

Teaching mathematics meaningfully with technology: Implications for professional development: A Namibian case

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Abstract. The practice of teaching and learning is being redefined while it is being imagined how to teach with and through technology. The process is characterised by non-linear interaction between local case-based efforts through reformed curricula, and general frameworks such as TPACK and SAMR developed through academic research. Locally, the demand on mathematics teachers to adapt their teaching to include technology is often mismatched with intended aims, rationales, and practices that they are held accountable for in curricula. Within this complex endeavour, professional development toward teaching mathematics with technology is a design process. This paper proposes that mathematics teaching with technology must be meaningful and not merely a response to societal adoption of digital tools. The concept meaningful is defined using Vygotsky's definition of meaningful teaching as well as Kilpatrick et al's five strands of mathematical proficiency. For the purpose of designing a professional development intervention, Activity Theory is used as the lens to compare goals, rules, and division of labour of current Namibian secondary school mathematics teaching in relation to an envisaged new activity system for teaching mathematics meaningfully with technology, that is, beyond task level and representation use. The article ends with design principles for professional development of secondary school mathematics teachers to teach mathematics meaningfully with technology.

Key words: *meaningful teaching; professional development; technology; Mathematics; Namibia*

Introduction

Teaching and learning with technology is seen as a hallmark of education for the 21st century. Schools with technology in the classroom are rated by parents as more desirable than schools without. Developing countries such as Namibia are expected to catch up and compete with developed countries in terms of curriculum goals, as well as teaching mathematics with technology. Namibian Education authorities have taken up the duty to provide professional development for teachers in this respect. The Ministry of Education Arts and Culture (MoEAC) in Namibia has ensured that approximately 1822 teachers were certified competent in Microsoft Office programs, online essentials, computer essentials and using an Operating System (OS) between 2007 and 2018 (Ministry of Education Arts and Culture, 2018). Yet, researchers found that many schools still lack facilities to teach with technology (Nchindo, 2019; Nendongo, 2018). Schools that could obtain funding, are now equipped with computer laboratories, or have at least a single computer and data projector in some classrooms.

The COVID 19 pandemic, which drove teaching to the internet wherever the infrastructure was available, brought to stark attention the affordances and problems relating to teaching and learning with technology. It has further spurred on the development of an extensive library of products for teaching and learning with technology (Mishra & Warr, 2021). There is no paucity of professionally developed mathematical software applications such as Geogebra, Desmos, Geometers Sketchpad (GSP), Microsoft mathematics, Gauthmath and PhotoMath. Social media platforms, such as Facebook, Twitter, WhatsApp and Telegram, and YouTube, are exploding with products by mathematics teachers and other people with a wide range of credentials who came to embrace technology affordances. Fewer teachers than ever before can be described as

technological aliens, and having dabbled with technology in their teaching should allow them to engage with professional development much deeper than before (Burns, 2006; Hamilton, Kapenda, Miranda, & Ngololo 2019; Kanandjebo & Ngololo, 2017). Invitations to various online professional development courses abound. These range from free webinars by university faculty or progressive schools to paid-for webinars by private enterprises.

Research about technology in mathematics education shows that people are agentic in the use of technological tools to achieve teaching and learning goals (Borba, 2021) and judge for themselves which of the multitude of available tools are relevant to their current teaching goals. Yet, through mere availability, technology is spurring on a shift in curriculum goals and in what it means to learn. Increasingly, cognitive work that would have been performed by humans is now shared with or left to technology. For example, calculators on cellular phones are ubiquitous and a habitual means of solving calculation tasks, to the extent that learners actively avoid memorization of basic calculations. It is apparent that technology is getting a share in the division of labour in the mathematics classroom, and this is becoming evident in curricula too. In particular, Pea (1987) argued that technology is changing what are considered as goals for mathematical understanding.

As technology continues to disrupt and reconfigure traditional teaching and learning practices in a growing number of classrooms, hindsight is not yet available to evaluate gains and losses. Research reports of teaching mathematics with technology is overwhelmingly positive and hardly ever consider possible downsides. It is self-evident that teachers' beliefs are disrupted as they experience a wide variety of alternative ways to teach even basic mathematics topics such as the function concept or the Theorem of Pythagoras, while high stakes assessments still hold them accountable for traditional goals. Therefore, this paper utilizes Activity Theory to consider changes in goals, tools, rules and organization of work toward mathematical thinking as the new object as it is afforded, catalyzed and uncovered by technology (Pea, 1987). From these considerations, a framework of principles is proposed for the professional development of mathematics teachers to teach meaningfully with technology.

Theoretical framework

Third generation Cultural Historic Activity Theory (CHAT) (Engeström, 2001) is used as a lens to compare goals, rules and division of labour, based on current Namibian secondary school mathematics teaching in relation to an envisaged new activity system for teaching with technology. The dialectic relationship between activity theory and human-technology interaction has a rich history. Already at the end of the 1900's the contribution of activity theory to a deeper understanding of the meaning of human-technology interaction (Engeström & Miettinen, 1999; Flavin, 2017) as well as to the emergence of new disciplines such as ergonomics, the science of design for effective use of tools (Kaptelinin & Nardi, 2018) was acknowledged, as it evoked the larger context of purposeful human activity. Third generation CHAT holds that understanding practice as meaningful and purposeful requires social mediation through tools, of how teachers think about their practices (Engeström, 2001). Figure 1 shows an activity system constructed from a wider network of activities that it remains part of. It depicts how human subjects operate within a system, use physical tools and 'abstract' resources as mediators, in pursuit of the production of objects or outcomes (Engeström, 2001; Flavin, 2017, p. 56). In the activity system (Figure 1), mediating artefacts can be conceptualised as both tools and signs, mediating between object, meaning making and the outcome of the system (Bloomfield & Nguyen, 2015). The depiction of an object with an oval indicates that object-oriented actions as they arise from relationships with the other nodes do not converge to

an objective certainty, but are characterised by sense making as well as potential for change (Engeström, 2001) and sharing.

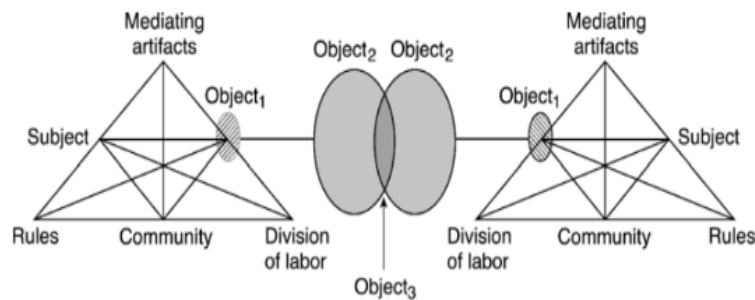


Figure 1. Two interacting activity system minimal model for the third generation of activity theory (Engeström, 2001, p. 136)

It is straight forward to interpret teachers-delivering-the-curriculum as an activity triangle aimed at achieving Object 1, and researchers-and-developers-teaching-with-technology as a second activity triangle as in Figure 1. Mathematics teachers as the subjects of one activity system and part of this paper’s anticipated professional development community, act on the object of their activity, which they colloquially describe as “delivering the mathematics curriculum”. Alongside their current activity system, teachers are confronted from outside with activity systems where researchers and curriculum developers as well as software developers strive towards teaching mathematics with technology. Such teachers experience a clear but weakly mediated change in rules in the Namibian government’s call for technology integration. True to the depiction in Figure 1 the intersection of Object 2 (the goal of the external activity system) hardly overlaps with the object of activity systems of most teachers in Namibia. In many instances even reform curricula fail to describe the changed object in ways that can expand teaching goals. According to Pea (1987), owing to technology, organizations such as the National Council of Teachers of Mathematics (NCTM) that represent mathematics educators, had been rethinking mathematical goals since the 1980’s and still these reforms are not evident in many classrooms. However, Namibian teachers may not be aware of the way the object of mathematics with technology has changed.

The change in teaching goals place teachers at the cusp of “important transformations of [their] personal lives and organizational practices” (Engeström, 2001, p. 138) since they are required to adopt new tools and to rethink the object of teaching and learning mathematics. Literally, all parts of their activity system are in flux, which brings opportunities and pitfalls that should be learnt as they arise. Engeström (2001) explains that activity in such instances cannot be mediated top-down by regulation but must necessarily emerge in new forms from expansive learning.

“The object of expansive learning activity is the entire activity system in which the learners are engaged. Expansive learning activity produces culturally new patterns of activity. Expansive learning at work produces new forms of work activity”. Engeström (2001, p.139)

Vague, locally mandated change in rules will not lead to expansive changes in teaching and learning mathematics with technology; of developing skills necessary to optimally function in a world dominated by technologies. To catalyse change, dynamics such as rules and goals need to be initiated that could cause serious transformational effort of the activity system (Engeström & Sannino, 2021). Consequently, for this study, the expanded object is named *meaningful teaching of mathematics with technology*. The next sub-sections discuss meaningful teaching

of mathematics as a goal of reform-curricula-as-intended even before the current pervasive emphasis on teaching with technology. The problematic of the enacted curriculum is discussed, and implications are drawn for Professional Development towards the expanded goal.

Expanded goal: Meaningful teaching of mathematics

The choice of the adjective ‘meaningful’, and not the adjective efficient, to characterize the teaching and learning of mathematics, is value laden and requires explanation. For this study, the definition of meaningful is modelled on Vygotsky’s interpretation in relation to teaching writing skills to young children (Vygotsky, 1978):

...Therefore, the issue of teaching writing in the preschool years necessarily entails a second requirement: writing must be "relevant to life"-in the same way that we require a "relevant" arithmetic. A second conclusion, then, is that writing should be meaningful for children, that an intrinsic need should be aroused in them, and that writing should be incorporated into a task that is necessary and relevant for life. Only then can we be certain that it will develop not as a matter of hand and finger habits but as a really new and complex form of speech. The third point that we are trying to advance as a practical conclusion is the requirement that writing be taught naturally. (p. 118)

Hence, meaningful teaching of mathematics is relevant to learners and arouses an intrinsic need to master mathematics and to engage with tasks that are *necessary and relevant* for life. Relevance is not a narrow requirement restricted only to real-life applications of mathematics; rather that teaching provides the means to engage with connections so that any topic or task in mathematics is relevant in the bigger body of mathematics with which learners engage. Vygotsky’s practical requirement that writing should be taught naturally refers to the preparation of skill to control the hand to write as well as the skill to use writing to accomplish *communication needs*. This can be applied to the meaningful teaching of mathematics to secondary school learners as well. Teaching that is premised on communication through mathematics creates opportunities for natural encounters with content and articulation through cognitive technology tools. Further, in the advent of widespread technology, the *necessity and relevance* to life of specific content is changing, but there is widespread agreement that the goals of teaching mathematics with technology must be the development of *mathematical thinking skills* (Noss & Hoyles, 1996; Pea, 1987). With this Vygotskian backing of meaningful teaching of mathematics in mind the widely adopted Five strands of mathematical Proficiency (Kilpatrick, Swafford & Bradford, 2001) is taken as the conceptual framework. Relevance is approached through strategic competence (formulate, represent and solve problems) when learners engage with tasks of which the relevance is premised on conceptual understanding of their relation to the larger body of mathematics; adaptive reasoning (logical thought, reflection, explanation and justification) provides a natural space for communication through mathematics; procedural competence (accuracy, efficiency, flexibility, appropriateness) is required to wield the mathematical thinking tool with confidence; and teaching that allows learners to see mathematics as sensible, useful and worth of their intellectual effort (productive dispositions) evidently arouses an intrinsic need to engage with mathematics meaningfully.

The five strands of mathematical proficiency are regarded by researchers to capture all aspects of expertise, competence, knowledge, and facility and motivation in school mathematics (Graven & Stott, 2012; National Research Council, 2001) and are a sensible goal to posit in the endeavour to expand teaching goals to incorporate technology. Mathematics curricula of developed countries, such as Singapore, Australia, and the USA, have embraced mathematical goals in line with Kilpatrick et al. (2001), but despite a widespread adoption of these

mathematical proficiency strands, Ally (2011) questions the extent to which opportunities for developing these competencies are present in teachers' pedagogies.

Tools, and teaching and learning goals of the Namibian Mathematics curriculum

The Namibian curriculum rules acknowledge that mathematics *“is more than an accumulation of facts, skills and knowledge. The learning of Mathematics involves conceptual structures and general strategies of problem solving and attitudes towards and appreciation of Mathematics”* (National Institute for Educational Development (NIED), 2020, p. 2). However, an analysis of the mathematics curriculum by identifying key proficiency terms (Sullivan, 2012) in a doctoral design-based study, shows that procedural fluency (Mateya, Utete & Ilukena, 2016) and conceptual understanding are best accommodated, with rare reference to the development of strategic competence and/or adaptive reasoning (Stephanus, 2014). There are no goals formulated for the development of productive disposition.

In terms of teaching and learning with technology the curriculum noted that *“today's learners will live in a world dominated by computers...”* (National Institute for Educational Development (NIED), 2018, p. 5, 2020, p. 2), but goals for the use of computers and other technology propose them not as cognitive tools, but as optional or “efficiency tool” (Olive, Makar, Hoyos, Kor, Kosheleva & Sträßer, 2010, p. 138) that complement pencil and paper calculations (Goos, 2012). Moreover, since the localisation of the national curriculum in 2007, school teachers have been expected to achieve the foundation level ICT certification, and at least two staff members at a school must have advanced level ICT literacy certification or a higher ICT qualification (Ministry of Education Arts and Culture, 2016). These rules place teachers at the substitution level of technology integration according to the Substitution, Augmentation, Modification, and Redefinition (SAMR) model (Puentedura, 2010) as well as at tasks level opportunities on a pedagogical map (Pierce & Stacey, 2010). At these levels, technology tools and mediating artefacts are narrowly used to improve speed and accuracy, and to provide a greater variety of visual representations of mathematics content. The danger is that such inclusion of technology in mathematics classrooms may limit the intended curriculum even further by using technology as a crutch rather than tools to expand mathematical thinking.

Professional development intervention to enable mathematics teaching with technology must engage with the current mismatch of aims, rationales, visions and goals between the Namibian curriculum and expanded goals for the inclusion of technology as tools that have the potential to expand goals for mathematical thinking and lead to new activity in a rapidly changing world.

Object of mathematics in the advent of technology as mediating artefacts

The object signifies the rationale behind the activity (Foot, 2014; Jonassen & Rohrer-Murphy, 1999). Understanding the purpose (object) holds the key for teachers to act meaningfully in the activity system. Without defining the new object clearly, teachers use technology toward their old objectives of delivering the curriculum.

Technology shifts the object of the activity leading to new ways of acting in the activity system (Hakkarainen, 2009; Hardman, 2015; Hokanson & Hooper, 2000). When a calculator can factorise large numbers at the press of a button, a large part of the traditional primary school curriculum is obsolete; when dynamic software can be used to drag a graph and read off the function formula a large part of the traditional lower secondary curriculum is obsolete. Yet technology also presents new areas for mathematical study and shifts learning outcomes to earlier grade levels (Nathan, 2010). For example, many reform curricula include the study of

geometric transformations and tessellation in primary school through hands-on manipulation which often remain at craft level. With the aid of dynamic geometry software that enables effortless experimentation with “virtual manipulatives” (Goldenberg, 2000; Trinidad, 2003), the topic of transformation can be pursued through problem solving to the systematization of formal rules and applied in the study of functions. When technology becomes a mathematical thinking tool, new habits of mind may emerge, such as expectations that objects are malleable and morphable through mathematics, and the need to learn to communicate with technology by adopting computational thinking and different symbol systems, e.g. denoting multiplication with * and exponentiation with ^.

The potential of technology to drive cognitive growth and change thinking activities (Pea, 1987) is premised on pedagogy aimed at meaningful mathematics as argued earlier. Tseng Tang and Morris (2016) argue that technology use characterised by static content without social interaction, results in ‘mundane’ courses (p.199). Technology as a mediating cognitive tool may make it harder for teachers with traditional pedagogies to engage with learners, as it may become difficult to hear and see how learners understand mathematical ideas.

Division of labour and emerging goals

Teachers’ labour includes choosing appropriate teaching approaches and teaching materials, setting and providing feedback on assessment tasks, and subject administration. In many traditional classrooms in Namibia teachers view their role as that of knowledgeable presenters and explainers. Learners’ roles are mostly to imitate attentively and practice accuracy. In traditional classrooms, the use of technology is strictly monitored mainly to its exclusion. Calculators can hardly be used in the class, but definitely not in assessments. When technology is integrated in teaching and learning, it shares in the division of labour when it carries out some roles. The labour of recalling and practicing cumbersome mathematical algorithms is now often displaced by selecting appropriate mathematical software and entering data (Pea, 1987) or setting up relationships between points on lines and planes in an interactive virtual representation system such as Geogebra or a similar Computer-aided design (CAD) programs.

Teachers find themselves with new and expanded roles when they allow learners to engage directly with tasks and technology. As learners explore and reason, the synchronisation of discussions and summarizing of key concepts and skills, often in retrospect is a new and unfamiliar role for most teachers. Thus, teachers need to learn how to divide labour with expanded roles and how to select appropriate functional technologies (Pea, 1987) and tasks toward curriculum attainment. With the expanded goal of meaningful teaching of mathematics with technology, the traditional community in which a teacher and the class find themselves, is extended to almost infinite internet-based communities that produce innumerable artefacts, often without any qualms, as they expect to be monitored by this vast community.

Meaningful teaching with technology

Meaningful teaching with technology encompasses the use of software tools to enhance the development of meaningful goals. Hakkarainen (2009) argued that the implementation of technology goes beyond mechanical assimilation of that which already exists, or the mere adaptation of technology for its affordances. Meaningful use of tools, should be beyond manipulating technology to suit existing pedagogies, as technology may conform and be consumed by traditional pedagogy (Flavin, 2017). It is clear that General ICT teacher training as rolled out on a large scale in Namibia, must be augmented with meaningful subject integration that enables teachers to develop learners’ technological skills along with their mathematical thinking. Such integration must empower teachers to view the world through mathematical and technology eyes to equip them to identify and include opportunities for

learners to apply knowledge and skills accurately and innovatively in everyday life. Numeracy skills are only meaningful when they become functional life skills applied to the world around us (Ministry of Education Arts and Culture, 2016).

Implications for Professional development for meaningful teaching with technology

Expanded educational goals demand reform of Namibian teachers' ongoing professional development. Various professional development frameworks parse knowledge needed to teach with technology, such as the overarching generic Technological Pedagogical and Content Knowledge (TPACK) (Mishra & Koehler, 2006) and Mathematical Knowledge for Teaching (MCK) (Ball, Thames & Phelps, 2008). Locally, as Namibia has just revised its secondary school curriculum, present professional development interventions are mostly restricted to mathematics content knowledge based on traditional pedagogies. Such interventions acknowledge the difficulty for teachers to learn new content at teacher level, but run the risk of continuing to separate content knowledge from the other requisites for meaningful learning and teaching proposed earlier. Sarason (1990) argued that teaching for reform competencies can only happen if teachers develop similar competencies. To design professional development as a series of interventions that must add to each other over time, runs the risk of failing to engage teachers in experiential learning-with-technology as a means to expand their epistemologies.

With the notion of expansive learning through the lens of activity theory, a framework is proposed for professional development of mathematics teachers to teach with technology. The proposed framework is intended as a bridge between current local and overarching general professional development frames. Teacher educators who provide professional development for teachers, are involved in a different activity system that must literally be imagined as it is constructed. Figure 2 shows a Professional development (PD) activity system that can be imagined, encompassing or holding the activity system of teachers. This organization suggests PD through direct nodal relationships, especially subject-subject relationships, community-community interaction and directly negotiated division of labour. In contrast, placing the PD activity system next to the teacher activity system, as in Figure 1, suggests essentially disconnected activities. The organisation envisaged in Figure 2, allows imagination of reciprocal engagements at every node as the expansion of the teacher's system will bring a node closer to that of the PD system, or resistance may result in the PD node to move closer to the teacher system. The effects of nodal movements on distances between nodes across the triangles also suggest interesting mental experiments. For example, if the subject node of the teacher system moves closer to the subject node of the PD system, teachers may find their knowledge of technology in teaching enhanced (the distance to the expanded tools node is shorter), but that they are not yet closer to realising expanded goals (The distance across to their current goals, as well as the expanded goals, is now longer).

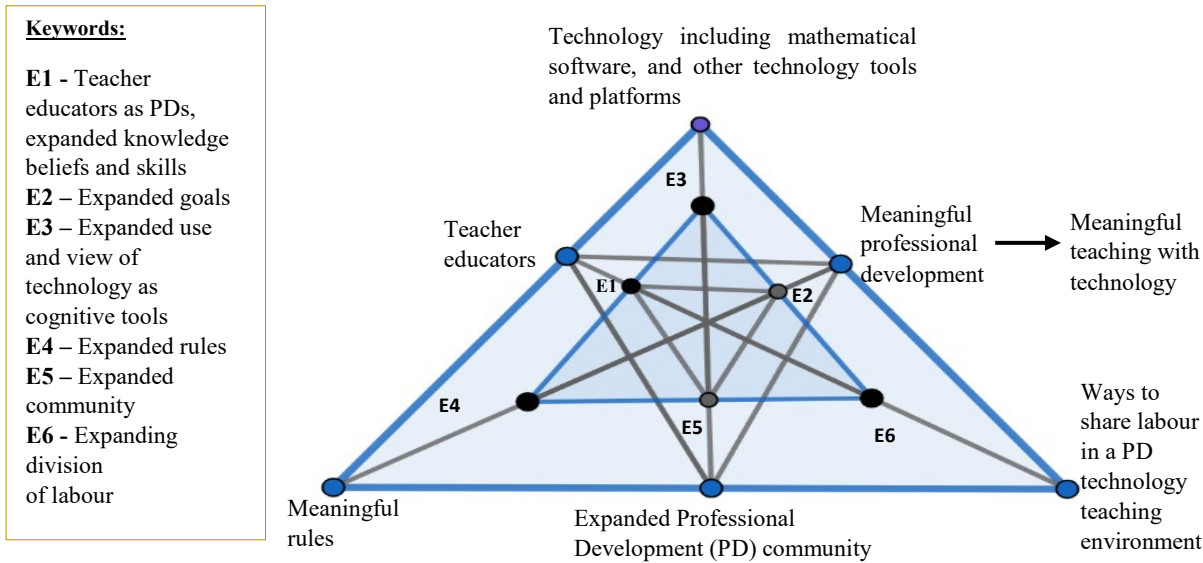


Figure 2. A different relationship between Professional Developers (expanded) and Teacher Activity systems

The professional development for expanded and emerging goals are guided by the following principles. These principles are not intended to be complete, but rather are emerging from ongoing theoretical research and reflection. They are also not intended to be a list, but to be used to develop meaningful PD along the argument in this paper. Hence, in relation to each of the principles, teachers must be engaged in opportunities for

- (a) making connections and thinking about relevance (conceptual understanding)
- (b) communication, including logical thinking, reflecting, explaining and justifying (adaptive reasoning)
- (c) formulating, representing and solving problems (strategic competence)
- (d) developing fluency with teaching mathematics meaningfully with technology (procedural fluency)
- (e) engaging in design, creating and trying out teaching with technology (productive disposition), especially engagement, productive struggle and communication.

The design principles, labelled with a prefix P, are linked to an ‘E’ which indicate the nodes that are expanded, as in Figure 2. The design principles are¹:

P1. Teacher educators (E1): Expanding the worldview of mathematics teachers

- (a) Determining mathematics teachers’ teaching need for emerging goals
- (b) Determining mathematics teachers’ views and beliefs on technology use, choices to use expanded pedagogy, as well as views on participation in professional development activities
- (c) Immerse teachers as learners in teaching-with-technology situations that stimulate their imagination of expanded personal goals.

P2. Expanded goals (E2): Stimulate teachers’ imagination to teach mathematics meaningfully

- (a) Promote reflection on teachers’ existing teaching and learning of mathematics
- (b) Provide opportunities to experience mathematics in a new way through:
 - i) Exploration and viewing the world through mathematical eyes
 - ii) Communicating mathematically
 - iii) Making objects with mathematical properties

¹ The numbering does not imply any linear order.

- (c) Expose teachers to practices from the larger mathematics education community where teaching and learning approaches the meaningful
 - i) Stimulate critical comparison and creative engagement with other practices.

P3. Expanded use and view of technology as cognitive tools (E3):

- (a) Provide many opportunities for teachers to collaborate and experiment with teaching mathematical concepts through the use of cognitive technology
- (b) Provide many opportunities to reflect on the practicality and appropriateness of traditional and new technology tools. Also reflect on the ethical obligation to the practice of teaching in Namibia in terms of access and equal opportunity for all to learn.

P4. Expanded rules for meaningful teaching with technology (E4):

- (a) Reflect on the mathematics curriculum and contribute to curriculum reform by participation in local (and international) conferences
- (b) Explore affordances and constraints of technology in relation to current and expanded assessment rules.

P5. Expanded community (E5): The program design should provide ongoing opportunities for critical reflection on teaching meaningfully with technology in Namibia

- (a) Expand teachers' communities of professional practice and development to include professional developers as well as education department officials to sustain long-term development
- (b) Expand teachers' communities of professional practice and development to include remote online professional communities with compatible goals to expand and sustain learning on demand over the long term.

P6. Expanded division of labour (E6): An expanded community enables expanded division of labour

- (a) Afford teachers an opportunity to plan, design, and showcase meaningful teaching experiences with technology on wider than local forums
- (b) PD must support teachers on an ongoing basis by developing a repository of tasks for meaningful teaching with technology in Namibia, and by creating and providing non-synchronous PD materials
- (c) PD with teachers must design prototype assessments with technology as central cognitive tool and stimulate reflection on their use.

Conclusion

The Namibian curriculum as intended is reticent to promote unrestrained meaningful teaching of mathematics, with or without technology. In particular, as technology is not envisaged as a cognitive tool in the current curriculum it is unlikely to lead to expanded teaching and learning that includes technology. The curriculum stands to be expanded from the bottom up as a result of teachers learning to teach mathematics meaningfully with technology as we re-interpreted the Five strands of mathematical proficiency to premise a pedagogy of mathematical thinking through communication. A professional development activity system is envisaged where each node of teachers' activity system is linked to a node in the expanded PD activity system. The

Disclosure statement

No potential conflict of interest is reported by the authors

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