GROWTH, YIELD AND ECONOMICS OF PINUS PATULA IN THE NATAL MIDLANDS.

By

N.D. CROWE.

Thesis submitted for the degree of Doctor of Science in Forestry at the University of Stellenbosch.

September, 1965.

ACKNOWLEDGEMENTS

I wish to record my gratitude to my promoter dr. A. van Laar for his outstanding guidance, advice and encouragement.

Further I wish to thank my internal examiner, professor C.L. Wicht and my external examiner professor J.H. Becking for their sound advice.

The willing co-operation and assistance of the following timber growers is much appreciated; Holley Bros., Turner Bros., Department of Forestry, Clan Syndicate, Greytown Municipality, Pietermaritzburg Municipality and Messrs. T.E.B. Hill, A.J. Hill, M.B. Hill, C.J.J. Crowe, P.J.J. Crowe, E.F.J. Küsel, I.MacKenzie and Sons, T.F. MacKenzie, R. Carbarns, C. Anderson, M. Anderson and P. Palmer.

Lastly, I would like to thank my parents for their encouragement throughout my studies.

****

SUMMARY

This thesis deals with the mensurational and economic studies in plantations of P.patula in the Natal Midlands.

In Chapter I the climate of P.patula areas in Natal is classified with the Thornthwaite method. The present silvicultural practice in the Natal Midlands is summarised.

In Chapter II the construction of Volume Yield tables is discussed. These tables are based on the measurement of stand characteristics of temporary sample plots which have been measured once. The past diameter and the basal area increment is determined from increment cores. A multiply regression analysis is used to determine the relationship between basal increment as dependent variable, stand density, age, site index and their interactions as independent variables. Site index is deleted but age and stand density are retained as explaining variables.

Site index curves, based on temporary sample plots are constructed, but the yield tables are based on the mean site index of 81.9 feet at 20 years.

The volume increment is estimated from separately determined height and basal area increment equations.

Four thinning models are introduced and for each of them a yield table is constructed. For these thinning regimes the diameter --, volume --, and log class volume distributions are investigated.

In Chapter IV the results of the economic analysis are presented. Production costs have been investigated through questionnaires.

Various types of rotations are calculated for each of the four thinning regimes. The profitability of different rotations and thinning regimes are compared.

The implications of these calculations on the practice of silviculture and management are discussed.

CONTENTS

INDEX OF TABLES.

15. Summary of Annual costs.
16. Costs of silvicultural operations.
17. Log class prices at roadside.
18. Profit per acre of Periodic Forest.
19. Profit expressed as a % of the Capital invested.
20. Distribution of production costs.

INDEX OF FIGURES

CHAPTER I

GENERAL INFORMATION AND FORMULATION OF PROBLEM.

1. Introduction of species.

P. patula, was introduced into South Africa from Mexico in 1907, and was first planted in the Natal Midlands at Cedara in March, 1914. It occurs naturally in a comparatively restricted range in the States of Queretaro, Hidalgo, Puebla and Vera Cruz in Mexico. According to Loock (1950) "It grows in pure, dense stands in the warmer to colder temperate zones between elevations of 6,000 to 9,000 feet with an annual rainfall of 40" or more and frequent mists."

It is an extremely fast-growing, intolerant species planted in even-aged, pure plantations in moist, summer-rain regions of South Africa.

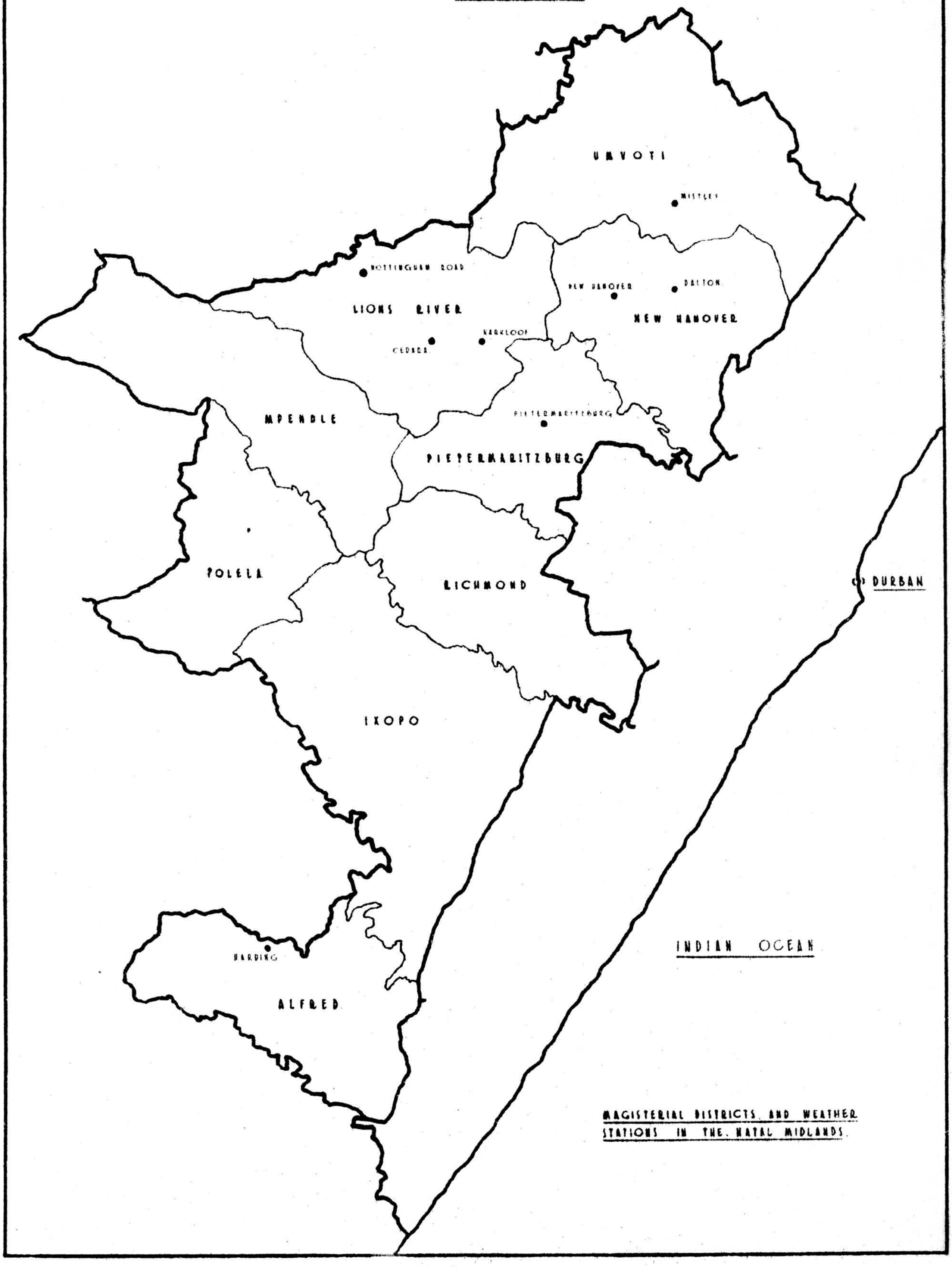
2. Area and Age Class distribution.

In South Africa pines occupy 898,000 acres or 40.4% of the total area of commercial plantations. The area under P.patula is 429,574 acres, i.e. 47.8% of the area planted with pines and 19.3% of the total area of commercial plantations.

The Natal Midlands, comprising the magisterial districts of Alfred, Impendle, Ixopo, Lions River, Pietermaritzburg, New Hanover, Richmond and Umvoti (Fig.1), has 67,060 acres of P.patula i.e. 15.6% of the total area under this species. Its distribution by magisterial districts and age classes is given in Table I.

2/........

FIGURE 1.



MAGISTERIAL DISTRICTS AND WEATHER STATIONS IN THE NATAL MIDLANDS.

In the magisterial district Alfred, P.patula plantations older than 20 years occupy 2813 acres, i.e. 75% of the area of this age class in the Midlands. The uneven distribution of this age class over magisterial districts has the following causes:

(a) The majority of state plantations in Natal are found in the district Alfred and as the Department of Forestry was the first to plant P.patula the older plantations are consequently owned by the state.

(b) The rotation in state plantations is generally longer than that in private plantations.

TABLE I

DISTRIBUTION P.PATULA IN THE NATAL MIDLANDS

Magisterial District	Age Class (years)					
	0-5	6-10	11-15	16-20	20+	TOTAL
Alfred	2,466	6,886	1,093	2,630	2,813	15,888
Pietermaritzburg/ Camperdown	773	1,306	378	427	6	2,890
Polela/Impendle	2,857	5,957	931	581	64	10,390
New Hanover	3,070	3,155	1,882	2,098	510	10,715
Ixopo	1,188	1,669	118	148	30	3,153
Lions River	4,219	1,832	1,279	514	141	7,985
Richmond	2,114	2,640	1,282	351	-	6,387
Umvoti	3,654	3,470	1,786	556	186	9,652
	20,341	26,915	8,749	7,305	3,750	67,060

3. Vegetation and Soil.

The vegetation of the region is characterized by two ecological types, the Ngongoni Veld and the Natal Mist belt

Ngongoni Veld (Acocks, 1953). These Ngongoni Veld types lie above the "Coastal Forest Belt" and below the "Highland Sourveld" at an altitude of between 1,500 and 4,500 feet above sea level. Acocks, referring to the ecological similarity of these two veld types recommends that they be grouped as one.

The outstanding feature of the Ngongoni Veld types is the predominance of the grass Ngongoni (*Aristida junciformis*) and the limited occurrence of other grasses. The climax vegetation in this region, however, is high and scrub forest of tropical affinity with *Prodocarpus* species dominating.

The nature of the soils in the Midlands is closely related to the parent material from which they have been derived. De Villiers (1962) recognised three major soil types:

(a) Sandy soils derived from Table Mountain sandstone in the Coastal forest veld type or the eastern parts of the Midlands.

(b) Concretionary soils derived from glacial and other Karroo sediments in the Ngongoni Veld types in the central areas of the region.

(c) Leached soils derived from Karroo rocks in the Highland Sourveld areas or the Western areas of the Midlands.

4. Climate.

The Natal Midlands has a temperate moist sub-humid climate with an average rainfall of between 775 mm. to 1,330 mm. per annum falling primarily in the summer months. Mist contributes considerably to the precipitation especially at high elevations.

The Thornthwaite method was used for a more definite classification of the climate of those areas in the Natal Midlands and Mexico where P.patula is grown. Thornthwaite (1948) introduced the term evapotranspiration, which is the sum of the evaporation (from the surface of land and vegetation) and transpiration of the vegetation. He distinguished between potential evapotranspiration (P.E.T.), i.e. the amount of evapotranspiration when sufficient water is available for transpiration and actual evapotranspiration (R.E.T.) when water is insufficient.

$$\text{P.E.T.} = 1.6\ (10t/I)^{a}$$

where I = Heat index figure, defined as the sum of monthly heat indices i,

where $i = (t/5)^{1.514}$

t = mean monthly temperature in degrees centigrade.

$a = 675 \times 10^{-9} I^{3} - 771 \times 10^{-7} I^{2} + 17921 \times 10^{-6} I + 49239 \times 10^{-5}$

P.E.T. = Potential evapotranspiration for a standard month of 30 days and day length of 12 hours.

Thornthwaite constructed a nomogram to read off P.E.T. for a given month when the mean monthly temperature, and the heat index I is known. Finally this estimate of P.E.T. must be corrected for the true length of days and months.

If P.E.T. exceeds rainfall in a given month a portion of the available ground water in the soil will be used by the plants. This water reserve is approximately equivalent to 100 mm. rainfall, but may vary according to soil texture. When P.E.T. exceeds the rainfall during several successive months, all available water will be used and a water deficit will occur. During the season with high rainfall the water reserve in the soil will be replenished and when the level of 100 mm. is reached, a water surplus will result.

The moisture index of a climate is defined as:-

$$Im = \frac{100S - 60d}{T.P.E.T.}$$

S = Annual total of monthly water surplusses.

d = Annual total of monthly water deficiencies.

T.P.E.T. = Annual total of monthly potential evapotranspiration.

The calculation of the moisture index for the weather station Mistley is given below in Table 2:-

TABLE 2

CALCULATION OF MOISTURE INDEX

STATION MISTLEY

Month	J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.	TOTAL
P.E.T. (mm)	95	82	81	62	45	32	34	44	53	73	79	96	776
Rainfall (mm)	132	121	123	59	29	20	20	23	48	87	130	139	931
Available water reserve in soil (mm)	100	100	100	97	81	69	55	34	29	43	94	100	
Water deficit (mm)	-	-	-	-	-	-	-	-	-	-	-	-	-
Water Surplus (mm)	37	39	42	-	-	-	-	-	-	-	-	37	155

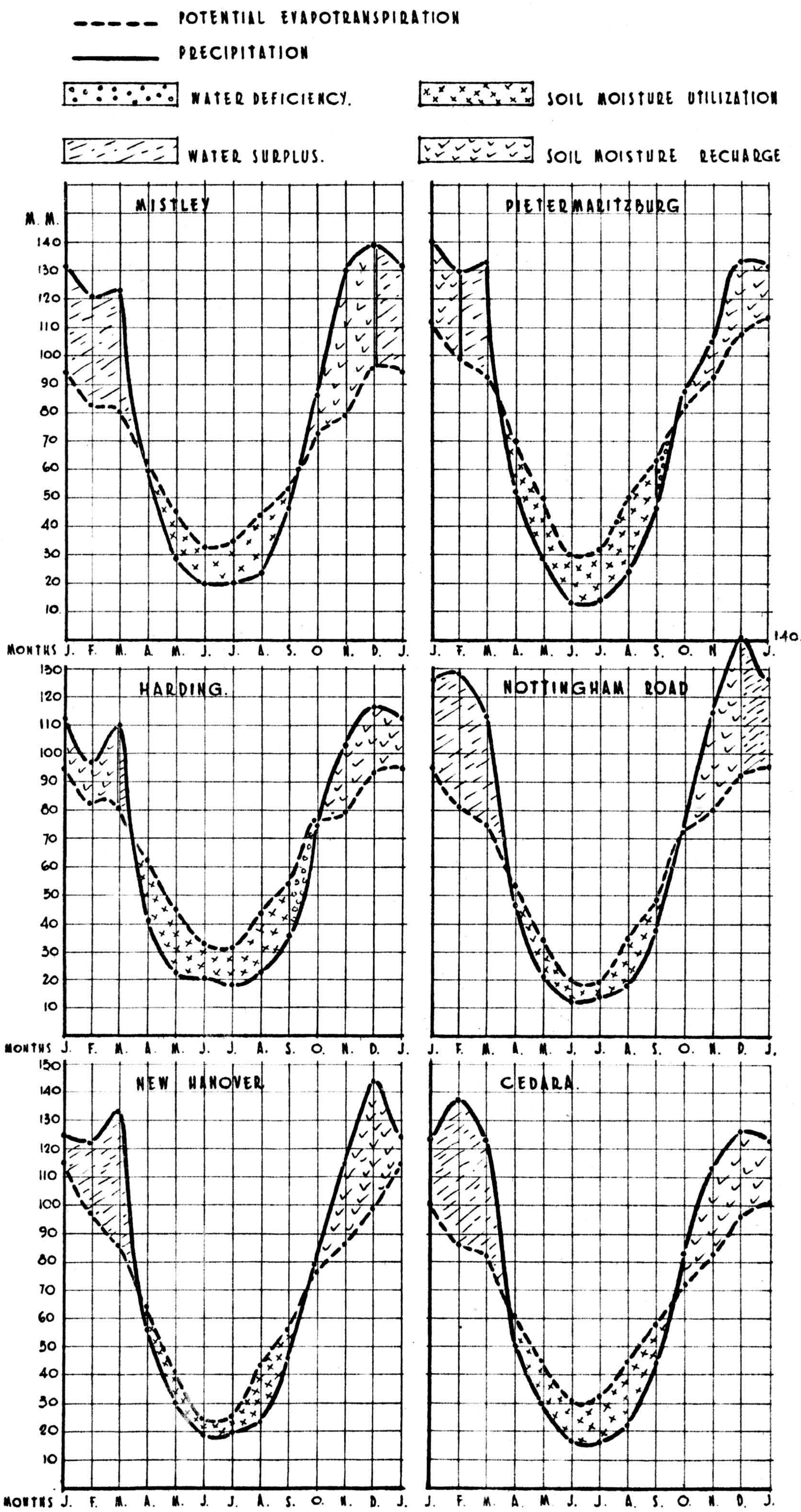
$$\text{The moisture index} = \frac{100 \times 155 - 60 \times 0}{776} = 20$$

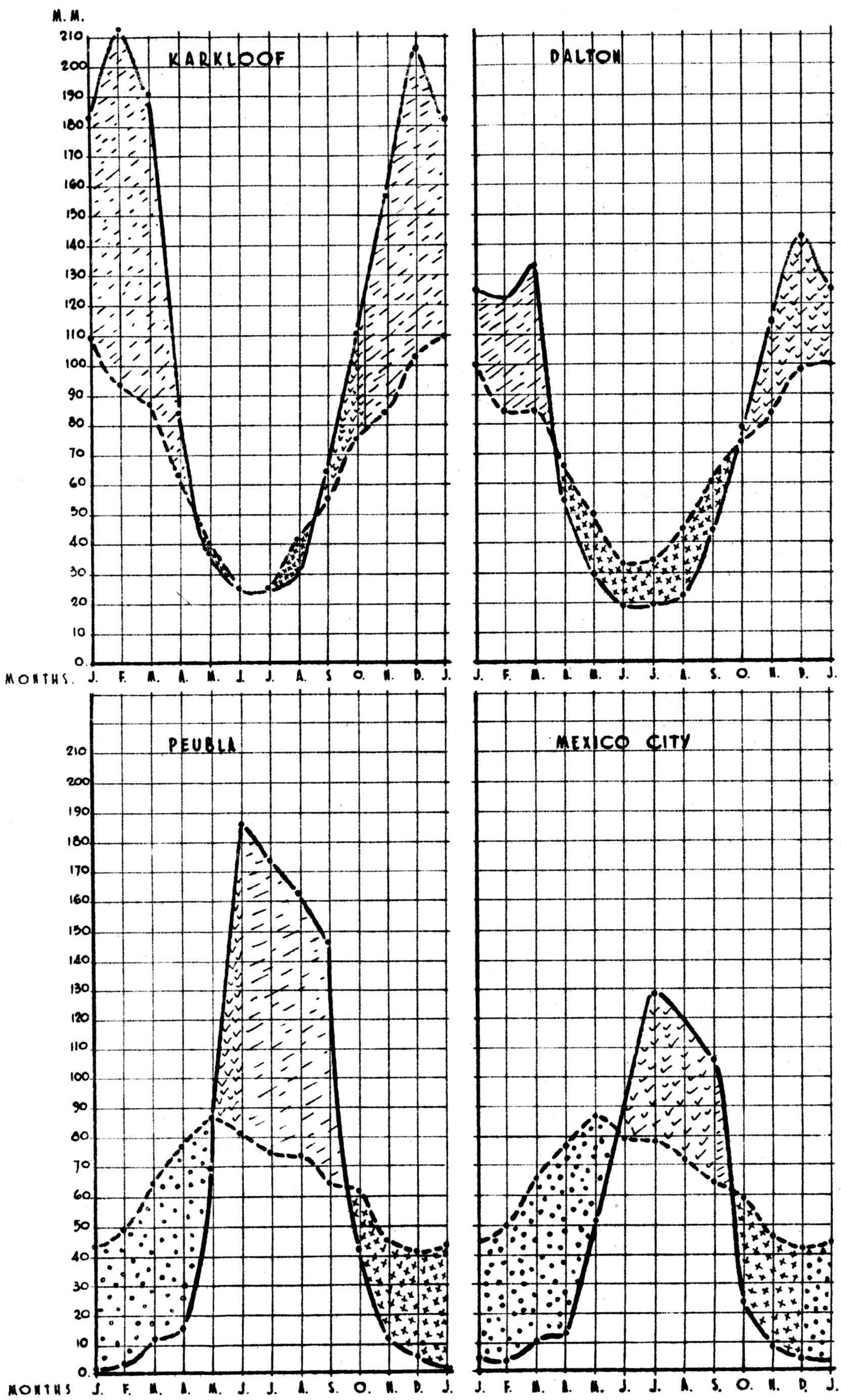
A moisture index of 20 is the boundary between a Moist sub-humid and a humid climate.

A graphical representation of the water balance of 8 weather stations in the Natal Midlands and 2 stations in Mexico is given in Fig. 2.

FIG. 2.

WATER BALANCE OF WEATHER STATIONS.




M.M.
KARKLOOF
DALTON
PEUBLA
MEXICO CITY
MONTHS
J. F. M. A. M. J. J. A. S. O. N. D. J.

The Moisture indices, elevations and mean annual rainfall for these stations are given in Table 3.

TABLE 3

DESCRIPTION OF CLIMATES

Weather Station	Elevation (Feet)	Rainfall per Annum (mm.)	Months with Rainfall <50 mm.	Moisture Index	Description of Climate
Mistley	3,199	931	May-Sept.	20	Moist sub-humid - humid
Pietermaritzburg	2,244	928	May-Sept.	6	Moist sub-humid
Harding	3,501	775	April - Sept.	0	Dry sub-humid - Moist sub-humid
New Hanover	2,595	911	May-Sept.	10	Moist sub-humid
Cedara	3,540	886	May-Sept.	12	Moist sub-humid
Nottingham Road	4,718	845	May-Sept.	19	Moist sub-humid
Dalton	3,428	911	May-Sept.	11	Moist sub-humid
Karkloof	3,999	1,330	June-Sept.	64	Humid
Puebla (M)	7,054	841	Oct.-April	20	Moist sub-humid
Mexico City (M)	7,411	569	Oct.-May	-14	Dry sub-humid

The climate in P.patula plantations in the Natal Midlands is thus generally moist sub-humid.

The Mexican station Puebla (7,054 feet above sea level) has a climate similar to Mistley (3,199 feet above sea level) in Natal namely moist sub-humid/humid although it has a higher altitude. The station Mexico City with a dry sub-humid

7/.......

climate is unsuited for the growing of P.patula.

5. The silviculture of P.patula in the Natal Midlands.

The species is grown in even-aged stands to produce sawtimber or pulpwood. In the plantations producing sawtimber, pulpwood is a by-product. Initial espacement, thinning regime and rotation vary according to the production goal of the plantations.

One pound of P.patula seed contains approximately 66,000 seeds. Usually the germination percentage is 85-90%. The seeds are sown in beds at the end of summer, i.e. in February or March. After six weeks the seedlings are pricked out into 13 x 10½ x 5 inch boxes with 25 to 30 seedlings per box. The seedlings are planted in the field after the first substantial spring rains until the end of the rainy season, i.e. from November to March. The transplants are then 10 to 12 months old and 3 to 6 inches high.

Soil preparation consists of either spot pitting or ploughing. The method used depends primarily on the terrain. Rough, steep and rocky areas are obviously unsuitable for ploughing. Terrains without these obstructions are either ploughed or pitted depending on the relative cost of each method. Ploughing activates the soil and consequently the trees will grow more rapidly particularly during the first years after ploughing. The disadvantages of ploughing are:-

(a) It is a more expensive operation.

(b) It stimulates weed growth. The subsequent cleaning operation is costly but necessary if the superior growth of the young trees is to be maintained after ploughing.

(c) It may lead to erosion.

8/......

A further consideration is the size of the plantation owned by the grower for as the area to be planted increases so mechanisation becomes more economical. Consequently in plantations owned by large companies and large private growers, ploughing is sometimes practised. Spot pitting is, however, the normal practice as a large proportion of the plantations are owned by small growers.

The espacement depends upon the objectives of management and varies between 7 x 7 to 9 x 9 feet.

Blanking follows within 3 to 5 weeks after planting. Mortality is dependent on the vigour of the transplants and weather conditions. It varies from 10 to 20%.

Live pruning is practised in all P.patula stands. The object of the operation is to ensure that the timber produced will not contain dead knots. Pruning is imperative in plantations grown for sawtimber. Although the pulp companies do not pay a higher price for pruned timber, plantations grown for pulpwood are also pruned. This is justified as the costs of pruning are not very much higher than the costs of debranching after the final felling of the trees. In addition, thinning operations in unpruned stands are seriously hampered by the heavy branches characteristic of unpruned P.patula.

Moreover pruned stands are also more readily accessible for management and fire protection operations.

Investigations by Sherry (1961) indicated that up to 33% of the living crown could be removed during each pruning operation without affecting the diameter growth. Lückhoff (1949 & 1956) found that 25% of the living crown could be removed without affecting either height or diameter growth.

Their investigations also revealed that the removal of 50% of the living crown retards the diameter and height growth significantly.

The pruning schedules recommended by Sherry and Lückhoff are given in Table 4. Sherry's recommendations are commonly used in privately owned plantations in the Midlands. Lückhoff's recommendations except for the last pruning from 22 to 36 feet at a mean height of 50 feet, are used by the state.

TABLE 4

SUMMARY OF PRUNING SCHEDULES

Sherry (1961)			Lückhoff (1949 & 1956)		
Mean height at time of Pruning (ft.)	Height of Pruning (ft.)	% living crown removed	Mean height at time of Pruning (ft.)	Height of Pruning (ft.)	% living crown removed
16	6	37.5	20	7	35
24	12	33.3	30	15	34
30	18	33.3	40	22	28
36	24	33.3	50	36	50
40	30	33.3			

Thinning regimes vary with the production goal of the plantation: sawtimber or pulpwood. Sawtimber prices increase rapidly with the increasing diameter and length of the sawlog and there are six price classes. However, for pulpwood the diameter and length of the poles is of far less importance and only two price classes, based on the diameter of the poles exist. Furthermore, the requirements for the quality of sawtimber are far more stringent than those for pulpwood.

The general tendency is to carry out early, repeated and severe thinnings for the growing of sawtimber and light thinnings for the growing of pulpwood. Three silvicultural regimes recommended for P.patula in South Africa, summarised in Table 5 are discussed below.

(1) In a significant publication Craib (1947) recommended a silvicultural system for the growing of sawtimber. This system is based on two important assumptions.

(a) A mean diameter of the final stand of 18 inches is technically and economically desirable for conversion into sawtimber.

(b) There is a fixed relationship between the values per cu.ft. of the sawlog classes A (logs 3 to 4.9 inch diameter at thin end), B (logs 5.0 to 7.9 inch diameter at thin end), C (logs 8 to 11.9 inch diameter at thin end) and D (logs 12 to 16 inch diameter at thin end). The ratio is; A:B:C:D = -1: + 1: + 3: + 5. The negative value of the log class A indicates that the costs of logging and subsequent manufacture of the logs are higher than the selling price of the sawn timber.

In order to reduce root competition and to promote the diameter growth of the trees, Craib recommended.

(a) an espacement of 9 x 9 feet on site quality I and II sites and 12 x 12 feet on site quality III sites.

(b) Increasingly severe thinnings on poorer sites.

(c) A rotation of 30 years on site quality I, 40 years on site quality II, and 50 years on site quality III, ensuring that the mean diameter of the final stand will then be 18 inches.

Due to additional research data, and changing markets and prices, Craib's regime was revised (De Villiers, Marsh, Sonntag and Van Wyk, 1961). Their research indicated that a site is not fully utilized by the trees when they are planted at a distance of 12 feet and consequently the total volume production will be reduced. In addition weed growth in these widely spaced stands is frequently excessive. For these reasons the authors recommended an espacement of 9 x 9 feet on all sites, lighter thinnings in site quality III stands, and a slight modification of the thinning regime in site quality II stands. The thinning regime recommended for site quality I stands by Craib remains unchanged.

Craib's thinning regime, with the 1961 revision, is widely used in the Midlands in plantations producing sawtimber.

(2) Kotze (1960) suggested a silvicultural system for growing of pulpwood with the object of maximizing the mean annual increment per acre. This can be realized when dense stocking is combined with a short rotation. Kotze recommends a 7 x 7 feet espacement. Further the author recommends that a first thinning be carried out at 11 years as a minimum age of 10 years is specified by pulpmillers for P.patula pulpwood. It is thought that the fibres of an 11-year-old-tree are sufficiently long for the manufacturing of chemical pulp. The second thinning is carried out at 16 years, and for a rotation of 25 years the estimated mean d.b.h. of the final stand on a mean site quality will be 9.5 inches.

Kotze's silvicultural system has been applied in the Natal Midlands for the growing of pulpwood.

(3) Marsh (1963) expresses the opinion that the production

of inferior juvenile wood by young trees cannot be influenced by the severity of early thinnings. The production of a core of juvenile wood, characterized by wide annual rings by young P.patula trees in South Africa is an inherent characteristic. This phenomenon is primarily related to the age of the trees.

Marsh felt further that sawlogs of superior quality can be produced in trees grown at a steady rate of 4 to 6 rings per inch, superimposed upon the 5 to 6 inch inherently fast grown core.

Marsh's thinning regime for sawlogs is based on these considerations. The stand is thinned so that a growth rate of 4 to 6 inches is maintained until maturity after the initial juvenile wood has been formed. The author estimates that such a thinning would give 2.2 rings to the inch to the six year, 6.1 rings to the inch from the tenth to the fourteenth year, 6.0 rings to the inch from the fourteenth to the 25th year, and 6.5 rings to the inch or less to the 40th year.

In the Natal Midlands two agencies cause injury to P.patula.

(1) Hail, chiefly occurring during the summer months, injures the trees. These wounds are frequently infected by the fungus Diploides pinea particularly when a severe hail storm is followed by hot humid days. Trees infected by Diploidea pinea frequently die. Poynton (1957) stated that trees attacked by the fungus tend to develop forked stems.

(2) Fire is a major threat to plantations in the Natal Midlands during dry winter months from June to September particularly when the area is fanned by hot and desiccating North Westerly Berg winds.

TABLE 5

SUMMARY OF P.PATULA THINNING REGIMES

Craib (1947)						Kotze (1960)	
Site quality I		Site quality II		Site quality III		Mean Site quality	
Age	Stems per acre	Age	Stems per acre	Age	Stems per acre	Age	Stems per acre
0	530	0	530	0	530	0	890
8	300	6	300	6	180	11	500
12	200	14	190	20	120	16	300
18	130	25	130	30	185	25	0
30	0	-	-	50	0		
		40	0				

De Villiers, Marsh, Sonntag, van Wyk (1961)						Marsh (1963)	
Site quality I		Site quality II		Site quality III		Mean Site quality	
Age	Stems per acre	Age	Stems per acre	Age	Stems per acre	Age	Stems per acre
0	530	0	530	0	530	0	620
8	300	6	300	6	300	11	300
12	200	14	200	14	150	14	200
18	130	20	130	20	100	18	150
30	0	25	100	50	0	21	120
		40	0			24	100
						27	80
						34	0

14/.......

6. Defining of the Problem.

P.patula is grown for either sawtimber or pulpwood. The silvicultural techniques vary, not only according to the production aims of the plantation, but also within the group of plantations, producing sawtimber and pulpwood. At present, it is more or less an arbitrary decision which of these products to grow and which silvicultural technique to employ, for until now no scientifically sound economic research has been done on this subject.

The purpose of the present study is to compare the economics of growing sawtimber and pulpwood with different silvicultural systems.

The logical sequence for these investigations is as follows:-

(a) To construct a variable density volume yield table, i.e. a table which presents the expected volume yield per unit area, mean d.b.h. and height at different ages, stand densities, and sites for a given species and geographical area.

(b) To estimate the production costs for growing of sawtimber and pulpwood with different silvicultural regimes.

(c) To construct a money yield table, i.e. a table which expresses the expected money yield per unit area for stands of different ages, stand densities and sites for a given species and geographical area.

(d) To study the economics of different silvicultural regimes and production goals. The results of these calculations may be used for decision making.

7. (a) Field-work.

200 Temporary rectangular sample plots were laid out

in P.patula plantations in the Natal Midlands with a minimum of 20 trees per sample plot.

The distribution over Magisterial districts is:-

MAGISTERIAL DISTRICT	NO. OF PLOTS
Alfred	44
Impendle/Polela	12
Ixopo	4
Lions River	29
New Hanover	33
Pietermaritzburg/Camperdown	25
Richmond	15
Umvoti	38
	200

The age class distribution of the plots is as follows:-

AGE CLASS (YEARS)	NO. OF PLOTS
5 - 9	31
10 - 14	37
15 - 19	50
20 - 24	24
25 - 29	25
30 - 34	18
35 - 40	15
	200

The distribution by Site Indices is:-

SITE INDEX (FEET)	NO. OF PLOTS
60 - 64	1
65 - 69	7
70 - 74	17
75 - 79	58
80 - 84	43
85 - 89	40

16/......

90 - 94	22
95 - 99	6
100 -104	2
105 -109	3
110 -114	1
	200

The stand density distribution is as follows:-

STAND DENSITY (S%)	NO. OF PLOTS
15 - 19.9	44
20 - 24.9	102
25 - 29.9	40
30 - 34.9	7
35 - 39.9	5
40 - 44.9	0
45 - 49.9	2
	200

In addition 10 sample plots with 125 trees on each plot were measured to investigate the nature of the diameter distribution in stands of varying ages.

The following measurements were made on each of the 200 sample plots.

(a) The diameter at breast height (d.b.h.) of each tree to the nearest 1/10th inch, with calipers.

(b) The height of ten trees, being a random sample from the trees in the sample plot, measured to the nearest foot with a Blume Leiss hypsometer.

(c) The diameter at stump height, i.e. at 6 inches above ground of a random sample of five trees per plot, to the nearest 1/10th inch.

(d) Bark thickness at breast height of these five trees, with the Swedish bark gauge. Readings were made to the nearest 1/10th cm. and then converted into inches.

(e) The radial growth at breast height of all trees in the sample plot. The diameter growth for two year periods since the most recent thinning was measured to the nearest 0.01 inch from one increment core per sample tree.

(f) The age of the stand, determined by counting the number of annual rings. This age was compared with plantation records.

(g) The age of the stand at the time of the most recent thinning, determined by ring counts on the most recent stumps. The results were compared with plantation records.

(h) The stump diameter of the trees removed during the most recent thinning operation. This diameter was measured at 6 inches above the ground to the nearest 1/10th inch. For stump heights, lower than 6 inches, the diameter at a height of 6 inches was estimated.

(i) A tree with a d.b.h. equal to the 75th percentile diameter in the sample plot was felled and sawn into 5 foot lengths. The number of annual rings on each cross section was counted.

7. (b) <u>Preliminary calculations.</u>

(i) The following equation was used to estimate d.b.h. from diameter at stump height,

$$Y = a + b X$$

where X = diameter at stump height

Y = d.b.h.

a and b constants.

The equation based on 998 observations is as follows:-

$Y = 0.9106 + 0.87565\ X$ - - - - (1)

The standard error of the regression coefficient is .005187.

(ii) In order to estimate d.b.h. under bark from d.b.h. over bark and age, the following equation was introduced:-

$$Y = a + b_1 X_1 + b_2 X_2$$

where X_1= d.b.h. over bark

X_2= age

Y = d.b.h. under bark

a, b_1, b_2,= constants.

The analysis of variance is as follows:-

Source of Variation	D.F.	M.S.	F.
Regr. Y on X_1	1	12,331	67,144.56**
Regr. Y on X_2 (X_1 fixed)	1	1.00	5.45*
Residuals	1003	1.8365	

The contribution of X_1 to the regression is significant of 0.01 and that of X_2 significant at 0.05. The resulting equation is:-

$Y = -0.43074 + 0.89165\ X_1 + 0.0075844\ X_2$----(2)

(iii) The height curve was computed for each sample plot separately. The relationship between d.b.h. and height is normally curvilinear, but a graphic representation of this relationship between d.b.h. and height in the P.patula sample plots indicated a linear relationship. This probably due to the relatively small number of height measurements in each sample plot.

(iv) Finally, the following stand characteristics were computed for each sample plot.

Present Stand

Mean d.b.h. (inches)

Basal area (sq. ft. per acre)

Stems per acre

Mean height (feet)

Top height (feet)

Stand density (S%)

Site index

Thinnings (from stumps)

Mean d.b.h. (inches)

Stems per acre

Stand after last thinning

Mean d.b.h. (inches)

Basal area (sq. ft. per acre)

Growth

Current Basal increment since last thinning (sq. ft. per acre per annum).

CHAPTER II

YIELD TABLE CONSTRUCTION

1. Site, Site Quality and Site Index.

(1) Site and Site Quality.

Site is the complex of physical and biological factors determining the productive capacity of an area for a given species. The productive capacity of a site is best expressed by the total volume yield or production. The total volume yield at a given age is made up of the present stand plus the volume from past thinnings. The yield from thinnings is normally unknown and consequently so is the total volume yield of a stand. The common procedure is thus to investigate in permanent sample plots located on various sites, the relationship between age and height, and between age and total volume yield. When this relationship is known, height may be used to estimate total volume yield. The basic assumption of this procedure is that a stand of a given age and height will always produce the same volume yield and that deviations will be of a random nature. If silvicultural techniques, in particular initial espacement and thinning regime vary widely, this assumption will not be valid. Climatological and other factors may also have a disturbing effect. Assman (1961) analysed yield data from two groups of sample plots in stands of P.abies in Sachsenried and Denklingen respectively, with a practically identical height development. At the age of 46 years the total mean annual volume increment of the Sachsenried sample plots was approximately 20% higher than that of the Denklingen plots. The regression curves for these two sites, expressing the relationship between total mean annual volume increment and height, differ appreciably both in level and slope.

Grandjean, van Soest and Stoffels (1956) compared the growth of Pinus silvestrus in Great Britain and in Holland. For site class II the mean height of the stands at 47 years was about 53 feet both in Great Britain and in Holland. At this age, however, the total mean annual increment in Holland was about 35% lower than in Great Britain. Differences were thought to be caused by differences in climate, soil and seed provenance.

Van Laar (1961) constructed volume yield tables for E.saligna in South Africa. He found that for a given age and mean height of the stand, the total volume yield in the coastal plain of Zululand was significantly higher than in the Transvaal.

These yield studies indicate that for a certain age, the relationship between total volume yield and height of the stand in climatologically different areas do vary, although the true nature of this discrepancy is not always known.

This yield study in stands of P.patula was confined to a limited geographical area in Natal. It may, therefore, be reasonably assumed that for a given age, site will not affect the relationship between height of the stand and total volume production, provided that the silvicultural techniques are similar.

(2) Site Index and Site Index Curves.

Site indexing is an arbitrary system used in forestry to classify the different productive capacities or qualities of site. Either height or mean annual volume increment may be used to differentiate between site classes.

The following indexing systems are in use:-

(a) The site indices are numbered from 1 to n and each number designates a different site index curve for the relationship between age and height. The older Middle-European yield tables are based on this principle.

(b) A system originating in the U.S.A. and now used in South Africa, Australia and New Zealand is to denote the site index of a stand by its expected height at a reference age. An age of 50 years is normally used in the U.S.A. In South Africa, however, because of the rapid growth and short rotations of *Pinus* species, a reference age of 20 years is used (Marsh 1957).

Essentially the methods (a) and (b) are similar; the site classes are classified on the basis of the height of the stand at a given age.

(c) A recently developed system, applied in Germany, Austria and Holland is to designate the site index of a stand by the expected total mean annual volume increment at a reference age. A reference age of 100 years was introduced by Wiedemann (1951) and Schober (1942), and an average of 80 years by Assmann (1961). A varying age, coinciding with the culmination of the total mean annual volume increment was recommended by Van Laar and Van Soest (1958). However, it should be borne in mind that the height of the stand must be known to estimate the site index expression in cu.ft./acre/annum.

Volume yield tables based on any of these three site indexing systems will eventually give identical information about the growth of stands.

The main advantage of system (c) is that the site index of the stand is expressed in a measure which is of direct and primary interest to forest management. In this study, system (b) now in use in South Africa, will be retained for continuity.

(3) Expressions of Stand Height used for Site Indexing.

In using the height of a stand for constructing site index curves, the distinction must be made between mean height and top height.

Mean height of a stand may be defined in any of the following ways:-

(a) The regression height corresponding to the arithmetic mean of the diameters at breast height.

(b) The regression height corresponding to the diameter of the tree with the mean volume in the stand.

(c) The regression height corresponding to the mean basal area of the stand.

(d) Lorey's mean height defined as the arithmetic mean of the heights of the trees, and a weight assigned to each individual height proportionate to its basal area.

Regression height (a) is never used since the volume of the tree- with d.b.h. equal to the arithmetic mean of the diameter distribution - multiplied by stem number underestimates the stand volume. The computation of Lorey's mean height is unduly complicated and consequently expressions (b) and (c) are preferable if mean height is to be used for site indexing.

Usually the mean height of a stand is inadequate for the construction of the site index curves as it is affected by an arithmetic shift in the mean diameter due to the removal of the smaller trees in thinnings. The mean height of a stand will thus tend to increase with the severity of thinning. The site index of individual stands may, therefore, be over- or underestimated if the thinning regime applied in these stands differs appreciably from that used for the construction of yield tables.

Top height is introduced to eliminate the effect of stand density on the site index.

The following are expressions of top height in use:-

(a) Hart's top height, defined as the mean height of 40 or 100 of the tallest trees per acre. An acre is subdivided into 40 or 100 blocks of equal size. The tallest tree in each block is measured and the mean of these tallest trees is calculated. This top height is practically independent of the severity of thinning since the tallest trees are seldom removed in thinning. There is usually one drawback to this assessment of top height, namely the tedious and time-consuming field measurement that it entails.

(b) Näslunds top height defined as the regression height of the tree with a d.b.h. of $D + 3\sigma$ where

D = arithmetic mean diameter

σ = standard deviation.

The relationship between d.b.h. and height is usually curvilinear. The increase of height with a unit increase in d.b.h. is inversely proportional to the diameter of the trees. Näslunds top height falls in the flat part of the height curve and consequently is almost independent of the severity of thinning.

(c) Weise's top height is defined as the regression height of the mean diameter of the thickest 20% of the trees in the stand. To calculate this top height the tally sheet of diameters is divided into 5 classes with an equal number of stems. The mean diameter in the largest diameter class is computed and the regression height read off the height curve. To a limited extent this top height will be affected by thinnings for with increasing severity of thinnings, the mean diameter of the highest class will tend to rise. Weise's top height, however, will also fall in the flatter part of the height curves and hence the effect of thinnings will be slight. Obviously the diameter used in Weise's top height can be more readily calculated than that used for Näslund's.

For a further study of the relationship between mean and top heights, the following heights were investigated:-

(a) Top height A. The 75th percentile diameter was computed for each sample plot separately, and a tree

26/......

with this d.b.h. was felled and its height measured. Damaged, broken and forked trees were excluded.

(b) Top height B. This height is similar to top height A, but the height of the 75th percentile diameter was read from the height curve.

(c) Top height C. The tally sheet of diameters was divided into 4 classes, the mean diameter by basal area of the highest class was computed and the height read from the height curve.

(d) Median height. The median of the diameter distribution was computed and the regression height of this diameter read off from the height curve.

The relationships between mean height, and top heights A.B.C., and median height respectively are reproduced in fig. 3a, b, c and d.

The figures indicate linear relationships between mean height and each of the other variables, and consequently linear regressions were fitted to the data.

The results of the analysis of variance applied to each of the regressions were as follows:-

Source of Variation	D.F.	Top ht. A M.S.	Top ht. B M.S.	Top ht. C M.S.	Median ht. M.S.
Total	199	354.55	378.45	405.73	352.82
Regression	1	67,597.57	74,596.14	78,355.29	69,966.27
Residuals	198	14.94	3.61	12.06	1.24
"F" value	1,198	4,523.9**	20,644.6**	6,504.7**	56,585.9**

The regression equations were:-

Top height A = 3.790 + 0.9788 Mean height........(3)
Top height B = 0.899 + 1.0286 Mean height........(4)
Top height C = 1.431 + 1.0538 Mean height........(5)
Mean Height =-0.155 + 0.9958 Mean height........(6)

FIG. 3a.

THE RELATIONSHIP BETWEEN MEAN HEIGHT AND TOP HEIGHT 'A'

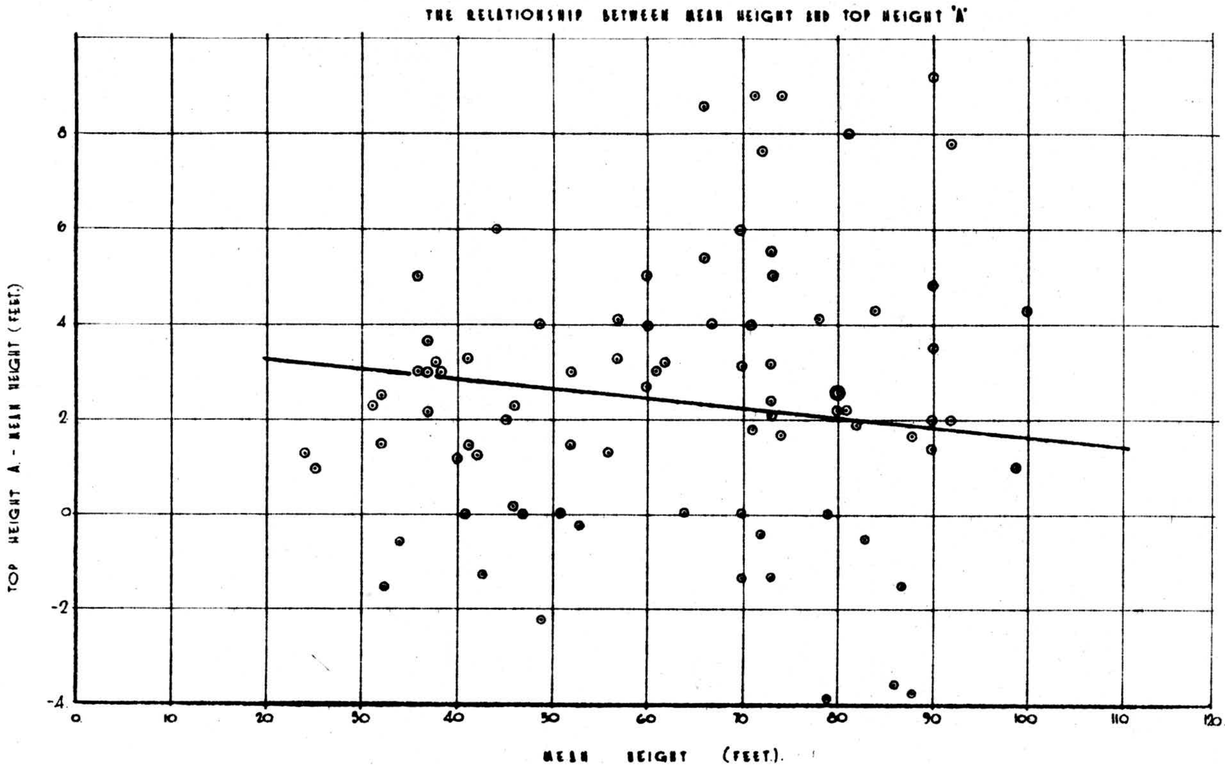


FIG 3.b.

RELATIONSHIP BETWEEN MEAN HEIGHT AND TOP HEIGHT B

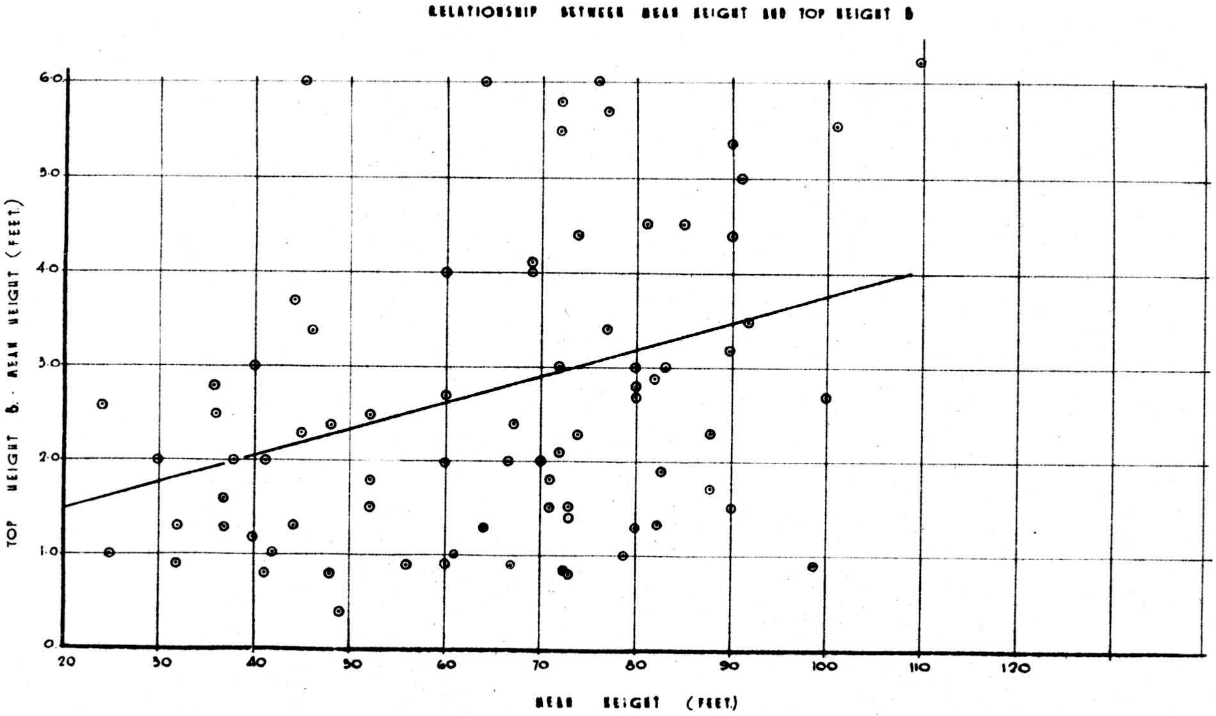


FIG 3c.

RELATIONSHIP BETWEEN MEAN HEIGHT AND TOP HEIGHT-C

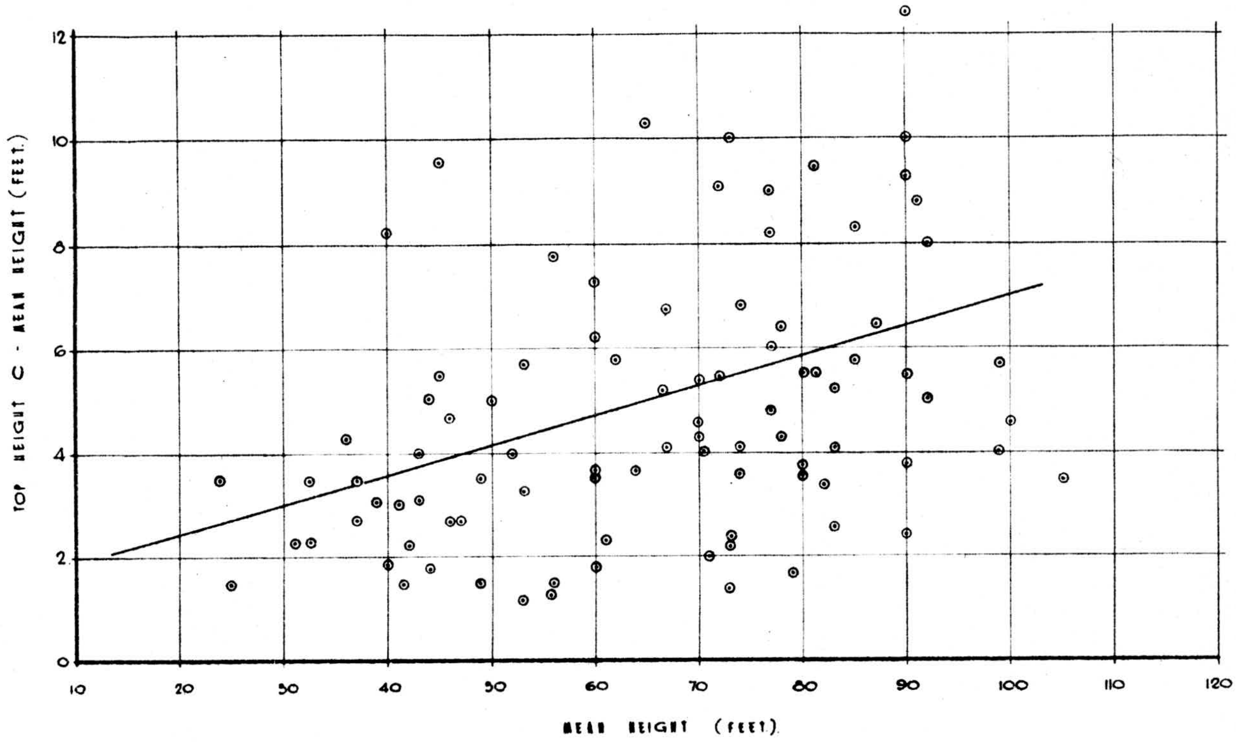
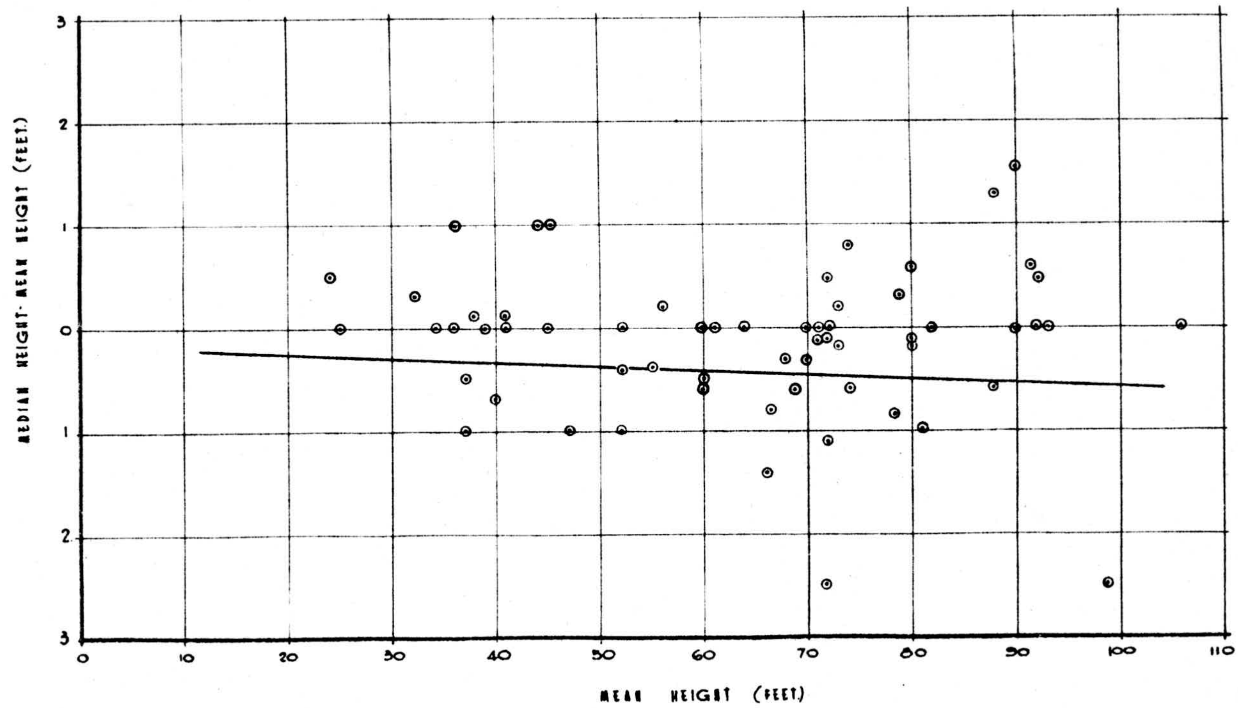


FIG. 3.d.

THE RELATIONSHIP BETWEEN MEAN HEIGHT AND MEDIAN HEIGHT.



A comparison between regression equations (3) and (4) shows a significantly greater variance around the regression of equation (3). The appropriate variance ratio is 4.1, distributed with 198, and 198 degrees of freedom. Both Top heights A and B are the regression heights of the 75th percentile diameter, but top height A is derived from a single tree, felled on the sample plot, and top height B from a height curve, based on 10 measurements of tree height. The smaller variance about the regression in equation (4) indicates a greater stability of top height B and should consequently be preferred to top height A as an expression of site index.

There is only a slight difference between the mean height and median height. For a mean height of 69.3 ft. the corresponding median height is 68.9 ft. It is smaller than the mean height because the median of a normal distribution of diameters is consistently smaller than the mean diameter, corresponding to the mean basal area. Thus, the difference between these height expressions is small and as a measure for site index, the median height has all the previously emphasized disadvantages of the mean height. Top height A and median height were thus discarded as expressions of site index and top heights B and C are left for comparison. For a mean height of 69.3 ft. the regression value of top height B is 72.2 ft. and that of top height C, 74.5 ft. The differences from mean height are 2.9 and 5.2 ft. respectively. The variances around the regressions are 3.6 for the top height B and 12.1 for top height C and the standard errors of estimate 1.9 and 3.5 ft. respectively.

The relative standard errors of estimate, expressed as percentages of these differences are 65% and 67% respectively. In other words the amount of fluctuation around regression line (5) is greater than that around regression line (4), but this is entirely attributable to the greater difference between top height C and mean height than between top height B and mean height. There are two reasons for giving preference to top height C rather than top height B.

(a) Top height C is the regression height of the mean diameter (of the largest 25% of the trees). The arithmetic shift of this mean diameter due to thinnings is less than that of the 75th percentile diameter and therefore, top height C will be less affected by severity of thinnings than top height B.

(b) The mean d.b.h. of the largest 25% of the trees is always greater than the d.b.h. of the 75th percentile tree. Consequently top height C will always be located in the flatter part of the height curve and will thus be less affected by severity of thinnings than top height B.

(4) Construction of site index curves.

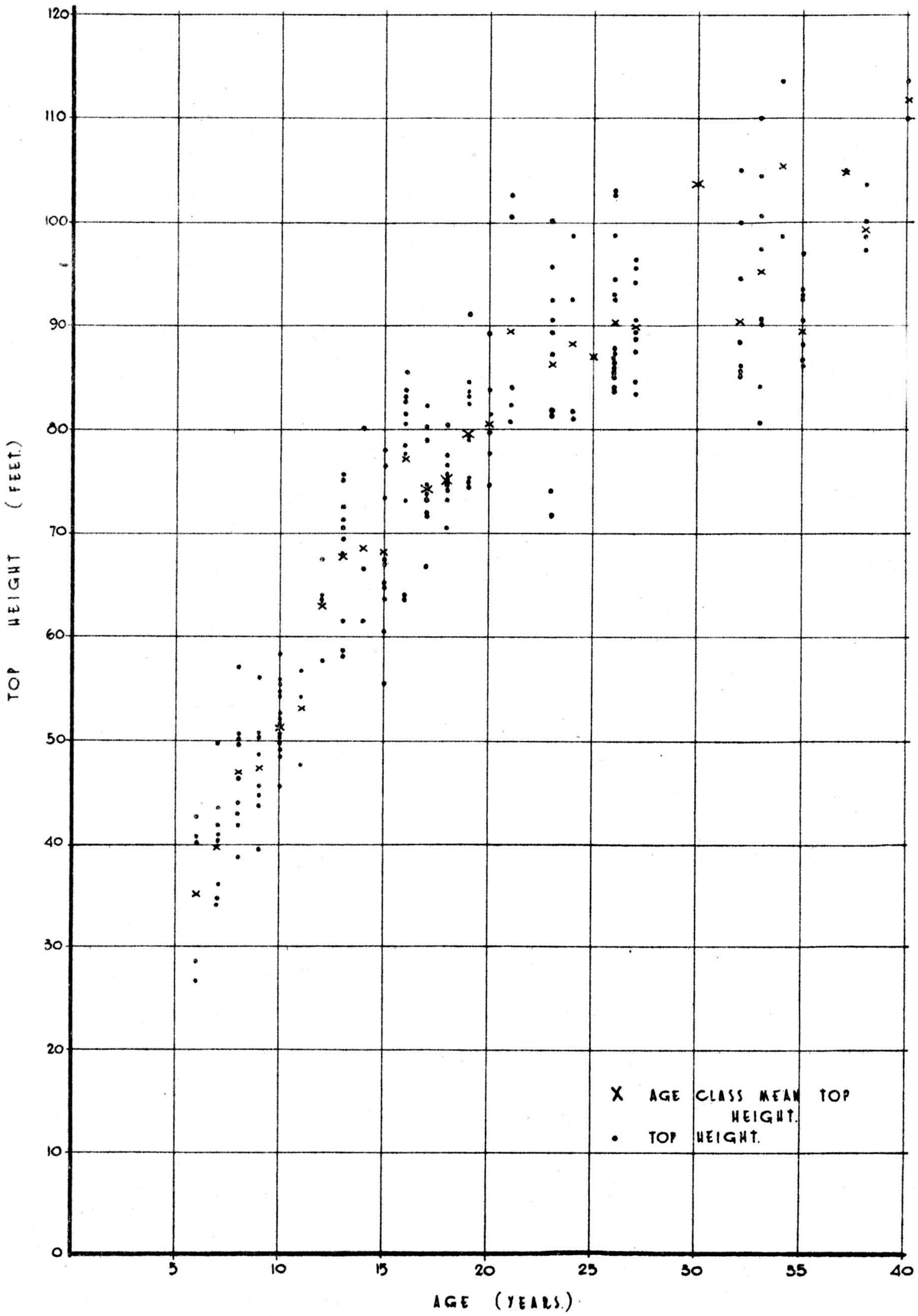
Top height, as previously defined, was used to construct site index curves, and a reference age of 20 years was introduced.

The relationship between top height and age is reproduced in fig. 4.

Transformation of the independent and dependent variables is necessary to obtain a linear regression. The re-

FIG 4.

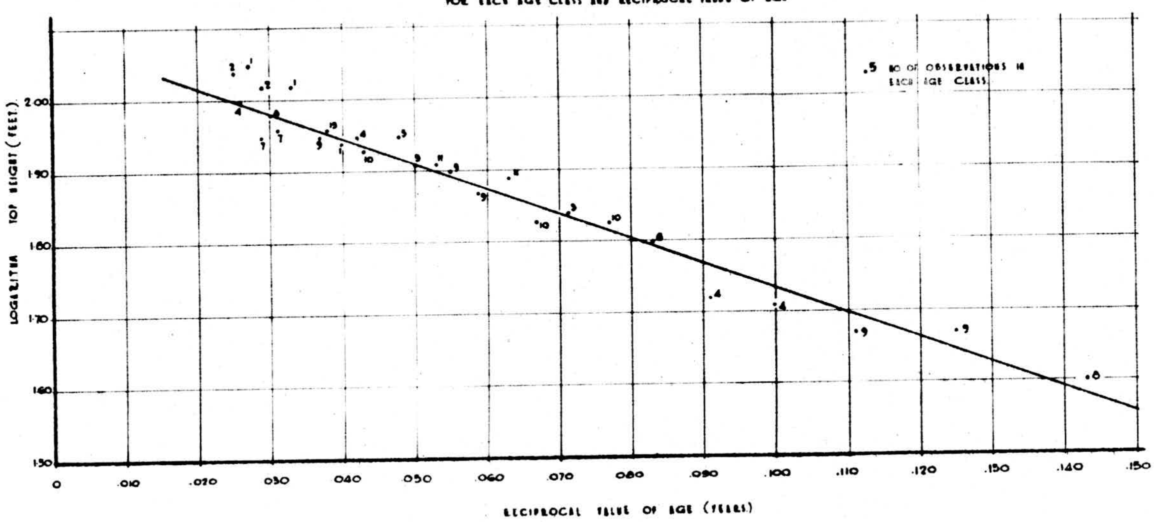
RELATIONSHIP BETWEEN TOP HEIGHT AND AGE.



lationship between log top height and the reciprocal value of age, for sample plots older than six years, is reproduced in fig. 5. One sample plot with the exceptionally large top height of 143 ft. at 35 years was excluded.

FIG 5

THE RELATIONSHIP BETWEEN THE LOGARITHM OF MEAN TOP HEIGHT FOR EACH AGE CLASS AND RECIPROCAL VALUE OF AGE



The following equation was fitted to the data:-

$$\text{Log (Y)} = a + b(\frac{1}{x})$$

where x = age

y = top height

a and b constants.

The result of the analysis of variance were as follows:-

Source of Variation	D.F.	M.S.	F.
Regression	1	2.46812	911.26**
Deviations from Regression	26	.002708	1.86279N.S.
Total amongst age classes	27	.124606	
Within age classes	166	.0014539	
Total	193	.0144036	

The analysis of variance indicates that deviations from linearity are not significant and consequently, the equation $\log (y) = a + b(\frac{1}{x})$ is a satisfactory approximation of the relationship between top height and age. The analysis of variance also shows that the regression is highly significant. The resulting equation was as follows:-

$$\text{Log (y)} = 2.08832 - 3.15515\ (1/x) \qquad (7)$$

This equation gives an estimate of the top height of stands of various ages, but this estimate applies to the sample of forest stands included in the present study. The resulting site index curve will also reflect the average development of top height with age, but is only true when the site quality of older stands does not differ appreciably from that of younger stands.

In this study an appropriate sampling procedure has been followed. In each age group a random sample of plots from the population of forest stands in the Natal Midlands was selected. It ensures that this sample will be unbiased but does not exclude the possibility that the site quality of stands of various ages differ. For example:-

(a) Stands from the older age classes have been planted on the better or poorer sites.

(b) The older stands originate from seed of a different provenance.

(c) Other methods of stand establishment or soil cultivation have been applied in older stands.

It is therefore necessary to compare the height growth curve derived from the top height of the stands at various ages of the sample plots with that derived from stem analysis. It should be borne in mind that the stem analysis was carried out on the tree with the 75th percentile diameter. One tree in each sample plot was felled, sawn into 5 foot lengths and the annual rings counted every 5 feet. These trees produced a varying number of observations of the relationship between age and height.

The number of counts on each tree depends obviously upon the actual height of the tree. For this investigation, the observations from all sample trees were combined and a single regression equation was computed for these data. The result was as follows:-

$$\text{Log } (y) = 2.0505 - 3.2511\ (1/x) \quad \ldots\ldots\ldots\ldots (8)$$

where x = age

and y = top height.

For comparison the observed present top heights of the sample trees were considered and an equation was computed to determine the relationship between age (x) and present top height (y). The result was as follows:-

$$\text{Log } (y) = 2.057939 - 3.29709 \, (1/x) \quad \ldots\ldots\ldots (9)$$

The following relationships were derived from these equations:-

Age	Top Height	
	Equation (8)	Equation (9)
10	53.1	53.5
20	77.3	78.2
30	87.5	88.7
40	93.1	94.5

It can be seen that there is a slight difference between the growth curves derived from equations (8) and (9). The slope of the regression line of equation (8) is steeper than that of equation (9). This may be due to the effect of repeated thinnings.

The relationship between age and top height of a stand after thinning cannot be represented by a continuous function because with every thinning the top height is subject to an arithmetic shift. Therefore, as a result of the thinning, the slope of the regression curve for the relationship between height after thinning and age will be slightly steeper than that of the biological growth curve expressed by the stem analysis equation. This explains the differences in the slopes of the regression curves.

It is, therefore, justified to base the site index curves on the present heights of sample plots. And consequently equation (7) can be accepted.

The next step in the construction of these curves is to investigate their shape. The sample plots measured in each age group are a random sample from the population of stands in that age groups and if the shapes of the site index curves are similar there will not be a correlation between the relative variation of the top heights in a given age class and age. In the present case the correlation coefficient had a value of 0.1584 with 23 degrees of freedom. This correlation is not significant, which indicates that the shapes of the curves are similar. In other words the height difference between the site index curves, expressed as a % of the height, does not change with age.

Seven site index curves were established with a difference of 5 feet between two successive curves at the reference age of 20 years. The equation of these site index curves was derived from equation (7): i.e.

$$\text{Log } (y) = 2.08832 - 3.15515 \ (1/x)$$

The mean top height for the sample plots is: 81.41 ft. at 20 years and 23.88 ft. at 5 years. The equation for site index curve 90 is computed as follows:-

At 20 years the top height is 90 ft., i.e. 1.105515 times the top height of site index 81.41. Consequently the top height at 5 years is 23.88 x 1.105515 = 26.40 ft.

Thus: $\text{Log } (90) = a + b \ (^1/20) \ldots\ldots\ldots\ldots(i)$

$$\text{Log } (26.40) = a + b \ (^1/5) \ldots\ldots\ldots(ii)$$

Solving equations (i) and (ii) simultaneously gives the following result:-

$$a = 2.1317$$

$$b = 3.5507$$

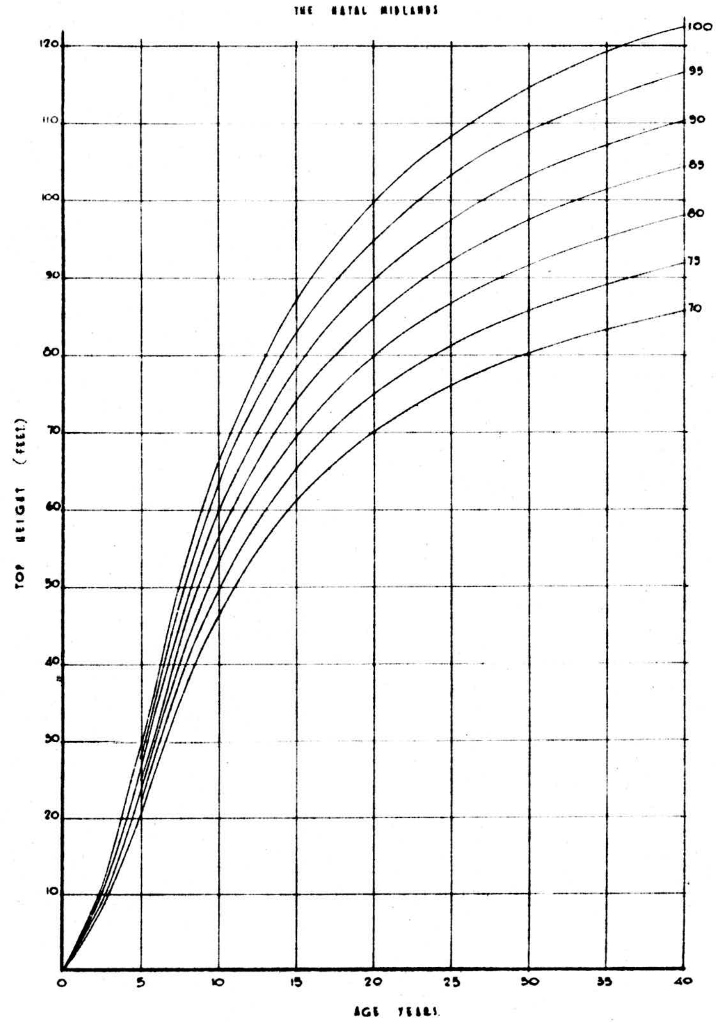
Thus, for site index 90, the equation is as follows:-

$$\text{Log }(y) = 2.1317 - 3.5507(^1/x)$$

A similar method was applied to compute the equations for site indices 70, 75, 80, 85, 90, 95, 100 and resulting site index curves are reproduced in fig. 6.

FIG 6.

SITE INDEX CURVES FOR P.PATULA IN THE NATAL MIDLANDS



2. Stand Density.

Stand density may be defined as the degree to which an area is utilized by trees and is measured by the stand density index. In a given stand of a species the stand density will decrease as the stand is more severely thinned but naturally the stand density is also affected by the age and site quality prior to thinning.

If a stand density index is used to express the severity of thinnings the effect of all other factors must be eliminated. Nevertheless it is advisable to select a stand density index which is virtually unrelated to the age and site quality of a stand. Moreover the index should be simple and objective.

The following measures of stand density are used:-

(a) Volume per acre.

(b) Basal area per acre.

(c) Number of trees per acre.

These stand characteristics are logical, readily understood and frequently used measures of stand density. However, they are all closely correlated with age and site quality, which is a serious drawback in thinning research. These measures might be used on a limited scale to compare the densities of stands of a given age and site quality.

(d) Reineke's stand density.

Forestry literature has an abundance of references to the relationship between mean d.b.h. and trees per acre. Reineke (1933), assuming that the distribution of diameters about their mean approximates the normal distribution curve, found a linear relationship between the number of trees per acre of fully stocked stands and the mean diameter when both are expressed in logarithms. The relationship is expressed by the equation:-

$$\text{Log } (Y) = b_o + b_1 \log (X)$$

where X = mean d.b.h.

Y = number of trees per acre.

b_1 = slope of the regression line, largely independent of species.

b_o = constant varying with species.

The stand density index is given by the equation:-

$$\text{Log } Z = \log Y + b_1 \log X - b_1$$

where Z = stand density index.

Reineke tested the index on 12 species. He also investigated the effect of age and site quality on the stand density index in a multiple correlation analysis and found no significant correlations. This indicates that Reineke's stand density index is independent of age and site quality.

Mulloy (1943) used this stand density index successfully in stands of red and white pine. Hummel (1953), criticizing Reineke's index, pointed out that the d.b.h. of the trees which is used to estimate the stand density index is affected by the severity of thinnings. In other words an independent variable is used to estimate the value of a dependent variable, which is affected by this variable. This is obviously a serious disadvantage. In addition, the assumption of normality of the d.b.h. distribution is not always satisfied.

(e) Chisman and Schumacher tree area equation.

Chisman and Schumacher (1940) assumed that the area occupied by a single tree is a function of the diameter of this tree.

The authors introduced the following second degree equation:-

$$Y = a + b_1 X + b_2 X^2$$

where X = d.b.h. of the tree.

Y = area occupied by the tree.

a and b constants.

For N trees per acre the equation becomes

$$\overset{n}{S}(y) = b_o + b_1 \overset{n}{S}(X) + b_2 \overset{n}{S} (X^2)$$

The value of the dependent variable is 1 because N trees per acre occupy a ground area of one acre.

Chisman and Schumacher tested this stand density measure on 133 sample plots of fully stocked loblolly pine and calculated the values of the constants by the method of least squares.

The stand density of the individual sample plots may now be calculated by inserting the appropriate d.b.h. data into the equation. The result, multiplied by 100 will represent the percentage stocking of this sample plot in proportion to the average stocking of the sample plots.

In order to analyse the influence of age and site quality on this tand density index, the authors assumed that the parameters b_o, b_1 and b_2 are linear functions of both age and site quality.

This resulted into a multiple regression equation, with diameter, diameter X diameter, age, site quality and the interactions between these factors as independent variables. In the analysis of variance they found that age and site quality did not reduce the error sum of squares significantly and consequently these factors could be disregarded.

(f) Schumacher and Coile's stocking percent.

Schumacher and Coile (1960) working with southern pines introduced the term stocking percent. This expression refers to the ground area which a stand of a given age dominant height and basal area occupies. The following regression equation was used for loblolly pine.

$$Y = b_o X_1 + b_1 \frac{X_2}{100} + b_2 \frac{100}{X_3} + b_3 \frac{X_2}{X_3}$$

where X_1 = basal area per acre.

X_2 = dominant height.

X_3 = age.

Y = stocking percent.

b_o, b_1, b_2 and b_3 = constants.

In each fully stocked sample plot, the basal area, age and dominant height were measured and a value of 100 (i.e. stocking percent = 100) assigned to the dependent variable Y. The values of the parameters b_o, b_1, b_2 and b_3 were computed by the method of the least squares.

In practical application the stocking of a particular stand is then expressed in proportion to the average stocking of 100%. This clearly demonstrates that this stand density index is a relative term.

Regression equations of a more or less similar nature were computed for other pine species.

(g) The Hart-Becking stand density index.

Hart (1928) introduced the S% defined as the mean distance between the trees expressed as a percentage of the top height of the stand. In order to determine the mean distance between the trees, the author assumed that trees in a stand, even after repeated thinnings are perfectly regularly spaced. For square spacing, the relationship between spacing and number of stems is as follows:-

$$N = \frac{43560}{a^2}$$

where N = stems per acre.

a = spacing.

$$\text{hence } a = \sqrt{\frac{43560}{N}} \qquad \ldots\ldots\ldots(10)$$

To compute the distance between the trees the stem number is determined, and the spacing derived from equation (10). For the calculation of the S% Hart counted only those trees which are taller than ¾ of the top height of the stand. Subdominant and suppressed trees which do not reach this height are disregarded.

In order to determine the top height of a stand, a hectare was subdivided into 100 blocks of 1/100 hectare each, and the tallest tree in each block measured. The top height was then defined as the arithmetic mean of these 100 heights. In sample plots with an area less than 1 hectare, the plot was subdivided into blocks of 1/100 acre and the mean of the tallest trees in the plot used as an estimate of the top height of the stand. Hart's index has been used extensively in thinning research in the Netherlands (Becking 1952).

In Great Britain the S% is defined as the average spacing expressed as a % of the top height, which is defined as the average height of the 100 tallest trees per acre (Hummel 1953). In small sample plots 100 trees per hectare produce too few trees for a precise estimate of the top height.

Wilson (1946) introduced a similar stand density index and defined an f-value as follows:-

$$f = \frac{\sqrt{\frac{43560}{N}}}{H}$$

where N = number of trees per acre.

H = average height of the dominant trees.

In this study Hart - Becking's S% has been used to express stand density, as this measure provides a simple index and because the optimum S% in plantations is not closely related to age and site quality. The index was modified by defining top height as the regression height of the mean diameter of the 25% largest trees, in order to avoid the time-consuming measurement of Hart's top height.

3. Basal Area Growth.

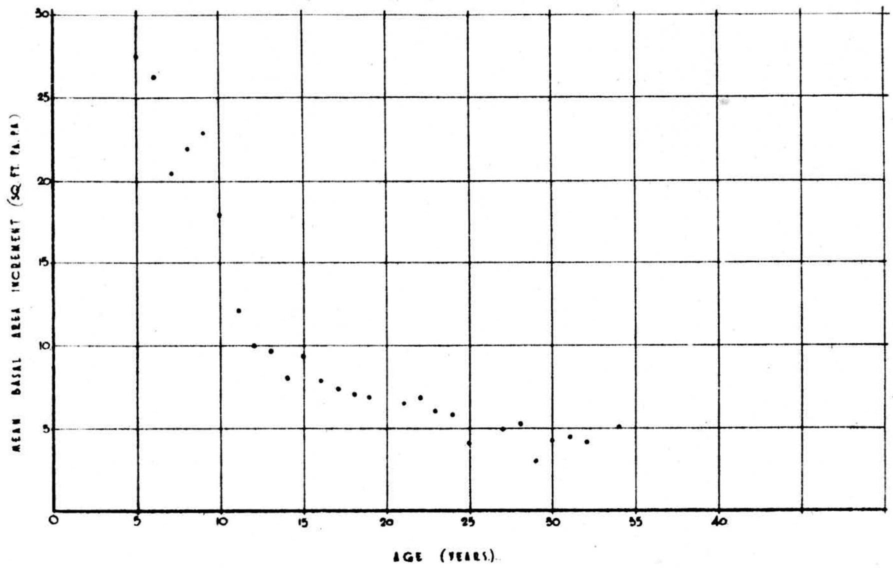
In this study basal area growth of the temporary sample plots, derived from increment cores was correlated with a number of stand characteristics. This basal area growth refers to the periodic current increment since the last thinning and was expressed in square feet per acre per annum.

A graphic analysis of the data revealed that the age of the stand was the most important variable affecting the current basal area increment. For this purpose the age of the stand is defined as the age at the middle of the period during which the periodic current increment is measured. If the last thinning was carried out n years ago, the reference age of the sample plot is ($A-\frac{1}{2}n$) years, where A = present age of the plot.

The relationship between age and basal area increment is reproduced in fig. 7.

FIG. 7.

THE RELATIONSHIP BETWEEN AGE AND MEAN BASAL AREA INCREMENT FOR EACH AGE CLASS.



The following equation was fitted to the data:-

$$Y = a_1 + b_1(^1/_{X_1}) \quad ....(11)$$

where $X_1 = {}^1/\text{age}$

Y = basal area increment

a_1 and b_1 regression constants,

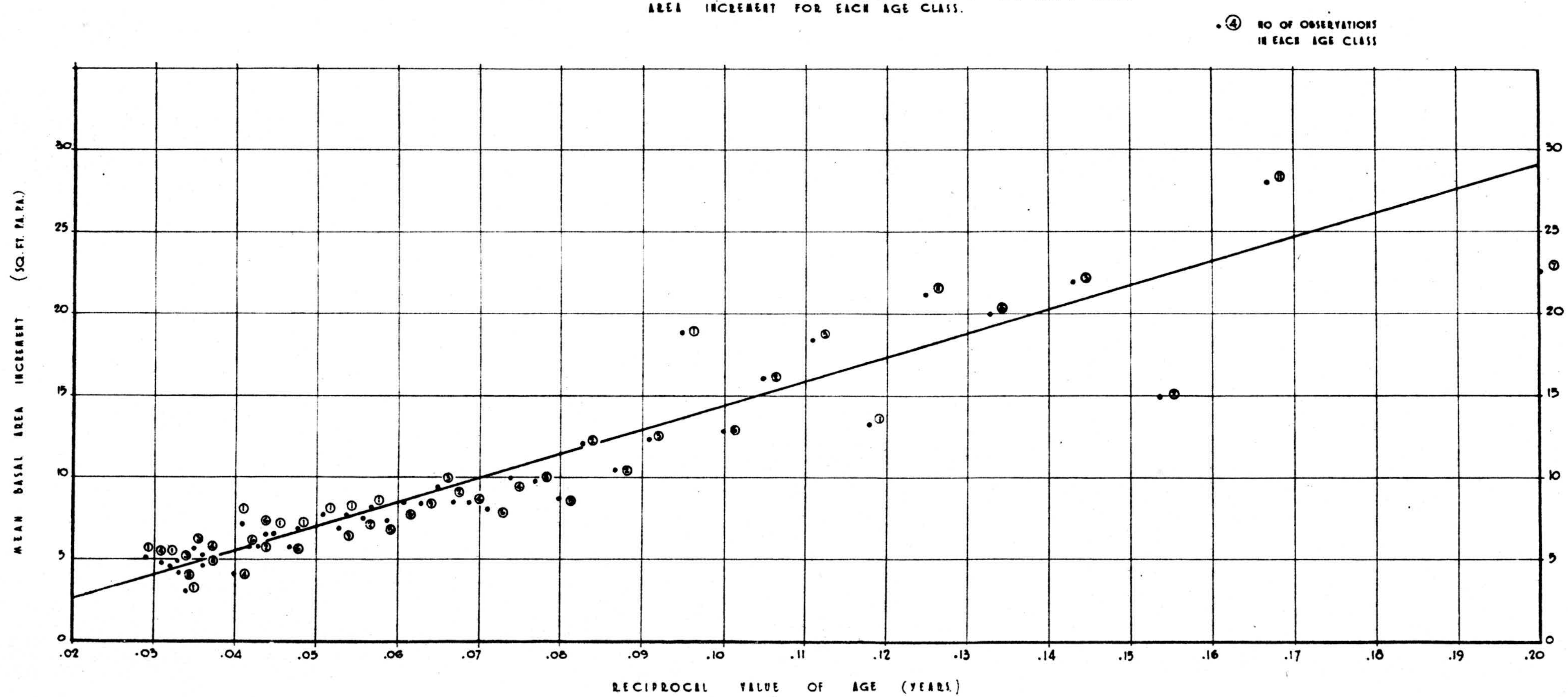
and was as follows:

$$Y = -0.545 + 148.76879 \left(\frac{1}{X_1}\right)........(12)$$

Fig. (8) shows that this equation fits satisfactorily and consequently the reciprocal value of age was introduced as an independent variable.

FIG 8.

THE RELATIONSHIP BETWEEN THE RECIPROCAL VALUE OF AGE AND MEAN BASAL AREA INCREMENT FOR EACH AGE CLASS.



A review of the literature on yield studies indicates that the basal area increment may also be affected by stand density and site quality. For a further investigation, stand density expressed in S% and site quality expressed in site index were introduced as independent variables. The logical procedure is to assume that the constants a_1 and b_1 in equation (11), i.e. the level and the slope of the regression line, expressing the relation between age (X_1) and increment are affected by site quality (X_2) and stand density (X_3).

The following equations were introduced

$$a_1 = a_2 + b_2 X_2 + b_3 X_3 \quad \ldots\ldots\ldots\ldots(i)$$

$$b_1 = a_3 + b_4 X_2 + b_5 X_3 \quad \ldots\ldots\ldots\ldots(ii)$$

Substituting of equation (i) and (ii) into equation (11) produces the following equation:-

$$Y = a_2 + b_2 X_2 + b_3 X_3 + a_3 \left(\frac{1}{X_1}\right) + b_4 \left(\frac{X_2}{X_1}\right) + b_5 \left(\frac{X_3}{X_1}\right)$$

For the sake of clarity the symbolic notation of the constants in this equation is slightly changed, as follows:-

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 \ldots\ldots(13)$$

where $X_1 = {}^1/\text{age}$

X_2= stand density (S%)

X_3= site quality (Site Index).

$X_4 = X_2/X_1$

$X_5 = X_3/X_1$

a, b_1, b_2, b_3, b_4 and b_5 are the regression constants.

The analysis of variance, indicating the contribution of each variable, independent of the other variables is as follows:-

Source of Variation	D.F.	Mean Square	"F" value
Y on X_1	1	8,510.6	733.6**
Y on X_5(X_1 fixed)	1	522.0	45.0**
Y on X_2(X_1,X_5 fixed)	1	225.7	19.4*
Y on X_4(X_1,X_5,X_2 fixed)	1	99.3	8.6*
Y on X_3(X_1,X_5,X_2,X_4 fixed)	1	4.5	0.4N.S.
Residuals	189	11.6	

It can be seen that the reduction in the sum of squares due to age and site quality is highly significant and consequently these variables are significantly correlated with basal area increment. The interaction between age and site quality is also significant. Density alone is not correlated with basal area increment but in combination with age it contributes significantly to the regression. In other words the effect of density on basal area increment is not additive but depends on the age of the stand.

Therefore, the variable X_3 (stand density) was deleted and the following equation computed.

$$Y = 2.175842 + 334.34528/X_1 - 0.001748X_2$$
$$- 1.006887X_4 - 3.31303X_5 \qquad \text{........(14)}$$

Equation (14) indicates that the basal area increment is negatively correlated with the site index of the stands. When stand density and age are held constant the relationship between basal area increment and site index is as follows:-

S% = 23, Age = 10, B.a. increment = 27.990401 - .102437 (Site Index)

S% = 23, Age = 20, B.a. increment = 15.083121 - .052092 (Site Index)

S% = 23, Age = 30, B.a. increment = 10.780694 - .035311 (Site Index)

This conclusion is illogical as it can be expected that the basal area increment will be higher on sites of better quality. A more detailed analysis of the relationship between basal area increment and site index was therefore necessary.

(1) The sample plots were divided into 4 age groups:-

Group	Age (years)
I	6 - 10
II	11 - 16
III	17 - 24
IV	over 24

In each age class the basal area increment of the sample plots was adjusted for the varying ages of the plots within these classes. For each sample plot the adjusted basal area increment is as follows:-

$$Ya = Y_1 + b(\frac{1}{X_j} - \frac{1}{X_i}) \quad ....(15)$$

where Y_i = actual basal area increment.

Y_a = adjusted basal area increment.

X_i = actual age of the sample plot

X_j = reference age at the middle of the age group. ($j = 1$ — — 4).

b = regression coefficient in equation (12) ($b = 148.76879$).

In other words the gross regression of basal area increment on age was used firstly to adjust the increment and secondly to estimate the expected increment for one particular age in each age group, with the effect of the varying ages within these groups being eliminated. For each age group basal area increment was plotted over the site index (fig. 9a, b, c, and d). The correlation coefficients in these groups were as follows:-

FIG 9a

THE RELATIONSHIP BETWEEN SITE INDEX AND BASAL AREA INCREMENT ADJUSTED FOR AGE (6-10 YEARS AGE GROUP)

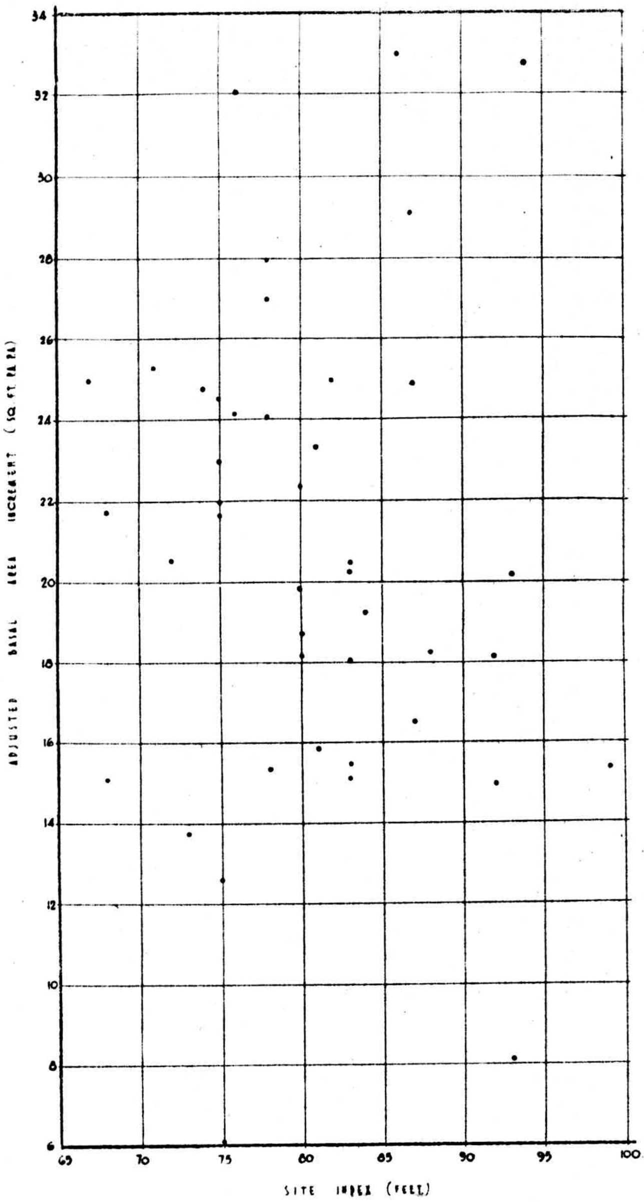


FIG 9b

THE RELATIONSHIP BETWEEN SITE INDEX AND BASAL AREA INCREMENT ADJUSTED FOR AGE (11-16 YEAR AGE GROUP)

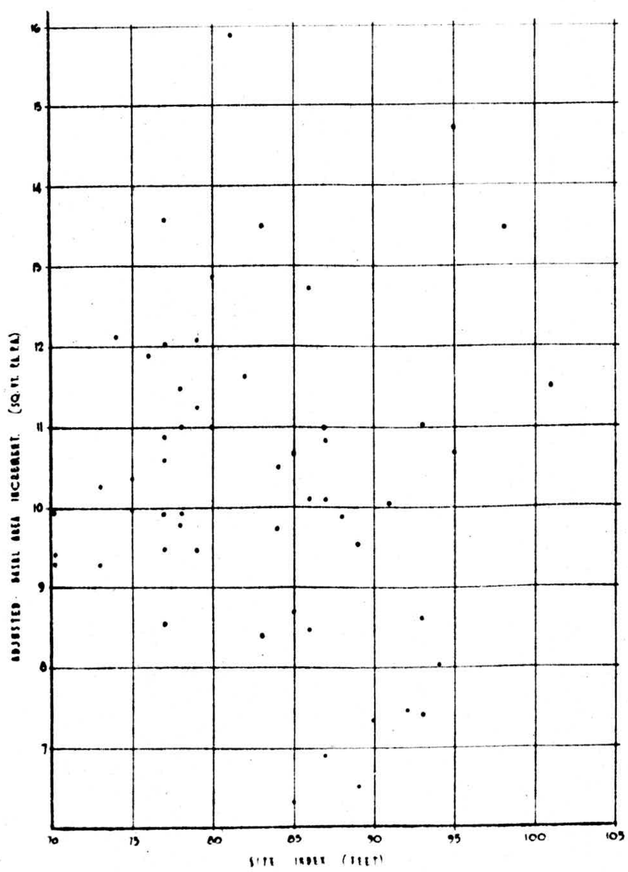


FIG. 9c.

THE RELATIONSHIP BETWEEN SITE INDEX AND BASAL AREA GROWTH ADJUSTED FOR AGE (17-24 YEAR AGE GROUP.)

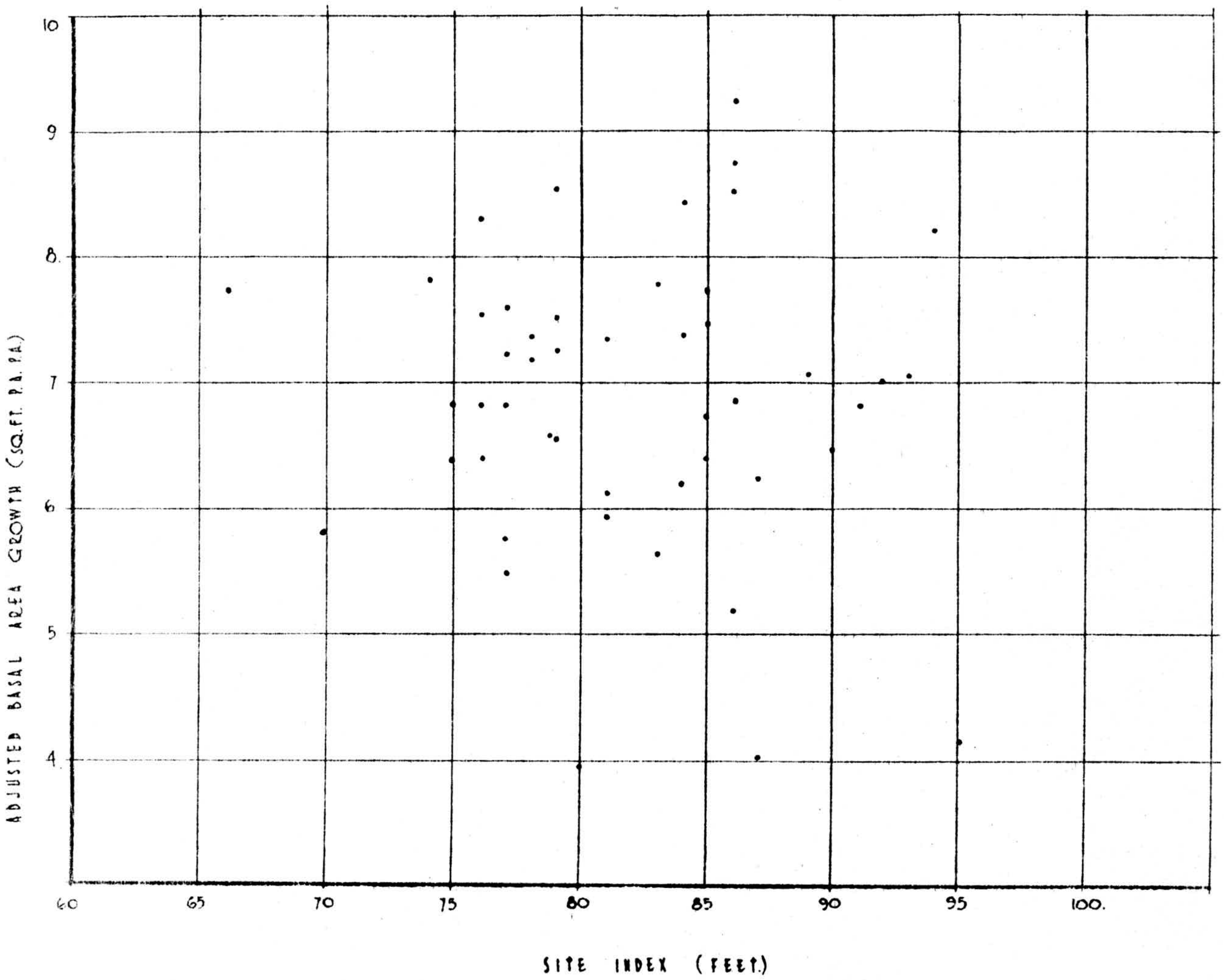
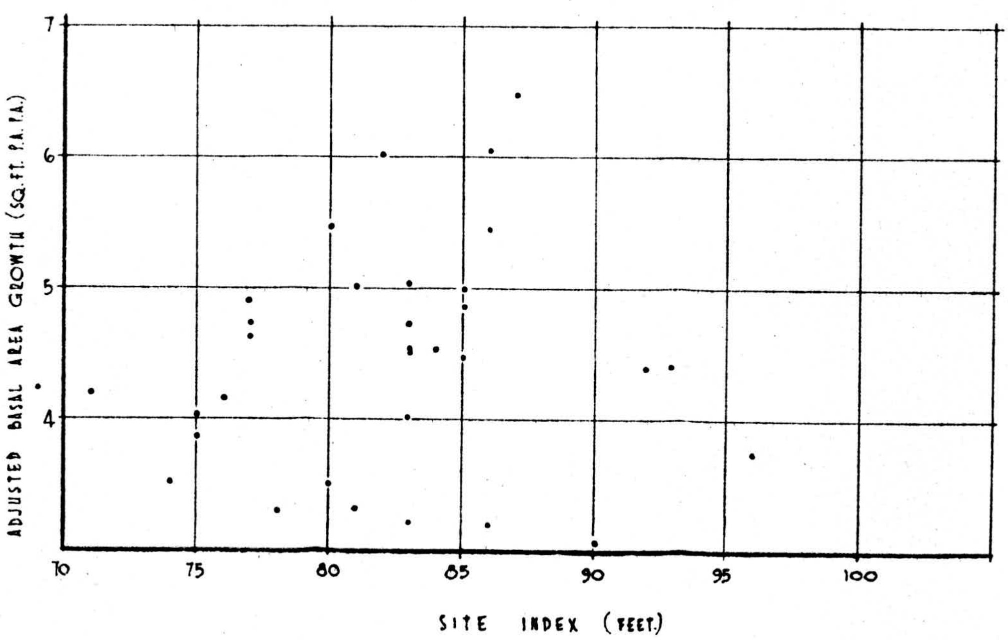


FIG. 9d.

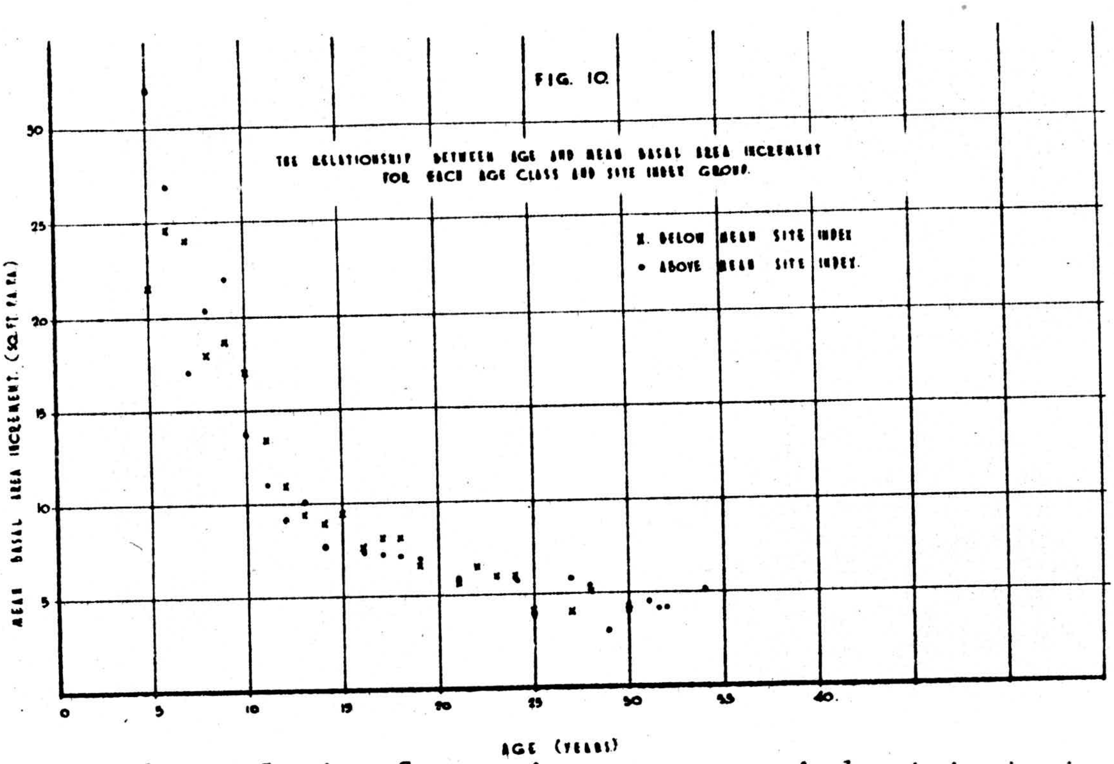
THE RELATIONSHIP BETWEEN SITE INDEX AND BASAL AREA GROWTH ADJUSTED FOR AGE (>24 YEAR AGE GROUP.)



Age group (yrs.)	Correlation coefficient.
6 - 10	.099 (not significant)
11 - 16	.047 (not significant)
17 - 24	.141 (not significant)
over 24	.052 (not significant)

They show that the basal area increment is not affected by the site index of the stands.

(2) The sample plots were divided into two groups containing those plots with a site index above and below the mean site index of 81.9 feet. For each group the basal area increment was plotted over age (fig. 10). In each group a regression $Y = a + bx$ was computed, where $X = {}^{1}/_{age}$ and Y = basal area increment.



The analysis of covariance was carried out to test the difference between regression coefficients and between adjusted mean basal areas increments.

The results are as follows:-

Source of Variation	D.F.	M.S.
Between adjusted means.	1	.22
Between Regression Coefficients.	1	15.40
Residuals around Regression.	44	12.65
Total	46	16.66

The difference between the adjusted mean basal area increments in these two groups is non-significant. This also indicates that the increment is not affected by the site index of the stand.

The multiple regression equation (14) however, showed a negative correlation between increment and site index. This is probably due to certain failures in the underlying assumptions of the regression analysis, for example, the site index of the stands is not measured error-free or for some other unknown reasons.

It is therefore justifiable to delete site index as an explaining variable in equation (14). The resultant equation becomes

$$Y = - 2.0538 + 240.418X_1 - 2.9825X_2 \text{......(16)}$$

where $X_1 = {}^1/\text{age}$

X_2 = stand density/age.

Y = basal area increment.

The analysis of variance for this regression is as follows:-

Source of Variation	D.F.	M.S.	"F"
Regression Y on X_1	1	8,510.7	689.1**
Regression Y on X_2(X_1 fixed)	1	552.04	42.2**
Residuals	193	12.35	

For a more detailed investigation into the effect of stand density on increment, within various age groups, the basal area increment in each age class was plotted over stand density. The increment figures were once again adjusted for the varying ages within these groups. The results are given in fig.(11a, b, c, and d). These figures indicate that the nature of the relationship changes with the age of the stand. In the age classes I and II, the increment decreases with increasing S%, in age class III the increment does not seem to be affected by stand density and in age class IV, the regression curve has a maximum. Consequently equations of the form $Y = a + bx$ (x = S%., Y = increment) were fitted to the observations in age groups, I, II and III and an equation $Y = a + b_1X + b_2X^2$ in group IV. The results are as follows:-

Group I $Y = 28.307 - 0.3171X$
II $Y = 13.167 - 0.1168X$
III $Y = 6.509 + 0.0179X$
IV $Y = -28.174 + 2.7525X - .05138X^2$

The standard error of the regression coefficients b in equations I, II and III, the t-values, computed to test the hypothesis that the population value β of these coefficients is 0 and the corresponding degrees of freedom are as follows:-

Group	b	Sb	$t = \frac{b-\beta}{Sb}$	D.F.
I	-.3171	.05253	6.0365	43
II	-.1168	.07707	1.5155	55
III	+.0179	.06462	0.2770	45

It can be seen that the basal area increment decreases with increasing S% with a probability of 99% in group I, and a probability of 90% in group II. In age group III

48/......

FIG IIa.

THE RELATIONSHIP BETWEEN S% AND BASAL AREA INCREMENT ADJUSTED FOR AGE (6-10 YEAR AGE GROUP.)

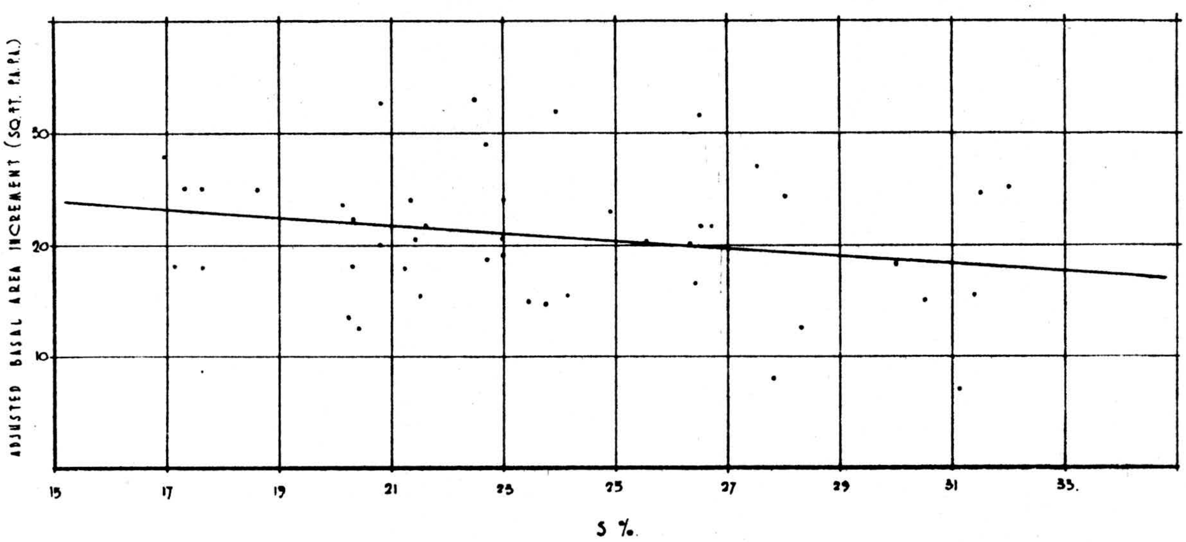


FIG. IIb.

THE RELATIONSHIP BETWEEN S% AND BASAL AREA INCREMENT ADJUSTED FOR AGE (11-16 YEAR AGE GROUP)

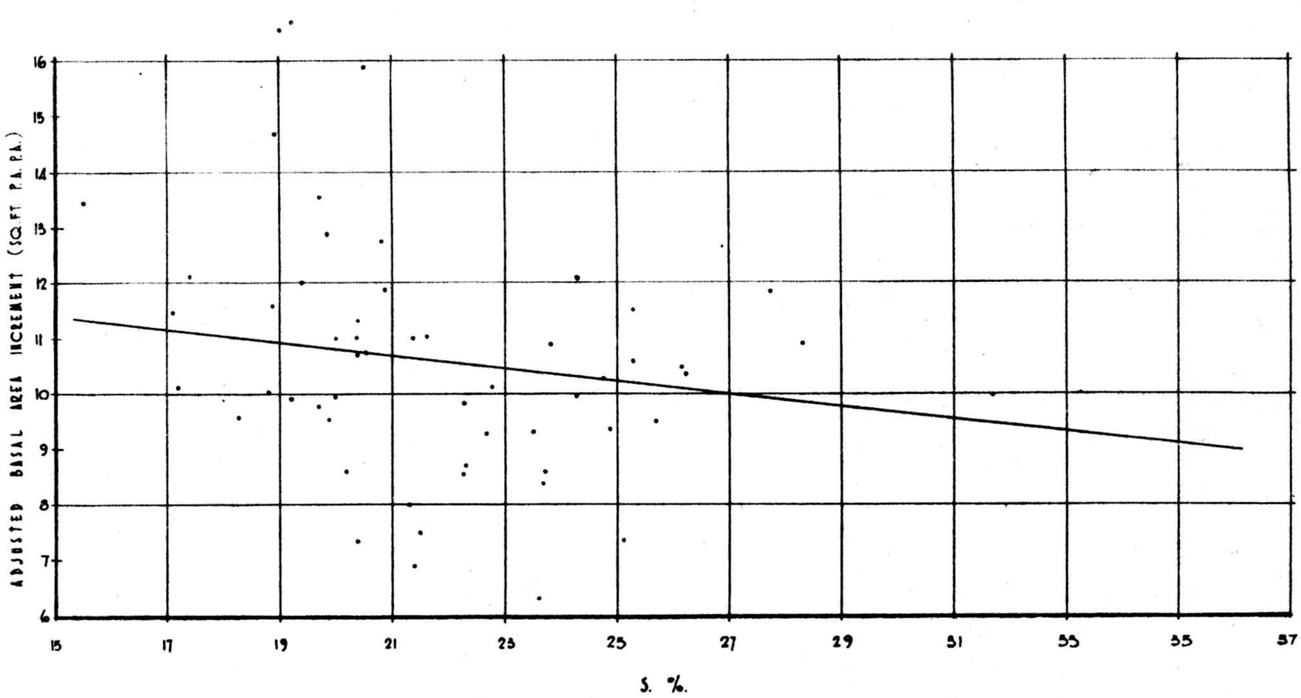


FIG. IIc.

THE RELATIONSHIP BETWEEN S% AND BASAL AREA INCREMENT ADJUSTED FOR AGE (17 - 24 YEAR AGE GROUP.)

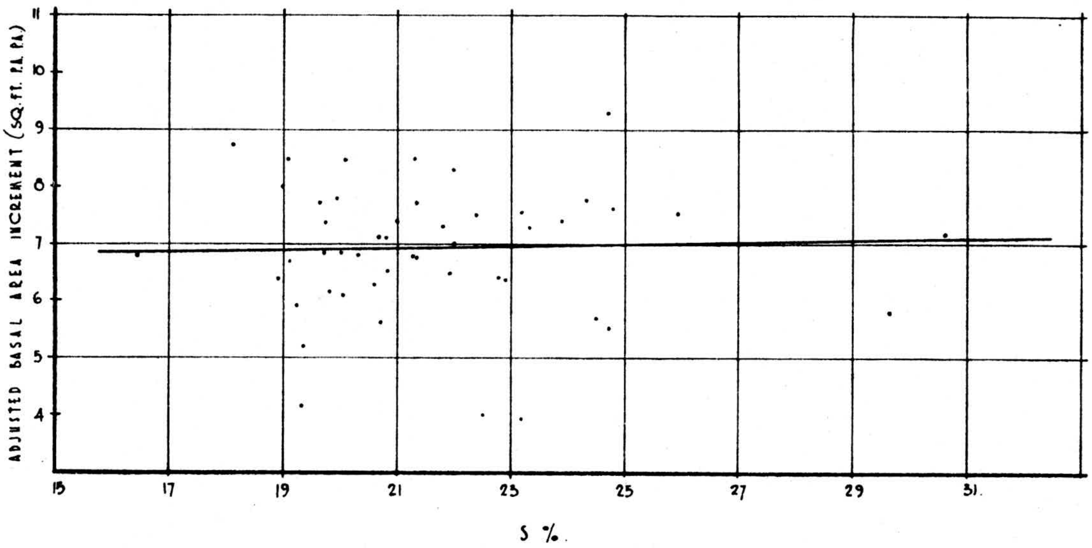
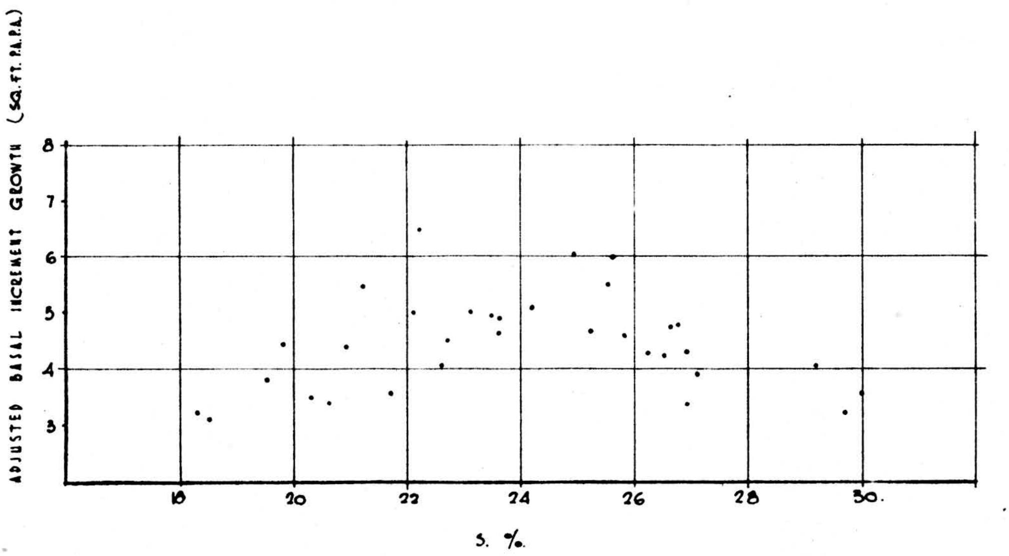


FIG. IId.

THE RELATIONSHIP BETWEEN S% AND BASAL AREA INCREMENT ADJUSTED FOR AGE (>24 YEAR AGE GROUP.)



there is no correlation between basal area increment and S%. In group IV there are two regression coefficients b_1 and b_2. The analysis of variance of the observations in this age class is as follows:-

Source of Variation	D.F.	M.S.	"F"
Regression Y on X^2	1	9.342	20.23**
Regression Y on X (X^2 fixed)	1	0.447	0.968N.S.
Residuals	29	.46172	

The contribution of X, i.e. the linear term is non-significant, that of X^2 i.e. the quadratic term is significant at the 0.01 level. The linear term (X) is thus deleted and the equation for Group IV becomes

$$Y = 4.14122 + .00057979X^2$$

The occurrence of a maximum in the highest age classes may be a true thinning effect, but it may also be of an accidental nature. For the purpose of this study it is assumed that the relationship between stand density and increment is linear but with the possibility that the rate of change in increment with unit change in stand density, varies with the age of the stand. This assumption was introduced in the form of equation (16):

$$Y = -2.0538 + 240.418X_1 - 2.9825X_2 \quad \ldots\ldots\ldots\ldots(16)$$

where $X_1 = {}^1/\text{age}$.

X_2 = stand density/age.

Y = basal area increment.

From this equation - the following relationship between S% and basal area increment at various ages may be derived.

Age = 8.0 years; $Y = 27.9984 - 0.3728X$
Age =13.0 years; $Y = 16.4399 - 0.2294X$
Age =20.5 years; $Y = 9.6739 - 0.1455X$
Age =27.0 years; $Y = 6.8506 - 0.1105X$
where X = stand density.
Y = basal area increment.

4. Construction of Yield Tables.

A. Introduction.

A variable density yield table presents for a given species, the expected volume yield and other stand parameters with age, stand density and site index as independent variables.

MacKinney, Schumacher & Chaiken (1937) based yield tables for *Pinus taeda* on an equation derived from the Pearl-Reid logistic growth curve:

$$\text{Log } \left(\frac{k-y}{y}\right) = a + b_1x + b_2x^2 + b_3x^3 \text{ -- } b_n$$

where x = age.
k = upper limit of volume yield per acre.
y = volume per acre.

The upper limit of the volume yield per acre was thought to be a function of stand density and site index. It is not directly affected by the age of the stand, but this maximum will be reached at a younger age, as the site index increases, i.e. the rate of approach towards the maximum yield is correlated with age. A first approximation of maximum yield was estimated from the equation

$$k = \frac{D.C.S.}{100}$$

where D = Reineke's stand density index.
C = basal area of *Pinus taeda* expressed as a percentage of the basal area of the total stand.
S = site index.

The authors realised that the rate of approach towards the maximum yield may also be affected by these factors and introduced them as independent variables:

$$\text{Log}\ \left(\frac{k-y}{y}\right) = a + b_1 A + b_2 A^2 + b_3 A^3 + b_4 D + b_5 C + b_6 S$$

where A = age.

A solution of this equation indicated the necessity to adjust the estimated maximum volume yield and to recalculate the equation.

Warrack (1959) constructed variable density yield tables for *Pseudotsuga taxifolia* from equations with basal area increment and height increment as dependent variables. The relationships between the basal area increment percent and 22 different combinations of stand variables were analysed statistically. The equation producing the highest multiple correlation coefficient was:

$$Y = a + b_1 A + b_2 \frac{100}{Ba_a} + b_3 \frac{100}{D_a} + b_4 H_{50} + b_5 \frac{d}{D}$$

where A = stand age

Ba_a = basal area after thinning.

D_a = mean d.b.h. after thinning.

H_{50} = site index at reference age 50 years.

$d./D$ = ratio of mean diameter of thinned trees/mean diameter after thinning.

y = basal area increment percent.

The height increase of a stand consists of two components. The biological growth component was estimated by the equation:

$$H = a + b_1 D_a + b_2 {}^{d}/D + b_3 DI.$$

where

DI = density index.

H = height increment.

The second component, i.e. the mechanical increment due to thinnings was expressed by the equation

$$h/H = a + b_1\ {}^{d}/D + b_2\ (\tfrac{d}{D})^2 + b_3 A$$

where h/H = mean height of thinned trees/mean height of trees after thinning.

Brender (1960) recommended the following equation for *Pinus taeda*.

$$y = a + b_1(B_a) + b_2\ (B_a)^2 + b_3/AS + b_4\ (\tfrac{BA}{S})$$

where B_a = basal area
A = stand age
S = site index
y = volume growth

Marsh (1957) constructed yield tables for *P.patula* by graphical methods using the records from permanent experimental sample plots. The relationship between total volume per acre and age for different stem numbers per acre was given for site index 75 feet. The range of stem numbers was between 50 and 1200 stems per acre.

His basic assumption was that the volume increment of stands of a given age, site index and stem number is not affected by past stand densities. This assumption is incorrect, but when applied within a limited range of densities, no serious errors will result.

In 1961 Marsh recommended the construction of basal area yield curves instead of volume yield curves, for constructing yield tables. Joubert (1964) constructed yield tables for *P.patula* based on these recommendations by Marsh.

B. Procedure.

In this study the basal area increment per acre and the mean height at various ages were estimated separately from previously given regression equations. The difference between the products of basal area and form-height at two successive ages will then produce an estimate for the volume growth during this period. In other words, the volume growth was not estimated directly but indirectly thourgh the components, basal area and height growth.

The following equations were used to construct the yield tables.

(a) Basal area increment over 5 years.

$$Y = 20538 + 240.418X_1 + 2.9825X_2 \quad .....(16)$$

where X_1 = 1/age

X_2 = stand density/age.

Y = basal area increment per acre per annum.

The equation refers to an average site index, of the sample plots of 81.9 feet at 20 years.

(b) To estimate the total basal area of stands from 4 to 6 years the following equation was introduced.

$$Y = a + b_1X_1 + b_2X_2 + b_3X_3$$

where X_1 = stems per acre.

X_2 = age.

X_3 = site index.

Y = basal area.

The analysis of variance was as follows:-

Source of Variation	D.F.	M.S.
Regression Y on X_1	1	26.57**
Regression Y on X_2(X_1 fixed)	1	10.69**
Regression Y on X_3(X_1X_2 fixed)	1	1.16N.S.
Residuals	32	357.52

The variable site index (X_3) was not significant and was therefore deleted from the equation. The resultant equation was:-

$$Y = 51.39212 + 14.843409X_1 + .0609588X_2 \quad \ldots\ldots(17)$$

To obtain an estimate of the basal area per acre at 5 years a value $X_1 = 5$ was substituted into equation (17):-

$$Y = 22.82493 + .0609588X_2 \quad \ldots\ldots\ldots\ldots\ldots(18)$$

(c) The relationship between the mean d.b.h. of trees removed in a thinning and that of the total stand before thinning is necessary to estimate the volume of the trees removed. The following equation was used:-

$$Y = a + b_1 X_1 + b_2 X_1 X_2 + b_3 X_2$$

where X_1 = mean d.b.h. before thinning.

X_2 = the number of trees removed in thinning expressed as a percentage of the number of trees before thinning.

Y = mean d.b.h. of trees removed in the thinning.

The analysis of variance indicated that the % trees removed (X_2) and its interaction with the mean d.b.h. before thinning (X_1) were not significant variables. They were deleted and the equation becomes:-

$$Y = .0026 + 0.92034X_1 \quad \ldots\ldots\ldots\ldots(19)$$

The standard error of the regression coefficient is 0.01909 associated with 182 degrees of freedom.

(d) Height increment.

The increase of mean height of trees in a stand is the sum of biological height growth and a mechanical increase due to thinnings. The biological height growth is a function of site

54/.......

quality and is graphically represented by site index curves but these curves express the relationship between age and top height. The mechanical increase in the mean height of a stand is a function of the difference between mean diameter before and after thinning and of the slope of the height curve.

In order to estimate the combined effect of these factors on the current height growth of a stand the following relationships were investigated.

(i) A preliminary graphical analysis revealed a linear relationship between diameter and height in the stand.

An equation.

$$Y = a + bX \qquad \ldots\ldots\ldots\ldots(20)$$

where X = d.b.h.

Y = tree height.

was fitted in each sample plot.

(ii) The correlation coefficient between the slopes b in the equation (20) and the mean height of each sample plot was 0.412 associated with 189 degrees of freedom. This correlation is significant at the 0.01 level. The next step was to compute a linear equation:

$$Y = a + bX$$

where X = mean height.

Y = slope of regression equation (20)

The result was

$$Y = 3.3998 - .024433X \qquad \ldots\ldots\ldots\ldots(21)$$

The standard error of the regression coefficient is .001243.

(iii) The relationship between top height and mean height was investigated so as to convert top height into mean height. The regression equation was:

$$Y = 0.728 + 0.920873X \qquad \text{............(22)}$$

where X = top height.

Y = mean height.

The construction of the yield tables is illustrated for yield table A (Table 6a).

(1) The first step is to introduce a certain initial espacement and thinning regime. In table A the initial stem number is 800 per acre and the thinning regime is as follows:-

Age (years)	Stem number before thinning	Stem number after thinning
10	800	500
14	500	300
18	300	200
30	200	0

(2) The basal area at 5 years is determined from equation (18).

$$Y = 22.8249 + 0.609588X$$

For $X = 800$, $Y = 71.3$ sq. ft. per acre at 5 years.

(3) The basal area at 7 years is determined as follows:- The mean age during the growth period is 6 years, the top height 31.4 ft. (Fig.6) and the number of trees per acre 800. This indicates a S% of 23.5 at this age. The basal area increment estimated from equation (16).

$$Y = 2.9825 + 240.418X_1 = 2.9825X_2$$

For $X_1 = {}^1/6$ and $X_2 = 23.5/6 = 26.3$ sq. ft. per acre per annum. For the growth period of 2 years the basal increment is 2 x 26.3 = 52.6 sq.ft./acre. Consequently the basal area at 7 years is 123.9 sq.ft./acre.

This procedure is repeated to determine the basal area at the ages of 9 and 10 years. The basal area at 9 years is 167 sq.ft./acre and at 10 years 186.3 sq.ft./acre.

(4) At 10 years a thinning is carried out and the stem number is reduced from 800 to 500 trees per acre. The basal area before thinning is 186.3 sq.ft./acre and the mean basal per tree is thus 0.2328 sq.ft. and the corresponding diameter 6.5 inches. The mean diameter of trees removed in thinning is estimated from equation (19).

$$Y = 0.0026 + 0.92034X$$

with X = 6.5, thus

$$Y = 6.0 \text{ inches.}$$

The top height at 10 years is 54.8 ft. (Fig.6) and the corresponding mean height is estimated from equation (22).

$$Y = 0.728 + 0.920873X$$

with X = 54.8 ft. hence Y = 51.3 ft.

The slope of the d.b.h.-height curve is estimated from equation (21).

$$Y = 3.3998 - .024433X$$

with X = 51.3 ft. thus Y = 2.1469

The difference between the mean d.b.h. before thinning and that of the thinned trees is 0.5 inches and consequently the mean height of the thinned trees is 51.30 - 0.5 (2.1469) = 50.2 feet.

The basal area of the thinned trees is 0.1963 x 300 = 58.9 sq. feet per acre and the basal area after thinning is (186.3 - 58.9) = 127.3 sq. ft./acre.

The mean volume of the trees removed in thinning - estimated from the alignment chart for P.patula - (Marsh 1957) is 3.75 cu.ft. - and the volume per acre is 300 x 3.75 = 1125 cu.ft./acre. The volume per acre before thinning also estimated with the alignment chart is 3584 cu.ft. per acre.

(5) Exactly, the same procedure is followed to determine basal area, volume, mean d.b.h. and mean height of the stand before thinning, and of the thinnings at 14 and 18 years. In addition the basal area, volume mean d.b.h. and mean height at 12, 16, 20, 22, 24, 26, 28 and 30 years are computed.

(6) Finally, the resultant volume growth curve is smoothed as follows: A preliminary estimate of the stand form factor is calculated, plotted over age and a regression curve fitted graphically. The adjusted stand form factor estimated from this regression curve is used to recalculate the volume of the remaining stand and thinnings.

The economics of various thinning regimes can be best illustrated where they differ appreciably. Four theoretical regimes were constructed and incorporated into yield tables A, B, C and D. These models varied according to the age and severity of the first and subsequent thinnings and also to the initial espacements of the stands.

(a) An initial stem number of 800 per acre was introduced in yield tables A and B and a stem number of 530 in tables C and D.

(b) The age of the first thinning was taken to be

Yield	table	A	:	10 years.
"	"	B	:	10 "
"	"	C	:	8 "
"	"	D	:	6 "

(c) Throughout the rotation, yield table A represents the highest stand density and table D the lowest density while tables B and C occupy intermediate positions. At the age of 30 years, the S% was as follows:-

Yield	table	A	:	15.6
"	"	B	:	20.2
"	"	C	:	19.4
"	"	D	:	24.7

The range of densities adopted in these tables covers the range found in the thinning practice in Natal. Light thinnings similar to those expressed in tables A and B are practised in plantations managed for the production of pulpwood. Yield tables A and B differ with respect to the severity of the third thinning and to the total number of thinnings. The thinning regime on which table D is based is slightly more severe than the current thinning practice in state plantations.

It should be recalled that the analyses of the multiple regression of age, site index and stand density on basal area increment did not reveal a significant correlation between site index and increment. It was thought that the basic study material was inadequate for detecting and measuring a possible relationship between stand density and increment. Consequently yield tables were constructed for the mean site index of 81.9 feet.

The initial espacements, thinning degrees and stand densities before and after thinnings are summarised in tables 6a, b, c and d. The relationship between S% and age is shown in figure 12. The yield tables are given in tables 7a, b, c and d. The relationship between age and mean and current annual increment respectively is reproduced in figures 13, a, b, c and d.

FIG 12

THE RELATION BETWEEN AGE AND STAND DENSITY

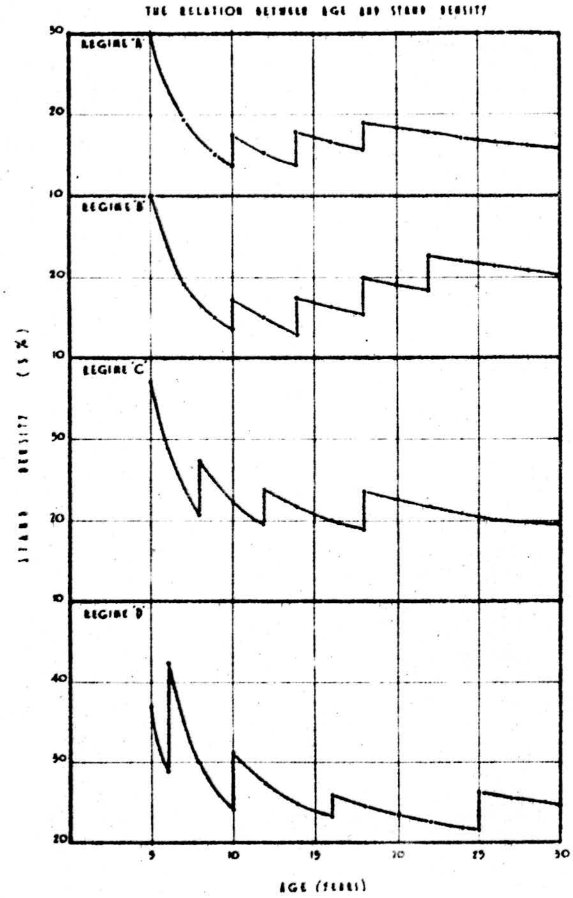


Table 6a

Thinning regime A.

Age (years)	Stems per acre		S%	
	Before thinning	After thinning	Before thinning	After thinning
0	800	-	-	-
10	800	500	13.5	17.1
14	500	300	13.6	17.6
18	300	200	15.4	18.8
30	200	0	15.6	-

Table 6b

Thinning regime B.

Age (years)	Stems per acre		S%	
	Before thinning	After thinning	Before thinning	After thinning
0	800	-	-	-
10	800	500	13.5	17.1
14	500	300	13.6	17.6
18	300	180	15.4	19.8
22	180	120	18.3	22.4
30	120	0	20.2	-

Table 6c

Thinning regime C.

Age (years)	Stems per acre		S%	
	Before thinning	After thinning	Before thinning	After thinning
0	530	-	-	-
8	530	300	20.5	27.2
12	300	200	19.4	23.7
18	200	130	18.9	23.4
30	130	0	19.4	-

Table 6d

Thinning regime D.

Age (years)	Stems per acre		S%	
	Before thinning	After thinning	Before thinning	After thinning
0	530	-	-	-
6	530	250	28.9	42.1
10	250	150	24.1	31.1
16	150	120	23.1	25.8
25	120	80	21.4	26.2
30	80	0	24.7	-

At the age of 30 years the total volume yield and the mean d.b.h. are:

Thinning regime	Total volume yield cu.ft. acre	Volume yield as a proportion of regime A	Mean d.b.h. inches
A	10970	100	13.2
B	10366	95	14.4
C	9127	83	15.9
D	8213	75	18.0

In 1957 Marsh devised the previously-described method of constructing variable density yield tables based on the records of C.C.T. experiments. This method, modified in 1961, was used by Joubert (1964) to compare the profitability of different thinning regimes. In this method the basal area per acre at the beginning and end of a growth period is derived from a basal area yield chart, expressing the relationship between age, stem number, and basal area in the C.C.T. sample plots. The difference between the basal areas at different ages produces an estimate of the current basal area increment during the growth period.

This procedure can be applied for any thinning regime, for example for thinning regimes A and D. The volume yields at 30 years, estimated from the Marsh-Joubert method are as follows:-

Thinning regime	Total volume yield	
	Cu.ft./acre	relative yield
A	11226	100
D	8146	73

These independent methods of estimation produce similar results and this indicates that these two methods are more or less equally reliable to estimate the volume yield of stands thinned according to different regimes.

Table 7a.

Yield table A — *P.patula* - Natal Midlands. — Site Index 81.9

	Remaining stand							Thinnings					Total stand						Volume increment		
Age	Stems per acre	Top ht. ft.	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ac.	Volume cu.ft./ac.	S%	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ac.	Volume cu.ft./ac.	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ac.	Volume cu.ft./ac.	S%	Total cu.ft./ac.	Current annual cu.ft./ac.	Mean annual cu.ft./ac.
0	800												800								
5	800	24.5	23.3	4.0	71.3	673	30.2						800	23.3	4.0	71.3	673	30.2	673		135
7	800	38.5	35.7	5.3	123.9	1712	19.2						800	35.7	5.3	123.9	1712	19.2	1712	520	245
9	800	50.0	46.8	6.2	167.5	2979	14.8						800	46.8	6.2	167.5	2979	14.8	2979	634	331
10	500	54.8	52.2	6.8	127.4	2489	17.1	300	50.2	6.0	58.9	1124	800	51.3	6.5	186.3	3613	13.5	3613	634	361
12	500	62.3	58.1	7.6	158.4	3474	15.0						500	58.1	7.6	158.4	3474	15.0	4598	492	383
14	300	68.7	64.9	8.7	123.3	3065	17.6	200	62.7	7.5	61.4	1451	500	64.0	8.2	184.7	4516	13.6	5640	521	403
16	300	74.0	68.9	9.4	144.5	3913	16.3						300	68.9	9.4	144.5	3913	16.3	6488	424	406
18	200	78.4	73.5	10.4	116.9	3505	18.8	100	71.7	9.2	46.2	1310	300	72.9	10.0	163.1	4815	15.4	7390	451	411
20	200	81.9	76.1	11.0	132.3	4168	18.0						200	76.1	11.0	132.3	4168	18.0	8053	333	403
22	200	85.3	79.3	11.6	146.1	4866	17.3						200	79.3	11.6	146.1	4866	17.3	8751	349	398
24	200	88.0	81.8	12.1	158.5	5526	16.8						200	81.8	12.1	158.5	5526	16.8	9411	330	392
26	200	90.2	83.6	12.5	169.7	6100	16.4						200	83.6	12.5	169.7	6100	16.4	9985	287	384
28	200	92.4	85.8	12.9	179.8	6618	16.0						200	85.8	12.9	179.8	6618	16.0	10503	259	375
30	200	94.5	87.5	13.2	189.2	7085	15.6						200	87.5	13.2	189.2	7085	15.6	10970	234	366

Table 7b.

Yield table B — *P.patula* - Natal Midlands. — Site Index 81.9

Remaining stand								Thinnings					Total stand						Volume increment		
Age	Stems per acre	Top ht. ft.	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ac.	Volume cu.ft./ac.	S%	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ac.	Volume cu.ft./ac.	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ac.	Volume cu.ft./ac.	S%	Total cu.ft./ac.	Current annual cu.ft./ac.	Mean annual cu.ft./ac.
0	800												800								
5	800	24.5	23.3	4.0	71.3	673	30.2						800	23.3	4.0	71.3	673	30.2	673		135
																				520	
7	800	38.5	35.7	5.3	123.9	1712	19.2						800	35.7	5.3	123.9	1712	19.2	1712		245
																				634	
9	800	50.0	46.8	6.2	167.5	2979	14.8						800	46.8	6.2	167.5	2979	14.8	2979		331
																				634	
10	500	54.8	52.2	6.8	127.4	2489	17.1	300	50.2	6.0	58.9	1124	800	51.3	6.5	186.3	3613	13.5	3613		361
																				492	
12	500	62.3	58.1	7.6	158.4	3474	15.0						500	58.1	7.6	158.4	3474	15.0	4598		383
																				521	
14	300	68.7	64.9	8.7	123.3	3065	17.6	200	62.7	7.5	61.4	1451	500	64.0	8.2	184.7	4516	13.6	5640		403
																				424	
16	300	74.0	68.9	9.4	144.5	3913	16.3						300	68.9	9.4	144.5	3913	16.3	6488		406
																				451	
18	180	78.4	73.7	10.5	107.7	3254	19.8	120	71.7	9.2	55.4	1561	300	72.9	10.0	163.1	4815	15.4	7390		411
																				312	
20	180	81.9	76.1	11.2	122.8	3878	19.0						180	76.1	11.2	122.8	3878	19.0	8014		401
																				333	
22	120	85.3	79.9	12.2	97.5	3287	22.4	60	77.8	10.9	38.9	1256	180	79.3	11.8	136.4	4543	18.3	8679		395
																				231	
24	120	88.0	81.8	12.9	108.6	3749	21.7						120	81.8	12.9	108.6	3749	21.7	9141		381
																				218	
26	120	90.2	83.6	13.5	118.6	4184	21.2						120	83.6	13.5	118.6	4184	21.2	9576		368
																				209	
28	120	92.4	85.8	14.0	127.7	4602	20.7						120	85.2	14.0	127.7	4602	20.7	9994		357
																				186	
30	120	94.5	87.5	14.4	136.0	4974	20.2						120	87.5	14.4	136.0	4974	20.2	10366		346

Table 7c.

Yield table C — *P.patula* - Natal Midlands. — Site Index 81.9

Remaining stand								Thinnings					Total stand						Volume increment		
Age	Stems per acre	Top ht. ft.	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft/ ac.	Volume cu.ft./ ac.	S%	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ ac.	Volume cu.ft./ ac.	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft./ ac.	Volume cu.ft./ ac.	S%	Total cu.ft./ ac.	Current annual cu.ft./ ac.	Mean annual cu.ft./ ac.
0	530												530								
5	530	24.5	23.3	4.4	55.8	530	37.1						530	23.3	4.4	55.8	530	37.1	530		106
8	300	44.3	42.5	7.0	79.1	1237	27.2	230	40.1	6.0	45.1	696	530	41.5	6.6	124.2	1933	20.5	1933	468	242
10	300	54.8	51.3	8.3	112.5	2124	22.0						300	51.3	8.3	112.5	2124	22.0	2820	444	282
12	200	62.3	58.7	9.6	101.0	2246	23.7	100	56.6	8.6	39.9	766	300	58.1	9.3	140.9	3012	19.4	3708	444	309
14	200	68.7	64.0	10.6	123.6	2942	21.5						200	64.0	10.6	123.6	2942	21.5	4404	348	315
16	200	74.0	68.9	11.5	143.4	3754	20.0						200	68.9	11.5	143.4	3754	20.0	5216	392	326
18	130	78.4	73.9	12.7	113.8	3303	23.4	70	71.3	11.1	47.0	1257	200	72.9	12.1	160.8	4560	18.9	6022	403	335
20	130	81.9	76.1	13.4	127.7	3839	22.3						130	76.1	13.4	127.7	3839	22.3	6558	268	328
22	130	85.3	79.3	14.1	140.3	4484	21.5						130	79.3	14.1	140.3	4484	21.5	7203	323	327
24	130	88.0	81.8	14.6	151.6	5022	20.8						130	81.8	14.6	151.6	5022	20.8	7741	269	323
26	130	90.2	83.6	15.1	161.8	5519	20.3						130	83.6	15.1	161.8	5519	20.3	8230	249	317
28	130	92.4	85.5	15.5	171.1	5983	19.8						130	85.5	15.5	171.1	5983	19.8	8702	232	311
30	130	94.5	87.5	15.9	179.5	6408	19.4						130	87.5	15.9	179.5	6408	19.4	9127	213	304

Table 7d.

Yield table D — *P.patula* - Natal Midlands. — Site Index 81.9

Remaining stand								Thinnings					Total stand						Volume increment		
Age	Stems per acre	Top ht. ft.	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft/ ac	Volume cu.ft./ ac.	S%	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft/ ac.	Volume cu.ft./ ac.	Stems per acre	Mean ht. ft.	Mean d.b.h. inc.	Basal area sq.ft/ ac.	Volume cu.ft./ ac.	S%	Total cu.ft./ ac.	Current annual cu.ft./ ac.	Mean annual cu.ft/ ac.
0	530												530								
5	530	24.5	23.3	4.4	55.9	530	37.1						530	23.3	4.4	55.9	530	37.1	530		106
																				399	
6	250	31.4	30.4	5.6	43.4	513	42.1	280	28.5	4.9	36.7	416	530	29.6	5.3	80.1	929	28.9	929		155
																				355	
8	250	44.3	41.5	7.6	78.4	1214	29.8						250	41.5	7.6	78.4	1214	29.8	1630		204
																				425	
10	150	54.8	52.2	9.4	72.6	1374	31.1	100	49.8	8.3	37.6	689	250	51.3	9.0	110.2	2063	24.1	2479		248
																				339	
12	150	62.3	58.1	10.9	96.5	2052	27.4						150	58.1	10.9	96.5	2062	27.4	3157		263
																				358	
14	150	68.7	64.0	12.0	117.5	2767	24.8						150	64.0	12.0	117.5	2767	24.8	3872		276
																				369	
16	120	74.0	69.2	13.1	112.8	2930	25.8	30	67.2	11.9	23.2	574	150	68.9	12.9	136.0	3505	23.1	4609		288
																				334	
18	120	78.4	72.9	14.0	128.2	3598	24.5						120	72.9	14.0	128.2	3598	24.5	5277		293
																				334	
20	120	81.9	76.1	14.7	141.9	4265	23.3						120	76.1	14.7	141.9	4265	23.3	5944		297
																				313	
22	120	85.3	79.3	15.4	154.2	4891	22.4						120	79.3	15.4	154.2	4891	22.4	6570		299
																				273	
24	120	88.0	81.8	15.9	165.3	5436	21.7						120	81.8	15.9	165.3	5436	21.7	7115		296
																				250	
25	80	89.3	84.0	16.8	122.6	4145	26.2	40	81.6	14.8	47.8	1541	120	83.0	16.1	170.4	5686	21.4	7365		295
																				161	
27	80	91.1	84.6	17.3	131.0	4466	25.6						80	84.6	17.3	131.0	4466	25.6	7686		285
																				181	
29	80	93.3	86.6	17.8	138.7	4828	25.0						80	86.6	17.8	138.7	4828	25.0	8048		278
																				165	
30	80	94.5	87.5	18.0	142.3	4993	24.7						80	87.5	18.0	142.3	4993	24.7	8213		274

FIG. 13a.

THE RELATIONSHIP BETWEEN MEAN.
AND CURRENT ANNUAL INCREMENT AND AGE
THINNING REGIME A.

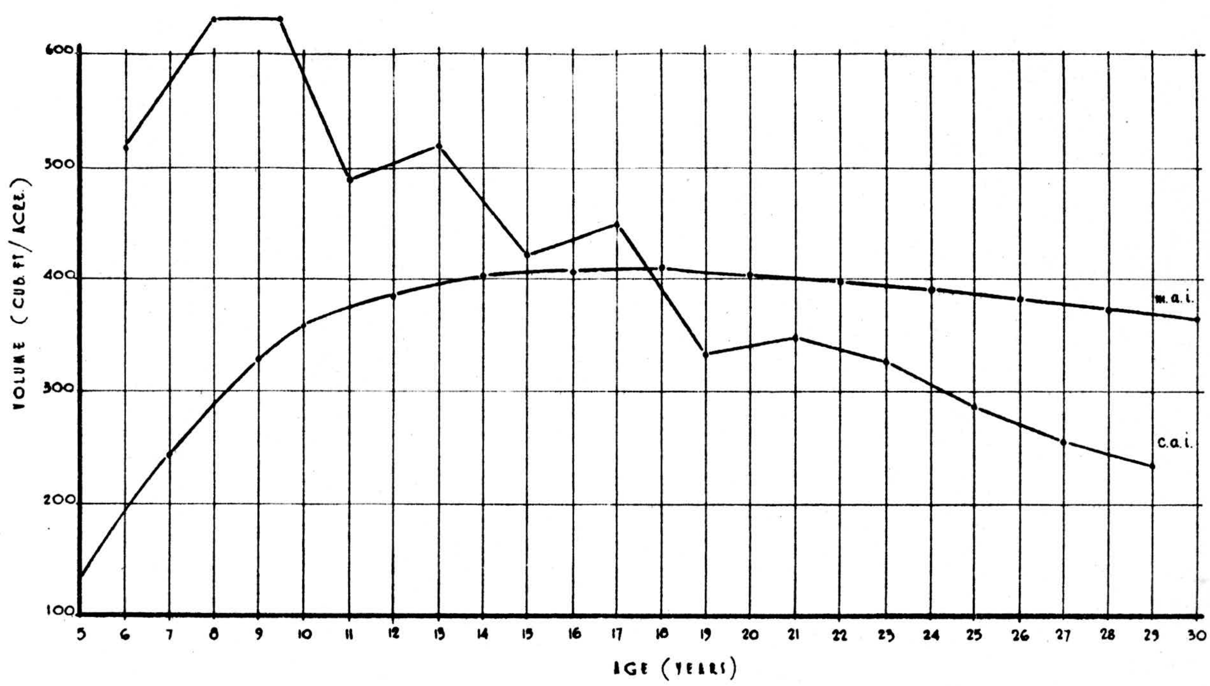


FIG 13b.

THE RELATIONSHIP BETWEEN MEAN AND CURRENT
ANNUAL INCREMENT AND AGE THINNING
REGIME B

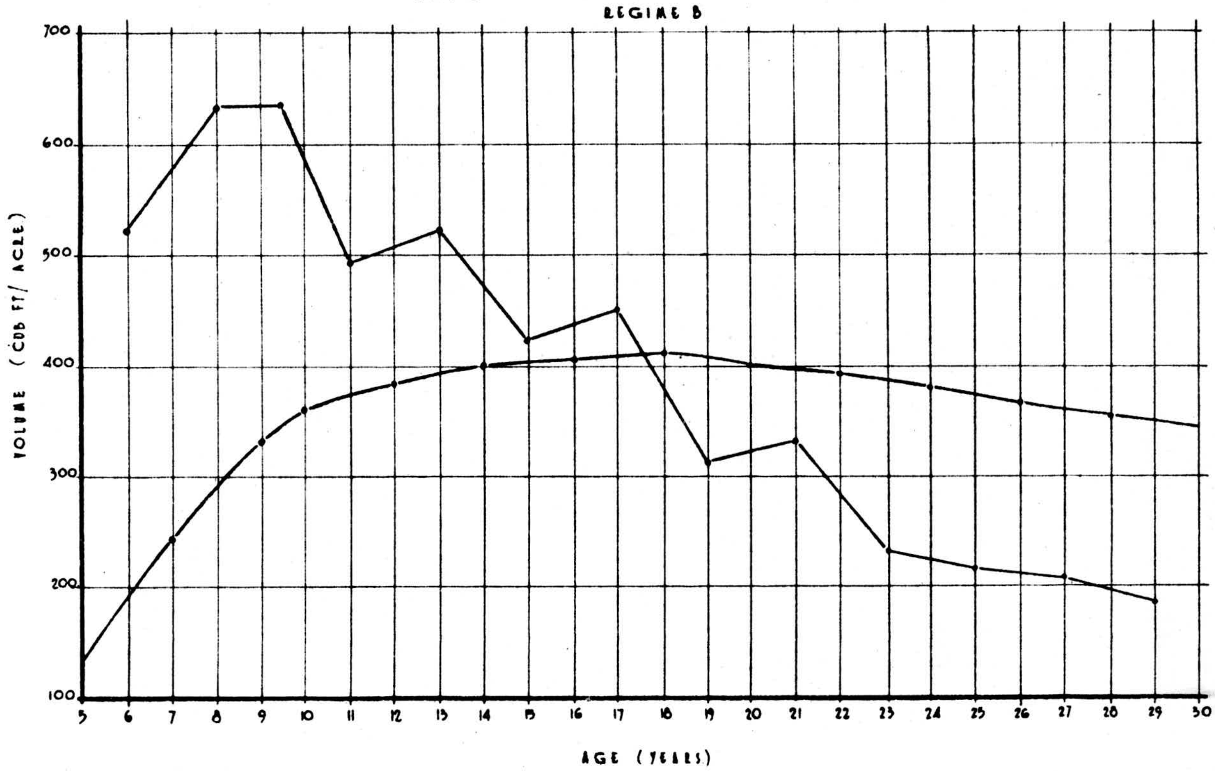


FIG 13c.

THE RELATIONSHIP BETWEEN MEAN AND CURRENT ANNUAL INCREMENT AND AGE. THINNING REGIME C.

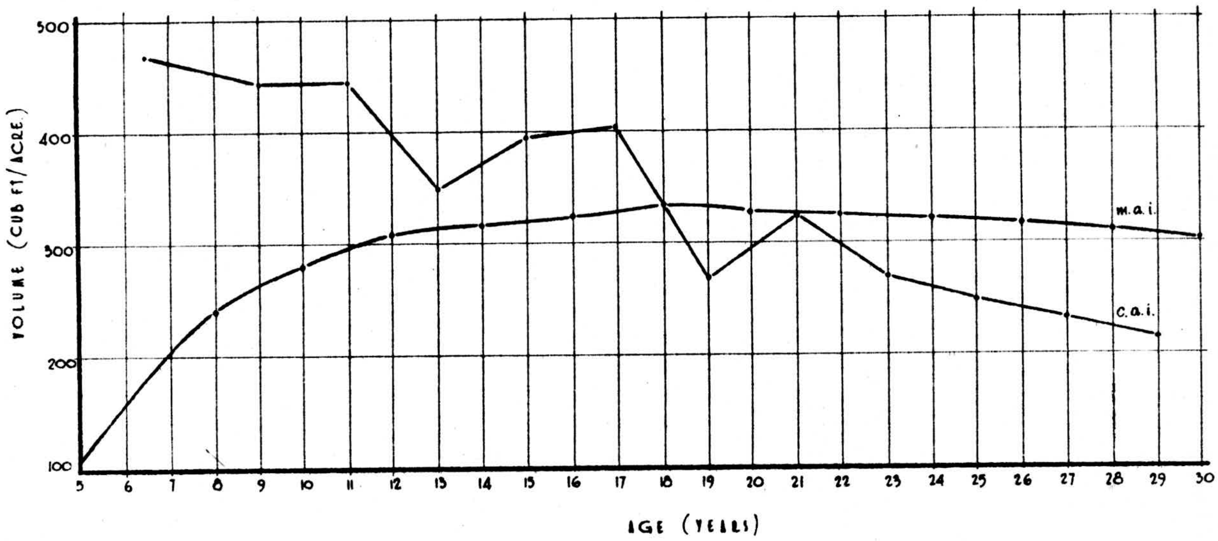
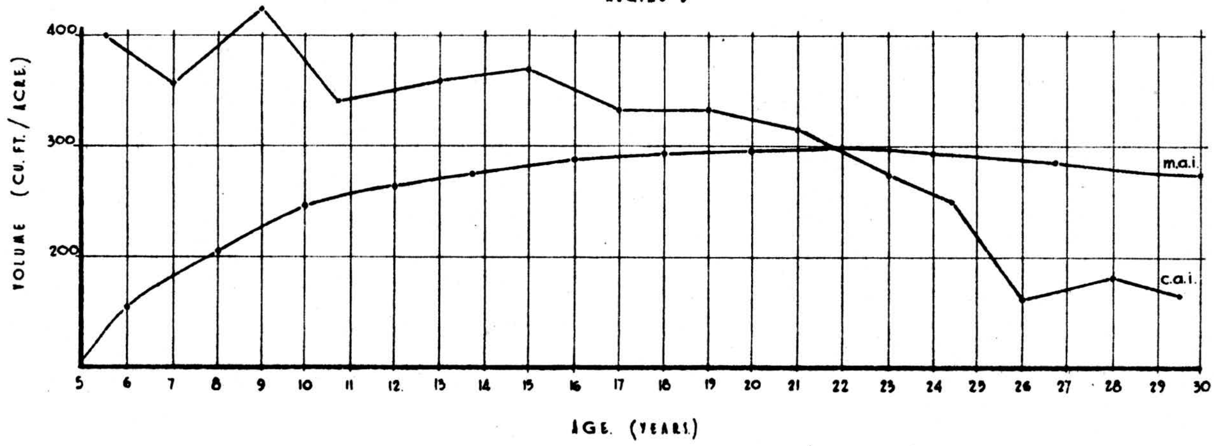


FIG 13d.

THE RELATIONSHIPS BETWEEN MEAN AND CURRENT ANNUAL INCREMENT AND AGE. THINNING. REGIME D



5. Diameter Distribution.

A knowledge of the range of diameter classes occurring in stands of <u>P.patula</u> is necessary for the economic analysis of the growing of sawtimber because the price of sawtimber is closely related to the size of the sawlogs. Various methods may be applied to describe the diameter distribution curves graphically or mathematically.

In this study the relative accuracy of 3 equations was investigated, i.e.:-

(a) The equation of the normal distribution.

(b) Charlier type A equation.

(c) Pearl - Reed population growth equation.

(a) The normal distribution.

The equation is:

$$Y = \frac{N}{S\sqrt{2\pi}}\, e^{-\frac{(x-\bar{x})^2}{2S^2}}$$

where Y = frequency of a given diameter class.

x = diameter class.

$\bar{x}$ = mean diameter.

S = standard deviation.

e = base of the Naperian system of logarithms.

In order to fit the normal distribution, the mean diameter and the standard deviation must be known. Tables for the cumulative distribution are then used to compute the frequency in each diameter class.

The fitting of the normal distribution is a simple method to describe the diameter distribution. In unthinned stands the results are normally satisfactory but thinnings may cause skewness of the distribution.

(b) Charlier's type A equation.

As stands become older so the distribution of diameters becomes increasingly skewed because dominated and suppressed trees with diameters below the mean diameter of the stand are normally removed in low thinnings. In addition, the frequency near the average of the distribution and in the tails, may be too high or too low. Consequently, it may be necessary to modify the equation of the normal distribution. The type A equation of Charlier involves the introduction of the function of normal distribution and the derivatives of this function in order to correct for deviations from normality. The equation is as follows:-

$$Y = N\ (f(x) + \beta_3 f^{III}(x) + \beta_4 f^{IV} x)$$

where Y = frequency of a given d.b.h. class.

N = total frequency.

$f(x)$ = equation of the normal distribution.

$f^{III}(x)$ & $f^{IV}(x)$

= third and fourth derivatives of this function.

β_3 = coefficient of asymmetry.

β_4 = coefficient of excess.

The values of β_3 and β_4 are as follows:-

$$\beta_3 = {}^1/_6\ \frac{M_3}{S_3}$$

$$\beta_4 = {}^1/_{24}\ (\frac{M_4}{S_4} - 3)$$

where M_3 = third moment of the normal distribution.

$= {}^1/_N \sum f_i (x_i - \bar{x})^3$

f_i = frequency of i th diameter class.

M_4 = fourth moment of the normal distribution.

$= {}^1/_N \sum f_i (x_i - \bar{x})^4$

The final distribution curve, resulting from the application of Charlier type A equation, thus consists of three components:-

$f(x)$ expressing the normal equation.

$\beta_3 f^{III}(x)$ to adjust for asymmetry and

$\beta_4 f^{IV}(x)$ to adjust for excess.

The coefficients of asymmetry and excess for the 8 sample plots investigated in this study are shown in table 8.

Table 8.

Diameter Distribution of 8 Sample Plots.

Plot No	Mean d.b.h.	Standard deviation	Coefficient of asymmetry	Coefficient of excess
1	5.4	0.9286	-.0080	+.0088
2	4.9	0.8808	+.0143	+.0027
3	13.9	1.7220	-.0987	-.0089
4	12.2	1.639	-.0939	+.0019
5	6.7	0.9820	+.0316	-.0945
6	6.4	0.800	+.0320	+.0047
7	12.3	1.493	-.0125	-.0043
8	5.6	.7746	+.0166	+.0027

(c) The Pearl - Reed population growth curve is of the form:

$$Y = C + \frac{k}{1 + me^{f(x)}} \qquad \text{..........(23)}$$

where Y = the population of a given area.

C = the lower and k + c the upper limiting asymptote of the population.

m = an arbitrary constant.

e = the base of the Naperian logarithm.

x = the time variable.

f(x) = the function of the form

$$b_1X + b_2X^2 + b_3X^3 + \text{.........} b_nX^n$$

This equation may be applied to the diameter distribution of even-aged forests when y is defined as the relative cumulative frequency and X as the corresponding diameter class. In this case the lower limit c = o and the upper limit k + c = 100. To simplify calculations the equation (23) may be transformed into the following form:-

$$\text{Log } \frac{(100-Y)}{Y} = a + b_1 \ x + b_2 \ x^2 \ldots\ldots\ldots b_n X^n$$

where Y = cumulative frequency of d.b.h. to the upper limit of a d.b.h. class.

x = corresponding upper limit.

$a, b_1, b_2, \ldots\ldots\ldots n_b$ constants of the equation.

If a polynomial of the third degree is fitted, the equation is:-

$$\text{Log } \frac{(100-Y)}{100} = a + b_1x + b_2x^2 + b_3x^3 \ldots\ldots\ldots)24)$$

The values of a, b_1, b_2 and b_3, computed by the method of least squares are given in table 9.

Table 9.

Value of Constants a_1, b_1, b_2, b_3, of Equation (24)

Plot No	Constants			
	a	b_1	b_2	b_3
1	1.7112	-.4954	+.04553	-.0009793
2	1.6312	-.4784	+.04678	-.0010113
3	8.6565	-1.5713	+.09798	-.0020419
4	9.3254	-1.9041	+1.32930	-.0030905
5	4.2620	-1.4597	+.17998	-.0075607
6	.9531	+.1437	-.07193	+.0053033
7	4.5315	-.7863	+.04789	-.0009747
8	.7323	+.2704	-.10897	+.0085535

The goodness of fit of the three equations for describing diameter distribution was examined by means of the chi-square test.

The chi-square criterion is:-

$$\chi^2 = \frac{\sum_{i=1}^{k}(F_{A_i} - F_{e_i})^2}{F_{e_i}}$$

distributed with (k-1) degrees of freedom.

where F_{A_i} = actual frequency in i th diameter class.

F_{A_i} = expected frequency in this class.

K = number of classes.

The values of χ^2 computed in this investigation for the 8 sample plots according to the 3 equations are given in table 10.

<u>Table 10.</u>

Values of Chi-square

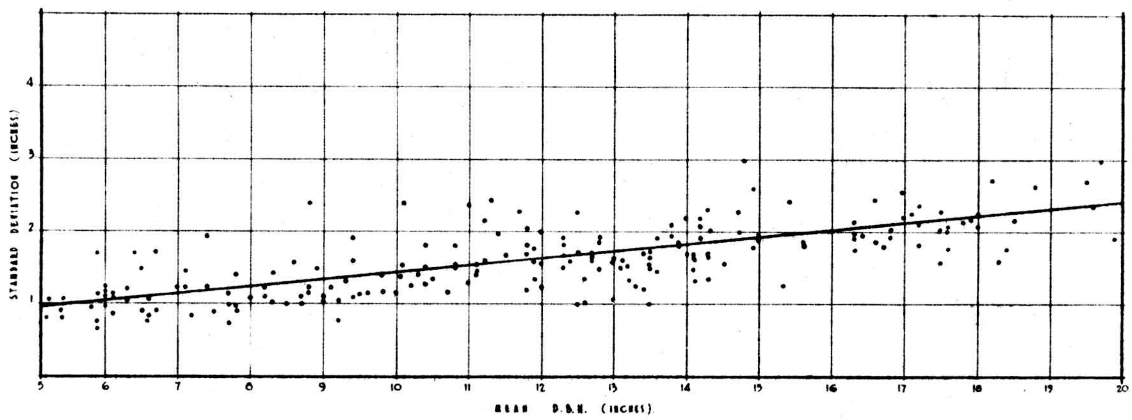
Plot No	Chi-square			Degrees of freedom
	Equation of the normal distribution	Charlier Type A	Pearl-Reed equation	
1	0.68	0.68	0.22	5
2	1.62	1.74	5.57	5
3	11.91	11.84	10.90	8
4	14.05	9.22	13.45	8
5	3.97	2.97	3.90	5
6	39.65	34.03	4.73	5
7	5.41	5.07	7.07	7
8	6.29	6.29	7.49	5

In all sample plots with the exception of plot No.6 the discrepancy between actual and expected frequency is non-significant. This applies to each of the distribution functions tested in this study. For the sake of simplicity the normal curve was selected to describe the diameter distribution in stands of P.patula.

The next step was to investigate the relationship between mean d.b.h. and standard deviation of the sample plots (fig. 14).

FIG 14.

THE RELATIONSHIP BETWEEN THE STANDARD DEVIATION AND THE MEAN DIAMETER



The following regression equation was introduced:-

$$Y = a + bX$$

where X = mean d.b.h. of the sample plot.

Y = standard deviation.

The equation, based on 200 observations is

$$Y = 0.5067 + 0.09362X \qquad \text{..............(25)}$$

The standard error of the regression coefficient b is 0.02799.

This distribution of diameters of any stand with known mean d.b.h. can now be computed with tables for cumulative frequencies of the normal distribution.

The computations may be carried out for each age class in yield tables A, B, C and D. The results are presented in tables 11a, b, c and d.

<u>Table 11a.</u>

Diameter Distribution.
(Yield table A)

Age	Stems per acre	Mean D.b.h. (inc.)	D.b.h.(inches)																		
			4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Number of trees per acre																		
10	800	6.5	32	112	256	256	112	32	-	-	-	-	-	-	-	-	-	-	-	-	-
12	500	7.6	-	20	75	140	150	85	30	-	-	-	-	-	-	-	-	-	-	-	-
14	500	8.2	-	-	45	100	150	130	55	20	-	-	-	-	-	-	-	-	-	-	-
16	300	9.4	-	-	-	27	51	81	78	42	21	-	-	-	-	-	-	-	-	-	-
18	300	10.0	-	-	-	12	33	63	84	63	33	12	-	-	-	-	-	-	-	-	-
20	200	11.0	-	-	-	-	10	22	42	52	42	22	10	-	-	-	-	-	-	-	-
22	200	11.6	-	-	-	-	6	12	32	46	46	34	18	6	-	-	-	-	-	-	-
24	200	12.1	-	-	-	-	-	12	20	40	46	42	26	14	-	-	-	-	-	-	-
26	200	12.5	-	-	-	-	-	8	16	30	46	46	30	16	8	-	-	-	-	-	-
28	200	12.9	-	-	-	-	-	-	16	26	40	46	38	22	12	-	-	-	-	-	-
30	200	13.2	-	-	-	-	-	-	12	20	38	44	40	28	12	6	-	-	-	-	-

Table 11b.

Diameter Distribution.
(Yield table B)

Age	Stems per acre	Mean D.b.h. (inc.)	D.b.h.(inches)																		
			4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Number of trees per acre																		
10	800	6.5	32	112	256	256	112	32	-	-	-	-	-	-	-	-	-	-	-	-	-
12	500	7.6	-	20	75	140	150	85	30	-	-	-	-	-	-	-	-	-	-	-	-
14	500	8.2	-	-	45	100	150	130	55	20	-	-	-	-	-	-	-	-	-	-	-
16	300	9.4	-	-	-	27	51	81	78	42	21	-	-	-	-	-	-	-	-	-	-
18	300	10.0	-	-	-	12	33	63	84	63	33	12	-	-	-	-	-	-	-	-	-
20	180	11.2	-	-	-	-	7	18	34	45	40	23	13	-	-	-	-	-	-	-	-
22	180	11.8	-	-	-	-	-	14	24	38	45	34	16	9	-	-	-	-	-	-	-
24	120	12.9	-	-	-	-	-	-	10	15	24	28	22	14	7	-	-	-	-	-	-
26	120	13.5	-	-	-	-	-	-	6	10	19	25	25	19	10	6	-	-	-	-	-
28	120	14.0	-	-	-	-	-	-	4	7	14	22	26	22	14	7	4	-	-	-	-
30	120	14.4	-	-	-	-	-	-	-	6	12	19	25	24	18	10	6	-	-	-	-

Table 11c.

Diameter Distribution.
(Yield table C)

Age	Stems per acre	Mean D.b.h. (inc.)	D.b.h.(inches)																	
			4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Number of trees per acre																		
10	300	8.3	-	-	24	54	90	81	39	12	-	-	-	-	-	-	-	-	-	-	-
12	300	9.3	-	-	-	30	54	84	75	45	12	-	-	-	-	-	-	-	-	-	-
14	200	10.6	-	-	-	-	16	30	48	52	34	14	6	-	-	-	-	-	-	-	-
16	200	11.5	-	-	-	-	6	14	32	48	48	32	14	6	-	-	-	-	-	-	-
18	200	12.1	-	-	-	-	-	12	20	40	46	42	26	14	-	-	-	-	-	-	-
20	130	13.4	-	-	-	-	-	-	6	12	21	29	27	20	10	5	-	-	-	-	-
22	130	14.1	-	-	-	-	-	-	3	8	14	23	28	25	16	13	-	-	-	-	-
24	130	14.6	-	-	-	-	-	-	-	6	10	20	26	26	22	12	8	-	-	-	-
26	130	15.1	-	-	-	-	-	-	-	4	8	14	23	26	25	16	9	5	-	-	-
28	130	15.5	-	-	-	-	-	-	-	-	8	12	19	26	26	19	12	8	-	-	-
30	130	15.9	-	-	-	-	-	-	-	-	5	9	17	23	26	22	15	8	5	-	-

Table 11d.

Diameter Distribution.
(Yield table D)

Age	Stems per acre	Mean D.b.h. (inc.)	D.b.h.(inches)																		
			4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
			Number of trees per acre																		
10	250	9.0	-	-	-	33	57	70	57	33	-	-	-	-	-	-	-	-	-	-	-
12	150	10.9	-	-	-	-	11	19	24	41	26	21	8	-	-	-	-	-	-	-	-
14	150	12.0	-	-	-	-	-	9	18	30	36	30	18	9	-	-	-	-	-	-	-
16	150	12.9	-	-	-	-	-	-	12	20	30	34	29	16	9	-	-	-	-	-	-
18	120	14.0	-	-	-	-	-	-	4	7	14	22	26	22	14	7	4	-	-	-	-
20	120	14.7	-	-	-	-	-	-	-	4	10	17	28	23	18	12	8	-	-	-	-
22	120	15.4	-	-	-	-	-	-	-	-	8	12	18	24	23	18	10	7	-	-	-
25	120	16.1	-	-	-	-	-	-	-	-	5	7	13	20	24	22	14	8	6	-	-
27	80	17.3	-	-	-	-	-	-	-	-	-	3	5	8	12	15	14	11	7	5	-
30	80	18.0	-	-	-	-	-	-	-	-	-	-	4	6	10	13	14	13	10	6	4

6. Log class volume distribution.

The generally accepted log class classification is given in table 12.

Table 12.

Log class classification.

Log class	Log length (ft.)	Diameter ub.at thin end (inc.)
b_1	6 - 11	7.0 - 10.5
b_2	12 +	7.0 - 10.5
C_1	6 - 11	10.6 - 13.5
C_2	12 +	10.6 - 13.5
d_1	6 - 11	13.6 +
d_2	12 +	13.6 +
P : Pulpwood. Stem sections thicker than 3 inches ub. and longer than 4 ft., but not classified as sawtimber because of defects.		
P_b : Pulpwood with a diameter 3 - 6 inches.		
P_a : Pulpwood with a diameter 6 - 7 inches.		

Schumacher and Deetlefs (1963) used the probit transformation to assess the volume of the log classes within the individual trees. Diameter at breast height and tree height were introduced as the independent variables and the following equation was fitted for each log class.

$$Y = a + b_1\left(\frac{10}{D}\right) + b_2\left(\frac{10}{D}\right)^2 + b_3\left(\frac{H}{100}\right) + b_4\left(\frac{H}{10D}\right)$$

where D = d.b.h.

H = height.

Y = probit of the cumulative percentage of the log class volume.

They expressed the cumulative log classes volume as a percentage of the merchantable volume but Joubert (1964) modified this procedure and constructed tables with the log class volume expressed as a percentage of the total volume.

An investigation into the log class volume distribution on a per acre basis is of major importance for the economic analysis of timber growing with different objectives of management. This distribution must be examined for different ages and thinning regimes.

(a) Stand before thinning.

Volume yield tables giving stand parameters for thinning regimes A, B, C and D were presented in tables 7a, b, c and d and the diameter distribution before thinning was reproduced in tables 11a, b, c and d.

The procedure for constructing tables which express the distribution of the stand volume before thinning over diameter and log classes will be illustrated for thinning regime A and is as follows:-

(i) The age, mean d.b.h., mean height, number of stems per acre and volume per acre of the stand before thinning were obtained from the yield table.

(ii) At the age of 10 years the mean d.b.h. is 6.5 inches and the diameter distribution as follows:-

D.b.h.(inches)	Stems per acre.
4	32
5	112
6	256
7	256
8	112
9	32

(iii) At 10 years and a mean height of 51.3 feet the average slope at the height curve estimated from equation (21) was 2.15. The diameters and corresponding heights were as follows:-

D.b.h.(inches).	Height (ft.).
4	45.9
5	48.1
6	50.2
7	52.4
8	54.5
9	56.7

(iv) The alinement chart for P.patula was used to estimate the volume per tree in each d.b.h. class. This volume was then multiplied by the number of trees in each class to determine the total volume in each d.b.h. class.

(v) This procedure was repeated in each age for the four thinning regimes. The volume distribution over diameter class is given in tables 13a, b, c and d.

(vi) The volumes of the log classes in each d.b.h. class were estimated from the tables constructed by Joubert (1964). The timber under 7 inches diameter was classified as pulpwood without further differentiation. Because of the present pulpwood classification, given in table 12, it was necessary to modify Joubert's method accordingly. These log class volumes were then summed and the results are given in tables 14a, b, c and d.

(b) Trees removed in thinning.

Joubert (1964) constructed a table giving the relationship between mean diameter of the trees removed in thinnings and the log class volume distribution on a per acre basis. This table was used to determine the volume distribution over log classes of the thinnings in yield tables A, B and C.

The results are given in tables 14a, b, c and d.

Table 13a.

Volume Distribution by Diameter Classes.
(Yield table A)

Age	Mean ht.	Mean d.b.h.	Total Volume.	D.b.h.(inches) 4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
10	51.3	6.5	3613	49	281	944	1335	771	233	-	-	-	-	-	-	-	-	-	-	-	-	-
12	58.1	7.6	3474	-	55	306	774	1135	825	379	-	-	-	-	-	-	-	-	-	-	-	-
14	64.0	8.2	4516	-	-	202	607	1222	1403	747	335	-	-	-	-	-	-	-	-	-	-	-
16	68.9	9.4	3913	-	-	-	181	449	946	1129	745	463	-	-	-	-	-	-	-	-	-	-
18	72.9	10.0	4815	-	-	-	87	318	789	1303	1213	770	335	-	-	-	-	-	-	-	-	-
20	76.1	11.0	4168	-	-	-	-	101	288	689	1047	1048	648	347	-	-	-	-	-	-	-	-
22	79.3	11.6	4866	-	-	-	-	59	165	550	962	1197	1042	640	251	-	-	-	-	-	-	-
24	81.8	12.1	5526	-	-	-	-	-	171	357	886	1230	1334	955	593	-	-	-	-	-	-	-
26	83.6	12.5	6100	-	-	-	-	-	119	294	686	1259	1510	1134	700	398	-	-	-	-	-	-
28	85.8	12.9	6618	-	-	-	-	-	-	304	617	1120	1525	1466	980	606	-	-	-	-	-	-
30	87.5	13.2	7085	-	-	-	-	-	-	236	478	1084	1472	1575	1268	612	360	-	-	-	-	-

Table 13b.

Volume Distribution by Diameter Classes.
(Yield table B)

Age	Mean ht.	Mean d.b.h.	Total Volume.	D.b.h.(inches) 4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
10	51.3	6.5	3613	49	281	944	1335	771	233	-	-	-	-	-	-	-	-	-	-	-	-	-
12	58.1	7.6	3474	-	55	306	774	1135	825	379	-	-	-	-	-	-	-	-	-	-	-	-
14	64.0	8.2	4516	-	-	202	607	1222	1403	747	335	-	-	-	-	-	-	-	-	-	-	-
16	68.9	9.4	3913	-	-	-	181	449	946	1129	745	463	-	-	-	-	-	-	-	-	-	-
18	72.9	10.0	4815	-	-	-	87	378	789	1303	1213	770	335	-	-	-	-	-	-	-	-	-
20	76.1	11.2	3878	-	-	-	-	70	234	553	909	987	675	449	-	-	-	-	-	-	-	-
22	79.3	11.8	4543	-	-	-	-	-	193	412	801	1169	1033	565	370	-	-	-	-	-	-	-
24	81.8	12.9	3749	-	-	-	-	-	-	175	326	638	873	805	596	336	-	-	-	-	-	-
26	83.6	13.5	4184	-	-	-	-	-	-	108	222	509	798	918	810	486	333	-	-	-	-	-
28	85.8	14.0	4602	-	-	-	-	-	-	72	158	384	713	973	952	688	399	263	-	-	-	-
30	87.5	14.4	4974	-	-	-	-	-	-	-	141	333	616	951	1054	901	576	401	-	-	-	-

Table 13c.

Volume Distribution by Diameter Classes.
(Yield table C)

Age	Mean ht.	Mean d.b.h.	Total Volume.	D.b.h.(inches) 4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
10	51.3	8.3	2124	-	-	79	251	551	668	417	158	-	-	-	-	-	-	-	-	-	-	-
12	58.1	9.3	3012	-	-	-	156	374	760	874	640	208	-	-	-	-	-	-	-	-	-	-
14	64.0	10.6	2942	-	-	-	-	120	294	603	806	634	321	164	-	-	-	-	-	-	-	-
16	68.9	11.5	3754	-	-	-	-	55	148	424	780	953	786	404	204	-	-	-	-	-	-	-
18	72.9	12.1	4560	-	-	-	-	-	139	288	692	998	1131	800	512	-	-	-	-	-	-	-
20	76.1	13.4	3839	-	-	-	-	-	86	212	458	765	867	757	442	252	-	-	-	-	-	-
22	79.3	14.1	4484	-	-	-	-	-	-	48	155	331	649	920	978	712	691	-	-	-	-	-
24	81.8	14.6	5022	-	-	-	-	-	-	-	123	253	594	886	1038	1006	630	492	-	-	-	-
26	83.6	15.1	5519	-	-	-	-	-	-	-	84	205	430	809	1064	1170	843	562	352	-	-	-
28	85.8	15.5	5983	-	-	-	-	-	-	-	-	213	374	686	1091	1243	1038	761	577	-	-	-
30	87.5	15.9	6408	-	-	-	-	-	-	-	-	134	286	632	968	1259	1226	904	587	412	-	-

Table 13d.

Volume Distribution by Diameter Classes.
(Yield table D)

Age	Mean ht.	Mean d.b.h.	Total Volume.	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
				D.b.h. (inches)																		
10	51.3	9.0	2063	-	-	-	149	342	562	586	424	-	-	-	-	-	-	-	-	-	-	-
12	58.1	10.9	2052	-	-	-	-	69	159	256	546	420	409	193	-	-	-	-	-	-	-	-
14	64.0	12.0	2767	-	-	-	-	-	86	210	439	633	643	474	282	-	-	-	-	-	-	-
16	68.9	12.9	3504	-	-	-	-	-	-	150	309	569	799	812	526	339	-	-	-	-	-	-
18	72.9	14.0	3598	-	-	-	-	-	-	55	115	285	555	756	747	557	319	209	-	-	-	-
20	76.1	14.7	4265	-	-	-	-	-	-	-	72	219	457	872	848	770	580	447	-	-	-	-
22	79.3	15.4	4891	-	-	-	-	-	-	-	-	179	327	577	862	983	874	566	523	-	-	-
25	83.0	16.1	5686	-	-	-	-	-	-	-	-	124	201	445	812	1096	1136	847	557	468	-	-
27	84.6	17.3	4466	-	-	-	-	-	-	-	-	-	87	171	318	540	779	847	754	458	512	-
30	87.5	18.0	4993	-	-	-	-	-	-	-	-	-	-	135	242	451	673	859	928	786	524	395

Table 14a.

Log Class Volume Distribution.
(Yield table A)

	Stand before thinning												Thinnings											
Age	Spa	Mean d.b.h. inc.	Mean ht. ft.	Volume	P_b	P_a	b_1	b_2	c_1	c_2	d_1	d_2	Spa	Mean d.b.h. inc.	Mean ht. ft.	Volume	P_b	P_a	b_1	b_2	c_1	c_2	d_1	d_2
				Cu.ft. per acre												Cu.ft. per acre								
10	800	6.5	51.3	3613	2143	875	312	-	-	-	-	-	300	6.0	50.2	1124	983	-	-	-	-	-	-	-
12	500	7.6	58.1	3474	1231	1047	835	131	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	500	8.2	64.0	4516	1375	1110	1375	539	5	-	-	-	200	7.5	62.7	1451	543	463	307	81	-	-	-	-
16	300	9.4	68.9	3913	731	723	1372	946	25	19	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	300	10.0	72.9	4815	741	761	1630	1422	70	65	-	-	100	9.2	71.7	1310	307	412	447	176	-	-	-	-
20	200	11.0	76.1	4168	484	492	1254	1484	124	234	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	200	11.6	79.3	4866	489	486	1314	1778	193	504	-	-	-	-	-	-	-	-	-	-	-	-	-	-
24	200	12.1	81.8	5526	506	495	1344	1988	262	805	15	-	-	-	-	-	-	-	-	-	-	-	-	-
26	200	12.5	83.6	6100	519	497	1361	2049	336	1159	34	40	-	-	-	-	-	-	-	-	-	-	-	-
28	200	12.9	85.8	6618	517	485	1388	2068	415	1536	51	60	-	-	-	-	-	-	-	-	-	-	-	-
30	200	13.2	87.5	7085	520	478	1393	2047	481	1890	89	136	-	-	-	-	-	-	-	-	-	-	-	-

86/......

Table 14b.

Log Class Volume Distribution.
(Yield table B)

Stand before thinning													Thinnings											
Age	Spa	Mean d.b.h. inc.	Mean ht. ft.	Vol-ume.	Log Class P_b	P_a	b_1	b_2	c_1	c_2	d_1	d_2	Spa	Mean d.b.h. inc.	Mean ht. ft.	Vol-ume.	Log Class P_b	P_a	b_1	b_2	c_1	c_2	d_1	d_2
				Cu.ft. per acre												Cu.ft. per acre								
10	800	6.5	51.3	3613	2143	875	312	-	-	-	-	-	300	6.0	50.2	1124	983	-	-	-	-	-	-	-
12	500	7.6	58.1	3474	1231	1047	835	131	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	500	8.2	64.0	4516	1375	1110	1375	539	5	-	-	-	200	7.5	62.7	1451	543	463	307	81	-	-	-	-
16	300	9.4	68.9	3913	731	723	1372	946	25	19	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	300	10.0	72.9	4815	741	761	1630	1422	70	65	-	-	120	9.2	71.7	1561	367	412	533	210	-	-	-	-
20	180	11.2	76.1	3878	431	437	1121	1403	133	263	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	180	11.8	79.3	4543	434	440	1183	1667	198	520	-	-	60	10.9	77.8	1256	153	159	450	424	23	33	-	-
24	120	12.9	81.8	3749	313	292	749	1283	224	737	29	32	-	-	-	-	-	-	-	-	-	-	-	-
26	120	13.5	83.6	4184	317	286	723	1201	270	1142	69	112	-	-	-	-	-	-	-	-	-	-	-	-
28	120	14.0	85.8	4602	321	285	718	1132	326	1438	115	227	-	-	-	-	-	-	-	-	-	-	-	-
30	120	14.4	87.5	4974	326	280	709	1080	377	1689	156	323	-	-	-	-	-	-	-	-	-	-	-	-

Table 14c.

Log Class Volume Distribution.

(Yield table C)

Stand before thinning													Thinnings											
Age	Spa	Mean d.b.h. inc.	Mean ht. ft.	Vol-ume.	P_b	P_a	b_1	b_2	c_1	c_2	d_1	d_2	Spa	Mean d.b.h.	Mean ht. ft.	Vol-ume.	P_b	P_a	b_1	b_2	c_1	c_2	d_1	d_2
				Cu.ft. per acre												Cu.ft. per acre								
10	300	8.3	51.3	2124	658	559	573	274	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	300	9.3	58.1	3012	631	634	860	699	18	-	-	-	100	8.6	56.6	766	260	270	162	43	-	-	-	-
14	300	10.6	64.0	2942	426	433	842	1027	53	120	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	200	11.5	68.9	3754	442	431	923	1447	120	349	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	200	12.1	72.9	4560	550	445	1015	1716	194	678	-	-	70	11.1	71.3	1257	178	171	412	433	17	28	-	-
20	130	13.4	76.1	3839	307	281	628	1187	235	1054	64	56	-	-	-	-	-	-	-	-	-	-	-	-
22	130	14.1	79.3	4484	330	289	643	1186	308	1404	111	178	-	-	-	-	-	-	-	-	-	-	-	-
24	130	14.6	81.8	5022	340	296	621	1133	363	1678	172	362	-	-	-	-	-	-	-	-	-	-	-	-
26	130	15.1	83.6	5519	365	308	631	1057	413	1902	247	579	-	-	-	-	-	-	-	-	-	-	-	-
28	130	15.5	85.8	5983	364	292	616	936	453	2060	295	774	-	-	-	-	-	-	-	-	-	-	-	-
30	130	15.9	87.6	6408	396	310	600	913	501	2217	364	1081	-	-	-	-	-	-	-	-	-	-	-	-

...../88

Table 14d.

Log Class Volume Distribution.
(Yield table D)

Stand before thinning													Thinnings											
Age	Spa	Mean d.b.h. inc.	Mean ht. ft.	Volume.	P_b	P_a	Log Class b_1	b_2	c_1	c_2	d_1	d_2	Spa	Mean d.b.h. inc.	Mean ht. ft.	Volume.	P_b	P_a	Log Class b_1	b_2	c_1	c_2	d_1	d_2
				Cu.ft. per acre												Cu.ft. per acre								
10	250	9.0	51.3	2063	493	496	608	413	-	-	-	-	100	8.3	49.8	689	247	247	129	41	-	-	-	-
12	150	10.9	58.1	2052	288	292	506	586	42	111	-	-	-	-	-	-	-	-	-	-		-	-	-
14	150	12.0	64.0	2767	302	305	582	1086	100	357	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	150	12.9	68.9	3504	349	304	609	1297	173	703	23	15	30	11.9	67.2	574	66	62	150	234	17	38	-	-
18	120	14.0	72.9	3598	289	254	483	1063	220	1047	94	130	-	-	-	-	-	-	-	-	-	-	-	-
20	120	14.7	76.1	4265	317	265	493	1071	289	1406	153	248	-	-	-	-	-	-	-	-	-	-	-	-
22	120	15.4	79.3	4891	330	267	476	921	358	1730	265	501	-	-	-	-	-	-	-	-	-	-	-	-
25	120	16.1	83.0	5686	364	275	413	787	447	1950	340	1021	40	14.8	81.6	1541	110	90	191	397	102	558	36	40
27	80	17.3	84.6	4466	254	192	306	348	360	1384	333	1292	-	-	-	-	-	-	-	-	-	-	-	-
30	80	18.0	87.5	4993	287	191	273	281	405	1425	390	1735	-	-	-	-	-	-	-	-	-	-	-	-

89/...

CHAPTER III

ECONOMIC ANALYSIS

1. Production cost.

The costs of production vary according to site, accessibility, size and slope of the plantation, availability of labour and the level of wages. The operations are carried out either mechanically or manually and both men and women are employed.

The task system is normally used for most of the operations and thus each operation is preferably expressed as the daily task per labourer.

In order to obtain an estimate of the production costs, a survey was carried out on ten of the more important timber estates in the Natal Midlands. The costs and/or task figures for each operation collected in the survey, refer to the working methods normally used in the practice of silviculture and exploitation.

The median cost value of each operation was calculated rather than the mean, since it is less affected by the excessively high or low cost estimates that occur sometimes. These can result from exceptional working conditions or faulty estimates in the survey.

Production costs and daily tasks are as follows:-

(a) Establishment.

The trees are planted by hand in previously-prepared soil pits, which have been marked according to the required espacement. The cost of a thousand plants delivered in boxes of thirty each is R6.00. Women are employed for planting out, their daily task fluctuates between 300 and 600 trees with a median of 350 plants. The pits are prepared by women with a task of 80 pits per day. The estimated costs for marking of these pits is R0.30 per acre.

Mortality depends upon the initial growth vigour of the plants and weather conditions before and after planting. The estimated average mortality is 15%.

The cost of cleaning and slashing during the first year after planting is R1.00 per acre and the cost of the same operation during the second year is R0.90 per acre.

(b) Pruning.

The cost of pruning a tree varies according to the number and height of the branches removed. The normal pruning schedule and daily tasks are:-

Pruning height (feet)	Mean height of tree (feet)	Task (No. trees pruned) per day.	Remarks
0 - 6	16	135	Women employed
6 - 12	24	90	Women employed
12 - 18	30	80	Men employed

(c) Annual costs.

The annual costs subdivided into categories are as follows:-

Table 15.

Summary of Annual Costs.

Category	Cost per acre per annum
(a) Fire protection.	R0.40
(b) Fire insurance.	R0.40
(c) Road maintenance.	R0.40
(d) Maintenance and depreciation of permanent improvements.	R0.30
(e) Overheads.	R3.00
Total annual costs.	R4.50 per acre.

(d) Transportation.

Lorries are normally used to transport the timber from roadside to sawmill or rail. The transport costs are 10 cents per ton mile. The distance of the plantations from rail or sawmill fluctuates between 2 and 12 miles and the median is 6 miles.

(e) Exploitation.

The total cost of exploitation when estimated from the median of the tasks for felling, cross-cutting debranching and skidding varies from 0.15 to 0.4 cents per cu.ft. (This refers to the volume recovered from the trees). However, the books of the timber estates indicate costs between 1 and 2 cents per cu.ft. For the purpose of this study a cost estimate of 1.25 cents per cu.ft. merchantable timber was used for the final stand and 1.50 cents per cu.ft. for the intermediate cuttings.

(f) Labour.

The wages of male labourers are 50 cents per day and those of women 30 cents per day with rations and housing included.

(g) Land.

The present value of bare land is R40.00 an acre. The estimated cost of constructing an adequate road system is R5.00 per acre.

One mile of plantation road, 15 feet wide, generally serves an area of 80 acres. This area amounts to 2.3% of the total plantation area. The area occupied by houses, other buildings, open ground for grazing etc. is approximately 1%. The proportion of land unsuitable for planting is estimated at 3%. This brings the price of land with permanent improvements included to R48.00 per plantable acre of land.

Summary of Costs.

The costs of the silvicultural operations for thinning regimes, A, B, C and D are as follows:-

Table 16.

Costs of Silvicultural Operations.

Silvicultural operations	Regime A	Regime B	Regime C	Regime D
	Cost (R./acre)			
(a) Marking of pits	0.30	0.30	0.30	0.30
(b) Preparation of pits	3.00	3.00	1.99	1.99
(c) Plants	4.80	4.80	3.18	3.18
(d) Planting	0.68	0.68	0.45	0.45
(e) Blanking	0.82	0.82	0.54	0.54
(f) Cleaning 1st year	1.00	1.00	1.00	1.00
(g) Cleaning 2nd year	0.90	0.90	0.90	0.90
(h) Pruning:				
I 0- 6 ft.	1.77	1.77	1.17	1.17
II 6-12 ft.	2.66	2.66	1.76	0.83
III 12-18 ft.	3.12	3.12	1.87	0.95

2. Stumpage Values and Yields.

The log prices at roadside for the previously-defined classes laid down by the Department of Forestry and generally accepted as a standard for timber sales by private owners are as follows:-

Table 17.

Log Class prices at Roadside.

Log Class	Price at roadside (cents per cu.ft.)
b_1	6.5
b_2	15.2
c_1	14.2
c_2	20.7
d_1	16.5
d_2	26.5

The prices for pulpwood six weeks air-dried debarked and delivered at the pulpmill are R6.50 and R7.50 a ton for grades B and A pulpwood respectively. The average cost of railage from plantations in the Midlands to the nearest pulpmill is R2.00 per ton. The cost of road transport over a distance of 6 miles at 10 cents per ton mile is R0.60 a ton. The prices of pulpwood at roadside are then R3.90 and R4.90 a ton for grades B and A respectively. A ton of pulpwood six weeks air-dried contains approximately 50 cu.ft. of timber. Thus the prices per cu.ft. are 7.8 and 9.8 cents for grade B and A respectively. The pulpwood is debarked at roadside at a cost of .4 cents a cu.ft. The price of pulpwood at roadside is therefore

94/........

as follows:-

Pulpwood Grade.	Price at roadside (cents per cu.ft.)
Pb	7.4
Pa	9.4

The price of the sawlog class b_1 is 6.5 cents a cu.ft. at roadside and hence falls below the price of pulpwood grade A. For the economic calculations, this log class was reclassified as grade A (Pa) pulpwood with a price of 9.4 cents a cu.ft. at roadside.

The gross income from final fellings and intermediate cuttings for each thinning regime is the product of the log class yields (tables 14a, b, c and d) and the appropriate log class prices per cu.ft.

The cost of exploitation of the final stand and thinnings is determined by multiplying the volume logged at each operation by 1.50 cents for thinnings and 1.25 cents for the final stand. This cost subtracted from the gross income gives the net income from thinnings and the final fellings.

The yield tables have been constructed for fully-stocked stands. Losses due to insects, diseases, wind, hail etc., reduce the expected yields. Consequently the volume yields estimated from the yield tables are reduced by 10%.

3. Economics of Thinnings and Rotations.

The economic aspects of thinnings and rotations are interrelated and these problems are therefore discussed simultaneously.

(a) Rotation of the highest volume yield.

This rotation corresponds to the age at which the mean annual increment culminates. The relationship between age and mean annual increment for regimes A, B, C and D is shown in figure 13a, b, c and d. The ages of culmination of the m.a.i. are:-

regime A - 18 years
" B - 17-18 years
" C - 18 years
" D - 22 years

The early culmination in regimes A, B and C is partly due to the effect of the thinnings at 18 years. If this thinning is delayed for four years or more the ages of culmination are as follows:-

regime A - 18-19 years
" B - 19 years
" C - 20 years
" D - 22 years

With decreasing severity of thinning the mean annual increment reaches a peak at an earlier age.

Because of price differences between logs of different sizes this type of rotation does not indicate the economic optimum and consequently it cannot be recommended for private timber growers.

(b) Financial rotation of the highes economic land value.

The economic land value is:-

$$S_e = \frac{Y_t + \sum D_a\ 1.op^{t-a} - C\ 1.op^{t} - \sum P_a 1.op^{t-a}}{1.op^{t}-1} - E$$

where Y_t = net income of final yield at t years.

D_a = net income from thinning at a years.

P_a = cost of pruning at a years.

C = cost of establishment.

t = length of rotation in years.

p = rate of interest.

E = capitalised annual cost.

S_e = economic land value.

The economic land value is the sum of the market value plus capitalised profit per acre. The market value of land of a given productive capacity and accessibility is constant and not related to the length of the rotation. Consequently the rotation of the highest economic land value will also maximise the profit per acre.

In the calculation of the economic land value a fixed rate of interest of 6%, being the present rate of money loans was used. The relationship between economic land value and age for thinning regimes A, B, C and D is shown in figure 15.

The economic land value reaches a peak at 22 years and this age of culminations does not seem to be related to the thinning regime.

(c) Financial rotation of the highest earned rate of interest.

This rotation maximises the profit expressed as a percentage of the capital invested in timber growing, in contradistinction to the rotation of the highest land expectation value, which maximises the profit per acre.

In order to calculate this rotation, the economic land value is calculated for various rates of interest and the relationship between this land value and its age of culmination is examined. The rotation of the highest earned rate of interest is then equal to the age giving a economic land value which is equal to the market value of the land. The relationships between economic land value and rotation length for rates of 12% and 13% are shown in figure 16a, b, c and d for thinning regimes A, B, C and D respectively.

The result of the calculation is:-

Thinning regime	Rotation of highest rate of interest (yrs.)	Rate earned (%)
A	16	12.4
B	16	12.4
C	16-17	12.5
D	16-18	11.9

The length of rotation is slightly or not affected by the severity of thinnings.

(d) The rotation of the lowest relative cost price.

This rotation minimises the cost price per cu.ft. of timber, expressed as a percentage of the market price. The relative cost price is as follows:-

$$CP_r = \frac{(S_m + E)(1.op^{t}-1) + \sum P_a 1.op^{t-a} + C\,1.op^{t}}{Y_t + \sum D_a 1.op^{t-a}}$$

where S_m = market value of land.

CP_r = Relative cost price of timber.

The relationships between the relative cost price and rotation length for the thinning regimes is reproduced in figure 17. The result is as follows:-

FIG. 15.

THE RELATIONSHIP BETWEEN ECONOMIC LAND VALUE AND ROTATION LENGTH. REGIMES 'A','B','C', AND 'D'

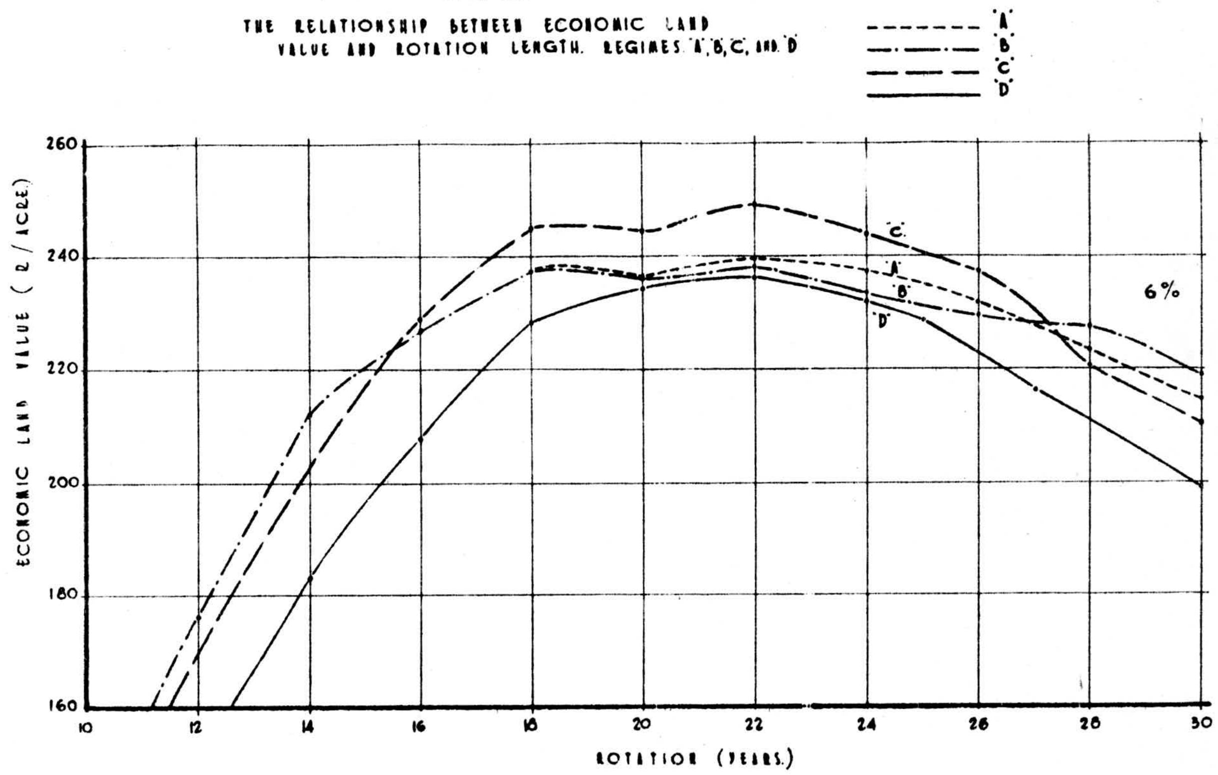


FIG. 17.

THE RELATIONSHIP BETWEEN RELATIVE COST PRICE AND ROTATION LENGTH REGIME 'A', 'B', 'C', AND 'D'

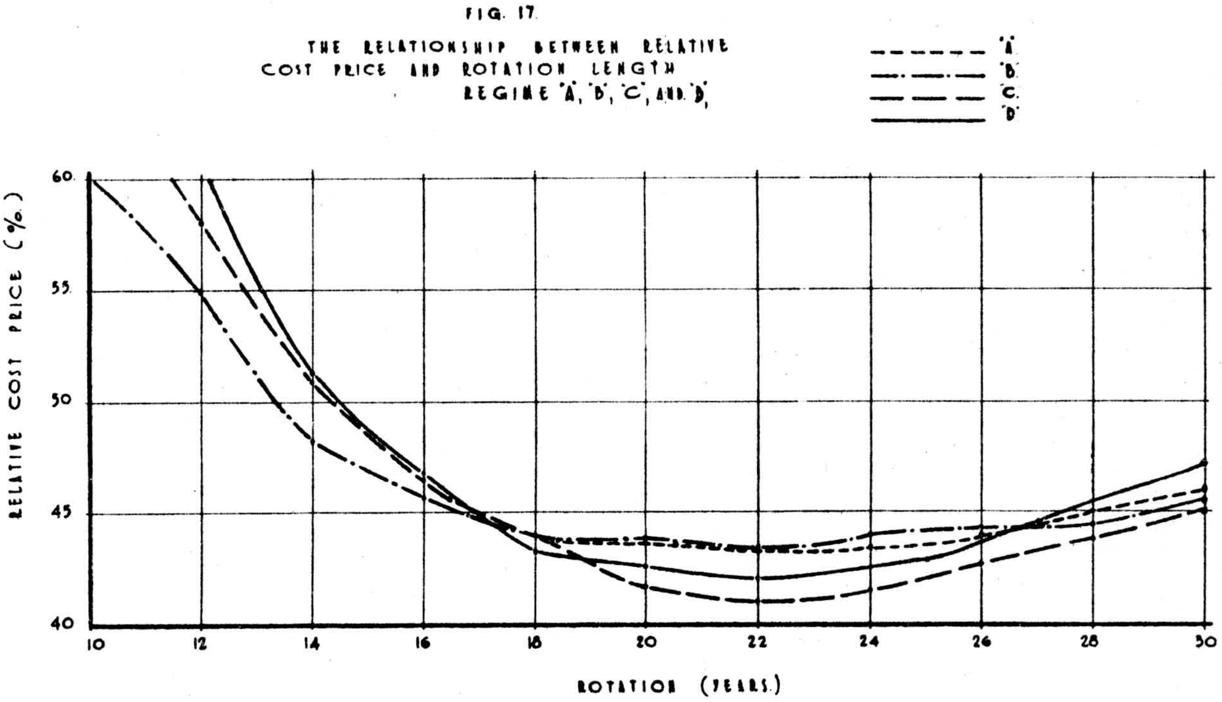


FIG 16a.

THE RELATIONSHIP BETWEEN ECONOMIC LAND VALUE
AND ROTATION LENGTH
REGIME 'A'

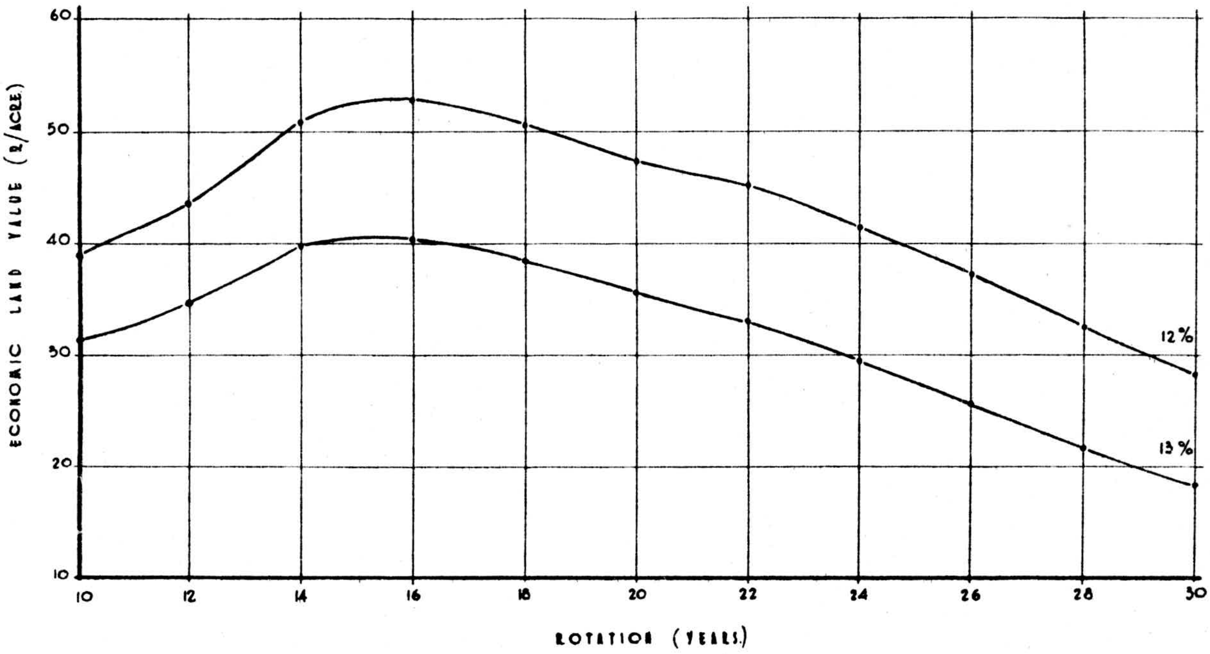


FIG 16b.

THE RELATIONSHIP BETWEEN ECONOMIC LAND
VALUE AND ROTATION LENGTH
REGIME 'B'

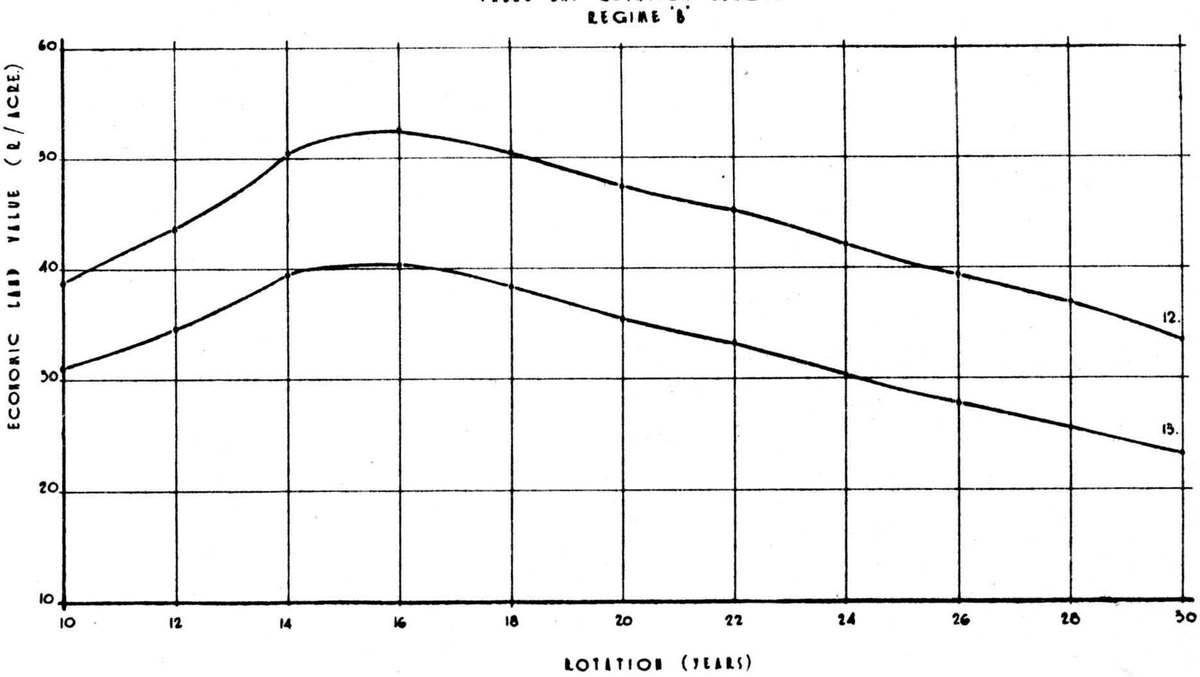


FIG 16c.

THE RELATIONSHIP BETWEEN ECONOMIC LAND VALUE AND ROTATION LENGTH

REGIME 'C'

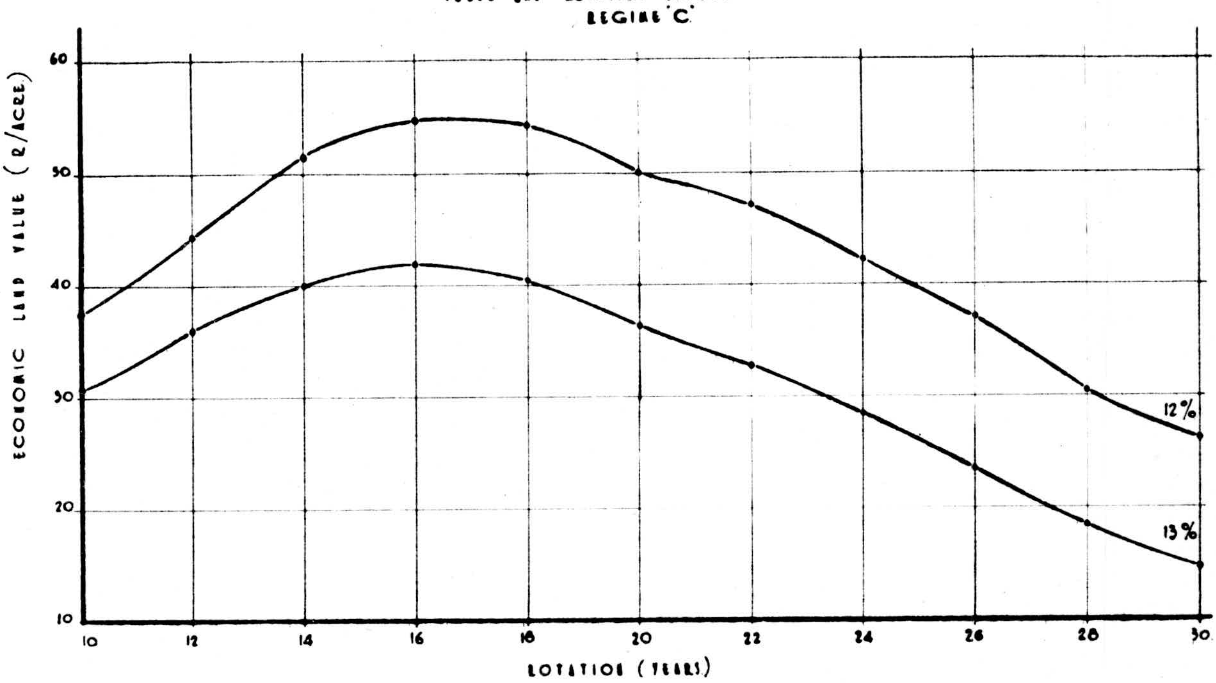
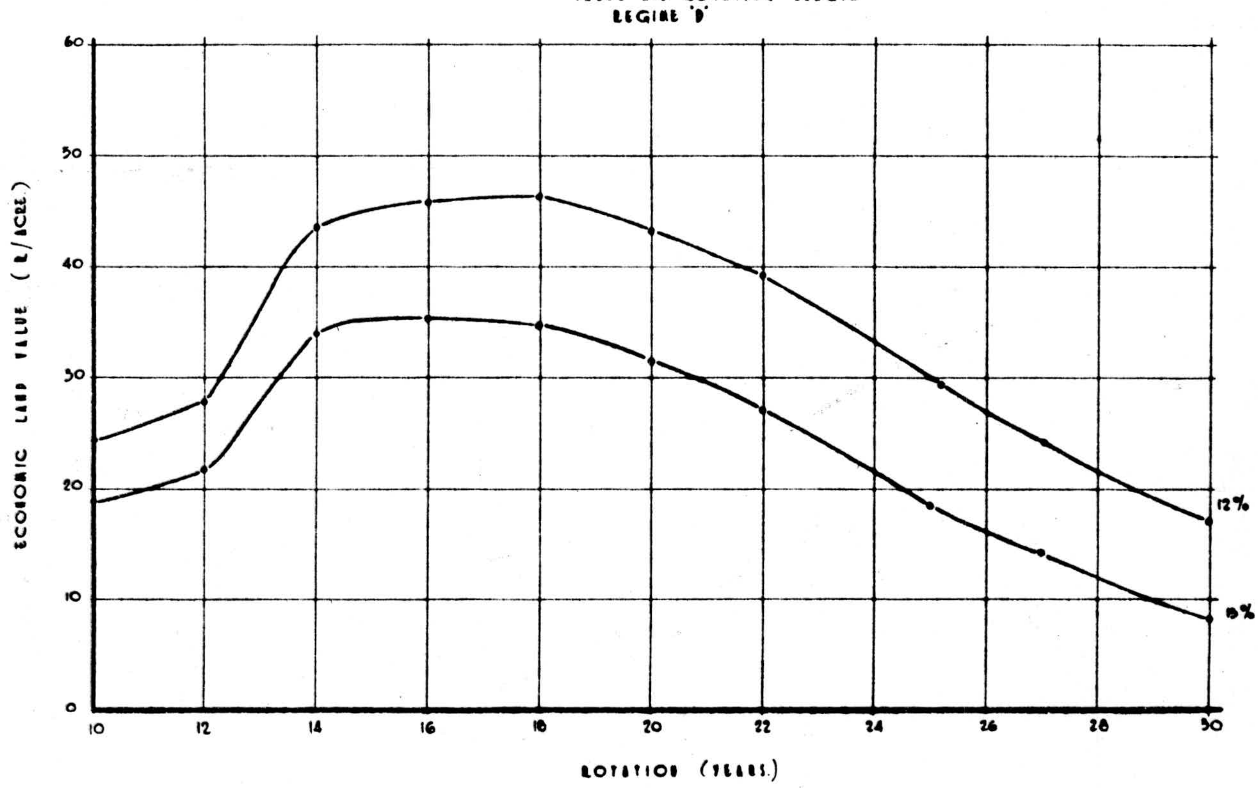


FIG. 16d

THE RELATIONSHIP BETWEEN ECONOMIC LAND VALUE AND ROTATION LENGTH

REGIME 'D'



Thinning regime	Rotation of the lowest relative cost price (yrs.)	Minimum relative cost price (%)
A	22	43.5
B	22	43.5
C	22	41.0
D	22	42.0

This rotation which coincides with that of the highest economic land value is not affected by the severity of thinning.

4. Profit.

Profit is defined as the remuneration for the entrepreneur and is equal to the gross income minus all production costs.

As most plantations in the Natal Midlands have been established in the last 15 years they do not have a normal age class distribution. For the calculation of the profit per acre they may be regarded as periodic forests. The profit is

$$Op = (S_e - S_m)\ o.op$$

The rotation of the highest economic land value will produce the highest entrepeneur's profit per acre. For rotations of 16, 22 and 30 years and thinning regimes A, B, C and D the profit is

Table 18.

Profit per acre of Periodic Forest.

Rotation (years)	Thinning regime			
	A	B	C	D
	Profit R/acre			
16	10.7	10.7	10.8	9.6
22	11.5	11.4	12.0	11.3
30	10.0	10.3	9.8	9.1

The highest profit per acre will not necessarily maximise the profit expressed as a percentage of the capital invested, since the amount of capital invested will tend to rise with increasing length of the rotation. The profit expressed as the difference between the earned rate of interest and the fixed rate of interest of 6%, are as follows:-

Table 19.

Profit expressed as a % of the Capital invested.

Rotation (years)	Thinning regime			
	A	B	C	D
	%			
16	6.4	6.4	6.5	5.9
22	5.8	5.8	5.9	5.3
30	3.0	4.5	3.8	2.5

5. Cost distribution.

For a critical examination of the production costs it is of interest to investigate the distribution of the production costs. These costs can be classified as:-

Land rent	(S_m)
Establishment	(C)
Pruning	(P_a)
Felling and extraction	(F)
Transportation	(T)
Overhead (annual)	(e)

Each cost item may be subdivided into direct costs and interest charges as follows:-

Cost item	Direct cost	Interest
	Total costs per acre at t years	
Land rent	$t.o.op.S_m$	$S_m(1.op^t-1) - t.o.op\ S_m$
Establishment	C	$C(1.op^t-1)$
Pruning	$\sum_{a=1}^{t} P_a$	$\sum_{a=1}^{t} P(1.op^{t-a}-1)$
Felling and extraction	$\sum_{a=1}^{t} F_a$	$\sum_{a=1}^{t} F_a(1.op^{t-a}-1)$
Transportation	$\sum_{a=1}^{t} T_a$	$\sum_{a=1}^{t} T_a(1.op^{t-a}-1)$
Overhead	$t.e$	$\frac{e}{o.op}(1.op^t-1) - t.e$

These results are presented in table 20 for thinning regimes A, B, C and D and in figure 18 for the two extreme regimes A and D.

FIG. 18.

DISTRIBUTION OF PRODUCTION COSTS.

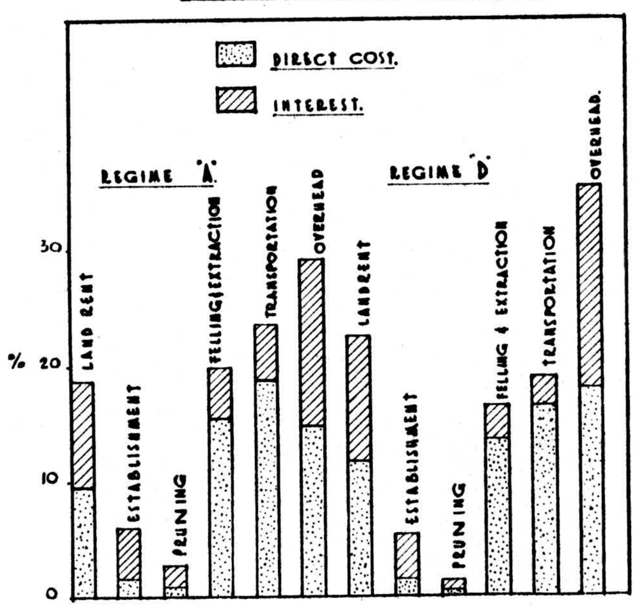


Table 20.

Distribution of Production Costs for a rotation of 22 years.

Cost item	Thinning regime							
	A		B		C		D	
	Direct cost	Interest	Direct cost	Interest	Direct cost	Interest	Direct cost	Interest
	%	%	%	%	%	%	%	%
Land rent	9.5	9.2	9.5	9.2	10.7	10.4	11.5	11.1
Establishment	1.7	4.4	1.7	4.5	1.4	3.7	1.5	3.9
Pruning	1.1	1.7	1.2	1.7	0.8	1.3	0.5	0.9
Felling and extraction	15.4	4.3	15.1	4.5	14.0	3.3	13.4	3.0
Transportation	18.8	4.8	18.3	5.0	17.6	3.9	16.4	2.5
Overhead + annual costs	14.8	14.3	14.9	14.4	16.7	16.2	17.9	17.4
Total	61.3	38.7	60.7	39.3	61.1	38.8	61.2	38.8

6. Discussion and Conclusion.

(a) Rotation.

Private growers will adopt either the rotation of the highest economic land value or that of the highest rate of return. When the rate of return is higher than the fixed rate of interest, the rotation of the highest earned rate of interest will be shorter than the rotation of the highest economic land value. As the difference between the fixed rate of interest and the earned rate of return increases, so will the difference between the lengths of these rotations increase.

Growing of P.patula in Natal yields a return of 12-13%. The fixed rate of interest which was introduced for calculating the economic land value is 6% and that explains why the rotation of the highest financial yield (16-17 years) is shorter than the rotation of the highest economic land value (22 years).

The rotation of the highest economic land value can be recommended for private growers whenever the amount of land available is limiting and that of the highest financial return when capital is a limiting factor and sufficient land is available to reinvest the profits, realised in timber growing.

In the Natal Midlands, afforestable land can still be purchased but in general these sites are less accessible, less fertile and their distances from rail greater than the average of the present timber estate in the Midlands.
With a further increase in the afforestations, the financial yields in the newly established plantations will thus tend

103/.......

to be less. This justifies a rotation longer than that of the highest financial yield. Economically, a rotation of approximately 20 years can be recommended for private timber growers in Natal.

The present rotation in private plantations in the Natal Midlands is about 25 years, and a rotation of 30-40 years for state plantations. With increasing length of the rotation the quality of the sawtimber improves: more layers of wood are imposed on the juvenile core of inferior quality, the proportion of knots is smaller and the timber is stronger. The present timber prices in state plantations are based on the recovery of sawn timber from sawlogs of different sizes from plantations of different ages and silvicultural treatments. These prices represent the average condition of the timber in state plantations. In the actual sales of timber however diameter and length of the sawlogs, independent of the age of the tree, are used as criteria. In other words, difference between the quality of timber of different ages is not recognised in timber sales from individual plantations and stands. A steeper price-size gradient does not solve the problem, for with an increasing price difference between small and big logs, the cost price of the final product recovered from the big logs will increase. Sawmillers, converting timber of large dimensions, will then not be able to compete with those converting smaller logs into the same end product having the same sale value in the open market. If it is desired to promote the quality of pine timber grown in South Africa it will be necessary to recognise the importance of age and to give a premium on sawlogs recovered from trees, over a certain age, for example 25 years. This will encourage timber growers to adopt rotations higher than 25 years.

In the adoption of a rotation it is also necessary to consider the distribution of the final yield and thinnings over log classes. For rotations of 20 and 30 years, the distribution is as follows:-

Rotation	Log class			
	Pulpwood	B_2	C	D
	Yield in % of utilizable volume			
20	36.0	24.7	32.0	7.3
30	26.3	12.3	32.9	28.5

A rotation of 30 years increases the proportion of the large-sized D-logs considerably, and reduces the production of pulpwood and B_2-logs as compared with a rotation of 20 years. At present the demand for D-logs is greater than for B-logs because a greater variety of products can be recovered from these logs. If the present price-size gradient does not change, the demand for large-sized timber may always be greater. A general adoption of a rotation of 20 years may therefore result in an overproduction of B-logs, particularly in view of the expected overproduction of sawtimber in South Africa.

(b) Stand density.

The economic analysis showed that the profit per acre and financial yield are not clearly related to initial espacement and severity of thinning. With increasingly severe thinnings and wider espacements the volume increment is reduced but the higher value of the timber per unit volume counterbalances this decrease in volume yield.

The results of the economic analysis depend on the estimated effect of thinning on volume yield. The regression analysis revealed a difference of 25% between the mean annual increment of thinning regimes A and D at 30 years, and a similar estimate was derived from the records of the Departmental C.C.T. sample plots. But thinning research in Europe has shown that the total increment of stands with extreme thinning regimes do not differ more than 5 to 8%.

The effect of thinning on volume increment in P.patula plantations in the Transvaal and Natal may differ appreciably from the effect of thinning on the growth of trees in Europe, because of climatological and soil differences between these areas. Alternatively, the effect of thinning on volume increment may have been over estimated in the present study, because the basic material was not sufficient to provide accurate estimates. This problem of the influence of thinning on volume increment and financial yield has not been solved and requires investigations on a much larger scale.

(c) Production goal.

The production goal in plantations of P.patula may be pulpwood or sawtimber. The pulpwood produced in plantations managed for sawtimber is a by-product. Plantations, managed for the production of pulpwood, will produce a limited amount of logs falling in the B_2, C and

D sawlog classification, and because of the higher prices of these sawlogs, they will be sold as such. But the proportion of pulpwood and sawlogs differs for different thinning regimes and rotations. For thinning regime A and a rotation of 16 years, the proportions of pulpwood and sawtimber are 83% and 17% respectively. The financial yield is 12.4% and the profit per acre R.10.7 per annum. For thinning regime D and a rotation of 20 years the proportions of pulpwood and sawtimber are 36% and 64% respectively, the financial yield is 11.5% and the profit per acre R.11.3 per annum. Thinning regime A and a rotation of 16 years produces a considerably higher proportion of pulpwood in comparison with regime D and a rotation of 20 years. The financial yield is higher but the profit per acre slightly lower. This indicates that the production of pulpwood in plantations lightly thinned and with a short rotation produces financial results, almost similar to those in stands, severely thinned and managed on a longer rotation. This comparison however is also impeded by the estimated effect of thinning on volume production. If further investigations show that the effect of thinning has been overestimated, the position changes in favour of the production of sawtimber in more severely thinned plantations.

REFERENCES

ACOCKS, J.P.H. (1953). Veld Types of South Africa. Dept. of Agric., Div. of Botany. Botanical Survey of S.A. Memoire No.28.

ASSMANN, E. (1961). Waldertragskunde. BLV.Verlagsgesellschaft.

BECKING, J.H. (1952). De Economie van een sterke dunning voor de Douglas. Nederlandsch Boschbouw Tijdschrift. 24, 257-269.

BRENDER, E.V. (1960). Growth predictions of Natural stands of Loblolly pine in the lower Piedmont. Georgia Forest Research Council. Report No.6.

CHISMAN, H.H. and SCHUMACHER, F.X. (1940). On the tree-area ratio and certain of its applications. Journ. of Forestry 38, 311-317.

CRAIB, I.J. (1939). Thinning, pruning and management studies on the main exotic conifers grown in South Africa. Science Bulletin No.196. Dept. of Agric. & Forestry. Govt. Printer, Pretoria.

CRAIB, I.J. (1947). The silviculture of exotic conifers in South Africa. British Empire Forestry Conference, 1947. Govt. Printer, Pretoria.

DE VILLIERS, J.M. (1962). A Study of soil formation in Natal. D.Sc. thesis, Univer. of Natal.

DE VILLIERS, P.C., MARSH, E.K., SONNTAG, A.E. and VAN WYK, J.H. (1961). The silviculture and management of Exotic Conifer plantations in South Africa. Forestry in S.A. No.1. 13-30.

GRANDJEAN, A.J., VAN SOEST, J. and STOFFELS, A. (1956). Groei van de groveden in Engeland en Nederland. Nederlandsch Boschbouw Tijdschrift. Vol.28, 273-284.

HART, H.M.J. (1928). Stamtal en dunning. Mededelingen van het Proefstation voor het Boschwezen. Dept. Landbouw en Handel. Bull. No.21.

HUMMEL, F.C. (1953). The definition of thinning treatments. International Union of Forest Research Organisations. 11th Congress, Rome. 1953: 582-588.

JOUBERT, A.F. (1964). An Analysis of the relative profitability of various thinning degrees in stands of *Pinus patula* in the Eastern Transvaal. M.Sc. thesis. Univ. of Stellenbosch. (Unpublished).

KOTZE, J.J. (1960). The management of pine stands for Pulpwood. Journal S.A. For. Ass. No.35, 3-9.

LOOCK, E.E.M. (1950). The pines of Mexico and British Honduras. Govt. Printer, Pretoria.

LÜCKHOFF, H.A. (1949). The effect of live pruning on the growth of *Pinus patula*, *P.caribaea*, and *P.taeda*. Journ. of S.A. For.Ass. No.18, 25-55.

LÜCKHOFF, H.A. (1956). High pruning in *Pinus patula* its feasibility, effect on growth and economics. Journ. of S.A. For. Ass. No.27, 55-71.

MacKINNEY, A.L., SCHUMACHER, F.X., CHAIKEN, L.E. (1937). Construction of yield tables for nonnormal Loblolly pine stands. Journ. of Agric. Res. Vol.54, 531-545.

MARSH, E.K. (1957). Some preliminary results from Oconnor's Correlated Curve trend (C.C.T.) Experiments on thinnings and espacements and their practical significance. Govt. Printer, Pretoria.

MARSH, E.K. (1961). Thinning results of C.C.T. plots. Letter to the Chief Conservator of Forests, Uganda. Secretary of Forestry's file R3100, minute dd.3.3.1961. Pretoria.

MARSH, E.K. (1963). The possibilities and limitations of silvicultural treatment in plantations in South Africa. Journ. S.A. For. Ass. No.47, 2-8.

POYNTON, R.J. (1960). Notes on exotic forest trees in South Africa. 2nd Ed. Rev. Rep. of S.Africa For. Bull.No.38.

REINEKE, L.H. (1933). Perfecting a stand-density index for even-aged forests. Journ. Agric. Res. 46: 627-638.

SCHOBER, R. (1942). Mitteilungen aus Forstwirtschaft und Forstwissenschaft, 8. From: Waldertragskunde by Assmann.

SCHUMACHER, F.X. and COILE, T.S. (1960). Growth and Yield of natural stands of the Southern Pines. T.S. Coil Inc., Durham. N.C.

SCHUMACHER, F.X. and DEETLEFS, P.P. du T. (1963). The assessment of log class volumes within trees by probit transformation. Forest Science No.9, 289-294.

SHERRY, S.P. (1961). Live-pruning of Pinus patula and P.elliottii in South Africa. Journ. of S.A. For. Ass. No.36, 18-22.

THORNTHWAITE, C.W. (1948). An approach towards a rational classification of climate. The Geograph. Review. Vol.38, 73-95.

VAN LAAR, A. (1961). Eucalyptus saligna in South Africa. An investigation into the silviculture and economics. Annale Universiteit van Stellenbosch. Vol.36, Serie A No.1.

110/.......

VAN LAAR, A. and VAN SOEST, J. (1959). De groei van de Douglas. Nederlandsch Boschbouw Tijdschrift Vol.30 118-129.

WARRACK, G.C. (1959). Forecast of Yield in relation to thinning regimes in Douglas Fir. Dept. of Lands and Forests. B.C. For. Service. Techn. Publ. T.51.

WIEDEMANN, E. (1951). Ertragskundliche und Waldbauliche Grundlagen der Forstwirtschaft.

WILSON, F.G. (1946). Numerical expressions of stocking in terms of height. Journ. For. 44, 758-761.