PERFORMANCE OPTIMIZATION OF ENGINEERING SYSTEMS WITH PARTICULAR REFERENCE TO DRY-COOLED POWER PLANTS

by

Antonie Eduard Conradie

Dissertation presented for the Degree of Doctor of Philosophy (Mechanical Engineering) at the University of Stellenbosch.

Promoters: Prof. D.G. Kröger (Mechanical Engineering Department)
Dr. J.D. Buys (Mathematics Department)

Department of Mechanical Engineering

University of Stellenbosch

South Africa

January 1995

DECLARATION

I, Antonie Eduard Conradie, the undersigned, hereby declare that the work contained in this dissertation is my own original work and has not previously, in its entirety or in part, been submitted at any university for a degree.

Signature: Date: 25 January 1995

SYNOPSIS

Computer simulation programs were developed for the analysis of dry-cooling systems for power plant applications. Both forced draft direct condensing air-cooled condensers and hyperbolic natural draft indirect dry-cooling towers are considered.

The results of a considerable amount of theoretical and experimental work are taken into account to model all the physical phenomena of these systems, to formulate the problems in formal mathematical terms and to design and apply suitable computational algorithms to solve these problems effectively and reliably.

The dry-cooling systems are characterized by equation-based models. These equations are simultaneously solved by a specially designed constrained nonlinear least squares algorithm to determine the performance characteristics of the dry-cooling systems under fixed prescribed operating conditions, or under varying operating conditions when coupled to a turbo-generator set. The solution procedure is very fast and effective.

A capital and operating cost estimation procedure, based on information obtained from dry-cooling system component manufacturers and the literature, is proposed. Analytical functions express the annual cost in terms of the various geometrical and operating parameters of the dry-cooling systems.

The simulation and the cost estimation procedures were coupled to a constrained nonlinear programming code which enable the design of minimum cost dry-cooling systems at fixed prescribed operating conditions, or dry-cooling systems which minimize the ratio of total annual cost to the annual net power output of the corresponding turbo-generator set. Since prevailing atmospheric conditions, especially the ambient temperature, influence the performance of dry-cooling systems, wide fluctuations in turbine back pressure occur. Therefore, in the latter case the optimal design is based on the annual mean hourly frequency of ambient temperatures, rather than a fixed value.

The equation-based models and the optimization problems are simultaneously solved along an infeasible path (infeasible path integrated approach). The optimization model takes into consideration all the parameters that may affect the capital and operating cost of the dry-cooling systems and does not prescribe any limits, other than those absolutely essential due to practical limitations and to simulate the systems effectively. The influence that changes of the constraint limits and some problem parameters have on the optimum solution, are evaluated (sensitivity analysis).

The Sequential Quadratic Programming (SQP) method is used as the basis in implementing nonlinear optimization techniques to solve the cost minimization problems. A stable dual active set algorithm for convex quadratic programming (QP) problems is implemented that makes use of the special features of the QP subproblems associated with the SQP methods. This QP algorithm is also used as part of the algorithm that solves the constrained nonlinear least squares problem. This particular implementation of the SQP method proved to be very reliable and efficient when applied to the optimization problems based on the infeasible path integrated approach.

However, as the nonlinear optimization problems become large, storage requirements for the Hessian matrix and computational expense of solving large quadratic programming (QP) subproblems become prohibitive. To overcome these difficulties, a reduced Hessian SQP decomposition strategy with coordinate bases was implemented. This method exploits the low dimensionality of the subspace of independent decision variables. The performance of this SQP decomposition is further improved by exploiting the mathematical structure of the engineering model, for example the block diagonal structure of the Jacobian matrix. Reductions of between 50-90% in the total CPU time are obtained compared to conventional SQP optimization methods. However, more function and gradient evaluations are used by this decomposition strategy.

The computer programs were extensively tested on various optimization problems and provide fast and effective means to determine practical trends in the manufacturing and construction of cost-optimal dry-cooling systems, as well as their optimal performance and operating conditions in power plant applications.

The dissertation shows that, through the proper application of powerful optimization strategies and careful tailoring of the well constructed optimization model, direct optimization of complex models does not need to be time consuming and difficult.

Recommendations for further research are made.

KEYWORDS

Dry-cooling; Power plants; Engineering optimization; Computer simulation; Cost estimation; Sequential Quadratic Programming; Quadratic Programming; Nonlinear Least Squares; Sensitivity analysis.

OPSOMMING

Rekenaar simulasie programme is ontwikkel vir die analise van droëverkoelingstelsels soos aangetref in die kragstasienywerheid. Beide geforseerde trek direkte lugverkoelde kondensors en hiperboliese natuurlike trek indirekte koeltorings word beskou.

Die resultate van 'n groot hoeveelheid teoretiese en eksperimentele werk word in ag geneem om die fisiese gedrag van die stelsel te modelleer, die probleme wiskundig korrek te formuleer en om geskikte algoritmes te ontwikkel en toe te pas vir die effektiewe oplossing van die probleme.

Droëverkoelingstelsels word gekarakteriseer deur vergelyking-gebaseerde wiskundige modelle. 'n Beperkte nie-lineêre kleinste kwadrate algoritme is ontwikkel om hierdie vergelykings gelyktydig op te los. Met hierdie metode kan die werkverrigting van droëverkoelingstelsels tydens vaste, sowel as veranderlike bedryfstoestande bepaal word. Laasgenoemde geval kom voor tydens die koppeling van die droëverkoelingstelsel met 'n turbogenerator eenheid.

'n Kapitaal- en bedryfskoste beramingsmetode, gebaseer op inligting uit die droëverkoelings-nywerheid en literatuur, word voorgestel. Analitiese funksies druk die jaarlikse koste in terme van die verskillende geometriese- en bedryfs parameters van die droëverkoelingstelsels uit.

Die simulasie en kosteberamingsmetodes word gekoppel aan 'n nie-lineêre optimeringskode vir die ontwerp van minimum koste droëverkoelingstelsels tydens vaste bedryfstoestande, of vir die ontwerp van stelsels om die verhouding van totale jaarlikse koste tot netto jaarlikse energie uitset van die turbogenerator eenheid waaraan die verkoelingstelsel gekoppel is, te minimeer. Aangesien heersende atmosferiese toestande, veral die omgewingstemperatuur, die werkverrigting van die droëverkoelingstelsels beïnvloed, fluktueer die terugdruk van die turbine oor 'n wye gebied. Gevolglik word die optimum ontwerp in hierdie geval gebaseer op die jaarlikse gemiddelde uurlikse frekwensie van die omgewingstemperatuur en nie op 'n vaste waarde nie.

Die wiskundige modelle en die optimeringsprobleme word gelyktydig opgelos langs 'n nie-toelaatbare pad (nie-toelaatbare pad geïntegreerde benadering). Die optimeringsprosedure neem al die veranderlikes in ag wat die kapitaal- en bedryfskoste beïnvloed en skryf geen grense voor, behalwe die wat absoluut noodsaaklik is weens praktiese oorwegings. Die invloed wat die versteuring van die grense van beperkings en sommige probleemparameters het op die optimum oplossing, word ook ondesoek (sensitiwiteitsanalise).

Die "Sequential Quadratic Programming" (SQP) metode word gebruik as die basis vir die implementering van die nie-lineêre optimeringstegnieke om die minimum koste probleme op te los. 'n Stabiele duale aktiewe basis algoritme vir konvekse kwadratiese programmerings probleme word geïmplimenteer. Hierdie algoritme maak gebruik van die spesiale kenmerke van die SQP-metode se kwadratiese subprobleme en word ook gebruik as deel van die oplosmetode vir die beperkte nie-lineêre kleinste kwadrate probleem. Die SQP-metode is baie betroubaar en effektief tydens die oplossing van die optimerinsprobleme gebaseer op die nie-toelaatbare pad geïntegreerde benadering.

Die verlangde storingskapasiteit vir die Hessiaan matriks en die berekeningskoste vir die oplos van die kwadratiese subprobleme word egter onaanvaarbaar hoog soos die dimensie van die optimerings-probleme toeneem. Hierdie probleme word oorkom deur die implementering van 'n gereduseerde Hessiaan SQP dekomposisie strategie wat gebruik maak van koördinaat basisse. Die metode benut die lae dimensie van die subruimte van onafhanklike veranderlikes. Die werkverrigting van die metode word verder verbeter deur die benutting van die wiskundige struktuur van die simulasie model, byvoorbeeld die blokdiagonale struktuur van die Jakobiaan matriks. 'n Vermindering van tussen 50-90% in die totale berekeningstyd, in vergelyking met konvensionele SQP-metodes, word ondervind. Die SQP dekomposisie strategie maak egter van meer iterasies en funksie evaluasies gebruik.

Die rekenaarprogramme is deeglik getoets op 'n verskeidenheid van optimeringsprobleme en voorsien 'n baie effektiewe metode om tendense vas te stel vir die ontwerp en vervaardiging van minimum koste droëverkoelingstelsels. Die optimale werkverrigting en bedryfstoestande vir die stelsels, wanneer toegepas in die kragstasienywerheid, kan ook bepaal word.

Die proefskrif toon aan dat die effektiewe toepassing van kragtige optimeringstegnieke en die benutting van die wiskundige struktuur van die optimeringsprobleem tot gevolg het dat komplekse probleme nie noodwendig moeilik hoef te wees en 'n groot hoeveelheid tyd in beslag hoef te neem nie.

Aanbevelings vir vedere navorsing word gemaak.

TREFWOORDE

Droëverkoeling; Kragstasie; Ingenieursoptimering; Rekenaar simulasie; Kosteberaming; "Sequential Quadratic Programming"; Kwadratiese programmering; Nie-lineêre kleinste kwadrate; Sensitiwiteits-analise.

ACKNOWLEDGMENTS

I would like to acknowledge the valued contributions and support of the following persons and organizations:

My promoters, Prof. D.G. Kröger and Dr. J.D. Buys for their enthusiasm, guidance and support throughout the project;

The other lecturers in the Mechanical Engineering Department and my fellow-students for many interesting and useful discussions;

The personnel of the engineering library and the computer center for their assistance;

The Foundation for Research Development, Water Research Commission, and Harry Crossley Trust for their financial support;

My family and friends for their moral support and encouragement;

My Creator, for His presence and motivation.

TABLE OF CONTENTS

NOMENCLATURE

A	Area, m^2
$\mathcal{A}$	Set of active constraints
A	Matrix, matrix of constraint normals, Jacobian matrix
a	Coefficient, or constant
a	Vector (usually a column vector)
B	Approximation for the Hessian matrix ($\mathbf{B} \approx \mathbf{G}$)
b	Coefficient, or constant
b	Vector
C	Cost, \$, or coefficient
C(**x**)	Constraint functions
C(**x**)	Vector of constraint functions
c	Constant
c(**x**)	Constraint function
c(**x**)	Vector of constraint function
c_p	Specific heat at constant pressure, J/kgK
D	Diagonal matrix
d	Diameter, m, or minimum of primal problem
d_e	Hydraulic or equivalent diameter, m
d	Vector
E	Energy
Ey	Characteristic pressure drop parameter, m^{-2}
e	Escalation rate, %, effectiveness, or constant
exp	Exponent, e^x, e = natural base of logarithms = 2.7182818285…
F	Correction factor, or force, N
F(**x**)	Objective function
f	Friction factor, or constant
f(**x**)	Objective function
f(**x**)	Vector function
G	Mass velocity, kg/sm^2
G(**x**)	Hessian matrix, or matrix

g	Gravitational acceleration, m/s^2, or constant
g(**x**)	Gradient vector
H	Height, m
H	Approximate inverse of the Hessian matrix ($\mathbf{H} \approx \mathbf{G}^{-1}$)
h	Heat transfer coefficient, W/m^{2}K, or step length
h	Error ($\mathbf{x}^{(k)}$ - $\mathbf{x}$*)
h(**x**)	Equality constraint
h(**x**)	Vector of equality constraints, or vector
I	Unit matrix
i	Interest rate, %
i_{fg}	Latent heat, J/kg
J(**x**)	Jacobian matrix
K	Loss coefficient, or correction factor
K	Convex set
k	Thermal conductivity, W/mK, or iteration counter
L	Length, m
L(**x**,Λ)	Lagrange function
ℓn	$\log_e$
M	Mass per unit length, kg/m, or Molecular weight, kg/mole
m	Mass flow rate, kg/s, or number of constraints
N	Revolutions per second, s^{-1}
Ny	Characteristic heat transfer parameter, m^{-1}
n	Number of variables, or number
P	Pitch, m, or power, W
P(g(**x**),σ)	Penalty function
P	Matrix
p	Pressure, N/m^2
Δp	Pressure differential, N/m^2
Q	Heat transfer rate, W
Q	Orthogonal matrix
Q(**x**)	Quadratic form
q	Heat flux, W/m^2

$q(\mathbf{x})$	Quadratic function
R	Universal gas constant, J/kgK, or thermal resistance, m^2K/W
Ry	Characteristic flow parameter, m^{-1}
r	Radius, m
$\mathbf{r}$	Vector
S	Shape factor, or sensitivity
$\mathbf{S}$	Matrix
$\mathbf{s}$	Search direction
s	Tip clearance, m
T	Temperature, °C or K
$\mathbf{T}$	Reversed lower triangular matrix
t	Thickness, m
U	Overall heat transfer coefficient, W/m^2K
$\mathbf{U}$	Upper triangular matrix
u	Lagrange multiplier
$\mathbf{u}$	Vector of Lagrange multipliers, or vector
V	Volume flow rate, m^3/s, or volume, m^3
v	Velocity, m/s, or Lagrange multiplier
$\mathbf{v}$	Vector of Lagrange multipliers, or vector
W	Width, m, or weighting factor
$\mathbf{W}$	Hessian of the Lagrange function
w	Humidity ratio, kg moisture/kg dry air
$\mathbf{w}$	Vector
X	Mole fraction
x	Co-ordinate, variable
$\mathbf{x}$	Vector of decision variables $(x_1, x_2, \ldots, x_n)$
$\mathbf{Y}$	Basis matrix
y	Co-ordinate, variable
$\mathbf{y}$	Vector of variables
$\mathbf{Z}$	Basis matrix
z	Co-ordinate
$\{\}$	Set

$\|\cdot\|$	Norm of a vector or matrix
$\vert\ \vert$	Absolute value
[a,b]	Closed interval
$\in$	Element in set
$\notin$	Not element in set
(a,b)	Open interval
0	Zero vector
∇	First derivative operator ($\partial/\partial x_i$)
$\nabla_{\mathbf{x}}$	Partial derivative operators with respect to **x**
∇^2	Second derivative operator ($\partial^2/\partial x_i \partial x_j$)
$\delta\mathbf{x}$, $\delta\Lambda$	Correction to x, Λ
$\Re^n$	n-dimensional space
$\Rightarrow$	Implies

GREEK SYMBOLS

α	Step length, or scalar parameter, or fraction
α_e	Kinetic energy coefficient
β	Scalar
γ	Constant
γ	Correction
δ	Correction
Δ	Differential
ε	Surface roughness, small positive constant, or convergence tolerance
η	Efficiency
η	Correction
ϑ	Constant
θ	Angle, °, or parameter
λ	Lagrange multiplier, scalar
Λ	Vector of Lagrange multipliers
μ	Dynamic viscosity, kg/ms
ν	Scalar
ξ	Length ratio, or feasibility variable

π	Pi = 3.1415926535...
ρ	Density, kg/m^3, or constant
σ	Area ratio
σ	Penalty parameter
Σ	Summation
τ	Time, s
ϕ	Angle, °
$\Phi(x)$	$y = \Phi(x)$
$\Psi(x)$	Merit function

DIMENSIONLESS GROUPS

Fr_D	Densimetric Froude number, $\rho v^2 / (\Delta\rho L g)$
Re	Reynolds number, $\rho v L/\mu$ or $\rho v d/\mu$ for a tube
Eu	Euler number, $\Delta p / (\rho v^2)$
Nu	Nusselt number, hL/k or hd/k for a tube
Pr	Prandtl number, $\mu c_p / k$

SUBSCRIPTS

a	Air, or assembly, or annual, or based on airside area
abs	Absolute
av	Mixture of dry air and water vapor, or average
A	Set of active constraints
b	Bundle, or bellmouth, or blade
c	Contraction, or casing, or condensate, or construction
ct	Cooling tower
ctc	Cooling tower contraction
cte	Cooling tower expansion
d	Diameter, or diagonal, or downstream
dep	Dependent variables
D	Drag, or D'Arcy
db	Drybulb
do	Downstream
e	Electricity, or effective, or expansion

eff	Effective
em	Electric motor
eq	Equality constraints
F	Fan
f	Fin, or fuel, or friction
Fb	Fan bays
Fd	Fan drive system
fix	Fixed
fl	Fin leading edge
Fr	Fan rows
fr	Frontal
ft	Fin tip, or finned tube
g	Gross, or galvanizing
gen	General
h	Hydraulic, or hub, or header
he	Heat exchanger
i	Inlet, or inside
id	Ideal
indep	Independent variables
ineq	Inequality constraints
iso	Isothermal
j	Jetting
l	Lower, or longitudinal, or land
lm	Logarithmic mean
m	Mean, or model, or maintenance
max	Maximum
min	Minimum
n	Net, or normal
o	Outlet, or outside
p	Pump
pl	Plenum, or platform
pv	Piping and valves
r	Root, or row

s	Static, or screen, or shell, or steam, or steel
sd	Steam duct
sr	Speed reducer
t	Throat, or tube, or transversal, or total, or turbine, or tower, or tip
T	Temperature
tb	Tubes per bundle
tg	Turbo-generator
tgc	Turbo-generator-condenser
ts	Tower support, or tube cross-section
u	Upper, or unit
up	Upstream
v	Vapor
V	Volumetric
w	Water, or walkway, or wall
wb	Wetbulb
wp	Water passes
ws	Wiring and switching
Y	Range space
z	Co-ordinate
Z	Null space
θ	Inclined

SUPERSCRIPTS

*	Optimum solution point
T	Transpose of vector or matrix
(k)	Iteration k

ABBREVIATIONS AND ACRONYMS

ABC	Activity based costing
ACC	Air-cooled condenser
ACHE	Air-cooled heat exchanger
AI	Artificial intelligence
BFGS	Broyden-Fletcher-Goldfard-Shanno method

CPU	Central processing unit
DALR	Dry adiabatic lapse rate
DFP	Davidon-Fletcher-Powell method
FCR	Fixed charge rate
IBM	International business Machines
LHS	Left-hand side
LP	Linear programming
max	Maximum
min	Minimum
NFEVAL	Number of function and gradient evaluations
NITER	Number of major SQP iterations
NLP	Nonlinear programming
NLS	Nonlinear least squares
PC	Personal computer
QP	Quadratic programming
QPCPU	CPU time related to QP problem solution
RAM	Random access memory
RHS	Right-hand side
SBL	Surface boundary layer
SI	International system of units (Système International d'Unités)
SQP	Sequential/successive quadratic programming
TOTCPU	Total CPU time
TTD	Terminal temperature difference

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

All the engineering disciplines are increasingly using some form of optimization to comply with the demands in the planning, design, analysis and control of projects. Financial pressures towards cost-effectiveness are increasing, resources are becoming limited and the restrictions imposed by social, environmental and technological requirements are much more stringent today. The recognition that optimization can be of invaluable assistance in aiding the intuition, skill and experience of the engineer in these actions can be widely seen in the present-day literature.

During the last twenty five years considerable numerical work has been done to show that nonlinear programming (optimization) methods can be used to optimize various engineering problems. Most optimization problems are so large and complex that efficient solution methods are essential in solving these problems. Engineering applications, like other practical areas, make special demands on optimization codes. As a direct result, various new optimization techniques and modifications to existing techniques have been developed to satisfy these needs.

A great step forward has been the development of Sequential Quadratic Programming (SQP) methods, which solve a sequence of simplified quadratic subproblems containing linearizations of the nonlinear constraints, yet capturing the essential features of the original problem. For optimization of complex, computationally intensive models with a small to moderate number of variables, SQP consistently requires fewer function evaluations to complete the optimization per iteration than other nonlinear optimization algorithms. SQP methods have been very successfully applied to various engineering and scientific problems.

In engineering applications the particular model to be optimized may possess a special structure. By adapting the basic SQP algorithm to exploit this model structure, a tailored SQP algorithm is obtained which performs (computationally) significantly better than the original method. Thus, through the application of powerful optimization strategies and careful tailoring of the model, engineering optimization problems can be efficiently solved.

1.2 THE DRY-COOLING SYSTEM OPTIMIZATION PROBLEM

Most industrial processes require the rejection of low-quality waste heat. In particular, steam-electric plants reject heat at approximately twice the rate at which electricity is generated. For a long time designers found once-through and evaporative cooling (open-cycle cooling systems) an efficient means to reject waste heat at a low cost. However, water shortages and stringent environmental regulations forced designers to consider less efficient and more expensive air-cooling, or dry-cooling as it is often termed (closed-cycle cooling systems). Dry-cooled plants offer potential economic advantages due to plant siting flexibility. Both natural draft and mechanical draft dry-cooling towers, equipped with air-cooled heat exchangers (extended airside surface area), are used.

The cooling system is a significant cost item in the power plant and affects the performance of the entire power cycle. If the cooling system does not provide adequate cooling, the overall plant efficiency decreases with serious economic consequences (e.g. decreased electricity production), i.e. a cost-performance trade-off exists. The selection of waste heat rejection systems for steam-electric power plants involves a trade-off among environmental, energy and water conservation, and economic factors, while achieving the required cooling rate.

The heat rejection performance of the dry-cooling tower and the thermodynamic performance of the turbine are the two most significant factors in the operation of a dry-cooling system. The complex relationships which exist between the condensing system and the turbine must be determined in order to predict the performance of a combination of the turbo-generator-condenser system. Since the performance of a particular dry-cooling system and the turbine which it serves are so closely related the complete condensing system and the turbine can best be considered as one integral unit in studies of economic comparisons for various combinations of dry-cooling towers and turbines.

It is therefore necessary to simulate and evaluate the performance and costs of dry-cooling systems at specified operating conditions and when coupled to a turbo-generator set. To remain competitive, the performance and design of these cooling systems should be optimized. In the present study two types of nonlinear optimization problems, related to natural draft indirect dry-cooling towers and forced draft direct air-cooled condensers as found in steam-electric power plant applications will be addressed:

(1) Design a dry-cooling system that satisfies the prescribed operating conditions at the minimum annual cost (combination of capital and operating cost).

(2) Design a dry-cooling system at a particular location for the minimum ratio of annual cost (combination of capital, fuel and operating cost) to annual net energy output of the turbo-generator set it is coupled to.

The SQP method is used as the basis in implementing optimization techniques to minimize the objective functions (annual cost or annual cost/kWh electricity generated) in the design of dry-cooling systems for steam-electric power plants.

1.3 SCOPE OF THE WORK

In order to apply numerical solution techniques of optimization theory to engineering problems, it is necessary to cover the following steps:

(1) Formulate the problem to be optimized in terms of the decision variables, objective function and the imposed constraints. The decision variables characterize the possible designs or operating conditions of the system. The objective function provides a criterion on the basis of which the performance or design of the system can be evaluated to select the optimum outcome. The constraints represent all the restrictions placed on the design or performance of the system.

(2) Construct the mathematical representation of the real system, called the model. The model consists of all the elements that must be considered in calculating a design or in predicting the performance of an engineering system. It describes the manner in which the problem variables are related and the way in which the objective function is influenced by the decision variables.

(3) Select a suitable optimization algorithm that will effectively and reliably solve the problem. The basic algorithm can be modified to satisfy specific needs. Choose or prepare an efficient computer implementation of this algorithm. Prepare the problem for solution and perform the computer runs to obtain numerical answers.

(4) Examine the solution and perform a post-optimality (sensitivity) analysis to critically evaluate the solution behavior to changes in model parameters, assumptions and constraints.

The different chapters and the corresponding appendices follow the general procedure outlined above and illustrate how each step is performed on the engineering optimization problems investigated in this dissertation.

Chapter 2 contains a comprehensive survey of literature on topics such as the basic fundamental concepts from optimization theory, basic solution techniques for unconstrained and constrained optimization, the application of optimization in engineering and the optimal design of air-cooled heat exchangers and dry-cooling systems.

Chapter 3 describes the process of formulating and modeling an engineering optimization problem and apply these techniques to dry-cooling systems for use in power plants. The results of a considerable amount of theoretical and experimental work from various sources are included in this study (Appendices A, B, C, D) to model all the physical phenomena of these systems and to formulate these problems in formal mathematical terms.

Chapter 4 contains the dry-cooling system economic analysis as well as the capital and operating cost estimating techniques that are used to set up the objective function required by the economic optimization. Appendix E contains the proposed dry-cooling system cost estimation model.

The methods for solving the dry-cooling system performance evaluation and optimization problems as well as the post-optimality analyses are described in Chapter 5. Detailed discussions, derivations and modifications of these algorithms are described in Appendices F, G, H, I and J. Different solution strategies are derived, investigated and implemented in order to improve the efficiency of the solution process when applied to engineering problems. These methods are tailored to take advantage of the special features of the problems.

Chapter 6 describes the different computer programs that were developed to implement all the procedures described in the above chapters and appendices. These programs deal with both forced draft direct air-cooled condensers and natural draft indirect dry-cooling towers with particular reference to power plant applications. The programs are capable of performing performance evaluation, design, cost and optimization calculations.

Chapter 7 illustrates the performance of the different computational algorithms described in Chapter 5 and implemented in the various computer programs discussed in Chapter 6. Two interesting examples on dry-cooling system performance evaluation, cost estimation, design and performance optimization are formulated and solved to illustrate the various capabilities of the programs and to compare the performance of the solution techniques. Printouts of program input and output are listed in Appendices K and L. The program results and performance of the solution methods are interpreted and discussed.

The dissertation concludes with Chapter 8. Conclusion and recommendations are summarized to emphasize what has been learned from this investigation and how this study can be extended for future applications.

1.4 CLOSING REMARKS

In this dissertation we investigate and apply the methodology applicable to the efficient solution of engineering optimization problems, as well as the computational and mathematical techniques that will expedite the solution of application problems.

One of the goals of the study is to demonstrate the applicability of optimization methodology to engineering problems and to introduce the powerful tool of mathematical optimization techniques which can be applied to the various engineering practices. Although similar motivation underlies optimization algorithms for almost all problems, it is very efficient to exploit the specialized properties of a particular problem, as shown in this dissertation.

The other goal is to perform a detailed economic optimization study on dry-cooling systems as found in power plants. The aim of this study is to develop computer programs which enable the user to very effectively obtain certain trends in the manufacturing and construction of cost-optimal dry-cooling systems, as well as their optimal performance and operating conditions. The results of these programs are strongly dependent on the quality of the input data.

The optimum engineering solution must not be confused with the ultimate minimum cost solution, for the ultimate answer may be that which provides significant non-quantifiable benefits of an engineering or socio-economic nature.

This study forms an integral part of an on-going research program on dry-cooling performed in the Department of Mechanical Engineering at the University of Stellenbosch.

CHAPTER 2

SURVEY OF LITERATURE

2.1 INTRODUCTION

This chapter evolves around the topic of the engineering application of numerical optimization techniques. It is not the aim to discuss all the different applications of the optimization concepts and techniques in this chapter. However, by discussing the fundamental principles and the basic techniques that can be used, the powerful tool of optimization is introduced which can be applied to various engineering practices.

In the first section the basic concepts from optimization theory are reviewed. Various definitions and solution techniques to the unconstrained and constrained optimization problems are described. These concepts provide the mathematical basis for the understanding of some algorithms to be discussed in later chapters. The rest of the chapter contains a literature survey on the application of optimization in engineering. A general overview of this topic is presented first to identify some problems where optimization has been used effectively. Thereafter, the discussion is limited to the optimal design of dry-cooling towers and air-cooled heat exchangers to review the concepts used and to gain some insight into these problem formulations and solutions. These literature surveys will provide valuable background for the investigation to be conducted.

2.2 MATHEMATICAL OPTIMIZATION: AN OVERVIEW

Introduction

The primary purpose of this section is to give a general overview of the mathematics involved in the process of optimization and to gain a basic knowledge of the many underlying principles involved in this field of study. It is not the purpose of this discussion to present a detailed analysis of all the methods and to discuss all the possible variations and refinements; references are cited for this purpose. Results from optimization theory are stated without formal proofs of the relevant theorems.

General problem statement

The general form of a mathematical optimization (programming) problem may be written as ([81GI1m, 83MC1m, 84LU1m, 87FL1m])

$$
\begin{aligned}
&\underset{\mathbf{x}}{\text{Minimize}}\ f(\mathbf{x}) && \text{objective function} \\
&\text{subject to the constraints} && \\
&c_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m_{eq} && \text{equality constraints} \qquad (2.1) \\
&c_i(\mathbf{x}) \geq 0, \quad i = m_{eq} + 1, \ldots, m && \text{inequality constraints}
\end{aligned}
$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a vector of variables, called the decision variables.

The functions $f(\mathbf{x})$ and $c_i(\mathbf{x}), i = 1, \ldots, m$ are real-valued functions used to formulate the problem in terms of the decision variables. Note that simple bounds on the variables such as $x_{il} \leq x_i \leq x_{iu}, i = 1, \ldots, n$, are included in the inequality constraints of problem (2.1). These bounds can also be treated separately in order to gain algorithmic advantages. The solution of this problem, referred to as $\mathbf{x}^*$, is found when the objective function is minimized and the constraints are satisfied. If any point $\mathbf{x}$ satisfies all the constraints stated in problem (2.1) it is said to be a feasible point and the set of all such points is referred to as the feasible region. Some problems may not have any constraints ($m = 0$) and are called unconstrained optimization problems in contrast with the constrained optimization problem stated above. The entire decision space is feasible for unconstrained optimization problems.

The above form of stating the optimization problem is not unique and various other statements equivalent to this are presented in the literature [87FL1m]. For example, maximization problems are easily handled by the transformation

$$\underset{\mathbf{x}}{\text{maximum}}\ f(\mathbf{x}) = -\underset{\mathbf{x}}{\text{minimum}}(-f(\mathbf{x})) \qquad (2.2)$$

The objective and the constraint functions may be linear or nonlinear functions of the variable $\mathbf{x}$ and these functions may be implicit or explicit in $\mathbf{x}$. If the objective and the constraint functions are linear in the variables $\mathbf{x}$, then the problem is called a linear programming problem. If any of these functions are nonlinear the problem is called a nonlinear programming problem. However, except for special classes of problems (non-smooth optimization) it is required that these functions are smooth, that is continuous and twice continuously differentiable in $\mathbf{x}$ [87FL1m]. The general problem formulation covers most types of problems having continuous decision variables, real valued constraint functions and a single real valued objective function. The condition that some variables x_i take only discrete values are not covered. This type of condition is covered in integer programming [81GI1m, 87FL1m, 89NE1m]. Practical procedures to treat problems with discrete or integer variables are discussed in [81GI1m, 89AR1e].

The existence of a general problem statement does not imply that all distinctions among problems should be ignored. It is highly advantageous to determine the special problem characteristics that allow it to be solved more efficiently by a specific solution method. The most obvious distinctions between problems involve variations in the mathematical characteristics of the objective and the constraint functions. Significant algorithmic advantage can be taken of these characteristics.

The following important points should be noted about the general problem statement:

(1) The number of independent equality constraints must be less than or equal to the number of variables, i.e. $m_{eq} \leq n$. When $m_{eq} > n$ the system of equations is over-determined; either there are some redundant equality constraints or the formulation is inconsistent. If $m_{eq} < n$ an optimum solution of the problem is possible. When $m_{eq} = n$ no optimization of the problem is necessary because solutions of the system of equality constraints are the only candidates for the optimum.

(2) There is no restriction on the number of independent inequality constraints. Some inequality constraints may be strictly satisfied (as equalities) at the optimal solution and are referred to as active constraints. The inactive constraints remain as strict inequalities at the optimal solution. The total number of active constraints (equality and active inequality constraints) at the optimal solution is usually less than or at least equal to the number of variables as mentioned previously. The active inequality constraints at the optimal solution are not known beforehand and are determined during the optimization process. A constraint is said to be violated if it is not satisfied. Furthermore, the theory used to solve these problems requires linear independence of the gradients of the active constraints at the optimum point.

(3) The objective function can be scaled by a positive constant or a constant can be added to it without changing the optimum solution point $\mathbf{x}^*$. The value of the objective function will, however, change. Similarly the constraints can be scaled by any positive constant without affecting the feasible region and the optimum solution.

Fundamental concepts

A thorough knowledge of linear algebra (vector and matrix operations) and basic calculus is essential mathematical background for optimization theory. Therefore, some of the most important fundamental concepts will be stated. In general it will be assumed that the problem functions, i.e. f(**x**) and $c_i(\mathbf{x})$, are twice continuously differentiable.

(1) Global and local minima [81GI1m, 84LU1m, 87FL1m, 89AR1e]

A function f(**x**) has a global (absolute) minimum at **x*** if f(**x***) ≤ f(**x**) for all **x** in the feasible region.

A function f(**x**) has a local (relative) minimum at **x*** if f(**x***) ≤ f(**x**) for all **x** in some small neighborhood of **x*** in the feasible region.

(2) Gradient vector [81GI1m, 84LU1m, 87FL1m, 89AR1e]

The gradient of f(**x**) at any point **x** is defined as the vector of first partial derivatives

$$\nabla f(\mathbf{x}) = \left[\frac{\partial f(\mathbf{x})}{\partial x_1} \frac{\partial f(\mathbf{x})}{\partial x_2} \cdots \frac{\partial f(\mathbf{x})}{\partial x_n} \right]^T = \mathbf{g}(\mathbf{x}) \tag{2.3}$$

where the superscript T denotes transpose of the row vector. Geometrically, the gradient vector is normal to the tangent plane at **x** and points in the direction of maximum increase in the function. Points satisfying $\nabla f(\mathbf{x}) = \mathbf{0}$ are called stationary points.

(3) Hessian matrix [81GI1m, 84LU1m, 87FL1m, 89AR1e]

If f(**x**) is twice continuously differentiable, then there exists a matrix of second partial derivatives, the Hessian matrix, defined as

$$\nabla^2 f(\mathbf{x}) = \left[\frac{\partial^2 f(\mathbf{x})}{\partial x_i \partial x_j} \right] = \mathbf{G}(\mathbf{x}) \quad i = 1, \ldots, n \text{ and } j = 1, \ldots, n \tag{2.4}$$

This matrix is square and symmetric.

(4) Jacobian matrix [83DE1m, 84LU1m, 87FL1m]

If $\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})\right)^T$, the Jacobian matrix of the vector function is defined as

$$\nabla \mathbf{f}(\mathbf{x}) = \left[\nabla f_1(\mathbf{x}), \nabla f_2(\mathbf{x}), \ldots, \nabla f_m(\mathbf{x})\right] = \mathbf{J}(\mathbf{x}) \tag{2.5}$$

and is a n × m matrix, the columns of which are the gradient vectors of f_i.

(5) Taylor series expansion [81GI1m, 84LU1m, 87FL1m, 89AR1e]

The Taylor series expansion of a smooth function in the neighborhood of any point **x** is

$$f(\mathbf{x}) = f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0)^T (\mathbf{x} - \mathbf{x}_0) + 0.5(\mathbf{x} - \mathbf{x}_0)^T \nabla^2 f(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0) + \cdots \tag{2.6}$$

Most methods for optimizing nonlinear differentiable functions of continuous variables rely heavily upon Taylor series expansions. When only the first two terms on the right hand side of equation (2.6) are used and the higher order terms are neglected, a linear approximation to the function is obtained. Similarly the first three terms on the right hand side represent a quadratic approximation.

(6) Form and definiteness of a matrix [81GI1m, 84LU1m, 87FL1m, 89AR1e]

For a symmetric matrix $\mathbf{A}$, a quadratic form may be defined as $Q(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x}$. Quadratic forms may be either positive, negative or zero for any fixed $\mathbf{x}$.

$Q(\mathbf{x})$ is positive definite when $\mathbf{x}^T\mathbf{A}\mathbf{x} > 0$ for all $\mathbf{x} \neq \mathbf{0}$.

$Q(\mathbf{x})$ is positive semidefinite when $\mathbf{x}^T\mathbf{A}\mathbf{x} \geq 0$ for all $\mathbf{x}$, and $Q(\mathbf{x}) = 0$ for at least one $\mathbf{x} \neq \mathbf{0}$.

Similar definitions for negative definite and negative semidefinite are obtained by reversing the sense of the inequality. A quadratic form which is positive for some vectors $\mathbf{x}$ and negative for others is called indefinite.

A symmetric matrix $\mathbf{A}$ is often referred to as positive definite, positive semidefinite, negative definite, negative semidefinite or indefinite if the quadratic form associated with $\mathbf{A}$ is positive definite, positive semidefinite, negative definite, negative semidefinite or indefinite, respectively. Methods for checking definiteness are discussed in the above-mentioned references.

These concepts are the keys to the necessary and sufficient optimality conditions discussed in the next section.

Optimality conditions

It is possible to check mathematically if $\mathbf{x}^*$ is a local minimum point of problem (2.1). The conditions that must be satisfied at the optimum point are called the necessary conditions. However, there can be non-optimum points that also satisfy the necessary conditions. The sufficient conditions provide a means to distinguish between optimum and non-optimum points. The necessary and sufficient conditions for unconstrained and constrained optimization are as follows:

(1) Unconstrained optimization [81GI1m, 83MC1m, 84LU1m, 87FL1m, 89AR1e, 89DE1m]

The location of a local minimum point for an unconstrained optimization problem is determined by the nature of the objective function. The first order necessary condition states that if $f(\mathbf{x})$ has a local minimum at $\mathbf{x}^*$, the gradient vector is zero, i.e. $\nabla f(\mathbf{x}^*) = \mathbf{0}$. The second order necessary condition states that if $f(\mathbf{x})$ has a local minimum at $\mathbf{x}^*$, then the Hessian matrix is positive semidefinite or positive definite at $\mathbf{x}^*$. If the Hessian matrix is positive semidefinite it is necessary to examine higher order derivatives to determine whether **x*** is a local minimizer or not. The second-order sufficient condition states that if the Hessian matrix is positive definite at the stationary point **x***, then **x*** is a local minimum point for the function f(**x**).

The above-mentioned necessary and sufficient conditions characterize a local minimum point and are obviously not sufficient to verify a global minimum point. In order to find the global optimum point of an unconstrained objective function, one first has to solve for all the local optimum points and then choose the best solution amongst these local optima. Additional assumptions, such as convexity, on the objective function are necessary to guarantee a global solution. Only a few solution methods for global optimization have been developed thus far and there is still a lot to be done before this field of study is complete. For a detailed discussion on this subject consult [89RI1m] and the references given therein. Simple practical advice to obtain a global solution is to solve the problem from different starting points and take the best local solution that is obtained [84VA1e, 87FL1m].

(2) Constrained optimization [81GI1m, 83MC1m, 84LU1m, 87FL1m, 89AR1e, 89GI1m]

Both the nature of the objective function and the constraints play a prominent role in determining the optimum solution of a constrained optimization problem. Of fundamental importance for nonlinear programming theory and the development of constrained optimization algorithms is the Lagrange function, i.e.

$$L(\mathbf{x},\Lambda) = f(\mathbf{x}) - \sum_{i=1}^{m} \lambda_i c_i(\mathbf{x}) \tag{2.7}$$

where $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)$ are the Lagrange multipliers associated with the constraints of problem (2.1).

Before we can state the necessary conditions for a constrained optimum point, we need to define what is meant by a regular point of the feasible region. A point **x*** satisfying the constraints is called a regular point of the feasible region if the gradient vectors of all the active constraints are linearly independent. Linear independence means that no two gradients are parallel to each other, and no gradient can be expressed as a linear combination of the others.

The first order necessary conditions, often referred to as the Kuhn-Tucker conditions, can be stated as follows:

If **x*** is a local minimizer of problem (2.1) and if **x*** is a regular point for the feasible region thereof, then there exist Lagrange multipliers Λ* such that **x*** and Λ* satisfy the following system of equations when (**x**, Λ) = (**x***, Λ*):

$$
\begin{aligned}
&\nabla_{\mathbf{x}} L(\mathbf{x}, \Lambda) = 0 \text{ or } \nabla f(\mathbf{x}) = \sum_{i=1}^{m} \lambda_i \nabla c_i(\mathbf{x}) \\
&c_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m_{eq} \\
&c_i(\mathbf{x}) \geq 0, \quad i = m_{eq}+1, \ldots, m \\
&\lambda_i \geq 0, \quad i = m_{eq}+1, \ldots, m \\
&\lambda_i c_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m
\end{aligned}
\tag{2.8}
$$

From the conditions stated in equation (2.8), it can be seen that at **x*** the gradient vector of the objective function is a linear combination of the gradients of the constraints with the Lagrange multipliers as the scalar parameters of the linear combination. Furthermore, the Lagrange multipliers of the active inequality constraints must be greater than or equal to zero. The Lagrange multipliers of the equality constraints have no sign restriction.

The constraint and objective function curvature play an important role in evaluating the second order conditions. The second order necessary conditions can be stated as follows:

Let **x*** satisfy the first order necessary conditions stated in equation (2.8). The Hessian matrix of the Lagrange function at $(\mathbf{x}^*, \Lambda^*)$ is

$$
\nabla_{\mathbf{x}}^2 L(\mathbf{x}^*, \Lambda^*) = \nabla^2 f(\mathbf{x}^*) - \sum_{i=1}^{m} \lambda_i^* \nabla^2 c_i(\mathbf{x}^*) \tag{2.9}
$$

A feasible direction, s, is defined as follows: $\nabla c_i^T(\mathbf{x}^*)\mathbf{s} = 0, \quad i \in A$. Thus, for a small move in this direction all the active constraints (equality and active inequality constraints) are not violated.

The second order necessary condition for a local minimizer is that

$$
\mathbf{s}\, \nabla_{\mathbf{x}}^2 L(\mathbf{x}^*, \Lambda^*)\, \mathbf{s} \geq 0 \tag{2.10}
$$

for all non-zero feasible directions at any feasible point **x***. This condition states that the Lagrange function must have non-negative curvature for all feasible directions at **x***. Any point that does not satisfy the second order necessary conditions cannot be a local minimum point.

The second order sufficient conditions can be stated as follows: Let **x*** satisfy the first order necessary conditions stated in equation (2.8). The Hessian matrix of the Lagrange function at **x*** is defined by equation (2.9). Let there be a non-zero direction, s, satisfying

$$\nabla c_i^T(\mathbf{x}^*)\,s = 0 \;\text{ for } i = 1,\ldots,\; m_{eq}$$

$$\nabla c_i^T(\mathbf{x}^*)\,s = 0 \;\text{ for } i = m_{eq}+1,\ldots, m \text{ with } \lambda_i > 0 \text{ (active inequality constraints)}$$

Also let

$$\nabla c_i^T(\mathbf{x}^*)\,s \geq 0 \;\text{ for } i = m_{eq}+1,\ldots, m \text{ with } \lambda_i = 0 \text{ (inactive inequality constraints)}$$

If it is found that

$$s\,\nabla_{\mathbf{x}}^2 L(\mathbf{x}^*, \Lambda^*)\,s > 0 \tag{2.11}$$

for all such vectors s, then **x*** is an isolated local minimum point (isolated means that there are no other minimum points in the neighborhood of **x***). Thus, if the Hessian of the Lagrange function is positive definite, the sufficiency conditions for an isolated local minimum point are satisfied. The second order sufficient condition should be interpreted in the sense that the Lagrange function projected on the subspace of the binding constraints is locally convex.

To treat the subject of global optimality for constrained optimization problems one needs to consider the topics of convexity and convex programming problems. The first fundamental concept is that of a convex set. A convex set *K* is a collection of points (vectors **x**) defined by the property that for all $\mathbf{x}_0, \mathbf{x}_1 \in K$, it follows that for any scalar $\lambda \in [0,1]$, the point

$$\mathbf{x}_\lambda = (1-\lambda)\mathbf{x}_0 + \lambda\mathbf{x}_1 \tag{2.12}$$

is also contained in *K*. The other fundamental idea is that of a convex function. A convex function f(**x**) is defined on a convex set *K*, i.e. the independent variables must lie in a convex set. A convex function f(**x**) is defined by the condition that for any $\mathbf{x}_0, \mathbf{x}_1 \in K$, and any scalar $\lambda \in [0,1]$, it follows that

$$f(\mathbf{x}_\lambda) \leq (1-\lambda)f(\mathbf{x}_0) + \lambda f(\mathbf{x}_1) \tag{2.13}$$

It can be proved that a function f(**x**) defined on a convex set *K* is convex if and only if the Hessian matrix of the function is positive semidefinite or positive definite at all the points in the set *K*. A strictly convex function is one for which the Hessian matrix is positive definite for all the points in the set *K*. A concave function is defined as one for which −f(**x**) is convex.

A convex programming problem is an optimization problem of the form shown in equation (2.1) in which f($\mathbf{x}$) is convex, the equality constraints are linear, and the inequality constraints are concave. It can be shown that the set defined by the constraints is a convex set. Examples of convex programming problems are linear programming (LP) and quadratic programming (QP) problems (the Hessian of the latter must at least be positive semidefinite). A fundamental property of a convex programming problem is that any local minimum $\mathbf{x}^*$ is a global minimum; furthermore, if $\nabla^2 f(\mathbf{x})$ is positive definite, $\mathbf{x}^*$ is unique. Another very useful property of the convex programming problem is that if f($\mathbf{x}$) is a convex objective function defined on a convex feasible region, the first order (Kuhn-Tucker) conditions are necessary as well as sufficient to characterize a global minimum point.

The analytical methods discussed in the preceding section are sometimes very cumbersome to use in practical applications. In practice one is not given a point and asked to check if the optimality conditions are satisfied. However, a thorough knowledge of optimality conditions is important to understand the performance and implementation of various numerical solution methods discussed in the next section. Optimality conditions are not algorithms; they are used to motivate algorithms and the corresponding convergence proofs.

Solution methods

Numerical methods to solve nonlinear optimization problems are of an iterative nature. An initial estimate for the optimum point is chosen and the estimate is improved in an iterative manner until, either some user-supplied convergence criteria become satisfied or the optimality conditions are satisfied. Almost all iterative optimization algorithms have the following main steps [84VA1e, 87FL1m, 89AR1e]:

(1) Determine a search direction. This process can involve function and gradient evaluations, the solution of a set of linear equations, or the solution of linear or quadratic programming subproblems.

(2) Solution of the subproblem. Several methods are available to solve the subproblem.

(3) A step size along the search direction. This usually involves evaluation of functions along the search direction.

The basic underlying principle in solving a nonlinear programming problem is that of replacing a difficult problem by an easier, solvable one. This leads to the formulation and solution of a sequence of subproblems, each of which is related to the original problem in a known way. The subproblems are based on local models of the objective and constraint functions (if present). The most common model functions originate from the Taylor series expansions of these nonlinear functions. Linear or quadratic models are used; the latter is the most frequent choice for the objective functions since it has attractive properties; near the optimum point every problem is approximately quadratic [87FL1m]. The subproblem is solved to obtain a search direction and a step length is chosen in order to ensure a decrease in a quantity that measures progress towards the solution.

Many iterative methods are based on the following iterative equation to generate a sequence of points $\{\mathbf{x}^{(k)}\}$ [87FL1m]

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)} \quad k = 0,1,2,\ldots \tag{2.14}$$

where the superscript k represents an iteration counter, $\alpha^{(k)}$is a positive scalar called the step size and $\mathbf{s}^{(k)}$ is the direction of search that depends on the local behavior of the objective and constraint functions at $\mathbf{x}^{(k)}$. A variety of computational algorithms exist, based on the way $\mathbf{s}^{(k)}$ and $\alpha^{(k)}$ are calculated. The vector $\mathbf{s}^{(k)}$ is usually a direction of descent for a function that is used to monitor progress towards the minimum point and is called the descent or merit function $\Psi(\mathbf{x})$ [89GI1m], i.e.

$$\Psi\left(\mathbf{x}^{(k+1)}\right) < \Psi\left(\mathbf{x}^{(k)}\right) \tag{2.15}$$

The descent function also has the property that its minimum value is the same as that of the original objective function. The step size $\alpha^{(k)}$ is chosen to ensure (2.15). The descent function is used to find the step size and is thus transformed into a one dimensional function of the scalar variable $\alpha^{(k)}$. For unconstrained optimization the objective function serves as the descent function. For constrained optimization several descent functions have been used and it is usually constructed from both the objective function and the constraints.

The numerical algorithms usually work well in the vicinity of the local solution point. Most of the effort spent in algorithms is to come close enough to a local solution from a poor starting point, in order that the algorithm can take advantage of the local convergence properties. Furthermore, the models derived from the Taylor series expansion neglect higher order terms and are therefore only valid in a neighborhood of unknown size of the current point. Trust region and line search methods are used to overcome these problems. Trust region methods are used to restrict the domain in which the model function is considered reliable. Line search methods are used to obtain the best choice of α along the direction of descent for the merit function. Both line search and trust region methods are used in optimization algorithms to ensure global convergence from poor starting points [81GI1m, 83DE1m, 87FL1m, 89DE1m].

An algorithm is said to be convergent if it approaches a minimum point starting from a given starting point $\mathbf{x}^{(0)}$. Convergence to a local minimum point irrespective of the starting point is called global convergence. A convergent algorithm usually has a descent function that monitors progress towards the minimum. The rate of convergence of an algorithm is usually measured by the number of iterations and function evaluations, or the CPU time taken to obtain an acceptable solution.

Most optimization methods are iterative and generate a sequence of estimates of the optimum point $\mathbf{x}^*$. The rate at which this sequence converges can be defined in terms of the error

$$\mathbf{h}^{(k)} = \left(\mathbf{x}^{(k)} - \mathbf{x}^*\right) \qquad (2.16)$$

If $\mathbf{h}^{(k)} \to 0$ (convergence) and the errors behave according to [87FL1m, 89DE1m]

$$\left\|\mathbf{h}^{(k+1)}\right\| \Big/ \left\|\mathbf{h}^{(k)}\right\|^{p} \le \nu, \;\; \nu \ge 0 \qquad (2.17)$$

then the order of convergence is defined to be p-th order. The most important cases are p = 1 (first order or linear convergence) and p = 2 (second order or quadratic convergence). In practice local quadratic convergence is quite fast as it implies that the number of significant digits in $\mathbf{x}^{(k)}$ as an approximation to $\mathbf{x}^*$ roughly doubles at each iteration once $\mathbf{x}^{(k)}$ is near $\mathbf{x}^*$. Linear convergence can be quite slow and much better results can be obtained when the rate constant, ν, tends to zero. This is known as superlinear convergence and algorithms with this property are also quickly convergent in practice.

Another important feature of an algorithm is the convergence test which is required to terminate the iterations. These tests have a major effect on the efficiency and reliability of the optimization process. Some of the termination criteria that are used, are the maximum number of iterations or function evaluations, the absolute or relative change in the objective function or decision variables, or satisfaction of the optimality conditions [84VA1e, 87FL1m].

In practical problems it may often be cumbersome to calculate the derivatives of a complex function analytically. In such cases it is possible to approximate the gradient vector or the Hessian matrix of a function by means of finite differences (forward difference, central difference, backward difference) [81GI1m, 83DE1m, 87FL1m, 89AR1e, 89DE1m]. The finite difference steps must be chosen large enough in order to minimize the finite precision cancellation errors, as well as small enough to ensure that the difference produce a good approximation to the derivative. If step sizes are properly selected, then finite difference methods give reliable results. The main disadvantage of these derivative approximations is the computational effort in evaluating them if the function evaluations are expensive. However, finite differences will not require more computational effort if the analytical derivatives of a complicated function require the same effort to evaluate as the function itself.

Some of the most important solution methods for unconstrained and constrained nonlinear programming will now be discussed.

(1) Unconstrained optimization

The early methods of optimization required the use of function values only (zero order methods) and were very reliable, easy to program and can often deal effectively with discontinuous functions and discrete values of the decision variables. However, these methods require many function evaluations, even for simple problems. The main difficulty in the formulation of these methods is how to search effectively for the optimum point. The different methods perform their searches in a random or in an ordered manner. Various methods are discussed and referred to in [83RE1e, 84VA1e, 87FL1m, 88ED1e]. The rest of the section will deal with techniques that use the mathematical nature of the problem in order to make the search for the optimum point as efficient as possible.

Iterative methods are composed of a search direction subproblem and a step size calculation, also referred to as the one dimensional line search problem. The line search involves the reduction of a multiple variable function to a function of one variable, the step size, and then finding the minimum point of the one dimensional problem, i.e.

$$\underset{\alpha}{\text{Minimize}} \; \Psi\left(\mathbf{x}^{(k)} + \alpha\, \mathbf{s}^{(k)}\right) = \Psi\left(\mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}\right) \tag{2.18}$$

where $\alpha^{(k)}$ is the step length and $\mathbf{s}^{(k)}$ is the search direction. If we assume that a search direction $\mathbf{s}^{(k)}$ is known at the current point $\mathbf{x}^{(k)}$, the descent function reduces to a function of one variable. This function $\Psi(\alpha) = \Psi\left(\mathbf{x}^{(k)} + \alpha\, \mathbf{s}^{(k)}\right)$ is assumed to be unimodal, i.e. its minimum exists and is unique in the interval of interest [87FL1m, 89AR1e]. Analytical methods can be used to determine the value α^* that minimizes Ψ (α). However, analytical methods may become cumbersome and therefore we rather turn to numerical methods which are in themselves iterative. The line search problem involves the finding of the interval $[0,\bar{\alpha}^{(k)}]$ in which the required α^* lies, and then the reduction of this interval until the desired accuracy for locating $\alpha^{(k)}$ is reached. These methods can be grouped into polynomial approximations (use function values and derivative information) and those which require function evaluations only [87FL1m]. The latter methods contain interval searches like the golden section search and the Fibonnaci section search. Any continuous function can be closely approximated by passing a polynomial of sufficient order through it and its minimum can be found explicitly. The minimum point of the approximating polynomial is often a good estimate of the exact minimum of $\Psi(\alpha)$ (equation (2.18)). Accurate line searches are very expensive to carry out and researchers developed a set of conditions for terminating the line search which would allow low accuracy line searches whilst still forcing global convergence [83DE1m, 87FL1m]. The line search thus aims to find a step $\alpha^{(k)}$ which gives a significant reduction in $\Psi(\alpha)$ on each iteration and which is not close to the extremes of the interval $[0,\bar{\alpha}^{(k)}]$.

In the following paragraphs various algorithms are presented for calculating the search direction, s. These methods are categorized as first order methods and second order methods.

First order methods utilize gradient information, supplied either analytically or by finite difference computations. The steepest descent method [81GI1m, 84LU1m, 87FL1m, 89AR1e] is the simplest and probably the best known method for unconstrained optimization. In this method the search direction, $\mathbf{s}^{(k)}$, is taken as the negative of the gradient of the objective function, i.e. $\mathbf{s}^{(k)} = -\mathbf{g}(\mathbf{x}^{(k)})$, and presents the direction of maximum decrease in $f(\mathbf{x})$. The convergence rate of this method is very poor due to the fact that the method does not utilize information from previous iterations. The steepest descent directions are also orthogonal to each other. This method is therefore not recommended for general applications and its principal importance is that it usually forms the starting point for more sophisticated first order methods.

The conjugate gradient [81GI1m, 83MC1m, 84LU1m, 87FL1m, 89AR1e] method requires a simple modification to the steepest descent algorithm that substantially improves its rate of convergence compared to the steepest descent method. The conjugate gradient directions, however, are not orthogonal to each other, but tend to cut diagonally through the orthogonal steepest descent directions. The initial search vector is the steepest descent vector and afterwards the conjugate direction is defined as

$$\mathbf{s}^{(k+1)} = -\mathbf{g}\left(\mathbf{x}^{(k+1)}\right) + \beta^{(k)}\mathbf{s}^{(k)} \tag{2.19}$$

with $\beta^{(0)} = 0$ and $\beta^{(k)} = \dfrac{\mathbf{g}\left(\mathbf{x}^{(k+1)}\right)^T \mathbf{g}\left(\mathbf{x}^{(k+1)}\right)}{\mathbf{g}\left(\mathbf{x}^{(k)}\right)^T \mathbf{g}\left(\mathbf{x}^{(k)}\right)}$

Information about previous iterations is carried forward in the optimization process by β. The conjugate gradient algorithm finds the minimum of a positive definite quadratic function having n variables in n iterations. For general functions it is recommended that the iterative process is restarted every (n+1) iterations if the minimum has not been found by then. This method is based on the calculation of both f(**x**) and **g**(**x**) and only requires storage of n-dimensional vectors (no n^2-dimensional arrays). This property makes the method very useful for computation of large dimensional problems.

Second order methods make use of the second derivatives of the objective function. Newton's method is the classical second order method [81GI1m, 83DE1m, 83MC1m, 84LU1m, 87FL1m, 89AR1e, 89DE1m]. The idea behind this method is that the function f(**x**) to be minimized, is approximated locally by a quadratic function and this approximate function is minimized exactly. Near $\mathbf{x}^{(k)}$, the function $f(\mathbf{x}^{(k)})$ can be approximated by the second order Taylor series expansion given in a slightly different form

$$f\left(\mathbf{x}^{(k)} + \mathbf{s}^{(k)}\right) \approx f\left(\mathbf{x}^{(k)}\right) + \mathbf{g}\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} + 0.5\,\mathbf{s}^{(k)T}\mathbf{G}\left(\mathbf{x}^{(k)}\right)\mathbf{s}^{(k)} \tag{2.20}$$

This method requires first and second order derivatives of f(**x**) at any point. Equation (2.20) has a unique minimizer if $\mathbf{G}(\mathbf{x}^{(k)})$ is positive definite and Newton's method is only well defined in these circumstances. By applying the first order necessary condition to equation (2.20), the k-th iteration of Newton's method can be written as

Solve $\mathbf{G}\left(\mathbf{x}^{(k)}\right)\mathbf{s}^{(k)} + \mathbf{g}\left(\mathbf{x}^{(k)}\right) = 0$ for $\mathbf{s}^{(k)}$

$$\text{Set} \quad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{s}^{(k)} \tag{2.21}$$

This involves the solution of a $n \times n$ system of linear equations. Considerable computational effort is usually needed to calculate the Hessian matrix. The classical method uses a step size of one in the search direction. For a true convex quadratic function this method will find the solution in only one iteration. Since $\mathbf{G}(\mathbf{x}^{(k)})$ may not be positive definite when $\mathbf{x}^{(k)}$ is far from the solution, the classical method is not suitable as a general purpose algorithm. If $\mathbf{G}(\mathbf{x}^{(k)})$ is positive definite, the method can be made globally convergent when Newton's method is used in combination with a line search. The main difficulty, however, is in modifying this algorithm when $\mathbf{G}(\mathbf{x}^{(k)})$ is not positive definite. A multiple of a unit matrix can be added to $\mathbf{G}(\mathbf{x}^{(k)})$ to make it positive definite [44LE1m, 63MA1m], thus giving the search direction a bias towards the steepest descent vector $-\mathbf{g}(\mathbf{x}^{(k)})$. The search direction is then computed from

$$\left(\mathbf{G}\left(\mathbf{x}^{(k)}\right) + \lambda^{(k)}\mathbf{I}\right)\mathbf{s}^{(k)} + \mathbf{g}\left(\mathbf{x}^{(k)}\right) = \mathbf{0}, \quad \lambda^{(k)} \geq 0 \tag{2.22}$$

When the scalar $\lambda^{(k)}$ is large the effect of $\mathbf{G}(\mathbf{x}^{(k)})$ essentially gets neglected and $s^{(k)}$ is essentially the steepest descent direction. As the iterative process proceeds, λ is reduced. When $\lambda^{(k)}$ becomes sufficiently small the Newton direction is obtained. If $s^{(k)}$ does not reduce the descent function, $\lambda^{(k)}$ is increased and the direction is recomputed.

Line search and trust region methods are used to make Newton's method globally convergent while retaining its excellent local convergence properties [81GI1m, 83DE1m, 87FL1m, 89DE1m]. In line search methods the quadratic model is used to compute the search direction and then a step length is chosen. In a trust region method a trial step length $h^{(k)}$ is chosen and then the quadratic model is used to select a step of at most this length, i.e.

$$\left\|s^{(k)}\right\| \le h^{(k)} \tag{2.23}$$

The trial step length is considered an estimate of the region of validity of the Taylor series expansion. These methods are generally applicable and globally convergent and retain the convergence rate of Newton's method. Trust region methods are characterized by solving equation (2.22) in order to determine $s^{(k)}$. The scalars $h^{(k)}$ and $\lambda^{(k)}$ are related to each other. Such methods were first suggested by Levenberg [44LE1m] and Marquardt [63MA1m] in the context of nonlinear least squares problems. The trust region can be adjusted during the iterations and should be as large as possible subject to a certain measure of agreement between the actual and approximated function. Both trust region and line search methods appear in modern software and neither appears to be consistently superior to the other in practice.

The main disadvantage of Newton's method, even when modified to ensure global convergence, is that the second derivative matrix, $\mathbf{G}(\mathbf{x}^{(k)})$, must be evaluated. Furthermore, Newton's method does not use information from previous iterations. A class of methods that overcomes these disadvantages is the quasi-Newton (variable metric) methods [81GI1m, 83DE1m, 83MC1m, 84LU1m 84VA1e, 87FL1m, 89AR1e, 89DE1m]. They require only first derivatives and previously calculated information to approximate the Hessian matrix or its inverse during each iteration. This approximation is updated during each iteration and these methods have convergence characteristics similar to second order methods. During updating, the properties of positive definiteness and symmetry are preserved. The initial matrix can be any positive definite matrix and is usually chosen as the unit matrix **I**. Two of the most popular methods are the Davidon-Fletcher-Powell method

(DFP) and the Broyden-Fletcher-Goldfarb-Shanno method (BFGS) [87FL1m]. The DFP method builds up the approximate inverse of the Hessian ($\mathbf{H} \approx \mathbf{G}^{-1}$) of f($\mathbf{x}$) using only first derivatives.

$$\mathbf{H}_{\mathrm{DFP}}^{(k+1)} = \mathbf{H}^{(k)} + \frac{\delta^{(k)}\delta^{(k)T}}{\delta^{(k)T}\gamma^{(k)}} + \frac{\mathbf{H}^{(k)}\gamma^{(k)}\,\gamma^{(k)T}\mathbf{H}^{(k)}}{\gamma^{(k)T}\mathbf{H}^{(k)}\gamma^{(k)}} \tag{2.24}$$

where

$$\delta^{(k)} = \alpha^{(k)}\mathbf{s}^{(k)} = \mathbf{x}^{(k+1)} - \mathbf{x}^{(k)} \tag{2.25}$$

$$\gamma^{(k)} = \mathbf{g}\left(\mathbf{x}^{(k+1)}\right) - \mathbf{g}\left(\mathbf{x}^{(k)}\right) \tag{2.26}$$

In the BFGS method the Hessian rather than its inverse is updated at every iteration

$$\mathbf{B}_{\mathrm{BFGS}}^{(k+1)} = \mathbf{B}^{(k)} + \frac{\gamma^{(k)}\gamma^{(k)T}}{\gamma^{(k)T}\delta^{(k)}} - \frac{\mathbf{B}^{(k)}\delta^{(k)}\,\delta^{(k)T}\mathbf{B}^{(k)}}{\delta^{(k)T}\mathbf{B}^{(k)}\delta^{(k)}} \tag{2.27}$$

where $\delta^{(k)}$ and $\gamma^{(k)}$ are defined by equation (2.25) and (2.26) respectively. The BFGS method has been found to work well in practice. Certain conditions must be complied with, in order to keep the updates positive definite [87FL1m].

If $\tilde{f}(\mathbf{x})$ is the sum of squares of nonlinear functions, special advantages can be taken of the problem structure to find the minimum solution [74LA1m, 81GI1m, 83DE1m, 87FL1m, 89DE1m]. The objective function of this so-called nonlinear least squares problem is written as

$$\tilde{f}(\mathbf{x}) = 0.5\sum_{i=1}^{m} f_i(\mathbf{x})^2 = \mathbf{f}(\mathbf{x})^T\mathbf{f}(\mathbf{x}) \tag{2.28}$$

and its derivatives are given by

$$\mathbf{g}(\mathbf{x}) = \mathbf{J}(\mathbf{x})\mathbf{f}(\mathbf{x}) \tag{2.29}$$

$$\mathbf{G}(\mathbf{x}) = \mathbf{J}(\mathbf{x})\mathbf{J}(\mathbf{x})^T + \sum_{i=1}^{m} f_i(\mathbf{x})\nabla^2 f_i(\mathbf{x}) \tag{2.30}$$

Problems of this type occur in nonlinear parameter estimation when fitting model functions to data. When m > n, least squares solutions to over-determined systems of equations are computed by minimizing (2.28). Exact solutions can be obtained for well-determined problems if m = n. Equation (2.29) can be used with quasi-Newton methods or both equations (2.29) and (2.30) can be used with a modified Newton method. A good approximation to $\mathbf{G}(\mathbf{x})$ can be obtained by assuming that the last term on the right hand side of equation (2.30) is small (small residual problem), i.e.

$$\mathbf{G}(\mathbf{x}) \approx \mathbf{J}(\mathbf{x})\mathbf{J}(\mathbf{x})^{T} \tag{2.31}$$

Whereas a quasi-Newton method might take n iterations to estimate $\mathbf{G}(\mathbf{x})$ satisfactorily, here the approximation is immediately available. The basic Newton method becomes the Gauss-Newton method when (2.31) is used to approximate $\mathbf{G}(\mathbf{x})$. The k-th iteration of the basic Gauss-Newton method can be written as

$$\text{Solve} \quad \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^{T}\mathbf{s}^{(k)} + \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right) = \mathbf{0} \quad \text{for } \mathbf{s}^{(k)}$$

$$\text{Set} \quad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{s}^{(k)} \tag{2.32}$$

The basic Gauss-Newton method is not necessarily globally convergent or sometimes not even locally convergent on problems that are very nonlinear or on large residual problems. This algorithm can be improved in two ways to make it locally and globally convergent, i.e. using it with line search (damped Gauss-Newton) or with a trust region strategy (refer to equation (2.22)). The trust region method is usually referred to as the Levenberg-Marquardt method [44LE1m, 63MA1m].

Unconstrained optimization methods can also be used to solve nonlinear constrained problems [81GI1m, 83MC1m, 83RE1e, 84LU1m, 84VA1e, 87FL1m, 89AR1e, 89GI1m]. The basic idea is to construct a composite function using the objective and the constraint functions. This composite function also contains penalty parameters that penalize it for constraint violations. Mathematically the problem can be stated as

$$\underset{\mathbf{x}}{\text{Minimize}} \;\; \Phi(\mathbf{x},\sigma) = f(\mathbf{x}) + P(c(\mathbf{x}),\sigma) \tag{2.33}$$

where σ is the penalty parameter and P is a real valued function (penalty function) whose form depends on the method used.

The composite function is constructed for a set of penalty parameters and solved using any of the unconstrained optimization techniques. The penalty parameters are then adjusted, the composite function is redefined and then minimized. The process is continued until no improvement in the estimated optimum point is obtained; thus the term sequential unconstrained minimizing techniques identifies these methods. These methods are often called transformation methods because a constrained problem is transformed into an unconstrained one. Examples of these methods are the penalty (exterior) and barrier (interior) function methods as well as the multiplier (augmented Lagrangian) methods.

The transformation methods conclude the discussion of the most important classes of unconstrained optimization solution methods. The techniques used in solving constrained problems rely heavily on the techniques for solving unconstrained problems.

(2) Constrained optimization

In this section methods used for directly solving the original constrained problem (primal methods) will be discussed. The algorithms for unconstrained and constrained optimization are based on the same iterative philosophy. There is one important difference for constrained problems however; constraints must be taken into account while determining the search direction as well as the step size. Furthermore, the starting point may be feasible or non-feasible with regard to the constraints. Different procedures for treating these issues give rise to different optimization algorithms.

An important concept that should be addressed is the status of the inequality constraints at the optimum solution. Active inequality constraints can be treated as equality constraints and the remaining inequality constraints can be ignored (locally). These inactive constraints can be perturbed by small amounts without affecting the local solution. Equality constraints must be exactly satisfied at the solution. Active constraints at any point $\mathbf{x}^{(k)}$ are defined by the index set

$$A^{(k)} = A\left(\mathbf{x}^{(k)}\right) = \left\{i : c_i\left(\mathbf{x}^{(k)}\right) = 0\right\} \tag{2.34}$$

called the active set (working set), so that any constraint is active at $\mathbf{x}^{(k)}$ if $\mathbf{x}^{(k)}$ is on the boundary of its feasible region [87FL1m]. Equality constraints are always present in the active set. If A^* is known, the problem can be solved as an equality constrained problem. When inequality constraints are present the active set A^* is unknown and must be predicted by some strategy in the solution method. The predicted $A^{(k)}$ is used to compute the search direction and to change the active set as the iterations proceed. $A^{(k)}$ usually changes during each iteration, except when $\mathbf{x}^{(k)}$ is near $\mathbf{x}^*$. It should be remembered that the constraints active at the solution are significant in the optimality conditions and that an algorithm will only succeed if the correct active set is identified. An active set strategy is easier to define for problems with linear constraints than those with nonlinear constraints (refer to [81GI1m]) and only the basic ideas involved in problems with linear constraints will be discussed. Linear constraints are of the form

$$c_i(\mathbf{x}) = \mathbf{a}_i^T\mathbf{x} - b_i \tag{2.35}$$

Assume that a feasible point $\mathbf{x}^{(k)}$ and a predicted active $A^{(k)}$ set are known. Next, the search direction $\mathbf{s}^{(k)}$ can be computed. Two situations are now possible:

(1) The point $\mathbf{x}^{(k)} + \mathbf{s}^{(k)}$ may violate a constraint, or several of them, not currently in the active set. Thus $A^{(k)}$ is not the correct active set and to remain feasible, a step length $0 \leq \alpha^{(k)} \leq 1$ is determined such that $\alpha^{(k)}$ is the largest step that retains feasibility. A constraint that becomes exactly satisfied at $\mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}$ is added to the active set and $A^{(k)}$ is enlarged. A new search direction is computed with this modified active set.

(2) The feasible point $\mathbf{x}^{(k)} + \mathbf{s}^{(k)}$ is the minimizer of the objective function and $A^{(k)}$ is treated as a set of equality constraints. The Lagrange multipliers of the active inequality constraints are then calculated to determine if the solution is a Kuhn-Tucker point. If the minimum of these multipliers is greater than or equal to zero, a feasible solution is found. Otherwise, the inequality constraint with the smallest Lagrange multiplier is removed from the active set ($A^{(k)}$is reduced) and thus becomes inactive. A new search direction is computed with this modified active set.

For linearly constrained problems, the largest step size $\alpha^{(k)}$ to retain feasibility during iteration k is obtained from solving

$$\alpha^{(k)} = \min\left(1;\ \min_{\mathbf{a}_i^T \mathbf{s}^{(k)} < 0} \frac{b_i - \mathbf{a}_i^T \mathbf{x}^{(k)}}{\mathbf{a}_i^T \mathbf{s}^{(k)}},\quad i \notin A^{(k)}\right) \tag{2.36}$$

If $\alpha^{(k)} < 1$ is obtained, then a new constraint becomes active and its index is added to the active set $A^{(k)}$. Various matrices must be updated during the adding and dropping of constraints from the active set [84GI1m].

This description outlines the strategies involved in a primal active set method for quadratic programming, because only feasible iterates are allowed. Dual active set methods are also used where it is not required to satisfy primal feasibility during each iteration [83GO1m, 85PO1m, 87FL1m]. For more detail regarding active set methods (e.g. the conditions under which the correct active set will be predicted), the reader should consult Gill, Murray and Wright [81GI1m, 89GI1m] and Fletcher [87FL1m] as well as the references stated therein.

Constrained optimization can be categorized into two parts, i.e. nonlinear or linear objective functions with linear constraints and linear or nonlinear objective functions with nonlinear constraints. The simplest type of constrained optimization is the linear programming (LP) problem. This subject is quite well developed and several textbooks and journal articles on the topic are available, e.g. [81GI1m, 84LU1m, 84VA1e, 87FL1m, 89AR1e, 89GO1m]. LP problems are the most thoroughly developed and understood optimization problems. Many optimization problems of practical interest are not of this form and linear programming is often overlooked in favor of the nonlinear programming optimization methods. However, linear programming techniques often form the basis for the development of more complex nonlinear programming algorithms and sometimes it is even possible to simplify a nonlinear programming problem by means of linearization and solve it with linear programming techniques [87FL1m, 89GI1m].

The simplex method is the earliest method for solving LP problems. The basic idea of this method is to proceed from one basic feasible solution to another in such a way as to continually decrease the value of the objective function until a minimum is reached. This method is still used today but in more sophisticated forms, e.g. the revised simplex method [84LU1m]. A LP problem is an example of a convex programming problem and if an optimum solution exists, it is also the global solution. The optimum solution will always lie on the boundary of a feasible region, i.e. there are always some constraints active at the solution. LP problems can always be solved in a finite number of steps.

The quadratic programming (QP) problem involves the minimization of a quadratic objective function subject to linear constraints. QP is of great interest in its own right, and also plays an important role in the solution of general nonlinear programming problems as a direction finding subproblem. A QP problem may be stated as follows [81GI1m, 87FL1m, 89GI1m]:

$$\underset{\mathbf{x}}{\text{Minimize}}\ q(\mathbf{x}) = 0.5\mathbf{x}^T\mathbf{G}\,\mathbf{x} + \mathbf{g}^T\mathbf{x} \qquad \text{quadratic objective function}$$

subject to the linear constraints

$$\mathbf{a}_i^T\mathbf{x} = b_i, \quad i = 1,\dots,m_{eq} \quad \left(\mathbf{A}_{eq}^T\mathbf{x} = \mathbf{b}\right) \qquad \text{linear equality constraints} \tag{2.37}$$

$$\mathbf{a}_i^T\mathbf{x} \geq b_i, \quad i = m_{eq}+1,\dots,m \quad \left(\mathbf{A}_{ineq}^T\mathbf{x} \geq \mathbf{b}\right) \qquad \text{linear inequality constraints}$$

where **G** is symmetric, $\nabla q(\mathbf{x}) = \mathbf{Gx} + \mathbf{g}$ and $\nabla^2 q(\mathbf{x}) = \mathbf{G}$. If the Hessian matrix **G** is positive semidefinite and the active constraints are linearly independent, **x*** is the global solution, and if **G** is positive definite, **x*** is also unique (convex programming problem). When **G** is indefinite, then local solutions which are not global can occur. QP problems can always be solved in a finite number of steps.

Equality constrained QP problems can be solved, mainly by using matrix manipulations and computations. The most straightforward method is to use the equality constraints to eliminate variables and then to minimize the resulting quadratic function. Fletcher [87FL1m] also presents a generalized elimination method with its various variations for solving these kinds of problems.

Elimination methods reduce the constrained problem to an unconstrained problem that must be solved for a minimizer. By using the feasibility and optimality conditions for problem (2.37), one is able to derive the solution **x*** and the associated Lagrange multipliers Λ* by the method of Lagrange multipliers for equality constraint problems (refer to equation (2.8)). This method is discussed in detail in Fletcher [87FL1m] and Gill et al. [89GI1m] and may be implemented either by null space or the range space methods.

QP problems containing inequality constraints are usually solved by active set methods as previously discussed (refer to [81GI1m, 83GO1m, 85PO1m, 87FL1m, 89GI1m]). The direction of movement is towards the solution of the corresponding equality constrained problem. The main difficulty in these methods is the prediction of the correct active set of constraints at the solution. If the correct active set is identified, the problem is solved as an equality constrained quadratic problem that involves many matrix calculations. It is therefore important to find an effective means of updating the relevant matrices when the set of active constraints changes [81GI1m]. A number of solution methods for QP problems that are extensions of the simplex method for linear programming have also been suggested [87FL1m, 89AR1e].

Another class of linearly constrained problems are those in which the objective function is general. These problems can in general no longer be solved finitely like linear and quadratic programming problems and the solution is obtained in the limit of some iterative sequence $\{\mathbf{x}^{(k)}\}$. The equality constrained problem can be handled by generalized elimination methods as discussed for quadratic programming. Inequality constrained problems can be handled by means of active set methods. In quadratic programming the point $\mathbf{x}^{(k)}$ either solves the equality constrained problem or a previously inactive constraint becomes active. In the more general case the solution of the equality constrained problem is only located in the limit of a sequence of iterations with the same active set. The line search procedure is also more complicated and the step length must not exceed an upper limit in order to retain feasibility. More detail of the general linearly constrained optimization problem can be found in Fletcher [87FL1m].

Nonlinear programming (NLP) problems arise when both the objective function and the constraints are nonlinear and is the most difficult of all the smooth optimization problems. The earliest methods used in solving these problems were the sequential minimization methods or transformation methods discussed in the section covering the topic of unconstrained optimization. NLP problems can also be solved by using exact penalty functions in which the minimizer of the penalty function and the solution of the NLP problem coincide. Sequential processes are not necessary to find the minimizer of this exact penalty function. The most popular exact penalty function is the non-smooth or non-differentiable L_1-penalty function [87FL1m, 89GI1m]. Smooth exact penalty functions are also possible.

NLP problems can also be solved by methods based directly on the first order optimality conditions stated in equation (2.8) [81GI1m, 84LU1m, 84VA1e, 87FL1m, 89AR1e, 89GI1m]. These methods perform well in the neighborhood of the solution. The idea of a quadratic model is very important in the most successful methods for unconstrained optimization. In constrained optimization the curvature of the Lagrange function should be considered and a quadratic model constructed thereof. The original nonlinear constraints are approximated as linear constraints (linearization by means of a Taylor series expansion). A sequence of approximations $\mathbf{x}^{(k)}$ and $\Lambda^{(k)}$ to the optimum solution vector, $\mathbf{x}^*$, and the optimum Lagrange multipliers, Λ^*, are generated in the process.

For the derivation of the quadratic subproblem, only the equality constrained problem will be considered. Inequality constraints can be easily incorporated into the subproblem. The subproblem is derived by writing the first order necessary conditions for the equality constrained problem and then it is solved by Newton's method for nonlinear equations. Each iteration of Newton's method can then be interpreted as the solution of a quadratic subproblem. We assume in the following derivations that all the functions are twice continuously differentiable and gradients of all the constraints are linearly independent.

The Kuhn-Tucker condition of the equality constrained problem states that

$$\nabla_{\mathbf{x},\Lambda} L(\mathbf{x}^*, \Lambda^*) = \mathbf{0} \tag{2.38}$$

where derivatives are evaluated both with regard to $\mathbf{x}$ and Λ. A linear Taylor series expansion of ∇L about $\mathbf{x}^{(k)}$ and $\Lambda^{(k)}$ gives rise to the system

$$\begin{bmatrix} \mathbf{W}^{(k)} & -\mathbf{A}_{eq}^{(k)} \\ -\mathbf{A}_{eq}^{(k)T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x}^{(k)} \\ \delta\Lambda^{(k)} \end{bmatrix} = \begin{bmatrix} -\mathbf{g}^{(k)} + \mathbf{A}_{eq}^{(k)}\Lambda^{(k)} \\ \mathbf{c}^{(k)} \end{bmatrix} \tag{2.39}$$

where $\mathbf{A}_{eq}^{(k)}$ is the Jacobian matrix of the equality constraint normals evaluated at $\mathbf{x}^{(k)}$, $\mathbf{W}^{(k)}$ is the Hessian matrix of the Lagrange function, $\Lambda^{(k+1)} = \Lambda^{(k)} + \delta\Lambda^{(k)}$and $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta\mathbf{x}^{(k)}$. Equation (2.39) can be converted into a slightly different form to solve for $\Lambda^{(k+1)}$ and $\delta\mathbf{x}^{(k)}$

$$\begin{bmatrix} \mathbf{W}^{(k)} & -\mathbf{A}_{eq}^{(k)} \\ -\mathbf{A}_{eq}^{(k)T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x}^{(k)} \\ \Lambda^{(k+1)} \end{bmatrix} = \begin{bmatrix} -\mathbf{g}^{(k)} \\ \mathbf{c}^{(k)} \end{bmatrix} \tag{2.40}$$

Initial approximations of $\mathbf{x}^{(k)}$ and $\Lambda^{(k)}$ are required and the iterative procedure is continued until the convergence criteria are satisfied. Just as with Newton's method for unconstrained optimization it is possible to restate the problem in terms of one in which the subproblem involves the minimization of a quadratic function, defined as

$$\underset{\delta\mathbf{x}}{\text{Minimize}}\ q^{(k)}\left(\delta\mathbf{x}^{(k)}\right) = 0.5\,\delta\mathbf{x}^{(k)T}\mathbf{W}^{(k)}\delta\mathbf{x}^{(k)} + \mathbf{g}^{(k)T}\delta\mathbf{x}^{(k)} + f^{(k)}$$

subject to the linearized constraints

$$\mathbf{A}_{eq}^{(k)T}\delta\mathbf{x}^{(k)} + \mathbf{c}^{(k)} = \mathbf{0} \tag{2.41}$$

The Kuhn-Tucker conditions of problem (2.41) are given by equation (2.40). If the Hessian matrix of the Lagrange function is positive definite on the subspace defined by $\left\{\delta\mathbf{x}\colon\ \mathbf{A}_{eq}^{(k)T}\delta\mathbf{x}^{(k)} = 0\right\}$, then $\delta\mathbf{x}^{(k)}$ minimizes equation (2.41). The QP subproblem for the general nonlinear constrained problem can thus be defined as

$$\underset{\delta\mathbf{x}}{\text{Minimize}}\ q^{(k)}(\delta\mathbf{x}) = 0.5\,\delta\mathbf{x}^{T(k)}\mathbf{W}^{(k)}\delta\mathbf{x}^{(k)} + \mathbf{g}^{T}\delta\mathbf{x}^{(k)} + f^{(k)}$$

subject to the linearized constraints

$$\mathbf{A}_{eq}^{(k)T}\delta\mathbf{x}^{(k)} + \mathbf{c}^{(k)} = \mathbf{0} \tag{2.42}$$

$$\mathbf{A}_{ineq}^{(k)T}\delta\mathbf{x}^{(k)} + \mathbf{c}^{(k)} \geq \mathbf{0}$$

For given initial estimates $\mathbf{x}^{(1)}$ and $\Lambda^{(1)}$, problem (2.42) can be solved to obtain $\delta\mathbf{x}^{(k)}$, and $\Lambda^{(k+1)}$ is the vector of Lagrange multipliers of the linear constraints. Update $\mathbf{x}^{(k)}$ according to $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta\mathbf{x}^{(k)}$ and continue this iterative process until convergence is reached. Thus, the nonlinear programming problem as defined by equation (2.1) can be solved by iteratively solving the quadratic subproblem defined by equation (2.42). This method of solving the nonlinear constrained programming problem is known as the sequential/successive quadratic programming (SQP) method or recursive quadratic programming (RQP) method.

A major disadvantage of the method is that second order derivatives of all the constraints and the objective function must be evaluated to construct the Hessian matrix of the Lagrange function. However, a major breakthrough was made when it was discovered that this matrix can be approximated by using only first order information [76HA1m, 77HA1m, 78PO1m, 78PO3m]. The idea is similar to that used for Hessian matrix updating in the quasi-Newton methods for unconstrained optimization. Both the DFP and the BFGS updating methods have been suggested. It is sometimes preferred to use the direct updating method given by the BFGS method. It is important to note that the updated Hessian should be kept positive definite, because then the resulting quadratic subproblem remains strictly convex. The standard BFGS updating method can lead to an indefinite or singular Hessian. Powell [78PO1m] suggested a modification to the standard formula to overcome this difficulty so that the solution to the subproblem is always well defined.

The derivation of QP based methods is based on conditions that hold only in a small neighborhood of the optimum and hence the significance of the quadratic subproblem is questionable when the current iterate is far from $\mathbf{x}^*$ (Λ is far from Λ^*). SQP methods converges locally at second order. In order to ensure that $\mathbf{x}^{(k+1)}$ is a better point than $\mathbf{x}^{(k)}$, the solution of the quadratic subproblem can be interpreted as a search direction. The next iterate is defined by equation (2.14), where $\mathbf{s}^{(k)}$ is the solution to the quadratic subproblem and $\alpha^{(k)}$ is a step length chosen to yield a sufficient decrease in a merit function Ψ that measures progress towards $\mathbf{x}^*$. Typically, a merit function is a combination of the objective and the constraint functions. Several different choices of merit functions have been used, namely quadratic penalty functions, exact penalty functions, and augmented Lagrangian functions [81GI1m, 87FL1m, 89GI1m]. The most successful implementations of the SQP methods use a modified BFGS updating method and either the L_1-exact penalty function or the augmented Lagrangian function. SQP methods are widely regarded as the most effective general methods of solving constrained NLP problems. These methods perform well in practice, even on problems that were formerly regarded as difficult.

Sequential linear constrained methods [89GI1m] minimize the Lagrange function subject to the linearization of the nonlinear constraints. A quadratic model of the Lagrange function is not constructed as in the case of the SQP methods, but the objective function is a general approximation to the Lagrange function. Thus, the subproblem involves the minimization of a general nonlinear function subject to linear constraints. These methods are widely used for large scale problems, because general purpose techniques for solving large scale linear constrained problems are better developed than general methods for large scale quadratic problems. However, issues like global convergence proofs, the definition of the objective function, the use of merit functions and the detection of inefficient computations are not yet satisfactorily solved.

There exist other methods to solve NLP problems as well. Sequential linear programming (SLP) methods [81GI1m, 84VA1e, 89AR1e] are based on the idea of linearizing (Taylor series) both the objective and the constraint functions about the current $\mathbf{x}^{(k)}$ and then to solve the resulting LP subproblem by means of efficient LP methods. This process is repeated until the optimum solution is achieved. The main difficulty of this method is that the solution of the LP subproblem will lie at a vertex of the linearized constraint set. This method can also give rise to non-feasible solutions and is recommended only when the curvature effects are negligible. For highly nonlinear programming problems these methods converge slowly and become unreliable. It is therefore recommended to rather use SQP methods that contain curvature information and possess excellent convergence properties.

Feasible direction methods [81GI1m, 84LU1m, 84VA1e, 87FL1m, 89AR1e] are based on the concept of moving from one feasible solution to an improved feasible solution. The feasible search direction that is calculated from the solution of a linear or quadratic programming subproblem reduces the value of the objective function and remains strictly feasible for a small step size. The disadvantages of this method are that a feasible starting point must be known and equality constraints are not easily implemented. However, these methods will work well in interactive mode [85BE1e].

Gradient projection [81GI1m, 84LU1m, 84VA1e, 87FL1m, 89AR1e] methods were developed in order to calculate the search direction without solving a linear or quadratic subproblem. The search direction vector is obtained from an explicit expression and is much easier to compute than in the case of the feasible direction methods. Non-feasible points are corrected by a series of correction steps to move back into the feasible region. Numerically, the method does not perform well and can be very inefficient, but is well suited for an interactive process [85BE1e, 89AR1e].

Reduced gradient methods [81GI1m, 84LU1m, 84VA1e, 87FL1m, 89AR1e] are based on variable elimination in that some variables are eliminated by using the constraints currently satisfied as equalities. The problem then becomes unconstrained in the remaining variables. The search direction is found, such that for any small move the current active constraints remain precisely active. If some active constraints are not precisely satisfied due to the nonlinearity of these functions, the Newton-Raphson method is used to return to the constraint boundary. Both equality and inequality nonlinear constraints can be treated. There is a definite relationship between reduced gradient methods and gradient projection methods [81GI1m, 85BE1e]. Special algorithms must be used to handle arbitrary starting points to obtain a feasible starting point in the feasible direction, gradient projection and the reduced gradient methods.

There are many other types of optimization solution techniques that were developed to address certain problem characteristics. Here are some examples:

(1) Integer programming is the study of optimization problems in which some of the variables are required to take on integer values.

(2) Network programming treats network flow problems such as encountered in many real life physical processes.

(3) Geometric programming is a nonlinear programming technique that deals with problems whose functions are all generalized polynomials. Its development was stimulated by the development of minimization techniques for engineering design.

(4) Non-differentiable optimization optimizes a function which fails to have derivatives for some values of the variables. Standard differentiable calculus must be replaced by methods that treat these problems effectively.

(5) Multiobjective (multicriterion) optimization deals with multiple criteria optimization problems.

(6) Stochastic programming considers problems in which some degree of uncertainty exists about the values assigned to certain parameters and these are given a probabilistic representation.

These special classes of problems will not be discussed in detail and the interested reader can consult Fletcher [87FL1m] and Nemhauser et al. [89NE1m] and the numerous references therein to obtain further information.

Duality

The concept of duality occurs widely in constrained mathematical programming [84LU1m, 84VA1e, 87FL1m]. The purpose of duality is to provide an alternative formulation of a mathematical programming problem which either has computational advantages or theoretical significance. Dual methods do not attack the original constrained problem, referred to as the primal problem, directly but instead attack an alternate problem, the dual problem, whose unknowns are the Lagrange multipliers. At the dual solution point the values of the multiplier vector Λ^* are associated with the solution of the primal problem, $\mathbf{x}^*$. The dual problem is thus related to the optimality conditions of the primal problem. The objective function used in the dual methods are related to the Lagrangian function (equation (2.7)).

Any function $f(\mathbf{x},\mathbf{y})$ is said to have a saddle point at $(\mathbf{x}^*,\mathbf{y}^*)$ if the following condition holds:

$$f(\mathbf{x}^*,\mathbf{y}) \leq f(\mathbf{x}^*,\mathbf{y}^*) \leq f(\mathbf{x},\mathbf{y}^*) \quad \text{for all } (\mathbf{x},\mathbf{y}) \tag{2.43}$$

The function $f(\mathbf{x},\mathbf{y})$ exhibits a minimum with respect to $\mathbf{x}$ and a maximum with respect to $\mathbf{y}$. In the case of the Lagrangian function, the point $(\mathbf{x}^*,\Lambda^*)$ is known to be a saddle point representing a minimum with respect to $\mathbf{x}$ and a maximum with respect to Λ. If it is assumed that the Hessian of the Lagrangian function is positive definite at the solution of the primal problem, $\mathbf{x}$ can be expressed in terms of Λ in the neighborhood of $(\mathbf{x}^*,\Lambda^*)$. Thus we can define the Lagrangian function in terms of Λ alone as

$$\hat{L}(\Lambda) = \min_{\mathbf{x}} L(\mathbf{x},\Lambda) \tag{2.44}$$

for a fixed Λ and subject to no constraints. But $\hat{L}(\Lambda) \leq d$ for Λ in the neighborhood of Λ^*, where d is the minimum of the primal problem stated in equation (2.1). Thus, Λ^* is the solution of the problem

$$\begin{aligned}&\underset{\Lambda}{\text{maximize}}\ L(\mathbf{x},\Lambda)\\ &\text{subject to}\\ &\nabla_{\mathbf{x}} L(\mathbf{x},\Lambda) = 0 \text{ and } \lambda_i \geq 0, \quad i = m_{eq}+1,\ldots m\end{aligned} \tag{2.45}$$

This formulation is known as the dual optimization problem and the simple bounds on the variables make it easier to solve. In the case of the QP problem stated in equation (2.37), its dual can be written as ($\mathbf{A}$ contains the constraint normals of both the equality and the inequality constraints)

$$\begin{aligned}&\underset{\mathbf{x},\Lambda}{\text{Maximize}}\ 0.5\mathbf{x}^T\mathbf{G}\mathbf{x} + \mathbf{g}^T\mathbf{x} - \Lambda^T\left(\mathbf{A}^T\mathbf{x} - \mathbf{b}\right)\\ &\text{subject to}\\ &\mathbf{G}\mathbf{x} + \mathbf{g} - \mathbf{A}\Lambda = \mathbf{0}\\ &\lambda_i \geq 0, \quad i = m_{eq}+1,\ldots,m\end{aligned} \tag{2.46}$$

or

$$\begin{aligned}&\underset{\Lambda}{\text{Maximize}}\ -0.5\Lambda^T\left(\mathbf{A}^T\mathbf{G}^{-1}\mathbf{A}\right)\Lambda + \Lambda^T\left(\mathbf{A}^T\mathbf{G}^{-1}\mathbf{g} + \mathbf{b}\right) - 0.5\mathbf{g}^T\mathbf{G}^{-1}\mathbf{g}\\ &\text{subject to}\\ &\lambda_i \geq 0, \quad i = m_{eq}+1,\ldots,m\end{aligned} \tag{2.47}$$

Furthermore, the minimum of the primal and the maximum of the dual function values are equal at $(\mathbf{x}^*,\Lambda^*)$. If one is able to solve the primal problem, then the dual variables Λ^* can be retrieved. Similarly, it may be possible in some cases to solve the dual problem and then retrieve the optimum primal solution, $\mathbf{x}^*$. The theory regarding duality is locally as well as globally (convex programming problems) applicable. Duality cannot be used as a general purpose solution technique and great care must be exercised, because in some cases the dual problem may have a solution from which one is not able to retrieve the primal solution. Dual methods have been used extensively in LP problems as a very efficient solution technique and are also used in QP and general NLP problems.

Closing remarks

Many methods and their variations for constrained optimization have been developed and evaluated. References such as Gill et al. [81GI1m], Dennis and Schnabel [83DE1m], Luenberger [84LU1m], and Fletcher [87FL1m] should be consulted.

The preceding discussion attempts to give a general overview of the basic mathematical programming techniques (optimization) and the theory involved in order to develop some understanding for the relative merits of the methods. The reader is referred to the references stated for more information on the theoretical development as well as the various modifications and refinements to ensure efficiency and numerical stability of these methods for practical use.

There is no one optimization technique that will solve all problems satisfactorily. Various methods exist which have all been modified in some or other way to take advantage of some problem characteristic or to avoid some unacceptable behavior in order to create more efficient solution techniques. Optimization problems can be categorized into different standard problem types. Solution algorithms can be formulated for each of these. It is the user's task to determine into which category his/her problem fits, and then to apply the appropriate optimization subroutine. Therefore one must have a basic understanding of the underlying principles of the various methods available.

2.3 APPLICATION OF OPTIMIZATION IN ENGINEERING

Many scientific and engineering problems can be posed in terms of optimization, i.e. seeking the optimum value of an objective function by varying some parameters. The regular use of optimization to solve practical problems started during and after World War II and was made possible by the creation of computer technology and the development of the simplex method to solve LP problems. LP is still one of the most widely used optimization techniques.

With the development of the computer technology, researchers were able to solve problems that were previously regarded as theoretically interesting, because of the amount of computational power necessary to solve them. Enormous progress has been made over the years in developing efficient optimization algorithms for solving nonlinear optimization problems and to enhance the performance of existing ones. Furthermore, these developments have led to the solution of real-world problems that were previously regarded as "unsolvable". The quasi-Newton methods for unconstrained optimization problems and the SQP methods for nonlinear optimization problems form part of the success story. In spite of all these developments, there are still problems that are intractable and cannot realistically be solved today [91WR1m].

As more and more of the optimization algorithms became known to the engineering and scientific communities, they were applied or adapted to exploit the structure of particular problems. The successful solution of real-world problems motivates problem formulators to seek and solve larger and more complicated problems in the same or closely related areas. The application of optimization in engineering is a very dynamic and relevant research topic as will be subsequently discussed. Several publications cover the theoretical development and the applications of optimization algorithms, for example, Mathematical Programming, Journal of Optimization Theory and Applications, SIAM Journal on Control and Optimization, Engineering Optimization, International Journal for Numerical Methods in Engineering, Journal of the Operations Research Society, Computers and Chemical Engineering, and many more.

Engineering versus mathematical optimization

Engineers and mathematicians view optimization in different ways. Mathematical optimization assumes that a problem can be formulated in formal mathematical terms and much of the theoretical and algorithmic development concentrates on methods for locating the solution as accurately and efficiently as possible. Engineers look from a very problem orientated viewpoint in which they require sensible, reliable and economic designs or plans to fulfill functional requirements. Engineers often experience difficulties in formulating the real-world problems in precise mathematical terms (equation (2.1) refers). The proper formulation (mathematical transcription) of a problem is an important step in the optimization of any system. The problem formulation process requires identification of the decision variables, an objective function and the constraints of the system for its reliable performance. It is critically important to formulate a problem by properly modeling the physical system. If the model of the system is inaccurate or its formulation is incorrect, the optimization process can lead to strange results, or no results at all.

Once the problem is transcribed into the form stated in equation (2.1), it can be solved by means of the numerical optimization methods available. Even if this is possible, it may sometimes occur that the problem formulation does not fit into one of the categories studied by mathematicians. The engineer is more than willing to accept a good approximation for the optimum as long as it can be obtained without too much effort. To perform engineering optimization in a very effective way, the engineer needs a solid background in engineering fundamentals, matrix algebra, a basic knowledge of numerical optimization algorithms (underlying principles) and computer programming. Textbooks, like those published by Leitmann [62LE1e, 67LE1e], Fox [71FO1e], Haug and Arora [79HA2e], Reklaitis et al. [83RE1e], Vanderplaats [84VA1e], Edgar and Himmelblau [88ED1e], Arora [89AR1e], provide valuable background on the principles that govern engineering optimization. The formulation of the optimization problem and the scrutinizing (verifying) of the results are the links between mathematical optimization and engineering.

The application of optimization in engineering is not limited to specific numerical optimization methods, but covers the whole spectrum of linear programming, constrained and unconstrained nonlinear programming, network programming, integer programming, multiobjective optimization, geometric programming, and stochastic programming just to name a few. Although engineering optimization is basically concerned with the application of existing techniques, new techniques have been developed that originate directly from the physical sciences, e.g. geometric programming [80EC1e, 85RI1e]. Optimization methods are fundamentally interdisciplinary in nature and one cannot in general dedicate a specific method to a specific engineering discipline. Some methods, however, are better suited to specific problems than others.

Engineers are sometimes confronted with problems or decisions in which more than one objective must be satisfied [83CL1e]. Multiobjective optimization methods are used to make the right decision in these conflicting situations. The theory and application of multiobjective optimization and decision making are discussed in detail in Osyczka [84OS1e] and Reeves and Lawrence [92RE1e]. Due to its relevance, multiobjective optimization and decision making is a very active research topic and the interested reader should consult the above-mentioned references and the references contained therein for further information.

Numerical optimization techniques have been applied widely to solve a great number of different engineering problems in the various application fields of design, planning and control.

Optimal design

Engineering design has emerged as a major field of application for numerical optimization techniques. Numerical optimization is to provide a computer tool to aid the designer in his/her task. In the past the conventional design process has more than once lead to uneconomical designs. The designer was able to freely use his experience and intuition in making decisions. The optimum design process forces the designer to identify a set of design variables, an objective function to be optimized, and constraint functions for the system and as such gain a better understanding of the problem. Human interaction is an essential part of the optimum design process and it is performed in a more formal and organized way than in the conventional design process. The design process is more organized, using trend information in a very systematized way. Computers play a very important role in the optimal design process, introducing more accuracy, efficiency and allow us to understand the behavior of complex systems.

The basic principles of optimal design are discussed in, for example, Avriel et al. [73AV1e], Vanderplaats [84VA1e], Arora and Thanedar [86AR2e] and Arora [89AR1e] and various design applications are illustrated. Major contributions in the field of optimum design originate from structural optimization, process equipment design, and mechanical design. A wide variety of design applications (in various engineering disciplines) to illustrate the integration of optimization as part of the design process can be found in, for example [71FO1e, 73AV1e, 78WI1e, 79AV1e, 79HA2e, 83RE1e, 84BE1e, 84VA1e, 85BE1e, 85GE1e, 86AR1e, 86AR2e, 86LI1e, 86TH1e, 88AR1e, 88ED1e, 88TS1e, 89AR1e, 91PE1e, 92BI1e]. The articles and textbooks give useful hints to assist the designer when faced with a practical design optimization problems. It is clearly illustrated that an efficient design process should allow for the designer's experience, intuition and creativity to go hand in hand with the optimization technique.

Optimal control

The optimal control problem is to find a time dependent feedback control function that assures optimal behavior of a system during its dynamic performance. The system has active elements that sense change in the output and the system controls get automatically adjusted to correct the situation and optimize a measure of performance. On the other hand, optimal design concerns the design of a system and its elements in order to optimize an objective function; the system then remains fixed for its entire life. The solution of optimal control problems relies heavily on the theory of calculus of variations and optimal control theory [62LE1e, 67LE1e, 79HA2e, 87FR1e]. Optimal control theory considers problems that cannot be solved by the calculus of variations.

Optimal control problems can also be transformed into optimum design methods and solved by the same numerical optimization methods [89AR1e, 92BI1e]. Control problems can be found in all the different engineering disciplines and were particularly relevant in many of the complex military and space programs that were initiated during the 1960's [87FR1e]. Optimal control problems are experienced in aerospace systems (e.g. trajectory optimization) [62LE1e, 67LE1e, 87HA2e, 93BE1e], process engineering (e.g. reactor design and nonsteady-state processing) [92BI1e], Operations Research and Economics (e.g. economical planning) [87FR1e], and the dynamic response control of both mechanical and structural systems (e.g. precision control of machines, control of structures under wind loads and earthquakes etc.) [79HA2e, 89AR1e].

Optimal planning

Optimization techniques play a vital role in effective planning, scheduling, and control of projects and production processes. The processes of planning, control and scheduling are in themselves optimum-seeking processes. The efficient use and proper scheduling of expensive human resources and equipment results in creating more with less input, resulting in great economical benefit [91GL1e]. Plant layout, plant and warehouse location, product distribution, operations planning, production and working schedules are just a few examples of the problems that can be addressed. Special numerical techniques have been developed to aid management to perform planning and control tasks in a more effective manner [92HO1e]. The role that optimization plays in these activities is illustrated both theoretically and practically in [87FR1e, 91HU1e, 92HO1e]. Clearly, the logical approach of mathematical optimization is very useful when applied to the many rational problems in decision-making.

Algorithms and software

The application of optimization in engineering needs efficient and robust algorithms. Researchers in the area of mathematical programming are constantly working on this aspect to develop and implement new algorithms and to improve old ones. Both general-purpose and specialized algorithms (and software) are needed to aid engineers in performing optimal design, control and planning. Software development for engineering applications is, however, lagging behind the mainstream of mathematical programming development.

Special-purpose algorithms and software are mostly needed to solve real-world problems. The various engineering disciplines have specialized software to address their specific needs [84VA1e, 88ED1e, 89AR1e]. Some general purpose design optimization software is becoming available, e.g. ADS (Automated Design Synthesis - Vanderplaats [86VA1e]) and IDESIGN (Interactive Design Optimization of Engineering Systems - Arora [88AR1e, 89AR1e]). Some other codes are discussed by Reklaitis et al. [83RE1e], Belegundu and Arora [85BE1e], and Edgar and Himmelblau [88ED1e].

Software that implements the various optimization algorithms discussed in section 2.2 is readily available. Some general-purpose libraries are available that contain subroutines implementing these. Examples are the NAG-library, IMSL-library and the Harwell-library. Furthermore, these libraries also contain the implementation of various other mathematical routines, e.g. linear algebra. The use of the general-purpose libraries is the only resort when specialized software to perform the task is not available. The user is responsible for the proper interaction between engineering problem formulation and modeling and the mathematical solution process. The main program and the other routines written by the user can become very complex and are prone to errors, so extreme care must be exercised during program development. This method of applying optimization to engineering requires that the user has some knowledge of the algorithm in use. Guide-lines on use of general-purpose software are discussed in Arora [89AR1e].

Artificial intelligence

The use of artificial intelligence (AI) in engineering design, planning and control is a fairly new development. Artificial intelligence concentrates on emulating human reasoning in order to solve problems. AI algorithms analyze the methodology of how a human being solves a problem and translates the thought process to the computer. The computer then approaches the human reasoning process to solve the problem in contrast to executing an ordered set of instructions as found in traditional computer programming. Expert systems (knowledge-based systems), neural networks, and fuzzy logic are alternative approaches within the artificial intelligence field. Artificial intelligence can couple the reasoning and judgment of the human mind with the power, speed and memory of the computer to solve various engineering problems. The general application of artificial intelligence to engineering problems is discussed in Arora and Baenziger [86AR1e], Winstanley [91WI1e], Gero [92GE1e], Rowe [92RO1e] and the references cited therein. The application of artificial intelligence in engineering is of great importance, especially the fact that the decision-making power of an expert can be emulated to perform optimal tasks.

Closing remarks

The various numerical optimization techniques present the engineer with a means to determine the best possible solution on a theoretical basis. Intangible factors and practical considerations may change the final recommendation to other than the calculated optimum. It is up to the engineer to apply proper judgment to take into account these practical factors. Thus, theoretical and economic principles must be combined with the understanding of the practical problems that will arise.

The preceding discussion presents just a few examples to illustrate how optimization techniques can be implemented in the engineers' never-ending search for improved ways of doing things. The references cited in this section are intended to serve as an introduction into the field of the application of optimization in the different engineering disciplines. Substantial literature exists for future education in this topic and more research and knowledge than presented here is required to fully understand these topics . The references contained in these books and articles can be used as further sources of information.

2.4 OPTIMAL DESIGN OF DRY-COOLING SYSTEMS AND AIR-COOLED HEAT EXCHANGERS

In this section literature relevant to the optimal design of dry-cooling systems and air-cooled heat exchangers are reviewed. The discussion on dry-cooling system optimization will be cast primarily in the context of power plant cooling to coincide with the illustration of the engineering optimization application of the present investigation.

Irreversibility and Thermoeconomic design optimization of heat exchangers

The second law of Thermodynamics involves the fact that real processes proceed in a certain direction and not in the opposite direction due to the thermodynamic irreversibilities that exist. London [82LO1e] gives a list of irreversibilities that exist in energy conversion systems. The second law further leads to a property, called entropy, which enables us to treat it quantitatively in terms of the operating conditions for processes. Bejan [89BE1e] states that if we are serious about constructing efficient energy systems and conserving energy, we have no other choice but to design for less and less entropy production.

The basic design problem is to determine the thermodynamically optimum size or operating regime of a certain engineering system, where by optimum we mean the condition in which the system destroys the least exergy (useful energy) while still performing its fundamental engineering function [87BE1e, 89BE1e]. The destruction of exergy is intimately tied to the generation of entropy in the various components of a system. The loss of exergy, or irreversibility, provides a generally applicable quantitative measure of process inefficiency. Analyzing a multi-component plant indicates the total plant irreversibility distribution and interaction among the plant components, pinpointing those contributing most to overall plant inefficiency [85KO1e, 87BE1e, 89TS1e]. A solid understanding of the mechanism of entropy generation in each of the systems components is essential in setting out a strategy for decreasing entropy generation by the entire system [89BE1e].

Different criteria based on the exergy concept are commonly used for the performance evaluation of heat exchangers. These include irreversibility generation minimization analysis [82BE1e, 87BE1e, 89AC1e, 89RA2e] and thermoeconomic analysis [82LO1e, 83LO2e, 85KO1e, 89RA2e, 89TS1e]. These criteria are used to define objective functions used for optimization of heat exchangers. The traditional irreversibility generation based objective function is simply expressed in terms of the irreversibilities occurring in the heat exchanger, while the thermoeconomic objective function is obtained by combining the pricing of the penalties of the thermodynamic irreversibilities with the capital costs.

The irreversibility of any heat exchanger is basically due to two factors, namely the transfer of heat across a stream-to-stream temperature difference and the pressure drop that accompanies the circulation of fluid through the apparatus [78BE1e, 82BE1e, 85KO1e, 89AC1e, 89RA2e]. The above-mentioned irreversibilities can systematically be reduced by slowing down the movement of the fluid through the heat exchanger. This technique is synonymous with employing larger heat exchangers. Thus, in order to build thermodynamically efficient heat exchangers, one has to build large units. Large heat exchangers, however, require large amounts of materials that, in order even to be produced, require large amounts of exergy for consumption during the manufacturing process. It is clear that an optimization program must include the exergy losses associated with both the operation of the heat exchanger and those invested (capital) in the hardware [82BE1e].

The thermoeconomic objective function is a combination of the capital cost and the costs associated with the heat exchanger irreversibilities (running/operating costs). The purpose of thermoeconomic optimization is to achieve, within a given system structure, a balance between expenditure on capital costs and exergy costs which will give a minimum cost of the plant product [85KO1e]. This objective function is preferred for commercial heat exchanger design and the global optimum yields a realistic result of a finite area heat exchanger.

Conventional heat exchanger optimization seeks the optimum trade-off between the capital costs and the operating costs for the entire system. Most thermoeconomic methods simplify the search for an optimum by making these trade-offs at system component level. The fundamental idea that justifies the irreversibility minimization at system component level is that the overall entropy generation rate of a system is the sum of all the system's components contributions [87BE1e]. The conventional design process can be seen as thermoeconomic optimization on a global scale, because the objective function tries to minimize the combination of capital and operation costs (minimum material consumption, minimum energy consumption). Contrary to thermoeconomic optimization, the real causes and sources of costs due to exergy destruction are not as clearly defined through the conventional design optimization process. Thermoeconomic design optimization indicates the optimal thermodynamic efficiency of a component from a cost viewpoint and therefore helps the designer to create improvements, both thermodynamically and economically.

Air-cooled heat exchanger optimization

Air cooling made a strong bid for application in the refining, petrochemical and natural gas industries during the 1950's when it was realized that air can be a more economical coolant, even in locations where water is plentiful. The design of air-cooled heat exchangers, especially during the 1950's and the 1960's, was mainly based on rules of thumb obtained from the recorded operating experience of existing installations or designers' intuition. For example, the average design face velocity for air-cooled heat exchangers was regarded as between 2 m/s and 3.5 m/s, and heat exchangers were usually equipped with four tube rows [59KE1e, 60RU1e, 66LO1e, 66LO2e]. However, it was realized that by changing the traditional design configurations and operating conditions, great economical advantages were possible due to the trade-off that exist between capital and operating costs [59KE1e, 59NA1e, 66LO1e, 66LO2e, 66SC1e, 66SC2e].

Commercially available equipment (e.g. finned tubes, fans) were often used in the search for the optimum air-cooled heat exchanger. As a result of this practice, the dimensions of the equipment were not optimized, but the configuration instead [66LO1e, 66LO2e]. The use of this commercially available equipment simplified the optimization process to a great extent. The techniques used in the search for the optimal heat exchanger design were mainly mathematical techniques (e.g. differential calculus - setting the first derivative equal to zero) [57FA1e, 59KE1e] or graphical presentations of the trade-off between heat transfer and fluid flow relations [59NA1e, 66LO1e, 66LO2e, 66SC2e, 69JE1e] . Schoonman [66SC1e] presented a more advanced approach by using a computer program to perform a cyclic variation of three variables, namely number of tube rows, fin spacing and fan power, through several combinations to find the optimum design. Although these approaches tend to limit the number of relevant parameters due to practical considerations or in order to simplify the problem, they can be seen as the forerunner of the more modern and complex optimal analyses that followed.

More sophisticated search techniques were used to locate optimal heat exchanger designs. Peters and Nicole [72PE1e] discuss the use of factorial searches of all the possible combinations of variables and come to the conclusion that they become unsatisfactory as the number of variables increases. Due to the large number of discrete variables encountered in air-cooled heat exchanger design (due to the use of commercially available equipment), they favored the use of heuristic algorithms (starting close to the optimum) to perform the optimal design process, specific to the equipment under consideration. Palen et al. [74PA1e] propose the application of the Box Complex Method [65BO1e] of systematic search for the optimal design of heat exchangers. Various aspects of optimal heat exchanger design (e.g. the criteria for the objective function and design variables) are discussed. Previously, the investment was related to the heat exchanger area only, but Palen et al. [74PA1e] proposed the calculation of the investment from the dimensions of the component parts and the manufacturing costs. Fontein and Groot Wassink [78FO1e] utilized the Simplex Method [65NE1e] and a steepest decent method to optimize a shell and tube heat exchanger by using the NTU-method (number of transfer units) [86HO1e].

Shah et al. [78SH1e, 81SH2e] recognize the heat exchanger optimization problem as one suitable for nonlinear programming (optimization) techniques. Several numerical optimization techniques (e.g. conjugate gradient methods, penalty function methods) are referred to in the paper as they have been developed specifically for computer application and are thus suitable for implementation on these design problems. Once the problem is transcribed into formal mathematical terms, the optimization method most suited to the specific problem formulation can be used. The authors suggest that a computer program containing several optimization methods and a heat exchanger performance analysis routine should be used for design purposes, as no single optimization method will be well-suited to all the problems. A heat exchanger optimization methodology is outlined and the process is illustrated by means of an example (also refer to [81SH1e, 88SH1e]). This paper is indeed a very valuable contribution to promote the use of nonlinear programming techniques in optimal heat exchanger design.

Kröger [79KR1e] presents a method to determine the cost optimized dimensions and operating conditions for circular finned heat exchanger tubes by maximizing the performance over cost ratio. The general practice in the previous optimal design processes of air-cooled heat exchanger was to choose a surface (described by its tube pitch, fin pitch, fin diameter, outside tube diameter and fin thickness) beforehand and use its experimentally obtained performance correlations. Kröger, however, uses the general relations for the heat transfer and pressure drop during flow across finned tube arrangements, expressed in terms of the various geometrical and flow parameters, as found in literature [45JA1e, 63BR1e, 66RO1e, 66VA1e, 74MI2e]. The optimization procedure takes into consideration all the parameters that may affect the capital and operating cost of the heat exchanger and does not prescribe any limits, other than those absolutely essential due to practical limitations. A specific capital and operation cost structure for the finned tubes is assumed and an analytic objective function is obtained that expresses the performance over cost ratio in terms of the various geometrical and operating parameters. The design method is illustrated with a practical example and the objective function maximization is performed by computer. The results of this study show that the optimum design and operating conditions differ considerably from conventional designs. For example, the fin thickness is found to be much less than that found in practice. Similarly, the stream velocities are also lower than those found in industrial applications. In part this is due to the cost structure chosen. However, the method illustrates trends in the search for optimum geometrical and operating parameters.

A similar procedure (refer to [79KR1e]) is proposed by Kröger [83KR1e] for the optimal design and operating conditions of a plate fin air-oil heat exchanger with tape inserts inside the tubes. Literature correlations are also used for the heat transfer and pressure drop calculations. An industrial air-oil heat exchanger is tested experimentally in order to verify the accuracy of the heat transfer and pressure drop equations employed and for the purposes of comparison. The results of the performance over cost ratio maximization again indicate geometrical and operating parameters that differ from those found in industrial applications.

Hedderich et al. [80HE1e, 82HE1e] developed a computer code for analysis of air-cooled heat exchangers and coupled it to a numerical optimization program (constrained function minimization) to obtain an air-cooled heat exchanger optimization and design procedure. The optimization program is based upon the method of feasible directions and the Augmented Lagrangian Multiplier Method [84VA1e] that can be used for different optimization applications. They used the heat transfer and pressure drop correlations provided by Briggs and Young [63BR1e] and Robinson and Briggs [66RO1e] which enabled them to vary the finned tube geometrical parameters in order to find the optimum heat transfer surface. The developed code has the capability to design for nine different configurations of triangular pitch banks of finned tubes. Air-cooled heat exchangers can be optimized according to the following objective functions: minimum volume, minimum heat transfer surface area, minimum fan power, minimum airside pressure drop, and minimum tubeside pressure drop. No economically based objective functions were considered in this study.

Pribis [81PR1e] describes the combination of heat exchanger design programs and optimization routines. The optimization routines are based on gradient search methods. Various surfaces can be accommodated in the program, e.g. bare tubes, plate fins, spine fins, and helical wound fins. The author stresses the fact that the heat exchanger must not be modeled on its own, but must be modeled in the system to get realistic optimization results.

Heat exchangers are used in essentially all the process industries and play a vital role in process design considerations. In their textbooks, Edgar and Himmelblau [88ED1e], and Peters and Timmerhaus [91PE1e] extensively treat the topic of optimal economic design of equipment as found in the chemical process industries. These books gives a clear concept of the important principles of design and economics from a practical point of view. The material presented will help the engineer in heat transfer equipment selection, design and optimization and various examples are given to illustrate the methodology used.

Computer programs for the design, performance evaluation and economic optimization of heat exchangers are extremely useful engineering tools. Most of the more complex optimal heat exchanger designs discussed in this section were obtained in this way. Breber [88BR1e] discuss the main features, such as program capabilities, data input, result output and program logic, required in these type of programs to perform reliable analyses. An overview of currently available programs is presented to aid the engineer in choosing a program for his/her specific requirements. The author concludes that computer programs cannot substitute engineering judgment which is needed in problem formulation and scrutinizing of the results. Taborek [91TA1e] discusses the use of expert systems in heat exchanger design which aims to establish more user interaction during the design optimization process.

Optimum industrial heat exchanger design differs from the thermodynamic design in that several additional criteria, such as demands and restrictions imposed by the interrelated design considerations, must be satisfied. The heat exchanger designer must obtain a global optimum within all the demands and restrictions with respect to initial and operating cost, reliability, manufacturability, and maintenance ease.

The literature cited in the survey above are by no means complete and aim to give an overview of the different optimization algorithms and analyses models used in the design of optimal air-cooled heat exchangers.

Dry-cooling system optimization

Between 1970 and 1980 major engineering economic studies of dry-cooled electrical generating plants were conducted in the USA with the object of minimizing the cost of electricity generation when dry-cooling towers are coupled to a turbo-generator [70RO1e, 75MI1e, 76FR1e, 78RO1e, 79CH2e, 79HA1e, 79NA1e]. Factors that affect the design and optimization of dry-cooled power plants are discussed in Miliaras [74MI1e]. Both natural and mechanical draft indirect dry-cooling towers are considered in these studies. At that time dry-cooling was successfully utilized in a number of steam-electric generating plants in Europe and the concept was relatively new to the USA electric generating plants [70RO1e]. Up to then, dry-cooling in the USA was extensively used in the process industries. The need for an alternative means of cooling was to promote water conservation through the industrial use of dry-cooling and to conform with the environmental regulations. Most of the studies compared dry-cooled steam-electric plants to wet-cooled plants in terms of the increased power generation costs. Dry-cooling towers have the inherent characteristic of performance degradation during high ambient temperature and subsequently the plant will suffer production losses during these conditions (lost performance). In the USA most of the electricity generating plants are privately owned and thus cannot afford production losses. The concept of lost performance is discussed in detail in section 4.7. Dry-cooling received a bad reputation and was sometimes severely penalized as a result of this inherent characteristic.

Due to the complexity of dry-cooling system economics, several assumptions are introduced into the reviewed studies to simplify the optimization problem. The optimization of some plant and cooling system operating parameters and/or the general cooling system is conducted. However, in most of the studies, optimization consists of matching components effectively. These particular studies are for state-of-the-art cooling systems and therefore use vendor data and fixed designs for the available dry-cooling tower equipment. This approach results in the use of ± 4–6 decision variables for the optimization process. Ard et al. [76AR1e] developed detailed costs and cost algorithms for dry-cooling systems. The equipment cost are expressed in terms of their dimensions or operating variables. These cost relationships present a means of varying equipment dimensions continuously and not discretely as is permitted by the use of standard equipment. Choi and Glicksman [79CH2e] used these cost algorithms in their optimization study.

In the above-mentioned studies, the basic underlying principle of the optimization process is the same. First, a plant location (weather data), a turbo-generator (performance characteristics) and an initial temperature difference (difference between the steam and the inlet ambient air temperature to the cooling tower) are chosen. The dry-cooling tower, that minimizes the cost of electricity generation throughout the year, is determined by varying the different decision variables that describe its geometry and operation. The cost of electricity generation is made up of the capital cost, operating cost (including fuel cost), and the cost due to lost performance. The process is repeated for all the initial temperature differences to be considered in the analysis. Thus, for each initial temperature difference an optimum dry-cooling system is calculated and the global optimum dry-cooling system is selected from these. Computer programs were written to perform the economic optimum designs. This method of optimization is quite a lengthy process and will not be adequate when many variables enter the optimization process. The Box Complex Method [65BO1e], double-shotgun-and-search method [72AN1e] are just two of the optimization techniques used [78RO1e, 79CH2e].

The goals of these reports were to reduce the cost of state-of-the-art dry-cooling systems and to try and develop new concepts with lower costs (refer to [79HA1e]). They differ mainly in the following aspects: the assumptions made to facilitate the analysis, the treatment of lost performance, cooling system design, plant type, plant location, turbine type, cost modeling, physical modeling, optimization procedure used, and parameters optimized. Most studies have shown that if dry-cooling is used, the cost of electricity will be 10% to 20% higher than when wet-cooling towers are used. The cost is higher due to the higher investment cost (extensive finned tube heat transfer area required) and the poorer plant thermal efficiency (higher turbine backpressure). Valuable information, regarding the advantages and disadvantages of dry-cooling and process modeling, is presented in these reports. The results present general tendencies rather than specific values. The reports indicate that dry-cooling systems require special turbines that can operate at higher back pressure than the conventional turbines that are found in applications where wet-cooling is used. Although the extended surface heat exchanger is the heart of any dry-cooling system, no author tries to optimize it in a rigorous manner. It is not clear from the above-mentioned studies if dry-cooling system equipment has been optimized, either by the vendors or by the studies performed. The coupled effects between the economic approaches, the underlying assumptions, the detailed designs and the overall system designs are not well defined in many respects. A summary of cooling tower selection and optimization as covered by these reports can be found in [85MA1e].

Hauser et al. [71HA1e] discuss the economic optimization of turbine and cooling system combinations. The object of the paper is to describe briefly the parameters and input to be considered during optimization and their effects on the overall system performance. The authors suggest that improved performance, coupled with lower costs, can be obtained in the cooling system design by using techniques to optimize the turbine and cooling system as an entity. The basic requirements for the optimum selection of a turbine and cooling system combination are:

(1) Accurate cost data for equipment, installation, system operation and deficiencies;

(2) Accurate determination of the system performance throughout its entire life;

(3) A comprehensive evaluating technique which can merge cost and performance factors into a single figure of merit, e.g. present worth of the total revenue requirements for a given system over its entire operating life.

An extensive list of input data that is required to perform such an integrated optimization is given and discussed. A sample turbine-cooling system optimization study is presented to illustrate the above principles.

Moore [72MO1e, 72MO2e, 73MO1e, 73MO2e] presents a series of reports on the minimum size of large dry-cooling towers. An analytical minimum size function is presented that relates the flow areas, tower height and fan power to parameters of the heat exchanger, assuming one dimensional flow through the tower. This was done with a large power plant in mind, with a view to minimize the size of a natural draft tower shell, and hence its visual impact and cost; or the fan power of a mechanical draft tower, and hence its cost and noise impact. The results show the importance of heat exchanger design on tower size and various requirements for minimum tower size are directly revealed by the size function (e.g. a very shallow heat exchanger). However, it is not obvious how such requirements could economically and practically be met.

Johnson and Dickinson [73JO1e] perform a similar study for forced draft dry-cooling towers as the one presented by Moore [72MO1e, 72MO2e, 73MO1e, 73MO2e]. Although no cost factors are considered in this analysis, an interrelationship of the parameters that will contribute most significantly in an optimum design is derived. The influence of these parameters on the minimum size and the heat exchanger frontal area is determined.

Andeen and Glicksman [72AN1e] describe a cost optimization procedure to design a dry-cooling tower. They state that the important cost to be optimized is the incremental increase in the cost of power generation, resulting from the use of a dry-cooling tower, over a plant operating at 40% efficiency. The incremental cost is derived from the capital and operating costs of the plant. The optimization problem is reduced to heat transfer and cost equations containing 6 variables and the double shot-gun method is used to solve the problem. The authors state that in order to make dry-cooling towers more economical, their cost must be minimized by optimizing the entire plant. Furthermore, the authors realize that the optimization must not be restricted to the adaptation of predesigned heat exchanger modules, because the module design may not be anywhere near its optimum.

Ecker and Wiebking [78EC1e] investigate the optimal economic design of a natural draft dry-cooling tower with a vertical heat exchanger bundle arrangement. The optimization model is developed by using the relevant physical laws and engineering design relations and transcribing them into the objective function that minimizes the annual cost (fixed charges and operating cost). The problem is then reformulated as a geometric programming problem according to the special mathematical formulation required by this method. The paper is concluded with a numerical example to illustrate the method. Geometric programming is also used in the design and performance optimization of a condenser and wet cooling tower combination as described by Stuart and Arnold [86ST1e].

A design method for optimal economical dry-cooling towers, using the existing literature and standard sourcebooks on heat exchanger design and performance, is presented by Vangala and Eaton [78VA1e]. The combination of all the different information is explained by means of flow diagrams and the method seems very cumbersome.

Montakhab [80MO1e, 80MO2e] published two papers that deal with the factors that affect the size and cost of dry cooling towers and the related heat transfer equipment. He aims to reduce the heat exchanger surface area, the tower size and the pumping power requirements by investigating a means to define suitable heat exchanger geometries and surfaces for application in dry-cooling tower applications.

Li and Priddy [85LI1e] published a textbook on the topic of power plant system design and discuss all the power plant components, economic aspects and design concepts. The topic of mathematical design optimization is also treated and illustrated with an example covering a turbine-cooling system combination. Three phases are identified for a proper optimization study, namely:

(1) System configuration design (specify the different configurations and combinations to obtain the initial costs);

(2) System simulation (determine the performance characteristics and the operating costs for the different configurations/combinations);

(3) Comparative economic evaluations (compare the different configurations/combinations to find the minimum cost).

The methodology presented in the above-mentioned textbook is also discussed and illustrated in the paper by Li and Sadiq [85LI2e].

Electricity generation costs, i.e. capital costs and operation costs, can form a large part of the total electricity cost and therefore emphasizes the importance of optimizing power plant design. Veck and Rubbers [87VE1e] discuss the important role that the cooling system plays in this optimization process and outlines the important choices that have to be made in order to perform an economic evaluation. The economics of evaluating cooling systems are closely related to the characteristics of the demand for electricity. A methodology is presented to determine the optimum size and type of cooling system based on the amount of net energy produced annually. The cooling system must be designed to maintain the turbine backpressure as close as possible to its maximum efficiency, where the electrical output is the maximum.

Buys and Kröger [89BU1e, 89BU2e] use a constrained variable metric method (SQP-method) to perform cost-optimal designs of new or existing (retrofit of heat exchanger bundles) natural draft dry-cooling towers. The SQP method is preferred to other algorithms due to its computational efficiency. Heat transfer and fluid flow relations, expressed in terms of the various geometrical parameters, are used to simulate the performance of the dry-cooling towers. A prescribed cost structure and practical constraints are employed in the problem formulation. The finned tube performance correlations of Briggs and Young [63BR1e] and Robinson and Briggs [66RO1e] are used to simulate the influence of finned tube geometrical parameter variation on their performance. Both articles are accompanied by a numerical example. Sensitivity analyses have also been performed to investigate the influence of the fixed parameters and design correlations on the optimum design. Although the results show geometrical parameters that differ from those usually found in practice, they provide certain trends in the manufacturing and construction of cost-optimal dry-cooling systems.

The optimal sizing of indirect natural draft dry-cooling towers for combined cycle power plants that corresponds to the minimum cost of electricity generation is discussed by Lovino et al. [90LO1e]. The authors identify the strong interaction between the turbine and the cooling system as one of the main reasons for such an optimization study to be performed. Simplified design criteria and cost evaluation techniques are employed, originating from preliminary design data. By considering all possible combinations of the tower heat exchanger and condenser geometry and steam turbine design, the optimization has been performed by a step by step variation of the cooling tower range and approach. Sensitivity analyses have been performed to investigate the influence on the optimal solution of different hypotheses concerning the tower investment and fuel cost. The results show that the optimum dry-cooling tower design is strongly influenced by the turbine characteristics.

Closing remarks

The application of optimization techniques in dry-cooling tower design has been reviewed in this section. Many different methodologies have been discussed; some of them are generally applicable, while others are confined to a specific application under consideration. One of the most important conclusions that can be drawn from this review is that the cooling system has a significant effect on the overall plant economics. The effect on both the investment cost and the plant overall thermodynamic efficiency make it necessary to perform optimization analyses that take the strong interaction between the cooling system and turbine into account (in the case of power plants). The matching between the performance characteristics of the dry-cooling tower type, i.e. natural draft indirect system or mechanical draft direct system, and the performance characteristics of the specific turbine type (differences in blade design and exit area), needs careful consideration [81MO1e, 87KN1e, 87TR1e, 87VE1e, 91SZ1e]. This requires the simulation of the interaction by means of a computer model.

CHAPTER 3

FORMULATION AND MODELING OF THE PERFORMANCE EVALUATION AND OPTIMIZATION PROBLEMS

3.1 INTRODUCTION

Formulation of an optimization problem involves transcribing a verbal description of the problem into a well-defined mathematical statement. The optimum problem formulation process requires the identification of the design variables, an objective function and the constraints imposed on the system for its reliable performance. It is very important to formulate a design problem by proper modeling of the physical system. If the model of the system is inaccurate or the formulation is incorrect, the mathematics of optimization can lead to strange results, or no results at all. Thus, the solution is only as good as the problem formulation and modeling.

The concepts of problem formulation and modeling will subsequently be discussed and applied to the performance evaluation and optimal design of both natural and forced draft dry-cooling towers.

3.2 PROBLEM FORMULATION

The formulation process can be broken down into three well-defined steps [83RE1e, 88ED1e, 89AR1e], i.e.

(1) Identification of the decision variables;

(2) Identification of the objective and expressing it as a function of the decision variables;

(3) Identification of all the constraints and expressing them as functions of the decision variables in order to be transcribed into mathematical expressions.

Each of these steps will subsequently be discussed.

Decision variables

The decision variables describe the system under consideration. There is a minimum number of variables required to formulate the problem properly, otherwise the formulation is incorrect or not possible at all. It should be kept in mind that these variables must be independent of each other as far as possible. Once these variables are assigned numerical values, the design of the system is known. The specified values of these variables must satisfy the imposed constraints in order to arrive at a feasible solution.

Objective function

An objective function is required to represent some criterion to compare the different designs. The objective function must depend on the decision variables (in an implicit or explicit way) that can be varied to achieve its optimum value. The selection of a proper objective function is a very important decision in any design process. The minimum value of single-objective functions will be determined in this study.

Constraints

All the restrictions placed on a design are called constraints. Each of the constraints must be influenced by the decision variables in an implicit or explicit way. Furthermore, these constraints must be independent of each other. Engineering problems may have equality or inequality constraints. Linear programming problems have only linear constraints, whereas general problems have nonlinear constraint functions as well. Both linear and nonlinear constraints are considered in this study.

A feasible design must satisfy all the equality constraints. In some designs, it is possible for some of the inequality constraints to be satisfied as equalities (active inequality constraints), while others remain as inequalities (inactive equality constraints). Although engineering problems have large numbers of inequality constraints, the majority of them are not active at the optimum. Inactive constraints have no influence on the optimum point. The set of active constraints at the optimum point is not known beforehand and must be determined as part of the solution process. The number of independent equality constraints must be less than or equal to the number of decision variables. There is, however, no restriction on the number of independent inactive inequality constraints.

Infeasible problems can be the result of conflicting requirements or inconsistent constraints. When too many constraints are considered, it may happen that there is no feasible solution. Therefore, one must be very careful in formulating a problem. A general mathematical model for optimum problem formulation is discussed in section 2.2. The construction of the mathematical model for a specific engineering problem is not easy in general. It involves a good understanding of the engineering system and reasonable skills in mathematics. Setting up the problem to the point where it can be solved, requires a major part of the total effort involved. Once the problems have been transcribed into mathematical statements using the standard notation, they all look alike. General guidelines for the proper formulation of design problems are described in Arora [86AR2e, 89AR1e]. The problem formulation concepts are illustrated by means of practical examples in, for example Reklaitis et al. [83RE1e], Vanderplaats [84VA1e], Edgar and Himmelblau [88ED1e], and Arora [89AR1e].

3.3 PROBLEM MODELING (SIMULATION)

A major component of any optimization study is system modeling or simulation. System modeling is the process in which the system performance characteristics are determined under various operating conditions. Modeling of the system enable designers to carry out economic and optimization analyses throughout the operating range and life. Existing systems can be improved in this way and a computer simulation of the system will provide the information needed for certain decisions to be made.

A system usually consists of one or more components related to each other to perform one particular task. Before a system modeling or simulation can be performed, a model for each of the components must be available. This model will define the output for a given set of input values. The model can be analytical or determined by suitable experiments. In the analytical approach the physical laws (e.g. conservation principles) are used to develop an analytical equation or a set of equations that will uniquely define the outputs for a set of input values. This is referred to as the equation-based approach. These equations must have certain features, for example differentiability, to be used in conjunction with optimization methods. Otherwise, simplification of these equations will be necessary by means of physical or mathematical approximations. The simultaneous solution of these linear or nonlinear equations can be direct or by means of an iterative procedure. Experimental investigations are usually used to obtain expressions (regression analysis) when the analytical model is complex and cannot be simplified or changed into a desirable form. The analytical approach is frequently used in engineering simulation.

As stated previously, system simulation is the process in which the system performance characteristics are determine under various operating conditions. Each of the system components must be modeled and these models must be combined into an integrated unit. In operation, these components will affect each other in performance. The purpose of the system simulation is to determine the performance of each component in the system environment and the performance of the system as an integrated unit. Most of the simulations encountered in real world problems are of the simultaneous type, characterized by the physical coupling of the various components in the system. In mathematical terms this means that a set of equations must be solved simultaneously. In order to prevent meaningless or misleading results, proper care must be exercised to take all the restrictions placed on the system into account. Restrictions placed on each component of the system are also restricting the system. The system model is simulated by solving the component equations and the related coupling equations. The solution methods will vary from case to case. Some engineering problems require very large analysis models for accuracy in prediction of response. Therefore the evaluation of the objective and constraint functions can require enormous calculational effort.

3.4 APPLICATION OF PROBLEM FORMULATION AND MODELING

The concepts discussed thus far are now applied to the performance evaluation and the optimum design of both natural draft and forced draft dry-cooling systems for use in steam-electric plants.

A mathematical structural characteristic of the performance evaluation and optimization problems considered in this study, is that it involves two distinct classes of variables: the geometric or independent variables and the operating or dependent variables. The geometric variables represent the physical dimensions of the dry-cooling systems, while the operating variables represent operating conditions such as temperatures and flow rates. The performance evaluation calculations determine the operating variables for a fixed geometry dry-cooling system. On the other hand, the optimization calculations determine the combination of geometric and operating variables that best satisfy the required objective.

An equation-based model, consisting of energy-balance equations, mass-balance equations, momentum-balance equations and engineering design relations, is used to model the dry-cooling systems. These equations can be evaluated for selected values of the independent variables. The equation-orientated models describe the system behavior using basic engineering principles. The equation-orientated models are most conveniently treated by the conventional nonlinear programming techniques.

Performance evaluation

The performance evaluation of dry-cooling systems rely heavily on the ability to model the physical phenomena of the system. A considerable amount of theoretical and experimental work is given in the form of correlations and equations in Appendices C and D (sections C.3, C.4, C.5, D.3, D.5) to describe the performance of these systems and to model the different system components. The formulation of the performance evaluation model is based on the calculation of the heat transfer rate and pressure drops (airside and process fluid side) occurring in the system. The heat rejected by the process fluid must be equal to the heat absorbed by the air (energy equation balance), and the pressure differential external to the cooling tower must equal the internal pressure differential (draft equation balance). The combination of the heat transfer and draft equations are used to simulate the dry-cooling system under various operating conditions. Two methods of dry-cooling tower performance evaluation are distinguished, namely operating point calculations and the interaction of a turbo-generator and dry-cooling system.

(1) Operating point calculations (sections C.6 and D.5)

The operating point (ability to reject heat) of the fixed geometry dry-cooling system is defined as that combination of operating variables (e.g. air mass flow rate, air outlet temperature) that will simultaneously satisfy the draft and heat transfer (energy) equations for fixed process fluid inlet and ambient air conditions. The operating variables that are varied until this requirement is satisfied, are identified in sections C.6 and D.5 for the forced and natural draft dry-cooling systems respectively.

These operating variables are subjected to certain bounds which prevent the violation of some physical laws inherent to the problem under consideration (feasibility inequalities). These inequality constraints will not be active at the operating point. The number of operating variables is equal to the number of equations to be satisfied (balances), in which case the system of equations is said to be well-determined and an exact solution can be expected. Stated in this form, the operating point of the dry-cooling system can be determined by the simultaneous solution of the balance equations.

(2) Power generation (sections C.7 and D.6)

The characteristics of a turbo-generator, i.e. the power generated and the heat to be rejected are expressed as functions of the turbine exhaust pressure or the corresponding saturated steam temperature. The dry-cooling system must be able to reject the required waste heat for a given turbine back pressure. Atmospheric conditions influence the performance of the dry-cooling systems, resulting in a wide fluctuation of turbine back pressure (and the corresponding steam temperature). Changes in the ambient temperatures are the most important reason for this. The mean annual hourly frequency of ambient air temperatures is considered to investigate the interaction between the dry-cooling system and the turbo-generator performance characteristics.

The operating point of the turbo-generator is determined by matching the operating point of the dry-cooling system and the performance characteristics of the turbo-generator at a specific ambient air temperature selected from the annual ambient temperature frequency set. This calculation involves the selection of turbine exhaust conditions such that the heat to be rejected by the heat exchanger after the turbo-generator, equals the heat absorbed by the air (heat rejected by the dry-cooling system), while satisfying the draft equation. The power generation problem is formulated as a sequence of operating point calculations.

At this point the power requirement of the fans (forced draft direct system) or cooling water pumps (natural draft indirect system) as well as the generator power output are known. By subtracting the total power consumed by the fans or pumps from the generator power output, the net power output of the plant is obtained. The net power output is multiplied by the corresponding number of operating hours to give the net energy output for this period. These calculations are repeated for each of the ambient temperatures listed and the results are added to obtain the total net annual energy output.

The operating variables, the bounds on these variables and the equations to be satisfied, are identified in sections C.7 and D.6 for the forced and natural draft dry-cooling systems respectively. A new set of operating variables with bounds are used to determine the turbo-generator's operating point for each ambient temperature in the frequency data set. The comments stated for the operating point calculations do also apply in this case.

The cooling system affects the performance of the entire power cycle in steam-generating plants, i.e. if the cooling system does not provide adequate cooling, the overall plant efficiency decreases with serious economic consequences. Since the performance of the dry-cooling system and the turbo-generator which it serves are so closely related, the selection, design and matching of these components are of the utmost importance in order to achieve effective operation and power output.

Performance evaluation of the dry-cooling system is required in order to obtain an initial feasible starting design point for the optimum design calculation. Furthermore, performance evaluation calculations are also used to perform detailed simulations of existing systems, to quickly investigate the effect of parameter adjustments on the overall system and to aid in decision-making regarding the components and the integrated system performance characteristics.

Optimization

The objective of the optimization process is to design a dry-cooling system that will perform its task effectively at the lowest possible annual cost, while satisfying all the imposed constraints. The derivation of the objective function in terms of the decision variables will be performed in Chapter 4 and in Appendix E. The optimal design of dry-cooling systems hinges on the performance evaluation calculations discussed above and in Appendices C and D. The optimization procedure takes into account all the parameters that will affect the capital and operating cost of the system.

Two distinct approaches can be used to optimize dry-cooling systems, namely a sequential approach or an integrated approach [85BI1e, 85BI2e, 86AR2e, 93SC1e]:

(1) Sequential approach (feasible path method)

In the sequential approach the independent variables are defined and fixed at some initial value. This allows the system of equations, given by the performance evaluation requirements, to be solved. The independent variables are then updated by a suitable objective function. The procedure is repeated until optimality is achieved. The sequential approach thus requires that the model equations be solved at each iteration; therefore, it is also referred to as a feasible path method. The simulation and optimization processes are thus performed sequentially.

(2) Integrated approach (infeasible path method)

With the integrated model, the performance evaluation model is included directly as a set of equality constraints in the problem formulation and the operating variables are also considered as decision variables. The model's equality constraints are required to be satisfied only at the optimal solution. All the variables are adjusted simultaneously and no solution satisfies the equations and constraints until the optimal solution is reached. Since optimization and simulation are simultaneously performed along an infeasible path, it leads to much more efficient computational performance. The dimensionality of the problem is substantially increased in this way and little information is recoverable if the solution algorithm fails.

The integrated approach will be used in this study, i.e. the operating variables with their bounds, as well as the balance equations will be formulated as part of the optimization process. The infeasible path integrated approach has superior performance over the sequential approach. The operating variables are dependent on the geometrical variables. This means of problem formulation tends to disguise the dependence of the operating variables on the geometrical variables. The results of the sequential and integrated methods will be the same.

The optimization of dry-cooling systems for steam electric power plants can be divided into the following categories:

(1) Operating point optimization (sections C.8 and D.7)

The operating point optimization involves the computational process of finding the combination of process and geometrical variables that will minimize the total annual cost of the dry-cooling system subject to fixed process fluid inlet and ambient air conditions and a specified heat transfer rate. All the imposed constraints must be satisfied. The objective function can be expressed as

$$C_{total} = C_{operating} + C_{maintenance} + C_{FCR} \quad (\$/\text{annum}) \tag{3.1}$$

The optimization (decision) variables, the various constraints and the objective function are discussed in sections C.8 and D.7. The operating variables, the equality (e.g. balance equations) and inequality constraints (e.g. bounds on the operating variables) introduced by the performance evaluation model must always be present during the optimization process. The operating variables cannot assume constant values, but are required to be variable. The other geometrical variables can either be variable or fixed at a constant value during the optimization process. The corresponding geometrical constraints can either be applicable or completely ignored. Various practically orientated geometrical constraints are prescribed. It should be noted that variations of the operating variables will only affect the operating constraints and not the geometrical constraints, whereas variations of the geometrical variables influence al the constraints. A minimum number of optimization variables can be considered, depending on the number of equality constraints. The feasibility inequality constraints introduced by the operating variables (bounds), will not be active at the optimum solution.

Both circular finned tubes and finned tubes with any fixed geometry are considered in the optimization. The performance correlations of these finned tubes are described in Appendix B. The limitations imposed on the heat transfer and pressure drop equations for circular finned tubes discussed in section B.1 are not used as constraints. The performance correlations for the circular finned tubes are explicit expressions for the airside heat transfer coefficient and airside pressure drop in terms of the geometrical and layout parameters. It is thus possible to vary these parameters during the optimization process in order to obtain the optimum dimensions and layout. For more sophisticated designs the experimentally obtained correlations for fixed geometry and layout can be used (section B.2).

Four different cases of operating point optimizations are investigated for the natural draft indirect dry-cooling system: (i) the mass flow rate and the temperature of the water that enters the cooling tower are fixed at some specified values; (ii) the temperature of the water that enters the cooling tower are fixed at some specified value; (iii) a constant waterside pressure drop is maintained when the existing bundles are replaced by optimally dimensioned ones (retrofit); (iv) and the condenser design is considered to be constant (constant thermal conductance). These cases are discussed in detail in section D.7.

In the case of the forced draft direct condensing air-cooled condenser, separate performance calculations are performed for each tube row. A maximum number of two tube rows are considered in the optimization study.

Various geometrical parameters are linked to the geometrical decision variables used in the optimization process. The assumptions and relations that are used to obtain these values are shown in sections C.8 and D.7. Several parameters are also considered to be constant during the optimization process and can be fixed beforehand. The sensitivity of the optimum design to variation in some of these parameters will be investigated.

The operating point optimization is useful when a minimum cost dry-cooling system has to be designed for a specified heat transfer rate and some fixed operating parameters. It also provides the necessary insight into the problem formulation and characteristics that are needed to extend the problem as discussed in the following section.

(2) Minimization of power generation cost (sections C.9 and D.8)

The minimization of the power generation cost attributed to the dry-cooling system's performance for the given temperature frequency data set, involves the variation of the dry-cooling tower operating and geometrical variables that will minimize the ratio of its total annual cost to the net energy output of the turbo-generator set it is coupled to. Thus, minimize the unit cost of generated electricity that is directly related to the performance of the dry-cooling system. The objective function is

$$C_{power} = C_{total}/E_{net} \quad (\$/\mathrm{kWh}) \tag{3.2}$$

The calculated values of the different variables must satisfy all the relevant constraints. Only one forced draft or natural draft dry-cooling tower must be designed to perform this task as effectively as possible. The optimization procedure takes into account all the parameters that will affect the capital and operating cost of the system.

The optimization (decision) variables, the various constraints and the objective function are discussed in sections C.9 and D.8. For each temperature data set there is a corresponding set of operating variables, equality and inequality constraints (operating constraints). These constraint sets must always be satisfied and the operating variables must always be present during the optimization process. The operating variables of a specific temperature data set effect the operating constraints belonging to the same temperature data set only. The operating variables do not effect the geometrical constraints. However, the geometrical variables effect all the operating constraints of the different temperature data sets. It is important to take advantage of this problem structure in performing the optimization process. Only one set of geometrical variables and constraints are defined in this case. The same comments regarding the variables and equations as previously discussed for the operating point optimization are applicable here.

The special structure of this problem formulation can be exploited in the following ways:

(i) When an operating variable belonging to a particular temperature frequency data set is changed, only the equality and inequality constraints (operating constraints) corresponding to the temperature data set under consideration, need to be evaluated. All the operating constraints belonging to the other temperature data sets, as well as the geometrical constraints, can be ignored.

(ii) When the constraints are differentiated in terms of an operating variable belonging to a particular temperature frequency data set, the differentiation only needs to be performed on the equality and inequality constraints (operating constraints) corresponding to the temperature data set under consideration. All the other derivatives can be set equal to zero and need not to be calculated.

(iii) The model simulation equations for each temperature data set can be used to exploit the distinction between the dependent (operating) and independent (geometric) variables. The combination of a suitable infeasible path integrated approach with a decomposition technique will result in great computational advantages (e.g. reduction of problem dimensionality).

The influence of the variation in a certain geometrical variable may be confined to one or a few geometrical constraints. However, this structure is not exploited due to relatively small number of geometrical variables and constraints in comparison to the number of operating variables and constraints.

3.5 CLOSING REMARKS

The existence of an optimum solution to an engineering model depends on its formulation. For example, no feasible solution may be obtained if the constraints are too restrictive. The mathematics of optimization methods can easily give rise to situations that are absurd or violate the laws of physics. So, to transcribe a physical problem correctly into a mathematical model, extreme care must be exercised. General guidelines regarding problem formulation, simulation, preparation and implementation can be found in [83RE1e, 84VA1e, 88ED1e, 89AR1e].

The performance evaluation and the optimization calculations are based on the problem formulation and the modeling equations stated in Appendices B, C, D and E. From these appendices it is evident that these problems have certain features that can be exploited during the solution phase.

CHAPTER 4

DRY-COOLING SYSTEM ECONOMIC AND COST ANALYSIS: DEFINITION OF THE OBJECTIVE FUNCTION

4.1 INTRODUCTION

Optimization requires the formulation of an objective function that must be minimized or maximized. In an economic analysis the objective is usually the minimum cost or maximum profit. To set up a cost function is not a trivial task and forms an integral part of any attempted economic optimization study. Various methods of estimating costs and the necessary economic concepts to perform an economic analysis are described.

A prime criterion for the selection of a power plant type or unit size is the production of electricity at the minimum cost. The cooling system is a significant cost item at a power plant and affects the performance of the entire power cycle. A simplified economic and cost analysis, based on all costs which will affect the choice of the optimum cooling tower to perform a certain duty, is presented in this chapter. Various cost components are discussed, namely the capital costs, operating and maintenance costs and the cost of lost performance as found in dry-cooling systems.

4.2 ECONOMIC CONCEPTS

Capital costs are incurred at the beginning of the project and concluded during completion of the entire project or during completion of a part of it. Each year during the life of a plant there may be different or variable operating costs. Since these expenditures take place over different periods in time, and since money has time value, it must be brought to a common reference. Engineering economy analysis provides a suitable means to bring the capital and operating costs to a common reference by applying certain engineering economy formulae. A summary of the most important compound interest formulae that find application in this study is presented below. A detailed analysis and derivation of these formulae is beyond the scope of this study and the reader is referred to Blank and Tarquin [83BL1e] or Stoll [89ST1e] for further information regarding the fundamental concepts that form the basis of engineering economic analysis. The nomenclature that will be used in these formulae is:

P = Present worth

F = Future worth

A = Uniform annual series

D = Cost in the first year

U = Uniform levelized annual equivalent of an escalating series

n = Number of years considered

i = Interest rate

e = Escalation rate

Single-payment factor

$$F = P(1+i)^n \tag{4.1}$$

Uniform series factors

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right], \quad i \neq 0 \tag{4.2}$$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right], \quad i \neq 0 \tag{4.3}$$

Escalating series factors

$$P = D\frac{\left[1-(1+e)^n/(1+i)^n\right]}{i-e}, \quad i \neq e \tag{4.4}$$

$$U = D\frac{\left[1-(1+e)^n/(1+i)^n\right]}{(i-e)}\,\frac{i(1+i)^n}{(1+i)^n - 1}, \quad i \neq e \tag{4.5}$$

4.3 COST ESTIMATING

The engineering profession is also governed by economic principles, especially during decision making, as is the rest of the business community. One of the basic economic skills that engineers often need, is to make cost estimates for the design, construction and manufacturing processes. Accurate cost estimating is essential if a firm is to stay in business. Unfortunately, cost estimating is not an exact science and in the best of circumstances will only provide an approximation of the cost that will actually be incurred. It is thus essential that the various tools available for this task be understood and be applied so that the degree of approximation will be minimized. A wide variety of topics concerned with cost accounting that provide valuable background can be found in Davidson and Weil [78DA1e] and Nicks [92NI1e].

Estimating concepts

Cost estimates are produced for various reasons such as whether or not to produce a newly designed product, to assist in make-or-buy decisions, to determine the selling price of a product and to check vendors' quotations. Estimates are developed according to the purpose of the estimate, the amount of time available and the complexity of what is being estimated. A variety of estimating methods do exist and their use depends on the amount of information known by the estimator. Some cost estimates rely on the estimator's experience and judgment when little detail is known. New estimates can also be based on past history and estimating information found in the literature. Depending on the risk and the amount of money involved in an estimate, the estimator will vary the amount of detail used in the estimating process. Detailed estimates for machinery operations, for example, would include calculations for speeds, feeds, cutting times, load and unload times and even machine manipulation. These time values are usually calculated from standard time tables and adjusted with an efficiency factor to predict the actual performance or from measured values. Parametric estimation is also widely used as a preliminary estimating method and statistical estimating is a special form of this. Parametric estimating formulas can be as simple as multipliers or as complex as regression models and are developed by relating cost data to the factors on which it depends. Project estimating is by far the most complex of all the estimating tasks and needs very careful planning, especially if the project spans a number of years. Inflation and risk analysis affect project estimating and must be carefully examined to quantify their effect on the estimation process.

In most companies the elements of the cost details that make up the estimating are obtained from the accounting department. The major cost elements are classified as direct labor, indirect labor, direct materials, indirect materials, and overheads. The direct costs are the costs that can be traced to a specific product whereas indirect costs are those costs that are considered not being traceable to a product, but are still required to run the company. Overhead costs usually includes salary and management cost and all the other costs elements such as insurance, machinery cost, office expenses, and other miscellaneous expenses not considered to be directly related to the product. Overhead costs are usually expressed as a percentage of direct labor or direct material cost for cost estimating purposes.

Estimators make use of standard time data to develop consistent estimates of the manual effort involved in performing specific tasks. These data can be obtained from past history or actual performance on jobs, time and motion studies or predetermined time standards. Experience has shown that it is easier to develop standard data for machining operations than for the manual effort required in most fabrication operations. Material cost can be estimated to a high degree of accuracy compared to the labor content in an estimate that is subjected to more error. If there is a bill of materials available from a detailed drawing, decisions must be made about what will be made or what will be purchased. For those items that are decided upon to be made, the estimator must determine the cost of the product by using his/her best judgment. The shop's actual performance will be compared to the estimated cost. The cost of the purchased components can be obtained from the vendors or catalogues. The estimation process is usually aided by sketches, line drawings or complete drawings. Nowadays computers are satisfactorily used in the estimating process and many different programs are available. The major advantage of computer estimating is the flexibility the programs offer the estimator to manipulate data once the problem has been specified. However, it should be realized that no one program can satisfy everyone's needs and as such the source codes should be customizable.

A very important component of the estimating process is a good feedback system that will permit the estimator to compare the actual and the estimated costs. This cost comparison enables the estimator to review the estimating process and improve the estimating accuracy. A good estimator will heavily rely on skills and intuition in making an estimate. The basic skills like cost accounting and the different estimating methods can be taught, but intuition comes only with experience and the feedback system.

Costs can be categorized into two major categories, namely capital cost and operation and maintenance cost.

Capital cost

Capital cost estimates may vary from predesign estimates based on very little information to a detailed estimate prepared from complete drawings and specifications. Between these two extremes there are various other estimates which vary in accuracy depending on the stage of development of the project. The American Association of Cost Engineers (AACE) uses the following five categories for describing the various estimate types at different stages of evaluation, design and procurement [83NO1e, 89GA1e, 91PE1e]:

(1) Order-of-magnitude estimate (ratio estimate) based on similar previous cost information and estimating charts; probable accuracy of estimate ± 40%.

(2) Study estimate (factored estimate) based on knowledge of major items of equipment, estimating charts and some vendor quotations; probable accuracy of estimate ± 25%.

(3) Preliminary estimate (budget authorization estimate, scope estimate) based on sufficient data to permit the estimate to be budgeted (detailed vendor quotes, recent experiences); probable accuracy of estimate ± 12%.

(4) Definite estimate (project control estimate) based on almost complete data but before completion of drawings and specifications (detailed quotes, labor, material estimates); probable accuracy of estimate ± 6%.

(5) Detailed estimate (firm estimate, contractor's estimate) based on complete engineering drawings, competitive vendor quotes, specifications, and site surveys; probable accuracy of estimate ± 3%.

As soon as the final design stage is completed, it becomes possible to make accurate cost estimations because detailed information is then available. No design project should proceed to the final stages before costs are considered and costs estimates should be made throughout the early stages of the design even when complete specifications are not available and revised as the project progresses. Predesign cost estimation (defined as order-of-magnitude, study, and preliminary estimates) provides company management a basis to decide if further capital should be invested in the project and to compare alternative designs. The validity of any cost estimation can only be tested when the completed project becomes operational. It should also be noted that the distinction between predesign and firm estimates gradually disappears as more detail is included in the estimate. A good cost estimator will be able to make remarkably accurate cost estimations even before the final project design is completed. Furthermore, the cost estimator must keep up to date on factors effecting investment and production costs, i.e. sources of equipment or services, price fluctuations, company policies, operating time and production rate, and government regulations.

Various methods of predesign capital cost and product cost estimation are used in the process industry (chemical engineering) and consist of labor and material indexes, standard cost ratios, estimating charts and special multiplication factors [64BA1e, 74GU1e, 74OS1e, 74ZA1e, 79CH1e, 83HO1e, 84CH1e, 88KH1e, 89GA1e, 91PE1e]. Capital costs are of two types, i.e. direct costs and indirect costs. Direct costs can be traced to the material and labor involved in the actual installation of the facility (e.g. equipment, installation, land), while indirect costs are the expenses which are not directly involved in the material and labor of the actual installation of the facility (e.g. engineering, supervision, contingencies). Direct capital costs are 70-85% of the capital investment and indirect cost 15-30% for chemical plants [91PE1e]. Major equipment costs (e.g. reactors, storage tanks, separators) play an important part in detailed plant cost estimates. Predesign cost estimates can be categorized according to the following basic principles:

(1) Inflation cost indexes

Most cost data which are available for use in a predesign estimate are based on conditions in the past. Due to the changes in the economic conditions some means of updating these costs, to be representative of conditions at a later time, must be used. This can be done by the use of inflation cost indexes.

$$\text{Present cost} = \text{Original cost}\left(\frac{\text{Index value at present time}}{\text{Index value at time when original cost was obtained}}\right) \tag{4.6}$$

Many different types of cost indexes are published regularly, e.g. Marshall and Swift all-industry and process industry equipment indexes, Engineering News-Record construction index [89GA1e, 91PE1e]. Some of these can be used for estimating equipment costs and others apply specifically to labor, construction, material and other specialized fields. These indexes are artificial and two indexes covering the same types of projects may give results that differ considerably. The most that any index can hope to do is to reflect average changes and great caution must be exercised when using these indexes [91PE1e].

(2) Cost factors

The cost of purchased equipment is the basis of several predesign methods for estimating capital investment. Sources of equipment prices, methods of adjusting equipment prices for capacity, and methods of estimating auxiliary process equipment are therefore essential to the estimator in making reliable cost estimates.

The most accurate method for determining equipment costs is to obtain quotations from the manufacturer. Often, fabricators can supply quick estimates which will be very close to bid price but will not involve too much time. Cost values from past purchases, corrected to current cost indexes, can also be used. The cost of one size of equipment can also be used to predict the cost of another size by using the following exponential scaling relationship

$$\text{Cost of equipment size a} = \text{Cost of equipment size b}\left(\frac{\text{Size of a}}{\text{Size of b}}\right)^{\text{Exponent}} \tag{4.7}$$

Good results can be obtained by using the six-tenths-factor-rule, i.e. replacing the value of the exponent in equation (4.7) by 0.6. However, the application of this rule of thumb is an oversimplification and the value of the exponent can vary from less than 0.2 to greater than 1.0 for different pieces of equipment [89GA1e, 91PE1e]. The 0.6 factor should only be used in the absence of other information. This concept should not be used beyond a tenfold range of capacity and only directly comparable pieces of equipment should be scaled.

The installation of the purchased equipment involves cost for labor, foundations, supports, platforms, construction and other factors directly related to the erection of purchased equipment and getting it to function properly. These costs are presented as a percentage of the purchased equipment costs [89GA1e, 91PE1e]. Garrett [89GA1e] states an average equipment installation factor of 63% of the equipment cost for chemical plants.

Once individual equipment costs are known from either manufacturer's price quotations or the estimating methods discussed, they can be used to form preliminary total plant cost estimates. These plant costs, such as piping, electrical, instrumentation and controls, land, buildings, utilities, engineering, supervision, contingencies etc. can be expressed either as a percentage of the total capital investment or as a percentage of the purchased equipment costs [79CH1e, 84CH1e, 89GA1e, 91PE1e].

Operation and maintenance costs

Operation and maintenance costs estimates are just as important as capital cost estimates. These estimates predict the expenses incurred in operating and maintaining the infrastructure to produce products or services. These costs occur over the life of the plant being operated and maintained and usually increase with time.

The total product cost can be divided into two major classes, namely the direct and the indirect product cost. The direct costs include the costs directly related to the manufacturing operation and consist, for example, of the cost of material, direct operating labor, operating supplies and utilities. Indirect costs include the costs not directly related to the manufacturing operation and include indirect material and labor cost, factory burden, machinery cost, general and administrative cost. These costs are usually classified as overhead costs. Overhead costs are usually defined to include the management cost, salary cost and other general expenses that are involved in the company's operation but not directly related to the manufacturing process. Overhead costs such as payroll overheads (e.g. pension funds, medical schemes etc.) and general plant overheads (e.g. general maintenance, office supplies, services, building, plant superintendence etc.) are always present if the complete plant is to function efficiently. Fixed charges are expenses which remain practically constant over the plant lifetime like depreciation, taxes, interest and insurance.

The direct costs are easier to predict than the indirect costs and definite estimates of the former are used to predict the indirect cost. Labor related indirect costs can be expressed as a percentage of the direct labor cost, capital related costs can be expressed as a percentage of the plant capital cost and sales related costs can be expressed as a percentage of the sales. Another useful source of information for use in the total product cost estimate is the use of data from similar or identical projects contained in the company records. Adjustments must however be made to relate the cost differences due to inflation, plant site and the geographical location. In the absence of detailed information, indexes and charts with information regarding the specific process under consideration can be consulted to obtain quick estimates [89GA1e, 91PE1e]. Detailed operating cost estimates are also possible and require the gathering of exact data on all the factors that influence the total product cost. The overhead costs must be assigned to a specific action in a more precise manner.

The maintenance required for any plant is a function of factors like the operating environment, the plant age, the management's maintenance policy and the original decision made in what type of equipment to use. Labor and material cost are the main contributors to make up maintenance expenses, the former being the main cost contributor. Maintenance expenses can vary widely and are usually between 2% and 10% of the total plant cost per year. Maintenance is very much part of the management policy and when properly managed, it can bring about major advantages for the company like, for example, significant cost savings. Nowadays, maintenance is far more than the frenetic rush to repair broken equipment, it also involves the art and science of preventative maintenance and reliability. Companies must keep performance and cost records of the various equipment used to be able to plan preventative maintenance, equipment replacement and to gauge reliability. A properly planned and administrated maintenance plan can also assist in the future decisions of replacing equipment.

The most widely used method to estimate maintenance cost is to express it as a percentage of the capital cost per year. As previously stated, the major cost components are those for labor and materials. Material cost estimates can be done more accurately than the labor cost estimates. Detailed estimates can be performed based on information stated in performance records, industry averages or ratios, and comparative and specific job standards. Good judgment on the side of the estimator also forms an essential ingredient in the maintenance cost estimating process. The accuracy of such an estimate will depend on the quality of the data available and the experience of the estimator. The interested reader is referred to Higgins [88HI1e] for a complete covering of the topic of maintenance engineering.

When labor costs constitute a major portion of the manufacturing cost it makes sense to use labor cost as the basis for allocating overhead costs. But in some highly automated manufacturing facilities labor cost makes up a small percentage of the total cost, whereas overhead costs will greatly contribute to the cost of goods sold due to the high cost of automation. Therefore it makes no sense to allocate overhead costs on the basis of labor cost. A solution to this problem is to analyze all the activities that make up the overhead cost and then to allocate these costs to the products to the extent that the products make use of these activities. This concept is known as activity based costing (ABC). In activity based costing the cost of the product or service equals the cost of the materials plus the sum of all the costs of every activity used to produce the product or service. In that way activity based costing is different from traditional costing which uses arbitrary allocation methods to estimate the overhead cost. The major objective of activity based costing is thus to relate the costs that are classified as overhead in the traditional cost estimating models directly to the products. Firstly, one identifies all the support activities needed for production and then determine how the product actually consumes these activities. Furthermore, this methods provide one with the ability to distinguish between non-value-added activities and value-added activities in order to eliminate or minimize non-value-added activities and reduce manufacturing costs. A detail discussion on activity based costing can be found in McCormick [92MC1e].

The cost to prepare cost estimates is difficult to predict and the estimate becomes more expensive as the amount of detail involved in the estimate increases. A detailed cost estimate can amount up to 5% of the total project cost. Realistic and accurate cost estimating is not a trivial task and requires a detailed analysis of all the different actions involved and material requirements for a specific project. After the completion of each design stage the cost estimate can be refined until a detailed analysis is performed. The skills needed to be a good cost estimator are a mix of business, finance, engineering, technical, manufacturing, planning, management and marketing skills [82ST1e, 92NI1e]. The quality of these combinations of skills has a great bearing in the overall credibility, accuracy, and completeness of the resulting cost estimate. These skills need to be combined with intuition that can only come from first hand experience. Over the last decade there have been significant technological advances in the design, manufacturing and service industries that require costing systems to correspond with these changes. In the past, cost estimating was mainly the task of cost accountants, but industry has since realized how well industrial engineers are equipped to fulfill the task.

4.4 DRY-COOLING SYSTEM COST ESTIMATION

The previous section discussed various detailed and approximate cost estimation methods. In the following sections a cost structure based on these principles, as well as cost information obtained from dry-cooling system component manufacturers, will be presented to estimate the cost of a dry-cooling system during the optimal design process.

The time value of money requires that costs should be brought to a common reference. Two methods can be used to comply with this requirement and are found in literature regarding power generation, namely the present worth of lifetime evaluated costs [71HA1e, 75CR1e, 80GU1e, 85LI1e, 85LI2e, 87VE1e] and the annualized cost method [70RO1e, 72AN1e, 78RO1e, 79CH2e, 79NA1e, 85LI1e, 89BU1e, 89BU2e]. The annualized cost method will be used in the economic analysis and requires that all the costs are evaluated on an annual basis.

If current costs are used in selecting equipment that will be purchased three to four years later, an escalation factor must be applied to make the cost estimates more realistic. Also, after the equipment has been purchased, it takes time to install the various components (construction takes a few years to complete). During this time an additional cost is incurred because the investment is not yielding a return. The factor for this cost is known as interest during construction [71HA1e, 85LI1e, 86HI1e]. Once the plant becomes operational, the fixed charge rate can be applied to the capital costs to give the annual expenditure required for the entire life of the plant. A detailed discussion of power plant system economics can be found in Li and Priddy [85LI1e].

Various engineering economic studies of dry-cooled electrical generating plants were conducted in the seventies with the major aim of minimizing the increased power generating cost attributed to dry-cooling (mostly indirect natural draft and mechanical draft systems) [70RO1e, 75MI1e, 76FR1e, 78RO1e]. These studies were extended to investigate the advantages of combined wet/dry-cooling systems for power plants [75CR1e, 76CR1e, 76ZA1e, 78LA2e, 79CH2e, 80GU1e]. In these references, detailed cost breakdowns of the optimization results for the utility power plants (conventional fossil fuel and nuclear power plants) considered, are given. Cost estimation is usually based on vendors' cost data or manufacturers' quotations and simplified generalized cost functions based on these data are used. Most of the equipment used in the cooling systems are predesigned vendor-offered equipment and thus have a fixed geometry. The optimization consists of varying the amount of standard equipment and components used, rather than their geometry. The results from these studies are not conclusive due to the many assumptions used and the simplified manner in which the complex optimization process is often performed (±5 variables are used to describe the cooling system). It cannot be concluded from these studies whether the dry-cooling equipment offered by the industry is near optimum.

Ard et al. [76AR1e] developed detailed cost algorithms (empirical predictions) for the components used in the indirect dry-cooling system with mechanical draft and surface condenser that make it possible to investigate the variation of cost with component dimensions. For an optimization study it is necessary to use component manufacturing cost models in evaluating system design and material alternatives. The components that are included in the study are the heat exchanger bundles, mechanical draft equipment, circulation system, condenser, tower structure, water quality control and the electrical system. Cost information was obtained from manufacturers and suppliers of equipment and materials as well as estimates by persons knowledgeable in the field. Some of the cost relations are based on trends experienced in a specific field by plotting cost against some controlling parameter.

We will try to present a similar cost structure that will adequately cover all the major cost components as far as possible. As we intend to change the component dimensions during the optimization calculations it is very important that such a model reflects the actual case as realistically as possible. Standardized component sizes will not be used in the analysis. In real life there is a definite relationship between product cost and product volume. However, we shall ignore such a relationship for the purpose of the optimization process.

Cost functions are proposed for the various capital cost and operating and maintenance cost items. Because some of the variables (e.g. tube dimensions) come in standard sizes the cost function tends to be highly discontinuous. However, classical optimization techniques require an objective function that must be continuous and differentiable. In this analysis all the variables used in the optimization process (defining the cost function) will be regarded as continuous and practical constraints are introduced to ensure realistic results.

Cost weighting factors are introduced on several occasion in the proposed cost structure to take into consideration estimation uncertainties, overheads and other related costs.

4.5 DRY-COOLING SYSTEM CAPITAL COST ESTIMATION

The capital cost of the dry-cooling system includes the equipment and construction cost of the cooling towers, water circulation system and air moving system and the indirect costs related to it.

Indirect costs are assumed to be 20% to 25% of the direct capital costs for the cooling tower [79CH2e, 79NA1e, 80GU1e].

For more accurate costing the capital cost is broken up into its individual components and the cost of each component is estimated.

Heat exchanger bundle cost

Various methods have been used to assign costs to heat exchanger bundles, the most simple of which is to base the cost on the heat exchanger area [59KE1e, 59NA1e, 89KO1e, 90LO1e]. Approximate costs are often quoted on the basis of cost per square meter of outside bare tube surface [83NO2e]. These rough approximations can be somewhat refined by using cost-multipliers to account for factors such as the number of tube rows, tube length, fin pitch and extended surface type.

Tube elements are the most significant cost factors of the air-cooled heat exchanger system. For this reason the choice of fin and tube material, the finned tube geometrical parameters and the method of finning should be carefully chosen to yield the maximum economy commensurate with an adequate and satisfactory life. One of the most important steps that can be taken to reduce cost of equipment is to design it so that it can be fabricated, assembled and tested in the shop rather than in the field [86HI1e].

The capital investment is calculated from the material cost (component dimensions) and the manufacturing cost. Material costs are divided according to the major cost contributors, e.g. tubes, fins and headers and defined in terms of the component dimensions [74PA1e, 78FO1e, 83PU1e]. This is the method that will be used in estimating the heat exchanger bundle costs. Some of the more useful heat exchanger bundle cost estimates that appear in the literature will now be discussed.

The report of Ard et al. [76AR1e] placed special emphasis on the development of cost algorithms for heat exchanger bundles. The heat exchanger bundle analysis covered the following major cost components, namely the heat transfer elements, heat exchanger bundle headers, heat exchanger bundle frames, bundle assembly, louvers and hail screens. The total heat exchanger bundle cost consists of these major cost components and are based on the manufacturer's material, labor, equipment and job overhead costs. The manufacturer's general overheads and profit are added as a fixed percentage. The reader is referred to the report for a detailed description of the cost estimates.

Buys and Kröger [89BU1e, 89BU2e] modeled the cost of the heat exchanger bundles by assuming a certain cost structure related to the material cost and other costs incurred by the manufacturer.

Based on the above-mentioned methods, the total heat exchanger bundle cost, C_{he}, can be expressed as

$$C_{he} = \left(C_{ft}L_t n_{tb} + C_h + C_{ba}\right)W_{he} \quad (\$/\text{bundle}) \tag{4.8}$$

The components that make up the total heat exchanger bundle cost are discussed in Appendix E.

Fan system capital cost

The fan system cost consists basically of the fan, inlet bell, fan shroud, inlet safety screen, electric motor, gearbox or belt drive and the electrical and control equipment cost.

Axial flow fans are usually used with diameters ranging from 1.2 m to 9.9 m. The number of blades range between 3 and 9. The fan blades are usually either cast aluminum, glass fiber reinforced polyester or epoxy laminates. The blades are fixed to a hub where the blade angle can be manually or automatically adjusted. Low noise blades are also available. Fan speed adjustment is performed by using multispeed motors and result in a stepped variation in the air flow.

The fan system is usually costed on a \$/kW basis plus a \$/m^2 of fan blade swept area [72AN1e, 76FR1e]. Ard et al. [76AR1e] developed a detailed model to describe the fan system cost which is also used by Choi and Glicksman [79CH2e]. They used the fan size and power requirements to develop a cost structure for the fan, fan equipment, fan plenum and fan velocity recovery stacks. Their fan cost algorithms are given below.

The fan cost, C_F, depends primarily on the fan diameter, number of blades and whether the fan has manual or auto-variable pitch control and the hub and seal cost.

$$C_F = n_{Fb}\left(d_F C_{Fb} + C_{Fbu}\right) + C_{Fh} \tag{4.9}$$

where C_{Fb} is the cost per fan blade, C_{Fbu} is the fan blade added unit cost and C_{Fh} is the hub cost. These costs were developed from a fan manufacturer's price list. The fan equipment cost, C_{FE}, consists of the electric motor cost, C_{em}, speed reducer cost, C_{sr} (either gear drives or belt drives), shaft, bearings and mountings costs ,C_{FEu}, and is dependent on the fan power requirement measured in Watt, P_e.

$$C_{FE} = P_e\left(C_{sr} + C_{em}\right) + P_e^{1.5} + C_{FEu} \tag{4.10}$$

The fan plenum type depends on whether the system is forced or induced draft and the cost is derived from the plenum and fan ring material weight used during the manufacturing process. Both horizontal and vertical plenums are investigated and the results are not applicable to the A-frame arrangement. Velocity recovery stacks are used to recover some of the kinetic energy at the discharge of induced draft fans and their cost depends mainly on the material used in the manufacturing process. The total fan system cost is the sum of all the above mentioned costs.

In this study it is assumed that $d_{Fc} \approx 1.005 d_F$, $d_{Fh} \approx 0.165 d_F$, the height of the fan casing is $0.1 d_{Fc}$, the inlet bell height is $0.15 d_{Fc}$ and the inlet bell diameter is $1.2 d_{Fc}$. Based on the above-mentioned methods, the total fan system cost, C_{Ft}, can be expressed as

$$C_{Ft} = \left(C_F + C_{Fc} + C_{Fs} + C_{em} + C_{sr} + C_{ws}\right) W_F \quad (\$/\text{fan}) \tag{4.11}$$

The components that make up the total fan system cost are discussed in Appendix E.

Forced draft cooling tower structure

The structural components must be adequate to support the tube bundles, the heat exchanger bundle structure, ducting, fluid load, fans and the fan drive equipment under a variety of conditions. Furthermore, the height of the fan platform above the ground level must be selected with due consideration of the total cooling air flow of all the fans, without exceeding the average air access velocity [83SH1e, 87KN1e, 94SA1e].

Ard et al. [76AR1e] developed cost algorithms for both round and rectangular mechanical draft dry-cooling towers. Initial cost estimates were based on commercial designs. A base cost was thus established from which costs could be derived for towers which were larger, smaller, or for which the load to be supported varied. For the rectangular tower with horizontal heat exchanger bundles, the structure cost per square meter of heat exchanger frontal area is expressed as a linear function of the load to be supported, M_S, i.e.

$$C_S = a + b M_S \quad (\$/\text{m}^2) \tag{4.12}$$

For the round tower the steel cost is expressed as a function of the tower volume and roof loading and the foundation cost as a function of the heat exchanger bundle weight and the tower area. The same correlations are employed by Choi and Glicksman [79CH2e]. Haberski and Bentz [79HA1e] use preliminary design layout drawings to estimate the construction and structural costs.

In this study it is proposed to express the structure, foundation and the land area requirement costs as a function of the heat exchanger bundles' plot area and the structure height.

The total construction cost, C_c, can be expressed as

$$C_c = \left(C_l + C_{cu} H_3 + C_{Fpl}\right) A_{Fpl} W_c \quad (\$) \tag{4.13}$$

The components that make up the total construction cost are discussed in Appendix E.

Natural draft cooling tower shell (Hyperbolic reinforced concrete)

The shape of the meridian curve of the cooling tower shell is a hyperbola described by [84ZE1e]

$$r(z) = r_o + \frac{a}{b}\sqrt{z^2 + b^2} \tag{4.14}$$

where a and b are constant values depending on the geometry, r_o is the offset between the hyperbolic axis and the cooling tower axis (axis of rotation). In most cooling tower designs more than one hyperbolic function is required to describe the meridian curve of the shell. Therefore, two hyperbolic functions are used as generating functions, one above and the other below the throat. The constants a and b can be calculated from the tower geometry.

If $t_{ct}(z)$ is the thickness of the shell at height z ($t_{ct} \geq 200$ mm [83SI1e]), and if $C_c(z)$ is the construction cost per unit volume of shell as a function of height z, then the total cost of the shell may be approximated by [75RE1e] (refer to figure D.2)

$$\begin{aligned} C_{ct} = & \int_0^{H_5 - H_t} 2\pi r_1(z_1)\sqrt{1 + \left(\frac{dr_1}{dz_1}\right)^2}\, t_{ct}(z_1) C_c(z_1) dz_1 \\ & + \int_0^{H_t - H_3} 2\pi r_2(z_2)\sqrt{1 + \left(\frac{dr_2}{dz_2}\right)^2}\, t_{ct}(z_2) C_c(z_2) dz_2 \end{aligned} \tag{4.15}$$

The above equation cannot be given in closed form and must be solved numerically.

The inlet and overall height, base diameter and the outlet diameter of the concrete shell of a cooling tower are determined mainly by thermal considerations, whereas the shape of the tower is primarily a function of strength and cost. The assessment of the cost of the hyperbolic concrete shell is thus a function of the concrete and steel reinforcement quantities used, which in turn are governed by the structural design. Due to the fact that we are only interested in thermal parameters, a detailed structural design optimization is outside the scope of the present study. We must therefore find a more simplified way of expressing the cost of the cooling tower shell. The cost information obtained in the literature is subsequently discussed.

Furzer [72FU1e] expressed the cooling tower shell cost as

$$C_{ct} = C_{cts} S \pi d_3 t_{ct} (H_5 - H_3) \tag{4.16}$$

where S is a shape factor depending on the hyperbolic contour and has the value 0.6667.

Ecker and Wiebking [78EC1e] considered a cylindrical tower shell and expressed the cost as

$$C_{ct} = C_{cts}\pi d_3 t_{ct}\left(H_5 - H_3\right) \tag{4.17}$$

In both the above expressions the shell cost, C_{cts}, is proportional to the volume of reinforced concrete in the shell.

Buys and Kröger [89BU2e] approximated the cooling tower shell as a conical frustum and express its cost as a function of the cooling tower shell area, i.e.

$$C_{ct} = 0.5\pi C_{ctc}\left(d_3 + d_5\right)\left[\left(H_5 - H_3\right)^2 + 0.25\left(d_3 - d_5\right)^2\right]^{0.5} \tag{4.18}$$

where C_{ctc} is the cooling tower construction cost per square meter of shell area.

Lovino et al. [90LO1e] expressed the cooling tower cost as

$$C_{ct} = a\ d_3^b \tag{4.19}$$

where a and b have been derived from preliminary proposals for several similar cooling tower designs.

A simplified method is proposed to estimate the costs involved in the cooling tower construction. The hyperbolic concrete cooling tower has a relatively thin shell of varying thickness which is greatest at the base. The use of stiffening rings can reduce the volume of reinforced concrete as the towers get larger [80ZE1e, 83SI1e]. However, it is not our intention to perform a structural design and optimization on the shell and therefore the construction cost will be presented as $\$/m^3$ of concrete used, assuming a constant mean shell thickness. The cooling tower shell will be approximated by means of two conical frustums, one above and the other below the throat.

The total construction cost, C_c, can be expressed as

$$C_c = \left(C_l + C_{ct} + C_{hepl} + C_{ts}\right)W_c \quad (\$) \tag{4.20}$$

The components that make up the total construction cost are discussed in Appendix E.

Steam/condensate distribution system costs

The steam/condensate distribution system cost (air-cooled condenser), C_{sd}, is expressed as a function of the total heat exchanger bundle and fan system costs as follows

$$C_{sd} = \left(C_{he} + C_{Ft}\right)W_{sd} \quad (\$) \tag{4.21}$$

The components that make up the total heat exchanger bundle and fan system costs are discussed in Appendix E.

Circulation system costs

The circulation system design is a function of the particular plant layout and requirements. The only way to determine the actual cost of building a large piping system would be to obtain bids with a detailed piping design [76AR1e] or to use a preliminary design layout to obtain costs [79HA1e].

Ard et al. [76AR1e] present a detailed piping model to define the cost of the optimized piping system for a 1000 MWe plant. Two cooling tower arrangements were considered for indirect dry-cooling systems: circular towers and rectilinear towers. Although an actual design will differ from the presented designs, the authors expect the model to be representative of the actual costs. They further conclude that the costs are not greatly dependent on the piping layout. A piping design and cost algorithm is also presented. Extensive tables with cost coefficients, obtained from catalogues and contractors, for calculating installed steel pipe and pipe fittings are provided (also refer to [79CH2e]). The installed cost is basically a function of the welding, material and other installation costs. Choi and Glicksman [79CH2e] illustrate the economic trade-off between the capital cost of piping and the cost of pumping power.

A linear relationship between pipe diameter and installed piping cost was developed [76AR1e, 79CH2e, 79NA1e, 80GU1e], i.e.

$$C_p = a + b\, d_p \tag{4.22}$$

Most of the piping cost models are based on the model of Ard et al. [76AR1e].

A distance of approximately 150 m between the condenser and the cooling tower is usually assumed and the optimum pipe velocity varies between 2.75 m/s and 3.7 m/s [79CH2e, 79NA1e, 80GU1e]. For large natural draft dry-cooling towers the pipe velocity may be as low as 1.75 m/s [85ES1e]. The literature cited are for the indirect dry-cooling tower system with water as the fluid.

An investigation into the optimum results of various reports ([70RO1e, 75MI1e, 77SU1e, 78LA1e, 78RO1e, 79CH2e, 79HA1e, 79NA1e, 80GU1e, 91SZ1e]) show a vast amount of cost data for the different designs (indirect dry-cooling towers, both natural and mechanical draft). The circulation system cost, including the piping, pumps, pipe fittings and other equipment can however be expressed as a percentage of the cooling tower cost including the heat exchanger bundles. The percentages vary from as high as 50% to as low as 15%.

The pump cost is a function of the power requirement as governed by the water flow rate and the total pressure drop in the cooling system. Various authors expressed the pump cost as a linear function of the pumping power [79CH2e, 79NA1e, 80GU1e, 90LO1e], i.e.

$$C_{pump} = a + b\,P_w \tag{4.23}$$

Crowley et al. [75CR1e] express the capital cost of the pump and piping as a function of the total water flow rate.

Buys and Kröger [89BU2e] express the capital cost of the pump and piping as a percentage of the sum of the cooling tower construction and the heat exchanger bundles capital cost.

For better part load performance it is preferable to divide circulating pump capacity between two 50%-duty units. Sometimes three 50%-duty pumps are installed so as to have a stand-by unit in emergencies; this solution makes it possible to bring in the third pump to assist the cooling effect under conditions of high ambient air temperatures [71HE1e, 87TR1e, 91SZ1e]. Two 50%-duty pump units will be considered in this analysis.

The total pump system cost, C_{pst}, can be expressed as

$$C_{pst} = \left(C_{pump} + C_{em} + C_{ws}\right)W_{ps} \quad (\$) \tag{4.24}$$

The piping and valves cost, C_{pv}, can be expressed as

$$C_{pv} = C_{he} n_b W_{pv} \quad (\$) \tag{4.25}$$

The components that make up the total pump, piping and valves costs are discussed in Appendix E.

4.6 DRY-COOLING SYSTEM OPERATING COST ESTIMATION

The operating costs that will be considered in this analysis are the fuel cost, the auxiliary power requirement cost of the fans and pumps, the fixed charges and the maintenance costs. These costs may be variable during each year of the life of the plant and are usually expressed on an annual basis. Levelized values (over the plant lifetime) of these variable cost factors are usually computed in order to simplify the calculations [85LI1e].

Auxiliary power requirements, such as the fan and pumping power, are calculated as the product of electricity cost and the equipment power requirements. The operating or running costs of pumps and fans are sometimes expressed in terms of fuel requirement by dividing the total input power requirements of the pumps and the fans by the plant efficiency [75CR1e, 85LI2e].

The cost of electricity, C_e, and the cost of fuel, C_f, will not remain constant over the operating lifetime of the plant. It is mostly preferred to work with the average value over the operating lifetime of the plant, C_{eav} [81SM1e, 83KR1e] and C_{fav} [85LI2e]. If it is known at which rate the initial costs, C_{el} and C_{fl}, will escalate over the plant operating lifetime, the average values can be determined according to equation (4.5).

Fuel cost

Fuel costs are a major portion of the cost of power generation, running at about half the total cost for coal-fired plants [86HI1e] and are therefore the largest operating expense. Fuel cost varies with the plant's efficiency, the unit fuel cost and the amount of electricity produced. The cooling system plays a major role in the efficient conversion of fuel to electricity. The turbine heat rate, defined as the ratio of heat supplied by the boiler to the turbine output (kJ/kWh), will increase with decreasing cooling tower performance during high ambient temperatures, thus wasting valuable fuel. The selection of a cooling system affects the overall plant performance, e.g. a large cooling system will result in a lower turbine back pressure and the plant will use less fuel than in the opposite case. It is for this reason that the plant fuel cost must be taken into consideration in the cooling system optimization.

The ratio of the heat supplied by the boiler per hour to the turbo-generator output (the heat rate) in kJ/kWh can be approximated as follows:

$$\text{Heat rate} \approx \frac{\left(P_g + Q\right)3600}{P_g} = \frac{1}{\eta_{plant}} \qquad (4.26)$$

where P_g is the gross power output of the turbo-generator and Q is the heat to be rejected by the cooling system. The unit cost of the fuel, C_{fav}, can be expressed in \$/kJ which is derived from the fuel cost per kilogram (\$/kg) and its calorific value (kJ/kg). The total annual fuel cost can be expressed as [85LI1e]

$$C_{ft} = C_{fav}\eta_{plant}^{-1}P_g\tau \qquad (4.27)$$

where τ is the number of operating hours (refer to Appendix E).

Fan operating cost

In mechanical draft air-cooled heat exchangers, either forced or induced draft, the required air flow is assured by fans. Axial flow fans are normally used. The operating point of the cooling system is where the system resistance curve intersects the fan characteristic curve at a specific air mass flow rate, i.e. $\Delta p_{system} = \Delta p_{Fs}$. System resistance includes the various flow resistances encountered, atmospheric conditions, heat exchanger dimensions and the heat exchanger bundle characteristics. Variation of the amount of air flow can be obtained by adjusting either the fan blade angle or the speed of rotation.

The operating cost of the fan is related to the required input electrical power to move the volume of air to properly cool the process fluid inside the heat exchanger bundles. It can be expressed as [80MO1e]

$$P_e = \frac{V_F \Delta p_{Fs}}{\eta_{Fs}\eta_{Fd}} \tag{4.28}$$

The annual operating cost of the fan can be expressed as [80GU1e]

$$C_{Fo} = C_{eav} P_e \tau \tag{4.29}$$

where τ is the number of operating hours (refer to Appendix E).

Various means of fan control that can aid in saving fan operating cost are possible [77SC1e, 80HE2e, 81KO1e]. However, expensive control equipment is needed to perform the control properly [85MO1e, 85MO2e, 93AD1e].

Pump operating cost

The running costs of the indirect dry-cooling system arises from the circulating pumps' power requirements and the fan power if mechanical draft is used. The pumping system provides the necessary pumping head to overcome the hydraulic pressure drops in the cooling system's distribution network. The total pumping power requirement is governed by the water flow rate and the total pressure drop of the cooling system's distribution network. For the indirect cooling system the total pressure drop is the sum of the pressure drop in the condenser, distribution piping and the heat exchanger bundles. The pressure loss in pipe fittings is believed to be 30% to 45% of the pressure drop in the pipes [79CH2e].

The total pressure drop in the cooling system can be expressed as [80MO1e]

$$\Delta p_w = 0.5 f_w \rho_w \frac{L_w}{d_i} v_w^2 \tag{4.30}$$

where L_w is the total equivalent length based on the heat exchanger tube geometry. The required pumping power is

$$P_e = \frac{m_w \Delta p_w}{\eta_p \eta_m \rho_w} \tag{4.31}$$

The ratio $L_w/(n_{tb} n_b L_t)$ is always greater then one due to the resistances of the pipes, headers, and fittings [80MO1e]. The annual operating cost of the pump is [66SC2e, 80GU1e, 89BU2e, 90LO1e]

$$C_{po} = P_e C_{eav} \tau \tag{4.32}$$

Pumping power requirements for the direct contact jet condensers are higher than those for surface condensers.

Fixed charges

The concept of fixed charge rates (FCR) is widely used in the utility industry [70RO1e, 71HA1e, 75MI1e, 76FR1e, 76ZA1e, 78RO1e, 79CH2e, 79NA1e, 80GU1e, 85LI1e, 89ST1e]. The electric utility must charge the lowest electric rates possible consistent with providing an acceptable rate of return on its investment and an acceptable quality of electric service. Fixed charge rate is defined as the annual owning costs of an investment as the percent of the investment. Once the plant becomes operational, the fixed charge rate can be applied to the capital costs to give the annual expenditure required for the entire life of the plant. It includes the following:

(1) Interest or cost of money

(2) Depreciation or amortization

(3) Insurance and taxes

(4) Interim replacements

The fixed charge rate has a yearly variation: it is largest when the plant is first installed and decreases as the plant ages. A uniform annual levelized fixed charge rate is often calculated, which is the present-worth levelized average value of the fixed charge rate (refer to equation (4.5)). For most economic analyses, the levelized annual (or average) fixed charge rate is much easier to apply because only one number is carried in the calculations. The levelized fixed charge rate will provide the same answer as the varying annual fixed charge rate (refer to Appendix E). A detailed analysis of the concept of fixed charge rate can be found in Stoll [89ST1e] and Li and Priddy [85LI1e].

Maintenance costs

Maintenance is defined as the activity/expenditure that keeps the operation and performance of equipment at the original as-built level. The operation (other than those for auxiliary power) and maintenance cost of the power plants vary substantially with both the size of the individual units and the number of units in the plant. The biggest cost is for personnel [86HI1e]. For the cooling system, the annual operating maintenance charge is estimated as a fixed percentage of its capital costs and a figure of 1% is generally quoted in literature [66SC1e, 70RO1e, 72AN1e, 72PE1e, 76FR1e, 79NA1e, 85MA1e].

Routine anticipated operation and maintenance costs would appear to be relatively easy to define. However the actual operation and maintenance costs would appear difficult to define exactly until several years of widespread use of dry-cooling have been logged. Furthermore, maintenance cost is also a function of plant life and generally becomes higher as the plant becomes older. The yearly variation in these costs can also be accounted for by using a levelized value over the plant lifetime [85LI1e]. Planned or unplanned (forced) outages can occur which can lead to either a complete shutdown or derating of a unit. When a component is being repaired as a result of an unscheduled outage, both direct and indirect costs are involved. The direct costs are those costs associated with the maintenance labor and the repaired or new parts. The indirect costs, however, are usually much larger than the direct costs. Purchased power to replace the power that the unit would have produced or the loss in income due to unit derating are examples of indirect costs [85LI1e, 91GU1e]. It is evident from the above that it is not easy to predict the operation and maintenance costs accurately.

In this study the maintenance cost of a specific component is assumed to be equal to the product of the component's capital cost and a specified maintenance cost weighting factor. The maintenance cost weighting factors are assumed to be levelized values over the operating lifetime of the plant. The maintenance costs of the different dry-cooling system components are discussed in Appendix E.

4.7 COST OF LOST PERFORMANCE

The economics of evaluating cooling systems for power plants are closely related to the characteristics of the demand for electricity, e.g. summer peak demand, winter peak demand or utility system.

Ideally, most power stations would prefer to produce a constant maximum power output (base load) without regard to the changes in the ambient conditions. The cooling system has a dominant effect on whether this goal can be realized or not. Once through cooling can provide essentially constant turbine back pressure since there are relatively small swings in the temperature of the water bodies used for cooling. Evaporative cooling can approximately provide a constant turbine back pressure except for combinations of both high drybulb and wetbulb temperatures. On the other hand, dry-cooling is very sensitive to variations of the ambient drybulb temperature. With increasing ambient temperature the dry-cooling system provides less cooling, causing the turbine back pressure to rise, thus reducing the plant efficiency and the plant performance. This is an inherent characteristic of dry-cooling systems and can be treated as such, or can be used to penalize these systems in optimization studies as will be subsequently discussed.

If there is a fixed demand that must be met, any deficit between the net power output and this demand must be provided by another power generating source. This deficit is defined as lost performance and is of relevance in the utility power industry [70RO1e, 71HA1e, 76FR1e 78RO1e, 79HA1e, 80GU1e, 85MA1e].

Lost performance is usually made up of two portions, namely back pressure effects and auxiliary power requirements. Larger more costly dry-cooling systems will reduce lost performance. There exists a tradeoff between capital cost and the cost of lost performance. Therefore one must try to determine the most cost effective manner to operate the dry-cooled plant within the utility system (tradeoff between excess cooling capacity on cold days and inadequate cooling capacity on hot days).

The costs involved in lost performance are determined both by how the capacity and energy losses are evaluated and by how their replacement is provided. Computationally, the output at any hour is calculated as a function of ambient conditions and cooling system and plant performance and then compared to the output of a reference base plant. Fryer [76FR1e] identifies the following methods for defining the base line:

(1) Fixed demand with a fixed heat source

(2) Fixed demand with scaleable steam supply and scaleable plant

(3) Negotiable demand with a fixed heat source

The following means can be employed to make up the lost capacity during high ambient temperatures [76FR1e, 78RO1e]:

(1) Gas turbine peaking unit

(2) Enlarged base units

(3) Purchase power from a power pool

The added generating facility requires both capital expenditure and operating revenues. These costs are, respectively, referred to as the penalty cost due to loss of generating capability and the penalty cost due to the loss of energy generation. The capability penalty is the amount paid for each kW of additional capacity when the unit is unable to produce its required capability and is considered as a capital cost item ($/kWe). The energy penalty is the amount paid for each kWh of additional energy when the unit is unable to produce its required capability and is considered as an operating cost ($/kWh).

Rozenman et al. [78RO1e] state the most commonly used rules to calculate the temperatures above which penalties are considered:

(1) 10 hour rule - the hottest 10 hours of the years are ignored and the highest temperature which is exceeded during this 10 hours is used.

(2) 1% rule - use 1% of the four hottest months which corresponds to 29 hours.

(3) 2.5% rule - use 2.5% of the four hottest months which corresponds to 72 hours.

Choi and Glicksman [79CH2e] concluded that the optimization is very sensitive to the methods of making up lost capacity and state that it is very important to have an accurate representation of the possible methods. The method available to make up lost capacity caused by the dry-cooling towers is very much dependent on the particular condition of a utility and generalizations are not valid without detailed examinations of these conditions.

The power requirements for the circulating pumps and the cooling tower fans present a loss in generating capability from the plant output to the grid and this cost is considered as an additional penalty cost. There are two ways whereby the auxiliary power required to power the fans and the circulating pumps [78RO1e] can be supplemented, namely:

(1) Consider the auxiliary power to be a loss similar to the loss of generating capability at high ambient back pressures. Thus the auxiliary power can be drawn from the same source as the loss in capability at high ambient temperature with the corresponding capacity loss and the energy usage charges [78RO1e, 79HA1e, 80GU1e, 85LI2e].

(2) Recognize that the auxiliary power will always be needed through the entire plant life and choose base plant capacity accordingly. Use base plant cost factors to charge capacity and energy.

In the present study we will not employ the methods (penalties) used by the utility systems to ensure constant power output, but rather recognize the inherent characteristics of the dry-cooling system over its operating range when designing a cost-optimal dry-cooled power generation system.

Auxiliary power (e.g. fans or pumps) is assumed to be purchased from the grid. It is further assumed that the plant's gross power output is designed to take these power requirements into account, because they are an integral part of the dry-cooling system. Gross power is defined as the power generated less all the station auxiliary power, except that of the cooling tower fans and/or the cooling tower pumps.

4.8 TOTAL COST

The total annual cost of the dry-cooling system is:

$$C_{total} = C_{operating} + C_{maintenance} + C_{FCR} \quad (\$/\text{annum}) \tag{4.33}$$

The cost of power generation (attributed to the dry-cooling system) is:

$$C_{power} = \frac{C_{total}}{E_{net}} \quad (\$/\text{kWh}) \tag{4.34}$$

where E_{net} is the net annual power output.
These annual costs present the required objective functions in terms of the geometrical and operating variables. Equation (4.33) is used for the operating point optimization, whereas equation (4.34) is used for the minimization of power generation cost.

4.9 CLOSING REMARKS

In this chapter scalar valued objective functions are derived in terms of the geometrical and operating variables of the dry-cooling systems under consideration. These functions are nonlinear and twice continuously differentiable.

Various aspects of cost estimating, as found in practice, are discussed and a cost estimation model for the dry-cooling systems is derived, based on these principles. Although this cost estimation structure is an approximation to the real-world problem, it will be considered adequate for the purposes of this study. It was not an easy task to obtain cost information from the industry, mainly due to the confidentiality of the matter. These procedures can still further be improved with the co-operation of dry-cooling system equipment manufacturers, vendors and cost and design engineers.

CHAPTER 5

COMPUTATIONAL ALGORITHMS TO SOLVE THE PERFORMANCE EVALUATION AND OPTIMIZATION PROBLEMS

5.1 INTRODUCTION

Engineering optimization is usually concerned with the application of existing techniques ("black box" approach). Realistic engineering design problems have special characteristics that are often not considered while developing strategies for numerical optimization and solution techniques. As a result the development of new techniques and the modification of existing ones are encouraged to make efficient and robust optimization methodology available to engineers.

In this chapter we discuss the computational algorithms used to solve the engineering problems formulated in Chapter 3. Various computational methods do exist for solving these problems (refer to Chapter 2). However, it will be extremely inefficient to choose a method without considering the features of the problem that allow it to be solved more efficiently. The most obvious distinctions between problems involve variations in the mathematical characteristics of the objective and the constraint functions. No one method will solve all problems satisfactorily, which further underlines the advantage of exploiting specialized properties of a particular instance rather than using a "black box" approach.

We shall exploit the specialized properties of the dry-cooling system performance evaluation and optimization problems in order to design efficient computational algorithms for solving these problems. The different computational algorithms will subsequently be discussed.

5.2 PERFORMANCE EVALUATION

The performance evaluation problem can mathematically be expressed as

$$\begin{aligned}&\text{Solve}\quad f_j(\mathbf{x}) = f_j(x_1, x_2, \ldots, x_n) = 0\,, \quad j = 1,\ldots,n\\&\text{subject to the constraints}\\&\mathbf{a}_i^T \mathbf{x} \geq b_i\,, \quad i = 1,\ldots,m\end{aligned} \tag{5.1}$$

where $\mathbf{x}$ presents the operating variables, $f_j(\mathbf{x})$, $j = 1,\ldots,n$ are the model simulation equations (balances) to be satisfied at the operating point and the linear inequality constraints are the infeasibility inequalities. The infeasibility inequalities usually actually reflect physical bounds which, if violated, may cause numerical difficulties. They will, however, not be satisfied as equalities at the solution point. The solution of the performance evaluation problems (operating point and power generation calculations) can thus be found by the simultaneous solution of n nonlinear equations in n variables.

The performance evaluation problem's structure makes it suitable to be solved as a nonlinear least squares (NLS) problem. However, a strategy has to be developed to incorporate the feasibility inequalities into the nonlinear least squares solution strategy. With this in mind, problem (5.1) can be reformulated as a nonlinear least squares problem as follows:

$$
\begin{aligned}
&\text{Solve} \quad \tilde{f}(\mathbf{x}) = 0.5\sum_{j=1}^{n} f_j(\mathbf{x})^2 = 0.5\,\mathbf{f}(\mathbf{x})^T\mathbf{f}(\mathbf{x}) = 0 \\
&\text{subject to the constraints} \qquad\qquad (5.2) \\
&\mathbf{a}_i^T\mathbf{x} \geq b_i\,, \quad i = 1,\ldots,m
\end{aligned}
$$

NLS problems have a special structure that can be effectively exploited during the solution process of problem (5.2). The development of a constrained nonlinear least squares algorithm for solving systems of nonlinear equations subjected to linear inequality constraints will subsequently be discussed.

Solution of systems of nonlinear equations by using a constrained nonlinear least squares approach

In Appendix F, section F.2, we investigate the solution of the standard NLS problem by considering an approach closely related to solving systems of nonlinear equations, i.e. the Gauss-Newton method. The Gauss-Newton method is only locally convergent and can fail or converge slowly from a poor starting point. There are however, two ways of improving the Gauss-Newton method, i.e. using it with a line search or with a trust region strategy. These two approaches lead to two algorithms that are used in practice, i.e. the damped Gauss-Newton method (line search) and the Levenberg-Marquardt method (trust region) [83DE1m]. When applying the Gauss-Newton method for NLS, the trust region method appears to be more robust and efficient than the line search methods [83DE1m, 89DE1m].

We develop an algorithm for the simultaneous solution of the nonlinear equations that is based on the combination of the Levenberg-Marquardt and the damped Gauss-Newton methods for solving NLS problems. Both these methods are used in the same computational algorithm to overcome the difficulties experienced by the basic Gauss-Newton method. The computational algorithm and its implementation are discussed in detail in Appendix F, sections F.3 and F.4. Only the underlying principles of the algorithm will be stated here.

Levenberg-Marquardt methods are characterized by solving the following system of equations at some stage to determine the search direction, $\mathbf{s}^{(k)}$ [87FL1m]

$$\left[\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^T + \lambda^{(k)}\mathbf{I}\right]\mathbf{s}^{(k)} + \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right) = 0 \;, \quad \lambda^{(k)} \geq 0 \tag{5.3}$$

where $\mathbf{J}(\mathbf{x})$ is the n×n Jacobian matrix, the columns of which are the first derivative vectors ∇f_i of the components of $\mathbf{f}$. $\lambda^{(k)}$ is changed during the course of the iterations in order for $\mathbf{J}(\mathbf{x}^{(k)})\mathbf{J}(\mathbf{x}^{(k)})^T + \lambda^{(k)}\mathbf{I}$ to remain positive definite.

By making use of equation (5.3), problem (5.2) can be rewritten as a quadratic programming (QP) problem in order to deal with the imposed linear inequality constraints, i.e.

$$\begin{aligned} &\underset{\mathbf{s}}{\text{Minimize}} \quad 0.5\,\mathbf{s}^{(k)T}\left[\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^T + \lambda^{(k)}\mathbf{I}\right]\mathbf{s}^{(k)} + \left(\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right)\right)^T\mathbf{s}^{(k)} \\ &\text{subject to the linear inequality constraints} \\ &\mathbf{a}_i^T\mathbf{s}^{(k)} \geq b_i - \mathbf{a}_i^T\mathbf{x}^{(k)} \;, \quad i = 1,\ldots,m \end{aligned} \tag{5.4}$$

The resulting QP can be solved by means of any active set method. We use the dual active set QP algorithm in Appendix G (refer to section 5.3). This QP algorithm will also be used to solve the QP subproblems that result from the solution method of the nonlinear constrained optimization problem.

$\lambda^{(k)}$ is decreased by a constant factor if a suitable search direction $\mathbf{s}^{(k)}$ is found which causes a reduction in the objective function. Otherwise $\lambda^{(k)}$ is increased by some factor until the required reduction in the objective function is achieved. Under certain conditions $\lambda^{(k)}$ can become very large, resulting in slow convergence or no convergence at all (bias towards the steepest descent search direction). In these cases we introduce a line search procedure to find a new point $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}$ that causes the objective function to decrease.

5.3 OPTIMIZATION

Realistic optimization models can be quite large and complex and the evaluation of the objective and constraint functions can require substantial calculation. The optimum design process is iterative, needing the same calculations to be performed during each iteration. Thus, the number of function evaluations is a measure of efficiency for an optimization algorithm for engineering problems.

The cost minimization problems considered in this study can all be classified as nonlinear constrained optimization problems. The computation of the objective and constraint equations as well as the physical modeling of the system involves a large number of numerical operations, thus requiring the inherent characteristics of the problem, as discussed in section 3.4, to be exploited.

The structure of the dry-cooling system optimization problems make them suitable to be optimized without repeatedly converging the performance evaluation model equations, i.e. the simulation model will be formulated as part of the optimization problem (infeasible path integrated approach).

The nonlinear programming (optimization) problem to be solved is formulated as [87FL1m]

$$\begin{aligned}
&\underset{\mathbf{x}}{\text{Minimize}} \quad f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n) \\
&\text{subject to the constraints} \\
&c_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m_{eq} \\
&c_i(\mathbf{x}) \geq 0, \quad i = m_{eq} + 1, \ldots, m
\end{aligned} \tag{5.5}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a vector of decision variables (including both the dependent and independent variables). $f(\mathbf{x})$ is the objective function and $\mathbf{c}(\mathbf{x})$ is the vector of constraint functions which includes the model's simulation equations. It is assumed that the bounds on the variables are incorporated into the inequality constraints.

In dry-cooling system design and performance optimization for power plants applications, typical examples of the objective function are minimum annual cost (capital and operational costs) and minimum cost of power generation (refer to Appendix E). The constraint equations on the other hand, usually include the equations describing the conservation of energy, momentum, bounds on the variables, design constraints, and others (refer to Appendices C and D). This formulation tends to disguise the dependence of the operating variables on the geometric variables. The optimization problem is solved in such a way that the equality constraints describing the physical model are only satisfied at the final iteration. In other words, during the intermediate iterations no dry-cooling system exists that will satisfy any operating point.

Problem (5.5) will be solved by means of the Successive Quadratic Programming (SQP) method, because the SQP method has been recognized as one of the most efficient algorithms for solving small to moderately sized nonlinear constrained optimization problems [81HO1m, 83RE1e]. In a computational environment where one function evaluation can correspond to a full simulation, the number of function evaluations is critical for a successful method. It is also known that SQP methods are in general favored in such modeling environments due to the fewer function and gradient evaluations required.

Successive quadratic programming algorithm

The SQP method is derived in section 2.2 as a Newton method for solving the optimality conditions of problem (5.5). The most basic step of the SQP method is the formulation and solution of a sequence of successive QP subproblems to find a search direction, $s^{(k)}$, and to provide estimates to the Lagrange multipliers, $\Lambda^{(k)}$. The QP subproblem has the form

$$\begin{aligned}
&\underset{s}{\text{Minimize}} \quad \nabla f\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} + 0.5\, \mathbf{s}^{(k)T} \mathbf{B}^{(k)} \mathbf{s}^{(k)} \\
&\text{subject to the linearized constraints} \\
&c_i\left(\mathbf{x}^{(k)}\right) + \nabla c_i\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} = 0, \quad i = 1,\ldots,m_{eq} \\
&c_i\left(\mathbf{x}^{(k)}\right) + \nabla c_i\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} \geq 0, \quad i = m_{eq}+1,\ldots,m
\end{aligned} \tag{5.6}$$

where **B** is an approximation to the Hessian matrix of the Lagrange function of problem (5.5).

SQP methods have very desirable computational properties:

(1) they are globally convergent since the search direction has descent for a suitable line search objective function;

(2) near the solution point they converge superlinearly for a step length of one, because the Hessian of the Lagrange function contains curvature information about the problem function;

(3) only first order information is used;

(4) and they are suited to inequality as well as equality constraints because a general QP problem can be solved during each iteration.

With the use of proper numerical procedures these methods have the potential of solving complex nonlinear constrained engineering optimization problems.

Although quite a few SQP algorithms have been proposed, only a few of those have been coded and are generally available for distribution [86THle, 88EDle]. When a computational philosophy is implemented via different numerical schemes, different performances may result in practice. There are several numerical aspects in which the different SQP codes differ that sometimes result in great computational inefficiencies [86THle, 87KIle].

We will use Powell's implementation of the SQP algorithm, subroutine VMCWD, as a basis for the implementation of the SQP method [82PO1m, 82PO2m]. The details of the implementation of this SQP algorithm are described in Appendix H. Several modifications were made to the VMCWD to address its computational inefficiencies as briefly outlined below:

(1) The most significant change to Powell's implementation is the change in the algorithm to solve the QP subproblem. Powell used a feasible point primal QP algorithm with his SQP implementation. We replace this algorithm by a suitably tailored dual QP algorithm for use with SQP methods (for strictly convex QP problems). This QP algorithm is derived in Appendix G and will be discussed in a subsequent section.

(2) The implementation of VMCWD requires that the gradients of the objective and constraint functions are calculated even during line search iterations, which is unnecessary. A modification was made so that these functions and their corresponding gradients are only calculated when required.

(3) VMCWD requires that the user provides routine for the calculation of the objective and constraint functions as well as their corresponding gradients. The natural structure of the optimization problem, as discussed in section 3.4, are exploited in these routines. An extremely efficient function and gradient evaluation procedure is obtained as a result.

(4) When the size of the nonlinear constrained optimization problem becomes large, the cost of the optimization is dominated by the computational overhead and storage requirements of solving the QP subproblems. The optimization problem that results from considering the minimum cost of power generation, falls into this category. Although the infeasible path integrated approach solves this problem satisfactorily, it consumes a large amount of computational time. A reduced Hessian SQP decomposition strategy is implemented to solve this problem. The treatment of large-scale SQP problems will subsequently be discussed.

Section H.3 contains a detailed discussion of all the modifications to VMCWD.

Decomposition of large-scale successive quadratic programming problems

When the number of variables is large, there are basically two ways to apply SQP methods: either decomposition techniques can be used or the natural problem structure can be exploited. The natural problem structure has been exploited to a certain degree in the infeasible path integrated approach. However, the reasons for advocating decomposition are that the QP subproblems can be very expensive to solve, the storage requirements can become quite large and the problems have a small number of degrees of freedom (difference between the number of variables and the number of active constraints). Decomposition techniques use the equality constraints to eliminate the dependent variables and reduce the size of the QP subproblem that must be solved at each iteration. The QP subproblem is formulated in the independent variables only.

Equation (5.5) can be reformulated by separating the dependent and independent variables as well as the model simulation equations and the general equality constraints:

$$\begin{aligned}
&\underset{\mathbf{x},\mathbf{y}}{\text{Minimize}} \quad f(\mathbf{x},\mathbf{y}) \\
&\text{subject to the constraints} \\
&c_i(\mathbf{x},\mathbf{y}) = 0, \quad i = 1,\ldots,m_{eq} \\
&c_i(\mathbf{x},\mathbf{y}) \geq 0, \quad i = m_{eq}+1,\ldots,m_t \\
&\mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0}
\end{aligned} \tag{5.7}$$

where

f objective function (e.g. cost function)

$\mathbf{x}$ vector of independent variables (e.g. geometrical variables)

$\mathbf{y}$ vector of dependent variables (e.g. operating variables)

c_i general constraints (e.g. geometric and/or physical constraints)

$\mathbf{h}$ vector of equality constraints (e.g. equations needed for model simulation)

m_{eq} number of equality constraints

m_t total number of general constraints

n total number of variables

For fixed **x** values (independent variables), the values of **y** (dependent variables) can be determined by solving the equality constraints, $\mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0}$ (feasible path approach). The reduced problem formulation (the dependent variables are expressed in terms of the independent variables) can be stated as follows:

Let

$$\mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0} \tag{5.8}$$

Solve for

$$y = \Phi(x) \tag{5.9}$$

where $\Phi(x)$ denotes implicit functions that define the dependent variables in terms of the independent variables. With the dependent variables, y, known, one is able to formulate the reduced problem in terms of x (independent variables) only, i.e.

$$\begin{aligned}
&\underset{x}{\text{Minimize}} \quad F(x) = f(x, \Phi(x)) \\
&\text{subject to the constraints} \\
&C_i(x) = c_i(x, \Phi(x)) = 0, \quad i = 1, \ldots, m_{eq} \\
&C_i(x) = c_i(x, \Phi(x)) \geq 0, \quad i = m_{eq} + 1, \ldots, m_t
\end{aligned} \tag{5.10}$$

Problem (5.10) has the advantage of reducing the number of variables in problem (5.7).

Studies have shown that performing an optimization study while repeatedly simulating the physical model requires prohibitive computational effort [85BI2e, 93SC1e]. The derivation of the reduced problem is based on the assumption that equations (5.8) can be satisfactorily solved at each iteration (feasible path approach). However, this is not always the case as combinations of independent variables can be found that do not correspond to a practical system, i.e. one is unable to solve for the dependent variables [83TA1e]. It can thus be expected that the method will perform well near the optimum solution and will most probably run into the above-mentioned difficulty far from the optimum point.

Due to the fact that the solution of the system of equations in (5.8) is an iterative process, a great degree of inaccuracy will be introduced if the derivatives of the objective and constraint functions with respect to the dependent variables are determined numerically. Gill et al. [85GI2m] warn against the defining of problem functions that are the result of some iterative procedure. The solution of the subproblems to full machine precision will require considerable computational effort, making the optimization process very inefficient.

The sequential approach (feasible path) discussed above is thus a very primitive and inefficient way to take advantage of the problem structure [93SC1e]. However, this approach forms the basis of more advanced decomposition techniques.

In the infeasible path integrated approach the optimization problem and the physical model are converged simultaneously. This gives rise to a much better-behaved problem. A potentially advantageous method is to combine the infeasible path integrated approach with a suitable decomposition technique in order to reduce the size and computational overheads of the resulting QP subproblems. The solution of the QP subproblems is usually the most time consuming step in conventional SQP methods.

In Appendix I a coordinate bases decomposition strategy is constructed that is well suited to take advantage of the underlying mathematical structure of the optimization problem. This method is based on Powell's original SQP implementation [78PO1m]. The bases are formulations based on a partitioning of the variables. The dependent variables are "eliminated" by using the model simulation equations. A much smaller QP subproblem with a reduced Hessian matrix of order equal to the number of independent variables is obtained as a result. The resulting reduced Hessian SQP method is solved in the independent variable space only, while the move in the dependent variable space is obtained directly from a Newton step for the solution of the model equations. Consequently the structure of the Jacobian matrix and any tailored procedure to calculate the Newton steps are fully exploited. The model equations are not exactly solved during each iteration, but an approximation to their solution is computed.

The merit function (L_1 – penalty function) of the SQP method is constructed to include the general constraints as well as the equations needed for model simulation (all the constraints in problem (5.7)). The construction of the merit function in this particular way ensures that the dependent variables are taken into account during the optimization process and converged together with the independent variables. A suitable line search step length parameter is found that scales the search direction in both the dependent and independent variable space. A step in the direction of the solution is given such that convergence of the model equations and the optimization problem will be reached simultaneously. The convergence criterion also includes contributions of both the dependent and independent variable spaces in order to force simultaneous convergence.

Dual active set algorithm for convex quadratic programming problems

The solution of the strictly convex (positive definite) QP subproblem is a major calculation in SQP methods and can affect their overall efficiency. Several numerical procedures can be used to solve the QP subproblem. Most implementations of SQP currently employ QP routines that are based on primal active set strategies. Considerable effort is expended in determining an initial feasible point for the QP. However, we will use the dual method for solving strictly convex QP problems which is particularly suitable for use with SQP methods for nonlinearly constrained optimization calculations [82GO1m]. The most obvious advantage of the dual method is that the unconstrained minimum of the QP objective function provides an initial feasible solution.

In this section we describe the dual quadratic programming algorithm for convex QP problems subject to general linear equality/inequality constraints, i.e. problems of the form

$$
\begin{aligned}
&\underset{\mathbf{x}}{\text{Minimize}} \quad q(\mathbf{x}) = 0.5\mathbf{x}^T\mathbf{G}\mathbf{x} + \mathbf{g}^T\mathbf{x} \\
&\text{subject to the constraints} \\
&\mathbf{a}_i^T\mathbf{x} = b_i\,, \quad i = 1,\ldots,m_{eq} \qquad \left(\mathbf{A}_{eq}^T\mathbf{x} = \mathbf{b}\right) \\
&\mathbf{a}_i^T\mathbf{x} \geq b_i\,, \quad i = m_{eq}+1,\ldots,m \qquad \left(\mathbf{A}_{ineq}^T\mathbf{x} \geq \mathbf{b}\right)
\end{aligned}
\tag{5.11}
$$

where $\mathbf{G}$ is required to be positive definite. In this case, a unique $\mathbf{x}$ solves the problem or the constraints are inconsistent. This new QP solution technique is based on the original algorithm by Goldfarb and Idnani [83GO1m], but substantially modified to account for the fact that the QP is only a subproblem of the SQP algorithm. This new QP algorithm is also used in the solution of the constrained nonlinear least squares problem discussed in section 5.2 and Appendix F. The dual QP method and its implementation, as well as the corresponding dual QP algorithm are described in Appendix G.

It is highly recommended to implement a specialized QP algorithm that make use of the features of the QP subproblems associated with the SQP methods. The modifications to the basic QP algorithm stated in Appendix G focus mainly on efficient procedures to treat the active constraint set that will change from one iteration to the next.

Efficient and stable updating procedures are used to modify the corresponding matrix factorizations used within the dual algorithm when a constraint is deleted from or added to the current set of active constraints. The computational penalty in dropping constraints can be considerable and care must be taken to minimize the number of incorrect constraints added to the active set. Powell [85PO1m] reports that the original implementation of the QP algorithm [83GO1m] can become unstable under certain conditions. As a result, we have decided not to implement the matrix updating schemes described by Goldfarb and Idnani [83GO1m].

The first few iterations of the SQP method is generally marked by considerable changes in the constraint active set from one iteration to the next. However, once the correct active set of the nonlinear programming problem has been identified, the work per iteration can be reduced by using the constraints active at the previous iteration as an initial guess for the current iteration. This is referred to as a "warm start" option.

Powell [83PO2m, 85PO1m] implemented the algorithm proposed by Goldfarb and Idnani [83GO1m] in a subroutine called ZQPCVX. ZQPCVX will also be used as a "black box" to solve the QP subproblems of the SQP method as well as the constrained NLS problem. The major advantages of the current QP algorithm over ZQPCVX will be discussed later.

Post-optimality analysis

Calculation of the change in the value of the optimal solution in response to changes in coefficients in the objective functions or constraints is known as post-optimality or sensitivity analysis. Information concerning the sensitivity of the optimum to changes or variations in a parameter is very important; sometimes more important than the solution itself. The status of the solution cannot be fully understood without such information.

Sensitivity information can be obtained from the optimal solution without actually recomputing the problem. The Lagrange multiplier for a given constraint indicates how much the objective function will change for a differential change in the constraint constant (limit) [84LU1m, 89AR1e]. The sensitivity of the objective function with respect to some parameter is obtained by evaluating the partial derivative of the optimum Lagrange function with respect to the parameter [83FI1m].

The post-optimality analysis as applied in the current investigation is described in detail in section J.3. A scale-invariant measure of sensitivity is derived such that sensitivity results can be directly compared.

5.4 SCALING

It is well known that properly scaled variables and constraints can dramatically improve the efficiency and the accuracy of optimization methods. Variables of the scaled problem should be of similar magnitude and of order unity in the region of interest. The constraints should also be scaled such that the value of deviations from zero are of the same order of magnitude.

The implementation of all the above-mentioned computational algorithms are based on properly scaled variables and constraints. We use information about the problem to scale the variables and the constraints. The topic of scaling and its application in the computational procedures are discussed in detail in section J.2.

5.5 CLOSING REMARKS

In this chapter various computational algorithms for solving the engineering optimization problems and model simulations are briefly discussed. The reader is referred to the corresponding appendices for detailed discussions, derivations and modifications. The basic algorithms are modified by employing practical modifications found in the literature or by exploiting the problem structure.

A global solution to an optimization problem depends on the convexity of the objective and constraint functions. Most of the engineering optimization problems are not convex or convexity is very difficult to prove. Thus global optimality of local solutions cannot be guaranteed and there are usually multiple local optimum solutions. From a practical standpoint, the optimization process must be started from various initial points to see if a consistent optimum is obtained. One can then be reasonably assured that this is the true optimum.

Gill et al. [81GI1m, 85GI2m] discuss basic modeling principles that influence the performance of optimization methods. These principles enable one to construct a well-behaved mathematical model and give insight into the formulation of robust algorithms. The algorithms are implemented by following these principles. A good optimization algorithm must be reliable, general, efficient, easy to use and must be properly implemented [86AR2e]. The various algorithms have also been tested on some of the test problems found in the literature (e.g. [81HO1m, 81MO1m]) and performed satisfactorily.

The next chapter will describe the computer codes that were developed to perform the optimization and performance evaluation calculations of the problems formulated in Chapter 3.

CHAPTER 6

COMPUTER PROGRAMS

6.1 INTRODUCTION

This chapter discusses the computer programs developed to implement the procedures described in the previous chapters. These programs deal with both forced draft direct air-cooled condensers and natural draft indirect dry-cooling towers with particular reference to power plant applications. The program listings are too long to be included in this dissertation (±10000 lines per program). The experimental results and theoretical work included in Appendices A-L provide all the detail included in the different computer programs.

The programs were written in double precision FORTRAN 77 and developed on the VAX 6000-410 and ALPHA 3000-800 computers of the University of Stellenbosch. The programs can also be implemented on 486-based IBM personal computers with at least 16 Mb RAM and a FORTRAN compiler that can access the extended memory.

6.2 PROGRAM STRUCTURE AND IMPLEMENTATION

The main program structure is shown in figure 6.1 and consists of a main program and various subroutines and functions to execute the performance evaluation and optimization calculations. The subroutines are used to perform operations that logically belong together, while the functions are used to calculate values that are used throughout the program. Most of the subroutines describe the problem and provide an interface between the user and the solution techniques. The programs are menu-driven and the main programs control interactive communication with the user.

The input data can be entered from the keyboard or extracted from previously stored data files, examined and changed, and saved in unique input files. The results can be displayed on the screen or saved in unique output files. Examples of the format of the input and result files are given in Appendices K and L.

Apart from the input data needed to describe and formulate the problem (Appendices C, D, E refer), the user must also specify internal parameters such as the convergence criterion (tolerance), the maximum number of function evaluations, and the finite difference step size used in the numerical differentiation. The number of variables, equality and inequality constraints are specified within or calculated by the program. The user can also choose between using round or non-round finned tubes, the type of fin material (Aluminum, Copper, Steel, Galvanized steel) and which finned tube performance correlations to use (refer to Appendix B).

Input data such as the fan performance characteristics and the turbo-generator characteristics are entered as polynomial regression functions. The user must supply the coefficients of a third or fourth order polynomial that best fit the corresponding data points.

All the input data are entered, either in SI base units or derived units. For example millimeters or meters are used for length measurements, depending on which is the most convenient in the specific application. The derived units are automatically converted to SI base units before any computations are performed.

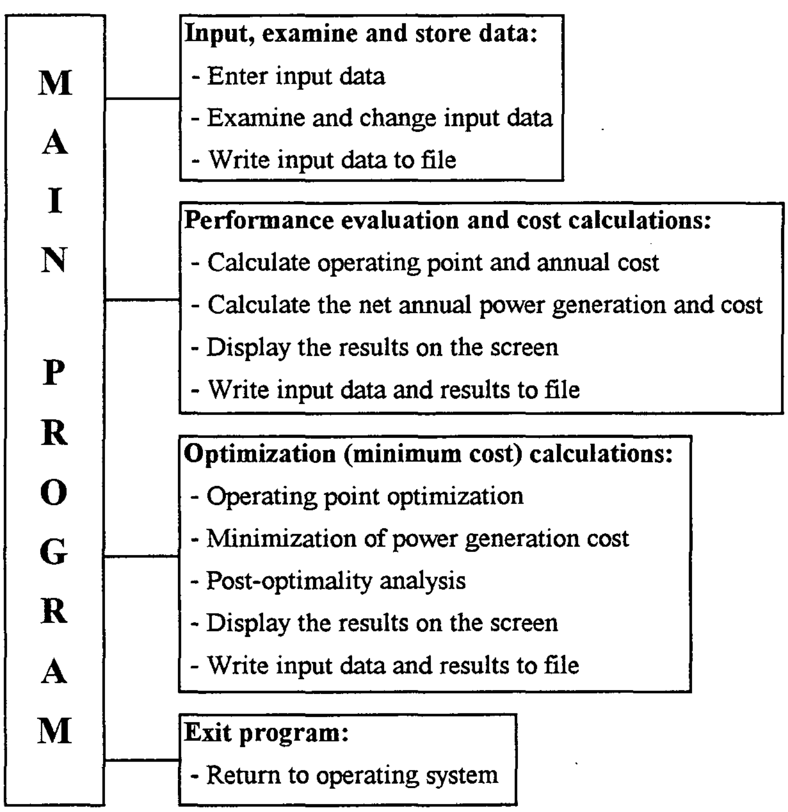

Figure 6.1: Main program structure.

In performing the numerical calculations, it is necessary to scale the decision variables, objective functions and the constraint functions. Using scaling, the efficiency of the solution processes will be enhanced. The programs are implemented in such a way that the solution algorithms only work with the scaled problem. Some scale factors are derived from the problem parameters internal to the program, while others are entered as input (lower and upper bounds on the variables). The rescaled parameters are used to generate the output results. Upper and lower bounds on some geometrical (independent) decision variables and geometrical constraints are also entered as input. These bounds provide the user with some control over the geometrical constraints imposed on the problem.

The computational procedures are of an iterative nature and thus require initial values for the decision variables. The choice of these initial values can have a substantial impact on the rate of convergence of an algorithm. In many practical applications a good starting point is available or can be obtained from some preliminary analyses. In the programs we have implemented a strategy to start with an initial feasible point, because this can substantially reduce the computational effort. Feasibility inequalities are also introduced to prevent the violation of the physical laws and the application region of the equations that influence the dry-cooling system simulation process.

Various error and warning messages are included to effectively guide the user during program execution. These messages are displayed on the screen. The input data are checked for obvious errors. If numerical difficulties are experienced by the solution techniques during program execution, error messages are displayed. This enables the user to take the required actions in order to continue with the computations. The warning messages guard against poor design practices that may occur, e.g. when the actual fan volume flow rate is less than its optimum value.

On completion of the optimization calculations, the following information, regarding the performance of the SQP solution method, is displayed: the total elapsed time, the total CPU time, the total CPU time consumed by the QP subproblems, the total number of iterations and the total number of function calls (objective function, constraint and gradient evaluations). This information is used to compare the performance of the different SQP and QP implementations with each other.

The user has the choice to create a convergence history data file in order to inspect the path followed by the solution technique to obtain convergence. Possible difficulties, such as poor scaling or solution technique inefficiencies, can be identified in this way.

6.3 PERFORMANCE EVALUATION

The performance evaluation of the dry-cooling systems consist of the following:

(1) operating point calculations;

(2) power generation calculations;

(3) annual cost estimation.

These calculations are performed by combining the following building-blocks in a logical way:

(1) the dry-cooling system simulation models, cost analysis and the corresponding input data to describe the problem (refer to Appendices A, B, C, D, E, K and L);

(2) the solution methods and scaling principles (refer to Appendices F, G, and J);

(3) output routines to display or store the computational results (refer to Appendices K and L).

The user-supplied input data must contain initial estimates for the final values of operating variables. An initial feasible point with regard to all the feasibility inequality constraints are determined before the main iterations are commenced.

The performance evaluation calculations can be used to investigate the effect on the dry-cooling systems performance by manually changing certain variables. This procedure is particularly suited to variables that take on discrete or integer variables, because the optimization process assumes all the variables to be continuous. The performance evaluation and optimization procedures are therefore interrelated.

6.4 OPTIMIZATION

The optimization calculations of the dry-cooling systems consist of the following:

(1) operating point optimization (minimum annual cost);

(2) minimum annual cost of power generation;

(3) post-optimality analysis.

These calculations are performed by combining the following building-blocks in a logical way:

(1) the dry-cooling system simulation models, cost analysis and the corresponding input data to describe the problem (refer to Appendices A, B, C, D, E, K and L);

(2) the optimization solution methods, post-optimality analysis and scaling principles (refer to Appendices G, H, I and J);

(3) output routines to display or store the computational results (refer to Appendices K and L).

The initial requirements before the operating point optimization and the minimization of power generation cost can be executed, are the completion of operating point and power generation calculations respectively. Thus the optimization calculations are started from an initial feasible starting point.

The optimization calculations are performed by following the infeasible path integrated approach. Both the modified SQP method (Appendix H) and the reduced Hessian SQP decomposition technique for large-scale problems (Appendix I) are implemented in separate programs. The decomposition technique is only applied to the problem about minimization of the power generation cost, because it can be classified as a large-scale optimization problem. Relatively little advantage will be obtained by applying the decomposition technique in the case of the operating point optimization problem.

The decision variables for the optimization calculations can be divided into dependent and independent variables. The dependent or operating variables are always present during the optimization process, whereas the independent or geometrical variables can be fixed at some value or varied during the optimization process. The fixing of independent variables can also result in disregarding some inequality constraints for a particular instance. Typical upper and lower limits on the decision variables are required for scaling purposes. The constraints are scaled by multiplication with a suitable constant. Upper and lower limits on some independent decision variables are also supplied as input in order to prevent unrealistic results.

The objective and constraint functions used in this study are continuous and differentiable. If a function is not continuous or differentiable then conventional optimization theory is not adequate. The constraints are also carefully formulated to eliminate the possibility of linear dependence, conflicting requirements and inconsistent constraint equations (infeasible problem).

In nonlinear programming (optimization) problems, the decision variables are often assumed to be continuous. In practice, however, discrete and integer variables often arise. The suggested procedure is to solve the problem assuming continuous decision variables. Then the nearest discrete/integer values are assigned to the variables. They are then held fixed and the optimization is performed again. This procedure is repeated until all the variables have proper values in order to obtain a feasible solution. Although this method requires additional computational effort, it is straightforward and does not require any additional software. In our case only the number of tubes per heat exchanger bundle assumes an integer value and the procedure is implemented satisfactorily. It is important to evaluate all the possible combinations when more than one variable assume integer or discrete values.

6.5 COMPUTER PROGRAMS

The computer programs that are described below are based on the theory and algorithms presented in Appendices A-J. All the different implementations require the same input data and produce the same output. The objective function, constraint and gradient evaluations as well as the model simulations are performed similarly. They only differ in the computational algorithms used to solve the performance evaluation and optimization problems. Each program is given a unique name which is used in all the following discussions. The following acronyms are used in the program names: FD denotes forced draft, while ND denotes natural draft. The differences between the program implementations are shown in table 6.1.

Table 6.1: Computer programs.

Program name	**Performance evaluation algorithms**	**Optimization algorithms**	**Remarks**
FDOPT1 NDOPT1	NLS - Appendix F QP - Appendix G	SQP - Appendix H [82PO1m, 82PO2m] QP - Appendix G	Performance evaluation: Constrained NLS approach Optimization: Infeasible path integrated approach
FDOPT2 NDOPT2	NLS - Appendix F QP - ZQPCVX [83PO2m, 85PO1m]	SQP - Appendix H [82PO1m, 82PO2m] QP - ZQPCVX [83PO2m, 85PO1m]	Performance evaluation: Constrained NLS approach Optimization: Infeasible path integrated approach
FDOPT3 NDOPT3	NLS - Appendix F QP - Appendix G	SQP - Appendix H, I [78PO1m, 82PO2m] QP - Appendix G	Performance evaluation: Constrained NLS approach Optimization: Infeasible path integrated approach, large-scale decomposition, improved coordinate bases algorithm [93SC1e]
FDOPT4 NDOPT4	NLS - Appendix F QP - Appendix G	SQP - Appendix H, I [78PO1m, 82PO2m] QP - Appendix G	Performance evaluation: Constrained NLS approach Optimization: Infeasible path integrated approach, large-scale decomposition, standard coordinate bases algorithm [83LO1e]

Powell's original SQP method [78PO1m] is implemented in such a way that both general NLP problems and large-scale NLP problems can be solved in the same program. In FDOPT3, FDOPT4, NDOPT3 and NDOPT4 the operating point optimization computations use Powell's original SQP implementation (Appendix H), while the minimization of the power generation cost problems use the large-scale SQP decomposition method based on this particular SQP implementation (Appendix I).

The term $\mathbf{Z}^T\mathbf{BY}s_Y$ is included in FDOPT3 and NDOPT3 (improved coordinate bases algorithm), whereas it is ignored in FDOPT4 and NDOPT4 (standard coordinate bases algorithm).

6.6 CLOSING REMARKS

The computer programs are implemented to perform the dry-cooling system performance evaluation, cost estimation and optimization calculations as efficiently as possible. The programs are capable of designing, costing and optimizing dry-cooling systems with particular reference to power plants. Various options are available to the user to facilitate decision making.

It is beyond the scope of this dissertation to provide a full description of the computer programs with specific reference to the functions and subroutines; all the relevant information is discussed throughout the dissertation. The solution techniques are easy to use, and implemented to perform efficiently and robustly on the application problems. These techniques exploit the mathematical structure of the problems to be solved.

The programs listed in table 6.1 were run on a set of test problems and the results are discussed in the next chapter.

CHAPTER 7

NUMERICAL RESULTS AND DISCUSSION

7.1 INTRODUCTION

In this chapter we present a numerical illustration of the computational algorithms discussed in Chapter 5 and implemented in the computer programs listed in table 6.1. The various programs are evaluated by solving two example problems and the results as well as their performance are compared. The example problems include performance evaluation and optimization problems on both forced draft direct air-cooled condensers and natural draft indirect dry-cooling towers. Evaluation criteria such as accuracy, efficiency and reliability are used to evaluate program performance. A discussion of the program results and the performance of the solution methods concludes the chapter.

7.2 NUMERICAL EXAMPLES AND RESULTS

Typical examples of both forced draft direct air-cooled condensers and natural draft indirect dry-cooling towers were chosen to illustrate the capabilities and compare the performance of the computer programs listed in table 6.1. Although the computational algorithms were also extensively tested on various mathematical programming problems stated in the literature (e.g. 81HO1m, 81MO1m, 83DE1m, 85NO1m) and dry-cooling system problems, the results will not be given in this dissertation.

The implementation of the solution methods and the principles followed during program execution are similar for the forced draft direct air-cooled condensers and natural draft indirect dry-cooling towers. The results depend strongly on the proposed dry-cooling system cost estimation procedure and the values allocated to the various cost coefficients (refer to Appendix E).

The following numerical calculations are performed:

(1) Operating point and annual cost calculations;

(2) Net annual power generation and cost calculations;

(3) Operating point optimization (minimum annual cost) and the corresponding post-optimality analyses;

(4) Minimization of power generation cost and the corresponding post-optimality analyses.

Evaluation criteria

The evaluation criteria that have been used in this dissertation to analyze the performance of the solution techniques are as follows:

(1) Accuracy: All the computations are performed in double precision and it is required that all the programs give the same results when applied to the same problem. The convergence criteria for both the performance evaluation and optimization computations are set to 10^{-7}. All the gradient calculations in this study have been done numerically (forward differences); a step length scaling parameter of 10^{-8} was used [83DE1m]. The same variable and constraint scaling methods and parameters were also employed in all the programs.

(2) Reliability: A reliable program implementation is expected to give a feasible and accurate solution of the problem under consideration. A failure to do so gives rise to an unreliable program.

(3) Efficiency: This criterion gives a measure of the convergence speed of the program implementation. The number of objective function, constraint and their corresponding gradient evaluations as well as the total computing (CPU) time are used to measure efficiency.

The following acronyms and definitions are introduced to measure efficiency:

(1) TOTCPU: The total CPU time (in seconds) used until the required convergence is reached.

(2) QPCPU: The total CPU time (in seconds) used to solve the QP subproblems of the SQP methods.

The difference between TOTCPU and QPCPU gives an indication of the total CPU time used to set up the optimization problem, perform function and gradient evaluations, Hessian updating, etc.

(3) NFEVAL: The number of objective function, constraint and their corresponding gradient evaluations performed during program execution. NFEVAL corresponds to the number of complete system simulations.

When a line search is performed during optimization (SQP method), the resulting increase in NFEVAL corresponds to objective function and constraint evaluations. The corresponding gradient evaluations are only performed once the line search has been successfully completed (refer to Appendix H).

(4) NITER: Number of major SQP iterations. NITER is always less than NFEVAL.

The maximum number of iterations for both performance evaluation and optimization problems can be set by the user and was not used to limit program execution. No CPU times are measured in the case of the performance evaluation computations, because the solution methods are only applied to small dimensional problems that consume very little CPU time.

Examples

Example 1: Forced draft direct air-cooled condensers

A forced draft direct air-cooled condenser consists of an array of $5\times6 = 30$ A-frame units as shown schematically in figure C.2. Each unit has a 9.145 m diameter axial flow fan. The fan drive is supported by a bridge structure, while a safety screen protects the inlet to the fan. The non-freestanding fan platform, supported by a steel and concrete structure, is 25 m above the ground level. Walkways of 0.4 m width are located between adjacent A-frames and the condenser is surrounded by windwalls.

The condenser consists of 2 rows of extruded bimetallic (steel tube, aluminum fins) round finned tubes with an equilateral triangular tube layout. The finned tube geometric details are listed in table K.1. The tubes are inclined at an angle of 60° with the horizontal and are 10 m long. Each tube row has different performance characteristics such that approximately the same amount of steam condenses in each tube row to ensure that noncondensables are not trapped in the condenser. Saturated steam at a design temperature of 60°C is supplied to the condenser by means of a steam header with an effective diameter of 1.25 m. The heat transfer and pressure drop correlations of Ganguli, Tung and Taborek [85GA1e] are used to evaluate the performance of the round tubes.

The condenser unit operates under ambient conditions where the design atmospheric pressure is $86400\ \mathrm{N/m^2}$ and the corresponding design air temperature is 15.6°C. The other geometric and operating details of this condenser, as well as initial approximations to the final values of the unknown operating variables (e.g. the air outlet temperatures) are listed in table K.1. The values of the different cost coefficients for the cost estimation procedure given in Appendix E can also be found there. The coefficients of polynomial curve fits for the fan performance characteristics are listed in table K.2 (refer to figure C.5).

The condenser forms part of a power plant that is erected at the location corresponding to the design atmospheric pressure stated above. The corresponding yearly temperature distribution (drybulb and wetbulb temperatures) is listed in table K.3 (refer to figure C.7). The air-cooled condenser is coupled to a turbo-generator set. The coefficients of polynomial curve fits for the turbo-generator performance characteristics can be found in table K.4 (refer to figure C.6).

With the above specifications and performance characteristics known, the following performance evaluation, cost and optimization calculations are performed:

(1) Operating point and annual cost calculations: Determine the operating point and annual cost of the air-cooled condenser at the design atmospheric and steam inlet conditions.

(2) Net annual power generation and cost calculations: Determine the annual net power output and the corresponding annual cost if the turbo-generator set is coupled to this air-cooled condenser. The fans rotate at a fixed speed of 100 rpm.

(3) Operating point optimization and the corresponding post-optimality analyses: Find the combination of operating and geometrical variables that will minimize the total annual cost of the air-cooled condenser that operates under the specified design operating conditions and rejects the same amount of heat as at the operating point. Determine the scale-invariant sensitivity of the optimal solution to variations in the cost coefficients, prescribed parameters and the geometrical constraints.

The following geometrical variables are varied during the optimization calculations (refer to table C.1): $H_3, d_F, \theta_F, N_F, n_{tb(max)}, L_t, \theta_b, d_o, t_t, t_r, d_f, t_f, P_{f1}, P_{f2}, P_t$.

(4) Minimization of power generation cost and the corresponding post-optimality analyses: Find the combination of operating and geometrical variables that will minimize the ratio of the total annual cost of the air-cooled condenser to the annual net energy output of the turbo-generator set it is coupled to. Determine the scale-invariant sensitivity of the optimal solution to variations in the cost coefficients, prescribed parameters and the geometrical constraints.

The following geometrical variables are varied during the optimization calculations: $H_3, d_F, \theta_F, N_F, n_{tb(max)}, L_t, \theta_b, d_o, t_t, t_r, d_f, t_f, P_{f1}, P_{f2}, P_t$ (refer to table C.1). Variable speed drives are used to find the most economical fan operating speed for each temperature data set.

<u>Example 2: Natural draft indirect dry-cooling towers</u>

A natural draft hyperbolic concrete cooling tower, as shown schematically in figure D.2, has the following dimension: $H_5 = 120$ m, $H_3 = 13.67$ m, $d_5 = 58$ m and $d_3 = 82.958$ m. The tower shell is supported by 60 tower supports of 0.5 m diameter. It is further assumed that the tower shell has an uniform thickness of 0.25 m.

142 heat exchanger bundles are arranged radially in the form of V-arrays at 30.75° bundle semi-apex angles in the base of the tower shell. Due to geometrical considerations, only 52.4 % of the tower inlet cross-sectional area is covered with heat exchanger bundles. Each bundle consists of 4 rows of finned tubes, 15 m long and with an effective width of 2.262 m. The headers are designed to allow two water passes through the 4 tube rows. Extruded bimetallic (steel tube, aluminum fins) round finned tubes with an equilateral triangular tube layout are used. The finned tube geometric details are listed in table L.1. The heat transfer and pressure drop correlations of Robinson, Briggs and Young [63BR1e, 66RO1e] are used to evaluate the performance of the round tubes.

Hot water at a mass flow rate of 4390 kg/s is pumped through the finned tubes. The design inlet water temperature is 61.45°C, while the design atmospheric pressure is 86400 $\mathrm{N/m^2}$ and the corresponding design air temperature is 15.6°C. The other geometric and operating details of this cooling tower, as well as initial approximations to the final values of the unknown operating variables (e.g. the air outlet temperature) are listed in table L.1. The values of the different cost coefficients for the cost estimation procedure given in Appendix E can also be found there.

The cooling tower forms part of a power plant that is erected at the location corresponding to the design atmospheric pressure stated above. The corresponding yearly temperature distribution (drybulb and wetbulb temperatures) is listed in table L.2 (refer to figure D.4). The cooling tower is coupled to a turbo-generator set. The coefficients of polynomial curve fits for the turbo-generator performance characteristics can be found in table L.3 (refer to figure D.3).

With the above specifications and performance characteristics known, the following performance evaluation, cost and optimization calculations are performed:

(1) Operating point and annual cost calculations: Determine the operating point and annual cost of the cooling tower at the design atmospheric and water inlet conditions.

(2) Net annual power generation and cost calculations: Determine the annual net power output and the corresponding annual cost if the turbo-generator set is coupled to this cooling tower.

(3) Operating point optimization and the corresponding post-optimality analyses: Find the combination of operating and geometrical variables that will minimize the total annual cost of the cooling tower that operates under the specified design operating conditions and rejects the same amount of heat as at the operating point. Determine the scale-invariant sensitivity of the

optimal solution to variations in the cost coefficients, prescribed parameters and the geometrical constraints.

The following geometrical variables are varied during the optimization calculations (refer to table D.1): $H_5, H_3, d_5, d_3, n_{tb(max)}, L_t, \theta_b, d_o, t_t, t_r, d_f, t_f, P_f, P_t$. This example corresponds to case 1 (m_w and T_{wi} fixed) described in section D.7. The other cases can also be illustrated in a similar manner.

(4) Minimization of power generation cost and the corresponding post-optimality analyses: Find the combination of operating and geometrical variables that will minimize the ratio of the total annual cost of the cooling tower to the annual net energy output of the turbo-generator set it is coupled to. Determine the scale-invariant sensitivity of the optimal solution to variations in the cost coefficients, prescribed parameters and the geometrical constraints.

The following geometrical variables are varied during the optimization calculations: (refer to table D.1): $H_5, H_3, d_5, d_3, n_{tb(max)}, L_t, \theta_b, d_o, t_t, t_r, d_f, t_f, P_f, P_t$.

Results

All the computations were performed on the ALPHA 3000-800 in double precision arithmetic (i.e. nearly 16 digits of accuracy) at the University of Stellenbosch. The input data and results of examples 1 and 2 are listed in Appendices K and L respectively.

(1) Operating point and annual cost calculations:

Example 1: The same results are obtained from FDOPT1 and FDOPT2. The results are listed in table K.5.

Example 2: The same results are obtained from NDOPT1 and NDOPT2. The results are listed in table L.4.

The solution methods usually require between 1 and 10 iterations to successfully determine the operating point of the particular dry-cooling system.

(2) Net annual power generation and cost calculations:

Example 1: The same results are obtained from FDOPT1 and FDOPT2. The results are listed in table K.6.

Example 2: The same results are obtained from NDOPT1 and NDOPT2. The results are listed in table L.5.

The solution methods usually require between 5 and 10 iterations per temperature data set to successfully determine the operating point of the particular dry-cooling and turbo-generator system.

The operating variables determined for a specific temperature data set are used as the initial approximations to the final values of the operating variables for the next temperature data set.

(3) Operating point optimization and the corresponding post-optimality analyses:

Example 1: The same results are obtained from FDOPT1, FDOPT2 and FDOPT3. The results are listed in tables K.7 and K.8.

For this example, find $n_t = 21$ and $m_t = 38$. The performance of the different optimization algorithms to reach convergence are shown in table 7.1.

Table 7.1: Performance of optimization algorithms (example 1).

Program name	NITER	NFEVAL	TOTCPU [s]	QPCPU [s]
FDOPT1	46	48	1.28	0.72
FDOPT2	46	48	1.05	0.48
FDOPT3	46	48	1.35	0.75

Example 2: The same results are obtained from NDOPT1, NDOPT2 and NDOPT3. The results are listed in tables L.6 and L.7.

For this example, find $n_t = 17$ and $m_t = 26$. The performance of the different optimization algorithms to reach convergence are shown in table 7.2.

Table 7.2: Performance of optimization algorithms (example 2).

Program name	NITER	NFEVAL	TOTCPU [s]	QPCPU [s]
NDOPT1	36	40	0.45	0.23
NDOPT2	36	41	0.38	0.16
NDOPT3	50	52	0.65	0.40

The operating point calculation performed in (1) serve as the initial feasible starting point for the optimization calculations.

(4) Minimization of power generation cost and the corresponding post-optimality analyses:

Example 1: The same results are obtained from FDOPT1, FDOPT2, FDOPT3 and FDOPT4. The results are listed in tables K.9, K.10 and K.11.

For this example, the number of independent variables is $n_{indep} = 48$ and the corresponding number of general constraints $m = 462$. The number of dependent variables is $n_{dep} = 238$, while the number of corresponding model simulation equations is $m_m = 238$. These equations are used to solve for the dependent variables. Thus, $n_t = 286$ and $m_t = 700$. The performance of the different optimization algorithms to reach convergence are shown in table 7.3.

Table 7.3: Performance of optimization algorithms (example 1).

Program name	NITER	NFEVAL	TOTCPU [s]	QPCPU [s]
FDOPT1	164	167	6094.8	5613.3
FDOPT2	141	147	4066.4	3638.1
FDOPT3	259	261	2261.9	11.36
FDOPT4	267	269	1234.5	12.32

Example 2: The same results are obtained from NDOPT1, NDOPT2, NDOPT3 and NDOPT4. The results are listed in tables L.8, L.9 and L.10.

For this example, the number of independent variables is $n_{indep} = 14$ and the corresponding number of general constraints $m = 355$. The number of dependent variables is $n_{dep} = 136$, while the number of corresponding model simulation equations is $m_m = 136$. These equations are used to solve for the dependent variables. Thus, $n_t = 150$ and $m_t = 491$. The performance of the different optimization algorithms to reach convergence are shown in table 7.4.

Table 7.4: Performance of optimization algorithms (example 2).

Program name	NITER	NFEVAL	TOTCPU [s]	QPCPU [s]
NDOPT1	83	89	339.33	272.66
NDOPT2	78	81	321.87	259.45
NDOPT3	143	145	264.82	1.17
NDOPT4	148	150	141.5	1.22

The net annual power generation calculations performed in (2) serve as the initial feasible starting point for the optimization calculations.

The values in all the above tables correspond to the total effort required to obtain proper values for the integer variables (e.g. the number of finned tubes) as explained in section 6.4.

7.3 DISCUSSION

Dry-cooling systems

The following discussions are strongly based on the proposed cost estimation model and the corresponding cost coefficients. However, this model is assumed to be adequate for the purpose of illustrating the programs' capabilities. The experimental and theoretical information needed to generate these results are contained in all the appendices.

The sensitivity of the optimal cost with respect to changes in the cost coefficients, prescribed parameters and the inequality constraints limits are presented in scale-invariant form in Appendices K and L. The sensitivity of the cost coefficients and prescribed parameters can be directly compared to each other. A 1% change in a cost coefficient or prescribed parameter, will result in a percentage change in the objective function equal to the particular sensitivity coefficient. A positive sensitivity coefficient indicates an increase in the objective function for an increase in the cost coefficient or prescribed parameter under consideration, while the opposite is true for a negative sensitivity coefficient.

Similarly, the sensitivity of changes in the constraint limits can be directly compared to each other. The sensitivity of the inequality constraints are directly linked to their corresponding Lagrange multipliers and must therefore always be greater than or equal to zero. A zero value indicates an inactive constraint satisfied as an inequality or a weakly active constraint, whereas a positive value indicates an active inequality constraint satisfied as an equality. Expanding the feasible region by relaxing an inequality constraint, will result in a decrease in the objective function, while contracting the feasible region by tightening an inequality constraint, will result in an increase in the objective function. By examining the inequality constraints it will be evident whether a positive constraint limit perturbation will increase or decrease the objective function.

<u>Example 1: Forced draft direct air-cooled condensers</u>

Table K.5 presents detailed airside and steamside information of the fixed geometry air-cooled condenser performing under specified atmospheric and steam inlet conditions. The total annual cost is broken down into its various components. Table K.6 presents the net annual power generation and cost calculations. The results listed in tables K.5 and K.6 are used as the reference case for the optimization computations that follow.

Comparison of the total costs listed in tables K.7 (operating point optimization) and K.5 indicate that the optimum air-cooled condenser consumes much less fan power, while its capital cost has increased. A 28% reduction in total annual cost is achieved as a result. The reduced operating cost is mainly a result of the reductions in both the fan operating speed and fan blade angle as well as the increase in fan system effectiveness. The increased capital cost can be attributed to the increased airside area of the finned tubes, the larger fan diameter, the higher fan platform and larger ground surface area requirement. The optimum finned tube has a realistic outside diameter and wall thickness, but a larger fin diameter and a slightly thinner fin than those usually found in practice. The fin pitches for the two tube rows are also smaller than those in the reference case. Approximately the same amount of steam condenses in each tube row.

From table K.8 it can be seen that an increase in the steam temperature will result in the largest reduction in the annual cost. Increasing the steam temperature results in an increased temperature difference between the steam and ambient air. A lower air mass flow rate is needed to reject heat to the atmosphere at the specified rate. Further significant cost decreases can also be achieved by increasing the fan drive system efficiency and operating the condenser at a lower altitude. The biggest cost increase is caused by an increase in the ambient air temperature, because of the reduction in the temperature potential between the steam and ambient air. Thus, operating conditions must be chosen with extreme care, because they have a pronounced effect on the optimal solution.

The optimal solution is also negatively influenced by increases in the interest parameters and the different cost coefficients. The negative sensitivity coefficient of the interest rate can clearly be explained by equation (4.5).

An increase in the constraint limits (lower limits) of the constraints numbered 1, 3, 6, 15, 16 and 18 in table K.8, will all contract the feasible region and thus result in an increase in the optimal cost and vice versa. An increase in the upper limit of the bundle semi-apex angle (constraint no. 11) will expand the feasible region and thus reduce the optimal cost and vice versa. Changes in constraint no. 3 will have the largest influence on the optimal solution.

Tables K.9 and K.10 show the results of the minimization of the power generation cost. The optimum geometry and performance characteristics of the air-cooled condenser ensure that the turbine performs in a very efficient region of its performance characteristics. When compared to table K.6, the results show an enormous decrease in the fan power consumption cost, an increase in the capital cost, while the total fuel cost remains almost constant. The total annual cost is slightly reduced by 1.5%, while the annual net power output is increased by 1%. These results give rise to a 2.5% reduction in the generated electricity cost. The optimum fan operating speed for each temperature data set can also be seen in table K.10. The same comments regarding the operating point optimization results (table K.7) are also applicable here.

Table K.11 shows that the optimal solution is very sensitive to changes in the fuel cost and its escalation rate. The positive effect that an increase in the interest rate has on the optimal solution can also clearly be seen from equation (4.5). The variation of the other cost coefficients and prescribed parameter will all have more or less the same influence on the optimal solution. The constraint sensitivity results show similar trends as those experienced during the operating point optimization.

The optimum geometries of the air-cooled condensers listed in tables K.7 and K.9 do not differ significantly from each other. For this particular instance, the operating cost optimization provides a reasonable approximation of the air-cooled condenser obtained by the minimization of power generation cost. However, this will not generally be true, because the operating point optimization is based on the assumption that a fixed ambient and a fixed steam temperature occur for all the operating hours throughout the year. No fuel cost, ambient and steam temperature variations or turbo-generator performance characteristics are taken into account when performing the operating point optimization.

Example 2: Natural draft indirect dry-cooling towers

Table L.4 presents detailed airside and waterside information of the fixed geometry dry-cooling tower performing under specified atmospheric and inlet water conditions. The total annual cost is broken down into its various components. Table L.5 presents the net annual power generation and cost calculations. The results listed in tables L.4 and L.5 are used as the reference case for the optimization computations that follow.

The operating point optimization (table L.6) reduces both the pump operating cost and the capital cost when compared to the results generated for the reference case in table L.4. A 30% reduction in total annual cost is achieved. The reduced capital cost can be attributed to the reduction in the heat exchanger bundle and circulation system cost. The cooling tower structural and construction cost have, however, increased due to larger cooling tower shell dimensions and ground area requirement. The reduction in the operating cost is caused by the combination of a larger finned tube inside diameter and shorter tube length.

The optimum finned tube has a realistic outside diameter and wall thickness, but a larger fin diameter and a slightly thinner fin than those usually found in practice. The fin pitch is also smaller than the reference case. The transversal tube pitch allows a rather wide gap of about 6.85 mm between the fin tips of the finned tubes. The bundle width is at its upper limit of 3 m.

The operating point optimizations performed by Buys and Kröger [89BU1e, 89BU2e] found the optimum fin thickness to be impractically thin. The present study uses a more extensive simulation model and cost estimation procedure which contribute to differences in the optimal solution.

From table L.7 it can be seen that an increase in the water inlet temperature will result in the largest reduction in the annual cost. Increasing the water inlet temperature at a constant water mass flow rate will require a smaller dry-cooling tower system to reject a constant amount of heat to the atmosphere. Further significant cost decreases can also be achieved by increasing the fraction of the tower base covered by bundles, the water mass flow rate and operating the cooling tower at a lower altitude. The biggest cost increase will be caused by an increase in the ambient air temperature (reduction in the temperature potential). The optimal solution is also negatively influenced by increases in the interest parameters and the different cost coefficients. The negative sensitivity coefficient of the interest rate can be explained by equation (4.5).

An increase in the constraint limits (lower limits) of the constraints numbered 6, 8 and 9 in table L.7, will all contract the feasible region and thus result in an increase in the optimal cost and vice versa. An increase in the upper limit of the bundle width and semi-apex angle (constraint no. 13 and 15 respectively) will expand the feasible region and thus reduce the optimal cost and vice versa. Constraint no. 8 will have the biggest influence on the optimal solution.

Tables L.8 and L.9 show the results of the minimization of the power generation cost. The optimum geometry and performance characteristics of the dry-cooling tower and the approximate halving of the pumping power requirements ensure that the turbine performs very effectively throughout the year. When compared to table L.5, the results show that the pumping power cost is approximately halved, the capital cost and maintenance costs are reduced, while the total fuel cost remains almost constant. The total annual cost is slightly reduced by 0.6%, while the annual net power output is increased by about 0.5%. These results give rise to a 1% reduction in the generated electricity cost.

The finned tube dimensions correspond fairly well to those found in practice, apart from the deviations in fin diameter and fin thickness. The gap between the fin tips is about 5 mm. The bundle width and bundle semi-apex angle are at their respective upper limits.

Table L.10 shows that the optimal solution is very sensitive to changes in the fuel cost and its escalation rate. The positive effect that an increase in the interest rate has on the optimal solution can also clearly be seen from equation (4.5). The variation of the other cost coefficients and prescribed parameter will all have more or less the same influence on the optimal solution. The constraint sensitivity results show similar trends as those experienced during the operating point optimization.

The optimum geometries of the dry-cooling tower systems listed in tables L.6 and L.8 do not differ significantly from each other. The cooling tower is slightly larger in the latter case, while the finned tube geometry and layout is more or less the same. Table L.8 shows that the heat exchanger bundles has more tubes of a longer length per row, and more heat exchanger bundles in the tower base. Smaller pumps are also used. The results in table L.8 correspond to a variation in water inlet and ambient air temperatures, the specified turbo-generator performance characteristics and fuel cost.

When suitable cost coefficients are available, the performance and annual costs of the optimal forced draft direct air-cooled condenser and the natural draft indirect dry-cooling tower can be directly compared to each other. For these particular examples, the natural draft indirect dry-cooling tower presents the preferred solution. It should, however, be kept in mind that the indirect system still requires an intermediate surface condenser that will increase the capital cost of the corresponding system. The surface condenser cost is not considered in this study.

The above examples illustrate that the operating point calculations provide the feature to generate detailed information of the air- and process side as well as the performance of the dry-cooling systems. The input data for these calculations can be obtained from the results of the power generation and optimization calculations. The operating point calculations thus forms an essential part of the optimization analyses.

In this study we recognize the fact that the performance of dry-cooling systems decrease at high ambient temperatures. Furthermore, we also take into account the power consumed by the axial flow fans and cooling water pumps. We use fixed turbo-generator-condenser characteristics in the case of an indirect dry-cooling system and fixed turbo-generator characteristics in the case of the direct dry-cooling system and find the cost optimal dry-cooling tower that causes the cheapest power to be produced for a given annual frequency of ambient temperatures, while taking the above-mentioned dry-cooling system characteristics into account. This can be viewed as a system optimization, because the interaction of the dry-cooling system and the turbo-generator unit forms an integral part of the optimization process. Designing a dry-cooling system in this way is different from the conventional means of design that is purely based on fixed ambient and process fluid conditions. Finding the best design over a range of operating conditions will definitely result in a more realistic design.

A practically similar procedure was followed in the design of the indirect dry-cooling system of the Kendal Power Station [87TR1e]. Previous dry-cooling system designs were based on a fixed operating point [76VA1e]. In the case of Kendal, the cooling system had to be dimensioned so that after taking the turbo-generator-condenser characteristics, the annual frequency of ambient air temperatures, the planned number of operating hours into account and having deducted the values for internal consumption, a certain net power output is achieved. The authors state that this method of design involves a considerable amount of computation but is the only correct way to compare different cooling systems and assess them from an economic point of view.

Solution methods

Based on the results presented above, the relative performance of the different computational algorithms are now discussed. The comparison is based on the accuracy of their solutions, reliability and efficiency. The CPU times measured in seconds should be considered as only approximations because they can vary depending on the operating system, programming habits, number of users and other factors.

<u>Performance evaluation calculations</u>

The performance evaluation calculations are performed by a specially designed constrained nonlinear least squares method. This method is applied to both the operating point and power generation calculations. Two different QP algorithms are used to solve the constrained subproblems: the algorithm in Appendix G (FDOPT1, NDOPT1) and ZQPCVX [83PO2m] (FDOPT2, NDOPT2). Both implementations have shown very good overall performance on various dry-cooling system performance evaluation calculations and are very easy to implement.

Exactly the same solutions are obtained when the different solution methods are applied to the same problem. These methods are very accurate and will solve well-formulated problems in 1 to 10 iterations only. The addition of the feasibility inequality constraints ensures that feasible solutions will be obtained, because the simulation equations will not be evaluated outside their application regions. None of these constraints will be active at the solution.

Failure of the solution methods to find solutions have occurred in a few instances. The main cause of this unreliable performance is created by a combination of variables that result in very large residual problems during the solution process (refer to Appendix F). The solution process then terminates prematurely and displays the reason for termination on the screen. These deficiencies can simply be overcome by resetting the operating parameters to different values and then restart the solution procedure.

ZQPCVX has failed to solve the resulting QP subproblems on quite a few occasions, whereas the QP algorithm listed in Appendix G experienced no difficulties when applied to the same problems. A possible explanation for this behavior is the fact that more stable matrix updating methods are used in the latter case than in ZQPCVX.

The process to find an initial feasible point that satisfies the feasibility inequalities must not be seen as a cure for the computational inefficiencies related to a poor starting point. The user must evaluate his/her initial choice of the approximation to the final values of the operating variables, as well as the combination of the different components. It is very important to make sure that the combination of geometrical and performance specifications will result in a practicable outcome. In most cases an experienced designer can select a much better starting design and obtain a solution more efficiently. The method of obtaining the initial approximations for the operating variables in the case of the power generation calculations performs very satisfactorily.

The solution process can be regarded as efficient because the results are obtained in very little computing time. Good global convergence is exhibited by the fact that both trust region and line search methods are used to force a decrease in the merit function. The line search procedure is only used in the case of potential difficulties. The performance of these solution techniques is very accurate, reasonably reliable and very efficient.

Optimization calculations

The methods of obtaining well-formulated and well-scaled initial feasible starting designs give rise to very satisfactory performance of the optimization algorithms.

(1) Operating point optimization

The operating point optimization was performed by using the three different computational algorithm combinations as implemented in the different programs listed in tables 7.1 and 7.2. These implementations have shown very good overall performance on various dry-cooling system operating point optimization calculations.

All the methods give the same solution when applied to the same dry-cooling system operating point optimization problem. Accurate and feasible solutions can be obtained in very little computing time. More or less the same number of iterations and function evaluations are used to obtain convergence on a specific problem. Tables 7.1 and 7.2 show differences in the TOTCPU and QPCPU values. FDOPT2 and NDOPT2 appear to be the most effective programs due to the smaller amount of time required to solve the QP subproblems. This in mainly due to the implementation of ZQPCVX

[83PO2m]. ZQPCVX uses very efficient, but unstable matrix updating schemes. Due to the small amount of computing time needed to solve the different optimization problems to a high degree of accuracy, the number of function and gradient evaluations are a more convenient means of measuring efficiency.

Failure of these optimization methods to find solutions have occurred in a few instances. Cause for this unreliable behavior can be traced to improper constraint and variable scaling, termination of the SQP method after 5 consecutive line search calls and failure of the QP algorithm to solve the resulting subproblem. The solution process then terminates prematurely and displays the reason for termination on the screen. These deficiencies can most of the time be overcome by changing the scaling parameters or some of the decision variables and then restart the optimization procedure. ZQPCVX has failed to solve the resulting QP subproblems on a few problems, whereas the QP algorithm listed in Appendix G experienced no difficulties.

The performance of these modified SQP algorithms in combination with the different QP algorithms are thus very accurate, reasonably reliable and very efficient to perform the operating point optimization calculations.

(2) Minimization of power generation cost

All the different programs listed in table 6.1 were used to minimize the power generation cost when a particular dry-cooling system is coupled to a specified turbo-generator set. All these programs are based on the infeasible path integrated approach to solve the optimization problem. The minimization of the power generation cost can be classified as a large-scale optimization problem. In FDOPT1, FDOPT2, NDOPT1 and NDOPT2 the conventional SQP method is implemented, while in FDOPT3, FDOPT4, NDOPT3 and NDOPT4 a reduced Hessian SQP decomposition strategy is implemented to reduce the size of the optimization problem. These implementations have shown very good overall performance on various power generation cost minimization calculations.

All the methods give nearly similar solutions when applied to the same optimization problem. The slight differences in these solutions can be attributed to computer rounding. The infeasible path approach ensures that a feasible solution is obtained when convergence is reached at the final iteration. Little information is recoverable if the algorithm fails to converge to a solution.

Failure of these optimization methods to find solutions can be attributed to the same reason already discussed above. Proper formulation and scaling of the optimization problem can reduce the potential of algorithmic failure.

The performance of the different methods on the two example problems are shown in tables 7.3 and 7.4. The efficiency trends observed in these tables can be explained as follows:

(i) FDOPT1 and NDOPT1: The optimization problems are solved in the total decision variable (dependent and independent) space and require the solution of large QP subproblems. The order of the Hessian of the Lagrange function is equal to the total number of decision variables. Large computational overhead and storage are need to solve these optimization problems. The solution of the QP subproblems consumes most of the CPU time required to solve the problem.

(ii) FDOPT2 and NDOPT2: These programs are similarly implemented as described above (see (i)), except for the different QP algorithm. FDOPT2 and NDOPT2 perform more efficiently than FDOPT1 and NDOPT1, partly due to the more efficient QP implementation. Although the QP algorithm employed in FDOPT1 and NDOPT1 is slower than ZQPCVX, it is more reliable.

(iii) FDOPT3 and NDOPT3: The optimization problems are solved in the independent variable space and significantly reduce the size of the QP subproblems. The order of the corresponding reduced Hessian matrix is equal to the number of independent variables. The computational overhead needed to solve the QP subproblems is drastically reduced. The solution of the QP subproblems requires between 100 and 500 times less CPU time than the programs using the conventional SQP implementation (refer to (i) and (ii)). Similar trends have been observed by [88VA1e, 93SC1e]. These drastic reductions in QPCPU indicate that little can be done to further improve the QP algorithm's performance.

Almost all the CPU time is consumed by the function and gradients evaluations as well as the computation of the term $\mathbf{Z}^T\mathbf{B}\mathbf{Y}\mathbf{s}_Y$ which requires an extra gradient evaluation at each SQP iteration. More SQP iterations and function and gradient evaluations are needed than in the conventional case. Although the order of the Hessian matrix is enormously reduced, additional storage is required for the vectors and matrices used in this decomposition method. The significant improvements that the reduced Hessian SQP method has over the original SQP methods illustrate the benefit of tailoring the solution algorithm to take advantage of the mathematical structure of the model.

(iv) FDOPT4 and NDOPT4: These programs are similarly implemented as described above (see (iii)), except for omission of the term $\mathbf{Z}^T\mathbf{B}\mathbf{Y}\mathbf{s}_Y$. Although QPCPU is slightly more than those recorded in (iii), TOTCPU is almost halved, because the extra term does not need to be evaluated. The maximum number of iterations and function evaluations are obtained in this case, because $\mathbf{Z}^T\mathbf{BY}s_Y$ will tend to increase the convergence rate and lower these values [93SC1e]. In general, FDOPT4 and NDOPT4 requires between 2 and 10 times less total CPU time when compared to the programs using the conventional SQP implementations (refer to (i) and (ii)) and the improved coordinate bases method (refer to (iii)). Similar trends have been observed by [88VA1e, 93SC1e]. Accurate and feasible solutions can be obtained in very little computing time when compared to the other programs.

Schmid and Biegler [93SC1e] included the $\mathbf{Z}^T\mathbf{BY}s_Y$ term to improve the convergence and reduce the dependence on variable partitioning of the original coordinate bases method [83LO1e]. The above results indicate that the optimization problems can be solved more efficiently by simply omitting $\mathbf{Z}^T\mathbf{BY}s_Y$. The coordinate bases method is thus not sensitive to the particular variable partitioning. The computation of this term was implemented via a finite difference scheme [93SC1e]. More elegant ways of implementing $\mathbf{Z}^T\mathbf{BY}s_Y$ are discussed in [93SC1e, 94BI1e].

The effect of the term, $\mathbf{Z}^T\mathbf{BY}s_Y$, was investigated on various dry-cooling systems and general optimization problems. The inclusion of $\mathbf{Z}^T\mathbf{BY}s_Y$ usually results in an increase in TOTCPU, and decreases in QPCPU, NITER and NFEVAL. None of the test problems failed when $\mathbf{Z}^T\mathbf{BY}s_Y$ was omitted. Whether the omission of $\mathbf{Z}^T\mathbf{BY}s_Y$ will have significant advantageous, will definitely depend on the type of problem to be solved, as well as the particular variable partitioning. The performance of these reduced Hessian SQP algorithms are thus very accurate, reliable and very efficient.

The post-optimality analyses are very important in implementing a solution on a real system. In many cases, detailed post-optimality analyses are as valuable as the optimal solution itself. The post-optimality analyses performed in this study supply detailed information to critically evaluate the optimal solution behavior to changes in model parameters, constraints and assumptions. These analyses are very straightforward and quick to perform.

It is possible that the algorithms may achieve acceptable low cost designs, from an engineering point of view, in much less iterations than that required to satisfy the prescribed mathematical convergence criterion of 10^{-7}. The strict convergence criterion is introduced to ensure that the engineering model's equations are properly satisfied at the optimum solution, because an infeasible path integrated approach is used.

7.4 CLOSING REMARKS

In this chapter two examples are studied to illustrate the various capabilities of the different computer programs. The illustration is by no means complete and the programs can be used in a variety of other applications concerning performance evaluation, cost and optimization calculations. In both examples, fairly realistic and practical results were obtained. It should again be stressed that realistic results depend very strongly on a realistic cost estimation model and cost coefficient values.

It is worthwhile to consider an approach that takes advantage of the problem structure during its solution, because of the potential to outperform the conventional approaches. The resulting tailored solution procedures are very effective for performing rather sophisticated performance evaluation and optimization computations in a relatively short time.

The degree of optimization that is ultimately achieved in such an economic optimization analysis and its value to a design engineer, are functions of the sophistication of the design program, the expertise of the user, the cost estimating procedure and the quality of the input data.

The next chapter contains the conclusions and recommendations of the performed study on engineering optimization.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 INTRODUCTION

In the preceding chapters we have concentrated on the analysis and solution of small to large-scale engineering optimization problems, with particular reference to dry-cooled power plants. The objective of these optimization studies was to obtain cost-optimal performance and designs of dry-cooling systems.

The basis of this study provide general information on the methodology to obtain an engineering solution in an efficient manner. The steps and considerations followed in conducting this study are: the definition of the optimization problem, the preparation for solution, the selection of suitable solution (optimization) algorithms, choosing or preparing efficient computer implementations of these algorithms, the execution of various computer runs, and after having obtained a reliable and feasible solution, the interpretation of this solution in terms of the real system and its implementation.

Dry-cooling systems are an environmentally sound alternative to wet cooling systems. However, due to their high capital and operating costs, it is justified to optimize their design and performance taking practical limitations into consideration as far as possible.

The conclusions and recommendations listed below serve to emphasize not only what has been learned, but what benefit could be achieved from additional effort in the future.

8.2 SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

In this study we develop and implement computational procedures which are very efficient and practical when applied to sophisticated design, performance evaluation and economic optimization computations of dry-cooling systems as found in power plant applications. The computer programs were extensively tested on a variety of dry-cooling system problems and provide a reliable practical tool with which realistic answers and trend information can be obtained.

Cost-optimal designs can be performed in the conventional way (fixed ambient and process fluid conditions), as well as for cases where the dry-cooling system is coupled to a specified turbo-generator set at a particular location. The latter method takes the dry-cooling system's inherent characteristics into account and will give a more realistic design over its operating range throughout the year. Although the latter method of optimal design involves a considerable amount of computation, the efficient computational algorithms developed in this study ensure that this procedure is not excessively time consuming.

The optimization process is found to be very sensitive to the cost estimation procedures. Realistic optimization results can only be obtained if the cost estimating procedure represents realistic operating and capital cost structures. The proposed cost estimation procedure and the corresponding cost coefficients adequately illustrate the various capabilities of the optimal design techniques.

These procedures can still further be refined with the co-operation of dry-cooling system equipment manufacturers, vendors or cost engineers. Since it is easy to modify the proposed cost estimation procedure to meet the requirements of a specific client, the cost-optimal design procedure is of great practical usefulness. An optimal design can be obtained using the proposed cost structure. Afterwards the results can be studied and the cost structure can be improved if necessary. The optimal design process can be repeated, the results studied and the cost structure improved. By continuing in this iterative manner it will be possible to arrive at a meaningful solution using an approximate cost structure.

The equation-based models of the dry-cooling systems have particular mathematical structures that are exploited by the computational algorithms. Our constrained NLS method is particularly well-suited for solving these equation-based models subject to feasibility inequalities and exhibits good global convergence properties. The simultaneous solution of the equation-based models and the optimization problems along an infeasible path proved to be very efficient and reliable. The excellent performance of these solution strategies can be mainly ascribed to the following modeling principles:

(1) Proper problem formulation and scaling of the objective function, variables and constraints are essential to obtain a reliable solution. A well constructed mathematical model minimizes the effects of ill-conditioning, degeneracy and inconsistent constraints.

(2) The method of obtaining initial feasible starting points guarantee good algorithmic performance far from the optimal solution. A good starting design will enhance the convergence of the solution procedure.

(3) The introduction of feasibility inequalities ensures that the model equations are not evaluated outside their limits and prevents the violation of physical laws.

(4) The overall performance of the computational algorithms will depend directly on their proper implementation. Attributes like reliability, efficiency, accuracy and ease of use are pursued in this study.

The performance evaluation and optimization problems are characterized by a nonlinear nature. Hence, failure to comply with the above requirements may result in the slow convergence or could cause the computational algorithms to experience numerical difficulties.

SQP methods have great potential for routinely solving complex engineering optimization problems. The infeasible path integrated approach was very reliable and efficient when applied to the optimization problems in this study. The modifications to Powell's SQP implementation, VMCWD [82PO2m], i.e. the new QP routine, more efficient derivative evaluation and the partial exploiting of the problem structure in the auxiliary routines all improved the performance of VMCWD. This modified version of VMCWD performed efficiently on dry-cooling system optimization problems with a small to moderate number of variables.

For larger optimization problems the storage requirements for the Hessian matrix as well as the computational expense involved in solving QP subproblems can become prohibitive. In this study we overcome these problems by developing and implementing both the original and improved coordinate bases SQP decomposition strategies. These decomposition strategies are particularly suited to take advantage of the block diagonal structure of the Jacobian matrix. VMCWD is modified to solve the resulting optimization problem in the independent decision variable space only. Numerical comparisons based on the solution of a number of dry-cooling system optimization problems demonstrate the effectiveness of the reduced SQP decomposition strategies, compared to conventional SQP implementations. Although more function and gradient evaluations are used by these decomposition strategies, the total CPU time is significantly reduced.

The original coordinate bases algorithm has a very superior performance effectiveness when compared to all the other SQP methods used in this study and suffers no ill-effects related to the fixed variable partitioning implemented in the dry-cooling system optimization problems. Although several authors claim that this method can give inconsistent results [88VA1e, 93SC1e, 94BI1e], it solves our large-scale optimization problems very accurately, efficiently and reliably.

The dual active set algorithm for convex QP problems implemented in this study, performs effectively and reliably in solving the QP subproblems of both the constrained NLS method and the SQP method. Matrix modifications are performed by stable matrix updating schemes. This QP algorithm is tailored to take advantage of the special features of the QP subproblems.

The optimization is not complete when the solution is obtained; in fact the solution only serves as the basis of the most important parts of the study: solution validation and sensitivity analysis. The information about the state of the system in the neighborhood of the solution provides key insights into the following important factors: the constraints active at the solution that limit further improvement of the system, the dominant cost terms that should be refined, and the prescribed parameters that can be improved.

In the interests of continued economic development, the cost of producing electricity should be kept to a minimum by rationalizing the utilization of both water and fuel resources. The optimal results and trends that one is able to obtain by means of the computational procedures developed in this dissertation, pose a challenge and establish new design and performance practices to designers, manufacturers and operators.

Finally, Varvarezos et al. [94VA1e] propose a new decomposition method for solving multiperiod design optimization problems based on SQP. The optimization problem concerning the minimization of power generation cost can be classified accordingly. The special mathematical structure of the multiperiod model can be effectively exploited. The authors state that the method is very efficient and robust when compared to the conventional SQP methods. A worthwhile future exercise will be to evaluate this method and compare its performance to the coordinate bases algorithms. Multiperiod design optimization will also be able to address topics like part-load performance of the turbo-generator unit according to a specified load plan.

REFERENCES

References are categorized under engineering and mathematical references. The references are identified by a code in square brackets which consists of the last two digits for the year of publication, the first two letters of the first author's name, a sequentially assigned digit to make the reference unique and an 'e' or 'm' to indicate whether it is an engineering or mathematical reference.

R.1 ENGINEERING REFERENCES

[33CO1e] Colburn, A.P., A method of correlating forced convection heat transfer data and a comparison with fluid friction, Trans. Am. Inst. Chem. Eng., Vol. 29, pp. 174-210, 1933; reprinted in International Journal of Heat and Mass Transfer, Vol. 7, pp. 1359-1384, 1964.

[45JA1e] Jameson, S.L., Tube spacing in finned-tube banks, ASME Transactions, Vol. 67, pp. 633-642, 1945.

[46SC1e] Schmidt, T.E., La Production Calorifique des Surfaces Munies Dailettes, Annexe Du Bulletin De L'Institut International Du Froid, Annex G-5, 1945-1946.

[50KA1e] Kays, W.M., Loss coefficients for abrupt changes in flow cross section with low Reynolds number flow in single and multiple-tube systems, Transactions of ASME, Vol. 72, No. 8, pp.1067-1074, 1950.

[57FA1e] Fax, D.H., and Mills, R.R., Generalized optimal heat-exchanger design, Transactions of ASME, Vol. 79, pp. 653-661, 1957.

[59KE1e] Kern, D.Q., Optimum air-fin cooler design, Chemical Engineering Progress Symposium Series, Vol. 55, No. 29, pp. 187-193, 1959.

[59NA1e] Nakayama, E.U., Find the best air fin cooler design, Petroleum Refiner, Vol. 38, No. 4, pp. 109-114, 1959.

[59PO1e] Potter, P.J., Power plant theory and design, 2nd edition, J. Wiley and Sons, new York, 1959.

[59WA1e] Ward, D.J., and Young, E.H., Heat transfer and pressure drop of air in forced convection across triangular pitch banks of finned tubes, Chemical Engineering Progress Symposium Series, Vol. 55, No. 29, pp. 37-43, 1959.

[60RU1e] Rubin, F.L., Design of air cooled heat exchangers, Chemical Engineering, Vol. 68, pp. 91-96, 1960.

[60SK1e] Skorotzki, B.G.A., and Vopat, W.A., Power station engineering and economy, McGraw-Hill Book Co., New York, 1960.

[62LE1e] Leitmann, G., Optimization techniques with applications to aerospace systems, Academic Press, New York, 1962.

[63BR1e] Briggs, D.E., and Young, E.H., Convection heat transfer and pressure drop of air flowing across triangular pitch banks of finned tubes, Chemical Engineering Progress Symposium Series, Vol. 59, No. 41, pp. 1-10, 1963.

[64BA1e] Bauman, H.C., Fundamentals of cost engineering in the chemical industry, Reinhold Publishing Corporation, New York, 1964.

[65BO1e] Box, M.J., A new method of constrained optimization and comparisons with other methods, Computer Journal, Vol. 8, pp. 42-52, 1965.

[65NE1e] Nelder, J.A., and Mead, R., A simplex method for function minimization, Computer Journal, Vol. 7, pp. 308-313, 1965.

[66LO1e] Lohrisch, F.W., What are optimum conditions for air-cooled exchangers?, Hydrocarbon Processing, Vol. 45, No. 6, pp.131-136, 1966.

[66LO2e] Lohrisch, F.W., How many tube rows for air-cooled exchangers?, Hydrocarbon Processing, Vol. 45, No. 6, pp. 137-140, 1966.

[66RO1e] Robinson, K.K., and Briggs, D.E., Pressure drop of air flowing across triangular pitch banks of finned tubes, Chemical Engineering Progress Symposium Series, Vol. 62, No. 64, pp. 177-184, 1966.

[66SC1e] Schoonman, W., Aircooler optimization aided by computer, Symposium on air-cooled heat exchangers, ASME, New York, pp. 86-102, 1966.

[66SC2e] Schulenberg, F.J., Finned elliptical tubes and their application in air-cooled heat exchangers, ASME Journal of Engineering for Industry, pp. 179-190, May 1966.

[66VA1e] Vampola, J., Heat transfer and pressure drop in flow of gases across finned tube banks, Strojirenstvi, Vol. 16, No. 7, pp. 501-507, 1966.

[66VD1e] Verein Deutscher Ingenieure, VDI-Richtlinien, Abnamhe- und Leistungsversuche an Ventilatoren, VDI 2044, Berlin und Köln, 1966.

[67LE1e] Leitmann, G., Topics in optimization, Academic Press, New York, 1967.

[69GO1e] Gould, P.L., Minimum weight design of hyperbolic cooling towers, ASCE Journal of the Structural Division, Vol. 95, pp. 203-208, 1969.

[69JE1e] Jenssen, S.K., Heat exchanger optimization, Chemical Engineering Progress, Vol. 65, No. 7, pp. 59-66, 1969.

[69RI1e] Richter, E., Untersuchung der Strömungsvorgänge am Austritt von Naturzugkühltürmen und deren Einfluss auf die Kühlwirkung, Dr. Ing. Thesis, TU Dresden, 1969.

[69SC1e] Schulenberg, F.J., Wärmeübergang und Druckänderung bei der Kondensation von strömendem Dampf in geneigten Rohren, Dr.-Ing. thesis, Universität Stuttgart, 1969.

[70RO1e] Rossie, J.P., and Cecil, E.A., Research on dry-type cooling towers for thermal electric generation: Part I and II, EPA Report, Water Pollution Control Research Series 16130 EES, November 1970.

[70SM1e] Smith, E.C., and Larinoff, M.W., Power plant siting, performance, and economics with dry cooling tower systems, Proceedings of the American Power Conference, Volume 32, pp. 544-572, 1970.

[71AV1e] Avriel, M., and Williams, A.C., An extension of geometric programming with applications in engineering optimization, Journal of Engineering Mathematics, Vol. 5, No. 3, pp. 187-194, 1971.

[71FO1e] Fox, R.L., Optimization methods for engineering design, Addison-Wesley Publishing Company, Reading Massachusetts, 1971.

[71HA1e] Hauser, L.G., Oleson, K.A., and Budenholzer, R.J., An advanced optimization technique for turbine, condenser, cooling system combinations, Proceeding of the American Power Conference, Vol. 33, pp. 427-445, 1971.

[71HE1e] Heeren, H., and Holly, L., Dry cooling eliminates thermal pollution, Energie, Vol. 23, 1971.

[72AN1e] Andeen, B.R., and Glicksman, L.R., Computer optimization of dry cooling tower heat exchangers, ASME Winter Annual Meeting, New York, November 26-30, 1972.

[72FU1e] Furzer, I.A., Very large hyperbolic cooling towers, The Institution of Engineers, Australia, Mechanical and Chemical Engineering Transactions, Vol. MC8, No. 2, pp. 122-124, 1972.

[72MO1e] Moore, F.K., On the minimum size of natural-draft dry cooling towers for large power plants, Presented at the ASME Winter Annual Meeting, New York, November 26-30, 1972.

[72MO2e] Moore, F.K., Scaling law for dry cooling towers with combined mechanical and natural draft, Report No. 72-19, Cornell Energy Project, 304 Olin Hall, Ithaca, N.Y., 1972.

[72PE1e] Peters, D.L., and Nicole, F.J.L., Efficient programming for cost-optimised heat exchanger design, The Chemical Engineer, No. 259, pp. 98-111, March 1972.

[73AV1e] Avriel, M., Rijckaert, M.J., and Wilde, D.J., Optimization and design, International summer school on the impact of optimization on technological design, Katholieke Universiteit te Leuven, Belgium, July 26-August 6, 1971, Prentice Hall, Englewood Cliffs, New Jersey, 1973.

[73JO1e] Johnson, B.M., and Dickinson, D.R., On the minimum size for forced draft dry cooling towers for power generating plants, in: R.L. Webb and R.E. Barry (eds.), Dry and wet/dry cooling towers for power plants, ASME Winter Annual Meeting, November 11-15, Detroit, Michigan, pp. 25-34, 1973.

[73MO1e] Moore, F.K., The minimization of air heat-exchange surface areas of dry cooling towers for large power plants, Heat Transfer Digest, Vol. 6, pp. 13-23, 1973.

[73MO2e] Moore, F.K., On the minimum size of large dry cooling towers with combined mechanical and natural draft, ASME Journal of Heat Transfer, pp. 383-389, August 1973.

[73RI1e] Rijckaert, M.J., Engineering applications of geometric programming, in: M.J. Rijckaert, and M. Wilde (eds.), Optimization and design, Prentice Hall, Englewood Cliffs, New Jersey, 1973.

[74GU1e] Guthrie, K.M., Process plant estimating, evaluation and control, Craftsman Book Company of America, Solana Beach, California,, 1974.

[74MI1e] Miliaras, E.S., Power plants with air-cooled condensing systems, MIT Press, Cambridge, Massachusetts, 1974.

[74MI2e] Mirkovic, Z., Heat transfer and flow resistance correlation for helically finned and staggered tube banks in crossflow, in: N.H. Afgan and E.U. Schlünder (eds.), Heat exchangers: Design and theory sourcebook, McGraw-Hill Book Co., pp. 559-584, 1974.

[74OS1e] Ostwald, P.F., Cost estimating for engineering and management, Prentice Hall Inc., Englewood Cliffs, New Jersey, 1974.

[74PA1e] Palen, J.W., Cham, T.P., and Taborek, J., Optimization of shell-and-tube heat exchangers by case study method, AIChE Symposium Series, Vol. 70, No. 138, pp. 205-214, 1974.

[74ZA1e] Zastrozny, E.P., Practical cost estimating for the South African chemical process industry, Revised edition, SASOL, South Africa, 1974.

[75CR1e] Croley, T.E., Patel, V.C., and Cheng, M., The water and total optimizations of wet and dry-wet cooling towers for electric power plants, IIHR Report No. 163, University of Iowa, Iowa City, January 1975.

[75GN1e] Gnielinski, V., Forsch. Ing. Wesen, Vol. 41, No. 1, 1975.

[75GO1e] Gordian Associates Inc., New York, A study of the relative economics and total energy requirements of natural draft and mechanical draft cooling towers, PB-256 762, December 1975.

[75MI1e] Mitchell, R.D., Method for optimizing and evaluating indirect dry-type cooling systems for large steam-electric generating plants, ERDA-74 Report, June 1975.

[75RE1e] Reinschmidt, K.F., and Narayanan, R., The optimum shape of cooling towers, Computers and Structures, Vol. 5, pp. 321-325, 1975.

[75RO1e] Roetzel, W., and Nicole, F.J.L., Mean temperature difference for heat exchanger design - a general approximate explicit equation, Journal of Heat Transfer, Trans. of ASME, Vol. 97 No. 1, February 1975.

[76AR1e] Ard, P.A., Henager, C.H., Pratt, D.R., and Wiles, L.E., Cost and cost algorithms for dry cooling tower systems, BNWL-2123, Richland, Washington, September 1976.

[76CR1e] Croley, T.E., Patel, V.C., and Cheng, M., Economics of dry-wet cooling towers, Transactions of the American Society of Civil Engineers, Journal of the Power division, Vol. 102, pp. 147-163, 1976.

[76FR1e] Fryer, B.C., A review and assessment of engineering economic studies of dry cooled electrical generating plants, BNWL, March 1976.

[76RO1e] Rozenman, T., Momoh, S.K., and Pundyk, J.M., Heat transfer and pressure drop characteristics of dry tower extended surfaces, Part II: Data analysis and correlation, PFR Engineering Systems Inc., Marina del Rey, California, June 1976.

[76VA1e] Van der Walt, N.T., West, L.A., Sheer, T.J., and Kuball, D., The design and operation of a dry cooling system for a 200 MW turbo-generator at Grootvlei Power Station, South Africa, The South African Mechanical Engineer, Vol. 26, pp. 498-511, 1976.

[76ZA1e] Zaloudek, F.R., Allemann, R.T., Faletti, D.W., Johnson, B.M., Parry, H.L., Smith, G.C., Tokarz, R.D., and Walter, R.A., A study of the comparative costs of five wet/dry cooling tower concepts, BNWL-2122, September 1976.

[77SC1e] Schulenberg. F., The air condenser for the 365 MW generating unit in Wyoming/USA, Special print from collected edition of the VGB-Conference: Power Station and Environment 1977.

[77SU1e] Surface, M.O., System designs for dry cooling towers, Power Engineering, pp. 42-50, September 1977.

[78BE1e] Bejan, A., General criterion for rating heat-exchanger performance, International Journal of Heat and Mass Transfer, Vol. 21, pp. 655-658, 1978

[78BR1e] Brown, R., Ganapathy, V., and Glass, J., Design of air-cooled exchangers, Chemical Engineering, pp. 106-124, March 1978.

[78DA1e] Davidson, S., and Weil, R.L., Handbook of cost accounting, McGraw-Hill Book Company, New York, 1978.

[78EC1e] Ecker, J.G., and Wiebking, R.D., Optimal design of a dry-type natural-draft cooling tower by geometric programming, Journal of Optimization Theory and Applications, Vol. 26, No. 2, 1978.

[78FO1e] Fontein, H.J., and Groot Wassink, J., The economically optimal design of heat exchangers, Engineering and Process Economics, Vol. 3, pp. 141-149, 1978.

[78LA1e] Larinoff, M.W., Moles, W.E., and Reichhelm, R., Design and specification of air-cooled steam condensers, Chemical Engineering, pp. 86-94, May 1978.

[78LA2e] Larinoff, M.W., Performance and capital costs of wet/dry cooling towers in power plant service, Combustion, Vol. 49, No. 11, pp. 9-19, 1978.

[78RO1e] Rozenman, T., Fake, J.M., and Pundyk, J.M., Optimization of design specifications for large dry cooling systems, PFR Engineering Systems, Inc., Marina del Rey, California, July 1978.

[78SH1e] Shah, R.K., Afimiwala, K.A., and Mayne, R.W., Heat exchanger optimization, Proceedings of the 6th International Heat Transfer Conference, 7-11 August, Toronto, Canada, Vol. 4, pp. 185-191, 1978.

[78VA1e] Vangala, G.K., and Eaton, T.E., A design method for dry cooling towers, Proceedings of the Conference on Waste Heat Management and Utilization, Miami Beach, Florida, December 4-6, 1978.

[78VD1e] VDI 2049, Thermal acceptance and performance tests on dry cooling towers, 1978

[78WI1e] Wilde, D.J., Globally optimal design, J. Wiley and Sons, New York, 1978.

[79AV1e] Avriel, M., and Dembo, R.S., Engineering optimization, Mathematical Programming Study 11, 1979.

[79CH1e] Chemical Engineering, Modern cost engineering: Methods and data, McGraw-Hill, New York, 1979.

[79CH2e] Choi, M., and Glicksman, L.R., Computer optimization of dry and wet/dry cooling tower systems for large fossil and nuclear power plants, MIT Energy Laboratory Report No. MIT-EL 79-034, Cambridge, Massachusetts, February 1979.

[79HA1e] Haberski, R.J., and Bentz, J.C., Conceptual designs and cost estimates of mechanical draft wet/dry and natural draft dry cooling systems using Curtiss-Wright integral fin-tube heat exchangers, Curtiss-Wright Corporation, Wood-Ridge, New Jersey, April 1979.

[79HA2e] Haug, E.J., and Arora, J.S., Applied optimal design-mechanical and structural systems, J. Wiley and Sons, New York, 1979.

[79KR1e] Kröger, D.G., Optimum dimensions and operating conditions for finned heat exchanger tubes, Fifteenth International Congress of Refrigeration, Venice, Paper No. B1-5, International Institute of Refrigeration, Paris, 1979.

[79NA1e] Najjar, K.F., Shaw, J.J., Adams, E.E., Jirka, G.H., and Harleman, D.R.F., An environmental and economic comparison of cooling system designs for steam-electric power plants, MIT Energy Laboratory Report No. MIT-EL 79-037, Cambridge Massachusetts, January 1979.

[80BE1e] Berna, T.J., Locke, M.H., and Westerberg, A.W., A new approach to optimization of chemical processes, AIChE Journal, Vol. 26, No. 1, pp. 37-43, 1980.

[80BS1e] British Standards Institution, Fans for general purposes, Part 1, Methods of testing performance, BS848, 1980.

[80EC1e] Ecker, J.G., Geometric programming: methods, computations and applications, SIAM Review, Vol. 22, No. 3, pp. 338-362, July, 1980.

[80GU1e] Guyer, E.C., and Brownell, D.L., Wet/Dry cooling for cycling steam-electric plants, EPRI CS-1474, Project 1182-1, August, 1980.

[80HE1e] Hedderich, C.P. Heat exchanger optimization, Master's thesis, Naval Postgraduate School Monterey, California, 1980.

[80HE2e] Hesse, G., Wyodak: A milestone in dry cooling, Power Engineering, August 1980.

[80KE1e] Kern, J., Zur Bewertung von Kompakt-Wärmeaustauschern, Wärme- und Stoffübertragung, Vol. 13, pp. 205-215, 1980.

[80MO1e] Montakhab, A., Waste heat disposal to air with mechanical and natural draft-some analytical design considerations, ASME Journal of Engineering for Power, Vol. 102, No. 3, pp. 719-727, 1980.

[80MO2e] Montakhab, A., Analysis of heat transfer surface geometries for dry cooling tower applications, ASME Journal of Engineering for Power, Vol. 102, No. 4, pp. 807-812, 1980.

[80RU1e] Rubin, F.L., Winterizing air-cooled heat exchangers, Hydrocarbon Processing, pp. 147-149, October 1980.

[80WE1e] Webb, R.L., Air-side heat transfer in finned tube heat exchangers, Heat Transfer Engineering, Vol. 1, No. 3, pp. 33-49, 1980.

[80ZE1e] Zerna, W., and Mungan, I., Construction and design of large cooling towers, ASCE Journal of the Structural Division, Vol. 106, pp. 531-544, 1980.

[81GI1e] Gianolio, E., and Cuti, F., Heat transfer coefficients and pressure drops for air coolers with different numbers of rows under induced and forced draft, Heat Transfer Engineering, Vol. 3, No. 1, pp. 38-47, 1981.

[81HU1e] Humphreys, K.K., and Katell, S., Basic cost engineering, Marcel Dekker Inc., New York, 1981.

[81KO1e] Kosten, G.J., Morgan, J.I., Burns, J.M., and Curlett, P.L., Operating experience and performance testing of the world's largest air-cooled condenser, Presented at the American Power Conference, Chicago, Illinois, April 27-29, 1981.

[81MO1e] Montakhab, A., A survey of dry cooling tower technology for power generation application, in S. Kakac, A.E. Bergles, and F. Mayinger (eds.), Heat Exchangers, Thermal-Hydraulic Fundamentals and Design, Hemisphere Publishing Corporation, New York, pp. 799-816, 1981.

[81PR1e] Pribis, P.B., Optimizing heat exchanger design-the pro and cons, AIChE Symposium Series, Vol. 77, No. 208, pp. 229-237, 1981.

[81SH1e] Shah, R.K., Heat exchanger design methodology - an overview, in S. Kakac, A.E. Bergles, and F. Mayinger (eds.), Heat Exchangers, Thermal-Hydraulic Fundamentals and Design, Hemisphere Publishing Corporation, New York, pp. 455-459, 1981.

[81SH2e] Shah, R.K., Compact heat exchanger design procedures, in S. Kakac, A.E. Bergles, and F. Mayinger (eds.), Heat Exchangers, Thermal-Hydraulic Fundamentals and Design, Hemisphere Publishing Corporation, New York, pp. 495-536, 1981.

[81SH3e] Shah, R.K., and Webb, R.L., Compact and enhanced heat exchangers, ICHMT, Yugoslavia, 1981.

[81SM1e] Smith, R.A., Economic velocity in heat exchangers, AIChE Symposium Series, Vol. 77, No. 208, pp. 221-228, 1981.

[81ST1e] Stierlin, K., and Tesar, A., Performance optimization at the cold-end of steam turbosets with air-cooled condensers, Combustion, pp. 30-38, March 1981.

[82BE1e] Bejan, A., Entropy generation through heat and fluid flow, J. Wiley and Sons, New York, 1982.

[82HE1e] Hedderich, C.P., Kelleher, M.D., and Vanderplaats, G.N., Design and optimization of air-cooled heat exchangers, ASME Journal of Heat Transfer, Vol. 104, pp. 683-690, 1982.

[82LO1e] London, A.L., Economics and the second law: an engineering view and methodology, International Journal of Heat and Mass Transfer, Vol. 25, No. 6, pp. 743-751, 1982.

[82ST1e] Stewart, R.D., Cost Estimating, J. Wiley and Sons, New York, 1982.

[83BL1e] Blank, L., and Tarquin, A., Engineering Economy, 2nd ed., McGraw-Hill Book Co., London, 1983.

[83CL1e] Clark, P.A., and Westerberg, A.W., Optimization for design problems having more than one objective, Computers and Chemical Engineering, Vol. 7, pp. 259-278, 1983.

[83HA1e] Haaland, S.E., Simple and explicit formulas for the friction factor in turbulent pipe flow, Transactions of ASME, Journal of Fluids Engineering, Vol. 105, No. 3, pp. 89-90, March 1983.

[83HO1e] Holland, F.A., Watson, F.A., and Wilkinson, J.K., Introduction to process economics, Second Edition, J. Wiley and Sons, New York, 1983.

[83KR1e] Kröger, D.G., Design optimisation of an air-oil heat exchanger, Chemical Engineering Science, Vol. 38, No. 2, pp. 329-333, 1983.

[83LO1e] Locke, M.H., Westerberg, A.W., and Edahl, R.H., An improved successive quadratic programming optimization algorithm for engineering design problems, AIChE Journal, Vol. 29, No. 5, pp. 871-874, 1983.

[83LO2e] London, A.L., and Shah, R.K., Cost of irreversibilities in heat exchanger design, Heat Transfer Engineering, Vol. 4, No. 2, pp. 59-73, 1983.

[83NO1e] Noe, B.J., and Strickler, G.L., Computerized cost estimation of heat exchangers, ASME Paper 83-HT-62, ASME, New York, 1983.

[83NO2e] North, C., Costing of air coolers, in VDI-Verlag, Heat exchanger design handbook, Section 4.8.3 Hemisphere Publishing Corporation, London, 1983.

[83PA1e] Paikert, P., Air-cooled heat exchangers, in VDI-Verlag, Heat exchanger design handbook, Section 3.8, Hemisphere Publishing Corporation, London, 1983.

[83PU1e] Purohit, G.P., Estimating costs of shell-and-tube heat exchangers, Chemical Engineering, pp. 56-67, August 1983.

[83RE1e] Reklaitis, G.V., Ravindran, A., and Ragsdell, K.M., Engineering optimization: Methods and applications, J. Wiley and Sons, New York, 1983.

[83SH1e] Shipes, K.V., Mechanical design of air-cooled heat exchangers, in VDI-Verlag, Heat exchanger design handbook, Section 4.4.1, Hemisphere Publishing Corporation, London, 1983.

[83SI1e] Singham, J.R., Cooling towers, in VDI-Verlag, Heat exchanger design handbook, Section 3.12, Hemisphere Publishing Corporation, London, 1983.

[83TA1e] Talukdar, S.N., Giras, T.C., and Kalyan, V.K., Decompositions for optimal power flows, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 12, pp. 3877-3884, 1983.

[83VA1e] Van der Walt, I.D., Water as a resource in power generation, President's invitation lecture, The Transactions of the SA Institute of Electrical Engineers, pp. 50-58, March 1983.

[84AL1e] Alowooja, K.T., Gould, P.L., Furzer, I., Optimization of large cooling towers, in: P.L. Gould, W.B. Krätzig, I. Mungan, and U. Wittek (eds.), Natural draught cooling towers, Proceedings of the 2nd International Symposium, Ruhr-Universität Bochum, Germany, September 5-7, 1984, Springer-Verlag, Berlin, pp. 413-429, 1984.

[84BE1e] Belegundu, A.D., and Arora, J.S., A recursive quadratic programming method with active set strategy for optimal design, International Journal for Numerical Methods in Engineering, Vol. 20, pp. 803-816, 1984.

[84CH1e] Chemical Engineering, Modern cost engineering: Methods and data, Vol. 2, McGraw-Hill, New York, 1984.

[84OS1e] Osyczka, A., Multicriterion optimization in engineering, Ellis Horwood Ltd., Chichester, 1984.

[84QI1e] Qian, L.X., Zhong, W.X., Cheng, K.T., and Sui, Y.K., An approach to structural optimization- sequential quadratic programming, SQP, Engineering Optimization, Vol. 8, pp. 83-100, 1984.

[84RO1e] Roetzel, W., Berechnung von Wärmeübertragern, VDI-Wärmeatlas, 4.Auflage, VDI-Verlag GmbH, Düsseldorf, pp. Ca1-Ca31, 1984.

[84VA1e] Vanderplaats, G.N., Numerical Optimization Techniques for Engineering Design with Applications, McGraw-Hill Book Company, New York, 1984.

[84VO1e] Von Cleve, H.-H., Die luftgekühlte Kondensationsanlange des 4000-MW-Kraftwerkes Matimba/Südafrika, VGB Kraftwerkstechnik 64, Heft 4, April 1984.

[84ZE1e] Zerna, W., Impulses of the research on the development of large cooling towers, in: P.L. Gould, W.B. Krätzig, I. Mungan, and U. Wittek (eds.), Natural draught cooling towers, Proceedings of the 2nd International Symposium, Ruhr-Universität Bochum, Germany, September 5-7, 1984, Springer-Verlag, Berlin, pp. 1-18, 1984.

[85BE1e] Belegundu, A.D., and Arora, J.S., A study of mathematical programming methods for structural optimization. Part I: Theory; Part II: Numerical results, International Journal for Numerical Methods in Engineering, Vol. 21, No. 9, pp. 1583-1623, 1985.

[85BI1e] Biegler, L.T., Improved infeasible path optimization for sequential modular simulators-I: the interface, Computers and Chemical Engineering, Vol. 9, No. 3, pp. 245-256, 1985.

[85BI2e] Biegler, L.T., and Cuthrell, J.E., Improved infeasible path optimization for sequential modular simulators-II: the optimization algorithm, Computers and Chemical Engineering, Vol. 9, No. 3, pp. 257-267, 1985.

[85EC1e] Eckels, P.W., and Rabas, T.J., Heat transfer and pressure drop of typical air cooler finned tubes, ASME Journal of Heat Transfer, Vol. 107, pp. 198-204, 1985.

[85ES1e] ESCOM, Kendal - breakthrough for new technology, Technical information, ESCOM, South Africa, 1985.

[85GA1e] Ganguli, A., Tung, S.S., and Taborek, J., Parametric study of air-cooled heat exchanger finned tube geometry, AIChE Symposium Series, Vol. 81, No. 245, pp. 122-128, 1985.

[85GE1e] Gero, J.S., Optimization in computer-aided design, Proceedings of the IFIP WG 5.2 Working Conference on Optimization in Computer-aided Design, Lyon, France, 24-26 October 1983, North-Holland, Amsterdam, 1985.

[85KO1e] Kotas, T.J., The exergy method of thermal plant analysis, Butterworths, London, 1985.

[85LI1e] Li, K.W., and Priddy, A.P., Power plant system design, J. Wiley and Sons, New York, 1985.

[85LI2e] Li, K.W., and Sadiq, W., Computer-aided optimization of cooling systems, ASME International Computers in Engineering Conference, Vol. 3, pp. 165-171, 1985.

[85MA1e] Maulbetsch, J.S., and Bartz, J.A., Cooling towers and cooling ponds, in: W.M. Rohsenow, J.P. Hartnett and E.N. Ganic (eds.), Handbook of Heat Transfer Applications, 2nd ed., McGraw-Hill Book Co., New York, 1985.

[85MO1e] Monroe, R.C., Minimizing fan energy costs, Part 1, Chemical Engineering, May 27, pp. 141-142, 1985.

[85MO2e] Monroe, R.C., Minimizing fan energy costs, Part 2, Chemical Engineering, June 24, pp. 57-58, 1985.

[85RI1e] Rijckaert, M.J., and Walraven, E.J.C., Reflections on geometric programming, in: K. Schittkowski (ed.),Computational Mathematical Programming, NATO ASI Series F: Computer and System Sciences, Vol. 15, Springer-Verlag, Berlin, 1985.

[85VE1e] Ventilatoren Stork Hengelo, General instructions for E-type fans, V.960874, 1985.

[86AR1e] Arora, J.S., and Baenziger, G., Uses of artificial intelligence in design optimization, Computer Methods in Applied Mechanics and Engineering, Vol. 54, No. 1, pp. 303-323, 1986.

[86AR2e] Arora, J.S., and Thanedar, P.B., Computational methods for optimum design of large complex systems, Computational Mechanics, Vol. 1, pp. 221-242, 1986.

[86GE1e] Geldenhuys, J.D., and Kröger, D.G., Aerodynamic inlet losses in natural draft cooling towers, Proceedings of the 5th IAHR cooling tower workshop, Monterey, 1986.

[86HO1e] Holman, J.P., Heat Transfer, McGraw-Hill Book Company, New York, 1986.

[86HI1e] Hicks, T.G., Power plant evaluation and design reference guide, McGraw-Hill Book Co., New York, 1986.

[86KO1e] Kotzé, J.C.B., Bellstedt, M.O., and Kröger, D.G., Pressure drop and heat transfer characteristics of inclined finned tube heat exchanger bundles, Proceedings of the 8th International Heat Transfer Conference, San Francisco, 1986.

[86KR1e] Kröger, D.G., Performance characteristics of industrial finned tubes presented in dimensional form, International Journal of Heat and Mass Transfer, Vol. 29, No. 8, pp. 1119-1125, 1986.

[86LI1e] Lim, O.K., and Arora, J.S., An active set RQP algorithm for engineering design optimization, Computer Methods in Applied Mechanics and Engineering 57, pp. 51-65, 1986.

[86ST1e] Stuart, D.O., and Arnold, R.C., Condenser, cooling tower system optimization using geometric programming, Presented at the Joint ASME/IEEE Power Generation Conference, Portland, Oregon, October 19-23, 1986.

[86TH1e] Thanedar, P.B., Arora, J.S., Tseng, C.H., Lim, O.K. and Park, G.J., Performance of some SQP algorithms on structural design problems, International Journal for Numerical Methods in Engineering, Vol. 23, pp. 2187-2203, 1986.

[86VA1e] Vanderplaats, G.N., and Sugimoto, H., A general-purpose optimization program for engineering design, Computers and Structures, Vol. 24, No. 1, pp. 13-21, 1986.

[87AD1e] Adibhalta, S., and Leigh, D.C., A comparison of optimization methods for engineering system optimization, Engineering Optimization, Vol. 12, pp. 13-23, 1987.

[87BE1e] Bejan, A., The thermodynamic design of heat and mass transfer processes and devices, Heat and Fluid Flow, Vol. 8, No. 4, pp. 258-276, 1987.

[87FR1e] Fryer, M.J., and Greenman, J.V., Optimisation Theory, Applications in OR and Economics, Edward Arnold Publishers, London, 1987.

[87HA1e] Ham, A.J., and West, L.A., ESCOM'S advance into dry cooling, VGB Conference, November 9-13 1987, Johannesburg, South Africa , pp. 33-43, 1987.

[87HA2e] Hargraves, C.R., and Paris, S.W., Direct trajectory optimization using nonlinear programming and collocation, Journal of Guidance, Control and Dynamics, Vol. 10, No. 4, pp. 338-342, 1987.

[87HO1e] Horngren, C.T., and Foster, G., Cost accounting, A managerial emphasis, 6th edition, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1987.

[87KI1e] Kisala, T.P., Trevino-Lozano, R.A., Boston, J.F., and Britt, H.I., Sequential modular and simultaneous modular strategies for process flowsheet optimization, Computers and Chemical Engineering, Vol. 11, No. 6, pp. 567-579, 1987.

[87KN1e] Knirsch, H., Design and construction of direct dry cooling units, VGB Conference, November 9-13 1987, Johannesburg, South Africa , pp. 54-69, 1987.

[87KR1e] Kröger, D.G., Dry cooling research, VGB Conference, November 9-13 1987, Johannesburg, South Africa, pp. 44-53, 1987.

[87OS1e] Ostrowski, E.T., and Queenan, P.T., Increase net plant output through selective operation of the heat-rejection system, Power, pp. 61-64, May 1987.

[87TR1e] Trage, B., and Hintzen, F.J., Design and construction of indirect dry cooling units, VGB Conference, November 9-13 1987, Johannesburg, South Africa , pp. 70-79, 1987.

[87VE1e] Veck, G.A., and Rubbers, P.J.E., Economic evaluation of dry cooling: the options, VGB Conference, November 9-13 1987, Johannesburg, South Africa , pp. 80-89, 1987.

[88AR1e] Arora, J.S., and Tseng, C.H., Interactive design optimization, Engineering Optimization, Vol. 13, pp. 173-188, 1988.

[88BI1e] Biegler, L.T., On the simultaneous solution and optimization of large scale engineering systems, Computers and Chemical Engineering, Vol. 12, No. 5, pp. 357-369, 1988.

[88BR1e] Breber, G., Computer programs for design of heat exchangers, in R.K. Shah, E.C. Subbarao, and R.A., Mashelkar (eds.), Heat transfer equipment design, Hemisphere Publishing Corporation, London, pp. 167-177, 1988.

[88ED1e] Edgar, T.F., and Himmelblau, D.M., Optimization of Chemical Processes, McGraw-Hill Book Co., New York, 1988.

[88HI1e] Higgins, R.L., Maintenance Engineering Handbook, Fourth Edition, McGraw-Hill Book Company, New York, 1988.

[88KH1e] Kharbanda, O.P., and Stallworthy, C.A., Capital cost estimating for the process industry, Buterworths, London, 1988.

[88SH1e] Shah, R.K., Heat exchanger design methodology, in R.K. Shah, E.C. Subbarao, and R.A., Mashelkar (eds.), Heat transfer equipment design, Hemisphere Publishing Corporation, London, pp. 17-22, 1988.

[88ST1e] Stasiulevicius, J., and Skrinska, A., Heat transfer of finned tube bundles in crossflow, (A. Zukauskas and G. Hewitt), Hemisphere Publishing Corporation, New York, 1988.

[88TS1e] Tseng, C.H., and Arora, J.S., On implementation of computational algorithms for optimal design. Part 1: Preliminary investigation; Part 2: Extensive numerical investigation, International Journal for Numerical Methods in Engineering, Vol. 26, No. 6, pp. 1365-1402, 1988.

[88VA1e] Vasantharajan, S., and Biegler, L.T., Large-scale decomposition for successive quadratic programming, Computers and Chemical Engineering, Vol. 12, No. 11, pp. 1087-1101, 1988.

[88ZU1e] Zukauskas, A., and Ulinskas, R., Heat transfer in tube banks in crossflow, Hemisphere Publishing Corporation, New York, 1988.

[89AC1e] Aceves-Saborio, S., Ranasinghe, J., and Reistad, G.M., An extension to irreversibility minimization analysis applied to heat exchangers, ASME Journal of Heat Transfer, Vol. 111, pp. 29-36, 1989.

[89AR1e] Arora, J.S., Introduction to optimum design, McGraw, New York, 1989.

[89BE1e] Bejan, A., Minimizing entropy in thermal systems, Mechanical Engineering, pp. 88-91, August 1989.

[89BU1e] Buys, J.D., and Kröger, D.G., Dimensioning heat exchangers for existing dry cooling towers, Energy Conversion Management, Vol. 29, No. 1, pp. 63-71, 1989.

[89BU2e] Buys, J.D., and Kröger, D.G., Cost-optimal design of dry cooling towers through mathematical programming techniques, ASME Journal of Heat Transfer, Vol. 111, No. 2, pp. 322-327, 1989.

[89GA1e] Garrett, D.E., Chemical engineering economics, Van Nostrand Reinhold, New York, 1989.

[89KO1e] Kovarik, M., Optimal heat exchangers, ASME Journal of Heat Transfer, Vol. 111, pp. 287-293, 1989.

[89PE1e] Peurifoy, R.L., and Oberlender, G.D., Estimating construction costs, 4th edition, McGraw-Hill, Inc. New York, 1989.

[89RA1e] Rabas, T.J., and Huber, F.V., Row number effects on the heat transfer performance of in-line finned tube banks, Heat Transfer Engineering, Vol. 10, No. 4, pp. 19-29, 1989.

[89RA2e] Ranasinghe, J., Aceves-Saborio, S., and Reistad, G.M., Irreversibility and thermoeconomics based design optimization of a ceramic heat exchanger, ASME Journal of Engineering for Gas Turbines and Power, Vol. 111, pp. 719-727, 1989.

[89ST1e] Stoll, H.G., Least-cost Electric Utility Planning, J. Wiley and Sons, New York, 1989.

[89TS1e] Tsatsaronis, G., and Valero, A., Thermodynamics meets economics, Mechanical Engineering, pp. 84-86, August 1989.

[89WI1e] Winchell, W., Realistic cost estimating for manufacturing, 2nd ed., Society of Manufacturing Engineers, Dearborn, Michigan, 1989.

[90LO1e] Lovino, G., Hattoni, H., Hazzocchi, L., Senis, R., and Vanzan, R., Optimal sizing of natural draft, dry cooling towers for ENEL combined cycle power plants, in: S.S. Stecco and M.J. Moran (eds.), A future for energy, FLOWERS '90, Proceedings of the Florence World Energy Research Symposium, Firenze, Italy, 28 May-1 June 1990, Pergamon Press, New York, pp. 669-680, 1990.

[90LU1e] Lucia, A., and Xu, J., Chemical process optimization using Newton-like methods, Computers and Chemical Engineering, Vol. 14, No. 2, pp. 119-138, 1990.

[90VA1e] Vasantharajan, S., Viswanathan, J., and Biegler, L.T., Reduced successive quadratic programming implementation for large-scale optimization problems with smaller degrees of freedom, Computers and Chemical Engineering, Vol. 14, No. 8, pp. 907-915, 1990.

[90VE1e] Venter, S.J., The influence of distorted flow patterns on the overall performance of axial flow fans, Ph. D. Thesis, University of Stellenbosch, 1990.

[91AS1e] ASME PTC 30-1991, Air cooled heat exchangers, Performance test codes, ASME, New York, 1991.

[91CO1e] Conradie , A.E., Evaluation of the performance characteristics of forced draft air-cooled heat exchangers and the effect of plume recirculation thereon, M. Eng. Thesis, University of Stellenbosch, South Africa, 1991.

[91CO2e] Conradie, T.A., and Kröger, D.G., Enhanced performance of a dry-cooled power plant through air precooling, ASME Paper 91-JPGC-Pwr-6 presented at the ASME International Joint Power Conference, San Diego, October 1991.

[91GL1e] Glimm, J.G., Mathematical Sciences, Technology and Economic Competitiveness, National Academy Press, Washington, D.C, 1991.

[91GU1e] Guha, M.K., and Singh, A., Economic benefits of availability improvement, Presented at the International Power Generation Conference, October 6-10, San Diego, CA, 1991.

[91HU1e] Humphreys, K.K., Jelen's cost and optimization engineering, third edition, McGraw-Hill, Inc., New York, 1991.

[91NI1e] Nir, A., Heat transfer and friction factor correlations for crossflow over staggered finned tube banks, Heat Transfer Engineering, Vol. 12, No. 1, pp. 43-58, 1991.

[91PE1e] Peters, M.S., and Timmerhaus, K.D., Plant design and economics for chemical engineers, 4th edition, McGraw-Hill, Inc. New York, 1991.

[91SZ1e] Szabó, Z., Why use 'Heller System'? Circuitry, characteristics and special features, Paper presented at the Symposium on Dry Cooling Towers, Tehran, 1991.

[91TA1e] Taborek, J., Industrial heat exchanger design practices, in: S.Kakac (ed.), Boilers, Evaporators, and Condensers, J. Wiley and sons, New York, pp. 143-177, 1991

[91VE1e] Venter, S.J., and Kröger, D.G., An evaluation of methods to predict the system effect present in air-cooled heat exchangers, Heat Recovery Systems and CHP, Vol. 11, No. 5, pp. 431-440, 1991.

[91WI1e] Winstanley, G., Artificial intelligence in engineering, John Wiley and Sons, New York, 1991.

[92BI1e] Biegler, L.T., Optimization strategies for complex process models, in: J. Wei (ed.), Advances in Chemical Engineering, Vol. 18, pp. 197-256, Academic Press, Inc., New York, 1992.

[92GE1e] Gero, J.S., Artificial intelligence in design '92, Kluwer Academic Publishers, Dordrecht, 1992.

[92HO1e] Hodson, W.K., Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, 1992.

[92MC1e] McCormick, E.J., Activity based costing and controls, in: W.K. Hodson, Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, pp. 9.73-9.106, 1992.

[92MC2e] McKetta, J.J., Heat transfer design methods, Marcel Dekker Inc., New York, 1992.

[92NI1e] Nicks, J.E., Cost estimating, in: W.K. Hodson, Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, pp. 9.107-9.120, 1992.

[92RA1e] Rajgopal, J., Nonlinear Programming, in: W.K. Hodson, Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, pp. 14.177-14.200, 1992.

[92RE1e] Reeves, G.R., and Lawrence, K., Multicriteria decision making, in: W.K. Hodson, Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, pp. 14.201-14.218, 1992.

[92RO1e] Rowe, A.J., Artificial intelligence and expert systems, in: W.K. Hodson, Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, pp. 12.87-12.100, 1992.

[92VE1e] Venter, S.J., and Kröger, D.G., The effect of tip clearance on the performance of an axial flow fan, Energy Conversion Management, Vol. 33, No. 2, pp. 89-97, 1992.

[92WA1e] Warren, H.E., Linear Programming, in: W.K. Hodson, Maynard's Industrial Engineering Handbook, 4th edition, McGraw-Hill, Inc. New York, pp. 14.151-14.175, 1992.

[93AD1e] Adams, S., and Stevens, J., Strategies for improved cooling tower economy, CTI Journal, Vol. 13, No. 1, pp. 6-13, 1993.

[93BE1e] Betts, J.T., and Huffman, W.P., Path-constrained trajectory optimization using sparse sequential quadratic programming, Journal of Guidance, control and Dynamics, Vol. 16, No. 1, pp. 59-68, Jan-Feb 1993

[93CI1e] Ciriani, T.A., and Leachman, R.C., Optimization in industry - Mathematical programming and modeling techniques in practice, J.Wiley and Sons, Chichester, 1993.

[93ER1e] Ertas, A., and Jones, J.J., The Engineering Design Process, J. Wiley and Sons, Inc., New York, 1993.

[93GR1e] Groenewald, W., Heat transfer and pressure change in an inclined air-cooled flattened tube during condensation of steam, M. Eng. Thesis, University of Stellenbosch, 1993

[93SC1e] Schmid, C., and Biegler, L.T., Acceleration of reduced Hessian methods for large-scale nonlinear programming, Computers and Chemical Engineering, Vol. 17, No. 5/6, pp. 451-463, 1993.

[93VA1e] Van Aarde, D.J., and Kröger, D.G., Flow losses through an array of A-frame heat exchangers, Heat Transfer Engineering, Vol. 14, No. 1, 1993.

[94BI1e] Biegler, L.T., Nocedal, J., and Schmid, C., A reduced Hessian method for large-scale constrained optimization, To appear, SIAM J. Optim., 1994.

[94DU1e] Du Preez, A.F., and Kröger, D.G., The influence of a buoyant plume on the performance of a natural draft cooling tower, Paper presented at the 9th IAHR Cooling Tower Conference, Brussels, September, 1994.

[94GR1e] Groenewald, W., and Kröger, D.G., Effect of suction on turbulent friction inside ducts, Submitted for publication, 1994.

[94KA1e] Kao, C., and Chen, S., A sequential quadratic programminc algorithm utilizing QR matrix factorization, Engineering Optimization, Vol. 22, pp. 283-296, 1994.

[94KR1e] Kröger, D.G., Personal communication, Department of Mechanical Engineering, University of Stellenbosch, 1994.

[94KR2e] Kröger, D.G., Fan performance in air-cooled steam condensers, Heat Recovery Systems and CHP, vol. 14, No. 4, pp. 391-399, 1994.

[94SA1e] Salta, C, and Kröger, D.G., Effect of inlet flow distortions on fan performance in forced draught air-cooled heat exchangers, Submitted for publication, 1994.

[94SC1e] Schmid, C., and Biegler, L.T., Quadratic programming methods for reduced Hessian SQP, Computers and Chemical Engineering, Vol. 18, No. 9, pp. 817-832, 1994.

[94VA1e] Varvarezos, D.K., Biegler, L.T., and Grossmann, I.E., Multiperiod design optimization with SQP decomposition, Computers and Chemical Engineering, Vol. 18, No. 7, pp. 579-595, 1994.

R.2 MATHEMATICAL REFERENCES

[44LE1m] Levenberg, K., A method for the solution of certain problems in least squares, Quart. Appl. Math., Vol. 2, pp. 164-168

[63MA1m] Marquardt, D.W., An algorithm for least-squares estimation of nonlinear parameters, J. Soc. Indust. Appl. Math., Vol. 11, No. 2, pp. 431-441, 1963.

[63WI1m] Wilson, R.B., A simplicial method for concave programming, Ph.D. Thesis, Graduate School of Business Administration, Harvard University, Cambridge, Massachusetts, 1963.

[72BI1m] Biggs, M.C., Constrained minimization using recursive equality quadratic programming, in: F.A. Lootsma (ed.), Numerical methods for Non-Linear Optimization, Academic press, new York, pp. 411-428, 1972.

[72OS1m] Osborne, M.R., Some aspects of non-linear least squares calculations, in: F.A. Lootsma (ed.), Numerical methods for non-linear optimization, Academic Press, London, 1972.

[74GI1m] Gill, P.E., Golub, G.H., Murray, W., and Saunders, M.A., Methods for modifying matrix factorizations, Mathematics of Computation, Vol. 28, No. 126, pp. 505-535, 1974.

[74LA1m] Lawson, C.L., and Hanson, R.J., Solving Least Squares Problems, Prentice-Hall, Englewood Cliffs, New Jersey, 1974.

[76HA1m] Han, S.P., Superlinearly convergent variable metric algorithms for general nonlinear programming problems, Mathematical Programming 11, pp. 263-282, 1976.

[76MU1m] Murray, W., Methods for constrained optimization, in: L.C.W. Dixon (ed.), Optimization in action, Academic Press, New York, 1976.

[77HA1m] Han, S.P. A globally convergent method for nonlinear programming, Journal of Optimization Theory and Applications, Vol. 22, No. 3, pp. 297-309, 1977.

[78DE1m] Dembo, R.S., Current state of the art of algorithms and computer software for geometric programming, Journal of Optimization Theory and Applications, Vol. 26, No. 2, 1978.

[78GI1m] Gill, P.E., and Murray, W., Numerically stable methods for quadratic programming, Mathematical Programming 14, pp. 349-372, 1978.

[78PO1m] Powell, M.J.D., A fast algorithm for nonlinearly constrained optimization calculations, in: G.A. Watson (ed.), Numerical Analysis, Dundee 1977, Lecture Notes in Mathematics 630, Springer-Verlag, Berlin, pp. 144-157, 1978.

[78PO2m] Powell, M.J.D., Algorithms for nonlinear constraints that use Lagrangian functions, Mathematical Programming 14, pp. 224-248, 1978.

[78PO3m] Powell, M.J.D., The convergence of variable metric methods for nonlinearly constrained optimization calculations, in: O.L. Mangasarian, R.R. Meyer and S.M. Robinson (eds.), Nonlinear Programming 3, Academic Press, New York, pp. 27-63, 1978.

[78RA1m] Ratner, M., Lasdon, L.S., and Jain, A., Solving geometric programs using GRG: Results and comparisons, Journal of Optimization Theory and Applications, Vol. 26, No. 2, 1978.

[78TA1m] Tapia, R.A., Quasi-Newton methods for constrained optimization: Equivalence of existing methods and a new implementation, in: O.L. Mangasarian, R.R. Meyer and S.M. Robinson (eds.), Nonlinear Programming 3, Academic Press, New York, pp. 125-164, 1978.

[79CH1m] Chamberlain, R.M., Some examples of cycling in variable metric methods for constrained minimization, Mathematical Programming 16, pp. 378-383, 1979.

[79HO1m] Holt, J.N., and Fletcher, R., An algorithm for constrained non-linear least-squares, Journal of the Institute of Mathematics and its Applications, Vol. 23, pp. 449-463, 1979.

[80BE1m] Betts, J.T., A compact algorithm for computing the stationary point of a quadratic function subject to linear constraints, ACM Transactions on Mathematical Software, Vol. 6, No. 3, pp. 391-397, 1980.

[80BE2m] Betts, J.T., Algorithm 559: The stationary point of a quadratic function subject to linear constraints, ACM Transactions on Mathematical Software, Vol. 6, No. 3, pp. 432-436, 1980.

[80EC1m] Ecker. J.G., Geometric programming: Methods, computations and applications, SIAM Review, Vol. 22, No. 3, pp. 338-362, 1980.

[80SC1m] Schittkowski, K., Nonlinear programming codes (Information, tests, performance), in: M. Beckmann, and H.P. Künzi (eds.), Lecture Notes in Economics and Mathematical Systems, Vol. 183, Springer-Verlag, Berlin, 1980.

[81FL1m] Fletcher, R., Numerical experiments with an exact L1 penalty function method, in: O.L. Mangasarian, R.R. Meyer, and S.M. Robinson (eds.), Nonlinear Programming 4, Academic Press, New York, pp. 99-129, 1981.

[81GI1m] Gill, P.E., Murray, W., and Wright, M.H., Practical optimization, Academic Press, London, 1981.

[81HO1m] Hock, W., and Schittkowski, K., Test examples for nonlinear programming codes, in: M. Beckmann, and H.P. Künzi (eds.), Lecture Notes in Economics and Mathematical Systems, Vol. 187, Springer-Verlag, Berlin, 1981.

[81MO1m] Moré, J.J., Garbow, B.S., and Hillstrom, K.E., Testing unconstrained optimization software, ACM Transactions on Mathematical Software, Vol. 7, No. 1, pp. 17-41, 1981.

[81SC1m] Schittkowski, K., The nonlinear programming method of Wilson, Han and Powell with an augmented Lagrangian type line search function, Numerische Mathematik 38, pp. 83-114, 1981.

[82BA1m] Bartholomew-Biggs, M.C., Recursive quadratic programming methods for nonlinear constraints, in: M.J.D. Powell (ed.), Nonlinear Programming 1981, Academic Press, New York, pp. 213-221, 1982.

[82CH1m] Chamberlain, R.M., Powell, M.J.D., Lemarechal, C., and Pederson, H.C., The watchdog technique for forcing convergence in algorithms for constrained optimization, Mathematical Programming Study 16, pp. 1-17, 1982.

[82GA1m] Gabay, D., Reduced quasi-Newton methods with feasibility improvement for nonlinearly constrained optimization, Mathematical Programming Study 16, pp. 18-44, 1982.

[82GI1m] Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., Linearly constrained optimization, in M.J.D. Powell (ed.), Nonlinear Optimization 1981, Academic Press, pp. 123-139, 1982.

[82GO1m] Goldfarb, D., Numerically stable approaches to linearly constrained optimization, in M.J.D. Powell (ed.), Nonlinear Optimization 1981, Academic Press, pp. 141-146, 1982.

[82PO1m] Powell, M.J.D., Extensions to subroutine VFO2AD, in: R.F. Drenick and F. Kozin (eds.), System Modeling and Optimization, Lecture notes in Control and Information Sciences, No. 38, Springer-Verlag, Berlin, pp. 529-538, 1982.

[82PO2m] Powell, M.J.D., VMCWD: A Fortran subroutine for constrained optimization, Report DAMTP - 1982/NA4, University of Cambridge, 1982.

[82SC1m] Schittkowski, K., Nonlinear programming methods with linear least squares subproblems, in: J.M. Mulvey (ed.), Evaluating Mathematical Programming Techniques, Lecture Notes in Economics and Mathematical Systems 199, Springer-Verlag, Berlin, pp. 200-213, 1982.

[83DE1m] Dennis, J.E., and Schnabel, R.B., Numerical methods for unconstrained optimization and nonlinear equations, Prentice-Hall, Englewood Cliffs, New Jersey, 1983.

[83FI1m] Fiacco, A.V., Introduction to sensitivity and stability analysis in nonlinear programming, Academic Press, New York, 1983.

[83GO1m] Goldfarb, D., and Idnani, A., A numerically stable dual method for solving strictly convex quadratic programs, Mathematical Programming 27, pp. 1-33, 1983.

[83KR1m] Kreyzig, E., Advanced Engineering Mathematics, 5th Ed., J. Wiley and Sons, New York, 1983.

[83MC1m] McCormick, G.P., Nonlinear programming - theory, algorithms, and applications, J. Wiley and Sons, New York, 1983.

[83PO1m] Powell, M.J.D., Variable metric methods for constrained optimization, in: A. Bachem, M. Grötschel and B. Korte (eds.), Mathematical Programming: The state of the art, Bonn, 1982, Springer-Verlag, pp. 288-311, 1983.

[83PO2m] Powell, M.J.D., ZQPCVX: a Fortran subroutine for convex quadratic programming, Report DAMTP/1983/NA17, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, 1983.

[83TO1m] Tone, K. Revisions of constraint approximations in the successive QP method for nonlinear programming, Mathematical Programming 26, pp. 144-152, 1983.

[84GI1m] Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., Procedures for optimization problems with a mixture of bounds and general linear constraints, ACM Transactions on Mathematical Software, Vol. 10, No. 3, pp. 282-298, 1984.

[84LU1m] Luenberger, D.G., Introduction to linear and nonlinear programming, Addison-Wesley, Reading Massachusetts, 1984.

[85GI1m] Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., Some issues in implementing a sequential quadratic programming algorithm, SIGNUM Newsletter, Vol. 20, No. 2, pp. 13-19, 1985.

[85GI2m] Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., Model building and practical aspects of nonlinear programming, in: K. Schittkowski (ed.),Computational Mathematical Programming, NATO ASI Series F: Computer and System Sciences, Vol. 15, Springer-Verlag, Berlin, pp. 209-247, 1985.

[85NO1m] Nocedal, J., and Overton, M.L., Projected Hessian updating algorithms for nonlinearly constrained optimization, SIAM Journal on Numerical Analysis 22, pp. 821-850, 1985.

[85PO1m] Powell, M.J.D., On the quadratic programming algorithm of Goldfarb and Idnani, Mathematical Programming Study 25, pp. 46-61, 1985.

[85ST1m] Stoer, J., Principles of sequential quadratic programming methods for solving nonlinear programs, in: K. Schittkowski (ed.),Computational Mathematical Programming, NATO ASI Series F: Computer and System Sciences, Vol. 15, Springer-Verlag, Berlin, pp. 165-207, 1985.

[86GI1m] Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., Considerations of numerical analysis in a sequential quadratic programming method, in: J.P. Hennart (ed.), Numerical Analysis, Guanajuato, 1984, Lecture Notes in Mathematics 1230, Springer-Verlag, Berlin, pp. 46-62, 1986.

[86PR1m] Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., Numerical recipes - The art of scientific computing, Cambridge University Press, Cambridge, 1986.

[86SC1m] Schittkowski, K., NLPQL: a FORTRAN subroutine solving constrained nonlinear programming problems, Annals of Operations Research, Vol. 5, pp. 485-500, 1986.

[87BA1m] Bartholomew-Biggs, M.C., Recursive quadratic programming methods based on the augmented Lagrangian, Mathematical Programming Study 31, pp. 21-41, 1987.

[87FL1m] Fletcher, R., Practical methods of optimization, 2nd Ed., J. Wiley and Sons, New York, 1987.

[87SA1m] Salane, D.E., A continuation approach for solving large-residual nonlinear least squares problems, SIAM J. Sci. Stat. Comput., Vol. 8, No. 4, pp. 655-671, 1987.

[87SC1m] Schittkowski, K., More test examples for nonlinear programming codes, in: M. Beckmann, and W. Krelle (eds.), Lecture Notes in Economics and Mathematical Systems, Vol. 282, Springer-Verlag, Berlin, 1987.

[88FA1m] Fan, Y., Sarkar, S., and Lasdon, L., Experiments with successive quadratic programming algorithms, Journal of Optimization Theory and Applications Vol. 56, No. 3, pp. 359-383, 1988.

[88GR1m] Grandinetti, L. and Conforti, D., Numerical comparisons of nonlinear programming algorithms on serial and vector processors using automatic differentiation, Mathematical Programming 42, pp. 375-389, 1988.

[88HA1m] Hager, W.W., Applied numerical linear algebra, Prentice Hall, Englewood Cliffs, New Jersey, 1988.

[89DE1m] Dennis, J.E., and Schnabel, R.B., A view of unconstrained optimization, in: G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd (eds.), Optimization, Vol. 1, North-Holland, Amsterdam, pp. 1-72, 1989.

[89GI1m] Gill, P.E., Murray, W., Saunders, M.A., and Wright, M.H., Constrained Nonlinear Programming, in: G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd (eds.), Optimization, Vol. 1, North-Holland, Amsterdam, pp. 171-210, 1989.

[89GO1m] Goldfarb, D., and Todd, M.J., Linear Programming, in: G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd (eds.), Optimization, Vol. 1, North-Holland, Amsterdam, pp. 73-170, 1989.

[89GU1m] Gurwitz, C.B., and Overton, M.L., Sequential quadratic programming methods based on approximating a projected Hessian matrix, SIAM Journal on Sci. Stat. Comput., Vol. 10, pp. 631-653, 1989.

[89KA1m] Kahaner, D., Moler, C., and Nash, S., Numerical methods and software, Prentice Hall, New Jersey, 1989.

[89LE1m] Lemaréchal, C., Nondifferential optimization, in: G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd (eds.), Optimization, Vol. 1, North-Holland, Amsterdam, pp. 529-572, 1989.

[89NE1m] Nemhauser, G.L., Rinnooy Kan, A.H.G., and Todd, M.J., Optimization, Vol. 1, North-Holland, Amsterdam, 1989.

[89PO1m] Powell, M.J.D., A tolerant algorithm for linearly constrained optimization calculations, Mathematical Programming 45, pp. 547-566, 1989.

[89RI1m] Rinnooy Kan, A.H.G., and Timmer, G.T., Global optimization, in: G.L. Nemhauser, A.H.G. Rinnooy Kan, and M.J. Todd (eds.), Optimization, Vol. 1, North-Holland, Amsterdam, pp. 631-662, 1989.

[89SH1m] Shanno, D.F., and Phua, K.H., Numerical experience with sequential quadratic programming algorithms for equality constrained nonlinear programming, ACM Transactions on Mathematical Software, Vol. 15, No. 1, pp. 49-63, 1989.

[89WR1m] Wright, S., Convergence of SQP-like methods for constrained optimization, SIAM Journal of Control and Optimization, Vol. 27, No. 1, pp. 13-26, 1989.

[90BY1m] Byrd, R.H., On the convergence of constrained optimization methods with accurate Hessian information on a subspace, SIAM Journal on Numerical Analysis 27, pp. 141-153, 1990.

[90SE1m] Sewell, G., Computational methods of linear algebra, Ellis Horwood Limited, New York, 1990.

[91BY1m] Byrd, R.H., and Nocedal, J., An analysis of reduced Hessian methods for constrained optimization, Mathematical Programming, Vol. 49, pp. 285-323, 1991.

[91VA1m] Van Huffel, S., and Vandewalle, J., The total least squares problem, Computational aspects and analysis, Society for Industrial and Applied Mathematics, Philadelphia, 1991.

[91WA1m] Watkins, D.S., Fundamentals of matrix computations, J Wiley and Sons, New York, 1991.

[91WR1m] Wright, M.H., Optimization and large-scale computation, in: J.P. Mesirov (ed.), Very large scale computation in the 21st century, Thinking Machines Corporation, pp. 250-272, 1991.

[91ZL1m] Zlatev, Z., Computational methods for general sparse matrices, Kluwer Academic Publishers, London, 1991.

[92BU1m] Buchanan, J.L., and Turner, P.R., Numerical methods and analysis, McGraw-Hill, Inc., New York, 1992.

[92JE1m] Jennings, A., and McKeown, J.J., Matrix Computation, Second Edition, J. Wiley and Sons, New York, 1992.

APPENDIX A

PROPERTIES OF FLUIDS

A.1 THE THERMOPHYSICAL PROPERTIES OF DRY AIR FROM 220 K TO 380 K

Density

$$\rho_a = p_a/(RT), \quad kg/m^3 \tag{A.1}$$
where $R = 287.08$ J/kgK

Specific heat

$$c_{pa} = a + bT + cT^2 + dT^3, \quad J/kgK$$
$$a = 1.045356 \times 10^3$$
$$b = -3.161783 \times 10^{-1} \tag{A.2}$$
$$c = 7.083814 \times 10^{-4}$$
$$d = -2.705209 \times 10^{-7}$$

Dynamic viscosity

$$\mu_a = a + bT + cT^2 + dT^3, \quad kg/ms$$
$$a = 2.287973 \times 10^{-6}$$
$$b = 6.259793 \times 10^{-8} \tag{A.3}$$
$$c = -3.131956 \times 10^{-11}$$
$$d = 8.150380 \times 10^{-15}$$

Thermal conductivity

$$k_a = a + bT + cT^2 + dT^3, \quad W/mK$$
$$a = -4.937787 \times 10^{-4}$$
$$b = 1.018087 \times 10^{-4} \tag{A.4}$$
$$c = -4.627937 \times 10^{-8}$$
$$d = 1.250603 \times 10^{-11}$$

A.2 THE THERMOPHYSICAL PROPERTIES OF SATURATED WATER VAPOR FROM 273.15 K TO 365 K

Vapor pressure

$$p_v = a\exp(-b/T), \quad N/m^2$$
$$a = 1.020472843 \times 10^{11} \tag{A.5}$$
$$b = 5149.6889682$$

Specific heat

$$c_{pv} = a + bT + cT^5 + dT^6, \quad J/kgK$$
$$a = 1.3605 \times 10^3$$
$$b = 2.31334 \tag{A.6}$$
$$c = -2.46784 \times 10^{-10}$$
$$d = 5.91332 \times 10^{-13}$$

Dynamic viscosity

$$\mu_v = a + bT + cT^2 + dT^3, \quad kg/ms$$
$$a = 2.562435 \times 10^{-6}$$
$$b = 1.816683 \times 10^{-8} \tag{A.7}$$
$$c = 2.579066 \times 10^{-11}$$
$$d = -1.067299 \times 10^{-14}$$

Thermal conductivity

$$k_v = a + bT + cT^2 + dT^3, \quad W/mK$$
$$a = 1.30460 \times 10^{-2}$$
$$b = -3.756191 \times 10^{-5} \tag{A.8}$$
$$c = 2.217964 \times 10^{-7}$$
$$d = -1.111562 \times 10^{-10}$$

Vapor density

$$\rho_v = a + bT + cT^2 + dT^3 + eT^4 + fT^5, \quad kg/m^3$$
$$a = -4.062329056$$
$$b = 0.10277044$$
$$c = -9.76300388 \times 10^{-4} \tag{A.9}$$
$$d = 4.475240795 \times 10^{-6}$$
$$e = -1.004596894 \times 10^{-8}$$
$$f = 8.9154895 \times 10^{-12}$$

A.3 THE THERMOPHYSICAL PROPERTIES OF MIXTURES OF DRY AIR AND WATER VAPOR

Density

$$\rho_{av} = (1+w)[1-w/(w+0.62198)](p_{abs}/RT), \quad kg/m^3 \tag{A.10}$$
where R = 287.08 J/kgK

Specific heat

$$c_{pav} = \left(c_{pa} + w\,c_{pv}\right)/(1+w), \quad J/kgK \tag{A.11}$$

Dynamic viscosity

$$\mu_{av} = \left(X_a \mu_a M_a^{0.5} + X_v \mu_v M_v^{0.5}\right) / \left(X_a M_a^{0.5} + X_v M_v^{0.5}\right), \quad kg/ms \tag{A.12}$$

Thermal conductivity

$$k_{av} = \left(X_a k_a M_a^{0.33} + X_v k_v M_v^{0.33}\right) / \left(X_a M_a^{0.33} + X_v M_v^{0.33}\right), \quad W/mK \tag{A.13}$$

where

$M_a = 28.97$ kg/mole

$M_v = 18.016$ kg/mole

$X_a = 1/(1+1.608\,w)$

$X_v = w/(w+0.622)$

Humidity ratio

$$w = \left(\frac{2501.6 - 2.3263(T_{wb} - 273.15)}{2501.6 + 1.8577(T_{db} - 273.15) - 4.184(T_{wb} - 273.15)}\right)\left(\frac{0.62509\,p_{vwb}}{p_{abs} - 1.005\,p_{vwb}}\right) - \left(\frac{1.00416(T_{db} - T_{wb})}{2501.6 + 1.8577(T_{db} - 273.15) - 4.184(T_{wb} - 273.15)}\right), \quad kg/kg \tag{A.14}$$

A.4 THE THERMOPHYSICAL PROPERTIES OF SATURATED WATER LIQUID FROM 273.15 K TO 380 K

Density

$$\rho_w = \left(a + bT + cT^2 + dT^6\right)^{-1}, \quad kg/m^3$$

$$a = 1.49343 \times 10^{-3}$$
$$b = -3.7164 \times 10^{-6} \qquad (A.15)$$
$$c = 7.09782 \times 10^{-9}$$
$$d = -1.90321 \times 10^{-20}$$

Specific heat

$$c_{pw} = a + bT + cT^2 + dT^6, \quad J/kgK$$

$$a = 8.15599 \times 10^3$$
$$b = -2.80627 \times 10 \qquad (A.16)$$
$$c = 5.11283 \times 10^{-2}$$
$$d = -2.17582 \times 10^{-13}$$

Dynamic viscosity

$$\mu_w = a\,10^{b/(T-c)}, \quad kg/ms$$

$$a = 2.414 \times 10^{-5} \qquad (A.17)$$
$$b = 247.8$$
$$c = 140$$

Thermal conductivity

$$k_w = a + bT + cT^2 + dT^4, \quad W/mK$$

$$a = -6.14255 \times 10^{-1}$$
$$b = 6.9962 \times 10^{-3} \qquad (A.18)$$
$$c = -1.01075 \times 10^{-5}$$
$$d = 4.74737 \times 10^{-12}$$

Latent heat of vaporization

$$i_{fg} = a + bT + cT^2 + dT^3, \quad J/kg$$

$$a = 3.4831814 \times 10^6$$
$$b = -5.8627703 \times 10^3 \qquad (A.19)$$
$$c = 1.2139568 \times 10$$
$$d = -1.40290431 \times 10^{-2}$$

APPENDIX B

PERFORMANCE CHARACTERISTICS OF FINNED TUBES

B.1 HEAT TRANSFER AND PRESSURE DROP CORRELATIONS

Several investigators have studied the air-side heat transfer and pressure drop characteristics of circular finned tube bundles. All the resulting correlations are empirical (based on multiple regression analysis) and attempt to define a power law dependence of the geometrical parameters and the basic nondimensional groups such as Reynolds and Prandtl numbers. Such correlations must account for the five geometric parameters, which include the tube outside diameter, d_o, the fin parameters, t_f, $(d_f - d_o)/2$, P_f and the tube layout, P_t and P_l. Typical dimensions of an extruded bimetallic finned tube are shown in figure B.1. Being empirical, the range of validity of such correlations is strictly dependent on the range of data from which the correlations were developed. A good correlation must maintain its accuracy when tested against a truly representative data set.

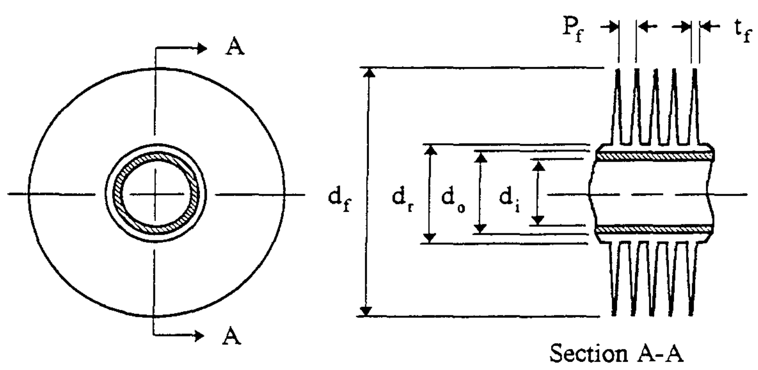

Figure B.1: Extruded finned tube.

In order to have the optimization program play a significant role in the selection of the optimum surface for a finned tube heat exchanger, explicit expressions for the airside heat transfer coefficient and the airside pressure drop in terms of the geometric and layout parameters are a necessity. The presented correlations fulfill this requirement to a greater or lesser extent. It is important not to exceed the limits of applicability of the different correlations.

When a more sophisticated analysis is required, laboratory tests must be conducted on a bundle of the particular finned tubes under consideration. Laboratory tests, if conducted with extreme care, present the most accurate and reliable way to correlate performance data. For finned tube heat exchanger bundles, the heat transfer to the air stream is dependent upon so many factors that reliable rating and performance information for any specific bundle design should be verified by actual tests.

The correlations that will subsequently be presented are not necessarily better than the others in the literature, but present a wide spectrum of data and have been applied in the design of practical systems. They are valid for a staggered, equilateral, circular finned tube arrangement ($P_l = 0.866P_t$). The thermophysical properties used in these equations are evaluated at the mean air temperature unless specified otherwise.

Heat transfer correlations

(1) Briggs and Young [63BR1e]

$$h_a = 0.134\,Re_a^{0.681}\,Pr_a^{0.333}\,\frac{k_a}{d_r}\left[\frac{2(P_f - t_f)}{d_f - d_r}\right]^{0.2}\left(\frac{P_f - t_f}{t_f}\right)^{0.1134} \tag{B.1}$$

where $Re_a = G_c d_r / \mu_a$

This correlation is valid for an equilateral, triangular tube layout with 6 tube rows within the following limits:

$1000 < Re_a < 18000$
$11.13\ \text{mm} < d_r < 40.89\ \text{mm}$
$19.35\ \text{mm} < d_f < 69.85\ \text{mm}$
$1.42\ \text{mm} < (d_f - d_r)/2 < 16.57\ \text{mm}$
$0.33\ \text{mm} < t_f < 2.02\ \text{mm}$
$1.3\ \text{mm} < P_f < 4.06\ \text{mm}$
$24.49\ \text{mm} < P_t < 111\ \text{mm}$
$0.13 < 2(P_f - t_f)/(d_f - d_r) < 0.63$
$1.01 < (P_f - t_f)/t_f < 6.62$

To take the row effect into account during heat transfer, the equation of Giagnolia and Cuti [81GI1e] is proposed:

$$h_{an}/h_{a6} = \left[1 + v_c \big/ \left(n_r^2\right)\right]^{-0.14} \tag{B.2}$$

(2) Ganguli, Tung and Taborek [85GA1e]

$$h_a = 0.38\,Re_a^{-0.4}\,Pr_a^{-0.667}\,G_c c_{pa}\left(A_a/A_r\right)^{-0.15} \qquad \text{(B.3)}$$

where $Re_a = G_c d_r/\mu_a$

$$A_a/A_r = 1 + \frac{(d_f - d_r)}{P_f}\left[1 + \frac{(d_f - d_r)/2 + t_f}{d_r}\right] \quad \text{(Finning factor)}$$

$$A_r = \pi d_r P_f$$

$$A_a = 0.5\pi\left(d_f^2 - d_r^2\right) + \pi d_f t_{ft} + \pi d_r (P_f - t_f)$$

This correlation is valid for an equilateral, triangular tube layout with more than 3 tube rows within the following limits:

$1 < A_a/A_r < \approx 50$

$1800 < Re_a < 100000, 190000$

$11.176\ \text{mm} < d_r < 50.8\ \text{mm}, 114.554\text{mm}$

$5.842\ \text{mm} < (d_f - d_r)/2 < 19.05\ \text{mm}, 22.987\ \text{mm}$

$2.3\ \text{mm} < P_f < 3.629\ \text{mm}, 6.18\ \text{mm}$

$0.254\ \text{mm} < t_f < 0.559\ \text{mm}, 1.524\ \text{mm}$

$27.432\ \text{mm} < P_t < 98.552\ \text{mm}, 171.45\ \text{mm}$

(3) Nir [91NI1e]

$$h_a = G_c c_{pa}\,Pr_a^{-0.667}\,Re_{d_f}^{-0.4}\,W^{-0.266} R_1^{-0.4} K_h \qquad \text{(B.4)}$$

where $Re_h = G_c d_h/\mu_a = 4G_c d_f/\mu_a W = 4\,Re_{d_f}/W$

$$W = \frac{P_f^{-1}\left[0.5\pi\left(d_f^2 - d_r^2\right) + \pi d_f t_{ft} + \pi d_r (P_f - t_f)\right]}{(d_f - d_r)(1 - t_f/P_f) + P_t - d_f}$$

$$R_1 = \frac{(d_f - d_r)(1 - t_f/P_f) + P_t - d_f}{(d_f - d_r)(1 - t_f/P_f)}$$

$$R_2 = \frac{2\left\{\left[0.25(P_t/d_f)^2 + (P_l/d_f)^2\right]^{0.5} - 1 + \left[(d_f - d_r)(1 - t_f/P_f)\right]/d_f\right\}}{P_t/d_f - 1 + \left[(d_f - d_r)(1 - t_f/P_f)\right]/d_f}$$

K_h = Row correction factor for heat transfer

$n_r = 2,\ K_h = 0.9$

$n_r = 3,\ K_h = 0.95$

$n_r \geq 4,\ K_h = 1$

This correlation is valid for an equilateral, triangular tube layout with 2-8 tube rows within the following limits:

$300 < Re_h < 10000$
$11.6 < W < 68.3$
$1 < R_1 < 3$
$1 < R_2 < 4.6$
$9.65 \text{ mm} < d_r < 50.8 \text{ mm}$
$21.9 \text{ mm} < d_f < 101.6 \text{ mm}$
$2.242 \text{ mm} < P_f < 4.95 \text{ mm}$

Pressure drop correlations

(1) Robinson and Briggs [66RO1e]

$$\Delta p_a = 18.93\, n_r\, Re_a^{-0.316} \frac{G_c^2}{\rho_a} \left(\frac{P_t}{d_r}\right)^{-0.927} \left(\frac{P_t}{P_d}\right)^{0.515} \tag{B.5}$$

where $Re_a = G_c d_r / \mu_a$

$$P_d = \left[(P_t/2)^2 + (P_l)^2\right]^{0.5} \text{ (diagonal pitch)}$$

This correlation is valid for an equilateral, triangular tube layout with 6 tube rows within the following limits:

$2000 < Re_a < 50000$
$18.64 \text{ mm} < d_r < 40.89 \text{ mm}$
$39.68 \text{ mm} < d_f < 69.85 \text{ mm}$
$10.52 \text{ mm} < (d_f - d_r)/2 < 14.48 \text{ mm}$
$0.4 \text{ mm} < t_f < 0.6 \text{ mm}$
$2.31 \text{ mm} < P_f < 3.22 \text{ mm}$
$42.85 \text{ mm} < P_t < 114.3 \text{ mm}$
$1.86 < P_t/d_r < 4.6$
$P_t/P_d \approx 1$

(2) Ganguli, Tung and Taborek [85GA1e]

$$\Delta p_a = \frac{2 G_c^2 n_r K}{\rho_a} \left(0.021 + \frac{27.2}{Re_{eff}} + \frac{0.29}{Re_{eff}^{0.2}}\right) \tag{B.6}$$

$$\text{where } Re_{eff} = (G_c d_r/\mu_a)\left[0.5(d_f - d_r)/(P_f - t_f)\right]^{-1}$$

$$K = 1 + \frac{2}{1+(P_t - d_f)/d_r} \exp\left[-0.25(P_t - d_f)/d_r\right]$$

This correlation is valid for an equilateral, triangular tube layout with more than 3 tube rows. The correlation is based on the literature data of Webb [80WE1e]. The following geometrical limit is imposed:

$$2.5 < (d_f - d_r)/\left[2(P_f - t_f)\right] < 12.5$$

(3) <u>Nir [91NI1e]</u>

$$\Delta p_a = 1.06\, Re_{d_f}^{-0.25}\, W^{0.45}\, K_p\, n_r G_c^2/\rho_a \tag{B.7}$$

$$\text{where } Re_h = G_c d_h/\mu_a = 4G_c d_f/\mu_a W = 4\, Re_{d_f}/W$$

$$W = \frac{P_f^{-1}\left[0.5\pi\left(d_f^2 - d_r^2\right) + \pi d_f t_{ft} + \pi d_r (P_f - t_f)\right]}{(d_f - d_r)(1 - t_f/P_f) + P_t - d_f}$$

$$R_1 = \frac{(d_f - d_r)(1 - t_f/P_f) + P_t - d_f}{(d_f - d_r)(1 - t_f/P_f)}$$

$$R_2 = \frac{2\left\{\left[0.25(P_t/d_f)^2 + (P_l/d_f)^2\right]^{0.5} - 1 + \left[(d_f - d_r)(1 - t_f/P_f)\right]/d_f\right\}}{P_t/d_f - 1 + \left[(d_f - d_r)(1 - t_f/P_f)\right]/d_f}$$

K_p = Row correction factor for pressure drop

$$K_p = 2.08 - 0.83 R_2, \quad 1 < R_2 < 1.3$$

$$K_p = 1, \quad R_2 > 1.3$$

This correlation is valid for an equilateral, triangular tube layout with 2-8 tube rows within the following limits:

$$400 < Re_h < 30000$$
$$8.5 < W < 57.4$$
$$9.65 \text{ mm} < d_r < 50.8 \text{ mm}$$
$$19.35 \text{ mm} < d_f < 114.3 \text{ mm}$$
$$2.217 \text{ mm} < P_f < 8.475 \text{ mm}$$

The correlations presented by Ganguli et al. [85GA1e] and Nir [91NI1e] are based on original and published experimental data, whereas those presented by Briggs, Young and Robinson [63BR1e, 66RO1e] are based on their own experimental data. It is difficult to recommend a single correlation. However, the Briggs and Young [63BR1e] (heat transfer) and the Robinson and Briggs [66RO1e] (pressure drop) correlations are widely used. The correlations by Ganguli et al. [85GA1e] and Nir [91NI1e] are included because they are based on a very wide range of data and therefore are assumed to give more accurate heat transfer and pressure drop correlations.

The arrangement (staggered vs. inline) of the finned tubes in the heat exchanger as well as the tube and fin geometry (tube pitch, fin pitch, fin height, fin thickness, number of tube rows) affect both the heat transfer and pressure drop. The interested reader is referred to the following literature for a detailed discussion on these topics [45JA1e, 59WA1e, 63BR1e, 66RO1e, 66VA1e, 74MI2e, 76RO1e, 80WE1e, 81GI1e, 81SH3e, 85EC1e, 85GA1e, 88ST1e, 88ZU1e, 89RA1e]. There exists a substantial amount of test data in the published literature. However, careful examination of the data and the method of experimentation is needed in each specific case to establish their usefulness in design. The few correlations that are listed present a wide spectrum of data and have been applied in the design of practical systems.

In the optimization process, one can vary the geometrical parameters of the finned tubes and the tube layout in the ranges of applicability. However, the optimum design/layout is done according to economic criteria and may not necessarily present those required for optimum performance. In any industrial application the selection of finned tubes is dictated by the combination of the operating and capital costs as well as a variety of technical constraints. For a finned tube module, the capital cost is primarily related to the size and layout of the tube bundle, whereas the operating cost is primarily related to the fan and/or the pumping power.

B.2 HEAT TRANSFER AND PRESSURE DROP CHARACTERISTICS PRESENTED IN DIMENSIONAL FORM

A method for presenting the experimentally obtained performance data of industrial finned tubes is evaluated by Kern [80KE1e] and modified by Kröger [86KR1e]. The method presents the performance characteristics of finned tube bundles in the form of dimensional heat transfer and pressure drop parameters. This form of data presentation eliminates uncertainties inherent in the evaluation of the airside heat transfer coefficient, fin efficiency, thermal contact resistance fouling and other thermal resistances.

Finned tube performance correlations contain Nusselt and Reynolds numbers. Both these numbers contain an equivalent or hydraulic diameter. Because of the relatively arbitrary nature of the definition of this quantity for finned surfaces, different definitions are found in the literature. In practice this often leads to confusion and makes any comparison of performance characteristics of different types of finned surface meaningless. The method of which a summarized derivation is presented below, eliminates potential sources of error in well conducted experiments.

In general, the overall heat transfer coefficient can be expressed as

$$U_a = \left(\frac{1}{h_a e_f} + \frac{A_a}{h_w A_w} + \sum_n \frac{A_a R_n}{A_n} \right)^{-1} \tag{B.8}$$

where the summation term represents all the thermal resistances other than the airside and waterside values. The heat transfer rate can be expressed in terms of the overall heat transfer coefficient and the logarithmic mean temperature difference as follows:

$$Q = U_a A_a F_T \Delta T_{lm} \tag{B.9}$$

where F_T is the cross flow temperature correction factor [86HO1e]. Substitute equation (B.9) into (B.8) and find

$$h_a = \left[e_f A_a \left(\frac{F_T \Delta T_{lm}}{Q} - \frac{1}{h_w A_w} - \sum_n \frac{R_n}{A_n} \right) \right]^{-1} \tag{B.10}$$

Rearrange equation (B.10) and define the effective heat transfer coefficient, h_{ae},based on the airside surface area as follows

$$h_{ae} A_a = \left(\frac{1}{h_a e_f A_a} + \sum_n \frac{R_n}{A_n} \right)^{-1} = \left(\frac{F_T \Delta T_{lm}}{Q} - \frac{1}{h_w A_w} \right)^{-1} \tag{B.11}$$

The value of $h_{ae} A_a$ can be determined experimentally.

According to Colburn [33CO1e] the heat transfer coefficient under conditions of forced convection through finned surfaces may be expressed in terms of dimensionless parameters as

$$\frac{Nu_a}{Re_a \, Pr_a^{0.33}} = f(Re_a) \quad \text{or} \quad Nu_a = a_1 \, Re_a^{b_1} \, Pr_a^{0.33} \tag{B.12}$$

In the absence of the equivalent diameter, equation (B.12) may be written as

$$\frac{h_{ae}}{k_a \, Pr_a^{0.33}} = a_2 Ry^{b_2} \tag{B.13}$$

where $Ry = m_a / (A_{fr} \mu_a)$ (characteristic flow parameter)

The effective finned surface area and the heat exchanger frontal area play a major role in comparing and optimizing heat exchangers. These geometric parameters may be introduced into equation (B.13) such that

$$Ny = \frac{h_{ae} A_a}{A_{fr} k_a \, Pr_a^{0.33}} = a_{Ny} Ry^{b_{Ny}} \quad \text{(characteristic heat transfer parameter)} \tag{B.14}$$

All the physical properties are evaluated at the arithmetic mean temperature.

The pressure drop across a finned tube heat exchanger during isothermal flow conditions may also be expressed in dimensionless form based on the free stream conditions as

$$Eu_{iso} = \Delta p_{iso} / \rho_a v_a^2 = a_3 \, Re^{b_3} \tag{B.15}$$

If the equivalent diameter is not included in this equation it may be written as

$$Ey_{iso} = \rho_a \Delta p_{iso} / \mu_a^2 = a_{Ey} \, Re^{b_{Ey}} = Eu_{iso} Ry^2 \quad \text{(characteristic pressure drop parameter)} \tag{B.16}$$

A corresponding pressure loss coefficient based on the total pressure difference across the heat exchanger can be defined as

$$K_{he} = \frac{\Delta p_t}{0.5 \, \rho_a v_a^2} = a_K Ry^{b_K} \tag{B.17}$$

The performance correlations presented in equations (B.14) and (B.17) are determined experimentally. These correlations enable one to compare different finned surfaces (of any geometry) directly and can be used in optimization studies where fixed finned surfaces are considered. The finned tube geometrical parameters are subsumed in the constants a and b.

It should however be stressed that to date no correlation exists that accurately predicts the performance over a wide spectrum of finned tube geometries and operating conditions. Discrepancies do exist in literature correlations [86KR1e]. Therefore, the final design of a costly air-cooled heat exchanger system cannot be based on approximate correlations. In such cases specific performance tests should be conducted on the finned tubes to be used.

APPENDIX C

FORCED DRAFT DIRECT AIR-COOLED CONDENSERS

C.1 INTRODUCTION

The performance prediction of air-cooled condensers are based on two sets of governing equations, i.e. the draft equation and the heat transfer equations. The governing heat transfer and draft equations will be derived in this appendix (also refer to [83SI1e, 91CO1e]). The coupling of the air-cooled condenser to a turbo-generator unit, as found in power plants, is also discussed. The results of a considerable amount of experimental and theoretical work are taken into account to model all the physical phenomena of such a system. The search for economically viable air-cooled condenser operation requires the proper formulation and modeling of the system. The variables and constraints required to perform this task are defined and discussed.

C.2 DESCRIPTION OF THE DIRECT AIR-COOLED CONDENSER

In the direct condensing air-cooled heat exchanger (ACHE), also referred to as air-cooled condensers (ACC), the process fluid (low pressure turbine exhaust steam in the case of power plants) is channeled directly to the air-cooled heat exchanger bundles, as shown in figure C.1. The heat exchanger bundles can consist of one or more rows of finned tubes. The steam pressure inside the exhaust duct and the ACC is lower than atmospheric pressure. The turbine exhaust steam duct has a large diameter and is required to be as short as possible to minimize pressure losses. In very large ACHEs the finned tube bundles are usually sloped at some angle with the horizontal (A-frame arrangement) in order to reduce the plot area. This arrangement is also frequently used in condensing plants due to good condensate discharge. Horizontal finned tube arrangements are also found in many industrial applications.

Steam enters the top of the air-cooled heat exchanger bundles (finned tubes) and condenses as it flows downward with the steam and the condensate flowing in the same direction. In actual installations, provisions are made for the removal of noncondensable gasses and air (dephlegmator) and for the prevention of freezing during cold weather (airflow control) [77SC1e, 77SU1e, 78LA1e, 80RU1e, 81KO1e, 83PA1e, 83SH1e, 85MO1e, 85MO2e, 87OS1e]. The airflow across the heat exchangers is created by means of axial flow fans, i.e. the steam is condensed by forced convection of air flowing over the heat exchanger bundles. The condensate that collects in the condensate tank is pumped back to the boiler feedwater circuit.

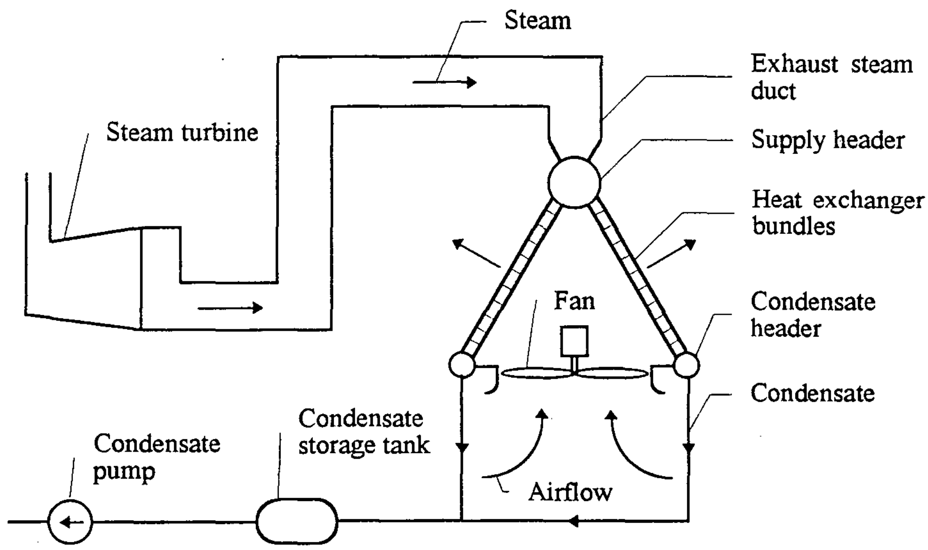

Figure C.1: Forced draft direct air-cooled condenser (A-frame arrangement).

C.3 HEAT TRANSFER AND PRESSURE DROP DURING CONDENSATION

The finned tube elements are the heart of any air-cooled condensing plant. The air-cooled heat exchanger may consist of one or more rows of finned tubes, each row having a different fin pitch. The transfer of heat from the process fluid to the air is influenced by a number of variables:

(1) The temperature difference between the process fluid and the air.

(2) The design and surface arrangement of the finned tube bundles.

(3) The velocity of the air flowing across the finned tubes.

(4) The velocity and physical properties of the process fluid.

Consider the forced draft air-cooled condenser shown schematically in figure C.2. In this configuration the heat exchanger bundles are arranged in the form of an A-frame to drain the condensate effectively, reduce the steam duct lengths and minimize the required ground surface area. A windwall is provided to reduce recirculation of the hot plume air.

The amount of heat transferred from the condensing process fluid to the air stream can be expressed as

$$Q_a = m_a c_{pam}\left(T_{a6} - T_{a5}\right) = m_c i_{fg} = Q_c \tag{C.1}$$

Subcooling of the condensate is neglected in the above relation and it is assumed that all the steam entering the finned tubes is condensed. The thermophysical properties of the air will be determined at the mean temperature and atmospheric pressure at ground level.

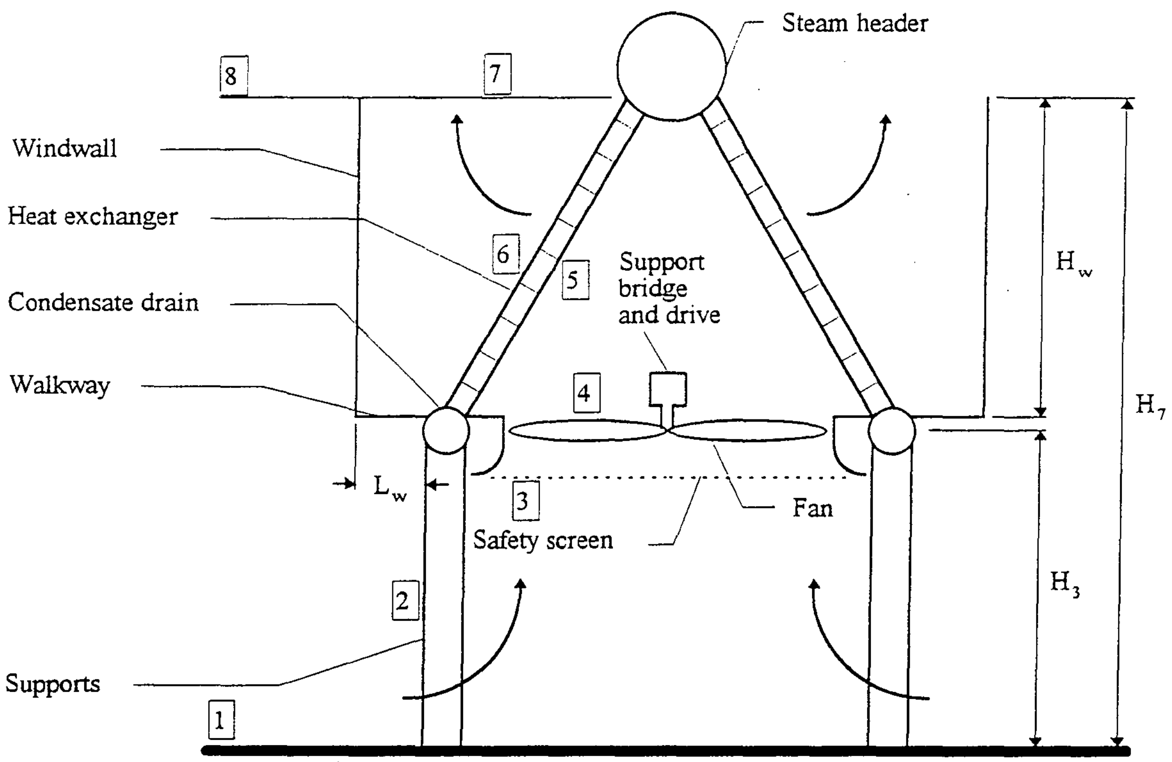

Figure C.2: Air-cooled condenser unit

If the geometry of the finned tubes changes in consecutive rows and performance data is available for the individual rows, the heat transfer equation may be written as

$$Q_a = \sum_{i=1}^{n_r} m_a c_{pam(i)} \left(T_{ao(i)} - T_{ai(i)} \right) = \sum_{i=1}^{n_r} m_{c(i)} i_{fg(i)} \tag{C.2}$$

Furthermore, the effectiveness of the condenser bundle or each tube row can be expressed as [86HO1e]

$$e_{(i)} = 1 - \exp\left[-(UA)_{(i)} / \left(m_a c_{pam(i)} \right) \right] \tag{C.3}$$

With this expression the heat transfer rate becomes

$$Q_{ac} = \sum_{i=1}^{n_r} m_a c_{pam(i)} \left(T_{vm(i)} - T_{ai(i)} \right) e_{(i)} \tag{C.4}$$

The product of the overall heat transfer coefficient and area for each tube row can be expressed as

$$(UA)_{(i)} = \left(\frac{1}{h_{ae(i)} A_{a(i)}} + \frac{1}{h_{c(i)} A_{c(i)}} \right)^{-1} \tag{C.5}$$

where

$$h_{ae(i)} A_{a(i)} = k_{am(i)} \, Pr_{am(i)}^{0.333} \, A_{fr} Ny_{(i)} \left(n_{tb(i)} / n_{tb(max)} \right) \tag{C.6}$$

and

$$Ry_{(i)} = m_a / \left(\mu_{am(i)} A_{fr} \, n_{tb(i)} / n_{tb(max)} \right) \tag{C.7}$$

for any finned tube geometry, and

$$h_{ae(i)} A_{a(i)} = \left(\frac{1}{h_{a(i)} e_{f(i)} A_{a(i)}} + \frac{\ell n(d_o/d_i)}{2\pi k_t L_t n_{tb(i)} n_b} + \frac{\ell n(d_r/d_o)}{2\pi k_f L_t n_{tb(i)} n_b} \right)^{-1} \tag{C.8}$$

for radially finned tubes. The frontal area of the heat exchanger bundle corresponds to the area covered by the maximum number of tubes per row in a multiple tube arrangement. Ry is corrected for this effect. Furthermore, Ny is based on heat exchanger bundle tests in which the maximum number of finned tubes are installed in the bundle. In actual heat exchanger bundles it is not practical to install half-tubes and a correction is made when determining the effective heat transfer coefficient.

The effectiveness of the circular finned surface is expressed in terms of the fin efficiency, i.e.

$$e_{f(i)} = 1 - A_{f(i)} \left(1 - \eta_{f(i)} \right) / A_{a(i)} \tag{C.9}$$

According to Schmidt [46SC1e], the fin efficiency for radial fins of uniform thickness can be determined approximately from

$$\eta_{f(i)} = \frac{\tanh\left(b d_r \, \Phi/2 \right)_{(i)}}{\left(b d_r \, \Phi/2 \right)_{(i)}} \tag{C.10}$$

where $\Phi_{(i)} = \left(d_f/d_r - 1 \right)\left[1 + 0.35 \ell n\left(d_f/d_r \right) \right]$ and $b_{(i)} = \left[\left(2\, h_{a(i)} \right) / \left(t_f k_f \right) \right]^{0.5}$.

For galvanized steel fins, the fin thickness can be expressed as $t_f = 2t_g + t_s$ and the thermal conductivity of the fin can be expressed as $k_f = \left(2t_g k_g + t_s k_s \right) / t_f$.

The performance characteristics of the finned tubes are discussed in Appendix B and correlations are stated to determine the airside heat transfer coefficient and the characteristic heat transfer parameter.

The pressure losses in the turbine exhaust steam duct and at the inlet to the finned tubes are expressed as

$$\Delta p_{sd} = 0.5\rho_{vi(1)}v_{vi(1)}^2\left(1-\sigma_c^2+K_c+K_{sd}\right) \tag{C.11}$$

where σ_c is the tube inlet contraction area ratio and the contraction loss coefficient, $K_c \approx 0.6$ (sharp inlet). K_{sd} is the loss coefficient for the steam duct system. The mean saturation pressure of the steam at the inlet of the finned tubes, p_{vi}, is calculated by subtracting Δp_{sd} (equation (C.11)) from the mean saturation steam pressure at the turbine outlet. The mean steam temperature at the inlet of the finned tubes, T_{vi}, is obtained from equation (A.5). The inlet conditions (thermophysical properties) of all the tube rows are thus assumed to be equal.

Due to pressure changes along the finned tube and in the direction of air flow, the condensation process will not take place at a constant temperature. The mean static pressure in the finned tubes can be determined from the correlations given by Groenewald and Kröger [94GR1e].

$$p_{vm(i)} = p_{vi} - \frac{0.1582\mu_{vi}^2 L_t}{\rho_{vi} d_e^3 Re_{vi(i)}}\left(0.267 a_{1(i)} Re_{vi(i)}^{2.75} + 0.364 a_{2(i)} Re_{vi(i)}^{1.75}\right) + \frac{2}{3}\rho_{vi}v_{vi(i)}^2 \tag{C.12}$$

The coefficients a_1 and a_2 are functions of the suction Reynolds number, Re_{vn}. For round tubes, these coefficients are

$$\begin{aligned} a_{1(i)} &= 1.0046 + 1.719\times10^{-3} Re_{vn(i)} - 9.7746\times10^{-6} Re_{vn(i)}^2 \\ a_{2(i)} &= 574.3115 + 24.2891 Re_{vn(i)} + 1.8515 Re_{vn(i)}^2 \end{aligned} \tag{C.13}$$

and the suction Reynolds number is expressed as

$$Re_{vn(i)} = \frac{\rho_{vi}v_{vi(i)}d_i}{\mu_{vi}}\frac{d_i}{4L_t} = Re_{vi(i)}\frac{d_i}{4L_t} \tag{C.14}$$

For non-round tubes (elliptical or flattened tubes or ducts) with a high aspect ratio, these coefficients are

$$\begin{aligned} a_{1(i)} &= 1.0649 + 1.0411\times10^{-3} Re_{vn(i)} - 2.011\times10^{-7} Re_{vn(i)}^3 \\ a_{2(i)} &= 290.1479 + 59.3153 Re_{vn(i)} + 1.5995\times10^{-2} Re_{vn(i)}^3 \end{aligned} \tag{C.15}$$

and the suction Reynolds number is expressed as

$$Re_{vn(i)} \approx \frac{\rho_{vi}v_{vi(i)}d_e}{\mu_{vi}}\frac{W_t}{2L_t} = Re_{vi(i)}\frac{W_t}{2L_t} \quad \left(0 \le Re_{vn(i)} \le 40\right) \tag{C.16}$$

It is assumed that condensation occurs at the mean steam temperature, $T_{vm(i)}$, corresponding to the mean steam pressure inside the tubes (refer to equation (A.5)).

It should be noted that the pressure drop in the different tube rows is usually not identical, with the result that backflow of steam will occur [78LA1e]. To avoid this and the corresponding accumulation of noncondensables, a dephlegmator is usually installed after the condenser.

For non-round tubes (elliptical or flattened tubes or ducts) the correlation of Groenewald [93GR1e] is employed to determine the mean condensation heat transfer coefficient, i.e.

$$h_{c(i)} = 0.9245\left[\frac{L_t k_{cm(i)}^3 \rho_{cm(i)}^2 g \cos(90° - \theta_b) i_{fg(i)}}{\mu_{cm(i)} m_{a1} c_{pam(i)}\left(T_{vm(i)} - T_{ai(i)}\right)\left[1 - \exp\left\{-\left(U_{c(i)} H_t L_t\right)/\left(m_{a1} c_{pam(i)}\right)\right\}\right]}\right]^{0.333} \quad \text{(C.17)}$$

By neglecting the thermal resistance of the condensate film, the approximate overall heat transfer coefficient based on the condensation surface area can be expressed as

$$U_{c(i)} H_t L_t = \left(h_{ae(i)} A_{a(i)}\right)/\left(2 n_{tb(i)} n_b\right) \quad \text{(C.18)}$$

The air mass flow rate flowing on one side of the finned tube is

$$m_{a1} = m_a/\left(2 n_{tb(i)} n_b\right) \quad \text{(C.19)}$$

For round tubes, the mean condensation heat transfer coefficient for inclined tubes according to Schulenburg [69SC1e] is used, i.e.

$$h_{c(i)} = 1.197\left(\sin\theta_b\right)^{0.175} \frac{k_{cm(i)}}{d_i}\left(\frac{\rho_{cm(i)}\mu_{vm(i)}}{\rho_{vm(i)}\mu_{cm(i)}}\right)^{0.5} Re_{vi(i)}^{0.325} \quad \text{(C.20)}$$

This equation is valid within the following ranges: $5° \leq \theta_b \leq 90°$ (angle of inclination with respect to the vertical), $10950 \leq Re_{vm} = Re_{vi}/2 \leq 14150$ and $6525\,N/m^2 \leq p_{vm} \leq 8085\,N/m^2$.

$A_{c(i)}$, referred to in equation (C.5), is the inside tube area of tube row (i) exposed to the condensing steam.

The energy balance for the forced draft air-cooled condenser thus requires that the following relationships be satisfied for each tube row:

$$m_a c_{pam(i)}\left(T_{ao(i)} - T_{ai(i)}\right) = m_{c(i)} i_{fg(i)} = m_a c_{pam(i)}\left(T_{vm(i)} - T_{ai(i)}\right) e_{(i)} \quad \text{(C.21)}$$

C.4 DERIVATION OF THE DRAFT EQUATION FOR A FORCED DRAFT AIR-COOLED CONDENSER

The draft equation describes the relation between the various flow resistances encountered, the atmospheric conditions, heat exchanger dimensions, the heat exchanger bundle performance characteristics and the fan performance characteristics at a given flow rate. The draft equation is derived similarly to the procedure described in Kröger [94KR2e]

Significant changes in the ambient air temperature occur near the ground level during any 24-hour period [94KR1e]. During the day, a temperature lapse rate of -0.00975 K/m, also known as the dry adiabatic lapse rate (DALR), is observed in the region of the surface boundary layer (SBL). For this analysis, the specified ambient air temperature at any elevation z, will be assumed to be given by the equation

$$T_{az} = T_{a1} - 0.00975\,z \tag{C.22}$$

To derive the draft equation, consider the variation with elevation of the pressure in the atmosphere external to the air-cooled condenser in a gravity field, i.e.

$$dp_a = -\rho_a g dz \tag{C.23}$$

Substitute equations (C.22) and (A.1), the perfect gas law, into equation (C.23) and integrate to find the pressure difference between point 1 and a point at elevation z external to the air-cooled condenser (refer to figure C.2)

$$p_{a1} - p_{az} = p_{a1}\left[1-\left(1-0.00975\,z/T_{a1}\right)^{102.564g/R}\right] \approx p_{a1}\left[1-\left(1-0.00975\,z/T_{a1}\right)^{3.5}\right] \tag{C.24}$$

According to equation (C.22), the approximate air temperature before the fan can be expressed as

$$T_{a3} = T_{a1} - 0.00975\,H_3 \tag{C.25}$$

The approximate temperature at the inlet to the heat exchanger bundle can be derived from the first law of Thermodynamics (conservation of energy) and can be expressed as [94KR1e, 94KR2e]

$$T_{a5} = T_{a1} - 0.00975\,H_5 + P_F \big/ \left(m_a c_{pa1}\right) \tag{C.26}$$

where $H_5 \approx H_6$ is the mean heat exchanger height above the ground level.

Stagnant ambient air at 1 accelerates and flows across the heat exchanger supports at 2 before reaching the fan at section 3, where upstream obstacles such as structural supports or a safety screen may be located. After leaving the fan at 4 where further downstream obstacles may be located, the flow experiences losses in the plenum before entering the heat exchanger bundle at 5 and exiting at 6. Additional flow losses are encountered due to the inclined flow approaching and leaving the heat exchanger bundles.

Taking into consideration all the flow losses, the pressure difference between sections 1 and 7 can be expressed as [94KR2e]

$$\begin{aligned} p_{a1} - p_{a7} = {} & p_{a1}\left[1-\left(1-0.00975 H_6/T_{a1}\right)^{3.5}\right] + K_{ts}\left(m_a/A_2\right)^2/\left(2\rho_{a2}\right) \\ & + K_{up}\left(m_a/A_3\right)^2/\left(2\rho_{a3}\right) - \rho_{a3} P_F/m_a + K_{do}\left(m_a/A_4\right)^2/\left(2\rho_{a4}\right) \\ & + K_{pl}\left(m_a/A_{Fc}\right)^2/\left(2\rho_{a4}\right) + K_{\theta t}\left(m_a/A_{fr}\right)^2/\left(2\rho_{a56}\right) \\ & + p_{a6}\left[1-\left\{1-0.00975\left(H_7 - H_6\right)/T_{a1}\right\}^{3.5}\right] \end{aligned} \qquad (C.27)$$

When evaluating the effective frontal area, A_{fr}, obstructions like straps or stiffening beams located up against the finned surface and thus impeding flow through it, must be taken into consideration. In all the heat transfer and pressure drop calculations, the effective finned tube length, L_t, is used as a result.

Fan performance characteristics incorporated in the $\rho_{a3} P_F/m_a$ term are obtained from standard installation tests. According to Venter [90VE1e], the total dynamic component after the fan is dissipated in the plenum when the fan is operating in its application range. For this configuration, $K_{pl} = \alpha_{eF}$ and it follows that

$$-\rho_{a3} P_F/m_a + K_{pl}\left(m_a/A_{Fc}\right)^2/\left(2\rho_{a4}\right) \approx -K_{Fs}\left(m_a/A_{Fc}\right)^2/\left(2\rho_{a3}\right) \qquad (C.28)$$

where the fan static coefficient is defined as

$$K_{Fs} = 2\Delta p_{Fs}\rho_{a3}/\left(m_a/A_{Fc}\right)^2 = 2P_F\rho_{a3}^2 A_{Fc}^2/m_a^3 \qquad (C.29)$$

A_{Fc} is the fan casing cross-sectional area and the fan static pressure, Δp_{Fs}, is obtained from fan performance tests conducted according to certain test codes [94KR1e, 94KR2e].

If the ambient air far from the heat exchanger is dry and the temperature distribution is according to the DALR, the difference in pressure between 1 and 8 is according to equation (C.24)

$$\begin{aligned} p_{a1} - p_{a8} &= p_{a1} - p_{a7} = \left(p_{a1} - p_{a6}\right) + \left(p_{a6} - p_{a7}\right) \\ &\approx p_{a1}\left[1-\left(1-0.00975 H_6/T_{a1}\right)^{3.5}\right] + p_{a6}\left[1-\left\{1-0.00975\left(H_8 - H_6\right)/T_{a1}\right\}^{3.5}\right] \end{aligned} \qquad (C.30)$$

where the ambient air temperature at elevation 6 is assumed to be approximately equal to T_{a1}. Although the air temperature distribution near ground level generally deviates considerably from the DALR, the error introduced by this assumption in equation (C.30) is small for large units.

Substitute equation (C.30) into equation (C.27) and find with equation (C.29) the draft equation for the air-cooled condenser shown in figure C.2 $(H_7 = H_8)$

$$\begin{aligned} & p_{a1}\left[\left\{1-0.00975(H_7-H_6)/T_{a6}\right\}^{3.5}-\left\{1-0.00975(H_7-H_6)/T_{a1}\right\}^{3.5}\right] \\ & = K_{ts}(m_a/A_2)^2/(2\rho_{a1})+K_{up}(m_a/A_{Fe})^2/(2\rho_{a3})-K_{Fs}(m_a/A_{Fc})^2/(2\rho_{a3}) \\ & +K_{do}(m_a/A_{Fe})^2/(2\rho_{a3})+K_{\theta t}(m_a/A_{fr})^2/(2\rho_{a56}) \end{aligned} \tag{C.31}$$

where it is assumed that $\rho_{a2} \approx \rho_{a1}$, $\rho_{a4} \approx \rho_{a3}$ and $\rho_{a7} \approx \rho_{a6}$. Furthermore, the area on which the upstream and downstream flow obstacles are based is defined as $A_{Fe} = A_3 = A_4 = (A_{Fc} - A_{Fh})$, where A_{Fc} and A_{Fh} are the fan casing and hub cross-sectional areas respectively.

The approximate air density at section 3 is

$$\rho_{a3} \approx p_{a1}/(RT_{a3}) \tag{C.32}$$

The density of the air immediately after the heat exchanger is

$$\rho_{a6} \approx p_{a1}/(RT_{a6}) \tag{C.33}$$

and the harmonic mean density through the heat exchanger is given by

$$\rho_{a56} \approx 2p_{a1}/[R(T_{a5}+T_{a6})] \tag{C.34}$$

In the above equations the thermophysical properties of dry air (Appendix A) are usually employed since the influence of moisture is a negligible factor in determining the forced draft air-cooled heat exchanger's draft.

The loss coefficient of the air-cooled condenser supports, K_{ts}, is based on the drag coefficient of these supports, i.e.

$$C_{Dts} = 2F_{Dts}/(\rho_{a1}v_{a2}^2A_{ts}) \tag{C.35}$$

The effective pressure drop across the air-cooled condenser supports is given by

$$\Delta p_{ats} = n_{ts}F_{Dts}/A_2 = 0.5\,\rho_{a1}v_{a2}^2\,C_{Dts}L_{ts}d_{ts}\,n_{ts}/A_2 \tag{C.36}$$

where L_{ts} is the support length and d_{ts} is its effective diameter or width, and n_{ts} is the number of supports, while A_2 is the corresponding free flow area into the air-cooled condenser. The corresponding loss coefficient based on these conditions at section 2 is

$$K_{ts} = 2\,\Delta p_{ats}\Big/\left(\rho_{a1} v_{a2}^2\right) = C_{Dts} L_{ts} d_{ts}\, n_{ts}/A_2 \qquad \text{(C.37)}$$

The loss coefficients due to the flow obstacles at both the fan suction (upstream) and discharge (downstream) sides are obtained from the bulk method proposed by Venter and Kröger [91VE1e]. The bulk method [85VE1e] is related to the total blockage area of the different flow distorting components, as well as the distance between the fan rotor and the respective components. These loss coefficients are obtained from figures C.3 and C.4 (copied from [85VE1e]).

The pressure loss across the heat exchanger bundles and the kinetic energy losses at the outlet elevation (section 7) are derived by Van Aarde and Kröger [93VA1e] for an A-frame heat exchanger array. For non-isothermal oblique flow through an A-frame heat exchanger bundle, the loss coefficient is

$$K_{\theta t} = K_{heiso} + \frac{2}{\sigma^2}\left(\frac{\rho_{a5}-\rho_{a6}}{\rho_{a5}+\rho_{a6}}\right) + \left(\frac{1}{\sin\theta_m} - 1\right)\left(\frac{1}{\sin\theta_m} - 1 + 2K_c^{0.5}\right)\left(\frac{2\rho_{a6}}{\rho_{a5}+\rho_{a6}}\right)\frac{1}{\sigma_{fl}^2} + K_d\left(\frac{2\rho_{a5}}{\rho_{a5}+\rho_{a6}}\right) \qquad \text{(C.38)}$$

where σ is the ratio of the minimum free flow area through the heat exchanger bundle to the free stream flow (frontal) area and σ_{fl} is the ratio of the fin leading edge frontal area to the free stream flow area.

The heat exchanger loss coefficient under normal non-isothermal flow conditions, including inlet-, frictional-, and exit losses as well as acceleration effects (due to heating), is defined as (refer to Appendix B)

$$K_{he} = a_K Ry^{b_K} + \frac{2}{\sigma^2}\left(\frac{\rho_{a5}-\rho_{a6}}{\rho_{a5}+\rho_{a6}}\right) \qquad \text{(C.39)}$$

for finned tube bundles with any finned tube geometry. For radially finned tubes the heat exchanger loss coefficient under normal non-isothermal flow conditions can, in general, be expressed as [86KR1e]

$$K_{he} = \frac{2}{\sigma^2}\left[Eu + \left(\frac{\rho_{a5} - \rho_{a6}}{\rho_{a5} + \rho_{a6}}\right)\right] = \frac{2}{\sigma^2}\left[\frac{\Delta p_a \rho_{am}}{G_c^2} + \left(\frac{\rho_{a5} - \rho_{a6}}{\rho_{a5} + \rho_{a6}}\right)\right] \quad \text{(C.40)}$$

The Euler number can be obtained from the pressure drop correlations discussed in Appendix B.

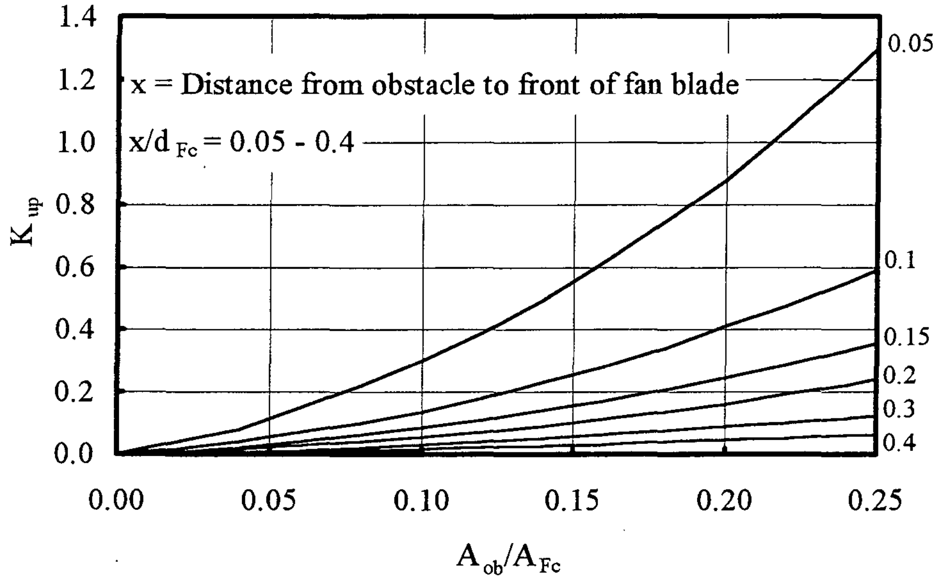

Figure C.3: Loss coefficients for flow obstacles at the fan suction side (upstream).

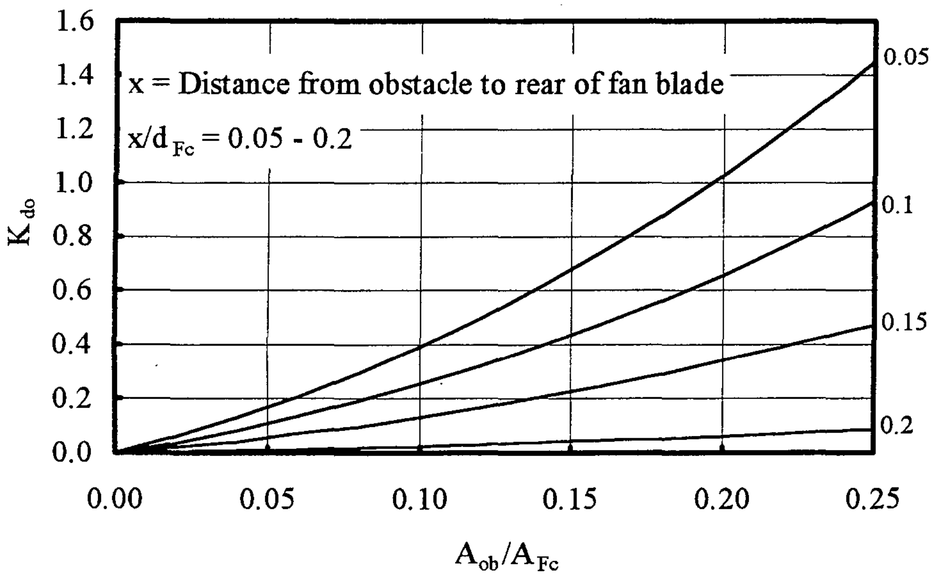

Figure C.4: Loss coefficients for flow obstacles at the fan discharge side (downstream).

The second term on the right hand side of equation (C.38) represents the loss due to acceleration effects during non-isothermal operation. The third term represents the loss due to the oblique flow at the inlet to the bundle. The entrance contraction loss coefficient, K_c, is according to Kays [50KA1e]

$$K_c = \left[\frac{1}{\sigma_{21}}\left(\frac{1}{\sigma_c} - 1\right)\right]^2 \tag{C.41}$$

where $\sigma_{21} = \left(P_f - t_f\right)/P_f$ is the ratio of the flow area between the fins to the free stream flow area at the inlet to the finned tube bundles. The contraction coefficient, σ_c, for flow between parallel plates is given by

$$\begin{aligned}\sigma_c = {} & 0.6144517 + 0.04566493\sigma_{21} - 0.3366510\sigma_{21}^2 + 0.4082743\sigma_{21}^3 + 2.672041\sigma_{21}^4 \\ & - 5.963169\sigma_{21}^5 + 3.558944\sigma_{21}^6\end{aligned} \tag{C.42}$$

For round and elliptical fins a value of $K_c \approx 0.05$ is assumed.

When the finned tube bundles are arranged in the form of A-frames, the curvature of the downstream flow patterns cause the actual mean flow incidence angle to differ from the heat exchanger bundle semi-apex angle θ_b. An empirical correlation for the mean flow incidence angle is presented by Kotzé et al. [86KO1e], i.e.

$$\theta_m = 0.0019\theta_b^2 + 0.9133\theta_b - 3.1558 \tag{C.43}$$

K_d is the downstream loss coefficient that consists of two components, namely the turning and jetting losses in the V-region, K_{dj}, and the loss of kinetic energy into the atmosphere, K_o. The following empirical correlations for these losses are obtained from Van Aarde and Kröger [93VA1e]

$$\begin{aligned}K_{dj} = {} & \left\{\left[-2.8919\frac{L_w}{L_t} + 2.9329\left(\frac{L_w}{L_t}\right)^2\right]\left[\sin\theta_b - \frac{d_{sh}}{2L_t} + \frac{L_w}{L_t}\right]^{-1}\left[1 - \frac{0.5\,d_{sh}/L_t}{\sin\theta_b + L_w/L_t}\right]^{-1}\right. \\ & \times\left[\frac{28}{\theta_b}\right]^{0.4} + \left(\exp\left(2.36987 + 5.8601\times10^{-2}\theta_b - 3.3797\times10^{-3}\theta_b^2\right)\right)^{0.5} \\ & \left.\times\left[1 - \frac{0.5\,d_{sh}/L_t}{\sin\theta_b + L_w/L_t}\right]^{0.5}\left[1 + \frac{L_w}{L_t\sin\theta_b}\right]^{-1}\right\}^2\end{aligned} \tag{C.44}$$

and

$$K_o = \left\{\left[-2.89188\frac{L_w}{L_t} + 2.93291\left(\frac{L_w}{L_t}\right)^2\right]\left[1 - \frac{0.5\,d_{sh}/L_t}{\sin\theta_b + L_w/L_t}\right]^3 + 1.9874 \right.$$
$$\left. - 3.02783\left[\frac{0.5\,d_{sh}/L_t}{\sin\theta_b + L_w/L_t}\right] + 2.0817\left[\frac{0.5\,d_{sh}/L_t}{\sin\theta_b + L_w/L_t}\right]^2\right\} \qquad (C.45)$$
$$\times\left[\sin\theta_b - \frac{d_{sh}}{2L_t} + \frac{L_w}{L_t}\right]^{-2}$$

where L_w is the half-width of the walkway between the A-frames. Thus, $K_d = K_{dj} + K_o$. Both equations (C.44) and (C.45) are valid within the following limits:

$$20° \le \theta_b \le 35°$$
$$0 \le \frac{0.5\,d_{sh}/L_t}{\sin\theta_b + L_w/L_t} \le 0.17886$$
$$0 \le L_w/L_t \le 0.09033$$
$$0 \le d_{sh}/L_t \le 0.303 \quad \text{for } L_w = 0$$
$$K_{he} \ge 30 \quad \text{(uniform normal velocity distribution)}$$

If performance correlations from oblique flow experiments are available, the pressure loss coefficient for the heat exchanger bundle under non-isothermal flow conditions has the following form

$$K_{\theta t} = a_{K\theta} Ry^{b_{K\theta}} + \frac{2}{\sigma^2}\left(\frac{\rho_{a5} - \rho_{a6}}{\rho_{a5} + \rho_{a6}}\right) \qquad (C.46)$$

Equation (C.31) is known as the draft equation for a forced draft air-cooled heat exchanger where the heat exchanger bundles are arranged in the formation of A-frames. This equation can be rewritten with all the loss coefficients based on the heat exchanger frontal area and the mean harmonic air density through the heat exchanger bundle. Multiply equation (C.31) by $2\rho_{a56}(A_{fr}/m_a)^2$ to obtain

$$2\rho_{a56}(A_{fr}/m_a)^2 p_{a1}\left[\left\{1 - 0.00975(H_7 - H_6)/T_{a6}\right\}^{3.5} - \left\{1 - 0.00975(H_7 - H_6)/T_{a1}\right\}^{3.5}\right]$$
$$= K_{ts}(A_{fr}/A_2)^2(\rho_{a56}/\rho_{a1}) + K_{up}(A_{fr}/A_{Fe})^2(\rho_{a56}/\rho_{a3}) - K_{Fs}(A_{fr}/A_{Fc})^2(\rho_{a56}/\rho_{a3}) \qquad (C.47)$$
$$+ K_{do}(A_{fr}/A_{Fe})^2(\rho_{a56}/\rho_{a3}) + K_{\theta t}$$

By means of equation (C.47) it is possible to compare the magnitude of the loss coefficients directly.

C.5 FAN PERFORMANCE CHARACTERISTICS

The choice of a suitable axial flow fan must be such that it will efficiently deliver a cooling air flow rate that will guarantee the desired heat transfer rate. In ACHE systems, the design of the fan system is as important as the heat exchanger. In order to achieve this, a series of resistances must be overcome [94KR2e]. It is, however, possible to rearrange the draft equation and obtain the system resistance curve as a function of the air mass flow rate. The point where the system resistance curve meets the fan performance curve at the desired air mass flow rate, defines the fan operating point.

The performance characteristics of an axial flow fan are determined experimentally according to certain fan performance test codes, e.g. the British Standards Institution BS848 [80BS1e]. Fans should be tested according to the method most closely representing the actual performance conditions. Fan test results are presented in the form of a performance curve as shown in figure C.5.

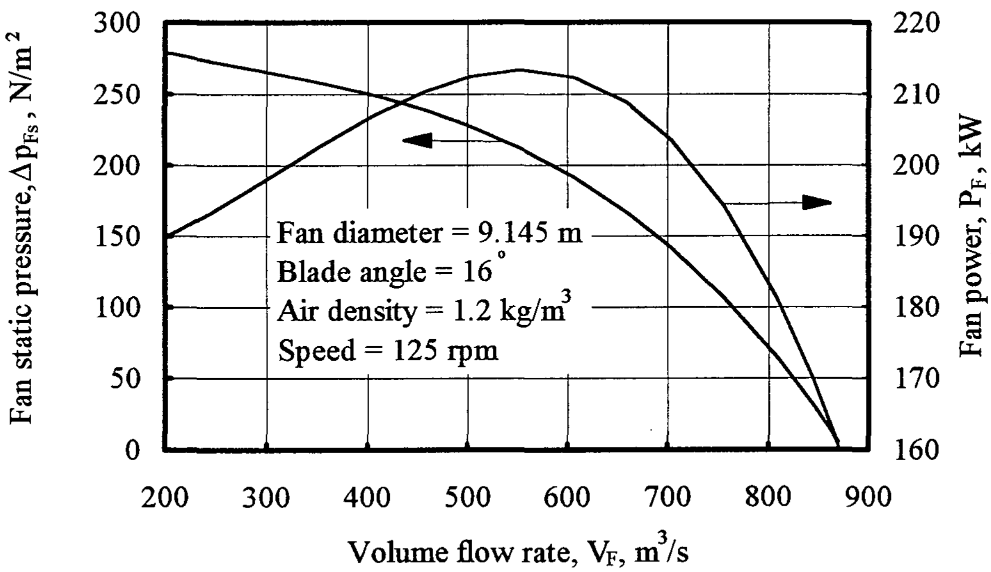

Figure C.5: Performance of an axial flow fan.

The fan static efficiency can be defined as

$$\eta_{Fs} = \frac{\Delta p_{Fs} V_F}{P_F} \tag{C.48}$$

In practice, it is preferred that the actual air volume flow rate, corresponding to the fan operating point, is larger than the volume flow rate at the optimum fan static efficiency. A decrease in air volume flow rate will thus result in an increase in fan static efficiency rather than a decrease in fan static efficiency.

The total electrical power requirement of the fans can be calculated as follows:

$$P_e = P_F/\eta_{Fd} \tag{C.49}$$

Model tests may be conducted if it is impractical to conduct full scale tests. The model must be geometrically similar to the actual fan in all the parts which affect the air flow and its dimensions must not exceed those of the actual fan. The actual full size fan may be expected to show a slight improvement in efficiency. However, no allowance is made for this effect [66VD1e, 80BS1e]. The model test results can be scaled, according to the fan laws [80BS1e], to obtain the prototype performance characteristics. The fan laws are as follows (for a fixed fan blade angle):

(1) Volume flow rate

$$\frac{V_F}{V_{Fm}} = \left(\frac{N_F}{N_{Fm}}\right)\left(\frac{d_F}{d_{Fm}}\right)^3 \tag{C.50}$$

(2) Fan static pressure rise

$$\frac{\Delta p_{Fs}}{\Delta p_{Fsm}} = \left(\frac{N_F}{N_{Fm}}\right)^2\left(\frac{d_F}{d_{Fm}}\right)^2\left(\frac{\rho_F}{\rho_{Fm}}\right) \tag{C.51}$$

(3) Power consumption

$$\frac{P_F}{P_{Fm}} = \left(\frac{N_F}{N_{Fm}}\right)^3\left(\frac{d_F}{d_{Fm}}\right)^5\left(\frac{\rho_F}{\rho_{Fm}}\right) \tag{C.52}$$

The tip clearance of the prototype fan which results in the same fan static efficiency as measured for the model fan is presented by the VDI fan test code [66VD1e] as

$$\frac{s_F}{s_{Fm}} = \left(\frac{\Delta p_{Fm}}{\Delta p_F}\right)^{0.1}\left(\frac{d_F}{d_{Fm}}\right)^{0.8} \tag{C.53}$$

According to this equation the tip clearance ratio s_F/d_F for the prototype fan is required to be smaller than the corresponding ratio for the model to ensure similar operating conditions. In practice, a small tolerance between the fan casing and the fan blade is difficult to guarantee due to the material properties and construction methods of the blades and fan casing. As a result, typical values for tip clearances of 0.5% to 1% of the fan diameter are employed by the manufacturers.

Performance curves for large fans are generally obtained under ideal conditions. However, in actual installations the efficiencies are always lower than those obtained under the ideal test conditions. The reduction can be ascribed to a number of reasons, such as a high air approach velocity when the fan is located too close to the ground level and distorted inlet flow conditions. In large cooling plants covering a considerable area and including numerous fans, some of the fans may be subjected to significant cross-flow which tends to distort inlet conditions to the fan, resulting in a corresponding reduction in performance. Flow separation at the inlet to the fan can be avoided by attaching a well designed inlet bell to the fan housing [85VE1e]. The effect of the inlet losses is correlated by Salta and Kröger [94SA1e] for more than two fan rows as a volumetric effectiveness, i.e.

$$\frac{V_F}{V_{Fid}} = 0.985 - \exp\left[-\frac{(1+45/n)H_3}{6.35\,d_F}\right] = e_V \tag{C.54}$$

where $n = n_{Fr}$ for a freestanding fan platform and $n = 2\,n_{Fr}$ for a non-freestanding fan platform. This correlation is applicable for $H_b/d_F = 0.19$ and $W_F/d_F = 1.27$, where H_b is the height of the bellmouth fan inlet from the fan platform and W_F is the fan pitch. n_{Fr} is the number of fan rows (number of fans per bay), n_{Fb} is the number of fan bays and the total number of fans are $n_F = n_{Fr}n_{Fb}$. V_F/V_{Fid} correlates the effect of the inlet losses on the whole fan system and states that $n_F m_a$ will be less than $n_F m_{a,id}$. When using this correlation, the layout of the fan platform must be fixed beforehand.

The actual airside heat transfer rate can be obtained by multiplying the maximum ideal airside heat transfer rate by the fan volumetric effectiveness, e_V. Thus, equation (C.21) changes to

$$m_a c_{pam(i)}\left(T_{ao(i)} - T_{ai(i)}\right) e_V = m_{c(i)} i_{fg(i)} = m_a c_{pam(i)}\left(T_{vm(i)} - T_{ai(i)}\right) e_{(i)} e_V \tag{C.55}$$

The fan motor must be selected to handle the maximum load, which will occur during operation at low ambient air temperatures (due to the higher air density). The fan motor must be oversized by 25-30% for fans that are not autovariable and 15% otherwise [83PA1e]. In large mechanical draft dry-cooling towers incorporating axial flow fans, it is essential that the required fan power be kept as low as possible. Ambient air is a coolant which exhibits large temperature fluctuations. Constant speed fan drives may result in a much higher air mass flow rate than needed at low ambient air temperatures, which can result in freezing (in the ACC) or throttling of the steam flow (in the turbine exit) when coupled to a steam turbine. At high ambient air temperatures the constant speed drives may result in an air mass flow rate that is inadequate to meet the cooling demands. Thus, it seems that the fan energy costs can be minimized by harmonizing the fan operating conditions with the external ambient air temperatures, and the turbine performance characteristics when applicable.

The variation of the air mass flow rate can be achieved by varying the fan blade angle during operation, the fan motor speed during operation (refer to the fan laws) or by means of louvers [77SC1e, 81KO1e, 81ST1e, 83PA1e, 85MO1e, 85MO2e, 93AD1e]. Automatically variable pitch (fan blade angle) fans are limited in diameter (up to ± 6 m) and automatically variable speed drives are expensive, although in some applications they may present a more cost effective proposition than other methods of flow control. The most common means of flow control is to switch single-speed electric motors on (full speed) or off, or to have two-speed electric motors that allow the fans to operate at full speed, half speed or to be switched off. Fan speed is usually limited by noise restrictions, available gear ratios, or the manufacturer's practice. To meet noise requirements, the rule of thumb is to limit the fan tip speed to 60 m/s [83SH1e].

C.6 OPERATING POINT CALCULATION

The operating point (ability to reject heat) of the fixed geometry ACC is defined as the combination of operating variables that will simultaneously satisfy the draft and heat transfer (energy) equations for specified turbine exhaust and ambient air conditions. This point is obtained from the intersection of the system resistance line and the fan performance curve.

Definition of the operating variables, operating constraints and equations to be satisfied

For a specified air-cooled condenser system geometry, specified turbine outlet conditions $\left(T_{tv}, p_{tv}\right)$ and specified ambient air conditions $\left(T_{db}, T_{wb}, p_a\right)$ the following variables, constraints and equations (balances) can be defined for the simultaneous solution of the operating point conditions:

(1) One tube row

The operating variables are: $m_a, T_{ao(1)}, T_{vi}, m_{c(1)}$

These variables must satisfy the following feasibility inequality constraints (bounds):

$$m_a \geq m_{al} + \varepsilon_1 \tag{C.56}$$

$$-m_a \geq -m_{au} + \varepsilon_1 \tag{C.57}$$

$$m_{c(1)} \geq \varepsilon_2 \tag{C.58}$$

$$T_{ao(1)} \geq T_{ai(1)} + \varepsilon_3 \tag{C.59}$$

$$-T_{ao(1)} \geq -T_{vi} + \varepsilon_3 \tag{C.60}$$

$$-T_{vi} \geq -T_{tv} + \varepsilon_4 \tag{C.61}$$

The following equations must be satisfied by the operating variables:

$$Q_{a(1)} = Q_{c(1)} \tag{C.62}$$

$$Q_{a(1)} = Q_{ac(1)} \tag{C.63}$$

$$T_{vi} = T(p_{vi}) \tag{C.64}$$

$$\Delta p_{system} = \Delta p_{Fs} \tag{C.65}$$

(2) Two tube rows

The operating variables are: m_a, $T_{ao(1)}$, $T_{ao(2)}$, T_{vi}, $m_{c(1)}$, $m_{c(2)}$

These variables must satisfy the following feasibility inequality constraints (bounds):

$$m_a \geq m_{al} + \varepsilon_1 \tag{C.66}$$

$$-m_a \geq -m_{au} + \varepsilon_1 \tag{C.67}$$

$$m_{c(1)} \geq \varepsilon_2 \tag{C.68}$$

$$m_{c(2)} \geq \varepsilon_2 \tag{C.69}$$

$$T_{ao(1)} \geq T_{ai(1)} + \varepsilon_3 \tag{C.70}$$

$$-T_{ao(1)} \geq -T_{ao(2)} + \varepsilon_3 \tag{C.71}$$

$$-T_{ao(2)} \geq -T_{vi} + \varepsilon_3 \tag{C.72}$$

$$-T_{vi} \geq -T_{tv} + \varepsilon_4 \tag{C.73}$$

The following equations must be satisfied by the operating variables:

$$Q_{a(1)} = Q_{c(1)} \tag{C.74}$$

$$Q_{a(2)} = Q_{c(2)} \tag{C.75}$$

$$Q_{a(1)} = Q_{ac(1)} \tag{C.76}$$

$$Q_{a(2)} = Q_{ac(2)} \tag{C.77}$$

$$T_{vi} = T(p_{vi}) \tag{C.78}$$

$$\Delta p_{system} = \Delta p_{Fs} \tag{C.79}$$

ε_i are small constants and are used to introduce the ≥ sign in the feasibility inequalities. The bounds on the operating variables are introduced to safeguard against the violation of the physical laws. The upper and lower limits on the air mass flow rate can be obtained from the fan performance characteristic curve.

C.7 POWER GENERATION

The turbo-generator characteristics of the direct condensing system, i.e. the heat to be rejected and the power output of the turbo-generator, are expressed in terms of the turbine back pressure, p_{tv}, or the corresponding saturated vapor temperature, T_{tv}. The performance characteristics of an example of such a system is shown in figure C.6. It is assumed that the power needed for the boiler feedpumps and other auxiliaries, excluding the fan operating power, is already subtracted from the generated power.

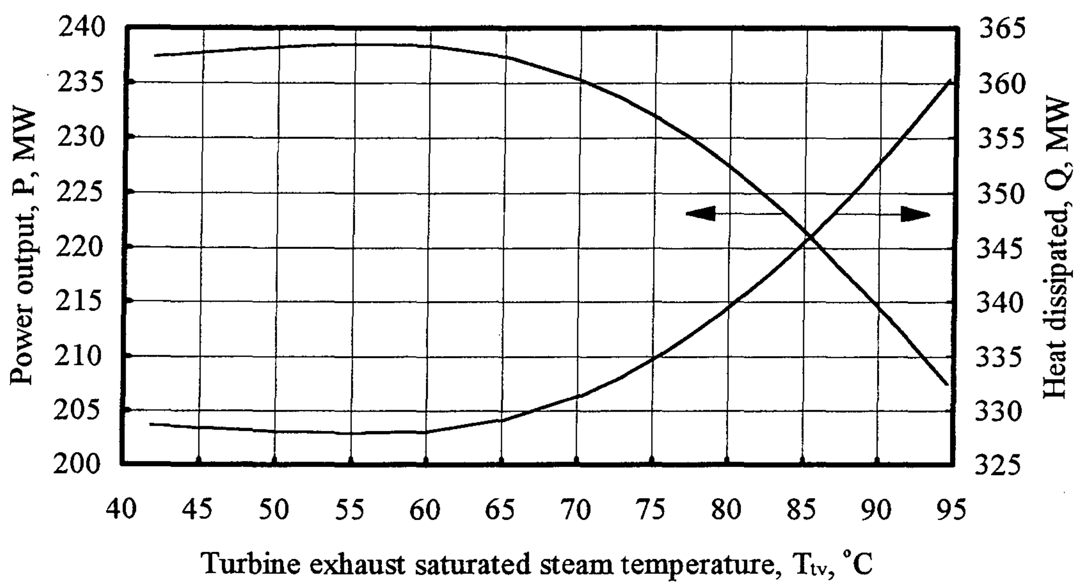

Figure C.6: Performance characteristics of a turbo-generator.

Atmospheric conditions influence the performance of air-cooled condensers, resulting in a wide fluctuation of turbine back pressure (and the corresponding saturated steam temperature). Changes in the ambient temperature are the most important reason for this, although other environmental effects such as wind, inversions, solar radiation, and rain all contribute to this behavior. The mean hourly frequency of ambient temperatures over a period of one year is normally supplied in tabulated or graphical form. An example of the frequency of the dry- and wetbulb temperatures 2m above the ground level at a particular location is shown in figure C.7. It should be noted that the air entering the air-cooled condenser may deviate considerably from these measured values and more detailed information on the actual ambient temperature distribution would be preferred for more sophisticated designs [94KR1e].

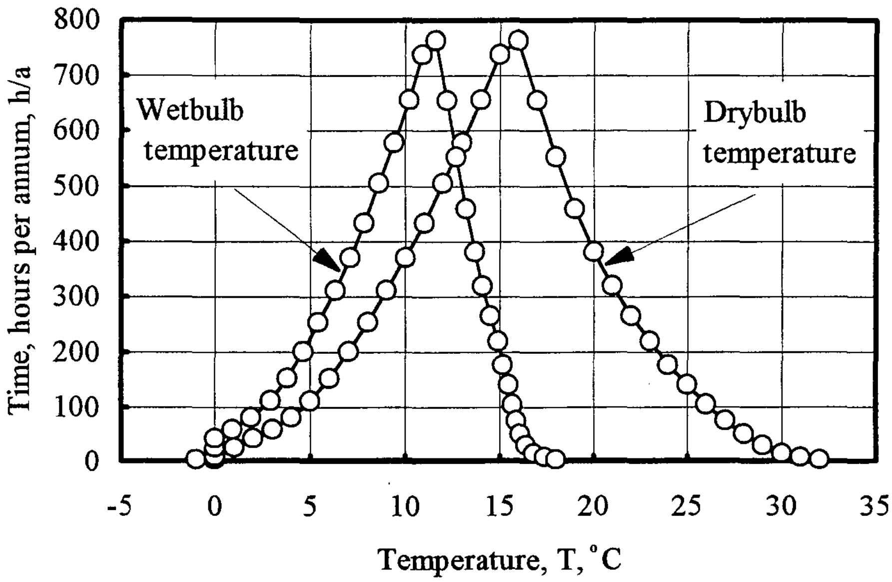

Figure C.7: Frequency of ambient dry- and wetbulb air temperatures.

The operating point of the turbo-generator is determined by matching the operating point of the air-cooled condenser and the performance characteristics of the turbo-generator (Figure C.6) at a specific ambient air temperature selected from Figure C.7. This calculation involves the selection of turbine exhaust conditions such that the heat to be rejected by the turbo-generator, equals the heat absorbed by the air (heat rejected by the air-cooled condenser), i.e.

$$\begin{aligned} & n_F \sum_{i=1}^{n_r} m_a c_{pam(i)} \left(T_{ao(i)} - T_{ai(i)}\right) e_V = n_F \sum_{i=1}^{n_r} m_{c(i)} i_{fg(i)} \\ & = n_F \sum_{i=1}^{n_r} m_a c_{pam(i)} \left(T_{vm(i)} - T_{ai(i)}\right) e_{(i)} e_V \\ & = Q_{tg} \end{aligned} \tag{C.80}$$

At this point the power requirement of the fans as well as the generator power output are known. Subtract the total power consumed by the fans from the generator power output to find the net power output of the plant. The net power output is multiplied by the corresponding number of operating hours to give the net energy output for this period. These calculations are repeated for each of the ambient temperatures listed to obtain the total annual net energy output.

In the cases when very low ambient temperatures are experienced, it is possible that the air-cooled condenser can dissipate more heat than the amount required by the turbo-generator characteristic at its bottom limit. As the turbine back pressure decreases, more power can be generated due to the more complete expansion of steam through the turbine. The ACC will tend to lower the turbine exhaust pressure under these conditions, thus giving rise the a phenomenon called "choking" [71HA1e, 71HE1e, 91SZ1e]. No further advantages are obtained by lowering the turbine back pressure and turbine losses may become excessive due to high steam velocities. As performance of the air-cooled condenser deteriorates with increasing ambient air temperatures, the turbine back pressure increases, resulting in a decrease in power generation. The maximum allowable turbine exhaust pressure is such that flow induced vibration of the turbine blades will not occur for lower values [91SZ1e]. When reaching the upper limit of the characteristic curve, the turbine load must be reduced. The variation of air mass flow rate, as discussed in section C.5, should be considered to overcome these difficulties and to give optimum utilization of the fans (auxiliary power needed) and the cooling demands to be met during annual operation [77SC1e, 81KO1e]. Air humidification can also be used as a means to increase the cooling capacity during high ambient air temperatures [83PA1e, 94KR1e].

Definition of the operating variables, operating constraints and equations to be satisfied

For a fixed air-cooled condenser system geometry, a fixed turbo-generator characteristic curve and an annual frequency of ambient temperatures (at a constant p_a), the following variables, constraints and equations (balances) can be defined for the simultaneous solution of the operating point conditions during the annual operation:

(1) One tube row

The operating variables are: m_a, $T_{ao(1)}$, T_{vi}, $m_{c(1)}$, T_{tv}

These variables must satisfy the following constraints (bounds):

Equations (C.56), (C.57), (C.58), (C.59), (C.60), (C.61), as well as

$$T_{tv} \geq T_{tvl} + \varepsilon_5 \tag{C.81}$$

$$-T_{tv} \geq -T_{tvu} + \varepsilon_5 \tag{C.82}$$

The upper and lower limits on the turbine exhaust steam temperature can be obtained from the turbo-generator characteristic curve.

The following equations must be satisfied by the operating variables:

Equations (C.62), (C.63), (C.64), (C.65) as well as

$$n_F Q_{a(1)} = Q_{tg} \tag{C.83}$$

(2) Two tube rows

The operating variables are: m_a, $T_{ao(1)}$, $T_{ao(2)}$, T_{vi}, $m_{c(1)}$, $m_{c(2)}$, T_{tv}

These variables must satisfy the following constraints (bounds):

Equations (C.66), (C.67), (C.68), (C.69), (C.70), (C.71), (C.72), (C.73) as well as

$$T_{tv} \geq T_{tvl} + \varepsilon_5 \tag{C.84}$$

$$-T_{tv} \geq -T_{tvu} + \varepsilon_5 \tag{C.85}$$

The following equations must be satisfied by the operating variables:

Equations (C.74), (C.75), (C.76), (C.77), (C.78), (C.79) as well as

$$n_F\left(Q_{a(1)} + Q_{a(2)}\right) = Q_{tg} \tag{C.86}$$

With the turbo-generator's operating point known, the net power output of the plant at that specific condition is calculated as follows:

$$P_{net} = P_{tg} - n_F P_F \tag{C.87}$$

For each temperature data set (consisting of a drybulb air temperature, a wetbulb air temperature and the annual duration of these temperatures), a new set of operating variables with bounds are defined and used to determine the turbo-generator's operating point. The net annual energy output of the plant is thus

$$E_{net} = \sum_{i=1}^{n} P_{net(i)} \tau_{(i)} \tag{C.88}$$

where n is the number of temperature frequency data sets and τ is the duration of these temperatures.

When the ambient conditions are such that the air-cooled condenser can dissipate more heat than required by the turbo-generator characteristic, fans are switched off to control the air mass flow rate in order to achieve the desired cooling capacity.

C.8 OPERATING POINT OPTIMIZATION

Operating point optimization involves the process of finding the combination of operating and geometrical variables that will minimize the total annual cost (capital and operating) of the ACC for specified turbine exhaust conditions (T_{tv}, p_{tv}), specified ambient air conditions (T_{db}, T_{wb}, p_a) and specified heat transfer rate Q_{ACC}, while satisfying all the imposed constraints.

Optimization variables

Table C.1: Optimization variables (✓= applicable; ×= not applicable)

Number	Variable	Round tubes with circular fins	Finned tubes with fixed geometry
1	m_a	✓	✓
2	$T_{ao(1)}$	✓	✓
3	$T_{ao(2)}$	✓	✓
4	T_{vi}	✓	✓
5	$m_{c(1)}$	✓	✓
6	$m_{c(2)}$	✓	✓
7	T_{tv}	✓	✓
8	H_3	✓	✓
9	d_F	✓	✓
10	θ_F	✓	✓
11	N_F	✓	✓
12	n_b	✓	✓
13	$n_{tb(max)}$	✓	✓
14	L_t	✓	✓
15	θ_b	✓	✓
16	d_o	✓	×
17	t_t	✓	×
18	t_r	✓	×
19	d_f	✓	×
20	t_f	✓	×
21	t_g	✓	×
22	$P_{f(1)}$	✓	×
23	$P_{f(2)}$	✓	×
24	P_t	✓	×

The optimization variables defined for the round tubes with circular fins use the airside performance correlations specified in Appendix B.1, while the variables defined for finned tubes with any geometry use the performance correlations defined in Appendix B.2. The optimization variables are defined for finned tube bundles having one or two tube rows. The operating variables are also considered as optimization variables because the ACC model is included as equality and inequality constraints (balances and bounds) in the minimization problem formulation (integrated approach). The operating variables (numbers 1-6) are, as a result, always present during optimization, whereas the geometrical variables (numbers 8-24) can be kept constant or varied during the optimization process.

Objective function

The objective function requires that the total annual cost be minimized. The construction of the objective function from its various capital and operating cost components is discussed in Appendix E. The total annual cost is

$$C_{total} = C_{operating} + C_{maintenance} + C_{FCR} \text{ (\$/annum)} \tag{C.89}$$

Constraints

(1) One tube row

The following equality constraints must always be satisfied:

Equations (C.62), (C.63), (C.64), (C.65) as well as

$$n_F Q_{a(1)} = Q_{ACC} \tag{C.90}$$

The following inequality constraints must always be satisfied:

Equations (C.56), (C.57), (C.58), (C.59), (C.60), (C.61), as well as

$$-v_{vi(1)} \geq -120 \quad \text{(The steam velocity at the tube inlet must be less than 120 m/s.)} \tag{C.91}$$

(2) Two tube rows

The following equality constraints must always be satisfied:

Equations (C.74), (C.75), (C.76), (C.77), (C.78), (C.79) as well as

$$n_F\left(Q_{a(1)} + Q_{a(2)}\right) = Q_{ACC} \tag{C.92}$$

The following inequality constraints must always be satisfied:

Equations (C.66), (C.67), (C.68), (C.69), (C.70), (C.71), (C.72), (C.73) as well as

$$-v_{vi(1)} \geq -120 \tag{C.93}$$

$$-v_{vi(2)} \geq -120 \tag{C.94}$$

The following general inequality constraints (geometrical constraints) must be satisfied, depending on whether or not the relevant optimization variables are varied during the optimization process:

The ratio of the fan unit's width and the fan unit's length to the fan diameter must be kept within practical limits to assure a reasonable heat exchanger normal approach velocity (2 to 4 m/s). The ratio of the total heat exchanger frontal area to the fan casing area usually ranges from 1.8 to 2.6.

$$2L_t \sin\theta_b - 1.2\, d_F \geq 0 \tag{C.95}$$

$$1.5\, d_F - 2L_t \sin\theta_b \geq 0 \tag{C.96}$$

$$0.5 n_b W_b - 1.2\, d_F \geq 0 \tag{C.97}$$

$$1.5\, d_F - 0.5 n_b W_b \geq 0 \tag{C.98}$$

The fan blade angle must lie within the limits imposed by the fan performance characteristics.

$$\theta_F - \theta_{F\,min} \geq 0 \tag{C.99}$$

$$\theta_{F\,max} - \theta_F \geq 0 \tag{C.100}$$

The heat exchanger bundle width is limited by transport requirements.

$$W_{bu} - W_b \geq 0 \tag{C.101}$$

The tip speed of the fan blade is usually limited to 60 m/s to control noise.

$$v_{Fbt} - \pi d_F N_F \geq 0 \tag{C.102}$$

The correlations of the downstream loss, K_d, are valid within the following limits:

$$\theta_b - 20° \geq 0 \tag{C.103}$$

$$35° - \theta_b \geq 0 \tag{C.104}$$

$$0.17886 - \frac{0.5\, d_{sh}/L_t}{\sin\theta_b + L_w/L_t} \geq 0 \tag{C.105}$$

$$0.09033 - L_w/L_t \geq 0 \tag{C.106}$$

$$0.303 - d_{sh}/L_t \geq 0 \quad (L_w = 0) \tag{C.107}$$

For round tubes with circular fins, the following inequality constraints are imposed (refer to Appendix B.1):

The finned tubes must be kept from touching.

$$P_t - d_f \geq 0 \qquad \text{(C.108)}$$

The fin diameter must exceed the fin root diameter (extended airside surface area is required).

$$d_f - d_r \geq 0 \qquad \text{(C.109)}$$

Due to structural considerations, the fin root thickness and the tube thickness must exceed their lower limits.

$$t_r - t_{rl} \geq 0 \qquad \text{(C.110)}$$

$$t_t - t_{tl} \geq 0 \qquad \text{(C.111)}$$

For effective cleaning and where airside fouling is of significance, the fin pitch must exceed the imposed lower limit.

$$P_{f(1)} - P_{fl} \geq 0 \qquad \text{(C.112)}$$

$$P_{f(2)} - P_{fl} \geq 0 \qquad \text{(C.113)}$$

The galvanizing thickness must lie within practical limits.

$$t_g - t_{gl} \geq 0 \qquad \text{(C.114)}$$

$$t_{gu} - t_g \geq 0 \qquad \text{(C.115)}$$

The fin thickness is limited by the manufacturing processes.

$$t_f - t_{fl} \geq 0 \qquad \text{(C.116)}$$

The fin pitch must exceed the fin thickness.

$$P_{f(1)} - t_f \geq 0 \qquad \text{(C.117)}$$

$$P_{f(2)} - t_f \geq 0 \qquad \text{(C.118)}$$

The application limits of the finned tube performance correlations, as discussed in Appendix B (section B.1), are not used as constraints during the optimization of circular finned tubes.

The following assumptions and relations are used to obtain values for the geometrical parameters linked to the optimization variables:

$n_{tb(1)} = n_{tb(max)}$ for 1 tube row

$n_{tb(2)} = n_{tb(max)}$ for 2 tube rows

$W_b = n_{tb(max)} P_t$ (bundle width)

$d_{Fc} = 1.005 d_F$ (fan casing diameter)

$d_{Fh} = 0.165 d_F$ (fan hub diameter)

$H_{Fc} = 0.1 d_{Fc}$ (fan casing height)

$H_b = 0.15 d_{Fc}$ (bellmouth height)

$d_b = 1.2 d_{Fc}$ (bellmouth diameter)

$H_w = L_t \cos\theta_b + 0.3 d_{Fc}$ (windwall height)

$P_l = 0.866 P_t$ (longitudinal tube pitch for circular finned tubes)

$d_i = d_o - 2t_t$ (round tube inside diameter)

$d_r = d_o + 2t_r$ (root diameter of circular finned tube)

The following parameters are assumed to be constant during the optimization process:

$L_w, K_{ts}, d_{sh}, K_{sd}, K_{up}, K_{do}, n_F, n_{Fb}, n_{Fr}, \eta_{Fd}, n_r, n_b$

C.9 MINIMIZATION OF POWER GENERATION COST

The minimization of the power generation cost attributed to the ACC performance for the given temperature frequency data set, involves the variation of the ACC operating and geometrical variables that will minimize the ratio of its total annual cost to the annual net power output of the turbo-generator set it is coupled to. The calculated values of the different variables must satisfy all the relevant constraints.

Optimization variables

The optimization variables are defined in Table C.1. Different operating variables (numbers 1-7) are defined for each temperature data set and they are always present during optimization. When variable speed fan drives are considered (for airflow control), a new fan operating speed, N_F, is also calculated for each temperature data set. The other geometrical variables can be varied or remain constant.

Objective function

The objective function requires that the ratio of the total annual cost of the ACC to the net energy produced by the turbo-generator be minimized, i.e.

$$C_{power} = C_{total}/E_{net} \quad (\$/kWh) \tag{C.119}$$

The construction of the total annual cost from its various cost components is discussed in Appendix E, while the topic of power generation is treated in section C.7.

Constraints

(1) One tube row

The following equality constraints must always be satisfied:

Equations (C.62), (C.63), (C.64), (C.65), (C.83)

The following inequality constraints must always be satisfied:

Equations (C.56), (C.57), (C.58), (C.59), (C.60), (C.61), (C.81), (C.82), (C.91)

(2) Two tube rows

The following equality constraints must always be satisfied:

Equations (C.74), (C.75), (C.76), (C.77), (C.78), (C.79, (C.86)

The following inequality constraints must always be satisfied:

Equations (C.66), (C.67), (C.68), (C.69), (C.70), (C.71), (C.72), (C.73), (C.84), (C.85), (C.93), (C.94)

The following general inequality constraints (geometrical constraints) must be satisfied, depending on whether or not the relevant optimization variables are varied during the optimization process:

Equations (C.95), (C.96), (C.97), (C.98), (C.99), (C.100), (C.101), (C.102), (C.103), (C.104), (C.105), (C.106), (C.107), (C.108), (C.109), (C.110), (C.111), (C.112), (C.113), (C.114), (C.115), (C.116), (C.117), (C.118)

When variable speed fan drives are considered (for airflow control), equation (C.102) must be satisfied for each temperature data set.

For each temperature data set there is a corresponding set of operating variables, equality and inequality constraints (operating constraints). These constraint sets must always be satisfied. The operating variables of a specific temperature data set effect the operating constraints belonging to the same temperature data set only. The operating variables do not effect the geometrical constraints. However, the geometrical variables effect all the operating constraints of the different temperature data sets.

APPENDIX D

NATURAL DRAFT INDIRECT DRY-COOLING TOWERS

D.1 INTRODUCTION

The performance prediction of dry-cooling towers are based on two sets of governing equations, i.e. the draft equation and the heat transfer equations. The governing heat transfer and draft equations will be derived in this appendix. The coupling of the indirect dry-cooling tower to a turbo-generator unit, as found in power plants, is also discussed. The results of a considerable amount of experimental and theoretical work are taken into account to model all the physical phenomena of such a system. The search for economically viable dry-cooling tower operation requires the proper formulation and modeling of the system. The variables and constraints required to perform this task are defined and discussed.

D.2 DESCRIPTION OF THE INDIRECT DRY-COOLING TOWER

Indirect dry-cooling systems make use, either of surface (conventional) condensers or spray (jet) condensers to condense the process fluid (steam). The secondary fluid (water) that is used to condense the process fluid is cooled by the cooling tower. Only the indirect dry-cooling system employing a surface condenser and water as the secondary cooling fluid will be considered in this study. As shown in figure D.1, cold water flows through the tubes of the condenser and removes heat from the steam passing over them. This heated water is pumped through the finned tubes of the heat exchanger bundles arranged in the natural draft cooling tower. This water is cooled by natural convection of air flowing over the extended surface heat exchanger bundles. The density of the heated air inside the tower shell is less than that of the atmosphere outside the tower, with the result that the pressure inside the tower is less than the external pressure at the same elevation. The pressure differential causes air to flow through the tower at a rate which is dependent on the various flow resistances encountered, the cooling tower dimensions and the heat exchanger characteristics.

D.3 HEAT TRANSFER AND PRESSURE DROP CALCULATION

The finned tube elements are the heart of any the indirect dry-cooling system. The air-cooled heat exchanger bundles may consist of one or more rows of finned tubes. Various heat exchanger bundle arrangements do exist, i.e. the cooling deltas can, for example, be installed either horizontally, conically or vertically at the tower's inlet section. For the purpose of this study the heat exchanger bundles are located horizontally at the inlet section of the hyperbolic natural draft dry-cooling tower shown schematically in figure D.2.

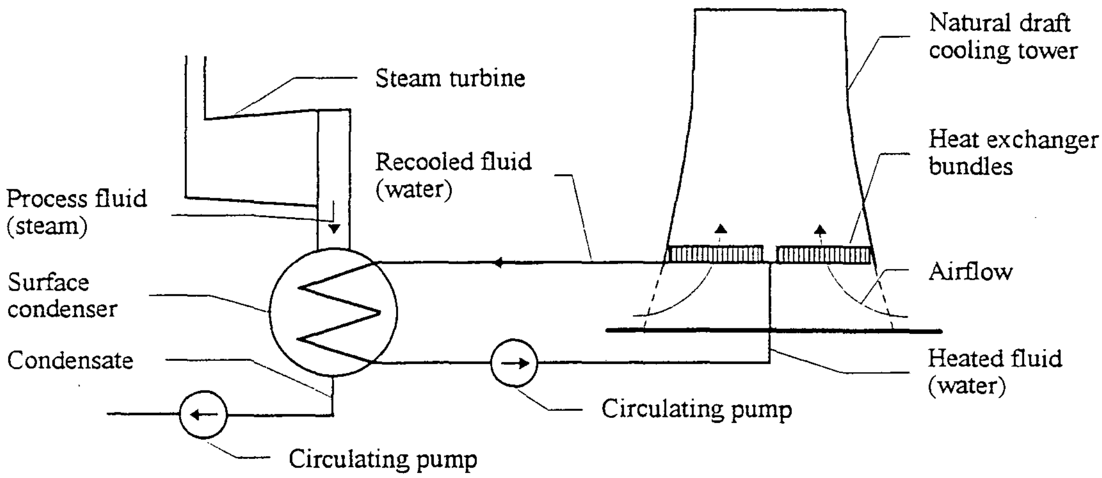

Figure D.1: Natural draft dry-cooling tower with surface condenser (indirect system).

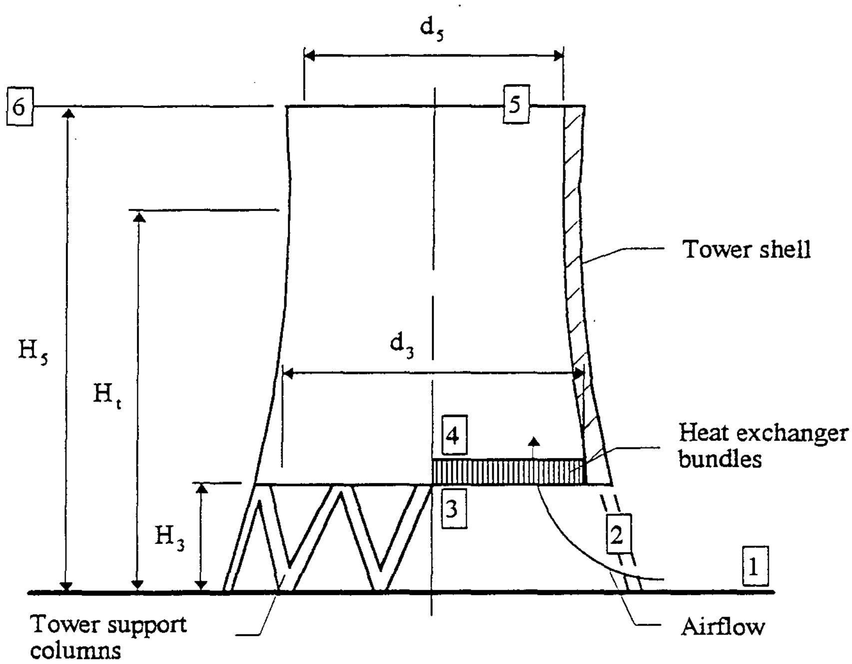

Figure D.2: Natural draft dry-cooling tower with horizontal heat exchanger.

The amount of heat transferred from the condenser cooling water to the air stream can be expressed as

$$Q_a = m_a c_{pam}(T_{a4} - T_{a3}) = \frac{UA\,F_T[(T_{wi} - T_{a4}) - (T_{wo} - T_{a3})]}{\ell n[(T_{wi} - T_{a4})/(T_{wo} - T_{a3})]} = m_w c_{pwm}(T_{wi} - T_{wo}) = Q_w \qquad (D.1)$$

The thermophysical properties of the air and water will be determined at their respective mean temperatures.

The product of the overall heat transfer coefficient and area can be expressed as

$$UA = \left(\frac{1}{h_{ae}A_a} + \frac{1}{h_w A_w} \right)^{-1} \tag{D.2}$$

where

$$h_{ae}A_a = k_{am}\, Pr_{am}^{0.333}\, A_{fr} Ny \left(n_{tb(actual)} / n_{tb(max)} \right) \tag{D.3}$$

and

$$Ry = m_a / \left(\mu_{am} A_{fr} \right) \tag{D.4}$$

for any finned tube geometry, and

$$h_{ae}A_a = \left(\frac{1}{h_a e_f A_a} + \frac{\ell n(d_o / d_i)}{2\pi k_t L_t n_{tb} n_b} + \frac{\ell n(d_r / d_o)}{2\pi k_f L_t n_{tb} n_b} \right)^{-1} \tag{D.5}$$

for radially finned tubes. Ny is based on heat exchanger bundle tests in which the maximum number of finned tubes are installed in the bundle. In the actual cooling tower bundle it is not practical to install half-tubes at the bundle ends and a correction is made when determining the effective heat transfer coefficient.

The effectiveness of the circular finned surface is expressed in terms of the fin efficiency, i.e.

$$e_f = 1 - A_f (1 - \eta_f) / A_a \tag{D.6}$$

According to Schmidt [46SC1e], the fin efficiency for radial fins of uniform thickness can be determined approximately from

$$\eta_f = \frac{\tanh(b d_r\, \Phi / 2)}{(b d_r\, \Phi / 2)} \tag{D.7}$$

where $\Phi = (d_f / d_r - 1)[1 + 0.35 \ell n(d_f / d_r)]$ and $b = [(2 h_a)/(t_f k_f)]^{0.5}$.

For galvanized steel fins, the fin thickness can be expressed as $t_f = 2t_g + t_s$ and the thermal conductivity of the fin can be expressed as $k_f = (2 t_g k_g + t_s k_s) / t_f$.

The performance characteristics of the finned tubes are discussed in Appendix B and correlations are stated to determine the airside heat transfer coefficient and the characteristic heat transfer parameter.

The Reynolds number of the water flowing inside the heat exchanger tubes is

$$Re_w = \frac{\rho_{wm} v_w d_e}{\mu_{wm}} = \frac{m_w n_{wp} d_e}{A_{ts} n_{tb} n_b \mu_{wm}} \tag{D.8}$$

The corresponding average water velocity in the heat exchanger tubes is

$$v_w = \frac{m_w n_{wp}}{A_{ts} n_{tb} n_b \rho_w} \tag{D.9}$$

The friction factor inside the tube for $\varepsilon_f / d_e > 10^{-4}$ is, according to Haaland [83HA1e]

$$f_{Dw} = 0.3086 \left[\log_{10} \left\{ \frac{6.9}{Re_w} + \left(\frac{\varepsilon_f / d_e}{3.7} \right)^{1.11} \right\} \right]^{-2} \tag{D.10}$$

The frictional pressure drop inside the tubes per unit length is

$$\Delta p_{fw} = f_{Dw} \frac{\rho_{wm} v_w^2}{2 d_e} \tag{D.11}$$

The total water pumping power is

$$P_w = A_{ts} \Delta p_{fw} v_w L_w / (\eta_p \eta_{em}) \tag{D.12}$$

where L_w is the total equivalent length, based on the heat exchanger tubing geometry, to make provision for additional flow resistances (e.g. bends, headers, valves). The ratio $L_w / (n_{tb} n_b L_t) \doteq \xi$ is always greater than one.

The waterside heat transfer coefficient is, according to Gnielinski [75GN1e]

$$h_w = \frac{k_{wm} (f_{Dw}/8)(Re_w - 1000) Pr_{wm} \left[1 + (d_e / L_t)^{0.67} \right]}{d_e \left[1 + 12.7 (f_{Dw}/8)^{0.5} \left(Pr_{wm}^{0.67} - 1 \right) \right]} \tag{D.13}$$

and A_w is the total waterside surface area.

According to Roetzel [84RO1e], the logarithmic mean temperature difference correction factor for crossflow conditions can be expressed as

$$F_T = 1 - \sum_{i=1}^{4} \sum_{k=1}^{4} a_{ik} (1 - \Phi_3)^k \sin[2i \arctan(\Phi_1/\Phi_2)] \tag{D.14}$$

where

$$\Phi_1 = \frac{T_{wi} - T_{wo}}{T_{wi} - T_{ai}}$$

$$\Phi_2 = \frac{T_{ao} - T_{ai}}{T_{wi} - T_{ai}}$$

$$\Phi_3 = \frac{\Phi_1 - \Phi_2}{\ell n[(1 - \Phi_2)/(1 - \Phi_1)]}$$

The values of a_{ik} are individual to each heat exchanger configuration [75RO1e].

The energy balance for the natural draft indirect dry cooling tower requires that the heat rejected by the cooling water must be absorbed by the air flowing over the heat exchanger bundles (equation (D.1) must be satisfied).

D.4 DERIVATION OF THE DRAFT EQUATION FOR A NATURAL DRAFT INDIRECT DRY-COOLING TOWER

The draft equation describes the relation between the various flow resistances encountered, the atmospheric conditions, cooling tower dimensions, and the heat exchanger bundle performance characteristics at a given flow rate.

Significant changes in the ambient air temperature occur near the ground level during any 24-hour period [94KR1e]. During the day, a temperature lapse rate of -0.00975 K/m, also known as the dry adiabatic lapse rate (DALR), is observed in the region of the surface boundary layer (SBL). Significant deviations do however occur at ground level, which, if not taken into consideration, may lead to erroneous design specifications or the incorrect interpretation of cooling tower acceptance test data.

For this analysis, the specified ambient air temperature at any elevation z, will be assumed to be given by the equation

$$T_{az} = T_{a1} - 0.00975\, z \tag{D.15}$$

where T_{a1} is the temperature at ground level, obtained by extrapolating the measured DALR to that elevation. T_{a1} will usually differ from the actual temperature measured at ground level. At elevation 6, which corresponds to the top of the cooling tower, the temperature of the ambient air is thus

$$T_{a6} = T_{a1} - 0.00975 H_5 \tag{D.16}$$

The approximate temperature at the inlet to the heat exchanger bundles can be derived from the first law of Thermodynamics (conservation of energy) in the absence of work interaction, and can be expressed as [94KR1e]

$$T_{a3} \approx T_{a1} - 0.00975 H_3 \tag{D.17}$$

To derive the draft equation, consider the variation with elevation of the pressure in the atmosphere external to the dry-cooling tower in a gravity field, i.e.

$$dp_a = -\rho_a g dz \tag{D.18}$$

Substitute equations (D.15) and (A.1), the perfect gas law, into equation (D.18) and integrate to find the pressure difference between point 1 and a point at elevation z external to the cooling tower (refer to figure D.2)

$$p_{a1} - p_{az} = p_{a1}\left[1 - \left(1 - 0.00975 z/T_{a1}\right)^{102.564 g/R}\right] \approx p_{a1}\left[1 - \left(1 - 0.00975 z/T_{a1}\right)^{3.5}\right] \tag{D.19}$$

The pressure external to the tower at section 6 is

$$p_{a6} = p_{a1}\left(1 - 0.00975 H_5/T_{a1}\right)^{3.5} \tag{D.20}$$

where $H_6 = H_5$ is the tower height.

Stagnant ambient air at 1 accelerates and flows across the tower supports at 2 before flowing through the heat exchanger bundles from 3 to 4. The flow is essentially isentropic from 4 to 5. In most practical towers, the change in kinetic energy between sections 4 and 5 is normally approximately an order of magnitude smaller than the corresponding change in potential energy. A total pressure balance between 1 and 5 yields [94KR1e]:

$$\begin{aligned} & p_{a1} - \left[p_{a5} + \alpha_{e5}\left(m_a/A_5\right)^2/\left(2\rho_{a5}\right)\right] \\ & = \left(K_{ts} + K_{ct} + K_{ctc} + K_{he} + K_{cte}\right)_{he}\left(m_a/A_{fr}\right)^2/\left(2\rho_{a34}\right) \\ & + p_{a1}\left[1 - \left\{1 - 0.00975\left(H_3 + H_4\right)/\left(2T_{a1}\right)\right\}^{3.5}\right] \\ & + p_{a4}\left[1 - \left\{1 - 0.00975\left(H_5 - H_3/2 - H_4/2\right)/T_{a4}\right\}^{3.5}\right] \end{aligned} \tag{D.21}$$

All the loss coefficients, K, are based on the frontal area of the heat exchanger and the mean air density through it. This form of the equation is useful for comparing the relative magnitudes of the flow losses. The frontal area is the projection of the effective finned surface as viewed from the upstream side. Stiffening beams, straps or other obstructions located up against the finned surface, thereby impeding flow through the heat exchanger must be considered when evaluating the effective frontal area, A_{fr}. The last two terms on the right hand side of equation (D.21) take into consideration static pressure differentials due to elevation between ground level and the mean heat exchanger elevation and the latter between the ground level and the tower outlet respectively.

Du Preez and Kröger [94DU1e] studied the velocity and pressure distribution in the outlet plane of hyperbolic natural draft cooling towers. They find that for $1/Fr_D \leq 3$, the velocity distribution is almost uniform, i.e. $\alpha_{e5} \approx 1$ for dry-cooling towers where the heat exchangers are located in the cross-section near the base of the tower. The mean pressure at the outlet plane is found to be slightly less than that of the ambient air at the same elevation, i.e.

$$p_{a5} = p_{a6} + \Delta p_{a56} = p_{a6} + K_{to}\left(m_a/A_5\right)^2 / \left(2\rho_{a5}\right) \tag{D.22}$$

For a hyperbolic tower with a cylindrical outlet the loss coefficient is given by

$$K_{to} = \Delta p_{a56} \Big/ \left(\rho_{a5} v_{a5}^2/2\right) = 2\rho_{a5}\,\Delta p_{a56} \Big/ \left(m_a/A_5\right)^2 = -0.28\,Fr_D^{-1} + 0.04\,Fr_D^{-1.5} \tag{D.23}$$

where $Fr_D = \left(m_a/A_5\right)^2 \Big/ \left[\rho_{a5}\left(\rho_{a6} - \rho_{a5}\right) g\, d_5\right]$. This equation is valid for $0.5 \leq d_5/d_3 \leq 0.85$ and $5 \leq K_{he} \leq 40$.

The approximate temperature at the tower can be derived from the first law of thermodynamics (conservation of energy) in the absence of work interaction, and can be expressed as [94KR1e]

$$T_{a5} \approx T_{a4} - 0.00975\left(H_5 - H_4\right) \tag{D.24}$$

From the perfect gas relation it thus follows for $p_{a5} \approx p_{a6}$, the density at the outlet of the tower is

$$\rho_{a5} = p_{a6} \Big/ \left[R\left\{T_{a4} - 0.00975\left(H_5 - H_4\right)\right\}\right] \tag{D.25}$$

The density of the ambient air at elevation 6 is

$$\rho_{a6} = p_{a6}/RT_{a6} \tag{D.26}$$

If dynamic effects are neglected, an approximate expression for p_{a4} may similarly obtained.

$$\begin{aligned} p_{a4} \approx\; & p_{a1}\left[1 - 0.00975\left(H_3 + H_4\right)/\left(2T_{a1}\right)\right]^{3.5} \\ & - \left(K_{ts} + K_{ct} + K_{ctc} + K_{he} + K_{cte}\right)_{he}\left(m_a/A_{fr}\right)^2 / \left(2\rho_{a34}\right) \end{aligned} \tag{D.27}$$

Substitute equations (D.20), (D.22) and (D.27) into equation (D.21) and find with $\alpha_{e5} = 1$

$$p_{a1}\left[\left\{1-0.00975\left(H_3+H_4\right)/\left(2T_{a1}\right)\right\}^{3.5}\left\{1-0.00975\left(H_5-H_3/2-H_4/2\right)/T_{a4}\right\}^{3.5}\right.$$
$$\left.-\left(1-0.00975H_5/T_{a1}\right)^{3.5}\right]$$
$$=\left(K_{ts}+K_{ct}+K_{ctc}+K_{he}+K_{cte}\right)_{he}\left(m_a/A_{fr}\right)^2/\left(2\rho_{a34}\right)$$
$$\times\left[1-0.00975\left(H_5-H_3/2-H_4/2\right)/T_{a4}\right]^{3.5}+\left(1+K_{to}\right)\left(m_a/A_5\right)^2/\left(2\rho_{a5}\right) \qquad \text{(D.28)}$$

This equation is known as the draft equation for a natural draft dry-cooling tower where the heat exchangers are arranged horizontally in the base of the tower. Thus, the left hand side of the draft equation, Δp_{LHS}, must equal the right hand side of the equation, Δp_{RHS}. If the heat exchangers are arranged in the form of A-frames or V-arrays, K_{he} (non-isothermal) is replaced by $K_{\theta t}$.

In determining the dry air density after the heat exchanger, the specified pressure at ground level can be employed in the perfect gas relation, i.e.

$$\rho_{a4} \approx p_{a1}/\left(RT_{a4}\right) \qquad \text{(D.29)}$$

The approximate air density at section 3 is

$$\rho_{a3} \approx p_{a1}/\left(RT_{a3}\right) \qquad \text{(D.30)}$$

The harmonic mean density through the heat exchanger is given by

$$\rho_{a34} \approx 2p_{a1}/\left[R\left(T_{a3}+T_{a4}\right)\right] \qquad \text{(D.31)}$$

The loss coefficient of the tower supports, K_{ts}, is based on the drag coefficient of these supports, i.e.

$$C_{Dts} = 2\,F_{Dts}/\left(\rho_{a1}v_{a2}^2A_{ts}\right) \qquad \text{(D.32)}$$

The effective pressure drop across the dry-cooling tower supports is given by

$$\Delta p_{ats} = n_{ts}F_{Dts}/A_2 = 0.5\rho_{a1}v_{a2}^2\,C_{Dts}L_{ts}d_{ts}\,n_{ts}/\left(\pi d_3H_3\right) \qquad \text{(D.33)}$$

where L_{ts} is the support length and d_{ts} is its effective diameter or width, and n_{ts} is the number of supports. The corresponding loss coefficient based on these conditions at section 2 is

$$K_{ts} = 2\,\Delta p_{ats}/\left(\rho_{a1}v_{a2}^2\right) = C_{Dts}L_{ts}d_{ts}\,n_{ts}/\left(\pi d_3H_3\right) \qquad \text{(D.34)}$$

For substitution into equation (D.28), this loss coefficient is required to be based on conditions at the heat exchanger, i.e.

$$K_{tshe} = 2\Delta p_{ats}\rho_{a34}/(m_a/A_{fr})^2 = \frac{\rho_{a34}C_{Dts}L_{ts}d_{ts}n_{ts}A_{fr}^2}{\rho_{a1}(\pi d_3 H_3)^3} \quad \text{(D.35)}$$

It is assumed that the air density and the velocity distribution through the supports is uniform. Since the distance between the tower supports is finite, the above approach tends to underestimate the magnitude of the loss coefficient.

Due to separation at the lower edge of the tower shell and distorted inlet flow patterns, a cooling tower loss coefficient K_{ct}, based on the tower cross-sectional area at 3, can be defined to take these effects into consideration. For dry-cooling towers where $K_{he} \geq 30$ and $5 \leq d_3/H_3 \leq 10$, Geldenhuys and Kröger [86GE1e] recommend the following expression

$$K_{ct} = 0.072(d_3/H_3)^2 - 0.34(d_3/H_3) + 1.7 \quad \text{(D.36)}$$

The cooling tower inlet loss coefficient based on conditions at the heat exchanger is

$$K_{cthe} = K_{ct}(\rho_{a34}/\rho_{a3})(A_{fr}/A_3)^2 \quad \text{(D.37)}$$

Depending on the heat exchanger bundle arrangement in the cooling tower base, only a portion of the available area is effectively covered due to the rectangular shape of the bundles. The reduction in effective flow area results in contraction, and subsequent expansion losses. The contraction losses can be approximated by loss coefficients based on the effective reduced flow area A_{e3} [50KA1e]

$$K_{ctc} = 1 - 2/\sigma_c + 1/\sigma_c^2 \quad \text{(D.38)}$$

The contraction coefficient, σ_c, is given by

$$\begin{aligned}\sigma_c = {} & 0.6144517 + 0.04566493\sigma_{e3} - 0.3366651\sigma_{e3}^2 + 0.4082743\sigma_{e3}^3 + 2.672041\sigma_{e3}^4 \\ & - 5.963169\sigma_{e3}^5 + 3.558944\sigma_{e3}^6\end{aligned} \quad \text{(D.39)}$$

The expansion losses can be approximated by

$$K_{cte} = (1 - A_{e3}/A_3)^2 = (1 - \sigma_{e3})^2 \quad \text{(D.40)}$$

The effective area, A_{e3}, corresponds to the frontal area of the heat exchanger bundles if they are installed horizontally. In the case of an array of A-frames, $A_{e3} = A_{fr}\sin\theta_b$ and thus corresponds to the projected frontal area of the bundles. Because of the essentially porous nature of the bundles, the actual contraction loss coefficient will be less than the value given by the above equation.

Based on the conditions at the heat exchanger, the above expressions become

$$K_{ctche} = K_{ctc}(\rho_{a34}/\rho_{a3})(A_{fr}/A_{e3})^2 \tag{D.41}$$

and

$$K_{ctehe} = K_{cte}(\rho_{a34}/\rho_{a4})(A_{fr}/A_{e3})^2 \tag{D.42}$$

For non-isothermal oblique flow through an array of V-bundles, the following relation holds for the loss coefficient [86KO1e]:

$$K_{\theta t} = K_{heiso} + \frac{2}{\sigma^2}\left(\frac{\rho_{a3}-\rho_{a4}}{\rho_{a3}+\rho_{a4}}\right) + \left(\frac{1}{\sin\theta_m} - 1\right)\left(\frac{1}{\sin\theta_m} - 1 + 2K_c^{0.5}\right)\left(\frac{2\rho_{a4}}{\rho_{a3}+\rho_{a4}}\right) + K_d\left(\frac{2\rho_{a3}}{\rho_{a3}+\rho_{a4}}\right) \tag{D.43}$$

where σ is the ratio of the minimum free flow area through the heat exchanger bundle to the free stream flow (frontal) area.

The heat exchanger loss coefficient under normal non-isothermal flow conditions, including inlet-, frictional-, and exit losses as well as acceleration effects (due to heating), is defined as (refer to Appendix B)

$$K_{he} = a_K Ry^{b_K} + \frac{2}{\sigma^2}\left(\frac{\rho_{a3}-\rho_{a4}}{\rho_{a3}+\rho_{a4}}\right) \tag{D.44}$$

for finned tube bundles with any finned tube geometry. For radially finned tubes the heat exchanger loss coefficient under normal non-isothermal flow conditions can, in general, be expressed as [86KR1e]

$$K_{he} = \frac{2}{\sigma^2}\left[Eu + \left(\frac{\rho_{a3}-\rho_{a4}}{\rho_{a3}+\rho_{a4}}\right)\right] = \frac{2}{\sigma^2}\left[\frac{\Delta p_a \rho_{am}}{G_c^2} + \left(\frac{\rho_{a3}-\rho_{a4}}{\rho_{a3}+\rho_{a4}}\right)\right] \tag{D.45}$$

The Euler number can be obtained from the pressure drop correlations discussed in Appendix B.

The second term on the right hand side of equation (D.43) represents the loss due to acceleration effects during non-isothermal operation. The third term represents the loss due to the oblique flow at the inlet to the bundle. The entrance contraction loss coefficient, K_c, is according to Kays [50KA1e]

$$K_c = \left[\frac{1}{\sigma_{21}}\left(\frac{1}{\sigma_c} - 1\right)\right]^2 \tag{D.46}$$

where $\sigma_{21} = (P_f - t_f)/P_f$ is the ratio of the flow area between the fins to the free stream flow area at the inlet to the finned tube bundles. The contraction coefficient, σ_c, is given by equation (D.39), with σ_{21} replacing σ_{e3}. For round and elliptical fins a value of $K_c \approx 0.05$ is assumed.

When the finned tube bundles are arranged in the form of A-frames, the curvature of the downstream flow patterns cause the actual mean flow incidence angle to differ from the heat exchanger bundle semi-apex angle θ_b. An empirical correlation for the mean flow incidence angle is presented by Kotzé et al. [86KO1e], i.e.

$$\theta_m = 0.0019\theta_b^2 + 0.9133\theta_b - 3.1558 \tag{D.47}$$

K_d is the downstream loss coefficient that includes the jetting and kinetic energy losses, and can be expressed in terms of the following relation [86KO1e]

$$K_d = \exp\left(5.488405 - 0.2131209\theta_b + 3.533265\times10^{-3}\theta_b^2 - 0.2901016\times10^{-4}\theta_b^3\right) \tag{D.48}$$

If performance correlations from oblique flow experiments are available, the pressure loss coefficient for the heat exchanger bundle under non-isothermal flow conditions has the following form

$$K_{\theta t} = a_{K\theta}Ry^{b_{K\theta}} + \frac{2}{\sigma^2}\left(\frac{\rho_{a5} - \rho_{a6}}{\rho_{a5} + \rho_{a6}}\right) \tag{D.49}$$

Natural draft cooling towers are prone to the inflow of cold air at the upper edge of the tower shell. Significant performance degradation is observed under these conditions. According to Richter [69RI1e], small disturbances may cause flow instabilities in cooling towers, resulting in cold air inflow under the following conditions

$$1/Fr_D \le 3.05 \tag{D.50}$$

Equation (D.50) will be employed in the design of natural draft dry-cooling towers as a safeguard against cold inflow.

In most of the hyperbolic cooling towers found in practice, the following typical relationships between the tower dimensions can be observed:

$$0.05 \le H_3/H_5 \le 0.15$$
$$0.64 \le d_3/(H_5 - H_3) \le 1.13$$
$$1.37 \le d_3/d_5 \le 1.77$$
$$0.08 \le H_3/d_3 \le 0.16$$

D.5 OPERATING POINT CALCULATION

The operating point (ability to reject heat) of the fixed geometry dry-cooling tower is defined as the combination of operating variables that will simultaneously satisfy the draft and heat transfer (energy) equations for specified condenser outlet and ambient air conditions

Definition of the operating variables, operating constraints and equations to be satisfied

For a specified hyperbolic dry-cooling system geometry, specified condenser outlet conditions (T_{wi}) and specified ambient air conditions (T_{db}, T_{wb}, p_a) the following variables, constraints and equations (balances) can be defined for the simultaneous solution of the operating point conditions:

The operating variables are: m_a, T_{a4}, T_{wo}

These variables must satisfy the following feasibility inequality constraints (bounds):

$$m_a \geq \varepsilon_1 \tag{D.51}$$

$$T_{a4} \geq T_{a1} + \varepsilon_2 \tag{D.52}$$

$$-T_{a4} \geq -T_{wi} + \varepsilon_2 \tag{D.53}$$

$$T_{wo} \geq T_{a1} + \varepsilon_3 \tag{D.54}$$

$$-T_{wo} \geq -T_{wi} + \varepsilon_3 \tag{D.55}$$

$$T_{a4} + T_{wo} \geq T_{a1} + T_{wi} + \varepsilon_4 \tag{D.56}$$

ε_i are small constants and are used to introduce the $\geq$ sign in the feasibility inequalities. Equation (D.56) is introduced to prevent the logarithmic temperature difference of being undefined. The bounds on the operating variables are introduced to safeguard against the violation of the physical laws.

The following equations must be satisfied by the operating variables:

$$Q_a = Q_{LMTD} \tag{D.57}$$

$$Q_a = Q_w \tag{D.58}$$

$$\Delta p_{LHS} = \Delta p_{RHS} \tag{D.59}$$

D.6 POWER GENERATION

The turbo-generator-condenser characteristics of the indirect condensing system, i.e. the heat to be rejected by the condenser and the power output of the turbo-generator, are expressed in terms of the turbine back pressure, p_{tv}, or the corresponding vapor temperature, T_{tv}. The performance characteristics of an example of such a system is shown in figure D.3. It is assumed that the power needed for the auxiliaries, excluding the cooling water pumping power, is already subtracted from the generated power.

The temperature of the heated water that leaves the surface condenser is not equal to the steam temperature. The temperature difference that exists between the condensing steam and the water leaving the condenser is known as the terminal temperature difference (TTD). Typical values for TTD range from $2°C$ to $4°C$. The relation between the heat rejection rate and the TTD can be approximated as follows:

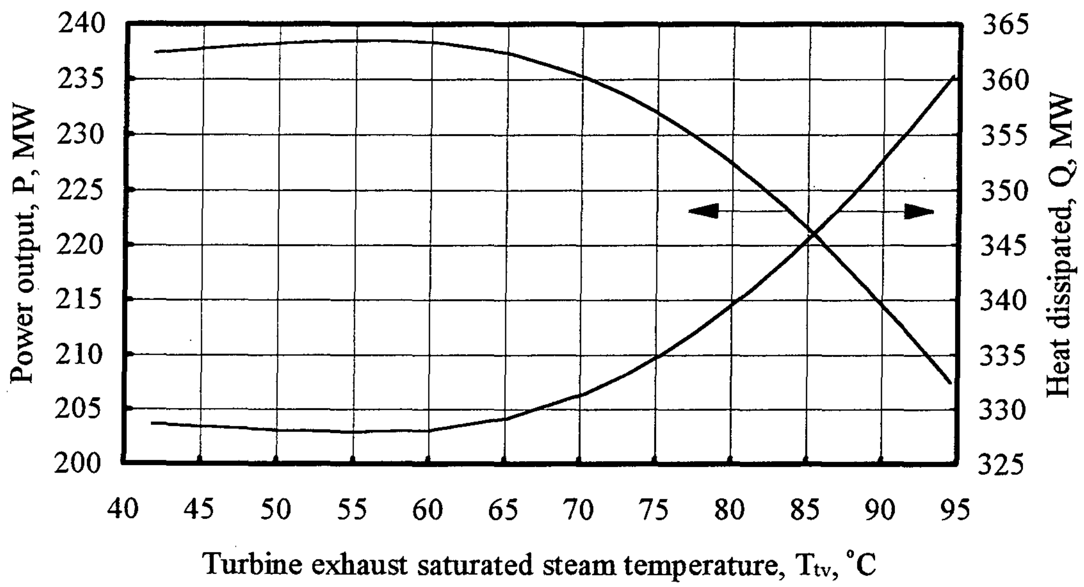

Figure D.3: Performance characteristics of a turbo-generator.

For a fixed condenser design the water mass flow rate , m_w, the heat transfer area, A, and the overall heat transfer coefficient, U, are constant. Thus,

$$\begin{aligned} Q &= UA\frac{(T_{tv} - T_{wo}) - (T_{tv} - T_{wi})}{\ell n\left[(T_{tv} - T_{wo})/(T_{tv} - T_{wi})\right]} \\ &= m_w c_{pwm}(T_{wi} - T_{wo}) \end{aligned} \tag{D.60}$$

By rearranging equation (D.60), find

$$\ell n\left[(T_{tv} - T_{wo})/(T_{tv} - T_{wi})\right] = UA/\left(m_w c_{pwm}\right) \approx \text{constant}$$

or

$$T_{tv} - T_{wo} = (T_{tv} - T_{wi})\,c_1 \tag{D.61}$$

where c_1 is a constant. Equation (D.61) can be simplified by using the equation for the water heat transfer rate, i.e.

$$T_{tv} - T_{wi} = TTD = c_2\, Q/(m_w c_{pwm}) \tag{D.62}$$

where $c_2 = -(1-c_1)^{-1}$.

For the design point, find

$$TTD_{design} = c_2\, Q_{design}/(m_w c_{pwm}) \tag{D.63}$$

For off-design conditions, find

$$TTD = c_2\, Q/(m_w c_{pwm}) \tag{D.64}$$

From equations (D.63) and (D.64), find

$$TTD \approx TTD_{design}\, Q/Q_{design} \tag{D.65}$$

With the aid of equation (D.65), the temperature of the water leaving the condenser (entering the cooling tower) can be calculated.

Atmospheric conditions influence the performance of indirect dry-cooling tower, resulting in a wide fluctuation of turbine back pressure (and the corresponding steam temperature). Changes in the ambient temperature are the most important reason for this, although other environmental effects such as wind, inversions and humidity must also be considered. The mean hourly frequency of ambient temperatures over a period of one year is normally supplied in tabulated or graphical form. An example of the frequency of the dry- and wetbulb temperatures 2m above the ground level at a particular location is shown in figure D.4. It should be noted that the air entering the dry-cooling tower may deviate considerably from these measured values and more detailed information on the actual ambient temperature distribution would be preferred for more sophisticated designs [94KR1e].

The operating point of the turbo-generator-condenser is determined by matching the operating point of the indirect dry-cooling tower and the performance characteristics of the turbo-generator-condenser (Figure D.3) at a specific ambient air temperature selected from Figure D.4. This calculation involves the selection of turbine exhaust conditions such that the heat to be rejected by the turbo-generator-condenser, equals the heat absorbed by the air (heat rejected by the dry-cooling tower), i.e.

$$Q_a = Q_w = Q_{LMTD} = Q_{tgc} \tag{D.66}$$

At this point the power requirement of the cooling water pumps as well as the generator power output are known. Subtract the total power consumed by the pump from the generator power output to find the net power output of the plant. The net power output is multiplied by the corresponding number of operating hours to give the net energy output for this period. These calculations are repeated for each of the ambient temperatures listed to obtain the total annual net energy output.

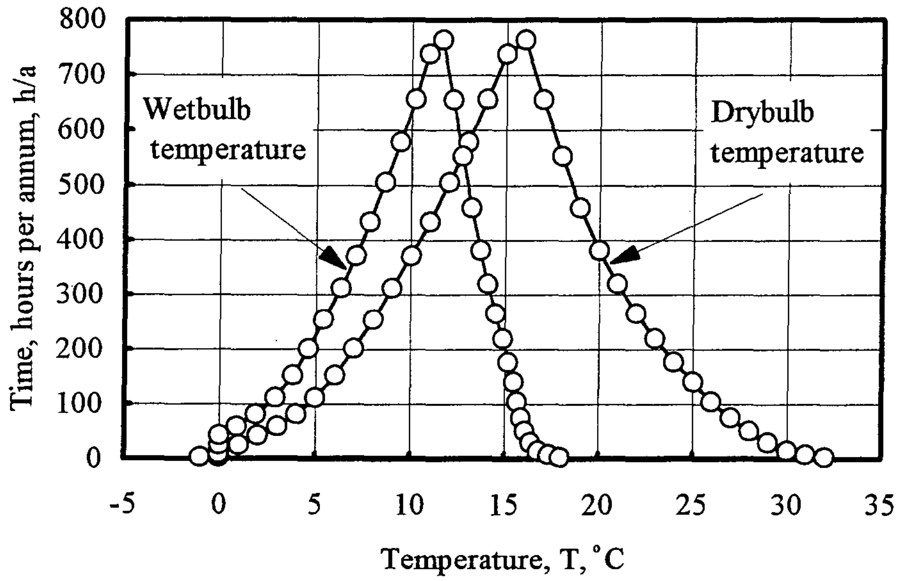

Figure D.4: Frequency of ambient dry- and wetbulb air temperatures.

In the cases when very low ambient temperatures are experienced, it is possible that the dry-cooling tower can dissipate more heat than the amount required by the turbo-generator characteristic at its bottom limit. As the turbine back pressure decreases, more power can be generated due to the more complete expansion of steam through the turbine. The dry-cooling tower will tend to lower the turbine exhaust pressure under these conditions, thus giving rise the a phenomenon called "choking" [71HA1e, 71HE1e, 91SZ1e]. No further advantages are obtained by lowering the turbine back pressure and turbine losses may become excessive due to high steam velocities. As performance of the dry-cooling tower deteriorates with increasing ambient air temperatures, the turbine back pressure increases, resulting in a decrease in power generation. The maximum allowable turbine exhaust pressure is such that flow induced vibration of the turbine blades will not occur for lower values [91SZ1e]. When reaching the upper limit of the characteristic curve, the turbine load must be reduced. Air humidification can also be used as a means to increase the cooling capacity during high ambient air temperatures [83PA1e, 91CO2e, 94KR1e].

For better part load performance it is preferable to divide circulating pump capacity between two 50%-duty units. Sometimes three 50%-duty pumps are installed so as to have a stand-by unit in emergencies; this solution makes it possible to bring in the third pump to assist the cooling effect under conditions of high ambient air temperatures [71HE1e, 87TR1e, 91SZ1e].

Definition of the operating variables, operating constraints and equations to be satisfied

For a fixed hyperbolic dry-cooling tower system geometry, a fixed turbo-generator-condenser characteristic curve and an annual frequency of ambient temperatures (at a constant p_a), the following variables, constraints and equations (balances) can be defined for the simultaneous solution of the operating point conditions during the annual operation:

The operating variables are: m_a, T_{a4}, T_{wo}, T_{tv}

These variables must satisfy the following constraints (bounds):

Equations (D.51), (D.52), (D.53), (D.54), (D.55), (D.56), as well as

$$T_{tv} \geq T_{tvl} + \varepsilon_5 \tag{D.67}$$

$$-T_{tv} \geq -T_{tvu} + \varepsilon_5 \tag{D.68}$$

The upper and lower limits on the turbine exhaust steam temperature can be obtained from the turbo-generator characteristic curve.

The following equations must be satisfied by the operating variables:

Equations (D.57), (D.58), (D.59), (D.66)

With the turbo-generator-condenser operating point known, the net power output of the plant at that specific conditions is calculated as follows:

$$P_{net} = P_{tgc} - P_w \tag{D.69}$$

For each temperature data set (consisting of a drybulb air temperature, a wetbulb air temperature and the annual duration of these temperatures), a new set of operating variables with bounds are defined and used to determine the turbo-generator-condenser operating point. The net annual energy output of the plant is thus

$$E_{net} = \sum_{i=1}^{n} P_{net(i)} \tau_{(i)} \tag{D.70}$$

where n is the number of temperature frequency data sets and τ is the duration of these temperatures.

D.7 OPERATING POINT OPTIMIZATION

Operating point optimization involves the process of finding the combination of operating and geometrical variables that will minimize the total annual cost (capital and operating) of the dry-cooling tower for specified operating conditions, specified ambient air conditions (T_{db}, T_{wb}, p_a) and specified heat transfer rate Q_{ct}, while satisfying all the imposed constraints.

Four different cases of operating point optimization calculations are investigated:

(1) Case 1: m_w and T_{wi} fixed

The mass flow rate and the temperature of the water that enters the dry-cooling tower are fixed at some specified values, while m_a, T_{a4}, and T_{wo} are variable.

(2) Case 2: T_{wi} fixed

The temperature of the water that enters the dry-cooling tower is fixed at a specified value, while m_a, T_{a4}, T_{wo} and m_w are variable..

(3) Case 3: Maintaining a constant waterside pressure drop (and pumping power) [89BU1e]

The replacement of ineffective finned tube bundles of an existing natural draft dry-cooling tower by optimally dimensioned units is investigated in this optimization study. No changes are allowed to the bundle base support, the water pipe layout, the water flow rate and the pressure drop through the heat exchanger bundles (i.e. the pumping power remains unchanged). Only round tubes with circular fins are considered in this investigation.

Equation (D.12) can be rewritten for round tubes as

$$P_w = m_w \Delta p_{fw} \xi / \left(\eta_p \eta_{em} \rho_{wm} \right) \tag{D.71}$$

Hence, if all the parameters on the right hand side of the above equation remain constant, so will P_w.

For a constant total pressure drop in a tube, find

$$\frac{f_{Dw}}{2\rho_{wm} d_i} \left[\frac{4 m_w n_{wp}}{\pi d_i^2 n_{tb} n_b} \right]^2 L_t n_{wp} = \frac{8 f_{Dw} m_w^2 n_{wp}^3 L_t}{\pi^2 \rho_{wm} d_i^5 n_{tb}^2 n_b^2} = \text{constant} = c_1 \tag{D.72}$$

Assuming that m_w, L_t, ρ_{wm}, and n_b remain constant, find

$$\frac{f_{Dw} n_{wp}^3}{d_i^5 n_{tb}^2} = \text{constant} = c_2$$

or

$$d_i = \left[\frac{f_{Dw} n_{wp}^3}{c_2 n_{tb}^2}\right]^{0.2} = d_o - 2t_t \tag{D.73}$$

The bundle base length must be kept constant, i.e.

$$W_b \sin\theta_b = \text{constant (original value)} \tag{D.74}$$

and

$$n_r P_l \cos\theta_b = \text{constant (original value)} \tag{D.75}$$

The constants c_1 and c_2 are determined from the original specifications. The optimization is performed by keeping the variables that describe the tower geometry, d_3, d_5, H_3 and H_5, as well as $m_w, T_{wi}, L_t, n_b, n_r, W_b \sin\theta_b$ and $n_r P_l \cos\theta_b$ at constant values, while m_a, T_{a4}, and T_{wo} are variable. The number of water passes, n_{wp}, can be varied in consideration with the number of tube rows, to get a realistic heat exchanger bundle configuration.

(4) <u>Case 4</u>: Fixed condenser design ($(UA)_{con}$ = constant) [89BU2e]

When the dry-cooling tower is coupled to a condenser having a specified thermal conductance, $(UA)_{con}$, the following relations can be derived for the specified cooling rate:

$$Q_{ct} = (UA)_{con} \frac{(T_{tv} - T_{wo}) - (T_{tv} - T_{wi})}{\ell n\left[(T_{tv} - T_{wo})/(T_{tv} - T_{wi})\right]} = m_w c_{pwm}(T_{wi} - T_{wo})$$

or

$$\frac{(UA)_{con}}{\ell n\left[(T_{tv} - T_{wo})/(T_{tv} - T_{wi})\right]} = m_w c_{pwm} \tag{D.76}$$

where $TTD = T_{tv} - T_{wi}$. Assume that both $(UA)_{con}$ and TTD are constant at their initial specified values. m_w, T_{wi} and T_{wo} are variable. In order to achieve the specified cooling capacity, the water mass flow rate must be calculated from:

$$m_w = \frac{(UA)_{con}/c_{pwm}}{\ell n\left[1+(T_{wi}-T_{wo})/TTD\right]} \tag{D.77}$$

Optimization variables

The optimization variables are shown in table D.1.

Table D.1: Optimization variables (✓= applicable; ×= not applicable)

Number	Variable	Round tubes with circular fins	Finned tubes with fixed geometry
1	m_a	✓	✓
2	T_{a4}	✓	✓
3	T_{wo}	✓	✓
4	m_w	✓	✓
5	T_{wi}	✓	✓
6	T_{tv}	✓	✓
7	H_3	✓	✓
8	H_5	✓	✓
9	d_3	✓	✓
10	d_5	✓	✓
11	$n_{tb(max)}$	✓	✓
12	L_t	✓	✓
13	θ_b	✓	✓
14	d_o	✓	×
15	t_t	✓	×
16	t_r	✓	×
17	d_f	✓	×
18	t_f	✓	×
19	t_g	✓	×
20	P_f	✓	×
21	P_t	✓	×

The optimization variables defined for the round tubes with circular fins use the airside performance correlations specified in Appendix B.1, while the variables defined for finned tubes with any geometry use the performance correlations defined in Appendix B.2. The operating variables are also considered as optimization variables because the dry-cooling tower model is included as equality and inequality constraints (balances and bounds) in the minimization problem formulation (integrated approach). The geometrical variables (numbers 7-21) can be kept constant or varied during the optimization process.

Objective function

The objective function requires that the total annual cost be minimized. The construction of the objective function from its various capital and operating cost components is discussed in Appendix E. The total annual cost is

$$C_{total} = C_{operating} + C_{maintenance} + C_{FCR} \ (\$/annum) \tag{D.78}$$

Constraints

The following <u>equality constraints</u> must always be satisfied:

Equations (D.57), (D.58), (D.59) as well as

$$Q_a = Q_{ct} = Q_w = Q_{LMTD} \tag{D.79}$$

The following <u>inequality constraints</u> must always be satisfied:

Equations (D.51), (D.52), (D.53), (D.54), (D.55), (D.56)

To prevent cold inflow, equation (D.50) is also a prescribed constraint. The velocity of the water flowing inside the tube can be specified as a fixed value and thus presented as an equality constraint, i.e.

$$v_w = v_{wfix} \tag{D.80}$$

In order to prevent fouling caused by a low water velocity inside the tubes, a lower velocity limit can also be specified and formulated by means of an inequality constant, i.e.

$$v_w \geq v_{wl} \tag{D.81}$$

Considering case 2, the following limit is introduced:

$$m_w \geq \varepsilon_6 \tag{D.82}$$

Considering case 3, the following equality constraints are introduced:

Equations (D.73), (D.74), (D.75)

Considering case 4, the following limits are introduced:

$$T_{wi} \geq T_{wil} + \varepsilon_7 \tag{D.83}$$

$$-T_{wi} \geq -T_{wiu} + \varepsilon_7 \tag{D.84}$$

The following <u>general inequality constraints</u> (geometrical constraints) must be satisfied, depending on whether or not the relevant optimization variables are varied during the optimization process:

The relationship between the hyperbolic cooling tower dimensions are limited to the practical values stated in section D.4.

$$H_3/H_5 \geq 0.05 \tag{D.85}$$

$$-H_3/H_5 \geq -0.15 \tag{D.86}$$

$$d_3/(H_5 - H_3) \geq 0.64 \tag{D.87}$$

$$-d_3/(H_5 - H_3) \geq -1.13 \tag{D.88}$$

$$d_3/d_5 \geq 1.37 \tag{D.89}$$

$$-d_3/d_5 \geq -1.77 \tag{D.90}$$

The heat exchanger bundle width is limited by transport requirements.

$$W_{bu} - W_b \geq 0 \tag{D.91}$$

The finned tube length is limited by structural, transport and construction requirements.

$$L_{tu} - L_t \geq 0 \tag{D.92}$$

The bundle semi-apex angle cannot exceed $90°$.

$$90° - \theta_b \geq 0 \tag{D.93}$$

For round tubes with circular fins, the following inequality constraints are imposed (refer to Appendix B.1):

The finned tubes must be kept from touching.

$$P_t - d_f \geq 0 \tag{D.94}$$

The fin diameter must exceed the fin root diameter (extended airside surface area is required).

$$d_f - d_r \geq 0 \tag{D.95}$$

Due to structural considerations, the fin root thickness and the tube thickness must exceed their lower limits.

$$t_r - t_{rl} \geq 0 \tag{D.96}$$

$$t_t - t_{tl} \geq 0 \tag{D.97}$$

For effective cleaning and where airside fouling is of significance, the fin pitch must exceed the imposed lower limit.

$$P_f - P_{fl} \geq 0 \tag{D.98}$$

The galvanizing thickness must lie within practical limits.

$$t_g - t_{gl} \geq 0 \tag{D.99}$$

$$t_{gu} - t_g \geq 0 \tag{D.100}$$

The fin thickness is limited by the manufacturing processes.

$$t_f - t_{fl} \geq 0 \tag{D.101}$$

The fin pitch must exceed the fin thickness.

$$P_f - t_f \geq 0 \tag{D.102}$$

The application limits of the finned tube performance correlations, as discussed in Appendix B (section B.1), are not used as constraints during the optimization of circular finned tubes.

The following assumptions and relations are used to obtain values for the geometrical parameters linked to the optimization variables:

The number of finned tube bundles is obtained from

$$n_b = \alpha_{ct}\pi d_3^2 \big/ \left[4L_t\left(W_b \sin\theta_b + n_r P_l \cos\theta_b\right)\right]$$

where α_{ct} is the fraction of the tower area covered by heat exchanger bundles.

$n_{tb} = n_{tb(max)} n_r - n_r/2$ (total number of tubes per bundle)

$W_b = n_{tb(max)} P_t$ (bundle width)

$P_l = 0.866\, P_t$ (longitudinal tube pitch for circular finned tubes)

$d_i = d_o - 2t_t$ (round tube inside diameter)

$d_r = d_o + 2t_r$ (root diameter of circular finned tube)

The following parameters are assumed to be constant during the optimization process:

$\phi_b, C_{Dts}, d_{ts}, n_{ts}, t_{cts}, \alpha_{ct}, n_r, \varepsilon_f, \xi$

D.8 MINIMIZATION OF POWER GENERATION COST

The minimization of the power generation cost attributed to the dry-cooling tower performance for the given temperature frequency data set, involves the variation of the dry-cooling tower operating and geometrical variables that will minimize the ratio of its total annual cost to the annual net power output of the turbo-generator-condenser set it is coupled to. The calculated values of the different variables must satisfy all the relevant constraints.

Optimization variables

The optimization variables are defined in Table D.1. Different operating variables (numbers 1,2,3 and 6) are defined for each temperature data set and they are always present during optimization. The water mass flow rate remains constant and the cooling tower inlet water temperature, T_{wi}, is calculated from equations (D.62) and (D.65). The other geometrical variables can be varied or remain constant.

Objective function

The objective function requires that the ratio of the total annual cost of the dry-cooling tower to the net energy produced by the turbo-generator be minimized, i.e.

$$C_{power} = C_{total}/E_{net} \quad (\$/kWh) \tag{D.103}$$

The construction of the total annual cost from its various cost components is discussed in Appendix E, while the topic of power generation is treated in section D.6.

Constraints

The following equality constraints must always be satisfied:

Equations (D.57), (D.58), (D.59), (D.66)

The following inequality constraints must always be satisfied:

Equations (D.50), (D.51), (D.52), (D.53), (D.54), (D.55), (D.56), (D.67), (D.68), (D.81)

The following general inequality constraints (geometrical constraints) must be satisfied, depending on whether or not the relevant optimization variables are varied during the optimization process:

Equations (D.85), (D.86), (D.87), (D.88), (D.89), (D.90), (D.91), (D.92), (D.93), (D.94), (D.95), (D.96), (D.97), (D.98), (D.99), (D.100), (D.101), (D.102)

For each temperature data set there is a corresponding set of operating variables, equality and inequality constraints (operating constraints). These constraint sets must always be satisfied. The operating variables of a specific temperature data set effect the operating constraints belonging to the same temperature data set only. The operating variables do not effect the geometrical constraints. However, the geometrical variables effect all the operating constraints of the different temperature data sets.

APPENDIX E

DRY–COOLING SYSTEM COST ESTIMATION

E.1 INTRODUCTION

In this appendix the cost components needed to define the objective functions are described. The objective functions to be minimized are the annual cost of the dry-cooling system under prescribed conditions or the annual cost of electricity production that is attributed to the dry-cooling system when coupled to a turbo-generator unit. Capital and operating cost estimation methods, based on the methods discussed in Chapter 4, are proposed for both forced draft and natural draft dry-cooling systems.

E.2 COOLING SYSTEM CAPITAL COST ESTIMATION

Heat exchanger bundle cost

(1) Tube cost per unit tube length, C_t

$$C_t = C_{tm}M_t + C_{tu} \quad (\$/m) \tag{E.1}$$

M_t is the tube mass per unit tube length (kg/m)

C_{tu} is the tube added unit cost ($/m)

C_{tm} is the tube material cost ($/kg)

(2) Cost of fins per unit tube length, C_f

$$C_f = C_{fm}M_f + C_{fu}L_{fb} \quad (\$/m) \tag{E.2}$$

C_{fu} is the fin material added unit cost per unit tube length ($/m)

C_{fm} is the fin material cost ($/kg)

M_f is the fin material mass per unit tube length (kg/m)

L_{fb} is the length of fin strip per unit tube length (m/m)

(3) Surface coating cost per unit tube length, C_{sc}

$$C_{sc} = C_{scm}M_{sc} + C_{scu}A_a \quad (\$/m) \tag{E.3}$$

C_{scm} is the surface coating material unit cost ($/kg)

C_{scu} is the surface coating added unit cost ($\$/m^2$)

M_{sc} is the mass of the surface coating material per unit tube length (kg/m)

A_a is the air side surface area per unit tube length (m^2/m)

(4) Total finned tube cost per unit tube length, C_{ft}

$$C_{ft} = (C_t + C_f + C_{sc})W_{ft} \quad (\$/m) \tag{E.4}$$

W_{ft} is the finned tube cost weighting factor

(5) Heat exchanger bundle header and frame cost, C_h

$$C_h = C_{ft}L_t n_{tb} W_h \quad (\$/\text{bundle}) \tag{E.5}$$

W_h is the header and frame cost weighting factor

(6) Bundle assembly cost, C_{ba}

$$C_{ba} = C_{bat} n_{tb} \quad (\$/\text{bundle}) \tag{E.6}$$

C_{bat} is the assembly cost per finned tube ($/finned tube)

(7) Total heat exchanger bundle cost, C_{he}

$$C_{he} = (C_{ft}L_t n_{tb} + C_h + C_{ba})W_{he} \quad (\$/\text{bundle}) \tag{E.7}$$

W_{he} is the heat exchanger bundle cost weighting factor (installation)

Fan system capital cost

It is assumed that $d_{Fc} \approx 1.005 d_F$, $d_{Fh} \approx 0.165 d_F$, the height of the fan casing is $0.1 d_{Fc}$, the inlet bell height is $0.15 d_{Fc}$ and the inlet bell diameter is $1.2 d_{Fc}$.

(1) Fan capital cost, C_F

$$C_F = C_{Fu}A_F + C_{Ff} \quad (\$/\text{fan}) \tag{E.8}$$

C_{Fu} is the fan unit cost ($\$/m^2$)

C_{Ff} is the fan fixed cost ($/fan)

A_F is the fan sweep area (m^2)

(2) Fan casing and inlet bell cost, C_{Fc}

$$C_{Fc} = \left(A_{Fc} + A_{Fb}\right)C_{Fcb} \quad (\$/\text{fan}) \tag{E.9}$$

C_{Fcb} is the fan casing and inlet bell unit cost ($\$/m^2$)

A_{Fc} is the fan casing circumference area (m^2)

A_{Fb} is the inlet bell circumference area, approximated by a conical frustum (m^2)

$$A_{Fc} = 0.1\pi d_{Fc}^2 \quad (m^2)$$

$$A_{Fb} \approx 1.1\pi d_{Fc}\sqrt{0.0225 d_{Fc}^2 - 0.01 d_{Fc}^2} = 0.122984\pi d_{Fc}^2 \quad (m^2)$$

(3) Fan safety screen cost, C_{Fs}

$$C_{Fs} = C_{Fss}A_{Fs} \quad (\$/\text{fan}) \tag{E.10}$$

C_{Fss} is the safety screen cost ($\$/m^2$)

A_{Fs} is the safety screen area (m^2)

$$A_{Fs} = 0.25\pi\left(1.2 d_{Fc}\right)^2 = 0.36\pi d_{Fc}^2 \quad (m^2)$$

(4) Electric motor cost, C_{em}

$$C_{em} = C_{emf} + s_{em}C_{emu}P_e \quad (\$/\text{motor}) \tag{E.11}$$

C_{emf} is the electric motor fixed cost (\$/motor)

C_{emu} is the electric motor unit cost (\$/kW)

P_e is the electric power input to the electric motor (kW)

A safety factor, s_{em}, against electric motor undersizing is also introduced.

(5) Speed reducer cost, C_{sr}

$$C_{sr} = \left(C_F + C_{em}\right)W_{sr} \quad (\$/\text{speed reducer}) \tag{E.12}$$

W_{sr} is the speed reducer cost weighting factor

(6) Electric wiring and switching cost, C_{ws}

$$C_{ws} = C_{em}W_{ws} \quad (\$/\text{fan}) \tag{E.13}$$

W_{ws} is the wiring and switching cost weighting factor

(7) Total fan system cost, C_{Ft}

$$C_{Ft} = \left(C_F + C_{Fc} + C_{Fs} + C_{em} + C_{sr} + C_{ws}\right)W_F \quad (\$/\text{fan}) \tag{E.14}$$

W_F is the fan system cost weighting factor

Forced draft cooling tower structure

(1) Cost of land, excavation, foundations and construction, C_c

$$C_c = \left(C_l + C_{cu}H_3 + C_{Fpl}\right)A_{Fpl}W_c \quad (\$) \tag{E.15}$$

C_l is the land, excavation and foundation cost ($\$/m^2$)

C_{cu} is the construction unit cost ($\$/m^3$)

C_{Fpl} is the fan platform cost ($\$/m^2$)

A_{Fpl} is the fan platform area (m^2)

W_c is the construction cost weighting factor

Natural draft cooling tower shell (Hyperbolic reinforced concrete)

The cooling tower shell will be approximated by means of two conical frustums, one above and the other below the throat. A constant shell thickness is assumed. The volume of the conical frustum can be calculated as follows:

$$V_{cf} = 0.083333\pi\left(H_5 - H_t\right)\left(d_t^2 + d_t d_5 + d_5^2\right) + 0.083333\pi\left(H_t - H_3\right)\left(d_t^2 + d_t d_3 + d_3^2\right) \tag{E.16}$$

with $H_5 - H_t = 0.25\left(H_5 - H_3\right)$, $H_t - H_3 = 0.75\left(H_5 - H_3\right)$ and $d_5 = 1.05\,d_t$. These relationships are typical in most of the existing hyperbolic cooling towers found in practice. The volume of concrete in the tower shell can be calculated with the aid of equation (E.16). It is further assumed that the tower base angle, ϕ_b, is fixed at 70° (which is the minimum limit for construction [84AL1e]) and that the tower support length can be expressed as $L_{ts} = H_3/\sin\left(\phi_b\right)$

(1) Cost of land, excavation and foundation, C_l

$$C_l = C_{lu}A_b \quad (\$) \tag{E.17}$$

C_{lu} is the land, excavation and foundation cost ($\$/m^2$)

A_b is the tower base area (m^2)

$$A_b = 0.25\pi\left(d_3 + 2H_3/\tan\phi_b\right)^2 \quad \text{(E.18)}$$

(2) Cost of the tower shell, C_{ct}

$$C_{ct} = C_{ctc}V_{ts} \quad (\$) \quad \text{(E.19)}$$

C_{ctc} is the cost of the reinforced concrete used in the shell ($\$/m^3$)

V_{ts} is the volume of reinforced concrete in the tower shell (m^3)

(3) Cost of the heat exchanger bundle platform, C_{hepl}

$$C_{hepl} = C_{plu}A_{hepl} \quad (\$) \quad \text{(E.20)}$$

C_{plu} is the platform unit cost ($\$/m^2$)

A_{hepl} is the heat exchanger platform area (m^2)

(4) Cost of the tower supports, C_{ts}

$$C_{ts} = C_{tsu}L_{ts}d_{ts}n_{ts} \quad (\$) \quad \text{(E.21)}$$

C_{tsu} is the tower support unit cost ($\$/m^3$)

(5) Total construction cost, C_c

$$C_c = \left(C_l + C_{ct} + C_{hepl} + C_{ts}\right)W_c \quad (\$) \quad \text{(E.22)}$$

W_c is the structural maintenance cost weighting factor

Steam/condensate distribution system costs (ACC)

(1) Distribution system cost, C_{sd}

$$C_{sd} = \left(C_{he} + C_{Ft}\right)W_{sd} \quad (\$) \quad \text{(E.23)}$$

W_{sd} is the steam/condensate distribution system cost weighting factor

Circulation system costs

Two 50%-duty pump units will be considered in this analysis.

(1) Pump capital cost, C_{pump}

$$C_{pump} = 2\left(C_{pf} + C_{pu}P_w\right) \quad (\$) \quad \text{(E.24)}$$

C_{pf} is the pump fixed cost ($/pump)

C_{pu} is the pump unit cost ($/kW)

P_w is the required pumping power (kW)

(2) Electric motor cost, C_{em}

$$C_{em} = 2\left(C_{emf} + s_{em} C_{emu} P_w\right) \quad (\$) \tag{E.25}$$

C_{emf} is the electric motor fixed cost ($/motor)

C_{emu} is the electric motor unit cost ($/kW)

A safety factor, s_{em}, against electric motor undersizing is also introduced.

(3) Electric wiring and switching cost, C_{ws}

$$C_{ws} = C_{em} W_{ws} \quad (\$) \tag{E.26}$$

W_{ws} is the wiring and switching cost weighting factor

(4) Total pump system cost, C_{pst}

$$C_{pst} = \left(C_{pump} + C_{em} + C_{ws}\right) W_{ps} \quad (\$) \tag{E.27}$$

W_{ps} is the pump system cost weighting factor

(5) Piping and valves cost, C_{pv}

$$C_{pv} = C_{he} n_b W_{pv} \quad (\$) \tag{E.28}$$

W_{pv} is the piping and valves cost weighting factor

E.3 COOLING SYSTEM OPERATION COST ESTIMATION

Fuel cost

The total annual fuel cost , C_{ft} ,can be expressed as

$$C_{ft} = C_{fav} \eta_{plant}^{-1} P_g \tau \tag{E.29}$$

η_{plant} is the overall plant efficiency

P_g is the gross power output of the turbo-generator (MW)

C_{fav}, levelized unit cost of the fuel ($/MJ)

τ is the number of operating hours (h)

Fan operating cost

The annual operating cost of the fan, C_{Fo}, can be expressed as

$$C_{Fo} = C_{eav} P_e \tau \tag{E.30}$$

C_{eav} levelized electricity cost (\$/kWh)

P_e is the input power to the fans (kW)

τ is the number of operating hours (h)

Pump operating cost

The annual operating cost of the pump, C_{po}, can be expressed as

$$C_{po} = P_e C_{eav} \tau \tag{E.31}$$

C_{eav} levelized electricity cost (\$/kWh)

P_e is the input power to the pumps (kW)

τ is the number of operating hours (h)

Fixed charges

The cost of the fixed charges, C_{FCR}, can be expressed as

$$C_{FCR} = FCR\, C_{capital} \tag{E.32}$$

FCR is the levelized fixed charge rate

$C_{capital}$ is the total capital cost (\$)

Maintenance costs

The maintenance cost weighting factors are assumed to be levelized values over the operating lifetime of the plant.

(1) Heat exchanger bundle maintenance cost, C_{hem}

$$C_{hem} = W_{hem} C_{he} \quad (\$/\text{bundle}) \tag{E.33}$$

W_{hem} is the heat exchanger bundle maintenance cost weighting factor

(2) Fan system maintenance cost, C_{Fm}

$$C_{Fm} = C_{Ft} W_{Fm} \quad (\$/\text{fan}) \tag{E.34}$$

W_{Fm} is the fan system maintenance cost weighting factor

(3) Structural maintenance cost, C_{cm}

$$C_{cm} = C_c W_{cm} \quad (\$) \tag{E.35}$$

W_{cm} is the structural maintenance cost weighting factor

(4) Distribution system maintenance cost, C_{sdm}

$$C_{sdm} = C_{sd} W_{sdm} \quad (\$) \tag{E.36}$$

W_{sdm} is the steam/condensate distribution system maintenance cost weighting factor

(5) Pump system maintenance cost, C_{pm}

$$C_{pm} = C_{pst} W_{pm} \quad (\$) \tag{E.37}$$

W_{pm} is the pump system maintenance cost weighting factor

(6) Piping maintenance cost, C_{pvm}

$$C_{pvm} = C_{pv} W_{pvm} \quad (\$) \tag{E.38}$$

W_{pvm} is the piping and valves maintenance cost weighting factor

E.4 TOTAL COSTS

The total annual cost of the cooling system is:

$$C_{total} = C_{operating} + C_{maintenance} + C_{FCR} \quad (\$/\text{annum}) \tag{E.39}$$

The cost of power generation (attributed to the cooling system) is:

$$C_{power} = \frac{C_{total}}{E_{net}} \quad (\$/\text{kWh}) \tag{E.40}$$

where E_{net} is the net annual power output.

Equations (E.39) and (E.40) present the required objective functions for the operating point optimization and the minimization of power generation cost respectively.

APPENDIX F

SOLUTION OF SYSTEMS OF NONLINEAR EQUATIONS BY USING A CONSTRAINED NONLINEAR LEAST SQUARES APPROACH

F.1 INTRODUCTION

The solution methods for unconstrained optimization and the nonlinear equations problem (simultaneous solution of n nonlinear equations in n unknowns) are closely related. Newton's method is the basic method for solving both these types of problems [83DE1m, 89DE1m]. A significant percentage of real-world unconstrained optimization problems in science and engineering arise from fitting model functions to data, i.e. nonlinear parameter estimation. Usually the number of data points, m, is greater than the number of independent variables, n, and it is not possible to obtain an exact solution. This gives rise to a special class of unconstrained optimization problems with a special structure, the so-called nonlinear least squares (NLS) problems where the objective function is a sum of squares.

In this section we consider the special case of the NLS problem where m = n. Such problems can be viewed as an attempt to obtain the simultaneous solution of n nonlinear equations in n unknowns. A globally convergent solution technique (convergence to a local minimizer from a poor starting point) for these problems will be explained. The problem is extended to include inequality constraints and bounds on the variables. The implementation of the modified NLS problem is presented by stating its algorithmic representation.

F.2 THE NONLINEAR LEAST SQUARES PROBLEM

If **f(x)** is the sum of squares of nonlinear functions, the objective function of the nonlinear least squares problem is written as [74LA1m, 81GI1m, 83DE1m, 87FL1m, 89DE1m]

$$\tilde{f}(\mathbf{x}) = 0.5\sum_{i=1}^{m} f_i(\mathbf{x})^2 = 0.5\,\mathbf{f}(\mathbf{x})^T\mathbf{f}(\mathbf{x}) \tag{F.1}$$

where $\mathbf{x}$ is an n-vector. $f_i(\mathbf{x}) = 0$, $i = 1,\ldots,m$ can be viewed as a system of m nonlinear equations. When m > n, least squares solutions to over-determined systems of equations are computed by minimizing expression (F.1).

The derivatives of the objective function are given by

$$\mathbf{g}(\mathbf{x}) = \mathbf{J}(\mathbf{x})\mathbf{f}(\mathbf{x}) \tag{F.2}$$

and

$$\mathbf{G}(\mathbf{x}) = \mathbf{J}(\mathbf{x})\mathbf{J}(\mathbf{x})^T + \sum_{i=1}^{m} f_i(\mathbf{x})\nabla^2 f_i(\mathbf{x}) \tag{F.3}$$

where $\mathbf{J}(\mathbf{x})$ is the n×m Jacobian matrix, the columns of which are the first derivative vectors ∇f_i of the components of f. We observe that the Hessian of the NLS objective function consists of a combination of first- and second-order information. Most solution methods assume that the second term on the right hand side of equation (F.3) is small compared to the first term, and it is simply omitted. The resulting problem is referred to as a small residual problem. Large residual problems do exist, but will not be considered in this analysis (refer to [81GI1m, 83DE1m, 89DE1m] for more information). Note that the linear least squares and zero residual problem is a special case of the above formulation.

A good approximation to the Hessian of the objective function, **G**(**x**), can be obtained by assuming that the residual is small, i.e.

$$\mathbf{G}(\mathbf{x}) \approx \mathbf{J}(\mathbf{x})\mathbf{J}(\mathbf{x})^T \tag{F.4}$$

Special purpose NLS solution techniques make use of the special problem structure. For example, equations (F.1), (F.2) and (F.4) can be used with Newton's basic method or with a quasi-Newton method. Whereas a quasi-Newton method might take n iterations to estimate **G**(**x**) satisfactorily, here the approximation is immediately available. The basic Newton method (equation (2.21)) becomes the Gauss-Newton method when (F.4) is used to approximate $\mathbf{G}(\mathbf{x}^{(k)})$. The Gauss-Newton direction, $\mathbf{s}^{(k)}$, is based on the minimization of the following quadratic model at iteration (k):

$$\underset{\mathbf{s}}{\text{Minimize}} \quad 0.5\,\mathbf{s}^{(k)T}\mathbf{G}\left(\mathbf{x}^{(k)}\right)\mathbf{s}^{(k)} + \mathbf{g}\left(\mathbf{x}^{(k)}\right)^T\mathbf{s}^{(k)} + \mathbf{f}\left(\mathbf{x}^{(k)}\right)^T\mathbf{f}\left(\mathbf{x}^{(k)}\right) \tag{F.5}$$

The k-th iteration of the basic Gauss-Newton method can thus be written as

$$\begin{aligned} &\text{Solve} \quad \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^T\mathbf{s}^{(k)} + \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right) = 0 \quad \text{for } \mathbf{s}^{(k)} \\ &\text{Set} \quad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{s}^{(k)} \end{aligned} \tag{F.6}$$

Gauss-Newton methods perform better on zero and small residual problems that are not too nonlinear than on large residual problems. Convergence of this method is slowed as the problem nonlinearity or the relative residual size increases; if either of these is too large, the method may not converge at all [83DE1m]. The Gauss-Newton method is also not necessarily globally convergent.

The Gauss-Newton method's global convergence and performance characteristics can be improved in two ways: using it with a line search or with a trust region strategy [83DE1m, 87FL1m]. The basic idea of both line search and trust region methods, is that they use the properties of the quickly convergent local methods (e.g. Newton's method) when the estimated solution is close to a minimizer, and when a solution estimate lies outside the convergence region of these methods, some reliable approach is used that gets them closer to the region where the local methods will work.

The k-th iteration of the Gauss-Newton method that incorporates a line search is simply

$$\begin{aligned}&\text{Solve}\quad \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^{T}\mathbf{s}^{(k)}+\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right)=0\quad \text{for } \mathbf{s}^{(k)}\\&\text{Set}\quad \mathbf{x}^{(k+1)}=\mathbf{x}^{(k)}+\alpha^{(k)}\mathbf{s}^{(k)}\end{aligned} \tag{F.7}$$

where the step length $\alpha^{(k)}$ is chosen by a line search procedure (see Dennis and Schnabel [83DE1m] or Fletcher [87FL1m]). We will refer to equation (F.7) as the damped Gauss-Newton method.

The idea of a line search algorithm is simply to calculate an acceptable point $\mathbf{x}^{(k+1)}$, given a descent direction, $\mathbf{s}^{(k)}$, and a positive step length $\alpha^{(k)}$. High accuracy line searches are very expensive to carry out and researchers have developed some conditions for low accuracy line searches while still obtaining global convergence [83DE1m, 87FL1m]. They have shown that line search methods can be globally convergent if each step size satisfies two simple conditions:

(1) The average rate of decrease from $\tilde{f}\left(\mathbf{x}^{(k)}\right)$ to $\tilde{f}\left(\mathbf{x}^{(k+1)}\right)$ must be at least some prescribed fraction of the initial rate of decrease in that direction; i.e. we pick an $\lambda \in (0, 0.5)$ that satisfies

$$\tilde{f}\left(\mathbf{x}^{(k+1)}\right)\le\tilde{f}\left(\mathbf{x}^{(k)}\right)+\alpha^{(k)}\lambda\,\mathbf{g}\left(\mathbf{x}^{(k)}\right)^{T}\mathbf{s}^{(k)} \tag{F.8}$$

(2) The steps must not be too short, i.e. we pick a $\beta \in (\lambda, 1)$ that satisfies

$$\mathbf{g}\left(\mathbf{x}^{(k+1)}\right)^{T}\mathbf{s}^{(k)}\ge\beta\,\mathbf{g}\left(\mathbf{x}^{(k)}\right)^{T}\mathbf{s}^{(k)} \tag{F.9}$$

Typical line search algorithms set $\lambda = 10^{-4}$ and β between 0.7 and 0.9 [83DE1m, 89DE1m]. These two conditions, when incorporated into a line search algorithm, lead to a practical and globally convergent method. In practice, equation (F.9) is generally not needed, because backtracking will avoid small steps being taken.

The common procedure is to try and use $\alpha^{(k)} = 1.0$ as the first step, and when it fails, backtracking is performed in a systematic way along the direction of search until an acceptable $\mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}$ is found. The backtracking strategy as well as line search algorithms that employ the concepts above are described in Dennis and Schnabel [83DE1m] and Fletcher [87FL1m].

Since the damped Gauss-Newton method always takes descent steps that satisfy the line search criteria, it is usually globally convergent on almost all NLS problems, including large residual and very nonlinear problems. However, it may still be slowly convergent or fail on the problems that the Gauss-Newton method had trouble with.

The other modification to the Gauss-Newton method is the use of globally convergent trust region strategies by solving the system (refer to chapter 2, section 2.2)

$$\left[\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^{T} + \lambda^{(k)}\mathbf{I}\right]\mathbf{s}^{(k)} + \mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right) = \mathbf{0} \ , \quad \lambda^{(k)} \geq 0 \tag{F.10}$$

This formulation was first suggested by Levenberg [44LE1m] and Marquardt [63MA1m] and is known as the Levenberg-Marquardt method. $\lambda^{(k)}$ is changed during the course of the iterations in order for $\mathbf{J}(\mathbf{x}^{(k)})\mathbf{J}(\mathbf{x}^{(k)})^{T} + \lambda^{(k)}\mathbf{I}$ to remain positive definite. Many versions of this algorithm do exist; some control the iterations using $\lambda^{(k)}$ directly, while others use the radius $h^{(k)}$ and choose $\lambda^{(k)}$ to satisfy $\left\|\mathbf{s}^{(k)}\right\| \leq h^{(k)}$ [83DE1m]. The local convergence properties of the Levenberg-Marquardt method are similar to those of the Gauss-Newton method. The Levenberg-Marquardt method may still find difficulty in solving some large residual or very nonlinear problems. There are, however, certain factors that make Levenberg-Marquardt methods preferable to damped Gauss-Newton methods on many problems, e.g. when the Gauss-Newton step is too long, the Levenberg-Marquardt step is close to being the steepest descent direction, and is often superior to the damped Gauss-Newton step [83DE1m, 89DE1m].

When m = n in the above analysis, the NLS problem can also be interpreted as the simultaneous solution of n nonlinear equations in n unknowns, i.e. $f_i(\mathbf{x}) = 0, \quad i = 1,\ldots,n$. The minimum of the well-determined NLS problem corresponds to the solution of the system of n nonlinear equations in n unknowns. The gradient of the NLS objective function vanishes when $\mathbf{f}(\mathbf{x})$ is zero. The Newton-Raphson method that is usually used to solve systems of nonlinear equations is equivalent to the basic Gauss-Newton method [81GI1m, 83DE1m]. Global strategies for solving systems of nonlinear equations are obtained from the global strategies of unconstrained optimization. Thus, the line search or trust region strategies can be used to ensure global convergence of this solution technique.

F.3 SIMULTANEOUS SOLUTION OF A SYSTEM OF NONLINEAR EQUATIONS

We have shown that exploiting the special form of the NLS problem can lead to substantial improvements over using standard solution methods for unconstrained minimization problems. Furthermore, this method can also be used to solve systems of nonlinear equations due to its efficiency. In this section a procedure based on the Levenberg-Marquardt method will be implemented to solve the system of nonlinear equations subject to linear inequality constraints.

The problem to be solved is

$$\begin{aligned}&\text{Solve} \quad f_j(\mathbf{x}) = 0, \quad j = 1,\dots,n\\&\text{subject to the constraints}\\&\mathbf{a}_i^T\mathbf{x} \geq b_i, \quad i = 1,\dots,m\end{aligned} \tag{F.11}$$

where the constraints are feasibility inequalities, i.e. lower and upper bounds on the variables as well as linear relationships between the variables. These constraints are not expected to be satisfied as equalities at the solution, but rather define the feasible region for the iterative solution method. Problem (F.11) can also be interpreted as a minimization of a NLS problem where the minimum of the objective function is known to be equal to zero, i.e.

$$\begin{aligned}&\text{Solve} \quad \tilde{f}(\mathbf{x}) = 0.5\sum_{j=1}^{n} f_j(\mathbf{x})^2 = 0.5\,\mathbf{f}(\mathbf{x})^T\mathbf{f}(\mathbf{x}) = 0\\&\text{subject to the constraints}\\&\mathbf{a}_i^T\mathbf{x} \geq b_i, \quad i = 1,\dots,m\end{aligned} \tag{F.12}$$

Holt and Fletcher [79HO1m] describe an algorithm for constrained NLS problem with special constraints for specific application in data fitting problems.

The Levenberg-Marquardt step is obtained by solving the system of equations stated in (F.10). A quadratic function of the following form presents a solution to this system of equations subject to the constraints:

$$\begin{aligned}&\underset{\mathbf{s}}{\text{Minimize}} \quad 0.5\,\mathbf{s}^{(k)T}\left[\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^T + \lambda^{(k)}\mathbf{I}\right]\mathbf{s}^{(k)} + \left(\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right)\right)^T\mathbf{s}^{(k)}\\&\text{subject to the linear inequality constraints}\\&\mathbf{a}_i^T\mathbf{s}^{(k)} \geq b_i - \mathbf{a}_i^T\mathbf{x}^{(k)}, \quad i = 1,\dots,m\end{aligned} \tag{F.13}$$

The NLS problem is thus rewritten as a quadratic programming (QP) problem that can deal with the imposed linear inequality constraints. This QP problem can be solved by means of any active set method, although none of the constraints will be active at the solution. We use the dual active set QP algorithm explained in Appendix G.

We shall form the approximation to the Hessian matrix of the objective function, $\mathbf{J}(\mathbf{x}^{(k)})\mathbf{J}(\mathbf{x}^{(k)})^T + \lambda^{(k)}\mathbf{I}$, directly and not by means of QR factorizations, because the problems to be considered are not large-dimensional (typically $n \leq 10$). QR factorizations can result in considerable savings if calculations with several values of $\lambda^{(k)}$ are required and also introduce computational stability (refer to [87FL1m]).

The Levenberg-Marquardt method uses the trust region strategy to obtain global convergence. The parameter $\lambda^{(k)}$ is decreased by a constant factor if a suitable search direction $\mathbf{s}^{(k)}$ is found which satisfies the following condition [83DE1m]

$$\tilde{f}\left(\mathbf{x}^{(k+1)}\right) = \tilde{f}\left(\mathbf{x}^{(k)} + \mathbf{s}^{(k)}\right) \leq \tilde{f}\left(\mathbf{x}^{(k)}\right) + 10^{-4}\mathbf{g}\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} \tag{F.14}$$

Otherwise $\lambda^{(k)}$ is increased by some factor and inner iterations are performed with this value. Subsequent inner iterations with consequent further increases in $\lambda^{(k)}$ may follow until condition (F.14) is satisfied. However, under certain conditions, $\lambda^{(k)}$ can become very large, resulting in slow convergence or no convergence at all (steepest descent search direction). In these cases, we introduce a line search procedure to find a new point $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}$ that satisfies equation (F.14). Previously, either the damped Gauss-Newton method or the Levenberg-Marquardt method was used to overcome some of the difficulties in the basic Gauss-Newton method. Here, we use a combination of both improvement schemes to obtain a reliable solution method for a system of nonlinear equations subject to linear inequality constraints.

The line search procedure using a backtracking procedure, as implemented by Dennis and Schnabel [83DE1m], is used. The line search fails if a point $\mathbf{x}^{(k+1)}$ sufficiently distinct from $\mathbf{x}^{(k)}$ cannot be found. A user-supplied maximum step length is also imposed to ensure feasibility in the domain of interest.

Due to the presence of constraints, the largest step size $\overline{\alpha}^{(k)}$ to retain feasibility during iteration k is obtained from solving [87FL1m]

$$\overline{\alpha}^{(k)} = \min\left(\min_{\mathbf{a}_i^T \mathbf{s}^{(k)} < 0} \frac{b_i - \mathbf{a}_i^T \mathbf{x}^{(k)}}{\mathbf{a}_i^T \mathbf{s}^{(k)}}, \quad i \notin \mathcal{A}^{(k)} \right) \tag{F.15}$$

If $\overline{\alpha}^{(k)} < 1$ is obtained (move towards a constraint), then a new constraint becomes active and $\overline{\alpha}^{(k)}$ is used as an upper limit for the step size in the line search. This procedure must be executed before the line search is performed in order to retain constraint feasibility during the line search procedure.

An initial feasible point is required for the algorithm because of the feasibility inequalities (e.g. physical laws) to be satisfied at each iteration. Such a point can be found by minimizing the distance between the given point, $\mathbf{x}_0$, and the nearest feasible point. This problem can be stated mathematically as a QP problem, i.e.

$$\begin{aligned} &\underset{\mathbf{x}}{\text{Minimize}} \quad 0.5\|\mathbf{x} - \mathbf{x}_0\|^2 \quad \equiv \quad \underset{\mathbf{s}}{\text{Minimize}} \quad 0.5\|\mathbf{s}\|^2 = 0.5\,\mathbf{s}^T \mathbf{I} \mathbf{s} \\ &\text{subject to the linear inequality constraints} \\ &\mathbf{a}_i^T(\mathbf{x}_0 + \mathbf{s}) \geq b_i\,, \quad i = 1, \ldots, m \end{aligned} \tag{F.16}$$

This problem can be solved, for example by the active set QP method explained in Appendix G. The initial feasible point is then calculated as $\mathbf{x}^{(0)} = \mathbf{x}_0 + \mathbf{s}$.

F.4 ALGORITHM

The algorithm given below implements a modified NLS method for the solution of a system of nonlinear equations where the variables are subjected to feasibility inequality constraints. The algorithm is based on the Levenberg-Marquardt method with line search.

Step 0: Initialization

(i) Choose a starting point $\mathbf{x}_0$.

(ii) Compute the constraints at the given point, $\mathbf{x}_0$ (equation (F.16)).

(iii) Solve the QP problem (F.16) and find s that satisfies the imposed constraints (refer to Appendix G). If the QP algorithm fails, the problem was not properly set up. Go to step 6. Otherwise, determine the initial feasible starting point $\mathbf{x}^{(0)} = \mathbf{x}_0 + \mathbf{s}$

(iv) Initialize the Levenberg-Marquardt parameter, $\lambda^{(0)}$ (typically 1.0).

Initialize the factors by which $\lambda^{(k)}$ will increase (typically 3.0) and decrease (typically 5.0).

Set the iteration counter k to 0

Specify a convergence tolerance, ε

Step 1: Solve the system of nonlinear equations subject to the imposed feasibility constraints

(i) Evaluate the objective function $\tilde{f}\left(\mathbf{x}^{(k)}\right)$ (equation (F.12)).

(ii) Calculate the Jacobian matrix, $\mathbf{J}\left(\mathbf{x}^{(k)}\right)$, the gradient vector, $\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{f}\left(\mathbf{x}^{(k)}\right)$, and the approximate Hessian matrix $\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^T$ of the objective function (equations (F.2) and (F.4) respectively). These first-order derivatives are calculated by means of the finite difference derivative approximations outlined in Dennis and Schnabel [83DE1m].

(iii) Set the iteration counter for the case when the objective function increases, $\hat{k}$, to zero.

(iv) Set up the constraints $\mathbf{a}_i^T\mathbf{s}^{(k)} \geq b_i - \mathbf{a}_i^T\mathbf{x}^{(k)}$, $i = 1,\ldots,m$ (equation (F.13)).

(v) Adjust the diagonal elements of the Hessian matrix, i.e. compute $\mathbf{J}\left(\mathbf{x}^{(k)}\right)\mathbf{J}\left(\mathbf{x}^{(k)}\right)^T + \lambda^{(k)}\mathbf{I}$.

(vi) Solve problem (F.13) to obtain the new search direction, $\mathbf{s}^{(k)}$ (refer to Appendix G). If the QP algorithm fails to find a solution, go to step 6.

(vii) Calculate the new point $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{s}^{(k)}$ and the directional derivative $\mathbf{g}\left(\mathbf{x}^{(k)}\right)^T\mathbf{s}^{(k)}$.

(viii) Evaluate the objective function $\tilde{f}\left(\mathbf{x}^{(k+1)}\right)$ (equation (F.12)).

Step 2: Test if the objective function increases or decreases

(i) If $\tilde{f}\left(\mathbf{x}^{(k+1)}\right) \leq \tilde{f}\left(\mathbf{x}^{(k)}\right) + 10^{-4}\mathbf{g}\left(\mathbf{x}^{(k)}\right)^T\mathbf{s}^{(k)}$, go to step 5.

(ii) If $\tilde{f}\left(\mathbf{x}^{(k+1)}\right) > \tilde{f}\left(\mathbf{x}^{(k)}\right) + 10^{-4}\mathbf{g}\left(\mathbf{x}^{(k)}\right)^T\mathbf{s}^{(k)}$, go to step 3.

Step 3: Increase $\lambda^{(k)}$, because the objective function has increased.

(i) If $\hat{k} \geq 4$ or $\lambda^{(k)} \geq 50.0$, go to step 4.

(ii) Set $\lambda^{(k)} = 3.0\lambda^{(k)}$ and $\hat{k} = \hat{k} + 1$. If $\hat{k} \geq 3$, set $\lambda^{(k)} = 1.5\lambda^{(k)}$.

(iii) Go to step 1 (v).

Step 4: Line search procedure (refer to [83DE1m] for details)

(i) Determine the maximum allowable step length, $\bar{\alpha}^{(k)}$, to remain feasible with regard to the constraints (an inactive constraints become active). If none of the constraints become active, the maximum allowable step length is set to a positive constant (typically 10.0).

(ii) Perform a line search and determine $\alpha^{(k)} \le \bar{\alpha}^{(k)}$ such that the objective function decreases (conditions (F.8) and (F.9) are satisfied). Set $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}$. Go to step 5.

If the line search fails to find a point, $\mathbf{x}^{(k+1)}$, sufficiently distinct from $\mathbf{x}^{(k)}$ which satisfies conditions (F.8) and (F.9), an error return occurs. Go to step 6.

Step 5: Test for convergence

(i) Determine the maximum component of the objective function and the maximum relative change in $\mathbf{x}^{(k)}$:

$$\bar{f}_{max} = \max_i f_i\left(\mathbf{x}^{(k+1)}\right)^2 \text{ and } \Delta\mathbf{x}_{max} = \max_i \left|\left(\mathbf{x}_i^{(k+1)} - \mathbf{x}_i^{(k)}\right)\right| / \left|\mathbf{x}_i^{(k)}\right|, \quad i = 1,\ldots,n$$

(ii) Set k = k + 1.

(iii) If the convergence tolerance is satisfied, i.e. $\bar{f}_{max} \le \varepsilon$, a feasible solution to the system of nonlinear equations has been found. Go to step 6.

(iv) If $\Delta\mathbf{x}_{max} > \varepsilon$ and $\hat{k} = 0$, set $\lambda^{(k)} = \lambda^{(k)}/5.0$ and go to step 1 (ii).

If $\Delta\mathbf{x}_{max} > \varepsilon$ and $\hat{k} \ne 0$, keep $\lambda^{(k)}$ constant and go to step 1 (ii).

If $\Delta\mathbf{x}_{max} \le \varepsilon$ for 3 consecutive iterations, negligible changes in $\mathbf{x}^{(k)}$ has occurred (very slow convergence). Go to step 6.

Step 6: Return to caller program

(i) Stop, return to the caller program and indicate the reason for the return. The following cases can occur:

(a) Successful completion of calculations.

(b) Modified nonlinear least squares algorithm failed to find a solution.

(c) Variables changed less than ε for three consecutive iterations.

(d) Line search failed to locate new point sufficiently distinct from the original point.

(e) QP algorithm failed to find a solution (refer to Appendix G for explanation).

APPENDIX G

DUAL ACTIVE SET ALGORITHM FOR CONVEX QUADRATIC PROGRAMMING PROBLEMS

G.1 INTRODUCTION

The quadratic programming (QP) problem involves the minimization of a quadratic objective function subject to linear constraints. QP methods can be used as an optimizing technique on its own and also play an important role in the solution of general nonlinear programming problems. In the latter case a series of QP problems (with different constraint sets) may be posed to approximate the actual problem behavior.

QP problems can contain linear equality and/or inequality constraints. The major difference between QP problems subject to only equality constraints and those subject to some inequality constraints is that the set of constraints active at the solution is unknown in the latter case. Therefore, QP algorithms must include a procedure, termed an active set strategy, that determines the correct set of active constraints at the solution. This is usually done by maintaining a working set that estimates the final active set. QP problems containing some inequality constraints are solved as a sequence of problems in which the constraints in the working set are treated as equalities.

The major difference among QP methods arise from the numerical procedures for solving the associated linear equations and the strategies that control the changes in the working set. Three methods for selecting the constraints for the working set can be identified, namely :

(1) primal active set methods allowing only feasible iterates (refer to chapter 2, section 2.2) [81GI1m, 85GI2m, 87FL1m, 89GI1m];

(2) dual active set methods (for convex QP) where it is not required to satisfy primal feasibility during each iteration [83GO1m, 85GI2m, 87FL1m];

(3) primal-dual active set methods (for convex QP) which allows both primal and dual infeasibilities [87FL1m].

In this appendix a dual active set method for convex QP problems that is based on the method proposed by Goldfarb and Idnani [83GO1m] will be developed. General QP problems containing both equality and inequality constraints will be considered. This method will primarily be used as an efficient QP subproblem minimization technique for the SQP method. Its secondary use will be as a part of a general solution technique for the NLS problem.

G.2 DUAL ACTIVE SET QUADRATIC PROGRAMMING METHOD

Currently, most implementations of SQP employ QP routines that are based on primal active set strategies. Considerable effort is expended in determining an initial feasible point for the primal QP solution. The most obvious advantage of the dual method is that the unconstrained minimum of the of the QP objective function provides an initial feasible solution. The dual algorithm then iterates until primal feasibility (i.e. dual optimality) is achieved, while maintaining the primal optimality of the intermediate subproblems (i.e. dual feasibility). This procedure is equivalent to solving the dual problem by a primal method [83GO1m, 87FL1m]. Some further advantages of the dual algorithm over feasible point methods are discussed in Goldfarb [82GO1m], Powell [85PO1m] and Fletcher [87FL1m].

The concept of duality is discussed in chapter 2 (section 2.2). The transformation of the primal quadratic programming problem into its dual is repeated here for convenience. The primal QP can be written as

$$\begin{aligned}
&\underset{\mathbf{x}}{\text{Minimize}} \quad q(\mathbf{x}) = 0.5\mathbf{x}^T\mathbf{G}\mathbf{x} + \mathbf{g}^T\mathbf{x} \\
&\text{subject to the constraints} \\
&\mathbf{a}_i^T\mathbf{x} = b_i\,, \quad i = 1,\ldots,m_{eq} \qquad \left(\mathbf{A}_{eq}^T\mathbf{x} = \mathbf{b}\right) \\
&\mathbf{a}_i^T\mathbf{x} \geq b_i\,, \quad i = m_{eq}+1,\ldots,m \qquad \left(\mathbf{A}_{ineq}^T\mathbf{x} \geq \mathbf{b}\right)
\end{aligned} \tag{G.1}$$

where $\mathbf{G}$ is required to be positive definite. The dual of problem (G.1) can be written as ($\mathbf{A}$ contains the constraint normals of both the equality and the inequality constraints)

$$\begin{aligned}
&\underset{\mathbf{x},\Lambda}{\text{Maximize}} \quad 0.5\mathbf{x}^T\mathbf{G}\mathbf{x} + \mathbf{g}^T\mathbf{x} - \Lambda^T\left(\mathbf{A}^T\mathbf{x} - \mathbf{b}\right) \\
&\text{subject to the constraints} \\
&\mathbf{G}\mathbf{x} + \mathbf{g} - \mathbf{A}\Lambda = \mathbf{0} \\
&\lambda_i \geq 0\,, \quad i = m_{eq}+1,\ldots,m
\end{aligned} \tag{G.2}$$

or

$$\begin{aligned}
&\underset{\Lambda}{\text{Maximize}} \quad -0.5\,\Lambda^T\left(\mathbf{A}^T\mathbf{G}^{-1}\mathbf{A}\right)\Lambda + \Lambda^T\left(\mathbf{A}^T\mathbf{G}^{-1}\mathbf{g} + \mathbf{b}\right) - 0.5\mathbf{g}^T\mathbf{G}^{-1}\mathbf{g} \\
&\text{subject to the constraints} \\
&\lambda_i \geq 0\,, \quad i = m_{eq}+1,\ldots,m
\end{aligned} \tag{G.3}$$

where the constraints are used to solve for $\mathbf{x}$, i.e. $\mathbf{x} = \mathbf{G}^{-1}\mathbf{A}\Lambda - \mathbf{G}^{-1}\mathbf{g}$ and λ_i, $i = 1,\ldots,m_{eq}$ have no sign restriction. Problem (G.3) has only simple constraints on the dual variables (Lagrange multipliers). The Kuhn-Tucker condition for the primal problem state (refer to equation (2.8)):

$$\lambda_i\left(\mathbf{a}_i^T\mathbf{x} - b_i\right) = 0, \quad i = 1,\ldots,m \tag{G.4}$$

Similarly, the Kuhn-Tucker conditions for the dual problem state:

$$u_i\lambda_i = 0, \quad i = m_{eq} + 1,\ldots,m \tag{G.5}$$

where u_i are the Lagrange multipliers of the dual problem. Thus, the values of the Lagrange multipliers of the dual problem, u_i, correspond to the values $\left(\mathbf{a}_i^T\mathbf{x} - b_i\right)$, $i = m_{eq} + 1,\ldots,m$ of the primal problem. (It should be noted, however, that this is not a formal proof.) The following points should be noted about the relationship between equations (G.4) and (G.5):

(1) If $u_i = 0$, $i = p$ and the corresponding element of λ_i is greater than zero, it indicates that the primal inequality constraint corresponding to the value of λ_i is active. If $u_i = 0$, $i = p$ and the corresponding element of λ_i is equal to zero (weakly active), it indicates that constraint p is superfluous.

(2) If $u_i > 0$, $i = p$, then the corresponding element of λ_i is equal to zero which indicates that the primal inequality constraint corresponding to the value of λ_i is inactive.

In the dual active set method, $\mathbf{x}^{(k)}$ is not primal feasible (some of the constraints are not satisfied), but the Lagrange multipliers are dual feasible, i.e. $\lambda_i \geq 0$, $i = m_{eq} + 1,\ldots,m$. Changes in the working set are performed to maintain non-negative Lagrange multipliers while moving to satisfy the violated constraints, i.e. obtain primal feasibility.

The dual active set strategy can be explained from a primal point of view as follows:

(1) Solve the unconstrained QP problem to obtain $\mathbf{x}^{(1)}$. This provides an initial feasible solution for the dual problem. All the Lagrange multipliers are equal to zero. Set k = 1.

(2) Test the linear constraints for primal feasibility. If the current point $\mathbf{x}^{(k)}$ satisfies all the constraints, a feasible solution is found; terminate with $\mathbf{x}^* = \mathbf{x}^{(k)}$. Otherwise, the most or any violated constraint not in the current active set is added, i.e. $\mathcal{A}^{(k)}$ is enlarged.

(3) Store the current values of the Lagrange multipliers of the inequality constraints, $\bar{\lambda}_i = \lambda_i^{(k)}$, $i = m_{eq}+1,\ldots,m$. Set k = k + 1.

(4) Solve the QP to obtain new values for $x^{(k)}$ and $\lambda_i^{(k)}$, $i = 1,\ldots,m$.

(5) Test the inequality constraints for dual feasibility, i.e. $\lambda_i^{(k)} \geq 0$, $i = m_{eq}+1,\ldots,m$. If the Lagrange multipliers of the inequality constraints are dual feasible, go to step 2. Otherwise one or more of the new Lagrange multipliers of the inequality constraints contained in the current active set is negative (dual infeasibility). Thus, $\mathcal{A}^{(k)}$ is not the correct active set. In order to remain dual feasible, a search direction,

$$\delta\lambda_i^{(k)} = \lambda_i^{(k)} - \bar{\lambda}_i,\; i \in \left[\left\{m_{eq}+1,\ldots,m\right\} \cap \mathcal{A}^{(k)}\right]$$

and the largest possible step length, $0 \leq \alpha^{(k)} \leq 1$, are determined such that the Lagrange multipliers of the inequality constraints in the current active set satisfy dual feasibility, i.e.

$$\bar{\lambda}_i = \lambda_i^{(k)} + \alpha^{(k)}\delta\lambda_i^{(k)} \geq 0,\; i \in \left[\left\{m_{eq}+1,\ldots,m\right\} \cap \mathcal{A}^{(k)}\right]$$

The inequality constraint corresponding to the first Lagrange multiplier that becomes zero is dropped from the active set and $\mathcal{A}^{(k)}$ is reduced (inequality constraint becomes inactive). Go to step 4.

The largest step size $\alpha^{(k)}$ to retain dual feasibility during iteration k is obtained from solving

$$\alpha^{(k)} = \min\left(\min_{\delta\lambda_i^{(k)}<0} \frac{0-\lambda_i^{(k)}}{\delta\lambda_i^{(k)}},\; i \in \left[\left\{m_{eq}+1,\ldots,m\right\} \cap \mathcal{A}^{(k)}\right]\right) \tag{G.6}$$

During the course of the QP solution the active set will change from one iteration to the next. Therefore, as constraints are added to and deleted from the active set, various matrices need to be updated, rather than recomputed.

G.3 SOLUTION OF THE EQUALITY CONSTRAINED QP PROBLEM

Equality constrained QP subproblems occur within many active set methods for general QP problems as well as in SQP methods. A variety of methods are available for solving these problems, e.g. [81GI1m, 87FL1m, 89GI1m]. The method that will subsequently be derived are based on the procedures proposed by Gill et al. [84GI1m] and Betts [80BE1m, 80BE2m].

Calculation of the search direction and the Lagrange multipliers

In this section a procedure is developed for solving the QP problem by means of an orthogonal decomposition of the constraint matrix. We shall be concerned with the numerical solution of the following QP problem:

$$\begin{aligned}&\underset{\mathbf{x}}{\text{Minimize}}\quad q(\mathbf{x}) = 0.5\mathbf{x}^T\mathbf{G}\mathbf{x} + \mathbf{g}^T\mathbf{x}\\ &\text{subject to the equality constraints}\\ &\mathbf{A}^T\mathbf{x} = \mathbf{b}\end{aligned} \tag{G.7}$$

where $\mathbf{A}^T$ is an m × n matrix with rank($\mathbf{A}$) = m, $\mathbf{G}$ an n × n positive definite symmetric matrix, $\mathbf{g}$ is an n-vector, $\mathbf{b}$ is an m-vector and $\mathbf{x}$ is an n-vector and $n \geq m$.

Let $\mathbf{Q}$ be an orthogonal n × n matrix ($\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$), and

$$\mathbf{A}^T\mathbf{Q} = [\mathbf{0} \ \vdots \ \mathbf{T}] \tag{G.8}$$

where $\mathbf{T}$ is an m × m reversed lower-triangular nonsingular matrix.

Let $\mathbf{Q}$ be partitioned as

$$\mathbf{Q} = [\mathbf{Z} \ \vdots \ \mathbf{Y}] \tag{G.9}$$

where $\mathbf{Z}$ is an n × (n-m) and $\mathbf{Y}$ is an n × m matrix respectively. Substitute equation (G.9) into equation (G.8) and find

$$\mathbf{A}^T\mathbf{Q} = [\mathbf{A}^T\mathbf{Z} \ \vdots \ \mathbf{A}^T\mathbf{Y}] = [\mathbf{0} \ \vdots \ \mathbf{T}] \tag{G.10}$$

hence

$$\mathbf{A}^T\mathbf{Z} = \mathbf{0} \quad \text{and} \quad \mathbf{A}^T\mathbf{Y} = \mathbf{T} \tag{G.11}$$

The equality constraints can be rewritten as

$$\mathbf{A}^T\mathbf{Q}\mathbf{Q}^T\mathbf{x} = \mathbf{b} \Rightarrow [\mathbf{0} \ \vdots \ \mathbf{T}]\mathbf{Q}^T\mathbf{x} = \mathbf{b} \tag{G.12}$$

Define the n-vector $\mathbf{p}$ as

$$\mathbf{p} = \mathbf{Q}^T\mathbf{x} = \begin{bmatrix}\mathbf{Z}^T\mathbf{x}\\ \cdots \\ \mathbf{Y}^T\mathbf{x}\end{bmatrix} = \begin{bmatrix}\mathbf{p}_Z\\ \cdots \\ \mathbf{p}_Y\end{bmatrix} \tag{G.13}$$

where $\mathbf{p}_Z$ is an (n-m)-vector and $\mathbf{p}_Y$ is an m-vector. Substitute equation (G.13) into equation (G.12) and find

$$\begin{bmatrix} 0 & \vdots & \mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{p}_Z \\ \cdots \\ \mathbf{p}_Y \end{bmatrix} = \mathbf{b} \Rightarrow \mathbf{T}\mathbf{p}_Y = \mathbf{b} \tag{G.14}$$

Since $\mathbf{T}$ is nonsingular, the m-vector $\mathbf{p}_Y$ can be solved by means of back substitution. Call the solution $\hat{\mathbf{p}}_Y$, i.e. $\hat{\mathbf{p}}_Y = \mathbf{T}^{-1}\mathbf{b}$.

Now, if

$$\mathbf{x} = \mathbf{Q}\mathbf{p} = \mathbf{Z}\mathbf{p}_Z + \mathbf{Y}\hat{\mathbf{p}}_Y \tag{G.15}$$

then problem (G.7) can be written as an unconstrained problem, i.e.

$$\begin{aligned} q(\mathbf{x}) &= 0.5(\mathbf{Z}\mathbf{p}_Z + \mathbf{Y}\hat{\mathbf{p}}_Y)^T \mathbf{G}(\mathbf{Z}\mathbf{p}_Z + \mathbf{Y}\hat{\mathbf{p}}_Y) + \mathbf{g}^T(\mathbf{Z}\mathbf{p}_Z + \mathbf{Y}\hat{\mathbf{p}}_Y) \\ &= 0.5\mathbf{p}_Z^T\mathbf{Z}^T\mathbf{G}\mathbf{Z}\mathbf{p}_Z + \left(\mathbf{Z}^T\mathbf{G}\mathbf{Y}\hat{\mathbf{p}}_Y + \mathbf{Z}^T\mathbf{g}\right)^T \mathbf{p}_Z + 0.5\hat{\mathbf{p}}_Y^T\mathbf{Y}^T\mathbf{G}\mathbf{Y}\hat{\mathbf{p}}_Z + \mathbf{g}^T\mathbf{Y}\hat{\mathbf{p}}_Y \end{aligned} \tag{G.16}$$

$q(\mathbf{x})$ is minimized when $\mathbf{p}_Z$ satisfies (gradient of equation (G.16) is set equal to zero)

$$\left(\mathbf{Z}^T\mathbf{G}\mathbf{Z}\right)\hat{\mathbf{p}}_Z = -\mathbf{Z}^T(\mathbf{G}\mathbf{Y}\hat{\mathbf{p}}_Y + \mathbf{g})$$

The solution of problem (G.7) can be calculated from

$$\hat{\mathbf{x}} = \mathbf{Z}\hat{\mathbf{p}}_Z + \mathbf{Y}\hat{\mathbf{p}}_Y \tag{G.17}$$

The Lagrange multipliers $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ can be obtained as follows:

$$\mathbf{A}\Lambda = \mathbf{G}\hat{\mathbf{x}} + \mathbf{g} \qquad (\nabla L(\hat{\mathbf{x}}) = 0) \tag{G.18}$$

or

$$\mathbf{Q}^T\mathbf{A}\Lambda = \mathbf{Q}^T\mathbf{G}\hat{\mathbf{x}} + \mathbf{Q}^T\mathbf{g} \Rightarrow \begin{bmatrix} \mathbf{0} \\ \cdots \\ \mathbf{T}^T \end{bmatrix} \Lambda = \mathbf{Q}^T\mathbf{G}\mathbf{Q}\hat{\mathbf{p}} + \mathbf{Q}^T\mathbf{g} \tag{G.19}$$

Now

$$\mathbf{Q}^T\mathbf{G}\mathbf{Q} = \begin{bmatrix} \mathbf{Z}^T \\ \cdots \\ \mathbf{Y}^T \end{bmatrix} \mathbf{G}\begin{bmatrix} \mathbf{Z} & \vdots & \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^T\mathbf{G}\mathbf{Z} & \vdots & \mathbf{Z}^T\mathbf{G}\mathbf{Y} \\ \cdots & \cdots & \cdots \\ \mathbf{Y}^T\mathbf{G}\mathbf{Z} & \vdots & \mathbf{Y}^T\mathbf{G}\mathbf{Y} \end{bmatrix}. \tag{G.20}$$

hence

$$\mathbf{T}^T\Lambda = \mathbf{Y}^T\mathbf{GZ}\hat{\mathbf{p}}_Z + \mathbf{Y}^T\mathbf{GY}\hat{\mathbf{p}}_Y + \mathbf{Y}^T\mathbf{g} \tag{G.21}$$

and Λ is determined by back substitution.

The original constrained problem is replaced by a lower dimensional unconstrained problem in the variables $\mathbf{p}_Z$ after choosing the variables $\mathbf{p}_Y$ to satisfy the constraints. A unique solution to the equality constrained QP problem will exist due the positive definite restriction placed on $\mathbf{G}$.

Changes in the working set

Unless the correct active set is known a priori, the working set must be modified during execution of an active set method by adding and deleting constraints. The corresponding matrix factorizations will be updated accordingly, because computing them ab initio would be too expensive. Numerically stable updating methods of a matrix when it is modified by adding or deleting a row are well known [74GI1m]. In our updating procedures we make use of Givens rotations. Sequences of plane rotations (Givens rotations) are used to introduce zeros into the appropriate positions of a vector or matrix [74GI1m, 84GI1m]. Basic linear algebra manipulations are then used to perform the matrix modifications (updates).

Before the changes in the working set are discussed, we define the following matrices and vectors and assume that they are available:

(i) Matrices $\mathbf{Q}$ and $\mathbf{T}$ such that $\mathbf{Q}$ is an orthogonal n × n matrix, i.e. $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$, and $\mathbf{T}$ is an m × m reversed lower-triangular nonsingular matrix. $\mathbf{A}^T\mathbf{Q} = [\mathbf{0} \;\vdots\; \mathbf{T}]$ and $\mathbf{Q}$ is partitioned as $\mathbf{Q} = [\mathbf{Z} \;\vdots\; \mathbf{Y}]$ ($\mathbf{Z}$ is an n × (n-m) and $\mathbf{Y}$ is an n × m matrix).

(ii) Matrices $\mathbf{U}$, $\mathbf{G}_{YY}$ and $\mathbf{G}_{ZY}$, where

(a) $\mathbf{U}$ is an (n-m) × (n-m) upper triangular matrix, $\mathbf{U}^T\mathbf{U} = \mathbf{Z}^T\mathbf{GZ}$

(b) $\mathbf{G}_{YY}$ is an m × m matrix, $\mathbf{G}_{YY} = \mathbf{Y}^T\mathbf{GY}$

(c) $\mathbf{G}_{ZY}$ is an (n-m) × m, $\mathbf{G}_{ZY} = \mathbf{Z}^T\mathbf{GY}$

(iii) Vector $\mathbf{g}_Q$, an n × 1 vector such that $\mathbf{g}_Q = \mathbf{Q}^T\mathbf{g} = \begin{bmatrix} \mathbf{Z}^T \\ \cdots \\ \mathbf{Y}^T \end{bmatrix}\mathbf{g} = \begin{bmatrix} \mathbf{Z}^T\mathbf{g} \\ \cdots \\ \mathbf{Y}^T\mathbf{g} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_Z \\ \cdots \\ \mathbf{g}_Y \end{bmatrix}$.

The partitioning of $\mathbf{g}_Q$ is such that $\mathbf{g}_Z$ is an (n-m)-vector and $\mathbf{g}_Y$ is an m-vector.

The following should also be noted:

(i) When m = 0, let $\mathbf{Q} = \mathbf{I}$, $\mathbf{Z} = \mathbf{I}$, $\mathbf{g_Q} = \mathbf{g}$ and $\mathbf{U}$ the Cholesky factor of $\mathbf{G}$ $\left(\mathbf{G} = \mathbf{U}^T\mathbf{U}\right)$. $\mathbf{T}$, $\mathbf{Y}$, $\mathbf{G}_{YY}$ and $\mathbf{G}_{ZY}$ are empty matrices.

(ii) The storage required for the following matrices are:

(a) $\mathbf{Q}$ $(n \times n)$

(b) $$\mathbf{G}_Q = \begin{bmatrix} \mathbf{U}^T\mathbf{U} & \vdots & \mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ & \vdots & \mathbf{G}_{YY} \end{bmatrix} \quad (n \times n)$$

$\mathbf{G}_Q$ is symmetric so only its upper triangular part is stored (refer to equation (G.20)).

(c) $\mathbf{T}$ $(m \times m)$

(1) <u>Adding a constraint</u>

When a constraint is added to the working set, its index can simply be placed at the end of the list of constraint indices of the working set. Therefore we assume that the new constraint is added at the last row of $\mathbf{A}^T$.

Suppose $\mathbf{Q}$, $\mathbf{T}$, $\mathbf{U}$, $\mathbf{G}_{YY}$, $\mathbf{G}_{ZY}$ and $\mathbf{g}_Q$ corresponding to the constrains $\mathbf{A}^T\mathbf{x} = \mathbf{b}$ are available and that the constraint $\mathbf{a}_{m+1}^T\mathbf{x} = b_{m+1}$ is added to the set of constraints. Furthermore, $\overline{\mathbf{Q}}, \overline{\mathbf{T}}, \overline{\mathbf{U}}, \overline{\mathbf{G}}_{YY}, \overline{\mathbf{G}}_{ZY}$ and $\overline{\mathbf{g}}_Q$ correspond to $\overline{\mathbf{A}}^T = \begin{bmatrix} \mathbf{A}^T \\ \mathbf{a}_{m+1}^T \end{bmatrix}$.

(a) Updating $\mathbf{Q}$ and $\mathbf{T}$:

Let $\mathbf{w}^T = \mathbf{a}_{m+1}^T\mathbf{Q}$, then

$$\overline{\mathbf{A}}^T\mathbf{Q} = \begin{bmatrix} \mathbf{A}^T\mathbf{Q} \\ \mathbf{a}_{m+1}^T\mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{T} \\ \mathbf{w}_Z^T & \mathbf{w}_Y^T \end{bmatrix}$$

where $\mathbf{w}_Z^T$ and $\mathbf{w}_Y^T$ represent the partitioning of the new row that is added. We see that a new matrix $\overline{\mathbf{Q}}$ can be obtained by applying a sequence of plane rotations (Givens rotations) on the right of $\mathbf{Q}$ to transform $\mathbf{w}_Z^T$ to suitable form. The sequence of rotations take linear combinations of the elements of $\mathbf{w}_Z^T$ to reduce it to a multiple (say γ) of a column of the identity matrix.

Let $\tilde{\mathbf{Q}}$ be an orthogonal matrix formed by the product of rotations in the planes (1,2), (2,3), (3,4),..., (n-m-1,n-m) such that the effect of the transformation can be expressed as

$$\overline{\mathbf{A}}^T\mathbf{Q}\tilde{\mathbf{Q}} = \begin{bmatrix} 0 & \vdots & 0 & \mathbf{T} \\ 0 & \vdots & \gamma & \mathbf{w}_Y^T \end{bmatrix} = \begin{bmatrix} 0 & \vdots & \overline{\mathbf{T}} \end{bmatrix}, \text{ where } \overline{\mathbf{T}} = \begin{bmatrix} 0 & \mathbf{T} \\ \gamma & \mathbf{w}_Y^T \end{bmatrix}.$$

Then $\overline{\mathbf{Q}} = \mathbf{Q}\tilde{\mathbf{Q}}$ and $\overline{\mathbf{A}}^T\overline{\mathbf{Q}} = \begin{bmatrix} 0 & \overline{\mathbf{T}} \end{bmatrix}$.

Note, however, that $\tilde{\mathbf{Q}}$ has the form $\begin{bmatrix} \mathbf{P} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{I} \end{bmatrix}$ with $\mathbf{P}$ an $(n-m)\times(n-m)$ matrix; hence the rotations affect only the first (n-m) columns of $\mathbf{Q}$ so that the last m columns of $\overline{\mathbf{Q}}$ are identical to those of $\mathbf{Q}$. The first (n-m) columns of $\overline{\mathbf{Q}}$ are linear combinations of the first (n-m) columns of $\mathbf{Q}$. $\overline{\mathbf{Q}}$ can be expressed as

$$\overline{\mathbf{Q}} = \begin{bmatrix} \mathbf{Z} & \vdots & \mathbf{Y} \end{bmatrix} \begin{bmatrix} \mathbf{P} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{ZP} & \vdots & \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{Z}} & \vdots & \mathbf{y} & \vdots & \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{Z}} & \vdots & \overline{\mathbf{Y}} \end{bmatrix}$$

where $\mathbf{ZP} = \begin{bmatrix} \overline{\mathbf{Z}} & \vdots & \mathbf{y} \end{bmatrix}$ and $\overline{\mathbf{Y}} = \begin{bmatrix} \mathbf{y} & \vdots & \mathbf{Y} \end{bmatrix}$ (the last column of $\mathbf{Z}$ becomes the first column of $\overline{\mathbf{Y}}$).

(b) Updating $\mathbf{U}, \mathbf{G}_{YY}$ and $\mathbf{G}_{ZY}$:

The matrix $\mathbf{Q}^T\mathbf{GQ}$ can be expressed as

$$\mathbf{G}_Q = \mathbf{Q}^T\mathbf{GQ} = \begin{bmatrix} \mathbf{Z}^T\mathbf{GZ} & \vdots & \mathbf{Z}^T\mathbf{GY} \\ \cdots & \cdots & \cdots \\ \mathbf{Y}^T\mathbf{GZ} & \vdots & \mathbf{Y}^T\mathbf{GY} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^T\mathbf{U} & \vdots & \mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ \mathbf{G}_{ZY}^T & \vdots & \mathbf{G}_{YY} \end{bmatrix}$$

The matrix $\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}}$ can be expressed as

$$\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}} = \begin{bmatrix} \mathbf{P}^T & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{U}^T\mathbf{U} & \vdots & \mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ \mathbf{G}_{ZY}^T & \vdots & \mathbf{G}_{YY} \end{bmatrix} \begin{bmatrix} \mathbf{P} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^T\mathbf{U}^T\mathbf{UP} & \vdots & \mathbf{P}^T\mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ \mathbf{G}_{ZY}^T\mathbf{P} & \vdots & \mathbf{G}_{YY} \end{bmatrix}$$

The matrix $\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}}$ must be repartitioned, because m increases by 1 and (n-m) decreases by 1.

Let $\mathbf{UP} = \left[\tilde{\mathbf{U}} \;\vdots\; \tilde{\mathbf{u}}\right]$, then $\mathbf{P}^T\mathbf{U}^T\mathbf{UP} = \begin{bmatrix} \tilde{\mathbf{U}}^T\tilde{\mathbf{U}} & \vdots & \tilde{\mathbf{U}}^T\tilde{\mathbf{u}} \\ \cdots & \cdots & \cdots \\ \tilde{\mathbf{u}}^T\tilde{\mathbf{U}} & \vdots & \tilde{\mathbf{u}}^T\tilde{\mathbf{u}} \end{bmatrix}$ and $\mathbf{P}^T\mathbf{G}_{ZY} = \begin{bmatrix} \tilde{\mathbf{G}}_{ZY} \\ \cdots \\ \tilde{\mathbf{h}}_{ZY}^T \end{bmatrix}$.

Furthermore $\overline{\mathbf{G}}_{ZY} = \left[\tilde{\mathbf{U}}^T\tilde{\mathbf{u}} \;\vdots\; \tilde{\mathbf{G}}_{ZY}\right]$ and $\overline{\mathbf{G}}_{YY} = \begin{bmatrix} \tilde{\mathbf{u}}^T\tilde{\mathbf{u}} & \vdots & \tilde{\mathbf{h}}_{ZY}^T \\ \cdots & \cdots & \cdots \\ \tilde{\mathbf{h}}_{ZY} & \vdots & \mathbf{G}_{YY} \end{bmatrix}$.

$\overline{\mathbf{U}}$ should satisfy $\overline{\mathbf{U}}^T\overline{\mathbf{U}} = \tilde{\mathbf{U}}^T\tilde{\mathbf{U}} = \tilde{\mathbf{U}}^T\tilde{\mathbf{P}}^T\tilde{\mathbf{P}}\tilde{\mathbf{U}}$ where $\tilde{\mathbf{P}}$ is an orthogonal matrix chosen to triangularize $\tilde{\mathbf{U}}$ (upper triangular form). $\tilde{\mathbf{U}}$ has subdiagonal elements (introduced by the plane rotations applied to **Z**) that must be eliminated. $\tilde{\mathbf{P}}$ is a product of rotations (Givens rotations) in the planes (1,2), (2,3),…, (m-1,m-1).

The following procedure must be performed to update $\mathbf{G}_Q$:

If $\mathbf{G}_Q = \begin{bmatrix} \mathbf{U} & \vdots & \mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ & \vdots & \mathbf{G}_{YY} \end{bmatrix}$, then

(i) Compute $\left[\tilde{\mathbf{U}} \;\vdots\; \tilde{\mathbf{u}}\right] = \mathbf{UP}$ and $\begin{bmatrix} \tilde{\mathbf{G}}_{ZY} \\ \cdots \\ \tilde{\mathbf{h}}_{ZY}^T \end{bmatrix} = \mathbf{P}^T\mathbf{G}_{ZY}$

(ii) Compute $\mathbf{v} = \tilde{\mathbf{U}}^T\tilde{\mathbf{u}}$ and $\vartheta = \tilde{\mathbf{u}}^T\tilde{\mathbf{u}}$

(iii) Form $\begin{bmatrix} \overline{\mathbf{G}}_{ZY} \\ \cdots \\ \overline{\mathbf{G}}_{YY} \end{bmatrix} = \begin{bmatrix} \mathbf{v} & \vdots & \tilde{\mathbf{G}}_{ZY} \\ \cdots & \cdots & \cdots \\ \vartheta & \vdots & \tilde{\mathbf{h}}_{ZY}^T \\ \cdots & \cdots & \cdots \\ \tilde{\mathbf{h}}_{ZY} & \vdots & \mathbf{G}_{YY} \end{bmatrix}$ (Both $\mathbf{G}_{YY}$ and $\overline{\mathbf{G}}_{YY}$ are symmetric matrices)

(iv) Form $\overline{\mathbf{U}} = \tilde{\mathbf{P}}\tilde{\mathbf{U}}$ where $\tilde{\mathbf{P}}$ triangularizes $\tilde{\mathbf{U}}$.

(v) $\overline{\mathbf{G}}_Q = \begin{bmatrix} \overline{\mathbf{U}} & \vdots & \overline{\mathbf{G}}_{ZY} \\ \cdots & \cdots & \cdots \\ & \vdots & \overline{\mathbf{G}}_{YY} \end{bmatrix}$

It can be shown that $\overline{\mathbf{Q}}^{\mathrm{T}}\mathbf{G}\overline{\mathbf{Q}} = \begin{bmatrix} \overline{\mathbf{Z}}^{\mathrm{T}}\mathbf{G}\overline{\mathbf{Z}} & \vdots & \overline{\mathbf{Z}}^{\mathrm{T}}\mathbf{G}\overline{\mathbf{Y}} \\ \cdots & \cdots & \cdots \\ \overline{\mathbf{Y}}^{\mathrm{T}}\mathbf{G}\overline{\mathbf{Z}} & \vdots & \overline{\mathbf{Y}}^{\mathrm{T}}\mathbf{G}\overline{\mathbf{Y}} \end{bmatrix}$ to illustrate that the repartitioning procedure is correct.

(c) Updating $\mathbf{g}_{\mathrm{Q}}$:

$$\overline{\mathbf{g}}_{\mathrm{Q}} = \overline{\mathbf{Q}}^{\mathrm{T}}\mathbf{g} = \widetilde{\mathbf{Q}}^{\mathrm{T}}\mathbf{Q}\mathbf{g} = \begin{bmatrix} \mathbf{P}^{\mathrm{T}} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{g}_{\mathrm{Z}} \\ \cdots \\ \mathbf{g}_{\mathrm{Y}} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{g}_{\mathrm{Z}} \\ \cdots \\ \mathbf{g}_{\mathrm{Y}} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{g}}_{\mathrm{Z}} \\ \cdots \\ \overline{\mathbf{g}}_{\mathrm{Y}} \end{bmatrix}$$

where $\overline{\mathbf{g}}_{\mathrm{Z}}$ is an (n-m-1) vector and $\overline{\mathbf{g}}_{\mathrm{Y}}$ is an (m+1) vector. Hence, compute $\mathbf{P}^{\mathrm{T}}\mathbf{g}_{\mathrm{Z}}$ and repartition.

(d) Linear dependent constraints

A linear dependent constraint can be expressed as a linear combination of all the other constraints in the active set.

Suppose that the current equality constraints are $\mathbf{A}^{\mathrm{T}}\mathbf{x} = \mathbf{b}$. If the new constraint, $\mathbf{a}_{m+1}^{\mathrm{T}}\mathbf{x} = b_{m+1}$, that is added to the active set is linear dependent on the other constraints, then $\mathbf{a}_{m+1}$ can be expressed in terms of the other constraint normals as

$$\mathbf{a}_{m+1} = \sum_{i=1}^{m} r_i \mathbf{a}_i = \mathbf{A}\mathbf{r}, \quad \mathbf{r} = \begin{bmatrix} r_1 & r_2 & \cdots & r_m \end{bmatrix}^{\mathrm{T}}.$$

where $\mathbf{r}$ is the vector of coefficients expressing the linear dependence.

It is known that

$$\mathbf{Q}^{\mathrm{T}}\mathbf{A} = \begin{bmatrix} 0 \\ \cdots \\ \mathbf{T}^{\mathrm{T}} \end{bmatrix}$$

hence

$$\mathbf{Q}^{\mathrm{T}}\mathbf{a}_{m+1} = \mathbf{Q}^{\mathrm{T}}\mathbf{A}\mathbf{r} = \begin{bmatrix} 0 \\ \cdots \\ \mathbf{T}^{\mathrm{T}}\mathbf{r} \end{bmatrix}$$

Let

$$\mathbf{w} = \mathbf{Q}^T \mathbf{a}_{m+1} = \begin{bmatrix} \mathbf{w}_Z \\ \cdots \\ \mathbf{w}_Y \end{bmatrix} = \begin{bmatrix} 0 \\ \cdots \\ \mathbf{T}^T \mathbf{r} \end{bmatrix}$$

Thus $\mathbf{w}_Z = 0$ and $\mathbf{r}$ can be determined from solving the following system of equations:

$$\mathbf{T}^T \mathbf{r} = \mathbf{w}_Y.$$

(2) <u>Deleting a constraint</u>

When a constraint is deleted from the working set, the row dimension of $\mathbf{A}^T$ and the dimension of $\mathbf{T}$ are decreased by one, while the column dimension of $\mathbf{Z}$ is increased by one.

Suppose $\mathbf{Q}$, $\mathbf{T}$,$\mathbf{U}$, $\mathbf{G}_{YY}$, $\mathbf{G}_{ZY}$ and $\mathbf{g}_Q$ corresponding to the constraints $\mathbf{A}^T\mathbf{x} = \mathbf{b}$ are available and that the constraint $\mathbf{a}_i^T\mathbf{x} = b_i$ is deleted from the active set of constraints. Let the new set of active constraints be $\overline{\mathbf{A}}^T\mathbf{x} = \mathbf{b}$.

(a) Updating $\mathbf{Q}$ and $\mathbf{T}$:

$\overline{\mathbf{A}}^T\mathbf{Q} = [0 \;\vdots\; \mathbf{S}]$, where $\mathbf{S}$ is an (m-1) $\times$ m matrix such that rows 1 to (i-1) are in the reverse triangular form and the remaining rows have one extra element above the reverse diagonal. In order to reduce these columns of $\mathbf{S}$ to the desired triangular form of $\overline{\mathbf{T}}$, a sequence of plane rotations (Givens rotations) are applied on the right of $\mathbf{S}$.

Let $\tilde{\mathbf{Q}}$ be an orthogonal matrix formed by the product of rotations in the planes (m-i+1,m-i),..., (2,1) such that $\mathbf{SP} = \overline{\mathbf{T}}$, where $\overline{\mathbf{T}}$ is in the reverse lower-triangular form.

Note, however, that $\tilde{\mathbf{Q}}$ has the form $\begin{bmatrix} \mathbf{I} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & \mathbf{P} \end{bmatrix}$ with $\mathbf{P}$ an $\mathrm{m} \times \mathrm{m}$ matrix; hence the rotations affect only the last m columns of $\mathbf{Q}$ so that the first m columns of $\overline{\mathbf{Q}} = \mathbf{Q}\tilde{\mathbf{Q}}$ are identical to those of $\mathbf{Q}$. Then

$$\overline{\mathbf{A}}^T\mathbf{Q}\tilde{\mathbf{Q}} = [0 \;\vdots\; \overline{\mathbf{T}}]$$

$\overline{\mathbf{Q}}$ can be expressed as

$$\overline{\mathbf{Q}} = \mathbf{Q}\tilde{\mathbf{Q}} = [\mathbf{Z} \;\vdots\; \mathbf{Y}]\begin{bmatrix} \mathbf{I} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{P} \end{bmatrix} = [\mathbf{Z} \;\vdots\; \mathbf{YP}] = [\mathbf{Z} \;\vdots\; \mathbf{z} \;\vdots\; \overline{\mathbf{Y}}] = [\overline{\mathbf{Z}} \;\vdots\; \overline{\mathbf{Y}}]$$

where $\mathbf{YP} = [\mathbf{z} \;\vdots\; \overline{\mathbf{Y}}]$ and $\overline{\mathbf{Z}} = [\mathbf{Z} \;\vdots\; \mathbf{z}]$ (the first column of $\mathbf{YP}$ becomes the last column of $\overline{\mathbf{Z}}$). z is a linear combination of the relevant columns of $\mathbf{Y}$.

(b) Updating $\mathbf{U}$, $\mathbf{G}_{YY}$ and $\mathbf{G}_{ZY}$:

The matrix $\mathbf{Q}^T\mathbf{GQ}$ can be expressed as

$$\mathbf{G}_Q = \mathbf{Q}^T\mathbf{GQ} = \begin{bmatrix} \mathbf{Z}^T\mathbf{GZ} & \vdots & \mathbf{Z}^T\mathbf{GY} \\ \cdots & \cdots & \cdots \\ \mathbf{Y}^T\mathbf{GZ} & \vdots & \mathbf{Y}^T\mathbf{GY} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^T\mathbf{U} & \vdots & \mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ \mathbf{G}_{ZY}^T & \vdots & \mathbf{G}_{YY} \end{bmatrix}$$

The matrix $\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}}$ can be expressed as

$$\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}} = \begin{bmatrix} \mathbf{I} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{P}^T \end{bmatrix}\begin{bmatrix} \mathbf{U}^T\mathbf{U} & \vdots & \mathbf{G}_{ZY} \\ \cdots & \cdots & \cdots \\ \mathbf{G}_{ZY}^T & \vdots & \mathbf{G}_{YY} \end{bmatrix}\begin{bmatrix} \mathbf{I} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^T\mathbf{U} & \vdots & \mathbf{G}_{ZY}\mathbf{P} \\ \cdots & \cdots & \cdots \\ \mathbf{P}^T\mathbf{G}_{ZY}^T & \vdots & \mathbf{P}^T\mathbf{G}_{YY}\mathbf{P} \end{bmatrix}$$

The matrix $\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}}$ must be repartitioned, because m decreases by 1 and (n-m) increases by 1, and $\overline{\mathbf{U}}$ must be retriangularized.

Let

$$\mathbf{G}_{ZY}\mathbf{P} = [\mathbf{v} \;\vdots\; \tilde{\mathbf{G}}_{ZY}] \text{ and } \mathbf{P}^T\mathbf{G}_{YY}\mathbf{P} = \begin{bmatrix} \vartheta & \vdots & \mathbf{h}_{YY}^T \\ \cdots & \cdots & \cdots \\ \mathbf{h}_{YY} & \vdots & \overline{\mathbf{G}}_{YY} \end{bmatrix}$$

It then follows that

$$\overline{\mathbf{G}}_{ZY} = \begin{bmatrix} \tilde{\mathbf{G}}_{ZY} \\ \cdots \\ \mathbf{h}_{YY}^T \end{bmatrix}$$

and $\overline{\mathbf{U}}$ should satisfy

$$\overline{\mathbf{U}}^{\mathrm{T}}\overline{\mathbf{U}} = \begin{bmatrix} \mathbf{U}^{\mathrm{T}}\mathbf{U} & \vdots & \mathbf{v} \\ \cdots & \cdots & \cdots \\ \mathbf{v}^{\mathrm{T}} & \vdots & \vartheta \end{bmatrix}$$

Suppose that the Cholesky factor can be defined as

$$\overline{\mathbf{U}} = \begin{bmatrix} \mathbf{U} & \vdots & \mathbf{u} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \rho \end{bmatrix}$$

then

$$\overline{\mathbf{U}}^{\mathrm{T}}\overline{\mathbf{U}} = \begin{bmatrix} \mathbf{U}^{\mathrm{T}}\mathbf{U} & \vdots & \mathbf{U}^{\mathrm{T}}\mathbf{u} \\ \cdots & \cdots & \cdots \\ \mathbf{u}^{\mathrm{T}}\mathbf{U} & \vdots & \mathbf{u}^{\mathrm{T}}\mathbf{u}+\rho^2 \end{bmatrix}$$

Hence choose $\mathbf{U}^{\mathrm{T}}\mathbf{u} = \mathbf{v}$ and $\mathbf{u}^{\mathrm{T}}\mathbf{u}+\rho^2 = \vartheta$ and solve for $\mathbf{u}$ and ρ. (If $\vartheta \leq \mathbf{u}^{\mathrm{T}}\mathbf{u}$, $\mathbf{G}$ cannot be positive definite.)

The following procedure must be performed to update $\mathbf{G}_{\mathrm{Q}}$:

If $\mathbf{G}_{\mathrm{Q}} = \begin{bmatrix} \mathbf{U} & \vdots & \mathbf{G}_{\mathrm{ZY}} \\ \cdots & \cdots & \cdots \\ & \vdots & \mathbf{G}_{\mathrm{YY}} \end{bmatrix}$, then

(i) Compute $\begin{bmatrix} \mathbf{G}_{\mathrm{ZY}}\mathbf{P} \\ \cdots \\ \mathbf{P}^{\mathrm{T}}\mathbf{G}_{\mathrm{YY}}\mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{v} & \vdots & \widetilde{\mathbf{G}}_{\mathrm{ZY}} \\ \cdots & \cdots & \cdots \\ \vartheta & \vdots & \widetilde{\mathbf{h}}_{\mathrm{YY}}^{\mathrm{T}} \\ \cdots & \cdots & \cdots \\ \widetilde{\mathbf{h}}_{\mathrm{YY}} & \vdots & \widetilde{\mathbf{G}}_{\mathrm{YY}} \end{bmatrix}$

(ii) Solve $\mathbf{U}^{\mathrm{T}}\mathbf{u} = \mathbf{v}$ for $\mathbf{u}$ and calculate $\rho = \sqrt{\vartheta - \mathbf{u}^{\mathrm{T}}\mathbf{u}}$

(iii) Form the new Cholesky factor $\overline{\mathbf{U}} = \begin{bmatrix} \mathbf{U} & \vdots & \mathbf{u} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \rho \end{bmatrix}$

It can again be shown that $\overline{\mathbf{Q}}^T\mathbf{G}\overline{\mathbf{Q}} = \begin{bmatrix} \overline{\mathbf{Z}}^T\mathbf{G}\overline{\mathbf{Z}} & \vdots & \overline{\mathbf{Z}}^T\mathbf{G}\overline{\mathbf{Y}} \\ \cdots & \cdots & \cdots \\ \overline{\mathbf{Y}}^T\mathbf{G}\overline{\mathbf{Z}} & \vdots & \overline{\mathbf{Y}}^T\mathbf{G}\overline{\mathbf{Y}} \end{bmatrix}$ which indicates that the repartitioning procedure is correct.

(c) Updating $\mathbf{g}_Q$:

$$\overline{\mathbf{g}}_Q = \overline{\mathbf{Q}}^T\mathbf{g} = \tilde{\mathbf{Q}}^T\mathbf{Q}\mathbf{g} = \begin{bmatrix} \mathbf{I} & \vdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{P}^T \end{bmatrix} \begin{bmatrix} \mathbf{g}_Z \\ \cdots \\ \mathbf{g}_Y \end{bmatrix} = \begin{bmatrix} \mathbf{g}_Z \\ \cdots \\ \mathbf{P}^T\mathbf{g}_Y \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{g}}_Z \\ \cdots \\ \overline{\mathbf{g}}_Y \end{bmatrix}$$

where $\overline{\mathbf{g}}_Z$ is an (n-m+1) vector and $\overline{\mathbf{g}}_Y$ is an (m-1) vector. Hence, compute $\mathbf{P}^T\mathbf{g}_Y$ and repartition.

Computational algorithm

The computational algorithm to find the solution ($\hat{\mathbf{x}}, \hat{\Lambda}$) of an equality constrained QP is explained below.

Step 0: Preliminaries

(i) When m = 0, let $\mathbf{Q} = \mathbf{I}$, $\mathbf{Z} = \mathbf{I}$, $\mathbf{g}_Q = \mathbf{g}$ and $\mathbf{U}$ the Cholesky factor of $\mathbf{G}$, i.e. $\mathbf{G} = \mathbf{U}^T\mathbf{U}$. $\mathbf{T}$, $\mathbf{Y}$, $\mathbf{G}_{YY}$ and $\mathbf{G}_{ZY}$ are empty matrices. Go to step 3.

(ii) When a constraint is added to the active set, update $\mathbf{Q}$, $\mathbf{T}$, $\mathbf{U}$, $\mathbf{G}_{YY}$, $\mathbf{G}_{ZY}$ and $\mathbf{g}_Q$.

(iii) When a constraint is deleted from the active set, update $\mathbf{Q}$, $\mathbf{T}$, $\mathbf{U}$, $\mathbf{G}_{YY}$, $\mathbf{G}_{ZY}$ and $\mathbf{g}_Q$.

(iv) Test for linear dependence of the constraints in the active set. If the constraints are linearly independent, go to step 2. Otherwise, calculate the coefficients, $\mathbf{r}$, that express the linear dependence. Go to step 9.

Step 1: Initialization

(i) Check for errors in the input dimensions. Go to step 9 if an error is encountered.

Step 2: Solve for $\hat{\mathbf{p}}_Y$

(i) Solve $\mathbf{T}\mathbf{p}_Y = \mathbf{b}$ for $\hat{\mathbf{p}}_Y$.

(ii) If m = n, go to step 5.

Step 3: Compute $\mathbf{G}_{ZY}\hat{\mathbf{p}}_Y + \mathbf{g}_Z$

(i) Let $\tilde{\mathbf{g}} = \mathbf{G}_{ZY}\hat{\mathbf{p}}_Y + \mathbf{g}_Z$.

Step 4: Solve for $\hat{\mathbf{p}}_Z$

(i) Solve $\left(\mathbf{U}^T\mathbf{U}\right)\mathbf{p}_Z = -\tilde{\mathbf{g}}$ for $\hat{\mathbf{p}}_Z$.

(Note: When m = 0, solve $\mathbf{Gx} = -\mathbf{g}$ for $\hat{\mathbf{x}}$.)

Step 5: Compute $\mathbf{G}_{ZY}^T\hat{\mathbf{p}}_Z + \mathbf{G}_{YY}^T\hat{\mathbf{p}}_Y + \mathbf{g}_Y$

(i) If m = 0, go to step 7.

(ii) Let $\mathbf{d} = \mathbf{G}_{ZY}^T\hat{\mathbf{p}}_Z + \mathbf{G}_{YY}^T\hat{\mathbf{p}}_Y + \mathbf{g}_Y$

Step 6: Solve for the Lagrange multipliers $\hat{\Lambda}$

(i) Solve $\mathbf{T}^T\Lambda = \mathbf{d}$ for $\hat{\Lambda}$

Step 7: Solve for $\hat{\mathbf{x}}$

(i) If it is not necessary to calculate $\hat{\mathbf{x}}$, go to step 8.

(ii) Calculate $\hat{\mathbf{x}} = \mathbf{Qp} = \mathbf{Z}\hat{\mathbf{p}}_Z + \mathbf{Y}\hat{\mathbf{p}}_Y$

Step 8: Calculate the objective function q($\hat{\mathbf{x}}$)

(i) Calculate $q(\hat{\mathbf{x}}) = 0.5\left(\hat{\mathbf{p}}_Y^T\mathbf{d} + \mathbf{g}_Q^T\hat{\mathbf{p}}\right) = 0.5\left(\mathbf{g}_Z^T\hat{\mathbf{p}}_Z + \left(\mathbf{g}_Y + \mathbf{d}\right)^T\hat{\mathbf{p}}_Y\right)$

Step 9: Exit subroutine

(i) Stop.

G.4 DUAL ACTIVE SET ALGORITHM

The algorithm given below follows the dual approach described in the previous sections for convex quadratic programming problems (refer to Goldfarb and Idnani [83GO1m] and Powell [83PO2m, 85PO1m]).

Step 0: Initialization

(i) Check for errors in the input dimensions (e.g. $m_{eq} > n$)

(ii) Find the reciprocals of the lengths of the constraint normals. Go to step 9 if a constraint is infeasible due to a zero normal.

(iii) Store the original **G**-matrix and the g-vector.

(iv) Form the Cholesky decomposition of **G**. If **G** is not positive definite, go to step 9.

(v) Find the unconstrained minimum of problem (G.1). Go to step 5.

Step 1: Solve the equality constrained QP problem (section G.3)

(i) Solve problem (G.7), the equality constrained QP problem.

(ii) If an error occurs in the input dimensions of the routine that solves the equality constrained QP problem, go to 9.

(iii) If the constraints in the active set are linearly dependent, go to step 7.

(iv) If the calculation is successful, go to step 2.

Step 2: Test for dual feasibility

(i) If all the Lagrange multipliers of the inequality constraints contained in the current active (working) set are greater than or equal to zero, dual feasibility is achieved. Go to step 5.

(ii) If one or more of the Lagrange multipliers contained in the current active set is negative, go to step 3.

Step 3: Determine the largest step size to retain dual feasibility (partial step)

(i) Perform a line search with the Lagrange multipliers of the inequality constraints contained in the current active set and determine the first inequality constraint to become inactive (Lagrange multiplier equals zero, refer to equation (G.6)).

(ii) Calculate the new Lagrange multipliers with the partial step taken in the dual space.

Step 4: Drop the inactive inequality constraint from the active set

(i) Drop the inequality constraint which becomes inactive (Lagrange multiplier equals zero) from the current active set (reduce the active set).

(ii) Store the values of the Lagrange multipliers of the inequality constraints in the reduced active set.

(iii) Go to step 1.

Step 5: Test for primal feasibility

(i) If the number of constraints in the active set equals the total number of constraints, the current solution is both feasible and optimal. Go to step 8.

(ii) Test all the inactive constraints for feasibility.

(iii) If no constraint violations occur, the current solution is both feasible and optimal. Go to step 8.

(iv) If one or more of the constraints is violated, a feasible solution has not yet be found. Store the index of the most violated constraint and go to step 6.

<u>Step 6</u>: Add the violated constraint to the active set

(i) Add the violated constraint to the active set (enlarge the active set).

(ii) Set the Lagrange multiplier of this constraint equal to zero.

(iii) Store the values of the Lagrange multipliers of the inequality constraints in the enlarged active set.

(iv) Go to step 1.

<u>Step 7</u>: Linear dependent active set

(i) If a violated equality constraint in the current active set is linear dependent on the other constraints in the active set, the QP problem is infeasible. (An equality constraint, once added to the active set, can never be dropped.) Go to step 9.

(ii) If the constraint violation and the coefficient expressing the linear dependence of the constraints have similar signs (refer to [83GO1m], theorem 2). The QP problem is thus infeasible. Go to step 9.

(iii) Drop the inequality constraint with the largest coefficient expressing the linear dependence from the current active set i.e. reduce the active set (step in the dual space).

(iv) Store the values of the Lagrange multipliers of the inequality constraints in the reduced active set.

(v) Go to step 1.

<u>Step 8</u>: Store the Lagrange multipliers

(i) Store the Lagrange multipliers of the active constraints. The inactive constraints have zero Lagrange multipliers.

<u>Step 9</u>: Return to caller program

(i) Stop, return to the caller program and indicate the reason for the return. The following cases can occur:

(a) Successful completion of calculations

(b) Error in input dimensions

(c) Inconsistent constraints

(d) Infeasible QP problem

(e) Matrix **G** is not positive definite

G.5 EXTENSIONS TO THE DUAL ACTIVE SET ALGORITHM

The dual algorithm described in the previous section is modified in several ways which result in computational advantageous for application with the SQP method and as a general solution technique. Gill et al. [85GI1m, 86GI1m] discuss the benefits (e.g. reduction in linear algebra computations) of designing and implementing a specialized QP algorithm intended for use within SQP methods. These developments are motivated by the special features of the QP subproblems associated with the SQP methods. A specialized QP algorithm can also be used on general problems with equal success. The modifications are:

(1) The **x**-values are only calculated when necessary. The Lagrange multipliers are used as the decision variables in the dual active set algorithm and the **x**-values are only needed for testing primal feasibility.

(2) Goldfarb and Idnani [83GO1m] remarked that the active set at the optimal solution of the QP subproblem in Powell's SQP method [82PO2m] tends not to change very much from one iteration to the next. This means that a good estimate of the active set of the QP subproblem will often be available before the QP subproblem is solved. The major work within a general QP algorithm is to identify the correct active set.

To take advantage of this, the dual algorithm is executed in two passes on every iteration of the SQP algorithm. An active set, containing only the equality constraints are assumed for the first iteration of the SQP method (initialization). In the first pass all the active constraints of the previous iteration of the SQP method are considered when testing for primal feasibility. The first violated (normalized violation), rather than the most violated constraint is added to the current active set. In the second pass all the inactive constraints are tested for primal feasibility and the dual algorithm is started from the optimal solution obtained in the first pass. The most violated constraint (normalized violation) is added to the active set in the second pass. The indices of the active constraints of the previous SQP iteration must thus be available for its next iteration.

This procedure where the active set at the solution of each QP subproblem is used as a prediction of the working set of the next, is referred to as the "warm start" procedure. Since the active sets eventually do not change, the effect of the warm start procedure is that later QP subproblems reach optimality in only one iteration. Using warm starts can greatly reduce the QP problem solution time [78GI1m, 83GO1m, 85BI1e, 85GI1m, 86GI1m].

(3) All the constraint equations are scaled to a magnitude of about unity by using some typical values of problem parameters. With this approach it is easier to check for linear dependence of the constraint gradients during the solution process. The QP subproblem is also solved in terms of the dual variables. These schemes make the QP problems well scaled and numerically stable. The normalized constraint violations, rather than the unscaled constraint violations, are compared with each other in order to determine which constraint to add to the active set.

(4) The constraint updating formulae use stable and efficient updating procedures (also refer to [94KA1e]). Givens rotations are used to introduce zeros in the required positions of the matrices to be updated when the active set changes from one iteration to the next. Powell [85PO1m] states that the Goldfarb and Idnani [83GO1m] implementation of the dual algorithm can become unstable under certain conditions. Powell [83PO2m, 85PO1m] implemented this dual QP algorithm for convex quadratic programming calculations.

Schmid and Biegler [94SC1e] discuss several modifications made to the dual algorithm of Goldfarb and Idnani [83GO1m] when it is used with reduced Hessian SQP methods. These modifications include, for example, a warm start option, the special treatment of the doubly-bounded constraints (e.g. variable bounds) and the handling of infeasible QP problems.

The dual QP algorithm explained in this appendix and ZQPCVX [83PO2m, 85PO1m] will be used in the computational procedures. The major difference between the two algorithms are the updating methods used to account for the addition to and removal of constraints from the active set. Our algorithm employs the methods proposed by Gill et al. [84GI1m], while ZQPCVX uses the methods of Goldfarb and Idnani [83GO1m]. Furthermore, ZQPCVX will be treated as a "black box", whereas our algorithm makes use of the special features of the QP subproblems associated with SQP methods. It is expected that ZQPCVX will be faster than our algorithm due to the matrix updating and transformation methods used. However, our implementation, using orthogonal transformations, will be more stable. ZQPCVX does not employ any of the modifications to the dual QP algorithm as discussed above.

APPENDIX H

SUCCESSIVE QUADRATIC PROGRAMMING ALGORITHM

H.1 INTRODUCTION

A variety of successive quadratic programming (SQP) methods have been developed and used for solving nonlinear programming problems, e.g. [63WI1m, 72BI1m, 76HA1m, 78PO1m, 85NO1m]. SQP methods usually require fewer function and gradient evaluations when compared to other nonlinear programming methods. Various different numerical implementations of the SQP algorithm can be found, e.g. [78PO1m, 82PO2m, 86LI1e, 86SC1m]. The fundamental differences between these algorithms and the numerical implementations based thereon, are the definition of the QP subproblem to be solved at each iteration and the descent function used during the step size calculation. The performances of these algorithms are mainly influenced by the implementation and execution of the above-mentioned differences. In this section we will discuss the numerical implementation of the SQP method as proposed by Powell [78PO1m, 82PO1m, 82PO2m]. Both implementations of Powell are used in this study: [78PO1m] for the reduced Hessian or general SQP methods and [82PO2m] for the general SQP method only.

H.2 ALGORITHM

Powell has written two FORTRAN subroutines VF02AD [78PO1m] and VMCWD [82PO1m, 82PO2m] that implement the Wilson, Han and Powell SQP algorithm. VMCWD is basically an extension of VF02AD to overcome the algorithmic disadvantages of cycling and the Maratos effect [79CH1m, 82CH1m, 82PO1m]. The SQP algorithm can be outlined as follows:

The nonlinear programming (optimization) problem to be solved is formulated as [87FL1m]

$$
\begin{aligned}
&\underset{\mathbf{x}}{\text{Minimize}} \quad f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n) \\
&\text{subject to the constraints} \\
&c_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m_{eq} \\
&c_i(\mathbf{x}) \geq 0, \quad i = m_{eq} + 1, \ldots, m
\end{aligned}
\tag{H.1}
$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a vector of variables, called the decision variables. It is assumed that the bounds on the variables are incorporated into the inequality constraints. The Lagrange function of the problem stated above has the form

$$L(\mathbf{x},\Lambda) = f(\mathbf{x}) - \sum_{i=1}^{m} \lambda_i c_i(\mathbf{x}) \tag{H.2}$$

where $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$ are the Lagrange multipliers (Kuhn-Tucker multipliers) of the constraints. The gradient of the Lagrange function is given by

$$\nabla_{\mathbf{x}} L(\mathbf{x},\Lambda) = \nabla_{\mathbf{x}} f(\mathbf{x}) - \sum_{i=1}^{m} \lambda_i \nabla_{\mathbf{x}} c_i(\mathbf{x}) \tag{H.3}$$

The Hessian of this Lagrange function is given by

$$\nabla_{\mathbf{x}}^2 L(\mathbf{x},\Lambda) = \nabla_{\mathbf{x}}^2 f(\mathbf{x}) - \sum_{i=1}^{m} \lambda_i \nabla_{\mathbf{x}}^2 c_i(\mathbf{x}) \tag{H.4}$$

We are now able to state Powell's implementations of the SQP algorithm (VF02AD and VMCWD):

<u>Step 0</u>: Initialization

Set the iteration counter k to 0

Initialize the approximation to the Hessian of the Lagrange function of the nonlinear programming problem, $\mathbf{B}^{(0)} = \mathbf{I}$

Choose a starting point $\mathbf{x} = \mathbf{x}^{(0)}$

Choose a convergence tolerance, ε

<u>Step 1</u>: Compute the objective function and its gradient, the constraints, and the Jacobian matrix

Evaluate $f^{(k)}, \nabla f^{(k)}, \mathbf{c}^{(k)}, \nabla \mathbf{c}^{(k)}$ at $\mathbf{x}^{(k)}$

If $k < 1$ go to step 3; otherwise go to step 2.

<u>Step 2</u>: Update the approximation to the Hessian of the Lagrange function

The BFGS quasi-Newton update formula with Powell's positive definite correction is used to update $\mathbf{B}$ [78PO1m]. The BFGS update builds the approximation to $\nabla_{\mathbf{x}}^2 L(\mathbf{x},\Lambda)$ directly using only first order information. On the first iteration this matrix is initialized to the identity matrix. Thereafter, the following formula is used:

$$\mathbf{B}^{(k)} = \mathbf{B}^{(k-1)} - \frac{\mathbf{B}^{(k-1)}\delta^{(k-1)}\delta^{(k-1)T}\mathbf{B}^{(k-1)}}{\delta^{(k-1)T}\mathbf{B}^{(k-1)}\delta^{(k-1)}} + \frac{\eta^{(k)}\eta^{(k)T}}{\delta^{(k-1)T}\eta^{(k)}}$$

where

k = the iteration counter

$\delta^{(k-1)} = \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} = \alpha^{(k-1)}\mathbf{s}^{(k-1)}$

$\gamma^{(k)} = \nabla_{\mathbf{x}} L\left(\mathbf{x}^{(k)}, \Lambda^{(k-1)}\right) - \nabla_{\mathbf{x}} L\left(\mathbf{x}^{(k-1)}, \Lambda^{(k-1)}\right)$

$\eta^{(k)} = \theta\gamma^{(k)} + (1.0 - \theta)\mathbf{B}^{(k-1)}\delta^{(k-1)}$

$\theta = 1.0 \quad \text{if} \quad \delta^{(k-1)T}\gamma^{(k)} \geq 0.2\,\delta^{(k-1)T}\mathbf{B}^{(k-1)}\delta^{(k-1)}$

$$\theta = \frac{0.8\,\delta^{(k-1)T}\mathbf{B}^{(k-1)}\delta^{(k-1)}}{\delta^{(k-1)T}\mathbf{B}^{(k-1)}\delta^{(k-1)} - \delta^{(k-1)T}\gamma^{(k)}} \quad \text{otherwise}$$

Step 3: Solve the QP subproblem and evaluate the Lagrange multipliers

Powell proposes the following QP subproblem to be solved at each iteration:

$$\underset{\mathbf{s}}{\text{Minimize}} \quad \nabla f\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} + 0.5\,\mathbf{s}^{(k)T}\mathbf{B}^{(k)}\mathbf{s}^{(k)}$$

subject to the linearized constraints

$$c_i\left(\mathbf{x}^{(k)}\right) + \nabla c_i\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} = 0, \quad i = 1,\ldots,m_{eq}$$

$$c_i\left(\mathbf{x}^{(k)}\right) + \nabla c_i\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} \geq 0, \quad i = m_{eq}+1,\ldots,m$$

The solution of the QP subproblem gives the search direction at the current iteration, $\mathbf{s}^{(k)}$, and the Lagrange multipliers, $\Lambda^{(k)}$. Powell's SQP implementations used a primal quadratic programming package contained in the Harwell subroutine library.

Powell observed that the linearized constraints can be inconsistent, even if the nonlinear constraints of the original problem are consistent and define a feasible region and a solution. Powell has recommended the introduction of a dummy (feasibility) variable, ξ, such that the QP subproblem becomes:

$$\underset{\mathbf{s},\xi}{\text{Minimize}} \quad \nabla f\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} + 0.5\,\mathbf{s}^{(k)T}\mathbf{B}^{(k)}\mathbf{s}^{(k)} - a\xi$$

subject to the linearized constraints

$$\xi c_i\left(\mathbf{x}^{(k)}\right) + \nabla c_i\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} = 0, \quad i = 1,\ldots,m_{eq}$$

$$\xi_i c_i\left(\mathbf{x}^{(k)}\right) + \nabla c_i\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)} \geq 0, \quad i = m_{eq}+1,\ldots,m$$

$$0 \leq \xi \leq 1$$

where a is set to a large positive number (10^6, for example). $\xi_i = 1$ for the inequality constraints that are satisfied, and $\xi_i = \xi$ otherwise. ξ is made as large as possible within the range $0 \leq \xi \leq 1$

by setting the gradient of the objective function with respect to ξ to a large negative number. The elements in the extra column and row of **B** for ξ are set to zero. If $\xi = 1$, the original problem is obtained. If no solution is found for the QP, $\xi < 1$ will allow additional freedom to find a search direction to the modified problem. If no solution can be found with $\xi > 0$, then the algorithm fails, i.e. $\xi = 0 \Rightarrow \mathbf{s}^{(k)} = 0$.

Step 4: Check for convergence

If the convergence criterion is satisfied, i.e. $\left[\left|\nabla f\left(\mathbf{x}^{(k)}\right)^T \mathbf{s}^{(k)}\right| + \left|\sum_{i=1}^{m} \lambda_i c_i\left(\mathbf{x}^{(k)}\right)\right|\right] \leq \varepsilon$, set $\mathbf{x}^* = \mathbf{x}^{(k)}$ and $\Lambda^* = \Lambda^{(k)}$. Go to step 6.

Step 5: Calculate the line search step length parameter

Perform a line search along the search direction, $\mathbf{s}^{(k)}$, to determine the value of $\alpha^{(k)} \in [0,1]$ such that the chosen merit function satisfies the following inequality:

$$\Psi\left(\mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}, \tilde{\lambda}^{(k)}\right) \leq \Psi\left(\mathbf{x}^{(k)}, \tilde{\lambda}^{(k)}\right) + 0.1\alpha^{(k)}\beta^{(k)}$$

where $\beta^{(k)}$ is an approximation to $\left(\frac{\partial \Psi}{\partial \alpha}\right)_{\alpha=0}$ $\left(\beta^{(k)} \approx \Psi\left(\mathbf{x}^{(k)} + \mathbf{s}^{(k)}, \tilde{\lambda}^{(k)}\right) - \Psi\left(\mathbf{x}^{(k)}, \tilde{\lambda}^{(k)}\right)\right)$.

In the line search a step length of one is tried initially, but it is reduced if the above inequality is not satisfied. A quadratic approximation of the merit function using two function values and its slope at the current point is constructed. A suitable value of $\alpha^{(k)}$ is obtained by means of quadratic interpolation and its value is not reduced by more than 10% per line search. If five step length reductions are insufficient, there is an error return from the subroutine. The line search uses a L_1-exact penalty function as a merit function, i.e.

$$\Psi(\mathbf{x}, \tilde{\lambda}) = f(\mathbf{x}) + \sum_{i=1}^{m_{eq}} \tilde{\lambda}_i |c_i(\mathbf{x})| + \sum_{i=m_{eq}+1}^{m} \tilde{\lambda}_i \left|\max[0, -c_i(\mathbf{x})]\right|$$

where $\tilde{\lambda}_i$ are the constraint weighting factors which are functions of the Lagrange multipliers at the solution of the QP. On the first iteration, they are set equal to the absolute value of the Lagrange or Kuhn-Tucker multipliers, i.e. $\tilde{\lambda}_i = |\lambda_i|$, $i = 1, \ldots, m$. After the first iteration,

$$\tilde{\lambda}_i^{(k)} = \max\left\{\left|\lambda_i^{(k)}\right|, 0.5\left(\tilde{\lambda}_i^{(k-1)} + \left|\lambda_i^{(k)}\right|\right)\right\}$$

The requirement of a decrease in the merit function at every iteration may sometimes inhibit the superlinear rate of convergence of the SQP method. This phenomenon is also known as the Maratos effect and causes slow convergence. Furthermore, the technique for adjusting the constraint weighting factors during line search may give rise to cycling. Powell [82CH1m, 82PO1m, 82PO2m] implemented the watchdog technique in VMCWD to remedy these disadvantages. Here the step size is chosen by reducing either the Lagrange function or the line search objective function during the line search. This technique allows the line search objective function to increase on some iterations. However, the line search objective function must be reduced every t iterations ($t \geq 2$); otherwise a restart is required from the previous point if no reduction occurs.

The choice and adjustment of the constraint weighting factors, $\tilde{\lambda}$, are also done according to different prescribed conditions in VMCWD: Before the first iteration all the values of $\tilde{\lambda}$ are given tiny positive values, in case the objective function is constant. If the search direction $s^{(k)}$ of an iteration satisfies the condition

$$\mathbf{s}^{(k)T}\nabla f\left(\mathbf{x}^{(k)}\right)+0.5\,\mathbf{s}^{(k)T}\mathbf{B}^{(k)}\mathbf{s}^{(k)}\leq 0$$

then no change is made to $\tilde{\lambda}^{(k)}$. Otherwise we require the inequality

$$\sum_{i=1}^{m_{eq}}\tilde{\lambda}_i^{(k)}\left|c_i\left(\mathbf{x}^{(k)}\right)\right|+\sum_{i=m_{eq}+1}^{m}\tilde{\lambda}_i^{(k)}\max\left[0,-c_i\left(\mathbf{x}^{(k)}\right)\right]-\sum_{i=1}^{m_{eq}}\tilde{\lambda}_i^{(k)}\left|c_i\left(\mathbf{x}^{(k)}\right)+\mathbf{s}^{(k)T}\nabla c_i\left(\mathbf{x}^{(k)}\right)\right|$$
$$-\sum_{i=m_{eq}+1}^{m}\tilde{\lambda}_i^{(k)}\max\left[0,-c_i\left(\mathbf{x}^{(k)}\right)-\mathbf{s}^{(k)T}\nabla c_i\left(\mathbf{x}^{(k)}\right)\right]\geq\beta^{(k)}\left|\mathbf{s}^{(k)T}\nabla f\left(\mathbf{x}^{(k)}\right)\right|$$

to hold, where $\beta^{(k)}$ is a positive constant. If the $\tilde{\lambda}^{(k)}$ that is set at the beginning of the iteration satisfies this condition when $\beta^{(k)}=1.5$, then $\tilde{\lambda}^{(k)}$ is not altered. Otherwise the following procedure is used to obtain a value of $\tilde{\lambda}^{(k)}$ such that the above inequality holds for $\beta^{(k)}=2.0$. The increase in $\beta^{(k)}$ ensures that at least one component of $\tilde{\lambda}^{(k)}$ is multiplied by a number that exceeds 1.333.

For $i = 1,\ldots,m$ we let λ_i be the Lagrange multiplier of the i-th constraint at the solution of the QP subproblem that determines $s^{(k)}$. For each i we leave $\tilde{\lambda}_i$ unchanged if increasing its value would not increase the left hand side of the above inequality. Otherwise $\tilde{\lambda}_i$ is given the value

$$\tilde{\lambda}_{i(new)} = \max\left[\tilde{\lambda}_{i(old)}, \nu\left|\lambda_i\right|\right]$$

where the positive parameter ν, which is independent of i, is determined by the condition that the above inequality is satisfied as an equation when $\beta^{(k)} = 2.0$.

After a suitable choice of $\alpha^{(k)}$, the next point, $\mathbf{x}^{(k+1)}$, is calculated from

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha^{(k)}\mathbf{s}^{(k)}$$

Increment the iteration counter k, i.e. k = k + 1

Go to step 1 and repeat the calculations.

Step 6: Return to the caller program

Stop, return to the caller program and indicate the reason for the return. The following cases can occur:

(a) Optimization completed successfully
(b) Maximum number of function calls reached
(c) Line search required 5 function calls
(d) Uphill search direction calculated
(e) Constraints seem to be inconsistent
(f) Not enough working space reserved
(g) QP algorithm failed to find a solution

Auxiliary subroutines

The user must provide a subroutine called CALCFG to define the objective and constraint functions and their gradients for any vector of variables. The user can also provide his/her own QP subroutine to solve the subproblems at each iteration. Powell's original SQP implementation uses a primal QP method from the Harwell library.

H.3 MODIFICATIONS TO THE SQP ALGORITHM

Several modifications were made to Powell's SQP algorithm, VMCWD [82PO2m], which will be discussed below.

(1) Quadratic programming routine

The solution of the QP subproblem is a major calculation in SQP methods and can affect the overall efficiency of the SQP method. SQP methods require the solution of strictly convex QP subproblems to determine the direction of search at each iteration. Powell [82PO2m] noted that a more accurate, efficient and reliable QP algorithm is desirable for solving the subproblems.

The most significant change to Powell's implementation is the change in the algorithm to solve the QP subproblem. The feasible point primal QP algorithm normally used with Powell's SQP implementation is replaced by a dual QP algorithm for convex quadratic problems based on the method of Goldfarb and Idnani [83GO1m] (refer to Appendix G). The dual QP subproblem has only simple constraints on the dual variables (Lagrange multipliers). The active set seems not to change very much from one iteration to the next iteration [83GO1m].

Gill et al. [85GI1m, 86GI1m] state that substantial gains in efficiency of the linear algebra calculations can result from a suitably tailored QP algorithm for use with SQP methods. The dual QP algorithm explained in Appendix G is extended and includes special features such as the warm start procedure. Since the iterations of a QP method are essentially a search for the correct active set, it is computationally highly desirable to exploit this information. Further discussions on specialized QP algorithms for SQP methods can be found in Gill et al. [85GI1m, 86GI1m].

(2) Evaluation of derivatives

VMCWD requires that the gradients of the objective and constraint functions be calculated even during line search iterations. Since this information is only required on the last line search point (the new point), a modification was made so that the values of the objective function and the constraints and their corresponding gradients can be calculated only when required.

(3) Modification of the objective function

The dual QP algorithm requires a strictly convex objective function. Therefore, the term $-10^6\xi$ added to the objective function in Powell's original code is replaced by $0.5\times10^6\xi^2-2.0\times10^6\xi$ as suggested by Goldfarb and Idnani [83GO1m]. The elements in the extra column and row of **B** for ξ are set to zero, except for the last element in this column and row which becomes 10^6.

(4) QP algorithm failed to find a solution

When the QP algorithm fails to find a solution, an error return from VMCWD occurs to enable the user to modify some input parameters and restart the optimization process.

(5) Auxiliary subroutines

The user is responsible for the calculation of the objective and constraint functions, as well as their gradients. Therefore, the problem structure should be exploited to perform these calculations as efficiently as possible and the corresponding subroutine should be developed with this goal in mind.

The first-order derivatives that are needed by the SQP method are calculated by means of the finite difference derivative approximations outlined in Dennis and Schnabel [83DE1m].

(6) Large-scale nonlinear optimization problems

Although the SQP method performs very effectively on small to moderately sized problems, the computational and storage requirement can become quite excessive for large sized problems. The SQP method however, can be extended to solve large-scale problems by means of reduced Hessian SQP decomposition methods. The structure and implementation of VMCWD make it too difficult to adjust and use with the reduced Hessian SQP decomposition methods. All the modifications to VF02AD, e.g. the watchdog technique etc., were removed from VMCWD to obtain a representation of Powell's original SQP implementation [78PO1m]. The resulting subroutine is used as the basis for the implementation of the reduced Hessian SQP decomposition methods. All the modifications discussed above are also contained in this SQP algorithm. Reduced Hessian SQP decomposition methods are treated in detail in Appendix I.

Arora [86AR2e, 89AR1e] and Thanedar et al. [86TH1e] discuss a potential constraint strategy in which only a subset of the original constraints is used to define the QP subproblem. This strategy is motivated by the fact that only a subset of the total number of constraints is active at the optimum. Such a strategy only requires gradient computations of those constraints that play a role in deciding the optimum solution. The number of constraints used to define the QP subproblem to be solved at each iteration are drastically reduced when compared to the original problem. These authors state that certain large-scale engineering optimization problems cannot be solved without a potential constraint strategy. This modification is not implemented in the current study, but may prove to be worthwhile for large-scale engineering optimization problems containing very large numbers of inequality constraints.

SQP methods usually require fewer function and gradient evaluations when compared to other nonlinear programming solution techniques. They have desirable properties such as, for example global convergence and only first order information is used in updating the Hessian of the Lagrange function. However, SQP methods must be computationally properly implemented, to enable one to fully exploit their advantages over other solution methods.

APPENDIX I

DECOMPOSITION OF LARGE-SCALE SUCCESSIVE QUADRATIC PROGRAMMING PROBLEMS

I.1 INTRODUCTION

The successive quadratic programming (SQP) method has emerged as the preferred algorithm for solving nonlinear programming (optimization) problems with a small to moderate number of variables. It consistently requires fewer function evaluations per iteration than other solution methods (e.g. reduced gradient methods). The basic step in the SQP algorithm is the formulation and solution of a quadratic programming (QP) problem. Aside from the effort required for function and gradient evaluations, this is the most time consuming step for the algorithm. This operation requires the storage and updating of the Hessian matrix of the Lagrange function of the original nonlinear programming problem at each iteration, which is of the order of the number of decision variables. Therefore, when the size of the nonlinear programming problem becomes large, considerable storage and computational overhead are required. To extend SQP to large systems, two approaches have emerged recently, i.e. either exploiting the natural problem structure or decomposition techniques [80BE1e, 82GA1m, 83LO1e, 85NO1m, 88VA1e, 89GU1m, 90LU1e, 90VA1e, 92BI1e, 93SC1e].

In the first approach, advantage has been taken of the quasi-Newton updates so that they are stored and evaluated in an efficient manner. The QP methods have also been tailored to take advantage of the system sparsity. The second approach exploits the fact that while the optimization problem with m active constraints and n variables can be very large, few degrees of freedom are generally present. Thus, decomposition strategies can be applied to reduce the size (dimensionality) of the quadratic subproblem. This approach considers a much smaller QP problem with a reduced Hessian matrix in the reduced space, where the number of variables is equal to the degrees of freedom of the problem, $n - m$. The reduced Hessian is expected to be positive definite at the solution and can consequently be approximated by positive definite Quasi-Newton formulae, such as BFGS [87FL1m].

This study will concentrate on decomposition strategies since they are better suited to general-purpose problems than the first approach. Powell's implementation of the SQP method [78PO1m] will be used as the basis for the implementation of the reduced Hessian SQP decomposition strategies.

I.2 DECOMPOSITION STRATEGIES FOR SQP

Several SQP decomposition algorithms have been proposed that reduce the size of the QP subproblem by eliminating dependent variables and equality constraints. Berna et al. [80BE1e] and Locke et al. [83LO1e] proposed decomposition strategies that are modifications of Powell's SQP algorithm [78PO1m]. The decomposition of Berna et al. [80BE1e] is difficult to implement and still requires a large amount of storage. The decomposition strategy of Locke et al. [83LO1e] overcomes these difficulties. The variables are partitioned into dependent and independent (decision) variables. The equality constraints are used to "eliminate" the dependent variables and the QP problem is solved in the decision variable space only. Thus the algorithm is similar to the generalized reduced gradient algorithm [87FL1m], which also eliminates the dependent variables and the equality constraints from the optimization problem. Unlike this method, the algorithm proposed by Locke et al. [83LO1e] does not converge the constraints before eliminating them. Rather, it "eliminates" the constraints based on a linear approximation of the constraints, thus performing one Newton-Raphson iteration towards converging the constraints. The decrease in the number of variables in the QP subproblem results in substantial savings in the storage requirements for the approximate Hessian matrix and in the computation time for updating the approximate Hessian matrix. This decomposition strategy uses coordinate bases matrices. While this decomposition method is in itself efficient, it frequently requires more iterations than the full SQP method and in some instances leads to inconsistent convergence results [88VA1e, 93SC1e]. The algorithm of Locke et al. [83LO1e] is also very sensitive to variable partitioning.

Vasantharajan and Biegler [88VA1e] proposed a decomposition strategy using orthogonal bases representations to remedy this difficulty. Although the computational effort per iteration is higher in this case, especially as the number of degrees of freedom increases, the resulting SQP performs better. However, orthogonal projections are not always easy to adapt to the mathematical structure of the model under consideration [93SC1e]. Coordinate bases decomposition strategies are the best suited to take advantage of the underlying mathematical structure of the model. Schmid and Biegler [93SC1e] improved the coordinate bases decomposition strategy of Locke et al. [83LO1e] to guarantee consistent convergence results and reduce its dependence on variable partitioning.

I.3 COORDINATE BASES DECOMPOSITION METHOD

The improved coordinate bases decomposition algorithm will subsequently be explained (refer to [88VA1e, 93SC1e]).

The nonlinear optimization problem to be solved can be formulated as

$$\begin{aligned}
&\underset{\mathbf{x},\mathbf{y}}{\text{Minimize}} \quad f(\mathbf{x},\mathbf{y}) \\
&\text{subject to the constraints} \\
&c_i(\mathbf{x},\mathbf{y}) = 0, \quad i = 1,\ldots,m_{eq} \\
&c_i(\mathbf{x},\mathbf{y}) \geq 0, \quad i = m_{eq}+1,\ldots,m_t \\
&\mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0}
\end{aligned} \tag{I.1}$$

where

f objective function (e.g. cost function)

x (n-r)-component vector of independent variables (e.g. geometrical variables)

y r-component vector of dependent variables (e.g. operating variables)

c_i general constraints (e.g. geometric constraints or feasibility inequalities)

h r-component vector of equality constraints (e.g. balance equations)

r number of equality constraints, **h**

m_{eq} number of general equality constraints

m_t total number of general constraints

n total number of variables

It is assumed that the problem is formulated in such a way that, for fixed **x** values, it is possible to determine **y** (r-component vector) by solving the equality constraints, $\mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0}$, that describe the physical model to be optimized. This procedure is known as the sequential or feasible path method. However, we use the infeasible path integrated approach where **x** and **y** are adjusted simultaneously and all the equations need only to be satisfied at the final solution. It is assumed that any bounds on the variables are incorporated into the inequality constraints.

The Lagrange function of the problem stated above has the form

$$L(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}) = f(\mathbf{x},\mathbf{y}) - \mathbf{u}^T\mathbf{c}(\mathbf{x},\mathbf{y}) - \mathbf{v}^T\mathbf{h}(\mathbf{x},\mathbf{y}) \tag{I.2}$$

where **v** and **u** are the Lagrange multipliers (Kuhn-Tucker multipliers) of the equality and general constraints respectively. SQP is motivated by a Newton method for the solution of the Kuhn-Tucker optimality conditions. This can be shown to be equivalent to the solution of a sequence of QP subproblems. At each major SQP iteration the values of $(\mathbf{x},\mathbf{y})$ are fixed and the resulting QP subproblem has the form:

$$
\begin{aligned}
&\underset{\mathbf{s}}{\text{Minimize}} \quad \nabla f(\mathbf{x},\mathbf{y})^{T}\mathbf{s} + 0.5\,\mathbf{s}^{T}\mathbf{B}\,\mathbf{s} \\
&\text{subject to the linearized constraints} \\
&c_i(\mathbf{x},\mathbf{y}) + \nabla c_i(\mathbf{x},\mathbf{y})^{T}\mathbf{s} = 0, \quad i = 1,\ldots,m_{eq} \\
&c_i(\mathbf{x},\mathbf{y}) + \nabla c_i(\mathbf{x},\mathbf{y})^{T}\mathbf{s} \geq 0, \quad i = m_{eq}+1,\ldots,m_t \\
&\mathbf{h}(\mathbf{x},\mathbf{y}) + \nabla \mathbf{h}(\mathbf{x},\mathbf{y})^{T}\mathbf{s} = \mathbf{0}
\end{aligned}
\tag{I.3}
$$

where **s** is the search direction and **B** is an approximation to the Hessian of the Lagrange function (equation (I.2)). Convexity of problem (I.3) is guaranteed by calculating **B** using the BFGS matrix update formula with Powell damping [78PO1m]. The Lagrange function of this QP subproblem is

$$
\begin{aligned}
\hat{L}(\mathbf{s},\hat{\mathbf{u}},\hat{\mathbf{v}}) = {} & \nabla f(\mathbf{x},\mathbf{y})^{T}\mathbf{s} + 0.5\,\mathbf{s}^{T}\mathbf{B}\,\mathbf{s} - \hat{\mathbf{u}}^{T}\left[\mathbf{c}(\mathbf{x},\mathbf{y}) + \nabla \mathbf{c}(\mathbf{x},\mathbf{y})^{T}\mathbf{s}\right] \\
& - \hat{\mathbf{v}}^{T}\left[\mathbf{h}(\mathbf{x},\mathbf{y}) + \nabla \mathbf{h}(\mathbf{x},\mathbf{y})^{T}\mathbf{s}\right]
\end{aligned}
\tag{I.4}
$$

where $\hat{\mathbf{v}}$ and $\hat{\mathbf{u}}$ are the Lagrange multipliers of the equality and general constraints respectively. These Lagrange multipliers approximate the values of **v** and **u** during the SQP iterations. At the optimal solution the Lagrange multipliers of the subproblem, $\hat{\mathbf{v}}$ and $\hat{\mathbf{u}}$, are exactly equal to the Lagrange multipliers of the original problem, **v** and **u**.

At each iteration, the following first-order Kuhn-Tucker optimality conditions for equation (I.3) (QP subproblem) must be satisfied:

$$
\begin{aligned}
&\nabla f(\mathbf{x},\mathbf{y}) + \mathbf{B}\,\mathbf{s} - \nabla \mathbf{c}(\mathbf{x},\mathbf{y})\,\hat{\mathbf{u}} - \nabla \mathbf{h}(\mathbf{x},\mathbf{y})\,\hat{\mathbf{v}} = 0 \\
&c_i(\mathbf{x},\mathbf{y}) + \nabla c_i(\mathbf{x},\mathbf{y})^{T}\mathbf{s} = 0, \quad i \in A \\
&\hat{u}_i\left[c_i(\mathbf{x},\mathbf{y}) + \nabla c_i(\mathbf{x},\mathbf{y})^{T}\mathbf{s}\right] = 0, \quad i = m_{eq}+1,\ldots,m_t \\
&\hat{u}_i \geq 0, \quad i = m_{eq}+1,\ldots,m_t \\
&\mathbf{h}(\mathbf{x},\mathbf{y}) + \nabla \mathbf{h}(\mathbf{x},\mathbf{y})^{T}\mathbf{s} = \mathbf{0}
\end{aligned}
\tag{I.5}
$$

where A is set of general constraints which are active at the current iteration (satisfied as equalities). The active set consists of m general active constraints. Let $\mathbf{c}_A$ present the vector of general active constraints at each iteration. The above first order necessary conditions for the QP subproblem can be presented in matrix form as follows:

$$\begin{bmatrix} \mathbf{B} & -\nabla \mathbf{c}_A & -\nabla \mathbf{h} \\ -\nabla \mathbf{c}_A^T & 0 & 0 \\ -\nabla \mathbf{h}^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{s} \\ \hat{\mathbf{u}} \\ \hat{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} -\nabla \mathrm{f} \\ \mathbf{c}_A \\ \mathbf{h} \end{bmatrix} \tag{I.6}$$

where $\nabla \mathbf{c}_A$ and $\nabla \mathbf{h}$ are $n \times m$ and $n \times r$ matrices respectively. **B** is a dense $n \times n$ matrix that must be updated, factored and stored at each iteration. As the size of the nonlinear programming problem in equation (I.1) increases, the solution of the QP subproblem becomes increasingly expensive. The performance of this method can be considerably improved through a suitable change of bases representation [88VA1e, 93SC1e]. The new bases vectors are obtained by partitioning the search space into two subspaces which are spanned by the columns of matrices **Z** and **Y**, respectively, where **Z** is chosen so that its columns span the null space of $\nabla \mathbf{h}(\mathbf{x},\mathbf{y})^T$, i.e. $\nabla \mathbf{h}(\mathbf{x},\mathbf{y})^T \mathbf{Z} = \mathbf{0}$ (where **Z** is a n×(n-r) matrix and **Y** is a n×r matrix). It is assumed that the rank of $\nabla \mathbf{h}(\mathbf{x},\mathbf{y})^T$ is r. Several distinct choices of the bases matrices, **Z** and **Y**, can be made such that $[\mathbf{Y} \quad \mathbf{Z}]$ is non-singular (spans the entire search space). These choices include orthonormal, orthogonal and coordinate bases [93SC1e]. After the decomposition, the matrix to be updated is a projection of **B** onto a space whose dimension is given by the number of degrees of freedom of the problem. The actual projected Hessian matrix is expected to be positive definite at the solution, which is sufficient to guarantee optimality.

The variables are partitioned into the dependent variables, $\mathbf{y} \in \Re^{r}$, and the independent variables, $\mathbf{x} \in \Re^{(n-r)}$. The decomposition method that corresponds to the coordinate bases method is obtained when the following choice is made for **Z** and **Y** (**Z** represents the influence of **h(x,y)** = **0** in the reduced QP subproblem):

$$\mathbf{Z} = \begin{bmatrix} \mathbf{I} \\ -\left(\nabla_{\mathbf{y}} \mathbf{h}^T\right)^{-1} \nabla_{\mathbf{x}} \mathbf{h}^T \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix} \tag{I.7}$$

The search direction, s, can be expressed as the sum of its components in the two subspaces, i.e.

$$\mathbf{s} = \mathbf{Y}\mathbf{s}_Y + \mathbf{Z}\mathbf{s}_Z \tag{I.8}$$

Let **Q** be a non-singular matrix of order (n + m + r) given by

$$\mathbf{Q}=\begin{bmatrix}\mathbf{Y} & \mathbf{Z} & 0 & 0\\ 0 & 0 & \mathbf{I} & 0\\ 0 & 0 & 0 & \mathbf{I}\end{bmatrix} \tag{I.9}$$

Premultiplying equation (I.6) by $\mathbf{Q}^{\mathrm{T}}$ and substituting for s (equation (I.8)) yields:

$$\begin{bmatrix}\mathbf{Y}^{\mathrm{T}}\mathbf{BY} & \mathbf{Y}^{\mathrm{T}}\mathbf{BZ} & -\mathbf{Y}^{\mathrm{T}}\nabla\mathbf{c}_A & -\mathbf{Y}^{\mathrm{T}}\nabla\mathbf{h}\\ \mathbf{Z}^{\mathrm{T}}\mathbf{BY} & \mathbf{Z}^{\mathrm{T}}\mathbf{BZ} & -\mathbf{Z}^{\mathrm{T}}\nabla\mathbf{c}_A & 0\\ -\nabla\mathbf{c}_A^{\mathrm{T}}\mathbf{Y} & -\nabla\mathbf{c}_A^{\mathrm{T}}\mathbf{Z} & 0 & 0\\ -\nabla\mathbf{h}^{\mathrm{T}}\mathbf{Y} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{s}_{\mathrm{Y}}\\ \mathbf{s}_{\mathrm{Z}}\\ \hat{\mathbf{u}}\\ \hat{\mathbf{v}}\end{bmatrix}=\begin{bmatrix}-\mathbf{Y}^{\mathrm{T}}\nabla\mathrm{f}\\ -\mathbf{Z}^{\mathrm{T}}\nabla\mathrm{f}\\ \mathbf{c}_A\\ \mathbf{h}\end{bmatrix} \tag{I.10}$$

The Lagrange multipliers of the equality constraints, $\hat{\mathbf{v}}$, can be obtained from the solution of (the first row of equation (I.10)):

$$\mathbf{Y}^{\mathrm{T}}\mathbf{BY}\,\mathbf{s}_{\mathrm{Y}}+\mathbf{Y}^{\mathrm{T}}\mathbf{BZ}\,\mathbf{s}_{\mathrm{Z}}-\mathbf{Y}^{\mathrm{T}}\nabla\mathbf{c}_A\,\hat{\mathbf{u}}-\mathbf{Y}^{\mathrm{T}}\nabla\mathbf{h}\,\hat{\mathbf{v}}=-\mathbf{Y}^{\mathrm{T}}\nabla\mathrm{f} \tag{I.11}$$

Exact values of these multipliers are only required at the solution of the nonlinear programming problem. As the algorithm converges, $\mathbf{s}\rightarrow 0$ and equation (I.11) can be simplified to

$$-\mathbf{Y}^{\mathrm{T}}\nabla\mathbf{h}\,\hat{\mathbf{v}}=-\mathbf{Y}^{\mathrm{T}}\nabla\mathrm{f}+\mathbf{Y}^{\mathrm{T}}\nabla\mathbf{c}_A\,\hat{\mathbf{u}} \tag{I.12}$$

The second and third rows of equation (I.10), namely

$$\begin{aligned}&\mathbf{Z}^{\mathrm{T}}\mathbf{BZ}\mathbf{s}_{\mathrm{Z}}+\left(\mathbf{Z}^{\mathrm{T}}\mathbf{BY}\mathbf{s}_{\mathrm{Y}}+\mathbf{Z}^{\mathrm{T}}\nabla\mathrm{f}\right)-\mathbf{Z}^{\mathrm{T}}\nabla\mathbf{c}_A\hat{\mathbf{u}}=0\\ &\mathbf{c}_{\mathrm{A}}+\nabla\mathbf{c}_{\mathrm{A}}^{\mathrm{T}}\mathbf{Y}\mathbf{s}_{\mathrm{Y}}+\nabla\mathbf{c}_{\mathrm{A}}^{\mathrm{T}}\mathbf{Z}\mathbf{s}_{\mathrm{Z}}=\mathbf{0}\end{aligned} \tag{I.13}$$

are the optimality conditions of the following reduced QP subproblem to be solved at each iteration:

$$\begin{aligned}&\underset{\mathbf{s}_{\mathrm{Z}}}{\text{Minimize}}\quad \left(\mathbf{Z}^{\mathrm{T}}\mathbf{BY}\mathbf{s}_{\mathrm{Y}}+\mathbf{Z}^{\mathrm{T}}\nabla\mathrm{f}\right)^{\mathrm{T}}\mathbf{s}_{\mathrm{Z}}+0.5\,\mathbf{s}_{\mathrm{Z}}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}}\mathbf{BZ}\mathbf{s}_{\mathrm{Z}}\\ &\text{subject to the constraints}\\ &\mathrm{c}_{\mathrm{i}}+\nabla\mathrm{c}_{\mathrm{i}}^{\mathrm{T}}\left(\mathbf{Z}\mathbf{s}_{\mathrm{Z}}+\mathbf{Y}\mathbf{s}_{\mathrm{Y}}\right)=0,\quad \mathrm{i}=1,\ldots,\mathrm{m}_{\mathrm{eq}}\\ &\mathrm{c}_{\mathrm{i}}+\nabla\mathrm{c}_{\mathrm{i}}^{\mathrm{T}}\left(\mathbf{Z}\mathbf{s}_{\mathrm{Z}}+\mathbf{Y}\mathbf{s}_{\mathrm{Y}}\right)\geq 0,\quad \mathrm{i}=\mathrm{m}_{\mathrm{eq}}+1,\ldots,\mathrm{m}_{\mathrm{t}}\end{aligned} \tag{I.14}$$

where $\mathbf{s}_{\mathrm{Y}}$ is obtained from the last row of equation (I.10):

$$\mathbf{s}_{\mathrm{Y}}=-\left(\nabla\mathbf{h}^{\mathrm{T}}\mathbf{Y}\right)^{-1}\mathbf{h} \tag{I.15}$$

The search direction for the dependent variables satisfies the set of linearized equality constraints.

Problem (I.14) may be solved to obtain $\mathbf{s}_{\mathrm{Z}}$ and $\hat{\mathbf{u}}$ (Lagrange multipliers of the general constraints).

The Jacobian matrix of the equality constraints, $\mathbf{h}$, can be written as

$$\nabla\mathbf{h} = \begin{bmatrix} \nabla_x \mathbf{h} \\ \nabla_y \mathbf{h} \end{bmatrix} \tag{I.16}$$

Due to the particular choice of $\mathbf{Y}$, $\mathbf{Y}^T\nabla\mathbf{h}$ is now simply equal to $\nabla_y\mathbf{h}$. Any sparsity and structure of the equations that describe the model is thus maintained and it is thus straightforward to tailor this algorithm to take advantage of the particular structure of certain problems classes which can greatly improve the performance of SQP. An additional benefit is that coordinate bases require less computational effort per iteration than the other decomposition methods.

Equation (I.12) now simplifies to

$$-\nabla_y \mathbf{h}\,\hat{\mathbf{v}} = -\nabla_y f + \nabla_y \mathbf{c}_{\mathcal{A}}\,\hat{\mathbf{u}} \tag{I.17}$$

and equation (I.15) simplifies to

$$\mathbf{s}_Y = -\left(\nabla_y \mathbf{h}^T\right)^{-1} \mathbf{h} \tag{I.18}$$

The coordinate bases decomposition strategy is computationally the cheapest decomposition method and allows us to exploit the particular structure of the model when calculating $\mathbf{s}_Y$ [93SC1e]. This method is therefore preferred for large-scale optimization problems.

The matrix $\mathbf{Z}^T\mathbf{BY}$ contained in the objective function (equation (I.14)) can become too large to store or to compute when r becomes large. The term $\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y$ can usually be neglected in the objective function when orthonormal or orthogonal bases are used [88VA1e, 93SC1e]. The original algorithm proposed by Locke et al. [83LO1e] also did not include this term. However, the performance of the coordinate bases method using an objective function for the QP subproblem that omits this term, depends to a large extent on the partitioning of the variables and can lead to inconsistent performance of the algorithm [87KI1e, 88VA1e, 93SC1e].

Schmid and Biegler [93SC1e] propose an improved coordinate bases SQP algorithm which constructs an approximation of the term $\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y$ to enhance the performance of the original algorithm. Three different approaches are considered, i.e. the finite difference correction, the Broyden update correction and the limited memory Broyden update correction. The finite difference correction method yields the most consistent results, while the Broyden updates are computationally cheaper. Schmid and Biegler [93SC1e] propose an update criterion which allows the algorithm to determine at which iteration it requires the finite difference correction or the Broyden update.

The finite difference correction method will be considered in this study. A first order approximation of $\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y$ is given by a Taylor series expansion about the current point $\mathbf{x}$:

$$\begin{aligned}\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y &\approx \mathbf{Z}^T\nabla^2 L(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v})\mathbf{Y}\mathbf{s}_Y \\ &\approx \mathbf{Z}^T\nabla L(\mathbf{x},\mathbf{y}+\mathbf{s}_Y,\mathbf{u},\mathbf{v}) - \mathbf{Z}^T\nabla L(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v})\end{aligned} \tag{I.19}$$

where

$$\nabla L(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}) \approx \nabla L(\mathbf{x},\mathbf{y},\hat{\mathbf{u}},\hat{\mathbf{v}}) = \nabla f(\mathbf{x},\mathbf{y}) - \nabla \mathbf{c}(\mathbf{x},\mathbf{y})\,\hat{\mathbf{u}} - \nabla \mathbf{h}(\mathbf{x},\mathbf{y})\,\hat{\mathbf{v}} \tag{I.20}$$

Since Z lies in the null space of $\nabla\mathbf{h}^T$, equation (I.19) becomes

$$\begin{aligned}\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y \approx\ & \mathbf{Z}^T\nabla f(\mathbf{x},\mathbf{y}+\mathbf{s}_Y) - \mathbf{Z}^T\nabla f(\mathbf{x},\mathbf{y}) - \left[\mathbf{Z}^T\nabla\mathbf{c}(\mathbf{x},\mathbf{y}+\mathbf{s}_Y) - \mathbf{Z}^T\nabla\mathbf{c}(\mathbf{x},\mathbf{y})\right]\hat{\mathbf{u}} \\ & -\mathbf{Z}^T\nabla\mathbf{h}(\mathbf{x},\mathbf{y}+\mathbf{s}_Y)\,\hat{\mathbf{v}}\end{aligned} \tag{I.21}$$

This correction improves the convergence rate of the coordinate bases method. A disadvantage of this correction method is the additional gradient evaluation to be performed at each iteration.

The Hessian matrix of the reduced QP subproblem, $\mathbf{Z}^T\mathbf{BZ}$, can be approximated by means of the BFGS quasi-Newton update formula as proposed by Powell [78PO1m, 83LO1e, 88VA1e]. $\mathbf{Z}^T\mathbf{BZ}$ is of much smaller dimension than the original full Hessian matrix $\mathbf{B}$. This matrix is initialized to the identity matrix and updated at each iteration.

The BFGS quasi-Newton update formula with Powell's positive definite correction is used to update $\mathbf{Z}^T\mathbf{BZ}$ [78PO1m]. On the first iteration, $\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(0)} = \mathbf{I}$. Thereafter,

$$\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k)} = \left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)} - \frac{\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}\delta^{(k-1)}\delta^{(k-1)T}\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}}{\delta^{(k-1)T}\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}\delta^{(k-1)}} + \frac{\eta^{(k)}\eta^{(k)T}}{\delta^{(k-1)T}\eta^{(k)}} \tag{I.22}$$

where (k) is the iteration counter and

$$\delta^{(k-1)} = \mathbf{x}^{(k)} - \mathbf{x}^{(k-1)} = \alpha^{(k-1)}\mathbf{s}_Z^{(k-1)}$$

$$\gamma^{(k)} = \left(\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y + \mathbf{Z}^T\nabla f\right)^{(k)} - \left(\mathbf{Z}^T\nabla \mathbf{c}_{\mathcal{A}}\right)^{(k)}\hat{\mathbf{u}}^{(k-1)}$$

$$-\left(\mathbf{Z}^T\mathbf{BY}\mathbf{s}_Y + \mathbf{Z}^T\nabla f\right)^{(k-1)} + \left(\mathbf{Z}^T\nabla \mathbf{c}_{\mathcal{A}}\hat{\mathbf{u}}\right)^{(k-1)}$$

$$\eta^{(k)} = \theta\gamma^{(k)} + (1.0-\theta)\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}\delta^{(k-1)}$$

$$\theta = 1.0 \quad \text{if} \quad \delta^{(k-1)T}\gamma^{(k)} \geq 0.2\,\delta^{(k-1)T}\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}\delta^{(k-1)}$$

$$\theta = \frac{0.8\,\delta^{(k-1)T}\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}\delta^{(k-1)}}{\delta^{(k-1)T}\left(\mathbf{Z}^T\mathbf{BZ}\right)^{(k-1)}\delta^{(k-1)} - \delta^{(k-1)T}\gamma^{(k)}} \quad \text{otherwise}$$

The line search step length parameter, $\alpha^{(k)}$, is selected from $\alpha^{(k)} \in [0,1]$ such that the chosen merit function satisfies the following inequality [78PO1m] (refer to Appendix H):

$$\Psi\left(\mathbf{x}^{(k+1)}, \mathbf{y}^{(k+1)}, \tilde{\mathbf{u}}^{(k)}, \tilde{\mathbf{v}}^{(k)}\right) \leq \Psi\left(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}, \tilde{\mathbf{u}}^{(k)}, \tilde{\mathbf{v}}^{(k)}\right) + 0.1\alpha^{(k)}\beta^{(k)} \tag{I.23}$$

where (k) is the iteration counter,

$$\begin{bmatrix} \mathbf{x}^{(k+1)} \\ \mathbf{y}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \mathbf{x}^{(k)} \\ \mathbf{y}^{(k)} \end{bmatrix} + \alpha^{(k)}\mathbf{s}^{(k)} = \begin{bmatrix} \mathbf{x}^{(k)} \\ \mathbf{y}^{(k)} \end{bmatrix} + \alpha^{(k)}\begin{bmatrix} \mathbf{s}_Z^{(k)} \\ \mathbf{s}_Y^{(k)} - \left(\nabla_y \mathbf{h}^{(k)T}\right)^{-1}\nabla_x \mathbf{h}^{(k)T}\mathbf{s}_Z^{(k)} \end{bmatrix} \tag{I.24}$$

and $\beta^{(k)} \approx \Psi\left(\mathbf{x}^{(k)} + \mathbf{s}^{(k)}, \tilde{\lambda}^{(k)}\right) - \Psi\left(\mathbf{x}^{(k)}, \tilde{\lambda}^{(k)}\right)$

The merit function has the form [78PO1m]

$$\Psi(\mathbf{x}, \mathbf{y}, \tilde{\mathbf{u}}, \tilde{\mathbf{v}}) = f(\mathbf{x}, \mathbf{y}) + \sum_{j=1}^{r}\tilde{\mathbf{v}}_j^T\left|\mathbf{h}_j(\mathbf{x}, \mathbf{y})\right| + \sum_{i=1}^{m_{eq}}\tilde{u}_i\left|c_i(\mathbf{x}, \mathbf{y})\right| + \sum_{i=m_{eq}+1}^{m_t}\tilde{u}_i\left|\max\left[0, -c_i(\mathbf{x}, \mathbf{y})\right]\right| \tag{I.25}$$

where $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$ are the constraint weighting factors. On the first iteration, they are set equal to the absolute value of the Lagrange or Kuhn-Tucker multipliers, i.e. $\tilde{\mathbf{v}} = |\hat{\mathbf{v}}|$ and $\tilde{\mathbf{u}} = |\hat{\mathbf{u}}|$. After the first iteration,

$$\begin{aligned} \tilde{\mathbf{u}}^{(k)} &= \max\left\{\left|\hat{\mathbf{u}}^{(k)}\right|, 0.5\left(\tilde{\mathbf{u}}^{(k-1)} + \left|\hat{\mathbf{u}}^{(k)}\right|\right)\right\} \\ \tilde{\mathbf{v}}^{(k)} &= \max\left\{\left|\hat{\mathbf{v}}^{(k)}\right|, 0.5\left(\tilde{\mathbf{v}}^{(k-1)} + \left|\hat{\mathbf{v}}^{(k)}\right|\right)\right\} \end{aligned} \tag{I.26}$$

where (k) is the iteration counter. A value of $\alpha^{(k)} = 1$ is tried initially and it is reduced in an ordered manner if the inequality (equation (I.23)) is not satisfied.

All the equality and inequality constraints are included in the merit function, equation (I.25), of the nonlinear programming problem, because a suitable $\alpha^{(k)}$ that scales the search direction in both $\mathbf{x}$ and $\mathbf{y}$ (satisfy equation (I.23)) must be found. A step in the direction of the solution is given such that convergence of the model equations and the optimization problem will be reached simultaneously.

I.4 APPLICATION TO PRACTICAL PROBLEM

The problem formulation presented in equation (I.1) can be classified as the infeasible path integrated approach, where the simulation (performance evaluation) model is included directly as a set of equality constraints, $\mathbf{h}(\mathbf{x},\mathbf{y}) = \mathbf{0}$, in the problem formulation and the operating variables, $\mathbf{y}$, are considered as decision variables [93SC1e]. These model equations are then solved as part of the optimization process. The operating variables, $\mathbf{y}$, depend on the independent geometrical variables, $\mathbf{x}$. This formulation tends to disguise the variable dependence.

For the particular practical application under consideration (minimization of the power generation cost), the vector of operating variables can be expanded as follows:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_p \end{bmatrix} = \begin{bmatrix} y_1^{(1)} & \cdots & y_1^{(q)} & y_2^{(1)} & \cdots & y_2^{(q)} & \cdots & y_p^{(1)} & \cdots & y_p^{(q)} \end{bmatrix}^T \tag{I.27}$$

where q is the number of variables contained in each vector, $\mathbf{y}_j$, $j = 1,\ldots,p$ and $p \times q = r$. The partitioning of the variables into dependent and independent variables is also fixed in this particular practical application. The performance evaluation model's equality constraints can be expanded as follows:

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_p \end{bmatrix}^T = \begin{bmatrix} h_1^{(1)} & \cdots & h_1^{(q)} & h_2^{(1)} & \cdots & h_2^{(q)} & \cdots & h_p^{(1)} & \cdots & h_p^{(q)} \end{bmatrix}^T \tag{I.28}$$

The structure of the performance evaluation model is such that $\mathbf{h}_j, j = 1,\ldots,p$, is a function of $(\mathbf{x},\mathbf{y}_j)$ only. This can be mathematically expressed as:

$$\mathbf{h}_j(\mathbf{x},\mathbf{y}_j) = 0, \text{ or } h_j^{(v)}\left(\mathbf{x}, y_j^{(1)}, y_j^{(2)}, \ldots, y_j^{(q)}\right) = 0, \quad j = 1,\ldots,p;\ v = 1,\ldots,q \tag{I.29}$$

Equation (I.1) can thus be reformulated as

$$\begin{aligned}
&\underset{\mathbf{x},\mathbf{y}_1,\ldots,\mathbf{y}_p}{\text{Minimize}} \quad f(\mathbf{x},\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_p) \\
&\text{subject to the constraints} \\
&c_i(\mathbf{x},\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_p) = 0, \quad i = 1,\ldots,m_{eq} \\
&c_i(\mathbf{x},\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_p) \geq 0, \quad i = m_{eq}+1,\ldots,m_t \\
&\mathbf{h}_j(\mathbf{x},\mathbf{y}_j) = 0, \quad j = 1,\ldots,p
\end{aligned} \tag{I.30}$$

Equation (I.16) can now be rewritten as

$$\begin{aligned}
\nabla\mathbf{h} &= \begin{bmatrix} \nabla_x\mathbf{h} \\ \nabla_y\mathbf{h} \end{bmatrix} = \begin{bmatrix} \nabla_x\mathbf{h}_1 & \nabla_x\mathbf{h}_2 & \cdots & \nabla_x\mathbf{h}_p \\ \nabla_y\mathbf{h}_1 & \nabla_y\mathbf{h}_2 & \cdots & \nabla_y\mathbf{h}_p \end{bmatrix} \\
&= \begin{bmatrix} \nabla_x\mathbf{h}_1 & \nabla_x\mathbf{h}_2 & \cdots & \nabla_x\mathbf{h}_p \\ \nabla_{y_1}\mathbf{h}_1 & \nabla_{y_1}\mathbf{h}_2 & \cdots & \nabla_{y_1}\mathbf{h}_p \\ \nabla_{y_2}\mathbf{h}_1 & \nabla_{y_2}\mathbf{h}_2 & \cdots & \nabla_{y_2}\mathbf{h}_p \\ \vdots & \vdots & \vdots & \vdots \\ \nabla_{y_p}\mathbf{h}_1 & \nabla_{y_p}\mathbf{h}_2 & \cdots & \nabla_{y_p}\mathbf{h}_p \end{bmatrix} = \begin{bmatrix} \nabla_x\mathbf{h}_1 & \nabla_x\mathbf{h}_2 & \cdots & \nabla_x\mathbf{h}_p \\ \nabla_{y_1}\mathbf{h}_1 & 0 & \cdots & 0 \\ 0 & \nabla_{y_2}\mathbf{h}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nabla_{y_p}\mathbf{h}_p \end{bmatrix}
\end{aligned} \tag{I.31}$$

and

$$\mathbf{Y}^T\nabla\mathbf{h} = \nabla_y\mathbf{h} = \begin{bmatrix} \nabla_{y_1}\mathbf{h}_1 & 0 & \cdots & 0 \\ 0 & \nabla_{y_2}\mathbf{h}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nabla_{y_p}\mathbf{h}_p \end{bmatrix} \tag{I.32}$$

Furthermore, $\mathbf{s}_Y$ can be obtained by solving the following systems of equations

$$\begin{bmatrix} \nabla_{y_1}\mathbf{h}_1^T & 0 & \cdots & 0 \\ 0 & \nabla_{y_2}\mathbf{h}_2^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nabla_{y_p}\mathbf{h}_p^T \end{bmatrix} \begin{bmatrix} \mathbf{s}_Y^{(1)} \\ \mathbf{s}_Y^{(2)} \\ \vdots \\ \mathbf{s}_Y^{(p)} \end{bmatrix} = -\begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_p \end{bmatrix} \tag{I.33}$$

which decomposes into p smaller systems: $\nabla_{y_i}\mathbf{h}_i^T\mathbf{s}_Y^{(i)} = -\mathbf{h}_i$, $i = 1,\ldots,p$.

$\mathbf{Z}$ can be determined by solving $\left(\nabla_y\mathbf{h}^T\right)\mathbf{A} = -\nabla_x\mathbf{h}^T$ for $\mathbf{A} = -\left(\nabla_y\mathbf{h}^T\right)^{-1}\nabla_x\mathbf{h}^T$, i.e.

$$\begin{bmatrix} \nabla_{y_1}\mathbf{h}_1^T & 0 & \cdots & 0 \\ 0 & \nabla_{y_2}\mathbf{h}_2^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nabla_{y_p}\mathbf{h}_p^T \end{bmatrix} \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_p \end{bmatrix} = - \begin{bmatrix} \nabla_x\mathbf{h}_1^T \\ \nabla_x\mathbf{h}_2^T \\ \vdots \\ \nabla_x\mathbf{h}_p^T \end{bmatrix} \tag{I.34}$$

Thus,

$$\mathbf{Z} = \begin{bmatrix} \mathbf{I} \\ \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_p \end{bmatrix} \tag{I.35}$$

The above problem simplifications are possible due to the specific structure of the original engineering optimization problem. The block diagonal structure in equations (I.33) and (I.34) provide great computational advantages (e.g. solution techniques and storage requirements).

I.5 ALGORITHM

The SQP algorithm of Powell with its modifications ([78POlm], also refer to Appendix H), is used as the basis for the implementation of the coordinate bases method. We are now able to state the coordinate bases algorithm:

Step 0: Initialization

Set the iteration counter k to 0

Initialize the projected Hessian $\left(\mathbf{Z}^T\mathbf{B}\mathbf{Z}\right)^{(0)} = \mathbf{I}$

Set $\left(\mathbf{Z}^T\mathbf{B}\mathbf{Y}\mathbf{s}_Y\right)^{(0)} = 0$

Choose a starting point $\mathbf{x} = \mathbf{x}^{(0)}, \mathbf{y} = \mathbf{y}^{(0)}$

Choose a convergence tolerance, ε

Step 1: Compute the objective function, constraints, Jacobian matrices, coordinate bases matrices, and the range space search direction

Increment the iteration counter k, i.e. k = k + 1

Evaluate $f^{(k)}, \nabla f^{(k)}, \mathbf{h}^{(k)}, \nabla\mathbf{h}^{(k)}, \mathbf{c}^{(k)}, \nabla\mathbf{c}^{(k)}$ at $\left(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}\right)$

Solve $\left(\nabla_{\mathbf{y}}\mathbf{h}^{(k)T}\right)\mathbf{A}^{(k)} = -\nabla_{\mathbf{x}}\mathbf{h}^{(k)T}$ for $\mathbf{A}^{(k)} = -\left(\nabla_{\mathbf{y}}\mathbf{h}^{(k)T}\right)^{-1}\nabla_{\mathbf{x}}\mathbf{h}^{(k)T}$

Set $\mathbf{Z}^{(k)} = \begin{bmatrix}\mathbf{I}\\ \mathbf{A}^{(k)}\end{bmatrix}$, $\mathbf{Y}^{(k)} = \begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}$

Solve $\left(\nabla_{\mathbf{y}}\mathbf{h}^{(k)T}\right)s_Y^{(k)} = -\mathbf{h}^{(k)}$ for $s_Y^{(k)}$. The above two systems of equations can be solved simultaneously by performing only one LU-decomposition of the augmented matrix $\left[\nabla_{\mathbf{y}}\mathbf{h}^{(k)T} \;\vdots\; -\nabla_{\mathbf{x}}\mathbf{h}^{(k)T} \;\vdots\; -\mathbf{h}^{(k)}\right]$.

Step 2: Evaluate the reduced gradients

Evaluate the reduced gradient: $\mathbf{Z}^{(k)T}\nabla f^{(k)} = \nabla_{\mathbf{x}} f^{(k)} + \mathbf{A}^{(k)T}\nabla_{\mathbf{y}} f^{(k)}$

Evaluate $\nabla\mathbf{c}^{(k)T}\mathbf{Z}^{(k)}$: $\nabla\mathbf{c}^{(k)T}\mathbf{Z}^{(k)} = \nabla_{\mathbf{x}}\mathbf{c}^{(k)T} + \nabla_{\mathbf{y}}\mathbf{c}^{(k)T}\mathbf{A}^{(k)}$

Evaluate $\nabla\mathbf{c}^{(k)T}\mathbf{Y}^{(k)}$: $\nabla\mathbf{c}^{(k)T}\mathbf{Y}^{(k)} = \nabla_{\mathbf{y}}\mathbf{c}^{(k)T}$

Step 3: Evaluate the finite difference correction and update the approximate Hessian of the Lagrange function

If $k < 2$ go to step 4.

Evaluate $\left(\mathbf{Z}^T\mathbf{B}\mathbf{Y}s_Y\right)^{(k)}$ by making use of equation (I.21).

Update $\left(\mathbf{Z}^T\mathbf{B}\mathbf{Z}\right)^{(k)}$ by making use of equation (I.22).

Step 4: Solve the reduced QP subproblem and evaluate the Lagrange multipliers

Solve the reduced QP subproblem, equation (I.14), to obtain the null space component of the search direction, $s_Z^{(k)}$, and the Lagrange multipliers of the general constraints, $\hat{\mathbf{u}}^{(k)}$:

$$\underset{s_Z^{(k)}}{\text{Minimize}} \quad \left(\left(\mathbf{Z}^T\mathbf{B}\mathbf{Y}\right)^{(k)}s_Y^{(k)} + \mathbf{Z}^{(k)T}\nabla f^{(k)}\right)^T s_Z^{(k)} + 0.5\, s_Z^{(k)T}\left(\mathbf{Z}^T\mathbf{B}\mathbf{Z}\right)^{(k)}s_Z^{(k)}$$

subject to the constraints

$$c_i^{(k)} + \nabla c_i^{(k)T}\left(\mathbf{Z}s_Z + \mathbf{Y}s_Y\right)^{(k)} = 0, \quad i = 1,\ldots,m_{eq}$$

$$c_i^{(k)} + \nabla c_i^{(k)T}\left(\mathbf{Z}s_Z + \mathbf{Y}s_Y\right)^{(k)} \geq 0, \quad i = m_{eq}+1,\ldots,m_t$$

Calculate the search direction, $\mathbf{s}^{(k)}$:

$$\mathbf{s}^{(k)} = \left(\mathbf{Z}\mathbf{s}_Z\right)^{(k)} + \left(\mathbf{Y}\mathbf{s}_Y\right)^{(k)} = \begin{bmatrix} \mathbf{s}_Z^{(k)} \\ \mathbf{s}_Y^{(k)} + \mathbf{A}^{(k)}\mathbf{s}_Z^{(k)} \end{bmatrix}$$

Step 5: Calculate the line search step length parameter

Solve equation (I.17) to obtain the Lagrange multipliers of the equality constraints, $\hat{\mathbf{v}}^{(k)}$, i.e.

$$-\nabla_{\mathbf{y}}\mathbf{h}^{(k)}\hat{\mathbf{v}}^{(k)} = -\nabla_{\mathbf{y}}\mathbf{f}^{(k)} + \nabla_{\mathbf{y}}\mathbf{c}_A^{(k)}\hat{\mathbf{u}}^{(k)}.$$

Perform a line search along the search direction, $\mathbf{s}^{(k)}$, to determine the value of $\alpha^{(k)} \in [0,1]$ which minimizes the merit function according to the procedure described in equations (I.23), (I.24), (I.25), and (I.26). With a suitable choice of $\alpha^{(k)}$, increment $\mathbf{x}^{(k)}$ and $\mathbf{y}^{(k)}$ as follows:

$$\begin{bmatrix} \mathbf{x}^{(k+1)} \\ \mathbf{y}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \mathbf{x}^{(k)} \\ \mathbf{y}^{(k)} \end{bmatrix} + \alpha^{(k)}\mathbf{s}^{(k)} = \begin{bmatrix} \mathbf{x}^{(k)} \\ \mathbf{y}^{(k)} \end{bmatrix} + \alpha^{(k)} \begin{bmatrix} \mathbf{s}_Z^{(k)} \\ \mathbf{s}_Y^{(k)} + \mathbf{A}^{(k)}\mathbf{s}_Z^{(k)} \end{bmatrix}$$

Step 6: Check for convergence

If the convergence criterion is satisfied, i.e. $\left[\left|\nabla\mathbf{f}^{(k)T}\mathbf{s}^{(k)}\right| + \left|\hat{\mathbf{u}}^{(k)T}\mathbf{c}^{(k)}\right| + \left|\hat{\mathbf{v}}^{(k)T}\mathbf{h}^{(k)}\right|\right] \leq \varepsilon$, stop.

Otherwise, go to step 1 and repeat the calculations.

In problem (I.30) the variable partitioning is fixed and its effect on the performance of the original coordinate bases algorithm is unknown. As a result, the algorithm stated above is implemented in such a way that the term $\left(\mathbf{Z}^T\mathbf{B}\mathbf{Y}\mathbf{s}_Y\right)^{(k)}$ can be included or ignored during the computational process. The inclusion of this term will improve the convergence rate of the algorithm, but increases its computational overhead.

APPENDIX J

SCALING AND POST-OPTIMALITY ANALYSIS

J.1 INTRODUCTION

Suitable scaling of variables and constraints can dramatically improve the efficiency and accuracy of optimization methods. The scale of a problem is the measure of the relative importance of the variables and constraints. In a properly scaled problem, equal weight will be assigned to each variable or constraint and as such, the implicit definitions of "large" and "small" are based on similar grounds. The discussion of scaling in this section will be restricted to simple transformations of the variables and constraints.

Post-optimality or sensitivity analysis is the study of the variation in the optimum solution as some of the original problem parameters are changed. This analysis is helpful to estimate the effect of parameter variation and changes in the constraint limits on the optimum solution without actually solving the optimization problem again. The effects that changes of the constraint limits and some problem parameters have on the optimum objective function will be discussed.

J.2 SCALING

In real world problems the dependent or independent variables may differ greatly in magnitude. Scaling of the variables by means of variable transformation to convert them to similar magnitudes may enhance the efficiency and the reliability of the numerical optimization procedures. A badly scaled problem is essentially an ill-conditioned problem. Badly scaled variables lead to ill-conditioned Hessian matrices, while badly scaled constraints give rise to near singular Jacobian matrices.

If typical values of all the variables are known, the variables are usually scaled to the order of unity in the region of interest, thus giving them equal "weight" during the optimization process. Variables can be scaled by means of a linear transformation of the form $\mathbf{x} = \mathbf{D}\mathbf{y}$, where $\mathbf{x}$ are the original variables, $\mathbf{y}$ are the transformed variables and $\mathbf{D}$ is a constant diagonal matrix [81GI1m, 83DE1m]. Unfortunately, this simple type of transformation has the disadvantages that some accuracy may be lost during scaling and that the magnitude of the variables may vary significantly during optimization, making the scaling harmful [81GI1m]. These disadvantages can be overcome if we know a realistic range of values that a variable is likely to assume during optimization. Suppose that the variable x_i will always lie in the range $a_i \le x_i \le b_i$. A new variable y_i can be defined as [81GI1m]

$$y_i = \frac{x_i - 0.5(a_i + b_i)}{0.5(b_i - a_i)}, \quad i = 1,\ldots,n \tag{J.1}$$

This transformation guarantees that $-1 \le y_i \le 1$ for all i, regardless of the value of x_i in the interval $[a_i, b_i]$. Transformation (J.1) can be written in matrix form as

$$\mathbf{x} = \mathbf{D}\mathbf{y} + \mathbf{c} \tag{J.2}$$

where $\mathbf{D}$ is a diagonal matrix and $\mathbf{c}$ a vector. The values of the derivatives of the objective function are also affected by this scaling method. Let $\mathbf{g}_\mathbf{y}$ and $\mathbf{G}_\mathbf{y}$ denote the gradient vector and Hessian matrix of the transformed problem's objective function respectively. The derivatives of the original and transformed problems are then related by [81GI1m, 87FL1m]

$$\mathbf{g}_\mathbf{y} = \mathbf{D}\mathbf{g} \;\text{ and }\; \mathbf{G}_\mathbf{y} = \mathbf{D}\mathbf{G}\mathbf{D} \tag{J.3}$$

Once the variables are transformed, the optimization can proceed in the scaled space. Once the optimization is complete, the variables are unscaled to provide the final solution in terms of the original variables.

The scaling of constraints has several effects on the computation of the solution and the interpretation of the results. For example, the Lagrange multiplier estimates are dependent on the condition number of the Jacobian matrix of the active constraints. Furthermore, scaling also plays a vital role in the choice of constraints to be added to, or deleted from the current active set. Constraints should thus have equal weight in the solution process.

Linear constraints can be defined, either in terms of scaled variables or in terms of the original variables. When using scaled variables to define the constraints, it should be noted that the different variables in the same constraint function should be scaled within the same limits, $[a_i, b_i]$. When using the original variables to define the constraint functions, the constraint functions can be divided by suitable constants to scale them to the order of unity.

The objective function can also be scaled easily by choosing an appropriate constant with which it is multiplied. As mentioned previously, scaling of the objective function and the constraints do not alter the solution. The values of the Lagrange multipliers for constrained optimization will be affected by these transformations. Consider the mathematical programming problem stated in equation (2.1). The Lagrange function of problem (2.1) at the optimum point is

$$L(\mathbf{x}^*, \Lambda^*) = f(\mathbf{x}^*) - \sum_{i=1}^{m} \lambda_i^* c_i(\mathbf{x}^*) \tag{J.4}$$

where $\Lambda^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_m^*)$ are the Lagrange multipliers associated with the constraints.

When the objective function is multiplied by a constant, the Lagrange multipliers also get multiplied by the same constant, i.e.

$$\begin{aligned} &\text{Replace } f(\mathbf{x}^*) \text{ with } a\, f(\mathbf{x}^*) \\ &\therefore \quad \bar{\lambda}_i^* = a\, \lambda_i^*, \quad i = 1, \ldots, m \end{aligned} \tag{J.5}$$

where $\bar{\lambda}_i^*$ is the optimum Lagrange multiplier of the scaled problem. When a constraint is multiplied by a constant, its Lagrange multiplier gets divided by the same constant, i.e.

$$\begin{aligned} &\text{Replace } c_i(\mathbf{x}^*) \text{ with } g_i c_i(\mathbf{x}^*) \\ &\therefore \quad \bar{\lambda}_i^* = \lambda_i^* / g_i, \quad i = 1, \ldots, m \end{aligned} \tag{J.6}$$

When both the objective and constraint functions are scaled, the values of the Lagrange multipliers of the original unscaled problem can be obtained from

$$\lambda_i^* = \bar{\lambda}_i^* \, g_i / a, \quad i = 1, \ldots, m \tag{J.7}$$

In practice, the objective function, constraints and their derivatives are usually calculated in the unscaled space and the optimization calculations performed in the scaled space. The above practices of variable and constraint scaling are used in the present study. The advantages, disadvantages and the pitfalls involved in the scaling of the variables, objective and constraint functions are discussed in detail in Gill et al. [81GI1m], Dennis and Schnabel [83DE1m] and Luenberger [84LU1m].

J.3 POST-OPTIMALITY OR SENSITIVITY ANALYSIS

In this section we consider methods of estimating the approximate effect that some changes in the problem parameters and constraints have on the optimum solution.

(1) The effect of variations of constraint limits on the optimum objective function

The Lagrange multipliers at the optimum solution can be used to investigate the effect of changing the constraint limits on the optimum objective function value. Thus the multipliers can be used to study the benefit of relaxing a constraint or the penalty of tightening it; relaxation enlarges the feasible region (constraint set), while tightening contracts it. Consider the modified problem

$$\underset{\mathbf{x}}{\text{Minimize}}\ f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n)$$

subject to the constraints

$$c_i(\mathbf{x}) = b_i\ , \quad i = 1, \ldots, m_{eq} \tag{J.8}$$

$$c_i(\mathbf{x}) \geq e_i\ , \quad i = m_{eq} + 1, \ldots, m$$

where b_i and e_i are small variations in the neighborhood of zero. The optimum solution of this perturbed problem depends on the vectors **b** and **e**, i.e. **x** = **x***(**b**,**e**). The optimum objective function will also depend on these vectors. However, the explicit dependence in this case is not known. The constraint variation sensitivity theorem states that [83FI1m, 84LU1m, 89AR1e]

$$\frac{\partial f(\mathbf{x}^*(0,0))}{\partial b_i} = \lambda_i^* \quad i = 1, \ldots, m_{eq} \tag{J.9}$$

$$\frac{\partial f(\mathbf{x}^*(0,0))}{\partial e_i} = \lambda_i^* \quad i = m_{eq} + 1, \ldots, m \tag{J.10}$$

and thus provides a means of calculating changes in the optimum objective function as b_i and e_i are changed. Using this theorem, one can estimate changes in the objective function if one decides to adjust the right-hand side of the constraints in the neighborhood of zero, i.e.

$$\Delta f^* = \sum_{i=1}^{m_{eq}} \lambda_i^* b_i + \sum_{i=m_{eq}+1}^{m} \lambda_i^* e_i \tag{J.11}$$

The magnitude of the optimum Lagrange multipliers (corresponding to the original unscaled problem) of the active constraints can also be compared with each other. The multipliers with the relatively larger values will have the most significant effect on the optimum objective function value if the corresponding constraint parameters are changed.

For inequality constraint right hand side perturbations (Δe_i), the sensitivity S_i, $i = 1, \ldots, m$ for the unscaled problem is

$$S_i = \lambda_i^* \approx \frac{\Delta f^*}{\Delta e_i} \tag{J.12}$$

A scale-invariant measure of the sensitivity, $\tilde{S}_i$, $i = 1, \ldots, m$, for inequality constraint right hand side perturbations (Δe_i) can be defined as the relative change in the objective function divided by the relative change in the constraint right hand side, i.e.

$$\widetilde{S}_i = S_i \frac{e_i}{f^*} = \frac{\Delta f^*}{f^*} \Big/ \frac{\Delta e_i}{e_i} \tag{J.13}$$

For problems where both the objective function and the constraints are scaled, scale-invariant measure of the sensitivity is

$$\widetilde{S}_i = \frac{\Delta \bar{f}^*}{\bar{f}^*} \Big/ \frac{\Delta \bar{e}_i}{\bar{e}_i} = \bar{\lambda}_i^* \frac{\bar{e}_i}{\bar{f}^*} = \frac{a}{g_i} \lambda_i^* \frac{g_i e_i}{a f^*} = \lambda_i^* \frac{e_i}{f^*} \tag{J.14}$$

Hence, for $i = 1, \ldots, m$

$$\frac{\Delta f^*}{f^*} = \widetilde{S}_i \frac{\Delta e_i}{e_i} \quad \text{or} \quad \frac{\Delta \bar{f}^*}{\bar{f}^*} = \widetilde{S}_i \frac{\Delta \bar{e}_i}{\bar{e}_i} \tag{J.15}$$

If $\Delta e_i < 0$ the feasible region will expand (relax constraints), resulting in a lower objective function value. If $\Delta e_i > 0$ the feasible region will contract (tighten constraints), resulting in a higher objective function value. Information about the sensitivity of the constraints is often very useful because constraints may not be rigidly defined within the context of the problem to be optimized [81GI1m, 89AR1e].

(2) The effect of variations of problem parameters on the optimum objective function

Parameter sensitivity enables one to determine the relative importance of specifying certain parameters in the problem statement. The theory regarding the sensitivity of the optimal solution with respect to the variation of some problem parameters are discussed in detail by Fiacco [83FI1m].

Mathematically the sensitivity of the objective function with respect to some parameter α, is obtained by evaluating the partial derivative of the objective function with respect to α. If the optimal solution of the problem is known, and the minimum value of the objective function as a function of a parameter α is denoted by $f^*(\alpha)$, then it is shown by Fiacco [83FI1m] that

$$\frac{\partial f^*(\alpha)}{\partial \alpha} = \frac{\partial L(\mathbf{x}^*, \Lambda^*)}{\partial \alpha} \tag{J.16}$$

where $L(\mathbf{x}^*, \Lambda^*)$ is the optimum Lagrange function. Once the optimal solution is known, it is a simple matter using equation (J.16) to calculate the sensitivity of the objective function with respect to the specified parameter. The scale-invariant measure of sensitivity can be expressed as

$$\hat{S} = \frac{\partial f^*}{\partial \alpha} \frac{\alpha}{f^*} \approx \frac{\Delta f^*}{f^*} \Big/ \frac{\Delta \alpha}{\alpha} = \frac{\Delta L^*}{f^*} \Big/ \frac{\Delta \alpha}{\alpha} \quad \text{or} \quad \frac{\Delta L^*}{f^*} = \hat{S} \frac{\Delta \alpha}{\alpha} \tag{J.17}$$

We use equation (J.17) to calculate the scale-invariant measure of sensitivity when some problem parameters are perturbed from their original values. Furthermore, we consider only positive perturbations of α, i.e. $\Delta\alpha > 0$. A positive value of $\hat{S}$ corresponds to an increase in the optimum value of the objective function, while a negative value of $\hat{S}$ corresponds to a decrease in the objective function.

APPENDIX K

FORCED DRAFT DIRECT AIR-COOLED CONDENSERS: PROGRAM INPUT AND OUTPUT

K.1 PROGRAM INPUT DATA

Table K.1: General input data

```
 FAN INSTALLATION SPECIFICATIONS
  H3    : Height of fan platform above ground level    30.000       m
  nfr   : Number of fan rows (No. of fans per bay)     5.0000
  nfb   : Number of fan bays . . . . . . . . . . .     6.0000
  free  : Freestanding fan platform (No=0, Yes=1) .   0.00000E+00
  df    : Fan diameter . . . . . . . . . . . . . .     9.1450       m
  dfc   : Fan casing diameter . . . . . . . . . .      9.1700       m
  dfh   : Fan hub diameter . . . . . . . . . . . .     1.4000       m
  Kos   : Loss coeff. for flow obstacles (suction)    0.29693       m
  Kod   : Loss coeff. for flow obst. (discharge)  .   0.39080       m
  THETf: Fan blade angle . . . . . . . . . . . . .     16.000       deg
  RPM   : Fan operating speed . . . . . . . . . .      100.00       rpm
  ETAfd: Efficiency of fan drive system . . . . .      90.000       %
  Lsw   : Half-width of walkway between A-frames  .   0.20000       m
  Kts   : ACC installation support loss coefficient    1.5000       m
  Hw    : Height of wind wall . . . . . . . . . .      8.2700       m
 ACC OPERATING CHARACTERISTICS
  pa1 : Barometric pressure at ground level. . . .     84600.       N/m^2
  Ta1 : Dry bulb air temperature at ground level .     15.600        C
  Twb : Wet bulb air temperature at ground level .    0.00000E+00    C
  g   : Gravitational acceleration . . . . . . . .     9.8000       m/s^2
  Tao1: Air outlet temperature (row 1) . . . . . .     23.375        C
  Tao2: Air outlet temperature (row 2) . . . . . .     30.605        C
  Ts  : Saturated steam supply temperature . . . .     60.000        C
  Tacc: Saturated steam temperature at ACC inlet .     59.545        C
  ma  : Air mass flow rate . . . . . . . . . . . .     665.94       kg/s
  mc1 : Condensate mass flow rate (row 1) . . . .      2.0396       kg/s
  mc2 : Condensate mass flow rate (row 2) . . . .      1.8935       kg/s
  dsh : Effective steam header diameter . . . . .      1.2500        m
  Ksd : Mean steam ducting loss coefficient . . .     0.60000
 FINNED TUBE BUNDLE SPECIFICATIONS
  nb    : No. of heat exchanger bundles above fan .    2.0000
  nr    : Number of tube rows (maximum 2) . . . . .    2.0000
  ntr1 : Number of tubes per row - row 1 . . . . .     152.00
  ntr2 : Number of tubes per row - row 2 (maximum)     153.00
  Lt    : Effective finned tube length . . . . . .     10.000       m
  Wb    : Width of heat exchanger bundle . . . . .     11.659       m
  THETb: Bundle semi-apex angle . . . . . . . . .      30.000       deg
 EXPERIMENTAL PERFORMANCE CHARACTERISTICS
  PC   : Correlation(normal flow=0,inclined flow=1)   0.00000E+00
  AKhe: Constant in Khe correlation . . . . . . .     0.00000E+00
  BKhe: Exponent in Khe correlation . . . . . . .     0.00000E+00
  ANy1: Constant in Ny correlation - row 1 . . . .    0.00000E+00
  ANy2: Constant in Ny correlation - row 2 . . . .    0.00000E+00
  BNy1: Exponent in Ny correlation - row 1 . . . .    0.00000E+00
  BNy2: Exponent in Ny correlation - row 2 . . . .    0.00000E+00
```

```
FINNED TUBE DIMENSIONS: GENERAL
 tt   : Tube thickness . . . . . . . . . . . . . . .    1.5000        mm
 tf   : Fin thickness (mean - core material) . . .    0.35000       mm
 tg   : Thickness of galvanizing material (mean) .    0.00000E+00   mm
 Pf1  : Fin pitch - row 1  . . . . . . . . . . . .    3.6300        mm
 Pf2  : Fin pitch - row 2  . . . . . . . . . . . .    2.5400        mm
 Pt   : Transversal tube pitch . . . . . . . . . .    76.200        mm
 Pl   : Longitudinal tube pitch  . . . . . . . . .    65.991        mm
FINNED TUBE DIMENSIONS: ROUND TUBES
 do   : Tube outside diameter  . . . . . . . . . .    38.100        mm
 tr   : Fin root thickness . . . . . . . . . . . .    1.1000        mm
 df   : Fin diameter . . . . . . . . . . . . . . .    69.900        mm
FINNED TUBE DIMENSIONS: NON-ROUND TUBES
 Peri    : Fin perimeter form (Ellip=1, Rect=2) . .   0.00000E+00
 de      : Tube hydraulic diameter  . . . . . . . .   0.00000E+00   mm
 Ht      : Inside height of tube  . . . . . . . . .   0.00000E+00   mm
 Wtn     : Inside width of tube . . . . . . . . . .   0.00000E+00   mm
 Atcs    : Tube cross-sectional area  . . . . . . .   0.00000E+00   mm^2
 Ati     : Tube inside perimeter length . . . . . .   0.00000E+00   mm
 Af1     : Surface area of one fin - row 1  . . . .   0.00000E+00   mm^2
 Af2     : Surface area of one fin - row 2  . . . .   0.00000E+00   mm^2
 Ar1     : Surface area of exposed root - row 1 . .   0.00000E+00   mm^2
 Ar2     : Surface area of exposed root - row 2 . .   0.00000E+00   mm^2
 Vf1     : Material volume of one fin - row 1 . . .   0.00000E+00   mm^3
 Vf2     : Material volume of one fin - row 2 . . .   0.00000E+00   mm^3
 SIGMf1: Fin leading edge fr. area/HE fr. area  .   0.00000E+00
 SIGM21: HE inlet contraction area ratio  . . . .   0.00000E+00
 SIGM    : Minimum HE free flow area/HE fr. area  .   0.00000E+00
FINNED TUBE PROPERTIES
 RHOt: Density of tube material . . . . . . . . .    7850.0        kg/m^3
 RHOf: Density of fin material  . . . . . . . . .    2707.0        kg/m^3
 RHOg: Density of galvanizing material  . . . . .    0.00000E+00   kg/m^3
 kt   : Thermal conductivity of tube material  . .    50.000        W/mC
 kf   : Thermal conductivity of fin material . . .    204.00        W/mC
 kg   : Thermal conductivity of galv. material . .    0.00000E+00   W/mC
COST FACTORS: GENERAL INFORMATION
 Ce   : Present electricity cost (self-generated)     5.0000        c/kWh
 ese  : Electricity cost escalation rate . . . . .    7.5000        %
 Cf   : Present fuel cost  . . . . . . . . . . . .    0.25000E-02   $/MJ
 esf  : Fuel cost escalation rate  . . . . . . . .    8.5000        %
 i    : Interest rate  . . . . . . . . . . . . . .    10.000        %
 NY   : Capital repayment period (plant life). . .    30.000        years
 FCR  : Levelized fixed charge rate  . . . . . . .    20.000        %
 Tau  : Running hours per annum  . . . . . . . . .    8760.0        h
FINNED TUBE BUNDLE COSTS
 Cb1  : Tube material unit cost  . . . . . . . . .    0.80000       $/kg
 Cb2  : Tubing fixed cost  . . . . . . . . . . . .    2.0000        $/m
 Cb3  : Fin material unit cost . . . . . . . . . .    4.0000        $/kg
 Cb4  : Finning fixed cost . . . . . . . . . . . .    0.20000       $/m
 Cb5  : Galvanizing material unit cost . . . . . .    0.00000E+00   $/kg
 Cb6  : Surface coating fixed cost . . . . . . . .    0.00000E+00   $/m^2
 Cb7  : Finned tube cost weighting factor  . . . .    1.3000
 Cb8  : Bundle frame and header cost factor  . . .    0.20000
 Cb9  : Tube assembly and end preparation cost . .    25.000        $/FT
 Cb10: Bundle cost weighting factor . . . . . . .    1.3000
 Cb11: Bundle maintenance cost factor . . . . . .    0.10000E-01
FAN SYSTEM COSTS
 Cf1  : Fan fixed cost . . . . . . . . . . . . . .    650.00        $/F
 Cf2  : Fan unit cost  . . . . . . . . . . . . . .    110.00        $/m^2
 Cf3  : Fan casing and inlet bell unit cost  . . .    10.000        $/m^2
```

```
 Cf4 : Fan safety screen unit cost . . . . . . . .   2.5000      $/m^2
 Cf5 : Electric motor fixed cost . . . . . . . . .   400.00      $/EM
 Cf6 : Electric motor unit cost . . . . . . . . . .  120.00      $/kW
 Cf7 : Electric motor safety factor (undersizing)    1.2500
 Cf8 : Speed reducer cost multiplier . . . . . . .  0.40000
 Cf9 : Electric wiring/switching cost multiplier    0.10000
 Cf10: Fan system cost weighting factor . . . . .    1.2500
 Cf11: Fan system maintenance cost factor . . . .   0.30000E-01
STRUCTURAL AND CONSTRUCTION COSTS
 Cs1 : Land, excavation and foundation unit cost     15.000      $/m^2
 Cs2 : Structural material/installation unit cost    17.500      $/m^3
 Cs3 : Fan platform unit cost . . . . . . . . . . .  75.000      $/m^2
 Cs4 : Structural cost weighting factor . . . . .    1.2500
 Cs5 : Structural maintenance cost factor . . . .   0.50000E-03
STEAM/CONDENSATE DISTRIBUTION COSTS
 Cd1 : Distribution system cost factor . . . . . .  0.35000
 Cd2 : Distribution system maintenance cost fact.   0.50000E-02
FIN MATERIAL PROPERTIES
 RHOal: Density of aluminum . . . . . . . . . . . .  2707.0      kg/m^3
 RHOcu: Density of copper . . . . . . . . . . . . .  8954.0      kg/m^3
 RHOst: Density of steel . . . . . . . . . . . . .   7850.0      kg/m^3
 RHOzn: Density of zinc . . . . . . . . . . . . . .  7144.0      kg/m^3
 kal  : Thermal conductivity of aluminum . . . .     204.00      W/mC
 kcu  : Thermal conductivity of copper . . . . .     386.00      W/mC
 kst  : Thermal conductivity of steel . . . . . .    50.000      W/mC
 kzn  : Thermal conductivity of zinc . . . . . .     116.00      W/mC
PARAMETERS FOR SUBROUTINE SOLVE
 H    : Step length for numerical differentiation   0.10000E-07
 ACCS : Stopping accuracy for SOLVE . . . . . . .   0.10000E-06
 ITMAX: Maximum no. of iterations for SOLVE . . .    50.000
 IPRNT: Controls intermediate printing of SOLVE .    1.0000
PARAMETERS FOR SUBROUTINE OPTIM
 IPRNT : Controls intermediate printing of OPTIM     1.0000
 MAXFUN: Maximum no. of function evaluations . .     500.00
 ACCO  : Stopping accuracy for OPTIM . . . . . . .  0.10000E-06
 H     : Step length for numerical differentiation  0.10000E-07
ALL FINNED TUBE TYPES: CONSTANT VALUES(=0.0)
 H3    : Height of fans above ground level . . . .   1.0000
 df    : Fan diameter . . . . . . . . . . . . . . .  1.0000
 THETf: Fan blade angle . . . . . . . . . . . . . .  1.0000
 RPM   : Fan operating speed . . . . . . . . . . .   1.0000
 nb    : No. of heat exchanger bundles above fan .  0.00000E+00
 ntr   : Number of tubes per row . . . . . . . . .   1.0000
 Lt    : Length of finned tubes . . . . . . . . .    1.0000
 THETb: Bundle semi-apex angle . . . . . . . . . .   1.0000
ALL FINNED TUBE TYPES: LOWER VALUES
 H3    : Height of fans above ground level . . . .   10.000      m
 df    : Fan diameter . . . . . . . . . . . . . . .  5.0000      m
 THETf: Fan blade angle . . . . . . . . . . . . . .  10.000      deg
 RPM   : Fan operating speed . . . . . . . . . . .   30.000      rpm
 nb    : No. of heat exchanger bundles above fan .   2.0000
 ntr   : Number of tubes per row . . . . . . . . .   100.00
 Lt    : Length of finned tubes . . . . . . . . .    5.0000      m
 THETb: Bundle semi-apex angle . . . . . . . . . .   20.000      deg
ALL FINNED TUBE TYPES: UPPER VALUES
 H3    : Height of fans above ground level . . . .   60.000      m
 df    : Fan diameter . . . . . . . . . . . . . . .  12.000      m
 THETf: Fan blade angle . . . . . . . . . . . . . .  30.000      deg
 RPM   : Fan operating speed . . . . . . . . . . .   150.00      rpm
 nb    : No. of heat exchanger bundles above fan .   10.000
```

```
 ntr   : Number of tubes per row . . . . . . . . . .    250.00
 Lt    : Length of finned tubes  . . . . . . . . . .    20.000        m
 THETb: Bundle semi-apex angle  . . . . . . . . . .    35.000        deg
ROUND TUBES: CONSTANT VALUES(=0.0)
 do   : Tube outside diameter  . . . . . . . . . .    1.0000
 tt   : Tube thickness . . . . . . . . . . . . . .    1.0000
 tr   : Fin root thickness . . . . . . . . . . . .    1.0000
 df   : Fin diameter . . . . . . . . . . . . . . .    1.0000
 tf   : Fin thickness  . . . . . . . . . . . . . .    1.0000
 tg   : Thickness of galvanizing material  . . . .  0.00000E+00
 Pf1  : Fin pitch (row 1). . . . . . . . . . . . .    1.0000
 Pf2  : Fin pitch (row 2)  . . . . . . . . . . . .    1.0000
 Pt   : Transversal tube pitch . . . . . . . . . .    1.0000
ROUND TUBES: LOWER VALUES
 do   : Tube outside diameter  . . . . . . . . . .    10.000        mm
 tt   : Tube thickness . . . . . . . . . . . . . .    1.0000        mm
 tr   : Fin root thickness . . . . . . . . . . . .    1.0000        mm
 df   : Fin diameter . . . . . . . . . . . . . . .    20.000        mm
 tf   : Fin thickness  . . . . . . . . . . . . . .  0.50000E-01     mm
 tg   : Thickness of galvanizing material  . . . .  0.30000E-01     mm
 Pf1  : Fin pitch (row 1)  . . . . . . . . . . . .    1.0000        mm
 Pf2  : Fin pitch (row 2)  . . . . . . . . . . . .    1.0000        mm
 Pt   : Transversal tube pitch . . . . . . . . . .    20.000        mm
ROUND TUBES: UPPER VALUES
 do   : Tube outside diameter  . . . . . . . . . .    90.000        mm
 tt   : Tube thickness . . . . . . . . . . . . . .    2.0000        mm
 tr   : Fin root thickness . . . . . . . . . . . .    2.0000        mm
 df   : Fin diameter . . . . . . . . . . . . . . .    120.00        mm
 tf   : Fin thickness  . . . . . . . . . . . . . .    1.0000        mm
 tg   : Thickness of galvanizing material  . . . .  0.70000E-01     mm
 Pf1  : Fin pitch (row 1)  . . . . . . . . . . . .    5.0000        mm
 Pf2  : Fin pitch (row 2)  . . . . . . . . . . . .    5.0000        mm
 Pt   : Transversal tube pitch . . . . . . . . . .    120.00        mm
OPTIMIZATION VARIABLES: GENERAL INFORMATION
 trL   : Lower bound for tr  . . . . . . . . . . .    1.0000        mm
 ttL   : Lower bound for tt  . . . . . . . . . . .    1.2000        mm
 tfL   : Lower bound for tf  . . . . . . . . . . .  0.50000E-01     mm
 tgL   : Lower bound for tg  . . . . . . . . . . .  0.30000E-04     mm
 tgU   : Upper bound for tg  . . . . . . . . . . .  0.70000E-04     mm
 PfL   : Lower bound for Pf  . . . . . . . . . . .    1.7500        mm
 WbU   : Upper bound for Wb  . . . . . . . . . . .    15.000        m
 WFL   : Lower bound for fan unit width ratio  . .    1.2000
 WFU   : Upper bound for fan unit width ratio  . .    1.3500
 LFL   : Lower bound for fan unit length ratio . .    1.2000
 LFU   : Upper bound for fan unit length ratio . .    1.3500
 THbL : Lower bound for bundle semi-apex angle  .    20.000        deg
 THbU : Upper bound for bundle semi-apex angle  .    35.000        deg
 vbtU : Upper bound for fan blade tip speed . . .    60.000        m/s
 Qacc : ACC specified heat transfer rate  . . . .    331.10        MW
CHOICE OF CORRELATIONS AND FORMULAE
Finned tube type . . . . . . . . . . . . . . . . : 1.0
 1 Round tubes (extruded fins)
 2 Non-round tubes
Round tube performance correlations  . . . . . : 3.0
 1 Ny vs. Ry and Khe vs. Ry
 2 Briggs, Young, Robinson
 3 Ganguli, Tung, Taborek
 4 Nir
Non-round tube performance correlations  . . . : 1.0
 1 Ny vs. Ry and Khe vs. Ry
```

```
Fin material . . . . . . . . . . . . . . . . . : 1.0
 1 Aluminum
 2 Copper
 3 Steel
 4 Galvanized fin (steel core)
```

Table K.2: Fan performance characteristics

```
FAN PERFORMANCE CHARACTERISTICS
 nbs   : Number of fan curves (maximum 5)  . . . .   5.0000
 RHOrf: Reference air density for characteristics    1.2000      kg/m^3
 dfm   : Fan diameter for characteristics  . . . .   9.1450      m
 Nfm   : Fan rotational speed for characteristics    125.00      rpm
 SPD   : Speed drive: Constant=0, Variable=1 . . .   1.0000
FAN PERFORMANCE CURVE No. 1
 THETf: Fan blade angle . . . . . . . . . . . . .    12.000      deg
 Vsmin: Minimum air volume flow rate  . . . . . .    150.00      m^3/s
 Vsmax: Maximum air volume flow rate  . . . . . .    770.00      m^3/s
FAN STATIC PRESSURE [N/m^2] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cspf0: Constant coefficient  . . . . . . . . . .    316.06
 Cspf1: First degree coefficient  . . . . . . . .  -0.28085
 Cspf2: Second degree coefficient . . . . . . . .   0.33046E-03
 Cspf3: Third degree coefficient  . . . . . . . .  -0.63900E-06
 Cspf4: Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN SHAFT POWER [kW] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cpf0 : Constant coefficient  . . . . . . . . . .    135.04
 Cpf1 : First degree coefficient  . . . . . . . .  -0.25995E-02
 Cpf2 : Second degree coefficient . . . . . . . .   0.33536E-03
 Cpf3 : Third degree coefficient  . . . . . . . .  -0.49629E-06
 Cpf4 : Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN PERFORMANCE CURVE No. 2
 THETf: Fan blade angle . . . . . . . . . . . . .    14.000      deg
 Vsmin: Minimum air volume flow rate  . . . . . .    150.00      m^3/s
 Vsmax: Maximum air volume flow rate  . . . . . .    825.00      m^3/s
FAN STATIC PRESSURE [N/m^2] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cspf0: Constant coefficient  . . . . . . . . . .    310.99
 Cspf1: First degree coefficient  . . . . . . . .  -0.20930
 Cspf2: Second degree coefficient . . . . . . . .   0.27940E-03
 Cspf3: Third degree coefficient  . . . . . . . .  -0.57410E-06
 Cspf4: Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN SHAFT POWER [kW] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cpf0 : Constant coefficient  . . . . . . . . . .    165.90
 Cpf1 : First degree coefficient  . . . . . . . .  -0.64406E-01
 Cpf2 : Second degree coefficient . . . . . . . .   0.45685E-03
 Cpf3 : Third degree coefficient  . . . . . . . .  -0.52120E-06
 Cpf4 : Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN PERFORMANCE CURVE No. 3
 THETf: Fan blade angle . . . . . . . . . . . . .    16.000      deg
 Vsmin: Minimum air volume flow rate  . . . . . .    150.00      m^3/s
 Vsmax: Maximum air volume flow rate  . . . . . .    870.00      m^3/s
FAN STATIC PRESSURE [N/m^2] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cspf0: Constant coefficient  . . . . . . . . . .    320.05
 Cspf1: First degree coefficient  . . . . . . . .  -0.29752
 Cspf2: Second degree coefficient . . . . . . . .   0.63515E-03
 Cspf3: Third degree coefficient  . . . . . . . .  -0.81400E-06
 Cspf4: Fourth degree coefficient . . . . . . . .   0.00000E+00
```

```
FAN SHAFT POWER [kW] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cpf0 : Constant coefficient  . . . . . . . . . .    186.65
 Cpf1 : First degree coefficient  . . . . . . . .  -0.59414E-01
 Cpf2 : Second degree coefficient . . . . . . . .   0.47617E-03
 Cpf3 : Third degree coefficient  . . . . . . . .  -0.50831E-06
 Cpf4 : Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN PERFORMANCE CURVE No. 4
 THETf: Fan blade angle . . . . . . . . . . . . .    18.000      deg
 Vsmin: Minimum air volume flow rate  . . . . . .    150.00      m^3/s
 Vsmax: Maximum air volume flow rate  . . . . . .    940.00      m^3/s
FAN STATIC PRESSURE [N/m^2] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cspf0: Constant coefficient  . . . . . . . . . .    305.12
 Cspf1: First degree coefficient  . . . . . . . .  -0.87519E-01
 Cspf2: Second degree coefficient . . . . . . . .   0.14541E-03
 Cspf3: Third degree coefficient  . . . . . . . .  -0.42190E-06
 Cspf4: Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN SHAFT POWER [kW] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cpf0 : Constant coefficient  . . . . . . . . . .    211.03
 Cpf1 : First degree coefficient  . . . . . . . .  -0.75730E-01
 Cpf2 : Second degree coefficient . . . . . . . .   0.47561E-03
 Cpf3 : Third degree coefficient  . . . . . . . .  -0.45873E-06
 Cpf4 : Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN PERFORMANCE CURVE No. 5
 THETf: Fan blade angle . . . . . . . . . . . . .    20.000      deg
 Vsmin: Minimum air volume flow rate  . . . . . .    150.00      m^3/s
 Vsmax: Maximum air volume flow rate  . . . . . .    980.00      m^3/s
FAN STATIC PRESSURE [N/m^2] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cspf0: Constant coefficient  . . . . . . . . . .    279.09
 Cspf1: First degree coefficient  . . . . . . . .   0.48486E-01
 Cspf2: Second degree coefficient . . . . . . . .  -0.25045E-04
 Cspf3: Third degree coefficient  . . . . . . . .  -0.29590E-06
 Cspf4: Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN SHAFT POWER [kW] vs. AIR VOLUME FLOW RATE [m^3/s]
 Cpf0 : Constant coefficient  . . . . . . . . . .    248.36
 Cpf1 : First degree coefficient  . . . . . . . .  -0.11954
 Cpf2 : Second degree coefficient . . . . . . . .   0.52012E-03
 Cpf3 : Third degree coefficient  . . . . . . . .  -0.42960E-06
 Cpf4 : Fourth degree coefficient . . . . . . . .   0.00000E+00
FAN SPEEDS FOR VARIABLE SPEED DRIVES
```

No	Speed [rpm]	No	Speed [rpm]
1	100.00	11	100.00
2	100.00	12	100.00
3	100.00	13	100.00
4	100.00	14	100.00
5	100.00	15	100.00
6	100.00	16	100.00
7	100.00	17	100.00
8	100.00	18	100.00
9	100.00	19	100.00
10	100.00	20	100.00
21	100.00	31	100.00
22	100.00	32	100.00
23	100.00	33	100.00
24	100.00	34	100.00
25	100.00	35	100.00
26	100.00	36	100.00
27	100.00	37	100.00
28	100.00	38	100.00
29	100.00	39	100.00
30	100.00	40	100.00

Table K.3: Frequency of dry- and wetbulb temperatures

MEAN HOURLY FREQUENCY OF AMBIENT TEMPERATURES
Number of data sets (maximum 40): 34
Tdb : Dry bulb temperature at ground level [C]
Twb : Wet bulb temperature at ground level [C]
h : Annual duration of these temperatures [h/a]

No	Tdb [C]	Twb [C]	h [h/a]
1	-1.000	0.000	4.000
2	0.000	0.000	10.000
3	1.000	0.000	26.000
4	2.000	0.000	43.000
5	3.000	0.900	59.000
6	4.000	1.900	82.000
7	5.000	2.900	112.000
8	6.000	3.800	152.000
9	7.000	4.600	201.000
10	8.000	5.400	254.000
11	9.000	6.300	312.000
12	10.000	7.100	371.000
13	11.000	7.800	434.000
14	12.000	8.600	506.000
15	13.000	9.400	578.000
16	14.000	10.200	656.000
17	15.000	10.900	738.000
18	16.000	11.600	764.000
19	17.000	12.200	655.000
20	18.000	12.700	553.000
21	19.000	13.200	459.000
22	20.000	13.700	381.000
23	21.000	14.100	320.000
24	22.000	14.500	265.000
25	23.000	14.900	219.000
26	24.000	15.200	177.000
27	25.000	15.500	140.000
28	26.000	15.700	105.000
29	27.000	15.900	76.000
30	28.000	16.100	51.000
31	29.000	16.400	30.000
32	30.000	16.800	15.000
33	31.000	17.400	8.000
34	32.000	18.000	4.000

Table K.4: Turbo-generator-condenser characteristic curves

TURBO-GENERATOR-CONDENSER PERFORMANCE CHARACTERISTICS

Tsmin:	Minimum saturated steam temperature . . .	42.000	C
Tsmax:	Maximum saturated steam temperature . . .	94.610	C

HEAT TO BE REJECTED [MW] vs. SATURATED STEAM TEMPERATURE [C]

Cq0	: Constant coefficient	336.40
Cq1	: First degree coefficient	0.18223
Cq2	: Second degree coefficient	-0.16010E-01
Cq3	: Third degree coefficient	0.17753E-03
Cq4	: Fourth degree coefficient	0.00000E+00
Cq5	: Fifth degree coefficient	0.00000E+00

```
GENERATOR POWER OUTPUT [MW] vs. SATURATED STEAM TEMPERATURE [C]
 Cp0  : Constant coefficient . . . . . . . . . . .   225.83
 Cp1  : First degree coefficient . . . . . . . . . -0.42994E-02
 Cp2  : Second degree coefficient . . . . . . . .   0.13317E-01
 Cp3  : Third degree coefficient . . . . . . . . . -0.16257E-03
 Cp4  : Fourth degree coefficient . . . . . . . .   0.00000E+00
 Cp5  : Fifth degree coefficient . . . . . . . . .  0.00000E+00
```

K.2 PROGRAM OUTPUT

Operating point calculations

Table K.5: Operating point output and cost data

```
COMPLETE OUTPUT DATA SET:

 AIRSIDE DATA:
    Air temperature:
       At inlet of fan, Tsa3                =     15.308 C
       At inlet of heat exchangers, Tsa5    =     15.432 C
       At outlet of heat exchangers, Tsa6 =      37.302 C
       Mean temperature (Row 1), Tsam   =     21.049 C
       Mean temperature (Row 2), Tsam   =     31.984 C
       Humidity ratio, w  =  0.0000000 kg moisture/kg dry air

    Air properties (evaluated at ground level air pressure):
    At fan inlet temperature:
       Density, RHOsa3                =      1.021611 kg/m^3
       Specific heat, cpsa3           =   1006.601792 J/kgK
    At HE inlet temperature:
       Density, RHOsa5                =      1.021169 kg/m^3
    At HE outlet temperature:
       Density, RHOsa6                =      0.949234 kg/m^3
    At mean bundle temperature:
       Harmonic mean density, RHOsa56 =      0.983888 kg/m^3
       Viscosity , MUsa56             =      0.000018 kg/ms

    Air properties (evaluated at ground level air pressure):
    At mean HE temperature (Row 1):
       Density, RHOsam                =      1.001672 kg/m^3
       Viscosity, MUsam               =      0.000018 kg/ms
       Specific heat, cpsam           =   1006.760748 J/kgK
       Thermal conductivity, ksam     =      0.025771 W/mK
       Prandtl number, PRAsam         =      0.711030
    At mean HE temperature (Row 2):
       Density, RHOsam                =      0.965776 kg/m^3
       Viscosity, MUsam               =      0.000019 kg/ms
       Specific heat, cpsam           =   1007.148846 J/kgK
       Thermal conductivity, ksam     =      0.026618 W/mK
       Prandtl number, PRAsam         =      0.707717

    Air flow data:
       Air mass flow rate per fan, msa          =    540.821 kg/s
       Air volume flow rate per fan, VsF        =    529.381 m^3/s
       Mean axial fan inlet velocity, vsnF      =      8.016 m/s
       HE normal approach velocity, vshen       =      2.271 m/s
```

```
Fan performance data:
   Fan blade angle, THETsF                =      16.000 deg
   Fan operating speed, NsF               =     100.000 rpm
   Air volume flow rate per fan, VsF      =     529.381 m^3/s
   Fan static pressure, DpsFs             =      90.137 N/m^2
   Fan coefficient, KsF                   =       2.746
   Fan power consumption, PsF             =      90.906 kW
   Electrical power input, Pse            =     101.007 kW
   Fan static efficiency, ETAsF           =      52.490 %
   Optimum fan static efficiency, ETAso   =      55.051 %
   Optimum air volume flow rate, VsFo     =     454.480 m^3/s

HEAT TRANSFER AND PRESSURE DROP CORRELATIONS:
   Correlation: Ganguli, Tung, Taborek
   Heat transfer (Row 1):
      Coefficient asHA                        =  0.261050
      Exponent bsHA                           = -0.400000
      Equivalent diameter, de                 =   40.3000      mm
      Air side Reynolds number, Re            =   11919.9
      Air side heat transfer coeff., hsa      =   41.5858      W/m^2K
      Fin efficiency, ETAsf                   =  0.894320
      Surface effectiveness, EPSIsf           =  0.902135
      Effective heat transfer coeff., hAsae   =   173376.      W/K
   Pressure drop (Row 1):
      Coefficient asEU                        =  0.486208
      Exponent bsEU                           =  0.000000E+00
      Equivalent diameter, de                 =   40.3000      mm
      Air Reynolds number, Re                 =   11919.9
      Euler number,Eu                         =  0.486208
      Pressure loss coefficient, Kshe         =   5.37343
   Heat transfer (Row 2):
      Coefficient asHA                        =  0.248362
      Exponent bsHA                           = -0.400000
      Equivalent diameter, de                 =   40.3000      mm
      Air side Reynolds number, Re            =   11966.9
      Air side heat transfer coeff., hsa      =   40.8973      W/m^2K
      Fin efficiency, ETAsf                   =  0.895852
      Surface effectiveness, EPSIsf           =  0.901124
      Effective heat transfer coeff., hAsae   =   237458.      W/K
   Pressure drop (Row 2):
      Coefficient asEU                        =  0.539793
      Exponent bsEU                           =  0.000000E+00
      Equivalent diameter, de                 =   40.3000      mm
      Air Reynolds number, Re                 =   11966.9
      Euler number,Eu                         =  0.539793
      Pressure loss coefficient, Kshe         =   6.39047

      Total bundle loss coefficient, Kshet    =      23.9768

EFFECTIVE OUTSIDE HEAT TRANSFER COEFFICIENTS:
      Row 1
      Effective outside heat transfer coeff.
      based on the inside area, hsaeff        =     517.1992     W/m^2K
      Row 2
      Effective outside heat transfer coeff.
      based on the inside area, hsaeff        =     703.7329     W/m^2K

PRESSURE LOSS COEFFICIENTS (based on HE frontal area and mean air density):
      Fan pressure rise coefficient, KsFs          =      32.970
      ACHE support loss coefficient, Ksts          =       1.445
```

```
     Obstacle loss coeff. (fan suction), Ksos   =      3.737
     Obstacle loss coeff. (fan discharge), Ksod =      4.918
     HE inlet loss coefficient, Kshi            =      2.145
     HE normal flow loss coefficient, Kshe      =     11.764
     HE outlet kinetic energy loss coeff., Kso  =      7.993
     HE outlet jetting loss coeff., Ksdj        =      2.075

PRESSURE LOSS TERMS (based on HE frontal area and mean air density):
     Fan pressure rise , dpsFs                  =     90.137 N/m^2
     ACHE support pressure loss , dpsts         =      3.951 N/m^2
     Obstacle press. loss (fan suction), dpsos  =     10.216 N/m^2
     Obstacle press. loss (fan discharge), dpsod=     13.445 N/m^2
     HE inlet press. loss, dpshi                =      5.863 N/m^2
     HE normal flow pressure loss, dpshe        =     32.161 N/m^2
     HE outlet kinetic energy press. loss, dpso =     21.852 N/m^2
     HE outlet jetting pressure loss, dpsdj     =      5.674 N/m^2

DRAFT EQUATION:
     Draft equation LHS  =      3.025 N/m^2
     Draft equation RHS  =      3.025 N/m^2

STEAMSIDE DATA:
   Steam temperatures and pressures:
       Turbine outlet steam temp., Tss =        60.000 C
       Turbine outlet steam pres., pss =     19754.440 N/m^2
       ACC inlet steam temp., Tsvi     =        59.372 C
       ACC inlet steam pres., psvi     =     19185.831 N/m^2

   Steam properties at ACC inlet temperature:
     Density, RHOsvi             =      0.126715 kg/m^3
     Viscosity, MUsvi            =      0.000011 kg/ms
     Specific heat, cpsvi        =   1925.844202 J/kgK
     Thermal conductivity, ksvi  =      0.020993 W/mK

   Water properties at mean steam temperature (Row 1):
     Mean condensation temp., Tsvm      =        59.121807 C
     Mean condensation pressure, psvm   =     18963.649381 N/m^2
     Density, RHOsw                     =       983.676422 kg/m^3
     Viscosity, MUsw                    =         0.000469 kg/ms
     Specific heat, cpsw                =      4183.534441 J/kgK
     Thermal conductivity, ksw          =         0.652337 W/mK
     Latent heat of vaporization, isfg  =   2360765.792716 J/kg
     Prandtl number, PRAsw              =         3.010293

   Water properties at mean steam temperature (Row 2):
     Mean condensation temp., Tsvm      =        59.143239 C
     Mean condensation pressure, psvm   =     18982.615384 N/m^2
     Density, RHOsw                     =       983.665265 kg/m^3
     Viscosity, MUsw                    =         0.000469 kg/ms
     Specific heat, cpsw                =      4183.547882 J/kgK
     Thermal conductivity, ksw          =         0.652358 W/mK
     Latent heat of vaporization, isfg  =   2360713.451617 J/kg
     Prandtl number, PRAsw              =         3.009211

   Pressure and temperature drop parameters (Row 1):
     Steam inlet velocity, vsvi            =       64.414289 m/s
     Steam inlet Reynolds number, Resvi =       25634.918538
     Normal flow Reynolds number, Resvn =          22.725272
     Constant a1                           =        1.038617
     Constant a2                           =     2082.472784
```

```
Pressure and temperature drop parameters (Row 2):
   Steam inlet velocity, vsvi            =      60.606801 m/s
   Steam inlet Reynolds number, Resvi =   24140.781771
   Normal flow Reynolds number, Resvn =      21.381995
   Constant a1                           =       1.036887
   Constant a2                           =    1940.147696

Condensation data (Row 1):
   Condensation heat transfer coeff., hsc =    7258.309      W/m^2K
   Condensate mass flow rate, msc         =    2.400981      kg/s
Condensation data (Row 2):
   Condensation heat transfer coeff., hsc =    7116.270      W/m^2K
   Condensate mass flow rate, msc         =    2.273923      kg/s

   Condensate mass flow rate per fan, msc =      4.6749 kg/s

SYSTEM DATA:
   System data (Row 1):
      Overall heat transfer coeff., UA     =     161843.4      W/K
      Heat exchanger effectiveness, e      =     25.71382      %
      Airside heat transfer rate, Qsa      =     5668155.      W
      Steamside heat transfer rate, Qsw    =     5668155.      W
      Heat transfer rate based on UA, Qsu  =     5668155.      W
   System data (Row 2):
      Overall heat transfer coeff., UA     =     216088.6      W/K
      Heat exchanger effectiveness, e      =     32.74778      %
      Airside heat transfer rate, Qsa      =     5368081.      W
      Steamside heat transfer rate, Qsw    =     5368081.      W
      Heat transfer rate based on UA, Qsu  =     5368081.      W

      Heat transfer rate per fan, Q =    0.1103624E+08 W

TOTAL SYSTEM DATA:
      Number of fans in operation, nsF         =    30.00000
      Air mass flow rate, msat                 =    16224.64      kg/s
      Air volume flow rate, VsFt               =    15881.42      m^3/s
      Fan power consumption, PsFt              =    2727.176      kW
      Electrical power input to fans, Pset     =    3030.195      kW
      Mass flow rate steam condensed, mswt     =    140.2471      kg/s
      Fan system effectiveness, esfs           =    92.66538      %
      Heat transfer rate, Qst                  =  0.3310871E+09 W

HEAT EXCHANGER BUNDLE COST:
      Cost of tubes, Cstm                  =  564213.372      $
      Cost of fins, Csfm                   =  2414586.86      $
      Cost of galvanizing, Csgm            = 0.000000000E+00 $
      Cost of finned tubes, Csft           =  3872440.30      $
      Cost of bundle frames/headers, Csbf  =  774488.060      $
      Cost of bundle assembly, Csba        =  457500.000      $
      Total HE bundle cost, Cshe           =  6635756.87      $

FAN SYSTEM COST:
      Cost of fans, Csfanc               =  236256.056      $
      Cost of fan casing/inlet bell, Csfc=  17671.8897      $
      Cost of fan safety screen, Csfsc   =  7132.67293      $
      Cost of electric motor, Csem       =  466529.295      $
      Cost of speed reducer, Cssr        =  281114.140      $
      Cost of electric wiring, Csew      =  46652.9295      $
      Total fan system cost, Csfant      =  1319196.23      $
```

STRUCTURAL, CONSTRUCTION AND DISTRIBUTION COSTS:

Cost of land, structure, construction, Csc	=	1888693.20	$
Cost of fan platform ,Csfp	=	262318.500	$
Total structural/construction cost, Cssc	=	2688764.63	$
Total steam/cond. distribution cost, Csdis	=	2784233.58	$

OPERATING AND MAINTENANCE COSTS:

Operating cost of fans, Csfo	=	2806057.52	$/a
HE bundles maintenance cost, Cshem	=	66357.5687	$/a
Fan system maintenance cost, Csfsm	=	39575.8869	$/a
Structural maintenance cost, Cssm	=	1344.38231	$/a
Distribution system maint. cost, Csdm	=	13921.1679	$/a
Fuel cost, Csfuel	=	0.000000000E+00	$/a

TOTAL COSTS:

Total power consumption cost	=	2806057.52	$/a
Total maintenance cost	=	121199.006	$/a
Total fuel cost	=	0.000000000E+00	$/a
Total capital cost	=	13427951.3	$
Total annual cost	=	5612846.78	$/a

Power generation calculations

Table K.6: Power generation output and cost data

RESULTS OF NET ANNUAL POWER OUTPUT

No	Tdb [C]	Twb [C]	h [h/a]	Ts [C]	ps [kPa]	Q [MW]	Pg [MW]	PF [MW]	Pn [MW]	nsF	FOS [rpm]
1	-1.00	0.00	4	44.42	9.26	328.46	237.67	3.203	234.46	30	100.00
2	0.00	0.00	10	45.18	9.62	328.33	237.83	3.193	234.63	30	100.00
3	1.00	0.00	26	45.96	10.01	328.19	237.98	3.182	234.80	30	100.00
4	2.00	0.00	43	46.77	10.42	328.06	238.13	3.171	234.96	30	100.00
5	3.00	0.90	59	47.60	10.87	327.95	238.27	3.159	235.11	30	100.00
6	4.00	1.90	82	48.45	11.34	327.84	238.39	3.147	235.25	30	100.00
7	5.00	2.90	112	49.32	11.84	327.74	238.51	3.135	235.37	30	100.00
8	6.00	3.80	152	50.22	12.38	327.66	238.61	3.124	235.49	30	100.00
9	7.00	4.60	201	51.13	12.94	327.59	238.69	3.112	235.58	30	100.00
10	8.00	5.40	254	52.06	13.54	327.54	238.76	3.101	235.66	30	100.00
11	9.00	6.30	312	53.00	14.18	327.52	238.81	3.089	235.72	30	100.00
12	10.00	7.10	371	53.96	14.85	327.51	238.83	3.078	235.75	30	100.00
13	11.00	7.80	434	54.94	15.56	327.53	238.83	3.066	235.76	30	100.00
14	12.00	8.60	506	55.93	16.32	327.57	238.80	3.055	235.75	30	100.00
15	13.00	9.40	578	56.93	17.11	327.64	238.75	3.044	235.71	30	100.00
16	14.00	10.20	656	57.95	17.95	327.74	238.66	3.033	235.63	30	100.00
17	15.00	10.90	738	58.98	18.84	327.88	238.55	3.022	235.53	30	100.00
18	16.00	11.60	764	60.02	19.78	328.05	238.39	3.011	235.38	30	100.00
19	17.00	12.20	655	61.08	20.77	328.26	238.20	3.000	235.20	30	100.00
20	18.00	12.70	553	62.15	21.81	328.50	237.98	2.989	234.99	30	100.00
21	19.00	13.20	459	63.22	22.91	328.79	237.70	2.979	234.73	30	100.00
22	20.00	13.70	381	64.31	24.07	329.12	237.39	2.969	234.42	30	100.00
23	21.00	14.10	320	65.41	25.29	329.51	237.03	2.959	234.07	30	100.00
24	22.00	14.50	265	66.53	26.59	329.94	236.62	2.949	233.67	30	100.00
25	23.00	14.90	219	67.65	27.95	330.42	236.15	2.939	233.21	30	100.00
26	24.00	15.20	177	68.79	29.39	330.96	235.63	2.929	232.70	30	100.00
27	25.00	15.50	140	69.94	30.91	331.56	235.05	2.919	232.14	30	100.00
28	26.00	15.70	105	71.10	32.51	332.23	234.42	2.910	231.51	30	100.00

29	27.00	15.90	76	72.27	34.21	332.96	233.71	2.901	230.81	30	100.00
30	28.00	16.10	51	73.45	36.00	333.76	232.94	2.891	230.04	30	100.00
31	29.00	16.40	30	74.65	37.89	334.64	232.09	2.882	229.21	30	100.00
32	30.00	16.80	15	75.87	39.89	335.60	231.16	2.872	228.29	30	100.00
33	31.00	17.40	8	77.09	42.01	336.64	230.16	2.862	227.29	30	100.00
34	32.00	18.00	4	78.34	44.25	337.77	229.06	2.852	226.21	30	100.00

```
NET ANNUAL POWER OUTPUT =   2058.351 GWh
ANNUAL HEAT DISSIPATED =   2877.138 GWh

 HEAT EXCHANGER BUNDLE COST:
       Cost of tubes, Cstm                 =  564213.372      $
       Cost of fins, Csfm                  =  2414586.86      $
       Cost of galvanizing, Csgm           = 0.000000000E+00 $
       Cost of finned tubes, Csft          =  3872440.30      $
       Cost of bundle frames/headers, Csbf =  774488.060      $
       Cost of bundle assembly, Csba       =  457500.000      $
       Total HE bundle cost, Cshe          =  6635756.87      $

 FAN SYSTEM COST:
       Cost of fans, Csfanc                  =  236256.056     $
       Cost of fan casing/inlet bell, Csfc=  17671.8897     $
       Cost of fan safety screen, Csfsc      =  7132.67293     $
       Cost of electric motor, Csem          =  465100.503     $
       Cost of speed reducer, Cssr           =  280542.624     $
       Cost of electric wiring, Csew         =  46510.0503     $
       Total fan system cost, Csfant         =  1316517.25     $

 STRUCTURAL, CONSTRUCTION AND DISTRIBUTION COSTS:
       Cost of land, structure, construction, Csc=  1888693.20     $
       Cost of fan platform ,Csfp                 =  262318.500     $
       Total structural/construction cost, Cssc   =  2688764.63     $
       Total steam/cond. distribution cost, Csdis=  2783295.94     $

 OPERATING AND MAINTENANCE COSTS:
       Operating cost of fans, Csfo           =  2797236.81     $/a
       HE bundles maintenance cost, Cshem     =  66357.5687     $/a
       Fan system maintenance cost, Csfsm     =  39495.5174     $/a
       Structural maintenance cost, Cssm      =  1344.38231     $/a
       Distribution system maint. cost, Csdm=  13916.4797     $/a
       Fuel cost, Csfuel                      =  106623684.     $/a

 TOTAL COSTS:
       Total power consumption cost      =  2797236.81     $/a
       Total maintenance cost            =  121113.948     $/a
       Total fuel cost                   =  106623684.     $/a
       Total capital cost                =  13424334.7     $
       Total annual cost                 =  112226902.     $/a
```

Operating point optimization

Table K.7: Optimization output and cost data

```
OPTIMIZATION OUTPUT DATA SET:
  FINNED TUBE TYPE: Round tubes (extruded fins)
  CORRELATION: Ganguli, Tung, Taborek
  OPERATING POINT OPTIMIZATION
```

```
VARIABLES FOR OPTIMIZATION:
ma Tao1 Tao2 Tacc mw1 mw2 H3 dfan THETF rpm Lt ntr THETB
do tt tr df tf Pf1 Pf2 Pt

  FAN INSTALLATION
   H3    : Height of fans above ground level=    35.3768      m
   nf    : Total number of fans             =    30.0000
   df    : Fan diameter                     =    10.1350      m
   THETF: Fan blade angle                   =    12.0000      deg
   rpm   : Fan operating speed              =    63.0471      rpm
   Qacc: Specified heat transfer rate       =  0.331100E+09 W
   Q    : Calculated heat transfer rate     =  0.331100E+09 W

  FINNED TUBES
   do : Tube outside diameter             =    37.9285      mm
   tt : Tube thickness                    =    1.20000      mm
   tr : Fin root thickness                =    1.00000      mm
   df : Fin diameter                      =    92.0346      mm
   tf : Fin thickness (core)              =  0.222671      mm
   Pf1: Fin pitch (row 1)                 =    3.19237      mm
   Pf2: Fin pitch (row 2)                 =    1.75000      mm
   Pt : Transversal tube pitch            =    93.5534      mm
   Pl : Longitudinal tube pitch           =    81.0173      mm

  HEAT EXCHANGER BUNDLE
   nr    : Number of tube rows              =    2.00000
   ntr1 : Number of tubes per row (row 1)  =    129.000
   ntr2 : Number of tubes per row (row 2)  =    130.000
   nb    : Number of heat exchanger bundles =    2.00000
   Lt    : Length of finned tubes           =    10.6019      m
   THETb: Bundle semi-apex angle            =    35.0000      deg
   Wb    : Width of heat exchanger bundle   =    12.1619      m

  HEAT EXCHANGER BUNDLE COST:
   Cost of tubes, Cstm                  =  472766.073      $
   Cost of fins, Csfm                   =  2998530.58      $
   Cost of galvanizing, Csgm            = 0.000000000E+00 $
   Cost of finned tubes, Csft           =  4512685.65      $
   Cost of bundle frames/headers, Csbf =  902537.131      $
   Cost of bundle assembly, Csba        =  388500.000      $
   Total HE bundle cost, Cshe           =  7544839.62      $

  FAN SYSTEM COST:
   Cost of fans, Csfanc               =  285724.149      $
   Cost of fan casing/inlet bell, Csfc=  21803.1915      $
   Cost of fan safety screen, Csfsc   =  8800.13607      $
   Cost of electric motor, Csem       =  147886.368      $
   Cost of speed reducer, Cssr        =  173444.207      $
   Cost of electric wiring, Csew      =  14788.6368      $
   Total fan system cost, Csfant      =  815558.359      $

  STRUCTURAL, CONSTRUCTION AND DISTRIBUTION COSTS:
   Cost of land, structure, construction, Csc=  2813720.04      $
   Cost of fan platform ,Csfp                 =  332804.076      $
   Total structural/construction cost, Cssc  =  3933155.15      $
   Total steam/cond. distribution cost, Csdis=  2926139.29      $

  OPERATING AND MAINTENANCE COSTS:
   Operating cost of fans, Csfo           =  838900.745      $/a
   HE bundles maintenance cost, Cshem    =  75448.3962      $/a
```

Fan system maintenance cost, Csfsm	=	24466.7508	$/a
Structural maintenance cost, Cssm	=	1966.57758	$/a
Distribution system maint. cost, Csdm	=	14630.6965	$/a
Fuel cost, Csfuel	=	0.000000000E+00	$/a

TOTAL COSTS:

Total power consumption cost	=	838900.745	$/a
Total maintenance cost	=	116512.421	$/a
Total fuel cost	=	0.000000000E+00	$/a
Total capital cost	=	15219692.4	$
Total annual cost	=	3999351.65	$/a

Table K.8: Post-optimality analysis

SCALE-INVARIANT MEASURE OF SENSITIVITY

GENERAL COST FACTORS

Ce	Present electricity cost (self-generated)	0.209759
ere	Electricity cost escalation rate	0.187191
Cf	Present fuel cost	0.000000E+00
erf	Fuel cost escalation rate	0.000000E+00
i	Interest rate	-0.880034E-01
FCR	Levelized fixed charge rate	0.761108
Tau	Running hours per annum	0.209759

FINNED TUBE BUNDLE COST

Cb1	Tube material unit cost	0.204677E-01
Cb2	Tubing fixed cost	0.470766E-01
Cb3	Fin material unit cost	0.166896
Cb4	Finning fixed cost	0.261505
Cb5	Galvanizing material unit cost	0.000000E+00
Cb6	Surface coating fixed cost	0.000000E+00
Cb7	Finned tube cost weighting factor	0.495946
Cb8	Bundle frame and header cost factor	0.826576E-01
Cb9	Tube assembly and end preparation cost	0.355802E-01
Cb10	Bundle cost weighting factor	0.531526
Cb11	Bundle maintenance cost factor	0.188652E-01

FAN SYSTEM COSTS

Cf1	Fan fixed cost	0.257472E-02
Cf2	Fan unit cost	0.351515E-01
Cf3	Fan casing and inlet bell unit cost	0.205631E-02
Cf4	Fan safety screen unit cost	0.829957E-03
Cf5	Electric motor fixed cost	0.169761E-02
Cf6	Electric motor unit cost	0.192236E-01
Cf7	Electric motor safety factor	0.192236E-01
Cf8	Speed reducer cost multiplier	0.163579E-01
Cf9	Electric wiring/switching cost multiplier	0.139475E-02
Cf10	Fan system cost weighting factor	0.615337E-01
Cf11	Fan system maintenance cost factor	0.611768E-02

STRUCTURAL AND CONSTRUCTION COSTS

Cs1	Land, excavation and foundation unit cost	0.417113E-02
Cs2	Structural material/installation unit cost	0.172155
Cs3	Fan platform unit cost	0.208556E-01
Cs4	Structural cost weighting factor	0.197181
Cs5	Structural maintenance cost factor	0.491714E-03

```
STEAM/CONDENSATE DISTRIBUTION COSTS
  Cd1 : Distribution system cost factor             0.149989
  Cd2 : Distribution system maintenance cost fact.  0.365826E-02

PRESCRIBED PARAMETERS
  Kos   : Loss coeff. for flow obstacles (suction)   0.197513E-01
  Kod   : Loss coeff. for flow obst. (discharge)     0.259953E-01
  ETAfd: Efficiency of fan drive system             -0.228983
  Lsw   : Half-width of walkway between A-frames    -0.529491E-02
  Kts   : ACC installation support loss coefficient  0.943268E-02
  pa1   : Barometric pressure at ground level       -0.500096
  dsh   : Effective steam header diameter            0.184559E-02
  Ksd   : Mean steam ducting loss coefficient        0.574610E-02
  Ts    : Saturated steam supply temperature        -8.62346
  Ta1   : Air temperature at ground level            7.38966

GEOMETRIC CONSTRAINT VARIATION SENSITIVITY
  CONSTRAINTS: ROUND TUBES
   1  (2*LT*SIN(THETB)/DFAN-WFL)                          0.661276E-01
   2  (WFU-2*LT*SIN(THETB)/DFAN)                          0.000000E+00
   3  (0.5*NB*WB/DFAN-LFL)                                0.666161E-01
   4  (LFU-0.5*NB*WB/DFAN)                                0.000000E+00
   5  (1.0-WB/WBU)                                        0.000000E+00
   6  (THETF-THETFMIN)                                    0.301764E-01
   7  (THETFMAX-THETF)                                    0.000000E+00
   8  (1.0-PI*DFAN*RPM/60/VBTU)                           0.000000E+00
   9  (0.09033-LSW/LT)                                    0.000000E+00
  10  (THETB/THBL-1.0)                                    0.000000E+00
  11  (1.0-THETB/THBU)                                    0.269489E-01
  12  (0.17886-(DSH/LT)/(2*(DSIN(THETB)+LSW/LT)))         0.000000E+00
  13  (PT/DF-1.0)                                         0.000000E+00
  14  (DF/DR-1.0)                                         0.000000E+00
  15  (TR/TRL-1.0)                                        0.380751E-01
  16  (TT/TTL-1.0)                                        0.379985E-01
  17  (PF(1)/PFL-1.0)                                     0.000000E+00
  18  (PF(2)/PFL-1.0)                                     0.612912E-02
  19  (TF/TFL-1.0)                                        0.000000E+00
  20  (PF(1)/TFIN-1.0)                                    0.000000E+00
  21  (PF(2)/TFIN-1.0)                                    0.000000E+00
```

Minimization of power generation cost

Table K.9: Optimization output and cost data

```
OPTIMIZATION OUTPUT DATA SET:
  FINNED TUBE TYPE: Round tubes (extruded fins)
  CORRELATION: Ganguli, Tung, Taborek

  ANNUAL NET POWER OUTPUT OPTIMIZATION

  VARIABLES FOR OPTIMIZATION:
  ma Tao1 Tao2 Tacc mw1 mw2 Ts rpm H3 dfan THETF Lt ntr THETB
  do tt tr df tf Pf1 Pf2 Pt

    FAN INSTALLATION
     H3   : Height of fans above ground level=   35.9088     m
     nf   : Total number of fans            =   30.0000
     df   : Fan diameter                    =   10.4918     m
```

THETF: Fan blade angle	=	12.0000	deg

FINNED TUBES

do : Tube outside diameter	=	36.8049	mm
tt : Tube thickness	=	1.20000	mm
tr : Fin root thickness	=	1.00000	mm
df : Fin diameter	=	92.3285	mm
tf : Fin thickness (core)	=	0.215643	mm
Pf1: Fin pitch (row 1)	=	3.06108	mm
Pf2: Fin pitch (row 2)	=	1.75000	mm
Pt : Transversal tube pitch	=	96.1082	mm
Pl : Longitudinal tube pitch	=	83.2297	mm

HEAT EXCHANGER BUNDLE

nr : Number of tube rows	=	2.00000	
ntr1 : Number of tubes per row (row 1)	=	130.000	
ntr2 : Number of tubes per row (row 2)	=	131.000	
nb : Number of heat exchanger bundles	=	2.00000	
Lt : Length of finned tubes	=	10.9752	m
THETb: Bundle semi-apex angle	=	35.0000	deg
Wb : Width of heat exchanger bundle	=	12.5902	m

HEAT EXCHANGER BUNDLE COST:

Cost of tubes, Cstm	=	488620.069	$
Cost of fins, Csfm	=	3099237.82	$
Cost of galvanizing, Csgm	=	0.000000000E+00	$
Cost of finned tubes, Csft	=	4664215.25	$
Cost of bundle frames/headers, Csbf	=	932843.050	$
Cost of bundle assembly, Csba	=	391500.000	$
Total HE bundle cost, Cshe	=	7785125.79	$

FAN SYSTEM COST:

Cost of fans, Csfanc	=	304802.242	$
Cost of fan casing/inlet bell, Csfc	=	23365.6468	$
Cost of fan safety screen, Csfsc	=	9430.76941	$
Cost of electric motor, Csem	=	107933.618	$
Cost of speed reducer, Cssr	=	165094.344	$
Cost of electric wiring, Csew	=	10793.3618	$
Total fan system cost, Csfant	=	776774.979	$

STRUCTURAL, CONSTRUCTION AND DISTRIBUTION COSTS:

Cost of land, structure, construction, Csc	=	3059628.50	$
Cost of fan platform ,Csfp	=	356653.405	$
Total structural/construction cost, Cssc	=	4270352.39	$
Total steam/cond. distribution cost, Csdis	=	2996665.27	$

OPERATING AND MAINTENANCE COSTS:

Operating cost of fans, Csfo	=	592250.607	$/a
HE bundles maintenance cost, Cshem	=	77851.2579	$/a
Fan system maintenance cost, Csfsm	=	23303.2494	$/a
Structural maintenance cost, Cssm	=	2135.17619	$/a
Distribution system maint. cost, Csdm	=	14983.3263	$/a
Fuel cost, Csfuel	=	106630383.	$/a

TOTAL COSTS:

Total power consumption cost	=	592250.607	$/a
Total maintenance cost	=	118273.010	$/a
Total fuel cost	=	106630383.	$/a
Total capital cost	=	15828918.4	$
Total annual cost	=	110506690.	$/a

```
NET ANNUAL POWER OUTPUT
 Net annual power output =   2079.41     GWh
 Annual heat dissipated  =   2877.25     GWh
 Cost per kWh            =  0.531434E-01 $/kWh
```

Table K.10: Power generation output

RESULTS OF NET ANNUAL POWER OUTPUT

No	Tdb [C]	Twb [C]	h [h/a]	Ts [C]	ps [kPa]	Q [MW]	Pg [MW]	PF [MW]	Pn [MW]	nsF	FOS [rpm]
1	-1.00	0.00	4	53.77	14.72	327.51	238.83	0.254	238.57	30	36.07
2	0.00	0.00	10	56.38	16.67	327.60	238.78	0.210	238.57	30	36.27
3	1.00	0.00	26	57.28	17.39	327.67	238.72	0.212	238.51	30	36.40
4	2.00	0.00	43	56.91	17.10	327.64	238.75	0.244	238.51	30	38.15
5	3.00	0.90	59	57.36	17.46	327.68	238.72	0.258	238.46	30	38.88
6	4.00	1.90	82	57.58	17.64	327.70	238.70	0.279	238.42	30	39.94
7	5.00	2.90	112	57.89	17.90	327.74	238.67	0.300	238.37	30	40.91
8	6.00	3.80	152	58.08	18.06	327.76	238.65	0.325	238.33	30	42.09
9	7.00	4.60	201	58.37	18.31	327.79	238.62	0.350	238.27	30	43.15
10	8.00	5.40	254	58.66	18.56	327.83	238.59	0.376	238.21	30	44.24
11	9.00	6.30	312	58.97	18.83	327.88	238.55	0.404	238.14	30	45.35
12	10.00	7.10	371	59.30	19.12	327.93	238.50	0.433	238.07	30	46.46
13	11.00	7.80	434	59.63	19.42	327.98	238.46	0.465	237.99	30	47.60
14	12.00	8.60	506	60.00	19.75	328.04	238.40	0.497	237.90	30	48.73
15	13.00	9.40	578	60.38	20.11	328.11	238.33	0.531	237.80	30	49.86
16	14.00	10.20	656	60.77	20.48	328.19	238.26	0.567	237.70	30	51.02
17	15.00	10.90	738	61.19	20.87	328.28	238.18	0.605	237.58	30	52.18
18	16.00	11.60	764	61.62	21.29	328.38	238.09	0.644	237.45	30	53.34
19	17.00	12.20	655	62.07	21.73	328.48	237.99	0.685	237.31	30	54.51
20	18.00	12.70	553	62.54	22.20	328.60	237.88	0.729	237.15	30	55.69
21	19.00	13.20	459	63.03	22.71	328.74	237.76	0.773	236.98	30	56.85
22	20.00	13.70	381	63.56	23.26	328.89	237.61	0.817	236.79	30	57.98
23	21.00	14.10	320	64.06	23.79	329.04	237.47	0.868	236.60	30	59.23
24	22.00	14.50	265	64.61	24.39	329.22	237.30	0.917	236.38	30	60.40
25	23.00	14.90	219	65.17	25.02	329.42	237.11	0.969	236.14	30	61.57
26	24.00	15.20	177	65.74	25.66	329.63	236.91	1.024	235.89	30	62.78
27	25.00	15.50	140	66.29	26.31	329.84	236.71	1.084	235.62	30	64.06
28	26.00	15.70	105	66.93	27.07	330.11	236.45	1.138	235.32	30	65.17
29	27.00	15.90	76	67.51	27.77	330.36	236.21	1.205	235.01	30	66.50
30	28.00	16.10	51	68.22	28.67	330.69	235.90	1.257	234.64	30	67.51
31	29.00	16.40	30	68.71	29.28	330.92	235.67	1.347	234.32	30	69.16
32	30.00	16.80	15	69.88	30.83	331.53	235.09	1.335	233.75	30	69.02
33	31.00	17.40	8	70.34	31.46	331.79	234.84	1.436	233.40	30	70.81
34	32.00	18.00	4	70.02	31.02	331.61	235.01	1.698	233.31	30	74.97

```
NET ANNUAL POWER OUTPUT =    2079.407 GWh
ANNUAL HEAT DISSIPATED =   2877.253 GWh
```

Table K.11: Post-optimality analysis

SCALE-INVARIANT MEASURE OF SENSITIVITY

GENERAL COST FACTORS

Ce : Present electricity cost (self-generated)	0.535940E-02
ere: Electricity cost escalation rate	0.478279E-02
Cf : Present fuel cost	0.964922
erf: Fuel cost escalation rate	1.01856
i : Interest rate	-0.466996
FCR: Levelized fixed charge rate	0.286479E-01

FINNED TUBE BUNDLE COST

Cb1 : Tube material unit cost	0.749104E-03
Cb2 : Tubing fixed cost	0.177735E-02
Cb3 : Fin material unit cost	0.628484E-02
Cb4 : Finning fixed cost	0.974015E-02
Cb5 : Galvanizing material unit cost	0.000000E+00
Cb6 : Surface coating fixed cost	0.000000E+00
Cb7 : Finned tube cost weighting factor	0.185515E-01
Cb8 : Bundle frame and header cost factor	0.309190E-02
Cb9 : Tube assembly and end preparation cost	0.129762E-02
Cb10: Bundle cost weighting factor	0.198491E-01
Cb11: Bundle maintenance cost factor	0.704481E-03

FAN SYSTEM COSTS

Cf1 : Fan fixed cost	0.931743E-04
Cf2 : Fan unit cost	0.136334E-02
Cf3 : Fan casing and inlet bell unit cost	0.797371E-04
Cf4 : Fan safety screen unit cost	0.321890E-04
Cf5 : Electric motor fixed cost	0.614365E-04
Cf6 : Electric motor unit cost	0.491158E-03
Cf7 : Electric motor safety factor	0.491158E-03
Cf8 : Speed reducer cost multiplier	0.563491E-03
Cf9 : Electric wiring/switching cost multiplier	0.368352E-04
Cf10: Fan system cost weighting factor	0.212105E-02
Cf11: Fan system maintenance cost factor	0.210866E-03

STRUCTURAL AND CONSTRUCTION COSTS

Cs1 : Land, excavation and foundation unit cost	0.161764E-03
Cs2 : Structural material/installation unit cost	0.677734E-02
Cs3 : Fan platform unit cost	0.808870E-03
Cs4 : Structural cost weighting factor	0.774800E-02
Cs5 : Structural maintenance cost factor	0.193201E-04

STEAM/CONDENSATE DISTRIBUTION COSTS

Cd1 : Distribution system cost factor	0.555909E-02
Cd2 : Distribution system maintenance cost fact.	0.135575E-03

PRESCRIBED PARAMETERS

Kos : Loss coeff. for flow obstacles (suction)	0.759359E-03
Kod : Loss coeff. for flow obst. (discharge)	0.999419E-03
ETAfd: Efficiency of fan drive system	-0.854487E-02
Lsw : Half-width of walkway between A-frames	-0.197914E-03
Kts : ACC installation support loss coefficient	0.362630E-03
pa1 : Barometric pressure at ground level	-0.192358E-01
dsh : Effective steam header diameter	0.676723E-04
Ksd : Mean steam ducting loss coefficient	0.208786E-03

GEOMETRIC CONSTRAINT VARIATION SENSITIVITY

CONSTRAINTS: ROUND TUBES

1	(2*LT*SIN(THETB)/DFAN-WFL)	0.247300E-02
2	(WFU-2*LT*SIN(THETB)/DFAN)	0.000000E+00
3	(0.5*NB*WB/DFAN-LFL)	0.283731E-02
4	(LFU-0.5*NB*WB/DFAN)	0.000000E+00
5	(1.0-WB/WBU)	0.000000E+00
6	(THETF-THETFMIN)	0.109201E-02
7	(THETFMAX-THETF)	0.000000E+00
8	(0.09033-LSW/LT)	0.000000E+00
9	(THETB/THBL-1.0)	0.000000E+00
10	(1.0-THETB/THBU)	0.724287E-03
11	(0.17886-(DSH/LT)/(2*(DSIN(THETB)+LSW/LT)))	0.000000E+00
12	(PT/DF-1.0)	0.000000E+00
13	(DF/DR-1.0)	0.000000E+00
14	(TR/TRL-1.0)	0.140311E-02
15	(TT/TTL-1.0)	0.139961E-02
16	(PF(1)/PFL-1.0)	0.000000E+00
17	(PF(2)/PFL-1.0)	0.213565E-03
18	(TF/TFL-1.0)	0.000000E+00
19	(PF(1)/TFIN-1.0)	0.000000E+00
20	(PF(2)/TFIN-1.0)	0.000000E+00

APPENDIX L

NATURAL DRAFT INDIRECT DRY-COOLING TOWERS: PROGRAM INPUT AND OUTPUT

L.1 PROGRAM INPUT DATA

Table L.1: General input data

```
COOLING TOWER DIMENSIONS
 H5    : Tower overall height . . . . . . . . . . .  120.00     m
 H3    : Tower inlet height . . . . . . . . . . . .  13.670     m
 d5    : Tower diameter at outlet . . . . . . . . .  58.000     m
 d3    : Tower diameter at inlet . . . . . . . . .   82.958     m
 PHIsb: Tower base angle . . . . . . . . . . . . .   70.000     deg
 CDts  : Tower support drag coefficient . . . . .    2.0000
 Lts   : Tower support length . . . . . . . . . .    14.547     m
 dts   : Tower support diameter/width . . . . . .   0.50000     m
 nts   : Number of tower supports . . . . . . . .    60.000
 tcts  : Tower shell thickness . . . . . . . . . .  0.25000     m
 ALPH  : Fraction of tower area covered by bundles 0.52379
COOLING TOWER OPERATING CHARACTERISTICS
 pa1  : Barometric pressure at ground level. . . .  84600.     N/m^2
 Ta1  : Dry bulb air temperature at ground level .  15.600      C
 Twb  : Wet bulb air temperature at ground level . 0.00000E+00  C
 g    : Gravitational acceleration . . . . . . . .  9.8000     m/s^2
 Ta4  : Air outlet temperature . . . . . . . . . .  45.475      C
 Twi  : Water inlet temperature . . . . . . . . .   61.450      C
 Two  : Water outlet temperature . . . . . . . . .  43.097      C
 ma   : Air mass flow rate . . . . . . . . . . . .  11203.     kg/s
 maL  : Lower bound for air mass flow rate . . . .  5000.0     kg/s
 maU  : Upper bound for air mass flow rate . . . .  20000.     kg/s
 mw   : Water mass flow rate . . . . . . . . . . .  4390.0     kg/s
 mwL  : Lower bound for water mass flow rate . . .  2000.0     kg/s
 mwU  : Upper bound for water mass flow rate . . .  7000.0     kg/s
FINNED TUBE BUNDLE SPECIFICATIONS
 nb   : Number of heat exchanger bundles . . . . .  142.00
 ntb  : Number of tubes per bundle . . . . . . . .  154.00
 nt/r: Number of tubes per row (maximum) . . . .    39.000
 nr   : Number of tube rows . . . . . . . . . . .   4.0000
 nwp  : Number of water passes . . . . . . . . . .  2.0000
 Lt   : Length of finned tubes . . . . . . . . . .  15.000     m
 Wb   : Width of heat exchanger bundle . . . . . .  2.2620     m
 THET: Bundle semi-apex angle . . . . . . . . . .   30.750     deg
 Ke   : Total flow resistance length ratio . . . .  2.0000
EXPERIMENTAL PERFORMANCE CHARACTERISTICS
 PC   : Correlation(normal flow=0,inclined flow=1) 0.00000E+00
 AKhe: Constant in Khe correlation . . . . . . . . 0.00000E+00
 BKhe: Exponent in Khe correlation . . . . . . . . 0.00000E+00
 ANy  : Constant in Ny correlation . . . . . . . . 0.00000E+00
 BNy  : Exponent in Ny correlation . . . . . . . . 0.00000E+00
FINNED TUBE DIMENSIONS: GENERAL
 tt   : Tube thickness . . . . . . . . . . . . . .  1.9000     mm
 tf   : Fin thickness (mean - core material) . . . 0.50000     mm
 tg   : Thickness of galvanizing material (mean) . 0.00000E+00 mm
```

```
 Pf   : Fin pitch  . . . . . . . . . . . . . . . .  2.8000      mm
 Pt   : Transversal tube pitch . . . . . . . . . .  58.000      mm
 Pl   : Longitudinal tube pitch  . . . . . . . . .  50.220      mm
 EPS  : Tube inside surface roughness  . . . . . . 0.11500E-01 mm
FINNED TUBE DIMENSIONS: ROUND TUBES
 do   : Tube outside diameter  . . . . . . . . . .  25.400      mm
 tr   : Fin root thickness . . . . . . . . . . . .  1.1000      mm
 df   : Fin diameter . . . . . . . . . . . . . . .  57.200      mm
FINNED TUBE DIMENSIONS: NON-ROUND TUBES
 Peri   : Fin perimeter form (Ellip=1, Rect=2) . . 0.00000E+00
 de     : Tube hydraulic diameter  . . . . . . . . 0.00000E+00 mm
 Atcs   : Tube cross-sectional area  . . . . . . . 0.00000E+00 mm^2
 Ati    : Tube inside surface area/unit length . . 0.00000E+00 mm
 Af1    : Surface area of one fin  . . . . . . . . 0.00000E+00 mm^2
 Aa1    : Surface area of one fin and exposed root 0.00000E+00 mm^2
 Vf1    : Material volume of one fin . . . . . . . 0.00000E+00 mm^3
 SIGMf1: Fin leading edge fr. area/HE fr. area   . 0.00000E+00
 SIGM21: HE inlet contraction area ratio   . . . . 0.00000E+00
 SIGM   : Minimum HE free flow area/HE fr. area  . 0.00000E+00
FINNED TUBE PROPERTIES
 RHOt: Density of tube material . . . . . . . . .  7850.0      kg/m^3
 RHOf: Density of fin material  . . . . . . . . .  2707.0      kg/m^3
 RHOg: Density of galvanizing material  . . . . . 0.00000E+00 kg/m^3
 kt   : Thermal conductivity of tube material  . .  50.000      W/mC
 kf   : Thermal conductivity of fin material . . .  204.00      W/mC
 kg   : Thermal conductivity of galv. material . . 0.00000E+00 W/mC
COST FACTORS: GENERAL INFORMATION
 Ce   : Present electricity cost (self-generated)   5.0000      c/kWh
 ese  : Electricity cost escalation rate . . . . .  7.5000      %
 Cf   : Present fuel cost  . . . . . . . . . . . . 0.25000E-02 $/MJ
 esf  : Fuel cost escalation rate  . . . . . . . .  8.5000      %
 i    : Interest rate  . . . . . . . . . . . . . .  10.000      %
 NY   : Capital repayment period . . . . . . . . .  30.000      years
 FCR  : Fixed charge rate  . . . . . . . . . . . .  20.000      %
 Tau  : Running hours per annum  . . . . . . . . .  8760.0      h
FINNED TUBE BUNDLE COSTS
 Cb1  : Tube material unit cost  . . . . . . . . . 0.80000     $/kg
 Cb2  : Tubing fixed cost  . . . . . . . . . . . .  2.0000      $/m
 Cb3  : Fin material unit cost . . . . . . . . . .  4.0000      $/kg
 Cb4  : Finning fixed cost . . . . . . . . . . . . 0.20000      $/m
 Cb5  : Galvanizing material unit cost . . . . . . 0.00000E+00 $/kg
 Cb6  : Surface coating fixed cost . . . . . . . . 0.00000E+00 $/m^2
 Cb7  : Finned tube cost weighting factor  . . . .  1.2000
 Cb8  : Bundle frame and header cost factor  . . . 0.20000
 Cb9  : Tube assembly and end preparation cost . .  25.000      $/FT
 Cb10: Bundle cost weighting factor . . . . . . .  1.2000
 Cb11: Bundle maintenance cost factor . . . . . . 0.10000E-01
CIRCULATION SYSTEM COSTS
 Cp1  : Pump fixed cost  . . . . . . . . . . . . .  2300.0      $/P
 Cp2  : Pump unit cost . . . . . . . . . . . . . .  180.00      $/kW
 Cp3  : Electric motor fixed cost  . . . . . . . .  400.00      $/EM
 Cp4  : Electric motor unit cost . . . . . . . . .  120.00      $/kW
 Cp5  : Electric motor safety factor (undersizing)  1.2500
 Cp6  : Electric wiring/switching cost multiplier  0.10000
 Cp7  : Pump system cost multiplier  . . . . . . .  1.2500
 Cp8  : Pump system maintenance cost factor  . . . 0.30000E-01
 Cp9  : Piping and valves cost factor  . . . . . . 0.25000
 Cp10: Piping and valves maintenance cost factor  0.50000E-02
```

```
COOLING TOWER CONSTRUCTION COSTS
 Cs1  : Land, excavation and foundation unit cost   70.000      $/m^2
 Cs2  : Cooling tower shell cost . . . . . . . . .  250.00      $/m^3
 Cs3  : HE Bundle support platform cost  . . . . .  100.00      $/m^2
 Cs4  : Cooling tower support cost . . . . . . . .  175.00      $/m^3
 Cs5  : Construction cost multiplier . . . . . . .  1.2500
 Cs6  : Maintenance cost factor  . . . . . . . . . 0.50000E-03
FIN MATERIAL PROPERTIES
 RHOal: Density of aluminium  . . . . . . . . . .  2707.0      kg/m^3
 RHOcu: Density of copper . . . . . . . . . . . .  8954.0      kg/m^3
 RHOst: Density of steel  . . . . . . . . . . . .  7850.0      kg/m^3
 RHOzn: Density of zinc . . . . . . . . . . . . .  7144.0      kg/m^3
 kal   : Thermal conductivity of aluminium . . . .  204.00      W/mC
 kcu   : Thermal conductivity of copper  . . . . .  386.00      W/mC
 kst   : Thermal conductivity of steel . . . . . .  50.000      W/mC
 kzn   : Thermal conductivity of zinc  . . . . . .  116.00      W/mC
PARAMETERS FOR SUBROUTINE SOLVE
 H     : Step length for numerical differentiation 0.10000E-07
 ACCS : Stopping accuracy for SOLVE . . . . . . . 0.10000E-06
 ITMAX: Maximum no. of iterations for SOLVE . . .  50.000
 IPRNT: Controls intermediate printing of SOLVE .  1.0000
PARAMETERS FOR SUBROUTINE OPTIM
 IPRNT : Controls intermediate printing of OPTIM   1.0000
 MAXFUN: Maximum no. of function evaluations  . .  250.00
 ACCO  : Stopping accuracy for OPTIM  . . . . . . 0.10000E-06
 H     : Step length for numerical differentiation 0.10000E-07
ALL FINNED TUBE TYPES: CONSTANT VALUES(=0.0)
 H5   : Tower overall height . . . . . . . . . . .  1.0000
 H3   : Tower inlet height . . . . . . . . . . . .  1.0000
 d5   : Tower diameter at outlet . . . . . . . . .  1.0000
 d3   : Tower diameter at inlet  . . . . . . . . .  1.0000
 nt/r: Number of tubes per row  . . . . . . . . .  1.0000
 Lt   : Length of finned tubes . . . . . . . . . .  1.0000
 THET: Bundle semi-apex angle . . . . . . . . . .  1.0000
ALL FINNED TUBE TYPES: LOWER VALUES
 H5   : Tower overall height . . . . . . . . . . .  80.000      m
 H3   : Tower inlet height . . . . . . . . . . . .  5.0000      m
 d5   : Tower diameter at outlet . . . . . . . . .  40.000      m
 d3   : Tower diameter at inlet  . . . . . . . . .  45.000      m
 nt/r: Number of tubes per row  . . . . . . . . .  10.000
 Lt   : Length of finned tubes . . . . . . . . . .  5.0000      m
 THET: Bundle semi-apex angle . . . . . . . . . .  10.000      deg
ALL FINNED TUBE TYPES: UPPER VALUES
 H5   : Tower overall height . . . . . . . . . . .  200.00      m
 H3   : Tower inlet height . . . . . . . . . . . .  25.000      m
 d5   : Tower diameter at outlet . . . . . . . . .  100.00      m
 d3   : Tower diameter at inlet  . . . . . . . . .  150.00      m
 nt/r: Number of tubes per row  . . . . . . . . .  100.00
 Lt   : Length of finned tubes . . . . . . . . . .  20.000      m
 THET: Bundle semi-apex angle . . . . . . . . . .  80.000      deg
ROUND TUBES: CONSTANT VALUES(=0.0)
 do   : Tube outside diameter  . . . . . . . . . .  1.0000
 tt   : Tube thickness . . . . . . . . . . . . . .  1.0000
 tr   : Fin root thickness . . . . . . . . . . . .  1.0000
 df   : Fin diameter . . . . . . . . . . . . . . .  1.0000
 tf   : Fin thickness  . . . . . . . . . . . . . .  1.0000
 tg   : Thickness of galvanizing material  . . . . 0.00000E+00
 Pf   : Fin pitch  . . . . . . . . . . . . . . . .  1.0000
 Pt   : Transversal tube pitch . . . . . . . . . .  1.0000
```

```
ROUND TUBES: LOWER VALUES
 do   : Tube outside diameter . . . . . . . . . .  10.000       mm
 tt   : Tube thickness . . . . . . . . . . . . . .  1.0000       mm
 tr   : Fin root thickness . . . . . . . . . . . .  1.0000       mm
 df   : Fin diameter . . . . . . . . . . . . . . .  20.000       mm
 tf   : Fin thickness  . . . . . . . . . . . . . . 0.50000E-01  mm
 tg   : Thickness of galvanizing material  . . . . 0.30000E-01  mm
 Pf   : Fin pitch  . . . . . . . . . . . . . . . .  1.0000       mm
 Pt   : Transversal tube pitch . . . . . . . . . .  20.000       mm
ROUND TUBES: UPPER VALUES
 do   : Tube outside diameter . . . . . . . . . .  80.000       mm
 tt   : Tube thickness . . . . . . . . . . . . . .  2.0000       mm
 tr   : Fin root thickness . . . . . . . . . . . .  2.0000       mm
 df   : Fin diameter . . . . . . . . . . . . . . .  100.00       mm
 tf   : Fin thickness  . . . . . . . . . . . . . .  1.0000       mm
 tg   : Thickness of galvanizing material  . . . . 0.70000E-01  mm
 Pf   : Fin pitch  . . . . . . . . . . . . . . . .  5.0000       mm
 Pt   : Transversal tube pitch . . . . . . . . . .  120.00       mm
OPTIMIZATION: GENERAL INFORMATION
 trL    : Lower bound for tr . . . . . . . . . . .  1.0000       mm
 ttL    : Lower bound for tt . . . . . . . . . . .  1.2000       mm
 tfL    : Lower bound for tf . . . . . . . . . . . 0.50000E-01  mm
 tgL    : Lower bound for tg . . . . . . . . . . . 0.20000E-01  mm
 tgU    : Upper bound for tg . . . . . . . . . . . 0.10000      mm
 PfL    : Lower bound for Pf . . . . . . . . . . .  2.0000       mm
 LtU    : Upper bound for Lt . . . . . . . . . . .  25.000       m
 WbU    : Upper bound for Wb . . . . . . . . . . .  3.0000       m
 HirL   : Lower bound for H3/H5-ratio  . . . . . . 0.50000E-01
 HirU   : Upper bound for H3/H5-ratio  . . . . . . 0.15000
 DirL   : Lower bound for d3/(H5-H3)-ratio . . . . 0.64000
 DirU   : Upper bound for d3/(H5-H3)-ratio . . . .  1.1300
 DrL    : Lower bound for d3/d5-ratio  . . . . . .  1.3700
 DrU    : Upper bound for d3/d5-ratio  . . . . . .  1.7700
 THbU   : Upper bound for bundle semi-apex angle .  60.000       deg
OPTIMIZATION: GENERAL INFORMATION
 vwL    : Lower bound for vw . . . . . . . . . . .  1.0000       m/s
 vwfx   : Fix vw  at this value, 0 otherwise . . . 0.00000E+00  m/s
 Qct    : Tower heat transfer rate . . . . . . . .  331.10       MW
 UAc    : Condenser thermal conductance  . . . . . 0.38480E+08  W/K
 TTDc   : Cond. terminal temperature difference  .  2.5000       C
 TsU    : Upper steam temperature limit  . . . . .  90.000       C
 TsL    : Lower steam temperature limit  . . . . .  30.000       C
CHOICE OF CORRELATIONS AND FORMULAE
 Finned tube type . . . . . . . . . . . . . . . : 1.0
  1 Round tubes (round fins)
  2 Non-round tubes
 Round tubes performance correlations . . . . . : 2.0
  1 Ny vs. Ry and Khe vs. Ry
  2 Briggs, Young, Robinson
  3 Ganguli, Tung, Taborek
  4 Nir
 Non-round tubes performance correlations . . . : 1.0
  1 Ny vs. Ry and Khe vs. Ry
 Fin material . . . . . . . . . . . . . . . . . : 1.0
  1 Aluminium
  2 Copper
  3 Steel
  4 Galvanized fin (steel core)
```

```
Optimization model (Operating point) . . . . . : 3.0
  1 Constant water pressure drop
  2 Constant condenser design
  3 Fixed mw and Twi
  4 Fixed Twi
```

Table L.2: Frequency of dry- and wetbulb temperatures

```
MEAN HOURLY FREQUENCY OF AMBIENT TEMPERATURES
 Number of data sets (maximum 40):   34
 Tdb : Dry bulb temperature at ground level [C]
 Twb : Wet bulb temperature at ground level [C]
 h   : Annual duration of these temperatures [h/a]
```

No	Tdb [C]	Twb [C]	h [h/a]
1	-1.000	0.000	4.000
2	0.000	0.000	10.000
3	1.000	0.000	26.000
4	2.000	0.000	43.000
5	3.000	0.900	59.000
6	4.000	1.900	82.000
7	5.000	2.900	112.000
8	6.000	3.800	152.000
9	7.000	4.600	201.000
10	8.000	5.400	254.000
11	9.000	6.300	312.000
12	10.000	7.100	371.000
13	11.000	7.800	434.000
14	12.000	8.600	506.000
15	13.000	9.400	578.000
16	14.000	10.200	656.000
17	15.000	10.900	738.000
18	16.000	11.600	764.000
19	17.000	12.200	655.000
20	18.000	12.700	553.000
21	19.000	13.200	459.000
22	20.000	13.700	381.000
23	21.000	14.100	320.000
24	22.000	14.500	265.000
25	23.000	14.900	219.000
26	24.000	15.200	177.000
27	25.000	15.500	140.000
28	26.000	15.700	105.000
29	27.000	15.900	76.000
30	28.000	16.100	51.000
31	29.000	16.400	30.000
32	30.000	16.800	15.000
33	31.000	17.400	8.000
34	32.000	18.000	4.000

Table L.3: Turbo-generator-condenser characteristic curves

```
TURBO-GENERATOR-CONDENSER PERFORMANCE CHARACTERISTICS
Tsmin: Minimum saturated steam temperature . . .  42.000    C
Tsmax: Maximum saturated steam temperature . . .  94.610    C
Qdes : Heat to be rejected at the design point .  327.63    MW
TTDd : Design point terminal temp. difference  .  2.5000    K
```

```
HEAT TO BE REJECTED [MW] vs. SATURATED STEAM TEMPERATURE [C]
Cq0   : Constant coefficient . . . . . . . . . . .  336.40
Cq1   : First degree coefficient  . . . . . . . . . 0.18223
Cq2   : Second degree coefficient . . . . . . . . .-0.16010E-01
Cq3   : Third degree coefficient  . . . . . . . . . 0.17753E-03
Cq4   : Fourth degree coefficient . . . . . . . . . 0.00000E+00
Cq5   : Fifth degree coefficient  . . . . . . . . . 0.00000E+00
GENERATOR POWER OUTPUT [MW] vs. SATURATED STEAM TEMPERATURE [C]
Cp0   : Constant coefficient . . . . . . . . . . .  225.83
Cp1   : First degree coefficient  . . . . . . . . .-0.42994E-02
Cp2   : Second degree coefficient . . . . . . . . . 0.13317E-01
Cp3   : Third degree coefficient  . . . . . . . . .-0.16257E-03
Cp4   : Fourth degree coefficient . . . . . . . . . 0.00000E+00
Cp5   : Fifth degree coefficient  . . . . . . . . . 0.00000E+00
```

L.2 PROGRAM OUTPUT

Operating point calculations

Table L.4: Operating point output and cost data

```
COMPLETE OUTPUT DATA SET:

 AIRSIDE DATA:
    Air temperature:
         At inlet of heat exchangers, Tsa3    =    15.46672 C
         At outlet of heat exchangers, Tsa4   =    45.29730 C
         Mean HE temperature, Tsam            =    30.38201 C

         Humidity ratio, w  =  0.0000000 kg moisture/kg dry air

    Air properties (evaluated at ground level air pressure):
    At HE inlet temperature:
         Density, RHOsa3              =     1.021047 kg/m^3
    At mean HE temperature:
         Density, RHOsa               = 0.970874     kg/m^3
         Viscosity, MUsa              = 0.186308E-04 kg/ms
         Specific heat, cpsa          =  1007.09     J/kgK
         Thermal conductivity, ksa    = 0.264944E-01 W/mK
         Prandtl number, PRAsa        = 0.708183
    At HE outlet temperature:
         Density, RHOsa4              =     0.925401 kg/m^3
    At cooling tower outlet:
         Air temperature, Tsa5        =     44.27954 C
         Density, RHOsa5              =     0.915264 kg/m^3
         Air pressure, psa5           =    83405.872 N/m^2

    Air flow data:
         Air mass flow rate, msa                  =   11020.0      kg/s
         Characteristic flow parameter, Ry        =   122766.      m^-1
         Effective heat transfer coeff., hAsae =  0.178831E+08 W/K
         1/Densimetric Froude number, 1/FrsD      =   2.84087

 WATERSIDE DATA:
    Water temperature:
         At inlet of cooling tower, Tswi    =    61.45000 C
         At outlet of cooling tower, Tswo   =    43.40813 C
```

```
Mean CT temperature, Tswm          =   52.42907 C

   Water properties at mean CT temperature:
   At mean CT temperature:
        Density, RHOsw              =  987.010     kg/m^3
        Viscosity, MUsw             = 0.522417E-03 kg/ms
        Specific heat, cpsw         =  4179.89     J/kgK
        Thermal conductivity, ksw   = 0.645492     W/mK
        Prandtl number, PRAsw       =  3.38292

   Water flow data:
        Water mass flow rate, msw              =   4390.00     kg/s
        Water mass velocity, Gsw               =   1095.69     kg/sm^2
        Water velocity inside tubes, vw        =   1.11011     m/s
        Bundle tube Reynolds number, Resw      =   45302.8
        Tube friction factor, fsw              =  0.227298E-01
        Pressure drop per unit length, dpsw    =   639.979     N/m^3
        Total water pumping power, Psw         =   211503.     W
        Waterside heat transf coeff., hsw      =   6969.06     W/m^2K

SYSTEM DATA:
        Overall heat transfer coeff., UA          =   0.160346E+08 W/K
        Log. mean temperature diff., LMTD         =    21.5114      C
        Cross-flow correction factor, Fst         =   0.959809
        Airside heat transfer rate, Qsa           =   0.331063E+09 W
        Waterside heat transfer rate, Qsw         =   0.331063E+09 W
        Heat transfer rate based on LMTD, Qsu     =   0.331063E+09 W

DRAFT EQUATION:
        Draft equation LHS  = 97.785     N/m^2
        Draft equation RHS  = 97.785     N/m^2

HEAT EXCHANGER GEOMETRIC DETAILS:
        Total tube length, Lstt                 =     328020.      m
        Total waterside surface area, Aswt      =     22258.9      m^2
        Total fin surface area, Asft            =     472428.      m^2
        Total air side surface area, Asat       =     495791.      m^2
        Total frontal bundle area, Asfr         =     4818.06      m^2
        Min. HE flow area/frontal area, SIGM    =    0.433005

HEAT TRANSFER AND PRESSURE DROP CORRELATIONS:
   Correlation: Briggs, Young, Robinson
   Heat transfer:
        Coefficient asHA                     =  0.109788
        Exponent bsHA                        =  0.681000
        Equivalent diameter, de              =   27.6000      mm
        Air side Reynolds number, Re         =   7825.19
        Air side heat transfer coeff., hsa   =   40.4130      W/m^2K
        Fin efficiency, ETAsf                =  0.917840
        Surface effectiveness, EPSIsf        =  0.921711
   Pressure drop:
        Coefficient asEU                        =   9.51000
        Exponent bsEU                           =  -0.316000
        Equivalent diameter, de                 =   27.6000     mm
        Air Reynolds number, Re                 =   7825.19
        Euler number,Eu                         =   2.23819
        Total bundle loss coefficient, Kshet    =   30.7376

PRESSURE LOSS COEFFICIENTS (based on HE frontal area and mean air density):
        Tower support loss coefficient, Ksts    = .42625
```

```
          Tower inlet loss coefficient, Ksct        = 1.7289
          Contraction loss coefficient, Kscon       = 1.2131
          HE inlet loss coefficient, Kshi           = 1.9442
          HE normal flow loss coefficient, Kshe     = 24.399
          HE downstream loss coefficient, Ksd       = 4.3944
          Expansion loss coefficient, Ksexp         = 1.1887
          Tower outlet loss coefficient, Kso        = 1.3972

PRESSURE LOSS TERMS (based on HE frontal area and mean air density):
          Tower support pressure loss, dpsts       = 1.1484     N/m^2
          Tower inlet pressure loss, dpsct         = 4.6579     N/m^2
          Tower contraction press. loss, dpscon = 3.2683     N/m^2
          HE inlet pressure loss, dpshi            = 5.2380     N/m^2
          HE normal flow pressure loss, dpshe      = 65.736     N/m^2
          HE downstream pressure loss, dpsd        = 11.839     N/m^2
          Tower expansion pressure loss, dpsexp = 3.2026     N/m^2
          Tower outlet pressure loss, dpso         = 3.7643     N/m^2

HEAT EXCHANGER BUNDLE COST:
      Cost of tubes, Cstm                          =  944995.725     $
      Cost of fins, Csfm                           =  3607200.59     $
      Cost of galvanizing, Csgm                    = 0.000000000E+00 $
      Cost of finned tubes, Csft                   =  5462635.58     $
      Cost of bundle frames/headers, Csbf          =  1092527.12     $
      Cost of bundle assembly, Csba                =  546700.000     $
      Total HE bundle cost, Cshe                   =  8522235.23     $

CIRCULATION SYSTEM COST:
      Cost of pumps, Cspc              =  80741.1488     $
      Cost of electric motors, Csem    =  32525.4787     $
      Cost of electric wiring, Csew    =  3252.54787     $
      Total pumping system cost, Cspt =  145648.969     $
      Cost of piping and valves, Cspv =  2130558.81     $

COOLING TOWER STRUCTURAL AND CONSTRUCTION COSTS:
      Cost of land, excavation, foundation, Csle=  474572.960     $
      Cost of cooling tower shell, Cscts          =  1382704.24     $
      Cost of HE bundle platform, Cspl            =  540513.354     $
      Cost of cooling tower supports, Csts        =  38186.6891     $
      Total structural, construction cost, Cssc =  3044971.56     $

OPERATING AND MAINTENANCE COSTS:
      Operating cost of pumps, Cspo                =  195858.702     $/a
      HE bundles maintenance cost, Cshem           =  85222.3523     $/a
      Pump system maintenance cost, Cspsm          =  4369.46907     $/a
      Piping and valves maint. cost, Cspvm         =  10652.7940     $/a
      Cooling tower struct. maint. cost, Cssm =  1522.48578     $/a
      Fuel cost, Csfuel                            = 0.000000000E+00 $/a

TOTAL COSTS:
      Total operating cost           =  195858.702     $/a
      Total maintenance cost         =  101767.101     $/a
      Total fuel cost                = 0.000000000E+00 $/a
      Total capital cost             =  13843414.6     $
      Total annual cost              =  3066308.72     $/a
```

Power generation calculations

Table L.5: Power generation output and cost data

RESULTS OF NET ANNUAL POWER OUTPUT

No	Tdb [C]	Twb [C]	h [h/a]	Twi [C]	Two [C]	Ts [C]	Q [MW]	Pg [MW]	Pp [MW]	Pn [MW]
1	-1.00	0.00	4	43.36	25.47	45.87	328.21	237.96	0.221	237.74
2	0.00	0.00	10	44.41	26.52	46.91	328.04	238.15	0.221	237.93
3	1.00	0.00	26	45.46	27.58	47.96	327.90	238.32	0.220	238.10
4	2.00	0.00	43	46.52	28.64	49.02	327.77	238.47	0.219	238.25
5	3.00	0.90	59	47.58	29.71	50.08	327.67	238.59	0.219	238.38
6	4.00	1.90	82	48.64	30.77	51.14	327.59	238.69	0.218	238.48
7	5.00	2.90	112	49.70	31.84	52.20	327.54	238.77	0.217	238.55
8	6.00	3.80	152	50.77	32.90	53.26	327.51	238.82	0.217	238.60
9	7.00	4.60	201	51.84	33.98	54.34	327.51	238.83	0.216	238.62
10	8.00	5.40	254	52.91	35.05	55.41	327.54	238.82	0.215	238.61
11	9.00	6.30	312	53.99	36.12	56.49	327.61	238.78	0.215	238.56
12	10.00	7.10	371	55.07	37.20	57.57	327.70	238.70	0.214	238.49
13	11.00	7.80	434	56.16	38.28	58.66	327.83	238.59	0.214	238.37
14	12.00	8.60	506	57.25	39.37	59.75	328.00	238.44	0.213	238.22
15	13.00	9.40	578	58.35	40.45	60.85	328.21	238.25	0.213	238.04
16	14.00	10.20	656	59.45	41.54	61.95	328.45	238.02	0.212	237.81
17	15.00	10.90	738	60.55	42.64	63.06	328.75	237.75	0.212	237.54
18	16.00	11.60	764	61.67	43.73	64.18	329.08	237.43	0.211	237.22
19	17.00	12.20	655	62.78	44.83	65.30	329.46	237.07	0.211	236.86
20	18.00	12.70	553	63.91	45.93	66.43	329.90	236.66	0.211	236.45
21	19.00	13.20	459	65.04	47.04	67.56	330.38	236.19	0.210	235.98
22	20.00	13.70	381	66.17	48.15	68.70	330.92	235.67	0.210	235.47
23	21.00	14.10	320	67.32	49.27	69.85	331.52	235.10	0.209	234.89
24	22.00	14.50	265	68.47	50.38	71.00	332.17	234.47	0.209	234.26
25	23.00	14.90	219	69.63	51.51	72.17	332.90	233.77	0.209	233.56
26	24.00	15.20	177	70.79	52.63	73.34	333.68	233.01	0.208	232.80
27	25.00	15.50	140	71.97	53.76	74.52	334.54	232.19	0.208	231.98
28	26.00	15.70	105	73.15	54.90	75.71	335.47	231.29	0.208	231.08
29	27.00	15.90	76	74.35	56.04	76.91	336.48	230.31	0.207	230.10
30	28.00	16.10	51	75.55	57.19	78.12	337.57	229.26	0.207	229.05
31	29.00	16.40	30	76.76	58.34	79.35	338.75	228.12	0.207	227.91
32	30.00	16.80	15	77.98	59.50	80.58	340.01	226.89	0.207	226.69
33	31.00	17.40	8	79.22	60.67	81.82	341.38	225.58	0.206	225.37
34	32.00	18.00	4	80.47	61.84	83.08	342.84	224.16	0.206	223.96

NET ANNUAL POWER OUTPUT = 2074.337 GWh
ANNUAL HEAT DISSIPATED = 2886.389 GWh

HEAT EXCHANGER BUNDLE COST:

Cost of tubes, Cstm	=	944995.725	$
Cost of fins, Csfm	=	3607200.59	$
Cost of galvanizing, Csgm	=	0.000000000E+00	$
Cost of finned tubes, Csft	=	5462635.58	$
Cost of bundle frames/headers, Csbf	=	1092527.12	$
Cost of bundle assembly, Csba	=	546700.000	$
Total HE bundle cost, Cshe	=	8522235.23	$

CIRCULATION SYSTEM COST:

Cost of pumps, Cspc	=	78781.4240	$
Cost of electric motors, Csem	=	31708.9267	$

```
     Cost of electric wiring, Csew    =  3170.89267     $
     Total pumping system cost, Cspt =  142076.554     $
     Cost of piping and valves, Cspv =  2130558.81     $

COOLING TOWER STRUCTURAL AND CONSTRUCTION COSTS:
     Cost of land, excavation, foundation, Csle=  474572.960     $
     Cost of cooling tower shell, Cscts          =  1382704.24     $
     Cost of HE bundle platform, Cspl            =  540513.354     $
     Cost of cooling tower supports, Csts        =  38186.6891     $
     Total structural, construction cost, Cssc =  3044971.56     $

OPERATING AND MAINTENANCE COSTS:
     Operating cost of pumps, Cspo             =  196280.725     $/a
     HE bundles maintenance cost, Cshem        =  85222.3523     $/a
     Pump system maintenance cost, Cspsm       =  4262.29662     $/a
     Piping and valves maint. cost, Cspvm      =  10652.7940     $/a
     Cooling tower struct. maint. cost, Cssm =  1522.48578     $/a
     Fuel cost, Csfuel                         =  106637256.     $/a

TOTAL COSTS:
     Total operating cost          =  196280.725     $/a
     Total maintenance cost        =  101659.929     $/a
     Total fuel cost               =  106637256.     $/a
     Total capital cost            =  13839842.2     $
     Total annual cost             =  109703165.     $/a
```

Operating point optimization

<u>Table L.6: Optimization output and cost data</u>

```
OPTIMIZATION OUTPUT DATA SET:

  FINNED TUBE TYPE: Round tubes (round fins)
  CORRELATION: Briggs, Young, Robinson

  OPERATING POINT OPTIMIZATION: Fixed mw and Twi

  VARIABLES FOR OPTIMIZATION:
    ma Ta4 Two H5 H3 d5 d3 Lt nt/r THET do tt tr df tf Pf Pt

    COOLING TOWER
     H5   : Tower overall height            =   127.857      m
     H3   : Tower inlet height              =   17.7755      m
     d5   : Tower diameter at outlet        =   67.3921      m
     d3   : Tower diameter at inlet         =   92.3271      m
     Lts  : Tower support length            =   18.9163      m
     nts  : Number of tower supports        =   60.0000
     Qct    : Specified heat transfer rate  =  0.331100E+09 W
     Q      : Calculated heat transfer rate =  0.331100E+09 W
     1/FrsD: 1/Densimetric Froude number    =   2.30787

    FINNED TUBES
     do  : Tube outside diameter   =   27.2636     mm
     tt  : Tube thickness          =   1.20000     mm
     tr  : Fin root thickness      =   1.00000     mm
     df  : Fin diameter            =   86.2928     mm
     tf  : Fin thickness (core)    =  0.206700     mm
     Pf  : Fin pitch               =   2.22286     mm
```

Pt	: Transversal tube pitch	=	100.000	mm
Pl	: Longitudinal tube pitch	=	86.6000	mm

HEAT EXCHANGER BUNDLE

nt/r	: Number of tubes per row	=	30.0000	
ntb	: Number of tubes per bundle	=	118.000	
nb	: Number of heat exchanger bundles	=	138.203	
nr	: Number of tube rows	=	4.00000	
nwp	: Number of water passes	=	2.00000	
Lt	: Length of finned tubes	=	9.15604	m
THET	: Bundle semi-apex angle	=	60.0000	deg
Wb	: Width of heat exchanger bundle	=	3.00000	m
vw	: Water velocity inside tubes	=	1.12345	m/s

HEAT EXCHANGER BUNDLE COST:

Cost of tubes, Cstm	=	390769.862	$
Cost of fins, Csfm	=	2156819.00	$
Cost of galvanizing, Csgm	=	0.000000000E+00	$
Cost of finned tubes, Csft	=	3057106.63	$
Cost of bundle frames/headers, Csbf	=	611421.329	$
Cost of bundle assembly, Csba	=	407699.886	$
Total HE bundle cost, Cshe	=	4891473.42	$

CIRCULATION SYSTEM COST:

Cost of pumps, Cspc	=	44495.3310	$
Cost of electric motors, Csem	=	17423.0546	$
Cost of electric wiring, Csew	=	1742.30546	$
Total pumping system cost, Cspt	=	79575.8638	$
Cost of piping and valves, Cspv	=	1222868.35	$

COOLING TOWER STRUCTURAL AND CONSTRUCTION COSTS:

Cost of land, excavation, foundation, Csle	=	609213.602	$
Cost of cooling tower shell, Cscts	=	1629478.63	$
Cost of HE bundle platform, Cspl	=	669497.010	$
Cost of cooling tower supports, Csts	=	49655.3150	$
Total structural, construction cost, Cssc	=	3697305.69	$

OPERATING AND MAINTENANCE COSTS:

Operating cost of pumps, Cspo	=	102623.192	$/a
HE bundles maintenance cost, Cshem	=	48914.7342	$/a
Pump system maintenance cost, Cspsm	=	2387.27591	$/a
Piping and valves maint. cost, Cspvm	=	6114.34177	$/a
Cooling tower struct. maint. cost, Cssm	=	1848.65285	$/a
Fuel cost, Csfuel	=	0.000000000E+00	$/a

TOTAL COSTS:

Total operating cost	=	102623.192	$/a
Total maintenance cost	=	59265.0047	$/a
Total fuel cost	=	0.000000000E+00	$/a
Total capital cost	=	9891223.33	$
Total annual cost	=	2140132.86	$/a

Table L.7: Post-optimality analysis

SCALE-INVARIANT MEASURE OF SENSITIVITY

GENERAL COST FACTORS

Ce	Present electricity cost (self-generated)	0.479518E-01
ere	Electricity cost escalation rate	0.427927E-01
Cf	Present fuel cost	0.000000E+00
erf	Fuel cost escalation rate	0.000000E+00
i	Interest rate	-0.201179E-01
FCR	Levelized fixed charge rate	0.924356
Tau	Running hours per annum	0.479518E-01

FINNED TUBE BUNDLE COST

Cb1	Tube material unit cost	0.194353E-01
Cb2	Tubing fixed cost	0.629938E-01
Cb3	Fin material unit cost	0.194427
Cb4	Finning fixed cost	0.260533
Cb5	Galvanizing material unit cost	0.000000E+00
Cb6	Surface coating fixed cost	0.000000E+00
Cb7	Finned tube cost weighting factor	0.537389
Cb8	Bundle frame and header cost factor	0.895648E-01
Cb9	Tube assembly and end preparation cost	0.597224E-01
Cb10	Bundle cost weighting factor	0.597111
Cb11	Bundle maintenance cost factor	0.228559E-01

CIRCULATION SYSTEM COSTS

Cp1	Pump fixed cost	0.617972E-03
Cp2	Pump unit cost	0.535945E-02
Cp3	Electric motor fixed cost	0.118237E-03
Cp4	Electric motor unit cost	0.245643E-02
Cp5	Electric motor safety factor (undersizing)	0.245643E-02
Cp6	Electric wiring/switching cost multiplier	0.234087E-03
Cp7	Pump system cost multiplier	0.855203E-02
Cp8	Pump system maintenance cost factor	0.111549E-02
Cp9	Piping and valves cost factor	0.117137
Cp10	Piping and valves maintenance cost factor	0.285700E-02

COOLING TOWER CONSTRUCTION COSTS

Cs1	Land, excavation and foundation unit cost	0.713433E-01
Cs2	Cooling tower shell cost	0.190824
Cs3	HE Bundle support platform cost	0.784030E-01
Cs4	Cooling tower support cost	0.581501E-02
Cs5	Construction cost multiplier	0.346385
Cs6	Maintenance cost factor	0.863804E-03

PRESCRIBED PARAMETERS

dts	Tower support diameter/width	0.140626E-01
ALPH	Fraction of tower area covered by bundles	-0.182248
pa1	Barometric pressure at ground level	-0.414740
Twi	Water inlet temperature	-9.45551
mw	Water mass flow rate	-0.192260
Ke	Total flow resistance length ratio	0.557677E-01
Ta1	Air temperature at ground level	8.37099

GEOMETRIC CONSTRAINT VARIATION SENSITIVITY

CONSTRAINTS: ROUND TUBES

1	(HIRU-H3/H5)	0.000000E+00
2	(H3/H5-HIRL)	0.000000E+00

3	(DIRU-D3/(H5-H3))	0.000000E+00
4	(D3/(H5-H3)-DIRL)	0.000000E+00
5	(DRU-D3/D5)	0.000000E+00
6	(D3/D5-DRL)	0.288041E-02
7	(DF/DR-1.0)	0.000000E+00
8	(TT/TTL-1.0)	0.535095E-01
9	(TR/TRL-1.0)	0.461663E-01
10	(PT/DF-1.0)	0.000000E+00
11	(TF/TFL-1.0)	0.000000E+00
12	(PF/PFL-1.0)	0.000000E+00
13	(1.0-WB/WBU)	0.159110E-01
14	(1.0-LT/LTU)	0.000000E+00
15	(1.0-THTE/THBU)	0.230760E-01

Minimization of power generation cost

Table L.8: Optimization output and cost data

```
OPTIMIZATION OUTPUT DATA SET:
  FINNED TUBE TYPE: Round tubes (round fins)
  CORRELATION: Briggs, Young, Robinson

  ANNUAL NET POWER OUTPUT OPTIMIZATION

  VARIABLES FOR OPTIMIZATION:
     ma Ta4 Two Ts H5 H3 d5 d3 Lt nt/r THET do tt tr df tf Pf Pt

     COOLING TOWER
     H5   : Tower overall height          =   136.398     m
     H3   : Tower inlet height            =   18.8443     m
     d5   : Tower diameter at outlet      =   70.7988     m
     d3   : Tower diameter at inlet       =   97.6356     m
     Lts  : Tower support length          =   20.0537     m
     nts  : Number of tower supports      =   60.0000

    FINNED TUBES
     do   : Tube outside diameter    =   27.2587     mm
     tt   : Tube thickness           =   1.20000     mm
     tr   : Fin root thickness       =   1.00000     mm
     df   : Fin diameter             =   86.6256     mm
     tf   : Fin thickness (core)     =  0.205304     mm
     Pf   : Fin pitch                =   2.22441     mm
     Pt   : Transversal tube pitch   =   96.7742     mm
     Pl   : Longitudinal tube pitch  =   83.8065     mm

     HEAT EXCHANGER BUNDLE
     nt/r: Number of tubes per row           =   31.0000
     ntb : Number of tubes per bundle        =   122.000
     nb  : Number of heat exchanger bundles  =   148.991
     nr  : Number of tube rows               =   4.00000
     nwp : Number of water passes            =   2.00000
     Lt  : Length of finned tubes            =   9.51700     m
     THET: Bundle semi-apex angle            =   60.0000     deg
     Wb  : Width of heat exchanger bundle    =   3.00000     m

 HEAT EXCHANGER BUNDLE COST:
       Cost of tubes, Cstm                   =   452703.627     $
       Cost of fins, Csfm                    =   2498644.72     $
```

```
    Cost of galvanizing, Csgm                  = 0.000000000E+00 $
    Cost of finned tubes, Csft                 =  3541618.02     $
    Cost of bundle frames/headers, Csbf        =  708323.605     $
    Cost of bundle assembly, Csba              =  454422.277     $
    Total HE bundle cost, Cshe                 =  5645236.68     $

CIRCULATION SYSTEM COST:
    Cost of pumps, Cspc              =  39007.2483     $
    Cost of electric motors, Csem    =  15136.3535     $
    Cost of electric wiring, Csew    =  1513.63535     $
    Total pumping system cost, Cspt =  69571.5464     $
    Cost of piping and valves, Cspv =  1411309.17     $

COOLING TOWER STRUCTURAL AND CONSTRUCTION COSTS:
    Cost of land, excavation, foundation, Csle=  681699.380     $
    Cost of cooling tower shell, Cscts        =  1833304.29     $
    Cost of HE bundle platform, Cspl          =  748697.217     $
    Cost of cooling tower supports, Csts      =  52641.0144     $
    Total structural, construction cost, Cssc =  4145427.38     $

OPERATING AND MAINTENANCE COSTS:
    Operating cost of pumps, Cspo              =  88506.1374     $/a
    HE bundles maintenance cost, Cshem         =  56452.3668     $/a
    Pump system maintenance cost, Cspsm        =  2087.14639     $/a
    Piping and valves maint. cost, Cspvm       =  7056.54585     $/a
    Cooling tower struct. maint. cost, Cssm =  2072.71369     $/a
    Fuel cost, Csfuel                          =  106623742.     $/a

TOTAL COSTS:
    Total operating cost          =  88506.1374     $/a
    Total maintenance cost        =  67668.7727     $/a
    Total fuel cost               =  106623742.     $/a
    Total capital cost            =  11271544.8     $
    Total annual cost             =  109034226.     $/a

NET ANNUAL POWER OUTPUT
  Net annual power output =   2083.90      GWh
  Annual heat dissipated  =   2877.21      GWh
  Cost per kWh            =  0.523222E-01 $/kWh
```

Table L.9: Power generation output

RESULTS OF NET ANNUAL POWER OUTPUT

No	Tdb [C]	Twb [C]	h [h/a]	Twi [C]	Two [C]	Ts [C]	Q [MW]	Pg [MW]	Pp [MW]	Pn [MW]
1	-1.00	0.00	4	40.50	22.58	43.01	328.75	237.35	0.100	237.25
2	0.00	0.00	10	41.49	23.57	43.99	328.55	237.57	0.100	237.47
3	1.00	0.00	26	42.47	24.57	44.98	328.36	237.78	0.099	237.69
4	2.00	0.00	43	43.46	25.57	45.97	328.19	237.98	0.099	237.88
5	3.00	0.90	59	44.46	26.57	46.96	328.04	238.16	0.099	238.06
6	4.00	1.90	82	45.46	27.57	47.96	327.90	238.32	0.098	238.22
7	5.00	2.90	112	46.45	28.58	48.96	327.78	238.46	0.098	238.36
8	6.00	3.80	152	47.46	29.58	49.96	327.68	238.58	0.098	238.48
9	7.00	4.60	201	48.46	30.59	50.96	327.60	238.68	0.098	238.58
10	8.00	5.40	254	49.47	31.61	51.97	327.55	238.76	0.097	238.66
11	9.00	6.30	312	50.48	32.62	52.98	327.52	238.81	0.097	238.71

12	10.00	7.10	371	51.50	33.64	54.00	327.51	238.83	0.097	238.73
13	11.00	7.80	434	52.52	34.65	55.01	327.53	238.83	0.096	238.73
14	12.00	8.60	506	53.54	35.68	56.04	327.58	238.80	0.096	238.70
15	13.00	9.40	578	54.57	36.70	57.07	327.65	238.74	0.096	238.64
16	14.00	10.20	656	55.60	37.73	58.10	327.76	238.65	0.096	238.55
17	15.00	10.90	738	56.63	38.76	59.13	327.90	238.53	0.096	238.43
18	16.00	11.60	764	57.67	39.79	60.18	328.08	238.37	0.095	238.27
19	17.00	12.20	655	58.72	40.82	61.22	328.29	238.18	0.095	238.08
20	18.00	12.70	553	59.77	41.86	62.27	328.53	237.95	0.095	237.85
21	19.00	13.20	459	60.82	42.90	63.33	328.82	237.68	0.095	237.58
22	20.00	13.70	381	61.88	43.94	64.39	329.15	237.37	0.094	237.27
23	21.00	14.10	320	62.94	44.99	65.46	329.52	237.01	0.094	236.92
24	22.00	14.50	265	64.01	46.04	66.53	329.94	236.61	0.094	236.52
25	23.00	14.90	219	65.09	47.09	67.61	330.40	236.17	0.094	236.08
26	24.00	15.20	177	66.17	48.15	68.70	330.92	235.68	0.094	235.58
27	25.00	15.50	140	67.26	49.21	69.79	331.48	235.13	0.094	235.04
28	26.00	15.70	105	68.35	50.27	70.89	332.10	234.53	0.093	234.44
29	27.00	15.90	76	69.45	51.34	71.99	332.78	233.88	0.093	233.79
30	28.00	16.10	51	70.56	52.41	73.11	333.52	233.17	0.093	233.08
31	29.00	16.40	30	71.68	53.48	74.23	334.32	232.40	0.093	232.30
32	30.00	16.80	15	72.80	54.56	75.36	335.19	231.56	0.093	231.47
33	31.00	17.40	8	73.93	55.65	76.50	336.12	230.66	0.093	230.56
34	32.00	18.00	4	75.07	56.74	77.65	337.13	229.68	0.093	229.59

NET ANNUAL POWER OUTPUT = 2083.902 GWh
ANNUAL HEAT DISSIPATED = 2877.214 GWh

Table L.10: Post-optimality analysis

SCALE-INVARIANT MEASURE OF SENSITIVITY

GENERAL COST FACTORS

Ce : Present electricity cost (self-generated)	0.811736E-03
ere: Electricity cost escalation rate	0.724399E-03
Cf : Present fuel cost	0.977892
erf: Fuel cost escalation rate	1.03225
i : Interest rate	-0.471335
FCR: Levelized fixed charge rate	0.206753E-01

FINNED TUBE BUNDLE COST

Cb1 : Tube material unit cost	0.441881E-03
Cb2 : Tubing fixed cost	0.143249E-02
Cb3 : Fin material unit cost	0.442586E-02
Cb4 : Finning fixed cost	0.591942E-02
Cb5 : Galvanizing material unit cost	0.000000E+00
Cb6 : Surface coating fixed cost	0.000000E+00
Cb7 : Finned tube cost weighting factor	0.122196E-01
Cb8 : Bundle frame and header cost factor	0.203662E-02
Cb9 : Tube assembly and end preparation cost	0.130658E-02
Cb10: Bundle cost weighting factor	0.135262E-01
Cb11: Bundle maintenance cost factor	0.517760E-03

CIRCULATION SYSTEM COSTS

Cp1 : Pump fixed cost	0.121373E-04
Cp2 : Pump unit cost	0.907324E-04
Cp3 : Electric motor fixed cost	0.232560E-05
Cp4 : Electric motor unit cost	0.415892E-04
Cp5 : Electric motor safety factor (undersizing)	0.415892E-04

Cp6	: Electric wiring/switching cost multiplier	0.398917E-05
Cp7	: Pump system cost multiplier	0.146767E-03
Cp8	: Pump system maintenance cost factor	0.191480E-04
Cp9	: Piping and valves cost factor	0.265348E-02
Cp10	: Piping and valves maintenance cost factor	0.647264E-04

COOLING TOWER CONSTRUCTION COSTS

Cs1	: Land, excavation and foundation unit cost	0.156695E-02
Cs2	: Cooling tower shell cost	0.421402E-02
Cs3	: HE Bundle support platform cost	0.172096E-02
Cs4	: Cooling tower support cost	0.120999E-03
Cs5	: Construction cost multiplier	0.762291E-02
Cs6	: Maintenance cost factor	0.190122E-04

PRESCRIBED PARAMETERS

dts	: Tower support diameter/width	0.289878E-03
ALPH	: Fraction of tower area covered by bundles	-0.401616E-02
pa1	: Barometric pressure at ground level	-0.912144E-02
Ke	: Total flow resistance length ratio	0.134576E-02
mw	: Water mass flow rate	-0.609977E-02
Qdes	: Design point heat rejection	-0.185092E-02
TTD	: Design point terminal temp. difference	0.185088E-02

GEOMETRIC CONSTRAINT VARIATION SENSITIVITY

CONSTRAINTS: ROUND TUBES

1	(HIRU-H3/H5)	0.000000E+00
2	(H3/H5-HIRL)	0.000000E+00
3	(DIRU-D3/(H5-H3))	0.000000E+00
4	(D3/(H5-H3)-DIRL)	0.000000E+00
5	(DRU-D3/D5)	0.000000E+00
6	(D3/D5-DRL)	0.000000E+00
7	(DF/DR-1.0)	0.000000E+00
8	(TT/TTL-1.0)	0.121021E-02
9	(TR/TRL-1.0)	0.104727E-02
10	(PT/DF-1.0)	0.000000E+00
11	(TF/TFL-1.0)	0.000000E+00
12	(PF/PFL-1.0)	0.000000E+00
13	(1.0-WB/WBU)	0.236809E-03
14	(1.0-LT/LTU)	0.000000E+00
15	(1.0-THTE/THBU)	0.350549E-03