

# A STOCHASTIC MODEL FOR SEWER BASE FLOWS USING MONTE CARLO SIMULATION

by  
Garth Flores



*Thesis presented in partial fulfilment of the requirements for the degree  
Master of Science in Engineering at the Stellenbosch University*

Supervisor: Professor H.E. Jacobs  
Faculty of Engineering  
Department of Civil Engineering  
Division of Water and Environmental Engineering

March 2015

# DECLARATION

By submitting this thesis electronically, I declare that the entirety of the work contained herein is my own, original work, that I am the owner of the copyright thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Signature: .....

G Flores

Date: .....

Copyright © 2015 Stellenbosch University

All rights reserved

# ABSTRACT

This thesis deals with understanding and quantifying the components that make up sewage base flows (SBF). SBF is a steady flow that is ubiquitous in sewers, and is clearly seen when measuring the flow rate in the sewer between 03:00 and 04:00. The components of SBF are:

- return flow from residential night use,
- return flow from leaking plumbing,
- groundwater infiltration,
- stormwater inflow.

By understanding each component of SBF, this research can answer the burning question as to how much of the SBF was due to plumbing leaks on residential properties. While previous work on SBF had been done, the work focused on groundwater ingress and stormwater inflows, and thus not much had been said about plumbing leaks. Furthermore, previous work focused on SBF as an isolated sewer related topic, whereas this research integrated SBF as both a sewer related topic and water conservation and demand management (WCDM) topic.

Due to the high variability in each of the SBF components, a method of quantifying each component was developed using residential end-use modelling and Monte Carlo simulations. The author constructed the Leakage, Infiltration and Inflow Technique Model (LIFT Model). This stochastic model was built in MS Excel using the @Risk software add-on. The LIFT Model uses probability distributions to model the inflow variability. The results of the stochastic model were analysed and the findings discussed.

This research can be used by water utilities as a tool to better understand the SBF in networks. Armed with this knowledge, water utilities could make informed decisions about how to best reduce the high SBF encountered in networks.

# OPSOMMING

Hierdie verhandeling bespreek die begrip en berekening van die komponente van riool nagvloei. Die nagvloei was duidelik wanneer die vloei in die rioolstelsel tussen 03:00 en 04:00 gemeet is. Die verskillende komponente van die nagvloei is:

- huishoudelike gebruik,
- lekkende krane en toilette,
- grondwaterinfiltrasie, en
- stormwaterinvloei.

'n Begrip van die komponente van nagvloei kan die brandende vraag van hoeveel nagvloei die gevolg van lekkende krane en toilette is, na aanleiding van die navorsing beantwoord. Vorige werk het op beter begrip van die grondwaterinfiltrasie en stormwaterinvloei gefokus en lekke het nie veel aandag geniet nie. Vorige werk het net op nagvloei as geïsoleerde rioolonderwerp gefokus, terwyl hierdie navorsing nagvloei as 'n onderwerp wat met riool verband hou, sowel as 'n waterverbruik- en behoeftebestuursonderwerp, ondersoek.

As gevolg van die groot verskil tussen elk van die komponente van die nagvloei, is 'n metode ontwikkel wat elke komponent kwantifiseer deur gebruik te maak van eindgebruik-modelle en Monte Carlo-simulasies. Die outeur het die *Leakage Infiltration and Inflow Technique Model (LIFT-Model)* gebou. Hierdie stogastiese model is in MS Excel, met behulp van die @Risk sagtewarebyvoeging gebou. Die *LIFT-Model* gebruik waarskynlikheidsverspreidings om invloeivariasie te modelleer. Die resultate van die stogastiese model is ontleed en die bevindinge bespreek.

Hierdie navorsing mag moontlik deur watervoorsieningsmaatskappe as instrument gebruik word om nagvloei in rioolstelsels beter te verstaan. Hierdie nuwe kennis kan watervoorsieningsmaatskappe in staat stel om ingeligte besluite te neem rakende die beste metodes om te volg om nagvloei te verminder.

## ACKNOWLEDGEMENTS

I could not have completed this work without the support of the following people:

My wife, Marietjie – for her persevering support and looking after our children when I was working on this thesis.

My parents – for funding to pay my first year's registration fees.

Professor Heinz Jacobs – for taking on the arduous task of being my supervisor, and patiently encouraging me to keep grinding away at the task at hand.

To Huibré Lombard and Ernéne Verster – for proof reading and language editing my document and fixing the mistakes that were inevitable working in the early hours of the morning.

*Soli Deo Gloria*

# TABLE OF CONTENTS

DECLARATION.....	i
ABSTRACT .....	ii
OPSOMMING .....	iii
ACKNOWLEDGEMENTS.....	iv
TABLE OF CONTENTS .....	v
LIST OF FIGURES.....	x
LIST OF TABLES.....	xi
LIST OF SYMBOLS .....	xiii
ABBREVIATIONS AND ACRONYMS.....	xiv
1 INTRODUCTION.....	1
1.1 Background.....	1
1.2 Problem statement .....	2
1.3 Research objectives .....	2
1.4 Scope of this project.....	3
1.5 Definition of terms and concepts .....	4
1.6 Significance.....	4
2 WATER CONSERVATION AND DEMAND MANAGEMENT .....	5
2.1 Introduction .....	5
2.2 Water sensitive urban design .....	5
2.2.1 Defining water sensitive urban design .....	5
2.2.2 WSUD water balance concept.....	8
2.3 Water conservation and demand management .....	9
2.4 Minimum night flow .....	14
2.4.1 Consumer night leakage .....	15
2.5 Cost of supplying water .....	16
3 SEWER BASE FLOWS.....	17
3.1 Introduction .....	17
3.2 Existing sewer design guidelines.....	17

3.3	Stormwater inflow.....	19
3.3.1	Flood calculations .....	19
3.3.2	Precipitation .....	20
3.3.3	Quantifying stormwater inflows.....	22
3.4	Groundwater infiltration .....	23
3.5	Normal domestic use .....	25
3.6	Leaking plumbing .....	25
3.6.1	Leakage theory .....	26
3.6.2	Extent of plumbing leaks .....	26
3.6.3	Substandard components and poor workmanship.....	26
3.6.4	Vandalism, theft and poor maintenance .....	27
3.6.5	Toilet leak flow rate .....	27
3.7	On-site leaks and non-revenue water .....	28
4	RESIDENTIAL END USES OF WATER .....	29
4.1	Residential end-use studies .....	29
4.2	REU parameters .....	31
4.3	Socio-demographic factors that influence water use .....	31
4.4	Embedded energy in water .....	32
4.5	Daily consumption patterns .....	32
4.6	Time of use .....	32
5	STATISTICAL CONCEPTS.....	33
5.1	Introduction .....	33
5.2	Stages in a statistical investigation .....	33
5.3	Analyse the data .....	33
5.4	Numerical description of data .....	34
5.5	Description of central tendency .....	34
5.5.1	Arithmetic mean .....	34
5.5.2	Mode .....	34
5.5.3	Median .....	35

5.6	Description of variability .....	35
5.6.1	Range .....	35
5.6.2	Percentiles .....	36
5.6.3	Variance.....	37
5.6.4	Standard deviation .....	37
5.7	Graphical representation of the data .....	37
5.7.1	Bar charts / histograms .....	38
5.7.2	Scattergraph .....	39
5.7.3	Frequency distribution .....	39
5.7.4	Cumulative distribution .....	40
5.8	Probability theory .....	41
5.8.1	Probability .....	41
5.8.2	Mutually exclusive outcomes.....	41
5.8.3	Adding probability.....	41
5.8.4	Independence .....	41
5.8.5	Multiplying probabilities .....	41
5.9	Probability distributions .....	42
5.9.1	Probability density function.....	42
5.9.2	Cumulative distribution function.....	42
5.9.3	Probability distribution parameters .....	42
5.9.4	Distribution fitting.....	43
6	USING MONTE CARLO SIMULATIONS.....	44
6.1	Brief history of the Monte Carlo method .....	44
6.2	Monte Carlo method in civil engineering.....	45
6.3	Reliability engineering .....	45
6.4	Typical probability distributions.....	46
6.5	Monte Carlo model.....	47
7	MODEL DEVELOPMENT.....	48
7.1	Case study site .....	48



7.2	Set up household size and income functions.....	49
7.3	Set up indoor end-use presence functions .....	52
7.4	Set up end-use volume and frequency functions.....	53
7.5	Calculate average annual daily demand per end use .....	55
7.6	Calculate the average annual dry weather flow per end use .....	55
7.7	Distribute the AADWF per end use over 24 hours .....	56
7.7.1	Option a: use 24-hour flow variation per end use .....	56
7.7.2	Option b: use Assessed Residential Night Consumption method .....	57
7.8	Summation of all the end uses to get a total residential AADWF .....	58
7.9	Add groundwater infiltration and stormwater ingress .....	58
7.9.1	Add groundwater infiltration.....	58
7.9.2	Add stormwater inflow .....	59
7.10	Measure flows to compare modelled flows to real flows .....	60
7.10.1	The hydraulic theory required for flow monitoring .....	60
7.10.2	Hardware/tools required for flow monitoring .....	61
7.10.3	Software/systems required for flow monitoring .....	63
7.10.4	General flow logging considerations.....	63
8	ANALYSIS AND RESULTS.....	64
8.1	Introduction .....	64
8.2	Results of household size and income functions .....	64
8.3	Results of indoor end-use presence functions.....	65
8.4	Results of end-use volume and frequency functions .....	66
8.4.1	End-use volumes .....	66
8.4.2	Frequency of end-use events.....	67
8.4.3	Presence of end use in household .....	69
8.5	Results of the AADD per end use.....	72
8.5.1	Breakdown of end uses.....	74
8.6	Calculate the AADWF per end use.....	77
8.7	Distribute the AADWF per end use over 24 hours .....	77

8.7.1	Diurnal pattern method.....	77
8.7.2	Assessed residential night consumption) .....	78
8.8	Summation of the end uses.....	80
8.9	Results of groundwater infiltration and stormwater ingress calculations .....	81
8.9.1	Groundwater infiltration .....	81
8.9.2	Stormwater inflow.....	81
8.10	Measure flows to compare modelled flows to real flows .....	82
8.10.1	Calibrating the model .....	86
9	CONCLUSION .....	87
9.1	Summary of findings .....	87
9.2	Suggestions for future research .....	90
10	REFERENCES .....	91
	APPENDIX A – PROBABILITY DISTRIBUTIONS .....	A1

# LIST OF FIGURES

Figure 1-1 Components of SBF to be modelled.....	1
Figure 2-1 Illustration of holistic approach to managing water in the urban environment. ....	5
Figure 2-2 Water and the urban context of WSUD (Ashley et al. 2013) .....	6
Figure 2-3 WSUD Components (Melbourne Water, 2011).....	7
Figure 2-4 The differences between the conventional urban water balance and the WSUD water balance (Shaffer, 2011) .....	9
Figure 2-5 Managing real losses (Mckenzie, 2011) .....	11
Figure 2-6 Managing apparent losses (Mckenzie, 2011) .....	12
Figure 3-1 Components of domestic sewage to be investigated.....	18
Figure 3-2 Outfall sewers in relation to water courses. ....	18
Figure 3-3 Typical unit hydrograph (adapted from Van der Spuy & Rademeyer, 2010) .....	22
Figure 3-4 Alternative drain pipe for groundwater drainage (Jayasooriya, 2013) .....	24
Figure 5-1 Boxplot of range and percentiles .....	36
Figure 5-2 Five figure diagram summary .....	36
Figure 5-3 Bar chart of daily consumption patterns for a household (Aquacraft, 2011).....	39
Figure 5-4 Frequency distribution of household size.....	40
Figure 5-5 Cumulative frequency of household size .....	40
Figure 6-1 Reliability engineering (Kleyner, 2013) .....	46
Figure 6-2 LIFT Model process.....	47
Figure 7-1 Google Earth image of proposed layout (Google, 2014).....	48
Figure 7-2 Screenshot of 'Bath' end-use calculator .....	55
Figure 7-3 My City / Flotron remote data logger (Vosloo, 2009) .....	62
Figure 7-4 Flow measuring at manholes.....	62
Figure 8-1 Assessed residential night consumption.....	79
Figure 8-2 Assessed residential night consumption (higher leak rates) .....	79
Figure 8-3 Groundwater infiltration result.....	81
Figure 8-4 SBF components during the rainy season .....	83
Figure 8-5 SBF components during the dry season.....	83
Figure 8-6 Dry season diurnal pattern (no significant leaks) .....	84
Figure 8-7 Dry season flow with 10% of hh having leaking toilets.....	84
Figure 8-8 Dry season flow with 20% of hh having leaking toilets.....	85
Figure 8-9 Dry season flow with 50% of hh having leaking toilets.....	85

# LIST OF TABLES

Table 2-1 IWA water balance (IWA, 2000) .....	10
Table 2-2 Minimum night flow balance (Fantozzi and Lambert, 2010) .....	15
Table 2-3 Comparison of night flows .....	16
Table 3-1 Rainfall processes in South Africa (concept presented by Van der Spuy & Rademeyer, 2010).....	21
Table 3-2 Stormwater inflow design guidelines.....	22
Table 3-3 Groundwater infiltration design guidelines .....	23
Table 4-1 Proposed indoor end uses and accompanying return flows .....	30
Table 4-2 Snapshot of REU reports in recent times .....	31
Table 5-1 Summary of numerical description of the data .....	34
Table 7-1 Household size function variables ( <i>number of people per household</i> ) .....	50
Table 7-2 Household size input distributions .....	51
Table 7-3 Household split for modelled area .....	52
Table 7-4 Probability of the presence of each end-use.....	53
Table 7-5 End-use volume and frequency function input variables .....	54
Table 7-6 End-use return factors .....	56
Table 7-7 Assessed night use calculator (based on Fantozzi and Lambert's method) .....	57
Table 7-8 Comparison of groundwater infiltration .....	59
Table 8-1 Household size results .....	64
Table 8-2 Mean end-use volumes (litres/event).....	66
Table 8-3 Frequency of events (number of events per day).....	68
Table 8-4 Goodness of fit results for end-use frequency .....	69
Table 8-5 Presence of events – high-income households.....	70
Table 8-6 Presence of events – middle-income households.....	71
Table 8-7 Presence of events – low-income households.....	72
Table 8-8 Results of AADD per household category using various end-use input functions based on the Jacobs and Haarhoff (2004) published values .....	73
Table 8-9 Results of AADD per household category using the Scheepers (2012) end-use functions.....	73
Table 8-10 Individual end-use mean values .....	74
Table 8-11 Summary of end-use results (l/hh/day) .....	76
Table 8-12 Base flows of end uses – large households.....	77
Table 8-13 Base flows of end uses – small households .....	78
Table 8-14 Total residential AADWF for area .....	80
Table 8-15 Total SBF for area .....	80

Table 9-1 End-use volumes and frequencies.....	88
Table 9-2 AADD and SBF comparison .....	89

## LIST OF SYMBOLS

A	area (normal measured in m <sup>2</sup> )
hh	Households
kl	kilolitre (equal to 1000 litres)
l	Litre
m	Metre
MI	mega litre (equal to 1000 kilo litres or 1 000 000 litres)
m <sup>2</sup>	square metre (standard unit of area)
m <sup>3</sup>	cubic metre (standard unit of volume, equal to 1 kl)
Q	flow rate (measured in m <sup>3</sup> /s or l/s or l/minute)
s	second (standard unit of time)
v	velocity (measured in m/s)
Ø	pipeline diameter (measured in m or mm)

## ABBREVIATIONS AND ACRONYMS

A-D	Anderson-Darling
AADD	Annual average daily demand
AADWF	Average annual dry weather flow
ACIP	Accelerated Community Infrastructure Programme.
CDF	Cumulative distribution function
DWA	Department of Water Affairs
EPA	The Environmental Protection Agency
FS	Free State
IDP	Integrated development plan
IWA	International Water Association
K-S	Kolmogorov-Smirnov
MDG	Millennium Development Goals
NRW	Non-revenue water
PDF	Probability density function
REU	Residential end-use
REUM	Residential End-Use Model
SSS	Separate sewer systems
SBF	Sewage base flow rate
WCDM	Water conservation and demand management
WRC	South African Water Research Commission
WSA	Water services authority
WSDP	Water services development plan
WSP	Water services provider
WTW	Water treatment works
WWTW	Wastewater treatment works

# 1 INTRODUCTION

## 1.1 Background

According to the initial findings of a survey done for the South African Water Research Commission (WRC), approximately 50% of toilets in low-cost housing had broken and were leaking within the first 18 months of being installed (Van Zyl et al. 2008). Water leaking from certain home appliances, such as the toilet, was wasted directly into the sewer. Thus, in areas with high plumbing leaks, a relatively high sewage flow rate would be expected. Randwater studied the real-time flow rate into residential water-supply areas and the related sewage flow rate out of the same areas. It was found that a relatively large fraction of water entering a supply zone would also flow out of the same zone. The conclusion was that there was a high level of inefficient water usage by end-users, because of suspected wasteful habits and on-site plumbing leakages (Maré, 2013). Leaking taps and toilets were reported to contribute to the high sewage base flow rates (SBF).

SBF is a term used to describe the relatively constant flow rate in sewer pipes. The following questions arose regarding the breakdown of the components of SBF:

- i. Which components contributed to the SBF?
- ii. What was the contribution of each component to the total SBF?

The components of SBF ( $Q_{\text{base}}$ ) were on-site plumbing leaks ( $Q_{\text{leaks}}$ ), groundwater infiltration ( $Q_{\text{gw}}$ ), stormwater inflow ( $Q_{\text{sw}}$ ) and normal residential usage ( $Q_{\text{res}}$ ). This is represented in Figure 1-1.

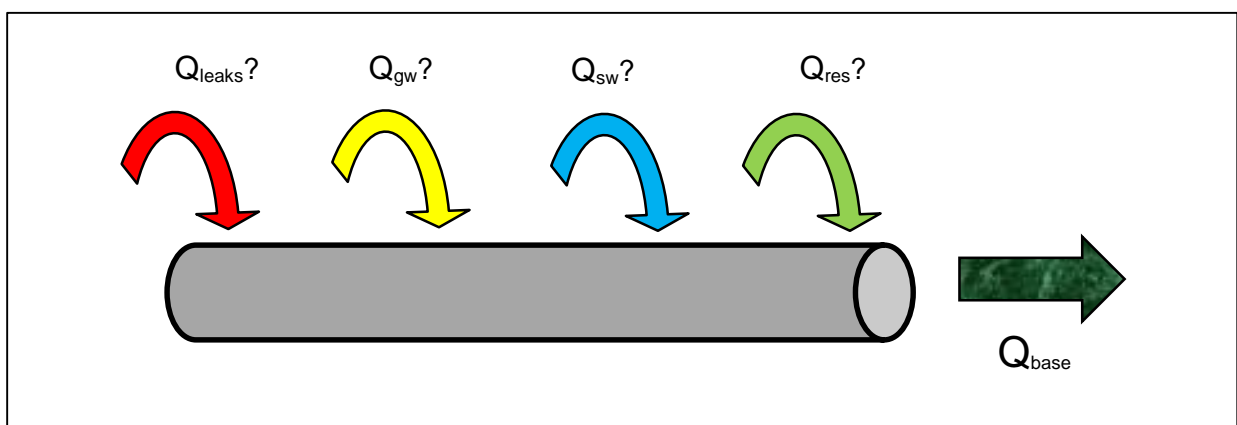


Figure 1-1 Components of SBF to be modelled



SBF can also be written mathematically as the following equation:

$$Q_{base} = Q_{leaks} + Q_{gw} + Q_{sw} + Q_{res}$$

Each of the terms on the right hand side of the equation should be properly described in mathematical terms. The mathematical science of stochastic modelling is a tool used to calculate a set of possible outcomes based on the probability of those outcomes occurring (Pinsky & Karlin, 2011).

Mckenzie et al. (2012) indicated that almost 37% of all purified water in South Africa ended up as non-revenue water (NRW). Water utilities had focused much attention on potable water night flow rates to determine where the major leaks in the system occurred (Fantozzi & Lambert, 2012). Calculating the amount of return flow entering the sewer system as a result of on-site plumbing leaks would provide useful information to water utility managers with regard to the networks and catchments that required water conservation and demanded management (WCDM) interventions. A study of minimum night flows (MNF) and SBF was of critical importance to determine the relationship between the high incidence of NRW and the relatively high SBF measured in South Africa.

## 1.2 Problem statement

Comparing simulated SBF to real-time measured base flow rates could be a useful tool to planners. The results could indicate excessive household plumbing leaks, groundwater infiltration or stormwater infiltration in the sewer's catchment area. There are, however, a great number of variables when simulating SBF.

It is relatively difficult to determine what component of SBF could be attributed to leaks and what component of base flow could be attributed to groundwater infiltration due to the high variability of these components.

## 1.3 Research objectives

An elegant method of modelling the flow is to use Monte Carlo simulation. This thesis explored the feasibility of using end-use modelling in conjunction with Monte Carlo simulation to determine the SBF. Water utilities could then compare the modelled sewer flow with the real-time measured flows. If significant differences were found, a utility could conduct further investigation to determine if the high SBF were linked to water loss on the consumers' stands.

The purpose of this research project was to:

- Conduct a literature review of the relevant subject matter.
- Construct an MS Excel-based end-use model for water use and wastewater flow that could be used to assess the SBF, with respect to the different base flow components.
- Analyse the simulated results and determine the sensitivity of the model parameters.

## **1.4 Scope of this project**

This project dealt with residential consumers who had yard connections for their water supply and flushing toilets that were connected to waterborne sewer systems. Furthermore, the sewers in South Africa are designed to operate separately from stormwater systems, but unwanted stormwater inflows can (and often do) end up in the sewer system.

The model used in this research was set up to describe SBF during the 'dry' season only. This research was conducted on the assumption that during the 'dry' season, stormwater ingress could be omitted from the model in order to simplify the calculations. Lastly, the model did not take into account vacant households or periods during which the houses could have been vacant.

The focus of this study was on residential consumers because the contribution to SBF from other land use types, such as business, commercial and industrial, might be entirely different to residential base flow. The findings of previous research on groundwater infiltration and stormwater ingress were used to populate the model.

## 1.5 Definition of terms and concepts

The following terms and concepts were used often in this thesis, and were included to familiarise the reader with the meaning of each term or concept.

**Average annual dry weather flow (AADWF):** the total daily flow in sewers during the driest months of the year measured in m<sup>3</sup>/day.

**Base flow:** the flow rate of wastewater in a sewer network during the night hours between 03:00 and 04:00 when flow should theoretically be at its lowest, measured in m<sup>3</sup>/s or m<sup>3</sup>/hour.

**Indigents:** low income households that qualify for subsidised water (normally the cost of the first 6 kl of water consumed per month is not borne by the consumers in that household).

**Minimum night flow (MNF):** the flow of potable water in the water network during the night hours, normally between 02:00 and 04:00 when flow should theoretically be at its lowest.

**Non-revenue water (NRW):** NRW is water that passes through the urban water supply system without any income from that water.

**Water services provider (WSP):** a municipality or utility company responsible for providing and distributing potable water, and for removing and treating wastewater.

## 1.6 Significance

If water services providers (WSPs) could better understand the composition of relatively high SBF, remedial actions required to reduce the SBF could be identified, such as repairing leaking toilets because of high plumbing leaks or replacing old pipes due to high groundwater infiltration. High SBF places an additional load on wastewater treatment works (WWTW), which in turn affects the plant's ability to treat the effluent (Van Vuuren & Van Dijk, 2009). If the WSP could reduce the SBF, the benefits would include lower operating costs for WWTWs, deferred capital investment for the upgrading of WWTWs and lower water loss (Stephenson & Barta, 2005). Similar statements on the benefits of reducing SBF had been made by the Environmental Protection Agency (2008) and Waldron (2013).

## 2 WATER CONSERVATION AND DEMAND MANAGEMENT

### 2.1 Introduction

The concept of a holistic or integrated approach to managing the urban water environment (Ashley et al. 2013; Roy et al. 2008) is generally called water sensitive urban design (WSUD). In this integrated approach, as illustrated in Figure 2-1, the topic of leaking toilets and high SBF relates to both WCDM and the sanitation spheres of water. Reducing SBF could thus reduce potable water consumption and downstream infrastructure capacity requirements.



Figure 2-1 Illustration of holistic approach to managing water in the urban environment.

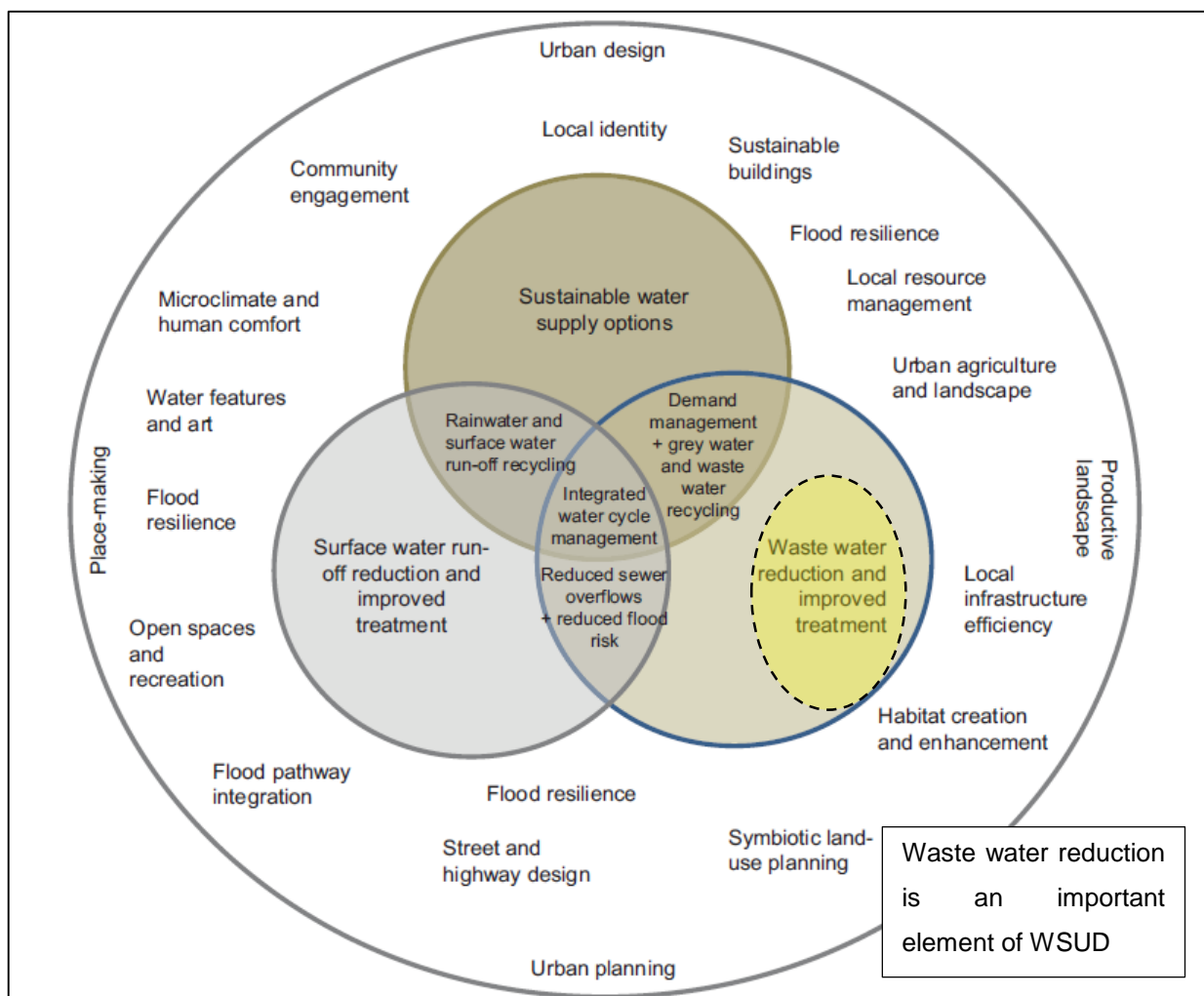
### 2.2 Water sensitive urban design

#### 2.2.1 Defining water sensitive urban design

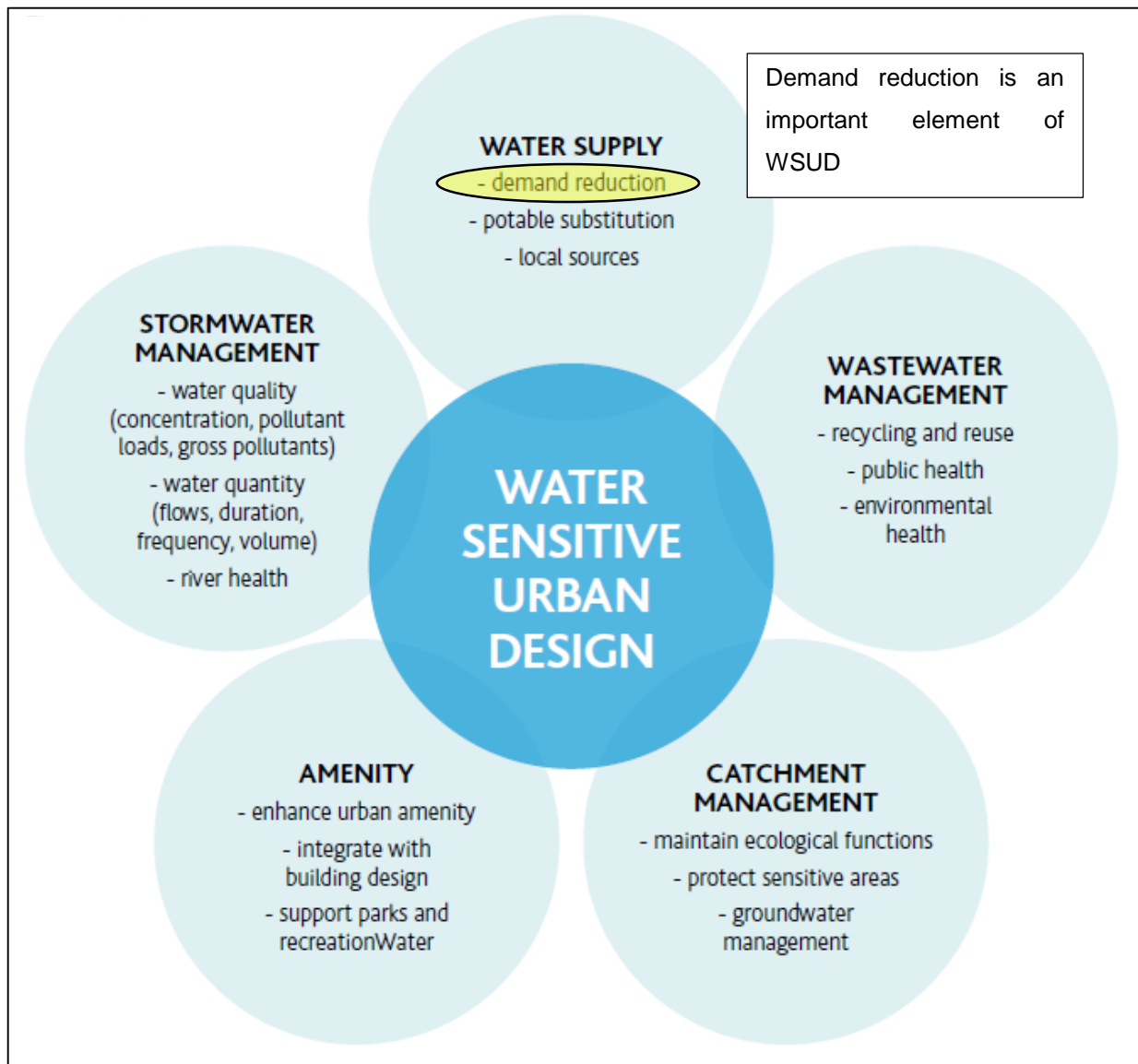
The Australian National Water Commission (2007) proposed the following definition for water sensitive urban design (WSUD): “WSUD is defined... as the integration of urban planning with the management, protection and conservation of the urban water cycle that ensures urban water management is sensitive to natural hydrological and ecological cycles.”

WSUD could also be viewed as a methodology aimed at “minimising the impacts of urban development on the water balance and the environment” (Australian National Water Commission, 2007).

Historically, planners and engineers investigated each water field in isolation to the other fields, in other word potable water versus wastewater versus stormwater (Melbourne Water, 2011). However, one of the key drivers of WSUD acknowledged that all these fields were inter-related and thus needed to be studied as a whole (Melbourne Water, 2011). Figure 2-2 and Figure 2-3 illustrate this interrelated concept.



**Figure 2-2 Water and the urban context of WSUD (Ashley et al. 2013)**



**Figure 2-3 WSUD Components (Melbourne Water, 2011)**

WCDM is one of the important tools used in WSUD. The aim of WCDM is to reduce the potable water requirement of an urban area. A reduction in potable water consumption is often accompanied by a reduction in the wastewater generated by the same urban area. The majority of stormwater aspects of WSUD fall outside the scope of this study (for example permeable paving, stormwater attenuation ponds and stormwater treatment in wetland systems). Stormwater ingress into sewers has relevance to this thesis and thus a paragraph reviewing how stormwater ingress influences separate sanitation systems was included in Chapter 3 of this report.

### 2.2.2 WSUD water balance concept

The concept of the WSUD water balance is based on the law of the conservation of mass. This means that the water entering the environment also has to leave the environment. Water enters the natural environment as precipitation (rain, hail, snow and fog) and rivers. Water enters the urban environment by precipitation, rivers and man-made means (piped potable water). Water leaves the natural environment via evapo-transpiration, groundwater infiltration and runoff. In the urban case water leaves the environment largely as wastewater discharge and stormwater run-off.

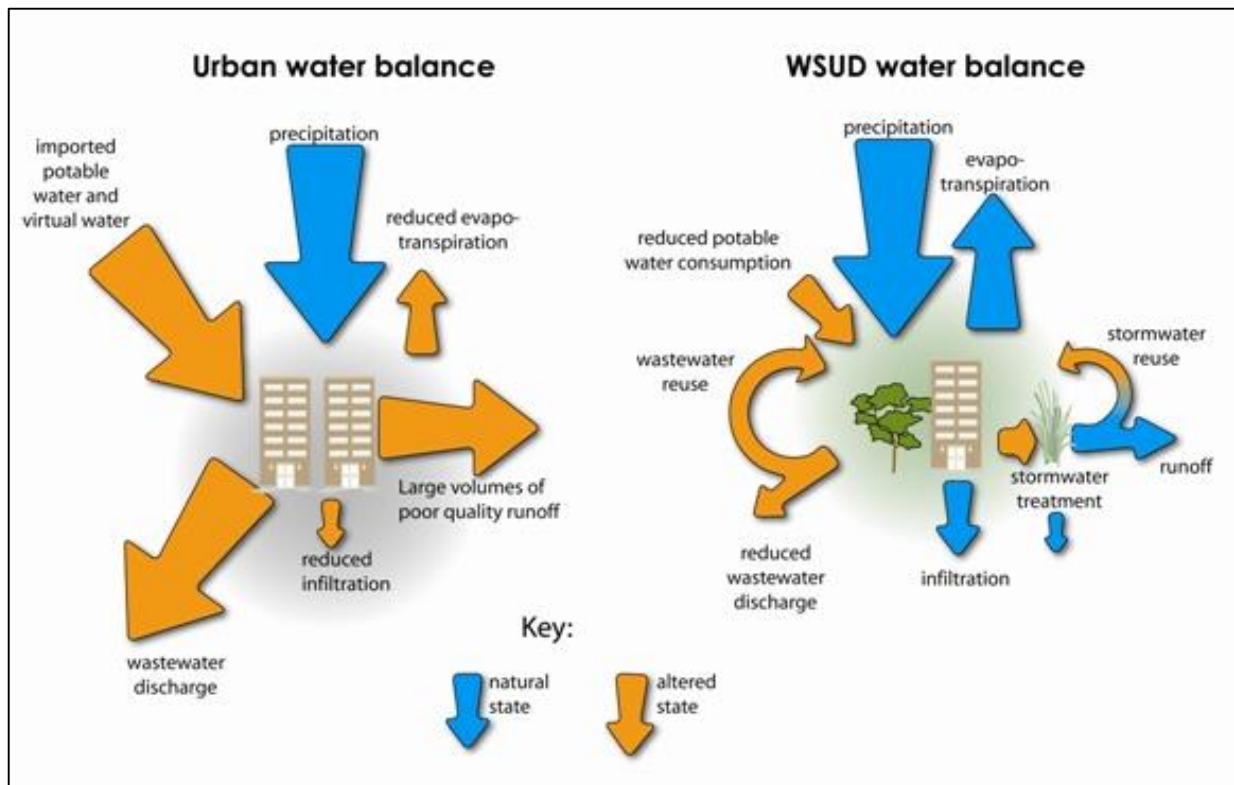
In the past, stormwater design focused on moving the run-off out of the urban area as quickly as possible and subsequently, very little water could be infiltrated and evapo-transpired. WSUD on the other hand, treats urban surface water run-off as a resource rather than a nuisance (Ashley et al. 2013).

In WSUD the negative impact on the water balance is reduced by reducing the potable water for the urban environment by:

- using water efficiently,
- reducing water network losses,
- harvesting rainwater, and
- re-using stormwater and wastewater where appropriate.

WSUD also investigated ways of improving the quality of water leaving the urban environment through stormwater treatment and reducing the unnaturally high run-off by designing structures that allow infiltration and evapo-transpiration and by harvesting rainwater. As soon as strategies for using water efficiently started reducing the demand on potable water, the wastewater discharge into the environment decreased. Wastewater discharge could further be reduced by re-using the wastewater.

The concepts discussed above are best illustrated in the WSUD water balance diagram depicted by Schaffer (2011) and shown in Figure 2-4.



**Figure 2-4 The differences between the conventional urban water balance and the WSUD water balance (Shaffer, 2011)**

## 2.3 Water conservation and demand management

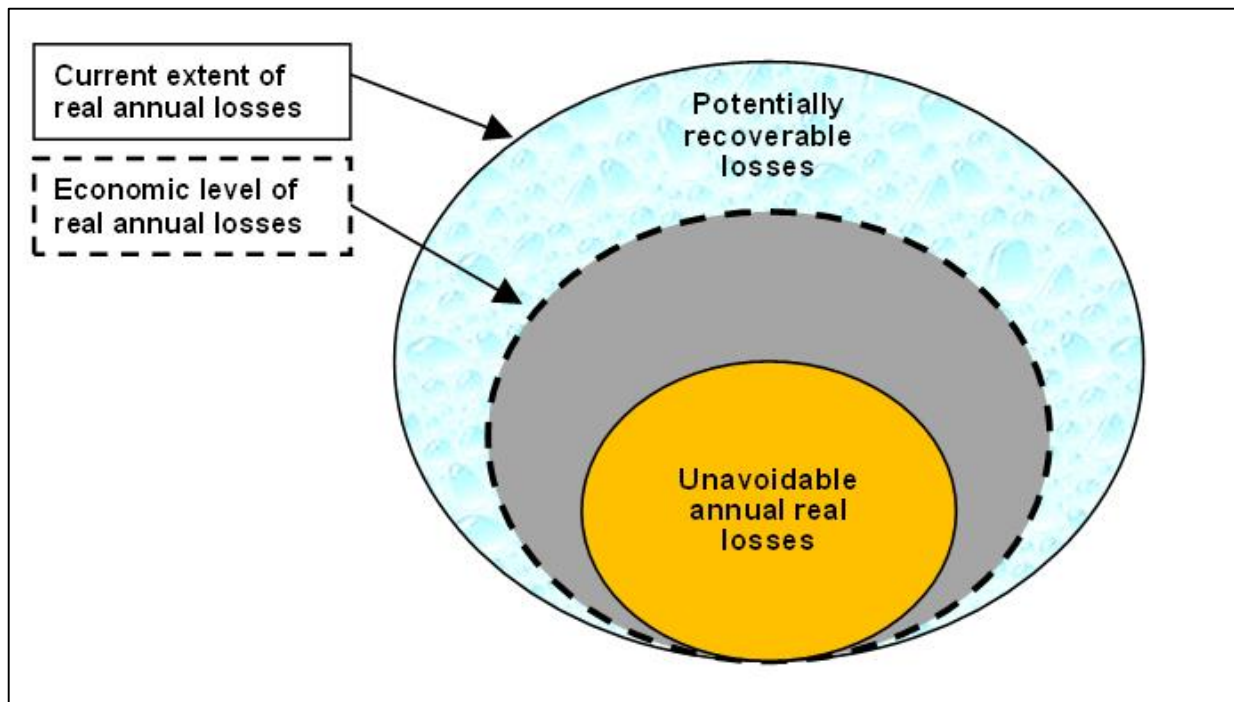
The International Water Association (IWA) developed a standard (potable) water balance table, which should not be confused with the WSUD water balance shown in Figure 2.4. The IWA water balance is also based on the law of conservation of mass, with standard descriptions of various forms of consumption and losses. The IWA water balance relates to potable water and not to precipitation and groundwater. Table 2-1 indicates the IWA water balance.



**Table 2-1 IWA water balance (IWA, 2000)**

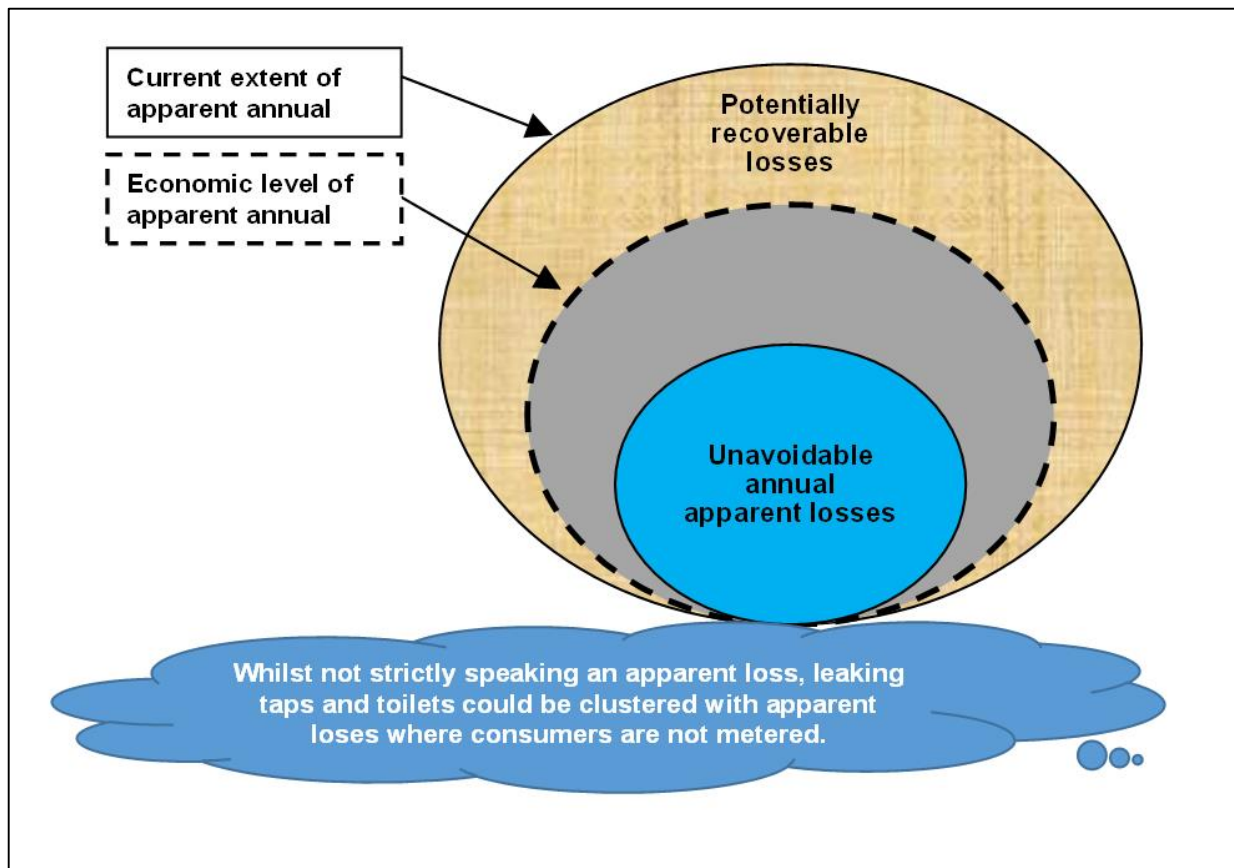
System input volume	Authorised consumption	Billed authorised consumption	Billed metered consumption	Revenue water	
			Billed unmetered consumption		
		Unbilled authorised consumption	Unbilled metered consumption	Non-revenue water	
			Unbilled unmetered consumption		
	Water loss	Apparent losses	Unauthorised consumption		
			Customer meter inaccuracies, billing and accounting errors		
		Real losses	Leakage on transmission and distribution mains		
			Leakage and overflows at reservoirs		
Leakage on service connections up to metering point					
Notes:					
Collect data from WTW / zone meters			Collect data from consumer meters, and calculate remainder based on industry acceptable practices		

A primary goal of WCDM is to increase a water utility's revenue water (RW) and decrease the utility's NRW. NRW comprises unbilled consumers and water loss. The economic level of real losses describes the point where it becomes more expensive to find and fix certain leaks than the financial loss of the water wasted from those leaks (Mckenzie, 2012). Some leaks are so small that they are difficult to detect using the available technology. These small leaks are classified as unavoidable annual real losses (UARL). Through research the IWA established a formula for calculating UARL. Figure 2-5 illustrates the concept of UARL and the economic level of water losses. The challenge for WCDM is to implement interventions (blue arrows) to get the current annual real losses as close to the economic level of real losses as possible.



**Figure 2-5 Managing real losses (Mckenzie, 2011)**

Similar to the real losses in the water network system, the IWA defined various levels of apparent losses, as indicated in Figure 2-6. Leaking taps and toilets in the South African context was a special case in that it was not strictly speaking a loss to the utility. It was a real loss to the consumer, and if the metering and billing system had been fully operational, the consumers would have been billed for the leaks on their properties. However, because a large portion of consumers were not metered, or if they were metered they were not billed, the utility had no way of recovering the expense of water wasted due to leaking taps and toilets. Leaking taps and toilets in low-cost housing townships could thus be clustered with apparent losses, though Mckenzie et al. (2012) proposed that the IWA water balance should be specifically adapted for the South African situation.



**Figure 2-6 Managing apparent losses (Mckenzie, 2011)**

In the South African context, there are a large number of billed, unmetered, and metered and unbilled consumers (DWAF, 2004). A large number of old meters also causes meter inaccuracies. Old meters are typically older than 15 years, and due to mechanical wear and tear on the internal mechanisms of the meter, the meter no longer registers low flow, typical of a dripping tap, or a toilet that does not completely seal when full (Couvelis et al. 2014).

The result of the large number of unmetered consumers and consumers with meters not reading low flows is that the water utility cannot accurately recognise what is happening in its supply areas, because there is no accurate monthly consumption data. Increased knowledge ensures informed decisions, and the potable water supply can thus be managed effectively. Knowing how much water is consumed enables a utility to compare itself to similar utilities. Mckenzie et al. (2012) found that many municipalities had either no records (36 municipalities or 15%), or poor records (40 municipalities or 17%). It is a matter of concern that only 45 municipalities (19%) had good records. Mckenzie et al. (2012) correctly stated that it was unacceptable for municipalities to not know how much water is consumed.

While a few utilities might have had accurate system input data (like a daily reading from the water treatment works), they could not accurately calculate the various other components of the water balance, because of either insufficient meter reading data or because they were not familiar with industry accepted means of determining 'unmeasurable' components.

Another problem involved recorded meter data, because the recording interval was often only monthly. The monthly consumption could be averaged out over the recording interval to determine daily consumption, but it was impossible to determine the way in which the consumption pattern varied over a 24 hour period from most domestic meters installed in South Africa. Internationally there is a trend in developed countries to install domestic 'smart meters' for residential consumers that can log domestic flows at desired intervals.

By modelling, measuring and studying the night flows of sewage generated from areas that are not metered, and allowing for stormwater ingress and groundwater infiltration, the following information can be calculated:

- billed unmetered consumption, and
- unbilled unmetered consumption.

Maré (2013) made the following statement in a presentation at the African Water Leakage Summit in 2013: "Water use efficiencies on end user properties must receive monitoring and evaluation. To do this it is necessary combine water supply and sewer discharge data to obtain a better assessment of the nature and extent of the problem."

Maré (2013) reported that a large portion of current urban inefficiency lay with the end users because of:

- wasteful consumptive use,
- wasteful use returned to sewers,
- excessive plumbing leakages, and
- excessive tap and toilet leakages (Maré, 2013).

The report by Mckenzie et al. (2012) stated that the high per capita consumption for South Africa points to inefficient and wasteful water usage patterns. This was supported by the findings made by Maré (2013).

While wasteful consumptive use might be difficult to quantify, excessive tap and toilet leakages will result in higher than expected SBF. Wasteful consumptive behaviour might be exacerbated by concepts such as free basic water and billed unmetered (flat rate consumption) scenarios. In these scenarios, a utility or municipality typically subsidises the consumers' first 6 kl consumption each month and might charge them a small fee if they had the means to pay. These consumers were often not metered, so if more than 6 kl was consumed, the municipality would be unable to determine this and collect the additional income (Mckenzie et al. 2012). It is important to note that wasteful behaviour can be changed by educating the consumers. Institutional interventions, such as water restrictions could also be implemented in extreme cases. Case studies of both behavioural change and institutional interventions were discussed in a report by Rabe et al. (2012).

## 2.4 Minimum night flow

MNF is the potable water flow that enters a demand zone during the period of low consumption. It is typically measured at a zonal reservoir by means of logging the flow every 15 minutes. Choi et al. (2012) identified the following components of NRW:

- exceptional night use,
- normal night use,
- background losses, and
- large pipe leakage (bursts).

The significance of measuring and analysing night flow was that it was one of the most frequently used method for determining and understanding real water losses in a district meter area (DMA) (Loureiro et al. 2012). Mckenzie (1999) stated that background losses for South Africa comprised flow rates of less than 0.25 m<sup>3</sup>/h (4 l/minute) and bursts flow rates in excess of 0.25 m<sup>3</sup>/h.

In a study of 2 844 smart meters installed in the Wide Bay area in Australia, Cole (2010) found that the MNF occurred between 03:00 and 04:00. The average flow per connection in litres per hour was found to be 4 l/hour during the MNF hour. Fantozzi and Lambert (2010) proposed that the MNF hour varied between midnight and 01:00, and between 05:00 and 06:00, depending on factors like climate, society, religion, age of residents, or presence of storage tanks. MNF showed a seasonal variation as well as a weekly variation (Cole, 2010).

Fantozzi and Lambert (2010) adjusted the IWA water balance table for MNF based on work by the IWA Water Loss Specialist Group (WLSG) Night Flow Team in 2010. Table 2-2 shows the adjusted water balance for MNF.

**Table 2-2 Minimum night flow balance (Fantozzi and Lambert, 2010)**

Minimum night flow (MNF)	Night consumption (NC)	Night use (NU)	Exceptional night use (ENU)	Consumer
			Assessed residential night use (ARNU)	
			Assessed non-residential night use (ANRNU)	
		Customer night leakage (CNL)	Inside building (CNLI)	
			Outside buildings (CNLO)	
	Utility night leakage (UNL)	Bursts (B)	Unreported bursts (UB)	Utility
			Reported bursts (UB) (not yet repaired)	
		Background leakage (BL)	On service connections (BLS)	
			On mains (BLM)	

While utility night leakage could end up in the sewer system through groundwater infiltration or through open manholes, the main components of night flow in the sewer would be the night consumption (NC) components, highlighted orange.

#### 2.4.1 Consumer night leakage

Fantozzi and Lambert (2010) itemised consumer night leakage (CNL) inside buildings as the following leaking components:

- toilets,
- taps,
- plumbing, and
- storage tanks (geysers).

Water from leaking taps and toilets entered the sewage systems, while the water from leaking plumbing outside the building and storage tanks (geysers) might not necessarily enter the sewage system.

Fantozzi and Lambert (2010) found that in their study sample, 123 (0.19%) of the 63 000 water meters had recorded leaks with a flow rate of less than 1.7 l/minute (100 l/hour) and 11 meters (0,017%) had leaks greater than 1.7 l/minute (100 l/hour). This was in stark contrast to a sample of 10 properties surveyed by Van Zyl et al. (2008) who found that 50% of properties in low-income areas had leaks. Proposed night flow figures are given in Table 2-3.

**Table 2-3 Comparison of night flows**

Source	Proposed night consumption	Units	Base hour	Comments
Lambert and Fantozzi (2010)	5.9	l/connection/hour	2:00 – 3:00	Base hour on weekdays.
Lambert and Fantozzi (2010)	3 – 6% of population	l/person/hour	n/a	Use end-use components (toilet flushes, shower, etc.).
UK Water Industry (1994)	1.7	l/household/hour	n/a	
Warren (2002)	1.5 – 2.3	l/household/hour	n/a	
Loureiro et al. (2012)	0,85	l/household/hour	n/a	
Choi et al. (2012)	14,58	l/connection/hour	n/a	5 l/connection/hour proposed for losses.
Couvelis et al. (2014)	20 – 40	l/hour	n/a	Household leaks.

Table 2.3 indicates that the night flow should be lowest at about 03:00. The design household flow rate could vary from 0.85 l/hour to 14 l/hour in more developed areas. In South Africa, with its high incidence of on-site leaks in low-cost housing areas, the night flow could be significantly influenced by leaks and be as high as 40 l/hour (Couvelis et al. 2014).

## 2.5 Cost of supplying water

Water utilities need to invest in infrastructure and incur the expense of pumping, treating, conveying and storing water before it reaches the consumer. Where the consumers are not metered and billed, the utility cannot recover the cost of the consumption. NRW costs South African water utilities R7 billion per annum (Mckenzie et al. 2012).

## 3 SEWER BASE FLOWS

### 3.1 Introduction

In the previous chapter, some key concepts of WCDM were reviewed. It was also noted that NRW costs South African WSPs billions of rands annually. High SBF in turn added to the costs of operating sewer pump stations and WWTW. The high base flows also increased the risk of sewage spills because the attenuation volume to accommodate the daily peaks had been depleted. De Swart and Barta (2008) identified the following interventions in the sanitation field as necessary:

- Practical methods for reducing infiltration and ingress in urban areas, incorporating technical, social, environmental and legal considerations.
- Opportunities for implementing benchmarking in municipalities (pilot projects).
- User education programmes regarding cleaning materials and system abuse.
- Impact investigation of inflow/infiltration on the WWTW treatment capacity and cost of treatment due to standard design allowance for extraneous flows
- Impact investigation of inflow and infiltration on the WW pump station capacity and cost of pumping (and temporary/emergency storage) due to standard design allowance for extraneous flows.
- Evaluation of the capability of treatment processes in urban WWTW with regard to changing the quality and nature of sewerage flows.

Good research had fortunately been conducted in the field of separate sanitation systems, and this chapter endeavours to highlight the work relevant to SBF.

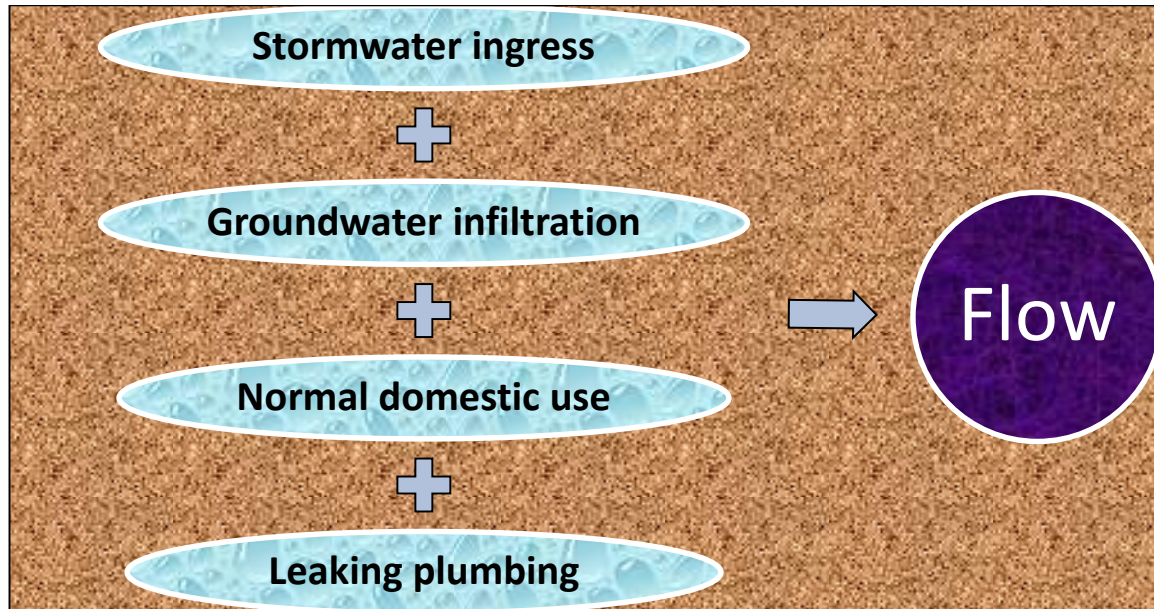
### 3.2 Existing sewer design guidelines

The following four main components of sewer flow were identified in Chapter 1 are discussed in further detail for the remainder of this chapter:

- stormwater inflows,
- groundwater infiltration,
- normal domestic use return flows, and
- leaking plumbing.

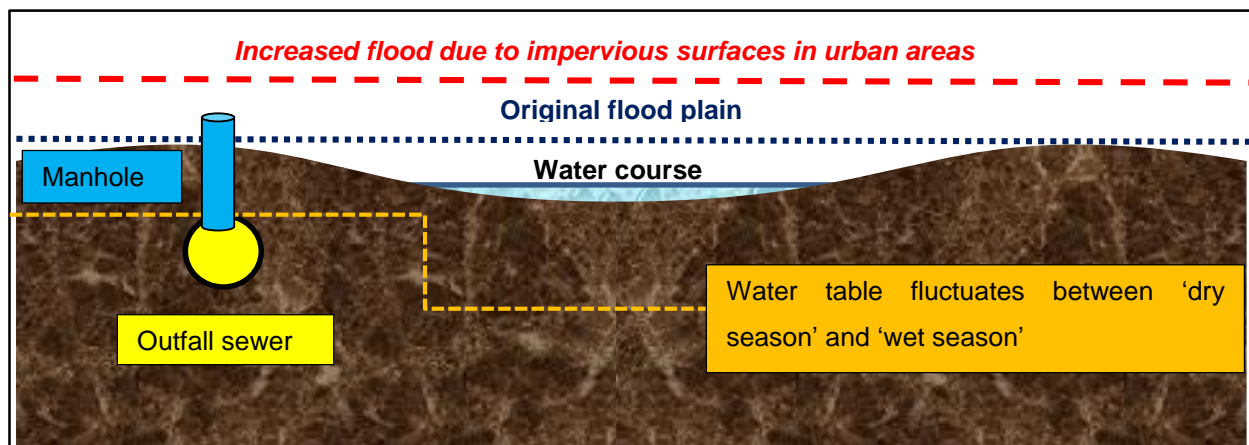


Commercial and industrial wastewater return flows were considered in this research. There was also the possibility of residential consumers discharging their swimming-pool water into the sewer system, but since these flows typically would occur when the users were awake, the author was convinced they seldom formed part of the SBF. Figure 3-1 shows the components of SBF investigated in this portion of the literature review.



**Figure 3-1 Components of domestic sewage to be investigated.**

Outfall sewers are often constructed next to a water course. During the wet season, the groundwater level could rise above the sewer, and during the dry season the groundwater level could drop below the sewer, as indicated in Figure 3-2. Furthermore, should the sewer be constructed within the flood plain of the water course, the tops of manholes would be underwater during flood events.



**Figure 3-2 Outfall sewers in relation to water courses.**

### 3.3 Stormwater inflow

Some parts of the world have drainage systems to accommodate both the sewer flows and the stormwater flows. These combined sewers are typically found in older cities in Europe and the United States of America. In South Africa, however, separate stormwater networks convey stormwater while sewer networks convey wastewater. Even though there are two separate systems, some portion of the stormwater run-off invariably ends up in the sewer system. Stormwater inflow, also called rainfall dependent inflow (Environmental Protection Agency, 2008), into the sewer system could be at manholes and where illegal roof run-off is discharged into the sewer system at cleaning eyes and gullies on consumers' properties (Stephenson & Barta, 2005). Stormwater depends on precipitation (in South Africa this is mostly rainfall but could include hail and snow in certain areas) and thus, where there is no precipitation, there is no stormwater ingress.

Stephenson and Barta (2005) stated that flood peaks tend to increase in developed areas, because of the increase in impervious surfaces. Manholes that were constructed outside the flood zone 20 years ago were, therefore, frequently flooded during heavy rainfall. The red line in Figure 3.2 shows the higher flood peaks that result from urbanisation.

With the advent of WSUD and sustainable urban drainage systems (SUDS), the trend is to attenuate stormwater and thus reduce the peaks. This could result in lower peaks and less stormwater inflow into the sewer network. The City of Cape Town has by-laws that dictate that no development may exacerbate the stormwater peaks (City of Cape Town, 2005).

#### 3.3.1 *Flood calculations*

In general, stormwater ingress flow patterns follow rainfall, and the volume of stormwater inflow is affected by the permeability of the soil, the intensity of the rainfall, and the number of openings into the sewer system at stolen manhole covers or illegal connections of gutters into the sewer gullies (Stephenson & Barta, 2005). The stormwater inflow at a certain point in the network would generally have the same pattern as the stormwater flood above ground, except that the value would be greatly reduced (Environmental Protection Agency, 2008). The Environmental Protection Agency also stated that using a synthetic unit hydrograph (SUH) method was most accurate for calculating stormwater inflows.

According the guidelines published by Van der Spuy and Rademeyr (2010) the SUH is a deterministic method for calculating flood peaks, and is used for catchments ranging from 15 km<sup>2</sup> to 5 000 km<sup>2</sup>. Unit hydrographs were developed for the following 9 catchment types:

- Coastal tropical forest,
- Schlerophyllous bush (Cape Fynbos),
- Mountain sourveld,
- Grasslands of the interior plateau,
- Highveld sourveld and Dohne sourveld,
- Karoo,
- False Karoo,
- Bushveld, and
- Tall sourveld.

The above-mentioned areas take various catchment characteristics into consideration, such as vegetation cover, soil type and permeability. The different rainfall patterns were also used to determine the different catchment zones (Van der Spuy & Rademeyer, 2010).

The rational method could just as easily be used to calculate storm peaks. The rational method is also a deterministic flood calculation method that is used for small catchment, up to 15 km<sup>2</sup> (Van der Spuy & Rademeyer, 2010).

### 3.3.2 *Precipitation*

The flood calculation guidelines compiled by Van der Spuy and Rademeyer (2010) stated that various attributes of the rainfall over the catchment are the primary driver of the flood severity. These attributes included the depth, areal spread, duration of the rainfall, and the variation in the intensity in space and time over the catchment. They defined four processes that determined the type of rainfall that would be experienced, namely orographic lifting, convection, low pressure and fronts.

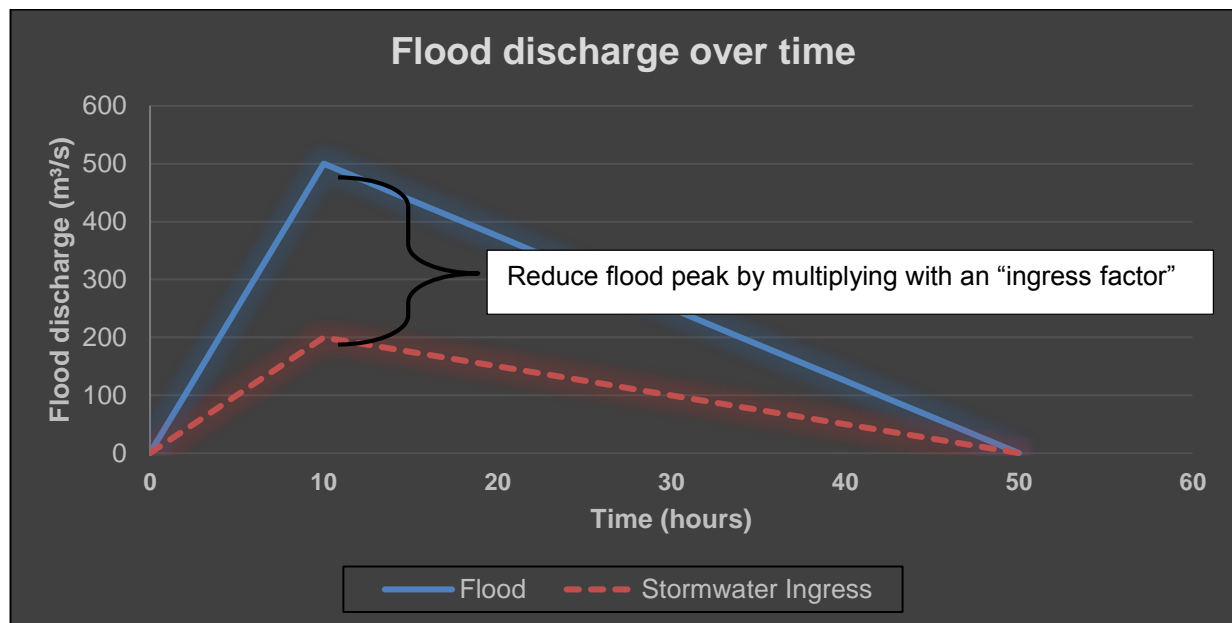
Table 3-1 contains a summary of the description of the processes and the resultant rainfall and flood peaks attributed to each process.

In South Africa, rainfall can occur in summer, in winter or all year round, depending on the location. Thus, for most parts of the country, there is a definite dry season and a wet season. In areas where there is year round rainfall, there is a wet season and a wetter season. During the dry season, there is little or no rainfall and stormwater inflow is zero during those periods.

**Table 3-1 Rainfall processes in South Africa (concept presented by Van der Spuy & Rademeyer, 2010)**

Process	Description	Resultant rainfall & floods
Orographic lifting	Moist air from the ocean moves up a mountain range. As the air rises, it cools down and this results in precipitation. This type of rainfall is typical along the Drakensberg escarpment and the Cape Fold Mountains in the Southern Cape.	Low intensity, long duration. Will produce a smaller but prolonged flood peak.
Convection	Differential heating leads to air pockets that rise rapidly. As the air rises, it becomes saturated and clouds and precipitation occur. This type of process is most often experienced in summer over the interior of the country.	High intensity, shorter durations. Thunderstorms and hail are associated with this process. Can produce large peaks of short duration, especially in built up areas.
Low pressure (cyclonic)	When a low pressure system drags moist air from the oceans over the land, precipitation occurs. This type of process affects the north eastern part of the country, but can also have devastating impacts on the Eastern and Southern Cape regions.	High intensity rainfall over a prolonged period. Cause major, catastrophic flood events.
Fronts	Cold air from the polar regions moves towards the Equator. The cold air moves faster than the warm air in front of it, and pushes the warm air up. As the warm air cools, it causes precipitation. This type of process typically affects the Cape Peninsula during winter.	High intensity rainfall of medium duration is experienced. This type of process often also results in snowfall. High flood peaks can be expected.

Figure 3-3 contains a typical unit hydrograph which shows how a short rainfall event has produced a flood peak flow event. The hydrograph can be modified by multiplying with some 'ingress factor' to determine the stormwater ingress into the sewer network.



**Figure 3-3 Typical unit hydrograph (adapted from Van der Spuy & Rademeyer, 2010)**

### 3.3.3 Quantifying stormwater inflows

Table 3-2 reviews various guidelines on stormwater inflows into sewer networks. For the purpose of the modelling, the following methods could be used to calculate the stormwater ingress:

- stormwater inflow as some percentage of ADWF
- reduce the flood peak of either the SUH or Rational Method by multiplying by some 'ingress factor' (note: the author did not find any literature on such an 'ingress factor'),
- omit stormwater from calculations during the dry season.

**Table 3-2 Stormwater inflow design guidelines**

Source	Allowance for stormwater	Comments
(Van Vuuren & Van Dijk, 2009)	30% to 40% of pipe flow area	Worst case scenario is 5 x ADWF
(CSIR, 2009)	1.15 x ADWF	Includes groundwater infiltration
(Department of Environment and Resource Management, 2005)	3.5 to 5 x ADWF	For Queensland, Australia
(Department of Public Works, 2004)	2.5 – 10 x ADWF	South African standard used in goals and military camps
(Environmental Protection Agency, 2008)	Use SUH and some appropriate return factor	Policy includes combined sewers

### 3.4 Groundwater infiltration

Groundwater infiltrates the sewer system through cracks in the sewer pipes and at the joints between pipe lengths. Groundwater can also infiltrate the sewer system at manholes and house-connection joints. There is no easy way to quantify how much groundwater infiltration is acceptable and each site needs to be evaluated to better understand groundwater infiltration. Outfall sewers often run parallel to water courses, and thus the presence of a high groundwater table is highly likely (Stephenson & Barta, 2005).

Sewers are normally designed as a gravity system with the pressure inside the pipes equal to atmospheric pressure. The pressure of groundwater is proportional to the pipe's depth under the water table, and hence the deeper the pipe, the higher the likelihood of groundwater infiltration (Environmental Protection Agency, 2008).

Leaking water mains in the vicinity of a sewer line could also result in high groundwater tables. Table 3-3 contains a summary of various design guidelines for groundwater infiltration.

Groundwater infiltration results in a steady flow pattern. Groundwater infiltration can show a seasonal variation and could also depend on precipitation (e.g. rainfall events), depending on the permeability of the soil.

**Table 3-3 Groundwater infiltration design guidelines**

Source	Allowance for storm water	Comments
(Van Vuuren & Van Dijk, 2009)	30% to 40% of pipe flow area	
(CSIR, 2009)	1.15 x ADWF	Includes stormwater infiltration
(Department of Environment and Resource Management, 2005)	Not specified	The guideline says that during the dry season the flow between midnight and 4:00 can be assumed to be ground water
(Department of Public Works, 2004)	0.012 – 0.02 l/s/100 m pipe	
(Environmental Protection Agency, 2008)	13 l/second/hectare	Converted from 2000 gal/day/acre
(CTMM, 2010)	0.04 l/minute/m diameter/m length	Includes stormwater infiltration

An alternative method to calculate groundwater infiltration is to use the same formula used to calculate the groundwater infiltration into rivers (Karpf & Krebs, 2004):

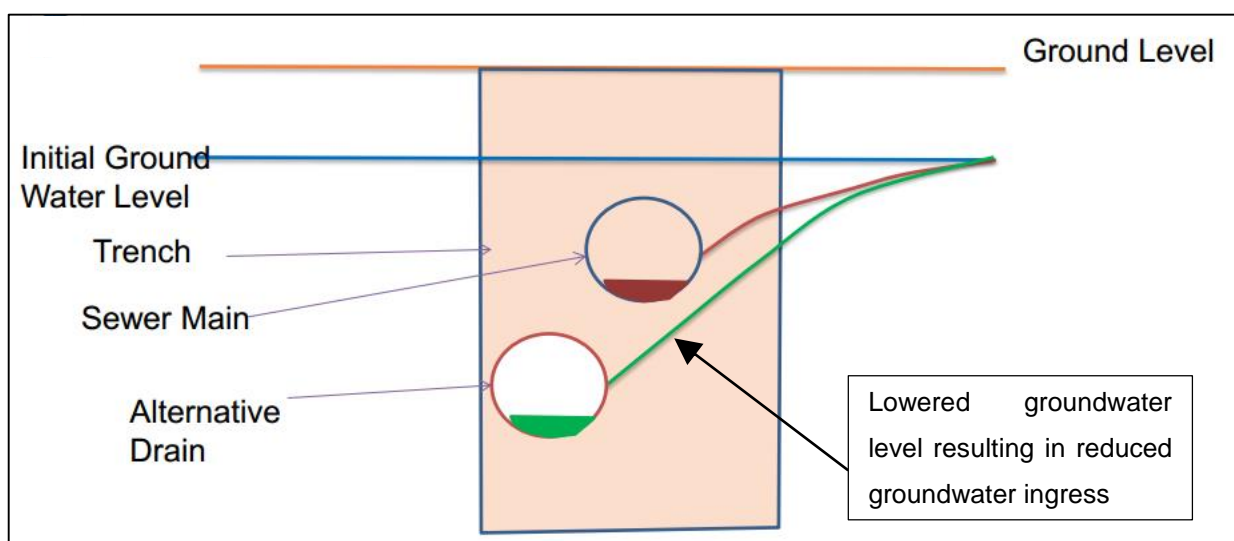
$$Q_{infiltration} = k_L \times A_s \times (h_g - h_s).$$

Where:

- $Q_{infiltration}$  = infiltration of groundwater ( $m^3/s$ )
- $A_s$  = groundwater influenced pipe surface ( $m^2$ )
- $h_s$  = water level of sewer pipes (m)
- $h_g$  = groundwater level (m)
- $k_L$  = leakage factor ( $s^{-1}$ ).

The formula is based on the fact that infiltration is relative to the pressure difference between the groundwater around the pipe and the pressure inside the pipe ( $h_g - h_s$ ). The groundwater leakage factor needs to be calibrated for the catchment in question. The author could not find any research that had been done on this in South Africa.

Some research had been conducted on groundwater infiltration and proposals submitted stating that constructing an alternative subsoil drainage system below the outfall sewer would lower the groundwater table in the vicinity of the pipe. Such a drain pipe would reduce groundwater infiltration significantly (Jayasooriya, 2013). This is shown in Figure 3-4.



**Figure 3-4 Alternative drain pipe for groundwater drainage (Jayasooriya, 2013)**



### 3.5 Normal domestic use

Normal domestic use is generally modelled as a percentage of the annual average of daily demand (AADD) and ranges between 500 l/day and 1 200 l/day. The flow pattern typically indicates very low flows during the early morning hours with peaks between 06:00 and 07:00 and 18:00 and 19:00. This typical pattern is called a diurnal flow distribution. Normal domestic use is discussed in more detail in Chapter 4.

### 3.6 Leaking plumbing

Stephenson and Barta (2005a) claimed that leakage (or base domestic flow) is a relatively insignificant contributor to residential wastewater flow. This was in stark contrast to Maré's (2013) statement that leaking toilets had a massive impact. The reason for the different views could be because Stephenson and Barta investigated leaks with regard to their influence on the sewage conveyance system. At the time, they thought the impact of leaks was minimal. At the time of writing their report, only 49% of all households in the country had waterborne sanitation (flushing toilets connected to a municipal sewer network). It could be that a large portion of those toilets were still new and in good working condition. Maré (2013), on the other hand, investigated leaking toilets from a water resource point of view. According to Maré (2013), leaking toilets consumed water unnecessarily. According to Statistics South Africa's Census 2011 data, 57% of households had waterborne sanitation (Statistics South Africa, 2012), thus an increase of 8% between Stephenson and Barta's report and Maré's findings. It should further be pointed out that the increase from 49% to 57% is an increase of 2 745 000 households that gained access to waterborne sanitation.

Another reason for the difference between Stephenson and Barta's results and Maré's findings could be that many of the toilets were still new during Stephenson and Barta's (2005a) investigation. The toilets could have started leaking after Stephenson and Barta's report was published. The same could be said for a portion of the 2 745 000 toilets that were constructed between 2005 and 2011. Studies by Van Zyl et al. (2008) found that there were serious problems with faulty plumbing fittings and workmanship, and that a large percentage of households had leaks present on the properties.



### 3.6.1 Leakage theory

According to the orifice theory, as presented by Van Zyl (2009), the derived equation for calculating the leak flow rate is:

$$q = C_d A H^\alpha$$

Where

- $q$  = leakage flow rate (m<sup>3</sup>/s)
- $C_d$  = leakage coefficient
- $A$  = area of hole in pipe (m<sup>2</sup>)
- $H$  = pressure head in pipeline (m)
- $\alpha$  = leakage exponent (varies from 0.5 to 2.79, median value of 1.15)  
(Coetzer et al. 2004).

This equation can be used to determine the flow rate of household plumbing leaks. In general, household connections are smaller than 30 mm Ø and thus  $A$  in the above equation has very little contribution to the leakage flow rate. The flow rate is proportional to the pressure in the network. Thus, a reduction in pressure would also result in a reduction in the leakage flow rate.

### 3.6.2 Extent of plumbing leaks

As part of a WRC project, Lugoma et al. (2009) conducted an assessment of the water meters on 182 properties in Johannesburg. There were 117 properties (67%) that had measurable on-site leakages and the average flow rate for those properties was found to be 40.7 l/hour or 30 kl/month. If this leakage is spread out over all 182 properties, the leakage rate averages out to 24.2 l/h or 17 kl/month.

### 3.6.3 Substandard components and poor workmanship

The study by Van Zyl et al. (2008) found that approximately 50% of all plumbing components were not meeting the SANS 10252 and 10254 requirements, and were thus illegal. The reasons for the high prevalence of sub-standard plumbing fittings were the following:

- The lack of competent building inspectors meant that there was no enforcement of legislation.
- South Africa currently does not prohibit the import and sale of non-compliant products.

- The non-compliance was found to be higher in low-income areas and in some cases, where government was the implementing agent for large housing developments, government had paid for these non-compliant products.

One of the major product concerns highlighted by Van Zyl et al. (2008) (that was of particular interest to this research), was that the non-compliant products were prone to fail more frequently and the life span of the products were shorter than compliant products. In houses built by government under the Reconstruction and Development Programme (RDP), the failure of most products occurred within 18 months of being installed. The result of the failure was that 50% of the toilets investigated had already been leaking less than two years after installation.

#### *3.6.4 Vandalism, theft and poor maintenance*

A national survey conducted by De Swart and Barta (2008) indicated that toilets were often vandalised and the ball float removed. The ball float is used to automatically close the inlet valve to the toilet cistern, and if it was missing, the valve never shut off, resulting in the water running constantly. In discussions on the matter with municipal maintenance teams, the author heard that the copper fittings were often stolen and sold as scrap metal.

In discussions with municipal workers regarding the high leakage rates of residential toilets in low income townships, the author was informed that many people in the community claimed that it was too expensive to get a plumber to fix their toilets. The people in the community believed that the water was free, and thus saw little reason to incur the expense to maintain the infrastructure. This attitude, coupled with poor quality products and workmanship, led to many leaking toilets in lower income township areas.

#### *3.6.5 Toilet leak flow rate*

An important question that needed to be answered by this report was how much water was lost through leaking toilets. To determine this, the toilet flow rate was required. Two variables required to calculate the toilet flow rate were the size of the pipe supplying the toilet and the pressure in the system.

A study by Scheepers (2014) found that the average toilet flow rate was 0.245 l/s. In toilets where the ball valve was vandalised or stolen, 2 kl of potable water could be flowing through the consumers' cistern and wasted on a daily basis. Where consumers were not being metered, the utility had no way of knowing the extent of the on-site leakage problem until the utility investigated and understood the night flows into the area and the SBF out of the area. Once the

utility had identified high night flow and base flow zones, the utility needed to conduct a household survey to determine the location of plumbing leaks.

### **3.7 On-site leaks and non-revenue water**

On-site water leaks had, until recently, been difficult to quantify and at most previous scholars had acknowledged that on-site leaks existed. A study by Couvelis et al. (2014) found that between 60% and 80% of low-income households in South Africa had on-site leaks. The trend for medium and high income households ranged between 20% and 70%.

Previously known attempts to quantify on-site leaks for residential properties are listed here:

- 25 l/hour (Lugoma, et al. (2008),
- 0.15 l/minute (GLS, 1997), and
- 0.06 l/minute (Hine & Stephenson, 1985).

Leaking taps and cisterns discharge directly into the sewer system and thus a leaking toilet relates to the sanitation field. Leaking toilets also relate to the field of WCDM. There is thus an intersection of different disciplines within the water sector.

## 4 RESIDENTIAL END USES OF WATER

### 4.1 Residential end-use studies

The standard practice is that residential consumers' water consumption is metered, and monthly readings are taken to determine the monthly consumption (in order to bill the user for that month's consumption). Researchers used questionnaires completed by residential consumers to determine daily water consumption patterns. However, in the past, researchers had to make estimated guesses when calculating the residential end-use patterns of consumption (Heinrich, 2006).

Technological advances in measuring and recording water consumption data had improved significantly since the early 2000s. Not only had the advance in technology made it possible to measure water consumption continuously (in some cases at 10 second intervals), but it had also become much more affordable to do so. Water researchers had been able to measure individual water use in houses (either by metering each individual water outlet, or by flow trace analysis) and this had led to a more scientific approach to measure end uses (Heinrich, 2006).

In the introduction to the Residential End-Use Measurement Guidebook, Giurco et al. (2008) defined residential end use (REU) as follows "Residential end-use measurement is concerned with understanding where and how water is used in the home". Jacobs and Haarhoff (2004) defined end use as "... the smallest identifiable use of water on a stand".

REU studies, therefore, break domestic water consumption down into the smallest practical measurable consumption for each end use by a residential consumer. The end uses referred to are typically the following: showering, bathing, flushing toilets, irrigating gardens and washing cars. Table 4-1 contains a list of various indoor end uses commonly proposed and studied in literature. The residential consumer could be living on a stand with a garden and swimming pool, or could be staying in a block of flats. Either scenario could be evaluated.

**Table 4-1 Proposed indoor end uses and accompanying return flows**

Indoor end uses	Water consumer per end use event (l/person/day)						Return flow (to sewer)
	Athuraliya et al. (2012) Summer 2012	Beal and Stewart (2011)	Aquacraft (2011) CALMAC – low income (%)	Gascon et al (2010) (l/hh/day)	Heinrich (2006) only quotes other sources e.g. AWWA	Jacobs & Haarhoff (2004)	
Bath	1.6	3.0	2%	n/a	2%	39-189	100%
Bathroom basin	20	3.0	n/a	129	18.2%	0.3-60	100%
Dishwasher	1.0	5.9	0.4%	2	1.7%	15.1-43	100%
Kitchen sink	n/a	n/a	22.9%	n/a	n/a	0.6-73	90%
Leaks	n/a	13.2	13.6%	30	n/a	27.4	100%
Miscellaneous indoor	1.5	n/a	3.6%	n/a	2.7%	-	90%
Shower	30.9	24.6	21.4%	66	19.4%	7.6-303	100%
Toilet flush large	19.0	16.4	17.7%	74	30.9%	8-26.5	100%
Toilet flush small	n/a	n/a	n/a	n/a	n/a	2-6.1	100%
Washing machine	18.3	28.2	18.4%	33	25.1%	60-200	100%

Giurco et al. (2011) stated that REU studies had been used for generating and understanding information about technological and behavioural aspects of household water use to determine how much of the overall water used by a household can be attributed to individual end-uses. A useful output of a REU study was accurate hourly demand patterns (Aquacraft, 2011). These patterns show how each household use varied over a typical day, and when the peak usage occurred and when the minimum usage occurred.

End-use investigations had gained increasing popularity in recent times, as can be seen from the snapshot of some recent end-use studies as presented in Table 4-2. It is important to state that this snapshot is not meant to give an exhaustive list of studies. It is interesting to note that, in their report, Beal and Stewart (2011) had a similar table with 10 completely different REU studies.

**Table 4-2 Snapshot of REU reports in recent times**

Region	Report Title	Authors	Number of End-Uses*	Date
Yarra Valley, Australia	Residential water use study	Athuraliya, Roberts and Brown	11 (9)	2012
Queensland, Australia	South East Queensland residential end use study	Beal and Stewart	8 (7)	2011
California, USA	End use water demand profiles	Aquacraft	8 (8)	2011
Spain	Urban water demand in Spanish cities by measuring end uses consumption patterns	Gascon et al.	6 (6)	2011
New Zealand	Residential water end use literature survey	Heinrich	10 (7)	2006
South Africa	Structure and data requirements of an end-use model for residential water demand and return flow	Jacobs and Haarhoff	16 (12)	2004

\* Note: the number in brackets indicates the number of indoor end uses.

Jacobs and Haarhoff (2004) had the most comprehensive list of end uses and the indoor end uses are listed in Table 4.1. There are generally eight or nine indoor end uses.

## 4.2 REU parameters

REU is affected by various factors. These factors vary from the event volume, to the number of events, to the socio-economic and demographic make-up of the end users. There are also seasonal variations in water use as well as weekly variations in water use (weekday versus weekend).

## 4.3 Socio-demographic factors that influence water use

The following factors influence household water consumption:

- household size and age of occupants,
- household income and employment status,
- garden size and irrigation behaviour, and
- rainfall patterns and surrounding climate (Heinrich, 2006).

## 4.4 Embedded energy in water

Embedded energy in water is “the amount of energy that is used to collect, convey, treat and distribute potable water to end-users (*sic*) and the amount of energy that is used to collect and transport used water for treatment prior to safe discharge of the effluent” (Aquacraft, 2011). There is growing support for the theory that saving water saves energy. REU studies were used to quantify the relationship between saving water and saving energy (Aquacraft, 2011).

Beal and Stewart (2011) called this the “Water-energy nexus”. They also mentioned that certain water consuming events also used energy (washing machine, dishwasher and heating water). For the purpose of this report, it was important to note that, should it be proven that significant numbers of toilets were continuously leaking, there was an opportunity to not only fix the toilets and save water, but also to save electricity (the embedded energy in the water).

## 4.5 Daily consumption patterns

The residential end-use studies also determined how the various end uses were spread over a typical day. Most studies ended up with a diurnal pattern and the study patterns were quite similar. The concept of end uses required the planner to model the number of times each appliance or end use was used per day irrespective of how many appliances might have been present. The results from the Aquacraft (2011) report produced an excellent indoor end-use hourly demand profile for low income households. These results were used in the model as discussed in Chapter 7.

## 4.6 Time of use

Some ambitious work is being undertaken in Australia to ascertain the possibility of charging water consumers on a time-of-use basis. The benefits of this could be changing consumer behaviour to reduce peaks, and thus reduce the size of required infrastructure, which was sized to accommodate the peak flow. By changing their consumption behaviour (the time they used appliances) consumers could influence the base flow in sewers.

## 5 STATISTICAL CONCEPTS

### 5.1 Introduction

In this chapter some of the basic statistical concepts that the reader needs to be conversant with in order to understand and compile a Monte Carlo simulation model are reviewed. Some readers may have the required background and choose to move straight on to the next chapter on Monte Carlo simulations.

### 5.2 Stages in a statistical investigation

According to Graham (1999), the field of statistics, generally speaking, seeks to answer questions about the world around us by going through the following stages:

- define the question,
- collect the required data,
- analyse the data, and
- interpret the results.

For example, when the size of an expected flood in a river is calculated, a measuring station could be built in the river to collect data of the flows in the river. The collected data could then be analysed and the expected flow could be estimated. Similarly, to determine what portion of the population in a catchment area regularly used a toilet between 02:00 and 03:00, a questionnaire could be developed and an appropriate number of residents could be interviewed for that catchment area. After the interviews, the researcher could analyse the questionnaires (data) and present some findings on the research.

The question posed in this research was what portion of base flow in sewers originated from leaking toilets. It might initially have seemed to be an impossible question to answer, but using the Monte Carlo simulation method the required data could be collected and analysed for an accurate estimation.

### 5.3 Analyse the data

Once the data had been collected, it needed to be processed and analysed. Data needed to be summarised, represented graphically and described numerically. The data could be referred to as the data set or the data distribution.



## 5.4 Numerical description of data

The first way of describing the data numerically was through some sort of typical value that best represented the data set. This typical value could also be called the average (Graham, 1999) or central tendency (Gravetter & Wallnau, 2000). There were three statistical ways of finding this value, namely the mean, the median and the mode (Graham, 1999).

The second way of describing the data was through the spread or variability of the data. Table 5-1 is a summary of the way the data could be described numerically.

**Table 5-1 Summary of numerical description of the data**

Data description	Central tendency	Variability
Most often used description to describe data characteristic	Mode Median Mean	Range Variance Deviation

## 5.5 Description of central tendency

### 5.5.1 Arithmetic mean

This is the best known way of describing the data, and hence many people refer to the arithmetic mean when using the word 'average'. The arithmetic mean is easily calculated by adding up all the values in the data set, and then dividing that total by the number of values in the data set. The arithmetic mean for a series is defined by the following equation:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

It is important to mention that the mean is sensitive to the outliers (outliers are very high or very low values in the data set). If the dataset is large, the effect out a few outliers on the mean is insignificant. However if the dataset is small, then the mean will be more sensitive to outliers.

### 5.5.2 Mode

The mode is the value that occurs most often in the data. It is possible that two or more modes could be present in any data set. The mode is not sensitive to outliers.

### 5.5.3 Median

If the values of the data set are arranged from smallest to largest, the median is the middlemost value in the re-arranged data set. The median is thus a value for which the probability of any number being smaller than the median is 0.5 (50%) and any number being larger than the median is 0.5 (50%).

For an even number of values the median is calculated as follows:

$$m = \frac{\left(\frac{n}{2}\right) + \left(\frac{n}{2} + 1\right)}{2}$$

For an uneven number of values the median is calculated as follows:

$$m = \left(\frac{n}{2} + 1\right)^{th} \text{ ordered}$$

The median is not sensitive to outliers.

## 5.6 Description of variability

### 5.6.1 Range

The range is a measure of the difference between the highest and the lowest value in a distribution. The highest value is called the upper extreme value ( $E_V$ ) and the lowest value is called the lower extreme value ( $E_L$ ). The range is thus  $E_V - E_L$ .

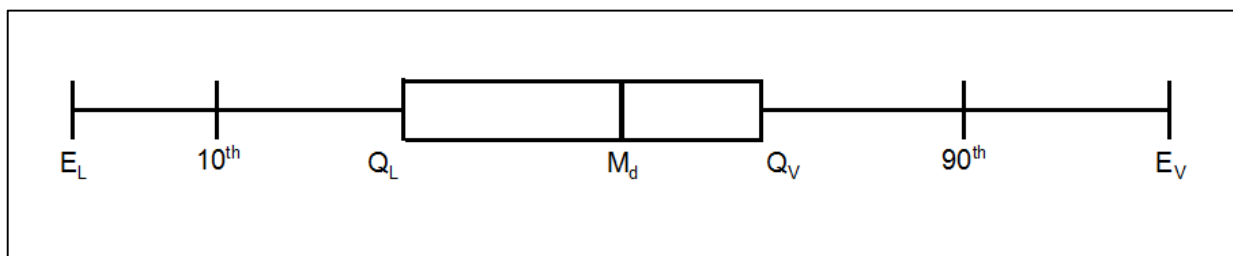
The range is sensitive to extreme values (outliers). Engineering judgement is required when dealing with outliers. The outlier could be an incorrect recording of data, for example an incorrect recording of a decimal place, or the outlier could be an accurate recording of an anomaly. If the outlier seems to be a recording error, then it makes sense to discard the outlier and work with the trimmed down dataset. If the outlier is an anomaly then it could (and in most cases should) be included. In the model results, discussed in Section 8.5, the shower end use had an extreme value of 8 576 l/hh/day. Whilst it is in theory possible for a household to consume that much water for showering, it could be argued that this upper extreme value can be safely discarded. Whether or not a researcher decides to eliminate outliers, the researcher should have valid reasons for either keeping, or eliminating the outliers (Researchgate, 2014).

### 5.6.2 Percentiles

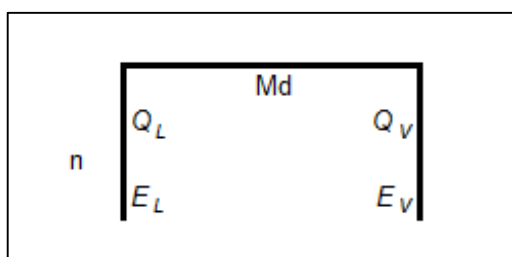
The data can be broken down into hundredths. A percentile is that portion of the data that lies below a specific value. The median is interestingly the 50<sup>th</sup> percentile, because 50% of the values in the data set lie below the median. Similarly, when 90% of the values lie below a certain value, the value is called the 90<sup>th</sup> percentile. Some common percentiles are:

- 5<sup>th</sup> percentile,
- 10<sup>th</sup> percentile,
- 25<sup>th</sup> percentile (or lower quartile ~  $Q_L$ ),
- 50<sup>th</sup> percentile (or median ~  $M_d$ ),
- 75<sup>th</sup> percentile (or upper quartile ~  $Q_V$ ),
- 90<sup>th</sup> percentile, and
- 95<sup>th</sup> percentile.

Percentiles and range are often summarised using 'boxplots' (Figure 5-1) and 'five figure summaries' (Figure 5-2).



**Figure 5-1 Boxplot of range and percentiles**



**Figure 5-2 Five figure diagram summary**

The inter-quartile range,  $d$ , is calculated by subtracting  $Q_L$  from  $Q_V$ . The total range is calculated by subtracting  $E_L$  (extreme lowest value) from  $E_V$  (extreme highest value). The standardised range is calculated by dividing the range by the median.

### 5.6.3 Variance

The variance is the measure of each value relative to the mean value. This measure relative to the mean can also be called the deviation from the mean. The deviation is thus:

$$d = X_i - \bar{X}$$

Thus, when the value in the data set is higher than the mean, the deviation is positive; when the value in the data set is lower than the mean, the deviation will be negative.

When all the deviations are small, the data lies close to the mean and there is little variability in the data set.

The variance is calculated as follows:

$$\sigma^2 = \frac{\sum (X_i - \bar{X})^2}{N - 1}$$

### 5.6.4 Standard deviation

The standard deviation is the square root of the variance and is calculated as follows:

$$\sigma = \sqrt{\sigma^2}$$

## 5.7 Graphical representation of the data

A very common way of representing the data is by means of a graph. Various types of graphs are used to represent different aspects of the data. The following are ways of representing the data:

- bar charts/histograms,
- histograms,
- frequency distributions, and
- cumulative distributions.

The above list of graphs are ordered in the way that data is typically analysed. Graham (1999) mentioned that the type of graph ultimately depended on two factors:

- the type of data being represented, and
- the type of statistical judgement being made.

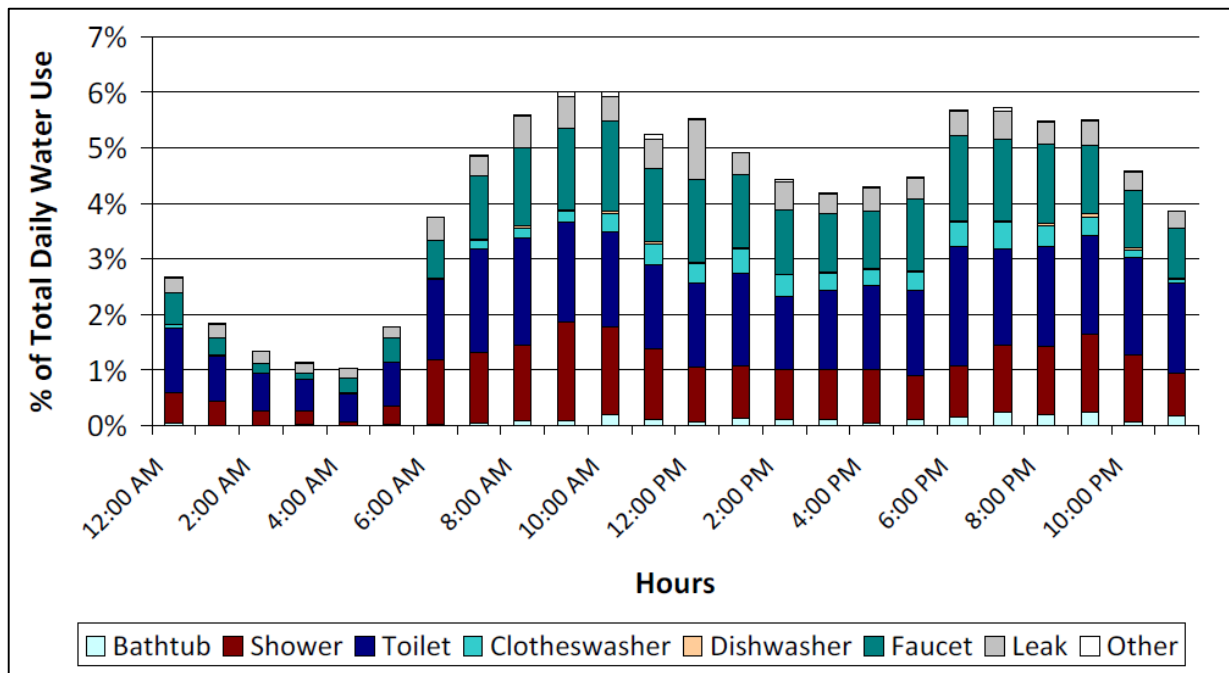
The data could be discrete or continuous. Household size is a typical value which is discrete – there could be either three or four people living in a house, but there cannot be 3.678 people living in a house. An example of continuous data is the volume of sewage flow passing through a manhole. While the accuracy of the measuring device may be limited, there is, in theory, an infinite number of flow rates that could be measured.

The three most common types of judgments made from analysing data are:

- **summarising** the data in a graph which will show the expected central tendency and the range of the data;
- **comparing** the data, e.g. the difference between the base flow data in summer and the base flow data in winter; and
- **relating** different variables to determine the possible relationship between those variables. One of those relationships could be the number of household appliances that generate sewage proportional to the household income.

### 5.7.1 *Bar charts / histograms*

The main distinction between a bar chart and a histogram is that the bar chart is used for discrete data and the histogram for continuous data (Graham, 1999). When two (or more) sets of bar charts are plotted on the same axis, an image is generated in which the reader could easily compare the different sets of data as seen in Figure 5-3.



**Figure 5-3 Bar chart of daily consumption patterns for a household (Aquacraft, 2011)**

### 5.7.2 Scattergraph

These graphs are a set of points plotted by using two attributes of each sample in the data set. Typically, the relationship between household income and the number of sewage generating appliances could be plotted. Scattergraphs are useful for conducting correlation analyses.

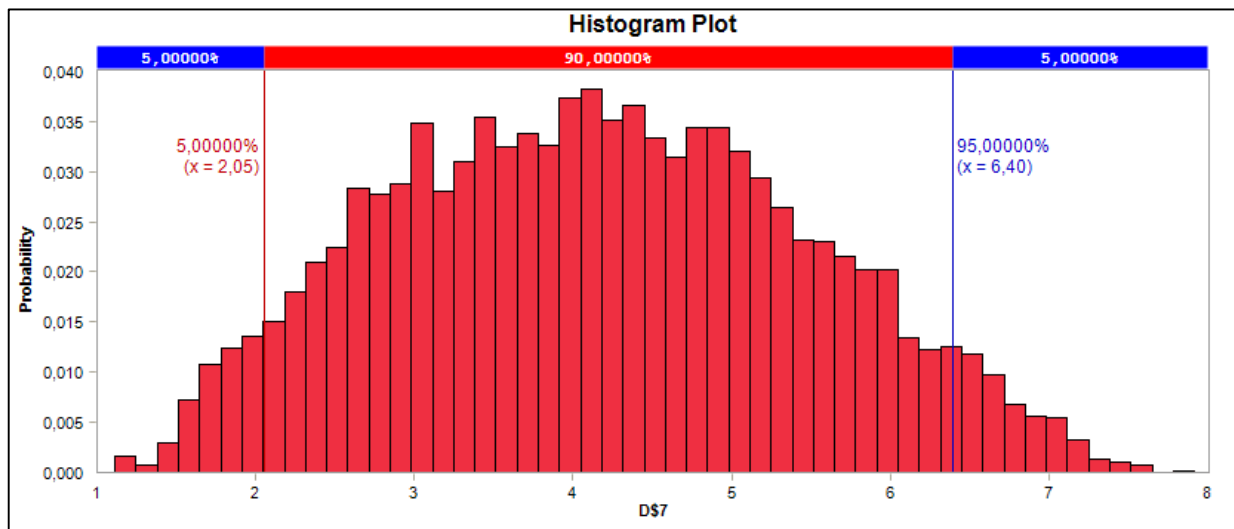
### 5.7.3 Frequency distribution

A frequency distribution is constructed from data by arranging values into classes and representing the frequency of occurrence in any class by the height of the bar (bar of the graph) (Palisade, 2013).

To produce a frequency distribution the data needs to be manipulated and re-arranged. The first step is to rank the data from smallest to largest and then to group the data in appropriate ranges (e.g. flows in 5 l/s ranges). The next step is to divide the number of readings in a particular range, by the total number of readings. For example, if there were three readings in the 0-4 l/s range from a total of 60 readings, the relative frequency of occurrence is 3/60 for flows in that range (Van der Spuy & Rademeyer, 2010). All the relative frequencies for the different ranges are then plotted. Van der Spuy and Rademeyer (2010) further stated that relative frequency of occurrence was a rough approximation of the probability of occurrence.

Frequency distributions, such as the one shown in Figure 5-4, once again summarise the range of the data and give a good indication of the values of central tendency. Frequency distributions

also indicate the *skewness* of the data. The skewness of the data is a measure of where the bulk of the data is in relation to the mean (Van der Spuy & Rademeyer, 2010).

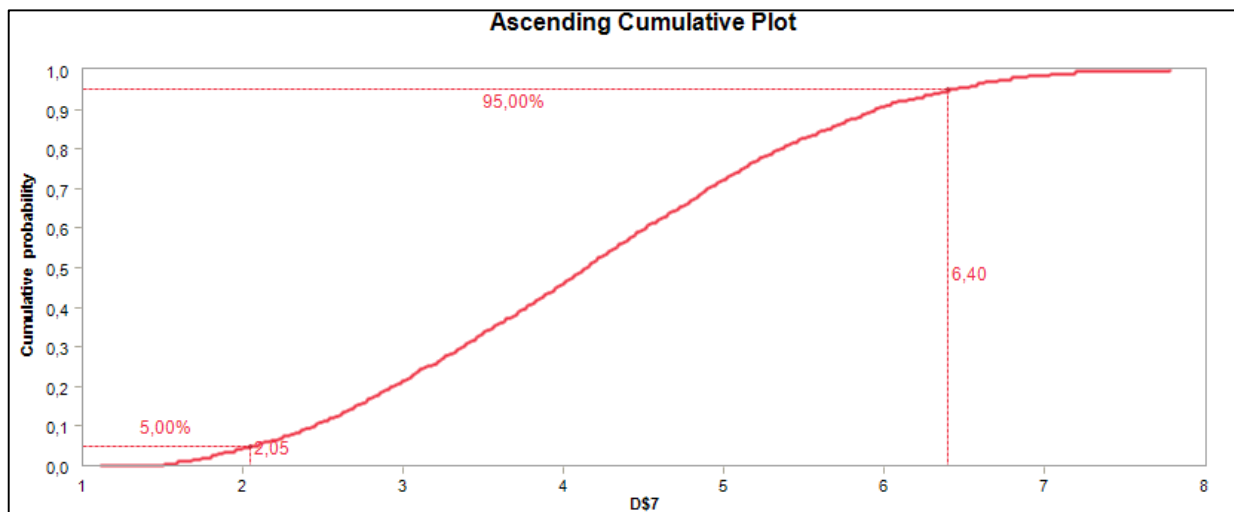


**Figure 5-4 Frequency distribution of household size**

Note: the 5<sup>th</sup> and 95<sup>th</sup> percentiles are indicated as the vertical red and blue lines respectively. The frequency distribution was generated using VOSE software add-on to MS Excel.

#### 5.7.4 Cumulative distribution

The cumulative frequency diagram (Figure 5-5) is obtained by summing the relative frequencies of the various ranges. The cumulative frequency diagram gives a good indication of where the median value is (Van der Spuy & Rademeyer, 2010).



**Figure 5-5 Cumulative frequency of household size**

## 5.8 Probability theory

### 5.8.1 Probability

Probability is a measure of how likely a value or event is to occur (Palisade, 2013).

Probability is a way of describing the likelihood of a particular event taking place. In statistics, probabilities are measured as numbers between zero (0) and one (1). If an event has a probability of zero ( $p=0$ ), then that event is impossible. If an event has a probability of one ( $p=1$ ), then that event is certain to occur (Graham, 1999).

### 5.8.2 Mutually exclusive outcomes

An outcome is mutually exclusive because, when it occurs, no other outcome could occur from that same event. An example could be that when a person woke up that person could either take a bath or shower, or neither of the two. If the person were to bath; showering and none of the two are excluded as options. Exclusivity is normally associated with a single event (Graham, 1999).

### 5.8.3 Adding probability

The sum of all the probability of the exclusive outcomes for an event must add to unity. Using the previous example, if 50% said they had showered and 40% said they had a bath, it stands to reason that 10% did neither.

### 5.8.4 Independence

Two outcomes are independent if the occurrence of one does not affect the likelihood of the other occurring. An example could be that whether a person took a bath or showered in the morning did not say anything as to the likelihood of them using a toilet. Independence is normally associated with the outcomes of two separate events (Graham, 1999).

### 5.8.5 Multiplying probabilities

When calculating the probability of two independent outcomes occurring, the probabilities of the two outcomes are multiplied to obtain the answer. Taking the example in Section 5.8.4 above, if the probability of having a bath is  $P(b)$  and the probability of going to the toilet is  $P(t)$ , the probability of bathing and going to the toilet is  $P(b) \times P(t)$ .



## 5.9 Probability distributions

### 5.9.1 Probability density function

The probability density function (PDF) is derived from the frequency distribution by applying a continuous mathematical function that best fits the frequency distribution. This mathematical function  $f(x)$  is known as the PDF. An important property of the PDF is that the area under the graph would be one (Van der Spuy & Rademeyer, 2010).

### 5.9.2 Cumulative distribution function

Similarly, the cumulative distribution function (CDF) is derived from the cumulative distribution by applying a continuous mathematical function that best fits the cumulative distribution. This mathematical function,  $F(x)$ , is known as the CDF. An important property of the CDF is that its minimum value is 0, and its maximum value is 1 (Van der Spuy & Rademeyer, 2010).

### 5.9.3 Probability distribution parameters

Mathematicians and statisticians had studied various distributions. Three important parameters help define some of the popular distributions:

- shape parameter ( $\alpha$ ),
- scale parameter ( $\beta$ ), and
- location parameter ( $\gamma$ ).

Some parameters might only apply to certain distributions, while other distributions might contain all three parameters.

The skewness of a distribution is a uniquely shaped parameter that measures the symmetry of the distribution (Van der Spuy & Rademeyer, 2010). Data could be either positively or negatively skewed. The closer the skewness is to zero, the closer the mean and the median are to each other.

#### 5.9.4 *Distribution fitting*

Once the data has been analysed, the researcher must ask if it can be defined by existing distribution models. To this end various goodness-of-fit tests have been developed. Three goodness-of-fit tests are often used:

- Chi-squared test,
- Anderson-Darling (A-D) test, and
- Kolmogorov-Smirnov (K-S) test.

Some of the more common distributions discussed in the next chapter are:

- normal distribution
- log-normal distribution
- exponential distribution
- gamma distribution
- Weibull distribution
- log-logistic distribution
- Rayleigh distribution
- triangular distribution
- PERT distribution
- uniform distribution
- continuous distribution.

## 6 USING MONTE CARLO SIMULATIONS

### 6.1 Brief history of the Monte Carlo method

In 1777, Count Buffon proposed a method of calculating  $\pi$  ( $\pi$ ) using needles dropped on a floor to generate random numbers. This method required a great number of computations and was thus not practical for common use (Lapeyre et al. 2003).

However, with the advent of computers and the means of easily computing the iterations, it was now easy to solve problems using the methods proposed by Count Buffon. It is widely accepted that Monte Carlo simulations were used in the Manhattan Project, with the result being the construction of the first atomic bombs at the end of World War II (Lapeyre et al. 2003). Nicholas Metropolis and Stanislaw Ulam, two mathematicians who worked on the atomic bomb design, published a paper called the “Monte Carlo Method” in 1949. They named this method after the capital of Monaco, which was a gambling hotspot (Dunn & Shultis, 2012).

Monte Carlo simulation was used by mechanical and chemical engineers to simulate the movement of gases and atoms. It is also suited for risk analysis, and thus widely used in the financial field. In recent years, water services planners have been using Monte Carlo simulations in calculating demand (Claassens, 2007).

Dunn and Shultis (2012) summarised the Monte Carlo method as “a methodology to use sample means to estimate population means”. Using the @RISK software, a computer generates a random number based on a pre-determined probability distribution. This number is used to calculate a result. The computer could be programmed to repeat this exercise any number of times, (say 10 000 if desired) using randomly generated numbers. The computer stores the results of all the iterations, thus creating the ‘sample means’. The sample is then used to create a new probability distribution that is used to calculate the ‘population means’.

Monte Carlo simulation is a virtual experiment that repeats a process or project or situation a large number of times, and generates a large number of random samples bound by a specific set of parameters.

Those random samples are collected and then organised and analysed to help understand the behaviour of a simple or complex system or process (Sich, 2013).

The value of using Monte Carlo simulation is that many 'unknowns' can be accommodated by means of a probability distribution that describes any particular unknown. The modelling requires thousands of iterations to build a data set that will provide results with a 95% certainty (Claassens, 2007). However, should the wrong data be entered into the model, the results would be meaningless and, more importantly, misleading.

## 6.2 Monte Carlo method in civil engineering

Monte Carlo simulations have been gaining popularity in civil engineering applications, especially in calculating demand. On their website, the manufacturers of @Risk present two case studies where @Risk was used to assist civil engineering planners with water demand calculations. Civil engineering students at the University of Stellenbosch had also used Monte Carlo simulations in postgraduate research to determine residential indoor demand (Scheepers, 2014), the benefits of rainwater harvesting (o'Brien, 2014) and residential outdoor demand (du Plessis, 2014).

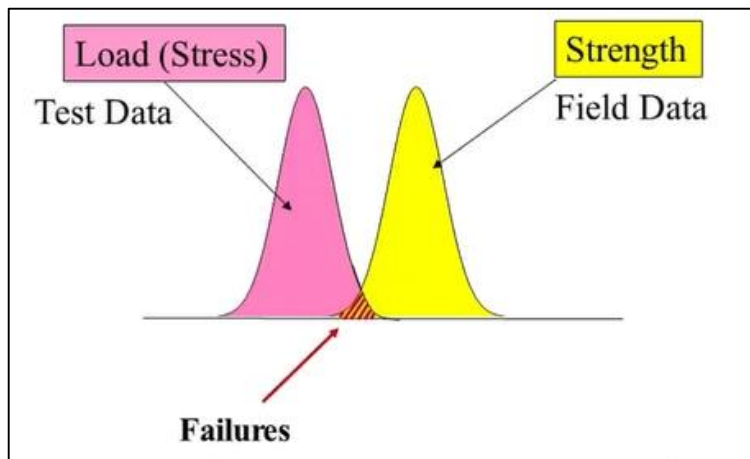
Chapters 3 and 4 demonstrated that there were many variables associated with calculating the SBF, namely:

- legitimate domestic consumption of potable water and the generation of wastewater,
- leaking plumbing fittings,
- stormwater inflow, and
- groundwater infiltration.

Chapter 7 describes the methodology for setting up the Monte Carlo model in MS Excel using the @Risk add-on. All of the above variables could be programmed into the model.

## 6.3 Reliability engineering

Mechanical engineering has a specialised field of reliability engineering. In this field, engineers determine the reliability of a vehicle component. In order to calculate the reliability, the engineer must determine the relationship between the load on the component and the strength of the component. When the strength is more than the load, the member is safe for use. When the load exceeds the strength, the member fails. This is illustrated in Figure 6-1, where both the load and the strength are described as probability functions. Where the two functions overlap, the risk of failure is high.



**Figure 6-1 Reliability engineering (Kleyner, 2013)**

If this concept is extrapolated to the civil engineering sphere, the load on a sewer pump station and WWTW is calculated during the design process. The pump station and WWTW are built to accommodate the load. However, if the toilets started leaking, this additional load was not included during the design stage, and hence the load on the infrastructure exceeded its design capacity. The result was that pump stations and WWTW overflowed (failed) and did not function correctly.

Another application could be if the modeller constructed and ran an SBF model for a certain catchment zone. The data would then be measured in the field. If the field measurements were significantly greater than the modelled results, the modeller could inform maintenance that there was a problem with high SBF.

## 6.4 Typical probability distributions

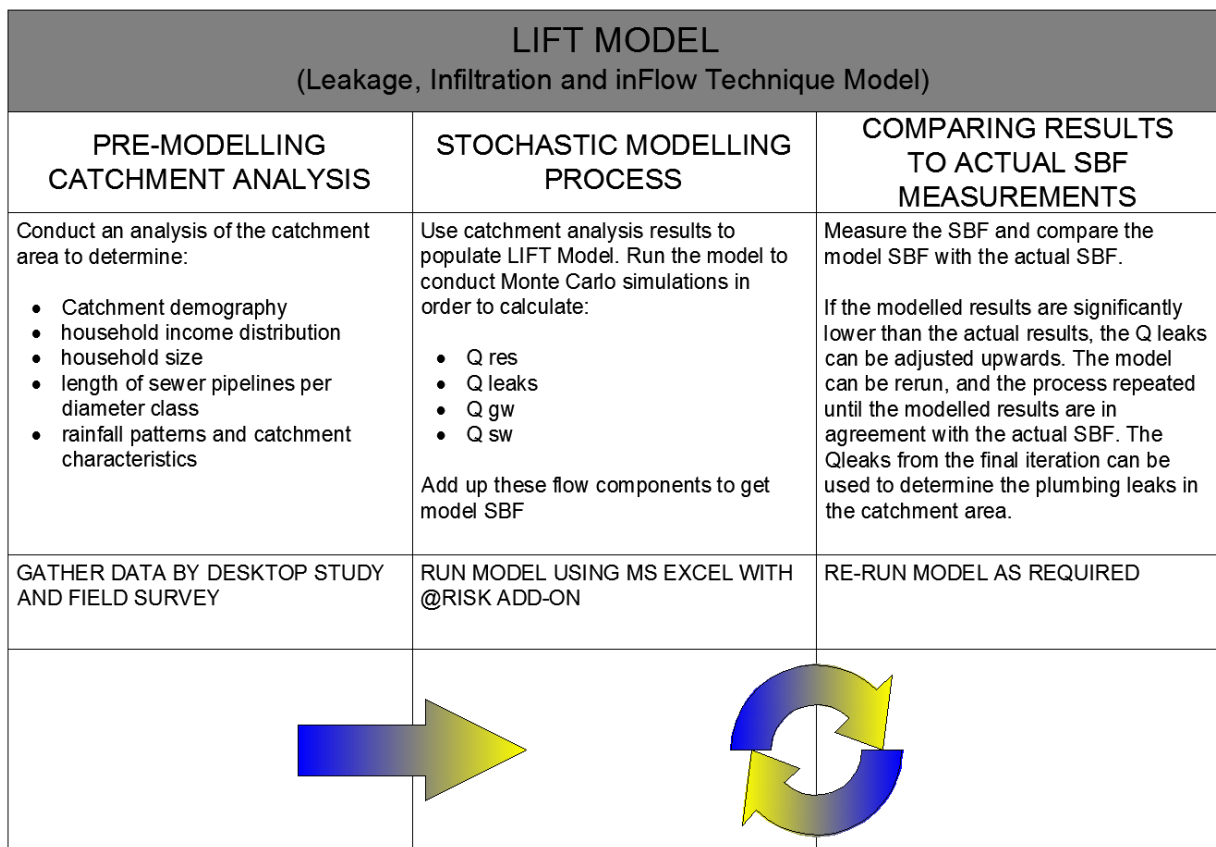
The @RISK software contains many different probability distributions. Eight probability distributions are used in the model, namely

- uniform distribution,
- triangle distribution,
- PERT distribution,
- log-normal distribution,
- log-logistic distribution,
- Rayleigh distribution,
- Weibull distribution, and
- gamma distribution.

Annexure A contains a brief discussion on each of these distributions. To the untrained eye, some of the distributions would look very similar.

## 6.5 Monte Carlo model

The model developed as a part of this research is called the Leakage, Infiltration and inFlow Technique (LIFT) Model. The LIFT Model integrates the concepts of residential end use (REU) modelling, groundwater inflow and stormwater ingress, by stochastically modelling each of the SBF components. Each SBF component is described as an inflow probability distribution function (PDF). Each iteration of the model selects a random sample from within the PDF to use for calculating the SBF for that iteration. Figure 6-2 contains a concept of the LIFT Model process.



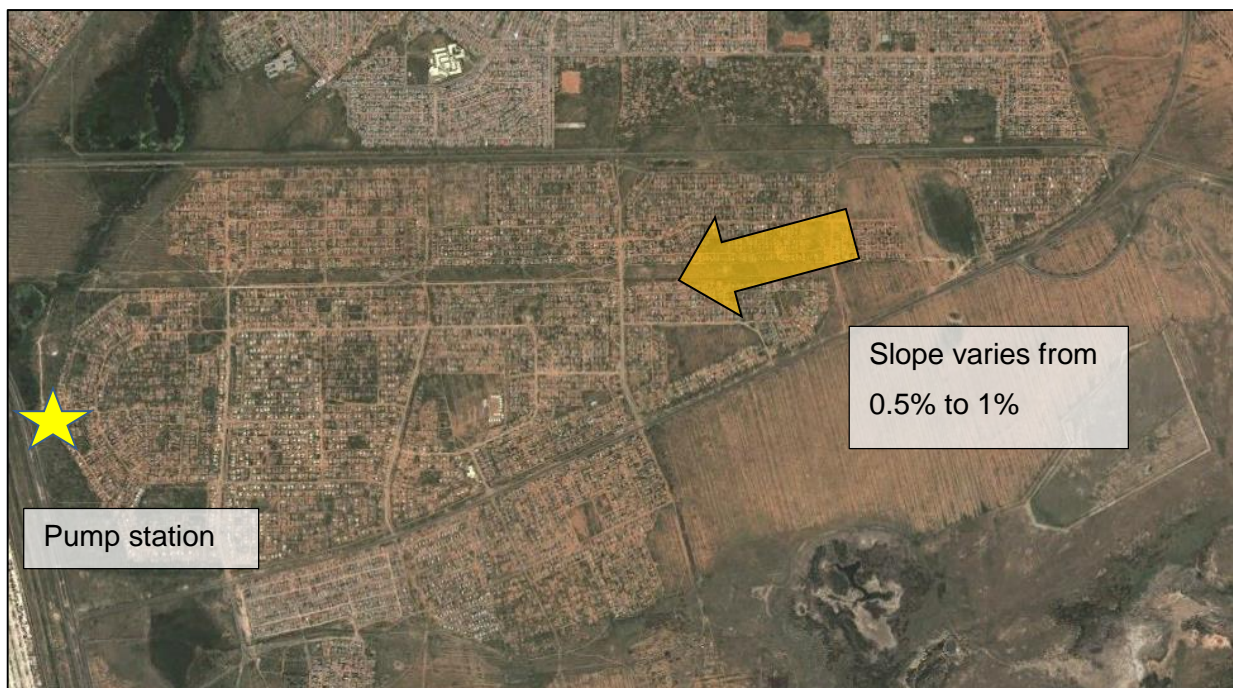
**Figure 6-2 LIFT Model process**

The first main process in the model is to determine the catchment characteristics, rainfall data, household data and rainfall patterns. This information is used to populate the model during the second main process. During the second process, the various inflow components are calculated using REU methods and Monte Carlo simulations. Once the SBF has been determined, compare the SBF to the modelled results; if they differ by more than 10%, adjust the leakage rates and run the model again. Repeat this iteration until the model results and the measured flow rate are within 10% of each other.

## 7 MODEL DEVELOPMENT

### 7.1 Case study site

A township in the Free State, as shown in Figure 7-1, was chosen to model the inflow into the sewer system. The township has 5 500 low-income residential stands and 55 other stands, including schools, businesses and other institutional stands. The stands currently have on-site sanitation (pit toilets) and the municipality plans to improve the level of service to full waterborne sanitation. The whole catchment will drain to a pump station which will be constructed to accommodate the peak flow.



**Figure 7-1 Google Earth image of proposed layout (Google, 2014)**

The proposed sewer design for the township will include the following length of pipes being installed:

- 160 mm Ø – 48 085 m
- 200 mm Ø – 9 175 m
- 250 mm Ø – 3 580 m
- 315 mm Ø – 2 040 m.

The area is relatively flat, with a gradient ranging from 0.5% to 1%. Relatively deep sewers would be required to drain the area. The groundwater table is high and the sewers would mostly



lie below the groundwater table, especially during the rainy season. The area has a summer rainfall pattern, with little or no rainfall during the winter months.

This township is typical of many informal townships that are being formalised in South Africa. The results should thus be suitable for benchmarking against other similar townships.

In Chapter 1, the following formula was presented to calculate the SBF rate:

$$Q_{base} = Q_{leaks} + Q_{gw} + Q_{sw} + Q_{res}$$

The following methodology for calculating the base flow rate ( $Q_{base}$ ) was developed by the author. Firstly, the normal residential flow ( $Q_{res}$ ) and plumbing leaks ( $Q_{leaks}$ ) were calculated using an end-use model and stochastic methods. Secondly, the groundwater infiltration ( $Q_{gw}$ ) and stormwater inflow ( $Q_{sw}$ ) were calculated. These calculations were executed in 8 steps. An additional 9<sup>th</sup> step to measure the future flow is also discussed in this chapter. The 9 steps were:

1. Set up household size and income functions.
2. Set up indoor end-use presence functions.
3. Set up end-use volume and frequency functions.
4. Calculate the AADD per end use.
5. Calculate the AADWF per end use.
6. Distribute the AADWF per end use over 24 hours.
7. Add up the end uses to get a total residential AADWF.
8. Add groundwater infiltration and stormwater ingress.
9. Measure flows to compare modelled flows to real flows.

Each of the above-mentioned steps is discussed in detail under its own sub-section in the remainder of this chapter. Steps 1-8 were modelled in MS Excel and the @RISK add-on to MS Excel. The @RISK add-on was used to generate 10 000 iterations for various modelled scenarios. Where practically possible, each step was allocated its own sheet in the MS Excel file. The 9<sup>th</sup> step was included so that the model could be tested to compare the actual flows with the modelled flows once the sewer network had been installed for the area.

## 7.2 Set up household size and income functions

The income level of the household would determine the type of end use present in the household and the number of people staying in the household would determine the number of



events per end use per day. Three different household income categories were chosen, namely high income, medium income and low income. Each income category was further split into small and large households. Table 7-1 below contains the six different types of households used in the model.

**Table 7-1 Household size function variables (*number of people per household*)**

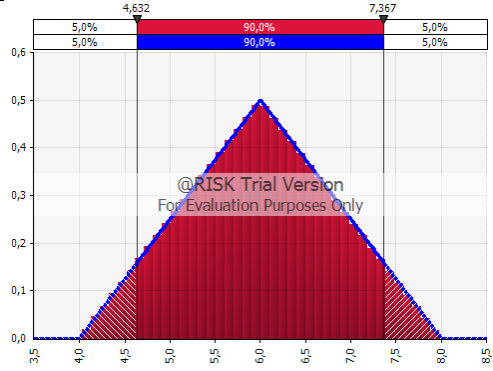
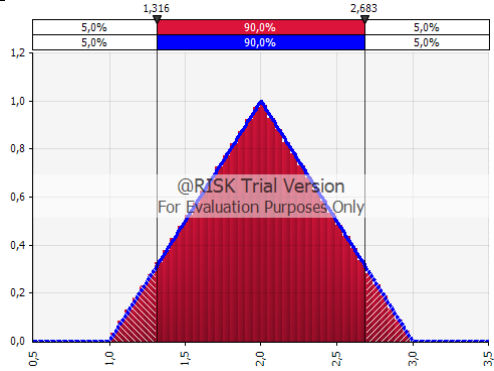
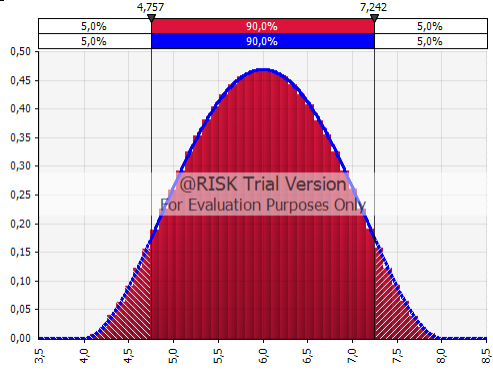
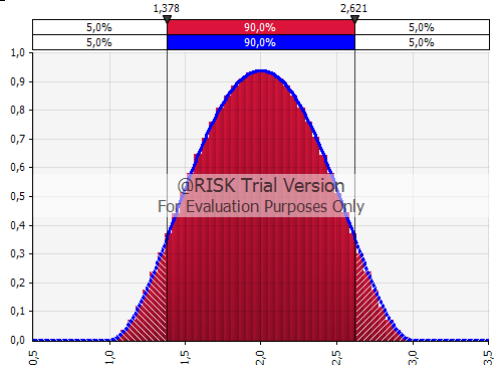
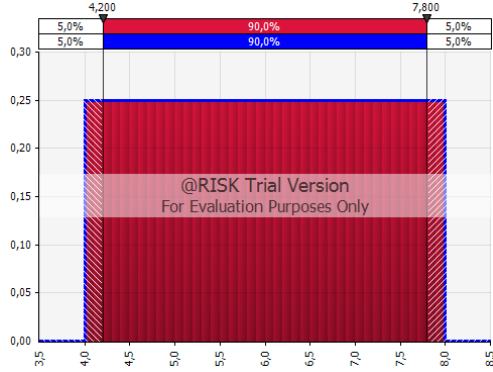
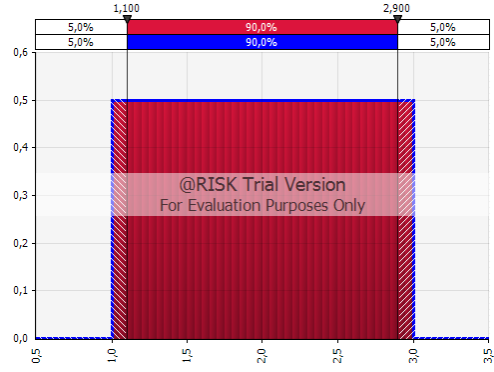
Income category	Household size	Low	Most likely	High
High income	Large	4	6	8
	Small	1	2	3
Medium income	Large	4	6	8
	Small	1	2	3
Low income	Large	4	6	8
	Small	1	2	3

The household size per income category was calculated using three different probability distributions in @RISK:

- Triangular distribution with lowest number of persons, most likely number of persons and highest number of persons per household.
- PERT distribution with lowest number of persons, most likely number of persons and highest number of persons per household.
- Uniform distribution with lowest and highest number of persons per household.

Table 7-2 summarises the input distributions for household size. The results of the three distributions were compared. While the mean of each of the above-mentioned three distributions may be the same for each set of household sizes, the standard deviation (Std dev) and 95<sup>th</sup> percentiles vary. The mean of the above-mentioned three results was used to calculate the final household size used in the rest of the model.

**Table 7-2 Household size input distributions**

	Large household	Small household
<b>Triangular distribution</b>	 <p>Mean 6.0</p> <p>Std dev 0.8165</p> <p>95<sup>th</sup> percentile 7.367</p>	 <p>Mean 2.0</p> <p>Std dev 0.4083</p> <p>95<sup>th</sup> percentile 2.683</p>
<b>PERT distribution</b>	 <p>Mean 6.0</p> <p>Std dev 0.7560</p> <p>95<sup>th</sup> percentile 7.242</p>	 <p>Mean 2.0</p> <p>Std dev 0.3780</p> <p>95<sup>th</sup> percentile 2.621</p>
<b>Uniform distribution</b>	 <p>Mean 6.0</p> <p>Std dev 1.1548</p> <p>95<sup>th</sup> percentile 7.8</p>	 <p>Mean 2.0</p> <p>Std dev 0.5774</p> <p>95<sup>th</sup> percentile 2.9</p>

The three income levels as determined from Census 2011 reports were split as follows for the region (Statistics SA, 2012):

- 10% of households earn more than R12 817 per month,
- 25% of households earn between R3 184 and R12 816 per month, and
- 65% of households earn less than R3 183 per month.

The author observed that the general trend for South Africa was that the household size was shrinking. According to the 1996 Census, the average household size was 4.5 people per household. In the 2011 Census, the average household size was 3.6 people per household. (Statistics SA, 2012). For the typical household sizes, the following two options are proposed:

- between 4 and 8 persons per household with the most likely size being 6 persons; and
- between 1 and 3 persons per household with the most likely size being 2 persons.

The author proposed that, for the area in question, the split between household sizes is that 40% of households are smaller and 60% of households are larger. The split of households for the township under evaluation is given in Table 7-3. Jacobs and Haarhoff (2004) proposed a low income household size of between 3.8 people and 8.2 people per household, with a most likely value of 6.2.

**Table 7-3 Household split for modelled area**

Income category	Household size	Number of households	Percentage
High income	Large	330	6%
	Small	220	4%
Middle income	Large	825	15%
	Small	550	10%
Low income	Large	2 145	39%
	Small	1 430	26%
<b>TOTAL</b>		<b>5 500</b>	<b>100%</b>

### 7.3 Set up indoor end-use presence functions

The household income influences the presence of end uses available within the household (for example a low-income household would have a much lower probability of owning a dishwashing machine than a high-income household). Each end use was given a uniform probability of being

present in the household depending on the household income grouping. There are nine types of indoor use and they are presented in Table 7-4 together with their upper and lower bounds. The same distribution is used for large and small households.

**Table 7-4 Probability of the presence of each end-use**

End Use	High income		Middle income		Low income	
	Low	High	Low	High	Low	High
Bath	95%	100%	60%	80%	30%	60%
Bathroom Basin	100%	100%	80%	100%	60%	90%
Dishwasher	60%	90%	30%	60%	0%	30%
Kitchen sink	100%	100%	80%	100%	60%	90%
Leaks	50%	80%	60%	90%	70%	100%
Miscellaneous indoor	60%	90%	60%	90%	60%	90%
Shower	90%	100%	60%	80%	30%	60%
Toilet	100%	100%	95%	100%	90%	95%
Washing machine	80%	100%	60%	90%	0%	50%

For the purpose of this model the probability of the end use being present depends only on household income category (in other words, the same distribution was used to determine if an end use is present for small and large households within an income category). The probability of an end use being present was something the designer would need to consider. The figures in Table 7-4 are proposed by the author, but should the designer require more accurate data, data gathered from community surveys would increase the level of accuracy. A uniform distribution was used to determine the probability of whether or not an end use was present.

## 7.4 Set up end-use volume and frequency functions

The model was run using four different sets of volume functions:

- The first model setup was based on input values provided by Jacobs and Haarhoff (2004) and these were used to define a triangular function.
- The second model setup was based on the same input values, but the function was changed to a PERT function.
- The third model set up was based on the same input values as the previous two setups, but this time a uniform distribution function was used.
- The end-use results of Scheepers (2012) were used lastly.

The reason for running four model setups was that the event volume depended on the input variable probability distribution. Running different models with different functions allowed the author to determine the sensitivity of the model to the different input functions.

The frequency of end use was based on Jacobs and Haarhoff's (2004) proposed frequency. Table 7-5 summarises the various model input variables for both end-use volume and end-use frequency.

**Table 7-5 End-use volume and frequency function input variables**

End Use	Volume (litres per event)			Frequency (events/person/day)			Distribution type	Distribution variables		
	L	T	H	L	T	H		$\alpha$	$\beta$	$\gamma$
	Jacobs and Haarhoff (2004)						Scheepers (2012)			
Bath	39	80	189	0.22	0.24	0.9	Rayleigh	n/a	65.985	n/a
Bathroom basin	0.3	3.8	60	3.4	3.6	3.8	Log-normal	1.064	0.276	n/a
Dish-washer	15.1	25	43	0.18	0.25	0.29	Log-logistic	4.319	7.101	0.0
Kitchen sink	0.6	6.7	73	2.0	2.0	2.1	Log-normal*	1.064	0.276	n/a
Leaks	0.0	27.4	27.4	1	1	1	Uniform*	0.000	30.000	n/a
Misc. indoor	0.0	2.0	2.0	1.0	1.0	1.0	Uniform*	0.000	2.000	n/a
Shower	7.6	59.1	303	0.19	0.31	0.68	Log-logistic	2.828	55.197	0.0
Toilet	8	14.3	26.5	1.7	3.7	10.3	Weibull	3.207	14.717	n/a
Washing machine	60	113.6	200	0.12	0.3	0.63	Weibull	0.823	32.226	n/a

Kitchen sinks, leaks and miscellaneous indoor end uses were not presented in Scheepers' (2012) report. The same distribution Scheepers used for bathroom basin was, therefore, applied to kitchen sink. Leaks and miscellaneous indoor end uses were modelled using a uniform distribution with an upper and lower limit as per Jacobs and Haarhoff (2004).

## 7.5 Calculate average annual daily demand per end use

The AADD function consists of the volume per event multiplied by the frequency of the event multiplied by the number of people per household. This was modelled using the following function:

$$AADD_{EU} = V_{EU}F_{EU}N$$

Where:

- $V_{EU}$  was the end use volume measured in litres, and
- $F_{EU}$  was the end use frequency measured in events per day
- $N$  was the number of people per household.

A screen shot of the MS Excel® of the BATH end-use is included in Figure 7-2 to illustrate to the reader what the programming looked like.

	A	B	C	D	E	F	G	H	I	J
1					<b>BATH</b>					
2					<b>Description</b>	<b>Low</b>	<b>Typical</b>	<b>High</b>	<b>Units</b>	
3					Volume	39	80	189	l/event	
4					Frequency	0,22	0,24	0,9	events/person/day	
5										
6										
7										
8	<b>Income Category</b>	<b>Household Size</b>	<b>People</b>		<b>Volume</b>	<b>Frequency</b>	<b>Presence</b>	<b>Random</b>	<b>Yes/No</b>	<b>AADD</b>
9	High income	Large	5,3		82,7	2,4	98%	0,5	1	200,0
10		Small	2,7		82,7	1,2	98%	0,5	1	100,0
11	Middle Income	Large	5,7		82,7	2,6	70%	0,5	1	212,4
12		Small	3,0		82,7	1,4	70%	0,5	1	112,5
13	Low Income	Large	6,7		82,7	3,0	45%	0,5	0	0,0
14		Small	3,0		82,7	1,4	45%	0,5	0	0,0
15										
16					<b>Distribution</b>					
17					Rayleigh					
18					<b>γ</b>	n/a				
19					<b>β</b>	65,985				
20					<b>α</b>	n/a				
21										

Figure 7-2 Screenshot of 'Bath' end-use calculator

## 7.6 Calculate the average annual dry weather flow per end use

This step entailed using a 'return factor' which calculated which portion of the end use ended up as wastewater discharged into the sewer system. In Table 7-6, the return factors proposed by Jacobs and Haarhoff (2004) have been used in the model.

**Table 7-6 End-use return factors**

End use	Return factor	End use	Return factor
Bath	1	Misc. indoor	0.9 – 1.0
Bathroom basin	1	Shower	1
Dishwasher	1	Toilet	1
Kitchen sink	1	Washing machine	1
Leaks	0.9 – 1.0	Pool filter	0 - 1

A return factor of 1 meant that 100% of the water used for an end use was returned as wastewater. Similarly, a return factor of 0.9 indicated that 90% of the water used in a particular end use was discharged into the sewer system. The pool filter was not used in the model as it was highly unlikely that the pool would discharge into the sewer system during the base flow hour, hence, the omission would not negatively impact the model. The mathematical function for calculating the AADWF per end use is represented by the following equation:

$$AADWF_{EU} = r_{EU} AADD_{EU}$$

Where:

- $r_{EU}$  was the return factor for that specific end use, and
- $AADD_{EU}$  was the AADD for the specific end use.

## 7.7 Distribute the AADWF per end use over 24 hours

Once the AADWF was known, the end use during the base flow hour needed to be calculated. There were two ways to execute this step. The first option was to apply a daily flow variation (diurnal pattern) per end use, and the second was to say that only 3 – 6% of the total number of households contributed to the night flow.

### 7.7.1 Option a: use 24-hour flow variation per end use

Since accurate diurnal flow patterns of low-income residential end-uses had been documented, the author used the patterns proposed in the Aquacraft (2011) report. There is a possibility that the low-income family unit in California was probably equivalent to a middle-income household in South Africa, but in the absence of formally published diurnal patterns for South Africa, the

author proposed that the Aquacraft (2011) patterns be used. The Aquacraft (2011) report did not have distributions for kitchen sinks or bathroom basins but instead used the term faucet.

### 7.7.2 Option b: use Assessed Residential Night Consumption method

Fantozzi and Lambert (2012) investigated MNF extensively. Based on their findings, they proposed the following formula to calculate residential night consumption:

$$ARNC = (A \times T_v + B)N_o + C \times N_s \times \frac{AZNP}{50}$$

Where:

- ARNC is the assessed residential night consumption (litres/connection/hour or litres/occupant/hour)
- A is the percentage occupants flushing the toilet during the MNF hour (varies between 3% and 6%)
- $T_v$  is the average toilet volume (litres)
- B is other indoor end use and small leaks (litres/occupant/hour)
- $N_o$  is the number of occupants in the zone (number of people)
- C is the leakage on the property after the meter (litres/connection/hour)
- $N_s$  is the number of connections (number of connections)
- AZNP is the average zone night pressure (meters of static pressure in the system).

The input variables used for ARNC are presented in Table 7-7.

**Table 7-7 Assessed night use calculator (based on Fantozzi and Lambert's method)**

Input Variables	Symbol	Units	Min	Most likely	Max
Household Size	$N_o$	No	1	4	8
Percentage occupants flushing toilets during MNF	A	%	3%	3%	6%
Other in-house use	B	l/person/hour	0	0.2	0.6
Leaks	C	l/connection/hour	0	0.2	0.6
Average Zone Pressure	AZNP	metres	20	40	60
Toilet flush volume (Scheepers, 2012)	$T_v$	litres	$\alpha$	$\beta$	
			3.207	14.717	



The ARNC was multiplied by a return factor of 1.0. The results of the Aquacraft diurnal distribution and the Fantozzi and Lambert method were compared, and the more conservative results were used. Thus, if the Aquacraft had a greater SBF rate, the Fantozzi and Lambert (ibid) results were not used in further calculations.

## 7.8 Summation of all the end uses to get a total residential AADWF

Once the AADWF for the individual end uses was distributed over the typical diurnal pattern, the end uses for each hour could be summarised for an hourly system input. At this stage, leaks should be kept separate from normal residential use.

The AADWF and leaks per household type could then be multiplied by the number of households in that household type. The total catchment AADWF was thus calculated by summing all the household types. Mathematically this is described by the following functions:

$$Q_{res} = x(BF_H - BF_{leaksH}) + y(BF_M - BF_{leaksM}) + z(BF_L - BF_{leaksL})$$

and

$$Q_{leaks} = x(BF_{leaksH}) + y(BF_{leaksM}) + z(BF_{leaksL})$$

where

- $Q_{res}$  is the normal residential sewage generated during the base flow hour (kl/hour)
- $x$ ,  $y$  and  $z$  are the number of high, middle and low income households respectively
- $BF_H$ ,  $BF_M$  and  $BF_L$  are the SBF rates for the base flow hour for high, middle and low income households respectively (kl/hour)
- $BF_{leaksH}$ ,  $BF_{leaksM}$  and  $BF_{leaksL}$  are the plumbing leakage flow rate for the base flow hour for high, middle and low income households respectively (kl/hour).

## 7.9 Add groundwater infiltration and stormwater ingress

### 7.9.1 Add groundwater infiltration

According to Stephenson and Barta (2005), the flow into the sewer as a result of groundwater infiltration varies between 0.01 and 0.5 l/minute/m-diameter/m-length. The City of Tshwane (2010) stated that 0.04 l/minute/m-diameter/m-length must be used for their sewer

networks. Three various distributions were tested to compare their results, namely triangular, PERT and uniform distributions. The inputs for each distribution are shown in Table 7-8

**Table 7-8 Comparison of groundwater infiltration**

Pipeline Type	Outside Diameter (m)	Length (m)	Inflow (l/min/m-Ø/m-l)			Inflow (l/min/m-Ø/m-l)		
			Lower	Likely	Upper	Triang	PERT	Uniform
160 mm ND	0.160	48 085	0.01	0.04	0.50	0.1833	0.1117	0.2550
200 mm ND	0.200	9 175	0.01	0.04	0.50	0.1833	0.1117	0.2550
250 mm ND	0.250	3 580	0.01	0.04	0.50	0.1833	0.1117	0.2550
315 mm ND	0.315	2 040	0.01	0.04	0.50	0.1833	0.1117	0.2550

There is a concern that the 0.5 at the upper end of the range distorts the distribution. For the purpose of calculating groundwater infiltration, the uniform distribution was chosen. The calculation of the groundwater infiltration is described by the following equation:

$$Q_{gw} = \frac{1}{1000} \sum_{1}^n (D_n L_n N)$$

Where

- $Q_{gw}$  is the groundwater infiltration (kl/hour).
- $D_n$  is the external pipe diameter (m).
- $L_n$  is the length of  $D_n$  pipe diameter installed (m).
- $N$  is the groundwater inflow factor (m-diameter/m-length).

### 7.9.2 Add stormwater inflow

To calculate the stormwater ingress, rainfall information for the area needs to be considered. The rainfall for the study area in question is negligible during June, July and August (with an average monthly rainfall of less than 10 mm per month during those months). Thus this step can safely be omitted during the winter months for this area. Taking stormwater inflow as zero for this study has simplified the model and reduced the number of calculations required.

## 7.10 Measure flows to compare modelled flows to real flows

This paragraph briefly discusses the method for measuring sewer flows. Sewer flows could be measured in a variety of ways:

- Measuring weirs/flumes at the inlet works to WWTW (manual or automatic flow logging).
- Measuring flume at the inlet works to pump station (manual or automatic flow logging).
- Permanent flow logger in selected manholes.
- Temporary flow logging at appropriate manholes.

Each of the measurements uses the height of the flow through the measuring point to calculate the flow using open channel flow calculations.

In the case of manual readings, the operator is required to write down the measurement of the flow depth every hour (or any other required time interval); in the case of automatic flow logging, an ultrasonic or level sensor takes readings at the desired time interval (varies from once a minute to every 15 minutes to every hour) and records the measurements. The measurements need to be downloaded onto a notebook computer, or in some cases, the measurements are transmitted via radio signals to an appropriate computer/internet site where they could be downloaded.

The theory behind sewer network flow monitoring is based on following three requirements:

- The hydraulic theory required for flow monitoring.
- The hardware/tools required for flow monitoring.
- The software/systems required for flow monitoring.

### *7.10.1 The hydraulic theory required for flow monitoring*

Van Vuuren and Van Dijk (2009) have a detailed chapter on the hydraulics of sewers in their design guideline published by the WRC in 2010. For sewer network flow monitoring the Manning formula is often used. The Manning formula calculates flow velocity in the open channel as follows:

$$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

Where:

- Q is the flow rate (m<sup>3</sup>/s)
- n is the Manning co-efficient of roughness (s/m<sup>1/3</sup>)
- A is the flow area (m<sup>2</sup>)
- P is the wetted perimeter (m)
- S is the slope of the energy grade line (m/m).

For a circular conduit, A and P can be re-written as a function of the flow height in the conduit (d).

$$Q = \frac{1}{n} \cdot \pi \cdot \frac{d^{8/3}}{4^{5/3}} S^{1/2}$$

The value of *n* varies and Van Vuuren and Van Dijk (2009) recommend the use of *n* = 0.013 for plastic pipes and *n* = 0.014 for vitrified clay. The value of *n* could also change as the pipe ages, and depending on how often the pipe is cleaned. It is interesting to note that, according to Vosloo (2009), measuring flows in manholes using the Manning formula produced results which were within 20% of the actual flow. For greater accuracy, properly constructed measuring weirs or flumes were recommended, and these would produce results which were within 5% of the actual flows.

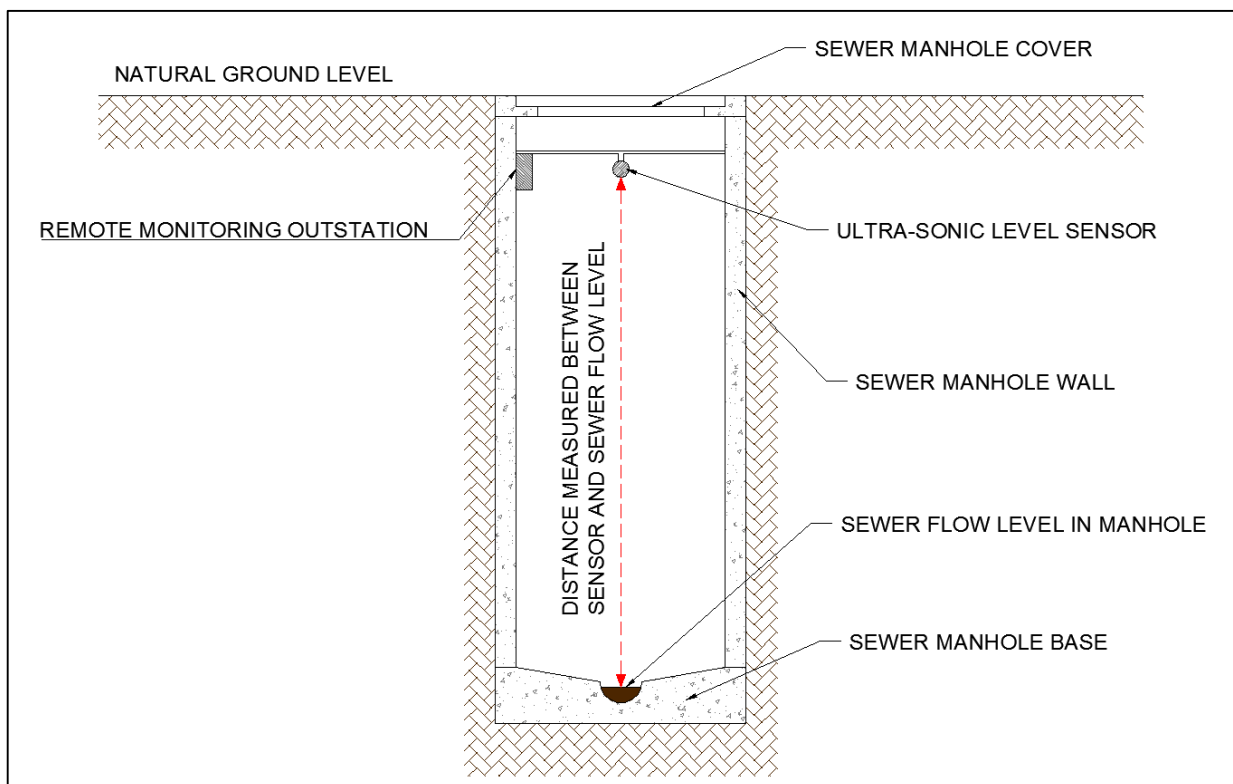
### *7.10.2 Hardware/tools required for flow monitoring*

The tools required to measure the flow vary from manually measuring the flow depth in the sewer pipe at a manhole (or at a flume or weir) using a calibrated measuring stick, to using ultra-sonic level sensors that log the water depth and convert this to a flow measurement at prescribed time intervals. Flotron is a company in South Africa that sells ultra-sonic level sensors connected to a My City remote monitoring outstation (RMO). The RMO contains a battery/energy cell and logs the level readings at regular intervals. The RMO also contains a 'cell phone' or GSM device which transmits the information to a remote system discussed in the next paragraph. The level sensor could be a permanent fixture at a measuring weir/flume at the inlet works to a pump station or WWTW, or the level sensor could be a temporary fixture installed in an appropriate manhole for a short period of time (at least 14 days). The frequency of readings

has an impact on the battery life, thus the battery would run flat quicker if the readings are taken every minute compared to readings taken every 30 minutes. Figure 7-3 shows a typical data logger used for flow logging. Figure 7-4 shows a schematic representation of a temporary flow measuring setup in a manhole.



**Figure 7-3 My City / Flotron remote data logger (Vosloo, 2009)**



**Figure 7-4 Flow measuring at manholes**

### *7.10.3 Software/systems required for flow monitoring*

Flotron developed the My City programme. This is a user-friendly internet-based remote monitoring system. This system captures and records the level readings and converts them to a flow using the hydraulic formulas briefly discussed in the preceding paragraph. The user is able to download the data in a comma separated (csv) file format and convert that to an MS Excel spreadsheet for further processing/analysis.

### *7.10.4 General flow logging considerations*

When the flow in a system is accurately measured, the analysed data could be used to make informed management decisions relating to maintenance, refurbishment, replacement or network upgrades. Analysing the flow data would allow planners to make informed decisions about their networks. However, the planner needs to know that the data from the field is accurate and can be reliable.

To ensure that the field readings are as accurate as possible, it is important that the manhole on the pipeline section being measured is:

- on a straight section of pipe (not on a bend),
- that the pipeline size coming into the manhole is the same as the pipeline size going out of the manhole,
- that there are no side junctions coming into the manhole,
- that there are no steps in the manhole invert level (i.e. the fall in the manhole must be on the same slope as the incoming and outgoing lines), and
- that the pipeline is cleaned before the equipment is installed.

When installing the equipment, the person in the field is required to insert the following data into the system:

- slope of the incoming pipe,
- incoming pipe diameter, and
- water depth at the time of installation (the sensor uses this depth to calculate future water depths so this reading needs to be as accurate as possible).

The equipment to temporarily measure flow is expensive and requires valuable man-hours to install. It is important that installations are done properly to ensure accurate readings and that the equipment does not fall into the sewer.

## 8 ANALYSIS AND RESULTS

### 8.1 Introduction

The results of the end-use model using Monte Carlo simulation are presented in this chapter. This chapter follows the same pattern as Chapter 7, and the results of each step are presented.

### 8.2 Results of household size and income functions

The household size consisted of two sets of results, the first set represented the large household size (between 4 and 8 person/hh), and the second set the small household size (between 1 and 3 persons/hh). The input distributions for each size of household were the same, irrespective of the income category (in other words, the large household distribution for a high-income household was the same as the large household distribution for a low-income household).

The uniform distribution yielded the highest 5<sup>th</sup> percentile, followed by the triangular distribution, while the PERT distribution yielded the lowest 95<sup>th</sup> percentile. The averages of the two sets of results are presented in Table 8-1. For both household sizes, the Beta-General distribution yielded the best fit.

**Table 8-1 Household size results**

	Large household	Small household
<b>Mean</b>	6.00	2.00
<b>Median</b>	6.00	2.00
<b>Std Dev</b>	0.54	0.27
<b>Skewness</b>	-0.04	-0.02
<b>95<sup>th</sup> percentile</b>	6.87	2.44
<b>Proposed distribution</b>	<b>Beta General</b>	<b>Beta General</b>
<b>Input parameters</b>	$\alpha_1 = 6.721$ $\alpha_2 = 6.376$ Minimum = 3.93 Maximum = 7.96	$\alpha_1 = 6.353$ $\alpha_2 = 6.107$ Minimum = 1.00 Maximum = 2.96

Table 8-1 continued on next page

	Large household	Small household
<b>Goodness of fit</b>	Chi-Squared : Weibull (74.46) Anderson-Darling: Beta-General (0.33) Kolmogorov-Smirnov: Beta-General (0.01)	Chi-Squared : Beta-General (50.89) Anderson-Darling: Beta-General (0.22) Kolmogorov-Smirnov: Beta-General (0.01)
<b>PDF</b>		
<b>CDF</b>		

### 8.3 Results of indoor end-use presence functions

The results of the indoor end-use functions are included in Table 7-4. Since all the distributions are uniform distributions, each distribution is symmetrical (skewness of zero) and the mean and median are the same.



## 8.4 Results of end-use volume and frequency functions

### 8.4.1 End-use volumes

The calculated mean volumes per indoor end use are summarised in Table 8-2.

**Table 8-2 Mean end-use volumes (litres/event)**

End use	Uniform distribution	Triangular distribution	PERT distribution	Scheepers' distribution	Difference (Scheepers – Mean of other 3)
Bath	114.0	102.7	91.3	82.7	-20.0
Bathroom basin	30.2	21.5	12.6	1.1	-20.3
Dishwasher	29.1	27.7	26.4	7.8	-19.9
Kitchen sink	36.8	26.8	16.7	2.1	-24.6
Leaks	13.7	18.3	22.8	15.0	-3.3
Misc. indoor	1.0	1.3	1.7	1.0	-0.3
Shower	155.3	123.2	91.2	68.4	-54.8
Toilet	17.3	16.3	15.3	13.2	-3.1
Washing machine	130.0	124.5	119.1	35.8	-88.7

The results calculated using input variables and distributions based on Scheepers' (2012) report produced values that were generally smaller than those modelled using the input variables proposed in Jacobs and Haarhoff's (2004) research paper. One of the reasons for this was that the values proposed by Jacobs and Haarhoff had some relatively high outliers which distorted the uniform and triangular distributions, and also negatively impacted the PERT distribution. The outliers are as follows:

- Bath – 189 l/event
- Bathroom basin – 60 l/event
- Kitchen sink – 73 l/event
- Shower – 303 l/event
- Washing machine – 200 l/event.

Furthermore, the values proposed by Jacobs and Haarhoff (2004) for the dishwasher and washing machine might be based on older appliances. The mean values of both the washing

machine and the dishwasher published by Scheepers (2012) were less than the respective minimum values published by Jacobs and Haarhoff (2004):

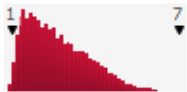
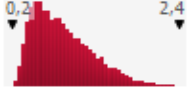

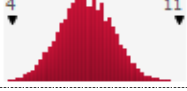
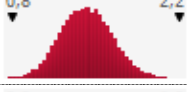
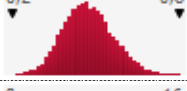
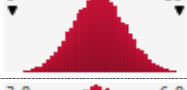
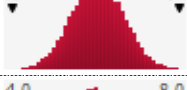



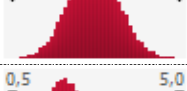

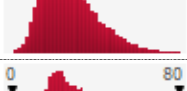
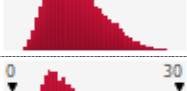
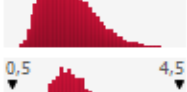


- Dishwasher: Scheepers' mean 7.8 l/event versus Jacobs and Haarhoff's minimum 15.1 l/event
- Washing machine: Scheepers' mean 35.8 l/event versus Jacobs and Haarhoff's minimum 60.0 l/event.

#### *8.4.2 Frequency of end-use events*

The calculation of the frequency of events is influenced by the number of people per household (calculated in Step 1) and whether or not the appliance or end use is present within the household (calculated in Step 2).

Tables 8-3 and 8-4 present the results of the modelling of frequency of events. Since the household sizes for high, middle and low incomes were the same, the frequency of events were initially the same per household size category, irrespective of income grouping (thus there were between 8.8 and 72.6 toilet flushes in both large high-income households and large low-income households). It is interesting to note that a beta-general distribution was the best fit distribution for all the frequency calculations.

**Table 8-3 Frequency of events (number of events per day)**

Name	Graph	Min	Mean	Max	Median	Standard Deviation	Skewness	95 <sup>th</sup> percentile
Large hh bath		1.1	2.7	6.1	2.5	1.0	0.6	4.6
Small hh bath		0.3	0.9	2.3	0.8	0.3	0.7	1.6
Large hh bath-room basin		15.4	21.6	28.5	21.6	2.0	0.0	24.9
Small hh bath-room basin		4.1	7.2	10.5	7.2	1.0	0.0	8.8
Large hh dish washer		0.8	1.4	2.1	1.4	0.2	0.1	1.8
Small hh dish washer		0.2	0.5	0.8	0.5	0.1	0.2	0.6
Large hh sink		8.9	12.2	15.7	12.2	1.1	0.0	14.0
Small hh sink		2.4	4.1	5.9	4.1	0.5	0.0	5.0
Large hh leaks		4.4	6.0	7.7	6.0	0.5	0.0	6.9
Small hh leaks		1.2	2.0	2.9	2.0	0.3	0.0	2.4
Large hh misc use		4.4	6.0	7.7	6.0	0.5	0.0	6.9
Small hh misc use		1.2	2.0	2.9	2.0	0.3	0.0	2.4
Large hh shower		0.9	2.4	4.8	2.3	0.7	0.5	3.6
Small hh shower		0.3	0.8	1.8	0.8	0.2	0.6	1.2
Large hh toilet		8.9	31.4	72.6	29.8	11.4	0.5	52.4
Small hh toilet		2.6	10.5	26.3	9.9	4.0	0.6	17.9
Large hh washing machine		0.6	2.1	4.4	2.0	0.7	0.4	3.3
Small hh washing machine		0.2	0.7	1.6	0.7	0.2	0.5	1.1

**Table 8-4 Goodness of fit results for end-use frequency**

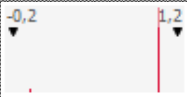




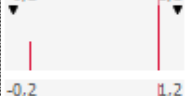
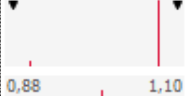
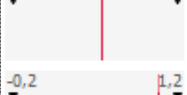
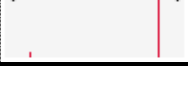
End-Use	$\alpha_1$	$\alpha_2$	Min	Max	*Chi <sup>2</sup>	*A-D	*K-S
<b>Large (L) Bath</b>	1.917	5.216	1.066	7.240	390.902	19.219	0.032
<b>Small (S) Bath</b>	2.21	8.519	0.317	3.184	214.368	12.702	0.026
<b>L Bathroom basin</b>	7.227	7.178	13.758	18.388	71.060	0.290	0.008
<b>S Bathroom basin</b>	6.857	7.479	3.575	11.154	60.522	0.184	0.006
<b>L Dishwasher</b>	8.264	9.825	0.688	2.334	75.218	0.283	0.006
<b>S Dishwasher</b>	8.795	12.901	0.167	0.940	45.160	0.301	0.006
<b>L Kitchen sink</b>	6.358	6.566	8.168	16.363	49.748	0.352	0.005
<b>S Kitchen sink</b>	7.103	7.183	1.937	6.221	56.866	0.177	0.006
<b>L Leaks</b>	5.389	5.624	4.176	7.904	85.934	0.900	0.009
<b>S Leaks</b>	6.824	6.733	0.967	3.019	77.498	0.629	0.009
<b>L Misc. indoor</b>	6.519	6.461	3.988	7.994	59.027	0.210	0.004
<b>S Misc. indoor</b>	6.824	6.733	0.967	3.019	77.498	0.629	0.009
<b>L Shower</b>	2.922	6.937	0.961	5.685	144.852	5.751	0.022
<b>S Shower</b>	3.406	11.203	0.277	2.462	112.884	3.120	0.014
<b>L Toilet</b>	2.324	5.257	9.335	81.737	122.726	4.191	0.019
<b>S Toilet</b>	2.800	8.220	2.557	33.686	134.714	4.600	0.019
<b>L Washing machine</b>	3.016	5.599	0.595	4.893	67.700	1.080	0.011
<b>S Washing machine</b>	3.374	8.123	0.171	1.976	68.396	0.920	0.009

\* Note: the goodness of fit test results are also presented in this table. Chi<sup>2</sup> is short for the Chi-squared test, A-D is short for Anderson-Darling test and K-S for the Kolmogorov-Smirnov test.

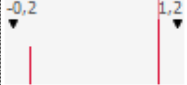
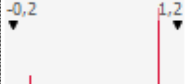
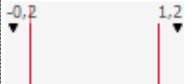
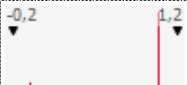

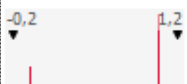
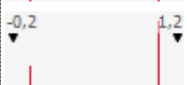

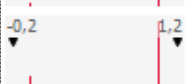
#### 8.4.3 Presence of end use in household

The results of the modelling for the presence of each end use per household income category are presented in Tables 8-5, 8-6 and 8-7. The modelling of presence of end uses resulted in binomial distributions.

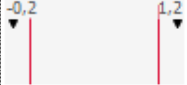
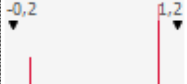
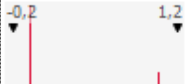
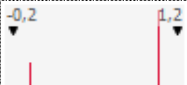

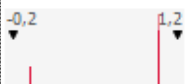
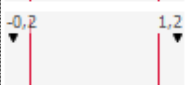

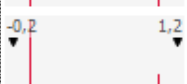
**Table 8-5 Presence of events – high-income households**

Name	Graph	Min	Mean	Max	Median	Std Dev	Skewness	95%
<b>High Income</b>								
Bath		-	0,98	1,00	1,00	0,15	-6,29199	1,00
Bathroom basin		1,00	1,00	1,00	1,00	-	n/a	1,00
Dishwasher		-	0,75	1,00	1,00	0,43	-1,137114	1,00
Kitchen Sink		1,00	1,00	1,00	1,00	-	n/a	1,00
Leaks		-	0,76	1,00	1,00	0,43	-1,202416	1,00
Misc Indoor		-	0,75	1,00	1,00	0,43	-1,146887	1,00
Shower		-	0,95	1,00	1,00	0,22	-4,017973	1,00
Toilet		1,00	1,00	1,00	1,00	-	n/a	1,00
Washing Machine		-	0,95	1,00	1,00	0,22	-4,199108	1,00

**Table 8-6 Presence of events – middle-income households**

Name	Graph	Min	Mean	Max	Median	Std Dev	Skewness	95%
<b>Medium Income</b>								
Bath		-	0,70	1,00	1,00	0,46	-0,8564491	1,00
Bathroom basin		-	0,90	1,00	1,00	0,30	-2,652328	1,00
Dishwasher		-	0,44	1,00	-	0,50	0,2266753	1,00
Kitchen Sink		-	0,91	1,00	1,00	0,29	-2,778853	1,00
Leaks		-	0,75	1,00	1,00	0,43	-1,16476	1,00
Misc Indoor		-	0,75	1,00	1,00	0,44	-1,124984	1,00
Shower		-	0,71	1,00	1,00	0,45	-0,9145299	1,00
Toilet		-	0,97	1,00	1,00	0,16	-5,8729	1,00
Washing Machine		-	0,76	1,00	1,00	0,43	-1,201146	1,00

**Table 8-7 Presence of events – low-income households**

Name	Graph	Min	Mean	Max	Median	Std Dev	Skewness	95%
<b>Low Income</b>								
Bath		-	0,55	1,00	1,00	0,50	-0,1848114	1,00
Bathroom basin		-	0,75	1,00	1,00	0,43	-1,148113	1,00
Dishwasher		-	0,15	1,00	-	0,35	2,012196	1,00
Kitchen Sink		-	0,74	1,00	1,00	0,44	-1,120158	1,00
Leaks		-	0,78	1,00	1,00	0,42	-1,34224	1,00
Misc Indoor		-	0,75	1,00	1,00	0,43	-1,162283	1,00
Shower		-	0,45	1,00	-	0,50	0,2055067	1,00
Toilet		-	0,93	1,00	1,00	0,26	-3,258033	1,00
Washing Machine		-	0,26	1,00	-	0,44	1,114746	1,00

## 8.5 Results of the AADD per end use

The results of the AADD calculations are presented in tables and figures on the following pages. Table 8-8 presents the results of using the uniform, triangular and PERT distributions to generate end-use functions. The results for large households using the uniform and triangular distributions with the Jacobs and Haarhoff (2004) input parameters were very high, bearing in mind that only indoor consumption was calculated.

In Table 8-9, the results using the Scheepers (2012) input distributions are presented. The mean of AADD for the large and small household categories using the Scheepers (2012) input distributions yielded results that fall within generally accepted AADD ranges.

**Table 8-8 Results of AADD per household category using various end-use input functions based on the Jacobs and Haarhoff (2004) published values**

Category	Uniform		Triangular		PERT	
	Mean (litres/hh/day)	Median (litres/hh/day)	Mean (litres/hh/day)	Median (litres/hh/day)	Mean (litres/hh/day)	Median (litres/hh/day)
High income						
Large	2 571.8	2 525.0	2 138.9	2 089.5	1 708.2	1 666.4
Small	856.6	835.1	713.9	690.0	569.3	551.3
Middle income						
Large	2 204.0	2 160.3	1 836.7	1 784.5	1 468.9	1 431.2
Small	736.2	709.2	610.6	584.8	489.0	470.9
Low income						
Large	1 730.9	1 672.4	1 442.2	1 385.1	1 151.6	1 103.7
Small	578.7	551.4	481.1	455.7	384.8	365.2

**Table 8-9 Results of AADD per household category using the Scheepers (2012) end-use functions**

Category	Mean (litres/hh/day)	Median (litres/hh/day)	Mode (litres/hh/day)	95 <sup>th</sup> (litres/hh/day)	Standard deviation (litres/hh/day)	Skewness
High income						
Large	901.1	855.8	724.7	1465.4	317.9	0.959
Small	300.2	282.2	221.0	518.7	118.1	1.244
Middle income						
Large	768.9	728.8	664.8	1341.7	325.2	1.303
Small	255.8	239.7	199.8	467.4	113.8	1.023
Low income						
Large	633.3	592.3	560.1	1184..9	306.0	1.073
Small	211.5	195.0	191.4	412.4	107.3	1.045



### 8.5.1 Breakdown of end uses

Tables 8-10 and 8-11 give a breakdown of the AADD per end use based on the Scheepers (2012) input distributions. The household size has a greater impact on the results of end uses than the household income, and thus there are two typical sets of results: the large household set of results and the small household set of results.

**Table 8-10 Individual end-use mean values**

End Use	High income (litres/hh/day)		Middle income (litres/hh/day)		Low income (litres/hh/day)	
	Large	Small	Large	Small	Large	Small
Bath	219.6	72.9	219.6	72.9	219.6	72.9
Bathroom Basin	23.0	7.7	23.0	7.7	23.0	7.7
Dishwasher	8.4	2.8	-	-	-	-
Kitchen sink	13.0	4.3	-	-	-	-
Leaks	66.7	22.6	66.7	22.6	66.7	22.6
Miscellaneous indoor	4.5	1.5	4.5	1.5	4.5	1.5
Shower	154.3	50.7	154.3	50.7	-	-
Toilet	352.5	117.8	352.5	117.8	352.5	117.8
Washing machine	71.7	23.7	71.7	23.7	-	-
<b>TOTAL</b>	<b>913.7</b>	<b>304.0</b>	<b>892.4</b>	<b>296.9</b>	<b>666.3</b>	<b>222.5</b>

**Bath end use:** the bath end use yielded results that varied from 0.0 l/hh/day (i.e. no bath present) to 1 133 l/hh/day for large households and 512 l/hh/day for small households.

**Bathroom basin end use:** the bathroom basin end use yielded results that varied from 6 to 60 l/hh/day for large households and 2 to 23 l/hh/day for small households.

**Dishwasher end use:** the dishwasher end use varied from 0.0 l/hh/day (i.e. no dishwasher present) to 77 l/hh/day for large households (high income) and 39 l/hh/day (high income).

**Kitchen sink end use:** the kitchen sink end use yielded results that varied from 4 to 36 l/hh/day for large households and 1 to 14 l/hh/day for small households.

**Leaks end use:** the leaks varied from 0.00 l/hh/day (i.e. no leak present) to 232 l/hh/day for large households and 86 l/hh/day for small households.

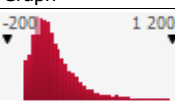
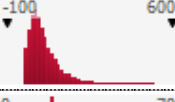
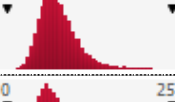
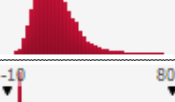

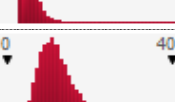
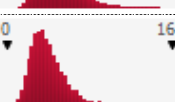
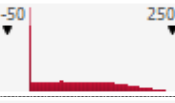


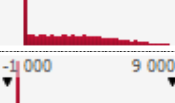


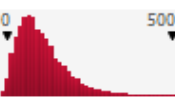




**Miscellaneous indoor end use:** the miscellaneous indoor use varied from 0.00 l/hh/day (i.e. no miscellaneous indoor use present) to 15 l/hh/day for large households and 5 l/hh/day for small households.

**Shower end use:** the shower use varied from 0.00 l/hh/day (i.e. no shower present) to 8 576 l/hh/day for large households and 1 111 l/hh/day for small households.

**Toilet end use:** the toilet end use yielded results that varied from 7 to 1 506 l/hh/day for large households and 4 to 497 l/hh/day for small households.

**Washing machine end use:** the washing machine end use varied from 0.00 l/hh/day (i.e. no washing machine present) to 1 711 l/hh/day for large households (high income) and 663 l/hh/day for small households (high income). These high values are outliers, and this could be seen in the big difference between the 95<sup>th</sup> percentile and the maximum value. The maximum value was six times higher than the 95<sup>th</sup> percentile. The 95<sup>th</sup> percentile results were used in steps 5 and 6.

**Table 8-11 Summary of end-use results (l/hh/day)**

Name	Graph	Min	Mean	Max	Median	Mode	Std Dev	Skewness	95%
HIL Bath		0.00	219.60	1133.09	187.14	0.00	153.89	1.34	515.61
HIS Bath		0.00	72.93	512.57	60.88	0.00	52.95	1.46	176.67
HIL BB		6.55	22.98	60.67	22.06	21.06	6.77	0.84	35.45
HIS BB		2.06	7.66	23.25	7.32	6.08	2.54	0.84	12.32
HIL DW		0.00	8.39	77.39	8.49	0.00	6.83	1.15	19.46
HIS DW		0.00	2.77	39.30	2.72	0.00	2.35	1.63	6.58
HIL KS		4.18	12.98	36.22	12.44	9.76	3.85	0.87	20.07
HIS KS		1.14	4.33	14.43	4.11	3.96	1.47	0.90	7.04
HIL Leaks		0.00	66.72	232.40	57.38	0.00	60.52	0.45	172.73
HIS Leaks		0.00	22.55	86.77	19.04	0.00	20.80	0.56	59.79
HIL MI		0.00	4.50	15.57	3.94	0.00	4.06	0.44	11.58
HIS MI		0.00	1.50	5.72	1.27	0.00	1.39	0.56	4.04
HIL Shower		0.00	154.32	8576.32	118.57	0.00	168.57	15.62	392.25
HIS Shower		0.00	50.71	1111.67	39.17	0.00	49.21	5.10	129.46
HIL Toilet		7.01	352.50	1506.05	319.29	237.42	181.33	1.04	696.30
HIS Toilet		4.61	117.80	497.41	104.92	79.36	64.30	1.14	241.64
HIL WM		0.00	71.73	1711.65	36.44	0.00	99.94	3.39	263.75
HIS WM		0.00	23.74	663.44	12.05	0.00	33.87	3.84	85.80

## 8.6 Calculate the AADWF per end use

For the purpose of this research, the return factor for all indoor uses was simplified to 1, thus the AADD for the indoor end use was equal to the AADWF. This simplification had no significant impact on the SBF.

## 8.7 Distribute the AADWF per end use over 24 hours

### 8.7.1 Diurnal pattern method

The end uses were distributed over 24 hours using the 95<sup>th</sup> percentile results spread over a typical day. The SBF rate results for the end uses in a large household are presented in Table 8-12. The minimum flow for a large household occurred between 03:00 and 04:00, and during that time, the leaks accounted for 33% of the residential flow.

**Table 8-12 Base flows of end uses – large households**

End-Use	Hour (litres/hh/hour)				
	00 – 01	01 – 02	02 – 03	03 – 04	04 – 05
Bath	2.0	1.0	0.0	0.0	0.0
Bathroom basin	0.3	0.2	0.2	0.1	0.2
Dishwasher	0.2	0.1	0.1	0.1	0.1
Kitchen sink	0.3	0.2	0.2	0.1	0.2
<i>Leaks</i>	<i>3.1</i>	<i>3.0</i>	<i>2.9</i>	<i>2.9</i>	<i>2.9</i>
Misc. indoor	0.4	0.3	0.3	0.1	0.1
Shower	1.8	0.8	0.5	1.5	6.0
Toilet	7.6	5.5	5.5	4.9	6.2
Washing machine	0.6	0.3	0.1	0.1	0.1
<b>TOTAL</b>	<b>16.0</b>	<b>11.2</b>	<b>9.5</b>	<b>8.7</b>	<b>14.1</b>

The SBF rate results for the end uses in a small household are presented in Table 8-13. For a small household, the minimum flow occurred between 03:00 and 04:00 and leaks once again accounted for 33% of the residential flow.

**Table 8-13 Base flows of end uses – small households**

End-Use	Hour (litres/hh/hour)				
	00 – 01	01 – 02	02 – 03	03 – 04	04 – 05
Bath	0.7	0.3	0.0	0.0	1.0
Bathroom basin	0.1	0.1	0.1	0.1	0.1
Dishwasher	0.1	0.0	0.0	0.0	0.0
Kitchen sink	1.0	0.0	0.0	0.0	0.0
<i>Leaks</i>	<i>1.0</i>	<i>1.0</i>	<i>1.0</i>	<i>1.0</i>	<i>1.0</i>
Misc. indoor	0.1	0.1	0.1	0.0	0.0
Shower	0.6	0.3	0.2	0.2	0.5
Toilet	2.5	1.8	1.8	1.6	2.1
Washing machine	0.2	0.1	0.0	0.0	0.0
<b>TOTAL</b>	<b>5.3</b>	<b>3.7</b>	<b>3.2</b>	<b>2.9</b>	<b>4.7</b>

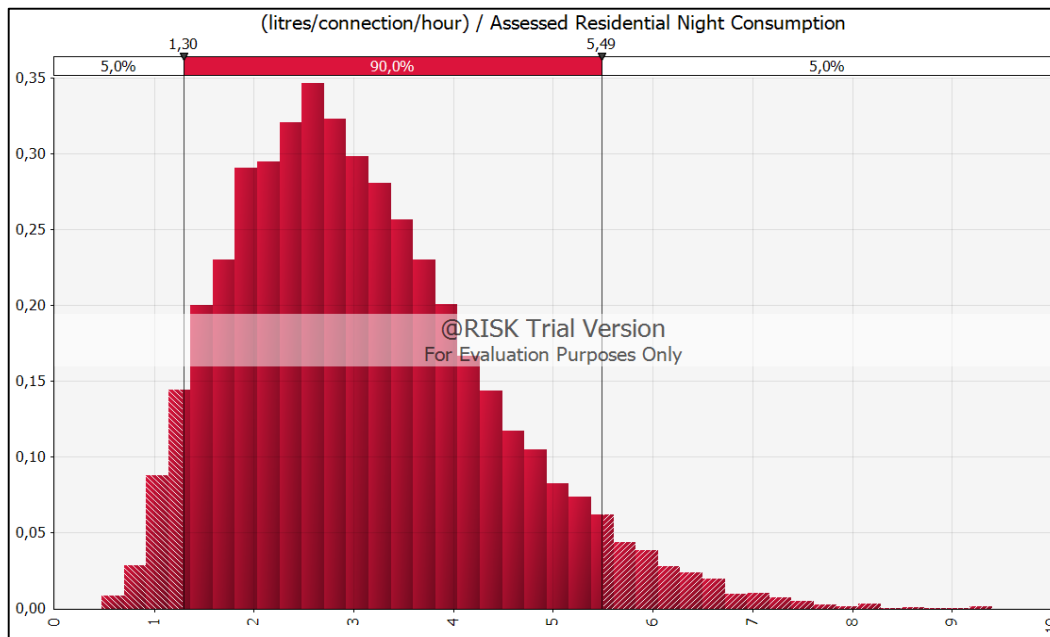
### 8.7.2 Assessed residential night consumption)

Using the Assessed Residential Night Consumption technique proposed by Fantozzi and Lambert (2012) the following values were obtained for the MNF:

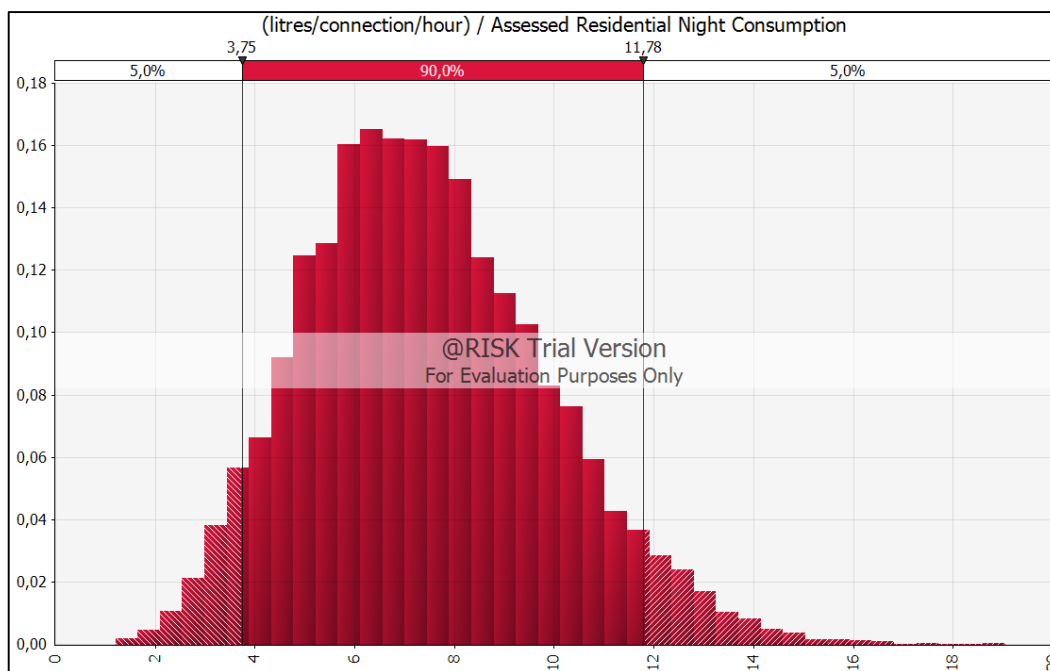
Mean	:	3.1 l/connection/hour
Median	:	2.9 l/connection/hour
95 <sup>th</sup> percentile	:	5.49 l/connection/hour
Standard deviation	:	1.3 l/connection/hour
Skewness	:	0.7.

Figure 8-1 shows the results of the initial ARNC modelling. Increasing the leakage flow rates to 5 l/connection/hour for most likely and 10 l/connection/hour for upper, yielded the following results (Figure 8-2 shows the increased leakage rate results):

Mean	:	7.4 l/connection/hour
Median	:	7.2 l/connection/hour
95 <sup>th</sup> percentile	:	11.8 l/connection/hour
Standard deviation	:	2.4 l/connection/hour
Skewness	:	0.4.



**Figure 8-1 Assessed residential night consumption**



**Figure 8-2 Assessed residential night consumption (higher leak rates)**

The results of the ARCN compared to the REU results were:

REU: Base flow of large hh : 8.7 l/connection/hour  
 REU: Base flow of small hh : 2.9 l/connection/hour  
 ARCN: Normal leaks : 2.9 l/connection/hour  
 ARCN: Higher leaks : 7.4 l/connection/hour

The REU results were more conservative than the ARNC results. For the remaining steps, the REU results were used.

## 8.8 Summation of the end uses

Since a return factor of one was used in the model; Tables 8-12 and 8-13 also present the results of the AADWF. Using the results obtained in Section 8.7, the total AADWF for the study area was 3 150.7 kl/day (or 3.151 MI/day). Table 8-14 gives a breakdown of the AADWF per household category. Similarly, Table 8-15 gives the SBF per household category. The SBF contained  $Q_{leaks}$  of 2.9 l/hh/hour and 1.0 l/hh/hour for large and small households respectively. The total SBF,  $Q_{base}$ , for the township was 33.2 kl/hour and the leakage flow,  $Q_{leaks}$ , for the township was 11.9 kl/hour, which was 35% of the SBF.

**Table 8-14 Total residential AADWF for area**

Income category	Household size	Number of households	AADWF (l/hh/day)	AADWF (kl/day)
High income	Large	330	945.1	311.877
	Small	220	315.0	69.306
Middle income	Large	825	933.9	770.464
	Small	550	311.3	171.214
Low income	Large	2 145	697.2	1 495. 544
	Small	1 430	232.4	332.343
<b>TOTAL</b>		<b>5 500</b>		<b>3 150.749</b>

**Table 8-15 Total SBF for area**

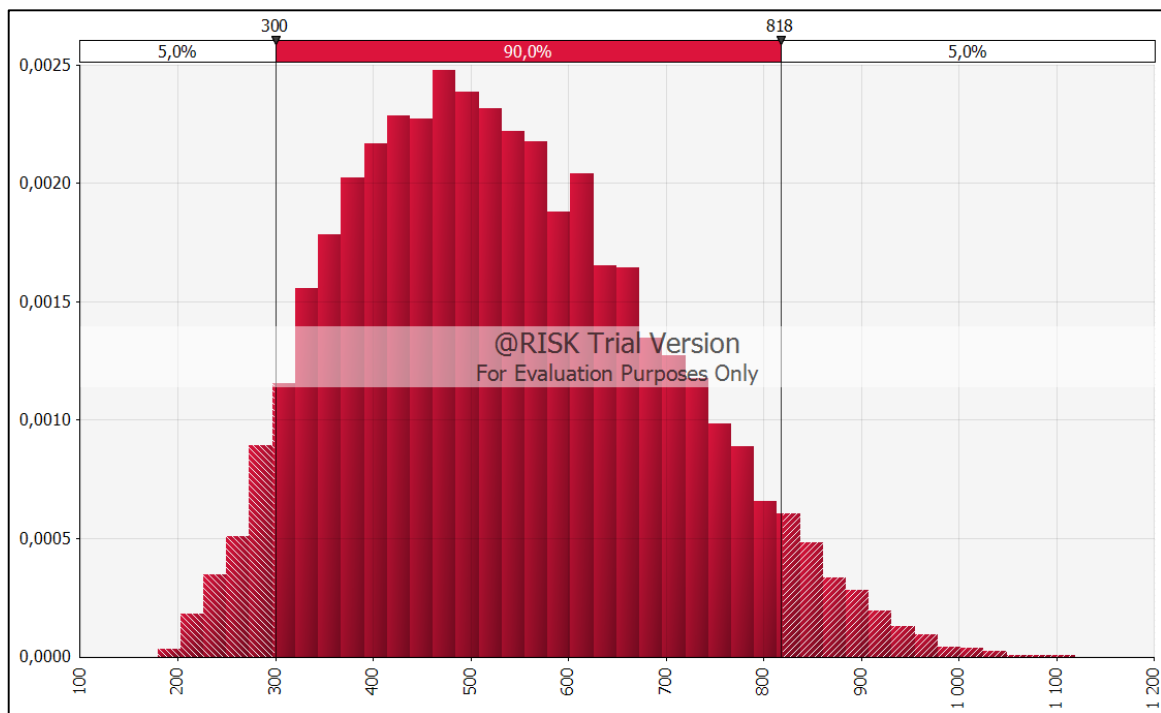
Income category	Household size	Number of households	Base flow (l/hh/hour)	Base flow (kl/hour)
High income	Large	330	8.7	2.885
	Small	220	2.9	0.641
Middle income	Large	825	8.6	7.112
	Small	550	2.9	1.580
Low income	Large	2 145	8.0	17.144
	Small	1 430	2.7	3.810
<b>TOTAL</b>		<b>5 500</b>		<b>33.172</b>

## 8.9 Results of groundwater infiltration and stormwater ingress calculations

### 8.9.1 Groundwater infiltration

Figure 8-3 shows the PDF of the groundwater infiltration ( $Q_{gw}$ ). The  $Q_{gw}$  results for the sewer network in question were as follows:

Mean	:	534.87 l/min (32.092 kl/hour)
Median	:	519.86 l/min
95 <sup>th</sup> percentile	:	818 l/min
Standard deviation	:	158.05 l/min
Skewness	:	0.3807.



**Figure 8-3 Groundwater infiltration result**

### 8.9.2 Stormwater inflow

In order to simplify the model, the  $Q_{sw}$  was taken as zero during the dry season.



## 8.10 Measure flows to compare modelled flows to real flows

In Chapter 1, the following equation was presented for the calculation of SBF:

$$Q_{base} = Q_{res} + Q_{sw} + Q_{gw} + Q_{leaks}$$

From the results of the modelling:

$$Q_{res} = 21.508 \text{ kl/hour.}$$

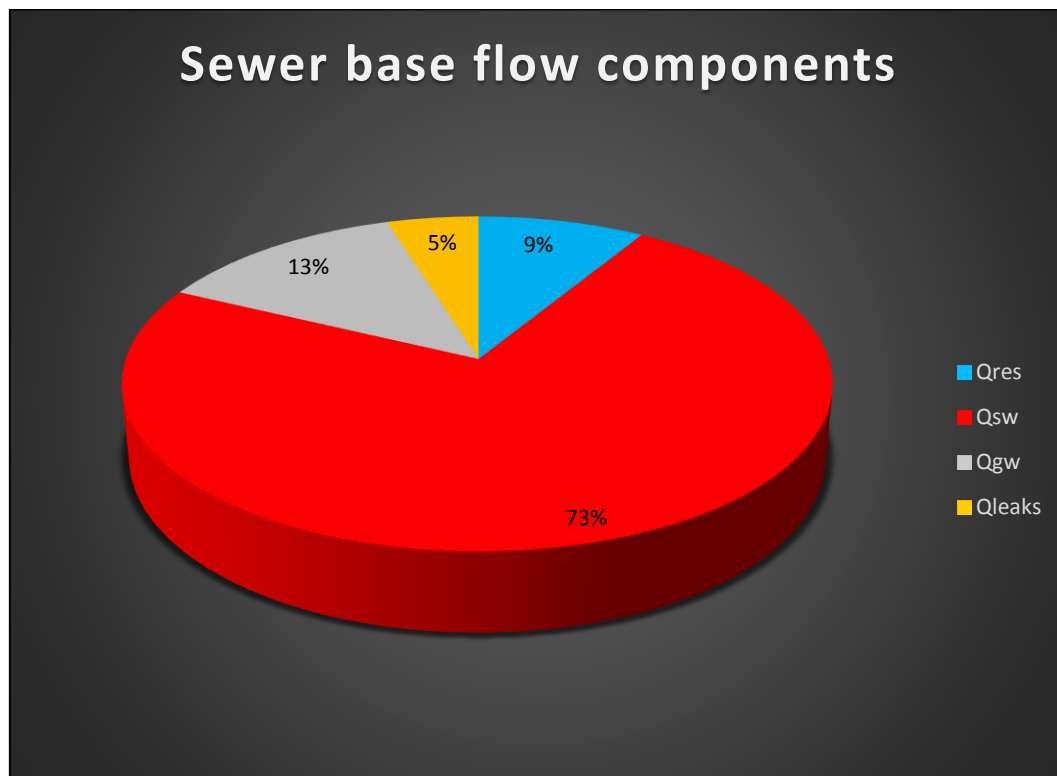
$$Q_{sw} = 180.148 \text{ kl/hour (during the wet season).}$$

$$Q_{sw} = 0.000 \text{ kl/hour (during the dry season).}$$

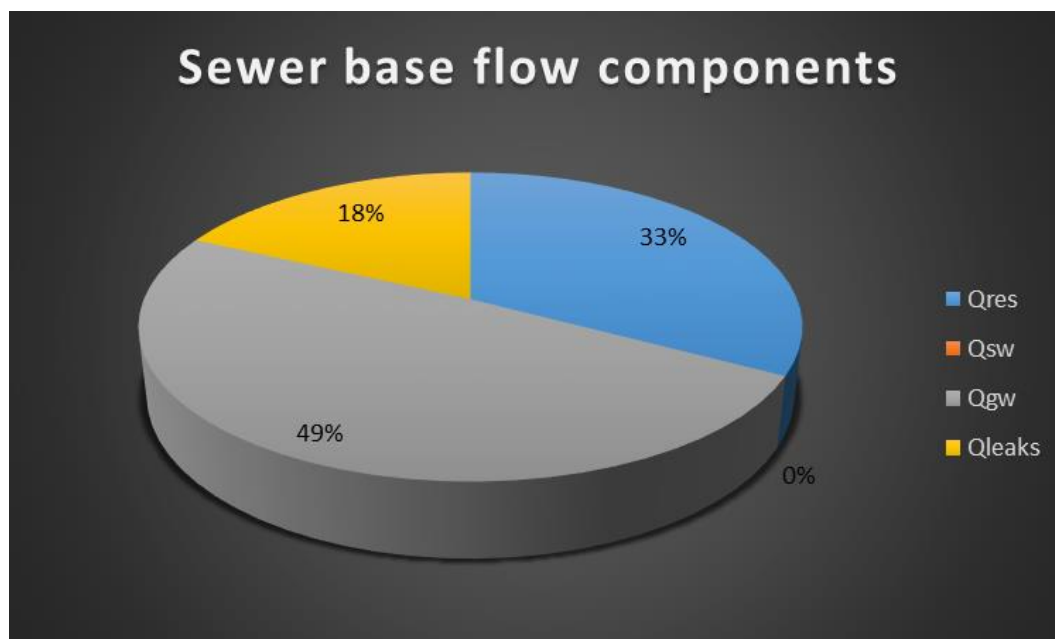
$$Q_{gw} = 32.092 \text{ kl/hour.}$$

$$Q_{leaks} = 11.761 \text{ kl/hour.}$$

Figures 8-4 and 8-5 show two different base flow scenarios. During the wet season, the main contributor to the base flow directly after a rainfall event would be stormwater inflow. However, during the dry season, the main contributor to SBF would be groundwater infiltration.

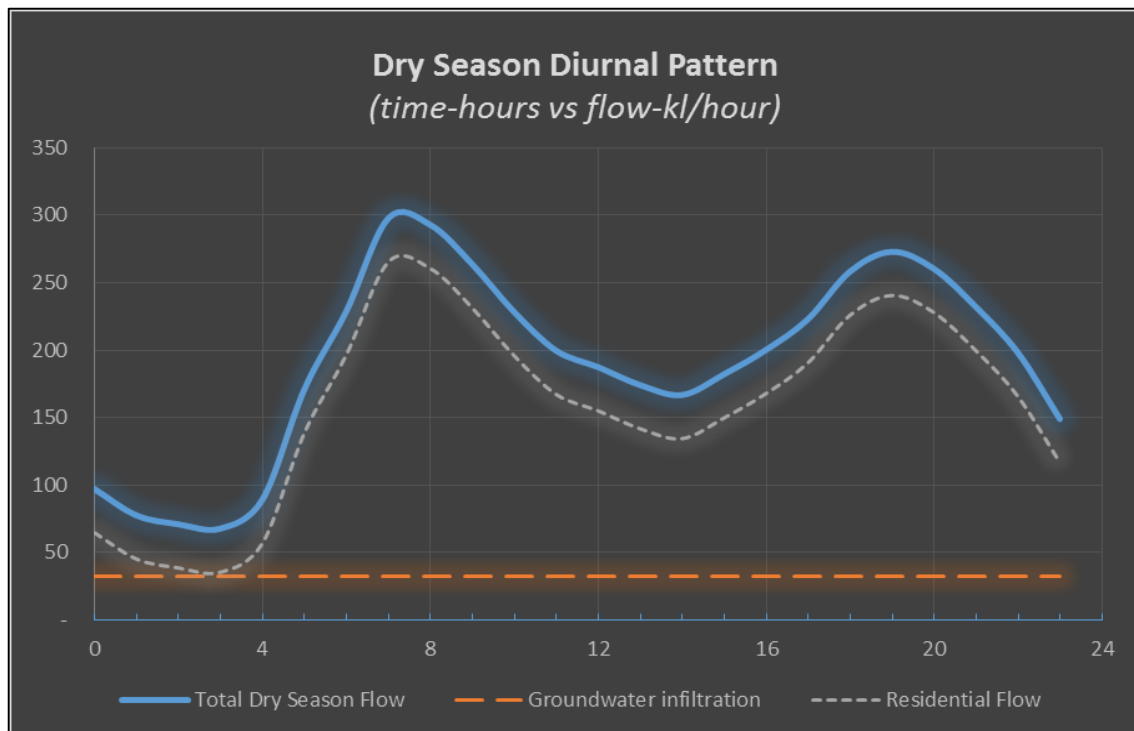


**Figure 8-4 SBF components during the rainy season**



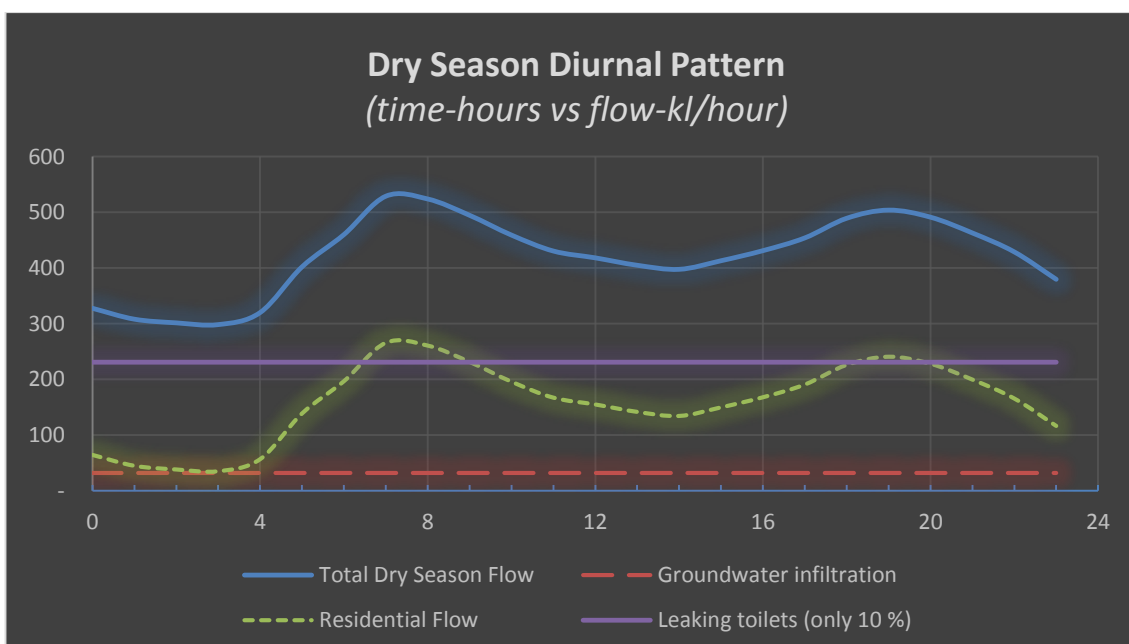
**Figure 8-5 SBF components during the dry season**

When analysing the diurnal pattern of the dry season flow, a distinct peak flow is noticeable, as can be seen in Figure 8-6. Figure 8-6 shows results with negligible leaks.



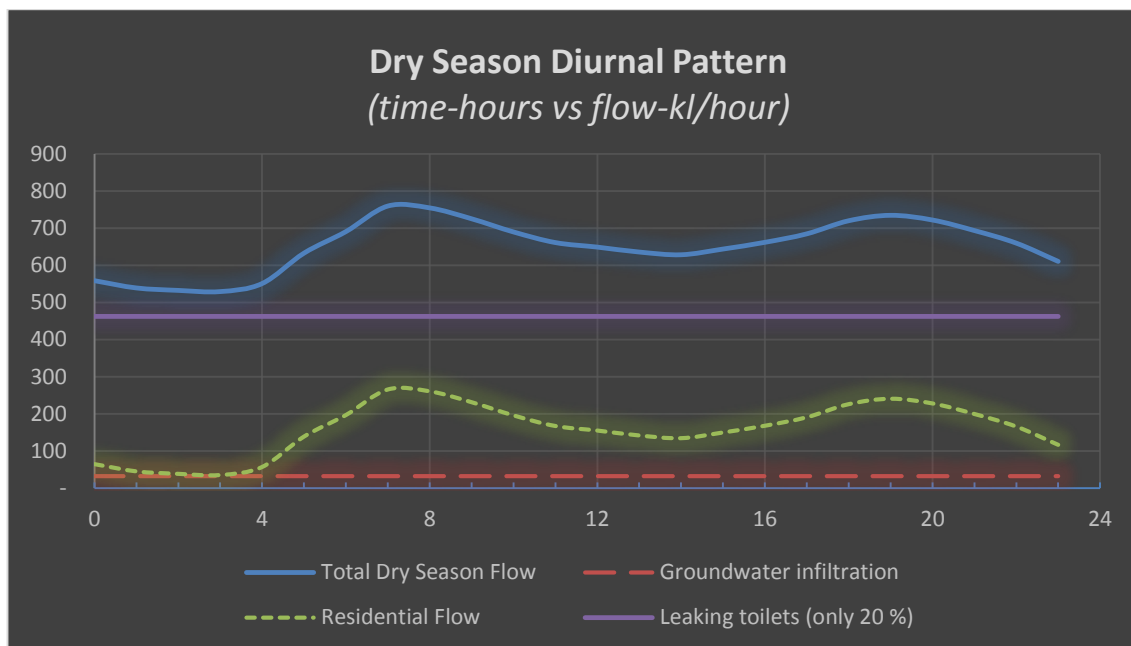
**Figure 8-6 Dry season diurnal pattern (no significant leaks)**

If the groundwater infiltration is doubled, and 10% of the households are given leaks with a flow rate 7 l/minute (half of Scheepers' (2012) proposed flow rate for toilet cisterns) the picture changes significantly. For this scenario, there are still, significant peaks visible, but they are less prominent as seen in Figure 8-7.

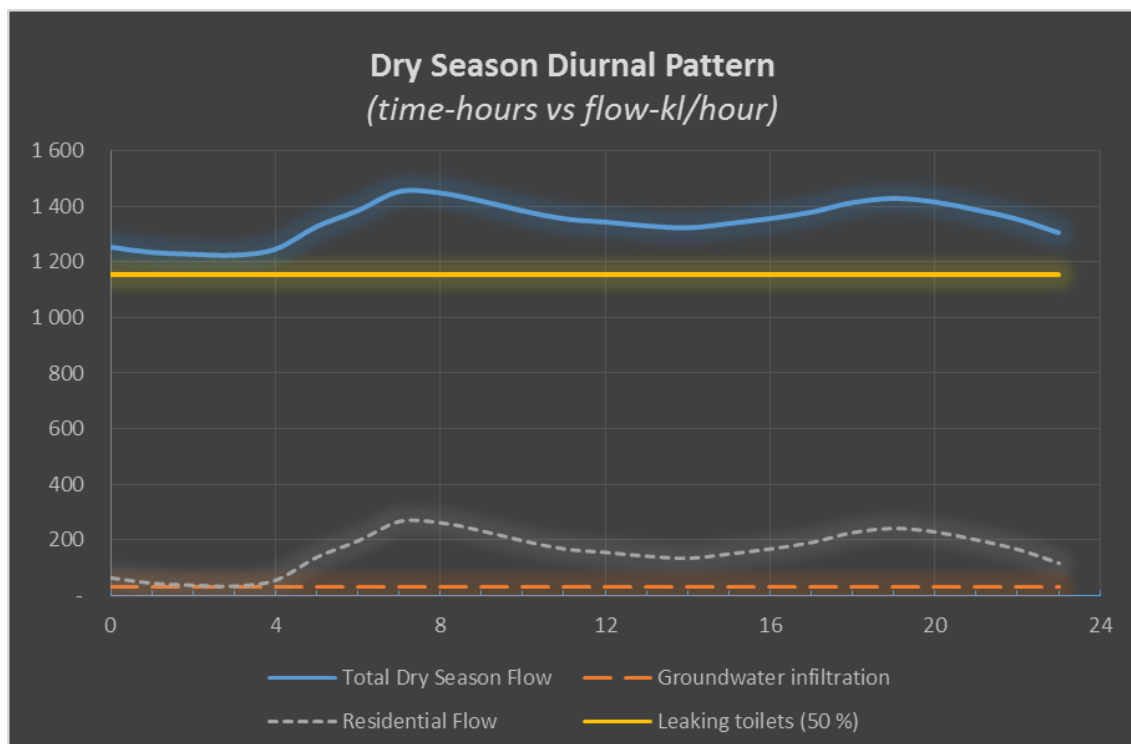


**Figure 8-7 Dry season flow with 10% of hh having leaking toilets**

Figures 8-8 and 8-9 show the situation with 20% and 50% of the households having leaking toilets respectively. At 50% the peaks have almost flattened out.



**Figure 8-8 Dry season flow with 20% of hh having leaking toilets**



**Figure 8-9 Dry season flow with 50% of hh having leaking toilets**

If the measured SBF results showed no distinctive peaks, then it would point to a situation where there was a high percentage of households with leaking plumbing. This informs the utility to prioritise maintenance work in that catchment.

#### *8.10.1 Calibrating the model*

This step is done once the sewer network is installed and is thus potentially the topic of a future final-year project. It is important to note that, even if a final-year student used the exact spreadsheet, the modelled results would differ slightly, since each time the model is run, the model produces a new set of results.

## 9 CONCLUSION

### 9.1 Summary of findings

Domestic sewer base flow ( $Q_{\text{base}}$ ) (SBF) comprises various flow components, namely:

- residential inflow ( $Q_{\text{res}}$ ),
- stormwater inflow ( $Q_{\text{sw}}$ ),
- groundwater infiltration ( $Q_{\text{gw}}$ ), and
- plumbing leaks ( $Q_{\text{leaks}}$ ).

Previous literature downplayed the impact of plumbing leaks into the sewer system, stating that the impact was negligible. However, as had been found by Van Zyl et al. (2008), 50% of toilets in low-income township households leaked within 18 months of being installed. This was further emphasised by studies of base flows by Maré (2013) which showed high levels of inefficient potable water use and plumbing leaks. This was a water conservation and demand management (WCDM) problem. It was also a sewer problem because the high SBF analysed by Maré (2013) put a heavy load on sewer pump stations and waste water treatment works (WWTW.)

The literature review built the case for the need for this study. The literature review also presented previous research related to this study. This study complements the existing literature.

After comparing various sewer design guidelines, the author found that there was a relatively high variability in ground water infiltration and stormwater inflow. Similarly, the normal household sewage (return flow) was found to be very variable. This research found that Monte Carlo simulation together with residential end use (REU) modelling techniques could be used to model the various components of SBF.

The Leakage, Infiltration and inFlow Technique (LIFT) Model was developed as a part of this research. The LIFT Model is a stochastic method of determining SBF using the various components of SBF and accommodated the variability in the domestic inflow (including leaks), stormwater inflow and groundwater infiltration. In the LIFT Model each SBF component and REU is modelled as a probability distribution with some of the key variables noted here:

**Household size and income:** the model allowed for a large and a small household. A large household was defined as more than three people per household but up to and including 8 people per household. A small household was defined between one and three people per household. There were three income categories namely low, medium and high.

**Indoor end-use presence:** the income category was used to determine if an end use is present in the household. It was found that low income households have a low probability of having appliances like a dishwasher. In the LIFT Model, household size did not influence the presence of an end-use (in other words, a high income large household had the same probability of having a dishwasher as a high income small household).

**End-use frequency and function:** using the REU principals proposed in a paper by Jacobs and Haarhof (2004), various domestic indoor end-uses were modelled and used to generate return flow into the sewer system. The end use frequencies were then also modelled using parameters proposed in a report by Scheepers (2012) and compared with the results using Jacobs and Haarhof (2004) input parameters. The mean of the end-use volumes using Scheepers (2012) input parameters are presented in Table 9-1.

**Table 9-1 End-use volumes and frequencies**

End use	Mean Even Volume (l/event)	Mean Event Frequency – Large Household (events/hh/day)	Mean Event Frequency – Small Household (events/hh/day)
Bath	82.7	2.7	0.9
Bathroom basin	1.1	21.6	7.2
Dishwasher	7.8	1.4	0.5
Kitchen sink	2.1	12.2	4.1
Leaks	15.0	6.0	2.0
Misc. indoor	1.0	6.0	2.0
Shower	68.4	2.4	0.8
Toilet	13.2	31.4	10.5
Washing machine	35.8	2.1	0.7

The total average annual daily demand (AADD) per household size and income was calculated using the event volumes, event frequency, and indoor end-use presence parameters. The results are listed in Table 9-2.

**End-use diurnal pattern:** the total end-use per day was split over a 24 hour period using a diurnal pattern published in a report by Aquacraft (2010). The minimum flow per household was found to be between 03:00 and 04:00. The SBF was found to be less sensitive to household income, this is due to appliances like dishwashers and washing machines (which are more prevalent in high income households) normally not operating during the SBF period. Table 9-2 lists the domestic and leakage components of SBF.

**Table 9-2 AADD and SBF comparison**

Household Type	AADD (l/hh/day)	$Q_{res}$ component of SBF (l/hh/hour)	$Q_{leaks}$ component of SBF (l/hh/hour)
High income – large household	913.7	5.9	2.9
High income – small household	304.0	1.9	1.0
Middle income – large household	892.4	5.9	2.9
Middle income – small household	296.9	1.9	1.0
Low income – large household	666.3	5.9	2.9
Low income – small household	222.5	1.9	1.0

According to the model results, the normal domestic return flow was 21.508 kl/hour (33 % of the study area's SBF). The leaks contribution was 11.761 kl/hour (18 % of the SBF).

**Groundwater and stormwater inflows:** the groundwater infiltration for the study area was modelled using the infiltration factor between 0.01 and 0.5 l/minute/m-diameter/m-length as proposed in a report by Stephenson and Barta (2005). The mean groundwater infiltration for the study area was 32.092 kl/hour. During the dry season, the stormwater ingress was 0 l/hour. The groundwater infiltration component is calculated to be 49% of the SBF during the dry season.

SBF was successfully modelled using a stochastic method as was seen in the results. The results of the LIFT Model indicated what acceptable SBF should be for a particular catchment. These results could be used as a benchmark to compare actual SBF with the modelled results. When the leakage component of the LIFT Model was increased manually, the distinct peaks in the diurnal pattern of the model results flattened out considerably. Real-time flow measurements



often present flat peaks (Maré, 2013) and this could be an indication of unacceptably high leakage rates on residential properties. The LIFT Model still needed to be tested against actual SBF measurements.

There is still much work to be done in gaining a better understanding of SBF. The research presented in this report has taken a small step towards improving the existing knowledge base of understanding the relationship between SBF and WCDM.

## **9.2 Suggestions for future research**

While conducting this research, the following future research opportunities were identified. Future research could include:

- Calibrating the groundwater infiltration into river models for sewers and revising the normal groundwater infiltration values.
- Calibrating this model with actual flows measured and building into the model cost-functionality that will further inform management on project prioritising.
- Calculating the non-revenue sewerage in a system.
- Extending the end use to other land use (for example schools and other institutional users).
- Identifying a sewer catchment where the water services provider suspects severe leaking plumbing fittings, measure flows prior to fixing the leaks, and measure the flow subsequent to fixing the leaks. Calculate the return on investment (if any) of fixing the leaks.
- How does the high sewer base flow affect WWTW treatment processes?
- Updating the residential end-use event volumes and modifying/simplifying the residential end-use model to differentiate between person-based end uses and household-based end uses (for example flushing a toilet is a person-based end use, whereas using a washing machine is a household-based end use).

## 10 REFERENCES

- Aquacraft, 2011. Embedded Energy in Water Studies: Study 3: End-use Water Demand Profiles. San Francisco, USA: California Public Utilities Commission.
- Ashley, R. et al. 2013. Water sensitive urban design: opportunities for the UK. *Municipal Engineer*, Volume 166(Issue ME2).
- Athuraliya, A., Roberts, P. & Brown, A., 2012. Yarra Valley future water: residential water use study: Volume 2 - summer 2012. Yarra Valley, Australia: Yarra Valley Water.
- Australian National Water Commission, 2007. Institutional and regulatory models for integrated urban water cycle management: issues and scoping paper. Canberra, Australia: Commonwealth of Australia.
- Beal, C. & Stewart, R. A., 2011. South Queensland Residential End Use Study: Final Report. City East, Australia: Urban Water Security Research Alliance.
- Choi, T., Koizumi, A. & Koo, J., 2012. Analysis of customer night use and night flow losses using minimum night flow analysis. s.l., s.n.
- City of Cape Town, 2005. By-law relating to stormwater management. Cape Town, South Africa: City of Cape Town.
- Claassens, T., 2007. Introduction to Monte Carlo Simulations. Pretoria: Bigen Africa Services (Pty) Ltd.
- Coetzer, A. J., Van Zyl, J. E. & Clayton, C. R. I., 2004. Experimental study of hydraulics of small circular holes in water pipes. s.l.: EWISA.
- Cole, G., 2010. Time of use tariffs. Canberra, Australia: s.n.
- Couvelis, F., van Zyl, J. & Bhagwan, J., 2014. Apparent losses due to meter under registration in South Africa. Vienna, Austria, IWA Water Loss Group.
- CSIR, 2009. Guidelines for Human Settlement Planning and Design. Pretoria, RSA: CSIR.
- CTMM, 2010. Guideline for the Design and Construction of Water and Sanitation Systems. Pretoria, South Africa: City of Tshwane Metropolitan Municipality.
- De Swardt, B. W. & Barta, B., 2008. A first order national audit of sewerage reticulation issues. Pretoria, South Africa: Water Research Commission.
- Department of Environment and Resource Management, 2005. Planning Guidelines for Water Supply and Sewerage. s.l.: Department of Environment and Resource Management.
- Department of Public Works, 2004. Civil Engineering Manual. Pretoria, RSA: Department of Public Works.
- Department of Water Affairs and Forestry, 2004. Water Conservation and water demand management strategy for the water services sector, Pretoria, South Africa: Department of Water Affairs and Forestry.
- du Plessis, J. L., 2014. Estimating domestic outdoor water demand for residential estates. Stellenbosch, University of Stellenbosch.

Dunn, W. L. & Shultis, J. K., 2012. Exploring Monte Carlo Methods. Amsterdam, The Netherlands: Elsevier B.V..

Environmental Protection Agency, 2008. Review of Sewer Design Criteria and RDII Prediction Methods, Washington DC, USA: United States Environmental Protection Agency.

Fantozzi, M. & Lambert, A., 2010. Legitimate night use component of minimum night flows initiative. Sao Paulo, International Water Association.

Fantozzi, M. & Lambert, A., 2012. Residential Night Consumption – Assessment, Choice of Scaling Units and Calculation of Variability. s.l., IWA Water Loss Group.

Gascon, L., Arregui, F., Cobacho, R. & Cabrera, E., 2010. Urban water demand in Spanish cities by measuring end uses consumption patterns. Sao Paulo, International Water Association.

Giurco, D., Carrard, N., McFallan, S. & Nalbantoglu, M., 2008. Residential end-use measurement Guidebook: a guide to study, design, sampling and technology. Victoria, Australia: s.n.

GLS Incorporated, 1997. Sewerage Masterplan for Moreletaspruit catchment area, Pretoria: GLS Incorporated.

Graham, A., 1999. Teach yourself statistics. 2nd ed. Chicago Illinois, USA: NTC/C Contemporary Publishing..

Gravetter, F. J. & Wallnau, L. B., 2000. Statistics for the behavioural sciences. 5th ed. Belmont (CA), USA: Wadsworth/Thomson Learning.

Heinrich, M., 2006. Residential Water End Use Literature Survey, Judgeford, New Zealand: Brans Ltd.

Hine, A. H. & Stephenson, D., 1985. Sewer flow hydrographs, Brighton: IFME.

International Water Association, 2000. Losses from water supply systems: standard terminology and recommended performance measures. London, United Kingdom: IWA

Jacobs, H. E. & Haarhoff, J., 2004. Application of a residential end-use model for estimating cold and hot water demand, wastewater flow and salinity. Water SA, 3 July, 30(3), pp. 305-316.

Jacobs, H. E. & Haarhoff, J., 2004. Structure and data requirements of an end-use model for residential water demand and return flow. Water SA, 3 July, 30(3), pp. 293-304.

Jayasooriya, M., 2013. Ground water infiltration: use of alternative drainage to reduce infiltration volume: a theoretical overview, s.l.: Central Highlands Water.

Karpf, C. & Krebs, P., 2004. Sewers as drainage systems - quantification of groundwater infiltration. NOVATECH, pp. 969-975.

Kleyner, A., 2013. @ RISK for engineers. Ithaca, Palisade.

Lapeyre, B., Pardoux, E. & Sentis, R., 2003. Introduction to Monte-Carlo Methods for Transport and Diffusion Equations. New York, USA: Oxford University Press.

Loureiro, D., Alegre, H., Coelho, S. T. & Borba, R., 2012. A new approach to estimating household night consumption at DMA level. s.l., IWA Water Loss Group.

Lugoma, M. F. T., van Zyl, J. E. & Ilemobade, A. A., 2009. The extent of on-site leakage in selected suburbs of Johannesburg, South Africa, Johannesburg, South Africa: University of Johannesburg.

Maré, K., 2013. Monitoring and Evaluation of Water and Sewer Flows. Midrand, South Africa, WRP Consulting Engineers.

Mckenzie, R., 1999. Development of a standardised approach to evaluate burst and background losses in water distribution systems in South Africa, Pretoria, South Africa: Water Research Commission.

Mckenzie, R., 2011. Introduction to water conservation and demand management. Midrand, WRP Consulting Engineers.

Mckenzie, R., Sigalaba, Z. N. & Wegelin, W. A., 2012. The state of non-revenue water in South Africa, Pretoria, South Africa: Water Research Commission.

Melbourne Water, 2011. Developing a strategic approach to WSUD implementation: guidelines for councils, Melbourne, Australia: Melbourne Water.

o'Brien, O., 2014. Domestic water demand for consumers with rainwater harvesting systems. Stellenbosch, University of Stellenbosch.

Palisade Corporation, 2013. @Risk Users Guide: Risk Analysis and Simulation Add-In for Microsoft Excel. Version 6, New York, USA: Palisade Corporation.

Pinsky, M. P. & Karlin, S., 2011. An Introduction to Stochastic Modeling. 4th ed. Burlington, USA: Academic Press.

Rabe, M., Maree, D., Ramano, R. & Price, G., 2012. Compendium of water conservation and demand management interventions at the municipal level in South Africa, South Africa: Water Research Commission.

ResearchGate, 2014. When is it justifiable to exclude 'outlier' data points from statistical analyses? [Online Forum Thread at ResearchGate]. Available at: [http://www.researchgate.net/post/When\\_is\\_it\\_justifiable\\_to\\_exclude\\_outlier\\_data\\_points\\_from\\_statistical\\_analyses](http://www.researchgate.net/post/When_is_it_justifiable_to_exclude_outlier_data_points_from_statistical_analyses) [Accessed January 2015].

Roy, A. H. et al. 2008. Impediments and solutions to sustainable, watershed-scale urban stormwater management: lessons from Australia and the United States. Environmental Management, 42(2), pp. 344-359.

Scheepers, H. M., 2012. Deriving peak factors for residential indoor water demand by means of a probability based end-use model, Stellenbosch: University of Stellenbosch.

Shaffer, P., 2011. Water Sensitive Urban Design – too good an opportunity too miss?. [Online] Available at: <http://www.engineeringnatureway.co.uk/2011/wsud-too-good-to-miss/> [Accessed 2013].

Sich, A. J., 2013. Monte Carlo Simulation and @RISK: A requirement for making and communicating defensible business decisions., s.l.: Palisade Corporation.

Statistics South Africa, 2012. Census 2011 Methodology and Highlights of key results, Pretoria, South Africa: Statistics South Africa.

Stephenson, D. & Barta, B., 2005. Guidelines on reduction of the impact of water infiltration into sewers, Pretoria, South Africa: Water Research Commission.

Stephenson, D. & Barta, B., 2005. Impacts of stormwater and groundwater ingress on municipal sanitation services, Pretoria, South Africa: Water Research Commission.

UK Water Industry, 1994. Managing leakage: interpreting measured night flows, United Kingdom: Report e, WRC .

Van der Spuy, D. & Rademeyer, P. F., 2010. Flood frequency estimation methods as applied in the Department of Water Affairs. Stellenbosch, South Africa, University of Stellenbosch.

Van Vuuren, S. J. & Van Dijk, M., 2009. Waterborne Sanitation Design Guide, Pretoria, South Africa: Water Research Commission.

van Zyl, J. E., 2009. Losses in water distribution systems. Stellenbosch, University of Stellenbosch.

Van Zyl, J. E., Lobanga, K. P., Lugoma, F. M. T. & Ilemobade, A. A., 2008. The state of plumbing in South Africa, Pretoria, South Africa: Water Research Commission.

Vose Software, 2014. Helpfile, s.l.: Vose Software.

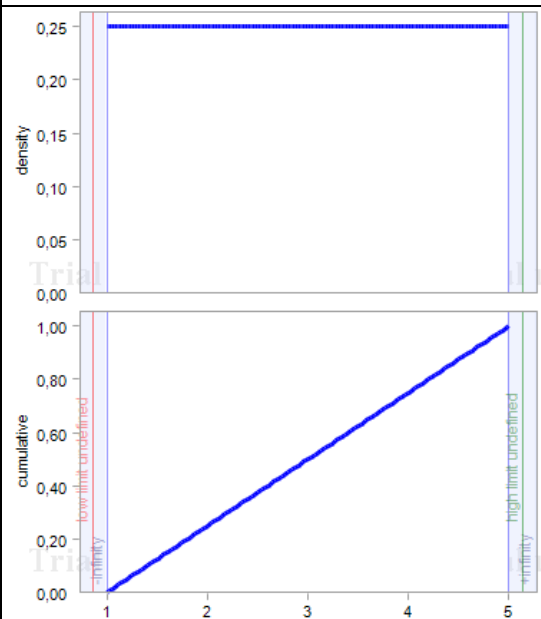
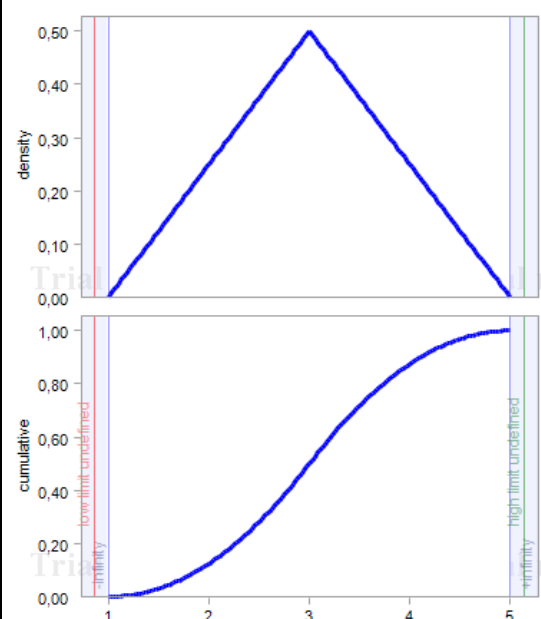
Vosloo, J., 2009. Real time sewer flow monitoring. Stellenbosch, University of Stellenbosch.

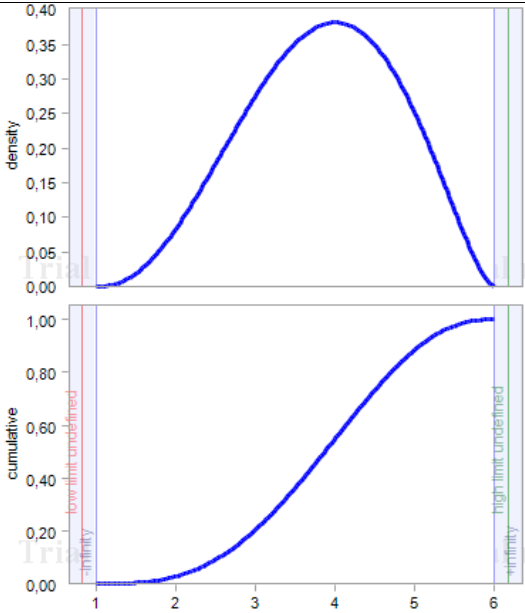
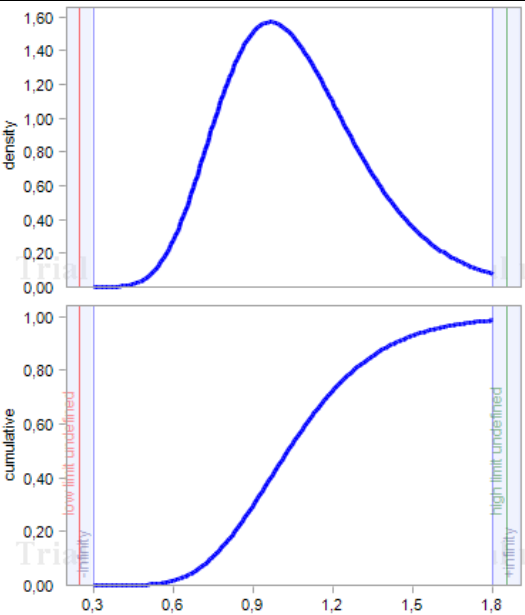
Waldron, T., 2014. A CEO's perspective of water loss. Midrand, WRP Consulting Engineers.

Warren, R., 2002. Household night consumption. London, UK: UK Water Industry Research.

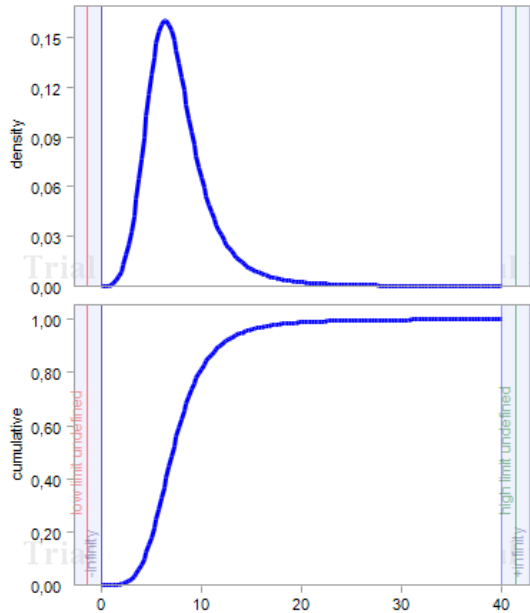
# APPENDIX A – PROBABILITY DISTRIBUTIONS

Discussion of various probability distributions (Vose Software, 2014)

Uniform distribution	
	<p>Origin: Type</p> <p><b>Typically used for:</b> The random function in MS Excel should yield a uniform distribution between 0 and 1. The uniform distribution is used as a very approximate model where there are very few or no available data. It is rarely a good reflection of the perceived uncertainty of a parameter since all values within the allowed range have the same constant probability density, but that density abruptly changes to zero at the minimum and maximum. However, it is sometimes useful for bringing attention to the fact that a parameter is very poorly known (Vose, 2014).</p>
Triangular distribution	
	<p>The triangle distribution is used as a rough modelling tool where the range (a to c) and the most likely value within the range (b) can be estimated. It has no theoretical basis but derives its statistical properties from its geometry.</p> <p>The triangle distribution offers considerable flexibility in its shape, coupled with the intuitive nature of its defining parameters and speed of use. It has therefore achieved a great deal of popularity among risk analysts. However, a and c are the absolute minimum and maximum estimated values for the variable and it is generally a difficult task to make estimates of these values.</p> <p>It should be noted that the triangle shape will also usually overemphasise the tails of the distribution and underemphasise the shoulders in comparison with other, more natural, distributions (Vose, 2014).</p>

PERT distribution	
 <p>The figure shows two plots for the PERT distribution. The top plot is the density function, which is a smooth, bell-shaped curve peaking at approximately 0.38 at x=4. The bottom plot is the cumulative distribution function, which is an S-shaped curve starting at 0 and ending at 1. Both plots have a horizontal axis from 1 to 6 and a vertical axis from 0.00 to 0.40 for density and 0.00 to 1.00 for cumulative. Vertical lines at x=1 and x=6 are labeled 'low limit undefined' and 'high limit undefined' respectively, with 'infinity' written below them.</p>	<p>The PERT distribution is used exclusively for modelling expert estimates, where one is given the expert's minimum, most likely and maximum guesses. The PERT distribution takes the same parameters as the triangle, but generally offers a more reasonable interpretation of the parameter values in modelling expert opinion.</p> <p>Caution: where the most likely value is more than 13% away from the extreme value, the tail of the distribution becomes almost meaningless (Vose 2014).</p>
Log-normal distribution	
 <p>The figure shows two plots for the log-normal distribution. The top plot is the density function, which is a smooth, bell-shaped curve peaking at approximately 1.55 at x=1.0. The bottom plot is the cumulative distribution function, which is an S-shaped curve starting at 0 and ending at 1. Both plots have a horizontal axis from 0.3 to 1.8 and a vertical axis from 0.00 to 1.60 for density and 0.00 to 1.00 for cumulative. Vertical lines at x=0.3 and x=1.8 are labeled 'low limit undefined' and 'high limit undefined' respectively, with 'infinity' written below them.</p>	<p>The log-normal distribution is useful for modelling naturally occurring variables that are the product of a number of other naturally occurring variables. Central Limit Theorem shows that the product of a large number of independent random variables is log normally distributed.</p> <p>Log-normal distributions often provide a good representation for a physical quantity that extend from zero to + infinity and is positively skewed (Vose, 2014).</p>

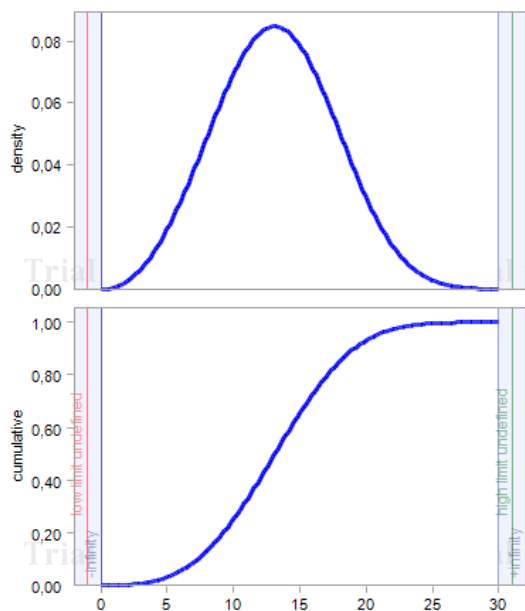
### Log-logistic distribution



The log-logistic distribution has the same relationship to the logistic distribution that the log-normal distribution has to the normal distribution. If one feels that a variable is driven by some process that is the product of a number of variables, then a natural distribution to use is the log-normal because of Central Limit Theorem. However, if one or two of these factors could be dominant, or correlated, so that the distribution is less spread than a log-normal, then the log-logistic may be an appropriate distribution to try.

The log-logistic distribution, which has a heavier tail than the gamma distribution, has been fit to quite a number of finance and insurance variables, for example: the duration of claim for income protection insurance (i.e. time until claimant returns to work); residuals for a time series regression of agricultural product values; insurance losses; natural catastrophe claims, etc.

### Weibull distribution

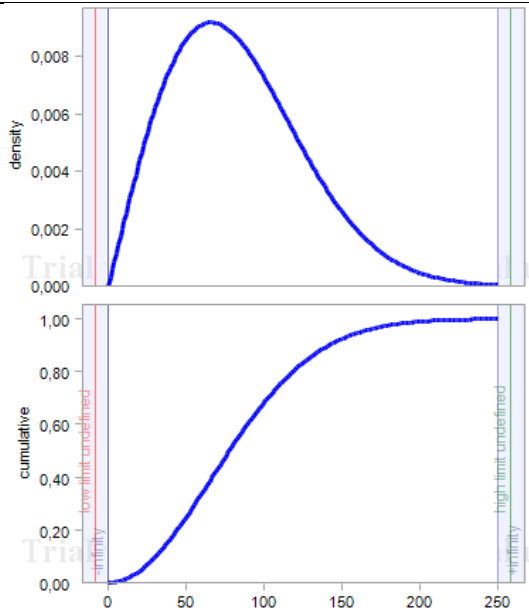


The Weibull distribution is often used to model the time until occurrence of an event where the probability of occurrence changes with time (the process has 'memory'), as opposed to the exponential distribution where the probability of occurrence remains constant ('memoryless'). It has also been used to model variation in wind speed at a specific site. Example: light bulbs.

The Weibull distribution becomes an exponential distribution when  $a = 1$ , i.e.  $\text{Weibull}(1, b) = \text{Expon}(b)$ . The Weibull distribution is very close to the normal distribution when  $b = 3.25$ . The Weibull distribution is named after the Swedish physicist Dr E.H. Wallodi Weibull (1887-1979) who used it to model the distribution of the breaking strengths of materials.



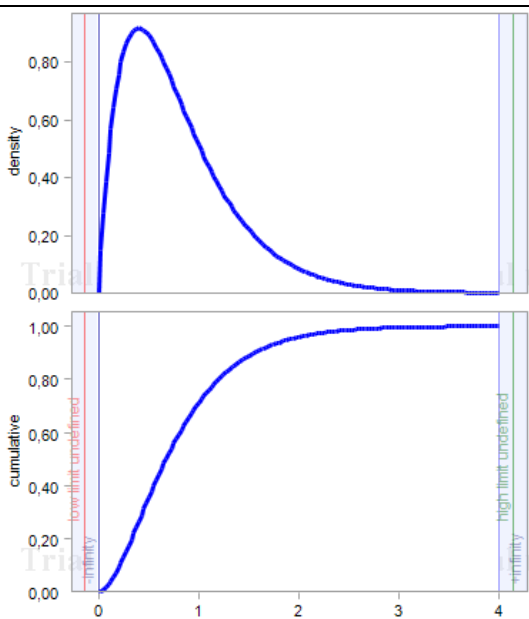
### Rayleigh distribution



The Rayleigh distribution is a special case of the Weibull distribution.

The Rayleigh distribution is frequently used to model wave heights in oceanography, and in communication theory to describe hourly median and instantaneous peak power of received radio signals. It has been used to model the frequency of different wind speeds over a year at wind turbine sites.

### Gamma distribution



The gamma distribution is right-skewed and bound at zero. It is a parametric distribution based on Poisson mathematics.

The gamma distribution is extremely important in risk analysis modelling.

The  $\text{Gamma}(a,b)$  distribution models the time required for  $a$  events to occur, given that the events occur randomly in a Poisson process with a mean time between events of  $b$ . For example, if we know that major flooding occurs in a town on average every six years,  $\text{gamma}(4,6)$  models how many years it will take before the next four floods could occur.