the notional of the bond, DF is each cash flows associated discount factor, SF is the probability of survival (i.e. the probability of being realised), RR is the retention rate in the event of default and DP_i is the probability of default occurring at time i.

The single factor of the Sovereign credit spread curve is calibrated which implies a specific Eskom credit curve, which implies an associated hazard rate curve, which in turn implies a specific theoretical value through the unique weighting of cash flows. The single factor of Sovereign is calibrated in such a way that the market observed value of the Eskom bonds equal the implied bond prices. The calibration is done simultaneously across all three bonds and the minimum sum of squared errors is used as the measure of calibration goodness.

Through the calibration described above, a single factor of 1.88 is calibrated as at the 29th of July 2016. This is a similar result than that of the previous section (1.85).

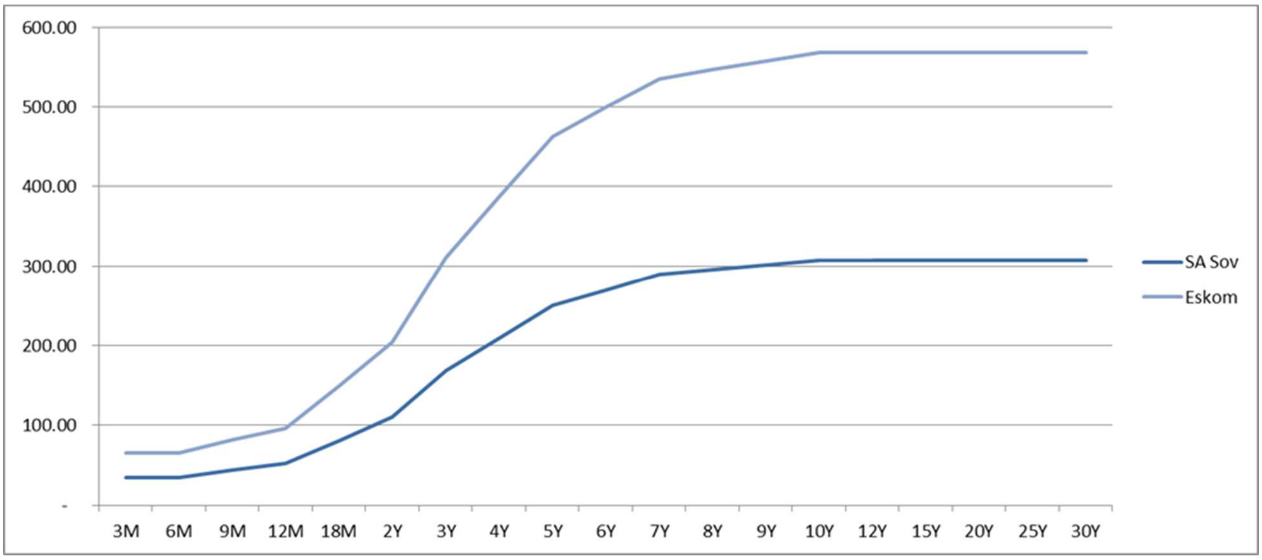

Figure 3.6: Full Revaluation Calibrated Eskom CDS Curve

3.5 LGD

Since bonds are the main debt source for most companies in the market, Altman (2006:25) used defaulted bonds to estimate a LGD. The estimation was done by calculating the mean of 2300 defaulted bond recoveries. They estimated a 37.8% LGD. However, there was a significant difference found between bonds that were classified as investment grade and non-investment grade.

The International Valuation Standards Council (2014:15) notes that different estimates of LGD should be used across different types of entities (i.e. municipalities, banks, corporate treasurers etc.). The council also suggests that estimates form credit rating agencies should be used, especially when historical modelling data is lacking. This also suits the new IFRS 13 guidelines which states that input variables for CVA calculation purposes should be taken as those that an average market participant is expected to take.

The Financial Services Board of South Africa (FSB) concluded in the third South African Quantitative Impact Study (SA QIS3) that an average LGD of 60% is sufficient. Therefore, a LGD of 60% will also be applied in this research, which implies a recovery assumption of 40% in the event of default. This will be applicable to Eskom as well as the banks considered in the following chapter.

3.6 COMPARISON AND SUMMARY

If is comforting to note that all the calibration methods yield similar results. The approximated Eskom credit spread curves are depicted below as calibrated on the 29th of July.

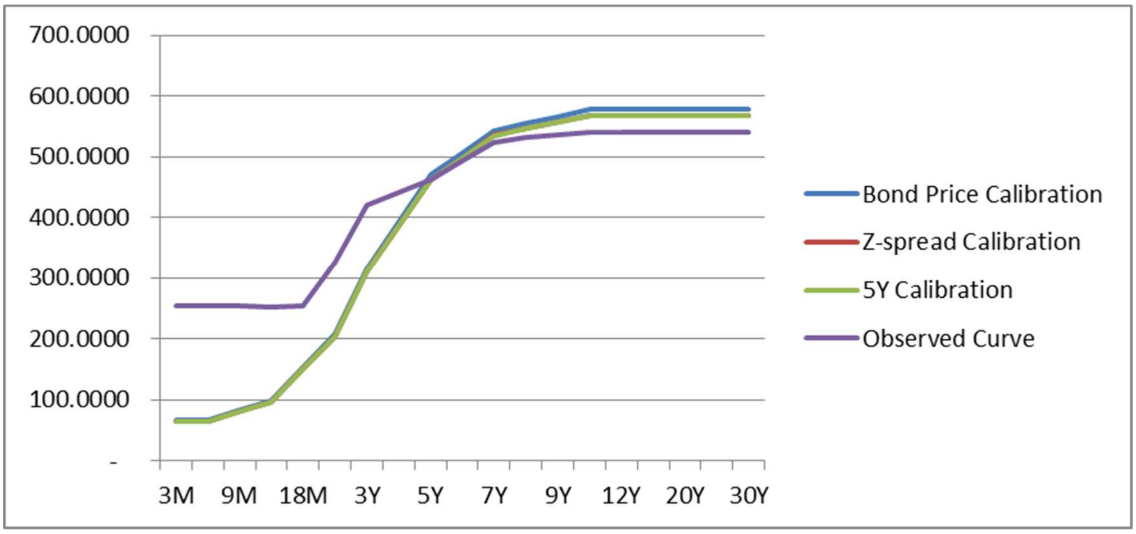

Figure 3.7: Calibrated Eskom Curve Comparison

is expected that the level of the factor changes over time as the relative credit quality between South Africa and Eskom changes. As can be seen the differences between the calibrated factors for different approximation methods remains relatively small.

Since the bond price calibration makes use of live bond price data instead of z-spreads, this method is chosen in this research. The probability of default can then be stripped from this credit curve. The model can be calibrated daily with newly observed end of day bond prices. The same methodology can be implied to other corporate treasurers or entities with liquidly traded debt.

CHAPTER 4
Counterparty Probability of Default

4.1 INTRODUCTION

In this chapter the methodology and results are presented for the estimation of counterparty probability of default from the perspective of a corporate treasurer in the South African market.

Corporate treasurers want to hedge their market risk by entering into a variety of derivative transactions. As previously mentioned, the only market-makers within the South African market to facilitate such transactions are the five biggest local investment banks: Standard Bank CIB, Investec, ABSA Capital and Rand Merchant Bank.

For corporate treasurers to value these derivative transactions according to IFRS guidelines, they need to account for CCR through CVA and DVA. In this chapter one of the building blocks of CVA will be considered; the probability of the corporate treasurer's counterparty, entering default.

At the start of the chapter available credit risk data for South African banks are discussed. This also includes an explanation regarding the market observed characteristics of banking credit spreads. In the second section alternative approximation methods of banks credit spread curves are explored, since observable CDS data on South African banks are non-existent. Thereafter different approximations for South African banks are analysed and in the final section a linear multiple-regression analysis is presented which provides a generic approximated credit curve across all large tier one South African banks. This approximation will be used to estimate probability of default estimates, which in turn will be used in the CVA calculations presented in Chapter 5.

4.2 SOUTH AFRICAN CORPORATE TREAUSERERS' CREDIT SPREADS

In Figure 4.1 below an example of a bank's credit curve is depicted. It consists of a sovereign portion and an idiosyncratic portion. The sovereign portion moves according to the credit quality of the associated sovereign and the idiosyncratic portion represents the credit risk that is associated with bank specific risks. As was mentioned in Chapter 3, the idiosyncratic portion of corporate treasurers can be ignored since most of the big corporate treasurers in South Africa are government owned. However, South African banks are privately owned and managed, therefore the behaviour of the idiosyncratic portion is significant and cannot be ignored. Thus the correlation between the performance of the government's credit spread and those of the banks are going to differ slightly due to this idiosyncratic portion. However, in South African banks, like banks in most developing markets, the main driver of the underlying credit spreads will be credit quality changes in the associated Sovereign. Evidence of this is also empirically provided in the multiple regression at the end of the chapter.

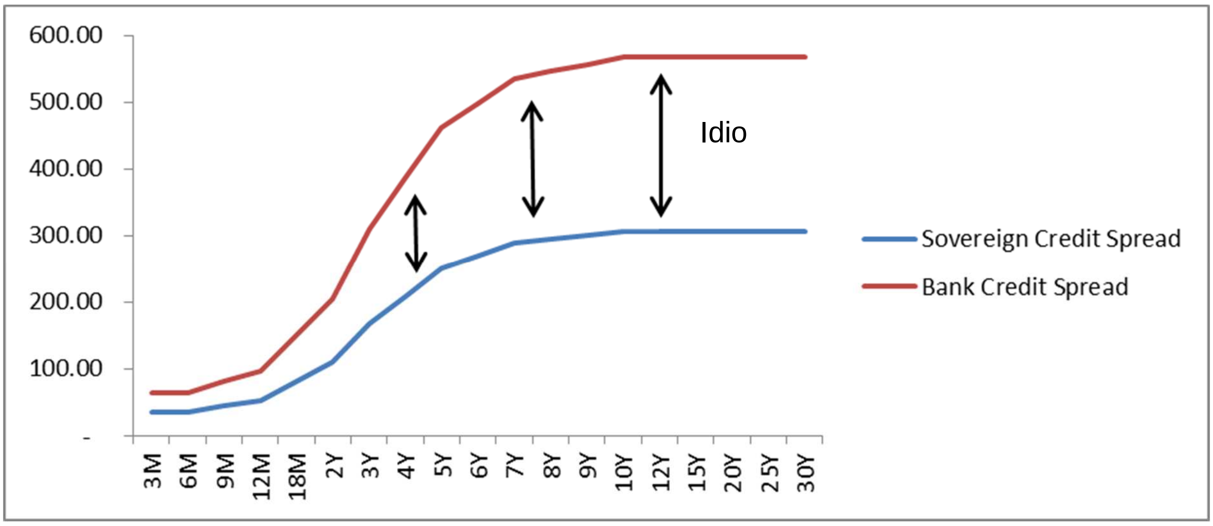

Figure 4.1: Bank Idiosyncratic Risk

In the previous chapter it was stated that Credit Rating and Internally Modelled probability of default measures are not preferred as inputs for CVA and DVA estimation by IFRS 13. As was discussed in Chapter 3, the most sought after credit risk metric would be live continuous CDS quotes in the market. However, similar to Eskom and all other corporate treasurers operating within the South African market, South Africa banks also do not have liquid CDS quotes available in the banking sector. The credit market for banks is so illiquid in South Africa that no CDS data is even available from a data provider like Bloomberg.

The next best thing to do is to use approximated curves for the local banks. Since the tier one South African banks are similar though their tier one status and net assets, a single generic credit spread curve will be approximated in the study which will represent the credit risk of all the local banks.

This assumption is made since the credit quality movements of all South African banks is notoriously closely to those of Sovereign and each other. This assumption is made from the perspective of a reasonable market participant familiar with South Africa's credit market as prescribed by IFRS 13. Therefore an approximation of the generic bank credit spread should contain a Sovereign linked and an idiosyncratic portion.

4.3 ALTERNATIVE CREDIT SPREAD CURVE PROXIES

South African banks do not have liquidly traded debt instruments trading in the secondary market to make use of ASW or z-spreads for approximation purposes. Therefore, some other approximation needs to be considered.

The IFRS guidelines states that observable market data should be used to estimate the probability of default if available. Since no observable data is available alternative peer observable data needs to be used. The only observable credit spread relationship between banks and Sovereigns are in foreign markets.

Similar countries to South Africa should be chosen which would serve as an approximation for market conditions in South Africa. The choice of country and associated banks will be discussed in the following two subsections.

The behaviour of the idiosyncratic portion of the banks' credit spread should also be modelled. In order to do this international credit indexes can be considered which would typically move to that of a South African bank's credit spread.

After the appropriate foreign bank versus local sovereign relationship and appropriate idiosyncratic representing credit indexes are chosen, these can be used in a multi-regression analysis to arrive at an approximated generic South African banking sector credit spread curve. A brief overview of regression is firstly presented in the next section.

4.4 INTRODUCTION TO MULTI LINEAR REGRESSION

Multiple-regression is a statistical tool that can be used to analyse the relationships and underlying associations between different variables. The relationships between different variables can take on many different forms. The simplest relationship that can be observed is a linear relationship.

When considering a complex process, it is not realistic to assume that a variable is only dependent on one other variable. Linear multiple regression can formally be introduced by the equation below:

$$Y = X_1B_1 + X_2B_2 + \cdots + X_NB_N + B_0$$

Where,

Y	$The\ dependable\ variable$
X_i	$The\ i^{th}\ independent\ varaible$
B_i	$The\ coefficient\ weighting\ associated\ with\ variable\ X_i$
B_0	$The\ constant\ intercept\ in\ the\ analysis$

In short some variable Y are explained by considering the values of N other dependent variables. It would be ideal if each of the independent variables can explain a different characteristic of Y. If this is the case each independent variable explains a different characteristic of Y. Therefore, ideally all independent variables should be as uncorrelated as possible. In section 4.8 different goodness of fit measurements are presented and used which helps with variable selection that insure that included variables in the analysis improves the overall fit. If too many highly correlated variables are included, the inherent multicollinearity can possibly reduce the effectiveness of the model.

4.5 CHOICE OF FOREIGN APPROXIMATION COUNTRY

Similar countries to South Africa should be chosen which would serve as an approximation for market conditions in South Africa. Aspects that need to be considered are availability of data, a country's credit rating, the underlying economic drivers and the available banks within chosen countries.

4.5.1 CREDIT RATING

To identify a country with a similar bank versus sovereign relationship it is important to consider developing countries with similar credit ratings. Developed countries' banks tend to be less correlated to their associated sovereigns in terms of movements in their underlying credit spreads. In developing countries, each bank form a far greater importance to the country's economy since there are only a small number of tier 1 banks.

As at July 2016, South Africa has been rated by Moody's as BBB- with a negative outlook. The poor rating is linked to political uncertainty in 2016 due to the firing of finance minister Nene at the end of 2015, government corruption allegations and upcoming municipal elections. This rating is only one notch above non-investment grade status and therefore a country with a similar credit outlook and rating should be considered. At the time of analysis, there were fears in the media and by local investment banks that South Africa will be downgraded in December 2016. Fortunately for South Africa that was not the case. However, this uncertain credit environment is an important characteristic to keep in mind when identifying countries with similar economic landscapes than South Africa.

Moody's credit rating is considered in this research since Moody's is one of three top three credit rating agencies in the financial world and it is also the chosen default credit rating presented in Bloomberg's description section. Below a ranking table of countries are given with respect to their credit ratings. The countries in the table below were identified as possible country approximations for South Africa. These were chosen on the bases that they have credit ratings similar or close to that of South Africa. The ratings below were sourced directly from Bloomberg.

Country	Rating
El Salvador	B+ stable
Hungary	BB+ positive
Portugal	BB+ stable
Russia	BB+ negative
Guatemala	BB stable
Bahrain	BB stable
Croatia	BB negative
Costa Rica	BB negative
Brazil	BB negative
Tunisia	BB- stable
Iceland	BBB+ stable
Mexico	BBB+ stable
Spain	BBB+ stable
Peru	BBB+ stable
Columbia	BBB+ stable
Thailand	BBB+ stable
Panama	BBB stable
Italy	BBB stable
Uruguay	BBB negative
Bulgaria	BBB- stable
Romania	BBB- stable
Morocco	BBB- stable
Indonesia	BBB- stable
South Africa	**BBB- negative**
Turkey	BBB- negative
Kazakhstan	BBB- negative

Table 4.1: Country Ratings

All the countries in the table above will be included in the analysis going forward, since all of them have credit qualities roughly similar to that of South Africa and sovereign credit spread data are available on them all in at least the 5-year point. In the next section the 5-year credit spreads will be compared between the listed countries.

4.5.2 CREDIT SPREAD

The credit spread level of a particular country also gives a good indication of the inherent credit risk. The 5-year credit spread levels are plotted in Figure 4.2 below for all countries identified in the previous section as at 11 July 2016. It gives a visual representation of the countries' spreads compared to that of South Africa sovereign. The goal of this comparison, as was with the previous section, is to identify countries with a similar creditworthiness and credit characteristics than South Africa. This will help to find a suitable country or countries to approximate the relationship between sovereigns and banks off. The 5-year point was chosen for this analysis, since it is the most liquid tenor point on most credit curves and the FRTB (2016:62) requires that all default measures be calibrated off this point in the case of single point calibration.

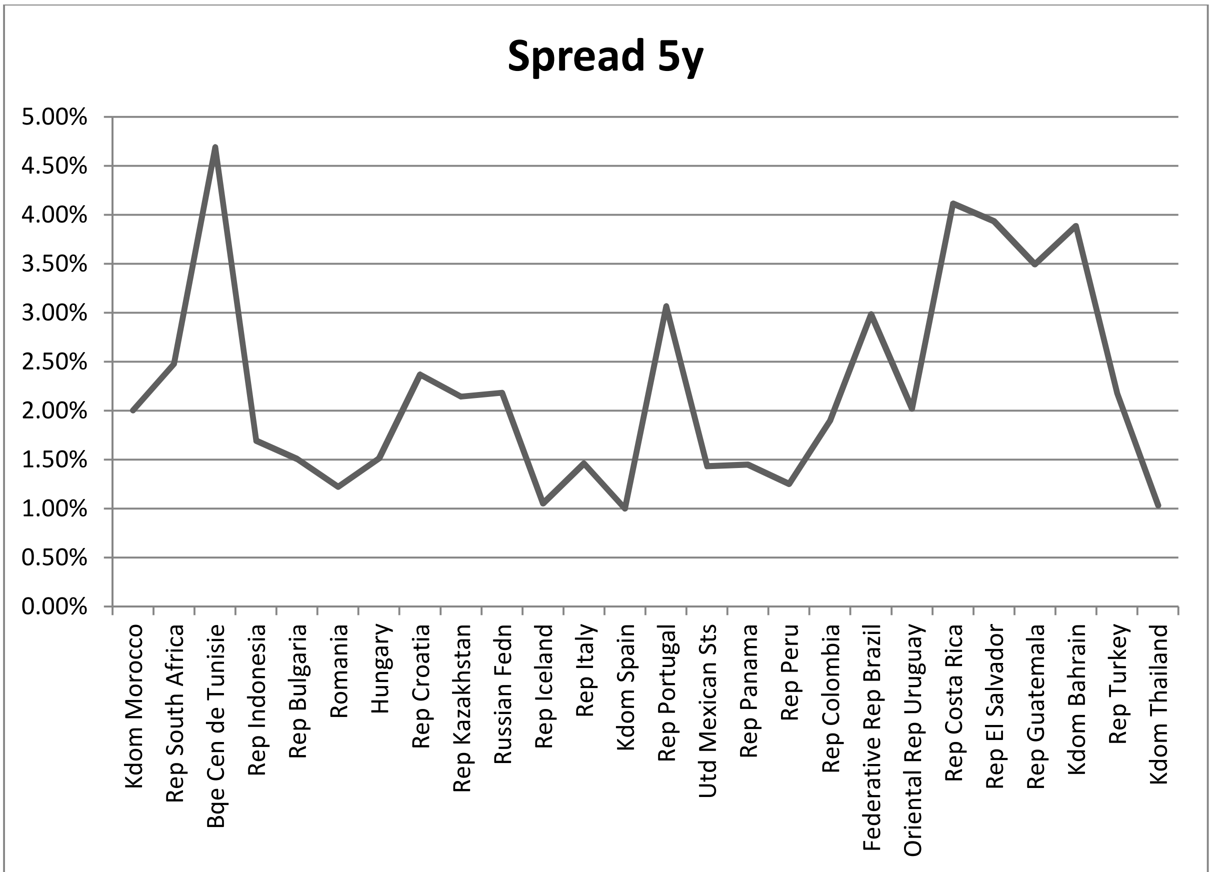

Figure 4.2: Country 5Y CDS Comparison

As can be seen in the graph there is quite a large amount of variability in the spread levels. Tunisia for example had a credit spread in the 4.75% region, while others like European Union member Spain are trading around the 1.00% level. At that point in time South Africa was trading at 2.5%. This 2.5% can be interpreted as the premium level in bps at which to buy 5-

year South African credit protection. Ideally countries with a similar credit premium levels will be included in the regression.

Since credit spreads are dynamic and quite volatile over time, this can only serve as a qualitative mechanism to sense check selected countries. When applying this approximation method over time, it is needed to recalibrate the model and check if selected countries remain sensible in terms of their credit environment.

The credit data available from Bloomberg is liquid enough for all the listed countries, except for Iceland. The 5-year credit spread is plotted in Figure 4.3. It can be seen that the credit spread seems to remain static for the third quarter of 2016. Iceland has an active credit market; however, the data available on Bloomberg seems to be of poor quality. The behaviours of the other 24 countries' 5-year credit spreads are as expected and are kept for further analysis.

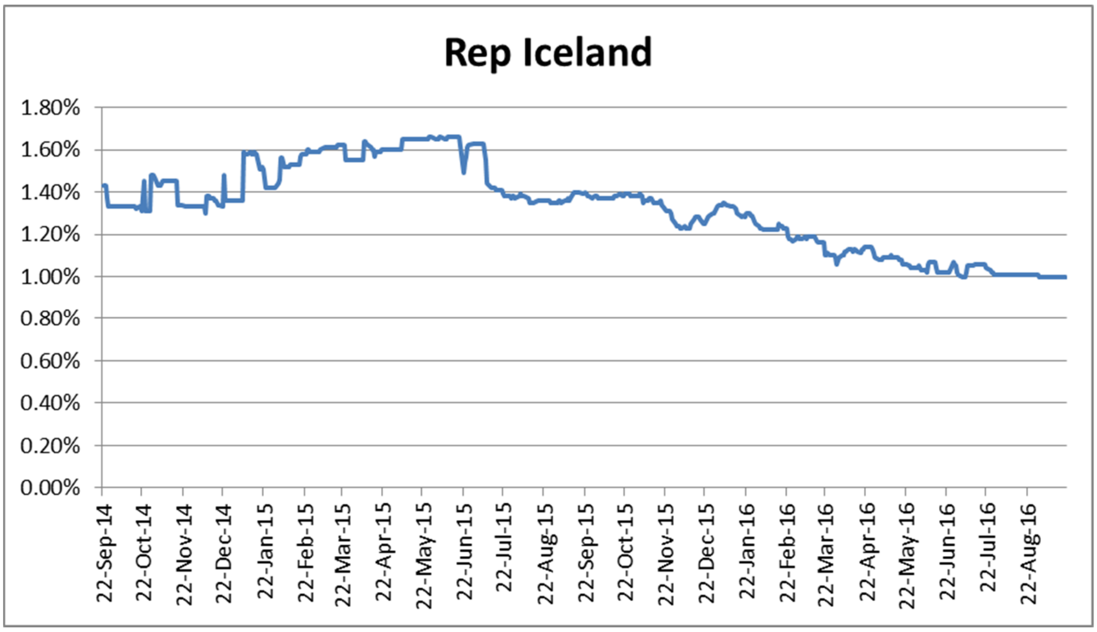

Figure 4.3: Stale Iceland CDS Curve

4.5.3 ECONOMIC FACTORS

When considering countries which would serve as good approximations for South Africa one need to look at more than just the credit quality. Underlying economic factors and drivers are also important measures to consider. Due to some fundamental economic drivers of a currency or regulations, the behaviour of bank's credit spreads and their correlation with associated sovereigns might differ drastically.

A factor such as the main economic growth driver of a country plays a big role. A country's credit quality might solely depend on their ability to deliver a valuable resource. Many Arabic countries and well as Russia is highly dependent on oil. Therefore, the value of this underlying resource will have a big influence on the credit quality of the country and its banks. South Africa has a large resources component which determines the growth of the economy. However, countries should be avoided that are heavily reliant on a single resource.

Another aspect that requires attention when considering proxies for South Africa and its banks is the ownership structure of the banks. All South African banks are privately managed and owned. Countries should be avoided, where the tier 1 banks are wholly government owned.

4.6 CHOICE OF FOREIGN APPROXIMATION BANKS

Corporate treasurers are only able to enter into financial derivative transactions with banks with the capability to do so. Not all banks have this ability. In South Africa there are about five investment banks, which are categorised as tier one banks within the country. Therefore, only tier one banks of the listed countries will be considered.

Data availability is again a major concern. Many of the countries are excluded purely due to lack of data availability for its banks. There are either no data available for any of its banks or the data is of poor quality. After the bank data of all the countries were considered Russia, Brazil, Turkey, Bahrain, Spain and Columbia were left over.

Russia

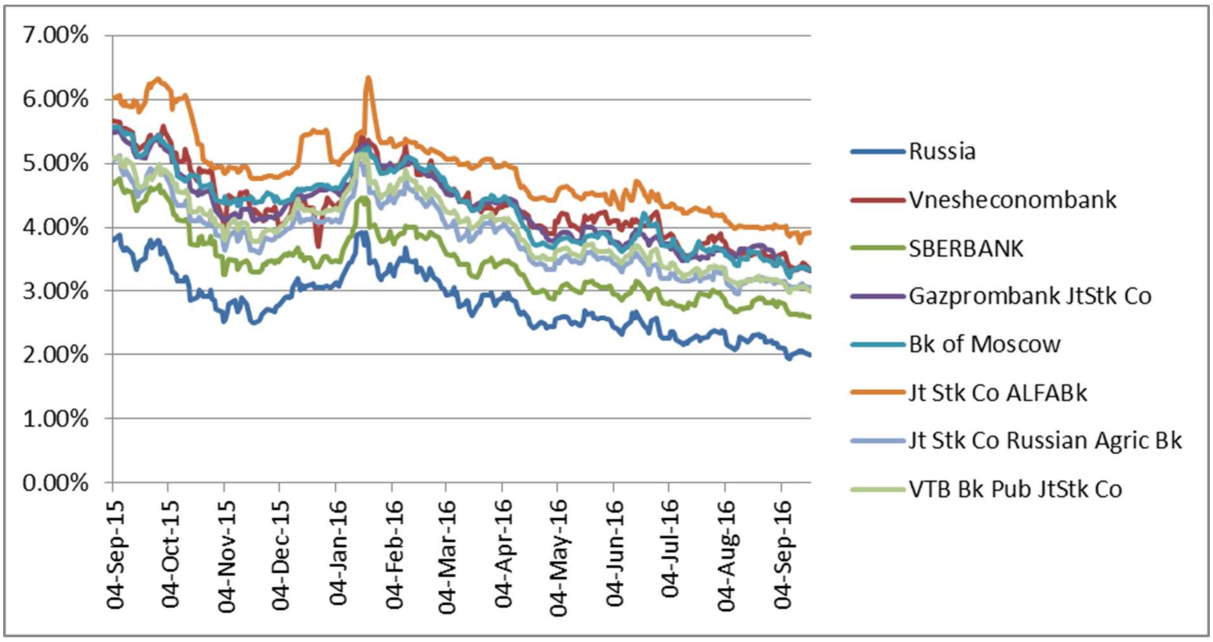

Figure 4.4: Russia CDS Curve

__Brazil__

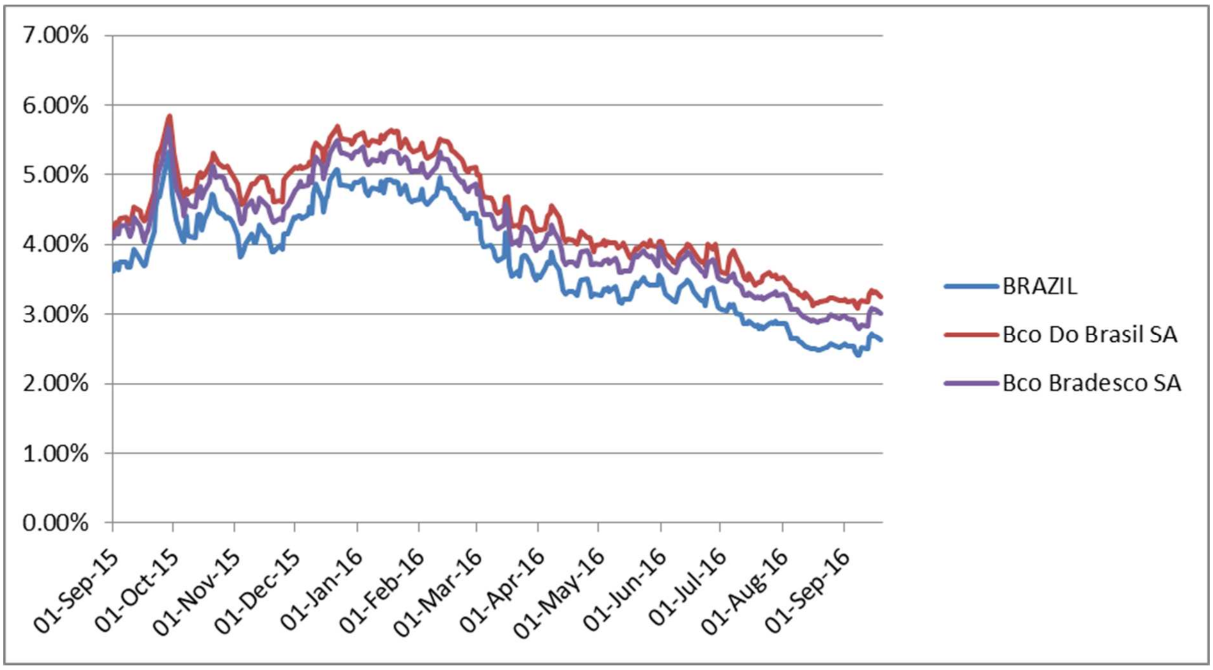

Figure 4.5: Brazil CDS Curve

__Turkey__

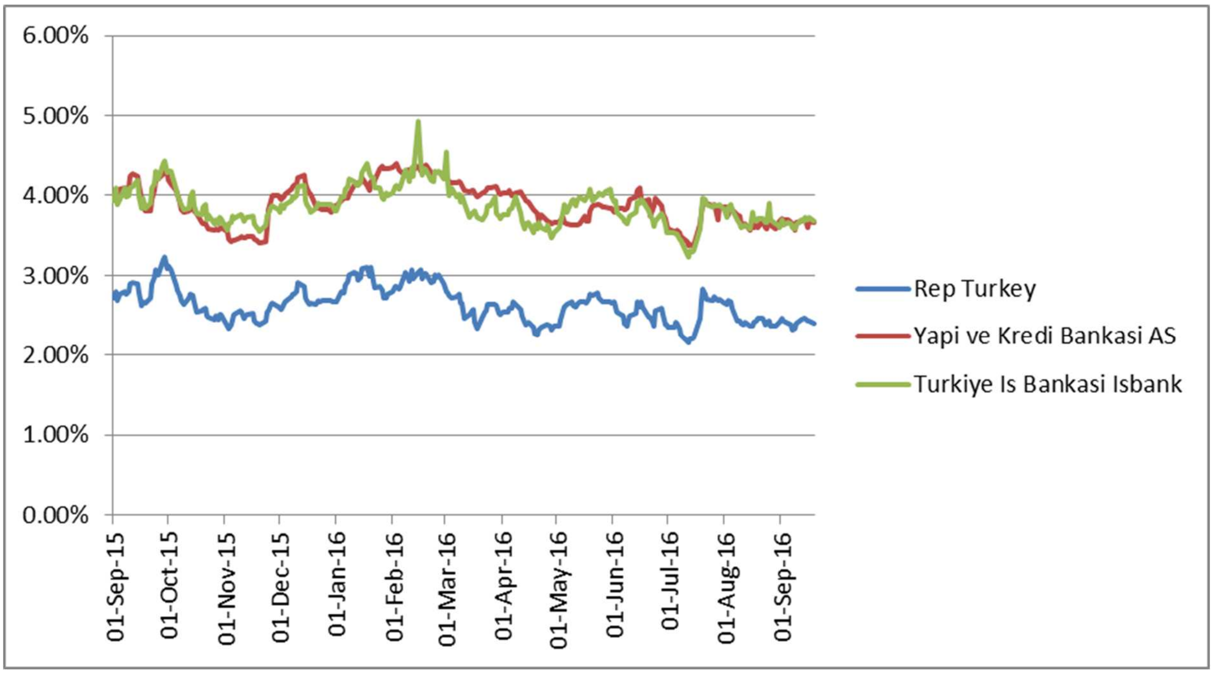

Figure 4.6: Turkey CDS Curve

Bahrain

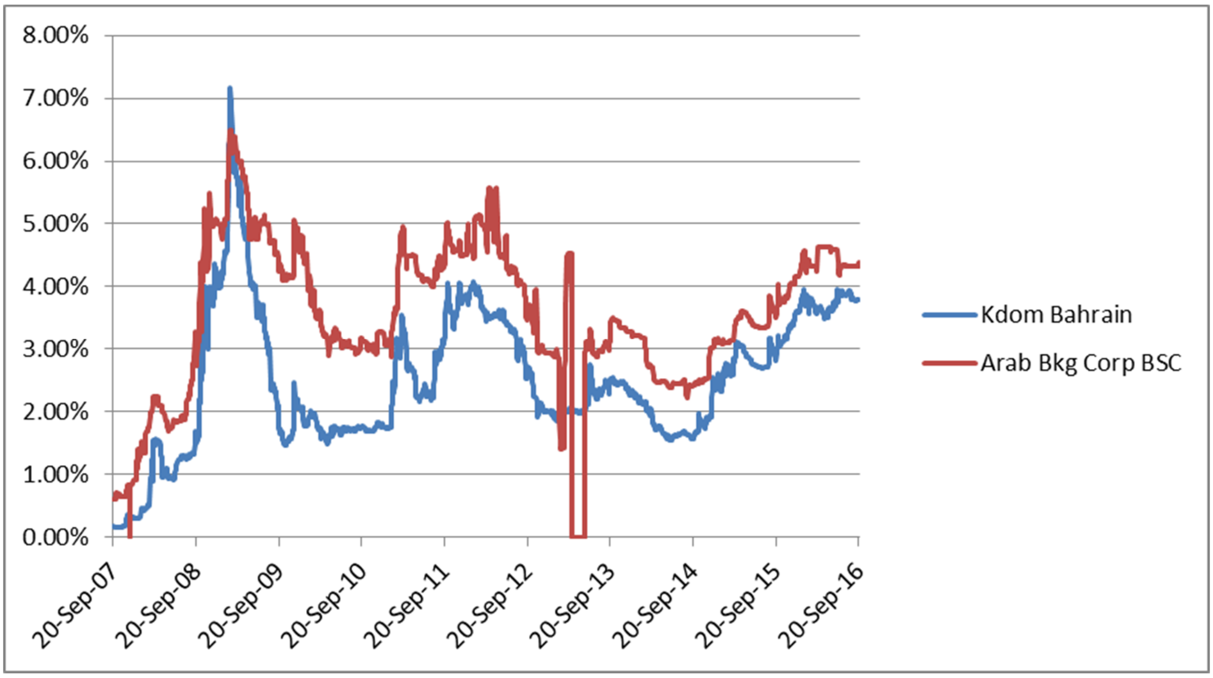

Figure 4.7: Bahrain CDS Curve

Spain

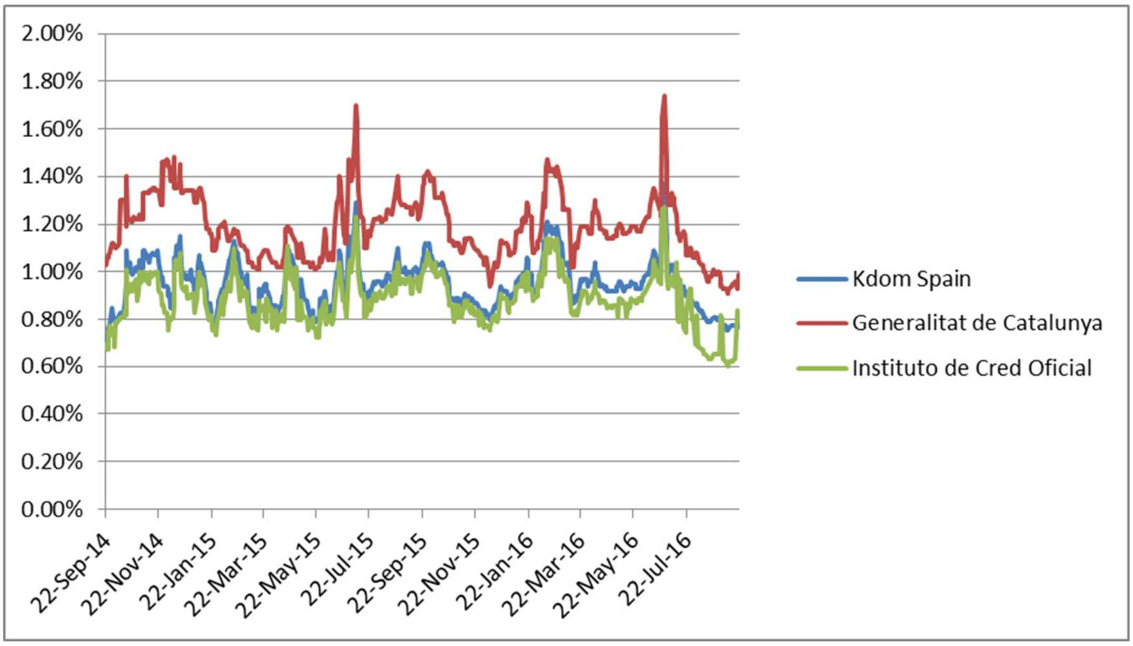

Figure 4.8: Spain CDS Curve

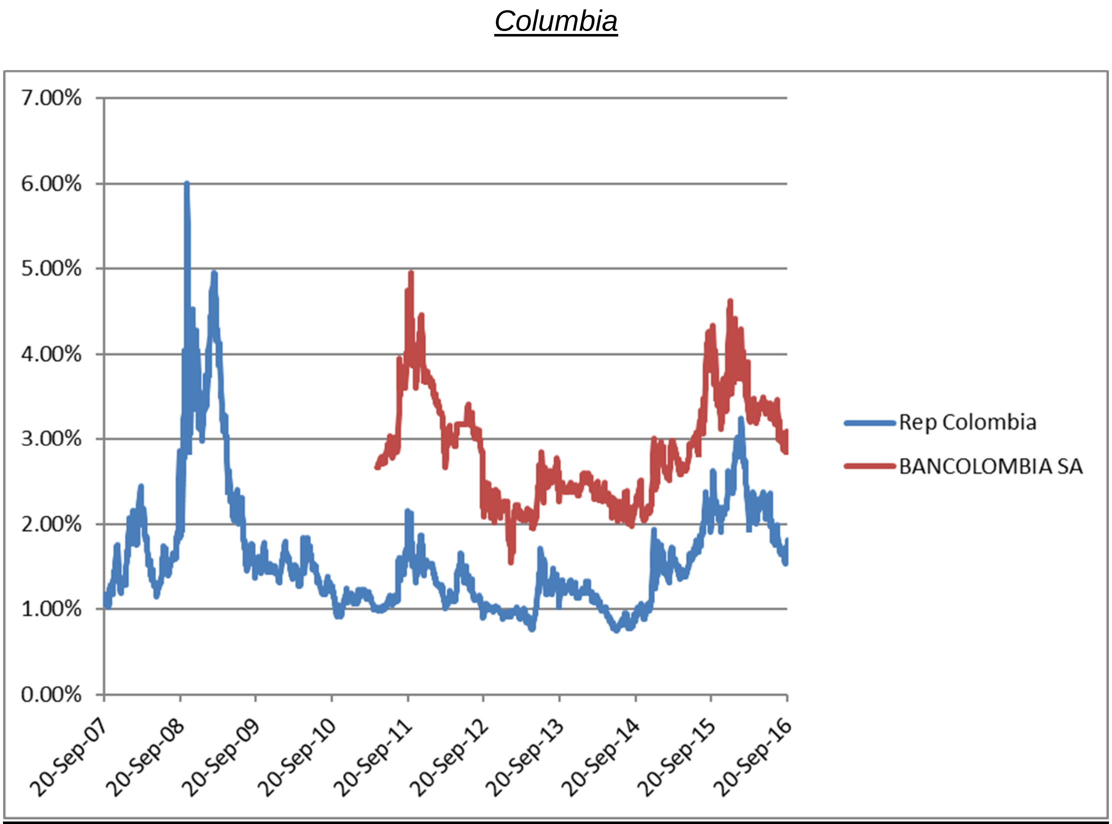

Figure 4.9: Columbia CDS Curve

Spain is however also left out, due to the developed market nature of their economy. The remaining five countries, together with their best rated bank in terms of credit spread level will be analysed in the linear multiple regression. In the regression the best country or combination of countries will be considered.

Country	Bank
Russia	Sberbank
Brazil	Bco Bradesco SA
Turkey	Turkiye Is Bankasi Isbank
Bahrain	Arab Bkg Corp BSC
Columbia	Bancolumbia SA

Table 4.2: Foreign Banks

4.7 IDIOSYNCRATRIC MODELLING

Thus far the sovereign component of South African banks credit curves has been considered. However, the idiosyncratic portion still needs to be considered. This portion will be modelled by including financially driven credit indexes in the regression scope.

In regression, underlying variables need to be independent of each other to achieve the best results. However, although the movements in financial indexes considered in this section are correlated to the movements of sovereign credit spreads, it will still add more explanatory power. The worldwide developing market is correlated to a certain extent due to risk-on versus risk-off cycles. Therefore, correlation between the sovereign and idiosyncratic portions is unavoidable. The newly introduced indexes to the regression are mostly financial and represent world market movements towards banks or other financial institutions in developing countries.

Itraxx Senior Financials, Itraxx Europe, CDX EM and CDX NA IG are included in the analysis to try and determine they can help improve the accuracy of the regression.

4.8 REGRESSION

In this section regression is applied to each country individually. Close of business 5-year credit spread data has been sourced from Bloomberg from the beginning of 2014 until 31 August 2016. In total 696 close-of-business 5-year credit spreads have been used in the analysis.

Firstly, the relationship between the sovereigns and their underlying banks are considered by a linear regression through the origin.

Country	Factor
Russia	1.28
Brazil	1.16
Turkey	1.5
Bahrain	1.23
Columbia	1.69
Average	1.372

Table 4.3: Sovereign Regression Coefficients

For all of the countries the linear regression had an overall p-value very close to zero and high R^2 values. These results make intuitively sense. It is expected that banks would trade at a slightly higher level than their associated sovereign due to banks being perceived as riskier. It is also expected that this calibrated factor would be less than the one calibrated for Eskom (1.85) in the previous chapter. Now that the isolated relationships between these banks and their sovereigns have been investigated, new variables are introduced to the regression to try and find the regression model for each country with the best explanatory power.

Normal regression goodness of fit measures will be considered to make that the models fit good enough. These include R^2 and normal $p\ values$. However, a backward selection method based on the Akaike Information Criterion (AIC) measure will be used to pick the most appropriate model for each country. After each run variables are excluded on the bases of the model's overall Akaike Information Criterion (AIC) measure. The AIC measure can be represented by the following equation,

$$AIC = -2log\hat{L} + 2k,$$

Where,

$\hat{L}$ $\quad$ $The\ maximum$ value of log $likelihood\ function$

k $\quad$ $The\ total\ number\ of\ regressors\ or\ independent\ variables$

The lower the value the better is the multiple regression fit. This measure cannot be interpreted as an absolute comparison measure between different regression analyses. However, it is an excellent way to compare models based off different combinations of independent variables. Therefore, for each of the countries a stepwise AIC regression analysis was performed to select the most suitable regression components per country.

Each regression was started off by considering the entire universe of independent variables identified in the previous sections. The starting regression formula for each country can be written as follow,

$$Country\ Banking\ Spread = B_1(Sovereign\ Spread) + B_2(Itraxx\ Fins) + B_3(CDXEM) + B_4(CDX\ NA\ IG)$$

Regression will be performed through the origin and therefore without an intercept variable. This is done because, for hedging purposes, it would be better to not have random credit spreads that cannot be hedged efficiently.

The R^2 goodness of fit measure represents the difference in fit between the proposed regression model and a model assumed to be the mean of the data. This measure takes on values between 0 and 1, where 1 indicates a perfect fit and 0 indicate complete randomness. The problem with the normal R^2 measure is that it increases as new predictors are included in the model. Therefore, the adjusted R^2 measure was used in this analysis, which adjusts measures according to the degrees of freedom (DF) underlying the regression model.

The goodness of independent predictor variables will be checked by considering their individual $p\ values$. All $p\ values < 0.05$ is taken as statistically significant predictors as per statistical norm. In the result below p $values$ are denoted as follow to indicate their goodness: *** ≈ 0, ** < 0.01 and * < 0.05.

The regression results for each country included in the analysis are presented in the table below. Where an independent predictor is marked as red it means that through the AIC backward stepwise analysis, this variable was removed.

Country	Foreign Bank	Sovereign	Itraxx SR Financials	Itraxx EUR	CDX EM	CDX NA IG	Bank Proxy
Russia	Sberbank	***		***	***	**	302
Brazil	Bco Bradesco SA	***	***	**	**		291
Turkey	Turkiye Is Bankasi	***	***	**	*		364
Bahrain	Arab Bkg Corp	***	**	**	*		309
Columbia	Bancolumbia SA	***	**	**	***	**	308

Table 4.4: Regression Results

The variables marked in red, was removed through the stepwise AIC analysis. The bank approximations in the last column were obtained by using each country's calibrated regression model and inserting 5-year credit spread data for the relevant variables as at the 29th of July 2016. South Africa was traded at this stage at 250bps, therefore the results make intuitively sense.

Turkey was removed in the analysis due to its high calibrated generic bank spread for the South African market. Also due to political uncertainty and threatening conflict within the country it was decided to be removed. An average of the remaining country's estimates was taken and averaged to obtain an approximated 5-year credit for South Africa's generic banking spread of 303bps. The ratio between the approximated spread and sovereign can then be calculated as 1.212. In order to extend the generic banking curve to other maturities this factor is multiplied by the South African sovereign curve. Again this is done to insure that the approximated shape of the banking credit spread curve is regular. Since all the inputs are liquidly trading in the market, approximations for the entire generic banking curve can be calculated daily. The regression assumptions should however be updated timeously to ensure that the calibrated regression fit remains accurate.

4.9 SUMMARY

In the graph depicted below, the observed South African credit curve is depicted against the calibrated Eskom and generic South African banking curve. The results are as expected, with

South Africa being the most creditworthy, followed by the banking curve and then Eskom. In the next section, probability of default estimates will be stripped out to use as an input for the CVA and DVA calculations presented in Chapter 5.

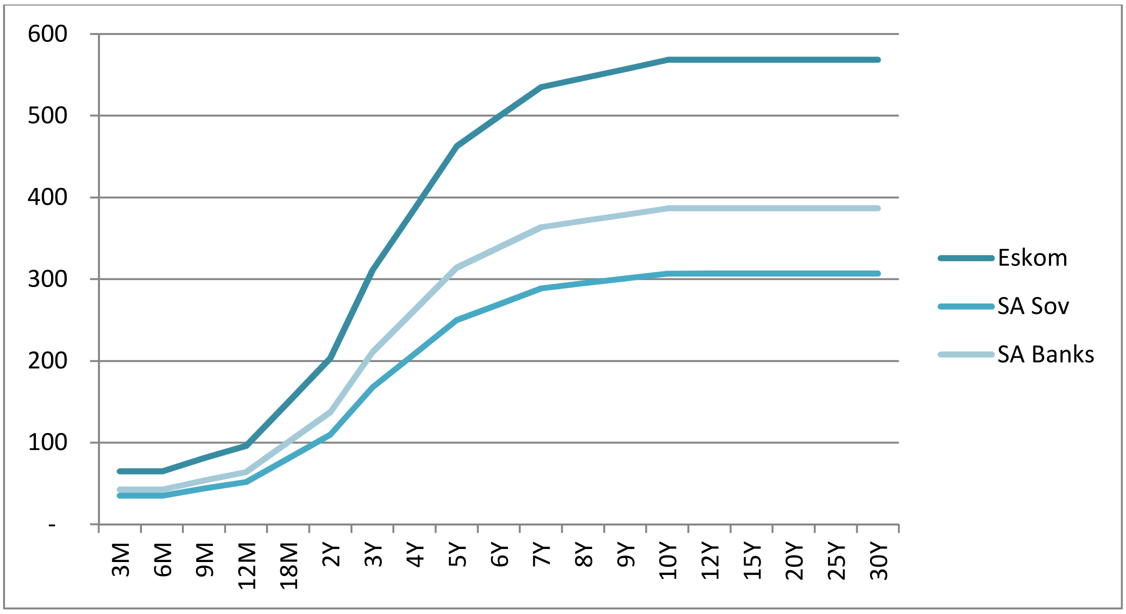

Figure 4.10: Calibrated Credit Curves

CHAPTER 5
Calculating CVA and DVA

In this chapter the different estimation methods of CVA and DVA, as introduced in Chapter 2, are implemented by making use of a hypothetical portfolio of interest rate swaps. In the previous two chapters the probability of default estimation methods were outlined through the approximation of credit spread curves. It was seen that the estimated underlying credit curves can be stripped to arrive at hazard rate curves, from which in turn probability of defaults can be calculated for any time period (t, T). Where T represents maturity and $t < T$. Recovery assumptions were also presented and defined in terms of loss given default.

The default probability and LGD estimates serve as inputs to the calculation of CVA/DVA in this chapter. Exposure is the only CVA/DVA component which practical application has not yet been addressed in the research. Exposure estimates are embedded in all of the CVA/DVA estimation methods presented in this chapter.

In the first section of this chapter the different Ernst and Young estimation methods are briefly introduced. The hypothetical portfolio that is used as a numerical example for CVA/DVA calculation is presented in the second section. This portfolio is seen as the entire set of derivative transactions Eskom has with a given South African investment bank. IFRS 13 guidelines require that the CCR embedded in these derivatives be estimated and included in the fair value of the derivatives as an adjustment. The portfolio consists of interest rate swaps with different maturities, moneyness and swap rates. In the third section each of the methods are discussed in detail and it is illustrated how CVA/DVA estimates can be obtained through numerical examples. The chapter will be concluded by a summary of results, together with a comparative discussion of the different methods presented.

5.2 HYPOTHETICAL PORTFOLIO

In the table below the different interest rate swaps are given. The interest rate swaps are across different maturities up to ten years. Since corporate treasurers almost solely hedge out floating interest rate risk with these types of transactions with a semi-annual payment schedule, it is assumed that Eskom will be paying the fixed and receiving the floating rate in all of the listed contracts. The notional are taken as R1bn on all the contracts to make CVA estimation comparison simpler between different estimation methods. For the numerical study the 29th of July 2016 will be used as the valuation date. Practically it would not make sense for Eskom to enter in the portfolio of contracts below; the contracts are chosen to present the impact of CVA and DVA on trades with differing characteristics.

Contract Number	Notional (R)	Maturity	Tenor	Annual Swap-rate
1	1 000 000 000	1-year	Semi-annually	2%
2	1 000 000 000	1-year	Semi-annually	12%
3	1 000 000 000	1-year	Quarterly	6%
4	1 000 000 000	3-year	Semi-annually	7%
5	1 000 000 000	5-year	Semi-annually	7%
6	1 000 000 000	10-year	Semi-annually	7%

Table 5.1: Hypothetical Portfolio

5.3 CVA/DVA METHODOLOGIES AND RESULTS

In the literature review different methods for estimating CVA and DVA are introduced as presented by Ernst & Young (2014). The methods are presented in terms of CVA. It is straightforward to adjust the methods for DVA by simply changing the counterparty perspective.

In this section the different methods are considered and implemented to calculate CVA and DVA.

5.3.1 EXPECTED FUTURE EXPOSURE APPROACH

Mathematically this method is denoted as follow as introduced in the literature review,

$$CVA = LGD \sum_{t=1}^{T} DF_t EPE_t PD_{(t,t+\delta)}$$

Where,

LGD $\quad$ $Deterministic\ assumed\ LGD$

EPE_t $\quad$ $Expected\ positive\ exposure\ at\ discrete\ time\ point\ t$

PD_t $\quad$ $Probability\ of\ defualt\ esitmate\ between\ discrete\ time\ points\ t\ and\ t+\delta$

DF_t $\quad$ $Applicable\ discount\ factor\ at\ time\ point\ t$

This method simulates expected positive exposure from the inception of the contract up to maturity T for discrete time points $t < T$. The more discrete time point used in the simulation, the more accurate is the CVA estimated according the LLN. CVA is then estimated by weighting expected exposures by the associated probability of default and LGD.

The above equation is simplified due to its discrete nature. The different time points are chosen as the different cash flow time points defined through the design of all the interest rate derivatives within the portfolio. Since all inputs to this model have been compared to those in the market and found to satisfactory, the resulting CVA and DVA estimates from this method is assumed to be correct. All following estimation methods will be compared to this one and subsequently measured for error.

The probability of default is then defined on the time interval between each cash flow point. The probability of default estimates for the generic bank counterparty and that of Eskom are presented in the table below after linearly interpolating unobservable tenors as estimated on 29 July 2016.

Time t (years)	Time $t+\delta$ (years)	Days	Eskom	Bank
0	0.25	91.25	0.27%	0.18%
0.25	0.5	182.5	0.27%	0.18%
0.5	0.75	273.75	0.48%	0.31%
0.75	1	365	0.59%	0.38%
1	1.5	547.5	2.12%	1.38%
1.5	2	730	2.93%	1.92%
2	2.5	912.5	3.66%	2.43%
2.5	3	1095	4.29%	2.90%
3	3.5	1277.5	4.07%	2.79%
3.5	4	1460	4.37%	3.06%
4	4.5	1642.5	4.60%	3.29%
4.5	5	1825	4.76%	3.48%
5	5.5	2007.5	3.68%	2.75%
5.5	6	2190	3.66%	2.80%
6	6.5	2372.5	3.63%	2.83%
6.5	7	2555	3.57%	2.85%
7	7.5	2737.5	2.70%	2.20%
7.5	8	2920	2.61%	2.17%
8	8.5	3102.5	2.52%	2.13%
8.5	9	3285	2.43%	2.09%
9	9.5	3467.5	2.33%	2.05%
9.5	10	3650	2.24%	2.01%
10	12	4380	6.69%	6.27%
12	15	5475	7.90%	8.06%
15	20	7300	9.02%	10.52%
20	25	9125	5.58%	7.72%
25	30	10950	3.45%	5.66%

Table 5.2: Stripped Probability of Defaults

Therefore PD_t can be taken as the probability of default of the counterparty between the previous cash flow time point and time point t. Through method design t is chosen as all the cash flow points and the expected exposure is estimated at each of these time points. LGD is taken as the deterministic value 60% as addressed in Chapter 3.

The way of simulation is through Monte Carlo simulation of interest rates as presented in Chapter 2 to arrive at simulated exposure estimates.

In the figure below the expected exposure profile of a far in-the-money interest rate swap paying a swap rate of 2% quarterly is plotted for increasing number of simulation. This provides empirical proof of the LLN presented in Chapter 2. For the purpose of this analysis 10 000 simulations are chosen the ideal number for the trade-off between estimation accuracy and programming time.

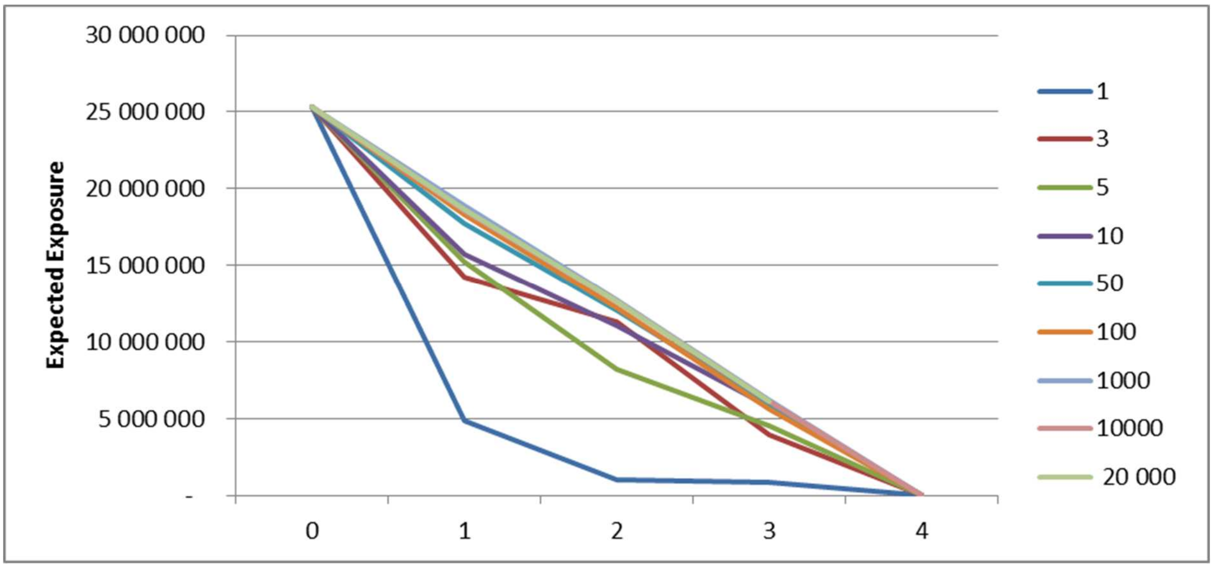

Figure 5.1: Monte Carlo Iterations

A brief revision of the Hull white interest rate model used in the research is presented below as introduced in Chapter 2. The Hull-White interest rate model will be used in this research with a mean reversion factor of 5% and volatility of 2%. These parameters were chosen so that the shape of resulting curve fit to that of the prevailing South African swap curve (Kapp: 2014)

The Hull-White (one-factor) model is a no-arbitrage model which extends the Vasicek model for which all the needed data is available under the South African context. It is characterised as a Vasicek model, but the mean reversion rate is dependent of time. This can be formulated as follow,

$$dr = [\theta(t) - ar]dt + \sigma dz$$

Where, a and σ are constants. The reversion rate within this model takes the form of $\frac{\theta(t)}{a}$, where r reverts back to the level at a rate a. The value of θ can be calculated by using the initial term structure of an assumed risk-free interest rate curve (for this research taken as the ZAR swap curve).

$$\theta(t) = F_t(0,t) + a\,F(0,t) + \frac{\sigma^2}{2a}\,(1 - \exp(-2at))$$

Where, $F(0,t)$ is defined as the instantaneous forward rate for a maturity t as seen at time zero. Since the present time is taken as zero, values for $F(0,t)$ is known for all values of t up to maturity say T. $F_t(0,t)$ is defined as the first partial derivative of the particular forward rate with respect to time t. The last term is significantly small and can be ignored in the calculation of θ.

Bond prices, within a risk-neutral framework, at time t is given by

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

Where,

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$

and

$$lnA(t,T) = ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aT} - e^{-at})^2(e^{2at} - 1)$$

The algorithm presented below was implemented in the R statistical computer software program, available freely on the internet. The programming code is outlined in the Appendix B.

Algorithm 5.1 (MC Simulation of Exposure)

1. *Calculate $\theta(t)$ for all values of t from the prevailing forward rate curve implied form the current ZAR swap curve and model input parameters.*
2. *Calculate $B(t,T)$ and $A(t,T)$ for all values of t from the model input parameters.*
3. *Estimate the short-rate vector of the short-rate by applying the Hull-White model.*
4. *Calculate the discount factors $P(t,T)$ for all values of t.*
5. *Use the contract specifics and the estimated values of $P(t,T)$ to calculate the expected exposure of the swap for all values of t.*
6. *Repeat steps 3 to 5 to get many sample values of the payoff form the derivative in a risk-neutral world over all values of t.*

7. Calculate the EPE, NPE and PFE for each contract.

The expected positive and negative exposure profiles are presented below.

Contract 1: 1-year 2% Semi-annual

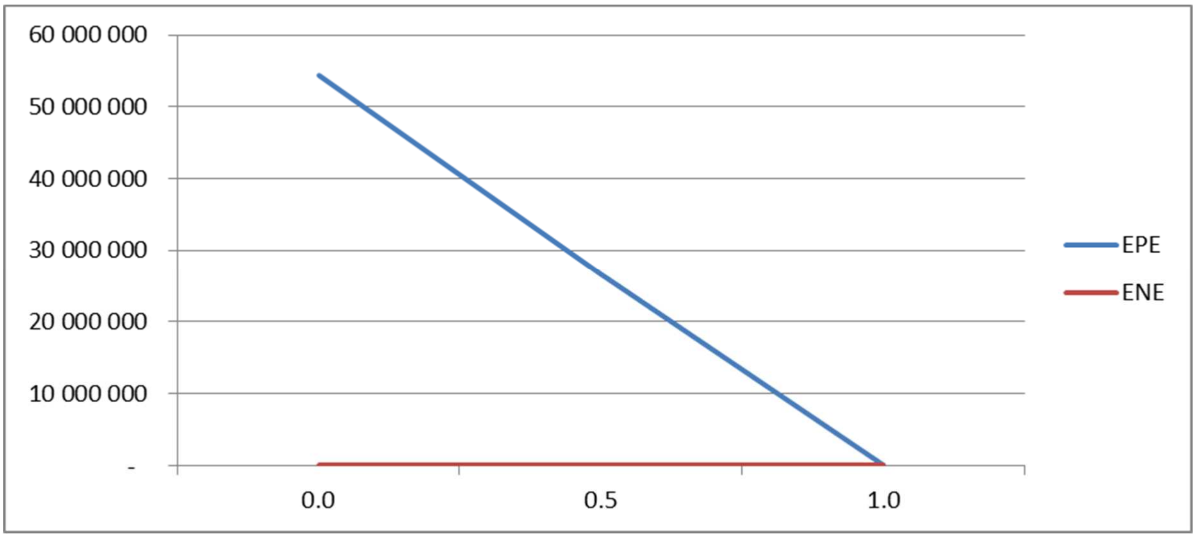

Figure 5.2: Contract 1 EE

It can be seen that this contract is far in-the-money for Eskom. Through the MC simulation no values were calculated for any payment time where the Bank might have exposure to Eskom. Therefore, the DVA on this contract can already be concluded to be zero.

Contract 2: 1-year 12% Semi-annual

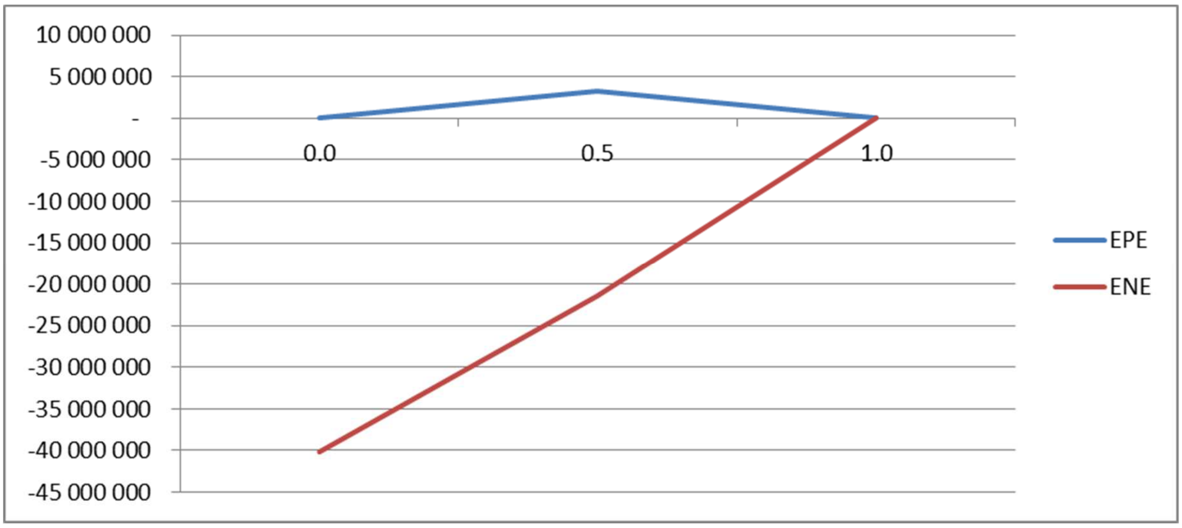

Figure 5.3: Contract 2 EE

This contract is far out-of-the money. All the characteristics are the same as the previous contract however in this case the fixed rate paid by Eskom is substantially larger. In this case there will be a small amount of CVA and a large DVA component.

Contract 3: 1-year 6% Quarterly

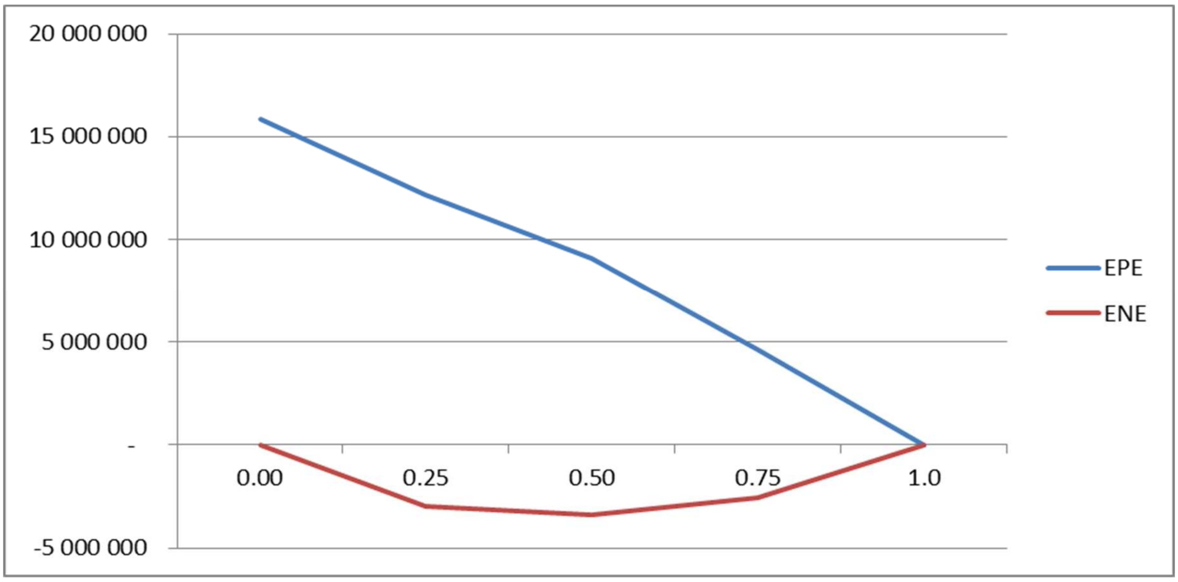

Figure 5.4: Contract 3 EE

In the exposure profile of the contract it can be seen that the 6% annual swap-rate is again favourable for Eskom. Estimated bank exposure is quite large with a small DVA related exposure also obtained throughout the 10 000 simulations.

Contract 4: 3-year 7% Semi-annual

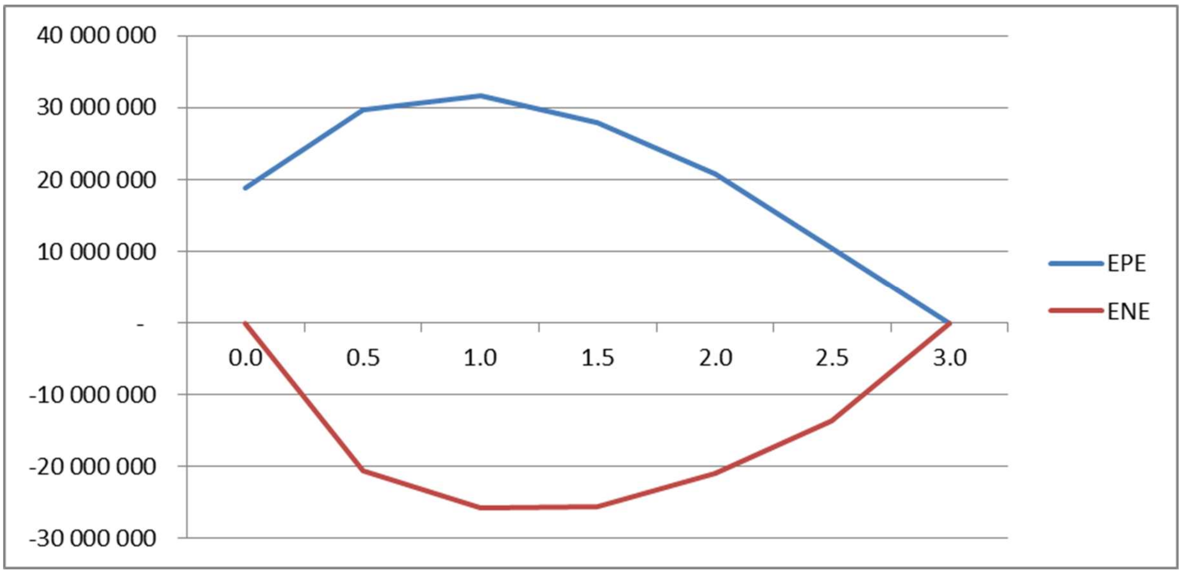

Figure 5.5: Contract 4 EE

Contract 5: 5-year 7% Semi-annual

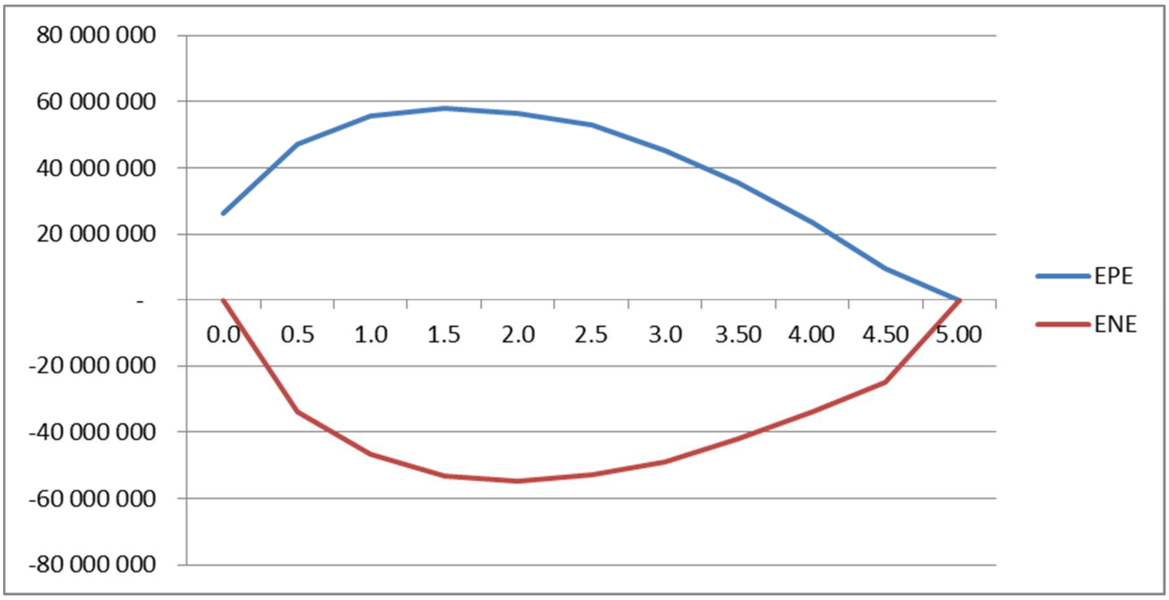

Figure 5.6: Contract 5 EE

Contract 5: 10-year 7% Semi-annual

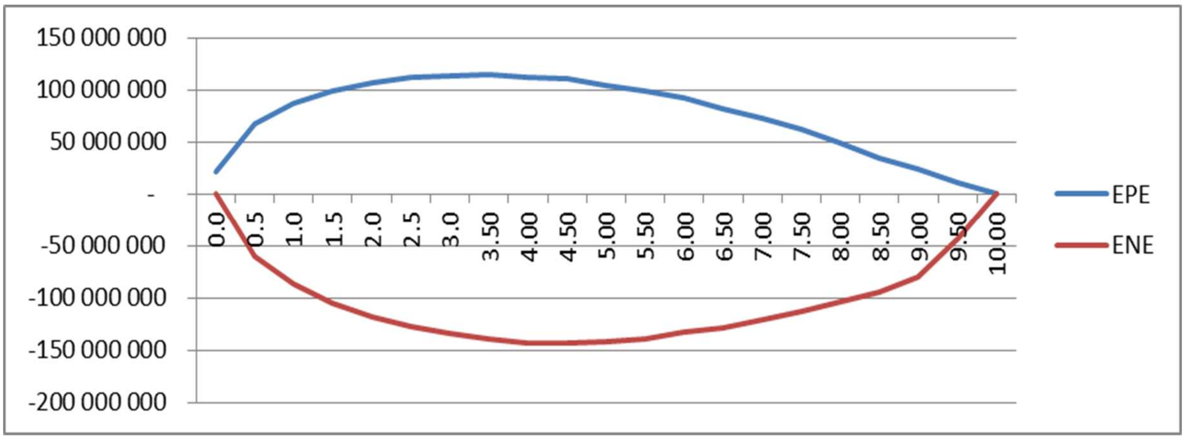

Figure 5.7: Contract 6 EE

The underlying swap characteristics of the last three contracts are all similar, except for their maturities. Firstly, it can be noted that the 7% swap-rate represents a rate very close to the fair value of the swaps. The CVA and DVA profiles look quite similar. It can however be noted that the gross exposure profiles exhibit a much larger exposure level due to the relatively longer duration the interest rate swaps.

Applying the PFE measure to these contracts provides a much more conservative measure. In this research a 99% PFE was applied to all positive and negative exposures in addition the mean exposure presented in the expected exposure profiles above. A comparison between the exposure profiles of the PFE and expected exposure measure is provided in the Figure 5.8 below for contract 4.

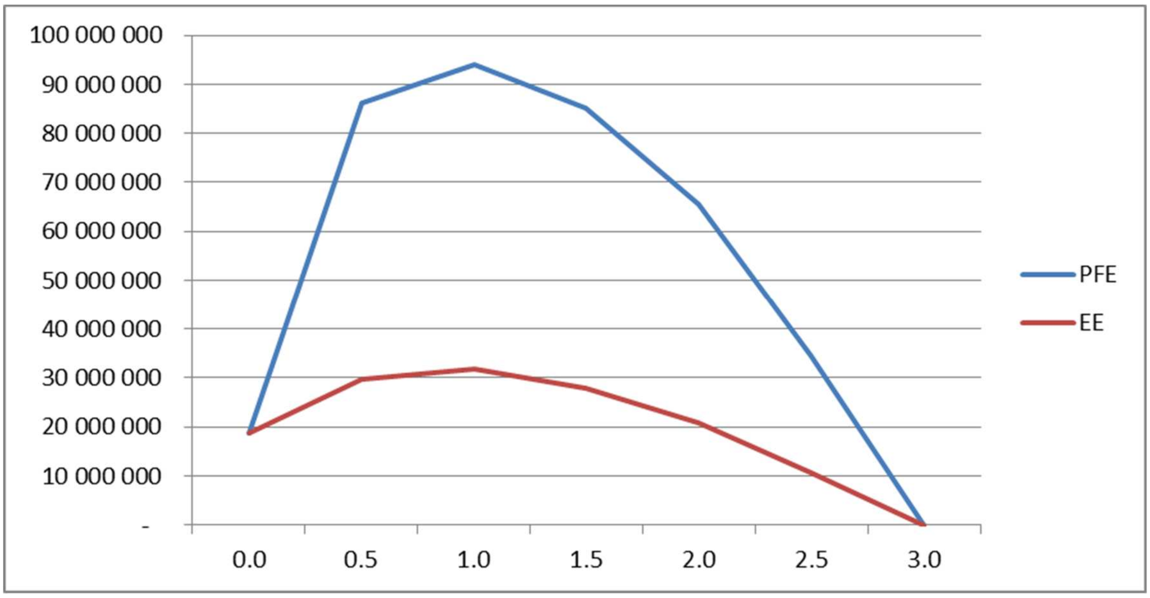

Figure 5.8: PFE vs EE

According to IFRS 13 the CVA and DVA exposures can be netted within each of the contracts for all cash payment dates. This reduces the gross credit adjustments needed significantly. In addition to this netting, further netting is possible if a proper netting agreement (NA) is set-up beforehand agreed upon by both counterparties. As will be seen in the CVA and DVA estimates presented below, this also provides a significant reduction in credit adjustments. Firstly, netting is only considered on trade level, after which a counterparty wide one is considered. Like cited in the literature CVA and DVA are not directly additive. When netting across contracts is taken it is important that the netting occurs on a term structure equivalent manner.

Collateral agreements can also be set up which provides protection on a portion of the exposure. In this research a haircut of 80% of exposure is applied. This collateral adjustment reduces exposure, which in turn reduces credit adjustments needed.

Specifications of the above mentioned exposure mitigating mechanisms are specified in a legally binding CSA, agreed upon by counterparties. Therefore, these mitigating methods are very customisable. The manner in which to handle collateral and netting often differs from contract to contract, even if the contracts are done with the same counterparty. For the purpose of this study the choice of netting and collateral specifications are rather arbitrary. The main focus lies on the comparing of CVA and DVA adjustments under consistent circumstances. It remains however a test for the different methods to see if they can handle different type of netting and collateral agreements and unilateral versus bilateral underlying implications of the hypothetical portfolio. All CVA and DVA estimates obtained from the expected future exposure approach are summarised in Table 5.3 below. Note that negative adjustments pertain to DVA.

	Uncollateralised CVA/DVA
Contract 1	-350 019
Contract 2	173 648
Contract 3	-88 388
Contract 4	-1 871 546
Contract 5	-7 681 125
Contract 6	-32 645 954

Table 5.3: Monte Carlo Results

5.3.2 SWAPTION APPROACH

Mathematically this method is denoted as follow as introduced Chapter 2,

$$CVA = LGD \sum_{t=1}^{T} PD_{(t-\delta,t)} Swaption_t$$

Where,

$Swaption_t$ $The\ value\ of\ a\ swaption\ contract\ with\ expiry\ at\ time\ point\ t$

This method models EPE as a series of swaptions. The optionality part of the valuations refers to either the scenario where the counterparty defaults or not at each time point. Each swaption is then interpreted as the cost of replacing the remainder of the swap that was lost due to counterparty default.

The inputs for valuing the swaptions over the differing time t are the following,

$$V_{pay}(0,t,T_n) = \sum_{i-1}^{n} P(0,T_i)\ L\ (s_0\Phi(d_1) - s_k\ \Phi(d_2))$$

Where,

$P(0,T_i)$ $Discount\ factor\ between\ time\ 0\ and\ T_i$

Φ $Cumulative\ probability\ function\ for\ a\ standardised\ normal\ distribution$

s_0 $Forward\ swap\ rate = \frac{P(0,t)-P(0,T_n)}{\sum_{i=0}^{N-1}(T_{i+1}-T_i)P(0,T_{i+1})}$

s_k $Swaption\ strike\ rate$

$$d_1 = \frac{\ln\left(\frac{S_0}{S_k}\right) + \frac{\sigma^2 T_n}{2}}{\sigma\sqrt{T_n}}$$

$$d_2 = d_1 - \sigma\sqrt{T_n}$$

The volatility σ is taken as the average volatility of the implied volatility matrix of 30 different at-the-money South African swaptions as presented by Kapp (2014). The swaptions differ in expiry and swap term length. The average of 20% will be used in this research. The volatility matrix considered can be found in the Appendix A4.

The CVA per contract is listed in the table below. Please note a negative amount refers to DVA.

	Uncollateralised DVA/CVA
Contract 1	89 723
Contract 2	16
Contract 3	-76 766
Contract 4	-1 313 337
Contract 5	-10 789 029
Contract 6	-42 222 823

Table 5.4: Swaption Results

5.3.2 CURRENT EXPOSURE LINEAR MODEL

$$CVA = LGD \sum_{t=1}^{T} EE_t \, PD_{t-\delta,t}$$

In this method the current exposure of the hypothetical portfolio is calculated. Thereafter an appropriate diminishing exposure model is applied to the current exposure over the lifetime of the portfolio. This will take on the form of a linear decreasing model, which will be weighted by the probability of default at each cash flow point.

	Uncollateralised CVA/DVA
Contract 1	-352 956
Contract 2	167 861
Contract 3	-82 193
Contract 4	-705 863
Contract 5	-2 206 075
Contract 6	-4 088 338

Table 5.5: Current Exposure Results

5.3.2 CONSTANT EXPOSURE MODEL

This method is similar to the previous one; the only difference is that the estimated exposure as measured at time zero is kept constant throughout the life of the contract.

.

	Uncollateralised CVA/DVA
Contract 1	-527 567
Contract 2	251 005
Contract 3	-153 387
Contract 4	-1 650 629
Contract 5	-5 133 998
Contract 6	-7 885 760

Table 5.6: Constant Exposure Results

5.3.2 DURATION MODEL

$$CVA = MtM\ x\ Credit\ Spread\ x\ Duration$$

This method makes use of duration to measure how much the fair value changes when credit spreads are introduced to the discounting rate. The duration serves as an estimate of the average lifetime of the exposure

	Uncollateralised CVA/DVA
Contract 1	-141 522
Contract 2	126 199
Contract 3	-51 433
Contract 4	-710 012
Contract 5	-2 310 377
Contract 6	-4 734 709

Table 5.7: Duration Results

5.4 SUMMARY

The summary of results can be seen below.

	MC	Swaption	Linear Model	Constant Exposure	Duration
Contract 1	-350 019	-89 723	-352 956	-527 567	-141 522
Contract 2	173 648	-16	167 861	251 005	126 199
Contract 3	-88 388	-76 766	-82 193	-153 387	-51 433
Contract 4	-1 871 546	-1 313 338	-705 863	-1 650 629	-710 012
Contract 5	-7 681 125	-10 789 030	-2 206 075	-5 133 998	-2 310 377
Contract 6	-32 645 954	-42 222 824	-4 088 338	-7 885 760	-4 734 709

Table 5.8: Summary of Results

As can be seen from the results there is quite a difference between the results from the assumed to be correct Monte Carlo and some of the other methods. For the shorter dated interest rate swaps the constant exposure model seems to perform the best. For the longer matuirties the swaption approach is the best compared to the MC. The results will be discussed in detail in the following chapter.

CHAPTER 6

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

Five different models were implemented for the purpose of calculating the CVA and DVA of a portfolio of interest rate swaps between the South African corporate treasurer Eskom and a generic South African tier 1 bank. The models differ from simple to complex. The MC model is assumed to be the most accurate, since it involves the simulation of expected exposure and the short-rate modelling. Corporate treasurers do not always have the necessary resources to calculate CVA by means of a sophisticated approach. Therefore, semi-analytic methods were also considered in the research.

From the results, it can be seen that in the majority of the interest rate contracts the generic bank has CCR towards Eskom. In practice this is standard. This is common in practice for banks to take some counterparty risk in order to facilitate structured hedge deals for corporate treasurer clients.

Both current exposure and the duration method are understating the amount of CCR. This makes intuitively sense, since these methods are unable to account for the upward curving swap of the expected exposure. This can easily be observed when considering the graphs is section 5.3.1. However, on shorter dated maturities these methods still seem to provide valid results. The swaption approach on the other hand seems to overstate the embedded CCR. The swaption approach proves to be the best alternative for CVA calculation, instead of the rigorous Monte Carlo simulation approach.

Short comings of all the alternative methods are also that they do not fully include the bilateral nature of interest rate derivatives. In the unilateral case these models are still able to provide sound estimates of CVA. It is the recommendation of this study that these methods should only be used to get a high level estimated figure of the CCR included in a deal. With the MC method it is also easier to handle all the complexities of collateral and netting agreements.

In future research the impact of netting and collateral can be fully included in model comparisons, since these agreements are often tailor-made for each specific derivatives contract.

CCR has become an important facet of the financial industry. As updated guidelines are introduced, market participants will constantly need to adjust the way they consider non-performance risk to remain competitive.

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APPENDIX A:

A1 INTEREST RATES

Interest rates are one of the most important underling variables when valuing the majority of derivative transactions. These types of derivatives fall under the category of fixed income financial products, where the price of financial products or derivatives depends on the value of underlying interest rates. To get a complete understanding of how derivatives are priced it is important to know how interest rates are measured and analysed.

In its simplest form an interest rate defines the amount of money a borrower must pay a lender for the use of his capital. There are many factors which influence the size of this interest amount and many ways in which it can be defined. The most important factors that influence interest rates are the term of borrowing, how frequently interest rate are compounded and how dependable or creditworthy is the borrower? The risk of the borrower defaulting, or inability to meet all financial obligations, is embedded within an interest rate through a credit spread. The higher the credit risk, the higher the interest charged to the borrower. Over different currencies, different types of interest rates are quoted corresponding to standardised exchange traded assets such as bonds, treasury bills and notes. These can refer to mortgage rates, repo rates, deposit rates, bond coupon rates or even more dynamic rates like swap rates which change inter daily.

From these quoted interest rates, it is possible to construct an interest rate curve over a term structure. For example, treasury rates are the rates governments agree on to borrow capital in their own currencies. These rates are linked to notes (loans) or other similar simple lending products over different time horizons. From these rates an interest rate curve can then be implied for missing maturities through interpolation. Since governments are seen to be fairly safe borrowers, these rates are assumed to be close to riskless. Governments are seen as riskless because when Sovereign entities operate in their own monetary region they can create local currency to meet local currency obligation. This is, however, not the case for foreign currency (FC) obligations and hence many sovereign entities have active FC debt that implies significant non-zero default risk. This small portion of inherent credit risk, which effects was observed during the European crisis, led to government interest rates being referred to as ‘basic risk’ measures that serve as proxies for riskless borrowing.

Another alternative which serve as a proxy for riskless borrowing is short-term inter-bank interest rates applied between large banks. These are the short term rates that a large bank is willing to except in exchange for a deposit made to another large bank in the same sovereign jurisdiction over some specified maturity. The average between all banks is then calculated and published as dynamic observable rate in the market. Since they are short term borrowing rates, the rates are normally only quoted up to one year. Standard tenors are 1-month, 3-month, 6-months and 12-months in most developed markets.

The best known inter-bank quoted rate across differing maturities is the LIBOR rate (London Interbank Offer Rate), which takes the average interbank rate in the United Kingdom. LIBOR is usually referenced as the underlying in the United Kingdom as well as in cross border international derivative transactions. Both treasury rates and interbank rates are seen as sufficiently liquid to be used as riskless proxies in the pricing of financial instruments. However, since LIBOR rates are only quoted up to 12-months, the market trend is to extend the riskless curve out to 2-years by making use of Eurodollar future rates. To extend it even further traders tend to make use of swap rates which are quoted for longer maturities as far out as fifty years.

Interest rates can be defined in a large number of different ways. The compounding frequency of interest rates refers to the equal recursive time period length on which interest is cumulatively calculated. For example, if an investor invests a R100 which earns 10% annually compounded, over a period of five years, how much does he earn? If he earned a simple interest rate of 10% over the five years then he would earn R10 (R100 x 10%). However, if the interest rate is compounded annually, gained interest is added to the R100 notional to calculate the interest earned over the next annual period. In layman's terms, your money works harder.

It's not only the compounding frequency that defines a quoted interest rate. Day count convention also plays a large role. A certain day-count convention specifies how the number of days is treated within the quoted interest rate calculation. This becomes a vital piece of information when the term of investment doesn't stretch over an exact number of compounding periods. For example some conventions make the assumption that there are 30-days in all months or others work according to the exact number of days. Next zero rates and forward rates are introduced.

The T year zero coupon (zero for short) rate is the interest rate earned over a T year investment period where the notional value and all accrued interest is exchanged at the end of the T year investment period. Define the continuously compounded zero rate as $R(t,T)$ and $P(0,T)$ as the investment amount that needs to be invested as time 0 to achieve a 1 unit return at the end of time T. Thus,

$$P(0,T) = 1 \times e^{-R(0,T)(T-t)}$$

$$\rightarrow R(0,T) = -\frac{1}{T-t}\ln\big(P(0,t)\big)$$

The above is illustrated graphically in Figure 2.1 through the depicted time line, where $R(0,t_1), R(0,t_2)$ and $R(0,t_3)$ denote the respective zero rates for time points t_1, t_2 and t_3:

It is possible to determine the rate implied by these zero rates between any of the points t_1, t_2 and t_3, while currently standing at time point 0. This rate is referred to as the forward rate between time say t_1 and t_2 as calculated from time 0 and denoted by $F_0(t_1,t_2)$. More formally the forward rate for any arbitrary time period between $T_{i-\delta}$ to T_i with time period length δ can be written as:

$$F(T_{i-\delta},T_i) = \frac{R(0,T_i)T_i - R(0,T_{i-\delta})T_{i-\delta}}{T_i - T_{i-\delta}}$$

Next the rationale behind bond pricing is briefly introduce. Bond pricing methodologies are widely used to price more complex derivatives. A bond owner receives a series of periodic coupon payments made by the seller to the buyer as well as a principal amount at maturity. The price of a bond is calculated by using a DCF model to discount all future cash flows the buyer will receive to the valuation date. Sometimes traders use a single discount rate for all cash flows points, this is referred to as the bond yield. However, the most widely implemented approach is to discount cash flows using the cash flows associated zero rates for each cash flow time point. Bonds can be valued by taken the sum of the present values of the coupon leg and the notional leg. The present values of these legs are given below,

$$Coupon\ Leg\ PV = \sum_{t=1}^{T} C\,(DF_t)\,L$$

$$Notional\ Leg\ PV = \ L\,(DF_T)$$

Where, C denotes the coupon rate, L the principal and $\mathrm{DF_t}$ denoted the discount factor at time t. In this research the value of a zero paying coupon bond at time t will be denoted by $P(t,T)$. The prices of zero coupon bonds are often used in derivative pricing to denote discount factors.

A2 RISK NEUTRAL VALUATION

One important assumption in this model must be considered. Which rate should one use to discount future payments? It is not easy to know the correct discount rate to apply in a world which has real world expectations. The value of a payoff in the future will have a different value at a different valuation dates. The discount factor should represent the expectation of interest gained in the transaction. This complicates the pricing of instruments since most instruments' earned interest is different through different risk-return design.

In risk neutral valuation a DCF model is used in an assumed risk neutral world where all market participants are indifferent towards risk. In other words, investors require no compensation for risk taken and the expected return on any instrument is assumed to be the risk-free rate (or a proxy thereof). In valuation terms this means that the assumption can be made to discount all future payments with the appropriate proxied riskless rate to arrive at the fair value of any financial instrument at any valuation date.

If the discount rate is assumed to be riskless; what can be considered to be riskless? Common market practice is to use some liquid observable 'basic rate' as proxy for riskless borrowing such as Treasury bill rates or interbank offered rates. When using such riskless rates for valuation one makes use of the principal referred to as risk-neutral valuation.

The risk-neutral valuation assumption arises from the Black-Scholes-Merton pricing differential model. A key property of this model is that none of the variable inputs of the model are related to the risk preference of a market participant. Only the risk-free rate comes into the equation, which is also not dependent on the risk preference of any market participant.

The manner in which to value any European style derivative can be summarised by the two step process described in the bullet points below. A European derivative is a derivative where the maturity of a derivative is known at transaction initiation unlike an American style derivative which has the optionality to be terminated before maturity.

- Calculate all expected future payments either through contract design, simulation or by using forward rates in estimated in a world assuming risk neutral expectation.
- Discount the payments by making use of an appropriate riskless discount rate for the time period from valuation date to payment date.

(Hull 2009: 287-288 ; Bingham 2009)

A3 STOCHASTIC PROCESSES

Financial variables are often modelled as stochastic processes. The basics around continuous time stochastic processes will now be covered. Results can easily be simplified to the discrete case where variables only change at certain fixed points in time. The results covered here also relate to continuous variable systems, where variables can take on any value and not only certain discrete values. Understanding the basics of stochastic modelling is an important foundation for the understanding of interest rate modelling and the pricing of options and other more complex derivative products.

It is important to note that stocks and most other financial variables are assumed to follow a Markov process. In Markov process modelling future predictions of an underling variable is only dependent on the market state or value the variable finds itself in currently. This corresponds to the weak form of market efficiency where the current stock price is a true reflection of the stock's worth and includes all relevant market information Fama (1976). The current price therefore already includes all information regarding a stock's historical path.

A Wiener process is a special type of Markov process which is often used in financial mathematics for the modelling of financial variables. A Wiener process is a Markov process where the modelled variable has a mean of zero and an annual variance of one. This process is also referred to as a Brownian motion process.

A variable z follows a Wiener process if it has the following properties.

- The change Δz over a short period of time Δt is defined as $\Delta z = \epsilon\sqrt{\Delta t}$, where ϵ has a standardised normal distribution $\varphi(0,1)$.
- The values of Δz for any two different short intervals of time, Δt, are independent (This follows from a Winer process's Markov quality).
- $\Delta z \sim Normal(0, \Delta t)$

In a stochastic process the mean change of the underlying variable is referred to as the drift rate and the variance per unit of time as the variance rate. If the short interval under consideration for the change in z tends to zero it can be written, as is normal Calculus practice, Δz as dz. Thus the process of the change in a variable x over a short time period $\Delta t \rightarrow \infty$ can be written as below, where z follows a Wiener process and a and b are constants representing the drift and variance rate constants in the process.

$$dx = a\,dt + b\,dz$$

One can expand a Wiener process further to an Ito process. A Ito process is defined as a Wiener process where the parameters a and b are in turn defined as functions of the value of the underlying variable x and time.

$$dx = a(x,t)dt + b(x,t)dz$$

A4 CVA CALCULATION INPUTS

Volatility matrix for swaptions.

Maturity		Tenor				
	3 Months	6 Months	1Y	4Y	10Y	20Y
6 Months	0.25	0.23	0.22	0.2	0.19	0.18
1Y	0.25	0.24	0.23	0.21	0.20	0.19
4Y	0.26	0.25	0.24	0.22	0.22	0.19
10Y	0.24	0.24	0.23	0.215	0.23	0.2
20Y	0.24	0.23	0.22	0.21	0.2	0.18

Volatility Maxtrix; Adapted from Kapp (2014)

APPENDIX B:

B1 CVA AND MC CODE

```
trade1 <<- data.frame(10,0.02,"Buy",0.5)
colnames(trade1) <- c("Ei","swap_rate","BuySell","tenor")
trade1


#trade2 <- data.frame(1.5,0.06,1,0.5)
#colnames(trade2) <- c("Ei","swap_rate","BuySell","tenor")
#Trade2

col <- data.frame(0.8)
colnames(col) <- c("ApplyThres")
col

trades <- list(trade1)
trades


PO_LGD = 0.6
cpty_LGD = 0.6

sim_data <- data.frame(1,0.05,0.02,0.99)
colnames(sim_data) <-
c("num_of_sims","mean_reversion_a","volatility","PFE_Percentile")
sim_data

spot_rates_x = c(tenors1)
spot_rates_y = c(IR_curve)
spot_rates <- data.frame(spot_rates_x,spot_rates_y)
spot_rates

credit_curve_cpty_x = c(tenors)
credit_curve_cpty_y = c(CS_curve_cpty)
credit_curve_cpty <-
data.frame(credit_curve_cpty_x,credit_curve_cpty_y)

credit_curve_PO_x = c(tenors)
credit_curve_PO_y = c(CS_curve_PO)
credit_curve_PO <- data.frame(credit_curve_PO_x,credit_curve_PO_y)

maturity <- max(as.numeric(lapply(trades, function(x) x$Ei)))
Interval <- min(as.numeric(lapply(trades, function(x) x$tenor)))
maturity
Interval

time_points <- seq(0,maturity,Interval)
time_points

num_of_points = length(time_points)
num_of_points
```

```
spot_curve <- approx(spot_rates_x,spot_rates_y,time_points)$y
spot_curve <- spot_curve [!is.na(spot_curve)]
spot_curve

cpty_spread <-
approx(credit_curve_cpty_x,credit_curve_cpty_y,time_points)$y
cpty_spread <- cpty_spread [!is.na(cpty_spread)]
cpty_spread

PO_spread <-
approx(credit_curve_PO_x,credit_curve_PO_y,time_points)$y
PO_spread <- PO_spread[!is.na(PO_spread)]
PO_spread

discount_factors = exp(-time_points * spot_curve)
discount_factors

CalcPD <- function (spread, LGD, time_points)
{
    num_of_points = length(time_points)
    spread = spread/10^4
  PD = exp(-(spread[1:(num_of_points - 1)] *
time_points[1:(num_of_points -
        1)])/LGD) - exp(-(spread[2:num_of_points] *
time_points[2:num_of_points])/LGD)
    PD[PD < 0] = 0
    return(PD)
}

PD_cpty = CalcPD(cpty_spread, cpty_LGD, time_points)
PD_PO = CalcPD(PO_spread, PO_LGD, time_points)
PD_cpty#PD_PO

CalcSimulatedExposure <- function (discount_factors, time_points,
spot_curve, col, trades,
    sim_data)
{
    num_of_points = length(time_points)
num_of_points
    num_of_trades = length(trades)
num_of_trades

    Swap_MtMs = matrix(0, nrow = sim_data$num_of_sims, ncol =
num_of_points)
Swap_MtMs
    Swap_MtMs_coll = matrix(0, nrow = sim_data$num_of_sims, ncol =
num_of_points)
Swap_MtMs_coll

    timesteps_diff = diff(time_points)
timesteps_diff
    spot_interest_rate = spot_curve[1]
    forward_curve <- (discount_factors[1:(num_of_points -
1)]/discount_factors[2:num_of_points] - 1)/timesteps_diff
```

```
    forward_curve = c(spot_interest_rate, forward_curve)
forward_curve
    forward_diff = diff(forward_curve)
forward_diff
    theta = forward_diff + sim_data$mean_reversion_a *
forward_curve[2:length(forward_curve)]
    set.seed(30269)
theta
    interest_rates = rep(0, num_of_points)
    interest_rates[1] = spot_interest_rate
    random_numbers = matrix(runif(sim_data$num_of_sims *
(num_of_points -
        1)), nrow = sim_data$num_of_sims, ncol = num_of_points -
        1)
random_numbers
interest_rates

index =1

        maturity = trades[[index]]$Ei
        swap_rate = trades[[index]]$swap_rate

  BuySell = ifelse(trades[[index]]$BuySell == "Buy", 1,
           -1)
BuySell
        time_points_temp = time_points[time_points <= maturity]
time_points_temp
        num_of_points_temp = length(time_points_temp)
num_of_points_temp
        A = rep(0, length(time_points_temp))
A
        B = (1 - exp(-sim_data$mean_reversion_a * (maturity -
            time_points_temp)))/sim_data$mean_reversion_a
       B
 disc_factors = matrix(0, nrow = num_of_points_temp, ncol =
num_of_points_temp)
  disc_factors
dt = maturity/(num_of_points_temp-1)
dt


  for (j in 1:sim_data$num_of_sims)
{
            Floating_leg = rep(0, num_of_points_temp)
Floating_leg

            for (i in 2:num_of_points_temp) interest_rates[i] =
interest_rates[i -
                1] + (theta[i - 1] - sim_data$mean_reversion_a *
interest_rates[i - 1]) * timesteps_diff[i - 1] + sim_data$volatility
* qnorm(random_numbers[j,
                  i - 1]) * sqrt(timesteps_diff[i - 1])
interest_rates
```

```
            for (i in 1:num_of_points_temp) A[i] =
discount_factors[num_of_points_temp]/discount_factors[i] *
                exp(B[i] * forward_curve[i] -
(sim_data$volatility^2/(4 *
                  sim_data$mean_reversion_a)) * (1 - exp(-2 *
                  sim_data$mean_reversion_a * time_points_temp[i]))
*
                  B[i]^2)
A

 for (i in 1:num_of_points_temp) disc_factors[i,
1:(num_of_points_temp -
                i + 1)] = A[num_of_points_temp - i + 1] * exp(-
B[num_of_points_temp -
                i + 1] * interest_rates[1:(num_of_points_temp -
                i + 1)])
disc_factors

            for (i in 0:(num_of_points_temp - 1)) Floating_leg[i +
                1] = 1 - disc_factors[num_of_points_temp - i,
                i + 1]
Floating_leg

        Fixed_leg = dt * swap_rate *
colSums(disc_factors[2:num_of_points_temp,], na.rm =TRUE)
Fixed_leg = dt * swap_rate * disc_factors[2:num_of_points_temp,]
Fixed_leg

            Swap_MtM = (Floating_leg - Fixed_leg) * BuySell
Swap_MtM

            Swap_MtMs[j, 1:num_of_points_temp] = Swap_MtMs[j,
1:num_of_points_temp] + Swap_MtM

Swap_MtMs_coll[j, 1:num_of_points_temp] = Swap_MtMs[j,
              1:num_of_points_temp]*col$ApplyThres
Swap_MtMs
}

   trades[[index]]$MtM = Swap_MtMs[1, 1]

    exposure_profile = list()
 exposure_profile$EE_uncoll = apply(apply(Swap_MtMs, c(1,
        2), function(x) ifelse(x >= 0, x, NA)), 2, mean, na.rm =
TRUE)
exposure_profile$EE_uncoll[is.na(exposure_profile$EE_uncoll)] = 0
exposure_profile$EE_uncoll


    exposure_profile$NEE_uncoll = apply(apply(Swap_MtMs, c(1,
        2), function(x) ifelse(x < 0, x, NA)), 2, mean, na.rm =
TRUE)
  exposure_profile$NEE_uncoll[is.na(exposure_profile$NEE_uncoll)] =
0
```

```
   exposure_profile$NEE_uncoll


 exposure_profile$PFE_uncoll = apply(apply(Swap_MtMs, c(1,
        2), function(x) ifelse(x > 0, x, NA)), 2, quantile,
sim_data$PFE_Percentile,
        na.rm = TRUE)
    exposure_profile$PFE_uncoll[is.na(exposure_profile$PFE_uncoll)]
= 0
exposure_profile$PFE_uncoll


    exposure_profile$EE = apply(apply(Swap_MtMs_coll, c(1, 2),
        function(x) ifelse(x >= 0, x, NA)), 2, mean, na.rm = TRUE)
    exposure_profile$EE[is.na(exposure_profile$EE)] = 0
exposure_profile$EE

    exposure_profile$NEE = apply(apply(Swap_MtMs_coll, c(1, 2),
        function(x) ifelse(x < 0, x, NA)), 2, mean, na.rm = TRUE)
    exposure_profile$NEE[is.na(exposure_profile$NEE)] = 0
exposure_profile$NEE

    exposure_profile$PFE = apply(apply(Swap_MtMs_coll, c(1, 2),
        function(x) ifelse(x > 0, x, NA)), 2, quantile,
sim_data$PFE_Percentile,
        na.rm = TRUE)
    exposure_profile$PFE[is.na(exposure_profile$PFE)] = 0
exposure_profile$PFE

    EEE = rep(0, num_of_points)
    EEE[1] = exposure_profile$EE[1]
    for (i in 2:(num_of_points)) EEE[i] = max(EEE[i - 1],
exposure_profile$EE[i])
    exposure_profile$EEE = EEE
    exposure_profile
```

B2 CVA AND MC CODE

```
RegDatas <- read.csv("RegDatas.csv")

colnames(RegDatas) <- c("Date", "Russia", "Russia_b" ,"Brazil",
"Brazil_b", "Turkey", "Turkey_b", "Bahrain", "Bahrain_b",
"Columbia", "Columbia_b", "Itraxx_Fins", "Itraxx_Eur", "CDXEM",
"CDX_NA")

summary(lm(RegDatas$Russia_b ~ 0 + RegDatas$Russia+
RegDatas$Itraxx_Eur+RegDatas$CDXEM+RegDatas$CDX_NA))
```

```
step(lm(Brazil_b ~ 0 + RegDatas$Brazil + RegDatas$Itraxx_Fins +
RegDatas$Itraxx_Eur + RegDatas$CDXEM +
RegDatas$CDX_NA),direction="backward")
```