Bargaining Competition and Vertical Mergers

by

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Dissertation presented for the degree of Doctor of Philosophy in Economics in the Faculty of Economic and Management Sciences at Stellenbosch University

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April 2022

Declaration

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Declaration with regard to parts of the dissertation in which other authors were involved

This dissertation includes limited contributions from two co-authors:

Chapter 2 section 2.3.3 (p. 22 - 26) contains the write-up of a demand function by Luke Froeb and Steven Tschantz. The write-up of the functions shown in the calibration section 2.4 is also the work of Luke Froeb and Steven Tschantz (p. 26 - 27).

The functions contained in chapter 4 section 4.4 (p. 66 - 67) were written up by Steven Tschantz.

Abstract

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Dissertation: PhD (Economics)

April 2022

Vertically related markets and vertical mergers are complex systems that comprise a number of distinct features. The modelling of such complex systems involves several modelling choices that affect the predicted model outcomes. We rely on simulation-based methods to consider how choices of key model parameters, assumptions and industry structures map onto competitive outcomes in vertically related markets and for vertical mergers. Our simulation results can help guide practitioners in selecting models that best characterise the features of a given vertical relationship, especially since assumptions that distinguish the models from one another — how and over what parties bargain — are typically not observed.

In particular, this dissertation studies vertically related markets and vertical mergers along three dimensions. Firstly, we focus on comparing alternative models of vertical competition, based on different assumptions regarding the nature and object of vertical contracting. As far as the nature of vertical contracting is concerned, models may assume upstream and downstream firms reaching agreement through take-it-or-leave-it offers, bargaining or recursive bargaining. As far as the object of vertical contracting is concerned, models may assume vertical contracting is over linear prices (a marginal wholesale price) or two-part prices (a marginal wholesale price and a fixed f ee). We systematically compare the corpus of models of vertically related markets across two simple industry structures ('1×2', one upstream and two downstream firms; and '2×1', two upstream and one downstream firm) to allow direct comparisons. Our comparisons show that in a linear pricing setting, a modelling choice between bargaining and recursive bargaining is irrelevant to the outcome. In two-part pricing, however, bargaining leads to a more competitive outcome than the joint profit maximising outcome under recursive bargaining.

Secondly, we study and compare models for vertical merger analysis, in order to investigate how assumptions regarding vertical contracting map onto observable merger effects. We also examine the extent to which predictions from models of vertical mergers are robust to different specifications of substitutability. In particular, we compare models calibrated to an increasing aggregate elasticity (i.e. the substitutability of the inside goods with the outside good) with models calibrated to the nest strength parameter of the demand function. Our results show that the predicted merger effects from different models are consistent for the two measures of substitutability. The results also illustrate that modelling choices such as the specification of the industry structure or object and nature of vertical contracting that determined outcomes in the pre-merger world, can also predetermine post-merger outcomes.

Lastly, we introduce a vertical merger simulator tool to allow an assessment of vertical merger scenarios in practice. We illustrate the utility of the simulator as a screening tool by reference to a number of examples reflecting modelling choices often faced by practitioners. In this regard, we illustrate three examples where the exogenous variables of interest are the marginal cost of the upstream firm and downstream firms, the market shares and the prices of the downstream firms respectively. We compare the simulator to incentive scoring methods (comprising of various upward pricing pressure indices), which have received extensive attention in literature and policy circles as a screening tool for merger effects, including for vertical mergers. While direct comparisons are challenging, it is evident that the data requirements of our vertical merger simulator are not particularly onerous compared to those of incentive scoring indices. The simulator offers the additional benefit of full equilibrium analysis, compared to the partial equilibrium focus of incentive scoring methods. We conclude that the simulator can be a useful complementary tool for vertical merger screening.

Uittreksel

Bargaining Competition and Vertical Mergers

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April 2022

Vertikaalverwante markte en vertikale samesmeltings behels komplekse sisteme wat uit verskeie unieke eienskappe bestaan. Die modellering van hierdie komplekse sisteme vereis verskeie modelleringskeuses wat die voorspelde uitkomste van hierdie modelle kan beïnvloed. Gevolglik steun ons op simulasie-gebaseerde metodes om te ondersoek hoe keuses rakende belangrike parameters, aannames en industrie strukture neerslag vind in mededingende uitkomste in vertikaalverwante markte, ook na aanleiding van vertikale samesmeltings. Ons simulasie resultate kan praktisyns lei om daardie modelle te kies wat die eienskappe van 'n gegewe vertikale verhouding ten beste belig. Dit is veral behulpsaam gegewe dat aannames wat verskillende modelle onderskei — aannames omtrent hoe en waaroor partye onderhandel — tipies nie waargeneem kan word nie.

Hierdie proefskrif ondersoek vertikaalverwante markte en vertikale samesmeltings vanuit drie oogpunte. Eerstens, fokus ons op die modellering van uitkomste in vertikaalverwante markte. Die literatuur stel 'n verskeidenheid modelle van vertikale mededinging voor, gegrond op verskillende aannames rakende die tipe en objek van vertikale kontraktering. Met betrekking tot die tipe vertikale kontraktering, kan modelle voorsiening maak vir geen onderhandeling, onderhandeling of rekursiewe onderhandeling tussen firmas wat in 'n vertikale verhouding verkeer. Met betrekking tot die objek van vertikale kontraktering, kan modelle voorsiening maak vir lineêre pryse ('n marginale groothandelprys) of tweestukpryse ('n marginale groothandelprys en 'n vaste fooi). Ons vergelyk die corpus van modelle vir twee eenvoudige industriestrukture ('1×2', een opstroom en twee afstroom firmas; en '2×1', twee opstroom en een afstroom firmas) wat direkte vergelykings moontlik maak. Ons vergelykings toon dat, sover dit lineêre pryse aangaan, 'n keuse tussen onderhandeling en rekursiewe onderhandeling irrelevant tot modeluitkomste is. Daarenteen lei onderhandeling tot 'n meer mededingende uitkoms as die gesamentlike winsmaksimerende uitkoms van rekursiewe onderhandeling, waar dit tweestukpryse aangaan.

Tweedens, vergelyk ons modelle van vertikale samesmeltings om te ondersoek hoe aannames rakende vertikale kontraktering in waarneembare samesmeltingeffekte figureer. Ons ondersoek ook hoe verskillende substitusiespesifikasies die vooruitskattings van modelle van vertikale samesmeltings beïnvloed. Ons vergelyk, in die besonder, modeluitkomste met 'n fokus op sg. totale elastisiteit (d.w.s. die

substitusie van alle produkte binne die mark met 'n nominale buite-produk) en dié met 'n fokus op die sg. nesparameter van die vraagfunksie. Ons bevind dat die voorspelde samesmeltingseffekte van die verskillende modelle konsekwent vir die twee alternatiewe maatstawwe van substitusie is. Die resultate dui ook aan dat modelleringskeuses vir die wêreld voor 'n samesmelting (insluitend keuses omtrent die spesifikasie van die industriestruktuur sowel as die objek en tipe van vertikale kontraktering) dikwels die voorspelde uitkomste van die samesmelting voorafbepaal.

Laastens, stel ons 'n vertikale-samesmelting simulator bekend wat die beoordeling van alternatiewe vertikale-samesmelting scenarios in die praktyk kan ondersteun. Ons illustreer die bruikbaarheid van die simulator as 'n keuringsinstrument deur te verwys na voorbeelde wat die gereelde modelleringskeuses van praktisyns reflekteer. In hierdie opsig, verwys ons na drie voorbeelde waar die eksogene veranderlikes van belang onderskeidelik die marginale koste van opstroom en afstroom firmas, die markaandele en die prys van die afstroom firmas is. Ons vergelyk die simulator met metodes vir insentieftellings (en bepaald verskeie sg. opwaartseprysdrukindekse), wat breedvoerige aandag as keuringsinstrument (ook vir vertikale samesmeltings) in die literatuur en onder beleidsmakers geniet. Alhoewel 'n direkte vergelyking uitdagend is, is dit duidelik dat die datavereistes van ons vertikale-samesmelting simulator nie noodwendig hoër as diè van opwaartseprysdrukindekse is nie. Die simulator bied die addisionele voordeel van 'n ontleding vanuit die oogpunt van totale markewewig, terwyl die resultate van opwaartseprysdrukindekse op parsiële markewewig staatmaak. Die simulator bied dus 'n nuttige komplementêre instrument vir die keuring van vertikale samesmeltings.

Acknowledgements

My life is certainly very different and I am a very different person compared to when I embarked on this journey three years ago. I have several things and people to thank for this. First, I thank God for giving me the ability to pursue this degree. When I look back at my life, I always seem to be exactly where I need to be at the right time. I know that this is no coincidence.

On an academic level, I would firstly like to thank my supervisor, Willem Boshoff. You believed in me and gave me tremendous opportunities throughout my studies. I have learned so much from you on many levels.

Secondly, I owe a huge deal of gratitude to my co-supervisor, Luke Froeb. In many ways, working with you has changed my life. Thank you for always being inspiring and kind. Thank you for being so open to sharing your resources and network with me.

On a personal level, at the start of my PhD, I have found someone that was willing to join me on this journey. Two years later, she signed up for an even greater journey — the rest of our lives. Tiana, your incredible love and support throughout the past two years were many times what kept me going. Thank you for being with me through the many lows and highs. Thank you for the words of encouragement and motivation in times when I needed it most.

To my unbelievable parents, I do not have the words to express my gratitude. You have given up so much for me to be where I am today and have always supported me throughout. Thank you for the many hysterical phone calls, encouragement, advice and unconditional love.

To my family (and by family I mean grandparents, aunts, uncles, future in-laws and friends that are like family), thank you for your unwavering support and encouragement. I cannot overstate how much every phone call, message, quick coffee, meal at a restaurant, lift to the exam and many more, means to me.

To my friends and colleagues at the Centre for Competition Law and Economics and the Department of Economics at Stellenbosch University, thank you for making this journey a little less lonely. Thank you for the support throughout my eight years of studies. Thank you for not making it easy, because that enabled me to become better every day. Thank you for the kind words in the corridor or outside of the Schumann building. Two people also helped a great deal when the coding got tricky. Steven Tschantz and Hanjo Odendaal, thank you for your tremendous help in this regard.

The GEMs program also has my gratitude for providing me the means to do a full-time PhD. Thank you to Jaco Franken for the incredible job you do. Thank you for the support and always having the perfect advice for any situation.

Dedications

To Tiana and my parents, Johan and Christelle.

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Chapter 1

Introduction and Research Questions

Ascertaining the likely effects of a vertical merger is by no means an easy task. Vertical mergers involve multi-level arrangements, which, compared with horizontal mergers, greatly increase the number of variables that may affect market outcomes. Moreover, as discussed below, the incentives for vertical mergers are often efficiency related, resulting in significant pro-competitive effects, while anti-competitive effects are brought on only in an indirect manner. Economists have developed several tools that shed light on these complex relationships, such as upward pricing indices and simulation models (Shapiro, 2021). However, findings from both theoretical models and empirical studies of such models, are often ambiguous or contradictory (Slade, 2020).

This dissertation contributes to our understanding of vertically related markets and vertical mergers by investigating three aspects of their analysis. First, we focus on modelling outcomes in vertically related markets. In particular, we systematically compare models of vertically related markets across different industry structures and different contractual forms. Second, we extend our focus to mergers in vertically related markets. Specifically, we compare the magnitude of predicted vertical merger effects across the corpus of models. Our results shed light on how assumptions about the nature and object of vertical contracting are mapped onto observable outcomes. Third, we introduce a merger simulation tool that can be used to screen vertical mergers and compare their predictions with those of incentive scoring methods.

The research presented in this dissertation is motivated by recent developments in both the theoretical and empirical literature on vertical merger modelling, as well as by developments in vertical merger policy. Indeed, as is argued below, the complexity of vertical merger analysis — and our evolving understanding of such mergers — is reflected in policy developments. This chapter first provides a brief overview of these developments, which provides the context for the research questions outlined later in the chapter.

1.1 Policy context

There is no historical policy consensus about the conditions under which vertical mergers may give rise to anti-competitive effects. The lack of policy consensus reflects two broad policy developments of the past decades. First, enforcement actions against vertical mergers have been limited in the main jurisdictions. Second, in contrast to horizontal mergers, most competition authorities historically have not issued guidelines for vertical mergers (Church, 2008). One argument is that vertical merger analysis is well understood and does not require scrutiny. Another is that vertical mergers may lead to various complex competitive effects, so that it would be too difficult to provide coherent guidelines (Salop and

Culley, 2016).

In recent years, vertical merger enforcement actions have become more prominent, which Slade (2020) ascribes to the move from industrial-focussed economies (where mergers may raise mostly horizontal competitive concerns) to knowledge economies (where transactions may raise more vertical concerns). Some of these recent high profile vertical merger cases in the United States and European Union have emphasised the uncertainty around quantifying vertical merger effects. These developments have also led to or coincided with amendments to, or publication of guidelines for vertical merger analysis. The following subsections provide a brief survey of these developments, by jurisdiction.

1.1.1 Developments in the United States of America

In the USA the first vertical merger guidelines were issued in 1968 and revised only once in 1984 (Salop and Culley, 2016). This left practitioners without clear principles (Church, 2008) concerning assessing vertical mergers for several decades. While this was perhaps adequate for the only 48 vertical merger enforcement actions in the USA in the 1994 – 2015 period (Salop and Culley, 2016), the recent increase in such actions (an additional 17 cases from 2016 – 2020 (Salop and Culley, 2020)) calls for greater guidance. The approach taken in the AT&T/TimeWarner merger case of 2018 is an exemplar of the problems facing vertical merger analysis.

The AT&T/TimeWarner merger case was, arguably, an important inflection point for vertical merger analysis in practice, at least in terms of the US jurisdiction. First, it was the first litigated US vertical merger case in 40 years (Underwood, 2020). Second, the Court was very sceptical of the economic testimony and evidence. Afterwards, the economic expert for the Department of Justice noted a 'distaste for economic models' and an 'open hostility towards experts in general' (Shapiro, 2021). In ruling for the defendants (Leon, 2018), the Judge called the economist's bargaining model a 'Rube Goldberg contraption' (Shapiro, 2018), questioning both its reliability and factual credibility, and hence its ability to predict future harm.

Shapiro (2021) views this decision as a rejection of basic bargaining theory. Hovenkamp (2020) agrees, calling it 'a serious misstep' to reject the assumption of profit maximisation — an assumption not only underpinning Nash bargaining methodologies, but also economics in general.

These proceedings coincided with the Department of Justice and Federal Trade Commission publishing Vertical Merger Guidelines in 2020 (Department of Justice and Federal Trade Commission, 2020b). The Guidelines do not appear to have resolved the uncertainties associated with how US policymakers approach vertical merger analysis. The initial draft of the Guidelines sparked more than 70 differing public comments from leading academics and practitioners on what the Guidelines should cover (Department of Justice and Federal Trade Commission, 2020a).

The published Vertical Merger Guidelines (Department of Justice and Federal Trade Commission, 2020b) depart from the Chicago School position, that vertical mergers are unambiguously welfare enhancing. The Guidelines deal extensively with unilateral effects, outlining how US agencies are likely to assess the *ability* and *incentive* of a vertically integrated firm to foreclose or raise rivals' costs. Even so, the Guidelines indicate a holistic approach to quantifying competitive effects, stating that the agencies will consider both potential competitive harm as well as the potential benefits to competition emanating from a proposed merger.

For an assessment of the net effect of a vertical merger, the Guidelines mandate a clear understanding of how the merged firm's incentives are altered post merger. It is clear that the agencies view the source of both potential pro-competitive, and anti-competitive effects to be the alignment of economic incentives between merging firms. It is emphasised that regardless of these effects emanating from the same source, they are offsetting, so that their relative magnitude has to be determined (Department of Justice and Federal Trade Commission, 2020b).

The Guidelines state that the agencies may employ merger simulation models to quantify the net effect on competition. The examples and discussion provided in the Guidelines suggests a need for understanding of how the characterisation of contracting between vertically related firms influence model outcomes. As discussed further on, the literature provides little guidance in this regard.

1.1.2 Developments in the European Union

In the EU, Article 2 on the control of concentrations between undertakings of 1990 governed merger decisions until the European Court of Justice established the new Council Regulation for merger control in 2004 (Mosso, 2007). The European Commission subsequently published Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings in 2008 (European Commission, 2008). These Guidelines cover both conglomerate and vertical mergers; we focus on the latter. Different from those of the US, these Guidelines precede recent major cases.

The EU Guidelines repeat the established policy view that non-horizontal mergers are generally less likely to adversely affect competition than horizontal mergers[1]. In particular, the Guidelines note that vertical mergers do not lead to the direct elimination of a competitor (the main source of anti-competitive effect in a horizontal merger) and that such mergers can provide substantial scope for efficiencies (European Commission, 2008).

Regardless, the Guidelines do identify circumstances in which vertical mergers are anti-competitive. As with the US Guidelines, foreclosure — including both input and customer foreclosure — is flagged as a potential competitive concern. The Guidelines pay particular attention to the scope for raising of rivals' cost as a form of customer foreclosure.

Similar to those of US Agencies, the Guidelines state that the *ability* and *incentive* of a merged firm to foreclose a rival will be examined. Emphasis is placed on the net effect of a vertical merger. In assessing the net effect, the agencies are to consider the potential pro- and anti-competitive effects of a proposed merger. The key concern is whether increased input prices lead to increased prices to consumers, but that this may be mitigated by the merged firm possibly decreasing its price.

The Guidelines mention that the effect on competition may be considered in light of countervailing factors. Specifically, the mention of countervailing buyer power, suggests an understanding of bargaining models to assess vertical mergers. However, no specific mention is made of bargaining models or any other quantitative techniques to assess vertical mergers. This perpetuates the uncertainty surrounding the characterisation and quantification of vertical merger effects.

A recent litigated case does shed more light on the agencies' approach to vertical merger enforcement. In the Telia/Bonnier Broadcasting merger case of 2019, the Commission employed a bargaining model to analyse the potential merger effects. The Commission relied on data from the parties' internal

[1]Albeit reminiscent of the Chicago School presumption that vertical mergers are unambiguously welfare enhancing, we do not view this as a strong policy statement. However, it is at least interesting that the EU Guidelines, different from the US Guidelines first discuss efficiencies relating to vertical mergers.

estimates of customer switching and a natural experiment to calibrate their bargaining model. The model predicted that the merged entity will substantially increase the input price (licensing fees for Bonnier Broadcasting's content) causing viewers to substitute away from the foreclosed rival towards the merged entity. These predictions enabled the Commission to successfully impose significant access remedies on the approved merger (Karlinger et al., 2020). It therefore appears that the Commission has been more willing to rely on bargaining models.

1.1.3 Vertical merger enforcement in South Africa

In South Africa, the Competition Act of 1998 introduced compulsory pre-merger notification for mergers exceeding certain thresholds[2] (Competition Tribunal, 2009). Consistent with the approach in other jurisdictions, the focus has been on mergers — including vertical mergers — by large firms. Even so, and of particular relevance to vertical merger analysis, the Competition Commission has recently expanded its focus to include selected small merger transactions in the digital sphere[3].

While SA has not seen the same increase in vertical merger enforcement activity in recent years as observed in other jurisdictions, merger enforcement nevertheless suggest an increasing concern with vertical issues. Already in 2009, in reviewing 10 years of enforcement under the new competition law regime, South African policymakers argued that vertical mergers may require closer scrutiny (Competition Tribunal, 2009). Policymakers flagged several features of the South African economy that may give rise to competition concerns associated with vertical mergers. SA (as with many other developing countries) has a relatively small and concentrated economy, characterised by extensive cross-holding between large firms. Moreover, as a result of the high levels of protection and extensive regulation enjoyed during the Apartheid era, multiple levels of key value chains are dominated by one or two firms (Competition Tribunal, 2009).

These features of the South African economy may give rise to more serious competition concerns associated with vertical mergers, especially in key sectors of the economy, and this is reflected in the merger enforcement activities of the South African authorities (Mncube et al., 2012). Specifically, the South African authorities' approach in assessing vertical merger effects has evolved from being sympathetic to the Chicago School (Saggers, 2008) towards a clear embrace of the Post-Chicago view of unilateral effects. Moreover, the South African Authorities' approach reflects concern for how a proposed vertical transaction may create a conducive environment for coordination[4].

A more hostile approach to vertical mergers first manifested in the Mondi/Kohler merger in 2002. In this precedent-setting case, the Tribunal accepted that the merger would likely lead to input and customer foreclosure (Competition Tribunal, 2009), despite criticism surrounding the quantitative rigour of the evidence that lead to the decision (Saggers, 2008). Whether merited or not, this decision clearly illustrates the need for a better understanding of the incentive (and not just ability) of the merging firm to foreclose rivals. In this regard, simple screening tools, such as the ones presented later in this dissertation, may go a long way in assisting competition authorities in both developing and developed country jurisdictions.

[2]The Competition Board, the predecessor of the modern South African competition authorities, relied on voluntary notification and information from public sources (Competition Tribunal, 2009).

[3]In this regard, the CC published draft guidelines on small merger notification in 2021 (Competition Commision, 2021).

[4]Indeed, the South African experience shows a substantial portion of collusion cases involve industries where vertical integration is prominent (Mncube et al., 2012).

1.1.4 The role of economists

As mentioned at the outset of this chapter, there is no consensus about the proper characterisation and quantitative assessment of vertical mergers. Recent vertical merger enforcement actions and policy developments across a range of jurisdictions have failed to clear up this uncertainty. This experience is not unique to vertical merger analysis, as the quantitative analyses applied to horizontal merger evaluation were also heavily disputed when first introduced (Hovenkamp, 2020). Since the publication of the Horizontal Merger Guidelines in 2010 in the US, we have learned a great deal about the quantitative analysis of horizontal mergers. Quantitative techniques that were initially a point of contention now constitute a key part of horizontal merger enforcement.

Similarly, policy approaches to evaluating vertical mergers are likely to evolve over time (Hovenkamp, 2020) as enforcers learn about the reliability and robustness of vertical merger analysis (Shapiro, 2021). This sentiment underpins the research presented in this dissertation. As such, our aim is to contribute to the body of work on vertically related markets and vertical mergers.

Economists have a pivotal role to play in answering questions that arise in the context of vertical mergers, including "Which model assumptions best capture the significant features of competition in vertically related markets?" (Werden et al., 2004); "Do these assumptions also capture the loss of such competition following a vertical merger?" (Werden et al., 2004); and "How can we effectively screen the likely effects of vertical mergers with limited data?" (Slade, 2020). In the subsections that follow, we investigate these questions in light of the extant literature and formalise the main and ancillary research questions covered in the dissertation.

1.2 The features of models of vertically related markets

Vertically related markets represent a complex system with many moving parts. In particular, vertically related markets can be described by reference to four characteristics: (i) a network of upstream suppliers and downstream retailers who (ii) contract vertically in the presence of externalities (iii) caused by the competition between firms (iv) over consumer demand. In the existing literature, these features have enjoyed a lot of attention and changing the modelling assumptions about any one of them, can lead to a variety of results.

As a result of the complexity of vertically related markets, it can be difficult to ascertain exactly which assumptions cause differences in the predicted results. Therefore, in the first chapter of this dissertation, we employ a methodology that allows the direct comparison of the predictions of different models of vertically related markets. We make assumptions and modelling decisions to control for three of the features: (i) the network of upstream suppliers and downstream retailers, (ii) competition between firms and (iii) consumer demand. The focus is then on the remaining element: vertical contracting in the presence of externalities.

From the existing literature, we observe that the assumptions about vertical contracting can differ on the basis of the *nature* as well as *object* of vertical contracting. In terms of the *nature* of vertical contracting, the question: "How do vertically related firms contract with each other?" is answered. In terms of the *object* of vertical contracting, the question: "Over what do vertically related firms contract?" is answered. Below, we consider several models that pose different answers to these questions.

The traditional model of vertically related markets is the successive monopoly model, in which buyers are price takers. The model assumes that the upstream firm(s) maximise their profit function to determine the optimal price at which to sell their product. Consequently, it makes a take-it-or-leave-it offer to a downstream firm(s) for the input equal to this price. In this model, the upstream firm(s) sells to final consumers through the downstream firm(s), and the demand for the product produced by the upstream firm is derived from the demand for the product produced by the downstream firm. Consequently, this model was renamed the 'derived demand model' in the literature. Because both firms maximise individual profits, both add their margin, selling their product and asking for the monopolistic mark-up over their costs (Motta, 2004). It is thus well-established that in the derived demand model, an outcome far below the joint profit maximising outcome (the outcome if the two firms jointly maximised profits) is achieved (O'Brien and Shaffer, 1992; Moresi et al., 2007).

However, scholars argue that bargaining models — where the seller does not have all the bargaining power, as in derived demand models — provide a more realistic assumption about the nature of vertical contracting. Muthoo (1999) describes a bargaining situation as a setup where two players have to overcome conflicting interests and co-operate to serve their common interest. Vertical contracting over the wholesale price of an input meets the requirements: The two players have conflicting interests since the upstream firm wants to charge the highest possible price for the input, while the downstream firm wants to pay the lowest possible price. However, it is in both players' best interest to co-operate in order to secure a profit.

Since the seminal contribution by Horn and Wolinsky (1988), bargaining theory has been applied to a wide range of industrial organisation problems. The Nash-equilibrium-in-Nash-bargains (Nash-in-Nash) approach, as in Horn and Wolinsky (1988), entails bilateral bargaining between firms where any agreement made does not affect the value of an agreement for the other players (Collard-Wexler et al., 2019). It has the benefit that it provides outcomes that are easily computable for complicated bargaining problems. Consequently, it has grown into something of an empirical 'workhorse' (Froeb et al., 2019).

Different interpretations of the Nash-in-Nash solution mean that it can be applied for different *objects* of vertical contracting. First, the solution can be interpreted as an agreement on certain terms of the contract resulting from maximising the bargaining problem over the feasible set of agreements (Froeb et al., 2019). The solution to this interpretation involves the argument that maximises the elements of the bargaining problem, for example, a wholesale price that would maximise upstream and downstream profit. This interpretation fits well when the *object* of vertical contracting is linear pricing. For this vertical contracting *object*, bargaining is over linear wholesale prices, and firms reach an agreement over a marginal wholesale price per input.

Second, the Nash-in-Nash bargaining solution can be viewed as a surplus division rule (Collard-Wexler et al., 2019). In this interpretation, a fixed fee component forms part of the agreement, so that the surplus gained from agreeing, is split equally (or proportionally to the asymmetric bargaining strength of the players) (Froeb et al., 2019). The terms of the contract (such as the wholesale price) are chosen so as to maximise the coalition profit, while the fixed fee characterises the splitting of this profit. Therefore, the solution to this interpretation of the bargaining problem is the fixed payment paid from one player to the other. This interpretation can be applied when the *object* of vertical contracting is two-part prices. For this type of contract, players reach an agreement over a wholesale price and a fixed fee.

Developments in bargaining theory have lead us to identify a third way of how firms can contract with each other. In contrast to Nash-in-Nash bargaining, parties can bargain in anticipation of how one agreement affect other potential agreements. When parties take account of bargaining externalities, e.g., with the ability to renegotiate or to specify contingent contracts, profit is split according to the Shapley value (Stole and Zwiebel, 1996; Inderst and Wey, 2003). This bargaining solution is called "Nash-in-Nash with Recursive Threat Points" (Yu and Waehrer, 2018) or "Nash-in-Shapley" (Froeb et al., 2019) to indicate that the outcome is a Shapley division of surplus determined by payoffs in noncooperative threat points. In the same way as in Nash-in-Nash, firms can reach agreement over a linear- or two-part pricing contract.

In summary, models of vertically related markets can be classified into three categories if based on the assumed *nature* of vertical contracting: 1) take-it-or-leave-it offers by the upstream firm (derived demand), (2) bilateral bargaining between upstream and downstream firms (Nash-in-Nash) as well as (3) bilateral bargaining with contingent contracts (Nash-in-Shapley). We also identify two *objects* of vertical contracts: (1) linear pricing and (2) two-part pricing.

The predictions from models making these different assumptions may give rise to a variety of results. We aim to ascertain how these different assumptions map onto observable outcomes. Thus, we perform a systematic comparison of the corpus of models of vertically related markets. This yields the first research question investigated in this dissertation.

1.2.1 Research question 1: Assumptions about the nature and object of vertical contracting

Chapter 2 explores the first main research question of the dissertation: Which model assumptions best capture the significant features of competition in vertically related markets?

Choosing the right model to characterise observed competition can be difficult for at least two reasons. First, the critical bargaining assumptions - the alternatives to agreement that determine the terms of agreement - are typically not observed. Second, the assumptions interact with each other. We argue that the best way to answer the research question is by doing a systematic comparison of the outcomes of different models of vertically related markets across different industry structures and different contractual forms.

From the main research question in this chapter, an important ancillary research question arises: How can different model predictions be compared directly? As mentioned, in order to answer this, we recognise models of vertically related markets as "complex systems" and simulate their effects (Wolfram, 2002). In these simulations, we keep constant the specification of competition and consumer demand and calibrate all the models that we consider to the same set of parameters. We then investigate how the assumptions about vertical contracting affect model predictions in two simple industry structures: "1×2", one upstream and two downstream firms; and "2×1", two upstream and one downstream firm.

Admittedly, these two industry structures restrict attention to a very specific set of markets. However, since we calibrate all the models to the same set of parameters, simple industry structures allow direct comparisons. We are able to attribute the differences in outcomes solely to the different assumptions about vertical contracting. Moreover, we conjecture that these results can inform decisions regarding more complex industries. For example, two major US vertical merger cases over the past ten years — the AT&T/TimeWarner merger (Shapiro, 2018) and the Comcast/NBCU merger (Rogerson, 2014) — have featured models assuming these industry structures.

The research questions answered in chapter 2 provide a solid foundation for further investigation of vertically related markets. Specifically, the following chapter in the dissertation explores vertical mergers in these markets.

1.3 Vertical mergers

A vertical merger typically involves the joining of two firms at different levels of production or distribution (Perry, 1989). Such a joining may alter any one of the four distinct features of vertically related markets discussed above. Most evident is the change that a merger brings about in the network of upstream and downstream firms. However, a vertical merger does not lead to the elimination of a direct competitor, as is the case in a horizontal merger. Consequently, the immediate welfare implications of a vertical merger are more ambiguous than for horizontal mergers.

A vertical merger is often employed to solve coordination problems between firms (Werden and Froeb, 2020), with the most distinguishing of these being the elimination of the need for inter-firm contracting (Salop and Culley, 2016). When firms vertically integrate, a market transaction is substituted with an internal transfer within the boundaries of a firm (Perry, 1989) and the double margin on that product is eliminated. Hence, a vertical merger also alters the vertical contracting feature of models of vertically related markets.

The predicted magnitude of the pro-competitive effect of the elimination of double marginalisation can differ significantly depending on the assumptions that the model makes regarding vertical contracting. The Chicago School literature presumes that vertical mergers are unambiguously pro-competitive. This is a direct result of the significant elimination of double marginalisation in the derived demand model — the empirical workhorse in this literature (Cooper et al., 2005).

As Moresi and Salop (2013) show analytically, in the derived demand model, the pro-competitive effect of the elimination of double marginalisation is so significant that it completely overshadows any potential anti-competitive effects. Subsequently, vertical mergers have historically been viewed as pro-competitive and beneficial to consumers (LaFontaine and Slade, 2007). However, recently, scholars have shown that vertical mergers can result in anti-competitive outcomes when taking cognisance of game-theoretic bargaining models (Motta, 2004), as described in the preceding section.

As alluded to earlier, the Nash-in-Nash approach quickly became the empirical workhorse as a result of its easily computable and tractable results (Froeb et al., 2019). In this regard, scholars such as Sheu and Taragin (2017) have employed it in simulation models of vertical mergers. However, this literature typically investigates one or a few industries, such as the health care market (Gowrisankaran et al., 2015) or television markets (Crawford et al., 2018), making inference outside of the context of individual research pieces difficult (Sheu and Taragin, 2017).

Additionally, very few studies have investigated how recursive bargaining — Nash-in-Shapley as in Froeb et al. (2019) — fares in vertical merger simulation models. As mentioned previously, predictions from models making different assumptions may give rise to a variety of results. Therefore, we want to compare the magnitude of merger predictions across different industries for models with different assumptions regarding vertical contracting. This yields the second research question of the dissertation.

1.3.1 Research question 2: Predicted vertical merger effects

Chapter 3 considers the second research question of the dissertation: Which model assumptions best capture the loss of competition following a vertical merger?

The research questions answered in chapter 2 provide a solid foundation for answering the main research question of chapter 3. Moreover, since the methodology for making direct comparisons is developed in chapter 2, we are able to extend this to answer an ancillary research question: How are vertical merger predictions related to different measures of substitutability?

We answer the ancillary research question by employing the same methodology and system of models but consider an alternative calibration of the demand model. This allows inference about how robust vertical merger predictions are to different specifications of substitutability. Subsequently, we are able to identify costs and their relation to substitutability as a major determinant of outcomes in models of vertically related markets. Because vertical mergers change cost structures through the elimination of double marginalisation and potential raising of rivals' cost, this can be a major driver of the competitive effects of mergers.

The research questions answered in chapter 3 can inform practitioners about which assumptions and relationships are important when considering vertical mergers. Ultimately though, we would want to enable practitioners to consider vertical merger simulation results for the variables pertinent to a specific case. Therefore, the following chapter introduces a practical tool for vertical merger simulation. Users of this tool are able to choose control variables to calibrate the models presented in this dissertation and produce predictions of pre- and post-merger outcomes.

1.4 Ex ante evaluation of vertical mergers

In many jurisdictions, firms are required to notify the relevant authorities of their intention of pursuing a merger if the proposed merger exceeds a prescribed size threshold (Asker and Nocke, 2021). The authorities are given time to investigate whether the proposed merger raises potential competitive concerns, before approving or blocking the transaction. As highlighted in previous sections, ascertaining the likely competitiveness of a vertical merger can be difficult as a result of the many complexities associated with the modelling of vertically related markets. This is particularly challenging given the forward-looking nature of merger analysis; authorities have to rely solely on pre-merger data to determine these effects (Slade, 2020). This suggests a role for tools that can assist authorities in screening mergers for which there is limited data.

Slade (2020) provides a survey of vertical merger screening tools and discuss how they are often adapted from horizontal merger screens. This practice can result in several problems. For this dissertation, two of these pitfalls are especially pertinent: availability of data and predicted equilibrium. We discuss these in terms of two categories of screening tools that are often employed to quantify the effects of vertical mergers: incentive scoring methods and merger simulation.

Upward pricing pressure tools (universally referred to as incentive scoring methods) aim to quantify the effect of merging parties taking the "cost of competing" into account post-merger (Valletti and Zenger, 2021). With this approach, the unilateral effects of a merger are calculated with simple and intuitive formulae using *inter alia* diversion of sales and margins (Miller and Sheu, 2021). In this vein, scholars (see for example Moresi and Salop (2013) and Rogerson (2020)) have developed a range of upward pricing pressure formulae to account for the intricacies of vertically related markets and all of the players that could be affected by a proposed merger.

There is no consensus on the use of vertical incentive scoring measures as screening tools for vertical mergers. While Shapiro (2021) argues that these tools can be highly informative and practical, Slade

(2020) advises against their use, citing complex data requirements and high probability of type 1 and 2 errors. Domnenko and Sibley (2020) provides further evidence for this, stating that some of the indices that comprise this approach are based on partial derivatives. This results in the indices always predicting an increase in price, which decreases their predictive ability.

Additionally, incentive scoring measures are only able to provide an estimate of a partial equilibrium. This is because, as acknowledged by Rogerson (2020), the formulae used to calculate these measures, ignore the effect that the elimination of double marginalisation will have on the equilibrium as well as any feedback effects between competitive effects. Therefore, these measures are not able to provide a final answer on the net effect on consumers following a vertical merger.

An alternative tool employed to quantify vertical merger effects is simulation models. Merger simulation aims to provide numerical predictions of price and quantity changes (Budzinski and Ruhmer, 2009) by calibrating specific economic models (Valletti and Zenger, 2021). It requires a system of equations to allow inference about the pre-merger and predicted post-merger equilibria (Miller and Sheu, 2021).

Unlike the primary goal of incentive scoring methods, that of vertical merger simulations is to determine the net welfare effect of the pro-competitive elimination of double marginalisation and anti-competitive foreclosure (Slade, 2020). A frequently made argument in this literature is that incentive scoring methods are simpler to use, and the data requirements are less than those of simulation models (Moresi and Salop, 2013). This argument is investigated in the third research question of the dissertation.

1.4.1 Research question 3: Screening vertical merger effects with limited data

Chapter 4 considers the third research question of the dissertation: Can simulation models form part of the toolkit to screen the likely effects of vertical mergers?

Ancillary to, and implicit in the main research question is the issue of the data requirements for vertical merger simulation. In order to answer this, we first investigate the typical data and estimation requirements of incentive scoring methods. We then develop a vertical merger simulator tool that calculates pre- and post-merger equilibria for the different models presented in this dissertation. This tool can be calibrated with just six control variables one of which is chosen as the exogenous variable over which the models are simulated.

After introducing the vertical merger simulator and illustrating its use with examples, we compare the data requirements of incentive scoring methods and the simulator. While a direct comparison is not possible, we argue that there is no conclusive evidence that the vertical merger simulator requires more data than what would be required to calculate incentive scoring methods. The added benefit of the simulator is that it provides us with a full equilibrium prediction, whereas incentive scoring gives only a partial equilibrium. Therefore, we argue for the inclusion of the vertical merger simulator in the toolkit for vertical merger screening, especially considering that competition authorities prefer multiple tools.

1.5 Summary

Selecting the right model to predict the likely effects of a vertical merger is difficult. The assumptions that distinguish bargaining models from one another, particularly the alternatives to agreement, are typically not observed. This dissertation builds on the existing research of vertically related markets and mergers by making systematic comparisons of their predicted effects, and develops a practical tool with which economists can do the same. As is evident from the discussion above, the three chapters that follow each consider a research question:

- Chapter 2 investigates research question 1: Which model assumptions best capture the significant features of competition in vertically related markets?
- Chapter 3 investigates research question 2: Which model assumptions best capture the loss of competition following a vertical merger?
- Chapter 4 investigates research question 3: Can simulation models form part of the toolkit to screen the likely effects of vertical mergers?

Chapter 2 therefore serves two goals: First, it provides a systematic comparison of the pre-merger outcomes of the corpus of models of vertically related markets. Crucially, these models differ in the assumptions made about vertical contracting. Second, chapter 2 develops the methodology that allows a direct comparison of these outcomes. Chapter 3 builds on this in two ways: First, we can ascertain how assumptions that determine pre-merger outcomes, map onto predicted merger effects. Second, we employ the methodology developed in chapter 2 to directly compare these outcomes for two measures of substitutability. Chapter 4 provides the final contribution of the dissertation: a practical vertical merger simulator tool that can be employed by practitioners and academics alike. Finally, chapter 5 provides a summary of the arguments presented in this dissertation and identifies possible future research avenues.

Chapter 2

Modelling Vertical Contracting in Vertically Related Markets

2.1 Introduction

Economists have well accepted models of price, quantity and bidding competition. However, there are many competing models of bargaining competition, each with different predictions about the effects of mergers. Church (2008) eloquently pin the frequent controversy over vertical merger cases pursued by the USA's Department of Justice or the EU's European Commission on the lack of 'clear and bright-line principles'. Indeed, when the US agencies published draft vertical merger guidelines early in 2020 (Department of Justice and Federal Trade Commission, 2020b), it sparked more than 70 differing public comments from leading academics and practitioners on what they should and should not have said (Department of Justice and Federal Trade Commission, 2020a).

The published 2020 vertical merger guidelines placed renewed focus on vertically related markets and how to model them. These markets are formally defined as a set of potential agreements (links) between upstream and downstream firms that sell to end consumers and earn an operating profit. From this definition we are able to determine that models of vertical relationships typically involve four distinct features: (i) a network of upstream suppliers and downstream retailers, (ii) who contract vertically in the presence of externalities, (iii) caused by the competition between firms (iv) over consumer demand.

In this chapter, we focus on the second feature of models of vertically related markets — vertical contracting in the presence of externalities[1]. The theoretical literature suggests that models assuming bilateral bargaining, as opposed to a take-it-or-leave-it scenario when settling vertical contracts, produce quite different price and output predictions. As discussed later, we therefore know that assumptions about vertical contracting matter considerably for models of vertically related markets. Even so, translating models into practice — and merger simulation in particular — is more challenging. For one, model selection is complicated, as the modeller is confronted with various choices related to bargaining parameters. Moreover, the assumptions that distinguish these models from one another are typically not observed. As we show below, these choices are not inconsequential, and carry significant implications for model predictions.

Therefore, in this chapter, we aim to address the challenging question of model selection by answer-

[1] We view vertical contracting as encompassing all forms of upstream firms contracting with downstream firms over an input. It thus includes models assuming take-it-or-leave-it (derived demand) offers by the upstream firm as well models assuming bilateral bargaining between firms.

ing the research question: "To what extent do assumptions about the nature of vertical contracting as well as the objects of vertical contracts determine model predictions?" To do this, we provide a systematic comparison of six different models of vertically related markets. The models differ in terms of either the assumption about the nature of vertical contracting (i.e. how upstream and downstream firms reach agreement) or the object of contracting (i.e. over what do they reach agreement). With regard to the nature of vertical contracting, we consider models that assume (1) take-it-or-leave-it offers by the upstream firm (derived demand model), (2) bilateral bargaining between upstream and downstream firms (Nash-in-Nash model) and (3) bilateral bargaining with contingent contracts (Nash-in-Shapley model). With regards to the object of contracts, we consider both linear pricing and two-part pricing contracts. The assumptions underlying each model are discussed in greater detail in section 2.3.

In comparing the magnitude of model predictions and their differences, we tacitly recognise these models as 'complex systems' and simulate their effects (Wolfram, 2002). Accordingly, we run a series of computational experiments where we keep downstream competition and demand features unchanged but vary the vertical contracting feature for two industry structures. Specifically, we assume that downstream competition is characterised by Nash equilibrium and a demand system that we call a 'rectangular logit' (discussed at length in section 2.3). For the network of upstream suppliers and downstream retailers (point (i) above), we limit our attention to two simple structures: 1×2 (one upstream and two downstream firms) and 2×1 (two upstream and one downstream firm). These simple structures are not only sufficient to illustrate the main differences between the models, but also allow us to identify exactly which assumptions cause these differences.

The simulation results in this chapter can help guide model selection, because they map assumptions about the nature and object of bargaining into observable outcomes. A systematic comparison of different models across different industry structures and different contractual forms provides novel insights into vertically related markets.

Among other things, we find that:

- In derived demand models, output is far below the level that would characterise monopoly, as a result of the take-it-or-leave-it linear wholesale price that the upstream firm(s) set.
- When parties bargain over linear wholesale prices, the differences between Nash-in-Shapley and Nash-in-Nash bargaining disappear, as there is only one instrument to both increase industry profit and split it up.
- When parties bargain over two-part prices in a 1×2 setting, Nash-in-Nash models show a more competitive outcome while the Nash-in-Shapley model follows the monopoly outcome.
- When parties bargain over two-part prices in a 2×1 industry setting, Nash-in-Nash and Nash-in-Shapley equilibria are equal.

This chapter presents first an overview of the literature pertinent to models of vertically related markets. This overview contextualizes the discussion of the different elements of the models simulated in section 2.3 of this chapter. Section 2.4 comprises a discussion on the methodology followed to calibrate the system of models, and section 2.5 presents the results.

2.2 Literature overview

There are no 'clear and bright-line principles' in the economic theories of vertical mergers (Church, 2008), stemming at least partly from how complex vertically related markets are to model. For economists, the core question is "which assumptions best capture the significant features of competition in vertically related markets, and the loss of such competition following a vertical merger?" (Werden et al., 2004). However, only after we understand how assumptions inhering in different models determine pre-merger outcomes, can we begin to investigate vertical mergers. Therefore, this chapter concerns the first part of this question and comprises a systematic comparison of the various models frequently employed to predict the outcomes in vertically related markets. Before the start of this systematic comparison, we provide an overview of the existing literature on traditional- and bargaining models.

The simplest (and older) model of vertical relationships is the successive monopoly (or derived demand) model, where upstream firms make a take-it-or-leave-it linear wholesale price offer to downstream firms. This wholesale price then contributes to the downstream firms' total marginal cost; and they then mark it up again. The double marginalisation in this model leads to an outcome far below monopoly (O'Brien and Shaffer, 1992). When employing this model to analyse a vertical merger, the elimination of the double margin always outweighs any potential anti-competitive effect (Moresi et al., 2007)[2]. However, this model is still employed in analyzing vertically related markets and vertical mergers. For example, Moresi and Salop (2013) depend on this modelling to derive a vertical merger screening measure called the vertical general upward pricing index[3].

The development of bargaining as a game-theoretic concept inspired a different approach to modelling economic problems. Muthoo (1999) describes a bargaining situation as a setup where two players have to overcome conflicting interests and co-operate to serve their common interest. Binmore et al. (1986) articulate the relationship between Nash's static axiomatic theory of bargaining (Nash, 1950) and the sequential strategic approach to bargaining. In doing so, Binmore et al. (1986) show how economists can improve their choice of how to model the outside option if they take the data for a specific economic situation into account when applying Nash's bargaining solution to a problem. For example, Binmore et al. (1986) show that the outside option can be modelled either as the outcome if no agreement is reached (when utility functions are derived from risk preferences) or as the status quo (when utility is derived from time preferences). This seminal piece contributed to the field by providing a manner in which the appropriate static representations can be selected for common bargaining situations. Subsequently, it enabled scholars to apply bargaining theory concepts in an ever-increasing range of economic problems.

In industrial organization literature applying bargaining theory is intuitive in situations where an upstream- and downstream firm have to contract over the terms of a contract. This setup meets the requirements for the bargaining situation set out in Muthoo (1999). Take, for example, the case where two firms in a vertical relationship bargain over the wholesale price of an input. They have conflicting interests because the upstream firm wants to charge the highest possible price for the input, while the downstream firm wants to pay the lowest possible price. However, it is in both players' best interest to co-operate in order to secure a profit. It is argued that this way of determining wholesale prices more accurately depicts reality especially since it is difficult to prove an anti-competitive merger with older models of vertical relationships, such as the derived demand model.

[2]This is discussed at length in chapter 3.

[3]See chapter 4 for a full discussion of this and other vertical merger screening measures.

Horn and Wolinsky (1988) provide the first application of bargaining models to a traditional industrial organization problem. The authors developed a model in which a duopoly acquires inputs through bilateral bargaining with a monopoly supplier. Horn and Wolinsky's approach entails a single player that is involved in all bargaining situations, but an agreement between any two players, does not affect the value of the agreement for the other players (Collard-Wexler et al., 2019). There is thus independent bilateral bargaining between the upstream- and downstream firms. The solution concept is deemed to be the Nash-in-Nash solution, as it is the 'Nash equilibrium in Nash bargains' (Collard-Wexler et al., 2019). The Nash-in-Nash solution is frequently employed by scholars and practitioners to model vertically related markets for various contractual forms.

One such Nash-in-Nash contractual form is bargaining over linear wholesale prices, where firms reach an agreement over a marginal wholesale price per input. This type of bargaining partially solves the double marginalisation problem of the derived demand model. As a result of countervailing bargaining power, downstream firms are able to negotiate a lower wholesale price than in the take-it-or-leave-it scenario of the derived demand model. However, firms are not able to realize the joint profit maximising outcome (which would also arise in the vertically integrated setting), as a result of the opportunistic behaviour of the upstream firm. An upstream firm contracting with several downstream firms has the incentive to restrict its supply to below joint profit maximising levels so as to extract higher profits (Hart et al., 1990).

As O'Brien and Shaffer (1992) illustrate, the crux of the opportunism problem lies with the equilibrium contract that induces the vertically integrated outcome. In this contract, the upstream- and downstream firm can agree to a lower wholesale price than the upstream firm agreed to with a rival, thus undercutting the retail price and increasing their profit. This behaviour induces what Collard-Wexler et al. (2019) call 'schizophrenia'. The upstream firm acts independently in each bilateral bargaining situation: for a given single bilateral contract, it commits to an outcome below the joint profit maximising outcome, because it believes the terms of this contract will be such that it undercuts the rival downstream firm. However, when we consider a different bilateral negotiation, the upstream firm behaves in exactly the same way. It is deemed 'schizophrenic' because there is a total disregard for how the independent behaviour in a single bilateral bargaining situation influences other contracts.

In a different contractual form — Nash-in-Nash bargaining over two-part prices — the opportunism problem is still present. O'Brien and Shaffer (1992) show that contracts specifying a wholesale price and fixed fee, are not sufficient to maximise joint profits. Moreover, the authors show that in the Nash-in-Nash bargaining equilibrium, marginal wholesale prices for each downstream firm are the same and are equal to the upstream firm's marginal cost. This result is again proven by Rey and Vergé (2019), who show that in a multilateral vertical contracting context, equilibrium wholesale prices are cost-based, in that they reflect the marginal cost of production.

Since the seminal contribution by Horn and Wolinsky (1988), a number of scholars have added to our understanding of the Nash-in-Nash model. Two main strands of literature have arisen — the first concerns the nesting of solutions. The subsequent literature in this strand has argued that Nash-in-Nash nests the Nash bargaining solution, a cooperative game theory concept, within a Nash equilibrium, which constitutes a noncooperative solution (Collard-Wexler et al., 2019). Scholars such as de Fontenay and Gans (2014) and Collard-Wexler et al. (2019) have aimed to address this criticism by providing a noncooperative foundation to the Nash-in-Nash bargaining solution. They have achieved this by specifying an alternating-offers game, in which the results get arbitrarily close to the Nash-in-Nash solution, by means of a limit equilibrium (Collard-Wexler et al., 2019).

The second strand of literature concerns the specification of the outside option. As Binmore et al. (1986) state: "the precise meaning of [the outside option] in Nash's model is somewhat vague". This is of great importance in the industrial organization literature, since the specification of the outside option impacts the terms of a contract being negotiated. Sheu and Taragin (2017) explain that in the Nash-in-Nash setting, firms view the terms of all other contracts as being fixed, which implies that the derivative with regard to the outside option in the maximisation problem will be zero. Thus, a firm contracting with multiple parties treats each contract separately. This leads to 'schizophrenia' (Collard-Wexler et al., 2019) as mentioned in previous paragraphs.

The schizophrenia of the pivotal negotiator is not a result of the model. Rather, it is a convenient assumption that ensures the existence of an equilibrium when downstream firms compete on prices (Rey and Vergé, 2019). While this assumption is restrictive, many scholars have accepted it in favour of simplified computational tractability and model calibration.

Scholars such as Froeb et al. (2020) and Yu and Waehrer (2018) argue that this should not be the consensus on how to model bargaining in the presence of competition-caused externalities (where the value of one agreement is conditional on the other agreements made). More specifically, they take issue with the fact that the Nash-in-Nash solution does not allow for contingent contracts or renegotiation following a breakdown in negotiation (Yu and Waehrer, 2018).

This reasoning follows earlier work by Stole and Zwiebel (1996) and Inderst and Wey (2003), who both consider cases where contracts can be renegotiated. Stole and Zwiebel (1996) investigated a case where workers or firms can renegotiate wages, while Inderst and Wey (2003) considered contingent contracts in a bilateral oligopoly environment. In both studies, the authors employed an axiomatic approach that gave rise to the Shapley value (de Fontenay and Gans, 2014).

Similar to de Fontenay and Gans (2014), Stole and Zwiebel (1996) and Inderst and Wey (2003), Froeb et al. (2020) refine the Nash-in-Nash solution by proving the existence of a concept they coined Nash-in-Shapley. The result relies on recursive treat points to solve the bargaining problem: a partially ordered set is defined with the order running from disagreement points to each agreement. Therefore, the elements of the set represents all of the possible combinations of bilateral agreements that can be formed. These elements portray the contingent contracts captured by Nash-in-Shapley since at every node of the set, the agreements made with other parties define the outside option.

Bedre-Defolie (2012) show that a contract arising from this nature of vertical contracting (one that allows renegotiation) is able to internalise the contracting externalities between an upstream and two downstream firms. In this 1×2 setting, firms are thus able to achieve the efficient joint profit maximising outcome.

The overview above illustrates that the literature on models of vertically related markets is vast. There are many different approaches to modelling these markets in the presence of externalities brought about by the added complexity of vertical contracting. The literature reviewed above alludes to two assumptions being particularly important: the assumptions about the nature and object of vertical contracting. Predictions based on models comprising different assumptions may give rise to a variety of results. Thus, a systematic comparison of the corpus of models of vertically related markets across different contractual forms and industries is a novel contribution to this literature. In what follows, we describe how the different models of vertically related markets that we consider in this chapter are set up and calibrated.

2.3 Models of vertically related markets

The literature review reveals that models of vertically related markets can be classified into three categories if based on the assumed nature of vertical contracting. We also identify three objects of vertical contracting. This provides the underpinning for the choice of models compared in this chapter. In accordance with the literature, we identify three groups according to which we classify the *nature* of vertical contracting: (1) take-it-or-leave-it offers by the upstream firm (derived demand), (2) bilateral bargaining between upstream and downstream firms (Nash-in-Nash) and (3) bilateral bargaining with contingent contracts (Nash-in-Shapley). With regards to the *object* of contracts, we consider (1) linear pricing, (2) two-part pricing, and (3) quantity fixing in two-part pricing[4]. Furthermore, to anchor our comparison of the magnitude of the predictions from these models, we also include what we consider to be benchmarks: perfect competition and monopoly outcomes. This brings the total number of models covered to eight.

Models of vertically related markets typically consists of four distinct features: (i) a network of upstream suppliers and downstream retailers (ii) who contract vertically in the presence of externalities (iii) caused by the competition between firms over (iv) consumer demand. We discuss each of these elements and the eight models separately below.

2.3.1 Network of bilateral trading relationships

We consider two simple industry structures:

- The 1×2 industry structure of one upstream firm, designated A, supplying two downstream firms, designated 1 and 2, with downstream firms competing for final consumers. Consumers have a choice of the product from A sold through 1, denoted $A@1$, or the same product sold through 2, denoted $A@2$.

- The 2×1 industry structure of two upstream firms, designated A and B, supplying one downstream firm, designated 1, with the downstream firm selling two products to final consumers. Consumers have a choice of the product from A or the product from B, sold through 1 in either case, denoted $A@1$ and $B@1$, respectively.

We imagine the upstream firm(s) as producing a product at some cost, agreeing to transfer the product to the downstream firm(s), which have additional costs in selling to final consumers. Each firm acts to maximise its own final profit.

Admittedly, the two industry structures considered are narrow and restrict attention to a very specific set of markets. However, the focus of this chapter is on ascertaining by how much the assumptions about vertical contracting (the second element of models of vertically related models) influence the magnitude of model predictions. Seeing that these models are so complex, with many moving parts, restricting our attention to two simple industry settings can be advantageous, for two reasons.

Firstly, because we use the same set of parameters to calibrate all the models, simple industry structures allows direct comparisons, and we are able to attribute the differences in outcomes solely to the different assumptions about vertical contracting. Secondly, when considering vertical mergers (as in

[4]The results for the quantity fixing contract are very close to the two-part pricing contract. We include the results for completeness but do not focus on discussing these results individually.

chapter 3), these simple industry structures mean that we only have to consider the vertical contracting between the vertically integrated and rival firm post-merger. This simplifies post-merger analysis and makes model outcomes more tractable. Lastly, in many antitrust cases, conclusions can be drawn from these simple structures. For example, two major US vertical merger cases over the past ten years (the AT&T/TimeWarner merger (Shapiro, 2018) and Comcast/NBCU merger (Rogerson, 2014)) have featured models assuming these industry structures.

2.3.2 Vertical contracting

As mentioned, we examine a total of eight models, two of which are included as benchmarks: perfect competition and monopoly. The remaining six models allow for different combinations of the nature and object of contracting assumed (schematically summarized in table 2.1). We examine a model that assumes no bargaining (derived demand), models that assume bilateral bargaining between upstream- and downstream firms for three objects of contracting (Nash-in-Nash linear pricing, Nash-in-Nash two-part pricing and Nash-in-Nash quantity) and models that assume bilateral bargaining with contingent contracts (also referred to as recursive bargaining) for two objects of contracting (Nash-in-Shapley linear pricing and Nash-in-Shapley two-part pricing).

Table 2.1: Schematic showing which combinations of nature and object of vertical contracting lead to which models

Nature ↓ / Object →	**Linear pricing**	**Two-part pricing**	**Quantity fixing**
Take-it-or-leave-it	Derived demand	/	/
Bargaining	Nash-in-Nash 1	Nash-in-Nash 2	Nash-in-Nash Q
Recursive bargaining	Nash-in-Shapley 1	Nash-in-Shapley 2	/

The three objects of contracts are identified as linear (one-part) pricing, assigning a marginal wholesale price; two-part pricing, specifying a marginal wholesale price and a fixed fee to equally split all profit over the threat point; and quantity fixing, specifying a quantity at a fixed price. For linear and two-part pricing, we assume the marginal wholesale price determines Nash equilibrium consumer prices, and thus product demands, and that demand is known. For specified quantity, we assume that consumer prices are set to sell the specified quantities.

We proceed below to briefly discuss the derivation of the analytical results for each of the eight models included in this chapter[5]. For the full derivations, refer to appendix A. These analytical results are the equations that are calibrated as in section 2.4 to yield the results in section 2.5.

2.3.2.1 General profit functions and setup

In the 1×2 industry structure for all the models, we define the downstream profit functions as $\pi_1 = (p_1 - mc_1 - w_1)q_1$ and $\pi_2 = (p_2 - mc_2 - w_2)q_2$. Then, for the given marginal wholesale prices w_1 and w_2 and demand levels q_1 and q_2 as functions of retail prices, retail price p_1 is set to maximise π_1 simultaneously with retail price p_2, set to maximise π_2, or so that the first-order conditions

[5]The exposition of the models is adapted from Tschantz (2019), which contains the write-up of a bargaining comparison tool. The simulations are built on the static models presented for this online tool, available at https://daag.shinyapps.io/b1x2/.

$$
\begin{aligned}
0 &= q_1 + (p_1 - mc_1 - w_1)\frac{\partial q_1}{\partial p_1} \\
0 &= q_2 + (p_2 - mc_2 - w_2)\frac{\partial q_2}{\partial p_2}
\end{aligned} \tag{2.1}
$$

are satisfied.

The upstream profit function in a 1×2 industry structure is defined as $\pi_A = (w_1 - mc_A)q_1 + (w_2 - mc_A)q_2$.

In the 2×1 industry structure for all models, downstream profit is defined as $\pi_1 = (p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B$. Then, for given marginal wholesale prices w_A and w_B and the demand q_A and q_B as functions of retail prices, retail prices p_A and p_B are set to maximise π_1 or so that the first-order conditions

$$
\begin{aligned}
0 &= q_A + (p_A - mc_1 - w_A)\frac{\partial q_A}{\partial p_A} + (p_B - mc_1 - w_B)\frac{\partial q_B}{\partial p_A} \\
0 &= q_B + (p_A - mc_1 - w_A)\frac{\partial q_A}{\partial p_B} + (p_B - mc_1 - w_B)\frac{\partial q_B}{\partial p_B}
\end{aligned} \tag{2.2}
$$

are satisfied.

The upstream profit functions are $\pi_A = (w_A - mc_A)q_A$ and $\pi_B = (w_B - mc_B)q_B$ respectively.

2.3.2.2 Benchmark Models: Perfect competition and Monopoly

To allow us to ascertain by how much assumptions about vertical contracting matter, we consider two benchmarks models. This provides the anchors against which model predictions can be compared. The first benchmark model, Perfect competition, comprises the case where the two goods are sold competitively given their respective total marginal costs. In a 1×2 industry structure, this entails the upstream firm supplying the input at marginal cost to the downstream firms. In a 2×1 industry structure, the downstream firm acts 'transparently', adding only its marginal cost.

For the second benchmark model, we estimate the monopoly outcome. In both industry structures, this means that the monopolist (the upstream firm in a 1×2 industry structure and the downstream firm in a 2×1 industry structure) maximises total industry profit to determine retail prices. After profit is calculated in a 1×2 industry structure, the fixed fees are set equal to the respective downstream profits so that the monopolist recovers all profit. In a 2×1 industry structure, the downstream monopolist pays no fixed fees to the upstream firms to retain monopoly profit.

2.3.2.3 No bargaining model: Derived demand

In the case of no bargaining (derived demand model), we imagine that the upstream firm simply dictates terms. In the case of two-part pricing, the upstream firm would set the marginal wholesale price to set a desired downstream price and then require a fixed fee that recovers all profit from the downstream firm, an unrealistic scenario. Instead, we consider only the case of simple linear pricing dictated by the upstream firm(s) maximising their own profit(s), resulting in double marginalisation.

In the 1×2 industry structure, the upstream firm wants to maximise its profit with respect to the wholesale prices charged to the downstream firms. However, quantities will vary as retail prices will adjust consistent with the first-order conditions. Therefore, we solve first-order conditions for the wholesale prices in terms of the retail prices and then treat upstream profit as a function of retail prices to be maximised.

In the 2×1 industry structure, the upstream firms maximise profits facing the derived demand determined in the downstream profit maximisation problem. Upstream firms maximise their respective profits with respect to wholesale prices simultaneously in Nash equilibrium.

2.3.2.4 Linear pricing bargaining models: Nash-in-Nash and Nash-in-Shapley

In the cases where upstream- and downstream firms bargain (Nash-in-Nash bargaining or Nash-in-Shapley bargaining), we need to consider scenarios where negotiations fail. Where an upstream firm fails to agree with a downstream firm, the product concerned is unavailable to the end consumer. Thus, in addition to a base case where there are two agreements and two products sold to consumers denoted by $\{A1, A2\}$ in the 1×2 case and $\{A1, B1\}$ in the 2×1 case), we consider also in the 1×2 case, where only A and 1 agree $\{A1\}$, or where only A and 2 agree $\{A2\}$, and in the 2×1 case, where only A and 1 agree $\{A1\}$ or where only B and 1 agree $\{B1\}$. If no agreements are reached $\{\emptyset\}$ and no products sold, we may assume some residual profit or liability for each firm, regarding which profits in other cases are measured, usually taking this to be simply a zero reference point. We consider outcomes of agreements, but not the process of arriving at agreements.

In bargaining models, we assume that each agreement negotiated results from a Nash bargaining solution with respect to the parties' total profits over some threat point. In the 1×2 industry structure, we suppose when pricing linearly, firms A and 1 would settle on a wholesale price such that their final net profits would maximise the Nash bargaining solution, $P_{A1} = (\pi_A - \pi_A^*)(\pi_1 - \pi_1^*)$. Simultaneously, firms A and 2 do the same with regards to $P_{A2} = (\pi_A - \pi_A^{**})(\pi_2 - \pi_2^{**})$. Subsequently, retail prices and quantities are determined by downstream Nash competition satisfying the usual first-order conditions.

In a 2×1 industry structure, the pivotal player is the downstream firm. The Nash bargaining solution maximised to obtain the wholesale prices, become $P_{A1} = (\pi_A - \pi_A^*)(\pi_1 - \pi_1^*)$ and $P_{B1} = (\pi_B - \pi_B^{**})(\pi_1 - \pi_1^{**})$ respectively.

The delineation between Nash-in-Nash and Nash-in-Shapley is the treatment of the threat point. For Nash-in-Nash, we assume that the threat point is given by the profits determined in the scenarios with all other agreements held fixed, e.g., Sheu and Taragin (2017); Collard-Wexler et al. (2019); Rey and Vergé (2019). Put simply, there is no difference in the terms of the contract when the pivotal player reaches an agreement with only one player versus more than one player. In the linear pricing scenario, this boils down to the wholesale price being equal across all possible agreements. For Nash-in-Shapley, we assume that the threat point is determined by profits, with all other agreements adjusted for the new set of agreements, with each of these being determined recursively from cases with fewer agreements, e.g., Froeb et al. (2020); Yu and Waehrer (2018). Thus, in a linear pricing setting, a different wholesale price is specified for when the pivotal player reaches one agreement versus two agreements, and so on.

2.3.2.5 Two-part pricing bargaining: Nash-in-Nash and Nash-in-Shapley

In two-part pricing, parties will agree to marginal wholesale pricing and fees, and these agreements also become the basis for the threat points to agreement. For a Nash bargaining solution with transferable utility, one can view this as maximising the total surplus over the threat point with this surplus then split so each party benefits equally over the threat point.

In a 1×2 industry structure, we suppose marginal wholesale prices are agreed to. Subsequently, retail price are set, and the total operating profits of each firm can be determined. Then, fixed fees are determined so that the final net profits are a fair split over the threat points. The relevant formulae are:

$$\begin{aligned} \pi_A &= \pi_A^o + f_1 + f_2 \\ \pi_1 &= \pi_1^o - f_1 \\ \pi_2 &= \pi_2^o - f_2 \\ \pi_A^* &= \pi_A^{o*} + f_2^{**} \\ \pi_1^* &= 0 \\ \pi_A^{**} &= \pi_A^{o**} + f_1^{**} \\ \pi_2^{**} &= 0 \end{aligned} \tag{2.3}$$

where operating profits at the given marginal wholesale prices are π_A^o, π_1^o, and π_2^o with both agreements, and for A, π_A^{o*} if product 2 is sold but product 1 is unavailable, and π_A^{0**} if product 1 is sold but product 2 is unavailable.

For a fair split of operating profits, we require that firms split equally the additional surplus from reaching an agreement; thus: $\pi_A - \pi_A^* = \pi_1 - \pi_1^*$ and $\pi_A - \pi_A^{**} = \pi_2 - \pi_2^{**}$.

As mentioned in the previous subsection, the delineation between Nash-in-Nash and Nash-in-Shapley entails the treatment of threat points. In two-part pricing, the threat point consists not only of the marginal wholesale price, but also of the wholesale fixed fee. In the same manner as in linear pricing, Nash-in-Nash determines this fee with all other agreements held fixed. Thus, in 2.3 above $f_1 = f_1^{**}$ and $f_2 = f_2^{**}$. However, for Nash-in-Shapley, these fees are determined recursively.

In the 2×1 industry structure, the same procedure is followed with the relevant profit formulae:

$$\begin{aligned} \pi_A &= \pi_A^o + f_A \\ \pi_B &= \pi_B^o + f_B \\ \pi_1 &= \pi_1^o - f_A - f_B \\ \pi_1^* &= \pi_1^{o*} + f_B^{**} \\ \pi_A^* &= 0 \\ \pi_1^{**} &= \pi_1^{o**} + f_A^{**} \\ \pi_B^{**} &= 0 \end{aligned} \tag{2.4}$$

where operating profits at the given marginal wholesale prices are π_A^o, π_B^o, and π_1^o, and for 1 in the alternatives π_1^{o*} if product A is unavailable and π_1^{o**} if product B is unavailable.

For most cases, specifying quantity is equivalent to two-part pricing. The exceptional scenario where quantity setting differs from two-part pricing is Nash-in-Nash, which assumes other agreements are held constant in the alternative to each agreement, hence the form of these contracts matter. The derivation follows as set out above in that operating profits are determined given the retail prices implied by the set quantities. Thereafter, the total price is determined so that the surplus is split equally between firms.

2.3.3 Downstream demand

One of the four elements of vertically related markets, is the modelling of downstream demand. In all the models considered, we imagine a downstream consumer demand for two products: in the 1×2 case, a choice of the product from A and sold through 1, denoted $A@1$, or else the same product sold through 2, denoted $A@2$; in the 2×1 case, a choice of the product from A or else the product from B sold through 1 in either case, denoted $A@1$ and $B@1$ respectively. When only one agreement is reached, consumers' decisions are reduced to a single choice, and the demand model is suitably adjusted. We assume a nested logit demand model, so eliminating one product corresponds simply to a limiting case where the price of that product goes to infinity. This demand makes it a simple matter to determine a change in consumer surplus between pre- and post-merger scenarios.

Underpinning bargaining where there are a number of simultaneous agreements, is how the surplus available in one agreement is affected by the other agreements. If each agreement were independent of the others, it would suffice to apply a separate bargaining model calculation to each. Instead, we expect the existence of other agreements to reflect an externality to be accounted for in bargaining over a given agreement. To illustrate such externalities, we use a readily understood example, where agreements allow for competing products to be sold to consumers. The sales of a product under one agreement will be negatively affected by a second agreement to sell a product in competition, whether it be the same product through an alternative retail outlet (the 1×2 case) or a different product through the same retail outlet (the 2×1 case). The other agreement represents a negative externality in these cases, but it is also possible for network effects to result in a positive externality.

The downstream consumer demand model that determines the negative externalities in agreements needs to define sales for the case where all products are available and cases where one or more products become unavailable in the alternative to agreement for those products. A demand model is usually calibrated to observed demand at current prices. However, the nature of the bargaining models is to consider alternatives without some products, and this circumstance is likely in itself to represent a considerable extrapolation from observations, effectively a new demand model for restricted sets of products. Instead of depending on extrapolation, we appeal to the internal logic of the demand model for guidance. A choice model of demand explains consumer demand as individuals choosing their most preferred alternative given price, averaging over the distribution of preferences in the population of potential consumers. Such a model naturally suggests what will happen when the alternatives are reduced. The nested logit demand model is a tractable choice model of demand for the purpose of giving a definite and consistent background of negative externalities for evaluating bargaining models.

2.3.3.1 Nested logit demand

Suppose there are n inside products, indexed 1 to n, together with an outside, no purchase, alternative indexed as 0. In our two industry structures, n will be at most 2, with products $A@1$ and $A@2$ in the 1×2 case or $A@1$ and $B@1$ in the 2×1 case. Let p_i be the price of the i-th inside goods, for each i, fixing $p_0 = 0$. Suppose that a consumer sees these products and prices and chooses one, with some total number of choices per specified time period, allowing for some scaling. If $(V_0, V_1, \ldots, V_n)$ is the $n+1$-tuple of values of a random consumer (nominated in the same units as prices), we suppose that the consumer will choose alternative i where, for all $j \neq i$, $V_i - p_i > V_j - p_j$ (ignoring possible ties). Let $X_i = V_i - p_i$ be the net value for alternative i; then the total demand for alternative i is

$q_i = M \; \Pr(X_i > X_j, \text{all } j \neq i)$ where M is the total number of consumer choices (during some period). The outside quantity q_0 represents those consumers not choosing any of the inside products which is usually not observable.

It is convenient to take the V_i to be marginal distributions that represent extreme value with the same scale parameter λ and various location parameters η_i. Then the marginal distribution of X_i is also an extreme value with scale parameter λ and location parameters $\eta_i^\dagger = \eta_i - p_i$, that is, with cumulative distribution function

$$F_i(t) = \Pr(X_i \leq t) = \exp\left(-\exp\left(-\frac{t-\eta_i^\dagger}{\lambda}\right)\right)$$

In fact, these distributions are power-related (Froeb et al., 2001). Anticipating the application below, write $F_i(t) = (F_{\text{max}}(t))^{s_i^{1/\theta}}$, for a parameter $\theta \geq 1$, where

$$F_{\text{max}}(t) = \exp\left(-\exp\left(-\frac{t-\eta_{\text{max}}^\dagger}{\lambda}\right)\right)$$

is the extreme value distribution function with scale parameter λ and location parameter $\eta_{\text{max}}^\dagger$, where $s_i = \exp(\theta\eta_i^\dagger/\lambda)/\exp(\theta\eta_{\text{max}}^\dagger/\lambda)$ and $\eta_{\text{max}}^\dagger$ is taken so $\sum_i s_i = 1$, i.e.,

$$\eta_{\text{max}}^\dagger = \frac{\lambda}{\theta}\log\left(\sum_{i=1}^{n}\exp\left(\frac{\theta\eta_i^\dagger}{\lambda}\right)\right)$$

A flat logit demand model results if the V_i (so X_i) are taken as being independent, but a more general model is only slightly more complex. A Gumbel copula combines nicely with power-related distributed marginals to give a model with V_i correlated, and these can be simply combined in nests of smaller nests of increasing strengths. For our purposes it suffices to imagine X_0 is independent of $X_1 \ldots X_n$, but to take these inside goods as a nest in a nested logit demand model. The Gumbel copula is the Archimedean copula given by generator $\psi_\theta(t) = (-\log(t))\theta$ for parameter $\theta \geq 1$ reflecting the strength of the correlation, with $\theta = 1$ being the limiting case of independence. For a nest of n variables, the copula is $C(u_1, \ldots, u_n; \theta) = \psi_\theta^{-1}(\psi_\theta(u_1) + \ldots + \psi_\theta(u_n))$, where $\psi_\theta^{-1}(t) = \exp(-t^{1/\theta})$, That is, C is a joint cumulative distribution function with uniform marginals, and the joint distribution function of the inside X_i is taken to be

$$\begin{aligned}
F_{1\ldots n}(t_1, \ldots, t_n) &= \Pr(X_i \leq t_i, \text{for all } i > 0) \\
&= C(F_1(t_1), \ldots, F_n(t_n); \theta) \\
&= C(F_{\text{max}}(t_1)^{s_1/\theta}, \ldots, F_{\text{max}}(t_n)^{s_n/\theta}; \theta) \\
&= \psi_\theta^{-1}(\psi_\theta(F_{\text{max}}(t_1)^{s_1^{1/\theta}}) + \ldots + \psi_\theta(F_{\text{max}}(t_n)^{s_n^{1/\theta}})) \\
&= \psi_\theta^{-1}(s_1\psi_\theta(F_{\text{max}}(t_1)) + \ldots + s_n\psi_\theta(F_{\text{max}}(t_n))) \;\; \text{and so}
\end{aligned}$$

$$\begin{aligned}
F_{1\ldots n}(t, \ldots, t) &= \Pr(\max_{i>0} X_i \leq t) \\
&= \psi_\theta^{-1}((s_1 + \ldots + s_n)\psi_\theta(F_{\text{max}}(t))) \\
&= F_{\text{max}}(t)
\end{aligned}$$

In general, the distribution of the maximum of n random variables having joint distribution given by the Gumbel copula being applied to power-related marginal distributions is also power-related, and taking this maximum distribution as the base distribution, the marginal distributions are $F_{\text{max}}(t_i)^{s_i/\theta}$ where the s_i sum to unity. Moreover (under mild conditions),

$$\begin{aligned}
\Pr(X_i > X_j, \text{all } 0 < j \neq i) &= \int_{\substack{t_i > t_j,\\ \text{all } j \neq i}} dF_{1\ldots n}(t_1, \ldots, t_n) \\
&= \int_{t_i} \frac{\partial}{\partial t_i} F_{1\ldots n}(t_1, \ldots, t_n)\bigg|_{t_1 = t_i, \ldots, t_n = t_i} dt_i \\
&= \int_{t_i} (\psi_\theta^{-1})'(s_1 \psi_\theta(F_{\text{max}}(t_i)) + \ldots + s_n \psi_\theta(F_{\text{max}}(t_i))) \\
&\qquad \cdot s_i \psi_\theta'(F_{\text{max}}(t_i)) F_{\text{max}}'(t_i)\, dt_i \\
&= s_i \int_{t_i} \frac{\partial}{\partial t_i} (\psi_\theta^{-1}(\psi_\theta(F_{\text{max}}(t_i))))\, dt_i \\
&= s_i
\end{aligned}$$

so the s_i reflect the probability that X_i is the maximum of $X_1, \ldots, X_n$. In fact, the distribution of the maximum of $X_1, \ldots, X_n$ is independent of the identity X_i that realizes that maximum. Hence,

$$\begin{aligned}
\Pr(X_i \leq t | X_i > X_j, \text{all } 0 < j \neq i) &= \frac{1}{s_i} \Pr(t \geq X_i > X_j, \text{all } 0 < j \neq i) \\
&= \frac{1}{s_i} \int_{\substack{t \geq t_i > t_j,\\ \text{all } j \neq i}} dF_{1\ldots n}(t_1, \ldots, t_n) \\
&= \frac{1}{s_i} s_i \int_{t \geq t_i} \frac{\partial}{\partial t_i} (\psi_\theta^{-1}(\psi_\theta(F_{\text{max}}(t_i))))\, dt_i \\
&= F_{\text{max}}(t)
\end{aligned}$$

Taking X_0 as being independent of the inside X_i combines X_0 with the maximum of the X_i in an outside nest with $\theta = 1$. The maximum of all X_i is thus extreme value distributed with scale parameter λ and location parameter

$$\eta^\dagger_{\text{max all}} = \lambda \log\left(\exp\left(\frac{\eta^\dagger_0}{\lambda}\right) + \exp\left(\frac{\eta^\dagger_{\text{max}}}{\lambda}\right)\right)$$

The probability that X_0 is greater than any other X_i is thus

$$\begin{aligned}
\pi_0 = \Pr(X_0 > X_i, \text{all } i > 0) &= \frac{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_{\text{max all}}}{\lambda}\right)} \\
&= \frac{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right) + \exp\left(\frac{\eta^\dagger_{\text{max}}}{\lambda}\right)}
\end{aligned}$$

The probability that X_i is greater than any X_j, $j \neq i$, is the probability that X_i is the maximum of X_j,

$j > 0$, times the probability that the maximum of $X_1, \ldots, X_n$ exceeds X_0,

$$\pi_i = \Pr(X_i > X_j, \text{all } j \neq i) = s_i \cdot \frac{\exp\left(\frac{\eta^\dagger_{\text{max}}}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_{\text{max all}}}{\lambda}\right)} = s_i \cdot \frac{\exp\left(\frac{\eta^\dagger_{\text{max}}}{\lambda}\right)}{\exp\left(\frac{\eta^\dagger_0}{\lambda}\right) + \exp\left(\frac{\eta^\dagger_{\text{max}}}{\lambda}\right)}$$

Adding a constant to all of the η_i adjusts $\eta^\dagger_{\text{max}}$ and $\eta^\dagger_{\text{max all}}$ by the same constant and hence leaves all of the choice probabilities π_i unchanged. We conventionally take $\eta_0 = 0$. The expected value of the maximum $X^\dagger_i$ — the expected difference between the value of the product chosen by a random consumer and the price paid for that choice — is not determined without reference to an actual η_i, but the change in this quantity between two prices represents the change in consumer surplus in this model.

Sampling from this joint distribution of consumer values is not trivial when $\theta > 1$. A method for sampling from the Gumbel copula follows from work of Marshall and Olkin (1967). Sample V from the type 1 stable distribution with stability parameter $\alpha = 1/\theta$, skewness parameter $\beta = 1$, scale parameter $\sigma = \cos(\pi/2/\theta)^\theta$ and location parameter $\mu = 0$. Take W_i as being the independent uniform $[0, 1]$. Then $U_i = \phi^{-1}_\theta(-\log(W_i)/V)$ are jointly distributed with distribution function $C(u_1, \ldots, u_n; \theta)$. From there we can take $X_i = F^{-1}_i(U_i)$ and $V_i = X_i + p_i$, with V_0 taken as an independent extreme value with scale parameter λ and location parameter $\eta_0 = 0$.

The correlation between variates defined by a copula is not independent of the marginal distributions but the Kendall rank correlation coefficient τ are, as they depend on rank orderings only. For two particular products, if p is the probability that between two random consumers — the one who values one product more highly will also be the one who values the other product more highly — then $\tau = 2p - 1$. For the Gumbel copula, $\tau = 1 - 1/\theta$ is between 0 (independent) and 1 (in the limit). Otherwise put, for a specified Kendall $\tau \in [0, 1)$, we may take $\theta = 1/(1 - \tau)$ as nest parameter.

To summarize, for a nested logit model with a nest around inside products having $\tau \in [0, 1)$ — so the nest parameter $\theta = 1/(1 - \tau)$, the demand for an inside product $i > 0$, and the total consumer surplus up to a constant — is given by

$$q_i = M \frac{\exp\left(\frac{\eta_i - p_i}{\lambda(1-\tau)}\right)}{S + S^\tau}, \quad q_0 = M \frac{S^\tau}{S + S^\tau} \quad \text{where } S = \sum_{i=1}^{n} \exp\left(\frac{\eta_i - p_i}{\lambda(1 - \tau)}\right)$$

$$CS = M\eta^\dagger_{\text{max all}} = M\lambda \log\left(1 + S^{1-\tau}\right)$$

2.3.3.2 Demand derivatives

For elasticities and first-order conditions, we need demand derivatives; for pass-through rates and other purposes, we need second derivatives of demand. To this end, let

$$s_i = \exp\left(\frac{\eta_i - p_i}{\lambda(1 - \tau)}\right), \quad S = \sum_{i=1}^{n} s_i, \quad \text{and} \quad f(x) = \frac{1}{x + x^\tau} \quad \text{so}$$

$$q_i = M s_i f(S)$$

Hence for $i, j, k \in \{1, \ldots, n\}$ with $\delta_{ij} = 1$ if $i = j$ and 0 otherwise,

$$\begin{aligned}
\frac{\partial s_i}{\partial p_j} &= -\frac{1}{\lambda(1-\tau)}\delta_{ij}s_i, \quad \frac{\partial S}{\partial p_j} = -\frac{1}{\lambda(1-\tau)}s_j \\
\frac{\partial q_i}{\partial p_j} &= -\frac{M}{\lambda(1-\tau)}\left(\delta_{ij}s_i f(S) + s_i f'(S)s_j\right) \\
&= -\frac{1}{\lambda(1-\tau)}\left(\delta_{ij}q_i + M s_i s_j f'(S)\right) \\
\frac{\partial^2 q_i}{\partial p_j \partial p_k} &= \frac{1}{\lambda^2(1-\tau)^2}\big(\delta_{ij}\delta_{ik}q_i + \delta_{ij}M s_i s_k f'(S) + \\
&\qquad \delta_{ik}M s_i s_j f'(S) + \delta_{jk}M s_i s_j f'(S) + M s_i s_j f''(S) s_k\big) \\
&= \frac{1}{\lambda^2(1-\tau)^2}\big(\delta_{ij}\delta_{ik}q_i + \\
&\qquad M(\delta_{ij}s_i s_k + \delta_{ik}s_i s_j + \delta_{jk}s_i s_j)f'(S) + M s_i s_j s_k f''(S)\big)
\end{aligned}$$

where

$$\begin{aligned}
f'(x) &= -\frac{x + \tau x^\tau}{x(x + x^\tau)^2} \\
f''(x) &= \frac{2x^2 + (5\tau - \tau^2)x^{1+\tau} + (\tau + \tau^2)x^{2\tau}}{x^2(x + x^\tau)^3}
\end{aligned}$$

2.4 Calibration

This chapter is aimed at making a systematic comparison of the magnitude of predictions from the six different models of vertically related markets discussed in the preceding section. We do this by choosing two simple industry structures and calibrating all of the models to the same set of parameters. This allows us to directly compare the outcomes of the models.

For the numerical simulations, we choose to simulate each model over an increasing ratio of the outside good in relation to the inside goods. Thus, we exogenously increase the quantity of the outside good as a ratio of the inside goods. We start with the outside good being equal to 10% of the sum of the inside goods and increase it by one percentage point at a time until it is 210% of the sum of the inside goods. In doing this, the total market size is increased, which causes aggregate elasticity in the market to increase. This then allows us to observe how the substitutability of the inside goods with the outside good affects the predictions of the system of models.

For the remaining parameters, we fix the scaling parameter (λ), initial prices ($p_{A@1}, p_{A@2}$ in the 1×2 setting and $p_{A@1}, p_{B@1}$ in the 2×1 setting), initial quantities of the inside goods ($q_{A@1}, q_{A@2}$ in the 1×2 setting and $q_{A@1}, q_{B@1}$ in the 2×1 setting) and the nest strength parameter (τ). This provides us with a list of parameters: a varying outside good quantity with corresponding varying market size and aggregate elasticity, each with the same fixed parameters as stated above.

The above parameters are sufficient to determine the location parameters (η_i) of the logit demand function:

$$\log\left(\frac{q_i}{\sum_{j=1}^n q_j}\right) = \frac{\eta_i - p_i}{\lambda(1-\tau)} - \log(S)$$

so

$$\eta_i = p_i + \lambda(1-\tau)\left(\log\left(\frac{q_i}{\sum_{j=1}^n q_j}\right) + \log(S)\right)$$

with

$$S = \left(\frac{q_0}{\sum_{i=1}^n q_i}\right)^{1/(\tau-1)}$$

Given this set of parameters, we can calibrate the demand model to prices and elasticities appropriate for the monopolist. We assume in the 1×2 and 2×1 case that the upstream firm(s) have zero marginal cost and that the marginal cost of the downstream firm(s) is inferred from the monopoly case, assuming that the above prices are optimal. Specifically, for two products with total marginal costs mc_{tot1}, mc_{tot2}, the monopolist maximises

$$\text{profit}_M = (p_1 - mc_{tot1})q_1 + (p_2 - mc_{tot2})q_2$$

so choosing prices that satisfy the first-order conditions

$$0 = q_1 + (p_1 - mc_{tot1})\frac{\partial q_1}{\partial p_1} + (p_2 - mc_{tot2})\frac{\partial q_2}{\partial p_1}$$
$$0 = q_2 + (p_1 - mc_{tot1})\frac{\partial q_1}{\partial p_2} + (p_2 - mc_{tot2})\frac{\partial q_2}{\partial p_2},$$

a system of two linear equations easily solved for the total marginal costs. For the nested logit model, there are certain simple relations. Substituting the derivative formulae

$$0 = q_1 + (p_1 - mc_{tot1})\left(q_1 + q_1 s_1\frac{f'(S)}{f(S)}\right) + (p_2 - mc_{tot2})\left(q_1 s_2\frac{f'(S)}{f(S)}\right)$$
$$0 = q_2 + (p_1 - mc_{tot1})\left(q_2 s_1\frac{f'(S)}{f(S)}\right) + (p_2 - mc_{tot2})\left(q_2 + q_2 s_2\frac{f'(S)}{f(S)}\right),$$

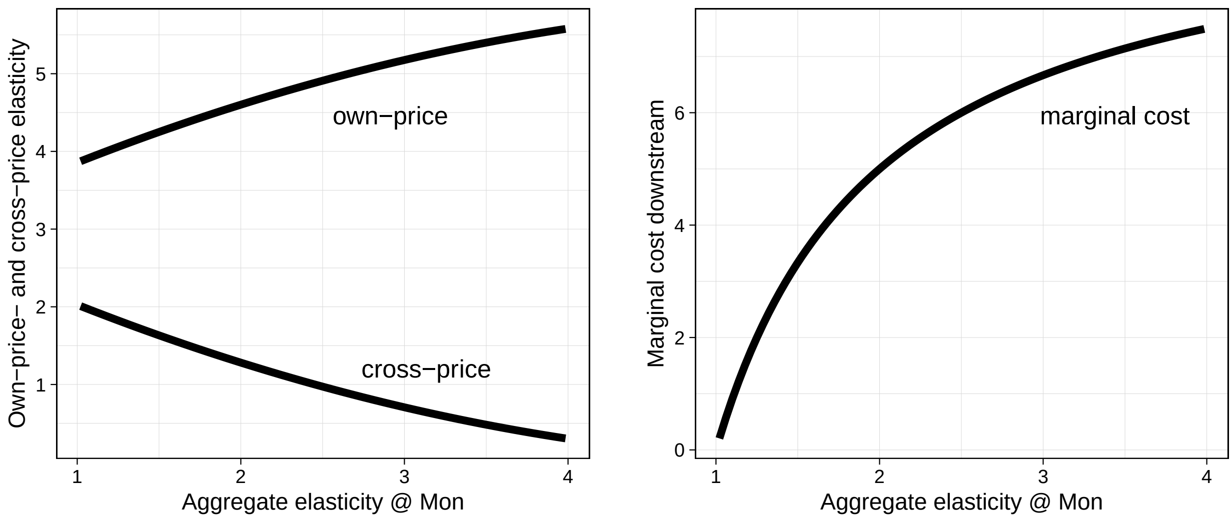
dividing by quantities, and subtracting shows $p_1 - mc_{tot1} = p_2 - mc_{tot2}$, i.e., any difference in pricing for the monopolist is due to differences in total marginal cost.

Specific units on prices will not change the results of our calculations, so that we take the quantity weighted average price to be $\bar{p} = 1$. The units on quantity similarly will not matter, so we may set the total quantity of inside products arbitrarily to $q_{tot} = 100$. We further assume that the prices, quantities and marginal cost of the two products are equal (i.e. they are balanced).

This list of parameters: initial prices, inside quantities, outside quantity, nest parameter, aggregate elasticity, scaling parameter and location parameters along with the conventions for $\bar{p} = 1$, $q_{tot} = 100$ and marginal costs inferred from monopoly pricing are enough to calibrate the demand model. Finally, a flat logit (i.e a nest strength parameter of zero) demand function is assumed for this calibration. These parameters are used to compute the results of each of the eight models.

Exogenously varying the outside quantity yields a list of parameters with which we calibrate the demand model. Since we compute the monopoly equilibrium after each exogenous increase of the outside quantity, some of the initial parameters in the parameter list vary along with the control variable. Specifically, the location parameters, elasticities and marginal cost. Figure 2.1(a) shows how the cross-price- and own-price elasticities relate to a changing aggregate elasticity. Exogenously increasing the quantity of the outside good to increase aggregate elasticity causes cross-price- and own-price elasticity to move in opposite directions. We observe cross-price elasticity decreasing and own-price elasticity increasing as aggregate elasticity increases. The increased substitutability of the inside goods with the outside good causes the inside goods to become less substitutable but also more responsive to changes in their own prices.

As a result of the initial prices being fixed, the change in aggregate elasticity also causes a change in marginal cost. The increased substitutability indicates that profit margins should decrease, which is achieved by an increasing marginal cost. This is clearly observed in figure 2.1(b). For the 1×2 case, figure 2.1(b) shows the symmetrical marginal cost for both downstream firms. In the 2×1 setting, figure 2.1(b) shows the marginal cost for the only downstream firm. In both cases, marginal cost approaches the price as the aggregate elasticity increases.



(a) Calibration of elasticities vs ae

(b) Calibration of marginal cost vs ae

Figure 2.1: Demand and cost parameter calibration

2.5 Results

The numerical simulations presents us with copious results to interpret. We summarize predicted quantities, prices, wholesale prices, wholesale fees, consumer welfare and profits graphically in appendix B and in the tables in appendix E. However, attempting a systematic comparison of the corpus of models of vertically related markets based on each individual outcome variable would be extremely difficult.

Fortunately, the benefit of employing simulations to analyze vertically related markets is that we are able to compute a full equilibrium[6] (Rogerson, 2020; Slade, 2020). The equilibrium is easily observed in a range of variables such as industry price, output, profits or consumer welfare. In figures 2.2 and 2.3, we choose to focus on the total quantity since this variable summarizes model-specific effects in a tractable manner[7].

[6]This is especially applicable when analyzing mergers in vertically related markets, as in chapter 3 where the net effect of mergers is examined. See also chapter 4 for a discussion of the different screening methods for vertical mergers and the benefits of employing simulations.

[7]A choice between total quantity and consumer welfare as the variable of interest is arbitrary. Full equilibrium effects are easily observed in either of these variables. However, when assessing vertical merger effects, it is readily understood that an increase in total quantity is pro-competitive, and a decrease in total quantity is anti-competitive. This is the focus of chapter 3 and thus, for consistency, we discuss net effects along total quantity.

As discussed in section 2.4, we calibrated the demand system to a varying ratio of the outside good to the sum of the inside goods. We also discuss how this calibration yields a list of parameters where the location parameters, elasticities and marginal costs vary along with the control variable. In figures 2.2 and 2.3, we choose to display aggregate elasticity on the x-axis for two reasons. Firstly, it provides a comprehensive measure of the overall movement in the calibration. Secondly, our base knowledge of the effects of elasticity on prices and quantities makes it easy to interpret as a measure of substitutability.

To make the comparisons more digestible, we discuss the results of the different models in groups. Perfect competition and monopoly (the two benchmarks) are grouped with the derived demand (DD) model, since no bargaining is assumed in these models. Nash-in-Nash (NiN1) and Nash-in-Shapley (NiS1) are grouped together, since both models assume a form of bargaining over linear wholesale prices. Finally, Nash-in-Nash(NiN2), Nash-in-Shapley(NiS2) and Nash-in-Nash quantity(NiNQ) are grouped together since there is some form of bargaining over two components (wholesale price and fixed fee or fixed quantity and total price). In figures 2.2 and 2.3 (and all the figures in the appendix) these groups are presented in three panels. In the subsections that follow, we discuss these results panel by panel.

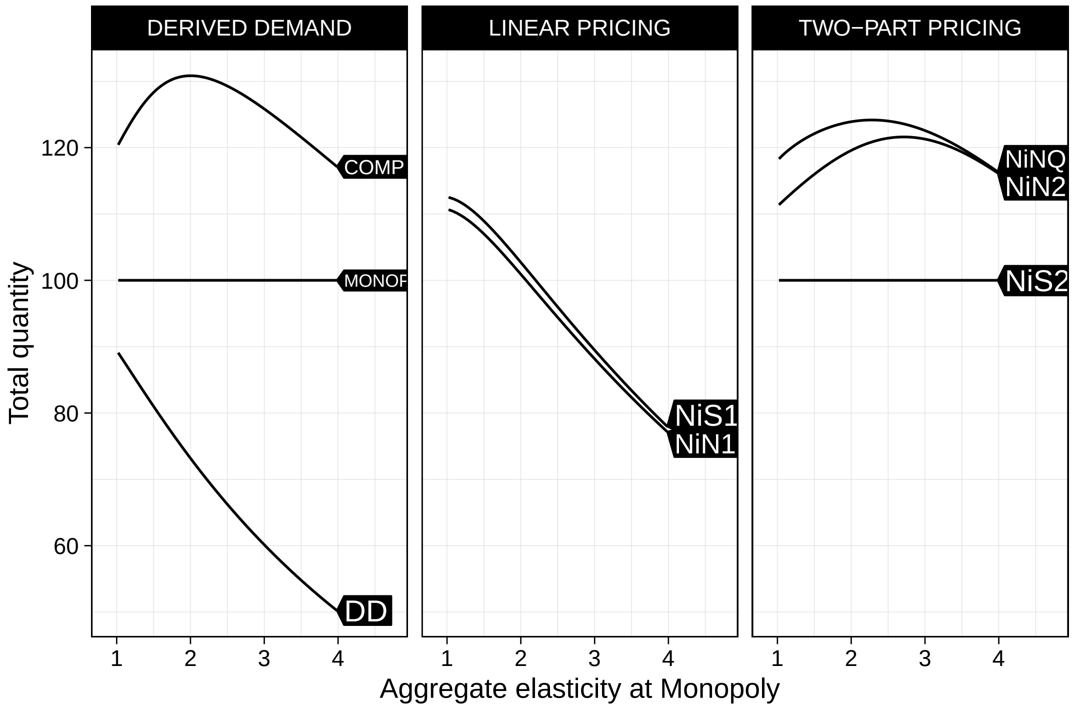


Figure 2.2: Total quantity vs ae 1×2 setting

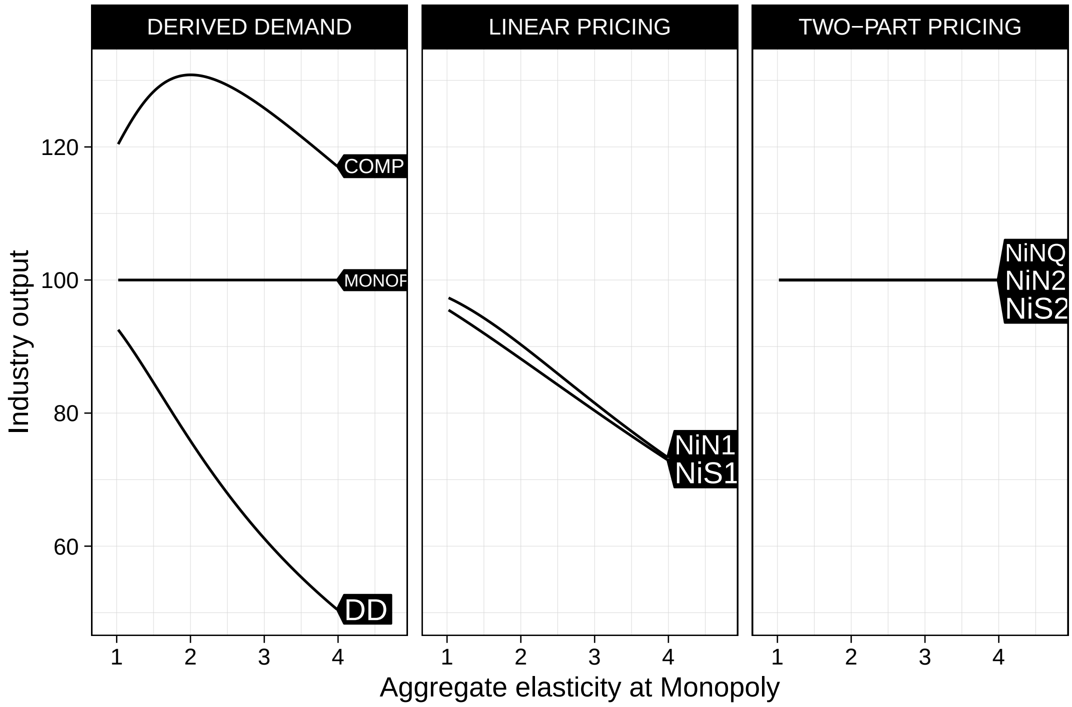


Figure 2.3: Total quantity vs ae 2×1 setting

2.5.1 Non-bargaining models of vertically related markets

As discussed in section 2.4, an increase in the outside quantity increases aggregate elasticity. It also renders the demand for products effectively independent, thus causing a decrease in cross-price elasticity. We expect that as the cross-price elasticity decreases, the substitutability between inside goods to also decrease, which means competition is weakened so that firms are able to increase prices and decrease quantities. These effects are observed in most model predictions (table E.1 and figures B.1(a) to B.4(d)). However, we see that the benchmark models each show a different relation to changing elasticity.

Total quantity in the monopoly model is observed to remain constant as we exogenously increase aggregate elasticity. Recall that we calibrate the system with the monopoly model, meaning that we choose initial parameters, as explained in section 2.4, and then calculate the prices and elasticities appropriate for the monopolist. We use these parameters to calibrate the demand model and then infer the marginal cost from this result. This exercise results in the predicted quantities of the monopoly model being equal to the initial quantities. Because we keep the initial quantities constant for our simulations, the predicted quantities also remain constant for all levels of aggregate elasticity.

The non-monotonicity in the quantity predicted by the perfect competition model can be explained by the trade-off in aggregate elasticity- and marginal cost effects. The locus is increasing where the percentage increase in aggregate elasticity is larger than the percentage increase in marginal cost. This causes an increase in the quantity sold by the downstream firms in the perfect competition model. Total quantity is decreasing where the percentage increase in aggregate elasticity is less than that of marginal cost[8].

[8]This is likely the same mechanism that causes the non-monotonicity in NiN2 and NiNQ. However, we were not able to relate the turning point to this exact point. This is likely because there is a non-linear relationship between aggregate elasticity and total marginal cost (wholesale price plus marginal cost).

Along the DD locus, the upstream firm makes a take-it-or-leave-it linear wholesale price offer to the downstream firm. Since there is no bargaining involved, this wholesale price is the highest of the models that we simulate. This contributes to the downstream firm's total marginal cost, and these firms then add their own margin. As a result, total output is way below the monopoly quantity and the lowest of all the models.

Table 2.2: Derived demand summary: Figures 2.2 and 2.3 panel 1

Model Description	**Assumptions**	1×2**:** Results	2×1**:** Results
DD: Derived Demand models generalise the old 'successive monopoly' models, by allowing a more general downstream game, e.g. Bertrand or Cournot.	Upstream firm(s) set wholesale linear prices, and downstream firm(s) play a noncooperative game, taking wholesale prices as given.	Output is far below monopoly output, due to big double marginalisation.	Output is far below monopoly output, due to big double marginalisation.

2.5.2 Linear pricing bargaining models

In the second panel of figures 2.2 and 2.3, we introduce bargaining over linear prices. Along the NiN1 and NiS1 loci, we assume bargaining over linear wholesale prices of goods. Downstream firms subsequently set retail prices in Nash equilibrium. Each bilateral bargain is assumed to be reached on the basis of a Nash bargaining solution relative to either a NiN1- or NiS1-specified threat point. For the NiN1 model, the threat point for each agreement (the outcome should an agreement not be reached) is taken to be the continuation of the other agreement at the same wholesale price. The NiS1 model allows for bilateral contracts that are contingent on which other agreements are made (renegotiation), and thus the threat point is specified for an agreement at a new wholesale price that satisfies the Nash bargaining solution for a single good.

Despite the differences in threat points, these two models are almost indistinguishable for all of the variables of interest. This is a result of inefficient bargaining. In linear pricing, there is only one instrument — the wholesale price — to achieve two conflicting goals. Seeing that in this setting the wholesale price comprises a significant part of the marginal cost of the downstream firms, it helps determine the downstream prices, quantities and ultimately profits. In turn, the downstream equilibrium determines the wholesale quantities that the upstream firm will sell. Thus, the wholesale price is firstly an instrument that the upstream firm can use to increase the industry profits. However, concurrently the wholesale price is also the only instrument with which the upstream firm takes its share of the profit. These two goals work in opposite directions since a lower wholesale price is better for the first goal, but a higher wholesale price fits the second goal. These counteracting effects thus diminish the difference in threat points between NiN1 and NiS1 so that we do not observe any discernible difference between these two models.

Apart from the conflicting goals of the upstream firm, introducing bargaining means that the downstream firms also influence the wholesale prices. Consequently, firms are able to bargain for a lower wholesale price than the take-it-or-leave-it scenario in a derived demand setting[9]. As a result, we ob-

[9]This can be observed when comparing panels 1 and 2 of figures B.2(a) and B.2(b) and figures B.5(a) and B.5(b).

serve total quantity to be higher than in a derived demand setting for both industry structures (figures 2.2 and 2.3).

Table 2.3: Linear pricing summary: Figures 2.2 and 2.3 panel 2

Model Description	**Assumptions**	1×2: Results	2×1: Results
NiN1: One instrument (linear wholesale price) performs two tasks, it determines the size of total profit and how the profit is split. NiN1 and NiN2 are very close.	Upstream and downstream players bargain bilaterally over linear wholesale price, taking other agreements as fixed. The threat point for one agreement is profits in the existing remaining agreements.	Output is above monopoly output for low aggregate elasticity.	Output is always below monopoly output in the range of parameters we consider. Not true in general.
NiS1: As above, only one instrument (linear wholesale price) performs two tasks, but the alternatives to agreement change.	Upstream and downstream players bargain bilaterally over linear wholesale price, but expect prices to change if agreements do not make. The threat points determined by re-negotiating remaining agreements.	Output is above monopoly output for low aggregate elasticity.	Output is always below monopoly output in the range of parameters we consider. Not true in general.

2.5.3 Two-part pricing: Threat points matter

In the third panel for all the figures, we introduce two-part pricing bargaining. In this setting, we assume bargaining over wholesale prices and fees. For both the industry structures and the specifications of bargaining objects, the incentives of bargaining players are aligned. This is because all parties wish to increase industry profits when negotiating over the wholesale price. Consider the 1×2 setting: downstream firms earn profit

$$\pi_i = (p_i - mc_i - w_i) * q_i - f_i, i = 1, 2$$

and pay upstream firm(s) $w_i * q_i + f_i$, where w_i is the marginal wholesale price and f_i is a fixed fee used to ensure an equal profit split over the threat points. The upstream firm earns profit

$$\pi_A = ((w_1 - mc_{A1}) * q_1 + f_1) + ((w_2 - mc_{A2}) * q_2 + f_2)$$

The mc_{A1} and mc_{A2} are the possibly different upstream marginal costs of supplying goods to M_1 and M_2 respectively. We assume that $mc_{A1} = mc_{A2} = 0$ for convenience. In a 1×2 two-part pricing setting, A and 1 negotiate agreement $A1 = (w_1, f_1)$ while taking into consideration agreement $A2$, and A and 2 negotiate agreement $A2 = (w_2, f_2)$ taking into consideration agreement $A1$. In these negotiations, the assumed status of the other agreement characterises the nature of bargaining.

In the 2×1 setting, we invert the bargaining set up by assuming firm 1 (downstream monopolist) contracts with A and B (upstream firms) to produce goods for sale by firm 1. As above, we look at the bargaining between 1 and A, who negotiate over contract $A1 = (w_1, f_1)$ while taking into consideration agreement $B1$, and 1 and B negotiate agreement $B1 = (w_2, f_2)$ taking into consideration agreement $A1$. Again, the assumptions we make about the other agreement characterise either NiN or NiS bargaining.

In an NiN setting, each surplus is maximised independently assuming that the terms of the other agreement are fixed. In contrast, NiS assumes that the total surplus from both agreements is maximised and that the effects of the other agreement are accounted for in computing the surplus from an agreement. In the second stage of the game, the maximised surplus is split between the parties with the fixed fee. Thus, there are now two different parameters to achieve two conflicting goals, as set out in section 2.5.2. In contrast to linear pricing bargaining, there is an incentive for the pivotal player (the firm involved in both bilateral bargains) to internalise competition in the opposing market (the upstream market in a 2×1 and the downstream market in a 1×2 setting) when determining the wholesale price. This incentive leads to different outcomes depending on the assumed nature of bargaining.

Table 2.4: Two-part pricing summary: Figures 2.2 and 2.3 panel 3

Model Description	**Assumptions**	1×2**:** Results	2×1**:** Results
NiN2: The NiN assumption that the other agreement is fixed makes the parties bargain as if they do not know that the bargain they strike in one deal has an effect on the profitability of the other.	Upstream and downstream players bargain bilaterally over two part prices, taking other agreements as fixed. The threat point for one agreement is profits in the existing remaining agreements.	Output is above monopoly output as the NiN assumption makes it appear as though the single upstream firm is bargaining against itself. Output is above monopoly output.	Output equals monopoly output because the downstream monopoly retailer internalises upstream competition.
NiNQ: The NiN assumption that the other agreement is fixed makes the parties bargain as if they do not know that the bargain they strike in one deal has an effect on the profitability of the other.	Upstream and downstream players bargain bilaterally over two part prices, taking other agreements as fixed. The threat point for one agreement is profits in the existing remaining agreements.	Output is above monopoly output as the NiN assumption makes it appear as though the single upstream firm is bargaining against itself. Output is above monopoly output.	Output equals monopoly output because the downstream monopoly retailer internalises upstream competition.
NiS2: The parties bargain as if they know that they will get a share of any improvement in profit in grand coalition (both agreements make). They are willing to, e.g., reduce the wholesale price if that leads to a higher total profit.	Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements do not make. The threat point determined by re-negotiating remaining agreements.	Output equals monopoly output.	Output equals monopoly output because the downstream monopoly retailer internalises upstream competition.

2.5.3.1 *Nash-in-Nash:* Bargaining against yourself

In the 1×2 NiN2 setting, 1 and A negotiate agreement $A1 = (w_1, f_1)$ by assuming that agreement $A2 = (w_2, f_2)$ is fixed. To compute equilibrium, we have to check the conditions under which A and 1 can increase joint profit by reaching a different agreement. This occurs only if a change leads to an increase in their joint profit.

$$\Delta(\pi_A + \pi_1) = \Delta(q_1 * (p_1 - mc_1) + w_2 * q_2) > 0$$

Note that the wholesale payments cancel each other out, as they are revenue to A but costs to 1.

Intuitively, A and 1 try to make themselves better off at the expense of 2. Of course, when A and 2 negotiate, they try to do the same thing. NiN equilibrium occurs at a point where it is no longer profitable for either of the pairs $(A, 1)$ or $(A, 2)$ to deviate from the agreement where $w_1^* = \text{argmax}_{mc_1}(\pi_A + \pi_1)$. The fixed fee f_1 is chosen to split surplus by maximising the product of the surpluses,

$$f_1^* = \underset{f_1}{\text{argmax}}(\pi_A - \pi_A^*)(\pi_1 - \pi_1^*).$$

In the case of transferable utility, this reduces to

$$\pi_A - \pi_A^* = \pi_1 - \pi_1^*,$$

which is exactly why both A and 1 want to maximise $\pi_A + \pi_1$ in the previous step. Here, $\pi_1^* = 0$ but $\pi_A^* = (w_2^* - mc_2) * q_2^* + f_2$, with only q_2^* manufactured by 2. What differentiates NiN2 from NiS2 is the suboptimal form of the NiN2 contracts, as they would remain fixed even if the other agreement does not make.

In the NiN2 equilibrium $\{A1^*, A2^*\}$, the suboptimal nature of the contractual form shows up in the joint output above $(q_1^* + q_2^*)$ and joint profit $(\pi_1^* + \pi_2^*)$ below monopoly levels. NiN leads the pair $(A, 1)$ to compete with $(A, 2)$, lowering wholesale prices to maximise (almost) independent profits in Nash equilibrium, with the dubious consequence that A in the two negotiations ends up competing with itself.

In the 2×1 NiN2 setting, we now look at the bargaining between 1 and A, who negotiate by assuming that agreement B1=(w_2, f_2) is fixed, where 1 pays $w_2 * q_2 + f_2$ to B, depending on the quantity q_2 that 1 chooses to sell at to maximise its final retail profit. In contrast to the 1×2 setting, the equilibrium is the joint profit maximising outcome.

To see this, we have to show that neither firm has an incentive to change wholesale prices from marginal costs, $w_i = mc_i, i = A, B$. If w_1 were lower than mc_A, 1's total operating profit would increase, but the total joint profit $\pi_1 + \pi_A = (p_1 - mc_A) * q_1 + (p_2 - mc_B) * q_2$ would decrease as 1 sets retail prices to maximise $\pi_1 = (p_1 - w_1) * q_1 + (p_2 - mc_B) * q_2$. This would result in a price lower than the monopoly price for p_1; and similarly, if 1 and A were to raise w_1 above mc_A.

We see that the pair $(1, A)$ has no incentive to deviate from the $w_1 = mc_A$ marginal price, and likewise $(1, B)$ will not deviate from $w_2 = mc_B$. The downstream firm 1 takes these wholesale prices as given and finds the monopoly retail prices and quantities maximising joint surplus $(= \pi_1 + f_1 + f_2)$ given that f_1 and f_2 are fixed.

From the above, it is clear that NiN2 bargaining leads to results that are dependent on the industry structure. While the pivotal player manages to completely internalise competition in the opposing market in the 2×1 setting, it is unable to do so in the 1×2 setting. In the latter, we see the NiN2 models showing a more competitive outcome than the NiS2 model. Moreover, fixing the quantity and total price instead of the wholesale price and fee, as in the NiNQ model, leads to an outcome even closer to competition. The difference between NiNQ and NiN2 here is that the joint profit functions for NiN2 for $(A, 1)$ includes q_2, and $(A, 2)$ includes a term of q_1. These terms 'internalises' some of the 'schizophrenia' (Collard-Wexler et al., 2019) associated with A bargaining against itself, as the pairs will not compete as vigorously against each other as in NiNQ, where there is no such dependence. In NiNQ the joint profit of $(A, 1)$ is not a function of q_2, and $(A, 2)$ is not a function of q_1. This leads to the more intense competition between the pairs.

2.5.3.2 *Nash-in-Shapley:* Internalising competition

When bargaining in NiS2, the pivotal player takes full cognisance of the externality that the other agreement imposes on this bargain. For example, in the 1×2 setting, we assert that firms A and 2 would anticipate the change in conditions if agreement $A1$ were to fail and would set a (w_2, f_2) for this contingency, different from the contract when agreement $A1$ makes. This changes the threat point in negotiation with 1. Moreover, we assert that firms A and 2 would anticipate how the split of profits determined by f_2 would change (through renegotiation) as they vary w_1. This would lead them to maximise joint profit, and hence each of their own profits. In the case where both agreements make, they should try to maximise the joint surplus of A, 1, and 2. This leads to higher wholesale prices signalling the downstream firms to price at the joint profit maximising level. Therefore, NiS2 achieves the joint profit maximising outcome.

In the 2×1 NiS2 case, the pair $(1, A)$ anticipates the split in profits 1 will realize with B, and so will set w_1 to maximise the total surplus $\pi_1 + \pi_A + \pi_B = q_1 * (p_1 - mc_A) + q_2 * (p_2 - mc_B)$. In like vein $(1, B)$ will maximise the same total surplus. Since 1 will set retail prices to maximise $q_1 * (p_1 - w_1) + q_2 * (p_2 - w_2)$, both A and B would be happy to set $w_1 = mc_A$ and $w_2 = mc_B$, leading to monopoly retail prices, and collect their share of the maximum possible total surplus.

It is not the result of the industry structure, where operating profits are earned or the marginal cost balance between upstream- and downstream firms that yields the NiS2 model equal to the monopoly outcome. Rather, it is a direct result of how the model characterises bargaining. NiS2 assumes that total surplus from both agreements is maximised and that the effects of the other agreement are accounted for in calculating the surplus. It is then exactly the monopoly outcome that maximises total surplus when determining the wholesale price. Hence, we observe that the NiS2 model follows the monopoly outcome in both industry settings (figure 2.2 and 2.3). This is robust against a change in the pivotal player, where the operating profit is earned and what the marginal cost balance between the pivotal- and other players is. The NiS model thus displays a consistency over different industry specifications.

2.6 Conclusion

In this chapter, we recognise that vertically related markets represent a complex system with many moving parts. We group these moving parts into what we identify as the four elements of models of vertically related markets. We make assumptions and modelling decisions to control for three of these elements: (i) the network of upstream suppliers and downstream retailers, (ii) downstream competition and (iii) consumer demand. The focus is thus on the remaining element: vertical contracting in the presence of externalities.

Vertical contracting is further divided into two categories: the nature of vertical contracting (i.e. how firms reach agreement) and the object of contracts (i.e. over what do firms reach agreement). From these categories, we identify that there are six models that are typically used to model vertically related markets. We add a further two models that serve as benchmarks for our comparisons.

The complex nature of these models motivates us to employ numerical simulations to gauge how the assumptions about vertical contracting influence model outcomes. The results are especially helpful because the simple industry structures and uniform calibration across models mean that the assumptions about vertical contracting are directly mapped onto observable outcomes. Further, it is helpful to have two benchmarks against which to compare these outcomes.

Our simulations show that as a result of the upstream firm(s) making a take-it-or-leave-it offer to downstream firm(s), the significant double margin yields output far below monopoly levels in the derived demand model for both industry structures. Comparing these outcomes against models where upstream and downstream firms bargain, shows that the derived demand model yields the lowest total quantity. From our review of the literature, this is expected.

In the 1×2 industry setting, the introduction of linear pricing bargaining (NiN1) moves us closer to competition for low aggregate elasticity and closer to monopoly for high aggregate elasticity. In two-part pricing, bargaining (NiN2 and NiNQ) leads to an outcome closer to competition that is always above monopoly. The case for recursive bargaining over linear prices (NiS1) is almost indistinguishable from the bargaining case. In the two-part pricing setting, however, recursive bargaining follows the monopoly outcome.

In the 2×1 setting, linear pricing bargaining and recursive bargaining (NiN1 and NiS1) leads to an outcome closer to, but always below monopoly. In the two-part pricing case for all bargaining models (NiN2, NiNQ and NiS2), the monopoly outcome is obtained.

The results discussed in this chapter can assist with the difficult question of model selection. For example, if a practitioner is analysing a vertically related market and observes evidence of linear pricing contracts, the simulation results suggests that a choice between bargaining and recursive bargaining (NiN versus NiS) is negligible. In such a case, the practitioner will spend more time on choosing the appropriate industry structure seeing that this has a greater effect on predicted outcomes than a refinement of the specification of threat points which characterise bargaining.

Suppose, however, that evidence suggests that two-part pricing contracts are negotiated. In such a case, the practitioner will have to take care in selecting the industry structure that best portrays the specific case. If a 2×1 industry structure is most appropriate, the specification of bargaining versus recursive bargaining does not matter since both predict the monopoly outcome. If, however, a 1×2 structure is most appropriate, further investigation is needed on how firms negotiate.

In the case of two-part pricing contracts, a practitioner may also first investigate how firms negotiate. If there is evidence of Nash-in-Nash-style bargaining, further investigation will be necessary into the appropriate industry structure. However, if the evidence suggests recursive bargaining (Nash-in-Shapley), a choice between a 1×2 and 2×1 industry structure will be redundant since predictions across structures are consistent for NiS.

The systematic and direct comparison of the corpus of models of vertically related markets presented in this chapter is a novel contribution to our understanding of these markets. This in-depth analysis of how the assumptions about vertical contracting effects outcomes improves our understanding of the impact of our modelling choices. Only after we better understand how to model pre-merger outcomes, can we begin to investigate vertical mergers in these complex markets.

Chapter 3

Vertical Mergers

3.1 Introduction

In the simplest case, a vertical merger is the joining of two firms at different levels of production or distribution. Predicting the likely effects of such a joining is more complex and less certain than for horizontal mergers. This stems partly from the incentive for vertical mergers being inherently efficiency related. Consequently, vertical mergers are accompanied by significant pro-competitive effects, but anti-competitive effects are brought on only in an indirect manner. This adds another complex component to the models of vertically related markets that we covered in chapter 2. Trying to understand how such a system works by looking at its components would be difficult.

The 2016 AT&T/TimeWarner merger challenge in the USA, the first litigated vertical merger challenge in the country in 40 years, showed the complexity of modelling vertical mergers. In this case, the judge called the plaintiff's model a 'Rube Goldberg' machine', before ruling for the defendants. This trial highlighted the uncertainty surrounding the characterisation of bargaining competition, which either lead to, or coincides with, recent academic interest in the topic[1]. It has also coincided with draft- and later published vertical merger guidelines by the USA agencies (Department of Justice and Federal Trade Commission, 2020b), which has resulted in 74 differing comments from leading academics and practitioners on what they should or should not say (Department of Justice and Federal Trade Commission, 2020a). However, a general theme seems to be that the guidelines are a 'missed opportunity' for providing clear guidance on analysing vertical mergers (Shapiro, 2021).

Against this background, we continue our treatment of models of vertically related markets as 'complex systems' that can best be understood by simulating their effects (Wolfram, 2002). However, in this chapter, we present our assessment of the effects of a vertical merger with each of the models presented in chapter 2. Now, we are able to ascertain how the assumptions made about the nature- and object of bargaining in the pre-merger models, affect the post-merger predictions. We also build on the methodology developed in chapter 2 by presenting an additional calibration for analysing post-merger outcomes.

Simulations (as opposed to analytical results) enable us to compute the magnitude of merger effects and to determine how various pieces of the system interact. The simulations presented in this chapter enables us to begin to address the difficult question of model selection for analysing vertical mergers. These results, along with the findings in chapter 2, can begin to answer the question: "for a given case,

[1] See, for example, Crawford et al. (2018); Froeb et al. (2020); Rogerson (2020); Sheu and Taragin (2017); Yu and Waehrer (2018)

which assumptions best capture the significant features of competition, and the loss of such competition following a vertical merger?" (Werden et al., 2004).

Choosing the right model is challenging because the assumptions that distinguish models of vertically related markets from one another, are typically not observed. The results in this chapter can help guide model selection as they map assumptions about the nature of bargaining and demand into observable outcomes and merger effects. A systematic comparison of the magnitude of predicted merger effects across different merger models, industry structures and contractual forms provides a novel contribution to our understanding of vertical mergers.

Among other things, we find:

- Compared to derived demand models, bargaining models increase the scope for anticompetitive outcomes because they reduce upstream margins and the magnitude of the elimination of double marginalisation.
- In bargaining models, a vertical merger gives the merged firm a better outside option, resulting in a larger profit share.
- When parties bargain over linear wholesale prices:
  - A lower aggregate elasticity increases the likelihood of an anti-competitive vertical merger.
  - Specifically in a 1×2 setting, the vertical merger model predicts an anti-competitive merger for an aggregate elasticity of below 2.5 and a nest strength of above 0.125.
- When parties bargain over two-part prices:
  - Nash-in-Nash and Nash-in-Shapley pre-merger equilibria in a 2×1 industry setting are equal, and both show no merger effects.
  - Nash-in-Nash and Nash-in-Shapley in a 1×2 setting give opposing predictions about whether vertical merger outcomes are anti-competitive.

This chapter sets out with an overview of the existing literature on vertical mergers and their analysis. We proceed to discuss the derivation of the vertical merger models simulated in this chapter in section 3.3. Section 3.4 discusses the methodology followed for the two different calibrations of the system of models, and section 3.5 presents the results. Finally, section 3.6 investigates the relationship between the two measures of substitutability presented in this chapter and vertical merger effects.

3.2 Literature overview

In the simplest case, a vertical merger is the joining of two firms at different levels of production or distribution (e.g. integration between two single-output production processes in which part or the entirety of the output of an upstream process is an input into the downstream process) (Perry, 1989). Analysing the effects of such a joining is more complex than for those of horizontal mergers. Church (2008) provides four reasons for this complexity: (i) the incentives for vertical integration are not to do with market power, but rather with efficiencies, (ii) vertical mergers often lead to lower prices as a result of the pro-competitive elimination of double marginalisation, (iii) anti-competitive effects of vertical mergers are only ever indirect, since there is no direct elimination of a competitor as in horizontal mergers, and (iv) assessing when a merger is anti-competitive is difficult since the aforementioned effects work concurrently and arise from the same source. The paragraphs below examine each of these points separately.

Church (2008) argues that it is often nonprice efficiencies that incentivise firms to vertically integrate. In this vain, Werden and Froeb (2020) define a vertical merger as a solution to coordination problems. There are many examples in the literature of such coordination problems where vertical integration improves efficiency. For example, vertical integration may improve technological efficiency when intermediate inputs can be replaced with primary inputs[2] (Perry, 1989). Vertical integration may also align the incentives of merging firms and improve communication (Salop and Culley, 2016). In this regard, Church (2008) explains that integrated firms are able for example to share information about market conditions.

Perhaps the most distinguishing feature of vertical mergers concerns the pro-competitive elimination of double marginalisation. Motta (2004) explains with a simple example: A manufacturer sells to final consumers through a retailer. Both firms maximise individual profits and thus add their margin, selling at the monopolistic mark-up over their costs. However, if the firms were vertically integrated, the end price would be chosen by adding only a mark-up over the single cost. Perry (1989) eloquently defines this as a market transaction being substituted with an internal transfer within the boundaries of a firm. Vertically integrated firms can thus coordinate and internalise the externality they imposed on each other (Motta, 2004).

The elimination of double marginalisation leads to lower prices of the product produced by the integrated firm. Consequently, it is suggested that vertical mergers mostly benefit consumers (LaFontaine and Slade, 2007). There is a widely held view that vertical mergers rarely raise competitive concerns and are likely pro-competitive (Salop and Culley, 2016). However, this may be a consequence of how we model vertically related markets and the difficulty in quantifying (often indirect) anti-competitive effects.

Anti-competitive effects of vertical mergers arise when the competitive constraints imposed on an integrated firm are relaxed post-transaction, increasing the market power of the integrated firm (Church, 2008). There are a number of theories on anti-competitive effects of vertical mergers[3], but often authorities focus on how vertical mergers can change the incentives and thereby increase the ability of the merged firm to undermine the competitive process (Salop and Culley, 2016). In this regard, modern theories of anti-competitive effects focus on how (input or customer) foreclosure raises rivals' cost or reduces rivals' revenue (Church, 2008).

The Chicago School maintained that the argument that a vertically integrated firm may foreclose its rivals, is incorrect (Motta, 2004). Church (2008) finds that when employing the workhorse model of that time, the successive monopoly vertical model (later named the derived demand model), there is empirical and theoretical support for the Chicago School's argument. This is in line with Moresi et al. (2007), who show analytically that in the derived demand model, the pro-competitive effect of the elimination of double marginalisation is so significant that it completely overshadows potential anti-competitive effects.

However, this is a direct result of how vertical contracting is modelled in the derived demand model. Recently, scholars have shown that vertical mergers can result in anti-competitive outcomes when taking

[2] An example presented by Perry (1989) is the energy saved from not having to reheat steel in the production of steel sheets.

[3] For example, Salop and Culley (2016) identify potential anti-competitive categories as "potential competition effects, exclusionary effects, unilateral effects, coordinated effects, regulatory evasion and facilitation of harmful price discrimination".

cognisance of modern game-theoretic models (Motta, 2004) and estimation techniques, as is discussed in subsection 3.2.1, below.

Assessing when a merger is anti-competitive is difficult since the pro- and anti-competitive effects described above, arise concurrently and from the same source (Church, 2008). When ascertaining the effects of vertical mergers theoretically, scholars frequently employ either analytical or numerical tools. The recent literature has shown that general (analytical) results can be obtained in vertical structures as complex as the ones we investigate in this chapter, and the appeal of such results is very clear[4]. However, for this chapter, we make use of numerical simulations to solve the models. The reason for this is twofold: Firstly, by employing numerical simulations, we are able to ascertain not only the direction of merger effects, but also the relative magnitude of these effects. Secondly, as stated by Sheu and Taragin (2017), numerical simulations make it possible to assess how bargaining models perform in a range of market conditions. Indeed, they (Sheu and Taragin, 2017) note that the merger evaluation literature typically looks at one or a few industries, making inference outside of the context of individual research pieces difficult[5].

Budzinski and Ruhmer (2009) ascribe the increased adoption of merger simulations to four causes. First, the evolution of pro- and anti-competitive effects calls for an overall assessment of welfare effects. Second, there is increased availability in market data, which, thirdly, allows using computational techniques for increasingly complex simulations. Fourth, competition policy has been receptive to innovative economic assessment instruments. For these reasons, we limit our attention to the literature on vertical merger simulation in the remainder of this section[6].

3.2.1 Evolution of vertical merger simulation models

As mentioned above, Church (2008) states that there is empirical and theoretical support for the frequent presumption that vertical mergers are welfare enhancing. This stems from the analytical results of the derived demand model, the workhorse model of the Chicago School (Moresi et al., 2007). In estimating a derived demand model by means of Monte Carlo simulations, however, Domnenko and Sibley (2020) show that an anti-competitive outcome is possible for the set of parameters they consider. Specifically, when the pre-merger market share of the acquired downstream firm increases above 40% to 50%, the incentive of the integrated firm to raise the cost of its rival dominates. For a market share lower than that, all prices fall and a merger is pro-competitive.

The model that Domnenko and Sibley (2020) consider, does not take account of bargaining. Since the seminal contribution of Horn and Wolinsky (1988), it has been believed that embedding the Nash bargaining solution in a Nash equilibrium is a refined way to model vertical contracting between upstream- and downstream firms[7]. In this regard, Sheu and Taragin (2017) and Crawford et al. (2018) simulate vertical mergers in a Nash-in-Nash setting.

[4]For example, in a two-part pricing setting, Nocke and Rey (2018) and Rey and Vergé (2019) analysed vertical mergers in a bilateral duopoly framework, including a general demand system and bilateral negotiations. For linear pricing, O'Brien (2014) analysed wholesale price discrimination in a 1×2 industry setting and Gaudin (2018) studied horizontal mergers in a $1 \times N$ setting with a general demand system and allowing for any distribution of bargaining power.

[5]For example, Gowrisankaran et al. (2015) simulated mergers in the health care market between hospitals and managed-care organisations. Crawford et al. (2018) studied the welfare effects of vertical mergers in television markets.

[6]For a complete overview of the general methodology of merger simulation see chapter 4 section 4.2.

[7]For a comprehensive review of bargaining models, refer to chapter 2 section 2.2.

Simulating vertical mergers for a varying bargaining power, Sheu and Taragin (2017) find that the potential for an anti-competitive vertical merger is highest when the downstream bargaining power is in an intermediate range. Intuitively, when the downstream firm(s) has less bargaining power, the upstream firm(s) is able to impose higher pre-merger wholesale prices. Following a vertical merger, the elimination of double marginalisation easily overshadows the raising of rivals' costs. When the downstream firm(s) has more bargaining power, it can resist the raising of rivals' costs post-merger so that the elimination of double marginalisation effect again easily dominates.

Crawford et al. (2018) similarly employed the Nash-in-Nash model to study the interactions in multichannel television markets. In a novel assumption, the authors determined the degree of internalisation of the internal transfer price of the integrated firm (Slade, 2020). The study found that \$0.79 of each dollar was internalised by the integrated firms.

Despite the contributions of these scholars, there is still not a consensus on how to properly model bargaining competition in the presence of externalities created by competition. This is evidenced by the strand of literature that focusses on contingent contracts or contracts that allow for renegotiation (de Fontenay and Gans, 2014; Stole and Zwiebel, 1996; Inderst and Wey, 2003; Yu and Waehrer, 2018; Froeb et al., 2020) as well as contested cases. Indeed, the 2016 AT&T/TimeWarner merger challenge in the USA, the first litigated vertical merger in the country in forty years, highlighted the uncertainty surrounding the characterisation of bargaining competition.

There is a vast literature on the theoretical effects of vertical mergers. However, empirical support for the magnitude of potential pro- and anti-competitive effects and the subsequent net effect is still lacking (Crawford et al., 2018). We believe that the computational experiments presented in this chapter — despite not employing existing data — do provide support for the quantitative magnitudes of merger effects.

In the remainder of this chapter, we provide a systematic comparison of the magnitude of vertical merger effects for a range of models. We use the same set of parameters to calibrate all the models and consider simple industry settings. This allows direct comparisons, and we are able to attribute the differences in outcomes solely to the different assumptions of the models. This systematic comparison of the corpus of models of vertical mergers across different contractual forms and different industry settings provides a novel contribution to our understanding of the magnitude of merger effects.

3.3 Vertical merger models

As in chapter 2, we group models of vertical mergers into three categories based on the assumed nature of vertical contracting. We also identify three objects of vertical contracting. Again, we identify three groups into which we classify the nature of vertical contracting: (1) take-it-or-leave-it offers by the upstream firm (derived demand), (2) bilateral bargaining between upstream and downstream firms (Nash-in-Nash), as well as (3) bilateral bargaining with contingent contracts (Nash-in-Shapley). With regard to the object of contracts, we consider (1) linear pricing, (2) two-part pricing and (3) quantity fixing in two-part pricing. Again, for comparison, we show two benchmarks: the perfect competition and monopoly outcome.

Recall that models of vertically related markets involve four distinct features: (i) a network of upstream suppliers and downstream retailers (ii) who contract vertically in the presence of externalities (iii) created by competition (iv) over consumer demand. We build on the framework set out in chapter 2, section 2.3 by assuming the same network of firms (section 2.3.1), vertical contracting (section 2.3.2) and demand specification (section 2.3.3). With this as the foundation, we then simulate a vertical merger with each of the six models presented in chapter 2.

In terms of the network of upstream suppliers and downstream retailers, the benefits of working in a 1×2 or 2×1 industry structure is highlighted when considering vertical mergers. When two firms merge vertically, we assume that they are automatically in agreement. This means that in a post-merger environment, we only have to consider the vertical contracting between the vertically integrated- and rival firm. This is especially beneficial in the models that consider bargaining between the upstream and downstream firms, seeing that it renders the NiN and NiS specified threat points equal[8]. There is thus no delineation between bargaining and recursive bargaining post-merger in the two industry structures that we consider. Therefore, we compute a vertical merger in the derived demand, linear pricing and two-part pricing settings, seeing that the post-merger predictions for Nash-in-Nash and Nash-in-Shapley are equal.

Below, we proceed to briefly discuss the derivation of analytical results for each of the vertical merger models presented in this chapter[9]. The complete derivations are contained in appendix C.

3.3.1 General setup

In both industry structures, we consider a vertical merger between upstream firm A and downstream firm 1. Thus, the 1×2 pre-merger industry structure changes to a structure with a vertically integrated firm (firm A and 1) and one downstream rival (firm 2). The 2×1 industry structure similarly changes to a structure with the vertically integrated firm (firm A and 1) and an upstream rival (firm B). Post-merger, we assume that the wholesale price of the merged firm is completely eliminated and the firm acts with respect to its total final profit.

3.3.2 Vertical merger in derived Demand

For a vertical merger in a 1×2 industry structure, we assume that the merged firm takes the rival downstream price as being given when determining the wholesale price. Subsequently, downstream price 1 is determined by the merged firm's first-order conditions.

In the 2×1 industry structure, we assume that the rival upstream firm sets the wholesale price to the merged firm. Subsequently, the merged firm determines downstream prices, solving the first-order conditions for its total profit.

3.3.3 Vertical merger in linear pricing

A vertical merger in a 1×2 linear pricing setting means that the merged firm negotiates a wholesale price with the rival downstream firm according to the Nash bargaining solution $P_{A12} = (\pi_{A1} - \pi^*_{A1})(\pi_2 -$

[8]Suppose in the 1×2 industry setting there is a vertical merger between firm A and 1. We then only have to consider the bargaining between the vertically integrated firm and firm 2. Firm 2's outside option remains the same as pre-merger i.e. it is zero since there is no alternative upstream firm with which to reach an agreement. The vertically integrated upstream firm's outside option changes. The agreement with downstream firm 1 (its vertically integrated downstream firm) is taken as being given. Since this the only contract to consider, there is no recursive bargaining that takes place. This renders the NiN and NiS outcomes equal. Precisely the same reasoning applies for a 2×1 case.

[9]The exposition of the models is adapted from Tschantz (2019) which contains the write-up of a bargaining comparison tool. The simulations are built on the static models presented in this online tool, available at https://daag.shinyapps.io/b1x2/.

π_2^*). The threat points, π_{A1}^* and π_2^*, are the profits without product 2. Since we only consider a single agreement, and thus, these are the only set of threat points to consider, the delineation between Nash-in-Nash and Nash-in-Shapley is rendered obsolete[10]. The usual first-order conditions determine the downstream prices.

In the 2×1 industry structure, the merged firm negotiates a wholesale price with the rival upstream firm, maximising the Nash bargaining solution $P_{AB1} = (\pi_{A1} - \pi_{A1}^*)(\pi_B - \pi_B^*)$. Again, the threat points, π_{A1}^* and π_B^*, are the profits without product B, and these are equal for Nash-in-Nash and Nash-in-Shapley.

3.3.4 Vertical merger in two-part pricing

In the two-part pricing setting, firms bargain over a marginal wholesale price and a fixed fee. As in the linear pricing setting, Nash-in-Nash and Nash-in-Shapley predictions are equal since there is only one agreement that is considered. In both industry structures, the wholesale price and fixed fee are determined according to the usual Nash bargaining solution.

3.4 Calibration

Ultimately, we want to compare the pre- and post-merger predictions of six different models of vertically related markets. As discussed in chapter 2, we choose two simple industry structures and calibrate all models to the same set of parameters. However, building on the simulation methodology developed in chapter 2, we consider an additional set of calibration parameters covered in this chapter. This allows us to make inferences about how substitutability influences merger predictions.

Recall that the calibration as covered in chapter 2, section 2.4 concerns the substitutability of the inside goods with the outside good. For this we exogenously increase the quantity of the outside good as a ratio of the inside goods. In doing this, the total market size is increased, which causes aggregate elasticity in the market to increase. This constitutes the first calibration considered in this chapter.

For the second calibration, we investigate a different measure of substitutability. This calibration is concerned with how the substitutability of only the inside goods influences the results of the models. For this purpose, the nest strength parameter of the logit demand function (τ) measures to what extent the inside goods are substitutable, with $\tau = 0$ being a weak nest, i.e. bad substitutes and $\tau = 1$ a strong nest, i.e. good substitutes.

For both calibrations, we fix the scaling parameter (λ), initial prices ($p_{A@1}, p_{A@2}$ in the 1×2 setting and $p_{A@1}, p_{B@1}$ in the 2×1 setting) and initial quantities of the inside goods ($q_{A@1}, q_{A@2}$ in the 1×2 setting and $q_{A@1}, q_{B@1}$ in the 2×1 setting). Additionally, for the second calibration, we also fix the aggregate elasticity and quantity of the outside good. We then exogenously increase the nest strength from 0 to 1, again producing a list of parameters.

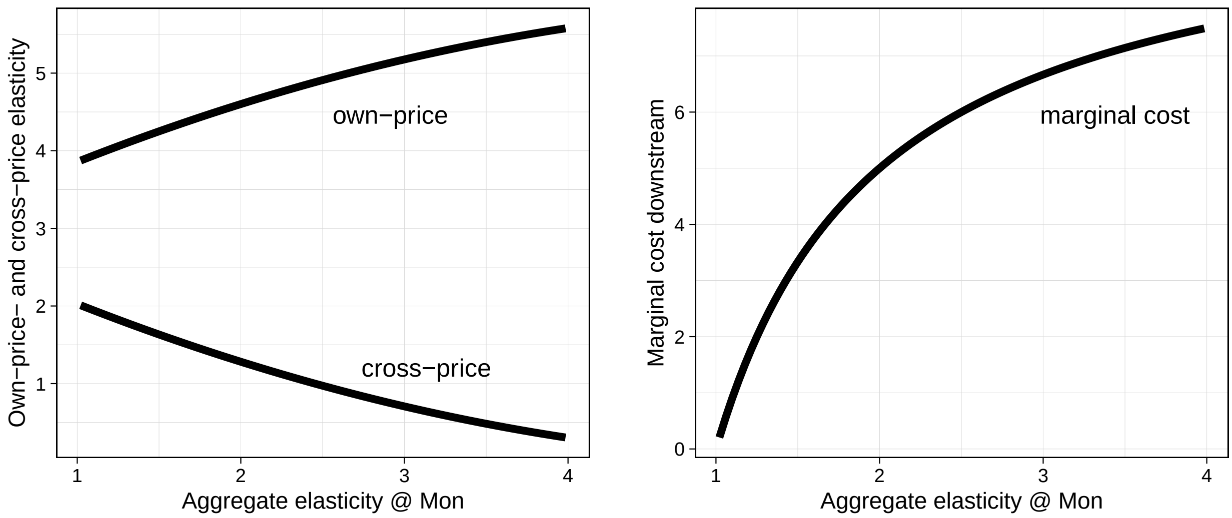
Employing the same equations as in the first calibration, the parameter list is used to determine the location parameters, allowing us to calibrate the demand model. We again assume in the 1×2 and 2×1 case that the upstream firm(s) have zero marginal cost and that the marginal cost of the downstream

[10] In chapter 2 section 2.3.2.4, we state that the delineation between Nash-in-Nash and Nash-in-Shapley is the treatment of the threat point. Nash-in-Nash assumes the threat point is given by the profits determined in the scenarios with all other agreements held fixed, while Nash-in-Shapley threat points are determined recursively.

firm(s) are inferred from monopoly, assuming that the above prices are optimal. We apply the same conventions as in the first calibration, that $\bar{p} = 1$ and $q_{tot} = 100$.

As in chapter 2 section 2.4, figure 3.1(a) shows how the cross-price and own-price elasticities are calibrated to a changing aggregate elasticity — the first calibration. Exogenously increasing the quantity of the outside good to increase aggregate elasticity causes cross-price and own-price elasticity to move in opposite directions. We observe cross-price elasticity decreasing and own-price elasticity increasing as aggregate elasticity increases. The increased substitutability of the inside goods with the outside good causes the inside goods to become less substitutable but also more responsive to a change in their own prices.

As a result of the initial prices being fixed, the change in aggregate elasticity also causes a change in marginal cost. The increased substitutability indicates that profit margins should decrease, so that this is achieved by an increasing marginal cost. This is clearly observed in figure 3.1(b). For the 1×2 case, figure 3.1(b) shows the symmetrical marginal cost for both downstream firms. In the 2×1 setting, figure 3.1(b) shows the marginal cost for the only downstream firm. In both cases, marginal cost approaches the price as the aggregate elasticity increases.



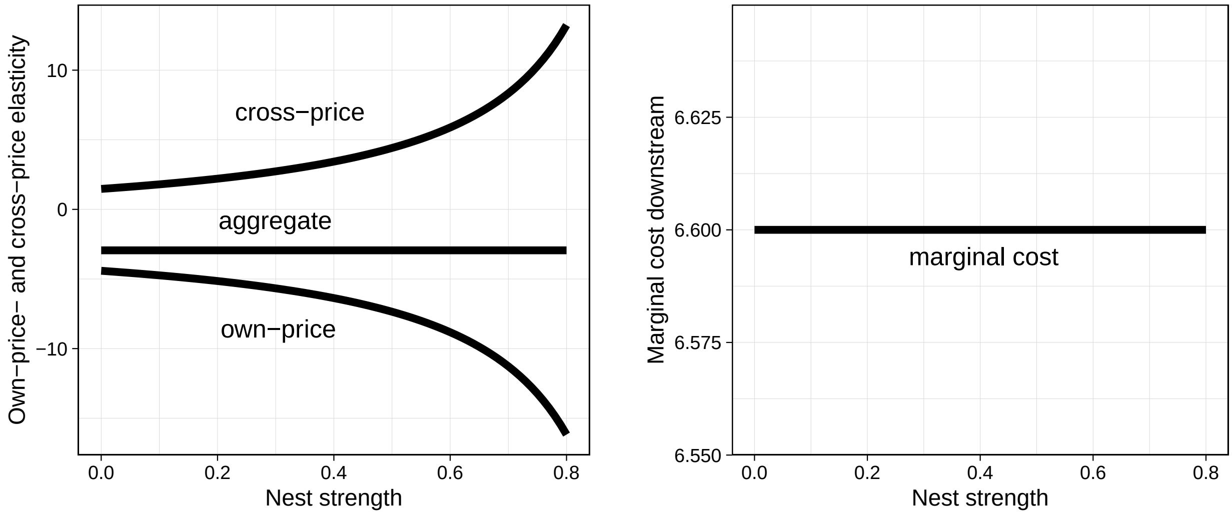
(a) Calibration of elasticities vs ae

(b) Calibration of marginal cost vs ae

Figure 3.1: Demand and cost parameter calibration for aggregate elasticity

Figure 3.2(a) shows how the elasticities are calibrated to an increasing nest strength — the second calibration. Increasing the nest strength renders the inside goods better substitutes, so that the cross-price elasticity increases. However, we keep the aggregate elasticity constant, so that the own-price elasticity decreases. It is important to note that the cross-price- and own-price elasticities move in the same direction, i.e. as the nest strength increases, both increase and the difference between them is always constant. This is in contrast to the preceding measure of substitutability, where as we increased the aggregate elasticity, cross-price- and own-price elasticity moved in opposite directions, and the difference between them increased.

Finally, because we keep the aggregate elasticity constant in this calibration of the system, the marginal cost stays constant for all levels of the nest strength. Figure 3.2(b) illustrates this, by showing the marginal cost for both firms (as with the preceding measure, we assume the marginal cost for downstream firms are symmetrical).



(a) Calibration of elasticities vs tau

(b) Calibration of marginal cost vs tau

Figure 3.2: Demand and cost parameter calibration for tau

3.5 Results

From the literature review in section 3.2, we know that a vertical merger leads to two opposing competitive effects. For the firm that is vertically integrating, elimination of double marginalisation takes place. This pro-competitive effect sees the wholesale price charged to the downstream firm pre-merger, being erased post-merger. For the post-merger rival firm, the anti-competitive raising of rivals' cost occurs in a 1×2 setting. This is when the vertically integrated upstream firm increases the wholesale price to its now rival. Similarly, in the 2×1 setting, a reduction of rivals' revenue occurs when the vertically integrated downstream firm decreases the wholesale price it pays to the upstream rival.

The aforementioned pro- and anti-competitive effects on wholesale price have obvious implications for individual prices and quantities. In both a 1×2 and a 2×1 setting, a vertical merger leads to an increase in the quantity and a decrease in the downstream price of the good of the vertically integrating firm. Concurrently, it leads to a decrease in quantity and an increase in the downstream price of the rival firm[11].

The simultaneous occurrence of two opposing competitive effects means that we have to evaluate the system as a whole before we can make a call on the likely merger effects. We therefore shift the focus to investigate total merger effects that are most easily observed in the figures for total quantity (figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d))[12]. This also illustrates the benefit of employing numerical simulations

[11] All of these effects can be observed in the appendix in figures D.2(a), D.5(a), D.2(b), D.5(b), D.1(c), D.1(d), D.4(c), D.4(d), D.1(a), D.1(b), D.4(a) and D.4(b); and figures D.8(a), D.11(a), D.8(b), D.11(b), D.7(c), D.7(d), D.10(c), D.10(d), D.7(a), D.7(b), D.10(a) and D.10(b)

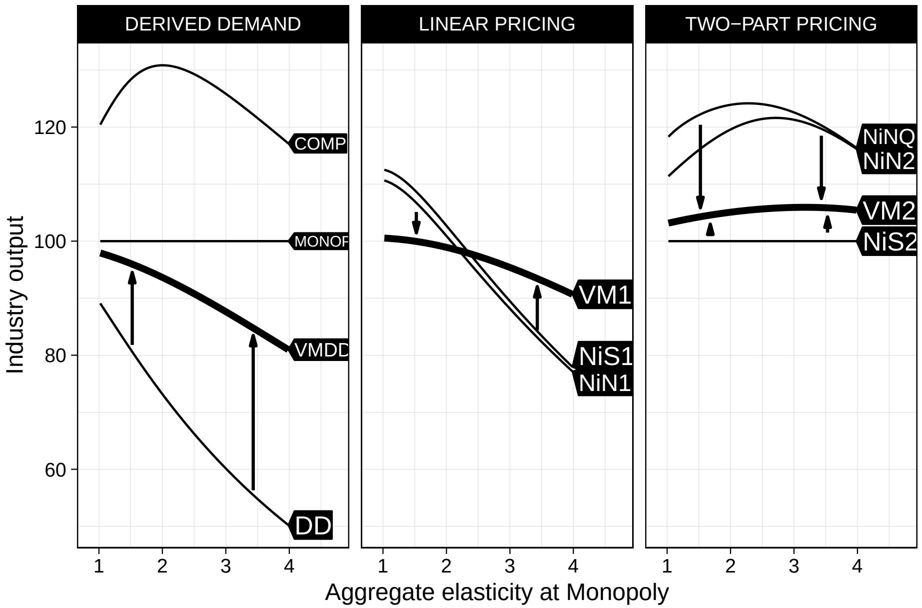
[12] The change in total quantity closely tracks the change in consumer surplus, so that welfare effects are inferred from either of these measures. In these figures, an arrow upward indicates an increase in total quantity following a merger, while an arrow downward indicates a decrease in quantity and, hence, an anti-competitive merger.

to analyse vertical mergers: we are able to compute a full equilibrium to ascertain the net effect of a merger (Rogerson, 2020; Slade, 2020).

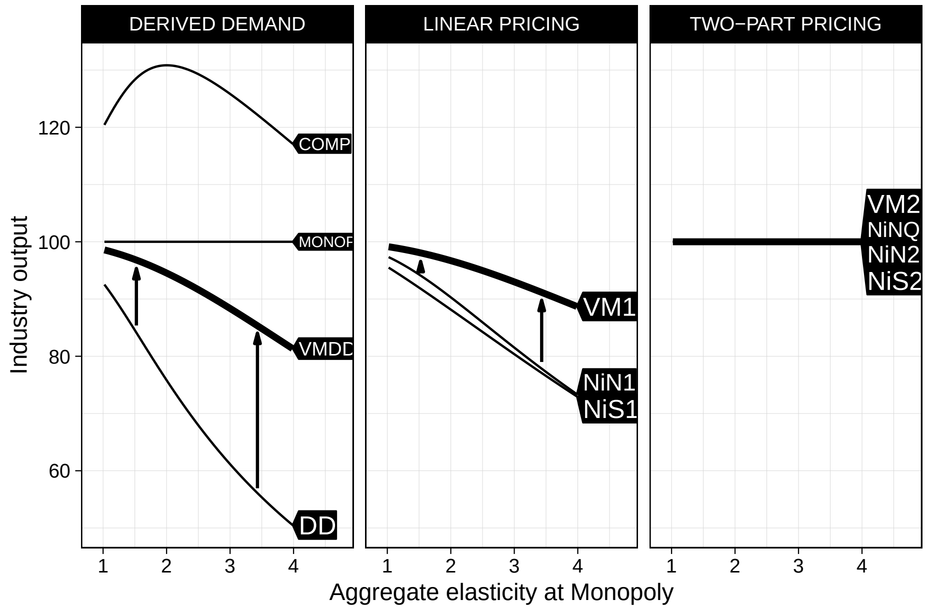
Henceforth, we discuss the predictions of six different models of vertical mergers[13]. For comparison, we also show the predictions of the perfect competition and monopoly models. We group the results with their pre-merger model in the same groups as in chapter 2. Perfect competition, monopoly, derived demand (DD) and vertical merger(VMDD) under derived demand are grouped together since no bargaining is assumed in these models. Nash-in-Nash (NiN1) and Nash-in-Shapley (NiS1) are grouped with their post-merger linear pricing vertical merger model (VM1). Finally, Nash-in-Nash(NiN2), Nash-in-Shapley(NiS2) and Nash-in-Nash quantity(NiNQ) are grouped with the post-merger two-part pricing vertical merger model (VM2).

As alluded to previously, the benefits of working in a 1×2 or 2×1 industry structure is that we have to consider only the agreement between the vertically integrated- and rival firm post-merger. This renders the NiN and NiS specified threat points equal, so that the post-merger equilibria of these models show the same results (discussed in detail below). This enables us to analyse key differences in NiN and NiS bargaining that we would otherwise not have been able to observe. These differences are consistent across our simulations for aggregate elasticity and the nest strength parameter. As such, the first subsection below focuses on these effects and can be observed in either set of results. We therefore refer to the figures or tables for aggregate elasticity first and for the nest strength parameter second.

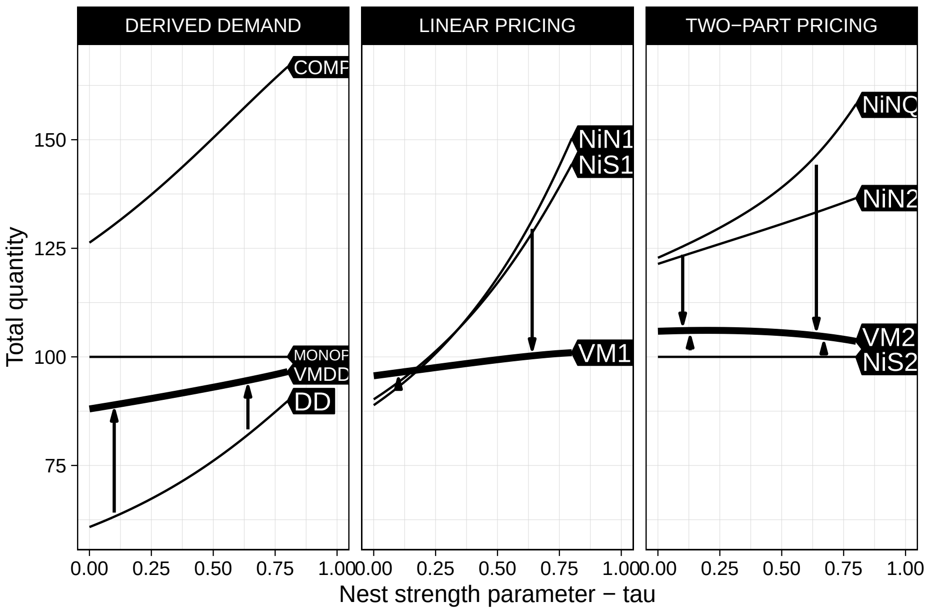
[13] As mentioned previously, the post-merger predictions of the Nash-in-Nash linear pricing model and Nash-in-Shapley linear pricing model are equal. Similarly, the post-merger predictions of the Nash-in-Nash two-part pricing, Nash-in-Shapley two-part pricing and Nash-in-Nash quantity setting models are also equal.



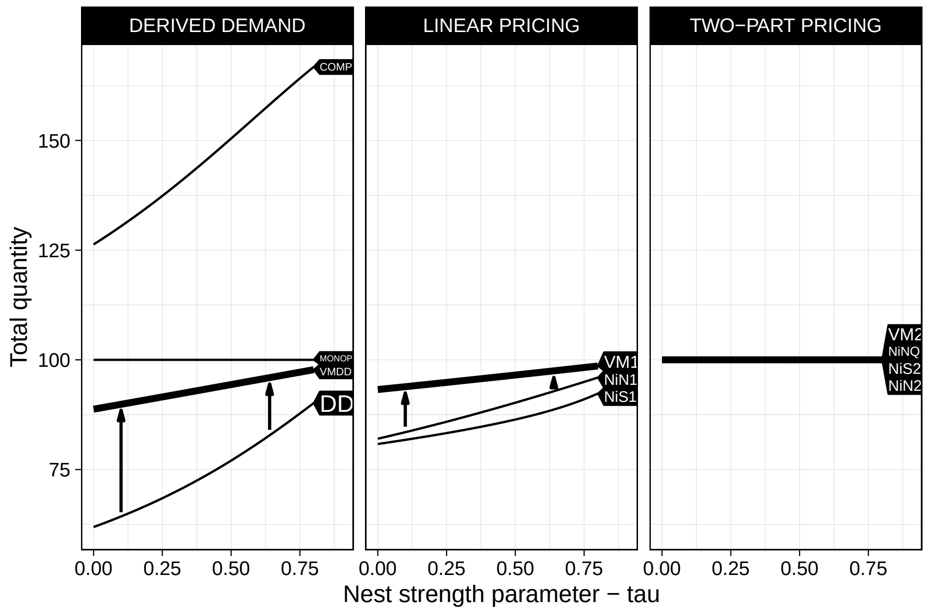
(a) Total quantity vs ae 1×2 setting



(b) Total quantity vs ae 2×1 setting



(c) Total quantity vs tau 1×2 setting



(d) Total quantity vs tau 2×1 setting

3.5.1 What do we know: Derived demand merger effects

In the first panel of figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d), we show the traditional derived demand model for a vertical relationship. We also show the perfect competition and monopoly cases as benchmark models. Along the DD locus, the upstream firm makes a take-it-or-leave-it linear wholesale price offer to the downstream firm. This contributes to the downstream firm's total marginal cost which then marks it up again. As a result, output is far below the monopoly output.

Along the VMDD locus, we have the post-vertical-merger world. The upstream firm still makes a take-it-or-leave-it offer to the unintegrated firm, but the downstream equilibrium is no longer symmetric. The captive downstream manufacturer gains share because the perceived margin on sales of its product is larger than the margin on sales to its rival. As a result, it can decrease the price of its own product, which increases its output but reduces the sales of rivals. The substantial increase in the vertically integrated quantity outweighs the decrease in the rival's quantity, leading to an increase in total output.

The up arrows from the DD to VMDD loci show us what we already know: A vertical merger in a derived demand setting moves the market closer to monopoly, by raising total quantity, which closely tracks the change in consumer surplus. The derived demand model is then only able to find a pro-competitive merger, as the elimination of double marginalisation always outweighs the anti-competitive effect (raising rivals' cost in a 1×2 setting, and reducing rivals' revenue in a 2×1 setting).

Table 3.1: Derived demand summary: Figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d) panel 1

Model Description	**Assumptions**	1×2**: Results**	2×1**: Results**
DD: Derived Demand models generalise the old 'successive monopoly' models, by allowing a more general downstream game, e.g. Bertrand or Cournot.	Upstream firm(s) set wholesale linear prices, and downstream firm(s) play a noncooperative game, taking wholesale prices as given.	Pre-merger output is far below monopoly output, due to big double marginalisation.	Pre-merger output is far below monopoly output, due to big double marginalisation.
VMDD: The merged firm eliminates double marginalisation (EDM) and raises rivals' cost (RRC) or reduces rivals' revenue (RRR) to the unintegrated retailer.	The vertically integrated firm sets a wholesale linear price to the unintegrated firm, then plays a noncooperative game with the same in downstream (or upstream) market.	Post-merger output slightly below monopoly output. Vertical merger raises output because EDM > RRC.	Post-merger output slightly below monopoly output. Vertical merger raises output because EDM > RRR.

3.5.2 Linear pricing bargaining vertical merger

In the second panel of figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d), we introduce bargaining over linear prices. The VM1 locus shows the post-vertical-merger world for both the NiN1 and NiS1 models. The merged firm negotiates a wholesale price with the now-rival firm in a Nash bargaining setting. Subsequently, asymmetric retail prices are determined in a Nash equilibrium. Again, the pro-competitive elimination of double marginalisation increases output for the vertically integrated firm. The anti-competitive raising of rivals' cost (1×2 setting) or reducing of rivals' revenue (2×1 setting) reduces output for the rival. The relative magnitude of these effects determines the ultimate predicted effects of a vertical merger.

The linear pricing 1×2 setting is the only case where the competitiveness of a merger is dependent on the level of the substitutability parameter. For a low aggregate elasticity, we find that the post-merger total quantity (figure 3.3(a)) in the market decreases. When aggregate elasticity is low, the pre-merger wholesale prices are high. Despite the fact that this results in a significant elimination of double marginalisation post-merger, the vertically integrated firm also manages to significantly raise its rivals' cost. This latter effect outweighs the pro-competitive effect and thus yield a welfare-decreasing (anti-competitive) vertical merger. At high aggregate elasticity, pre-merger wholesale prices are significantly lower, so that the elimination of double marginalisation post-merger is smaller. However, it still outweighs the smaller increase in the rival's wholesale price, so that the model predicts a welfare-increasing (pro-competitive) vertical merger.

The 1×2 linear pricing setting results for the nest strength (figure 3.3(c)) correspond with those for aggregate elasticity. For a strong nest ($\tau \approx 1$), the model predicts an anti-competitive merger, while for a weak nest, it yields a pro-competitive merger. This corresponds with the prediction of aggregate elasticity, in that a vertical merger is more pro-competitive the lower cross-price elasticity is. The competitiveness is again dependent on whether the elimination of double marginalisation or the raising of rivals' cost effect is dominant.

The linear pricing 2×1 setting shows a pro-competitive merger for all levels of aggregate elasticity and nest strength. However, it does relate to the 1×2 case, in that a merger is more pro-competitive at a higher aggregate elasticity and a low nest strength.

Table 3.2: Linear pricing summary: Figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d) panel 2

Model Description	**Assumptions**	1×2**: Results**	2×1**: Results**
NiN1: One instrument (linear wholesale price) performs two tasks: determines the size of total profit and how profit is split. NiN1 and NiN2 are very close.	Upstream and downstream players bargain bilaterally over the linear wholesale price, taking other agreements as fixed. The threat point for one agreement is profits in the remaining existing agreements.	Output is above monopoly output for low aggregate elasticity and a strong nest strength.	Output is always below monopoly output in the range of parameters we consider. Not true in general.
NiS1: As above, only one instrument (linear wholesale price) performs two tasks, but the alternatives to agreement change.	Upstream and downstream players bargain bilaterally over a linear wholesale price, but expect prices to change if agreements are not reached. The threat point is determined by re-negotiating the remaining agreements.	Output is above monopoly output for low aggregate elasticity and a strong nest strength.	Output is always below monopoly output in the range of parameters we consider. Not true in general.
VM1: The merged firm eliminates double marginalisation.	The vertically integrated firm bargains over linear wholesale prices to the unintegrated firm.	In a case with low aggregate elasticity or with a strong nest strength, vertical mergers have anti-competitive effects (RRC>EDM). The first factor results in fewer lost sales to the "no purchase" alternative; the second makes it easier to capture lost sales from the unintegrated retailer. Integrated firm profit increases due to a better post-merger threat point.	Output is slightly below monopoly output in the range of parameters we consider and vertical mergers always have beneficial effects, as EDM>RRR. Neither is true in general. The integrated firm profit increases due to a better post-merger threat point.

3.5.3 Two-part pricing: Threat points matter

In the third panel of all the figures, we introduce two-part pricing bargaining. In this setting, we assume bargaining over wholesale prices and fees. Downstream firms subsequently set retail prices in Nash equilibrium. Each bargain is assumed to be reached on the basis of a Nash bargaining solution relative to either a NiN2-, NiNQ- or NiS2-specified threat point. For the NiN2 and NiS2 models, the wholesale price is set to maximise surplus from agreement and the fee is set so that profit over the threat point is split equally. For the NiNQ model, the quantity is set to maximise surplus from agreement, and the total price splits the profit over the threat point equally. The threat points for NiN and NiS are as described in section 3.3, in summary, NiS allows for renegotiation and NiN assumes continuation at the same wholesale price or quantity.

3.5.3.1 Merger effects and drivers in two-part pricing

The VM2 locus in the third panel of figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d) shows the NiN2, NiNQ and NiS2 models for a vertical merger. Along this locus, the merged firm negotiates a wholesale price and fee with the now-rival firm in a Nash bargaining setting. Subsequently, retail prices (now asymmetric) are determined in a Nash equilibrium. Again, the pro-competitive elimination of double marginalisation increases output for the vertically integrated firm. The anti-competitive raising of rivals' cost in a 1×2 setting and reducing rivals' revenue in a 2×1 setting, reduces output for the rival. The relative magnitude of these effects determines the ultimately predicted competitiveness of a vertical merger.

Moving away from joint profit maximising

A vertical merger usually means firms are able to achieve the joint profit maximising outcome. However, the NiS2 model in both industry structures, and NiN models in a 2×1 setting are already at joint profit maximisation pre-merger. Following a merger, it is only the NiS2 model in a 1×2 setting that moves away from joint profit maximisation towards a more competitive outcome.

Post-merger in a NiS2 1×2 model, the double margin of the vertically integrated firm is eliminated, but the pivotal player (formerly the upstream firm) also inherits the marginal cost from its vertically integrated downstream firm[14]. It now acts as if this, and not the transfer price (which is zero by assumption), is its true marginal cost. In contrast, the rival downstream firm's total marginal cost is partly determined by negotiation with the vertically integrated firm. In this negotiation, the vertically integrated firm cannot effect a commitment to raise retail prices to joint profit maximising prices, as we assume firms are prohibited from setting retail prices as part of their negotiations. Moreover, the vertically integrated firm cannot credibly commit to the price at the joint profit maximising level because of the change in its marginal cost. Consequently, the vertically integrated firm reduces its price and increases quantity (figures D.1(c) and D.1(a)).

A vertical merger in a two-part pricing setting shows the lowest post-merger wholesale prices of all the models allowing a vertical merger in the 1×2 case (figure 3.4(a)). Because the NiS2 pre-merger wholesale price is higher than the NiN counterparts, the raising rivals' cost effect for NiS is diminished. As such, we observe some of the smallest increases in wholesale price in figure D.2(b). Hence, the decrease in post-merger quantity for downstream firm 2 (figure D.1(b)) is also not as significant as for

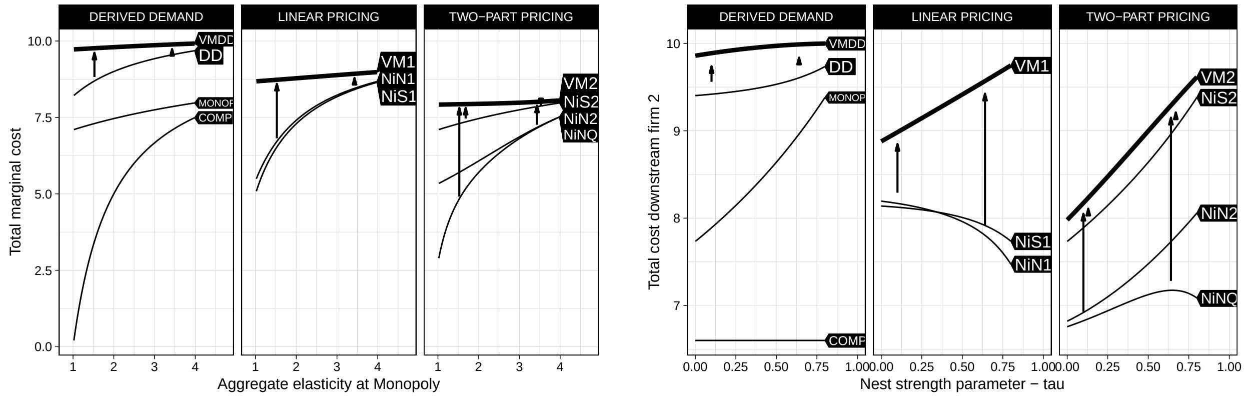
[14] Recall that we fixed the pre-merger marginal cost of the upstream firm to zero, and it was able to induce monopoly prices and quantities by setting the wholesale prices to both downstream firms.

Table 3.3: Two-part pricing summary: Figures 3.3(a), 3.3(b), 3.3(c) and 3.3(d) panel 3

Model Description	**Assumptions**	1×2: Results	2×1: Results
NiN2: The NiN assumption that the other agreement is fixed makes the parties bargain as if they do not know that the bargain they make in one deal has an effect on the profitability of the other.	Upstream and downstream players bargain bilaterally over two-part prices, taking other agreements as fixed. The threat point for one agreement is profits in the extant remaining agreements.	Output above monopoly output as NiN assumption makes it appear that the single upstream firm is bargaining against itself. Output is above monopoly output.	Pre-merger output equals monopoly output because the downstream monopoly retailer internalises upstream competition.
NiNQ: The NiN assumption that the other agreement is fixed makes the parties bargain as if they do not know that the bargain they make in one deal has an effect on the profitability of the other.	Upstream and downstream players bargain bilaterally over the fixed wholesale price and quantity, taking the other agreements as fixed. The threat point for one agreement is profits in the extant remaining agreements.	Output above monopoly output as NiN assumption makes it appear that the single upstream firm is bargaining against itself. Output is above monopoly output.	Pre-merger output equals monopoly output because the downstream monopoly retailer internalises upstream competition.
NiS2: Parties bargain as if they know that they will get a share of any improvement in profit in grand coalition (both agreements make). They are willing to, e.g., reduce the wholesale price if that leads to higher total profit.	Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements do not make. The threat point is determined by re-negotiating the remaining agreements.	Output equals monopoly profit.	Pre-merger output equals monopoly output because the downstream monopoly retailer internalises upstream competition.
VM2: The merged firm eliminates double marginalisation, favours its captive downstream retailer in 1×2 case, but not 1x2 cases.	Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements do not make. The threat points are determined recursively by profits in sets of agreements without current agreement.	Output above monopoly profit because of what Church (2008) calls 'inefficient contracting,' i.e., the increased margin on the integrated product due to EDM gives the integrated firm an incentive to increase its sales. NiN2: Vertical mergers have a big negative effect. NiNQ: Vertical mergers have a big negative effect. NiS2: Vertical mergers have a small positive effect.	Post-merger output equals monopoly output because the downstream monopoly retailer internalises upstream competition. Vertical mergers have no effect.

some other models. This combined with a greater increase in the quantity of the vertically integrated firm sees the total quantity increasing following a vertical merger in a NiS2 setting.

Because the pivotal player is downstream and it does not inherit a marginal cost post-merger, we do not see a merger effect on total quantity in the 2×1 setting. All three models remain at the monopoly equilibrium, so that there is no pro-competitive elimination of double marginalisation or anti-competitive reducing of rival's revenue in this setting.



(a) Total marginal cost of firm 2 vs ae 1×2 setting (b) Total marginal cost of firm 2 vs tau 1×2 setting

Figure 3.4: Total marginal cost of firm 2 in a 1×2 setting for two substitutability parameters

Moving towards joint profit maximising

The NiN two-part pricing models in a 1×2 setting are the only models in two-part pricing that show an anti-competitive merger. This is attributed to the pivotal player in the NiN models bargaining against themself, as discussed in detail in chapter 2 section 2.5.3.1. As a result of this, we observe the lowest total marginal costs (figure 3.4(a)) of all the models.

Following a vertical merger, the incentive of the upstream firm to internalise competition between its vertically integrated downstream firm and the rival firm is eliminated. However, it cannot reach the monopoly outcome as in the 2×1 case, for two reasons. Firstly, the pivotal player is now upstream, and thus it cannot impose monopoly retail prices because of the order in which profit maximisation takes place[15]. Secondly, the vertically integrated firm has an incentive to raise its rivals' cost, so that we observe a substantial percentage increase in the post-merger wholesale price.

The 1×2 two-part pricing setting is unique in that the specification of the nature of bargaining (NiN or NiS) completely predetermines the competitiveness of a vertical merger.

3.6 Vertical mergers and substitutability

In the previous section, we discussed merger effects across different objects of bargaining that were consistent in our simulations for aggregate elasticity and nest strength parameter. We now focus on the relationships with the parameters of substitutability. In the sections that follow, we continue to group the results according to the object of bargaining assumed. However, it is often also useful to refer to

[15] In the 2×1 case, the pivotal player is downstream and retail prices are determined after wholesale prices. It is thus possible to end up at the monopoly outcome post-merger as well.

benchmark models (monopoly and perfect competition); vertically separated models, i.e. models that do not allow a vertical merger (DD, NiN1, NiS1, NiN2, NiS2 and NiNQ); and vertically integrated models, i.e. models allowing a vertical merger (VMDD, VM1 and VM2).

3.6.1 Relationship with aggregate elasticity

As discussed in section 3.4, for the first calibration, an increase in the outside quantity increases aggregate elasticity. It also renders the demand for products effectively independent, thus causing a decrease in cross-price elasticity. We expect that as the cross-price elasticity decreases, the substitutability between inside goods to also decrease, which means competition is weakened, so that firms are able to increase prices and decrease quantities. These effects are observed in most vertically separated models (table E.1 and figures D.1(a) to D.1(d)).

The cross-price elasticity effect also carries through to the wholesale price (table E.2 columns 2 and 3 and figure D.2(a) and D.2(b)). For all models apart from perfect competition, the wholesale price is decreasing in aggregate elasticity[16]. As cross-price elasticity decreases, the inside goods become less substitutable, so that the margin that downstream firms are able to charge increases. It is thus the same mechanism that made downstream prices an increasing function of aggregate elasticity that makes wholesale prices (a part of the downstream firms' marginal cost) a decreasing function.

As discussed, we observe the anticipated merger effects for respective prices, quantities and wholesale price. A vertical merger in derived demand, linear pricing or two-part NiN bargaining increases quantity and decreases price for the vertically integrated firm, while it decreases quantity and increases prices for the now-rival firm. Subsequently, we explore how these prices and quantities react to a changing aggregate elasticity.

Following a merger in a derived demand setting, price and quantity for the now-rival firm shows almost no response to a changing aggregate elasticity (figures D.1(d) and D.1(b)). We explain this by two counteracting effects: In the first stage, the merged firm sets the wholesale price to downstream firm 2. As already discussed, the merged firm manages to increase its rival's cost (figure D.2(b)). Total marginal cost (marginal cost plus wholesale price) for firm 2 is increased to borderline monopoly equilibrium initial price to which the system is calibrated (10 in these models), as shown in figure 3.4(a). In the second stage, retail prices are determined in Nash equilibrium. Firm 2 wishes to respond to the weakened competition as aggregate elasticity increases, but the high marginal cost forces them to raise their price and decrease quantity, so that we observe no response to changing elasticity.

With a merger in a linear pricing setting, there is again almost no response of the post-merger quantity and prices of downstream firm 2 to a changing aggregate elasticity. However, we do see quantity increasing and price decreasing a bit more (figures D.1(b) and D.1(d)) than VMDD, resulting in a higher profit than in derived demand (figure D.3(d)). We ascribe this to the total marginal cost of firm 2 (figure 3.4(a)) being lower, since the firm is able to bargain for a lower wholesale cost.

In the two-part case, price is a decreasing function, and quantity an increasing function of aggregate elasticity (figures D.1(d) and D.1(b)). We ascribe this to the diminished raising rivals' cost effect in two-part pricing, so that the rival downstream firm's total cost in this scenario is the lowest for all the models of vertical integration (figure 3.4(a)). Downstream firm 2 is now able to respond to its increased

[16] In the perfect competition model, wholesale prices are shown to be zero as it is expected that the upstream firm sells at marginal cost (which we assume to be zero).

own-price elasticity and reduce its price towards monopoly level, explaining the decreasing relationship with aggregate elasticity.

The effect of firm 2's reaction to its increased own-price elasticity is clearly observable in figure D.3(d). By decreasing its price as aggregate elasticity increases, it manages to posit a greater percentage increase in quantity. In doing so, it secures an almost stable profit across all levels of aggregate elasticity and sees a smaller decrease in profit at high levels of aggregate elasticity (figure D.3(d)).

3.6.2 Relationship with nest strength

In contrast to the previous subsection, cross-price elasticity is now increasing in our substitutability parameter. As discussed in section 3.4, an increase in the nest strength while keeping aggregate elasticity constant, causes an increase in cross-price elasticity. We therefore expect prices to be a decreasing function and quantities to be an increasing function of the nest strength. These effects are observed in most vertically separated models (table E.4 and figures D.7(a) to D.7(d))[17].

As the nest strength and substitutability increase, we expect the upstream firm to internalise more and more of the competition between the downstream firms. It does so by increasing the pre-merger wholesale price, as seen in figures D.8(a) and D.8(b). However, for the linear pricing panel, we see the opposite effect. Keeping the marginal cost of all firms and the substitutability of the inside goods with the outside good constant while increasing the nest strength and subsequently the cross-price elasticity causes the wholesale price for the NiN1 and NiS1 models to decrease.

Albeit in the upstream firm's best interest to internalise the competition and increase the wholesale price, it also faces a conflicting goal — increasing its share of the industry profit. In the linear pricing case, the upstream firm has only one instrument with which to increase industry profits, but also to take its share, as discussed in chapter 2 section 2.5. For the linear pricing case, the incentive to decrease the wholesale price outweighs the incentive to raise it so that we see the wholesale price being a decreasing function of the nest strength (figures D.8(a) and D.8(b)). Although it seems that the upstream firm is not exercising its increased bargaining power as the nest strength increases, we do see that it is indeed capitalising on this in that its profits in the linear pricing panel in figure D.9(c) is increasing.

Following a vertical merger, we observe the wholesale price as an increasing function of the nest strength (figure D.8(b)). This is because post-merger, downstream firm 2 has to bargain with its rival over the wholesale price of the input necessary for its production. The vertically integrated firm no longer has the incentive to internalise the competition between its vertically integrated downstream firm and rival. Furthermore, the bargaining power of the rival firm is diminished as the nest strength increases, and the goods become perfectly substitutable. These effects lead to raising of rivals' cost which increases as the nest strength increases (figure D.8(b)).

As a result of the raising of rivals' cost, the post-merger price of downstream firm 2 significantly increases, and the quantity decreases (figures D.7(b) and D.7(d)). We also observe the post-merger quantity being a decreasing function of the nest strength as its bargaining power decreases. Juxtaposed to this, the vertically integrated firm can significantly decrease its price post-merger as a result of the elimination of double marginalisation (figure D.7(c)). We also observe its price being an increasing

[17] We again see that the monopoly and NiS2 models show no relation to a changing nest strength. Recall that the system is calibrated to the monopoly outcome so that this explains the non-reaction.

function of the nest strength, as it is able to increase its price towards the monopoly price as it gains an increasing share of the market, by raising its rivals' cost.

3.7 Conclusion

In this chapter, we recognize that vertically related markets represent a complex system with many moving parts. This system becomes even more complex when we allow for a vertical merger that alters the incentives to trade. We attempt to unpack this complexity and identify the drivers of competitive effects by simulating vertical mergers. From these simulations, we are able to show how the assumptions that we make about the nature and object of vertical contracting affect the predictions.

In a 1×2 industry setting, we show that refining the specification of threat points that characterise bargaining has little effect when firms bargain over linear wholesale prices. However, the differences in Nash-in-Nash and Nash-in-Shapley bargaining are highlighted in a two-part pricing bargaining scenario. We show that Nash-in-Nash is more competitive, while Nash-in-Shapley follows the monopoly outcome. This predetermines the predicted competitiveness of a vertical merger in that Nash-in-Nash is always welfare reducing while Nash-in-Shapley is welfare increasing for the range of aggregate elasticity and nest strength that we consider.

In a 2×1 setting, the predicted merger effects are smaller. Specifically in a two-part pricing setting, we do not observe any merger effects on total quantity. However, the 2×1 still gives us an important distinction between Nash-in-Nash and Nash-in-Shapley bargaining, which is the consistency of the Nash-in-Shapley model predictions across different industry structures. This property makes it an appealing model for vertically related markets.

The results presented in this chapter show that the relationships between elasticities, prices and quantities are important in the simulation of vertical mergers. We identify costs as a major driver of the outcomes in vertically related markets. Because vertical mergers change the cost structures in the market through the elimination of double marginalisation and raising of rivals' cost, we find that these relationships can change after a vertical merger. This is most clear in a linear pricing setting, where the competitiveness of a vertical merger depends on substitutability. A merger is pro-competitive when the outside good is a good substitute for the inside goods and when the inside goods are bad substitutes for each other. It is anti-competitive when the outside good is a bad substitute for the inside goods and when the inside goods are good substitutes.

The chapter illustrates that specifying the industry structure and object of contracts determines outcomes in the pre-merger equilibrium and that this can predetermine outcomes in the post-merger world. This systematic and direct comparison of the magnitude of different vertical merger models' predictions provides a novel contribution to our understanding of vertical mergers.

Chapter 4

A Simulation Tool for Vertical Merger Screening

4.1 Introduction

The toolkit of economists for screening the likely effects of vertical mergers comprises few instruments tailored to deal with the additional complexity of vertically related markets. This chapter assesses said toolkit and investigates how these tools compare with regard to data requirements as well as their ability to predict the effects of vertical mergers ex ante.

We narrow the focus to those tools that aim to quantify the unilateral effects arising from mergers. Werden and Froeb (2007) define these as the effects that arise from internalising competition between merging firms. In the case of vertical mergers, this relates to diminished competition between one merging firm and rivals that trade with, or could trade with, the other merging firm (Department of Justice and Federal Trade Commission, 2020a).

An oft-used tool employed to quantify unilateral effects is incentive scoring methods, which quantify the effect of firms taking the "cost of competing" into account post-merger. This is done with simple and intuitive formulae that make use of data that is argued, is readily available (Rogerson, 2020). As such, a range of measures has been developed in the literature in the incentive scoring methods category. These measures differ according to the product for which the upward pricing pressure is calculated.

Another tool at the disposal of economists attempting to quantify the unilateral effects of vertical mergers is merger simulation. This approach aims to provide numerical predictions of price and quantity changes by calibrating an economic model. It requires a system of equations used to make inference about the pre-merger equilibrium and make predictions about the post-merger equilibrium. However, merger simulation is less often employed to screen the likely effects of vertical mergers, as a result of the data and computational intensity associated with the method.

In this chapter, we introduce a vertical merger simulator to address the accessibility of merger simulation as a screening method. The simulator can be calibrated with just six control variables and calculates pre- and post-merger equilibria for six different models. We argue, in line with chapter 2 and 3, that the strength of the simulator is that it is a comparison tool. As in previous chapters, the same parameters are used to calibrate all of the models. Thus, as a screening tool, it is not only able to compare pre- and post-merger equilibria but also to directly compare these equilibria for a range of different models.

There is an argument frequently made in the existing literature that incentive scoring measures generally require less data than merger simulations. However, directly comparing data requirements is difficult since the methods differ substantially in what is necessary for their calculation and its equilibria. Despite this, we find no conclusive evidence that the vertical merger simulator requires more data than what would be required to calculate incentive scoring methods. Moreover, the simulator has the added benefit in that it provides us with a full equilibrium prediction, whereas incentive scoring gives only a partial equilibrium.

The aim of this chapter is not to discredit any screening tools or advocate for the use of one tool over an other. The sole aim is to add to the toolkit available to practitioners, especially given that the competition authorities prefer multiple limited-information tools (Boshoff, 2011). In this regard, the vertical merger simulator may well be a useful addition to the toolkit. The onus is then on practitioners to assess the merits of the case before them and choose the appropriate tools.

This chapter first presents an overview of the literature pertaining to the quantitative assessment of vertical merger effects. This overview provides the context for the discussion on the incentive scoring method for vertical merger screening reffered to in section 4.3. In this section, we summarise the different upward pricing pressure formulae that comprise the approach. Also investigated is how a practitioner may go about estimating the key variables necessary for its calculation. Then in section 4.4, we introduce the vertical merger simulator and present three illustrative examples. Finally, in section 4.5, we compare the incentive scoring method and the vertical merger simulator in terms of their data requirements and their predictions.

4.2 Literature overview

Recently, merger review have shifted towards a greater emphasis on the unilateral effects of mergers (Baltzopoulos et al., 2015). Slade (2020) identifies two techniques that aim to quantify and predict such effects — upward pricing pressure tools (universally categorised as incentive scoring measures) and merger simulations. Valletti and Zenger (2021) eloquently summarise the rationale for price pressure tools as quantifying the effect of merging parties taking the 'cost of competing' into account post-merger. With this approach, the unilateral effects of a merger are calculated with formulae using inter alia diversion of sales and profit margins (Miller and Sheu, 2021). Alternatively, merger simulation aims to provide numerical predictions of price and quantity changes (Budzinski and Ruhmer, 2009) by calibrating specific economic models (Valletti and Zenger, 2021). It requires a system of equations that is used to make inference about the pre-merger equilibrium and make predictions about the post-merger equilibrium (Miller and Sheu, 2021).

Slade (2020) provides an excellent discussion on these vertical merger screening tools and how they are adapted from their horizontal merger screening counterparts. The remainder of the literature overview similarly provides a discussion on how the literature has adapted upward pricing pressure tools and merger simulations to analyse vertical merger.

In an adaptation of the horizontal gross upward pricing pressure index (GUPPI) measure, Moresi and Salop (2013) developed a range of vertical GUPPIs specifically tailored to address the intricacies of vertical markets. The authors developed a different GUPPI for every player affected by a proposed vertical merger: the merged upstream firm (vGUPPIu), the merged downstream firm (vGUPPId) and the rival downstream firm (vGUPPIr). The biggest drawback of the measures developed by Moresi and Salop (2013) is that they are based on a derived demand model where the upstream firm(s) has all the bargaining power and makes a take-it-or-leave-it offer to the downstream firm(s).

Taking into account that the vertical contracting between upstream and downstream firms is more realistically modelled by bargaining models, Rogerson (2020) developed the bargaining analogue of the vGUPPI, called the bargaining leverage over rivals (the BLR) measure. Because a vertical merger improves the outside option (the alternative if contract negotiations break down) of the merging firm, its bargaining leverage over its rival is increased. The merged firm can then increase the price of inputs charged to the rival downstream firm — the effect of which is quantified by the BLR measure.

Slade (2020) notes two shortcomings of vertical GUPPIs/BLR measures as their extensive data requirements and that they are a measure of only a partial equilibrium. The first shortcoming does not disqualify the use of these measures to predict merger effects, since most analyses require sufficient data. However, the second shortcoming may require serious consideration. As acknowledged by Rogerson (2020), the formulae used to calculate the BLR measure ignore the effect that the elimination of double marginalisation will have on the equilibrium as well as any feedback effects between the two competitive effects. Therefore, this measure is not able to provide a final answer with respect to the net effect on consumers following a vertical merger. Moreover, Das Varma and De Stefano (2020) prove that failing to take the link between the two effects into account leads to unreliable predicted effects.

The use of vertical GUPPIs/BLR measures as screening tools for vertical mergers is not universally accepted. While Shapiro (2021) states that price pressure tools can be highly informative and practical (despite showing only a partial equilibrium), Slade (2020) advises against the use of these measures, citing complex data requirements and high probability of type 1 and 2, errors as also evidenced by Domnenko and Sibley (2020). Moresi and Zenger (2017) also note that these methods are difficult to apply in practice, due to the diversion ratio (an integral input to the calculation of these measures) being unknown. Perhaps indicative of the caution with regard to these measures, the Department of Justice makes no mention in their Vertical Merger Guidelines (Department of Justice and Federal Trade Commission, 2020b) of these measures as a way to evaluate or screen vertical mergers.

The Vertical Merger Guidelines do make mention of merger simulations (Department of Justice and Federal Trade Commission, 2020b). However, they do so very cautiously: “The Agencies do not treat merger simulation evidence as conclusive in itself, and they place more weight on whether merger simulations using reasonable models consistently predict substantial price increases than on the precise prediction of any single simulation."

Unlike incentive scoring methods, the primary goal of vertical merger simulations is to determine the net welfare effect emanating from the tension between the pro-competitive elimination of double marginalisation and anti-competitive foreclosure (Slade, 2020). Budzinski and Ruhmer (2009) ascribe the recent increase in the adoption of merger simulations to four reasons: first, the evolution of pro- and anti-competitive effects calling for an overall assessment of welfare effects; second, the increased availability of market data that; third, allows the use of computational techniques for increasingly complex simulations; fourth, the receptiveness of competition policy to innovative economic assessment instruments.

Another type of analysis, which has not gained widespread acceptance, has its foundation in the structure-conduct-performance paradigm. For concentration indices, market share data is used to calculate concentration ratios, such as the Hirschman-Herfindahl index (HHI). In this vein, Gans (2007) derived the vertical HHI, which reflects the degree of distortion following a vertical merger that emanates both from the horizontal concentration and the degree of vertical integration in the market. Gans’s approach takes cognisance of the evolution of the theory of vertical contracting[1] and thus considers bilateral bargaining between the upstream- and downstream firms.

Gans (2007) argues that the vertical HHI can provide a baseline for the level of concentration in the entire vertical chain against which vertical mergers can be evaluated. However, Moresi and Salop (2013) state that this measure suffers from the same shortcoming as its horizontal counterpart: it assumes homogenous goods, so that its application in mergers with differentiated products is inappropriate.

In the remainder of this chapter, we narrow the focus to incentive scoring and merger simulation methods. In the following section, we investigate the different upward pricing pressure formulae that comprise the incentive scoring approach. We also investigate how a practitioner may go about estimating key variables necessary for its calculation. This allows us to compare the data requirements of this method with those of the vertical merger simulator that we introduce.

4.3 Incentive scoring for vertical merger screening

As described in the literature review, the rationale behind the incentive scoring method is to quantify the effect of merging parties taking the 'cost of competing' into account post-merger (Valletti and Zenger, 2021). In the subsections below, we provide an overview of different upward pricing pressure indices and how they are calculated. Specifically, the indices differ based on the players involved and the good for which the upward pricing pressure is measured. Subsequently, we assess the typical data, estimation and assumptions required to calculate these indices.

4.3.1 Moresi and Salop's vertical upward pricing pressure indices

Moresi and Salop (2013) developed a range of indices to gauge the incentive of a merging firm to foreclose its rival. Each index corresponds to a different player affected by a proposed vertical merger and their incentive to raise the price of the good they produce following the merger, i.e. the merged upstream firm (vGUPPIu), the rival downstream firm (vGUPPIr) and the merged downstream firm (vGUPPId). The authors then elaborates on these indices and provide variants for each one, which take into account possible input substitution following a vertical merger. The methodology and data requirements of these measures are discussed below.

vGUPPIu: The upstream merging partner's price

The first index measures the incentive of the merging firm to raise the input price to each targeted manufacturer,[2] and is calculated as:

$$vGUPPIu = DR_{UD} \times M_D \times P_D/W_R \tag{4.1}$$

where DR_{UD} gives the vertical diversion ratio, i.e. the volume of output gained by the integrated downstream firm as a fraction of the volume of input sales to the rival firm lost by the integrated upstream firm. M_D gives the integrated downstream firm's profit margin, calculated as $M_D = (P_D - C_D)/P_D$, with P_D being the price and C_D the cost. Finally, W_R indicates the wholesale price to the targeted rival downstream firm. W_R is calculated as the rival firm's total payments to the upstream firm, divided by the total quantity of output that uses the upstream firm's input.

[1] See chapter 2, section 2.2 for a discussion on the evolution of the models of vertically related markets.
[2] Note that the vGUPPIu is calculated for each downstream competitor separately.

vGUPPIr: The rival firm's downstream price

The second index calculates the incentive of the rival firm to increase its downstream price in response to an increase in its marginal cost from the increased input price. It is calculated as:

$$vGUPPIr = vGUPPIu \times PTR_U \times W_R/P_R \tag{4.2}$$

where PTR_U gives the cost pass-through rate of the upstream firm and P_R the downstream price of the rival firm.

vGUPPId1: The downstream merging partner's price

The $vGUPPId1$ index measures the incentive of the downstream merged firm to increase its price in order to urge customers to substitute towards the rival's product. This increases the input sales of the integrated upstream firm to the rival downstream firm and is calculated as:

$$vGUPPId1 = DR_{DU} \times M_U \times W_U/P_D \tag{4.3}$$

with DR_{DU} denoting the vertical diversion ratio from the integrated downstream firm to the upstream firm, M_U the integrated upstream firm's average profit margin across all customers excluding the integrated downstream firm, and P_D the integrated downstream price.

vGUPPId2: Accounting for elimination of double marginalisation

Vertical mergers lead to the elimination of double marginalisation. This would reduce the marginal cost of the integrated downstream firm and thus decrease the vGUPPId. To account for this, Moresi and Salop (2013) calculated an adjusted vGUPPId as:

$$vGUPPId2 = vGUPPId1 - M_{UD} \times W_D/P_D \tag{4.4}$$

with M_{UD} and W_D giving the margin and price of the upstream firm on input sales to the integrated downstream firm.

vGUPPI's accounting for input substitution

Finally, Moresi and Salop (2013) also accounted for downstream firms being able to substitute away from the input of the integrated upstream firm. Each index is appropriately adjusted as follows:

$$vGUPPIu^* = \frac{DR_{RD} \times M_D \times P_D/W_R}{1 + M_R \times E_{SR}/E_P} \tag{4.5}$$

$$vGUPPIr^* = vGUPPIu^* \times PTR_U \times W_R/P_R \times S_{UR}^{post}/S_{UR} \tag{4.6}$$

$$vGUPPId3 = vGUPPId2 - E_{SD} \times (M_{UD})^2 \times W_D/P_D \tag{4.7}$$

where E_{SR} measures the extent to which the rival downstream firm can substitute away from the integrated upstream firm's input, and E_P is the elasticity of the rival downstream price with regard to an increase in the input price. S_{UR}^{post}/S_{UR} is the fraction of the integrated upstream firm's share of the rival's total purchases of the relevant input pre- and post-merger. E_{SD} denotes the pre-merger elasticity of the integrating upstream firm's share of the integrating downstream firm's input purchase with regard to the input price.

4.3.2 Rogerson's bargaining vertical UPP

Rogerson (2020) calculated the bargaining analogue of Moresi and Salop's vGUPPI. As with the previous indices, a vertical merger changes the incentives of the upstream firm, as it now takes the profit of its integrated downstream firm into account (Rogerson, 2020). However, Rogerson's index quantifies this, by focussing on the ability of the upstream firm to raise input prices as a result of how the vertical merger altered the threat point of the upstream firm in the bargaining game. The resulting vGUPPI is called the bargaining leverage over rivals (BLR) effect, and is calculated as:

$$vGUPPI_{BLR} = (1 - \delta) \times v \times d \times \pi \tag{4.8}$$

where δ is the Nash bargaining strength of the upstream firm; v is the diversion rate that measures the share of customers that shifts from the rival firm to the integrated downstream firm; d is the departure rate, which gives the share of the rival firm's customers that departs if input foreclosure takes place; and π is the profit margin of the integrated downstream firm.

4.3.3 Data, estimation and assumptions requirements

One of the goals of this chapter is to provide a comparison of the data requirements and predictions of different vertical merger screening tools. Therefore, in this section we discuss how the eight vGUPPIs discussed above may be employed by a practitioner attempting to ascertain the likely effects of a vertical merger. We discuss the general data, necessary estimations and assumptions required to employ these measures.

Generally, the vGUPPIs of Moresi and Salop (2013) and Rogerson (2020) require data on the marginal costs and prices of the upstream and downstream firms in order to calculate profit margins. A possible benefit of employing these measures is that this data is required only for the merging parties. It is only when estimating the $vGUPPIr$ or $vGUPPIr^*$ that this data for the rival firm(s) is also required.

In addition to these data requirements, estimates are needed for diversion ratios, cost pass-through rates and elasticities. Each of these are discussed in the subsections below.

4.3.3.1 Diversion and departure ratios

In a vertically related market, diversion ratios measure the share of the upstream firm's volume which is lost when raising the input price to a rival, that is gained by its vertically integrated downstream firm (Slade, 2020). While Moresi and Salop (2013) assume values for the different diversion ratios in their examples illustrating how to employ the vGUPPIs, it may not be as simple in practice. This data may not always be available, and a number of methods can be employed to estimate diversion ratios.

Rogerson (2014) argues that it is reasonable to make the proportional switching assumption about the diversion ratios: consumers divert to other downstream firms in proportion to their market shares. This will require data on the upstream firm's as well as the rival firms' market shares. However, in the Comcast/NBCU vertical merger case of 2011 (Commission, 2011), Murphy (2010) argued against this approach, instead proposing a method based on the survey data of customers leaving a downstream service provider. While helpful in this particular case, such data may not be readily available for many other cases, and estimation may be required.

Rossi et al. (2019) compared two approaches in estimating diversion ratios in hospital mergers. For the first, data on historical market shares were employed to estimate a measure of closeness of the competition. The second approach used econometric demand estimation in order to uncover the diversion ratio. However, this approach was more data and resource intensive[3]. Ultimately, Rossi et al. (2019) found that these different approaches yielded largely consistent estimates of the diversion ratio.

Moresi and Zenger (2017) argue that the estimation of individual diversion ratios requires knowledge of the aggregate diversion ratios in the market. The authors warn that ad hoc assumptions about the aggregate diversion ratio can severely bias the predicted price effects of a merger. Therefore, they explored the relationship between aggregate diversion ratios and the price elasticity of market demand to yield sensible estimations of the aggregate and individual diversion ratio. Moresi and Zenger (2017) show analytically that the aggregate diversion ratio can be estimated with only market shares, profit margins and elasticity of demand — data, they argue, that is routinely requested early on in merger proceedings.

4.3.3.2 Cost pass-through rates

Slade (2020) notes that one of the shortcomings of UPP indices for horizontal merger cases is their inability to incorporate cost pass-through which depends on higher order properties of demand. While the vertical analogues overcome this disadvantage, it comes at the cost of greater data requirements.

To calculate the merger screening measures of either Moresi and Salop (2013) or Rogerson (2020), an element of cost pass-through is necessary. For the vGUPPI of Moresi and Salop (2013), the rate at which upstream cost increases are passed through to input costs is used to calculate $vGUPPIr$ and $vGUPPIr^*$. For the $vGUPPI_{BLR}$ of Rogerson (2020), the share of the cost increase to serve the rival downstream firm that is passed through in the bargaining game is necessary.

The advantage of using Rogerson's measure is that the pass-through in a Nash bargaining game is simply equal to the share of $(1-\theta)$, with θ being the bargaining strength. Thus it is reliant on the assumptions made about, or estimation of the bargaining strength and is not related to the specific functional form or curvature of a demand function.

For the incentive scoring measures in Moresi and Salop (2013), however, an estimation of the cost pass-through rate is required, which depend on the specification of the demand function. In their examples, Moresi and Salop (2013) use a cost pass-through rate of 50%, in line with the default when demand is linear (Baltzopoulos et al., 2015). However, Froeb et al. (2005) show that more convex demand specifications lead to higher pass-through rates, which can vary substantially[4].

4.3.3.3 Elasticities

Two of the vGUPPIs by Moresi and Salop (2013), $vGUPPIu^*$ and $vGUPPId3$, require data on elasticities for their calculation. The authors discuss that these elasticities may be inferred from other variables, given profit maximisation assumptions. Specifically, this depends on the availability of an estimate for the market elasticity, E. If an estimate for E is available and there is an estimate for the input price pass-

[3]For example, Rossi et al. (2019) collected data on *inter alia* the number of hospitals, sites and patients; the distance patients were from the site of treatment; age; severity of conditions; income level; and area.

[4]For example, Kim and Cotterill (2008) determine the cost pass-through rate in the processed cheese market in the USA (a differentiated product market). The authors estimated a mixed logit demand model with panel data on volumes, prices, product characteristics and consumer demographic variables. They found that under Nash-Bertrand price competition, pass-through rates range between 73% and 103%.

through elasticity, E_P, the values for E_{SR} (the elasticity of the upstream firm's share of the rival firm's total volume of inputs with respect to an increase in the price) and E_{SD} (the elasticity of the upstream firm's share of the integrating downstream firm's total input purchase with regards to the input price) can be calculated with:

$$E_{SR} = E_{SD} = 1/M_U - E \times E_P \tag{4.9}$$

If an estimate for E is not available, it can be assumed to be 1, and if an estimate for E_P is not available, it can be approximated with:

$$E_P = PTR_R \times W_R/P_R \tag{4.10}$$

In the empirical literature on elasticities, Clements (2008) provides a review of studies estimating price elasticities. The author found that elasticities for branded products (i.e. those products which are more narrowly defined) differed fundamentally from broader products (i.e. broad aggregates of products). The author also establishes an empirical regularity for price elasticities of demand for such broad groups. Through an utility-maximising theory of consumer demand under the conditions of preference independence, the author found elasticities of broader products to be scattered around -0.5. Such a regularity can hence be employed for the calculation of vGUPPIs where there is limited data on the price sensitivity of goods.

4.4 A simulator for vertical merger screening

Incentive scoring measures are relatively simple to implement and can provide highly informative results (Shapiro, 2021). However, their partial equilibrium is a drawback and suggests a role for merger simulations. Merger simulations, on the other hand, are often avoided due to data and computational requirements.

To improve the accessibility of merger simulations, we developed a tool for performing vertical merger simulations based on the models and numerical simulations presented in chapters 2 and 3. It can calculate six different models, each characterised by its assumption regarding vertical contracting, as explained in chapters 2 and 3. This section serves as an introduction to its use[5].

At its core, the vertical merger simulator is a comparison tool. Its interface allows a user to compare pre- and post-merger predictions in derived demand, linear pricing or two-part pricing settings. On the landing page, shown in figure 4.1, a panel on the left-hand side (highlighted in a red block marked A) hosts three tabs: 'Inputs', 'Tables' and 'Graphs' tabs. Selecting the 'Inputs'-tab prompts the Inputs dialogue box (block B in figure 4.1) to display.

[5]The aim is to host the vertical merger simulator online, so that practitioners and academics may have access to it.

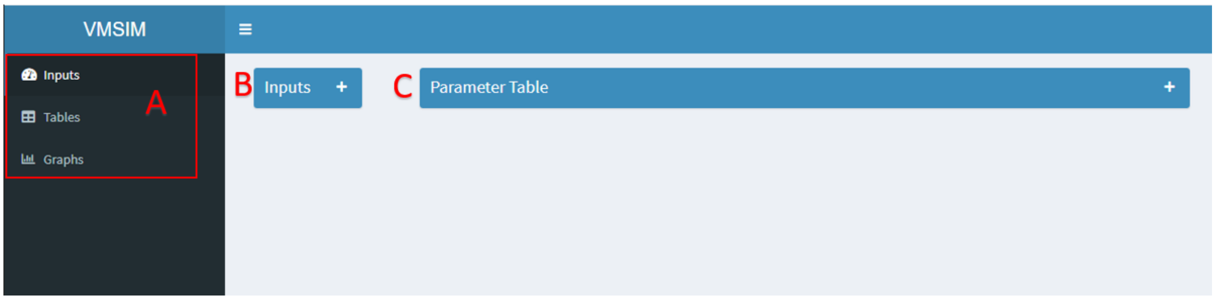


Figure 4.1: Landing page of the vertical merger simulator

On the Inputs dialogue box, a user is able to switch the industry structure from the default 1×2 to a 2×1 structure,[6] as shown in figure 4.2. Thereafter, a user may choose one of six control variables over which to simulate. After a control variable has been selected, the interface updates to display the relevant slider input for choosing the range over which to simulate. The choice of control variables and ranges[7] are:

- Inside/Outside quantity balance (out_{bal}); range: $0.1 \rightarrow 2.1$
- Price balance (p_{bal}); range: $0.1 \rightarrow 0.9$
- Quantity balance (q_{bal}); range: $0.1 \rightarrow 0.9$
- Marginal cost balance (mc_{bal}); range: $0 \rightarrow 1$
- Logit nest parameter (τ); range: $0.1 \rightarrow 0.8$
- Aggregate elasticity (ae); range: $-5.1 \rightarrow -1.1$

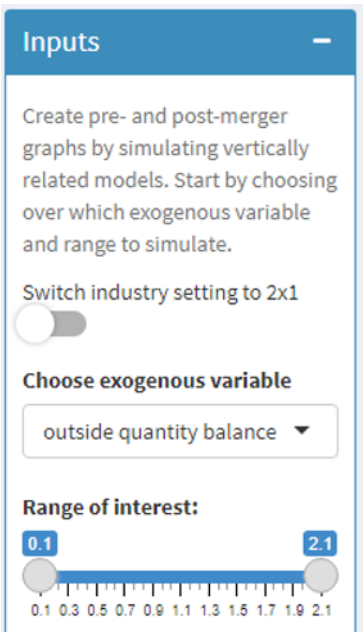


Figure 4.2: Choosing industry structure and exogenous variable range for simulation

[6] As done previously, we focus again on two simple industries: the 1×2 and 2×1 structures (as explained in chapter 2, section 2.3 and chapter 3, section 3.3).

[7] These ranges are chosen so as to increase the applicability of the simulator to various situations.

Users can choose any subset of the range over which to simulate models. Subsequently, the user has to specify the levels of the remaining five control variables, above. Figure 4.3 shows the remaining five control variables if the Inside/Outside quantity balance is chosen as the variable over which to simulate. Note that this list updates if a different control variable is chosen as the exogenous variable.

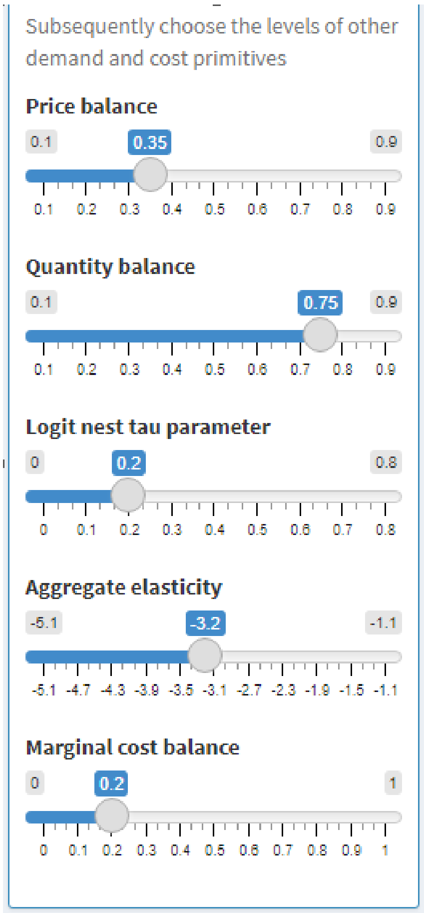


Figure 4.3: Remaining inputs on the Inputs dialogue box for Inside/Outside quantity balance as exogenous variable

Together, the six control variables (one simulation control variable and five remaining level control variables) are used to determine the parameters that calibrate the system of models. As in previous chapters, specific units on prices will not change the results of our calculations, so that we take the quantity-weighted average price to be $\bar{p} = 1$. The units on quantity similarly will not matter, so we may set the total quantity of inside products arbitrarily to $q_{tot} = 100$. These conventions allow the four ‘balance’ control variables to be used to calculate firm-specific initial parameters.

We use q_{bal} to calculate initial quantities $q_1 = q_{tot}q_{bal}$ and $q_2 = q_{tot}(1 - q_{bal})$. Initial prices are calculated with p_{bal} as $p_1 = \frac{p_{bal}(q_1+q_2)}{q_1 p_{bal}+q_2(1-p_{bal})}\bar{p}$ and $p_2 = \frac{(1-p_{bal})(q_1+q_2)}{q_1 p_{bal}+q_2(1-p_{bal})}\bar{p}$. The unobserved outside quantity q_0 is specified as a multiple of the total inside quantity $q_0 = q_{tot}out_{bal}$. Finally, total marginal costs are inferred from monopoly pricing, but mc_{bal} determines the ratio of how it is split between upstream and downstream firms. In the 1×2 case, we assume a common upstream marginal cost mc_A and two downstream marginal costs mc_1 and mc_2, $mc_{tot1} = mc_A + mc_1$ and $mc_{tot2} = mc_A + mc_2$

with $mc_A(q_1 + q_2)/(q_1 mc_1 + q_2 mc_2) = mc_{bal}/(1 - mc_{bal})$, i.e.,

$$mc_A = mc_{bal} \frac{q_1 mc_{tot1} + q_2 mc_{tot2}}{q_1 + q_2}$$
$$mc_1 = (1 - mc_{bal}) \frac{q_1 mc_{tot1} + q_2 mc_{tot2}}{q_1 + q_2} + (mc_{tot1} - mc_{tot2}) \frac{q_2}{q_1 + q_2}$$
$$mc_2 = (1 - mc_{bal}) \frac{q_1 mc_{tot1} + q_2 mc_{tot2}}{q_1 + q_2} - (mc_{tot1} - mc_{tot2}) \frac{q_1}{q_1 + q_2}$$

In the 2×1 case, we assume upstream marginal costs mc_A and mc_B but a common downstream marginal cost mc_1, $mc_{tot1} = mc_A + mc_1$, $mc_{tot2} = mc_B + mc_1$ with $(q_1 mc_A + q_2 mc_2)/(q_1 + q_2)/mc_1 = mc_{bal}/(1 - mc_{bal})$, i.e.

$$mc_A = mc_{bal} \frac{q_1 mc_{tot1} + q_2 mc_{tot2}}{q_1 + q_2} + (mc_{tot1} - mc_{tot2}) \frac{q_2}{q_1 + q_2}$$
$$mc_B = mc_{bal} \frac{q_1 mc_{tot1} + q_2 mc_{tot2}}{q_1 + q_2} - (mc_{tot1} - mc_{tot2}) \frac{q_1}{q_1 + q_2}$$
$$mc_1 = (1 - mc_{bal}) \frac{q_1 mc_{tot1} + q_2 mc_{tot2}}{q_1 + q_2}$$

After the industry setting, control variable, simulation range and levels of the remaining control variables are selected, the initial parameters are calculated with the above formulae. This leads to a list of parameters: the initial prices, inside quantities, outside quantity, nest parameter, aggregate elasticity, scaling parameter and location parameters along with the conventions for $\bar{p} = 1$, $q_{tot} = 100$ and marginal costs inferred from monopoly pricing. The result are displayed in a parameters table on the 'Inputs'-page in the Parameters Table dialogue box (block C in figure 4.1).

The first row in the table corresponds with the parameter list for the minimum of the selected range of the exogenous variable. Each subsequent row shows the calculated parameter list for an incremental increase in the chosen control variable until the maximum in the specified range is reached. In figure 4.4, we show an example of a calculated parameters table.

Parameter Table

Show 10 entries | Search:

b2x1	pbal	qbal	qout	ae	tau	mcbal	p1	p2	q1	q2	qout1	qtot	lambda	tau1	eta1	eta2	ae1	mcA	mc1	mc2	e11	e12	e21	
false	0.35	0.75	0.86	-3.2	0.2	0.2	8.24	15.29	75	25	86	186	1.44	0.2	8.12	13.91	-3.2	1.37	3.74	10.79	-3.76	2.08	3.37	-
false	0.35	0.75	0.87	-3.2	0.2	0.2	8.24	15.29	75	25	87	187	1.45	0.2	8.1	13.88	-3.2	1.37	3.74	10.79	-3.75	2.06	3.33	-
false	0.35	0.75	0.88	-3.2	0.2	0.2	8.24	15.29	75	25	88	188	1.46	0.2	8.09	13.86	-3.2	1.38	3.74	10.79	-3.74	2.04	3.3	-
false	0.35	0.75	0.89	-3.2	0.2	0.2	8.24	15.29	75	25	89	189	1.47	0.2	8.07	13.83	-3.2	1.38	3.74	10.79	-3.73	2.02	3.27	-
false	0.35	0.75	0.9	-3.2	0.2	0.2	8.24	15.29	75	25	90	190	1.48	0.2	8.05	13.81	-3.2	1.37	3.74	10.79	-3.72	2.01	3.24	-
false	0.35	0.75	0.91	-3.2	0.2	0.2	8.24	15.29	75	25	91	191	1.49	0.2	8.03	13.78	-3.2	1.37	3.74	10.79	-3.7	1.99	3.21	-
false	0.35	0.75	0.92	-3.2	0.2	0.2	8.24	15.29	75	25	92	192	1.5	0.2	8.02	13.76	-3.2	1.37	3.74	10.79	-3.7	1.97	3.18	
false	0.35	0.75	0.93	-3.2	0.2	0.2	8.24	15.29	75	25	93	193	1.51	0.2	8	13.73	-3.2	1.38	3.74	10.79	-3.69	1.95	3.15	-
false	0.35	0.75	0.94	-3.2	0.2	0.2	8.24	15.29	75	25	94	194	1.51	0.2	7.98	13.71	-3.2	1.38	3.74	10.79	-3.68	1.93	3.12	-
false	0.35	0.75	0.95	-3.2	0.2	0.2	8.24	15.29	75	25	95	195	1.52	0.2	7.96	13.68	-3.2	1.38	3.74	10.79	-3.67	1.92	3.09	-

Showing 1 to 10 of 55 entries | Previous 1 2 3 4 5 6 Next

Figure 4.4: A calculated parameters table

The parameter list in each row is sufficient to calibrate the demand model employed for the system of models[8]. After choosing their preferred controls and inspecting how this alters the initial parameters,

[8] As in chapters 2 and 3, we use a rectangular logit demand model. For a full exposition of this model, refer to chapter 2 section 2.3

users can choose either to view the results of the calculated models in tables or graphically. Users exercise their choice by selecting either the 'Tables' or 'Graphs' tab on the side panel (Block A, figure 4.1).

On the 'Tables' tab the parameters are used to simulate a vertical merger in either a derived demand, linear pricing or two-part pricing setting. Users select their choice by expanding the appropriate dialogue box, as shown in figure 4.5. The *Derived Demand* box will show the outcomes for the derived demand and vertical merger in a derived demand setting; the *Linear Pricing* box will show the Nash-in-Nash linear pricing, Nash-in-Shapley linear pricing and vertical merger in linear pricing models; the *Two-part Pricing* box will show the Nash-in-Nash two-part pricing, Nash-in-Nash quantity, Nash-in-Shapley two-part pricing and vertical merger in two-part pricing models. The derivation and discussion of these models are in chapter 2, section 2.3 and chapter 3, section 3.3.

An example of a calculated table is shown in figure 4.6. Table 4.1 summarises the variables calculated by the models and presented in each table.

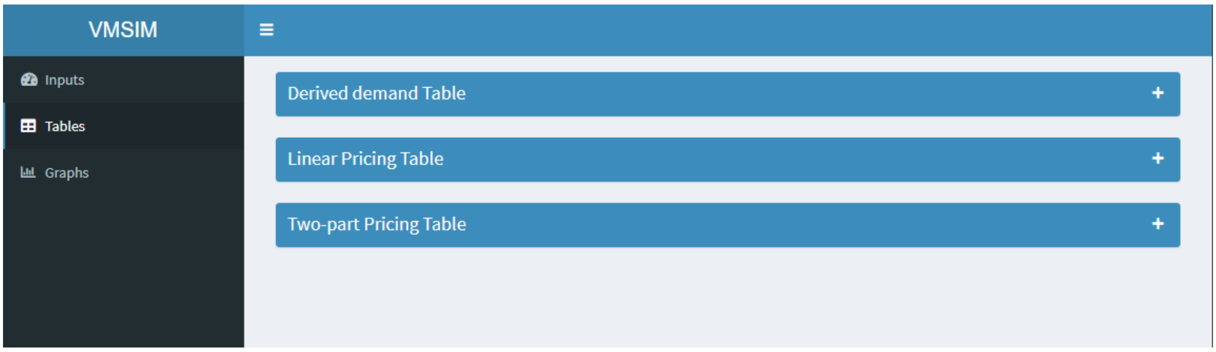
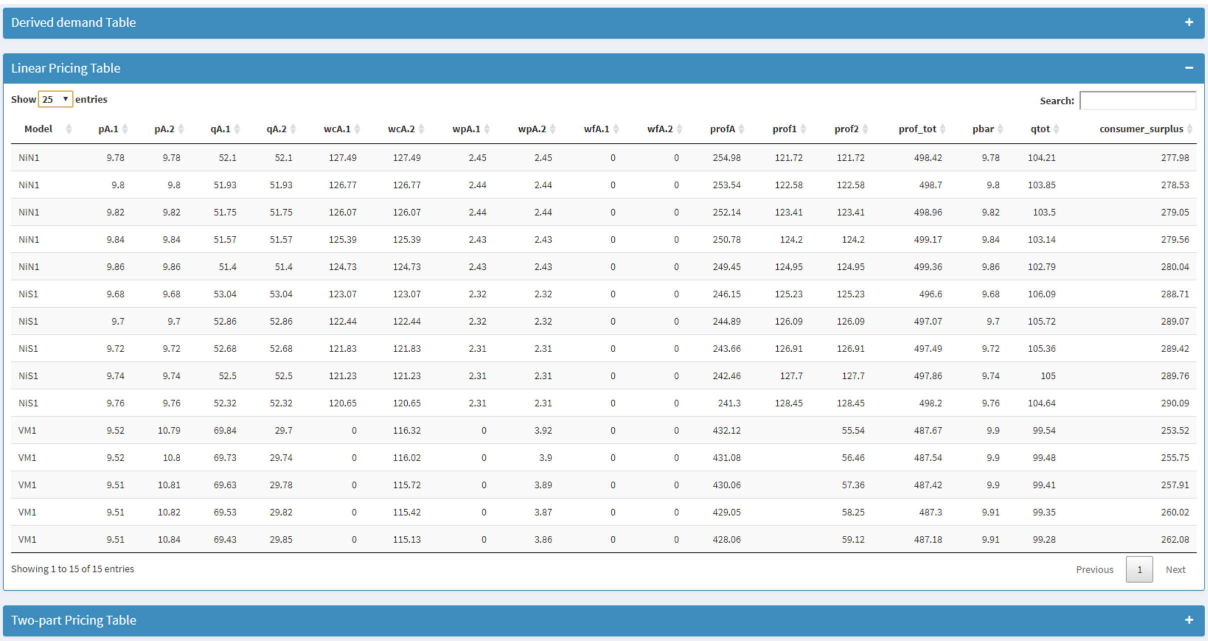


Figure 4.5: The landing page of the 'Tables'-tab


Derived demand Table

Linear Pricing Table

Show 25 entries Search:

Model	pA.1	pA.2	qA.1	qA.2	wcA.1	wcA.2	wpA.1	wpA.2	wfA.1	wfA.2	profA	prof1	prof2	prof_tot	pbar	qtot	consumer_surplus
NiN1	9.78	9.78	52.1	52.1	127.49	127.49	2.45	2.45	0	0	254.98	121.72	121.72	498.42	9.78	104.21	277.98
NiN1	9.8	9.8	51.93	51.93	126.77	126.77	2.44	2.44	0	0	253.54	122.58	122.58	498.7	9.8	103.85	278.53
NiN1	9.82	9.82	51.75	51.75	126.07	126.07	2.44	2.44	0	0	252.14	123.41	123.41	498.96	9.82	103.5	279.05
NiN1	9.84	9.84	51.57	51.57	125.39	125.39	2.43	2.43	0	0	250.78	124.2	124.2	499.17	9.84	103.14	279.56
NiN1	9.86	9.86	51.4	51.4	124.73	124.73	2.43	2.43	0	0	249.45	124.95	124.95	499.36	9.86	102.79	280.04
NiS1	9.68	9.68	53.04	53.04	123.07	123.07	2.32	2.32	0	0	246.15	125.23	125.23	496.6	9.68	106.09	288.71
NiS1	9.7	9.7	52.86	52.86	122.44	122.44	2.32	2.32	0	0	244.89	126.09	126.09	497.07	9.7	105.72	289.07
NiS1	9.72	9.72	52.68	52.68	121.83	121.83	2.31	2.31	0	0	243.66	126.91	126.91	497.49	9.72	105.36	289.42
NiS1	9.74	9.74	52.5	52.5	121.23	121.23	2.31	2.31	0	0	242.46	127.7	127.7	497.86	9.74	105	289.76
NiS1	9.76	9.76	52.32	52.32	120.65	120.65	2.31	2.31	0	0	241.3	128.45	128.45	498.2	9.76	104.64	290.09
VM1	9.52	10.79	69.84	29.7	0	116.32	0	3.92	0	0	432.12		55.54	487.67	9.9	99.54	253.52
VM1	9.52	10.8	69.73	29.74	0	116.02	0	3.9	0	0	431.08		56.46	487.54	9.9	99.48	255.75
VM1	9.51	10.81	69.63	29.78	0	115.72	0	3.89	0	0	430.06		57.36	487.42	9.9	99.41	257.91
VM1	9.51	10.82	69.53	29.82	0	115.42	0	3.87	0	0	429.05		58.25	487.3	9.91	99.35	260.02
VM1	9.51	10.84	69.43	29.85	0	115.13	0	3.86	0	0	428.06		59.12	487.18	9.91	99.28	262.08

Showing 1 to 15 of 15 entries Previous 1 Next

Two-part Pricing Table

Figure 4.6: Example of a calculated table

Table 4.1: Variables calculated by vertical merger simulator

Column name	Description	Column name	Description
Model	Model identifier	wf_1	Wholesale fee of good 1. Denoted $wf_{A@1}$ in 1×2 and 2×1 structure
p_1	Price of good 1. Denoted by $p_{A@1}$ in 1×2 and 2×1 structures	wf_2	Wholesale fee of good 2. Denoted by $wf_{A@2}$ in a 1×2 structure and $wf_{B@1}$ in a 2×1 structure
p_2	Price of good 2. Denoted by $p_{A@2}$ in a 1×2 structure and $p_{B@1}$ in a 2×1 structure	$ProfA$	Profit of upstream firm A
q_1	Quantity of good 1. Denoted by $q_{A@1}$ in 1×2 and 2×1 structures	$Prof1$	Profit of downstream firm 1
q_2	Quantity of good 2. Denoted by $q_{A@2}$ in a 1×2 structure and $q_{B@1}$ in a 2×1 structure	$ProfB/Prof2$	Profit of non-merging upstream firm if in a 2×1 structure or profit of non-merging downstream firm if in a 1×2 structure
wc_1	Total wholesale cost of good 1. Denoted $wc_{A@1}$ in a 1×2 and 2×1 structure	$Proftot$	Total industry profit
wc_2	Total wholesale cost of good 2. Denoted $wc_{A@2}$ in a 1×2 structure and $wc_{B@1}$ in a 2×1 structure	$pbar$	Average price of the two goods
wp_1	Wholesale price of good 1. Denoted $wp_{A@1}$ in 1×2 and 2×1 structures	$qtot$	Total quantity in the market (excludes outside good)
wp_2	Wholesale price of good 2. Denoted $wp_{A@2}$ in a 1×2 structure and $wp_{B@1}$ in a 2×1 structure	cons_surplus	Total consumer surplus

If users choose the ‘Graphs’-tab on the side panel (Block A figure 4.1), they can plot the simulation results of their control variable choices. Plots are again grouped as derived demand, linear pricing and two-part pricing. Users select their choice by expanding the appropriate dialogue box, as shown in figure 4.7. They can choose to plot any of the variables calculated by the models (in table 4.1) against any one of the parameters in the parameter list. Figure 4.8 shows an example of a graph generated by the vertical merger simulator.

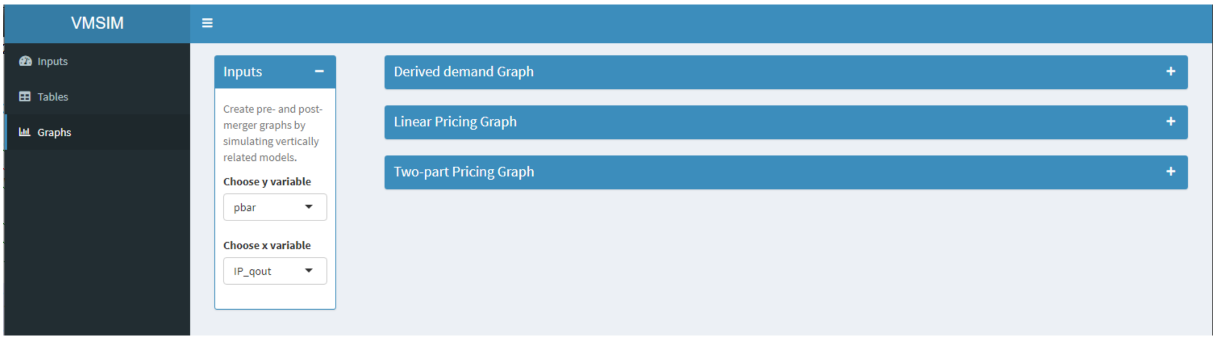


Figure 4.7: The landing page of the ‘Graphs’-tab

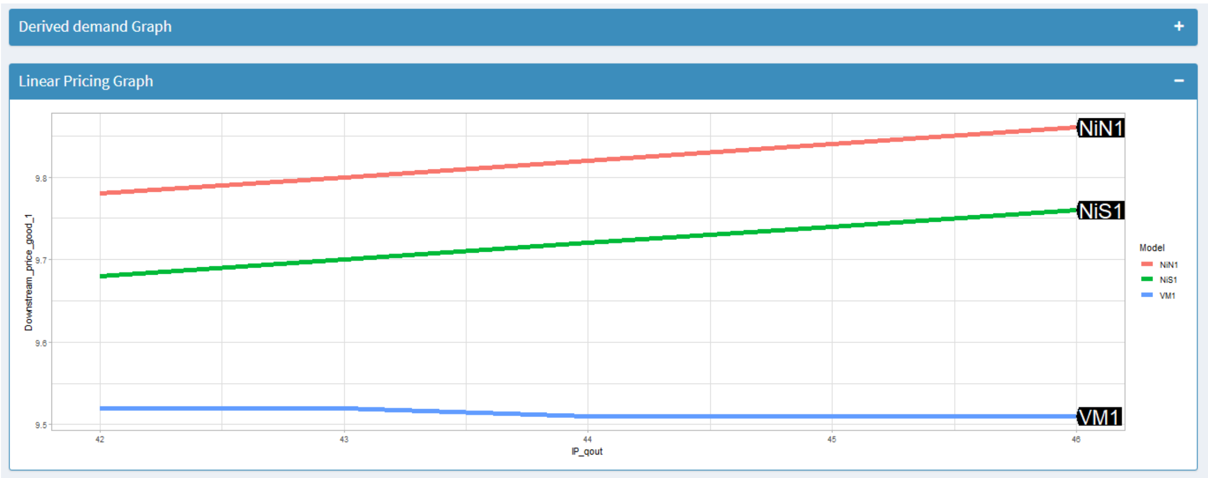


Figure 4.8: Example of a generated graph

4.4.1 Examples

The vertical merger simulator presents both practitioners and academics with copious possibilities for the analysis of vertically related markets and mergers. In the following subsections, we present three examples of how the simulator may be employed.

4.4.1.1 Marginal cost

Suppose that a practitioner is faced with a vertical merger where data on prices, market shares and substitutability are readily available. Specifically, there is evidence that suggests that prices and market shares of the firms under investigation are fairly balanced. Then, the practitioner may choose the price balance (p_{bal}) and quantity balance (q_{bal}) control variables to be 0.5. Moreover, suppose there is evidence that there is low substitutability in this industry. The practitioner may choose the inside/outside quantity balance (out_{bal}), logit nest parameter (τ) and aggregate elasticity (ae) to be 0.3, 0 and -1 respectively — all relating to relatively low substitutability.

The remaining control variable, the marginal cost balance (mc_{bal}) can then be chosen as the exogenous variable over which to simulate. In figure 4.9 we show how the inputs dialogue box will look for this scenario and summarise the predicted results for two industry structures. The summary table is constructed from the graphs presented in appendix F, figures F.1 and F.2.

As shown in figure 4.9, in a 1×2 industry structure, the derived demand and Nash-in-Shapley two-part pricing models predict a pro-competitive merger, while the other models predict an anti-competitive merger. If a practitioner can find conclusive evidence of linear pricing contracts in the industry in question, a modelling choice between Nash-in-Nash- and Nash-in-Shapley-specified vertical contracting is eliminated.

If the practitioner is investigating a 2×1 industry structure, figure 4.9 shows that there is not a concern about anti-competitive effects given the selected control variables. For all levels of the marginal cost balance, the models predict either a pro-competitive merger or no effect. Therefore, a choice between the different models is irrelevant to the conclusion.

4.4.1.2 Market shares

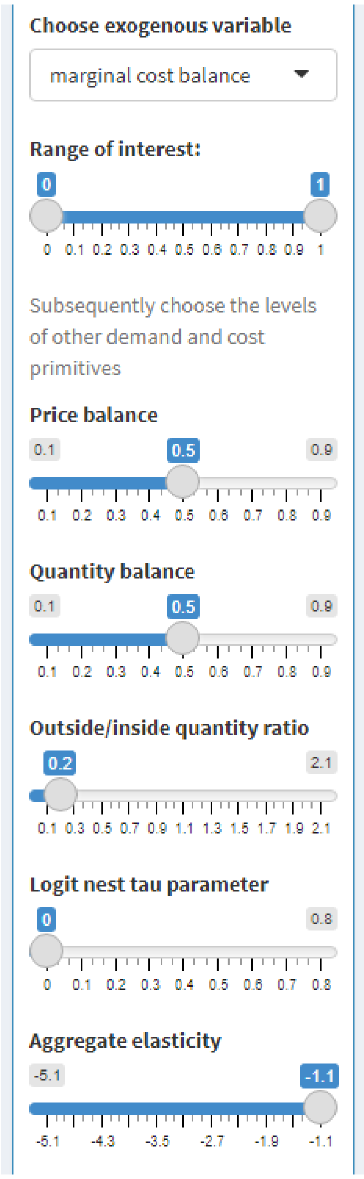
Suppose a practitioner is investigating a vertical merger in a market where market shares are uncertain. The practitioner can then choose the quantity balance (q_{bal}) control variable as the exogenous variable. This will provide an approximation of how market shares influence vertical merger predictions.

Assume that there is evidence that the downstream price of the merging firm is lower than the rival's price, say $p_{bal} = 0.25$. Also assume that the practitioner finds that the industry is characterised by relatively high elasticity. The inside/outside quantity balance (out_{bal}), logit nest parameter (τ) and aggregate elasticity (ae) can subsequently be set to 2, 0.2 and -4 respectively. Finally, suppose the practitioner makes the modelling judgement that the upstream firm(s) bears no marginal cost, so that $mc_{bal} = 0$. The appropriate input dialogue box for this situation is shown in figure 4.10.

Figure 4.10 shows the summary table of the results that is constructed from the graphs presented in appendix F, figures F.3 and F.4. In a 1×2 industry structure, the results suggest that more investigation will be necessary into the market shares if it is believed that a linear pricing model will accurately model the market in question. Specifically, the critical point seems to be a quantity balance of 0.15. A modelling choice between Nash-in-Nash and Nash-in-Shapley if in a linear pricing scenario can be eliminated, since these models show almost identical results.

In the two-part pricing scenario in a 1×2 industry, Nash-in-Nash and Nash-in-Shapley predict opposite merger effects. Thus, the practitioner will have to either make a modelling judgement regarding Nash-in-Nash versus Nash-in-Shapley bargaining or find qualitative evidence on the nature of bargaining that usually takes place in this market.

If the merger case involves a 2×1 industry structure, the linear pricing scenario predicts a pro-competitive merger for all levels of market shares. It is only in a two-part pricing setting where the competitiveness of a merger depends on market shares when following a Nash-in-Nash quantity setting model. In this scenario, the downstream merging firm would be able to provide qualitative evidence on the terms of typical contracts negotiated with the rival firm. If there is not evidence on contracts



Model	Merger	1x2 Industry	2x1 Industry
DD	VMDD	Pro-competitive	Pro-competitive
NiN1	VM1	Anti-competitive	Pro-competitive
NiS1	VM1	Anti-competitive	Pro-competitive
NiN2	VM2	Anti-competitive	No merger effect
NiNQ	VM2	Anti-competitive	Pro-competitive, NiNQ decreasing; thus merger effect grows as upstream mc increases
NiS2	VM2	Pro-competitive	No merger effect

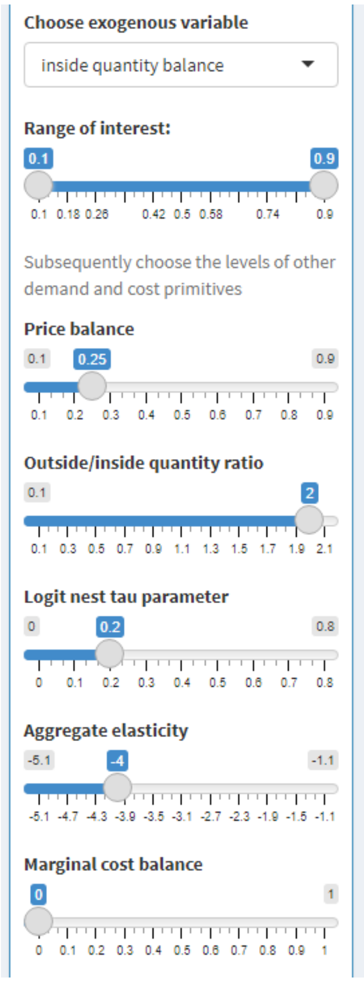
Figure 4.9: The chosen controls that lead to the results of the merger models

specifying quantities and total price, there will be no merger effect in a two-part pricing setting and further investigation into market shares can be avoided.

4.4.1.3 Prices

The benefit of the vertical merger simulator is that it calculates the full equilibrium for each model. Thus, it enables practitioners to analyse more than just the net effect of mergers. Suppose that a practitioner is interested in how a vertical merger will influence the average price in the industry. Assume the price balance (p_{bal}) control variable is chosen as exogenous, and the five remaining control variables are selected as shown in figure 4.11. The practitioner is then able to plot the quantity-weighted price average ($pbar$ in the Inputs dialogue box, on the 'Graphs'-tab) against the initial downstream price of good 1. The graphic results of these choices are shown in appendix F figures F.5 and F.6.

Figure 4.11 summarises the observed effects for the different models for the two industry structures considered. For the two-part pricing 2×1 industry models, there is no effect post-merger, apart from

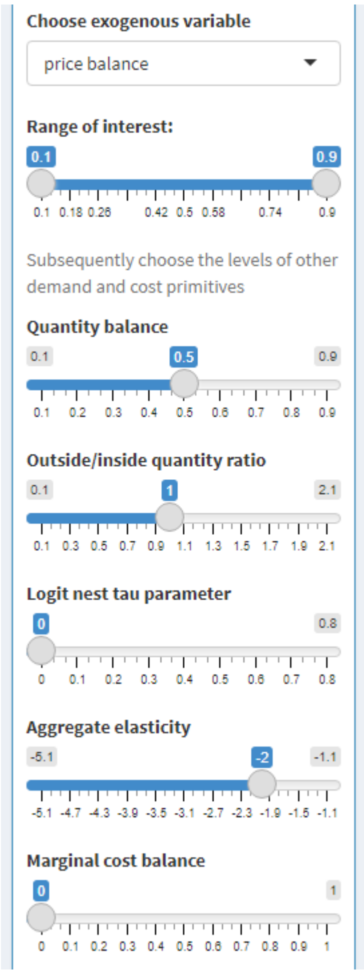


Model	Merger	1x2 Industry	2x1 Industry
DD	VMDD	Pro-competitive, DD locus non-monotonic, VMDD locus increasing in q1	Pro-competitive, DD locus non-monotonic, VMDD locus increasing in q1
NiN1	VM1	Pro-competitive for quantity balance greater than approximately 0.15, NiN1 locus non-monotonic, VM1 locus increasing in q1	Pro-competitive, NiN1 locus non-monotonic, VM1 locus increasing in q1
NiS1	VM1	Pro-competitive for quantity balance greater than approximately 0.15, NiS1 locus non-monotonic, VM1 locus increasing in q1	Pro-competitive, NiS1 locus non-monotonic, VM1 locus increasing in q1
NiN2	VM2	Anti-competitive, NiN2 locus non-monotonic, VM2 locus non-monotonic	No merger effect
NiNQ	VM2	Anti-competitive, NiNQ locus non-monotonic, VM2 locus non-monotonic	Anti-competitive for q1 less than 37, NiNQ decreasing in q1
NiS2	VM2	Pro-competitive	No merger effect

Figure 4.10: The chosen controls that lead to the results of the merger models

for the Nash-in-Nash quantity setting model. For the remaining models across both industry structures, the initial price balance determines whether the industry price increases or decreases post-merger. For all these models, the post-merger industry price is increasing in downstream price 1 (and hence the price balance control variable).

Investigating the critical points where the post-merger industry price intersects with the pre-merger industry price yields further interesting results. For all the models apart from the Nash-in-Nash models in the two-part pricing 1×2 industry setting, the critical point is where $p_{bal} > 0.5$. This means that when the initial downstream price of the merging firm is greater than initial downstream price of good 2 (the rival firm), the industry price increases post-merger. For the Nash-in-Nash models, in the two-part pricing 1×2 setting, the critical point is where $p_{bal} < 0.5$. Thus, when the merging firm's downstream price is lower than the rival firm, the industry price increases post-merger.



Model	Merger	1x2 Industry	2x1 Industry
DD	VMDD	Industry price increases post-merger for downstream price 1 greater than 13	Industry price increases post-merger for downstream price 1 greater than 13
NiN1	VM1	Industry price increases post-merger for downstream price 1 greater than 11	Industry price increases post-merger for downstream price 1 greater than 12.5
NiS1	VM1	Industry price increases post-merger for downstream price 1 greater than 11	Industry price increases post-merger for downstream price 1 greater than 12.5
NiN2	VM2	Industry price increases post-merger for downstream price 1 greater than 6	No effect on industry price
NiNQ	VM2	Industry price increases post-merger for downstream price 1 greater than 6	Variable
NiS2	VM2	Industry price increases post-merger for downstream price 1 greater than 12	No effect on industry price

Figure 4.11: The chosen controls that lead to the results of the merger models

4.5 Comparing incentive scoring and the vertical merger simulator

Both incentive scoring and the vertical merger simulator offer tools for vertical merger screening. Therefore, it is useful to consider how these methods compare. This section offers such a comparative assessment from two perspectives. Firstly, we compare their data requirements, assumptions and conclusions. Secondly, we employ the data simulated in the examples in section 4.4.1 and calculate the incentive scoring measures discussed in section 4.3. This allows a (if somewhat limited) comparison of the two methods.

4.5.1 Data requirements and equilibria

In the preceding sections, we discussed at length the data requirements for the incentive scoring methods. We also showed the six control variables necessary to calibrate the vertical merger simulator. In this section, we delve deeper into these data requirements, with the aim of comparing the methods.

To enable easy comparisons, we create a table with *data* and *assumptions* sections. Under the *data* section, we include rows for downstream prices and quantities, wholesale prices, total market size, aggregate elasticity, marginal costs, nest strength and bargaining strength. Under the *assumptions* section, we include rows for the assumed industry setting, demand specification, number of goods, pass-through rates and elimination of double marginalisation[9]. If a measure requires the data or assumption in a specific row, we mark it with an x or provide a short description of the requirement. In tables 4.2 to 4.5 we populate these rows for the different upward pricing indices discussed in section 4.3 and for the vertical merger simulator in section 4.4.

Populating the table for the upward pricing indices requires meeting two distinct sets of data requirements. The first set relates to the direct requirements, as is evident from the formulae discussed earlier. These include variables such as prices, wholesale prices and margins. The second set of data requirements relates to the underlying variables on which the formulae are based. This relates to the variables used for the calculation of *inter alia* diversion ratios and cost pass-through rates, as are evident from the discussion of the relevant literature in section 4.3.3. For the *assumptions* section, we summarise the assumptions discussed in Moresi and Salop (2013) and Rogerson (2020).

We populate the table for the vertical merger simulator by ascertaining the data that would be necessary to calculate the control variables discussed in section 4.4. A practitioner presented with a case and data may employ the formulae used to calculate the initial parameters, to uncover the control variables that fits the data (for example, prices may be used to calculate the price balance control variable etc.). Finally, we summarise the assumptions made by the simulator in the *assumptions* section of table 4.5.

Tables 4.2 to 4.5 offer two key insights in relation to the comparative data requirements of incentive scoring methods and the vertical merger simulator. Firstly, our results challenge the argument of Moresi and Salop (2013) and Rogerson (2020) that incentive scoring methods 'require less data than merger simulation models'. Tables 4.2 to 4.5 show that this is not the case. Consistent with earlier arguments by Slade (2020), tables 4.2 to 4.5 show that incentive scoring has its own extensive data and estimation requirements. In particular, a comparison of any one of the upward pricing index columns with the vertical merger simulator column does not suggest a marked difference in data requirements. Moreover, incentive scoring consists of an array of indices, each tailored to measure a related but distinct part of the effect of a vertical merger. The tables show that the various indices each have their own set of data requirements. A practitioner attempting to ascertain the incentives of the different players affected by a vertical merger would not be able to calculate only one such index, meaning that data requirements can quickly accumulate.

[9]These rows reflect the typical aggregate requirements of the two methods considered.

Table 4.2: Comparison of data requirements

		$vGUPPIu$	$vGUPPIr$
Calculation		$DR_{UD} \times M_D \times P_D/W_R$	$vGUPPIu \times PTR_U \times W_R/P_R$
Data requirements	Downstream price	x	x
	Rival downstream price		x
	Downstream quantity	Diversion ratio requires output volume gained after price increase	Diversion ratio requires output volume gained after price increase
	Rival downstream quantity	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input.	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input.
	Wholesale price		Cost pass-through rate of upstream firm needed
	Rival wholesale price	x	x
	Total market size		
	Aggregate elasticity		
	Marginal cost downstream	x	x
	Marginal cost rival downstream		
	Marginal cost upstream		Cost pass-through rate of upstream firm needed
	Nest strength		
	Bargaining strength	N/A	N/A
Assumptions	Industry	Different index for each rival firm. Assume all other prices constant	Different index for each rival firm. Assume all other prices constant
	Demand		50% PTR based on linear demand
	Number of goods	Cannot substitute away from inputs purschased from rival upstream suppliers	Cannot substitute away from inputs purschased from rival upstream suppliers
	Cost pass-through rate		50% used as default unless contrary evidence in a specific case
	Merger specific EDM	No	No

Table 4.3: Comparison of data requirements

		$vGUPPId1$	$vGUPPId2$
Calculation		$DR_{DU} \times M_U \times W_U/P_D$	$vGUPPId1 - M_{UD} \times W_D/P_D$
Data	Down price	x	x
	Rival Down price		
	Down quantity	Diversion ratio requires output volume gained after price increase	Diversion ratio requires output volume gained after price increase
	Rival down quantity	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input.	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input.
	Whole price	x	x
	Whole price rival	x	x
	Total market size		
	Aggregate elasticity		
	MC down		
	MC rival down		
	MC upstream	x	x
	Nest strength		
	Bargaining strength	N/A	N/A
Assumptions	Industry	Different index for each rival firm. Assume all other prices constant	Different index for each rival firm. Assume all other prices constant
	Demand		
	Number of goods	Cannot substitute away from inputs purschased from rival upstream suppliers	Cannot substitute away from inputs purschased from rival upstream suppliers
	PTR		
	Merger specific EDM	No	Yes

Table 4.4: Comparison of data requirements

		$vGUPPIu^*$	$vGUPPIr^*$
Calculation		$\frac{DR_{RD} \times M_D \times P_D / W_R}{1 + M_R \times E_{SR} / E_P}$	$vGUPPIu^* \times PTR_U \times W_R/P_R \times S_{UR}^{post}/S_{UR}$
Data	Down price	x	x
	Rival Down price	x	x
	Down quantity	Diversion ratio requires output volume gained after price increase. Quantity shares also needed	Diversion ratio requires output volume gained after price increase. Quantity shares also needed
	Rival down quantity	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input. Quantity shares also needed	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input. Quantity shares also needed
	Whole price	x	Cost pass-through rate of upstream firm needed
	Whole price rival	x	x
	Total market size		
	Aggregate elasticity	x	
	MC down	x	x
	MC rival down	x	
	MC upstream		Cost pass-through rate of upstream firm needed
	Nest strength		
	Bargaining strength	N/A	N/A
Assumptions	Industry	Different index for each rival firm. Assume all other prices constant	Different index for each rival firm. Assume all other prices constant
	Demand		50% PTR based on linear demand
	Number of goods		
	PTR		50% used as default unless contrary evidence in a specific case
	Merger specific EDM	No	No

Table 4.5: Comparison of data requirements

		$vGUPPId3$	$vGUPPI_{BLR}$	Vertical merger simulator
Calculation		$vGUPPId2 - E_{SD} \times (M_{UD})^2 \times W_D/P_D$	$(1-\delta) \times v \times d \times \pi$	
Data	Down price	x	Profit margin required	Price balance required
	Rival Down price			Price balance required
	Down quantity	Diversion ratio requires output volume gained after price increase	Diversion ratio requires share of departing customers from rival shifting to merging downstream firm	Quantity balance required
	Rival down quantity	Diversion ratio requires input volume lost after price increase. Wholesale price calculation requires total quantity produced with the upstream firm's input.	Departure rate requires share of rival firm's customers that would leave if the merging firm refused supply of input	Quantity balance required
	Whole price	x	Profit margin required	
	Whole price rival	x		
	Total market size			Inside/outside quantity balance required
	Aggregate elasticity	x		x
	MC down		Profit margin required	Marginal cost balance required
	MC rival down			Marginal cost balance required
	MC upstream	x		Marginal cost balance required
	Nest strength			x
	Bargaining strength	N/A	x	Assumed to be 0.5
Assumptions	Industry	Different index for each rival firm. Assume all other prices constant	Input and output prices are set simultaneously	1×2 or 2×1
	Demand			Rectangular logit
	Number of goods			Two
	PTR			
	Merger specific EDM	Yes		Yes

The second insight is that, as acknowledged by Moresi and Salop (2013) and Rogerson (2020), incentive scoring offer measures of partial, rather than general, equilibria. In contrast, the vertical merger simulator calculates a full equilibrium, duly taking note of how opposing competitive effects of vertical mergers influences this equilibrium. Moreover, practitioners are able to compare the outcomes of six different models of vertical mergers, across two industry structures and for a choice of six control variables. The graphical comparisons also allow practitioners to ascertain which assumptions matter and by how much.

A direct comparison of incentive scoring and the vertical merger simulator is challenging and tables 4.2 to 4.5 represent an attempt to pin down the data and assumptions required for each. Following the discussion above, we are not convinced by the motivations for the use of one method over the other based purely on data or estimation requirements. Ultimately, a practitioner faces a difficult trade-off when deciding on a screening method for vertical mergers since both methods have intensive data requirements and assumptions. However, as Friedman (1953) reminds us, it is not whether the assumptions of the model are descriptively realistic, for they never are, but 'sufficiently good approximations for the purpose in hand.' To be clear, we are not arguing for the substitutability, but rather for the complementarity of the tools. The fact that the vertical merger simulator has similar data requirements to those of incentive scoring methods, but is able to provide a full-equilibrium outcome, warrants its use as a complementary tool for vertical merger analysis.

4.5.2 Practical comparisons

In this subsection, we employ the data simulated in the examples in section 4.4.1 and calculate the incentive scoring formulae discussed in section 4.3. As mentioned in section 4.3, pre-merger data on the marginal costs and prices of the upstream and downstream firms, diversion ratios, cost pass-through rates and elasticities are required to calculate the incentive scoring measures.

Since we employ simulated data, pre-merger diversion ratios and cost pass-through rates are not available. Therefore, we need to either make estimates or assumptions regarding these values. For the cost pass-through rate, we follow the convention in Moresi and Salop (2013) and assume a rate of 50%. For the diversion ratios, recall that vertical diversion ratios measure the share of the upstream firm's volume lost when raising the input price to a rival, which is gained by its vertically integrated downstream firm (Slade, 2020). In this vein, the merger simulation provides a suitable approximation of how quantities adjust to an increase in the input price. Therefore, we calculate the diversion ratio of the rival firm using the pre- and post-merger quantities and make the proportional switching assumption for the diversion ratio of the merging firm. However, we are not able to use this data to calculate the departure rate necessary for the $vGUPPI_{BLR}$, and thus we exclude this incentive scoring measure from the comparisons below.

In calculating the incentive scoring indices, we limit our attention to the 1×2 industry structure since, the indices are not suited for situations where the pivotal player is the downstream firm as in a 2×1 industry structure. Recall that in a 1×2 industry structure, there is only one upstream firm, so that downstream firms are not able to substitute inputs. Hence, we do not include $vGUPPIu^{*}$, $vGUPPIr^{*}$ or $vGUPPId3$ in our comparisons below. This leaves us with four incentive scoring indices: $vGUPPIu$, $vGUPPIr$, $vGUPPId1$ and $vGUPPId2$.

The remaining incentive scoring indices can each be compared to the the relevant increase in the price predicted by the vertical merger simulator. As such, we compare the $vGUPPIu$ to the predicted increase in the wholesale price to the rival downstream firm, the $vGUPPIr$ to the predicted increase in the downstream price of the rival firm and $vGUPPId1$ and $vGUPPId2$ to the predicted change in the merged firm's downstream price.

Below, we compare the predictions of the vertical merger simulator and incentive scoring measures for each of the examples in section 4.4. Since the indices developed by Moresi and Salop (2013) assume a derived demand model, we focus on this model in the discussions below; however, the graphical results for the remaining five models are presented in appendix G.

4.5.2.1 Marginal cost example

Figure 4.12 shows the graphical results for the marginal cost example. Both the $vGUPPIu$ and $vGUPPIr$ correctly predict an increase in the wholesale price to the rival and the downstream price of the rival respectively. However, the increase predicted by the incentive scoring indices is at least double what is observed in the simulated data at every level of the marginal cost balance.

The $vGUPPId1$ incorrectly predicts an increase in the downstream price of the merging firm. When adjusting the $vGUPPId1$ for the elimination of double marginalisation, the $vGUPPId2$ correctly predicts the decrease in price, but again this prediction is more than double the observed decrease.

Observing the predictions for the remaining five models, in appendix G figures G.1(a) to G.3(b) yields further insights. Specifically, figure G.3(a), displaying the results for the NiS2 model, shows exactly the same pattern for the $vGUPPIu$, $vGUPPIr$, $vGUPPId1$ and $vGUPPId2$ as described above for the derived demand model[10] . Interestingly, the derived demand and NiS2 models are the only models that indicate a pro-competitive merger in figure 4.9. For the remaining models, the incentive scoring measures predict a lower increase compared with the simulated data.

[10]The $vGUPPIu$ and $vGUPPIr$ correctly predict an increase, but more than double that observed in the simulated data. The $vGUPPId1$ incorrectly predicts an increase, and when adjusting to the $vGUPPId2$, the predicted decrease is more than double the observed decrease.

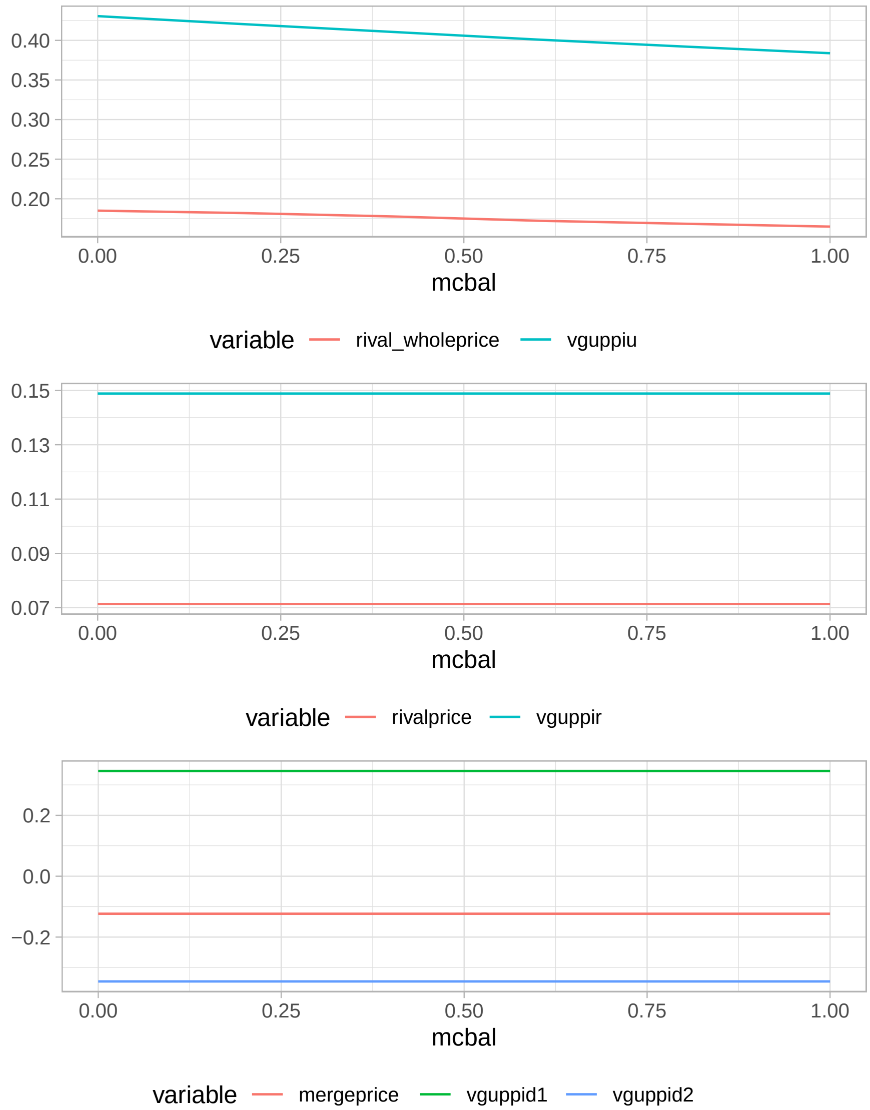


Figure 4.12: Comparison of simulation and incentive scoring predictions

4.5.2.2 Market share example

Figure 4.13 shows the results for the market share example. The simulated data shows that at every level of quantity balance, the upstream firm does not have the incentive to increase the wholesale price to the rival firm. Subsequently, the rival downstream firm does not increase its downstream price.

The predictions from the $vGUPPIu$ vary substantially over the quantity balance, ranging from a predicted 500% decrease to a 1500% increase in the wholesale price to the rival firm. This can be attributed to the large input/output ratio in the simulated data. The predicted $vGUPPIr$ shows similar variation over the quantity balance, but the predictions range between a 60% decrease and a 60% increase in the downstream rival price.

Comparing the $vGUPPId1$ and $vGUPPId2$ predictions, an adjustment for elimination of double marginalisation improves the prediction of the decrease in the downstream price of the merging firm. Specifically, the $vGUPPId2$ closely tracks the observed decrease in the downstream price.

The robustness of the vertical merger simulator as a comparison tool is highlighted by this example. The simulator shows stable predictions across the entire quantity balance range. In contrast, predictions from the incentive scoring methods vary substantially, with extreme predictions at the endpoints of the range[11].

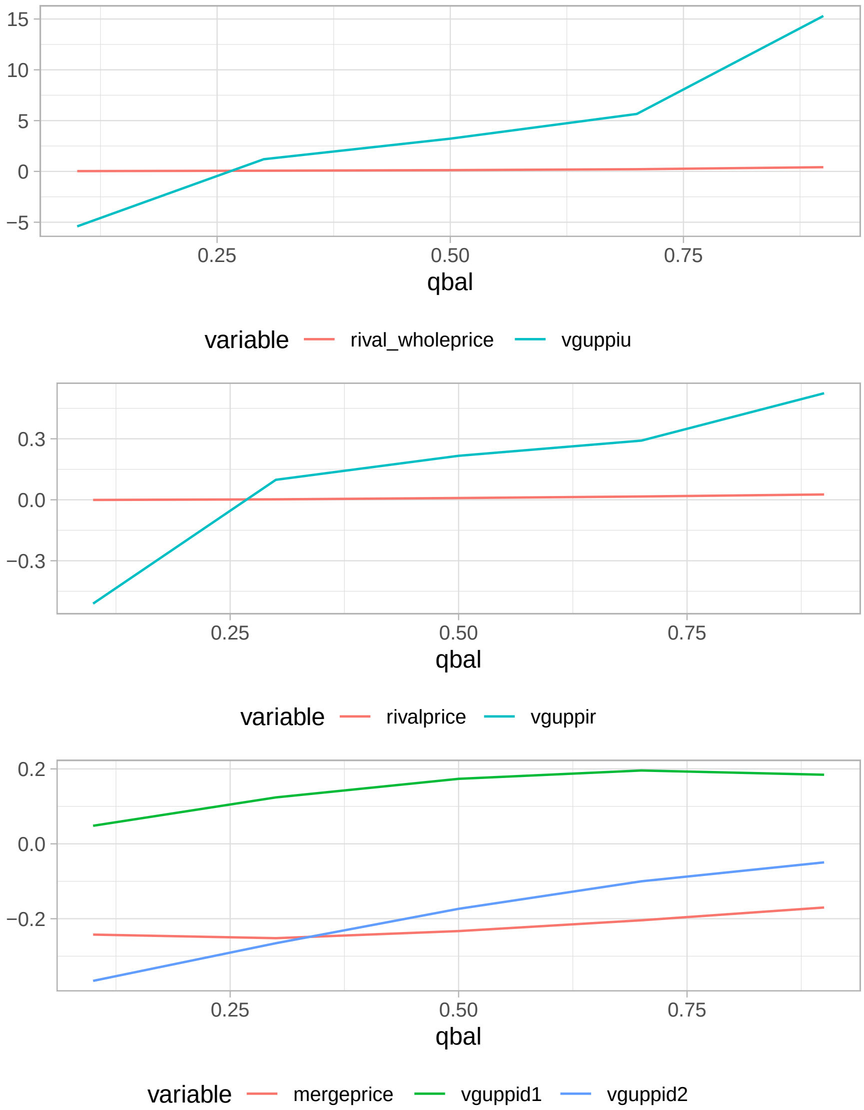


Figure 4.13: Comparison of simulation and incentive scoring predictions

4.5.2.3 Price balance example

Figure 4.14 shows that the $vGUPPIu$ and $vGUPPIr$ predict increases substantially higher than what are observed in the wholesale price and downstream price of the rival firm respectively. This over-prediction is observed for most of the remaining models in appendix G figures G.7(a) to G.9(b). However, the NiN2 (figure G.8(b)) and NiNQ (figure G.9(b)) models show the opposite, in that the incentive scoring predictions are lower than the observed increase. These were also the two models in section 4.4.1, for which the critical point where the post-merger industry price intersects with the pre-merger industry price, was below 0.5.

[11]The predictions for the remaining five models in figures G.4(a) to G.6(b) in appendix G show very similar results to the derived demand model discussed above.

Adjusting $vGUPPId1$ for the elimination of double marginalisation to yield $vGUPPId2$, results in an accurate prediction of the decrease in the downstream price of the merging firm. Figure 4.14 shows that $vGUPPId2$ almost perfectly tracks the observed decrease. This pattern is observed for most of the remaining models, again barring the exception of NiN2 and NiNQ.

It is puzzling that the predictions from the different indices vary so substantially despite being based on the same simulated data. The $vGUPPIu$ grossly over-predicts an increase of more than 250% in the wholesale price to the rival, while the $vGUPPId2$ almost perfectly predicts the 60% to 30% decrease in the merged firm's downstream price. In contrast, the predictions from the vertical merger simulator are in a consistent range for the different prices.

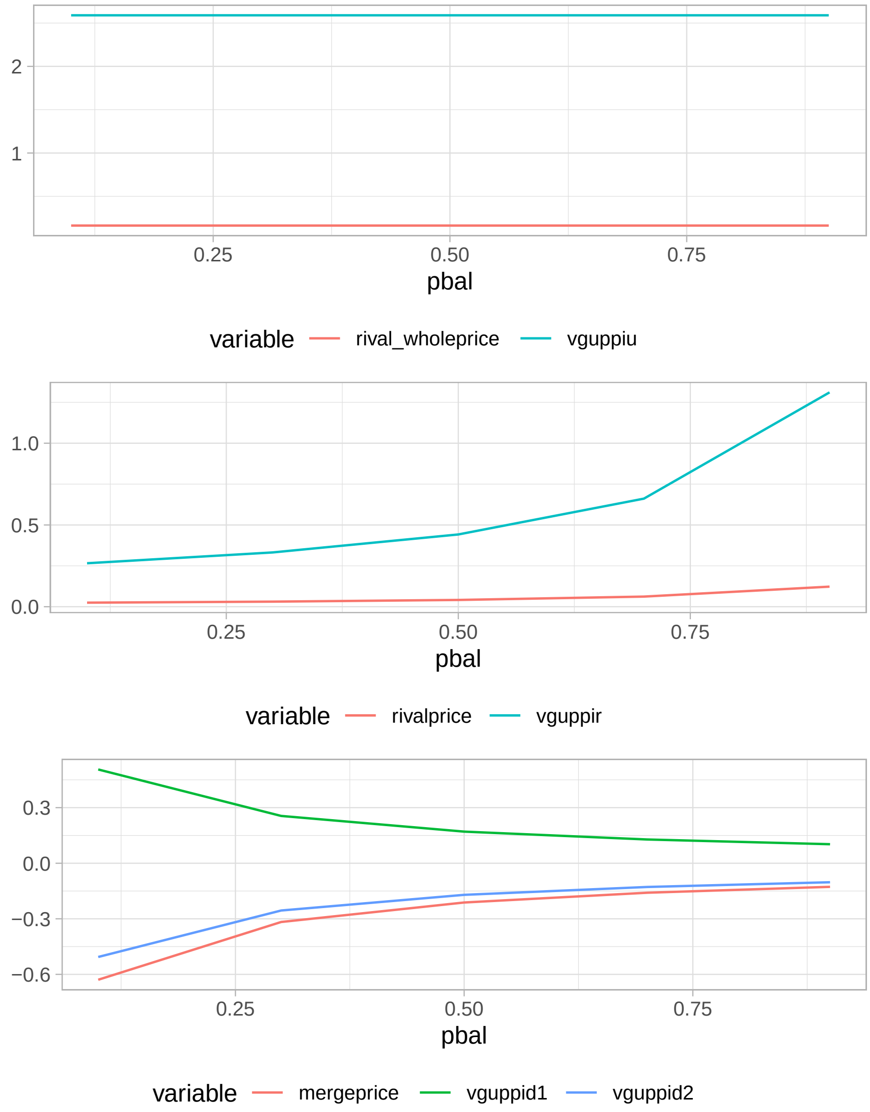


Figure 4.14: Comparison of simulation and incentive scoring predictions

4.6 Conclusion

In this chapter we assessed the toolkit for vertical merger screening. The existing literature shows that vertical merger assessments are often derived from their horizontal counterparts. However, the additional interaction between vertically related firms brings about greater data and estimation requirements and can often mean compromising the tractability of these measures.

In line with the shift of the emphasis of merger review, we focus on two techniques to predict the unilateral effects of vertical mergers. We reviewed incentive scoring techniques that rely on simple and intuitive formulae that gauge the upward pricing pressure induced by a merger. It is argued that the data requirements of these measures are less intensive than simulation models. However, these measures have a partial equilibrium focus.

We introduce a vertical merger simulator tool that can be calibrated with six control variables, which work well as a screening tool. Even in the early stages of an investigation, when limited data is available, the flexibility of the simulator can offer important insights. As illustrated by the examples, the vertical merger simulator provides results for a range of modelling choices. If a practitioner observes predictions to be constant for a specific modelling choice (for example, across different characterisations of vertical contracting, for both industry structures or over the entire range of a control variable), it may be deemed irrelevant to the conclusion. This property emphasises the simulator tool's usefulness as a screening tool, since vertical merger effects can be predicted with limited evidence.

Although a direct comparison of the data requirements for the two methods is challenging, we argue that the data necessary to calibrate the vertical merger simulator is no more extensive than that of the incentive scoring methods. Additionally, the vertical merger simulator has the advantage of providing full equilibrium results. However, the examples also illustrate that with the simulator, graphical results can be obtained for any chosen variable of interest (such as prices, quantities, profits, etc.). The examples further highlight the robustness and consistency of such predictions across a range of specifications.

In this chapter, we present several insights in relation to the comparison between the incentive scoring method and the vertical merger simulator. However, the aim of providing such comparisons is not to argue for the substitutability, but rather for the complementarity of the tools. We show that the vertical merger simulator provides a useful complementary addition to our toolkit of quantitative screening methods for vertical mergers.

Chapter 5

Conclusion

The academic literature on the characterisation of vertical relationships and the quantification of vertical merger effects remains unsettled. As suggested in the introductory chapter of this dissertation, vertical merger policy remains similarly unsettled. There have been few litigated vertical merger cases in the main jurisdictions from which to establish legal precedent. Furthermore, guidelines on the appropriate analysis of vertical mergers do not yet provide practitioners with clear principles.

The state of affairs is reflected in the 2018 AT&T/TimeWarner merger case in the USA. This case, the first litigated vertical merger in decades (Underwood, 2020), gave rise to bitterly contested court proceedings. Afterwards, the economic expert for the Department of Justice, Carl Shapiro, noted a 'distaste for economic models' and 'open hostility towards experts in general' (Shapiro, 2021). In Court, the Judge questioned the reliability and factual credibility of the model used, which led to a rejection of model-based conclusions as persuasive evidence (Leon, 2018). Practitioners have viewed this as a serious misstep: not only does it suggest a rejection of the established economic theory of bargaining (Shapiro, 2021), but also a rejection of the assumption of profit maximisation (Hovenkamp, 2020).

As explained in chapter 1, the experience in the EU has not been better. Guidelines on the assessment of non-horizontal mergers, issued by the European Commission in 2008 (European Commission, 2008), has left many questions surrounding vertical merger analysis unanswered. Despite the Guidelines covering different unilateral effects associated with vertical mergers, no specific mention is made of the quantitative techniques that the European Commission may employ in assessing vertical mergers. The lack of guidance on appropriate economic tools for the analysis of vertical mergers is concerning, given that quantitative techniques were a major point of contention in the AT&T/TimeWarner merger case, as noted above.

Based on a survey of recent developments in vertical merger enforcement and the evolution of the theoretic and empirical literature on vertical merger effects, this dissertation poses three research questions:

- Research question 1: Which model assumptions best capture the significant features of competition in vertically related markets?

- Research question 2: Which model assumptions best capture the loss of competition following a vertical merger?

- Research question 3: Can simulation models form part of the toolkit to screen the likely effects of vertical mergers?

The first two research questions are closely related but require separate consideration. Research question 1 aims at ascertaining how assumptions regarding vertical contracting affect the outcomes in vertically related markets. This is an important first step in any analysis, as assumptions can predetermine results. Research question 2 builds on this, but the independent focus is on the predicted effects of vertical mergers. In answering this research question, careful attention is paid to how a vertical merger alters the manner in which firms contract with one another. It is convenient that the predictions from research question 1 can be relied upon as representing the pre-merger outcomes for this analysis. The final research question concerns whether simulation models, usually employed in more in-depth investigations of vertical mergers, can be used as a screening tools for likely effects.

Several challenges arise when attempting to answer the main research questions above. This leads to various ancillary research questions, of which we narrow the focus to three:

- Ancillary research question 1: How can different model predictions be compared directly?
- Ancillary research question 2: How are vertical merger predictions related to different measures of substitutability?
- Ancillary research question 3: How do vertical merger simulation models compare to other screening methods in terms of the data requirements and predictions?

The first ancillary research question provides a crucial foundation for the research presented in this dissertation. We wish to study the link between differences in the predictions of models and the differences in the assumptions on which they are based. To this end, we perform numerical simulations with three distinct features. Firstly, we specify the competition between firms and consumer demand to be identical for all of the models that we consider. Secondly, we calibrate the demand function to the same parameters for all models. Finally, we consider only two simple industry structures: "1×2", one upstream and two downstream firms; and "2×1", two upstream and one downstream firm. Together, these three features allow direct comparison of the predictions for a variety of models.

We are able to build on the foundation set by the first ancillary research question and extend the methodology to answer ancillary research question 2. We consider an alternative calibration of the demand model, which allows inference about the robustness of vertical merger predictions to different specifications of substitutability. Ancillary research question 1 also provides the foundation for the vertical merger simulator developed in the answering of main research question 3. The final ancillary research question then compares the data requirements and predictions of the vertical merger simulator with incentive scoring measures.

The main and ancillary research questions posed here correspond with the three core chapters of this dissertation. The remainder of this chapter provides a summary of the findings and contributions from each subsequent chapter: Chapters 2, 3 and 4 are summarised in sections 5.1, 5.2 and 5.3, below. This chapter concludes by suggesting future research avenues in the literature fields explored in the dissertation.

5.1 Modelling vertical contracting in vertically related markets

Chapter 2 explores the features of models of vertically related markets. Our ancillary focus involves how the predictions from different models can be compared directly.

We identified four prominent features of models of vertically related markets: (i) the network of upstream and downstream firms, (ii) the competition between firms, (iii) consumer demand, and (iv) vertical contracting in the presence of externalities. We made assumptions and modelling decisions to control for the first three of these features. The emphasis then fell on the remaining element, vertical contracting, which is the focus of this dissertation.

An overview of relevant existing literature revealed that models can be differentiated based on their assumptions regarding the *object* or *nature* of vertical contracting. In this regard, we specifically focussed on the difference between *Nash-in-Nash* and *Nash-in-Shapley* bargaining. In these different *natures* of vertical contracting, the treatment of threat points differ. For the *Nash-in-Nash*, we assume the threat point is given by the profits determined in the scenarios with all other agreements held fixed — e.g. as in Sheu and Taragin (2017); Collard-Wexler et al. (2019); Rey and Vergé (2019). Put simply, there is no difference in the terms of the contract when the pivotal player reaches an agreement with only one player versus more than one player. For *Nash-in-Shapley*, we assume that the threat point is determined by profits, with all other agreements adjusted for the new set of agreements, each of these determined recursively from cases with fewer agreements — e.g., Froeb et al. (2020); Yu and Waehrer (2018).

Regarding the *object* of vertical contracting, we focus on linear pricing and two-part pricing models. For the former, firms agree on a marginal wholesale price for the input supplied by an upstream firm to the downstream firm. For the latter, firms contract over both the marginal wholesale price and a fixed fee.

For direct comparisons, all models are calibrated to the same set of parameters. The calibration comprise three features. First, we fixed a subset of the parameters required to calibrate the demand model. Second, we chose the outside quantity as the exogenous variable — increasing the ratio of the outside good in relation to the inside goods. Third, this meant that the remaining subset of parameters that was not fixed previously varies along with the exogenous variable. These three features produced the list of parameters used to calibrate the demand function for all models.

Our results showed that assumptions regarding vertical contracting matter greatly. Firstly, predictions may vary substantially if the *nature* of vertical contracting is kept constant, but the *object* of vertical contracting is altered. This is observed, for example, in the predictions for Nash-in-Shapley in a linear pricing versus two-part pricing setting. Secondly, predictions for a given *object* of vertical contracting can differ significantly for the different *natures* of vertical contracting. An example of this is the predictions for Nash-in-Nash versus Nash-in-Shapley in a two-part pricing setting.

The results for linear pricing vertical contracting show that the predictions for the models assuming different *natures* of vertical contracting are almost indistinguishable. We attribute this to the inefficiency of linear pricing contracts since the wholesale price is the only instrument at the disposal of players who aim to achieve two conflicting goals. The wholesale price is primarily an instrument that the upstream firm employs to increase industry profits. However, concurrently it is also the only instrument with which the upstream firm takes its share of that profit. These effects are counteracting, since a low wholesale price fits the first goal, while a high one fits the second. This internal conflict faced by the upstream firm diminishes the difference between the Nash-in-Nash and Nash-in-Shapley bargaining models.

In a two-part pricing vertical contracting setting, players have two instruments at their disposal to achieve two conflicting goals. Firstly, firms can bargain for a wholesale price so as to increase industry profit. Secondly, upstream firms are able to bargain over a fixed fee to recoup their share of this profit. In this setting, the results from different industry settings clearly show how assumptions regarding the *nature* of vertical contracting affect predictions.

In a two-part pricing setting, the Nash-in-Nash model leads to different predictions in different industry settings. In a 1×2 industry, the predictions resemble the perfect competition outcome. We attribute this to the "schizophrenic" behaviour of the pivotal player in this *nature* of vertical contracting. However, in a 2×1 industry, the pivotal player is able to internalise competition in the opposing market, and the joint profit maximising outcome is achieved.

In contrast, the Nash-in-Shapley model predicts identical results regardless of the industry structure. In both industries that we consider, the model achieves the joint profit maximising outcome. We attribute this directly to the characterisation of threat points in this model. When bargaining according to the Nash-in-Shapley *nature* of vertical contracting, pivotal players take full cognisance of the externality that another agreement imposes on a contract. Therefore, wholesale prices are set to maximise total joint surplus, and subsequent fixed fees are set so that this surplus is split equally.

When modelling vertically related markets, assumptions regarding vertical contracting should be made based on how well they capture the observed characteristics of the considered case. However, in practice, it seem to be based on the computability and tractability of the Nash-in-Nash model. The results presented in chapter 2 hold an important lesson for policymakers in this regard. The systematic and direct comparison of the corpus of models of vertically related markets illustrates the impact that our modelling choices may have. Such insights can be of great importance for vertical merger analysis.

5.2 Vertical mergers

Chapter 3 considers vertical merger analysis. Specifically, we investigated how assumptions regarding vertical contracting map onto observable effects. Our ancillary focus involved how merger effects relate to different measures of substitutability.

The existing literature confirms that vertical mergers are fundamentally different from horizontal mergers. Church (2008) provides four reasons for this; (i) the incentives for vertical integration are often efficiency-related rather than to do with market power; (ii) vertical mergers often lead to lower prices as a result of the pro-competitive elimination of double marginalisation; (iii) the anti-competitive effects of vertical mergers are only ever indirect since there is not the direct elimination of a competitor as in horizontal mergers; and (iv) assessing when a merger is anti-competitive is difficult since the aforementioned effects work concurrently and arise from the same source.

The methodology developed in chapter 2 enables the direct comparison of predicted merger effects from different models. Since we employ calibrated simulation models, we are able to assess the pro- and anti-competitive effects by observing the full equilibrium. An additional calibration enabled a further comparison of merger effects for different measures of substitutability. As a first measure, we considered aggregate elasticity (as in chapter 2), which concerns the substitutability of the inside goods with the outside good. A second substitutability measure considered is the nest strength of the demand function, which concerns the substitutability between the inside goods (a weak nest corresponding to bad substitutes, and a strong nest to good substitutes).

In a linear pricing vertical contracting setting in a 1×2 industry structure, the predicted competitiveness depends on the level of substitutability. For both measures of substitutability, a vertical merger is pro-competitive when the cross-price elasticity is low. At low cross-price elasticity, negotiated wholesale prices are lower resulting in a smaller increase in the rival's wholesale price post-merger.

The pro-competitive elimination of double marginalisation effect then dominates, resulting in a welfare-enhancing vertical merger.

The results in a 2×1 industry structure for linear pricing show a pro-competitive vertical merger for all levels of the substitutability measures. However, the 2×1 structure corresponds to the 1×2 structure in that the elimination of double marginalisation effect dominates more when cross-price elasticity is low.

A vertical merger implies that the merging firms are able to achieve the joint profit maximising outcome. We especially expect firms to be able to achieve this in a two-part pricing setting, since there are two instruments with which to achieve profit maximising goals. In the 2×1 industry in the two-part pricing setting, our results show that firms do operate at the joint profit maximising level pre- and post-merger. However, the results for the 1×2 industry yields interesting insights.

Following a vertical merger in the Nash-in-Shapley two-part pricing model in a 1×2 industry structure, we observed total quantity moving away from joint profit maximising towards a more competitive outcome. We ascribe this to the pivotal player inheriting the marginal cost from its vertically integrated downstream firm. This leads to an increase in the quantity produced by the vertically integrated firm post-merger, and the total quantity subsequently increases.

In contrast, the post-merger equilibrium in the Nash-in-Nash setting shows a move towards the joint profit maximising outcome. However, the post-merger equilibrium output is still above the joint profit maximising output. Following a vertical merger, the incentive of the upstream firm to internalise competition between its vertically integrated downstream firm and the rival firm is eliminated. Therefore, we observe substantial raising of rivals' cost, resulting in an anti-competitive merger.

The results presented in chapter 3 provide useful insights to practitioners. They illustrate that modelling choices such as specifying the industry structure or *object* and *nature* of vertical contracting could predetermine post-merger outcomes. They also highlighted the importance of the relationship between vertical merger effects and elasticities. The results showed that because vertical mergers alter cost structures in industries, the relationship with elasticities can change. Therefore, the characterisation of substitutability can be a crucial modelling choice.

5.3 A simulation tool for vertical merger screening

Predicting the likely effects of a vertical merger is challenging. Often the data required to inform a modelling choice between assumptions that distinguish bargaining models from one another is not available. The insights and lessons presented in chapters 2 and 3 are then of little use. To address this, we develop a tool that enables practitioners to compare predictions from different bargaining models for the parameters that are relevant to them.

Chapter 4 introduces a vertical merger simulator that is calibrated with six control variables. As a screening tool, the strength of the simulator lies in its ability to provide predictions even with limited data. Furthermore, these predictions are of the net effects of a vertical merger, and thus provides a full equilibrium result.

Given the exposition of the simulator as a screening tool, it is useful to consider how it compares with other screening tools. To this end, in chapter 4 we focus on incentive scoring methods. These simple and intuitive indices quantify the effect of merging parties, taking the cost of competing into account post-merger (Valletti and Zenger, 2021). We summarise eight indices that differ based on the good for which they calculate upward pricing pressure.

We challenge the argument frequently made in the existing literature (Moresi and Salop, 2013; Rogerson, 2020) that incentive scoring methods generally require less data than vertical merger simulation models. We present two counterarguments. First, the incentive scoring method comprises an array of indices, each tailored to measure a distinct but related part of the effect of a vertical merger. A practitioner wanting to ascertain the effect of a merger on prices other than just the wholesale price to the rival firm (for example, the downstream price of the rival firm or integrated firm) would have to calculate an index for each such price. Since each index has its own data requirements, the data necessary for such an analysis can accumulate quickly. Second, when we compared the data requirements for any of the incentive scoring indices with those of the vertical merger simulator, we found no conclusive evidence that the indices generally require less data.

Ultimately, "economists have to be realistic about what can and cannot be quantified in an informative and reliable manner," (Shapiro, 2021). The simulator and incentive scoring methods both have extensive data requirements and assumptions which can be restrictive. We view the full equilibrium focus of the simulator, as opposed to the partial equilibrium focus of incentive scoring methods, as an advantage. However, the aim of chapter 4 is not to discredit any screening tools or advocate for the use of one tool over the other. The sole aim is to add an complementary (rather than substitute) tool to the toolkit available to practitioners. The onus is then on practitioners to assess the merits of the case before them, and choose the appropriate tools.

5.4 Future research

The research presented in this dissertation fits within the framework of the literature on *inter alia* bargaining theory, contract theory, vertical mergers, merger screening methods and simulation models. In this regard, our research suggests several further avenues for future research.

A natural extension of the work in chapters 2 and 3 would be to consider more complex industry structures. As discussed in chapter 2, this dissertation restricts its attention to two simple industry structures. As argued previously, this focus offers several advantages. Firstly, as we use the same set of parameters to calibrate all the models, simple industry structures allow direct comparisons, enabling us to attribute differences in outcomes to assumptions about vertical contracting. Secondly, when considering vertical mergers, these simple industry structures mean that we only have to consider the vertical contracting between the vertically integrated and rival firms post-merger. This simplifies post-merger analysis and makes model outcomes more tractable.

The Nash-in-Nash linear pricing as well as the Nash-in-Nash and Nash-in-Shapley two-part pricing models are computable for $M \times N$ environments (i.e. M upstream firms and N downstream firms). However, computing the equilibrium for the Nash-in-Shapley linear pricing model requires solving a system of non-linear equations. The convergence to equilibrium of the Nash-in-Shapley linear pricing model will depend on the appropriate starting values. In this regard, their existence or uniqueness are not guaranteed. Future research on this matter could decrease the costs of comparative static exercises in more complex bargaining environments[1].

[1] The discussion presented in the paragraphs on extending the industry structures is based on email correspondence with Prof Froeb.

Furthermore, the vertical merger simulator introduced in chapter 4, also creates several research opportunities. For example, the simulator enables comparative studies (such as those presented in chapters 2 and 3), with any one of the six control variables at the centre of the research question (for example, the price balance between the merging and rival downstream firms). The simulator accommodates six different models across two industry structures, with six control variables, allowing for a variety of combinations.

Further research is also necessary on how incentive scoring methods and merger simulation models can be employed collaboratively. This could be pertinent for practitioners wishing to present complementary results from an array of quantitative techniques.

Appendices

Appendix A

Derivation of Analytical Results of Bargaining Models

Below we proceed to derive the analytical results for each of the eight models included in chapter 2. The exposition of the models is adapted from Tschantz (2019) which contains the write-up of a bargaining comparison tool. The simulations are built on the static models presented on this online tool available at https://daag.shinyapps.io/b1x2/. These analytical results are the equations that are calibrated as presented in section 2.4 to yield the results in section 2.5.

A.0.1 Benchmark models: Perfect competition and Monopoly

To allow us to ascertain by how much assumptions about vertical contracting matter, we consider two benchmarks models. This provides the anchors against which model predictions can be compared. First we consider a model of perfect competition: in the 1×2 case, downstream firms acquire products at the upstream firm's marginal cost with competition for consumers producing Nash equilibrium pricing. In the 2×1 case, we imagine upstream firms competing for final consumers through a "transparent" downstream outlet (Froeb et al., 2017).

Secondly, we consider the monopoly model, where prices to consumers are set to maximise the total profit of all firms.

For the 1×2 case in both the perfect competition and monopoly models, given marginal wholesale prices w_1 and w_2 and demands q_1 and q_2 as functions of retail prices, retail price p_1 is set to maximise $\pi_1 = (p_1 - mc_1 - w_1)q_1$ simultaneously with retail price p_2 set to maximise $\pi_2 = (p_2 - mc_2 - w_2)q_2$, or so the first-order conditions

$$
\begin{aligned}
0 &= q_1 + (p_1 - mc_1 - w_1)\frac{\partial q_1}{\partial p_1} \\
0 &= q_2 + (p_2 - mc_2 - w_2)\frac{\partial q_2}{\partial p_2}
\end{aligned}
\tag{A.1}
$$

are satisfied. The competition model yields $w_1 = w_2 = mc_A$.

For the monopoly case, retail prices should maximise total profit

$$\pi_{tot} = (p_1 - mc_1 - mc_A)q_1 + (p_2 - mc_2 - mc_A)q_2$$

or equivalently satisfy the first-order conditions

$$0 = q_1 + (p_1 - mc_1 - mc_A)\frac{\partial q_1}{\partial p_1} + (p_2 - mc_2 - mc_A)\frac{\partial q_2}{\partial p_1} \tag{A.2}$$
$$0 = q_2 + (p_1 - mc_1 - mc_A)\frac{\partial q_1}{\partial p_2} + (p_2 - mc_2 - mc_A)\frac{\partial q_2}{\partial p_2}$$

with the reported marginal wholesale prices determined by conditions (A.1). The fees are then determined by $f_1 = \pi_1$ and $f_2 = \pi_2$ so no profit is left at downstream firms.

In the 2×1 case for the benchmark models, given marginal wholesale prices w_A and w_B and demands q_A and q_B as functions of retail prices, retail prices p_A and p_B are set to maximise

$$\pi_1 = (p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B$$

or so the first-order conditions

$$0 = q_A + (p_A - mc_1 - w_A)\frac{\partial q_A}{\partial p_A} + (p_B - mc_1 - w_B)\frac{\partial q_B}{\partial p_A} \tag{A.3}$$
$$0 = q_B + (p_A - mc_1 - w_A)\frac{\partial q_A}{\partial p_B} + (p_B - mc_1 - w_B)\frac{\partial q_B}{\partial p_B}$$

are satisfied. For the competition model, the retail price p_A is set to maximise $\pi_{Atot} = (p_A - mc_1 - mc_A)q_A$ while simultaneously p_B is set to maximise $\pi_{Btot} = (p_B - mc_1 - mc_B)q_B$ or so the first order conditions

$$0 = q_A + (p_A - mc_1 - mc_A)\frac{\partial q_A}{\partial p_A} \tag{A.4}$$
$$0 = q_B + (p_B - mc_1 - mc_B)\frac{\partial q_B}{\partial p_B}$$

are satisfied. The marginal wholesale prices that would induce the downstream firm to set perfectly competitive prices are determined from conditions (A.3). The fees are then set to $f_A = (p_A - mc_1 - w_A)q_A$ and $f_B = (p_B - mc_1 - w_B)q_B$ leaving no profit with the downstream firm. For the monopoly model, $w_A = mc_A$ and $w_B = mc_B$ with fees set to zero.

A.0.2 Derived demand

In the derived demand 1×2 setting, the upstream firm wants to maximise its profit $\pi_A = (w_1 - mc_A)q_1 + (w_2 - mc_A)q_2$ with respect to its marginal wholesale prices w_1 and w_2 knowing that quantities q_1 and q_2 will vary as retail prices will adjust consistent with conditions (A.1). These retail pricing conditions can be implicitly differentiated to determine the pass-through rates from wholesale prices to retail prices that can be used in corresponding first-order conditions. However a slightly different strategy seems fruitful. Solve conditions (A.1) for the wholesale prices in terms of the retail prices and then treat π_A as a function of p_1 and p_2 to be maximised. The inverse pass-through rates from retail to wholesale prices are then determined by solving

$$\begin{aligned}
0 &= 2\frac{\partial q_1}{\partial p_1} + (p_1 - mc_1 - w_1)\frac{\partial^2 q_1}{\partial p_1^2} - \frac{\partial w_1}{\partial p_1}\frac{\partial q_1}{\partial p_1} \\
0 &= \frac{\partial q_2}{\partial p_1} + (p_2 - mc_2 - w_2)\frac{\partial^2 q_2}{\partial p_1 \partial p_2} - \frac{\partial w_2}{\partial p_1}\frac{\partial q_2}{\partial p_2} \\
0 &= \frac{\partial q_1}{\partial p_2} + (p_1 - mc_1 - w_1)\frac{\partial^2 q_1}{\partial p_1 \partial p_2} - \frac{\partial w_1}{\partial p_2}\frac{\partial q_1}{\partial p_1} \\
0 &= 2\frac{\partial q_2}{\partial p_2} + (p_2 - mc_2 - w_2)\frac{\partial^2 q_2}{\partial p_2^2} - \frac{\partial w_2}{\partial p_2}\frac{\partial q_2}{\partial p_2}
\end{aligned}$$

The first-order conditions for retail prices are then

$$\begin{aligned}
0 &= \frac{\partial w_1}{\partial p_1} q_1 + (w_1 - mc_A)\frac{\partial q_1}{\partial p_1} + \frac{\partial w_2}{\partial p_1} q_2 + (w_2 - mc_A)\frac{\partial q_2}{\partial p_1} \\
0 &= \frac{\partial w_1}{\partial p_2} q_1 + (w_1 - mc_A)\frac{\partial q_1}{\partial p_2} + \frac{\partial w_2}{\partial p_2} q_2 + (w_2 - mc_A)\frac{\partial q_2}{\partial p_2}
\end{aligned}$$

In the 2×1 case, the upstream firms maximise their profits in the face of a derived demand determined by the downstream firm maximising profit consistent with conditions (A.3). This means maximising $\pi_A = (w_A - mc_A)q_A$ with respect to w_A holding w_B fixed while simultaneously maximising $\pi_B = (w_B - mc_B)q_B$ with respect to w_B holding w_A fixed, in Nash equilibrium. We need the pass-through rates from wholesale prices to retail prices calculated from (A.3) by solving

$$\begin{aligned}
0 = {} & \frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A} + (p_A - mc_1 - w_A)\left(\frac{\partial^2 q_A}{\partial p_A^2}\frac{\partial p_A}{\partial w_A} + \frac{\partial^2 q_A}{\partial p_A \partial p_B}\frac{\partial p_B}{\partial w_A}\right) \\
& (p_B - mc_1 - w_B)\left(\frac{\partial^2 q_B}{\partial p_A^2}\frac{\partial p_A}{\partial w_A} + \frac{\partial^2 q_B}{\partial p_A \partial p_B}\frac{\partial p_B}{\partial w_A}\right) + \\
& \frac{\partial p_A}{\partial w_A}\frac{\partial q_A}{\partial p_A} + \frac{\partial p_B}{\partial w_A}\frac{\partial q_B}{\partial p_A} - \frac{\partial q_A}{\partial p_A} \\
0 = {} & \frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_A} + (p_A - mc_1 - w_A)\left(\frac{\partial^2 q_A}{\partial p_A \partial p_B}\frac{\partial p_A}{\partial w_A} + \frac{\partial^2 q_A}{\partial p_B^2}\frac{\partial p_B}{\partial w_A}\right) \\
& (p_B - mc_1 - w_B)\left(\frac{\partial^2 q_B}{\partial p_A \partial p_B}\frac{\partial p_A}{\partial w_A} + \frac{\partial^2 q_B}{\partial p_B^2}\frac{\partial p_B}{\partial w_A}\right) + \\
& \frac{\partial p_A}{\partial w_A}\frac{\partial q_A}{\partial p_B} + \frac{\partial p_B}{\partial w_A}\frac{\partial q_B}{\partial p_B} - \frac{\partial q_A}{\partial p_B} \\
0 = {} & \frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B} + (p_A - mc_1 - w_A)\left(\frac{\partial^2 q_A}{\partial p_A^2}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_A}{\partial p_A \partial p_B}\frac{\partial p_B}{\partial w_B}\right) \\
& (p_B - mc_1 - w_B)\left(\frac{\partial^2 q_B}{\partial p_A^2}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_B}{\partial p_A \partial p_B}\frac{\partial p_B}{\partial w_B}\right) + \\
& \frac{\partial p_A}{\partial w_B}\frac{\partial q_A}{\partial p_A} + \frac{\partial p_B}{\partial w_B}\frac{\partial q_B}{\partial p_A} - \frac{\partial q_B}{\partial p_A} \\
0 = {} & \frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B} + (p_A - mc_1 - w_A)\left(\frac{\partial^2 q_A}{\partial p_A \partial p_B}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_A}{\partial p_B^2}\frac{\partial p_B}{\partial w_B}\right) \\
& (p_B - mc_1 - w_B)\left(\frac{\partial^2 q_B}{\partial p_A \partial p_B}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_B}{\partial p_B^2}\frac{\partial p_B}{\partial w_B}\right) + \\
& \frac{\partial p_A}{\partial w_B}\frac{\partial q_A}{\partial p_B} + \frac{\partial p_B}{\partial w_B}\frac{\partial q_B}{\partial p_B} - \frac{\partial q_B}{\partial p_B}
\end{aligned}$$

The first-order conditions for the upstream competition becomes

$$0 = q_A + (w_A - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right)$$
$$0 = q_B + (w_B - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)$$

A.0.3 Nash-in-Nash linear pricing

In the 1×2 case, suppose without product 1, firm A would net π_A^* and firm 1 would make π_1^*, and similarly suppose without product 2, firm A would make π_A^{**} and firm 2 would make π_2^{**}. Then we suppose when pricing linearly, firms A and 1 would settle on a wholesale price w_1 such that final net profits π_A and π_1 would maximise $P_{A1} = (\pi_A - \pi_A^*)(\pi_1 - \pi_1^*)$ while at the same time firms A and 2 would settle on a wholesale price w_2 such that final net profits π_A and π_2 would maximise $P_{A2} = (\pi_A - \pi_A^{**})(\pi_2 - \pi_2^{**})$, these agreements in a sort of Nash equilibrium (even though A is party to both, and is thus viewed as in competition with itself). Now $\pi_A = (w_1 - mc_A)q_1 + (w_2 - mc_A)q_2$, $\pi_1 = (p_1 - mc_1 - w_1)q_1$ and $\pi_2 = (p_2 - mc_2 - w_2)q_2$ where retail prices and quantities are determined by downstream Nash competition satisfying conditions (A.1). Again, we might back out the marginal costs assuming given eventual retail prices, but then we need to account for how p_1 and p_2 move together varying w_1 but holding w_2 fixed and vice-versa. Instead supposing pass-through rates from wholesale to retail rates, the first-order conditions for Nash bargaining solutions in the linear pricing scenario are

$$\begin{aligned}
0 = &\Bigg(q_1 + (w_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) + \\
&\qquad (w_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right)\Bigg)\cdot \\
&\quad ((p_1 - mc_1 - w_1)q_1 - \pi_1^*) + \\
&((w_1 - mc_A)q_1 + (w_2 - mc_A)q_2 - \pi_A^*)\cdot \\
&\quad \left(\left(\frac{\partial p_1}{\partial w_1} - 1\right)q_1 + (p_1 - mc_1 - w_1)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right)\right) \\
0 = &\Bigg(q_2 + (w_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right) + \\
&\qquad (w_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)\Bigg)\cdot \\
&\quad ((p_2 - mc_2 - w_2)q_2 - \pi_2^{**}) + \\
&((w_1 - mc_A)q_1 + (w_2 - mc_A)q_2 - \pi_A^{**})\cdot \\
&\quad \left(\left(\frac{\partial p_2}{\partial w_2} - 1\right)q_2 + (p_2 - mc_2 - w_2)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)\right)
\end{aligned} \tag{A.5}$$

In the Nash-in-Nash scenario, threat points π_A^*, π_1^*, π_A^{**}, and π_2^{**} are assumed determined by the same wholesale prices as will be negotiated when both products are available. Let p_2^* maximise $\pi_2^* = (p_2^* - mc_2 - w_2)q_2^*$ where q_2^* is q_2 at this price when product 1 is unavailable. The first-order condition is then

$$0 = q_2^* + (p_2^* - mc_2 - w_2)\frac{\partial q_2^*}{\partial p_2^*}$$

Then $\pi_1^* = 0$, and $\pi_A^* = (w_2 - mc_A)q_2^*$. Likewise, let p_1^{**} maximise $\pi_1^{**} = (p_1^{**} - mc_1 - w_1)q_1^{**}$ where q_1^{**} is q_1 at this price when product 2 is unavailable. The first-order condition is

$$0 = q_1^{**} + (p_1^{**} - mc_1 - w_1)\frac{\partial q_1^{**}}{\partial p_1^{**}}$$

Then $\pi_2^{**} = 0$ and $\pi_A^{**} = (w_1 - mc_A)q_1^{**}$.

In the 2×1 setting, suppose without product A, firm A would make π_A^* and firm 1 would make π_1^*, and similarly suppose without product B, firm B would make π_B^{**} and firm 1 would make π_1^{**}. Then for linear pricing we suppose firms A and 1 would settle on a wholesale price w_A such that final net profits π_A and π_1 would maximise $P_{A1} = (\pi_A - \pi_A^*)(\pi_1 - \pi_1^*)$ while at the same time firms B and 1 would settle on a wholesale price w_B such that final net profits π_B and π_1 would maximise $P_{B1} = (\pi_B - \pi_B^{**})(\pi_1 - \pi_1^{**})$. Now $\pi_A = (w_A - mc_A)q_A$, $\pi_B = (w_B - mc_B)q_B$, and $\pi_1 = (p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B$ where retail prices and quantities are determined by the downstream firm satisfying conditions (A.2). Assuming pass-through rates, the first-order conditions for the Nash bargaining solutions are

$$\begin{aligned}
0 = &\left(q_A + (w_A - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right)\right)\cdot \\
&\quad ((p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B - \pi_1^*) + \\
&((w_A - mc_A)q_A - \pi_A^*)\cdot \\
&\quad \left(\left(\frac{\partial p_A}{\partial w_A} - 1\right)q_A + \frac{\partial p_B}{\partial w_A}q_B + \right. \\
&\qquad (p_A - mc_1 - w_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right) + \\
&\qquad \left.(p_B - mc_1 - w_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right)\right) \\
0 = &\left(q_B + (w_B - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)\right)\cdot \\
&\quad ((p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B - \pi_1^{**}) + \\
&((w_B - mc_B)q_B - \pi_B^{**})\cdot \\
&\quad \left(\frac{\partial p_A}{\partial w_B}q_A + \left(\frac{\partial p_B}{\partial w_B} - 1\right)q_B + \right. \\
&\qquad (p_A - mc_1 - w_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right) + \\
&\qquad \left.(p_B - mc_1 - w_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)\right)
\end{aligned} \tag{A.6}$$

Similar to the 1×2 setting, threat points π_A^*, π_1^*, π_B^{**}, π_1^{**} are assumed determined by the same wholesale prices as will be negotiated when both products are available. Now $\pi_A^* = 0$ and $\pi_B^{**} = 0$. Let p_B^* maximise $\pi_1^* = (p_B^* - mc_1 - w_B)q_B^*$ where q_B^* is q_B at this price when product A is unavailable. The first-order condition for this is

$$0 = q_B^* + (p_B^* - mc_1 - w_B)\frac{\partial q_B^*}{\partial p_B^*}$$

Let p_A^{**} maximise $\pi_1^{**} = (p_A^{**} - mc_1 - w_A)q_A^{**}$ where q_A^{**} is q_A at this price when product B is unavailable. The first-order condition for this is

$$0 = q_A^{**} + (p_A^{**} - mc_1 - w_A)\frac{\partial q_A^{**}}{\partial p_A^{**}}$$

A.0.4 Nash-in-Shapley linear pricing

For the Nash-in-Shapley model there's more to calculate, but what there is to calculate is somewhat simpler. In the 1×2 case, wholesale prices w_1 and w_2 are to be determined with w_1 maximising $P_{A1} = (\pi_A - \pi_A^*)(\pi_1 - \pi_1^*)$ and w_2 maximising $P_{A2} = (\pi_A - \pi_A^{**})(\pi_2 - \pi_2^{**})$ where $\pi_A = (w_1 - mc_A)q_1 + (w_2 - mc_A)q_2$, $\pi_1 = (p_1 - mc_1 - w_1)q_1$ and $\pi_2 = (p_2 - mc_2 - w_2)q_2$ at retail prices and quantities maximising π_1 and π_2 in Nash equilibrium, and π_A^*, $\pi_1^* = 0$ is the threat point without product 1 and π_A^{**}, $\pi_2^{**} = 0$ is the threat point without product 2. These threat points in turn are determined by negotiated wholesale prices w_2^* maximising $P_{A2}^* = \pi_A^* \pi_2^*$ (zero threat points) with $\pi_A^* = (w_2^* - mc_A)q_2^*$ and $\pi_2^* = (p_2^* - mc_2 - w_2^*)q_2^*$, and w_1^{**} maximising $P_{A1}^{**} = \pi_A^{**}\pi_1^{**}$ (zero threat points) with $\pi_A^{**} = (w_1^{**} - mc_A)q_1^{**}$ and $\pi_1^{**} = (p_1^{**} - mc_2 - w_1^{**})q_1^{**}$, again with implied retail prices and quantities. The first-order conditions are

$$\begin{aligned}
0 = &\left(q_1 + (w_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) + \right.\\
&\left.(w_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right)\right)\cdot\\
&((p_1 - mc_1 - w_1)q_1 - \pi_1^*) +\\
&((w_1 - mc_A)q_1 + (w_2 - mc_A)q_2 - \pi_A^*)\cdot\\
&\left(\left(\frac{\partial p_1}{\partial w_1} - 1\right)q_1 + (p_1 - mc_1 - w_1)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right)\right)\\
0 = &\left(q_2 + (w_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right) + \right.\\
&\left.(w_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)\right)\cdot\\
&((p_2 - mc_2 - w_2)q_2 - \pi_2^{**}) +\\
&((w_1 - mc_A)q_1 + (w_2 - mc_A)q_2 - \pi_A^{**})\cdot\\
&\left(\left(\frac{\partial p_2}{\partial w_2} - 1\right)q_2 + (p_2 - mc_2 - w_2)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)\right)\\
0 = &\left(q_2^* + (w_2^* - mc_A)\frac{\partial q_2^*}{\partial p_2^*}\frac{\partial p_2^*}{\partial w_2^*}\right)\cdot(p_2^* - mc_2 - w_2^*)q_2^* +\\
&(w_2^* - mc_A)q_2^*\cdot\left(\left(\frac{\partial p_2^*}{\partial w_2^*} - 1\right)q_2^* + (p_2^* - mc_2 - w_2^*)\frac{\partial q_2^*}{\partial p_2^*}\frac{\partial p_2^*}{\partial w_2^*}\right)\\
0 = &\left(q_1^{**} + (w_1^{**} - mc_A)\frac{\partial q_1^{**}}{\partial p_1^{**}}\frac{\partial p_1^{**}}{\partial w_1^{**}}\right)\cdot(p_1^{**} - mc_1 - w_1^{**})q_1^{**} +\\
&(w_1^{**} - mc_A)q_1^{**}\cdot\left(\left(\frac{\partial p_1^{**}}{\partial w_1^{**}} - 1\right)q_1^{**} + (p_1^{**} - mc_1 - w_1^{**})\frac{\partial q_1^{**}}{\partial p_1^{**}}\frac{\partial p_1^{**}}{\partial w_1^{**}}\right)
\end{aligned}$$

In the 2×1 case, wholesale prices w_A and w_B are to be determined with w_A maximising $P_{A1} = (\pi_A - \pi_A^*)(\pi_1 - \pi_1^*)$ and w_B maximising $P_{B1} = (\pi_B - \pi_B^{**})(\pi_1 - \pi_1^{**})$ where $\pi_A = (w_A - mc_A)q_A$, $\pi_B = (w_B - mc_B)q_B$, and $\pi_1 = (p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B$ at retail prices and quantities maximising π_1, and $\pi_A^* = 0$, π_1^* is the threat point without product A and $\pi_B^{**} = 0$, π_1^{**} is the threat point without product B. These threat points in turn are determined by negotiated wholesale prices w_B^* maximising $P_{B1}^* = \pi_B^* \pi_1^*$ (zero threat points) with $\pi_B^* = (w_B^* - mc_B)q_B^*$ and $\pi_1^* = (p_B^* - mc_1 - w_B^*)q_B^*$, and w_A^{**} maximising $P_{A1}^{**} = \pi_A^{**}\pi_1^{**}$ (zero threat points) with $\pi_A^{**} = (w_A^{**} - mc_A)q_A^{**}$ and $\pi_1^{**} =$

$(p_A^{**} - mc_1 - w_A^{**})q_A^{**}$, again with implied retail prices and quantities. The first-order conditions are

$$0 = \left(q_A + (w_A - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right)\right) \cdot$$
$$((p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B - \pi_1^*) +$$
$$((w_A - mc_A)q_A - \pi_A^*) \cdot$$
$$\left(\left(\frac{\partial p_A}{\partial w_A} - 1\right) q_A + \frac{\partial p_B}{\partial w_A} q_B + \right.$$
$$(p_A - mc_1 - w_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right) +$$
$$\left.(p_B - mc_1 - w_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right)\right)$$
$$0 = \left(q_B + (w_B - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)\right) \cdot$$
$$((p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B - \pi_1^{**}) +$$
$$((w_B - mc_B)q_B - \pi_B^{**}) \cdot$$
$$\left(\frac{\partial p_A}{\partial w_B} q_A + \left(\frac{\partial p_B}{\partial w_B} - 1\right) q_B + \right.$$
$$(p_A - mc_1 - w_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right) +$$
$$\left.(p_B - mc_1 - w_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)\right)$$

$$0 = \left(q_B^* + (w_B^* - mc_B)\frac{\partial q_B^*}{\partial p_B^*}\frac{\partial p_B^*}{\partial w_B^*}\right) \cdot (p_B^* - mc_1 - w_B^*)q_B^* +$$
$$(w_B^* - mc_B)q_B^* \cdot \left(\left(\frac{\partial p_B^*}{\partial w_B^*} - 1\right) q_B^* + (p_B^* - mc_1 - w_B^*)\frac{\partial q_B^*}{\partial p_B^*}\frac{\partial p_B^*}{\partial w_B^*}\right)$$
$$0 = \left(q_A^{**} + (w_A^{**} - mc_A)\frac{\partial q_A^{**}}{\partial p_A^{**}}\frac{\partial p_A^{**}}{\partial w_A^{**}}\right) \cdot (p_A^{**} - mc_1 - w_A^{**})q_A^{**} +$$
$$(w_A^{**} - mc_A)q_A^{**} \cdot \left(\left(\frac{\partial p_A^{**}}{\partial w_A^{**}} - 1\right) q_A^{**} + (p_A^{**} - mc_1 - w_A^{**})\frac{\partial q_A^{**}}{\partial p_A^{**}}\frac{\partial p_A^{**}}{\partial w_A^{**}}\right)$$

A.0.5 Nash-in-Nash two-part pricing

In two-part pricing, parties will agree to marginal wholesale pricing and fees, and these agreements also become the basis for the threat points to agreement. For a Nash bargaining solution with transferable utility, one can view this as maximising the total surplus over the threat point with this surplus then split so each party benefits equally over the threat point.

Consider first the 1×2 industry structure. Suppose marginal wholesale prices w_1 and w_2 are agreed to, so that downstream firms can determine how to set retail price, thus determining total operating profits of each firm to be split, in both the two agreement case and in the alternatives to agreement. Suppose operating profits at the given marginal wholesale prices are π_A^o, π_1^o, and π_2^o with both agreements, and for A, π_A^{o*} if product 2 is sold but product 1 is unavailable, and π_A^{0**} if product 1 is sold but product 2 is unavailable. The fees f_1 and f_2 are supposed to be set so the final net profits are a fair split over the threat points, but the threat points aren't fixed and also depend on the fees. We have $\pi_A = \pi_A^o + f_1 + f_2$, $\pi_1 = \pi_1^o - f_1$, $\pi_2 = \pi_2^o - f_2$, $\pi_A^* = \pi_A^{o*} + f_2$, $\pi_1^* = 0$, $\pi_A^{**} = \pi_A^{o**} + f_1$, and $\pi_2^{**} = 0$. Then we require

$$\pi_A - \pi_A^* = \pi_1 - \pi_1^* \text{ and } \pi_A - \pi_A^{**} = \pi_2 - \pi_2^{**}$$

so

$$\pi_A^o - \pi_A^{o*} + f_1 = \pi_1^o - f_1 \text{ and } \pi_A^o - \pi_A^{o**} + f_2 = \pi_2^o - f_2$$

hence

$$\begin{aligned} f_1 &= \frac{1}{2}(\pi_1^o - \pi_A^o + \pi_A^{o*}) \\ f_2 &= \frac{1}{2}(\pi_2^o - \pi_A^o + \pi_A^{o**}) \\ \pi_A &= \frac{1}{2}(\pi_1^o + \pi_2^o + \pi_A^{o*} + \pi_A^{o**}) \\ \pi_1 &= \frac{1}{2}(\pi_1^o + \pi_A^o - \pi_A^{o*}) \\ \pi_2 &= \frac{1}{2}(\pi_2^o + \pi_A^o - \pi_A^{o**}) \end{aligned}$$

So the fees can be assigned to split surplus consistently, but it is clear that, even treating the operating profits in the alternatives as fixed and not changing with the negotiated marginal wholesale prices, the firms do not have the same interests in setting those marginal wholesale prices. While 1 wants to set w_1 to maximise $\pi_A^o + \pi_1^o$, A has a slightly different interest because of his interest in sales through both outlets, i.e., because of an externality to agreement $A1$, the sales through 2, A would actually like to maximise $\pi_1^o + \pi_2^o$, Of course, this is neglecting the impact of negotiated w_1 on the threat point for agreement $A2$. We must simply assume firms A and 1 are naively choosing w_1 to maximise $\pi_A^o + \pi_1^o$ assuming w_2 is fixed, while simultaneously A and 2 are choosing w_2 to maximise $\pi_A^o + \pi_2^o$ assuming w_1 is fixed, in Nash equilibrium, even though this puts A in competition with itself. This does allow A and 1 to account for the effect of A's loss of sales through 2, it just does not allow for coordinating marginal wholesale pricing.

The first-order conditions on w_1 maximising $\pi_A^o + \pi_1^o = (p_1 - mc_1 - mc_A)q_1 + (w_2 - mc_A)q_2$ and w_2 maximising $\pi_A^o + \pi_2^o = (w_1 - mc_A)q_1 + (p_2 - mc_2 - mc_A)q_2$ is then

$$\begin{aligned} 0 = \frac{\partial p_1}{\partial w_1} q_1 + (p_1 - mc_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) + \\ (w_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) \\ 0 = \frac{\partial p_1}{\partial w_2} q_2 + (w_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) + \\ (p_2 - mc_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) \end{aligned}$$

For this w_1 and w_2, the operating profit $\pi_A^{o*} = (w_2 - mc_A)q_2^*$ is determined by 2 maximising $\pi_2^{o*} = (p_2^* - mc_2 - w_2)q_2^*$ while the operating profit $\pi_A^{o**} = (w_1 - mc_A)q_1^{**}$ is determined by 1 maximising $\pi_1^{o**} = (p_A^{**} - mc_1 - w_1)q_1^{**}$.

In the 2×1 case, suppose w_A and w_B are set, determining retail pricing, quantities, and operating profits π_A^o, π_B^o, and π_1^o, and for 1 in the alternatives π_1^{o*} if product A is unavailable and π_1^{o**} if product B is unavailable. We solve for fees f_A and f_B so with $\pi_A = \pi_A^o + f_A$, $\pi_B = \pi_B^o + f_B$, $\pi_1 = \pi_1^o - f_A - f_B$,

$\pi_1^* = \pi_1^{o*} - f_B$, $\pi_A^* = 0$, $\pi_1^{**} = \pi_1^{o**} - f_A$, and $\pi_B^{**} = 0$, we have $\pi_A - \pi_A^* = \pi_1 - \pi_1^*$ and $\pi_B - \pi_B^{**} = \pi_1 - \pi_1^{**}$. Then

$$\begin{aligned} f_A &= \frac{1}{2}(\pi_1^o - \pi_1^{o*} - \pi_A^o) \\ f_B &= \frac{1}{2}(\pi_1^o - \pi_1^{o**} - \pi_B^o) \\ \pi_A &= \frac{1}{2}(\pi_1^o + \pi_A^o - \pi_1^{o*}) \\ \pi_B &= \frac{1}{2}(\pi_1^o + \pi_B^o - \pi_1^{o**}) \\ \pi_1 &= \frac{1}{2}(\pi_A^o + \pi_B^o + \pi_1^{o*} + \pi_1^{o**}) \end{aligned}$$

Again, though 1 is competing against itself, we suppose w_A and w_B are set in Nash equilibrium, w_A maximising the total of $\pi_A^o = (w_A - mc_A)q_A$ and $\pi_1^o = (p_A - mc_1 - w_A)q_A + (p_B - mc_1 - w_B)q_B$ simultaneously with w_B maximising the total of $\pi_B^o = (w_B - mc_B)q_B$ and π_1^o, where retail prices and quantities are determined maximising π_1^o. The first order conditions on w_A and w_B are then

$$\begin{aligned} 0 = & \frac{\partial p_A}{\partial w_A} q_A + \frac{\partial p_B}{\partial w_A} q_B + (p_A - mc_1 - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right) + \\ & (p_B - mc_1 - w_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right) \\ 0 = & \frac{\partial p_A}{\partial w_B} q_A + \frac{\partial p_B}{\partial w_B} q_B + (p_A - mc_1 - w_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right) + \\ & (p_B - mc_1 - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right) \end{aligned}$$

For this w_A and w_B, the operating profit $\pi_1^{o*} = (p_B - mc_1 - w_B)q_B^*$ is determined by 1 maximising $\pi_1^{o*} = (p_B^* - mc_1 - w_B)q_B^*$ while the operating profit $\pi_1^{o**} = (p_A - mc_1 - w_A)q_A^{**}$ is determined by 1 maximising $\pi_1^{o**} = (p_A^{**} - mc_1 - w_A)q_A^{**}$.

A.0.6 Nash-in-Nash fixed quantity contract

Linear pricing agreements (i.e. contracts that specify a marginal wholesale price) are supposed to imply to all parties the final retail prices and quantities that will result. But specifying that quantities will remain fixed in the alternatives to agreements will be different than specifying marginal wholesale prices are held constant in alternatives in the Nash-in-Nash setting. Now it would also seem possible to specify one marginal wholesale price and fee at the desired quantities when both agreements are in force, but specify a different marginal wholesale price and fee when only one product is for sale with the same fixed variable discount contract. But the effect then is to specify marginal wholesale price and fee contingent on which agreements are reached, contrary to the spirit of Nash-in-Nash and instead appropriate for the Nash-in-Shapley model. Given quantities, we will infer what marginal wholesale prices would result in the same sales, but for the current calculations, take wholesale price to be zero and take only a fixed payment to be specified by contract.

In the 1×2 case, suppose A negotiates quantity q_1 and total price t_1 with 1 and simultaneously negotiates quantity q_2 and total price t_2 with 2. Let $\pi_A^o = -mc_A(q_1 + q_2)$, $\pi_1^o = (p_1 - mc_1)q_1$, and $\pi_2^o = (p_2 - mc_2)q_2$ be the operating profits given the retail prices implied by the quantities. Take threat point operating profits $\pi_A^{o*} = -mc_A q_2$, and $\pi_1^{o*} = 0$ without product 1, and $\pi_A^{o**} = -mc_A q_1$, and $\pi_2^{o**} = 0$ without product 2. With payments, net profits are $\pi_A = \pi_A^o + t_1 + t_2$, $\pi_1 = \pi_1^o - t_1$, $\pi_2 = \pi_2^o - t_2$, $\pi_A^* = \pi_A^{o*} + t_2$, $\pi_1^* = 0$, $\pi_A^{**} = \pi_A^{o**} + t_1$, $\pi_2^{**} = 0$. Now agreement with 1 will want to maximise $\pi_A^o + \pi_1^o$ and then split the surplus equally so $\pi_A - \pi_A^* = \pi_1 - \pi_1^*$, i.e., $\pi_A^o + t_1 - \pi_A^{o*} = \pi_1^o - t_1$, and similarly with 2 so $\pi_A^o + t_2 - \pi_A^{o**} = \pi_2^o - t_2$ so

$$\begin{aligned} t_1 &= \frac{1}{2}(\pi_1^o - \pi_A^o + \pi_A^{o*}) \\ t_2 &= \frac{1}{2}(\pi_2^o - \pi_A^o + \pi_A^{o**} \\ \pi_A &= \frac{1}{2}(\pi_1^o + \pi_2^o + \pi_A^{o*} + \pi_A^{o**}) \\ \pi_1 &= \frac{1}{2}(\pi_1^o + \pi_A^o - \pi_A^{o*}) \\ \pi_2 &= \frac{1}{2}(\pi_2^o + \pi_A^o - \pi_A^{o**}) \end{aligned}$$

The first-order conditions on q_1, maximising $\pi_A^o + \pi_1^o = (p_1 - mc_1 - mc_A)q_1 - mc_A q_2$, and q_2, maximising $\pi_A^o + \pi_2^o = -mc_A q_1 + (p_2 - mc_2 - mc_A)q_2$, accounting for inverse demand slopes, are given by

$$\begin{aligned} 0 &= \frac{\partial p_1}{\partial q_1} q_1 + (p_1 - mc_1 - mc_A) \\ 0 &= \frac{\partial p_2}{\partial q_2} q_2 + (p_2 - mc_2 - mc_A) \end{aligned}$$

Now consider the 2×1 case. Firm 1 negotiates quantity q_A for payment t_A from A and quantity q_B for payment t_B from B. Then operating profits are $\pi_A^o = -mc_A q_A$, $\pi_B^o = -mc_B q_B$, and $\pi_1^o = (p_1 - mc_1)q_A + (p_B - mc_1)q_B$ with retail prices determined from quantities. Take threat point operating profits $\pi_1^{o*} = (p_B^* - mc_1)q_B$ and $\pi_A^{o*} = 0$, for price p_B^* without product A, and $\pi_1^{o**} = (p_A^{**} - mc_1)q_A$ and $\pi_B^{o**} = 0$, for price p_A^{**} without product B. With payments, net profits are $\pi_1 = \pi_1^o - t_A - t_B$, $\pi_A = \pi_A^o + t_A$, $\pi_B = \pi_B^o + t_B$, $\pi_1^* = \pi_1^{o*} - t_B$, $\pi_A^* = 0$, $\pi_1^{**} = \pi_1^{o**} - t_A$, $\pi_B^{**} = 0$. Now agreement with A will want to maximise $\pi_A^o + \pi_1^o$ and then split the surplus equally so $\pi_A - \pi_A^* = \pi_1 - \pi_1^*$, i.e., $\pi_A^o + t_A = \pi_1^o - t_A - \pi_1^{o*}$, and similarly with B so $\pi_B^o + t_B = \pi_1^o - t_B - \pi_1^{o**}$ so

$$\begin{aligned} t_A &= \frac{1}{2}(\pi_1^o - \pi_A^o - \pi_1^{o*}) \\ t_B &= \frac{1}{2}(\pi_1^o - \pi_B^o - \pi_1^{o**}) \\ \pi_A &= \frac{1}{2}(\pi_1^o + \pi_A^o - \pi_1^{o*}) \\ \pi_B &= \frac{1}{2}(\pi_1^o + \pi_B^o - \pi_1^{o**}) \\ \pi_1 &= \frac{1}{2}(\pi_A^o + \pi_B^o + \pi_1^{o*} + \pi_1^{o**}) \end{aligned}$$

The first-order conditions on q_1, maximising $\pi_A^o + \pi_1^o = (p_A - mc_1 - mc_A)q_A + (p_B - mc_1)q_B$, and q_2, maximising $\pi_B^o + \pi_1^o = (p_A - mc_1)q_A + (p_B - mc_1 - mc_B)q_B$, accounting for inverse demand slopes, are given by

$$\begin{aligned} 0 &= \frac{\partial p_A}{\partial q_A} q_A + \frac{\partial p_B}{\partial q_A} q_B + (p_A - mc_1 - mc_A) \\ 0 &= \frac{\partial p_A}{\partial q_B} q_A + \frac{\partial p_B}{\partial q_B} q_B + (p_B - mc_1 - mc_B) \end{aligned}$$

A.0.7 Nash-in-Shapley two-part pricing

Finally, suppose that both marginal wholesale price and fee are negotiated over a threat point recursively defined by a negotiated marginal wholesale price and fee. Now suppose that the marginal wholesale price maximises the total surplus and the fees split this surplus. In the 1×2 case, negotiating w_1 and f_1 with 1 and w_2 and f_2 with 2, we require $\pi_A - \pi_A^* = \pi_1 - \pi_1^*$ and $\pi_A - \pi_A^{**} = \pi_2 - \pi_2^{**}$ where $\pi_A = \pi_A^o + f_1 + f_2$, $\pi_1 = \pi_1^o - f_1$, $\pi_2 = \pi_2^o - f_2$, with threat point $\pi_A^* = \pi_A^{o*} + f_2^*$, $\pi_1^* = 0$, without product 1 and threat point $\pi_A^{**} = \pi_A^{o**} + f_1^{**}$, $\pi_2^{**} = 0$ without product 2. The threat points are determined independently from negotiated fees f_2^* and f_1^{**} so $\pi_A^* = \pi_2^* = \pi_2^{o*} - f_2^*$ and $\pi_A^{**} = \pi_1^{**} = \pi_1^{o**} - f_1^{**}$ respectively (these with zero threat points). Thus $f_2^* = (-\pi_A^{o*} + \pi_2^{o*})/2$, and $f_1^{**} = (-\pi_A^{o**} + \pi_1^{o**})/2$, with $\pi_A^* = \pi_2^* = (\pi_A^{o*} + \pi_2^{o*})/2$ and $\pi_A^{**} = \pi_1^{**} = (\pi_A^{o**} + \pi_1^{o**})/2$, and

$$
\begin{aligned}
f_1 &= \frac{1}{3}(-\pi_A^o + 2\pi_1^o - \pi_2^o + 2\pi_A^{o*} - \pi_A^{o**}) \\
f_2 &= \frac{1}{3}(-\pi_A^o - \pi_1^o + 2\pi_2^o - \pi_A^{o*} + 2\pi_A^{o**}) \\
\pi_A &= \frac{1}{3}(\pi_A^o + \pi_1^o + \pi_2^o + \pi_A^{o*} + \pi_A^{o**}) \\
\pi_1 &= \frac{1}{3}(\pi_A^o + \pi_1^o + \pi_2^o - 2\pi_A^{o*} + \pi_A^{o**}) \\
\pi_2 &= \frac{1}{3}(\pi_A^o + \pi_1^o + \pi_2^o + \pi_A^{o*} - 2\pi_A^{o**})
\end{aligned}
$$

Now we observe that the interests of A and 1 are aligned when it comes to setting w_1. To maximise their surplus, after fees, they want to maximise the sum of operating profits $\pi_A^o + \pi_1^o + \pi_2^o = (p_1 - mc_1 - mc_A)q_1 + (p_2 - mc_2 - mc_A)q_2$ (up to a constant). We do not assume that A and 1 take some myopic point of view and maximise only their own combined operating profits. Instead, we suppose they can see past their immediate interest to what is in their common interest when A is also negotiating with 2. Whether A and 1 can write some clause into a contract to guarantee that 1 gets a fair split of whatever A can get from 2 is another matter entirely, and outside our scope, since we consider a criterion only for how agreements end up and not how they are negotiated, imagining in any case that agreement that are not balanced would be renegotiated. The same is true for A and 2 with exactly the same total operating profit at stake. The first order conditions on w_1 and w_2 are then

$$
\begin{aligned}
0 = \frac{\partial p_1}{\partial w_1} q_1 + \frac{\partial p_2}{\partial w_1} q_2 + (p_1 - mc_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) + \\
(p_2 - mc_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_1} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_1}\right) \\
0 = \frac{\partial p_1}{\partial w_2} q_1 + \frac{\partial p_2}{\partial w_2} q_2 + (p_1 - mc_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right) + \\
(p_2 - mc_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)
\end{aligned}
$$

The first-order conditions for maximising the surplus in the threat points defining w_2^* and w_1^{**} are

$$
\begin{aligned}
0 &= \frac{\partial p_2^*}{\partial w_2^*} q_2^* + (p_2^* - mc_2 - mc_A)\frac{\partial q_2^*}{\partial p_2^*}\frac{\partial p_2^*}{\partial w_2^*} \\
0 &= \frac{\partial p_1^{**}}{\partial w_1^{**}} q_1^{**} + (p_1^{**} - mc_1 - mc_A)\frac{\partial q_1^{**}}{\partial p_1^{**}}\frac{\partial p_1^{**}}{\partial w_1^{**}}
\end{aligned}
$$

In the 2×1 case, negotiating w_A and f_A with A and w_B and f_B with B, we require $\pi_A-\pi_A^* = \pi_1-\pi_1^*$ and $\pi_B - \pi_B^{**} = \pi_1 - \pi_1^{**}$ where $\pi_A = \pi_A^o + f_A$, $\pi_B = \pi_B^o + f_B$, $\pi_1 = \pi_1^o - f_A - f_B$, with threat point $\pi_A^* = 0$, $\pi_1^* = \pi_1^{o*} - f_B^*$, without product A and threat point $\pi_B^{**} = 0$, $\pi_1^{**} = \pi_1^{o**} - f_A^{**}$, without product B. The threat points are determined independently from negotiated fees f_B^* and f_A^{**} so $\pi_1^* = \pi_B^* = \pi_B^{o*} + f_B^*$ and $\pi_1^{**} = \pi_A^{**} = \pi_A^{o**} + f_A^{**}$ respectively (these with zero threat points). Thus $f_B^* = (-\pi_B^{o*} + \pi_1^{o*})/2$, and $f_A^{**} = (-\pi_A^{o**} + \pi_1^{o**})/2$, with $\pi_1^* = \pi_B^* = (\pi_B^{o*} + \pi_1^{o*})/2$ and $\pi_1^{**} = \pi_A^{**} = (\pi_A^{o**} + \pi_1^{o**})/2$, and

$$
\begin{aligned}
f_A &= \frac{1}{3}(\pi_1^o - 2\pi_A^o + \pi_B^o - 2\pi_1^{o*} + \pi_1^{o**}) \\
f_B &= \frac{1}{3}(\pi_1^o + \pi_A^o - 2\pi_B^o + \pi_1^{o*} - 2\pi_1^{o**}) \\
\pi_A &= \frac{1}{3}(\pi_1^o + \pi_A^o + \pi_B^o - 2\pi_1^{o*} + \pi_1^{o**}) \\
\pi_B &= \frac{1}{3}(\pi_1^o + \pi_A^o + \pi_B^o + \pi_1^{o*} - 2\pi_1^{o**}) \\
\pi_1 &= \frac{1}{3}(\pi_1^o + \pi_A^o + \pi_B^o + \pi_1^{o*} + \pi_1^{o**})
\end{aligned}
$$

Again, the interests of A and 1 are aligned when it comes to setting w_A. To maximise their surplus, after fees, they want to maximise the sum of operating profits $\pi_1^o+\pi_A^o+\pi_B^o = (p_A-mc_1-mc_A)q_A+(p_B - mc_1 - mc_B)q_B$ (up to a constant). The same is true for B and 1 with exactly the same total operating profit at stake. The first order conditions on w_A and w_B are then

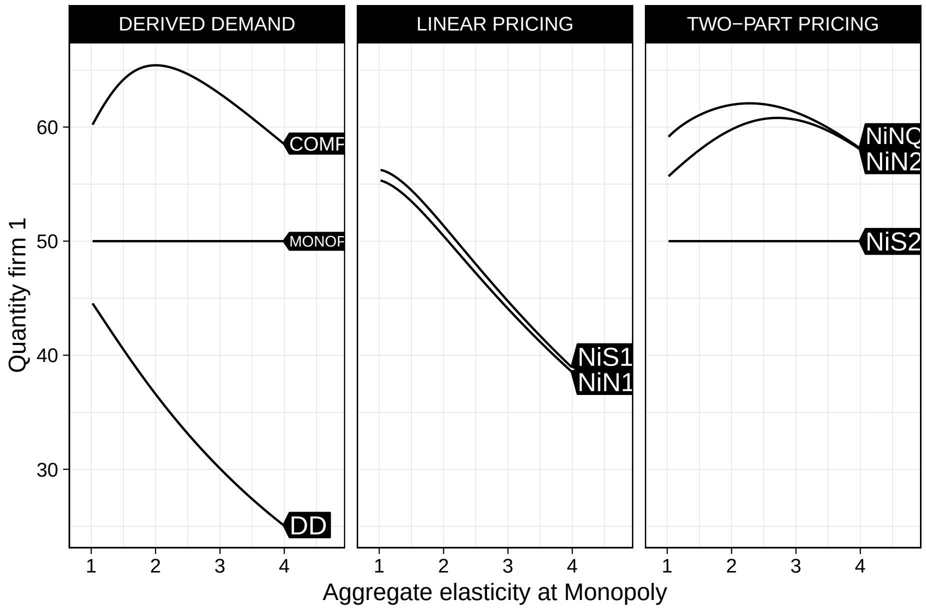
$$
\begin{aligned}
0 = \frac{\partial p_A}{\partial w_A}q_A + \frac{\partial p_B}{\partial w_A}q_B + (p_A - mc_1 - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right) + \\
(p_B - mc_1 - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_A} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_A}\right) \\
0 = \frac{\partial p_A}{\partial w_B}q_A + \frac{\partial p_B}{\partial w_B}q_B + (p_A - mc_1 - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right) + \\
(p_B - mc_1 - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)
\end{aligned}
$$

The first-order conditions for maximising the surplus in the threat points defining w_B^* and w_A^{**} are

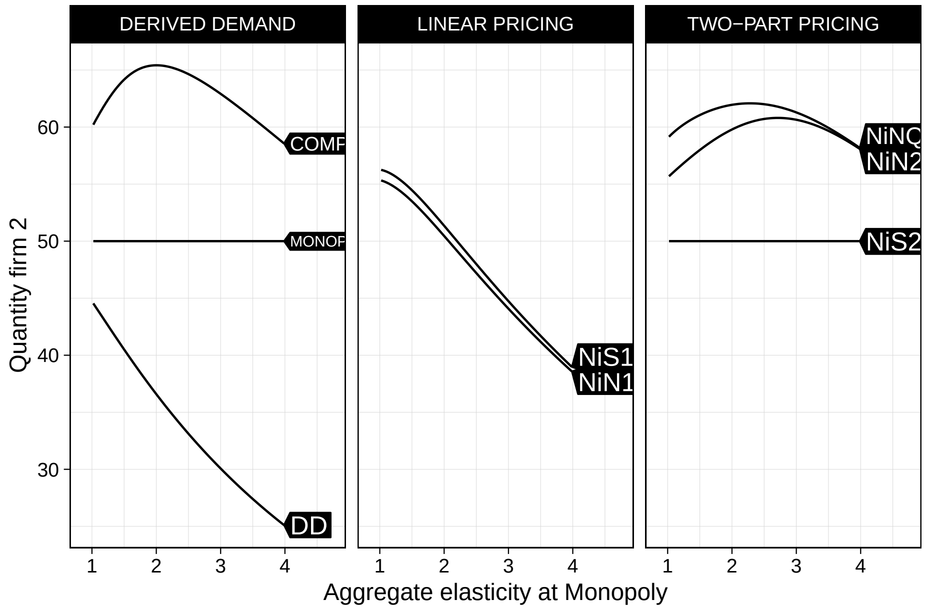
$$
\begin{aligned}
0 &= \frac{\partial p_B^*}{\partial w_B^*}q_B^* + (p_B^* - mc_1 - mc_B)\frac{\partial q_B^*}{\partial p_B^*}\frac{\partial p_B^*}{\partial w_B^*} \\
0 &= \frac{\partial p_A^{**}}{\partial w_A^{**}}q_A^{**} + (p_A^{**} - mc_1 - mc_A)\frac{\partial q_A^{**}}{\partial p_A^{**}}\frac{\partial p_A^{**}}{\partial w_A^{**}}
\end{aligned}
$$

Appendix B

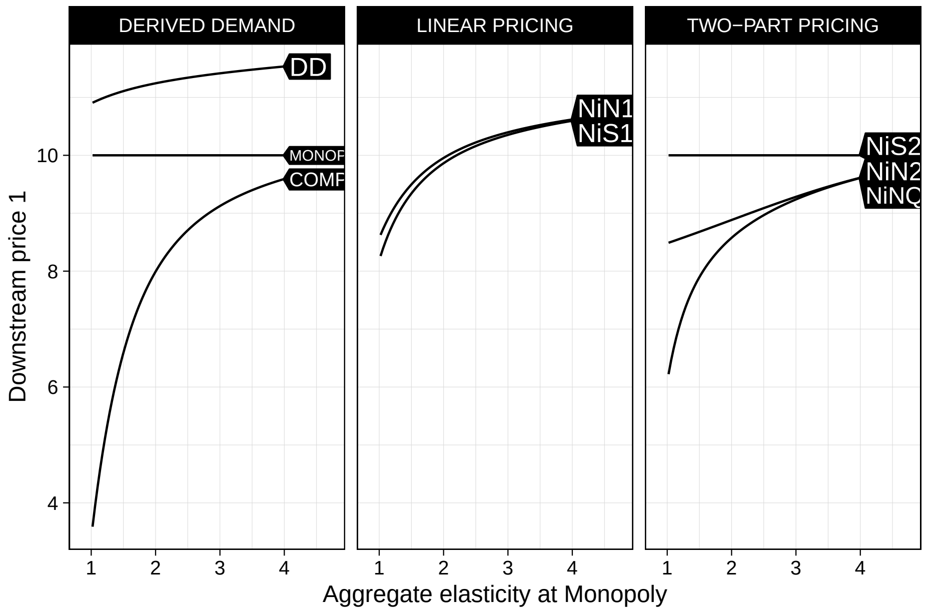
Bargaining Graphs



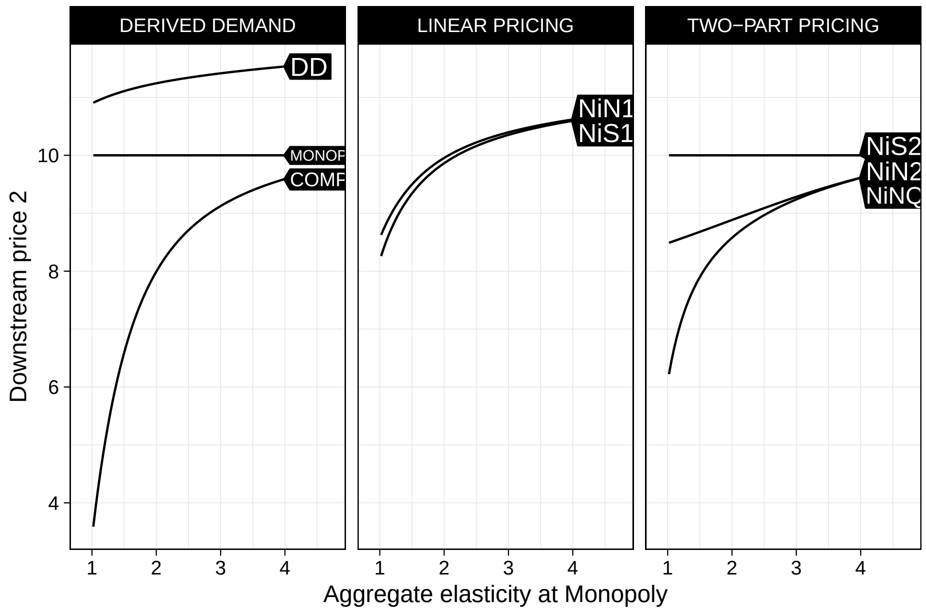
(a) Downstream quantity firm 1 vs ae



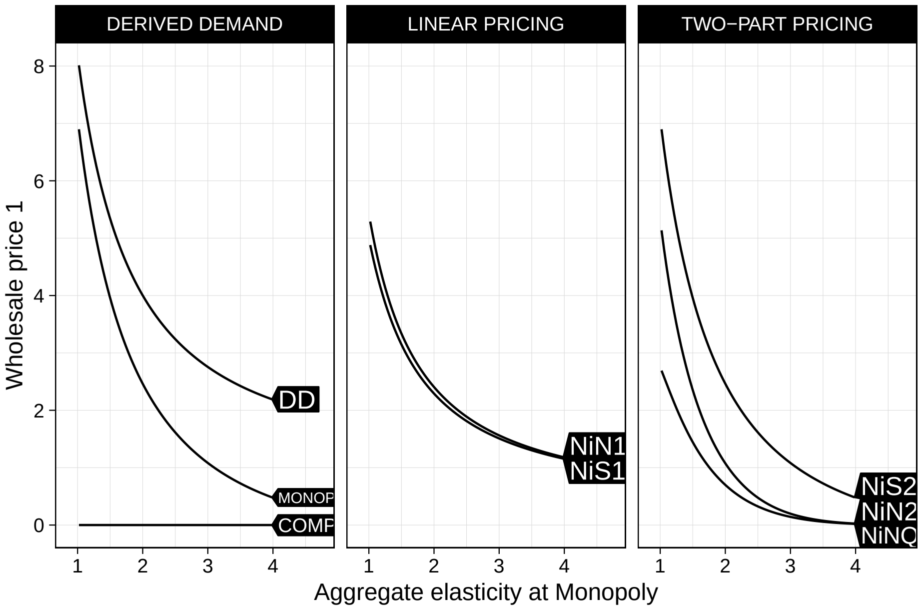
(b) Downstream quantity firm 2 vs ae



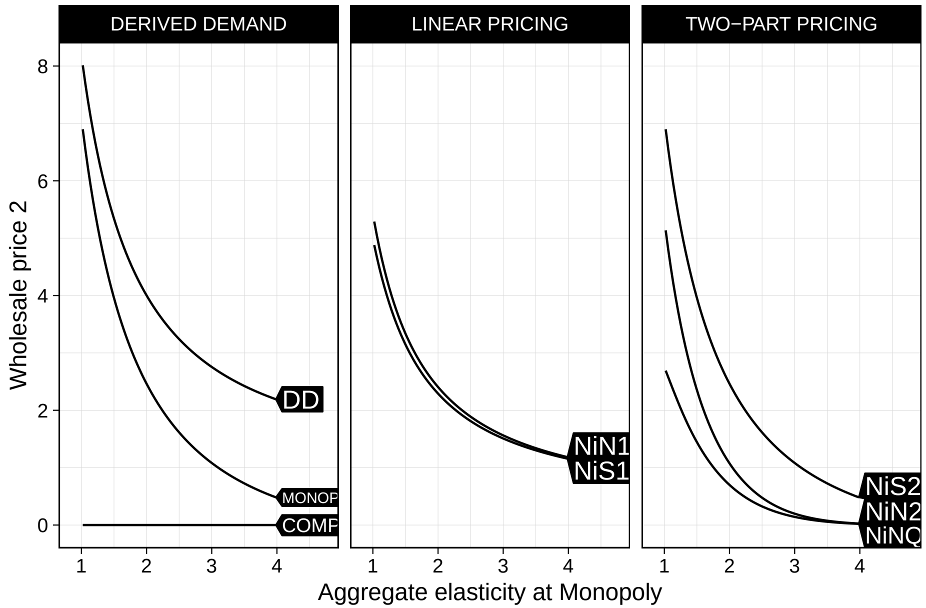
(c) Downstream price firm 1 vs ae



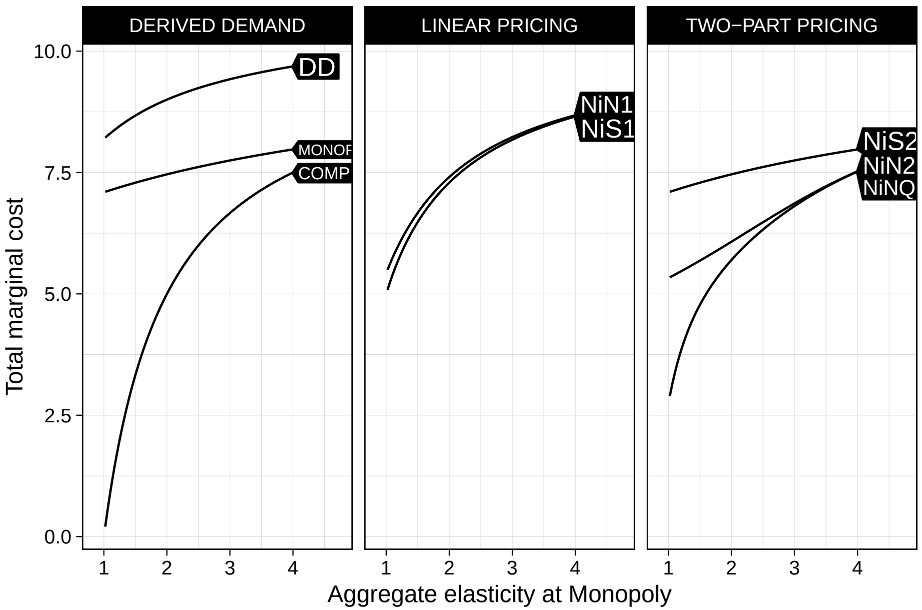
(d) Downstream price firm 2 vs ae



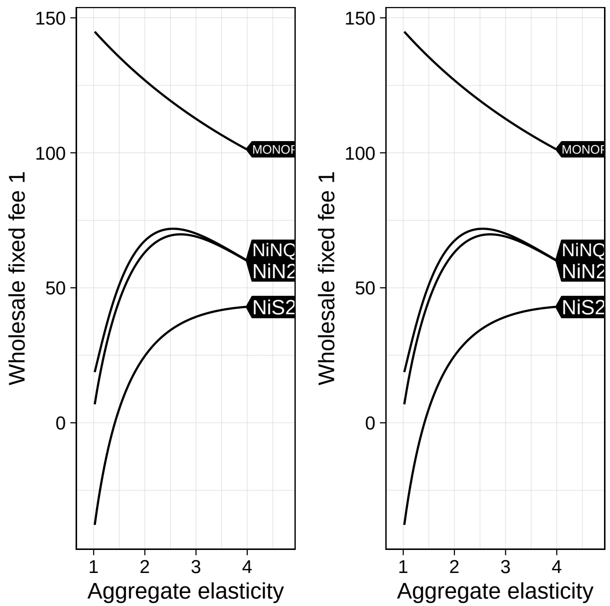
(a) Wholesale price firm 1 vs ae



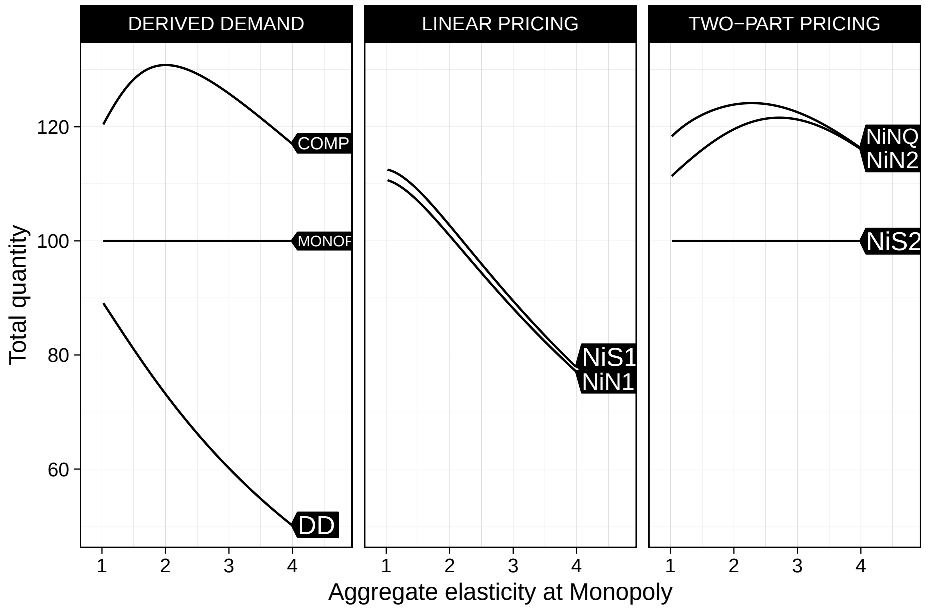
(b) Wholesale price firm 2 vs ae



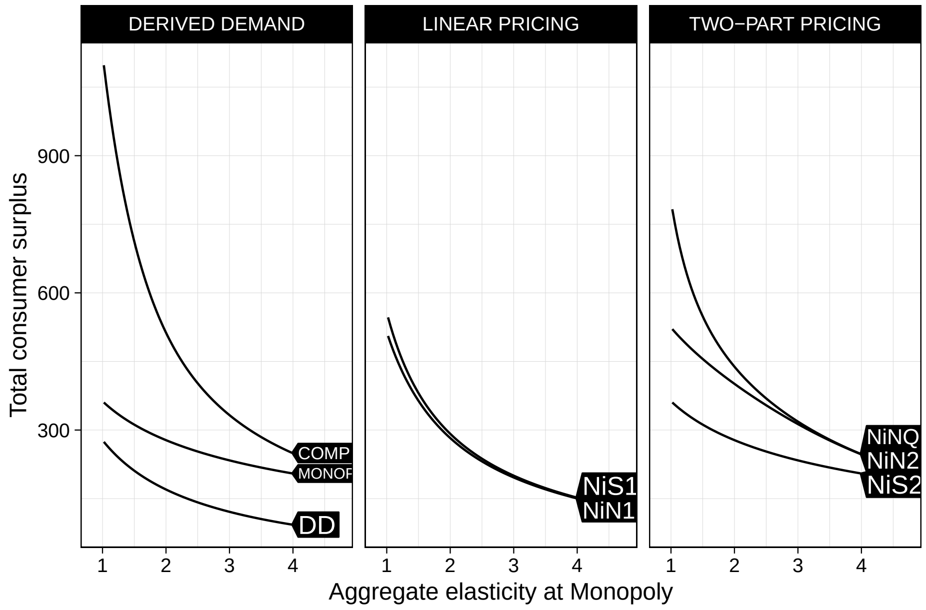
(c) Total marginal cost downstream firm 2 vs ae



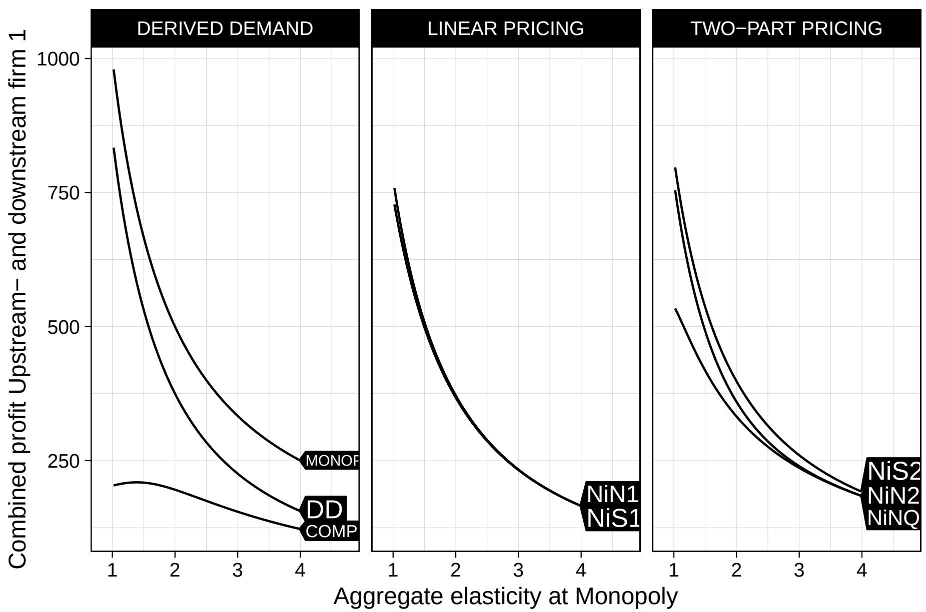
(d) Wholesale fees for firm 1 and 2



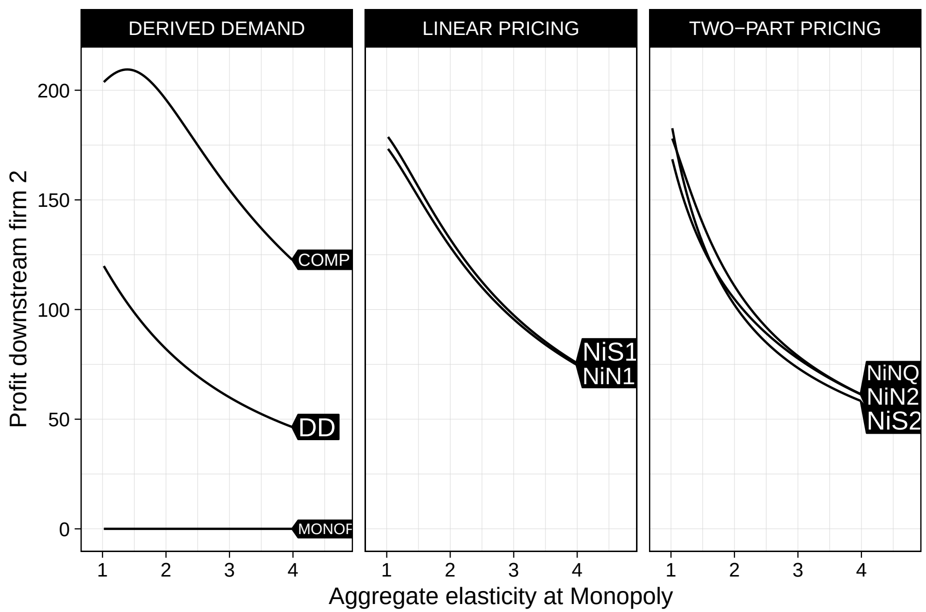
(a) Total quantity vs ae



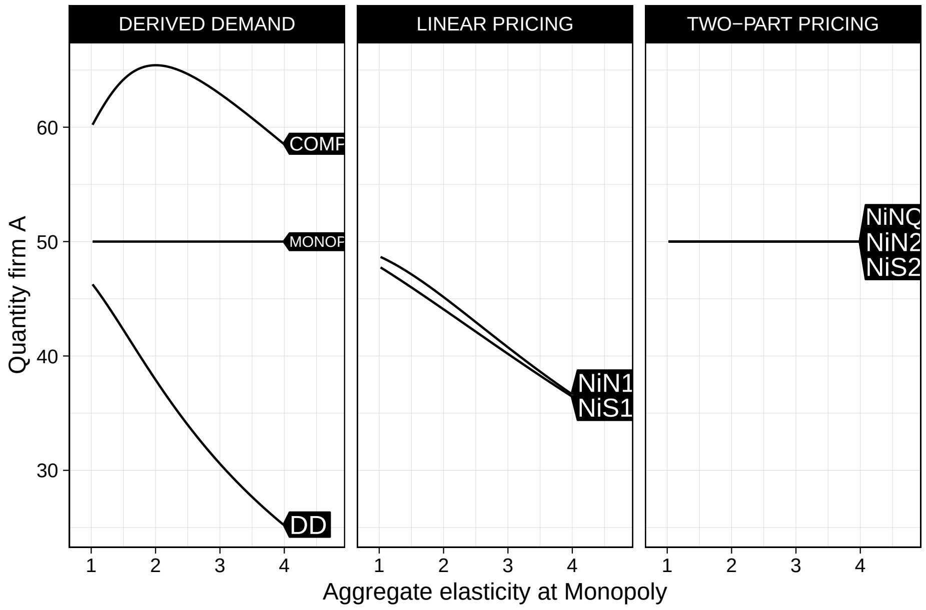
(b) Consumer surplus vs ae



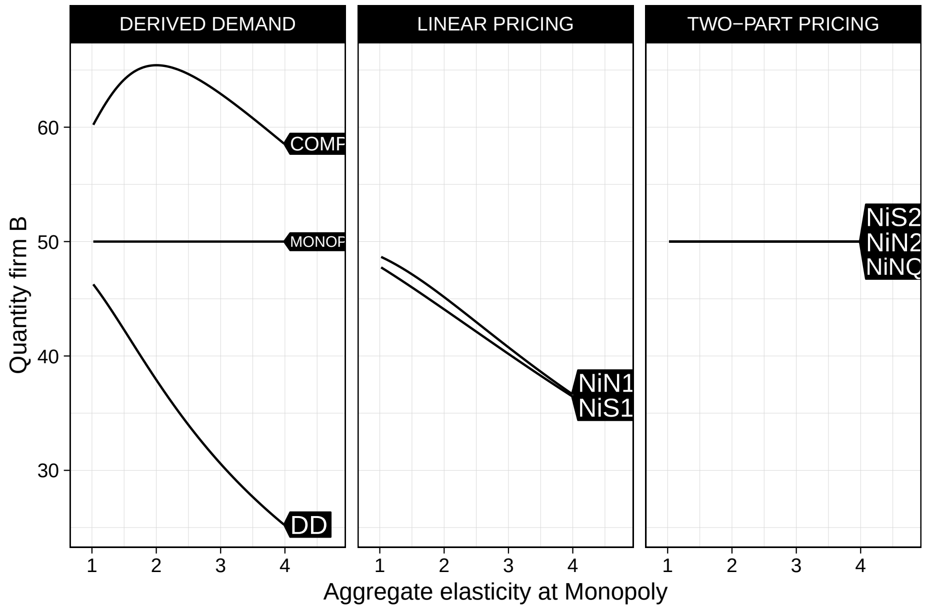
(c) Combined profits firm A and 1 vs ae



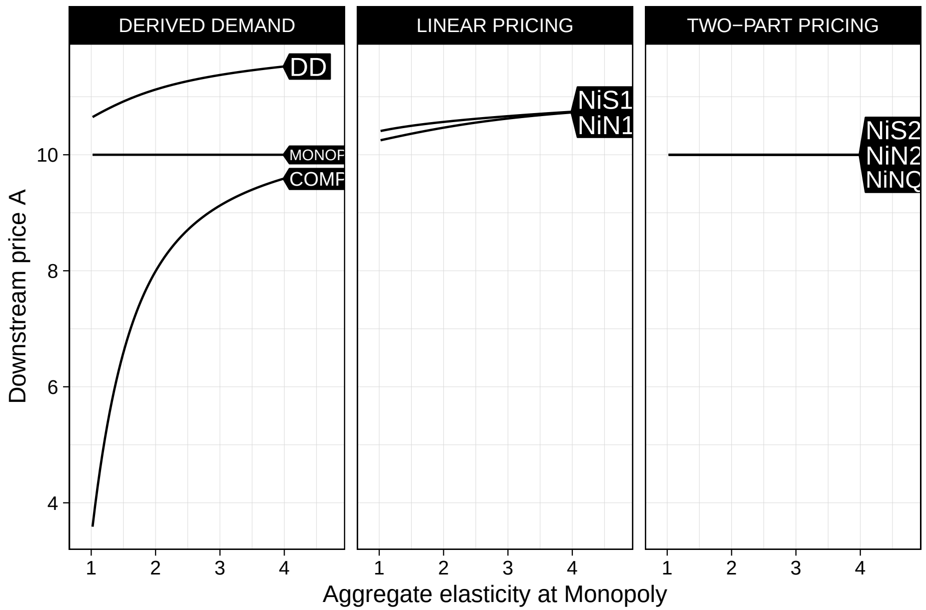
(d) Profit downstream firm 2 vs ae



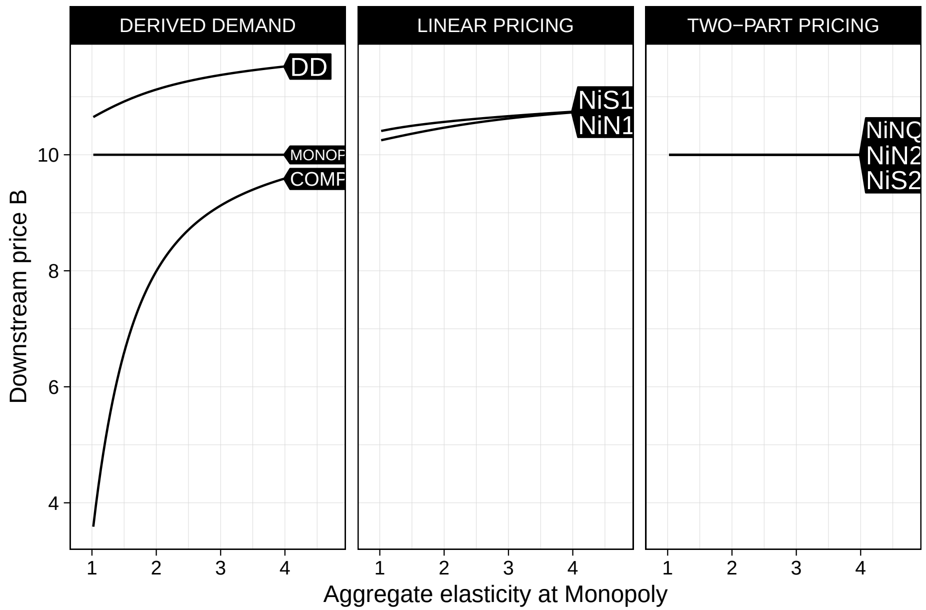
(a) Downstream quantity firm 1 vs ae



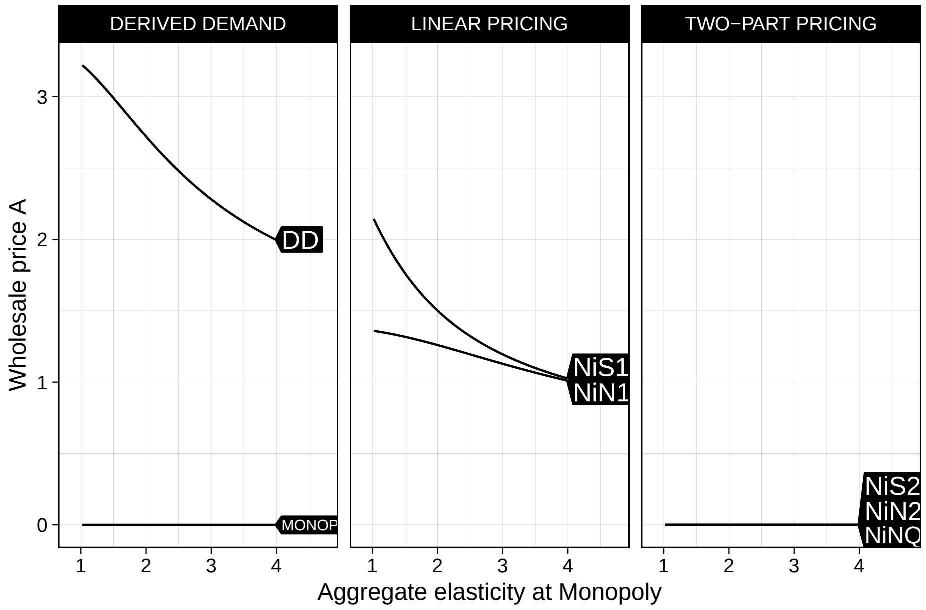
(b) Downstream quantity firm 2 vs ae



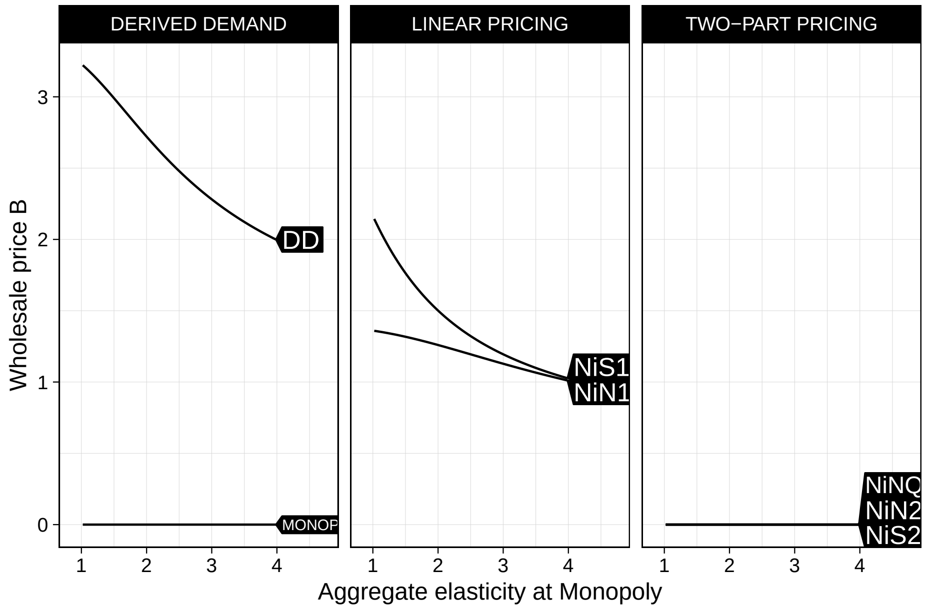
(c) Downstream price firm 1 vs ae



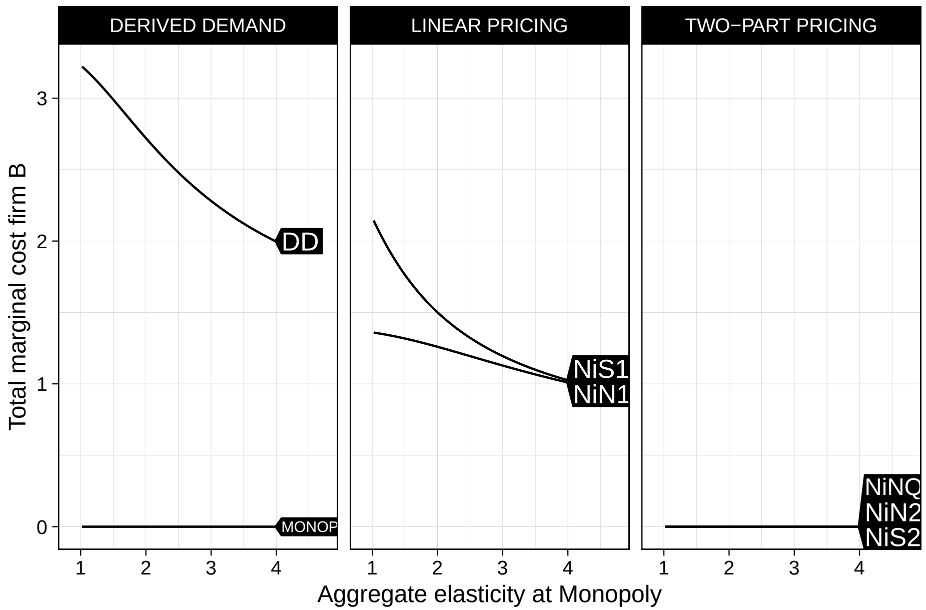
(d) Downstream price firm 2 vs ae



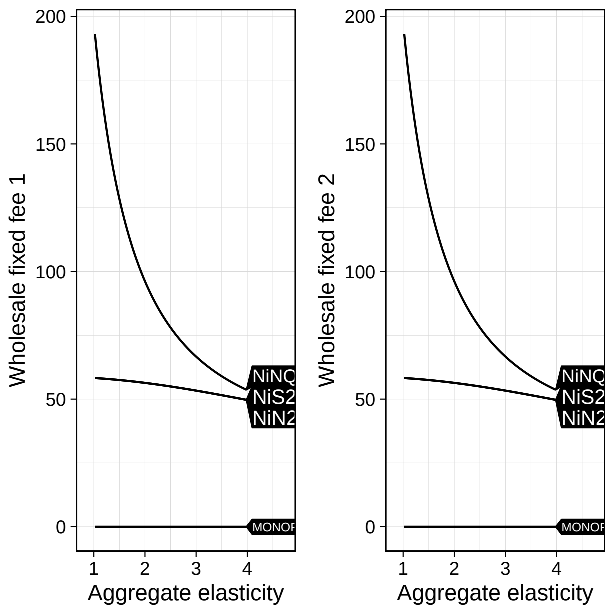
(a) Wholesale price firm 1 vs ae



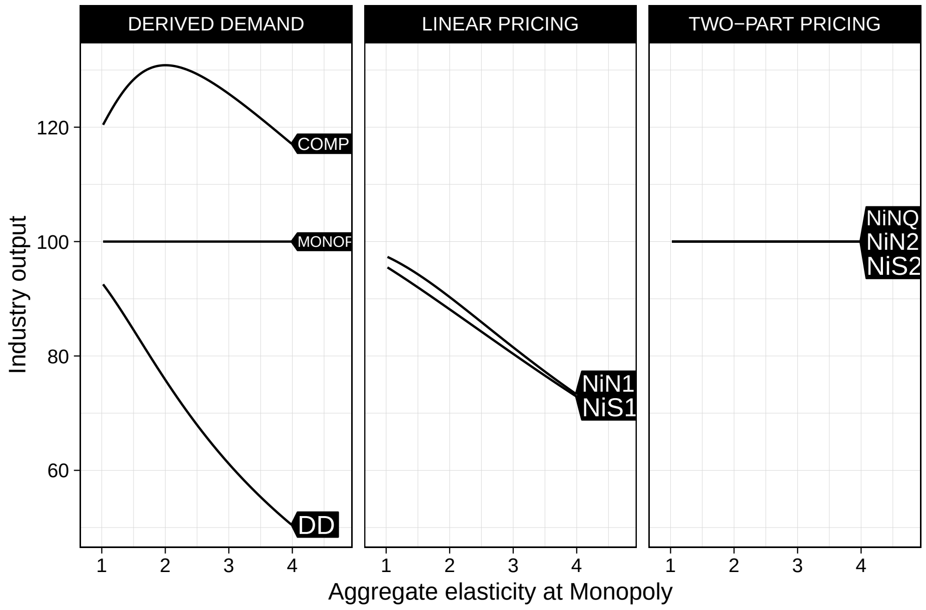
(b) Wholesale price firm 2 vs ae



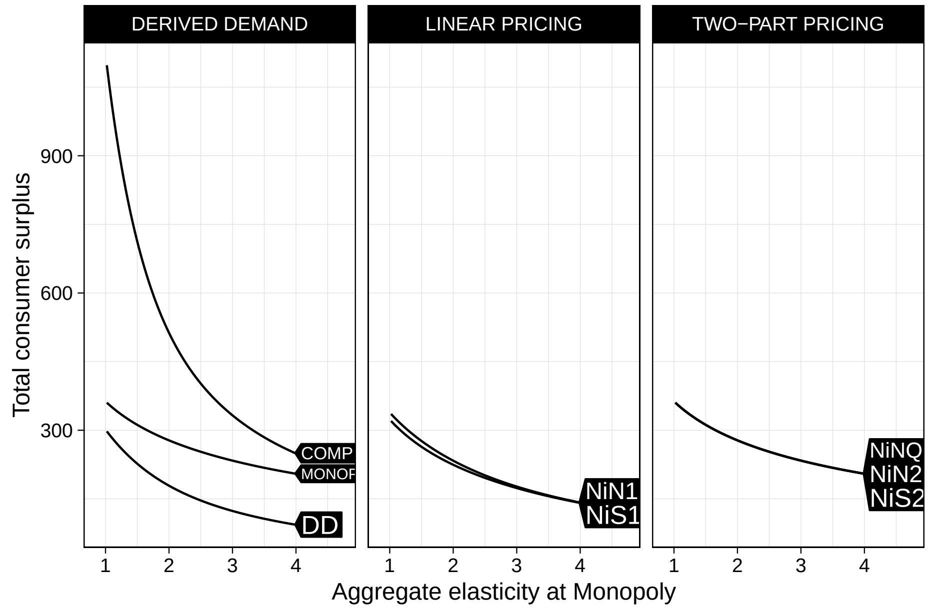
(c) Total marginal cost downstream firm 2 vs ae



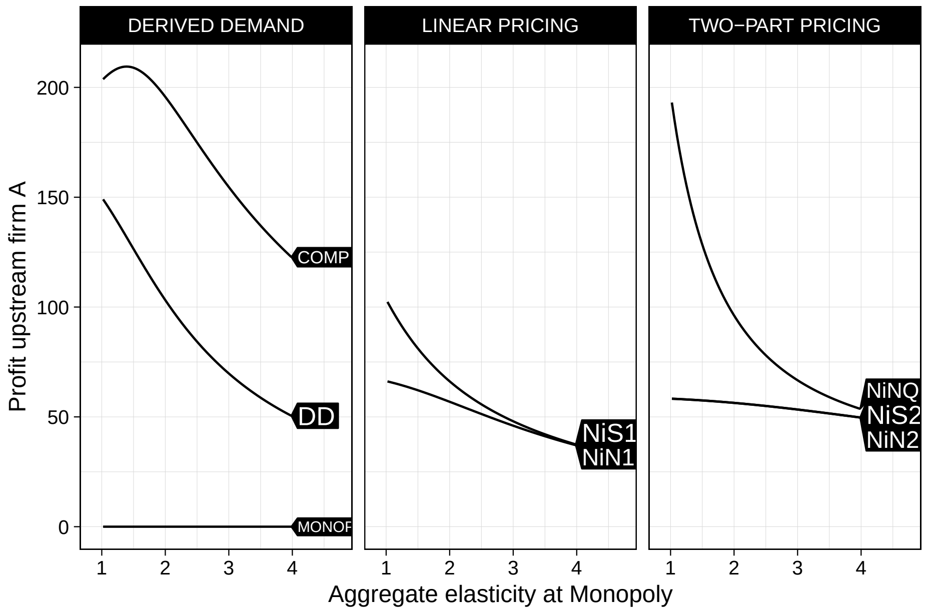
(d) Wholesale fees for firm A and B



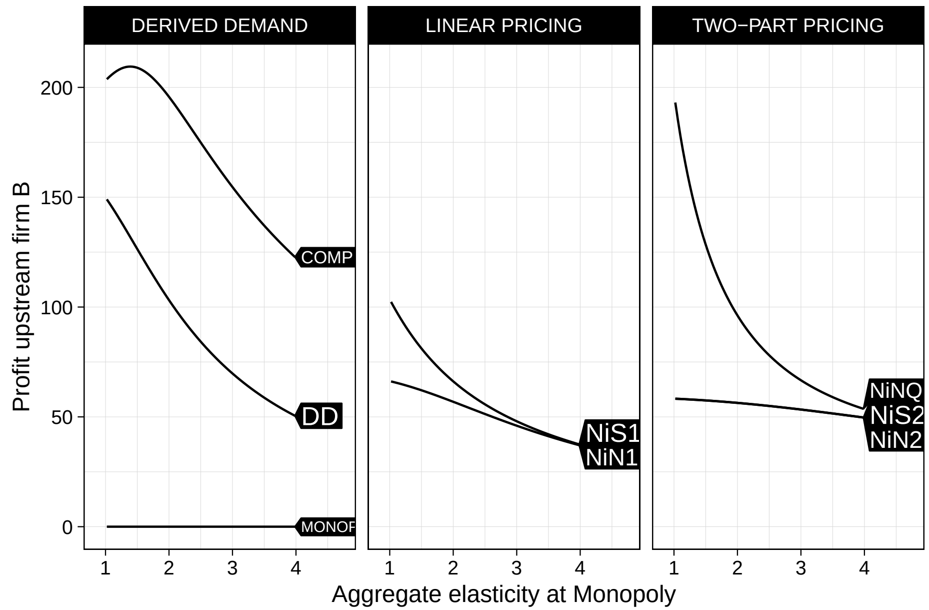
(a) Total quantity vs ae



(b) Consumer surplus vs ae



(c) Profit firm A and 1 vs ae



(d) Profit firm B vs ae

Appendix C

Derivation of Analytical Results of Vertical Merger Models

In this appendix we derive the analytical results for each of the three models included in chapter 3. The exposition of the models is adapted from Tschantz (2019) which contains the write-up of a bargaining comparison tool. The simulations are built on the static models presented on this online tool available at https://daag.shinyapps.io/b1x2/. These analytical results are the equations that are calibrated as presented in section 3.4 to yield the results in section 3.5.

C.0.1 Vertical merger in derived demand

Consider a vertical merger between A and 1 in a 1×2 industry structure. Once a marginal wholesale price w_2 for the second product is determined, the merged firm sets p_1 so as to maximise

$$\pi_{A1} = (p_1 - mc_1 - mc_A)q_1 + (w_2 - mc_A)q_2$$

while the second downstream firm simultaneously sets p_2 so as to maximise

$$\pi_2 = (p_2 - mc_2 - w_2)q_2$$

or equivalently satisfying first-order conditions

$$\begin{aligned} 0 &= q_1 + (p_1 - mc_1 - mc_A)\frac{\partial q_1}{\partial p_1} + (w_2 - mc_A)\frac{\partial q_2}{\partial p_1} \\ 0 &= q_2 + (p_2 - mc_2 - w_2)\frac{\partial q_2}{\partial p_2} \end{aligned} \tag{C.1}$$

The merged firm will initially set w_2 so as to maximise its eventual profit π_{A1} given quantities are determined from retail prices adjusting with w_2 to satisfy conditions (C.1). Although perhaps not quite as elegant, again it seems easier to work the other way around. Given p_2, w_2 and then p_1 are determined from conditions (C.1), the latter by a non-linear condition, but still approximable. Then π_{A1} is understood as a function of p_2 to be maximised. The pass-through rates from p_2 to w_2 and p_1 are found by solving

$$\begin{aligned} 0 = \frac{\partial q_1}{\partial p_2} + \frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial p_2} + (p_1 - mc_1 - mc_A)\left(\frac{\partial^2 q_1}{\partial p_1 \partial p_2} + \frac{\partial^2 q_1}{\partial p_1^2}\frac{\partial p_1}{\partial p_2}\right) + \\ (w_2 - mc_A)\left(\frac{\partial^2 q_2}{\partial p_1 \partial p_2} + \frac{\partial^2 q_2}{\partial p_1^2}\frac{\partial p_1}{\partial p_2}\right) + \frac{\partial p_1}{\partial p_2}\frac{\partial q_1}{\partial p_1} + \frac{\partial w_2}{\partial p_2}\frac{\partial q_2}{\partial p_1} \\ 0 = \frac{\partial q_2}{\partial p_2} + \frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial p_2} + (p_2 - mc_2 - w_2)\left(\frac{\partial^2 q_2}{\partial p_2^2} + \frac{\partial^2 q_2}{\partial p_1 \partial p_2}\frac{\partial p_1}{\partial p_2}\right) - \frac{\partial w_2}{\partial p_2}\frac{\partial q_2}{\partial p_2} \end{aligned}$$

The first-order condition for setting w_2 is then

$$0 = \frac{\partial p_1}{\partial p_2} q_1 + (p_1 - mc_1 - mc_A)\left(\frac{\partial q_1}{\partial p_2} + \frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial p_2}\right) + \frac{\partial w_2}{\partial p_2} q_2 + (w_2 - mc_A)\left(\frac{\partial q_2}{\partial p_2} + \frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial p_2}\right)$$

Next consider a vertical merger in the 2×1 case. The second upstream firms sets a wholesale price w_B then the merged firm sets retail prices to maximise

$$\pi_{A1} = (p_A - mc_1 - mc_A)q_A + (p_B - mc_1 - w_B)q_B$$

so satisfying first-order conditions

$$\begin{aligned} 0 &= q_A + (p_A - mc_1 - mc_A)\frac{\partial q_A}{\partial p_A} + (p_B - mc_1 - w_B)\frac{\partial q_B}{\partial p_A} \\ 0 &= q_B + (p_A - mc_1 - mc_A)\frac{\partial q_A}{\partial p_B} + (p_B - mc_1 - w_B)\frac{\partial q_B}{\partial p_A} \end{aligned} \tag{C.2}$$

The second upstream firm maximises $\pi_B = (w_B - mc_B)q_B$ supposing the derived demand determined by these conditions. The pass-through rates from w_B to p_A and p_B are determined from solving

$$\begin{aligned} 0 = {} & \frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B} + \\ & (p_A - mc_1 - mc_A)\left(\frac{\partial^2 q_A}{\partial p_A^2}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_A}{\partial p_A \partial p_B}\frac{\partial p_B}{\partial w_B}\right) + \\ & (p_B - mc_1 - w_B)\left(\frac{\partial^2 q_B}{\partial p_A^2}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_B}{\partial p_A \partial p_B}\frac{\partial p_B}{\partial w_B}\right) + \\ & \frac{\partial p_A}{\partial w_B}\frac{\partial q_A}{\partial p_A} + \frac{\partial p_B}{\partial w_B}\frac{\partial q_B}{\partial p_A} - \frac{\partial q_B}{\partial p_A} \\ 0 = {} & \frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B} + \\ & (p_A - mc_1 - mc_A)\left(\frac{\partial^2 q_A}{\partial p_A \partial p_B}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_A}{\partial p_B^2}\frac{\partial p_B}{\partial w_B}\right) + \\ & (p_B - mc_1 - w_B)\left(\frac{\partial^2 q_B}{\partial p_A \partial p_B}\frac{\partial p_A}{\partial w_B} + \frac{\partial^2 q_B}{\partial p_B^2}\frac{\partial p_B}{\partial w_B}\right) + \\ & \frac{\partial p_A}{\partial w_B}\frac{\partial q_A}{\partial p_B} + \frac{\partial p_B}{\partial w_B}\frac{\partial q_B}{\partial p_B} - \frac{\partial q_B}{\partial p_B} \end{aligned}$$

maximising π_B reduces to the first-order condition

$$0 = q_B + (w_B - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)$$

C.0.2 Vertical merger in linear pricing

Consider a vertical merger of A and 1 in the 1×2 linear pricing setting. The merged firm negotiates a wholesale price w_2 maximising $P_{A12} = (\pi_{A1} - \pi_{A1}^*)(\pi_2 - \pi_2^*)$ given profits π_{A1}^* and π_2^* without product 2, where $\pi_{A1} = (p_1 - mc_1 - mc_A)q_1 + (w_2 - mc_A)q_2$ and $\pi_2 = (p_2 - w_2)q_2$ are maximised in

Nash equilibrium determining retail prices and quantities according to conditions (C.1). The first-order condition is

$$0=\left(\frac{\partial p_1}{\partial w_2}q_1+(p_1-mc_1-mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_2}+\frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)+\right.$$
$$\left.(w_2-mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2}+\frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)\right)\cdot$$
$$((p_2-w_2)q_2-\pi_2^*)+$$
$$((p_1-mc_1-mc_A)q_1+(w_2-mc_A)q_2-\pi_{A1}^*)\cdot$$
$$\left(\left(\frac{\partial p_2}{\partial w_2}-1\right)q_2+(p_2-w_2)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2}+\frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)\right)$$

The threat points are determined, independent of the w_2 agreed to, to maximise $\pi_{A1}^*=(p_1^*-mc_1-mc_A)q_1^*$ with q_1^* the demand at price p_1^* with product 2 unavailable, with $\pi_2^*=0$. The first-order condition is

$$0=q_1^*+(p_1^*-mc_1-mc_A)\frac{\partial q_1^*}{\partial p_1^*}$$

This is the same for a vertical merger in the Nash-in-Shapley case.

In the 2×1 case of a vertical merger between A and 1, the merged firm negotiates a wholesale price w_B maximising $P_{AB1}=(\pi_{A1}-\pi_{A1}^*)(\pi_B-\pi_B^*)$ given π^*A1 and π_B^* without product B, where $\pi_{A1}=(p_A-mc_1-mc_A)q_A+(p_B-mc_1-w_B)q_B$ and $\pi_B=(w_B-mc_B)q_B$ for retail prices and quantities maximising π_{A1} satisfying conditions (C.2). The first-order condition is

$$0=\left(\frac{\partial p_A}{\partial w_B}q_A+\left(\frac{\partial p_B}{\partial w_B}-1\right)q_B+\right.$$
$$(p_A-mc_1-mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B}+\frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)+$$
$$\left.(p_B-mc_1-w_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B}+\frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)\right)\cdot$$
$$((w_B-mc_B)q_B-\pi_B^*)+$$
$$((p_A-mc_1-mc_A)q_A+(p_B-mc_1-w_B)q_B-\pi_{A1}^*)\cdot$$
$$\left(q_B+(w_B-mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B}+\frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)\right)$$

The threat points, independent of w_B, are $\pi_B^*=0$ and the maximum of $\pi_{A1}^*=(p_A^*-mc_1-mc_A)q_A^*$ satisfying first-order condition

$$0=q_A^*+(p_A^*-mc_1-mc_A)\frac{\partial q_A^*}{\partial p_A^*}$$

This is the same for a vertical merger in the Nash-in-Shapley case.

C.0.3 Vertical merger in two-part pricing

In the two-part pricing setting, parties will agree to marginal wholesale pricing and fees, and these agreements also become the basis for the threat points to agreement. In case of a vertical merger of A and 1 in the 1×2 setting, the merged firm negotiates with 2 over a marginal wholesale price w_2 and fee f_2, with threat point the usual result for A and 1 cooperating to sell only product 1, maximising

$\pi^*_{A1} = (p^*_1 - mc_1 - mc_A)q^*_1$ with $\pi^*_2 = 0$. Given w_2, retail prices are determined in Nash equilibrium, p_1 maximising $\pi^o_{A1} = (p_1 - mc_1 - mc_A)q_1 + (w_2 - mc_A)q_2$ holding p_2 fixed, with simultaneously p_2 maximising $\pi^o_2 = (p_2 - mc_2 - w_2)q_2$, Then w_2 is chosen to maximise the sum of these given how retail prices will adjust. The first-order condition is

$$0 = \frac{\partial p_1}{\partial w_2}q_1 + \frac{\partial p_2}{\partial w_2}q_2 + (p_1 - mc_1 - mc_A)\left(\frac{\partial q_1}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right) + (p_2 - mc_2 - mc_A)\left(\frac{\partial q_2}{\partial p_1}\frac{\partial p_1}{\partial w_2} + \frac{\partial q_2}{\partial p_2}\frac{\partial p_2}{\partial w_2}\right)$$

The fee f_2 is taken so $\pi_{A1} = \pi^o_{A1} + f_2$ and $\pi_2 = \pi^o_2 - f_2$ satisfy $\pi_{A1} - \pi^*_{A1} = \pi_2 - \pi^*_2$, i.e., $f_2 = (\pi^o_2 - \pi^o_{A1} + \pi^*_{A1})/2$. This is the same for a vertical merger in the Nash-in-Shapley and Nash-in-Nash quantity cases.

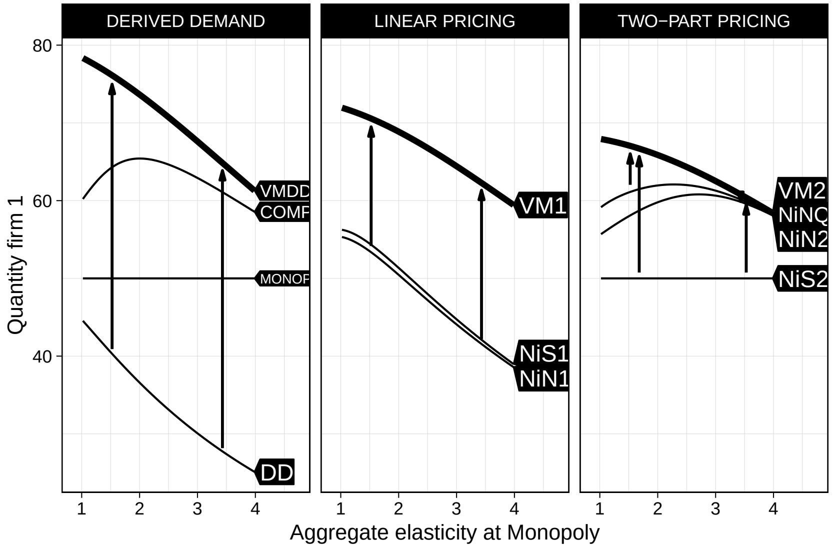
In the 2×1 case, the merged firm negotiates with B over a marginal wholesale price w_B and fee f_B, with threat point the usual result for A and 1 cooperating to sell only product A, maximising $\pi^*_{A1} = (p^*_A - mc_1 - mc_A)q^*_A$ with $\pi^*_B = 0$. Given w_B, retail prices are determined maximising $\pi^o_{A1} = (p_A - mc_1 - mc_A)q_A + (p_B - mc_1 - w_B)q_B$. With $\pi^o_B = (w_B - mc_B)q_B$, w_B is chosen to maximise $\pi^o_{A1} + \pi^o_B = (p_A - mc_1 - mc_A)q_A + (p_B - mc_1 - mc_B)q_B$ given how retail prices will adjust. The first-order condition is

$$0 = \frac{\partial p_A}{\partial w_B}q_A + \frac{\partial p_B}{\partial w_B}q_B + (p_A - mc_1 - mc_A)\left(\frac{\partial q_A}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_A}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right) + (p_B - mc_1 - mc_B)\left(\frac{\partial q_B}{\partial p_A}\frac{\partial p_A}{\partial w_B} + \frac{\partial q_B}{\partial p_B}\frac{\partial p_B}{\partial w_B}\right)$$

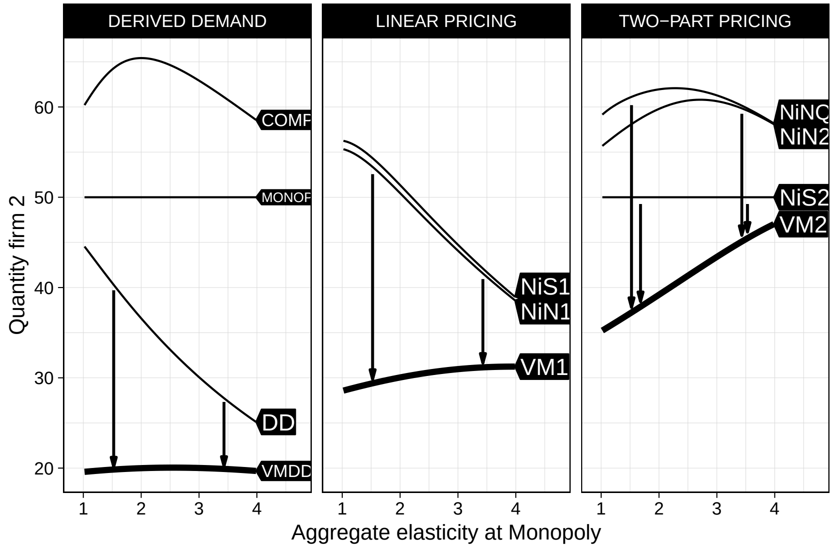
The fee f_B is taken so $\pi_{A1} = \pi^o_{A1} - f_B$ and $\pi_B = \pi^o_B + f_B$ satisfy $\pi_{A1} - \pi^*_{A1} = \pi_B - \pi^*_B$, i.e., $f_B = (\pi^o_{A1} - \pi^*_{A1} - \pi^o_B)/2$. This is the same for a vertical merger in the Nash-in-Shapley and Nash-in-Nash quantity cases.

Appendix D

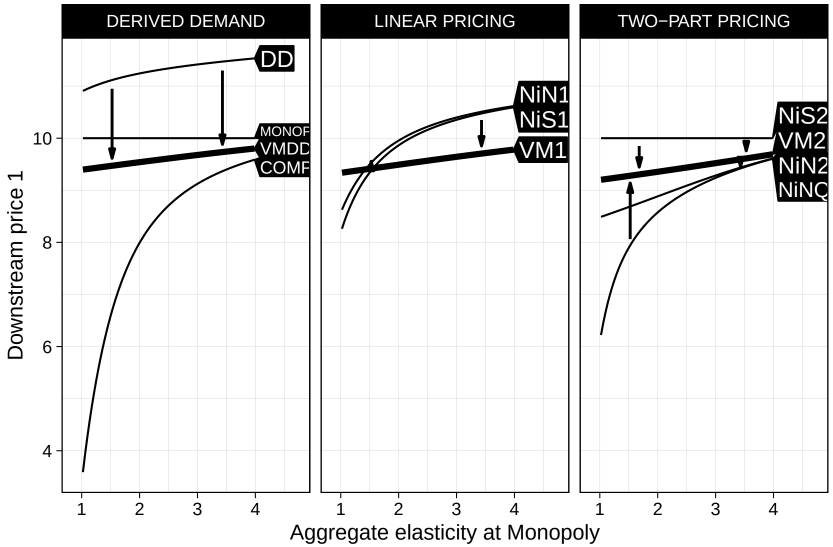
Vertical Mergers Graphs



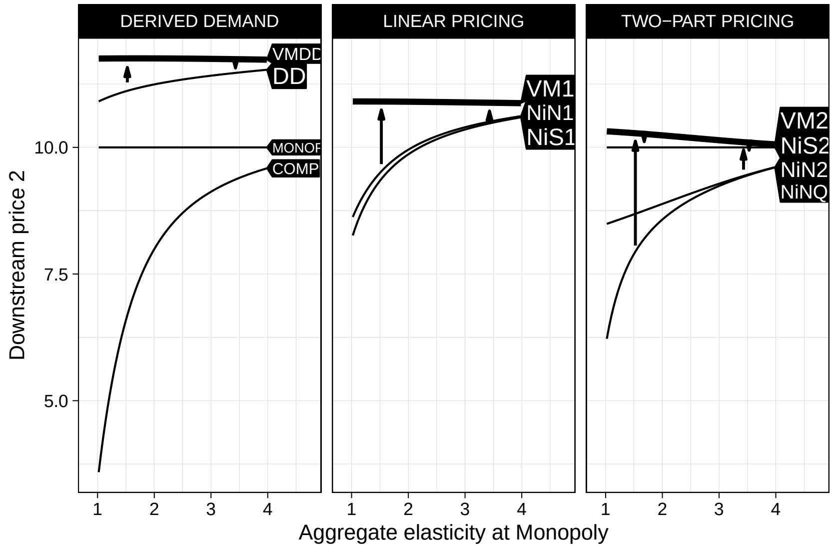
(a) Downstream quantity firm 1 vs ae



(b) Downstream quantity firm 2 vs ae

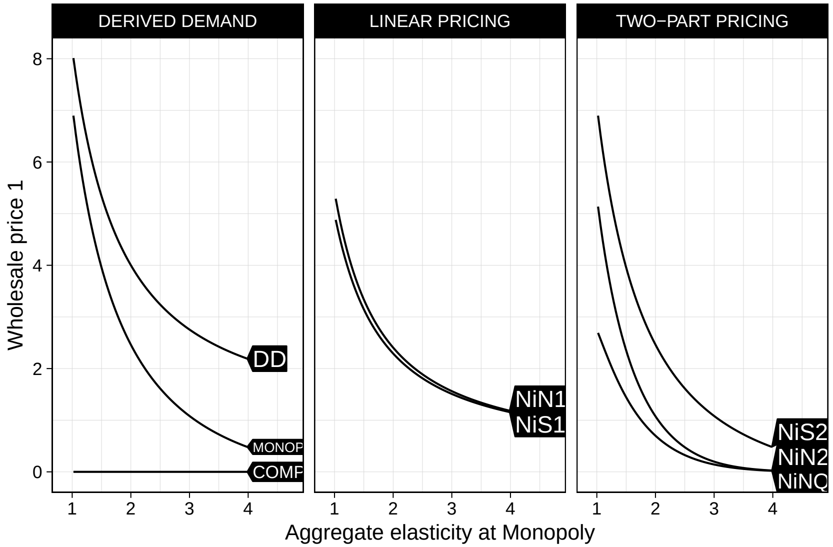


(c) Downstream price firm 1 vs ae

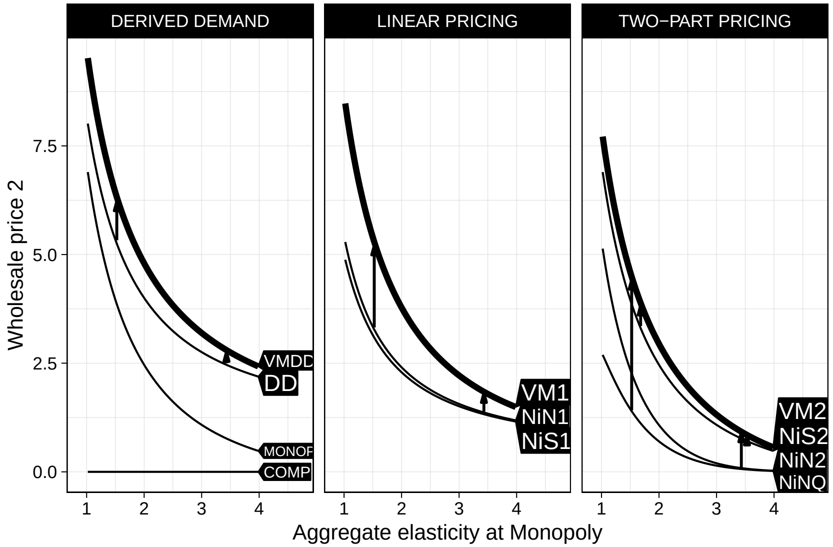


(d) Downstream price firm 2 vs ae

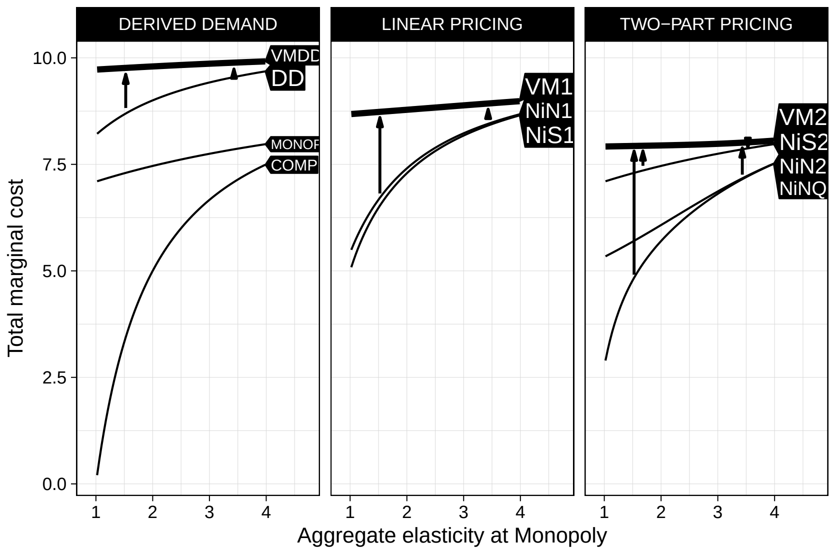
Figure D.1: Retail prices and quantities for a 1x2 setting react in the anticipated manner: (a) shows an increase in the vertically integrated firm's quantity; (b) shows a decrease in the rival firm's quantity; (c) shows an increase in the vertically integrated firm's price; and, (d) shows a decrease in the rival firm's price



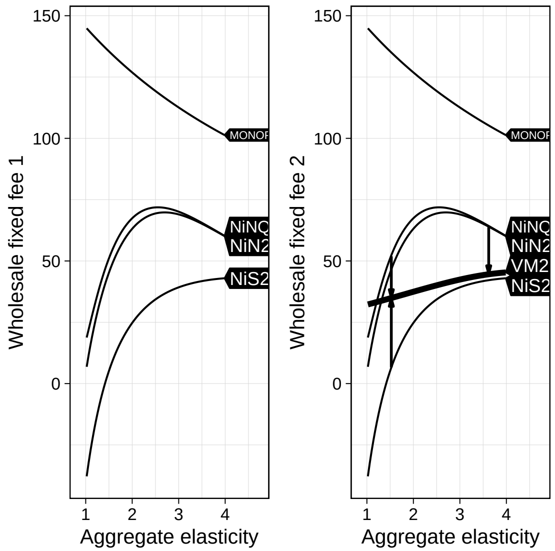
(a) Wholesale price firm 1 vs ae



(b) Wholesale price firm 2 vs ae

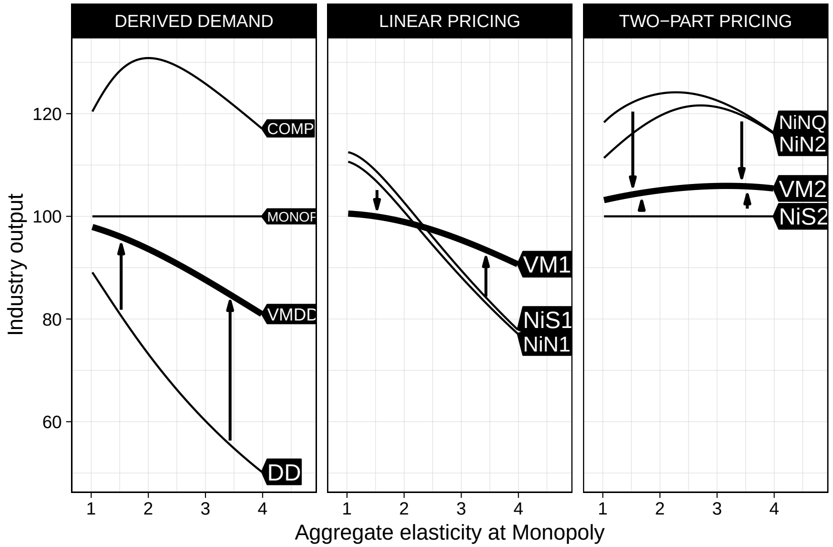


(c) Total marginal cost downstream firm 2 vs ae

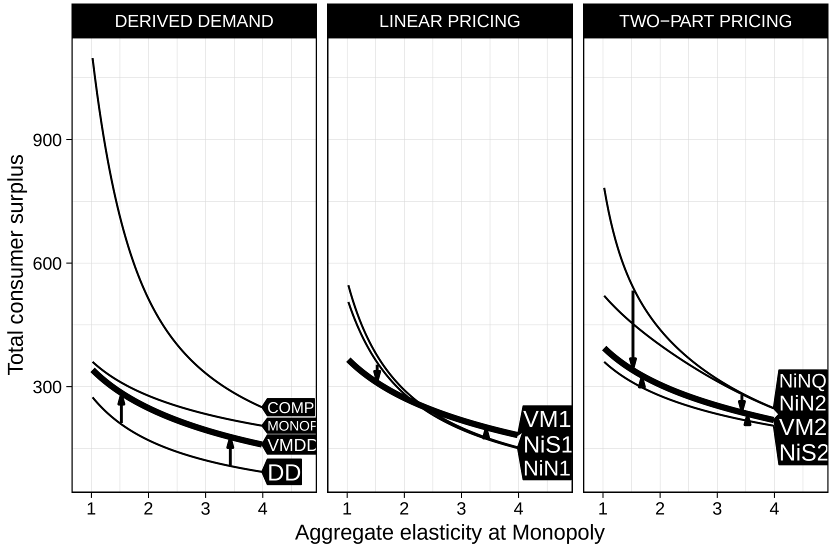


(d) Wholesale fees for firm 1 and 2

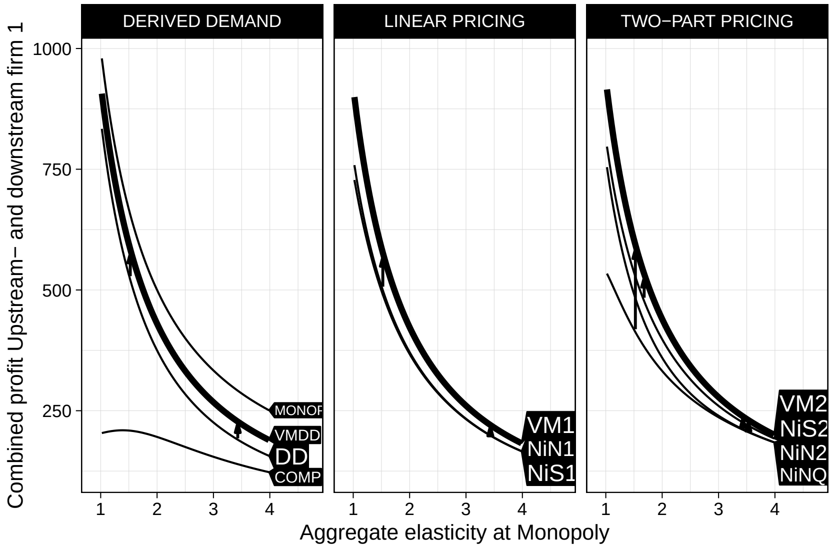
Figure D.2: Wholesale prices and fees for a 1x2 setting: (a) shows the elimination of double marginalisation for the vertically integrated firm; (b) shows the raising of rival's cost for the rival firm; (c) shows the increase in total marginal cost for the rival firm; and, (d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting



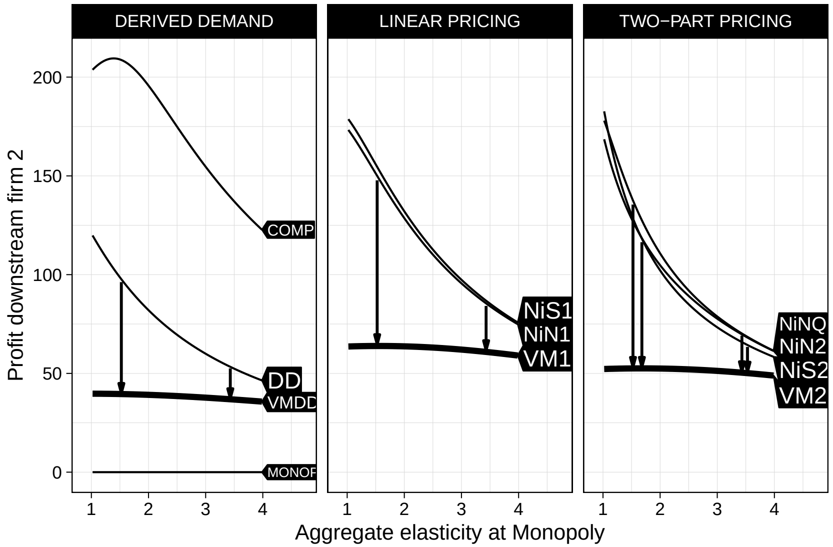
(a) Total quantity vs ae



(b) Consumer surplus vs ae

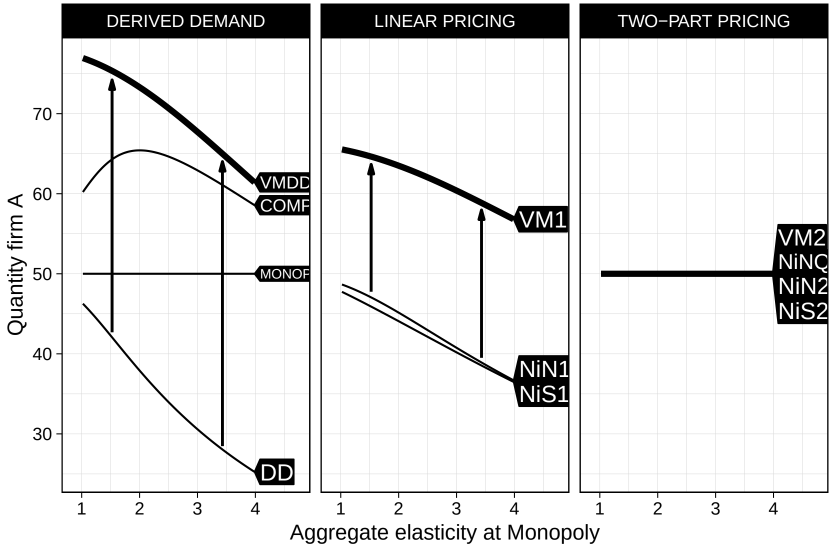


(c) Combined profits firm A and 1 vs ae

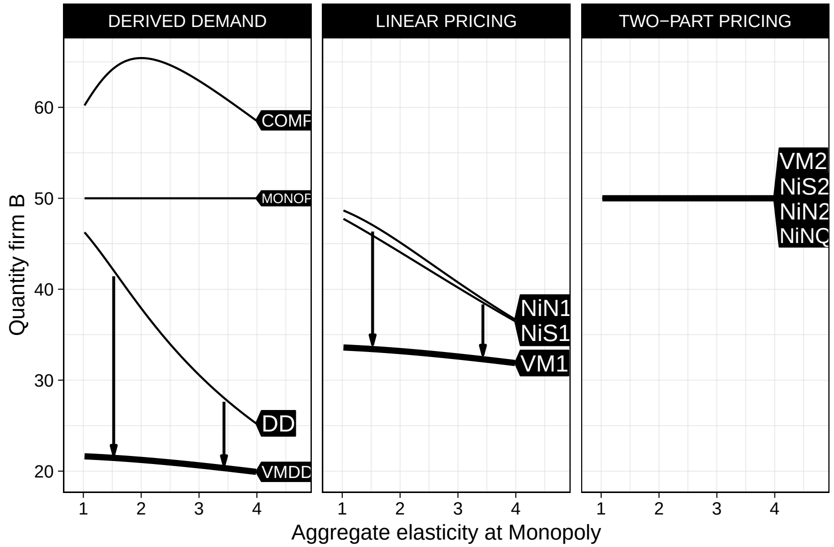


(d) Profit downstream firm 2 vs ae

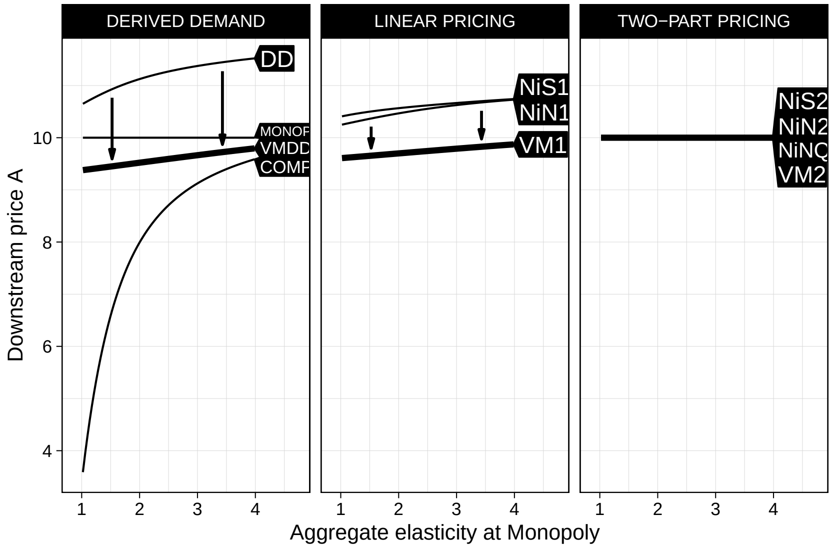
Figure D.3: Profit and welfare in a 1x2 setting: (a) shows total quantity; (b) shows that consumer surplus closely follows total quantity; (c) shows the combined profits of the upstream firm and downstream firm 1 - post-merger profit is always higher than pre-merger combined profit; and, (d) shows the profit of firm 2 - post-merger profit is always lower than pre-merger profit



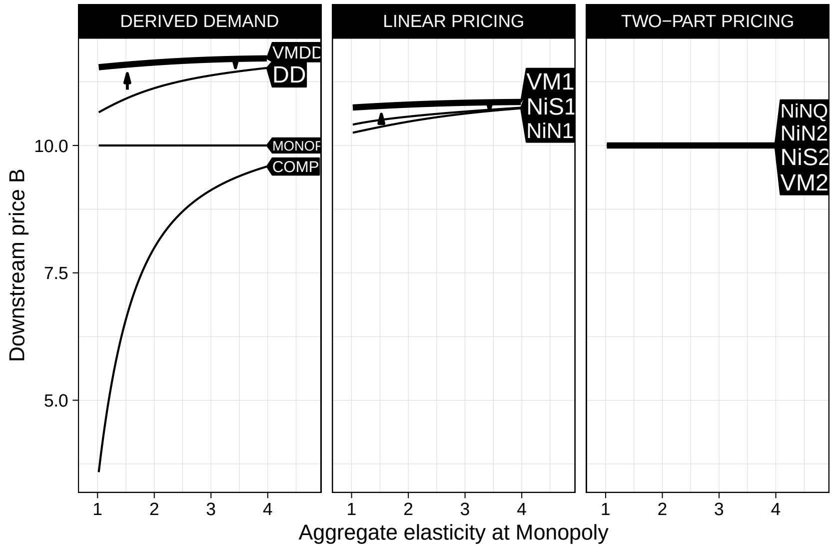
(a) Downstream quantity firm 1 vs ae



(b) Downstream quantity firm 2 vs ae

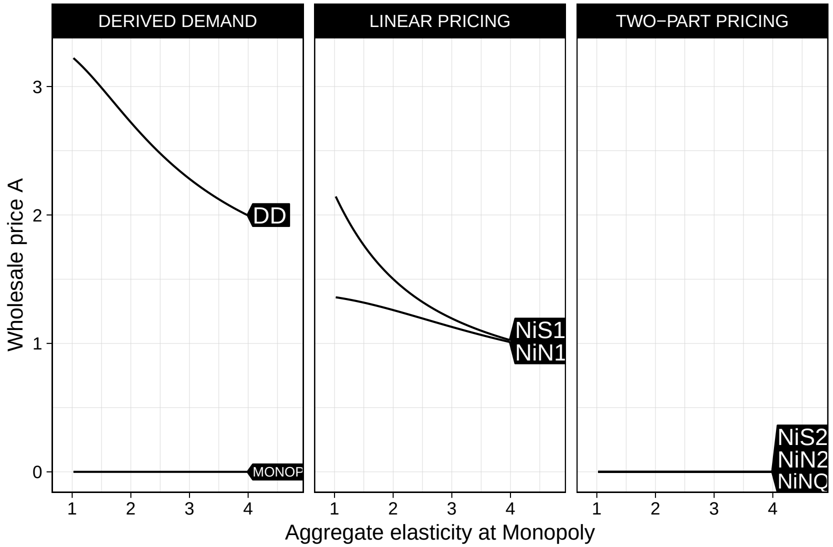


(c) Downstream price firm 1 vs ae

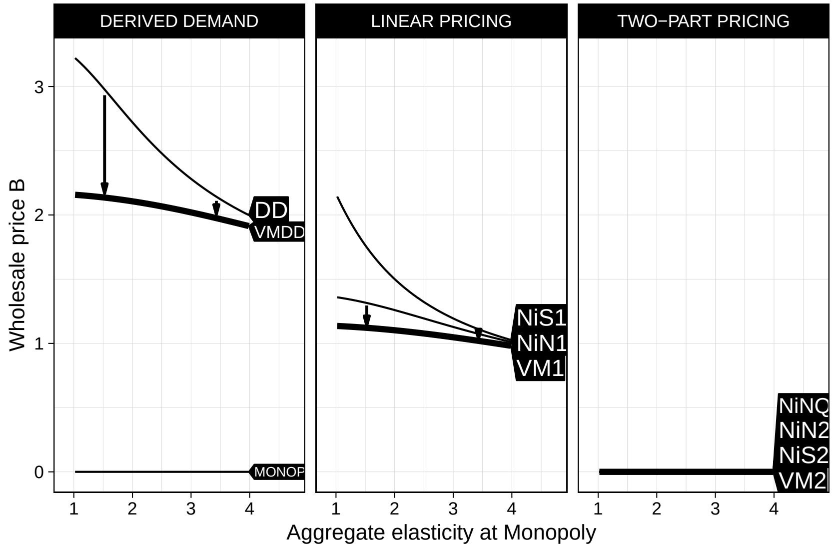


(d) Downstream price firm 2 vs ae

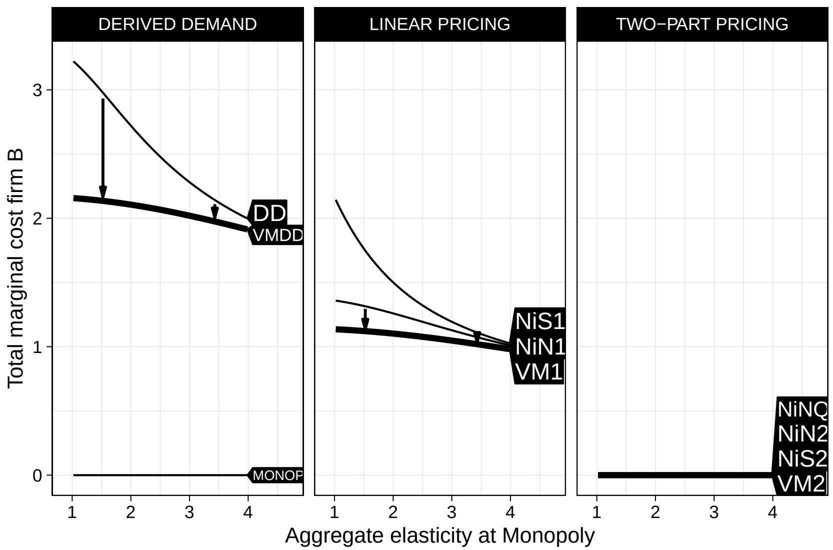
Figure D.4: Retail prices and quantities for a 2x1 setting react in the anticipated manner: (a) shows an increase in the vertically integrated firm's quantity; (b) shows a decrease in the rival firm's quantity; (c) shows an increase in the vertically integrated firm's price; and, (d) shows a decrease in the rival firm's price



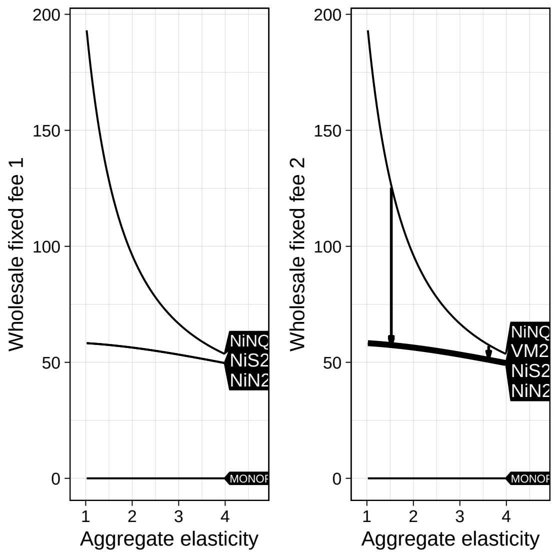
(a) Wholesale price firm 1 vs ae



(b) Wholesale price firm 2 vs ae

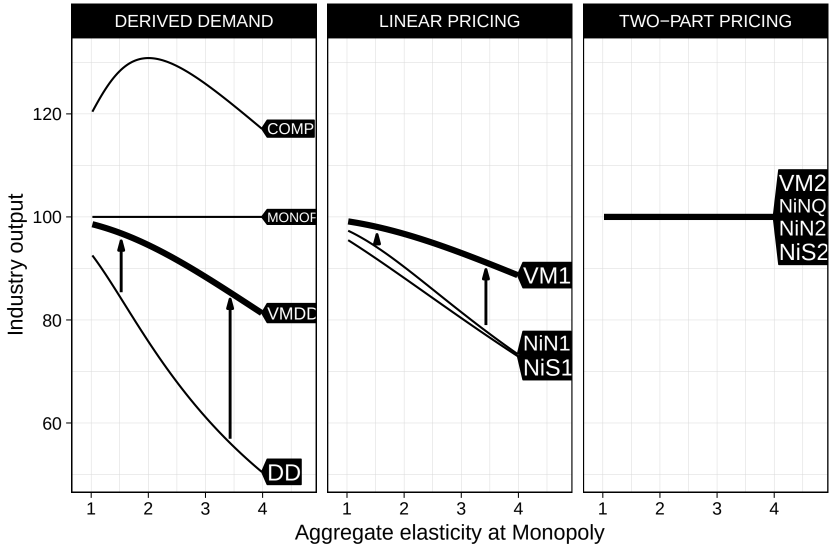


(c) Total marginal cost downstream firm 2 vs ae

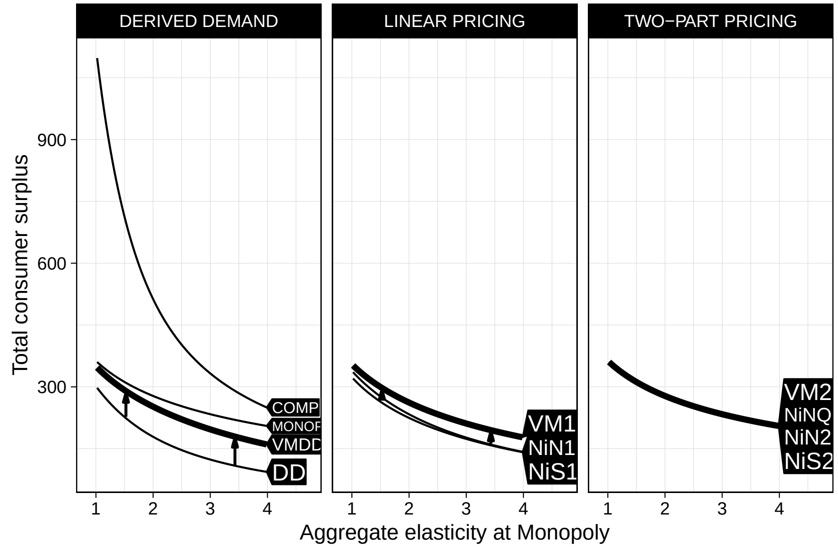


(d) Wholesale fees for firm A and B

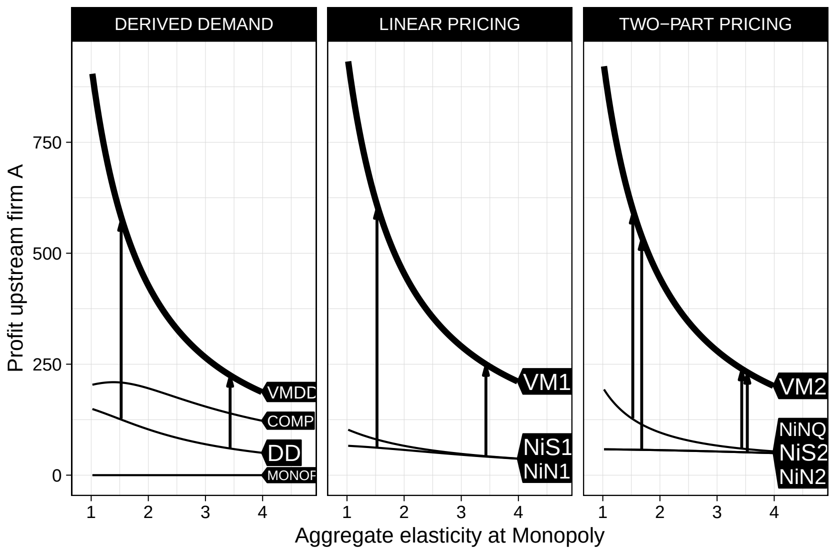
Figure D.5: Wholesale prices and fees for a 2x1 setting: (a) shows the elimination of double marginalisation for the vertically integrated firm; (b) shows the reducing of rival's revenue for the rival firm; (c) shows the decrease in total marginal cost for the rival firm as a result of reducing of rival's revenue; and, (d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting



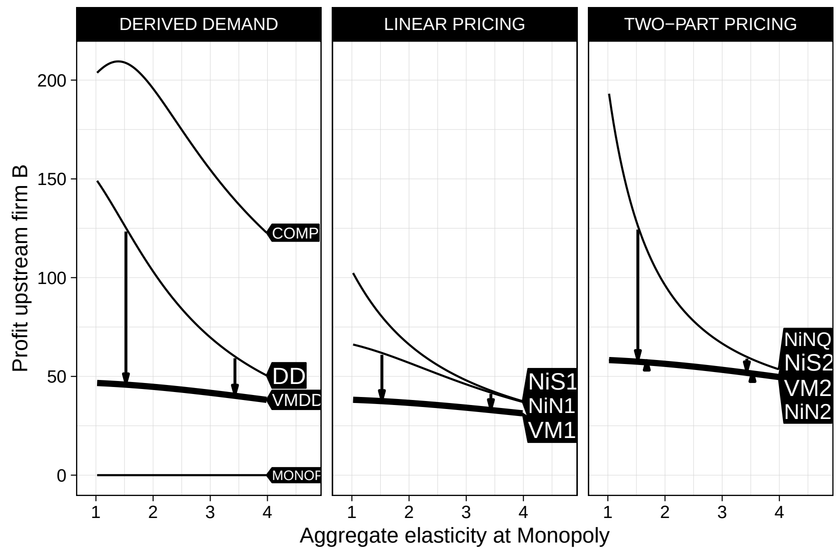
(a) Total quantity vs ae



(b) Consumer surplus vs ae

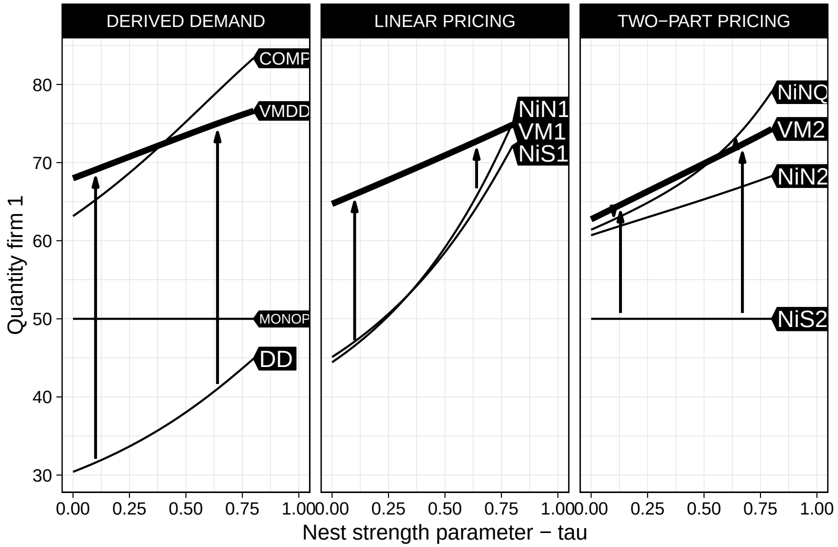


(c) Profit firm A and 1 vs ae

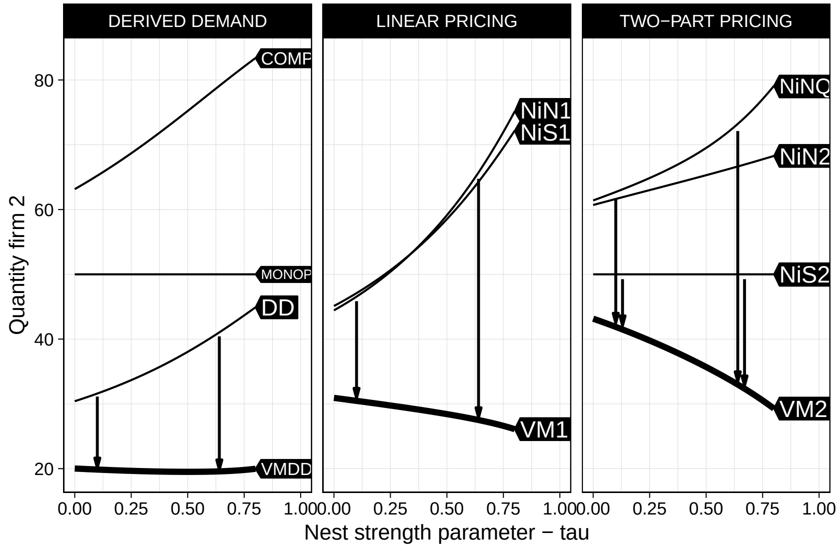


(d) Profit firm B vs ae

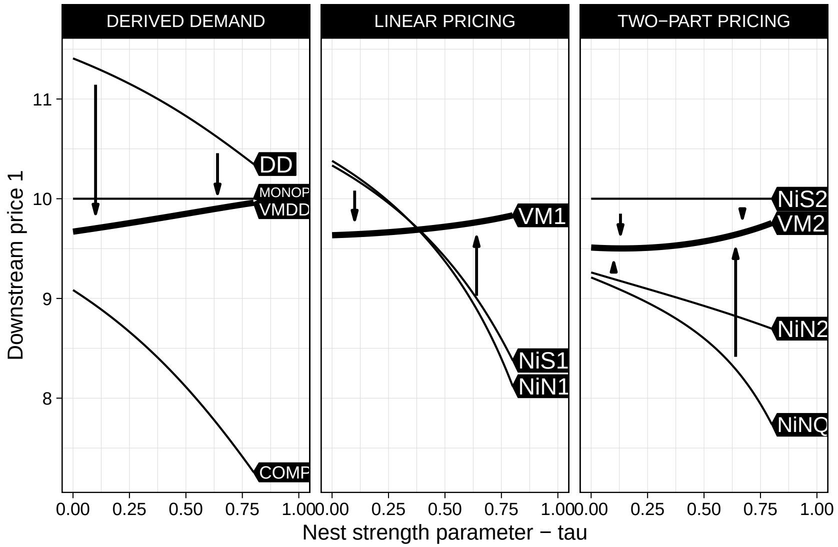
Figure D.6: Profit and welfare in a 2x1 setting: (a) shows total quantity; (b) shows that consumer surplus closely follows total quantity; (c) shows the profit of firm A - post-merger profit is always higher than pre-merger profit; and, (d) shows the profit of firm B - post-merger profit is always lower than pre-merger profit



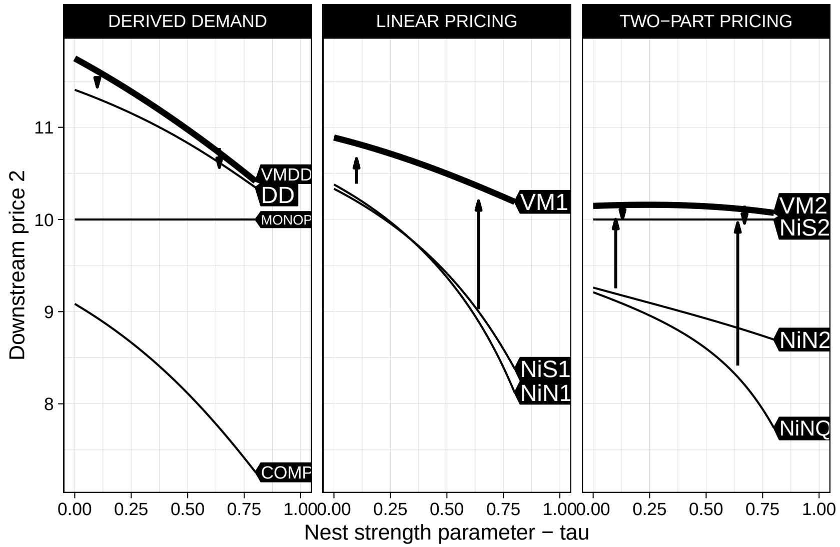
(a) Downstream quantity firm 1 vs tau



(b) Downstream quantity firm 2 vs tau

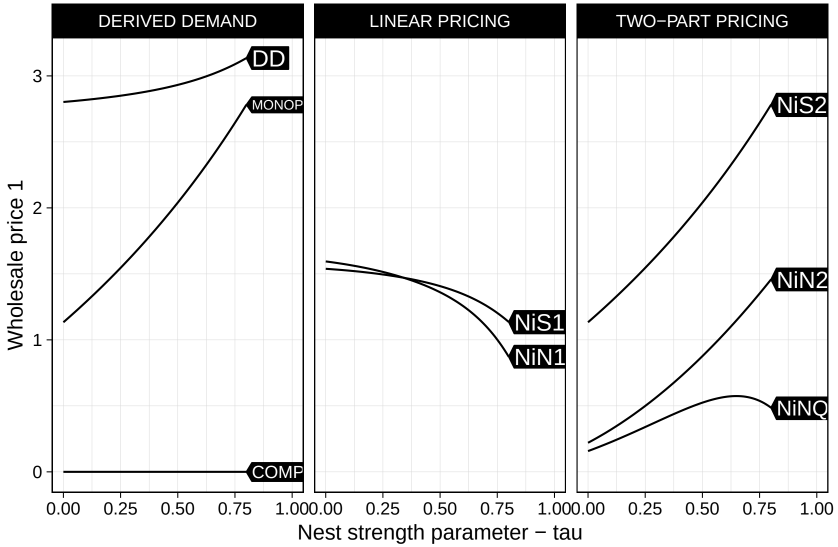


(c) Downstream price firm 1 vs tau

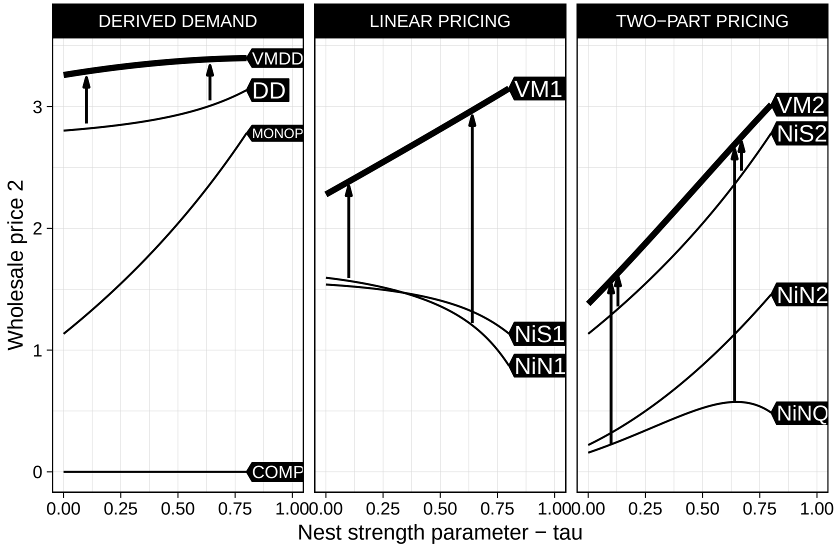


(d) Downstream price firm 2 vs tau

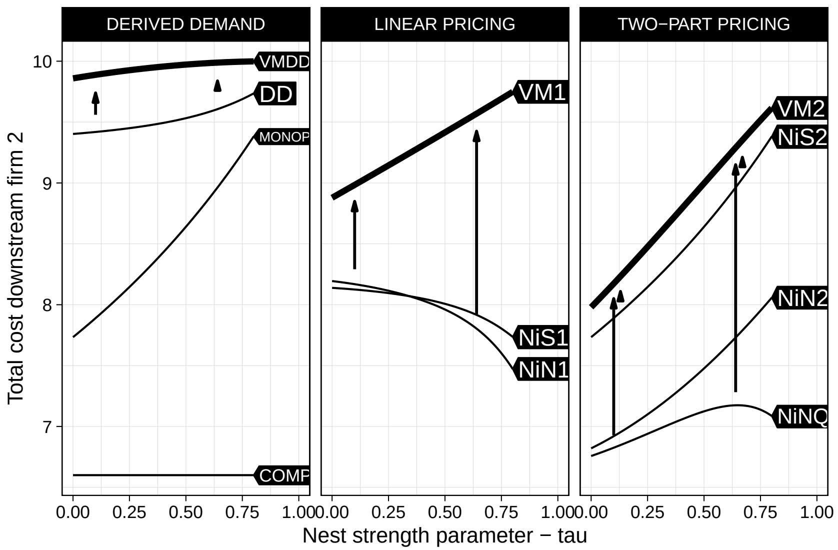
Figure D.7: Retail prices and quantities for a 1x2 setting react in the anticipated manner: (a) shows an increase in the vertically integrated firm's quantity; (b) shows a decrease in the rival firm's quantity; (c) shows an increase in the vertically integrated firm's price; and, (d) shows a decrease in the rival firm's price



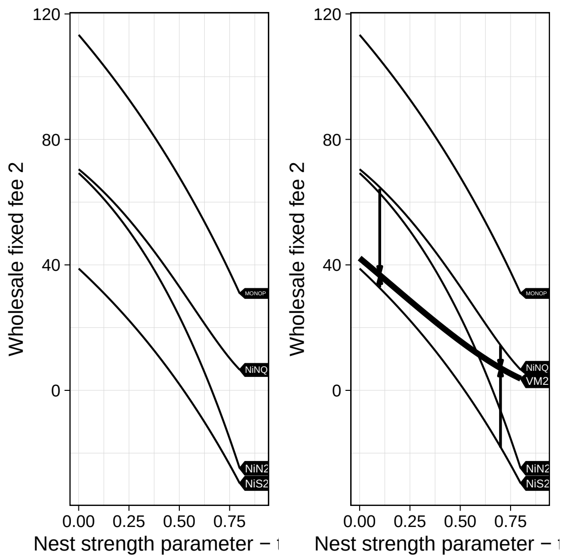
(a) Wholesale price firm 1 vs tau



(b) Wholesale price firm 2 vs tau

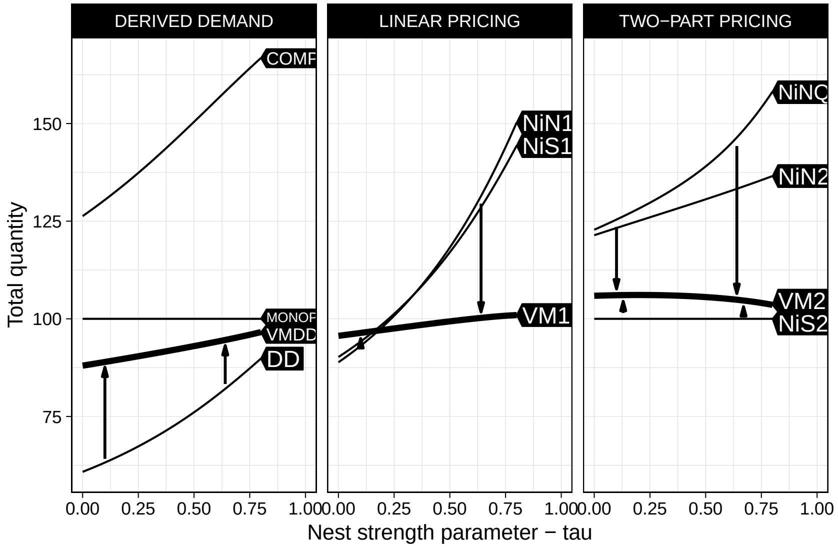


(c) Total marginal cost downstream firm 2 vs tau

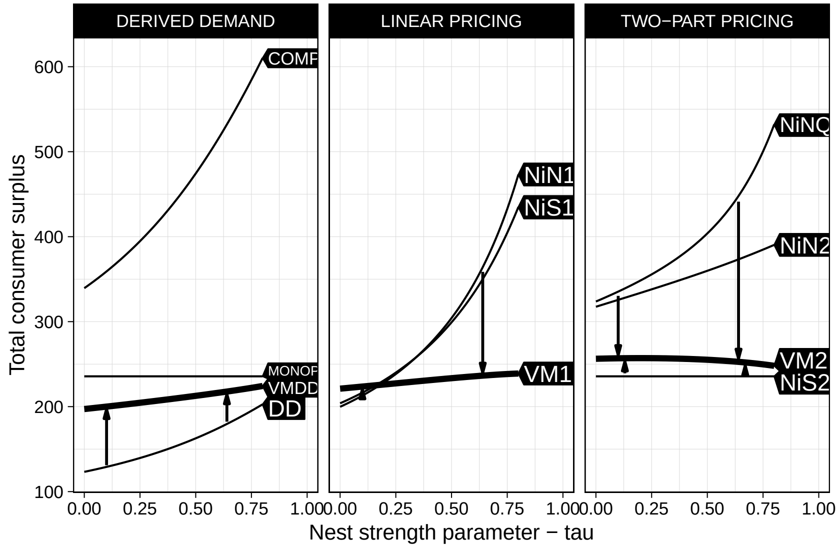


(d) Wholesale fees for firm 1 and 2

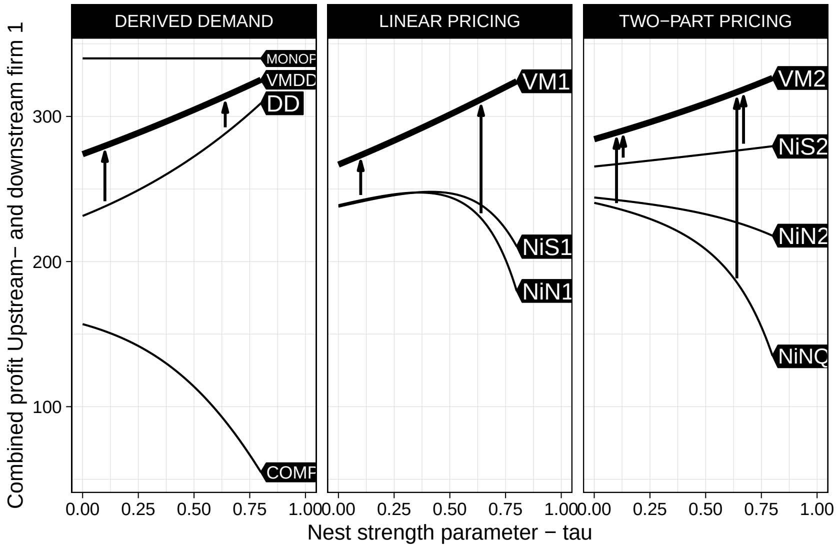
Figure D.8: Wholesale prices and fees for a 1x2 setting: (a) shows the elimination of double marginalisation for the vertically integrated firm; (b) shows the raising of rival's cost for the rival firm; (c) shows the increase in total marginal cost for the rival firm; and, (d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting



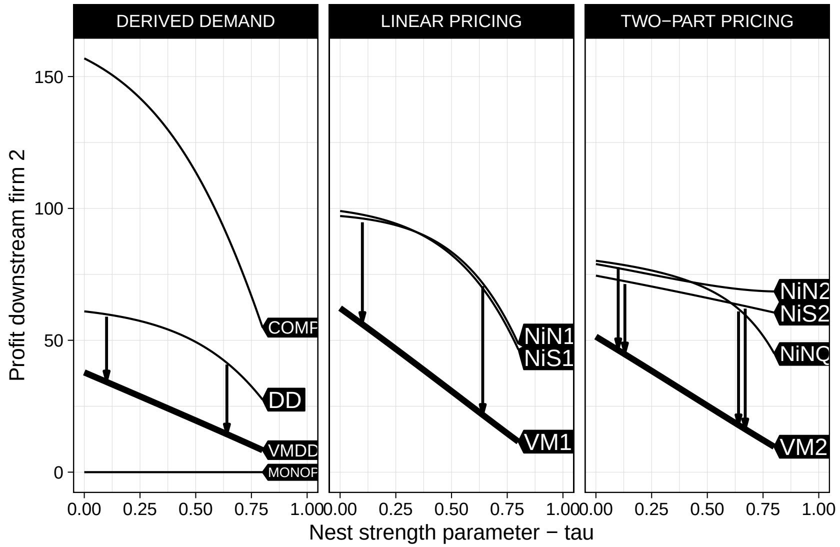
(a) Total quantity vs tau



(b) Consumer surplus vs tau

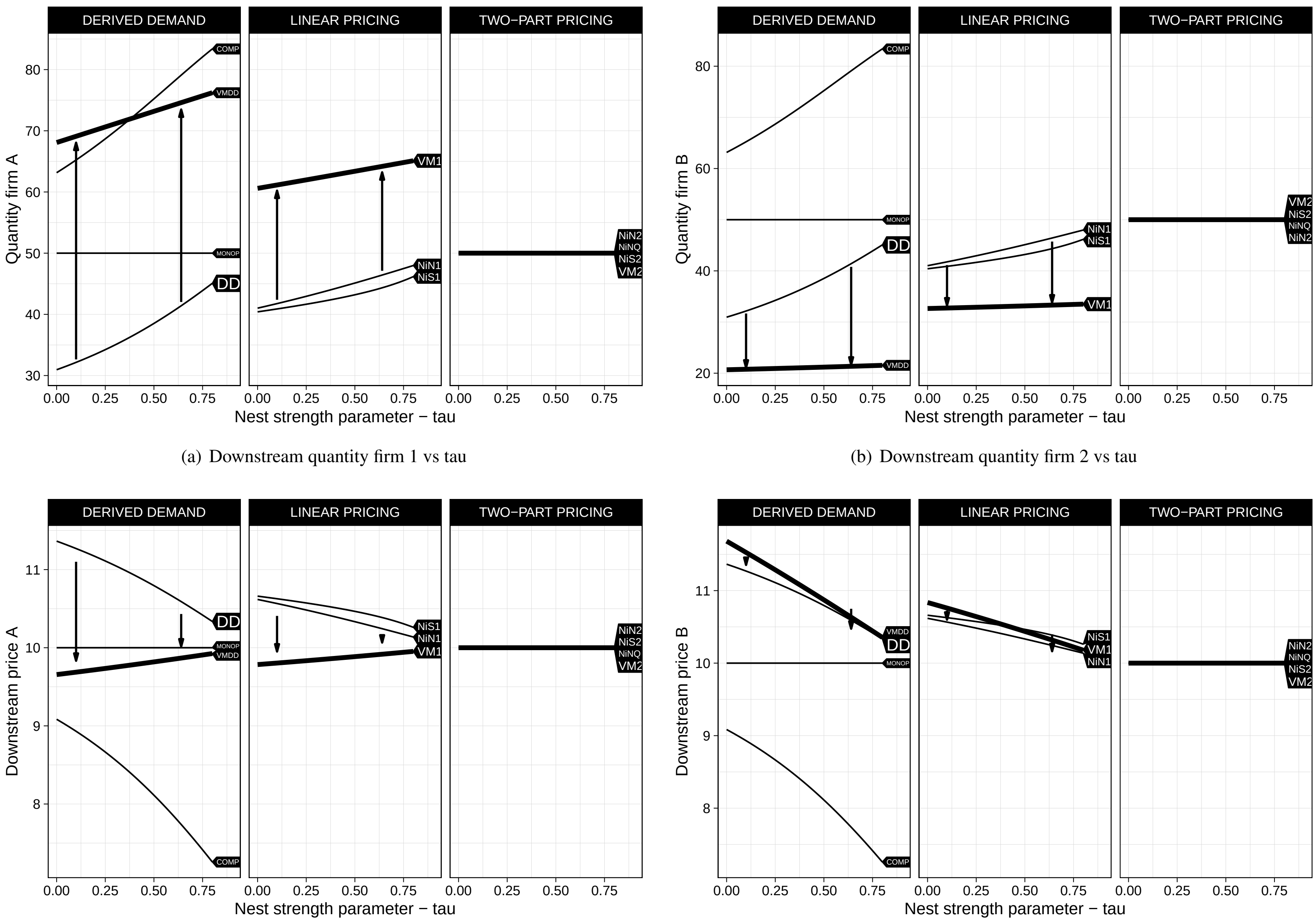


(c) Combined profits firm A and 1 vs tau



(d) Profit downstream firm 2 vs tau

Figure D.9: Profit and welfare in a 1x2 setting: (a) shows total quantity; (b) shows that consumer surplus closely follows total quantity; (c) shows the combined profits of the upstream firm and downstream firm 1 - post-merger profit is always higher than pre-merger combined profit; and, (d) shows the profit of firm 2 - post-merger profit is always lower than pre-merger profit



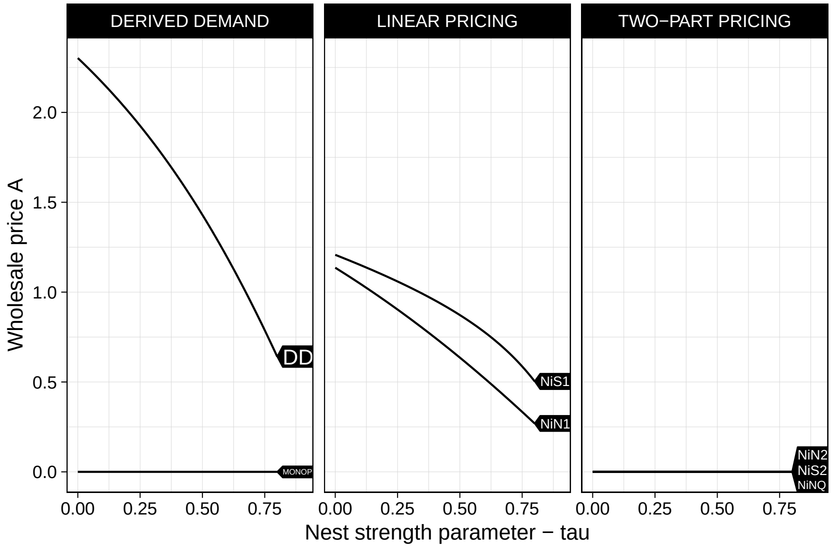
(a) Downstream quantity firm 1 vs tau

(b) Downstream quantity firm 2 vs tau

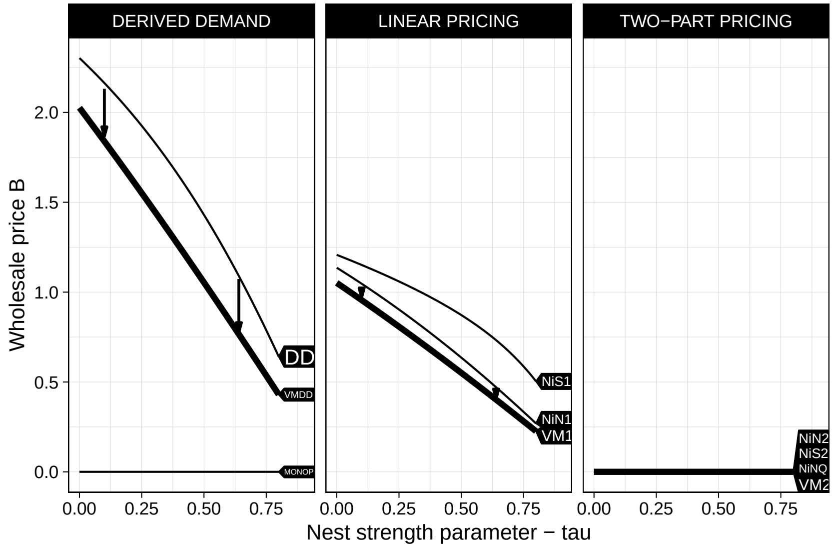
(c) Downstream price firm 1 vs tau

(d) Downstream price firm 2 vs tau

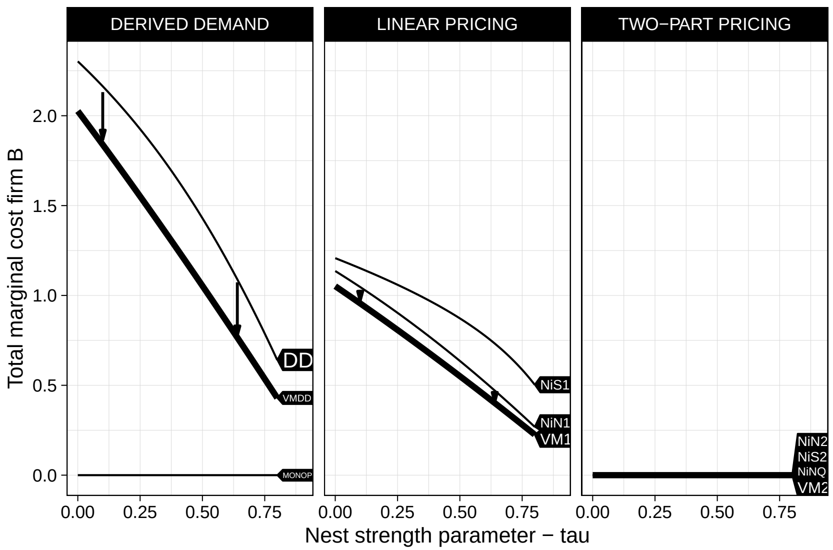
Figure D.10: Retail prices and quantities for a 2x1 setting react in the anticipated manner: (a) shows an increase in the vertically integrated firm's quantity; (b) shows a decrease in the rival firm's quantity; (c) shows an increase in the vertically integrated firm's price; and, (d) shows a decrease in the rival firm's price



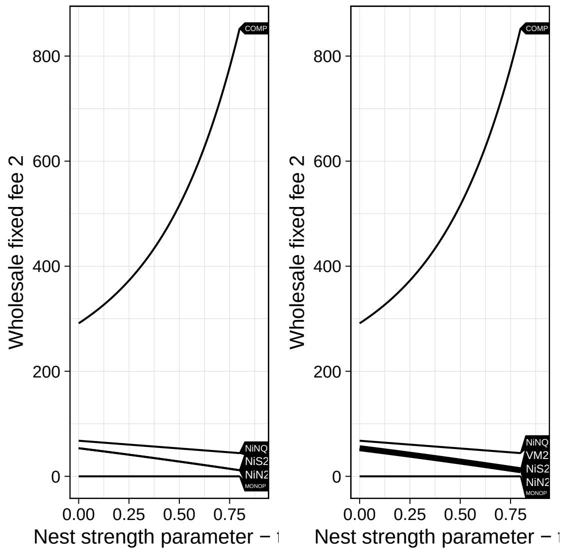
(a) Wholesale price firm 1 vs tau



(b) Wholesale price firm 2 vs tau

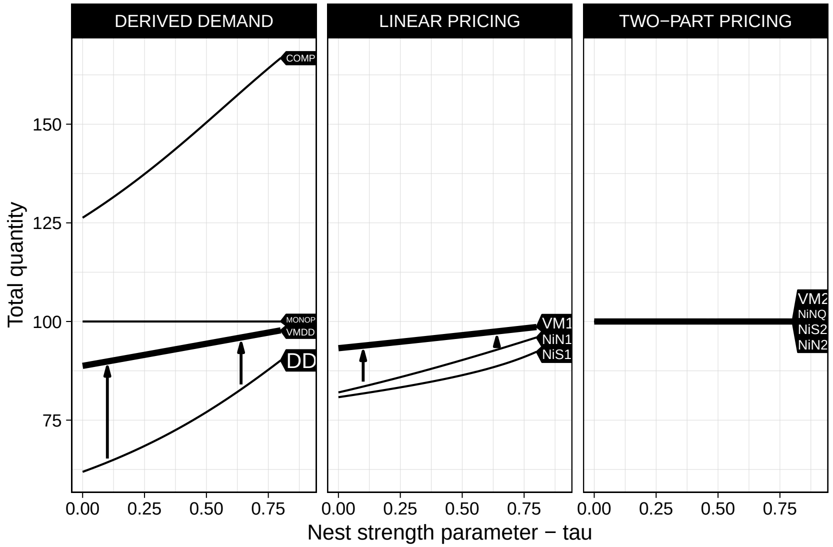


(c) Total marginal cost downstream firm 2 vs tau

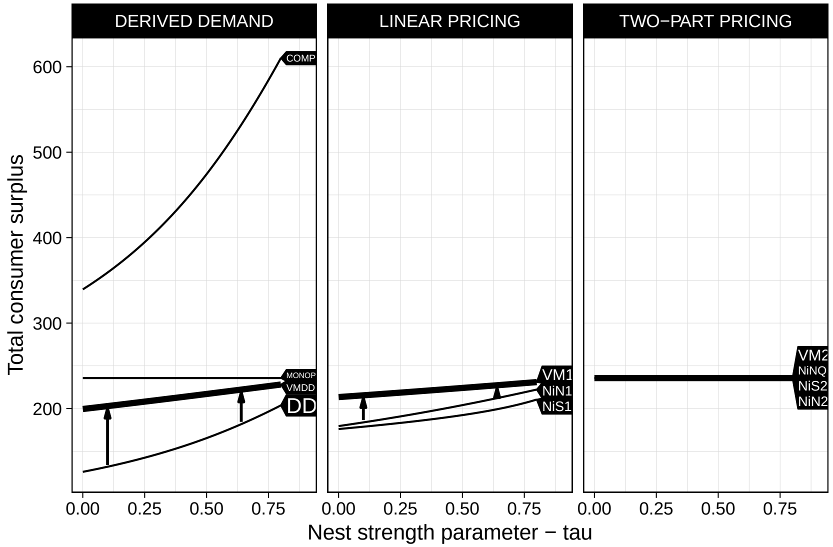


(d) Wholesale fees for firm A and B

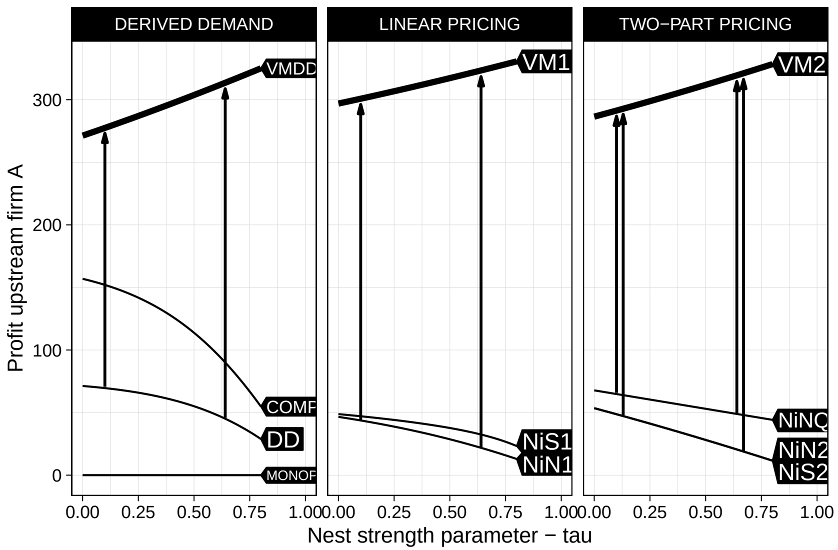
Figure D.11: Wholesale prices and fees for a 2x1 setting: (a) shows the elimination of double marginalisation for the vertically integrated firm; (b) shows the reducing of rival's revenue for the rival firm; (c) shows the decrease in total marginal cost for the rival firm as a result of reducing of rival's revenue; and, (d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting



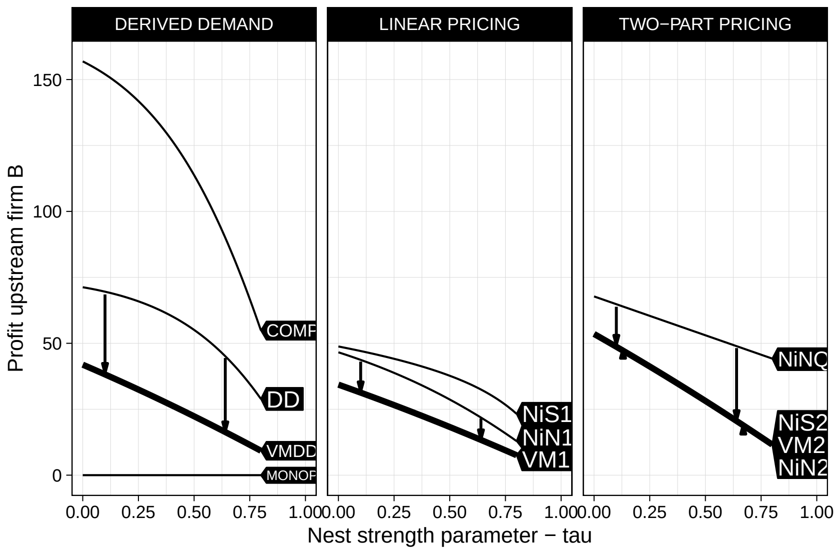
(a) Total quantity vs tau



(b) Consumer surplus vs tau



(c) Profit firm A and 1 vs tau



(d) Profit firm B vs tau

Figure D.12: Profit and welfare in a 2x1 setting: (a) shows total quantity; (b) shows that consumer surplus closely follows total quantity; (c) shows the profit of firm A - post-merger profit is always higher than pre-merger profit; and, (d) shows the profit of firm B - post-merger profit is always lower than pre-merger profit

Appendix E

Summary Tables

Table E.1: Summary for Aggregate Elasticity 1×2 industry: Downstream variables of interest

Model	q1	q2	p1	p2
Monopoly	Constant	Constant	Constant	Constant
Competition	Hump-shaped; Increasing where %deltaAE > %deltaMC; decreasing vice versa	Hump-shaped; Increasing where %deltaAE > %deltaMC; decreasing vice versa	Increasing; below Monopoly	Increasing; below Monopoly
DD	Decreasing; below Monopoly	Decreasing; below Monopoly	Increasing; above Monopoly	Increasing; above Monopoly
VMDD	Decreasing; above DD, Monopoly and Competition	Almost constant; below DD, Monopoly and Competition	Increasing; below Monopoly and DD; above Competition	Decreasing; above Monopoly, DD and Competition
NiN1	Decreasing; above Monopoly for low AE; below Monopoly for high AE; marginally below NiS1	Decreasing; above Monopoly for low AE; below Monopoly for high AE; marginally below NiS1	Increasing; below Monopoly for low AE; above Monopoly for high AE; marginally above NiS1	Increasing; below Monopoly for low AE; above Monopoly for high AE; marginally above NiS1
NiS1	Decreasing; above Monopoly for low AE; below Monopoly for high AE; marginally above NiN1	Decreasing; above Monopoly for low AE; below Monopoly for high AE; marginally above NiN1	Increasing; below Monopoly for low AE; above Monopoly for high AE; marginally below NiN1	Increasing; below Monopoly for low AE; above Monopoly for high AE; marginally below NiN1
VM1	Decreasing; above NiN1 & NiS1	Increasing; below Monopoly, NiN1 & NiS1; above VMDD	Increasing; below Monopoly; below NiN1 and NiS1 for low AE; above NiN1 and NiS1 for high AE	Decreasing; above Monopoly, NiN1, NiS1 and Competition; below VMDD
NiN2	Hump-shaped; above Monopoly	Hump-shaped; above Monopoly	Increasing; below Monopoly	Increasing; below Monopoly
NiS2	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly
NiNQ	Hump-shaped; above Monopoly, NiN2 and NiS2; below Competition	Hump-shaped; above Monopoly, NiN2 and NiS2; below Competition	Increasing; below Monopoly, NiN2, NiS2 and Competition	Increasing; below Monopoly, NiN2, NiS2 and Competition
VM2	Decreasing; above Monopoly, NiN2, NiS2 & NiNQ	Increasing; below Monopoly, NiN2, NiS2 & NiNQ	Increasing; below Monopoly; above NiN2 & NiNQ	Decreasing; above Monopoly, NiN2,NiS2 & NiNQ

Table E.2: Summary for Aggregate Elasticity 1×2 industry: Wholesale variables of interest

Model	wp1	wp2	wf1	wf2
Monopoly	Decreasing	Decreasing	Decreasing	Decreasing
Competition	Zero	Zero	Zero	Zero
DD	Decreasing, above Monopoly	Decreasing, above Monopoly	N/A	N/A
VMDD	Eliminated	Decreasing; above Monopoly and DD	N/A	N/A
NiN1	Decreasing; above Monopoly and NiNQ; marginally above NiS1	Decreasing; above Monopoly and NiNQ; marginally above NiS1	N/A	N/A
NiS1	Decreasing; above Monopoly and NiNQ; marginally below NiN1	Decreasing; above Monopoly and NiNQ; marginally below NiN1	N/A	N/A
VM1	Eliminated	Decreasing; above NiN1 & NiS1	N/A	N/A
NiN2	Decreasing; below NiS2	Decreasing; below NiS2	Hump-shaped; above NiS2	Hump-shaped; above NiS2
NiS2	Decreasing; follows Monopoly; above NiN2	Decreasing; follows Monopoly; above NiN2	Hump-shaped; below NiN2	Hump-shaped; below NiN2
NiNQ	Decreasing; below NiN2 and NiS2	Decreasing; below NiN2 and NiS2	Hump-shaped; just above NiN2; above NiS2	Hump-shaped; just above NiN2; above NiS2
VM2	Eliminated	Decreasing; above NiN2, NiS2 & NiNQ	Eliminated	Increasing; above NiS2; above NiN2 & NiNQ for low AE; below for high AE

Table E.3: Summary for Aggregate Elasticity 1×2 industry: Merger effects and welfare

Model	**Total quantity**	**Consumer welfare**	**Profit A and 1**	**Profit Down2**
Monopoly	Constant	Decreasing	Decreasing	Zero
Competition	Hump-shaped; Increasing where %deltaAE > %deltaMC; decreasing vice versa	Decreasing; above Monopoly	Decreasing; below Monopoly	Decreasing
DD	Decreasing; below Monopoly	Decreasing; below Monopoly and Competition	Decreasing; below Monopoly; above Competition	Decreasing
VMDD	Decreasing; below Monopoly; above DD	Decreasing; below Monopoly and Competition; above DD	Decreasing; below Monopoly; above DD	Decreasing; below DD
NiN1	Decreasing; above Monopoly for low AE; below Monopoly for high AE; marginally below NiS1	Decreasing; very close to NiS1	Decreasing; very close to NiS1	Decreasing
NiS1	Decreasing; above Monopoly for low AE; below Monopoly for high AE; marginally above NiN1	Decreasing; very close to NiN1	Decreasing; very close to NiN1	Decreasing
VM1	Decreasing; below NiN1 & NiS1 for low AE; below for high AE	Decreasing; below NiN1 & NiS1 for low AE; above for high AE	Decreasing; above NiN1, NiS1 and VMDD	Slightly decreasing; below NiN & NiS1; above VMDD
NiN2	Hump-shaped; above Monopoly and NiS2	Decreasing; above NiS2	Decreasing; below NiS2	Decreasing
NiS2	Constant; follows Monopoly	Decreasing; below NiN2	Decreasing; above NiN2	Decreasing
NiNQ	Hump-shaped; above Monopoly, NiN2, NiS2 and Competition	Decreasing above Monopoly, NiN2 & NiS2	Decreasing; below NiN2 & NiS2	Decreasing; above NiN2 & NiS2
VM2	Below NiN2 & NiNQ; above NiS2	Decreasing; above NiS2; below NiN2 & NiNQ	Decreasing; above NiN2, NiS2 & NiNQ; below Monopoly	Decreasing; below NiN2, NiS2 & NiNQ; below VM1

Table E.4: Summary for Nest strength parameter 1×2 industry: Downstream variables of interest

Model	**q1**	**q2**	**p1**	**p2**
Monopoly	Constant	Constant	Constant	Constant
Competition	Increasing	Increasing	Decreasing; below Monopoly	Decreasing; below Monopoly
DD	Increasing; below Monopoly	Increasing; below Monopoly	Decreasing; above Monopoly	Decreasing; above Monopoly
VMDD	Increasing; above DD and Monopoly; above Competition for low tau, vice verca for high tau	Almost constant; below DD, Monopoly and Competition	Increasing; below Monopoly and DD; above Competition	Decreasing; above Monopoly, DD and Competition
NiN1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiS1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiS2	Decreasing; above Monopoly for low tau; below Monopoly for high tau; very close to NiS1	Decreasing; above Monopoly for low tau; below Monopoly for high tau; very close to NiS1
NiS1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiN1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiN2	Decreasing; above Monopoly for low tau; below Monopoly for high tau; very close to NiN1	Decreasing; above Monopoly for low tau; below Monopoly for high tau; very close to NiN1
VM1	Increasing; above NiN1 and NiS1	Decreasing; below Monopoly, NiN1 & NiS1; above VMDD	Increasing; below Monopoly; below NiN1 and NiS1 for low tau; above NiN1 and NiS1 for high tau	Decreasing; above Monopoly, NiN1, NiS1 & Competition; below VMDD
NiN2	Increasing; above Monopoly	Increasing; above Monopoly	Decreasing; below Monopoly	Decreasing; below Monopoly
NiS2	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly
NiNQ	Increasing; above Monopoly, NiN2 & NiS2; below Competition	Increasing; above Monopoly, NiN2 & NiS2; below Competition	Decreasing; below Monopoly, NiN2 & NiS2	Decreasing; below Monopoly, NiN2 & NiS2
VM2	Increasing; above Monopoly, NiN2 and NiS2; above NiNQ for low tau, above vice versa	Decreasing; below Monopoly, NiN2, NiS2 & NiNQ; above VM1 and VMDD	Increasing; below Monopoly; above NiN2 & NiNQ	Decreasing; above Monopoly, NiN2, NiS2 & NiNQ

Table E.5: Summary for Nest strength parameter 1×2 industry: Wholesale variables of interest

Model	**wp1**	**wp2**	**wf1**	**wf2**
Monopoly	Increasing	Increasing	Decreasing	Decreasing
Competition	Zero	Zero	Zero	Zero
DD	Increasing, above Monopoly	Increasing, above Monopoly	N/A	N/A
VMDD	Eliminated	Increasing; above Monopoly and DD	N/A	N/A
NiN1	Decreasing; above Monopoly and NiS1 for low tau; below vice verca	Decreasing; above Monopoly and NiS1 for low tau; below vice verca	N/A	N/A
NiS1	Decreasing; above Monopoly for low tau; below vice verca; below NiN1 for low tau, above vice verca	Decreasing; above Monopoly for low tau; below vice verca; below NiN1 for low tau, above vice verca	N/A	N/A
VM1	Eliminated	Increasing; above NiN1 & NiS1	N/A	N/A
NiN2	Increasing; below NiS2	Increasing; below NiS2	Decreasing; above NiS2	Decreasing; above NiS2
NiS2	Increasing; above NiN2	Increasing; above NiN2	Decreasing; below NiN2	Decreasing; below NiN2
NiNQ	Increasing; below NiN2 & NiS2	Increasing; below NiN2 & NiS2	Decreasing; above NiN2 & NiS2	Decreasing; above NiN2 & NiS2
VM2	Eliminated	Increasing; above NiN2, NiS2 & NiNQ	Eliminated	Decreasing; above NiS2; below NiNQ; below NiN2 for low tau; above NiN2 for high tau

Table E.6: Summary for Nest strength parameter 1×2 industry: Merger effects and profit

Model	Total quantity	Consumer welfare	Profit A	Profit Down2
Monopoly	Constant	Constant	Constant	Zero
Competition	Increasing; above Monopoly	Increasing; above Monopoly	Zero	Decreasing
DD	Increasing; below Monopoly	Increasing; below Monopoly	Increasing; below Monopoly; above Competition	Decreasing
VMDD	Increasing; below Monopoly; above DD; approaches Monopoly	Increasing; below Monopoly; above DD	Increasing; below Monopoly; above DD	Decreasing; below DD
NiN1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiS1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiS1	Decreasing; very close to NiS1	Decreasing
NiS1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiN1	Increasing; below Monopoly for low tau; above Monopoly for high tau; very close to NiN1	Decreasing; very close to NiN1	Decreasing
VM1	Increasing; above NiN1 & NiS1 for low tau; below for high tau	Increasing; above NiN1 & NiS1 for low tau; above for high tau	Increasing; above NiN1 & NiS1; below VMDD	Decreasing; below NiN1 & NiS1; above VMDD
NiN2	Increasing; above Monopoly and NiS2	Increasing; above Monopoly and NiS2	Decreasing; below NiS2	Decreasing
NiS2	Constant; follows Monopoly	Constant; follows Monopoly	Increasing; above NiN2	Decreasing
NiNQ	Increasing; above Monopoly, NiN2, NiS2 and Competition	Increasing; below Monopoly, NiN2, NiS2 and Competition	Decreasing; below NiN2 & NiS2	Decreasing; above NiN1 & NiS1 for low tau; above vice versa
VM2	Below NiN2 & NiNQ; above NiS2	Decreasing; above NiS2; below NiN2 & NiNQ	Increasing; above NiN2, NiS2 & NiNQ; below Monopoly	Decreasing; below NiN2, NiS2 & NiNQ; below VM1

Table E.7: Summary for Aggregate Elasticity 2 $\times$ 1 industry: Downstream variables of interest

Model	q1	q2	p1	p2
Monopoly	Constant	Constant	Constant	Constant
Competition	Hump-shaped; Increasing where %deltaAE > %deltaMC; decreasing vice versa	Hump-shaped; Increasing where %deltaAE > %deltaMC; decreasing vice versa	Increasing; below Monopoly	Increasing; below Monopoly
DD	Decreasing; below Monopoly	Decreasing; below Monopoly	Increasing; above Monopoly	Increasing; above Monopoly
VMDD	Decreasing; above DD, Monopoly and Competition	Slightly decreasing; below DD, Monopoly and Competition	Slightly ncreasing; below Monopoly and DD; above Competition	Almost constant; above Monopoly, DD and Competition
NiN1	Decreasing; below Monopoly; marginally above NiS1	Decreasing; below Monopoly; marginally above NiS1	Slightly increasing; above Monopoly; marginally below NiS1	Slightly increasing; above Monopoly; marginally below NiS1
NiS1	Decreasing; below Monopoly; marginally below NiN1	Decreasing; below Monopoly; marginally below NiN1	Slightly increasing; above Monopoly; marginally above NiN1	Slightly increasing; above Monopoly; marginally above NiN1
VM1	Decreasing; above Monopoly, NiN1 & NiS1	Decreasing; below Monopoly, NiN1 & NiS1; above VMDD	Increasing; below Monopoly; below NiN1 and NiS1	Almost constant; above Monopoly, NiN1, NiS1 and Competition; below VMDD
NiN2	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly
NiS2	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly
NiNQ	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly
VM2	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly	Constant; follows Monopoly

Table E.8: Summary for Aggregate Elasticity 2×1 industry: Wholesale variables of interest

Model	wp1	wp2	wf1	wf2
Monopoly	0	0	0	0
Competition	Converges to zero	Converges to zero	Converges to zero	Converges to zero
DD	Decreasing, above Monopoly	Decreasing, above Monopoly	N/A	N/A
VMDD	Eliminated	Decreasing; above Monopoly; below DD	N/A	N/A
NiN1	Decreasing; above Monopoly; marginally below NiS1	Decreasing; above Monopoly; marginally below NiS1	N/A	N/A
NiS1	Decreasing; above Monopoly; marginally above NiN1	Decreasing; above Monopoly; marginally above NiN1	N/A	N/A
VM1	Eliminated	Decreasing; below NiN1 & NiS1	N/A	N/A
NiN2	Constant	Constant	Decreasing	Decreasing
NiS2	Constant	Constant	Decreasing	Decreasing
NiNQ	Constant	Constant	Decreasing	Decreasing
VM2	Eliminated	Constant	Eliminated	Decreasing

Table E.9: Summary for Aggregate Elasticity 2×1 industry: Merger effects and welfare

Model	Total quantity	Consumer welfare	Profit A	Profit B
Monopoly	Constant	Decreasing	Zero	Zero
Competition	Hump-shaped; Increasing where %deltaAE > %deltaMC; decreasing vice versa	Decreasing; above Monopoly	Decreasing; above Monopoly	Decreasing; above Monopoly
DD	Decreasing; below Monopoly	Decreasing; below Monopoly	Decreasing; above Monopoly	Decreasing; above Monopoly
VMDD	Decreasing; below Monopoly; above DD	Decreasing; below Monopoly and Competition; above DD	Decreasing; above Monopoly; above DD	Decreasing; above Monopoly; below DD
NiN1	Decreasing; below Monopoly; marginally above NiS1	Decreasing; very close to NiS1	Decreasing; very close to NiS1	Decreasing
NiS1	Decreasing; below Monopoly; marginally below NiN1	Decreasing; very close to NiN1	Decreasing; very close to NiN1	Decreasing
VM1	Decreasing; above NiN1 & NiS1	Decreasing; above NiN1 & NiS1	Decreasing; above NiN1, NiS1 and VMDD	Slightly decreasing; below NiN & NiS1
NiN2	Constant	Decreasing	Constant	Decreasing
NiS2	Constant; follows Monopoly	Decreasing	Constant	Decreasing
NiNQ	Constant	Decreasing	Decreasing; above NiN2 & NiS2	Decreasing; above NiN2 & NiS2
VM2	Constant	Decreasing	Decreasing; above NiN2, NiS2 & NiNQ	Decreasing

Table E.10: Summary for Nest strength parameter 2×1 industry: Downstream variables of interest

Model	q1	q2	p1	p2
Monopoly	Constant	Constant	Constant	Constant
Competition	Increasing	Increasing	Decreasing; below Monopoly	Decreasing; below Monopoly
DD	Increasing; below Monopoly	Increasing; below Monopoly	Decreasing; above Monopoly	Decreasing; above Monopoly
VMDD	Increasing; above DD and Monopoly; above Competition for low tau, vice verca for high tau	Almost constant; below DD, Monopoly and Competition	Increasing; below Monopoly and DD; above Competition	Decreasing; above Monopoly, DD and Competition
NiN1	Increasing; below Monopoly; very close to NiS1	Increasing; below Monopoly; very close to NiS2	Decreasing; above Monopoly; very close to NiS1	Decreasing; above Monopoly; very close to NiS1
NiS1	Increasing; below Monopoly; very close to NiN1	Increasing; below Monopoly; very close to NiN2	Decreasing; above Monopoly; very close to NiN1	Decreasing; above Monopoly; very close to NiN1
VM1	Increasing; above NiN1 and NiS1	Almost constant; below Monopoly, NiN1 & NiS1; above VMDD	Increasing; below Monopoly; below NiN1 and NiS1	Decreasing; above Monopoly; above NiN1 & NiS1 for low tau, below for high tau; below VMDD
NiN2	Constant	Constant	Constant	Constant
NiS2	Constant	Constant	Constant	Constant
NiNQ	Constant	Constant	Constant	Constant
VM2	Constant	Constant	Constant	Constant

Table E.11: Summary for Nest strength parameter 2×1 industry: Wholesale variables of interest

Model	**wp1**	**wp2**	**wf1**	**wf2**
Monopoly	Zero	Zero	Zero	Zero
Competition	Convergent to zero	Convergent to zero	Increasing	Increasing
DD	Decreasing, above Monopoly	Decreasing, above Monopoly	N/A	N/A
VMDD	Eliminated	Decreasing; above Monopoly; below DD	N/A	N/A
NiN1	Decreasing; above Monopoly; below NiS1	Decreasing; above Monopoly; below NiS1	N/A	N/A
NiS1	Decreasing; above Monopoly; above NiN1	Decreasing; above Monopoly; above NiN1	N/A	N/A
VM1	Eliminated	Decreasing; below NiN1 & NiS1	N/A	N/A
NiN2	Constant	Constant	Decreasing	Decreasing
NiS2	Constant	Constant	Decreasing	Decreasing
NiNQ	Constant	Constant	Decreasing	Decreasing
VM2	Constant	Constant	Eliminated	Decreasing

Table E.12: Summary for Nest strength parameter 2×1 industry: Merger effects and profit

Model	Total quantity	Consumer welfare	Profit A	Profit B
Monopoly	Constant	Constant	Zero	Zero
Competition	Increasing; above Monopoly	Increasing; above Monopoly	Decreasing	Decreasing
DD	Increasing; below Monopoly	Increasing; below Monopoly	Decreasing; below Monopoly	Decreasing; below Monopoly
VMDD	Increasing; below Monopoly; above DD; approaches Monopoly	Increasing; below Monopoly; above DD; approaches Monopoly	Increasing; above DD	Decreasing; below DD
NiN1	Increasing; below Monopoly; above NiS1	Increasing; below Monopoly; above NiS1	Decreasing; below NiS1	Decreasing, below NiS1
NiS1	Increasing; below Monopoly; below NiN1	Increasing; below Monopoly; below NiN1	Decreasing; above NiN1	Decreasing; above NiN1
VM1	Increasing; above NiN1 & NiS1	Increasing; above NiN1 & NiS1	Increasing; above NiN1 & NiS1	Decreasing; below NiN1 & NiS1
NiN2	Constant	Constant	Decreasing	Decreasing
NiS2	Constant	Constant	Decreasing	Decreasing
NiNQ	Constant	Constant	Decreasing	Decreasing
VM2	Constant	Constant	Increasing; above NiN2, NiS2 & NiNQ	Decreasing

Appendix F

Graphic Results for Vertical Merger Simulator Examples

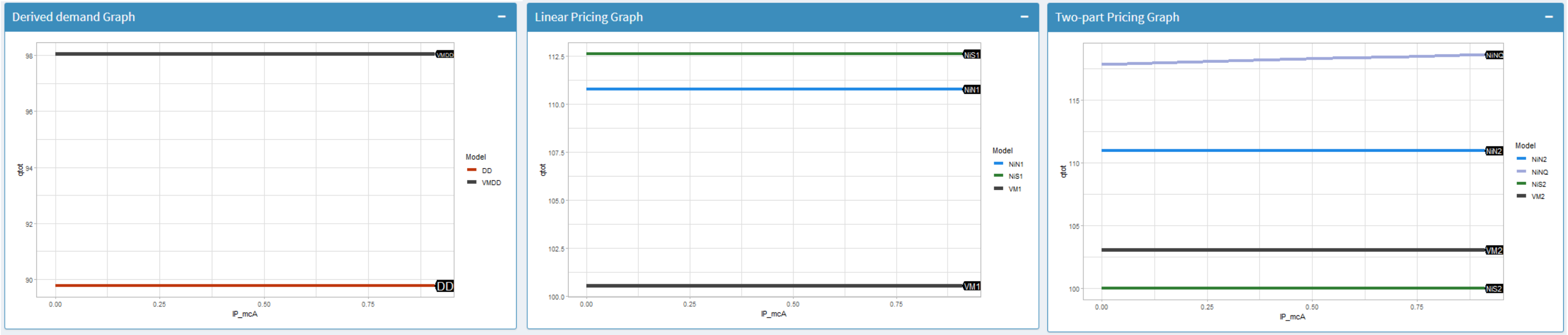


Figure F.1: Graphic results for marginal cost example: 1×2 industry structure

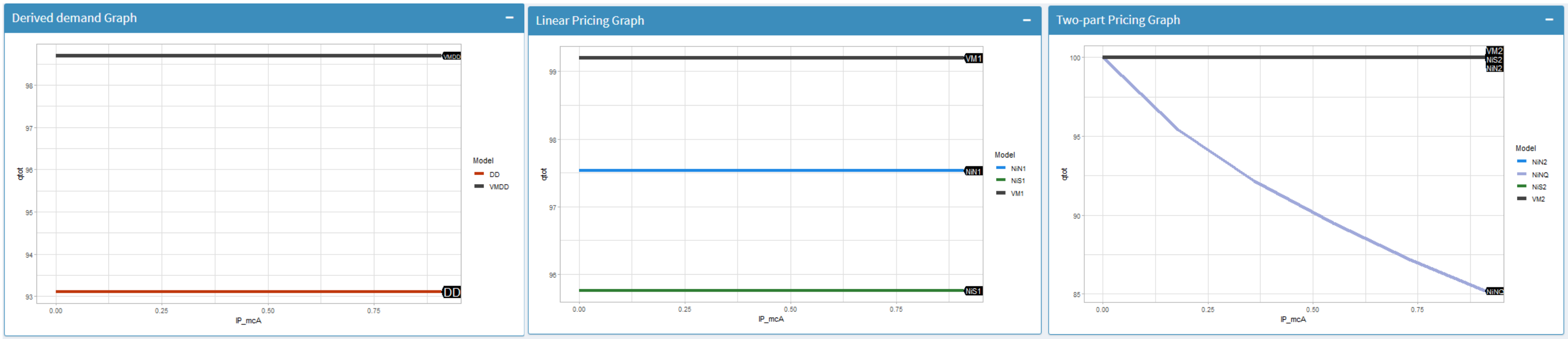


Figure F.2: Graphic results for marginal cost example: 2×1 industry structure

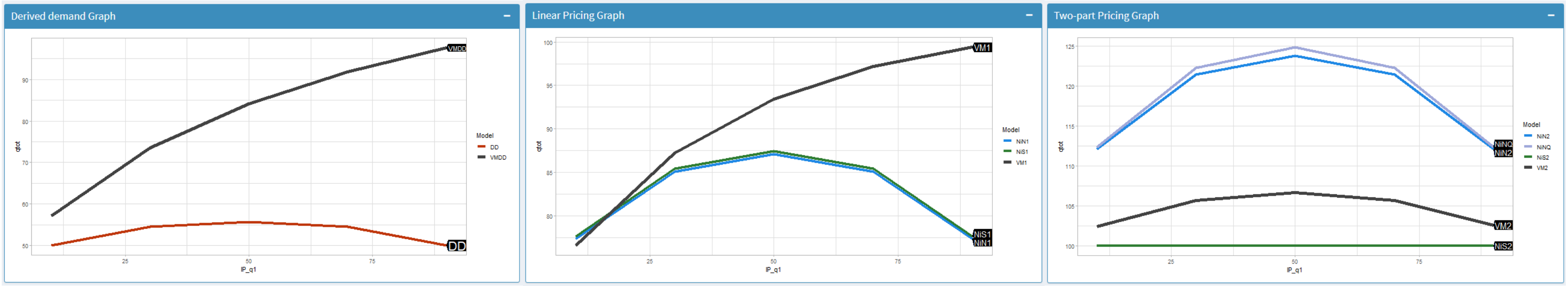


Figure F.3: Graphic results for market share example: 1×2 industry structure

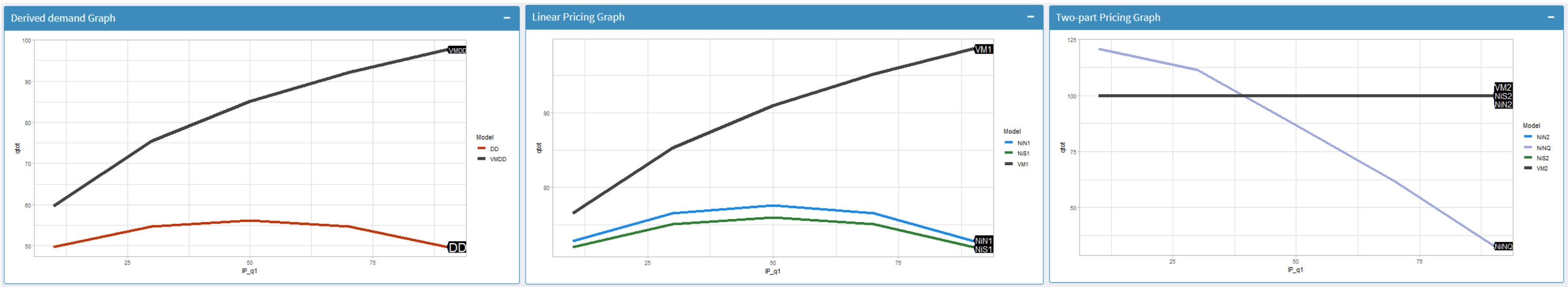


Figure F.4: Graphic results for market size example: 2×1 industry structure

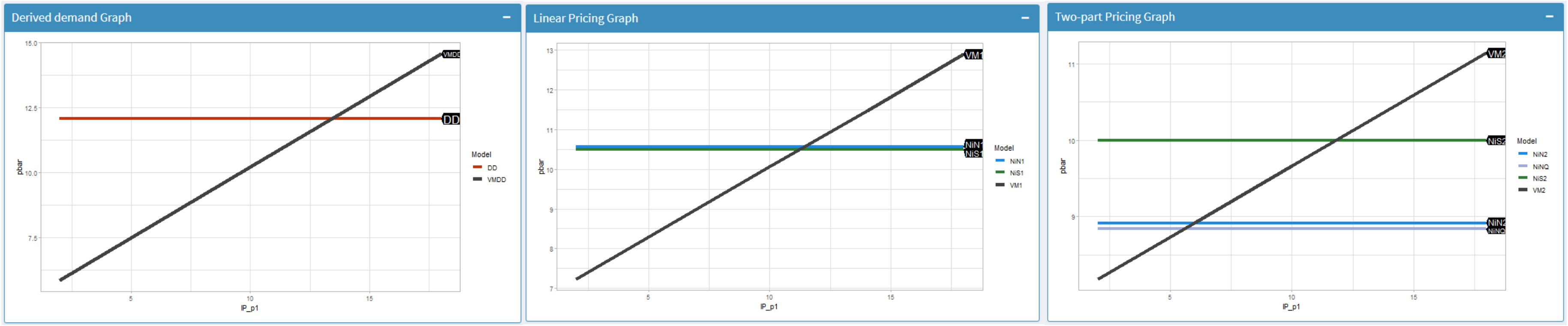


Figure F.5: Graphic results for price balance example: 1×2 industry structure

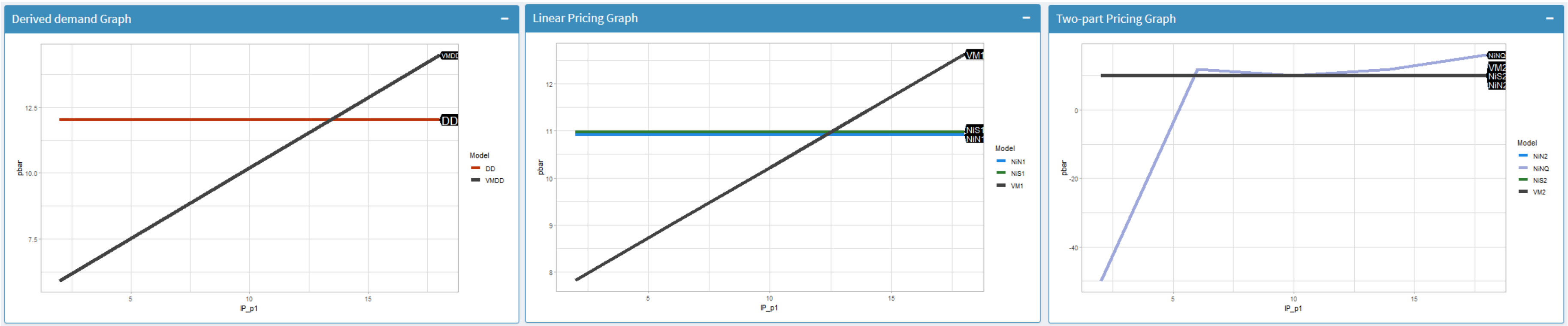
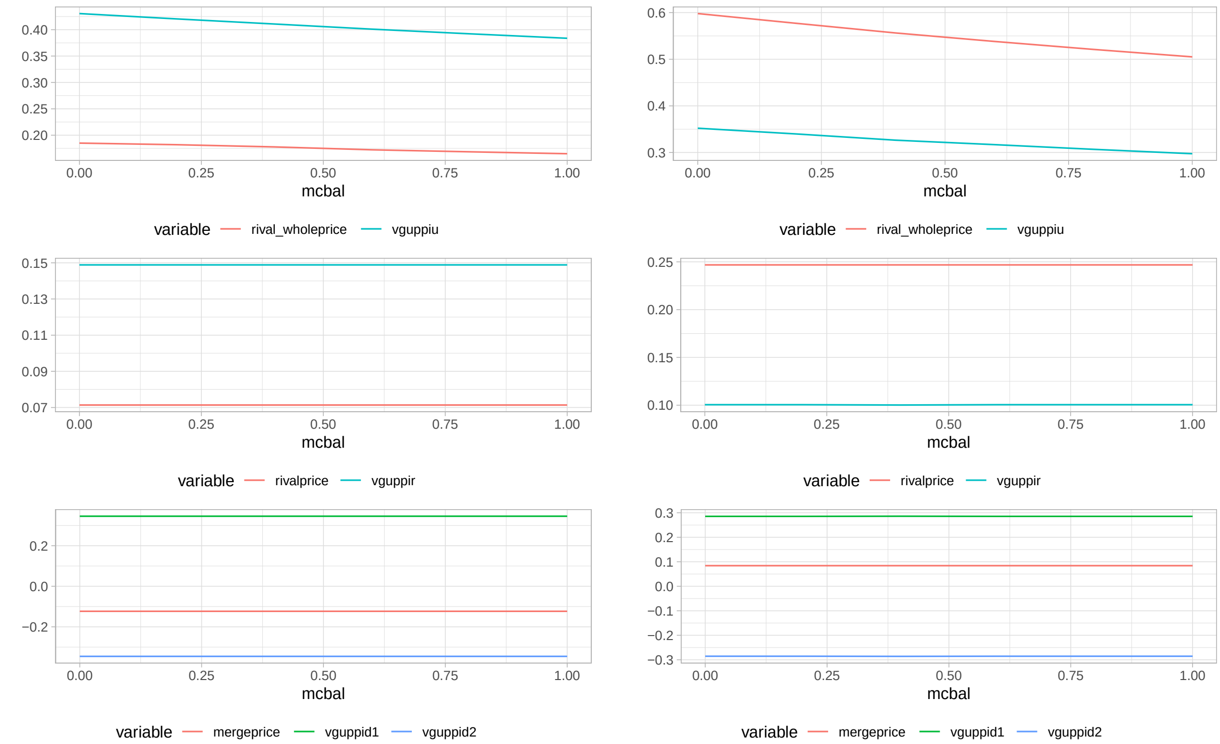


Figure F.6: Graphic results for price balance example: 2×1 industry structure

Appendix G

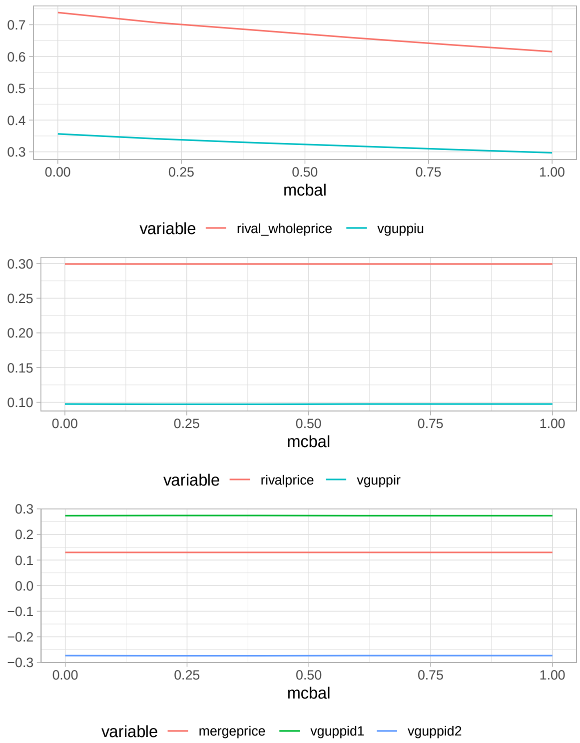
Comparison Examples



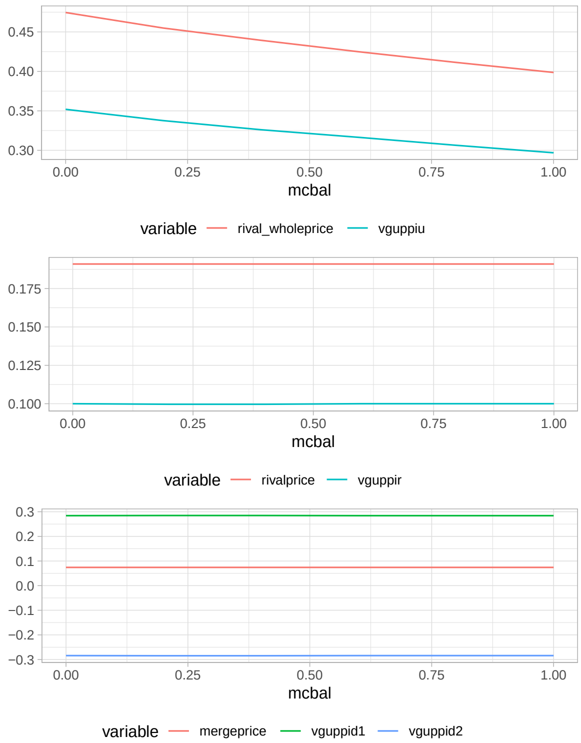
(a) Marginal cost example: derived demand model

(b) Marginal cost example: NiN1 model

Figure G.1: Comparisons for marginal cost example: derived demand and NiN1 models

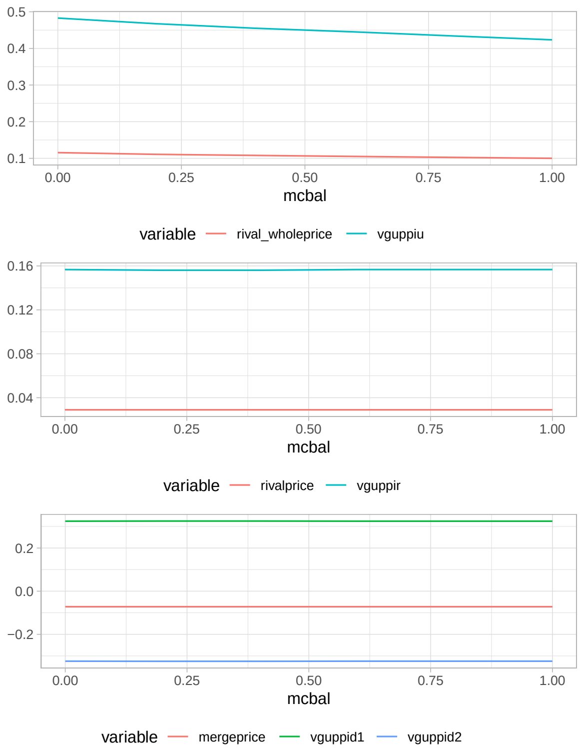


(a) Marginal cost example: NiS1 model

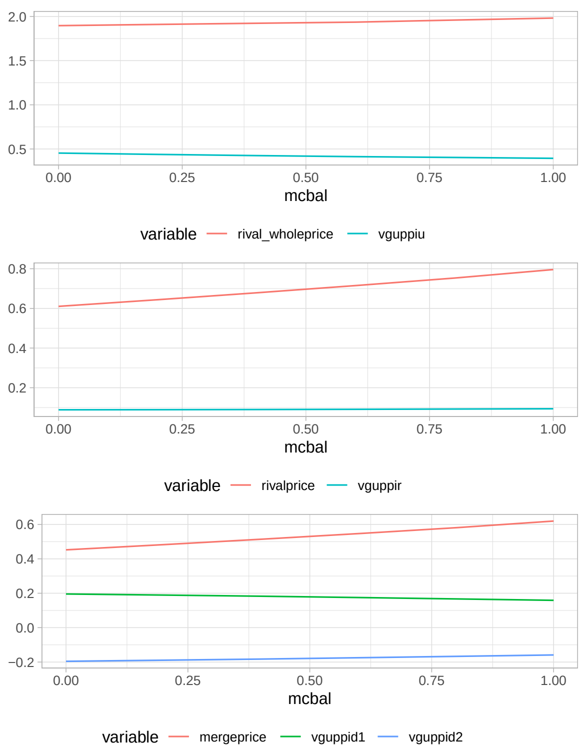


(b) Marginal cost example: NiN2 model

Figure G.2: Comparisons for marginal cost example: NiS1 and NiN2 models

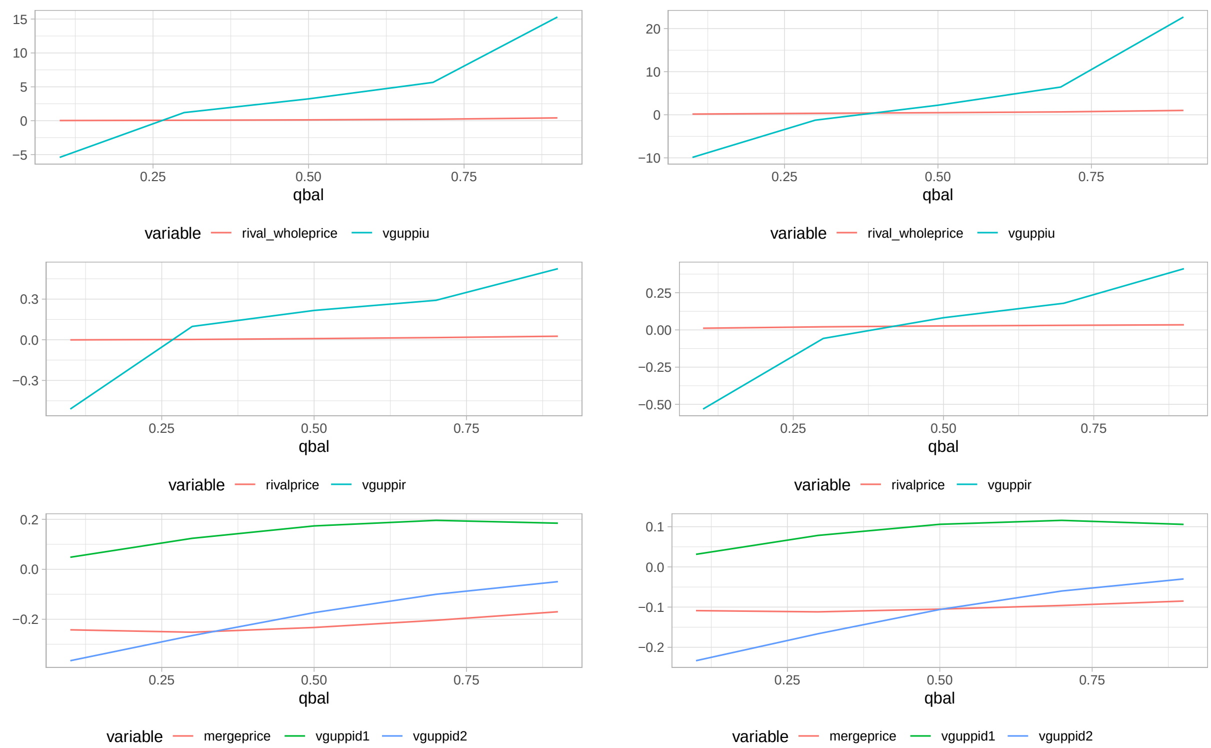


(a) Marginal cost example: NiS2 model



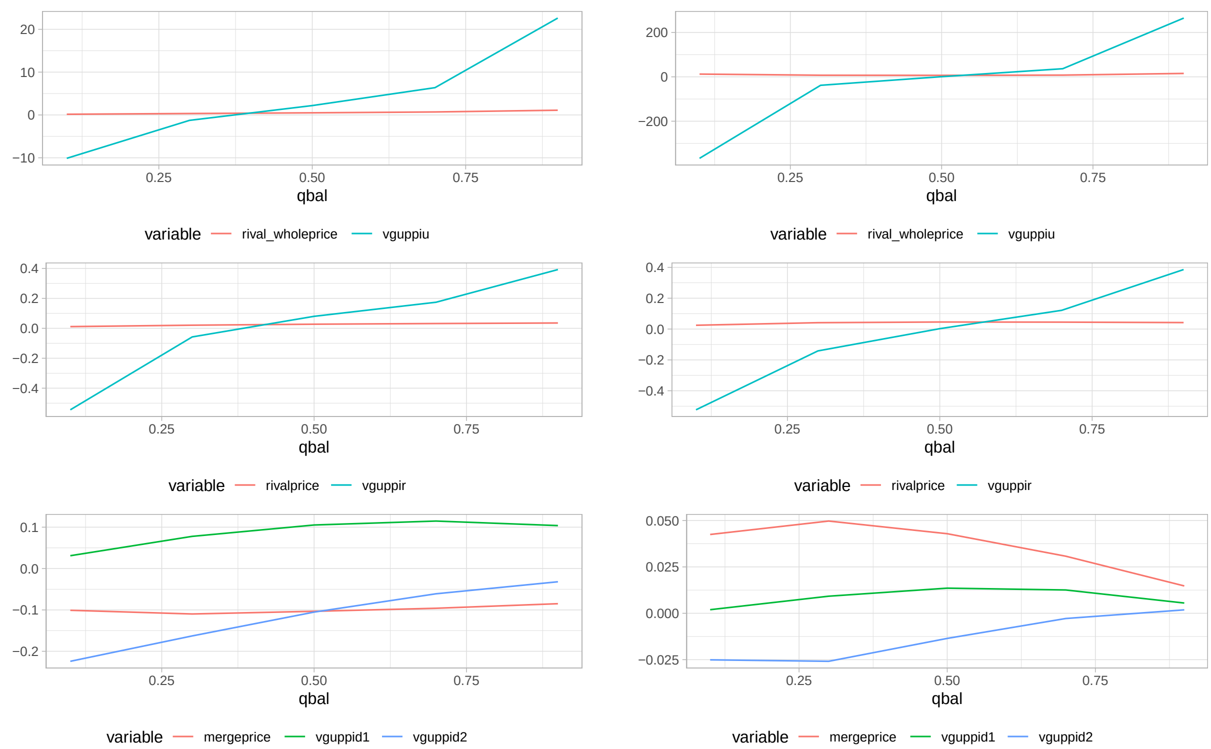
(b) Marginal cost example: NiNQ model

Figure G.3: Comparisons for marginal cost example: NiS2 and NiNQ models



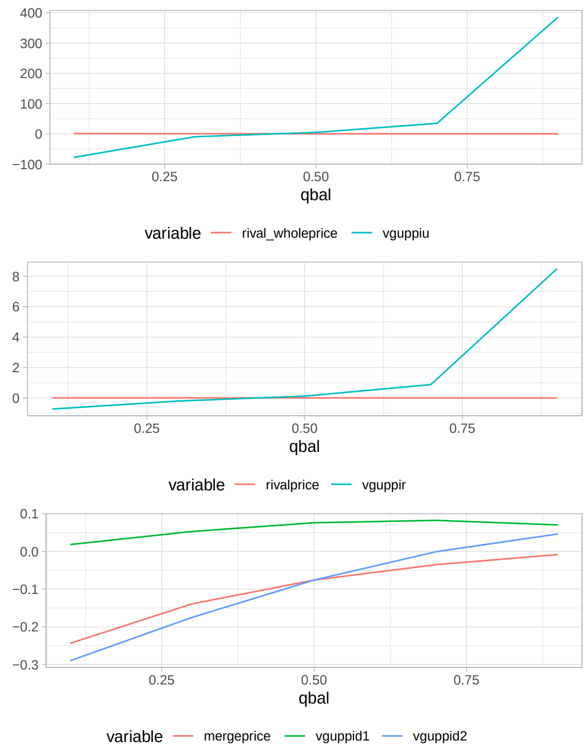
(a) Market share example: derived demand model (b) Market share example: NiN1 model

Figure G.4: Comparisons for market share example: derived demand and NiN1 models

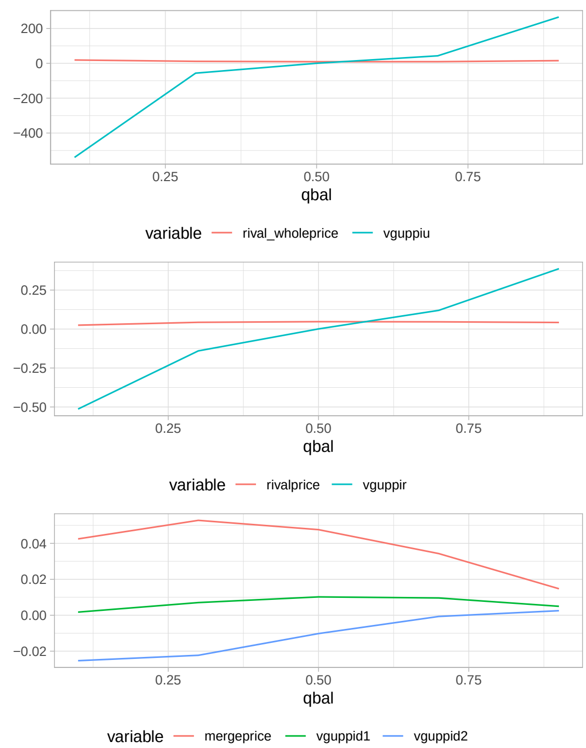


(a) Market share example: NiS1 model (b) Market share example: NiN2 model

Figure G.5: Comparisons for market share example: NiS1 and NiN2 models

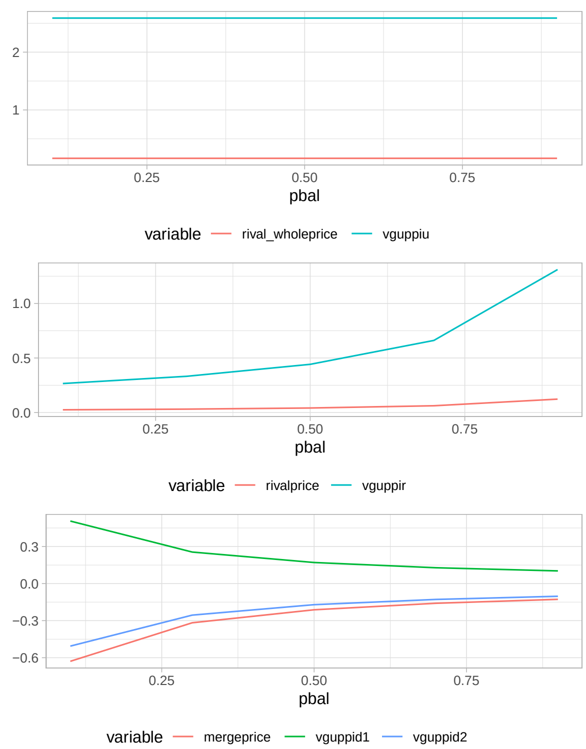


(a) Market share example: NiS2 model

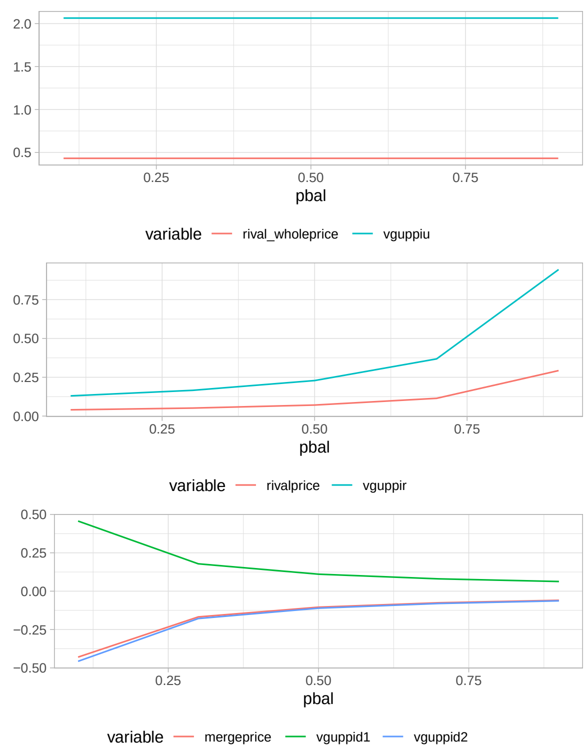


(b) Market share example: NiNQ model

Figure G.6: Comparisons for market share example: NiS2 and NiNQ models

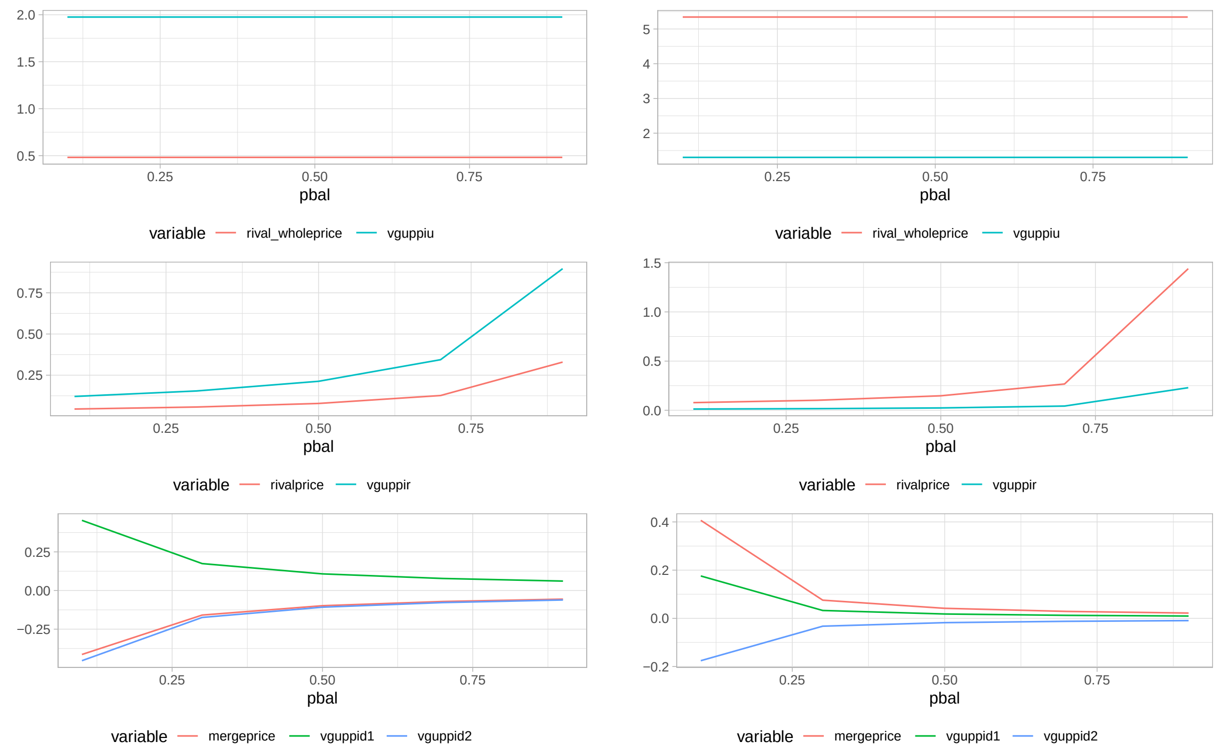


(a) Price balance example: derived demand model



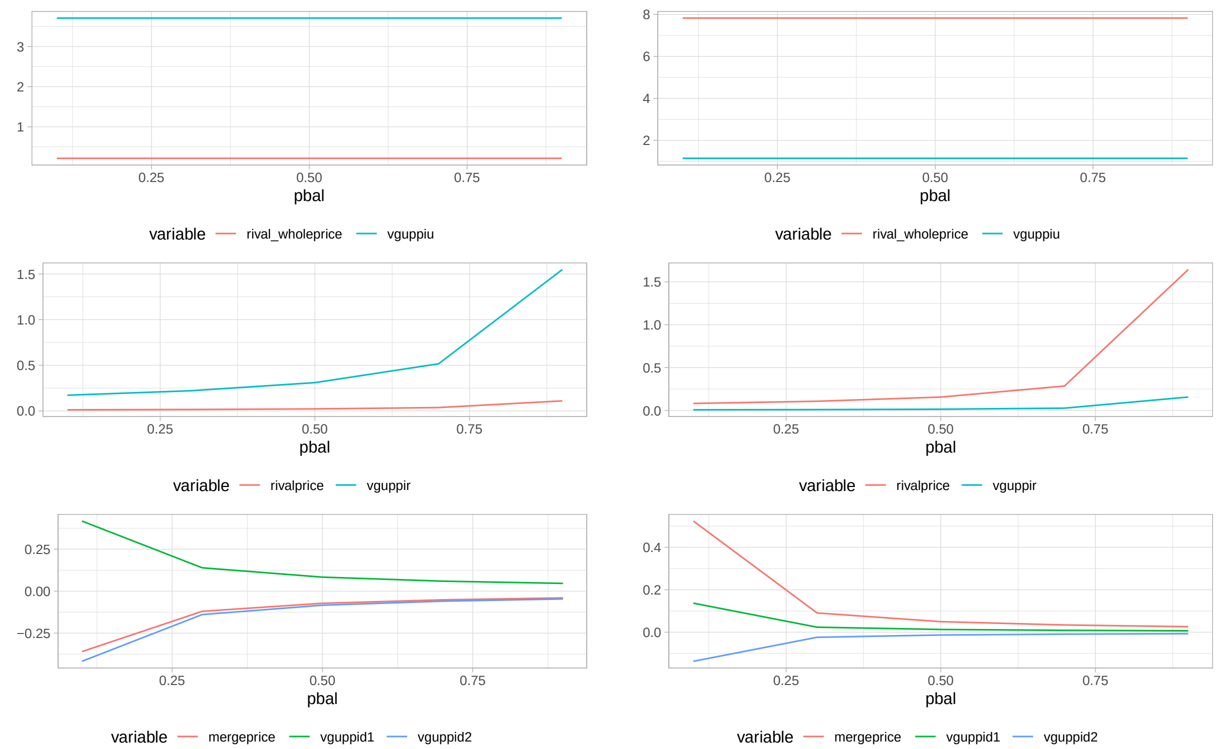
(b) Price balance example: NiN1 model

Figure G.7: Comparisons for price balance example: derived demand and NiN1 models



(a) Price balance example: NiS1 model (b) Price balance example: NiN2 model

Figure G.8: Comparisons for price balance example: NiS1 and NiN2 models



(a) Price balance example: NiS2 model (b) Price balance example: NiNQ model

Figure G.9: Comparisons for price balance example: NiS2 and NiNQ models

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