Essays on Dynamic Macroeconomics

by

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Declaration

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Abstract

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In the first essay of this thesis, a medium scale DSGE model is developed and estimated for the South African economy. When used for forecasting, the model is found to outperform private sector economists when forecasting CPI inflation, GDP growth and the policy rate over certain horizons.

In the second essay, the benchmark DSGE model is extended to include the yield on South African 10-year government bonds. The model is then used to decompose the 10-year yield spread into (1) the structural shocks that contributed to its evolution during the inflation targeting regime of the South African Reserve Bank, as well as (2) an expected yield and a term premium. In addition, it is found that changes in the South African term premium may predict future real economic activity.

Finally, the need for DSGE models to take account of financial frictions became apparent during the recent global financial crisis. As a result, the final essay incorporates a stylised banking sector into the benchmark DSGE model described above. The optimal response of the South African Reserve Bank to financial shocks is then analysed within the context of this structural model.

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Dedications

This thesis is dedicated to Almie, Reuben, Waldo and Mieke.

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Chapter 1

Introduction

During the past two decades, New Keynesian dynamic stochastic general equilibrium (DSGE) models have emerged as a new and increasingly prominent approach to quantitative macroeconomic modelling. In fact, this methodology has not only "taken centre stage in academic macroeconomic research" (Del Negro and Schorfheide, 2003), but has also become the *modus operandi* of policy analysis and forecasting in central banks across the globe.[1]

Until the 1970s, the structural, simultaneous equation models based on Keynesian theory and pioneered by the Cowles Commission were at the frontier of quantitative macroeconomic modelling, both in policymaking and academic institutions.[2] Within these models, the structure of each equation was loosely guided by theory, while the parameter estimates were based on statistical relationships of the past. However, the popularity of the Cowles Commission-style models waned during the 1970s, as they were unable to predict both the magnitude and persistence of the high inflation and unemployment that characterised the United States during this period. Inflation had become so entrenched that a 10 per cent reduction in annual GDP was required to reduce inflation by 1 percentage point, according to Okun (1978). Evidently, a structural change had occured in the US economy. Frank E. Morris (1978), the then-president of the Federal Reserve Bank of Boston, describes the practitioner's frustrations at the time as follows:

> I look back with nostalgia on those years in the early sixties when we used, with remarkable success, small econometric models to make fairly exact estimates of what we needed to produce a given result in the economy. Now we have much more elaborate econometric models that are coming up with estimates in which we have much less confidence.

Failure of the Cowles Commission's style of models opened the door to serious critique, which ultimately had far reaching implications. Lucas (1976) argued convincingly that these models would by way of construction fail. Having been estimated on reduced-form historical macroeconomic relationships, they fail once any structural change or changes in policy occur which alter the nature of the macroeconomic relationships on which the model was based. To circumvent this fundamental problem, Lucas (1976) proposed that macroeconomic models should be derived from microeconomic first principles, where the parameters reflect the behavioural aspects of economic agents, such as their tastes and preferences. Accordingly, these so-called *deep* parameters should not be affected by changes in policy or macroeconomic structure. Moreover, Lucas (1972, 1976) showed methodologically how to model agents' expectations about the future, while the Cowles Commission's

[1] See Tovar (2008) for a comprehensive review of the use of DSGE models in central banks.

[2] Perhaps the most striking evidence of the high regard in which this style of modelling was held during the 1950s and 1960s, was the awarding of the first Nobel Prize in Economics to Ragnar Frisch and Jan Tinbergen. Ragnar Frisch was one of the first members of the Cowles Commision and is credited for having founded the discipline of econometrics, while Jan Tinbergen was the first to develop a comprehensive macroeconomic model, initially for the Netherlands in 1936, but later he applied it to the United States and the United Kingdom – see Tinbergen (1939, 1951, 1959).

style of models were largely backward-looking. Based on Lucas' proposals, modelling the forward-looking, rational behaviour of agents implies that in the event of a change in policy or structure, the model's predictions would remain valid. Agents within the model would correctly anticipate the impact of the change and adjust their behaviour accordingly.

The critique of Lucas is regarded as a turning point in the history of macroeconomic theory, as the rational expectations theory had led to a revolution in the way in which macroeconomic models were formulated – a process to which Sargent (1980) refers as the "reconstruction" of macroeconomics. A seminal contribution to this reconstruction of the macroeconomic discipline was made by Kydland and Prescott (1982). Their paper found that technology shocks contributed to business cycle fluctuation, as opposed to the Keynesian tenet that these fluctuations resulted from shocks to aggregate demand. In addition, the model they used successfully addressed the critique of Lucas, as agents exhibited rational expectations and their behaviour was determined by microfounded decision rules based on tastes and preferences. The parameters that reflect these preferences, or *deep* parameters, were either calibrated to values largely found in microeconomic studies, or else values were selected such that the theoretical moments implied by the model would match those observed in the data. Kydland and Prescott's (1982) successful application of these microeconomic principles to a real world macroeconomic phenomenon – the business cycle – spawned a literature which soon became known as real business cycle (RBC) theory. Other key contributions to this literature were made by Hansen (1985) and King, Plosser and Rebelo (1988).

In retrospect, the RBC literature was the first step towards the modern day New Keynesian DSGE models that are widely in use today. However, RBC models abstracted from nominal factors by assuming a perfectly competitive environment where prices are fully flexible, and hence there was no role for monetary policy to stabilise economic activity. A first stab at combining the RBC framework with nominal factors, such as sticky prices and wages, was made by Blanchard and Kiyotaki (1987). They achieved this by allowing for imperfect competition within the RBC framework. Interestingly, Blanchard and Kiyotaki (1987) mention that their results are "tantalisingly close to traditional Keynesian models", as they find a significant role for demand shocks in output fluctuations, which stands in contrast to the early RBC literature. Another step in the road towards the New Keynesian DSGE literature had been taken, and the already-modified RBC framework was now ripe for the inclusion of monetary policy. Goodfriend and King (1997) and Clarida, Gali and Gertler (1999), amongst others, heeded the call. These studies analysed the implications of the framework for monetary policy, with Clarida, Gali and Gertler (1999) focusing specifically on optimal monetary policy. Hereafter, a number of studies made important contributions, but the scale and theoretical rigour of Michael Woodford's *Interest and Prices* (2003) deserves special mention. Being widely regarded as a landmark volume in modern monetary theory, McCallum (2005) at the time described the book as "the most important treatise on monetary economics" of the last 50 years. *Interest and Prices* provided the theoretical foundations of rule-based monetary policy within a microfounded, rational expectations framework. The literature had matured to a point where "[t]he consensus model [incorporated] classical features such as intertemporal optimization, rational expectations, and a real business cycle core, together with Keynesian features such as monopolistically competitive firms, staggered sticky nominal price setting, and a central role for monetary stabilization policy" (Goodfriend, 2007). In essence, the theoretical framework for New Keynesian DSGE models had finally been established.

While the theoretical progression from the first RBC models in the early 1980s to the eventual New Keynesian DSGE models of the early 2000s was taking place, two separate yet equally important developments were occurring in the field of statistics and computing. During this time, statisticians were developing crucial simulation techniques such as the Markov Chain Monte Carlo (MCMC) sampling method which is used during Bayesian estimation. In addition, computing power increased exponentially, in tandem with cost declines (Fernández-Villaverde and Rubio-Ramírez, 2006). Together, these advancements made it possible to take DSGE models to actual data. Parameter values could be estimated using full information methods, as opposed to the then existing practice of calibrating them. Suddenly, a whole new realm of uses and applications for DSGE models had been opened up. Given the theoretical consistency that these New Keynesian DSGE models brought to the table, the central banking community recognised their value for policy analysis and forecasting once they were estimated.

A seminal contribution in this regard came from Smets and Wouters (2003), who at the time were economists at the European Central Bank and National Bank of Belgium, respectively. The authors presented a richly specified DSGE model of the Euro area that incorporates ten structural shocks as well as a number of nominal and real frictions, such as Calvo (1983) staggered price and wage setting, habit formation in consumption, investment adjustment costs, and variable capacity utilisation. Moreover, the model had been estimated with Bayesian techniques, while using a total of seven macroeconomic variables.[3] They then display the usefulness of this estimated DSGE model for monetary policy analysis by (1) decomposing inflation and output into the structural shocks that contributed to their historical paths; (2) determining the degree of stickiness in Euro area wages and inflation; (3) assessing the uncertainty surrounding output gap estimates; and (4) analysing the transmission of monetary policy shocks to the rest of the economy. In a follow-up paper (Smets and Wouters, 2004), the authors compare forecasts from the estimated DSGE model for the seven macroeconomic variables used in estimation, to forecasts for these variables that were generated by vector autoregressions (VARs) and Bayesian VARs of differing lag lengths. According to their results, the estimated DSGE model outperforms both the VARs and Bayesian VARS over the 1 to 4 quarter forecasting horizon. In addition, the forecasting accuracy increased along with

[3] The Euro area variables used by Smets and Wouters (2003): Gross domestic product (GDP), consumption, investment, GDP deflator, real wage, employment and the short-term interest rate.

the horizon.

Taken in conjunction, Smets and Wouters (2003) and Smets and Wouters (2004) had proven to the central banking community that DSGE models were an essential part of the forecasting and policy analysis toolkit. Consequently, within the space of a few years, most of the advanced economies' central banks, as well as some emerging markets, had adopted DSGE models into their modelling suites.[4]

However, the benchmark New Keynesian DSGE model at the time did have a serious shortcoming: it ignored the role played by the financial sector in the evolution of the business cycle (Tovar, 2008). The role of financial intermediation had been deemed to be irrelevant for the transmission of monetary policy (Blanchard *et al.*, 2010). This weakness was brutally exposed by the global financial crisis of 2008. Given the models at their disposal, policymakers were neither able to coherently assess nor fully comprehend the macroeconomic implications of the financial instability brought about by the crisis (Blanchflower, 2009). As a result, the agenda of the DSGE literature shifted toward the inclusion of financial frictions within the benchmark framework. Initially, this new direction of the literature largely built on the financial accelerator work done almost a decade earlier by Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). According to Kiyotaki and Moore's (1997) collateral constraint framework, lenders require collateral from borrowers before granting a loan, while the value of this collateral co-moves with the business cycle. During a recession borrowers would have limited access to financing, which serves to amplify the business cycle. In a similar vein, Bernanke *et al.* (1999) assume borrowers pay a premium on external finance which increases with leverage, *i.e.* the size of the loan relative to their net worth. As before, the adverse impact of a recession on borrower net worth would increase the external finance premium and hence amplify the business cycle. Examples within the financial-friction New Keynesian DSGE literature that have built on this financial accelerator framework include Christensen and Dib (2008) and Iacoviello and Neri (2010). More specifically, Christensen and Dib (2008) find that the inclusion of the external finance premium within the New Keynesian DSGE framework improves the model's fit of observed data. A similar conclusion was reached by Iacoviello and Neri (2010). By incorporating the collateral constraint framework under the assumption that households offer housing stock as collateral when borrowing, the authors find that the model is able to match the observed fluctuations in household consumption.

Neither the collateral constraints nor external finance premium framework assign an explicit role for banks, as they largely focus on the demand for credit. Since the banking sector played a key role in the financial crisis, its explicit inclusion into the benchmark DSGE framework became an integral part of the new agenda. Accordingly, Cúrdia and Woodford (2009, 2010) introduce banks into the New

[4] Amongst others, these central banks include the Bank of Canada, Bank of England, US Federal Reserve, Norges Bank, Sveriges Riksbank, Swiss National Bank, Central Bank of Brazil, Central Bank of Chile, Central Bank of Colombia, Central Bank of Iceland, the Reserve Bank of New Zealand and the International Monetary Fund.

Keynesian DSGE framework by assuming that banks receive deposits from saving households which are then converted into loans for borrowing households. The deposits are remunerated at the policy rate, while loans are repaid at a spread over the policy rate. Moreover, the authors assume that a proportion of loans turn out to be non-performing, and the lending spread is a positive function of the amount of non-performing loans. During an economic downturn, when non-performing loans tend to increase, the lending spread increases as well, which in turn amplifies the downturn – a phenomenon which was widely observed in the global financial crisis of 2008. Cúrdia and Woodford (2010) find that it is optimal for the central bank to react to these rising lending spreads in the event of such a financial shock. Another seminal contribution in incorporating a banking sector in the New Keynesian DSGE framework was made by Gerali, Neri, Sessa and Signoretti (2010). The authors extend the literature by allowing for banks to own capital, and find that shocks to bank capital have macroeconomic consequences. Moreover, Gerali *et al.* (2010) show that the decline in real activity during the global financial crisis can largely be explained by shocks that originated within the banking sector.

Given the development of the DSGE literature to date, the aim of this thesis is firstly to develop an operational DSGE model of the South African economy – similar to the DSGE models used in the numerous central banks discussed above. What modern central banks generally require from operational DSGE models, is the ability to: (1) simulate various macroeconomic scenarios in order to facilitate and stimulate policy discussions; (2) shed light on the structural shocks that are driving relevant macroeconomic variables; and (3) forecast these macroeconomic variables over the policy-relevant horizon with sufficient accuracy. Consequently, by addressing these requirements in this thesis, the model could potentially form part of the South African Reserve Bank's current "suite of models" that are used for monetary policy analysis and forecasting. Secondly, the rational expectations solution of DGSE models make them an ideal framework to answer questions about any variable whose current value is largely determined by expectations of its underlying future values. Here bond yields serve as a perfect example, since the expectations hypothesis theory posits that any bond yield reflects the expectation of average short-term interest rates over the maturity period of the bond. Therefore, the DSGE model developed in this thesis is extended to incorporate the South African term structure, whereafter the structural shocks that have contributed to its evolution are analysed. Finally, given the evident need for central bank models to incorporate the role of financial frictions in the monetary policy transmission mechanism – as was highlighted by the recent global financial crisis – a banking sector is introduced into the model. The model is then used to analyse the role of a small open-economy's central bank in the face of financial shocks. In the light of the abovementioned aims, the remainder of the thesis is laid out as follows: In Chapter 2 a benchmark small-open economy New Keynesian DSGE model is derived and then estimated for the South African economy. Thereafter, Chapter 3 incorporates South Africa's term structure of interest rates into the DSGE framework. A banking sector is introduced into the benchmark model in Chapter 4, before Chapter 5 provides a brief summary of the thesis.

Chapter 2

A medium-sized open economy DSGE model of South Africa

2.1 Introduction

The past two decades have seen the emergence of dynamic stochastic general equilibrium (DSGE) models – a new approach to macroeconometric modelling which has "taken centre stage in academic macroeconomic research" (Del Negro and Schorfheide, 2003). Given their theoretical consistency and inherently forward-looking nature, DSGE models offer a serious alternative to the Cowles Commission tradition of structural simultaneous equation models. In fact, the differences between DSGE models and the Cowles Commission tradition are so fundamental that the development of the DSGE approach has been described as a paradigm shift in macro-econometric modelling, which Fernández-Villaverde (2010) aptly calls the "New Macroeconometrics".

DSGE models are not only confined to academic circles, but are also being developed and used for actual monetary policy analysis and forecasting in many central banks, including the Bank of Canada (Murchison and Rennison, 2006), the Bank of England (Harrison *et al.*, 2005), the Czech National Bank (Andrle *et al.*, 2009), the European Central Bank (Christoffel *et al.*, 2008), the Reserve Bank of New Zealand (Beneš *et al.*, 2009) and the Swedish Riksbank (Adolfson *et al.*, 2007*a*), among others.

A number of DSGE models of the South African economy have lately been developed. Liu and Gupta (2007) calibrated the RBC model of Hansen (1985) to match South African data. The model is used to generate forecasts for a number of macroeconomic variables, which are then compared with the forecasts of a Bayesian and classical VAR. Steinbach *et al.* (2009) used Bayesian methods to estimate a small open economy DSGE model on South African data, while Ortiz and Sturzenegger (2007) used a version of the Gali and Monacelli (2003) model and Bayesian techniques to estimate the policy reaction function of the South African Reserve Bank (SARB). Finally, Alpanda *et al.* (2010*a*, 2010*b*) explored the role of the exchange rate in South African monetary policy, before evaluating the forecasting properties of the model in Alpanda *et al.* (2011).

This chapter extends the South African literature by developing a DSGE model which could be operationalised in a policy institution such as the South African Reserve Bank. This is achieved by adding a number of variables and frictions to the standard small open economy DSGE model structure. First of all, as an extension to the aggregate demand components in Steinbach *et al.* (2009), investment (capital accumulation), exports and imports and their corresponding price deflators are added to the framework. In addition, the model includes a number of additional real and nominal frictions, such as investment adjustment costs, costly variation in the utilisation of capital and imperfect pass-through of import and export prices, to mention a few. The inclusion of these friction help make the model estimable. The model then is estimated with Bayesian methods, before its usefulness as a potential tool in a policy-making environment such as the South African Reserve Bank is assessed through a decomposition of historical developments in key variables, as well as the model's forecasting ability.

The chapter is laid out as follows. The structure of the model is presented in Section 2.2. Section 2.3 describes the detail surrounding the estimation procedure, as well as the results thereof. Thereafter, the historical decomposition, the dynamic behaviour of the model and its forecasting ability are discussed in Sections 2.4 and 2.5, before Section 2.6 concludes the chapter.

2.2 The model

The small open economy model structure largely follows the lines of Adolfson *et al.* (2007*a*), which in turn extended the models of Christiano *et al.* (2005) and Altig *et al.* (2011) into the open economy setting. More specifically, the general structure of Adolfson *et al.* (2007*a*) is ideal for the purposes of this chapter, as it forms the backbone of an operational DSGE model that is used for actual forecasting and policy analysis in an inflation-targeting central bank.[1] Nevertheless, the model laid out below departs from Adolfson *et al.* (2007*a*) in four key aspects. Firstly, households do not derive utility from holding money.[2] Secondly, allowance is made for the fact that on average, inflation in South Afica exceeds that of its trading partners. In the context of the model, this is achieved by assuming that South Africa has a higher steady state inflation rate. By implication, these differential inflation rates yield a nominal exchange rate depreciation in steady state, as predicted by purchasing parity theory. Thirdly, it is assumed that there is no cost channel of monetary policy, hence firms do not borrow their wage bill.[3] Finally, apart from lump-sum transfers, the role of taxes in the model is disregarded.

Households consume both domestic and imported goods, whilst exhibiting habit formation in consumption. They have the option to save in domestic or foreign bonds. In addition, being the owners thereof, households rent capital to firms and decide how much to invest in each period. Changes to the rate of investment are subject to adjustment costs. Households can also vary the rate at which capital is utilised, subject to adjustment costs. Following Erceg *et al.* (2000), each household supplies a differentiated labour service to firms which enables them to set their wage in a Calvo (1983) manner.

In the model there are three types of firms: domestic producers, importers and exporters. Domestic firms employ labour and capital in production, whilst being exposed to both transitory and permanent technology shocks as in Altig *et al.* (2011). A differentiated good is produced by each type of firm, and subsequently prices are set following Calvo's (1983) model, with a variation of Rabanal and Rubio-Ramírez (2005) which allows for indexation to past inflation. The incorporation of

[1] RAMSES, the DSGE model used for forecasting and policy analysis at the Sveriges Riksbank is based on Adolfson *et al.* (2007*a*).

[2] Money is introduced as an asset in Chapter 3, where the model is extended to include the term structure of interest rates.

[3] Liu (2013) finds evidence that the cost channel of monetary policy is at play in the South African economy. However, for the sake of simplicity this channel is not included here.

these nominal rigidities in the price-setting behaviour of importing and exporting firms enables incomplete pass-through of exchange rate changes in the short-run.

The central bank is assumed to follow a Taylor-type rule in setting the short-term policy interest rate. And finally, consistent with the small-open economy setup, the foreign economy is assumed to be exogenous to developments in the domestic economy.

2.2.1 Firms

2.2.1.1 Domestic firms

Final good producers A final good producer transforms intermediate goods into a final homogeneous good, which in turn is used by households for either consumption or investment purposes. The transformation process of intermediate goods into the final good takes the CES form

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_{d,t}}} di \right]^{\lambda_{d,t}}, \tag{2.1}$$

where λ_t^d is the time-varying markup for domestic goods that is assumed to follow an AR(1) process

$$\lambda_t^d = (1-\rho_{\lambda^d})\lambda^d + \rho_{\lambda^d}\lambda_{t-1}^d + \epsilon_t^{\lambda^d}, \tag{2.2}$$

with λ^d being the steady-state level of the domestic goods markup, while ρ_{λ^d} measures the degree of persistence and $\epsilon_t^{\lambda^d} \sim N(0, \sigma_{\lambda^d})$. Profit maximisation by the final good firm yields the demand function for intermediate goods

$$Y_{i,t} = \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} Y_t, \tag{2.3}$$

and the price of the final good as an index of intermediate goods' prices:

$$P_t = \left[\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_{d,t}}} di \right]^{1-\lambda_{d,t}}. \tag{2.4}$$

Intermediate good producers A continuum of intermediate good producers (indexed by i, where $i \in [0,1]$) operate in a monopolistically competitive environment and produce differentiated goods according to the production function:

$$Y_{i,t} = \varepsilon_t \left(K_{i,t}^s\right)^{\alpha} (z_t H_{i,t})^{1-\alpha} - z_t\phi, \tag{2.5}$$

where z_t and ε_t are permanent and transitory technology shocks respectively. $K_{i,t}^s$ represents capital services that are rented from households, $H_{i,t}$ is a homogenised labour input, and ϕ captures fixed costs that grow in line with technology. Capital services K_t^s may differ from the actual capital stock K_t as a result of variation in the utilisation rate of capital, u_t, where $K_t^s = u_t K_t$. It is further assumed that the respective technology shocks follow autoregressive processes:

$$\begin{aligned}\frac{z_t}{z_{t-1}} &= \mu_t^z \\ &= (1-\rho_{\mu^z})\mu^z + \rho_{\mu^z}\mu_{t-1}^z + \epsilon_t^{\mu^z}, \end{aligned} \tag{2.6}$$

and

$$\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \varepsilon_t^\varepsilon, \tag{2.7}$$

where μ^z is the steady-state growth rate of technology, $E(\varepsilon_t) = 1$ and $\hat{\varepsilon}_t = (\varepsilon_t - 1)/1$. Since $\mu_t^z > 1$, the presence of the permanent technology shock z_t in the model implies that all real variables contain a unit root. To render the model stationary, real variables are therefore detrended with the permanent technology shock. Let the notational convention be such that lower case letters indicate detrended variables. Then, as an example, the detrended capital stock is expressed as $k_{t+1} = K_{t+1}/z_t$. Nominal variables also contain a stochastic trend, as the price level is non-stationary, and hence are detrended by the domestic price level P_t^d. Note that the nominal wage W_t contains both the permanent technology and nominal price level trend, as nominal wages grow in line with changes in the price level and technology. Therefore, the detrended real wage is expressed in the model as $w_t = W_t/(z_t P_t^d)$.

The intermediate firm rents capital services at the gross nominal rate R_t^k and compensates the homogenous labour service at the nominal wage rate W_t. Accordingly, the intermediate firm's cost-minimisation problem is as follows:

$$\min_{K_{i,t}^s, H_{i,t}} W_t H_{i,t} + R_t^k K_{i,t}^s + \lambda_t P_{i,t}^d \left[Y_{i,t} - \varepsilon_t \left(K_{i,t}^s \right)^\alpha (z_t H_{i,t})^{1-\alpha} + z_t \phi \right]. \tag{2.8}$$

Optimization of Equation (2.8) with respect to $K_{i,t}^s$ and $H_{i,t}$ yields the familiar first-order conditions:

$$R_t^k = \alpha \lambda_t P_{i,t} z_t^{1-\alpha} \epsilon_t \left(K_{i,t}^s \right)^{\alpha-1} H_{i,t}^{1-\alpha} \tag{2.9}$$

and

$$W_t = (1-\alpha)\lambda_t P_{i,t} z_t^{1-\alpha} \epsilon_t \left(K_{i,t}^s \right)^\alpha H_{i,t}^{-\alpha}, \tag{2.10}$$

that equate the marginal returns of capital and labour to the cost of their compensation.

When combining Equations (2.9) and (2.10), the stationary real rental rate of capital is expressed as:

$$r_t^k = \frac{\alpha}{1-\alpha} \bar{w}_t \mu_t^z \left(\frac{H_t}{k_t} \right) \tag{2.11}$$

and real marginal cost as

$$mc_t = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha \epsilon_t^{-1} \left(r_t^k \right)^\alpha (\bar{w}_t)^{1-\alpha}, \tag{2.12}$$

where the Lagrange multiplier in Equation (2.8), $\lambda_t P^d_{i,t}$, is interpreted as nominal marginal cost MC_t.

Domestic price setting It is assumed that intermediate good firms set prices in a staggered manner as proposed by Calvo (1983). In his model a firm gets the opportunity to adjust its price with a probability of $(1-\theta_d)$ in every period. Thus, in a given period t, not all firms are able to react to supply shocks immediately, which implies that the higher θ_d, the more sticky is the price adjustment process. In addition, following Adolfson *et al.* (2007*a*), it is assumed that the intermediate good firms who do not receive the Calvo signal to change their price, index their price in $t+1$ to period t's inflation rate and the current inflation target, as follows:

$$P^d_{t+1} = \left(\pi^d_t\right)^{\kappa_d} \left(\bar{\pi}^c_{t+1}\right)^{1-\kappa_d} P^d_t, \tag{2.13}$$

where $\pi^d_t = P^d_t / P^d_{t-1}$ is the gross inflation rate, $\bar{\pi}^c_t$ the inflation target and κ_d the degree of indexation to past inflation.[4,5] As it aims to maximise its expected discounted profit, the intermediate firm i's intertemporal optimisation problem is therefore:

$$\begin{aligned} \max_{\tilde{P}_t} E_t \sum_{s=0}^{\infty} (\beta\theta_d)^s \upsilon_{t+s} \Bigg\{ \Bigg[\left(\prod_{k=1}^{s} \pi_{t+k-1} \right)^{\kappa_d} \left(\prod_{k=1}^{s} \bar{\pi}^c_{t+k} \right)^{1-\kappa_d} \tilde{P}_t \Bigg] Y_{i,t+s} \\ - MC_{i,t+s} \left(Y_{i,t+s} + z_{t+s}\phi \right) \Bigg\}, \end{aligned} \tag{2.14}$$

where $(\beta\theta_d)^s \upsilon_{t+s}$ is the stochastic discount factor. In addition, the price index of Equation (2.4) can be expressed as a weighted average of the new optimal price chosen by the firms that do receive the Calvo signal, and the backward indexed price set by the remaining firms:

$$P_t = \left[\theta_d \left(\left(\pi_{t-1}\right)^{\kappa_d} \left(\bar{\pi}^c_t\right)^{1-\kappa_d} P_{t-1} \right)^{\frac{1}{1-\lambda_{d,t}}} + (1-\theta_d) \tilde{P}_t^{\frac{1}{1-\lambda_{d,t}}} \right]^{1-\lambda_{d,t}}. \tag{2.15}$$

Optimising Equation (2.14), whilst taking account of the demand for intermediate goods in Equation (2.3), linearising the result and combing it with the linearised Equation (2.15), yields the New Keynesian Phillips curve for the domestic good:

$$\begin{aligned} \hat{\pi}_t - \hat{\bar{\pi}}^c_t &= \frac{\beta}{1+\kappa_d\beta} \left(E_t \hat{\pi}_{t+1} - \rho_\pi \hat{\bar{\pi}}^c_t \right) + \frac{\kappa_d}{1+\kappa_d\beta} \left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}^c_t \right) - \frac{\kappa_d \beta (1-\rho_\pi)}{1+\kappa_d\beta} \hat{\bar{\pi}}^c_t \\ &+ \frac{(1-\theta_d)(1-\beta\theta_d)}{(1+\kappa_d\beta)\theta_d} \left(\hat{mc}_t + \hat{\lambda}^d_t \right). \end{aligned} \tag{2.16}$$

[4]The time-varying inflation target is analogous to a flexible inflation targeting regime. More specifically, as discussed below in Section (2.3.1), it's role in this model is to facilitate the transition from high inflation and interest rates in the 1990s – prior to South Africa's implementation of an inflation targeting regime in February 2000 – to lower inflation and interest rates thereafter.

[5]$\kappa_d = 1$ implies that indexation is completely backward-looking.

2.2.1.2 Importing firms

There are two types of importing firms: importing consumption and importing investment firms. Both of these importing firms purchase a homogeneous good in the world market at the international price P_t^*. Thereafter, the importing consumption firm turns the homogeneous good into a differentiated consumption good $C_{i,t}^m$, while a differentiated investment good $I_{i,t}^m$ is created by the importing investment firm. Let $J_t \in \{C_t^m, I_t^m\}$ denote aggregate quantities of the imported consumption and investment good, and $j \in \{c, i\}$, then the final imported good can be expressed as a CES composite of the differentiated import goods:

$$J_t = \left[\int_0^1 (J_{i,t})^{\frac{1}{\lambda_t^{m,j}}} di \right]^{\lambda_t^{m,j}}. \tag{2.17}$$

The demand function faced by each importing firm i is given by:

$$J_{i,t} = \left(\frac{P_{i,t}^{m,j}}{P_t^{m,j}} \right)^{-\frac{\lambda_t^{m,j}}{\lambda_t^{m,j}-1}} J_t, \tag{2.18}$$

while the time-varying markup for the imported consumption and investment goods is:

$$\lambda_t^{m,j} = (1-\rho_{\lambda^{m,j}})\lambda^{m,j} + \rho_{\lambda^{m,j}}\lambda_{t-1}^{m,j} + \epsilon_{\lambda^{m,j},t}. \tag{2.19}$$

As with domestic firms, it is assumed that importing firms face a Calvo probability when setting their price. Hence, importing consumption firms may change their price with probability $(1-\theta_{m,c})$ and investment firms with probability $(1-\theta_{m,i})$. Firms who cannot reoptimise, index their price in period $t+1$ to a combination of the previous period's imported price inflation rate $\pi_t^{m,j}$ and the current inflation target $\bar{\pi}_{t+1}^c$ as follows:

$$P_{t+1}^{m,j} = \left(\pi_t^{m,j}\right)^{\kappa_{m,j}} \left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_{m,j}} P_t^{m,j}. \tag{2.20}$$

In a similar vein to the domestic intermediate firm, the respective importing firm's optimisation problem is therefore given by:

$$\max_{\tilde{P}_t^{m,j}} E_t \sum_{s=0}^{\infty} \left(\beta\theta_{m,j}\right)^s \upsilon_{t+s} \left\{ \left[\left(\prod_{k=1}^{s} \pi_{t+k-1}^{m,j} \right)^{\kappa_{m,j}} \left(\prod_{k=1}^{s} \bar{\pi}_{t+k}^{j} \right)^{1-\kappa_{m,j}} \tilde{P}_t^{m,j} \right] J_{i,t+s} \right.$$
$$\left. -S_{t+s} P_{t+s}^* \left(J_{i,t+s} + z_{t+s}\phi^{m,j} \right) \right\}, \tag{2.21}$$

where S_t is the nominal exchange rate expressed as the number of domestic currency units needed to buy one unit of the foreign currency, and hence, $S_t P_t^*$ is the importing firm's marginal cost. The respective aggregate imported goods price indices in period t are therefore a weighted average of firms who reoptimise and firms who set their price to the indexing scheme of Equation (2.20):

$$\begin{aligned} P_t^{m,j} &= \left[\int_0^1 \left(P_{i,t}^{m,j}\right)^{\frac{1}{1-\lambda_t^{m,j}}} di\right]^{1-\lambda_t^{m,j}} \\ &= \left[\theta_{m,j}\left(P_{t-1}^{m,j}\left(\pi_{t-1}^{m,j}\right)_{m,j}^{\kappa}\left(\bar{\pi}_t^j\right)^{1-\kappa_{m,j}}\right)^{\frac{1}{1-\lambda_t^{m,j}}} + \left(1-\theta_{m,j}\right)\left(\tilde{P}_t^{m,j}\right)^{\frac{1}{1-\lambda_t^{m,j}}}\right]^{1-\lambda_t^{m,j}} \quad (2.22) \end{aligned}$$

Combining Equations (2.21) and (2.18), linearising the result before inserting it in the linearised Equation (2.22), yields dynamic inflation equations for imported consumption and investment goods:

$$\begin{aligned} \hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_{m,j}\beta}\left(E_t\hat{\pi}_{t+1}^{m,j} - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta}\left(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_{m,j}\beta(1-\rho_\pi)}{1+\kappa_{m,j}\beta}\hat{\bar{\pi}}_t^c \\ &+ \frac{\left(1-\theta_{m,j}\right)\left(1-\beta\theta_{m,j}\right)}{\left(1+\kappa_{m,j}\beta\right)\theta_{m,j}}\left(\hat{mc}_t^{m,j} + \hat{\lambda}_t^{m,j}\right), \quad (2.23) \end{aligned}$$

where $j = \{c, i\}$ and the importing firms' real marginal cost deviation from its steady state is given by $\hat{mc}_t^j = \hat{s}_t + \hat{p}_t^* - \hat{p}_t^{m,j}$.

2.2.1.3 Exporting firms

Exporting firms purchase the final good, differentiate it and then sell this continuum of differentiated export goods to households abroad. The demand faced by the individual exporting firm is given by:

$$\tilde{X}_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{-\frac{\lambda_t^x}{\lambda_t^x-1}} \tilde{X}_t, \quad (2.24)$$

where P_t^x is the foreign currency price of exports, and the time-varying markup for the exporting firm is:

$$\lambda_t^x = (1-\rho_{\lambda^x})\lambda^x + \rho_{\lambda^x}\lambda_{t-1}^x + \varepsilon_{\lambda^x,t}. \quad (2.25)$$

We assume that exporters also set their prices in a staggered manner as proposed by Calvo (1983), and that the proportion of firms who cannot reoptimise in a given period, index their price to the previous period's export price inflation rate, as follows:[6]

$$P_{t+1}^x = \pi_t^x P_t^x. \quad (2.26)$$

[6] Since exporting firms set their prices for the foreign market, they do not consider the domestic inflation target when indexing.

Hence, the optimisation problem of the individual exporting firm is given by:

$$\max_{\tilde{P}_t^x} E_t \sum_{s=0}^{\infty} (\beta\theta_x)^s \upsilon_{t+s} \left\{ \left(\prod_{k=1}^{s} \pi_{t+k-1}^x \tilde{P}_t^x \right) \tilde{X}_{i,t+s} - \frac{P_{t+s}}{S_{t+s}} \left(\tilde{X}_{i,t+s} + z_{t+s}\phi^x \right) \right\}, \tag{2.27}$$

where P_{t+s}/S_{t+s} is the nominal marginal cost of the exporting firm as it buys the final good at the domestic price P_t^d before differentiating it and selling it in the foreign market's currency. The aggregate export price is once again a weighted combination of the two pricing schemes which exporting firms face: reoptimise with probability $(1-\theta_x)$, or else index with probability θ_x. Hence,

$$\begin{aligned} P_t^x &= \left[\int_0^1 \left(P_{i,t}^x \right)^{\frac{1}{1-\lambda_t^x}} di \right]^{1-\lambda_t^x} \\ &= \left[\theta_x \left(\pi_{t-1}^x P_{t-1}^x \right)^{\frac{1}{1-\lambda_t^x}} + (1-\theta) \left(\tilde{P}_t^x \right)^{\frac{1}{1-\lambda_t^x}} \right]^{1-\lambda_t^x}. \end{aligned} \tag{2.28}$$

As before, optimising the combination of Equations (2.27) and (2.24), and thereafter linearising the result and inserting the linearised Equation (2.22), yields the dynamic inflation equation for exported goods:

$$\hat{\pi}_t^x = \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1}^x + \frac{1}{1+\beta} \hat{\pi}_{t-1}^x + \frac{(1-\theta_x)(1-\beta\theta_x)}{(1+\beta)\theta_x} \left(\hat{mc}_t^x + \hat{\lambda}_t^x \right), \tag{2.29}$$

where $\hat{mc}_t^x = \hat{p}_t^d - \hat{s}_t - \hat{p}_t^x$ is the real marginal cost of the exporting firm.

In the foreign economy, the exported good may either be used for consumption C_t^* or investment I_t^*. The assumption that the domestic economy is so small that its contribution to aggregate demand in the foreign economy becomes negligible, allows us to express foreign demand for the exported consumption and investment goods as

$$C_t^x = \left[\frac{P_t^x}{P_t^*} \right]^{-\eta_f} C_t^* \quad \text{and} \quad I_t^x = \left[\frac{P_t^x}{P_t^*} \right]^{-\eta_f} I_t^*. \tag{2.30}$$

Since we assume that the elasticiy of substitution η_f is the same for both the exported consumption and investment good, their respective contributions to aggregate exports is irrelevant. Therefore, the individual demand functions for the exported consumption and investment goods can be simplified in terms of aggregate exports and aggregate foreign demand as follows:

$$\begin{aligned} \tilde{X}_t &= C_t^x + I_t^x = \left[\frac{P_t^x}{P_t^*} \right]^{-\eta_f} (C_t^* + I_t^*) \\ &= \left[\frac{P_t^x}{P_t^*} \right]^{-\eta_f} Y_t^*. \end{aligned} \tag{2.31}$$

2.2.2 Households

A continuum of infinitely-lived households (indexed by j, where $j \in [0,1]$) populate the domestic economy. They derive utility from consuming a basket of imported and domestic consumption goods and holding cash balances, while they exhibit disutility in supplying labour services. In every period, the j^{th} household maximises expected lifetime utility according to following intertemporal utility function

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\xi_t^c \ln\left(C_{j,t} - bC_{j,t-1}\right) - \xi_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} \right] \tag{2.32}$$

where $C_{j,t}$ denotes consumption by the household, $h_{j,t}$ is the labour it supplies. The parameter β represents the household's subjective discount factor, b captures the degree of habit formation in consumption, A_L pins down the steady state level of disutility from supplying labour, while σ_L is the inverted Frisch elasticity of labour supply. ξ_t^c and ξ_t^h represent consumption preference and labour supply shocks, respectively, and are assumed to follow AR(1) processes as follows:

$$\xi_t^c = \rho_c \xi_{t-1}^c + \varepsilon_t^c$$
$$\xi_t^h = \rho_c \xi_{t-1}^h + \varepsilon_t^h$$

Consumption The aggregate consumption basket from which households derive utility is given by the CES index:

$$C_t = \left[(1-\vartheta_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c - 1}{\eta_c}} + \vartheta_c^{\frac{1}{\eta_c}} \left(C_t^m\right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}, \tag{2.33}$$

where C_t^d and C_t^m denote domestic and imported consumption goods, η_c is the substitution elasticity between the two goods and ϑ_c is the imports share in aggregate consumption. The respective demand functions for the domestic and imported consumption goods are given by

$$C_t^d = (1-\vartheta_c) \left[\frac{P_t^d}{P_t^c} \right]^{-\eta_c} C_t \quad \text{and} \quad C_t^m = \vartheta_c \left[\frac{P_t^{m,c}}{P_t^c} \right]^{-\eta_c} C_t, \tag{2.34}$$

and the price index for the consumption basket (CPI) is:

$$P_t^c = \left[(1-\vartheta_c)(P_t^d)^{1-\eta_c} + \vartheta_c (P_t^{m,c})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \tag{2.35}$$

Labour supply and wage setting The differentiated labour service $h_{j,t}$ that is supplied by each household, is transformed by a labour aggregating firm into a homogeneous input good H_t as follows:

$$H_t = \left[\int_0^1 \left(h_{j,t} \right)^{\frac{1}{\lambda_w}} \right]^{\lambda_w}, \tag{2.36}$$

where H_t is then used by intermediate firms in production. By supplying a differentiated labour service, each household has monopoly power when setting its nominal wage $W_{j,t}$. However, in doing so it faces the following demand for its labour services:

$$h_{j,t} = \left[\frac{W_{j,t}}{W_t}\right]^{\frac{\lambda_w}{1-\lambda_w}} H_t, \tag{2.37}$$

where W_t is the aggregated nominal wage rate for the homogeneous labour input good H_t, expressed as the CES aggregate:

$$W_t = \left[\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} dj\right]^{1-\lambda_w}. \tag{2.38}$$

Moreover, it is assumed that a household cannot optimally set its wage in every period, but rather faces a Calvo probability $1-\theta_w$ of doing so. Hence, with probability θ_w household j will not be able to change its wage in period t, and as such will index its wage in period $t+1$ to a combination of the previous period's CPI inflation rate, the current inflation target and the current economy-wide technology growth rate, as follows:

$$W_{j,t+1} = (\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w)} \mu_{t+1}^z W_{j,t}, \tag{2.39}$$

where κ_w is the degree of indexation to CPI inflation.

$$\max_{\tilde{W}_{j,t}} E_t \sum_{s=0}^{\infty} (\beta\theta_w)^s \left\{ \begin{array}{c} -\xi_{t+s}^h A_L \frac{(h_{j,t+s})^{1+\sigma_L}}{1+\sigma_L} \\ +\upsilon_{t+s} h_{j,t+s} \left[\left(\prod_{k=1}^s \pi_{t+k-1}^c\right)^{\kappa_w} \left(\prod_{k=1}^s \bar{\pi}_{t+k}^c\right)^{(1-\kappa_w)} \left(\prod_{k=1}^s \mu_{t+k}^z\right) \tilde{W}_{j,t}\right] \end{array} \right\}, \tag{2.40}$$

where $\tilde{W}_{j,t}$ is the optimal reset wage. Optimisation of Eq. (2.40) subject to the demand for individual household labour given by Eq. (2.37), yields first-order condition for wage setting

$$E_t \sum_{s=0}^{\infty} (\beta\theta_w)^s h_{j,t+s} \left\{ \begin{array}{c} \xi_{t+s}^h A_L \left(h_{j,t+s}\right)^{\sigma_L} \\ +\frac{\tilde{W}_t}{z_t P_t} \frac{z_{t+s}\upsilon_{t+s} P_{t+s}}{\lambda_w} \frac{\left(\frac{P_{t+s-1}^c}{P_{t-1}^c}\right)^{\kappa_w} \left(\prod_{k=1}^s \bar{\pi}_{t+k}^c\right)^{(1-\kappa_w)}}{\frac{P_{t+s}^d}{P_t^d}} \end{array} \right\} = 0, \tag{2.41}$$

where we make use of the fact that in equilibrium, all households choose the same optimal reset wage $\tilde{W}_t$. In addition, the aggregate wage index from Eq. (2.38) can be expressed as a weighted average of households who reoptimise their wage in period t and those that set their wage to the indexing scheme of Eq. (2.39):

$$W_t = \left[\theta_w \left(\left(\pi_{t-1}^c\right)^{\kappa_w} (\bar{\pi}_t^c)^{1-\kappa_w} \mu_t^z W_{t-1}\right)^{\frac{1}{1-\lambda_w}} + (1-\theta_w)\tilde{W}_t^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}. \tag{2.42}$$

Combining the loglinearised versions of Eqs. (2.41) and (2.42), whilst also stationarising the nominal wage such that $w_t = W_t/P_t^d z_t$ is the real wage, yields the wage equation

$$\hat{w}_t = -\frac{1}{\eta_1}\left[\begin{array}{c} \eta_0\hat{w}_{t-1} + \eta_2 E_t\hat{w}_{t+1} + \eta_3\left(\hat{\pi}_t^d - \hat{\bar{\pi}}_t^c\right) + \eta_4\left(E_t\hat{\pi}_{t+1}^d - \rho_\pi\hat{\bar{\pi}}_t^c\right) \\ +\eta_5\left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \eta_7\hat{\psi}_t^z + \eta_8\hat{H}_t + \eta_9\hat{\xi}_t^h \end{array}\right] \tag{2.43}$$

where $b_w = \frac{\lambda_w\sigma_L - (1-\lambda_w)}{(1-\beta\theta_w)(1-\theta_w)}$

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \end{pmatrix} = \begin{pmatrix} b_w\theta_w \\ \left[\lambda_w\sigma_L - b_w(1+\beta\theta_w^2)\right] \\ b_w\beta\theta_w \\ -b_w\theta_w \\ b_w\beta\theta_w \\ b_w\theta_w\kappa_w \\ -b_w\beta\theta_w\kappa_w \\ (1-\lambda_w) \\ -(1-\lambda_w)\sigma_L \\ -(1-\lambda_w) \end{pmatrix} \tag{2.44}$$

and ψ_t^z is the stationarised Lagrange multiplier.

Asset holdings Households allocate their wealth among domestic and foreign risk-free bonds, B_t and B_t^*. The prices of these bonds are inversely proportional to their respective gross nominal interest rates, R_t and R_t^*, while they have a maturity of one period. However, as in Benigno (2009), the interest rate at which households purchase foreign bonds is adjusted with a risk premium that depends on the domestic economy's indebtedness in the international asset market, as measured by its net foreign asset position:

$$A_t \equiv \frac{S_t B_t^*}{P_t^d}. \tag{2.45}$$

Schmitt-Grohé and Uribe (2003) show that the inclusion of this debt-elastic risk premium is crucial for the determination of a well-defined steady state in small open economy models. In addition, following Adolfson *et al.* (2008), we assume that the risk premium is not only a function of the net foreign asset position, but also the expected depreciation of the domestic currency, S_{t+1}/S_{t-1}. The inclusion of the expected exchange rate in the risk premium aims to account for the "forward premium puzzle": an empirical anomaly according to which currencies with higher risk premiums *ex ante* often tend to appreciate *ex post*, and hence a negative relationship exists between risk premia and expected depreciations. Consequently, it is assumed that the risk premium has the following functional form

$$\Phi(\frac{A_t}{z_t}, S_t, \tilde{\phi}_t) = \exp\left\{-\tilde{\phi}_a(a_t - a) - \tilde{\phi}_s\left[\frac{E_t S_{t+1}}{S_t}\frac{S_t}{S_{t-1}} - \left(\frac{\pi}{\pi^*}\right)^2\right] + \tilde{\phi}_t\right\}, \tag{2.46}$$

such that households will pay a premium on the foreign interest rate if the domestic economy is a net borrower in the international asset market, and conversely they receive a lower remuneration if the domestic economy is a net lender. In addition, the negative sign on the expected change in the exchange rate could be interpreted as a willingness by households to accept a lower return on their foreign bond holdings, if they expected the exchange rate depreciation to exceed the steady state inflation differential $\frac{\pi}{\pi^*}$, as a depreciation would increase the domestic currency return of their foreign assets.[7] Finally, the term $\tilde{\phi}_t$ in Eq. (4.6) represents an AR(1) shock to the risk premium, while in the steady state, the risk premium has the property $\Phi(0,0,0) = 1$.

Investment and capital accumulation Households own the capital stock, and as a result, in every period t they make a decision on how much to invest, I_t. As with consumption, households may purchase domestic (I_t^d) or imported investment goods (I_t^m), which is given by the CES aggregate:

$$I_t = \left[(1-\vartheta_i)^{\frac{1}{\eta_i}}\left(I_t^d\right)^{\frac{\eta_i-1}{\eta_i}} + \vartheta_i^{\frac{1}{\eta_i}}\left(I_t^m\right)^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}}, \tag{2.47}$$

where η_i is the substitution elasticity between domestic and imported investment goods and ϑ_i is the share of imports in aggregate investment. The respective demand functions for domestic and imported investment goods are given by

$$I_t^d = (1-\vartheta_i)\left[\frac{P_t^d}{P_t^i}\right]^{-\eta_i} I_t \quad \text{and} \quad I_t^m = \vartheta_i\left[\frac{P_t^{m,i}}{P_t^i}\right]^{-\eta_i} I_t, \tag{2.48}$$

and subsequently, the price deflator for aggregate investment is:

$$P_t^i = \left[(1-\vartheta_i)(P_t^d)^{1-\eta_i} + \vartheta_i(P_t^{m,i})^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \tag{2.49}$$

Note that the domestic consumption and investment good share the same price P_t^d, while differences between the CPI and investment price deflators emanate from the fact that the imported consumption good's price $P_t^{m,c}$ may differ from the imported investment good's $P_t^{m,i}$. Given the household's investment decision, the capital stock K_{t+1} accumulates as follows:

$$K_t = (1-\delta)K_{t-1} + \xi_t^i F(I_t, I_{t-1}) + \Delta_t, \tag{2.50}$$

where ξ_t^i is an investment specific technology shock, with the property $E[\xi_t^i] = 1$, that follows the AR(1) process:

$$\hat{\xi}_t^i = \rho_c \hat{\xi}_{t-1}^i + \varepsilon_t^i,$$

[7] Adolfson *et al.* (2008) assume that the domestic and foreign inflation rates are identical in steady state, hence $\frac{\pi}{\pi^*} = 1$.

with $\hat{\xi}^i_t = (\xi^i_t - 1)/1$. Δ_t represents installed capital that households may purchase in the secondary market from other households. Although $\Delta_t = 0$ in equilibrium, as all households make indentical capital accumulation decisions, its inclusion facilitates the calculation of the price of installed capital $P^{k'}_t$. The term $F(I_t, I_{t-1})$ in Eq. (2.50) captures the investment adjustment cost that is paid by households whenever the rate of change in the level of investment deviates from the economy-wide steady state growth rate μ^z. Christiano *et al.* (2005) specify this adjustment cost function as follows:

$$F(I_t, I_{t-1}) = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{2.51}$$

where

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_i}{2}\left(\frac{I_t}{I_{t-1}} - \mu^z\right)^2, \tag{2.52}$$

such that in steady state $S(\cdot)$ satisfies $S(\mu^z) = S'(\mu^z) = 0$ and $S''(\mu^z) \equiv \phi_i$, with $\phi_i > 0$. In addition to the investment decision, households may also choose to vary the rate at which the current capital stock is utilised, u_t. The effective capital stock that is rented to firms, K^s_t, is therefore defined as:

$$K^s_t = u_t K_{t-1}. \tag{2.53}$$

However, as with investment, households pay a capital adjustment cost $a(u_t)$ when varying the level of capital utilisation. It is assumed that the utilisation adjustment cost function has the following properties in steady state: $a(1) = 0$, $a'(1) = r^k$ and $a''(1) \geq 0$.[8]

Budget constraint Given the set of variables introduced above, the household's budget constraint can be formulated as follows:

$$\begin{aligned} &\frac{B_{j,t}}{R_t} + \frac{S_t B^*_{j,t}}{R^*_t \Phi\left(\frac{A_t}{z_t}, S_t, \tilde{\phi}_t\right)} + P^c_t C_{j,t} + P^i_t I_{j,t} + P^d_t \left[a(u_{j,t}) K_{j,t} + P^{k'}_t \Delta_t\right] \\ &\quad = B_{j,t-1} + S_t B^*_{j,t-1} + W_{j,t} h_{j,t} + R^k_t u_{j,t} K_{j,t-1} + \Pi_t - T_t \end{aligned} \tag{2.54}$$

where the expression on the left of the equality represents nominal expenditure by the household in period t, while to the right we have nominal income earned by the household in period t as well as wealth carried over from $t-1$. Hence, households purchase new domestic and foreign assets, nominal consumption goods, nominal investment goods, they pay adjustment costs on capital utilisation and also purchase installed capital. The wealth households carry over from $t-1$ consists of

[8] $u = 1$ in steady state, since $K^s = K$.

their portfolio of domestic and foreign bond holdings. Households are remunerated for the labour they supply and the capital services they rent to firms. In addition, they receive profits from firm ownership, Π_t, while they pay lump-sum taxes to the government, T_t.

First-order conditions Optimisation of the household's utility function, Eq. (3.3), subject to the budget constraint and capital's law of motion, Eqs. (3.4) and (2.50), yields the following set of first-order conditions with respect to each of the choice variables:[9]

Consumption, c_t

$$\frac{\xi_t^c}{c_t - bc_{t-1}\frac{1}{\mu_t^z}} - \beta b E_t \frac{\xi_{t+1}^c}{c_{t+1}\mu_{t+1}^z - bc_t} - \psi_t^z \frac{P_t^c}{P_t} = 0 \tag{2.55}$$

Investment, i_t

$$-\psi_t^z \frac{P_t^i}{P_t} + \psi_t^z P_t^{k'} \xi_t^i F_1\left(i_t, i_{t-1}, \mu_t^z\right) + \beta E_t \left[\frac{\psi_{t+1}^z}{\mu_{t+1}^z} P_{t+1}^{k'} \xi_{t+1}^i F_2\left(i_{t+1}, i_t, \mu_{t+1}^z\right)\right] = 0 \tag{2.56}$$

Capital stock, k_t

$$-\psi_t^z P_t^{k'} + \beta E_t \left[\frac{\psi_{t+1}^z}{\mu_{t+1}^z}\left(r_{t+1}^k u_{t+1} + (1-\delta) P_{t+1}^{k'} - a(u_{t+1})\right)\right] = 0 \tag{2.57}$$

Installed capital, Δ_t

$$-\psi_t^z P_t^{k'} + \omega_t = 0 \tag{2.58}$$

Capital utilisation, u_t

$$\psi_t^z \left[r_t^k - a'(u_t)\right] = 0 \tag{2.59}$$

Domestic bond holdings, b_t

$$-\psi_t^z + \beta E_t \left[\frac{\psi_{t+1}^z}{\mu_{t+1}^z} \frac{R_t}{\pi_{t+1}}\right] = 0 \tag{2.60}$$

Foreign bond holdings, b_t^*

$$-\psi_t^z S_t + \beta E_t \left[\frac{\psi_{t+1}^z}{\mu_{t+1}^z \pi_{t+1}} \left(S_{t+1} R_t^* \Phi(a_t, S_t, \tilde{\phi}_t)\right)\right] = 0 \tag{2.61}$$

where all trending variables have been rendered stationary, as represented by their lower case counterparts, and $\psi_t^z = z_t P_t^d \upsilon_t$ is the stationary Lagrange multiplier. In

[9]Since all households make identical decisions in equilibrium, the subscript j is no longer needed.

addition, the log-linearised combination of the first-order conditions for domestic assets and foreign bond holdings, Eqs. (2.60) and (2.61), yield the UIP condition

$$\hat{R}_t - \hat{R}_t^* = (1-\tilde{\phi}_s)E_t\Delta\hat{S}_{t+1} - \tilde{\phi}_s\Delta\hat{S}_t - \tilde{\phi}_a\hat{a}_t + \hat{\tilde{\phi}}_t, \tag{2.62}$$

such that an increase (decrease) in the net foreign asset position of the domestic economy – *ceteris paribus* – leads to an appreciation (depreciation) of its currency.[10]

2.2.3 The Central Bank

When setting the short-term interest rate, it is assumed that the central bank responds to the expected deviation of year-on-year CPI inflation $\hat{\pi}_{t+1}^{c,4}$ from its target as well as the current quarter's change in the price level, $\hat{\pi}_t^c$. In addition, the central bank also takes into account the current level and rate of change in output. Based on the findings of Alpanda *et al.* (2010*b*) for South Africa, it is assumed that the central bank's policy rule does not respond to fluctuations in the real exchange rate – in contrast with studies such as Smets and Wouters (2003). Consequently, the monetary policy rule is specified as follows:

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + \phi_\pi\left(\hat{\pi}_{t+1}^{c,4} - \hat{\bar{\pi}}_t^c\right) + \phi_{\Delta\pi}\hat{\pi}_t^c + \phi_y\hat{y}_t + \phi_{\Delta y}\Delta\hat{y}_t\right] + \varepsilon_t^R \tag{2.63}$$

where year-on-year CPI inflation is defined as $\hat{\pi}_t^{c,4} = \frac{1}{4}\prod_{j=1}^4 \pi_{t+1-j}$.

2.2.4 Market clearing

In equilibrium, quantities demanded equal quantities supplied to ensure that markets clear. This applies to both the domestic final goods market and the foreign bond market.

Goods market Clearing in the domestic final goods market implies that the supply of the final good firm should match the demand from households, government and the export market, as follows:

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x \leq \varepsilon_t (K_t^s)^\alpha (z_t H_t)^{1-\alpha} - z_t\phi - a(u_t)K_{t-1}, \tag{2.64}$$

where government spending G_t is assumed to be determined exogenously. Stationarising Eq. (4.27), after having substituted the relevant demand functions from Eqs. (2.30), (2.34) and (2.48), yields

$$\begin{aligned} &(1-\vartheta_c)\left[\frac{P_t^c}{P_t^d}\right]^{\eta_c} c_t + (1-\vartheta_i)\left[\frac{P_t^i}{P_t^d}\right]^{\eta_i} i_t + g_t + \left[\frac{P_t^x}{P_t^*}\right]^{\eta_f} y_t^* \frac{z_t^*}{z_t} \\ \leq\; &\varepsilon_t\left(\frac{k_t^s}{\mu_t^z}\right)^\alpha H_t^{1-\alpha} - \phi - a(u_t)\left(\frac{k_{t-1}}{\mu_t^z}\right), \end{aligned} \tag{2.65}$$

[10] If $\tilde{\phi}_s = 0$ the standard UIP condition is obtained.

where $Y_t^* = C_t^* + I_t^*$ and since Y_t^* is detrended with the level of permanent technology in the foreign economy, z_t^*, the term $\frac{z_t^*}{z_t}$ captures temporary asymmetry in the relative technological progress between the foreign and domestic economy. Let $\tilde{z}_t^* = \frac{z_t^*}{z_t}$, and assuming that permananent technology growth in the domestic and foreign economy is equal in steady state, *i.e.*, $\mu^{z*} = \mu^z$, then $\tilde{z}^* = 1$.[11] The asymmetric technology shock is assumed to follow an AR(1) process as follows:

$$\hat{\tilde{z}}_t^* = \rho_{\tilde{z}^*}\hat{\tilde{z}}_{t-1}^* + \varepsilon_t^{\tilde{z}^*}, \tag{2.66}$$

where $\hat{\tilde{z}}_t^* = (\tilde{z}_t^* - 1)/1$.

Foreign bond market Clearing in the foreign bond market requires foreign bond holdings by households to equal the combined net position of importing and exporting firms. As such, the balance of payments identity for the evolution of (nominal) net foreign assets may be formulated as follows:

$$\frac{S_t B_{j,t}^*}{R_t^* \Phi\left(\frac{A_t}{z_t}, S_t, \tilde{\phi}_t\right)} - S_t B_{t-1}^* = S_t P_t^x (C_t^x + I_t^x) + S_t P_t^* (C_t^m + I_t^m). \tag{2.67}$$

As before, the stationary (real) net foreign asset position is given by $a_t \equiv \frac{S_t B_t^*}{P_t^d z_t}$.

2.2.5 Relative prices

In addition to the model's real variables, the various price levels also need to be rendered stationary. This is achieved by dividing these price levels through a numeraire. In the domestic economy, prices are rendered stationary by dividing with the domestic price level P_t^d, while prices that are relevant for the foreign economy, are divided with the foreign price level P_t^*. As a result, the following relative prices are defined:

Relative prices of consumption and investment goods:

$$\gamma_t^{c,d} \equiv \frac{P_t^c}{P_t^d} \tag{2.68}$$

$$\gamma_t^{i,d} \equiv \frac{P_t^i}{P_t^d}. \tag{2.69}$$

Relative prices of imported consumption and investment goods:

$$\gamma_t^{mc,d} \equiv \frac{P_t^{m,c}}{P_t^d} \tag{2.70}$$

$$\gamma_t^{mi,d} \equiv \frac{P_t^{m,i}}{P_t^d}. \tag{2.71}$$

[11] To hold, this result implicitly assumes $z_0^* = z_0$.

Relative price of exported goods:

$$\gamma_t^{x,*} \equiv \frac{P_t^x}{P_t^*}. \tag{2.72}$$

In addition, it is convenient to express both the importing and exporting firms' marginal cost as functions of the domestic-foreign relative price γ_t^f. Hence, let

$$\gamma_t^f \equiv \frac{P_t^d}{S_t P_t^*}. \tag{2.73}$$

Consequently, the marginal cost of the importing consumption and investment good firms are given as:

$$mc_t^{m,c} \equiv \frac{S_t P_t^*}{P_t^{m,c}} = \left(\gamma_t^f \gamma_t^{mc,d}\right)^{-1} \tag{2.74}$$

$$mc_t^{m,i} \equiv \frac{S_t P_t^*}{P_t^{m,i}} = \left(\gamma_t^f \gamma_t^{mi,d}\right)^{-1}, \tag{2.75}$$

while that of the exporting firm is given as:

$$mc_t^x = \frac{\gamma_t^f}{\gamma_t^{x,*}}. \tag{2.76}$$

2.2.6 Foreign economy

Being exogenous, the foreign economy is modelled as a standard three-equation closed economy DSGE model which is broadly similar to the log-linearised structure of An and Schorfheide (2007):

$$\hat{y}_t^* = E_t \hat{y}_{t+1}^* - \frac{1}{\sigma^*}\left(\hat{R}_t^* - E_t \hat{\pi}_{t+1}^* + \xi_t^{y,*}\right) \tag{2.77}$$

$$\hat{\pi}_t^* = \beta E_t \hat{\pi}_{t+1}^* + \kappa^* \hat{y}_t^* + \xi_t^{\pi,*} \tag{2.78}$$

$$\hat{R}_t^* = \rho_R^* \hat{R}_{t-1}^* + (1-\rho_R^*)\left[\phi_{\hat{\pi}}^* \pi_t^* + \phi_y^* \hat{y}_t^*\right] + \varepsilon_t^{R,*}, \tag{2.79}$$

where $\hat{y}_t^*$, $\hat{\pi}_t^*$ and $\hat{R}_t^*$ represent output, inflation and the policy rate of the foreign economy. $\xi_t^{y,*}$ and $\xi_t^{\pi,*}$ are AR(1) shock processes.

2.2.7 The model in state space form

In order to solve the model, its equations are log-linearised.[12] It is then possible to write the solved model in state space form, as follows:

$$\mathbf{S_t} = \mathbf{F S_{t-1}} + \mathbf{Q} \epsilon_\mathbf{t}, \tag{2.80}$$

$$\mathbf{Y_t} = \mathbf{M} + \mathbf{H S_t} + \eta_\mathbf{t}, \tag{2.81}$$

[12] See the Appendix for the entire set of log-linearised equations.

with

$$\begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} \sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}\right), \tag{2.82}$$

where the m dimensional state vector $\mathbf{S_t}$ contains the model's endogenous variables, while $\mathbf{Y_t}$ is an n dimensional vector of observable data which is discussed in greater detail in the following section. The matrices $\mathbf{F}$ and $\mathbf{Q}$ are functions of the model's parameters, $\mathbf{M}$ holds the steady state information of the observed data, and $\mathbf{H}$ serves to map the endogenous variables of the model to the data. ϵ_t is a vector of innovations to the model's structural shocks, while η_t is a vector of measurement errors, with $\mathbf{R} = E(\eta_t \eta_t')$.

2.3 Estimation

2.3.1 Data

In order to estimate the model, a total of fifteen observable domestic and international macro-economic time series for the sample period 2000Q1 to 2012Q4 are used.[13] The choice of sample period yields 52 quarterly observations and coincides with the inflation targeting regime of the South African Reserve Bank (SARB), which officially commenced in the first quarter of 2000.[14]

Data for the South African economy was largely obtained from the SARB Quarterly Bulletin, however, CPI and producer price inflation were obtained from StatsSA. GDP, inflation and the interest rate of the foreign economy are all calculated as trade-weighted averages of South Africa's main trading partner countries. The data for South Africa's trading partners was sourced from the Global Projection Model of the Center for Economic Research and its Applications (CEPREMAP).[15] In order to calculate the trade weights, bilateral trade data from the South African Revenue Service's Customs and Excise was used. The trade weight for each country j was calculated as the sum of imports and exports between South Africa and country j as a share of total South African exports and imports from January 2006 to December 2010. Table (4.1) lists the time series used, as well as their respective sources.

[13]Data plots of the fifteen series and their corresponding model predictions are in Figure (A.3) of the Appendix.

[14]Having announced its intention to adopt the inflation targeting framework in August 1999, it was officially implemented by the SARB in February 2000.

[15]In partnership with the Modelling Unit at the International Monetary Fund (IMF), the CEPREMAP modelling team have developed the Global Projection Model (GPM) – a quarterly model of around 35 countries which have been aggregated into 6 regions(see Carabenciov *et al.*, 2012).

Table 2.1: Observable variables

Variable	Series	Source
South Africa		
$\Delta\ln(\tilde{Y}_t)$	Real GDP	South African Reserve Bank
$\Delta\ln(\tilde{C}_t)$	Private consumption	
$\Delta\ln(\tilde{I}_t)$	Total fixed investment	
$\Delta\ln(\tilde{X}_t)$	Total exports	
$\Delta\ln(\tilde{M}_t)$	Total imports	
$\Delta\ln(\tilde{S}_t)$	Nominal effective exchange rate	
$\Delta\ln(\tilde{E}_t)$	Non-agricultural employment	
$\Delta\ln(\tilde{W}_t)$	Compensation of employees	
$\tilde{R}_t$	Repo rate	
$\tilde{\pi}^i_t$	Fixed investment deflator	
$\tilde{\pi}^c_t$	CPI inflation	StatsSA
$\tilde{\pi}^d_t$	PPI inflation, domestic manufacturing	
$\tilde{\bar{\pi}}^c_{t+1}$	Inflation target midpoint	Author's own calculations
Foreign economy		
$\Delta\ln(\tilde{Y}^*_t)$	Real GDP (trade weighted)	GPM, CEPREMAP
$\tilde{\pi}^*_t$	CPI inflation (trade weighted)	
$\tilde{R}^*_t$	Policy interest rates (trade weighted)	

2.3.1.1 Reconciling the high inflation of the early 2000s with the model structure

Given the legacy of high inflation (and interest rates) which characterised the 1990s, as well as a severe adverse exchange rate shock in December 2001, the measure of CPI inflation then targeted by the SARB only entered the 3 to 6 per cent target range for the first time in the fourth quarter of 2003 - almost four years after the implementation of inflation targeting.[16] Therefore, in order to reconcile the excessively high inflation rates at the start of the sample with the model's steady state inflation rate of 4.5 per cent (i.e. the midpoint of the inflation target range), it is assumed that the *unofficial* midpoint of the inflation target band most likely exceeded 4.5 per cent over this initial period. As such, the model's inflation target variable $\tilde{\bar{\pi}}^c_{t+1}$ is utilised as an additional observable variable and is calculated by means of a Hodrick-Prescott filter which then converges to the 4.5 per cent midpoint in 2004. Similarly, Klein (2012) estimates that the implicit inflation target of the SARB only reached the midpoint of the target band three years after the inflation targeting framework was adopted. Figure (A.2) in the Appendix plots the estimated inflation target midpoint

[16]After having averaged 10 per cent during the 1990s, CPI inflation had declined to 7.6 per cent by February 2000, but accelerated to a peak of 12.7 per cent in Novemeber 2002.

and CPI inflation.

2.3.2 Measurement of observable variables

Since the theoretical model is stationary, the observable variables need to be rendered stationary before matching them to their model counterparts. To this end, all trending observable variables are loaded as first differences. In addition, the construction of the observable variables may differ from that of their theoretical counterparts in the model. For example, the data on consumption ($\tilde{C}_t$) is constructed as the sum of imported and domestic consumption:

$$\tilde{C}_t = C_t^m + C_t^d, \tag{2.83}$$

where the ˜ above a variable denotes that it is observable. However, the theoretical measure of consumption in the model is a CES aggregate of imported and domestic consumption, and hence the observed measure of consumption needs to be adjusted in order to take account of the relative prices included in the theoretical measure. As a result, Eq. (2.83) is expressed as:

$$\tilde{C}_t = \left((1-\vartheta_c) \left[\frac{P_t^d}{P_t^c} \right]^{-\eta_c} + \vartheta_c \left[\frac{P_t^{m,c}}{P_t^c} \right]^{-\eta_c} \right) C_t. \tag{2.84}$$

Consequently, the need to account for relative prices also applies to observable investment, imports and exports, as follows:

$$\begin{aligned} \tilde{I}_t &= I_t^m + I_t^d \\ &= \left((1-\vartheta_i) \left[\frac{P_t^d}{P_t^i} \right]^{-\eta_i} + \vartheta_i \left[\frac{P_t^{m,i}}{P_t^i} \right]^{-\eta_i} \right) I_t, \end{aligned} \tag{2.85}$$

$$\begin{aligned} \tilde{M}_t &= C_t^m + I_t^m \\ &= \vartheta_c \left[\frac{P_t^{m,c}}{P_t^c} \right]^{-\eta_i} C_t + \vartheta_i \left[\frac{P_t^{m,i}}{P_t^i} \right]^{-\eta_i} I_t, \end{aligned} \tag{2.86}$$

$$\begin{aligned} \tilde{X}_t &= C_t^x + I_t^x \\ &= \left[\frac{P_t^x}{P_t^*} \right]^{-\eta_f} Y_t^*. \end{aligned} \tag{2.87}$$

Moreover, the aggregate resource constraint from Eq. (4.27) can be expressed as:

$$\begin{aligned} &\left(C_t^d + C_t^m\right) + \left(I_t^d + I_t^m\right) + G_t + (C_t^x + I_t^x) - (C_t^m + I_t^m) \\ &\qquad \leq \varepsilon_t \left(K_t^s\right)^{\alpha} (z_t H_t)^{1-\alpha} - z_t \phi - a(u_t) K_t. \end{aligned} \tag{2.88}$$

The presence of capital utilisation costs in Eq. (2.88) implies that observable GDP is not directly comparable with its theoretical counterpart and, as a result, the measurement equation for observed GDP needs to account for them. Appendix A contains the full set of log-linearised measurement equations. Of the fifteen observable variables, nine are included with measurement error, to allow for the fact that the data is merely an approximation of the actual underlying series.[17] Following Jääskelä and Nimark (2011), **R** in Eq. (2.82) is calibrated such that 10 per cent of the variation in the observed data is explained by measurement error.

2.3.3 Estimation methodology

The model is estimated with Bayesian techniques, as this approach offers a number of advantages. An and Schorfheide (2007) highlight some of them: First, Bayesian analysis is system based and therefore fits the complete solved DSGE model to actual data, as opposed to GMM, which estimates individual equilibrium relationships of the model. Second, it allows for the incorporation of additional information in parameter estimation by means of prior distributions which are specified by the researcher, whereas structural parameter estimates generated through maximum likelihood estimation are often significantly different from the additional prior information that the researcher might have. Therefore, Bayesian estimation serves as a bridge between pure calibration and maximum likelihood. Lubik and Schorfheide (2005) also emphasise the benefit of Bayesian estimation from a practical perspective, along with Sims (2008) who believes that the use of Bayesian methods can greatly improve macro-econometric modelling in central banks. In the light of these findings, the parameters of the model are estimated with Bayesian techniques.

2.3.4 Calibration

Although the model is estimated with Bayesian methods, a large number of parameters are nevertheless still calibrated. The need to calibrate certain parameters may either depend on specific steady state ratios which have to be pinned down, or result from insufficient identification of a specific parameter.[18] Table (4.2) lists the calibrated parameters.

The discount factor β is calibrated to 0.9975. Although this value is higher than 0.99 that is standard in the literature, its high value is crucial to ensure that the steady state nominal interest rate does not become unplausibly high. The deprecation rate δ is set to 0.025, which implies an annual depreciation of capital of 10 per cent. The constant in the disutility of labour, A_L, is calibrated to 7.5 which implies that households devote more or less 30 per cent of their time to working, while the calibration of the inverted Frisch elasticity of labour supply at 5 follows Martínez-García *et al.* (2012). Altig *et al.* (2011) estimate the parameter that governs the

[17] It is assumed that $\tilde{\pi}_t^c$, $\tilde{\pi}_t^d$, $\tilde{\pi}_t^*$, $\tilde{R}_t$, $\tilde{R}_t^*$ and $\Delta \ln(\tilde{S}_t)$ are free from measurement error.

[18] Identification analysis of the model's parameters was carried out using the identification toolbox in Dynare, which is largely based on Iskrev (2010*a*, 2010*b*) as well as Andrle (2010).

Table 2.2: Calibrated parameters

β	Discount factor	0.9975	δ	Depreciation rate	0.025
A_L	Labour disutility constant	7.5	σ_L	Labour supply elasticity	5
σ_a	Capital utilisation cost	10	α	Capital share in production	0.23
ϑ_c	Consumption imports share	0.36	ϑ_i	Investment imports share	0.48
θ_w	Calvo: wage setting	0.69	κ_w	Indexation: wage setting	0.5
λ_w	Wage setting markup	1.05	λ_d	Domestic price markup	1.1
η_c	Subst. elasticity: consumption	1.5	η_i	Subst. elasticity: investment	1.5
η_f	Subst. elasticity: foreign	1.25	μ^z	Permanent technology growth	1.0085
π	Steady state inflation	1.0114	g_y	Government spending to GDP	0.197
ρ_g	Government spending persistence	0.815	π^*	Foreign inflation	1.005

adjustment cost of capital utilisation, σ_a, at 2.02, while Adolfson *et al.* (2007*a*) calibrate it to 1,000,000 – which effectively removes the capital utilisation channel from the model. Based on a comparison of the model's log marginal likelihood using both Altig *et al.* (2011) and Adolfson *et al.*'s 2007*a* capital utilisation parameter values, as well as some intermediate ones, the parameter is ultimately set to 10. The share of capital used in production α is set to 0.23. This value is lower than its actual sample mean, but is necessary to ensure that the model's steady state ratios for both consumption and investment to GDP match their sample means of 60 and 20 per cent, respectively. Similarly, the shares of imports in aggregate consumption and investment, ϑ_c and ϑ_i, are calibrated to values slightly higher than their sample means. However, these calibrations ensure that the model's steady state ratios of total imports and exports to GDP match their sample means of roughly 27 per cent. The parameters that guide the persistence in wage setting, θ_w and κ_w, are not identified and as a result are both calibrated to 0.75 – implying that wage contracts are re-optimised once every four quarters, with a high degree of indexation to past inflation. The steady state wage markup follows Adolfson *et al.* (2007*a*) and is set at 1.05, while the markup for domestic prices is calibrated to 1.1. Estimates of the substitution elasticities for consumption, investment and foreign goods generally vary between 1 and 2, and are therefore calibrated to 1.5, 1.5 and 1.25 respectively. The steady state growth rate of the model's stochastic trend, μ^z, is set to 1.0085, which implies a steady state economy-wide growth rate of 3.4 per cent – roughly the average growth rate of GDP over the sample. Steady state growth of money, μ^m, is set to 1.02, *i.e.* an annualised rate of 8 per cent. Moreover, the steady state rate of inflation π in the model is calibrated to yield an annual rate of 4.5 per cent. The nominal interest rate in steady state is $R = (\pi\mu^z)/\beta$. Hence, the calibrations for β, μ^z and μ^m together imply an annualised steady state nominal interest rate of 8.9 per cent. The steady state ratio of government spending to GDP, g_y, matches its sample mean, while the persistence of government spending is set to an OLS estimate of the AR(1) coefficient for government spending. The calibration for steady state foreign inflation implies an annualised rate of 2 per cent.

2.3.5 Prior distributions

The prior means and their corresponding distributions are summarised in Table (2.3) and largely follow Adolfson *et al.* (2007*a*) and Smets and Wouters (2003), where exceptions pertain to specifics of the South African economy. Consequently, the prior for the investment adjustment cost parameter ϕ_i, is assumed to follow a normal distribution around a mean of 7.694. The degree of habit persistence – being bounded between zero and unity – is assumed to follow a beta distribution around 0.65.

The Calvo price-setting parameters (θ's) as well as those governing backward indexation (κ's) are also bounded to lie between zero and one and are assumed to follow beta distributions. Moreover, the prior means for the Calvo parameters reflect the view that South African inflation is fairly sticky, such that domestic prices are re-optimised once every 3 to 4 quarters. Moreover, the firms that do not reset are assumed to place an equal weight on the previous period's inflation rate and the current inflation target. The elasticity of the risk premium in the UIP condition is assumed to follow an inverse-gamma distribution around a mean of 0.01, which equals Alpanda *et al.*'s 2010*b* calibration of this parameter. Given the lack of prior information on ϕ_s – the parameter that guides the expected exchange rate modification in the UIP condition – it is assumed to follow a uniform distribution and hence, may take any value between zero and one.

Following Smets and Wouters (2003), the priors for the Taylor rule parameters are fairly standard. However, a larger weight is placed on both output parameters in order to allow for a more flexible approach to inflation targeting, especially during the period following the global financial crisis of 2008.

The persistence of structural shocks are all assumed to follow a beta distribution around a mean of 0.75 with standard deviation of 0.1, while the standard deviations of the shocks themselves are assumed to follow inverse-gamma distributions around means that are more or less in line with Adolfson *et al.* (2007*a*). However, the risk premium shock allows for a larger standard deviation, largely due to South Africa's emerging market status and the consequent exposure of the Rand to bouts of global risk aversion.

2.3.6 Estimation results

The posterior estimation results are summarised in Table (2.3), while Figure (A.1) in the Appendix contains the prior and posterior distributions. From the posterior results it can firstly be seen that investment adjustment costs are substantially higher than the prior mean, which implies an elasticity of investment of around 0.1 to a one per cent change in the price of installed capital. At 0.757, the degree of habit formation is found to be higher than Adolfson *et al.* (2007*a*), but in-line with the estimate of Jääskelä and Nimark (2011) for Australia.

The Calvo parameter estimates indicate that import and export price contracts are generally reoptimised every 4 quarters, while domestic contracts are reoptimised

Table 2.3: Priors and posterior estimation results

Parameter description		Prior			Posterior	
		Density[a]	Mean	Std. Dev.	Mean	90% interval
Adjustment costs						
ϕ_i	Investment	N	7.694	1.5	10.517	[8.49 ; 12.6]
Consumption						
b	Habit formation	B	0.65	0.1	0.808	[0.75 ; 0.87]
Calvo parameters						
θ_d	Domestic prices	B	0.715	0.05	0.699	[0.62 ; 0.78]
θ_{mc}	Imported consumption prices	B	0.675	0.1	0.762	[0.66 ; 0.87]
θ_{mi}	Imported investment prices	B	0.675	0.1	0.805	[0.74 ; 0.87]
θ_x	Export prices	B	0.675	0.1	0.640	[0.55 ; 0.73]
θ_E	Employment	B	0.675	0.1	0.633	[0.53 ; 0.73]
Indexation						
κ_d	Domestic prices	B	0.5	0.15	0.502	[0.31 ; 0.70]
κ_{mc}	Imported consumption prices	B	0.5	0.15	0.329	[0.14 ; 0.49]
κ_{mi}	Imported investment prices	B	0.5	0.15	0.283	[0.11 ; 0.44]
Exchange rate						
ϕ_a	Risk premium	IG	0.01	Inf	0.006	[0.00 ; 0.01]
ϕ_s	Modified UIP	U	0.5	[0,1]	0.192	[0.09 ; 0.30]
Taylor Rule						
ρ_R	Smoothing	B	0.8	0.05	0.830	[0.79 ; 0.87]
ϕ_π	Inflation	G	1.7	0.15	1.728	[1.49 ; 1.95]
$\phi_{\Delta\pi}$	Inflation (change)	G	0.3	0.1	0.271	[0.13 ; 0.41]
ϕ_y	Output gap	G	0.25	0.05	0.249	[0.17 ; 0.33]
$\phi_{\Delta y}$	Output gap (change)	G	0.125	0.05	0.170	[0.07 ; 0.27]
Persistence parameters						
ρ_{μ^z}	Permanent technology	B	0.75	0.1	0.835	[0.73 ; 0.93]
ρ_ε	Transitory technology	B	0.75	0.1	0.765	[0.62 ; 0.92]
ρ_i	Investment technology	B	0.75	0.1	0.786	[0.70 ; 0.88]
$\rho_{\tilde{z}^*}$	Asymmetric technology	B	0.75	0.1	0.783	[0.63 ; 0.94]
ρ_c	Consumption preference	B	0.75	0.1	0.682	[0.54 ; 0.84]
ρ_H	Labour supply	B	0.75	0.1	0.486	[0.35 ; 0.62]
ρ_a	Risk premium	B	0.75	0.1	0.699	[0.59 ; 0.81]
ρ_{λ^d}	Imported cons. price markup	B	0.75	0.1	0.648	[0.49 ; 0.80]
$\rho_{\lambda^{mc}}$	Imported cons. price markup	B	0.75	0.1	0.816	[0.66 ; 0.97]
$\rho_{\lambda^{mi}}$	Imported invest. price markup	B	0.75	0.1	0.651	[0.47 ; 0.83]
ρ_{λ^x}	Export price markup	B	0.75	0.1	0.591	[0.42 ; 0.77]
Structural shocks						
σ_{μ^z}	Permanent technology	IG	0.4	Inf	0.298	[0.20 ; 0.39]
σ_ε	Transitory technology	IG	0.7	Inf	1.548	[0.79 ; 2.27]
σ_i	Investment technology	IG	0.4	Inf	0.303	[0.21 ; 0.39]
$\sigma_{\tilde{z}^*}$	Asymmetric technology	IG	0.4	Inf	0.237	[0.11 ; 0.37]
σ_c	Consumption preference	IG	0.4	Inf	0.130	[0.09 ; 0.17]
σ_H	Labour supply	IG	0.2	Inf	0.355	[0.26 ; 0.46]
σ_a	Risk premium	IG	0.5	Inf	1.507	[0.97 ; 2.02]
σ_d	Domestic price markup	IG	0.3	Inf	0.648	[0.48 ; 0.82]
σ_{mc}	Imported cons. price markup	IG	0.3	Inf	0.942	[0.63 ; 1.25]
σ_{mi}	Imported invest. price markup	IG	0.3	Inf	0.646	[0.33 ; 0.95]
σ_x	Export price markup	IG	0.3	Inf	1.528	[1.06 ; 1.98]
σ_R	Monetary policy	IG	0.15	Inf	0.137	[0.11 ; 0.16]

[a] B – Beta, G – Gamma, IG – Inverse Gamma, N – Normal, U – Uniform

at a lower frequency – between 2 and 3 quarters. The Calvo estimate for domestic contracts compares favourably with Creamer, Farrell and Rankin (2012) who find that the average producer price duration in South Africa is around 6 months. The inflation indexation parameters are all estimated to be around 0.5, which implies that an equal weight is placed on past inflation and the current inflation target during indexation. Although the posterior estimate of the risk premium elasticity ϕ_a is lower than its prior, the data nevertheless to some degree favours the endogenous persistence in the risk premium induced by ϕ_s.

Turning to the estimates for Taylor rule parameters, it appears as if the SARB places a high weight on interest rate stabilisation. In addition, its reaction to changes in inflation and the output gap are less pronounced than what is indicated by the prior on these two parameters.

The estimates for the persistence of shocks indicate that the various technology shocks are most persistent, while export and imported investment markup shocks are least persistent. The standard deviations of the innovations to these shocks vary substantially. Consistent with the high weight placed on interest rate stabilisation, monetary policy shocks exhibit low volatility. However, export markup shocks are the most volatile, which possibly reflects the large weight of commodities in South Africa's export basket.

2.3.7 Model fit: moments, cross- and autocorrelations

The theoretical standard deviations, cross correlations and autocorrelations implied by the model are compared to those of the observed variables in order to assess how well the model structure conforms to the data.[19] A comparison of the standard deviations in Table (A.1) indicates that the model generally predicts a slightly greater degree of volatility than is observed in the actual data. Nevertheless, the relative magnitudes of the standard deviations correspond. Moreover, notoriously volatile variables such as imports, exports and especially the nominal exchange rate, are accurately portrayed by the model. The second column of Table (A.1) contains the cross correlation of the selected variables with the Repo rate.[20] Here there is a large degree of similarity – both in terms of sign and magnitude. More specifically, the model matches both GDP growth and CPI inflation's correlation with the Repo rate. Finally, the first and second coefficients of autocorrelation in Table (A.1) compare the model-implied persistence with the actual persistence observed in the data.[21] Apart from exports and wages, the model generally succeeds in matching the persistence observed in the remaining variables.

[19]This is standard practise in especially the RBC literature – see for instance Cooley (1995).

[20]Table (A.2) contains the cross-correlations of all the observed variables.

[21]Figure (A.4) displays up to the fifth coefficient of autocorrelation.

2.3.8 Variance decomposition

Table (A.3) reports the contribution of the structural shocks' innovations to the variation in the model's key endogenous variables. Innovations to temporary technology $\varepsilon_t^{\varepsilon}$, as well to the domestic and imported price markup shocks (ε_t^d, $\varepsilon_t^{m,c}$ and $\varepsilon_t^{m,i}$) are regarded as *supply* shocks, while innovations to the components of aggregate demand (ε_t^c, ε_t^i and ε_t^g) are grouped as *demand* shocks. Columns 8 to 13 in Table (A.3) contains the individual contributions of the remaining shocks.[22]

Variation in the Repo rate is dominated by innovations to domestic and imported consumption price markups, developments in the labour market and the exchange rate. These shocks also explain a significant proportion of the variation in CPI inflation. Variation in output is dominated by shocks to the labour market. In the light of the adverse impact that widespread labour market turmoil during 2012 is perceived to have had on economic activity, this is a highly intuitive result. In addition, domestic and export price markups are also of importance. The latter likely reflects the impact of variations in international commodity prices – more specifically precious metals – on domestic economic activity. Shocks to imported consumption markups, demand, labour and permanent technology explain the majority of variation in consumption and investment. Not surprisingly, labour market shocks explain a large proportion of the variation in employment. However, domestic price markups also play a significant role, which intuitively reflects the adverse impact that pressure on firms' profit margins has on employment. Innovations to the country risk premium and imported consumption markups dominate variation in both the nominal and real exchange rate. The significant role of the risk premium reflects the Rand's well-documented exposure to global risk aversion, whilst the role of price markups likely points to the theoretical underpinning of purchasing power parity. Innovations to export markups are the largest contributor to export variation, while labour market shocks also play a role. Although imported consumption markups explain the majority of the variation in imports, innovations to domestic price markups, investment and the exchange rate risk premium also contribute. Interestingly, innovations to domestic price markups dominate variation in the real wage, possibly reflecting the high degree of indexation to inflation during the setting of wage agreements.

2.3.9 Historical shock decomposition

Given the parameter estimates and the state space representation of the model in Equations (2.80) and (2.81), the historical evolution of the unobservable variables of the model, as well as the innovations to the structural shocks may be obtained through the Kalman filter.[23] An analysis of the contibutions of these structural

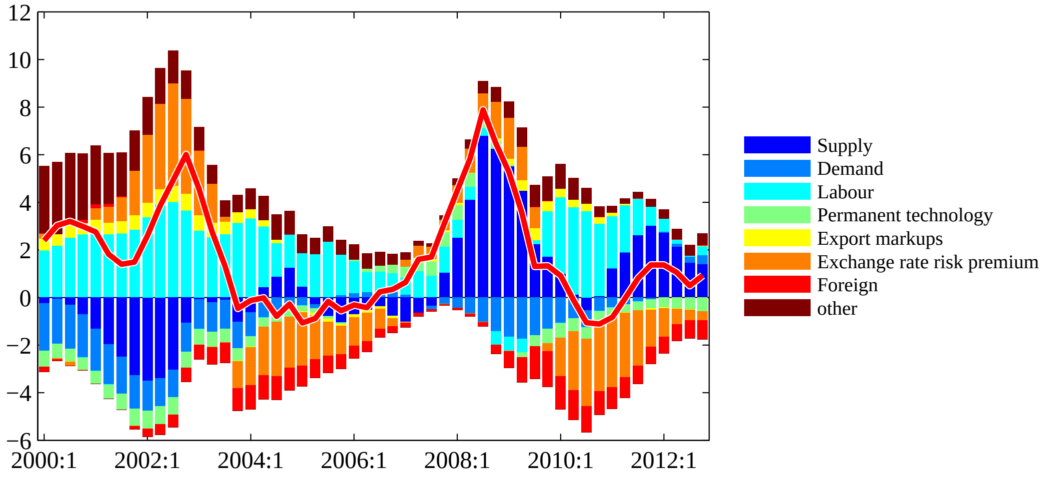
[22] Innovations to the assymetric technology and inflation target shock, $\varepsilon_t^{\tilde{z}^*}$ and $\varepsilon_t^{\bar{\pi}^c}$, have negligible contributions to the variation in the key variables and are therefore not reported in Table (A.3).

[23] The historical evolution of the individual structural shocks and their innovations are in Figure (A.5) of the Appendix.

shocks to CPI inflation and GDP growth (both year-on-year) may shed some light on the model's interpretation of historical developments in these variables.

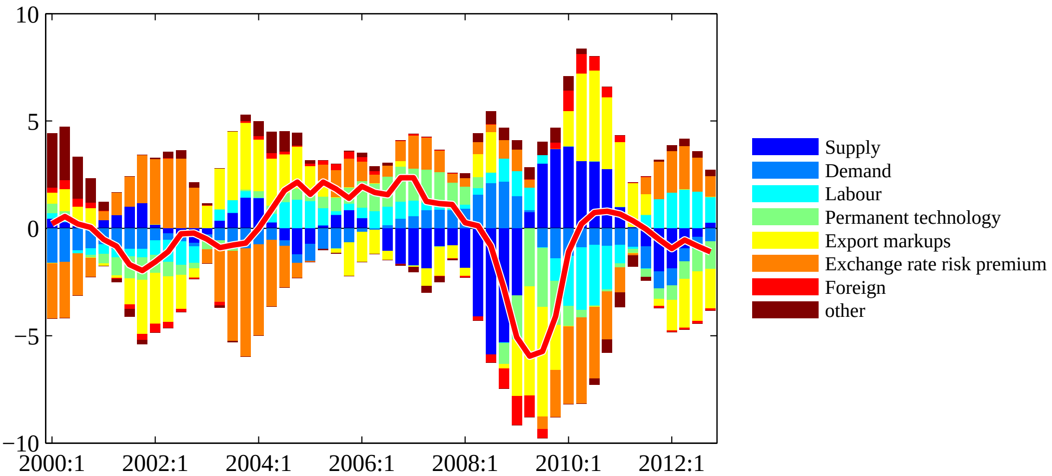
Applying a similar grouping as seen in the variance decomposition in Table A.3, the historical shock decomposition of CPI inflation in Figure 2.1 highlights the main shocks that contributed to inflation's deviations from the midpoint of the inflation target band during the inflation targeting regime. In the context of the model, the rise in inflation following the Rand's sudden depreciation towards the end of 2001 could be attributed to risk premium shocks and the ensuing domestic cost push shocks following the depreciation. The decline in inflation from 2003 to 2005 is partly attributed to reductions in the risk premium which led to the Rand's appreciation over this period. Favourable global economic conditions also contributed to the lowering of CPI inflation over this period. Nevertheless, throughout both of these periods the labour market has placed upward pressure on inflation. The model largely ascribes the rise in inflation from 2006 to 2008 to supply shocks, which possibly reflect the rising international oil price and subsequent rise in domestic fuel prices over this period. The onset of the global financial crisis in late 2008 led to a sudden depreciation of the Rand, a fall in international commodity prices, and a sharp decline in demand – global and domestic. The impact thereof can clearly be seen, as the falling commodity prices (more specifically oil prices) and adverse demand shocks contributed to CPI inflation's sudden decline during 2009. This decline in inflation would have been even steeper were it not for the depreciated exchange rate over this period. Nevertheless, by late 2009 a protracted reversal in the currency had begun, which – along with weak global conditions – had a favourable impact on inflation throughout the remainder of the sample period. However, this downward pressure was largely countered to the upside by supply shocks owing to renewed increases in international commodity prices, as well as adverse shocks to the domestic labour market.

Figure 2.1: CPI inflation: historical shock decomposition



Decomposing the model's estimate of year-on-year GDP growth indicates that developments surrounding the exchange rate dominated South African GDP during the first 4 to 5 years of the sample (see Figure 2.2). From 2006 to 2008, innovations to demand and permanent technology – the economy's trend growth rate – contributed favourably to growth, while being countered by significant adverse supply shocks during this period. Around the time of the onset of the financial crisis, the adverse impact of global developments becomes evident. Firstly through a decline in global demand, but also through a shock to export markups. It seems plausible that these export markup shocks reflect the substantial fall in international commodity prices at the time, and the subsequent impact thereof on South Africa's terms of trade. It also appears as if the economy's growth potential was adversely affected by some negative shocks to permanent technology that lasted from the end of 2008 to the beginning of 2010. In addition, whereas demand shocks contributed positively to growth while supply shocks hampered growth in the build-up to the financial crisis, their respective roles reversed during 2009 and 2010. Moreover, the strengthening of the Rand as well as unfavourable labour market conditions placed further pressure on economic growth during the wake of the financial crisis.

Figure 2.2: GDP growth: historical shock decomposition



2.4 Model dynamics

In order to analyse the dynamic reaction of the model in response to shocks, we discuss the impact of a selected number of structural shocks. In response to a 100 basis point increase in the Repo rate (see Figure A.6), the exchange rate (both real and nominal) appreciates on impact by a similar magnitude. The appreciation reduces imported inflation which lowers CPI inflation. In addition, the higher nominal interest rate, coupled with falling inflation, implies that the real interest rate increase exceeds that of the Repo rate. The higher real interest rate slows down consumption and investment, and hence output. This slowdown in the real economy reduces domestic inflation, which lowers CPI inflation even further. Moreover, the relative price change brought about by the exchange rate appreciation leads to a substantial decline in net exports, as imports surge while exports fall. This serves to amplify the decline in output. The fall in output peaks after 3 quarters at around -0.3 per cent, followed by the year-on-year fall in CPI inflation which peaks in the fourth quarter at roughly -0.2 per cent.

A one percentage point (annualised) shock to the risk premium depreciates the exchange rate by almost 3/4 of a per cent on impact (see Figure A.7). This sudden depreciation leads to a rise in imported inflation, and subsequently CPI inflation. In addition, exports rise in response to the favourable exchange rate, while the opposite holds for imports. Output rises as a result of the improvement in net exports, which in turn has a positive impact on employment. The central bank responds to rising inflation and output by increasing the Repo rate. This tightening of the policy rate cools down the domestic economy, as can be seen from the declines in consumption and investment. After more or less 16 quarters, output and CPI inflation return to their pre-shock levels.

Increasing transitory technology by 1 per cent increases output by almost half a per cent after one year in Figure (A.8). Simultaneously, this positive supply shock reduces domestic inflation. The gain in international competitiveness caused by falling domestic inflation lead to a real exchange rate depreciation, which in turn improves exports and reduces import demand. This general improvement in net exports improves the net foreign asset position which, through a reduction in the risk premium, leads an appreciation of the nominal exchange rate. The combination of lower import demand and an appreciated nominal exchange rate induce a lowering of imported consumer inflation, which further lowers CPI inflation. Monetary policy accommodates the falling CPI inflation, and after 16 quarters the economy has more-or-less returned to its steady state.

As expected, an annualised 1 percentage point shock to permanent technology μ_t^z – the economy's trend growth rate – leads to a permanent increase in all real variables. Given the unique nature of this shock, as well as its high degree of persistence, the model's long-run (25 years) reaction to the shock is shown in Figure (A.9). Although all inflation rates increase during the initial periods, largely as a result of rising real wages and a depreciating nominal exchange rate, they return to their steady state values in the long-run.

An adverse labour supply shock in Figure (A.10) increases the real wage, which in turn raises domestic inflation and subsequently CPI inflation. Rising inflation in the domestic economy appreciates the real exchange rate, fuelling imports while constraining exports. The fall in net exports deteriorates the economy's net foreign asset position, which puts pressure on the nominal exchange rate. Imported inflation rises in response to the nominal depreciation, and further contributes to the rise in CPI inflation. Monetary policy reacts by raising the Repo rate in order to contain rising inflation. The combined effect of contractionary policy and declining net exports cause a substantial decline in output.

Figure (A.11) shows the response to a sudden 1 per cent increase in foreign output. The overheating of the foreign economy leads to a rise in foreign inflation which necessitates appropriate policy reaction by the foreign central bank. The rise in foreign inflation implies that the real exchange rate in the domestic economy depreciates. As a result there are now two channels at play in the domestic economy: an income effect owing to increased foreign demand; and a price effect caused by the depreciating real exchange rate. Hence, exports rise substantially, which has a direct impact on domestic output. Moreover, the real depreciation reduces import demand which is of further benefit to output. Nevertheless, higher inflation abroad is reflected in higher imported inflation domestically. Monetary policy contracts in response to higher inflation and output, and as such cools down the domestic economy. After 20 quarters the economy has returned to its steady state level.

Finally, a 100 basis point increase in the foreign economy's policy rate depreciates the domestic exchange rate in Figure (A.12). Following the depreciation, inflation ensues which in turn leads the central bank to tighten the Repo rate. Contractionary monetary policy causes a slowdown in the real economy, as is evident from the responses of consumption and investment. The contraction in imports – largely on account of the depreciated exchange rate – is aggravated by the slowdown in domestic real economic activity. Exports fall in response to slower demand abroad following the increase in the foreign policy rate. However, after three quarters the price-effect of a weaker currency dominates the income-effect of slower foreign demand and, as a result, exports increase. The eventual increase in exports causes output to increase after its initial decline.

2.5 Forecasting performance

According to Del Negro *et al.* (2007), improvements in the time-series fit of DSGE models have contributed substantially to their increasing popularity in policy-making institutions such as central banks. Consequently, in order to gauge the usefulness of the DSGE model developed in this paper as a potential forecasting tool, its forecasting ability is assessed. Adolfson, Lindé and Villani (2007*b*), Alpanda *et al.* (2011) and Christoffel *et al.* (2010) compare the forecasting ability of open economy DSGE models with other reduced form models, and find that the DSGE models perform favourably. More specifically, the results of Alpanda *et al.* (2011) are based on a DSGE model that is estimated for South Africa.[24] In this paper we compare the DSGE model's forecasts of CPI inflation, GDP growth (quarter-on-quarter, annualised) and the Repo rate to both a random walk and consensus forecasts of private sector economists as polled by Reuters over the period 2006Q1 to 2012Q3. To this end, the model is re-estimated recursively every four quarters – once per year – where the first recursive estimation spans the sample 1993Q1 to 2005Q4, and the

[24] Alpanda *et al.* (2011) abstract from the role of capital, and as a result have fewer frictions than the model in this paper.

last is from 1999Q1 to 2011Q4. The model is then forecast 7 quarters ahead at each quarter.[25] Since the actual observations end in 2012Q4, there are 28 one-quarter-ahead and 22 seven-quarter-ahead forecast errors.

Table 2.4: Forecasting performance of the DSGE model

	Quarters ahead						
Relative RMSE statistics	1	2	3	4	5	6	7
CPI inflation, year-on-year							
DSGE/Reuters	1.549	1.066	**0.980**	1.029	**0.913**	**0.798**	**0.704**
DSGE/Random walk	**0.720**	**0.733**	**0.747**	**0.799**	**0.701**	**0.597**	**0.523**
GDP growth (quarter-on-quarter, ann.)							
DSGE/Reuters	1.520	1.528	1.200	1.078	1.012	**0.991**	**0.945**
DSGE/Random walk	1.226	**0.963**	**0.788**	**0.715**	**0.695**	**0.706**	**0.689**
Repo rate							
DSGE/Reuters	3.203	2.092	1.447	1.253	1.166	1.096	1.052
DSGE/Random walk	1.007	**0.967**	**0.941**	**0.970**	**0.991**	**0.978**	**0.938**

Accordingly, these forecast errors from the DSGE model are compared to the corresponding errors of the Reuters consensus poll of private sector economists as well as a random walk (see Table 3.4). The relative RMSE statistics indicate that the consensus forecasts of CPI inflation from the private sector outperform the DSGE model over the first two quarters of the forecast horizon. At the third quarter the DSGE model becomes competitive, and after the fifth quarter is consistently superior. In addition, the DSGE model's inflation forecasts outperform the random walk over all seven quarters of the forecast horizon. Turning to GDP growth, the consensus forecasts once again outperform the DSGE model over the near term, while the DSGE model is superior at a horizon of six and seven quarters. Moreover, the DSGE model outperforms the random walk from the second quarter onwards. When compared to consensus forecasts of the Repo rate, the DSGE model is less successful. Consensus forecasts are superior over all seven quarters of the forecast horizon, although this superiority decreases as the horizon increases. Nevertheless, the DSGE model is marginally superior to the random walk forecasts of the Repo rate from the second quarter onwards. This general ability of the DSGE model to forecast key macroeconomic variables over the medium to longer term affirms the increasing popularity and value of these models in policy-making institutions.

[25]The Reuters poll of consensus forecasts covers a seven quarter horizon.

2.6 Conclusion

In recent years, dynamic stochastic general equilibrium (DSGE) models have become an intergral part of the toolbox of models used in policymaking institutions. This chapter estimates an open economy New Keynesian DSGE model – that includes a large variety of frictions and structural shocks – for South Africa. The general structure of the model is similar to operational DSGE models used for forecasting and policy analysis in other central banks. Through the use of Bayesian methods, prior information pertaining to the South African economy is incorporated into the parameter estimates. It is found that the estimated model is able to decompose historical developments in variables of interest in a coherent and useful manner. In addition, the model is able to outperform professional forecasts of CPI inflation and GDP growth, especially at longer horizons. The estimated model is clearly suitable for storytelling as well as forecasting in the South African context and would be valuable to a policy institution such as the South African Reserve Bank.

Chapter 3

A structural decomposition of the South African yield curve

3.1 Introduction

The behaviour of interest rates at various maturities, more formally known as the *yield curve*, has intrigued economists from as early as Mitchell (1913). After Kessel (1965) had noted how movements in the yield curve corresponded to business cycle peaks and troughs, a substantial literature developed around the yield curve's ability to predict changes in economic activity. Although there exists some variation over time and across countries, the general consensus that has emerged from this literature is that the *yield spread* (*i.e.* the difference between interest rates on long-term and short-term bonds) has the ability to predict both the future level of output as well as the timing of turning points in the business cycle.[1]

Formal models aimed at characterising the yield curve itself first emerged from within the finance literature, where Vasicek (1977), Cox *et al.* (1985) and Nelson and Siegel (1987) are regarded as landmarks. Generally, these models relate the entire yield curve to three latent factors that capture its level, slope and curvature, while the latent factors themselves are usually functions of the yields too. Although the value of these models from an asset pricing perspective cannot be denied, they are fairly silent with respect to the actual macroeconomic dynamics that are driving the shape of the yield curve.[2] Addressing this issue, Evans and Marshall (1998) make use of a structural vector autoregression (SVAR) to identify the reaction of the yield curve in response to monetary policy shocks. The authors find that these shocks do affect the yield curve, especially at the short end. Yet, monetary policy only accounts for a small proportion of the yield curve's variance. Similarly, Ang and Piazzesi (2003) extend the traditional latent-factor approach to include a wider set of macroeconomic variables. Their results indicate that inflation is the overriding determinant of movements in the yield curve. In addition, they find that the inclusion of macroeconomic variables significantly improves the model's ability to forecast yields.

In recent years a number of studies have turned to the lens of a New Keynesian DSGE model in order to analyse the impact of macroeconomic dynamics on the yield curve. Within the standard New Keynesian setting, the rational expectations solution to the model facilitates the derivation of expected future short-term interest rates. This allows for a characterisation of the yield curve that is consistent with the expectations hypothesis, which in turn states that the yield of a given maturity should equal the average of expected short term rates over the period until maturity. Accordingly, De Graeve *et al.* (2009) use the Smets and Wouters (2007) model in this manner to provide a macroeconomic interpretation of historical developments in the US yield curve. However, the expectations hypothesis posits that bonds of different maturities are perfect substitutes which, in turn, implies that the slope of the yield curve is flat on average. Empirically, however, the yield curve tends to slope upwards on average (see Mishkin, 2007).

[1] See Wheelock and Wohar (2009) for a comprehensive survey.

[2] See, for instance Duffie and Kan (1996).

This empirical shortcoming of the expectations hypothesis in explaining the upward-sloping nature of the yield curve is addressed by the liquidity premium theory. It extends the expectations hypothesis by assuming that bonds of different maturities are not perfect substitutes, as investors generally prefer to hold shorter-term bonds given the increasing interest rate risk that they face when holding longer-term bonds. As a result, in order to be induced to hold longer-term bonds, investors require a *liquidity* or *term premium* that will compensate them for the additional risk. It is this term premium – which increases along with the maturity of the bond – that explains the tendency of the yield curve to slope upwards.

In order to create endogenous deviations from the expectations hypothesis within the New Keynesian framework, or *term premiums*, Andrés *et al.* (2004) assume imperfect asset substitutability *a la* Tobin (1969) between money and bonds of different maturities. Moreover, the authors show that within this expanded model structure where longer-term yields deviate from those dictated by the expectations hypothesis, central bank operations have an effect on the relative prices of financial assets. In turn, these changes in relative prices affect longer-term yields – the *unconventional* channel of monetary policy which Bernanke had already alluded to in 2002. With a similar motive – to create endogenous term premiums – Rudebusch and Swanson (2012) assume that households exhibit Epstein-Zin preferences, as opposed to the standard preference specification typically found in New Keynesian models.[3] This specification is sufficient to generate the large and time-varying term premiums that are often observed in US data. In addition, from a structural point of view, Rudebusch and Swanson (2012) find that shocks which drive output and prices in opposite directions, for instance shocks to technology and supply, explain a significant proportion of the movement in the US yield spread.

The South African literature on the yield curve has to date largely focused on its relationship with real economic activity. Using cointegration analysis, Nel (1996) compiled the first empirical study on the matter and found that a statistically-significant positive relationship existed between the yield spread on South African 10-year government bonds and GDP. Building on this proven empirical relationship, Moolman (2002) investigated the ability of the yield spread to predict turning points in the South African business cycle with the use of a probit model. The model correctly predicted 7 of the 8 eight turning points that occurred between 1979 and 2001. On average, the probit model predicted these turning points with a lead of two quarters. Moreover, Khomo and Aziakpono (2007) extended Moolman's (2002) result by comparing the predictive power of the term spread to that of other potential indicators of economic activity, such as real growth in M3 money supply, the All-Share index of the Johannesburg Securities Exchange (JSE), and the leading indicator of the South African Reserve Bank (SARB). Apart from the superiority of the SARBs leading indicator in the four months prior to the turning

[3]Under the standard habit preferences, households are only concerned with sudden changes in consumption, while under Epstein-Zin preferences, it is changes over the medium to longer term that matter as well.

point, their results indicate that, over longer horizons the yield spread is the more reliable predictor of recessions. Finally Bonga-Bonga (2010) – diverting from the yield spread-GDP literature – used a structural vector autoregression (SVAR) to examine the reaction of the yield curve to demand, supply and monetary policy shocks. The study concludes that, following a demand or monetary policy shock, short and long-term rates move in the same direction. However, short-term rates rise while long-term rates fall in response to a positive supply shock, which contradicts the prediction of Ellingsen and Söderström's (2001) structural model: that supply shocks will cause these rates to move in the same direction.

This paper extends the existing body of South African literature on the yield curve in five respects: Firstly, the macroeconomic shocks that have contributed to developments in the yield curve during the inflation targeting regime of the South African Reserve Bank are analysed within the context of a structural New Keynesian DSGE model. Secondly, the model is used to decompose the yield on South African 10-year government bonds into a term premium and a component related to the expected future short-term rates. Thirdly, whereas the literature to date has studied the predictive power of the aggregate yield spread with respect to economic activity, this paper goes further by distinguishing between the predictive ability of its subcomponents, i.e. the expected spread and the term premium. Finally, the model's ability to forecast South African 10-year government bond yields will be evaluated against the accuracy of professional forecasters.

The remainder of the paper is laid out as follows: In Section 3.2 the yield curve extension to the DSGE model is discussed. Section 3.3 covers both the estimation methodology and results. Thereafter, the model's interpretation of the structural shocks that contributed to historical developments in the 10-year yield spread is analysed. In Section 3.5, the rational expectations solution of the model is used to decompose the 10-year yield into its expectations-hypothesis component and the term premium, whereafter the dynamic reaction of the yield curve and its subcomponents are discussed. Finally, before concluding, there are two sections that analyse the DSGE model's ability to forecast the yield curve, as well as the yield curve's ability to predict future GDP.

3.2 The model

The model developed in Chapter 2 is extended to incorporate the South African yield curve into the rigorous structure of a small open economy DSGE model. Consequently, whereas the asset portfolio of households in Chapter 2 consisted solely of domestic and foreign one-period bonds, they are now assumed to hold two additional assets: money and L-period zero-coupon bonds ($B_{L,t}$).[4] As before, the one-period bond pays a gross return of R_t while, following Andrés *et al.* (2004), it

[4]For the purposes of this paper, $L = 40$ such that the L-period bonds represent South African 10 year government bonds.

is assumed that households hold their long-term bonds until they mature in period $t+L$, at which point these bonds yield a gross return of $(R_{L,t})^L$.[5]

In order to ensure positive holdings of both one-period and L-period bonds in equilibrium – irrespective of differences in yield – the model incorporates imperfect substitutabilty among assets, largely motivated by the work of Tobin (1958, 1969 and 1982). As a result, following Marzo *et al.* (2008) and Zagaglia (2009), it is assumed that bond trading is costly for the household, and hence, it pays the following quadratic adjustment cost when purchasing the long-term bond:

$$AC_t^b = \frac{\phi_L}{2}\left(\frac{b_{L,t}}{b_{L,t-1}}\right)^2 y_t. \tag{3.1}$$

The adjustment cost is measured in terms of stationary real bond holdings and may be interpreted as transaction costs on bond trading, which are paid in terms of output.[6] This formulation facilitates variations in the spread between the one-period and long-term bond, both in equilibrium and over time. The magnitude of the adjustment cost parameter ϕ_L reflects the opportunity cost associated with holding a bond of longer maturity. As such, $\phi_L > 0$, and $R_L > R$.

Moreover, the household's money holdings are directly affected by its holding of long-term bonds. Andrés *et al.* (2004) argue that households experience a loss of liquidity when purchasing bonds of maturities in excess of one period. As a result, they compensate for this loss of liquidity by holding additional money. This friction can therefore be represented as an adjustment cost function between the relative holdings of money and the L-period bond, as follows:

$$AC_t^m = \frac{\nu_L}{2}\left(\frac{m_t}{b_{L,t}}\kappa_L - 1\right)^2 y_t \tag{3.2}$$

where κ_L is the inverse of the steady state ratio m/b_L, such that the adjustment cost is zero in the steady state.

Consequently, against the background of its expanded asset portfolio, the representative household maximises the following intertemporal utility function:[7]

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\xi_t^c \ln\left(C_t - bC_{t-1}\right) - \xi_t^h A_L \frac{(h_t)^{1+\sigma_L}}{1+\sigma_L} + A_m \frac{m_t^{1-\sigma_m}}{1-\sigma_m}\right] \tag{3.3}$$

[5] The assumption that L-period bonds are zero-coupon corresponds to their general treatment in macroeconomic models (see, amongst others, Svensson (2000), Andrés *et al.* (2004) and Bekaert *et al.* (2010)). As a further simplification it is then assumed that these bonds are held until maturity. Without these two simplifying assumptions, both the budget constraint and first-order condition for long-term bond holdings would contain an additional $2L$ terms.

[6] $b_{L,t} = B_{L,t}/(z_t P_t^d)$.

[7] Since households make identical aggregate choices in equilibrium, the subscript j used to denote the representative household in Chapter 2 is dropped here for notational convenience.

subject to the budget constraint:

$$\begin{aligned} M_t(1+AC_t^m) + \frac{B_t}{R_t} + \frac{B_{L,t}}{(R_{L,t})^L}(1+AC_t^b) + \frac{S_t B_{j,t}^*}{R_t^* \Phi\left(\frac{A_t}{z_t}, S_t, \tilde{\phi}_t\right)} \\ + P_t^c C_t + P_t^i I_t + P_t^d \left[a(u_t)K_{t-1} + P_t^{k'} \Delta_t\right] \\ = M_{t-1} + B_{t-1} + B_{L,t-L} + S_t B_{t-1}^* + W_t h_t + R_t^k u_t K_t + \Pi_t - T_t. \end{aligned} \tag{3.4}$$

Within the utility function, C_t and h_t denote household consumption and labour supply, while $m_t = M_t/(z_t P_t^d)$ denotes its stationary real cash holdings. The additional parameters A_m and σ_m respectively pin down steady state money holdings and determine the curvature of money demand. Moreover, the expression on the left of the equality in Equation (3.4) represents nominal expenditure by the household in period t, while the right-hand side of the equality captures nominal income earned by the household in period t as well as wealth carried over from $t-1$. Hence, households purchase new domestic and foreign assets (where the bond prices are inversely proportional to their respective gross nominal interest rates), nominal consumption goods, nominal investment goods, they pay adjustment costs on capital utilisation and also purchase installed capital. The wealth households carry over from $t-1$ consists of cash holdings as well as their maturing domestic and foreign bond portfolio. Households are remunerated for the labour they supply and the capital services they rent to firms. In addition, they receive profits from firm ownership, Π_t, while they pay nominal lump-sum taxes to the government, T_t.

First-order conditions Optimising Equations (3.3) and (3.4) with respect to the two additional assets – money and L-period bonds – yields the following first-order conditions:

Money holdings, m_t

$$E_t\left[\frac{\beta\psi_{t+1}^z}{\pi_{t+1}\mu_{t+1}^z}\right] + A_m m_t^{-\sigma_m} - \psi_t^z\left\{1 + AC_t^m + \left[\nu_L\kappa_L\left(\frac{m_t}{b_{L,t}}\kappa_L - 1\right)\frac{m_t}{b_{L,t}}\right]y_t\right\} = 0 \tag{3.5}$$

Holdings of L-period bonds, $b_{L,t}$

$$\begin{aligned} E_t\left[\frac{\psi_{t+L}^z(\beta R_{L,t})^L}{\prod_{k=1}^L\left(\pi_{t+k}\mu_{t+k}^z\right)} + \beta\phi_L\psi_{t+1}^z\left(\frac{R_{L,t}}{R_{L,t+1}}\right)^L\left(\frac{b_{L,t+1}}{b_{L,t}}\right)^3 y_{t+1}\right] \\ -\psi_t^z\left[1 + \frac{3}{2}\phi_L\left(\frac{b_{L,t}}{b_{L,t-1}}\right)^2 y_t - \nu_L\kappa_L(R_{L,t})^L\left(\frac{m_t}{b_{L,t}}\kappa_L - 1\right)\left(\frac{m_t}{b_{L,t}}\right)^2 y_t\right] = 0 \end{aligned} \tag{3.6}$$

3.2.1 Government

In every period, the government finances its expenditure by issuing new one-period and L-period bonds, as well as raising taxes. Its period expenses consist of nominal general government expenditure $P_t^d G_t$ and also the repayment of maturing bonds. In addition, government controls the money supply. Consequently, the real (stationary) budget constraint of the government is expressed as follows:

$$\frac{\frac{B_t}{R_t}+\frac{B_{L,t}}{(R_{L,t})^L}+M_t+T_t}{z_t P_t^d}=\frac{B_{t-1}+B_{L,t-L}+M_{t-1}+P_t^d G_t}{z_t P_t^d} \tag{3.7}$$

Moreover, let the government's total liabilities, ℓ_t, be defined as follows:

$$\ell_t=\frac{1}{z_t P_t^d}\left(B_{t-1}+B_{L,t-L}+M_{t-1}\right). \tag{3.8}$$

In order to ensure dynamic stability, where inflation does not emerge as a fiscal phenomenon (see Leeper, 1991) , it is assumed that taxation by government is determined by the deviation of its outstanding liabilities from their steady state values:

$$\tau_t=\psi_0+\psi_1\left(\ell_t-\ell\right) \tag{3.9}$$

where $\tau_t = T_t/(z_t P_t^d)$. Accordingly, Equation (3.9) implies that taxes cannot be set independently from the level of outstanding government debt. This, in turn, rules out any possibility of an explosive path for government debt. Finally, both government expenditure and the supply of L-period bonds are assumed to follow AR(1) processes.

3.2.2 The central bank

As a further extension to Chapter 2, it is assumed that the central bank does not only consider inflation and ouput when setting the policy rate, but it also takes account of the growth rate in money:

$$\hat{R}_t=\rho_R\hat{R}_{t-1}+(1-\rho_R)\left[\hat{\bar{\pi}}_t^c+\phi_\pi\left(\hat{\pi}_{t+1}^{c,4}-\hat{\bar{\pi}}_t^c\right)+\phi_{\Delta\pi}\hat{\pi}_t^c+\phi_y\hat{y}_t+\phi_{\Delta y}\Delta\hat{y}_t+\phi_{\Delta m}\Delta\hat{m}_t\right]+\varepsilon_t^R. \tag{3.10}$$

The inclusion of money in the Taylor rule follows Andrés *et al.* (2004) and is largely motivated by the fact that growth in money not only serves as a potential precursor to future inflation, but also reflects asset market pressure.

3.2.3 Aggregate demand

Finally, clearing in the domestic final goods market requires that the supply of the final good firm matches the demand from households, government and the export market, after taking account of the additional adjustment costs on L-period bonds and money that are paid in terms of output:

$$y_t=\varepsilon_t\left(\frac{k_t^s}{\mu_t^z}\right)^\alpha H_t^{1-\alpha}-\phi-a(u_t)\left(\frac{k_{t-1}}{\mu_t^z}\right)-\left[\frac{\phi_l}{2}\left(\frac{b_{L,t}}{b_{L,t-1}}\right)^2+\frac{\nu_L}{2}\left(\frac{m_t}{b_{L,t}}\kappa_L-1\right)^2\right]y_t, \tag{3.11}$$

where $y_t = c_t^d + c_t^d + g_t + c_t^x + i_t^x$.[8]

3.3 Estimation

3.3.1 Data

In addition to the fifteen observable domestic and international macro-economic time series used to estimate the model in Chapter 2, two additional variables are now included. They are the yield on South African 10 year government bonds and M1 money supply. As before, the dataset spans the period from 2000Q1 to 2012Q4, which coincides with the inflation targeting regime of the South African Reserve Bank (SARB). Table 3.1 contains a summary of the data series used, as well as their respective sources.

Table 3.1: Observable variables

Variable	Series	Source
South Africa		
$\Delta\ln(\tilde{Y}_t)$	Real GDP	South African Reserve Bank
$\Delta\ln(\tilde{C}_t)$	Private consumption	
$\Delta\ln(\tilde{I}_t)$	Total fixed investment	
$\Delta\ln(\tilde{X}_t)$	Total exports	
$\Delta\ln(\tilde{M}_t)$	Total imports	
$\Delta\ln(\tilde{S}_t)$	Nominal effective exchange rate	
$\Delta\ln(\tilde{E}_t)$	Non-agricultural employment	
$\Delta\ln(\tilde{W}_t)$	Compensation of employees	
$\tilde{\pi}_t^i$	Fixed investment deflator	
$\tilde{R}_t$	Repo rate	
$\tilde{R}_{L,t}$	10 year government bond yield	
$\Delta\ln(\tilde{M}1_t)$	M1 money supply	
$\tilde{\pi}_t^c$	CPI inflation	StatsSA
$\tilde{\pi}_t^d$	PPI inflation, domestic manufacturing	
$\tilde{\bar{\pi}}_{t+1}^c$	Inflation target midpoint	Author's own calculations
Foreign economy		
$\Delta\ln(\tilde{Y}_t^*)$	Real GDP (trade weighted)	GPM, CEPREMAP
$\tilde{\pi}_t^*$	CPI inflation (trade weighted)	
$\tilde{R}_t^*$	Policy interest rates (trade weighted)	

[8] See the Appendix for the model's entire set of log-linearised equations.

3.3.2 Measurement issues

Based on the assumption that a proportion of the fluctuation in government bond yields are driven by exogenous factors (see Tobin, 1982) which cannot be explained by the macroeconomic structure of the model, the 10 year yield's measurement equation includes a measurement error $\eta_{L,t}$ as in De Graeve *et al.* (2009):

$$\tilde{R}_{L,t} = \ln(R_L) + \hat{R}_{L,t} + \eta_{L,t} \tag{3.12}$$

The standard deviation of the measurement error is calibrated such that 10 per cent of the variation in the 10 year government bond yield is accounted for by *exogenous* factors – a standard calibration for measurement errors.[9]

3.3.3 Calibration

As before, the model is estimated with Bayesian techniques, however, where necessary parameter values are calibrated. The calibrated parameter values are similar

Table 3.2: Calibrated parameters

β	Discount factor	0.9975	δ	Depreciation rate	0.025
A_L	Labour disutility constant	7.5	σ_L	Labour supply elasticity	5
σ_a	Capital utilisation cost	10	α	Capital share in production	0.23
ϑ_c	Consumption imports share	0.36	ϑ_i	Investment imports share	0.48
θ_w	Calvo: wage setting	0.69	κ_w	Indexation: wage setting	0.5
λ_w	Wage setting markup	1.05	λ_d	Domestic price markup	1.1
η_c	Subst. elasticity: consumption	1.5	η_i	Subst. elasticity: investment	1.5
η_f	Subst. elasticity: foreign	1.25	μ^z	Permanent technology growth	1.0085
π	Steady state inflation	1.0114	g_y	Government spending to GDP	0.197
ρ_g	Government spending persistence	0.815	π^*	Foreign inflation	1.005
Yield curve					
ϕ_L	Long bond adjustment cost	0.09	κ_L	Steady state L-bonds/money	0.2861

to Chapter 2, except for the two parameters related to the yield curve. Firstly, the parameter that determines the L-period bond adjustment cost, ϕ_L, is calibrated at 0.09 in order to match the 9.38 per cent sample mean of the 10 year government bond yield. It can be shown that in the steady state, while making use of the fact that $R = (\pi\mu^z)/\beta$, the first-order condition for L-period bond holdings in Equation (3.6) is:

$$R_L = R\left[1 + \phi_L\left(\frac{3}{2} - \beta\right)y\right]^{\frac{1}{L}}$$

Hence setting the adjustment cost parameter ϕ_L to 0.09 implies a steady state (annualised) 10 year government bond yield of 9.35 per cent and, in turn, a steady state

[9]The full set of 17 measurement equations are reported in the Appendix.

term premium of 45 basis points. Secondly, the steady state ratio of L-period bonds to money holdings, κ_L, is calibrated to reflect the historical average of outstanding government bonds with a maturity of 10 years and over as a share of the total.

3.3.4 Priors

The yield curve extension of the model implies that four additional parameters need to be estimated, *i.e.* when compared to the set of parameter estimates from Chapter 2. Firstly, the prior for the elasticity of substitution between money and bonds, ν_L, is assumed to follow a gamma distribution around a mean of 0.2 in Table 3.3. Marzo *et al.* (2008) calibrate this parameter to 0.5 for the US, while Zagaglia (2009) estimates it at 0.3 for the Euro Area. Secondly, the weight on money growth in the Taylor rule is assumed to be gamma distributed around a mean of 1.38 – Andrés *et al.*'s (2004) estimate for this parameter. Finally, the persistence of the AR(1) process for L-period bond supply is assumed to follow a beta distribution around 0.75, while the standard error of its shock has an inverse-gamma distribution with a mean of 1.65.[10]

The remaining parameters' prior means and densities are similar to Chapter 2.

3.3.5 Posterior estimates

The posterior estimation results are summarised in Table (3.3), while Figure (B.1) in the Appendix contains the prior and posterior distributions. From the posterior results it can firstly be seen that investment adjustment costs are substantially higher than the prior mean, which implies an elasticity of investment of around 0.1 to a one per cent change in the price of installed capital. At 0.828, the degree of habit formation is found to be higher than Adolfson *et al.* (2007*a*), but in-line with the estimate of Jääskelä and Nimark (2011) for Australia.

The Calvo parameter estimates indicate that import and export price contracts are generally reoptimised every 4 quarters, while domestic contracts are reoptimised at a lower frequency. The inflation indexation parameters are all estimated to be around 0.3 or lower, which implies that a higher weight is placed on the current inflation target than on past inflation during indexation. Although the posterior estimate of the risk premium elasticity ϕ_a is lower than its prior, the data nevertheless to some degree favours the endogenous persistence in the risk premium induced by ϕ_s.

Turning to the estimates for Taylor rule parameters, the posterior mean of 0.895 for the degree of interest rate smoothing closely matches Alpanda *et al.*'s (2010*b*) estimate of 0.916. It appears as if the SARB places a high weight on interest rate stabilisation. In addition, its reaction to the level of the output gap is more pronounced than what is indicated by the prior on this parameter. This result echoes the finding

[10] Using the total domestic marketable bonds of the South African government as an off-model proxy for bond supply, the magnitude of the prior mean for the standard deviation of the bond supply shock is loosely guided by the OLS residual standard error of an AR(1) process for this time-series.

Table 3.3: Priors and posterior estimation results

Parameter description		Prior			Posterior	
		Density[a]	Mean	Std. Dev.	Mean	90% interval
Adjustment costs						
ϕ_i	Investment	N	7.694	1.5	10.177	[8.020 ; 12.08]
Consumption						
b	Habit formation	B	0.65	0.1	0.828	[0.776 ; 0.884]
Calvo parameters						
θ_d	Domestic prices	B	0.715	0.05	0.872	[0.840 ; 0.904]
θ_{mc}	Imported consumption prices	B	0.675	0.1	0.814	[0.742 ; 0.882]
θ_{mi}	Imported investment prices	B	0.675	0.1	0.804	[0.747 ; 0.865]
θ_x	Export prices	B	0.675	0.1	0.620	[0.530 ; 0.717]
θ_E	Employment	B	0.675	0.1	0.795	[0.742 ; 0.849]
Indexation						
κ_d	Domestic prices	B	0.5	0.15	0.073	[0.029 ; 0.121]
κ_{mc}	Imported consumption prices	B	0.5	0.15	0.309	[0.133 ; 0.480]
κ_{mi}	Imported investment prices	B	0.5	0.15	0.263	[0.104 ; 0.412]
Exchange rate						
ϕ_a	Risk premium	IG	0.01	Inf	0.007	[0.003 ; 0.012]
ϕ_s	Modified UIP	U	0.5	[0,1]	0.211	[0.081 ; 0.336]
Taylor Rule						
ρ_R	Smoothing	B	0.8	0.05	0.895	[0.869 ; 0.923]
ϕ_π	Inflation	G	1.7	0.15	1.704	[1.472 ; 1.928]
$\phi_{\Delta\pi}$	Inflation (change)	G	0.3	0.1	0.281	[0.145 ; 0.423]
ϕ_y	Output gap	G	0.25	0.05	0.383	[0.292 ; 0.472]
$\phi_{\Delta y}$	Output gap (change)	G	0.125	0.05	0.130	[0.050 ; 0.212]
$\phi_{\Delta m}$	Money growth	G	1.38	0.27	0.391	[0.282 ; 0.507]
L-period bond						
ν_l	Money/bonds substitution	G	0.2	0.05	0.235	[0.158 ; 0.311]
Persistence parameters						
ρ_{μ^z}	Permanent technology	B	0.75	0.1	0.826	[0.748 ; 0.907]
ρ_ε	Transitory technology	B	0.75	0.1	0.916	[0.883 ; 0.952]
ρ_i	Investment technology	B	0.75	0.1	0.751	[0.653 ; 0.860]
$\rho_{\tilde{z}^*}$	Asymmetric technology	B	0.75	0.1	0.840	[0.698 ; 0.949]
ρ_c	Consumption preference	B	0.75	0.1	0.962	[0.929 ; 0.988]
ρ_H	Labour supply	B	0.75	0.1	0.341	[0.227 ; 0.446]
ρ_a	Risk premium	B	0.75	0.1	0.659	[0.543 ; 0.789]
ρ_{λ^d}	Imported cons. price markup	B	0.75	0.1	0.199	[0.130 ; 0.267]
$\rho_{\lambda^{mc}}$	Imported cons. price markup	B	0.75	0.1	0.754	[0.611 ; 0.898]
$\rho_{\lambda^{mi}}$	Imported invest. price markup	B	0.75	0.1	0.693	[0.531 ; 0.865]
ρ_{λ^x}	Export price markup	B	0.75	0.1	0.665	[0.494 ; 0.833]
ρ_L	L-period bond supply	IG	0.75	0.1	0.845	[0.739 ; 0.951]
Structural shocks						
σ_{μ^z}	Permanent technology	IG	0.4	Inf	0.286	[0.195 ; 0.372]
σ_ε	Transitory technology	IG	0.7	Inf	3.391	[2.041 ; 4.809]
σ_i	Investment technology	IG	0.4	Inf	0.353	[0.253 ; 0.456]
$\sigma_{\tilde{z}^*}$	Asymmetric technology	IG	0.4	Inf	0.404	[0.168 ; 0.600]
σ_c	Consumption preference	IG	0.4	Inf	0.126	[0.086 ; 0.166]
σ_H	Labour supply	IG	0.2	Inf	0.461	[0.355 ; 0.560]
σ_a	Risk premium	IG	0.5	Inf	1.672	[1.196 ; 2.163]
σ_d	Domestic price markup	IG	0.3	Inf	1.223	[1.002 ; 1.441]
σ_{mc}	Imported cons. price markup	IG	0.3	Inf	0.797	[0.576 ; 0.998]
σ_{mi}	Imported invest. price markup	IG	0.3	Inf	0.555	[0.268 ; 0.820]
σ_x	Export price markup	IG	0.3	Inf	1.635	[1.090 ; 2.153]
σ_R	Monetary policy	IG	0.15	Inf	0.159	[0.127 ; 0.188]
σ_L	L-period bond supply	IG	1.65	Inf	3.020	[2.214 ; 3.794]

[a] B – Beta, G – Gamma, IG – Inverse Gamma, N – Normal, U – Uniform

of Ortiz and Sturzenegger (2007) – that the SARB is more concerned about output than many of its emerging market peers. The parameter on money growth is substantially lower than the prior, which likely indicates the low weight that is placed on potential inflationary and asset market signals emanating from growth in money supply.

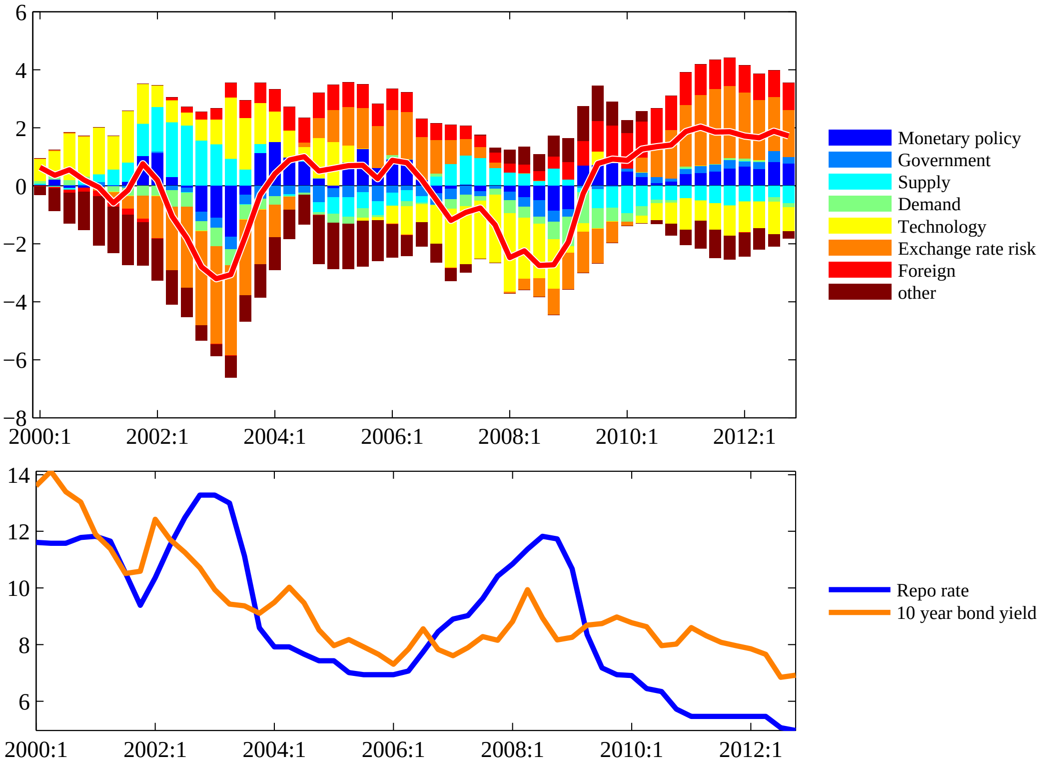
The estimates for the persistence of shocks indicate that consumption and transitory technology shocks are most persistent, while labour and imported consumption markup shocks are least persistent. The standard deviations of the innovations to these shocks vary substantially. Consistent with the heigh weight placed on interest rate stabilisation, monetary policy shocks exhibit low volatility. However, transitory technology and the supply of long-term bonds are the most volatile, with the latter possibly reflecting the volatility of government yields.

3.4 Historical shock decomposition of the 10 year yield spread

The historical evolution of the unobservable variables of the model, as well as the innovations to the structural shocks may be obtained through the Kalman filter. As such, an analysis of the contributions of these structural shocks to the model's measure of the slope of the yield curve – the spread between the 10 year government bond and the Repo rate – may shed some light on the model's interpretation of its historical evolution. For the sake of visual clarity in Figure 3.1, certain structural shocks have been grouped, as follows: *Government* represents shocks to government expenditure as well as L-period bond supply; *Supply* consists of domestic price markup shocks as well as imported consumption and investment markup shocks; *Demand* consists of shocks to consumption and investment; *Technology* includes both transitory and permanent technology shocks; and finally, *Foreign* includes shocks to foreign output, inflation and policy shocks.

In the context of the model, the inversion of the yield curve following the depreciation of the Rand in late 2001 was largely driven by exchange rate risk. In addition, the Repo rate was rising during 2002 and 2003, whilst the 10 year yield was declining, hence this policy tightening exacerbated the inversion, as can be seen from the contributions of monetary policy shocks over this period. Nevertheless, the decline in the 10 year yield most likely reflected the financial market's anticipation of future policy easing, which eventually commenced toward the second half of 2003. Hereafter, the positive slope of the yield curve during 2004 and 2005 was once again largely driven by a reduction in exchange rate risk, which kept CPI inflation low and facilitated expansionary monetary policy. At this point in time, global policy interest rates were mostly at very low levels too, which further contributed the steepening of the yield curve. During 2006 and 2007, adverse technology shocks were one of the main contributors to the flattening (and eventual inversion) of the yield curve. In addition, rising international oil prices are reflected

Figure 3.1: 10 year government bond yield spread: historical shock decomposition

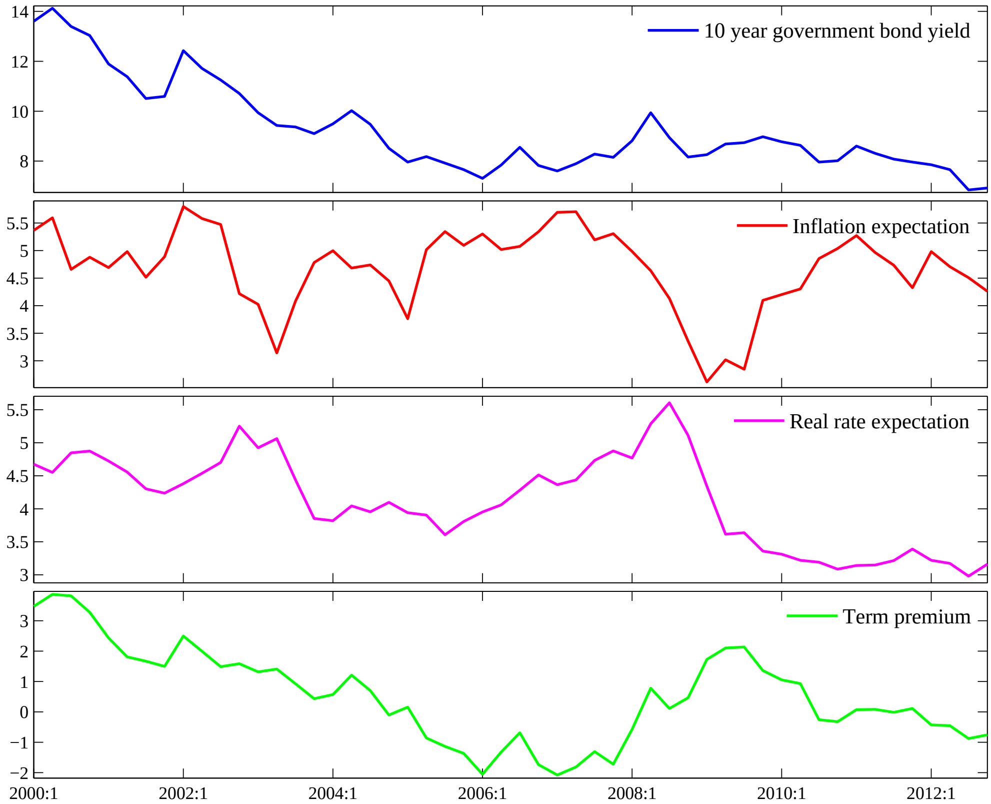


by the contribution of supply shocks over this period. At first glance, the positive contribution of these adverse supply shocks seems counter-intuitive. However, it appears as if the significant rise in inflation caused by supply shocks over this period did not only lead to a policy tightening at the short end, but also raised the long end of the yield curve. Hence the net effect entailed a steepening of the curve. At the inversion's peak in 2008Q2 and 2008Q3, monetary policy shocks were one of the main contributors. From the literature's point of view – where an inverted yield curve is associated with a future recession – a slowdown in economic activity was imminent. From late 2008 and onwards, the impact of the global financial crisis and its ensuing recession led to a rapid steepening of the yield curve. In the wake of the financial crisis, the model ascribes this change in direction of the slope to three highly intuitive shocks: monetary policy, global economic conditions and the eventual strengthening of the exchange rate.

3.5 The expectations hypothesis and the term premium

The rational expectations solution to the structural model allows for a decomposition of the yield on an L-period bond into various *unobserved* components of interest. Firstly, it may be expressed as the sum of an expected yield – as defined by

Figure 3.2: 10 year government bond yield: Expectation components and the term premium



the expectations hypothesis – and a term premium, *i.e.* the additional compensation investors require to bear the increasing interest rate risk associated with holding the longer-maturity bond. As mentioned before, the expectations hypothesis states that the bond yield of a given maturity reflects a weighted average of expected future short term rates over the period to maturity:

$$R^E_{L,t} = \frac{1}{L} E_t \left[R_t + R_{t+1} + R_{t+2} + ... + R_{t+L-1} \right] \tag{3.13}$$

where $R^E_{L,t}$ represents the bond yield that is consistent with the hypothesis. It then follows that for an L-period bond, the deviation in actual bond yields from the level determined by the expectations hypothesis represents the *term premium*, $\zeta^{TP}_{L,t}$:

$$R_{L,t} = R^E_{L,t} + \zeta^{TP}_{L,t}. \tag{3.14}$$

De Graeve *et al.* (2009) argue that the rigorous specification of the macroeconomy within a DSGE framework allows for a more rigorous modelling of the formation of expectations, as opposed to competing macro-finance models.

Secondly, Equation (3.13) may be expressed in Fisher-equation form:

$$R^E_{L,t} = \frac{1}{L} E_t \left[(r_t + \pi^c_{t+1}) + (r_{t+1} + \pi^c_{t+2}) + ... + (r_{t+L-1} + \pi^c_{t+L}) \right], \tag{3.15}$$

where r_t is the real interest rate. This formulation facilitates the decomposition of the yield on an L-period bond into three components: (1) a real interest rate expectation, (2) an expectation of inflation over the life of the bond, and (3) the term premium:

$$R_{L,t} = \frac{1}{L}\sum_{j=0}^{L-1} E_t r_{t+j} + \frac{1}{L}\sum_{j=0}^{L-1} E_t \pi^c_{t+j+1} + \zeta^{TP}_{L,t} \tag{3.16}$$

By applying Equation 3.16 to the South African 10 year yield on Government bonds, Figure 3.2 shows its decomposition into an expected inflation and real interest rate component as well as the term premium. After having largely fluctuated within the upper half of the inflation target band, 10-year inflation expectations briefly fell to below 3 per cent at the height of the global recession in early 2009. Since then it has risen toward the 5 per cent level. Similarly, real interest rate expectations fell from a pre-crisis average of between 4 and 5 per cent to its current level of around 3 per cent. These two components add up to the expected 10 year Repo rate average, and hence their levels toward the end of the sample point to an average Repo rate expectation of between $7\frac{1}{2}$ and 8 per cent.

Finally, the bottom panel in Figure 3.2 indicates that the model-implied term premium declined throughout the first seven years of the inflation targeting regime. This declining trend implies that holders of 10 year government bonds required less compensation for the risk associated with holding these longer term bonds as opposed to short term securities. However, from late 2007 – the build-up to the financial crisis – the term premium started rising again, and eventually peaked in mid 2009. This pattern largely corroborates Bernanke's (2006) view that the term premium is inversely related with expected future economic activity. As such, a declining term premium would indicate improving economic activity and *vice versa*.

3.6 Comparative dynamics

In order to analyse the dynamic reaction of the model, but more specifically the yield curve and its various sub-components, we discuss the impulse responses following an increase in the Repo rate, higher government spending, a temporary improvement in technology and an increase in the foreign policy rate.

3.6.1 Monetary policy

A 100 basis point shock to the Taylor rule in Figure B.2 of the Appendix raises the Repo rate by 85 basis points and the long bond rate by approximately 21 basis points on impact. Consequently, the yield spread narrows by the difference of these two impacts. Regarding the sub-components, both the term premium and expected L-period real interest rate increase, while the reaction of the expected inflation component is almost negligible. Nevertheless, expected inflation initially falls, but increases as CPI inflation is foreseen to overshoot in the medium to longer term. Similar to (Andrés *et al.*, 2004) a liquidity effect is evident as money holdings and interest rates move in opposite directions.

The dynamic reaction of the remaining macroeconomic variables is as expected. CPI inflation falls, with the year-on-year rate peaking at around -0.18 per cent after four quarters. Both the nominal and real exchange rate appreciate, domestic demand falls, as does net exports (in response to the stronger currency). Ultimately output declines to a low in excess of -0.4 per cent after four quarters, and the net foreign asset position of the economy deteriorates.

3.6.2 Government spending

A 5 per cent increase in government spending increases interest rates across the spectrum and reduces the yield spread. Almost half of the rise in the long bond rate in Figure B.3 is driven by the increase in the term premium. In addition, both the expected real interest rate and inflation components of the bond yield increase, while the higher interest rates induce a decline in the holdings of money. Moreover, in response to the higher interest rates, consumption falls and investment is crowded out, which in turn reduce inflation. Hence, the rise in output of almost 1 per cent is a direct effect of the increased government spending, which accounts for almost 20 per cent of GDP.

3.6.3 Technology

A 1 per cent improvement in transitory technology reduces the Repo rate as inflation falls while output increases in Figure B.4. The long bond rate also declines, but to a lesser extent than the Repo rate, which steepens the yield curve. Both expected real interest rates and inflation decline as a result of the lowering of interest rates and inflation. Money holdings decline, as the liquidity effect is most likely counteracted by the impact of lower inflation. The wealth effect brought about by the improvement in technology induces an increase in consumption, at the expense of lower investment. Both net exports and the net foreign asset position benefit from the depreciated real exchange rate, and ultimately the increase in output peaks at 0.4 per cent after 7 quarters.

3.6.4 Foreign monetary policy

Following a 100 basis point increase in the foreign policy rate in Figure B.5, the domestic exchange rate depreciates. This reaction has inflationary consequences which leads to an increase in the domestic policy rate by the central bank. The long-bond rate rises in response to the rising policy rate. Nevertheless, the slope of the yield curve flattens, as the magnitude of the Repo rate increase at the short-end of the curve exceeds the increase in the long-bond rate. Similar to the response in Chapter 2, the domestic economy slows down as a result of rising interest rates.

However, net exports benefit from the weaker exchange rate after 2 quarters, which in turn counters the slowdown in domestic demand to the extent that output ultimately increases.

3.7 Forecasting the yield curve

Forecasts of interest rates at various maturities do not only play a central role in policy formulation at central banks, but they are also used extensively in the private sector by large corporations and financial institutions, where the yield curve serves as a key input in the pricing of assets and the calculation reference rates for interbank lending, amongst others.

Against this background, the yield-curve extended DSGE model's value as a tool to forecast the South African 10 year government bond rate is assessed by comparing its forecasts to the Reuters poll of professional forecasters over the period 2006Q1 to 2012Q3. To this end, the model is re-estimated recursively every four quarters – once per year – where the first recursive estimation spans the sample 1993Q1 to 2005Q4, and the last is from 1999Q1 to 2011Q4. The model is then forecast 7 quarters ahead at each quarter.[11] Since the actual observations end in 2012Q4, there are 28 one-quarter-ahead and 22 seven-quarter-ahead forecast errors.

Accordingly, when these forecast errors from the yield-curve DSGE model (DSGE-Y hereafter) are compared to the corresponding errors of the Reuters consensus poll in Table 3.4, the relative RMSE statistics indicate that professional forecasts of the 10 year government bond yield are superior at the one-quarter horizon. However, from the second quarter onwards, the DSGE-Y model outperforms the Reuters poll.

Given the model's ability to compete with professional forecasts of the 10 year government bond yield, a natural extension of this forecasting exercise would be to establish whether the inclusion of long term bonds in the DSGE model structure affects the model's already established ability to forecast CPI inflation, GDP and the Repo rate. Hence, the remaining rows in Table 3.4 compare RMSE statistics of the DSGE-Y model's forecasts of these three macroeconomic variables to both the DSGE model developed in Chapter 2, as well as to professional forecasts polled by Reuters. When comparing the CPI inflation forecasts (year-on-year), the results are mixed. Although the DSGE-Y model is superior to Reuters forecasts after 4 quarters, when compared to the DSGE model of Chapter 2, the DSGE-Y model still fares worse over the first three quarters of the forecasting horizon, as well as at quarters 6 and 7. GDP growth forecasts (quarter-on-quarter, annualised) from the DSGE-Y model outperform the Reuters poll of professional forecasters after 2 quarters, while the DSGE model in Chapter 2 was only superior to the Reuters poll at quarters 6 and 7 of the forecast horizon. When the GDP growth forecasts of the DSGE-Y model are compared to the model of Chapter 2, the DSGE-Y model is superior at all 7 forecast horizons. Finally, when comparing the models' forecasts

[11] The Reuters poll of consensus forecasts covers a seven quarter horizon.

Table 3.4: Forecasting performance of DSGE model with yield curve extension

	Quarters ahead						
Relative RMSE statistics	1	2	3	4	5	6	7
10 government bond yield							
DSGE-Y / Reuters	1.569	**0.988**	**0.902**	**0.994**	**0.914**	**0.788**	**0.746**
CPI inflation, year-on-year							
DSGE / Reuters	1.549	1.066	**0.980**	1.029	**0.913**	**0.798**	**0.704**
DSGE-Y / Reuters	1.934	1.208	1.023	1.012	**0.879**	**0.832**	**0.827**
DSGE-Y / DSGE	1.248	1.133	1.043	**0.984**	**0.963**	1.043	1.176
GDP growth (quarter-on-quarter, annualised)							
DSGE / Reuters	1.520	1.528	1.200	1.078	1.012	**0.991**	**0.945**
DSGE-Y / Reuters	1.413	1.205	**0.929**	**0.892**	**0.871**	**0.914**	**0.847**
DSGE-Y / DSGE	**0.929**	**0.789**	**0.774**	**0.827**	**0.861**	**0.922**	**0.897**
Repo rate							
DSGE / Reuters	3.203	2.092	1.447	1.253	1.166	1.096	1.052
DSGE-Y / Reuters	3.049	1.882	1.312	1.137	1.066	1.039	1.071
DSGE-Y / DSGE	**0.952**	**0.900**	**0.906**	**0.907**	**0.914**	**0.947**	1.019

of the Repo rate, neither the model from Chapter 2 nor the DSGE-Y are able to improve on the Reuters forecasts. Nevertheless, a direct comparison between the two models indicates that Repo rate forecasts from the DSGE-Y model are superior over the first six quarters of the forecast horizon.

3.8 Do the yield spread and term premium predict future GDP growth

During the last three decades an immense body of literature has developed around this central question: does the term spread predict changes in economic activity?[12] Some have focused on the term spread's ability to forecast output growth, and even more have estimated its ability to predict actual turning points in the business cycle, more specifically recessions. The consensus result has been convincing: it does.

A number of studies that have considered the capacity of the yield spread in predicting output growth – such as Estrella and Hardouvelis (1991), Estrella and Mishkin (1997) and Haubrich *et al.* (1996) – have largely used the following specification:

$$\ln(Y_{t+4} - Y_t) \quad = \quad \alpha_0 + \alpha_1 \ln(Y_t - Y_{t-4}) + \alpha_2(R_{L,t} - R_t) + \epsilon_t, \tag{3.17}$$

[12]See Wheelock and Wohar (2009) for a comprehensive survey.

where $\ln(Y_{t+4} - Y_t)$ is the year-on-year change in GDP, four quarters ahead, and $R_{L,t} - R_t$ is the yield spread, i.e., the current difference between the long and short rate. Generally, α_2 is found to be both significant and positive, leading to the inference that an upward-sloping yield curve precedes improved economic growth, while an inverted yield curve tends to be followed by an economic slowdown.

However, since the long-term rate also includes a term premium, movements in the yield spread may originate from either changes in expected future short-term rates, or a changing term premium. This becomes clear when rewriting Equation (3.14) in terms of the yield spread:

$$R_{L,t} - R_t = \left(R^E_{L,t} - R_t\right) + \zeta^{TP}_{L,t}. \tag{3.18}$$

Whether or not the term premium in itself contains any predictive ability with respect to GDP, its sign has been a contentious issue. A case in point would be Greenspan's (2005) famous bond yield "conundrum". From June 2004 to February 2005, the Federal Open Market Committee (FOMC) had increased the federal funds rate by 150 basis points, while United States 10 year government bond yields remained almost unchanged over this period. This sideways trend in US 10 year government bond yields continued throughout the eventual 425 basis points increase in the federal funds rate. It was later believed that a declining term premium was the culprit in the conundrum, masking the increase in expected future short rates that would have followed the rising federal funds rate under normal circumstances. Bernanke (2006) contended that the impact of a declining term premium would be "stimulative", and as such, from a practitioner's point of view its coefficient should exhibit a negative sign with respect to economic activity. One of the first studies that investigated the individual predictive ability of the term premium was Hamilton and Kim (2002). The authors achieved this by substituting Equation (3.18)'s expression for the yield spread into Equation (3.17), as follows:

$$\ln(Y_{t+4} - Y_t) = \alpha_0 + \alpha_1 \ln(Y_t - Y_{t-4}) + \alpha_2 (R^E_{L,t} - R_t) + \alpha_3 \zeta^{TP}_{L,t} + \epsilon_t, \tag{3.19}$$

where $\alpha_2 = \alpha_3$. By allowing $\alpha_2 \neq \alpha_3$, it then becomes possible to test the individual predictive power of each of the two subcomponents of the yield spread. Hamilton and Kim (2002) find that both α_2 and α_3 are statistically significant and positive. Hence, Bernanke's (2006) view that a declining term premium would stimulate economic activity contradicted Hamilton and Kim's (2002) finding of a positive α_3. Moreover, to intensify the lack of consensus, Ang *et al.* (2006) also isolated the term premium, but found that it does not bear any predictive power in the context of Equation (3.19).

A slightly different approach is taken by Rudebusch *et al.* (2007). The authors argue that the term premium is nearly nonstationary. As such, by respecifying Equation (3.19) in terms of differences, as follows:

$$\begin{aligned} \ln(Y_{t+4} - Y_t) &= \alpha_0 + \alpha_1 \ln(Y_t - Y_{t-4}) + \alpha_2 \left[(R^E_{L,t} - R_t) - (R^E_{L,t-4} - R_{t-4})\right] \\ &+ \alpha_3 \left(\zeta^{TP}_{L,t} - \zeta^{TP}_{L,t-4}\right) + \epsilon_t, \end{aligned} \tag{3.20}$$

they find that the change in the term premium does predict future economic activity. In addition, α_3 is found to be negative, which adds credence to the view of Bernanke (2006) that the declining term premium in the US would stimulate economic activity.

Table 3.5: Predictive power of the South African term spread

	(1)	(2)	(3)
$\ln(Y_t - Y_{t-4})$	0.2 [2.03]**	-0.08 [-0.79]	0.10 [0.67]
$R_{L,t} - R_t$	0.38 [2.05]**		
$R^E_{L,t} - R_t$		0.48 [2.33]**	
$\zeta^{TP}_{L,t}$		-0.16 [-0.95]	
$(R^E_{L,t} - R_t) - (R^E_{L,t-4} - R_{t-4})$			0.30 [4.54]***
$\zeta^{TP}_{L,t} - \zeta^{TP}_{L,t-4}$			-0.67 [-2.51]**

The coefficients' corresponding Newey-West t-statistics are reported in parentheses, where *,** and *** indicate 90, 95 and 99 per cent confidence intervals.

South African literature has to date largely focused on the predictive ability of the aggregate yield spread and not its sub-components. Having derived a measure of the term premium for the South African 10 year government bond yield, a natural extension is to test whether these results for the US term premium also hold for the South African economy. Table 3.5 contains the estimated coefficients for Equations (3.17), (3.19) and (3.20) in columns (1) and (2) and (3), respectively.

From column (1) it is evident that – as has been proven before – the standard result holds: the South African yield spread is positively correlated with future economic activity. Column (2), which contains the estimated coefficients for the subcomponents of the yield spread, mirrors the findings of both Ang *et al.* (2006) and Rudebusch *et al.* (2007) for the US: the *level* of the South African term premium has no predictive power. However, when the *change* in the term premium is considered, it is found to be a significant predictor of future output, and its negative sign accords with the view of Bernanke (2006) and the finding of Rudebusch *et al.* (2007).

3.9 Conclusion

The rational expectations solution to the New Keynesian DSGE model provides a credible and consistent characterisation of the expectations formation process - a key component of yield curve dynamics. Moreover, the theoretical rigour of these models have also made them a highly desirable tool with which to analyse the macroeconomic dynamics that are driving developments in the yield curve. As a result, this chapter extends the standard New Keynesian DSGE model framework to incorporate long-term government bond yields, in order to analyse the macroeconomic forces that have shaped the yield spread between South African 10 year government bonds and the Repo rate over the inflation targeting regime of the SARB. Shocks to the exchange rate, technology and monetary policy were found to be some of the key drivers of yield curve dynamics over this period. In addition, the model's forecasts of the 10 year government yield are compared to a Reuters poll of professional forecasters and found to be superior after the one quarter forecast horizon. Finally, the model allows for a decomposition of the yield spread into an expected component and a term premium. This decomposition facilitates the investigation of the predictive power of the individual subcomponents of the yield spread with respect to future economic activity. Although the sign and significance of the term premium's predictive ability has been contentious, changes in the term premium are found to be a significant predictor of future economic activity in South Africa.

Chapter 4

Monetary policy and financial shocks in an empirical small open-economy DSGE model

4.1 Introduction

During the build-up to the global financial crisis, most macroeconomic models (especially those used for forecasting by central banks[1]) had to a large extent excluded the financial sector. The models of the day generally had only one interest rate, which was the policy rate, and fluctuations of actual market interest rates around the policy rate were not accounted for, as the role of financial intermediation was deemed to be irrelevant for the transmission of monetary policy (Blanchard *et al.*, 2010). Up until that point, developments in modern macroeconomics and finance had been largely disjointed, and, as a result, policymakers were neither able to coherently assess nor fully comprehend the macroeconomic implications of the financial instability induced by the crisis.[2]

Fortunately, recent years have seen both significant interest and progress in closing this chasm between modern macroeconomics and finance.[3] As an example within the DSGE literature, Goodfriend and McCallum (2007) adapt the standard New Keynesian framework by assuming that households need to borrow from banks in order to consume. The banking sector is modelled as a Cobb-Douglas loan "production function" with factor inputs being a combination of collateral and loan monitoring. Collateral is represented by the effective value of capital owned by households, and the loan monitoring is done by the proportion of labour being supplied to the banking sector by households. Similarly, Christiano *et al.* (2010) augment the standard DSGE framework to include a financial sector, where banks pay interest on household deposits, and in turn use these deposits in order to provide loans to firms and entrepeneurs. These studies assume that banks operate in a perfectly competitive environment and are therefore not able to set loan and deposit rates. More recent studies, like Andrés and Arce (2012), Aslam and Santoro (2008), Gerali *et al.* (2010) allow for monopolistic competition within the banking sector, which allows banks to set their deposit and loan rates. In addition, the loan dynamics are further enriched as households accumulate housing stock, which also serves as collateral when borrowing.

With regard to the monetary policy transmission mechanism, the studies discussed above accommodate two competing effects for the role played by the banking sector in the transmission of monetary policy: a "banking accelerator" effect; and a "banking attenuator" effect. The former follows from the notion that expansionary monetary policy stimulates employment and output, and hence the value of collateral in the economy. The rise in the value of collateral leads to a fall in the lending premium and therefore raises the demand for loans. The attenuator effect has the opposite impact, as banks need to increase their employment to meet the rise in loan demand. An increased wage bill translates into higher marginal costs on behalf of the bank, which raises the lending premium and therefore counteracts the

[1] See Tucker (2009).

[2] See Blanchflower (2009).

[3] See Cochrane (2006) for a compilation of studies that focus on the "intersection" of macroeconomics and finance.

initial expansionary effect. Goodfriend and McCallum (2007) find that for reasonable calibrations, either effect may dominate, and banks may subsequently either amplify or dampen monetary policy shocks when compared to a benchmark model without a banking sector.

Turning from the role of the financial sector in amplifying monetary policy shocks, the global financial crisis of 2008 ignited a rather different debate: the role of monetary policy in curtailing the impact of financial shocks. The aftermath of the crisis saw central banks across the globe reducing their policy rates by unprecedented margins. At the same time, commercial banks were increasing their lending rates in order to protect their crisis-induced fragile balance sheets. To a large extent, these opposing reactions reduced the efficacy of standard expansionary monetary policy in accommodating the substantial decline in aggregate demand seen at the time. As a result, the appropriate reaction of monetary policy to the increase in lending rates, or rather widening credit spreads, was at the centre of attention. Both McCulley and Toloui (2008) and Taylor (2008) suggested that the central bank follow a Taylor rule that yields a one-for-one reduction in the policy rate in response to increases in credit spreads. However, from the vantage point of a structural model, Cúrdia and Woodford (2010) found that a less than one-for-one reduction would be optimal. In their model, households are assumed to be either borrowers or savers, which creates a role for financial intermediation as banks take deposits from saving households, convert these deposits to loans, and then lend them to borrowing households at a spread over the deposit rate. However, it is assumed that a proportion of loans are not repaid in the end, and that the spread is an increasing function of this proportion of non-performing loans. Hence, the financial shock originates from an increase in non-performing loans which then leads to higher credit spreads.

To date, the majority of the literature that has aimed to incorporate financial frictions into DSGE models has been within the context of a closed economy, where parameters have mostly been calibrated. This chapter incorporates the Cúrdia and Woodford (2010) framework into the small open-economy model developed in Chapter 2. As an additional contribution to the existing literature, the model parameters are estimated using a dataset of 17 observable variables, which includes two variables that are specifically related to financial intermediaries: the effective lending rate and the ratio of non-performing loans to total assets of the South African banking sector. The suggestions of McCulley and Toloui (2008), Taylor (2008) and Cúrdia and Woodford (2010), *i.e.* that the central bank should respond to changes in credit spreads, are then analysed within the small open-economy context of South Africa. This is done through a loss-function comparison where the central bank includes credit spread deviations in a Taylor rule setting, as opposed to following the standard Taylor rule that focuses only on inflation and the output gap.

The remainder of the chapter is laid out as follows: Section 2 discusses the inclusion of a banking sector in the existing model. Thereafter, issues relating to the data as well as the estimation results are discussed in Section 3. The macroeconomic reaction to a financial shock is portrayed in Section 4, while the optimal response to such is shock is calculated in Section 5. Finally, Section 6 concludes.

4.2 The model

The general structure of the model largely builds on the small open economy model of Chapter 2. However, in order to analyse the role of monetary policy in the face of financial disturbances, two key extensions to the model are introduced: (1) heterogeneous households; and (2) financial intermediaries. These extensions broadly follow Cúrdia and Woodford (2009, 2010) where, based on their differing degrees of impatience to consume, households are classified as either savers or borrowers. In turn, this heterogeneity creates a role for financial intermediation in the model.

4.2.1 Heterogeneous households

The economy is populated by a mass of savers s and borrowers b, where the marginal utility of savers with respect to consumption is assumed to be lower than that of borrowers. In addition, the spending by these households in every period is allowed to differ from their income. As such, *savers* may either purchase risk-free government bonds or deposit funds at the financial intermediary if their income were to exceed their expenditure, while *borrowers* may borrow funds from the intermediary if their expenditure were to exceed their income. In turn, government bonds and deposits are remunerated at the prevailing gross policy rate R_t, while funds are borrowed at the gross lending rate R_t^b, where $R_t^b > R_t$. Furthermore, it is assumed that the households' types (*i.e.* saver or borrower) may change over time. Accordingly, their types evolve as two-state Markov chains, where in every period an event occurs with probability $1-\chi$ that renders the household eligible for the draw of a new type. At the draw, type s is drawn with probability p_s and type b with probability p_b, where $p_s + p_b = 1$. More specifically, χ is set to 0.975 and $p_s = p_b = 0.5$, such that there is an equal share of savers and borrowers and, on average, a household is expected to be eligible for the draw of a new type once every 10 years.

It is assumed that households maximise their expected discounted utility as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u^{\tau_t(i)} (C_t(i); \xi_t^c) - v^{\tau_t(i)} \left(h_t(i); \xi_t^h \right) \right], \tag{4.1}$$

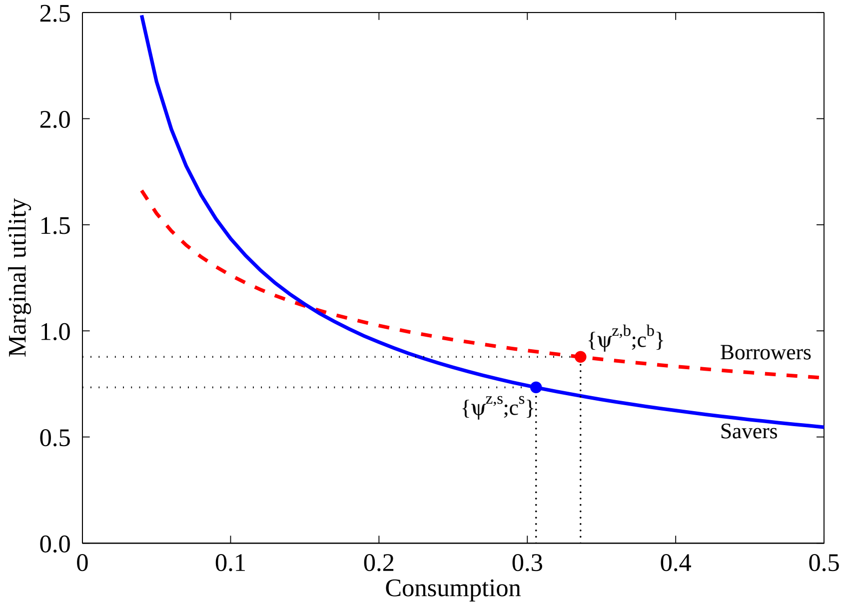
where $\tau_t(i) \in \{s, b\}$, while the utility derived from consumption and disutility from supplying labour are respectively given by:

$$u^{\tau_t(i)}(C_t(i); \xi_t^c) \equiv \xi_t^c \frac{\left(C_t(i) - b_\tau C_{t-1}^\tau \right)^{1-\sigma_\tau}}{1-\sigma_\tau} \quad \text{and} \quad v^{\tau_t(i)}(h_t(i); \xi_t^h) \equiv \xi_t^h A_L \frac{h_t(i)^{1+\sigma_L}}{1+\sigma_L}. \tag{4.2}$$

The degree of habit formation is denoted by b_τ, σ_τ is the (inverse) intertemporal elasticity of substitution, A_L pins down labour supply in the steady state and σ_L

represents the (inverse) Frisch elasticity[4]. Moreover, it is assumed that $\sigma_s > \sigma_b$ and $b_s > b_b$, such that savers' expenditure is less sensitive to changes in interest rates than that of borrowers. This further implies that in equilibrium the marginal utility of consumption for savers is less than for borrowers, as can be seen in Figure 4.1.[5] Finally, the aggregate preference shocks ξ_t^c and ξ_t^h in Equation (4.2) affect the preferences of both type s and b households.[6]

Figure 4.1: Marginal utilities of consumption



Let $A_t(i)$ denote the beginning-of-period domestic net financial wealth of household i, as follows:

$$A_t(i) = [B_{t-1}(i)]^{+} R_{t-1} + [B_{t-1}(i)]^{-} R^b_{t-1} + \Pi_t^{int} \tag{4.3}$$

where B_{t-1} is its (domestic) net financial wealth at the end of period $t-1$, $[B]^{+} \equiv \max(B,0)$ and $[B]^{-} \equiv \min(B,0)$, such that positive asset balances are remunerated

[4]The functional form of the utility function differs from Cúrdia and Woodford (2009) with respect to habit formation. In their specification, household i's consumption does not depend on a measure of lagged consumption. The specification chosen here is consistent with Chapter 2 and assumes that household's exhibit habit formation with respect to the aggregate consumption of their current type in $t-1$.

[5]Figure 4.1 is derived from the parameter values in Section 4.3.

[6]In order to ensure that households' expected marginal utilities of income do not diverge as a result of their differing type histories, it is assumed that households are able to sign state-contingent insurance contracts with one another which insures them against the risks (aggregate and idiosyncratic) associated with the random draw of a new type. However, households may only receive these insurance transfers periodically. For convenience, it is assumed that households may receive these transfers coincides with them becoming eligible for the draw of a new type. In addition, the fact that households have access to this insurance contract facilitates model aggregation.

at the gross policy rate R_{t-1}, while the (gross) borrowing rate R_t^b applies to negative balances. Hence, if D_t and B_t^g denote aggregate deposits and risk-free government bonds at the end of period t, while L_t denotes aggregate borrowing from financial intermediaries, then from Equation (4.3) it follows that:

$$D_t + B_t^g = \int_{\mathcal{S}_t} A_t(i)di \quad \text{and} \quad L_t = -\int_{\mathcal{B}_t} A_t(i)di, \tag{4.4}$$

where $\mathcal{S}_t$ and $\mathcal{B}_t$ represent the sets of households for whom $A_t(i) \geq 0$ and $A_t(i) < 0$, respectively. Moreover, households are assumed to be the owners of financial intermediaries and, as a result, the profits from this sector, Π_t^{int}, are distributed equally among all households in Equation (4.3).

In addition to domestic financial assets, households may also invest in foreign risk-free bonds, B_t^*. However, as in Benigno (2009), the interest rate on the foreign bond is subjected to a risk premium that is an increasing function of the domestic economy's indebtedness in the international asset market, as measured by its net foreign asset position:

$$a_t^* \equiv \frac{S_t B_t^*}{z_t P_t^d}, \tag{4.5}$$

where S_t is the nominal exchange rate, z_t represents the economy-wide real stochastic trend, and P_t^d is the domestic price deflator. Schmitt-Grohé and Uribe (2003) show that the inclusion of this debt-elastic risk premium is crucial for the determination of a well-defined steady state in small open economy models. Consequently, it is assumed that the risk premium has the following functional form

$$\Phi(a_t^*, \tilde{\phi}_t) = \exp\left\{-\tilde{\phi}_a(a_t - a) + \tilde{\phi}_t\right\}. \tag{4.6}$$

where $\tilde{\phi}_t$ represents an AR(1) shock to the risk premium, while in the steady state, the risk premium has the property $\Phi(0,0) = 1$.[7]

Given the above exposition of its asset holdings, the household budget constraint may be formulated as follows:

$$\begin{aligned} B_t(i) + S_t B_t^*(i) &= A_t(i) + S_t B_{t-1}^*(i) R_{t-1}^* \Phi\left(a_{t-1}^*, \tilde{\phi}_{t-1}\right) + W_t(i)h_t(i) + R_t^k K_{t-1}(i) \\ &- P_t^c C_t(i) - P_t^i I_t(i) + \Pi_t - T_t. \end{aligned} \tag{4.7}$$

Accordingly, households have at their disposal their beginning-of-period financial wealth which, depending on their type history, may be positive or negative, as well as foreign bonds. In addition, they earn wages $W_t(i)$ and return R_t^k on the labour and capital they supply to firms, as well as profits Π_t from firm ownership. Their income, combined with their beginning-of-period financial wealth, enables them to

[7] For simplicity it is assumed that only savers participate in international capital markets, which in turn ensures that uncovered interest rate parity (UIP) holds between domestic and foreign policy rates only.

purchase nominal consumption and investment goods and pay lump sum taxes. If the household is of type s, its resources may exceed its expenditure in period t, and it will either deposit the difference with the financial intermediary, purchase a risk-free domestic government bond, a foreign bond, or all of the above. If the household is of type b, its expenditure may exceed its resources, and it will borrow the difference from the financial intermediary. In addition, households are assumed to own the capital stock $K_t(i)$, and given their investment decision, the aggregate capital stock accumulates as follows:

$$K_t = (1-\delta)K_{t-1} + \xi_t^i I_t - S\left(\frac{I_t}{K_{t-1}}\right), \tag{4.8}$$

where δ is the rate of capital depreciation, while

$$S\left(\frac{I_t}{K_{t-1}}\right) = \frac{\phi_k}{2}\left(\frac{I_t}{K_{t-1}} - \delta^*\right)^2 K_{t-1}, \tag{4.9}$$

and $\delta^* = \frac{I}{K}$ such that in steady state, $S(\cdot) = S'(\cdot) = 0$ and $S''(\cdot) \equiv \phi_k$, with $\phi_k > 0$.[8] Moreover, similar to Greenwood *et al.* (1988), ξ_t^i is an investment specific technology shock that follows an AR(1) process.

Optimality conditions Optimisation of the household's utility function, Eq. (4.1), subject to the budget constraint and capital's law of motion, Eqs. (4.7) and (4.8), yields the following set of first-order conditions with respect to each of the choice variables for households of type $\tau \in \{s, b\}$:

Consumption, c_t^τ

$$\xi_t^c\left(c_t^\tau - b_\tau c_{t-1}^\tau \frac{1}{\mu_t^z}\right)^{-\sigma_\tau} - \beta b_\tau E_t \xi_{t+1}^c\left(c_{t+1}^\tau \mu_{t+1}^z - b c_t^\tau\right)^{-\sigma_\tau} - \psi_t^{z,\tau}\frac{P_t^c}{P_t^d} = 0 \tag{4.10}$$

Investment, i_t^τ

$$P_t^{k',\tau}\left[\xi_t^i - \phi_k\left(\frac{i_t^\tau}{k_{t-1}^\tau}\mu_t^z - \delta^*\right)\right] - \frac{P_t^i}{P_t^d} = 0 \tag{4.11}$$

Capital stock, k_t^τ

$$\begin{aligned}&\beta\chi E_t \psi_{t+1}^{z,\tau} P_{t+1}^{k',\tau}\left[(1-\delta) - \frac{\phi_k}{2}\left(\frac{i_{t+1}^\tau}{k_t^\tau}\mu_{t+1}^z - \delta^*\right)^2 + \phi_k\left(\frac{i_{t+1}^\tau}{k_t^\tau}\mu_{t+1}^z - \delta^*\right)\frac{i_{t+1}^\tau}{k_t^\tau}\mu_{t+1}^z\right] \\ &-\psi_t^{z,\tau} P_t^{k',\tau}\mu_{t+1}^z + \beta\chi E_t \psi_{t+1}^{z,\tau} r_{t+1}^k + \beta \sum_{\tau' \in \{b,s\}} (1-\chi) p_{\tau'} \psi_{t+1}^{z,\tau'} f_t = 0\end{aligned} \tag{4.12}$$

[8] The functional form of the capital adjustment cost function follows Avdjiev (2011), which builds on earlier work by Hayashi (1982) and Abel and Blanchard (1983), amongst others.

where

$$f_t = r^k_{t+1} + P^{k',\tau'}_{t+1}\left[(1-\delta) - \frac{\phi_k}{2}\left(\frac{i^{\tau'}_{t+1}}{k^{\tau'}_t}\mu^z_{t+1} - \delta^*\right)^2 + \phi_k\left(\frac{i^{\tau'}_{t+1}}{k^{\tau'}_t}\mu^z_{t+1} - \delta^*\right)\frac{i^{\tau}_{t+1}}{k^{\tau'}_t}\mu^z_{t+1}\right] \quad (4.13)$$

Borrowing, l_t

$$-\psi^{z,b}_t + \beta E_t\left[\frac{R^b_t}{\mu^z_{t+1}\pi_{t+1}}\left\{[\chi + (1-\chi)p_b]\psi^{z,b}_{t+1} + (1-\chi)p_s\psi^{z,s}_{t+1}\right\}\right] = 0 \quad (4.14)$$

Domestic deposit and bond holdings, $(d_t + b_t)$

$$-\psi^{z,s}_t + \beta E_t\left[\frac{R_t}{\mu^z_{t+1}\pi_{t+1}}\left\{(1-\chi)p_b\psi^{z,b}_{t+1} + [\chi + (1-\chi)p_s]\psi^{z,s}_{t+1}\right\}\right] = 0 \quad (4.15)$$

Foreign bond holdings, b^*_t

$$-\psi^{z,s}_t + \beta E_t\left[\frac{S_{t+1}}{S_t}\frac{R^*_t\Phi(a^*_t,\tilde{\phi}_t)}{\mu^z_{t+1}\pi_{t+1}}\left\{(1-\chi)p_b\psi^{z,b}_{t+1} + [\chi + (1-\chi)p_s]\psi^{z,s}_{t+1}\right\}\right] = 0 \quad (4.16)$$

where all trending variables have been rendered stationary, as represented by their lower case counterparts, and $\psi^{z,\tau}_t = z_t P^d_t \upsilon^\tau_t$ is the stationary Lagrange multiplier. In addition, the log-linearised combination of the first-order conditions for domestic assets and foreign bond holdings, Eqs. (4.15) and (2.61), yield the UIP condition

$$\hat{R}_t - \hat{R}^*_t = E_t\hat{S}_{t+1} - \hat{S}_t - \tilde{\phi}_a\hat{a}_t + \hat{\tilde{\phi}}_t, \quad (4.17)$$

such that an increase (decrease) in the net foreign asset position of the domestic economy – *ceteris paribus* – leads to an appreciation (depreciation) of its currency.

Evolution of household borrowing At the beginning of every period, a fraction χ of borrowers are not eligible for the draw of a new type and hence they remain borrowers with existing real debt to the value of $\chi l_{t-1}R^b_{t-1}/(\mu^z_t\pi^d_t)$. From the fraction $1-\chi$ of borrowers who are in the draw, p_b remain borrowers as well, with existing debt that amounts to $(1-\chi)p_b l_{t-1}R^b_{t-1}/(\mu^z_t\pi^d_t)$. In turn, a fraction $(1-\chi)$ of savers became eligible for the draw of a new type, where they learn that they are now type b. These new borrowers, who were savers in period $t-1$, own assets to the value of $(1-\chi)p_b[(d_{t-1} + b^g_{t-1})R_t + S_t b^*_{t-1}R^*_{t-1}\Phi(\cdot)]/(\mu^z_t\pi^d_t)$. As a result, end-of-period borrowing is given as:

$$\begin{aligned} b_t &= [\chi + (1-\chi)p_b]l_{t-1}R^b_{t-1} - (1-\chi)p_b[(d_{t-1} + b^g_{t-1})R_t + S_t b^*_{t-1}R^*_{t-1}\Phi(\cdot)]/(\mu^z_t\pi^d_t) \\ &+ p_b\left[\gamma^{c,d}_t c^b_t + \gamma^{i,d}_t i^b_t - w^b_t h^b_t - r^k_t\frac{k^b_{t-1}}{\mu^z_t} - \Pi^r_t + \tau_t\right] \end{aligned} \quad (4.18)$$

where $\Phi(\cdot) = \Phi\left(a^*_{t-1}, \tilde{\phi}_{t-1}\right)$, while $\gamma^{c,d}_t = P^c_t/P^d_t$ and $\gamma^{i,d}_t = P^i_t/P^d_t$

4.2.2 Financial intermediaries

Financial intermediaries take real deposits d_t from households and convert them into real loans l_t. However, the intermediary makes provision for the fact that a fraction $\zeta_t(l_t)$ of loans will not be repaid. As such, the period t real profits of financial intermediaries may be expressed as:

$$\Pi_t^{int,r} = d_t - l_t - \zeta_t(l_t), \tag{4.19}$$

where $\zeta_t(l_t) = \zeta_t l_t^{1+\eta_\zeta}$. Moreover, it is assumed that the activities of financial intermediaries are confined to the domestic economy. This assumption obviously simplifies the analysis, but it is also justified by the relatively low foreign currency exposure of the South African banking sector, where foreign currency deposits as a ratio to total liabilities averaged 4.5 per cent from January 2008 to December 2012, while the ratio of foreign currency loans to total assets averaged 5.8 per cent over the same period.[9]

Although loans extended in period t had a value of $l_t + \zeta_t(l_t)$, eventual repayment of these loans in $t+1$ will only amount to $l_t R_t^b$, which equals the remuneration on deposits of $d_t R_t$. Let the gross spread by which the financial intermediary sets the loan rate be denoted by ω_t, such that

$$R_t^b = \omega_t R_t. \tag{4.20}$$

Assuming financial intermediaries are able to lend at the spread ω_t, optimising profits in Equation (4.19) by choosing l_t yields the following first order condition for the gross spread:

$$\omega_t = 1 + (1+\eta_\zeta)\zeta_t l_t^{\eta_\zeta} + \eta_\Theta \Theta_t l_t^{\eta_\Theta - 1}. \tag{4.21}$$

Accordingly, the magnitude of the spread between the loan and deposit rate is an increasing function of both the rate of non-performing loans ζ_t, and the volume of loans l_t (when $\eta_\Theta > 0$). Here the positive role played by borrowing in the determination of the credit spread reflects the additional resource cost incurred by the financial intermediary when lending volumes increase. Hence, increased lending activity implies a greater need for loan origination and monitoring, which, in turn, are costly for the financial intermediary. Moreover, while Cúrdia and Woodford (2009) assume that ζ_t follows an exogenous process, in this paper it is assumed that non-performing loans are a function of real economic conditions:

$$\zeta_t = \zeta_{t-1}^{\rho_\zeta} y_t^{-\theta_\zeta} \varepsilon_{\zeta,t}, \tag{4.22}$$

where $\theta_\zeta > 0$ and $\varepsilon_{\zeta,t}$ is an exogenous shock. The link between loan performance and economic activity has been well documented in the literature.[10] During economic downturns, the balance sheets of borrowers are adversely affected by falling

[9] These ratios were taken from the South African Reserve Bank's BA900 returns.

[10] See, for instance, Beck *et al.* (2013), Glen and Mondragón-Vélez (2011) and Nkusu (2011).

asset prices and rising unemployment. These impaired balance sheets affect the ability of borrowers to repay their loans which, in turn, leads to an increase in non-performing loans on the balance sheets of financial intermediaries. Moreover, the inclusion of this link between non-performing loans and real economic activity introduces the so-called adverse feedback loop into the model, whereby "weakening real and financial economic conditions become mutually reinforcing." (Bernanke, 2009). Hence, the increase in non-performing loans caused by the deteriorating real economy leads to a higher lending spread ω_t. Higher lending rates, in turn, exacerbate the slowdown in economic activity, which translates into further loan losses on the balance sheets of financial intermediaries.

4.2.3 Government

In every period, the government finances its expenditure by issuing new one-period bonds and raising taxes. Its period expenses consist of nominal general government expenditure $P_t^d G_t$ and also the repayment of maturing one-period bonds. Consequently, the real (stationary) budget constraint of the government is expressed as follows:

$$b_t^g + \tau_t = \ell_t + g_t, \tag{4.23}$$

where the government's total liabilities, ℓ_t, is defined as:

$$\ell_t = (b_{t-1}^g R_{t-1})/(\mu_t^z \pi_t^d). \tag{4.24}$$

In order to ensure dynamic stability, where inflation does not emerge as a fiscal phenomenon (see Leeper, 1991) , it is assumed that taxation by government is determined by the deviation of its outstanding liabilities from their steady state values:

$$\tau_t = \psi_0 + \psi_1 (\ell_t - \ell) \tag{4.25}$$

Accordingly, Equation (4.25) implies that taxes cannot be set independently from the level of outstanding government debt. This, in turn, rules out any possibility of an explosive path for government debt. Finally, government expenditure is assumed to follow an AR(1) process.

4.2.4 The central bank

Following Chapter 2, it is assumed that the central bank sets the policy rate in response to the expected deviation of year-on-year CPI inflation $\hat{\pi}_{t+1}^{c,4}$ from its target as well as the current quarter's change in the price level, $\hat{\pi}_t^c$. In addition, the central bank also takes into account the current level and rate of change in output. Consequently, the monetary policy rule is specified as follows:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + \phi_\pi\left(\hat{\pi}_{t+1}^{c,4} - \hat{\bar{\pi}}_t^c\right) + \phi_{\Delta\pi}\hat{\pi}_t^c + \phi_y \hat{y}_t + \phi_{\Delta y}\Delta\hat{y}_t\right] + \varepsilon^R, \tag{4.26}$$

where year-on-year CPI inflation is defined as $\hat{\pi}_t^{c,4} = \frac{1}{4}\prod_{j=1}^{4} \pi_{t+1-j}$.

4.2.5 Aggregate demand

Finally, clearing in the domestic final goods market requires that the supply of the final good firm matches the demand from households, government and the export market, after taking account of the additional adjustment costs on L-period bonds and money that are paid in terms of output:

$$y_t = \varepsilon_t \left(\frac{k_t^s}{\mu_t^z} \right)^{\alpha} H_t^{1-\alpha} - \phi, \tag{4.27}$$

where $y_t = (p_s c_t^s + p_b c_t^b) + (p_s i_t^s + p_b i_t^b) + g_t + nx_t$.[11] The remainder of the model structure is similar to Chapter 2, and the entire set of log-linearised equations is in the Appendix.

4.3 Estimation

4.3.1 Data

In addition to the fifteen observable domestic and international macro-economic time series used to estimate the model in du Plessis *et al.* (2014), two additional variables are now included that relate to the South African banking sector. Firstly, a measure of the effective interest rate paid on outstanding debt is included, where the difference between this lending rate and the Repo rate yields the (observable) lending spread ω_t. This effective lending rate is approximated by dividing the monthly interest income that South African banks receive from mortgage loans, credit-card debt, instalment sales and overdrafts by the end-of-month balances of these various loan books.[12] The second additional variable – a measure of the ratio of non-performing loans to total lending by South African banks – allows for a quantification of the impact that non-performing loans (ζ_t) have on lending spreads in Equation (4.21), which is a key propagation channel in the model.[13] Moreover, the inclusion of non-performing loans as an observable variable assists in the quantification of the interrelationship between ζ_t and y_t in Equation (4.22) – the channel through which the adverse feedback loop functions in the model. As before, the dataset spans the period from 2000Q1 to 2012Q4, which coincides with the inflation targeting regime of the South African Reserve Bank (SARB). Table 4.1 contains a summary of the data series used, as well as their respective sources.

[11] See the Appendix for the model's entire set of log-linearised equations.

[12] Data for South African banks' interest income and loan book balances over the period January 2008 to December 2012 (January 2000 to December 2007) were taken from the South African Reserve Bank's BA120 (DI200) and BA100 (DI100) returns, respectively.

[13] Non-performing loans are calculated as the ratio of the banking sector's specific provisions in respect of loans and advances to its total loans and advances

Table 4.1: Observable variables

Variable	Series	Source
South Africa		
$\Delta\ln(\tilde{Y}_t)$	Real GDP	South African Reserve Bank
$\Delta\ln(\tilde{C}_t)$	Private consumption	
$\Delta\ln(\tilde{I}_t)$	Total fixed investment	
$\Delta\ln(\tilde{X}_t)$	Total exports	
$\Delta\ln(\tilde{M}_t)$	Total imports	
$\Delta\ln(\tilde{S}_t)$	Nominal effective exchange rate	
$\Delta\ln(\tilde{E}_t)$	Non-agricultural employment	
$\Delta\ln(\tilde{W}_t)$	Compensation of employees	
$\tilde{\pi}^i_t$	Fixed investment deflator	
$\tilde{R}_t$	Repo rate	
$\tilde{\zeta}_t$	Non-performing loans to total assets	
$\tilde{\pi}^c_t$	CPI inflation	StatsSA
$\tilde{\pi}^d_t$	PPI inflation, domestic manufacturing	
$\tilde{R}^b_t$	Effective lending rate	Author's own calculations
$\tilde{\bar{\pi}}^c_{t+1}$	Inflation target midpoint	
Foreign economy		
$\Delta\ln(\tilde{Y}^*_t)$	Real GDP (trade weighted)	GPM, CEPREMAP
$\tilde{\pi}^*_t$	CPI inflation (trade weighted)	
$\tilde{R}^*_t$	Policy interest rates (trade weighted)	

4.3.2 Measurement issues

The effective lending rate discussed above (the interest income received by banks divided by the value of their loan books), most likely exhibits some degree of noise that emanates from specific seasonal loan repayment patterns in the data.[14] As a result, the lending rate's measurement equation includes a measurement error η^b_t:

$$\tilde{R}^b_t = \ln(R^b) + \hat{R}^b_t + \eta^b_t \tag{4.28}$$

The standard deviation of the measurement error is calibrated such that 10 per cent of the variation in the lending rate accounts for these *exogenous* factors.[15]

[14] For example, during February the repayment of loans (and thus interest income) in the South African banking sector is generally lower than in other months of the year. This likely pertains to the fact that February not only follows the festive season, but also the start of the new school year, which often puts household balance sheets under pressure during this period.

[15] The full set of 17 measurement equations are reported in the Appendix.

4.3.3 Calibration

As before, the model is estimated with Bayesian techniques, while certain parameters are calibrated where necessary.

Table 4.2: Calibrated parameters

β	Discount factor	0.9975	δ	Depreciation rate	0.025
A_L	Labour disutility constant	7.5	σ_L	Labour supply elasticity	5
ϕ_k	Capital adjustment cost	1150	α	Capital share in production	0.23
ϑ_c	Consumption imports share	0.36	ϑ_i	Investment imports share	0.48
θ_w	Calvo: wage setting	0.69	κ_w	Indexation: wage setting	0.5
λ_w	Wage setting markup	1.05	λ_d	Domestic price markup	1.1
η_c	Subst. elasticity: consumption	1.5	η_i	Subst. elasticity: investment	1.5
η_f	Subst. elasticity: foreign	1.25	ϕ_a	NFA/exchange rate elasticity	0.006
μ^z	Permanent technology growth	1.0085	π	Steady state inflation	1.0114
ρ_g	Govt. spending persistence	0.815	g_y	Govt. spending to GDP	0.197
π^*	Steady state foreign inflation	1.005			
Heterogeneous households					
p_s	Share of savers	0.5	χ	p(no type draw)	0.975
σ_s^{-1}	Cons. elasticity: savers	1.667	σ_b^{-1}	Cons. elasticity: borrowers	3.333
Financial intermediaries					
ω	Steady state gross spread	$1.025^{1/4}$	ζ	Steady state NPL	0.06
η_ζ	NPL elasticity	1.0	ρ_ζ	NPL persistence	0.7

The parameters that correspond to the baseline DSGE model from Chapter 2 are calibrated to identical values, while four parameters that govern the dynamics of savers and borrowers are also calibrated. Firstly, the share of savers p_s is set to 0.5, which implies that $p_b = 0.5$. The probability of a new type being drawn, $1-\chi$, is calibrated such that on average, a household could expect to remain of the same type for 10 years, after which there is a 50 per cent chance of changing its type. Accordingly, a household discounts the expected path of the policy or lending rate – depending on whether it is of type s or type b – over the next ten years when making decisions at time t. Expectations beyond that point are essentially irrelevant. The intertemporal elasticities of substitution for consumption by borrowers and savers, σ_b^{-1} and σ_b^{-1}, are set to 3.333 and 1.667 respectively, such that the ratio $\sigma_s/\sigma_b = 2$. Cúrdia and Woodford (2010) calibrate this pairing to 13.8 and 2.76, such that $\sigma_s/\sigma_b = 5$. However, their relatively high calibration for these elasticities of substitution yields an excessive reaction of aggregate consumption in response to interest rate changes, which is largely at odds with the South African experience.

The remaining parameters pertain to the financial intermediary. As such, the steady state lending spread is calibrated to 250 basis points, such that the lending rate R_t^b equals 11.4 per cent in steady state – its sample average. The steady state of the non-performing loan ratio ζ_t is set to 6 per cent. This is higher than its sample average of 2 per cent, but ensures that the impact of non-performing loans on the lending spread matches its empirical relationship observed in the data, where a 1 percentage point increase in non-performing loans leads to an annualised lending spread increase of 50 basis points. The non-performing loan elasticity η_ζ is set to 1 such that in the log-linear solution of the model, increases of similar magnitudes in lending volumes and non-performing loans have an identical impact on the lending spread.

4.3.4 Priors

The financial intermediary extension of the model requires two additional parameters to be estimated, *i.e.* when compared to the set of parameter estimates from Chapter 2: the elasticity of non-performing loans with respect to output θ_ζ, and the standard deviation of non-performing loan shocks σ_ζ. The elasticity of non-performing loans with respect to output in Table 3.3 is assumed to follow a fairly tight beta distribution around a mean of 0.408. The prior mean for this parameter is guided by Glen and Mondragón-Vélez's (2011) estimate of the contemporaneous impact of GDP growth on non-performing loans in a panel of 22 major developing economies, which includes South Africa. The standard deviation of the non-performing loans structural shock is assumed to follow an inverse-gamma distribution around a mean of 0.5, which is guided by the magnitude of the standard deviation of the non-performing loan series.

Apart from these two additional parameter estimates, the estimates for habit persistence and capital adjustment costs also differ from their counterparts in Chapter 2. Firstly, based on the assumption that savers and borrowers have different degrees of habit persistence, the degree of savers' habit persistence is estimated. Borrower habit persistence is calibrated to equal half of the savers' persistence, such that $b_s/b_b = 2$. While the prior for overall habit persistence in Chapter 2 followed a beta distribution around a mean of 0.65, the prior mean for saver habit persistence is adjusted by $(0.65r)/(1+\frac{1}{r}) = 0.867$, where $r = b_s/b_b$, such that the implied mean of "overall" habit persistence remains at 0.65. Secondly, the capital adjustment cost parameter is assumed to follow a gamma distribution around a mean of 0.588 with standard deviation 0.174. The prior for this parameter is based on the capital adjustment cost estimate by Christensen and Dib (2008) for a New Keynesian model that includes a financial accelerator mechanism.

The remaining parameters' prior means and densities are similar to Chapter 2.

4.3.5 Posterior estimates

The posterior estimation results are summarised in Table 4.3, while Figure C.1 in the Appendix contains the prior and posterior distributions. From the posterior results it can firstly be seen that the estimate of the elasticity of non-performing loans with respect to output at 0.367 is in-line with the estimate of Glen and Mondragón-Vélez (2011). The degree of habit formation by savers is estimated at 0.93, which implies that at the aggregate, habit formation by all households roughly equals 0.7,

Table 4.3: Priors and posterior estimation results

Parameter description		Prior			Posterior	
		Density[a]	Mean	Std. Dev.	Mean	90% interval
Consumption						
b_s	Habit formation by savers	B	0.867	0.05	0.930	[0.910 ; 0.948]
Investment						
ϕ_k	Capital adjustment cost	G	0.588	0.174	1.513	[1.121 ; 1.906]
Calvo parameters						
θ_d	Domestic prices	B	0.715	0.05	0.674	[0.596 ; 0.747]
θ_{mc}	Imported consumption prices	B	0.675	0.1	0.794	[0.679 ; 0.896]
θ_{mi}	Imported investment prices	B	0.675	0.1	0.831	[0.770 ; 0.888]
θ_x	Export prices	B	0.675	0.1	0.681	[0.596 ; 0.764]
θ_E	Employment	B	0.675	0.1	0.394	[0.305 ; 0.484]
Indexation						
κ_d	Domestic prices	B	0.5	0.15	0.510	[0.302 ; 0.739]
κ_{mc}	Imported consumption prices	B	0.5	0.15	0.327	[0.134 ; 0.498]
κ_{mi}	Imported investment prices	B	0.5	0.15	0.286	[0.114 ; 0.439]
Non-performing loans						
θ_ζ	NPL/output elasticity	G	0.408	0.025	0.367	[0.330 ; 0.401]
Taylor Rule						
ρ_R	Smoothing	B	0.8	0.05	0.858	[0.824 ; 0.889]
ϕ_π	Inflation	G	1.7	0.15	1.658	[1.451 ; 1.892]
$\phi_{\Delta\pi}$	Inflation (change)	G	0.3	0.1	0.265	[0.114 ; 0.399]
ϕ_y	Output gap	G	0.25	0.05	0.103	[0.072 ; 0.131]
$\phi_{\Delta y}$	Output gap (change)	G	0.125	0.05	0.176	[0.065 ; 0.288]
Persistence parameters						
ρ_{μ^z}	Permanent technology	B	0.75	0.1	0.738	[0.629 ; 0.847]
ρ_ε	Transitory technology	B	0.75	0.1	0.841	[0.772 ; 0.899]
ρ_i	Investment technology	B	0.75	0.1	0.865	[0.832 ; 0.899]
$\rho_{\tilde{z}^*}$	Asymmetric technology	B	0.75	0.1	0.755	[0.590 ; 0.915]
ρ_c	Consumption preference	B	0.75	0.1	0.842	[0.794 ; 0.893]
ρ_H	Labour supply	B	0.75	0.1	0.331	[0.210 ; 0.444]
ρ_a	Risk premium	B	0.75	0.1	0.881	[0.826 ; 0.935]
ρ_{λ^d}	Imported cons. price markup	B	0.75	0.1	0.571	[0.422 ; 0.717]
$\rho_{\lambda^{mc}}$	Imported cons. price markup	B	0.75	0.1	0.738	[0.531 ; 0.942]
$\rho_{\lambda^{mi}}$	Imported invest. price markup	B	0.75	0.1	0.742	[0.582 ; 0.892]
ρ_{λ^x}	Export price markup	B	0.75	0.1	0.455	[0.304 ; 0.606]
Structural shocks						
σ_ζ	Non-performing loans	IG	0.5	Inf	0.139	[0.103 ; 0.175]
σ_{μ^z}	Permanent technology	IG	0.4	Inf	0.243	[0.177 ; 0.303]
σ_ε	Transitory technology	IG	0.7	Inf	0.937	[0.647 ; 1.203]
σ_i	Investment technology	IG	0.4	Inf	1.419	[1.029 ; 1.761]
$\sigma_{\tilde{z}^*}$	Asymmetric technology	IG	0.4	Inf	0.226	[0.108 ; 0.335]
σ_c	Consumption preference	IG	0.4	Inf	0.835	[0.607 ; 1.068]
σ_H	Labour supply	IG	0.2	Inf	0.438	[0.333 ; 0.536]
σ_a	Risk premium	IG	0.5	Inf	0.945	[0.613 ; 1.273]
σ_d	Domestic price markup	IG	0.3	Inf	0.790	[0.587 ; 0.996]
σ_{mc}	Imported cons. price markup	IG	0.3	Inf	0.929	[0.638 ; 1.210]
σ_{mi}	Imported invest. price markup	IG	0.3	Inf	0.464	[0.215 ; 0.700]
σ_x	Export price markup	IG	0.3	Inf	1.348	[0.907 ; 1.764]
σ_R	Monetary policy	IG	0.15	Inf	0.218	[0.177 ; 0.262]

[a] B – Beta, G – Gamma, IG – Inverse Gamma, N – Normal, U – Uniform

which is a fairly standard calibration of this parameter in the literature. The capital adjustment cost parameter's estimate of 1.513 is higher than the 0.588 estimate by Christensen and Dib (2008). Nevertheless, both estimates exceed the "reasonable" range of 0 to 0.5 suggested by Bernanke *et al.* (1999).

The Calvo parameter estimates indicate price contracts are generally reoptimised every 4 quarters, with the reoptimisation of domestic prices being most frequent and imported investment prices least frequent. The inflation indexation parameter for domestic prices is estimated to be around 0.5, which implies that an equal weight is placed on the current inflation target and past inflation during indexation. The degree of indexation for imported consumption and investment prices are both estimated to be around 0.3. As a result, a higher weight is placed on the current inflation target relative to past inflation during indexation.

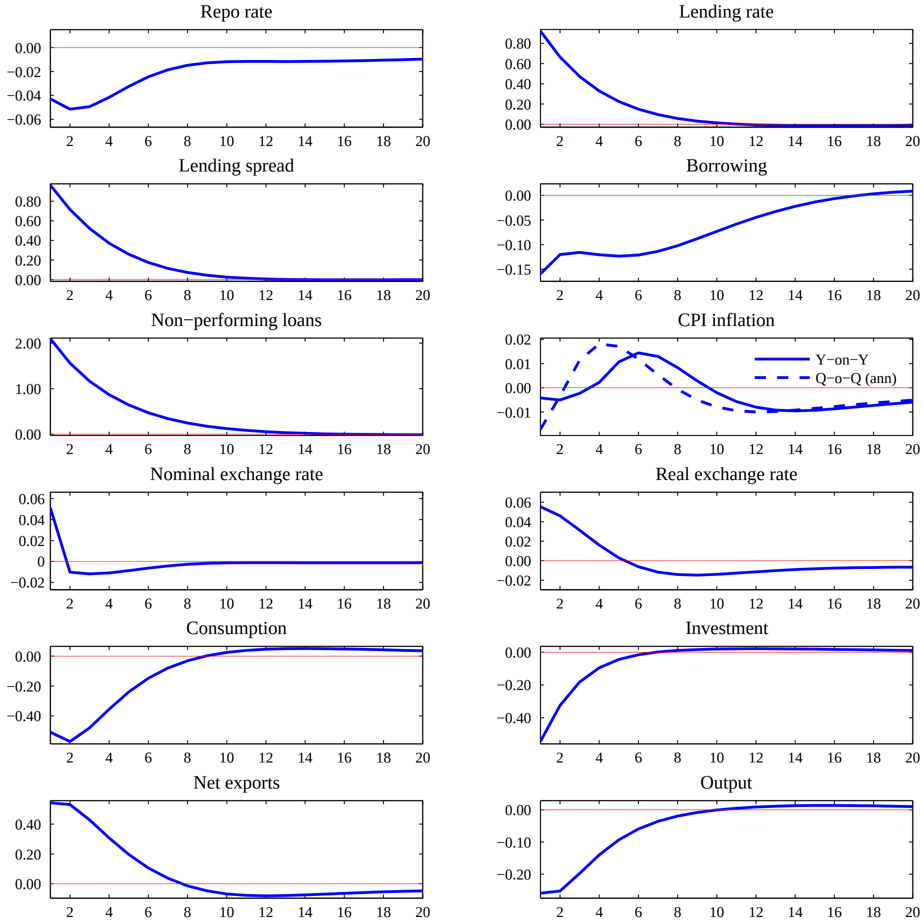
Turning to the estimates for Taylor rule parameters, the posterior mean of 0.858 for the degree of interest rate smoothing is slightly lower than Alpanda *et al.*'s (2010*b*) estimate of 0.916. Nevertheless, it appears as if the SARB places a high weight on interest rate stabilisation. In addition, its reactions to inflation, the change in inflation and the level of the output gap are slightly lower than what was indicated by the prior. However, the policy reaction to a change in the output gap - a proxy for GDP growth – is slightly more pronounced.

Estimates for persistence of the shocks indicate that the exchange rate risk premium, investment technology and consumption shocks are most persistent, while labour supply shocks are least persistent. The standard deviations of the innovations to these shocks vary substantially. Consistent with the high weight placed on interest rate stabilisation, monetary policy shocks exhibit low volatility. In turn, export and investment shocks are the most volatile. Finally, the standard deviation of shocks to non-performing loans is lower than its prior and the least volatile of all the structural shocks.

4.4 Dynamics of a financial shock

The macroeconomic impact of a financial shock – represented by an increase in non-performing loans – is shown in Figure 4.2. The shock is calibrated to ensure that the lending spread increases by 1 percentage point as a result of an increase in non-performing loans. In turn it leads to a concomitant rise in the lending rate faced by borrowers. A higher lending rate slows down the real economy, as both consumption and investment decline by around half a per cent. The combined impact of higher lending rates, lower consumption and lower investment reduces borrowing. The domestic slowdown does however improve net exports. Nevertheless, output ultimately declines by around 0.3 per cent. With the repo rate then responding to the real economic slowdown it is lowered slightly. Lowering the repo rate induces a nominal exchange rate depreciation, which counters the downward pressure on CPI inflation brought about by the slowing real economy. As a result, CPI inflation rises slightly in response to the financial shock, although in terms of magnitude it remains

Figure 4.2: Impulse response of a financial shock



largely unchanged. It is important to highlight here that the impact of the exchange rate channel on inflation fundamentally distinguishes the results in this paper from those of Cúrdia and Woodford (2010), where in response to a non-performing loan shock, both output and inflation fall. However, in the open economy dimension, as can be seen here, the reaction of the exchange rate significantly alters the response of inflation.

4.5 Optimal response to financial shocks

In order to determine the optimal response of the central bank in the event of a financial shock, the Taylor rule in Equation 4.26 is adjusted such that the central bank also considers lending spreads with weight ϕ_ω when setting the policy rate:

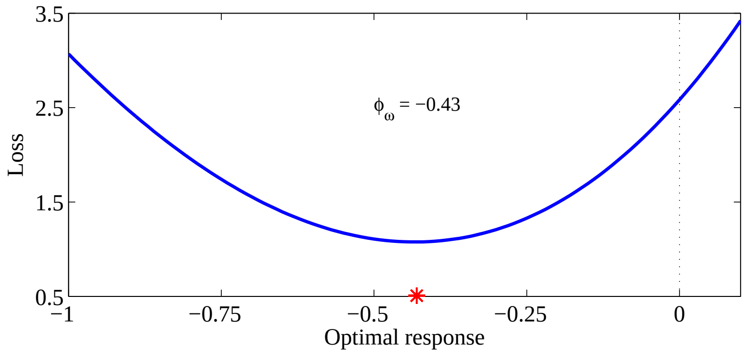
$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + \phi_\pi\left(\hat{\pi}_{t+1}^{c,4} - \hat{\bar{\pi}}_t^c\right) + \phi_{\Delta\pi}\hat{\pi}_t^c + \phi_y \hat{y}_t + \phi_{\Delta y}\Delta\hat{y}_t + \phi_\omega \hat{\omega}_t\right] + \varepsilon_t^R, \tag{4.29}$$

It is important to note here that optimal policy is not characterised by the typical Ramsey-type welfare-maximising competitive equilibrium.[16] Rather, the estimated Taylor rule without the lending spread is seen to represent the standard policy that has been followed by the South African Reserve Bank to date, and is therefore regarded as the benchmark against which to gauge alternative policy rules. Hence, it is assumed that the central bank aims to minimise the expected deviations of year-on-year CPI inflation and output when setting the policy rate, which can be represented by the following linear-quadratic loss function:

$$L = \sum_{t=1}^{\infty} \beta^t \left[(\hat{\pi}_t^{c,4})^2 + \lambda \hat{y}_t^2 \right], \tag{4.30}$$

where $\lambda = 0.5$. Accordingly, determining whether reacting to lending spreads in the event of a financial shock would be optimal, requires the comparison of the value of the loss function under the standard Taylor rule of Equation (4.26) to its value if Equation (4.29)'s rule above is followed. By varying the size of ϕ_ω in Equation (4.29), Figure 4.3 indicates that $\phi_\omega = -0.43$ minimises the central bank's loss function. This improves on the value of the loss function if $\phi_\omega = 0$. As such,

Figure 4.3: Optimal response to rising credit spreads that emanate from a financial shock



it is optimal for the central bank to reduce the repo rate by a further 0.43 basis points for every 100 basis point rise in the lending spread. This result compares favourably with Cúrdia and Woodford (2010), who find that the optimal $\phi_\omega = -0.66$, as opposed to McCulley and Toloui (2008) and Taylor (2008) who suggest a one-for-one reduction of the policy rate in reaction to rising credit spreads.

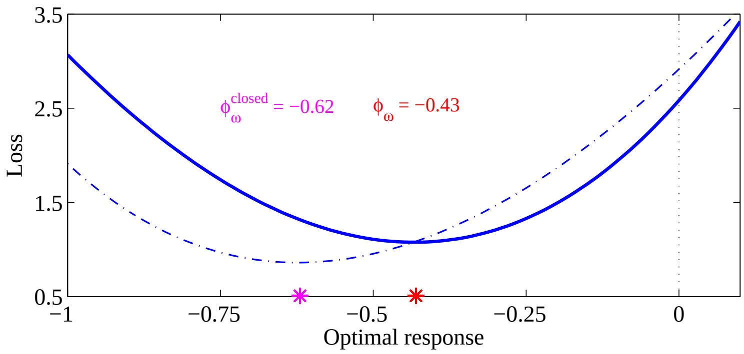
4.5.1 The role of the open-economy dimension

Moving aside the richer model structure, a key difference between this study and the stylised model of Cúrdia and Woodford (2010) lies in the inclusion of the open-

[16] Ramsey optimal policy is often derived in highly stylised and simple DSGE models. The large number of variables and frictions in this model would substantially complicate the calculation of such an optimal policy.

economy dimension.[17] As such, in the open-economy setting, a reduction in the policy rate leads to a depreciation of the exchange rate, which adds an additional channel of inflationary pressure. In order to control for the impact of the open-economy dimension on the magnitude of the optimal response coefficient, all the open-economy channels in the model are closed down such that the model approximates a closed economy. Assuming that the central bank follows the same Taylor rule as before, the optimal response coefficient ϕ_{ω}^{closed} is then found to be -0.62, which is remarkably closer to Cúrdia and Woodford's (2010) closed-economy response coefficient of -0.66, as opposed to the -0.43 found in the open-economy setting (see Figure 4.4). Hence, it is the reaction of the exchange rate in response to accommodative monetary policy that creates an additional channel of inflation that is not present in a closed economy. As a result, an inflation-targeting central bank operating in a small open economy has limited scope to ease policy in the event of a financial shock, when compared to its closed economy counterpart.

Figure 4.4: Open-economy impact on optimal response coefficient

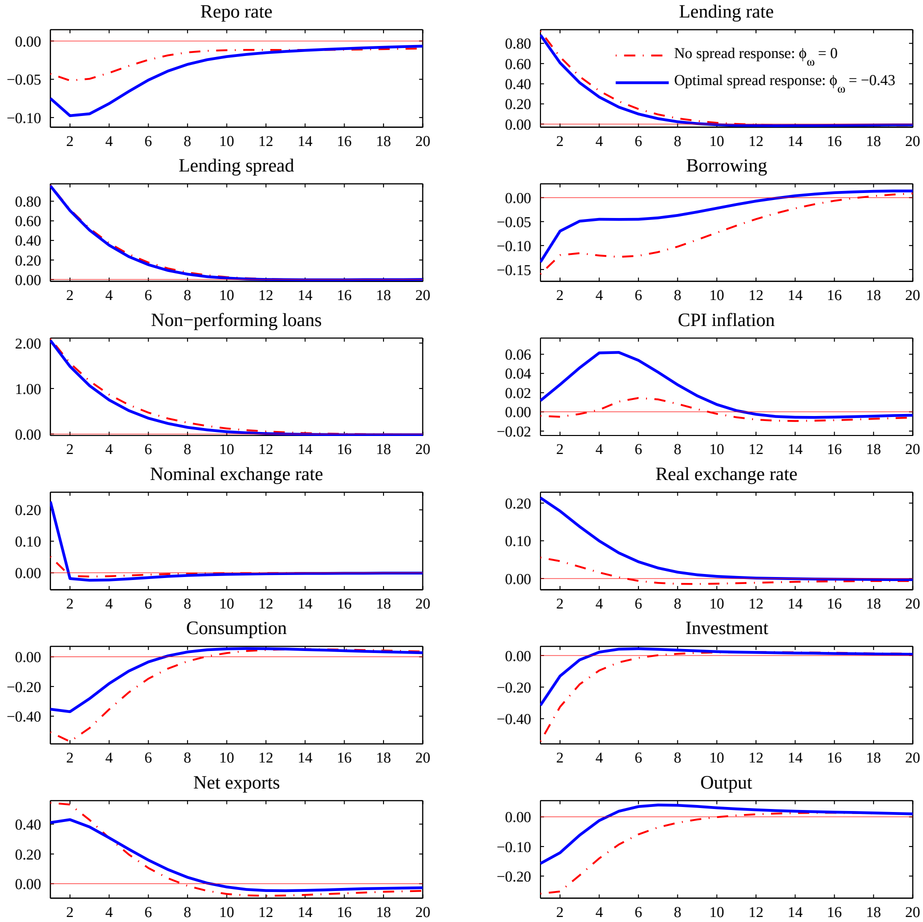


4.5.2 Dynamics of a financial shock under an optimal response

Having determined the magnitude of the ideal response to rising credit spreads in the event of a financial shock, Figure 4.5 compares the macroeconomic impact if the central bank were to follow the Taylor rule with $\phi_{\omega} = -0.43$, as opposed to the "no reaction" response where $\phi_{\omega} = 0$. Strikingly, the decline in output induced by the financial shock is almost halved by the central bank's reaction to the rising spread. The fall in consumption and investment is also substantially reduced, which in turn limits the decline in borrowing. However, inflation rises to a greater extent than before, largely as a result of the stronger depreciation of the nominal exchange

[17] Another key structural difference between this study and Cúrdia and Woodford (2010) is the inclusion of the adverse feedback loop – where the real economic slowdown leads to additional increases in non-performing loans. When closing down this channel, the absolute value of the optimal response coefficient declines with 0.02. This decrease in the parameter can be attributed to a less severe real economic slowdown in the absence of the adverse feedback loop, which in turn requires less policy accommodation.

Figure 4.5: Impulse response of a financial shock when ϕ_ω = -0.43



rate. The improved real economic performance is beneficial to the level of non-performing loans of financial intermediaries, and although the effect is marginal, borrowers also enjoy lower lending rates.

4.6 Concluding remarks

Financial shocks, such as the one experienced during the recent global financial crisis, generally lead to higher credit spreads and lower real economic activity. In a bid to lessen the decline in real activity, central banks follow expansionary monetary policy by reducing their policy rates. However, their actions are countered by the higher credit spreads, such that the effective lending rate remains largely unchanged and monetary policy loses its efficacy. As a result, some have argued that the central bank reduce the policy rate on a one-for-one basis in response to the rising credit spreads induced by a financial shock (see McCulley and Toloui (2008) and Taylor (2008)), while Cúrdia and Woodford (2010) find that a less-than-unitary reaction coefficient is optimal. This paper contributes to the debate by analysing the optimal response of the central bank to rising credit spreads induced by a financial shock within the context of a small open economy. It does so by incorporating a banking sector, which lends at a spread above the policy rate, into the standard New Keynesian DSGE model developed in Chapter 2. Moreover, within the model these spreads increase during times of financial distress. The optimal reaction coefficient that minimises the central bank's loss function is found to be less than one. However, when compared to the closed economy result of Cúrdia and Woodford (2010), the reaction coefficient is slightly smaller – partly reflecting the additional inflation cost brought about by the exchange rate depreciation that is associated with a policy rate reduction.

Chapter 5

Summary

Dynamic stochastic general equilibrium (DSGE) models have gained increasing prominence during the past two decades – not only in academic circles, but also in policymaking institutions such as central banks. The literature surrounding these models was birthed in the aftermath of Lucas's (1976) famous critique of the econometric models in use at the time. According to Lucas, these econometric models would fail once any changes in policy occur which alter the nature of the historical macroeconomic relationships on which the model was estimated. In fact, what was needed was a model where the parameters reflect the behavioural aspects of economic agents, such as their tastes and preferences, as these parameters would be policy invariant. Moreover, if these agents were rational and forward-looking, they would correctly anticipate the impact of the policy change and adjust their behaviour accordingly.

DSGE models largely succeeded in addressing the concerns of Lucas. Within these models, the macroeconomic relationships are derived from the microeconomics foundations of agents' intertemporal preferences. Moreover, agents' rational expectations play a central role in determining the macroeconomic outcomes. This theoretical consistency has created a highly credible tool in the hand of the policymaker, and therefore many central banks have adopted DSGE models into their policy analysis and forecasting frameworks.[1]

However, DSGE models have not escaped critique either. The most serious of these was lodged during the global financial crisis. The DSGE models in use at the time had largely ignored the role played by the financial sector in the evolution of the business cycle. This shortcoming was brutally exposed during the financial crisis, as policymakers were unable to coherently assess the macroeconomic implications of the financial instability induced by the crisis. In recognition of this shortcoming, including the necessary financial frictions in the benchmark DSGE model has become a primary objective of the DSGE research agenda since the crisis.

This thesis contributes to the existing body of literature in three respects. Firstly, in Chapter 2 a medium scale DSGE model of the South African economy is derived and estimated with Bayesian techniques. The model includes a vast array of real and nominal frictions, and is the first DSGE model of the South African economy that models all five components of aggregate demand. Chapter 3 exploits the rational expectations solution of the model by incorporating the yield on South Africa's 10 year government bond. Given the expectations hypothesis theory of the term structure, which states that the yield of any bond can be expressed as the average expected short-term interest rate over the period to maturity of the bond, the inclusion of longer-term interest rates becomes a natural extension of the model. Within this context, the structural shocks that have contributed to the evolution of the South African yield curve over the inflation targeting regime are analysed. Moreover, the predictive content of the yield spread's subcomponents – the term premium and expected short-term interest rate – with respect to GDP is analysed. Finally, against

[1] See Chapter 1 for an exhaustive list.

the background of the critique levelled at DSGE models during the global financial crisis, a banking sector is introduced in Chapter 4, and the implications of a financial shock for monetary policy is analysed.

The individual results of each chapter are summarised below.

A medium-sized open economy DSGE model

The small open economy model structure largely follows the lines of existing DSGE models that are operational in central banks, such as Adolfson *et al.* (2007*a*).

Essentially, the model consists of households, firms and the central bank. Households maximise their expected lifetime utility by choosing how much to consume in each period, as well as which amount of labour to supply to firms in order to finance their consumption. In addition, households own the capital stock, which in turn is rented to firms. Since households are the owners of the capital stock, they also decide how much to invest in every period. In addition, households may save in domestic or foreign one-period bonds. These bonds are remunerated at the domestic and foreign interest rate, respectively. Domestic intermediate goods firms employ the labour and capital that is supplied by the household, and remunerate these inputs through wages and the rental rate of capital. A final goods producer then transforms these intermediate goods into a final homogeneous good. There are also importing and exporting firms, that supply imports to the domestic and exports to the foreign economy. All firms are assumed to set their prices in a Calvo (1983) staggered manner, while exhibiting some degree of indexation to past inflation. The central bank follows a Taylor-type rule in setting the short-term policy interest rate, based on the level of inflation as well as output. And finally, the foreign economy is assumed to be exogenous to developments in the domestic economy.

The model parameters are estimated with Bayesian techniques, and where necessary certain parameters are calibrated – either to pin down a specific steady state value or due to lack of identification. The estimation sample covers the inflation targeting regime of the South African Reserve Bank (2000Q1 to 2012Q4), and includes 15 observable macroeconomic time-series. Thereafter, the estimation is validated through a comparison of the model's moments, cross- and autocorrelations with those of the data.

By writing the DSGE model in state-space form, the Kalman filter may be used to decompose the model's variables into the structural shocks that have driven them. Focusing on year-on-year CPI inflation, it is found that, amongst others, upward pressure on inflation could often be attributed to labour market and exchange rate risk premium shocks. Similarly, developments in the exchange rate appear to have dominated GDP growth – especially during the first few years of the inflation targeting regime. Thereafter, at the onset of the global financial crisis, adverse shocks to permanent technology seems to have reduced the economy's growth potential. In addition, further downward pressure on economic growth in the wake of the crisis appears to have emanated from unfavourable labour market conditions.

Finally, the model's forecasting ability up to seven quarters ahead is compared to a random walk and to a consensus of professional forecasters, as surveyed by Reuters. With respect to year-on-year CPI inflation, the DSGE model outperforms the professional forecasters over the 5 to 7 quarter horizon, while it outperforms a random walk over the entire horizon. When forecasting GDP growth (quarter-on-quarter, annualised), a similar result is obtained: the model outperforms the professional forecasters over the outer quarters of the forecast horizon, while it outperforms the random walk over almost the entire horizon. However, when forecasting the Repo rate, the DSGE model fails to outperform the professional forecasters. Nevertheless, the model's Repo rate forecasts are in general more accurate than those of a random walk.

A structural decomposition of the South African yield curve

The rational expectations solution of DGSE models make them an ideal framework for analysing the term structure of interest rates. More specifically, given that the solution of the model produces theoretically consistent expectations of future short-term interest rates, it is straightforward to derive the yield curve that is consistent with the expectations hypothesis theory.

Consequently, the yield on South Africa's 10 year government bond is introduced into the benchmark model developed in Chapter 2. This is done by assuming households do not only save in one-period bonds, but also in L-period bonds, where $L = 40$ in order to match the maturity of the 10 year government bond. However, Andrés *et al.* (2004) argue that households experience a loss of liquidity when holding bonds in excess of one period, while they compensate for this loss by holding additional money. As a result, money is introduced as an additional asset that is held by households.

The inclusion of the yield curve adds a number of additional parameters to the model. As before, the model parameters are estimated with Bayesian techniques, while certain parameters are calibrated. The yield-curve extension of the model is then validated through a forecast comparison between a consensus of professional forecasters (as polled by Reuters), as well as the benchmark DSGE model of Chapter 2. When forecasting the South African 10 year government bond yield over a forecast horizon of seven quarters, the yield-curve extended DSGE model outperforms the consensus of professional forecasters from the second to the seventh quarter. When comparing the extended model's forecasts of year-on-year CPI inflation to the benchmark model, there is no significant improvement to be gained from adding the yield curve to the DSGE model. However, when forecasting GDP growth (quarter-on-quarter, annualised), the addition of the yield curve to the DSGE model improves the model's forecasts of GDP growth over all seven quarters of the forecast horizon. This likely reflects the well-documented informational content of the yield curve with respect to future real economic activity. Similarly, it seems as if the addition of the yield curve to the DSGE framework also enhances the model's accuracy in predicting the Repo rate.

The extended model is then used to decompose the yield spread – defined as the 10 year yield minus the Repo rate – into the structural shocks that have contributed to its evolution during the inflation targeting regime of the SARB. Accordingly, it appears as if shocks to the exchange rate, monetary policy and the foreign economy have been significant contributors to movements in the yield spread over this period.

Building on an established body of literature that relates the slope of the yield curve to future economic activity, the model is used to decompose the 10 year yield into two subcomponents: the term premium and the expected short-term interest rate. This exercise is motivated by an open question in the international literature for which there have been ambiguous findings: does the term premium on its own contain any predictive content with respect to GDP? For example Bernanke (2006) argues that a declining term premium would point to a coming economic expansion, Hamilton and Kim (2002) state the opposite, while Ang *et al.* (2006) find that no relationship exists. Rudebusch *et al.* (2007) argue that the equation used in these analyses is misspecified, since the term premium is non-stationary. When considering the first difference of the term premium, Rudebusch *et al.* (2007) find a negative sign. When applying this question to the South African term premium, it is found that the level of the term premium does not predict future GDP – echoing Ang *et al.* (2006). However, similar to the finding of Rudebusch *et al.* (2007), changes in the South African term premium do predict future GDP, and a negative change would point to an economic expansion.

Monetary policy and financial shocks in an empirical small open-economy DSGE model

During the global financial crisis, banks were increasing their lending spreads as they experienced severe losses on their balance sheets. At the same time, central banks were reducing their policy rates in order to accommodate the substantial declines in aggregate demand caused by the financial crisis. However, the increases in lending spreads partly countered the declines in policy rates, and in turn reduced the efficacy of monetary policy. Consequently, a debate developed around the appropriate response of monetary policy when rising spreads induced by a financial shock counteract the expansionary policy followed by the central bank. McCulley and Toloui (2008) and Taylor (2008) argued that the central bank should reduce the policy rate by an additional 100 basis points for every 100 basis point rise in the credit spread. Using a stylised DSGE model calibrated to the US economy, Cúrdia and Woodford (2010) found that a 66 basis point reduction would be optimal.

This chapter contributes to the debate by determining the optimal response to such a financial shock in the context of a small open economy.

In order to create a role for financial intermediation in the benchmark DSGE model, heterogeneity is required on behalf of the households. This is achieved by assuming that a share of households are savers, and the remaining households are borrowers – largely following the methodology of Cúrdia and Woodford (2009, 2010). Banks are then introduced as financial intermediaries, who convert the deposits of saving households into loans for borrowers. However, it is assumed that a proportion of the loans given to borrowers are not repaid. This potential for non-performing loans on the bank's balance sheet, makes lending risky for the bank. As a result, the lending rate exceeds the deposit rate by a spread that increases as the number of non-performing loans increases.

A financial shock is simulated as an increase in non-performing loans, which leads to an increase in the lending spread and a real economic slowdown. The spread is then included in the Taylor rule that is followed by the SARB. Assuming that the SARB's loss function is quadratic in CPI inflation and the output gap, the magnitude of the parameter on the spread in the Taylor rule that minimises the loss function of the central bank is found to be -0.43. Hence, in the event of a financial shock, the SARB should reduce the Repo rate by 43 basis points for every 100 basis point rise in lending spreads.

In absolute terms, the optimal reaction coefficient of the SARB is smaller than Cúrdia and Woodford's (2010) estimate of -0.66. However, the authors use a closed-economy DSGE model in their analysis. Within an open economy, the exchange rate depreciates following a reduction in the policy rate, which creates an additional channel of inflation that is not present in a closed economy. In order to determine the impact that the open economy dimension has on the magnitude of the SARB's reaction coefficient, all the open economy channels are closed down to the point where the model approximates a closed economy. When simulating the financial shock in this setting, the optimal spread reaction coefficient in the Taylor rule is found to be -0.62, which is remarkably close to the result of Cúrdia and Woodford. Hence, it is evident that the reaction of the exchange rate in response to policy easing limits the scope with which an inflation-targeting central bank of a small open economy may reduce the policy rate in the event of a financial shock.

Appendices

Appendix A

A medium-sized open economy DSGE model of South Africa

A.1 The linearised model

Firms

Domestic goods

Production

$$\hat{y}_t = \lambda^d \left(\hat{\varepsilon}_t + \alpha \left(\hat{k}_t^s - \hat{\mu}_t^z\right) + (1-\alpha)\,\hat{H}_t\right) \tag{A.1}$$

Rental rate of capital

$$\hat{r}_t^k = \hat{w}_t + \hat{\mu}_t^z - \hat{k}_t^s + \hat{H}_t \tag{A.2}$$

Real marginal cost

$$\hat{mc}_t^d = \alpha\,\hat{r}_t^k + (1-\alpha)\,(\hat{w}_t) - \hat{\varepsilon}_t \tag{A.3}$$

New Keynesian Phillips curve

$$\begin{aligned}\hat{\pi}_t^d - \hat{\bar{\pi}}_t^c \;=\;& \frac{\beta}{1+\beta\kappa_d}\left(E_t\hat{\pi}_{t+1}^d - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_d}{1+\beta\kappa_d}\left(\hat{\pi}_{t-1}^d - \hat{\bar{\pi}}_t^c\right) - \frac{\beta\kappa_d\,(1-\rho_\pi)}{1+\beta\kappa_d}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_d)\,(1-\beta\theta_d)}{(1+\beta\kappa_d)\,\theta_d}\left(\hat{mc}_t^d + \hat{\lambda}_t^d\right)\end{aligned} \tag{A.4}$$

Imported goods

New Keynesian Phillips curve: imported consumption goods

$$\begin{aligned}\hat{\pi}_t^{m,c} - \hat{\bar{\pi}}_t^c \;=\;& \frac{\beta}{1+\beta\kappa_{m,c}}\left(E_t\hat{\pi}_{t+1}^{m,c} - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_{m,c}}{1+\beta\kappa_{m,c}}\left(\hat{\pi}_{t-1}^{m,c} - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_{m,c}\beta(1-\rho_\pi)}{1+\beta\kappa_{m,c}}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{m,c})(1-\beta\theta_{m,c})}{(1+\beta\kappa_{m,c})\,\theta_{m,c}}\left(\hat{mc}_t^{m,c} + \hat{\lambda}_t^{m,c}\right)\end{aligned} \tag{A.5}$$

Marginal cost: imported consumption goods

$$\hat{mc}_t^{m,c} = -\hat{\gamma}_t^f - \hat{\gamma}_t^{mc,d} \tag{A.6}$$

New Keynesian Phillips curve: imported investment goods

$$\begin{aligned}\hat{\pi}_t^{m,i} - \hat{\bar{\pi}}_t^c &= +\frac{\beta}{1+\beta\kappa_{m,i}}\left(E_t\hat{\pi}_{t+1}^{m,i} - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_{m,i}}{1+\beta\kappa_{m,i}}\left(\hat{\pi}_{t-1}^{m,i} - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_{m,i}\beta(1-\rho_\pi)}{1+\beta\kappa_{m,i}}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{m,i})(1-\beta\theta_{m,i})}{(1+\beta\kappa_{m,i})\theta_{m,i}}\left(\hat{mc}_t^{m,i} + \hat{\lambda}_t^{m,i}\right)\end{aligned} \tag{A.7}$$

Marginal cost: imported investment goods

$$\hat{mc}_t^{m,i} = -\hat{\gamma}_t^f - \hat{\gamma}_t^{mi,d} \tag{A.8}$$

Exported goods

New Keynesian Phillips curve: exported goods

$$\begin{aligned}\hat{\pi}_t^x - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_x}\left(E_t\hat{\pi}_{t+1}^x - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}_{t-1}^x - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_x\beta(1-\rho_\pi)}{1+\beta\kappa_x}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_x)(1-\beta\theta_x)}{(1+\beta\kappa_x)\theta_x}\left(\hat{mc}_t^x + \hat{\lambda}_t^x\right)\end{aligned} \tag{A.9}$$

Marginal cost: exported goods

$$\hat{mc}_t^x = \hat{mc}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta\hat{S}_t \tag{A.10}$$

Households

Wage setting

$$\hat{w}_t = -\frac{1}{\eta_1}\left[\begin{array}{l}\eta_0\hat{w}_{t-1} + \eta_2 E_t\hat{w}_{t+1} + \eta_3\left(\hat{\pi}_t^d - \hat{\bar{\pi}}_t^c\right) + \eta_4\left(E_t\hat{\pi}_{t+1}^d - \rho_\pi\hat{\bar{\pi}}_t^c\right) \\ +\eta_5\left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \eta_7\hat{\psi}_t^z + \eta_8\hat{H}_t + \eta_9\hat{\xi}_t^h\end{array}\right] \tag{A.11}$$

Consumption Euler equation

$$\begin{aligned}\hat{c}_t &= \frac{\mu^z b}{(\mu^z)^2+\beta b^2}\hat{c}_{t-1} + \frac{\beta\mu^z b}{(\mu^z)^2+\beta b^2}E_t\hat{c}_{t+1} - \frac{\mu^z b}{(\mu^z)^2+\beta b^2}\left(\hat{\mu}_t^z - \beta E_t\hat{\mu}_{t+1}^z\right) \\ &- \frac{(\mu^z-b)(\mu^z-\beta b)}{(\mu^z)^2+\beta b^2}\left(\hat{\psi}_t^z + \hat{\gamma}_t^{c,d}\right) + \frac{\mu^z - b}{(\mu^z)^2+\beta b^2}\left(\mu^z\hat{\xi}_t^c - \beta b E_t\hat{\xi}_{t+1}^c\right)\end{aligned} \tag{A.12}$$

Investment Euler equation

$$\hat{i}_t = \frac{1}{1+\beta}\left[\beta E_t\hat{i}_{t+1} + \hat{i}_{t-1} + \beta E_t\hat{\mu}_{t+1}^z - \mu_t^z\right] + \frac{1}{(\mu^z)^2\phi_i(1+\beta)}\left(\hat{P}_t^k - \hat{\gamma}_t^{i,d} + \hat{\xi}_t^i\right) \tag{A.13}$$

Price of installed capital

$$\hat{P}_t^k = E_t\left[\frac{(1-\delta)\beta}{\mu^z}\hat{P}_{t+1}^k + \hat{\psi}_{t+1}^z - \hat{\psi}_t^z - \hat{\mu}_{t+1}^z + \frac{\mu^z-(1-\delta)\beta}{\mu^z}\hat{r}_{t+1}^k\right] \tag{A.14}$$

Capital's law-of-motion

$$\hat{k}_{t+1} = \frac{1-\delta}{\mu^z}\left(\hat{k}_t - \hat{\mu}_t^z\right) + \left(1 - \frac{1-\delta}{\mu^z}\right)\left(\hat{i}_t + \hat{\xi}_t^i\right) \quad \text{(A.15)}$$

Capital utilisation

$$\hat{u}_t = \frac{1}{\sigma_a}\hat{r}_t^k \quad \text{(A.16)}$$

Capital services

$$\hat{k}_t^s = \hat{k}_t + \hat{u}_t \quad \text{(A.17)}$$

Optimal asset holdings

$$\hat{\psi}_t^z = E_t\left(\hat{\psi}_{t+1}^z - \hat{\mu}_{t+1}^z\right) + \left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d\right) \quad \text{(A.18)}$$

Modified UIP condition

$$\hat{R}_t - \hat{R}_t^* = (1-\tilde{\phi}_s)E_t\Delta\hat{S}_{t+1} - \tilde{\phi}_s\Delta\hat{S}_t - \tilde{\phi}_a\hat{a}_t + \hat{\tilde{\phi}}_t, \quad \text{(A.19)}$$

The Central Bank

Taylor rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + \phi_\pi\left(\hat{\pi}_{t+1}^{c,4} - \hat{\bar{\pi}}_t^c\right) + \phi_{\Delta\pi}\hat{\pi}_t^c + \phi_y\hat{y}_t + \phi_{\Delta y}\Delta\hat{y}_t\right] + \varepsilon_t^R. \quad \text{(A.20)}$$

where CPI inflation is given by

$$\hat{\pi}_t^c = (1-\vartheta_c)\left(\frac{1}{\gamma^{c,d}}\right)^{1-\eta_c}\hat{\pi}_t^d + \vartheta_c\left(\gamma^{mc,c}\right)^{1-\eta_c}\hat{\pi}_t^{m,c} \quad \text{(A.21)}$$

Relative prices

Consumption and investment goods

$$\hat{\gamma}_t^{c,d} = \hat{\gamma}_{t-1}^{i,d} + \hat{\pi}_t^c - \hat{\pi}_t^d \quad \text{(A.22)}$$

$$\hat{\gamma}_t^{i,d} = \hat{\gamma}_{t-1}^{i,d} + \hat{\pi}_t^i - \hat{\pi}_t^d \quad \text{(A.23)}$$

Imported consumption and investment goods

$$\hat{\gamma}_t^{mc,d} = \hat{\gamma}_{t-1}^{mc,d} + \hat{\pi}_t^{m,c} - \hat{\pi}_t^d \quad \text{(A.24)}$$

$$\hat{\gamma}_t^{mi,d} = \hat{\gamma}_{t-1}^{mi,d} + \hat{\pi}_t^{m,i} - \hat{\pi}_t^d \quad \text{(A.25)}$$

Export goods

$$\hat{\gamma}_t^{x,*} = \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \quad \text{(A.26)}$$

Domestic-foreign goods relative price

$$\hat{\gamma}_t^f = \hat{mc}_t^x + \hat{\gamma}_t^{x,*} \quad \text{(A.27)}$$

Real exchange rate

$$\hat{\gamma}_t^s = -\vartheta_c\left(\frac{1}{\gamma^{mc,c}}\right)^{\eta_c-1}\hat{\gamma}_t^{mc,d} - \hat{\gamma}_t^{x,*} - \hat{mc}_t^x \quad \text{(A.28)}$$

Market clearing

Domestic goods market

$$\begin{aligned}\hat{y}_t &= (1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c}\frac{c}{y}\left(\hat{c}_t+\eta_c\hat{\gamma}_t^{c,d}\right)+(1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i}\frac{i}{y}\left(\hat{i}_t+\eta_i\hat{\gamma}_t^{i,d}\right)\\ &+ g_y\hat{g}_t+\frac{y^*}{y}\left(\hat{y}_t^*-\eta_f\hat{\gamma}_t^{x,*}+\hat{\tilde{z}}_t^*\right)+\frac{r^k}{\mu^z}\frac{k}{y}\left(\hat{k}_t^s-\hat{k}_t\right) \qquad (A.29)\end{aligned}$$

Foreign bond market

$$\begin{aligned}\hat{a}_t &= -y^*\hat{mc}_t^x-\eta_f y^*\hat{\gamma}_t^{x,*}+y^*\hat{y}_t^*+y^*\hat{\tilde{z}}_t^*+(c^m+i^m)\hat{\gamma}_t^f\\ &- \left[c^m\left(-\eta_c(1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c-1}\right)\hat{\gamma}_t^{mc,d}+\hat{c}_t\right]\\ &- \left[i^m\left(-\eta_i(1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i-1}\right)\hat{\gamma}_t^{mi,d}+\hat{i}_t\right]\\ &+ \frac{\pi^*}{\pi}\frac{1}{\beta}\hat{a}_{t-1} \qquad (A.30)\end{aligned}$$

AR(1) shock processes

$$\Xi_t=\rho\Xi_{t-1}+\Gamma_t \qquad (A.31)$$

where

$$\begin{aligned}\Xi_t &= [\hat{\xi}_t^c \;\; \hat{\xi}_t^i \;\; \hat{\tilde{\phi}}_t \;\; \hat{\varepsilon}_t \;\; \hat{\xi}_t^H \;\; \hat{\lambda}_t^x \;\; \hat{\lambda}_t^d \;\; \hat{\lambda}_t^{m,c} \;\; \hat{\lambda}_t^{m,i} \;\; \hat{\tilde{z}}_t^* \;\; \hat{\mu}_t^z \;\; \hat{g}_t \;\; \hat{\bar{\pi}}_t^c]'\\ \rho &= [\rho_c \;\; \rho_i \;\; \rho_{\tilde{\phi}} \;\; \rho_\varepsilon \;\; \rho_H \;\; \rho_{\lambda^x} \;\; \rho_d \;\; \rho_{\lambda^{m,c}} \;\; \rho_{\lambda^{m,i}} \;\; \rho_{\tilde{z}^*} \;\; \rho_{\mu^z} \;\; \rho_g \;\; \rho_{\bar{\pi}^c}]'\\ \Gamma_t &= [\varepsilon_t^c \;\; \varepsilon_t^i \;\; \varepsilon_t^{\tilde{\phi}} \;\; \varepsilon_t^\varepsilon \;\; \varepsilon_t^H \;\; \varepsilon_t^x \;\; \varepsilon_t^d \;\; \varepsilon_t^{m,c} \;\; \varepsilon_t^{m,i} \;\; \varepsilon_t^{\tilde{z}^*} \;\; \varepsilon_t^{\mu^z} \;\; \varepsilon_t^g \;\; \varepsilon_t^{\bar{\pi}^c}]'\end{aligned}$$

Measurement equations

Output

$$\Delta\ln\left(\tilde{Y}_t\right)=\hat{y}_t-\hat{y}_{t-1}+\hat{\mu}_t^z+\ln(\mu^z) \qquad (A.32)$$

Consumption

$$\begin{aligned}\Delta\ln\left(\tilde{C}_t\right) &= \left(\frac{\eta_c}{c^d+c^m}\right)\left[c_d\vartheta_c(\gamma^{c,mc})^{\eta_c-1}-c^m(1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c-1}\right]\left(\hat{\pi}_t^{m,c}-\hat{\pi}_t^d\right)\\ &+ \hat{c}_t-\hat{c}_{t-1}+\mu_t^z+\ln(\mu^z) \qquad (A.33)\end{aligned}$$

Investment

$$\begin{aligned}\Delta\ln\left(\tilde{I}_t\right) &= \left(\frac{\eta_i}{i^d+i^m}\right)\left[i_d\vartheta_i\left(\gamma^{i,mi}\right)^{\eta_i-1}-i^m(1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i-1}\right]\left(\hat{\pi}_t^{m,c}-\hat{\pi}_t^d\right)\\ &+ \hat{i}_t-\hat{i}_{t-1}+\hat{\mu}_t^z+\ln(\mu^z) \qquad (A.34)\end{aligned}$$

Exports

$$\Delta\ln\left(\tilde{X}_t\right) = -\eta_f(\hat{\pi}_t^x-\hat{\pi}_t^*)+\hat{y}_t^*-\hat{y}_{t-1}^*+\hat{\tilde{z}}_t-\hat{\tilde{z}}_{t-1}+\hat{\mu}_t^z+\ln(\mu^z) \qquad (A.35)$$

Imports

$$\begin{aligned}\Delta\ln\left(\tilde{M}_t\right) &= \left(\frac{c^m}{c^m+i^m}\right)\left[\eta_c\left(1-\vartheta_c\right)\left(\gamma^{c,d}\right)^{\eta_c-1}\left[\hat{\pi}^d_t-\hat{\pi}^{m,c}_t\right]+\hat{c}_t-\hat{c}_{t-1}\right]\\ &+ \left(\frac{i^m}{c^m+i^m}\right)\left[\eta_i\left(1-\vartheta_i\right)\left(\gamma^{i,d}\right)^{\eta_i-1}\left[\hat{\pi}^d_t-\hat{\pi}^{m,i}_t\right]+\hat{i}_t-\hat{i}_{t-1}\right]\\ &+ \hat{\mu}^z_t+\ln\left(\mu^z\right)\end{aligned} \tag{A.36}$$

Foreign GDP

$$\Delta\ln\left(\tilde{Y}^*_t\right) = \hat{y}^*_t-\hat{y}^*_{t-1}+\hat{\tilde{z}}_t-\hat{\tilde{z}}_{t-1}+\hat{\mu}^z_t+\ln\left(\mu^z\right) \tag{A.37}$$

Wages

$$\Delta\ln\left(\tilde{W}_t\right) = \hat{w}_t-\hat{w}_{t-1}+\hat{\pi}^d_t+\hat{\mu}^z_t+\ln\left(\mu^m\right) \tag{A.38}$$

Employment

$$\Delta\ln\left(\tilde{E}_t\right) = \hat{E}_t-\hat{E}_{t-1} \tag{A.39}$$

CPI inflation

$$\tilde{\pi}^c_t = \hat{\pi}^c_t+\ln\left(\pi\right) \tag{A.40}$$

Producer price inflation

$$\tilde{\pi}^d_t = \hat{\pi}^d_t+\ln\left(\pi\right) \tag{A.41}$$

Investment deflator

$$\tilde{\pi}^i_t = \hat{\pi}^i_t+\ln\left(\pi\right) \tag{A.42}$$

Foreign inflation

$$\tilde{\pi}^*_t = \hat{\pi}^*_t+\ln\left(\pi^*\right) \tag{A.43}$$

Nominal exchange rate

$$\Delta\ln\left(\tilde{S}_t\right) = \Delta\hat{S}_t+\ln\left(\frac{\pi}{\pi^*}\right) \tag{A.44}$$

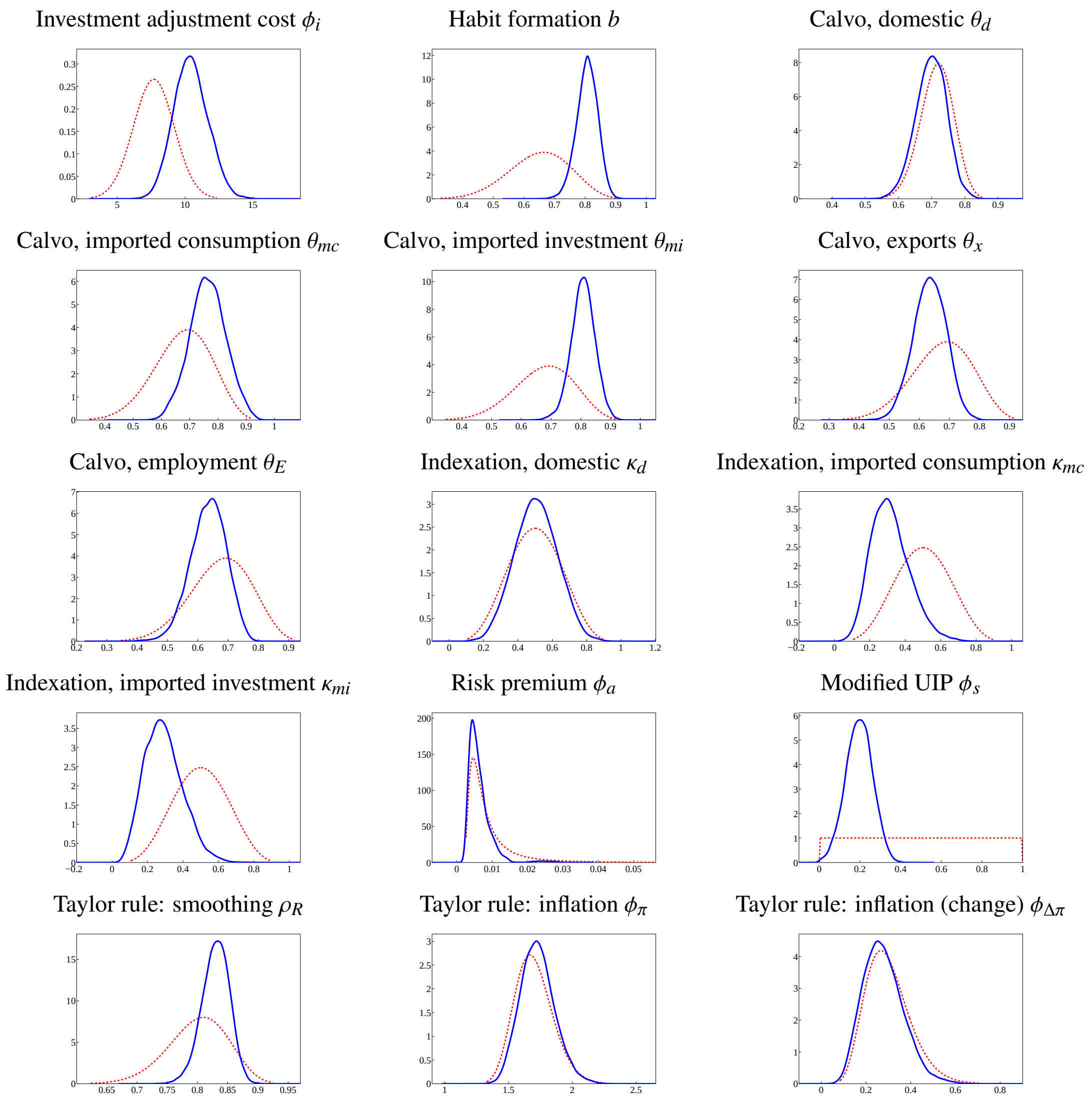
Repo rate

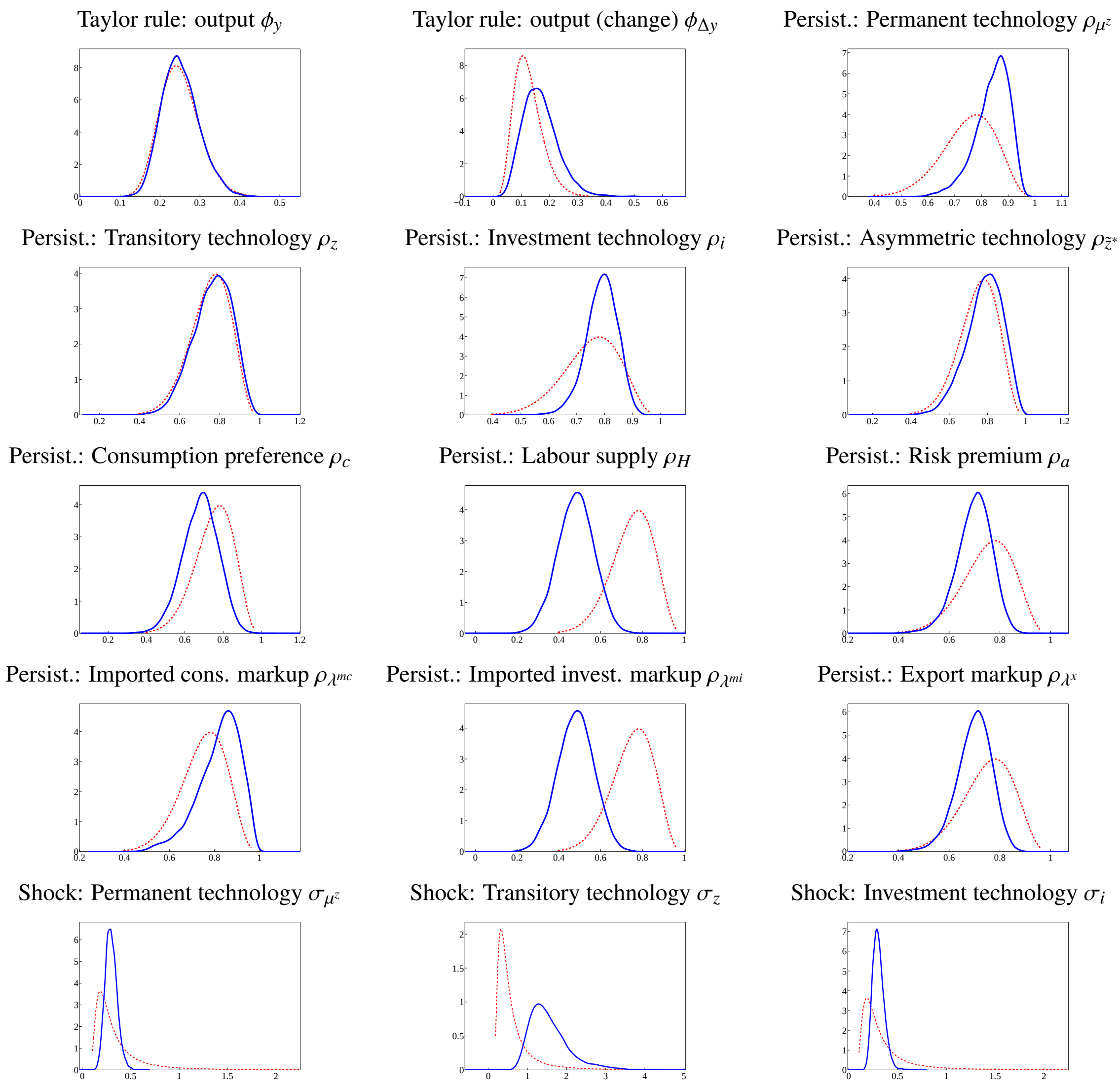
$$\tilde{R}^*_t = \hat{R}_t+\ln\left(R\right) \tag{A.45}$$

Foreign interest rate

$$\tilde{R}^*_t = \hat{R}^*_t+\ln\left(R^*\right) \tag{A.46}$$

Figure A.1: Prior and posterior density plots





Taylor rule: output ϕ_y
Taylor rule: output (change) $\phi_{\Delta y}$
Persist.: Permanent technology ρ_{μ^z}
Persist.: Transitory technology ρ_z
Persist.: Investment technology ρ_i
Persist.: Asymmetric technology $\rho_{\tilde{z}^*}$
Persist.: Consumption preference ρ_c
Persist.: Labour supply ρ_H
Persist.: Risk premium ρ_a
Persist.: Imported cons. markup $\rho_{\lambda^{mc}}$
Persist.: Imported invest. markup $\rho_{\lambda^{mi}}$
Persist.: Export markup ρ_{λ^x}
Shock: Permanent technology σ_{μ^z}
Shock: Transitory technology σ_z
Shock: Investment technology σ_i

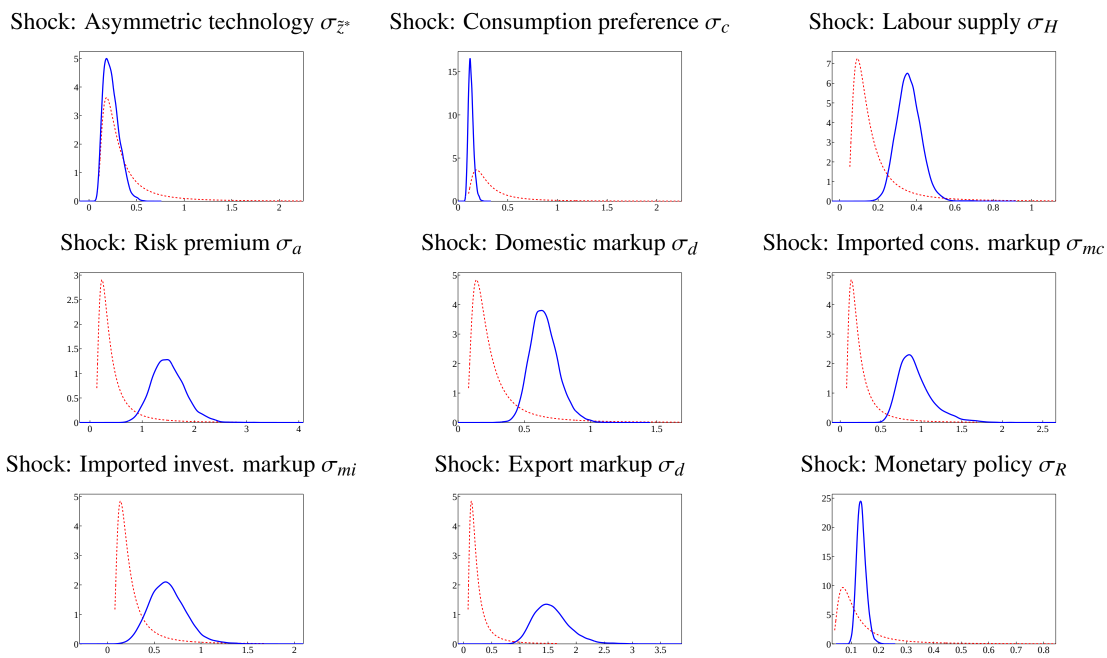


Figure A.2: Inflation target midpoint estimate: Early 2000s and before

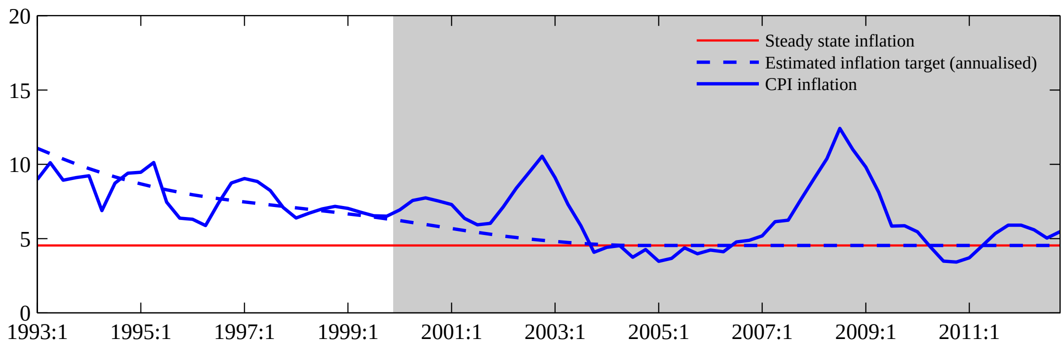


Figure A.3: Data plots with corresponding values predicted by the model

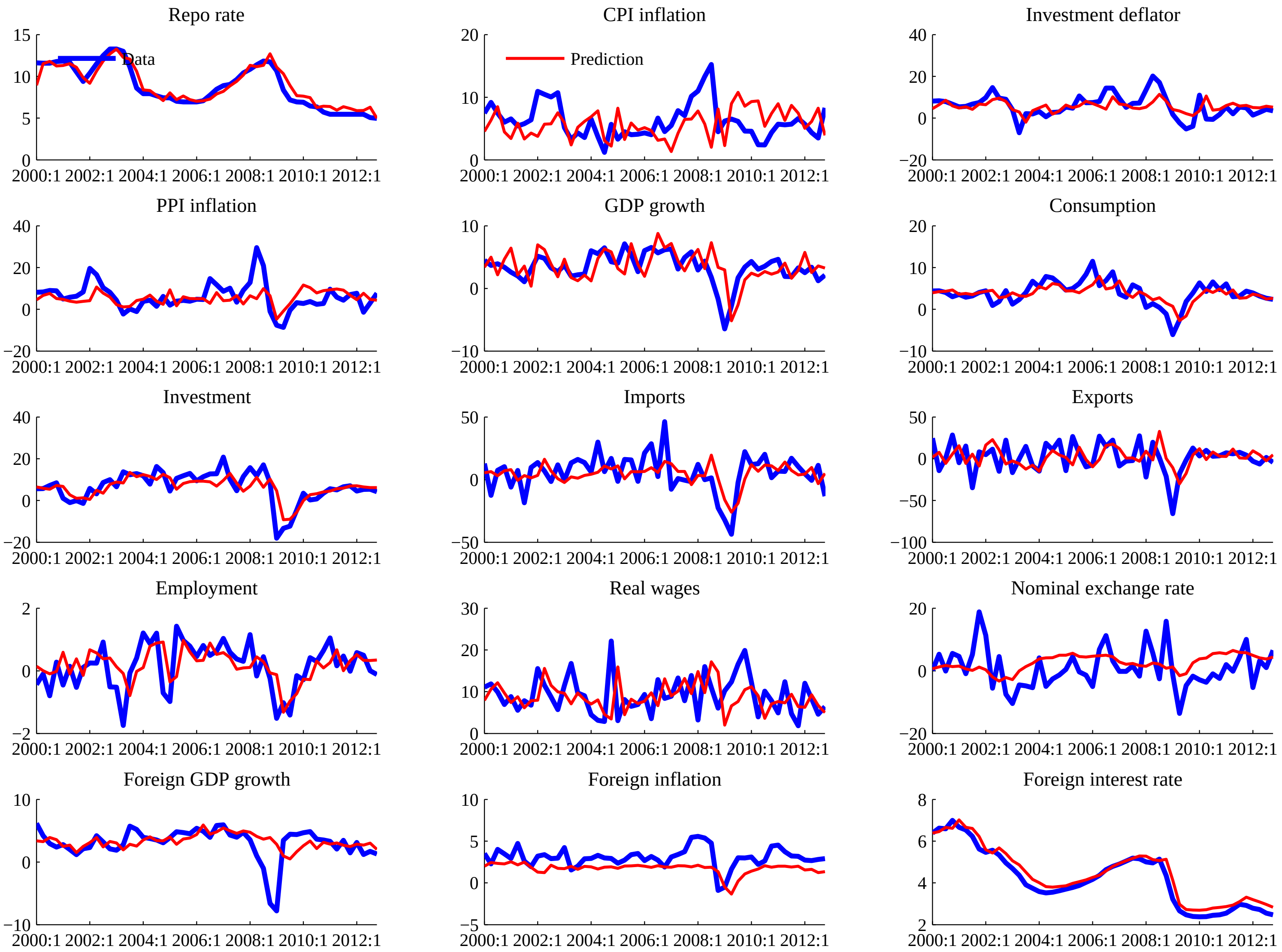


Table A.1: Second moments, cross- and autocorrelations: model and data

	Standard deviation	$c(.,\tilde{R}_t)$	$ac(1)$	$ac(2)$
$\tilde{R}_t$	1.57	1	0.97	0.88
	0.63		*0.92*	*0.79*
$\tilde{\pi}_t^d$	2.14	0.69	0.82	0.61
	0.70	*0.49*	*0.60*	*0.39*
$\Delta\ln(\tilde{Y}_t)$	1.96	-0.15	0.77	0.44
	0.60	*-0.16*	*0.66*	*0.33*
$\Delta\ln(\tilde{C}_t)$	1.00	-0.20	0.71	0.50
	0.72	*-0.37*	*0.68*	*0.45*
$\Delta\ln(\tilde{I}_t)$	2.74	0.02	0.73	0.56
	1.85	*0.10*	*0.69*	*0.42*
$\Delta\ln(\tilde{X}_t)$	5.90	-0.13	0.70	0.35
	4.35	*-0.10*	*-0.03*	*0.05*
$\Delta\ln(\tilde{M}_t)$	2.99	-0.03	0.59	0.33
	3.72	*-0.20*	*0.23*	*0.24*
$\Delta\ln(\tilde{S}_t)$	5.96	0.20	0.18	0.00
	6.28	*0.00*	*0.22*	*-0.01*
$\Delta\ln(\tilde{E}_t)$	1.44	-0.20	0.76	0.51
	0.72	*-0.33*	*0.40*	*0.24*
$\Delta\ln(\tilde{W}_t)$	2.32	0.61	0.72	0.52
	1.12	*0.19*	*-0.03*	*-0.02*
$\tilde{\pi}_t^i$	1.92	0.62	0.78	0.57
	1.34	*0.32*	*0.63*	*0.31*
$\tilde{\pi}_t^d$	2.38	0.60	0.80	0.53
	1.63	*0.25*	*0.60*	*0.13*

Statistics for the data are in italics

Table A.2: Matrix of variable cross correlations: model and data

Variables	$\tilde{R}_t$	$\tilde{\pi}^c_t$	$\Delta\ln(\tilde{Y}_t)$	$\Delta\ln(\tilde{C}_t)$	$\Delta\ln(\tilde{I}_t)$	$\Delta\ln(\tilde{E}_t)$	$\Delta\ln(\tilde{S}_t)$	$\Delta\ln(\tilde{X}_t)$	$\Delta\ln(\tilde{M}_t)$	$\Delta\ln(\tilde{W}_t)$	$\tilde{\pi}^i_t$	$\tilde{\pi}^d_t$
$\tilde{R}_t$	1	0.69 *0.49*	-0.15 *-0.16*	-0.20 *-0.37*	0.02 *0.10*	-0.20 *-0.33*	0.20 *0.00*	-0.13 *-0.10*	-0.03 *-0.20*	0.61 *0.19*	0.62 *0.32*	0.60 *0.25*
$\tilde{\pi}^c_t$	0.69 *0.49*	1	-0.44 *-0.05*	-0.51 *-0.37*	-0.15 *0.11*	-0.49 *-0.03*	0.04 *0.19*	-0.31 *0.01*	-0.04 *-0.23*	0.71 *0.26*	0.73 *0.60*	0.89 *0.72*
$\Delta\ln(\tilde{Y}_t)$	-0.15 *-0.16*	-0.44 *-0.05*	1	0.41 *0.76*	0.23 *0.61*	0.84 *0.59*	0.08 *0.10*	0.84 *0.63*	-0.12 *0.64*	-0.30 *-0.07*	-0.25 *0.29*	-0.50 *0.43*
$\Delta\ln(\tilde{C}_t)$	-0.20 *-0.37*	-0.51 *-0.37*	0.41 *0.76*	1	0.30 *0.47*	0.33 *0.46*	0.08 *-0.02*	0.17 *0.50*	0.32 *0.67*	-0.19 *-0.14*	-0.19 *0.06*	-0.39 *0.10*
$\Delta\ln(\tilde{I}_t)$	0.02 *0.10*	-0.15 *0.11*	0.23 *0.61*	0.30 *0.47*	1	0.15 *0.54*	0.02 *0.16*	0.00 *0.37*	0.30 *0.43*	0.03 *-0.15*	-0.19 *0.45*	-0.15 *0.43*
$\Delta\ln(\tilde{E}_t)$	-0.20 *-0.33*	-0.49 *-0.03*	0.84 *0.59*	0.33 *0.46*	0.15 *0.54*	1	0.10 *0.24*	0.75 *0.28*	-0.11 *0.38*	-0.46 *-0.44*	-0.27 *0.33*	-0.51 *0.30*
$\Delta\ln(\tilde{S}_t)$	0.20 *0.00*	0.04 *0.19*	0.08 *0.10*	0.08 *-0.02*	0.02 *0.16*	0.10 *0.24*	1	0.09 *-0.01*	0.05 *0.03*	0.07 *-0.24*	0.25 *0.41*	0.09 *0.35*
$\Delta\ln(\tilde{X}_t)$	-0.13 *-0.10*	-0.31 *0.01*	0.84 *0.63*	0.17 *0.50*	0.00 *0.37*	0.75 *0.28*	0.09 *-0.01*	1	0.04 *0.67*	-0.22 *0.09*	-0.07 *0.14*	-0.23 *0.37*
$\Delta\ln(\tilde{M}_t)$	-0.03 *-0.20*	-0.04 *-0.23*	-0.12 *0.64*	0.32 *0.67*	0.30 *0.43*	-0.11 *0.38*	0.05 *0.03*	0.04 *0.67*	1	0.18 *-0.08*	0.18 *0.08*	0.33 *0.24*
$\Delta\ln(\tilde{W}_t)$	0.61 *0.19*	0.71 *0.26*	-0.30 *-0.07*	-0.19 *-0.14*	0.03 *-0.15*	-0.46 *-0.44*	0.07 *-0.24*	-0.22 *0.09*	0.18 *-0.08*	1	0.61 *-0.11*	0.71 *0.15*
$\tilde{\pi}^i_t$	0.62 *0.32*	0.73 *0.60*	-0.25 *0.29*	-0.19 *0.06*	-0.19 *0.45*	-0.27 *0.33*	0.25 *0.41*	-0.07 *0.14*	0.18 *0.08*	0.61 *-0.11*	1	0.82 *0.76*
$\tilde{\pi}^d_t$	0.60 *0.25*	0.89 *0.72*	-0.50 *0.43*	-0.39 *0.10*	-0.15 *0.43*	-0.51 *0.30*	0.09 *0.35*	-0.23 *0.37*	0.33 *0.24*	0.71 *0.15*	0.82 *0.76*	1

Statistics for the data are in italics

Figure A.4: Autocorrelations of the model compared to the data

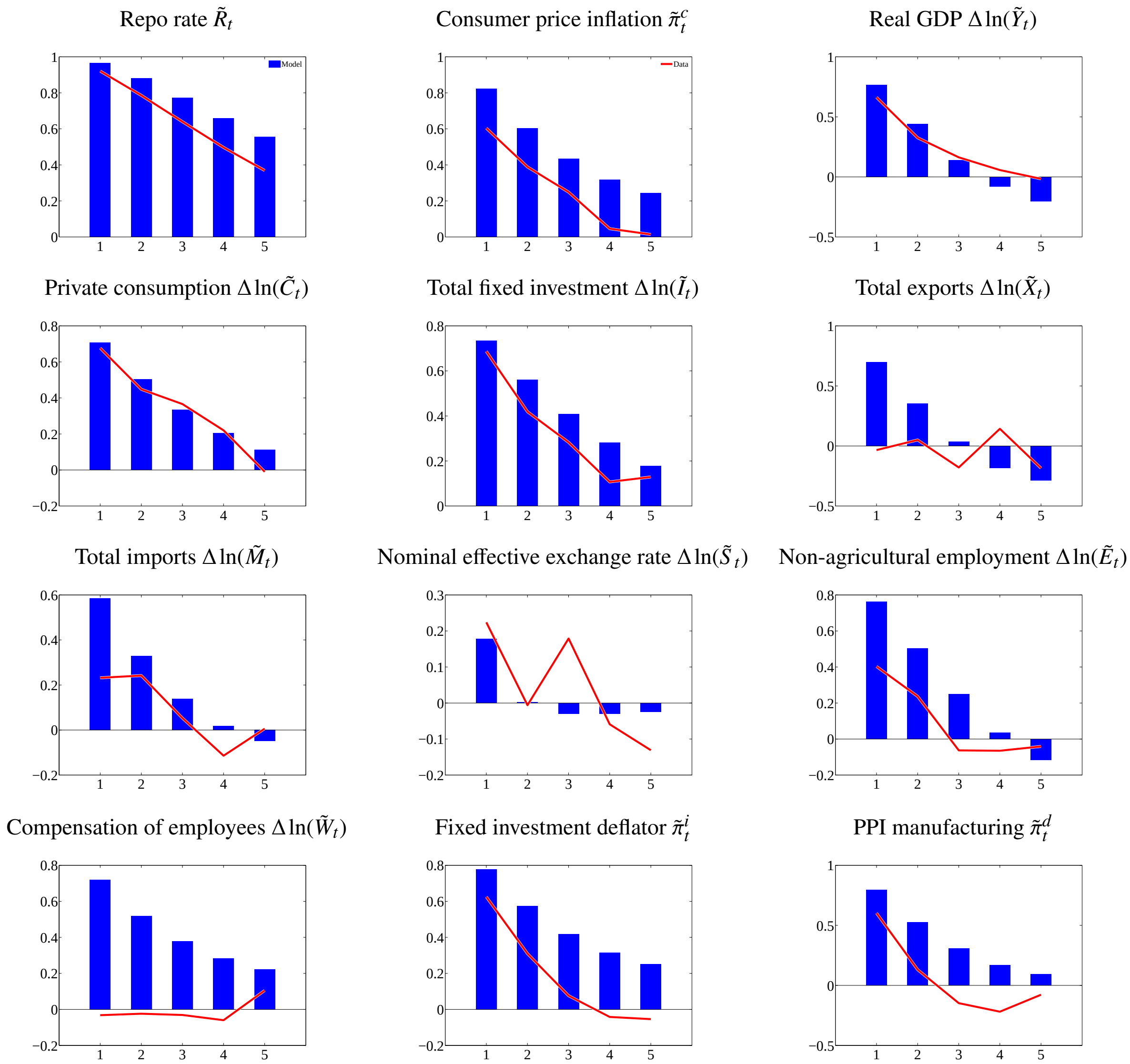
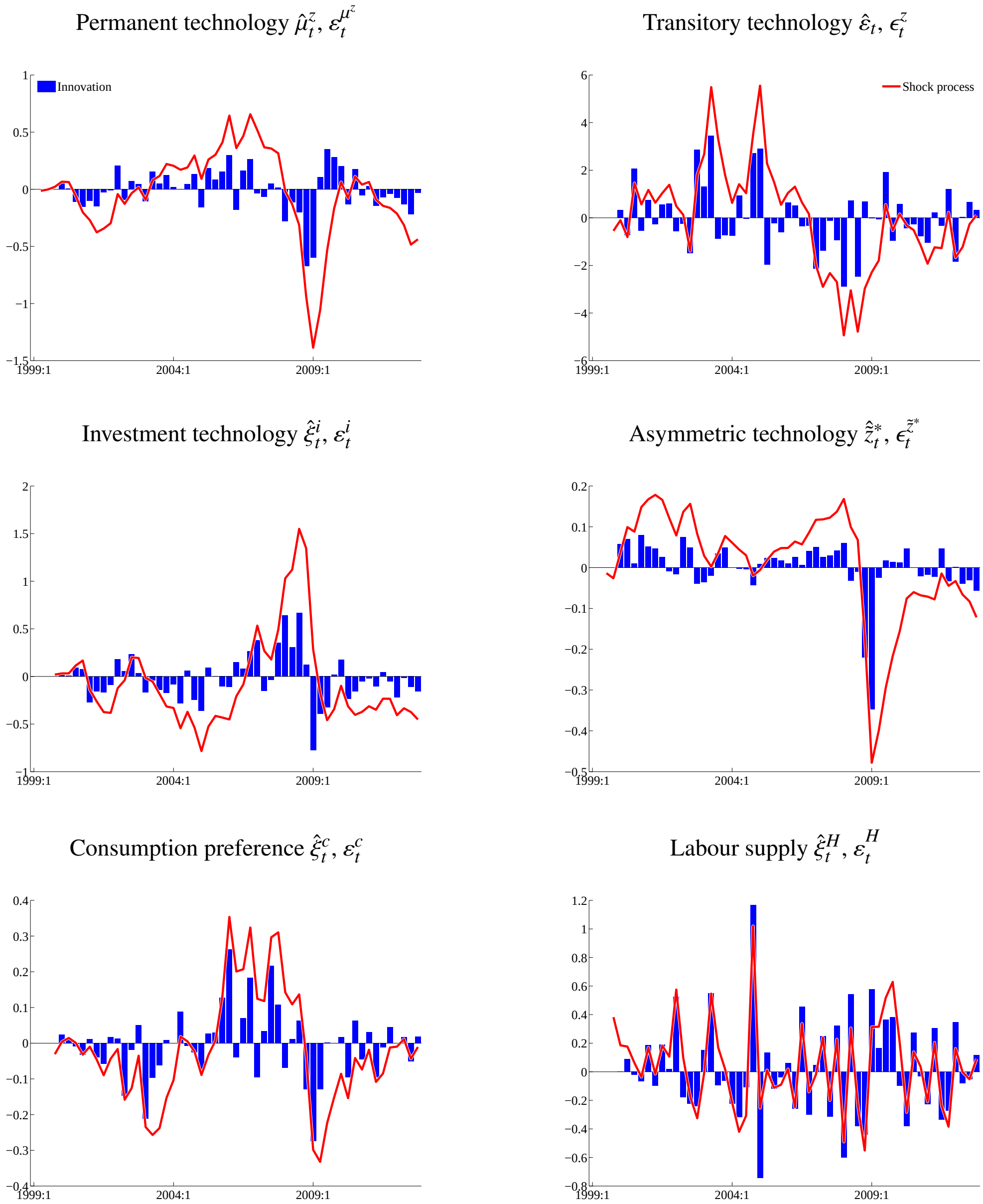


Table A.3: Variance decomposition

	Supply				Demand			Labour	Permanent technology	Export markups	Exchange rate risk	Monetary policy	Foreign shocks
	ε_t^{ϵ}	ε_t^{d}	$\varepsilon_t^{m,c}$	$\varepsilon_t^{m,i}$	ε_t^{c}	ε_t^{i}	ε_t^{g}	ε_t^{H}	$\varepsilon_t^{\mu^z}$	ε_t^{x}	$\varepsilon_t^{\tilde{\phi}}$	ε_t^{r}	$\varepsilon_t^{i,*}$
Repo rate	2.4	12.7	35.9	0.0	0.2	1.7	0.0	28.6	3.3	2.9	9.9	1.1	1.2
CPI inflation	4.0	25.4	29.7	0.0	0.0	2.7	0.0	32.9	2.0	0.2	2.7	0.1	0.3
Output	3.6	19.3	10.0	0.2	0.3	5.9	0.1	43.6	2.6	10.8	3.0	0.2	0.2
Consumption	1.0	3.4	26.0	1.1	7.2	13.3	0.0	23.0	20.9	0.5	2.5	0.1	1.0
Investment	0.8	3.7	17.9	8.7	0.7	37.8	0.0	9.2	12.2	0.4	5.2	0.1	3.3
Employment	1.7	16.3	9.1	0.0	0.2	1.3	0.0	62.3	0.6	6.5	1.8	0.2	0.1
Nominal exch. Rate	0.4	2.0	24.7	0.1	0.1	0.3	0.0	3.6	0.4	3.8	62.5	0.8	1.5
Real exch. Rate	0.7	3.8	66.0	0.2	0.1	0.7	0.0	7.0	0.7	3.0	17.1	0.2	0.5
Exports	1.3	6.5	28.5	0.4	0.1	1.9	0.0	15.6	1.4	36.0	7.2	0.1	1.0
Imports	1.6	10.6	44.1	5.4	1.3	6.1	0.0	14.5	3.2	1.1	8.9	0.0	3.3
Real wage	7.4	49.9	5.2	0.3	0.1	10.2	0.0	19.5	6.7	0.5	0.2	0.0	0.1

Figure A.5: Structural shock processes and their innovations



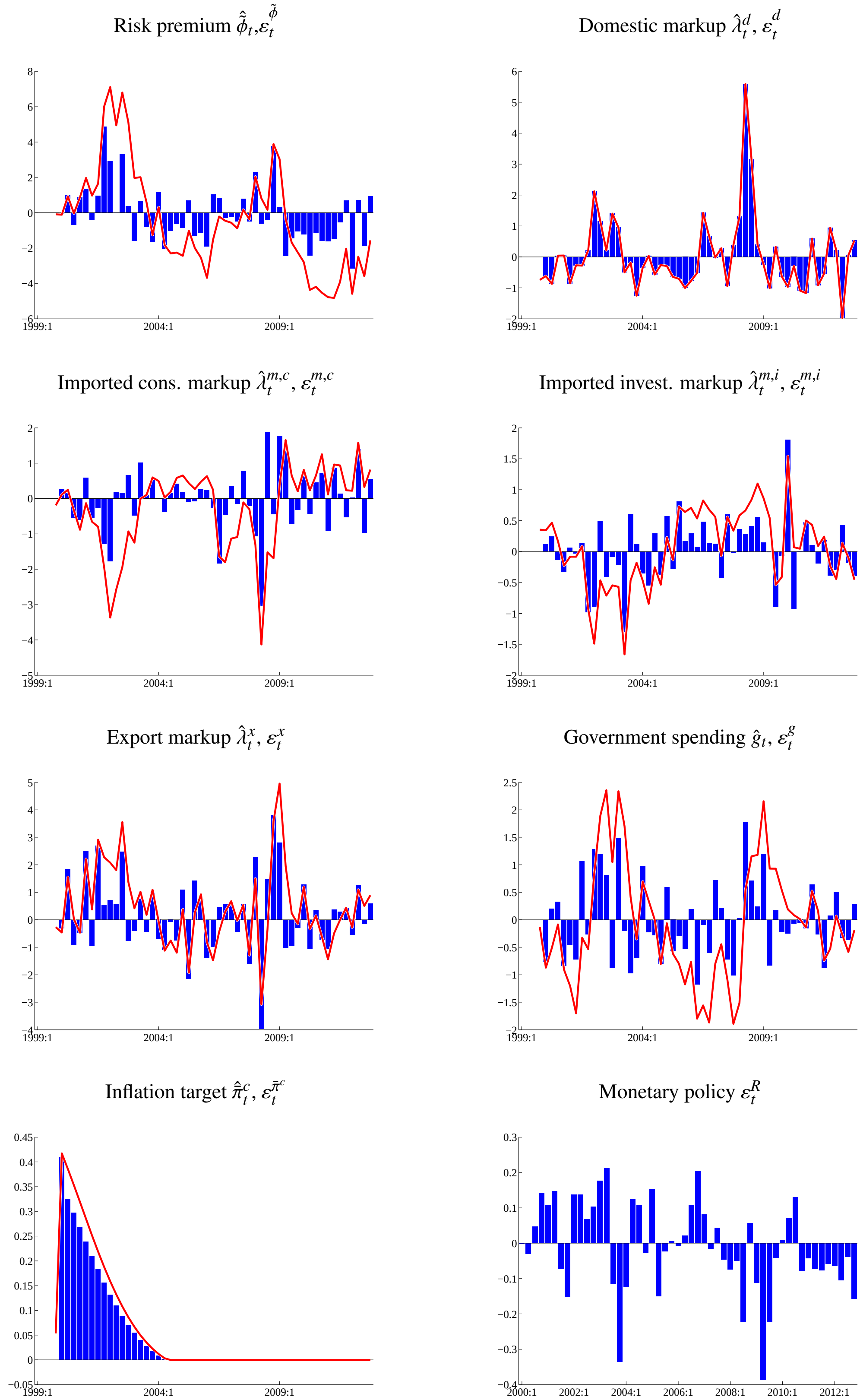

Risk premium $\hat{\tilde{\phi}}_t, \varepsilon_t^{\tilde{\phi}}$
Domestic markup $\hat{\lambda}_t^d$, ε_t^d
Imported cons. markup $\hat{\lambda}_t^{m,c}$, $\varepsilon_t^{m,c}$
Imported invest. markup $\hat{\lambda}_t^{m,i}$, $\varepsilon_t^{m,i}$
Export markup $\hat{\lambda}_t^x$, ε_t^x
Government spending $\hat{g}_t$, ε_t^g
Inflation target $\hat{\bar{\pi}}_t^c$, $\varepsilon_t^{\bar{\pi}^c}$
Monetary policy ε_t^R
1999:1
2004:1
2009:1
2000:1
2002:1
2004:1
2006:1
2008:1
2010:1
2012:1

Figure A.6: Monetary policy shock

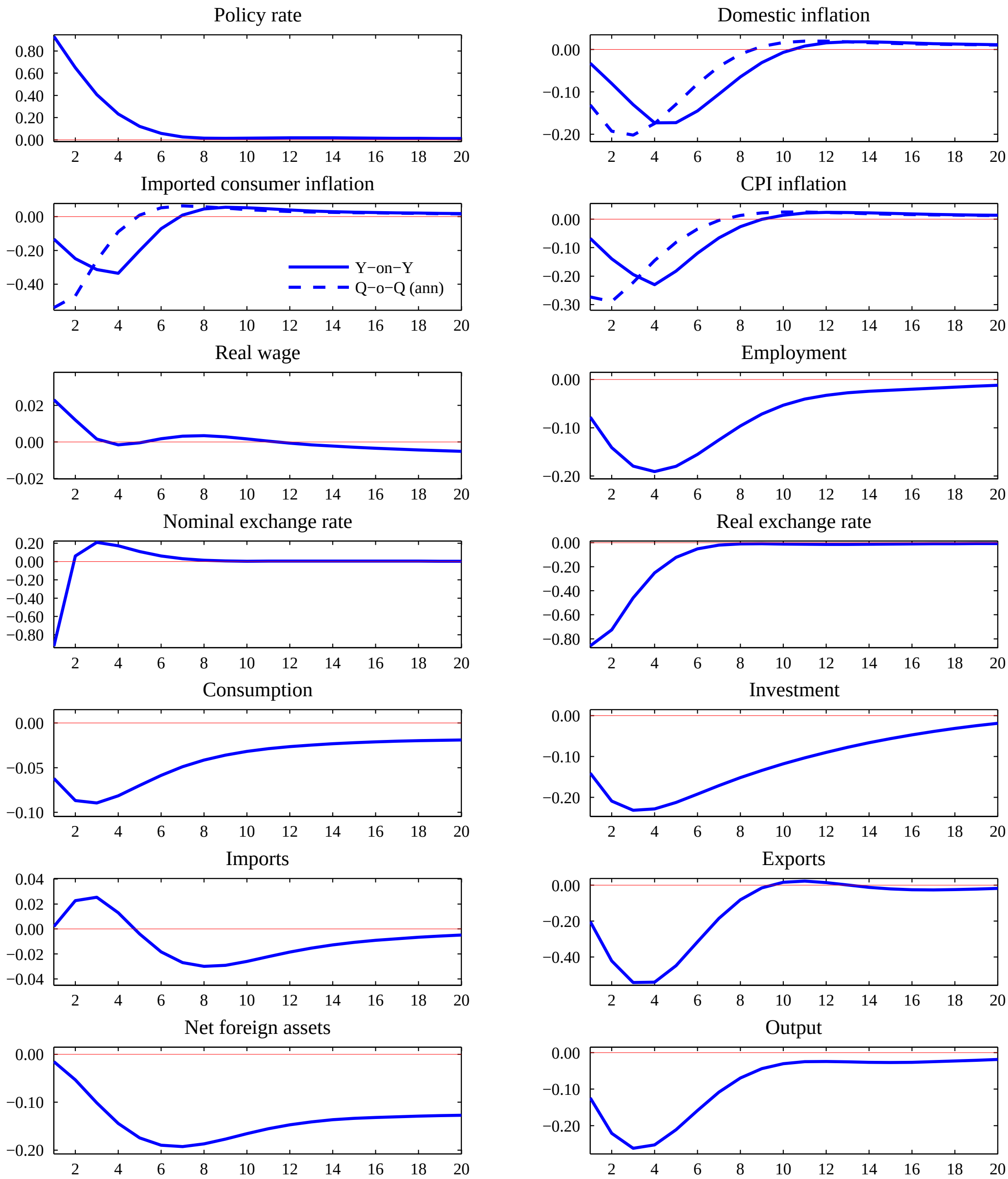


Figure A.7: Risk premium shock

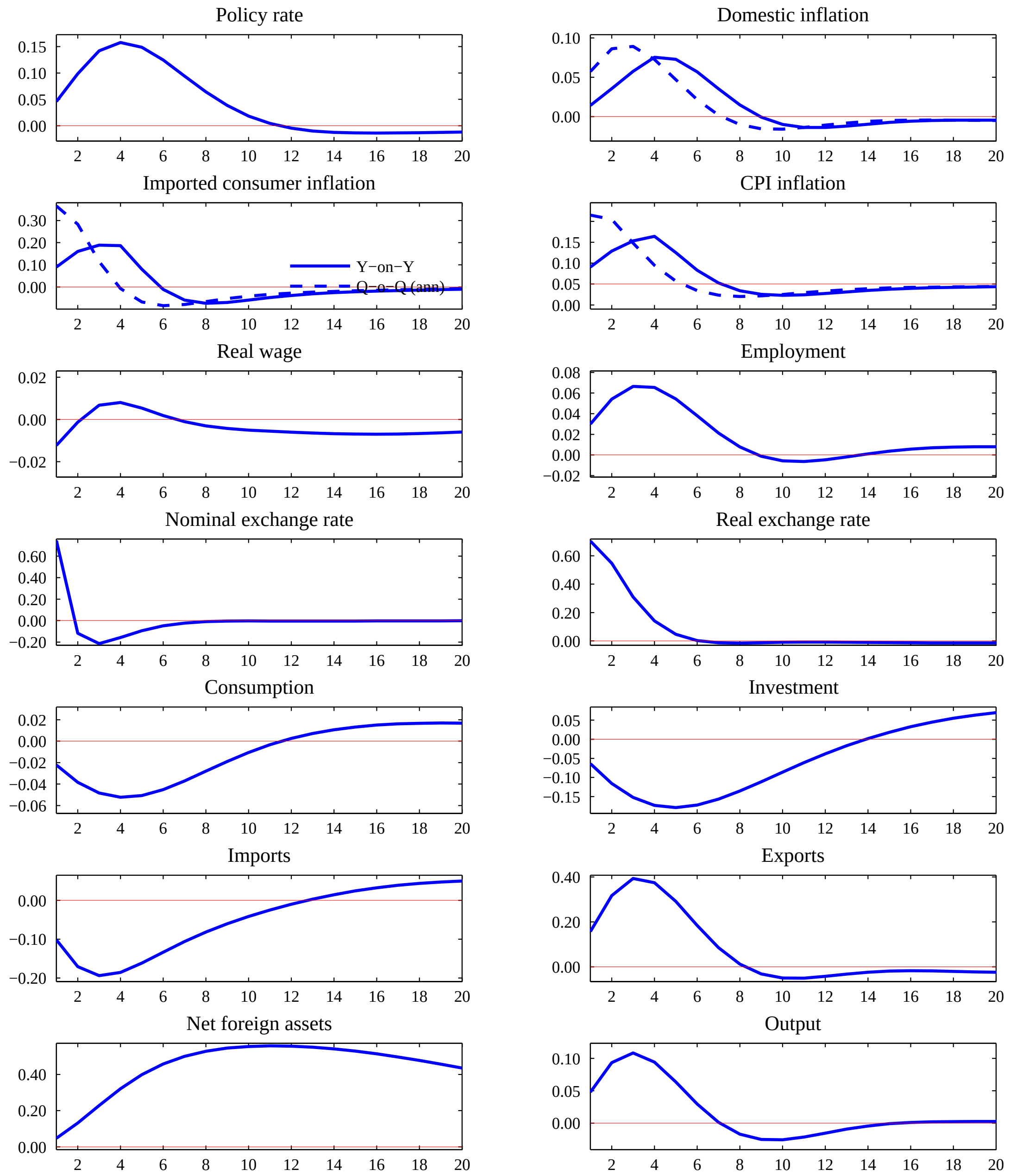


Figure A.8: Transitory technology shock

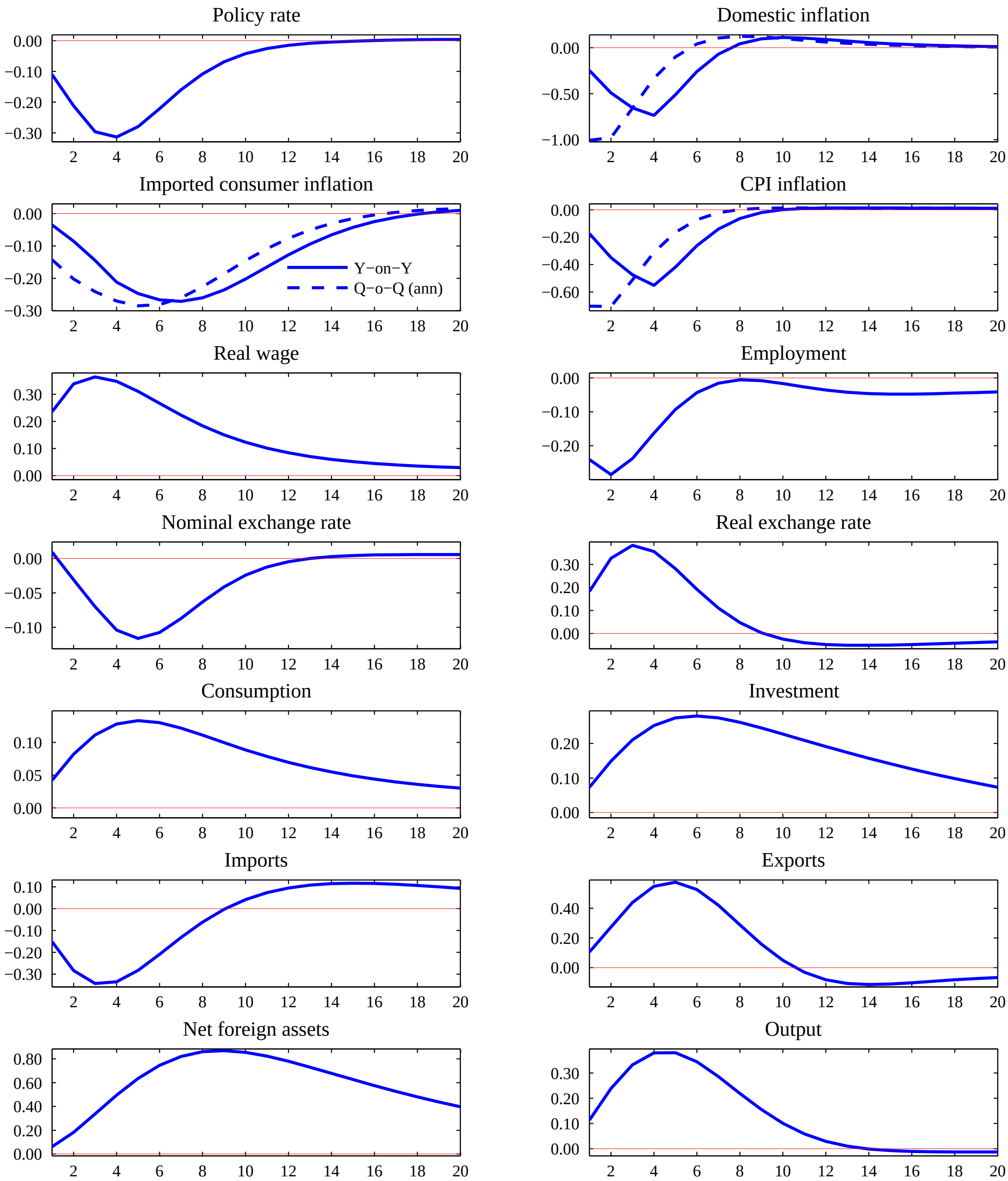


Figure A.9: Permanent technology premium shock

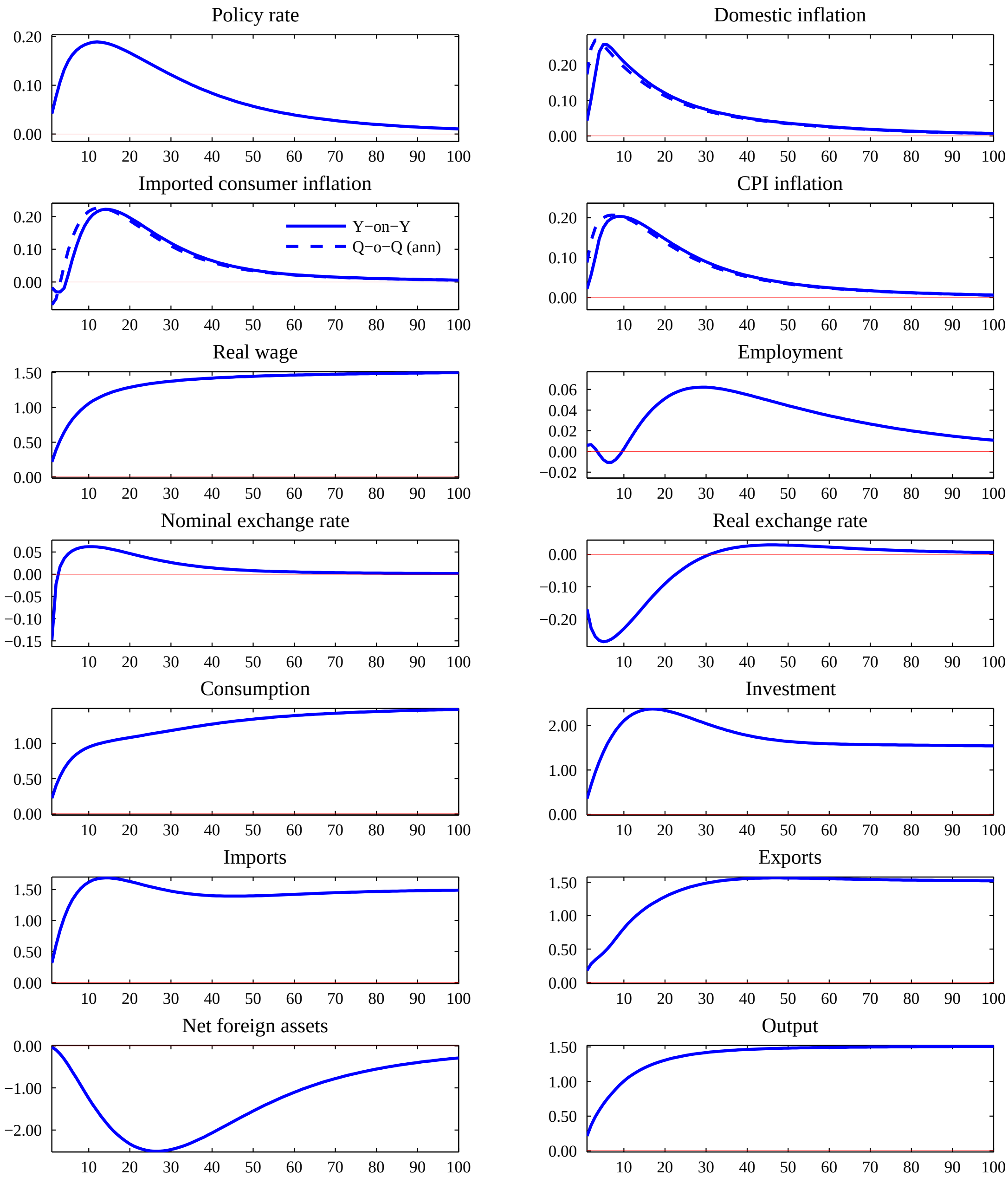


Figure A.10: Labour supply shock

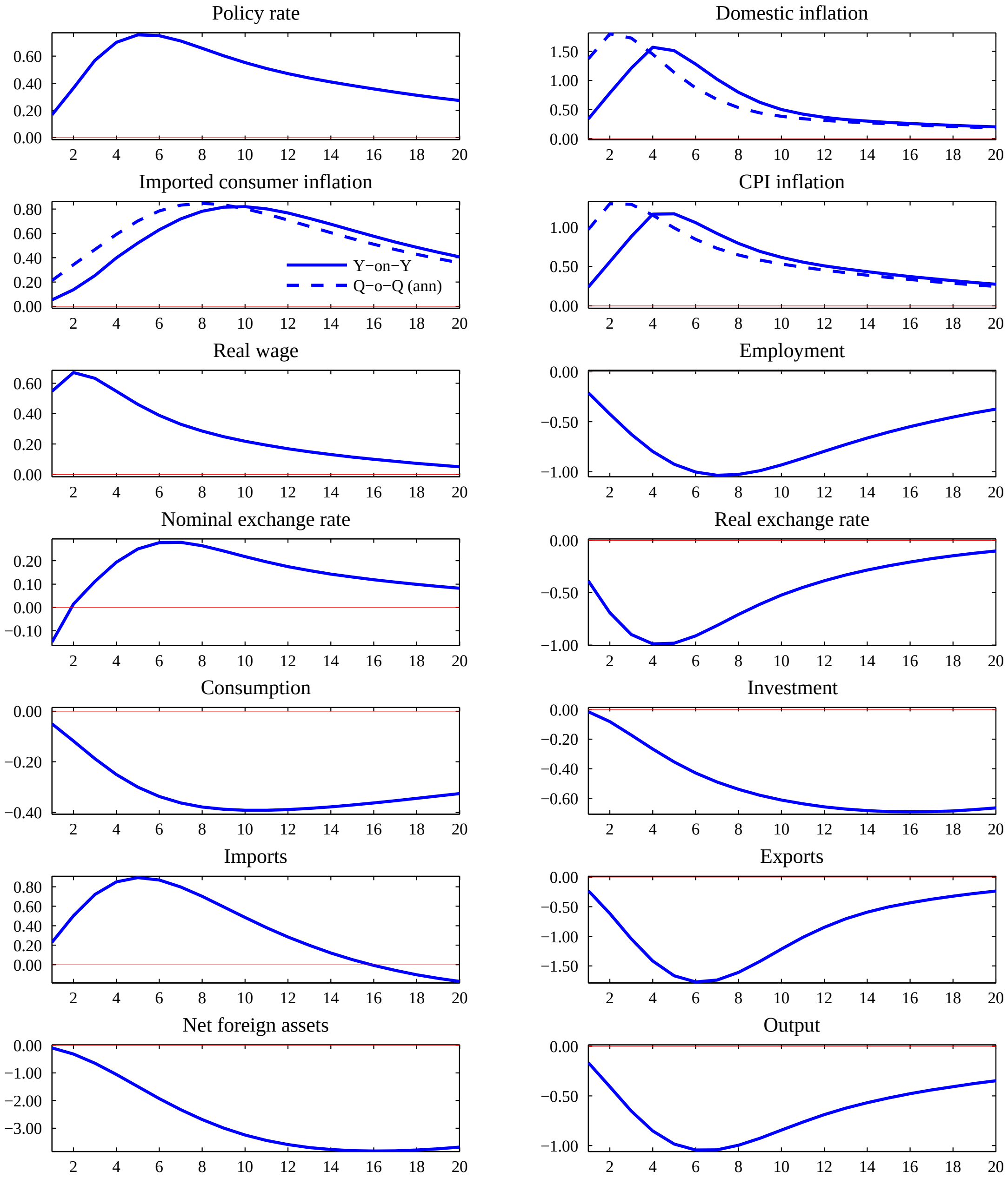


Figure A.11: Foreign output shock

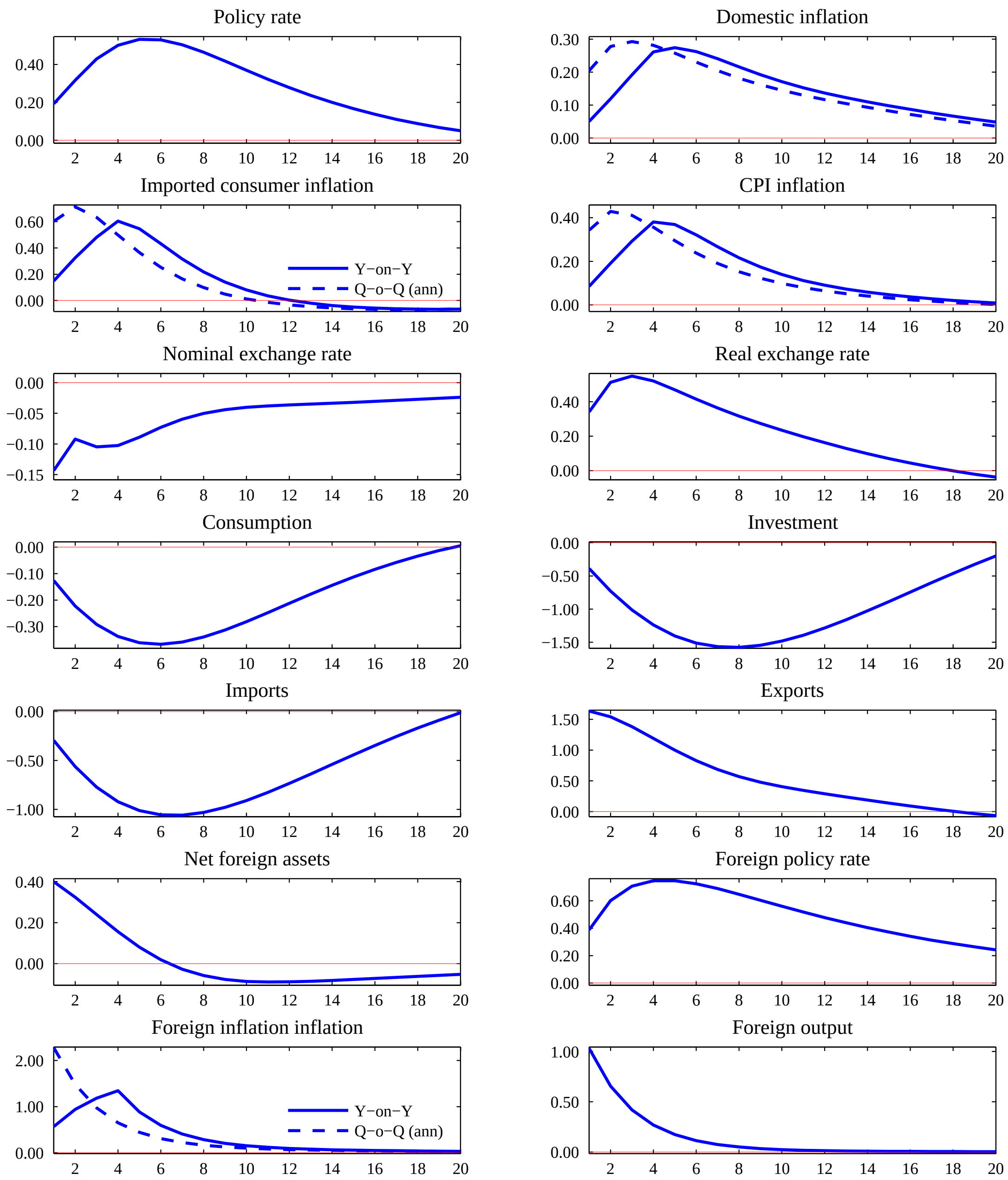
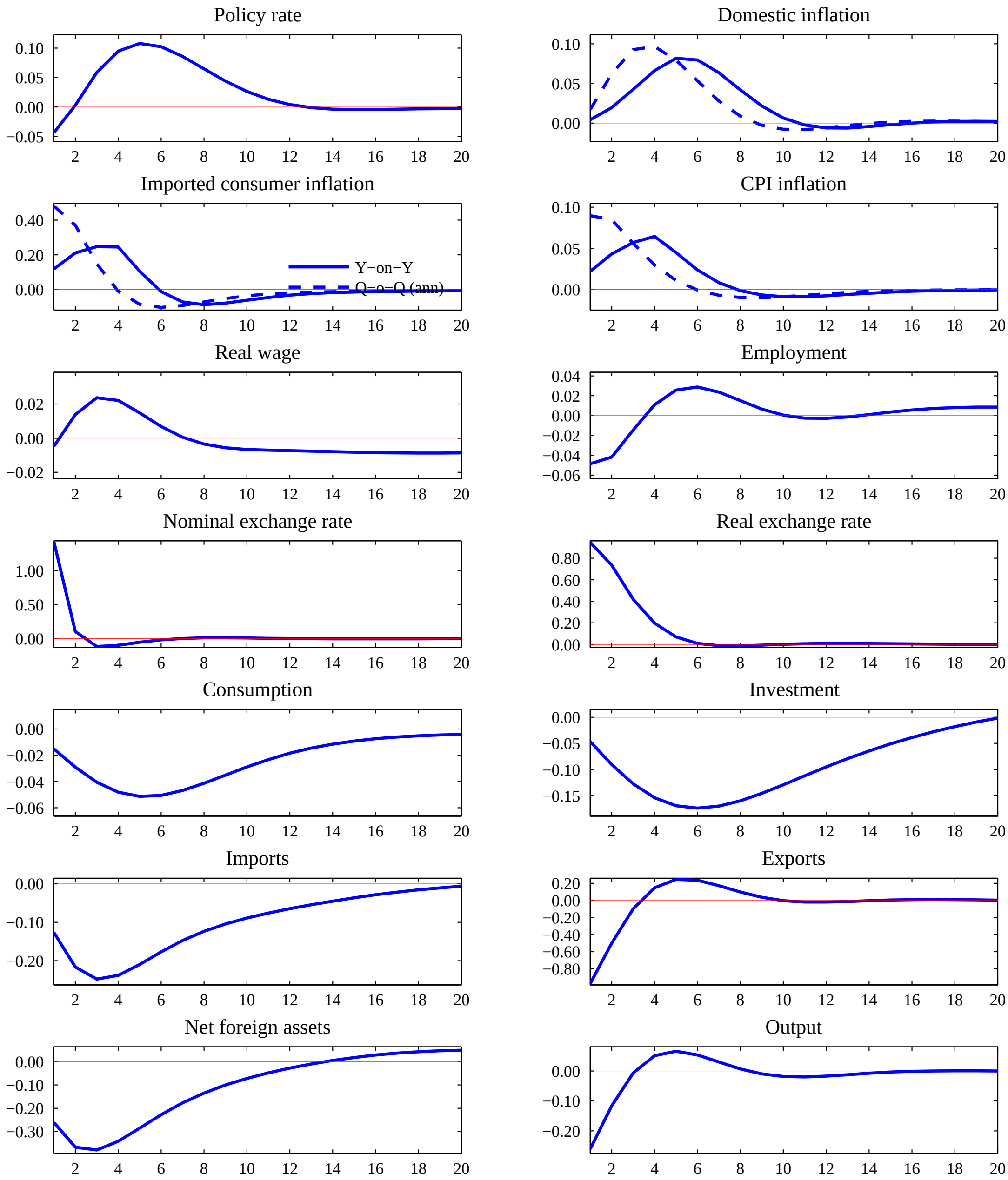


Figure A.12: Foreign policy shock



Appendix B

A structural decomposition of the South African yield curve

B.1 The linearised model

Households

Optimal L-period bond holdings

$$\hat{R}_{L,t} = \frac{1}{L}\left[\left(\frac{R_L}{R}\right)^L + \beta\phi_L y\right]^{-1}\Bigg\{\beta\phi_L y E_t\left[L\hat{R}_{L,t+1} - \hat{\psi}^z_{t+1} - 3\left(\hat{b}_{L,t+1} - \hat{b}_{L,t}\right) - y_{t+1}\right]$$
$$+\left(1+\frac{3}{2}\phi_L y\right)\hat{\psi}^z_t + \frac{3}{2}\phi_L y\left[2\left(\hat{b}_{L,t} - \hat{b}_{L,t-1}\right) + y_t\right] - \frac{\nu_L}{\kappa_L}(R_L)^L y\left[\hat{m}_t - \hat{b}_{L,t}\right]$$
$$+\left(\frac{R_L}{R}\right)^L E_t\left[\sum_{k=1}^{L}\hat{\mu}^z_{t+k} + \sum_{k=1}^{L}\hat{\pi}^d_{t+k} - \psi^{\hat{z}}_{t+L}\right]\Bigg\} \tag{B.1}$$

Money holdings

$$\hat{m}_t = \left[\frac{A_m \sigma^m m^{-\sigma_m}}{\psi^z} + \nu_L y\right]^{-1}\left\{\frac{1}{R}E_t\left[\hat{\psi}^z_{t+1} - \hat{\pi}^d_{t+1} - \hat{\mu}^z_{t+1}\right] - \hat{\psi}^z_t + \nu_L y\hat{b}_{L,t}\right\} \tag{B.2}$$

Wage setting

$$\hat{w}_t = -\frac{1}{\eta_1}\left[\begin{array}{l} \eta_0\hat{w}_{t-1} + \eta_2 E_t\hat{w}_{t+1} + \eta_3\left(\hat{\pi}^d_t - \hat{\bar{\pi}}^c_t\right) + \eta_4\left(E_t\hat{\pi}^d_{t+1} - \rho_\pi\hat{\bar{\pi}}^c_t\right) \\ +\eta_5\left(\hat{\pi}^c_{t-1} - \hat{\bar{\pi}}^c_t\right) + \eta_6\left(\hat{\pi}^c_t - \rho_\pi\hat{\bar{\pi}}^c_t\right) + \eta_7\hat{\psi}^z_t + \eta_8\hat{H}_t + \eta_9\hat{\xi}^h_t \end{array}\right] \tag{B.3}$$

Consumption Euler equation

$$\begin{aligned} \hat{c}_t &= \frac{\mu^z b}{(\mu^z)^2 + \beta b^2}\hat{c}_{t-1} + \frac{\beta\mu^z b}{(\mu^z)^2 + \beta b^2}E_t\hat{c}_{t+1} - \frac{\mu^z b}{(\mu^z)^2 + \beta b^2}\left(\hat{\mu}^z_t - \beta E_t\hat{\mu}^z_{t+1}\right) \\ &- \frac{(\mu^z - b)(\mu^z - \beta b)}{(\mu^z)^2 + \beta b^2}\left(\hat{\psi}^z_t + \hat{\gamma}^{c,d}_t\right) + \frac{\mu^z - b}{(\mu^z)^2 + \beta b^2}\left(\mu^z\hat{\xi}^c_t - \beta b E_t\hat{\xi}^c_{t+1}\right) \end{aligned} \tag{B.4}$$

Investment Euler equation

$$\hat{i}_t = \frac{1}{1+\beta}\left[\beta E_t \hat{i}_{t+1} + \hat{i}_{t-1} + \beta E_t \hat{\mu}^z_{t+1} - \mu^z_t\right] + \frac{1}{(\mu^z)^2\phi_i(1+\beta)}\left(\hat{P}^k_t - \hat{\gamma}^{i,d}_t + \hat{\xi}^i_t\right) \tag{B.5}$$

Price of installed capital

$$\hat{P}^k_t = E_t\left[\frac{(1-\delta)\beta}{\mu^z}\hat{P}^k_{t+1} + \hat{\psi}^z_{t+1} - \hat{\psi}^z_t - \hat{\mu}^z_{t+1} + \frac{\mu^z-(1-\delta)\beta}{\mu^z}\hat{r}^k_{t+1}\right] \tag{B.6}$$

Capital's law-of-motion

$$\hat{k}_{t+1} = \frac{1-\delta}{\mu^z}\left(\hat{k}_t - \hat{\mu}^z_t\right) + \left(1 - \frac{1-\delta}{\mu^z}\right)\left(\hat{i}_t + \hat{\xi}^i_t\right) \tag{B.7}$$

Capital utilisation

$$\hat{u}_t = \frac{1}{\sigma_a}\hat{r}^k_t \tag{B.8}$$

Capital services

$$\hat{k}^s_t = \hat{k}_t + \hat{u}_t \tag{B.9}$$

Optimal 1-period holdings

$$\hat{\psi}^z_t = E_t\left(\hat{\psi}^z_{t+1} - \hat{\mu}^z_{t+1}\right) + \left(\hat{R}_t - E_t\hat{\pi}^d_{t+1}\right) \tag{B.10}$$

Modified UIP condition

$$\hat{R}_t - \hat{R}^*_t = (1-\tilde{\phi}_s)E_t\Delta\hat{S}_{t+1} - \tilde{\phi}_s\Delta\hat{S}_t - \tilde{\phi}_a\hat{a}_t + \hat{\tilde{\phi}}_t, \tag{B.11}$$

Firms

Domestic goods

Production

$$\hat{y}_t = \lambda^d\left(\hat{\varepsilon}_t + \alpha\left(\hat{k}^s_t - \hat{\mu}^z_t\right) + (1-\alpha)\,\hat{H}_t\right) \tag{B.12}$$

Rental rate of capital

$$\hat{r}^k_t = \hat{w}_t + \hat{\mu}^z_t - \hat{k}^s_t + \hat{H}_t \tag{B.13}$$

Real marginal cost

$$\hat{mc}^d_t = \alpha\,\hat{r}^k_t + (1-\alpha)\,(\hat{w}_t) - \hat{\varepsilon}_t \tag{B.14}$$

New Keynesian Phillips curve

$$\begin{aligned}\hat{\pi}^d_t - \hat{\bar{\pi}}^c_t &= \frac{\beta}{1+\beta\kappa_d}\left(E_t\hat{\pi}^d_{t+1} - \rho_\pi\hat{\bar{\pi}}^c_t\right) + \frac{\kappa_d}{1+\beta\kappa_d}\left(\hat{\pi}^d_{t-1} - \hat{\bar{\pi}}^c_t\right) - \frac{\beta\kappa_d\,(1-\rho_\pi)}{1+\beta\kappa_d}\hat{\bar{\pi}}^c_t \\ &+ \frac{(1-\theta_d)\,(1-\beta\theta_d)}{(1+\beta\kappa_d)\,\theta_d}\left(\hat{mc}^d_t + \hat{\lambda}^d_t\right)\end{aligned} \tag{B.15}$$

Imported goods

New Keynesian Phillips curve: imported consumption goods

$$\begin{aligned}\hat{\pi}_t^{m,c} - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_{m,c}}\left(E_t\hat{\pi}_{t+1}^{m,c} - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_{m,c}}{1+\beta\kappa_{m,c}}\left(\hat{\pi}_{t-1}^{m,c} - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_{m,c}\beta(1-\rho_\pi)}{1+\beta\kappa_{m,c}}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{m,c})(1-\beta\theta_{m,c})}{(1+\beta\kappa_{m,c})\theta_{m,c}}\left(\hat{mc}_t^{m,c} + \hat{\lambda}_t^{m,c}\right)\end{aligned} \tag{B.16}$$

Marginal cost: imported consumption goods

$$\hat{mc}_t^{m,c} = -\hat{\gamma}_t^f - \hat{\gamma}_t^{mc,d} \tag{B.17}$$

New Keynesian Phillips curve: imported investment goods

$$\begin{aligned}\hat{\pi}_t^{m,i} - \hat{\bar{\pi}}_t^c &= +\frac{\beta}{1+\beta\kappa_{m,i}}\left(E_t\hat{\pi}_{t+1}^{m,i} - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_{m,i}}{1+\beta\kappa_{m,i}}\left(\hat{\pi}_{t-1}^{m,i} - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_{m,i}\beta(1-\rho_\pi)}{1+\beta\kappa_{m,i}}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{m,i})(1-\beta\theta_{m,i})}{(1+\beta\kappa_{m,i})\theta_{m,i}}\left(\hat{mc}_t^{m,i} + \hat{\lambda}_t^{m,i}\right)\end{aligned} \tag{B.18}$$

Marginal cost: imported investment goods

$$\hat{mc}_t^{m,i} = -\hat{\gamma}_t^f - \hat{\gamma}_t^{mi,d} \tag{B.19}$$

Exported goods

New Keynesian Phillips curve: exported goods

$$\begin{aligned}\hat{\pi}_t^{x} - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_{x}}\left(E_t\hat{\pi}_{t+1}^{x} - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_{x}}{1+\beta\kappa_{x}}\left(\hat{\pi}_{t-1}^{x} - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_{x}\beta(1-\rho_\pi)}{1+\beta\kappa_{x}}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{x})(1-\beta\theta_{x})}{(1+\beta\kappa_{x})\theta_{x}}\left(\hat{mc}_t^{x} + \hat{\lambda}_t^{x}\right)\end{aligned} \tag{B.20}$$

Marginal cost: exported goods

$$\hat{mc}_t^x = \hat{mc}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta\hat{S}_t \tag{B.21}$$

Government

Liabilities

$$\ell\hat{\ell}_t = \frac{1}{\mu^z\pi}\left(b\hat{b}_{t-1} + m\hat{m}_{t-1} - (b+m)\left[\hat{\mu}_t^z + \hat{\pi}_t^d\right]\right) + \frac{b_L}{(\mu^z\pi)^L}\left(\hat{b}_{L,t-L} - \sum_{k=0}^{L-1}\hat{\pi}_{t-k}^d - \sum_{k=0}^{L-1}\hat{\mu}_{t-k}^z\right) \tag{B.22}$$

Budget constraint

$$g\hat{g}_t + \ell\hat{\ell}_t = \tau\hat{\tau}_t + m\hat{m}_t + \frac{b}{R}\left(\hat{b}_t - \hat{R}_t\right) + \frac{b_L}{(R_L)^L}\left(\hat{b}_{L,t} - L\hat{R}_{L,t}\right) \tag{B.23}$$

Fiscal rule

$$\hat{\tau}_t = \psi_1\frac{\ell}{\tau}\hat{\ell}_t \tag{B.24}$$

The Central Bank

Taylor rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + \phi_\pi\left(\hat{\pi}_{t+1}^{c,4} - \hat{\bar{\pi}}_t^c\right) + \phi_{\Delta\pi}\hat{\pi}_t^c + \phi_y \hat{y}_t + \phi_{\Delta y}\Delta\hat{y}_t\right] + \varepsilon_t^R. \tag{B.25}$$

where CPI inflation is given by

$$\hat{\pi}_t^c = (1-\vartheta_c)\left(\frac{1}{\gamma^{c,d}}\right)^{1-\eta_c}\hat{\pi}_t^d + \vartheta_c\left(\gamma^{mc,c}\right)^{1-\eta_c}\hat{\pi}_t^{m,c} \tag{B.26}$$

Relative prices

Consumption and investment goods

$$\hat{\gamma}_t^{c,d} = \hat{\gamma}_{t-1}^{i,d} + \hat{\pi}_t^c - \hat{\pi}_t^d \tag{B.27}$$

$$\hat{\gamma}_t^{i,d} = \hat{\gamma}_{t-1}^{i,d} + \hat{\pi}_t^i - \hat{\pi}_t^d \tag{B.28}$$

Imported consumption and investment goods

$$\hat{\gamma}_t^{mc,d} = \hat{\gamma}_{t-1}^{mc,d} + \hat{\pi}_t^{m,c} - \hat{\pi}_t^d \tag{B.29}$$

$$\hat{\gamma}_t^{mi,d} = \hat{\gamma}_{t-1}^{mi,d} + \hat{\pi}_t^{m,i} - \hat{\pi}_t^d \tag{B.30}$$

Export goods

$$\hat{\gamma}_t^{x,*} = \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \tag{B.31}$$

Domestic-foreign goods relative price

$$\hat{\gamma}_t^f = \hat{mc}_t^x + \hat{\gamma}_t^{x,*} \tag{B.32}$$

Real exchange rate

$$\hat{\gamma}_t^s = -\vartheta_c\left(\frac{1}{\gamma^{mc,c}}\right)^{\eta_c-1}\hat{\gamma}_t^{mc,d} - \hat{\gamma}_t^{x,*} - \hat{mc}_t^x \tag{B.33}$$

Market clearing

Domestic goods market

$$\begin{aligned}\hat{y}_t &= \frac{1}{1-\frac{\phi_L}{2}}\Bigg[(1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c}\frac{c}{y}\left(\hat{c}_t + \eta_c\hat{\gamma}_t^{c,d}\right) + (1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i}\frac{i}{y}\left(\hat{i}_t + \eta_i\hat{\gamma}_t^{i,d}\right)\\ &+ g_y\hat{g}_t + \frac{y^*}{y}\left(\hat{y}_t^* - \eta_f\hat{\gamma}_t^{x,*} + \hat{\tilde{z}}_t^*\right) + \frac{r^k}{\mu^z}\frac{k}{y}\left(\hat{k}_t^s - \hat{k}_t\right) + \phi_L\left(\hat{b}_{L,t} - \hat{b}_{L,t-1}\right)\Bigg]\end{aligned} \tag{B.34}$$

Foreign bond market

$$\begin{aligned}\hat{a}_t &= -y^*\hat{mc}_t^x - \eta_f y^*\hat{\gamma}_t^{x,*} + y^*\hat{y}_t^* + y^*\hat{\tilde{z}}_t^* + \left(c^m + i^m\right)\hat{\gamma}_t^f\\ &- \left[c^m\left(-\eta_c(1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c-1}\right)\hat{\gamma}_t^{mc,d} + \hat{c}_t\right]\\ &- \left[i^m\left(-\eta_i(1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i-1}\right)\hat{\gamma}_t^{mi,d} + \hat{i}_t\right]\\ &+ \frac{\pi^*}{\pi}\frac{1}{\beta}\hat{a}_{t-1}\end{aligned} \tag{B.35}$$

AR(1) shock processes

$$\Xi_t = \rho \Xi_{t-1} + \Gamma_t \tag{B.36}$$

where

$$\begin{array}{lclcccccccccccccc}
\Xi_t & = & [\hat{\xi}_t^c & \hat{\xi}_t^i & \hat{\tilde{\phi}}_t & \hat{\varepsilon}_t & \hat{\xi}_t^H & \hat{\lambda}_t^x & \hat{\lambda}_t^d & \hat{\lambda}_t^{m,c} & \hat{\lambda}_t^{m,i} & \hat{\tilde{z}}_t^* & \hat{\mu}_t^z & \hat{g}_t & \hat{\bar{\pi}}_t^c & \hat{b}_{L,t}]' \\
\rho & = & [\rho_c & \rho_i & \rho_{\tilde{\phi}} & \rho_\varepsilon & \rho_H & \rho_{\lambda^x} & \rho_d & \rho_{\lambda^{m,c}} & \rho_{\lambda^{m,i}} & \rho_{\tilde{z}^*} & \rho_{\mu^z} & \rho_g & \rho_{\bar{\pi}^c} & \rho_L]' \\
\Gamma_t & = & [\varepsilon_t^c & \varepsilon_t^i & \varepsilon_t^{\tilde{\phi}} & \varepsilon_t^\varepsilon & \varepsilon_t^H & \varepsilon_t^x & \varepsilon_t^d & \varepsilon_t^{m,c} & \varepsilon_t^{m,i} & \varepsilon_t^{\tilde{z}^*} & \varepsilon_t^{\mu^z} & \varepsilon_t^g & \varepsilon_t^{\bar{\pi}^c} & \varepsilon_t^L]'
\end{array}$$

Measurement equations

Output

$$\Delta \ln\left(\tilde{Y}_t\right) = \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_t^z + \ln(\mu^z) \tag{B.37}$$

Consumption

$$\begin{aligned}
\Delta \ln\left(\tilde{C}_t\right) &= \left(\frac{\eta_c}{c^d + c^m}\right)\left[c_d \vartheta_c (\gamma^{c,mc})^{\eta_c - 1} - c^m (1 - \vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c - 1}\right]\left(\hat{\pi}_t^{m,c} - \hat{\pi}_t^d\right) \\
&+ \hat{c}_t - \hat{c}_{t-1} + \mu_t^z + \ln(\mu^z)
\end{aligned} \tag{B.38}$$

Investment

$$\begin{aligned}
\Delta \ln\left(\tilde{I}_t\right) &= \left(\frac{\eta_i}{i^d + i^m}\right)\left[i_d \vartheta_i \left(\gamma^{i,mi}\right)^{\eta_i - 1} - i^m (1 - \vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i - 1}\right]\left(\hat{\pi}_t^{m,c} - \hat{\pi}_t^d\right) \\
&+ \hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^z + \ln(\mu^z)
\end{aligned} \tag{B.39}$$

Exports

$$\Delta \ln\left(\tilde{X}_t\right) = -\eta_f \left(\hat{\pi}_t^x - \hat{\pi}_t^*\right) + \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\tilde{z}}_t - \hat{\tilde{z}}_{t-1} + \hat{\mu}_t^z + \ln(\mu^z) \tag{B.40}$$

Imports

$$\begin{aligned}
\Delta \ln\left(\tilde{M}_t\right) &= \left(\frac{c^m}{c^m + i^m}\right)\left[\eta_c (1 - \vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c - 1}\left[\hat{\pi}_t^d - \hat{\pi}_t^{m,c}\right] + \hat{c}_t - \hat{c}_{t-1}\right] \\
&+ \left(\frac{i^m}{c^m + i^m}\right)\left[\eta_i (1 - \vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i - 1}\left[\hat{\pi}_t^d - \hat{\pi}_t^{m,i}\right] + \hat{i}_t - \hat{i}_{t-1}\right] \\
&+ \hat{\mu}_t^z + \ln(\mu^z)
\end{aligned} \tag{B.41}$$

Foreign GDP

$$\Delta \ln\left(\tilde{Y}_t^*\right) = \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\tilde{z}}_t - \hat{\tilde{z}}_{t-1} + \hat{\mu}_t^z + \ln(\mu^z) \tag{B.42}$$

Wages

$$\Delta \ln\left(\tilde{W}_t\right) = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t^d + \hat{\mu}_t^z + \ln(\mu^m) \tag{B.43}$$

Employment

$$\Delta \ln\left(\tilde{E}_t\right) = \hat{E}_t - \hat{E}_{t-1} \tag{B.44}$$

CPI inflation

$$\tilde{\pi}_t^c = \hat{\pi}_t^c + \ln(\pi) \tag{B.45}$$

Producer price inflation

$$\tilde{\pi}_t^d = \hat{\pi}_t^d + \ln(\pi) \tag{B.46}$$

Investment deflator

$$\tilde{\pi}_t^i = \hat{\pi}_t^i + \ln(\pi) \tag{B.47}$$

Foreign inflation

$$\tilde{\pi}_t^* = \hat{\pi}_t^* + \ln(\pi^*) \tag{B.48}$$

Nominal exchange rate

$$\Delta \ln\left(\tilde{S}_t\right) = \Delta \hat{S}_t + \ln\left(\frac{\pi}{\pi^*}\right) \tag{B.49}$$

M1 Money

$$\Delta \ln\left(\tilde{M}_t\right) = \hat{m}_t - \hat{m}_{t-1} + \mu_t^z + \ln(\mu^z) + \pi_t^d + \ln(\pi) \tag{B.50}$$

Repo rate

$$\tilde{R}_t = \hat{R}_t + \ln(R) \tag{B.51}$$

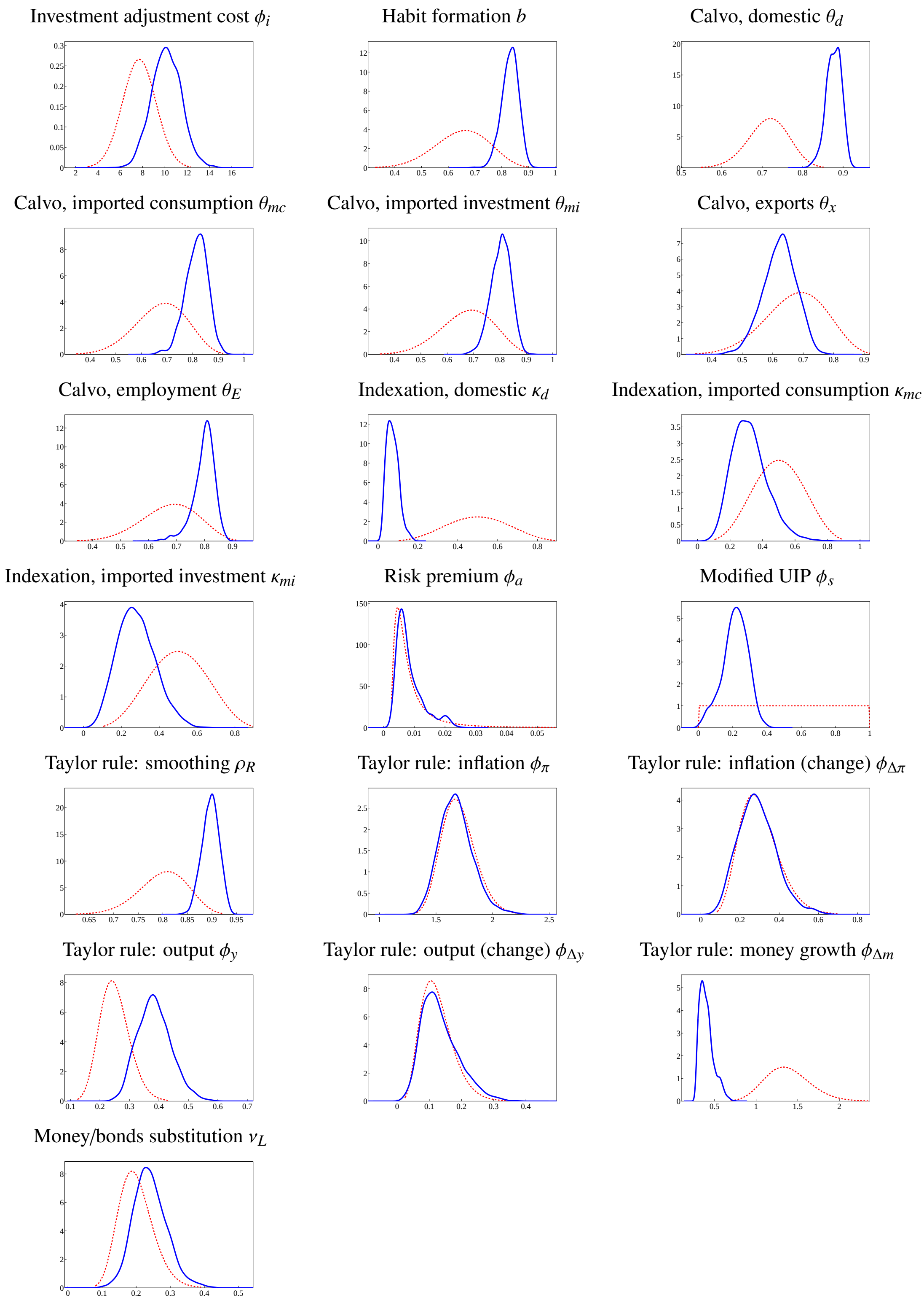
SA 10 year government bond yield

$$\tilde{R}_{L,t} = \ln(R_L) + \hat{R}_{L,t} + \eta_{L,t} \tag{B.52}$$

Foreign interest rate

$$\tilde{R}_t^* = \hat{R}_t^* + \ln(R^*) \tag{B.53}$$

Figure B.1: Prior and posterior density plots



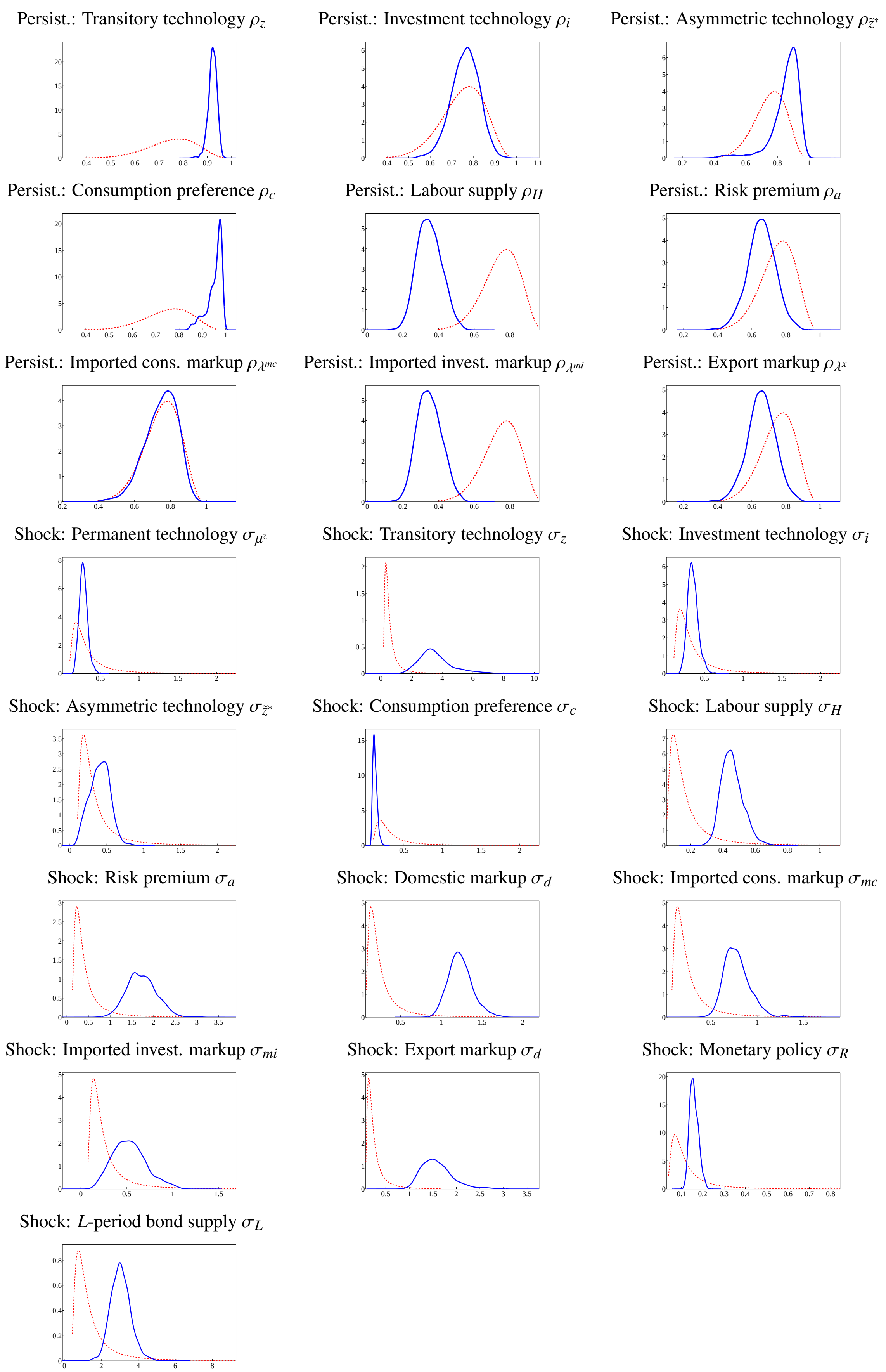

Persist.: Transitory technology ρ_z
Persist.: Investment technology ρ_i
Persist.: Asymmetric technology $\rho_{\tilde{z}^*}$
Persist.: Consumption preference ρ_c
Persist.: Labour supply ρ_H
Persist.: Risk premium ρ_a
Persist.: Imported cons. markup $\rho_{\lambda^{mc}}$
Persist.: Imported invest. markup $\rho_{\lambda^{mi}}$
Persist.: Export markup ρ_{λ^x}
Shock: Permanent technology σ_{μ^z}
Shock: Transitory technology σ_z
Shock: Investment technology σ_i
Shock: Asymmetric technology $\sigma_{\tilde{z}^*}$
Shock: Consumption preference σ_c
Shock: Labour supply σ_H
Shock: Risk premium σ_a
Shock: Domestic markup σ_d
Shock: Imported cons. markup σ_{mc}
Shock: Imported invest. markup σ_{mi}
Shock: Export markup σ_d
Shock: Monetary policy σ_R
Shock: L-period bond supply σ_L

Figure B.2: Monetary policy shock

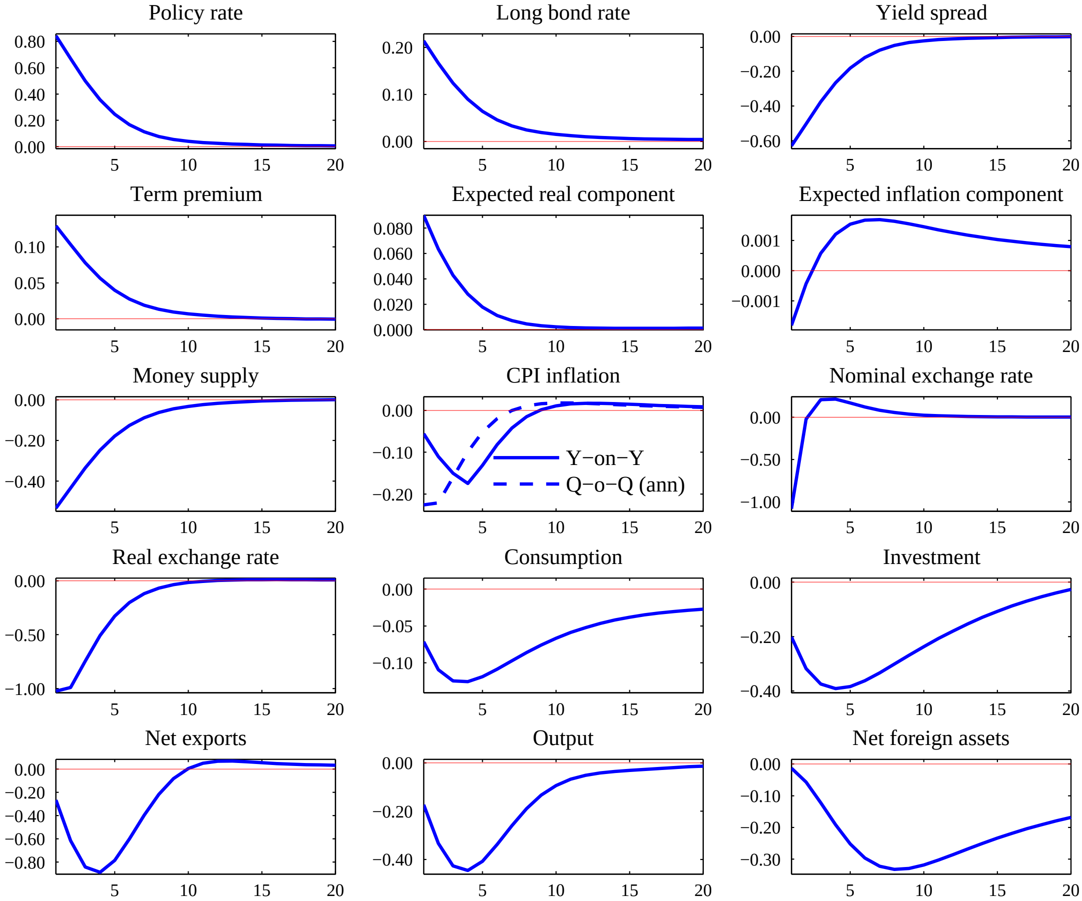


Figure B.3: Government spending shock

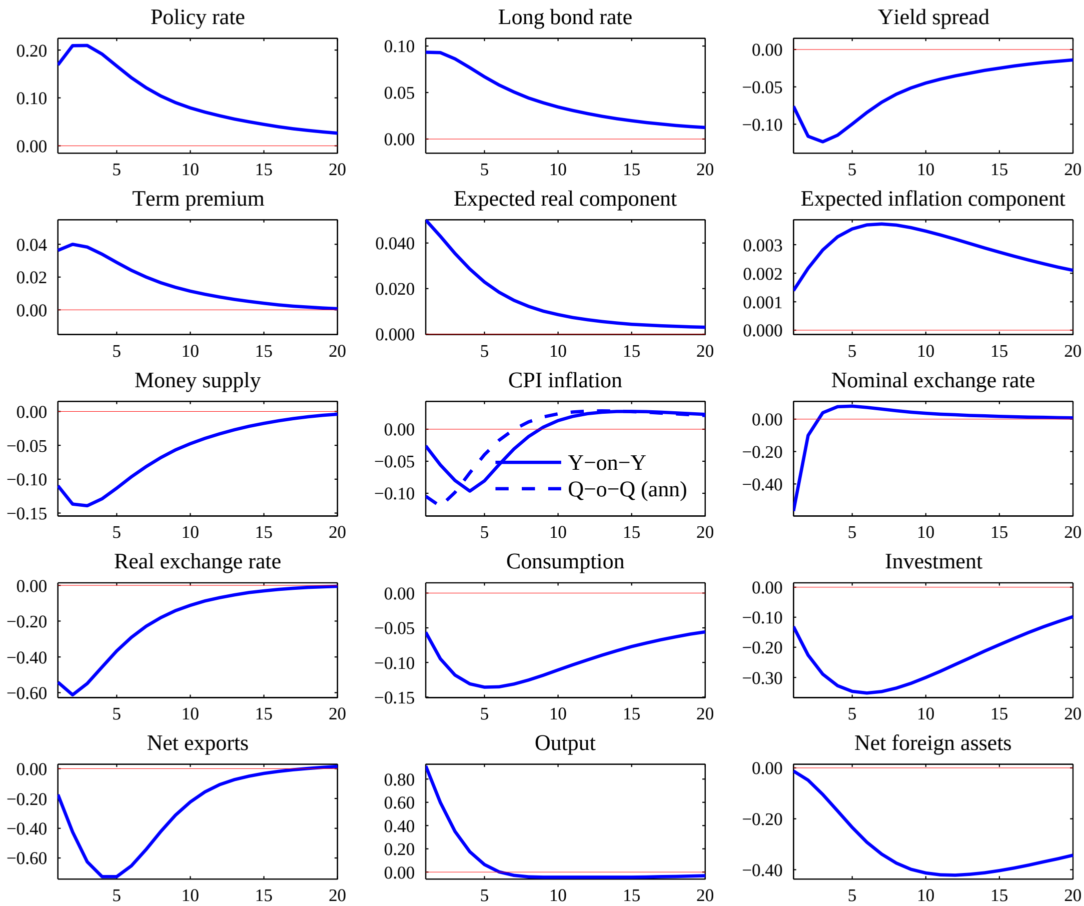


Figure B.4: Transitory technology shock

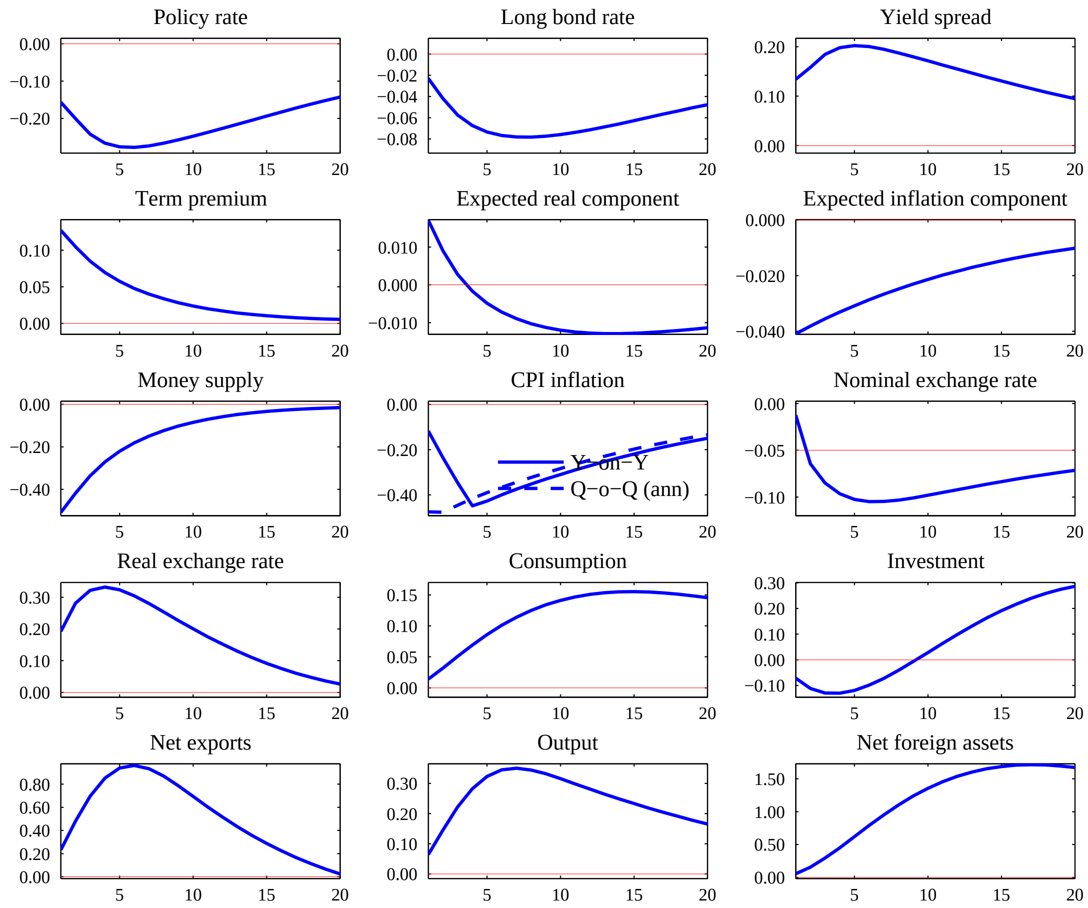
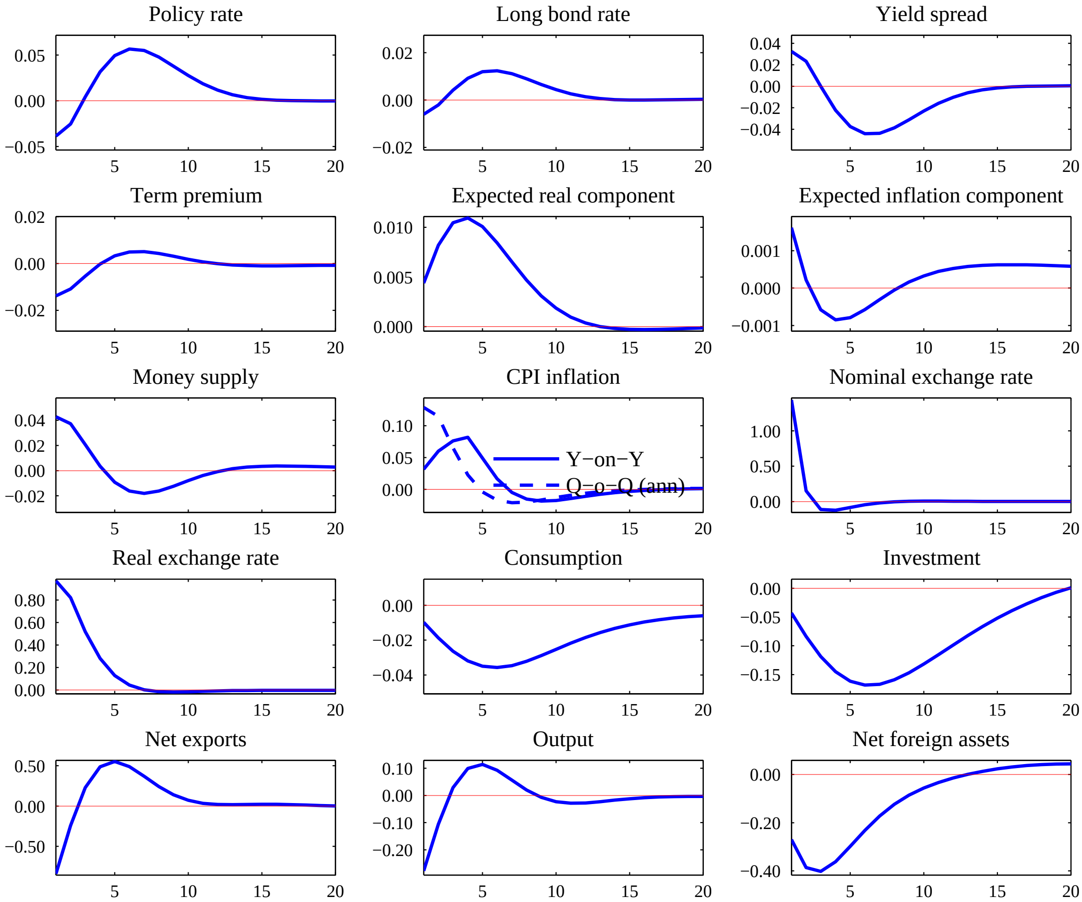


Figure B.5: Foreign monetary policy shock



Appendix C

To react or not: monetary policy and financial shocks in an empirical small open-economy DSGE model

C.1 The linearised model

Financial intermediaries

Lending spread

$$\hat{\omega}_t = \omega^{-1}\left[(1+\eta_\zeta)\zeta l_\zeta^\eta\right](\hat{\zeta}_t + \eta_\zeta \hat{l}_t) \tag{C.1}$$

Lending rate

$$\hat{R}_t^b = \hat{R}_t + \hat{\omega}_t \tag{C.2}$$

Non-performing loans

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} - \theta_\zeta \hat{y}_t + \varepsilon_{\zeta,t} \tag{C.3}$$

Deposits

$$\hat{d}_t = \frac{l}{d}\hat{l}_t + \frac{\zeta l^(1+\eta_\zeta)}{d}\left[\hat{\zeta}_t + (1+\eta_\zeta)\hat{l}_t\right] \tag{C.4}$$

Heterogeneous households, $\tau \in \{s, b\}$

Consumption Euler equations

$$\begin{aligned} \hat{c}_t^\tau \quad = \quad & \frac{\mu^z b_\tau}{(\mu^z)^2 + \beta b_\tau^2}\hat{c}_{t-1}^\tau + \frac{\beta \mu^z b_\tau}{(\mu^z)^2 + \beta b_\tau^2} E_t \hat{c}_{t+1}^\tau - \frac{(\mu^z - b_\tau)\,(\mu^z - \beta b_\tau)}{\sigma_\tau\left[(\mu^z)^2 + \beta b_\tau^2\right]}\left(\hat{\psi}_t^{z,\tau} + \hat{\gamma}_t^{c,d}\right) \\ + \quad & \frac{(\sigma_\tau - 1)\left[b_\tau \beta \mu^z - b_\tau^2 \beta - (\mu^z)^2\right] - b_\tau \mu^z}{\sigma_\tau\left[(\mu^z)^2 + \beta b_\tau^2\right]}\hat{\mu}_t^z, + \frac{(\sigma_\tau - 1)\left[b_\tau \mu^z - (\mu^z)^2\right] + \sigma_\tau b_\tau \beta \mu^z}{\sigma_\tau\left[(\mu^z)^2 + \beta b^2\right]} E_t \hat{\mu}_{t+1}^z \\ - \quad & + \frac{\mu^z - b_\tau}{\sigma_\tau\left[(\mu^z)^2 + \beta b_\tau^2\right]}\left(\mu^z \hat{\xi}_t^c - \beta b_\tau E_t \hat{\xi}_{t+1}^c\right) \end{aligned} \tag{C.5}$$

Investment Euler equation

$$\hat{P}_t^{k,\tau} = \phi_k \left(\frac{i^\tau}{k^\tau}\mu^z\right)\left(\hat{i}_t^\tau - \hat{k}_{t-1}^\tau + \hat{\mu}_t^z\right) + \hat{\gamma}_t^{i,d} + \hat{\xi}_t^i \tag{C.6}$$

Price of installed capital

$$\begin{aligned}
\hat{P}_t^{k,\tau} &= \frac{\beta}{\mu^z} E_t \Big\{ \chi \Big[(1-\delta)\hat{P}_{t+1}^{k,\tau} + \frac{r^k}{P^k}\hat{r}_{+1}^k + \phi_k \left(\frac{i^\tau}{k^\tau}\mu^z\right)(\hat{i}_{t+1}^\tau - \hat{k}_t^\tau + \hat{\mu}_{t+1}^z) \\
&+ \left[(1-\delta)P^k + r^k\right](\hat{\psi}_{t+1}^{z,\tau} - \hat{\mu}_{t+1}^z)\Big] \\
&+ \sum_{\tau' \in \{b,s\}} (1-\chi) p_{\tau'} \Big[(1-\delta)\hat{P}_{t+1}^{k,\tau'} + \frac{r^k}{P^k}\hat{r}_{+1}^k + \phi_k \left(\frac{i^{\tau'}}{k^{\tau'}}\mu^z\right)(\hat{i}_{t+1}^{\tau'} - \hat{k}_t^{\tau'} + \hat{\mu}_{t+1}^z) \\
&+ \left[(1-\delta)P^k + r^k\right](\hat{\psi}_{t+1}^{z,\tau'} - \hat{\mu}_{t+1}^z)\Big]\Big\} - \hat{\psi}_t^{z,\tau}
\end{aligned} \tag{C.7}$$

Capital's law-of-motion

$$\hat{k}_{t+1}^\tau = \frac{1-\delta}{\mu^z}\left(\hat{k}_t^\tau - \hat{\mu}_t^z\right) + \left(1 - \frac{1-\delta}{\mu^z}\right)\left(\hat{i}_t^\tau + \hat{\xi}_t^i\right) \tag{C.8}$$

Wage setting

$$\hat{w}_t^\tau = -\frac{1}{\eta_1}\left[\begin{array}{c} \eta_0 \hat{w}_{t-1} + \eta_2 E_t \hat{w}_{t+1} + \eta_3\left(\hat{\pi}_t^d - \hat{\bar{\pi}}_t^c\right) + \eta_4\left(E_t\hat{\pi}_{t+1}^d - \rho_\pi \hat{\bar{\pi}}_t^c\right) \\ +\eta_5\left(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \eta_7\hat{\psi}_t^{z,\tau} + \eta_8\hat{H}_t + \eta_9\hat{\xi}_t^h \end{array}\right] \tag{C.9}$$

Optimal borrowing

$$\begin{aligned}
\hat{\psi}_t^{z,b} &= \frac{R^b}{R}\Big\{\left[\chi + (1-\chi)p_b + (1-\chi)p_s\frac{\psi^{z,s}}{\psi^{z,b}}\right]\left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d - \hat{\mu}_{t+1}^z\right) \\
&+ [\chi + (1-\chi)p_b]\hat{\psi}_{t+1}^{z,b} + (1-\chi)p_s\frac{\psi^{z,s}}{\psi^{z,b}}\hat{\psi}_{t+1}^{z,s}\Big\}
\end{aligned} \tag{C.10}$$

Evolution of borrowing

$$\begin{aligned}
l_t &= \frac{(\chi + (1-\chi)p_b)R^b}{\pi\mu^z}\left(\hat{l}_{t-1} + \hat{R}_{t-1}^b - \hat{\mu}_t^z - \hat{\pi}_t^d\right) \\
&- \frac{(1-\chi)p_b(d+b)R}{\pi\mu^z l}\left(\frac{d}{d+b}\hat{d}_{t-1} + \frac{b}{d+b}b_{t-1} + R_{t-1} - \hat{\mu}_t^z - \hat{\pi}_t^d\right) \\
&- \frac{(1-\chi)p_b a^* R^*}{\pi\mu^z l}\left(\left(1-\tilde{\phi}_a\right)\hat{a}_{t-1} + \hat{\tilde{\phi}}_{t-1} - \hat{\pi}_t^d - \hat{\mu}_t^z\right) \\
&+ \frac{1}{l}\Big\{p_b\gamma^{c,d}c^b\left(\hat{\gamma}_t^{c,d} + \hat{c}_t^b\right) + p_b\gamma^{i,d}c^b\left(\hat{\gamma}_t^{i,d} + \hat{i}_t^b\right) \\
&- p_b wH\left(\hat{w}_t^b + \hat{H}_t\right) - \frac{p_b r^k k^b}{\mu^z}\left(r_t^k + k_{t-1}^b - \mu_t^z\right)\Big\}
\end{aligned} \tag{C.11}$$

Optimal deposit and bond holdings

$$\begin{aligned}
\hat{\psi}_t^{z,s} &= \left[\chi + (1-\chi)p_s + (1-\chi)p_b\frac{\psi^{z,b}}{\psi^{z,s}}\right]\left(\hat{R}_t - E_t\hat{\pi}_{t+1}^d - \hat{\mu}_{t+1}^z\right) \\
&+ [\chi + (1-\chi)p_s]\hat{\psi}_{t+1}^{z,s} + (1-\chi)p_b\frac{\psi^{z,b}}{\psi^{z,s}}\hat{\psi}_{t+1}^{z,b}
\end{aligned} \tag{C.12}$$

UIP condition

$$\hat{R}_t - \hat{R}_t^* = E_t \Delta \hat{S}_{t+1} - \tilde{\phi}_a \hat{a}_t + \hat{\tilde{\phi}}_t, \tag{C.13}$$

Firms

Domestic goods

Production

$$\hat{y}_t = \lambda^d \left(\hat{\varepsilon}_t + \alpha \left(\hat{k}_{t-1} - \hat{\mu}_t^z \right) + (1-\alpha) \hat{H}_t \right) \tag{C.14}$$

Rental rate of capital

$$\hat{r}_{t-1}^k = \hat{w}_t + \hat{\mu}_t^z - \hat{k}_t^s + \hat{H}_t \tag{C.15}$$

Real marginal cost

$$\hat{mc}_t^d = \alpha \hat{r}_t^k + (1-\alpha)(\hat{w}_t) - \hat{\varepsilon}_t \tag{C.16}$$

New Keynesian Phillips curve

$$\begin{aligned} \hat{\pi}_t^d - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_d} \left(E_t \hat{\pi}_{t+1}^d - \rho_\pi \hat{\bar{\pi}}_t^c \right) + \frac{\kappa_d}{1+\beta\kappa_d} \left(\hat{\pi}_{t-1}^d - \hat{\bar{\pi}}_t^c \right) - \frac{\beta\kappa_d (1-\rho_\pi)}{1+\beta\kappa_d} \hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_d)(1-\beta\theta_d)}{(1+\beta\kappa_d)\theta_d} \left(\hat{mc}_t^d + \hat{\lambda}_t^d \right) \end{aligned} \tag{C.17}$$

Imported goods

New Keynesian Phillips curve: imported consumption goods

$$\begin{aligned} \hat{\pi}_t^{m,c} - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_{m,c}} \left(E_t \hat{\pi}_{t+1}^{m,c} - \rho_\pi \hat{\bar{\pi}}_t^c \right) + \frac{\kappa_{m,c}}{1+\beta\kappa_{m,c}} \left(\hat{\pi}_{t-1}^{m,c} - \hat{\bar{\pi}}_t^c \right) - \frac{\kappa_{m,c}\beta(1-\rho_\pi)}{1+\beta\kappa_{m,c}} \hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{m,c})(1-\beta\theta_{m,c})}{(1+\beta\kappa_{m,c})\theta_{m,c}} \left(\hat{mc}_t^{m,c} + \hat{\lambda}_t^{m,c} \right) \end{aligned} \tag{C.18}$$

Marginal cost: imported consumption goods

$$\hat{mc}_t^{m,c} = -\hat{\gamma}_t^f - \hat{\gamma}_t^{mc,d} \tag{C.19}$$

New Keynesian Phillips curve: imported investment goods

$$\begin{aligned} \hat{\pi}_t^{m,i} - \hat{\bar{\pi}}_t^c &= +\frac{\beta}{1+\beta\kappa_{m,i}} \left(E_t \hat{\pi}_{t+1}^{m,i} - \rho_\pi \hat{\bar{\pi}}_t^c \right) + \frac{\kappa_{m,i}}{1+\beta\kappa_{m,i}} \left(\hat{\pi}_{t-1}^{m,i} - \hat{\bar{\pi}}_t^c \right) - \frac{\kappa_{m,i}\beta(1-\rho_\pi)}{1+\beta\kappa_{m,i}} \hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_{m,i})(1-\beta\theta_{m,i})}{(1+\beta\kappa_{m,i})\theta_{m,i}} \left(\hat{mc}_t^{m,i} + \hat{\lambda}_t^{m,i} \right) \end{aligned} \tag{C.20}$$

Marginal cost: imported investment goods

$$\hat{mc}_t^{m,i} = -\hat{\gamma}_t^f - \hat{\gamma}_t^{mi,d} \tag{C.21}$$

Exported goods

New Keynesian Phillips curve: exported goods

$$\begin{aligned}\hat{\pi}_t^x - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\beta\kappa_x}\left(E_t\hat{\pi}_{t+1}^x - \rho_\pi\hat{\bar{\pi}}_t^c\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}_{t-1}^x - \hat{\bar{\pi}}_t^c\right) - \frac{\kappa_x\beta(1-\rho_\pi)}{1+\beta\kappa_x}\hat{\bar{\pi}}_t^c \\ &+ \frac{(1-\theta_x)(1-\beta\theta_x)}{(1+\beta\kappa_x)\theta_x}\left(\hat{mc}_t^x + \hat{\lambda}_t^x\right) \end{aligned} \tag{C.22}$$

Marginal cost: exported goods

$$\hat{mc}_t^x = \hat{mc}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta\hat{S}_t \tag{C.23}$$

Government

Budget constraint

$$b\hat{b}_t = \ell\hat{\ell}_t + g\hat{g}_t - \tau\hat{\tau}_t \tag{C.24}$$

Liabilities

$$\ell\hat{\ell}_t = \frac{bR}{\mu^z\pi}\left(\hat{b}_{t-1} + \hat{R}_{t-1} - \hat{\mu}_t^z - \hat{\pi}_t^d\right) \tag{C.25}$$

Fiscal rule

$$\hat{\tau}_t = \psi_1\frac{\ell}{\tau}\hat{\ell}_t \tag{C.26}$$

The Central Bank

Taylor rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\hat{\bar{\pi}}_t^c + \phi_\pi\left(\hat{\pi}_{t+1}^{c,4} - \hat{\bar{\pi}}_t^c\right) + \phi_{\Delta\pi}\hat{\pi}_t^c + \phi_y\hat{y}_t + \phi_{\Delta y}\Delta\hat{y}_t\right] + \varepsilon_t^R. \tag{C.27}$$

where CPI inflation is given by

$$\hat{\pi}_t^c = (1-\vartheta_c)\left(\frac{1}{\gamma^{c,d}}\right)^{1-\eta_c}\hat{\pi}_t^d + \vartheta_c\left(\gamma^{mc,c}\right)^{1-\eta_c}\hat{\pi}_t^{m,c} \tag{C.28}$$

Relative prices

Consumption and investment goods

$$\hat{\gamma}_t^{c,d} = \hat{\gamma}_{t-1}^{i,d} + \hat{\pi}_t^c - \hat{\pi}_t^d \tag{C.29}$$

$$\hat{\gamma}_t^{i,d} = \hat{\gamma}_{t-1}^{i,d} + \hat{\pi}_t^i - \hat{\pi}_t^d \tag{C.30}$$

Imported consumption and investment goods

$$\hat{\gamma}_t^{mc,d} = \hat{\gamma}_{t-1}^{mc,d} + \hat{\pi}_t^{m,c} - \hat{\pi}_t^d \tag{C.31}$$

$$\hat{\gamma}_t^{mi,d} = \hat{\gamma}_{t-1}^{mi,d} + \hat{\pi}_t^{m,i} - \hat{\pi}_t^d \tag{C.32}$$

Export goods

$$\hat{\gamma}_t^{x,*} = \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \tag{C.33}$$

Domestic-foreign goods relative price

$$\hat{\gamma}_t^f = \hat{mc}_t^x + \hat{\gamma}_t^{x,*} \tag{C.34}$$

Real exchange rate

$$\hat{\gamma}_t^s = -\vartheta_c \left(\frac{1}{\gamma^{mc,c}}\right)^{\eta_c - 1} \hat{\gamma}_t^{mc,d} - \hat{\gamma}_t^{x,*} - \hat{mc}_t^x \tag{C.35}$$

Market clearing

Domestic goods market

$$\begin{aligned} \hat{y}_t &= \frac{1}{1-\frac{\phi_L}{2}} \Big[(1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c} \frac{c}{y}\left(\hat{c}_t + \eta_c \hat{\gamma}_t^{c,d}\right) + (1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i} \frac{i}{y}\left(\hat{i}_t + \eta_i \hat{\gamma}_t^{i,d}\right) \\ &+ g_y \hat{g}_t + \frac{y^*}{y}\left(\hat{y}_t^* - \eta_f \hat{\gamma}_t^{x,*} + \hat{\tilde{z}}_t^*\right) + \frac{r^k}{\mu^z}\frac{k}{y}\left(\hat{k}_t^s - \hat{k}_t\right) + \phi_L\left(\hat{b}_{L,t} - \hat{b}_{L,t-1}\right)\Big] \end{aligned} \tag{C.36}$$

Foreign bond market

$$\begin{aligned} \hat{a}_t &= -y^* \hat{mc}_t^x - \eta_f y^* \hat{\gamma}_t^{x,*} + y^* \hat{y}_t^* + y^* \hat{\tilde{z}}_t^* + (c^m + i^m)\hat{\gamma}_t^f \\ &- \left[c^m \left(-\eta_c (1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c - 1}\right)\hat{\gamma}_t^{mc,d} + \hat{c}_t\right] \\ &- \left[i^m \left(-\eta_i (1-\vartheta_i)\left(\gamma^{i,d}\right)^{\eta_i - 1}\right)\hat{\gamma}_t^{mi,d} + \hat{i}_t\right] \\ &+ \frac{\pi^*}{\pi}\frac{1}{\beta}\hat{a}_{t-1} \end{aligned} \tag{C.37}$$

AR(1) shock processes

$$\Xi_t = \rho \Xi_{t-1} + \Gamma_t \tag{C.38}$$

where

$$\begin{array}{lcl} \Xi_t &=& [\hat{\xi}_t^c \quad \hat{\xi}_t^i \quad \hat{\tilde{\phi}}_t \quad \hat{\varepsilon}_t \quad \hat{\xi}_t^H \quad \hat{\lambda}_t^x \quad \hat{\lambda}_t^d \quad \hat{\lambda}_t^{m,c} \quad \hat{\lambda}_t^{m,i} \quad \hat{\tilde{z}}_t^* \quad \hat{\mu}_t^z \quad \hat{g}_t \quad \hat{\bar{\pi}}_t^c \quad \hat{b}_{L,t}]' \\ \rho &=& [\rho_c \quad \rho_i \quad \rho_{\tilde{\phi}} \quad \rho_\varepsilon \quad \rho_H \quad \rho_{\lambda^x} \quad \rho_d \quad \rho_{\lambda^{m,c}} \quad \rho_{\lambda^{m,i}} \quad \rho_{\tilde{z}^*} \quad \rho_{\mu^z} \quad \rho_g \quad \rho_{\bar{\pi}^c} \quad \rho_L]' \\ \Gamma_t &=& [\varepsilon_t^c \quad \varepsilon_t^i \quad \varepsilon_t^{\tilde{\phi}} \quad \varepsilon_t^\varepsilon \quad \varepsilon_t^H \quad \varepsilon_t^x \quad \varepsilon_t^d \quad \varepsilon_t^{m,c} \quad \varepsilon_t^{m,i} \quad \varepsilon_t^{\tilde{z}^*} \quad \varepsilon_t^{\mu^z} \quad \varepsilon_t^g \quad \varepsilon_t^{\bar{\pi}^c} \quad \varepsilon_t^L]' \end{array}$$

Measurement equations

Output

$$\Delta \ln\left(\tilde{Y}_t\right) = \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_t^z + \ln(\mu^z) \tag{C.39}$$

Consumption

$$\begin{aligned} \Delta \ln\left(\tilde{C}_t\right) &= \left(\frac{\eta_c}{c^d + c^m}\right)\left[c_d \vartheta_c (\gamma^{c,mc})^{\eta_c - 1} - c^m (1-\vartheta_c)\left(\gamma^{c,d}\right)^{\eta_c - 1}\right]\left(\hat{\pi}_t^{m,c} - \hat{\pi}_t^d\right) \\ &+ \hat{c}_t - \hat{c}_{t-1} + \mu_t^z + \ln(\mu^z) \end{aligned} \tag{C.40}$$

Investment

$$\begin{aligned}\Delta\ln\left(\tilde{I}_t\right) &= \left(\frac{\eta_i}{i^d+i^m}\right)\left[i_d\vartheta_i\left(\gamma^{i,mi}\right)^{\eta_i-1} - i^m\left(1-\vartheta_i\right)\left(\gamma^{i,d}\right)^{\eta_i-1}\right]\left(\hat{\pi}_t^{m,c}-\hat{\pi}_t^d\right) \\ &+ \hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^z + \ln\left(\mu^z\right) \qquad \text{(C.41)}\end{aligned}$$

Exports

$$\Delta\ln\left(\tilde{X}_t\right) = -\eta_f\left(\hat{\pi}_t^x - \hat{\pi}_t^*\right) + \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\tilde{z}}_t - \hat{\tilde{z}}_{t-1} + \hat{\mu}_t^z + \ln\left(\mu^z\right) \qquad \text{(C.42)}$$

Imports

$$\begin{aligned}\Delta\ln\left(\tilde{M}_t\right) &= \left(\frac{c^m}{c^m+i^m}\right)\left[\eta_c\left(1-\vartheta_c\right)\left(\gamma^{c,d}\right)^{\eta_c-1}\left[\hat{\pi}_t^d - \hat{\pi}_t^{m,c}\right] + \hat{c}_t - \hat{c}_{t-1}\right] \\ &+ \left(\frac{i^m}{c^m+i^m}\right)\left[\eta_i\left(1-\vartheta_i\right)\left(\gamma^{i,d}\right)^{\eta_i-1}\left[\hat{\pi}_t^d - \hat{\pi}_t^{m,i}\right] + \hat{i}_t - \hat{i}_{t-1}\right] \\ &+ \hat{\mu}_t^z + \ln\left(\mu^z\right) \qquad \text{(C.43)}\end{aligned}$$

Foreign GDP

$$\Delta\ln\left(\tilde{Y}_t^*\right) = \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{\tilde{z}}_t - \hat{\tilde{z}}_{t-1} + \hat{\mu}_t^z + \ln\left(\mu^z\right) \qquad \text{(C.44)}$$

Wages

$$\Delta\ln\left(\tilde{W}_t\right) = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t^d + \hat{\mu}_t^z + \ln\left(\mu^m\right) \qquad \text{(C.45)}$$

Employment

$$\Delta\ln\left(\tilde{E}_t\right) = \hat{E}_t - \hat{E}_{t-1} \qquad \text{(C.46)}$$

CPI inflation

$$\tilde{\pi}_t^c = \hat{\pi}_t^c + \ln\left(\pi\right) \qquad \text{(C.47)}$$

Producer price inflation

$$\tilde{\pi}_t^d = \hat{\pi}_t^d + \ln\left(\pi\right) \qquad \text{(C.48)}$$

Investment deflator

$$\tilde{\pi}_t^i = \hat{\pi}_t^i + \ln\left(\pi\right) \qquad \text{(C.49)}$$

Foreign inflation

$$\tilde{\pi}_t^* = \hat{\pi}_t^* + \ln\left(\pi^*\right) \qquad \text{(C.50)}$$

Nominal exchange rate

$$\Delta\ln\left(\tilde{S}_t\right) = \Delta\hat{S}_t + \ln\left(\frac{\pi}{\pi^*}\right) \qquad \text{(C.51)}$$

Foreign interest rate

$$\tilde{R}_t^* = \hat{R}_t^* + \ln\left(R^*\right) \qquad \text{(C.52)}$$

Repo rate

$$\tilde{R}_t = \hat{R}_t + \ln(R) \tag{C.53}$$

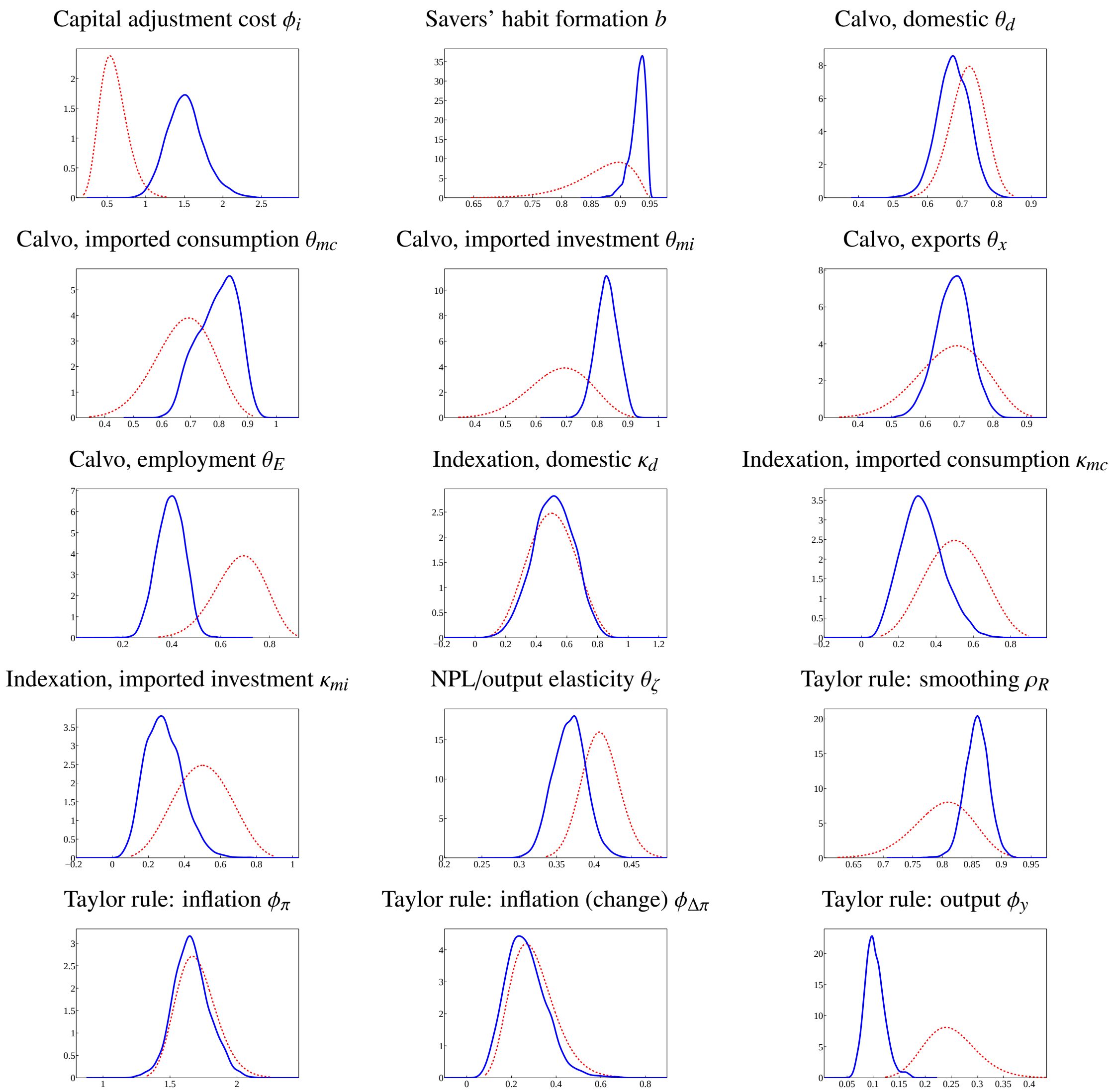
Effective lending rate

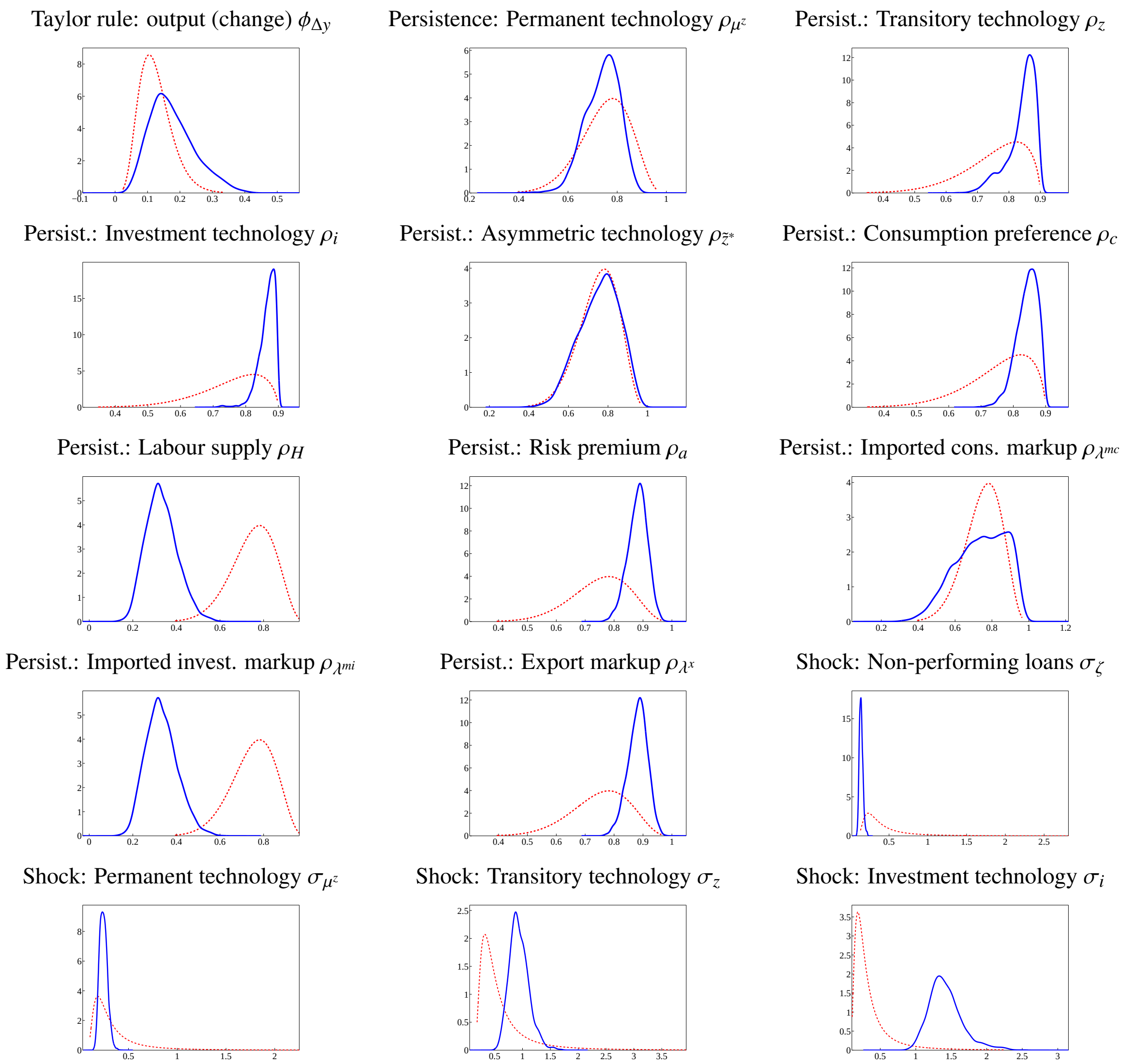
$$\tilde{R}_t^b = \ln(R^b) + \hat{R}_t^b + \eta_t^b \tag{C.54}$$

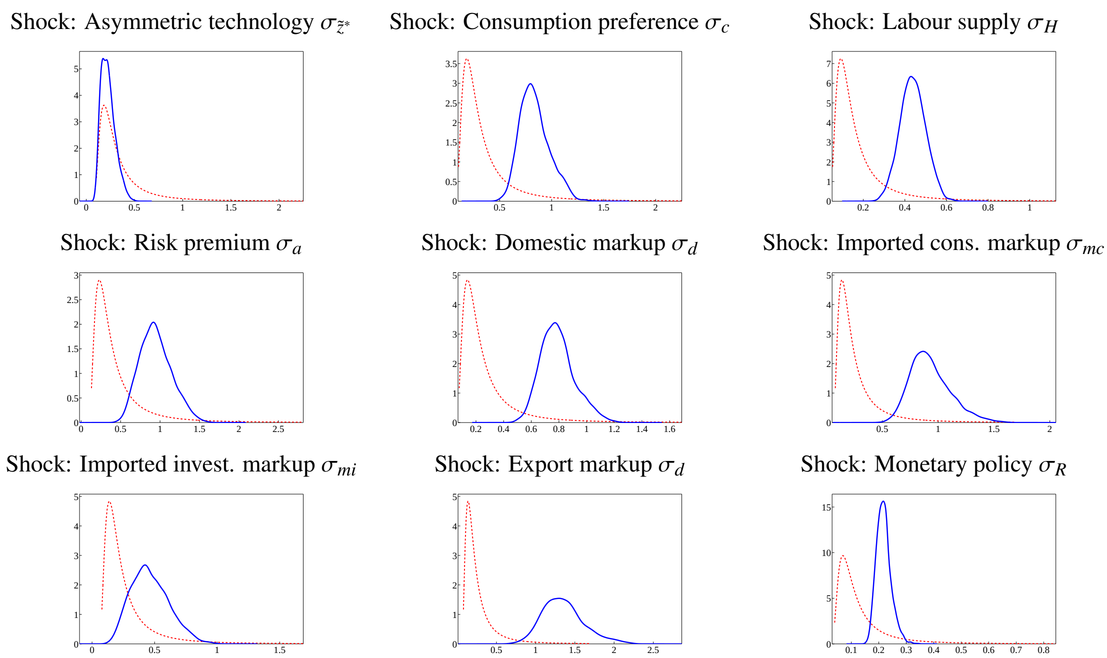
Non-performing loan ratio

$$\tilde{\zeta}_t = \ln(\zeta + 1) + \hat{\zeta}_t \tag{C.55}$$

Figure C.1: Prior and posterior density plots




Taylor rule: output (change) $\phi_{\Delta y}$
Persistence: Permanent technology ρ_{μ^z}
Persist.: Transitory technology ρ_z
Persist.: Investment technology ρ_i
Persist.: Asymmetric technology $\rho_{\tilde{z}^*}$
Persist.: Consumption preference ρ_c
Persist.: Labour supply ρ_H
Persist.: Risk premium ρ_a
Persist.: Imported cons. markup $\rho_{\lambda^{mc}}$
Persist.: Imported invest. markup $\rho_{\lambda^{mi}}$
Persist.: Export markup ρ_{λ^x}
Shock: Non-performing loans σ_ζ
Shock: Permanent technology σ_{μ^z}
Shock: Transitory technology σ_z
Shock: Investment technology σ_i


Shock: Asymmetric technology $\sigma_{\tilde{z}^*}$
Shock: Consumption preference σ_c
Shock: Labour supply σ_H
Shock: Risk premium σ_a
Shock: Domestic markup σ_d
Shock: Imported cons. markup σ_{mc}
Shock: Imported invest. markup σ_{mi}
Shock: Export markup σ_d
Shock: Monetary policy σ_R

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