

Dynamical Critical Scaling of Long-Range Interacting Quantum Magnets


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Slow quenches of the magnetic field across the paramagnetic-ferromagnetic phase transition of spin systems produce heat. In systems with short-range interactions the heat exhibits universal power-law scaling as a function of the quench rate, known as Kibble-Zurek scaling. In this work we analyze slow quenches of the magnetic field in the Lipkin-Meshkov-Glick (LMG) model, which describes fully connected quantum spins. We analytically determine the quantum contribution to the residual heat as a function of the quench rate δ by means of a Holstein-Primakoff expansion about the mean-field value. Unlike in the case of short-range interactions, scaling laws in the LMG model are only found for a ramp starting or ending at the critical point. If instead the ramp is symmetric, as in the typical Kibble-Zurek scenario, then the number of excitations exhibits a crossover behavior as a function of δ and tends to a constant in the thermodynamic limit. Previous, and seemingly contradictory, theoretical studies are identified as specific limits of this dynamics. Our results can be tested on several experimental platforms, including quantum gases and trapped ions.

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The development of a comprehensive statistical mechanics description of out-of-equilibrium systems is a quest of relevance across disciplines, including biology, physics, computer science and financial markets [1]. For systems with long-range interactions insights so far are mostly based on numerical simulations [2], which become increasingly involved when the dynamics is deep in the quantum regime [3,4]. An open question concerns understanding the interplay between time evolution, interactions, quantum, and thermal fluctuations, and specifically the connection between dynamical and equilibrium properties of quantum critical systems [5]. Theoretical and experimental studies of many-body critical dynamics after sudden variations of control fields have identified features which are reminiscent of the behavior of thermodynamic functions at transition points [6,7]. Yet, the relation between dynamical scaling and equilibrium critical phenomena is elusive and often only conjectured.

In this framework, it is believed that the thermodynamics of slow variations (quenches) of control fields across a critical point can be cast in terms of the so-called Kibble-Zurek (KZ) scaling [8–11]. The KZ scaling predicts that the heat produced by slow quenches scales with the quench rate as a power law determined by the equilibrium critical exponents [12–14]. This theory has a strong predictive power and has been experimentally tested in a variety of physical systems [15–29]. The KZ hypothesis can be explained as follows: Assume a system of interacting spins in presence of a magnetic field h , as illustrated in Fig. 1(a),

and let the magnitude of the magnetic field h vary slowly from the paramagnetic to the ferromagnetic phase across the critical point h_c . The evolution is adiabatic if the rate of change $\gamma_h = |\dot{h}|/|h - h_c|$ is smaller than the energy gap $\Delta(h)$, while in the opposite regime nonadiabatic effects are expected. Figure 1(b) displays the energy gap $\Delta(h)$ as a function of h : The gap vanishes as $\Delta(h) \sim |h - h_c|^{z\nu}$ at the

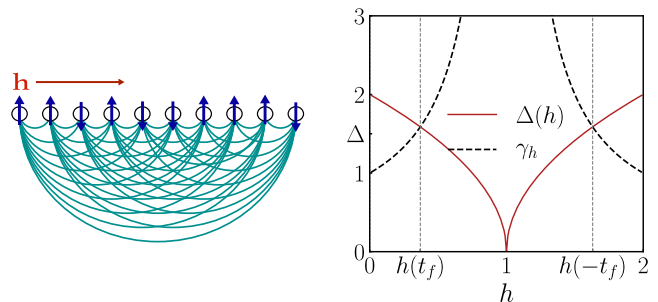


FIG. 1. Left: The dynamics of a spin-1/2 chain is analyzed when the strength of the magnetic field h is slowly varied across the paramagnetic-ferromagnetic transition. The lines connecting the sites illustrate that each spin interacts with equal strength J with the rest of the chain. Right: The energy gap $\Delta(h)$ between the ground and the first excited state of the chain is displayed as a function of $h = h_c - \delta t$ (solid red line). The dashed black line shows the rate $\gamma_h = |\dot{h}|/|h - h_c|$ with which the magnetic field is varied in time. The freezing time t_f is defined such that $\gamma_h(\pm t_f) = \Delta[h(\pm t_f)]$. We determine the scaling with δ of the quantum heat generated by the quench.

critical point, with equilibrium critical exponents ν and z . The KZ theory identifies the timescale $\pm t_f$, where $\Delta(h) = \gamma_h$ and assumes that in the time window $-t_f < t < t_f$ [namely, when $\Delta(h) < \gamma_h$] the dynamics is frozen. Then the heat Q produced by the quench scales as $Q \sim \xi_f^{-z}$, where $\xi_f \sim |h(t_f) - h_c|^{-\nu}$ is the average size of the spin domains formed at the time $t = -t_f$ in the adiabatic part of the dynamics, yielding $Q \sim |h(t_f) - h_c|^{z\nu}$. For a quench where the magnetic field varies linearly with time as $h = h_c - \delta t$ ($\delta > 0$) one obtains [11–13,30]

$$Q \sim \delta^{z\nu/(1+z\nu)}. \quad (1)$$

Although this separation between adiabatic and frozen (impulse) regime may seem oversimplified, it describes well the behavior found in isolated systems, where the relaxation time is determined by the instantaneous gap between the ground state and the first excited state, and for short-range interactions [31,32]. The validity of the KZ scaling (1) has been extensively verified in integrable fermionic systems [30,33–36]. Even at finite temperatures the KZ scaling is a good working hypothesis [37,38].

However the KZ theory seems to be valid only when the coherence length diverges with a power law at an isolated critical point, but is well defined otherwise [39]. This is not the case for systems with strongly long-ranged interactions where the two-body interaction potential $V(r) \propto r^{-\alpha}$ decays as a power law with the distance r , with $0 \leq \alpha < d$ in d spatial dimensions [2,42]. For such long-range interactions the dynamics is often radically different from short-range interacting systems [43–45] and it is therefore important to find a paradigm that allows one to extend the KZ scaling hypothesis also to these systems, where very few analytical solutions exist for the out-of-equilibrium dynamics. Moreover, such a paradigm would be important for the development of quantum devices based on quantum annealing, where one aims at preparing many-body quantum states by adiabatic transformations [46]. Yet, attempts to find the KZ scaling in the Lipkin-Meshkov-Glick (LMG) model [47], a prototype of a strong-long-range spin system, have led to seemingly contradicting results [48–50], which we detail below.

In this Letter we derive a scaling theory that encompasses different types of ramps across the critical region of the LMG model. We derive an exact solution which unifies previous findings [48–51] and thus provides an important benchmark for numerical studies of the out-of-equilibrium dynamics of quantum strong-long-range systems.

The LMG model describes a system of N spin-1/2 degrees of freedom with all-to-all ferromagnetic interactions in a transverse magnetic field h , as illustrated in Fig. 1(a). The Hamiltonian reads

$$H = -J \left(\frac{1}{N} \sum_{i < j} \sigma_i^x \sigma_j^x + h(t) \sum_i \sigma_i^z \right), \quad (2)$$

where σ_i^μ denotes the Pauli matrix of the spin at site i , and the prefactor $1/N$ in front of the interaction term warrants that the energy is extensive [2]. The parameter $J > 0$ scales the energy in units of the interaction strength, and we consider energy and time in units of J and J^{-1} , respectively. When the magnetic field h is constant in time, the LMG model displays equilibrium quantum phase transitions (QPTs) in the thermodynamic limit at $h_c = \pm 1$ between a symmetric phase ($|h| > h_c$) fully polarized along x , and a symmetry-broken phase ($|h| < h_c$) with two degenerate ground states of opposite macroscopic polarization along the z direction. The universality class of the QPT is the same as that of the Dicke model [52,53] and is given by mean-field theory with critical exponents $z = 1/3$ and $z\nu = 1/2$ [54–56]. Experimental realizations include trapped ions [57–59] and spinor Bose-Einstein condensates [60]. The Dicke model, moreover, has been used to describe quenches in the BEC-BCS crossover regime [61] as well as the self-organization transition of ultracold bosonic gases in optical cavities [62,63].

We now consider a continuous ramp of the control field $h(t) = 1 - \delta t$ with quench rate $\delta > 0$. The protocol starts deep in the paramagnetic phase and can end at the quantum critical point (half-ramp) or far in the symmetry-broken phase (full ramp). According to the KZ hypothesis, the quench generates a quantum contribution to the heat with the power-law scaling $Q \sim \delta^{1/3}$, consistent with $z\nu = 1/2$ in Eq. (1). However, this scaling of the heat was not found for full ramps in the numerical studies of Refs. [48,49]. In Ref. [48] the authors extracted the exponent $3/2$, while in Ref. [49] the authors could not identify any power-law scaling. Furthermore, power-law scaling seems inconsistent with the calculation in Ref. [51] for a system that is equivalent to the quantum dynamics of the LMG in the thermodynamic limit $N \rightarrow \infty$. In this reference it was shown that the heat produced at the end of the ramp is independent of the quench rate. On the other hand, in Ref. [50] the KZ scaling $Q \sim \delta^{1/3}$ was reported using a heuristic application of adiabatic perturbation theory for a quench to the critical point, which is expected to hold also in the thermodynamic limit.

To resolve this puzzle, we solve the Schrödinger equation governed by Hamiltonian (2) for $h(t) = 1 - \delta t$ with small quench rates δ . For this purpose we rewrite Eq. (2) in terms of a single collective spin of length N , namely, $S_\mu = \sum_i \sigma_i^\mu / 2$ and $S_\pm = S_x \pm iS_y$, such that [64]

$$H = -\frac{1}{N} (S^2 - S_z^2 - N/2) - 2h(t)S_z - \frac{1}{2N} (S_+^2 + S_-^2). \quad (3)$$

We then perform a $1/N$ expansion around the ground state of the mean-field model [65,66], which we detail in the Supplemental Material [67]. The expansion is obtained by first rotating the spin operators to align them with the

semiclassical magnetization and by applying a Holstein-Primakoff transformation [65] to quadratic order. With this approximation we write $S_z = N/2 - a^\dagger a$, $S_+ = S_-^\dagger = \sqrt{N}a$, where the operators a and a^\dagger satisfy bosonic commutation relations $[a, a^\dagger] = 1$. Following Ref. [64], we then use a Bogoliubov transformation to obtain the diagonal form

$$H_0 = N e_0(h) + \delta e(h) + \Delta(h) b^\dagger b \quad (4)$$

in the new bosonic operators b and b^\dagger . Here e_0 is the thermodynamic mean-field energy density, δe is a constant mean-field shift, and the quantum fluctuations are described by the quadratic term whose frequency is the gap Δ . The derivation of Eq. (4) and the explicit expressions for the parameters and for the operators in terms of the spin operators are reported in the Supplemental Material [67]. The quantum part of the Hamiltonian (4) is strictly valid only in the thermodynamic limit. However, by means of the continuous unitary transformation approach [70], the LMG Hamiltonian can be cast in the form (4) also at leading order in the $1/N$ expansion [64,71]. In this case the gap is given by [64,71]

$$\Delta = \begin{cases} 2\sqrt{h(h-1)} + \mathcal{F}(N, h) & \text{for } h > 1, \\ 2\sqrt{(1-h^2)} + \mathcal{F}(N, h) & \text{for } h < 1, \end{cases} \quad (5)$$

where for large N and $h \neq 1$ the function $\mathcal{F}(N, h) \sim 1/N$, while at the critical point the gap scales as $\Delta \propto 1/N^{1/3}$ [64,71]. The Hamiltonian (4) corresponds to a single harmonic oscillator with time-dependent frequency $\Omega(t) = \Delta[h(t)]$. It can be exactly solved in terms of the dynamical basis

$$\psi_n(x, t) = \left(\frac{e^{-i4\phi(t)}}{2\pi\xi^2(t)} \right)^{\frac{1}{4}} \frac{e^{-\tilde{\Omega}(t)x^2/2}}{\sqrt{2^n n!}} H_n \left(\frac{x}{\sqrt{2}\xi(t)} \right), \quad (6)$$

where H_n is the Hermite polynomial of degree n , $\phi(t)$ is a phase factor, $\tilde{\Omega}(t) = 1/2\xi^2 + i\dot{\xi}/\xi$ is the effective frequency and $\xi(t)$ is a time dependent scale factor which obeys the Ermakov-Milne equation [72–76]

$$\ddot{\xi}(t) + \Omega(t)^2 \xi(t) = \frac{1}{4\xi(t)^3}. \quad (7)$$

The wave function evolves from the time $t = -t_0 = -1/\delta$ until the final time $t = t_0$ (full ramp) or $t = 0$ (half ramp). In the adiabatic limit the $\psi_n(x, t)$ coincide with the instantaneous eigenstates $\psi_n^{\text{ad}}(x, t)$ of Hamiltonian $H(t)$, which are the solutions of Eq. (6) after setting $\dot{\xi} = \ddot{\xi} = 0$ in Eq. (7) and thus $\xi(t)^2 = 1/[\sqrt{2}\Omega(t)]$ in Eq. (6). We denote the overlap integral between $\psi_0(x, t)$ and the eigenfunctions $\psi_n^{\text{ad}}(x, t)$ of the adiabatic basis by $c_n(t) = \int \psi_n^{\text{ad}*}(x, t) \psi_0(x, t) dx$. Its explicit expression is derived in the Supplemental Material [67] and in Ref. [77]. By means of the fidelity $f(t) = |c_0(t)|^2$ we verify that the initial state $\psi_0(x, -t_0)$ coincides with the ground state of the Hamiltonian H at $h = 0$ [see Fig. 2(a) inset]. The heat (or excess energy) generated at time $t > -t_0$ is proportional to the excitation number $n_{\text{exc}}(t)$, $Q(t) = \Omega(t)n_{\text{exc}}(t)$, where

$$n_{\text{exc}}(t) = \sum_{n=1}^{\infty} n |c_n(t)|^2. \quad (8)$$

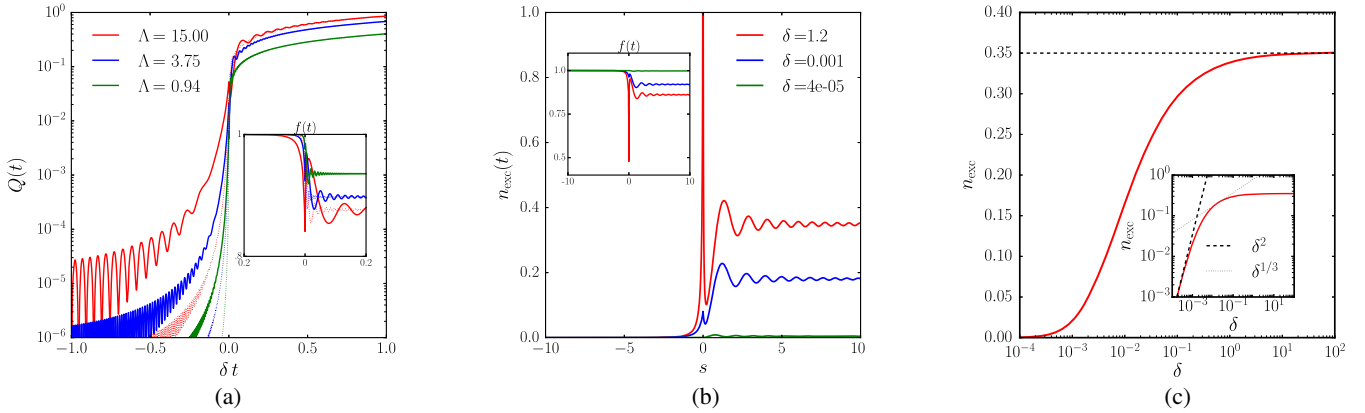


FIG. 2. (a) Heat Q generated by the quench, in units of J , as a function of time t , in units of $1/(J\delta)$. The heat is determined from Eq. (8) using Eqs. (6) and (7). The plot shows different values of $\Lambda = N\delta$ (indicated in the legend), chosen to be the same as in Ref. [49]. Solid lines correspond to $N = 2^9$, dashed lines to $N = 2^{12}$. The inset reports the corresponding fidelity $f(t)$. (b) The average number of excitations $n_{\text{exc}}(t)$ and the fidelity $f(t)$ are reported as functions of the rescaled time $s = \delta^{1/3}t$ for $N = 2^{12}$ and $\delta = 4 \times 10^{-5}, 10^{-3}, 1.2$, corresponding to $\Lambda = 2 \times 10^{-1}, 6, 6 \times 10^3$, respectively. (c) The number of excitations at the end of the quench, $n_{\text{exc}}(t_0)$, is reported as a function of δ for $N = 500$ (the behavior for a different system size N' is obtained by rescaling the δ axis by the factor N'/N). The horizontal dashed line indicates the constant value $n_{\text{exc}}(t_0) = 0.35$ of the thermodynamic limit. The inset shows $n_{\text{exc}}(t_0)$ on a logarithmic scale. The dotted line represents the KZ scaling prediction $\delta^{1/3}$. For the full ramp, there is only an accidental match in the crossover regime and no actual KZ scaling is found.

Figure 2(a) displays the time evolution of the heat Q obtained from Eqs. (6) and (7) for the parameters of Ref. [49]. The curves in Fig. 2(a) reproduce the ones found numerically in Ref. [49] by direct numerical computation of the dynamics of $2^9 - 2^{11}$ spins with Hamiltonian (2). As in Ref. [49], we observe a drop of the fidelity at the critical point, indicating the loss of adiabaticity. For the chosen parameter values, however, the dynamics remains close to adiabatic with fidelity $f > 80\%$. No numerical evidence of KZ scaling was found in Ref. [49], and it was conjectured that this may be due to universal finite-size scaling functions at the critical point. We now analyze the scaling of the gap and show that it depends only on the dimensionless parameter

$$\Lambda = N\delta. \quad (9)$$

To this end, we approximate the oscillator frequency as $\Omega(t)^2 = -4\delta t + 1/N^{2z_{\text{eff}}}$ for $t < 0$ and $\Omega(t)^2 = 8\delta t + 1/N^{2z_{\text{eff}}}$ for $t \geq 0$, up to nonuniversal intensive factors. We have verified numerically that further terms are irrelevant as they become subleading in the critical stage ($t \simeq 0$) of the dynamics. The overall effect of finite-size fluctuations is captured by an effective finite-size scaling exponent z_{eff} in the range $1/3 < z_{\text{eff}} < 1$. The full numerical solution of Eq. (7) indicates that finite-size corrections only become important at $t \simeq 0$; hence we assume $z_{\text{eff}} = 1/3$ for the purpose of our discussion. We identify the scaling relations by performing the transformation $\xi = \delta^{-1/6}\tilde{\xi}$ and $t = \delta^{-1/3}s$ [note that, apart from a factor $\Lambda^{2/3}$, s is the same rescaled time variable as in Fig. 2(c) of Ref. [49]]. This transformation leads to the Schrödinger equation of a quantum harmonic oscillator with effective frequency $\Omega(s)$, where

$$\Omega(s)^2 = \begin{cases} -4s + \Lambda^{-2/3} & \text{for } s < 0, \\ 8s + \Lambda^{-2/3} & \text{for } s > 0. \end{cases} \quad (10)$$

Hence, Λ is now the sole physical parameter which encodes the quench rate δ and the only scale which determines the dynamical behavior at the critical point. We can identify two asymptotic regimes. (i) The limit $\Lambda \ll 1$, where the quench rate is much smaller than the gap and thus the dynamics is expected to be adiabatic. This regime is expected to provide the Landau-Zener scaling $n_{\text{exc}} \sim \delta^2$, and corrections to adiabaticity scale with Λ^2 [12,30,34,36]. (ii) For $\Lambda \gg 1$, instead, the system approaches the thermodynamic limit where the dynamics is independent of Λ to leading order in $1/\Lambda$. In this limit, therefore, excitations and fidelity are expected to be independent of δ . This result is consistent with the prediction of Ref. [51] for a slow quench of the frequency of a single harmonic oscillator, albeit with a different power law in time.

Figure 2(b) shows the time evolution of f and n_{exc} for values of Λ in the two asymptotic regimes as well as in the

intermediate regime. The final value $n_{\text{exc}}(t_0)$, which we extract from these calculations, is reported in Fig. 2(c) as a function of δ for fixed N . We observe the Landau-Zener scaling $n_{\text{exc}} \sim \delta^2$ for $\delta \ll 1/N$, in agreement with our scaling arguments. For $\delta \gg 1/N$ the excitation number tends to a constant value, which we obtain in the thermodynamic limit as $n_{\text{exc},\infty} \approx 0.35$. Even though there is no power-law scaling in the thermodynamic limit, the final number of defects $n_{\text{exc}}(t_0)$ still depends on the scaling of the gap at $s \rightarrow 0$. This number is therefore universal and hints at a connection between out-of-equilibrium dynamics and universal equilibrium properties. Since the slope of the curve n_{exc} varies continuously as a function of δ , it contains also an interval of values with scaling $\delta^{1/3}$. This scaling, which would agree with the KZ prediction, is clearly only a crossover.

A very different result is found for a half ramp which starts or ends at the quantum critical point (QCP). As we show in the Supplemental Material [67], in that case $n_{\text{exc}} \sim \Lambda^{1/3}$. If the ramp ends exactly at the QCP where the gap scales as $1/N^{1/3}$, the heat scales as $Q \sim \delta^{1/3}$ also in the thermodynamic limit. This result is in agreement with the predictions of Refs. [30,50] and the KZ hypothesis. However, it occurs only for a half ramp ending exactly at the QCP and thus depends sensitively on the end point. For any other quench Q exhibits a functional dependence on Λ (and thus, for N fixed, of δ) which can be reduced to a power law only for finite systems in the adiabatic Landau-Zener limit. This behavior is markedly different from the one found in short-range interacting systems. It shows that the hypothesis of an impulse regime, where the system is expected to freeze in the time interval when the gap is smaller than the quench rate, $t \in [-t_f, t_f]$ of Fig. 1, does not hold for the LMG model, and thus strictly speaking the KZ scaling does not apply. These results are also valid for the Dicke model, whose finite-size corrections to the gap have the same scaling with N [78]. More generally, such behavior is expected to be valid also for long-range interacting spin-1/2 chains with power-law interaction $1/r^\alpha$ and $0 \leq \alpha < 1$. In fact, in this case the spin wave approximation can be cast in the form of a $1/N$ expansion [79,80] and the spin wave spectrum remains gapped, such that, even for finite α , only the zero-mode fluctuation contributes when $\delta \rightarrow 0$ [81].

Our analytical theory describes quantum contributions to the heat. These are due to excitations on top of the mean-field spin, and are therefore valid when the nonadiabatic corrections of the mean-field energy are smaller than the quantum heat. Assuming that the semiclassical evolution is analytical in δ and that, in a cyclic process, no work is done on the system in the adiabatic limit $\delta \rightarrow 0$, the semiclassical contribution to the heat would scale as $N\delta^2$ [82]. In this perspective the classical motion follows adiabatically the drive and the quench dynamics is independent of the quench direction [83]. The nonadiabatic quantum contribution

dominates the dynamics for $\delta \lesssim N^{-2/5}$, at least for the half ramp. These observations also suggest that the scaling $N\delta^2$ found for slower quenches [48] is dominated by mean-field dynamics. Our predictions can be experimentally tested in assemblies of trapped ions with all-to-all interactions [7,57,84], in atomic ensembles in optical resonators [63,85–89], and in spinor Bose-Einstein condensates [60]. The regime corresponding to $\Lambda \gg 1$, where the quantum residual energy tends to a constant, could be observed in simple systems such as the Rabi model [50].

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Note added.—Recently, a paper by Ming Xue, Shuai Yin, and Li You [90] appeared on arXiv. The authors describe the universal dynamics across the quantum critical point of a ferromagnetic spinor atomic Bose-Einstein condensate, whose universal behavior is equivalent to the one of the LMG model. Our analytical predictions agree with these numerical results for finite system size.

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